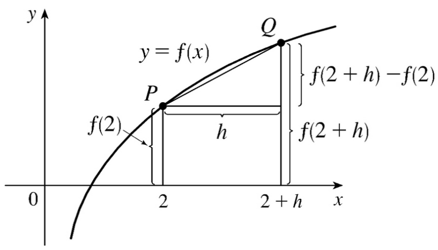
1.



The line from $P(2,f(2))$ to $Q(2+h,f(2+h))$ is the line that has slope $\dfrac{f(2+h)-f(2)}{h}$

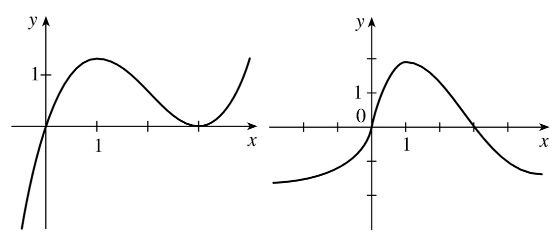
2. As h decreases, the line PQ becomes steeper, so its slope increases. So
$0<\dfrac{f(4)-f(2)}{4-2}<\dfrac{f(3)-f(2)}{3-2}<\lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2}$. Thus, $0<\dfrac{1}{2}[f(4)-f(2)]<f(3)-f(2)<f'(2)$.

3. $g'(0)$ is the only negative value. The slope at $x=4$ is smaller than the slope at $x=2$ and both are smaller than the slope at $x=-2$. Thus, $g'(0)<0<g'(4)<g'(2)<g'(-2)$.

4. Since (4,3) is on $y=f(x)$, $f(4)=3$. The slope of the tangent line between (0,2) and (4,3) is $\dfrac{1}{4}$, so $f'(4)=\dfrac{1}{4}$.

5.

We begin by drawing a curve through the origin at a slope of 3 to satisfy $f(0)=0$ and $f'(0)=3$. Since $f'(1)=0$, we will round off our figure so that there is a horizontal tangent directly over $x=1$. Lastly, we make sure that the curve has a slope of -1 as we pass over $x=2$. Two of the many possibilities are shown.



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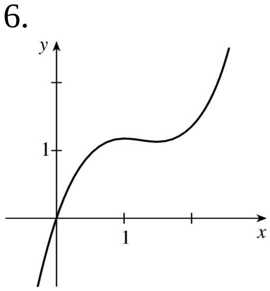


7. Using Definition 2 with

$f(x)=3x^2-5x$ and the point $(2,2)$, we have

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{[3(2+h)^2-5(2+h)]-2}{h}$$

$$=\lim_{h\to 0}\frac{(12+12h+3h^2-10-5h)-2}{h}=\lim_{h\to 0}\frac{3h^2+7h}{h}=\lim_{h\to 0}(3h+7)=7\ .$$

So an equation of the tangent line at $(2,2)$ is $y-2=7(x-2)$ or $y=7x-12$.

8. Using Definition 2 with $g(x)=1-x^3$ and the point $(0,1)$, we have

$$g'(0)=\lim_{h\to 0}\frac{g(0+h)-g(0)}{h}=\lim_{h\to 0}\frac{[1-(0+h)^3]-1}{h}=\lim_{h\to 0}\frac{(1-h^3)-1}{h}=\lim_{h\to 0}(-h^2)=0\ .$$

So an equation of the tangent line is $y-1=0(x-0)$ or $y=1$.

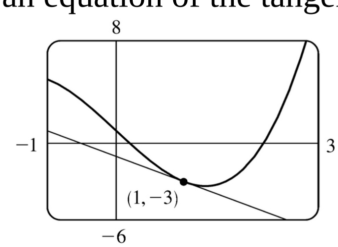
9. **(a)** Using Definition 2 with $F(x)=x^3-5x+1$ and the point $(1,-3)$, we have

$$F'(1)=\lim_{h\to 0}\frac{F(1+h)-F(1)}{h}=\lim_{h\to 0}\frac{[(1+h)^3-5(1+h)+1]-(-3)}{h}$$

$$=\lim_{h\to 0}\frac{(1+3h+3h^2+h^3-5-5h+1)+3}{h}=\lim_{h\to 0}\frac{h^3+3h^2-2h}{h}$$

$$=\lim_{h\to 0}\frac{h(h^2+3h-2)}{h}=\lim_{h\to 0}(h^2+3h-2)=-2$$

So an equation of the tangent line at $(1,-3)$ is $y-(-3)=-2(x-1)\Leftrightarrow y=-2x-1$.

