SOLUTION MANUAL CONTENTS

12–1.

A baseball is thrown downward from a 50-ft tower with an initial speed of 18 ft/s. Determine the speed at which it hits the ground and the time of travel.

SOLUTION

$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$

$v_2^2 = (18)^2 + 2(32.2)(50 - 0)$

$v_2 = 59.532 = 59.5 \text{ ft/s}$ **Ans.**

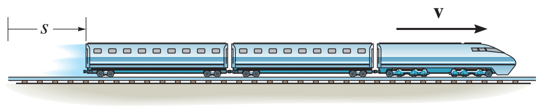
$v_2 = v_1 + a_c t$

$59.532 = 18 + 32.2(t)$

$t = 1.29 \text{ s}$ **Ans.**

12–2.

When a train is traveling along a straight track at 2 m/s, it begins to accelerate at $a = (60\,v^{-4})\ \text{m/s}^2$, where v is in m/s. Determine its velocity v and the position 3 s after the acceleration.



SOLUTION

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$3 = \frac{1}{300}\left(v^5 - 32\right)$$

$$v = 3.925\ \text{m/s} = 3.93\ \text{m/s}$$ **Ans.**

$$ads = vdv$$

$$ds = \frac{vdv}{a} = \frac{1}{60}v^5\,dv$$

$$\int_0^s ds = \frac{1}{60}\int_2^{3.925} v^5\,dv$$

$$s = \frac{1}{60}\left(\frac{v^6}{6}\right)\Bigg|_2^{3.925}$$

$$= 9.98\ \text{m}$$ **Ans.**

12–3.

From approximately what floor of a building must a car be dropped from an at-rest position so that it reaches a speed of 80.7 ft/s (55 mi/h) when it hits the ground? Each floor is 12 ft higher than the one below it. (*Note:* You may want to remember this when traveling 55 mi/h.)

SOLUTION

$(+\downarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$$80.7^2 = 0 + 2(32.2)(s - 0)$$

$$s = 101.13 \text{ ft}$$

$$\# \text{ of floors} = \frac{101.13}{12} = 8.43$$

The car must be dropped from the 9th floor. **Ans.**

***12–4.**

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

SOLUTION

$v = v_1 + a_c t$

$120 = 70 + 6000(t)$

$t = 8.33(10^{-3})\text{ hr} = 30\text{ s}$ **Ans.**

$v^2 = v_1^2 + 2\,a_c(s - s_1)$

$(120)^2 = 70^2 + 2(6000)(s - 0)$

$s = 0.792\text{ km} = 792\text{ m}$ **Ans.**

12–5.

A bus starts from rest with a constant acceleration of 1 m/s^2. Determine the time required for it to attain a speed of 25 m/s and the distance traveled.

SOLUTION

***Kinematics*:**

$v_0 = 0$, $v = 25$ m/s, $s_0 = 0$, and $a_c = 1\ \text{m/s}^2$.

$$\left(\overset{+}{\rightarrow}\right) \qquad v = v_0 + a_c t$$

$$25 = 0 + (1)t$$

$$t = 25\ \text{s}$$ **Ans.**

$$\left(\overset{+}{\rightarrow}\right) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$25^2 = 0 + 2(1)(s - 0)$$

$$s = 312.5\ \text{m}$$ **Ans.**

12–6.

A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.

SOLUTION

$$+\downarrow s = s_1 + v_1 t + \frac{1}{2}a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(32.2)(2)^2$$

$$s_A = 64.4 \text{ ft}$$

$$s_A = 0 + 0 + \frac{1}{2}(32.2)(1)^2$$

$$s_B = 16.1 \text{ ft}$$

$$\Delta s = 64.4 - 16.1 = 48.3 \text{ ft}$$ **Ans.**

12–7.

A bicyclist starts from rest and after traveling along a straight path a distance of 20 m reaches a speed of 30 km/h. Determine his acceleration if it is *constant*. Also, how long does it take to reach the speed of 30 km/h?

SOLUTION

$v_2 = 30 \text{ km/h} = 8.33 \text{ m/s}$

$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$

$(8.33)^2 = 0 + 2\,a_c(20 - 0)$

$a_c = 1.74 \text{ m/s}^2$ **Ans.**

$v_2 = v_1 + a_c\,t$

$8.33 = 0 + 1.74(t)$

$t = 4.80 \text{ s}$ **Ans.**

***■12–8.**

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})\ \text{m/s}^2$, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use Simpson's rule to evaluate the integral.

SOLUTION

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

$$v = 1.29\ \text{m/s} \qquad \textbf{Ans.}$$

12–9.

If it takes 3 s for a ball to strike the ground when it is released from rest, determine the height in meters of the building from which it was released. Also, what is the velocity of the ball when it strikes the ground?

SOLUTION

Kinematics:

$v_0 = 0,\ a_c = g = 9.81\ \text{m/s}^2,\ t = 3\ \text{s}$, and $s = h$.

$(+\downarrow) \qquad v = v_0 + a_c t$

$$= 0 + (9.81)(3)$$

$$= 29.4\ \text{m/s} \qquad \textbf{Ans.}$$

$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$h = 0 + 0 + \frac{1}{2}(9.81)(3^2)$$

$$= 44.1\ \text{m} \qquad \textbf{Ans.}$$

12–10.

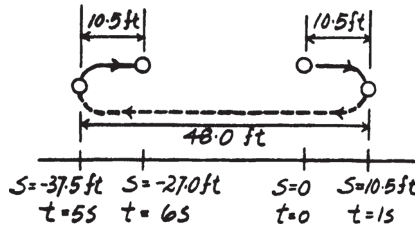
The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

SOLUTION

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft}$$ **Ans.**

Total DistanceTraveled: The velocity of the particle can be determined by applying Eq. 12–1.



$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$$t = 1 \text{ s} \qquad \text{and} \qquad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

$$s|_{t=0\text{ s}} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s|_{t=1\text{ s}} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$

$$s|_{t=5\text{ s}} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{\text{tot}} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft}$$ **Ans.**

12–11.

If a particle has an initial velocity of $v_0 = 12$ ft/s to the right, at $s_0 = 0$, determine its position when $t = 10$ s, if $a = 2$ ft/s^2 to the left.

SOLUTION

$(\overset{+}{\rightarrow})$

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$= 0 + 12(10) + \frac{1}{2}(-2)(10)^2$$

$$= 20 \text{ ft} \qquad \textbf{Ans.}$$

***12–12.**

Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at 1.5 m/s^2 and decelerate at 2 m/s^2.

SOLUTION

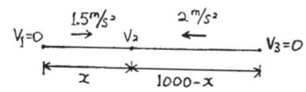
Using formulas of constant acceleration:

$$v_2 = 1.5\, t_1$$

$$x = \frac{1}{2}(1.5)(t_1^2)$$

$$0 = v_2 - 2\, t_2$$

$$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$$



Combining equations:

$$t_1 = 1.33\, t_2; \quad v_2 = 2\, t_2$$

$$x = 1.33\, t_2^2$$

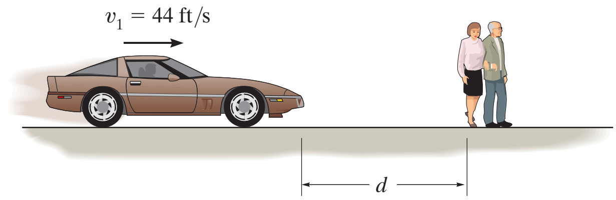
$$1000 - 1.33\, t_2^2 = 2\, t_2^2 - t_2^2$$

$$t_2 = 20.702 \text{ s}; \qquad t_1 = 27.603 \text{ s}$$

$$t = t_1 + t_2 = 48.3 \text{ s}$$ **Ans.**

12–13.

Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral*: If you must drink, please don't drive!



SOLUTION

Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 33.0)$$

$$d = 517 \text{ ft} \qquad \textbf{Ans.}$$

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

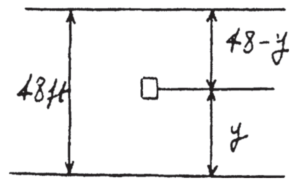
$$0^2 = 44^2 + 2(-2)(d - 132)$$

$$d = 616 \text{ ft} \qquad \textbf{Ans.}$$

12–14.

A car is to be hoisted by elevator to the fourth floor of a parking garage, which is 48 ft above the ground. If the elevator can accelerate at 0.6 ft/s^2, decelerate at 0.3 ft/s^2, and reach a maximum speed of 8 ft/s, determine the shortest time to make the lift, starting from rest and ending at rest.

SOLUTION



$+\uparrow \quad v^2 = v_0^2 + 2\,a_c\,(s - s_0)$

$v_{max}^2 = 0 + 2(0.6)(y - 0)$

$0 = v_{max}^2 + 2(-0.3)(48 - y)$

$0 = 1.2\,y - 0.6(48 - y)$

$y = 16.0 \text{ ft}, \qquad v_{max} = 4.382 \text{ ft/s} < 8 \text{ ft/s}$

$+\uparrow \quad v = v_0 + a_c\,t$

$4.382 = 0 + 0.6\,t_1$

$t_1 = 7.303 \text{ s}$

$0 = 4.382 - 0.3\,t_2$

$t_2 = 14.61 \text{ s}$

$t = t_1 + t_2 = 21.9 \text{ s}$ **Ans.**

12–15.

A train starts from rest at station A and accelerates at $0.5\ \text{m/s}^2$ for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at $1\ \text{m/s}^2$ until it is brought to rest at station B. Determine the distance between the stations.

SOLUTION

Kinematics: For stage (1) motion, $v_0 = 0$, $s_0 = 0$, $t = 60$ s, and $a_c = 0.5\ \text{m/s}^2$. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(60^2) = 900\ \text{m}$$

$$\left(\overset{+}{\rightarrow}\right) \qquad v = v_0 + a_c t$$

$$v_1 = 0 + 0.5(60) = 30\ \text{m/s}$$

For stage (2) motion, $v_0 = 30\ \text{m/s}$, $s_0 = 900\ \text{m}$, $a_c = 0$ and $t = 15(60) = 900\ \text{s}$. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_2 = 900 + 30(900) + 0 = 27\,900\ \text{m}$$

For stage (3) motion, $v_0 = 30\ \text{m/s}$, $v = 0$, $s_0 = 27\,900\ \text{m}$ and $a_c = -1\ \text{m/s}^2$. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad v = v_0 + a_c t$$

$$0 = 30 + (-1)t$$

$$t = 30\ \text{s}$$

$$\overset{+}{\rightarrow} \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_3 = 27\,900 + 30(30) + \frac{1}{2}(-1)(30^2)$$

$$= 28\,350\ \text{m} = 28.4\ \text{km} \qquad \textbf{Ans.}$$

***12–16.**

A particle travels along a straight line such that in 2 s it moves from an initial position $s_A = +0.5$ m to a position $s_B = -1.5$ m. Then in another 4 s it moves from s_B to $s_C = +2.5$ m. Determine the particle's average velocity and average speed during the 6-s time interval.

SOLUTION

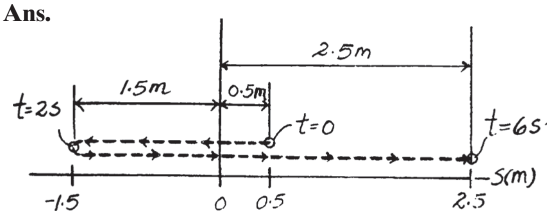
$$\Delta s = (s_C - s_A) = 2 \text{ m}$$

$$s_T = (0.5 + 1.5 + 1.5 + 2.5) = 6 \text{ m}$$

$$t = (2 + 4) = 6 \text{ s}$$

$$v_{avg} = \frac{\Delta s}{t} = \frac{2}{6} = 0.333 \text{ m/s}$$ **Ans.**

$$(v_{sp})_{avg} = \frac{s_T}{t} = \frac{6}{6} = 1 \text{ m/s}$$ **Ans.**



12–17.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 1\ \text{m}$ and $v = 2\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6\ \text{s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$$\int_2^v dv = \int_0^t (2\,t - 1)\,dt$$

$$v = t^2 - t + 2$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2)\,dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2\,t + 1$$

When $t = 6$ s,

$$v = 32\ \text{m/s} \qquad \textbf{Ans.}$$

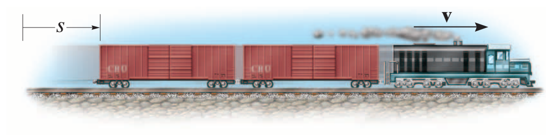
$$s = 67\ \text{m} \qquad \textbf{Ans.}$$

Since $v \neq 0$ then

$$d = 67 - 1 = 66\ \text{m} \qquad \textbf{Ans.}$$

12–18.

A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.



SOLUTION

$$v = 60(1 - e^{-t})$$

$$\int_0^s ds = \int v\,dt = \int_0^3 60(1 - e^{-t})dt$$

$$s = 60(t + e^{-t})\Big|_0^3$$

$s = 123$ ft **Ans.**

$$a = \frac{dv}{dt} = 60(e^{-t})$$

At $t = 3$ s

$a = 60e^{-3} = 2.99\ \text{ft/s}^2$ **Ans.**

12–19.

A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its position when $t = 6$ s if $s = 5$ m when $t = 0$.

SOLUTION

$$\frac{ds}{dt} = \frac{5}{4 + s}$$

$$\int_5^s (4 + s)\, ds = \int_0^t 5\, dt$$

$$4\,s + 0.5\,s^2 - 32.5 = 5\,t$$

When $t = 6$ s,

$$s^2 + 8\,s - 125 = 0$$

Solving for the positive root

$s = 7.87$ m **Ans.**

***12–20.**

The velocity of a particle traveling along a straight line is $v = (3t^2 - 6t)$ ft/s, where t is in seconds. If $s = 4$ ft when $t = 0$, determine the position of the particle when $t = 4$ s. What is the total distance traveled during the time interval $t = 0$ to $t = 4$ s? Also, what is the acceleration when $t = 2$ s?

SOLUTION

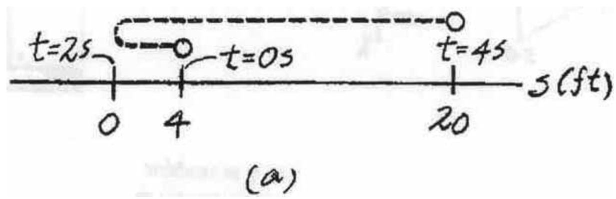
Position: The position of the particle can be determined by integrating the kinematic equation $ds = v\,dt$ using the initial condition $s = 4$ ft when $t = 0$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_{4\text{ ft}}^{s} ds = \int_0^t \left(3t^2 - 6t\right)dt$$

$$s\Big|_{4\text{ ft}}^{s} = \left(t^3 - 3t^2\right)\Big|_0^t$$

$$s = \left(t^3 - 3t^2 + 4\right)\text{ ft}$$



When $t = 4$ s,

$$s|_{4\text{ s}} = 4^3 - 3(4^2) + 4 = 20\text{ ft}$$ **Ans.**

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$v = 3t^2 - 6t = 0$$

$$t(3t - 6) = 0$$

$$t = 0 \text{ and } t = 2\text{ s}$$

The position of the particle at $t = 0$ and 2 s is

$$s|_{0\text{ s}} = 0 - 3\left(0^2\right) + 4 = 4\text{ ft}$$

$$s|_{2\text{ s}} = 2^3 - 3\left(2^2\right) + 4 = 0$$

Using the above result, the path of the particle shown in Fig. *a* is plotted. From this figure,

$$s_{\text{Tot}} = 4 + 20 = 24\text{ ft}$$ **Ans.**

Acceleration:

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}\left(3t^2 - 6t\right)$$

$$a = (6t - 6)\text{ ft/s}^2$$

When $t = 2$ s,

$$a|_{t=2\text{ s}} = 6(2) - 6 = 6\text{ ft/s}^2 \rightarrow$$ **Ans.**

12–21.

If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]\ \text{m/s}^2$, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{(1)}$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5\ \text{m/s} \qquad \textbf{Ans.}$$

b) If $t \rightarrow \infty$, $\dfrac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. (1)

$$v_{\max} = 100\ \text{m/s} \qquad \textbf{Ans.}$$

12–22.

The position of a particle on a straight line is given by $s = (t^3 - 9t^2 + 15t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint*: Plot the path to determine the total distance traveled.

SOLUTION

$s = t^3 - 9t^2 + 15t$

$v = \dfrac{ds}{dt} = 3t^2 - 18t + 15$

$v = 0$ when $t = 1$ s and $t = 5$ s

$t = 0,\ s = 0$

$t = 1$ s, $\quad s = 7$ ft

$t = 5$ s, $s = -25$ ft

$t = 6$ s, $s = -18$ ft **Ans.**

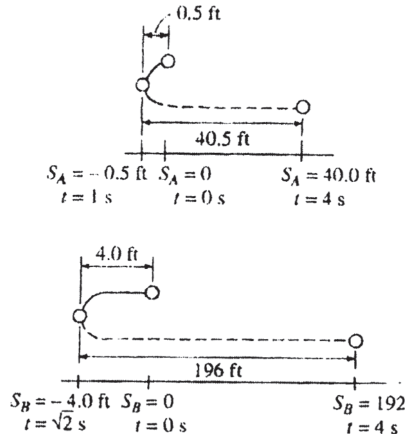
$s_T = 7 + 7 + 25 + (25 - 18) = 46$ ft **Ans.**

12–23.

Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)\ \text{ft/s}^2$ and $a_B = (12t^2 - 8)\ \text{ft/s}^2$, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

SOLUTION

Velocity: The velocity of particles A and B can be determined using Eq. 12-2.



$$dv_A = a_A dt$$

$$\int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$

$$v_A = 3t^2 - 3t$$

$$dv_B = a_B dt$$

$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$

$$v_B = 4t^3 - 8t$$

The times when particle A stops are

$3t^2 - 3t = 0 \qquad t = 0$ s and $= 1$ s

The times when particle B stops are

$4t^3 - 8t = 0 \quad t = 0$ s and $t = \sqrt{2}$ s

Position: The position of particles A and B can be determined using Eq. 12-1.

$$ds_A = v_A dt$$

$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t)dt$$

$$s_A = t^3 - \frac{3}{2}t^2$$

$$ds_B = v_B dt$$

$$\int_0^{s_B} ds_B = \int_0^t (4t^3 - 8t)dt$$

$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1$ s and 4 s are

$$s_A|_{t=1\,s} = 1^3 - \frac{3}{2}(1^2) = -0.500\ \text{ft}$$

$$s_A|_{t=4\,s} = 4^3 - \frac{3}{2}(4^2) = 40.0\ \text{ft}$$

Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0\ \text{ft}$$ **Ans.**

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4\ \text{ft}$$

$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192\ \text{ft}$$

Particle B has traveled

$$d_B = 2(4) + 192 = 200\ \text{ft}$$ **Ans.**

At $t = 4$ s the distance beween A and B is

$$\Delta s_{AB} = 192 - 40 = 152\ \text{ft}$$ **Ans.**

***12–24.**

A particle is moving along a straight line such that its velocity is defined as $v = (-4s^2)$ m/s, where s is in meters. If $s = 2$ m when $t = 0$, determine the velocity and acceleration as functions of time.

SOLUTION

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2}\, ds = \int_0^t -4\, dt$$

$$-s^{-1}\Big|_2^s = -4t\Big|_0^t$$

$$t = \frac{1}{4}\left(s^{-1} - 0.5\right)$$

$$s = \frac{2}{8t + 1}$$

$$v = -4\left(\frac{2}{8t + 1}\right)^2 = -\frac{16}{(8t + 1)^2}\ \text{m/s} \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \frac{256}{(8t + 1)^3}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–25.

A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

SOLUTION

Velocity: $v_0 = 27$ m/s at $t_0 = 0$ s. Applying Eq. 12–2, we have

$$(+\downarrow) \qquad dv = a\,dt$$

$$\int_{27}^{v} dv = \int_0^t -6t\,dt$$

$$v = \left(27 - 3t^2\right)\ \text{m/s} \qquad \textbf{(1)}$$

At $v = 0$, from Eq. (1)

$$0 = 27 - 3t^2 \qquad t = 3.00\ \text{s}$$

Distance Traveled: $s_0 = 0$ m at $t_0 = 0$ s. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$$(+\downarrow) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right) dt$$

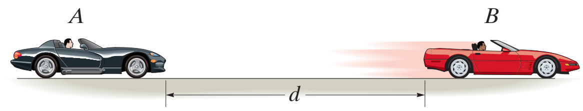
$$s = \left(27t - t^3\right)\ \text{m} \qquad \textbf{(2)}$$

At $t = 3.00$ s, from Eq. (2)

$$s = 27(3.00) - 3.00^3 = 54.0\ \text{m} \qquad \textbf{Ans.}$$

12–26.

When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B, respectively. If B maintains its constant speed, while A begins to decelerate at a_A, determine the distance d between the cars at the instant A stops.



SOLUTION

Motion of car A:

$$v = v_0 + a_c t$$

$$0 = v_A - a_A t \qquad t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B:

$$s_B = v_B t = v_B\left(\frac{v_A}{a_A}\right) = \frac{v_A v_B}{a_A}$$

The distance between cars A and B is

$$s_{BA} = |s_B - s_A| = \left|\frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A}\right| = \left|\frac{2v_A v_B - v_A^2}{2a_A}\right| \qquad \textbf{Ans.}$$

12–27.

A particle is moving along a straight line such that when it is at the origin it has a velocity of 4 m/s. If it begins to decelerate at the rate of $a = (-1.5v^{1/2})\ \text{m/s}^2$, where v is in m/s, determine the distance it travels before it stops.

SOLUTION

$$a = \frac{dv}{dt} = -1.5v^{\frac{1}{2}}$$

$$\int_4^v v^{-\frac{1}{2}}\,dv = \int_0^t -1.5\,dt$$

$$2v^{\frac{1}{2}}\Big|_4^v = -1.5t\Big|_0^t$$

$$2\left(v^{\frac{1}{2}} - 2\right) = -1.5t$$

$$v = (2 - 0.75t)^2\ \text{m/s} \qquad \textbf{(1)}$$

$$\int_0^s ds = \int_0^t (2 - 0.75t)^2\,dt = \int_0^t (4 - 3t + 0.5625t^2)\,dt$$

$$s = 4t - 1.5t^2 + 0.1875t^3 \qquad \textbf{(2)}$$

From Eq. (1), the particle will stop when

$$0 = (2 - 0.75t)^2$$

$$t = 2.667\ \text{s}$$

$$s\big|_{t=2.667} = 4(2.667) - 1.5(2.667)^2 + 0.1875(2.667)^3 = 3.56\ \text{m} \qquad \textbf{Ans.}$$

***12–28.**

A particle travels to the right along a straight line with a velocity $v = [5/(4 + s)]$ m/s, where s is in meters. Determine its deceleration when $s = 2$ m.

SOLUTION

$$v = \frac{5}{4 + s}$$

$$v\,dv = a\,ds$$

$$dv = \frac{-5\,ds}{(4 + s)^2}$$

$$\frac{5}{(4 + s)}\left(\frac{-\,5\,ds}{(4 + s)^2}\right) = a\,ds$$

$$a = \frac{-\,25}{(4 + s)^3}$$

When $s = 2$ m

$$a = -0.116 \text{ m/s}^2$$ **Ans.**

12–29.

A particle moves along a straight line with an acceleration $a = 2v^{1/2}\ \text{m/s}^2$, where v is in m/s. If $s = 0$, $v = 4$ m/s when $t = 0$, determine the time for the particle to achieve a velocity of 20 m/s. Also, find the displacement of particle when $t = 2$ s.

SOLUTION

Velocity:

$$\left(\xrightarrow{+}\right) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{2v^{1/2}}$$

$$t\Big|_0^t = v^{1/2}\Big|_4^v$$

$$t = v^{1/2} - 2$$

$$v = (t + 2)^2$$

When $v = 20$ m/s,

$$20 = (t + 2)^2$$

$$t = 2.47\ \text{s}$$ **Ans.**

Position:

$$\left(\xrightarrow{+}\right) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t (t + 2)^2\,dt$$

$$s\Big|_0^s = \frac{1}{3}(t + 2)^3\Big|_0^t$$

$$s = \frac{1}{3}[(t + 2)^3 - 2^3]$$

$$= \frac{1}{3}t(t^2 + 6t + 12)$$

When $t = 2$ s,

$$s = \frac{1}{3}(2)[(2)^2 + 6(2) + 12]$$

$$= 18.7\ \text{m}$$ **Ans.**

12–30.

As a train accelerates uniformly it passes successive kilometer marks while traveling at velocities of 2 m/s and then 10 m/s. Determine the train's velocity when it passes the next kilometer mark and the time it takes to travel the 2-km distance.

SOLUTION

Kinematics: For the first kilometer of the journey, $v_0 = 2$ m/s, $v = 10$ m/s, $s_0 = 0$, and $s = 1000$ m. Thus,

$$(\xrightarrow{+})\qquad v^2 = v_0{}^2 + 2a_c\,(s - s_0)$$

$$10^2 = 2^2 + 2a_c\,(1000 - 0)$$

$$a_c = 0.048\text{ m/s}^2$$

For the second kilometer, $v_0 = 10$ m/s, $s_0 = 1000$ m, $s = 2000$ m, and $a_c = 0.048$ m/s^2. Thus,

$$(\xrightarrow{+})\qquad v^2 = v_0{}^2 + 2a_c\,(s - s_0)$$

$$v^2 = 10^2 + 2(0.048)(2000 - 1000)$$

$$v = 14\text{ m/s} \qquad \textbf{Ans.}$$

For the whole journey, $v_0 = 2$ m/s, $v = 14$ m/s, and $a_c = 0.048$ m/s^2 Thus,

$$(\xrightarrow{+})\qquad v = v_0 + a_c t$$

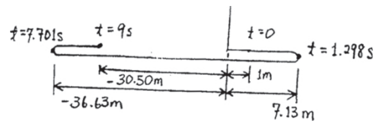
$$14 = 2 + 0.048t$$

$$t = 250\text{ s} \qquad \textbf{Ans.}$$

12–31.

The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION



$a = 2t - 9$

$$\int_{10}^{v} dv = \int_{0}^{t} (2t - 9)\, dt$$

$$v - 10 = t^2 - 9\,t$$

$$v = t^2 - 9\,t + 10$$

$$\int_{1}^{s} ds = \int_{0}^{t} (t^2 - 9t + 10)\, dt$$

$$s - 1 = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t$$

$$s = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t + 1$$

Note when $v = t^2 - 9\,t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298$ s, $\quad s = 7.13$ m

When $t = 7.701$ s, $\quad s = -36.63$ m

When $t = 9$ s, $\quad s = -30.50$ m

(a) $\quad s = -30.5$ m **Ans.**

(b) $\quad s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$\quad s_{Tot} = 56.0$ m **Ans.**

(c) $\quad v = 10$ m/s **Ans.**

***12–32.**

The acceleration of a particle traveling along a straight line is $a = \frac{1}{4}s^{1/2}$ m/s^2, where s is in meters. If $v = 0$, $s = 1$ m when $t = 0$, determine the particle's velocity at $s = 2$ m.

SOLUTION

Velocity:

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_1^s \frac{1}{4}s^{1/2}ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left.\frac{1}{6}s^{3/2}\right|_1^s$$

$$v = \frac{1}{\sqrt{3}}(s^{3/2} - 1)^{1/2} \text{ m/s}$$

When $s = 2$ m, $v = 0.781$ m/s. **Ans.**

12–33.

At $t = 0$ bullet A is fired vertically with an initial (muzzle) velocity of 450 m/s. When $t = 3$ s, bullet B is fired upward with a muzzle velocity of 600 m/s. Determine the time t, after A is fired, as to when bullet B passes bullet A. At what altitude does this occur?

SOLUTION

$$+\uparrow\ s_A = (s_A)_0 + (v_A)_0\, t + \frac{1}{2} a_c\, t^2$$

$$s_A = 0 + 450\, t + \frac{1}{2}(-9.81)\, t^2$$

$$+\uparrow s_B = (s_B)_0 + (v_B)_0\, t + \frac{1}{2}\, a_c\, t^2$$

$$s_B = 0 + 600(t - 3) + \frac{1}{2}(-9.81)(t - 3)^2$$

Require $s_A = s_B$

$$450\, t - 4.905\, t^2 = 600\, t - 1800 - 4.905\, t^2 + 29.43\, t - 44.145$$

$t = 10.3$ s **Ans.**

$h = s_A = s_B = 4.11$ km **Ans.**

12–34.

A boy throws a ball straight up from the top of a 12-m high tower. If the ball falls past him 0.75 s later, determine the velocity at which it was thrown, the velocity of the ball when it strikes the ground, and the time of flight.

SOLUTION

***Kinematics*:** When the ball passes the boy, the displacement of the ball in equal to zero.

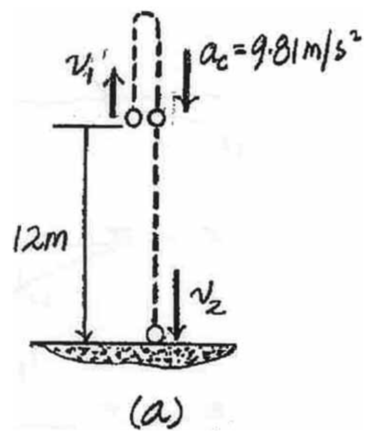
Thus, $s = 0$. Also, $s_0 = 0$, $v_0 = v_1$, $t = 0.75$ s, and $a_c = -9.81\text{ m/s}^2$.

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$0 = 0 + v_1(0.75) + \frac{1}{2}(-9.81)(0.75^2)$$

$$v_1 = 3.679\text{ m/s} = 3.68\text{ m/s}$$ **Ans.**

When the ball strikes the ground, its displacement from the roof top is $s = -12$ m. Also, $v_0 = v_1 = 3.679$ m/s, $t = t_2$, $v = v_2$, and $a_c = -9.81\text{ m/s}^2$.



$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-12 = 0 + 3.679t_2 + \frac{1}{2}(-9.81)t_2^{\,2}$$

$$4.905t_2^{\,2} - 3.679t_2 - 12 = 0$$

$$t_2 = \frac{3.679 \pm \sqrt{(-3.679)^2 - 4(4.905)(-12)}}{2(4.905)}$$

Choosing the positive root, we have

$$t_2 = 1.983\text{ s} = 1.98\text{ s}$$ **Ans.**

Using this result,

$(+\uparrow)$ $$v = v_0 + a_c t$$

$$v_2 = 3.679 + (-9.81)(1.983)$$

$$= -15.8\text{ m/s} = 15.8\text{ m/s}\downarrow$$ **Ans.**

12–35.

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2{}_f)(v^2{}_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

SOLUTION

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^{2'}} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f + v}{v_f - v}\right)\Bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v}{v_f - v}\right)$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v_f/2}{v_f - v_f/2}\right)$$

$$t = 0.549\left(\frac{v_f}{g}\right)$$ **Ans.**

***12–36.**

A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

SOLUTION

$$a = \frac{dv}{dt} = -kv^3$$

$$\int_{v_0}^{v} v^{-3}\,dv = \int_0^t -k\,dt$$

$$-\frac{1}{2}(v^{-2} - v_0^{-2}) = -kt$$

$$v = \left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \left.\frac{2\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}{2k}\right|_0^t$$

$$s = \frac{1}{k}\left[\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{v_0}\right] \qquad \textbf{Ans.}$$

12–37.

As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R+y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81 \text{ m/s}^2$ and $R = 6356 \text{ km}$, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \to \infty$.

SOLUTION

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R+y)^2}$$

$$\left.\frac{v^2}{2}\right|_v^0 = \left.\frac{g_0 R^2}{R+y}\right|_0^\infty$$

$$v = \sqrt{2g_0 R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167 \text{ m/s} = 11.2 \text{ km/s} \qquad \textbf{Ans.}$$

12–38.

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–37), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–37.

SOLUTION

From Prob. 12–37,

$$(+\uparrow) \qquad a = -g_0 \frac{R^2}{(R+y)^2}$$

Since $a\,dy = v\,dv$

then

$$-g_0 R^2 \int_{y_0}^{y} \frac{dy}{(R+y)^2} = \int_0^v v\,dv$$

$$g_0 R^2 \left[\frac{1}{R+y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R+y} - \frac{1}{R+y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R+y)(R+y_0)}}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356+500)(10^6)}}$$

$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow$ **Ans.**

12–39.

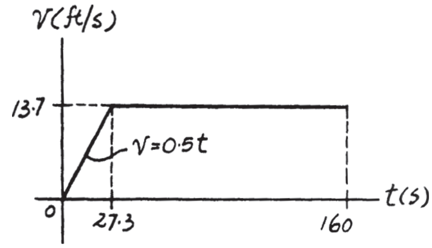
A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2. After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v–t graph for the motion.

SOLUTION

Total Distance Traveled: The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12–5 with $s_0 = 0$ and $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(t')^2 = 0.25(t')^2$$



The velocity at time t can be obtained by applying Eq. 12–4 with $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t = 0 + 0.5t = 0.5t \qquad \textbf{(1)}$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is traveling at a constant velocity of $v = 0.5t'$ (Eq. (1)). Thus, the distance for this part of motion is

$$(\overset{+}{\rightarrow}) \qquad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{\text{Tot}} = 2000$, then

$$s_{\text{Tot}} = s_1 + s_2$$

$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

$$0.25(t')^2 - 80t' + 2000 = 0$$

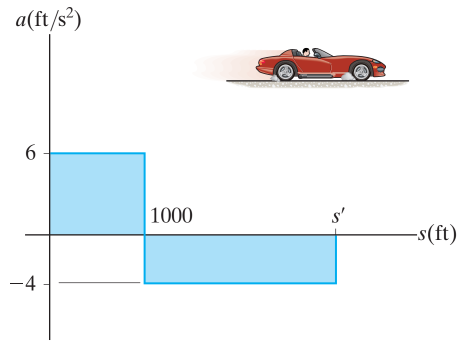
Choose a root that is less than 160 s, then

$$t' = 27.34 \text{ s} = 27.3 \text{ s} \qquad \textbf{Ans.}$$

v–t Graph: The equation for the velocity is given by Eq. (1). When $t = t' = 27.34$ s, $v = 0.5(27.34) = 13.7$ ft/s.

***12–40.**

A sports car travels along a straight road with an acceleration-deceleration described by the graph. If the car starts from rest, determine the distance s' the car travels until it stops. Construct the $v-s$ graph for $0 \le s \le s'$.



SOLUTION

v – s Graph: For $0 \le s < 1000$ ft, the initial condition is $v = 0$ at $s = 0$.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s 6\,ds$$

$$\frac{v^2}{2} = 6s$$

$$v = \left(\sqrt{12}s^{1/2}\right)\text{ ft/s}$$

When $s = 1000$ ft,

$$v = \sqrt{12}(1000)^{1/2} = 109.54 \text{ ft/s} = 110 \text{ ft/s}$$

For 1000 ft $< s \le s'$, the initial condition is $v = 109.54$ ft/s at $s = 1000$ ft.

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_{109.54 \text{ ft/s}}^{v} v\,dv = \int_{1000 \text{ ft}}^{s} -4\,ds$$

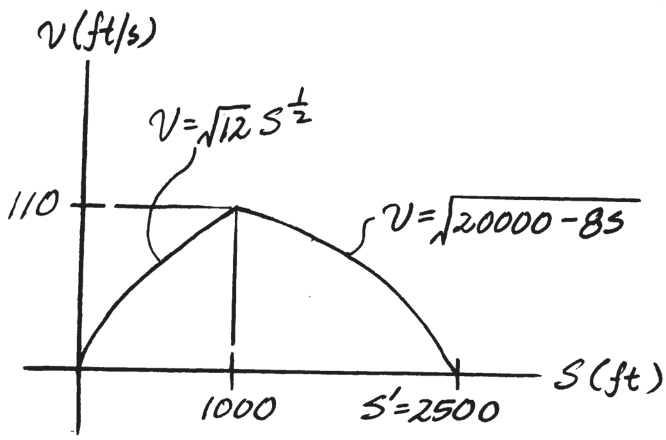
$$\left.\frac{v^2}{2}\right|_{109.54 \text{ ft/s}}^{v} = -4s\Big|_{1000 \text{ ft}}^{s}$$

$$v = \left(\sqrt{20\,000 - 8s}\right)\text{ ft/s}$$

When $v = 0$,

$$0 = \sqrt{20\,000 - 8s'} \qquad\qquad s' = 2500 \text{ ft} \qquad\qquad \textbf{Ans.}$$

The v–s graph is shown in Fig. a.



(a)

12–41.

A train starts from station A and for the first kilometer, it travels with a uniform acceleration. Then, for the next two kilometers, it travels with a uniform speed. Finally, the train decelerates uniformly for another kilometer before coming to rest at station B. If the time for the whole journey is six minutes, draw the $v-t$ graph and determine the maximum speed of the train.

SOLUTION

For stage (1) motion,

$$\left(\xrightarrow{+}\right) \qquad v_1 = v_0 + (a_c)_1 t$$

$$v_{\max} = 0 + (a_c)_1 t_1$$

$$v_{\max} = (a_c)_1 t_1 \qquad \textbf{(1)}$$

$$\left(\xrightarrow{+}\right) \qquad v_1^2 = v_0^2 + 2(a_c)_1(s_1 - s_0)$$

$${v_{\max}}^2 = 0 + 2(a_c)_1(1000 - 0)$$

$$(a_c)_1 = \frac{{v_{\max}}^2}{2000} \qquad \textbf{(2)}$$

Eliminating $(a_c)_1$ from Eqs. (1) and (2), we have

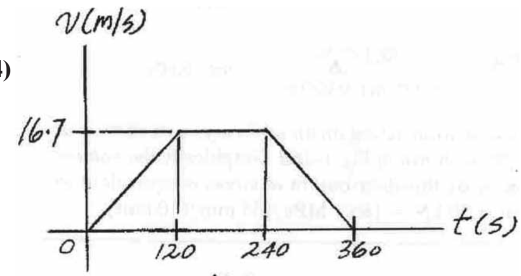
$$t_1 = \frac{2000}{v_{\max}} \qquad \textbf{(3)}$$

For stage (2) motion, the train travels with the constant velocity of $v_{\max}$ for $t = (t_2 - t_1)$. Thus,

$$\left(\xrightarrow{+}\right) \qquad s_2 = s_1 + v_1 t + \frac{1}{2}(a_c)_2 t^2$$

$$1000 + 2000 = 1000 + v_{\max}(t_2 - t_1) + 0$$

$$t_2 - t_1 = \frac{2000}{v_{\max}} \qquad \textbf{(4)}$$



(a)

For stage (3) motion, the train travels for $t = 360 - t_2$. Thus,

$$\left(\xrightarrow{+}\right) \qquad v_3 = v_2 + (a_c)_3 t$$

$$0 = v_{\max} - (a_c)_3(360 - t_2)$$

$$v_{\max} = (a_c)_3(360 - t_2) \qquad \textbf{(5)}$$

$$\left(\xrightarrow{+}\right) \qquad v_3^2 = v_2^2 + 2(a_c)_3(s_3 - s_2)$$

$$0 = {v_{\max}}^2 + 2[-(a_c)_3](4000 - 3000)$$

$$(a_c)_3 = \frac{{v_{\max}}^2}{2000} \qquad \textbf{(6)}$$

Eliminating $(a_c)_3$ from Eqs. (5) and (6) yields

$$360 - t_2 = \frac{2000}{v_{\max}} \qquad \textbf{(7)}$$

Solving Eqs. (3), (4), and (7), we have

$$t_1 = 120 \text{ s} \qquad t_2 = 240 \text{ s}$$

$$v_{\max} = 16.7 \text{ m/s} \qquad \textbf{Ans.}$$

Based on the above results, the $v-t$ graph is shown in Fig. a.

12–42.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \le t \le 4$ s.

SOLUTION

a–t Graph:

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$

$$a = (2t - 4)\ \text{m/s}^2$$

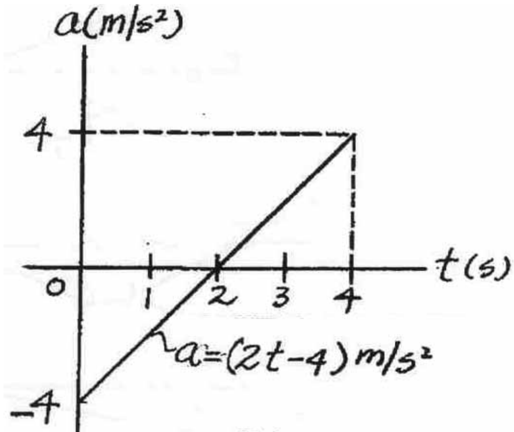
Thus,

$$a|_{t=0} = 2(0) - 4 = -4\ \text{m/s}^2$$

$$a|_{t=2} = 0$$

$$a|_{t=4\text{ s}} = 2(4) - 4 = 4\ \text{m/s}^2$$

The $a-t$ graph is shown in Fig. a.



(a)

v–t Graph: The slope of the $v-t$ graph is zero when $a = \dfrac{dv}{dt} = 0$. Thus,

$$a = 2t - 4 = 0 \qquad t = 2\text{ s}$$

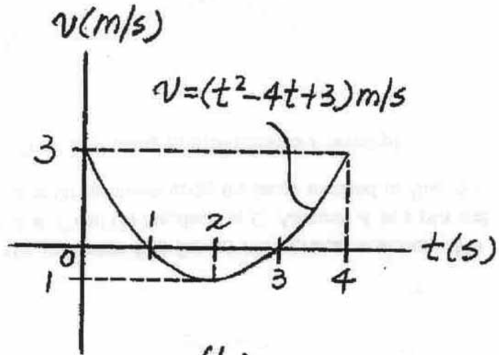
The velocity of the particle at $t = 0$ s, 2 s, and 4 s are

$$v|_{t=0\text{ s}} = 0^2 - 4(0) + 3 = 3\ \text{m/s}$$

$$v|_{t=2\text{ s}} = 2^2 - 4(2) + 3 = -1\ \text{m/s}$$

$$v|_{t=4\text{ s}} = 4^2 - 4(4) + 3 = 3\ \text{m/s}$$

The $v-t$ graph is shown in Fig. b.

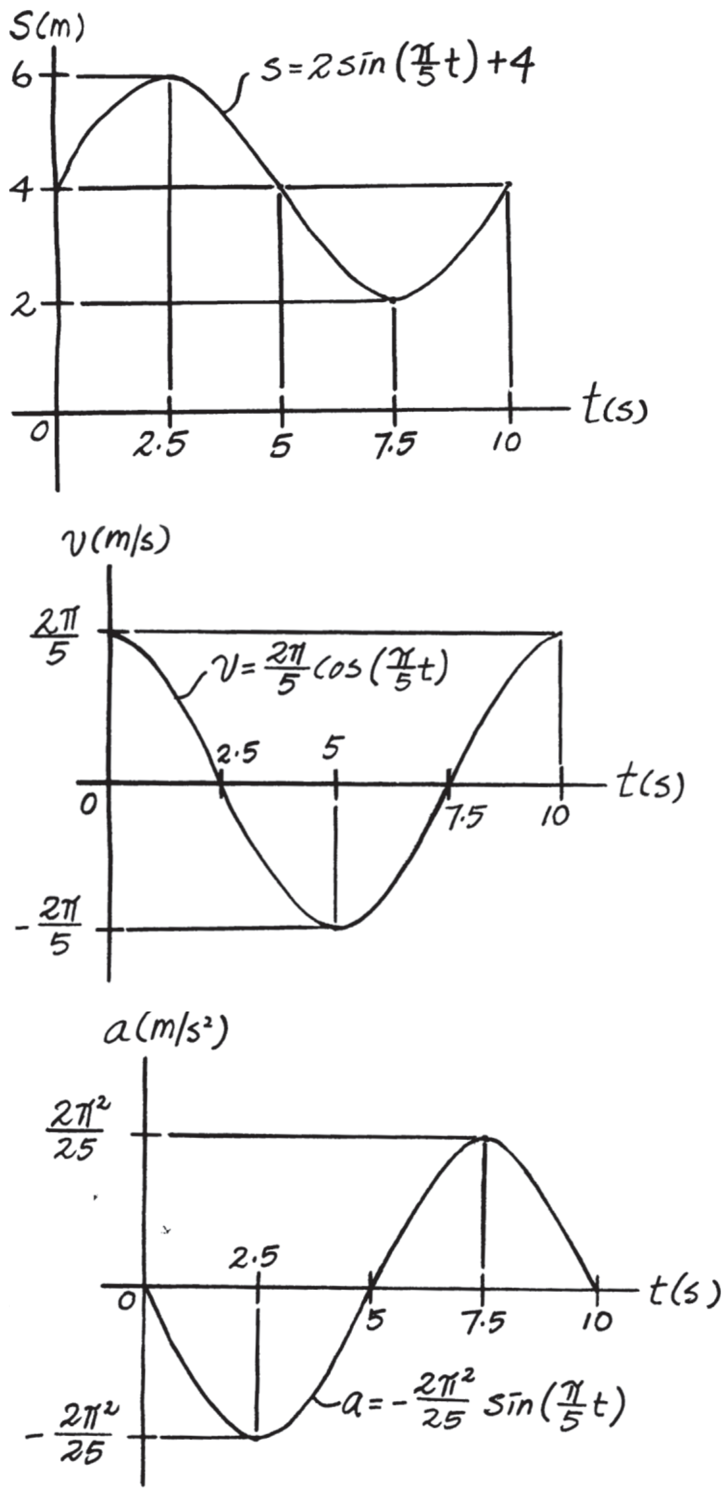


(b)

12–43.

If the position of a particle is defined by $s = [2 \sin [(\pi/5)t] + 4]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \leq t \leq 10$ s.

SOLUTION



***12–44.**

An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s^2 until it reaches a constant speed of 220 mi/h. Draw the *s–t*, *v–t*, and *a–t* graphs that describe the motion.

SOLUTION

$v_1 = 0$

$$v_2 = 162\frac{\text{mi}}{\text{h}}\,\frac{(1\text{h})\ 5280\text{ ft}}{(3600\text{ s})(1\text{ mi})} = 237.6\text{ ft/s}$$

$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$

$(237.6)^2 = 0^2 + 2(a_c)(5000 - 0)$

$a_c = 5.64538\text{ ft/s}^2$

$v_2 = v_1 + a_c t$

$237.6 = 0 + 5.64538\,t$

$t = 42.09 = 42.1\text{ s}$

$$v_3 = 220\,\frac{\text{mi}}{\text{h}}\,\frac{(1\text{h})\ 5280\text{ ft}}{(3600\text{ s})(1\text{ mi})} = 322.67\text{ ft/s}$$

$v_3^2 = v_2^2 + 2a_c(s_3 - s_2)$

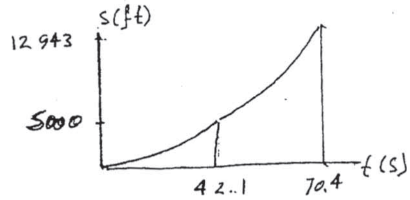
$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$

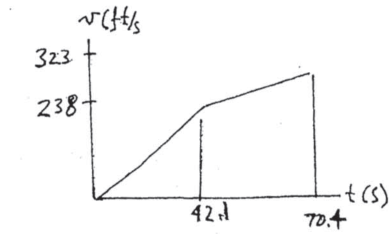
$s = 12\,943.34\text{ ft}$

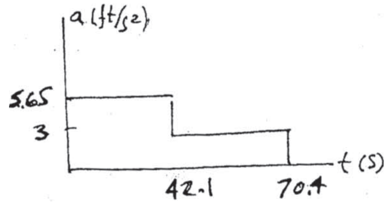
$v_3 = v_2 + a_c t$

$322.67 = 237.6 + 3\,t$

$t = 28.4\text{ s}$

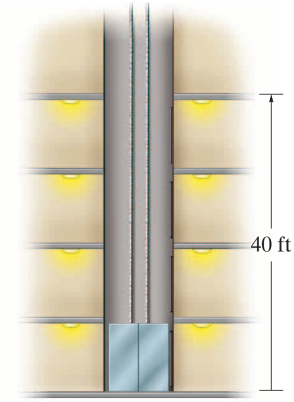






12–45.

The elevator starts from rest at the first floor of the building. It can accelerate at 5 ft/s^2 and then decelerate at 2 ft/s^2. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the a–t, v–t, and s–t graphs for the motion.



SOLUTION

$$+\uparrow \; v_2 = v_1 + a_c t_1$$

$$v_{max} = 0 + 5\, t_1$$

$$+\uparrow \; v_3 = v_2 + a_c t$$

$$0 = v_{max} - 2\, t_2$$

Thus

$$t_1 = 0.4\, t_2$$

$$+\uparrow \; s_2 = s_1 + v_1 t_1 + \frac{1}{2} a_c t_1^2$$

$$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5\, t_1^2$$

$$+\uparrow \; 40 - h = 0 + v_{max} t_2 - \frac{1}{2}(2)\, t_2^2$$

$$+\uparrow \; v^2 = v_1^2 + 2\, a_c (s - s_1)$$

$$v_{max}^2 = 0 + 2(5)(h - 0)$$

$$v_{max}^2 = 10h$$

$$0 = v_{max}^2 + 2(-2)(40 - h)$$

$$v_{max}^2 = 160 - 4h$$

Thus,

$$10\, h = 160 - 4h$$

$$h = 11.429 \text{ ft}$$

$$v_{max} = 10.69 \text{ ft/s}$$

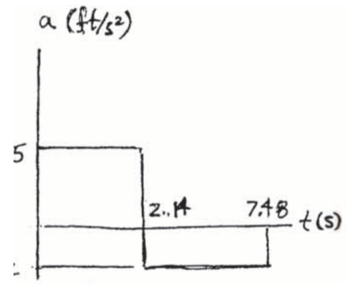
$$t_1 = 2.138 \text{ s}$$

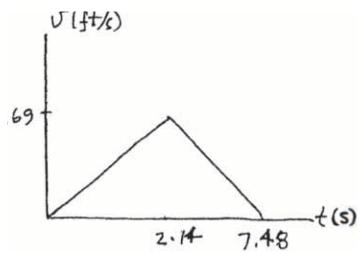
$$t_2 = 5.345 \text{ s}$$

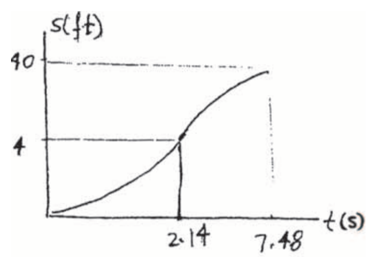
$$t = t_1 + t_2 = 7.48 \text{ s}$$ **Ans.**

When $t = 2.145$, $v = v_{max} = 10.7$ ft/s

and $h = 11.4$ ft.

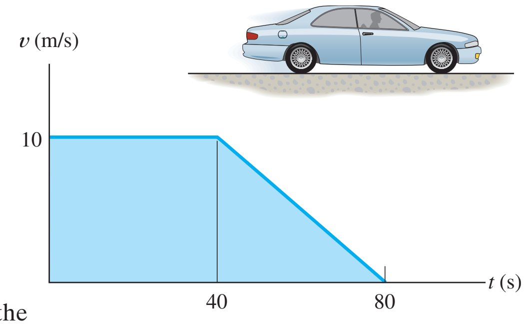






12–46.

The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops ($t = 80$ s). Construct the a–t graph.



SOLUTION

Distance Traveled: The total distance traveled can be obtained by computing the area under the $v - t$ graph.

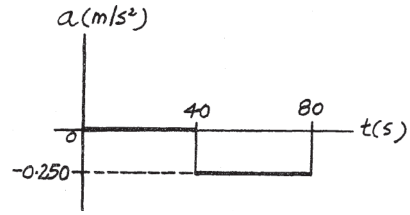
$$s = 10(40) + \frac{1}{2}(10)(80 - 40) = 600 \text{ m} \qquad \textbf{Ans.}$$

a – t Graph: The acceleration in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval $0 \text{ s} \leq t < 40$ s,

$$a = \frac{dv}{dt} = 0$$

For time interval $40 \text{ s} < t \leq 80$ s, $\frac{v - 10}{t - 40} = \frac{0 - 10}{80 - 40}$, $v = \left(-\frac{1}{4}t + 20\right)$ m/s.

$$a = \frac{dv}{dt} = -\frac{1}{4} = -0.250 \text{ m/s}^2$$

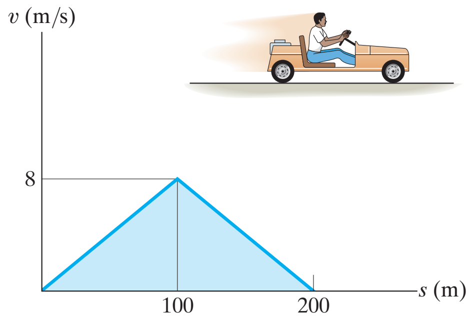


For $0 \leq t < 40$ s, $a = 0$.

For $40 \text{ s} < t \leq 80$, $a = -0.250 \text{ m/s}^2$.

12–47.

The v–s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at $s = 50$ m and $s = 150$ m. Draw the a–s graph.



SOLUTION

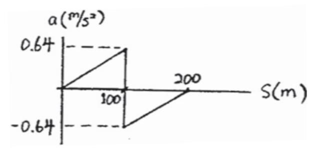
For $0 \leq s < 100$

$v = 0.08\,s, \qquad dv = 0.08\,ds$

$a\,ds = (0.08\,s)(0.08\,ds)$

$a = 6.4(10^{-3})\,s$

At $s = 50$ m, $\quad a = 0.32\text{ m/s}^2$ **Ans.**



For $100 < s < 200$

$v = -0.08\,s + 16,$

$dv = -0.08\,ds$

$a\,ds = (-0.08\,s + 16)(-0.08\,ds)$

$a = 0.08(0.08\,s - 16)$

At $s = 150$ m, $\quad a = -0.32\text{ m/s}^2$ **Ans.**

Also,

$v\,dv = a\,ds$

$a = v\left(\frac{dv}{ds}\right)$

At $s = 50$ m,

$a = 4\left(\frac{8}{100}\right) = 0.32\text{ m/s}^2$ **Ans.**

At $s = 150$ m,

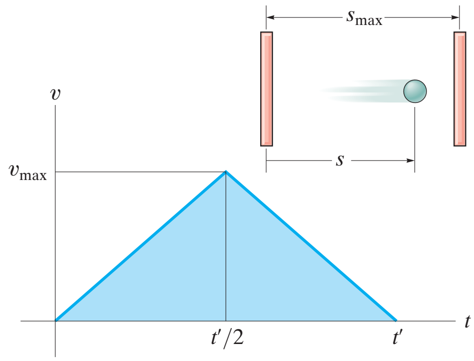
$a = 4\left(\frac{-8}{100}\right) = -0.32\text{ m/s}^2$ **Ans.**

At $s = 100$ m, a changes from $a_{max} = 0.64\text{ m/s}^2$

to $a_{min} = -0.64\text{ m/s}^2$.

***12–48.**

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of $4\ \text{m/s}^2$. If the plates are spaced 200 mm apart, determine the maximum velocity v_{max} and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.



SOLUTION

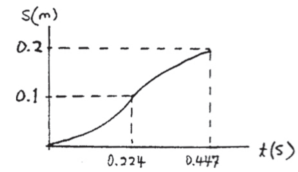
$a_c = 4\ \text{m/s}^2$

$\frac{s}{2} = 100\ \text{mm} = 0.1\ \text{m}$

$v^2 = v_0^2 + 2\,a_c(s - s_0)$

$v_{max}^2 = 0 + 2(4)(0.1 - 0)$

$v_{max} = 0.89442\ \text{m/s} \qquad = 0.894\ \text{m/s}$ **Ans.**



$v = v_0 + a_c\,t'$

$0.89442 = 0 + 4\left(\frac{t'}{2}\right)$

$t' = 0.44721\ \text{s} \qquad = 0.447\ \text{s}$ **Ans.**

$s = s_0 + v_0\,t + \frac{1}{2}\,a_c\,t^2$

$s = 0 + 0 + \frac{1}{2}\,(4)(t)^2$

$s = 2\,t^2$

When $t = \frac{0.44721}{2} = 0.2236 = 0.224$ s,

$s = 0.1$ m

$\int_{0.894}^{v} ds = -\int_{0.2235}^{t} 4\,dt$

$v = -4\,t + 1.788$

$\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t + 1.788)\,dt$

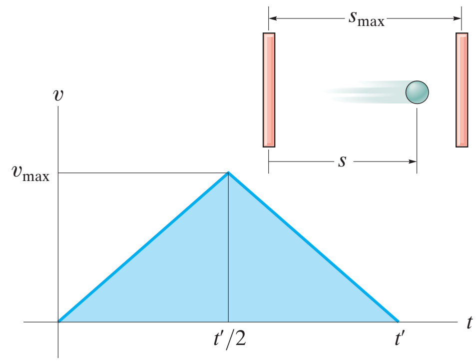
$s = -2\,t^2 + 1.788\,t - 0.2$

When $t = 0.447$ s,

$s = 0.2$ m

12–49.

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



SOLUTION

For $0 < t < 0.1$ s,

$v = 100\,t$

$a = \dfrac{dv}{dt} = 100$

$ds = v\,dt$

$\displaystyle\int_0^s ds = \int_0^t 100\,t\,dt$

$s = 50\,t^2$

When $t = 0.1$ s,

$s = 0.5$ m

For $0.1\text{ s} < t < 0.2$ s,

$v = -100\,t + 20$

$a = \dfrac{dv}{dt} = -100$

$ds = v\,dt$

$\displaystyle\int_{0.5}^s ds = \int_{0.1}^t (-100t + 20)\mathrm{d}t$

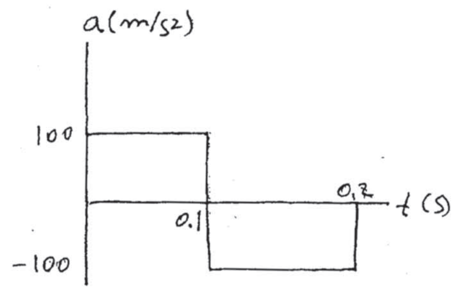
$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$

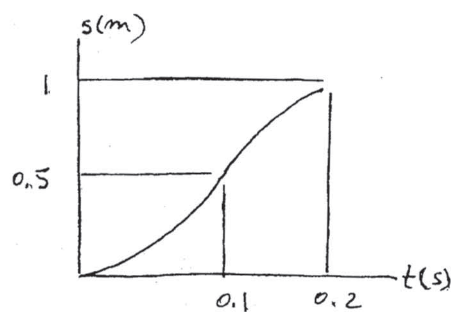
$s = -50\,t^2 + 20\,t - 1$

When $t = 0.2$ s,

$s = 1$ m

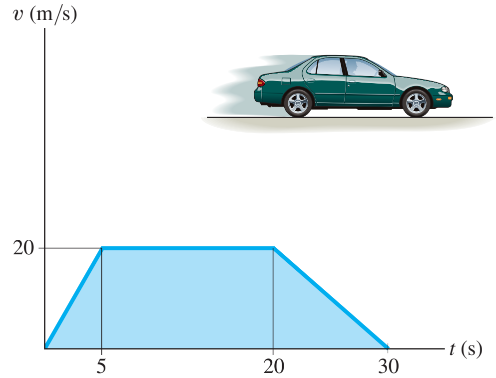
When $t = 0.1$ s, $s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When t = 0.2 s, $s = 1$ m.





12–50.

The $v-t$ graph of a car while traveling along a road is shown. Draw the $s-t$ and $a-t$ graphs for the motion.



SOLUTION

$0 \leq t \leq 5 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20}{5} = 4 \text{ m/s}^2$

$5 \leq t \leq 20 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{20 - 20}{20 - 5} = 0 \text{ m/s}^2$

$20 \leq t \leq 30 \qquad a = \dfrac{\Delta v}{\Delta t} = \dfrac{0 - 20}{30 - 20} = -2 \text{ m/s}^2$

From the $v-t$ graph at $t_1 = 5$ s, $t_2 = 20$ s, and $t_3 = 30$ s,

$$s_1 = A_1 = \frac{1}{2}(5)(20) = 50 \text{ m}$$

$$s_2 = A_1 + A_2 = 50 + 20(20 - 5) = 350 \text{ m}$$

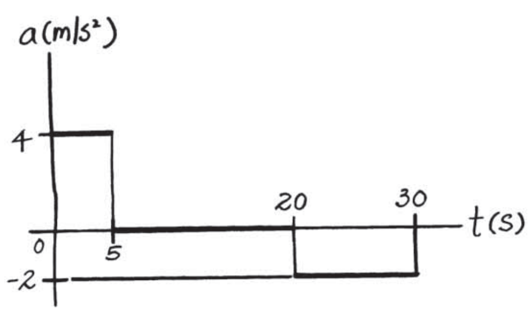
$$s_3 = A_1 + A_2 + A_3 = 350 + \frac{1}{2}(30 - 20)(20) = 450 \text{ m}$$

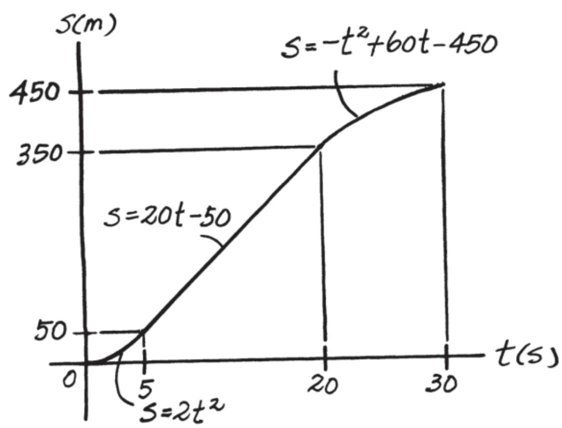
The equations defining the portions of the s–t graph are

$0 \leq t \leq 5 \text{ s} \qquad v = 4t; \qquad ds = v\,dt; \qquad \displaystyle\int_0^s ds = \int_0^t 4t\,dt; \qquad s = 2t^2$

$5 \leq t \leq 20 \text{ s} \qquad v = 20; \qquad ds = v\,dt; \qquad \displaystyle\int_{50}^s ds = \int_5^t 20\,dt; \qquad s = 20t - 50$

$20 \leq t \leq 30 \text{ s} \qquad v = 2(30 - t); \qquad ds = v\,dt; \qquad \displaystyle\int_{350}^s ds = \int_{20}^t 2(30 - t)\,dt; \qquad s = -t^2 + 60t - 450$





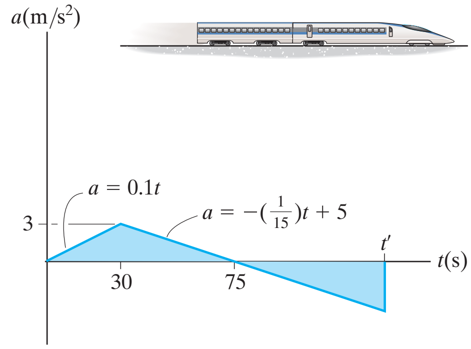
For $0 \leq t < 5$ s, $a = 4 \text{ m/s}^2$.

For $20 \text{ s} < t \leq 30$ s, $a = -2 \text{ m/s}^2$.

At $t = 5$ s, $s = 50$ m. At $t = 20$ s, $s = 350$ m.

12–51.

The a–t graph of the bullet train is shown. If the train starts from rest, determine the elapsed time t' before it again comes to rest. What is the total distance traveled during this time interval? Construct the v–t and s–t graphs.



SOLUTION

$v - t$ Graph: For the time interval $0 \leq t < 30$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$(\xrightarrow{+}) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.1t\,dt$$

$$v = \left(0.05t^2\right) \text{ m/s}$$

When $t = 30$ s,

$$v\big|_{t=30\text{ s}} = 0.05\left(30^2\right) = 45 \text{ m/s}$$

or the time interval $30\text{ s} < t \leq t'$, the initial condition is $v = 45$ m/s at $t = 30$ s.

$$(\xrightarrow{+}) \qquad dv = a\,dt$$

$$\int_{45\text{ m/s}}^v dv = \int_{30\text{ s}}^t \left(-\frac{1}{15}t + 5\right)dt$$

$$v = \left(-\frac{1}{30}t^2 + 5t - 75\right) \text{ m/s}$$

Thus, when $v = 0$,

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

Choosing the root $t' > 75$ s,

$$t' = 133.09\text{ s} = 133\text{ s} \qquad \textbf{Ans.}$$

Also, the change in velocity is equal to the area under the a–t graph. Thus,

$$\Delta v = \int a\,dt$$

$$0 = \frac{1}{2}(3)(75) + \frac{1}{2}\left[\left(-\frac{1}{15}t' + 5\right)(t' - 75)\right]$$

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

This equation is the same as the one obtained previously.

The slope of the v–$t$graph is zero when $t = 75$ s, which is the instant $a = \dfrac{dv}{dt} = 0$. Thus,

$$v\big|_{t=75\text{ s}} = -\frac{1}{30}\left(75^2\right) + 5(75) - 75 = 112.5 \text{ m/s}$$

12–51. continued

The v–t graph is shown in Fig. a.

$s-t$ Graph: Using the result of v, the equation of the s–t graph can be obtained by integrating the kinematic equation $ds = vdt$. For the time interval $0 \le t < 30$ s, the initial condition $s = 0$ at $t = 0$ s will be used as the integration limit. Thus,

$$(\overset{+}{\rightarrow}) \qquad ds = vdt$$

$$\int_0^s ds = \int_0^t 0.05t^2\,dt$$

$$s = \left(\frac{1}{60}t^3\right)\text{ m}$$

When $t = 30$ s,

$$s\big|_{t=30\text{ s}} = \frac{1}{60}\left(30^3\right) = 450\text{ m}$$

For the time interval $30\text{ s} < t \le t' = 133.09$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = vdt$$

$$\int_{450\text{ m}}^s ds = \int_{30\text{ s}}^t \left(-\frac{1}{30}t^2 + 5t - 75\right)dt$$

$$s = \left(-\frac{1}{90}t^3 + \frac{5}{2}t^2 - 75t + 750\right)\text{ m}$$

When $t = 75$ s and $t' = 133.09$ s,

$$s\big|_{t=75\text{ s}} = -\frac{1}{90}\left(75^3\right) + \frac{5}{2}\left(75^2\right) - 75(75) + 750 = 4500\text{ m}$$

$$s\big|_{t=133.09\text{ s}} = -\frac{1}{90}\left(133.09^3\right) + \frac{5}{2}\left(133.09^2\right) - 75(133.09) + 750 = 8857\text{ m} \qquad \textbf{Ans.}$$

The s–t graph is shown in Fig. b.

When $t = 30$ s,

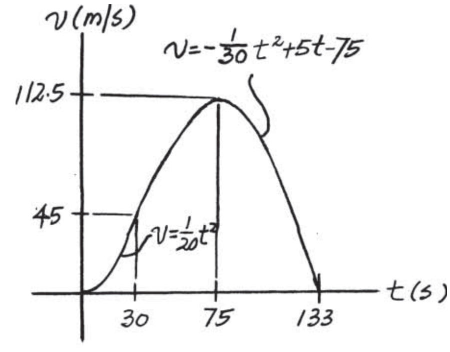
$v = 45$ m/s and $s = 450$ m.

When $t = 75$ s,

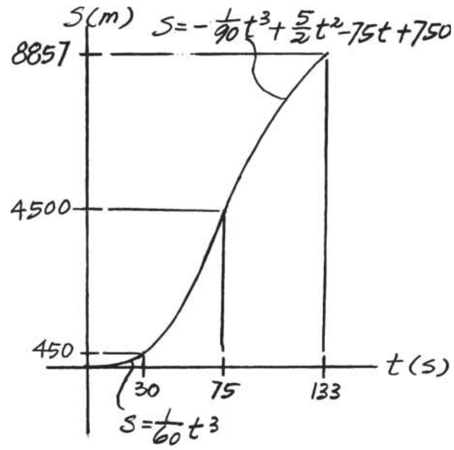
$v = v_{\max} = 112.5$ m/s and $s = 4500$ m.

When $t = 133$ s,

$v = 0$ and $s = 8857$ m.



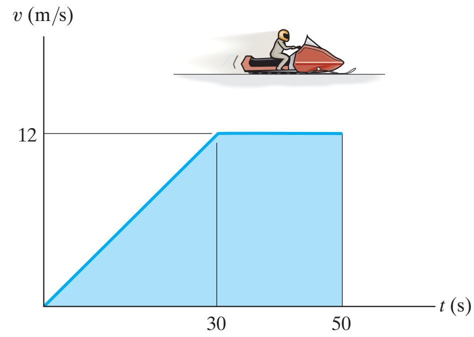
(a)



(b)

***12–52.**

The snowmobile moves along a straight course according to the v–t graph. Construct the s–t and a–t graphs for the same 50-s time interval. When $t = 0$, $s = 0$.



SOLUTION

***s–t* Graph:** The position function in terms of time t can be obtained by applying $v = \dfrac{ds}{dt}$. For time interval $0 \text{ s} \leq t < 30 \text{ s}$, $v = \dfrac{12}{30}t = \left(\dfrac{2}{5}t\right)$ m/s.

$$ds = v dt$$

$$\int_0^s ds = \int_0^t \frac{2}{5} t dt$$

$$s = \left(\frac{1}{5}t^2\right) \text{ m}$$

At $t = 30$ s, $$s = \frac{1}{5}\left(30^2\right) = 180 \text{ m}$$

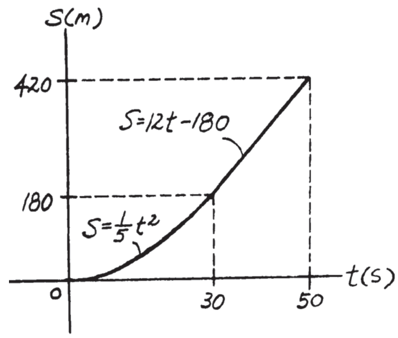
For time interval **$30 \text{ s} < t \leq 50 \text{ s}$**,

$$ds = v dt$$

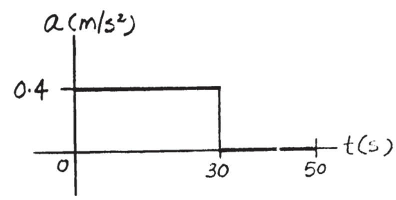
$$\int_{180 \text{ m}}^s ds = \int_{30 \, s}^t 12 dt$$

$$s = (12t - 180) \text{ m}$$

At $t = 50$ s, $$s = 12(50) - 180 = 420 \text{ m}$$

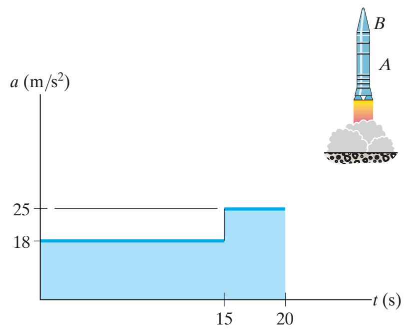


***a–t* Graph:** The acceleration function in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval **$0 \text{ s} \leq t < 30 \text{ s}$** and **$30 \text{ s} < t \leq 50 \text{ s}$**, $a = \dfrac{dv}{dt} = \dfrac{2}{5} = 0.4 \text{ m/s}^2$ and $a = \dfrac{dv}{dt} = 0$, respectively.



12–53.

A two-stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage A burns out and the second stage B ignites. Plot the $v-t$ and $s-t$ graphs which describe the two-stage motion of the missile for $0 \le t \le 20$ s.



SOLUTION

Since $v = \int a\,dt$, the constant lines of the a–t graph become sloping lines for the v–t graph.

The numerical values for each point are calculated from the total area under the a–t graph to the point.

At $t = 15$ s, $\quad v = (18)(15) = 270$ m/s

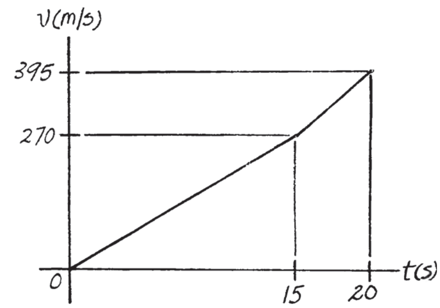
At $t = 20$ s, $\quad v = 270 + (25)(20 - 15) = 395$ m/s

Since $s = \int v\,dt$, the sloping lines of the v–t graph become parabolic curves for the s–t graph.

The numerical values for each point are calculated from the total area under the v–t graph to the point.

At $t = 15$ s, $\quad s = \frac{1}{2}(15)(270) = 2025$ m

At $t = 20$ s, $\quad s = 2025 + 270(20 - 15) + \frac{1}{2}(395 - 270)(20 - 15) = 3687.5 \text{ m} = 3.69 \text{ km}$



Also:

$0 \le t \le 15$:

$$a = 18 \text{ m/s}^2$$

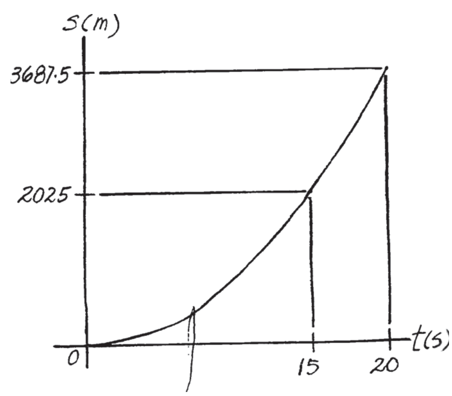
$$v = v_0 + a_c t = 0 + 18t$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 0 + 0 + 9t^2$$

When $t = 15$:

$$v = 18(15) = 270 \text{ m/s}$$

$$s = 9(15)^2 = 2025 \text{ m} = 2.025 \text{ km}$$



$15 \le t \le 20$:

$$a = 25 \text{ m/s}^2$$

$$v = v_0 + a_c t = 270 + 25(t - 15)$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2 = 2025 + 270(t - 15) + \frac{1}{2}(25)(t - 15)^2$$

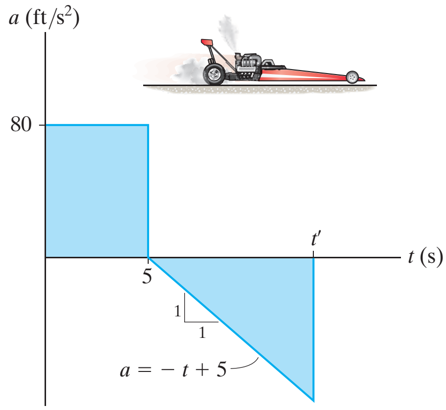
When $t = 20$:

$$v = 395 \text{ m/s}$$

$$s = 3687.5 \text{ m} = 3.69 \text{ km}$$

12–54.

The dragster starts from rest and has an acceleration described by the graph. Determine the time t' for it to stop. Also, what is its maximum speed? Construct the $v-t$ and $s-t$ graphs for the time interval $0 \le t \le t'$.



SOLUTION

v–t Graph: For the time interval $0 \le t < 5$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$\left(\overset{+}{\rightarrow}\right) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 80\,dt$$

$$v = (80t) \text{ ft/s}$$

The maximum speed occurs at the instant when the acceleration changes sign when $t = 5$ s. Thus,

$$v_{\max} = v|_{t=5\,\text{s}} = 80(5) = 400 \text{ ft/s} \qquad \textbf{Ans.}$$

For the time interval $5 < t \le t'$, the initial condition is $v = 400$ ft/s when $t = 5$ s.

$$\left(\overset{+}{\rightarrow}\right) \qquad dv = a\,dt$$

$$\int_{400\,\text{ft/s}}^{v} dv = \int_{5\,\text{s}}^{t} (-t + 5)\,dt$$

$$v = \left(-\frac{t^2}{2} + 5t + 387.5\right) \text{ft/s}$$

Thus when $v = 0$,

$$0 = -\frac{t'^2}{2} + 5t' + 387.5$$

Choosing the positive root,

$$t' = 33.28 \text{ s} = 33.3 \text{ s} \qquad \textbf{Ans.}$$

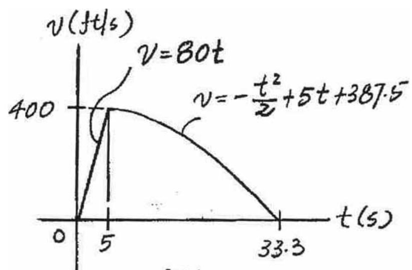
Also, the change in velocity is equal to the area under the $a-t$ graph. Thus

$$\Delta v = \int a\,dt$$

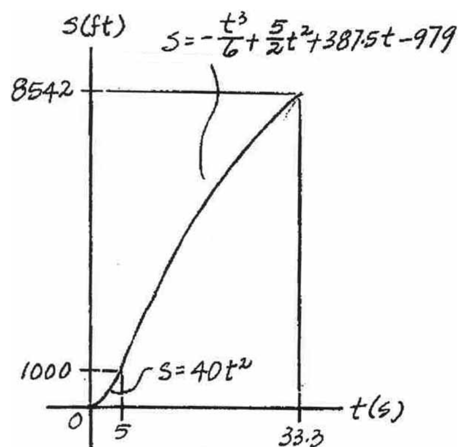
$$0 = 80(5) + \left\{\frac{1}{2}[(-t' + 5)(t' - 5)]\right\}$$

$$0 = -\frac{t'^2}{2} + 5t' + 387.5$$

This quadratic equation is the same as the one obtained previously. The $v-t$ graph is shown in Fig. a.



(a)



(b)

12–54. continued

s–t Graph: For the time interval $0 \leq t < 5\,\text{s}$, the initial condition is $s = 0$ when $t = 0\,\text{s}$.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t 80\,dt$$

$$s = (40t^2)\text{ ft}$$

When $t = 5\,\text{s}$,

$$s|_{t=5\,\text{s}} = 40(5^2) = 1000\text{ ft}$$

For the time interval $5\,\text{s} < t \leq t' = 45\,\text{s}$, the initial condition is $s = 1000\text{ ft}$ when $t = 5\,\text{s}$.

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_{1000\,\text{ft}}^s ds = \int_{5\,\text{s}}^t \left(-\frac{t^2}{2} + 5t + 387.5\right)dt$$

$$s = \left(-\frac{t^3}{6} + \frac{5}{2}t^2 + 387.5t - 979.17\right)\text{ft}$$

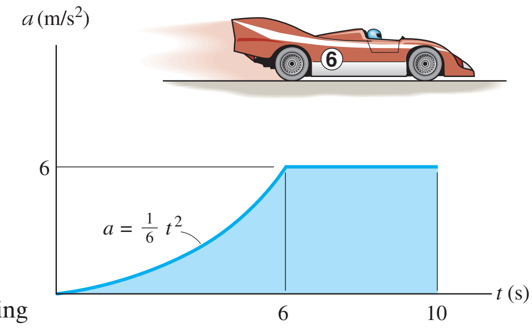
When $t = t' = 33.28\text{ s}$,

$$s|_{t=33.28\,\text{s}} = -\frac{33.28^3}{6} + \frac{5}{2}(33.28^2) + 387.5(33.28) - 979.17 = 8542\text{ ft}$$

The s–t graph is shown in Fig. *b*.

12–55.

A race car starting from rest travels along a straight road and for 10 s has the acceleration shown. Construct the v–t graph that describes the motion and find the distance traveled in 10 s.



SOLUTION

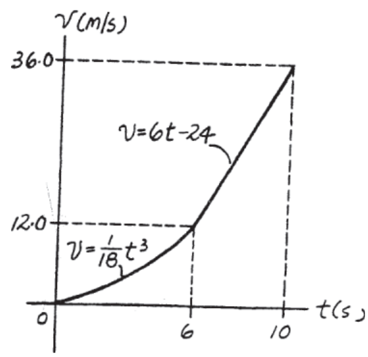
v–t Graph: The velocity function in terms of time t can be obtained by applying formula $a = \dfrac{dv}{dt}$. For time interval $0\text{ s} \le t < 6\text{ s}$,

$$dv = adt$$

$$\int_0^v dv = \int_0^t \frac{1}{6}t^2 dt$$

$$v = \left(\frac{1}{18}t^3\right)\text{ m/s}$$

At $t = 6$ s, $\quad v = \dfrac{1}{18}\left(6^3\right) = 12.0\text{ m/s},$



For time interval $6\text{ s} < t \le 10\text{ s}$,

$$dv = adt$$

$$\int_{12.0\text{m/s}}^v dv = \int_{6s}^t 6dt$$

$$v = (6t - 24)\text{ m/s}$$

At $t = 10$ s, $\quad v = 6(10) - 24 = 36.0\text{ m/s}$

Position: The position in terms of time t can be obtained by applying $v = \dfrac{ds}{dt}$. For time interval $0\text{ s} \le t < 6\text{ s}$,

$$ds = vdt$$

$$\int_0^s ds = \int_0^t \frac{1}{18}t^3\,dt$$

$$s = \left(\frac{1}{72}t^4\right)\text{m}$$

When $t = 6$ s, $v = 12.0$ m/s and $s = \dfrac{1}{72}\left(6^4\right) = 18.0$ m.

For time interval $6\text{ s} < t \le 10\text{ s}$,

$$ds = vdt$$

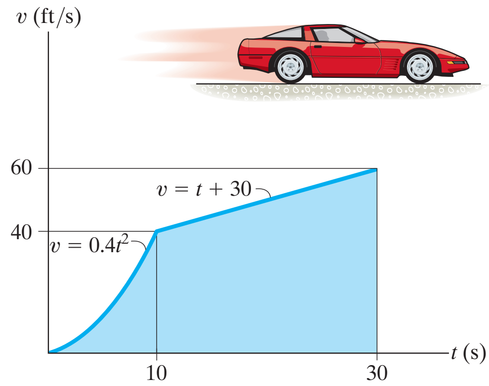
$$\int_{18.0\text{ m}}^s dv = \int_{6s}^t (6t - 24)dt$$

$$s = \left(3t^2 - 24t + 54\right)\text{m}$$

When $t = 10$ s, $v = 36.0$ m/s and $s = 3\left(10^2\right) - 24(10) + 54 = 114$ m **Ans.**

***12–56.**

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the a–t graph and determine the maximum acceleration during the 30-s time interval. The car starts from rest at $s = 0$.



SOLUTION

For $t < 10$ s:

$$v = 0.4t^2$$

$$a = \frac{dv}{dt} = 0.8t$$

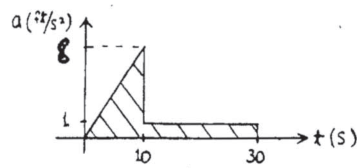
At $t = 10$ s:

$$a = 8 \text{ ft/s}^2$$

For $10 < t \leq 30$ s:

$$v = t + 30$$

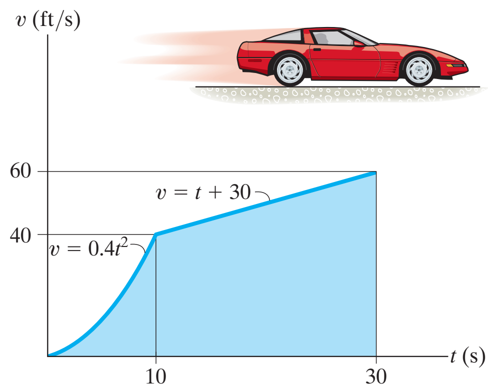
$$a = \frac{dv}{dt} = 1$$



$$a_{max} = 8 \text{ ft/s}^2$$ **Ans.**

12–57.

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t graph and determine the average speed and the distance traveled for the 30-s time interval. The car starts from rest at $s = 0$.



SOLUTION

For $t < 10$ s,

$$v = 0.4t^2$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 0.4t^2\,dt$$

$$s = 0.1333t^3$$

At $t = 10$ s,

$$s = 133.3 \text{ ft}$$

For $10 < t < 30$ s,

$$v = t + 30$$

$$ds = v\,dt$$

$$\int_{133.3}^s ds = \int_{10}^t (t + 30)\,dt$$

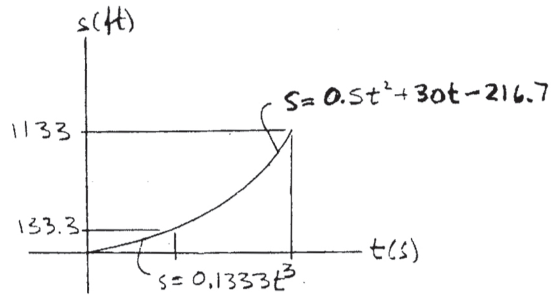
$$s = 0.5t^2 + 30t - 216.7$$

At $t = 30$ s,

$$s = 1133 \text{ ft}$$

$$(v_{sp})_{\text{Avg}} = \frac{\Delta s}{\Delta t} = \frac{1133}{30} = 37.8 \text{ ft/s}$$ **Ans.**

$$s_T = 1133 \text{ ft} = 1.13(10^3) \text{ ft}$$ **Ans.**

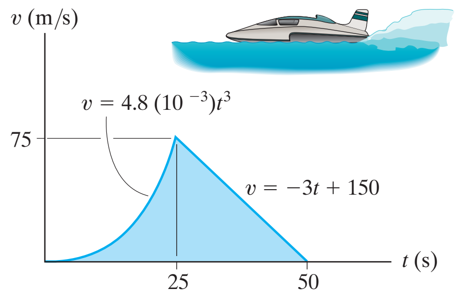


When $t = 0$ s, $s = 133$ ft.

When $t = 30$ s, $s = s_I = 1.33\ (10^3)$ ft

12–58.

The jet-powered boat starts from rest at $s = 0$ and travels along a straight line with the speed described by the graph. Construct the $s-t$ and $a-t$ graph for the time interval $0 \leq t \leq 50$ s.



SOLUTION

s–t Graph: The initial condition is $s = 0$ when $t = 0$.

$$(\overset{+}{\rightarrow}) \qquad ds = vdt$$

$$\int_0^s ds = \int_0^t 4.8(10^{-3})t^3\,dt$$

$$s = [1.2(10^{-3})t^4]\text{m}$$

At $t = 25$ s,

$$s|_{t=25\text{ s}} = 1.2(10^{-3})(25^4) = 468.75\text{ m}$$

For the time interval $25\text{ s} < t \leq 50\text{ s}$, the initial condition $s = 468.75$ m when $t = 25$ s will be used.

$$(\overset{+}{\rightarrow}) \qquad ds = vdt$$

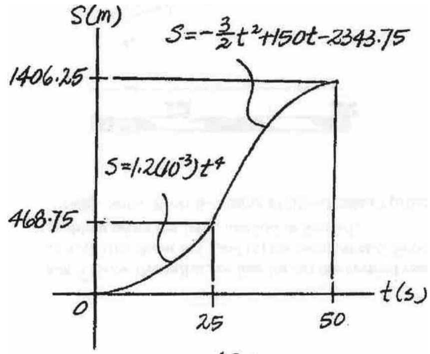
$$\int_{468.75\text{ m}}^s ds = \int_{25\text{ s}}^t (-3t + 150)\,dt$$

$$s = \left(-\frac{3}{2}t^2 + 150t - 2343.75\right)\text{m}$$

When $t = 50$ s,

$$s|_{t=50\text{ s}} = -\frac{3}{2}(50^2) + 150(50) - 2343.75 = 1406.25\text{ m}$$

The $s-t$ graph is shown in Fig. *a*.



(a)

a–t Graph: For the time interval $0 \leq t < 25$ s,

$$a = \frac{dv}{dt} = \frac{d}{dt}[4.8(10^{-3})t^3] = (0.0144t^2)\text{ m/s}^2$$

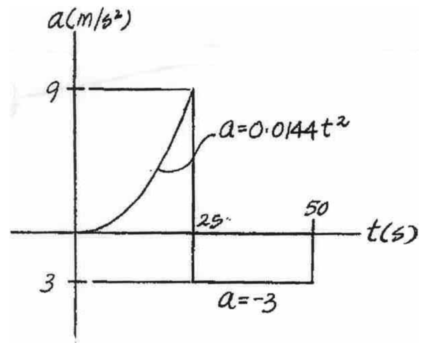
When $t = 25$ s,

$$a|_{t=25\text{ s}} = 0.0144(25^2)\text{ m/s}^2 = 9\text{ m/s}^2$$

For the time interval $25\text{ s} < t \leq 50\text{ s}$,

$$a = \frac{dv}{dt} = \frac{d}{dt}(-3t + 150) = -3\text{ m/s}^2$$

The $a-t$ graph is shown in Fig. *b*.



(b)

When $t = 25$ s,

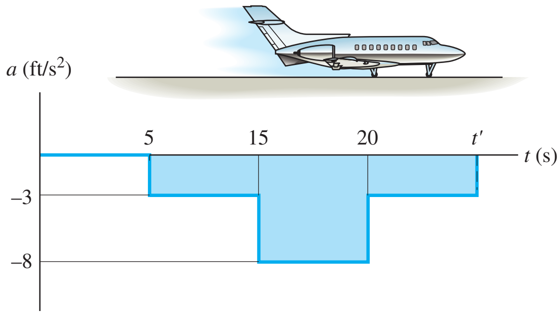
$$a = a_{max} = 9\text{ m/s}^2 \text{ and } s = 469\text{ m}.$$

When $t = 50$ s,

$$s = 1406\text{ m}.$$

12–59.

An airplane lands on the straight runway, originally traveling at 110 ft/s when $s = 0$. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane and construct the s–t graph for the motion.



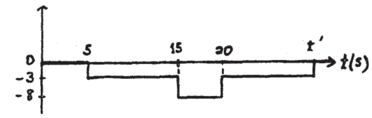
SOLUTION

$v_0 = 110\text{ ft/s}$

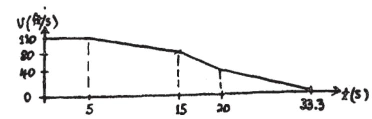
$\Delta v = \int a\, dt$

$0 - 110 = -3(15 - 5) - 8(20 - 15) - 3(t' - 20)$

$t' = 33.3\text{ s}$ **Ans.**



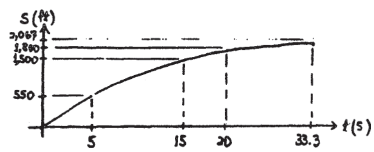
$s\big|_{t = 5\text{s}} = 550\text{ ft}$



$s\big|_{t = 15\text{s}} = 1500\text{ ft}$

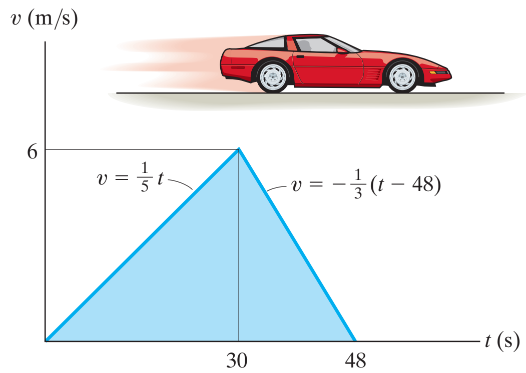
$s\big|_{t = 20\text{s}} = 1800\text{ ft}$

$s\big|_{t = 33.3\text{s}} = 2067\text{ ft}$



***12–60.**

A car travels along a straight road with the speed shown by the $v-t$ graph. Plot the $a-t$ graph.



SOLUTION

a–t Graph: For $0 \leq t < 30\,\text{s}$,

$$v = \frac{1}{5}t$$

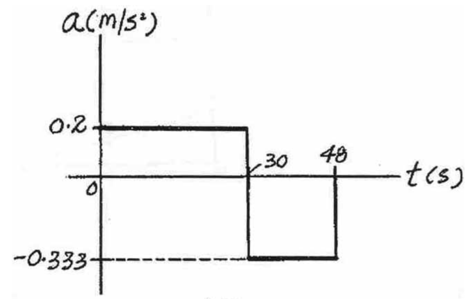
$$a = \frac{dv}{dt} = \frac{1}{5} = 0.2 \text{ m/s}^2$$

For $30 \text{ s} < t \leq 48 \text{ s}$

$$v = -\frac{1}{3}(t - 48)$$

$$a = \frac{dv}{dt} = -\frac{1}{3}(1) = -0.333 \text{ m/s}^2$$

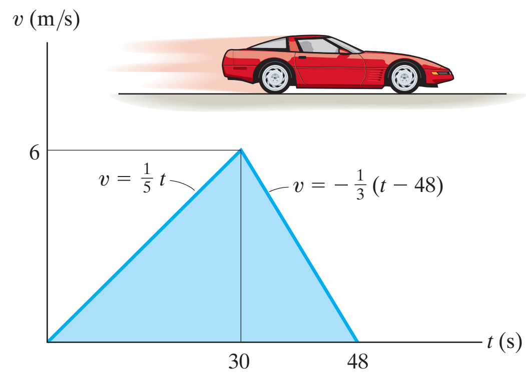
Using these results, $a-t$ graph shown in Fig. a can be plotted.



(a)

12–61.

A car travels along a straight road with the speed shown by the v–t graph. Determine the total distance the car travels until it stops when $t = 48$ s. Also plot the s–t and a–t graphs.



SOLUTION

For $0 \leq t \leq 30$ s,

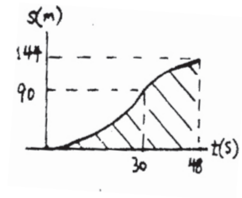
$$v = \frac{1}{5}t$$

$$a = \frac{dv}{dt} = \frac{1}{5}$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t \frac{1}{5}t\,dt$$

$$s = \frac{1}{10}t^2$$



When $t = 30$ s, $\quad s = 90$ m,

$$v = -\frac{1}{3}(t - 48)$$

$$a = \frac{dv}{dt} = -\frac{1}{3}$$

$$ds = v\,dt$$

$$\int_{90}^s ds = \int_{30}^t -\frac{1}{3}(t - 48)dt$$

$$s = -\frac{1}{6}t^2 + 16t - 240$$

When $t = 48$ s,

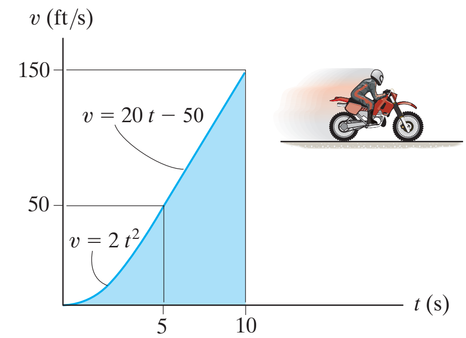
$$s = 144 \text{ m}$$ **Ans.**

Also, from the v–t graph

$$\Delta s = \int v\,dt \quad s - 0 = \frac{1}{2}(6)(48) = 144 \text{ m}$$ **Ans.**

12–62.

A motorcyclist travels along a straight road with the velocity described by the graph. Construct the $s-t$ and $a-t$ graphs.



SOLUTION

s–t Graph: For the time interval $0 \leq t < 5\text{s}$, the initial condition is $s = 0$ when $t = 0$.

$$\left(\stackrel{+}{\rightarrow}\right) \qquad ds = vdt$$

$$\int_0^s ds = \int_0^t 2t^2 dt$$

$$s = \left(\frac{2}{3}t^3\right)\text{ft}$$

When $t = 5\text{s}$,

$$s = \frac{2}{3}(5^3) = 83.33\text{ ft} = 83.3\text{ ft and } a = 20\text{ ft/s}^2$$

For the time interval $5\text{s} < t \leq 10\text{s}$, the initial condition is $s = 83.33$ ft when $t = 5$ s.

$$\left(\stackrel{+}{\rightarrow}\right) \qquad ds = vdt$$

$$\int_{83.33\text{ ft}}^s ds = \int_{5\text{ s}}^t (20t - 50)dt$$

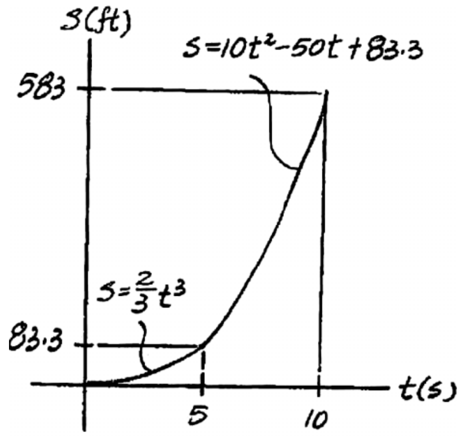
$$s\Big|_{83.33\text{ ft}}^{s} = (10t^2 - 50t)\Big|_{5\text{ s}}^{t}$$

$$s = (10t^2 - 50t + 83.33)\text{ ft}$$

When $t = 10$ s,

$$s|_{t=10\text{ s}} = 10(10^2) - 50(10) + 83.33 = 583\text{ ft}$$

The $s-t$ graph is shown in Fig. *a*.



(a)

a–t Graph: For the time interval $0 \leq t < 5$ s,

$$\left(\stackrel{+}{\rightarrow}\right) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(2t^2) = (4t)\text{ ft/s}^2$$

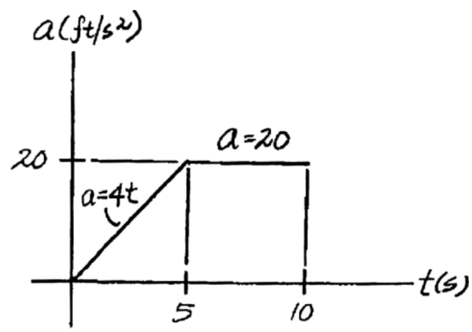
When $t = 5\text{s}$,

$$a = 4(5) = 20\text{ ft/s}^2$$

For the time interval $5\text{s} < t \leq 10$ s,

$$\left(\stackrel{+}{\rightarrow}\right) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(20t - 50) = 20\text{ ft/s}^2$$

The $a-t$ graph is shown in Fig. *b*.



(b)

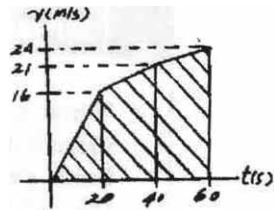
12–63.

