1. Product Rule:

$y=(x^2+1)(x^3+1)\Rightarrow$

$y'=(x^2+1)(3x^2)+(x^3+1)(2x)=3x^4+3x^2+2x^4+2x=5x^4+3x^2+2x$.

Multiplying first: $y=(x^2+1)(x^3+1)=x^5+x^3+x^2+1\Rightarrow y'=5x^4+3x^2+2x$ (equivalent).

2. Quotient Rule: $F(x)=\dfrac{x-3x\sqrt{x}}{\sqrt{x}}=\dfrac{x-3x^{3/2}}{x^{1/2}}\Rightarrow$

$$F'(x)=\frac{x^{1/2}\left(1-\frac{9}{2}x^{1/2}\right)-\left(x-3x^{3/2}\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}\right)^2}$$

$$=\frac{x^{1/2}-\frac{9}{2}x-\frac{1}{2}x^{1/2}+\frac{3}{2}x}{x}=\frac{\frac{1}{2}x^{1/2}-3x}{x}=\frac{1}{2}x^{-1/2}-3$$

Simplifying first: $F(x)=\dfrac{x-3x\sqrt{x}}{\sqrt{x}}=\sqrt{x}-3x=x^{1/2}-3x\Rightarrow F'(x)=\frac{1}{2}x^{-1/2}-3$ (equivalent).

For this problem, simplifying first seems to be the better method.

3. By the Product Rule, $f(x)=x^2e^x\Rightarrow f'(x)=x^2\frac{d}{dx}(e^x)+e^x\frac{d}{dx}(x^2)=x^2e^x+e^x(2x)=xe^x(x+2)$.

4. By the Product Rule, $g(x)=\sqrt{x}\,e^x=x^{1/2}e^x\Rightarrow g'(x)=x^{1/2}(e^x)+e^x\left(\frac{1}{2}x^{-1/2}\right)=\frac{1}{2}x^{-1/2}e^x(2x+1)$.

5. By the Quotient Rule, $y=\dfrac{e^x}{x^2}\Rightarrow y'=\dfrac{x^2\frac{d}{dx}(e^x)-e^x\frac{d}{dx}(x^2)}{(x^2)^2}=\dfrac{x^2(e^x)-e^x(2x)}{x^4}=\dfrac{xe^x(x-2)}{x^4}=\dfrac{e^x(x-2)}{x^3}$

6. By the Quotient Rule, $y=\dfrac{e^x}{1+x}\Rightarrow y'=\dfrac{(1+x)e^x-e^x(1)}{(1+x)^2}=\dfrac{e^x+xe^x-e^x}{(x+1)^2}=\dfrac{xe^x}{(x+1)^2}$.

7. $g(x)=\dfrac{3x-1}{2x+1}\overset{QR}{\Rightarrow}g'(x)=\dfrac{(2x+1)(3)-(3x-1)(2)}{(2x+1)^2}=\dfrac{6x+3-6x+2}{(2x+1)^2}=\dfrac{5}{(2x+1)^2}$

8. $f(t)=\dfrac{2t}{4+t^2}\overset{QR}{\Rightarrow}f'(t)=\dfrac{(4+t^2)(2)-(2t)(2t)}{(4+t^2)^2}=\dfrac{8+2t^2-4t^2}{(4+t^2)^2}=\dfrac{8-2t^2}{(4+t^2)^2}$

9. $V(x)=(2x^3+3)(x^4-2x) \overset{PR}{\Rightarrow}$

$V'(x)=(2x^3+3)(4x^3-2)+(x^4-2x)(6x^2)=(8x^6+8x^3-6)+(6x^6-12x^3)=14x^6-4x^3-6$

10. $Y(u)=(u^{-2}+u^{-3})(u^5-2u^2)\overset{PR}{\Rightarrow}$

$Y'(u)=(u^{-2}+u^{-3})(5u^4-4u)+(u^5-2u^2)(-2u^{-3}-3u^{-4})$

$=(5u^2-4u^{-1}+5u-4u^{-2})+(-2u^2-3u+4u^{-1}+6u^{-2})=3u^2+2u+2u^{-2}$

11. $F(y)=\left(\frac{1}{y^2}-\frac{3}{y^4}\right)(y+5y^3)=\left(y^{-2}-3y^{-4}\right)\left(y+5y^3\right)\overset{PR}{\Rightarrow}$

