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4 Stiffness and weight: density and elastic moduli

4.1 Stress, strain and moduli

Stress The *tensile stress* σ is defined as the normal of a *force* F per unit *area* A, as seen in figure 1a.

$$\sigma = \frac{F}{A} \tag{4.1}$$

The units are N/m^2 or Pa. Because a stress of 1 N/m^2 is very tiny, we use the units of MPa, MN/m^2 or N/mm^2.
If, instead, the *force* F_s lies parallel to the face of the element, one plane carries the *shear stress* τ of

$$\tau = \frac{F_s}{A} \tag{4.2}$$

The units, as before, are MPa.
When there are equal tensile or compressive forces to all six face of a cubic element, the state of stress is called *hydrostatic pressure* p, again with units of MPa. When there is a positive pressure, there are compressive forces.

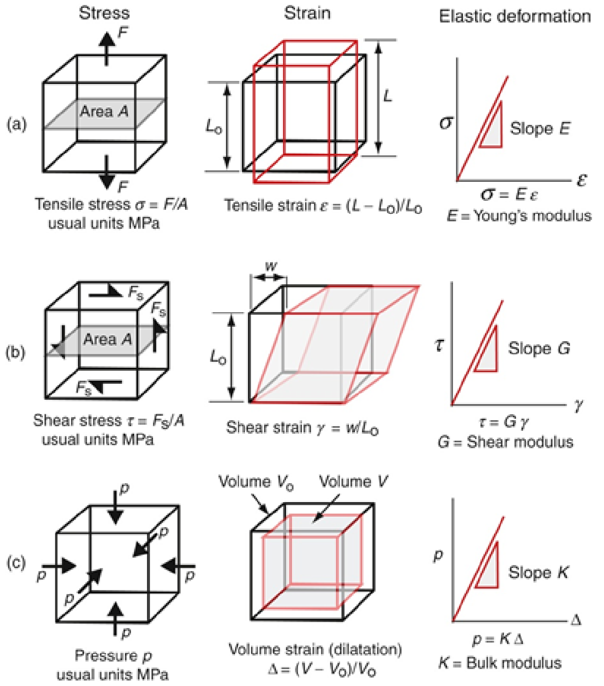


Figure 1: The definitions of stress, strain and elastic moduli

Strain The response of materials to stress is strain. When a tensile stress σ is applied, the element begins to stretch. If the original length L_0 stretches by $\delta L = L - L_0$, the nominal *tensile strain* is

$$\epsilon = \frac{\delta L}{L_0} \tag{4.3}$$

Since strain is the ratio between two lengths, it is dimensionless.
A shear stress causes *shear strain* γ. If the element shears by a distance w, the shear strain

$$tan(\gamma) = \frac{w}{L_0} \approx \gamma \tag{4.4}$$

Finally, the strain, or *dilatation*, for a hydrostatic pressure with volume V which changes by δV, is given by

$$\Delta = \frac{\delta V}{V} \tag{4.5}$$

Stress-strain curves and moduli When looking at a stress-strain curve, the initial part is linear (Hooke's law), this is until the yield point σ_y. The relation is defined by *Young's Modulus* or E:

$$\sigma = E\epsilon \tag{4.6}$$

Similarly, the shear strain γ is proportional to the shear stress τ by *Shear Modulus* G:

$$\tau = G\gamma \tag{4.7}$$

and the dilatation Δ is proportional to the pressure p by *Bulk Modulus* K:

$$p = K\Delta \tag{4.8}$$

The *Poisson's ratio* υ gives the ratio between elongation in the tensile direction and the contraction in perpendicular directions, so ϵ is positive and ϵ_t is negative.

$$\upsilon = -\frac{\epsilon_t}{\epsilon} \tag{4.9}$$

In an isotropic material the moduli are related in the following ways:

$$G = \frac{E}{2(1+\upsilon)}; \qquad K = \frac{E}{3(1-2\upsilon)} \tag{4.10}$$

For most metals, polymers and ceramics $\upsilon \approx 1/3$ so that

$$G \approx \frac{3}{8}E \ \ and \ \ K \approx E \tag{4.11a}$$

For elastomers $\upsilon \approx 1/2$, so

$$G \approx \frac{1}{3}E \ \ and \ \ K \gg E \tag{4.11b}$$

Hooke's Law in three dimensions Due to the Poisson's ratio, the strain of a material is three-dimensional. Consider a cubic element with three unequal stresses. The strain in three dimensions is:

$$\begin{aligned} \epsilon_1 &= \frac{1}{E}(\sigma_1 - \upsilon\sigma_2 - \upsilon\sigma_3) \\ \epsilon_2 &= \frac{1}{E}(-\upsilon\sigma_1 + \sigma_2 - \upsilon\sigma_3) \\ \epsilon_3 &= \frac{1}{E}(-\upsilon\sigma_1 - \upsilon\sigma_2 + \sigma_3) \end{aligned} \tag{4.13}$$

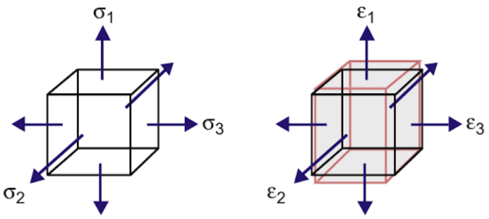


Figure 2: General 3-dimensional states of stress and strain

Elastic Energy If you stretch an elastic band, energy is stored in it. To calculate this energy we use work (W) calculations; work is force times displacement per unit volume, or:

$$dW = \frac{FdL}{AL} = \sigma d\epsilon \tag{4.15}$$

With unit J/m^3. If we integrate, we can calculate the work done per unit volume as the stress is raised from zero to a final value $\sigma^\star$ is the area under the stress-strain curve:

$$W = \int_0^{\sigma^\star} \sigma d\epsilon = \int_0^{\sigma^\star} \frac{\sigma d\sigma}{E} = \frac{1}{2}\frac{(\sigma^\star)^2}{E} \tag{4.16}$$

Stress-free strain Beside stress there are other factor that can contribute in strain of a material. A example is thermal expansion: strain caused by the change of temperature. The thermal strain ϵ_T is linearly related to the temperature change ΔT by the expansion coefficient, α:

$$\epsilon_T = \alpha \Delta T \tag{4.17}$$

4.2 Material property charts

The modulus-density chart In figure 3 you can see the modulus density chart. The chart can be used to see where different families and their member lie in the $E - \rho$ space. This can be usefull for material selection for stiffness-limited application in which weight must be minimized.

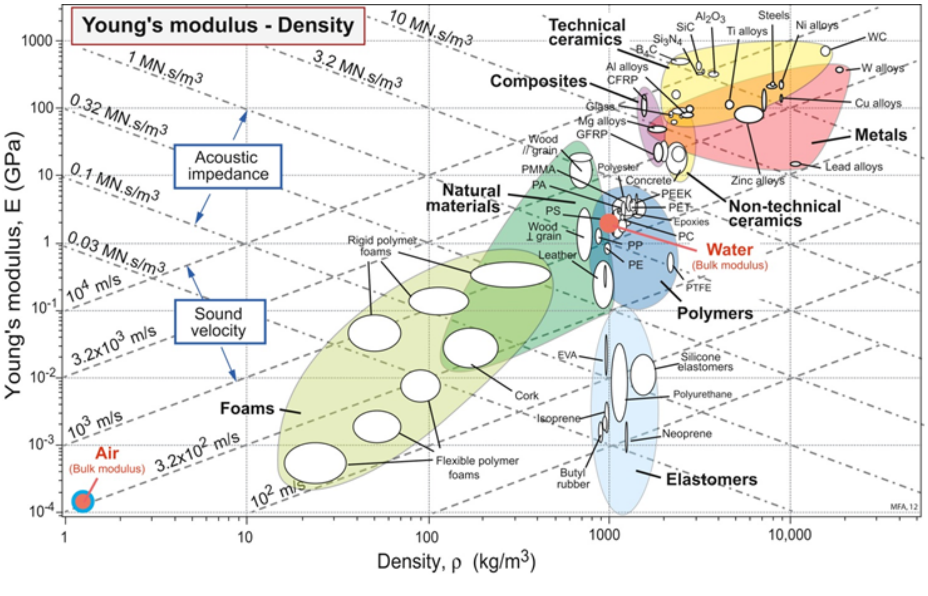


Figure 3: The modulus-density chart

The modulus-relative cost chart Sometimes minimizing cost, not weight is the objective. The modulus-relative cost chart come into play here, as seen in figure 4.