(b) 

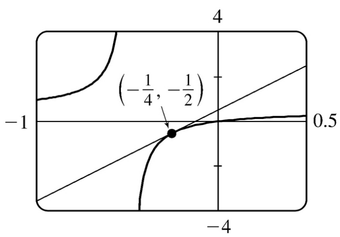
10. **(a)**

$$G'(a)=\lim_{h\to 0}\frac{G(a+h)-G(a)}{h}=\lim_{h\to 0}\frac{\frac{a+h}{1+2(a+h)}-\frac{a}{1+2a}}{h}$$

$$=\lim_{h\to 0}\frac{a+2a^2+h+2ah-a-2a^2-2ah}{h(1+2a+2h)(1+2a)}=\lim_{h\to 0}\frac{1}{(1+2a+2h)(1+2a)}=(1+2a)^{-2}$$

So the slope of the tangent at the point $(-\frac{1}{4},-\frac{1}{2})$ is $m=\left[1+2(-\frac{1}{4})\right]^{-2}=4$, and thus an equation is $y+\frac{1}{2}=4(x+\frac{1}{4})$ or $y=4x+\frac{1}{2}$.

(b)



11. **(a)** $f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{3^{1+h}-3^1}{h}$.

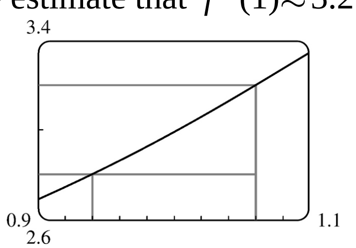
So let $F(h)=\frac{3^{1+h}-3}{h}$. We calculate:

h	$F(h)$
0.1	3.484
0.01	3.314
0.001	3.298
0.0001	3.296

h	$F(h)$
–0.1	3.121
–0.01	3.278
–0.001	3.294
–0.0001	3.296

We estimate that $f'(1)\approx 3.296$.

(b)



From the graph, we estimate that the slope of the tangent is about $\frac{3.2-2.8}{1.06-0.94}=\frac{0.4}{0.12}\approx 3.3$.

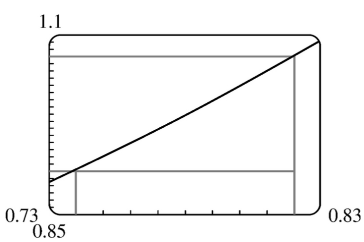
12. **(a)**

$$g'\left(\frac{\pi}{4}\right)=\lim_{h\to 0}\frac{g\left(\frac{\pi}{4}+h\right)-g\left(\frac{\pi}{4}\right)}{h}=\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{4}+h\right)-\tan\left(\frac{\pi}{4}\right)}{h}\ .$$

So let $G(h)=\dfrac{\tan\left(\frac{\pi}{4}+h\right)-1}{h}$. We calculate:

h	$G(h)$
0.1	2.2305
0.01	2.0203
0.001	2.0020
0.0001	2.0002

h	$G(h)$
–0.1	1.8237
–0.01	1.9803
–0.001	1.9980
–0.0001	1.9998

(b) 

From the graph, we estimate that the slope of the tangent is about $\dfrac{1.07-0.91}{0.82-0.74}=\dfrac{0.16}{0.08}=2$.

13. Use Definition 2 with $f(x)=3-2x+4x^2$.

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{[3-2(a+h)+4(a+h)^2]-(3-2a+4a^2)}{h}$$

$$=\lim_{h\to 0}\frac{[3-2a-2h+4a^2+8ah+4h^2]-(3-2a+4a^2)}{h}=\lim_{h\to 0}\frac{-2h+8ah+4h^2}{h}$$

$$=\lim_{h\to 0}\frac{h(-2+8a+4h)}{h}=\lim_{h\to 0}(-2+8a+4h)=-2+8a$$

14.