$F'(y)=\left(y^{-2}-3y^{-4}\right)\left(1+15y^2\right)+\left(y+5y^3\right)\left(-2y^{-3}+12y^{-5}\right)$

$=\left(y^{-2}+15-3y^{-4}-45y^{-2}\right)+(-2y^{-2}+12y^{-4}-10+60y^{-2})$

$=5+14y^{-2}+9y^{-4}$ or $5+14/y^2+9/y^4$

12. $R(t)=(t+e^t)(3-\sqrt{t}\,)=$

$R'(t)=(t+e^t)(-\frac{1}{2}t^{-1/2})+(3-\sqrt{t}\,)(1+e^t)$

$=\left(-\frac{1}{2}t^{1/2}-\frac{1}{2}t^{-1/2}e^t\right)+(3+3e^t-\sqrt{t}-\sqrt{t}\,e^t)=3+3e^t-\frac{3}{2}\sqrt{t}-\sqrt{t}\,e^t-e^t/(2\sqrt{t}\,)$

13. $y=\frac{t^2}{3t^2-2t+1}\overset{QR}{\Rightarrow}$

$y'=\frac{(3t^2-2t+1)(2t)-t^2(6t-2)}{(3t^2-2t+1)^2}=\frac{2t[3t^2-2t+1-t(3t-1)]}{(3t^2-2t+1)^2}$

$=\frac{2t(3t^2-2t+1-3t^2+t)}{(3t^2-2t+1)^2}=\frac{2t(1-t)}{(3t^2-2t+1)^2}$

14. $y=\frac{t^3+t}{t^4-2}\overset{QR}{\Rightarrow}y'=\frac{(t^4-2)(3t^2+1)-(t^3+t)(4t^3)}{(t^4-2)^2}=\frac{(3t^6+t^4-6t^2-2)-(4t^6+4t^4)}{(t^4-2)^2}$

$=\frac{-t^6-3t^4-6t^2-2}{(t^4-2)^2}=-\frac{t^6+3t^4+6t^2+2}{(t^4-2)^2}$

15. $y=(r^2-2r)e^r = y'=(r^2-2r)(e^r)+e^r(2r-2)=e^r(r^2-2r+2r-2)=e^r(r^2-2)$

16. $y=\dfrac{1}{s+ke^s} = y'=\dfrac{(s+ke^s)(0)-(1)(1+ke^s)}{(s+ke^s)^2}=-\dfrac{1+ke^s}{(s+ke^s)^2}$

17. $y=\dfrac{v^3-2v\sqrt{v}}{v}=v^2-2\sqrt{v}=v^2-2v^{1/2}\Rightarrow y'=2v-2\left(\dfrac{1}{2}\right)v^{-1/2}=2v-v^{-1/2}$.

We can change the form of the answer as follows: $2v-v^{-1/2}=2v-\dfrac{1}{\sqrt{v}}=\dfrac{2v\sqrt{v}-1}{\sqrt{v}}=\dfrac{2v^{3/2}-1}{\sqrt{v}}$

18. $z=w^{3/2}(w+ce^w)=w^{5/2}+cw^{3/2}e^w\Rightarrow z'=\dfrac{5}{2}w^{3/2}+c\left(w^{3/2}\cdot e^w+e^w\cdot\dfrac{3}{2}w^{1/2}\right)=\dfrac{5}{2}w^{3/2}+\dfrac{1}{2}cw^{1/2}e^w(2w+3)$

19. $y=\dfrac{1}{x^4+x^2+1}\Rightarrow y'=\dfrac{(x^4+x^2+1)(0)-1(4x^3+2x)}{(x^4+x^2+1)^2}=-\dfrac{2x(2x^2+1)}{(x^4+x^2+1)^2}$

20. $y=\dfrac{\sqrt{x}-1}{\sqrt{x}+1}\Rightarrow y'=\dfrac{(\sqrt{x}+1)\left(\dfrac{1}{2\sqrt{x}}\right)-(\sqrt{x}-1)\left(\dfrac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+1)^2}=\dfrac{\dfrac{1}{2}+\dfrac{1}{2\sqrt{x}}-\dfrac{1}{2}+\dfrac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2}=\dfrac{1}{\sqrt{x}(\sqrt{x}+1)^2}$