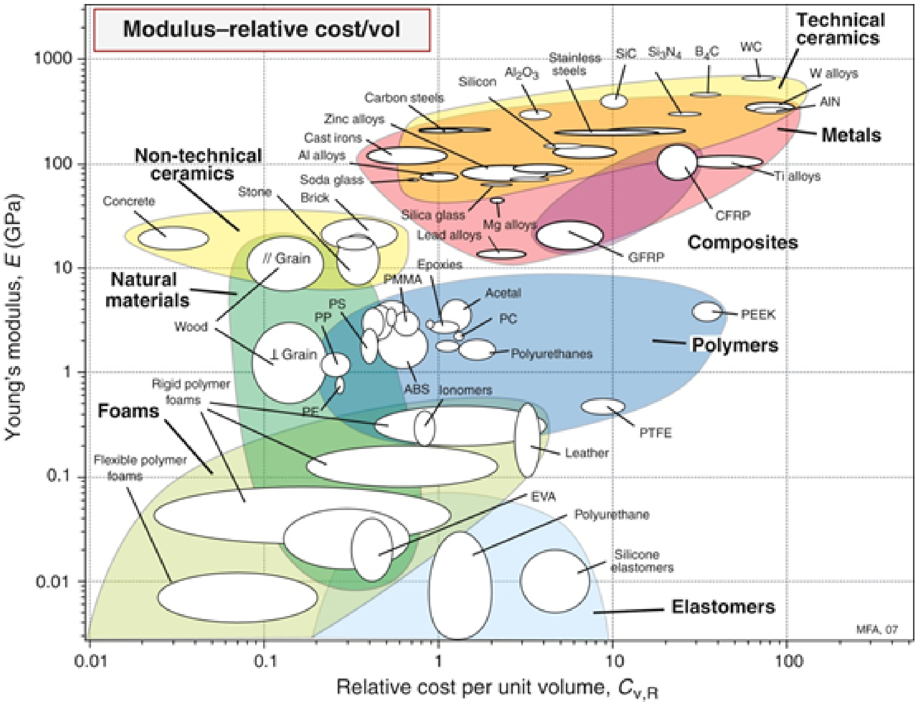


Figure 4: The modulus-relative cost chart

4.3 The science of stiffness and density

Atom packing in metals and the unit cell When looking at a close-packed layer of atoms in 2D (figure 5a), we see that the atoms cannot pack any closer. When adding another 2D close-packed layer on top of the first one, to create a 3D layer, we can create two different packing orders; layer B on layer A to create ABABAB... (figure 5b). This is called CPH packing, or *close-packed hexagonal.* A different way is; layer C on layer B on layer A, (figure 5c). This is called FCC or *face-centred cubic.*

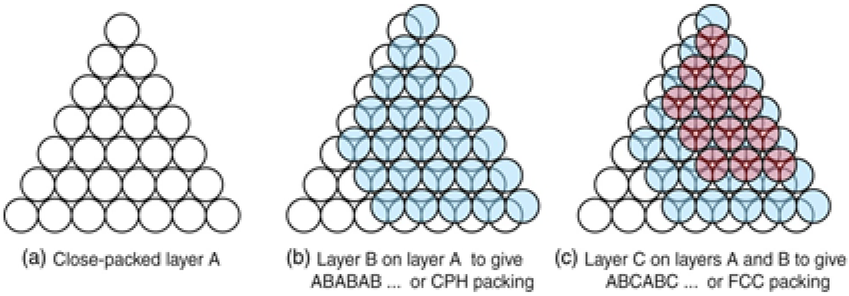

(a) Close-packed layer A
(b) Layer B on layer A to give ABABAB ... or CPH packing
(c) Layer C on layers A and B to give ABCABC ... or FCC packing

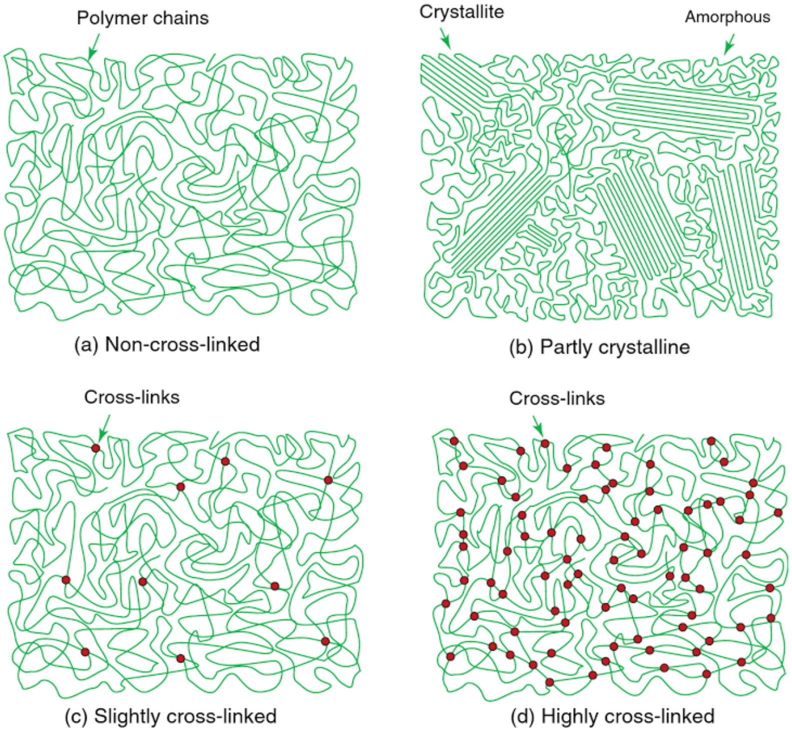
Figure 5: Layer packing of atoms

Not all structures are close-packed. These structure have a lower packing density and are usually BCC of *body-centred cubic* structures.
When the packing of atoms is regular, we can call it a crystal. When the packing is irregular, it is non-crystallographic, and is called an *amorphous* structure.

The size of metallic crystals is in the order of 1-100 µm. Metallic products are bigger than this, so metals are poly-crystals; which contain grains and grain boundaries.

Atom packing in ceramics Ceramics are made up of a metallic and non-metallic atom structure, the non-metallic atoms have places in the unit cells between the metallic atoms.

Atom packing in polymers Polymers are a collection of long covalent-bonded carbon atoms, to which atom groups are attached. Between these chains there can be weak hydrogen bonds (figure 6a, thermoplastics), when there are crystallites, the polymer is partly crystalline (figure 6b). Crystallites scatter light, so they cannot be transparent.
When the carbon chain are crossl-linked with C-C bonds, the polymers cannot be reformed, and thus not recycled. Figure 6c are called *elastomers*, and figure 6d are called *thermosets*.



(a) Non-cross-linked (b) Partly crystalline

(c) Slightly cross-linked (d) Highly cross-linked

Figure 6: Types of polymers

Stiffness The stiffness of a bond between atoms can be seen as a spring; a stretch of δ under a *force* F has a *stiffness* S of;

$$S = \frac{F}{\delta} \tag{4.18}$$

Looking into the stiffness between atoms, which are separated by distance a_0, compared to the *strain* ϵ, we get:

$$\sigma = \frac{S}{a_0}\epsilon \tag{4.19}$$

Comparing this with equation 4.6, gives us the E-modulus:

$$E = \frac{S}{a_0} \tag{4.20}$$

The glass transition temperature Crystalline solids have a well-defined melting point. Polymers on the otherhand have weak hydrogen bonds which start to melt at the *glass transition temperature*, T_g. The crystallites melt at a much higher temperature, typically $1.5 \times T_g$. Elastomers and thermosets have a T_g, but do not melt, because of the cross-links. They decompose and burn instead.
In figure 7a we can see the Young's modulus changing at different temperatures for thermoplastics. In figure 7b we see this for elastomers and thermosets.

