Automatic Flight Control - Exam April 2008 - Problems and Solutions

1 Automatic control (2 points)

Root locus plots are used to analyse graphically the influence of the open loop gain on the position of the closed loop poles. Alternatively, root contours can be used to analyse the influence of open loop zero positions on the closed loop poles.

Derive the expression for the modified open loop transfer function which can be used for this purpose. The open loop transfer function can be expressed as follows:

$$G(s)H(s) = (s - z_{var})P(s) \tag{1.1}$$

where the variable zero is situated in $s = z_{var}$ and $P(s)$ groups all the other non-varying terms.

Hint: include eq. (1.1) *in the closed loop transfer function and rewrite the latter one such that z_{var} appears as a common factor in the numerator for the modified open loop transfer function: $GH_{OL_{mod}}(s) = z_{var}\frac{NUM(s)}{DEN(s)}$. DEN(s) .*

1 Solution

This is just a matter of working out equations. We want something of the form

$$F_{CL}(s) = \frac{[\text{something}]}{1 + z_{var}[\text{something}]}. \tag{1.2}$$

So, we write down the closed loop transfer function, and try to work towards this form. We start with

$$F_{CL} = \frac{G}{1 + (s - z_{var})P} = \frac{G}{1 + sP - z_{var}P}. \tag{1.3}$$

Note that, for writing simplicity, we don't write the (s) behind the transfer functions. Now it can be seen that, if we divide both the numerator and the denominator by $1 + sP$, we reach the desired form. So,

$$F_{CL} = \frac{\frac{G}{1+sP}}{\frac{1+sP-z_{var}P}{1+sP}} = \frac{\frac{G}{1+sP}}{1 + z_{var}\frac{-P}{1+sP}}. \tag{1.4}$$

It follows that

$$G_{mod} = \frac{G}{1 + sP} \qquad \text{and} \qquad GH_{mod} = z_{var}\frac{-P}{1 + sP}. \tag{1.5}$$

Working things out a bit further thus gives us that

$$H_{mod} = -z_{var}\frac{P}{G}. \tag{1.6}$$

2 Reduced short period equations of motion (2 points)

Consider the longitudinal dynamics of the McDonnell Douglas F-4 Phantom II military aircraft. For the purpose of the determination and analysis of the handling qualities it is sufficient to study the part of the linearized equations of motion for constant trim speed $V = U_0$, the short period dynamics of the aircraft given by the following equations:

$$\begin{aligned} \dot{w} &= Z_w w + U_0 q + Z_{\delta_e}\delta_e \\ \dot{q} &= M_w w + M_q q + M_{\delta_e}\delta_e \end{aligned} \tag{2.1}$$

Here w[m/s] and q[rad/s] represent the perturbations of the vertical body rate and pitch rate respectively, while δ_e[rad] is the elevator deflection.

Determine from the set of equations (2.1) the so-called characteristic polynomial (commonly defined as $\Delta(s)$), expressed in terms of the short period damping ζ_s and short period frequency ω_s. Show how the system dynamics vary with the dimensional stability derivatives.

Determine, given the expression of the so-called characteristic polynomial (commonly defined as $\Delta(s)$), determined from the set of equations (2.1), the eigenvalues λ_i for $i = 1, 2$. Express the real part and imaginary part of the eigenvalues in terms of the previously calculated terms ζ_s and ω_s. It is assumed that the system is stable and underdamped.

Determine from the set of equations (2.1) the following transfer functions expressed in T_{θ_2}, k_q, k_α and $\Delta(s)(\zeta_s, \omega_s)$, when further the trim airspeed $V = U_0$ and the acceleration of gravity g[m/s^2] are given, and the following assumptions can be made for use of simplifications:

$$\begin{aligned} M_w Z_{\delta_e} &<< M_{\delta_e} Z_w \\ M_q Z_{\delta_e} &<< U_0 M_{\delta_e}, \quad \alpha \approx \frac{w}{V} \end{aligned} \tag{2.2}$$

$$\left| -\frac{Z_{\delta_e}}{U_0} \right| << |-M_{\delta_e} Z_w| \tag{2.3}$$

- Show that the transfer function from elevator to pitch rate:

$$\frac{q(s)}{\delta_e(s)} = k_q \frac{1 + T_{\theta_2} s}{\Delta(s)} \tag{2.4}$$

  when is given that $k_q = -M_{\delta_e} Z_w$ and $T_{\theta_2} = -\frac{1}{Z_w}$.

