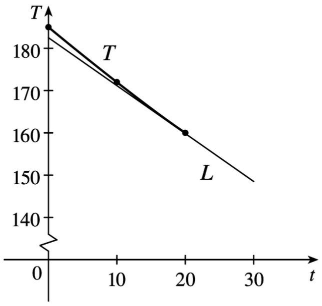
1. As in Example 1, $T(0)=185$, $T(10)=172$, $T(20)=160$, and
$T^{/}(20)\approx\dfrac{T(10)-T(20)}{10-20}=\dfrac{172-160}{-10}=-1.2^{\circ}$ F / min. $T(30)\approx T(20)+T^{/}(20)(30-20)\approx 160-1.2(10)=148^{\circ}$ F.

We would expect the temperature of the turkey to get closer to 75° F

as time increases. Since the temperature decreased 13° F in the first 10 minutes and 12° F in the second 10 minutes, we can assume that the slopes of the tangent line are increasing through negative values: $-1.3,-1.2,\ldots$. Hence, the tangent lines are under the curve and 148° F



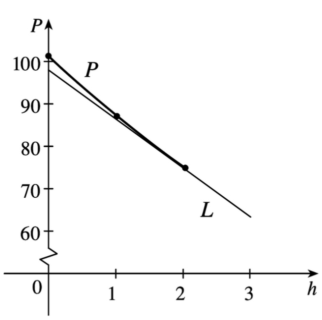
is an underestimate. From the figure, we estimate the slope of the tangent line at $t=20$ to be
$\dfrac{184-147}{0-30}=-\dfrac{37}{30}$.

Then the linear approximation becomes $T(30)\approx T(20)+T^{/}(20)\cdot 10\approx 160-\dfrac{37}{30}(10)=147\dfrac{2}{3}\approx 147.7$.

2. $P^{/}(2)\approx\dfrac{P(1)-P(2)}{1-2}=\dfrac{87.1-74.9}{-1}=-12.2$ kilopascals / km.

$P(3)\approx P(2)+P^{/}(2)(3-2)\approx 74.9-12.2(1)=62.7$ kPa.

From the figure, we estimate the slope of the tangent line at $h=2$ to be $\dfrac{98-63}{0-3}=-\dfrac{35}{3}$. Then the linear approximation becomes $P(3)\approx P(2)+P^{/}(2)\cdot 1\approx 74.9-\dfrac{35}{3}\approx 63.23$ kPa.



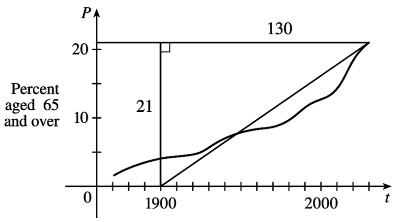
3. Extend the tangent line at the point (2030,21) to the t –axis. Answers will vary based on this approximation–—we'll use $t=1900$ as our t –intercept. The linearization is then

$P(t)\quad\approx P(2030)+P^{/}(2030)(t-2030)$

$$\approx 21+\frac{21}{130}(t-2030)$$

$$P(2040)=21+\frac{21}{130}(2040-2030)\approx 22.6\%$$

$$P(2050)=21+\frac{21}{130}(2050-2030)\approx 24.2\%$$



These predictions are probably too high since the tangent line lies above the graph at $t=2030$.

4. Let $A=\frac{N(1980)-N(1985)}{1980-1985}=\frac{15.0-17.0}{-5}=0.4$ and $B=\frac{N(1990)-N(1985)}{1990-1985}=\frac{19.3-17.0}{5}=0.46$. Then $N'(1985)=\lim_{t\to 1985}\frac{N(t)-N(1985)}{t-1985}\approx\frac{A+B}{2}=0.43$ million / year. So

$N(1984)\approx N(1985)+N'(1985)(1984-1985)\approx 17.0+0.43(-1)=16.57$ million.

$N'(2000)\approx\frac{N(1995)-N(2000)}{1995-2000}=\frac{22.0-24.9}{-5}=0.58$ million / year.

$N(2006)\approx N(2000)+N'(2000)(2006-2000)\approx 24.9+0.58(6)=28.38$ million.

