1. Let $u=g(x)=4x$ and $y=f(u)=\sin u$. Then $\dfrac{dy}{dx}=\dfrac{dy}{du}\,\dfrac{du}{dx}=(\cos u)(4)=4\cos 4x$.

2. Let $u=g(x)=4+3x$ and $y=f(u)=\sqrt{u}=u^{1/2}$. Then $\dfrac{dy}{dx}=\dfrac{dy}{du}\,\dfrac{du}{dx}=\dfrac{1}{2}\,u^{-1/2}(3)=\dfrac{3}{2\sqrt{u}}=\dfrac{3}{2\sqrt{4+3x}}$.

3. Let $u=g(x)=1-x^2$ and $y=f(u)=u^{10}$. Then $\dfrac{dy}{dx}=\dfrac{dy}{du}\,\dfrac{du}{dx}=(10u^9)(-2x)=-20x(1-x^2)^9$.

4. Let $u=g(x)=\sin x$ and $y=f(u)=\tan u$. Then $\dfrac{dy}{dx}=\dfrac{dy}{du}\,\dfrac{du}{dx}=(\sec^2 u)(\cos x)=(\sec^2 \mathrm{u})(\sin x)\cdot\cos x$, or equivalently, $[\sec(\sin x)]^2\cos x$.

5. Let $u=g(x)=\sin x$ and $y=f(u)=\sqrt{u}$. Then $\dfrac{dy}{dx}=\dfrac{dy}{du}\,\dfrac{du}{dx}=\dfrac{1}{2}\,u^{-1/2}\cos x=\dfrac{\cos x}{2\sqrt{u}}=\dfrac{\cos x}{2\sqrt{\sin x}}$.

6. Let $u=g(x)=e^x$ and $y=f(u)=\sin u$. Then $\dfrac{dy}{dx}=\dfrac{dy}{du}\,\dfrac{du}{dx}=(\cos u)(e^x)=e^x\cos e^x$.

7. $F(x)=(x^3+4x)^7\Rightarrow F'(x)=7(x^3+4x)^6(3x^2+4)$ [or $7x^6(x^2+4)^6(3x^2+4)$]

8. $F(x)=(x^2-x+1)^3\Rightarrow F'(x)=3(x^2-x+1)^2(2x-1)$

9. $F(x)=\sqrt[4]{1+2x+x^3}=(1+2x+x^3)^{1/4}\Rightarrow$

$$F'(x)=\frac{1}{4}\,(1+2x+x^3)^{-3/4}\cdot\frac{d}{dx}(1+2x+x^3)=\frac{1}{4(1+2x+x^3)^{3/4}}\cdot(2+3x^2)$$

$$=\frac{2+3x^2}{4(1+2x+x^3)^{3/4}}=\frac{2+3x^2}{4\sqrt[4]{(1+2x+x^3)^3}}$$

10. $f(x)=(1+x^4)^{2/3}\Rightarrow f'(x)=\dfrac{2}{3}(1+x^4)^{-1/3}(4x^3)=\dfrac{8x^3}{3\sqrt[3]{1+x^4}}$

11. $g(t)=\dfrac{1}{(t^4+1)^3}=(t^4+1)^{-3}\Rightarrow g'(t)=-3(t^4+1)^{-4}(4t^3)=-12t^3(t^4+1)^{-4}=\dfrac{-12t^3}{(t^4+1)^4}$

12.

$$f(t)=\sqrt[3]{1+\tan t}=(1+\tan t)^{1/3}\Rightarrow f'(t)=\frac{1}{3}(1+\tan t)^{-2/3}\sec^2 t=\frac{\sec^2 t}{3\sqrt[3]{(1+\tan t)^2}}$$

13. $y=\cos(a^3+x^3)\Rightarrow y'=-\sin(a^3+x^3)\cdot 3x^2$ [a^3 is just a constant] $=-3x^2\sin(a^3+x^3)$

14. $y=a^3+\cos^3 x\Rightarrow y'=3(\cos x)^2(-\sin x)$ [a^3 is just a constant] $=-3\sin x\cos^2 x$

15. $y=e^{-mx}\Rightarrow y'=e^{-mx}\dfrac{d}{dx}(-mx)=e^{-mx}(-m)=-me^{-mx}$

16. $y=4\sec 5x\Rightarrow y'=4\sec 5x\tan 5x(5)=20\sec 5x\tan 5x$

17. $g(x)=(1+4x)^5(3+x-x^2)^8\Rightarrow$

$$\begin{aligned}g'(x)&=(1+4x)^5\cdot 8(3+x-x^2)^7(1-2x)+(3+x-x^2)^8\cdot 5(1+4x)^4\cdot 4\\&=4(1+4x)^4(3+x-x^2)^7[2(1+4x)(1-2x)+5(3+x-x^2)]\\&=4(1+4x)^4(3+x-x^2)^7[(2+4x-16x^2)+(15+5x-5x^2)]\\&=4(1+4x)^4(3+x-x^2)^7(17+9x-21x^2)\end{aligned}$$

18. $h(t)=(t^4-1)^3(t^3+1)^4\Rightarrow$

$$\begin{aligned}h'(t)&=(t^4-1)^3\cdot 4(t^3+1)^3(3t^2)+(t^3+1)^4\cdot 3(t^4-1)^2(4t^3)\\&=12t^2(t^4-1)^2(t^3+1)^3[(t^4-1)+t(t^3+1)]=12t^2(t^4-1)^2(t^3+1)^3(2t^4+t-1)\end{aligned}$$

