AE1101 Formula Overview

Aeronautics

Lift for a Balloon.
Q=volume

$$L_B = \omega_{at} Q = \rho_{at} g Q$$

$$L_N = L_B - W_{gas} = gQ(\rho_{at} - \rho_{gas})$$

Hot air balloon

$$L_N = \rho_{at} gQ\left(1 - \frac{\rho_{gas}}{\rho_{at}}\right) = \rho_{at} gQ\left(1 - \frac{T_{at}}{T_{gas}}\right) = \rho_{at} gQ \frac{\Delta T}{T_{at} + \Delta T}$$

$$L_N = \rho_{at} gQ\left(1 - \frac{\rho_{gas}}{\rho_{at}}\right) = L_N = \rho_{at} gQ\left(1 - \frac{\hat{M}_{gas}}{\hat{M}_{at}}\right)$$

$\hat{M}$ =molair mass

Ideal gas law

$p \cdot V = n \cdot R \cdot T \qquad R = 8.3145 j / molK$

$p = \rho \cdot R_{air} \cdot T \qquad M_{air} = 28.97 g / mol$

$p = \rho RT \qquad R_{air} = 8.3145 / 0.02897 = 287.0 J / kgK$

$$g = g0\left(\frac{r}{h_a}\right)^2 = g0\left(\frac{r}{r + h_G}\right)^2$$

$$dp = -\rho g dh_G = -\rho g0 dh \longrightarrow 1 = \frac{g0}{g}\frac{dh}{dh_G}$$

$$dh = \frac{g}{g0} dh_G$$

$$dh = \frac{r^2}{(r + h_G)^2} dh_G$$

$$\int_0^h dh = \int_0^{h_G} \frac{r^2}{(r + h_G)^2} dh_G$$

$$h = r^2\left(\frac{-1}{(r + h_G)^2}\right)_0^{h_G} = r^2\left(\frac{-1}{r + h_G} + \frac{1}{r}\right) = r^2\left(\frac{-r + r + h_G}{(r + h_G) r}\right)$$

$$h = \frac{r}{r + h_G} h_G$$

ISA

$$\frac{p}{p1} = \left(\frac{T}{T1}\right)^{-g0/aR}$$

$$\frac{\rho}{\rho 1} = \left(\frac{T}{T1}\right)^{-((g0/aR)+1)}$$

Lift

$$L = C_L \cdot \frac{1}{2}\rho V^2 \cdot S$$

Drag

$$D = C_D \cdot \frac{1}{2}\rho V^2 \cdot S$$

$$C_D = C_{D0} + C_{Di}$$

$$C_D = C_{D0} + \frac{C_L{}^2}{\pi e AR}$$

$$C_{Di} = \frac{L^2}{\pi e AR \frac{1}{4}\rho^2 V^4 S^2}$$

$$D_i = C_{Di} \cdot \frac{1}{2}\rho V^2 \cdot S = \frac{L^2}{\pi e AR \frac{1}{2}\rho V^2 S}$$

Wing aspect ratio

$$AR = \frac{b^2}{S}$$

Pressure cabin

$$\sigma_{circ} = \frac{(p2 - p1) \cdot r}{t}$$

$$\sigma_{long} = \frac{(p2 - p1) \cdot r}{2t}$$

$$ratio \frac{\sigma_{circ}}{\sigma_{long}} = 2$$

Momentum equation

$$T = \dot{m} \cdot (V_j - V_0)$$

Jet efficiency

$$P_j = \frac{1}{2}\dot{m}\cdot\left(V_j{}^2 - V_0{}^2\right)$$

$$P_a = TV_0$$

$$\eta_j = \frac{P_a}{P_j} = \frac{TV_0}{\frac{1}{2}\dot{m}\cdot\left(V_j{}^2 - V_0{}^2\right)} = \frac{2}{1+\frac{V_j}{V_0}}$$

Mach number

$$\sin\mu = 1/M$$

Equivalent airspeed

$$\frac{1}{2}\rho 0 V_{EAS}{}^2 = \frac{1}{2}\rho V_{TAS}{}^2$$

$$C_L \frac{1}{2}\rho 0 V_{EAS}{}^2 S = C_L \frac{1}{2}\rho V_{TAS}{}^2 S$$

Astronautics

Earth gravity

$$F_g = \frac{GM_{S/C}M_E}{\left(R_E + h_{orbit}\right)^2} = M_{S/C}\cdot g$$

$$g = \frac{GM_E}{\left(R_E + h_{orbit}\right)^2} = \frac{\mu}{\left(R_E + h_{orbit}\right)^2} = g0\frac{R_E^2}{\left(R_E + h_{orbit}\right)^2}$$

