

$$1. f(x)=x-3\sin x \Rightarrow f'(x)=1-3\cos x$$

$$2. f(x)=x\sin x \Rightarrow f'(x)=x \cdot \cos x+(\sin x) \cdot 1=x\cos x+\sin x$$

$$3. y=\sin x+10\tan x \Rightarrow y'=\cos x+10\sec^2 x$$

$$4. y=2x+5\cos x \Rightarrow y'=-2x\cot x-5\sin x$$

$$5. g(t)=t^3 \cos t \Rightarrow g'(t)=t^3(-\sin t)+(\cos t) \cdot 3t^2=3t^2 \cos t-t^3 \sin t \text{ or } t^2(3\cos t-t\sin t)$$

$$6. g(t)=4\sec t+\tan t \Rightarrow g'(t)=4\sec t \tan t+\sec^2 t$$

$$7. h(\theta)=\csc \theta + e^\theta \cot \theta \Rightarrow$$

$$h'(\theta)=-\csc \theta \cot \theta + e^\theta (-\csc^2 \theta) + (\cot \theta) e^\theta = -\csc \theta \cot \theta + e^\theta (\cot \theta - \csc^2 \theta)$$

$$8. y=e^u(\cos u+cu) \Rightarrow y' = e^u(-\sin u+c) + (\cos u+cu)e^u = e^u(\cos u - \sin u + cu + c)$$

$$9. y = \frac{x}{\cos x} \Rightarrow y' = \frac{(\cos x)(1) - (x)(-\sin x)}{(\cos x)^2} = \frac{\cos x + x\sin x}{\cos^2 x}$$

$$10. y = \frac{1+\sin x}{x+\cos x} \Rightarrow$$

$$y' = \frac{(x+\cos x)(\cos x) - (1+\sin x)(1-\sin x)}{(x+\cos x)^2} = \frac{x\cos x + \cos^2 x - (1-\sin^2 x)}{(x+\cos x)^2}$$

$$= \frac{x\cos x + \cos^2 x - (\cos^2 x)}{(x+\cos x)^2} = \frac{x\cos x}{(x+\cos x)^2}$$

$$11. f(\theta) = \frac{\sec \theta}{1+\sec \theta} \Rightarrow$$

$$f'(\theta) = \frac{(1+\sec \theta)(\sec \theta \tan \theta) - (\sec \theta)(\sec \theta \tan \theta)}{(1+\sec \theta)^2} = \frac{(\sec \theta \tan \theta)[(1+\sec \theta) - \sec \theta]}{(1+\sec \theta)^2} = \frac{\sec \theta \tan \theta}{(1+\sec \theta)^2}$$

$$12. y = \frac{\tan x - 1}{\sec x} \Rightarrow$$

$$\frac{dy}{dx} = \frac{\sec x \sec^2 x - (\tan x - 1) \sec x \tan x}{\sec^2 x} = \frac{\sec x (\sec^2 x - \tan^2 x + \tan x)}{\sec^2 x} = \frac{1 + \tan x}{\sec x}$$

Another method: Simplify y first: $y = \sin x - \cos x \Rightarrow y' = \cos x + \sin x$.

$$13. y = \frac{\sin x}{x^2} \Rightarrow y' = \frac{x^2 \cos x - (\sin x)(2x)}{(x^2)^2} = \frac{x(x \cos x - 2 \sin x)}{x^4} = \frac{x \cos x - 2 \sin x}{x^3}$$

$$14. y = \theta (\theta + \cot \theta) \Rightarrow$$

$$\begin{aligned} y' &= \theta (1 - \csc^2 \theta) + (\theta + \cot \theta)(-\theta \cot \theta) = \theta (1 - \csc^2 \theta - \theta \cot \theta - \cot^2 \theta) \\ &= \theta (-\cot^2 \theta - \theta \cot \theta - \cot^2 \theta) \quad \{1 + \cot^2 \theta = \csc^2 \theta\} \\ &= \theta (-\theta \cot \theta - 2 \cot^2 \theta) = -\theta \cot \theta (\theta + 2 \cot \theta) \end{aligned}$$

$$15. y = \sec \theta \tan \theta \Rightarrow y' = \sec \theta (\sec^2 \theta) + \tan \theta (\sec \theta \tan \theta) = \sec \theta (\sec^2 \theta + \tan^2 \theta)$$