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{[(a+h)^4-5(a+h)]-(a^4-5a)}{h}$$

$$=\lim_{h\to 0}\frac{(a^4+4a^3h+6a^2h^2+4ah^3+h^4-5a-5h)-(a^4-5a)}{h}$$

$$=\lim_{h\to 0}\frac{4a^3h+6a^2h^2+4ah^3+h^4-5h}{h}=\lim_{h\to 0}\frac{h(4a^3+6a^2h+4ah^2+h^3-5)}{h}$$

$$=\lim_{h\to 0}(4a^3+6a^2h+4ah^2+h^3-5)=4a^3-5$$

15. Use Definition 2 with $f(t)=(2t+1)/(t+3)$.

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{\frac{2(a+h)+1}{(a+h)+3}-\frac{2a+1}{a+3}}{h}$$

$$=\lim_{h\to 0}\frac{(2a+2h+1)(a+3)-(2a+1)(a+h+3)}{h(a+h+3)(a+3)}$$

$$=\lim_{h\to 0}\frac{(2a^2+6a+2ah+6h+a+3)-(2a^2+2ah+6a+a+h+3)}{h(a+h+3)(a+3)}$$

$$=\lim_{h\to 0}\frac{5h}{h(a+h+3)(a+3)}=\lim_{h\to 0}\frac{5}{(a+h+3)(a+3)}=\frac{5}{(a+3)^2}$$

16.

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{\frac{(a+h)^2+1}{(a+h)-2}-\frac{a^2+1}{a-2}}{h}$$

$$=\lim_{h\to 0}\frac{(a^2+2ah+h^2+1))(a-2)-(a^2+1)(a+h-2)}{h(a+h-2)(a-2)}$$

$$=\lim_{h\to 0}\frac{(a^3-2a^2+2a^2h-4ah+ah^2-2h^2+a-2)-(a^3+a^2h-2a^2+a+h-2)}{h(a+h-2)(a-2)}$$

$$=\lim_{h\to 0}\frac{a^2h-4ah+ah^2-2h^2-h}{h(a+h-2)(a-2)}=\lim_{h\to 0}\frac{h(a^2-4a+ah-2h-1)}{h(a+h-2)(a-2)}$$

$$=\lim_{h\to 0}\frac{a^2-4a+ah-2h-1}{(a+h-2)(a-2)}=\frac{a^2-4a-1}{(a-2)^2}$$

17. Use Definition 2 with $f(x)=1/\sqrt{x+2}$.

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{\dfrac{1}{\sqrt{(a+h)+2}}-\dfrac{1}{\sqrt{a+2}}}{h}$$

$$=\lim_{h\to 0}\frac{\dfrac{\sqrt{a+2}-\sqrt{a+h+2}}{\sqrt{a+h+2}\,\sqrt{a+2}}}{h}=\lim_{h\to 0}\left[\frac{\sqrt{a+2}-\sqrt{a+h+2}}{h\sqrt{a+h+2}\,\sqrt{a+2}}\cdot\frac{\sqrt{a+2}+\sqrt{a+h+2}}{\sqrt{a+2}+\sqrt{a+h+2}}\right]$$

$$=\lim_{h\to 0}\frac{(a+2)-(a+h+2)}{h\sqrt{a+h+2}\,\sqrt{a+2}(\sqrt{a+2}+\sqrt{a+h+2})}$$

$$=\lim_{h\to 0}\frac{-h}{h\sqrt{a+h+2}\,\sqrt{a+2}(\sqrt{a+2}+\sqrt{a+h+2})}$$

$$=\lim_{h\to 0}\frac{-1}{\sqrt{a+h+2}\,\sqrt{a+2}(\sqrt{a+2}+\sqrt{a+h+2})}$$

$$=\frac{-1}{(\sqrt{a+2})^2(2\sqrt{a+2})}=-\frac{1}{2(a+2)^{3/2}}$$

18.

