Aerodynamics B Formula Overview

Navier-Stokes Equations

Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{1}$$

Momentum equation

$$-\frac{\partial p}{\partial x} = \nabla \cdot (\rho u \mathbf{V}) \tag{2}$$

$$-\frac{\partial p}{\partial y} = \nabla \cdot (\rho v \mathbf{V}) \tag{3}$$

$$-\frac{\partial p}{\partial z} = \nabla \cdot (\rho w \mathbf{V}) \tag{4}$$

Energy equation

$$\nabla \cdot \left(\rho \left(e + \frac{V^2}{2}\right) \mathbf{V}\right) = -\nabla \cdot (p\mathbf{V}) \tag{5}$$

Substantial derivative

$$\frac{Dx}{Dt} = \frac{\partial x}{\partial t} + (\nabla \cdot \mathbf{V})\, x \tag{6}$$

Divergence of velocity

$$\nabla \cdot \mathbf{V} = \frac{1}{\nu}\frac{D\nu}{Dt} \tag{7}$$

Lines and Equations

Two-Dimensional streamline condition

$$v\, dx = u\, dy \tag{8}$$

Vorticity

$$\xi = \nabla \times \mathbf{V} \tag{9}$$

Circulation

$$\Gamma = -\oint_C \mathbf{V} \cdot \mathbf{ds} \tag{10}$$

Stream function

$$\rho u = \frac{\partial \bar{\psi}}{\partial y}, \qquad \rho v = -\frac{\partial \bar{\psi}}{\partial x} \tag{11}$$

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{12}$$

$$V_r = \frac{1}{r}\frac{d\psi}{d\theta}, \qquad V_\theta = -\frac{d\psi}{dr} \tag{13}$$

Velocity potential

$$u = \frac{\partial \phi}{\partial x}, \qquad v = \frac{\partial \phi}{\partial y}, \qquad w = \frac{\partial \phi}{\partial z} \tag{14}$$

$$V_r = \frac{d\phi}{dr}, \qquad V_\theta = \frac{1}{r}\frac{d\phi}{d\theta} \tag{15}$$

Elementary Flows

Uniform flow

$$u = V_\infty, \qquad v = 0 \tag{16}$$

$$\phi = V_\infty x, \qquad \psi = V_\infty y \tag{17}$$

Source/Sink

$$V_r = \frac{\Lambda}{2\pi r}, \qquad V_\theta = 0 \tag{18}$$

$$\phi = \frac{\Lambda}{2\pi}\ln r, \qquad \psi = \frac{\Lambda}{2\pi}\theta \tag{19}$$

Doublet

$$V_r = -\frac{\kappa}{2\pi}\frac{\cos\theta}{r^2}, \qquad V_\theta = -\frac{\kappa}{2\pi}\frac{\sin\theta}{r^2} \tag{20}$$

$$\phi = \frac{\kappa}{2\pi}\frac{\cos\theta}{r}, \qquad \psi = -\frac{\kappa}{2\pi}\frac{\sin\theta}{r} \tag{21}$$

Vortex

$$V_r = 0, \qquad V_\theta = -\frac{\Gamma}{2\pi r} \tag{22}$$

$$\phi = -\frac{\Gamma}{2\pi}\theta, \qquad \psi = \frac{\Gamma}{2\pi}\ln r \tag{23}$$

Elementary Flow Applications

Nonlifting flow over a cylinder

$$R = \sqrt{\frac{\kappa}{2\pi V_\infty}} \tag{24}$$

$$\psi = V_\infty r \sin\theta \left(1 - \frac{R^2}{r^2}\right) \tag{25}$$

$$V_r = V_\infty \cos\theta \left(1 - \frac{R^2}{r^2}\right) \tag{26}$$

$$V_\theta = -V_\infty \sin\theta \left(1 + \frac{R^2}{r^2}\right) \tag{27}$$

Lifting flow over a cylinder

$$\psi = (V_\infty r \sin\theta)\left(1 - \frac{R^2}{r^2}\right) + \frac{\Gamma}{2\pi}\ln\frac{r}{R} \tag{28}$$

$$V_r = V_\infty \cos\theta \left(1 - \frac{R^2}{r^2}\right) \tag{29}$$

$$V_\theta = -V_\infty \sin\theta \left(1 + \frac{R^2}{r^2}\right) - \frac{\Gamma}{2\pi r} \tag{30}$$

Two-Dimensional Airfoils

Vortex Sheets

$$\Gamma = \int_a^b \gamma \, ds \tag{31}$$

$$L' = \rho_\infty V_\infty \Gamma \tag{32}$$

Thin airfoil theory

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\varepsilon)\, d\varepsilon}{x - \varepsilon} = V_\infty \left(\alpha - \frac{dz}{dx}\right) \tag{33}$$

Symmetric airfoils

$$\gamma(\theta) = 2\alpha V_\infty \frac{1 + \cos\theta}{\sin\theta} \tag{34}$$

$$c_l = 2\pi\alpha, \qquad c_{m,c/4} = 0 \tag{35}$$

Cambered airfoils

$$\gamma(\theta) = 2V_\infty \left(A_0 \frac{1 + \cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right) \tag{36}$$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx} d\theta_0 \tag{37}$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos n\theta_0 \, d\theta_0 \tag{38}$$

$$c_l = \pi(2A_0 + A_1), \qquad c_{m,c/4} = \frac{\pi}{4}(A_2 - A_1) \tag{39}$$

Camber line design

$$\frac{dz}{dx} = \alpha - A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta_0 \tag{40}$$

$$\alpha_{opt} = \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}\theta_0 \tag{41}$$

$$(c_l)_{design} = \pi A_1 = 2\int_0^\pi \frac{dz}{dx}\cos\theta_0 \, d\theta_0 \tag{42}$$

Three-Dimensional Wings

Vortex filaments

$$\mathbf{dV} = \frac{\Gamma}{4\pi}\frac{\mathbf{dl} \times \mathbf{r}}{|\mathbf{r}|^3} \tag{43}$$

Angles of attack

$$\alpha = \alpha_{eff} + \alpha_i \tag{44}$$

$$\alpha_i(y_0) = \frac{1}{4\pi V_\infty}\int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\left(\frac{d\Gamma}{dy}\right) dy}{(y_0 - y)} \tag{45}$$

$$\alpha_{eff}(y_0) = \frac{\Gamma(y_0)}{\pi V_\infty c(y_0)} + \alpha_{L=0} \tag{46}$$

Aspect ratio

$$A = \frac{b^2}{S} \tag{47}$$

Lift coefficient

$$C_L = A_1 \pi A \tag{48}$$

Induced drag

$$D_i'(y) = L'\alpha_i = \rho_\infty V_\infty \Gamma \alpha_i \tag{49}$$

$$C_{D,i} = \frac{C_L^2}{\pi A e} \tag{50}$$

$$e = A_1^2 \left(\sum_1^N n A_n^2\right)^{-1} \tag{51}$$