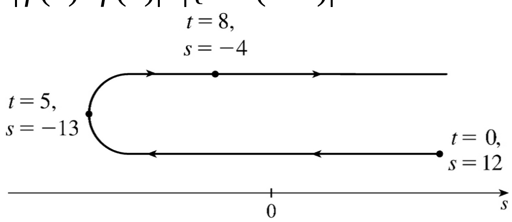
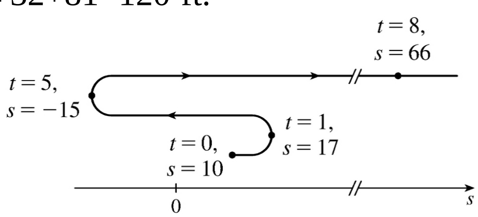
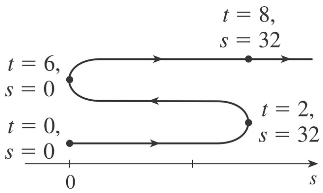
1. **(a)** $s=f(t)=t^2-10t+12 \Rightarrow v(t)=f'(t)=2t-10$
(b) $v(3)=2(3)-10=-4$ ft / s
(c) The particle is at rest when $v(t)=0 \Leftrightarrow 2t-10=0 \Leftrightarrow t=5$ s.
(d) The particle is moving in the positive direction when $v(t)>0 \Leftrightarrow 2t-10>0 \Leftrightarrow 2t>10 \Leftrightarrow t>5$.
(e) Since the particle is moving in the positive direction and in the negative direction, we need to calculate the distance traveled in the intervals $[0,5]$ and $[5,8]$ separately. $|f(5)-f(0)|=|-13-12|=25$ ft and $|f(8)-f(5)|=|\{-4-(-13)|=9$ ft. The total distance traveled during the first 8 s is $25+9=34$ ft.
(f)


2. **(a)** $s=f(t)=t^3-9t^2+15t+10 \Rightarrow v(t)=f'(t)=3t^2-18t+15=3(t-1)(t-5)$

(b) $v(3)=3(2)(-2)=-12$ ft / s
(c) $v(t)=0 \Leftrightarrow t=1$ s or 5 s
(d) $v(t)>0 \Leftrightarrow 0 \leq t<1$ or $t>5$
(e) $|f(1)-f(0)|=|17-10|=7$, $|f(5)-f(1)|=|-15-17|=32$, and $|f(8)-f(5)|=|66-(-15)|=81$. Total distance $=7+32+81=120$ ft.
(f)


3. **(a)** $s=f(t)=t^3-12t^2+36t \Rightarrow v(t)=f'(t)=3t^2-24t+36$
(b) $v(3)=27-72+36=-9$ ft / s

(c) The particle is at rest when $v(t)=0$. $3t^2-24t+36=0 \Rightarrow 3(t-2)(t-6)=0 \Rightarrow t=2$ s or 6 s.
(d) The particle is moving in the positive direction when $v(t)>0$. $3(t-2)(t-6)>0 \Leftrightarrow 0 \leq t<2$ or $t>6$.
(e) Since the particle is moving in the positive direction and in the negative direction, we need to calculate the distance traveled in the intervals $(0,2)$, $(2,6)$, and $[6,8]$ separately.
$|f(2)-f(0)|=|32-0|=32$.
$|f(6)-f(2)|=|0-32|=32$.
$|f(8)-f(6)|=|32-0|=32$.
The total distance is $32+32+32=96$ ft.
(f)



4. **(a)** $s=f(t)=t^4-4t+1 \Rightarrow v(t)=f'(t)=4t^3-4$

(b) $v(3)=4(3)^3-4=104$ ft / s

(c) It is at rest when $v(t)=4(t^3-1)=4(t-1)(t^2+t+1)=0 \Leftrightarrow t=1$ s.

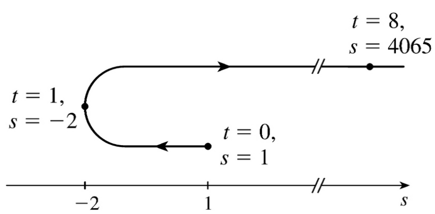
(d) It moves in the positive direction when $4(t^3-1)>0 \Leftrightarrow t>1$.