Using the identity $1 + \tan^2 \theta = \sec^2 \theta$, we can write alternative forms of the answer as $\sec \theta (1 + 2 \tan^2 \theta)$ or $\sec \theta (2 \sec^2 \theta - 1)$

$$16. \text{Recall that if } y = fgh, \text{ then } y' = f'gh + fg'h + fgh'. y = x \sin x \cos x \Rightarrow$$

$$\frac{dy}{dx} = \sin x \cos x + x \cos x \cos x + x \sin x (-\sin x) = \sin x \cos x + x \cos^2 x - x \sin^2 x$$

$$17. \frac{d}{dx} (\csc(x)) = \frac{d}{dx} \left(\frac{1}{\sin x} \right) = \frac{(\sin x)(0) - 1(\cos x)}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -x \cot x$$

$$18. \frac{d}{dx} (\sec x) = \frac{d}{dx} \left(\frac{1}{\cos x} \right) = \frac{(\cos x)(0) - 1(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x$$

$$19. \frac{d}{dx} (\cot x) = \frac{d}{dx} \left(\frac{\cos x}{\sin x} \right) = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

$$20. f(x) = \cos x \Rightarrow$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \left(\cos x \frac{\cos h - 1}{h} - \sin x \frac{\sin h}{h} \right) = \cos x \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} - \sin x \lim_{h \rightarrow 0} \frac{\sin h}{h} \\
 &= (\cos x)(0) - (\sin x)(1) = -\sin x
 \end{aligned}$$

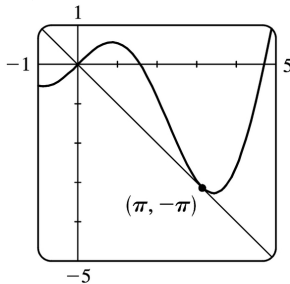
21. $y = \tan x \Rightarrow y' = \sec^2 x \Rightarrow$ the slope of the tangent line at $\left(\frac{\pi}{4}, 1\right)$ is $\sec^2 \frac{\pi}{4} = (\sqrt{2})^2 = 2$ and an equation of the tangent line is $y - 1 = 2\left(x - \frac{\pi}{4}\right)$ or $y = 2x + 1 - \frac{\pi}{2}$.

22. $y = e^x \cos x \Rightarrow y' = e^x(-\sin x) + (\cos x)e^x = e^x(\cos x - \sin x) \Rightarrow$ the slope of the tangent line at $(0, 1)$ is $e^0(\cos 0 - \sin 0) = 1(1 - 0) = 1$ and an equation is $y - 1 = 1(x - 0)$ or $y = x + 1$.

23. $y = x + \cos x \Rightarrow y' = 1 - \sin x$. At $(0, 1)$, $y' = 1$, and an equation of the tangent line is $y - 1 = 1(x - 0)$, or $y = x + 1$.

24. $y = \frac{1}{\sin x + \cos x} \Rightarrow y' = -\frac{\cos x - \sin x}{(\sin x + \cos x)^2}$ [Reciprocal Rule]. At $(0, 1)$, $y' = -\frac{1 - 0}{(0 + 1)^2} = -1$, and an equation of the tangent line is $y - 1 = -1(x - 0)$, or $y = -x + 1$.

25. (a) $y = x \cos x \Rightarrow y' = x(-\sin x) + \cos x(1) = \cos x - x \sin x$. So the slope of the tangent at the point $(\pi, -\pi)$ is $\cos \pi - \pi \sin \pi = -1 - \pi(0) = -1$, and an equation is $y + \pi = -(x - \pi)$ or $y = -x$.