- Show that the transfer function from elevator to angle of attack:

$$\frac{\alpha(s)}{\delta_e(s)} = k_\alpha \frac{1 + T_\alpha s}{\Delta(s)} \tag{2.5}$$

  when is given that $k_\alpha = M_{\delta_e}$ and $T_\alpha = \frac{Z_{\delta_e}}{U_0 M_{\delta_e}}$.

- Show that the transfer function from attitude to flight-path is given by:

$$\frac{\gamma(s)}{\theta(s)} = \frac{1}{1 + T_{\theta_2} s} \tag{2.6}$$

  when is given that $T_{\theta_2} = -\frac{1}{Z_w}$.

2 Solution

Let's start by putting the equations in state space form. This gives us

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & U_0 \\ M_w & M_q \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \left[\delta_e\right]. \tag{2.7}$$

Now let's take the system matrix, and find the characteristic equation. This gives us

$$\Delta(s) = (Z_w - s)(M_q - s) - U_0 M_w = s^2 - (Z_w + M_q)s + (Z_w M_q - U_0 M_w) = s^2 - 2\zeta_s \omega_s s + \omega_s^2. \quad (2.8)$$

Solving for ζ_s and ω_s gives

$$\omega_s = \sqrt{Z_w M_q - U_0 M_w} \qquad \text{and} \qquad \zeta_s = -\frac{Z_w + M_q}{2\omega_s}. \quad (2.9)$$

We can also solve for the eigenvalues $\lambda_{1,2}$ of the characteristic equations. This gives us

$$\lambda_{1,2} = \frac{(Z_w + M_q) \pm \sqrt{(Z_w + M_q)^2 - 4(Z_w M_q - U_0 M_w)}}{2} = -\zeta_s \omega_s \pm \sqrt{\zeta_s^2 \omega_s^2 - \omega_s^2}. \quad (2.10)$$

Since the system is stable and underdamped, we have $0 < \zeta_s < 1$. The part in the root is thus negative. This gives us

$$\lambda_{1,2} = -\zeta_s \omega_s \pm i\omega_s \sqrt{1 - \zeta_s^2}. \quad (2.11)$$

The damped frequency is thus equal to $\omega_d = \omega_s \sqrt{1 - \zeta_s^2}$, which is also what we would expect it to be.

Now let's start to examine transfer functions. Turning the equations of motion to transfer function form gives

$$sw = Z_w w + U_0 q + Z_{\delta_e} \delta_e, \quad (2.12)$$

$$sq = M_w w + M_q q + M_{\delta_e} \delta_e. \quad (2.13)$$

Solving for w and q gives

$$w = \frac{U_0 q + Z_{\delta_e} \delta_e}{s - Z_w}, \quad (2.14)$$

$$q = \frac{M_w w + M_{\delta_e} \delta_e}{s - M_q}. \quad (2.15)$$

Inserting these two equations into the two equations before them and solving them gives

$$w = \frac{\frac{U_0 M_{\delta_e}}{s - M_q} + Z_{\delta_e}}{s - Z_w - \frac{U_0 M_w}{s - M_q}} \delta_e = \frac{U_0 M_{\delta_e} - Z_{\delta_e} M_q + Z_{\delta_e} s}{s^2 - (M_q + Z_w)s + M_q Z_w - U_0 M_w} \delta_e, \quad (2.16)$$

$$q = \frac{\frac{M_w Z_{\delta_e}}{s - Z_w} + M_{\delta_e}}{s - M_q - \frac{U_0 M_w}{s - Z_w}} \delta_e = \frac{M_w Z_{\delta_e} - M_{\delta_e} Z_w + Z_{\delta_e} s}{s^2 - (M_q + Z_w)s + M_q Z_w - U_0 M_w} \delta_e. \quad (2.17)$$

Note that in the first of the above two equations, the part $Z_{\delta_e} M_q$ is negligible. In the second equation, $M_w Z_{\delta_e}$ is negligible. These terms can thus be removed.