21. $f(x)=\dfrac{x}{x+c/x}\Rightarrow f'(x)=\dfrac{(x+c/x)(1)-x(1-c/x^2)}{\left(x+\dfrac{c}{x}\right)^2}=\dfrac{x+c/x-x+c/x}{\left(\dfrac{x^2+c}{x}\right)^2}=\dfrac{2c/x}{\dfrac{(x^2+c)^2}{x^2}}\cdot\dfrac{x^2}{x^2}=\dfrac{2cx}{(x^2+c)^2}$

22. $f(x)=\dfrac{ax+b}{cx+d}\Rightarrow f'(x)=\dfrac{(cx+d)(a)-(ax+b)(c)}{(cx+d)^2}=\dfrac{acx+ad-acx-bc}{(cx+d)^2}=\dfrac{ad-bc}{(cx+d)^2}$

23. $y=\dfrac{2x}{x+1}\Rightarrow y'=\dfrac{(x+1)(2)-(2x)(1)}{(x+1)^2}=\dfrac{2}{(x+1)^2}$. At $(1,1)$, $y'=\dfrac{1}{2}$, and an equation of the tangent line is $y-1=\dfrac{1}{2}(x-1)$, or $y=\dfrac{1}{2}x+\dfrac{1}{2}$.

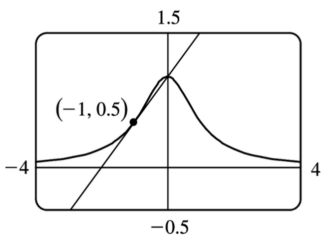
24. $y=\dfrac{\sqrt{x}}{x+1}\Rightarrow$

$$y' = \frac{(x+1)\left(\frac{1}{2\sqrt{x}}\right) - \sqrt{x}\,(1)}{(x+1)^2} = \frac{(x+1)-(2x)}{2\sqrt{x}\,(x+1)^2} = \frac{1-x}{2\sqrt{x}\,(x+1)^2}$$. At $(4,0.4)$, $y' = \frac{-3}{100} = -0.03$, and an equation of the tangent line is $y-0.4=-0.03(x-4)$, or $y=-0.03x+0.52$.

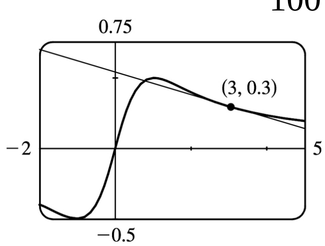
25. $y=2xe^x \Rightarrow y' = 2(x\cdot e^x + e^x \cdot 1) = 2e^x(x+1)$. At $(0, 0)$, $y' = 2e^0(0+1) = 2\cdot 1\cdot 1 = 2$, and an equation of the tangent line is $y-0=2(x-0)$, or $y=2x$.

26. $y=\frac{e^x}{x} \Rightarrow y' = \frac{x\cdot e^x - e^x\cdot 1}{x^2} = \frac{e^x(x-1)}{x^2}$. At $(1, e)$, $y'=0$, and an equation of the tangent line is $y-e=0(x-1)$, or $y=e$.

27. **(a)** $y=f(x)=\frac{1}{1+x^2} \Rightarrow f'(x) = \frac{(1+x^2)(0)-1(2x)}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2}$. So the slope of the tangent line at the point $\left(-1, \frac{1}{2}\right)$ is $f'(-1) = \frac{2}{2^2} = \frac{1}{2}$ and its equation is $y - \frac{1}{2} = \frac{1}{2}(x+1)$ or $y = \frac{1}{2}x + 1$.

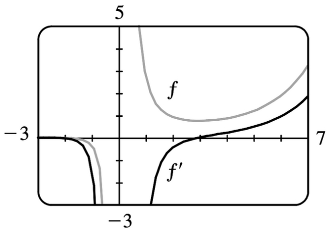
(b) 

28. **(a)** $y=f(x)=\frac{x}{1+x^2} \Rightarrow f'(x) = \frac{(1+x^2)1-x(2x)}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$. So the slope of the tangent line at the point $(3,0.3)$ is $f'(3) = \frac{-8}{100}$ and its equation is $y-0.3=-0.08(x-3)$ or $y=-0.08x+0.54$.

(b) 

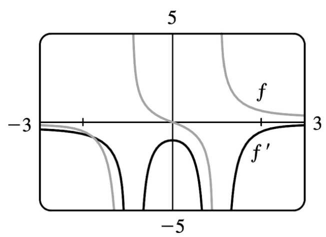
29. **(a)** $f(x)=\frac{e^x}{x^3} \Rightarrow f'(x) = \frac{x^3(e^x)-e^x(3x^2)}{(x^3)^2} = \frac{x^2e^x(x-3)}{x^6} = \frac{e^x(x-3)}{x^4}$

(b)



$f^{\prime}=0$ when f has a horizontal tangent line, $f^{\prime}$ is negative when f is decreasing, and $f^{\prime}$ is positive when f is increasing.