19. $y=(2x-5)^4(8x^2-5)^{-3}\Rightarrow$

$$\begin{aligned}y'&=4(2x-5)^3(2)(8x^2-5)^{-3}+(2x-5)^4(-3)(8x^2-5)^{-4}(16x)\\&=8(2x-5)^3(8x^2-5)^{-3}-48x(2x-5)^4(8x^2-5)^{-4}\end{aligned}$$

20. $y=(x^2+1)(x^2+2)^{1/3}\Rightarrow y'=2x(x^2+2)^{1/3}+(x^2+1)\left(\dfrac{1}{3}\right)(x^2+2)^{-2/3}(2x)=2x(x^2+2)^{1/3}\left[1+\dfrac{x^2+1}{3(x^2+2)}\right]$

21. $y=xe^{-x^2}\Rightarrow y'=xe^{-x^2}(-2x)+e^{-x^2}\cdot 1=e^{-x^2}(-2x^2+1)=e^{-x^2}(1-2x^2)$

22. $y=e^{-5x}\cos 3x \Rightarrow y'=e^{-5x}(-3\sin 3x)+(\cos 3x)(-5e^{-5x})=-e^{-5x}(3\sin 3x+5\cos 3x)$

23. $y=e^{x\cos x} \Rightarrow y'=e^{x\cos x}\cdot \frac{d}{dx}(x\cos x)=e^{x\cos x}[x(-\sin x)+(\cos x)\cdot 1]=e^{x\cos x}(\cos x-x\sin x)$

24. Using Formula 5 and the Chain Rule, $y=10^{1-x^2} \Rightarrow y'=10^{1-x^2}(\ln 10)\cdot \frac{d}{dx}(1-x^2)=-2x(\ln 10)10^{1-x^2}$.

25. $F(z)=\sqrt{\frac{z-1}{z+1}}=\left(\frac{z-1}{z+1}\right)^{1/2} \Rightarrow$

$$F'(z)=\frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-1/2}\cdot\frac{d}{dz}\left(\frac{z-1}{z+1}\right)=\frac{1}{2}\left(\frac{z+1}{z-1}\right)^{1/2}\cdot\frac{(z+1)(1)-(z-1)(1)}{(z+1)^2}$$

$$=\frac{1}{2}\,\frac{(z+1)^{1/2}}{(z-1)^{1/2}}\cdot\frac{z+1-z+1}{(z+1)^2}=\frac{1}{2}\,\frac{(z+1)^{1/2}}{(z-1)^{1/2}}\cdot\frac{2}{(z+1)^2}=\frac{1}{(z-1)^{1/2}(z+1)^{3/2}}$$

26. $G(y)=\frac{(y-1)^4}{(y^2+2y)^5} \Rightarrow$

$$G'(y)=\frac{(y^2+2y)^5\cdot 4(y-1)^3\cdot 1-(y-1)^4\cdot 5(y^2+2y)^4(2y+2)}{[(y^2+2y)^5]^2}$$

$$=\frac{2(y^2+2y)^4(y-1)^3[2(y^2+2y)-5(y-1)(y+1)]}{(y^2+2y)^{10}}$$

$$=\frac{2(y-1)^3[(2y^2+4y)+(-5y^2+5)]}{(y^2+2y)^6}=\frac{2(y-1)^3(-3y^2+4y+5)}{(y^2+2y)^6}$$

27. $y=\frac{r}{\sqrt{r^2+1}} \Rightarrow$

$$y'=\frac{\sqrt{r^2+1}\,(1)-r\cdot\frac{1}{2}(r^2+1)^{-1/2}(2r)}{\left(\sqrt{r^2+1}\right)^2}=\frac{\sqrt{r^2+1}-\frac{r^2}{\sqrt{r^2+1}}}{\left(\sqrt{r^2+1}\right)^2}=\frac{\frac{\sqrt{r^2+1}\,\sqrt{r^2+1}-r^2}{\sqrt{r^2+1}}}{\left(\sqrt{r^2+1}\right)^2}$$

$$=\frac{(r^2+1)-r^2}{\left(\sqrt{r^2+1}\right)^3}=\frac{1}{(r^2+1)^{3/2}}\text{ or }(r^2+1)^{-3/2}$$

Another solution: Write y as a product and make use of the Product Rule. $y=r(r^2+1)^{-1/2}\Rightarrow$

$$y'=r\cdot-\frac{1}{2}(r^2+1)^{-3/2}(2r)+r^2+1)^{-1/2}\cdot 1$$

$$=(r^2+1)^{-3/2}[-r^2+(r^2+1)^1]=(r^2+1)^{-3/2}(1)=(r^2+1)^{-3/2}$$

The step that students usually have trouble with is factoring out $(r^2+1)^{-3/2}$. But this is no different than factoring out x^2 from x^2+x^5; that is, we are just factoring out a factor with the *smallest* exponent that appears on it. In this case, $-\frac{3}{2}$ is smaller than $-\frac{1}{2}$.

