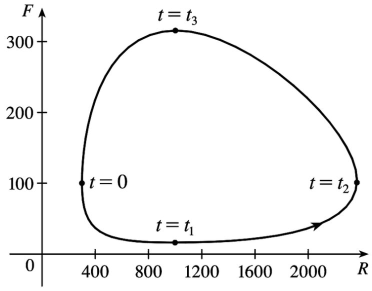
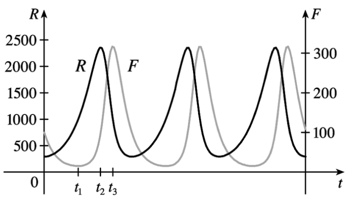
1. **(a)** $dx/dt=-0.05x+0.0001xy$ . If $y=0$ , we have $dx/dt=-0.05x$ , which indicates that in the absence of $y$ , $x$ declines at a rate proportional to itself. So $x$ represents the predator population and $y$ represents the prey population. The growth of the prey population, $0.1y$ (from $dy/dt=0.1y-0.005xy$ ), is restricted only by encounters with predators (the term $-0.005xy$ ). The predator population increases only through the term $0.0001xy$ ; that is, by encounters with the prey and not through additional food sources.
**(b)** $dy/dt=-0.015y+0.00008xy$ . If $x=0$ , we have $dy/dt=-0.015y$ , which indicates that in the absence of $x$ , $y$ would decline at a rate proportional to itself. So $y$ represents the predator population and $x$ represents the prey population. The growth of the prey population, $0.2x$ (from $dx/dt=0.2x-0.0002x^2-0.006xy=0.2x(1-0.001x)-0.006xy$ ), is restricted by a carrying capacity of 1000 and by encounters with predators (the term $-0.006xy$ ). The predator population increases only through the term $0.00008xy$ ; that is, by encounters with the prey and not through additional food sources.

2. **(a)** $dx/dt=0.12x-0.0006x^2+0.00001xy$ . $dy/dt=0.08y+0.00004xy$ .
The $xy$ terms represent encounters between the two species $x$ and $y$ . An increase in $y$ makes $dx/dt$ (the growth rate of $x$ ) larger due to the positive term $0.00001xy$ . An increase in $x$ makes $dy/dt$ (the growth rate of $y$ ) larger due to the positive term $0.00004xy$ . Hence, the system describes a cooperation model.
**(b)** $dx/dt=0.15x-0.0002x^2-0.0006xy=0.15x\left(1-x/750\right)-0.0006xy$ .
$dy/dt=0.2y-0.00008y^2-0.0002xy=0.2y\left(1-y/2500\right)-0.0002xy$ .
The system shows that $x$ and $y$ have carrying capacities of 750 and 2500 . An increase in $x$ reduces the growth rate of $y$ due to the negative term $-0.0002xy$ . An increase in $y$ reduces the growth rate of $x$ due to the negative term $-0.0006xy$ . Hence, the system describes a competition model.

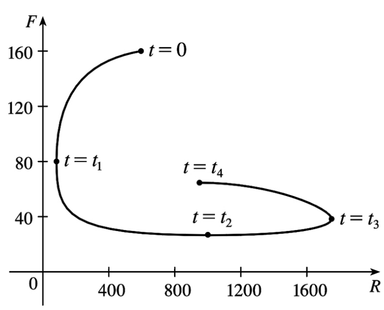
3. **(a)** At $t=0$ , there are about 300 rabbits and 100 foxes. At $t=t_1$ , the number of foxes reaches a minimum of about 20 while the number of rabbits is about 1000 . At $t=t_2$ , the number of rabbits reaches a maximum of about 2400 , while the number of foxes rebounds to 100 . At $t=t_3$ , the number of rabbits decreases to about 1000 and the number of foxes reaches a maximum of about 315 . As $t$ increases, the number of foxes decreases greatly to 100 , and the number of rabbits decreases to 300 (the initial populations), and the cycle starts again.