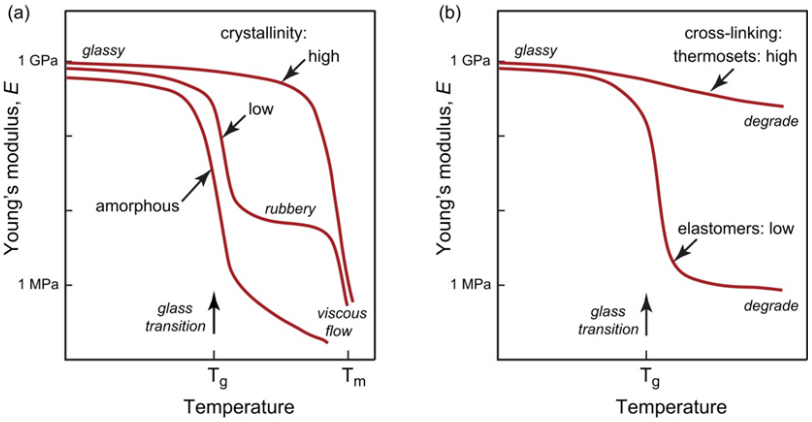


Figure 7: Young's modulus over temperature for polymers

4.4 Manipulating the modulus and density

Composites Composites are made by embedding fibres or particles in a continuous matrix of a polymer, metal or ceramic. When a volume fraction f of a reinforcement r (density ρ_r) is mixed with a matrix m (density ρ_m), the density $\tilde{\rho}$ is defined as:

$$\tilde{\rho}_{comp} = f\rho_r + (1-f)\rho_m \tag{4.23}$$

The modulus can be defined in two ways; the upper limit $\tilde{E}_u$ and lower limit $\tilde{E}_l$. The upper limit is load is inline with the fibres.

$$\tilde{E}_u = fE_r + (1-f)E_m \tag{4.25}$$

The lower limit is when load is perpendicular to the fibres:

$$\tilde{E}_l = \frac{E_r E_m}{fE_m + (1-f)E_r} \tag{4.25}$$

Foams A foam consists of solid edges surrounding an empty void/gas, then (with $t \ll L$);

$$\frac{\rho_{f\tilde{o}am}}{\rho_{solid}} = \left(\frac{t}{L}\right)^2 \tag{4.26}$$

When loaded, the foam modulus $\tilde{E}$ can be determined.

$$\frac{\tilde{E}_{foam}}{E_{solid}} = \left(\frac{\rho_{f\tilde{o}am}}{\rho_{solid}}\right)^2 \tag{4.27}$$

4.5 Acoustic properties

Sound transmitted through materials as elastic waves has a wavelength λ and frequency f which are related by the velocity at which the sound is travelling.

$$f = \frac{V}{\lambda} \tag{4.28}$$

Longitudinal sound velocity in a long rod of solid material ($t \ll \lambda$):

$$V_1 = \sqrt{\frac{E}{\rho}} \tag{4.29}$$

If $t \gg \lambda$, then;

$$V_B = \sqrt{\frac{E(1-\upsilon}{(1-\upsilon-2\upsilon^2)\rho}}$$

Anisotropic materials have different velocities along ($V_\parallel$) and perpendicular ($V_\perp$) to the grain/fibres.

$$\frac{V_\parallel}{V_\perp} = \sqrt{\frac{E_\parallel}{E_\perp}} \tag{4.30}$$

Soft, porous materials absorb airborne sound waves. Low-density polymeric foams, plaster, fibre glass and woven polymers (carpets & curtains) all absorb sound well too.

Sound-absorption coefficient: amount of sound a material absorbs. Heavier materials insulate better against airborne noise, but this does not work against vibrations in a solid material. Low modulus materials are useful here.

Sound wave impedance and radiation of sound energy If a sound wave is transmitted from one material to another, part of the amplitude is reflected back into the first material.

Impedance:

$$Z = \sqrt{\rho E}$$

Reflection coefficient R: fraction of acoustic energy that is reflected.

$$R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 \tag{4.32}$$

The energy that is not reflected is transmitted, so transmission coefficient T is:

$$T = 1 - R = \frac{4Z_1Z_2}{(Z_2+Z_1)^2} \tag{4.33}$$

So if the impedances are about equal, most of the acoustic energy is transmitted.

The intensity of sound radiation (important for musical instruments)

$$I \propto \sqrt{\frac{E}{\rho^3}} \tag{4.34}$$

Where $\sqrt{\frac{E}{\rho^3}}$ is called the radiation factor. For anisotropic materials: $\bar{E} = \sqrt{E_\parallel E_\perp}$

5 Flex sag and wobble: stiffness-limited design

5.1 Standard solutions to elastic problems

Elastic extension or compression Extension δ of a body under load F:

$$\delta = \frac{L_0 F}{AE} \tag{5.1}$$

Stiffness S is defined as

$$S = \frac{F}{\delta} = \frac{AE}{L_0} \tag{5.2}$$

Elastic bending of beams When a beam is loaded by a bending moment M, its neutral axis is deformed to curvature κ:

$$\kappa = \frac{d^2u}{dx^2} = \frac{1}{R}$$

$$\frac{\sigma}{y} = \frac{M}{I} = E\kappa \tag{5.3}$$

For a beam of length L with a transverse load F, the stiffness is

$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3} \tag{5.5}$$

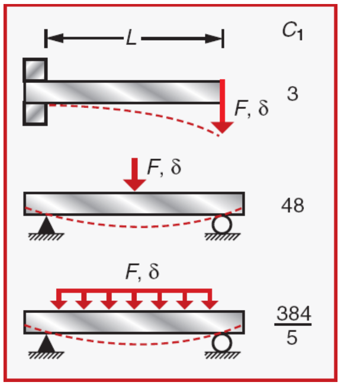


Figure 8: Different deflections of beams clamped in different ways with corresponding C_1 constant

Torsion of shafts When a torque T is applied to the ends of an isotropic bar with constant cross section generates a shear stress τ.

$$\frac{\tau}{r} = \frac{T}{K} = \frac{G\theta}{L} \tag{5.6}$$

With K as the resistance to twisting. For circular section it is equal to the polar second moment of area (J), otherwise it is smaller than J. We also have G as the shear modulus. The torsional rigidity is given to be $GK = T/\theta$.

Buckling of columns and plates Load at which buckling occurs: F_{crit}.

$$F_{crit} = \frac{n^2\pi^2 EI}{L^2} \tag{5.9}$$

With n as a constant, defined in the following figure.

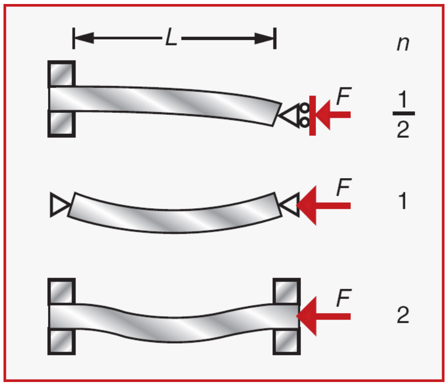


Figure 9: Buckling loads of beams clamped in different ways with corresponding n constant

Vibrating beams and plates The lowest natural frequency of a simple mass and spring system with mass m and spring stiffness k;

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

For higher frequencies:

$$f = \frac{C_2}{2\pi}\sqrt{\frac{EI}{m_0 L^4}} = \frac{C_2}{2\pi}\sqrt{\frac{I}{AL^4}}\sqrt{\frac{E}{\rho}}$$

With C_2 as a constant, defined by the following figure, and m_0 as mass per unit length, or $A\rho$.

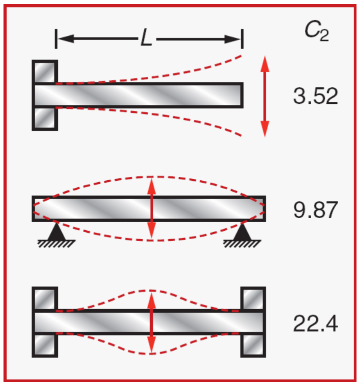


Figure 10: Natural vibration modes of beams clamped in different ways with corresponding C_2 constant

5.2 Material indices for elastic design

Minimizing weight: a light, stiff tie Mass of a tie rod; $m = AL_0\rho$.
The required stiffness: $S^* = \frac{AE}{L_0}$

Combining these equations, we get

$$m = S^* L_0^2 \left(\frac{\rho}{E} \right)$$

So the material index M_t is $M_t = E/\rho$ which needs to be maximized to be optimal.

