Aerodynamics C - Exam June 2008 Problems and Solutions

1 A Supersonic Airfoil

1. Consider a thin airfoil, immersed in a uniform supersonic stream at $M_{\infty} = 3.0$ at an angle of attack of $\alpha = 2.5$ deg. The airfoil has a thickness of 2h and has a parabolic upper profile given by:

$$\frac{y}{h} = 4\frac{x}{c}\left(1 - \frac{x}{c}\right). \tag{1.1}$$

The bottom profile is made out of two straight parts. The half-thickness to cord ratio is h/c = 0.087.

- (a) Draw on two diagrams the pressure coefficient distribution along the airfoil on the upper and lower side.
- (b) Compute the lift and drag coefficient of the airfoil using linearized theory.
- (c) Evaluate the pressure coefficient on the lower side by shock-expansion theory and compare the results with those obtained by linearized theory.
- 2. Demonstrate that the total temperature remains constant across a stationary normal shock wave. Also demonstrate that the total pressure decreases across the normal shock wave.

1 Solution

1. Let's define θ_u as the angle of incidence at the upper side. We also have $\theta_{l,1}$ and $\theta_{l,2}$ for the two parts of the lower side.

We know that

$$\theta_u = \frac{dy}{dx} - \alpha = 4\frac{h}{c} - 8\frac{hx}{c^2} - \alpha, \qquad (1.2)$$

$$\theta_{l,1} = \alpha - \frac{dy}{dx} = \alpha + 2\frac{h}{c},\tag{1.3}$$

$$\theta_{l,2} = \alpha - \frac{dy}{dx} = \alpha - 2\frac{h}{c}.$$
(1.4)

If we use linearized theory, we will have to use

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}.\tag{1.5}$$

Now it's time to look at the first question.

(a) For the upper side we know that

$$C_{p,u} = 2 \frac{4\frac{h}{c} - 8\frac{hx}{c^2} - \alpha}{\sqrt{M_{\infty}^2 - 1}}.$$
(1.6)

We see that this is a linearly decreasing function. It crosses the x-axis if

$$\frac{x}{c} = \frac{4(h/c) - \alpha}{8(h/c)} = 0.437.$$
(1.7)

I'm sure you can plot it now.

But what about the lower side? Well, this time the pressure coefficient is just constant. We have $2(a + a^{2}b)$

$$C_{p,l,1} = \frac{2\left(\alpha + 2\frac{n}{c}\right)}{\sqrt{M_{\infty}^2 - 1}},$$
(1.8)

$$C_{p,l,2} = \frac{2\left(\alpha - 2\frac{h}{c}\right)}{\sqrt{M_{\infty}^2 - 1}}.$$
(1.9)

Plot the two parts and you're done.

(b) To find c_l and c_d we need to find c_n and c_a . So let's do that. First, we know that

$$c_n = \frac{1}{c} \int_0^c \left(C_{p,l} - C_{p,u} \right) \, dx. \tag{1.10}$$

This is equal to

$$\frac{2}{c\sqrt{M_{\infty}^2 - 1}} \left(\int_0^c \alpha \, dx - \int_0^c \left(4\frac{h}{c} - 8\frac{xh}{c^2} - \alpha \right) \, dx \right). \tag{1.11}$$

The result will be

$$\frac{4\alpha}{\sqrt{M_{\infty}^2 - 1}}.$$
(1.12)

Finding c_a is a bit more difficult. This time we use

$$c_{a} = \frac{1}{c} \int_{0}^{c} \left(C_{p,u} - C_{p,l} \right) \, dy = \frac{1}{c} \int_{0}^{c} \left(C_{p,u} \left(\frac{dy}{dx} \right) - C_{p,l} \left(\frac{dy}{dx} \right) \right) \, dx.$$
(1.13)

Working this out will initially give

$$c_{a} = \frac{2}{c\sqrt{M_{\infty}^{2} - 1}} \left(\int_{0}^{c} \left(4\frac{h}{c} - 8\frac{xh}{c^{2}} - \alpha \right) \left(4\frac{h}{c} - 8\frac{xh}{c^{2}} \right) \, dx \right) - \dots$$
(1.14)

$$\dots - \frac{2}{c\sqrt{M_{\infty}^2 - 1}} \left(\int_0^{\frac{1}{2}c} \left(\alpha + \frac{2h}{c} \right) \left(-\frac{2h}{c} \right) dx + \int_{\frac{1}{2}c}^c \left(\alpha - \frac{2h}{c} \right) \left(\frac{2h}{c} \right) dx \right).$$
(1.15)