The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

Plot the $v-t$ graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

SOLUTION



The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m} \qquad \textbf{Ans.}$$

***12–64.**

A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v–t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

SOLUTION

For package:

$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s_2 - s_0)$

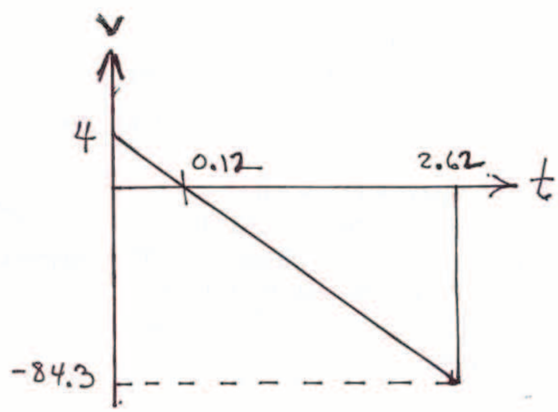
$$v^2 = (4)^2 + 2(-32.2)(0 - 100)$$

$$v = 80.35 \text{ ft/s} \downarrow$$

$(+\uparrow)\qquad v = v_0 + a_c t$

$$-80.35 = 4 + (-32.2)t$$

$$t = 2.620 \text{ s}$$



For elevator:

$(+\uparrow)\qquad s_2 = s_0 + vt$

$$s = 100 + 4(2.620)$$

$$s = 110 \text{ ft}$$

Ans.

12–65.

Two cars start from rest side by side and travel along a straight road. Car A accelerates at 4 m/s^2 for 10 s and then maintains a constant speed. Car B accelerates at 5 m/s^2 until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?

SOLUTION

Car A:

$v = v_0 + a_c t$

$v_A = 0 + 4t$

At $t = 10$ s, $\quad v_A = 40$ m/s

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$

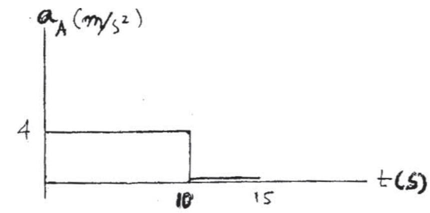
At $t = 10$ s, $\quad s_A = 200$ m

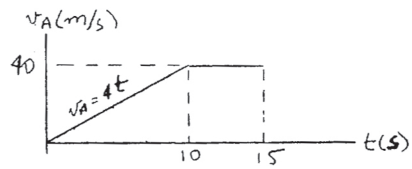
$t > 10$ s, $\quad ds = v\,dt$

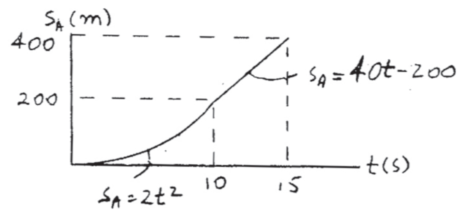
$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\,dt$$

$$s_A = 40t - 200$$

At $t = 15$ s, $\quad s_A = 400$ m







Car B:

$$v = v_0 + a_c t$$

$$v_B = 0 + 5t$$

When $v_B = 25$ m/s, $\quad t = \frac{25}{5} = 5$ s

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$$

When $t = 10$ s, $v_A = (v_A)_{max} = 40$ m/s and $s_A = 200$ m.

When $t = 5$ s, $s_B = 62.5$ m.

When $t = 15$ s, $s_A = 400$ m and $s_B = 312.5$ m.

12–65. continued

At $t = 5$ s, $\quad s_B = 62.5$ m

$t > 5$ s, $\quad ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_5^t 25\,dt$$

$$s_B - 62.5 = 25t - 125$$

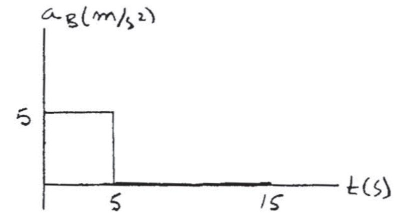
$$s_B = 25t - 62.5$$

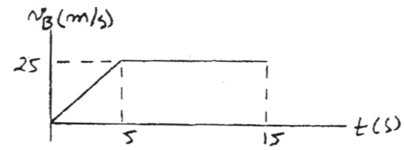
When $t = 15$ s, $\quad s_B = 312.5$

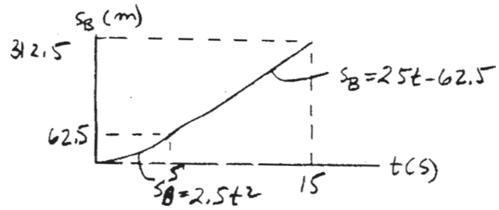
Distance between the cars is

$\Delta s = s_A - s_B = 400 - 312.5 = 87.5$ m **Ans.**

Car A is ahead of car B.

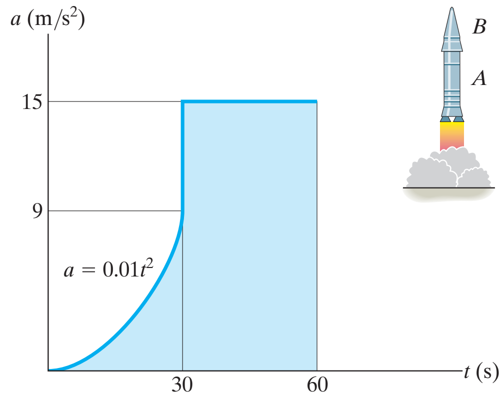






12–66.

A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After 30 s the first stage A burns out and the second stage B ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \leq t \leq 60$ s.



SOLUTION

For $0 \leq t \leq 30$ s

$$\int_0^v dv = \int_0^t 0.01\, t^2\, dt$$

$$v = 0.00333t^3$$

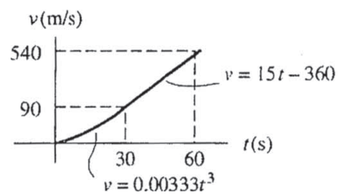
When $t = 30$ s, $\quad v = 90$ m/s

For 30 s $\leq t \leq 60$ s

$$\int_{90}^v dv = \int_{30}^t 15dt$$

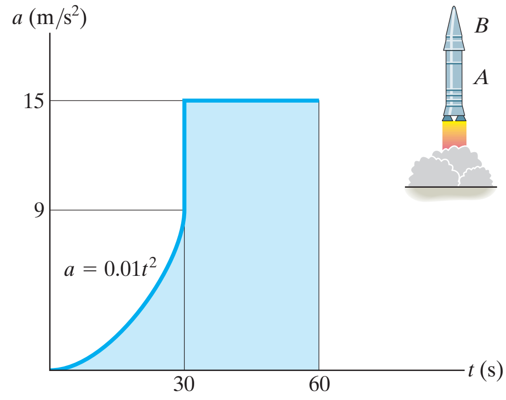
$$v = 15t - 360$$

When $t = 60$ s, $\quad v = 540$ m/s



12–67.

A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After 30 s the first stage A burns out and the second stage B ignites. Plot the s–t graph which describes the motion of the second stage for $0 \leq t \leq 60$ s.



SOLUTION

***v–t* Graph:** When $t = 0$, $v = 0$. For $0 \leq t \leq 30$ s,

$(+\uparrow)$ $\quad dv = a\,dt$

$$\int_0^v dv = \int_0^t 0.01t^2 dt$$

$$v\Big|_0^v = \frac{0.01}{3}t^3\Big|_0^t$$

$$v = \{0.003333t^3\}\text{ m/s}$$

When $t = 30$ s, $v = 0.003333(30^3) = 90$ m/s

For 30 s $< t \leq 60$ s,

$(+\uparrow)$ $\quad dv = a\,dt$

$$\int_{90\text{ m/s}}^v dv = \int_{30\text{ s}}^t 15\,dt$$

$$v\Big|_{90\text{ m/s}}^v = 15t\Big|_{30\text{ s}}^t$$

$$v - 90 = 15t - 450$$

$$v = \{15t - 360\}\text{ m/s}$$

When $t = 60$ s, $v = 15(60) - 360 = 540$ m/s

***s–t* Graph:** When $t = 0$, $s = 0$. For $0 \leq t \leq 30$ s,

$(+\uparrow)$ $\quad ds = vdt$

$$\int_0^s ds = \int_0^t 0.003333t^3 dt$$

$$s\Big|_0^s = 0.0008333t^4\Big|_0^t$$

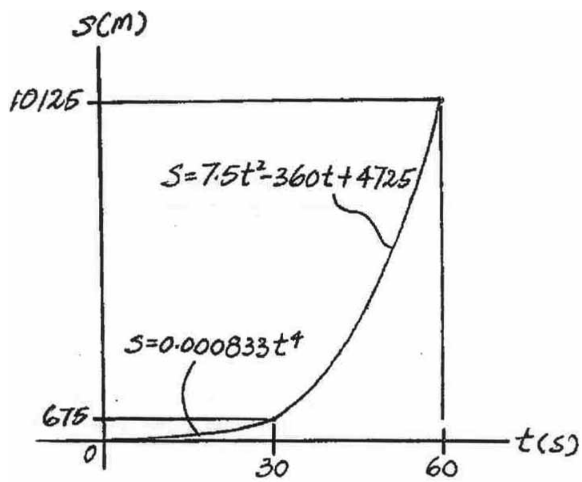
$$s = \{0.0008333\,t^4\}\text{ m}$$

When $t = 30$ s, $s = 0.0008333(30^4) = 675$ m

For 30 s $< t \leq 60$ s,

$(+\uparrow)$ $\quad ds = vdt$

$$\int_{675\text{ m}}^s ds = \int_{30\text{ s}}^t (15t - 360)\,dt$$



(a)

12–67. continued

$$s\Big|_{675\text{ m}}^{s} = (7.5t^2 - 360t)\Big|_{30\text{ s}}^{t}$$

$$s - 675 = (7.5t^2 - 360t) - [7.5(30^2) - 360(30)]$$

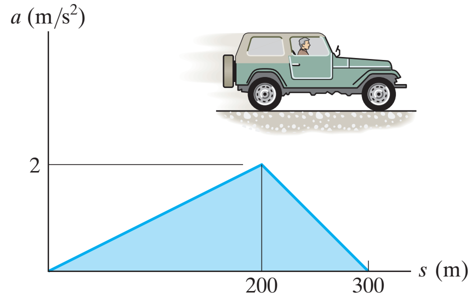
$$s = \{7.5t^2 - 360t + 4725\}\text{ m}$$

When $t = 60$ s, $s = 7.5(60^2) - 360(60) + 4725 = 10\,125$ m

Using these results, the s–t graph shown in Fig. a can be plotted.

***12–68.**

The a–s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v–s graph. At $s = 0, v = 0$.



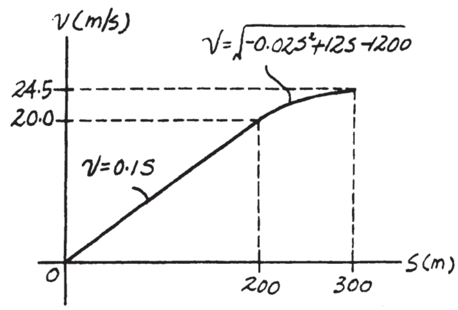
SOLUTION

***a–s* Graph**: The function of acceleration a in terms of s for the interval $0 \text{ m} \leq s < 200 \text{ m}$ is

$$\frac{a - 0}{s - 0} = \frac{2 - 0}{200 - 0} \qquad a = (0.01s) \text{ m/s}^2$$

For the interval $200 \text{ m} < s \leq 300 \text{ m}$,

$$\frac{a - 2}{s - 200} = \frac{0 - 2}{300 - 200} \qquad a = (-0.02s + 6) \text{ m/s}^2$$



***v–s* Graph**: The function of velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval $\mathbf{0\ m \leq s < 200\ m}$,

$$vdv = ads$$

$$\int_0^v vdv = \int_0^s 0.01sds$$

$$v = (0.1s) \text{ m/s}$$

At $s = 200$ m, $\qquad v = 0.100(200) = 20.0 \text{ m/s}$

For the interval $\mathbf{200\ m < s \leq 300\ m}$,

$$vdv = ads$$

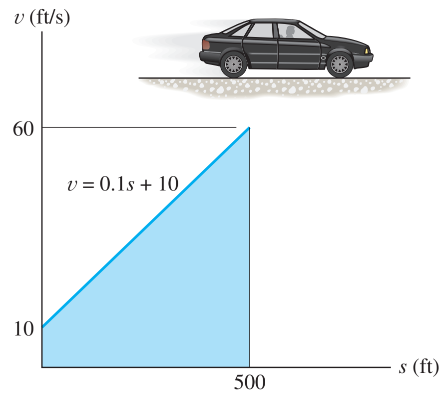
$$\int_{20.0 \text{ m/s}}^v vdv = \int_{200 \text{ m}}^s (-0.02s + 6)ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right) \text{ m/s}$$

At $s = 300$ m, $\qquad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5 \text{ m/s}$

12–69.

The v–s graph for the car is given for the first 500 ft of its motion. Construct the a–s graph for $0 \leq s \leq 500$ ft. How long does it take to travel the 500-ft distance? The car starts at $s = 0$ when $t = 0$.

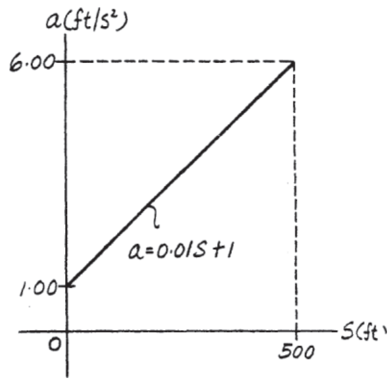


SOLUTION

a – s Graph: The acceleration a in terms of s can be obtained by applying $v dv = a ds$.

$$a = v\frac{dv}{ds} = (0.1s + 10)(0.1) = (0.01s + 1)\ \text{ft/s}^2$$

At $s = 0$ and $s = 500$ ft, $a = 0.01(0) + 1 = 1.00\ \text{ft/s}^2$ and $a = 0.01(500) + 1 = 6.00\ \text{ft/s}^2$, respectively.



Position: The position s in terms of time t can be obtained by applying $v = \dfrac{ds}{dt}$.

$$dt = \frac{ds}{v}$$

$$a\big|_{s=0} = 100\ \text{ft/s}^2$$

$$a\big|_{s=500\text{ft}} = 6.00\ \text{ft/s}^2$$

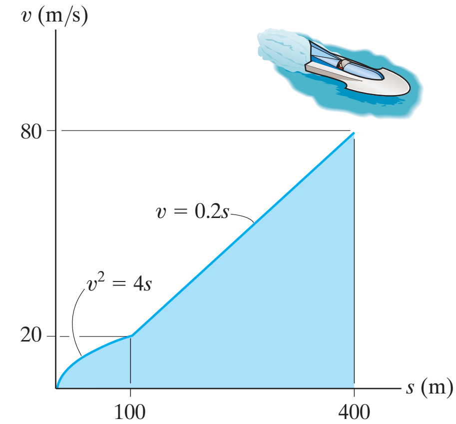
$$\int_0^t dt = \int_0^s \frac{ds}{0.1s + 10}$$

$$t = 10\ln(0.01s + 1)$$

When $s = 500$ ft, $\quad t = 10\ln[0.01(500) + 1] = 17.9$ s **Ans.**

12–70.

The boat travels along a straight line with the speed described by the graph. Construct the s–t and $a-s$ graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.



SOLUTION

$s-t$ Graph: For $0 \leq s < 100$ m, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{2s^{1/2}}$$

$$t = s^{1/2}$$

$$s = \left(t^2\right) \text{ m}$$

When $s = 100$ m,

$$100 = t^2 \qquad t = 10 \text{ s}$$

For $100 \text{ m} < s \leq 400$ m, the initial condition is $s = 100$ m when $t = 10$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_{10\text{ s}}^t dt = \int_{100\text{ m}}^s \frac{ds}{0.2s}$$

$$t - 10 = 5\ln\frac{s}{100}$$

$$\frac{t}{5} - 2 = \ln\frac{s}{100}$$

$$e^{t/5-2} = \frac{s}{100}$$

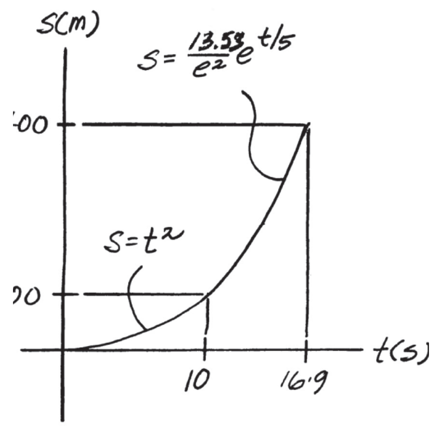
$$\frac{e^{t/5}}{e^2} = \frac{s}{100}$$

$$s = \left(13.53e^{t/5}\right) \text{ m}$$

When $s = 400$ m,

$$400 = 13.53e^{t/5}$$

$$t = 16.93 \text{ s} = 16.9 \text{ s} \qquad \textbf{Ans.}$$



(a)

The s–t graph is shown in Fig. a.

$a-s$ Graph: For $0 \text{ m} \leq s < 100$ m,

$$a = v\frac{dv}{ds} = \left(2s^{1/2}\right)\left(s^{-1/2}\right) = 2 \text{ m/s}^2$$

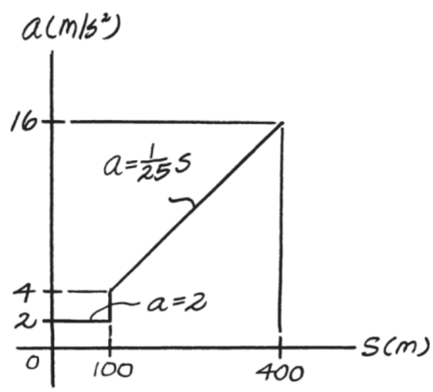
For $100 \text{ m} < s \leq 400$ m,

$$a = v\frac{dv}{ds} = (0.2s)(0.2) = 0.04s$$

When $s = 100$ m and 400 m,

$$a\big|_{s=100\text{ m}} = 0.04(100) = 4 \text{ m/s}^2$$

$$a\big|_{s=400\text{ m}} = 0.04(400) = 16 \text{ m/s}^2$$

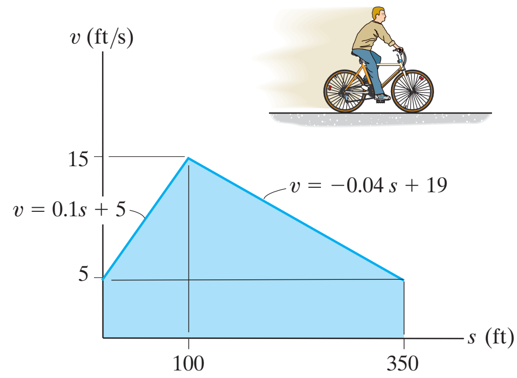


(b)

The a–s graph is shown in Fig. b.

12–71.

The $v-s$ graph of a cyclist traveling along a straight road is shown. Construct the $a-s$ graph.



SOLUTION

a–s Graph: For $0 \le s < 100$ ft,

$$\left(\xrightarrow{+}\right) \qquad a = v\frac{dv}{ds} = (0.1s + 5)(0.1) = (0.01s + 0.5)\ \text{ft/s}^2$$

Thus at $s = 0$ and 100 ft

$$a|_{s=0} = 0.01(0) + 0.5 = 0.5\ \text{ft/s}^2$$

$$a|_{s=100\text{ ft}} = 0.01(100) + 0.5 = 1.5\ \text{ft/s}^2$$

For $100\text{ ft} < s \le 350$ ft,

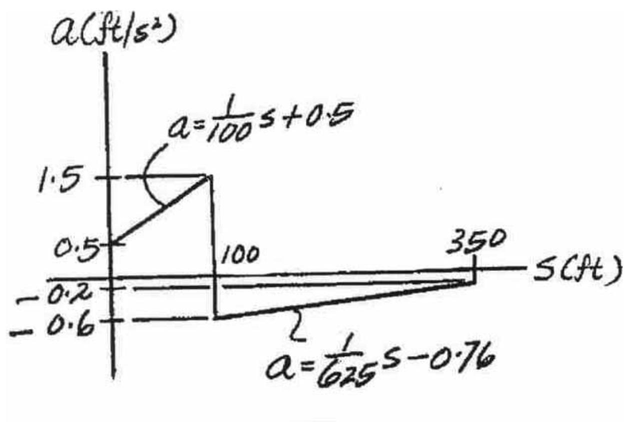
$$\left(\xrightarrow{+}\right) \qquad a = v\frac{dv}{ds} = (-0.04s + 19)(-0.04) = (0.0016s - 0.76)\ \text{ft/s}^2$$

Thus at $s = 100$ ft and 350 ft

$$a|_{s=100\text{ ft}} = 0.0016(100) - 0.76 = -0.6\ \text{ft/s}^2$$

$$a|_{s=350\text{ ft}} = 0.0016(350) - 0.76 = -0.2\ \text{ft/s}^2$$

The $a-s$ graph is shown in Fig. a.



(a)

Thus at $s = 0$ and 100 ft

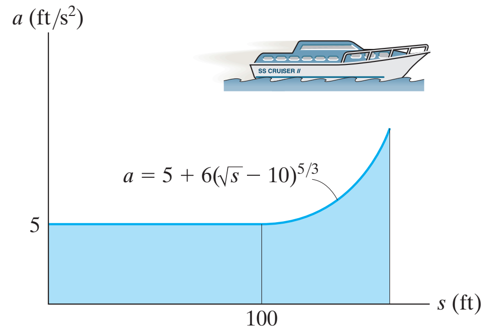
$$a|_{s=0} = 0.01(0) + 0.5 = 0.5\ \text{ft/s}^2$$

$$a|_{s=100\text{ ft}} = 0.01(100) + 0.5 = 1.5\ \text{ft/s}^2$$

At $s = 100$ ft, a changes from $a_{max} = 1.5\ \text{ft/s}^2$ to $a_{min} = -0.6\ \text{ft/s}^2$.

*■ **12–72.**

The a–s graph for a boat moving along a straight path is given. If the boat starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



SOLUTION

Velocity: The velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval **0 ft ≤ *s* < 100 ft**,

$$vdv = ads$$

$$\int_0^v vdv = \int_0^s 5ds$$

$$v = \sqrt{10s} = \text{ft/s}$$

At $s = 75$ ft, $\quad v = \sqrt{10(75)} = 27.4$ ft/s **Ans.**

At $s = 100$ ft, $\quad v = \sqrt{10(100)} = 31.62$ ft/s **Ans.**

For the interval **100 ft < *s* ≤ 125 ft**,

$$vdv = ads$$

$$\int_{31.62\text{ ft/s}}^{v} vdv = \int_{100\text{ ft}}^{125\text{ ft}} [5 + 6(\sqrt{s} - 10)^{5/3}]ds$$

Evaluating the integral on the right using Simpson's rule, we have

$$\left.\frac{v^2}{2}\right|_{31.62\text{ ft/s}}^{v} = 201.032$$

At $s = 125$ ft, **Ans.**

$$v = 37.4\text{ ft/s}$$

12–73.

The position of a particle is defined by $\mathbf{r} = \{5 \cos 2t\, \mathbf{i} + 4 \sin 2t\, \mathbf{j}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ s. Also, prove that the path of the particle is elliptical.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{-10 \sin 2t\mathbf{i} + 8 \cos 2t\mathbf{j}\}\ \text{m/s}$$

When $t = 1$ s, $v = -10 \sin 2(1)\mathbf{i} + 8 \cos 2(1)\mathbf{j} = \{-9.093\mathbf{i} - 3.329\mathbf{j}\}$ m/s. Thus, the magnitude of the velocity is

$$\mathbf{v} = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: The acceleration expressed in Cartesian vector from can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{-20 \cos 2t\mathbf{i} - 16 \sin 2t\mathbf{j}\}\ \text{m/s}^2$$

When $t = 1$ s, $\mathbf{a} = -20 \cos 2(1)\mathbf{i} - 16 \sin 2(1)\mathbf{j} = \{8.323\mathbf{i} - 14.549\mathbf{j}\}$ m/s^2. Thus, the magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8.323^2 + (-14.549)^2} = 16.8\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Traveling Path: Here, $x = 5 \cos 2t$ and $y = 4 \sin 2t$. Then,

$$\frac{x^2}{25} = \cos^2 2t \qquad \textbf{(1)}$$

$$\frac{y^2}{16} = \sin^2 2t \qquad \textbf{(2)}$$

Adding Eqs (1) and (2) yields

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

However, $\cos^2 2t + \sin^2 2t = 1$. Thus,

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \qquad \textbf{(Equation of an Ellipse) (Q.E.D.)}$$

12–74.

The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

SOLUTION

Position: The position $\mathbf{r}$ of the particle can be determined by integrating the kinematic equation $d\mathbf{r} = \boldsymbol{v}dt$ using the initial condition $\mathbf{r} = \mathbf{0}$ at $t = 0$ as the integration limit. Thus,

$$d\mathbf{r} = \boldsymbol{v}dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left[3\mathbf{i} + (6 - 2t)\mathbf{j}\right]dt$$

$$\mathbf{r} = \left[3t\mathbf{i} + \left(6t - t^2\right)\mathbf{j}\right]\text{m}$$

When $t = 1$ s and 3 s,

$$r\big|_{t=1\text{ s}} = 3(1)\mathbf{i} + \left[6(1) - 1^2\right]\mathbf{j} = [3\mathbf{i} + 5\mathbf{j}]\text{ m/s}$$

$$r\big|_{t=3\text{ s}} = 3(3)\mathbf{i} + \left[6(3) - 3^2\right]\mathbf{j} = [9\mathbf{i} + 9\mathbf{j}]\text{ m/s}$$

Thus, the displacement of the particle is

$$\Delta\mathbf{r} = \mathbf{r}\big|_{t=3\text{ s}} - \mathbf{r}\big|_{t=1\text{ s}}$$

$$= (9\mathbf{i} + 9\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})$$

$$= \{6\mathbf{i} + 4\mathbf{j}\}\text{ m}$$ **Ans.**

12–75.

A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s^2. Determine the particle's position (x, y, z) at $t = 1$ s.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$dv = \mathrm{a}dt$$

$$\int_0^v dv = \int_0^t (6t\mathbf{i} + 12t^2\mathbf{k})\,dt$$

$$v = \{3t^2\mathbf{i} + 4t^3\mathbf{k}\}\text{ ft/s}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$dr = \mathrm{v}dt$$

$$\int_{\mathrm{r}_1}^{\mathrm{r}} dr = \int_0^t (3t^2\mathbf{i} + 4t^3\mathbf{k})\,dt$$

$$\mathrm{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

$$\mathrm{r} = \{(t^3 + 3)\,\mathbf{i} + 2\mathbf{j} + (t^4 + 5)\mathbf{k}\}\text{ ft}$$

When $t = 1\ s$, $\mathrm{r} = (1^3 + 3)\mathbf{i} + 2\mathbf{j} + (1^4 + 5)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}$ ft.

The coordinates of the particle are

(4 ft, 2 ft, 6 ft) **Ans.**

***12–76.**

The velocity of a particle is given by $\boldsymbol{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

SOLUTION

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\boldsymbol{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12\left(2^2\right)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \boldsymbol{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right]\ \text{m}$$

When $t = 2$ s,

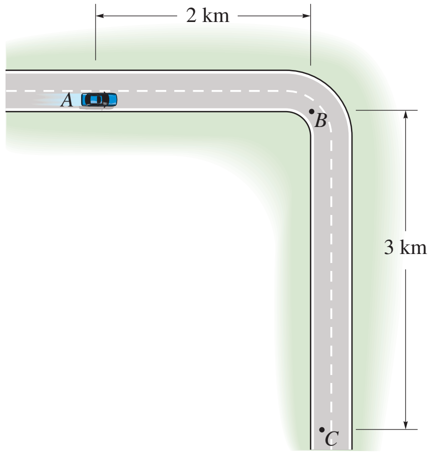
$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\}\ \text{m}.$$

Thus, the coordinate of the particle is

$$(42.7, 16.0, 14.0)\ \text{m} \qquad \textbf{Ans.}$$

12–77.

The car travels from A to B, and then from B to C, as shown in the figure. Determine the magnitude of the displacement of the car and the distance traveled.



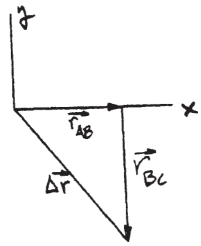
SOLUTION

Displacement: $\Delta\mathbf{r} = \{2\mathbf{i} - 3\mathbf{j}\}\ \text{km}$

$$\Delta r = \sqrt{2^2 + 3^2} = 3.61\ \text{km} \qquad \textbf{Ans.}$$

Distance Traveled:

$$d = 2 + 3 = 5\ \text{km} \qquad \textbf{Ans.}$$



12–78.

A car travels east 2 km for 5 minutes, then north 3 km for 8 minutes, and then west 4 km for 10 minutes. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

SOLUTION

Total Distance Traveled and Displacement: The total distance traveled is

$$s = 2 + 3 + 4 = 9 \text{ km} \qquad \textbf{Ans.}$$

and the magnitude of the displacement is

$$\Delta r = \sqrt{(2 - 4)^2 + 3^2} = 3.606 \text{ km} = 3.61 \text{ km} \qquad \textbf{Ans.}$$

Average Velocity and Speed: The total time is $\Delta t = 5 + 8 + 10 = 23 \text{ min} = 1380 \text{ s}$ The magnitude of average velocity is

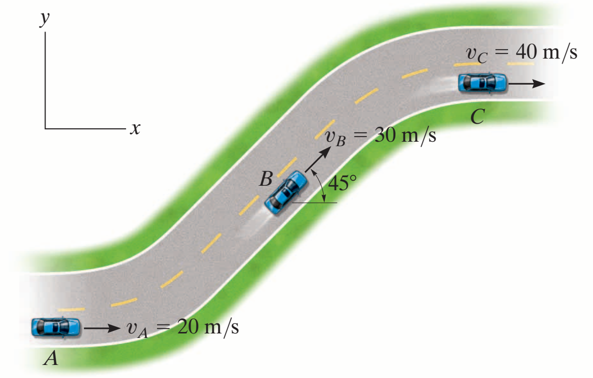
$$v_{\text{avg}} = \frac{\Delta r}{\Delta t} = \frac{3.606\left(10^3\right)}{1380} = 2.61 \text{ m/s} \qquad \textbf{Ans.}$$

and the average speed is

$$\left(v_{sp}\right)_{\text{avg}} = \frac{s}{\Delta t} = \frac{9\left(10^3\right)}{1380} = 6.52 \text{ m/s} \qquad \textbf{Ans.}$$

12–79.

A car traveling along the straight portions of the road has the velocities indicated in the figure when it arrives at points A, B, and C. If it takes 3 s to go from A to B, and then 5 s to go from B to C, determine the average acceleration between points A and B and between points A and C.



SOLUTION

$\mathbf{v}_A = 20\,\mathbf{i}$

$\mathbf{v}_B = 21.21\,\text{i} + 21.21\mathbf{j}$

$\mathbf{v}_C = 40\mathbf{i}$

$$\mathbf{a}_{AB} = \frac{\Delta\,\mathbf{v}}{\Delta\,t} = \frac{21.21\,\mathbf{i} + 21.21\,\mathbf{j} - 20\,\mathbf{i}}{3}$$

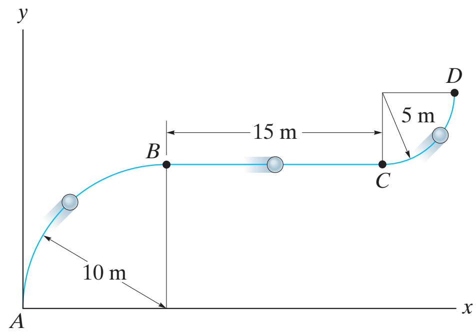
$\mathbf{a}_{AB} = \{\,0.404\,\mathbf{i} + 7.07\,\mathbf{j}\,\}\ \text{m/s}^2$ **Ans.**

$$\mathbf{a}_{AC} = \frac{\Delta\,\mathbf{v}}{\Delta\,t} = \frac{40\,\mathbf{i} - 20\,\mathbf{i}}{8}$$

$\mathbf{a}_{AC} = \{\,2.50\,\mathbf{i}\,\}\ \text{m/s}^2$ **Ans.**

***12–80.**

A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



SOLUTION

$$s_T = \frac{1}{4}(2\pi)(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sP} = \frac{s_T}{t_t} = \frac{38.56}{2 + 4 + 3} = 4.28 \text{ m/s}$$ **Ans.**

12–81.

The position of a crate sliding down a ramp is given by $x = (0.25t^3)$ m, $y = (1.5t^2)$ m, $z = (6 - 0.75t^{5/2})$ m, where t is in seconds. Determine the magnitude of the crate's velocity and acceleration when $t = 2$ s.

SOLUTION

Velocity: By taking the time derivative of x, y, and z, we obtain the x, y, and z components of the crate's velocity.

$$v_x = \dot{x} = \frac{d}{dt}\left(0.25t^3\right) = \left(0.75t^2\right)\text{ m/s}$$

$$v_y = \dot{y} = \frac{d}{dt}\left(1.5t^2\right) = \left(3t\right)\text{ m/s}$$

$$v_z = \dot{z} = \frac{d}{dt}\left(6 - 0.75t^{5/2}\right) = \left(-1.875t^{3/2}\right)\text{ m/s}$$

When $t = 2$ s,

$$v_x = 0.75\left(2^2\right) = 3\text{ m/s} \qquad v_y = 3(2) = 6\text{ m/s} \qquad v_z = -1.875\left(2\right)^{3/2} = -5.303\text{ m/s}$$

Thus, the magnitude of the crate's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2 + v_z{}^2} = \sqrt{3^2 + 6^2 + (-5.303)^2} = 8.551\text{ ft/s} = 8.55\text{ ft} \qquad \textbf{Ans.}$$

Acceleration: The x, y, and z components of the crate's acceleration can be obtained by taking the time derivative of the results of v_x, v_y, and v_z, respectively.

$$a_x = \dot{v}_x = \frac{d}{dt}\left(0.75t^2\right) = (1.5t)\text{ m/s}^2$$

$$a_y = \dot{v}_y = \frac{d}{dt}(3t) = 3\text{ m/s}^2$$

$$a_z = \dot{v}_z = \frac{d}{dt}\left(-1.875t^{3/2}\right) = \left(-2.815t^{1/2}\right)\text{ m/s}^2$$

When $t = 2$ s,

$$a_x = 1.5(2) = 3\text{ m/s}^2 \qquad a_y = 3\text{ m/s}^2 \qquad a_z = -2.8125\left(2^{1/2}\right) = -3.977\text{ m/s}^2$$

Thus, the magnitude of the crate's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} = \sqrt{3^2 + 3^2 + (-3.977)^2} = 5.815\text{ m/s}^2 = 5.82\text{ m/s} \qquad \textbf{Ans.}$$

12–82.

A rocket is fired from rest at $x = 0$ and travels along a parabolic trajectory described by $y^2 = [120(10^3)x]$ m. If the x component of acceleration is $a_x = \left(\frac{1}{4}t^2\right)$ m/s^2, where t is in seconds, determine the magnitude of the rocket's velocity and acceleration when $t = 10$ s.

SOLUTION

Position: The parameter equation of x can be determined by integrating a_x twice with respect to t.

$$\int dv_x = \int a_x dt$$

$$\int_0^{v_x} dv_x = \int_0^t \frac{1}{4}t^2 dt$$

$$v_x = \left(\frac{1}{12}t^3\right) \text{m/s}$$

$$\int dx = \int v_x dt$$

$$\int_0^x dx = \int_0^t \frac{1}{12}t^3 dt$$

$$x = \left(\frac{1}{48}t^4\right) \text{m}$$

Substituting the result of x into the equation of the path,

$$y^2 = 120\left(10^3\right)\left(\frac{1}{48}t^4\right)$$

$$y = \left(50t^2\right) \text{m}$$

Velocity:

$$v_y = \dot{y} = \frac{d}{dt}\left(50t^2\right) = \left(100t\right) \text{m/s}$$

When $t = 10$ s,

$$v_x = \frac{1}{12}\left(10^3\right) = 83.33 \text{ m/s} \qquad v_y = 100(10) = 1000 \text{ m/s}$$

Thus, the magnitude of the rocket's velocity is

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}} = \sqrt{83.33^2 + 1000^2} = 1003 \text{ m/s}$$ **Ans.**

Acceleration:

$$a_y = \dot{v}_y = \frac{d}{dt}(100t) = 100 \text{ m/s}^2$$

When $t = 10$ s,

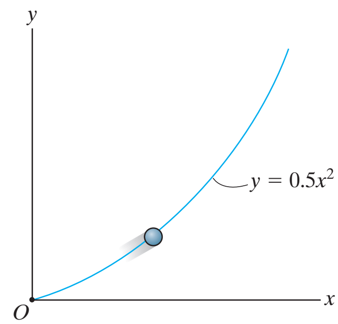
$$a_x = \frac{1}{4}\left(10^2\right) = 25 \text{ m/s}^2$$

Thus, the magnitude of the rocket's acceleration is

$$a = \sqrt{a_x^{\,2} + a_y^{\,2}} = \sqrt{25^2 + 100^2} = 103 \text{ m/s}^2$$ **Ans.**

12–83.

The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



SOLUTION

Position: The x position of the particle can be obtained by applying the $v_x = \dfrac{dx}{dt}$.

$$dx = v_x dt$$

$$\int_0^x dx = \int_0^t 5t\,dt$$

$$x = \left(2.50t^2\right)\ \text{ft}$$

Thus, $y = 0.5\left(2.50t^2\right)^2 = \left(3.125t^4\right)$ ft. At $t = 1$ s, $x = 2.5\left(1^2\right) = 2.50$ ft and $y = 3.125\left(1^4\right) = 3.125$ ft. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50 - 0)^2 + (3.125 - 0)^2} = 4.00\ \text{ft} \qquad \textbf{Ans.}$$

Acceleration: Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x} \qquad \textbf{(1)}$$

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. (1) becomes

$$a_y = v_x^2 + x a_x \qquad \textbf{(2)}$$

When $t = 1$ s, $v_x = 5(1) = 5$ ft/s $a_x = \dfrac{dv_x}{dt} = 5$ ft/s^2, and $x = 2.50$ ft. Then, from Eq. (2)

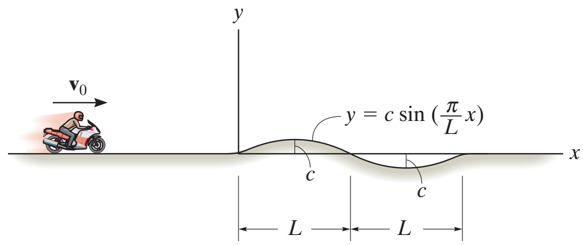
$$a_y = 5^2 + 2.50(5) = 37.5\ \text{ft/s}^2$$

Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

***12–84.**

The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



SOLUTION

$$y = c \sin\left(\frac{\pi}{L}x\right)$$

$$\dot{y} = \frac{\pi}{L}c\left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c\, v_x\left(\cos\frac{\pi}{L}x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

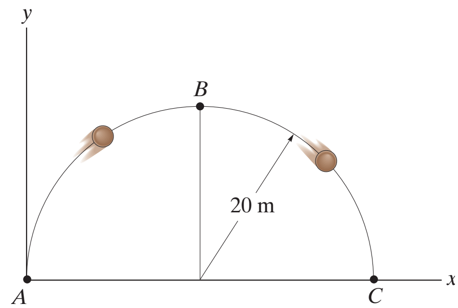
$$v_0^2 = v_x^2\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]$$

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

12–85.

A particle travels along the curve from A to B in 1 s. If it takes 3 s for it to go from A to C, determine its *average velocity* when it goes from B to C.



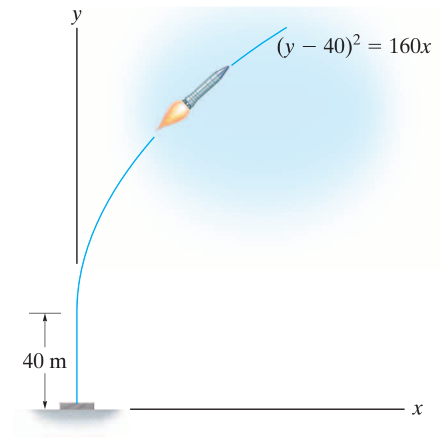
SOLUTION

Time from B to C is $3 - 1 = 2$ s

$$\mathbf{v}_{avg} = \frac{\Delta \mathrm{r}}{\Delta t} = \frac{(\mathbf{r}_{AC} - \mathbf{r}_{AB})}{\Delta t} = \frac{40\mathbf{i} - (20\mathbf{i} + 20\mathbf{j})}{2} = \{10\mathbf{i} - 10\mathbf{j}\}\ \mathrm{m/s} \qquad \textbf{Ans.}$$

12–86.

When a rocket reaches an altitude of 40 m it begins to travel along the parabolic path $(y - 40)^2 = 160x$, where the coordinates are measured in meters. If the component of velocity in the vertical direction is constant at $v_y = 180$ m/s, determine the magnitudes of the rocket's velocity and acceleration when it reaches an altitude of 80 m.



SOLUTION

$v_y = 180 \text{ m/s}$

$(y - 40)^2 = 160\, x$

$2(y - 40)v_y = 160 v_x$ **(1)**

$2(80 - 40)(180) = 160 v_x$

$v_x = 90 \text{ m/s}$

$v = \sqrt{90^2 + 180^2} = 201 \text{ m/s}$ **Ans.**

$$a_y = \frac{d\, v_y}{dt} = 0$$

From Eq. 1,

$2\, v_y^2 + 2(y - 40)a_y = 160\, a_x$

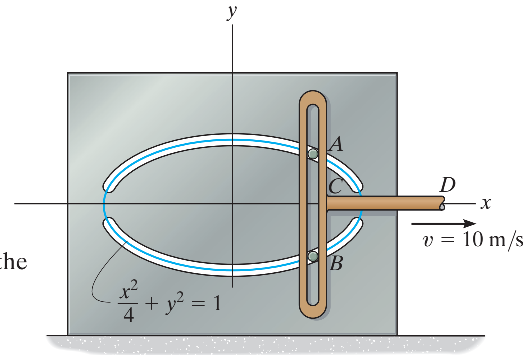
$2(180)^2 + 0 = 160\, a_x$

$a_x = 405 \text{ m/s}^2$

$a = 405 \text{ m/s}^2$ **Ans.**

12–87.

Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg A when $x = 1$ m.



SOLUTION

Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\frac{x^2}{4} + y^2 = 1$$

$$\frac{1}{4}(2x\dot{x}) + 2y\dot{y} = 0$$

$$\frac{1}{2}x\dot{x} + 2y\dot{y} = 0$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \qquad \textbf{(1)}$$

At $x = 1$ m,

$$\frac{(1)^2}{4} + y^2 = 1 \qquad\qquad y = \frac{\sqrt{3}}{2}\text{ m}$$

Here, $v_x = 10$ m/s and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \qquad\qquad v_y = -2.887\text{ m/s} = 2.887\text{ m/s}\downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{10^2 + 2.887^2} = 10.4\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) = 0$$

$$\frac{1}{2}\left(\dot{x}^2 + x\ddot{x}\right) + 2\left(\dot{y}^2 + y\ddot{y}\right) = 0$$

or

$$\frac{1}{2}\left(v_x{}^2 + xa_x\right) + 2\left(v_y{}^2 + ya_y\right) = 0 \qquad \textbf{(2)}$$

Since v_x is constant, $a_x = 0$. When $x = 1$ m, $y = \frac{\sqrt{3}}{2}$ m, $v_x = 10$ m/s, and $v_y = -2.887$ m/s. Substituting these values into Eq. (2),

$$\frac{1}{2}\left(10^2 + 0\right) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] = 0$$

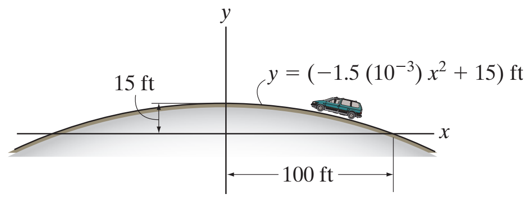
$$a_y = -38.49\text{ m/s}^2 = 38.49\text{ m/s}^2\downarrow$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{0^2 + (-38.49)^2} = 38.5\text{ m/s}^2 \qquad \textbf{Ans.}$$

***12–88.**

The van travels over the hill described by $y = (-1.5(10^{-3})\,x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when $x = 50$ ft.



SOLUTION

Velocity: The x and y components of the van's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = -1.5\left(10^{-3}\right)x^2 + 15$$

$$\dot{y} = -3\left(10^{-3}\right)x\dot{x}$$

or

$$v_y = -3\left(10^{-3}\right)xv_x$$

When $x = 50$ ft,

$$v_y = -3\left(10^{-3}\right)(50)v_x = -0.15v_x \qquad \textbf{(1)}$$

The magnitude of the van's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} \qquad \textbf{(2)}$$

Substituting $v = 75$ ft/s and Eq. (1) into Eq. (2),

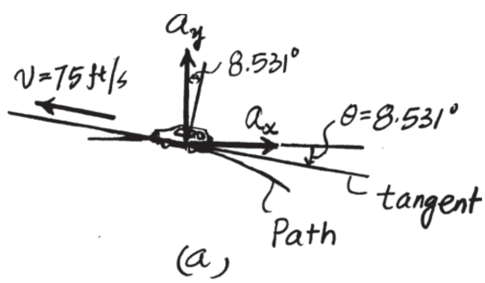
$$75 = \sqrt{v_x{}^2 + (-0.15v_x)^2}$$

$$v_x = 74.2 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Substituting the result of v_x into Eq. (1), we obtain

$$v_y = -0.15(-74.17) = 11.12 \text{ ft/s} = 11.1 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the van's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.



$$\ddot{y} = -3\left(10^{-3}\right)(\dot{x}\dot{x} + x\ddot{x})$$

or

$$a_y = -3\left(10^{-3}\right)\left(v_x{}^2 + xa_x\right)$$

When $x = 50$ ft, $v_x = -74.17$ ft/s. Thus,

$$a_y = -3\left(10^{-3}\right)\left[(-74.17)^2 + 50a_x\right]$$

$$a_y = -(16.504 + 0.15a_x) \qquad \textbf{(3)}$$

Since the van travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at $x = 50$ ft is $\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\Big|_{x=50\text{ ft}} = \tan^{-1}\left[-3\left(10^{-3}\right)x\right]\Big|_{x=50\text{ ft}} = \tan^{-1}(-0.15) = -8.531°$.
Thus, from the diagram shown in Fig. a,

$$a_x \cos 8.531° - a_y \sin 8.531° = 0 \qquad \textbf{(4)}$$

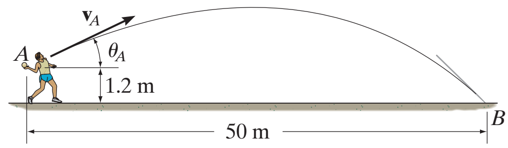
Solving Eqs. (3) and (4) yields

$$a_x = -2.42 \text{ ft/s} = 2.42 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$a_y = -16.1 \text{ ft/s} = 16.1 \text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

12–89.

It is observed that the time for the ball to strike the ground at B is 2.5 s. Determine the speed v_A and angle θ_A at which the ball was thrown.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A.

x-Motion: Here, $(v_A)_x = v_A \cos\theta_A$, $x_A = 0$, $x_B = 50$ m, and $t = 2.5$ s. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad x_B = x_A + (v_A)_x t$$

$$50 = 0 + v_A \cos\theta_A(2.5)$$

$$v_A \cos\theta_A = 20 \qquad \textbf{(1)}$$

y-Motion: Here, $(v_A)_y = v_A \sin\theta_A$, $y_A = 0$, $y_B = -1.2$ m, and $a_y = -g = -9.81\ \text{m/s}^2$. Thus,

$$\left(+\uparrow\right) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1.2 = 0 + v_A \sin\theta_A\,(2.5) + \frac{1}{2}(-9.81)\left(2.5^2\right)$$

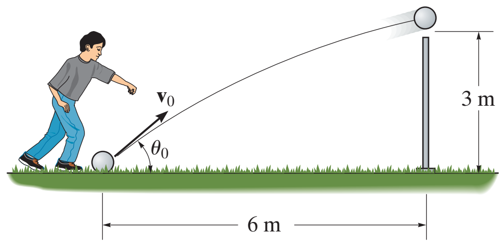
$$v_A \sin\theta_A = 11.7825 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta_A = 30.5^\circ \qquad v_A = 23.2\ \text{m/s} \qquad \textbf{Ans.}$$

12–90.

Determine the minimum initial velocity v_0 and the corresponding angle θ_0 at which the ball must be kicked in order for it to just cross over the 3-m high fence.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set so that its origin coincides with the ball's initial position.

x-Motion: Here, $(v_0)_x = v_0 \cos\theta$, $x_0 = 0$, and $x = 6$ m. Thus,

$$(\xrightarrow{+}) \qquad x = x_0 + (v_0)x^t$$

$$6 = 0 + (v_0 \cos\theta)t$$

$$t = \frac{6}{v_0 \cos\theta} \qquad \textbf{(1)}$$

y-Motion: Here, $(v_0)_x = v_0 \sin\theta$, $a_y = -g = -9.81 \text{ m/s}^2$, and $y_0 = 0$. Thus,

$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$3 = 0 + v_0 (\sin\theta) t + \frac{1}{2}(-9.81)t^2$$

$$3 = v_0 (\sin\theta) t - 4.905t^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$v_0 = \sqrt{\frac{58.86}{\sin 2\theta - \cos^2\theta}} \qquad \textbf{(3)}$$

From Eq. (3), we notice that v_0 is minimum when $f(\theta) = \sin 2\theta - \cos^2\theta$ is maximum. This requires $\frac{df(\theta)}{d\theta} = 0$

$$\frac{df(\theta)}{d\theta} = 2\cos 2\theta + \sin 2\theta = 0$$

$$\tan 2\theta = -2$$

$$2\theta = 116.57°$$

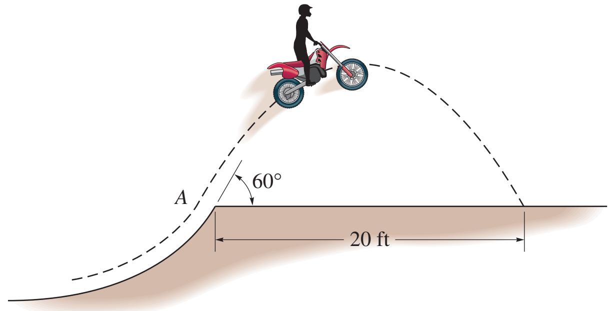
$$\theta = 58.28° = 58.3° \qquad \textbf{Ans.}$$

Substituting the result of θ into Eq. (2), we have

$$(v_0)_{min} = \sqrt{\frac{58.86}{\sin 116.57° - \cos^2 58.28°}} = 9.76 \text{ m/s} \qquad \textbf{Ans.}$$

12–91.

During a race the dirt bike was observed to leap up off the small hill at A at an angle of 60° with the horizontal. If the point of landing is 20 ft away, determine the approximate speed at which the bike was traveling just before it left the ground. Neglect the size of the bike for the calculation.



SOLUTION

$(\overset{+}{\rightarrow})\; s = s_0 + v_0 t$

$$20 = 0 + v_A \cos 60°\, t$$

$(+\uparrow)\; s = s_0 + v_0 + \frac{1}{2} a_c t^2$

$$0 = 0 + v_A \sin 60°\, t + \frac{1}{2}(-32.2)\, t^2$$

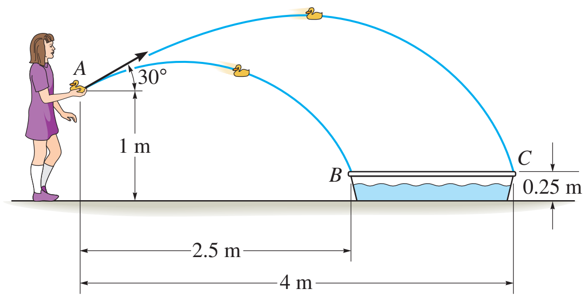
Solving

$t = 1.4668 \text{ s}$

$v_A = 27.3 \text{ ft/s}$ **Ans.**

***12–92.**

The girl always throws the toys at an angle of $30°$ from point A as shown. Determine the time between throws so that both toys strike the edges of the pool B and C at the same instant. With what speed must she throw each toy?



SOLUTION

To strike B:

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$2.5 = 0 + v_A \cos 30°\ t$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$0.25 = 1 + v_A \,\text{siv}\, 30°\ t - \frac{1}{2}(9.81)t^2$

Solving

$t = 0.6687 \text{ s}$

$(v_A)_B = 4.32 \text{ m/s}$ **Ans.**

To strike C:

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$4 = 0 + v_A \cos 30°\ t$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$0.25 = 1 + v_A \,\text{siv}\, 30°\ t - \frac{1}{2}(9.81)t^2$

Solving

$t = 0.790 \text{ s}$

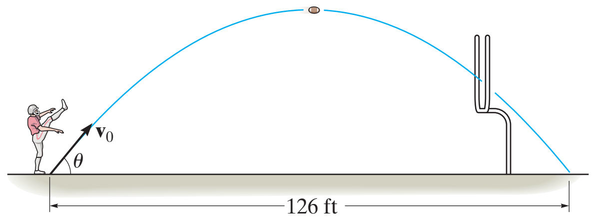
$(v_A)_C = 5.85 \text{ m/s}$ **Ans.**

Time between throws:

$\Delta t = 0.790 \text{ s} - 0.6687 \text{ s} = 0.121 \text{ s}$ **Ans.**

12–93.

The player kicks a football with an initial speed of $v_0 = 90$ ft/s. Determine the time the ball is in the air and the angle θ of the kick.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set with its origin coinciding with starting point of the football.

x-motion: Here, $x_0 = 0$, $x = 126$ ft, and $(v_0)_x = 90 \cos\theta$

$$(\overset{+}{\rightarrow}) \qquad x = x_0 + (v_0)_x t$$

$$126 = 0 + (90\cos\theta)t$$

$$t = \frac{126}{90\cos\theta} \tag{1}$$

y-motion: Here, $y_0 = y = 0$, $(v_0)_y = 90\sin\theta$, and $a_y = -g = -32.2$ ft. Thus,

$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$O = 0 + (90\sin\theta)t + \frac{1}{2}(-32.2)t^2$$

$$O = (90\sin\theta)t - 16.1t^2 \tag{2}$$

Substitute Eq. (1) into (2) yields

$$O = 90\sin\theta\left(\frac{126}{90\cos\theta}\right) - 16.1\left(\frac{126}{90\cos\theta}\right)^2$$

$$O = \frac{126\sin\theta}{\cos\theta} - \frac{31.556}{\cos^2\theta}$$

$$O = 126\sin\theta\cos\theta - 31.556 \tag{3}$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$, Eq. (3) becomes

$$63\sin 2\theta = 31.556$$

$$\sin 2\theta = 0.5009$$

$$2\theta = 30.06 \text{ or } 149.94$$

$$\theta = 15.03° = 15.0° \text{ or } \theta = 74.97° = 75.0° \qquad \textbf{Ans.}$$

If $\theta = 15.03°$,

$$t = \frac{126}{90\cos 15.03°} = 1.45\text{ s} \qquad \textbf{Ans.}$$

If $\theta = 74.97°$,

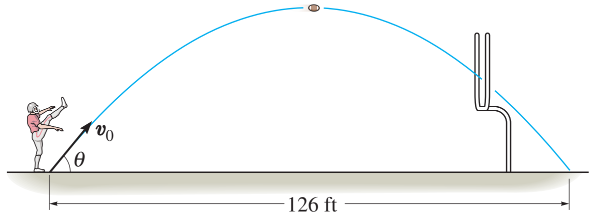
$$t = \frac{126}{90\cos 74.97°} = 5.40\text{ s} \qquad \textbf{Ans.}$$

Thus, $\quad \theta = 15.0°, \; t = 1.45$ s

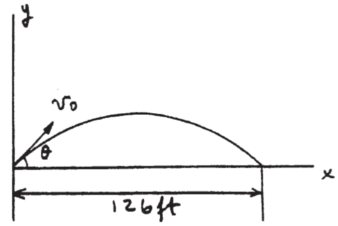
$\theta = 75.0°, \; t = 5.40$ s

12–94.

From a videotape, it was observed that a pro football player kicked a football 126 ft during a measured time of 3.6 seconds. Determine the initial speed of the ball and the angle θ at which it was kicked.



SOLUTION



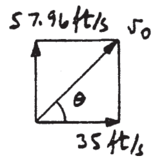
$(\overset{+}{\rightarrow})$ $\qquad s = s_0 + v_0 t$

$$126 = 0 + (v_0)_x (3.6)$$

$$(v_0)_x = 35 \text{ ft/s}$$

$(+\uparrow)$ $\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$O = 0 + (v_0)_y (3.6) + \frac{1}{2}(-32.2)(3.6)^2$$



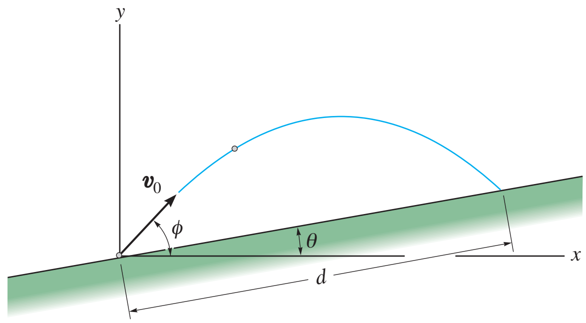
$$(v_0)_y = 57.96 \text{ ft/s}$$

$v_0 = \sqrt{(35)^2 + (57.96)^2} = 67.7 \text{ ft/s}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{57.96}{35}\right) = 58.9°$ **Ans.**

12–95.

A projectile is given a velocity $\mathbf{v}_0$ at an angle ϕ above the horizontal. Determine the distance d to where it strikes the sloped ground. The acceleration due to gravity is g.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$d \cos\theta = 0 + v_0(\cos\phi)t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$d \sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d \sin\theta = v_0 \sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

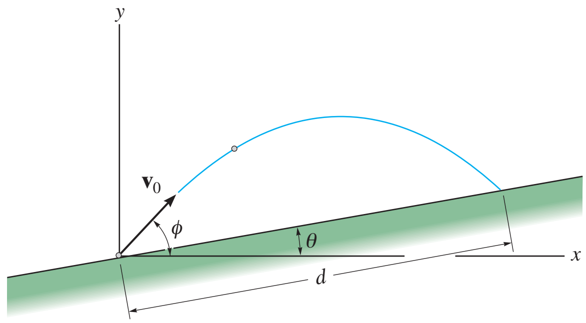
$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right) \qquad \textbf{Ans.}$$

***12–96.**

A projectile is given a velocity $\mathbf{v}_0$. Determine the angle ϕ at which it should be launched so that d is a maximum. The acceleration due to gravity is g.



SOLUTION

$(\overset{+}{\rightarrow})\qquad s_x = s_0 + v_0 t$

$$d\cos\theta = 0 + v_0(\cos\phi)t$$

$(+\uparrow)\qquad s_y = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$d\sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d\sin\theta = v_0\sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$

Require:

$$\frac{d(d)}{d\phi} = \frac{v_0^2}{g\cos\theta}\left[\cos 2\phi(2) - 2\tan\theta(2\cos\phi)(-\sin\phi)\right] = 0$$

$$\cos 2\phi + \tan\theta\sin 2\phi = 0$$

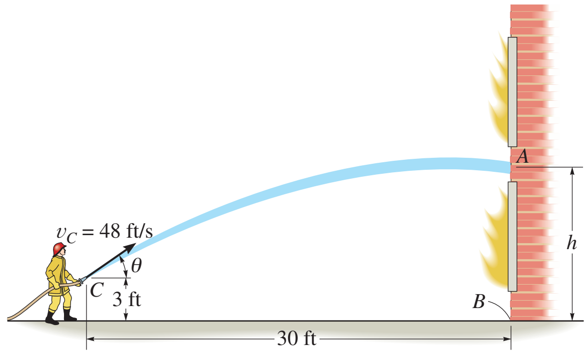
$$\frac{\sin 2\phi}{\cos 2\phi}\tan\theta + 1 = 0$$

$$\tan 2\phi = -\text{ctn}\,\theta$$

$$\phi = \frac{1}{2}\tan^{-1}(-\text{ctn}\,\theta)$$ **Ans.**

12–97.

Determine the maximum height on the wall to which the firefighter can project water from the hose, if the speed of the water at the nozzle is $v_C = 48$ ft/s.



SOLUTION

$(+\uparrow)\ v = v_0 + a_c t$

$$0 = 48 \sin\theta - 32.2\,t$$

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$$30 = 0 + 48\,(\cos\theta)(t)$$

$$48 \sin\theta = 32.2 \frac{30}{48\cos\theta}$$

$$\sin\theta\cos\theta = 0.41927$$

$$\sin 2\theta = 0.83854$$

$$\theta = 28.5^\circ$$

$$t = 0.7111 \text{ s}$$

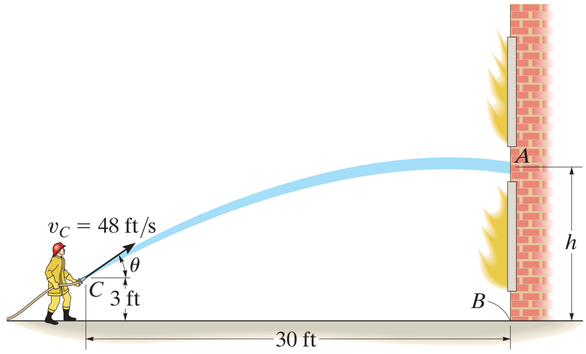
$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$h - 3 = 0 + 48 \sin 28.5^\circ\,(0.7111) + \frac{1}{2}(-32.2)(0.7111)^2$$

$h = 11.1$ ft **Ans.**

■ **12–98.**

Determine the smallest angle θ, measured above the horizontal, that the hose should be directed so that the water stream strikes the bottom of the wall at B. The speed of the water at the nozzle is $v_C = 48$ ft/s.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$30 = 0 + 48 \cos\theta\, t$$

$$t = \frac{30}{48\cos\theta}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$0 = 3 + 48\sin\theta\, t + \frac{1}{2}(-32.2)t^2$$

$$0 = 3 + \frac{48\sin\theta\,(30)}{48\cos\theta} - 16.1\left(\frac{30}{48\cos\theta}\right)^2$$

$$0 = 3\cos^2\theta + 30\sin\theta\cos\theta - 6.2891$$

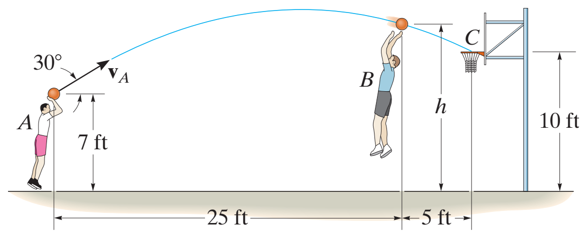
$$3\cos^2\theta + 15\sin 2\theta = 6.2891$$

Solving

$\theta = 6.41°$ or $77.9°$ **Ans.**

12–99.

Measurements of a shot recorded on a videotape during a basketball game are shown. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



SOLUTION

$(\overset{+}{\rightarrow})$ $$s = s_0 + v_0 t$$

$$30 = 0 + v_A \cos 30° \, t_{AC}$$

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$10 = 7 + v_A \sin 30° \, t_{AC} - \frac{1}{2}(32.2)(t_{AC}^2)$$

Solving

$v_A = 36.73 = 36.7$ ft/s **Ans.**

$t_{AC} = 0.943$ s

$(\overset{+}{\rightarrow})$ $$s = s_0 + v_0 t$$

$$25 = 0 + 36.73 \cos 30° \, t_{AB}$$

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$h = 7 + 36.73 \sin 30° \, t_{AB} - \frac{1}{2}(32.2)(t_{AB}^2)$$

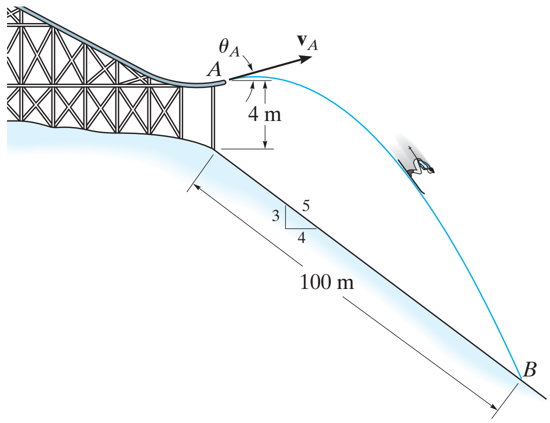
Solving

$t_{AB} = 0.786$ s

$h = 11.5$ ft **Ans.**

***12–100.**

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the time of flight t_{AB}.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

$$100\left(\frac{4}{5}\right) = v_A \cos 25° t_{AB}$$

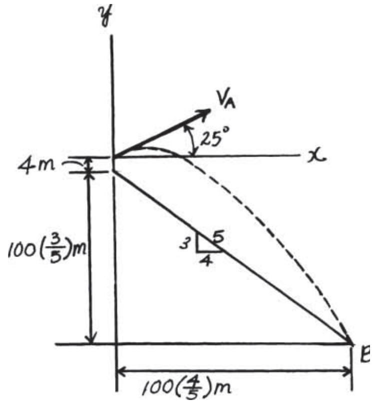
$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25° t_{AB} + \frac{1}{2}(-9.81)t_{AB}^2$$

Solving,

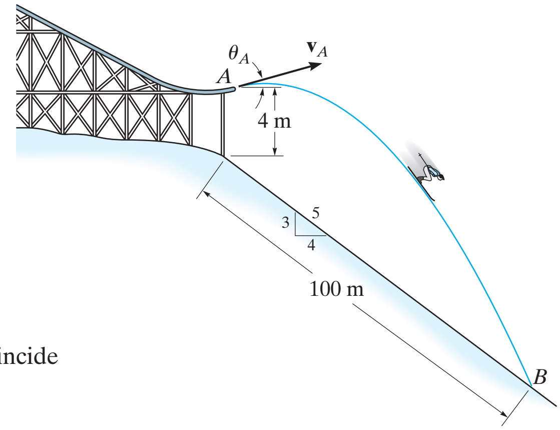
$$v_A = 19.4 \text{ m/s} \qquad \textbf{Ans.}$$

$$t_{AB} = 4.54 \text{ s} \qquad \textbf{Ans.}$$



12–101.

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the speed at which he strikes the ground.



SOLUTION

Coordinate System: $x-y$ coordinate system will be set with its origin to coincide with point A as shown in Fig. *a*.

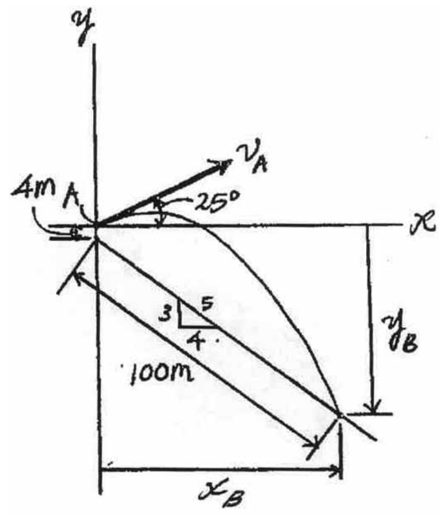
x-motion: Here, $x_A = 0,\ x_B = 100\left(\dfrac{4}{5}\right) = 80$ m and $(v_A)_x = v_A \cos 25°$.

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$80 = 0 + (v_A \cos 25°)t$$

$$t = \frac{80}{v_A \cos 25°} \qquad \textbf{(1)}$$

y-motion: Here, $y_A = 0,\ y_B = -\left[4 + 100\left(\dfrac{3}{5}\right)\right] = -64$ m and $(v_A)_y = v_A \sin 25°$ and $a_y = -g = -9.81\ \text{m/s}^2$.



(a)

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$-64 = 0 + v_A \sin 25°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - v_A \sin 25°\, t = 64 \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2) yieldS

$$4.905\left(\frac{80}{v_A \cos 25°}\right)^2 = v_A \sin 25°\left(\frac{80}{v_A \cos 25°}\right) = 64$$

$$\left(\frac{80}{v_A \cos 25°}\right)^2 = 20.65$$

$$\frac{80}{v_A \cos 25°} = 4.545$$

$$v_A = 19.42\ \text{m/s} = 19.4\ \text{m/s} \qquad \textbf{Ans.}$$

Substitute this result into Eq. (1),

$$t = \frac{80}{19.42 \cos 25°} = 4.54465$$

12–101. continued

Using this result,

$(+\uparrow)$ $$\begin{aligned}(v_B)_y &= (v_A)_y + a_y t \\ &= 19.42 \sin 25° + (-9.81)(4.5446) \\ &= -36.37 \text{ m/s } = 36.37 \text{ m/s} \downarrow\end{aligned}$$

And

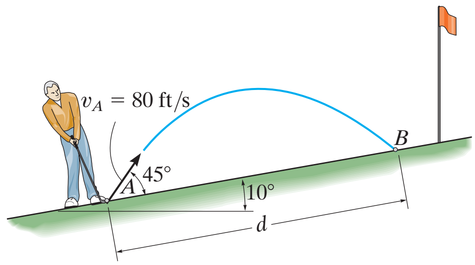
$(\overset{+}{\rightarrow})$ $$(v_B)_x = (v_A)_x = v_A \cos 25° = 19.42 \cos 25° = 17.60 \text{ m/s} \rightarrow$$

Thus,

$$\begin{aligned}v_B &= \sqrt{(v_B)_x^2 + (v_B)_y^2} \\ &= \sqrt{36.37^2 + 17.60^2} \\ &= 40.4 \text{ m/s}\end{aligned}$$ **Ans.**

12–102.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



SOLUTION

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 80 \cos 55° = 45.89$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = d \cos 10°$, respectively.

$$\left(\xrightarrow{+}\right) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$d \cos 10° = 0 + 45.89t \tag{1}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 80 \sin 55° = 65.53$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = d \sin 10°$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$d \sin 10° = 0 + 65.53t + \frac{1}{2}(-32.2)t^2 \tag{2}$$

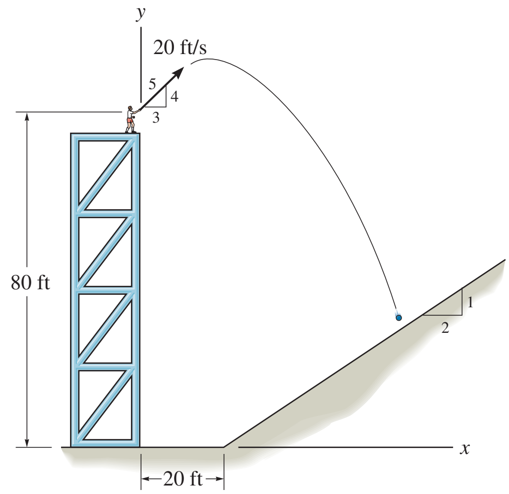
Solving Eqs. (1) and (2) yields

$$d = 166 \text{ ft} \qquad \textbf{Ans.}$$

$$t = 3.568 \text{ s}$$

12–103.

The ball is thrown from the tower with a velocity of 20 ft/s as shown. Determine the x and y coordinates to where the ball strikes the slope. Also, determine the speed at which the ball hits the ground.



SOLUTION

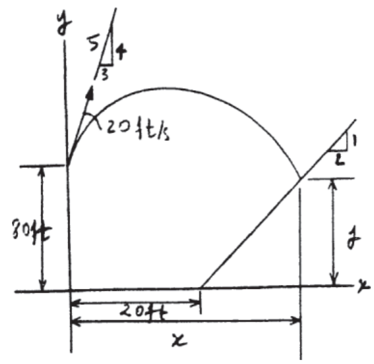
Assume ball hits slope.

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$x = 0 + \frac{3}{5}(20)t = 12t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 80 + \frac{4}{5}(20)t + \frac{1}{2}(-32.2)t^2 = 80 + 16t - 16.1t^2$$



Equation of slope: $y - y_1 = m(x - x_1)$

$$y - 0 = \frac{1}{2}(x - 20)$$

$$y = 0.5x - 10$$

Thus,

$$80 + 16t - 16.1t^2 = 0.5(12t) - 10$$

$$16.1t^2 - 10t - 90 = 0$$

Choosing the positive root:

$t = 2.6952$ s

$x = 12(2.6952) = 32.3$ ft **Ans.**

Since 32.3 ft > 20 ft, assumption is valid.

$y = 80 + 16(2.6952) - 16.1(2.6952)^2 = 6.17$ ft **Ans.**

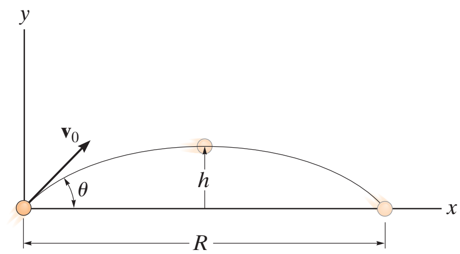
$(\overset{+}{\rightarrow})$ $\quad v_x = (v_0)_x = \frac{3}{5}(20) = 12$ ft/s

$(+\uparrow)$ $\quad v_y = (v_0)_y + a_c t = \frac{4}{5}(20) + (-32.2)(2.6952) = -70.785$ ft/s

$v = \sqrt{(12)^2 + (-70.785)^2} = 71.8$ ft/s **Ans.**

***12–104.**

The projectile is launched with a velocity $\mathbf{v}_0$. Determine the range R, the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0. The acceleration due to gravity is g.



SOLUTION

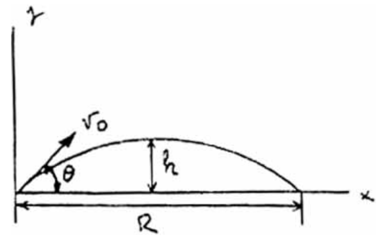
$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$$R = 0 + (v_0 \cos\theta)t$$

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$0 = 0 + (v_0 \sin\theta)\, t + \frac{1}{2}(-g)t^2$$

$$0 = v_0 \sin\theta - \frac{1}{2}(g)\left(\frac{R}{v_0 \cos\theta}\right)$$



$$R = \frac{v_0^2}{g}\sin 2\theta \qquad \textbf{Ans.}$$

$$t = \frac{R}{v_0\cos\theta} = \frac{v_0^2\,(2\sin\theta\cos\theta)}{v_0\, g\cos\theta}$$

$$= \frac{2v_0}{g}\sin\theta \qquad \textbf{Ans.}$$

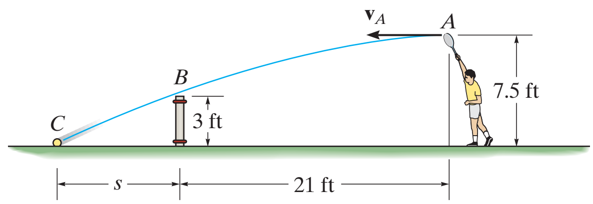
$(+\uparrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (v_0\sin\theta)^2 + 2(-g)(h - 0)$$

$$h = \frac{v_0^2}{2g}\sin^2\theta \qquad \textbf{Ans.}$$

12–105.

Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B. Also, find the distance s where the ball strikes the ground.



SOLUTION

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 0$. For the ball to travel from A to B, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 3$ ft, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$3 = 7.5 + 0 + \frac{1}{2}(-32.2)t_1^2$$

$$t_1 = 0.5287 \text{ s}$$

For the ball to travel from A to C, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 0$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 7.5 + 0 + \frac{1}{2}(-32.2)t_2^2$$

$$t_2 = 0.6825 \text{ s}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A$. For the ball to travel from A to B, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 21$ ft, respectively. The time is $t = t_1 = 0.5287$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 = 0 + v_A(0.5287)$$

$$v_A = 39.72 \text{ ft/s} = 39.7 \text{ ft/s} \qquad \textbf{Ans.}$$

For the ball to travel from A to C, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (21 + s)$ ft, respectively. The time is $t = t_2 = 0.6825$ s.

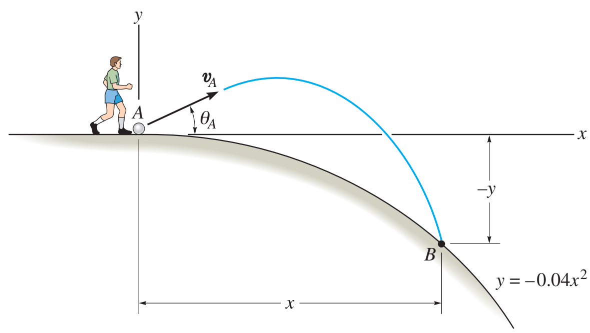
$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 + s = 0 + 39.72(0.6825)$$

$$s = 6.11 \text{ ft} \qquad \textbf{Ans.}$$

12–106.

The ball at A is kicked with a speed $v_A = 8$ ft/s and at an angle $\theta_A = 30°$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.



SOLUTION

$(v_A)_x = 8 \cos 30° = 6.928$ ft/s

$(v_A)_y = 8 \sin 30° = 4$ ft/s

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$$x = 0 + 6.928\, t \qquad \textbf{(1)}$$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$y = 0 + 4t + \frac{1}{2}(-32.2)t^2 \qquad \textbf{(2)}$$

$y = -0.04\, x^2$

From Eqs. (1) and (2):

$y = 0.5774\, x - 0.3354\, x^2$

$-0.04\, x^2 = 0.5774x - 0.3354\, x^2$

$0.2954\, x^2 = 0.5774x$

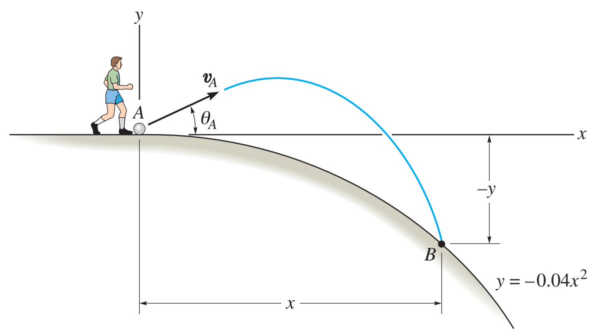
$x = 1.95$ ft **Ans.**

Thus,

$y = -0.04(1.954)^2 = -0.153$ ft **Ans.**

12–107.

The ball at A is kicked such that $\theta_A = 30°$. If it strikes the ground at B having coordinates $x = 15$ ft, $y = -9$ ft, determine the speed at which it is kicked and the speed at which it strikes the ground.



SOLUTION

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$$15 = 0 + v_A \cos 30°\, t$$

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-9 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-32.2)t^2$$

$v_A = 16.5$ ft/s **Ans.**

$t = 1.047$ s

$(\xrightarrow{+})\ (v_B)_x = 16.54 \cos 30° = 14.32$ ft/s

$(+\uparrow)\ v = v_0 + a_c t$

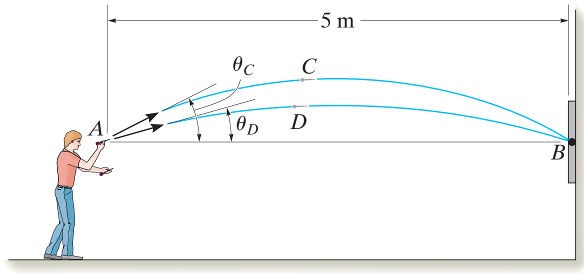
$(v_B)_y = 16.54 \sin 30° + (-32.2)(1.047)$

$= -25.45$ ft/s

$v_B = \sqrt{(14.32)^2 + (-25.45)^2} = 29.2$ ft/s **Ans.**

***12–108.**

The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at θ_C $(>\theta_D)$, then the second dart is thrown at θ_D.



SOLUTION

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$5 = 0 + (10 \cos \theta)\, t \qquad \textbf{(1)}$$

$(+\uparrow)$ $\quad v = v_0 + a_c t$

$$-10 \sin \theta = 10 \sin \theta - 9.81\, t$$

$$t = \frac{2(10 \sin \theta)}{9.81} = 2.039 \sin \theta$$

From Eq. (1),

$$5 = 20.39 \sin \theta \cos \theta$$

Since $\quad \sin 2\theta = 2 \sin \theta \cos \theta$

$$\sin 2\theta = 0.4905$$

The two roots are $\theta_D = 14.7°$ **Ans.**

$\theta_C = 75.3°$ **Ans.**

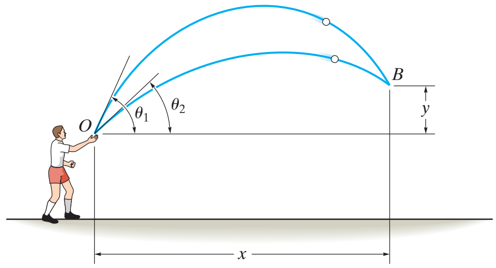
From Eq. (1): $t_D = 0.517$ s

$t_C = 1.97$ s

So that $\Delta t = t_C - t_D = 1.45$ s **Ans.**

12–109.

A boy throws a ball at O in the air with a speed v_0 at an angle θ_1. If he then throws another ball with the same speed v_0 at an angle $\theta_2 < \theta_1$, determine the time between the throws so that the balls collide in mid air at B.



SOLUTION

Vertical Motion: For the first ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin\theta_1$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin\theta_1 t_1 + \frac{1}{2}(-g)t_1^2 \qquad \textbf{(1)}$$

For the second ball, the vertical component of initial velocity is $(v_0)_y = v_0 \sin\theta_2$ and the initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + v_0 \sin\theta_2 t_2 + \frac{1}{2}(-g)t_2^2 \qquad \textbf{(2)}$$

Horizontal Motion: For the first ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos\theta_1$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos\theta_1\, t_1 \qquad \textbf{(3)}$$

For the second ball, the horizontal component of initial velocity is $(v_0)_x = v_0 \cos\theta_2$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + v_0 \cos\theta_2\, t_2 \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4), we have

$$t_2 = \frac{\cos\theta_1}{\cos\theta_2} t_1 \qquad \textbf{(5)}$$

Equating Eqs. (1) and (2), we have

$$v_0 t_1 \sin\theta_1 - v_0 t_2 \sin\theta_2 = \frac{1}{2} g\left(t_1^2 - t_2^2\right) \qquad \textbf{(6)}$$

Solving Eq. [5] into [6] yields

$$t_1 = \frac{2v_0 \cos\theta_2 \sin(\theta_1 - \theta_2)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

$$t_2 = \frac{2v_0 \cos\theta_1 \sin(\theta_1 - \theta_2)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

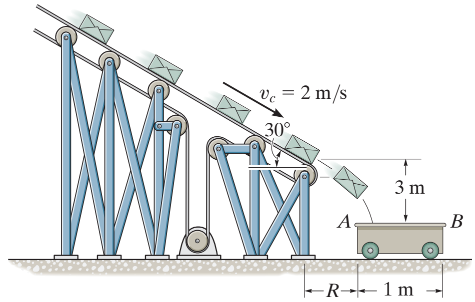
Thus, the time between the throws is

$$\Delta t = t_1 - t_2 = \frac{2v_0 \sin(\theta_1 - \theta_2)(\cos\theta_2 - \cos\theta_1)}{g(\cos^2\theta_2 - \cos^2\theta_1)}$$

$$= \frac{2v_0 \sin(\theta_1 - \theta_2)}{g(\cos\theta_2 + \cos\theta_1)} \qquad \textbf{Ans.}$$

12–110.

Small packages traveling on the conveyor belt fall off into a l-m-long loading car. If the conveyor is running at a constant speed of $v_C = 2$ m/s, determine the smallest and largest distance R at which the end A of the car may be placed from the conveyor so that the packages enter the car.



SOLUTION

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 2\sin 30° = 1.00$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 3$ m, respectively.

$$(+\downarrow) \qquad s_y = (s_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y\, t^2$$

$$3 = 0 + 1.00(t) + \frac{1}{2}(9.81)\left(t^2\right)$$

Choose the positive root $t = 0.6867$ s

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 2\cos 30° = 1.732$ m/s and the initial horizontal position is $(s_0)_x = 0$. If $s_x = R$, then

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

$$R = 0 + 1.732(0.6867) = 1.19 \text{ m}$$ **Ans.**

If $s_x = R + 1$, then

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x\, t$$

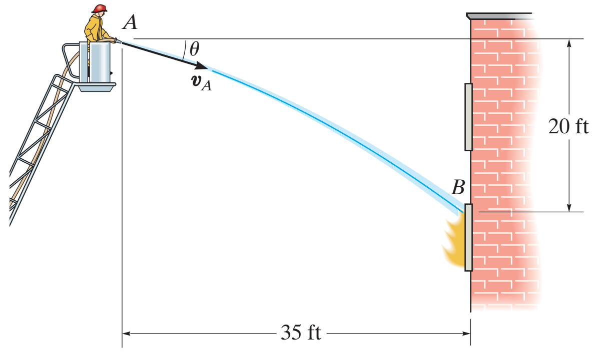
$$R + 1 = 0 + 1.732(0.6867)$$

$$R = 0.189 \text{ m}$$ **Ans.**

Thus, $R_{\min} = 0.189$ m, $R_{\max} = 1.19$ m **Ans.**

12–111.

The fireman wishes to direct the flow of water from his hose to the fire at B. Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at $v_A = 80$ ft/s.



SOLUTION

$(\xrightarrow{+})$ $\qquad s = s_0 + v_0 t$

$$35 = 0 + (80)(\cos\theta)t$$

$(+\uparrow)$ $\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-20 = 0 - 80\,(\sin\theta)t + \frac{1}{2}(-32.2)t^2$$

Thus,

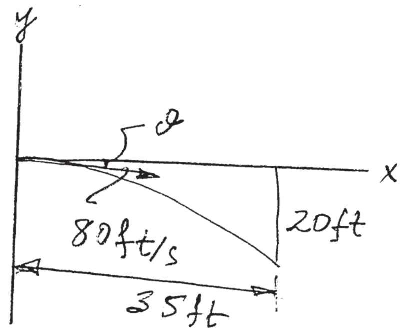
$$20 = 80 \sin\theta \frac{0.4375}{\cos\theta} t + 16.1\left(\frac{0.1914}{\cos^2\theta}\right)$$

$$20\cos^2\theta = 17.5 \sin 2\theta + 3.0816$$

Solving,

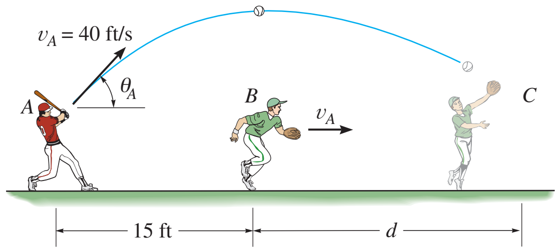
$\theta_1 = 24.9°$ (below the horizontal) **Ans.**

$\theta_2 = 85.2°$ (above the horizontal) **Ans.**



***12–112.**

The baseball player A hits the baseball at $v_A = 40$ ft/s and $\theta_A = 60^\circ$ from the horizontal. When the ball is directly overhead of player B he begins to run under it. Determine the constant speed at which B must run and the distance d in order to make the catch at the same elevation at which the ball was hit.



SOLUTION

Vertical Motion: The vertical component of initial velocity for the football is $(v_0)_y = 40 \sin 60^\circ = 34.64$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 0$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 0 + 34.64t + \frac{1}{2}(-32.2)\, t^2$$

$$t = 2.152 \text{ s}$$

Horizontal Motion: The horizontal component of velocity for the baseball is $(v_0)_x = 40 \cos 60^\circ = 20.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = R$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$R = 0 + 20.0(2.152) = 43.03 \text{ ft}$$

The distance for which player B must travel in order to catch the baseball is

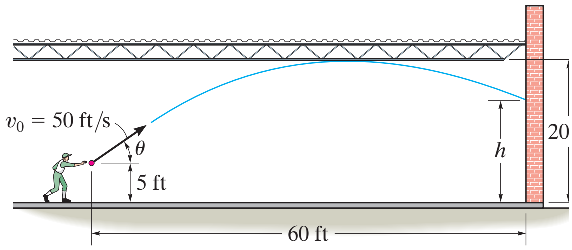
$d = R - 15 = 43.03 - 15 = 28.0$ ft **Ans.**

Player B is required to run at a same speed as the horizontal component of velocity of the baseball in order to catch it.

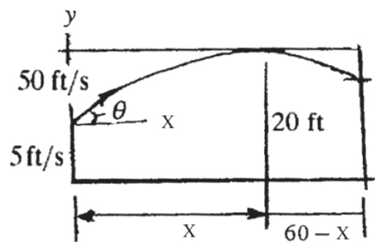
$v_B = 40 \cos 60^\circ = 20.0$ ft/s **Ans.**

12–113.

The man stands 60 ft from the wall and throws a ball at it with a speed $v_0 = 50$ ft/s. Determine the angle θ at which he should release the ball so that it strikes the wall at the highest point possible. What is this height? The room has a ceiling height of 20 ft.



SOLUTION



$v_x = 50 \cos \theta$

$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$

$$x = 0 + 50 \cos \theta \, t \qquad \textbf{(1)}$$

$(+\uparrow) \qquad v = v_0 + a_c t$

$$v_y = 50 \sin \theta - 32.2\, t \qquad \textbf{(2)}$$

$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$y = 0 + 50 \sin \theta \, t - 16.1\, t^2 \qquad \textbf{(3)}$$

$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$$v_y^2 = (50 \sin \theta)^2 + 2(-32.2)(s - 0)$$

$$v_y^2 = 2500 \sin^2 \theta - 64.4\, s \qquad \textbf{(4)}$$

Require $v_y = 0$ at $s = 20 - 5 = 15$ ft

$0 = 2500 \sin^2 \theta - 64.4\,(15)$

$\theta = 38.433° = 38.4°$ **Ans.**

From Eq. (2)

$0 = 50 \sin 38.433° - 32.2\, t$

$t = 0.9652$ s

From Eq. (1)

$x = 50 \cos 38.433°(0.9652) = 37.8$ ft

Time for ball to hit wall

From Eq. (1),

$60 = 50(\cos 38.433°)t$

$t = 1.53193$ s

From Eq. (3)

$y = 50 \sin 38.433°(1.53193) - 16.1(1.53193)^2$

$y = 9.830$ ft

$h = 9.830 + 5 = 14.8$ ft **Ans.**

12–114.

A car is traveling along a circular curve that has a radius of 50 m. If its speed is 16 m/s and is increasing uniformly at 8 m/s^2, determine the magnitude of its acceleration at this instant.

SOLUTION

$$v = 16 \text{ m/s}$$

$$a_t = 8 \text{ m/s}^2$$

$$r = 50 \text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(16)^2}{50} = 5.12 \text{ m/s}^2$$

$$a = \sqrt{(8)^2 + (5.12)^2} = 9.50 \text{ m/s}^2$$ **Ans.**

12–115.

Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

SOLUTION

Acceleration: Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$7.5 = \frac{v^2}{200}$$

$$v = 38.7 \text{ m/s} \qquad \textbf{Ans.}$$

***12–116.**

A car moves along a circular track of radius 250 ft such that its speed for a short period of time, $0 \leq t \leq 4$ s, is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of its acceleration when $t = 3$ s. How far has it traveled in $t = 3$ s?

SOLUTION

$v = 3(t + t^2)$

$$a_t = \frac{dv}{dt} = 3 + 6\,t$$

When $t = 3$ s, $\quad a_t = 3 + 6(3) = 21 \text{ ft/s}^2$

$$a_n = \frac{[3(3 + 3^2)]^2}{250} = 5.18 \text{ ft/s}^2$$

$a = \sqrt{(21)^2 + (5.18)^2} = 21.6 \text{ ft/s}^2$ **Ans.**

$$\int ds = \int_0^3 3(t + t^2)dt$$

$$\Delta s = \frac{3}{2}t^2 + t^3\Big|_0^3$$

$\Delta s = 40.5$ ft **Ans.**

12–117.

A car travels along a horizontal circular curved road that has a radius of 600 m. If the speed is uniformly increased at a rate of $2000\ \text{km/h}^2$, determine the magnitude of the acceleration at the instant the speed of the car is 60 km/h.

SOLUTION

$$a_t = \left(\frac{2000\ \text{km}}{\text{h}^2}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\text{h}}{3600\ \text{s}}\right)^2 = 0.1543\ \text{m/s}^2$$

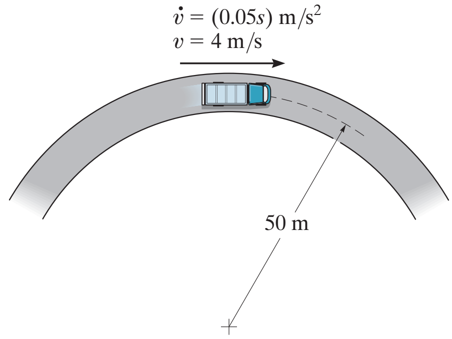
$$v = \left(\frac{60\ \text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\text{h}}{3600\ \text{s}}\right) = 16.67\ \text{m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{600} = 0.4630\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.1543^2 + 0.4630^2} = 0.488\ \text{m/s}^2$$ **Ans.**

12–118.

The truck travels in a circular path having a radius of 50 m at a speed of $v = 4$ m/s. For a short distance from $s = 0$, its speed is increased by $\dot{v} = (0.05s)$ m/s^2, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



SOLUTION

$$v\,dv = a_t\,ds$$

$$\int_4^v v\,dv = \int_0^{10} 0.05s\,ds$$

$$0.5v^2 - 8 = \frac{0.05}{2}(10)^2$$

$$v = 4.583 = 4.58 \text{ m/s} \qquad \textbf{Ans.}$$

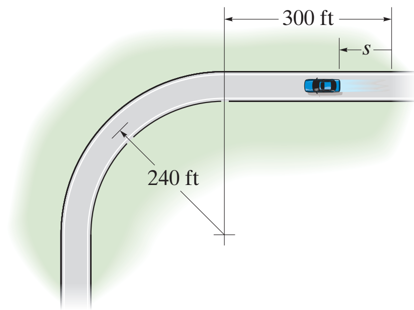
$$a_n = \frac{v^2}{\rho} = \frac{(4.583)^2}{50} = 0.420 \text{ m/s}^2$$

$$a_t = 0.05(10) = 0.5 \text{ m/s}^2$$

$$a = \sqrt{(0.420)^2 + (0.5)^2} = 0.653 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–119.

The automobile is originally at rest at $s = 0$. If its speed is increased by $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration when $t = 18$ s.



SOLUTION

$a_t = 0.05t^2$

$$\int_0^v dv = \int_0^t 0.05\, t^2\, dt$$

$v = 0.0167\, t^3$

$$\int_0^s ds = \int_0^t 0.0167\, t^3\, dt$$

$s = 4.167(10^{-3})\, t^4$

When $t = 18$ s, $\quad s = 437.4$ ft

Therefore the car is on a curved path.

$v = 0.0167(18^3) = 97.2$ ft/s **Ans.**

$$a_n = \frac{(97.2)^2}{240} = 39.37 \text{ ft/s}^2$$

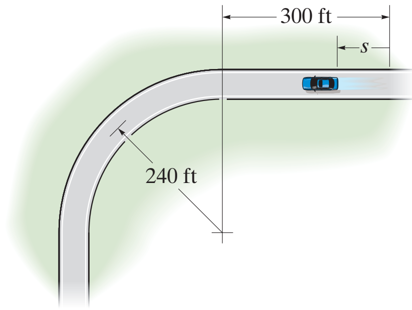
$a_t = 0.05(18^2) = 16.2$ ft/s^2

$a = \sqrt{(39.37)^2 + (16.2)^2}$

$a = 42.6$ ft/s^2 **Ans.**

***12–120.**

The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = (0.05t^2)\ \text{ft/s}^2$, where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550$ ft.



SOLUTION

The car is on the curved path.

$$a_t = 0.05\, t^2$$

$$\int_0^v dv = \int_0^t 0.05\, t^2\, dt$$

$$v = 0.0167\, t^3$$

$$\int_0^s ds = \int_0^t 0.0167\, t^3\, dt$$

$$s = 4.167(10^{-3})\, t^4$$

$$550 = 4.167(10^{-3})\, t^4$$

$$t = 19.06 \text{ s}$$

So that

$$v = 0.0167(19.06)^3 = 115.4$$

$$v = 115 \text{ ft/s} \qquad \textbf{Ans.}$$

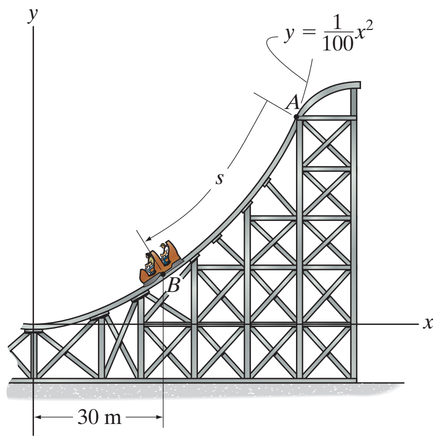
$$a_n = \frac{(115.4)^2}{240} = 55.51 \text{ ft/s}^2$$

$$a_t = 0.05(19.06)^2 = 18.17 \text{ ft/s}^2$$

$$a = \sqrt{(55.51)^2 + (18.17)^2} = 58.4 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–121.

When the roller coaster is at B, it has a speed of 25 m/s, which is increasing at $a_t = 3\ \text{m/s}^2$. Determine the magnitude of the acceleration of the roller coaster at this instant and the direction angle it makes with the x axis.



SOLUTION

Radius of Curvature:

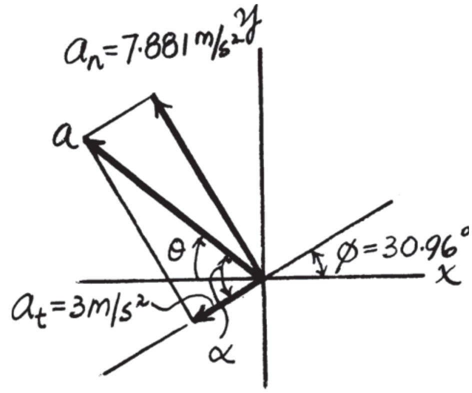
$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\text{ m}} = 79.30\text{ m}$$

Acceleration:



(a)

$$a_t = \dot{v} = 3\ \text{m/s}^2$$

$$a_n = \frac{v_B{}^2}{\rho} = \frac{25^2}{79.30} = 7.881\ \text{m/s}^2$$

The magnitude of the roller coaster's acceleration is

$$a = \sqrt{a_t{}^2 + a_n{}^2} = \sqrt{3^2 + 7.881^2} = 8.43\ \text{m/s}^2 \qquad \textbf{Ans.}$$

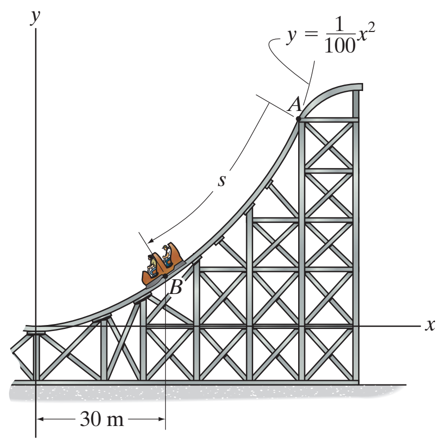
The angle that the tangent at B makes with the x axis is $\phi = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=30\text{ m}}\right) = \tan^{-1}\left[\frac{1}{50}(30)\right] = 30.96°$.