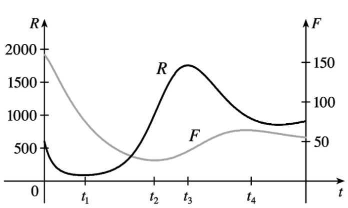

**(b)**



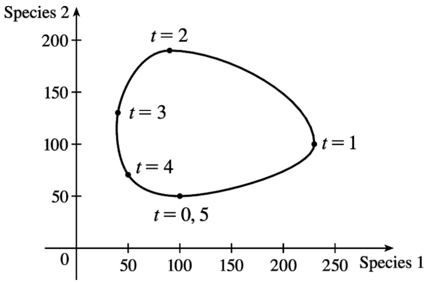

4. **(a)** At $t=0$ , there are about 600 rabbits and 160 foxes. At $t=t_1$ , the number of rabbits reaches a minimum of about 80 and the number of foxes is also 80 . At $t=t_2$ , the number of foxes reaches a minimum of about 25 while the number of rabbits rebounds to 1000 . At $t=t_3$ , the number of foxes has increased to 40 and the rabbit population has reached a maximum of about 1750 . The curve ends at $t=t_4$ , where the number of foxes has increased to 65 and the number of rabbits has decreased to about 950 .



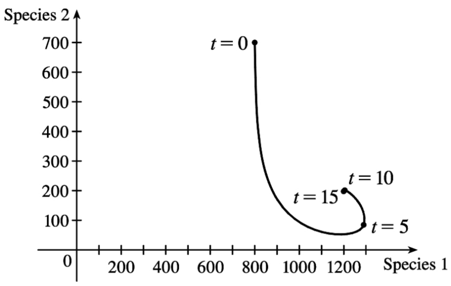

**(b)**




5.




6.




7. $\frac{dW}{dR}=\frac{-0.02W+0.00002RW}{0.08R-0.001RW}\Leftrightarrow(0.08-0.001W)R\,dW=(-0.02+0.00002R)W\,dR\Leftrightarrow$

$\frac{0.08-0.001W}{W}\,dW=\frac{-0.02+0.00002R}{R}\,dR\Leftrightarrow\int\left(\frac{0.08}{W}-0.001\right)dW=\int\left(-\frac{0.02}{R}+0.00002\right)dR\Leftrightarrow$

$0.08\ln|W|-0.001W=-0.02\ln|R|+0.00002R+K\Leftrightarrow0.08\ln W+0.02\ln R=0.001W+0.00002R+K\Leftrightarrow$

$\ln\left(W^{0.08}R^{0.02}\right)=0.00002R+0.001W+K\Leftrightarrow W^{0.08}R^{0.02}=e^{0.00002R+0.001W+K}\Leftrightarrow$

$R^{0.02}W^{0.08}=Ce^{0.00002R}e^{0.001W}\Leftrightarrow\frac{R^{0.02}W^{0.08}}{e^{0.00002R}e^{0.001W}}=C$ .

In general, if $\frac{dy}{dx}=\frac{-ry+bxy}{kx-axy}$ , then $C=\frac{x^{r}y^{k}}{e^{bx}e^{ay}}$ .

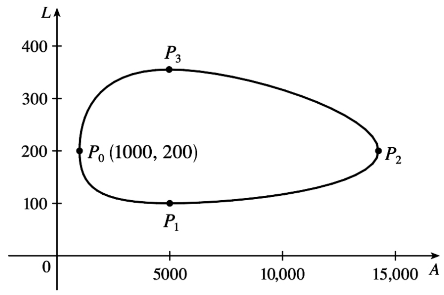
8. **(a)** $A$ and $L$ are constant $\Rightarrow A'=0$ and $L'=0\Rightarrow\left\{\begin{array}{l}0=2A-0.01AL\\0=-0.5L+0.0001AL\end{array}\right\}\Rightarrow$

$\left\{\begin{array}{l}0=A(2-0.01L)\\0=L(-0.5+0.0001A)\end{array}\right.$

So either $A=L=0$ or $L=\frac{2}{0.01}=200$ and $A=\frac{0.5}{0.0001}=5000$ . The trivial solution $A=L=0$ just says that if there aren’t any aphids or ladybugs, then the populations will not change. The non–trivial solution, $L=200$ and $A=5000$ , indicates the population sizes needed so that there are no changes in either the number of aphids or the number of ladybugs.