Working things out even further will get you

$$c_{a} = \frac{2}{c\sqrt{M_{\infty}^{2} - 1}} \left(\int_{0}^{c} \frac{16h^{2}}{c^{2}} \, dx - \int_{0}^{c} \frac{64xh^{2}}{c^{3}} \, dx + \int_{0}^{c} \frac{64x^{2}h^{2}}{c^{4}} \, dx - \int_{0}^{c} \frac{4h}{c} \alpha \, dx + \int_{0}^{c} \frac{8xh}{c^{2}} \alpha \, dx + \int_{0}^{c} \frac{4h^{2}}{c^{2}} \, dx \right)$$
(1.16)

Finally this should give you

$$c_a = \frac{56}{3} \frac{h^2}{c^2} \frac{1}{\sqrt{M_\infty^2 - 1}}.$$
(1.17)

Now let's move on to find the lift and drag coefficients. The lift coefficient can be found using

$$c_l = c_n - \alpha c_a = \frac{4\alpha - \frac{56}{3} \frac{h^2}{c^2} \alpha}{\sqrt{M_{\infty}^2 - 1}} = 0.0595.$$
 (1.18)

You may note that the part containing $\frac{h^2}{c^2}\alpha$ is very small. Indeed, this part may be neglected. This means that we could have calculated c_l in a much easier way. We just could have used

$$c_l = c_n = \frac{4\alpha}{\sqrt{M_{\infty}^2 - 1}}.$$
 (1.19)

Calculating c_l in this way would give you $c_l = 0.0617$, which would have been close enough. The drag coefficient can be found using

$$c_d = c_a + \alpha c_n = \frac{\frac{56}{3}\frac{h^2}{c^2} + 4\alpha^2}{\sqrt{M_{\infty}^2 - 1}} = 0.0526.$$
(1.20)

(c) It's time to look at shock waves. First let's take a look at what exactly happens. At the free stream we have a flow at Mach number $M_{\infty} = 3$. This flow will be deflected by the wing. This is done by an oblique shock wave.

After a little while the flow reaches the middle of the airfoil. Now it needs to bend around the small angle in the airfoil. This is done using expansion waves.

So, first let's calculate the shock wave properties. Let's call the situation after the shock wave situation 1. First of all, the deflection angle is given by $\theta = \alpha + dy/dx = \alpha + 2h/c = 12.47$ deg. The corresponding wave angle is $\beta = 30$ deg. (It can be found using the $\theta - \beta - M$ diagram.) This implies that the Mach number normal to the shock wave is given by

$$M_{n,\infty} = M_{\infty} \sin\beta = 3.0 \sin 30 = 1.5. \tag{1.21}$$

It follows that $p_1/p_{\infty} = 2.458$ and also that $M_{n,1} = 0.7011$. This implies that $M_1 = M_{n,1}/\sin(\beta - \theta) = 2.328$.

Using p_1 and M_1 we can find the pressure coefficient $C_{p,l,1}$ at the first part of the lower surface of the airfoil. We know that the pressure coefficient is given by

$$C_p = \frac{p - p_{\infty}}{q_{\infty}} = \frac{2(p - p_{\infty})}{\gamma p_{\infty} M_{\infty}^2} = \frac{2(p_1/p_{\infty} - 1)}{\gamma M_{\infty}^2}.$$
 (1.22)

It follows that

$$C_{p,l,1} = \frac{2(2.458 - 1)}{1.4 \cdot 3.0^2} = 0.23.$$
(1.23)

The linearized theory predicted 0.154, so it's off by about 33%. Now let's look at the second part of the airfoil. To arrive at this point, the flow has to bend over an angle of $\theta = 4h/c$. We can insert this θ in

$$\theta = \nu(M_2) - \nu(M_1). \tag{1.24}$$

Strangely enough, θ has to be inserted in degrees. So we have $\theta = 4h/c = 4 \cdot 0.087 \cdot 180/\pi = 19.94 \text{ deg}$. From appendix C we find that $\nu(M_1) = 34.98$. This gives

$$\nu(M_2) = \theta + \nu(M_1) = 19.94 + 34.98 = 54.92.$$
(1.25)

The corresponding Mach number M_2 is given by $M_2 = 3.283$.

