1. As x approaches 2, $f(x)$ approaches 5. [Or, the values of $f(x)$ can be made as close to 5 as we like by taking x sufficiently close to 2 (but $x \neq 2$).] Yes, the graph could have a hole at $(2,5)$ and be defined such that $f(2)=3$.

2. As x approaches 1 from the left, $f(x)$ approaches 3; and as x approaches 1 from the right, $f(x)$ approaches 7. No, the limit does not exist because the left– and right–hand limits are different.

3. **(a)** $\lim_{x \to -3} f(x) = \infty$ means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to -3 (but not equal to -3).
(b) $\lim_{x \to 4^+} f(x) = -\infty$ means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to 4 through values larger than 4.

4. **(a)** $\lim_{x \to 0} f(x) = 3$
(b) $\lim_{x \to 3^-} f(x) = 4$
(c) $\lim_{x \to 3^+} f(x) = 2$
(d) $\lim_{x \to 3} f(x)$ does not exist because the limits in part (b) and part (c) are not equal.
(e) $f(3)=3$

5. **(a)** $f(x)$ approaches 2 as x approaches 1 from the left, so $\lim_{x \to 1^-} f(x) = 2$.
(b) $f(x)$ approaches 3 as x approaches 1 from the right, so $\lim_{x \to 1^+} f(x) = 3$.
(c) $\lim_{x \to 1} f(x)$ does not exist because the limits in part (a) and part (b) are not equal.
(d) $f(x)$ approaches 4 as x approaches 5 from the left and from the right, so $\lim_{x \to 5} f(x) = 4$.
(e) $f(5)$ is not defined, so it doesn't exist.

6. **(a)** $\lim_{x \to -2^-} g(x) = -1$
(b) $\lim_{x \to -2^+} g(x) = 1$
(c) $\lim_{x \to -2} g(x)$ doesn't exist
(d) $g(-2)=1$
(e) $\lim_{x \to 2^-} g(x) = 1$

(f) $\lim_{x \to 2^+} g(x) = 2$

(g) $\lim_{x \to 2} g(x)$ doesn't exist

(h) $g(2) = 2$

(i) $\lim_{x \to 4^+} g(x)$ doesn't exist

(j) $\lim_{x \to 4^-} g(x) = 2$

(k) $g(0)$ doesn't exist

(l) $\lim_{x \to 0} g(x) = 0$

7. **(a)** $\lim_{t \to 0^-} g(t) = -1$

(b) $\lim_{t \to 0^+} g(t) = -2$

(c) $\lim_{t \to 0} g(t)$ does not exist because the limits in part (a) and part (b) are not equal.

(d) $\lim_{t \to 2^-} g(t) = 2$

(e) $\lim_{t \to 2^+} g(t) = 0$

(f) $\lim_{t \to 2} g(t)$ does not exist because the limits in part (d) and part (e) are not equal.

(g) $g(2) = 1$

(h) $\lim_{t \to 4} g(t) = 3$

8. **(a)** $\lim_{x \to 2} R(x) = -\infty$

(b) $\lim_{x \to 5} R(x) = \infty$

(c) $\lim_{x \to -3^-} R(x) = -\infty$

(d) $\lim_{x \to -3^+} R(x) = \infty$

(e) The equations of the vertical asymptotes are $x = -3$, $x = 2$, and $x = 5$.

9. **(a)** $\lim_{x \to -7} f(x) = -\infty$

(b) $\lim_{x \to -3} f(x) = \infty$

(c)

$\lim_{x\to 0} f(x)=\infty$

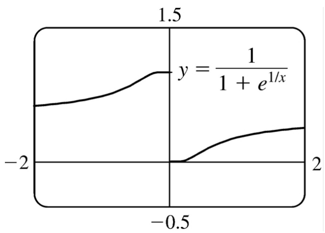
(d) $\lim_{x\to 6^-} f(x)=-\infty$

(e) $\lim_{x\to 6^+} f(x)=\infty$

(f) The equations of the vertical asymptotes are $x=-7$, $x=-3$, $x=0$, and $x=6$.

10. $\lim_{t\to 12^-} f(t)=150$ mg and $\lim_{t\to 12^+} f(t)=300$ mg. These limits show that there is an abrupt change in the amount of drug in the patient's bloodstream at $t=12$ h. The left–hand limit represents the amount of the drug just before the fourth injection. The right–hand limit represents the amount of the drug just after the fourth injection.