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{\sqrt{3(a+h)+1}-\sqrt{3a+1}}{h}$$

$$=\lim_{h\to 0}\frac{(\sqrt{3a+3h+1}-\sqrt{3a+1})(\sqrt{3a+3h+1}+\sqrt{3a+1})}{h(\sqrt{3a+3h+1}+\sqrt{3a+1})}$$

$$=\lim_{h\to 0}\frac{(3a+3h+1)-(3a+1)}{h(\sqrt{3a+3h+1}+\sqrt{3a+1})}=\lim_{h\to 0}\frac{3h}{h(\sqrt{3a+3h+1}+\sqrt{3a+1})}$$

$$=\lim_{h\to 0}\frac{3}{\sqrt{3a+3h+1}+\sqrt{3a+1}}=\frac{3}{2\sqrt{3a+1}}$$

19. By Definition 2, $\lim_{h\to 0}\dfrac{(1+h)^{10}-1}{h}=f'(1)$, where $f(x)=x^{10}$ and $a=1$. *Or:* By Definition 2, $\lim_{h\to 0}\dfrac{(1+h)^{10}-1}{h}=f'(0)$, where $f(x)=(1+x)^{10}$ and $a=0$.

20. By Definition 2,

$\lim_{h\to 0}\dfrac{\sqrt[4]{16+h}-2}{h}=f'(16)$, where $f(x)=\sqrt[4]{x}$ and $a=16$. *Or* : By Definition 2, $\lim_{h\to 0}\dfrac{\sqrt[4]{16+h}-2}{h}=f'(0)$, where $f(x)=\sqrt[4]{16+x}$ and $a=0$.

21. By Equation 3, $\lim_{x\to 5}\dfrac{2^x-32}{x-5}=f'(5)$, where $f(x)=2^x$ and $a=5$.

22. By Equation 3, $\lim_{x\to \pi/4}\dfrac{\tan x-1}{x-\pi/4}=f'(\pi/4)$, where $f(x)=\tan x$ and $a=\pi/4$.

23. By Definition 2, $\lim_{h\to 0}\dfrac{\cos(\pi+h)+1}{h}=f'(\pi)$, where $f(x)=\cos x$ and $a=\pi$. *Or* : By Definition 2, $\lim_{h\to 0}\dfrac{\cos(\pi+h)+1}{h}=f'(0)$, where $f(x)=\cos(\pi+x)$ and $a=0$.

24. By Equation 3, $\lim_{t\to 1}\dfrac{t^4+t-2}{t-1}=f'(1)$, where $f(t)=t^4+t$ and $a=1$.

25.

$$v(2)=f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{[(2+h)^2-6(2+h)-5]-[2^2-6(2)-5]}{h}$$

$$=\lim_{h\to 0}\frac{(4+4h+h^2-12-6h-5)-(-13)}{h}=\lim_{h\to 0}\frac{h^2-2h}{h}=\lim_{h\to 0}(h-2)=-2\,m/s$$

26.

$$v(2)=f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$=\lim_{h\to 0}\frac{[2(2+h)^3-(2+h)+1]-[2(2)^3-2+1]}{h}$$

$$=\lim_{h\to 0}\frac{(2h^3+12h^2+24h+16-2-h+1)-15}{h}$$

$$=\lim_{h\to 0}\frac{2h^3+12h^2+23h}{h}=\lim_{h\to 0}(2h^2+12h+23)=23m/s$$

27. **(a)** $f'(x)$ is the rate of change of the production cost with respect to the number of ounces of gold produced. Its units are dollars per ounce.
(b) After 800 ounces of gold have been produced, the rate at which the production cost is increasing is \$17/ ounce. So the cost of producing the 800 th (or 801 st) ounce is about \$17 .
(c) In the short term, the values of $f'(x)$ will decrease because more efficient use is made of start–up costs as x increases. But eventually $f'(x)$ might increase due to large–scale operations.

28. **(a)** $f'(5)$ is the rate of growth of the bacteria population when $t=5$ hours. Its units are bacteria per hour.
(b) With unlimited space and nutrients, f' should increase as t increases; so $f'(5)<f'(10)$. If the supply of nutrients is limited, the growth rate slows down at some point in time, and the opposite may be true.

29. **(a)** $f'(v)$ is the rate at which the fuel consumption is changing with respect to the speed. Its units are $(gal / h)/(mi / h)$.
(b) The fuel consumption is decreasing by $0.05(gal/\mathrm{h})/(mi/h)$ as the car's speed reaches 20 mi/h . So if you increase your speed to 21 mi/h , you could expect to decrease your fuel consumption by about $0.05(gal/\mathrm{h})/(mi/h)$.