As shown in Fig. a, $\mathbf{a}_n$ is always directed towards the center of curvature of the path. Here, $\alpha = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{7.881}{3}\right) = 69.16°$. Thus, the angle θ that the roller coaster's acceleration makes with the x axis is

$$\theta = \alpha - \phi = 38.2° \qquad \textbf{Ans.}$$

12–122.

If the roller coaster starts from rest at A and its speed increases at $a_t = (6 - 0.06s)\ \text{m/s}^2$, determine the magnitude of its acceleration when it reaches B where $s_B = 40$ m.



SOLUTION

Velocity: Using the initial condition $v = 0$ at $s = 0$,

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^s (6 - 0.06s)\,ds$$

$$v = \left(\sqrt{12s - 0.06s^2}\right)\ \text{m/s} \qquad \textbf{(1)}$$

Thus,

$$v_B = \sqrt{12(40) - 0.06(40)^2} = 19.60\ \text{m/s}$$

Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\ \text{m}} = 79.30\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 6 - 0.06(40) = 3.600\ \text{m/s}^2$$

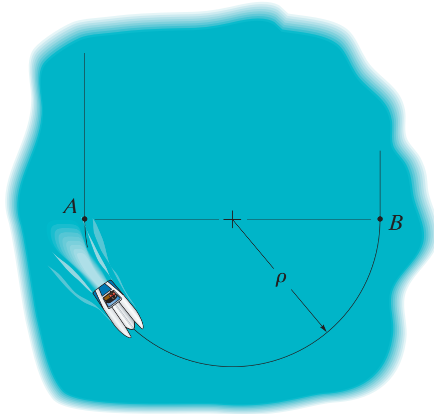
$$a_n = \frac{v^2}{\rho} = \frac{19.60^2}{79.30} = 4.842\ \text{m/s}^2$$

The magnitude of the roller coaster's acceleration at B is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{3.600^2 + 4.842^2} = 6.03\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–123.

The speedboat travels at a constant speed of 15 m/s while making a turn on a circular curve from A to B. If it takes 45 s to make the turn, determine the magnitude of the boat's acceleration during the turn.



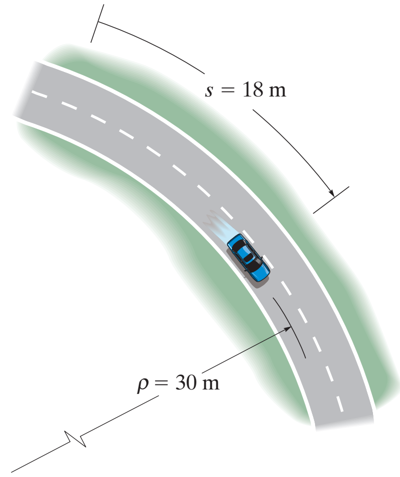
SOLUTION

Acceleration: During the turn, the boat travels $s = vt = 15(45) = 675$ m. Thus, the radius of the circular path is $\rho = \dfrac{s}{\pi} = \dfrac{675}{\pi}$ m. Since the boat has a constant speed, $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho} = \frac{15^2}{\left(\dfrac{675}{\pi}\right)} = 1.05\ \text{m/s}^2 \qquad \textbf{Ans.}$$

***12–124.**

The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18$ m starting from rest. Neglect the size of the car.



SOLUTION

$$\int_0^v dv = \int_0^t 0.5e^t\, dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving,

$$t = 3.7064\ \text{s}$$

$$v = 0.5(e^{3.7064} - 1) = 19.85\ \text{m/s} = 19.9\ \text{m/s} \qquad \textbf{Ans.}$$

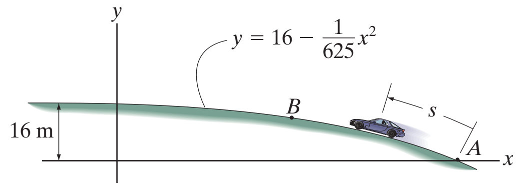
$$a_t = \dot{v} = 0.5e^t\big|_{t=3.7064\ \text{s}} = 20.35\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–125.

The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point B, where $s = 51.5$ m.



SOLUTION

Velocity: The speed of the car at B is

$$v_B = \left[25 - 0.15(51.5)\right] = 17.28 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=50\text{ m}} = 324.58 \text{ m}$$

Acceleration:

$$a_n = \frac{{v_B}^2}{\rho} = \frac{17.28^2}{324.58} = 0.9194 \text{ m/s}^2$$

$$a_t = v\frac{dv}{ds} = (25 - 0.15s)(-0.15) = (0.225s - 3.75) \text{ m/s}^2$$

When the car is at B $(s = 51.5 \text{ m})$

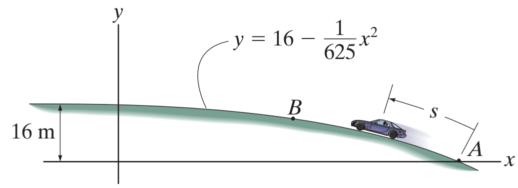
$$a_t = \left[0.225(51.5) - 3.75\right] = -2.591 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.591)^2 + 0.9194^2} = 2.75 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–126.

If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5\ \text{m/s}^2$, determine the magnitude of the car's acceleration when $s = 100$ m.



SOLUTION

Velocity: The speed of the car at C is

$$v_C{}^2 = v_A{}^2 + 2a_t(s_C - s_A)$$

$$v_C{}^2 = 20^2 + 2(0.5)(100 - 0)$$

$$v_C = 22.361\ \text{m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=0} = 312.5\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 0.5\ \text{m/s}$$

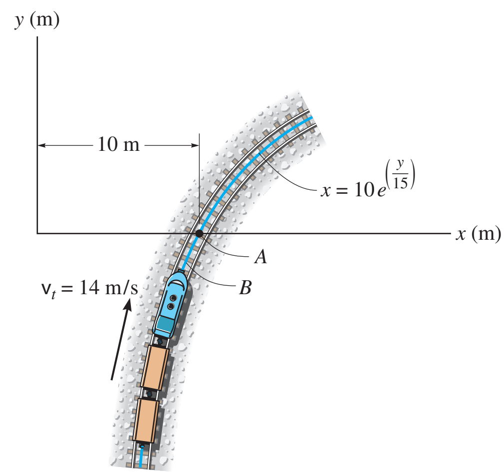
$$a_n = \frac{v_C{}^2}{\rho} = \frac{22.361^2}{312.5} = 1.60\ \text{m/s}^2$$

The magnitude of the car's acceleration at C is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68\ \text{m/s}^2$$ **Ans.**

12–127.

A train is traveling with a constant speed of 14 m/s along the curved path. Determine the magnitude of the acceleration of the front of the train, B, at the instant it reaches point A ($y = 0$).



SOLUTION

$$x = 10e^{\left(\frac{y}{15}\right)}$$

$$y = 15 \ln\left(\frac{x}{10}\right)$$

$$\frac{dy}{dx} = 15\left(\frac{10}{x}\right)\left(\frac{1}{10}\right) = \frac{15}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}$$

At $x = 10$,

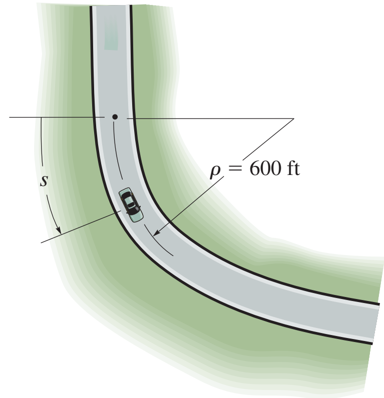
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (1.5)^2\right]^{\frac{3}{2}}}{|-0.15|} = 39.06 \text{ m}$$

$$a_t = \frac{dv}{dt} = 0$$

$$a_n = a = \frac{v^2}{\rho} = \frac{(14)^2}{39.06} = 5.02 \text{ m/s}^2$$ **Ans.**

***12–128.**

When a car starts to round a curved road with the radius of curvature of 600 ft, it is traveling at 75 ft/s. If the car's speed begins to decrease at a rate of $\dot{v} = (-0.06t^2)\ \text{ft/s}^2$, determine the magnitude of the acceleration of the car when it has traveled a distance of $s = 700$ ft.



SOLUTION

Velocity: Using the initial condition $v = 75$ ft/s when $t = 0$ s,

$$\int dt = \int a_t dt$$

$$\int_{v=75\ \text{ft/s}}^{v} dv = \int_0^t -0.06t^2 dt$$

$$v = (75 - 0.02t^3)\ \text{ft/s}$$

Position: Using the initial condition $s = 0$ at $t = 0$ s,

$$ds = vdt$$

$$\int_0^s ds = \int_0^t (75 - 0.02t^3)dt$$

$$s = [75t - 0.005t^4]\ \text{ft}$$

At $s = 700$ ft,

$$700 = 75t - 0.005t^4$$

Solving the above equation by trial and error,

$t = 10$ s and $t = 20$ s. Pick the first solution.

Acceleration: When $t = 10$ s, $a_t = \dot{v} = -0.06(10^2) = -6\ \text{ft/s}^2$ and $v = 75 - 0.02(10^3) = 55$ ft/s

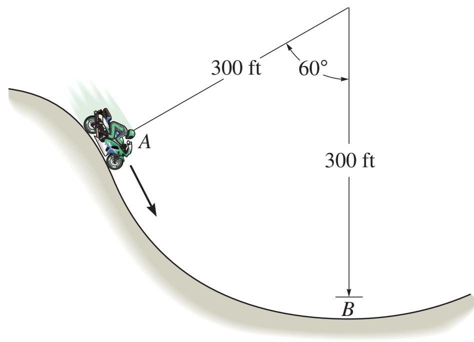
$$a_n = \frac{v^2}{\rho} = \frac{55^2}{600} = 5.042\ \text{ft/s}^2$$

Thus, the magnitude of the truck's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-6)^2 + 5.042^2} = 7.84\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–129.

When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of $\dot{v} = (0.3t)\ \text{ft/s}^2$, where t is in seconds. If he starts from rest at A, determine the magnitudes of his velocity and acceleration when he reaches B.



SOLUTION

$$\int_0^v dv = \int_0^t 0.3t\,dt$$

$$v = 0.15t^2$$

$$\int_0^s ds = \int_0^t 0.15t^2\,dt$$

$$s = 0.05t^3$$

When $s = \frac{\pi}{3}(300)$ ft, $\qquad \frac{\pi}{3}(300) = 0.05t^3 \qquad t = 18.453\ \text{s}$

$v = 0.15(18.453)^2 = 51.08\ \text{ft/s} = 51.1\ \text{ft/s}$ **Ans.**

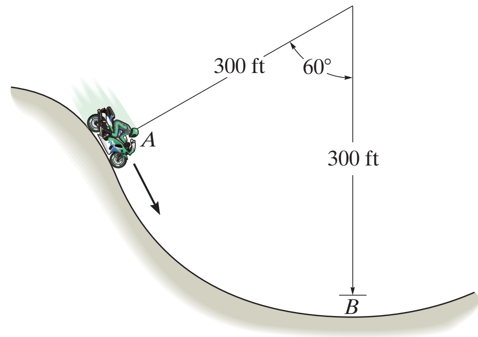
$a_t = \dot{v} = 0.3t|_{t=18.453\ \text{s}} = 5.536\ \text{ft/s}^2$

$$a_n = \frac{v^2}{\rho} = \frac{51.08^2}{300} = 8.696\ \text{ft/s}^2$$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(5.536)^2 + (8.696)^2} = 10.3\ \text{ft/s}^2$ **Ans.**

12–130.

When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of $\dot{v} = (0.04s)\ \text{ft/s}^2$ where s is in ft. If he starts at $v_A = 2\ \text{ft/s}$ where $s = 0$ at A, determine the magnitude of his velocity when he reaches B. Also, what is his initial acceleration?



SOLUTION

Velocity: At $s = 0, v = 2$. Here, $a_c = \dot{v} = 0.045$. Then

$$\int v\,dv = \int a_t\,ds$$

$$\int_2^v v\,dv = \int_0^s 0.04s\,ds$$

$$\left.\frac{v^2}{2}\right|_2^v = 0.025^2\Big|_0^s$$

$$\frac{v^2}{2} - 2 = 0.025^2$$

$$v^2 = 0.045^2 + 4 = 0.04(s^2 + 100)$$

$$v = 0.2\sqrt{s^2 + 100}$$

At B, $s = r\theta = 300\left(\frac{\pi}{3}\right) = 100\pi$ ft. Thus

$$v\Big|_{s=100\pi\ \text{ft}} = 0.2\sqrt{(100\pi)^2 + 100} = 62.9\ \text{ft/s}$$ **Ans.**

Acceleration: At $t = 0, s = 0$, and $v = 2$.

$$a_t = \dot{v} = 0.04\ \text{s}$$

$$a_t\Big|_{s=0} = 0$$

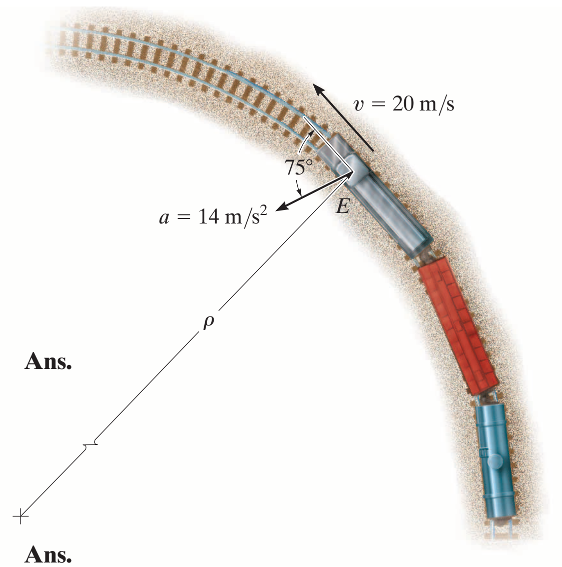
$$a_n = \frac{v^2}{\rho}$$

$$a_n\Big|_{s=0} = \frac{(2)^2}{300} = 0.01333\ \text{ft/s}^2$$

$$a = \sqrt{(0)^2 + (0.01333)^2} = 0.0133\ \text{ft/s}^2$$

12–131.

At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



SOLUTION

$a_t = 14 \cos 75° = 3.62$ m/s^2 **Ans.**

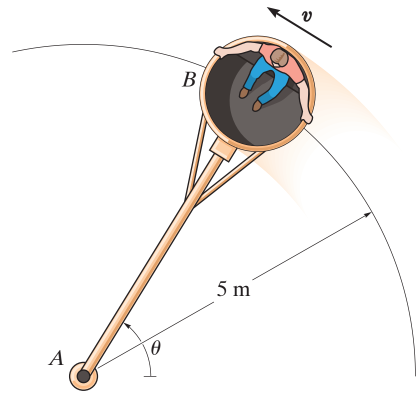
$a_n = 14 \sin 75°$

$$a_n = \frac{(20)^2}{\rho}$$

$\rho = 29.6$ m **Ans.**

***12–132.**

Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when the arm AB rotates $\theta = 30°$. Neglect the size of the car.



SOLUTION

Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = adt$$

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5\left(e^t - 1\right) \qquad \textbf{(1)}$$

When $\theta = 30°$, the car has traveled a distance of $s = r\theta = 5\left(\dfrac{30°}{180°}\pi\right) = 2.618$ m. The time required for the car to travel this distance can be obtained by applying $v = \dfrac{ds}{dt}$.

$$ds = vdt$$

$$\int_0^{2.618\text{ m}} ds = \int_0^t 0.5\left(e^t - 1\right)dt$$

$$2.618 = 0.5\left(e^t - t - 1\right)$$

Solving by trial and error $\quad t = 2.1234$ s

Substituting $t = 2.1234$ s into Eq. (1) yields

$$v = 0.5\left(e^{2.1234} - 1\right) = 3.680\ \text{m/s} = 3.68\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: The tangential acceleration for the car at $t = 2.1234$ s is $a_t = 0.5e^{2.1234} = 4.180\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

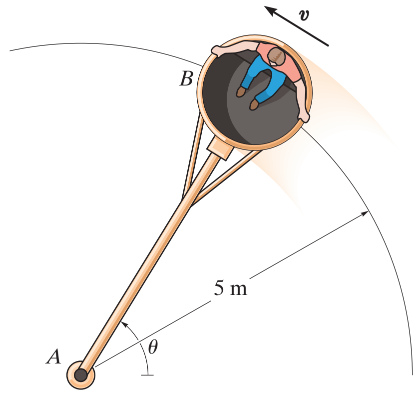
$$a_n = \frac{v^2}{\rho} = \frac{3.680^2}{5} = 2.708\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4.180^2 + 2.708^2} = 4.98\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–133.

Car B turns such that its speed is increased by $(a_t)_B = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. If the car starts from rest when $\theta = 0°$, determine the magnitudes of its velocity and acceleration when $t = 2$ s. Neglect the size of the car.



SOLUTION

Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5\left(e^t - 1\right)$$

When $t = 2$ s, $\quad v = 0.5\left(e^2 - 1\right) = 3.195\ \text{m/s} = 3.19\ \text{m/s}$ **Ans.**

Acceleration: The tangential acceleration of the car at $t = 2$ s is $a_t = 0.5e^2 = 3.695\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{3.195^2}{5} = 2.041\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3.695^2 + 2.041^2} = 4.22\ \text{m/s}^2$$ **Ans.**

12–134.

A boat is traveling along a circular curve having a radius of 100 ft. If its speed at $t = 0$ is 15 ft/s and is increasing at $\dot{v} = (0.8t)$ ft/s^2, determine the magnitude of its acceleration at the instant $t = 5$ s.

SOLUTION

$$\int_{15}^{v} dv = \int_{0}^{5} 0.8t\,dt$$

$$v = 25 \text{ ft/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{100} = 6.25 \text{ ft/s}^2$$

At $t = 5$ s, $$a_t = \dot{v} = 0.8(5) = 4 \text{ ft/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4^2 + 6.25^2} = 7.42 \text{ ft/s}^2$$ **Ans.**

12–135.

A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5\text{ m/s}$ and the rate of increase in the speed is $\dot{v} = 2\text{ m/s}^2$.

SOLUTION

$$a_t = 2\text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25\text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36\text{ m/s}^2 \qquad \textbf{Ans.}$$

***■12–136.**

Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

SOLUTION

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t)dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $\quad 3 = 0.03t^3 + 0.05t^2$

Solving,

$t = 4.147$ s

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

$$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96 \text{ m/s}$$ **Ans.**

$$a_t = \frac{dv}{dt} = 0.18t + 0.1\Big|_{t=4.147\text{ s}} = 0.8465 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930 \text{ m/s}^2$$ **Ans.**

12–137.

A particle travels around a circular path having a radius of 50 m. If it is initially traveling with a speed of 10 m/s and its speed then increases at a rate of $\dot{v} = (0.05\,v)\ \text{m/s}^2$, determine the magnitude of the particle's acceleraton four seconds later.

SOLUTION

Velocity: Using the initial condition $v = 10$ m/s at $t = 0$ s,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{10\ \text{m/s}}^{v} \frac{dv}{0.05v}$$

$$t = 20\ln\frac{v}{10}$$

$$v = \left(10e^{t/20}\right)\ \text{m/s}$$

When $t = 4$ s,

$$v = 10e^{4/20} = 12.214\ \text{m/s}$$

Acceleration: When $v = 12.214$ m/s ($t = 4$ s),

$$a_t = 0.05(12.214) = 0.6107\ \text{m/s}^2$$

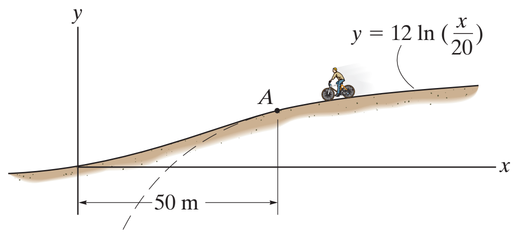
$$a_n = \frac{v^2}{\rho} = \frac{(12.214)^2}{50} = 2.984\ \text{m/s}^2$$

Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6107^2 + 2.984^2} = 3.05\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–138.

When the bicycle passes point A, it has a speed of 6 m/s, which is increasing at the rate of $\dot{v} = (0.5)\ \text{m/s}^2$. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = 12 \ln\left(\frac{x}{20}\right)$$

$$\frac{dy}{dx} = 12\left(\frac{1}{x/20}\right)\left(\frac{1}{20}\right) = \frac{12}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{12}{x^2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{12}{x}\right)^2\right]^{3/2}}{\left|-\frac{12}{x^2}\right|}\right|_{x=50\text{ m}} = 226.59\text{ m}$$

Acceleration:

$$a_t = v = 0.5\ \text{m/s}^2$$

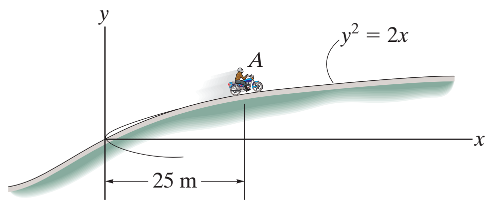
$$a_n = \frac{v^2}{\rho} = \frac{6^2}{226.59} = 0.1589\ \text{m/s}^2$$

The magnitude of the bicycle's acceleration at A is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{0.5^2 + 0.1589^2} = 0.525\ \text{m/s}^2 \qquad \textbf{Ans.}$$

12–139.

The motorcycle is traveling at a constant speed of 60 km/h. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = \sqrt{2}x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{2}x^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{2}x^{-3/2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{2}\sqrt{2}x^{-1/2}\right)^2\right]^{3/2}}{\left|-\frac{1}{4}\sqrt{2}x^{-3/2}\right|}\right|_{x=25\text{ m}} = 364.21\text{ m}$$

Acceleration: The speed of the motorcycle at a is

$$v = \left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 16.67\text{ m/s}$$

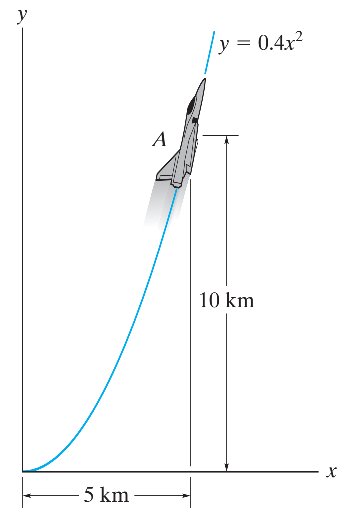
$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{364.21} = 0.7627\text{ m/s}^2$$

Since the motorcycle travels with a constant speed, $a_t = 0$. Thus, the magnitude of the motorcycle's acceleration at A is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{0^2 + 0.7627^2} = 0.763\text{ m/s}^2$$ **Ans.**

***12–140.**

The jet plane travels along the vertical parabolic path. When it is at point A it has a speed of 200 m/s, which is increasing at the rate of 0.8 m/s^2. Determine the magnitude of acceleration of the plane when it is at point A.



SOLUTION

$$y = 0.4x^2$$

$$\frac{dy}{dx} = 0.8x\bigg|_{x=5\text{ km}} = 4$$

$$\frac{d^2y}{dx^2} = 0.8$$

$$\rho = \frac{[1 + (4)^2]^{3/2}}{0.8} = 87.62\text{ km}$$

$$a_t = 0.8\text{ m/s}^2$$

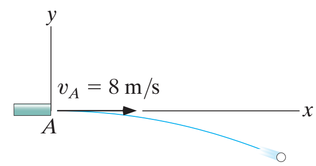
$$a_n = \frac{(0.200)^2}{87.62} = 0.457(10^{-3})\text{ km/s}^2$$

$$a_n = 0.457\text{ km/s}^2$$

$$a = \sqrt{(0.8)^2 + (0.457)^2} = 0.921\text{ m/s}^2$$ **Ans.**

12–141.

The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.

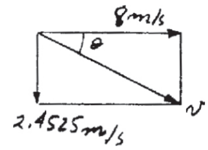


SOLUTION

$v_x = 8$ m/s

$(\overset{+}{\rightarrow})$ $\quad s = v_0 t$

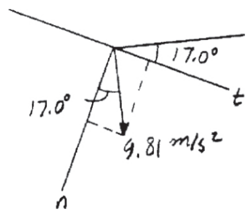
$x = 8t$



$(+\uparrow)$ $\quad s = s_0 + v_0\, t + \frac{1}{2} a_c t^2$

$$y = 0 + 0 + \frac{1}{2}(-9.81)t^2$$

$$y = -4.905t^2$$



$$y = -4.905\left(\frac{x}{8}\right)^2$$

$$y = -0.0766x^2 \qquad \text{(Parabola)}$$ **Ans.**

$v = v_0 + a_c t$

$v_y = 0 - 9.81t$

When $t = 0.25$ s,

$v_y = \; -2.4525$ m/s

$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37$ m/s **Ans.**

$$\theta = \tan^{-1}\left(\frac{2.4525}{8}\right) = 17.04°$$

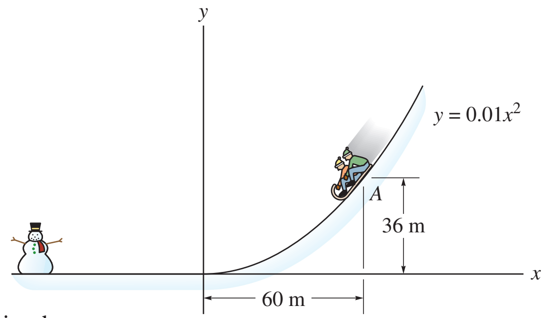
$a_x = 0 \quad a_y = 9.81\ \text{m/s}^2$

$a_n = 9.81 \cos 17.04° = 9.38\ \text{m/s}^2$ **Ans.**

$a_t = 9.81 \sin 17.04° = 2.88\ \text{m/s}^2$ **Ans.**

12–142.

A toboggan is traveling down along a curve which can be approximated by the parabola $y = 0.01x^2$. Determine the magnitude of its acceleration when it reaches point A, where its speed is $v_A = 10$ m/s, and it is increasing at the rate of $\dot{v}_A = 3$ m/s^2.



SOLUTION

Acceleration: The radius of curvature of the path at point A must be determined first. Here, $\dfrac{dy}{dx} = 0.02x$ and $\dfrac{d^2y}{dx^2} = 0.02$, then

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|} = \left.\frac{\left[1 + (0.02x)^2\right]^{3/2}}{|0.02|}\right|_{x=60\text{ m}} = 190.57\text{ m}$$

To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{10^2}{190.57} = 0.5247\text{ m/s}^2$$

Here, $a_t = \dot{v}_A = 3$ m/s. Thus, the magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 0.5247^2} = 3.05\text{ m/s}^2$$ **Ans.**

12–143.

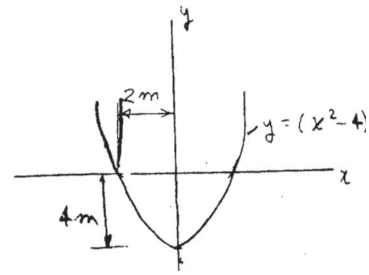
A particle P moves along the curve $y = (x^2 - 4)$ m with a constant speed of 5 m/s. Determine the point on the curve where the maximum magnitude of acceleration occurs and compute its value.

SOLUTION

$y = (x^2 - 4)$

$$a_t = \frac{dv}{dt} = 0,$$

To obtain maximum $a = a_n$, ρ must be a minimum.



This occurs at:

$x = 0, \quad y = -4$ m **Ans.**

Hence,

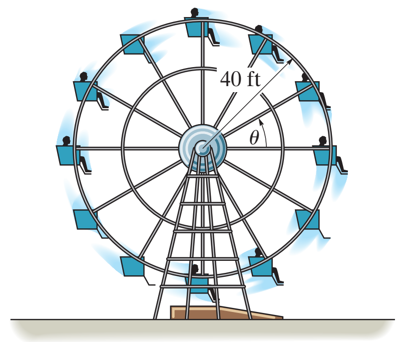
$$\left.\frac{dy}{dx}\right|_{x=0} = 2x = 0; \quad \frac{d^2y}{dx^2} = 2$$

$$\rho_{min} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + 0]^{\frac{3}{2}}}{|2|} = \frac{1}{2}$$

$$(a)_{max} = (a_n)_{max} = \frac{v^2}{\rho_{min}} = \frac{5^2}{\frac{1}{2}} = 50 \text{ m/s}^2$$ **Ans.**

***12–144.**

The Ferris wheel turns such that the speed of the passengers is increased by $\dot{v} = (4t)$ ft/s^2, where t is in seconds. If the wheel starts from rest when $\theta = 0°$, determine the magnitudes of the velocity and acceleration of the passengers when the wheel turns $\theta = 30°$.



SOLUTION

$$\int_0^v dv = \int_0^t 4t\,dt$$

$$v = 2t^2$$

$$\int_0^s ds = \int_0^t 2t^2\,dt$$

$$s = \frac{2}{3}t^3$$

When $s = \frac{\pi}{6}(40)$ ft, $\quad \frac{\pi}{6}(40) = \frac{2}{3}t^3 \quad t = 3.1554$ s

$$v = 2(3.1554)^2 = 19.91 \text{ ft/s} = 19.9 \text{ ft/s}$$ **Ans.**

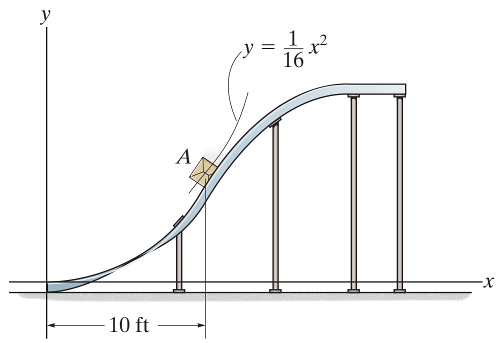
$$a_t = \dot{v} = 4t\,\big|_{t=3.1554\text{ s}} = 12.62 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.91^2}{40} = 9.91 \text{ ft/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{12.62^2 + 9.91^2} = 16.0 \text{ ft/s}^2$$ **Ans.**

12–145.

If the speed of the crate at A is 15 ft/s, which is increasing at a rate $\dot{v} = 3\ \text{ft/s}^2$, determine the magnitude of the acceleration of the crate at this instant.



SOLUTION

Radius of Curvature:

$$y = \frac{1}{16}x^2$$

$$\frac{dy}{dx} = \frac{1}{8}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{8}$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{8}x\right)^2\right]^{3/2}}{\left|\frac{1}{8}\right|}\right|_{x=10\text{ ft}} = 32.82\text{ ft}$$

Acceleration:

$$a_t = \dot{v} = 3\text{ft/s}^2$$

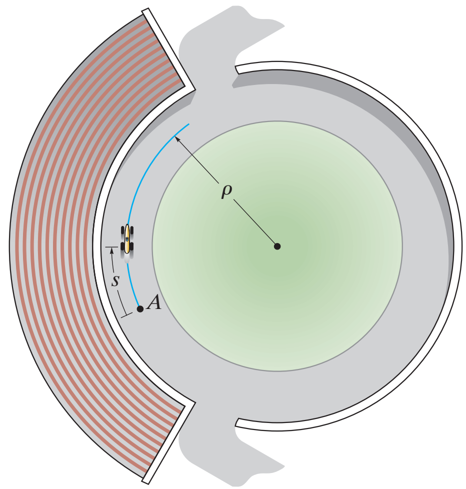
$$a_n = \frac{v^2}{\rho} = \frac{15^2}{32.82} = 6.856\text{ ft/s}^2$$

The magnitude of the crate's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 6.856^2} = 7.48\text{ ft/s}^2$$ **Ans.**

12–146.

The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



SOLUTION

$$a_t = 0.4s = \frac{v\,dv}{ds}$$

$$a\,ds = v\,dv$$

$$\int_0^s 0.4s\,ds = \int_{15}^v v\,dv$$

$$\left.\frac{0.4s^2}{2}\right|_0^s = \left.\frac{v^2}{2}\right|_{15}^v$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$$

$$1\text{n}\,(s + \sqrt{s^2 + 562.5})\Big|_0^s = 0.632\,456t$$

$$1\text{n}\,(s + \sqrt{s^2 + 562.5}) - 3.166\,196 = 0.632\,456t$$

At $s = 20$ m,

$t = 1.21$ s **Ans.**

12–147.

A boy sits on a merry-go-round so that he is always located at $r = 8$ ft from the center of rotation. The merry-go-round is originally at rest, and then due to rotation the boy's speed is increased at $2\ \text{ft/s}^2$. Determine the time needed for his acceleration to become $4\ \text{ft/s}^2$.

SOLUTION

$$a = \sqrt{a_n^2 + a_t^2}$$

$$a_t = 2$$

$$v = v_0 + a_c t$$

$$v = 0 + 2t$$

$$a_n = \frac{v^2}{\rho} = \frac{(2t)^2}{8}$$

$$4 = \sqrt{(2)^2 + \left(\frac{(2t)^2}{8}\right)^2}$$

$$16 = 4 + \frac{16\,t^4}{64}$$

$$t = 2.63\ \text{s}$$ **Ans.**

***12–148.**

A particle travels along the path $y = a + bx + cx^2$, where a, b, c are constants. If the speed of the particle is constant, $v = v_0$, determine the x and y components of velocity and the normal component of acceleration when $x = 0$.

SOLUTION

$$y = a + bx + cx^2$$

$$\dot{y} = b\dot{x} + 2\,c\,x\,\dot{x}$$

$$\ddot{y} = b\ddot{x} + 2c\,(\dot{x})^2 + 2c\,x\ddot{x}$$

When $x = 0$, $\quad \dot{y} = b\,\dot{x}$

$$v_0^2 + \dot{x}^2 + b^2\,\dot{x}^2$$

$$v_x = \dot{x} = \frac{v_0}{\sqrt{1 + b^2}}$$ **Ans.**

$$v_y = \frac{v_0\,b}{\sqrt{1 + b^2}}$$ **Ans.**

$$a_n = \frac{v_0^2}{\rho}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$\frac{dy}{dx} = b + 2c\,x$$

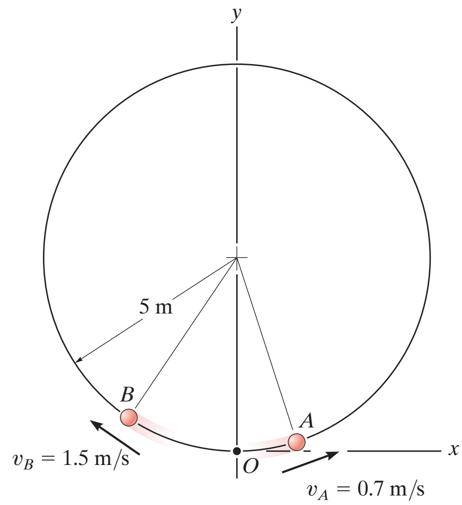
$$\frac{d^2y}{dx^2} = 2\,c$$

At $x = 0$, $\quad \rho = \dfrac{(1 + b^2)^{3/2}}{2\,c}$

$$a_n = \frac{2\,c\,v_0^2}{(1 + b^2)^{3/2}}$$ **Ans.**

12–149.

The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7\text{ m/s}$ and $v_B = 1.5\text{ m/s}$, respectively. Determine in $t = 2\text{ s}$, (a) the displacement along the path of each particle, (b) the position vector to each particle, and (c) the shortest distance between the particles.



SOLUTION

(a) $s_A = 0.7(2) = 1.40\text{ m}$ **Ans.**

$s_B = 1.5(2) = 3\text{ m}$ **Ans.**

(b) $\theta_A = \dfrac{1.40}{5} = 0.280\text{ rad.} = 16.04^\circ$

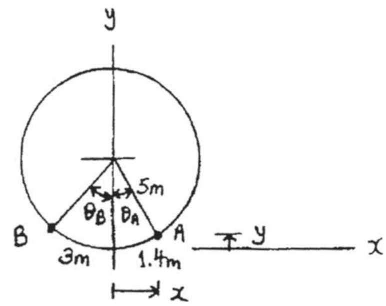
$\theta_B = \dfrac{3}{5} = 0.600\text{ rad.} = 34.38^\circ$

For A

$x = 5\sin 16.04^\circ = 1.382 = 1.38\text{ m}$

$y = 5(1 - \cos 16.04^\circ) = 0.1947 = 0.195\text{ m}$

$\mathbf{r}_A = \{1.38\mathbf{i} + 0.195\mathbf{j}\}\text{ m}$ **Ans.**



For B

$x = -5\sin 34.38^\circ = -2.823 = -2.82\text{ m}$

$y = 5(1 - \cos 34.38^\circ) = 0.8734 = 0.873\text{ m}$

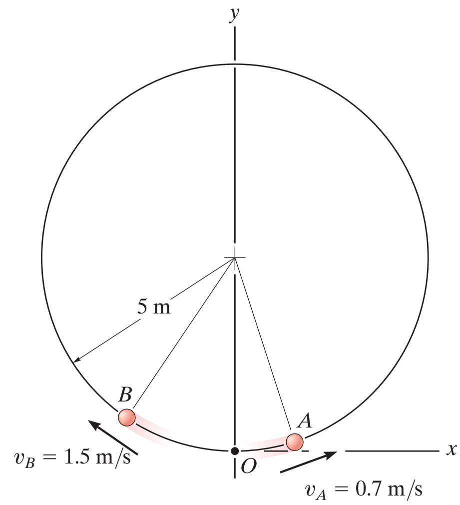
$\mathbf{r}_B = \{-2.82\mathbf{i} + 0.873\mathbf{j}\}\text{ m}$ **Ans.**

(c) $\Delta\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = \{-4.20\mathbf{i} + 0.678\mathbf{j}\}\text{ m}$

$\Delta r = \sqrt{(-4.20)^2 + (0.678)^2} = 4.26\text{ m}$ **Ans.**

12–150.

The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ m/s and $v_B = 1.5$ m/s, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.



SOLUTION

$$s_t = 2\pi(5) = 31.4159 \text{ m}$$

$$s_A = 0.7\,t$$

$$s_B = 1.5\,t$$

Require

$$s_A + s_B = 31.4159$$

$$0.7\,t + 1.5\,t = 31.4159$$

$$t = 14.28 \text{ s} = 14.3 \text{ s} \qquad \textbf{Ans.}$$

$$a_B = \frac{v_B^2}{\rho} = \frac{(1.5)^2}{5} = 0.45 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–151.

The position of a particle traveling along a curved path is $s = (3t^3 - 4t^2 + 4)$ m, where t is in seconds. When $t = 2$ s, the particle is at a position on the path where the radius of curvature is 25 m. Determine the magnitude of the particle's acceleration at this instant.

SOLUTION

Velocity:

$$v = \frac{d}{dt}\left(3t^3 - 4t^2 + 4\right) = \left(9t^2 - 8t\right)\text{m/s}$$

When $t = 2$ s,

$$v|_{t=2\text{ s}} = 9\left(2^2\right) - 8(2) = 20 \text{ m/s}$$

Acceleration:

$$a_t = \frac{dv}{ds} = \frac{d}{dt}\left(9t^2 - 8t\right) = \left(18t - 8\right)\text{m/s}^2$$

$$a_t|_{t=2\text{ s}} = 18(2) - 8 = 28 \text{ m/s}^2$$

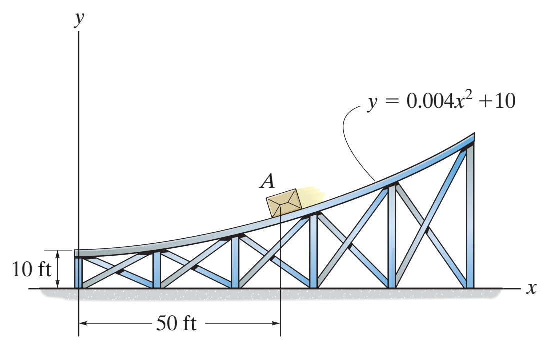
$$a_n = \frac{\left(v|_{t=2\text{s}}\right)^2}{\rho} = \frac{20^2}{25} = 16 \text{ m/s}^2$$

Thus,

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{28^2 + 16^2} = 32.2 \text{ m/s}^2 \qquad \textbf{Ans.}$$

***12–152.**

If the speed of the box at point A on the track is 30 ft/s which is increasing at the rate of $\dot{v} = 5\ \text{ft/s}^2$, determine the magnitude of the acceleration of the box at this instant.



SOLUTION

Radius of Curvature:

$$y = 0.004x^2 + 10$$

$$\frac{dy}{dx} = 0.008x$$

$$\frac{d^2y}{dx^2} = 0.008$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + (0.008x)^2\right]^{3/2}}{|0.008|}\right|_{x=50\ \text{ft}} = 156.17\ \text{ft}$$

Acceleration:

$$a_n = \frac{v^2}{\rho} = \frac{30^2}{156.17} = 5.763\ \text{ft/s}^2$$

$$a_t = \dot{v} = 5\ \text{ft/s}^2$$

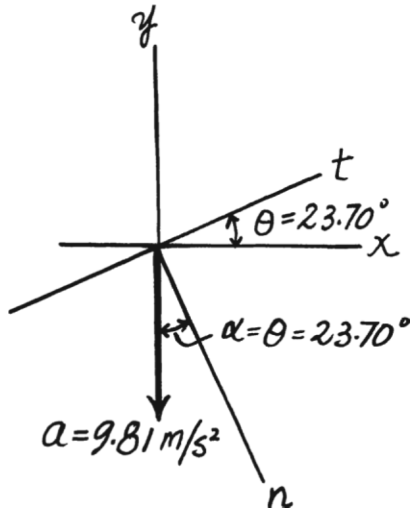
The magnitude of the box's acceleration at A is therefore

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{5^2 + 5.763^2} = 7.63\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

■ **12–153.**

A go-cart moves along a circular track of radius 100 ft such that its speed for a short period of time, $0 \le t \le 4$ s, is $v = 60(1 - e^{-t^2})$ ft/s. Determine the magnitude of its acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s? Use Simpson's rule with $n = 50$ to evaluate the integral.

SOLUTION



$$v = 60(1 - e^{-t^2})$$

$$a_t = \frac{dv}{dt} = 60(-e^{-t^2})(-2t) = 120\,t\,e^{-t^2}$$

$$a_t\big|_{t=2} = 120(2)e^{-4} = 4.3958$$

$$v\big|_{t=2} = 60(1 - e^{-4}) = 58.9011$$

$$a_n = \frac{(58.9011)^2}{100} = 34.693$$

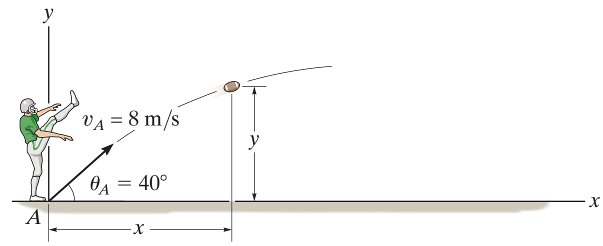
$$a = \sqrt{(4.3958)^2 + (34.693)^2} = 35.0 \text{ m/s}^2$$ **Ans.**

$$\int_0^s ds = \int_0^2 60(1 - e^{-t^2})\,dt$$

$$s = 67.1 \text{ ft}$$ **Ans.**

■ 12–154.

The ball is kicked with an initial speed $v_A = 8$ m/s at an angle $\theta_A = 40°$ with the horizontal. Find the equation of the path, $y = f(x)$, and then determine the normal and tangential components of its acceleration when $t = 0.25$ s.



SOLUTION

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 8\cos 40° = 6.128$ m/s and the initial horizontal and final positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

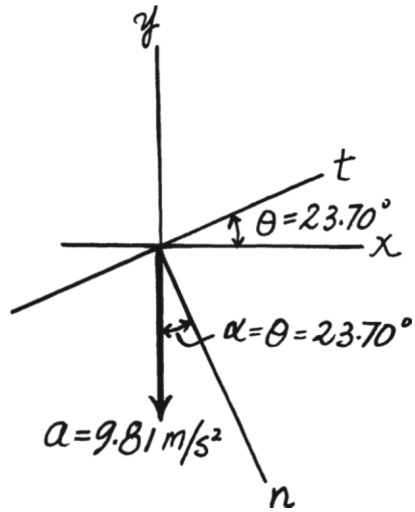
$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + 6.128t \qquad \textbf{(1)}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 8\sin 40° = 5.143$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + 5.143t + \frac{1}{2}(-9.81)\left(t^2\right) \qquad \textbf{(2)}$$



Eliminate t from Eqs (1) and (2),we have

$$y = \{0.8391x - 0.1306x^2\}\text{ m} = \{0.839x - 0.131x^2\}\text{ m} \qquad \textbf{Ans.}$$

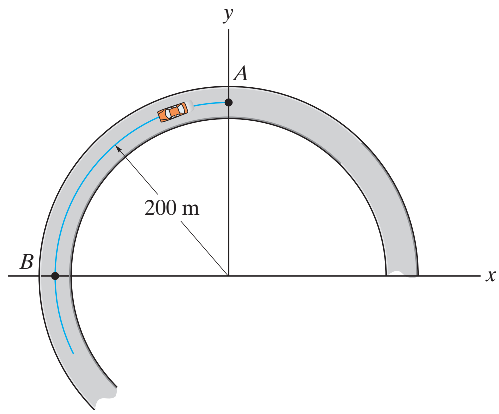
Acceleration: When $t = 0.25$ s, from Eq. (1), $x = 0 + 6.128(0.25) = 1.532$ m. Here, $\frac{dy}{dx} = 0.8391 - 0.2612x$. At $x = 1.532$ m, $\frac{dy}{dx} = 0.8391 - 0.2612(1.532) = 0.4389$ and the tangent of the path makes an angle $\theta = \tan^{-1} 0.4389 = 23.70°$ with the x axis. The magnitude of the acceleration is $a = 9.81$ m/s^2 and is directed downward. From the figure, $\alpha = 23.70°$. Therefore,

$$a_t = -a\sin\alpha = -9.81\sin 23.70° = -3.94\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = a\cos\alpha = 9.81\cos 23.70° = 8.98\text{ m/s}^2 \qquad \textbf{Ans.}$$

12–155.

The race car travels around the circular track with a speed of 16 m/s. When it reaches point A it increases its speed at $a_t = (\frac{4}{3}v^{1/4})$ m/s^2, where v is in m/s. Determine the magnitudes of the velocity and acceleration of the car when it reaches point B. Also, how much time is required for it to travel from A to B?



SOLUTION

$$a_t = \frac{4}{3}v^{\frac{1}{4}}$$

$$dv = a_t\ dt$$

$$dv = \frac{4}{3}v^{\frac{1}{4}}\ dt$$

$$\int_{16}^{v} 0.75\frac{dv}{v^{\frac{1}{4}}} = \int_0^t dt$$

$$v^{\frac{3}{4}}\Big|_{16}^{v} = t$$

$$v^{\frac{3}{4}} - 8 = t$$

$$v = (t+8)^{\frac{4}{3}}$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t (t+8)^{\frac{4}{3}}\,dt$$

$$s = \frac{3}{7}(t+8)^{\frac{7}{3}}\Big|_0^t$$

$$s = \frac{3}{7}(t+8)^{\frac{7}{3}} - 54.86$$

For $s = \frac{\pi}{2}(200) = 100\pi = \frac{3}{7}(t+8)^{\frac{7}{3}} - 54.86$

$t = 10.108\text{ s} = 10.1\text{ s}$ **Ans.**

$v = (10.108+8)^{\frac{4}{3}} = 47.551 = 47.6\text{ m/s}$ **Ans.**

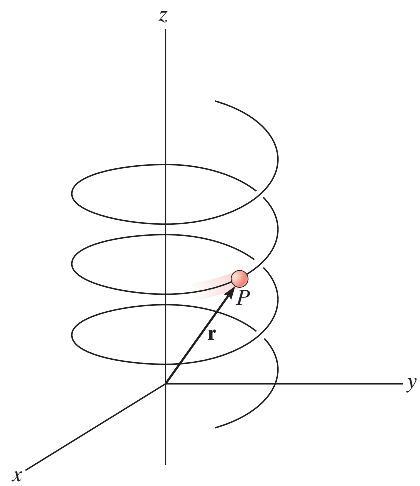
$$a_t = \frac{4}{3}(47.551)^{\frac{1}{4}} = 3.501\text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(47.551)^2}{200} = 11.305\text{ m/s}^2$$

$a = \sqrt{(3.501)^2 + (11.305)^2} = 11.8\text{ m/s}^2$ **Ans.**

***12–156.**

A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



SOLUTION

$$\mathbf{r}_P = 2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + 2t\mathbf{k}$$

$$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2\sin(0.1t)\mathbf{i} + 0.15\cos(0.1t)\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02\cos(0.1t)\mathbf{i} - 0.015\sin(0.1t)\mathbf{j}$$

When $t = 8$ s,

$$\mathbf{v}_P = -0.2\sin(0.8\text{ rad})\mathbf{i} + 0.15\cos(0.8\text{ rad})\mathbf{j} + 2\mathbf{k} = -0.143\,47\mathbf{i} + 0.104\,51\mathbf{j} + \mathbf{2k}$$

$$\mathbf{a}_P = -0.02\cos(0.8\text{ rad})\mathbf{i} - 0.015\sin(0.8\text{ rad})\mathbf{j} = -0.013\,934\mathbf{i} - 0.010\,76\mathbf{j}$$

Since the binormal vector is perpendicular to the plane containing the n–t axis, and $\mathbf{a}_p$ and $\mathbf{v}_p$ are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.14\,347 & 0.104\,51 & 2 \\ -0.013\,934 & -0.010\,76 & 0 \end{vmatrix} = 0.021\,52\mathbf{i} - 0.027\,868\mathbf{j} + 0.003\mathbf{k}$$

$$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035\,338$$

$$\mathbf{u}_b = 0.608\,99\mathbf{i} - 0.788\,62\mathbf{j} + 0.085\mathbf{k}$$

$\alpha = \cos^{-1}(0.608\,99) = 52.5°$ **Ans.**

$\beta = \cos^{-1}(-0.788\,62) = 142°$ **Ans.**

$\gamma = \cos^{-1}(0.085) = 85.1°$ **Ans.**

Note: The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_{b'} = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_p \times \mathbf{v}_p$.

For this case, $\alpha = 128°$, $\beta = 37.9°$, $\gamma = 94.9°$ **Ans.**

12–157.

The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

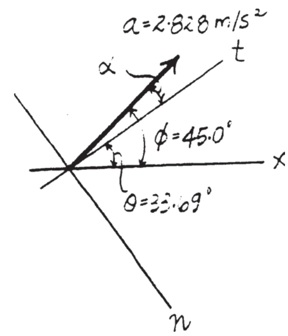
Velocity: Here, $\mathbf{r} = \left\{\left(2t + t^2\right)\mathbf{i} + t^2\mathbf{j}\right\}$ m. To determine the velocity $\mathbf{v}$, apply Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2 + 2t)\,\mathbf{i} + 2t\mathbf{j}\}\text{ m/s}$$

When $t = 2$ s, $\mathbf{v} = [2 + 2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2 + 4^2}$ $= 7.21$ m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \qquad \text{and} \qquad v_t = 7.21\text{ m/s} \qquad \textbf{Ans.}$$

The velocity $\mathbf{v}$ makes an angle $\theta = \tan^{-1}\frac{4}{6} = 33.69°$ with the x axis.



Acceleration: To determine the acceleration $\mathbf{a}$, apply Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\}\text{ m/s}^2$$

Then

$$a = \sqrt{2^2 + 2^2} = 2.828\text{ m/s}^2$$

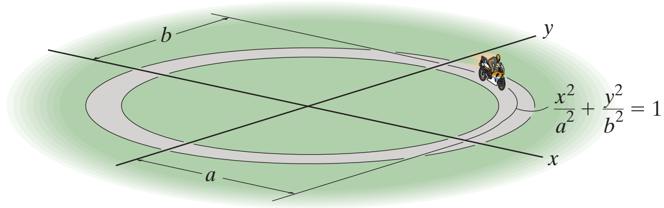
The acceleration $\mathbf{a}$ makes an angle $\phi = \tan^{-1}\frac{2}{2} = 45.0°$ with the x axis. From the figure, $\alpha = 45° - 33.69 = 11.31°$. Therefore,

$$a_n = a\sin\alpha = 2.828\sin 11.31° = 0.555\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_t = a\cos\alpha = 2.828\cos 11.31° = 2.77\text{ m/s}^2 \qquad \textbf{Ans.}$$

12–158.

The motorcycle travels along the elliptical track at a constant speed v. Determine the greatest magnitude of the acceleration if $a > b$.



SOLUTION

Acceleration: Differentiating twice the expression $y = \frac{b}{a}\sqrt{a^2 - x^2}$, we have

$$\frac{dy}{dx} = -\frac{bx}{a\sqrt{a^2 - x^2}}$$

$$\frac{d^2y}{dx^2} = -\frac{ab}{(a^2 - x^2)^{3/2}}$$

The radius of curvature of the path is

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{bx}{a\sqrt{a^2 - x^2}}\right)^2\right]^{3/2}}{\left|-\frac{ab}{(a^2 - x^2)^{3/2}}\right|} = \frac{\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2}}{\frac{ab}{(a^2 - x^2)^{3/2}}} \qquad \textbf{(1)}$$

To have the maximum normal acceleration, the radius of curvature of the path must be a minimum. By observation, this happens when $y = 0$ and $x = a$. When $x \rightarrow a$, $\frac{b^2x^2}{a^2(a^2 - x^2)} >> 1$. Then, $\left[1 + \frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} \rightarrow \left[\frac{b^2x^2}{a^2(a^2 - x^2)}\right]^{3/2} = \frac{b^3x^3}{a^3(a^2 - x^2)^{3/2}}$.

Substituting this value into Eq. [1] yields $\rho = \frac{b^2}{a^4}x^3$. At $x = a$,

$$\rho = \frac{b^2}{a^4}\left(a^3\right) = \frac{b^2}{a}$$

To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_{\max} = \frac{v^2}{\rho} = \frac{v^2}{b^2/a} = \frac{a}{b^2}v^2$$

Since the motorcycle is traveling at a constant speed, $a_t = 0$. Thus,

$$a_{\max} = (a_n)_{\max} = \frac{a}{b^2}v^2 \qquad \textbf{Ans.}$$

12–159.

A particle is moving along a circular path having a radius of 4 in. such that its position as a function of time is given by $\theta = \cos 2t$, where θ is in radians and t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = 30°$.

SOLUTION

When $\theta = \frac{\pi}{6}$ rad, $\quad \frac{\pi}{6} = \cos 2t \quad t = 0.5099$ s

$$\dot{\theta} = \frac{d\theta}{dt} = -2 \sin 2t \bigg|_{t=0.5099\text{ s}} = -1.7039 \text{ rad/s}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = -4 \cos 2t \bigg|_{t=0.5099\text{ s}} = -2.0944 \text{ rad/s}^2$$

$r = 4 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(-1.7039)^2 = -11.6135 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(-2.0944) + 0 = -8.3776 \text{ in./s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-11.6135)^2 + (-8.3776)^2} = 14.3 \text{ in./s}^2$$ **Ans.**

***12–160.**

A particle travels around a limaçon, defined by the equation $r = b - a\cos\theta$, where a and b are constants. Determine the particle's radial and transverse components of velocity and acceleration as a function of θ and its time derivatives.

SOLUTION

$$r = b - a\cos\theta$$

$$\dot{r} = a\sin\theta\dot{\theta}$$

$$\dot{r} = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta}$$

$$v_r = \dot{r} = a\sin\theta\dot{\theta} \qquad \textbf{Ans.}$$

$$v_\theta = r\theta = (b - a\cos\theta)\dot{\theta} \qquad \textbf{Ans.}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = a\cos\theta\dot{\theta}^2 + a\sin\theta\ddot{\theta} - (b - a\cos\theta)\dot{\theta}^2$$

$$= (2a\cos\theta - b)\,\dot{\theta}^2 + a\sin\theta\ddot{\theta} \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (b - a\cos\theta)\ddot{\theta} + 2\left(a\sin\theta\dot{\theta}\right)\dot{\theta}$$

$$= (b - a\cos\theta)\ddot{\theta} + 2a\dot{\theta}^2\sin\theta \qquad \textbf{Ans.}$$

12–161.

If a particle's position is described by the polar coordinates $r = 4(1 + \sin t)$ m and $\theta = (2e^{-t})$ rad, where t is in seconds and the argument for the sine is in radians, determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

When $t = 2$ s,

$r = 4(1+\sin t) = 7.637$

$\dot{r} = 4\cos t = -1.66459$

$\ddot{r} = -4\sin t = -3.6372$

$\theta = 2\,e^{-t}$

$\dot{\theta} = -2\,e^{-t} = -0.27067$

$\ddot{\theta} = 2\,e^{-t} = 0.270665$

$v_r = \dot{r} = -1.66\text{ m/s}$ **Ans.**

$v_\theta = r\dot{\theta} = 7.637(-0.27067) = -2.07\text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -3.6372 - 7.637(-0.27067)^2 = -4.20\text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.637(0.270665) + 2(-1.66459)(-0.27067) = 2.97\text{ m/s}^2$ **Ans.**

12–162.

An airplane is flying in a straight line with a velocity of 200 mi/h and an acceleration of 3 mi/h^2. If the propeller has a diameter of 6 ft and is rotating at an angular rate of 120 rad/s, determine the magnitudes of velocity and acceleration of a particle located on the tip of the propeller.

SOLUTION

$$v_{Pl} = \left(\frac{200\text{ mi}}{\text{h}}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 293.3\text{ ft/s}$$

$$a_{Pl} = \left(\frac{3\text{ mi}}{\text{h}^2}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2 = 0.001\,22\ \text{ft/s}^2$$

$$v_{Pr} = 120(3) = 360\text{ ft/s}$$

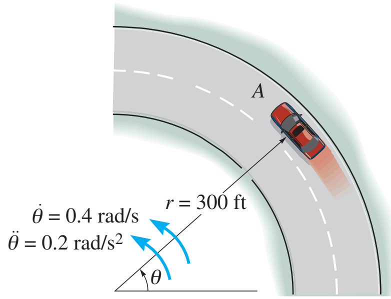
$$v = \sqrt{v_{Pl}^2 + v_{Pr}^2} = \sqrt{(293.3)^2 + (360)^2} = 464\text{ ft/s} \qquad \textbf{Ans.}$$

$$a_{Pr} = \frac{v_{Pr}^2}{\rho} = \frac{(360)^2}{3} = 43\,200\text{ ft/s}^2$$

$$a = \sqrt{a_{Pl}^2 + a_{Pr}^2} = \sqrt{(0.001\,22)^2 + (43\,200)^2} = 43.2(10^3)\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–163.

A car is traveling along the circular curve of radius $r = 300$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.4$ rad/s, which is increasing at the rate of $\ddot{\theta} = 0.2$ rad/s^2. Determine the magnitudes of the car's velocity and acceleration at this instant.



SOLUTION

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 300(0.4) = 120 \text{ ft/s}$$

Thus, the magnitude of the velocity of the car is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 120^2} = 120 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300\left(0.4^2\right) = -48.0 \text{ ft/s}^2$$

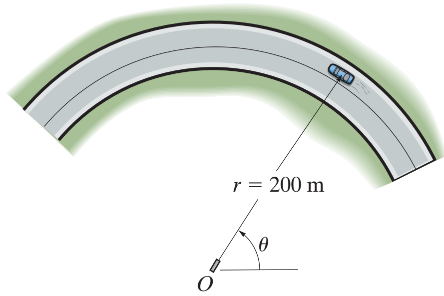
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(0.2) + 2(0)(0.4) = 60.0 \text{ ft/s}^2$$

Thus, the magnitude of the acceleration of the car is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-48.0)^2 + 60.0^2} = 76.8 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

***12–164.**

A radar gun at O rotates with the angular velocity of $\dot{\theta} = 0.1$ rad/s and angular acceleration of $\ddot{\theta} = 0.025$ rad/s^2, at the instant $\theta = 45°$, as it follows the motion of the car traveling along the circular road having a radius of $r = 200$ m. Determine the magnitudes of velocity and acceleration of the car at this instant.



SOLUTION

Time Derivatives: Since r is constant,

$$\dot{r} = \ddot{r} = 0$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 200(0.1) = 20 \text{ m/s}$$

Thus, the magnitude of the car's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{0^2 + 20^2} = 20 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \dot{r} - r\dot{\theta}^2 = 0 - 200(0.1^2) = -2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 200(0.025) + 0 = 5 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-2)^2 + 5^2} = 5.39 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–165.

If a particle moves along a path such that $r = (2 \cos t)$ ft and $\theta = (t/2)$ rad, where t is in seconds, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$r = 2 \cos t \quad \dot{r} = -2 \sin t \qquad \ddot{r} = -2 \cos t$

$\theta = \dfrac{t}{2} \qquad \dot{\theta} = \dfrac{1}{2} \qquad \ddot{\theta} = 0$

$v_r = \dot{r} = -2 \sin t$ **Ans.**

$v_\theta = r\dot{\theta} = (2 \cos t)\left(\dfrac{1}{2}\right) = \cos t$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = -2 \cos t - (2 \cos t)\left(\dfrac{1}{2}\right)^2 = -\dfrac{5}{2} \cos t$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2 \cos t(0) + 2(-2 \sin t)\left(\dfrac{1}{2}\right) = -2 \sin t$ **Ans.**

12–166.

If a particle's position is described by the polar coordinates $r = (2 \sin 2\theta)$ m and $\theta = (4t)$ rad, where t is in seconds, determine the radial and transverse components of its velocity and acceleration when $t = 1$ s.

SOLUTION

When $t = 1$ s,

$\theta = 4\,t = 4$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

$r = 2 \sin 2\theta = 1.9787$

$\dot{r} = 4 \cos 2\theta\, \dot{\theta} = -2.3280$

$\ddot{r} = -8 \sin 2\theta(\dot{\theta})^2 + 8 \cos 2\theta\, \ddot{\theta} = -126.638$

$v_r = \dot{r} = -2.33$ m/s **Ans.**

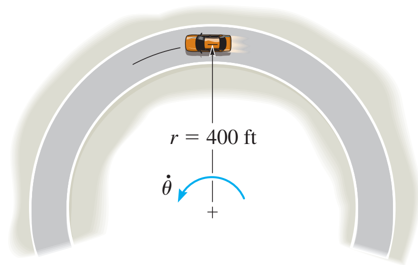
$v_\theta = r\dot{\theta} = 1.9787(4) = 7.91$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = -126.638 - (1.9787)(4)^2 = -158$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.9787(0) + 2(-2.3280)(4) = -18.6$ m/s^2 **Ans.**

12–167.

The car travels along the circular curve having a radius $r = 400$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.025$ rad/s, which is decreasing at the rate $\ddot{\theta} = -0.008$ rad/s^2. Determine the radial and transverse components of the car's velocity and acceleration at this instant and sketch these components on the curve.



SOLUTION

$r = 400 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

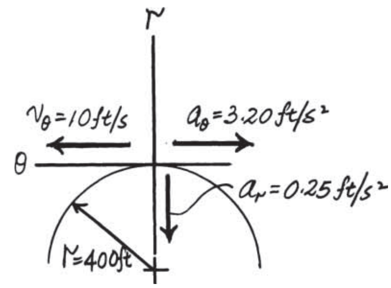
$\dot{\theta} = 0.025 \qquad \theta = -0.008$

$v_r = \dot{r} = 0$ **Ans.**

$v_\theta = r\dot{\theta} = 400(0.025) = 10$ ft/s **Ans.**

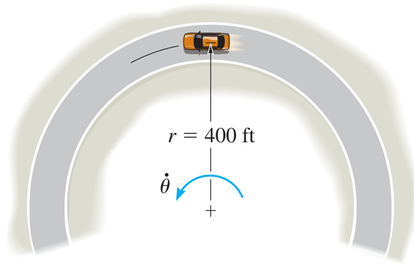
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.025)^2 = -0.25$ ft/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(-0.008) + 0 = -3.20$ ft/s^2 **Ans.**



***12–168.**

The car travels along the circular curve of radius $r = 400$ ft with a constant speed of $v = 30$ ft/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the car's acceleration.



SOLUTION

$r = 400 \text{ ft} \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 400\left(\dot{\theta}\right)$

$$v = \sqrt{(0)^2 + \left(400\,\dot{\theta}\right)^2} = 30$$

$\dot{\theta} = 0.075 \text{ rad/s}$ **Ans.**

$\ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.075)^2 = -2.25 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(0) + 2(0)(0.075) = 0$

$a = \sqrt{(-2.25)^2 + (0)^2} = 2.25 \text{ ft/s}^2$ **Ans.**

12–169.

The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

SOLUTION

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^{2}\right)\mathbf{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{u}_\theta + \ddot{z}\,\mathbf{u}_z$$

$$\ddot{\mathbf{a}} = \left(\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(\ddot{r} - r\dot{\theta}^2\right)\dot{\mathbf{u}}_r + \left(\dot{r}\ddot{\theta} + r\dddot{\theta} + 2\ddot{r}\dot{\theta} + 2\dot{r}\ddot{\theta}\right)\mathbf{u}_\theta + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\dot{\mathbf{u}}_\theta + \dddot{z}\,\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

But, $\mathbf{u}_r = \dot{\theta}\mathbf{u}_\theta \quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad \dot{\mathbf{u}}_z = 0$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = \left(\dddot{r} - 3r\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3\right)\mathbf{u}_\theta + \left(\dddot{z}\right)\mathbf{u}_z \qquad \textbf{Ans.}$$

12–170.

A particle is moving along a circular path having a radius of 6 in. such that its position as a function of time is given by $\theta = \sin 3t$, where θ is in radians, the argument for the sine are in radians, and t is in seconds. Determine the acceleration of the particle at $\theta = 30°$. The particle starts from rest at $\theta = 0°$.

SOLUTION

$r = 6 \text{ in.}, \qquad \dot{r} = 0, \qquad \ddot{r} = 0$

$\theta = \sin 3t$

$\dot{\theta} = 3 \cos 3t$

$\ddot{\theta} = -9 \sin 3t$

At $\theta = 30°$,

$\frac{30°}{180°}\pi = \sin 3t$

$t = 10.525 \text{ s}$

Thus,

$\dot{\theta} = 2.5559 \text{ rad/s}$

$\ddot{\theta} = -4.7124 \text{ rad/s}^2$

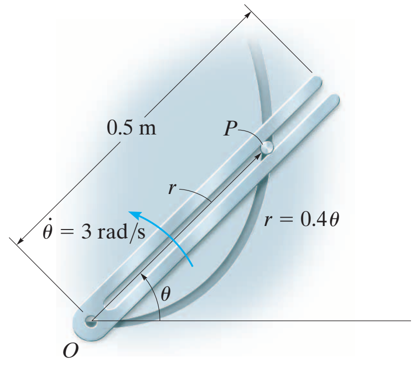
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 6(2.5559)^2 = -39.196$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 6(-4.7124) + 0 = -28.274$

$a = \sqrt{(-39.196)^2 + (-28.274)^2} = 48.3 \text{ in./s}^2$ **Ans.**

12–171.

The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\theta)$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3$ rad.



SOLUTION

$\dot{\theta} = 3$ rad/s $\quad r = 0.4\,\theta$

$$\dot{r} = 0.4\,\dot{\theta}$$

$$\ddot{r} = 0.4\,\ddot{\theta}$$

At $\theta = \dfrac{\pi}{3}, \quad r = 0.4189$

$$\dot{r} = 0.4(3) = 1.20$$

$$\ddot{r} = 0.4(0) = 0$$

$v = \dot{r} = 1.20$ m/s **Ans.**

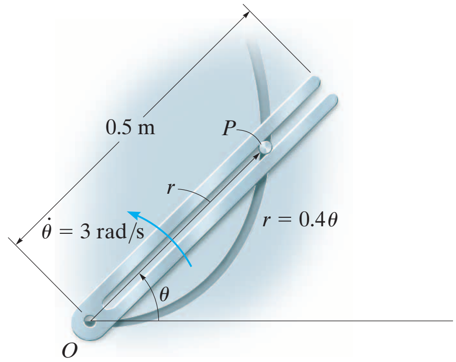
$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.4189(3)^2 = -3.77$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20$ m/s^2 **Ans.**

12–172.

Solve Prob. 12–171 if the slotted link has an angular acceleration $\ddot{\theta} = 8 \text{ rad/s}^2$ when $\dot{\theta} = 3$ rad/s at $\dot{\theta} = \pi/3$ rad.



SOLUTION

$\dot{\theta} = 3$ rad/s $\qquad r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\theta = \dfrac{\pi}{3}$

$\dot{\theta} = 3$

$\ddot{\theta} = 8$

$r = 0.4189$

$\dot{r} = 1.20$

$\ddot{r} = 0.4(8) = 3.20$

$v_r = \dot{r} = 1.20$ m/s **Ans.**

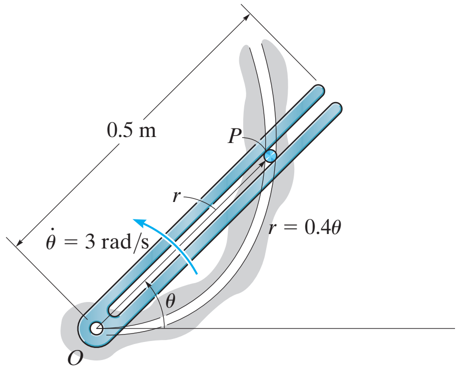
$v_\theta = r\,\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 3.20 - 0.4189(3)^2 = -0.570 \text{ m/s}^2$ **Ans.**

$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0.4189(8) + 2(1.20)(3) = 10.6 \text{ m/s}^2$ **Ans.**

12–173.

The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\theta)$ m, where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5$ m.



SOLUTION

$r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\dot{\theta} = 3$

$\ddot{\theta} = 0$

At $r = 0.5$ m,

$$\theta = \frac{0.5}{0.4} = 1.25 \text{ rad}$$

$\dot{r} = 1.20$

$\ddot{r} = 0$

$v_r = \dot{r} = 1.20 \text{ m/s}$ **Ans.**

$v_\theta = r\,\dot{\theta} = 0.5(3) = 1.50 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 0.5(3)^2 = -4.50 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20 \text{ m/s}^2$ **Ans.**

12–174.

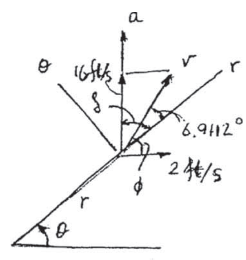
A particle moves in the x–y plane such that its position is defined by $\mathbf{r} = \{2t\mathbf{i} + 4t^2\mathbf{j}\}$ ft, where t is in seconds. Determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

$\mathbf{r} = 2t\mathbf{i} + 4\,t^2\mathbf{j}|_{t=2} = 4\mathbf{i} + 16\mathbf{j}$

$\mathbf{v} = 2\mathbf{i} + 8\,t\mathbf{j}|_{t=2} = 2\mathbf{i} + 16\mathbf{j}$

$\mathbf{a} = 8\mathbf{j}$



$$\theta = \tan^{-1}\left(\frac{16}{4}\right) = 75.964°$$

$$v = \sqrt{(2)^2 + (16)^2} = 16.1245 \text{ ft/s}$$

$$\phi = \tan^{-1}\left(\frac{16}{2}\right) = 82.875°$$

$a = 8 \text{ ft/s}^2$

$\phi - \theta = 6.9112°$

$v_r = 16.1245 \cos 6.9112° = 16.0 \text{ ft/s}$ **Ans.**

$v_\theta = 16.1245 \sin 6.9112° = 1.94 \text{ ft/s}$ **Ans.**

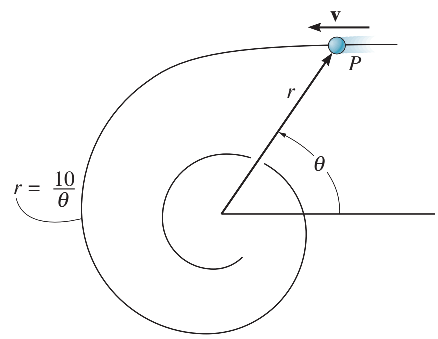
$\delta = 90° - \theta = 14.036°$

$a_r = 8 \cos 14.036° = 7.76 \text{ ft/s}^2$ **Ans.**

$a_\theta = 8 \sin 14.036° = 1.94 \text{ ft/s}^2$ **Ans.**

12–175.

A particle P moves along the spiral path $r = (10/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 20$ ft/s, determine the magnitudes v_r and v_θ as functions of θ and evaluate each at $\theta = 1$ rad.



SOLUTION

$$r = \frac{10}{\theta}$$

$$\dot{r} = -\left(\frac{10}{\theta^2}\right)\dot{\theta}$$

Since $v^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)\dot{\theta}^2 + \left(\frac{10^2}{\theta^2}\right)\dot{\theta}^2$$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)(1 + \theta^2)\dot{\theta}^2$$

Thus, $\dot{\theta} = \dfrac{2\theta^2}{\sqrt{1 + \theta^2}}$

$$v_r = \dot{r} = -\left(\frac{10}{\theta^2}\right)\left(\frac{2\theta^2}{\sqrt{1 + \theta^2}}\right) = -\frac{20}{\sqrt{1 + \theta^2}} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = \left(\frac{10}{\theta}\right)\left(\frac{2\theta^2}{\sqrt{1 + \theta^2}}\right) = \frac{20\theta}{\sqrt{1 + \theta^2}} \qquad \textbf{Ans.}$$

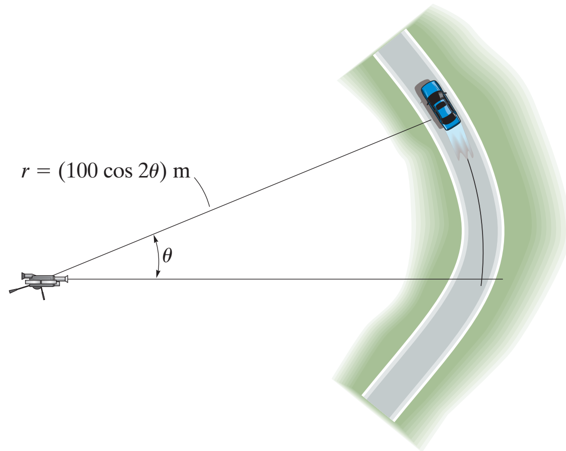
When $\theta = 1$ rad

$$v_r = \left(-\frac{20}{\sqrt{2}}\right) = -14.1 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = \left(\frac{20}{\sqrt{2}}\right) = 14.1 \text{ ft/s} \qquad \textbf{Ans.}$$

***12–176.**

The driver of the car maintains a constant speed of 40 m/s. Determine the angular velocity of the camera tracking the car when $\theta = 15°$.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\dot{\theta}) \text{ m/s}$$

At $\theta = 15°$,

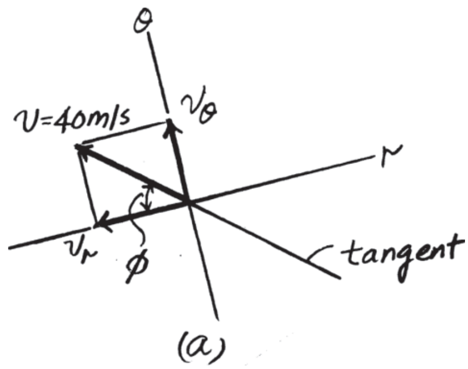
$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60 \text{ m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta} \text{ m/s}$$

Velocity: Referring to Fig. a, $v_r = -40 \cos \phi$ and $v_\theta = 40 \sin \phi$.

$$v_r = \dot{r}$$

$$-40 \cos \phi = -100\dot{\theta} \qquad \textbf{(1)}$$



and

$$v_\theta = r\dot{\theta}$$

$$40 \sin \phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$

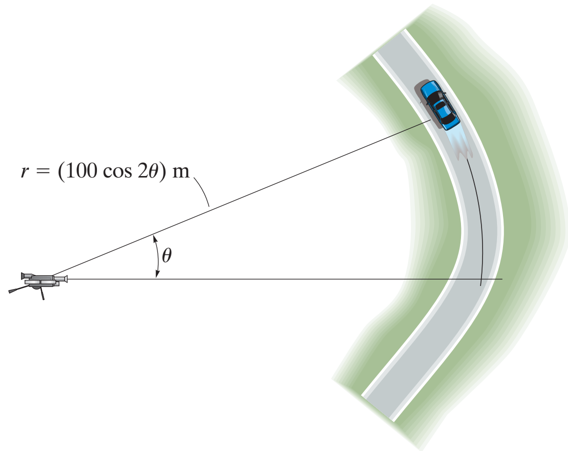
Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.3024 \text{ rad/s} = 0.302 \text{ rad/s} \qquad \textbf{Ans.}$$

12–177.

When $\theta = 15°$, the car has a speed of 50 m/s which is increasing at 6 m/s^2. Determine the angular velocity of the camera tracking the car at this instant.



SOLUTION

Time Derivatives:

$$r = 100 \cos 2\theta$$

$$\dot{r} = (-200 (\sin 2\theta)\,\dot{\theta})\ \text{m/s}$$

$$\ddot{r} = -200\left[(\sin\ 2\theta)\,\ddot{\theta} + 2\,(\cos 2\theta)\,\dot{\theta}^2\right]\ \text{m/s}^2$$

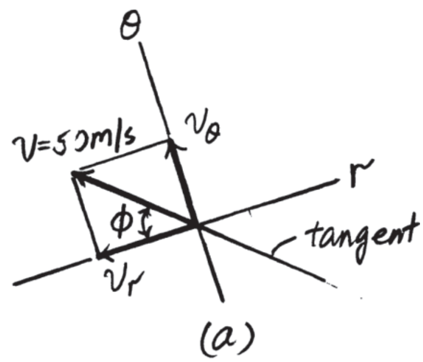
At $\theta = 15°$,

$$r\big|_{\theta=15°} = 100 \cos 30° = 86.60\ \text{m}$$

$$\dot{r}\big|_{\theta=15°} = -200 \sin 30°\dot{\theta} = -100\dot{\theta}\ \text{m/s}$$

$$\ddot{r}\big|_{\theta=15°} = -200\left[\sin 30°\,\ddot{\theta} + 2 \cos 30°\,\dot{\theta}^2\right] = \left(-100\ddot{\theta} - 346.41\dot{\theta}^2\right)\ \text{m/s}^2$$

Velocity: Referring to Fig. a, $v_r = -50 \cos\phi$ and $v_\theta = 50 \sin\phi$. Thus,



$$v_r = \dot{r}$$

$$-50 \cos\phi = -100\dot{\theta} \qquad \textbf{(1)}$$

and

$$v_\theta = r\dot{\theta}$$

$$50 \sin\phi = 86.60\dot{\theta} \qquad \textbf{(2)}$$

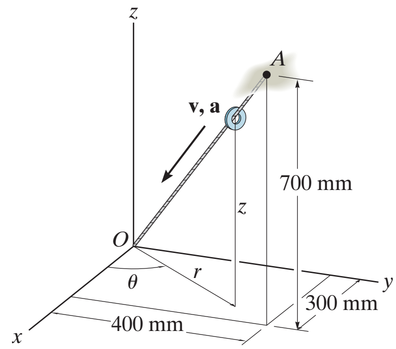
Solving Eqs. (1) and (2) yields

$$\phi = 40.89°$$

$$\dot{\theta} = 0.378\ \text{rad/s} \qquad \textbf{Ans.}$$

12–178.

The small washer slides down the cord OA. When it is at the midpoint, its speed is 200 mm/s and its acceleration is 10 mm/s^2. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.



SOLUTION

$$OA = \sqrt{(400)^2 + (300)^2 + (700)^2} = 860.23 \text{ mm}$$

$$OB = \sqrt{(400)^2 + (300)^2} = 500 \text{ mm}$$

$$v_r = (200)\left(\frac{500}{860.23}\right) = 116 \text{ mm/s}$$

$$v_\theta = 0$$

$$v_z = (200)\left(\frac{700}{860.23}\right) = 163 \text{ mm/s}$$

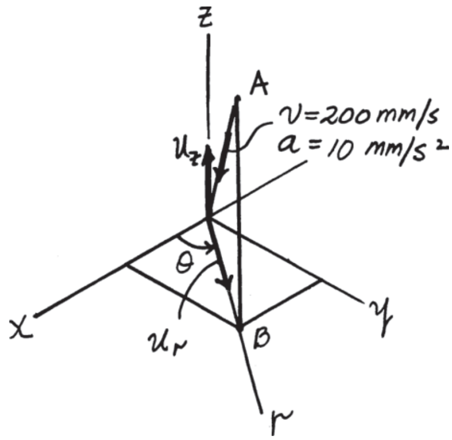
Thus, $\mathbf{v} = \{-116\mathbf{u}_r - 163\mathbf{u}_z\}$ mm/s **Ans.**

$$a_r = 10\left(\frac{500}{860.23}\right) = 5.81$$

$$a_\theta = 0$$

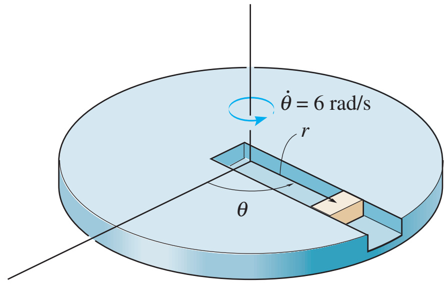
$$a_z = 10\left(\frac{700}{860.23}\right) = 8.14$$

Thus, $\mathbf{a} = \{-5.81\mathbf{u}_r - 8.14\mathbf{u}_z\}$ mm/s^2 **Ans.**



12–179.

A block moves outward along the slot in the platform with a speed of $\dot{r} = (4t)$ m/s, where t is in seconds. The platform rotates at a constant rate of 6 rad/s. If the block starts from rest at the center, determine the magnitudes of its velocity and acceleration when $t = 1$ s.



SOLUTION

$\dot{r} = 4t|_{t=1} = 4 \qquad \ddot{r} = 4$

$\dot{\theta} = 6 \qquad \ddot{\theta} = 0$

$$\int_0^1 dr = \int_0^1 4t\, dt$$

$r = 2t^2\Big]_0^1 = 2$ m

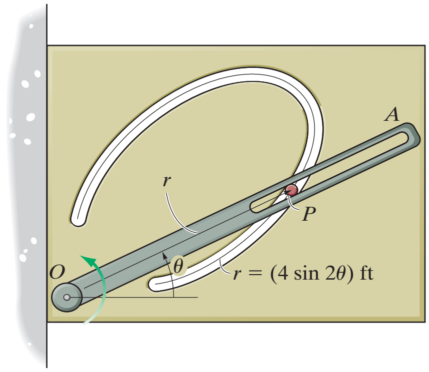
$v = \sqrt{\left(\dot{r}\right)^2 + \left(r\dot{\theta}\right)^2} = \sqrt{(4)^2 + [2(6)]^2} = 12.6$ m/s **Ans.**

$a = \sqrt{\left(\ddot{r} - r\dot{\theta}^2\right)^2 + \left(r\ddot{\theta} + 2\,\dot{r}\dot{\theta}\right)^2} = \sqrt{[4 - 2(6)^2]^2 + [0 + 2(4)(6)]^2}$ **Ans.**

$= 83.2$ m/s^2

***12–180.**

Pin P is constrained to move along the curve defined by the lemniscate $r = (4 \sin 2\theta)$ ft. If the slotted arm OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 1.5$ rad/s, determine the magnitudes of the velocity and acceleration of peg P when $\theta = 60°$.



SOLUTION

Time Derivatives:

$$r = 4\sin 2\theta$$

$$\dot{r} = (8(\cos 2\theta)\dot{\theta})\ \text{ft/s} \qquad \dot{\theta} = 1.5\ \text{rad/s}$$

$$\ddot{r} = 8[(\cos 2\theta)\ddot{\theta} - 2\sin 2\theta(\dot{\theta})^2]\ \text{ft/s}^2 \qquad \ddot{\theta} = 0$$

When $\theta = 60°$,

$$r|_{\theta=60°} = 4\sin 120° = 3.464\ \text{ft}$$

$$\dot{r}|_{\theta=60°} = 8\cos 120°(1.5) = -6\ \text{ft/s}$$

$$\ddot{r}|_{\theta=60°} = 8[0 - 2\sin 120°(1.5^2)] = -31.18\ \text{ft/s}^2$$

Velocity:

$$v_r = \dot{r} = -6\ \text{ft/s} \qquad v_\theta = r\dot{\theta} = 3.464(1.5) = 5.196\ \text{ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-6)^2 + 5.196^2} = 7.94\ \text{ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -31.18 - 3.464(1.5^2) = -38.97\ \text{ft/s}^2$$

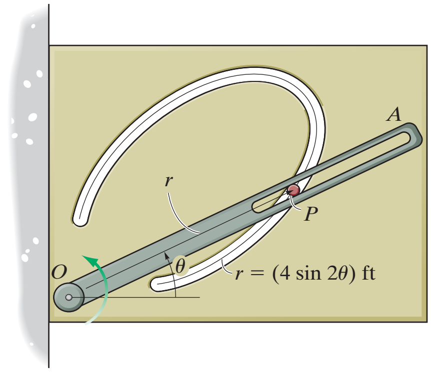
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-6)(1.5) = -18\ \text{ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-38.97)^2 + (-18)^2} = 42.9\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–181.

Pin P is constrained to move along the curve defined by the lemniscate $r = (4\sin 2\theta)$ ft. If the angular position of the slotted arm OA is defined by $\theta = (3t^{3/2})$ rad, determine the magnitudes of the velocity and acceleration of the pin P when $\theta = 60°$.



SOLUTION

Time Derivatives:

$$r = 4\sin 2\theta$$

$$\dot{r} = (8(\cos 2\theta)\dot{\theta})\text{ ft/s}$$

$$\ddot{r} = 8[(\cos 2\theta)\ddot{\theta} - 2(\sin 2\theta)\dot{\theta}^2]\text{ ft/s}^2$$

When $\theta = 60° = \dfrac{\pi}{3}$ rad,

$$\frac{\pi}{3} = 3t^{3/2} \qquad\qquad t = 0.4958\text{ s}$$

Thus, the angular velocity and angular acceleration of arm OA when $\theta = \dfrac{\pi}{3}$ rad ($t = 0.4958$ s) are

$$\dot{\theta} = \frac{9}{2}t^{1/2}\Big|_{t=0.4958\text{ s}} = 3.168\text{ rad/s}$$

$$\ddot{\theta} = \frac{9}{4}t^{1/2}\Big|_{t=0.4958\text{ s}} = 3.196\text{ rad/s}^2$$

Thus,

$$r|_{\theta=60°} = 4\sin 120° = 3.464\text{ ft}$$

$$\dot{r}|_{\theta=60°} = 8\cos 120°(3.168) = -12.67\text{ ft/s}$$

$$\ddot{r}|_{\theta=60°} = 8[\cos 120°(3.196) - 2\sin 120°(3.168^2)] = -151.89\text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = -12.67\text{ ft/s} \qquad\qquad v_\theta = r\dot{\theta} = 3.464(3.168) = 10.98\text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-12.67)^2 + 10.98^2} = 16.8\text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = -151.89 - 3.464(3.168^2) = -186.67\text{ ft/s}^2$$

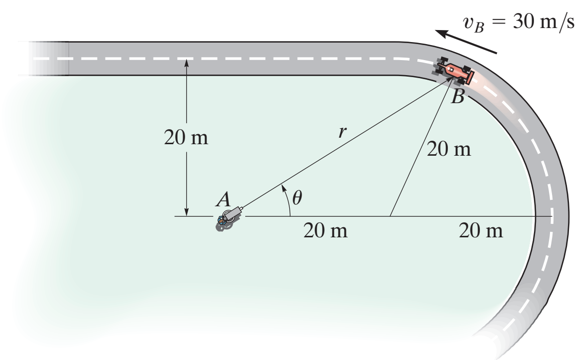
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.464(3.196) + 2(-12.67)(3.168) = -69.24\text{ ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-186.67)^2 + (-69.24)^2} = 199\text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–182.

A cameraman standing at A is following the movement of a race car, B, which is traveling around a curved track at a constant speed of 30 m/s. Determine the angular rate $\dot{\theta}$ at which the man must turn in order to keep the camera directed on the car at the instant $\theta = 30°$.



SOLUTION

$r = 2(20)\cos\theta = 40\cos\theta$

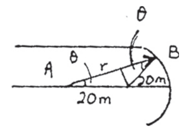
$\dot{r} = -(40\sin\theta)\dot{\theta}$

$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\dot{\theta}\,\mathbf{u}_\theta$

$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

$(30)^2 = (-40\sin\theta)^2(\dot{\theta})^2 + (40\cos\theta)^2(\dot{\theta})^2$

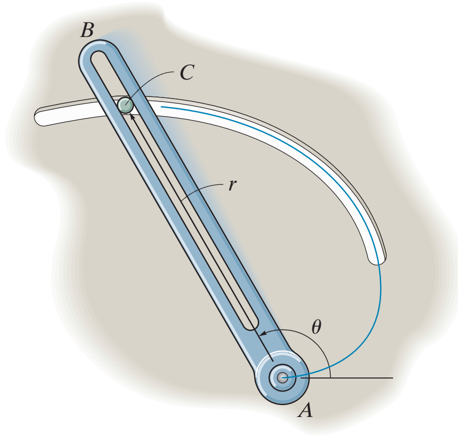
$(30)^2 = (40)^2[\sin^2\theta + \cos^2\theta](\dot{\theta})^2$



$\dot{\theta} = \dfrac{30}{40} = 0.75$ rad/s **Ans.**

12–183.

The slotted arm AB drives pin C through the spiral groove described by the equation $r = a\theta$. If the angular velocity is constant at $\dot{\theta}$, determine the radial and transverse components of velocity and acceleration of the pin.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$.

$$r = a\theta \qquad \dot{r} = a\dot{\theta} \qquad \ddot{r} = a\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = a\dot{\theta}$$ **Ans.**

$$v_\theta = r\dot{\theta} = a\theta\dot{\theta}$$ **Ans.**

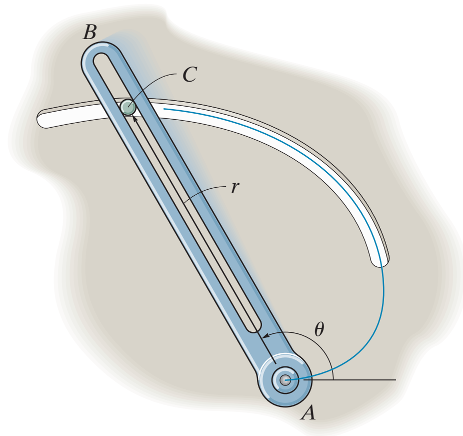
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - a\theta\dot{\theta}^2 = -a\theta\dot{\theta}^2$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(a\dot{\theta})(\dot{\theta}) = 2a\dot{\theta}^2$$ **Ans.**

***12–184.**

The slotted arm AB drives pin C through the spiral groove described by the equation $r = (1.5\,\theta)$ ft, where θ is in radians. If the arm starts from rest when $\theta = 60^\circ$ and is driven at an angular velocity of $\dot{\theta} = (4t)$ rad/s, where t is in seconds, determine the radial and transverse components of velocity and acceleration of the pin C when $t = 1$ s.



SOLUTION

Time Derivatives: Here, $\dot{\theta} = 4t$ and $\ddot{\theta} = 4$ rad/s^2.

$$r = 1.5\theta \qquad \dot{r} = 1.5\dot{\theta} = 1.5(4t) = 6t \qquad \ddot{r} = 1.5\ddot{\theta} = 1.5(4) = 6 \text{ ft/s}^2$$

Velocity: Integrate the angular rate, $\int_{\frac{\pi}{3}}^{\theta} d\theta = \int_0^t 4t\,dt$, we have $\theta = \frac{1}{3}(6t^2 + \pi)$ rad.

Then, $r = \left\{\frac{1}{2}(6t^2 + \pi)\right\}$ ft. At $t = 1$ s, $r = \frac{1}{2}\left[6(1^2) + \pi\right] = 4.571$ ft, $\dot{r} = 6(1) = 6.00$ ft/s.

and $\dot{\theta} = 4(1) = 4$ rad/s. Applying Eq. 12–25, we have

$$v_r = \dot{r} = 6.00 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4.571\,(4) = 18.3 \text{ ft/s} \qquad \textbf{Ans.}$$

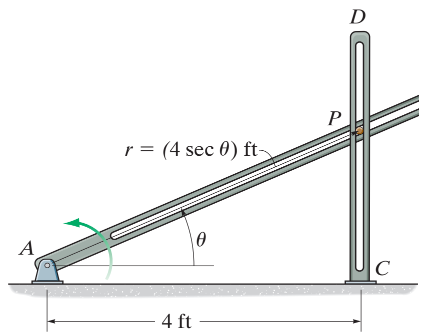
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 4.571\left(4^2\right) = -67.1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.571(4) + 2(6)\,(4) = 66.3 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–185.

If the slotted arm AB rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 2$ rad/s, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30°$. The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB.



SOLUTION

Time Derivatives:

$$r = 4\sec\theta$$

$$\dot{r} = (4\sec\theta(\tan\theta)\dot{\theta})\text{ ft/s} \qquad \dot{\theta} = 2\text{ rad/s}$$

$$\ddot{r} = 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}(\sec\theta(\sec^2\theta)\dot{\theta} + \tan\theta\ \sec\theta(\tan\theta)\dot{\theta})] \qquad \ddot{\theta} = 0$$

$$= 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}^2(\sec 3\theta + \tan^2\theta\ \sec\theta)]\text{ ft/s}^2$$

When $\theta = 30°$,

$$r|_{\theta=30°} = 4\ \sec 30° = 4.619\text{ ft}$$

$$\dot{r}|_{\theta=30°} = (4\ \sec 30°\ \tan 30°)(2) = 5.333\text{ ft/s}$$

$$\ddot{r}|_{\theta=30°} = 4[0 + 2^2(\sec^3 30° + \tan^2 30°\ \sec 30°)] = 30.79\text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.333\text{ ft/s} \qquad v_\theta = r\dot{\theta} = 4.619(2) = 9.238\text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.333^2 + 9.238^2} = 10.7\text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 30.79 - 4.619(2^2) = 12.32\text{ ft/s}^2$$

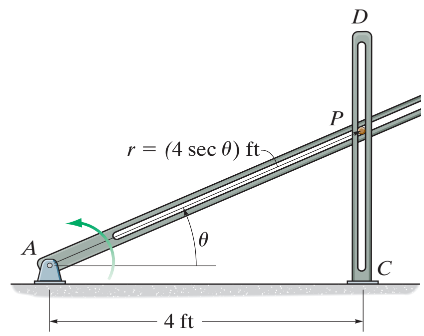
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(5.333)(2) = 21.23\text{ ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{12.32^2 + 21.23^2} = 24.6\text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–186.

The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB. When $\theta = 30°$, the angular velocity and angular acceleration of arm AB are $\dot{\theta} = 2\text{ rad/s}$ and $\ddot{\theta} = 3\text{ rad/s}^2$, respectively. Determine the magnitudes of the velocity and acceleration of the peg P at this instant.



SOLUTION

Time Derivatives:

$$r = 4\sec\theta$$

$$\dot{r} = (4\sec\theta(\tan\theta)\dot{\theta})\text{ ft/s} \qquad \dot{\theta} = 2\text{ rad/s}$$

$$\ddot{r} = 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}(\sec\theta\sec^2\theta\dot{\theta} + \tan\theta\sec\theta(\tan\theta)\dot{\theta})] \qquad \ddot{\theta} = 3\text{ rad/s}^2$$

$$= 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}^2(\sec^3\theta° + \tan^2\theta°\sec\theta°)]\text{ ft/s}^2$$

When $\theta = 30°$,

$$r|_{\theta=30°} = 4\sec 30° = 4.619\text{ ft}$$

$$\dot{r}|_{\theta=30°} = (4\sec 30°\tan 30°)(2) = 5.333\text{ ft/s}$$

$$\ddot{r}|_{\theta=30°} = 4[(\sec 30°\tan 30°)(3) + 2^2(\sec^3 30° + \tan^2 30°\sec 30°)] = 38.79\text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.333\text{ ft/s} \qquad v_\theta = r\dot{\theta} = 4.619(2) = 9.238\text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.333^2 + 9.238^2} = 10.7\text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 38.79 - 4.619(2^2) = 20.32\text{ ft/s}^2$$

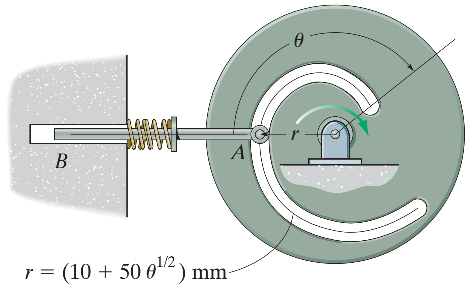
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.619(3) + 2(5.333)(2) = 35.19\text{ ft/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{20.32^2 + 35.19^2} = 40.6\text{ ft/s}^2 \qquad \textbf{Ans.}$$

12–187.

If the circular plate rotates clockwise with a constant angular velocity of $\dot{\theta} = 1.5$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod AB when $\theta = 2/3\pi$ rad.



SOLUTION

Time Derivaties:

$$r = \left(10 + 50\theta^{1/2}\right)\text{ mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta}\text{ mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\ddot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right]\text{mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$ rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36\text{ mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91\text{ mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[0 - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = -9.279\text{ mm/s}^2$$

Velocity: The radial component gives the rod's velocity.