Now it can be seen that the transfer function $q(s)/\delta_e(s)$ is given by

$$\frac{q(s)}{\delta_e(s)} = \frac{q(s)}{\delta_e(s)} = k_q \frac{1 + T_{\theta_2} s}{\Delta(s)}. \quad (2.18)$$

In this equation, $k_q = -M_{\delta_e} Z_w$ and $T_{\theta_2} = -\frac{1}{Z_w}$. Now let's turn our attention to w. We know that $w = \alpha U_0$. This teaches us that

$$\frac{\alpha(s)}{\delta_e(s)} = k_\alpha \frac{1 + T_\alpha s}{\Delta(s)}, \quad (2.19)$$

where $k_\alpha = M_{\delta_e}$ and $T_\alpha = \frac{Z_{\delta_e}}{U_0 M_{\delta_e}}$.

Finally, we have to find an expression for $\gamma(s)/\theta(s)$. For this, we can use the fact that $\theta(s) = q(s)/s$ and that $\theta = \gamma + \alpha$. So, we have

$$\frac{\gamma(s)}{\theta(s)} = 1 - \frac{\alpha(s)}{\theta(s)} = 1 - \frac{\alpha(s)/\delta_e(s)}{\theta(s)/\delta_e(s)} = 1 - s\frac{\alpha(s)/\delta_e(s)}{q(s)/\delta_e(s)}. \tag{2.20}$$

Filling in the values for this will give us

$$\frac{\gamma(s)}{\theta(s)} = 1 - s\frac{k_\alpha \frac{1+T_\alpha s}{\Delta(s)}}{k_q \frac{1+T_{\theta_2} s}{\Delta(s)}} = 1 - s\frac{M_{\delta_e}}{-M_{\delta_e} Z_w}\frac{1 + \frac{Z_{\delta_e}}{U_0 M_{\delta_e}} s}{1 - \frac{1}{Z_w} s} = 1 + s\frac{1 + \frac{Z_{\delta_e}}{U_0 M_{\delta_e}} s}{Z_w - s} = \frac{Z_w + \frac{Z_{\delta_e}}{U_0 M_{\delta_e}} s^2}{Z_w - s}. \tag{2.21}$$

Note that the right term of the numerator is negligible. So, we can simplify the above to

$$\frac{\gamma(s)}{\theta(s)} = \frac{1}{1 - \frac{1}{Z_w} s} = \frac{1}{1 + T_{\theta_2} s}. \tag{2.22}$$

And this is exactly what we needed to show.

3 Stability Augmentation System (1.5 point)

Give the three different kinds of dampers and explain in detail their purpose and in which channels they act. Give block diagrams of the closed loop structures and explain all blocks in the diagram. Give also transfer functions where possible and explain them.

3 Solution

The first damper we'll examine is the yaw damper, which controls the yaw rate. Its purpose is mainly to increase the damping ratio of the Dutch roll. The yaw damper uses the measured yaw rate as feedback and sends a signal to the rudder servo. The feedforward path G of the system consists of three blocks, being the yaw damper itself, the rudder servo and the aircraft. The feedback block H consists of the yaw rate gyro.

The second damper is the pitch damper, which controls the pitch rate. Its purpose is to increase the damping ratio of the short period motion. It uses the measured pitch rate as feedback and sends a signal to the elevator servo. The feedforward path G of the system consists of the pitch damper, the elevator servo and the aircraft. The feedback block H consists of the pitch rate gyro.

The third damper is the phugoid damper, which controls the velocity. Its purpose is to increase the damping ratio of the phugoid. It uses the measured velocity as feedback and sends a signal to the elevator. The feedforward path G consists of the phugoid damper and the aircraft. The elevator servo can also be part of the system, if it has not already been modelled in the pitch damper system. (If it has been incorporated in the pitch damper system, then the phugoid damper sends its signal to the pitch damper system instead, which then controls the elevator deflection. This pitch damper system is usually put inside of the already existing 'aircraft' block.)