5. $f(x)=x^3\Rightarrow f'(x)=3x^2$, so $f(1)=1$ and $f'(1)=3$. With $a=1$, $L(x)=f(a)+f'(a)(x-a)$ becomes $L(x)=f(1)+f'(1)(x-1)=1+3(x-1)=3x-2$.

6. $f(x)=\ln x\Rightarrow f'(x)=1/x$, so $f(1)=0$ and $f'(1)=1$. Thus, $L(x)=f(1)+f'(1)(x-1)=0+1(x-1)=x-1$.

7. $f(x)=\cos x\Rightarrow f'(x)=-\sin x$, so $f\left(\frac{\pi}{2}\right)=0$ and $f'\left(\frac{\pi}{2}\right)=-1$. Thus,
$L(x)=f\left(\frac{\pi}{2}\right)+f'\left(\frac{\pi}{2}\right)\left(x-\frac{\pi}{2}\right)=0-1\left(x-\frac{\pi}{2}\right)=-x+\frac{\pi}{2}$.

8. $f(x)=\sqrt[3]{x}=x^{1/3}\Rightarrow f'(x)=\frac{1}{3}x^{-2/3}$, so $f(-8)=-2$ and $f'(-8)=\frac{1}{12}$. Thus,
$L(x)=f(-8)+f'(-8)(x+8)=-2+\frac{1}{12}(x+8)=\frac{1}{12}x-\frac{4}{3}$.

9. $f(x)=\sqrt{1-x}\Rightarrow$

$f'(x)=\frac{-1}{2\sqrt{1-x}}$, so $f(0)=1$ and $f'(0)=-\frac{1}{2}$. Therefore,

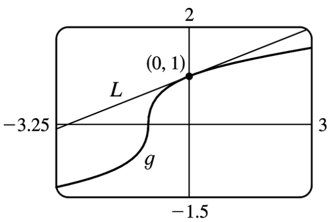
$$\sqrt{1-x}=f(x)\approx f(0)+f'(0)(x-0)$$
$$=1+\left(-\frac{1}{2}\right)(x-0)=1-\frac{1}{2}x$$

So $\sqrt{0.9}=\sqrt{1-0.1}\approx 1-\frac{1}{2}(0.1)=0.95$ and $\sqrt{0.99}=\sqrt{1-0.01}\approx 1-\frac{1}{2}(0.01)=0.995$.

10. $g(x)=\sqrt[3]{1+x}=(1+x)^{1/3}\Rightarrow g'(x)=\frac{1}{3}(1+x)^{-2/3}$,

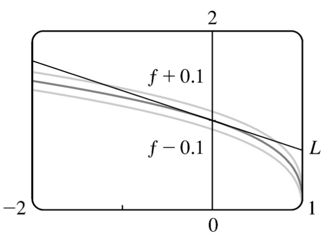
so $g(0)=1$ and $g'(0)=\frac{1}{3}$. Therefore, $\sqrt[3]{1+x}=g(x)\approx g(0)+g'(0)(x-0)=1+\frac{1}{3}x$. So

$\sqrt[3]{0.95}=\sqrt[3]{1+(-0.05)}\approx 1+\frac{1}{3}(-0.05)=0.98\overline{3}$, and $\sqrt[3]{1.1}=\sqrt[3]{1+0.1}\approx 1+\frac{1}{3}(0.1)=1.0\overline{3}$.

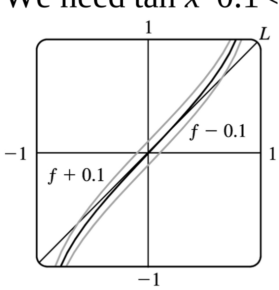


11. $f(x)=\sqrt[3]{1-x}=(1-x)^{1/3}\Rightarrow f'(x)=-\frac{1}{3}(1-x)^{-2/3}$, so $f(0)=1$ and $f'(0)=-\frac{1}{3}$. Thus,

$f(x)\approx f(0)+f'(0)(x-0)=1-\frac{1}{3}x$. We need $\sqrt[3]{1-x}-0.1<1-\frac{1}{3}x<\sqrt[3]{1-x}+0.1$, which is true when $-1.204<x<0.706$.