Minimizing weight: a light, stiff panel Mass of a panel; $m = AL\rho = bhL\rho$.
The required stiffness: $S^* = \frac{C_1 EI}{L^3}$ with $I = (bh^3)/12$

Combining these equations, we get

$$m = \left(\frac{12S^*}{C_1 b} \right)^{1/3} (bL^2) \left(\frac{\rho}{E^{1/3}} \right)$$

So the material index M_p is $M_p = E^{1/3}/\rho$ which needs to be maximized to be optimal.

Minimizing weight: a light, stiff beam Mass of a beam; $m = AL\rho = b^2 L\rho$.
The required stiffness: $S^* = \frac{C_1 EI}{L^3}$ with $I = (b^4)/12$

Combining these equations, we get

$$m = \left(\frac{12S^* L^3}{C_1} \right)^{1/2} (L) \left(\frac{\rho}{E^{1/2}} \right)$$

So the material index M_b is $M_b = E^{1/2}/\rho$ which needs to be maximized to be optimal.

6 Beyond elasticity: plasticity, yielding and ductility

6.1 Strength, plastic work and ductility: definition and measurement

Platic strain, ϵ_{pl}, is the permanent strain resulting from plasticity; thus total strain minus the elastic part:

$$\epsilon_{pl} = \epsilon_{tot} - \frac{\sigma}{E} \tag{6.1}$$

Ductility: measure of how much plastic strain a material can tolerate, expressed as how much can elongate until sample breaks as percentage.

Plastic work: work done in deforming a material permanently by yielding or crushing.

$$dW_{pl} = \frac{FdL}{V} = \frac{F}{A_0}\frac{dL}{L_0} = \sigma d\epsilon_{pl}$$

So,

$$W_{pl} = \int_0^{\epsilon_f} \sigma d\epsilon_{pl} \tag{6.2}$$

Which is the area under the stress-strain curve.

Elastic limit, σ_{el}, is the stress at the end of the elastic part of the stress-strain curve (not true yield strength). This is used for ceramics and glasses, because their yield strength is never reached.

The hardness test is a test which presses a small pyramidal diamond or hardened steel ball into sample. Resistance is $H = F/A$, where $H > \sigma_y$ (in practice $H \approx 3\sigma_y$).
For the Vicker's scale $H_v \approx \sigma_y/3$.

True stress, σ_t, and true strain, ϵ_t Taking into account change of area (in comparison to the starting area).

Nominal stress does not take the area change into account.

$$\sigma_n = \frac{F}{A_0} = \frac{F}{A}\left(\frac{L_0}{L}\right) \tag{6.7}$$

$$\epsilon_n = \frac{\delta L}{L_0} = \left(\frac{L - L_0}{L_0}\right) = \left(\frac{L}{L_0}\right) - 1 \tag{6.8}$$

So,

$$\sigma_t = \sigma_n(1 + \epsilon_n) \tag{6.9}$$

For true strain;

$$\epsilon_t = \int_{L_0}^{L} \frac{dL}{L} = ln\left(\frac{L}{L_0}\right) = ln(1 + \epsilon_n) \tag{6.10}$$

6.2 Drilling down: the origins of strength and ductility

Atomic bond is broken when stretched over 10% of original length. So the force needed to break a atomic bond:

$$F \approx \frac{Sa_0}{10}$$

With S as bond stiffness, and a_0 as the equilibrium spacing of 2 atoms. The ideal strength of a solid should be roughly

$$\sigma_{ideal} = \frac{F_{max}}{a_0^2} = \frac{S}{10a_0} = \frac{E}{10}$$

Crystalline imperfection: defects in metals and ceramics There are several kinds of imperfections in crystals:

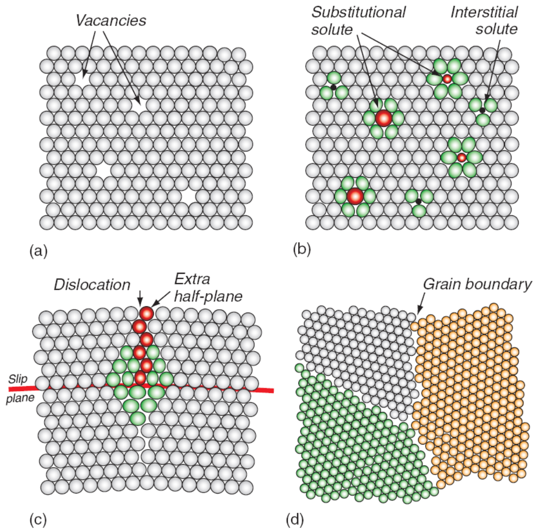


Figure 11: Defects in crystals:

(a) Vacancies - missing atoms
(b) Impurities (distortion in surrounding area because of different size).

 - Substitutional solid solution: one atom replaced.

 - Interstitial solid solution: atom added.
(c) Dislocation - "accidental" extra (half-) row of atoms.
(d) Grain boundaries

Dislocations and plastic flow Dislocations can make crystals deform at stresses far below ideal. Dislocations lead to shear strain γ.
Screw dislocation: works similar to edge dislocation but the lattice is displaced parallel to the dislocation line, not normal to it.
Slip plane: preferred plane to break bonds in.
Slip direction: preferred direction of slip in these planes.

Work done by slip plane;

$$W = \tau L_1 L_2 b$$

The force, τb, per unit length is equal to resistance, f, per unit length. With b as displacement parallel to the slip plane. This holds for any dislocation provided that $\tau > f/b$

Line tension As energy per unit length

$$T = \frac{1}{2} E b^2$$

The lattice resistance Lattice resistance, f_i, is the intrinsic resistance of the crystal structure to plastic shear.
When lattice resistance is low (metals), the material can be strengthened by introducing obstacles to by slip, this is done by adding alloying elements (solid solution hardening, f_{ss}), precipitates or

dispersed particles (precipitation hardening, f_{ppt}), other dislocations (work hardening, f_{wh}) or grain boundaries (grain-size hardening, f_{gb}).

Plastic flow in polymers At low temperatures ($T < 0.75T_g$) polymers are elastic-brittle, like ceramics, and fail by propagation of a dominant flaw.
Drawing: when pulled in tension, the chains slide over each other, unraveling so that they become aligned with the direction of stretch. It is harder to start drawing than to keep going so the zone where it starts drawing draws down completely before propagating further along the sample. These materials are stronger ($\sim \times 8$), but limited in geometry.
Craze: Polymers with $T_g >$ room temperature do not draw at room temperature; they craze. Which is a small crack shaped region within the polymers draw down. Crazes scatter light, so they cause whitening. When continued stretching, crazes can develop proper cracks and eventually fractures.
Shear banding: When compressing a polymer shear bands develop. Within each band, shear takes place with the same consequences for the shape of the sample as shear by dislocation motion.

6.3 Manipulating strength

Strengthening metals Each individual obstacle exerts a pinning force, p, on the dislocation line. Then the added contribution to the shear stress needed to make the dislocation move is

$$\Delta\tau = \frac{p}{bL}$$

With L as the spacing between a obstacle and the slip plane. The shear stress needed to force a dislocation through a field of obstacles;

$$\tau = \alpha\frac{Eb}{L}$$

With α as a dimensionless constant characterizing the obstacle strength.

Solution hardening By adding somewhat larger atoms into a existing lattice, they distort it and roughens the slip plane. The concentration of solute is on average

$$c = \frac{b^2}{L^2}$$

So,

$$\tau_{ss} = \alpha E c^{1/2} \tag{6.18}$$

Dispersion and precipitate strengthening When adding larger precipitate particles, the dislocation slip plane will escape when

$$\tau_{ppt} = \frac{2T}{bL} = \frac{Eb}{L} \tag{6.19}$$

Work hardening Dislocation creates jogs that exert a pinning force $p = Eb^2/2$. Which gives us

$$\tau_{wh} = \frac{Eb}{2}\sqrt{\rho_d} \tag{6.20}$$

With ρ_d as the dislocation density.