Several transfer functions can be given. For the gyroscopes, we usually have

$$H_{gyro}(s) = \frac{1}{s + \omega_{br}} \approx 1, \tag{3.1}$$

where ω_{br} is the break frequency. For the velocity sensor, we also usually have $H_{sensor} \approx 1$. For servos, we use a lag transfer function, being

$$H_{servo}(s) = \frac{K_{servo}}{1 + T_{servo} s}. \tag{3.2}$$

(The value of T_{servo} depends on the type of servo.) Additionally, the aircraft block also has a transfer function. This usually follows from the aircraft model. Finally, there's the transfer function of the damper block. This needs to be determined by the designer. But usually, it has the form

$$H_{damper}(s) = \frac{K_I + K_p s + K_D s^2}{s}. \tag{3.3}$$

Additionally, a washout filter of the form

$$H_{washout}(s) = \frac{\tau s}{\tau s + 1} \tag{3.4}$$

can be added to it.

4 Control Augmentation System (2.5 points)

Part A (1.5 point)

Explain in depth how a heading angle hold mode works in an aircraft. Your description should include at least the following topics enumerated below.

- Explain the purpose of a heading angle hold mode and describe which type of control device (surface) and type of feedback loop structure (sensor) are needed.
- Illustrate the working principle by drawing:
  1. a simple block diagram with the outer loop only.
  2. a comprehensive block diagram with all possible inner loops (the system can be considered as decoupled, i.e. you only have to consider the lateral situation).
- Explain the blocks of the outer loop in detail. Provide transfer functions and give drawings where applicable.

4A Solution

The purpose of the heading angle hold mode is mainly to reduce the pilot workload. Also, the airplane will most likely fly in a more accurate manner. To use this hold mode, the pilot needs to enter a desired heading angle in the mode control panel. The system then tries to hold this heading angle. As input, it uses the measured heading angle, which is usually obtained from a directional gyro. (We can model this gyro again as $H_{gyro}(s) \approx 1$.) The signal that is generated by the heading angle hold mode is sent to the (coordinated) roll angle hold mode system. This system then uses the ailerons to achieve a desired roll angle, which in turn causes the heading angle to change.

The heading angle hold mode basically consists of only three blocks. The feedback path H consists of just the directional gyro. The feedforward path G consists of two blocks. The first is the heading controller block, having a transfer function of the form

$$H_{controller}(s) = \frac{K_I + K_p s + K_D s^2}{s}. \tag{4.1}$$

The second block is the (coordinated) roll angle hold mode system. We can examine it in more detail. The normal roll angle hold mode block consists of another feedback loop. It gets its input from the roll angle gyro. The roll angle hold mode block uses the feedback signal from this gyro to determine the required aileron deflection to achieve this roll angle. This desired aileron deflection is then sent to the aileron servo, which in turn effects the aircraft.

If we are dealing with a coordinated roll angle hold mode, then the roll angle hold mode is also extended by another feedback loop. This feedback loop uses the sideslip angle as feedback signal. This signal is measured by the sideslip sensor, and also passes through the sideslip filter. This filter then determines the yaw rate that is necessary to reduce the sideslip angle. This desired yaw rate is then sent to the yaw damper (of the SAS of the aircraft). The yaw damper then again has a yaw rate gyro block, a yaw damper block and a rudder servo block.

Part B (1.0 point)

How can the heading angle ψ be calculated from the roll angle ϕ? For this purpose, consider an aircraft in a flat turn with constant turn radius R_t and determine the equations of motion. Rewrite these expressions so that you obtain the result $\psi = \psi(\phi)$.