30. $f(x)=\dfrac{x}{x^2-1} \Rightarrow f^{\prime}(x)=\dfrac{(x^2-1)1-x(2x)}{(x^2-1)^2}=\dfrac{-x^2-1}{(x^2-1)^2}=-\dfrac{x^2+1}{(x^2-1)^2}$ Notice that the slopes of all tangents to f are negative and $f^{\prime}(x)<0$ always.



31. We are given that $f(5)=1$, $f^{\prime}(5)=6$, $g(5)=-3$, and $g^{\prime}(5)=2$.

(a) $(fg)^{\prime}(5)=f(5)g^{\prime}(5)+g(5)f^{\prime}(5)=(1)(2)+(-3)(6)=2-18=-16$

(b) $\left(\dfrac{f}{g}\right)^{\prime}(5)=\dfrac{g(5)f^{\prime}(5)-f(5)g^{\prime}(5)}{[g(5)]^2}=\dfrac{(-3)(6)-(1)(2)}{(-3)^2}=-\dfrac{20}{9}$

(c) $\left(\dfrac{g}{f}\right)^{\prime}(5)=\dfrac{f(5)g^{\prime}(5)-g(5)f^{\prime}(5)}{[f(5)]^2}=\dfrac{(1)(2)-(-3)(6)}{(1)^2}=20$

32. We are given that $f(3)=4$, $g(3)=2$, $f^{\prime}(3)=-6$, and $g^{\prime}(3)=5$.

(a) $(f+g)^{\prime}(3)=f^{\prime}(3)+g^{\prime}(3)=-6+5=-1$

(b) $(fg)^{\prime}(3)=f(3)g^{\prime}(3)+g(3)f^{\prime}(3)=(4)(5)+(2)(-6)=20-12=8$

(c) $\left(\dfrac{f}{g}\right)^{\prime}(3)=\dfrac{g(3)f^{\prime}(3)-f(3)g^{\prime}(3)}{[g(3)]^2}=\dfrac{(2)(-6)-(4)(5)}{(2)^2}=\dfrac{-32}{4}=-8$

(d)

$$\left(\frac{f}{f-g}\right)'(3)=\frac{[f(3)-g(3)]f'(3)-f(3)[f'(3)-g'(3)]}{[f(3)-g(3)]^2}$$
$$=\frac{(4-2)(-6)-4(-6-5)}{(4-2)^2}=\frac{-12+44}{2^2}=8$$

33. $f(x)=e^x g(x)\Rightarrow f'(x)=e^x g'(x)+g(x)e^x=e^x\left[g'(x)+g(x)\right]$.
$f'(0)=e^0\left[g'(0)+g(0)\right]=1(5+2)=7$

34. $\frac{d}{dx}\left[\frac{h(x)}{x}\right]=\frac{xh'(x)-h(x)\cdot 1}{x^2}\Rightarrow \frac{d}{dx}\left[\frac{h(x)}{x}\right]_{x=2}=\frac{2h'(2)-h(2)}{2^2}=\frac{2(-3)-(4)}{4}=\frac{-10}{4}=-2.5$

35. **(a)** From the graphs of f and g, we obtain the following values: $f(1)=2$ since the point (1,2) is on the graph of f; $g(1)=1$ since the point (1,1) is on the graph of g; $f'(1)=2$ since the slope of the line segment between (0,0) and (2,4) is $\frac{4-0}{2-0}=2$; $g'(1)=-1$ since the slope of the line segment between (–2,4) and (2,0) is $\frac{0-4}{2-(-2)}=-1$. Now $u(x)=f(x)g(x)$, so $u'(1)=f(1)g'(1)+g(1)f'(1)=2\cdot(-1)+1\cdot 2=0$.

(b) $v(x)=f(x)/g(x)$, so $v'(5)=\frac{g(5)f'(5)-f(5)g'(5)}{[g(5)]^2}=\frac{2\left(-\frac{1}{3}\right)-3\cdot\frac{2}{3}}{2^2}=\frac{-\frac{8}{3}}{4}=-\frac{2}{3}$

36. **(a)** $P(x)=F(x)G(x)$, so $P'(2)=F(2)G'(2)+G(2)F'(2)=3\cdot\frac{2}{4}+2\cdot 0=\frac{3}{2}$.