(e) Distance in positive direction $=|f(8)-f(1)|=|4065-(-2)|=4067$ ft

Distance in negative direction $=|f(1)-f(0)|=|-2-1|=3$ ft

Total distance traveled $=4067+3=4070$ ft

(f)



5. **(a)** $s=\dfrac{t}{t^2+1} \Rightarrow v(t)=s'(t)=\dfrac{(t^2+1)(1)-t(2t)}{(t^2+1)^2}=\dfrac{1-t^2}{(t^2+1)^2}$

(b) $v(3)=\dfrac{1-(3)^2}{(3^2+1)^2}=\dfrac{1-9}{10^2}=\dfrac{-8}{100}=-\dfrac{2}{25}$ ft / s

(c) It is at rest when $v=0 \Leftrightarrow 1-t^2=0 \Leftrightarrow t=1$ s [$t\neq -1$ since $t\geq 0$].

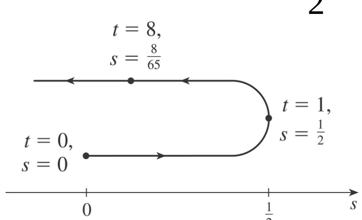
(d) It moves in the positive direction when $v>0 \Leftrightarrow 1-t^2>0 \Leftrightarrow t^2<1 \Leftrightarrow 0\leq t<1$.

(e) Distance in positive direction $=|s(1)-s(0)|=\left|\dfrac{1}{2}-0\right|=\dfrac{1}{2}$ ft

Distance in negative direction $=|s(8)-s(1)|=\left|\dfrac{8}{65}-\dfrac{1}{2}\right|=\dfrac{49}{130}$ ft

Total distance traveled $=\dfrac{1}{2}+\dfrac{49}{130}=\dfrac{57}{65}$ ft

(f)



6. **(a)** $s=\sqrt{t}(3t^2-35t+90)=3t^{5/2}-35t^{3/2}+90t^{1/2}\Rightarrow$
$v(t)=s'(t)=\frac{15}{2}t^{3/2}-\frac{105}{2}t^{1/2}+45t^{-1/2}=\frac{15}{2}t^{-1/2}(t^2-7t+6)=\frac{15}{2\sqrt{t}}(t-1)(t-6)$

(b) $v(3)=\frac{15}{2\sqrt{3}}(2)(-3)=-15\sqrt{3}$ ft / s

(c) It is at rest when $v=0\Leftrightarrow t=1$ s or 6 s.

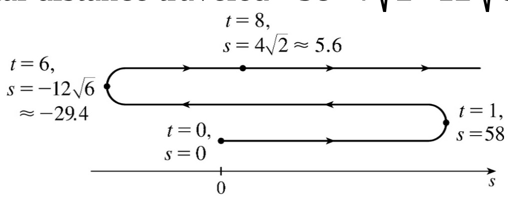
(d) It moves in the positive direction when $v>0\Leftrightarrow(t-1)(t-6)>0\Leftrightarrow 0\le t<1$ or $t>6$.

(e)

Distance in positive direction $=|s(1)-s(0)|+|s(8)-s(6)|=|58-0|+|4\sqrt{2}-(-12\sqrt{6})|$
$=58+4\sqrt{2}+12\sqrt{6}\approx 93.05$ ft

Distance in negative direction $=|s(6)-s(1)|=|-12\sqrt{6}-58|=58+12\sqrt{6}\approx 87.39$ ft
Total distance traveled $=58+4\sqrt{2}+12\sqrt{6}+58+12\sqrt{6}=116+4\sqrt{2}+24\sqrt{6}\approx 180.44$ ft

(f)



7. $s(t)=t^3-4.5t^2-7t\Rightarrow v(t)=s'(t)=3t^2-9t-7=5\Leftrightarrow 3t^2-9t-12=0\Leftrightarrow 3(t-4)(t+1)=0\Leftrightarrow t=4$ or -1 . Since $t\ge 0$, the particle reaches a velocity of 5 m / s at $t=4$ s.

8. **(a)** $s=5t+3t^2\Rightarrow v(t)=\frac{ds}{dt}=5+6t$, so $v(2)=5+6(2)=17$ m / s.