28. $y=\dfrac{e^{2u}}{e^u+e^{-u}}\Rightarrow$

$$y'=\frac{(e^u+e^{-u})(e^{2u}\cdot 2)-e^{2u}(e^u-e^{-u})}{(e^u+e^{-u})^2}=\frac{e^{2u}(2e^u+2e^{-u}-e^u+e^{-u})}{(e^u+e^{-u})^2}=\frac{e^{2u}(e^u+3e^{-u})}{(e^u+e^{-u})^2}$$

Another solution: Eliminate negative exponents by first changing the form of y.

$$y=\frac{e^{2u}}{e^u+e^{-u}}\cdot\frac{e^u}{e^u}=\frac{e^{3u}}{e^{2u}+1}\Rightarrow$$

$$y'=\frac{(e^{2u}+1)(3e^{3u})-e^{3u}(2e^{2u})}{(e^{2u}+1)^2}=\frac{e^{3u}(3e^{2u}+3-2e^{2u})}{(e^{2u}+1)^2}=\frac{e^{3u}(e^{2u}+3)}{(e^{2u}+1)^2}$$

29. $y=\tan(\cos x)\Rightarrow y'=\sec^2(\cos x)\cdot(-\sin x)=-\sin x\cdot\sec^2(\cos x)$

30. $y=\dfrac{\sin^2 x}{\cos x}\Rightarrow$

$$y'=\frac{\cos x(2\sin x\cdot\cos x)-\sin^2 x(-\sin x)}{\cos^2 x}=\frac{\sin x(2\cos^2 x+\sin^2 x)}{\cos^2 x}=\frac{\sin x(1+\cos^2 x)}{\cos^2 x}$$

$$=\sin x(1+\sec^2 x)$$

Another method: $y=\tan x\cdot\sin x\Rightarrow$

$y' = \sec^2 x\cdot \sin x + \tan x\cdot \cos x = \sec^2 x\cdot \sin x + \sin x$

31. Using Formula 5 and the Chain Rule, $y=2^{\sin \pi x} \Rightarrow$

$y' = 2^{\sin \pi x}(\ln 2)\cdot \frac{d}{dx}(\sin \pi x) = 2^{\sin \pi x}(\ln 2)\cdot \cos \pi x\cdot \pi = 2^{\sin \pi x}(\pi \ln 2)\cos \pi x$

32. $y=\tan^2(3\theta)=(\tan 3\theta)^2 \Rightarrow y' = 2(\tan 3\theta)\cdot \frac{d}{d\theta}(\tan 3\theta) = 2\tan 3\theta\cdot \sec^2 3\theta\cdot 3 = 6\tan 3\theta \sec^2 3\theta$

33. $y=(1+\cos^2 x)^6 \Rightarrow y' = 6(1+\cos^2 x)^5\cdot 2\cos x(-\sin x) = -12\cos x\sin x(1+\cos^2 x)^5$

34. $y = x\sin\frac{1}{x} \Rightarrow y' = \sin\frac{1}{x} + x\cos\frac{1}{x}\left(-\frac{1}{x^2}\right) = \sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x}$

35. $y=\sec^2 x+\tan^2 x=(\sec x)^2+(\tan x)^2 \Rightarrow$

$y' = 2(\sec x)\cdot(\sec x\tan x)+2(\tan x)(\sec^2 x) = 2\sec^2 x\cdot \tan x + 2\sec^2 x\cdot \tan x = 4\sec^2 x\cdot \tan x$

36. $y=e^{k\tan\sqrt{x}} \Rightarrow y' = e^{k\tan\sqrt{x}}\cdot \frac{d}{dx}(k\tan\sqrt{x}) = e^{k\tan\sqrt{x}}\left(k\sec^2\sqrt{x}\cdot \frac{1}{2}x^{-1/2}\right) = \frac{k\sec^2\sqrt{x}}{2\sqrt{x}}e^{k\tan\sqrt{x}}$

37. $y=\cot^2(\sin\theta)=[\cot(\sin\theta)]^2 \Rightarrow$

$y' = 2[\cot(\sin\theta)]\cdot \frac{d}{d\theta}[\cot(\sin\theta)] = 2\cot(\sin\theta)\cdot[-\csc^2(\sin\theta)\cdot \cos\theta] = -2\cos\theta\cdot \cot(\sin\theta)\cdot \csc^2(\sin\theta)$

38. $y=\sin(\sin(\sin x)) \Rightarrow y' = \cos(\sin(\sin x))\frac{d}{dx}(\sin(\sin x)) = \cos(\sin(\sin x))\cdot \cos(\sin x)\cdot \cos x$

39. $y=\sqrt{x+\sqrt{x}} \Rightarrow y' = \frac{1}{2}(x+\sqrt{x})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right) = \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)$

40. $y=\sqrt{x+\sqrt{x+\sqrt{x}}} \Rightarrow y' = \frac{1}{2}\left(x+\sqrt{x+\sqrt{x}}\right)^{-1/2}\left[1+\frac{1}{2}(x+\sqrt{x})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)\right]$

41. $y=\sin(\tan\sqrt{\sin x}) \Rightarrow$

$y' = \cos(\tan\sqrt{\sin x})\cdot \frac{d}{dx}(\tan\sqrt{\sin x}) = \cos(\tan\sqrt{\sin x})\cdot \sec^2\sqrt{\sin x}\cdot \frac{d}{dx}(\sin x)^{1/2}$

$= \cos(\tan\sqrt{\sin x})\sec^2\sqrt{\sin x}\cdot \frac{1}{2}(\sin x)^{-1/2}\cdot \cos x$

$$=\cos\left(\tan\sqrt{\sin x}\right)\left(\sec^2\sqrt{\sin x}\right)\left(\frac{1}{2\sqrt{\sin x}}\right)(\cos x)$$

42. $y=2^{3^{x^2}} \Rightarrow y'=2^{3^{x^2}}(\ln 2)\dfrac{d}{dx}\left(3^{x^2}\right)=2^{3^{x^2}}(\ln 2)3^{x^2}(\ln 3)(2x)$

43. $y=(1+2x)^{10}\Rightarrow y'=10(1+2x)^9\cdot 2=20(1+2x)^9$. At $(0,1)$, $y'=20(1+0)^9=20$, and an equation of the tangent line is $y-1=20(x-0)$, or $y=20x+1$.