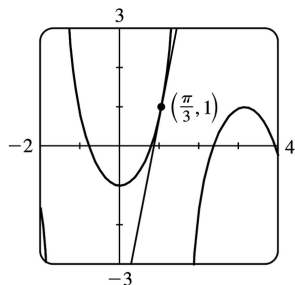


(b)

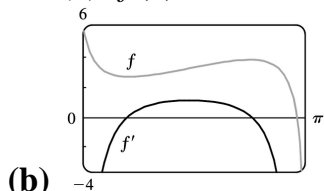
26. (a) $y = \sec x - 2 \cos x \Rightarrow y' = \sec x \tan x + 2 \sin x \Rightarrow$

the slope of the tangent line at $\left(\frac{\pi}{3}, 1\right)$ is $\sec \frac{\pi}{3} \tan \frac{\pi}{3} + 2 \sin \frac{\pi}{3} = 2 \cdot \sqrt{3} + 2 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$ and an equation is $y - 1 = 3\sqrt{3}\left(x - \frac{\pi}{3}\right)$ or $y = 3\sqrt{3}x + 1 - \pi\sqrt{3}$.

(b)



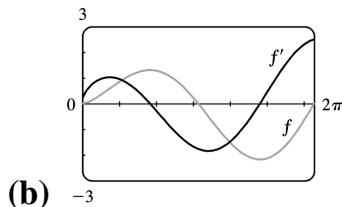
27. (a) $f(x) = 2x + \cot x \Rightarrow f'(x) = 2 - \csc^2 x$



Notice that $f'(x) = 0$ when f has a horizontal tangent.

f' is positive when f is increasing and f' is negative when f is decreasing. Also, $f'(x)$ is large negative when the graph of f is steep.

28. (a) $f(x) = \sqrt{x} \sin x \Rightarrow f'(x) = \sqrt{x} \cos x + (\sin x) \left(\frac{1}{2} x^{-1/2} \right) = \sqrt{x} \cos x + \frac{\sin x}{2\sqrt{x}}$



Notice that $f'(x) = 0$ when f has a horizontal tangent.

f' is positive when f is increasing and f' is negative when f is decreasing.

29. $f(x) = x + 2\sin x$ has a horizontal tangent when $f'(x) = 0 \Leftrightarrow 1 + 2\cos x = 0 \Leftrightarrow \cos x = -\frac{1}{2} \Leftrightarrow$
 $x = \frac{2\pi}{3} + 2\pi n$ or $\frac{4\pi}{3} + 2\pi n$, where n is an integer. Note that $\frac{4\pi}{3}$ and $\frac{2\pi}{3}$ are $\pm \frac{\pi}{3}$ units from π .

This allows us to write the solutions in the more compact equivalent form $(2n+1)\pi \pm \frac{\pi}{3}$, n an integer.

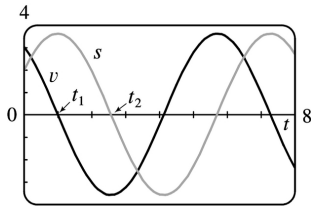
30. $y = \frac{\cos x}{2 + \sin x} \Rightarrow y' = \frac{(2 + \sin x)(-\sin x) - \cos x \cos x}{(2 + \sin x)^2} = \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2 + \sin x)^2} = \frac{-2\sin x - 1}{(2 + \sin x)^2} = 0$ when

$-2\sin x - 1 = 0 \Leftrightarrow \sin x = -\frac{1}{2} \Leftrightarrow x = \frac{11\pi}{6} + 2\pi n$ or $x = \frac{7\pi}{6} + 2\pi n$, n an integer. So $y = \frac{1}{\sqrt{3}}$ or $y = -\frac{1}{\sqrt{3}}$ and the points on the curve with horizontal tangents are: $\left(\frac{11\pi}{6} + 2\pi n, \frac{1}{\sqrt{3}}\right)$, $\left(\frac{7\pi}{6} + 2\pi n, -\frac{1}{\sqrt{3}}\right)$, n an integer.