(b) $v(t)=35\Rightarrow 5+6t=35\Rightarrow 6t=30\Rightarrow t=5$ s.

9. **(a)** $h=10t-0.83t^2\Rightarrow v(t)=\frac{dh}{dt}=10-1.66t$, so $v(3)=10-1.66(3)=5.02$ m / s.

(b) $h=25\Rightarrow 10t-0.83t^2=25\Rightarrow 0.83t^2-10t+25=0\Rightarrow t=\frac{10\pm\sqrt{17}}{1.66}\approx 3.54$ or 8.51 .
The value $t_1=(10-\sqrt{17})/1.66$ corresponds to the time it takes for the stone to rise 25 m and $t_2=(10+\sqrt{17})/1.66$ corresponds to the time when the stone is 25 m high on the way down. Thus, $v(t_1)=10-1.66[(10-\sqrt{17})/1.66]=\sqrt{17}\approx 4.12$ m / s.

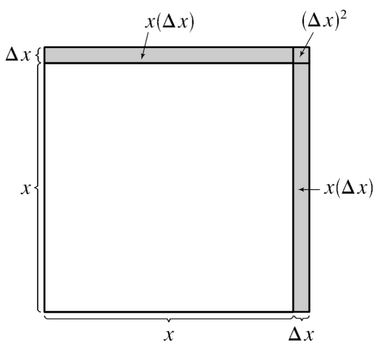
10. **(a)** At maximum height the velocity of the ball is 0 ft / s. $v(t)=s'(t)=80-32t=0\Leftrightarrow 32t=80\Leftrightarrow t=\frac{5}{2}$.
So the maximum height is

$s\left(\frac{5}{2}\right)=80\left(\frac{5}{2}\right)-16\left(\frac{5}{2}\right)^2=200-100=100$ ft.

(b) $s(t)=80t-16t^2=96 \Leftrightarrow 16t^2-80t+96=0 \Leftrightarrow 16(t^2-5t+6)=0 \Leftrightarrow 16(t-3)(t-2)=0$.
So the ball has a height of 96 ft on the way up at $t=2$ and on the way down at $t=3$. At these times the velocities are $v(2)=80-32(2)=16$ ft / s and $v(3)=80-32(3)=-16$ ft / s, respectively.

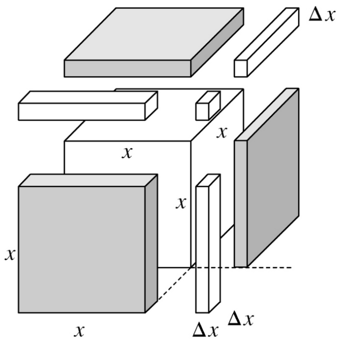
11. **(a)** $A(x)=x^2 \Rightarrow A'(x)=2x$. $A'(15)=30$ mm ^{2}/ mm is the rate at which the area is increasing with respect to the side length as x reaches 15 mm.

(b) The perimeter is $P(x)=4x$, so $A'(x)=2x=\frac{1}{2}(4x)=\frac{1}{2}P(x)$. The figure suggests that if Δx is small, then the change in the area of the square is approximately half of its perimeter (2 of the 4 sides) times Δx . From the figure, $\Delta A=2x(\Delta x)+(\Delta x)^2$. If Δx is small, then $\Delta A\approx 2x(\Delta x)$ and so $\Delta A/\Delta x\approx 2x$.



12. **(a)** $V(x)=x^3 \Rightarrow \frac{dV}{dx}=3x^2$. $\left.\frac{dV}{dx}\right|_{x=3}=3(3)^2=27$ mm ^{3}/ mm is the rate at which the volume is increasing as x increases past 3 mm.

(b) The surface area is $S(x)=6x^2$, so $V'(x)=3x^2=\frac{1}{2}(6x^2)=\frac{1}{2}S(x)$. The figure suggests that if Δx is small, then the change in the volume of the cube is approximately half of its surface area (the area of 3 of the 6 faces) times Δx . From the figure, $\Delta V=3x^2(\Delta x)+3x(\Delta x)^2+(\Delta x)^3$. If Δx is small, then $\Delta V\approx 3x^2(\Delta x)$ and so $\Delta V/\Delta x\approx 3x^2$.