We know that expansion waves are isentropic. So to find p_2 we can simply use isentropic flow relations, stating that

$$\frac{p_2}{p_1} = \left(\frac{2 + (\gamma - 1)M_1^2}{2 + (\gamma - 1)M_2^2}\right)^{\frac{\gamma}{\gamma - 1}}.$$
(1.26)

It follows that $p_2/p_1 = 0.234$. This, in turn, implies that

$$\frac{p_2}{p_{\infty}} = \frac{p_2}{p_1} \frac{p_1}{p_{\infty}} = 0.234 \cdot 2.458 = 0.575.$$
(1.27)

This results in

$$C_{p,l,2} = \frac{2(0.575 - 1)}{1.4 \cdot 3.0^2} = -0.0674.$$
(1.28)

During the previous question, we found a $C_{p,l,2}$ of -0.092. So the deviation is only about 27%.

2. First we'll show that the total temperature stays constant along a shock wave. Using conservation of energy, it can be shown that the quantity $u + h^2/2$ stays constant along a shock wave. This implies that the total enthalpy h_t stays constant. So also $c_p T_t$ stays constant. And thus also T_t stays constant.

Now let's show that the total pressure decreases. We know that

$$\frac{p_{t,2}}{p_{t,1}} = e^{\frac{s_1 - s_2}{R}},\tag{1.29}$$

with $p_{t,1}$ and s_1 the total pressure and the entropy before the shock wave. (Situation 2 indicates the properties after the shock wave.) The entropy always increases, so $s_1 - s_2$ is negative. Thus $e^{\frac{s_1-s_2}{R}} < 1$, meaning that $p_{t,1} > p_{t,2}$. This implies that the total pressure has decreased across the shock wave.

2 Designing a Supersonic Wind Tunnel

- 1. Consider the flow of air through a convergent-divergent nozzle with a throat cross-section $A_t = 0.4m^2$ and an exit-to-throat area ratio of 1.616. Assuming that the reservoir conditions are $p_0 = 5.5 \cdot 10^5 Pa$ and $T_0 = 550^{\circ}C$, determine the exit Mach number M_e and the mass flow \dot{m} through the nozzle when the pressure outside the reservoir p_a attains the following values:
 - (a) $p_a = 5.0 \cdot 10^5 Pa$.
 - (b) $p_a = 2.2 \cdot 10^5 Pa$.
 - (c) $p_a = 0.4 \cdot 10^5 Pa$.
 - (d) Draw the pattern of shocks and expansion waves at the nozzle exit for the last case.
- 2. In low-speed incompressible flow, the peak pressure coefficient (at the minimum pressure point) is $C_p = -0.48$. Estimate the critical Mach number for the given airfoil, using the Prandtl-Glauet rule.

2 Solution

1. For such questions, it's usually convenient to calculate the boundaries first. You may wonder, "what boundaries?" Well, there are specific exit pressures at which special things occur. These pressures are $p_{e,3}$, $p_{e,5}$ and $p_{e,6}$.

If $p_e > p_{e,3}$, then the flow is entirely subsonic. Since there are no shock waves, normal isentropic relations apply.

If $p_{e,3} > p_e > p_{e,5}$, then there must be a normal shock wave between the throat and the exit. Such situations are very hard to calculate with. So you'd better pray that you don't get a question about this on your exam. (Luckily, they usually don't ask questions about it, because it's so difficult.)

If $p_{e,5} > p_e > p_{e,6}$, then the flow is overexpanded. When exiting the wind tunnel, the flow will contract. Oblique shock waves will appear.

If $p_{e,6} > p_e$, then the flow is underexpanded. When exiting the wind tunnel, the flow will expand. This is caused by the so-called expansion waves.

To calculate $p_{e,3}$ we assume that there are sonic conditions in the throat, but that the rest of the flow is subsonic. Looking up the fully subsonic line of $A/A^* = 1.616$ in appendix A, we find that $p_0/p_{e,3} = 1.112$. So $p_{e,3} = 4.946 \cdot 10^5 Pa$.