44. $y=\sin x+\sin^2 x\Rightarrow y'=\cos x+2\sin x\cos x$. At $(0,0)$, $y'=1$, and an equation of the tangent line is $y-0=1(x-0)$, or $y=x$.

45. $y=\sin(\sin x)\Rightarrow y'=\cos(\sin x)\cdot\cos x$. At $(\pi,0)$, $y'=\cos(\sin\pi)\cdot\cos\pi=\cos(0)\cdot(-1)=1(-1)=-1$, and an equation of the tangent line is $y-0=-1(x-\pi)$, or $y=-x+\pi$.

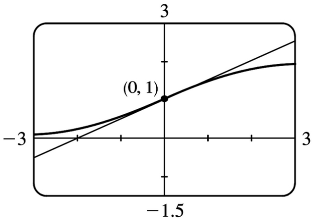
46. $y=x^2e^{-x}\Rightarrow y'=x^2(-e^{-x})+e^{-x}(2x)=2xe^{-x}-x^2e^{-x}$. At $\left(1,\ \dfrac{1}{e}\right)$, $y'=2e^{-1}-e^{-1}=\dfrac{1}{e}$. So an equation of the tangent line is $y-\dfrac{1}{e}=\dfrac{1}{e}(x-1)$ or $y=\dfrac{1}{e}x$.

47. **(a)** $y=\dfrac{2}{1+e^{-x}}\Rightarrow y'=\dfrac{(1+e^{-x})(0)-2(-e^{-x})}{(1+e^{-x})^2}=\dfrac{2e^{-x}}{(1+e^{-x})^2}$.

At $(0,\,1)$, $y'=\dfrac{2e^0}{(1+e^0)^2}=\dfrac{2(1)}{(1+1)^2}=\dfrac{2}{2^2}=\dfrac{1}{2}$.

So an equation of the tangent line is $y-1=\dfrac{1}{2}(x-0)$ or $y=\dfrac{1}{2}x+1$.

(b)



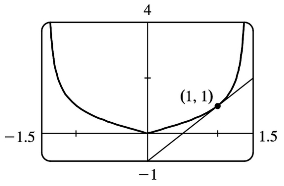
48. **(a)** For $x>0$, $|x|=x$, and $y=f(x)=\dfrac{x}{\sqrt{2-x^2}}\Rightarrow$

$$f'(x) = \frac{\sqrt{2-x^2}\,(1)-x\left(\frac{1}{2}\right)(2-x^2)^{-1/2}(-2x)}{\left(\sqrt{2-x^2}\right)^2}\cdot\frac{(2-x^2)^{1/2}}{(2-x^2)^{1/2}}$$

$$= \frac{(2-x^2)+x^2}{(2-x^2)^{3/2}} = \frac{2}{(2-x^2)^{3/2}}$$

So at $(1,1)$, the slope of the tangent line is $f'(1)=2$ and its equation is $y-1=2(x-1)$ or $y=2x-1$.

(b)

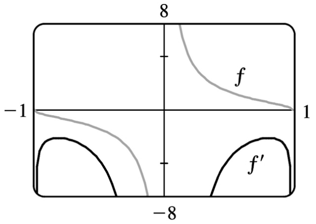


49. **(a)** $f(x)=\dfrac{\sqrt{1-x^2}}{x}\ \Rightarrow$

$$f'(x) = \frac{x\cdot\frac{1}{2}\,(1-x^{-1/2}(-2x)-\sqrt{1-x^2}\,(1)}{x^2}\cdot\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

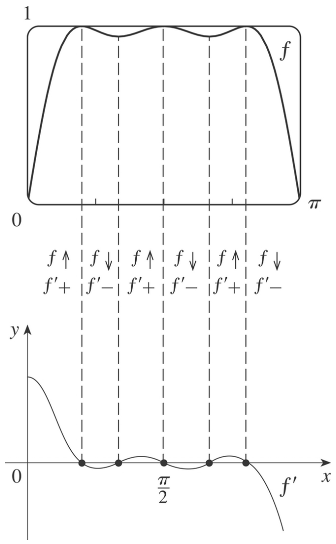
$$= \frac{-x^2-(1-x^2)}{x^2\sqrt{1-x^2}} = \frac{-1}{x^2\sqrt{1-x^2}}$$

(b)



Notice that all tangents to the graph of f have negative slopes and $f'(x)<0$ always.

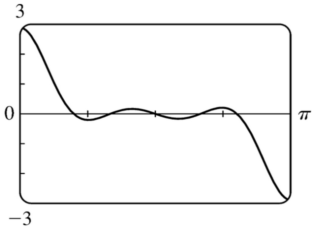
50. **(a)**



From the graph of f, we see that there are 5 horizontal tangents, so there must be 5 zeros on the graph of f'. From the symmetry of the graph of f, we must have the graph of f' as high at $x=0$ as it is low at $x=\pi$. The intervals of increase and decrease as well as the signs of f' are indicated in the figure.

(b)

$$\begin{aligned} f(x) &= \sin(x+\sin 2x) \Rightarrow \\ f'(x) &= \cos(x+\sin 2x)\cdot \frac{d}{dx}(x+\sin 2x) \\ &= \cos(x+\sin 2x)(1+2\cos 2x) \end{aligned}$$



51. For the tangent line to be horizontal, $f'(x)=0$. $f(x)=2\sin x+\sin^2 x \Rightarrow f'(x)=2\cos x+2\sin x\cos x=0$ $\Leftrightarrow 2\cos x\,(1+\sin x)=0 \Leftrightarrow \cos x=0$ or $\sin x=-1$, so $x=\frac{\pi}{2}+2n\pi$ or $\frac{3\pi}{2}+2n\pi$, where n is any integer. Now $f\left(\frac{\pi}{2}\right)=3$ and $f\left(\frac{3\pi}{2}\right)=-1$, so the points on the curve with a horizontal tangent are $\left(\frac{\pi}{2}+2n\pi, 3\right)$ and $\left(\frac{3\pi}{2}+2n\pi, -1\right)$, where n is any integer.