13. **(a)**

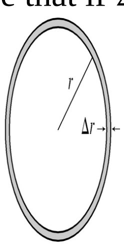
(i) $\dfrac{A(3)-A(2)}{3-2}=\dfrac{9\pi-4\pi}{1}=5\pi$

(ii) $\dfrac{A(2.5)-A(2)}{2.5-2}=\dfrac{6.25\pi-4\pi}{0.5}=4.5\pi$

(iii) $\dfrac{A(2.1)-A(2)}{2.1-2}=\dfrac{4.41\pi-4\pi}{0.1}=4.1\pi$

(b) $A(r)=\pi r^2 \Rightarrow A'(r)=2\pi r$, so $A'(2)=4\pi$.

(c) The circumference is $C(r)=2\pi r=A'(r)$. The figure suggests that if Δr is small, then the change in the area of the circle (a ring around the outside) is approximately equal to its circumference times Δr . Straightening out this ring gives us a shape that is approximately rectangular with length $2\pi r$ and width Δr , so $\Delta A\approx 2\pi r(\Delta r)$. Algebraically, $\Delta A=A(r+\Delta r)-A(r)=\pi(r+\Delta r)^2-\pi r^2=2\pi r(\Delta r)+\pi(\Delta r)^2$. So we see that if Δr is small, then $\Delta A\approx 2\pi r(\Delta r)$ and therefore, $\Delta A/\Delta r\approx 2\pi r$.



14. **(a)** $A'(1)=7200\pi\ \text{cm}^2/\text{s}$

(b) $A'(3)=21,600\pi\ \text{cm}^2/\text{s}$

(c) $A'(5)=36,000\pi\ \text{cm}^2/\text{s}$

15. **(a)** $S'(1)=8\pi\ \text{ft}^2/\text{ft}$

(b) $S'(2)=16\pi\ \text{ft}^2/\text{ft}$

(c) $S'(3)=24\pi\ \text{ft}^2/\text{ft}$

16. **(a)**

(a)
$$\frac{V(8)-V(5)}{8-5}=\frac{\frac{4}{3}\pi(512)-\frac{4}{3}\pi(125)}{3}=172\pi\ \mu\text{ m}^3/\mu\text{ m}$$

(b)
$$\frac{V(8)-V(5)}{8-5}=\frac{\frac{4}{3}\pi(512)-\frac{4}{3}\pi(125)}{3}=172\pi\ \mu\text{ m}^3/\mu\text{ m}$$

(c)
$$\frac{V(6)-V(5)}{6-5}=\frac{\frac{4}{3}\pi(216)-\frac{4}{3}\pi(125)}{1}=121.\overline{3}\pi\ \mu\text{ m}^3/\mu\text{ m}$$

(d)
$$\frac{V(6)-V(5)}{6-5}=\frac{\frac{4}{3}\pi(216)-\frac{4}{3}\pi(125)}{1}=121.\overline{3}\pi\ \mu\text{ m}^3/\mu\text{ m}$$

(e)
$$\frac{V(5.1)-V(5)}{5.1-5}=\frac{\frac{4}{3}\pi(5.1)^3-\frac{4}{3}\pi(5)^3}{0.1}=102.01\overline{3}\pi\ \mu\text{ m}^3/\mu\text{ m}$$

(f)
$$\frac{V(5.1)-V(5)}{5.1-5}=\frac{\frac{4}{3}\pi(5.1)^3-\frac{4}{3}\pi(5)^3}{0.1}=102.01\overline{3}\pi\ \mu\text{ m}^3/\mu\text{ m}$$

(b) $V'(r)=4\pi r^2$, so $V'(5)=100\pi\ \mu\text{ m}^3/\mu\text{ m}$.

(c) $V(r)=\frac{4}{3}\pi r^3\Rightarrow V'(r)=4\pi r^2=S(r)$. By analogy with Exercise 13(c) , we can say that the change in the volume of the spherical shell, ΔV , is approximately equal to its thickness, Δr , times the surface area of the inner sphere. Thus, $\Delta V\approx4\pi r^2(\Delta r)$ and so $\Delta V/\Delta r\approx4\pi r^2$.