To calculate $p_{e,6}$ we assume that the entire flow is supersonic. Looking up the fully supersonic line of $A/A^* = 1.616$ in appendix A, we find that $p_0/p_{e,6} = 7.211$. So $p_{e,6} = 7.627 \cdot 10^4 Pa$. (Corresponding is the Mach number M = 1.9474.)

To find $p_{e,5}$, we assume that there is a normal shock wave at the exit. This shock wave has an initial Mach number M = 1.9474. Its initial pressure (before the shock wave) is $p_{e,6}$. Its final pressure (after the shock wave) must be $p_{e,5}$. So we can look up the pressure ratio $p_{e,5}/p_{e,6} = 4.258$. It finally follows that $p_0/p_{e,5} = (p_0/p_{e,6})(p_{e,6}/p_{e,5}) = 7.211/4.258 = 1.694$. So $p_{e,5} = 3.247 \cdot 10^5 Pa$.

(a) Finally, we can start with the first case! We know that $p_a > p_{e,3}$, so the flow remains subsonic. In this case no shock waves appear, so we can use the isentropic flow relations. We can find the Mach number at the throat M_a using

$$\frac{p_0}{p_a} = \left(\frac{2 + (\gamma - 1) M_a^2}{2 + (\gamma - 1) M_0^2}\right)^{\frac{\gamma}{\gamma - 1}},\tag{2.1}$$

where M_0 is the Mach number in the reservoir. Since the flow in the reservoir moves very slowly, we have $M_0 = 0$. Since $p_0/p_a = 1.1$ we have $M_a = 0.372$. To find the mass flow, we use the equation

$$\dot{m} = \rho A V = \frac{P}{RT} A \sqrt{\gamma RT} M = P A M \sqrt{\frac{\gamma}{RT}}.$$
(2.2)

So we need to find the temperature at the exit. We can once more use isentropic flow relations. So we use

$$\frac{T_0}{T_a} = \left(\frac{p_0}{p_a}\right)^{\frac{\gamma-1}{\gamma}}.$$
(2.3)

It follows that $T_a = 800.9K$. So the mass flow becomes

$$\dot{m} = 5.0 \cdot 10^5 \cdot 0.4 \cdot 1.616 \cdot 0.372 \cdot \sqrt{\frac{1.4}{287 \cdot 800.9}} = 296.7 kg/s.$$
 (2.4)

(b) Time to look at the second case. We see that $p_{e,5} > p_a > p_{e,6}$. So we will have a completely supersonic flow, which is overexpanded.

Since the flow is completely supersonic, the Mach number at the exit can be found quite simply. Just look next to A/A^* in appendix A. You'll find M = 1.9474.

To find the mass flow, we use the equation

$$\dot{m} = PAM\sqrt{\frac{\gamma}{RT}}.$$
(2.5)

This time, however, we'll apply it at the throat. At this point we know that M = 1 and $A = 0.40m^2$. We can also find P and T using isentropic flow relations. We find that $P = 2.9055 \cdot 10^5 Pa$ and T = 685.96K. So the mass flow becomes

$$\dot{m} = PAM\sqrt{\frac{\gamma}{RT}} = 2.9055 \cdot 10^5 \cdot 0.4 \cdot 1 \cdot \sqrt{\frac{1.4}{287 \cdot 685.96}} = 309.9 kg/s.$$
(2.6)

(c) Time for the third case. In this case we find that $p_{e,6} > p_a$. This means that the flow is underexpanded. As soon as the flow exits the wind tunnel, it will expand.

However, the flow in the wind tunnel is still entirely supersonic. This means that the exit Mach number and the mass flow are the same as in the previous case.

(d) The pattern of shock waves and expansion waves of the previous case can be seen in Anderson, figure 10.16.f.

2. Since C_p is the peak pressure coefficient, we know, according to the Prandtl-Glauert rule, that

$$C_{p,cr} = \frac{C_p}{\sqrt{1 - M_{\infty}^2}}.$$
 (2.7)

We can also derive that

$$C_{p,cr} = \frac{2}{\gamma M_{\infty}^2} \left(\left(\frac{2 + (\gamma - 1) M_{\infty}^2}{2 + (\gamma - 1)} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right).$$
(2.8)

(See page 676 of Anderson.) We can now insert a couple of values for M_{∞} . It turns out that for $M_{\infty} = 0.72$ the two equations above give rather similar values. So this must (approximately) be the critical Mach number.