Grain boundary hardening Almost all metals are poly-crystalline, small grains meeting at grain boundaries. The grain size, D, is typically 10-100 µm.

$$\tau_{gb} = \frac{k_p}{\sqrt{D}} \tag{6.21}$$

Where k_p is called the Petch constant. For normal grain sizes, τ_{gb} is insignificant, but for micro- and nanocrystalline materials it is a significant source of strength.

Relationship between dislocation strength and yield strength A approximation is that the strengthening mechanisms add up, giving the shear yield strength:

$$\tau_y = \tau_i + \tau_{ss} + \tau_{ppt} + \tau_{wh} + \tau_{gb} \tag{6.22}$$

For a poly-crystalline materials in tension we use;

$$\tau = \frac{F \sin\theta}{A/\cos\theta} = \sigma \sin\theta \cos\theta$$

Where the maximum value of τ is at an angle of 45 degrees, when $\tau = \sigma/2$. With tha Taylor factor (1.5) we see the tensile stress to cause yielding of a sample that has many grains is approximately three time the shear strength of a single crystal: $\sigma_y \approx 3\tau_y$.

7 Bend and crush: strength-limited design

7.1 Standard solution to plastic problems

Yielding of ties and columns A **tie** is rod loaded in tension, a **column** is a rod loaded in compression. With both a uniform state of stress. If the stress σ exceeds σ_y it will plastically deform.

Yielding of beams and panels From chapter 5 we know the stress state in bending, $\sigma/y = M/I = E\kappa$. The maximum longitudinal stress σ_{max} occurs at the surface, at greatest distance y_m from the neutral axis.

$$\sigma_{max} = \frac{My_m}{I} = \frac{M}{Z_e} \tag{7.2}$$

With $Z_e = I/y_m$, which is called the elastic section modulus.
If $\sigma_{max} > \sigma_y$, small zones of plasticity appear at the surface where stress is the highest. The beam is no longer elastic and is thus damaged (not by definition failed).
When the stress is further increased, the linear profile is truncated; σ_{max} remains equal to σ_y and plastic zones grow inward. The yielded parts still carry load. When the zones grow so far that they penetrate through the section of the beam, it forms a plastic hinge. This is the maximum moment that can be carried before it will collapse by rotating about the plastic hinge.

$$M_f = Z_p\sigma_y \tag{7.3}$$

With Z_p as the plastic section modulus. The ratio Z_p/Z_e is always greater than 1 and is a measure of the safety margin between initial yielding and collapse.

Yielding of shafts In chapter 5 we saw that a torque T produces a shear stress, defined $\tau = Tr/K = G\theta r/K$. When the maximum surface stress exceeds the yield strength, failure occurs.

$$\tau_{max} = \frac{TR}{K} = \frac{\sigma_y}{2}$$

The maximum torque a shaft can carry occurs at $\tau = k$ over the whole section. Any greater torque will lead to collapse.
For a solid circular section, the collapse torque is

$$T = \frac{2}{3}\pi R^3 k$$

For helical springs, when loaded in tension, the individual turns twist. The spring stiffness is

$$S = \frac{F}{u} = \frac{Gd^4}{64nR^3}$$

If the spring has n turns of wire of shear modulus G, each of diameter d, wound to a radius R. At the onset of plasticity, the elastic extension stops.

$$F_{crit} = \frac{\pi}{32}\frac{d^3\sigma_y}{R}$$

Spinning disks (flywheels) Flywheels store kinetic energy U. Centrifugal forces generate a radial tensile stress inside the disk. A disk of density ρ, radius R, and thickness t, rotating at angular velocity ω, (when $\upsilon = 1/3$) gives

$$U = \frac{\pi}{4}\rho t\omega^2 R^4 \quad \textit{and} \quad \sigma_{max} = 0.42\rho\omega^2 R^2 \tag{7.8}$$

The disk yields when σ_{max} exceeds σ_y.

Contact stresses A sphere with radius R pressed against a flat surface, has a contact area

$$a \approx 0.7\left(\frac{FR}{E}\right)^{1/3}$$

and a relative displacement of the two bodies of

$$u \approx -\left(\frac{F^2}{E^2 R}\right)^{1/3}$$

$$\tau_{max} = \frac{F}{2\pi a^2}$$

If this exceeds the shear yield strength $k = \sigma_y/2$, a plastic zone appears beneath the centre of the contact.

Stress concentrations

$$K_{sc} = \frac{\sigma_{max}}{\sigma_{nom}} = 1 + \alpha\left(\frac{c}{\rho_{sc}}\right)^{1/2}$$

With c as the characteristic dimension associated with the half length of a notch. The factor α is roughly 2 for tension, and 1/2 for torsion and bending.

7.2 Case studies

Full plasticity: metal forming There are different processes to form metal, one is rolling. You have an initial thickness t_0, and emerges from the rolls with a lesser thickness t_1, so $\Delta t = t_0 - t_1$. The torque T and power P required to do this, is

$$T = \frac{1}{2}R\sigma_y\Delta t \tag{7.24}$$

$$P = 2T\omega = R\omega\sigma_y\Delta t \tag{7.25}$$

Torque and power increase with σ_y, so hot-rolling takes less power than cold-rolling.

8 Fracture and fracture toughness

8.1 Strength and toughness

Strength: A materials resistance to plastic flow
Toughness: A materials resistance to propagation of a crack.

Tests for toughness Notch a material and yank it till it breaks, measuring the energy to do so. However, this does not measure material properties.

8.2 The mechanics of fracture

Cracks contain a very sharp radius of curvature (essentially 0), and by equation 7.12 giving a infinite stress. So there is a different approach:

$$\sigma_{local} = \sigma\left(1 + Y\sqrt{\frac{\pi c}{2\pi r}}\right) \tag{8.1}$$

Where c as the length of a sharp crack, r as the distance from the tip of the crack. Y is a constant that depends weakly on geometry of a cracked body.
Far from the crack, $r \gg c$, the local stress falls to the value of σ. Near the tip of the crack, $r \ll c$, local stress rises steeply as

$$\sigma_{local} = Y\frac{\sigma\sqrt{\pi c}}{\sqrt{2\pi r}} \tag{8.2}$$

So for any given r, the local stress scales with $\sigma\sqrt{\pi c}$, which is a measure of intensity. The *mode 1 stress intensity* (tensile loading perpendicular to crack) is

$$K_1 = Y\sigma\sqrt{\pi c} \tag{8.3}$$

As $r \to 0$ this gives a infinite stress. Instead a small, contained zone will be damaged, such as by micro-cracking.
When K_1 exceeds the critical value, the *fracture toughness* K_{1c}, the crack propagates.

For a centre-cracked plate; $Y = 1$ (if $c \ll w$). This means

$$K_{1c} = K_1 = Y\sigma^*\sqrt{\pi c} \approx \sigma^*\sqrt{\pi c} \tag{8.4}$$

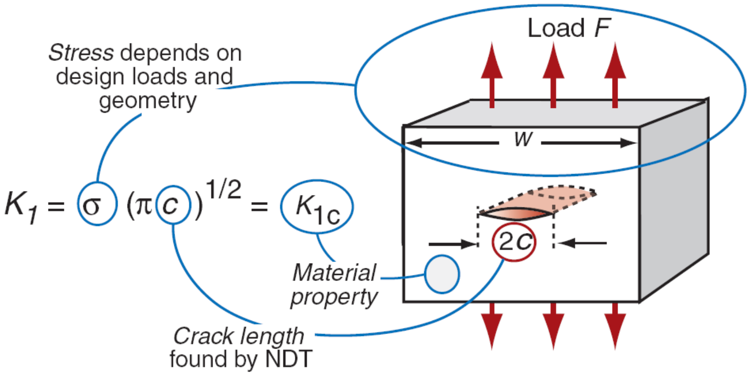


Figure 12: The key players with fracture

Energy release rate and toughness Surface energy γ: when a new surface is created, it contains energy $[J/m^2]$. With a energy release rate G of

$$G \geq 2\gamma \tag{8.5}$$

Energy stored in a slab of material of unit thickness carrying a stress σ, then the elastic energy per unit volume stored in it is;

$$U_v = \frac{1}{2}\frac{\sigma^2}{E} \tag{8.6}$$

Now with crack of crack length c;

$$U(c) = \frac{1}{2}\frac{\sigma^2}{E}\frac{1}{2}\pi c^2$$

If the crack extends by δc, some elastic energy is released. This energy must pay for the extra surface created, the cost is $G_c \delta c$, with G_c as the toughness.

$$\delta U = \frac{\sigma^2 \pi c}{2E}\delta c = \frac{K_{1c}^2}{2E} = G_c \delta c \tag{8.9}$$

So,

$$K_{1c} = \sqrt{EG_c} \tag{8.10}$$

The crack tip plastic zone The process zone;

- a plastic zone in ductile solids
- a zone of micro-cracking in ceramics
- a zone of delamination, debonding and fibre pull-out in composites.