17. **(a)** $\rho(1)=6$ kg / m
(b) $\rho(2)=12$ kg / m
(c) $\rho(3)=18$ kg / m

18. **(a)** $V'(5)=-250\left(1-\frac{5}{40}\right)=-218.75$ gal / min
(b) $V'(10)=-250\left(1-\frac{10}{40}\right)=-187.5$ gal / min
(c) $V'(20)=-250\left(1-\frac{20}{40}\right)=-125$ gal / min
(d)

$V'(40)=-250\left(1-\frac{40}{40}\right)=0$ gal / min

19. **(a)** $Q'(0.5)=3(0.5)^2-4(0.5)+6=4.75$ A

(b) $Q'(1)=3(1)^2-4(1)+6=5$ A

20. **(a)** $F=\frac{GmM}{r^2}=(GmM)r^{-2}\Rightarrow \frac{dF}{dr}=-2(GmM)r^{-3}=-\frac{2GmM}{r^3}$, which is the rate of change of the force with respect to the distance between the bodies. The minus sign indicates that as the distance r between the bodies increases, the magnitude of the force F exerted by the body of mass m on the body of mass M is decreasing.

(b) Given $F'(20\ ,000)=-2$, find $F'(10\ ,000)$. $-2=-\frac{2GmM}{20{,}000^3}\Rightarrow GmM=20\ ,000^3$.

$F'(10{,}000)=-\frac{2\left(20{,}000^3\right)}{10{,}000^3}=-2\cdot 2^3=-16$ N / km

21. **(a)** To find the rate of change of volume with respect to pressure, we first solve for V in terms of P .

$PV=C\Rightarrow V=\frac{C}{P}\Rightarrow \frac{dV}{dP}=-\frac{C}{P^2}$.

(b) From the formula for dV/dP in part (a), we see that as P increases, the absolute value of dV/dP decreases. Thus, the volume is decreasing more rapidly at the beginning.

(c) $\beta=-\frac{1}{V}\frac{dV}{dP}=-\frac{1}{V}\left(-\frac{C}{P^2}\right)=\frac{C}{(PV)P}=\frac{C}{CP}=\frac{1}{P}$

22. **(a)**

(i)

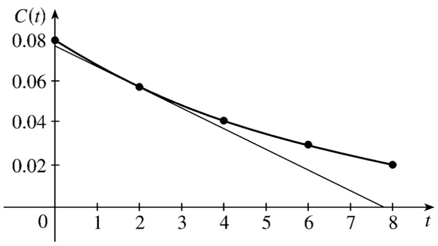
$$\frac{C(6)-C(2)}{6-2}=\frac{0.0295-0.0570}{4}=-0.006875\,(\text{moles/L})/\ \text{min}$$

$$\frac{C(4)-C(2)}{4-2}=\frac{0.0408-0.0570}{2}=-0.008\,(\text{moles/L})/\ \text{min}$$

(iii) $\frac{C(2)-C(0)}{2-0} = \frac{0.0570-0.0800}{2}$

$= -0.0115 \, (\text{moles/L}) / \text{ min}$

(b) Slope $= \frac{\Delta C}{\Delta t} \approx -\frac{0.077}{7.8} \approx -0.01 \, (\text{moles/L}) / \text{ min}$



23. **(a)** **1920:** $m_1 = \frac{1860-1750}{1920-1910} = \frac{110}{10} = 11$, $m_2 = \frac{2070-1860}{1930-1920} = \frac{210}{10} = 21$,
$(m_1+m_2)2 = (11+21)/2 = 16$ million / year

1980: $m_1 = \frac{4450-3710}{1980-1970} = \frac{740}{10} = 74$, $m_2 = \frac{5280-4450}{1990-1980} = \frac{830}{10} = 83$,
$(m_1+m_2)2 = (74+83)/2 = 78.5$ million / year

(b) $P(t) = at^3 + bt^2 + ct + d$ (in millions of people), where $a \approx 0.0012937063$, $b \approx -7.061421911$, $c \approx 12{,}822.97902$, and $d \approx -7{,}743{,}770.396$.