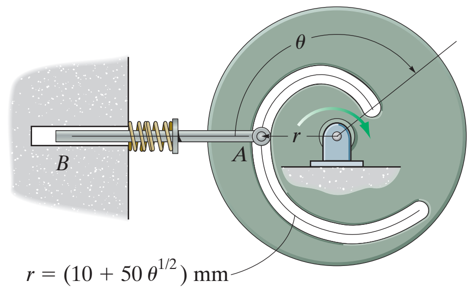
$$v_r = \dot{r} = 25.9\text{ mm/s}$$ **Ans.**

Acceleration: The radial component gives the rod's acceleration.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -9.279 - 82.36\left(1.5^2\right) = -195\text{ mm/s}^2$$ **Ans.**

***12–188.**

When $\theta = 2/3\pi$ rad, the angular velocity and angular acceleration of the circular plate are $\dot{\theta} = 1.5$ rad/s and $\ddot{\theta} = 3\ \text{rad/s}^2$, respectively. Determine the magnitudes of the velocity and acceleration of the rod AB at this instant.



SOLUTION

Time Derivatives:

$$r = (10 + 50\theta^{1/2})\ \text{mm}$$

$$\dot{r} = 25\theta^{-1/2}\dot{\theta}\ \text{mm/s}$$

$$\ddot{r} = 25\left[\theta^{-1/2}\ddot{\theta} - \frac{1}{2}\theta^{-3/2}\dot{\theta}^2\right]\text{mm/s}^2$$

When $\theta = \dfrac{2\pi}{3}$ rad,

$$r|_{\theta=\frac{2\pi}{3}} = \left[10 + 50\left(\frac{2\pi}{3}\right)^{1/2}\right] = 82.36\ \text{mm}$$

$$\dot{r}|_{\theta=\frac{2\pi}{3}} = 25\left(\frac{2\pi}{3}\right)^{-1/2}(1.5) = 25.91\ \text{mm/s}$$

$$\ddot{r}|_{\theta=\frac{2\pi}{3}} = 25\left[\left(\frac{2\pi}{3}\right)^{-1/2}(3) - \frac{1}{2}\left(\frac{2\pi}{3}\right)^{-3/2}(1.5^2)\right] = 42.55\ \text{mm/s}^2$$

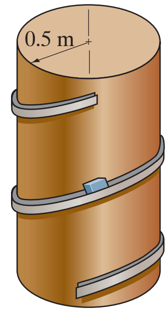
For the rod,

$$v = \dot{r} = 25.9\ \text{mm/s}$$ **Ans.**

$$a = \ddot{r} = 42.5\ \text{mm/s}^2$$ **Ans.**

12–189.

The box slides down the helical ramp with a constant speed of $v = 2$ m/s. Determine the magnitude of its acceleration. The ramp descends a vertical distance of 1 m for every full revolution. The mean radius of the ramp is $r = 0.5$ m.



SOLUTION

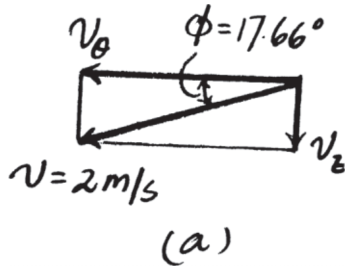
Velocity: The inclination angle of the ramp is $\phi = \tan^{-1}\dfrac{L}{2\pi r} = \tan^{-1}\left[\dfrac{1}{2\pi(0.5)}\right] = 17.66°$.

Thus, from Fig. *a*, $v_\theta = 2\cos 17.66° = 1.906$ m/s and $v_z = 2\sin 17.66° = 0.6066$ m/s. Thus,

$$v_\theta = r\dot{\theta}$$

$$1.906 = 0.5\dot{\theta}$$

$$\dot{\theta} = 3.812 \text{ rad/s}$$



Acceleration: Since $r = 0.5$ m is constant, $\dot{r} = \ddot{r} = 0$. Also, $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$. Using the above results,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(3.812)^2 = -7.264 \text{ m/s}^2$$

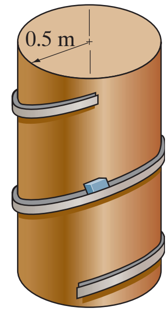
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(0) + 2(0)(3.812) = 0$$

Since $\mathbf{v}_z$ is constant $a_z = 0$. Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r{}^2 + a_\theta{}^2 + a_z{}^2} = \sqrt{(-7.264)^2 + 0^2 + 0^2} = 7.26 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–190.

The box slides down the helical ramp which is defined by $r = 0.5$ m, $\theta = (0.5t^3)$ rad, and $z = (2 - 0.2t^2)$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the box at the instant $\theta = 2\pi$ rad.



SOLUTION

Time Derivatives:

$$r = 0.5 \text{ m}$$

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = \left(1.5t^2\right) \text{ rad/s} \qquad \ddot{\theta} = (3t) \text{ rad/s}^2$$

$$z = 2 - 0.2t^2$$

$$\dot{z} = (-0.4t) \text{ m/s} \qquad \ddot{z} = -0.4 \text{ m/s}^2$$

When $\theta = 2\pi$ rad,

$$2\pi = 0.5t^3 \qquad t = 2.325 \text{ s}$$

Thus,

$$\dot{\theta}\big|_{t=2.325\text{ s}} = 1.5(2.325)^2 = 8.108 \text{ rad/s}$$

$$\ddot{\theta}\big|_{t=2.325\text{ s}} = 3(2.325) = 6.975 \text{ rad/s}^2$$

$$\dot{z}\big|_{t=2.325\text{ s}} = -0.4(2.325) = -0.92996 \text{ m/s}$$

$$\ddot{z}\big|_{t=2.325\text{ s}} = -0.4 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 0.5(8.108) = 4.05385 \text{ m/s}$$

$$v_z = \dot{z} = -0.92996 \text{ m/s}$$

Thus, the magnitude of the box's velocity is

$$v = \sqrt{v_r^{\,2} + v_\theta^{\,2} + v_z^{\,2}} = \sqrt{0^2 + 4.05385^2 + (-0.92996)^2} = 4.16 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(8.108)^2 = -32.867 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(6.975) + 2(0)(8.108)^2 = 3.487 \text{ m/s}^2$$

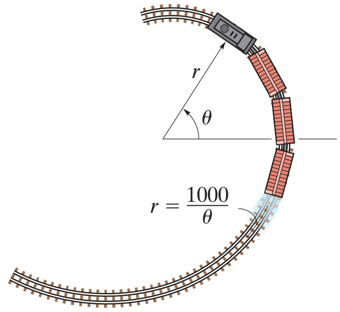
$$a_z = \ddot{z} = -0.4 \text{ m/s}^2$$

Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2} + a_z^{\,2}} = \sqrt{(-32.867)^2 + 3.487^2 + (-0.4)^2} = 33.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–191.

For a short distance the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If it maintains a constant speed $v = 20$ m/s, determine the radial and transverse components of its velocity when $\theta = (9\pi/4)$ rad.



SOLUTION

$$r = \frac{1000}{\theta}$$

$$\dot{r} = -\frac{1000}{\theta^2}\dot{\theta}$$

Since

$$v^2 = (\dot{r})^2 + (r\,\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(\dot{\theta})^2 + \frac{(1000)^2}{\theta^2}(\dot{\theta})^2$$

$$(20)^2 = \frac{(1000)^2}{\theta^4}(1 + \theta^2)(\dot{\theta})^2$$

Thus,

$$\dot{\theta} = \frac{0.02\theta^2}{\sqrt{1 + \theta^2}}$$

At $\theta = \dfrac{9\pi}{4}$

$$\dot{\theta} = 0.140$$

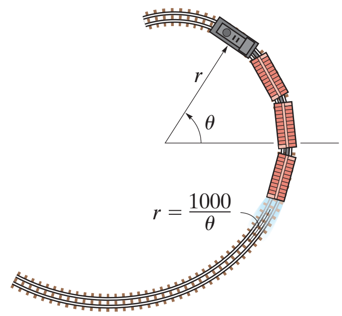
$$\dot{r} = \frac{-1000}{(9\pi/4)^2}(0.140) = -2.80$$

$v_r = \dot{r} = -2.80$ m/s **Ans.**

$v_\theta = r\dot{\theta} = \dfrac{1000}{(9\pi/4)}(0.140) = 19.8$ m/s **Ans.**

12–192.

For a *short distance* the train travels along a track having the shape of a spiral, $r = (1000/\theta)$ m, where θ is in radians. If the angular rate is constant, $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of its velocity and acceleration when $\theta = (9\pi/4)$ rad.



SOLUTION

$$\dot{\theta} = 0.2$$

$$\ddot{\theta} = 0$$

$$r = \frac{1000}{\theta}$$

$$\dot{r} = -1000(\theta^{-2})\dot{\theta}$$

$$\ddot{r} = 2000(\theta^{-3})(\dot{\theta})^2 - 1000(\theta^{-2})\ddot{\theta}$$

When $\theta = \dfrac{9\pi}{4}$

$$r = 141.477$$

$$\dot{r} = -4.002812$$

$$\ddot{r} = 0.226513$$

$$v_r = \dot{r} = -4.00 \text{ m/s}$$ **Ans.**

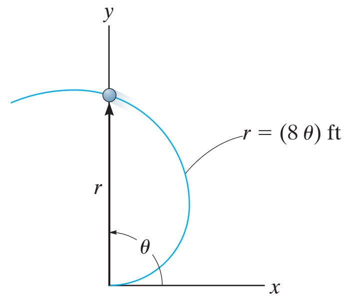
$$v_\theta = r\dot{\theta} = 141.477(0.2) = 28.3 \text{ m/s}$$ **Ans.**

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 0.226513 - 141.477(0.2)^2 = -5.43 \text{ m/s}^2$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-4.002812)(0.2) = -1.60 \text{ m/s}^2$$ **Ans.**

12–193.

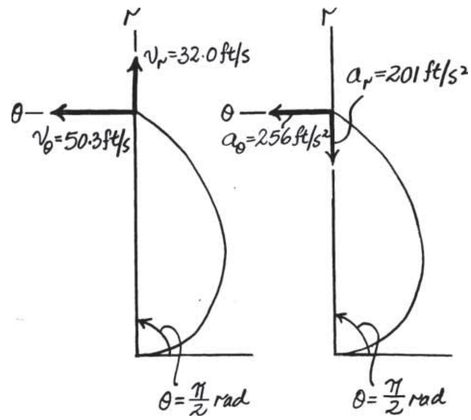
A particle moves along an Archimedean spiral $r = (8\theta)$ ft, where θ is given in radians. If $\dot{\theta} = 4$ rad/s (constant), determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \pi/2$ rad. Sketch the curve and show the components on the curve.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s} \qquad \ddot{r} = 8\ddot{\theta} = 0$$



Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 32.0 \text{ ft/s}$$ **Ans.**

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s}$$ **Ans.**

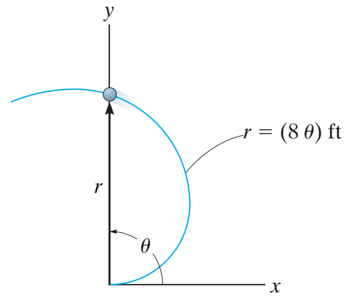
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4\pi\left(4^2\right) = -201 \text{ ft/s}^2$$ **Ans.**

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(32.0)(4) = 256 \text{ ft/s}^2$$ **Ans.**

12–194.

Solve Prob. 12–193 if the particle has an angular acceleration $\ddot{\theta} = 5\ \text{rad/s}^2$ when $\dot{\theta} = 4$ rad/s at $\theta = \pi\ \ 2$ rad.



SOLUTION

Time Derivatives: Here,

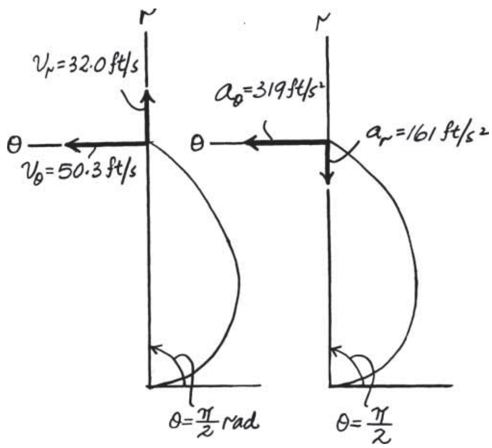
$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi\ \text{ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0\ \text{ft/s}$$

$$\ddot{r} = 8\ddot{\theta} = 8(5) = 40\ \text{ft/s}^2$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 32.0\ \text{ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3\ \text{ft/s} \qquad \textbf{Ans.}$$



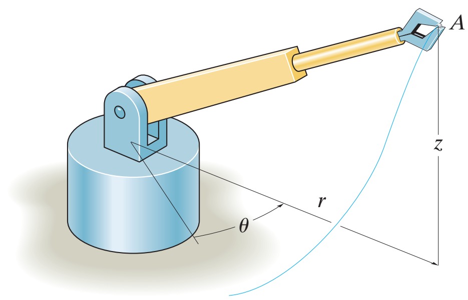
Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 40 - 4\pi\left(4^2\right) = -161\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4\pi(5) + 2(32.0)(4) = 319\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

12–195.

The arm of the robot has a length of $r = 3$ ft grip A moves along the path $z = (3 \sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t \qquad r = 3 \qquad z = 3 \sin 2t$

$\dot{\theta} = 0.5 \qquad \dot{r} = 0 \qquad \dot{z} = 6 \cos 2t$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = -12 \sin 2t$

At $t = 3$ s,

$z = -0.8382$

$\dot{z} = 5.761$

$\ddot{z} = 3.353$

$v_r = 0$

$v_\theta = 3(0.5) = 1.5$

$v_z = 5.761$

$v = \sqrt{(0)^2 + (1.5)^2 + (5.761)^2} = 5.95 \text{ ft/s}$ **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

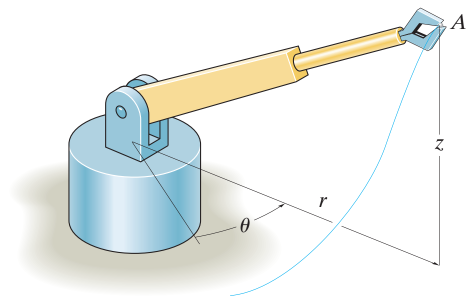
$a_\theta = 0 + 0 = 0$

$a_z = 3.353$

$a = \sqrt{(-0.75)^2 + (0)^2 + (3.353)^2} = 3.44 \text{ ft/s}^2$ **Ans.**

***12–196.**

For a short time the arm of the robot is extending at a constant rate such that $\dot{r} = 1.5$ ft/s when $r = 3$ ft, $z = (4t^2)$ ft, and $\theta = 0.5t$ rad, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the grip A when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t$ rad $\qquad r = 3$ ft $\qquad z = 4\,t^2$ ft

$\dot{\theta} = 0.5$ rad/s $\qquad \dot{r} = 1.5$ ft/s $\qquad \dot{z} = 8\,t$ ft/s

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$ ft/s^2

At $t = 3$ s,

$\theta = 1.5 \qquad r = 3 \qquad z = 36$

$\dot{\theta} = 0.5 \qquad \dot{r} = 1.5 \qquad \dot{z} = 24$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$

$v_r = 1.5$

$v_\theta = 3(0.5) = 1.5$

$v_z = 24$

$v = \sqrt{(1.5)^2 + (1.5)^2 + (24)^2} = 24.1$ ft/s **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

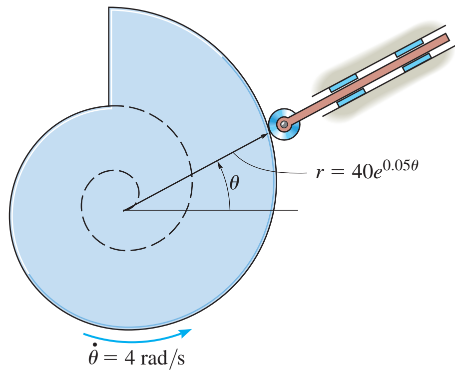
$a_\theta = 0 + 2(1.5)(0.5) = 1.5$

$a_z = 8$

$a = \sqrt{(-0.75)^2 + (1.5)^2 + (8)^2} = 8.17$ ft/s^2 **Ans.**

12–197.

The partial surface of the cam is that of a logarithmic spiral $r = (40e^{0.05\theta})$ mm, where θ is in radians. If the cam is rotating at a constant angular rate of $\dot{\theta} = 4$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod at the instant $\theta = 30°$.



SOLUTION

$$r = 40e^{0.05\,\theta}$$

$$\dot{r} = 2e^{0.05\theta}\,\dot{\theta}$$

$$\ddot{r} = 0.1e^{0.05\,\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\,\theta}\ddot{\theta}$$

$$\theta = \frac{\pi}{6}$$

$$\dot{\theta} = -4$$

$$\ddot{\theta} = 0$$

$$r = 40e^{\,0.05\left(\frac{\pi}{6}\right)} = 41.0610$$

$$\dot{r} = 2e^{\,0.05\left(\frac{\pi}{6}\right)}(-4) = -8.2122$$

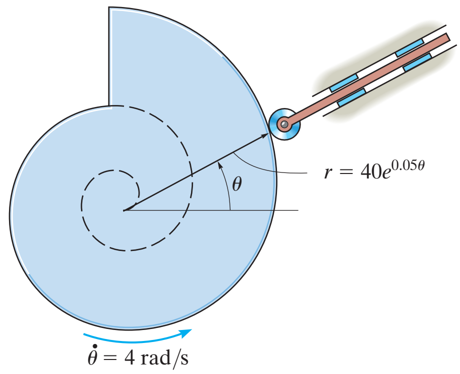
$$\ddot{r} = 0.1e^{\,0.05\left(\frac{\pi}{6}\right)}(-4)^2 + 0 = 1.64244$$

$v = \dot{r} = -8.2122 = 8.21$ mm/s **Ans.**

$a = \ddot{r} - r\dot{\theta}^2 = 1.642\,44 - 41.0610(-4)^2 = -665.33 = -665$ mm/s^2 **Ans.**

12–198.

Solve Prob. 12–197, if the cam has an angular acceleration of $\ddot{\theta} = 2\ \text{rad/s}^2$ when its angular velocity is $\dot{\theta} = 4\ \text{rad/s}$ at $\dot{\theta} = 30°$.



SOLUTION

$r = 40e^{0.05\theta}$

$\dot{r} = 2e^{0.05\theta}\dot{\theta}$

$\ddot{r} = 0.1e^{0.05\theta}\left(\dot{\theta}\right)^2 + 2e^{0.05\theta}\ddot{\theta}$

$\theta = \dfrac{\pi}{6}$

$\dot{\theta} = -4$

$\ddot{\theta} = -2$

$r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{0.05\left(\left(\frac{\pi}{6}\right)\right.}(-4) = -8.2122$

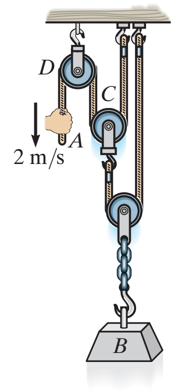
$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(-4)^2 + 2e^{0.05\left(\frac{\pi}{6}\right)}(-2) = -2.4637$

$v = \dot{r} = 8.2122 = 8.21\ \text{mm/s}$

$a = \ddot{r} - r\dot{\theta}^2 = -2.4637 - 41.0610(-4)^2 = -659\ \text{mm/s}^2$

12–199.

If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



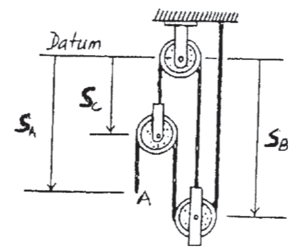
SOLUTION

Position-Coordinate Equation: Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B, and s_C respectively. Since the system consists of two cords, two position-coordinate equations can be derived.

$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \qquad \textbf{(1)}$$

$$s_B + s_C = l_2 \qquad \textbf{(2)}$$

Eliminating s_C from Eqs. (1) and (2) yields



$$s_A + 4s_B = l_1 = 2l_2$$

Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad \textbf{(3)}$$

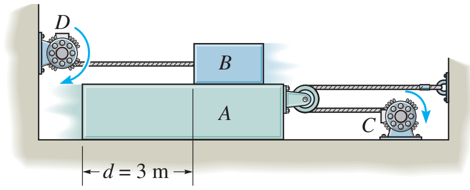
Since $v_A = 2$ m/s, from Eq. (3)

$(+\downarrow)$ $$2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

***12–200.**

The motor at C pulls in the cable with an acceleration $a_C = (3t^2)\ \text{m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5\ \text{m/s}^2$. If both motors start at the same instant from rest when $d = 3\ \text{m}$, determine (a) the time needed for $d = 0$, and (b) the relative velocity of block A with respect to block B when this occurs.



SOLUTION

For A:

$s_A + (s_A - s_C) = l$

$2v_A = v_C$

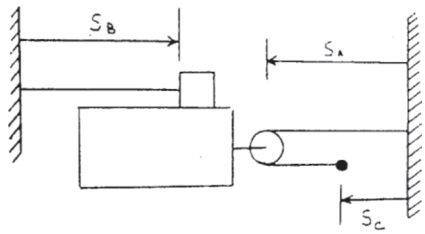
$2a_A = a_C = -3t^2$

$a_A = -1.5t^2 = 1.5t^2 \quad \rightarrow$

$v_A = 0.5t^3 \quad \rightarrow$

$s_A = 0.125\,t^4 \quad \rightarrow$

For B:



$a_B = 5\ \text{m/s}^2 \quad \leftarrow$

$v_B = 5t \quad \leftarrow$

$s_B = 2.5t^2 \quad \leftarrow$

Require $s_A + s_B = d$

$0.125t^4 + 2.5t^2 = 3$

Set $u = t^2 \qquad 0.125u^2 + 2.5u = 3$

The positive root is $u = 1.1355$. Thus,

$t = 1.0656 = 1.07\ \text{s}$ **Ans.**

$v_A = 0.5(1.0656)^3 = 0.6050$

$v_B = 5(1.0656) = 5.3281\ \text{m/s}$

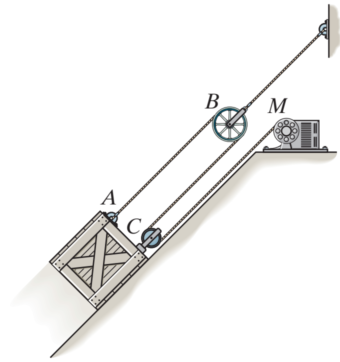
$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$0.6050\mathbf{i} = -5.3281\mathbf{i} + v_{A/B}\,\mathbf{i}$

$v_{A/B} = 5.93\ \text{m/s} \quad \rightarrow$ **Ans.**

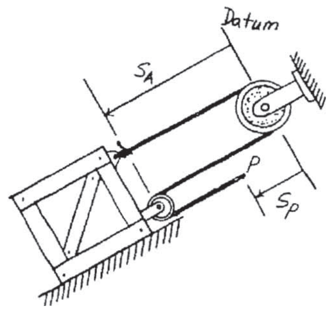
12–201.

The crate is being lifted up the inclined plane using the motor M and the rope and pulley arrangement shown. Determine the speed at which the cable must be taken up by the motor in order to move the crate up the plane with a constant speed of 4 ft/s.



SOLUTION

Position-Coordinate Equation: Datum is established at fixed pulley B. The position of point P and crate A with respect to datum are s_P and s_A, respectively.



$$2s_A + (s_A - s_P) = l$$

$$3s_A - s_P = 0$$

Time Derivative: Taking the time derivative of the above equation yields

$$3v_A - v_P = 0 \qquad \textbf{(1)}$$

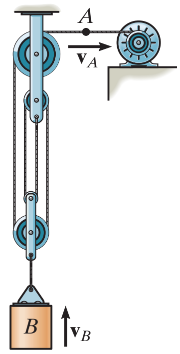
Since $v_A = 4$ ft/s, from Eq. [1]

$(+\ )$ $$3(4) - v_P = 0$$

$$v_P = 12 \text{ ft/s} \qquad \textbf{Ans.}$$

12–202.

Determine the time needed for the load at B to attain a speed of 8 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 0.2 m/s^2.



SOLUTION

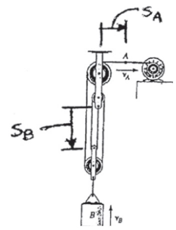
$$4s_B + s_A = l$$

$$4v_B = -v_A$$

$$4a_B = -a_A$$

$$4a_B = -0.2$$

$$a_B = -0.05 \text{ m/s}^2$$



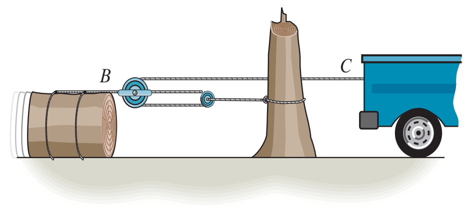
$(+\downarrow)$ $$v_B = (v_B)_0 + a_B t$$

$$-8 = 0 - (0.05)(t)$$

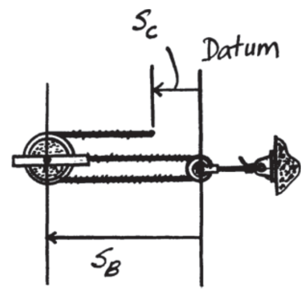
$$t = 160 \text{ s}$$ **Ans.**

12–203.

Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



SOLUTION



$2s_B + (s_B - s_C) = l$

$3s_B - s_C = l$

$3\Delta s_B - \Delta s_C = 0$

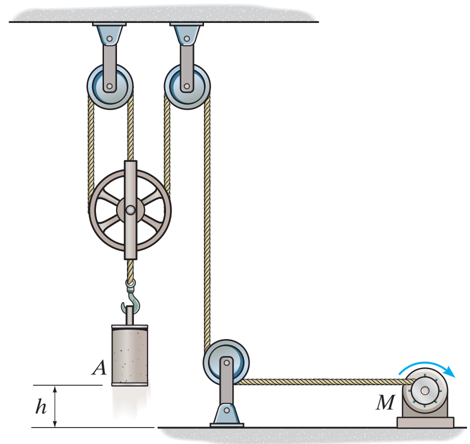
Since $\Delta s_C = -4$, then

$3\Delta s_B = -4$

$\Delta s_B = -1.33 \text{ ft} = 1.33 \text{ ft} \rightarrow$ **Ans.**

***12–204.**

Determine the speed of cylinder A, if the rope is drawn towards the motor M at a constant rate of 10 m/s.



SOLUTION

Position Coordinates: By referring to Fig. a, the length of the rope written in terms of the position coordinates s_A and s_M is

$$3s_A + s_M = l$$

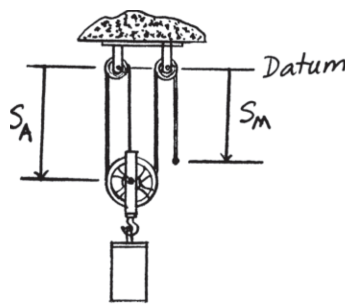
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad 3v_A + v_M = 0$$

Here, $v_M = 10\text{ m/s}$. Thus,

$$3v_A + 10 = 0$$

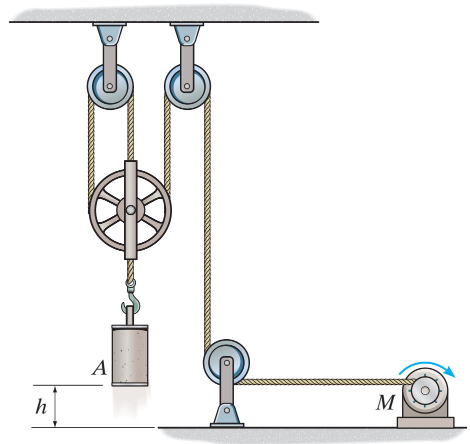
$$v_A = -3.33\text{ m/s} = 3.33\text{ m/s} \uparrow$$ **Ans.**



(a)

12–205.

If the rope is drawn toward the motor M at a speed of $v_M = (5t^{3/2})$ m/s, where t is in seconds, determine the speed of cylinder A when $t = 1$ s.



SOLUTION

Position Coordinates: By referring to Fig. *a*, the length of the rope written in terms of the position coordinates s_A and s_M is

$$3s_A + s_M = l$$

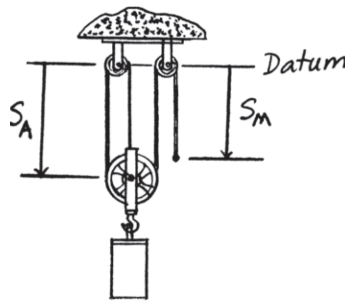
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad 3v_A + v_M = 0$$

Here, $v_M = \left(5t^{3/2}\right)$ m/s. Thus,

$$3v_A + 5t^{3/2} = 0$$

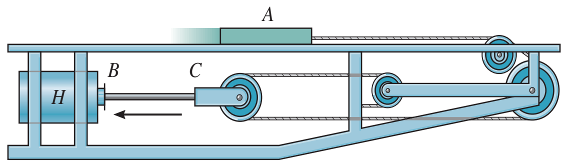
$$v_A = \left(-\frac{5}{3}t^{3/2}\right)\text{ m/s} = \left(\frac{5}{3}t^{3/2}\right)\text{ m/s}\Big|_{t=1\text{ s}} = 1.67\text{ m/s} \qquad \textbf{Ans.}$$



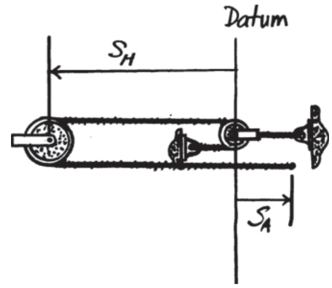
(a)

12–206.

If the hydraulic cylinder H draws in rod BC at 2 ft/s, determine the speed of slider A.



SOLUTION



$2s_H + s_A = l$

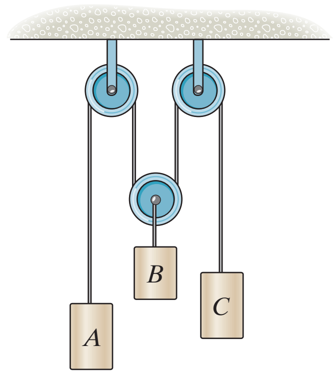
$2v_H = -v_A$

$2(2) = -v_A$

$v_A = -4 \text{ ft/s} = 4 \text{ ft/s} \leftarrow$ **Ans.**

12–207.

If block A is moving downward with a speed of 4 ft/s while C is moving up at 2 ft/s, determine the speed of block B.



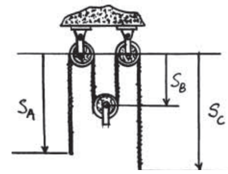
SOLUTION

$s_A + 2s_B + s_C = l$

$v_A + 2v_B + v_C = 0$

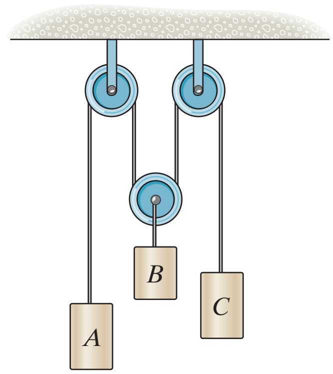
$4 + 2v_B - 2 = 0$

$v_B = -1 \text{ ft/s} = 1 \text{ ft/s} \uparrow$ **Ans.**



***12–208.**

If block A is moving downward at 6 ft/s while block C is moving down at 18 ft/s, determine the speed of block B.



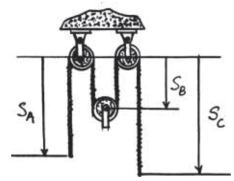
SOLUTION

$s_A + 2s_B + s_C = l$

$v_A + 2v_B + v_C = 0$

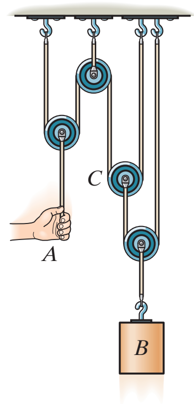
$6 + 2v_B + 18 = 0$

$v_B = -12 \text{ ft/s} = 12 \text{ ft/s} \uparrow$ **Ans.**



12–209.

Determine the displacement of the block B if A is pulled down 4 ft.



SOLUTION

$2s_A + 2s_C = l_1$

$\Delta s_A = -\Delta s_C$

$s_B - s_C + s_B = l_2$

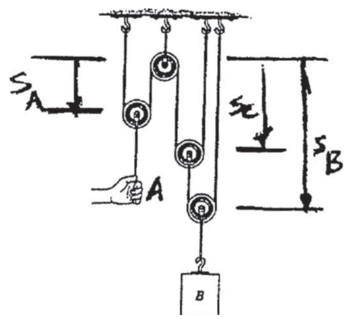
$2\,\Delta s_B = \Delta s_C$

Thus,

$2\,\Delta s_B = -\Delta s_A$

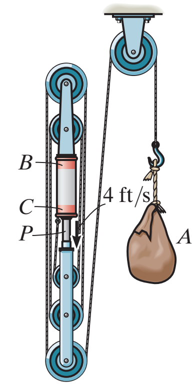
$2\,\Delta s_B = -4$

$\Delta s_B = \;-2\text{ ft} = 2\text{ ft}\uparrow$ **Ans.**



12–210.

The pulley arrangement shown is designed for hoisting materials. If *BC remains fixed* while the plunger P is pushed downward with a speed of 4 ft/s, determine the speed of the load at A.



SOLUTION

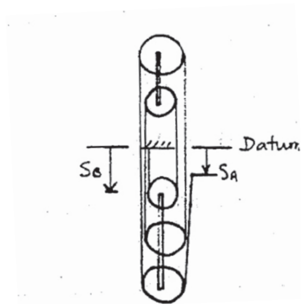
$5\,s_B + (s_B - s_A) = l$

$6\,s_B - s_A = l$

$6\,v_B - v_A = 0$

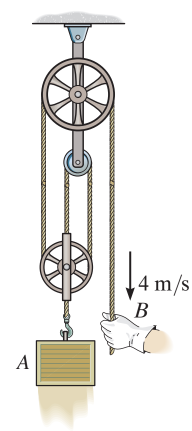
$6(4) = v_A$

$v_A = 24$ ft/s **Ans.**



12–211.

Determine the speed of block A if the end of the rope is pulled down with a speed of 4 m/s.



SOLUTION

Position Coordinates: By referring to Fig. a, the length of the cord written in terms of the position coordinates s_A and s_B is

$$s_B + s_A + 2(s_A - a) = l$$

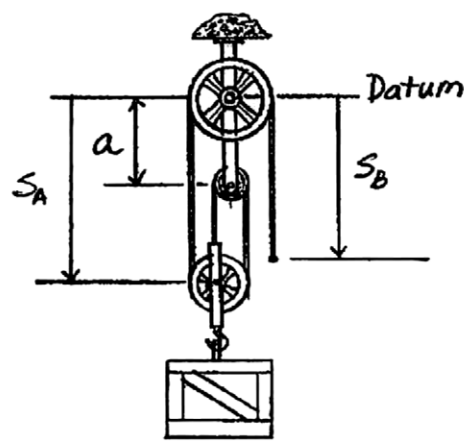
$$s_B + 3s_A = l + 2a$$

Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad v_B + 3v_A = 0$$

Here, $v_B = 4$ m/s. Thus,

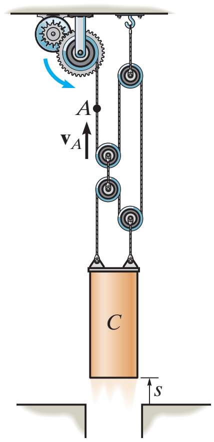
$$4 + 3v_A = 0 \qquad v_A = -133 \text{ m/s} = 1.33 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

***12–212.**

The cylinder C is being lifted using the cable and pulley system shown. If point A on the cable is being drawn toward the drum with a speed of 2 m/s, determine the speed of the cylinder.



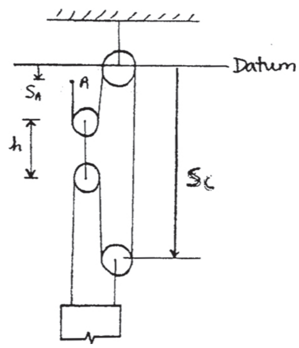
SOLUTION

$$l = s_C + (s_C - h) + (s_C - h - s_A)$$

$$l = 3s_C - 2h - s_A$$

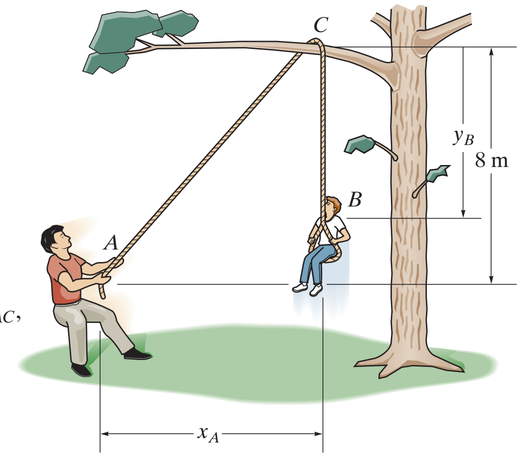
$$0 = 3v_C - v_A$$

$$v_C = \frac{v_A}{3} = \frac{-2}{3} = -0.667 \text{ m/s} = 0.667 \text{ m/s} \uparrow$$ **Ans.**



12–213.

The man pulls the boy up to the tree limb C by walking backward at a constant speed of 1.5 m/s. Determine the speed at which the boy is being lifted at the instant $x_A = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



SOLUTION

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1) and realizing that $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}} \frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}} v_A \qquad \textbf{(2)}$$

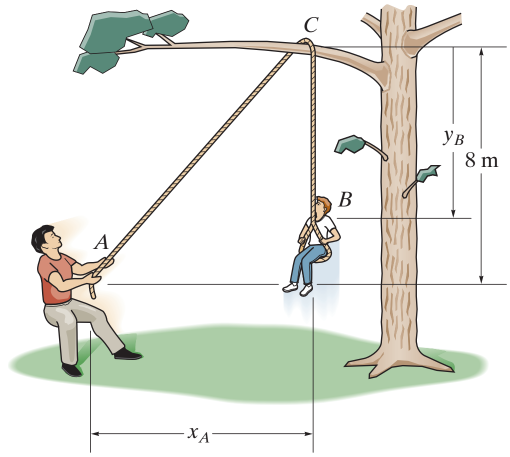
At the instant $x_A = 4$ m, from Eq. [2]

$$v_B = -\frac{4}{\sqrt{4^2 + 64}}(1.5) = -0.671 \text{ m/s} = 0.671 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

12–214.

The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with a constant acceleration $a_A = 0.2\ \text{m/s}^2$, determine the speed of the boy at the instant $y_B = 4$ m. Neglect the size of the limb. When $x_A = 0$, $y_B = 8$ m, so that A and B are coincident, i.e., the rope is 16 m long.



SOLUTION

Position-Coordinate Equation: Using the Pythagorean theorem to determine l_{AC}, we have $l_{AC} = \sqrt{x_A^2 + 8^2}$. Thus,

$$l = l_{AC} + y_B$$

$$16 = \sqrt{x_A^2 + 8^2} + y_B$$

$$y_B = 16 - \sqrt{x_A^2 + 64} \qquad \textbf{(1)}$$

Time Derivative: Taking the time derivative of Eq. (1) Where $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dy_B}{dt}$, we have

$$v_B = \frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 64}}\frac{dx_A}{dt}$$

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 64}}v_A \qquad \textbf{(2)}$$

At the instant $y_B = 4$ m, from Eq. (1), $4 = 16 - \sqrt{x_A^2 + 64}$, $x_A = 8.944$ m. The velocity of the man at that instant can be obtained.

$$v_A^2 = (v_0)_A^2 + 2(a_c)_A\left[s_A - (s_0)_A\right]$$

$$v_A^2 = 0 + 2(0.2)(8.944 - 0)$$

$$v_A = 1.891\ \text{m/s}$$

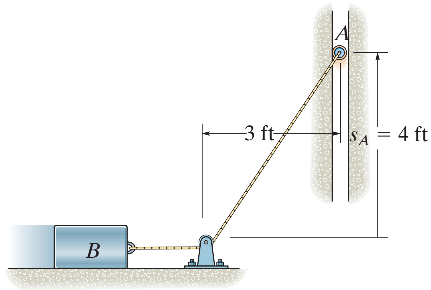
Substitute the above results into Eq. (2) yields

$$v_B = -\frac{8.944}{\sqrt{8.944^2 + 64}}(1.891) = -1.41\ \text{m/s} = 1.41\ \text{m/s}\uparrow \qquad \textbf{Ans.}$$

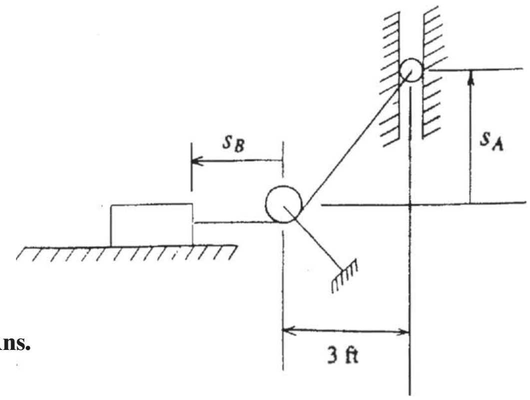
Note: The negative sign indicates that velocity v_B is in the opposite direction to that of positive y_B.

12–215.

The roller at A is moving upward with a velocity of $v_A = 3$ ft/s and has an acceleration of $a_A = 4\text{ ft/s}^2$ when $s_A = 4$ ft. Determine the velocity and acceleration of block B at this instant.



SOLUTION



$$s_B + \sqrt{(s_A)^2 + 3^2} = l$$

$$\dot{s}_B + \frac{1}{2}\left[(s_A)^2 + 3^2\right]^{-\frac{1}{2}}(2s_A)\,\dot{s}_A = 0$$

$$\dot{s}_B + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A\dot{s}_A\right) = 0$$

$$\ddot{s}_B - \left[(s_A)^2 + 9\right]^{-\frac{3}{2}}\left(s_A^2\,\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left[s_A^2 + 9\right]^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$$

At $s_A = 4$ ft, $\dot{s}_A = 3$ ft/s, $\ddot{s}_A = 4\text{ ft/s}^2$

$$\dot{s}_B + \left(\frac{1}{5}\right)(4)(3) = 0$$

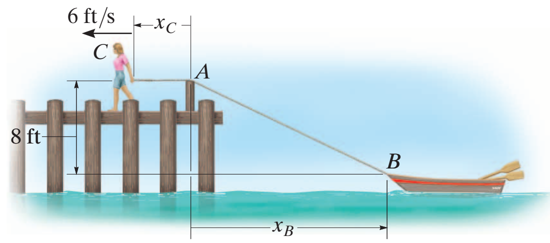
$$v_B = -2.4\text{ ft/s} = 2.40\text{ ft/s} \rightarrow$$ **Ans.**

$$\ddot{s}_B - \left(\frac{1}{5}\right)^3(4)^2(3)^2 + \left(\frac{1}{5}\right)(3)^2 + \left(\frac{1}{5}\right)(4)(4) = 0$$

$$a_B = -3.85\text{ ft/s}^2 = 3.85\text{ ft/s}^2 \rightarrow$$ **Ans.**

***12–216.**

The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at a constant speed of 6 ft/s. Determine how fast the boat approaches the pier at the instant the rope length AB is 50 ft.



SOLUTION

The length l of cord is

$$\sqrt{(8)^2 + x_B^2} + x_C = l$$

Taking the time derivative:

$$\frac{1}{2}[(8)^2 + x_B^2]^{-1/2}\, 2\, x_B \dot{x}_B + \dot{x}_C = 0 \qquad \textbf{(1)}$$

$$\dot{x}_C = 6 \text{ ft/s}$$

When $AB = 50$ ft,

$$x_B = \sqrt{(50)^2 - (8)^2} = 49.356 \text{ ft}$$

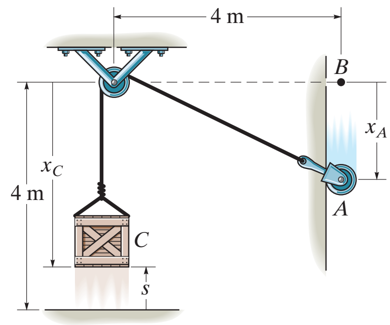
From Eq. (1)

$$\frac{1}{2}[(8)^2 + (49.356)^2]^{-1/2}\, 2(49.356)(\dot{x}_B) + 6 = 0$$

$$\dot{x}_B = -6.0783 = 6.08 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

12–217.

The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2\ \text{m/s}$ along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1\ \text{m}$. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.

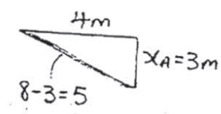


SOLUTION

$$x_C + \sqrt{x_A^2 + (4)^2} = l$$

$$\dot{x}_C + \frac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)(\dot{x}_A) = 0$$

$$\ddot{x}_C - \frac{1}{2}(x_A^2 + 16)^{-3/2}(2x_A^2)(\dot{x}_A^2) + (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(x_A)(\ddot{x}_A) = 0$$



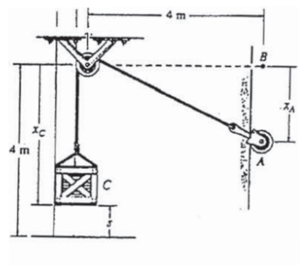
$l = 8\ \text{m}$, and when $s = 1\ \text{m}$,

$x_C = 3\ \text{m}$

$x_A = 3\ \text{m}$

$v_A = \dot{x}_A = 2\ \text{m/s}$

$a_A = \ddot{x}_A = 0$



Thus,

$v_C + [(3)^2 + 16]^{-1/2}(3)(2) = 0$

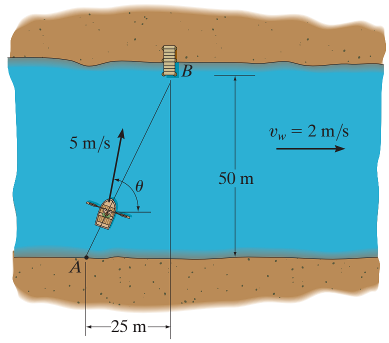
$v_C = -1.2\ \text{m/s} = 1.2\ \text{m/s} \uparrow$ **Ans.**

$a_C - [(3)^2 + 16]^{-3/2}(3)^2(2)^2 + [(3)^2 + 16]^{-1/2}(2)^2 + 0 = 0$

$a_C = -0.512\ \text{m/s}^2 = 0.512\ \text{m/s}^2 \uparrow$ **Ans.**

12–218.

The man can row the boat in still water with a speed of 5 m/s. If the river is flowing at 2 m/s, determine the speed of the boat and the angle θ he must direct the boat so that it travels from A to B.



SOLUTION

Solution I

Vector Analysis: Here, the velocity $\mathbf{v}_b$ of the boat is directed from A to B. Thus, $\phi = \tan^{-1}\left(\frac{50}{25}\right) = 63.43°$. The magnitude of the boat's velocity relative to the flowing river is $v_{b/w} = 5$ m/s. Expressing $\mathbf{v}_b$, $\mathbf{v}_w$, and $\mathbf{v}_{b/w}$ in Cartesian vector form, we have $\mathbf{v}_b = v_b \cos 63.43\mathbf{i} + v_b \sin 63.43\mathbf{j} = 0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j}$, $\mathbf{v}_w = [2\mathbf{i}]$ m/s, and $\mathbf{v}_{b/w} = 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_b = \mathbf{v}_w + \mathbf{v}_{b/w}$$

$$0.4472v_b\mathbf{i} + 0.8944v_b\mathbf{j} = 2\mathbf{i} + 5\cos\theta\mathbf{i} + 5\sin\theta\mathbf{j}$$

$$0.4472v_b\mathbf{i} + 0.8944v_b\,\mathbf{j} = (2 + 5\cos\theta)\mathbf{i} + 5\sin\theta\mathbf{j}$$

Equating the **i** and **j** components, we have

$$0.4472v_b = 2 + 5\cos\theta \qquad \textbf{(1)}$$

$$0.8944v_b = 5\sin\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_b = 5.56 \text{ m/s} \qquad \theta = 84.4° \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Referring to the velocity diagram shown in Fig. a and applying the law of cosines,

$$5^2 = 2^2 + v_b{}^2 - 2(2)(v_b)\cos 63.43°$$

$$v_b{}^2 - 1.789v_b - 21 = 0$$

$$v_b = \frac{-(-1.789) \pm \sqrt{(-1.789)^2 - 4(1)(-21)}}{2(1)}$$

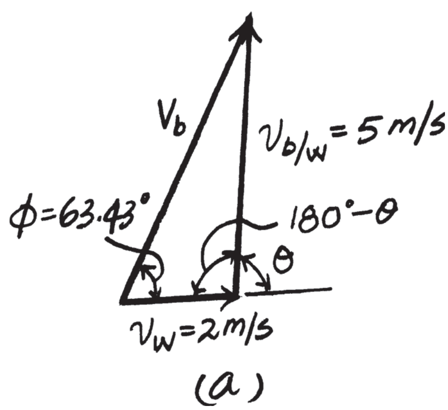
Choosing the positive root,

$$v_b = 5.563 \text{ m/s} = 5.56 \text{ m/s} \qquad \textbf{Ans.}$$

Using the result of v_b and applying the law of sines,

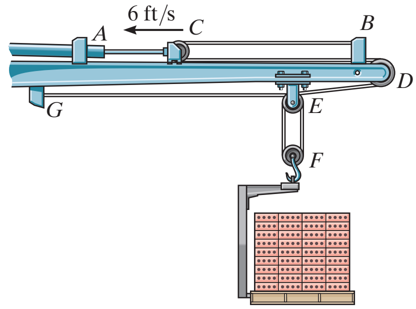
$$\frac{\sin(180° - \theta)}{5.563} = \frac{\sin 63.43°}{5}$$

$$\theta = 84.4° \qquad \textbf{Ans.}$$



12–219.

Vertical motion of the load is produced by movement of the piston at A on the boom. Determine the distance the piston or pulley at C must move to the left in order to lift the load 2 ft. The cable is attached at B, passes over the pulley at C, then D, E, F, and again around E, and is attached at G.



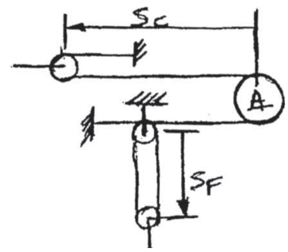
SOLUTION

$2\,s_C + 2\,s_F = l$

$2\,\Delta s_C = -2\,\Delta s_F$

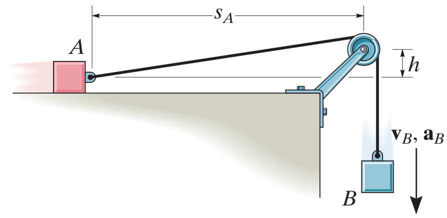
$\Delta s_C = -\Delta s_F$

$\Delta s_C = -(-2\text{ ft}) = 2\text{ ft}$ **Ans.**



12–220.

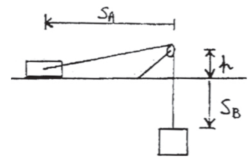
If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.



SOLUTION

$$l = s_B + \sqrt{s_B^2 + h^2}$$

$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2}\, 2s_A\, \dot{s}_A$$



$$v_A = \dot{s}_A = \frac{-\dot{s}_B(s_A^2 + h^2)^{1/2}}{s_A}$$

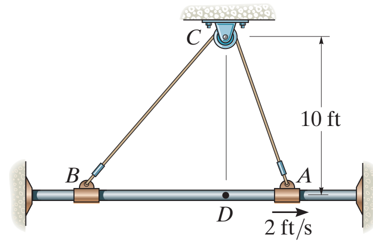
$$v_A = -v_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} \qquad \textbf{Ans.}$$

$$a_A = \dot{v}_A = -\dot{v}_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} - v_B\left(\frac{1}{2}\right)\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2}(h^2)(-2)(s_A)^{-3}\dot{s}_A$$

$$a_A = -a_B\left(1+\left(\frac{h}{s_B}\right)^2\right)^{1/2} + \frac{v_A v_B h^2}{s_A^3}\left(1+\left(\frac{h}{s_A}\right)^2\right)^{-1/2} \qquad \textbf{Ans.}$$

12–221.

Collars A and B are connected to the cord that passes over the small pulley at C. When A is located at D, B is 24 ft to the left of D. If A moves at a constant speed of 2 ft/s to the right, determine the speed of B when A is 4 ft to the right of D.



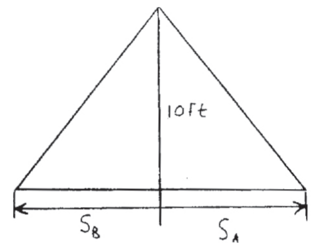
SOLUTION

$$l = \sqrt{(24)^2 + (10)^2} + 10 = 36 \text{ ft}$$

$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + s_A^2} = 36$$

$$\frac{1}{2}(100 + s_B^2)^{-\frac{1}{2}}(2s_B\dot{s}_B) + \frac{1}{2}(100 + s_A^2)^{-\frac{1}{2}}(2s_A\dot{s}_A) = 0$$

$$\dot{s}_B = -\left(\frac{s_A\dot{s}_A}{s_B}\right)\left(\frac{100 + s_B^2}{100 + s_A^2}\right)^{\frac{1}{2}}$$



At $s_A = 4$,

$$\sqrt{(10)^2 + s_B^2} + \sqrt{(10)^2 + (4)^2} = 36$$

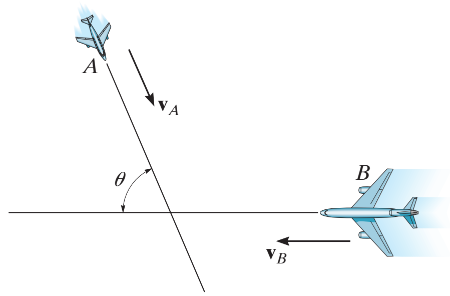
$$s_B = 23.163 \text{ ft}$$

Thus,

$$\dot{s}_B = -\left(\frac{4(2)}{23.163}\right)\left(\frac{100 + (23.163)^2}{100 + 4^2}\right)^{\frac{1}{2}} = -0.809 \text{ ft/s} = 0.809 \text{ ft/s} \rightarrow$$ **Ans.**

12–222.

Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 600$ km/h and $v_B = 500$ km/h such that the angle between their straight-line courses is $\theta = 75°$, determine the velocity of plane B with respect to plane A.



SOLUTION

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$[500 \leftarrow] = [600 \overset{75°}{\measuredangle}_{\theta}] + v_{B/A}$$

$$(\overset{+}{\leftarrow}) \qquad 500 = -600 \cos 75° + (v_{B/A})_x$$

$$(v_{B/A})_x = 655.29 \leftarrow$$

$$(+\uparrow) \qquad 0 = -600 \sin 75° + (v_{B/A})_y$$

$$(v_{B/A})_y = 579.56 \uparrow$$

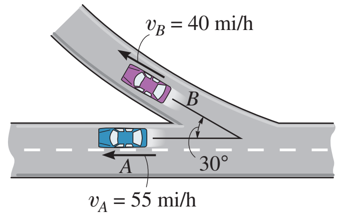
$$(v_{B/A}) = \sqrt{(655.29)^2 + (579.56)^2}$$

$$v_{B/A} = 875 \text{ km/h} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{579.56}{655.29}\right) = 41.5° \qquad \textbf{Ans.}$$

12–223.

At the instant shown, cars A and B are traveling at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.5 mi.



SOLUTION

$$v_B = -40\cos 30°\mathbf{i} + 40\sin 30°\mathbf{j} = \{-34.64\mathbf{i} + 20\mathbf{j}\}\text{ mi/h}$$

$$v_A = \{-55\mathbf{i}\}\text{ mi/h}$$

$$v_{B/A} = \nu_B - \nu_A$$

$$= (-34.64\mathbf{i} + 20\mathbf{j}) - (-55\mathbf{i}) = \{20.36\mathbf{i} + 20\mathbf{j}\}\text{ mi/h}$$

$$v_{B/A} = \sqrt{20.36^2 + 20^2} = 28.5\text{ mi/h}$$ **Ans.**

$$\theta = \tan^{-1}\frac{20}{20.36} = 44.5°$$ **Ans.**

$$(a_B)_n = \frac{v_A^2}{\rho} = \frac{40^2}{0.5} = 3200\text{ mi/h}^2 \qquad (a_B)_t = 1200\text{ mi/h}^2$$

$$\mathbf{a}_B = (3200\cos 60° - 1200\cos 30°)\mathbf{i} + (3200\sin 60° + 1200\sin 30°)\mathbf{j}$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\quad\text{mi/h}^2$$

$$\mathbf{a}_A = 0$$

$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$

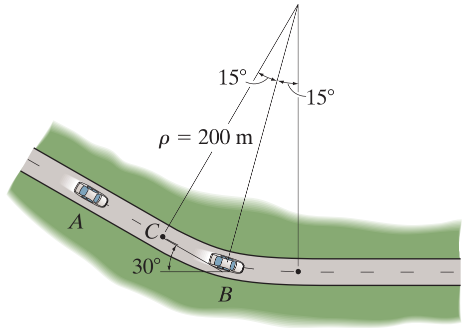
$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\} - 0 = \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\quad\text{mi/h}^2$$

$$a_{B/A} = \sqrt{(560.77)^2 + (3371.28)^2} = 3418\text{ mi/h}^2$$ **Ans.**

$$\theta = \tan^{-1}\frac{3371.28}{560.77} = 80.6°$$ **Ans.**

12–224.

At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.

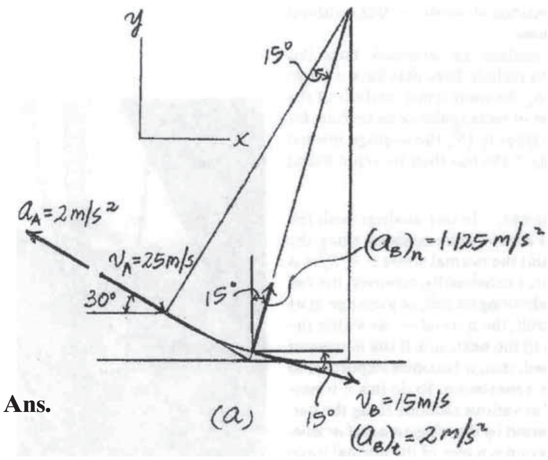


SOLUTION

Velocity: Referring to Fig. a, the velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [25 \cos 30° \, \mathbf{i} - 25 \sin 30° \, \mathbf{j}] \text{ m/s} = [21.65\mathbf{i} - 12.5\mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_B = [15 \cos 15° \, \mathbf{i} - 15 \sin 15° \, \mathbf{j}] \text{ m/s} = [14.49\mathbf{i} - 3.882\mathbf{j}] \text{ m/s}$$



Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$14.49\mathbf{i} - 3.882\mathbf{j} = 21.65\mathbf{i} - 12.5\mathbf{j} + \mathbf{v}_{B/A}$$

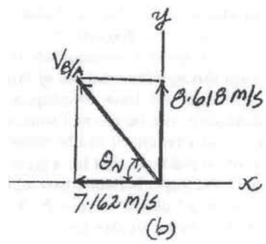
$$\mathbf{v}_{B/A} = [-7.162\mathbf{i} + 8.618\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is given by

$$v_{B/A} = \sqrt{(-7.162)^2 + 8.618^2} = 11.2 \text{ m/s} \qquad \textbf{Ans.}$$

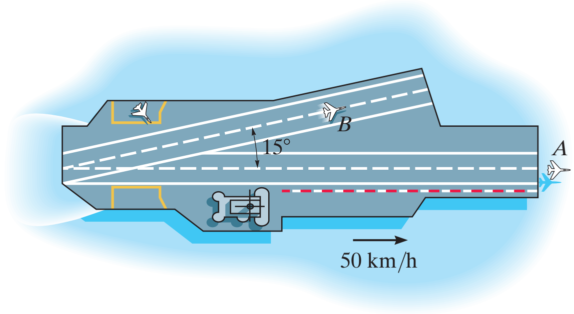
The direction angle θ_v of $\mathbf{v}_{B/A}$ measured down from the negative x axis, Fig. b is

$$\theta_v = \tan^{-1}\left(\frac{8.618}{7.162}\right) = 50.3° \qquad \textbf{Ans.}$$



12–225.

An aircraft carrier is traveling forward with a velocity of 50 km/h. At the instant shown, the plane at A has just taken off and has attained a forward horizontal air speed of 200 km/h, measured from still water. If the plane at B is traveling along the runway of the carrier at 175 km/h in the direction shown, determine the velocity of A with respect to B.



SOLUTION

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$\mathbf{v}_B = 50\mathbf{i} + 175\cos 15°\mathbf{i} + 175\sin 15°\mathbf{j} = 219.04\mathbf{i} + 45.293\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

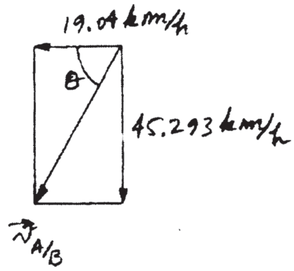
$$200\mathbf{i} = 219.04\mathbf{i} + 45.293\mathbf{j} + (v_{A/B})_x\mathbf{i} + (v_{A/B})_y\mathbf{j}$$

$$200 = 219.04 + (v_{A/B})_x$$

$$0 = 45.293 + (v_{A/B})_y$$

$$(v_{A/B})_x = -19.04$$

$$(v_{A/B})_y = -45.293$$



$$v_{A/B} = \sqrt{(-19.04)^2 + (-45.293)^2} = 49.1 \text{ km/h}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{45.293}{19.04}\right) = 67.2° \measuredangle$$ **Ans.**

12–226.

A car is traveling north along a straight road at 50 km/h. An instrument in the car indicates that the wind is directed toward the east. If the car's speed is 80 km/h, the instrument indicates that the wind is directed toward the north-east. Determine the speed and direction of the wind.

SOLUTION

Solution I

Vector Analysis: For the first case, the velocity of the car and the velocity of the wind relative to the car expressed in Cartesian vector form are $\mathbf{v}_c = [50\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_1\,\mathbf{i}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 50\mathbf{j} + (v_{w/c})_1\,\mathbf{i}$$

$$\mathbf{v}_w = (v_{w/c})_1\mathbf{i} + 50\mathbf{j} \qquad \textbf{(1)}$$

For the second case, $v_C = [80\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_2 \cos 45°\mathbf{i} + (v_{W/C})_2 \sin 45°\,\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 80\mathbf{j} + (v_{w/c})_2 \cos 45°\mathbf{i} + (v_{w/c})_2 \sin 45°\,\mathbf{j}$$

$$\mathbf{v}_w = (v_{w/c})_2 \cos 45°\,\mathbf{i} + \left[80 + (v_{w/c})_2 \sin 45°\right]\mathbf{j} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) and then the **i** and **j** components,

$$(v_{w/c})_1 = (v_{w/c})_2 \cos 45° \qquad \textbf{(3)}$$

$$50 = 80 + (v_{w/c})_2 \sin 45° \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$(v_{w/c})_2 = -42.43 \text{ km/h} \qquad (v_{w/c})_1 = -30 \text{ km/h}$$

Substituting the result of $(v_{w/c})_1$ into Eq. (1),

$$\mathbf{v}_w = [-30\mathbf{i} + 50\mathbf{j}] \text{ km/h}$$

Thus, the magnitude of $\mathbf{v}_W$ is

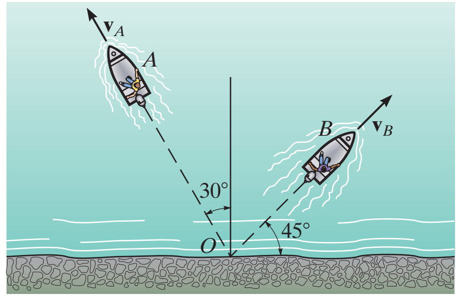
$$v_w = \sqrt{(-30)^2 + 50^2} = 58.3 \text{ km/h} \qquad \textbf{Ans.}$$

and the directional angle θ that $\mathbf{v}_W$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{50}{30}\right) = 59.0° \;\measuredangle \qquad \textbf{Ans.}$$

12–227.

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20\,\text{ft/s}$ and $v_B = 15\,\text{ft/s}$, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 800 ft apart?



SOLUTION

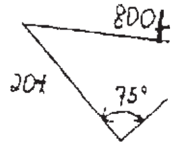
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-20\sin 30°\mathbf{i} + 20\cos 30°\mathbf{j} = 15\cos 45°\mathbf{i} + 15\sin 45°\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-20.61\mathbf{i} + 6.714\mathbf{j}\}\ \text{ft/s}$$

$$v_{A/B} = \sqrt{(-20.61)^2 + (+6.714)^2} = 21.7\ \text{ft/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{6.714}{20.61}\right) = 18.0°$$ **Ans.**



$$(800)^2 = (20\,t)^2 + (15\,t)^2 - 2(20\,t)(15\,t)\cos 75°$$

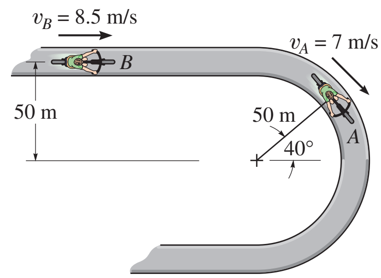
$$t = 36.9\ \text{s}$$ **Ans.**

Also

$$t = \frac{800}{v_{A/B}} = \frac{800}{21.68} = 36.9\ \text{s}$$ **Ans.**

***12–228.**

At the instant shown, the bicyclist at A is traveling at 7 m/s around the curve on the race track while increasing his speed at 0.5 m/s^2. The bicyclist at B is traveling at 8.5 m/s along the straight-a-way and increasing his speed at 0.7 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



SOLUTION

$$v_A = v_B + v_{A/B}$$

$$[7 \searrow_{40^\circ}] = [8.5 \rightarrow] + [(v_{A/B})_x \rightarrow] + [(v_{A/B})_y \downarrow]$$

$$(\overset{+}{\rightarrow}) \qquad 7 \sin 40^\circ = 8.5 + (v_{A/B})_x$$

$$(+\downarrow) \qquad 7 \cos 40^\circ = (v_{A/B})_y$$

Thus,

$$(v_{A/B})_x = 4.00 \text{ m/s} \leftarrow$$

$$(v_{A/B})_y = 5.36 \text{ m/s} \downarrow$$

$$(v_{A/B}) = \sqrt{(4.00)^2 + (5.36)^2}$$

$$v_{A/B} = 6.69 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{5.36}{4.00}\right) = 53.3^\circ \qquad \textbf{Ans.}$$

$$(a_A)_n = \frac{7^2}{50} = 0.980 \text{ m/s}^2$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$[0.980]\,\overset{\theta}{_{40^\circ}} + [0.5] \searrow_{40^\circ} = [0.7 \rightarrow] + [(a_{A/B})_x \rightarrow] + [(a_{A/B})_y \downarrow]$$

$$(+\rightarrow) \qquad -0.980 \cos 40^\circ + 0.5 \sin 40^\circ = 0.7 + (a_{A/B})_x$$

$$(a_{A/B})_x = 1.129 \text{ m/s}^2 \leftarrow$$

$$(+\downarrow) \qquad 0.980 \sin 40^\circ + 0.5 \cos 40^\circ = (a_{A/B})_y$$

$$(a_{A/B})_y = 1.013 \text{ m/s}^2 \downarrow$$

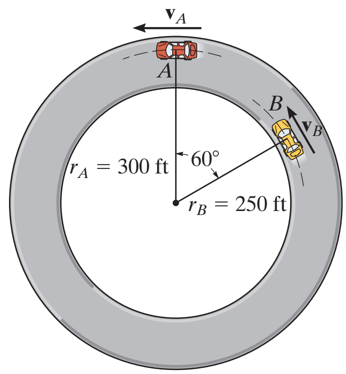
$$(a_{A/B}) = \sqrt{(1.129)^2 + (1.013)^2}$$

$$a_{A/B} = 1.52 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{1.013}{1.129}\right) = 41.9^\circ \qquad \textbf{Ans.}$$

12–229.

Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 90 ft/s and is increasing its speed at the rate of 15 ft/s^2, whereas B has a speed of 105 ft/s and is decreasing its speed at 25 ft/s^2. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



SOLUTION

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-90\mathbf{i} = -105 \sin 30°\,\mathbf{i} + 105 \cos 30°\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-37.5\mathbf{i} - 90.93\mathbf{j}\} \text{ ft/s}$$

$$v_{A/B} = \sqrt{(-37.5)^2 + (-90.93)^2} = 98.4 \text{ ft/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{90.93}{37.5}\right) = 67.6°$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$-15\mathbf{i} - \frac{(90)^2}{300}\mathbf{j} = 25 \cos 60°\mathbf{i} - 25 \sin 60°\mathbf{j} - 44.1 \sin 60°\mathbf{i} - 44.1 \cos 60°\mathbf{j} + \mathbf{a}_{A/B}$$

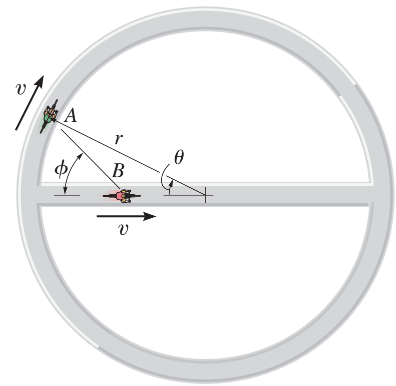
$$\mathbf{a}_{A/B} = \{10.69\mathbf{i} + 16.70\mathbf{j}\} \text{ ft/s}^2$$

$$a_{A/B} = \sqrt{(10.69)^2 + (16.70)^2} = 19.8 \text{ ft/s}^2$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{16.70}{10.69}\right) = 57.4°$$ **Ans.**

12–230.

The two cyclists A and B travel at the same constant speed v. Determine the speed of A with respect to B if A travels along the circular track, while B travels along the diameter of the circle.



SOLUTION

$$\mathbf{v}_A = v \sin \theta \mathbf{i} + v \cos \theta \mathbf{j} \qquad \mathbf{v}_B = v\mathbf{i}$$

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (v \sin \theta \mathbf{i} + v \cos \theta \mathbf{j}) - v\mathbf{i}$$

$$= (v \sin \theta - v)\mathbf{i} + v \cos \theta \mathbf{j}$$

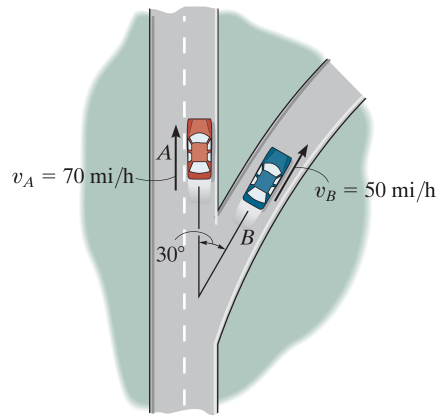
$$v_{A/B} = \sqrt{(v \sin \theta - v)^2 + (v \cos \theta)^2}$$

$$= \sqrt{2v^2 - 2v^2 \sin \theta}$$

$$= v\sqrt{2(1 - \sin \theta)} \qquad \textbf{Ans.}$$

12–231.

At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is increasing its speed by 1100 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.7 mi.



SOLUTION

Relative Velocity:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$50 \sin 30°\mathbf{i} + 50 \cos 30°\mathbf{j} = 70\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{25.0\mathbf{i} - 26.70\mathbf{j}\} \text{ mi/h}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{25.0^2 + (-26.70)^2} = 36.6 \text{ mi/h} \qquad \textbf{Ans.}$$

The direction of the relative velocity is the same as the direction of that for relative acceleration. Thus

$$\theta = \tan^{-1}\frac{26.70}{25.0} = 46.9° \qquad \textbf{Ans.}$$

Relative Acceleration: Since car B is traveling along a curve, its normal acceleration is $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{50^2}{0.7} = 3571.43$ mi/h^2. Applying Eq. 12–35 gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(1100 \sin 30° + 3571.43 \cos 30°)\mathbf{i} + (1100 \cos 30° - 3571.43 \sin 30°)\mathbf{j} = 0 + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{3642.95\mathbf{i} - 833.09\mathbf{j}\} \text{ mi/h}^2$$

Thus, the magnitude of the relative velocity $\mathbf{a}_{B/A}$ is

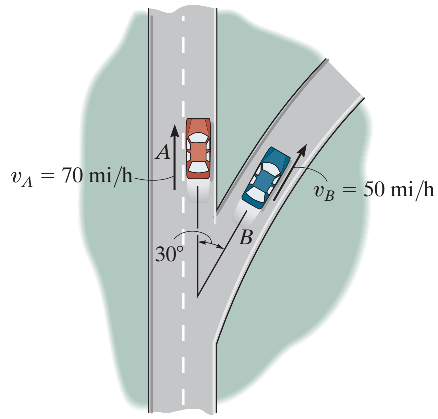
$$a_{B/A} = \sqrt{3642.95^2 + (-833.09)^2} = 3737 \text{ mi/h}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\phi = \tan^{-1}\frac{833.09}{3642.95} = 12.9° \qquad \textbf{Ans.}$$

12–232.

At the instant shown, cars A and B travel at speeds of 70 mi/h and 50 mi/h, respectively. If B is decreasing its speed at 1400 mi/h^2 while A is increasing its speed at 800 mi/h^2, determine the acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.7 mi.



SOLUTION

Relative Acceleration: Since car B is traveling along a curve, its normal acceleration is $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{50^2}{0.7} = 3571.43$ mi/h^2. Applying Eq. 12–35 gives

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$(3571.43 \cos 30° - 1400 \sin 30°)\mathbf{i} + (-1400 \cos 30° - 3571.43 \sin 30°)\mathbf{j} = 800\mathbf{j} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{2392.95\mathbf{i} - 3798.15\mathbf{j}\} \text{ mi/h}^2$$

Thus, the magnitude of the relative acc. $\mathbf{a}_{B/A}$ is

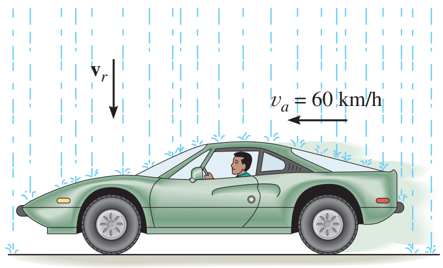
$$a_{B/A} = \sqrt{2392.95^2 + (-3798.15)^2} = 4489 \text{ mi/h}^2 \qquad \textbf{Ans.}$$

And its direction is

$$\phi = \tan^{-1}\frac{3798.15}{2392.95} = 57.8° \qquad \textbf{Ans.}$$

12–233.

A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant)velocity $\mathbf{v}_r$ of the rain if it is assumed to fall vertically.



SOLUTION

$$v_r = v_a + v_{r/a}$$

$$-v_r\mathbf{j} = -60\mathbf{i} + v_{r/a}\cos 30°\mathbf{i} - v_{r/a}\sin 30°\mathbf{j}$$

$$(\overset{+}{\rightarrow}) \qquad 0 = -60 + v_{r/a}\cos 30°$$

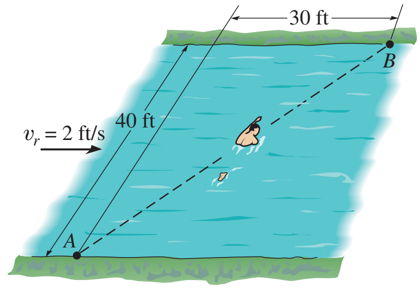
$$(+\uparrow) \qquad -v_r = 0 - v_{r/a}\sin 30°$$

$$v_{r/a} = 69.3 \text{ km/h}$$

$$v_r = 34.6 \text{ km/h}$$ **Ans.**

12–234.

A man can swim at 4 ft/s in still water. He wishes to cross the 40-ft-wide river to point B, 30 ft downstream. If the river flows with a velocity of 2 ft/s, determine the speed of the man and the time needed to make the crossing. *Note*: While in the water he must not direct himself toward point B to reach this point. Why?

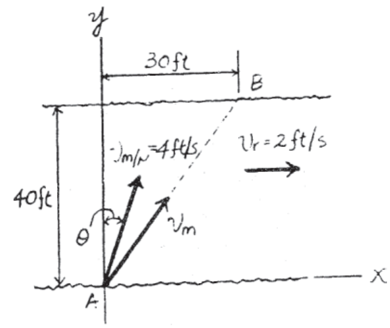


SOLUTION

Relative Velocity:

$$v_m = v_r + v_{m/r}$$

$$\frac{3}{5}v_m\mathbf{i} + \frac{4}{5}v_m\mathbf{j} = 2\mathbf{i} + 4\sin\theta\mathbf{i} + 4\cos\theta\mathbf{j}$$



Equating the i and j components, we have

$$\frac{3}{5}v_m = 2 + 4\sin\theta \qquad \textbf{(1)}$$

$$\frac{4}{5}v_m = 4\cos\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta = 13.29°$$

$$v_m = 4.866 \text{ ft/s} = 4.87 \text{ ft/s} \qquad \textbf{Ans.}$$

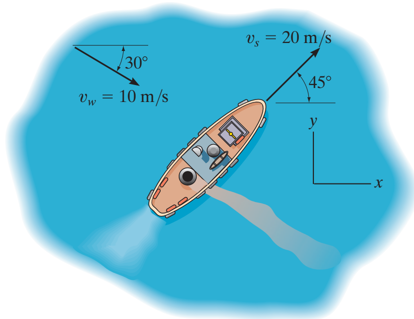
Thus, the time t required by the boat to travel from points A to B is

$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{40^2 + 30^2}}{4.866} = 10.3 \text{ s} \qquad \textbf{Ans.}$$

In order for the man to reached point B, the man has to direct himself at an angle $\theta = 13.3°$ with y axis.

12–235.

The ship travels at a constant speed of $v_s = 20$ m/s and the wind is blowing at a speed of $v_w = 10$ m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



SOLUTION

Solution I

Vector Analysis: The velocity of the smoke as observed from the ship is equal to the velocity of the wind relative to the ship. Here, the velocity of the ship and wind expressed in Cartesian vector form are $\mathbf{v}_s = [20 \cos 45° \,\mathbf{i} + 20 \sin 45° \,\mathbf{j}]$ m/s $= [14.14\mathbf{i} + 14.14\mathbf{j}]$ m/s and $\mathbf{v}_w = [10 \cos 30° \,\mathbf{i} - 10 \sin 30° \,\mathbf{j}] = [8.660\mathbf{i} - 5\mathbf{j}]$ m/s. Applying the relative velocity equation,

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$

$$8.660\mathbf{i} - 5\mathbf{j} = 14.14\mathbf{i} + 14.14\mathbf{j} + \mathbf{v}_{w/s}$$

$$\mathbf{v}_{w/s} = [-5.482\mathbf{i} - 19.14\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{w/s}$ is given by

$$v_w = \sqrt{(-5.482)^2 + (-19.14)^2} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ that $\mathbf{v}_{w/s}$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{19.14}{5.482}\right) = 74.0° \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Applying the law of cosines by referring to the velocity diagram shown in Fig. *a*,

$$v_{w/s} = \sqrt{20^2 + 10^2 - 2(20)(10)\cos 75°}$$

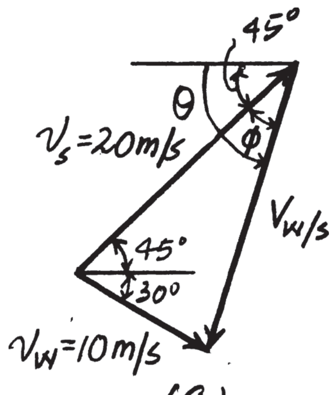
$$= 19.91 \text{ m/s} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

Using the result of $v_{w/s}$ and applying the law of sines,

$$\frac{\sin \phi}{10} = \frac{\sin 75°}{19.91} \qquad \phi = 29.02°$$

Thus,

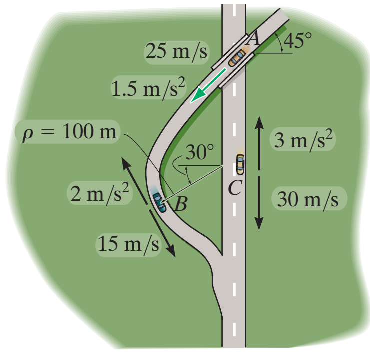
$$\theta = 45° + \phi = 74.0° \qquad \textbf{Ans.}$$



(a)

***12–236.**

Car A travels along a straight road at a speed of 25 m/s while accelerating at 1.5 m/s^2. At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car A relative to car C.



SOLUTION

Velocity: The velocity of cars A and C expressed in Cartesian vector form are

$$\mathbf{v}_A = [-25\cos 45^\circ\mathbf{i} - 25\sin 45^\circ\mathbf{j}]\text{ m/s} = [-17.68\mathbf{i} - 17.68\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\text{ m/s}$$

Applying the relative velocity equation, we have

$$\mathbf{v}_A = \mathbf{v}_C + \mathbf{v}_{A/C}$$

$$-17.68\mathbf{i} - 17.68\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{A/C}$$

$$\mathbf{v}_{A/C} = [-17.68\mathbf{i} + 12.32\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{A/C}$ is given by

$$v_{A/C} = \sqrt{(-17.68)^2 + 12.32^2} = 21.5\text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{A/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{12.32}{17.68}\right) = 34.9^\circ \qquad \textbf{Ans.}$$

Acceleration: The acceleration of cars A and C expressed in Cartesian vector form are

$$\mathbf{a}_A = [-1.5\cos 45^\circ\mathbf{i} - 1.5\sin 45^\circ\mathbf{j}]\text{ m/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}]\text{ m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\text{ m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$$

$$-1.061\mathbf{i} - 1.061\mathbf{j} = 3\mathbf{j} + \mathbf{a}_{A/C}$$

$$\mathbf{a}_{A/C} = [-1.061\mathbf{i} - 4.061\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{A/C}$ is given by

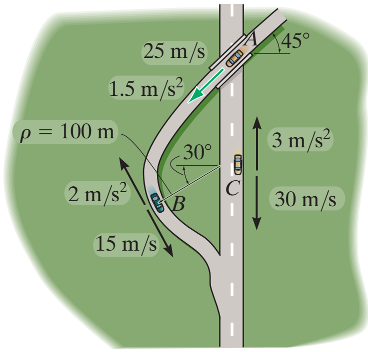
$$a_{A/C} = \sqrt{(-1.061)^2 + (-4.061)^2} = 4.20\text{ m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{A/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{4.061}{1.061}\right) = 75.4^\circ \qquad \textbf{Ans.}$$

12–237.

Car B is traveling along the curved road with a speed of 15 m/s while decreasing its speed at 2 m/s². At this same instant car C is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s². Determine the velocity and acceleration of car B relative to car C.



SOLUTION

Velocity: The velocity of cars B and C expressed in Cartesian vector form are

$$\mathbf{v}_B = [15\cos 60°\,\mathbf{i} - 15\sin 60°\,\mathbf{j}]\ \text{m/s} = [7.5\mathbf{i} - 12.99\mathbf{j}]\ \text{m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\ \text{m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$7.5\mathbf{i} - 12.99\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{B/C}$$

$$\mathbf{v}_{B/C} = [7.5\mathbf{i} + 17.01\mathbf{j}]\ \text{m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/C}$ is given by

$$v_{B/C} = \sqrt{7.5^2 + 17.01^2} = 18.6\ \text{m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{B/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{17.01}{7.5}\right) = 66.2° \ \measuredangle \qquad \textbf{Ans.}$$

Acceleration: The normal component of car B's acceleration is $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{15^2}{100} = 2.25\ \text{m/s}^2$. Thus, the tangential and normal components of car B's acceleration and the acceleration of car C expressed in Cartesian vector form are

$$(\mathbf{a}_B)_t = [-2\cos 60°\,\mathbf{i} + 2\sin 60°\mathbf{j}] = [-1\mathbf{i} + 1.732\mathbf{j}]\ \text{m/s}^2$$

$$(\mathbf{a}_B)_n = [2.25\cos 30°\,\mathbf{i} + 2.25\sin 30°\,\mathbf{j}] = [1.9486\mathbf{i} + 1.125\mathbf{j}]\ \text{m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\ \text{m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$(-1\mathbf{i} + 1.732\mathbf{j}) + (1.9486\mathbf{i} + 1.125\mathbf{j}) = 3\mathbf{j} + \mathbf{a}_{B/C}$$

$$\mathbf{a}_{B/C} = [0.9486\mathbf{i} - 0.1429\mathbf{j}]\ \text{m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/C}$ is given by

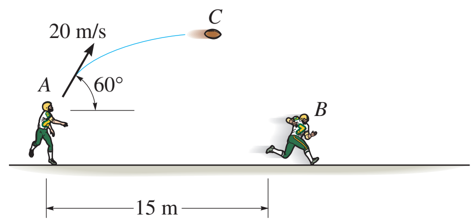
$$a_{B/C} = \sqrt{0.9486^2 + (-0.1429)^2} = 0.959\ \text{m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{B/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{0.1429}{0.9486}\right) = 8.57° \qquad \textbf{Ans.}$$

12–238.

At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



SOLUTION

Ball:

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$s_C = 0 + 20\cos 60°\, t$

$(+\uparrow)\qquad v = v_0 + a_c t$

$-20\sin 60° = 20\sin 60° - 9.81\, t$

$t = 3.53\text{ s}$

$s_C = 35.31\text{ m}$

Player B:

$(\overset{+}{\rightarrow})\quad s_B = s_0 + v_B t$

Require,

$35.31 = 15 + v_B(3.53)$

$v_B = 5.75\text{ m/s}$ **Ans.**

At the time of the catch

$(v_C)_x = 20\cos 60° = 10\text{ m/s} \rightarrow$

$(v_C)_y = 20\sin 60° = 17.32\text{ m/s} \downarrow$

$v_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$10\mathbf{i} - 17.32\mathbf{j} = 5.751\mathbf{i} + (v_{C/B})_x\mathbf{i} + (v_{C/B})_y\mathbf{j}$

$(\overset{+}{\rightarrow})\qquad 10 = 5.75 + (v_{C/B})_x$

$(+\uparrow)\qquad -17.32 = (v_{C/B})_y$

$(v_{C/B})_x = 4.25\text{ m/s} \rightarrow$

$(v_{C/B})_y = 17.32\text{ m/s} \downarrow$

$v_{C/B} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8\text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{17.32}{4.25}\right) = 76.2°$ **Ans.**

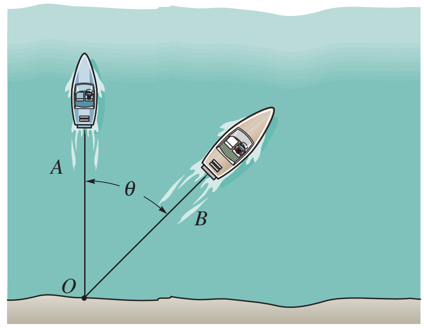
$a_C = \mathbf{a}_B + \mathbf{a}_{C/B}$

$-9.81\,\mathbf{j} = 0 + \mathbf{a}_{C/B}$

$a_{C/B} = 9.81\text{ m/s}^2\downarrow$ **Ans.**

12–239.

Both boats A and B leave the shore at O at the same time. If A travels at v_A and B travels at v_B, write a general expression to determine the velocity of A with respect to B.



SOLUTION

Relative Velocity:

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$v_A\mathbf{j} = v_B \sin\theta\mathbf{i} + v_B \cos\theta\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = -v_B \sin\theta\mathbf{i} + (v_A - v_B\cos\theta)\mathbf{j}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-v_B\sin\theta)^2 + (v_A - v_B\cos\theta)^2}$$

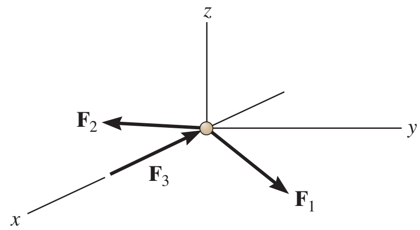
$$= \sqrt{v_A^2 + v_B^2 - 2v_Av_B\cos\theta} \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{v_A - v_B\cos\theta}{v_B\sin\theta}\right) \qquad \textbf{Ans.}$$

13–1.

The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{-2t\mathbf{i}\}$ lb, where t is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.



SOLUTION

$$\Sigma \mathbf{F} = m\mathbf{a}; \quad (2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}) + (t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}) - 2t\mathbf{i} - 6\mathbf{k} = \left(\frac{6}{32.2}\right)(\mathbf{a}_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

Equating components:

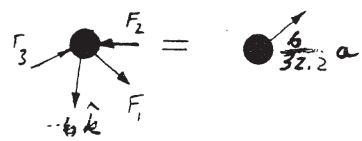
$$\left(\frac{6}{32.2}\right)a_x = t^2 - 2t + 2 \quad \left(\frac{6}{32.2}\right)a_y = -4t + 6 \quad \left(\frac{6}{32.2}\right)a_z = -2t - 7$$

Since $dv = a\,dt$, integrating from $v = 0, t = 0$, yields

$$\left(\frac{6}{32.2}\right)v_x = \frac{t^3}{3} - t^2 + 2t \qquad \left(\frac{6}{32.2}\right)v_y = -2t^2 + 6t \qquad \left(\frac{6}{32.2}\right)v_z = -t^2 - 7t$$

Since $ds = v\,dt$, integrating from $s = 0, t = 0$ yields

$$\left(\frac{6}{32.2}\right)s_x = \frac{t^4}{12} - \frac{t^3}{3} + t^2 \quad \left(\frac{6}{32.2}\right)s_y = -\frac{2t^3}{3} + 3t^2 \quad \left(\frac{6}{32.2}\right)s_z = -\frac{t^3}{3} - \frac{7t^2}{2}$$



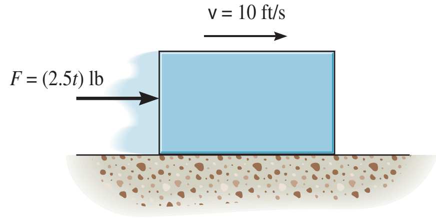
When $t = 2$ s then, $s_x = 14.31$ ft, $\quad s_y = 35.78$ ft $\quad s_z = -89.44$ ft

Thus,

$$s = \sqrt{(14.31)^2 + (35.78)^2 + (-89.44)^2} = 97.4 \text{ ft}$$ **Ans.**

13–2.

The 10-lb block has an initial velocity of 10 ft/s on the smooth plane. If a force $F = (2.5t)$ lb, where t is in seconds, acts on the block for 3 s, determine the final velocity of the block and the distance the block travels during this time.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 2.5t = \left(\frac{10}{32.2}\right)a$$

$$a = 8.05t$$

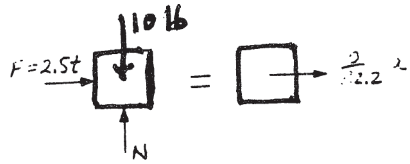
$$dv = a\,dt$$

$$\int_{10}^{v} dv = \int_0^t 8.05t\,dt$$

$$v = 4.025t^2 + 10$$

When $t = 3$ s,

$$v = 46.2 \text{ ft/s} \qquad \textbf{Ans.}$$



$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t (4.025t^2 + 10)\,dt$$

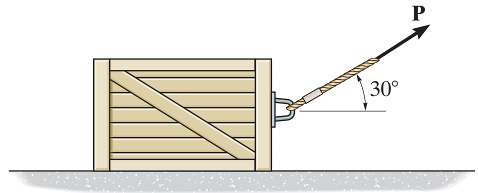
$$s = 1.3417t^3 + 10t$$

When $t = 3$ s,

$$s = 66.2 \text{ ft} \qquad \textbf{Ans.}$$

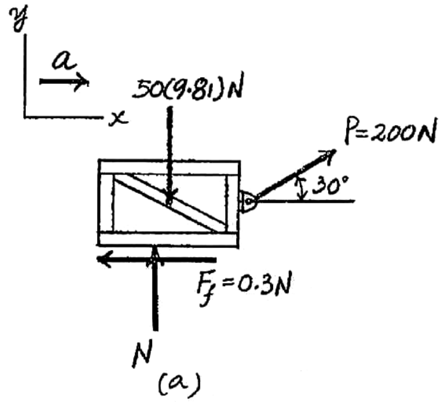
13–3.

If the coefficient of kinetic friction between the 50-kg crate and the ground is $\mu_k = 0.3$, determine the distance the crate travels and its velocity when $t = 3$ s. The crate starts from rest, and $P = 200$ N.



SOLUTION

Free-Body Diagram: The kinetic friction $F_f = \mu_k N$ is directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.



Equations of Motion: Here, $a_y = 0$. Thus,

$$+\uparrow \Sigma F_y = 0; \qquad N - 50(9.81) + 200 \sin 30° = 0$$

$$N = 390.5 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \quad 200 \cos 30° - 0.3(390.5) = 50a$$

$$a = 1.121 \text{ m/s}^2$$

Kinematics: Since the acceleration **a** of the crate is constant,

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t$$

$$v = 0 + 1.121(3) = 3.36 \text{ m/s}$$ **Ans.**

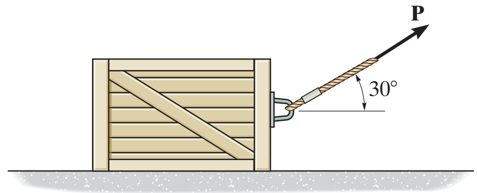
and

$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(1.121)\left(3^2\right) = 5.04 \text{ m}$$ **Ans.**

***13–4.**

If the 50-kg crate starts from rest and achieves a velocity of $v = 4\text{ m/s}$ when it travels a distance of 5 m to the right, determine the magnitude of force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$.



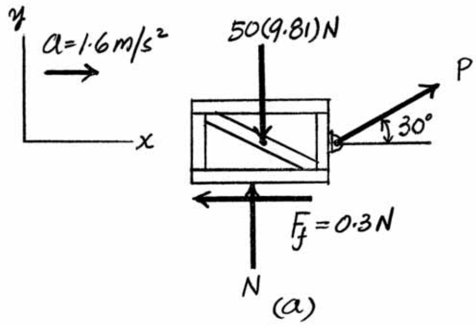
SOLUTION

Kinematics: The acceleration **a** of the crate will be determined first since its motion is known.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$4^2 = 0^2 + 2a(5 - 0)$$

$$a = 1.60\text{ m/s}^2 \rightarrow$$



Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.3N$ is required to be directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.

Equations of Motion:

$$+\uparrow \Sigma F_y = ma_y; \qquad N + P\sin 30° - 50(9.81) = 50(0)$$

$$N = 490.5 - 0.5P$$

Using the results of **N** and **a**,

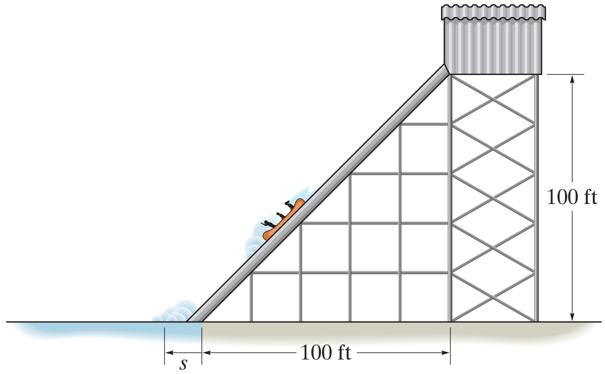
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad P\cos 30° - 0.3(490.5 - 0.5P) = 50(1.60)$$

$$P = 224\text{ N}$$

Ans.

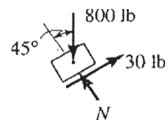
13–5.

The water-park ride consists of an 800-lb sled which slides from rest down the incline and then into the pool. If the frictional resistance on the incline is $F_r = 30$ lb, and in the pool for a short distance $F_r = 80$ lb, determine how fast the sled is traveling when $s = 5$ ft.



SOLUTION

$+\swarrow \sum F_x = ma_x;$ $\quad 800 \sin 45° - 30 = \dfrac{800}{32.2} a$



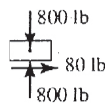
$$a = 21.561 \text{ ft/s}^2$$

$$v_1^2 = v_0^2 + 2a_c(s - s_0)$$

$$v_1^2 = 0 + 2(21.561)(100\sqrt{2} - 0))$$

$$v_1 = \ = 78.093 \text{ ft/s}$$

$\overset{+}{\leftarrow} \sum F_x = ma_x;$ $\quad -80 = \dfrac{800}{32.2} a$



$$a = -3.22 \text{ ft/s}^2$$

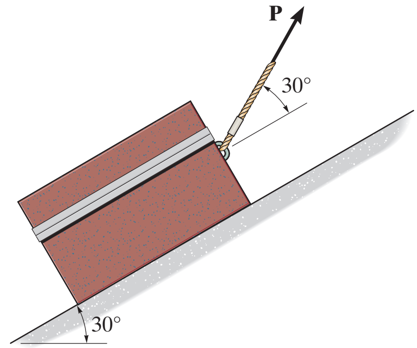
$$v_2^2 = v_1^2 + 2a_c(s_2 - s_1)$$

$$v_2^2 = (78.093)^2 + 2(-3.22)(5 - 0)$$

$$v_2 = 77.9 \text{ ft/s}$$ **Ans.**

13–6.

If $P = 400$ N and the coefficient of kinetic friction between the 50-kg crate and the inclined plane is $\mu_k = 0.25$, determine the velocity of the crate after it travels 6 m up the plane. The crate starts from rest.



SOLUTION

Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.25N$ is required to be directed down the plane to oppose the motion of the crate which is assumed to be directed up the plane. The acceleration **a** of the crate is also assumed to be directed up the plane, Fig. *a*.

Equations of Motion: Here, $a_{y'} = 0$. Thus,

$$\Sigma F_{y'} = ma_{y'}; \qquad N + 400 \sin 30° - 50(9.81) \cos 30° = 50(0)$$

$$N = 224.79 \text{ N}$$

Using the result of **N**,

$$\Sigma F_{x'} = ma_{y'}; \qquad 400 \cos 30° - 50(9.81) \sin 30° - 0.25(224.79) = 50a$$

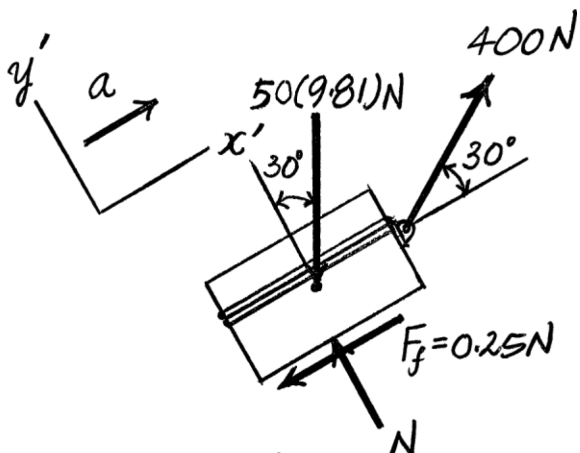
$$a = 0.8993 \text{ m/s}^2$$

Kinematics: Since the acceleration **a** of the crate is constant,

$$v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$v^2 = 0 + 2(0.8993)(6 - 0)$$

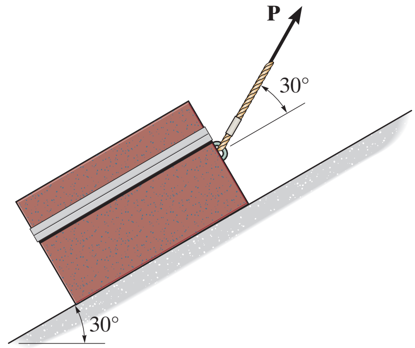
$$v = 3.29 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

13–7.