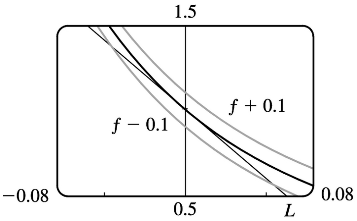


12. $f(x)=\tan x\Rightarrow f'(x)=\sec^2 x$, so $f(0)=0$ and $f'(0)=1$. Thus, $f(x)\approx f(0)+f'(0)(x-0)=0+1(x-0)=x$. We need $\tan x-0.1<x<\tan x+0.1$, which is true when $-0.63<x<0.63$.



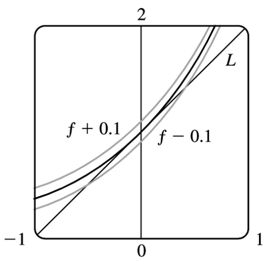
13. $f(x)=\dfrac{1}{(1+2x)^4}=(1+2x)^{-4}\Rightarrow f'(x)=-4(1+2x)^{-5}(2)=\dfrac{-8}{(1+2x)^5}$, so $f(0)=1$ and $f'(0)=-8$. Thus, $f(x)\approx f(0)+f'(0)(x-0)=1+(-8)(x-0)=1-8x$.

We need $1/(1+2x)^4-0.1<1-8x<1/(1+2x)^4+0.1$, which is true when $-0.045<x<0.055$.



14. $f(x)=e^x\Rightarrow f'(x)=e^x$, so $f(0)=1$ and $f'(0)=1$. Thus, $f(x)\approx f(0)+f'(0)(x-0)=1+1(x-0)=1+x$.

We need $e^x-0.1<1+x<e^x+0.1$, which is true when $-0.483<x<0.416$.



15. If $y=f(x)$, then the differential dy is equal to $f'(x)\,dx$. $y=x^4+5x\Rightarrow dy=(4x^3+5)dx$.

16. $y=\cos \pi x\Rightarrow dy=-\sin \pi x\cdot \pi\,dx=-\pi\sin \pi x\,dx$

17. $y=x\ln x\Rightarrow dy=\left(x\cdot \dfrac{1}{x}+\ln x\cdot 1\right)dx=(1+\ln x)\,dx$

18. $y=\sqrt{1+t^2}\Rightarrow dy=\dfrac{1}{2}(1+t^2)^{-1/2}(2t)\,dt=\dfrac{t}{\sqrt{1+t^2}}\,dt$

19. $y=\dfrac{u+1}{u-1}\Rightarrow dy=\dfrac{(u-1)(1)-(u+1)(1)}{(u-1)^2}\,du=\dfrac{-2}{(u-1)^2}\,du$

20. $y=(1+2r)^{-4}\Rightarrow dy=-4(1+2r)^{-5}\cdot 2\,dr=-8(1+2r)^{-5}\,dr$

21. **(a)** $y=x^2+2x\Rightarrow dy=(2x+2)\,dx$

(b) When $x=3$ and

$dx=\frac{1}{2}$, $dy=[2(3)+2]\left(\frac{1}{2}\right)=4$.

22. **(a)** $y=e^{x/4} \Rightarrow dy=\frac{1}{4}e^{x/4}dx$

(b) When $x=0$ and $dx=0.1$, $dy=\left(\frac{1}{4}e^{0}\right)(0.1)=0.025$.

23. **(a)** $y=\sqrt{4+5x} \Rightarrow dy=\frac{1}{2}(4+5x)^{-1/2}\cdot 5dx=\frac{5}{2\sqrt{4+5x}}dx$

(b) When $x=0$ and $dx=0.04$, $dy=\frac{5}{2\sqrt{4}}(0.04)=\frac{5}{4}\cdot\frac{1}{25}=\frac{1}{20}=0.05$.

24. **(a)** $y=1/(x+1) \Rightarrow dy=-\frac{1}{(x+1)^2}dx$

(b) When $x=1$ and $dx=-0.01$, $dy=-\frac{1}{2^2}(-0.01)=\frac{1}{4}\cdot\frac{1}{100}=\frac{1}{400}=0.0025$.