$$g0 = \frac{GM_E}{R_E^2} = \frac{\mu}{R_E^2}$$

$$F_g = M_{S/C}\cdot g = g0\frac{R_E^2}{\left(R_E + h_{orbit}\right)^2}$$

$$F_C = M_{S/C}\cdot\frac{V_{orbit}^2}{R_E + h_{orbit}}$$

$$F_g = F_C \Rightarrow V_{orbit} = V\left(h_{orbit}\right) = R_E\sqrt{\frac{g0}{R_E + h_{orbit}}}$$

$$T = \frac{2\pi\left(R_E + h_{orbit}\right)}{V_{orbit}}$$

$$F = \frac{Torque}{L}$$

$$a = \frac{F}{M_{Payload}}$$

Orbits

$$r_{sat} = \frac{a(1-e^2)}{1+e\cos(\theta)}$$

Ground systems

$$D = \frac{k \cdot d}{f}\sqrt{\frac{b}{p}}$$

$f = frequency$

$b = bitrate$

$D = diameter \cdot antenna$

$d = distsat$

$$E = \frac{P}{4\pi r^2} A$$

$E = received$

$P = transmitted$

$r = disttransmitter \Leftrightarrow receiver$

$A = areaantenna$

Atmosphere

$$a = C_D \frac{1}{2}\rho V^2 S / M_{sat}$$

Reduction of semi-major axis: $\Delta a_{2\pi} = -2\pi \frac{C_D S}{M_{sat}} a^2 \rho_p \exp(-c)[I_0 + 2eI_1]$

Reduction of eccentricity: $\Delta e_{2\pi} = -2\pi \frac{C_D S}{M_{sat}} a\rho_p \exp(-c)\left[I_1 + \frac{e}{2}(I_0 + I_1)\right]$

Limit lifetime**:** $L = \frac{H}{\Delta a_{2\pi}}$

Radiation

Total energy radiated by black body: $E_{tot} = \sigma T^4$

$$\sin \lambda = \frac{R_E}{a}$$

$$\lambda = \sin^{-1}\left(\frac{R_E}{a}\right)$$

Eclipse fraction (%): $T_{eclipse} = \frac{2\lambda}{360} \times 100$

Eclipse length(s): $T_{eclipse} = \frac{2\lambda}{360} \times T_{orbit} = \frac{2\lambda}{360} \times 2\pi\sqrt{\frac{a^3}{\mu}}$

$a = semi - major - axis$

Solar panel size/solar heat capacity

$$A_a = \frac{P_{req}}{P_\delta}$$

$$P_\delta = \eta \cdot S$$

$$\Delta T = \frac{SA}{CM}\Delta t$$

η=solar panel efficiency
S=solar intensity
C=heat capacity
A=area exposed
M=mass

Aerodynamics

$$KE = \frac{3}{2}kT$$

$$k = 1.38 \times 10^{-23} J / K$$

Equation of state for actual gas

$$\frac{p}{\rho RT} 1 + \frac{ap}{T} - \frac{bp}{T^3}$$

Specific volume

$$v = \frac{1}{\rho}$$

$$pv = RT$$

Continuity

$$\dot{M}_{in} = \dot{M}_{out}$$

$$\dot{M} = \rho AV$$

$$A_1 V_1 = A_2 V_2$$

Euler equation

$$F = m \cdot a$$

$$F = pdydz - \left(p + \frac{dp}{dx} dx \right) dydz$$

$$F = -\frac{dp}{dx} dxdydz$$

$$m = \rho V = \rho dxdydz$$

$$a = \frac{dV}{dt} = \frac{dVdx}{dxdt} = \frac{dV}{dx} V$$

$$F = m \cdot a$$

$$-\frac{dp}{dx} dxdydz = \rho (dxdydz) V \frac{dV}{dx} \Rightarrow$$

$$dp = -\rho V dV$$

Bernoulli's law

$$\int_{p1}^{p2} dp + \int_{V1}^{V2} \rho V dV = 0 \rightarrow$$

$$(p2 - p1) + \rho \left(\frac{1}{2} V_2^2 - \frac{1}{2} V_1^2 \right) = 0$$

$$p1 + \frac{1}{2} \rho V_1^2 = p2 + \frac{1}{2} \rho V_2^2$$

First Law thermodynamics

$$de = \delta q + \delta w$$

$$\delta w = \int_A pdAs = p \int_A sdA = -pdv$$

$$\delta q = de - \delta w = de + pdv$$

$$enthalpy = h = e + pv$$

$$dh = de + pdv + vdp$$

$$\delta q = de + pdv = dh - pdv - vdp + pdv$$

$$\delta q = dh - vdp$$

Specific heat

For constant volume dv=0: $de = c_v dT$, $e = c_v T$

For constant pressure dp=0: $dh = c_p dT$, $h = c_p T$

Isotropic flow relations

Since $\delta q = 0 \Rightarrow \delta q = de + pdv = 0 \Rightarrow -pdv = de = c_v dT$