**(b)** $\frac{dL}{dA}=\frac{dL/dt}{dA/dt}=\frac{-0.5L+0.0001AL}{2A-0.01AL}$

**(c)** C100708cC100708c.texC100708c.tex

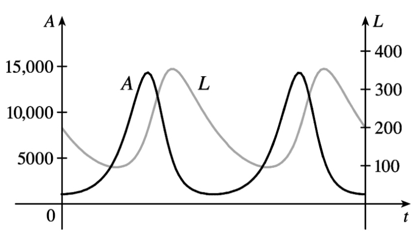



**(d)**

At $P_0(1000,200)$ , $dA/dt=0$ and $dL/dt=-80<0$ , so the number of ladybugs is decreasing and hence, we are proceeding in a counterclockwise direction. At $P_0$ , there aren't enough aphids to support the ladybug population, so the number of ladybugs decreases and the number of aphids begins to increase. The ladybug population reaches a minimum at $P_1(5000,100)$ while the aphid population increases in a dramatic way, reaching its maximum at $P_2(14{,}250,200)$ .

Meanwhile, the ladybug population is increasing from $P_1$ to $P_3(5000,355)$ , and as we pass through $P_2$ , the increasing number of ladybugs starts to deplete the aphid population. At $P_3$ the ladybugs reach a maximum population, and start to decrease due to the reduced aphid population. Both populations then decrease until $P_0$ , where the cycle starts over again.

**(e)** Both graphs have the same period and the graph of $L$ peaks about a quarter of a cycle after the graph of $A$ .



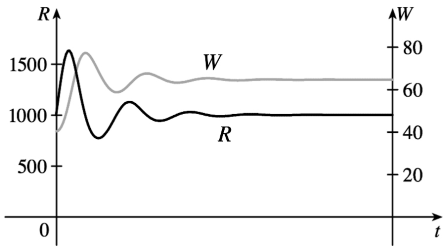

9. **(a)** Letting $W=0$ gives us $dR/dt=0.08R(1-0.0002R)$ . $dR/dt=0 \Leftrightarrow R=0$ or $5000$ . Since $dR/dt>0$ for $0<R<5000$ , we would expect the rabbit population to *increase* to $5000$ for these values of $R$ . Since $dR/dt<0$ for $R>5000$ , we would expect the rabbit population to *decrease* to $5000$ for these values of $R$ . Hence, in the absence of wolves, we would expect the rabbit population to stabilize at $5000$ .

**(b)** $R$ and $W$ are constant $\Rightarrow R'=0$ and $W'=0 \Rightarrow$

$$\left\{\begin{array}{l} 0=0.08R(1-0.0002R)-0.001RW \\ 0=-0.02W+0.00002RW \end{array}\right\} \Rightarrow \left\{\begin{array}{l} 0=R\left[0.08(1-0.0002R)-0.001W\right] \\ 0=W(-0.02+0.00002R) \end{array}\right.$$

The second equation is true if $W=0$ or

$R=\dfrac{0.02}{0.00002}=1000$ . If $W=0$ in the first equation, then either $R=0$ or $R=\dfrac{1}{0.0002}=5000$ . If $R=1000$ , then $0=1000\left[0.08(1-0.0002\cdot 1000)-0.001W\right]\Leftrightarrow 0=80(1-0.2)-W\Leftrightarrow W=64$ .
*Case (i):* $W=0$ , $R=0$ : both populations are zero
*Case (ii):* $W=0$ , $R=5000$ : see part (a)
*Case (iii):* $R=1000$ , $W=64$ : the predator/prey interaction balances and the populations are stable.
**(c)** The populations of wolves and rabbits fluctuate around 64 and 1000 , respectively, and eventually stabilize at those values.