31. (a) $x(t) = 8\sin t \Rightarrow v(t) = x'(t) = 8\cos t$

(b) The mass at time $t = \frac{2\pi}{3}$ has position $x\left(\frac{2\pi}{3}\right) = 8\sin\frac{2\pi}{3} = 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$ and velocity $v\left(\frac{2\pi}{3}\right) = 8\cos\frac{2\pi}{3} = 8\left(-\frac{1}{2}\right) = -4$. Since $v\left(\frac{2\pi}{3}\right) < 0$, the particle is moving to the left.

32. (a) $s(t) = 2\cos t + 3\sin t \Rightarrow v(t) = -2\sin t + 3\cos t$

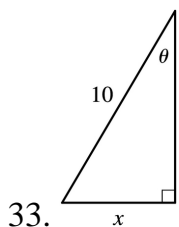


(b) -4

(c) $s = 0 \Rightarrow t_2 \approx 2.55$. So the mass passes through the equilibrium position for the first time when $t \approx 2.55$ s.

(d) $v = 0 \Rightarrow t_1 \approx 0.98$, $s(t_1) \approx 3.61$ cm. So the mass travels a maximum of about 3.6 cm (upward and downward) from its equilibrium position.

(e) The speed $|v|$ is greatest when $s = 0$; that is, when $t = t_2 + n\pi$, n a positive integer.

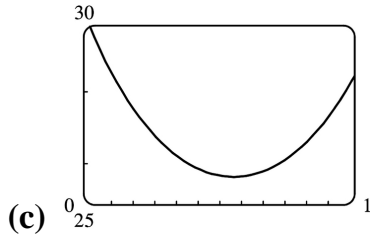


33. From the diagram we can see that $\sin \theta = x/10 \Leftrightarrow x = 10\sin \theta$. We want to find the rate of change of x with respect to θ ; that is, $dx/d\theta$. Taking the derivative of the above expression, $dx/d\theta = 10(\cos \theta)$. So when $\theta = \frac{\pi}{3}$, $\frac{dx}{d\theta} = 10\cos\left(\frac{\pi}{3}\right) = 10 \cdot \left(\frac{1}{2}\right) = 5$ ft/rad.

34. (a) $F = \frac{\mu W}{\mu \sin \theta + \cos \theta} \Rightarrow \frac{dF}{d\theta} = \frac{(\mu \sin \theta + \cos \theta)(0) - \mu W (\mu \cos \theta - \sin \theta)}{(\mu \sin \theta + \cos \theta)^2} = \frac{\mu W (\sin \theta - \mu \cos \theta)}{(\mu \sin \theta + \cos \theta)^2}$

(b)

$$\frac{dF}{d\theta} = 0 \Rightarrow \mu W (\sin \theta - \mu \cos \theta) = 0 \Rightarrow \sin \theta = \mu \cos \theta \Rightarrow \tan \theta = \mu \Rightarrow \theta = \tan^{-1} \mu$$



From the graph of $F = \frac{0.6(50)}{0.6 \sin \theta + \cos \theta}$ for $0 \leq \theta \leq 1$, we see that $\frac{dF}{d\theta} = 0 \Rightarrow \theta \approx 0.54$. Checking this with part (b) and $\mu = 0.6$, we calculate $\theta = \tan^{-1} 0.6 \approx 0.54$. So the value from the graph is consistent with the value in part (b).

35.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 3x}{x} &= \lim_{x \rightarrow 0} \frac{3 \sin 3x}{3x} \quad [\text{multiply numerator and denominator by 3}] \\ &= 3 \lim_{3x \rightarrow 0} \frac{\sin 3x}{3x} \quad [\text{as } x \rightarrow 0, 3x \rightarrow 0] \\ &= 3 \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \quad [\text{let } \theta = 3x] \\ &= 3(1) \quad [\text{Equation 2}] \\ &= 3 \end{aligned}$$