Since $\delta q = 0 \Rightarrow \delta q = dh - vdp = 0 \Rightarrow vdp = dh = c_p dT$

$$\frac{-pdv}{vdp}=\frac{c_v}{c_p}\Rightarrow\frac{dp}{p}=-\left(\frac{c_v}{c_p}\right)\frac{dv}{v}\Rightarrow\frac{dp}{p}=-\gamma\frac{dv}{v}$$

$$\frac{c_v}{c_p}=\gamma=1.4\,for-air$$

$$\int_1^2\frac{dp}{p}=-\gamma\int_1^2\frac{dv}{v}\Rightarrow[\ln p]_1^2=-\gamma[\ln v]_1^2\Rightarrow$$

$$\ln p2-\ln p1=-\gamma[\ln v2-\ln v1]$$

$$\ln\left(\frac{p2}{p1}\right)=-\gamma\ln\left(\frac{v2}{v1}\right)\Rightarrow\left(\frac{p2}{p1}\right)=\left(\frac{v2}{v1}\right)^{-\gamma}$$

$$v1=\frac{1}{\rho1}$$ so $$v2=\frac{1}{\rho2}$$

$$\left(\frac{p2}{p1}\right)=\left(\frac{\rho1}{\rho2}\right)^{-\gamma}\Rightarrow\frac{p2}{p1}=\left(\frac{\rho2}{\rho1}\right)^{\gamma}$$

with p/ρ=RT we write

$$\frac{p2}{p1}=\left(\frac{p2}{RT2}\frac{RT1}{p1}\right)^{\gamma}=\left(\frac{p2}{p1}\frac{T1}{T2}\right)^{\gamma}\Rightarrow\left(\frac{p2}{p1}\right)^{1-\gamma}=\left(\frac{T1}{T2}\right)^{\gamma}$$

$$\left(\frac{p2}{p1}\right)^{1-\gamma}=\left(\frac{T2}{T1}\right)^{-\gamma}\Rightarrow\frac{p2}{p1}=\left(\frac{T2}{T1}\right)^{\frac{\gamma}{\gamma-1}}$$

Energy equation

$$\delta q=dh-vdp$$

for adiabatic δq =0

$$dh-vdp=0$$

$$dp=-\rho VdV$$

Combining gives:

$$0=dh+v\rho VdV$$

$$dh+VdV=0$$

Now integrating

$$\int_1^2 dh+\int_1^2 VdV=0\Rightarrow h2-h1+\frac{1}{2}\left(V_2^2-V_1^2\right)=0\Rightarrow$$

h=CpT

$$h2+\frac{1}{2}V_2^2=h1+\frac{1}{2}V_1^2$$

$$C_pT+\frac{1}{2}V^2=\text{constant}$$

Isentropic flow relations second form

$$C_p T + \frac{1}{2}V^2 = \text{constant}$$

$$C_p T1 + \frac{1}{2}V1^2 = C_p T0 + \frac{1}{2}V0^2 \Rightarrow V0 = 0$$

$$C_p T1 + \frac{1}{2}V1^2 = C_p T0 \Rightarrow \frac{To}{T1} = 1 + \frac{V1^2}{2C_p T1} \Rightarrow$$

$$\frac{To}{T1} = 1 + \frac{\gamma - 1}{2}\frac{V1^2}{\gamma RT1}$$

$$a_1^2 = \gamma RT1$$

$$\frac{To}{T1} = 1 + \frac{\gamma - 1}{2}\frac{V1^2}{a_1^2} = 1 + \frac{\gamma - 1}{2}M_1^2$$

now with isentropic relations

$$\frac{p0}{p1} = \left(\frac{\rho 0}{\rho 1}\right)^{\gamma} = \left(\frac{To}{T1}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{p0}{p1} = \left(1 + \frac{\gamma - 1}{2}M_1^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho 0}{\rho 1} = \left(1 + \frac{\gamma - 1}{2}M_1^2\right)\frac{1}{\gamma - 1}$$

Reynolds number

$$\text{Re}_x = \frac{\rho_\infty V_\infty x}{\mu_\infty}$$

Boundary layer thickness

$$\delta = \frac{5.2x}{\sqrt{\text{Re}_x}}$$

Total skin drag coefficient

$$C_f = \frac{1.328}{\sqrt{\text{Re}_x}}$$