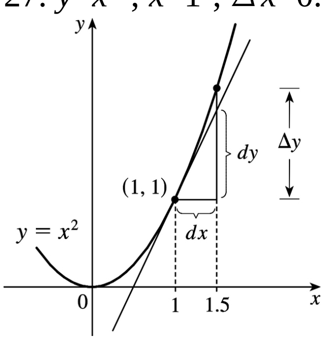
25. **(a)** $y=\tan x \Rightarrow dy=\sec^2 x\,dx$

(b) When $x=\pi/4$ and $dx=-0.1$, $dy=[\sec(\pi/4)]^2(-0.1)=(\sqrt{2})^2(-0.1)=-0.2$.

26. **(a)** $y=\cos x \Rightarrow dy=-\sin x\,dx$

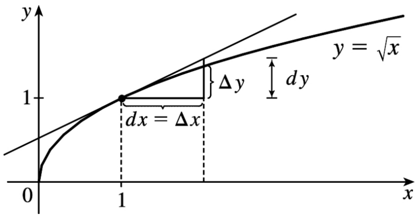
(b) When $x=\pi/3$ and $dx=0.05$, $dy=-\sin(\pi/3)(0.05)=-0.5\sqrt{3}(0.05)=-0.025\sqrt{3}\approx -0.043$.

27. $y=x^2$, $x=1$, $\Delta x=0.5 \Rightarrow \Delta y=(1.5)^2-1^2=1.25$. $dy=2x\,dx=2(1)(0.5)=1$

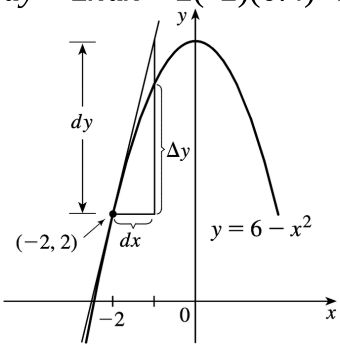


28. $y=\sqrt{x}$, $x=1$, $\Delta x=1 \Rightarrow \Delta y=\sqrt{2}-\sqrt{1}=\sqrt{2}-1\approx 0.414$

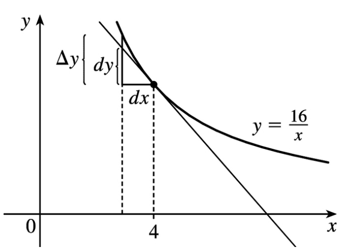
$dy=\frac{1}{2\sqrt{x}}dx=\frac{1}{2}(1)=0.5$



29. $y=6-x^2$, $x=-2$, $\Delta x=0.4 \Rightarrow \Delta y=(6-(-1.6)^2)-(6-(-2)^2)=1.44$
$dy=-2x\,dx=-2(-2)(0.4)=1.6$



30. $y=\frac{16}{x}$, $x=4$, $\Delta x=-1 \Rightarrow \Delta y=\frac{16}{3}-\frac{16}{4}=\frac{4}{3}$. $dy=-\left(\frac{16}{x^2}\right)dx=-\left(\frac{16}{4^2}\right)(-1)=1$



31. $y=f(x)=x^5 \Rightarrow dy=5x^4\,dx$. When $x=2$ and $dx=0.001$, $dy=5(2)^4(0.001)=0.08$, so
$(2.001)^5=f(2.001)\approx f(2)+dy=32+0.08=32.08$.

32. $y=f(x)=\sqrt{x} \Rightarrow dy=\frac{1}{2\sqrt{x}}\,dx$. When $x=100$ and $dx=-0.2$, $dy=\frac{1}{2\sqrt{100}}(-0.2)=-0.01$, so
$\sqrt{99.8}=f(99.8)\approx f(100)+dy=10-0.01=9.99$.

33. $y=f(x)=x^{2/3} \Rightarrow dy=\frac{2}{3\sqrt[3]{x}}\,dx$. When $x=8$ and $dx=0.06$, $dy=\frac{2}{3\sqrt[3]{8}}(0.06)=0.02$, so
$(8.06)^{2/3}=f(8.06)\approx f(8)+dy=4+0.02=4.02$.