Within the process zone, work is done against plastic and frictional forces (which accounts for the difference between G_c and 2γ).

The radius of the plastic zone is given by the following equation.

$$r_y = 2\left(\frac{\sigma^2 \pi c}{2\pi\sigma_y^2}\right) = \frac{K_1^2}{\pi\sigma_y^2} \tag{8.11}$$

When cracks are small, materials yield before they fracture. When they are large, the opposite is true.

When cracks are small, tensile failure stress is equal to yield stress. When it is large;

$$\sigma_f = \frac{K_{1c}}{\sqrt{\pi c}}$$

The transition from yield to fracture is smooth, but occurs when $\sigma_f = \sigma_y$, giving the transition crack length;

$$c_{crit} = \frac{K_{1c}^2}{\pi\sigma_y^2} \tag{8.13}$$

8.3 Drilling down: the origins of toughness

Surface energy As touched before, surface energy.
To cut a cube you will need this much energy

$$2\gamma = \frac{1}{6}H_c \cdot 4r_0 \quad or \quad \gamma = \frac{1}{3}H_c \cdot r_0$$

With H_c as the cohesive energy, an energy per unit volume.

Brittle 'cleavage' fracture High yield strength → no way to relieve crack tip stresses by plastic flow. This means that near the tip stresses reaches the ideal strength (about E/15), enough to break atomic bonds. Since $K_1 = Y\sigma^*\sqrt{\pi c}$, an increase in c means an increase in K_1, causing the crack to accelerate until it reaches the speed of sound (this is why brittle materials fails with a bang).

Tough 'ductile' fracture Pure metal → metal thin down until cross-section goes to zero.
Non-pure metals build up stress at inclusions, which act as stress concentrations, basically tiny holes. They grow as strain increases, they combine and end in ductile fracture.
Polymers craze in stretched regions, which nucleate, grow and link.
This plastic deformation absorbs energy, increasing the toughness G_c.

Ductile-to-brittle transition Some materials (mostly metals) become brittle at low(er) temperatures.

Embrittlement of other kinds Chemical segregation creates a network of low-toughness paths through the material (using grain boundaries).

8.4 Compressive and tensile failure of ceramics

Ceramics contain micro-cracks. In tension the "worst flaw" propagates to failure.
In compression cracks extend in a stable manner parallel to the applied compression. Crack growth is stable, they only grow when stress is increased. Multiple cracking therefore occurs, with different amounts of crack growth depending on size and orientation.

Tensile failure of ceramics - Weibull statistics Because of the worst flaw propagates to failure; the smaller the sample size, the greater the strength appears to be.
Consider a reference volume, V_0, loaded in uni-axial tension. The probability of survival P_s, with reference stress σ_0, is

$$P_s(V_0) = exp\left\{-\left(\frac{\sigma}{\sigma_0}\right)^m\right\} \tag{8.14}$$

Here is m the *Weibull modulus*, which controls the sensitivity of the probability to stress: with high values there is a greater confidence in survival below the reference stress.
Values for m and σ_0 are determined empirically.

If $V = n \cdot V_0$, then

$$P_s(V) = [P_s(V_0)]^n = [P_s(V_0)]^{V/V_0} = \left[exp\left\{-\left(\frac{\sigma}{\sigma_0}\right)^m\right\}\right]^{V/V_0} = exp\left\{-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right\}$$

A non-uniform stress load (such as in bending) the part in compression can effectively be ignored. Then the probability of survival is

$$P_s(V) = exp\left\{-\frac{1}{V_0\sigma_0^m}\int_V \sigma^m dV\right\}$$

Comparing the maximum bending stress σ_b and the strength of the same sample in tension σ_t gives us

$$\frac{\sigma_b}{\sigma_t} = [2(m+1)]^{1/m}$$

8.5 Manipulating properties: the strength-toughness trade-off

Metals When alloying metals, you increase the yield strength, which causes the plastic zone at crack tip to shrink. The smaller the zone, the smaller the toughness. Conventional casting introduces unwanted voids in the material, and those voids can grow and nucleate at inclusions. This makes the material less tough.

Polymers and composites Polymers properties are manipulated by cross-linking and adjusting the degree of crystallinity. By blending, adding fillers and by reinforcement with chopped or continuous fibres to form composites, there are more drastic changes possible.
Fibres in polymers have a high strength, and remain intact when a crack grows in the matrix. They bridge the crack, each fibre contributing its own energy and thereby raising the dissipation.

9 Shake, rattle and roll: cyclic loading, damage and failure

9.1 Vibration and resonance: the damping coefficient

By high damping we mean to kill vibrations, low damping means that there is a small to no contribution to stop vibrations.
The *mechanical loss coefficient* or *damping coefficient*, η, is the degree to which a material dissipates vibrational energy.
The *loss coefficient*, η is the fraction of elastic energy that is not returned in unloading. Bells have a low η, while if you want to damp vibrations, you choose materials with high η.

9.2 Fatigue

Low-cycle fatigue only survives a few cycles, usually $\sigma > \sigma_y$.
High-cycle fatigue has many cycles, usually $\sigma < \sigma_y$.
In a fatigue life, the components start undamaged, containing no cracks. Most of the fatigue life is spent generating the crack, called *initiation-controlled* fatigue. When there is already a crack (or assumed to be there), the fatigue life depends on crack growth. This is called *propagation controlled.*

The S-N curve In a stress to number of cycles to cause fracture there are different parameters needed. A sample is cyclically stressed with amplitude $\Delta\sigma/2$ about mean value σ_m. The number of cycles to cause failure is N_f.

$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{9.1}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \tag{9.2}$$

Fatigue data are usually reported for a specified R-value:

$$R = \frac{\sigma_{min}}{\sigma_{max}} \tag{9.3}$$

With R = -1 there is mean stress of zero. An R-value of 0 means the stress cycles from 0 to σ_{max}.

High-cycle fatigue For many materials there exists a *fatigue or endurance limit*, σ_e. It is defined as the stress amplitude, about 0 mean stress below which fracture does not occur at all ($N_f > 10^7$). High-cycle fatigue life is approximately related to the stress range by what is called Basquin's law:

$$\Delta\sigma N_f^b = C_1 \tag{9.4}$$

Where C_1 and b are constants, typically with $b = 0.07$ or $b = 0.13$. To get $\Delta\epsilon$, we divide $\Delta\sigma$ by E:

$$\Delta\epsilon = \frac{\Delta\sigma}{E} = \frac{C_1/E}{N_f^b} \tag{9.5}$$

or, by taking logs,

$$log(\Delta\epsilon) = -b \cdot log(N_f) + log(C_1/E)$$

High-cycle fatigue has a slope of $log(\Delta\epsilon) - log(N_f)$ of $2b$.

Low-cycle fatigue In low-cycle fatigue the peak stress exceeds yield, so the entire sample is plastic. Here we need the Coffin's law:

$$\Delta\epsilon^{pl} = \frac{C_2}{N_f^c} \tag{9.6}$$

Where $\Delta\epsilon^{pl}$ means the plastic strain range (total strain minus elastic strain). C_2 and c are constants, typically with $c = 0.5$.