11.

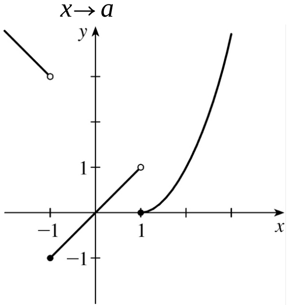


(a) $\lim_{x\to 0^-} f(x)=1$

(b) $\lim_{x\to 0^+} f(x)=0$

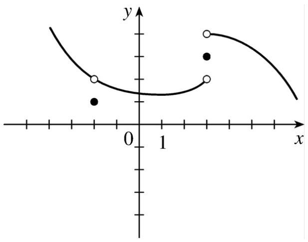
(c) $\lim_{x\to 0} f(x)$ does not exist because the limits in part (a) and part (b) are not equal.

12. $\lim_{x\to a} f(x)$ exists for all a except $a=\pm 1$.

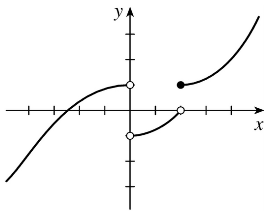


13. $\lim_{x\to 3^+} f(x)=4$, $\lim_{x\to 3^-} f(x)=2$,

$\lim_{x \to -2} f(x) = 2$, $f(3) = 3$, $f(-2) = 1$



14. $\lim_{x \to 0^-} f(x) = 1$, $\lim_{x \to 0^+} f(x) = -1$, $\lim_{x \to 2^-} f(x) = 0$, $\lim_{x \to 2^+} f(x) = 1$, $f(2) = 1$, $f(0)$ is undefined



15. For $f(x) = \dfrac{x^2 - 2x}{x^2 - x - 2}$:

x	$f(x)$
2.5	0.714286
2.1	0.677419
2.05	0.672131
2.01	0.667774
2.005	0.667221
2.001	0.666778

x	$f(x)$
1.9	0.655172
1.95	0.661017
1.99	0.665552
1.995	0.666110
1.999	0.666556

It appears that $\lim_{x \to 2} \dfrac{x^2 - 2x}{x^2 - x - 2} = 0.\overline{6} = \dfrac{2}{3}$.

16. For $f(x)=\dfrac{x^2-2x}{x^2-x-2}$:

x	$f(x)$
0	0
−0.5	−1
−0.9	−9
−0.95	−19
−0.99	−99
−0.999	−999

x	$f(x)$
−2	2
−1.5	3
−1.1	11
−1.01	101
−1.001	1001

It appears that $\lim\limits_{x\to -1}\dfrac{x^2-2x}{x^2-x-2}$ *does not exist* since $f(x)\to -\infty$ as $x\to -1^-$ and $f(x)\to \infty$ as $x\to -1^+$.

17. For $f(x)=\dfrac{e^x-1-x}{x^2}$:

x	$f(x)$
1	0.718282
0.5	0.594885
0.1	0.517092
0.05	0.508439
0.01	0.501671

x	$f(x)$
−1	0.367879

–0.5	0.426123
–0.1	0.483742
–0.05	0.491770
–0.01	0.498337

It appears that $\lim_{x\to 0} \frac{e^x-1-x}{x^2}=0.5=\frac{1}{2}$.

18. For $f(x)=x\ln\left(x+x^2\right)$:

x	$f(x)$
1	0.693147
0.5	–0.143841
0.1	–0.220727
0.05	–0.147347
0.01	–0.045952
0.005	–0.026467
0.001	–0.006907

It appears that $\lim_{x\to 0^+} x\ln\left(x+x^2\right)=0$.

19. For $f(x)=\frac{\sqrt{x+4}-2}{x}$:

x	$f(x)$
1	0.236068
0.5	0.242641
0.1	0.248457
0.05	0.249224
0.01	0.249844

x	$f(x)$
–1	0.267949

–0.5	0.258343
–0.1	0.251582
–0.05	0.250786
–0.01	0.250156

It appears that $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} = 0.25 = \frac{1}{4}$.

20. For $f(x) = \frac{\tan 3x}{\tan 5x}$:

x	$f(x)$
± 0.2	0.439279
± 0.1	0.566236
± 0.05	0.591893
± 0.01	0.599680
± 0.001	0.599997

It appears that $\lim_{x\to 0} \frac{\tan 3x}{\tan 5x} = 0.6 = \frac{3}{5}$.