52. $f(x)=\sin 2x-2\sin x \Rightarrow$

$f'(x)=2\cos 2x-2\cos x=4\cos^2 x-2\cos x-2$, and $4\cos^2 x-2\cos x-2=0 \Leftrightarrow (\cos x-1)(4\cos x+2)=0 \Leftrightarrow$ $\cos x=1$ or $\cos x=-\frac{1}{2}$. So $x=2n\pi$ or $(2n+1)\pi \pm \frac{\pi}{3}$, n any integer.

53. $F(x)=f(g(x)) \Rightarrow F'(x)=f'(g(x))\cdot g'(x)$,
so $F'(3)=f'(g(3))\cdot g'(3)=f'(6)\cdot g'(3)=7\cdot 4=28$. Notice that we did not use $f'(3)=2$.

54. $w=u\circ v \Rightarrow w(x)=u(v(x)) \Rightarrow w'(x)=u'(v(x))\cdot v'(x)$, so
$w'(0)=u'(v(0))\cdot v'(0)=u'(2)\cdot v'(0)=4\cdot 5=20$. The other pieces of information, $u(0)=1$, $u'(0)=3$, and $v'(2)=6$, were not needed.

55. **(a)** $h(x)=f(g(x)) \Rightarrow h'(x)=f'(g(x))\cdot g'(x)$, so $h'(1)=f'(g(1))\cdot g'(1)=f'(2)\cdot 6=5\cdot 6=30$.
(b) $H(x)=g(f(x)) \Rightarrow H'(x)=g'(f(x))\cdot f'(x)$, so $H'(1)=g'(f(1))\cdot f'(1)=g'(3)\cdot 4=9\cdot 4=36$.

56. **(a)** $F(x)=f(f(x)) \Rightarrow F'(x)=f'(f(x))\cdot f'(x)$, so $F'(2)=f'(f(2))\cdot f'(2)=f'(1)\cdot 5=4\cdot 5=20$.
(b) $G(x)=g(g(x)) \Rightarrow G'(x)=g'(g(x))\cdot g'(x)$, so $G'(3)=g'(g(3))\cdot g'(3)=g'(2)\cdot 9=7\cdot 9=63$.

57. **(a)** $u(x)=f(g(x)) \Rightarrow u'(x)=f'(g(x))g'(x)$. So $u'(1)=f'(g(1))g'(1)=f'(3)g'(1)$. To find $f'(3)$, note that f is linear from $(2,4)$ to $(6,3)$, so its slope is $\frac{3-4}{6-2}=-\frac{1}{4}$. To find $g'(1)$, note that g is linear from $(0,6)$ to $(2,0)$, so its slope is $\frac{0-6}{2-0}=-3$. Thus, $f'(3)g'(1)=\left(-\frac{1}{4}\right)(-3)=\frac{3}{4}$.
(b) $v(x)=g(f(x)) \Rightarrow v'(x)=g'(f(x))f'(x)$. So $v'(1)=g'(f(1))f'(1)=g'(2)f'(1)$, which does not exist since $g'(2)$ does not exist.
(c) $w(x)=g(g(x)) \Rightarrow w'(x)=g'(g(x))g'(x)$. So $w'(1)=g'(g(1))g'(1)=g'(3)g'(1)$. To find $g'(3)$, note that g is linear from $(2,0)$ to $(5,2)$, so its slope is $\frac{2-0}{5-2}=\frac{2}{3}$. Thus, $g'(3)\cdot g'(1)=\left(\frac{2}{3}\right)(-3)=-2$
.

58. **(a)** $h(x)=f(f(x)) \Rightarrow h'(x)=f'(f(x))f'(x)$.
So $h'(2)=f'(f(2))f'(2)=f'(1)f'(2)\approx(-1)(-1)=1$.
(b) $g(x)=f(x^2) \Rightarrow g'(x)=f'(x^2)\cdot \frac{d}{dx}(x^2)=f'(x^2)(2x)$. So $g'(2)=f'(2^2)(2\cdot 2)=4f'(4)\approx 4(1.5)=6$.

59. $h(x)=f(g(x))\Rightarrow h'(x)=f'(g(x))g'(x)$. So $h'(0.5)=f'(g(0.5))g'(0.5)=f'(0.1)g'(0.5)$. We can estimate the derivatives by taking the average of two secant slopes.

For $f'(0.1)$: $m_1=\dfrac{14.8-12.6}{0.1-0}=22$, $m_2=\dfrac{18.4-14.8}{0.2-0.1}=36$. So $f'(0.1)\approx\dfrac{m_1+m_2}{2}=\dfrac{22+36}{2}=29$.

For $g'(0.5)$: $m_1=\dfrac{0.10-0.17}{0.5-0.4}=-0.7$, $m_2=\dfrac{0.05-0.10}{0.6-0.5}=-0.5$. So $g'(0.5)\approx\dfrac{m_1+m_2}{2}=-0.6$.

Hence, $h'(0.5)=f'(0.1)g'(0.5)\approx(29)(-0.6)=-17.4$.

60. $g(x)=f(f(x))\Rightarrow g'(x)=f'(f(x))f'(x)$. So $g'(1)=f'(f(1))f'(1)=f'(2)f'(1)$.