(c) $P(t) = at^3 + bt^2 + ct + d \Rightarrow P'(t) = 3at^2 + 2bt + c$ (in millions of people per year)

(d)

$P'(1920) = 3(0.0012937063)(1920)^2 + 2(-7.061421911)(1920) + 12{,}822.97902$

≈ 14.48 million / year

$P'(1980) \approx 75.29$ million / year (smaller, but close)

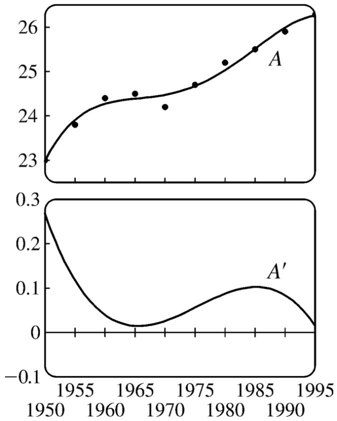
(e) $P'(1985) \approx 81.62$ million / year, so the rate of growth in 1985 was about 81.62 million / year.

24. **(a)** $A(t) = at^4 + bt^3 + ct^2 + dt + e$, where $a = -5.8275058275396 \times 10^{-6}$, $b = 0.0460458430461$, $c = -136.43277039706$, $d = 179 , 661.02676871$, and $e = -88 , 717 , 597.060767$.

(b) $A(t) = at^4 + bt^3 + ct^2 + dt + e \Rightarrow A'(t) = 4at^3 + 3bt^2 + 2ct + d$

(c)

$A'(1990) \approx 0.0833$ years of age per year

(d) 

25. **(a)** $[\mathrm{C}]=\dfrac{a^2kt}{akt+1} \Rightarrow$ rate of reaction

$$=\frac{d[\mathrm{C}]}{dt}=\frac{(akt+1)(a^2k)-(a^2kt)(ak)}{(akt+1)^2}=\frac{a^2k(akt+1-akt)}{(akt+1)^2}=\frac{a^2k}{(akt+1)^2}$$

(b) If $x=[\mathrm{C}]$, then $a-x=a-\dfrac{a^2kt}{akt+1}=\dfrac{a^2kt+a-a^2kt}{akt+1}=\dfrac{a}{akt+1}$.

So $k(a-x)^2=k\left(\dfrac{a}{akt+1}\right)^2=\dfrac{a^2k}{(akt+1)^2}=\dfrac{d[\mathrm{C}]}{dt}=\dfrac{dx}{dt}$.

26. **(a)** After an hour the population is $n(1)=3\cdot 500$; after two hours it is $n(2)=3(3\cdot 500)=3^2\cdot 500$; after three hours, $n(3)=3(3^2\cdot 500)=3^3\cdot 500$; after four hours, $n(4)=3^4\cdot 500$. From this pattern, we see that the population after t hours is $n(t)=3^t\cdot 500=500\cdot 3^t$.

(b) From (5) in Section 3.1, we have $\dfrac{d}{dx}(3^x)\approx(1.10)3^x$. Thus, for $n(t)=500\cdot 3^t$,

$\dfrac{dn}{dt}=500\dfrac{d}{dt}(3^t)\approx 500(1.10)3^t \Rightarrow \left.\dfrac{dn}{dt}\right|_{t=6}\approx 500(1.10)3^6\approx 400,950$ bacteria / hour.

27. **(a)** Using $v=\dfrac{P}{4\eta l}(R^2-r^2$ with $R=0.01$, $l=3$, $P=3000$, and $\eta=0.027$, we have v as a function of r :

$v(r)=\dfrac{3000}{4(0.027)3}(0.01^2-r^2)$. $v(0)=0.\overline{925}$ cm / s, $v(0.005)=0.69\overline{4}$ cm / s, $v(0.01)=0$.

(b) $v(r)=\dfrac{P}{4\eta l}(R^2-r^2) \Rightarrow v'(r)=\dfrac{P}{4\eta l}(-2r)=-\dfrac{Pr}{2\eta l}$. When $l=3$, $P=3000$, and $\eta=0.027$, we have

$v'(r)=-\dfrac{3000r}{2(0.027)3}$. $v'(0)=0$, $v'(0.005)=-92.\overline{592}$(cm/s)/ cm, and $v'(0.01)=-185.\overline{185}$(cm/s)/ cm.
(c) The velocity is greatest where $r=0$ (at the center) and the velocity is changing most where $r=R=0.01$ cm (at the edge).