If the 50-kg crate starts from rest and travels a distance of 6 m up the plane in 4 s, determine the magnitude of force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



SOLUTION

Kinematics: Here, the acceleration **a** of the crate will be determined first since its motion is known.

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$6 = 0 + 0 + \frac{1}{2}a(4^2)$$

$$a = 0.75\ \text{m/s}^2$$

Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.25N$ is required to be directed down the plane to oppose the motion of the crate which is directed up the plane, Fig. *a*.

Equations of Motion: Here, $a_{y'} = 0$. Thus,

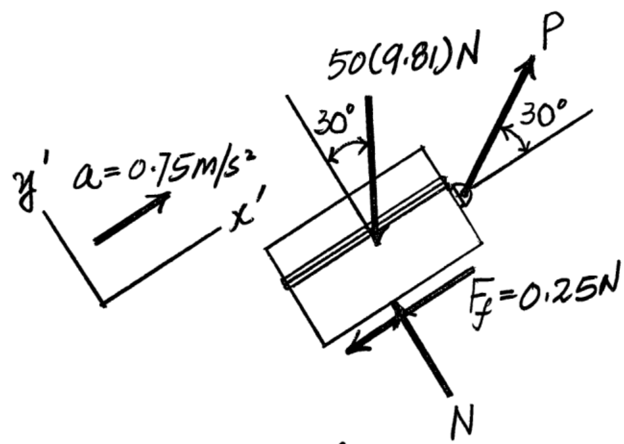
$\Sigma F_{y'} = ma_{y'};$ $\qquad N + P\sin 30° - 50(9.81)\cos 30° = 50(0)$

$$N = 424.79 - 0.5P$$

Using the results of **N** and **a**,

$\Sigma F_{x'} = ma_{x'};$ $\qquad P\cos 30° - 0.25(424.79 - 0.5P) - 50(9.81)\sin 30° = 50(0.75)$

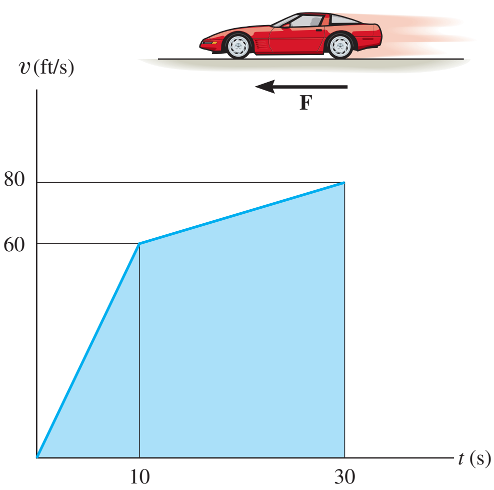
$$P = 392\ \text{N}$$ **Ans.**



(a)

***13–8.**

The speed of the 3500-lb sports car is plotted over the 30-s time period. Plot the variation of the traction force **F** needed to cause the motion.



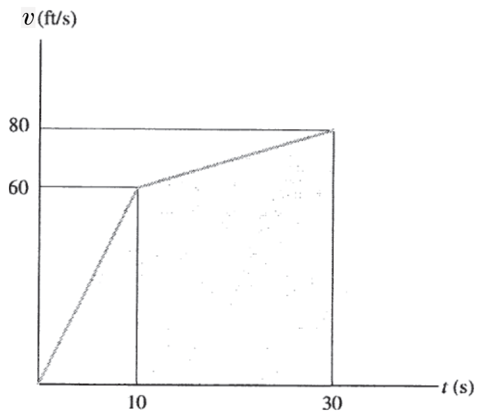
SOLUTION

Kinematics: For $\mathbf{0} \leq \mathbf{t} < \mathbf{10\ s}$. $v = \frac{60}{10}t = \{6t\}$ ft/s. Applying equation $a = \frac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 6 \text{ ft/s}^2$$

For $\mathbf{10} < t \leq \mathbf{30\ s}$, $\frac{v - 60}{t - 10} = \frac{80 - 60}{30 - 10}$, $v = \{t + 50\}$ ft/s. Applying equation $a = \frac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 1 \text{ ft/s}^2$$



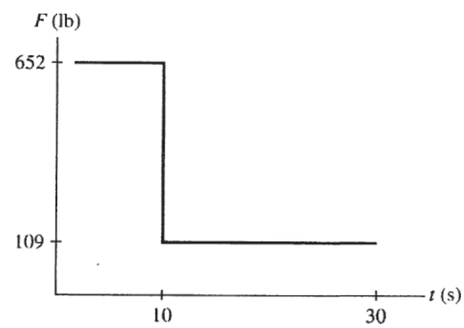
Equation of Motion:

For $\mathbf{0} \leq \mathbf{t} < \mathbf{10\ s}$

$$\overset{+}{\leftarrow} \sum F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(6) = 652 \text{ lb}$$ **Ans.**

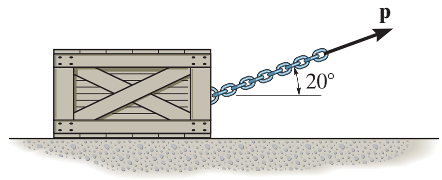
For $\mathbf{10} < \boldsymbol{t} \leq \mathbf{30\ s}$

$$\overset{+}{\leftarrow} \sum F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(1) = 109 \text{ lb}$$ **Ans.**



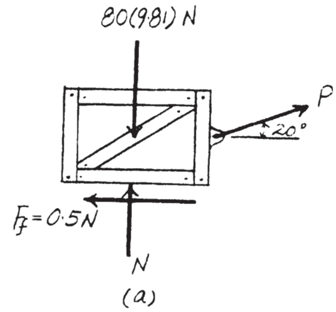
13–9.

The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of **P** is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



SOLUTION

Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.5N$. From FBD(a),



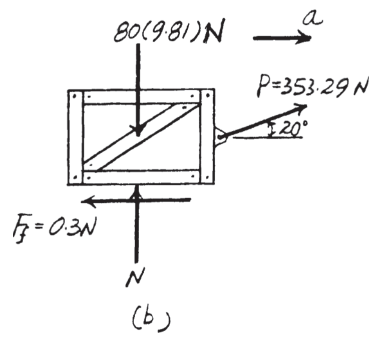
$$+\uparrow \Sigma F_y = 0; \qquad N + P \sin 20° - 80(9.81) = 0 \tag{1}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P \cos 20° - 0.5N = 0 \tag{2}$$

Solving Eqs.(1) and (2) yields

$$P = 353.29 \text{ N} \qquad N = 663.97 \text{ N}$$

Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),



$$+\uparrow \Sigma F_y = ma_y; \qquad N - 80(9.81) + 353.29 \sin 20° = 80(0)$$

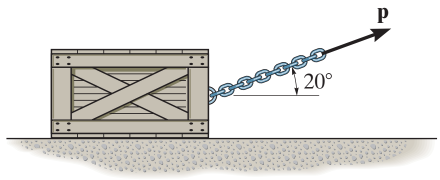
$$N = 663.97 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 353.29 \cos 20° - 0.3(663.97) = 80a$$

$$a = 1.66 \text{ m/s}^2 \qquad \textbf{Ans.}$$

13–10.

The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in $t = 2$ s if the coefficient of static friction is $\mu_s = 0.4$, the coefficient of kinetic friction is $\mu_k = 0.3$, and the towing force is $P = (90t^2)$ N, where t is in seconds.



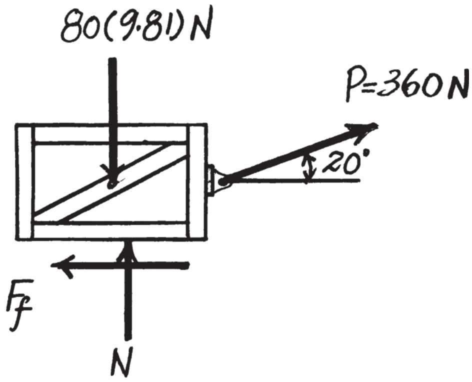
SOLUTION

Equations of Equilibrium: At $t = 2$ s, $P = 90\left(2^2\right) = 360$ N. From FBD(a)

$+\uparrow \Sigma F_y = 0; \quad N + 360 \sin 20° - 80(9.81) = 0 \quad N = 661.67 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \quad 360 \cos 20° - F_f = 0 \quad F_f = 338.29 \text{ N}$

Since $F_f > (F_f)_{\max} = \mu_s N = 0.4(661.67) = 264.67$ N, the crate accelerates.



(a)

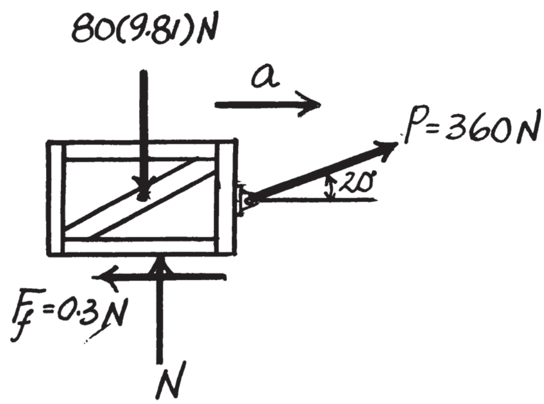
Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

$+\uparrow \Sigma F_y = ma_y; \quad N - 80(9.81) + 360 \sin 20° = 80(0)$

$N = 661.67 \text{ N}$

$\xrightarrow{+} \Sigma F_x = ma_x; \quad 360 \cos 20° - 0.3(661.67) = 80a$

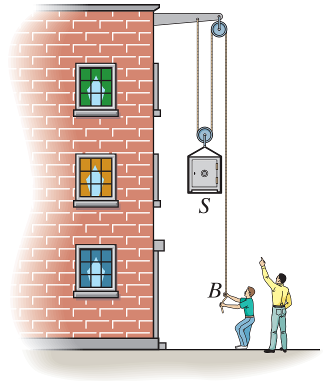
$a = 1.75 \text{ m/s}^2$ **Ans.**



(b)

13–11.

The safe S has a weight of 200 lb and is supported by the rope and pulley arrangement shown. If the end of the rope is given to a boy B of weight 90 lb, determine his acceleration if in the confusion he doesn't let go of the rope. Neglect the mass of the pulleys and rope.

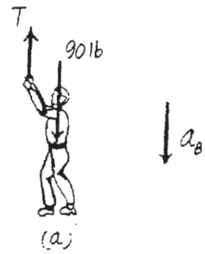


SOLUTION

Equation of Motion: The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys.

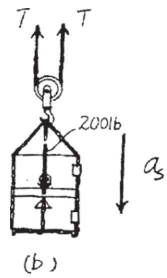
From FBD(a),

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 90 = -\left(\frac{90}{32.2}\right)a_B \qquad \textbf{(1)}$$



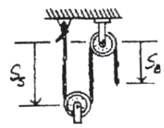
From FBD(b),

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 200 = -\left(\frac{200}{32.2}\right)a_S \qquad \textbf{(2)}$$



Kinematic: Establish the position-coordinate equation, we have

$$2s_S + s_B = l$$



Taking time derivative twice yields

$$(+\downarrow) \qquad 2a_S + a_B = 0 \qquad \textbf{(3)}$$

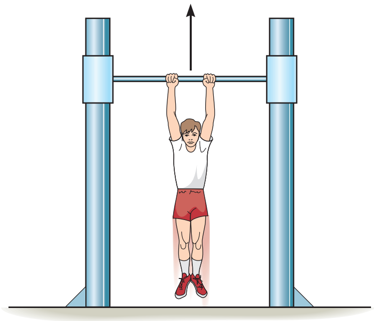
Solving Eqs.(1),(2), and (3) yields

$$a_B = -2.30 \text{ ft/s}^2 = 2.30 \text{ ft/s}^2 \quad \uparrow \qquad \textbf{Ans.}$$

$$a_S = 1.15 \text{ ft/s}^2 \downarrow \qquad T = 96.43 \text{ lb}$$

***13–12.**

The boy having a weight of 80 lb hangs uniformly from the bar. Determine the force in each of his arms in $t = 2$ s if the bar is moving upward with (a) a constant velocity of 3 ft/s, and (b) a speed of $v = (4t^2)$ ft/s, where t is in seconds.



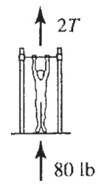
SOLUTION

(a) $T = 40$ lb **Ans.**

(b) $v = 4t^2$

$a = 8t$

$$+\uparrow \sum F_y = ma_y; \qquad 2T - 80 = \frac{80}{32.2}(8t)$$

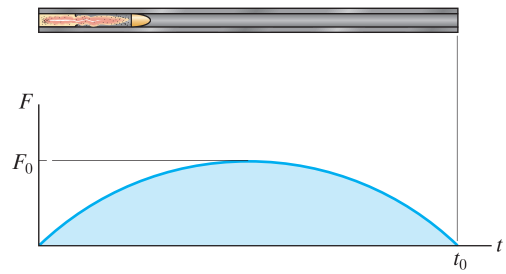


At $t = 2$ s.

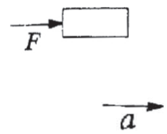
$T = 59.9$ lb **Ans.**

13–13.

The bullet of mass m is given a velocity due to gas pressure caused by the burning of powder within the chamber of the gun. Assuming this pressure creates a force of $F = F_0 \sin(\pi t/t_0)$ on the bullet, determine the velocity of the bullet at any instant it is in the barrel. What is the bullet's maximum velocity? Also, determine the position of the bullet in the barrel as a function of time.



SOLUTION



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 \sin\left(\frac{\pi t}{t_0}\right) = ma$$

$$a = \frac{dv}{dt} = \left(\frac{F_0}{m}\right) \sin\left(\frac{\pi t}{t_0}\right)$$

$$\int_0^v dv = \int_0^t \left(\frac{F_0}{m}\right) \sin\left(\frac{\pi t}{t_0}\right) dt \qquad v = -\left(\frac{F_0 t_0}{\pi m}\right) \cos\left(\frac{\pi t}{t_0}\right)\Bigg]_0^t$$

$$v = \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)$$ **Ans.**

v_{max} occurs when $\cos\left(\frac{\pi t}{t_0}\right) = -1$, or $t = t_0$.

$$v_{max} = \frac{2F_0 t_0}{\pi m}$$ **Ans.**

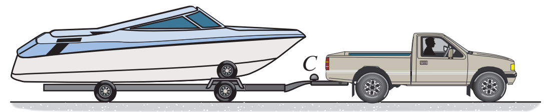
$$\int_0^s ds = \int_0^t \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right) dt$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left[\mathrm{t} - \frac{t_0}{\pi} \sin\left(\frac{\pi t}{t_0}\right)\right]_0^t$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left(t - \frac{t_0}{\pi} \sin\left(\frac{\pi t}{t_0}\right)\right)$$ **Ans.**

13–14.

The 2-Mg truck is traveling at 15 m/s when the brakes on all its wheels are applied, causing it to skid for a distance of 10 m before coming to rest. Determine the constant horizontal force developed in the coupling C, and the frictional force developed between the tires of the truck and the road during this time. The total mass of the boat and trailer is 1 Mg.



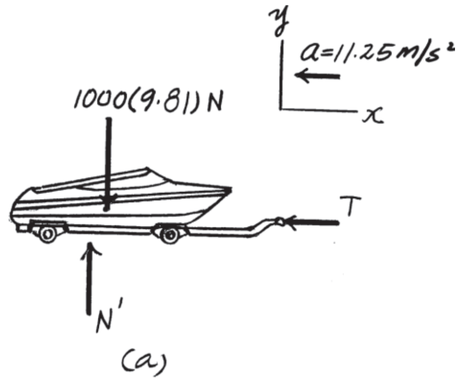
SOLUTION

Kinematics: Since the motion of the truck and trailer is known, their common acceleration **a** will be determined first.

$$\left(\xrightarrow{+}\right) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$0 = 15^2 + 2a(10 - 0)$$

$$a = -11.25\ \text{m/s}^2 = 11.25\ \text{m/s}^2 \ \leftarrow$$

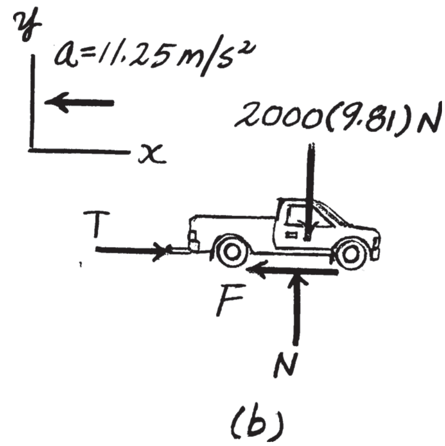


Free-Body Diagram: The free-body diagram of the truck and trailer are shown in Figs. (a) and (b), respectively. Here, **F** representes the frictional force developed when the truck skids, while the force developed in coupling C is represented by **T**.

Equations of Motion: Using the result of **a** and referrning to Fig. (a),

$$\xrightarrow{+} \Sigma F_x = ma_x\,; \qquad -T = 1000(-11.25)$$

$$T = 11\,250\ \text{N} = 11.25\ \text{kN} \qquad \textbf{Ans.}$$



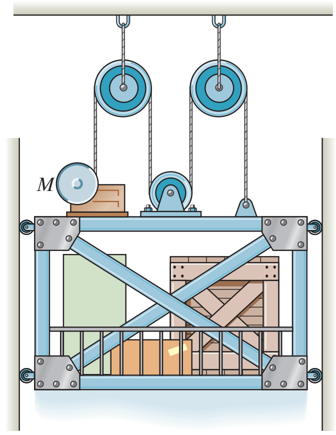
Using the results of **a** and **T** and referring to Fig. (b),

$$+\uparrow \Sigma F_x = ma_x\,; \qquad 11\,250 - F = 2000(-11.25)$$

$$F = 33\,750\ \text{N} = 33.75\ \text{kN} \qquad \textbf{Ans.}$$

13–15.

A freight elevator, including its load, has a mass of 500 kg. It is prevented from rotating by the track and wheels mounted along its sides. When $t = 2$ s, the motor M draws in the cable with a speed of 6 m/s, *measured relative to the elevator.* If it starts from rest, determine the constant acceleration of the elevator and the tension in the cable. Neglect the mass of the pulleys, motor, and cables.



SOLUTION

$$3s_E + s_P = l$$

$$3v_E = -v_P$$

$$(+\downarrow) \qquad v_P = v_E + v_{P/E}$$

$$-3v_E = v_E + 6$$

$$v_E = -\frac{6}{4} = -1.5 \text{ m/s} = 1.5 \text{ m/s} \uparrow$$

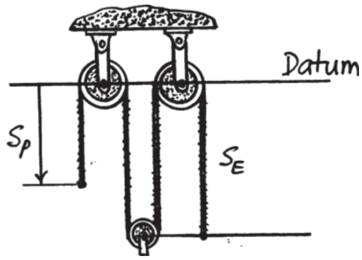
$$(+\uparrow) \qquad v = v_0 + a_c t$$

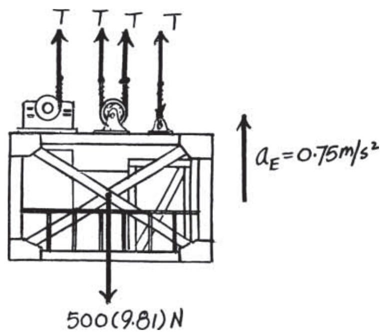
$$1.5 = 0 + a_E(2)$$

$$a_E = 0.75 \text{ m/s}^2 \uparrow$$ **Ans.**

$$+\uparrow \Sigma F_y = ma_y; \qquad 4T - 500(9.81) = 500(0.75)$$

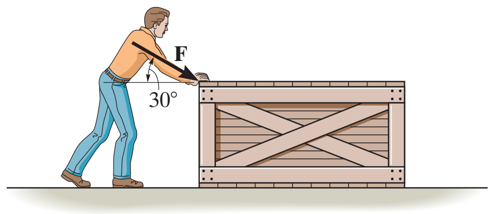
$$T = 1320 \text{ N} = 1.32 \text{ kN}$$ **Ans.**





***13–16.**

The man pushes on the 60-lb crate with a force **F**. The force is always directed down at 30° from the horizontal as shown, and its magnitude is increased until the crate begins to slide. Determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.6$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



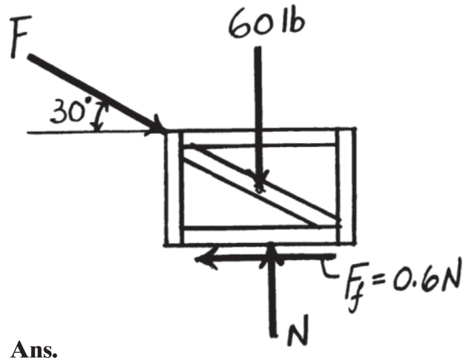
SOLUTION

Force to produce motion:

$\xrightarrow{+} \Sigma F_x = 0; \qquad F\cos 30° - 0.6N = 0$

$+\uparrow \Sigma F_y = 0; \qquad N - 60 - F\sin 30° = 0$

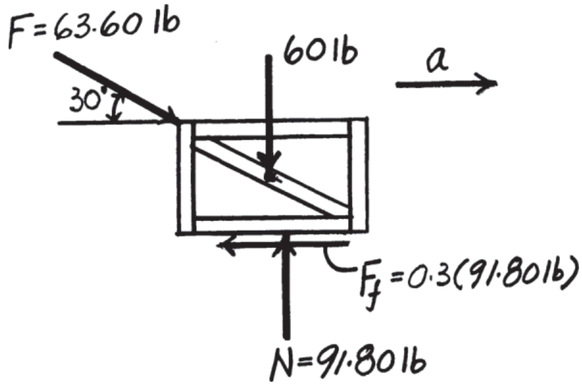
$N = 91.80 \text{ lb} \qquad F = 63.60 \text{ lb}$



Since $N = 91.80$ lb,

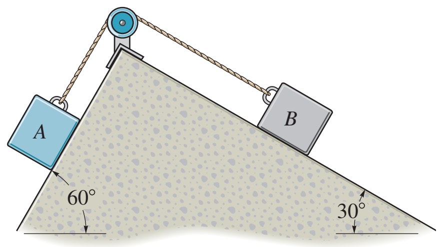
$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 63.60\cos 30° - 0.3(91.80) = \left(\frac{60}{32.2}\right)a$

$a = 14.8 \text{ ft/s}^2$ **Ans.**



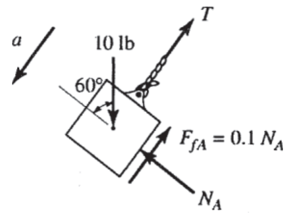
13–17.

The double inclined plane supports two blocks A and B, each having a weight of 10 lb. If the coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.1$, determine the acceleration of each block.



SOLUTION

Equation of Motion: Since blocks A and B are sliding along the plane, the friction forces developed between the blocks and the plane are $(F_f)_A = \mu_k N_A = 0.1\,N_A$ and $(F_f)_B = \mu_k N_B = 0.1 N_B$. Here, $a_A = a_B = a$. Applying Eq. 13–7 to FBD(a), we have

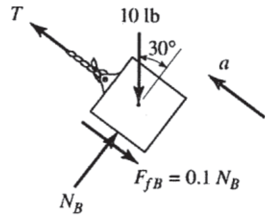


(a)

$$\nwarrow + \sum F_{y'} = ma_{y'}; \quad N_A - 10\cos 60° = \left(\frac{10}{32.2}\right)(0) \quad N_A = 5.00 \text{ lb}$$

$$\nearrow + \sum F_{x'} = ma_{x'}; \qquad T + 0.1(5.00) - 10\sin 60° = -\left(\frac{10}{32.2}\right)a \tag{1}$$

From FBD(b),



(b)

$$\nearrow + \sum F_{y'} = ma_{y'}; \quad N_B - 10\cos 30° = \left(\frac{10}{32.2}\right)(0) \quad N_B = 8.660 \text{ lb}$$

$$\nwarrow + \sum F_{x'} = ma_{x'}; \quad T - 0.1(8.660) - 10\sin 30° = \left(\frac{10}{32.2}\right)a \tag{2}$$

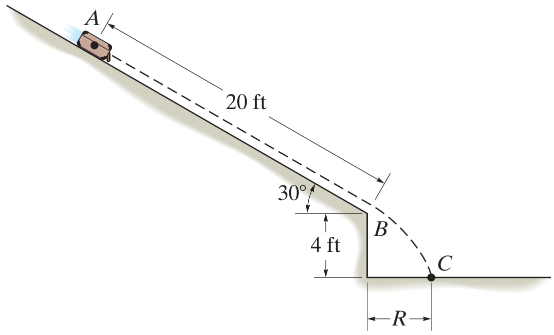
Solving Eqs. (1) and (2) yields

$$a = 3.69 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$T = 7.013 \text{ lb}$$

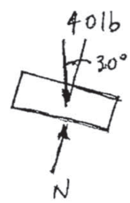
13–18.

A 40-lb suitcase slides from rest 20 ft down the smooth ramp. Determine the point where it strikes the ground at C. How long does it take to go from A to C?



SOLUTION

$+\searrow \Sigma F_x = m\,a_x; \qquad 40 \sin 30° = \dfrac{40}{32.2}a$



$a = 16.1 \text{ ft/s}^2$

$(+\searrow) v^2 = v_0^2 + 2\,a_c(s - s_0);$

$v_B^2 = 0 + 2(16.1)(20)$

$v_B = 25.38 \text{ ft/s}$

$(+\searrow)\ v = v_0 + a_c\,t;$

$25.38 = 0 + 16.1\,t_{AB}$

$t_{AB} = 1.576 \text{ s}$

$(\overset{+}{\rightarrow}) s_x = (s_x)_0 + (v_x)_0\,t$

$R = 0 + 25.38 \cos 30°(t_{BC})$

$(+\downarrow)\ s_y = (s_y)_0 + (v_y)_0\,t + \dfrac{1}{2}a_c t^2$

$4 = 0 + 25.38 \sin 30°\ t_{BC} + \dfrac{1}{2}(32.2)(t_{BC})^2$

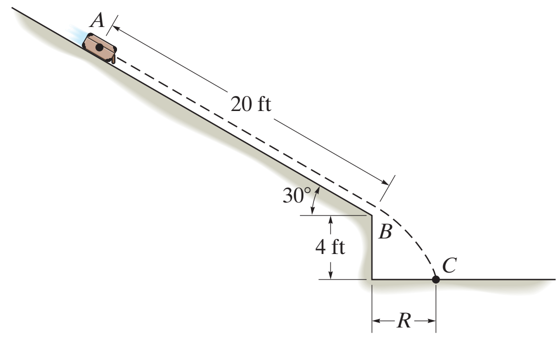
$t_{BC} = 0.2413 \text{ s}$

$R = 5.30 \text{ ft}$ **Ans.**

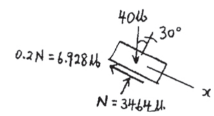
$\text{Total time} = t_{AB} + t_B{}^C = 1.82 \text{ s}$ **Ans.**

13–19.

Solve Prob. 13–18 if the suitcase has an initial velocity down the ramp of $v_A = 10\ \mathrm{ft/s}$ and the coefficient of kinetic friction along AB is $\mu_k = 0.2$.



SOLUTION



$+\searrow \Sigma F_x = ma_x; \qquad 40 \sin 30° - 6.928 = \frac{40}{32.2} a$

$a = 10.52\ \mathrm{ft/s^2}$

$(+\searrow)\ v^2 = v_0^2 + 2\,a_c(s - s_0);$

$v_B^2 = (10)^2 + 2(10.52)(20)$

$v_B = 22.82\ \mathrm{ft/s}$

$(+\searrow)\ v = v_0 + a_c\,t;$

$22.82 = 10 + 10.52\,t_{AB}$

$t_{AB} = 1.219\ \mathrm{s}$

$(\overset{+}{\rightarrow})\ s_x = (s_x)_0 + (v_x)_0\,t$

$R = 0 + 22.82 \cos 30°\ (t_{BC})$

$(+\downarrow)\ s_y = (s_y)_0 + (v_y)_0\,t + \frac{1}{2} a_c\,t^2$

$4 = 0 + 22.82 \sin 30°\ t_{BC} + \frac{1}{2}(32.2)(t_{BC})^2$

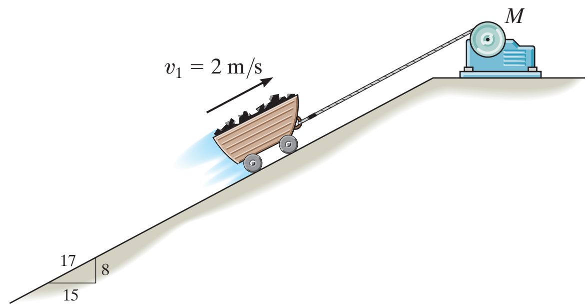
$t_{BC} = 0.2572\ \mathrm{s}$

$R = 5.08\ \mathrm{ft}$ **Ans.**

Total time $= t_{AB} + t_{BC} = 1.48\ \mathrm{s}$ **Ans.**

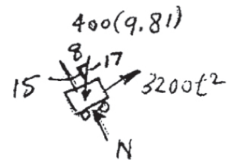
***13–20.**

The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s when $t = 0$, determine its velocity when $t = 2$ s.



SOLUTION

$$\nearrow + \Sigma F_{x'} = ma_{x'}; \qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$



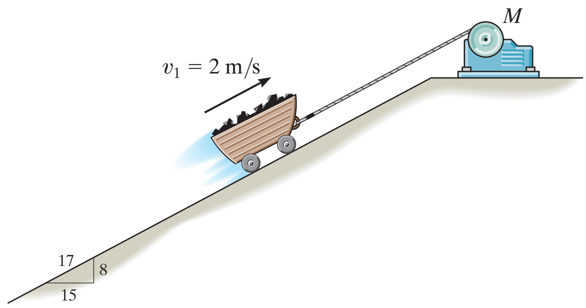
$$dv = a\,dt$$

$$\int_2^v dv = \int_0^2 (8t^2 - 4.616)\,dt$$

$$v = 14.1 \text{ m/s} \qquad \textbf{Ans.}$$

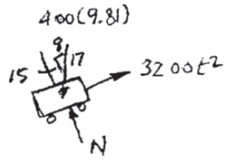
13–21.

The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s at $s = 0$ and $t = 0$, determine the distance it moves up the plane when $t = 2$ s.



SOLUTION

$$\nearrow + \Sigma F_{x'} = ma_{x'}; \qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$



$$dv = a\,dt$$

$$\int_2^v dv = \int_0^t (8t^2 - 4.616)\, dt$$

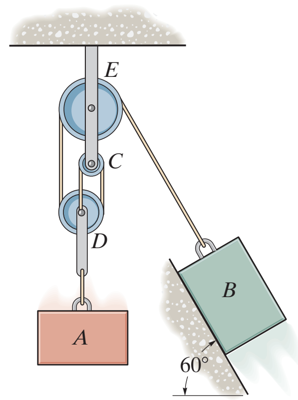
$$v = \frac{ds}{dt} = 2.667t^3 - 4.616t + 2$$

$$\int_0^s ds = \int_0^2 (2.667\,t^3 - 4.616t + 2)\, dt$$

$$s = 5.43 \text{ m}$$ **Ans.**

13–22.

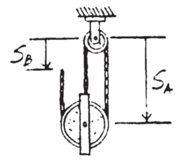
Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B a distance of 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.



SOLUTION

Kinematic: Applying equation $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$, we have

$$(\nwarrow +) \qquad 0.75 = 0 + 0 + \frac{1}{2} a_B \left(2^2\right) \qquad a_B = 0.375 \text{ m/s}^2$$

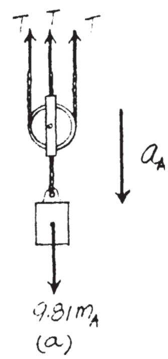


Establishing the position - coordinate equation, we have

$$2s_A + (s_A - s_B) = l \qquad 3s_A - s_B = l$$

Taking time derivative twice yields

$$3a_A - a_B = 0 \tag{1}$$



From Eq.(1),

$$3a_A - 0.375 = 0 \qquad a_A = 0.125 \text{ m/s}^2$$

Equation of Motion: The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(b),

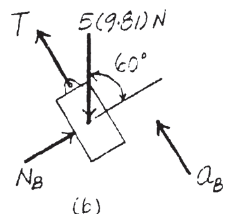
$$\nwarrow + \Sigma F_{y'} = m a_{y'}; \qquad T - 5(9.81)\sin 60° = 5(0.375)$$

$$T = 44.35 \text{ N}$$

From FBD(a),

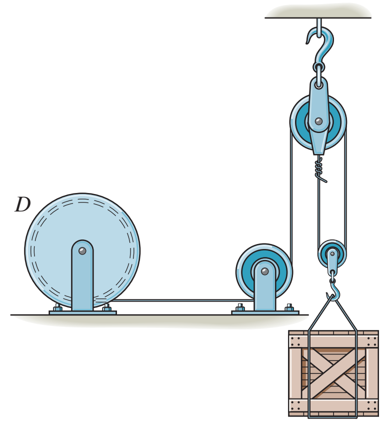
$$+\uparrow \Sigma F_y = m a_y; \qquad 3(44.35) - 9.81 m_A = m_A(-0.125)$$

$$m_A = 13.7 \text{ kg} \qquad \textbf{Ans.}$$



13–23.

The winding drum D is drawing in the cable at an accelerated rate of $5\ \text{m/s}^2$. Determine the cable tension if the suspended crate has a mass of 800 kg.



SOLUTION

$$s_A + 2\,s_B = l$$

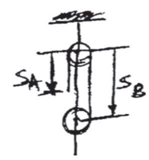
$$a_A = -2\,a_B$$

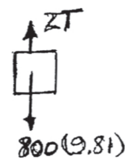
$$5 = -2\,a_B$$

$$a_B = -2.5\ \text{m/s}^2 = 2.5\ \text{m/s}^2 \uparrow$$

$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 800(9.81) = 800(2.5)$

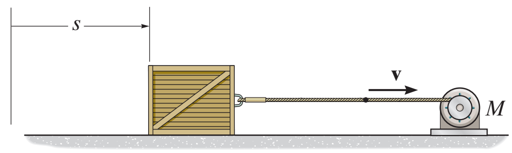
$$T = 4924\ \text{N} = 4.92\ \text{kN}$$ **Ans.**





***13–24.**

If the motor draws in the cable at a rate of $v = (0.05s^{3/2})$ m/s, where s is in meters, determine the tension developed in the cable when $s = 10$ m. The crate has a mass of 20 kg, and the coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



SOLUTION

Kinematics: Since the motion of the create is known, its acceleration **a** will be determined first.

$$a = v\frac{dv}{ds} = \left(0.05s^{3/2}\right)\left[(0.05)\left(\frac{3}{2}\right)s^{1/2}\right] = 0.00375s^2 \text{ m/s}^2$$

When $s = 10$ m,

$$a = 0.00375(10^2) = 0.375 \text{ m/s}^2 \rightarrow$$

Free-Body Diagram: The kinetic friction $F_f = \mu_k N = 0.2N$ must act to the left to oppose the motion of the crate which is to the right, Fig. a.

Equations of Motion: Here, $a_y = 0$. Thus,

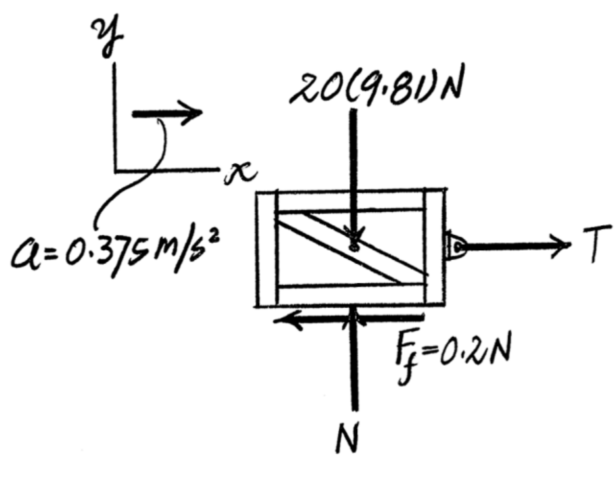
$$+\uparrow \Sigma F_y = ma_y; \qquad N - 20(9.81) = 20(0)$$

$$N = 196.2 \text{ N}$$

Using the results of **N** and **a**,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad T - 0.2(196.2) = 20(0.375)$$

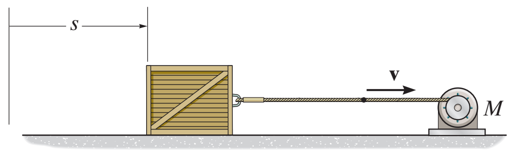
$$T = 46.7 \text{ N}$$ **Ans.**



(a)

13–25.

If the motor draws in the cable at a rate of $v = (0.05t^2)$ m/s, where t is in seconds, determine the tension developed in the cable when $t = 5$ s. The crate has a mass of 20 kg and the coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



SOLUTION

Kinematics: Since the motion of the crate is known, its acceleration **a** will be determined first.

$$a = \frac{dv}{dt} = 0.05(2t) = (0.1t)\ \text{m/s}^2$$

When $t = 5$ s,

$$a = 0.1(5) = 0.5\ \text{m/s}^2 \rightarrow$$

Free-Body Diagram: The kinetic friction $F_f = \mu_k N = 0.2N$ must act to the left to oppose the motion of the crate which is to the right, Fig. a.

Equations of Motion: Here, $a_y = 0$. Thus,

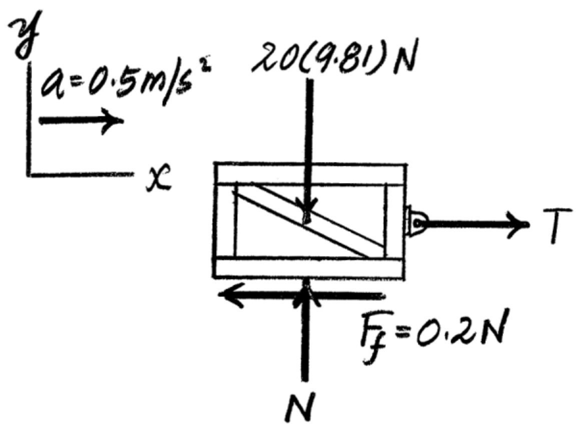
$$+\uparrow \Sigma F_y = ma_y; \qquad N - 20(9.81) = 0$$

$$N = 196.2\ \text{N}$$

Using the results of **N** and **a**,

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad T - 0.2(196.2) = 20(0.5)$$

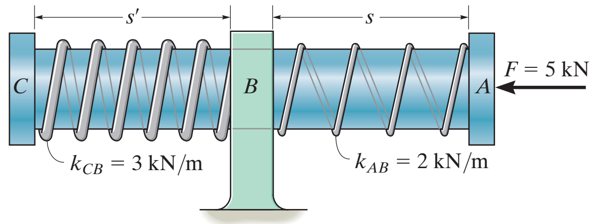
$$T = 49.2\ \text{N}$$ **Ans.**



(a)

13–26.

The 2-kg shaft CA passes through a smooth journal bearing at B. Initially, the springs, which are coiled loosely around the shaft, are unstretched when no force is applied to the shaft. In this position $s = s' = 250$ mm and the shaft is at rest. If a horizontal force of $F = 5$ kN is applied, determine the speed of the shaft at the instant $s = 50$ mm, $s' = 450$ mm. The ends of the springs are attached to the bearing at B and the caps at C and A.



SOLUTION

$F_{CB} = k_{CB}x = 3000x \qquad F_{AB} = k_{AB}x = 2000x$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad 5000 - 3000x - 2000x = 2a$

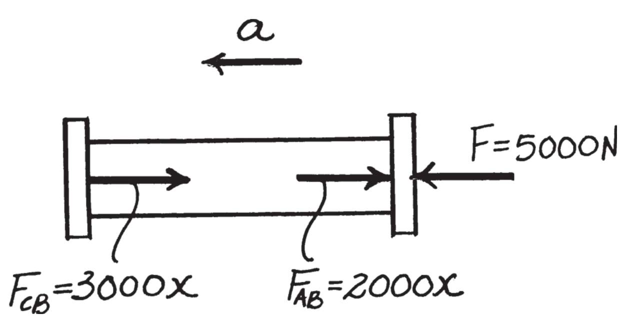
$$2500 - 2500x = a$$

$a\,dx - v\,dv$

$$\int_0^{0.2} (2500 - 2500x)\,dx = \int_0^v v\,dv$$

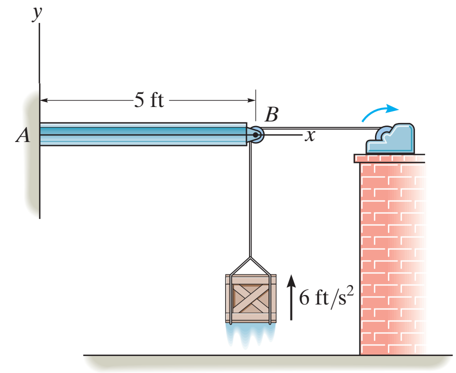
$$2500(0.2) - \left(\frac{2500(0.2)^2}{2}\right) = \frac{v^2}{2}$$

$v = 30$ m/s **Ans.**



13–27.

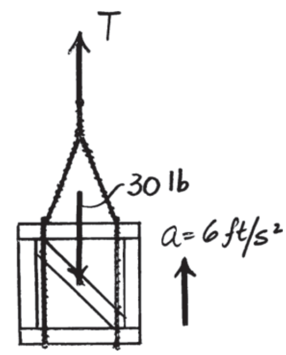
The 30-lb crate is being hoisted upward with a constant acceleration of 6 ft/s^2. If the uniform beam AB has a weight of 200 lb, determine the components of reaction at the fixed support A. Neglect the size and mass of the pulley at B. *Hint:* First find the tension in the cable, then analyze the forces in the beam using statics.



SOLUTION

Crate:

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 30 = \left(\frac{30}{32.2}\right)(6) \qquad T = 35.59 \text{ lb}$$

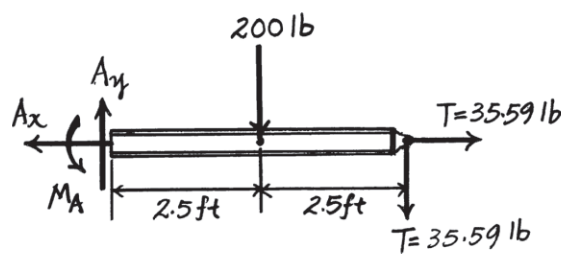


Beam:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -A_x + 35.59 = 0 \qquad A_x = 35.6 \text{ lb} \qquad \textbf{Ans.}$$

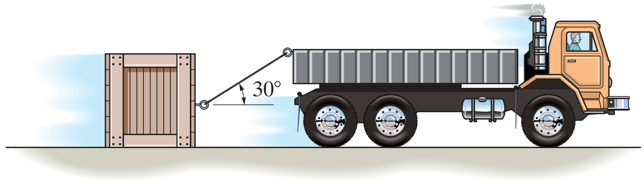
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 200 - 35.59 = 0 \qquad A_y = 236 \text{ lb} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - 200(2.5) - (35.59)(5) = 0 \qquad M_A = 678 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$



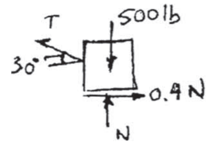
***13–28.**

The driver attempts to tow the crate using a rope that has a tensile strength of 200 lb. If the crate is originally at rest and has a weight of 500 lb, determine the greatest acceleration it can have if the coefficient of static friction between the crate and the road is $\mu_s = 0.4$, and the coefficient of kinetic friction is $\mu_k = 0.3$.



SOLUTION

Equilibrium: In order to slide the crate, the towing force must overcome static friction.

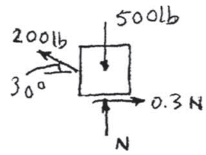


$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -T\cos 30° + 0.4N = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F = 0; \qquad N + T\sin 30° - 500 = 0 \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2) yields:

$$T = 187.6 \text{ lb} \qquad N = 406.2 \text{ lb}$$



Since $T < 200$ lb, the cord will not break at the moment the crate slides.

After the crate begins to slide, the kinetic friction is used for the calculation.

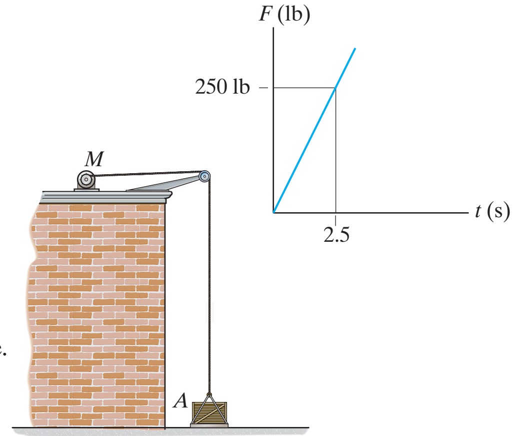
$$+\uparrow \Sigma F_y = ma_y; \qquad N + 200\sin 30° - 500 = 0 \qquad N = 400 \text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 200\cos 30° - 0.3(400) = \frac{500}{32.2}a$$

$$a = 3.43 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

13–29.

The force exerted by the motor on the cable is shown in the graph. Determine the velocity of the 200-lb crate when $t = 2.5$ s.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. *a*.

Equilibrium: For the crate to move, force **F** must overcome the weight of the crate. Thus, the time required to move the crate is given by

$$+\uparrow \Sigma F_y = 0; \qquad 100t - 200 = 0 \qquad t = 2\text{ s}$$

Equation of Motion: For $2\text{ s} < t < 2.5\text{ s}$, $F = \dfrac{250}{2.5}t = (100t)$ lb. By referring to Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad 100t - 200 = \frac{200}{32.2}a$$

$$a = (16.1t - 32.2)\text{ ft/s}^2$$

Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = a\,dt$. For $2\text{ s} \le t < 2.5\text{ s}$, $v = 0$ at $t = 2$ s will be used as the lower integration limit. Thus,

$$(+\uparrow) \qquad \int dv = \int a\,dt$$

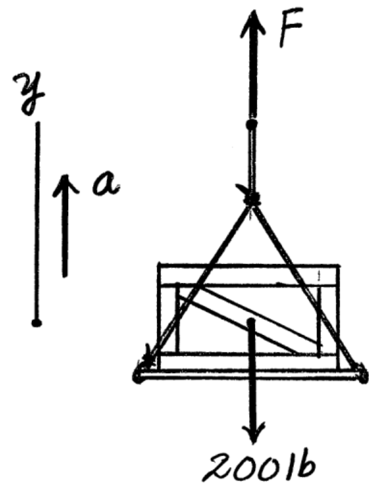
$$\int_0^v dv = \int_{2\text{ s}}^t (16.1t - 32.2)dt$$

$$v = \left(8.05t^2 - 32.2t\right)\Big|_{2\text{ s}}^t$$

$$= \left(8.05t^2 - 32.2t + 32.2\right)\text{ft/s}$$

When $t = 2.5$ s,

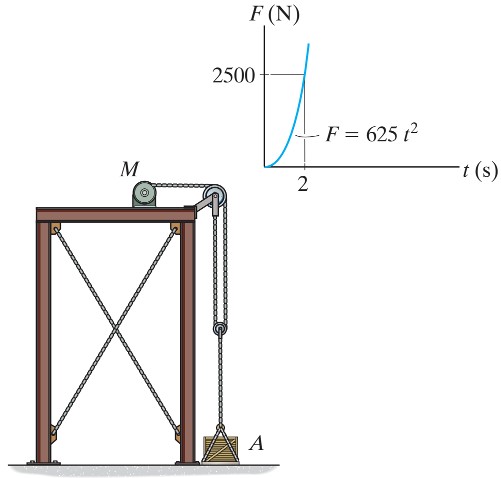
$$v = 8.05(2.5^2) - 32.2(2.5) + 32.2 = 2.01\text{ ft/s}$$ **Ans.**



(a)

13–30.

The force of the motor M on the cable is shown in the graph. Determine the velocity of the 400-kg crate A when $t = 2$ s.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. a.

Equilibrium: For the crate to move, force 2**F** must overcome its weight. Thus, the time required to move the crate is given by

$$+\uparrow \Sigma F_y = 0; \qquad 2(625t^2) - 400(9.81) = 0$$

$$t = 1.772 \text{ s}$$

Equations of Motion: $F = \left(625t^2\right)$ N. By referring to Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2\left(625t^2\right) - 400(9.81) = 400a$$

$$a = (3.125t^2 - 9.81) \text{ m/s}^2$$

Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = a\,dt$. For $1.772 \text{ s} \le t < 2$ s, $v = 0$ at $t = 1.772$ s will be used as the lower integration limit. Thus,

$$(+\uparrow) \qquad \int dv = \int a\,dt$$

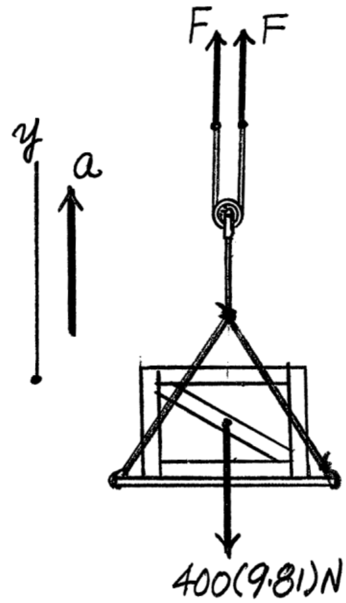
$$\int_0^v dv = \int_{1.772 \text{ s}}^t \left(3.125t^2 - 9.81\right)dt$$

$$v = \left(1.0417t^3 - 9.81t\right)\Big|_{1.772 \text{ s}}^t$$

$$= \left(1.0417t^3 - 9.81t + 11.587\right) \text{ m/s}$$

When $t = 2$ s,

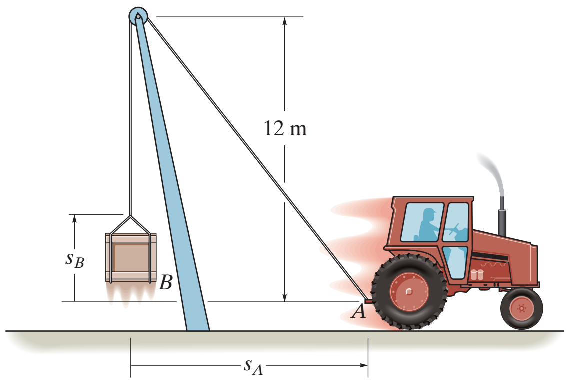
$$v = 1.0417(2^3) - 9.81(2) + 11.587 = 0.301 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

13–31.

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.



SOLUTION

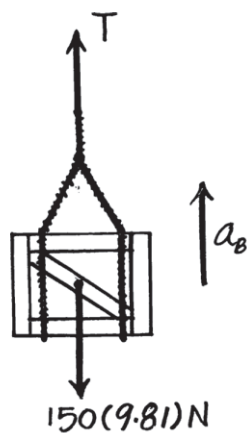
$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-s_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$

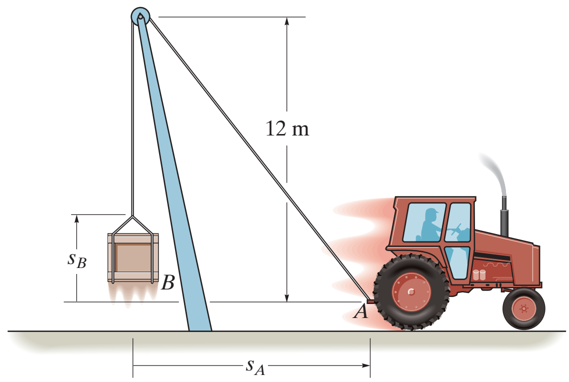


$+\uparrow \Sigma F_y = ma_y;\qquad T - 150(9.81) = 150(1.0487)$

$T = 1.63$ kN **Ans.**

***13–32.**

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



SOLUTION

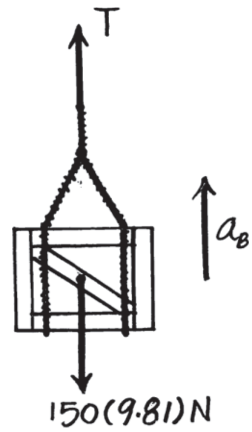
$$12 = s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \frac{1}{2}\left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(2 s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2\,\dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + (5)(3)}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$$

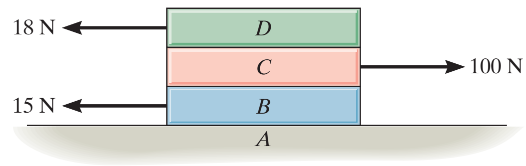


$+\uparrow \Sigma F_y = m a_y;\qquad T - 150(9.81) = 150(2.2025)$

$T = 1.80$ kN **Ans.**

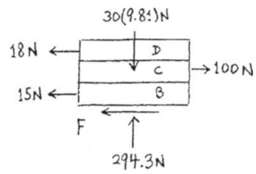
13–33.

Each of the three plates has a mass of 10 kg. If the coefficients of static and kinetic friction at each surface of contact are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively, determine the acceleration of each plate when the three horizontal forces are applied.



SOLUTION

Plates B, C and D



$\xrightarrow{+} \Sigma F_x = 0; \qquad 100 - 15 - 18 - F = 0$

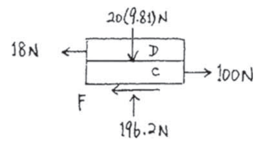
$F = 67 \text{ N}$

$F_{max} = 0.3(294.3) = 88.3 \text{ N} > 67 \text{ N}$

Plate B will not slip.

$a_B = 0$ **Ans.**

Plates D and C



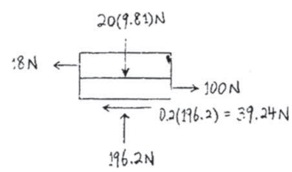
$\xrightarrow{+} \Sigma F_x = 0; \qquad 100 - 18 - F = 0$

$F = 82 \text{ N}$

$F_{max} = 0.3(196.2) = 58.86 \text{ N} < 82\text{N}$

Slipping between B and C.

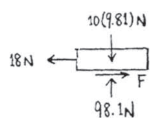
Assume no slipping between D and C,



$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 100 - 39.24 - 18 = 20\, a_x$

$a_x = 2.138 \text{ m/s}^2 \rightarrow$

Check slipping between D and C.



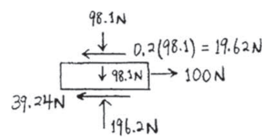
$\xrightarrow{+} \Sigma F_x = m\, a_x; \qquad F - 18 = 10(2.138)$

$F = 39.38 \text{ N}$

$F_{max} = 0.3(98.1) = 29.43 \text{ N} < 39.38 \text{ N}$

Slipping between D and C.

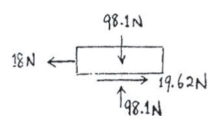
Plate C:



$\xrightarrow{+} \Sigma F_x = m\, a_x; \qquad 100 - 39.24 - 19.62 = 10\, a_c$

$a_c = 4.11 \text{ m/s}^2 \rightarrow$ **Ans.**

Plate D:

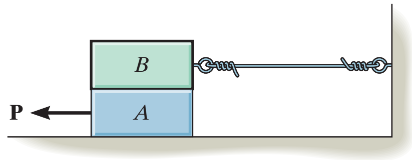


$\xrightarrow{+} \Sigma F_x = m\, a_x; \qquad 19.62 - 18 = 10\, a_D$

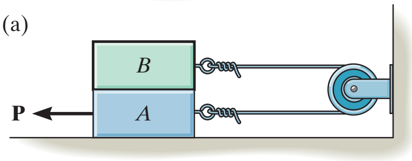
$a_D = 0.162\text{m/s}^2 \rightarrow$ **Ans.**

13–34.

Each of the two blocks has a mass m. The coefficient of kinetic friction at all surfaces of contact is μ. If a horizontal force **P** moves the bottom block, determine the acceleration of the bottom block in each case.



(a)



(b)

SOLUTION

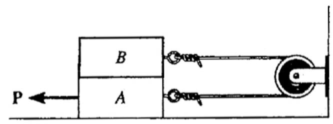
Block A:

(a) $\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - 3\mu mg = m\,a_A$

$$a_A = \frac{P}{m} - 3\mu g \qquad \textbf{Ans.}$$

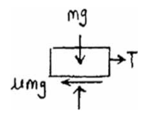
(b) $s_B + s_A = l$

$$a_A = -a_B \qquad \textbf{(1)}$$



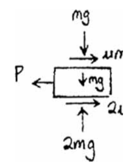
Block A:

$$\overset{+}{\leftarrow} \Sigma F_x = m a_x; \qquad P - T - 3\mu mg = m\,a_A \qquad \textbf{(2)}$$



Block B:

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad \mu mg - T = ma_B \qquad \textbf{(3)}$$



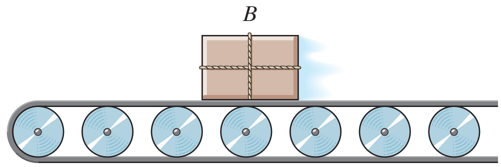
Subtract Eq.(3) from Eq.(2):

$$P - 4\mu mg = m\,(a_A - a_B)$$

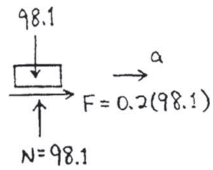
Use Eq.(1); $\qquad a_A = \dfrac{P}{2m} - 2\mu g \qquad$ **Ans.**

13–35.

The conveyor belt is moving at 4 m/s. If the coefficient of static friction between the conveyor and the 10-kg package B is $\mu_s = 0.2$, determine the shortest time the belt can stop so that the package does not slide on the belt.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m\,a_x;$ $\qquad 0.2(98.1) = 10\,a$

$$a = 1.962 \text{ m/s}^2$$

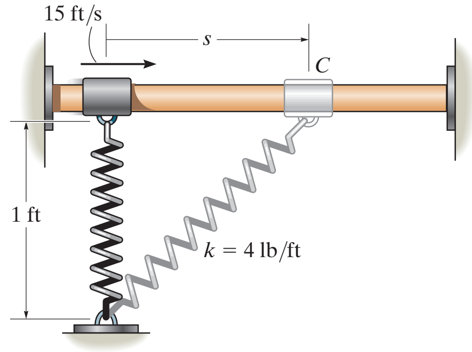
$(\xrightarrow{+})\, v = v_0 + a_c t$

$$4 = 0 + 1.962\,t$$

$$t = 2.04 \text{ s}$$ **Ans.**

***13–36.**

The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



SOLUTION

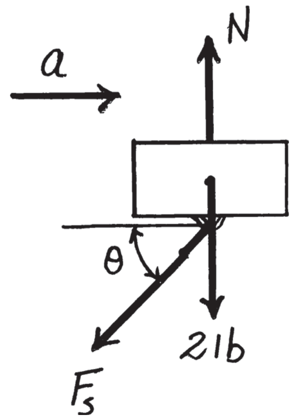
$F_s = kx; \qquad F_s = 4\left(\sqrt{1+s^2}-1\right)$

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -4\left(\sqrt{1+s^2}-1\right)\left(\dfrac{s}{\sqrt{1+s^2}}\right) = \left(\dfrac{2}{32.2}\right)\left(v\dfrac{dv}{ds}\right)$

$$-\int_0^1 \left(4s\,ds - \frac{4s\,ds}{\sqrt{1+s^2}}\right) = \int_{15}^{v}\left(\frac{2}{32.2}\right)v\,dv$$

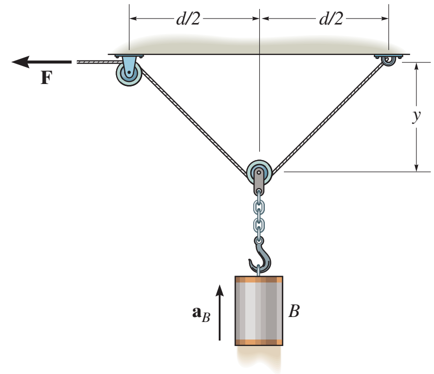
$$-\left[2s^2 - 4\sqrt{1+s^2}\right]_0^1 = \frac{1}{32.2}\left(v^2 - 15^2\right)$$

$v = 14.6$ ft/s **Ans.**



13–37.

Cylinder B has a mass m and is hoisted using the cord and pulley system shown. Determine the magnitude of force $\mathbf{F}$ as a function of the cylinder's vertical position y so that when $\mathbf{F}$ is applied the cylinder rises with a constant acceleration $\mathbf{a}_B$. Neglect the mass of the cord, pulleys, hook and chain.

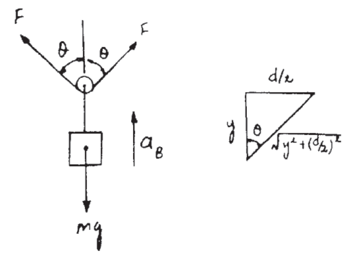


SOLUTION

$+\uparrow \Sigma F_y = ma_y;\qquad 2F\cos\theta - mg = ma_B \qquad \text{where } \cos\theta = \dfrac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}$

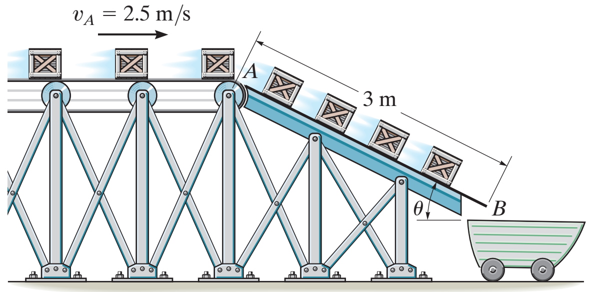
$$2F\left(\frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}\right) - mg = ma_B$$

$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y} \qquad \textbf{Ans.}$$



13–38.

The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5$ m/s, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs. Take $\theta = 30°$.



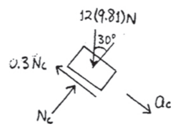
SOLUTION

$\nearrow + \Sigma F_y = ma_y;$ $\quad N_C - 12(9.81)\cos 30° = 0$

$N_C = 101.95$ N

$+\searrow \Sigma F_x = ma_x;$ $\quad 12(9.81)\sin 30° - 0.3(101.95) = 12\,a_C$

$a_C = 2.356 \text{ m/s}^2$



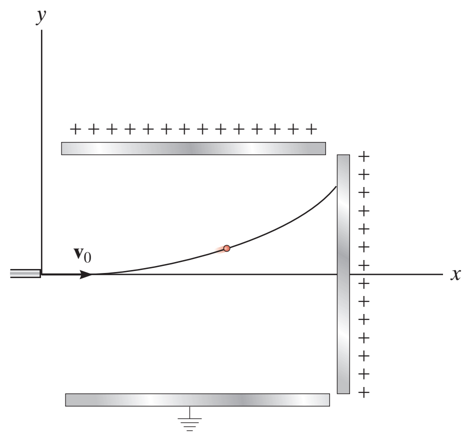
$(+\searrow)$ $\quad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$

$v_B^2 = (2.5)^2 + 2(2.356)(3 - 0)$

$v_B = 4.5152 = 4.52$ m/s **Ans.**

13–39.

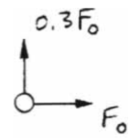
An electron of mass m is discharged with an initial horizontal velocity of $\mathbf{v}_0$. If it is subjected to two fields of force for which $F_x = F_0$ and $F_y = 0.3F_0$, where F_0 is constant, determine the equation of the path, and the speed of the electron at any time t.



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 = ma_x$

$+\uparrow \Sigma F_y = ma_y; \qquad 0.3\,F_0 = ma_y$



Thus,

$$\int_{v_0}^{v_x} dv_x = \int_0^t \frac{F_0}{m}\,dt$$

$$v_x = \frac{F_0}{m}t + v_0$$

$$\int_0^{v_y} dv_y = \int_0^t \frac{0.3F_0}{m}\,dt \qquad v_y = \frac{0.3F_0}{m}t$$

$$v = \sqrt{\left(\frac{F_0}{m}t + v_0\right)^2 + \left(\frac{0.3F_0}{m}t\right)^2}$$

$$= \frac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0tmv_0 + m^2v_0^2} \qquad \textbf{Ans.}$$

$$\int_0^x dx = \int_0^t \left(\frac{F_0}{m}t + v_0\right) dt$$

$$x = \frac{F_0 t^2}{2m} + v_0\,t$$

$$\int_0^y dy = \int_0^t \frac{0.3F_0}{m}t\,dt$$

$$y = \frac{0.3F_0 t^2}{2m}$$

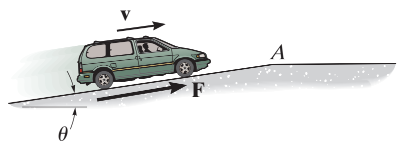
$$t = \left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$

$$x = \frac{F_0}{2m}\left(\frac{2m}{0.3F_0}\right)y + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$

$$x = \frac{y}{0.3} + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}} \qquad \textbf{Ans.}$$

***13–40.**

The engine of the van produces a constant driving traction force **F** at the wheels as it ascends the slope at a constant velocity **v**. Determine the acceleration of the van when it passes point A and begins to travel on a level road, provided that it maintains the *same* traction force.



SOLUTION

Free-Body Diagram: The free-body diagrams of the van up the slope and on the level road are shown in Figs. a and b, respectively.

Equations of Motion: Since the van is travelling up the slope with a constant velocity, its acceleration is $a = 0$. By referring to Fig. a,

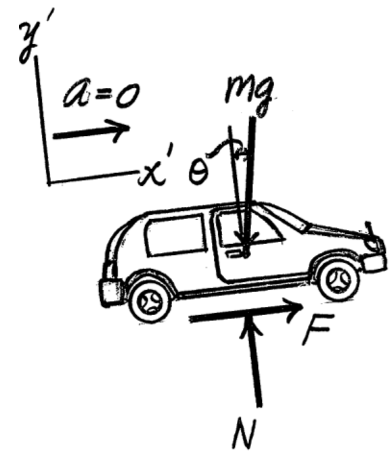
$$\Sigma F_{x'} = ma_{x'}; \qquad F - mg\sin\theta = m(0)$$

$$F = mg\sin\theta$$

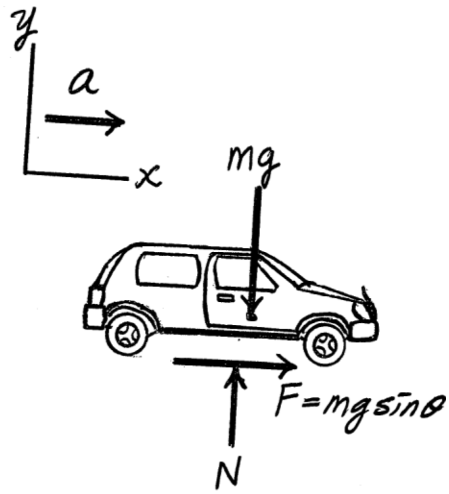
Since the van maintains the same tractive force **F** when it is on level road, from Fig. b,

$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \qquad mg\sin\theta = ma$$

$$a = g\sin\theta \qquad \textbf{Ans.}$$



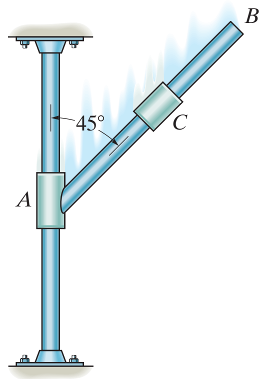
(a)



(b)

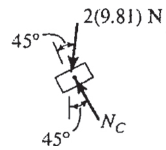
13–41.

The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if (a) the shaft is fixed from moving, (b) collar A, which is fixed to shaft AB, moves downward at constant velocity along the vertical rod, and (c) collar A is subjected to a downward acceleration of $2\ \text{m/s}^2$. In all cases, the collar moves in the plane.



SOLUTION

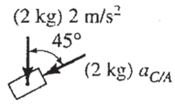
(a) $+\swarrow \Sigma F_{x'} = ma_{x'};\qquad 2(9.81)\sin 45^\circ = 2a_C \qquad a_C = 6.94\ \text{m/s}^2$



(b) From part (a) $\mathbf{a}_{C/A} = 6.94\ \text{m/s}^2$

$$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A} \qquad \text{Where } \mathbf{a}_A = 0$$

$$= 6.94\ \text{m/s}^2$$



(c)

$$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A}$$

$$= \underset{\downarrow}{2} + a_{C/A}\swarrow \qquad \textbf{(1)}$$

$$+\swarrow \Sigma F_{x'} = ma_{x'};\quad 2(9.81)\sin 45^\circ = 2(2\cos 45^\circ + a_{C/A}) \qquad a_{C/A} = 5.5225\ \text{m/s}^2 \swarrow$$

From Eq.(1)

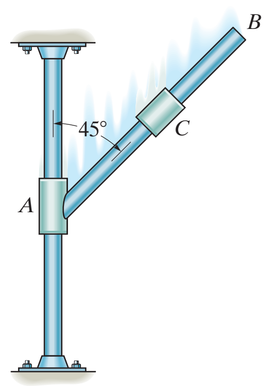
$$\mathbf{a}_C = \underset{\downarrow}{2} + \underset{\swarrow}{5.5225} = \underset{\leftarrow}{3.905} + \underset{\downarrow}{5.905}$$

$$a_C = \sqrt{3.905^2 + 5.905^2} = 7.08\ \text{m/s}^2 \qquad \textbf{Ans.}$$

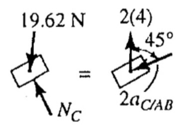
$$\theta = \tan^{-1}\frac{5.905}{3.905} = 56.5^\circ\ \theta \qquad \textbf{Ans.}$$

13–42.

The 2-kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of collar C if collar A is subjected to an upward acceleration of $4\ \text{m/s}^2$.



SOLUTION



$\xleftarrow{+}\ \Sigma F_x = ma_x;\qquad N \sin 45° = 2a_{C/AB} \sin 45°$

$$N = 2\,a_{C/AB}$$

$+\uparrow \Sigma F_y = ma_y;\qquad N\cos 45° - 19.62 = 2(4) - 2a_{C/AB}\cos 45°$

$$a_{C/AB} = 9.76514$$

$$\mathbf{a}_C = \mathbf{a}_{AB} + \mathbf{a}_{C/AB}$$

$$(a_C)_x = 0 + 9.76514 \sin 45° = 6.905 \leftarrow$$

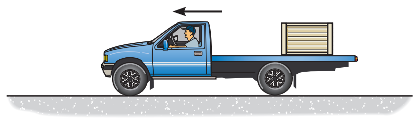
$$(a_C)_y = 4 - 9.76514 \cos 45° = 2.905 \downarrow$$

$$a_C = \sqrt{(6.905)^2 + (2.905)^2} = 7.49\ \text{m/s}^2$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{2.905}{6.905}\right) = 22.8°\ \theta$$ **Ans.**

13–43.

The coefficient of static friction between the 200-kg crate and the flat bed of the truck is $\mu_s = 0.3$. Determine the shortest time for the truck to reach a speed of 60 km/h, starting from rest with constant acceleration, so that the crate does not slip.



SOLUTION

Free-Body Diagram: When the crate accelerates with the truck, the frictional force F_f develops. Since the crate is required to be on the verge of slipping, $F_f = \mu_s N = 0.3N$.

Equations of Motion: Here, $a_y = 0$. By referring to Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 200(9.81) = 200(0)$$

$$N = 1962 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -0.3(1962) = 200(-a)$$

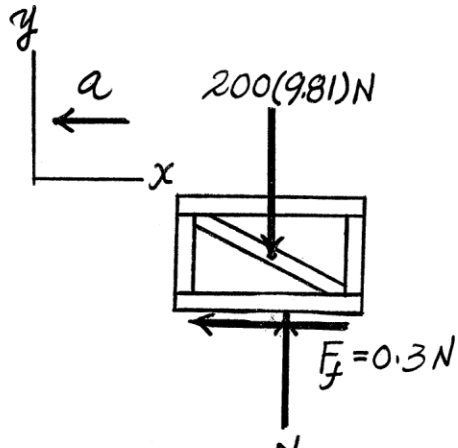
$$a = 2.943 \text{ m/s}^2 \leftarrow$$

Kinematics: The final velocity of the truck is $v = \left(60\dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1000 \text{ m}}{1 \text{ km}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right) =$ 16.67 m/s. Since the acceleration of the truck is constant,

$$(\overset{+}{\leftarrow}) \qquad v = v_0 + a_c t$$

$$16.67 = 0 + 2.943t$$

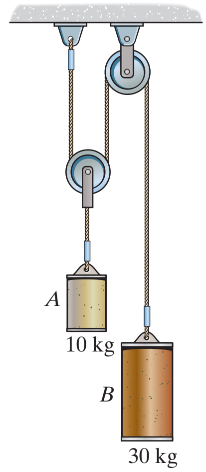
$$t = 5.66 \text{ s}$$ **Ans.**



(a)

***13–44.**

When the blocks are released, determine their acceleration and the tension of the cable. Neglect the mass of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of blocks A and B are shown in Figs. b and c, respectively. Here, $\mathbf{a}_A$ and $\mathbf{a}_B$ are assumed to be directed downwards so that they are consistent with the positive sense of position coordinates s_A and s_B of blocks A and B, Fig. a. Since the cable passes over the smooth pulleys, the tension in the cable remains constant throughout.

Equations of Motion: By referring to Figs. b and c,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 10(9.81) = -10a_A \tag{1}$$

and

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 30(9.81) = -30a_B \tag{2}$$

Kinematics: We can express the length of the cable in terms of s_A and s_B by referring to Fig. a.

$$2s_A + s_B = l$$

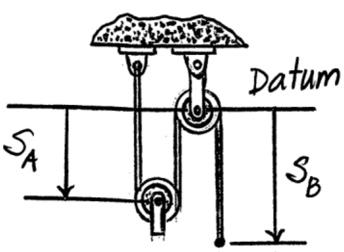
The second derivative of the above equation gives

$$2a_A + a_B = 0 \tag{3}$$

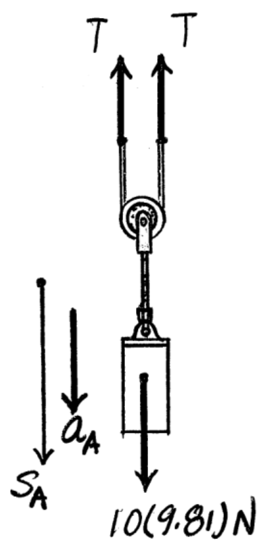
Solving Eqs. (1), (2), and (3) yields

$$a_A = -3.773\ \text{m/s}^2 = 3.77\ \text{m/s}^2 \uparrow \qquad a_B = 7.546\ \text{m/s}^2 = 7.55\ \text{m/s}^2 \downarrow \qquad \textbf{Ans.}$$

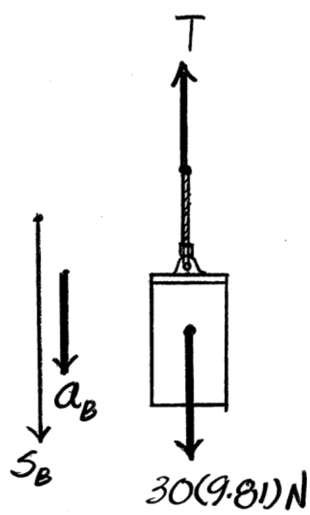
$$T = 67.92\ \text{N} = 67.9\ \text{N} \qquad \textbf{Ans.}$$



(a)



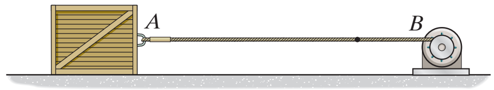
(b)



(c)

13–45.

If the force exerted on cable AB by the motor is $F = (100t^{3/2})$ N, where t is in seconds, determine the 50-kg crate's velocity when $t = 5$ s. The coefficients of static and kinetic friction between the crate and the ground are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. Initially the crate is at rest.



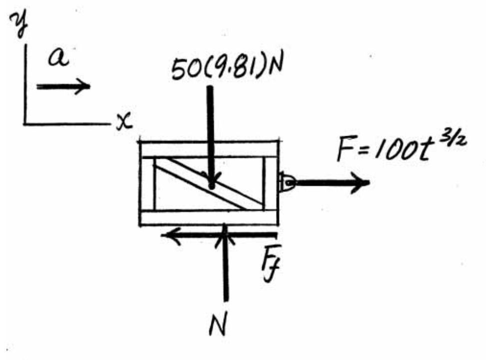
SOLUTION

Free-Body Diagram: The frictional force $\mathbf{F}_f$ is required to act to the left to oppose the motion of the crate which is to the right.

Equations of Motion: Here, $a_y = 0$. Thus,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 50(9.81) = 50(0)$$

$$N = 490.5 \text{ N}$$



(a)

Realizing that $F_f = \mu_k N = 0.3(490.5) = 147.15$ N,

$$+\uparrow \Sigma F_x = ma_x; \qquad 100t^{3/2} - 147.15 = 50a$$

$$a = \left(2t^{3/2} - 2.943\right) \text{ m/s}$$

Equilibrium: For the crate to move, force **F** must overcome the static friction of $F_f = \mu_s N = 0.4(490.5) = 196.2$ N. Thus, the time required to cause the crate to be on the verge of moving can be obtained from.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 100t^{3/2} - 196.2 = 0$$

$$t = 1.567 \text{ s}$$

Kinematics: Using the result of **a** and integrating the kinematic equation $dv = a\,dt$ with the initial condition $v = 0$ at $t = 1.567$ as the lower integration limit,

$$(\overset{+}{\rightarrow}) \qquad \int dv = \int a\,dt$$

$$\int_0^v dv = \int_{1.567 \text{ s}}^t \left(2t^{3/2} - 2.943\right)dt$$

$$v = \left(0.8t^{5/2} - 2.943t\right)\Big|_{1.567 \text{ s}}^t$$

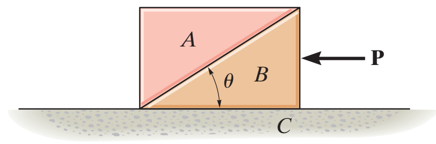
$$v = \left(0.8t^{5/2} - 2.943t + 2.152\right) \text{ m/s}$$

When $t = 5$ s,

$$v = 0.8(5)^{5/2} - 2.943(5) + 2.152 = 32.16 \text{ ft/s} = 32.2 \text{ ft/s} \qquad \textbf{Ans.}$$

13–46.

Blocks A and B each have a mass m. Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that A will not move relative to B. All surfaces are smooth.

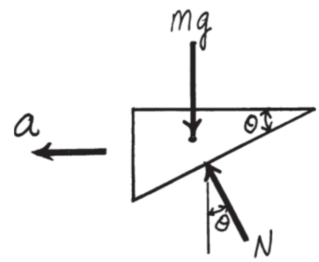


SOLUTION

Require

$$a_A = a_B = a$$

Block A:

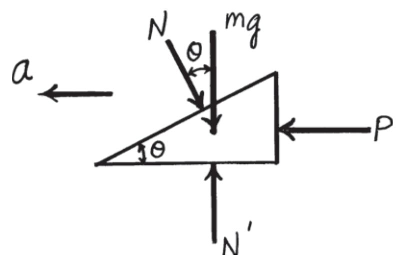


$$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - mg = 0$$

$$\xleftarrow{+} \Sigma F_x = ma_x; \qquad N\sin\theta = ma$$

$$a = g\tan\theta$$

Block B:



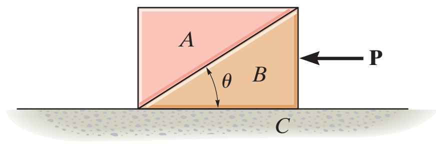
$$\xleftarrow{+} \Sigma F_x = ma_x; \qquad P - N\sin\theta = ma$$

$$P - mg\tan\theta = mg\tan\theta$$

$$P = 2mg\tan\theta \qquad \textbf{Ans.}$$

13–47.

Blocks A and B each have a mass m. Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that A will not slip on B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.

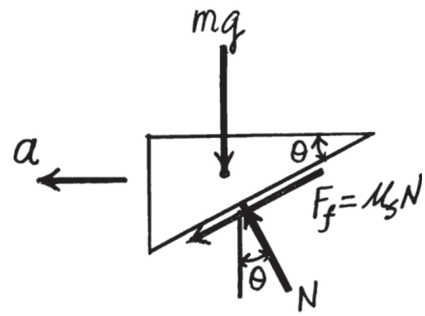


SOLUTION

Require

$a_A = a_B = a$

Block A:



$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - \mu_s N\sin\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad N\sin\theta + \mu_s N\cos\theta = ma$

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

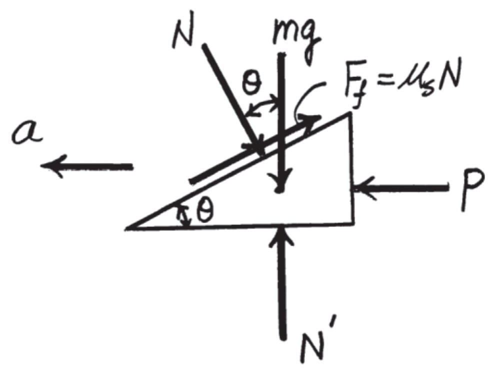
$$a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

Block B:

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - \mu_s N\cos\theta - N\sin\theta = ma$

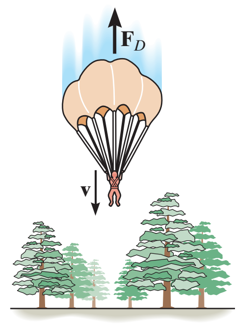
$$P - mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$ **Ans.**



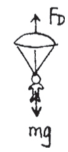
***13–48.**

A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.



SOLUTION

$$+\downarrow \Sigma F_z = m\,a_z; \qquad mg - kv^2 = m\frac{dv}{dt}$$



$$m\int_0^v \frac{m\,dv}{(mg - kv^2)} = \int_0^t dt$$

$$\frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k} - v^2} = t$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right)\ln\left[\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}\right]_0^v = t$$

$$\frac{k}{m}t\left(2\sqrt{\frac{mg}{k}}\right) = \ln\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$e^{2t\sqrt{\frac{mg}{k}}} = \frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

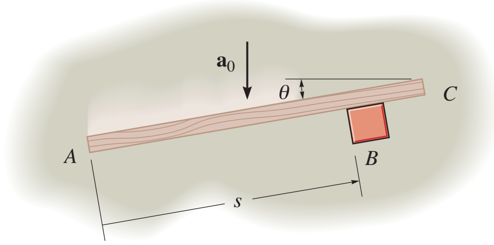
$$\sqrt{\frac{mg}{k}}\,e^{2t\sqrt{\frac{mg}{k}}} - v\,e^{2t\sqrt{\frac{mg}{k}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{mg}{k}}} - 1}{e^{2t\sqrt{\frac{mg}{k}}} + 1}\right] \qquad \textbf{Ans.}$$

When $t \rightarrow \infty$ $\qquad v_t = \sqrt{\frac{mg}{k}}$ **Ans.**

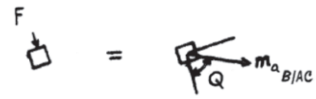
13–49.

The smooth block B of negligible size has a mass m and rests on the horizontal plane. If the board AC pushes on the block at an angle θ with a constant acceleration $\mathbf{a}_0$, determine the velocity of the block along the board and the distance s the block moves along the board as a function of time t. The block starts from rest when $s = 0$, $t = 0$.



SOLUTION

$\nearrow + \Sigma F_x = m\,a_x;\qquad 0 = m\,a_B \sin\phi$



$$\mathbf{a}_B = \mathbf{a}_{AC} + \mathbf{a}_{B/AC}$$

$$\mathbf{a}_B = \mathbf{a_0} + \mathbf{a}_{B/AC}$$

$\nearrow +\qquad a_B\ \sin\phi = -a_0\ \sin\theta + a_{B/AC}$

Thus,

$$0 = m(-a_0 \sin\theta + a_{B/AC})$$

$$a_{B/AC} = a_0\ \sin\theta$$

$$\int_0^{v_{B/AC}} dv_{B/AC} = \int_0^t a_0\ \sin\theta\ dt$$

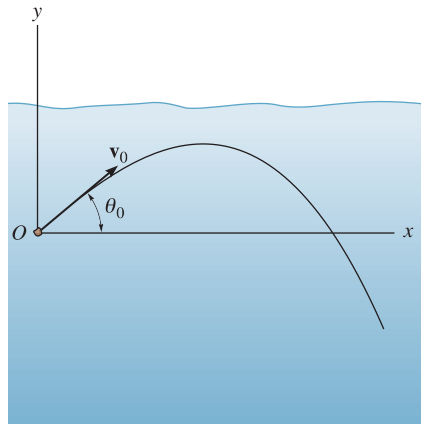
$$v_{B/AC} = a_0 \sin\theta\ t \qquad \textbf{Ans.}$$

$$s_{B/AC} = s = \int_0^t a_0\ \sin\theta\ t\ dt$$

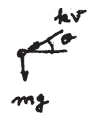
$$s = \frac{1}{2} a_0 \sin\theta\ t^2 \qquad \textbf{Ans.}$$

13–50.

A projectile of mass m is fired into a liquid at an angle θ_0 with an initial velocity $\mathbf{v}_0$ as shown. If the liquid develops a frictional or drag resistance on the projectile which is proportional to its velocity, i.e., $F = kv$, where k is a constant, determine the x and y components of its position at any instant. Also, what is the maximum distance x_{max} that it travels?



SOLUTION



$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -kv\cos\theta = m\,a_x$

$+\uparrow \Sigma F_y = m\,a_y; \qquad -mg - kv\sin\theta = m\,a_y$

or

$$-k\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$-mg - k\frac{dy}{dt} = m\frac{d^2y}{dt^2}$$

Integrating yields

In $\dot{x} = \dfrac{-k}{m}t + C_1$

In $(\dot{y} + \dfrac{mg}{k}) = \dfrac{k}{m}t + C_2$

When $t = 0$, $\dot{x} = v_0\cos\theta_0$, $\qquad \dot{y} = v_0\sin\theta_0$

$$\dot{x} = v_0\cos\theta_0\, e^{-(k/m)t}$$

$$\dot{y} = -\frac{mg}{k} + (v_0\sin\theta_0 + \frac{mg}{k})e^{-(k/m)t}$$

Integrating again,

$$x = \frac{-mv_0}{k}\cos\theta_0\, e^{-(k/m)t} + C_3$$

$$y = -\frac{mg}{k}t - (v_0\sin\theta_0 + \frac{mg}{k})(\frac{m}{k})e^{-(k/m)t}$$

When $t = 0$, $x = y = 0$, thus

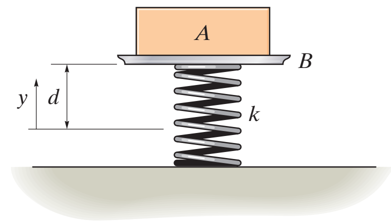
$$x = \frac{mv_0}{k}\cos\theta_0(1 - e^{-(k/m)t})$$ **Ans.**

$$y = -\frac{mgt}{k} + \frac{m}{k}(v_0\sin\theta_0 + \frac{mg}{k})(1 - e^{-(k/m)t})$$ **Ans.**

As $t \rightarrow \infty$

$$x_{max} = \frac{mv_0\cos\theta_0}{k}$$ **Ans.**

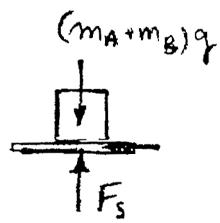
13–51.



The block A has a mass m_A and rests on the pan B, which has a mass m_B. Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.

SOLUTION

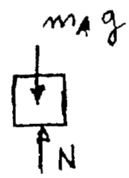
For Equilibrium



$$+\uparrow \Sigma F_y = ma_y; \qquad F_s = (m_A + m_B)g$$

$$y_{eq} = \frac{F_s}{k} = \frac{(m_A + m_B)g}{k}$$

Block:



$$+\uparrow \Sigma F_y = ma_y; \qquad -m_A g + N = m_A a$$

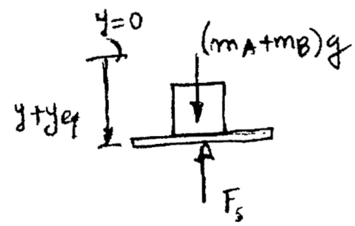
Block and pan

$$+\uparrow \Sigma F_y = ma_y; \qquad -(m_A + m_B)g + k(y_{eq} + y) = (m_A + m_B)a$$

Thus,

$$-(m_A + m_B)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = (m_A + m_B)\left(\frac{-m_A g + N}{m_A}\right)$$

Require $y = d, N = 0$



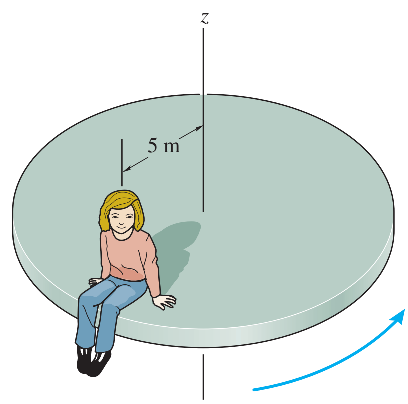
$$kd = -(m_A + m_B)g$$

Since d is downward,

$$d = \frac{(m_A + m_B)g}{k} \qquad \textbf{Ans.}$$

***13–52.**

A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.



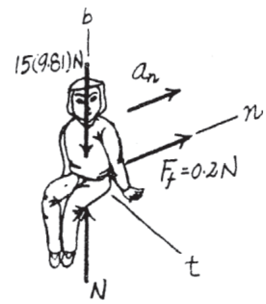
SOLUTION

Equation of Motion: Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15 \text{ N}$$

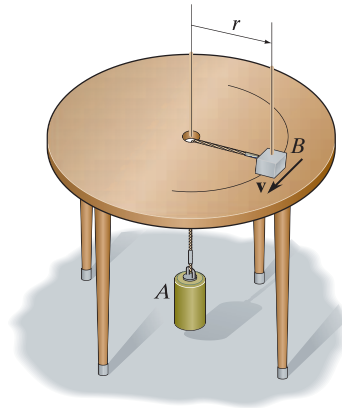
$$\Sigma F_n = ma_n; \qquad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$$

$$v = 3.13 \text{ m/s}$$ **Ans.**



13–53.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block is given a speed of $v = 10\,\text{m/s}$, determine the radius r of the circular path along which it travels.



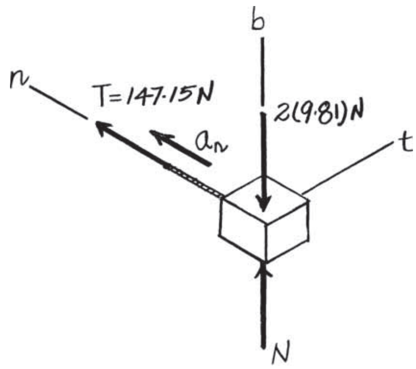
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{ N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{10^2}{r}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{10^2}{r}\right)$$

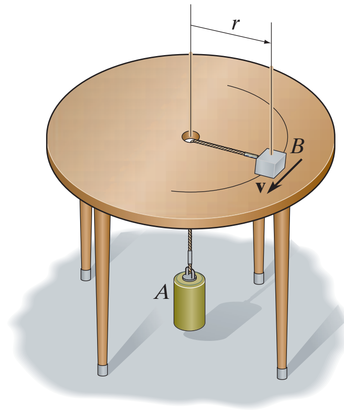
$$r = 1.36\text{ m} \qquad \textbf{Ans.}$$



(a)

13–54.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5\,\text{m}$, determine the speed of the block.



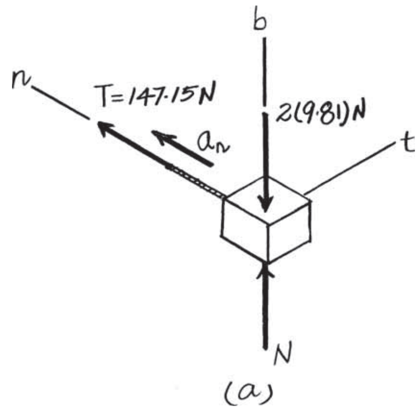
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{ N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{r} = \dfrac{v^2}{1.5}$ and referring to Fig. (a),

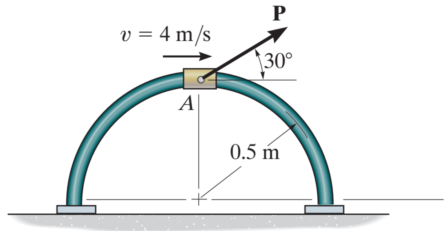
$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{v^2}{1.5}\right)$$

$$v = 10.5\text{ m/s} \qquad \textbf{Ans.}$$

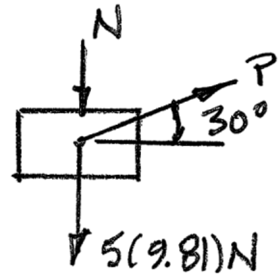


13–55.

The 5-kg collar A is sliding around a smooth vertical guide rod. At the instant shown, the speed of the collar is $v = 4\text{ m/s}$, which is increasing at 3 m/s^2. Determine the normal reaction of the guide rod on the collar, and force $\mathbf{P}$ at this instant.



SOLUTION



$\xrightarrow{+} \Sigma F_t = ma_t;$ $\quad P \cos 30° = 5(3)$

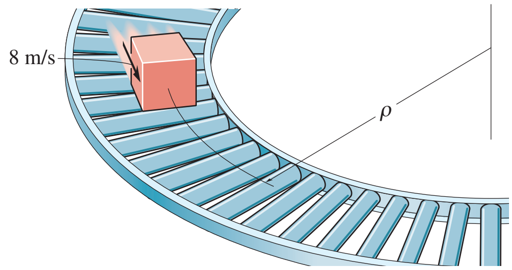
$P = 17.32\text{ N} = 17.3\text{ N}$ **Ans.**

$+\downarrow \Sigma F_n = ma_n;$ $\quad N + 5(9.81) - 17.32\ \sin 30° = 5\left(\frac{4^2}{0.5}\right)$

$N = 119.61\text{ N} = 120\text{ N}\downarrow$ **Ans.**

***13–56.**

Cartons having a mass of 5 kg are required to move along the assembly line at a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.



SOLUTION

$+\uparrow \Sigma F_b = m\,a_b; \qquad N - W = 0$

$$N = W$$

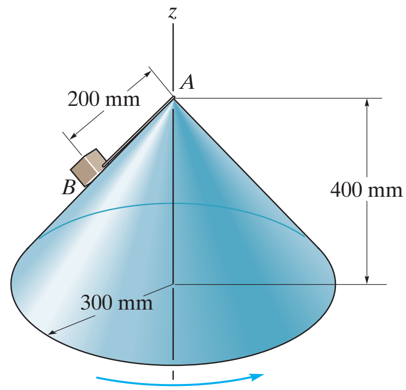
$$F_x = 0.7W$$

$\overset{+}{\leftarrow} \Sigma F_n = m a_n; \qquad 0.7W = \frac{W}{9.81}\left(\frac{8^2}{\rho}\right)$

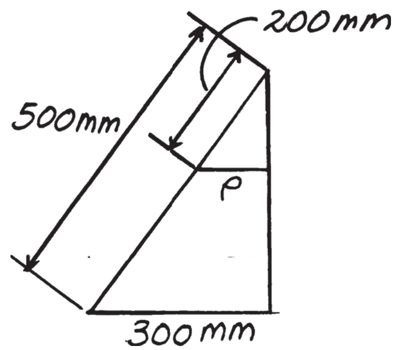
$$\rho = 9.32 \text{ m}$$ **Ans.**

13–57.

The block B, having a mass of 0.2 kg, is attached to the vertex A of the right circular cone using a light cord. If the block has a speed of 0.5 m/s around the cone, determine the tension in the cord and the reaction which the cone exerts on the block and the effect of friction.



SOLUTION



$$\frac{\rho}{200} = \frac{300}{500}; \qquad \rho = 120 \text{ mm} = 0.120 \text{ m}$$

$$+\nearrow\Sigma F_y = ma_y; \qquad T - 0.2(9.81)\left(\frac{4}{5}\right) = \left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{3}{5}\right)$$

$$T = 1.82 \text{ N} \qquad \textbf{Ans.}$$

$$+\nwarrow\Sigma F_x = ma_x; \qquad N_B - 0.2(9.81)\left(\frac{3}{5}\right) = -\left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{4}{5}\right)$$

$$N_B = 0.844 \text{ N} \qquad \textbf{Ans.}$$

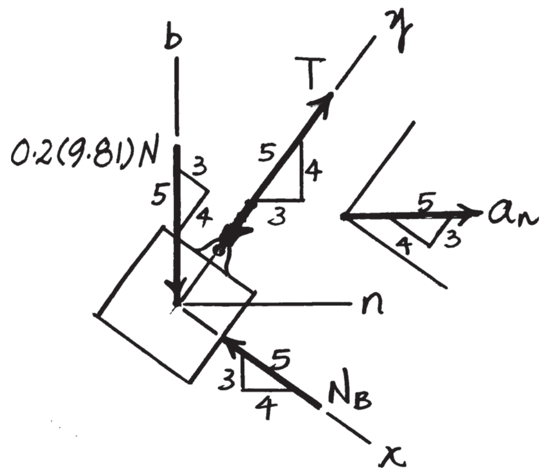
Also,

$$\overset{+}{\rightarrow}\Sigma F_n = ma_n; \qquad T\left(\frac{3}{5}\right) - N_B\left(\frac{4}{5}\right) = 0.2\left(\frac{(0.5)^2}{0.120}\right)$$

$$+\uparrow\Sigma F_b = 0; \qquad T\left(\frac{4}{5}\right) + N_B\left(\frac{3}{5}\right) - 0.2(9.81) = 0$$

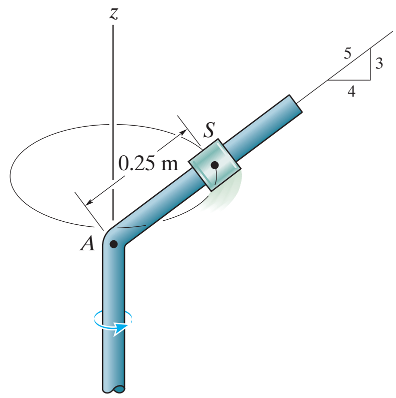
$$T = 1.82 \text{ N} \qquad \textbf{Ans.}$$

$$N_B = 0.844 \text{ N} \qquad \textbf{Ans.}$$



13–58.