High-cycle fatigue: mean stress and variable amplitude Basquin's and Coffin's law describe the fatigue failure of uncracked component cycled at constant amplitude and about mean stress of zero. Goodman's rule related the stress range $\Delta\sigma_{\sigma_m}$ for failure under a mean stress σ_m to that for failure at zero mean stress $\Delta\sigma_{\sigma_0}$:

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0}\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right) \tag{9.7}$$

With σ_{ts} as the tensile strength. We can eventually put $\Delta\sigma_{\sigma_0}$ into Basquin's law.

Miner's rule of cumulative damage: If N_1 are spent at $\Delta\sigma_1$, a fraction $N_1/N_{f,1}$ of the available life is used up. The sum of fractions of different stress amplitude will fail when they reach 1;

$$\sum_{i=1}^{n} \frac{N_i}{N_{f,i}} = 1$$

Fatigue loading of cracked components In structure we assume that there are already some crack are present which we cannot initially see. The cyclic stress intensity range is defined as;

$$\Delta K = K_{max} - K_{min} = \Delta\sigma\sqrt{\pi c}$$

Because the crack grows in length, ΔK increases with time under constant cyclic stress. The crack growth is defined as follows in the Paris law.

$$\frac{dc}{dN} = A\Delta K^m \tag{9.10}$$

Where K and m are constants. At high ΔK the growth accelerates as the K_{max} approaches K_{1c}

9.3 Drilling down: the origins of damping and fatigue

In polymers the dissipating energy depends on the ratio of temperature T to the glass temperature, T_g, of the polymer. When $T/T_g < 1$, η is low, the modulus high and the damping thus relatively low. When $T/T_g > 1$ the polymer chain can slide against each other, and dissipate energy. The η (damping) is high and the modulus low.

In high-cycle fatigue, once a crack is present it propagates by forming a small plastic zone at the crack tip, stretching it open. On the compressive part of the cycle, the crack closes again and leaving tiny ripples on the crack face. These are called striations, and are characteristic of a fatigue failure. In low-cycle fatigue the plastic zones are larger, sometime large enough to cause voids to nucleate, grow and link. Just like ductile fracture, discussed in chapter 8.

9.4 Manipulating resistance to fatigue

To enhance fatigue life, there are three options:
1. Choosing materials that are strong
2. Making sure they contain as few defects as possible
3. Providing a compressive surface stress.

Choosing materials that are strong Most design is based on yield strength, so we define the fatigue ratio as

$$F_r = \frac{\sigma_e}{\sigma_y}$$

Making sure they contain as few defects as possible Using X-ray imaging or ultrasonic sensing to detect dangerous defects.

Providing a compressive surface stress Crack propagate during the tensile part of a stress cycle. Giving an compressive surface stress, the cracks will remain closed even when the average stress is tensile.

10 Keeping it all together: fracture-limited design

To maximize the load in a structure, we want materials with highest values of $M_1 = K_{1c}$. But not all designs are load limited, some are *energy limited* while others are *deflection limited.*

Energy-limited design The elastic energy U_e stored in a structure is

$$U_e = \frac{1}{2}\sigma\epsilon = \frac{1}{2}\frac{\sigma^2}{E}$$

The maximum energy per unit volume a structure can store is

$$U_e^{max} = \frac{Y^2}{2\pi c}\left(\frac{K_{1c}^2}{E}\right)$$

The energy is maximized by choosing materials with large values of

$$M_2 = \frac{K_{1c}^2}{E} = G_c \tag{10.5}$$

Displacement-limited design For snap-on bottle tops, there is a displacement limit; they must allow sufficient elastic displacement. The failure strain is

$$\epsilon_f = \frac{C}{\sqrt{\pi c_{max}}}\frac{K_{1c}}{E}$$

The best materials for displacement-limited design are those with large values of

$$M_3 = \frac{K_{1c}}{E} \tag{10.6}$$

Yield-before-break criterion Where yielding is needed to avoid a catastrophe.

$$c_{max} \leq \frac{1}{Y^2\pi}\left[\frac{K_{1c}}{\sigma_y}\right]^2$$

The tolerable crack size is maximized by choosing a material with the largest value of

$$M_4 = \frac{K_{1c}}{\sigma_y} \tag{10.11}$$

In a pressure vessel, the stress is defined as

$$\sigma_y = \frac{pR}{t}$$

substituting for the thickness in equation 10.10 we get

$$p \leq \frac{1}{Y^2\pi R}\left(\frac{K_{1c}^2}{\sigma_y}\right)$$

So the pressure is carried most safely by the material with the greatest value of

$$M_5 = \frac{K_{1c}^2}{\sigma_y} \tag{10.13}$$

12 Agitated atoms: materials and heat

12.1 Thermal properties: definition and measurement

Pure crystalline solids have a sharp melting point, T_m, abruptly changing state from solid to low-viscosity liquid. Alloys usually melt over a temperature range.
Non-crystalline molecular materials (thermoplastics) have a more gradual transition from solid to very viscous liquid, with the onset of change given by the glass transition temperature, T_g.
It is help full in engineering to define the maximum and minimum service temperatures, T_{max} & T_{min}. Above T_{max} oxidation, chemical change or excessive distortion become a problem, under T_{min} a material becomes brittle or unsafe to use.

The specific heat, C_p, is the amount of energy needed to heat up 1 kg of a material by 1 K, units $[J/kg \cdot K]$. For gasses we use the heat capacity at constant volume, C_v.
When materials are heated, they expand, the thermal strain per degree of dT is called the linear thermal expansion coefficient;

$$\alpha = \frac{1}{L}\frac{dL}{dT} \tag{12.1}$$

Thermal conductivity, λ, is the rate at which heat is conducted through a solid. Heat flux, q, is the energy flowing through a surface.

$$q = -\lambda\frac{dT}{dx} = \lambda\frac{(T_1 - T_2)}{x} \tag{12.2}$$

The property governing transient heat flow is thermal diffusivity, a.

$$a = \frac{\lambda}{\rho C_p} \tag{12.3}$$

12.2 Drilling down: the physics of thermal properties

Heat capacity Heat is basically atoms that are vibrating. The shortest possible wavelength, λ_1, is just twice the atomic spacing. The average wavelength has $k_B T$, where k_B is Boltzmann's constant (1.38×10^{-23} J/K).
The volume occupied by an atom is Ω, then the number of atoms per unit volume is $N = 1/\Omega$, and the total thermal energy per unit volume in the material is $3k_B T/\Omega$. The heat capacity per unit volume is

$$\rho C_p = \frac{3k_B}{\Omega} \approx 3 \times 10^6 J/m^3 \cdot K \tag{12.4}$$

Thermal expansion If a solids expands when heated it must be, because the atoms are moving farther apart. Materials with high modulus have low expansion coefficient, while those with low modulus have high expansion. A good approximation is

$$\alpha \approx \frac{1.6 \times 10^{-3}}{E}$$

All crystalline solid expand by about the same amount on heating from absolute zero to their melting point by about 2%. So,

$$\alpha \approx \frac{0.02}{T_m}$$

Thermal conductivity Heat is carried by phonons that move through the material at the speed of sound, $c_o = \sqrt{E/\rho}$. Phonons can travel in 6 direction and carry an energy $\rho C_p(T + \Delta T)$ in the $+x$-direction. In the $-x$-direction this is $\rho C_p(T - \Delta T)$.
The distance travelled by a phonon before bouncing of something is l_m. In the $+x$- and $-x$-direction the energy flux q is

$$q = -\frac{1}{6}\rho C_p c_o \left(T + \frac{dT}{dx} l_m\right) + \frac{1}{6}\rho C_p c_o \left(T - \frac{dT}{dx} l_m\right) = -\frac{1}{3}\rho C_p l_m c_o \frac{dT}{dx}$$

So,

$$\lambda = \frac{1}{3}\rho C_p l_m c_o$$

12.3 Design and manufacture: using thermal properties

Managing thermal stresses Thermal stress is caused by two materials that bonded together with different thermal expansions. The stress is defined as

$$\sigma_1 = \frac{E_1}{(1-\upsilon)}(\alpha_1 - \alpha_2)\Delta T$$

So when the thermal expansion coefficient differ greatly, the stress increases. We can avoid this by adding one or more material with expansion that lies between the two. The graded joint spreads the mismatch, lowering the stress and the risk of damage.