For $f'(2)$: $m_1=\dfrac{3.1-2.4}{2.0-1.5}=1.4$, $m_2=\dfrac{4.4-3.1}{2.5-2.0}=2.6$. So $f'(2)\approx\dfrac{m_1+m_2}{2}=2$.

For $f'(1)$: $m_1=\dfrac{2.0-1.8}{1.0-0.5}=0.4$, $m_2=\dfrac{2.4-2.0}{1.5-1.0}=0.8$. So $f'(1)\approx\dfrac{m_1+m_2}{2}=0.6$.

Hence, $g'(1)=f'(2)f'(1)\approx(2)(0.6)=1.2$.

61. **(a)** $F(x)=f(e^x)\Rightarrow F'(x)=f'(e^x)\dfrac{d}{dx}(e^x)=f'(e^x)e^x$

(b) $G(x)=e^{f(x)}\Rightarrow G'(x)=e^{f(x)}\dfrac{d}{dx}f(x)=e^{f(x)}f'(x)$

62. **(a)** $F(x)=f(x^\alpha)\Rightarrow F'(x)=f'(x^\alpha)\dfrac{d}{dx}(x^\alpha)=f'(x^\alpha)\alpha x^{\alpha-1}$

(b) $G(x)=[f(x)]^\alpha\Rightarrow G'(x)=\alpha[f(x)]^{\alpha-1}f'(x)$

63. **(a)** $f(x)=L(x^4)\Rightarrow f'(x)=L'(x^4)\cdot 4x^3=(1/x^4)\cdot 4x^3=4/x$ for $x>0$.

(b) $g(x)=L(4x)\Rightarrow g'(x)=L'(4x)\cdot 4=(1/(4x))\cdot 4=1/x$ for $x>0$.

(c) $F(x)=[L(x)]^4\Rightarrow F'(x)=4[L(x)]^3\cdot L'(x)=4[L(x)]^3\cdot(1/x)=4[L(x)]^3/x$

(d) $G(x)=L(1/x)\Rightarrow G'(x)=L'(1/x)\cdot(-1/x^2)=(1/(1/x))\cdot(-1/x^2)=x\cdot(-1/x^2)=-1/x$ for $x>0$.

64. $r(x)=f(g(h(x)))\Rightarrow r'(x)=f'(g(h(x)))\cdot g'(h(x))\cdot h'(x)$, so
$r'(1)=f'(g(h(1)))\cdot g'(h(1))\cdot h'(1)=f'(g(2))\cdot g'(2)\cdot 4=f'(3)\cdot 5\cdot 4=6\cdot 5\cdot 4=120$

65. $s(t)=10+\dfrac{1}{4}\sin(10\pi t)\Rightarrow$ the velocity after t seconds is

$v(t)=s'(t)=\frac{1}{4}\cos(10\pi t)(10\pi)=\frac{5\pi}{2}\cos(10\pi t)$ cm / s.

66. **(a)** $s=A\cos(\omega t+\delta)\Rightarrow$ velocity $=s'=-\omega A\sin(\omega t+\delta)$.

(b) If $A\neq 0$ and $\omega\neq 0$, then $s'=0\Leftrightarrow\sin(\omega t+\delta)=0\Leftrightarrow\omega t+\delta=n\pi\Leftrightarrow t=\frac{n\pi-\delta}{\omega}$, n an integer.

67. **(a)** $B(t)=4.0+0.35\sin\frac{2\pi t}{5.4}\Rightarrow\frac{dB}{dt}=\left(0.35\cos\frac{2\pi t}{5.4}\right)\left(\frac{2\pi}{5.4}\right)=\frac{0.7\pi}{5.4}\cos\frac{2\pi t}{5.4}=\frac{7\pi}{54}\cos\frac{2\pi t}{5.4}$

(b) At $t=1$, $\frac{dB}{dt}=\frac{7\pi}{54}\cos\frac{2\pi}{5.4}\approx 0.16$.

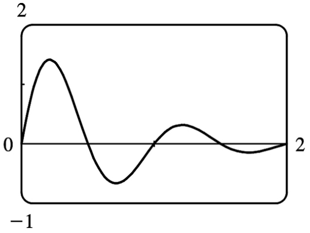
68. $L(t)=12+2.8\sin\left(\frac{2\pi}{365}(t-80)\right)\Rightarrow L'(t)=2.8\cos\left(\frac{2\pi}{365}(t-80)\right)\left(\frac{2\pi}{365}\right)$.

On March 21, $t=80$, and $L'(80)\approx 0.0482$ hours per day. On May 21, $t=141$, and $L'(141)\approx 0.02398$, which is approximately one–half of $L'(80)$.

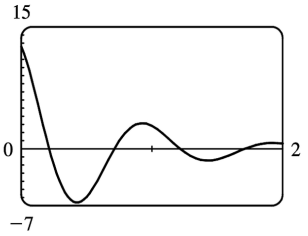
69. $s(t)=2e^{-1.5t}\sin 2\pi t\Rightarrow$

$v(t)=s'(t)=2\left[e^{-1.5t}(\cos 2\pi t)(2\pi)+(\sin 2\pi t)e^{-1.5t}(-1.5)\right]=2e^{-1.5t}(2\pi\cos 2\pi t-1.5\sin 2\pi t)$

Graph of position



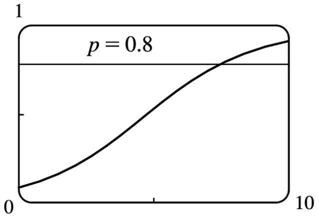
Graph of velocity



70. **(a)** $\lim_{t\to\infty}p(t)=\lim_{t\to\infty}\frac{1}{1+ae^{-kt}}=\frac{1}{1+a\cdot 0}=1$, since $k>0\Rightarrow -kt\to-\infty\Rightarrow e^{-kt}\to 0$.