**(d)** 

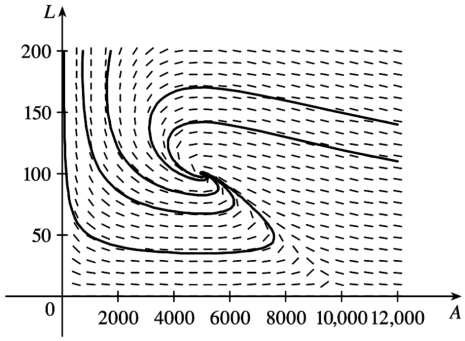

10. **(a)** If $L=0$ , $dA/dt=2A\left(1-0.0001A\right)$ , so $dA/dt=0\Leftrightarrow A=0$ or $A=\dfrac{1}{0.0001}=10,000$ . Since $dA/dt>0$ for $0<A<10,000$ , we expect the aphid population to *increase* to 10 , 000 for these values of $A$ . Since $dA/dt<0$ for $A>10,000$ , we expect the aphid population to *decrease* to 10 , 000 for these values of $A$ . Hence, in the absence of ladybugs we expect the aphid population to stabilize at 10 , 000 .

**(b)** $A$ and $L$ are constant $\Rightarrow A'=0$ and $L'=0\Rightarrow$

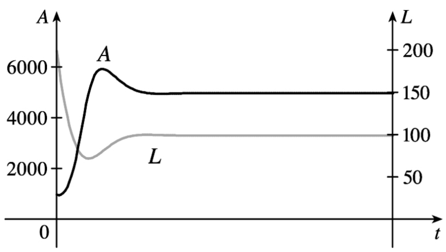
$$\left\{\begin{array}{l} 0=2A(1-0.0001A)-0.01AL \\ 0=-0.5L+0.0001AL \end{array}\right\} \Rightarrow \left\{\begin{array}{l} 0=A\left[2(1-0.0001A)-0.01L\right] \\ 0=L(-0.5+0.0001A) \end{array}\right.$$

The second equation is true if $L=0$ or $A=\dfrac{0.5}{0.0001}=5000$ . If $L=0$ in the first equation, then either $A=0$ or $A=\dfrac{1}{0.0001}=10,000$ . If $A=5000$ , then $0=5000\left[2(1-0.0001\cdot 5000)-0.01L\right]\Leftrightarrow 0=10,000(1-0.5)-50L\Leftrightarrow 50L=5000\Leftrightarrow L=100$ . The equilibrium solutions are: (i) $L=0, A=0$ (ii) $L=0, A=10,000$ (iii) $A=5000, L=100$

**(c)** $\dfrac{dL}{dA}=\dfrac{dL/dt}{dA/dt}=\dfrac{-0.5L+0.0001AL}{2A(1-0.0001A)-0.01AL}$

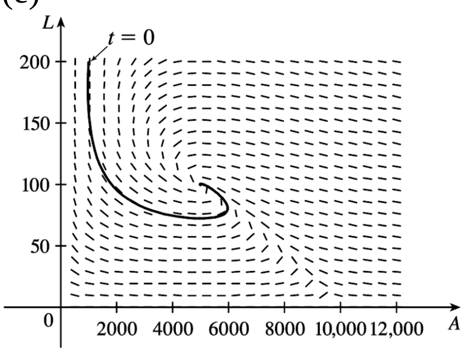



(d) All of the phase trajectories spiral tightly around the equilibrium solution $(5000,100)$ .




The graph of $A$ peaks just after the graph of $L$ has a minimum.

(e)




At $t=0$ , the ladybug population decreases rapidly and the aphid population decreases slightly before beginning to increase. As the aphid population continues to increase, the ladybug population reaches a minimum at about $(5000,75)$ . The ladybug population starts to increase and quickly stabilizes at $100$ , while the aphid population stabilizes at $5000$ .