28. **(a)**

(a) $$f=\frac{1}{2L}\sqrt{\frac{T}{\rho}}=\left(\frac{1}{2}\sqrt{\frac{T}{\rho}}\right)L^{-1}\Rightarrow\frac{df}{dL}=-\left(\frac{1}{2}\sqrt{\frac{T}{\rho}}\right)L^{-2}=-\frac{1}{2L^2}\sqrt{\frac{T}{\rho}}$$

(b) $$f=\frac{1}{2L}\sqrt{\frac{T}{\rho}}=\left(\frac{1}{2}\sqrt{\frac{T}{\rho}}\right)L^{-1}\Rightarrow\frac{df}{dL}=-\left(\frac{1}{2}\sqrt{\frac{T}{\rho}}\right)L^{-2}=-\frac{1}{2L^2}\sqrt{\frac{T}{\rho}}$$

(c) $$f=\frac{1}{2L}\sqrt{\frac{T}{\rho}}=\left(\frac{1}{2L\sqrt{\rho}}\right)T^{1/2}\Rightarrow\frac{df}{dT}=\frac{1}{2}\left(\frac{1}{2L\sqrt{\rho}}\right)T^{-1/2}=\frac{1}{4L\sqrt{T\rho}}$$

(d) $$f=\frac{1}{2L}\sqrt{\frac{T}{\rho}}=\left(\frac{1}{2L\sqrt{\rho}}\right)T^{1/2}\Rightarrow\frac{df}{dT}=\frac{1}{2}\left(\frac{1}{2L\sqrt{\rho}}\right)T^{-1/2}=\frac{1}{4L\sqrt{T\rho}}$$

(e) $$f=\frac{1}{2L}\sqrt{\frac{T}{\rho}}=\left(\frac{\sqrt{T}}{2L}\right)\rho^{-1/2}\Rightarrow\frac{df}{d\rho}=-\frac{1}{2}\left(\frac{\sqrt{T}}{2L}\right)\rho^{-3/2}=-\frac{\sqrt{T}}{4L\rho^{3/2}}$$

(f) $$f=\frac{1}{2L}\sqrt{\frac{T}{\rho}}=\left(\frac{\sqrt{T}}{2L}\right)\rho^{-1/2}\Rightarrow\frac{df}{d\rho}=-\frac{1}{2}\left(\frac{\sqrt{T}}{2L}\right)\rho^{-3/2}=-\frac{\sqrt{T}}{4L\rho^{3/2}}$$

(b)

(i) $\dfrac{df}{dL}<0$ and L is decreasing $\Rightarrow f$ is increasing $\Rightarrow$ higher note

(ii) $\dfrac{df}{dT}>0$ and T is increasing $\Rightarrow f$ is increasing $\Rightarrow$ higher note

(iii) $\dfrac{df}{d\rho}<0$ and ρ is increasing $\Rightarrow f$ is decreasing $\Rightarrow$ lower note

29. **(a)** $C(x)=2000+3x+0.01x^2+0.0002x^3\Rightarrow C'(x)=3+0.02x+0.0006x^2$
(b) $C'(100)=3+0.02(100)+0.0006(10,000)=3+2+6=\$11$/ pair. $C'(100)$ is the rate at which the cost is increasing as the 100 th pair of jeans is produced. It predicts the cost of the 101 st pair.
(c) The cost of manufacturing the 101 st pair of jeans is

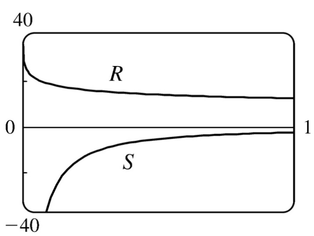
$$C(101)-C(100) = (2000+303+102.01+206.0602)-(2000+300+100+200) = 11.0702 \approx \$11.07$$

30. **(a)** $C(x)=84+0.16x-0.0006x^2+0.000003x^3 \Rightarrow C'(x)=0.16-0.0012x+0.000009x^2 \Rightarrow C'(100)=0.13$. This is the rate at which the cost is increasing as the 100 th item is produced.
(b) $C(101)-C(100)=97.13030299-97 \approx \$0.13$.