3 Analyzing a Power Plant

Consider the power plant below. The working fluid is air.



Figure 1: An overview of the power plant.

The following assumptions are made.

- 1. The transformations hosted by the compressor and the turbines are adiabatic and the isentropic efficiency is 0.82 for the compressor and 0.87 for both turbines.
- 2. The power delivered by the first turbine (operating between point 4 and point 5) directly equals that used by the compressor.
- 3. The regenerator is an open system with two inlets and two outlets hosting an isobaric $(p_3 = p_2$ and $p_7 = p_6)$ heat exchange process.
- 4. The gas expands in the second turbine (operating between points 5 and 6) delivering a power output of 150kW.
- 5. The transformation in the burner takes place at constant pressure.
- 6. $p_1 = 100kPa$, $T_1 = 300K$, $p_2/p_1 = 6$, $p_7 = 100kPa$, $T_4 = 1600K$.

Answer the following questions:

1. Draw the evolution of the cycle in the T-s plane and later verify the correspondence between the graphical representation and the obtained numerical values of the state variables.

- 2. Determine the temperature of the air T_2 at the exit of the compressor.
- 3. Determine the temperature of the air T_5 at the first turbine. (Hint: apply the first principle to both the compressor and the first turbine and then make use of the assumption that the power delivered by the first turbine is equal to the power used by the compressor.)
- 4. Determine the value of the pressure p_5 after the first turbine.
- 5. Determine the temperature of the air T_6 at the exit of the power turbine.
- 6. Determine the net work per unit mass produced by the power plant.
- 7. Explain the effect of the regenerator on the thermal efficiency of the power plant.

3 Solution

1. Let's draw a T - s diagram. (You can already see the final diagram below.) First of all we should draw two lines of constant pressure (one for p = 100kPa and one for p = 600kPa).



Figure 2: The T - s graph of the process.

Let's start at point 1. Point 1 represents the incoming flow. The pressure is p = 100kPa, so the point lies on the line p = 100kPa. Also, the temperature is still rather low, so we draw the point on the bottom left.

Now let's continue with point 2. The pressure has increased to p = 600kPa, so we know we have to move to the other line. But where on the other line? If the process would be isentropic, then s would have been constant. In this case point 2 should be directly above point 1. However, the isentropic efficiency is not 1, so the process is not isentropic. Point two therefore has a slightly higher entropy than point 1.

Point 3 has the same pressure as point 2. However, the temperature has increased by a bit, due to the regenerator. So we just follow the line of p = 600kPa for a little while.

Point 4 has still the same pressure as the previous two points. However, the burner has increased the temperature by quite a bit. So we follow the line of p = 600kPa for quite a distance, until we say we've arrived at point 4.

Now let's evaluate the turbines. At point 6 we have a pressure of p = 100kPa again. So point 6 should be on the line of p = 100kPa. However, how do we go from point 4 to point 6? We don't go

straight down, because the isentropic efficiency is not 1. Instead, we go a bit to the right as well. (Because the entropy always increases.) This is how we find point 6. Point 5 is simply somewhere between the points 4 and 6.

Finally we need to find point 7. We know that $p_7 = 100kPa$, so we stay on the isobaric line (line of constant pressure). In this part of the generator the temperature decreases. (Heat is taken from the flow between points 6 and 7, and added to the flow between points 2 and 3.) So we move to the bottom left, along the line, for a little while.

You may wonder, how can it be that the entropy decreases between points 6 and 7? That is because a cold sink is present. The entropy can only decrease if heat is discarded to a cold sink. This doesn't violate the second law of thermodynamics, because the entropy of the entire system still increases. The increase in entropy between points 2 and 3 is simply bigger than the decrease in entropy between points 6 and 7.

There are a few things we should have incorporated in our diagram as well. Let's take a look at the generator. An amount of energy is taken from process 67. This energy is added to process 23. Conservation of energy now implies that the temperature difference in these two processes is the same. So in our graph we should have $T_6 - T_7 = T_3 - T_3$.

Also, heat can only be moving from process 67 to process 23, if the temperature in process 67 is higher. So we should definitely have $T_6 \ge T_3$, and of course also $T_7 \ge T_2$.