21. For $f(x) = \frac{x^6-1}{x^{10}-1}$:

x	$f(x)$
0.5	0.985337
0.9	0.719397
0.95	0.660186
0.99	0.612018
0.999	0.601200

x	$f(x)$
1.5	0.183369
1.1	0.484119
1.05	0.540783
1.01	0.588022

1.001	0.598800

It appears that $\lim_{x \to 1} \frac{x^6 - 1}{x^{10} - 1} = 0.6 = \frac{3}{5}$.

22. For $f(x) = \frac{9^x - 5^x}{x}$:

x	$f(x)$
0.5	1.527864
0.1	0.711120
0.05	0.646496
0.01	0.599082
0.001	0.588906

x	$f(x)$
–0.5	0.227761
–0.1	0.485984
–0.05	0.534447
–0.01	0.576706
–0.001	0.586669

It appears that $\lim_{x \to 0} \frac{9^x - 5^x}{x} = 0.59$. Later we will be able to show that the exact value is $\ln(9/5)$.

23. $\lim_{x \to 5^+} \frac{6}{x-5} = \infty$ since $(x-5) \to 0$ as $x \to 5^+$ and $\frac{6}{x-5} > 0$ for $x > 5$.

24. $\lim_{x \to 5^-} \frac{6}{x-5} = -\infty$ since $(x-5) \to 0$ as $x \to 5^-$ and $\frac{6}{x-5} < 0$ for $x < 5$.

25. $\lim_{x \to 1} \frac{2-x}{(x-1)^2} = \infty$ since the numerator is positive and the denominator approaches 0 through positive values as $x \to 1$.

26.

$\lim_{x \to 0} \frac{x-1}{x^2(x+2)} = -\infty$ since $x^2 \to 0$ as $x \to 0$ and $\frac{x-1}{x^2(x+2)} < 0$ for $0 < x < 1$ and for $-2 < x < 0$.

27. $\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)} = -\infty$ since $(x+2) \to 0$ as $x \to -2^+$ and $\frac{x-1}{x^2(x+2)} < 0$ for $-2 < x < 0$.

28. $\lim_{x \to \pi^-} \csc x = \lim_{x \to \pi^-} (1/\sin x) = \infty$ since $\sin x \to 0$ as $x \to \pi^-$ and $\sin x > 0$ for $0 < x < \pi$.

29. $\lim_{x \to (-\pi/2)^-} \sec x = \lim_{x \to (-\pi/2)^-} (1/\cos x) = -\infty$ since $\cos x \to 0$ as $x \to (-\pi/2)^-$ and $\cos x < 0$ for $-\pi < x < -\pi/2$.

30. $\lim_{x \to 5^+} \ln(x-5) = -\infty$ since $x-5 \to 0^+$ as $x \to 5^+$.

31. **(a)** $f(x) = 1/(x^3 - 1)$

x	$f(x)$
0.5	–1.14
0.9	–3.69
0.99	–33.7
0.999	–333.7
0.9999	–3333.7
0.99999	–33 , 333.7

x	$f(x)$
1.5	0.42
1.1	3.02
1.01	33.0
1.001	333.0
1.0001	3333.0
1.00001	33 , 333.3

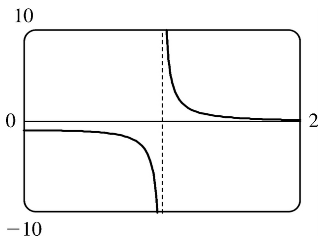
From these calculations, it seems that $\lim_{x \to 1^-} f(x) = -\infty$ and $\lim_{x \to 1^+} f(x) = \infty$.

(b) If x is slightly smaller than 1 , then

x^3-1 will be a negative number close to 0 , and the reciprocal of x^3-1 , that is, $f(x)$, will be a negative number with large absolute value. So $\lim_{x\to 1^-} f(x)=-\infty$.

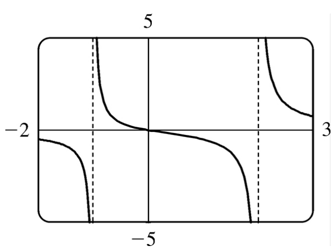
If x is slightly larger than 1 , then x^3-1 will be a small positive number, and its reciprocal, $f(x)$, will be a large positive number. So $\lim_{x\to 1^+} f(x)=\infty$.