31. **(a)** $A(x)=\dfrac{p(x)}{x} \Rightarrow A'(x)=\dfrac{xp'(x)-p(x)\cdot 1}{x^2}=\dfrac{xp'(x)-p(x)}{x^2}$. $A'(x)>0 \Rightarrow A(x)$ is increasing; that is, the average productivity increases as the size of the workforce increases.
(b) $p'(x)$ is greater than the average productivity $\Rightarrow p'(x)>A(x) \Rightarrow p'(x)>\dfrac{p(x)}{x} \Rightarrow$
$xp'(x)>p(x) \Rightarrow xp'(x)-p(x)>0 \Rightarrow \dfrac{xp'(x)-p(x)}{x^2}>0 \Rightarrow A'(x)>0$.

32. **(a)**

$$S=\frac{dR}{dx}=\frac{\left(1+4x^{0.4}\right)\left(9.6x^{-0.6}\right)-\left(40+24x^{0.4}\right)\left(1.6x^{-0.6}\right)}{\left(1+4x^{0.4}\right)^2}$$
$$=\frac{9.6x^{-0.6}+38.4x^{-0.2}-64x^{-0.6}-38.4x^{-0.2}}{\left(1+4x^{0.4}\right)^2}=-\frac{54.4x^{-0.6}}{\left(1+4x^{0.4}\right)^2}$$

(b)


At low levels of brightness, R is quite large and is quickly decreasing, that is, S is negative with large absolute value. This is to be expected: at low levels of brightness, the eye is more sensitive to slight changes than it is at higher levels of brightness.

33. $PV=nRT \Rightarrow T=\dfrac{PV}{nR}=\dfrac{PV}{(10)(0.0821)}=\dfrac{1}{0.821}(PV)$. Using the Product Rule, we have
$\dfrac{dT}{dt}=\dfrac{1}{0.821}[P(t)V'(t)+V(t)P'(t)]=\dfrac{1}{0.821}[(8)(-0.15)+(10)(0.10)] \approx -0.2436$ K / min.

34. **(a)** If $dP/dt=0$, the population is stable (it is constant).

(b) $\frac{dP}{dt}=0 \Rightarrow \beta P=r_0\left(1-\frac{P}{P_c}\right)P \Rightarrow \frac{\beta}{r_0}=1-\frac{P}{P_c} \Rightarrow \frac{P}{P_c}=1-\frac{\beta}{r_0} \Rightarrow P=P_c\left(1-\frac{\beta}{r_0}\right)$.

If $P_c=10,000$, $r_0=5\%=0.05$, and $\beta=4\%=0.04$, then $P=10,000\left(1-\frac{4}{5}\right)=2000$.

(c) If $\beta=0.05$, then $P=10,000\left(1-\frac{5}{5}\right)=0$. There is no stable population.

35. **(a)** If the populations are stable, then the growth rates are neither positive nor negative; that is, $\frac{dC}{dt}=0$ and $\frac{dW}{dt}=0$.

(b) ‘‘The caribou go extinct’’ means that the population is zero, or mathematically, $C=0$.

(c) We have the equations $\frac{dC}{dt}=aC-bCW$ and $\frac{dW}{dt}=-cW+dCW$. Let $dC/dt=dW/dt=0$, $a=0.05$, $b=0.001$, $c=0.05$, and $d=0.0001$ to obtain $0.05C-0.001CW=0$ **(1)** and $-0.05W+0.0001CW=0$ **(2)** . Adding 10 times **(2)** to **(1)** eliminates the CW –terms and gives us $0.05C-0.5W=0 \Rightarrow C=10W$. Substituting $C=10W$ into **(1)** results in

$0.05(10W)-0.001(10W)W=0 \Leftrightarrow 0.5W-0.01W^2=0 \Leftrightarrow 50W-W^2=0 \Leftrightarrow W(50-W)=0 \Leftrightarrow W=0$ or 50 . Since $C=10W$, $C=0$ or 500 . Thus, the population pairs (C,W) that lead to stable populations are (0,0) and (500,50) . So it is possible for the two species to live in harmony.