(b) $p(t)=(1+ae^{-kt})^{-1}\Rightarrow\frac{dp}{dt}=-(1+ae^{-kt})^{-2}(-kae^{-kt})=\frac{kae^{-kt}}{(1+ae^{-kt})^{2}}$

(c)



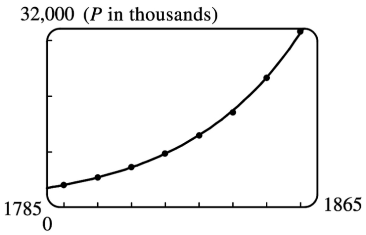
From the graph of $p(t)=(1+10e^{-0.5t})^{-1}$, it seems that $p(t)=0.8$ (indicating that 80% of the population has heard the rumor) when $t\approx 7.4$ hours.

71. **(a)** Using a calculator or CAS, we obtain the model $Q=ab^{t}$ with $a=100.0124369$ and $b=0.000045145933$. We can change this model to one with base e and exponent $\ln b$ [$b^{t}=e^{t\ln b}$ from precalculus mathematics or from Section 7.3]: $Q=ae^{t\ln b}=100.012437e^{-10.005531t}$.

(b) Use $Q^{/}(t)=ab^{t}\ln b$ or the calculator command nDeriv(Y_1, X, .04) with $Y_1=ab^{x}$ to get

$Q^{/}(0.04)\approx -670.63\mu A$. The result of Example 2 in Section 2.1 was $-670\mu A$.

72. **(a)** $P=ab^{t}$ with $a=4.502714\times 10^{-20}$ and $b=1.029953851$,
where P is measured in thousands of people. The fit appears to be very good.



(b) For 1800: $m_1=\dfrac{5308-3929}{1800-1790}=137.9$, $m_2=\dfrac{7240-5308}{1810-1800}=193.2$.

So $P^{/}(1800)\approx(m_1+m_2)/2=165.55$ thousand people / year.

For 1850: $m_1=\dfrac{23{,}192-17{,}063}{1850-1840}=612.9$, $m_2=\dfrac{31{,}443-23{,}192}{1860-1850}=825.1$.

So $P^{/}(1850)\approx(m_1+m_2)/2=719$ thousand people / year.

(c) Use the calculator command nDeriv(Y_1, X, .04) with $Y_1=ab^{x}$ to get

$P^{/}(1800)\approx 156.85$ and $P^{/}(1850)\approx 686.07$. These estimates are somewhat less than the ones in part (b).

(d) $P(1870)\approx 41{,}946.56$. The difference of 3.4 million people is most likely due to the Civil War (1861––1865).

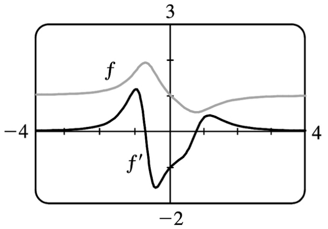
73. **(a)** Derive gives $g'(t)=\dfrac{45(t-2)^8}{(2t+1)^{10}}$ without simplifying. With either Maple or Mathematica, we first get $g'(t)=9\dfrac{(t-2)^8}{(2t+1)^9}-18\dfrac{(t-2)^9}{(2t+1)^{10}}$, and the simplification command results in the above expression.

(b) Derive gives $y'=2(x^3-x+1)^3(2x+1)^4(17x^3+6x^2-9x+3)$ without simplifying.
With either Maple or Mathematica, we first get $y'=10(2x+1)^4(x^3-x+1)^4+4(2x+1)^5(x^3-x+1)^3(3x^2-1)$. If we use Mathematica's **Factor** or **Simplify** , or Maple's **factor** , we get the above expression, but Maple's **simplify** gives the polynomial expansion instead. For locating horizontal tangents, the factored form is the most helpful.

74. **(a)** $f(x)=\left(\dfrac{x^4-x+1}{x^4+x+1}\right)^{1/2}$. Derive gives $f'(x)=\dfrac{(3x^4-1)\sqrt{\dfrac{x^4-x+1}{x^4+x+1}}}{(x^4+x+1)(x^4-x+1)}$ whereas either Maple or Mathematica give $f'(x)=\dfrac{3x^4-1}{\sqrt{\dfrac{x^4-x+1}{x^4+x+1}}\,(x^4+x+1)^2}$ after simplification.

(b) $f'(x)=0\Leftrightarrow 3x^4-1=0\Leftrightarrow x=\pm\sqrt[4]{\dfrac{1}{3}}\approx\pm 0.7598$.

(c) $f'(x)=0$ where f has horizontal tangents. f' has two maxima and one minimum where f has inflection points.



75. **(a)** If f is even, then $f(x)=f(-x)$. Using the Chain Rule to differentiate this equation, we get $f'(x)=f'(-x)\dfrac{d}{dx}(-x)=-f'(-x)$. Thus, $f'(-x)=-f'(x)$, so f' is odd.

(b) If f is odd, then $f(x)=-f(-x)$. Differentiating this equation, we get $f'(x)=-f'(-x)(-1)=f'(-x)$, so f' is even.