(b) $Q(x)=F(x)/G(x)$, so $Q'(7)=\frac{G(7)F'(7)-F(7)G'(7)}{[G(7)]^2}=\frac{1\cdot\frac{1}{4}-5\cdot\left(-\frac{2}{3}\right)}{1^2}=\frac{1}{4}+\frac{10}{3}=\frac{43}{12}$

37. **(a)** $y=xg(x)\Rightarrow y'=xg'(x)+g(x)\cdot 1=xg'(x)+g(x)$

(b) $y=\frac{x}{g(x)}\Rightarrow y'=\frac{g(x)\cdot 1-xg'(x)}{[g(x)]^2}=\frac{g(x)-xg'(x)}{[g(x)]^2}$

(c) $y=\frac{g(x)}{x}\Rightarrow y'=\frac{xg'(x)-g(x)\cdot 1}{(x)^2}=\frac{xg'(x)-g(x)}{x^2}$

38. **(a)**

$y=x^2 f(x) \Rightarrow y'=x^2 f'(x)+f(x)(2x)$

(b) $y=\dfrac{f(x)}{x^2} \Rightarrow y'=\dfrac{x^2 f'(x)-f(x)(2x)}{(x^2)^2}=\dfrac{xf'(x)-2f(x)}{x^3}$

(c) $y=\dfrac{x^2}{f(x)} \Rightarrow y'=\dfrac{f(x)(2x)-x^2 f'(x)}{[f(x)]^2}$

(d) $y=\dfrac{1+xf(x)}{\sqrt{x}} \Rightarrow$

$$y'=\frac{\sqrt{x}[xf'(x)+f(x)]-[1+xf(x)]\dfrac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$$

$$=\frac{x^{3/2}f'(x)+x^{1/2}f(x)-\frac{1}{2}x^{-1/2}-\frac{1}{2}x^{1/2}f(x)}{x}\cdot\frac{2x^{1/2}}{2x^{1/2}}=\frac{xf(x)+2x^2f'(x)-1}{2x^{3/2}}$$

39. If $P(t)$ denotes the population at time t and $A(t)$ the average annual income, then $T(t)=P(t)A(t)$ is the total personal income. The rate at which $T(t)$ is rising is given by $T'(t)=P(t)A'(t)+A(t)P'(t)\Rightarrow$

$$T'(1999)=P(1999)A'(1999)+A(1999)P'(1999)=(961{,}400)(\$1400/\text{yr})+(\$30{,}593)(9200/\text{yr})$$
$$=\$1{,}345{,}960{,}000/\text{yr}+\$281{,}455{,}600/\text{yr}=\$1{,}627{,}415{,}600/\text{yr}$$

So the total personal income was rising by about \$ 1.627 billion per year in 1999.

The term $P(t)A'(t)\approx\$1.346$ billion represents the portion of the rate of change of total income due to the existing population's increasing income. The term $A(t)P'(t)\approx\$281$ million represents the portion of the rate of change of total income due to increasing population.

40. **(a)** $f(20)=10{,}000$ means that when the price of the fabric is \$20/ yard, 10 , 000 yards will be sold.

$f'(20)=-350$ means that as the price of the fabric increases past \$20/ yard, the amount of fabric which will be sold is decreasing at a rate of 350 yards per (dollar per yard).

(b) $R(p)=pf(p)\Rightarrow R'(p)=pf'(p)+f(p)\cdot 1\Rightarrow R'(20)=20f'(20)+f(20)\cdot 1=20(-350)+10{,}000=3000$. This means that as the price of the fabric increases past \$20/ yard, the total revenue is increasing at \$3000/(\$/yard). Note that the Product Rule indicates that we will lose \$7000/(\$/yard) due to selling less fabric, but that that loss is more than made up for by the additional revenue due to the increase in price.

41. If $y=f(x)=\frac{x}{x+1}$, then $f^{\prime}(x)=\frac{(x+1)(1)-x(1)}{(x+1)^2}=\frac{1}{(x+1)^2}$. When $x=a$, the equation of the tangent line is $y-\frac{a}{a+1}=\frac{1}{(a+1)^2}(x-a)$. This line passes through (1,2) when $2-\frac{a}{a+1}=\frac{1}{(a+1)^2}(1-a)\Leftrightarrow$ $2(a+1)^2-a(a+1)=1-a\Leftrightarrow 2a^2+4a+2-a^2-a-1+a=0\Leftrightarrow a^2+4a+1=0$.

