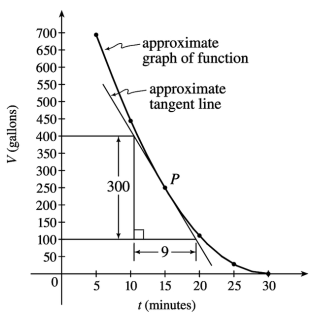
1. **(a)** Using $P(15,250)$, we construct the following table:

t	Q	slope=m_{PQ}
5	$(5,694)$	$\frac{694-250}{5-15}=-\frac{444}{10}=-44.4$
10	$(10,444)$	$\frac{444-250}{10-15}=-\frac{194}{5}=-38.8$
20	$(20,111)$	$\frac{111-250}{20-15}=-\frac{139}{5}=-27.8$
25	$(25,28)$	$\frac{28-250}{25-15}=-\frac{222}{10}=-22.2$
30	$(30,0)$	$\frac{0-250}{30-15}=-\frac{250}{15}=-16.\overline{6}$

(b) Using the values of t that correspond to the points closest to P ($t=10$ and $t=20$), we have

$$\frac{-38.8+(-27.8)}{2}=-33.3$$

(c) From the graph, we can estimate the slope of the tangent line at P to be $\frac{-300}{9}=-33.\overline{3}$.



2.

(a) Slope $=\frac{2948-2530}{42-36}=\frac{418}{6}\approx 69.67$ (b) Slope $=\frac{2948-2661}{42-38}=\frac{287}{4}=71.75$

(c) Slope $=\frac{2948-2806}{42-40}=\frac{142}{2}=71$ (d) Slope $=\frac{3080-2948}{44-42}=\frac{132}{2}=66$

From the data, we see that the patient's heart rate is decreasing from 71 to 66 heartbeats / minute after 42 minutes. After being stable for a while, the patient's heart rate is dropping.

3. **(a)** For the curve $y=x/(1+x)$ and the point $P\left(1, \frac{1}{2}\right)$

	x	Q	m_{PQ}
(i)	0.5	$(0.5,0.333333)$	0.333333
(ii)	0.9	$(0.9,0.473684)$	0.263158
(iii)	0.99	$(0.99,0.497487)$	0.251256
(iv)	0.999	$(0.999,0.499750)$	0.250125
(v)	1.1	$(1.5,06)$	0.2
(vi)	1.5	$(1.1,0.523810)$	0.238095
(vii)	1.01	$(1.01,0.502488)$	0.248756
(viii)	1.001	$(1.001,0.500250)$	0.249875

(b) The slope appears to be $\frac{1}{4}$.

(c) $y-\frac{1}{2}=\frac{1}{4}(x-1)$ or $y=\frac{1}{4}x+\frac{1}{4}$.

4. For the curve $y=\ln x$ and the point $P(2,\ln 2)$:

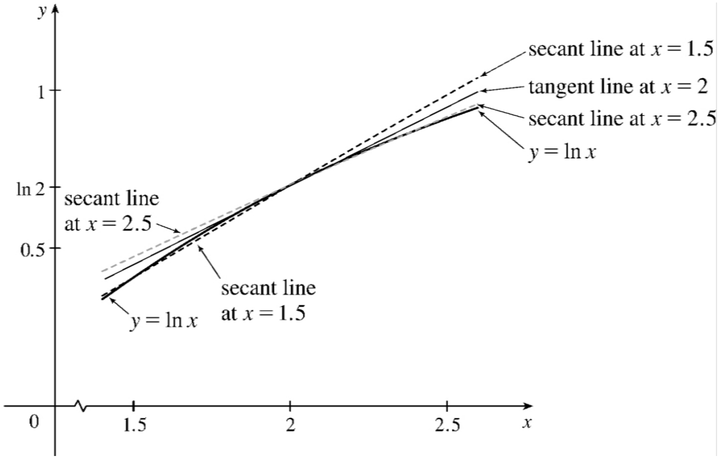
(a)

	x	Q	m_{PQ}
(i)	1.5	$(1.5,0.405465)$	0.575364
(ii)	1.9	$(1.9,0.641854)$	0.512933
(iii)	1.99	$(1.99,0.688135)$	0.501254
(iv)	1.999	$(1.999,0.692647)$	0.500125
(v)	2.5	$(2.5,0.916291)$	0.446287
(vi)	2.1	$(2.1,0.741937)$	0.487902
(vii)	2.01	$(2.01,0.698135)$	0.498754
(viii)	2.001	$(2.001,0.693647)$	0.499875

(b) The slope appears to be $\frac{1}{2}$.

(c) $y-\ln 2=\frac{1}{2}(x-2)$ or $y=\frac{1}{2}x-1+\ln 2$

(d)



5. **(a)** $y=y(t)=40t-16t^2$. At $t=2$, $y=40(2)-16(2)^2=16$. The average velocity between times 2 and $2+h$ is $v_{ave}=\dfrac{y(2+h)-y(2)}{(2+h)-2}=\dfrac{\left[40(2+h)-16(2+h)^2\right]-16}{h}=\dfrac{-24h-16h^2}{h}=-24-16h$, if $h\neq 0$.