Thermal sensing and actuation Bonding two materials with different expansion coefficients α_1 and α_2, will expand at different rates, causing the strip to bend. This bending can be measured.

Thermal shock resistance Image a hot, bulk material dropped in cold water. The surface adjusts to to the temperature of the bath, it wants to contract while the core does not strain at all. The stress induced is

$$\sigma = \frac{E_1}{(1-\upsilon)}\alpha\Delta T \tag{12.14}$$

The thermal shock resistance ΔT_s is the maximum sudden change of temperature to which asuch a material can be subjected without damage.

13 Running hot: using materials at high temperatures

13.1 The temperature dependence of material properties

Linear and non-linear temperature dependence Some properties depend on temperature in a linear way, meaning

$$P \approx P_0 \left(1 + \beta \frac{T}{T_m}\right) \tag{13.1}$$

Where P is the value of the property, P_0 its low-temperature value and β is a constant. Density, modulus, resistivity and refractive index are examples of linear temperature dependence.
Strength on the other hand falls in a much more sudden way and the rate of creep increases exponentially.

Strain rate When a substance flows, the particles shear relative to each other. The shear rate under shear stress τ is

$$\dot{\gamma} = \frac{\tau}{\eta}$$

Where η is the viscosity, $[Pa \cdot s]$. To convert this from shear rate to strain rate, we get;

$$\dot{\epsilon} = \frac{d\epsilon}{dt} = \frac{\sigma}{3\eta} \tag{13.2}$$

The fraction 1/3 is because of the conversion from shear to normal stress σ.

Creep Creep is defined as slow, continuous deformation with time under the 'low-temperature' response, or 'high-temperature' response. In creep curves we can see the strain-to-time graph. First, there is the initial elastic and the primary creep, where strains occur quickly and can be treated as elastic deflection. Thereafter, the strain increases steadily with time in what is called the secondary creep or steady-state creep regime. At constant T, we have a steady-state creep rate of

$$\dot{\epsilon}_{ss} = B\sigma^n \tag{13.3}$$

Where n is the creep exponent. When plotting the natural logarithm of $\dot{\epsilon_{ss}}$ against the reciprocal of the absolute temperature (1/T) at constant σ, we find

$$\dot{\epsilon}_{ss} = C \cdot exp - \left(\frac{Q_c}{\bar{R}T}\right) \tag{13.4}$$

Where C is the Monkman-Grant constant (typically 0.05 - 0.3), $\bar{R}$ as the gas constant, and Q_c is called the activation energy for creep.
As creep continues, damage accumulates (voids or internal cracks that slowly expand). With these damages, the cross-section decreases and the stress to rise. This makes the creep rate accelerate in the tertiary creep.

Melting point As a general rule, it is found that creep starts when $T \approx 0.35 \cdot T_m$ for metals and $T \approx 0.45 \cdot T_m$ for ceramics. Alloying can raise this temperature significantly.

Diffusion The flux per second of atoms in a crystal that have enough energy across unit area from left to right, known as Fick's law:

$$J = -D\frac{dc}{dx} \tag{13.11}$$

where D is called the diffusion coefficient given by

$$D = D_0 \cdot exp - \left(\frac{Q_d}{\bar{R}T}\right) \tag{13.12}$$

17 Durability: oxidation, corrosion, degradation

17.1 Oxidation, flammability and photo-degradation

Definition and measurement The most stable state of most elements is as an oxide. Oxidation is the process when an oxide adheres to a material's surface. The weight gain is either linear ($\Delta m \propto t$) or parabolic ($\Delta m \propto t^{1/2}$). When the oxide is volatile, the material loses weight ($\Delta m \propto -t$).

Flammability Polymers oxidize too, but here flammability is the result. To characterize flammability, the *Limiting oxygen index* (LOI) is used. It is the % oxygen required to maintain steady burning, fresh air has about 21% oxygen in it. A polymer with an LOI lower than 21% will burn freely in air, above 21 % LOI it will extinguish itself.

Photo-degradation Polymers and elastomers age when exposed to UV and oxygen, causing loss of strength, stiffness and toughness. UV creates free radicals which trigger a change in the physical and optical properties of the polymer; making them brittle and turning from transparent to white or gray.

17.2 Oxidation mechanisms

Oxidation is a reaction with metal and oxygen:

$$M(metal) + O(oxygen) \rightarrow MO(oxide) + energy$$

The reaction occurs in two steps, first the metal forms an ion and releases electron, and secondly the electrons are absorbed by oxygen to give an oxygen ion.
When a metal oxidizes, there forms a ultra-thin surface film of oxide on top of the metal, separating the metal from the oxygen. To react further, oxygen most diffuse inward. The weight gain per unit area of surface is parabolic, the oxidation rate is then;

$$\frac{d(\Delta m)}{dt} = \frac{k_p}{\Delta m} \quad giving \quad \Delta m^2 = k_p t \tag{17.3}$$

where k_p $[kg^2/m^4.s]$ is the *parabolic kinetic constant.*
Oxidation rates rise with rising temperatures.

Combustion involves reactions of free-radicals. Flame-retardants tie up free radicals or create protective layer between solid and gas which enables combustion.

17.3 Drilling down: mechanisms of corrosion

The pH scale defines the amount of hydrogen ions (H^+) by

$$pH = -log_{10}[H^+] \tag{17.9}$$

where $[H^+]$ is the molar concentration of hydrogen ions. For pure water, pH = 7.

Fresh and impure water Via an anodic reaction, iron placed in a conducting solution, like salt water, will dissociates into ions, releasing electrons.

$$Fe \leftrightarrow Fe^{2+} + 2e^- \tag{17.13}$$

If current is allowed, electrons flow from the anode to the cathode, causing corrosion of the anode. The reduction potential changes between two half-cells is described by the Nernst equation;

$$E = E_0 + \frac{0.059}{z} log_{10}(C_{ion}) \tag{17.15}$$

With E as the reduction potential when the molar concentration is $M = C_{ion}$, and z as the valence of the ion.

When two metals are joined together in sea water one will be the anode (and corrode), and the other will be the cathode. The weight of metal corroded away, w, as a function of time is:

$$w = \frac{ItM}{zF} \ \ [kg] \tag{17.16}$$

With I as the current [amps], t as time [seconds], M the atomic mass of the metal and F as Faraday's constant (96,500 Coulombs).

Selective corrosion Here is a list of localized corrosion:
- *Intergranular corrosion* occurs because grain boundaries have different chemical properties than the grain. At the grain boundary there are creating cracks.
- *Pitting corrosion* is preferential attack that can occur at breaks in the natural oxide film on metals or at precipitated alloys in certain compounds.
- *Galvanic attack* appears in alloys with two-phase micro-structure in which the two regions lie at different point on the reduction potential scale.
- *Stress corrosion cracking* is accelerated corrosion, localized at cracks in loaded components.
- *Corrosion fatigue* refers to the accelerated rate at which fatigue crack grow in corrosive environment.

17.4 Fighting corrosion

There are four broad strategies; good design, protective coatings, inhibitors and monitoring.

Anti corrosion design rules
- Allow for uniform attack
- Avoid fluid trapping
- Suppress galvanic attack
- Avoid crevices
- Consider cathodic protection
- Beware of stress corrosion and corrosion fatigue
- Design for inspection and maintenance

Coatings Passive coatings separate the material from the corrosive environment and are inherently corrosion resistant. When damaged, the coating does not work anymore.
Active coatings work even when damaged. This coating sacrificially corrode to protect the metal from corrosion.
Self-generated coatings rely on alloying, where a film forms on the metal surface that protects the metal. When damaged, the film re-grows.

Corrosion inhibitors Corrosion inhibitors are chemicals that, when dissolved or dispersed in a corrosive medium, reduce the rate of attack. The choice of inhibitor depends on both the material and the environment.

Monitoring, maintenance and replacement Regular inspection allows early indications of corrosion to be detected. Maintenance can then be carried out, minimizing down-time and risk of failure. The system must thus be designed such that it allows inspection.