(c) It appears from the graph of f that $\lim_{x\to 1^-} f(x)=-\infty$ and $\lim_{x\to 1^+} f(x)=\infty$.



32. **(a)** $y=\dfrac{x}{x^2-x-2}=\dfrac{x}{(x-2)(x+1)}$. Therefore, as $x\to -1^+$ or $x\to 2^+$, the denominator approaches 0 , and $y>0$ for $x<-1$ and for $x>2$, so $\lim_{x\to -1^+} y=\lim_{x\to 2^+} y=\infty$. Also, as $x\to -1^-$ or $x\to 2^-$, the denominator approaches 0 and $y<0$ for $-1<x<2$, so $\lim_{x\to -1^-} y=\lim_{x\to 2^-} y=-\infty$.

(b)

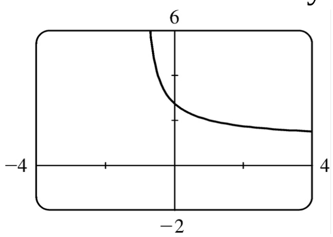


33. **(a)** Let $h(x)=(1+x)^{1/x}$.

x	$h(x)$
–0.001	2.71964
–0.0001	2.71842
–0.00001	2.71830
–0.000001	2.71828
0.000001	2.71828
0.00001	2.71827

0.0001	2.71815
0.001	2.71692

It appears that $\lim_{x\to 0}(1+x)^{1/x}\approx 2.71828$, which is approximately e . In Section 7.4 we will see that the value of the limit is exactly e .

(b) 

34. For the curve $y=2^x$ and the points $P(0,1)$ and $Q\left(x,2^x\right)$:

x	Q	m_{PQ}
0.1	(0.1,1.0717735)	0.71773
0.01	(0.01,1.0069556)	0.69556
0.001	(0.001,1.0006934)	0.69339
0.0001	(0.0001,1.0000693)	0.69317

The slope appears to be about 0.693 .

35. **(a)**

x	$f(x)$
1	0.998000
0.8	0.638259
0.6	0.358484
0.4	0.158680
0.2	0.038851
0.1	0.008928
0.05	0.001465

It appears that $\lim_{x\to 0} f(x)=0$.

(b)

x	$f(x)$
0.04	0.000572

0.02	–0.000614
0.01	–0.000907
0.005	–0.000978
0.003	–0.000993
0.001	–0.001000

It appears that $\lim_{x\to 0} f(x) = -0.001$.

36. $h(x) = \dfrac{\tan x - x}{x^3}$

(a)

x	$h(x)$
1.0	0.55740773
0.5	0.37041992
0.1	0.33467209
0.05	0.33366700
0.01	0.33334667
0.005	0.33333667

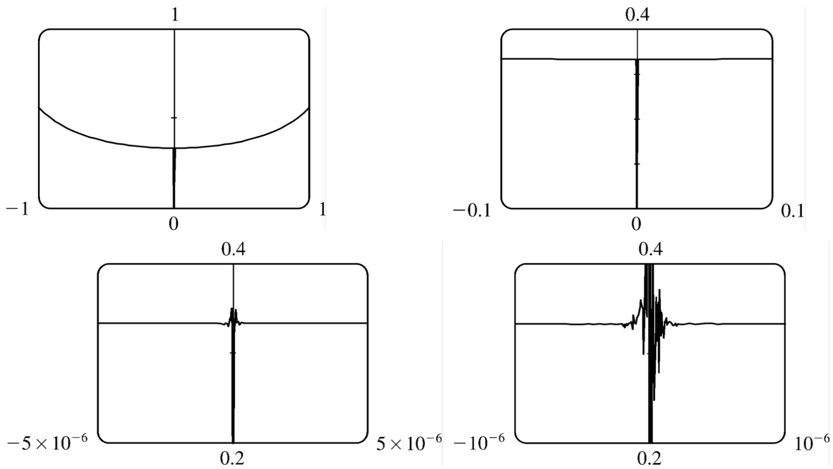
(b) It seems that $\lim_{x\to 0} h(x) = \dfrac{1}{3}$.