2. We know that $p_1 = 100kPa$, $p_2 = 600kPa$ and $T_1 = 300K$. To find T_2 , we have to use the isentropic efficiency.

Suppose that the compression was performed isentropically. Then a temperature $T_{2,is}$ would have been reached at the end of compression. This value $T_{2,is}$ can be found using the isentropic relations, being

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}.$$
(3.1)

It follows that $T_{2,is} = 500.6K$. The isentropic efficiency η_{is} for compression now is defined as

$$\eta_{is} = \frac{T_{2,is} - T_1}{T_2 - T_1}.\tag{3.2}$$

In words, the isentropic efficiency for compression is the temperature difference in the isentropic case, divided by the actual temperature difference. Using $\eta_{is} = 0.82$ we can find that $T_2 = 544.6K$. (Let me tell you a good thing to remember when using isentropic efficiency. The actual temperature is always bigger than the temperature in the isentropic case. So in this case this rule states that $T_2 > T_{2,is}$, which is, indeed, correct.)

3. We know that the work produced by the first turbine is equal to the work used by the compressor. So let's look at the amount of work used by the compressor.

The amount of energy entering the compressor is equal to $e_{in} = h_1 + W_{comp} = c_p T_1 + W_{comp}$. The amount of energy leaving the compressor is equal to $e_{out} = h_2 = c_p T_2$. Since the compressor is working adiabatically, no other energy can enter/leave the system. This means that $e_{in} = e_{out}$ and thus that $W_{comp} = c_p (T_2 - T_1)$.

The work done by the first turbine W_{turb} is equal to the work done by the compressor W_{comp} . However, we can also find that $W_{turb} = c_p (T_4 - T_5)$. So, using $W_{comp} = W_{turb}$, we find that

$$T_5 = T_4 + T_1 - T_2 = 1600 + 300 - 544.6 = 1355.4K.$$
(3.3)

4. We know that $p_4 = p_3 = p_2 = 600 kPa$. To find p_5 , we would like to use the isentropic relations. But we can't, since the turbine doesn't work isentropically. However, if it would work isentropically, then we would have a temperature $T_{5,is}$ after the turbine. In this case we would have

$$\frac{p_5}{p_4} = \left(\frac{T_{5,is}}{T_4}\right)^{\frac{\gamma}{\gamma-1}}.$$
(3.4)

So we need to find $T_{5,is}$. We use the isentropic efficiency for that. The definition of isentropic efficiency for expansion is

$$\eta_{is} = \frac{T_5 - T_4}{T_{5,is} - T_4}.\tag{3.5}$$

Using $\eta_{is} = 0.87$ we find that $T_{5,is} = 1318.9K$. Using the isentropic relations, we then find that $p_5 = 305.1kPa$.

- 5. We know that $p_5 = 305.1kPa$, $p_6 = 100kPa$ and $T_5 = 1355.4K$. Using the isentropic relations, we can find that $T_{6,is} = 985.5K$. If we insert this into the definition of isentropic efficiency for expansion, we find T_6 . The result is $T_6 = 1033.6K$.
- 6. The work done by the power plant "leaves" the power plant at the second turbine (between points 5 and 6). So if we want to know how much work leaves the power plant, we should evaluate energy for the second turbine. The energy per unit mass entering this turbine is $e_{in} = c_p T_5$. The energy per unit mass leaving this turbine through channel 6 is $e_{out} = c_p T_6$. So the work that leaves this turbine per unit mass must be equal to

$$W_{out} = e_{in} - e_{out} = c_p \left(T_5 - T_6 \right) = 323kJ/kg.$$
(3.6)

7. Why would the regenerator be useful in this power plant? To find that out, we have to look at the definition of efficiency.

Efficiency is "useful energy out" divided by "energy in". The place where energy enters the power plant is the burner. Now what is the effect of the regenerator on this?

The regenerator transfers heat from the hot exhaust gases to the cold gases leaving the compressor. So the generator increases the temperature of the gas entering the heater. The gas entering the heater now has a higher temperature. So the heater needs less energy to increase the gas temperature to the right level. Thus the energy going into the system at the heater decreases.

We have found that "energy in" decreases. Since "energy in" is in the denominator of the efficiency, it must mean that the efficiency itself increases. The regenerator thus causes the efficiency of the power plant to increase.