4B Solution

Let's examine an aircraft which is in a steady horizontal turn. It has a roll angle ϕ. The vertical forces must be in equilibrium. So, we find that

$$W = mg = L\cos\phi \qquad \Rightarrow \qquad L = \frac{mg}{\cos\phi}. \tag{4.2}$$

The horizontal forces must cause the centrifugal acceleration. Let's denote the rotation rate of the aircraft by $\omega = \dot{\psi}$. If we also use that $V = \omega R_t$, then we find that

$$m\frac{V^2}{R_t} = m\omega^2 R_t = m\omega V = L\sin\phi = \frac{mg\sin\phi}{\cos\phi} = mg\tan\phi. \tag{4.3}$$

We can now turn this equation to the Laplace domain, using $\omega = \dot{\psi} = s\psi$. Solving for ψ then gives

$$\psi = \frac{g\tan\phi}{sV} \approx \frac{g\phi}{sV}. \tag{4.4}$$

Alternatively, if this equation shouldn't contain V, but R_t instead, the result would be

$$\psi = \frac{1}{s}\sqrt{\frac{g}{R_t}\tan\phi}. \tag{4.5}$$

5 Autopilot navigational mode (2 points)

Describe the glideslope hold mode of a general autopilot in detail. Your description should include at least the following topics enumerated below.

- A clear picture of the situation in which the to-be-controlled parameter can be found.
- Assumptions which are made in this figure.
- A procedure which shows clearly how the control law is determined.
- A block diagram which represents the control law determined earlier.
- An explanation of the nature of each block in the diagram, and definitions of models or transfer functions of all these blocks.

After you described the glideslope hold mode in detail, explain the influence of the slant range R on the closed loop performance. Give three ways how this can be compensated.

5 Solution

In the glide slope hold mode, the autopilot tries to control the deviation d from the glide slope. It should be kept at zero. To accomplish this, the airplane makes use of a glide slope antenna. We assume that this antenna is positioned at the aircraft CG. We also assume that the aircraft attempts to let the CG follow the glide slope path. Finally, we assume that velocity control and pitch attitude control is present in the aircraft.

We also need a control law. We assume that the glide slope is at 3° with respect to the horizon. Based on this, we find that

$$\dot{d} = V \sin(\gamma + 3^\circ) \approx V(\gamma + 3^\circ)\frac{\pi}{180} \qquad \Rightarrow \qquad d(s) = \frac{V}{s}\frac{\pi}{180} L(\gamma + 3^\circ). \tag{5.1}$$

This is the control law that we're looking for. Next to this, we also require feedback. The glide slope antenna on board of the aircraft measures the glide slope error angle Γ. This angle is related to the deviation d, according to

$$\Gamma \approx \sin\Gamma = \frac{d}{R}\frac{180}{\pi}. \tag{5.2}$$

Now let's examine the block diagram. The feedback comes from the glide slope receiver ($H_{receiver}(s) \approx 1$). This block measures Γ. It is compared to a reference value of Γ (which is actually always zero). Their difference is sent to the glide slope coupler. This coupler calculates the desired pitch angle θ to fix the deviation. Its transfer function is

$$H_{coupler}(s) = K_c\left(1 + \frac{W_1}{s}\right), \tag{5.3}$$

where we usually have $W_1 \approx 0.1$. The desired pitch angle is then sent to the aircraft, having pitch attitude control. From this aircraft model, we can extract the resulting value of the actual pitch angle θ. But we don't want to know θ. Instead, we want to know γ. So, we add another block, having the transfer function $\gamma(s)/\theta(s)$. This block gives us the flight path angle γ. We then start to apply the control law. First, we add 3° to γ. We then multiply it by $\frac{V}{s}\frac{\pi}{180}$ to find d. And in the next block, we divide by R to find the resulting Γ. That concludes the block diagram.

It must be noted that the open loop transfer function has the term $1/R$ in it. This means that decreasing R is like increasing the gain of the open loop transfer function. If R decreases enough, we might get into an unstable situation. To compensate for this, we can use a DME beacon to measure R and adjust the open loop gain accordingly. (This is actually some form of gain scheduling.) Alternatively, if we don't have a DME beacon, we can also use the time to adjust the gain of the open loop transfer function. Or, if we want things even simpler, we can simply add a compensator. This compensator should be built such that instability will only occur for unimportantly small values of R.