76.

$$\left[\frac{f(x)}{g(x)}\right]' = \left\{f(x)[g(x)]^{-1}\right\}' = f'(x)[g(x)]^{-1} + (-1)[g(x)]^{-2}g'(x)f(x)$$

$$= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{[g(x)]^2} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

77. **(a)**

$$\begin{aligned}\frac{d}{dx}\left(\sin^n x\cos nx\right) &= n\sin^{n-1}x\cos x\cos nx + \sin^n x(-n\sin nx) && \text{[Product Rule]}\\ &= n\sin^{n-1}x(\cos nx\cos x - \sin nx\sin x) && \text{[factor out } n\sin^{n-1}x\text{]}\\ &= n\sin^{n-1}x\cos(nx+x) && \text{[Addition Formula for cosine]}\\ &= n\sin^{n-1}x\cos[(n+1)x] && \text{[factor out } x\text{]}\end{aligned}$$

(b)

$$\begin{aligned}\frac{d}{dx}\left(\cos^n x\cos nx\right) &= n\cos^{n-1}x(-\sin x)\cos nx + \cos^n x(-n\sin nx) && \text{[Product Rule]}\\ &= -n\cos^{n-1}x(\cos nx\sin x + \sin nx\cos x) && \text{[factor out } n\cos^{n-1}x\text{]}\\ &= -n\cos^{n-1}x\sin(nx+x) && \text{[Addition Formula for sine]}\\ &= -n\cos^{n-1}x\sin[(n+1)x] && \text{[factor out } x\text{]}\end{aligned}$$

78. ‘‘The rate of change of y^5 with respect to x is eighty times the rate of change of y with respect to x ’’ $\Leftrightarrow \frac{d}{dx}y^5 = 80\frac{dy}{dx} \Leftrightarrow 5y^4\frac{dy}{dx} = 80\frac{dy}{dx} \Leftrightarrow 5y^4 = 80$ (Note that $\frac{dy}{dx} \neq 0$ since the curve never has a horizontal tangent) $\Leftrightarrow y^4 = 16 \Leftrightarrow y = 2$ (since $y > 0$ for all x)

79. Since $\theta^\circ = \left(\frac{\pi}{180}\right)\theta$ rad, we have

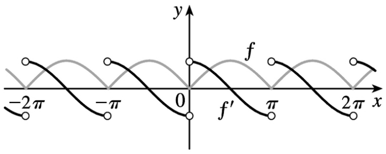
$$\frac{d}{d\theta}\left(\sin\theta^\circ\right) = \frac{d}{d\theta}\left(\sin\frac{\pi}{180}\theta\right) = \frac{\pi}{180}\cos\frac{\pi}{180}\theta = \frac{\pi}{180}\cos\theta^\circ .$$

80. **(a)** $f(x) = |x| = \sqrt{x^2} = (x^2)^{1/2} \Rightarrow f'(x) = \frac{1}{2}(x^2)^{-1/2}(2x) = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$ for $x \neq 0$.

f is not differentiable at $x = 0$.

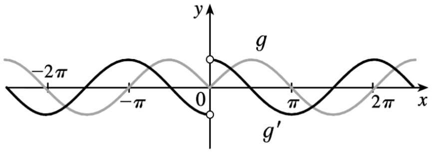
(b) $f(x)=|\sin x|=\sqrt{\sin^2 x} \Rightarrow$

$$f'(x)=\frac{1}{2}(\sin^2 x)^{-1/2}\cdot 2\sin x\cdot \cos x=\frac{\sin x}{|\sin x|}\cos x=\begin{cases} \cos x & \text{if } \sin x>0 \\ -\cos x & \text{if } \sin x<0 \end{cases}$$



f is not differentiable when $x=n\pi$, n an integer.

(c) $g(x)=\sin|x|=\sin\sqrt{x^2} \Rightarrow g'(x)=\cos|x|\cdot\frac{x}{|x|}=\frac{x}{|x|}\cos x=\begin{cases} \cos x & \text{if } x>0 \\ -\cos x & \text{if } x<0 \end{cases}$



g is not differentiable at 0 .

81. First note that products and differences of polynomials are polynomials and that the derivative of a polynomial is also a polynomial. When $n=1$,

$$f^{(1)}(x)=\left(\frac{P(x)}{Q(x)}\right)'=\frac{Q(x)P'(x)-P(x)Q'(x)}{[Q(x)]^2}=\frac{A_1(x)}{[Q(x)]^{1+1}},\ \text{where } A_1(x)=Q(x)P'(x)-P(x)Q'(x).$$

Suppose the result is true for $n=k$, where $k\geq 1$. Then $f^{(k)}(x)=\dfrac{A_k(x)}{[Q(x)]^{k+1}}$, so

$$f^{(k+1)}(x)=\left(\frac{A_k(x)}{[Q(x)]^{k+1}}\right)'=\frac{[Q(x)]^{k+1}A_k'(x)-A_k(x)\cdot(k+1)[Q(x)]^k\cdot Q'(x)}{\{[Q(x)]^{k+1}\}^2}$$

$$=\frac{[Q(x)]^{k+1}A_k'(x)-(k+1)A_k(x)[Q(x)]^kQ'(x)}{[Q(x)]^{2k+2}}$$

$$=\frac{[Q(x)]^{k\,1}\{A_k'(x)-(k+1)A_k(x)Q'(x)\}}{[Q(x)]^k[Q(x)]^{k+2}}=\frac{Q(x)A_k'(x)-(k+1)A_k(x)Q'(x)}{[Q(x)]^{k+2}}$$

$$=A_{k+1}(x)/[Q(x)]^{k+2},\ \text{where } A_{k+1}(x)=Q(x)A_k'(x)-(k+1)A_k(x)Q'(x)\ .$$

We have shown that the formula holds for $n=1$, and that when it holds for $n=k$ it also holds for $n=k+1$. Thus, by mathematical induction, the formula holds for all positive integers n.