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.



SOLUTION

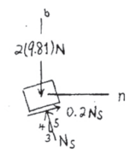
$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2\text{ m}$$

$\overset{+}{\leftarrow}\Sigma F_n = m\,a_n;$ $$N_s\left(\frac{3}{5}\right) - 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

$+\uparrow\Sigma F_b = m\,a_b;$ $$N_s\left(\frac{4}{5}\right) + 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

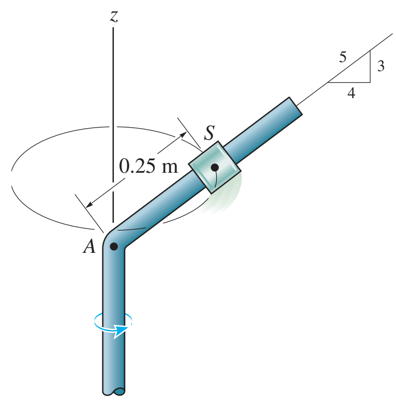
$$N_s = 21.3\text{ N}$$

$$v = 0.969\text{ m/s}$$ **Ans.**



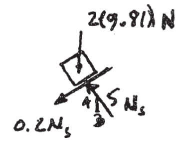
13–59.

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.



SOLUTION

$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$



$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $\quad N_s\left(\frac{3}{5}\right) + 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$

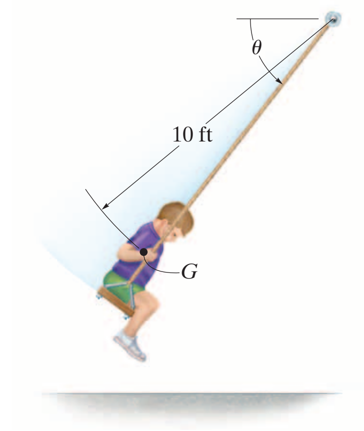
$+\uparrow \Sigma F_b = m\,a_b;$ $\quad N_s\left(\frac{4}{5}\right) - 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$

$$N_s = 28.85 \text{ N}$$

$$v = 1.48 \text{ m/s}$$ **Ans.**

***13–60.**

At the instant $\theta = 60°$, the boy's center of mass G has a downward speed $v_G = 15\text{ ft/s}$. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



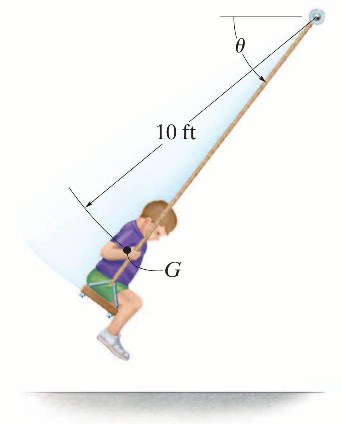
SOLUTION

$+\searrow \Sigma F_t = ma_t;\qquad 60 \cos 60° = \dfrac{60}{32.2} a_t \qquad a_t = 16.1\text{ ft/s}^2$ **Ans.**

$\nearrow + \Sigma F_n = ma_n;\qquad 2T - 60 \sin 60° = \dfrac{60}{32.2}\left(\dfrac{15^2}{10}\right) \qquad T = 46.9\text{ lb}$ **Ans.**

13–61.

At the instant $\theta = 60°$, the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = 90°$. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



SOLUTION

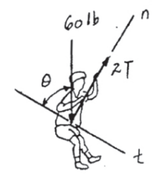
$$+\searrow \Sigma_t = ma_t; \qquad 60 \cos\theta = \frac{60}{32.2} a_t \qquad a_t = 32.2 \cos\theta$$

$$\nearrow + \Sigma F_n = ma_n; \qquad 2T - 60 \sin\theta = \frac{60}{32.2}\left(\frac{v^2}{10}\right) \qquad \textbf{(1)}$$

$$v\, dv = a\, ds \qquad \text{however } ds = 10 d\theta$$

$$\int_0^v v\, dv = \int_{60°}^{90°} 322 \cos\theta\, d\theta$$

$$v = 9.289 \text{ ft/s} \qquad \textbf{Ans.}$$

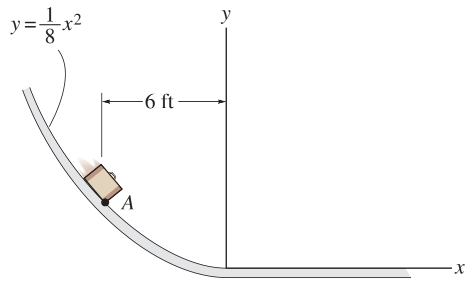


From Eq. (1)

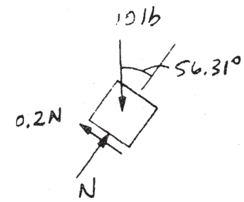
$$2T - 60 \sin 90° = \frac{60}{32.2}\left(\frac{9.289^2}{10}\right) \qquad T = 38.0 \text{ lb} \qquad \textbf{Ans.}$$

13–62.

The 10-lb suitcase slides down the curved ramp for which the coefficient of kinetic friction is $\mu_k = 0.2$. If at the instant it reaches point A it has a speed of 5 ft/s, determine the normal force on the suitcase and the rate of increase of its speed.



SOLUTION



$$\nu = \frac{1}{8}x^2$$

$$\frac{dy}{dx} = \tan\theta = \frac{1}{4}x\bigg|_{x=-6} = -1.5 \qquad \theta = -56.31^\circ$$

$$\frac{d^2y}{dx^2} = \frac{1}{4}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (-1.5)^2\right]^{\frac{3}{2}}}{\left|\frac{1}{4}\right|} = 23.436 \text{ ft}$$

$+\nearrow\Sigma F_n = ma_n;\qquad N - 10\cos 56.31^\circ = \left(\frac{10}{32.2}\right)\left(\frac{(5)^2}{23.436}\right)$

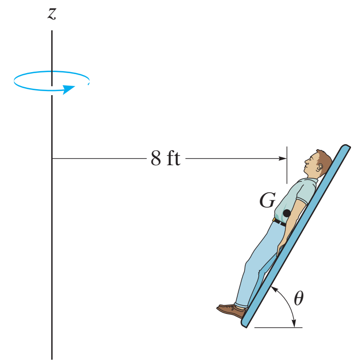
$N = 5.8783 = 5.88 \text{ lb}$ **Ans.**

$+\searrow\Sigma F_t = ma_t;\qquad -0.2(5.8783) + 10\sin 56.31^\circ = \left(\frac{10}{32.2}\right)a_t$

$a_t = 23.0 \text{ ft/s}^2$ **Ans.**

13–63.

The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60°$.



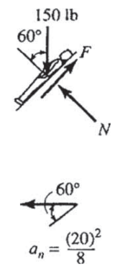
SOLUTION

$$+\nwarrow\sum F_y = m(a_n)_y; \qquad N - 150\cos 60° = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\sin 60°$$

$$N = 277 \text{ lb} \qquad \textbf{Ans.}$$

$$+\swarrow\sum F_x = m(a_n)_x; \qquad -F + 150\sin 60° = \frac{150}{32.2}\left(\frac{20^2}{8}\right)\cos 60°$$

$$F = 13.4 \text{ lb} \qquad \textbf{Ans.}$$

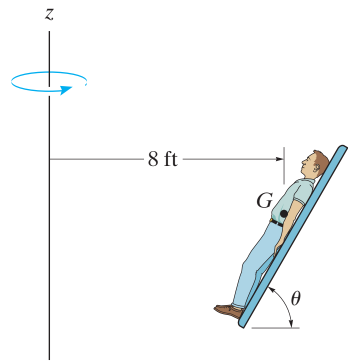


Note: No slipping occurs

Since $\mu_s N = 138.4 \text{ lb} > 13.4 \text{ lb}$

***13–64.**

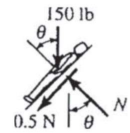
The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip off.



SOLUTION

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;\qquad 0.5N\cos\theta + N\sin\theta = \dfrac{150}{32.2}\left(\dfrac{(30)^2}{8}\right)$

$+\uparrow \Sigma F_b = 0;\qquad -150 + N\cos\theta - 0.5\,N\sin\theta = 0$



$$N = \frac{150}{\cos\theta - 0.5\sin\theta}$$

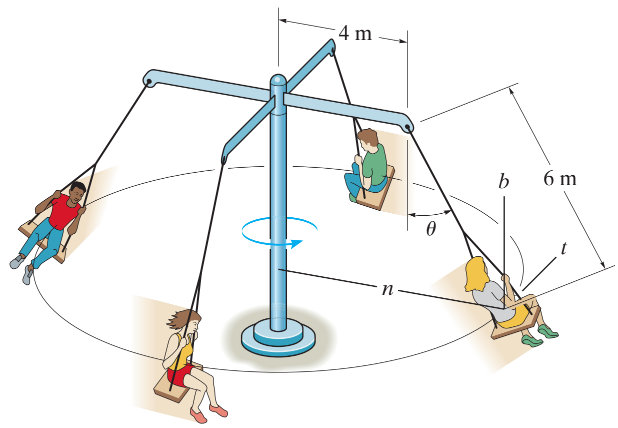
$$\frac{(0.5\cos\theta + \sin\theta)150}{(\cos\theta - 0.5\sin\theta)} = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

$$0.5\cos\theta + \sin\theta = 3.49378\cos\theta - 1.74689\sin\theta$$

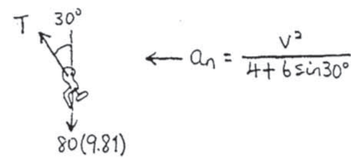
$\theta = 47.5°$ **Ans.**

13–65.

Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30^\circ$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the n, t, and b directions which the chair exerts on a 50-kg passenger during the motion?



SOLUTION



$\xleftarrow{+}\ \Sigma F_n = m\,a_n;\qquad T\sin 30^\circ = 80\left(\dfrac{v^2}{4 + 6\sin 30^\circ}\right)$

$+\uparrow \Sigma F_b = 0;\qquad T\cos 30^\circ - 80(9.81) = 0$

$T = 906.2\text{ N}$

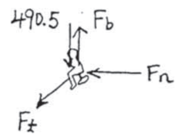
$v = 6.30\text{ m/s}$ **Ans.**

$\Sigma F_n = m\,a_n;\qquad F_n = 50\left(\dfrac{(6.30)^2}{7}\right) = 283\text{ N}$ **Ans.**

$\Sigma F_t = m\,a_t;\qquad F_t = 0$ **Ans.**

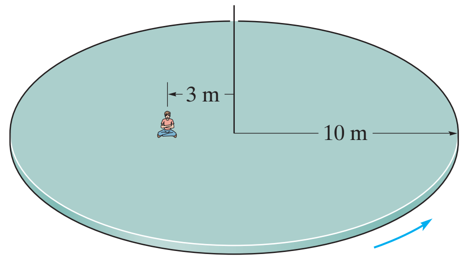
$\Sigma F_b = m\,a_b;\qquad F_b - 490.5 = 0$

$F_b = 490\text{ N}$ **Ans.**



13–66.

The man has a mass of 80 kg and sits 3 m from the center of the rotating platform. Due to the rotation his speed is increased from rest by $\dot{v} = 0.4\ \text{m/s}^2$. If the coefficient of static friction between his clothes and the platform is $\mu_s = 0.3$, determine the time required to cause him to slip.

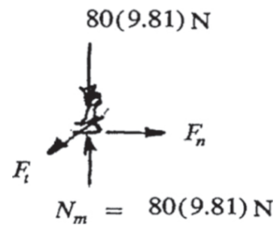


SOLUTION

$\Sigma F_t = m\,a_t;\qquad F_t = 80(0.4)$

$F_t = 32\ \text{N}$

$\Sigma F_n = m\,a_n;\qquad F_n = (80)\dfrac{v^2}{3}$



$F = \mu_s N_m = \sqrt{(F_t)^2 + (F_n)^2}$

$0.3(80)(9.81) = \sqrt{(32)^2 + ((80)\dfrac{v^2}{3})^2}$

$55\,432 = 1024 + (6400)(\dfrac{v^4}{9})$

$v = 2.9575\ \text{m/s}$

$a_t = \dfrac{dv}{dt} = 0.4$

$\displaystyle\int_0^v dv = \int_0^t 0.4\,dt$

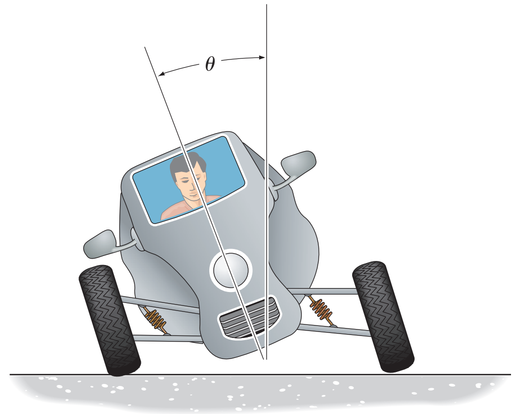
$v = 0.4\,t$

$2.9575 = 0.4\,t$

$t = 7.39\ \text{s}$ **Ans.**

13–67.

The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.



SOLUTION

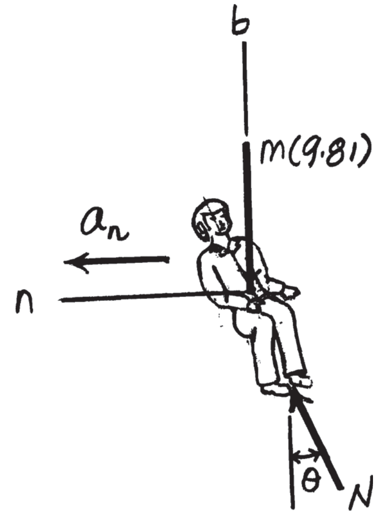
Free-Body Diagram: The free-body diagram of the passenger is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: The speed of the passenger is $v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}$. Thus, the normal component of the passenger's acceleration is given by $a_n = \frac{v^2}{\rho} = \frac{22.22^2}{100} = 4.938\text{ m/s}^2$. By referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad N\cos\theta - m(9.81) = 0 \qquad N = \frac{9.81m}{\cos\theta}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad \frac{9.81m}{\cos\theta}\sin\theta = m(4.938)$$

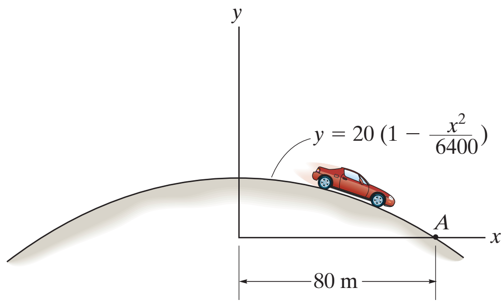
$$\theta = 26.7^\circ$$ **Ans.**



(a)

***13–68.**

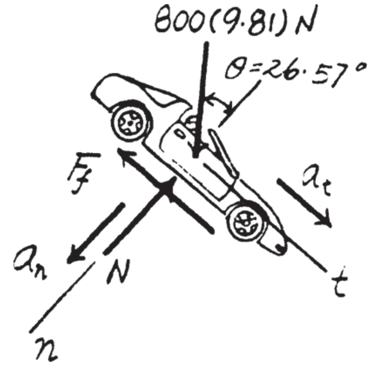
The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.



SOLUTION

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57°$$



and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$

Equations of Motion: Here, $a_t = 0$. Applying Eq. 13–8 with $\theta = 26.57°$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57° - F_f = 800(0)$$

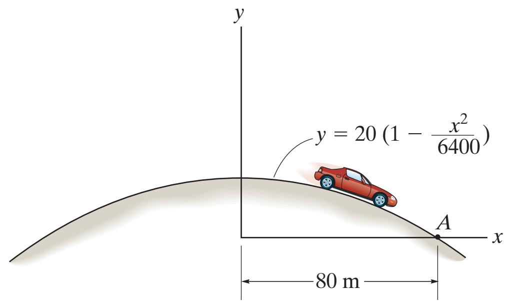
$$F_f = 3509.73\text{ N} = 3.51\text{ kN} \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57° - N = 800\left(\frac{9^2}{223.61}\right)$$

$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

13–69.

The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.



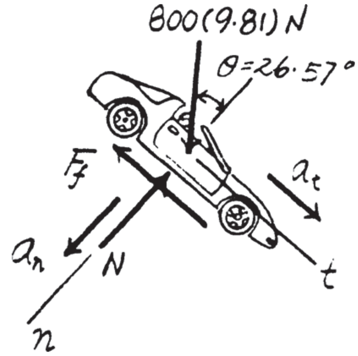
SOLUTION

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \frac{dy}{dx}\bigg|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$

and the radius of curvature at point A is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \frac{\left[1 + (-0.00625x)^2\right]^{3/2}}{|-0.00625|}\Bigg|_{x=80\text{ m}} = 223.61\text{ m}$$



Equation of Motion: Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57^\circ - F_f = 800(3)$$

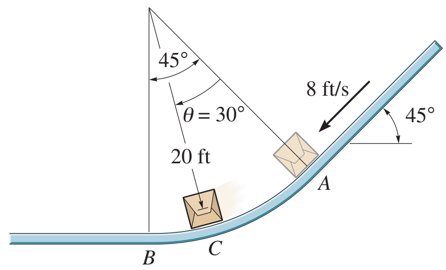
$$F_f = 1109.73\text{ N} = 1.11\text{ kN} \qquad \textbf{Ans.}$$

$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$$

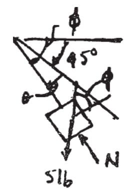
$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

13–70.

The package has a weight of 5 lb and slides down the chute. When it reaches the curved portion AB, it is traveling at 8 ft/s ($\theta = 0°$). If the chute is smooth, determine the speed of the package when it reaches the intermediate point C ($\theta = 30°$) and when it reaches the horizontal plane ($\theta = 45°$). Also, find the normal force on the package at C.



SOLUTION



$+\swarrow \Sigma F_t = ma_t; \qquad 5 \cos \phi = \dfrac{5}{32.2} a_t$

$$a_t = 32.2 \cos \phi$$

$+\nwarrow \Sigma F_n = ma_n; \qquad N - 5 \sin \phi = \dfrac{5}{32.2} \left(\dfrac{v^2}{20}\right)$

$$v\,dv = a_t\,ds$$

$$\int_g^v v\,dv = \int_{45°}^{\phi} 32.2 \cos \phi \,(20\,d\phi)$$

$$\frac{1}{2}v^2 - \frac{1}{2}(8)^2 = 644\,(\sin \phi - \sin 45°)$$

At $\phi = 45° + 30° = 75°$,

$$v_C = 19.933 \text{ ft/s} = 19.9 \text{ ft/s} \qquad \textbf{Ans.}$$

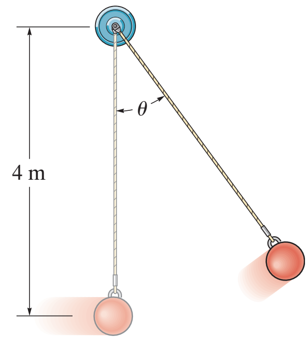
$$N_C = 7.91 \text{ lb} \qquad \textbf{Ans.}$$

At $\phi = 45° + 45° = 90°$

$$v_B = 21.0 \text{ ft/s} \qquad \textbf{Ans.}$$

13–71.

If the ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$, determine the tension in the cord at this instant. Also, determine the angle θ to which the ball swings and momentarily stops. Neglect the size of the ball.



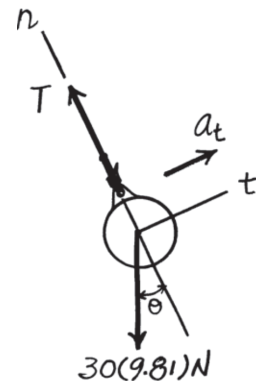
SOLUTION

$+\uparrow \Sigma F_n = ma_n; \qquad T - 30(9.81) = 30\left(\frac{(4)^2}{4}\right)$

$T = 414 \text{ N}$ **Ans.**

$+\nearrow \Sigma F_t = ma_t; \qquad -30(9.81)\sin\theta = 30a_t$

$a_t = -9.81 \sin\theta$



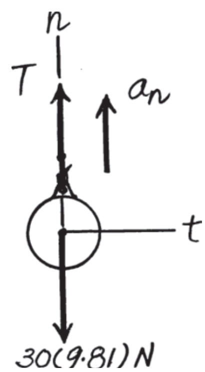
$a_t\, ds = v\, dv$ Since $ds = 4\, d\theta$, then

$$-9.81\int_0^{\theta} \sin\theta(4\, d\theta) = \int_4^0 v\, dv$$

$$\left[9.81(4)\cos\theta\right]_0^{\theta} = -\frac{1}{2}(4)^2$$

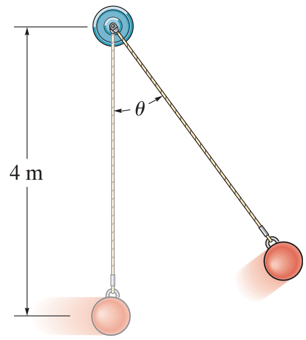
$$39.24(\cos\theta - 1) = -8$$

$\theta = 37.2°$ **Ans.**

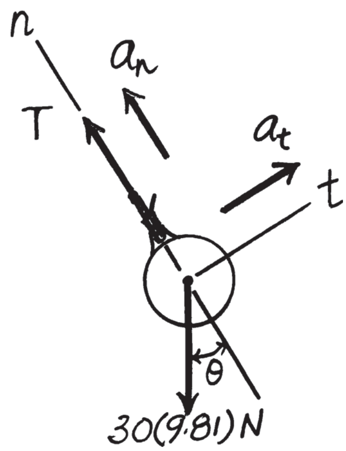


***13–72.**

The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



SOLUTION



$+\nwarrow \Sigma F_n = ma_n;\qquad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$

$+\nearrow \Sigma F_t = ma_t;\qquad -30(9.81)\sin\theta = 30a_t$

$$a_t = -9.81\sin\theta$$

$a_t\, ds = v\, dv$ Since $ds = 4\, d\theta$, then

$$-9.81\int_0^{\theta}\sin\theta\,(4\,d\theta) = \int_4^{v} v\,dv$$

$$9.81(4)\cos\theta\Big|_0^{\theta} = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$$

$$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$$

At $\theta = 20°$

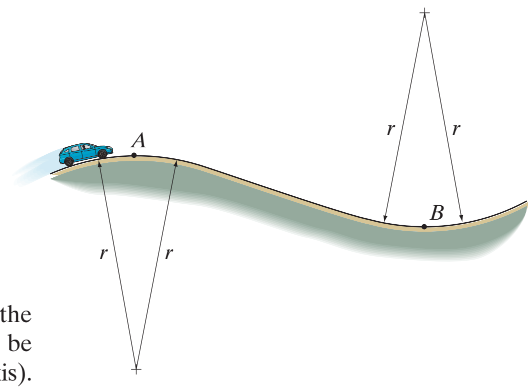
$v = 3.357$ m/s

$a_t = -3.36\text{ m/s}^2 = 3.36\text{ m/s}^2 \swarrow$ **Ans.**

$T = 361$ N **Ans.**

13–73.

Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?

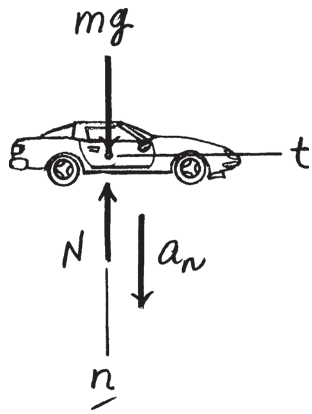


SOLUTION

Free-Body Diagram: The free-body diagram of the car at the top and bottom of the vertical curved road are shown in Figs. (a) and (b), respectively. Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the vertical curved road (positive n axis).

Equations of Motion: When the car is on top of the vertical curved road, it is required that its tires are about to lose contact with the road surface. Thus, $N = 0$. Realizing that $a_n = \frac{v^2}{\rho} = \frac{v^2}{r}$ and referring to Fig. (a),

$$+\downarrow \Sigma F_n = ma_n; \qquad mg = m\left(\frac{v^2}{r}\right) \qquad v = \sqrt{gr} \qquad \textbf{Ans.}$$

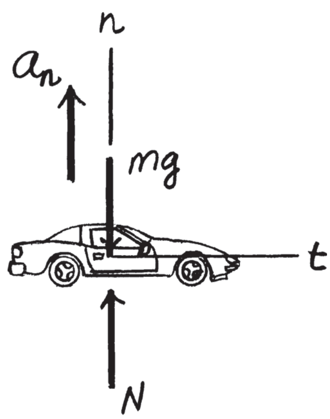


(a)

Using the result of v, the normal component of car acceleration is $a_n = \frac{v^2}{\rho} = \frac{gr}{r} = g$ when it is at the lowest point on the road. By referring to Fig. (b),

$$+\uparrow \Sigma F_n = ma_n; \qquad N - mg = mg$$

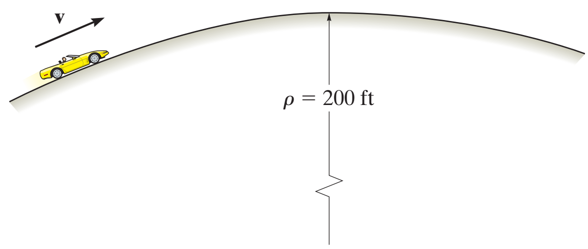
$$N = 2mg \qquad \textbf{Ans.}$$



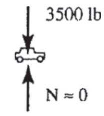
(b)

13–74.

If the crest of the hill has a radius of curvature $\rho = 200$ ft, determine the maximum constant speed at which the car can travel over it without leaving the surface of the road. Neglect the size of the car in the calculation. The car has a weight of 3500 lb.



SOLUTION

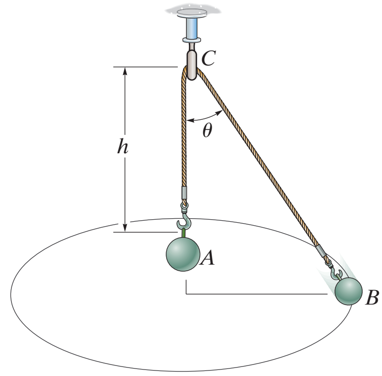


$$+\downarrow \Sigma F_n = ma_n; \qquad 3500 = \frac{3500}{32.2}\left(\frac{v^2}{200}\right)$$

$$v = 80.2 \text{ ft/s} \qquad \textbf{Ans.}$$

13–75.

Bobs A and B of mass m_A and m_B $(m_A > m_B)$ are connected to an inextensible light string of length l that passes through the smooth ring at C. If bob B moves as a conical pendulum such that A is suspended a distance of h from C, determine the angle θ and the speed of bob B. Neglect the size of both bobs.



SOLUTION

Free-Body Diagram: The free-body diagram of bob B is shown in Fig. a. The tension developed in the string is equal to the weight of bob A, i.e., $T = m_A g$. Here, $\mathbf{a}_n$ must be directed towards the center of the horizontal circular path (positive n axis).

Equations of Motion: The radius of the horizontal circular path is $r = (l - h)\sin\theta$. Thus, $a_n = \dfrac{v^2}{\rho} = \dfrac{v_B{}^2}{(l-h)\sin\theta}$. By referring to Fig. a,

$$+\uparrow \Sigma F_b = 0; \qquad m_A g\cos\theta - m_B g = 0$$

$$\theta = \cos^{-1}\left(\frac{m_B}{m_A}\right) \qquad \textbf{Ans.}$$

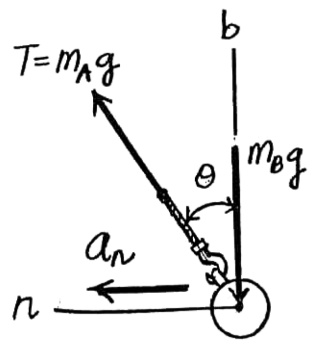
$$\overset{+}{\leftarrow} \Sigma F_n = m a_n; \qquad m_A g\sin\theta = m_B\left[\frac{v_B{}^2}{(l-h)\sin\theta}\right]$$

$$v_B = \sqrt{\frac{m_A g(l-h)}{m_B}}\sin\theta \qquad \textbf{(1)}$$

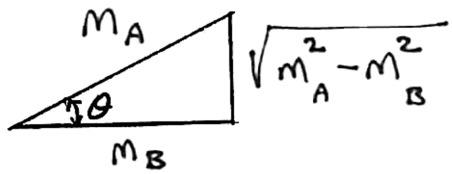
From Fig. b, $\sin\theta = \dfrac{\sqrt{m_A{}^2 - m_B{}^2}}{m_A}$. Substituting this value into Eq. (1),

$$v_B = \sqrt{\frac{m_A g(l-h)}{m_B}}\left(\frac{\sqrt{m_A{}^2 - m_B{}^2}}{m_A}\right)$$

$$= \sqrt{\frac{g(l-h)(m_A{}^2 - m_B{}^2)}{m_A m_B}} \qquad \textbf{Ans.}$$



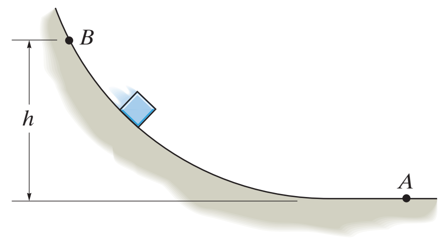
(a)



(b)

***13–76.**

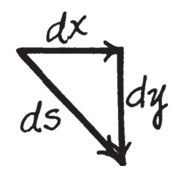
Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



SOLUTION

$+\searrow \Sigma F_t = ma_t; \qquad mg \sin\theta = ma_t \qquad a_t = g \sin\theta$

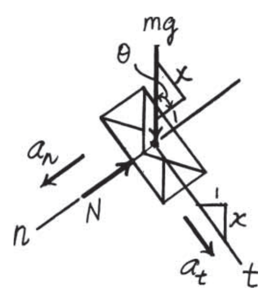
$$v\,dv = a_t\,ds = g\sin\theta\,ds \qquad \text{However} \quad dy = ds \sin\theta$$

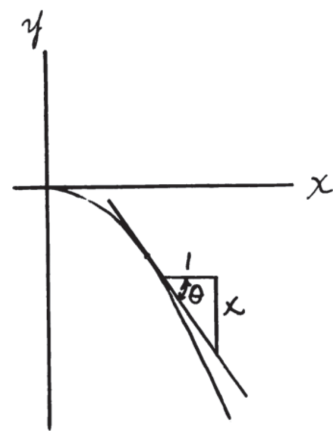


$$\int_0^v v\,dv = \int_0^h g\,dy$$

$$\frac{v^2}{2} = gh$$

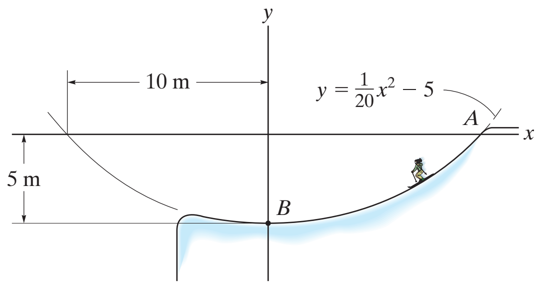
$$v = \sqrt{2gh}$$ **Q.E.D.**





13–77.

The skier starts from rest at $A(10\text{ m}, 0)$ and descends the smooth slope, which may be approximated by a parabola. If she has a mass of 52 kg, determine the normal force the ground exerts on the skier at the instant she arrives at point B. Neglect the size of the skier. *Hint:* Use the result of Prob. 13–76.



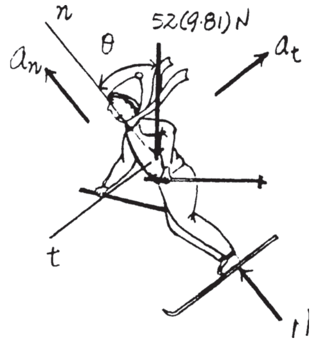
SOLUTION

Geometry: Here, $\dfrac{dy}{dx} = \dfrac{1}{10}x$ and $\dfrac{d^2y}{dx^2} = \dfrac{1}{10}$. The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=0\text{ m}} = 0 \qquad \theta = 0°$$

and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{|1/10|}\right|_{x=0\text{ m}} = 10.0\text{ m}$$



Equations of Motion:

$$+\swarrow\Sigma F_t = ma_t; \qquad 52(9.81)\sin\theta = -52a_t \qquad a_t = -9.81\sin\theta$$

$$+\nwarrow\Sigma F_n = ma_n; \qquad N - 52(9.81)\cos\theta = m\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$

Kinematics: The speed of the skier can be determined using $v\,dv = a_t\,ds$. Here, a_t must be in the direction of positive ds. Also, $ds = \sqrt{1 + (dy/dx)^2}dx = \sqrt{1 + \frac{1}{100}x^2}dx$

Here, $\tan\theta = \dfrac{1}{10}x$. Then, $\sin\theta = \dfrac{x}{10\sqrt{1 + \frac{1}{100}x^2}}$.

$$(+\ ) \qquad \int_0^v v\,dv = -9.81\int_{10\text{ m}}^{0}\left(\frac{x}{10\sqrt{1 + \frac{1}{100}x^2}}\right)\left(\sqrt{1 + \frac{1}{100}x^2}dx\right)$$

$$v^2 = 9.81\text{ m}^2/\text{s}^2$$

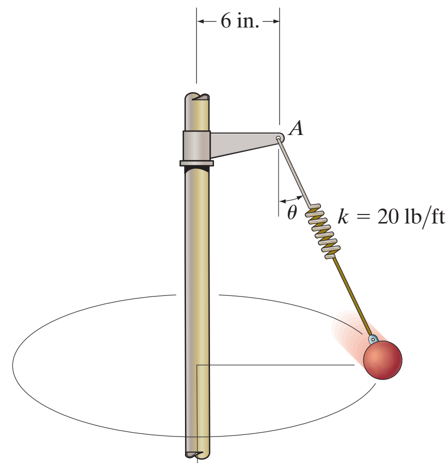
Substituting $v^2 = 98.1\text{ m}^2/\text{s}^2$, $\theta = 0°$, and $\rho = 10.0$ m into Eq.(1) yields

$$N - 52(9.81)\cos 0° = 52\left(\frac{98.1}{10.0}\right)$$

$$N = 1020.24\text{ N} = 1.02\text{ kN} \qquad \textbf{Ans.}$$

13–78.

A spring, having an unstretched length of 2 ft, has one end attached to the 10-lb ball. Determine the angle θ of the spring if the ball has a speed of 6 ft/s tangent to the horizontal circular path.



SOLUTION

Free-Body Diagram: The free-body diagram of the bob is shown in Fig. (a). If we denote the stretched length of the spring as l, then using the springforce formula, $F_{sp} = ks = 20(l - 2)$ lb. Here, $\mathbf{a}_n$ must be directed towards the center of the horizontal circular path (positive n axis).

Equations of Motion: The radius of the horizontal circular path is $r = 0.5 + l \sin\theta$. Since $a_n = \dfrac{v^2}{r} = \dfrac{6^2}{0.5 + l\sin\theta}$, by referring to Fig. (a),

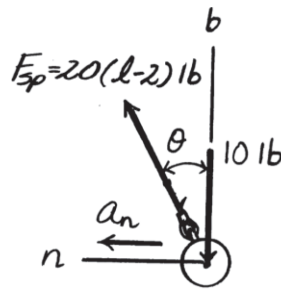
$$+\uparrow \Sigma F_b = 0; \qquad 20(l - 2)\cos\theta - 10 = 0 \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad 20(l - 2)\sin\theta = \frac{10}{32.2}\left(\frac{6^2}{0.5 + l\sin\theta}\right) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta = 31.26^\circ = 31.3^\circ \qquad \textbf{Ans.}$$

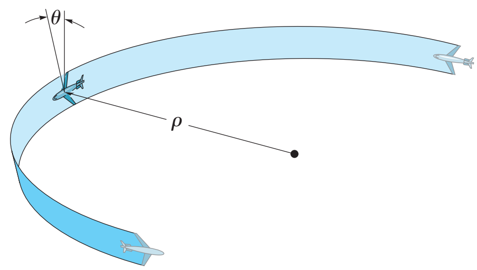
$$l = 2.585 \text{ ft} \qquad \textbf{Ans.}$$



(a)

13–79.

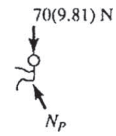
The airplane, traveling at a constant speed of 50 m/s, is executing a horizontal turn. If the plane is banked at $\theta = 15°$, when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has a mass of 70 kg.



SOLUTION

$+\uparrow \sum F_b = ma_b; \qquad N_P \sin 15° - 70(9.81) = 0$

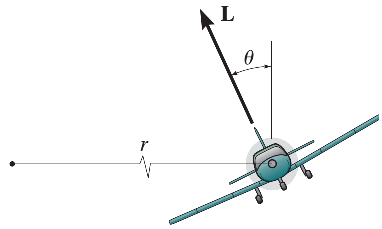
$N_P = 2.65 \text{ kN}$ **Ans.**



$\overset{+}{\leftarrow} \sum F_n = ma_n; \qquad N_P \cos 15° = 70\left(\dfrac{50^2}{\rho}\right)$

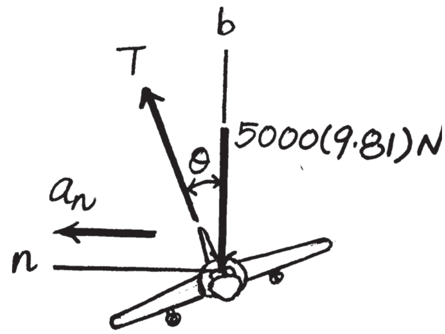
$\rho = 68.3 \text{ m}$ **Ans.**

***13–80.**



A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path of radius $r = 3000\,\text{m}$. Determine the uplift force **L** acting on the airplane and the banking angle θ. Neglect the size of the airplane.

SOLUTION



(a)

Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

Equations of Motion: The speed of the airplane is $v = \left(350\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 97.22\text{ m/s}$. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{3000} = 3.151\text{ m/s}^2$ and referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad T\cos\theta - 5000(9.81) = 0 \qquad \textbf{(1)}$$

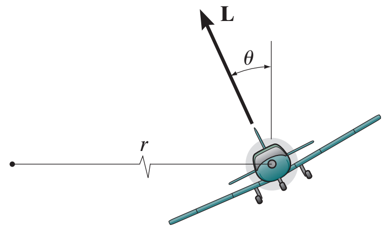
$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad T\sin\theta = 5000(3.151) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$\theta = 17.8°$ $\qquad T = 51517.75 = 51.5\text{ kN}$ **Ans.**

13–81.

A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path. If the banking angle $\theta = 15°$, determine the uplift force **L** acting on the airplane and the radius r of the circular path. Neglect the size of the airplane.



SOLUTION

Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

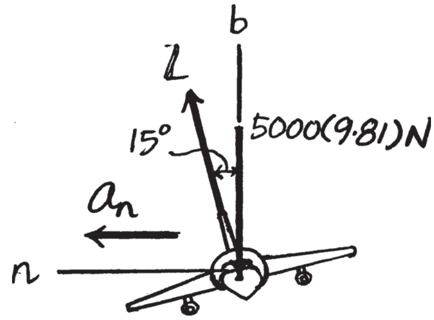
Equations of Motion: The speed of the airplane is $v = \left(350\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 97.22$ m/s. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{r}$ and referring to Fig. (a),

$+\uparrow \Sigma F_b = 0;$ $\quad L\cos 15° - 5000(9.81) = 0$

$L = 50780.30\text{ N} = 50.8\text{ kN}$ **Ans.**

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\quad 50780.30 \sin 15° = 5000\left(\frac{97.22^2}{r}\right)$

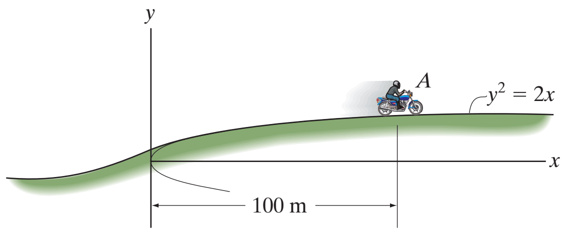
$r = 3595.92\text{ m} = 3.60\text{ km}$ **Ans.**



(a)

13–82.

The 800-kg motorbike travels with a constant speed of 80 km/h up the hill. Determine the normal force the surface exerts on its wheels when it reaches point A. Neglect its size.



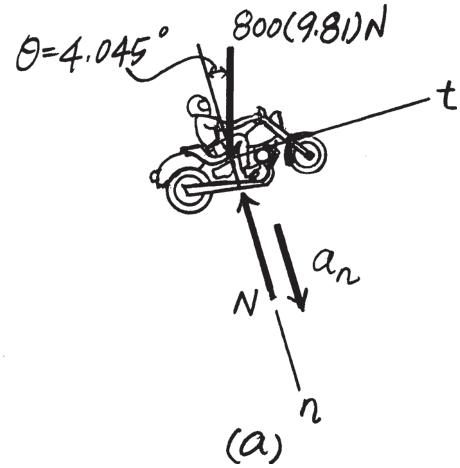
SOLUTION

Geometry: Here, $y = \sqrt{2}x^{1/2}$. Thus, $\dfrac{dy}{dx} = \dfrac{\sqrt{2}}{2x^{1/2}}$ and $\dfrac{d^2y}{dx^2} = -\dfrac{\sqrt{2}}{4x^{3/2}}$. The angle that the hill slope at A makes with the horizontal is

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\bigg|_{x=100\text{ m}} = \tan^{-1}\left(\frac{\sqrt{2}}{2x^{1/2}}\right)\bigg|_{x=100\text{ m}} = 4.045^\circ$$

The radius of curvature of the hill at A is given by

$$\rho_A = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|}\Bigg|_{x=100\text{ m}} = \frac{\left[1 + \left(\dfrac{\sqrt{2}}{2(100^{1/2})}\right)^2\right]^{3/2}}{\left|-\dfrac{\sqrt{2}}{4(100^{3/2})}\right|} = 2849.67\text{ m}$$



Free-Body Diagram: The free-body diagram of the motorcycle is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

Equations of Motion: The speed of the motorcycle is

$$v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}$$

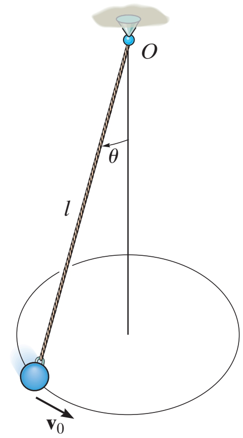
Thus, $a_n = \dfrac{v^2}{\rho_A} = \dfrac{22.22^2}{2849.67} = 0.1733\text{ m/s}^2$. By referring to Fig. (a),

$$\searrow + \Sigma F_n = ma_n; \qquad 800(9.81)\cos 4.045^\circ - N = 800(0.1733)$$

$$N = 7689.82\text{ N} = 7.69\text{ kN}$$ **Ans.**

13–83.

The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan\theta\sin\theta = v_0^2/gl$. Neglect air resistance and the size of the ball.



SOLUTION

$$\overset{+}{\rightarrow}\Sigma F_n = ma_n; \qquad T\sin\theta = m\left(\frac{v_0^2}{r}\right)$$

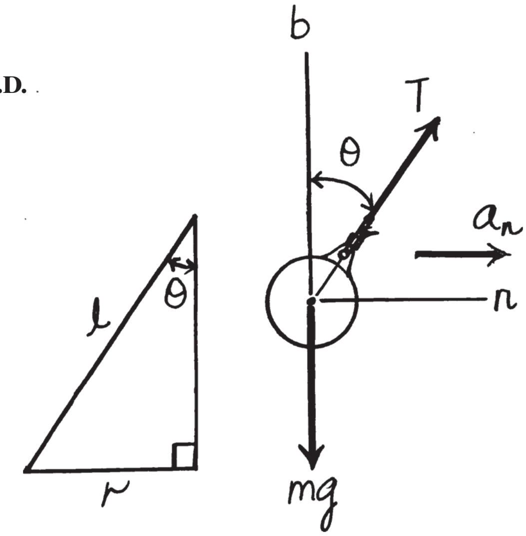
$$+\uparrow\Sigma F_b = 0; \qquad T\cos\theta - mg = 0$$

$$\text{Since } r = l\sin\theta \qquad T = \frac{mv_0^2}{l\sin^2\theta}$$

$$\left(\frac{mv_0^2}{l}\right)\left(\frac{\cos\theta}{\sin^2\theta}\right) = mg$$

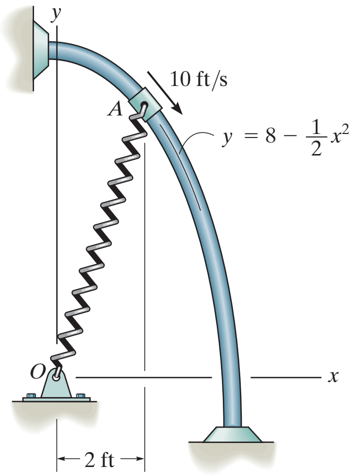
$$\tan\theta\sin\theta = \frac{v_0^2}{gl}$$

Q.E.D.



***13–84.**

The 5-lb collar slides on the smooth rod, so that when it is at A it has a speed of 10 ft/s. If the spring to which it is attached has an unstretched length of 3 ft and a stiffness of $k = 10$ lb/ft, determine the normal force on the collar and the acceleration of the collar at this instant.



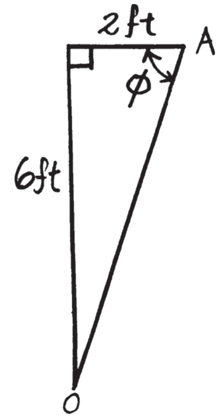
SOLUTION

$$y = 8 - \frac{1}{2}x^2$$

$$-\frac{dy}{dx} = \tan\theta = x\Big|_{x=2} = 2 \qquad \theta = 63.435^\circ$$

$$\frac{d^2y}{dx^2} = -1$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{(1 + (-2)^2)^{\frac{3}{2}}}{|-1|} = 11.18 \text{ ft}$$



$$y = 8 - \frac{1}{2}(2)^2 = 6$$

$$OA = \sqrt{(2)^2 + (6)^2} = 6.3246$$

$$F_s = kx = 10(6.3246 - 3) = 33.246 \text{ lb}$$

$$\tan\phi = \frac{6}{2}; \ \phi = 71.565^\circ$$

$+\swarrow \Sigma F_n = ma_n;$ $\quad 5\cos 63.435^\circ - N + 33.246\cos 45.0^\circ = \left(\frac{5}{32.2}\right)\left(\frac{(10)^2}{11.18}\right)$

$N = 24.4$ lb **Ans.**

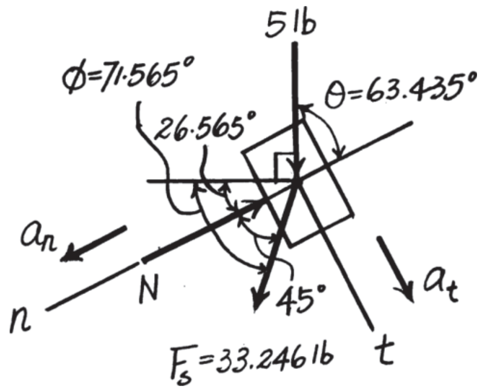
$+\searrow \Sigma F_t = ma_t;$ $\quad 5\sin 63.435^\circ + 33.246\sin 45.0^\circ = \left(\frac{5}{32.2}\right)a_t$

$a_t = 180.2 \text{ ft/s}^2$

$a_n = \frac{v^2}{\rho} = \frac{(10)^2}{11.18} = 8.9443 \text{ ft/s}^2$

$a = \sqrt{(180.2)^2 + (8.9443)^2}$

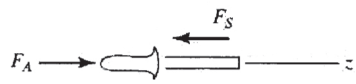
$a = 180 \text{ ft/s}^2$ **Ans.**



13–85.

The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin \theta)$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the force at the end A of the follower when $\theta = 90°$. In this position the spring is compressed 0.4 ft. Neglect friction at the bearing C.

SOLUTION



$$z = 0.1 \sin 2\theta$$

$$\dot{z} = 0.2 \cos 2\theta\dot{\theta}$$

$$\ddot{z} = -0.4 \sin 2\theta\dot{\theta}^2 + 0.2 \cos 2\theta\ddot{\theta}$$

$$\dot{\theta} = 6 \text{ rad/s}$$

$$\ddot{\theta} = 0$$

$$\ddot{z} = -14.4 \sin 2\theta$$

$$\sum F_z = ma_z; \qquad F_A - 12(z + 0.3) = m\ddot{z}$$

$$F_A - 12(0.1 \sin 2\theta + 0.3) = \frac{0.75}{32.2}(-14.4 \sin 2\theta)$$

For $\theta = 45°$,

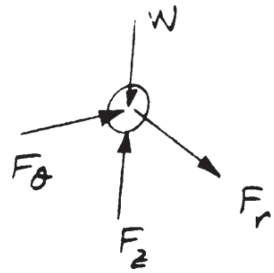
$$F_A - 12(0.4) = \frac{0.75}{32.2}(-14.4)$$

$F_A = 4.46$ lb **Ans.**

13–86.

Determine the magnitude of the resultant force acting on a 5-kg particle at the instant $t = 2$ s, if the particle is moving along a horizontal path defined by the equations $r = (2t + 10)$ m and $\theta = (1.5t^2 - 6t)$ rad, where t is in seconds.

SOLUTION



$r = 2t + 10|_{t=2\text{ s}} = 14$

$\dot{r} = 2$

$\ddot{r} = 0$

$\theta = 1.5t^2 - 6t$

$\dot{\theta} = 3t - 6|_{t=2\,s} = 0$

$\ddot{\theta} = 3$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 14(3) + 0 = 42$

Hence,

$\Sigma F_r = ma_r; \qquad F_r = 5(0) = 0$

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 5(42) = 210\text{ N}$

$F = \sqrt{(F_r)^2 + (F_\theta)^2} = 210\text{ N}$ **Ans.**

13–87.

The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the resultant force acting on the particle when $t = 2$ s.

SOLUTION

$$r = 2t + 1\big|_{t=2\text{ s}} = 5\text{ ft} \qquad \dot{r} = 2\text{ ft/s} \qquad \ddot{r} = 0$$

$$\theta = 0.5t^2 - t\big|_{t=2\text{ s}} = 0\text{ rad} \qquad \dot{\theta} = t - 1\big|_{t=2\text{ s}} = 1\text{ rad/s} \qquad \ddot{\theta} = 1\text{ rad/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1)^2 = -5\text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(1) + 2(2)(1) = 9\text{ ft/s}^2$$

$$\Sigma F_r = ma_r; \qquad F_r = \frac{5}{32.2}(-5) = -0.7764\text{ lb}$$

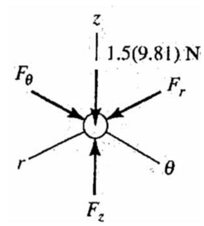
$$\Sigma F_\theta = ma_\theta; \qquad F_\theta = \frac{5}{32.2}(9) = 1.398\text{ lb}$$

$$F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(-0.7764)^2 + (1.398)^2} = 1.60\text{ lb}$$ **Ans.**

***13–88.**

A particle, having a mass of 1.5 kg, moves along a path defined by the equations $r = (4 + 3t)$ m, $\theta = (t^2 + 2)$ rad, and $z = (6 - t^3)$ m, where t is in seconds. Determine the r, θ, and z components of force which the path exerts on the particle when $t = 2$ s.

SOLUTION



$r = 4 + 3t|_{t=2\,s} = 10$ m $\qquad \dot{r} = 3$ m/s $\qquad \ddot{r} = 0$

$\theta = t^2 + 2 \qquad \dot{\theta} = 2t|_{t=2\text{ s}} = 4$ rad/s $\qquad \ddot{\theta} = 2$ rad/s^2

$z = 6 - t^3 \qquad \dot{z} = -3t^2 \qquad \ddot{z} = -6t|_{t=2\text{ s}} = -12$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 10(4)^2 = -160$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10(2) + 2(3)(4) = 44$ m/s^2

$a_z = \ddot{z} = -12$ m/s^2

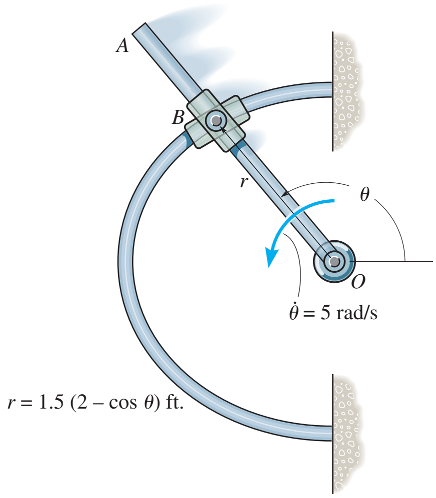
$\Sigma F_r = ma_r; \qquad F_r = 1.5(-160) = -240$ N **Ans.**

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 1.5(44) = 66$ N **Ans.**

$\Sigma F_z = ma_z; \qquad F_z - 1.5(9.81) = 1.5(-12) \qquad F_z = -3.28$ N **Ans.**

13–89.

Rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5\text{ rad/s}$. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other slides over the *horizontal* curved rod, of which the shape is described by the equation $r = 1.5(2 - \cos\theta)$ ft. If both collars weigh 0.75 lb, determine the normal force which the curved rod exerts on one collar at the instant $\theta = 120°$. Neglect friction.



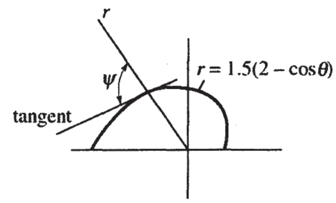
SOLUTION

Kinematic: Here, $\dot{\theta} = 5\text{ rad/s}$ and $\ddot{\theta} = 0$. Taking the required time derivatives at $\theta = 120°$, we have

$$r = 1.5(2 - \cos\theta)|_{\theta=120°} = 3.75\text{ ft}$$

$$\dot{r} = 1.5\sin\theta\dot{\theta}|_{\theta=120°} = 6.495\text{ ft/s}$$

$$\ddot{r} = 1.5(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^{\,2})|_{\theta=120°} = -18.75\text{ ft/s}^2$$



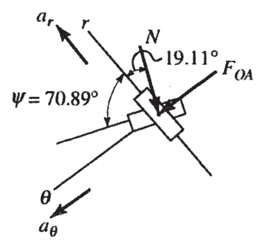
Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -18.75 - 3.75(5^2) = -112.5\text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.75(0) + 2(6.495)(5) = 64.952\text{ ft/s}^2$$

Equation of Motion: The angle ψ must be obtained first.

$$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{1.5(2 - \cos\theta)}{1.5\sin\theta}\right|_{\theta=120°} = 2.8867 \qquad \psi = 70.89°$$



Applying Eq. 13–9, we have

$$\sum F_r = ma_r; \qquad -N\cos 19.11° = \frac{0.75}{32.2}(-112.5)$$

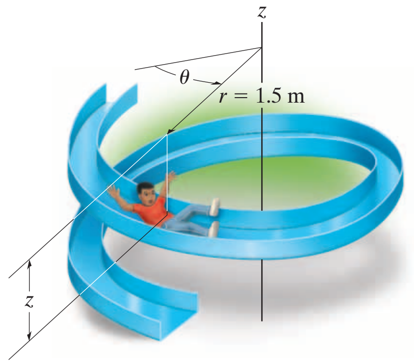
$$N = 2.773\text{ lb} = 2.77\text{ lb} \qquad \textbf{Ans.}$$

$$\sum F_\theta = ma_\theta; \qquad F_{OA} + 2.773\sin 19.11° = \frac{0.75}{32.2}(64.952)$$

$$F_{OA} = 0.605\text{ lb}$$

13–90.

The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.



SOLUTION

$r = 1.5 \qquad \theta = 0.7t \qquad z = -0.5t$

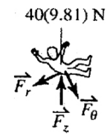
$\dot{r} = \ddot{r} = 0 \qquad \dot{\theta} = 0.7 \qquad \dot{z} = -0.5$

$\ddot{\theta} = 0 \qquad \ddot{z} = 0$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = 0$



$\Sigma F_r = ma_r; \qquad F_r = 40(-0.735) = -29.4 \text{ N}$ **Ans.**

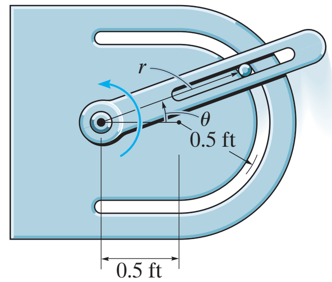
$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 0$ **Ans.**

$\Sigma F_z = ma_z; \qquad F_z - 40(9.81) = 0$

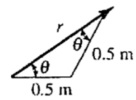
$F_z = 392 \text{ N}$ **Ans.**

13–91.

The 0.5-lb particle is guided along the circular path using the slotted arm guide. If the arm has an angular velocity $\dot{\theta} = 4$ rad/s and an angular acceleration $\ddot{\theta} = 8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the guide on the particle. Motion occurs in the *horizontal plane*.



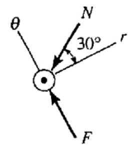
SOLUTION



$r = 2(0.5 \cos\theta) = 1 \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\bar{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$



At $\theta = 30°, \dot{\theta} = 4$ rad/s and $\ddot{\theta} = 8$ rad/s^2

$r = 1 \cos 30° = 0.8660$ ft

$\dot{r} = -\sin 30°(4) = -2$ ft/s

$\ddot{r} = -\cos 30°(4)^2 - \sin 30°(8) = -17.856$ ft/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -17.856 - 0.8660(4)^2 = -31.713$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(8) + 2(-2)(4) = -9.072$ ft/s^2

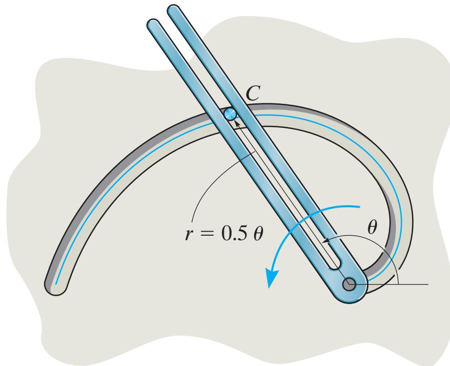
$\nearrow + \Sigma F_r = ma_r; \qquad -N\cos 30° = \dfrac{0.5}{32.2}(-31.713) \qquad N = 0.5686$ lb

$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad F - 0.5686 \sin 30° = \dfrac{0.5}{32.2}(-9.072)$

$F = 0.143$ lb **Ans.**

***13–92.**

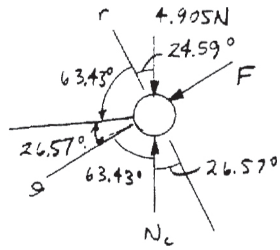
Using a forked rod, a smooth cylinder C having a mass of 0.5 kg is forced to move along the *vertical slotted* path $r = (0.5\theta)$ m, where θ is in radians. If the angular position of the arm is $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force of the rod on the cylinder and the normal force of the slot on the cylinder at the instant $t = 2$ s. The cylinder is in contact with only *one edge* of the rod and slot at any instant.



SOLUTION

$r = 0.5\theta \qquad \mathrm{r} = 0.5\dot{\theta} \qquad \ddot{r} = 0.5\ddot{\theta}$

$\theta = 0.5t^2 \qquad \dot{\theta} = t \qquad \ddot{\theta} = 1$



At $t = 2$ s,

$\theta = 2 \text{ rad} = 114.59° \qquad \dot{\theta} = 2 \text{ rad/2} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$

$r = 1 \text{ m} \qquad \dot{r} = 1 \text{ m/s} \qquad \ddot{r} = 0.5 \text{ m/s}^2$

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.5(2)}{0.5} \qquad \psi = 63.43°$$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0.5 - 1(2)^2 = -3.5$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1(1) + 2(1)(2) = 5$

$+\nwarrow\Sigma F_r = ma_r; \qquad N_C \cos 26.57° - 4.905 \cos 24.59° = 0.5(-3.5)$

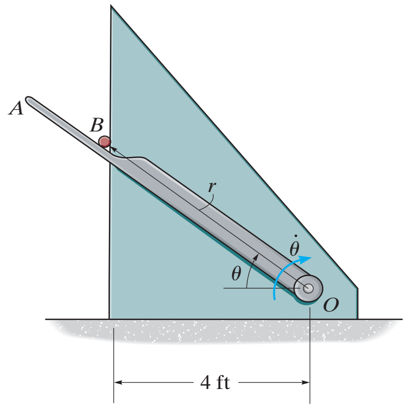
$N_C = 3.030 = 3.03 \text{ N}$ **Ans.**

$+\swarrow\Sigma F_\theta = ma_\theta; \qquad F - 3.030 \sin 26.57° + 4.905 \sin 24.59° = 0.5(5)$

$F = 1.81 \text{ N}$ **Ans.**

13–93.

If arm OA rotates with a constant clockwise angular velocity of $\dot{\theta} = 1.5$ rad/s. determine the force arm OA exerts on the smooth 4-lb cylinder B when $\theta = 45°$.



SOLUTION

Kinematics: Since the motion of cylinder B is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. Here, $\frac{4}{r} = \cos\theta$ or $r = 4\,\sec\theta$ ft. The value of r and its time derivatives at the instant $\theta = 45°$ are

$$r = 4\sec\theta\big|_{\theta=45°} = 4\sec 45° = 5.657\text{ ft}$$

$$\dot{r} = 4\sec\theta(\tan\theta)\dot{\theta}\big|_{\theta=45°} = 4\sec 45°\tan 45°(1.5) = 8.485\text{ ft/s}$$

$$\ddot{r} = 4\left[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}\left(\sec\theta\ \sec^2\theta\dot{\theta} + \tan\theta\sec\theta\tan\theta\dot{\theta}\right)\right]$$

$$= 4\left[\sec\theta(\tan\theta)\ddot{\theta} + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan^2\theta\dot{\theta}^2\right]\Big|_{\theta=45°}$$

$$= 4\left[\sec 45°\tan 45°(0) + \sec^3 45°(1.5)^2 + \sec 45°\tan^2 45°(1.5)^2\right]$$

$$= 38.18\text{ ft/s}^2$$

Using the above time derivatives,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 38.18 - 5.657\left(1.5^2\right) = 25.46\text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} - 2\dot{r}\dot{\theta} = 5.657(0) + 2(8.485)(1.5) = 25.46\text{ ft/s}^2$$

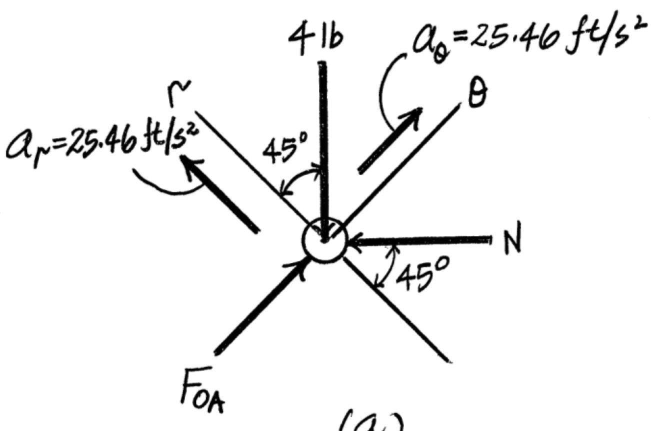
Equations of Motion: By referring to the free-body diagram of the cylinder shown in Fig. a,

$\Sigma F_r = ma_r;$ $\qquad N\cos 45° - 4\cos 45° = \frac{4}{32.2}(25.46)$

$N = 8.472\text{ lb}$

$\Sigma F_\theta = ma_\theta;$ $\qquad F_{OA} - 8.472\sin 45° - 4\sin 45° = \frac{4}{32.2}(25.46)$

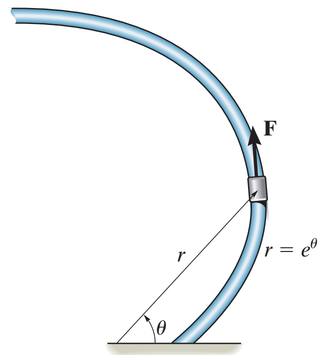
$F_{OA} = 12.0\text{ lb}$ **Ans.**



(a)

13–94.

The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 90°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



SOLUTION

$r = e^{\theta}$

$\dot{r} = e^{\theta}\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 90°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

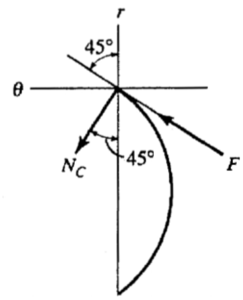
$r = 4.8105$

$\dot{r} = 9.6210$

$\ddot{r} = 19.242$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 19.242 - 4.8105(2)^2 = 0$

$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(9.6210)(2) = 38.4838 \text{ m/s}^2$



$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$$

$\psi = 45°$

$+\uparrow \sum F_r = ma_r; \qquad -N_C \cos 45° + F \cos 45° = 2(0)$

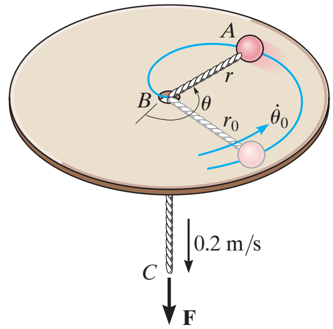
$\overset{+}{\leftarrow} \sum F_{\theta} = ma_{\theta}; \qquad F \sin 45° + N_C \sin 45° = 2(38.4838)$

$N_C = 54.4$ N **Ans.**

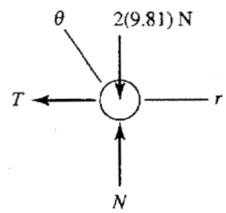
$F = 54.4$ N **Ans.**

13–95.

The ball has a mass of 2 kg and a negligible size. It is originally traveling around the horizontal circular path of radius $r_0 = 0.5$ m such that the angular rate of rotation is $\dot{\theta}_0 = 1$ rad/s. If the attached cord ABC is drawn down through the hole at a constant speed of 0.2 m/s, determine the tension the cord exerts on the ball at the instant $r = 0.25$ m. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)(d(r^2\dot{\theta})/dt) = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



SOLUTION



$$\sum F_\theta = ma_\theta; \qquad 0 = m[r\ddot{\theta} + 2\dot{r}\dot{\theta}] = m\left[\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\right] = 0$$

Thus,

$$d(r^2\dot{\theta}) = 0$$

$$r^2\dot{\theta} = C$$

$$(0.5)^2(1) = C = (0.25)^2\dot{\theta}$$

$$\dot{\theta} = 4.00 \text{ rad/s}$$ **Ans.**

Since $\dot{r} = -0.2$ m/s, $\quad \ddot{r} = 0$

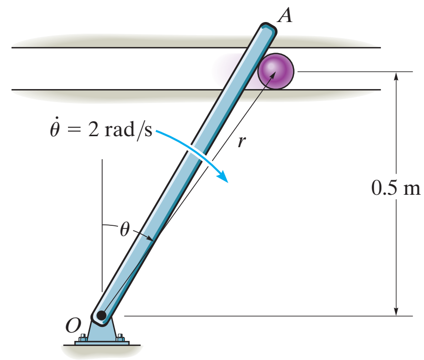
$$a_r = \ddot{r} - \dot{r}(\theta)^2 = 0 - 0.25(4.00)^2 = -4 \text{ m/s}^2$$

$$\sum F_r = ma_r; \quad -T = 2(-4)$$

$$T = 8 \text{ N}$$ **Ans.**

***13–96.**

The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



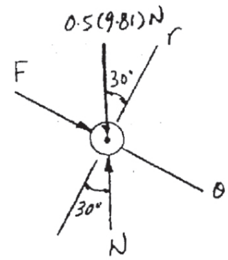
SOLUTION

$$r = \frac{0.5}{\cos\ \theta} = 0.5 \sec\theta$$

$$\dot{r} = 0.5 \sec\theta \tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta}\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta} + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$



When $\theta = 30°, \dot{\theta} = 2$ rad/s and $\ddot{\theta} = 0$

$$r = 0.5 \sec 30° = 0.5774 \text{ m}$$

$$\dot{r} = 0.5 \sec 30° \tan 30°(2) = 0.6667 \text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30° \tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30° \tan 30°(0)\right]$$

$$= 3.849 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667 \text{ m/s}^2$$

$\nearrow + \Sigma F_r = ma_r;\qquad N\ \cos 30° - 0.5(9.81)\ \cos 30° = 0.5(1.540)$

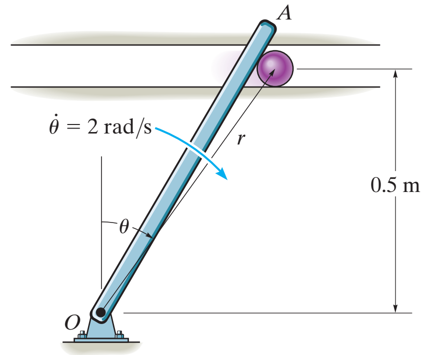
$$N = 5.79 \text{ N}$$ **Ans.**

$+\searrow \Sigma F_\theta = ma_\theta;\qquad F + 0.5(9.81)\sin 30° - 5.79 \sin 30° = 0.5(2.667)$

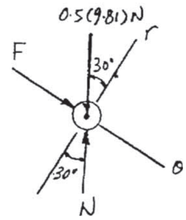
$$F = 1.78 \text{ N}$$ **Ans.**

13–97.

Solve Problem 13–96 if the arm has an angular acceleration of $\ddot{\theta} = 3\ \text{rad/s}^2$ and $\dot{\theta} = 2\ \text{rad/s}$ at this instant. Assume the particle contacts only one side of the slot at any instant.



SOLUTION



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta})\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30^\circ$, $\dot{\theta} = 2\ \text{rad/s}$ and $\ddot{\theta} = 3\ \text{rad/s}^2$

$$r = 0.5\sec 30^\circ = 0.5774\ \text{m}$$

$$\dot{r} = 0.5\sec 30^\circ\tan 30^\circ(2) = 0.6667\ \text{m/s}$$

$$\ddot{r} = 0.5\left[\sec 30^\circ\tan^2 30^\circ(2)^2 + \sec^3 30^\circ(2)^2 + \sec 30^\circ\tan 30^\circ(3)\right]$$

$$= 4.849\ \text{m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4.849 - 0.5774(2)^2 = 2.5396\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(3) + 2(0.6667)(2) = 4.3987\ \text{m/s}^2$$

$\nearrow + \Sigma F_r = ma_r;$ $\quad N\cos 30^\circ - 0.5(9.81)\cos 30^\circ = 0.5(2.5396)$

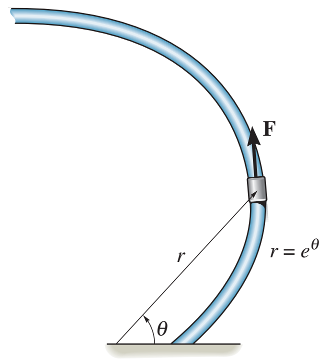
$$N = 6.3712 = 6.37\ \text{N}$$ **Ans.**

$+\searrow \Sigma F_\theta = ma_\theta;$ $\quad F + 0.5(9.81)\sin 30^\circ - 6.3712\sin 30^\circ = 0.5(4.3987)$

$$F = 2.93\ \text{N}$$ **Ans.**

13–98.

The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 45°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



SOLUTION

$r = e^{\theta}$

$\dot{r} = e^{\theta}\,\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 45°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 2.1933$

$\dot{r} = 4.38656$

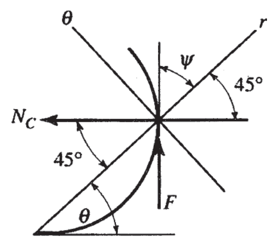
$\ddot{r} = 8.7731$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 8.7731 - 2.1933(2)^2 = 0$

$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(4.38656)(2) = 17.5462 \text{ m/s}^2$

$$\tan\psi = \frac{r}{\left(\dfrac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$$

$\psi = \theta = 45°$



$\nearrow + \sum F_r = ma_r; \qquad -N_C\cos 45° + F\cos 45° = 2(0)$

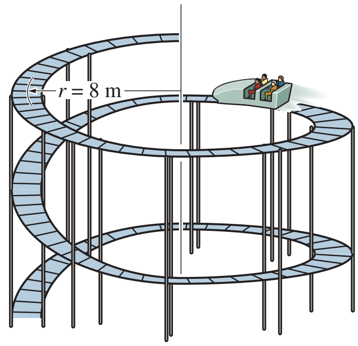
$+\nwarrow \sum F_{\theta} = ma_{\theta}; \qquad F\sin 45° + N_C\sin 45° = 2(17.5462)$

$N = 24.8 \text{ N}$ **Ans.**

$F = 24.8 \text{ N}$ **Ans.**

13–99.

For a short time, the 250-kg roller coaster car is traveling along the spiral track such that its position measured from the top of the track has components $r = 8$ m, $\theta = (0.1t + 0.5)$ rad, and $z = (-0.2t)$ m, where t is in seconds. Determine the magnitudes of the components of force which the track exerts on the car in the r, θ, and z directions at the instant $t = 2$ s. Neglect the size of the car.



SOLUTION

Kinematic: Here, $r = 8$ m, $\dot{r} = \ddot{r} = 0$. Taking the required time derivatives at $t = 2$ s, we have

$$\theta = 0.1t + 0.5|_{t=2s} = 0.700 \text{ rad} \qquad \dot{\theta} = 0.100 \text{ rad/s} \qquad \ddot{\theta} = 0$$

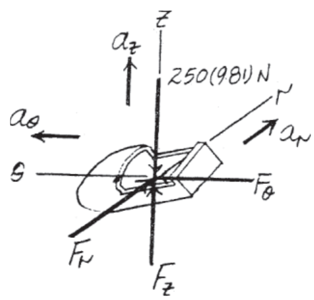
$$z = -0.2t|_{t=2\,s} = -0.400 \text{ m} \qquad \dot{z} = -0.200 \text{ m/s} \qquad \ddot{z} = 0$$

Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 8(0.100^2) = -0.0800 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 8(0) + 2(0)(0.200) = 0$$

$$a_z = \ddot{z} = 0$$



Equation of Motion:

$$\Sigma F_r = ma_r; \qquad F_r = 250(-0.0800) = -20.0 \text{ N} \qquad \textbf{Ans.}$$

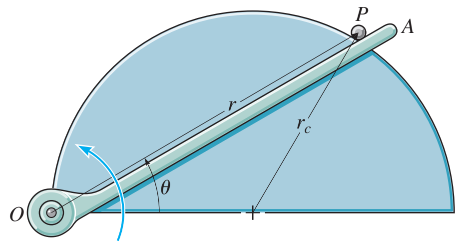
$$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 250(0) = 0 \qquad \textbf{Ans.}$$

$$\Sigma F_z = ma_z; \qquad F_z - 250(9.81) = 250(0)$$

$$F_z = 2452.5 \text{ N} = 2.45 \text{ kN} \qquad \textbf{Ans.}$$

***13–100.**

The 0.5-lb ball is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4$ rad/s and an angular acceleration $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_c = 0.4$ ft.



SOLUTION

$r = 2(0.4)\cos\theta = 0.8\cos\theta$

$\dot{r} = -0.8\sin\theta\dot{\theta}$

$\ddot{r} = -0.8\cos\theta\dot{\theta}^2 - 0.8\sin\theta\ddot{\theta}$

At $\theta = 30°$, $\dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8$ rad/s^2

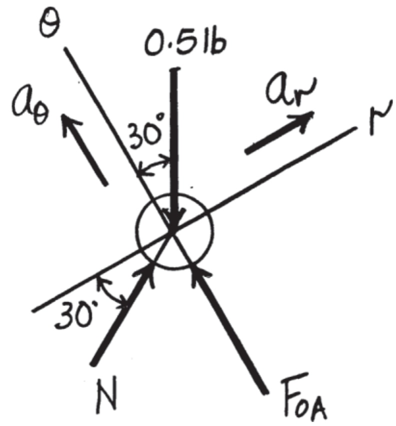
$r = 0.8\cos 30° = 0.6928$ ft

$\dot{r} = -0.8\sin 30°(0.4) = -0.16$ ft/s

$\ddot{r} = -0.8\cos 30°(0.4)^2 - 0.8\sin 30°(0.8) = -0.4309$ ft/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.4309 - 0.6928(0.4)^2 = -0.5417$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6928(0.8) + 2(-0.16)(0.4) = 0.4263$ ft/s^2



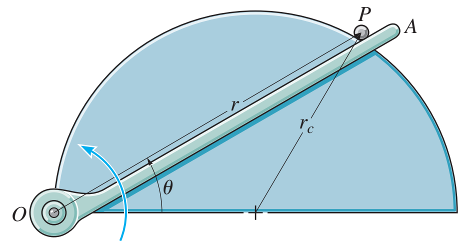
$+\nearrow\Sigma F_r = ma_r;\qquad N\cos 30° - 0.5\sin 30° = \dfrac{0.5}{32.2}(-0.5417)\qquad N = 0.2790$ lb

$\nwarrow + \Sigma F_\theta = ma_\theta;\qquad F_{OA} + 0.2790\sin 30° - 0.5\cos 30° = \dfrac{0.5}{32.2}(0.4263)$

$F_{OA} = 0.300$ lb **Ans.**

13–101.

The ball of mass m is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has a constant angular velocity $\dot{\theta}_0$, determine the angle $\theta \le 45°$ at which the ball starts to leave the surface of the semicylinder. Neglect friction and the size of the ball.



SOLUTION

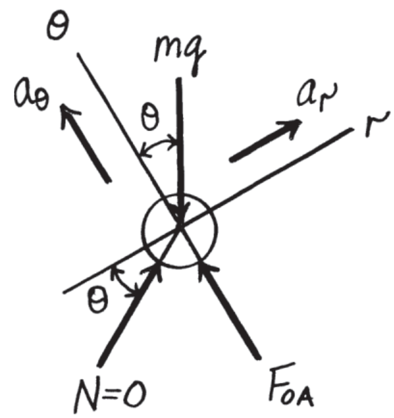
$r = 2r_c \cos\theta$

$\dot{r} = -2r_c \sin\theta\dot{\theta}$

$\ddot{r} = -2r_c \cos\theta\dot{\theta}^2 - 2r_c \sin\theta\ddot{\theta}$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2r_c \cos\theta\dot{\theta}_0^2 - 2r_c \cos\theta\dot{\theta}_0^2 = -4r_c \cos\theta\dot{\theta}_0^2$$

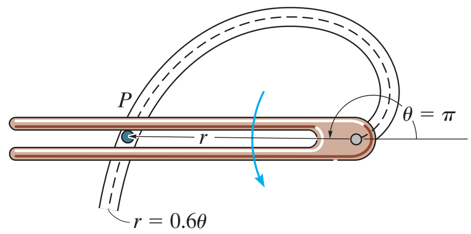


$+\nearrow\Sigma F_r = ma_r;\qquad -mg\sin\theta = m(-4r_c\cos\theta\dot{\theta}_0^2)$

$$\tan\theta = \frac{4r_c\,\dot{\theta}_0^2}{g}\qquad\qquad \theta = \tan^{-1}\left(\frac{4r_c\,\dot{\theta}_0^2}{g}\right)\qquad \textbf{Ans.}$$

13–102.

Using a forked rod, a smooth cylinder P, having a mass of 0.4 kg, is forced to move along the *vertical slotted* path $r = (0.6\theta)$ m, where θ is in radians. If the cylinder has a constant speed of $v_C = 2$ m/s, determine the force of the rod and the normal force of the slot on the cylinder at the instant $\theta = \pi$ rad. Assume the cylinder is in contact with only *one edge* of the rod and slot at any instant. *Hint:* To obtain the time derivatives necessary to compute the cylinder's acceleration components a_r and a_θ, take the first and second time derivatives of $r = 0.6\theta$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$, to determine $\ddot{\theta}$.



SOLUTION

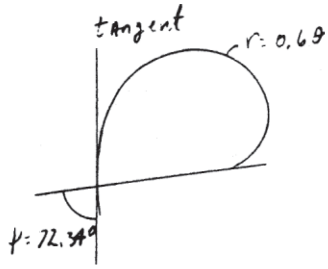
$r = 0.6\theta \qquad \dot{r} = 0.6\theta \qquad \ddot{r} = 0.6\ddot{\theta}$

$v_r = \dot{r} = 0.6\dot{\theta} \qquad v_\theta = r\dot{\theta} = 0.6\theta\dot{\theta}$

$v^2 = \dot{r}^2 + \left(r\theta\right)^2$

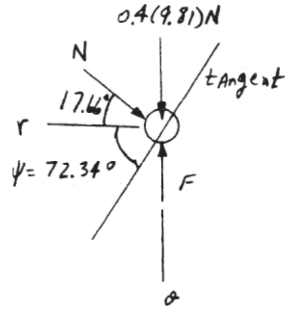
$2^2 = \left(0.6\dot{\theta}\right)^2 + \left(0.6\theta\dot{\theta}\right)^2 \qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1+\theta^2}}$

$0 = 0.72\dot{\theta}\ddot{\theta} + 0.36\left(2\theta\dot{\theta}^3 + 2\theta^2\dot{\theta}\ddot{\theta}\right) \qquad \ddot{\theta} = -\dfrac{\theta\dot{\theta}^2}{1+\theta^2}$



At $\theta = \pi$ rad, $\qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1+\pi^2}} = 1.011$ rad/s

$\ddot{\theta} = -\dfrac{(\pi)(1.011)^2}{1+\pi^2} = -0.2954$ rad/s^2



$r = 0.6(\pi) = 0.6\,\pi$ m $\qquad \dot{r} = 0.6(1.011) = 0.6066$ m/s

$\ddot{r} = 0.6(-0.2954) = -0.1772$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1772 - 0.6\pi(1.011)^2 = -2.104$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6\pi(-0.2954) + 2(0.6066)(1.011) = 0.6698$ m/s^2

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.6\theta}{0.6} = \theta = \pi \qquad \psi = 72.34°$

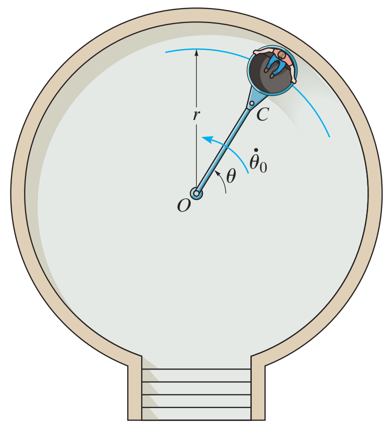
$\overset{+}{\leftarrow}\Sigma F_r = ma_r; \qquad -N\cos 17.66° = 0.4(-2.104) \qquad N = 0.883$ N **Ans.**

$+\downarrow\Sigma F_\theta = ma_\theta; \qquad -F + 0.4(9.81) + 0.883\sin 17.66° = 0.4(0.6698)$

$F = 3.92$ N **Ans.**

13–103.

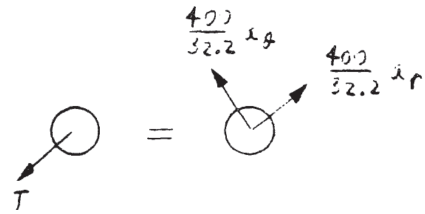
A ride in an amusement park consists of a cart which is supported by small wheels. Initially the cart is traveling in a circular path of radius $r_0 = 16$ ft such that the angular rate of rotation is $\dot{\theta}_0 = 0.2$ rad/s. If the attached cable OC is drawn inward at a constant speed of $\dot{r} = -0.5$ ft/s, determine the tension it exerts on the cart at the instant $r = 4$ ft. The cart and its passengers have a total weight of 400 lb. Neglect the effects of friction. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)\, d(r^2\dot{\theta})/dt = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



SOLUTION

$$+\nearrow\Sigma F_r = ma_r; \qquad -T = \left(\frac{400}{32.2}\right)\left(\ddot{r} - r\dot{\theta}^2\right) \qquad \textbf{(1)}$$

$$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad 0 = \left(\frac{400}{32.2}\right)\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) \qquad \textbf{(2)}$$



From Eq. (2), $\left(\frac{1}{r}\right)\frac{d}{dt}\left(r^2\dot{\theta}\right) = 0 \qquad r^2\dot{\theta} = c$

Since $\dot{\theta}_0 = 0.2$ rad/s when $r_0 = 16$ ft, $c = 51.2$.

Hence, when $r = 4$ ft,

$$\dot{\theta} = \left(\frac{51.2}{(4)^2}\right) = 3.2 \text{ rad/s}$$

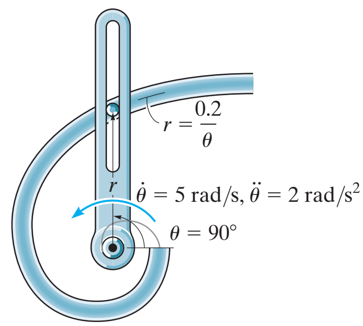
Since $r = -0.5$ ft/s, $\ddot{r} = 0$, Eq. (1) becomes

$$-T = \left(\frac{400}{32.2}\right)\left(0 - (4)(3.2)^2\right)$$

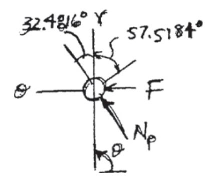
$T = 509$ lb **Ans.**

***13–104.**

The arm is rotating at a rate of $\dot{\theta} = 5 \text{ rad/s}$ when $\ddot{\theta} = 2 \text{ rad/s}^2$ and $\theta = 90°$. Determine the normal force it must exert on the 0.5-kg particle if the particle is confined to move along the slotted path defined by the *horizontal* hyperbolic spiral $r\theta = 0.2$ m.



SOLUTION



$$\theta = \frac{\pi}{2} = 90°$$

$$\dot{\theta} = 5 \text{ rad/s}$$

$$\ddot{\theta} = 2 \text{ rad/s}^2$$

$$r = 0.2/\theta = 0.12732 \text{ m}$$

$$\dot{r} = -0.2\,\theta^{-2}\,\dot{\theta} = -0.40528 \text{ m/s}$$

$$\ddot{r} = -0.2[-2\theta^{-3}\,(\dot{\theta})^2 + \theta^{-2}\ddot{\theta}] = 2.41801$$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 2.41801 - 0.12732(5)^2 = -0.7651 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0.12732(2) + 2(-0.40528)(5) = -3.7982 \text{ m/s}^2$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \frac{0.2/\theta}{-0.2\theta^{-2}}$$

$$\psi = \tan^{-1}\left(-\frac{\pi}{2}\right) = -57.5184°$$

$+\uparrow \Sigma F_r = m\,a_r;\qquad N_p \cos 32.4816° = 0.5(-0.7651)$

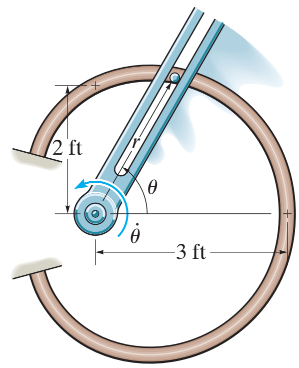
$\overset{+}{\leftarrow} \Sigma F_\theta = ma_\theta;\qquad F + N_p \sin 32.4816° = 0.5(-3.7982)$

$$N_P = -0.453 \text{ N}$$

$$F = -1.66 \text{ N}$$ **Ans.**

13–105.

The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If at all times $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the particle at the instant $\theta = 90°$. The fork and path contact the particle on only one side.



SOLUTION

$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 90°$, $\dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0$

$r = 2 + \cos 90° = 2$ ft

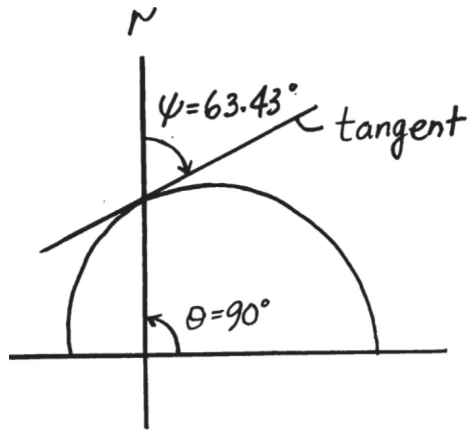
$\dot{r} = -\sin 90°(0.5) = -0.5$ ft/s

$\ddot{r} = -\cos 90°(0.5)^2 - \sin 90°(0) = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 2(0.5)^2 = -0.5 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(-0.5)(0.5) = -0.5 \text{ ft/s}^2$

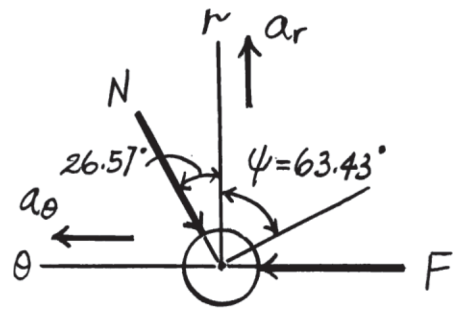
$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2 + \cos\theta}{-\sin\theta}\bigg|_{\theta=90°} = -2 \qquad \psi = -63.43°$$



$+\uparrow\Sigma F_r = ma_r;\qquad -N\cos 26.57° = \dfrac{2}{32.2}(-0.5) \qquad N = 0.03472$ lb

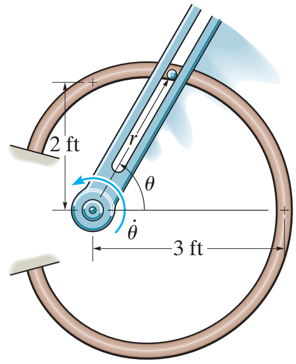
$\overset{+}{\leftarrow}\Sigma F_\theta = ma_\theta;\qquad F - 0.03472\ \sin 26.57° = \dfrac{2}{32.2}(-0.5)$

$F = -0.0155$ lb **Ans.**



13–106.

The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If at all times $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the particle at the instant $\theta = 60°$. The fork and path contact the particle on only one side.



SOLUTION

$r = 2 + \cos\theta$

$\dot{r} = -\sin\theta\dot{\theta}$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta}$

At $\theta = 60°, \dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0$

$r = 2 + \cos 60° = 2.5$ ft

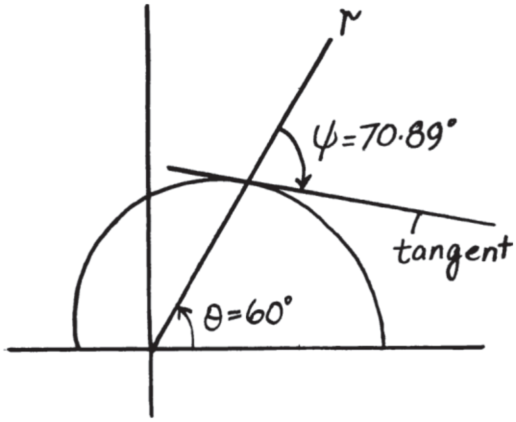
$\dot{r} = -\sin 60°(0.5) = -0.4330$ ft/s

$\ddot{r} = -\cos 60°(0.5)^2 - \sin 60°(0) = -0.125$ ft/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.125 - 2.5(0.5)^2 = -0.75$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.5(0) + 2(-0.4330)(0.5) = -0.4330$ ft/s^2

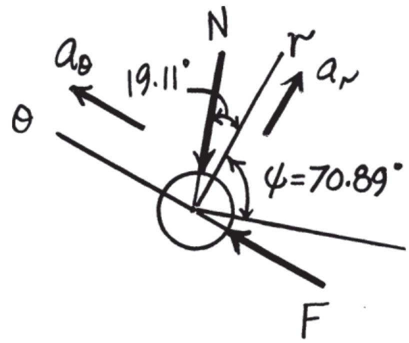
$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2 + \cos\theta}{-\sin\theta}\bigg|_{\theta = 60°} = -2.887 \qquad \psi = -70.89°$$



$+\nearrow\Sigma F_r = ma_r; \qquad -N\cos 19.11° = \dfrac{2}{32.2}(-0.75) \qquad N = 0.04930$ lb

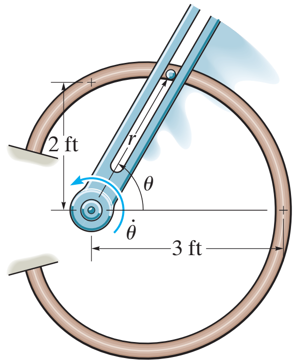
$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 0.04930\sin 19.11° = \dfrac{2}{32.2}(-0.4330)$

$F = -0.0108$ lb **Ans.**



13–107.

The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.

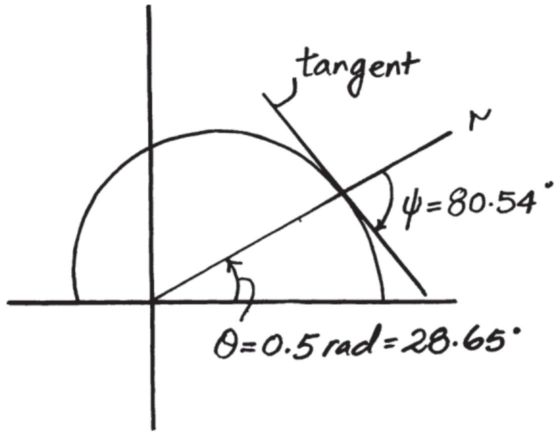


SOLUTION

$r = 2 + \cos\theta \qquad \theta = 0.5t^2$

$\dot{r} = -\sin\theta\theta \qquad \dot{\theta} = t$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} \qquad \ddot{\theta} = 1\ \text{rad/s}^2$



At $t = 1$ s, $\theta = 0.5$ rad, $\theta = 1$ rad/s, and $\ddot{\theta} = 1\ \text{rad/s}^2$

$r = 2 + \cos 0.5 = 2.8776$ ft

$\dot{r} = -\sin 0.5(1) = -0.4974\ \text{ft/s}^2$

$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357\ \text{ft/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -1.375 - 2.8776(1)^2 = -4.2346\ \text{ft/s}^2$

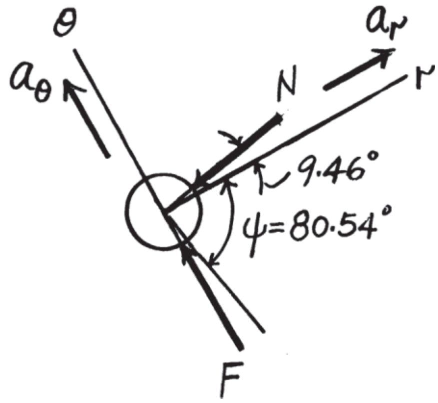
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1) = 1.9187\ \text{ft/s}^2$

$$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{2 + \cos\theta}{-\sin\theta}\right|_{\theta = 0.5\ \text{rad}} = -6.002 \qquad \psi = -80.54^\circ$$

$+\nearrow\Sigma F_r = ma_r; \qquad -N\cos 9.46^\circ = \dfrac{2}{32.2}(-4.2346) \qquad N = 0.2666$ lb

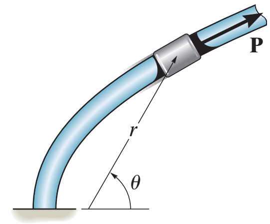
$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 0.2666\sin 9.46^\circ = \dfrac{2}{32.2}(1.9187)$

$F = 0.163$ lb **Ans.**



***13–108.**

The collar, which has a weight of 3 lb, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = 4/(1 - \cos\theta)$, where θ is in radians and r is in feet. If the collar's angular rate is constant and equals $\dot{\theta} = 4$ rad/s, determine the tangential retarding force P needed to cause the motion and the normal force that the collar exerts on the rod at the instant $\theta = 90°$.



SOLUTION

$$r = \frac{4}{1 - \cos\theta}$$

$$\dot{r} = \frac{-4\sin\theta\,\dot{\theta}}{(1 - \cos\theta)^2}$$

$$\ddot{r} = \frac{-4\sin\theta\,\ddot{\theta}}{(1 - \cos\theta)^2} + \frac{-4\cos\theta\,(\dot{\theta})^2}{(1 - \cos\theta)^2} + \frac{8\sin^2\theta\,\dot{\theta}^2}{(1 - \cos\theta)^3}$$

At $\theta = 90°$, $\quad \dot{\theta} = 4, \quad \ddot{\theta} = 0$

$$r = 4$$

$$\dot{r} = -16$$

$$\ddot{r} = 128$$

$$a_r = \ddot{r} - r(\theta)^2 = 128 - 4(4)^2 = 64$$

$$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0 + 2(-16)(4) = -128$$

$$r = \frac{4}{1 - \cos\theta}$$

$$\frac{dr}{d\theta} = \frac{-4\sin\theta}{(1 - \cos\theta)^2}$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \left.\frac{\frac{4}{1 - \cos\theta)}}{\frac{-4\sin\theta}{(1 - \cos\theta)^2}}\right|_{\theta = 90^\circ} = \frac{4}{-4} = -1$$

$$\psi = -45° = 135°$$

$$+\uparrow \Sigma F_r = m\,a_r; \qquad P\sin 45° - N\cos 45° = \frac{3}{32.2}(64)$$

$$\overset{+}{\leftarrow} \Sigma F_\theta = ma_\theta; \qquad -P\cos 45° - N\sin 45° = \frac{3}{32.2}(-128)$$

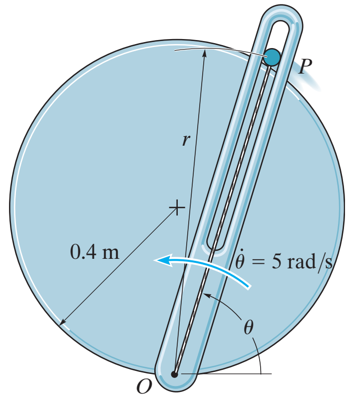
Solving,

$$P = 12.6 \text{ lb} \qquad \textbf{Ans.}$$

$$N = 4.22 \text{ lb} \qquad \textbf{Ans.}$$

13–109.