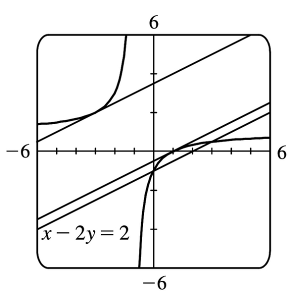
The quadratic formula gives the roots of this equation as $a=\frac{-4\pm\sqrt{4^2-4(1)(1)}}{2(1)}=\frac{-4\pm\sqrt{12}}{2}=-2\pm\sqrt{3}$, so there are two such tangent lines. Since

$$f(-2\pm\sqrt{3})=\frac{-2\pm\sqrt{3}}{-2\pm\sqrt{3}+1}=\frac{-2\pm\sqrt{3}}{-1\pm\sqrt{3}}\cdot\frac{-1\mp\sqrt{3}}{-1\mp\sqrt{3}}$$
$$=\frac{2\pm2\sqrt{3}\mp\sqrt{3}-3}{1-3}=\frac{-1\pm\sqrt{3}}{-2}=\frac{1\mp\sqrt{3}}{2},$$

the lines touch the curve at $A\left(-2+\sqrt{3},\frac{1-\sqrt{3}}{2}\right)\approx(-0.27,-0.37)$ and $B\left(-2-\sqrt{3},\frac{1+\sqrt{3}}{2}\right)\approx(-3.73,1.37)$.

42. $y=\frac{x-1}{x+1}\Rightarrow y^{\prime}=\frac{(x+1)(1)-(x-1)(1)}{(x+1)^2}=\frac{2}{(x+1)^2}$. If the tangent intersects the curve when $x=a$, then its slope is $2/(a+1)^2$. But if the tangent is parallel to $x-2y=2$, that is, $y=\frac{1}{2}x-1$, then its slope is $\frac{1}{2}$. Thus, $\frac{2}{(a+1)^2}=\frac{1}{2}\Rightarrow(a+1)^2=4\Rightarrow a+1=\pm2\Rightarrow a=1$ or -3 . When $a=1$, $y=0$ and the equation of the tangent is $y-0=\frac{1}{2}(x-1)$ or $y=\frac{1}{2}x-\frac{1}{2}$.

When $a=-3$, $y=2$ and the equation of the tangent is $y-2=\frac{1}{2}(x+3)$ or $y=\frac{1}{2}x+\frac{7}{2}$.



43. **(a)** $(fgh)^{\prime}=[(fg)h]^{\prime}=(fg)^{\prime}h+(fg)h^{\prime}=(f^{\prime}g+fg^{\prime})h+(fg)h^{\prime}=f^{\prime}gh+fg^{\prime}h+fgh^{\prime}$

(b) Putting $f=g=h$ in part (a) , we have

$$\frac{d}{dx}[f(x)]^3=(fff)'=f'ff+ff'f+fff'=3fff'=3[f(x)]^2f'(x).$$

(c) $\frac{d}{dx}(e^{3x})=\frac{d}{dx}(e^x)^3=3(e^x)^2e^x=3e^{2x}e^x=3e^{3x}$

44. **(a)**

$$\frac{d}{dx}\left(\frac{1}{g(x)}\right)=\frac{g(x)\cdot\frac{d}{dx}(1)-1\cdot\frac{d}{dx}[g(x)]}{[g(x)]^2}\quad\text{[Quotient Rule]}$$

$$=\frac{g(x)\cdot 0-1\cdot g'(x)}{[g(x)]^2}=\frac{0-g'(x)}{[g(x)]^2}=-\frac{g'(x)}{[g(x)]^2}$$

(b) $y=\frac{1}{x^4+x^2+1}\Rightarrow y'=-\frac{4x^3+2x}{(x^4+x^2+1)^2}$ or $\frac{-2x(2x^2+1)}{(x^4+x^2+1)^2}$

(c) $\frac{d}{dx}(x^{-n})=\frac{d}{dx}\left(\frac{1}{x^n}\right)=-\frac{(x^n)'}{(x^n)^2}$ [by the Reciprocal Rule] $=-\frac{nx^{n-1}}{x^{2n}}=-nx^{n-1-2n}=-nx^{-n-1}$