34. $y=f(x)=1/x \Rightarrow dy=(-1/x^2)dx$. When $x=1000$ and $dx=2, dy=[-1/(1000)^2](2)=-0.000002$, so
$1/1002=f(1002)\approx f(1000)+dy=1/1000-0.000002=0.000998$

35. $y=f(x)=\tan x \Rightarrow dy=\sec^2 x\,dx$. When $x=45^\circ$ and $dx=-1^\circ$, $dy=\sec^2 45^\circ(-\pi/180)=(\sqrt{2})^2(-\pi/180)=-\pi/90$, so $\tan 44^\circ=f(44^\circ)\approx f(45^\circ)+dy=1-\pi/90\approx 0.965$.

36. $y=f(x)=\ln x \Rightarrow dy=\frac{1}{x}\,dx$. When $x=1$ and $dx=0.07$, $dy=\frac{1}{1}(0.07)=0.07$, so $\ln 1.07=f(1.07)\approx f(1)+dy=0+0.07=0.07$.

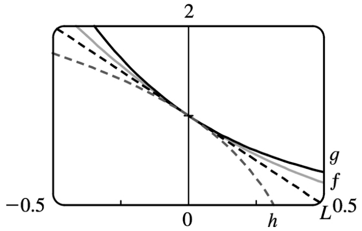
37. $y=f(x)=\sec x \Rightarrow f'(x)=\sec x\tan x$, so $f(0)=1$ and $f'(0)=1\cdot 0=0$. The linear approximation of f at 0 is $f(0)+f'(0)(x-0)=1+0(x)=1$. Since 0.08 is close to 0 , approximating $\sec 0.08$ with 1 is reasonable.

38. If $y=x^6$, $y'=6x^5$ and the tangent line approximation at $(1,1)$ has slope 6 . If the change in x is 0.01 , the change in y on the tangent line is 0.06 , and approximating $(1.01)^6$ with 1.06 is reasonable.

39. $y=f(x)=\ln x \Rightarrow f'(x)=1/x$, so $f(1)=0$ and $f'(1)=1$. The linear approximation of f at 1 is $f(1)+f'(1)(x-1)=0+1(x-1)=x-1$. Now $f(1.05)=\ln 1.05\approx 1.05-1=0.05$, so the approximation is reasonable.

40. **(a)** $f(x)=(x-1)^2 \Rightarrow f'(x)=2(x-1)$, so $f(0)=1$ and $f'(0)=-2$.
Thus, $f(x)\approx L_f(x)=f(0)+f'(0)(x-0)=1-2x$.
$g(x)=e^{-2x} \Rightarrow g'(x)=-2e^{-2x}$, so $g(0)=1$ and $g'(0)=-2$.
Thus, $g(x)\approx L_g(x)=g(0)+g'(0)(x-0)=1-2x$.
$h(x)=1+\ln(1-2x) \Rightarrow h'(x)=\dfrac{-2}{1-2x}$, so $h(0)=1$ and $h'(0)=-2$.
Thus, $h(x)\approx L_h(x)=h(0)+h'(0)(x-0)=1-2x$.
Notice that $L_f=L_g=L_h$. This happens because f , g , and h have the same function values and the same derivative values at $a=0$.

(b)



The linear approximation appears to be the best for the function f since it is closer to f for a larger domain than it is to g and h . The approximation looks worst for h since h moves away from L faster than f and g do.

41. **(a)** If x is the edge length, then $V=x^3 \Rightarrow dV=3x^2 dx$. When $x=30$ and $dx=0.1$, $dV=3(30)^2(0.1)=270$, so the maximum possible error in computing the volume of the cube is about 270 cm ^{3} . The relative error is calculated by dividing the change in V , ΔV , by V . We approximate ΔV with dV .

Relative error = $\frac{\Delta V}{V} \approx \frac{dV}{V} = \frac{3x^2 dx}{x^3} = 3\frac{dx}{x} = 3\left(\frac{0.1}{30}\right) = 0.01$.

Percentage error = relative error$\times 100\% = 0.01 \times 100\% = 1\%$.

(b) $S=6x^2 \Rightarrow dS=12x\,dx$. When $x=30$ and $dx=0.1$, $dS=12(30)(0.1)=36$, so the maximum possible error in computing the surface area of the cube is about 36 cm ^{2} .

Relative error = $\frac{\Delta S}{S} \approx \frac{dS}{S} = \frac{12x dx}{6x^2} = 2\frac{dx}{x} = 2\left(\frac{0.1}{30}\right) = 0.00\overline{6}$.