The smooth particle has a mass of 80 g. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = (0.8 \sin \theta)$ m. If the cord has a stiffness $k = 30$ N/m and an unstretched length of 0.25 m, determine the force of the guide on the particle when $\theta = 60°$. The guide has a constant angular velocity $\dot{\theta} = 5$ rad/s.



SOLUTION

$r = 0.8 \sin \theta$

$\dot{r} = 0.8 \cos \theta \, \dot{\theta}$

$\ddot{r} = -0.8 \sin \theta \, (\dot{\theta})^2 + 0.8 \cos \theta \ddot{\theta}$

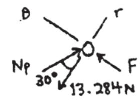
$\dot{\theta} = 5, \qquad \ddot{\theta} = 0$

At $\theta = 60°$, $\qquad r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -17.321$

$a_r = \ddot{r} - r(\dot{\theta})^2 = -17.321 - 0.6928(5)^2 = -34.641$



$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0 + 2(2)(5) = 20$

$F_s = ks; \qquad F_s = 30(0.6928 - 0.25) = 13.284 \text{ N}$

$\nearrow + \Sigma F_r = m a_r; \qquad -13.284 + N_P \cos 30° = 0.08(-34.641)$

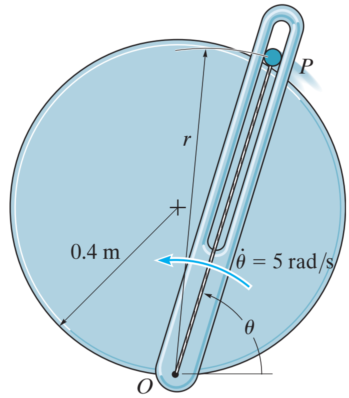
$\nwarrow + \Sigma F_\theta = m a_\theta; \qquad F - N_P \sin 30° = 0.08(20)$

$F = 7.67 \text{ N}$ **Ans.**

$N_P = 12.1 \text{ N}$

13–110.

The smooth particle has a mass of 80 g. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the horizontal circular path $r = (0.8 \sin \theta)$ m. If the cord has a stiffness $k = 30$ N/m and an unstretched length of 0.25 m, determine the force of the guide on the particle when $\ddot{\theta} = 2$ rad/s^2, $\dot{\theta} = 5$ rad/s, and $\theta = 60°$.



SOLUTION

$r = 0.8 \sin \theta$

$\dot{r} = 0.8 \cos \theta\, \dot{\theta}$

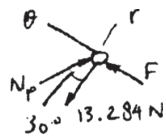
$\ddot{r} = -0.8 \sin \theta\, (\dot{\theta})^2 + 0.8 \cos \theta \ddot{\theta}$

$\dot{\theta} = 5, \qquad \ddot{\theta} = 2$

At $\theta = 60°$, $\qquad r = 0.6928$

$\dot{r} = 2$

$\ddot{r} = -16.521$



$a_r = \ddot{r} - r(\dot{\theta})^2 = -16.521 - 0.6928(5)^2 = -33.841$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6925(2) + 2(2)(5) = 21.386$

$F_s = ks; \qquad F_s = 30(0.6928 - 0.25) = 13.284$ N

$\nearrow + \Sigma F_r = m a_r; \qquad -13.284 + N_P \cos 30° = 0.08(-33.841)$

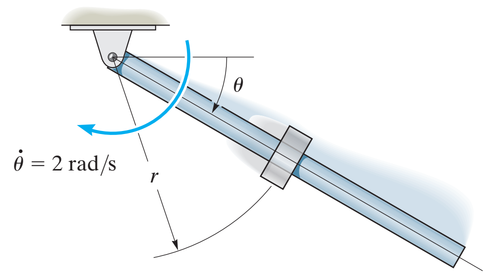
$+\nwarrow \Sigma F_\theta = m a_\theta; \qquad F - N_P \sin 30° = 0.08(21.386)$

$F = 7.82$ N **Ans.**

$N_P = 12.2$ N

13–111.

A 0.2-kg spool slides down along a smooth rod. If the rod has a constant angular rate of rotation $\dot{\theta} = 2$ rad/s in the vertical plane, show that the equations of motion for the spool are $\ddot{r} - 4r - 9.81 \sin\theta = 0$ and $0.8\dot{r} + N_s - 1.962\cos\theta = 0$, where N_s is the magnitude of the normal force of the rod on the spool. Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-2t} + C_2 e^{2t} - (9.81/8)\sin 2t$. If r, $\dot{r}$, and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 to determine r at the instant $\theta = \pi/4$ rad.

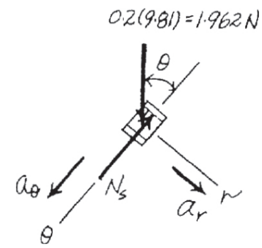


SOLUTION

Kinematic: Here, $\dot{\theta}. = 2$ rad/s and $\dot{\theta} = 0$. Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - r(2^2) = \ddot{r} - 4r$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(0) + 2\dot{r}(2) = 4\dot{r}$$



Equation of Motion: Applying Eq. 13–9, we have

$\Sigma F_r = ma_r;$ $\qquad 1.962 \sin\theta = 0.2(\ddot{r} - 4r)$

$$\ddot{r} - 4r - 9.81\sin\theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(1)}$$

$\Sigma F_\theta = ma_\theta;$ $\qquad 1.962\cos\theta - N_s = 0.2(4\dot{r})$

$$0.8\dot{r} + N_s - 1.962\cos\theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(2)}$$

Since $\theta. = 2$ rad/s, then $\int_0^\theta \dot{\theta} = \int_0^1 2dt, \theta = 2t$. The solution of the differential equation (Eq.(1)) is given by

$$r = C_1\, e^{-2t} + C_2\, e^{2t} - \frac{9.81}{8}\sin 2t \qquad \textbf{(3)}$$

Thus,

$$\dot{r} = -2\,C_1\,e^{-2t} + 2C_2\,e^{2t} - \frac{9.81}{4}\cos 2t \qquad \textbf{(4)}$$

At $t = 0, r = 0$. From Eq.(3) $0 = C_1(1) + C_2(1) - 0$ **(5)**

At $t = 0, \dot{r} = 0$. From Eq.(4) $0 = -2\,C_1(1) + 2C_2(1) - \frac{9.81}{4}$ **(6)**

Solving Eqs. (5) and (6) yields

$$C_1 = -\frac{9.81}{16} \qquad C_2 = \frac{9.81}{16}$$

Thus,

$$r = -\frac{9.81}{16}e^{-2t} + \frac{9.81}{16}e^{2t} - \frac{9.81}{8}\sin 2t$$

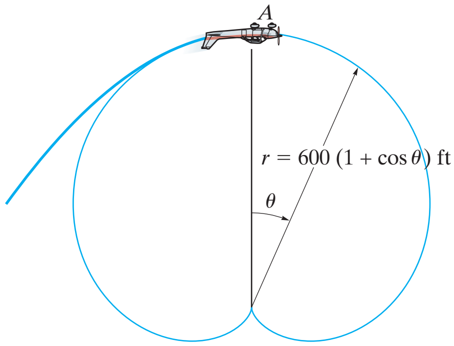
$$= \frac{9.81}{8}\left(\frac{-e^{-2t} + e^{2t}}{2} - \sin 2t\right)$$

$$= \frac{9.81}{8}(\sin\text{h}\, 2t - \sin 2t)$$

At $\theta = 2t = \frac{\pi}{4}$, $\qquad r = \frac{9.81}{8}\left(\sin\text{h}\frac{\pi}{4} - \sin\frac{\pi}{4}3\right) = 0.198$ m **Ans.**

***13–112.**

The pilot of an airplane executes a vertical loop which in part follows the path of a cardioid, $r = 600(1 + \cos\theta)$ ft. If his speed at A $(\theta = 0°)$ is a constant $v_P = 80$ ft/s, determine the vertical force the seat belt must exert on him to hold him to his seat when the plane is upside down at A. He weighs 150 lb.



SOLUTION

$$r = 600(1 + \cos\theta)\big|_{\theta=0°} = 1200 \text{ ft}$$

$$\dot{r} = -600 \sin\theta\dot{\theta}\big|_{\theta=0°} = 0$$

$$\ddot{r} = -600 \sin\theta\ddot{\theta} - 600\cos\theta\dot{\theta}^2\big|_{\theta=0°} = -600\dot{\theta}^2$$

$$v_p^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$$

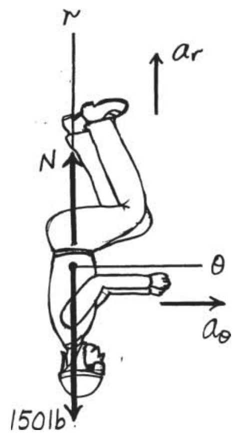
$$(80)^2 = 0 + \left(1200\dot{\theta}\right)^2 \qquad \dot{\theta} = 0.06667$$

$$2v_p v_p = 2\dot{r}\ddot{r} + 2\left(r\dot{\theta}\right)\left(\dot{r}\theta + r\ddot{\theta}\right)$$

$$0 = 0 + 0 + 2r^2\,\theta\ddot{\theta} \qquad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -600(0.06667)^2 - 1200(0.06667)^2 = -8 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$$



$$+\uparrow\Sigma F_r = ma_r; \qquad N - 150 = \left(\frac{150}{32.2}\right)(-8) \qquad N = 113 \text{ lb}$$ **Ans.**

13–113.

The earth has an orbit with eccentricity $e = 0.0821$ around the sun. Knowing that the earth's minimum distance from the sun is $151.3(10^6)$ km, find the speed at which the earth travels when it is at this distance. Determine the equation in polar coordinates which describes the earth's orbit about the sun.

SOLUTION

$$e = \frac{Ch^2}{GM_S} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0\, v_0^2}\right) \text{ and } h = r_0\, v_0$$

$$e = \frac{1}{GM_S\, r_0}\left(1 - \frac{GM_S}{r_0\, v_0^2}\right)(r_0 v_0)^2 \qquad e = \left(\frac{r_0\, v_0^2}{GM_S} - 1\right) \qquad \frac{r_0 v_0^2}{GM_S} = e + 1$$

$$v_0 = \sqrt{\frac{GM_S\,(e+1)}{r_0}}$$

$$= \sqrt{\frac{66.73(10^{-12})(1.99)(10^{30})(0.0821 + 1)}{151.3(10^9)}} = 30818 \text{ m/s} = 30.8 \text{ km/s} \qquad \textbf{Ans.}$$

$$\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0\, v_0^2}\right)\cos\theta + \frac{GM_S}{r_0^2 v_0^2}$$

$$\frac{1}{r} = \frac{1}{151.3(10^9)}\left(1 - \frac{66.73(10^{-12})(1.99)(10^{30})}{151.3(10^9)(30818)^2}\right)\cos\theta + \frac{66.73(10^{-12})(1.99)(10^{30})}{\left[151.3(10^9)\right]^2 (30818)^2}$$

$$\frac{1}{r} = 0.502(10^{-12})\cos\theta + 6.11(10^{-12}) \qquad \textbf{Ans.}$$

13–114.

A communications satellite is in a circular orbit above the earth such that it always remains directly over a point on the earth's surface. As a result, the period of the satellite must equal the rotation of the earth, which is approximately 24 hours. Determine the satellite's altitude h above the earth's surface and its orbital speed.

SOLUTION

The period of the satellite around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$T = \frac{2\pi r_0}{v_s}$$

$$24(3600) = \frac{2\pi\left[h + 6.378(10^6)\right]}{v_s}$$

$$v_s = \frac{2\pi\left[h + 6.378(10^6)\right.}{86.4(10^3)} \qquad \textbf{(1)}$$

The velocity of the satellite orbiting around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$v_S = \sqrt{\frac{GM_e}{r_0}}$$

$$v_S = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{h + 6.378(10^6)}} \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2),

$h = 35.87(10^6)\text{ m} = 35.9\text{ Mm}$ $\qquad$ $v_S = 3072.32\text{ m/s} = 3.07\text{ km/s}$ **Ans.**

13–115.

The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

SOLUTION

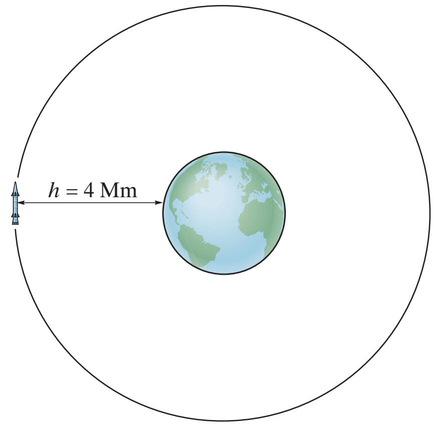
For a 800-km orbit

$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800 + 6378)(10^3)}}$$

$$= 7453.6 \text{ m/s} = 7.45 \text{ km/s} \qquad \textbf{Ans.}$$

***13–116.**

A rocket is in circular orbit about the earth at an altitude of $h = 4$ Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



SOLUTION

Circular Orbit:

$$v_C = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 6198.8 \text{ m/s}$$

Parabolic Orbit:

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 8766.4 \text{ m/s}$$

$$\Delta v = v_e - v_C = 8766.4 - 6198.8 = 2567.6 \text{ m/s}$$

$$\Delta v = 2.57 \text{ km/s}$$ **Ans.**

13–117.

Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

SOLUTION

From Eq. 13–19,

$$\frac{1}{r} = C\cos\theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$,

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C, from Eqs. 13–28 and 13–29,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13–31,

$$T = \frac{\pi}{h}(2a)(b)$$

Thus,

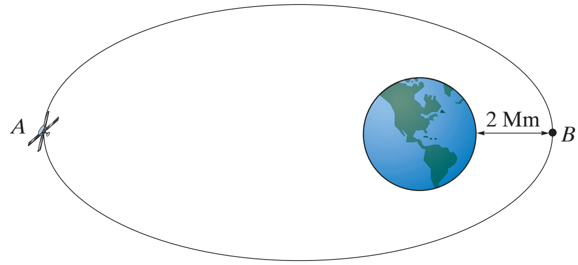
$$b^2 = \frac{T^2h^2}{4\pi^2a^2}$$

$$\frac{4\pi^2a^3}{T^2h^2} = \frac{GM_s}{h^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3$$ **Q.E.D.**

13–118.

The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.



SOLUTION

$$e = \frac{Ch^2}{GM_e}$$

where $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ and $h = r_0\, v_0$.

$$e = \frac{1}{GM_e\, r_0}\left(1 - \frac{GM_e}{r_0\, v_0^2}\right)(r_0\, v_0)^2$$

$$e = \left(\frac{r_0\, v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0\, v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e\,(e+1)}{r_0}}$$

where $r_0 = r_p = 2\left(10^6\right) + 6378\left(10^3\right) = 8.378\left(10^6\right)$ m.

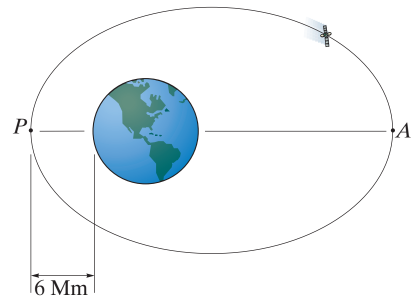
$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.25+1)}{8.378(10^6)}} = 7713 \text{ m/s} = 7.71 \text{ km/s} \qquad \textbf{Ans.}$$

$$r_a = \frac{r_0}{\frac{2GM_e}{r_0\, v_0} - 1} = \frac{8.378(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)(7713)^2} - 1} = 13.96\left(10^6\right) \text{ m}$$

$$v_A = \frac{r_p}{r_a}\, v_B = \frac{8.378(10^6)}{13.96(10^6)}(7713) = 4628 \text{ m/s} = 4.63 \text{ km/s} \qquad \textbf{Ans.}$$

13–119.

The elliptical orbit of a satellite orbiting the earth has an eccentricity of $e = 0.45$. If the satellite has an altitude of 6 Mm at perigee, determine the velocity of the satellite at apogee and the period.



SOLUTION

Here, $r_O = r_P = 6(10^6) + 6378(10^3) = 12.378(10^6)$ m.

$$h = r_P v_P$$

$$h = 12.378(10^6) v_P \qquad \textbf{(1)}$$

and

$$C = \frac{1}{r_P}\left(1 - \frac{GM_e}{r_P {v_P}^2}\right)$$

$$C = \frac{1}{12.378(10^6)}\left[1 - \frac{66.73(10^{-12})(5.976)(10^{24})}{12.378(10^6){v_P}^2}\right]$$

$$C = 80.788(10^{-9}) - \frac{2.6027}{{v_P}^2} \qquad \textbf{(2)}$$

Using Eqs. (1) and (2),

$$e = \frac{Ch^2}{GM_e}$$

$$0.45 = \frac{\left[80.788(10^{-9}) - \dfrac{2.6027}{{v_P}^2}\right]\left[12.378(10^6)v_P\right]^2}{66.73(10^{-12})(5.976)(10^{24})}$$

$$v_P = 6834.78 \text{ m/s}$$

Using the result of v_P,

$$r_a = \frac{r_P}{\dfrac{2GM_e}{r_P\, {v_P}^2} - 1}$$

$$= \frac{12.378(10^6)}{\dfrac{2(66.73)(10^{-12})(5.976)(10^{24})}{12.378(10^6)(6834.78^2)} - 1}$$

$$= 32.633(10^6) \text{ m}$$

Since $h = r_P v_P = 12.378(10^6)(6834.78) = 84.601(10^9)$ m^2/s is constant,

$$r_a v_a = h$$

$$32.633(10^6) v_a = 84.601(10^9)$$

$$v_a = 2592.50 \text{ m/s} = 2.59 \text{ km/s} \qquad \textbf{Ans.}$$

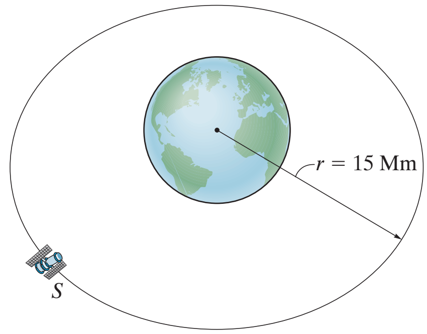
Using the result of h,

$$T = \frac{\pi}{h}(r_P + r_a)\sqrt{r_P r_a}$$

$$= \frac{\pi}{84.601(10^9)}\left[12.378(10^6) + 32.633(10^6)\right]\sqrt{12.378(10^6)(32.633)(10^6)}$$

***13–120.**

Determine the constant speed of satellite S so that it circles the earth with an orbit of radius $r = 15$ Mm. *Hint:* Use Eq. 13–1.



SOLUTION

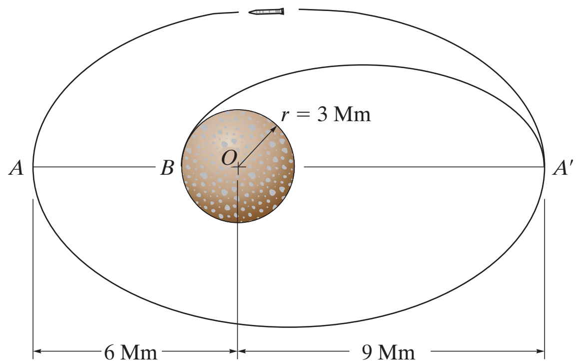
$$F = G\frac{m_s m_e}{r^2} \qquad \text{Also} \qquad F = m_s\left(\frac{v_s^2}{r}\right) \qquad \text{Hence}$$

$$m_s\left(\frac{v_0^2}{r}\right) = G\frac{m_s m_e}{r^2}$$

$$v = \sqrt{G\frac{m_e}{r}} = \sqrt{66.73(10^{-12})\left(\frac{5.976(10^{24})}{15(10^6)}\right)} = 5156 \text{ m/s} = 5.16 \text{ km/s} \qquad \textbf{Ans.}$$

13–121.

The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.



SOLUTION

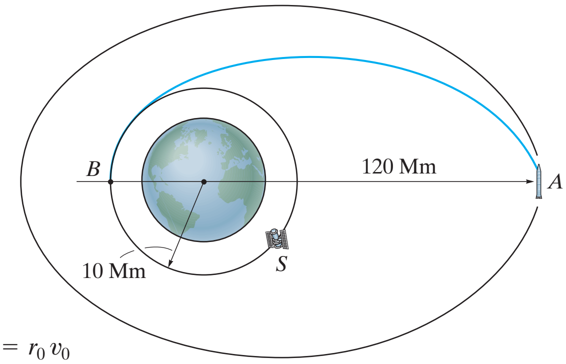
Central-Force Motion: Use $r_a = \dfrac{r_0}{(2\,GM/r_0\,v_0^2)-1}$, with $r_0 = r_p = 6\left(10^6\right)$ m and $M = 0.70M_e$, we have

$$9\left(10^6\right) = \frac{6(10)^6}{\left(\dfrac{2(66.73)\,(10^{-12})\,(0.7)\,[5.976(10^{24})]}{6(10^6)v_P^2}\right) - 1}$$

$$v_A = 7471.89 \text{ m/s} = 7.47 \text{ km/s} \qquad \textbf{Ans.}$$

13–122.

A satellite S travels in a circular orbit around the earth. A rocket is located at the apogee of its elliptical orbit for which $e = 0.58$. Determine the sudden change in speed that must occur at A so that the rocket can enter the satellite's orbit while in free flight along the blue elliptical trajectory. When it arrives at B, determine the sudden adjustment in speed that must be given to the rocket in order to maintain the circular orbit.



SOLUTION

Central-Force Motion: Here, $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13–21] and $h = r_0 v_0$ [Eq. 13–20]. Substitute these values into Eq. 13–17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0^2 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad \textbf{(1)}$$

Rearrange Eq. (1) gives

$$\frac{1}{1 + e} = \frac{GM_e}{r_0 v_0^2} \qquad \textbf{(2)}$$

Rearrange Eq. (2), we have

$$v_0 = \sqrt{\frac{(1 + e)GM_e}{r_0}} \qquad \textbf{(3)}$$

Substitute Eq. (2) into Eq. 13–27, $r_a = \dfrac{r_0}{\left(2\,GM_e/r_0 v_0^2\right) - 1}$, we have

$$r_a = \frac{r_0}{2\left(\frac{1}{1 + e}\right) - 1} \qquad \text{or} \qquad r_0 = \left(\frac{1 - e}{1 + e}\right)r_a \qquad \textbf{(4)}$$

or the first elliptical orbit $e = 0.58$, from Eq. (4)

$$(r_p)_1 = r_0 = \left(\frac{1 - 0.58}{1 + 0.58}\right)\left[120(10^6)\right] = 31.899(10^6)\text{ m}$$

Substitute $r_0 = (r_p)_1 = 31.899(10^6)$ m into Eq. (3) yields

$$(v_p)_1 = \sqrt{\frac{(1 + 0.58)(66.73)(10^{-12})(5.976)(10^{24})}{31.899(10^6)}} = 4444.34\text{ m/s}$$

Applying Eq. 13–20, we have

$$(v_a)_1 = \left(\frac{r_p}{r_a}\right)(v_p)_1 = \left[\frac{31.899(10^6)}{120(10^6)}\right](4444.34) = 1181.41\text{ m/s}$$

When the rocket travels along the second elliptical orbit, from Eq. (4), we have

$$10(10^6) = \left(\frac{1 - e}{1 + e}\right)\left[120(10^6)\right] \qquad e = 0.8462$$

Substitute $r_0 = (r_p)_2 = 10(10^6)$ m into Eq. (3) yields

$$(v_p) = \sqrt{\frac{(1 + 0.8462)(66.73)(10^{-12})(5.967)(10^{24})}{10(10^6)}} = 8580.25\text{ m/s}$$

13–122. continued

And in Eq. 13–20, we have

$$(v_a)_2 = \left[\frac{(r_p)_2}{(r_a)_2}\right](v_p)_2 = \left[\frac{10(10^6)}{120(10^6)}\right](8580.25) = 715.02 \text{ m/s}$$

For the rocket to enter into orbit two from orbit one at A, its speed must be decreased by

$$\Delta v = (v_a)_1 - (v_a)_2 = 1184.41 - 715.02 = 466 \text{ m/s} \qquad \textbf{Ans.}$$

If the rocket travels in a circular free-flight trajectory, its speed is given by Eq. 13–25.

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{10(10^6)}} = 6314.89 \text{ m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = (v_p)_2 - v_e = 8580.25 - 6314.89 = 2265.36 \text{ m/s} = 2.27 \text{ km/s} \qquad \textbf{Ans.}$$

13–123.

An asteroid is in an elliptical orbit about the sun such that its perihelion distance is $9.30(10^9)$ km. If the eccentricity of the orbit is $e = 0.073$, determine the aphelion distance of the orbit.

SOLUTION

$$r_p = r_0 = 9.30(10^9)\text{ km}$$

$$e = \frac{ch^2}{GM_s} = \frac{1}{r_0}\left(1 - \frac{GM_s}{r_0 v_0^2}\right)\left(\frac{r_0 v_0^2}{GM_s}\right)$$

$$e = \left(\frac{r_0\, v_0^2}{GM_s} - 1\right)$$

$$\frac{r_0\, v_0^2}{GM_s} = e + 1 \qquad \textbf{(1)}$$

$$\frac{GM_s}{r_0 v_0^2} = \left(\frac{1}{e + 1}\right)$$

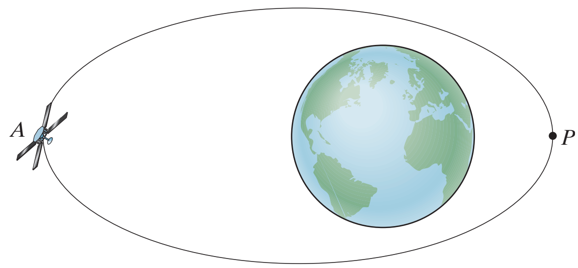
$$r_a = \frac{r_0}{\frac{2GM_s}{r_0\, v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e + 1}\right) - 1} \qquad \textbf{(2)}$$

$$r_a = \frac{r_0(e + 1)}{(1 - e)} = \frac{9.30(10^9)(1.073)}{0.927}$$

$$r_a = 10.8(10^9)\text{ km} \qquad \textbf{Ans.}$$

***13–124.**

An elliptical path of a satellite has an eccentricity $e = 0.130$. If it has a speed of 15 Mm/h when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?



SOLUTION

$$e = 0.130$$

$$v_p = v_0 = 15\text{ Mm/h} = 4.167\text{ km/s}$$

$$e = \frac{Ch^2}{GM_e} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_e}\right)$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e+1$$

$$r_0 = \frac{(e+1)GM_e}{v_0^2}$$

$$= \frac{1.130(66.73)(10^{-12})(5.976)(10^{24})}{\left[4.167(10^3)\right]^2}$$

$$= 25.96\text{ Mm}$$

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1}$$

$$r_A = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e+1}\right) - 1}$$

$$r_A = \frac{r_0(e+1)}{1-e}$$

$$= \frac{25.96(10^6)(1.130)}{0.870}$$

$$= 33.71(10^6)\text{ m} = 33.7\text{ Mm}$$

$$v_A = \frac{v_0 r_0}{r_A}$$

$$= \frac{15(25.96)(10^6)}{33.71(10^6)}$$

$$= 11.5\text{ Mm/h} \qquad \textbf{Ans.}$$

$$d = 33.71(10^6) - 6.378(10^6)$$

$$= 27.3\text{ Mm} \qquad \textbf{Ans.}$$

13–125.

A satellite is launched with an initial velocity $v_0 = 2500$ mi/h parallel to the surface of the earth. Determine the required altitude (or range of altitudes) above the earth's surface for launching if the free-flight trajectory is to be (a) circular, (b) parabolic, (c) elliptical, and (d) hyperbolic. Take $G = 34.4(10^{-9})(\text{lb}\cdot\text{ft}^2)/\text{slug}^2$, $M_e = 409(10^{21})$ slug, the earth's radius $r_e = 3960$ mi, and 1 mi = 5280 ft.

SOLUTION

$$v_0 = 2500 \text{ mi/h} = 3.67(10^3) \text{ ft/s}$$

(a) $$e = \frac{C^2 h}{GM_e} = 0 \qquad \text{or } C = 0$$

$$1 = \frac{GM_e}{r_0\, v_0^2}$$

$$GM_e = 34.4(10^{-9})(409)(10^{21})$$

$$= 14.07(10^{15})$$

$$r_0 = \frac{GM_e}{v_0^2} = \frac{14.07(10^{15})}{[3.67(10^{13})]^2} = 1.046(10^9) \text{ ft}$$

$$r = \frac{1.047(10^9)}{5280} - 3960 = 194(10^3) \text{ mi}$$ **Ans.**

(b) $$e = \frac{C^2 h}{GM_e} = 1$$

$$\frac{1}{GM_e}(r_0^2 v_0^2)\left(\frac{1}{r_0}\right)\left(1 - \frac{GM_e}{r_0\, v_0^2}\right) = 1$$

$$r_0 = \frac{2GM_e}{v_0^2} = \frac{2(14.07)(10^{15})}{[3.67(10^3)]^2} = 2.09(10^9) \text{ ft} = 396(10^3) \text{ mi}$$

$$r = 396(10^3) - 3960 = 392(10^3) \text{ mi}$$ **Ans.**

(c) $$e < 1$$

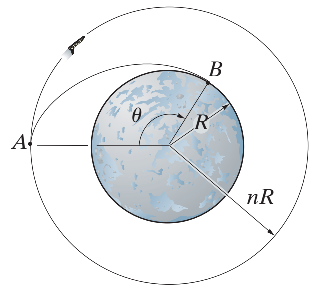
$$194(10^3) \text{ mi} < r < 392(10^3) \text{ mi}$$ **Ans.**

(d) $$e > 1$$

$$r > 392(10^3) \text{ mi}$$ **Ans.**

13–126.

A probe has a circular orbit around a planet of radius R and mass M. If the radius of the orbit is nR and the explorer is traveling with a constant speed v_0, determine the angle θ at which it lands on the surface of the planet B when its speed is reduced to kv_0, where $k < 1$ at point A.



SOLUTION

When the probe is orbiting the planet in a circular orbit of radius $r_O = nR$, its speed is given by

$$v_O = \sqrt{\frac{GM}{r_O}} = \sqrt{\frac{GM}{nR}}$$

The probe will enter the elliptical trajectory with its apoapsis at point A if its speed is decreased to $v_a = kv_O = k\sqrt{\dfrac{GM}{nR}}$ at this point. When it lands on the surface of the planet, $r = r_B = R$.

$$\frac{1}{r} = \frac{1}{r_P}\left(1 - \frac{GM}{r_P v_P{}^2}\right)\cos\theta + \frac{GM}{r_P{}^2 v_P{}^2}$$

$$\frac{1}{R} = \left(\frac{1}{r_P} - \frac{GM}{r_P{}^2 v_P{}^2}\right)\cos\theta + \frac{GM}{r_P{}^2 v_P{}^2} \qquad \textbf{(1)}$$

Since $h = r_a v_a = nR\left(k\sqrt{\dfrac{GM}{nR}}\right) = k\sqrt{nGMR}$ is constant,

$$r_P v_P = h$$

$$v_P = \frac{k\sqrt{nGMR}}{r_P} \qquad \textbf{(2)}$$

Also,

$$r_a = \frac{r_P}{\dfrac{2GM}{r_P v_P{}^2} - 1}$$

$$nR = \frac{r_P}{\dfrac{2GM}{r_P v_P{}^2} - 1}$$

$$v_P{}^2 = \frac{2nGMR}{r_p(r_p + nR)} \qquad \textbf{(3)}$$

Solving Eqs.(2) and (3),

$$r_p = \frac{k^2 n}{2 - k^2}R \qquad v_p = \frac{2 - k^2}{k}\sqrt{\frac{GM}{nR}}$$

Substituting the result of r_p and v_p into Eq. (1),

$$\frac{1}{R} = \left(\frac{2 - k^2}{k^2 nR} - \frac{1}{k^2 nR}\right)\cos\theta + \frac{1}{k^2 nR}$$

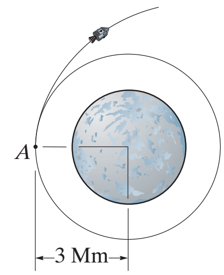
$$\theta = \cos^{-1}\left(\frac{k^2 n - 1}{1 - k^2}\right)$$

Here θ was measured from periapsis. When measured from apoapsis, as in the figure then

$$\theta = \pi - \cos^{-1}\left(\frac{k^2 n - 1}{1 - k^2}\right) \qquad \textbf{Ans.}$$

13–127.

Upon completion of the moon exploration mission, the command module, which was originally in a circular orbit as shown, is given a boost so that it escapes from the moon's gravitational field. Determine the necessary increase in velocity so that the command module follows a parabolic trajectory. The mass of the moon is 0.01230 M_e.



SOLUTION

When the command module is moving around the circular orbit of radius $r_0 = 3(10^6)$ m, its velocity is

$$v_c = \sqrt{\frac{GM_m}{r_0}} = \sqrt{\frac{66.73(10^{-12})(0.0123)(5.976)(10^{24})}{3(10^6)}}$$
$$= 1278.67 \text{ m/s}$$

The escape velocity of the command module entering into the parabolic trajectory is

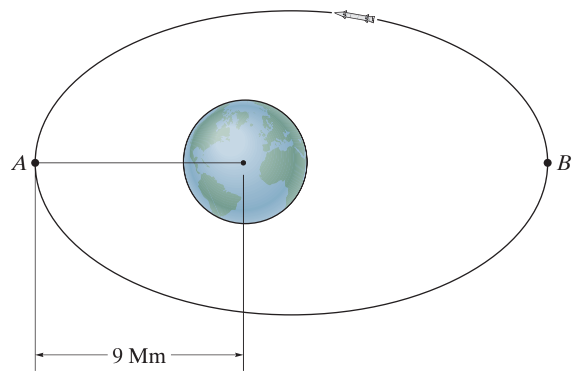
$$v_e = \sqrt{\frac{2GM_m}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})(0.0123)(5.976)(10^{24})}{3(10^6)}}$$
$$= 1808.31 \text{ m/s}$$

Thus, the required increase in the command module is

$$\Delta v = v_e - v_c = 1808.31 - 1278.67 = 529.64 \text{ m/s} = 530 \text{ m/s} \qquad \textbf{Ans.}$$

***13–128.**

The rocket is traveling in a free-flight elliptical orbit about the earth such that $e = 0.76$ and its perigee is 9 Mm as shown. Determine its speed when it is at point B. Also determine the sudden decrease in speed the rocket must experience at A in order to travel in a circular orbit about the earth.



SOLUTION

Central-Force Motion: Here $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ [Eq. 13–21] and $h = r_0 v_0$ [Eq. 13–20] Substitute these values into Eq. 13–17 gives

$$e = \frac{ch^2}{GM_e} = \frac{\frac{1}{r_0}\left(1 - \frac{GM_e}{r_0^2 v_0^2}\right)\left(r_0^2 v_0^2\right)}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1 \qquad \textbf{(1)}$$

Rearrange Eq.(1) gives

$$\frac{1}{1 + e} = \frac{GM_e}{r_0 v_0^2} \qquad \textbf{(2)}$$

Rearrange Eq.(2), we have

$$v_0 = \sqrt{\frac{(1 + e)\, GM_e}{r_0}} \qquad \textbf{(3)}$$

Substitute Eq.(2) into Eq. 13–27, $r_a = \dfrac{r_0}{(2GM_e/r_0 v_0^2) - 1}$, we have

$$r_a = \frac{r_0}{2\left(\frac{1}{1 + e}\right) - 1} \qquad \textbf{(4)}$$

Rearrange Eq.(4), we have

$$r_a = \left(\frac{1 + e}{1 - e}\right) r_0 = \left(\frac{1 + 0.76}{1 - 0.76}\right)\left[9(10^6)\right] = 66.0(10^6)\ \text{m}$$

Substitute $r_0 = r_p = 9\left(10^6\right)$ m into Eq.(3) yields

$$v_p = \sqrt{\frac{(1 + 0.76)(66.73)(10^{-12})(5.976)\,(10^{24})}{9(10^6)}} = 8830.82\ \text{m/s}$$

Applying Eq. 13–20, we have

$$v_a = \left(\frac{r_p}{r_a}\right) v_p = \left[\frac{9(10^6)}{66.0(10^6)}\right](8830.82) = 1204.2\ \text{m/s} = 1.20\ \text{km/s} \qquad \textbf{Ans.}$$

If the rocket travels in a circular free-flight trajectory, its speed is given by Eq. 13–25.

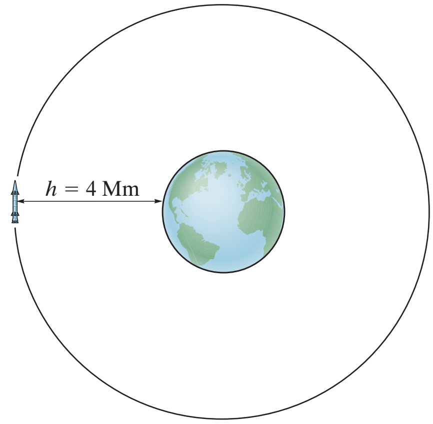
$$v_e = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{9(10^6)}} = 6656.48\ \text{m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = v_p - v_c = 8830.82 - 6656.48 = 2174.34\ \text{m/s} = 2.17\ \text{km/s} \qquad \textbf{Ans.}$$

13–129.

A rocket is in circular orbit about the earth at an altitude above the earth's surface of $h = 4$ Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



SOLUTION

Circular orbit:

$$v_C = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 6198.8 \text{ m/s}$$

Parabolic orbit:

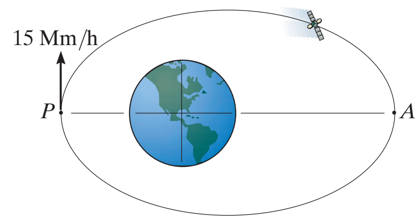
$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2(66.73)(10^{-12})5.976(10^{24})}{4000(10^3) + 6378(10^3)}} = 8766.4 \text{ m/s}$$

$$\Delta v = v_e - v_C = 8766.4 - 6198.8 = 2567.6 \text{ m/s}$$

$$\Delta v = 2.57 \text{ km/s}$$ **Ans.**

13–130.

The satellite is in an elliptical orbit having an eccentricity of $e = 0.15$. If its velocity at perigee is $v_P = 15\,\text{Mm/h}$, determine its velocity at apogee A and the period of the satellite.



SOLUTION

Here, $v_P = \left[15(10^6)\,\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 4166.67\text{ m/s}.$

$$h = r_P v_P$$

$$h = r_P\,(4166.67) = 4166.67 r_p \qquad \textbf{(1)}$$

and

$$C = \frac{1}{r_P}\left(1 - \frac{GM_e}{r_P\,{v_P}^2}\right)$$

$$C = \frac{1}{r_p}\left[1 - \frac{66.73(10^{-12})(5.976)(10^{24})}{r_p(4166.67^2)}\right]$$

$$C = \frac{1}{r_P}\left[1 - \frac{22.97(10^6)}{r_P}\right] \qquad \textbf{(2)}$$

$$e = \frac{Ch^2}{GM_e}$$

$$0.15 = \frac{\dfrac{1}{r_P}\left[1 - \dfrac{22.97(10^6)}{r_P}\right](4166.67\,r_P)^2}{66.73(10^{-12})(5.976)(10^{24})}$$

$$r_P = 26.415(10^6)\text{ m}$$

Using the result of r_p

$$r_A = \frac{r_P}{\dfrac{2GM_e}{r_P\,{v_P}^2} - 1}$$

$$= \frac{26.415(10^6)}{\dfrac{2(66.73)(10^{-12})(5.976)(10^{24})}{26.415(10^6)(4166.67^2)} - 1}$$

$$= 35.738(10^6)\text{ m}$$

Since $h = r_P\,v_P = 26.415(10^6)(4166.67^2) = 110.06(10^9)\text{ m}^2\text{/s}$ is constant,

$$r_A\,v_A = h$$

$$35.738(10^6)v_A = 110.06(10^9)$$

$$v_A = 3079.71\text{ m/s} = 3.08\text{ km/s} \qquad \textbf{Ans.}$$

Using the results of h, r_A, and r_P,

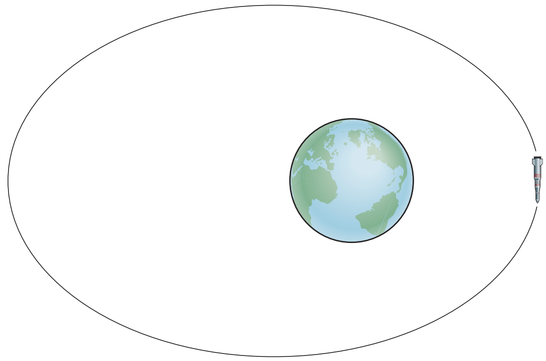
$$T = \frac{\pi}{6}\left(r_P + r_A\right)\sqrt{r_P\,r_A}$$

$$= \frac{\pi}{110.06(10^9)}\left[26.415(10^6) + 35.738(10^6)\right]\sqrt{26.415(10^6)(35.738)(10^6)}$$

$$= 54\,508.43\text{ s} = 15.1\text{ hr} \qquad \textbf{Ans.}$$

13–131.

A rocket is in a free-flight elliptical orbit about the earth such that the eccentricity of its orbit is e and its perigee is r_0. Determine the minimum increment of speed it should have in order to escape the earth's gravitational field when it is at this point along its orbit.



SOLUTION

To escape the earth's gravitational field, the rocket has to make a parabolic trajectory.

ParabolicTrajectory:

$$v_e = \sqrt{\frac{2GM_e}{r_0}}$$

Elliptical Orbit:

$$e = \frac{Ch^2}{GM_e} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0\, v_0$$

$$e = \frac{1}{GM_e\, r_0}\left(1 - \frac{GM_e}{r_0\, v_0^2}\right)(r_0\, v_0)^2$$

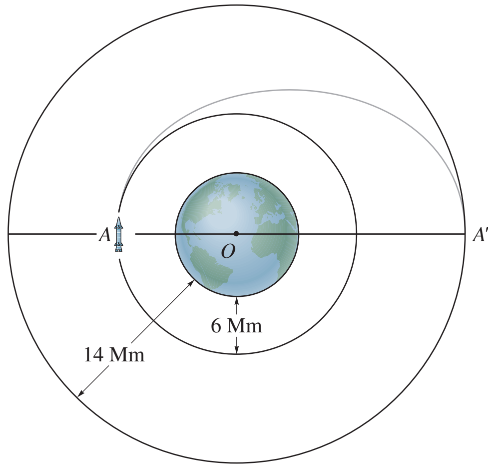
$$e = \left(\frac{r_0\, v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0\, v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e\,(e+1)}{r_0}}$$

$$\Delta v = \sqrt{\frac{2GM_e}{r_0}} - \sqrt{\frac{GM_e\,(e+1)}{r_0}} = \sqrt{\frac{GM_e}{r_0}}\left(\sqrt{2} - \sqrt{1+e}\right) \qquad \textbf{Ans.}$$

***13–132.**

The rocket shown is originally in a circular orbit 6 Mm above the surface of the earth. It is required that it travel in another circular orbit having an altitude of 14 Mm. To do this, the rocket is given a short pulse of power at A so that it travels in free flight along the gray elliptical path from the first orbit to the second orbit. Determine the necessary speed it must have at A just after the power pulse, and at the time required to get to the outer orbit along the path AA'. What adjustment in speed must be made at A' to maintain the second circular orbit?



SOLUTION

Central-Force Motion: Substitute Eq. 13–27, $r_a = \dfrac{r_0}{(2GM/r_0 v_0^2) - 1}$, with $r_a = (14 + 6.378)(10^6) = 20.378(10^6)$ m and $r_0 = r_p = (6 + 6.378)(10^6) = 12.378(10^6)$ m, we have

$$20.378(10^6) = \frac{12.378(10^6)}{\left(\dfrac{2(66.73)(10^{-12})[5.976(10^{24})]}{12.378(10^6)v_p^2}\right) - 1}$$

$$v_p = 6331.27 \text{ m/s}$$

Applying Eq. 13–20. we have

$$v_a = \left(\frac{r_p}{r_a}\right)v_p = \left[\frac{12.378(10^6)}{20.378(10^6)}\right](6331.27) = 3845.74 \text{ m/s}$$

Eq. 13–20 gives $h = r_p v_p = 12.378(10^6)(6331.27) = 78.368(10^9)\ \text{m}^2/\text{s}$. Thus, applying Eq. 13–31, we have

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{78.368(10^9)}[(12.378 + 20.378)(10^6)]\sqrt{12.378(20.378)}(10^6)$$

$$= 20854.54 \text{ s}$$

The time required for the rocket to go from A to A' (half the orbit) is given by

$$t = \frac{T}{2} = 10427.38 \text{ s} = 2.90 \text{ hr} \qquad \textbf{Ans.}$$

In order for the satellite to stay in the second circular orbit, it must achieve a speed of (Eq. 13–25)

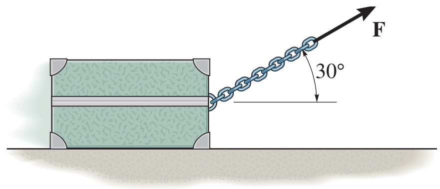
$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{20.378(10^6)}} = 4423.69 \text{ m/s} = 4.42 \text{ km/s} \qquad \textbf{Ans.}$$

The speed for which the rocket must be increased in order to enter the second circular orbit at A' is

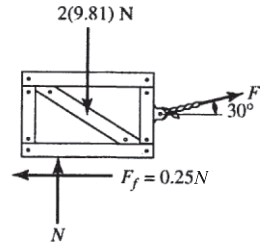
$$\Delta v = v_c - v_a = 4423.69 - 3845.74 = 578 \text{ m/s} \qquad \textbf{Ans.}$$

14–1.

The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



SOLUTION



Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25N$. Applying Eq. 13–7, we have

$$+\uparrow \sum F_y = ma_y; \qquad N + 100 \sin 30° - 20(9.81) = 20(0)$$

$$N = 146.2 \text{ N}$$

Principle of Work and Energy: The horizontal component of force F which acts in the direction of displacement does *positive* work, whereas the friction force $F_f = 0.25(146.2) = 36.55$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force F and the weight of the crate do not displace hence do no work. Applying Eq.14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

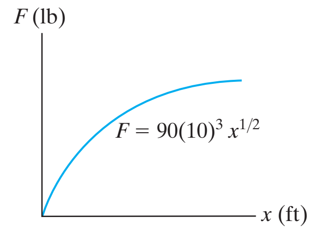
$$\frac{1}{2}(20)(8^2) + \int_{15 \text{ m}}^{25 \text{ m}} 100 \cos 30° \, ds$$

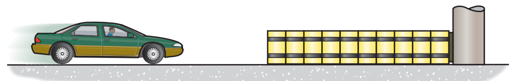
$$- \int_{15 \text{ m}}^{25 \text{ m}} 36.55 \, ds = \frac{1}{2}(20)v^2$$

$v = 10.7$ m/s **Ans.**

14–2.

For protection, the barrel barrier is placed in front of the bridge pier. If the relation between the force and deflection of the barrier is $F = (90(10^3)x^{1/2})$ lb, where x is in ft, determine the car's maximum penetration in the barrier. The car has a weight of 4000 lb and it is traveling with a speed of 75 ft/s just before it hits the barrier.





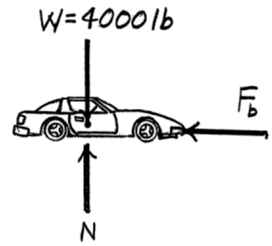
SOLUTION

Principle of Work and Energy: The speed of the car just before it crashes into the barrier is $v_1 = 75$ ft/s. The maximum penetration occurs when the car is brought to a stop, i.e., $v_2 = 0$. Referring to the free-body diagram of the car, Fig. *a*, **W** and **N** do no work; however, $\mathbf{F}_b$ does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(75^2) + \left[-\int_0^{x_{max}} 90(10^3)x^{1/2}dx\right] = 0$$

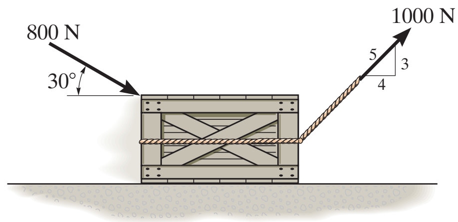
$$x_{max} = 3.24 \text{ ft} \qquad \textbf{Ans.}$$



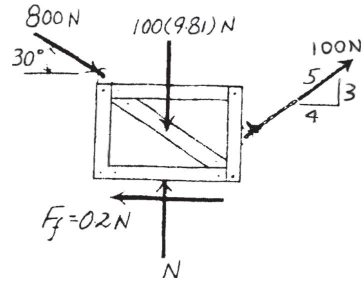
(a)

14–3.

The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



SOLUTION



Equations of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.2N$. Applying Eq. 13–7, we have

$$+\uparrow \Sigma F_y = ma_y; \qquad N + 1000\left(\frac{3}{5}\right) - 800 \sin 30° - 100(9.81) = 100(0)$$

$$N = 781 \text{ N}$$

Principle of Work and Energy: The horizontal components of force 800 N and 1000 N which act in the direction of displacement do *positive* work, whereas the friction force $F_f = 0.2(781) = 156.2$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of 800 N and 1000 N force and the weight of the crate do not displace, hence they do no work. Since the crate is originally at rest, $T_1 = 0$. Applying Eq. 14–7, we have

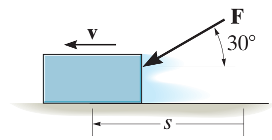
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 800 \cos 30°(s) + 1000\left(\frac{4}{5}\right)s - 156.2s = \frac{1}{2}(100)\left(6^2\right)$$

$$s = 1.35\text{m} \qquad \textbf{Ans.}$$

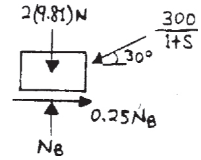
***14–4.**

The 2-kg block is subjected to a force having a constant direction and a magnitude $F = (300/(1 + s))$ N, where s is in meters. When $s = 4$ m, the block is moving to the left with a speed of 8 m/s. Determine its speed when $s = 12$ m. The coefficient of kinetic friction between the block and the ground is $\mu_k = 0.25$.



SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad N_B = 2(9.81) + \frac{150}{1 + s}$$



$$T_1 + \Sigma U_{1-2} = T_2$$

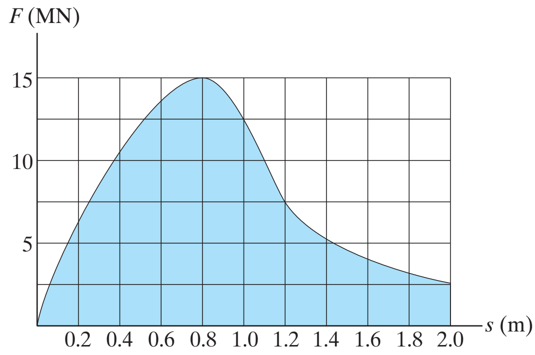
$$\frac{1}{2}(2)(8)^2 - 0.25[2(9.81)(12 - 4)] - 0.25\int_4^{12} \frac{150}{1 + s}\,ds + \int_4^{12}\left(\frac{300}{1 + s}\right)ds\,\cos 30^\circ = \frac{1}{2}(2)(v_2^2)$$

$$v_2^2 = 24.76 - 37.5\ln\left(\frac{1 + 12}{1 + 4}\right) + 259.81\ln\left(\frac{1 + 12}{1 + 4}\right)$$

$$v_2 = 15.4 \text{ m/s}$$ **Ans.**

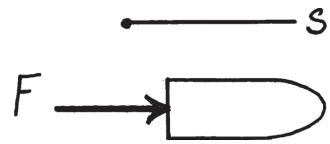
14–5.

When a 7-kg projectile is fired from a cannon barrel that has a length of 2 m, the explosive force exerted on the projectile, while it is in the barrel, varies in the manner shown. Determine the approximate muzzle velocity of the projectile at the instant it leaves the barrel. Neglect the effects of friction inside the barrel and assume the barrel is horizontal.



SOLUTION

The work done is measured as the area under the force–displacement curve. This area is approximately 31.5 squares. Since each square has an area of $2.5\left(10^6\right)(0.2)$,



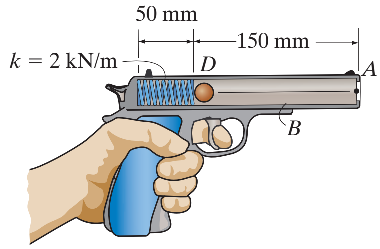
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[(31.5)(2.5)\left(10^6\right)(0.2)\right] = \frac{1}{2}(7)(v_2)^2$$

$v_2 = 2121 \text{ m/s} = 2.12 \text{ km/s}$ (approx.) **Ans.**

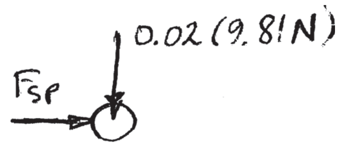
14–6.

The spring in the toy gun has an unstretched length of 100 mm. It is compressed and locked in the position shown. When the trigger is pulled, the spring unstretches 12.5 mm, and the 20-g ball moves along the barrel. Determine the speed of the ball when it leaves the gun. Neglect friction.



SOLUTION

Principle of Work and Energy: Referring to the free-body diagram of the ball bearing shown in Fig. *a*, notice that $\mathbf{F}_{sp}$ does positive work. The spring has an initial and final compression of $s_1 = 0.1 - 0.05 = 0.05\text{ m}$ and $s_2 = 0.1 - (0.05 + 0.0125) = 0.0375\text{ m}$.



(a)

$$T_1 + \Sigma U_{1-2} = T_2$$

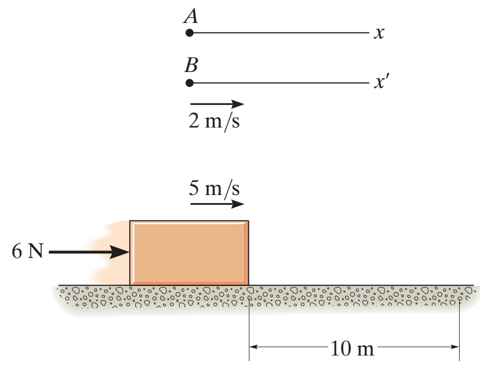
$$0 + \left[\frac{1}{2}ks_1{}^2 - \frac{1}{2}ks_2{}^2\right] = \frac{1}{2}mv_A{}^2$$

$$0 + \left[\frac{1}{2}(2000)(0.05)^2 - \frac{1}{2}(2000)(0.0375^2)\right] = \frac{1}{2}(0.02)v_A{}^2$$

$$v_A = 10.5\text{ m/s}$$ **Ans.**

14–7.

As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of 5 m/s and travels 10 m, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of 2 m/s relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.



SOLUTION

Observer A:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10)(5)^2 + 6(10) = \frac{1}{2}(10)v_2^2$$

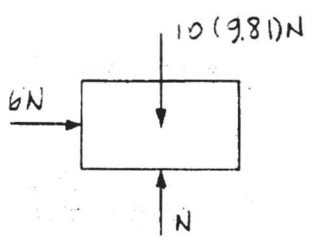
$$v_2 = 6.08 \text{ m/s} \qquad \textbf{Ans.}$$

Observer B:

$$F = ma$$

$$6 = 10a \qquad a = 0.6 \text{ m/s}^2$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$



$$10 = 0 + 5t + \frac{1}{2}(0.6)t^2$$

$$t^2 + 16.67t - 33.33 = 0$$

$$t = 1.805 \text{ s}$$

At $v = 2$ m/s, $s' = 2(1.805) = 3.609$ m

Block moves $10 - 3.609 = 6.391$ m

Thus

$$T_1 + \Sigma U_{1-2} = T_2$$

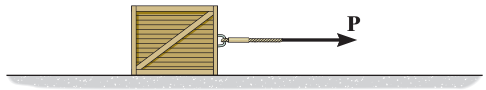
$$\frac{1}{2}(10)(3)^2 + 6(6.391) = \frac{1}{2}(10)v_2^2$$

$$v_2 = 4.08 \text{ m/s} \qquad \textbf{Ans.}$$

Note that this result is 2 m/s less than that observed by A.

***14–8.**

If the 50-kg crate is subjected to a force of $P = 200$ N, determine its speed when it has traveled 15 m starting from rest. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$.



SOLUTION

Free-Body Diagram: Referring to the free-body diagram of the crate, Fig. *a*,

$+\uparrow F_y = ma_y;\qquad N - 50(9.81) = 50(0)\qquad N = 490.5\text{ N}$

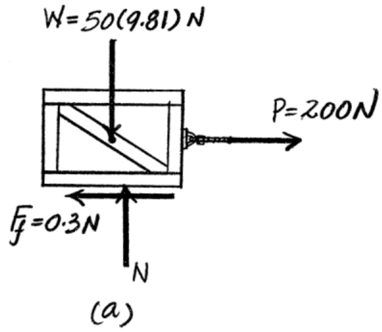
Thus, the frictional force acting on the crate is $F_f = \mu_k N = 0.3(490.5) = 147.15$ N.

Principle of Work and Energy: Referring to Fig. *a*, only $\mathbf{P}$ and $\mathbf{F}_f$ do work. The work of $\mathbf{P}$ will be positive, whereas $\mathbf{F}_f$ does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

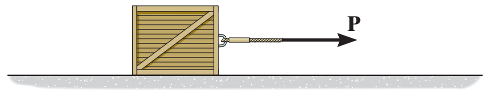
$$0 + 200(15) - 147.15(15) = \frac{1}{2}(50)v^2$$

$$v = 5.63\text{ m/s}$$ **Ans.**



14–9.

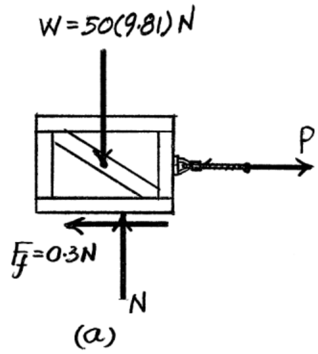
If the 50-kg crate starts from rest and attains a speed of 6 m/s when it has traveled a distance of 15 m, determine the force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$.



SOLUTION

Free-Body Diagram: Referring to the free-body diagram of the crate, Fig. *a*,

$$+\uparrow F_y = ma_y; \qquad N - 50(9.81) = 50(0) \qquad N = 490.5\text{ N}$$



Thus, the frictional force acting on the crate is $F_f = \mu_k N = 0.3(490.5) = 147.15\text{ N}$.

Principle of Work and Energy: Referring to Fig. *a*, only **P** and $\mathbf{F}_f$ do work. The work of **P** will be positive, whereas $\mathbf{F}_f$ does negative work.

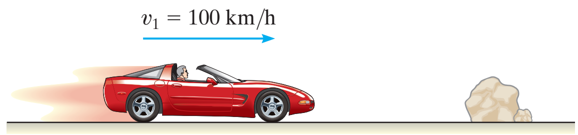
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + P(15) - 147.15(15) = \frac{1}{2}(50)(6^2)$$

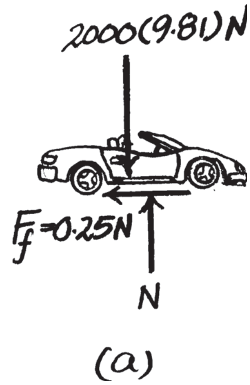
$$P = 207\text{ N}$$ **Ans.**

14–10.

The 2-Mg car has a velocity of $v_1 = 100\,\text{km/h}$ when the driver sees an obstacle in front of the car. If it takes 0.75 s for him to react and lock the brakes, causing the car to skid, determine the distance the car travels before it stops. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.25$.



SOLUTION



Free-Body Diagram: The normal reaction **N** on the car can be determined by writing the equation of motion along the y axis. By referring to the free-body diagram of the car, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 2000(9.81) = 2000(0) \qquad N = 19\,620\text{ N}$$

Since the car skids, the frictional force acting on the car is $F_f = \mu_k N = 0.25(19620) = 4905N$.

Principle of Work and Energy: By referring to Fig. a, notice that only $\mathbf{F}_f$ does work, which is negative. The initial speed of the car is $v_1 = \left[100(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) =$ 27.78 m/s. Here, the skidding distance of the car is denoted as s'.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2000)(27.78^2) + (-4905s') = 0$$

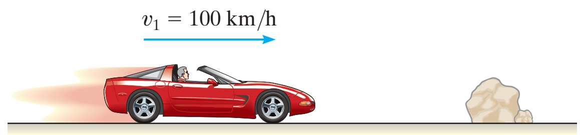
$$s' = 157.31\text{ m}$$

The distance traveled by the car during the reaction time is $s'' = v_1 t = 27.78(0.75) = 20.83$ m. Thus, the total distance traveled by the car before it stops is

$$s = s' + s'' = 157.31 + 20.83 = 178.14\text{ m} = 178\text{ m} \qquad \textbf{Ans.}$$

14–11.

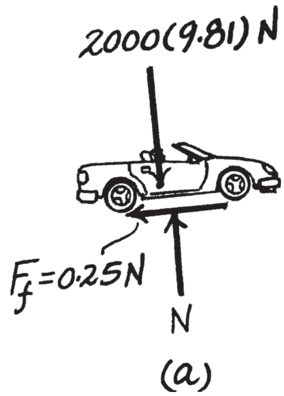
The 2-Mg car has a velocity of $v_1 = 100\,\text{km/h}$ when the driver sees an obstacle in front of the car. It takes 0.75 s for him to react and lock the brakes, causing the car to skid. If the car stops when it has traveled a distance of 175 m, determine the coefficient of kinetic friction between the tires and the road.



SOLUTION

Free-Body Diagram: The normal reaction **N** on the car can be determined by writing the equation of motion along the y axis and referring to the free-body diagram of the car, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 2000(9.81) = 2000(0) \qquad N = 19\,620\text{ N}$$



Since the car skids, the frictional force acting on the car can be computed from $F_f = \mu_k N = \mu_k(19\,620)$.

Principle of Work and Energy: By referring to Fig. a, notice that only $\mathbf{F}_f$ does work, which is negative. The initial speed of the car is $v_1 = \left[100(10^3)\,\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) =$ 27.78 m/s. Here, the skidding distance of the car is s'.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2000)(27.78^2) + \left[-\mu_k(19\,620)s'\right] = 0$$

$$s' = \frac{39.327}{\mu_k}$$

The distance traveled by the car during the reaction time is $s'' = v_1 t = 27.78(0.75) = 20.83$ m. Thus, the total distance traveled by the car before it stops is

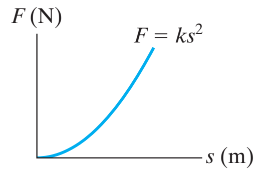
$$s = s' + s''$$

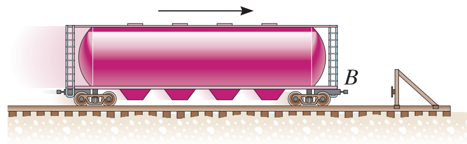
$$175 = \frac{39.327}{\mu_k} + 20.83$$

$$\mu_k = 0.255 \qquad \textbf{Ans.}$$

***14–12.**

Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.



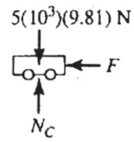


SOLUTION

$$\frac{1}{2}(5000)(4)^2 - \int_0^{0.2} ks^2\, ds = 0$$

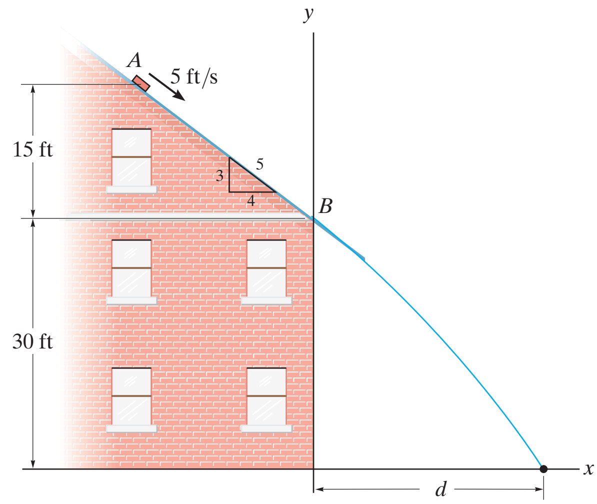
$$40\,000 - k\frac{(0.2)^3}{3} = 0$$

$$k = 15.0\ \text{MN/m}^2$$ **Ans.**



14–13.

The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the brick just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.

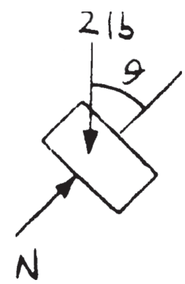


SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

$$v_B = 31.48 \text{ ft/s} = 31.5 \text{ ft/s}$$



$$\left(\xrightarrow{+}\right) \qquad s = s_0 + v_0 t$$

$$d = 0 + 31.48\left(\frac{4}{5}\right)t$$

$$(+\downarrow) \qquad s = s_0 + v_0 t - \frac{1}{2}a_c t^2$$

$$30 = 0 + 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(32.2)t^2$$

$$16.1t^2 + 18.888t - 30 = 0$$

Solving for the positive root,

$$t = 0.89916 \text{ s}$$

$$d = 31.48\left(\frac{4}{5}\right)(0.89916) = 22.6 \text{ ft}$$ **Ans.**

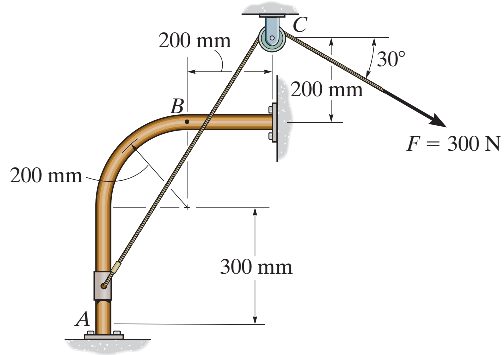
$$T_A + \Sigma U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2$$

$$v_C = 54.1 \text{ ft/s}$$ **Ans.**

14–14.

If the cord is subjected to a constant force of $F = 300$ N and the 15-kg smooth collar starts from rest at A, determine the velocity of the collar when it reaches point B. Neglect the size of the pulley.



SOLUTION

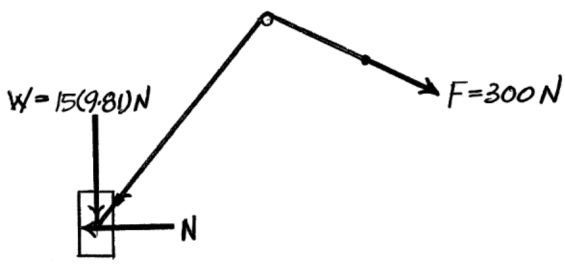
Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: Referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces vertically upward a distance $h = (0.3 + 0.2)$ m $= 0.5$ m, while force F displaces a distance of $s = AC - BC = \sqrt{0.7^2 + 0.4^2} - \sqrt{0.2^2 + 0.2^2} = 0.5234$ m. Here, the work of **F** is positive, whereas **W** does negative work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 300(0.5234) + [-15(9.81)(0.5)] = \frac{1}{2}(15)v_B^2$$

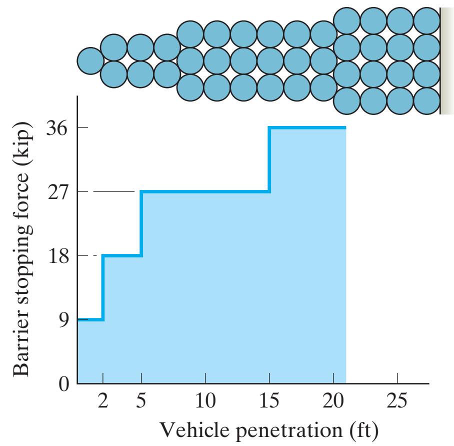
$$v_B = 3.335 \text{ m/s} = 3.34 \text{ m/s}$$ **Ans.**



(a)

14–15.

The crash cushion for a highway barrier consists of a nest of barrels filled with an impact-absorbing material. The barrier stopping force is measured versus the vehicle penetration into the barrier. Determine the distance a car having a weight of 4000 lb will penetrate the barrier if it is originally traveling at 55 ft/s when it strikes the first barrel.



SOLUTION

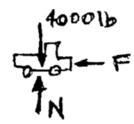
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(55)^2 - Area = 0$$

$$Area = 187.89 \text{ kip} \cdot \text{ft}$$

$$2(9) + (5 - 2)(18) + x(27) = 187.89$$

$x = 4.29 \text{ ft} < (15 - 5) \text{ ft}$ **(O.K!)**

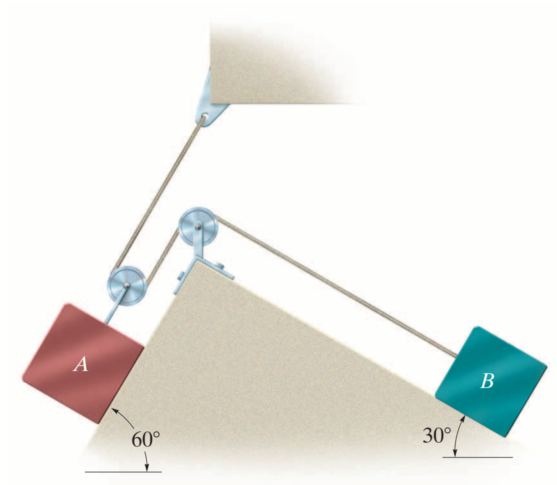


Thus

$s = 5 \text{ ft} + 4.29 \text{ ft} = 9.29 \text{ ft}$ **Ans.**

***14–16.**

Determine the velocity of the 60-lb block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



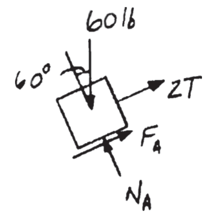
SOLUTION

Block A:

$+\nwarrow \Sigma F_y = ma_y;\qquad N_A - 60\cos 60° = 0$

$N_A = 30\text{ lb}$

$F_A = 0.1(30) = 3\text{ lb}$



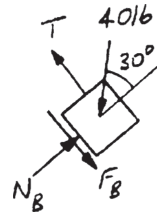
Block B:

$+\nearrow \Sigma F_y = ma_y;\qquad N_B - 40\cos 30° = 0$

$N_B = 34.64\text{ lb}$

$F_B = 0.1(34.64) = 3.464\text{ lb}$

Use the system of both blocks. N_A, N_B, T, and R do no work.



$T_1 + \Sigma U_{1-2} = T_2$

$$(0 + 0) + 60\sin 60°|\Delta s_A| - 40\sin 30°|\Delta s_B| - 3|\Delta s_A| - 3.464|\Delta s_B| = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)v_B^2$$

$2s_A + s_B = l$

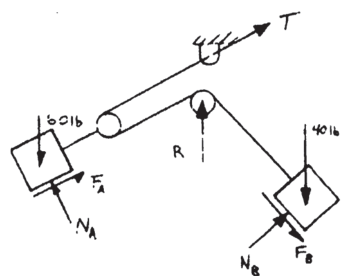
$2\Delta s_A = -\Delta s_B$

When $|\Delta s_B| = 2$ ft, $|\Delta s_A| = 1$ ft

Also,

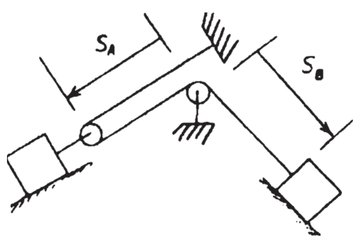
$2v_A = -v_B$

Substituting and solving,



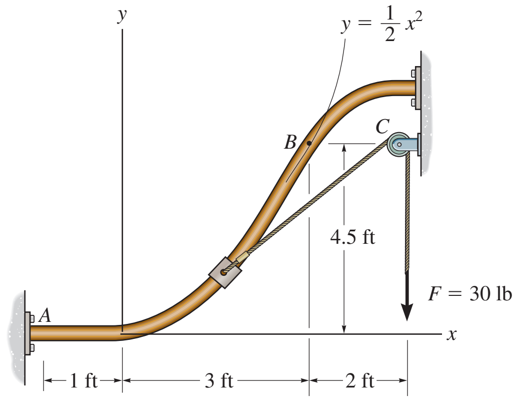
$v_A = 0.771\text{ ft/s}$ **Ans.**

$v_B = -1.54\text{ ft/s}$



14–17.

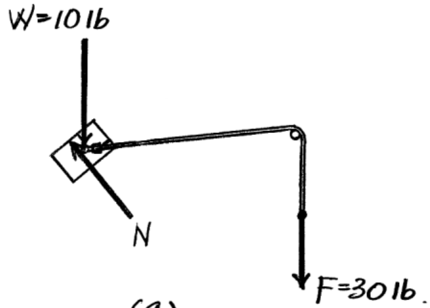
If the cord is subjected to a constant force of $F = 30$ lb and the smooth 10-lb collar starts from rest at A, determine its speed when it passes point B. Neglect the size of pulley C.



SOLUTION

Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: By referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces upward through a distance $h = 4.5$ ft, while force **F** displaces a distance of $s = AC - BC = \sqrt{6^2 + 4.5^2} - 2 = 5.5$ ft. The work of **F** is positive, whereas **W** does negative work.



(a)

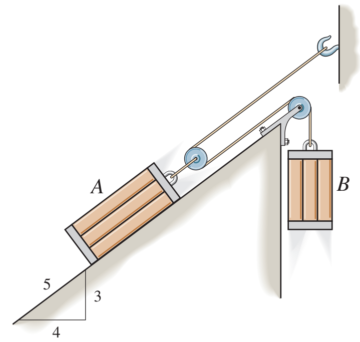
$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 30(5.5) + [-10(4.5)] = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$v_B = 27.8 \text{ ft/s}$$ **Ans.**

14–18.

The two blocks A and B have weights $W_A = 60$ lb and $W_B = 10$ lb. If the kinetic coefficient of friction between the incline and block A is $\mu_k = 0.2$, determine the speed of A after it moves 3 ft down the plane starting from rest. Neglect the mass of the cord and pulleys.



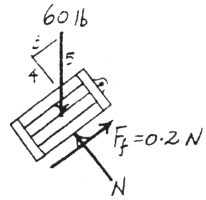
SOLUTION

Kinematics: The speed of the block A and B can be related by using position coordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

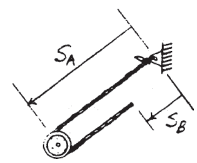
$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2(3) = 6 \text{ ft}$$

$$2v_A - v_B = 0 \qquad \textbf{(1)}$$



Equation of Motion: Applying Eq. 13–7, we have

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 60\left(\frac{4}{5}\right) = \frac{60}{32.2}(0) \qquad N = 48.0 \text{ lb}$$



Principle of Work and Energy: By considering the whole system, W_A which acts in the direction of the displacement does *positive* work. W_B and the friction force $F_f = \mu_k N = 0.2(48.0) = 9.60$ lb does *negative* work since they act in the opposite direction to that of displacement Here, W_A is being displaced vertically (downward) $\frac{3}{5}\Delta s_A$ and W_B is being displaced vertically (upward) Δs_B. Since blocks A and B are at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_A\left(\frac{3}{5}\Delta s_A\right) - F_f \Delta s_A - W_B \Delta s_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B \; v_B^2$$

$$60\left[\frac{3}{5}(3)\right] - 9.60(3) - 10(6) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$1236.48 = 60v_A^2 + 10v_B^2 \qquad \textbf{(2)}$$

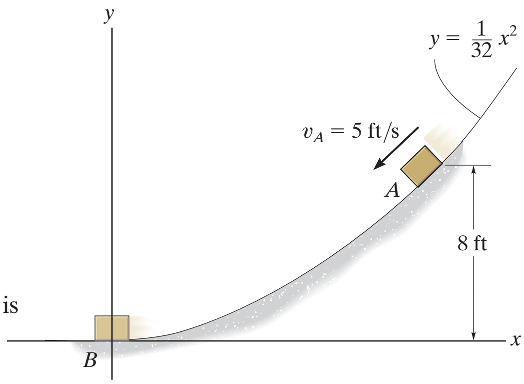
Eqs. (1) and (2) yields

$$v_A = 3.52 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_B = 7.033 \text{ ft/s}$$

14–19.

If the 10-lb block passes point A on the smooth track with a speed of $v_A = 5$ ft/s, determine the normal reaction on the block when it reaches point B.



SOLUTION

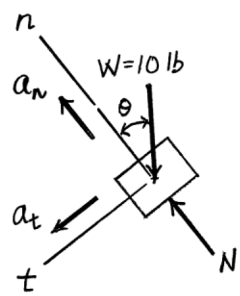
Free-Body Diagram: The free-body diagram of the block at an arbitrary position is shown in Fig. a.

Principle of Work and Energy: Referring to Fig. a, **N** does no work since it always acts perpendicular to the motion. When the block slides down the track from position A to position B, **W** displaces vertically downward $h = 8$ ft and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(5^2) + 10(8) = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^{\,2}$$

$$v_B = 23.24 \text{ ft/s}$$



(a)

Equation of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. By referring to Fig. a,

$$\Sigma F_n = ma_n; \qquad N - 10\cos\theta = \left(\frac{10}{32.2}\right)\left(\frac{v^2}{\rho}\right)$$

$$N = \frac{10}{32.2}\left(\frac{v^2}{\rho}\right) + 10\cos\theta \qquad \textbf{(1)}$$

Geometry: Here, $\dfrac{dy}{dx} = \dfrac{1}{16}x$ and $\dfrac{d^2y}{dx^2} = \dfrac{1}{16}$. The slope that the track at position B makes with the horizontal is $\theta_B = \tan^{-1}\left(\dfrac{dx}{dy}\right)\Bigg|_{x=0} = \tan^{-1}(0) = 0°$. The radius of curvature of the track at position B

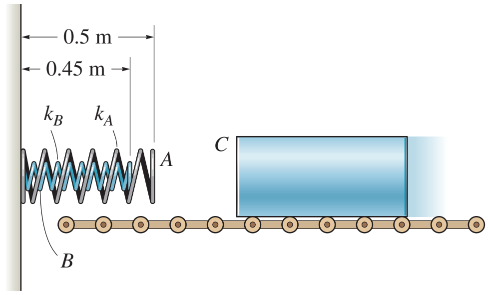
$$\rho_B = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(\dfrac{1}{16}x\right)^2\right]^{3/2}}{\left|\dfrac{1}{16}\right|}\Bigg|_{x=0} = 16 \text{ ft}$$

Substituting $\theta = \theta_B = 0°$, $v = v_B = 23.24$ ft/s, and $\rho = \rho_B = 16$ ft into Eq. (1),

$$N_B = \frac{10}{32.2}\left[\frac{23.24^2}{16}\right] + 10\cos 0° = 20.5 \text{ lb} \qquad \textbf{Ans.}$$

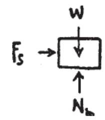
***14–20.**

The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed $v = 0.5$ m/s when it collides with the “nested” spring assembly. Determine the maximum deflection in each spring needed to stop the motion of the ingot. Take $k_A = 5$ kN/m, $k_B = 3$ kN/m.



SOLUTION

Assume both springs compress



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(1800)(0.5)^2 - \frac{1}{2}(5000)s^2 - \frac{1}{2}(3000)(s - 0.05)^2 = 0$$

$$225 - 2500\,s^2 - 1500(s^2 - 0.1\,s + 0.0025) = 0$$

$$s^2 - 0.0375\,s - 0.05531 = 0$$

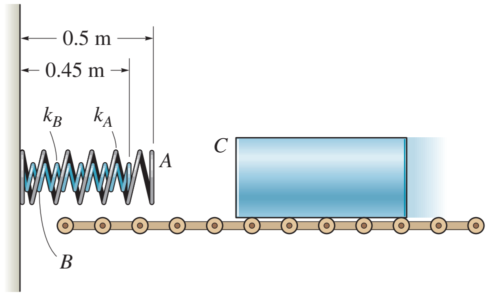
$s = 0.2547 \text{ m} > 0.05 \text{ m}$ **(O.K!)**

$s_A = 0.255$ m **Ans.**

$s_B = 0.205$ m **Ans.**

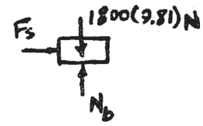
14–21.

The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed $v = 0.5$ m/s when it collides with the "nested" spring assembly. If the stiffness of the outer spring is $k_A = 5$ kN/m, determine the required stiffness k_B of the inner spring so that the motion of the ingot is stopped at the moment the front, C, of the ingot is 0.3 m from the wall.



SOLUTION

$T_1 + \Sigma U_{1-2} = T_2$

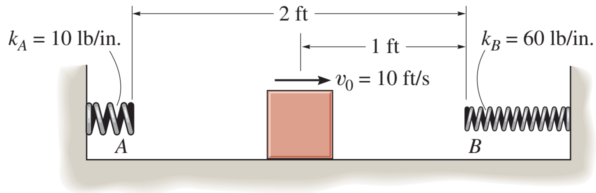


$$\frac{1}{2}(1800)(0.5)^2 - \frac{1}{2}(5000)(0.5 - 0.3)^2 - \frac{1}{2}(k_B)(0.45 - 0.3)^2 = 0$$

$k_B = 11.1$ kN/m **Ans.**

14–22.

The 25-lb block has an initial speed of $v_0 = 10$ ft/s when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.



SOLUTION

Principle of Work and Energy: Here, the friction force $F_f = \mu_k N = 0.4(25) = 10.0$ lb. Since the friction force is always opposite the motion, it does negative work. When the block strikes spring B and stops momentarily, the spring force does *negative* work since it acts in the opposite direction to that of displacement. Applying Eq. 14–7, we have

$$T_l + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{25}{32.2}\right)(10)^2 - 10(1 + s_1) - \frac{1}{2}(60)s_1^2 = 0$$

$$s_1 = 0.8275 \text{ ft}$$

Assume the block bounces back and stops without striking spring A. The spring force does *positive* work since it acts in the direction of displacement. Applying Eq. 14–7, we have

$$T_2 + \sum U_{2-3} = T_3$$

$$0 + \frac{1}{2}(60)(0.8275^2) - 10(0.8275 + s_2) = 0$$

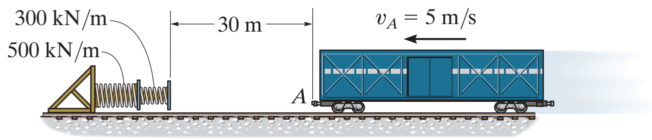
$$s_2 = 1.227 \text{ ft}$$

Since $s_2 = 1.227 \text{ ft} < 2$ ft, the block stops before it strikes spring A. Therefore, the above assumption was correct. Thus, the total distance traveled by the block before it stops is

$$s_{\text{Tot}} = 2s_1 + s_2 + 1 = 2(0.8275) + 1.227 + 1 = 3.88 \text{ ft} \qquad \textbf{Ans.}$$

14–23.

The train car has a mass of 10 Mg and is traveling at 5 m/s when it reaches A. If the rolling resistance is 1/100 of the weight of the car, determine the compression of each spring when the car is momentarily brought to rest.



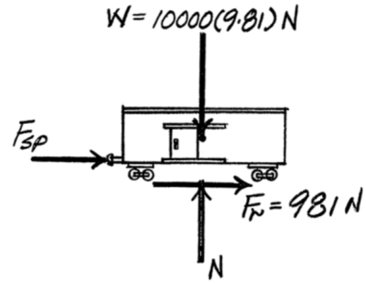
SOLUTION

Free-Body Diagram: The free-body diagram of the train in contact with the spring is shown in Fig. a. Here, the rolling resistance is $F_r = \frac{1}{100}[10\,000(9.81)] = 981$ N. The compression of springs 1 and 2 at the instant the train is momentarily at rest will be denoted as s_1 and s_2. Thus, the force developed in springs 1 and 2 are $(F_{sp})_1 = k_1 s_1 = 300(10^3)s_1$ and $(F_{sp})_2 = 500(10^3)s_2$. Since action is equal to reaction,

$$(F_{sp})_1 = (F_{sp})_2$$

$$300(10^3)s_1 = 500(10^3)s_2$$

$$s_1 = 1.6667 s_2$$



(a)

Principle of Work and Energy: Referring to Fig. a, **W** and **N** do no work, and $\mathbf{F}_{sp}$ and $\mathbf{F}_r$ do negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10\,000)(5^2) + [-981(30 + s_1 + s_2)] + \left\{-\frac{1}{2}\left[300(10^3)\right]s_1^2\right\} + \left\{-\frac{1}{2}\left[500(10^3)\right]s_2^2\right\} = 0$$

$$150(10^3)s_1^2 + 250(10^3)s_2^2 + 981(s_1 + s_2) - 95570 = 0$$

Substituting Eq. (1) into Eq. (2),

$$666.67(10^3)s_2^2 + 2616 s_2 - 95570 = 0$$

Solving for the positive root of the above equation,

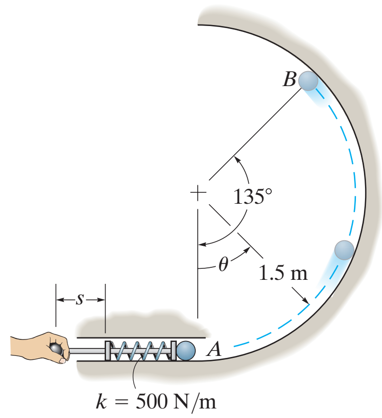
$$s_2 = 0.3767 \text{ m} = 0.377 \text{ m}$$

Substituting the result of s_2 into Eq. (1),

$$s_1 = 0.6278 \text{ m} = 0.628 \text{ m}$$ **Ans.**

***14–24.**

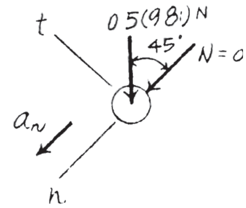
The 0.5-kg ball is fired up the smooth vertical circular track using the spring plunger. The plunger keeps the spring compressed 0.08 m when $s = 0$. Determine how far s it must be pulled back and released so that the ball will begin to leave the track when $\theta = 135°$.



SOLUTION

Equations of Motion:

$$\Sigma F_n = ma_n; \qquad 0.5(9.81)\cos 45° = 0.5\left(\frac{v_B^2}{1.5}\right) \qquad v_B^2 = 10.41\ \text{m}^2/\text{s}^2$$



Principle of Work and Energy: Here, the weight of the ball is being displaced vertically by $s = 1.5 + 1.5\sin 45° = 2.561$ m and so it does *negative* work. The spring force, given by $F_{sp} = 500(s + 0.08)$, does positive work. Since the ball is at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

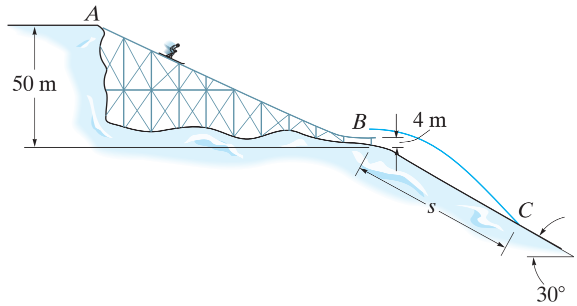
$$T_A + \sum U_{A-B} = T_B$$

$$0 + \int_0^s 500(s + 0.08)\,ds - 0.5(9.81)(2.561) = \frac{1}{2}(0.5)(10.41)$$

$$s = 0.1789\ \text{m} = 179\ \text{mm} \qquad \textbf{Ans.}$$

14–25.

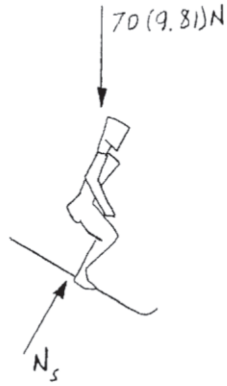
The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 70(9.81)(46) = \frac{1}{2}(70)(v_B)^2$$



$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s}$ **Ans.**

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$s \cos 30° = 0 + 30.04t$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$s \sin 30° + 4 = 0 + 0 + \frac{1}{2}(9.81)t^2$$

Eliminating t,

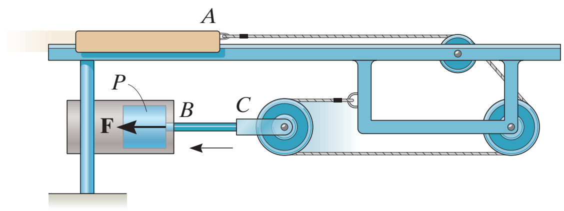
$$s^2 - 122.67s - 981.33 = 0$$

Solving for the positive root

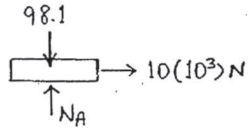
$s = 130 \text{ m}$ **Ans.**

14–26.

The catapulting mechanism is used to propel the 10-kg slider A to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies a constant force $F = 20$ kN to rod BC such that it moves it 0.2 m, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.



SOLUTION

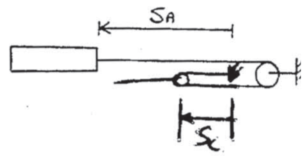


$$2\,s_C + s_A = l$$

$$2\,\Delta\,s_C + \Delta\,s_A = 0$$

$$2(0.2) = -\,\Delta\,s_A$$

$$-0.4 = \Delta\,s_A$$



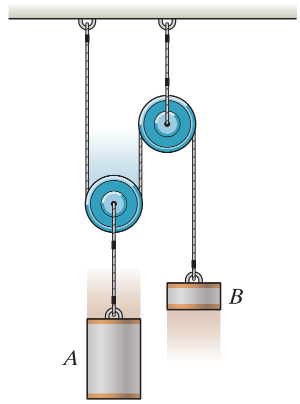
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (10\,000)(0.4) = \frac{1}{2}(10)(v_A)^2$$

$$v_A = 28.3 \text{ m/s} \qquad \textbf{Ans.}$$

14–27.

Block A has a weight of 60 lb and B has a weight of 10 lb. Determine the distance A must descend from rest before it obtains a speed of 8 ft/s. Also, what is the tension in the cord supporting A? Neglect the mass of the cord and pulleys.



SOLUTION

$2\,s_A + s_B = l$

$2\,\Delta\,s_A = -\,\Delta\,s_B$

$2\,v_A = -v_B$

For $v_A = 8$ ft/s, $\quad v_B = -16$ ft/s

For the system:

$T_1 + \Sigma U_{1-2} = T_2$

$$[0 + 0] + [60(s_A) - 10(2s_A)] = \frac{1}{2}\left(\frac{60}{32.2}\right)(8)^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(-16)^2$$

$s_A = 2.484 = 2.48$ ft **Ans.**

For block A:

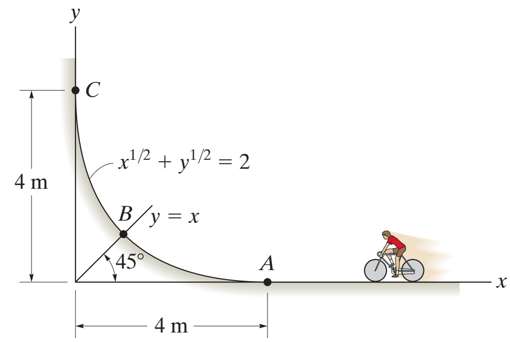
$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 60(2.484) - T_A(2.484) = \frac{1}{2}\left(\frac{60}{32.2}\right)(8)^2$$

$T_A = 36.0$ lb **Ans.**

***14–28.**

The cyclist travels to point A, pedaling until he reaches a speed $v_A = 4\ \text{m/s}$. He then coasts freely up the curved surface. Determine how high he reaches up the surface before he comes to a stop. Also, what are the resultant normal force on the surface at this point and his acceleration? The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.



SOLUTION

$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(4)^2 - 75(9.81)(y) = 0$$

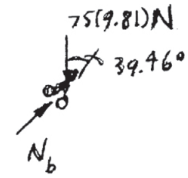
$$y = 0.81549\ \text{m} = 0.815\ \text{m} \qquad \textbf{Ans.}$$

$$x^{1/2} + (0.81549)^{1/2} = 2$$

$$x = 1.2033\ \text{m}$$

$$\tan\theta = \frac{dy}{dx} = \frac{-(1.2033)^{-1/2}}{(0.81549)^{-1/2}} = -0.82323$$

$$\theta = -39.46^\circ$$



$\nearrow + \Sigma F_n = m\,a_n;\qquad N_b - 9.81(75)\cos 39.46^\circ = 0$

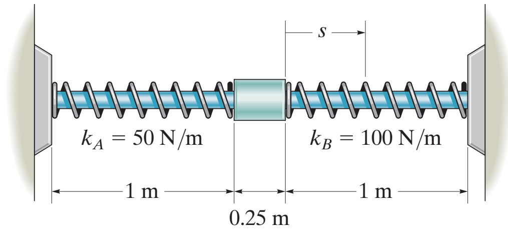
$N_b = 568\ \text{N}$ **Ans.**

$+\searrow \Sigma F_t = m\,a_t;\qquad 75(9.81)\sin 39.46^\circ = 75\,a_t$

$a = a_t = 6.23\ \text{m/s}^2$ **Ans.**

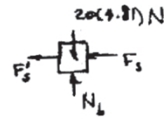
14–29.

The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

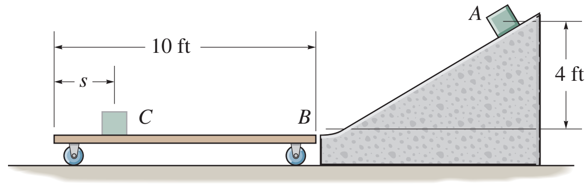


$$\frac{1}{2}(20)(2)^2 - \frac{1}{2}(50)(s)^2 - \frac{1}{2}(100)(s)^2 = 0$$

$s = 0.730$ m **Ans.**

14–30.

The 30-lb box A is released from rest and slides down along the smooth ramp and onto the surface of a cart. If the cart is *prevented from moving*, determine the distance s from the end of the cart to where the box stops. The coefficient of kinetic friction between the cart and the box is $\mu_k = 0.6$.



SOLUTION

Principle of Work and Energy: W_A which acts in the direction of the vertical displacement does *positive* work when the block displaces 4 ft vertically. The friction force $F_f = \mu_k N = 0.6(30) = 18.0$ lb does *negative* work since it acts in the opposite direction to that of displacement Since the block is at rest initially and is required to stop, $T_A = T_C = 0$. Applying Eq. 14–7, we have

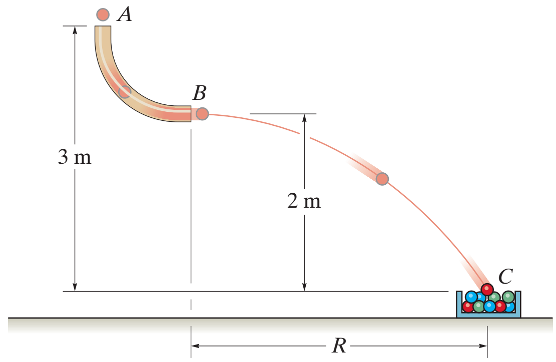
$$T_A + \sum U_{A-C} = T_C$$

$$0 + 30(4) - 18.0s' = 0 \qquad s' = 6.667 \text{ ft}$$

Thus, $$s = 10 - s' = 3.33 \text{ ft}$$ **Ans.**

14–31.

Marbles having a mass of 5 g are dropped from rest at A through the smooth glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.

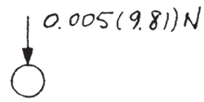


SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + [0.005(9.81)(3 - 2)] = \frac{1}{2}(0.005)v_B^2$$

$$v_B = 4.429 \text{ m/s}$$



$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$2 = 0 + 0 = \frac{1}{2}(9.81)t^2$$

$$t = 0.6386 \text{ s}$$

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$R = 0 + 4.429(0.6386) = 2.83 \text{ m}$$ **Ans.**

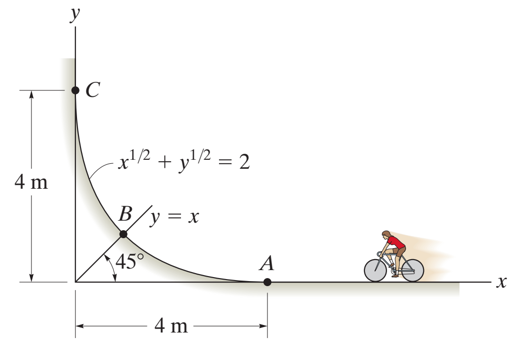
$$T_A + \Sigma U_{A-C} = T_1$$

$$0 + [0.005(9.81)(3) = \frac{1}{2}(0.005)v_C^2$$

$$v_C = 7.67 \text{ m/s}$$ **Ans.**

***14–32.**

The cyclist travels to point A, pedaling until he reaches a speed $v_A = 8$ m/s. He then coasts freely up the curved surface. Determine the normal force he exerts on the surface when he reaches point B. The total mass of the bike and man is 75 kg. Neglect friction, the mass of the wheels, and the size of the bicycle.