(i) $[2,2.5]$: $h=0.5$, $v_{ave}=-32$ ft / s (ii) $[2,2.1]$: $h=0.1$, $v_{ave}=-25.6$ ft / s

(iii) $[2,2.05]$: $h=0.05$, $v_{ave}=-24.8$ ft / s (iv) $[2,2.01]$: $h=0.01$, $v_{ave}=-24.16$ ft / s

(b) The instantaneous velocity when $t=2$ (h approaches 0) is -24 ft / s.

6. The average velocity between t and $t+h$ seconds is

$$\frac{58(t+h)-0.83(t+h)^2-\left(58t-0.83t^2\right)}{h}=\frac{58h-1.66th-0.83h^2}{h}=58-1.66t-0.83h \text{ if } h\neq 0 .$$

(a) Here $t=1$, so the average velocity is $58-1.66-0.83h=56.34-0.83h$.

(i) $[1,2]$: $h=1, 55.51$ m / s (ii) $[1,1.5]$: $h=0.5$, 55.925 m / s

(iii) $[1,1.1]$: $h=0.1$, 56.257 m / s (iv) $[1,1.01]$: $h=0.01$, 56.3317 m / s

(v) $[1,1.001]$: $h=0.001$, 56.33917 m / s

(b) The instantaneous velocity after 1 second is 56.34 m / s.

7. **(a)**

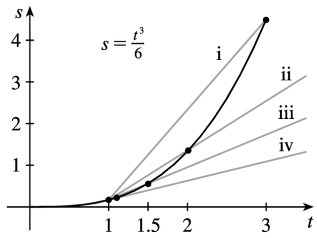
(i) $[1,3]$: $h=2$, $v_{ave}=\dfrac{13}{6}$ ft / s (ii) $[1,2]$: $h=1$, $v_{ave}=\dfrac{7}{6}$ ft / s

(iii) $[1,1.5]$: $h=0.5$, $v_{ave}=\dfrac{19}{24}$ ft / s (iv) $[1,1.1]$: $h=0.1$, $v_{ave}=\dfrac{331}{600}$ ft / s

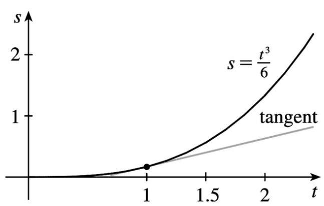
(b) As h approaches 0 , the velocity approaches

$\frac{3}{6}=\frac{1}{2}$ ft / s.

(c)



(d)



8. Average velocity between times $t=2$ and $t=2+h$ is given by $\frac{s(2+h)-s(2)}{h}$.

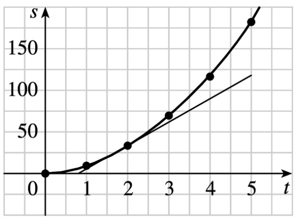
(a)

(i) $h=3 \Rightarrow v_{\text{av}}=\frac{s(5)-s(2)}{5-2}=\frac{178-32}{3}=\frac{146}{3}\approx 48.7$ ft / s

(ii) $h=2 \Rightarrow v_{\text{av}}=\frac{s(4)-s(2)}{4-2}=\frac{119-32}{2}=\frac{87}{2}=43.5$ ft / s

(iii) $h=1 \Rightarrow v_{\text{av}}=\frac{s(3)-s(2)}{3-2}=\frac{70-32}{1}=38$ ft / s

(b) Using the points $(0.8,0)$ and $(5,118)$ from the approximate tangent line, the instantaneous velocity at $t=2$ is about $\frac{118-0}{5-0.8}\approx 28$ ft / s.



9. For the curve $y=\sin\ (10\pi/x)$ and the point $P(1,0)$:

(a)

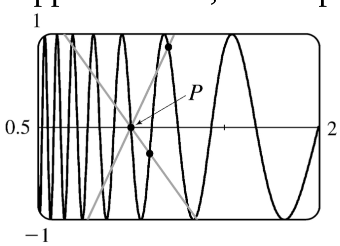
x	Q	m_{PQ}
2	$(2,0)$	0

1.5	$(1.5,0.8660)$	1.7321
1.4	$(1.4,–0.4339)$	–1.0847
1.3	$(1.3,–0.8230)$	–2.7433
1.2	$(1.2,0.8660)$	4.3301
1.1	$(1.1,–0.2817)$	–2.8173

x	Q	m_{PQ}
0.5	$(0.5,0)$	0
0.6	$(0.6,0.8660)$	–2.1651
0.7	$(0.7,0.7818)$	–2.6061
0.8	$(0.8,1)$	–5
0.9	$(0.9,–0.3420)$	3.4202

As x approaches 1 , the slopes do not appear to be approaching any particular value.

(b)



We see that problems with estimation are caused by the frequent oscillations of the graph. The tangent is so steep at P that we need to take x –values much closer to 1 in order to get accurate estimates of its slope.

(c) If we choose $x=1.001$, then the point Q is $(1.001,–0.0314)$ and $m_{PQ}\approx–31.3794$. If $x=0.999$, then Q is $(0.999,0.0314)$ and $m_{PQ}=–31.4422$. The average of these slopes is –31.4108 . So we estimate that the slope of the tangent line at P is about –31.4 .