Percentage error = relative error$\times 100\% = 0.00\overline{6} \times 100\% = 0.\overline{6}\%$.

42. **(a)** $A=\pi r^2 \Rightarrow dA=2\pi r\,dr$. When $r=24$ and $dr=0.2$, $dA=2\pi(24)(0.2)=9.6\pi$, so the maximum possible error in the calculated area of the disk is about $9.6\pi \approx 30$ cm ^{2} .

(b) Relative error = $\frac{\Delta A}{A} \approx \frac{dA}{A} = \frac{2\pi r dr}{\pi r^2} = \frac{2dr}{r} = \frac{2(0.2)}{24} = \frac{0.2}{12} = \frac{1}{60} = 0.01\overline{6}$.

Percentage error = relative error$\times 100\% = 0.01\overline{6} \times 100\% = 1.\overline{6}\%$.

43. **(a)** For a sphere of radius r , the circumference is $C=2\pi r$ and the surface area is $S=4\pi r^2$, so $r=C/(2\pi) \Rightarrow S=4\pi(C/2\pi)^2 = C^2/\pi \Rightarrow dS=(2/\pi)C\,dC$. When $C=84$ and $dC=0.5$, $dS=\frac{2}{\pi}(84)(0.5)=\frac{84}{\pi}$, so the maximum error is about $\frac{84}{\pi} \approx 27$ cm ^{2} . Relative error $\approx \frac{dS}{S} = \frac{84/\pi}{84^2/\pi} = \frac{1}{84} \approx 0.012$

(b) $V=\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{C}{2\pi}\right)^3 = \frac{C^3}{6\pi^2} \Rightarrow dV=\frac{1}{2\pi^2}C^2 dC$. When $C=84$ and $dC=0.5$, $dV=\frac{1}{2\pi^2}(84)^2(0.5)=\frac{1764}{\pi^2}$, so the maximum error is about $\frac{1764}{\pi^2} \approx 179$ cm ^{3} . The relative error is approximately $\frac{dV}{V} = \frac{1764/\pi^2}{(84)^3/(6\pi^2)} = \frac{1}{56} \approx 0.018$.

44. For a hemispherical dome,

$V=\frac{2}{3}\pi r^3 \Rightarrow dV=2\pi r^2 dr$. When $r=\frac{1}{2}(50)=25$ m and $dr=0.05$ cm $=0.0005$ m, $dV=2\pi(25)^2(0.0005)=\frac{5\pi}{8}$, so the amount of paint needed is about $\frac{5\pi}{8}\approx 2$ m ^{3} .

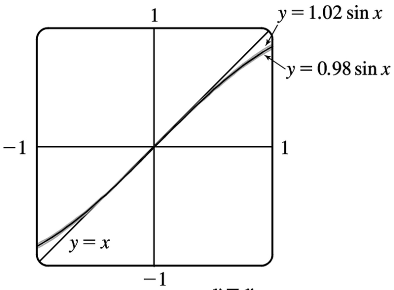
45. **(a)** $V=\pi r^2 h \Rightarrow \Delta V\approx dV=2\pi rh\,dr=2\pi rh\,\Delta r$
(b) The error is

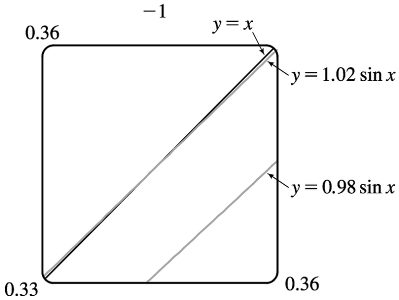
$$\begin{aligned}\Delta V-dV &=[\pi(r+\Delta r)^2 h-\pi r^2 h]-2\pi rh\,\Delta r=\pi r^2 h+2\pi rh\,\Delta r+\pi(\Delta r)^2 h-\pi r^2 h-2\pi rh\,\Delta r\\ &=\pi(\Delta r)^2 h\end{aligned}$$

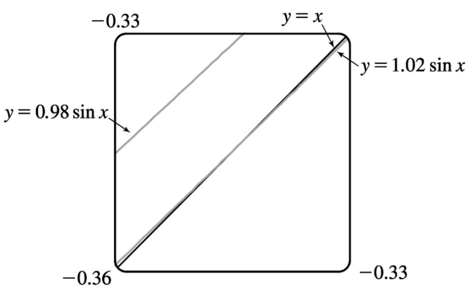
46. $F=kR^4 \Rightarrow dF=4kR^3 dR \Rightarrow \frac{dF}{F}=\frac{4kR^3 dR}{kR^4}=4\left(\frac{dR}{R}\right)$. Thus, the relative change in F is about 4 times the relative change in R . So a 5% increase in the radius corresponds to a 20% increase in blood flow.