(c)

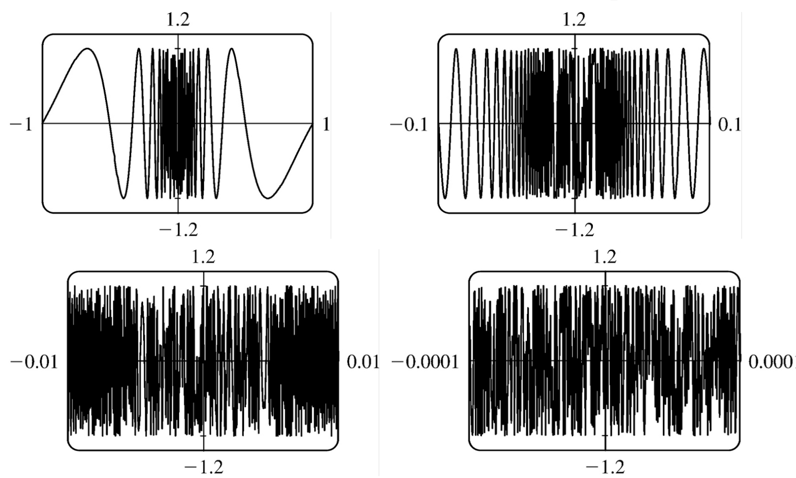
x	$h(x)$
0.001	0.33333350
0.0005	0.33333344
0.0001	0.33333000
0.00005	0.33333600
0.00001	0.33300000
0.000001	0.00000000

Here the values will vary from one calculator to another. Every calculator will eventually give *false values* .

(d) As in part (c), when we take a small enough viewing rectangle we get incorrect output.

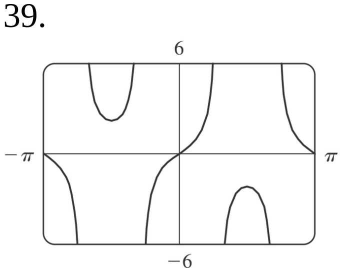


37. No matter how many times we zoom in toward the origin, the graphs of $f(x)=\sin(\pi/x)$ appear to consist of almost–vertical lines. This indicates more and more frequent oscillations as $x\to0$.



38. $\lim_{v\to c^-} m=\lim_{v\to c^-} \dfrac{m_0}{\sqrt{1-v^2/c^2}}$. As $v\to c^-$, $\sqrt{1-v^2/c^2}\to0^+$, and $m\to\infty$.

39.

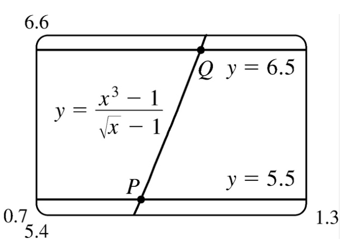


There appear to be vertical asymptotes of the curve $y=\tan(2\sin x)$ at $x\approx\pm0.90$ and $x\approx\pm2.24$. To find the exact equations of these asymptotes, we note that the graph of the tangent function has vertical asymptotes at $x=\frac{\pi}{2}+\pi n$. Thus, we must have $2\sin x=\frac{\pi}{2}+\pi n$, or equivalently, $\sin x=\frac{\pi}{4}+\frac{\pi}{2}n$. Since $-1\le \sin x\le 1$, we must have $\sin x=\pm\frac{\pi}{4}$ and so $x=\pm\sin^{-1}\frac{\pi}{4}$ (corresponding to $x\approx\pm 0.90$).

Just as 150° is the reference angle for 30° , $\pi-\sin^{-1}\frac{\pi}{4}$ is the reference angle for $\sin^{-1}\frac{\pi}{4}$. So $x=\pm\left(\pi-\sin^{-1}\frac{\pi}{4}\right)$ are also equations of the vertical asymptotes (corresponding to $x\approx\pm 2.24$).

40. **(a)** Let $y=\left(x^3-1\right)/\left(\sqrt{x}-1\right)$.

x	y
0.99	5.92531
0.999	5.99250
0.9999	5.99925
1.01	6.07531
1.001	6.00750
1.0001	6.00075



From the table and the graph, we guess that the limit of y as x approaches 1 is 6 .

(b) We need to have $5.5<\frac{x^3-1}{\sqrt{x}-1}<6.5$. From the graph we obtain the approximate points of intersection $P\left(0.9313853,5.5\right)$ and $Q\left(1.0649004,6.5\right)$. Now $1-0.9313853\approx 0.0686$ and $1.0649004-1\approx 0.0649$, so by requiring that x be within 0.0649 of 1 , we ensure that y is within 0.5 of 6 .