36.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 6x} &= \lim_{x \rightarrow 0} \left(\frac{\sin 4x}{x} \cdot \frac{x}{\sin 6x} \right) = \lim_{x \rightarrow 0} \frac{4 \sin 4x}{4x} \cdot \lim_{x \rightarrow 0} \frac{6x}{6 \sin 6x} \\ &= 4 \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \cdot \frac{1}{6} \lim_{x \rightarrow 0} \frac{6x}{\sin 6x} = 4(1) \cdot \frac{1}{6} (1) = \frac{2}{3} \end{aligned}$$

37.

$$\begin{aligned} \lim_{t \rightarrow 0} \frac{\tan 6t}{\sin 2t} &= \lim_{t \rightarrow 0} \left(\frac{\sin 6t}{t} \cdot \frac{1}{\cos 6t} \cdot \frac{t}{\sin 2t} \right) = \lim_{t \rightarrow 0} \frac{6 \sin 6t}{6t} \cdot \lim_{t \rightarrow 0} \frac{1}{\cos 6t} \cdot \lim_{t \rightarrow 0} \frac{2t}{2 \sin 2t} \\ &= 6 \lim_{t \rightarrow 0} \frac{\sin 6t}{6t} \cdot \lim_{t \rightarrow 0} \frac{1}{\cos 6t} \cdot \frac{1}{2} \lim_{t \rightarrow 0} \frac{2t}{\sin 2t} = 6(1) \cdot \frac{1}{1} \cdot \frac{1}{2} (1) = 3 \end{aligned}$$

38.

$$\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\sin \theta} = \lim_{\theta \rightarrow 0} \frac{\frac{\cos \theta - 1}{\theta}}{\frac{\sin \theta}{\theta}} = \frac{\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta}}{\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}} = \frac{0}{1} = 0$$

$$39. \lim_{\theta \rightarrow 0} \frac{\sin(\cos \theta)}{\sec \theta} = \frac{\sin\left(\lim_{\theta \rightarrow 0}(\cos \theta)\right)}{\lim_{\theta \rightarrow 0}(\sec \theta)} = \frac{\sin 1}{1} = \sin 1$$

40.

$$\begin{aligned} \lim_{t \rightarrow 0} \frac{\sin^2 3t}{t^2} &= \lim_{t \rightarrow 0} \left(\frac{\sin 3t}{t} \cdot \frac{\sin 3t}{t} \right) = \lim_{t \rightarrow 0} \frac{\sin 3t}{t} \cdot \lim_{t \rightarrow 0} \frac{\sin 3t}{t} \\ &= \left(\lim_{t \rightarrow 0} \frac{\sin 3t}{t} \right)^2 = \left(3 \lim_{t \rightarrow 0} \frac{\sin 3t}{3t} \right)^2 = (3 \cdot 1)^2 = 9 \end{aligned}$$

41.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\cot 2x}{x} &= \lim_{x \rightarrow 0} \frac{\cos 2x \cdot \sin x}{\sin 2x} = \lim_{x \rightarrow 0} \left(\cos 2x \left[\frac{(\sin x)/x}{(\sin 2x)/2x} \right] \right) = \lim_{x \rightarrow 0} \left(\cos 2x \left[\frac{\lim_{x \rightarrow 0}[(\sin x)/x]}{2 \lim_{x \rightarrow 0}[(\sin 2x)/2x]} \right] \right) \\ &= 1 \cdot \frac{1}{2 \cdot 1} = \frac{1}{2} \end{aligned}$$

42.

$$\begin{aligned} \lim_{x \rightarrow \pi/4} \frac{\sin x - \cos x}{\cos 2x} &= \lim_{x \rightarrow \pi/4} \frac{\sin x - \cos x}{\cos^2 x - \sin^2 x} = \lim_{x \rightarrow \pi/4} \frac{\sin x - \cos x}{(\cos x + \sin x)(\cos x - \sin x)} \\ &= \lim_{x \rightarrow \pi/4} \frac{-1}{\cos x + \sin x} = \frac{-1}{\cos \frac{\pi}{4} + \sin \frac{\pi}{4}} = \frac{-1}{\sqrt{2}} \end{aligned}$$