47. **(a)** $dc=\frac{dc}{dx}dx=0\,dx=0$
(b) $d(cu)=\frac{d}{dx}(cu)dx=c\frac{du}{dx}dx=c\,du$
(c) $d(u+v)=\frac{d}{dx}(u+v)dx=\left(\frac{du}{dx}+\frac{dv}{dx}\right)dx=\frac{du}{dx}dx+\frac{dv}{dx}dx=du+dv$
(d) $d(uv)=\frac{d}{dx}(uv)dx=\left(u\frac{dv}{dx}+v\frac{du}{dx}\right)dx=u\frac{dv}{dx}dx+v\frac{du}{dx}dx=u\,dv+v\,du$
(e) $d\left(\frac{u}{v}\right)=\frac{d}{dx}\left(\frac{u}{v}\right)dx=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}dx=\frac{v\frac{du}{dx}dx-u\frac{dv}{dx}dx}{v^2}=\frac{v\,du-u\,dv}{v^2}$
(f) $d(x^n)=\frac{d}{dx}(x^n)dx=nx^{n-1}dx$

48. **(a)** $f(x)=\sin x \Rightarrow f'(x)=\cos x$, so $f(0)=0$ and $f'(0)=1$. Thus, $f(x)\approx f(0)+f'(0)(x-0)=0+1(x-0)=x$.
(b)







We want to know the values of x for which $y=x$ approximates $y=\sin x$ with less than a 2% difference; that is, the values of x for which

$$\left|\frac{x-\sin x}{\sin x}\right|<0.02 \Leftrightarrow -0.02<\frac{x-\sin x}{\sin x}<0.02 \Leftrightarrow$$

$$\begin{cases} -0.02\sin x<x-\sin x<0.02\sin x & \text{if } \sin x>0 \\ -0.02\sin x>x-\sin x>0.02\sin x & \text{if } \sin x<0 \end{cases} \Leftrightarrow \begin{cases} 0.98\sin x<x<1.02\sin x & \text{if } \sin x>0 \\ 1.02\sin x<x<0.98\sin x & \text{if } \sin x<0 \end{cases}$$

In the first figure, we see that the graphs are very close to each other near $x=0$. Changing the viewing rectangle and using an intersect feature (see the second figure) we find that $y=x$ intersects $y=1.02\sin x$ at $x\approx 0.344$. By symmetry, they also intersect at $x\approx -0.344$ (see the third figure.). Converting 0.344 radians to degrees, we get $0.344\left(\frac{180^\circ}{\pi}\right)\approx 19.7^\circ\approx 20^\circ$, which verifies the statement.

49. **(a)** The graph shows that $f'(1)=2$, so $L(x)=f(1)+f'(1)(x-1)=5+2(x-1)=2x+3$. $f(0.9)\approx L(0.9)=4.8$ and $f(1.1)\approx L(1.1)=5.2$.

(b) From the graph, we see that $f'(x)$ is positive and decreasing. This means that the slopes of the tangent lines are positive, but the tangents are becoming less steep. So the tangent lines lie *above* the curve. Thus, the estimates in part (a) are too large.

50. **(a)** $g'(x)=\sqrt{x^2+5} \Rightarrow g'(2)=\sqrt{9}=3$. $g(1.95)\approx g(2)+g'(2)(1.95-2)=-4+3(-0.05)=-4.15$. $g(2.05)\approx g(2)+g'(2)(2.05-2)=-4+3(0.05)=-3.85$.

(b) The formula $g'(x)=\sqrt{x^2+5}$ shows that $g'(x)$ is positive and increasing. This means that the slopes of the tangent lines are positive and the tangents are getting steeper. So the tangent lines lie *below* the graph of g . Hence, the estimates in part (a) are too small.