30. **(a)** $f'(8)$ is the rate of change of the quantity of coffee sold with respect to the price per pound when the price is \$8 per pound. The units for $f'(8)$ are pounds /(dollars / pound) .
(b) $f'(8)$ is negative since the quantity of coffee sold will decrease as the price charged for it increases. People are generally less willing to buy a product when its price increases.

31. $T'(10)$ is the rate at which the temperature is changing at 10:00 A.M. To estimate the value of $T'(10)$, we will average the difference quotients obtained using the times $t=8$ and $t=12$. Let $A=\frac{T(8)-T(10)}{8-10}=\frac{72-81}{-2}=4.5$ and $B=\frac{T(12)-T(10)}{12-10}=\frac{88-81}{2}=3.5$. Then
$T'(10)=\lim_{t\to 10}\frac{T(t)-T(10)}{t-10}\approx\frac{A+B}{2}=\frac{4.5+3.5}{2}=4^{\circ}F/h$.

32. **For 1910:** We will average the difference quotients obtained using the years 1900 and 1920.

Let $A=\dfrac{E(1900)-E(1910)}{1900-1910}=\dfrac{48.3-51.1}{-10}=0.28$ and
$B=\dfrac{E(1920)-E(1910)}{1920-1910}=\dfrac{55.2-51.1}{10}=0.41$. Then
$E'(1910)=\lim_{t\to 1910}\dfrac{E(t)-E(1910)}{t-1910}\approx\dfrac{A+B}{2}=0.345$. This means that life expectancy at birth was increasing at about 0.345 year / year in 1910.
For 1950: Using data for 1940 and 1960 in a similar fashion, we obtain
$E'(1950)\approx[0.31+0.10]/2=0.205$. So life expectancy at birth was increasing at about 0.205 year / year in 1950.

33. **(a)** $S'(T)$ is the rate at which the oxygen solubility changes with respect to the water temperature. Its units are (mg / L)/° C .
(b) For $T=16°$ C , it appears that the tangent line to the curve goes through the points (0,14) and (32,6) . So $S'(16)\approx\dfrac{6-14}{32-0}=-\dfrac{8}{32}=-0.25$(mg / L)/° C . This means that as the temperature increases past 16° C , the oxygen solubility is decreasing at a rate of 0.25(mg / L)/° C .

34. **(a)** $S'(T)$ is the rate of change of the maximum sustainable speed of Coho salmon with respect to the temperature. Its units are (cm / s)/° C .
(b) For $T=15°$ C, it appears the tangent line to the curve goes through the points (10,25) and (20,32) . So $S'(15)\approx\dfrac{32-25}{20-10}=0.7$(cm / s)/° C . This tells us that at $T=15°$ C, the maximum sustainable speed of Coho salmon is changing at a rate of 0.7 (cm / s)/° C . In a similar fashion for $T=25°$ C, we can use the points (20,35) and (25,25) to obtain $S'(25)\approx\dfrac{25-35}{25-20}=-2$ (cm / s) /° C . As it gets warmer than 20° C, the maximum sustainable speed decreases rapidly.

35. Since $f(x)=x\sin(1/x)$ when $x\neq 0$ and $f(0)=0$, we have
$f'(0)=\lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\dfrac{h\sin(1/h)-0}{h}=\lim_{h\to 0}(sin(1/h))$. This limit does not exist since $sin(1/h)$ takes the values -1 and 1 on any interval containing 0 . (Compare with Example 4 in Section 2.2.)

36. Since $f(x)=x^2 sin(1/x)$ when $x\neq 0$ and $f(0)=0$, we have
$f'(0)=\lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\dfrac{h^2\sin(1/h)-0}{h}=\lim_{h\to 0}(h\sin(1/h))$. Since $-1\leq sin\,\dfrac{1}{h}\leq 1$, we have

$-|h| \leq |h| \sin \frac{1}{h} \leq |h| \Rightarrow -|h| \leq h\sin \frac{1}{h} \leq |h|$. Because $\lim_{h \to 0}(-|h|)=0$ and $\lim_{h \to 0}|h|=0$, we know that $\lim_{h \to 0}(h \sin \frac{1}{h})=0$ by the Squeeze Theorem. Thus, $f^{/}(0)=0$.