43. Divide numerator and denominator by θ . ($\sin(\theta)$ also works.)

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta} = \lim_{\theta \rightarrow 0} \frac{\frac{\sin \theta}{\theta}}{1 + \frac{\sin \theta}{\theta} \cdot \frac{1}{\cos \theta}} = \frac{\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}}{1 + \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \lim_{\theta \rightarrow 0} \frac{1}{\cos \theta}} = \frac{1}{1 + 1 \cdot 1} = \frac{1}{2}$$

44.

$$\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2+x-2} = \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x+2)(x-1)} = \lim_{x \rightarrow 1} \frac{1}{x+2} \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x-1} = \frac{1}{3} \cdot 1 = \frac{1}{3}$$

$$45. \text{(a)} \quad \frac{d}{dx} \tan x = \frac{d}{dx} \frac{\sin x}{\cos x} \Rightarrow \sec^2 x = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}. \text{ So } \sec^2 x = \frac{1}{\cos^2 x}.$$

$$\text{(b)} \quad \frac{d}{dx} \sec x = \frac{d}{dx} \frac{1}{\cos x} \Rightarrow \sec x \tan x = \frac{(\cos x)(0) - 1(-\sin x)}{\cos^2 x}. \text{ So } \sec x \tan x = \frac{\sin x}{\cos^2 x}.$$

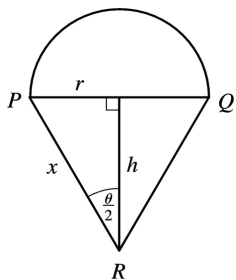
$$\text{(c)} \quad \frac{d}{dx} (\sin x + \cos x) = \frac{d}{dx} \frac{1 + \cot x}{x} \Rightarrow$$

$$\begin{aligned} \cos x - \sin x &= \frac{x(-\csc^2 x) - (1 + \cot x)(-x \cot x)}{\csc^2 x} = \frac{x[-\csc^2 x + (1 + \cot x) \cot x]}{\csc^2 x} \\ &= \frac{-\csc^2 x + \cot^2 x + \cot x}{x} = \frac{-1 + \cot x}{x} \end{aligned}$$

$$\text{So } \cos x - \sin x = \frac{\cot x - 1}{x}.$$

46. Let $|PR| = x$. Then we get the following formulas for r and h in terms of θ and x : $\sin \frac{\theta}{2} = \frac{r}{x} \Rightarrow$
 $r = x \sin \frac{\theta}{2}$ and $\cos \frac{\theta}{2} = \frac{h}{x} \Rightarrow h = x \cos \frac{\theta}{2}$. Now $A(\theta) = \frac{1}{2} \pi r^2$ and $B(\theta) = \frac{1}{2} (2r)h = rh$. So

$$\begin{aligned} \lim_{\theta \rightarrow 0^+} \frac{A(\theta)}{B(\theta)} &= \lim_{\theta \rightarrow 0^+} \frac{\frac{1}{2} \pi r^2}{rh} = \frac{1}{2} \pi \lim_{\theta \rightarrow 0^+} \frac{r}{h} = \frac{1}{2} \pi \lim_{\theta \rightarrow 0^+} \frac{x \sin(\theta/2)}{x \cos(\theta/2)} \\ &= \frac{1}{2} \pi \lim_{\theta \rightarrow 0^+} \tan(\theta/2) = 0. \end{aligned}$$



47. By the definition of radian measure, $s = r\theta$, where r is the radius of the circle.

By drawing the bisector of the angle θ , we can see that $\sin \frac{\theta}{2} = \frac{d/2}{r} \Rightarrow d = 2r \sin \frac{\theta}{2}$.

$$\text{So } \lim_{\theta \rightarrow 0^+} \frac{s}{d} = \lim_{\theta \rightarrow 0^+} \frac{r\theta}{2r \sin(\theta/2)} = \lim_{\theta \rightarrow 0^+} \frac{2 \cdot (\theta/2)}{2 \sin(\theta/2)} = \lim_{\theta \rightarrow 0} \frac{\theta/2}{\sin(\theta/2)} = 1.$$