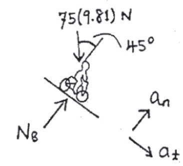


SOLUTION

$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$$

$$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$$



For $y = x$,

$$2\,x^{\frac{1}{2}} = 2$$

$x = 1, y = 1$ (Point B)

Thus,

$$\tan\theta = \frac{dy}{dx} = -1$$

$$\theta = -45°$$

$$\frac{dy}{dx} = (-x^{-\frac{1}{2}})(y^{\frac{1}{2}})$$

$$\frac{d^2y}{dx^2} = y^{\frac{1}{2}}\left(\frac{1}{2}x^{-\frac{3}{2}}\right) - x^{-\frac{1}{2}}\left(\frac{1}{2}\right)\left(y^{-\frac{1}{2}}\right)\left(\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}y^{\frac{1}{2}}x^{-\frac{3}{2}} + \frac{1}{2}\left(\frac{1}{x}\right)$$

For $x = y = 1$

$$\frac{dy}{dx} = -1, \qquad \frac{d^2y}{dx^2} = 1$$

$$\rho = \frac{[1 + (-1)^2]^{3/2}}{1} = 2.828 \text{ m}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(75)(8)^2 - 75(9.81)(1) = \frac{1}{2}(75)(v_B^2)$$

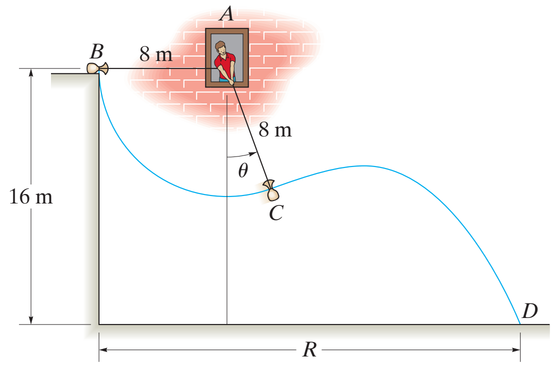
$$v_B^2 = 44.38$$

$\nearrow + \Sigma F_n = ma_n;\qquad N_B - 9.81(75)\cos 45° = 75\left(\dfrac{44.38}{2.828}\right)$

$N_B = 1.70$ kN **Ans.**

14–33.

The man at the window A wishes to throw the 30-kg sack on the ground. To do this he allows it to swing from rest at B to point C, when he releases the cord at $\theta = 30°$. Determine the speed at which it strikes the ground and the distance R.



SOLUTION

$T_B + \Sigma U_{B-C} = T_C$

$$0 + 30(9.81)8 \cos 30° = \frac{1}{2}(30)v_C^2$$

$v_C = 11.659$ m/s

$T_B + \Sigma U_{B-D} = T_D$

$$0 + 30(9.81)(16) = \frac{1}{2}(30)\,v_D^2$$

$v_D = 17.7$ m/s **Ans.**

During free flight:

$$(+\downarrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$16 = 8 \cos 30° - 11.659 \sin 30° t + \frac{1}{2}(9.81)t^2$$

$t^2 - 1.18848\,t - 1.8495 = 0$

Solving for the positive root:

$t = 2.0784$ s

$(\overset{+}{\rightarrow})\ s = s_0 + v_0\, t$

$s = 8 \sin 30° + 11.659 \cos 30°(2.0784)$

$s = 24.985$ m

Thus,

$R = 8 + 24.985 = 33.0$ m **Ans.**

Also,

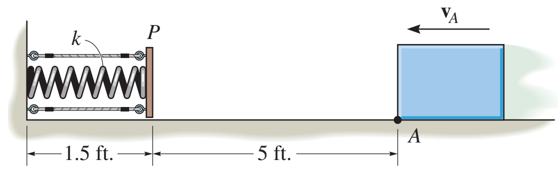
$(v_D)_x = 11.659 \cos 30° = 10.097$ m/s

$(+\downarrow)\ (v_D)_x = -11.659 \sin 30° + 9.81(2.0784) = 14.559$ m/s

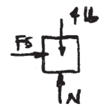
$v_D = \sqrt{(10.097)^2 + (14.559)^2} = 7.7$ m/s **Ans.**

14–34.

The spring bumper is used to arrest the motion of the 4-lb block, which is sliding toward it at $v = 9$ ft/s. As shown, the spring is confined by the plate P and wall using cables so that its length is 1.5 ft. If the stiffness of the spring is $k = 50$ lb/ft, determine the required unstretched length of the spring so that the plate is not displaced more than 0.2 ft after the block collides into it. Neglect friction, the mass of the plate and spring, and the energy loss between the plate and block during the collision.



SOLUTION



$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}\left(\frac{4}{32.2}\right)(9)^2 - \left[\frac{1}{2}(50)(s - 1.3)^2 - \frac{1}{2}(50)(s - 1.5)^2\right] = 0$$

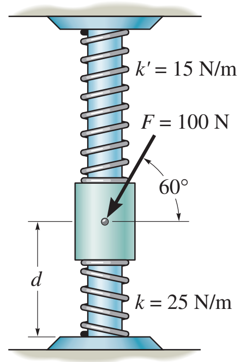
$$0.20124 = s^2 - 2.60\,s + 1.69 - (s^2 - 3.0\,s + 2.25)$$

$$0.20124 = 0.4\,s - 0.560$$

$s = 1.90$ ft **Ans.**

14–35.

The collar has a mass of 20 kg and is supported on the smooth rod. The attached springs are undeformed when $d = 0.5$ m. Determine the speed of the collar after the applied force $F = 100$ N causes it to be displaced so that $d = 0.3$ m. When $d = 0.5$ m the collar is at rest.



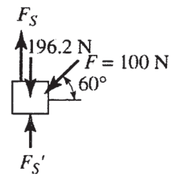
SOLUTION

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 100 \sin 60°(0.5 - 0.3) + 196.2(0.5 - 0.3) - \frac{1}{2}(15)(0.5 - 0.3)^2$$

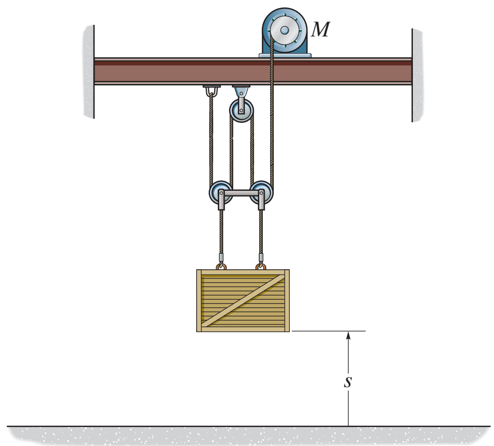
$$- \frac{1}{2}(25)(0.5 - 0.3)^2 = \frac{1}{2}(20)v_C^2$$

$$v_C = 2.36 \text{ m/s}$$ **Ans.**



***14–36.**

If the force exerted by the motor M on the cable is 250 N, determine the speed of the 100-kg crate when it is hoisted to $s = 3$ m. The crate is at rest when $s = 0$.



SOLUTION

Kinematics: Expressing the length of the cable in terms of position coordinates s_C and s_P referring to Fig. a,

$$3s_C + (s_C - s_P) = l$$

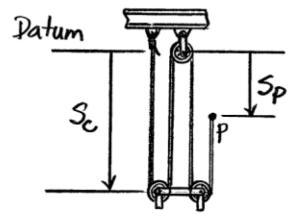
$$4s_C - s_P = l \tag{1}$$

Using Eq. (1), the change in position of the crate and point P on the cable can be written as

$$(+\downarrow) \qquad 4\Delta s_C - \Delta s_P = 0$$

Here, $\Delta s_C = -3$ m. Thus,

$$(+\downarrow) \qquad 4(-3) - \Delta s_P = 0 \qquad\qquad \Delta s_p = -12 \text{ m} = 12 \text{ m} \uparrow$$



(a)

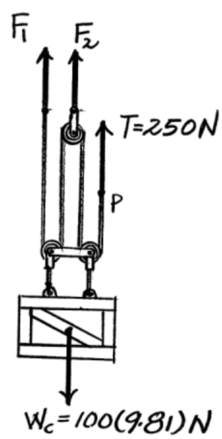
Principle of Work and Energy: Referring to the free-body diagram of the pulley system, Fig. b, $\mathbf{F}_1$ and $\mathbf{F}_2$ do no work since it acts at the support; however, $\mathbf{T}$ does positive work and $\mathbf{W}_C$ does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + T\Delta s_P + [-W_C \Delta s_C] = \frac{1}{2} m_C v^2$$

$$0 + 250(12) + [-100(9.81)(3)] = \frac{1}{2}(100)v^2$$

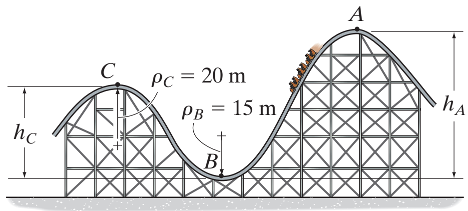
$$v = 1.07 \text{ m/s} \qquad\qquad \textbf{Ans.}$$



(b)

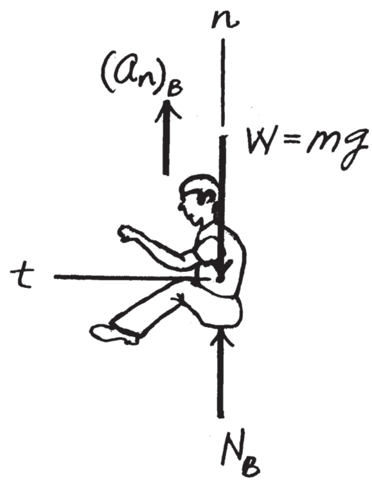
14–37.

If the track is to be designed so that the passengers of the roller coaster do not experience a normal force equal to zero or more than 4 times their weight, determine the limiting heights h_A and h_C so that this does not occur. The roller coaster starts from rest at position A. Neglect friction.



SOLUTION

Free-Body Diagram: The free-body diagram of the passenger at positions B and C are shown in Figs. a and b, respectively.



(a)

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. The requirement at position B is that $N_B = 4mg$. By referring to Fig. a,

$$+\uparrow \Sigma F_n = ma_n; \qquad 4mg - mg = m\left(\frac{v_B{}^2}{15}\right)$$

$$v_B{}^2 = 45g$$

At position C, N_C is required to be zero. By referring to Fig. b,

$$+\downarrow \Sigma F_n = ma_n; \qquad mg - 0 = m\left(\frac{v_C{}^2}{20}\right)$$

$$v_C{}^2 = 20g$$

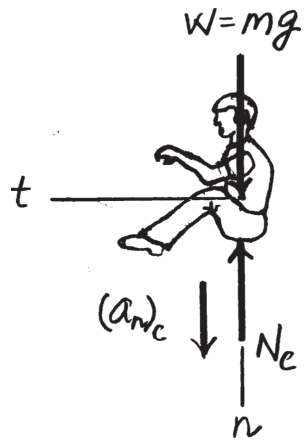
Principle of Work and Energy: The normal reaction **N** does no work since it always acts perpendicular to the motion. When the rollercoaster moves from position A to B, **W** displaces vertically downward $h = h_A$ and does positive work.

We have

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mgh_A = \frac{1}{2}m(45g)$$

$$h_A = 22.5 \text{ m} \qquad \textbf{Ans.}$$



(b)

When the rollercoaster moves from position A to C, **W** displaces vertically downward $h = h_A - h_C = (22.5 - h_C)$ m.

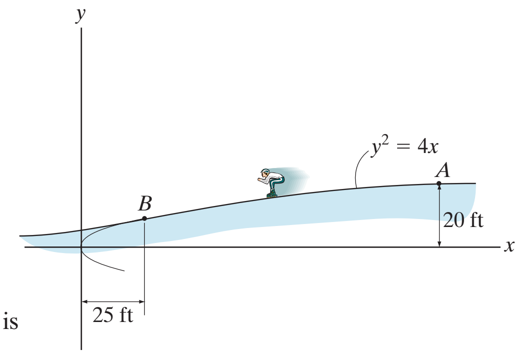
$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mg(22.5 - h_C) = \frac{1}{2}m(20g)$$

$$h_C = 12.5 \text{ m} \qquad \textbf{Ans.}$$

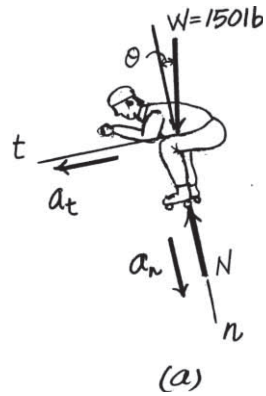
14–38.

The 150-lb skater passes point A with a speed of 6 ft/s. Determine his speed when he reaches point B and the normal force exerted on him by the track at this point. Neglect friction.



SOLUTION

Free-Body Diagram: The free-body diagram of the skater at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, notice that **N** does no work since it always acts perpendicular to the motion. When the skier slides down the track from A to B, **W** displaces vertically downward $h = y_A - y_B = 20 - \left[2(25)^{1/2}\right] = 10$ ft and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{150}{32.2}\right)(6^2) + \left[150(10)\right] = \frac{1}{2}\left(\frac{150}{32.2}\right)v_B{}^2$$

$$v_B = 26.08 \text{ ft/s} = 26.1 \text{ ft/s}$$ **Ans.**

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. By referring to Fig. *a*,

$$\searrow + \Sigma F_n = ma_n; \qquad 150 \cos\theta - N = \frac{150}{32.2}\left(\frac{v^2}{\rho}\right)$$

$$N = 150 \cos\theta - \frac{150}{32.2}\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$

Geometry: Here, $y = 2x^{1/2}$, $\dfrac{dy}{dx} = \dfrac{1}{x^{1/2}}$, and $\dfrac{d^2y}{dx^2} = -\dfrac{1}{2x^{3/2}}$. The slope that the track at position B makes with the horizontal is $\theta_B = \tan^{-1}\left(\dfrac{dx}{dy}\right)\Bigg|_{x=25\text{ ft}}$ $= \tan\left(\dfrac{1}{x^{1/2}}\right)\Bigg|_{x=25\text{ ft}} = 11.31°$. The radius of curvature of the track at position B is given by

$$\rho_B = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\dfrac{1}{x^{1/2}}\right)^2\right]^{3/2}}{\left|-\dfrac{1}{2x^{3/2}}\right|}\right|_{x=25\text{ ft}} = 265.15 \text{ ft}$$

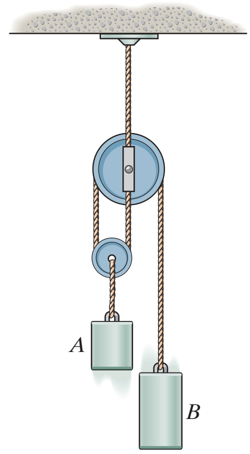
Substituting $\theta = \theta_B = 11.31°$, $v = v_B = 26.08$ ft/s, and $\rho = \rho_B = 265.15$ ft into Eq. (1),

$$N_B = 150 \cos 11.31° - \frac{150}{32.2}\left(\frac{26.08^2}{265.15}\right)$$

$$= 135 \text{ lb}$$ **Ans.**

14–39.

The 8-kg cylinder A and 3-kg cylinder B are released from rest. Determine the speed of A after it has moved 2 m starting from rest. Neglect the mass of the cord and pulleys.



SOLUTION

Kinematics: Express the length of cord in terms of position coordinates s_A and s_B by referring to Fig. *a*

$$2s_A + s_B = l \qquad \textbf{(1)}$$

Thus

$$2\Delta s_A + \Delta s_B = 0 \qquad \textbf{(2)}$$

If we assume that cylinder A is moving downward through a distance of $\Delta s_A = 2$ m, Eq. (2) gives

$$(+\downarrow) \qquad 2(2) + \Delta s_B = 0 \qquad \Delta s_B = -4\text{ m} = 4\text{ m}\uparrow$$

Taking the time derivative of Eq. (1),

$$(+\downarrow) \qquad 2v_A + v_B = 0 \qquad \textbf{(3)}$$

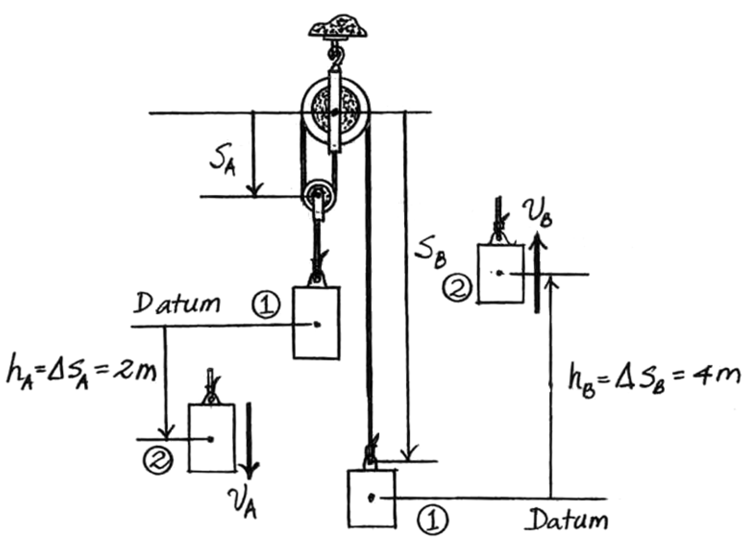
$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

$$0 + 8(2)9.81 - 3(4)9.81 = \frac{1}{2}(8)v_A^2 + \frac{1}{2}(3)v_B^2$$

Positive net work on left means assumption of A moving down is correct. Since $v_B = -2v_A$,

$$v_A = 1.98\text{ m/s}\downarrow \qquad \textbf{Ans.}$$

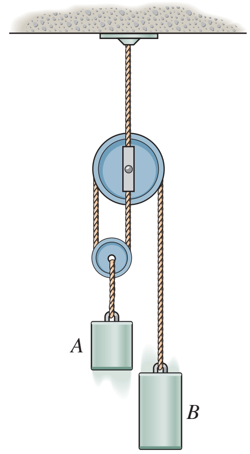
$$v_B = -3.96\text{ m/s} = 3.96\text{ m/s}\uparrow$$



(a)

***14–40.**

Cylinder A has a mass of 3 kg and cylinder B has a mass of 8 kg. Determine the speed of A after it moves upwards 2 m starting from rest. Neglect the mass of the cord and pulleys.



SOLUTION

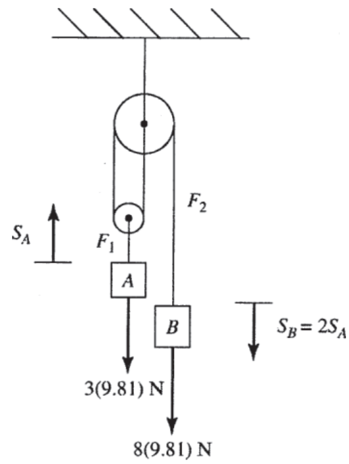
$$\sum T_1 + \sum U_{1-2} = \sum T_2$$

$$0 + 2[F_1 - 3(9.81)] + 4[8(9.81) - F_2] = \frac{1}{2}(3)v_A^2 + \frac{1}{2}(8)v_B^2$$

Also, $v_B = 2v_A$, and because the pulleys are massless, $F_1 = 2F_2$. The F_1 and F_2 terms drop out and the work-energy equation reduces to

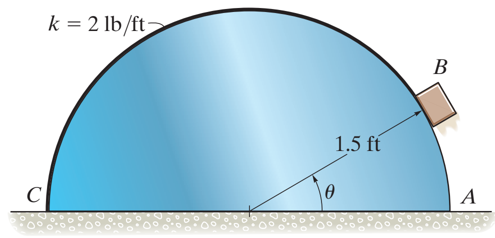
$$255.06 = 17.5v_A^2$$

$$v_A = 3.82 \text{ m/s}$$ **Ans.**

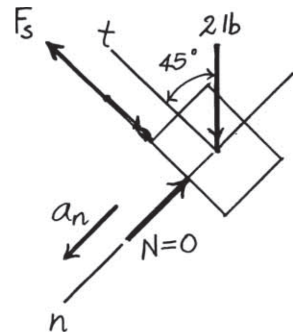


14–41.

A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2\ \text{lb/ft}$ is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at $A\ (\theta = 0°)$, determine the unstretched length of the cord so that the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.



SOLUTION



$$+\swarrow \Sigma F_n = ma_n; \qquad 2 \sin 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right)$$

$$v = 5.844\ \text{ft/s}$$

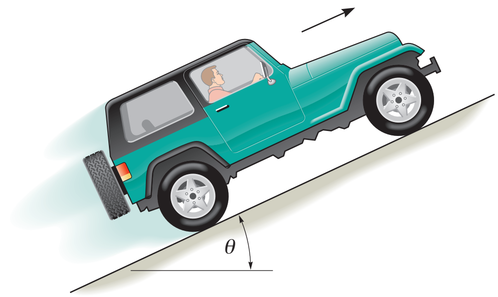
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \frac{1}{2}(2)\left[\pi(1.5) - l_0\right]^2 - \frac{1}{2}(2)\left[\frac{3\pi}{4}(1.5) - l_0\right]^2 - 2(1.5 \sin 45°) = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2$$

$l_0 = 2.77\ \text{ft}$ **Ans.**

14–42.

The jeep has a weight of 2500 lb and an engine which transmits a power of 100 hp to *all* the wheels. Assuming the wheels do not slip on the ground, determine the angle θ of the largest incline the jeep can climb at a constant speed $v = 30$ ft/s.

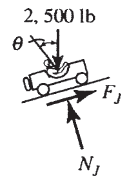


SOLUTION

$P = F_J v$

$100(550) = 2500 \sin \theta(30)$

$\theta = 47.2°$ **Ans.**



14–43.

Determine the power input for a motor necessary to lift 300 lb at a constant rate of 5 ft/s. The efficiency of the motor is $\epsilon = 0.65$.

SOLUTION

Power: The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 300(5) = 1500 \text{ ft} \cdot \text{lb/s}$$

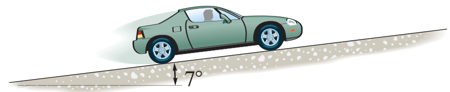
Using Eq. 14–11, the required power input for the motor to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\epsilon}$$

$$= \frac{1500}{0.65} = 2307.7 \text{ ft} \cdot \text{lb/s} = 4.20 \text{ hp} \qquad \textbf{Ans.}$$

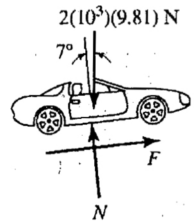
***14–44.**

An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100$ km/h. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\epsilon = 0.65$.



SOLUTION

Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.



$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 2(10^3)(9.81)\sin 7^\circ = 2(10^3)(0)$$

$$F = 2391.08\text{ N}$$

Power: Here, the speed of the car is $v = \left[\dfrac{100(10^3)\text{ m}}{\text{h}}\right] \times \left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 27.78\text{ m/s}$.

The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F}\cdot\mathbf{v} = 2391.08(27.78) = 66.418(10^3)\text{ W} = 66.418\text{ kW}$$

Using Eq. 14–11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{66.418}{0.65} = 102\text{ kW} \qquad \textbf{Ans.}$$

14–45.

The Milkin Aircraft Co. manufactures a turbojet engine that is placed in a plane having a weight of 13000 lb. If the engine develops a constant thrust of 5200 lb, determine the power output of the plane when it is just ready to take off with a speed of 600 mi/h.

SOLUTION

At 600 ms/h.

$$P = 5200(600)\left(\frac{88\text{ ft/s}}{60\text{ m/h}}\right)\frac{1}{550} = 8.32\ (10^3)\text{ hp} \qquad \textbf{Ans.}$$

14–46.

To dramatize the loss of energy in an automobile, consider a car having a weight of 5000 lb that is traveling at 35 mi/h. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy. (1 mi = 5280 ft.)

SOLUTION

Energy: Here, the speed of the car is $v = \left(\frac{35\text{ mi}}{\text{h}}\right) \times \left(\frac{5280\text{ ft}}{1\text{ mi}}\right) \times \left(\frac{1\text{ h}}{3600\text{ s}}\right) =$ 51.33 ft/s. Thus, the kinetic energy of the car is

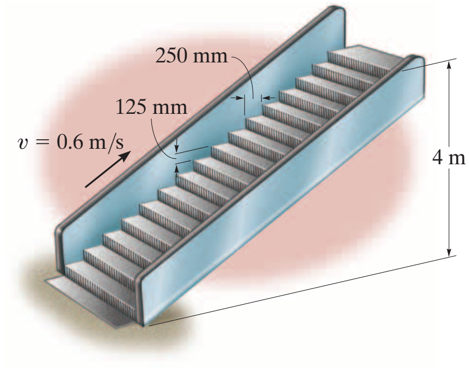
$$U = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{5000}{32.2}\right)\left(51.33^2\right) = 204.59\left(10^3\right)\text{ ft}\cdot\text{lb}$$

The power of the bulb is $P_{bulb} = 100\text{ W} \times \left(\frac{1\text{ hp}}{746\text{ W}}\right) \times \left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) =$ 73.73 ft · lb/s. Thus,

$$t = \frac{U}{P_{bulb}} = \frac{204.59(10^3)}{73.73} = 2774.98\text{ s} = 46.2\text{ min} \qquad \textbf{Ans.}$$

14–47.

The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.

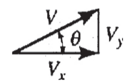


SOLUTION

Step height: 0.125 m

The number of steps: $\dfrac{4}{0.125} = 32$

Total load: $32(150)(9.81) = 47\ 088$ N



If load is placed at the center height, $h = \dfrac{4}{2} = 2$ m, then

$$U = 47\ 088\left(\frac{4}{2}\right) = 94.18\text{ kJ}$$

$$\nu_y = \nu \sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683\text{ m/s}$$

$$t = \frac{h}{\nu_y} = \frac{2}{0.2683} = 7.454\text{ s}$$

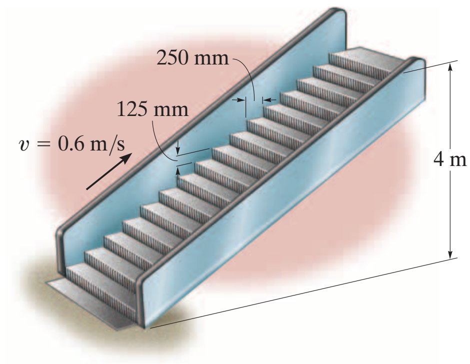
$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6\text{ kW}$$ **Ans.**

Also,

$$P = \mathbf{F}\cdot\mathbf{v} = 47\ 088(0.2683) = 12.6\ \text{kW}$$ **Ans.**

***14–48.**

If the escalator in Prob. 14–47 is not moving, determine the constant speed at which a man having a mass of 80 kg must walk up the steps to generate 100 W of power—the same amount that is needed to power a standard light bulb.



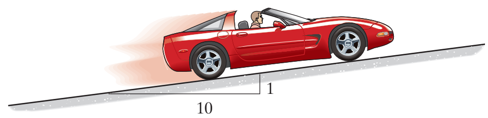
SOLUTION

$$P = \frac{U_{1-2}}{t} = \frac{(80)(9.81)(4)}{t} = 100 \qquad t = 31.4\text{ s}$$

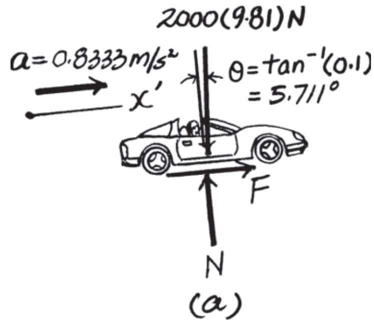
$$v = \frac{s}{t} = \frac{\sqrt{(32(0.25))^2 + 4^2}}{31.4} = 0.285\text{ m/s}$$ **Ans.**

14–49.

The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\epsilon = 0.8$. Also, find the average power supplied by the engine.



SOLUTION



Kinematics: The constant acceleration of the car can be determined from

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$25 = 0 + a_c(30)$$

$$a_c = 0.8333 \text{ m/s}^2$$

Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,

$$\Sigma F_{x'} = ma_{x'}; \qquad F - 2000(9.81)\sin 5.711° = 2000(0.8333)$$

$$F = 3618.93\text{N}$$

Power: The maximum power output of the motor can be determined from

$$(P_{\text{out}})_{\text{max}} = \mathbf{F} \cdot \mathbf{v}_{\text{max}} = 3618.93(25) = 90\,473.24 \text{ W}$$

Thus, the maximum power input is given by

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{90473.24}{0.8} = 113\,091.55 \text{ W} = 113 \text{ kW} \qquad \textbf{Ans.}$$

The average power output can be determined from

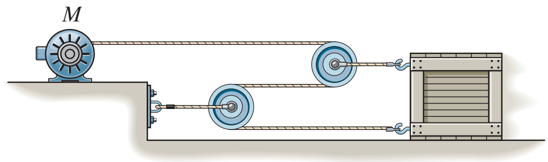
$$(P_{\text{out}})_{\text{avg}} = \mathbf{F} \cdot \mathbf{v}_{\text{avg}} = 3618.93\left(\frac{25}{2}\right) = 45\,236.62 \text{ W}$$

Thus,

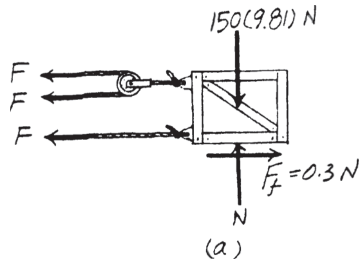
$$(P_{\text{in}})_{\text{avg}} = \frac{(P_{\text{out}})_{\text{avg}}}{\varepsilon} = \frac{45236.62}{0.8} = 56\,545.78 \text{ W} = 56.5 \text{ kW} \qquad \textbf{Ans.}$$

14–50.

The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively. If the motor M supplies a cable force of $F = (8t^2 + 20)$ N, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



SOLUTION

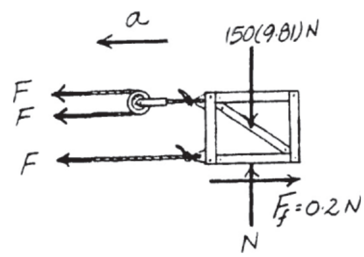


Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.3N$. From FBD(a),

$$+\uparrow \Sigma F_y = 0; \qquad N - 150(9.81) = 0 \qquad N = 1471.5 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3(1471.5) - 3\left(8t^2 + 20\right) = 0 \qquad t = 3.9867 \text{ s}$$

Equations of Motion: Since the crate moves 3.9867 s later, $F_f = \mu_k N = 0.2N$. From FBD(b),



$$+\uparrow \Sigma F_y = ma_y; \qquad N - 150(9.81) = 150(0) \qquad N = 1471.5 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0.2(1471.5) - 3\left(8t^2 + 20\right) = 150(-a)$$

$$a = \left(0.160t^2 - 1.562\right) \text{ m/s}^2$$

Kinematics: Applying $dv = a\,dt$, we have

$$\int_0^v dv = \int_{3.9867 \text{ s}}^{5} \left(0.160t^2 - 1.562\right) dt$$

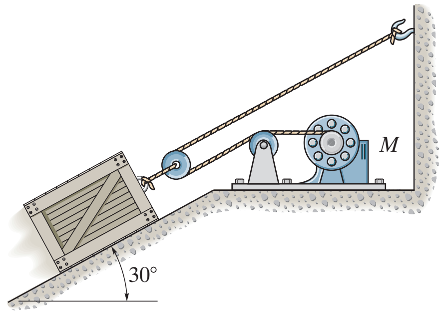
$$v = 1.7045 \text{ m/s}$$

Power: At $t = 5$ s, $F = 8\left(5^2\right) + 20 = 220$ N. The power can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 3(220)(1.7045) = 1124.97 \text{ W} = 1.12 \text{ kW} \qquad \textbf{Ans.}$$

14–51.

The 50-kg crate is hoisted up the 30° incline by the pulley system and motor M. If the crate starts from rest and, by constant acceleration, attains a speed of 4 m/s after traveling 8 m along the plane, determine the power that must be supplied to the motor at the instant the crate has moved 8 m. Neglect friction along the plane. The motor has an efficiency of $\epsilon = 0.74$.

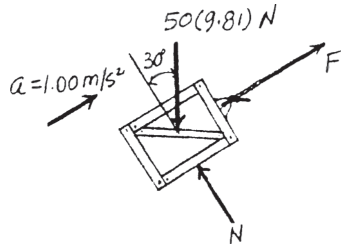


SOLUTION

Kinematics: Applying equation $v^2 = v_0^2 + 2a_c(s - s_0)$, we have

$$4^2 = 0^2 + 2a(8 - 0) \qquad a = 1.00 \text{ m/s}^2$$

Equations of Motion:



$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 50(9.81)\sin 30° = 50(1.00) \qquad F = 295.25 \text{ N}$$

Power: The power output at the instant when $v = 4$ m/s can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 295.25\,(4) = 1181 \text{ W} = 1.181 \text{ kW}$$

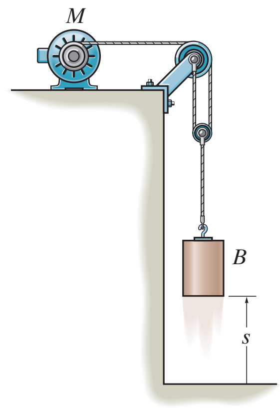
Using Eq. 14–11, the required power input to the motor in order to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{1.181}{0.74} = 1.60 \text{ kW} \qquad \textbf{Ans.}$$

***14–52.**

The 50-lb load is hoisted by the pulley system and motor M. If the motor exerts a constant force of 30 lb on the cable, determine the power that must be supplied to the motor if the load has been hoisted $s = 10$ ft starting from rest. The motor has an efficiency of $\epsilon = 0.76$.



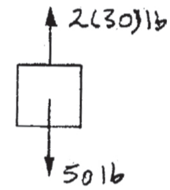
SOLUTION

$+\uparrow \Sigma F_y = m\,a_y;\qquad 2(30) - 50 = \dfrac{50}{32.2}a_B$

$$a_B = 6.44 \text{ m/s}^2$$

$(+\uparrow)\ v^2 = v_0^2 + 2a_c\,(s - s_0)$

$$v_B^2 = 0 + 2(6.44)(10 - 0)$$



$v_B = 11.349$ ft/s

$2s_B + s_M = l$

$2v_B = -\,v_M$

$v_M = -2(11.349) = 22.698$ ft/s

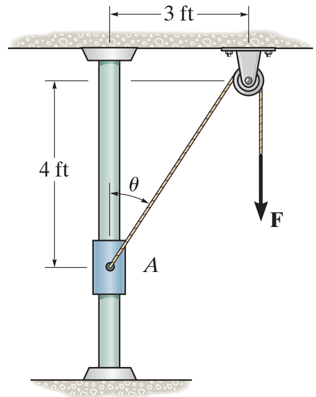
$P_o = \mathbf{F}\cdot\mathbf{v} = 30(22.698) = 680.94 \text{ ft}\cdot\text{lb/s}$

$P_i = \dfrac{680.94}{0.76} = 895.97 \text{ ft}\cdot\text{lb/s}$

$P_i = 1.63$ hp **Ans.**

14–53.

The 10-lb collar starts from rest at A and is lifted by applying a constant vertical force of $F = 25$ lb to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = 60°$.



SOLUTION

Work of **F**

$U_{1-2} = 25(5 - 3.464) = 38.40 \text{ lb} \cdot \text{ft}$

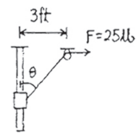
$T_1 + \Sigma U_{1-2} = T_2\text{s}$

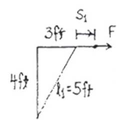
$$0 + 38.40 - 10(4 - 1.732) = \frac{1}{2}\left(\frac{10}{32.2}\right)v^2$$

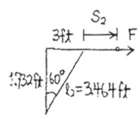
$v = 10.06 \text{ ft/s}$

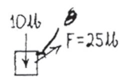
$P = \mathbf{F} \cdot \mathbf{v} = 25 \cos 60°(10.06) = 125.76 \text{ ft} \cdot \text{lb/s}$

$P = 0.229 \text{ hp}$ **Ans.**



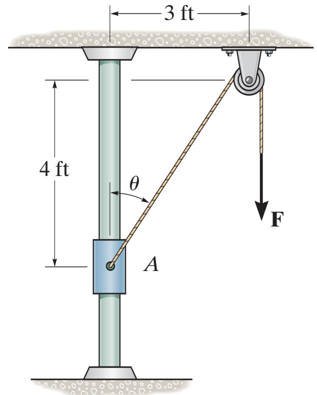






14–54.

The 10-lb collar starts from rest at A and is lifted with a constant speed of 2 ft/s along the smooth rod. Determine the power developed by the force **F** at the instant shown.



SOLUTION

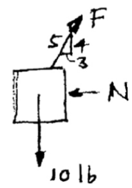
$$+\uparrow \Sigma F_y = m\,a_y; \qquad F\left(\frac{4}{5}\right) - 10 = 0$$

$$F = 12.5 \text{ lb}$$

$$P = \mathbf{F} \cdot \mathbf{v} = 12.5\left(\frac{4}{5}\right)(2) = 20 \text{ lb} \cdot \text{ft/s}$$

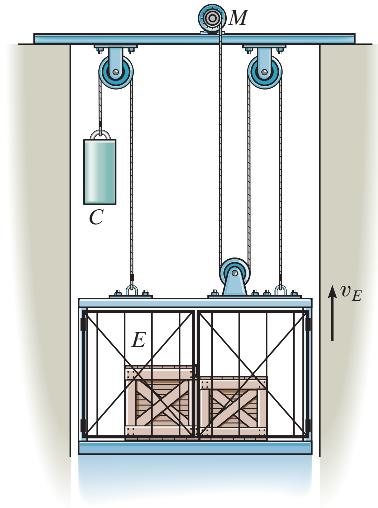
$$= 0.0364 \text{ hp}$$

Ans.



14–55.

The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\epsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.



SOLUTION

Elevator:

Since $a = 0$,

$$+\uparrow \Sigma F_y = 0; \qquad 60(9.81) + 3T - 400(9.81) = 0$$

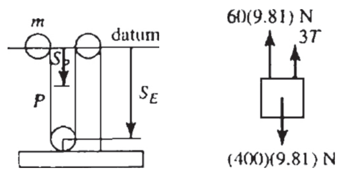
$$T = 1111.8 \text{ N}$$

$$2s_E + (s_E - s_P) = l$$

$$3v_E = v_P$$

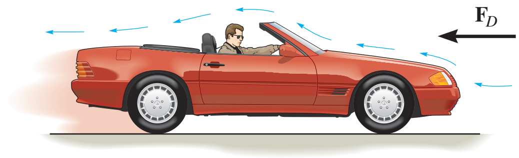
Since $v_E = -4$ m/s, $\qquad v_P = -12$ m/s

$$P_i = \frac{\mathbf{F} \cdot \mathbf{v}_P}{e} = \frac{(1111.8)(12)}{0.6} = 22.2 \text{ kW}$$ **Ans.**

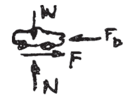


***14–56.**

The sports car has a mass of 2.3 Mg, and while it is traveling at 28 m/s the driver causes it to accelerate at 5 m/s^2. If the drag resistance on the car due to the wind is $F_D = (0.3v^2)$ N, where v is the velocity in m/s, determine the power supplied to the engine at this instant. The engine has a running efficiency of $\epsilon = 0.68$.



SOLUTION



$$\xrightarrow{+} \Sigma F_x = m\,a_x; \qquad F - 0.3v^2 = 2.3(10^3)(5)$$

$$F = 0.3v^2 + 11.5(10^3)$$

At $v = 28$ m/s

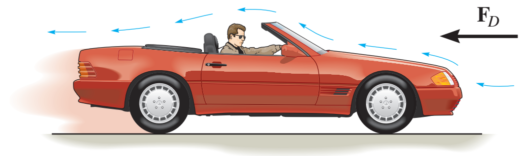
$$F = 11\,735.2 \text{ N}$$

$$P_O = (11\,735.2)(28) = 328.59 \text{ kW}$$

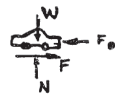
$$P_i = \frac{P_O}{e} = \frac{328.59}{0.68} = 438 \text{ kW}$$ **Ans.**

14–57.

The sports car has a mass of 2.3 Mg and accelerates at 6 m/s^2, starting from rest. If the drag resistance on the car due to the wind is $F_D = (10v)$ N, where v is the velocity in m/s, determine the power supplied to the engine when $t = 5$ s. The engine has a running efficiency of $\epsilon = 0.68$.



SOLUTION



$\overset{+}{\rightarrow} \Sigma F_x = m\,a_x; \qquad F - 10v = 2.3(10^3)(6)$

$$F = 13.8(10^3) + 10\,v$$

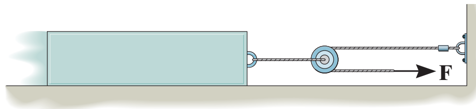
$(\overset{+}{\rightarrow})\ v = v_0 + a_c\,t$

$$v = 0 + 6(5) = 30 \text{ m/s}$$

$P_O = \mathbf{F} \cdot \mathbf{v} = [13.8(10^3) + 10(30)](30) = 423.0 \text{ kW}$

$P_i = \dfrac{P_O}{e} = \dfrac{423.0}{0.68} = 622 \text{ kW}$ **Ans.**

14–58.



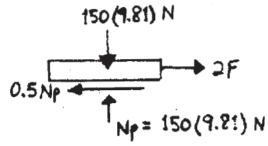
The block has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. If a force $F = (60t^2)$ N, where t is in seconds, is applied to the cable, determine the power developed by the force when $t = 5$ s. *Hint:* First determine the time needed for the force to cause motion.

SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F - 0.5(150)(9.81) = 0$

$$F = 367.875 = 60t^2$$

$$t = 2.476 \text{ s}$$

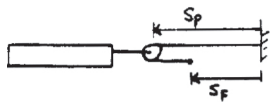


$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 2(60t^2) - 0.4(150)(9.81) = 150a_p$

$$a_p = 0.8t^2 - 3.924$$

$dv = a\,dt$

$$\int_0^v dv = \int_{2.476}^{5} \left(0.8t^2 - 3.924\right) dt$$



$$v = \left(\frac{0.8}{3}\right)t^3 - 3.924t\Big|_{2.476}^{5} = 19.38 \text{ m/s}$$

$s_P + (s_P - s_F) = l$

$2v_P = v_F$

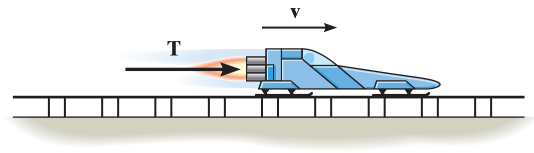
$v_F = 2(19.38) = 38.76$ m/s

$F = 60(5)^2 = 1500$ N

$P = \mathbf{F}\cdot\mathbf{v} = 1500(38.76) = 58.1$ kW **Ans.**

14–59.

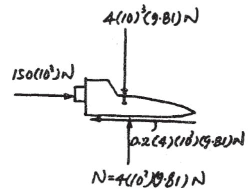
The rocket sled has a mass of 4 Mg and travels from rest along the horizontal track for which the coefficient of kinetic friction is $\mu_k = 0.20$. If the engine provides a constant thrust $T = 150$ kN, determine the power output of the engine as a function of time. Neglect the loss of fuel mass and air resistance.



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x;$ $\quad 150(10)^3 - 0.2(4)(10)^3(9.81) = 4(10)^3\, a$

$a = 35.54 \text{ m/s}^2$



$\left(\xrightarrow{+}\right) v = v_0 + a_c t$

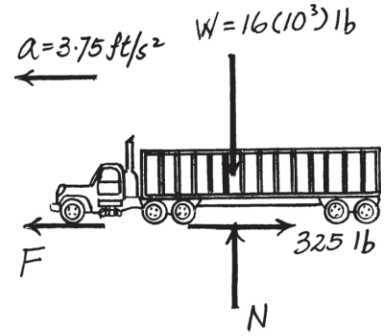
$= 0 + 35.54t = 35.54t$

$P = \mathbf{T} \cdot \mathbf{v} = 150(10)^3(35.54t) = 5.33t \text{ MW}$ **Ans.**

***14–60.**

A loaded truck weighs $16(10^3)$ lb and accelerates uniformly on a level road from 15 ft/s to 30 ft/s during 4 s. If the frictional resistance to motion is 325 lb, determine the maximum power that must be delivered to the wheels.

SOLUTION



$$a = \frac{\Delta v}{\Delta t} = \frac{30 - 15}{4} = 3.75 \text{ ft/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad F - 325 = \left(\frac{16(10^3)}{32.2}\right)(3.75)$$

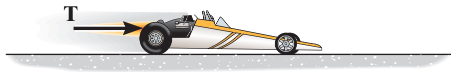
$$F = 2188.35 \text{ lb}$$

$$P_{\max} = \mathbf{F} \cdot \mathbf{v}_{\max} = \frac{2188.35(30)}{550} = 119 \text{ hp}$$

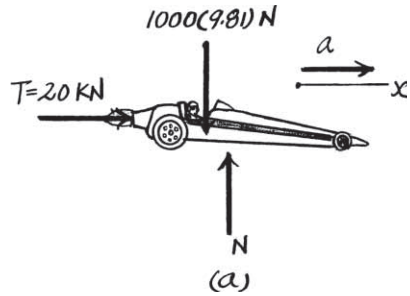
Ans.

14–61.

If the jet on the dragster supplies a constant thrust of $T = 20$ kN, determine the power generated by the jet as a function of time. Neglect drag and rolling resistance, and the loss of fuel. The dragster has a mass of 1 Mg and starts from rest.



SOLUTION



Equations of Motion: By referring to the free-body diagram of the dragster shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 20(10^3) = 1000(a) \qquad a = 20 \text{ m/s}^2$$

Kinematics: The velocity of the dragster can be determined from

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t$$

$$v = 0 + 20t = (20t) \text{ m/s}$$

Power:

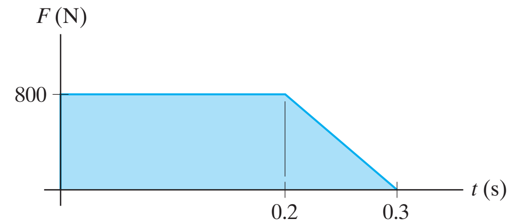
$$P = \mathbf{F} \cdot \mathbf{v} = 20(10^3)(20t)$$

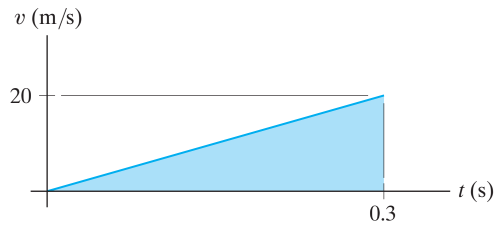
$$= \left[400(10^3)t\right] \text{ W}$$

Ans.

14–62.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3$ s.





SOLUTION

For $0 \leq t \leq 0.2$

$$F = 800 \text{ N}$$

$$v = \frac{20}{0.3}t = 66.67t$$

$$\boldsymbol{P} = \mathbf{F} \cdot \mathbf{v} = 53.3t \text{ kW}$$ **Ans.**

For $0.2 \leq t \leq 0.3$

$$F = 2400 - 8000t$$

$$v = 66.67t$$

$$P = \mathbf{F} \cdot \mathbf{v} = (160t - 533t^2) \text{ kW}$$ **Ans.**

$$u = \int_0^{0.3} P\, dt$$

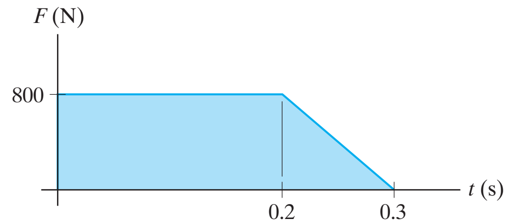
$$u = \int_0^{0.2} 53.3t\, dt + \int_{0.2}^{0.3} (160t - 533t^2)\, dt$$

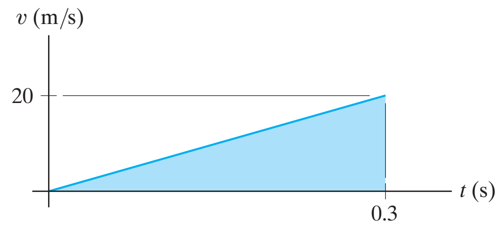
$$= \frac{53.3}{2}(0.2)^2 + \frac{160}{2}[(0.3)^2 - (0.2)^2] - \frac{533}{3}[(0.3)^3 - (0.2)^3]$$

$$= 1.69 \text{ kJ}$$ **Ans.**

14–63.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the 0.3-second time period.





SOLUTION

See solution to Prob. 14–62.

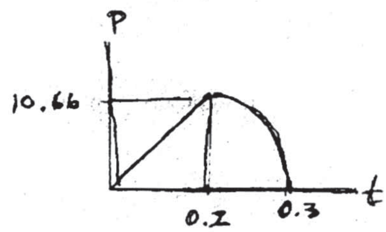
$P = 160\,t - 533\,t^2$

$$\frac{dP}{dt} = 160 - 1066.6\,t = 0$$

$t = 0.15 \text{ s} < 0.2 \text{ s}$

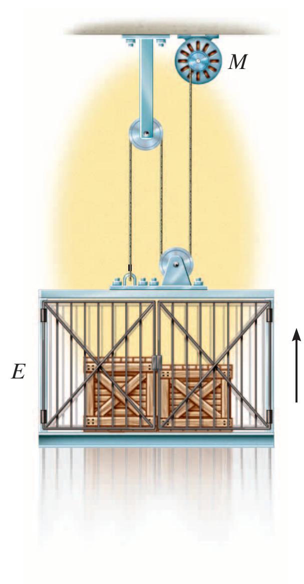
Thus maximum occurs at $t = 0.2$ s

$P_{max} = 53.3(0.2) = 10.7 \text{ kW}$ **Ans.**



***14–64.**

The 500-kg elevator starts from rest and travels upward with a constant acceleration $a_c = 2\ \text{m/s}^2$. Determine the power output of the motor M when $t = 3$ s. Neglect the mass of the pulleys and cable.



SOLUTION

$$+\uparrow \Sigma F_y = m\,a_y; \qquad 3T - 500(9.81) = 500(2)$$

$$T = 1968.33\ \text{N}$$

$3s_E - s_P = l$

$3\,v_E = v_P$

When $t = 3$ s,

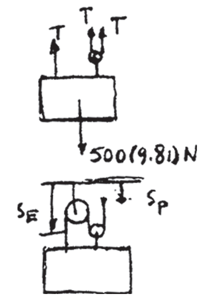
$(+\uparrow)\ \ v_0 + a_c\,t$

$$v_E = 0 + 2(3) = 6\ \text{m/s}$$

$$v_P = 3(6) = 18\ \text{m/s}$$

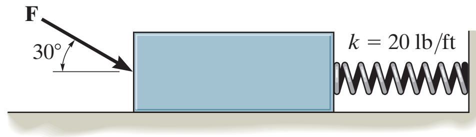
$$P_O = 1968.33(18)$$

$$P_O = 35.4\ \text{kW}$$ **Ans.**

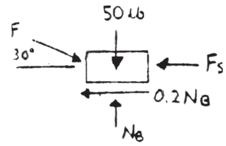


14–65.

The 50-lb block rests on the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force $F = (40 + s^2)$ lb, where s is in ft, acts on the block in the direction shown. If the spring is originally unstretched ($s = 0$) and the block is at rest, determine the power developed by the force the instant the block has moved $s = 1.5$ ft.



SOLUTION



$+\uparrow \Sigma F_y = 0; \qquad N_B - \left(40 + s^2\right)\sin 30^\circ - 50 = 0$

$$N_B = 70 + 0.5s^2$$

$T_1 + \Sigma U_{1-2} + T_2$

$$0 + \int_0^{1.5} \left(40 + s^2\right)\cos 30^\circ\, ds - \frac{1}{2}(20)(1.5)^2 - 0.2\int_0^{1.5}\left(70 + 0.5s^2\right) ds = \frac{1}{2}\left(\frac{50}{32.2}\right)v_2^2$$

$$0 + 52.936 - 22.5 - 21.1125 = 0.7764\, v_2^2$$

$$v_2 = 3.465 \text{ ft/s}$$

When $s = 1.5$ ft,

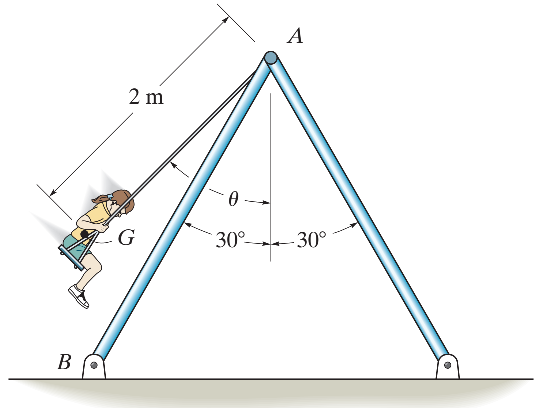
$$F = 40 + (1.5)^2 = 42.25 \text{ lb}$$

$$P = \mathbf{F}\cdot\mathbf{v} = (42.25\cos 30^\circ)(3.465)$$

$P = 126.79 \text{ ft}\cdot\text{lb/s} = 0.231 \text{ hp}$ **Ans.**

14–66.

The girl has a mass of 40 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 60°$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.



SOLUTION

The maximum tension in the cable occurs when $\theta = 0°$.

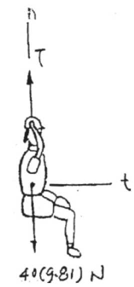
$$T_1 + V_1 = T_2 + V_2$$

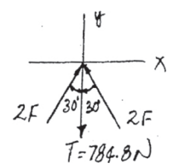
$$0 + 40(9.81)(-2\cos 60°) = \frac{1}{2}(40)v^2 + 40(9.81)(-2)$$

$$v = 4.429 \text{ m/s}$$

$+\uparrow \Sigma F_n = ma_n;\qquad T - 40(9.81) = (40)\left(\frac{4.429^2}{2}\right)\qquad T = 784.8 \text{ N}$

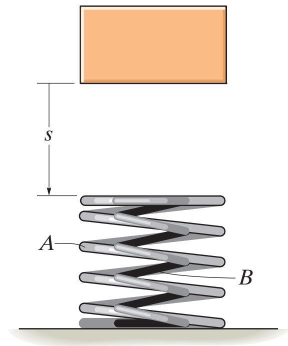
$+\uparrow \Sigma F_y = 0;\qquad 2(2F)\cos 30° - 784.8 = 0\qquad F = 227 \text{ N}$ **Ans.**





14–67.

Two equal-length springs are "nested" together in order to form a shock absorber. If it is designed to arrest the motion of a 2-kg mass that is dropped $s = 0.5$ m above the top of the springs from an at-rest position, and the maximum compression of the springs is to be 0.2 m, determine the required stiffness of the inner spring, k_B, if the outer spring has a stiffness $k_A = 400$ N/m.



SOLUTION

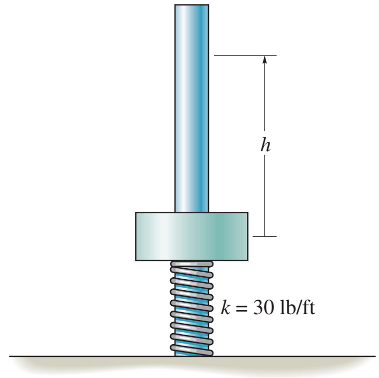
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 - 2(9.81)(0.5 + 0.2) + \frac{1}{2}(400)(0.2)^2 + \frac{1}{2}(k_B)(0.2)^2$$

$$k_B = 287 \text{ N/m}$$ **Ans.**

***14–68.**

The collar has a weight of 8 lb. If it is pushed down so as to compress the spring 2 ft and then released from rest $(h = 0)$, determine its speed when it is displaced $h = 4.5$ ft. The spring is not attached to the collar. Neglect friction.



SOLUTION

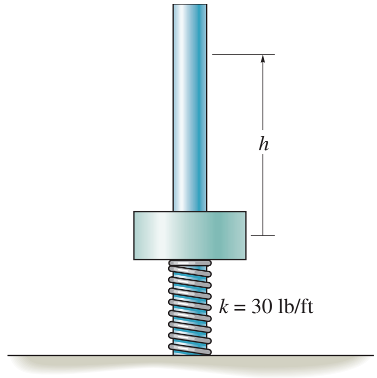
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(30)(2)^2 = \frac{1}{2}\left(\frac{8}{32.2}\right)v_2^2 + 8(4.5)$$

$$v_2 = 13.9 \text{ ft/s}$$ **Ans.**

14–69.

The collar has a weight of 8 lb. If it is released from rest at a height of $h = 2$ ft from the top of the uncompressed spring, determine the speed of the collar after it falls and compresses the spring 0.3 ft.



SOLUTION

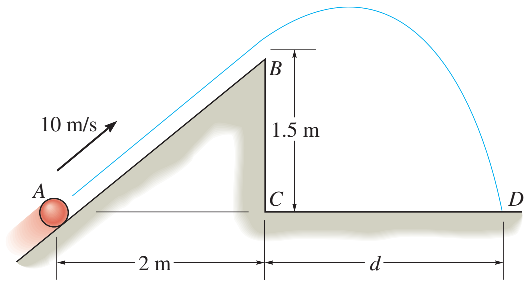
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{8}{32.2}\right)v_2^2 - 8(2.3) + \frac{1}{2}(30)(0.3)^2$$

$$v_2 = 11.7 \text{ ft/s}$$ **Ans.**

14–70.

The 2-kg ball of negligible size is fired from point A with an initial velocity of 10 m/s up the smooth inclined plane. Determine the distance from point C to where it hits the horizontal surface at D. Also, what is its velocity when it strikes the surface?



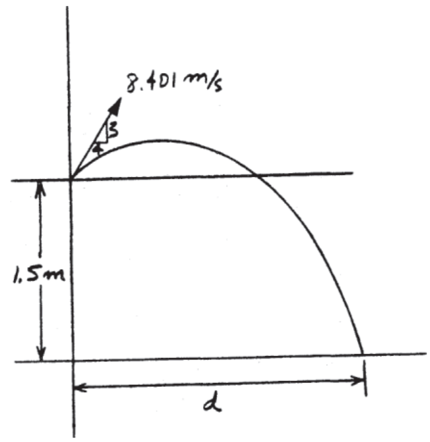
SOLUTION

Datum at A:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_B)^2 + 2(9.81)(1.5)$$

$$v_B = 8.401 \text{ m/s}$$



$$(\xrightarrow{+}) \quad s = s_0 + v_0 t$$

$$d = 0 + 8.401\left(\frac{4}{5}\right)t$$

$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-1.5 = 0 + 8.401\left(\frac{3}{5}\right)t + \frac{1}{2}(-9.81)t^2$$

$$-4.905t^2 + 5.040t + 1.5 = 0$$

Solving for the positive root,

$$t = 1.269 \text{ s}$$

$$d = 8.401\left(\frac{4}{5}\right)(1.269) = 8.53 \text{ m}$$ **Ans.**

Datum at A:

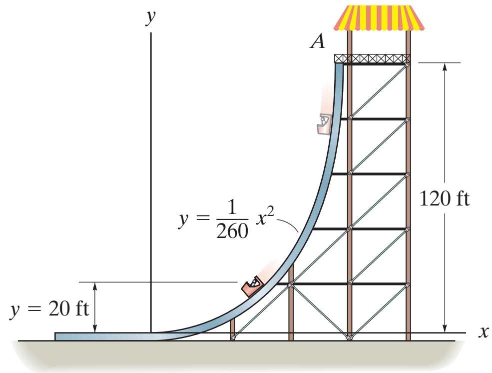
$$T_A + V_A = T_D + V_D$$

$$\frac{1}{2}(2)(10)^2 + 0 = \frac{1}{2}(2)(v_D)^2 + 0$$

$$v_D = 10 \text{ m/s}$$ **Ans.**

14–71.

The ride at an amusement park consists of a gondola which is lifted to a height of 120 ft at A. If it is released from rest and falls along the parabolic track, determine the speed at the instant $y = 20$ ft. Also determine the normal reaction of the tracks on the gondola at this instant. The gondola and passenger have a total weight of 500 lb. Neglect the effects of friction and the mass of the wheels.



SOLUTION

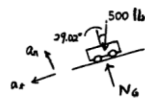
$$y = \frac{1}{260}x^2$$

$$\frac{dy}{dx} = \frac{1}{130}x$$

$$\frac{d^2y}{dx2} = \frac{1}{130}$$

At $y = 120 - 100 = 20$ ft

$$x = 72.11 \text{ ft}$$



$$\tan\theta = \frac{dy}{dx} = 0.555, \qquad \theta = 29.02^\circ$$

$$\rho = \frac{[1 + (0.555)^2]^{3/2}}{\frac{1}{130}} = 194.40 \text{ ft}$$

$$+\nwarrow \Sigma F_n = ma_n; \qquad N_G - 500\cos 29.02^\circ = \frac{500}{32.2}\left(\frac{v^2}{194.40}\right) \qquad \textbf{(1)}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{500}{32.2}\right)v^2 - 500(100)$$

$$v^2 = 6440$$

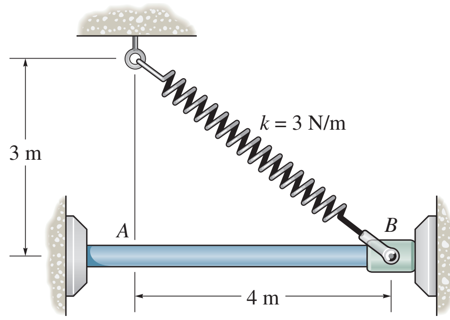
$$v = 80.2 \text{ ft/s} \qquad \textbf{Ans.}$$

Substituting into Eq. (1) yields

$$N_G = 952 \text{ lb} \qquad \textbf{Ans.}$$

***14–72.**

The 2-kg collar is attached to a spring that has an unstretched length of 3 m. If the collar is drawn to point B and released from rest, determine its speed when it arrives at point A.



SOLUTION

Potential Energy: The initial and final elastic potential energy are $\frac{1}{2}(3)\left(\sqrt{3^2 + 4^2} - 3\right)^2 = 6.00$ **J** and $\frac{1}{2}(3)(3 - 3)^2 = 0$, respectively. The gravitational potential energy remains the same since the elevation of collar does not change when it moves from B to A.

Conservation of Energy:

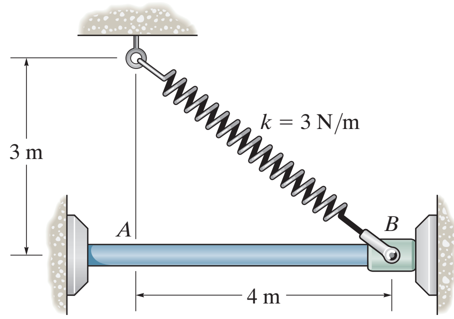
$$T_B + V_B = T_A + V_A$$

$$0 + 6.00 = \frac{1}{2}(2)\,v_A^2 + 0$$

$$v_A = 2.45 \text{ m/s}$$ **Ans.**

14–73.

The 2-kg collar is attached to a spring that has an unstretched length of 2 m. If the collar is drawn to point B and released from rest, determine its speed when it arrives at point A.



SOLUTION

Potential Energy: The stretches of the spring when the collar is at B and A are $s_B = \sqrt{3^2 + 4^2} - 2 = 3$ m and $s_A = 3 - 2 = 1$ m, respectively. Thus, the elastic potential energy of the system at B and A are

$$(V_e)_B = \frac{1}{2}ks_B{}^2 = \frac{1}{2}(3)(3^2) = 13.5 \text{ J}$$

$$(V_e)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(3)(1^2) = 1.5 \text{ J}$$

There is no change in gravitational potential energy since the elevation of the collar does not change during the motion.

Conservation of Energy:

$$T_B + V_B = T_A + V_A$$

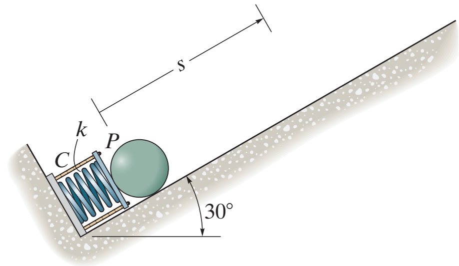
$$\frac{1}{2}mv_B^2 + (V_e)_B = \frac{1}{2}mv_A{}^2 + (V_e)_A$$

$$0 + 13.5 = \frac{1}{2}(2)v_A{}^2 + 1.5$$

$$v_A = 3.46 \text{ m/s}$$ **Ans.**

14–74.

The 0.5-lb ball is shot from the spring device shown. The spring has a stiffness $k = 10$ lb/in. and the four cords C and plate P keep the spring compressed 2 in. when no load is on the plate. The plate is pushed back 3 in. from its initial position. If it is then released from rest, determine the speed of the ball when it travels 30 in. up the smooth plane.



SOLUTION

Potential Energy: The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft · lb. The initial and final elastic potential energy are $\frac{1}{2}(120)\left(\frac{2+3}{12}\right)^2 = 10.42$ ft · lb and $\frac{1}{2}(120)\left(\frac{2}{12}\right)^2 = 1.667$ ft · lb, respectively.

Conservation of Energy:

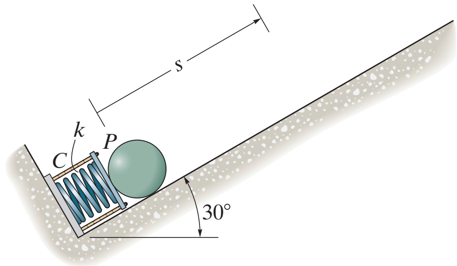
$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 10.42 = \frac{1}{2}\left(\frac{0.5}{32.2}\right)v^2 + 0.625 + 1.667$$

$$v = 32.3 \text{ ft/s} \qquad \textbf{Ans.}$$

14–75.

The 0.5-lb ball is shot from the spring device shown. Determine the smallest stiffness k which is required to shoot the ball a maximum distance of 30 in. up the smooth plane after the spring is pushed back 3 in. and the ball is released from rest. The four cords C and plate P keep the spring compressed 2 in. when no load is on the plate.



SOLUTION

Potential Energy: The datum is set at the lowest point (compressed position). Finally, the ball is $\frac{30}{12}\sin 30° = 1.25$ ft *above* the datum and its gravitational potential energy is $0.5(1.25) = 0.625$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(k)\left(\frac{2+3}{12}\right)^2 = 0.08681k$ and $\frac{1}{2}(k)\left(\frac{2}{12}\right)^2 = 0.01389k$, respectively.

Conservation of Energy:

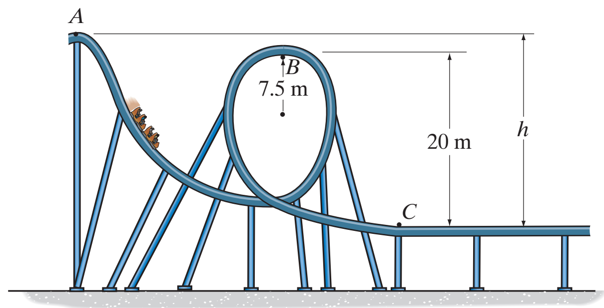
$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + 0.08681k = 0 + 0.625 + 0.01389k$$

$$k = 8.57 \text{ lb/ft}$$ **Ans.**

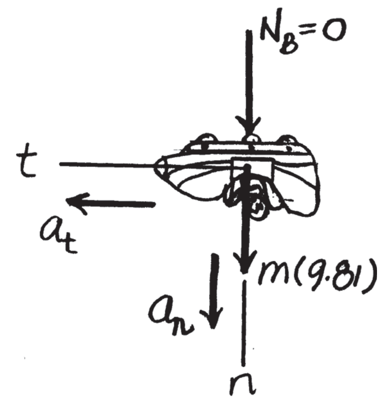
***14–76.**

The roller coaster car having a mass m is released from rest at point A. If the track is to be designed so that the car does not leave it at B, determine the required height h. Also, find the speed of the car when it reaches point C. Neglect friction.



SOLUTION

Equation of Motion: Since it is required that the roller coaster car is about to leave the track at B, $N_B = 0$. Here, $a_n = \dfrac{v_B^2}{\rho_B} = \dfrac{v_B^2}{7.5}$. By referring to the free-body diagram of the roller coaster car shown in Fig. a,



(a)

$$\Sigma F_n = ma_n; \qquad m(9.81) = m\left(\frac{v_B^2}{7.5}\right) \quad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$

Potential Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the rollercoaster car at positions A, B, and C are $(V_g)_A = mgh_A = m(9.81)h = 9.81mh$, $(V_g)_B = mgh_B = m(9.81)(20) = 196.2$ m, and $(V_g)_C = mgh_C = m(9.81)(0) = 0$.

Conservation of Energy: Using the result of v_B^2 and considering the motion of the car from position A to B,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + (V_g)_A = \frac{1}{2}mv_B^2 + (V_g)_B$$

$$0 + 9.81mh = \frac{1}{2}m(73.575) + 196.2m$$

$$h = 23.75 \text{ m}$$ **Ans.**

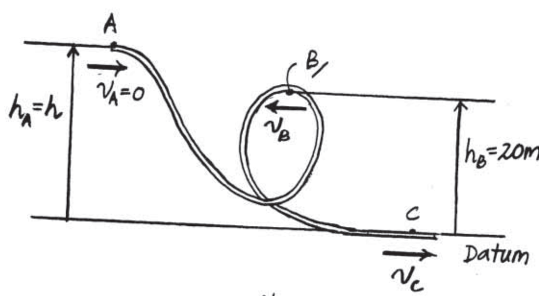
Also, considering the motion of the car from position B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 + (V_g)_B = \frac{1}{2}mv_C^2 + (V_g)_C$$

$$\frac{1}{2}m(73.575) + 196.2m = \frac{1}{2}mv_C^2 + 0$$

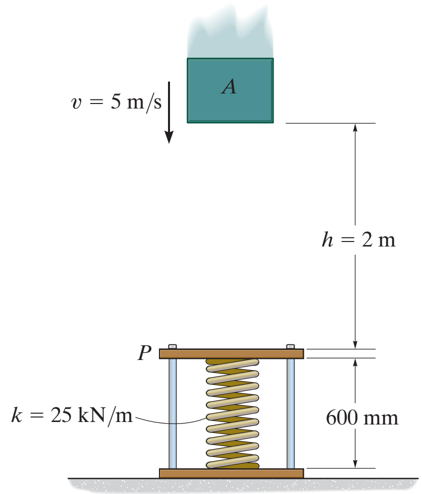
$$v_C = 21.6 \text{ m/s}$$ **Ans.**



(b)

14–77.

A 750-mm-long spring is compressed and confined by the plate P, which can slide freely along the vertical 600-mm-long rods. The 40-kg block is given a speed of $v = 5$ m/s when it is $h = 2$ m above the plate. Determine how far the plate moves downwards when the block momentarily stops after striking it. Neglect the mass of the plate.



SOLUTION

Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the block at positions (1) and (2) are $(V_g)_1 = mgh_1 = 40(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 40(9.81)[-(2 + y)] = [-392.4(2 + y)]$, respectively. The compression of the spring when the block is at positions (1) and (2) are $s_1 = (0.75 - 0.6) = 0.15$ m and $s_2 = s_1 + y = (0.15 + y)$ m. Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2}ks_1^2 = \frac{1}{2}(25)(10^3)(0.15^2) = 281.25 \text{ J}$$

$$(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(25)(10^3)(0.15 + y)^2$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

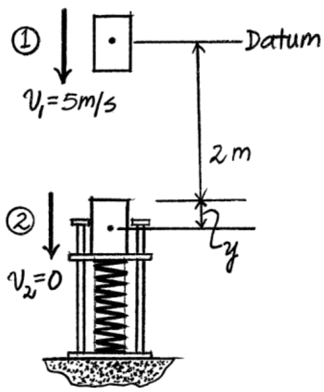
$$\frac{1}{2}mv_1^2 + \left[(V_g)_1 + (V_e)_1\right] = \frac{1}{2}mv_2^2 + \left[(V_g)_2 + (V_e)_2\right]$$

$$\frac{1}{2}(40)(5^2) + (0 + 281.25) = 0 + [-392.4(2 + y)] + \frac{1}{2}(25)(10^3)(0.15 + y)^2$$

$$12500y^2 + 3357.6y - 1284.8 = 0$$

Solving for the positive root of the above equation,

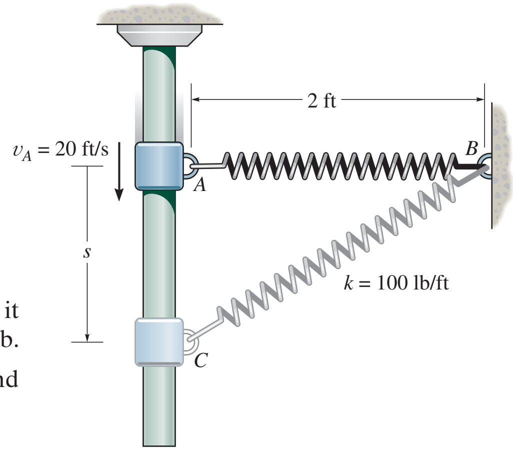
$$y = 0.2133 \text{ m} = 213 \text{ mm}$$ **Ans.**



(a)

14–78.

The 2-lb block is given an initial velocity of 20 ft/s when it is at A. If the spring has an unstretched length of 2 ft and a stiffness of $k = 100$ lb/ft, determine the velocity of the block when $s = 1$ ft.



SOLUTION

Potential Energy: Datum is set along AB. The collar is 1 ft *below* the datum when it is at C. Thus, its gravitational potential energy at this point is $-2(1) = -2.00$ ft · lb. The initial and final elastic potential energy are $\frac{1}{2}(100)(2 - 2)^2 = 0$ and $\frac{1}{2}(100)\left(\sqrt{2^2 + 1^2} - 2\right)^2 = 2.786$ ft · lb, respectively.

Conservation of Energy:

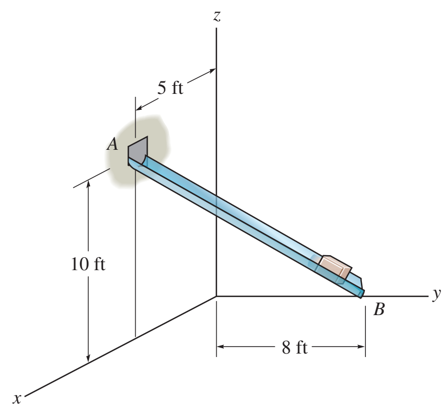
$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)\left(20^2\right) + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2 + 2.786 + (-2.00)$$

$$v_C = 19.4 \text{ ft/s}$$ **Ans.**

14–79.

The block has a weight of 1.5 lb and slides along the smooth chute AB. It is released from rest at A, which has coordinates of $A(5\text{ ft}, 0, 10\text{ ft})$. Determine the speed at which it slides off at B, which has coordinates of $B(0, 8\text{ ft}, 0)$.



SOLUTION

Datum at B:

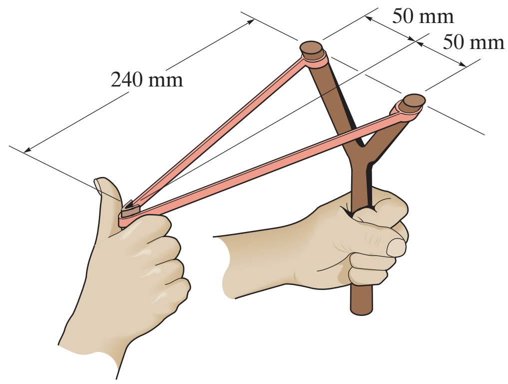
$$T_A + V_A = T_B + V_B$$

$$0 + 1.5(10) = \frac{1}{2}\left(\frac{1.5}{32.2}\right)(v_B)^2 + 0$$

$$v_B = 25.4\text{ ft/s}$$ **Ans.**

***14–80.**

Each of the two elastic rubber bands of the slingshot has an unstretched length of 200 mm. If they are pulled back to the position shown and released from rest, determine the speed of the 25-g pellet just after the rubber bands become unstretched. Neglect the mass of the rubber bands. Each rubber band has a stiffness of $k = 50$ N/m.



SOLUTION

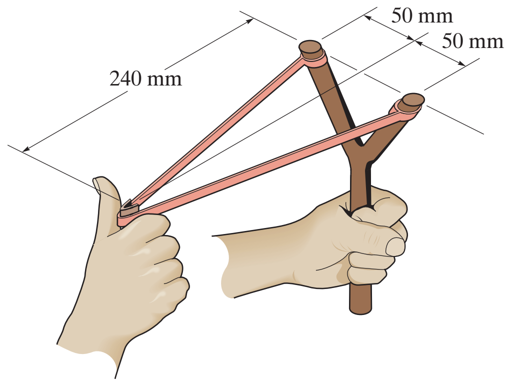
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (2)\left(\frac{1}{2}\right)(50)\left[\sqrt{(0.05)^2 + (0.240)^2} - 0.2\right]^2 = \frac{1}{2}(0.025)v^2$$

$v = 2.86$ m/s **Ans.**

14–81.

Each of the two elastic rubber bands of the slingshot has an unstretched length of 200 mm. If they are pulled back to the position shown and released from rest, determine the maximum height the 25-g pellet will reach if it is fired vertically upward. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness $k = 50$ N/m.



SOLUTION

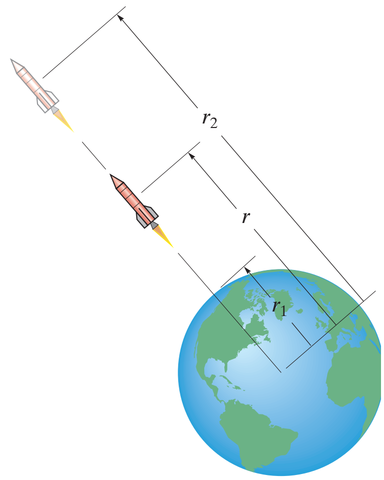
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left(\frac{1}{2}\right)(50)\left[\sqrt{(0.05)^2 + (0.240)^2} - 0.2\right]^2 = 0 + 0.025(9.81)h$$

$$h = 0.416 \text{ m} = 416 \text{ mm}$$ **Ans.**

14–82.

If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_e m/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_e m/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \to \infty$. Also, prove that F is a conservative force.



SOLUTION

The work is computed by moving F from position r_1 to a farther position r_2.

$$V_g = -U = -\int F\,dr$$

$$= -G\,M_e\,m \int_{r1}^{r2} \frac{dr}{r^2}$$

$$= -G\,M_e\,m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

As $r_1 \to \infty$, let $r_2 = r_1$, $F_2 = F_1$, then

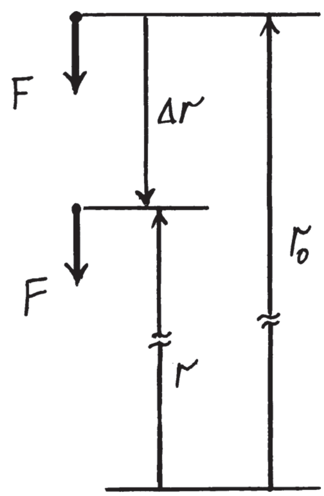
$$V_g \to \frac{-G\,M_e\,m}{r}$$

To be conservative, require

$$F = -\nabla V_g = -\frac{\partial}{\partial r}\left(-\frac{G\,M_e\,m}{r}\right)$$

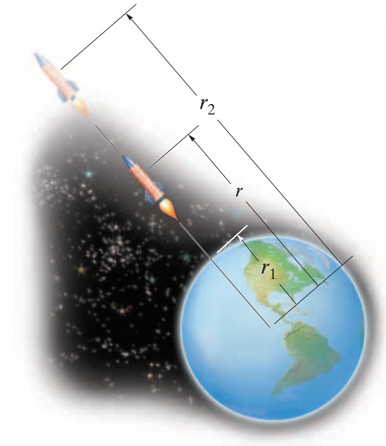
$$= \frac{-G\,M_e\,m}{r^2}$$

Q.E.D.



14–83.

A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.



SOLUTION

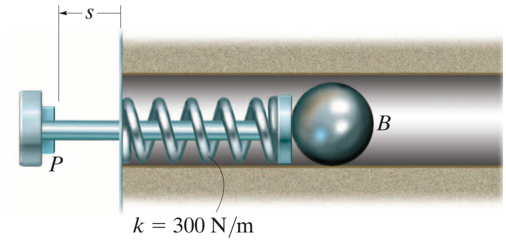
$$F = G\frac{M_e m}{r^2}$$

$$F_{1-2} = \int F\,dr = GM_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \textbf{Ans.}$$

***14–84.**

The firing mechanism of a pinball machine consists of a plunger P having a mass of 0.25 kg and a spring of stiffness $k = 300$ N/m. When $s = 0$, the spring is compressed 50 mm. If the arm is pulled back such that $s = 100$ mm and released, determine the speed of the 0.3-kg pinball B *just before* the plunger strikes the stop, i.e., $s = 0$. Assume all surfaces of contact to be smooth. The ball moves in the horizontal plane. Neglect friction, the mass of the spring, and the rolling motion of the ball.



SOLUTION

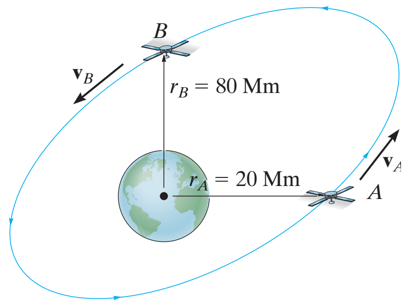
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(300)(0.1 + 0.05)^2 = \frac{1}{2}(0.25)(v_2)^2 + \frac{1}{2}(0.3)(v_2)^2 + \frac{1}{2}(300)(0.05)^2$$

$$v_2 = 3.30 \text{ m/s}$$ **Ans.**

14–85.

A 60-kg satellite travels in free flight along an elliptical orbit such that at A, where $r_A = 20$ Mm, it has a speed $v_A = 40$ Mm/h. What is the speed of the satellite when it reaches point B, where $r_B = 80$ Mm? *Hint:* See Prob. 14–82, where $M_e = 5.976(10^{24})$ kg and $G = 66.73(10^{-12})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$.



SOLUTION

$v_A = 40\ \text{Mm/h} = 11\,111.1\ \text{m/s}$

Since $V = -\dfrac{GM_e m}{r}$

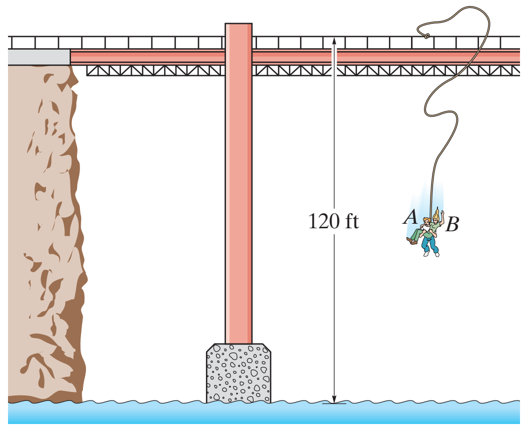
$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}(60)(11\,111.1)^2 - \frac{66.73(10)^{-12}(5.976)(10)^{23}(60)}{20(10)^6} = \frac{1}{2}(60)v_B^2 - \frac{66.73(10)^{-12}(5.976)(10)^{24}(60)}{80(10)^6}$$

$v_B = 9672\ \text{m/s} = 34.8\ \text{Mm/h}$ **Ans.**

14–86.

Just for fun, two 150-lb engineering students A and B intend to jump off the bridge from rest using an elastic cord (bungee cord) having a stiffness $k = 80$ lb/ft. They wish to just reach the surface of the river, when A, attached to the cord, lets go of B at the instant they touch the water. Determine the proper unstretched length of the cord to do the stunt, and calculate the maximum acceleration of student A and the maximum height he reaches above the water after the rebound. From your results, comment on the feasibility of doing this stunt.

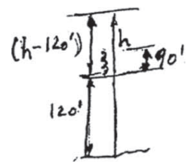


SOLUTION

$T_1 + V_1 = T_2 + V_2$

$$0 + 2(150)(120) = 0 + \frac{1}{2}(80)(x)^2$$

$x = 30$ ft



Unstretched length of cord.

$120 = l + 30$

$l = 90$ ft **Ans.**

When A lets go of B.

$T_2 + V_2 = T_3 + V_3$

$$0 + \frac{1}{2}(80)(30)^2 = 0 + (150)\,h$$

$h = 240$ ft

This is not possible since the 90 ft cord would have to stretch again, i.e., $h_{max} = 120 + 90 = 210$ ft.

Thus, $h > 120 + 90 = 210$ ft

$T_2 + V_2 = T_3 + V_3$

$$0 + \frac{1}{2}(80)(30)^2 = 0 + 150\,h + \frac{1}{2}(80)[(h - 120) - 90]^2$$

$36\,000 = 150\,h + 40(h^2 - 420\,h + 44\,100)$

$h^2 - 416.25\,h + 43\,200 = 0$

Choosing the root > 210 ft

$h = 219$ ft **Ans.**

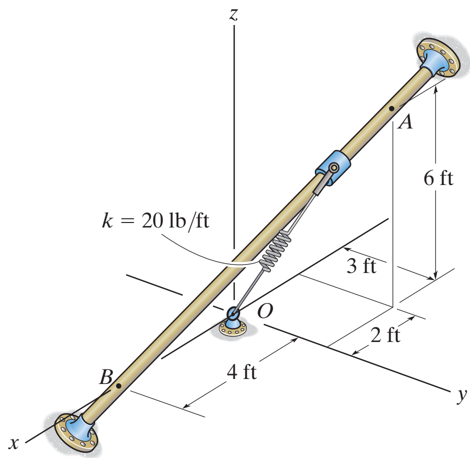
$+\uparrow \Sigma F_y = ma_y; \qquad 800(30) - 150 = \dfrac{150}{32.2}a$

$a = 483 \text{ ft/s}^2$ **Ans.**

It would not be a good idea to perform the stunt since $a = 15$ g which is excessive and A rises $219' - 120' = 99$ ft above the bridge!

14–87.

The 20-lb collar slides along the smooth rod. If the collar is released from rest at A, determine its speed when it passes point B. The spring has an unstretched length of 3 ft.



SOLUTION

$\mathbf{r}_{OA} = \{-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}\}$ ft, $\qquad r_{OA} = 7$ ft

$\mathbf{r}_{OB} = \{4\mathbf{i}\}$ ft, $\qquad r_{OB} = 4$ ft

Put datum at x–y plane

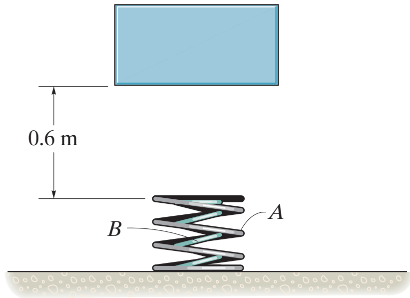
$$T_A + V_A = T_B + V_B$$

$$0 + (20\text{ lb})(6\text{ ft}) + \frac{1}{2}(20\text{ lb/ft})(7\text{ ft} - 3\text{ ft})^2 = \frac{1}{2}\left(\frac{20}{32.2}\right) v_B^2 + 0 + \frac{1}{2}(20\text{ lb/ft})(4\text{ ft} - 3\text{ ft})^2$$

$$v_B = 29.5\text{ ft/s} \qquad \textbf{Ans.}$$

***14–88.**

Two equal-length springs having a stiffness $k_A = 300$ N/m and $k_B = 200$ N/m are "nested" together in order to form a shock absorber. If a 2-kg block is dropped from an at-rest position 0.6 m above the top of the springs, determine their deformation when the block momentarily stops.



SOLUTION

Datum at initial position:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 - 2(9.81)(0.6 + x) + \frac{1}{2}(300 + 200)(x)^2$$

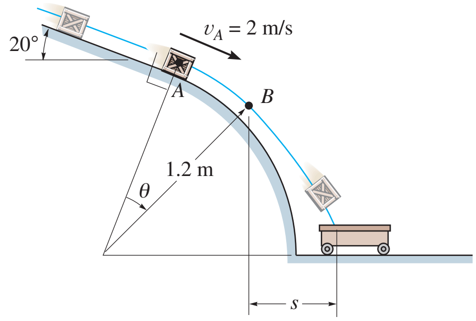
$$250x^2 - 19.62x - 11.772 = 0$$

Solving for the positive root,

$x = 0.260$ m **Ans.**

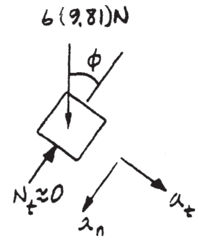
14–89.

When the 6-kg box reaches point A it has a speed of $v_A = 2$ m/s. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.



SOLUTION

At point B:



$$+\swarrow \Sigma F_n = ma_n; \qquad 6(9.81)\cos\phi = 6\left(\frac{v_B^2}{1.2}\right) \qquad \textbf{(1)}$$

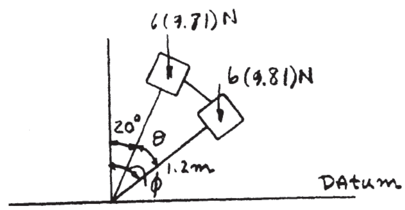
Datum at bottom of curve:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(6)(2)^2 + 6(9.81)(1.2\cos 20°) = \frac{1}{2}(6)(v_B)^2 + 6(9.81)(1.2\cos\phi)$$

$$13.062 = 0.5v_B^2 + 11.772\cos\phi \qquad \textbf{(2)}$$

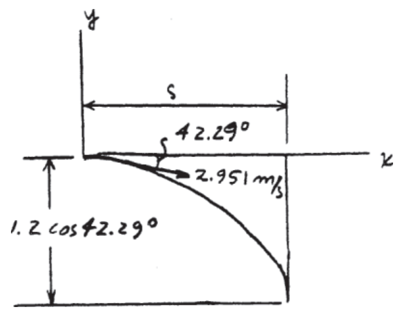
Substitute Eq. (1) into Eq. (2), and solving for v_B,



$$v_B = 2.951 \text{ m/s}$$

Thus, $$\phi = \cos^{-1}\left(\frac{(2.951)^2}{1.2(9.81)}\right) = 42.29°$$

$$\theta = \phi - 20° = 22.3° \qquad \textbf{Ans.}$$



$(+\uparrow)$ $$s = s_0 + v_0 t + \tfrac{1}{2}a_c t^2$$

$$-1.2\cos 42.29° = 0 - 2.951(\sin 42.29°)t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 + 1.9857t - 0.8877 = 0$$

Solving for the positive root: $t = 0.2687$ s

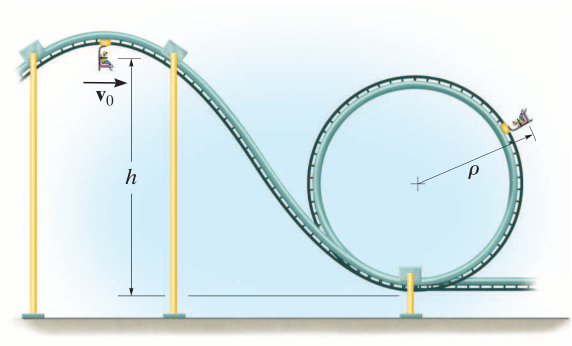
$(\overset{+}{\rightarrow})$ $$s = s_0 + v_0\, t$$

$$s = 0 + (2.951\cos 42.29°)(0.2687)$$

$$s = 0.587 \text{ m} \qquad \textbf{Ans.}$$

14–90.

The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. Determine the minimum speed v_0 at which the cars should coast down from the top of the hill, so that passengers can just make the loop without leaving contact with their seats. Neglect friction, the size of the car and passenger, and assume each passenger and car has a mass m.



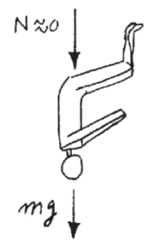
SOLUTION

Datum at ground:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_1^2 + mg2\rho$$

$$v_1 = \sqrt{v_0^2 + 2g(h-2\rho)}$$



$$+\downarrow \Sigma F_n = ma_n; \qquad mg = m\left(\frac{v_1^2}{\rho}\right)$$

$$v_1 = \sqrt{g\rho}$$

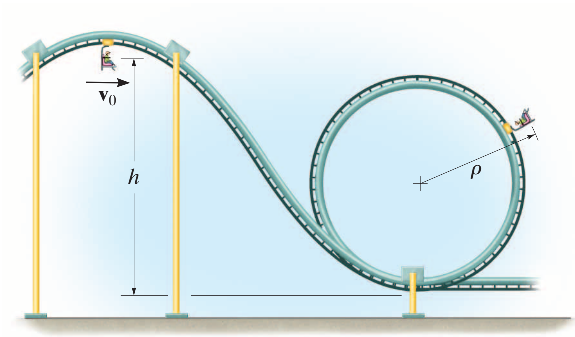
Thus,

$$g\rho = v_0^2 + 2gh - 4g\rho$$

$$v_0 = \sqrt{g(5\rho - 2h)}$$ **Ans.**

14–91.

The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. If the cars travel at $v_0 = 4$ m/s when they are at the top of the hill, determine their speed when they are at the top of the loop and the reaction of the 70-kg passenger on his seat at this instant. The car has a mass of 50 kg. Take $h = 12$ m, $\rho = 5$ m. Neglect friction and the size of the car and passenger.



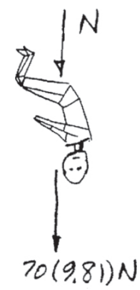
SOLUTION

Datum at ground:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(120)(4)^2 + 120(9.81)(12) = \frac{1}{2}(120)(v_1)^2 + 120(9.81)(10)$$

$$v_1 = 7.432 \text{ m/s}$$ **Ans.**

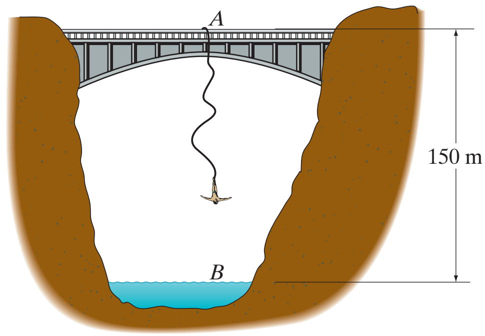


$$+\downarrow \Sigma F_n = ma_n; \qquad 70(9.81) + N = 70\left(\frac{(7.432)^2}{5}\right)$$

$$N = 86.7 \text{ N}$$ **Ans.**

***14–92.**

The 75-kg man bungee jumps off the bridge at A with an initial downward speed of 1.5 m/s. Determine the required unstretched length of the elastic cord to which he is attached in order that he stops momentarily just above the surface of the water. The stiffness of the elastic cord is $k = 80$ N/m. Neglect the size of the man.



SOLUTION

Potential Energy: With reference to the datum set at the surface of the water, the gravitational potential energy of the man at positions A and B are $\left(V_g\right)_A = mgh_A = 75(9.81)(150) = 110362.5$ J and $\left(V_g\right)_B = mgh_B = 75(9.81)(0) = 0$. When the man is at position A, the elastic cord is unstretched ($s_A = 0$), whereas the elastic cord stretches $s_B = \left(150 - l_0\right)$ m, where l_0 is the unstretched length of the cord. Thus, the elastic potential energy of the elastic cord when the man is at these two positions are $\left(V_e\right)_A = \frac{1}{2}ks_A{}^2 = 0$ and $\left(V_e\right)_B = \frac{1}{2}ks_B{}^2 = \frac{1}{2}(80)(150 - l_0)^2 = 40(150 - l_0)^2$.

Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

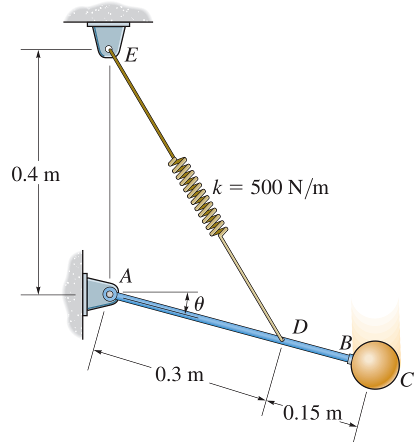
$$\frac{1}{2}mv_A{}^2 + \left[\left(V_g\right)_A + \left(V_e\right)_A\right] = \frac{1}{2}mv_B{}^2 + \left[\left(V_g\right)_B + \left(V_e\right)_B\right]$$

$$\frac{1}{2}(75)(1.5^2) + \left(110362.5 + 0\right) = 0 + \left[0 + 40(150 - l_0)^2\right]$$

$$l_0 = 97.5 \text{ m} \qquad \textbf{Ans.}$$

14–93.

The 10-kg sphere C is released from rest when $\theta = 0°$ and the tension in the spring is 100 N. Determine the speed of the sphere at the instant $\theta = 90°$. Neglect the mass of rod AB and the size of the sphere.



SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the sphere at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 10(9.81)(0.45) = 44.145\text{ J}$ and $\left(V_g\right)_2 = mgh_2 = 10(9.81)(0) = 0$. When the sphere is at position (1), the spring stretches $s_1 = \dfrac{100}{500} = 0.2\text{ m}$. Thus, the unstretched length of the spring is $l_0 = \sqrt{0.3^2 + 0.4^2} - 0.2 = 0.3\text{ m}$, and the elastic potential energy of the spring is $\left(V_e\right)_1 = \dfrac{1}{2}ks_1{}^2 = \dfrac{1}{2}(500)(0.2^2) = 10\text{ J}$. When the sphere is at position (2), the spring stretches $s_2 = 0.7 - 0.3 = 0.4\text{ m}$, and the elastic potential energy of the spring is $\left(V_e\right)_2 = \dfrac{1}{2}ks_2{}^2 = \dfrac{1}{2}(500)(0.4^2) = 40\text{ J}$.

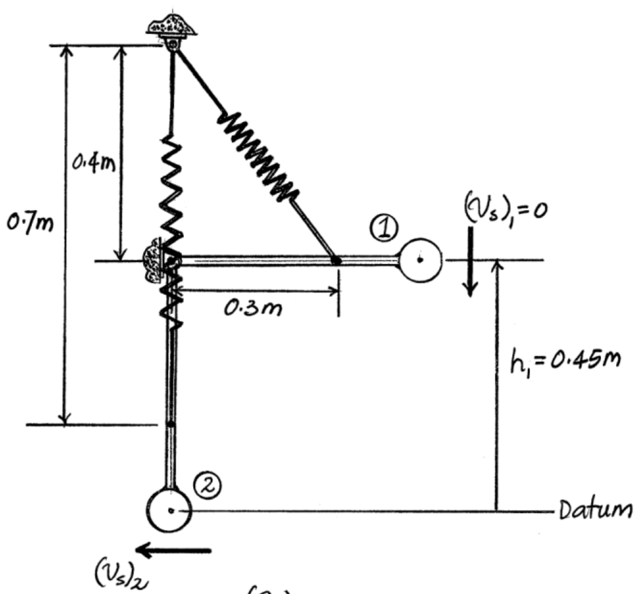
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_s\left(v_s\right)_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}m_s\left(v_s\right)_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

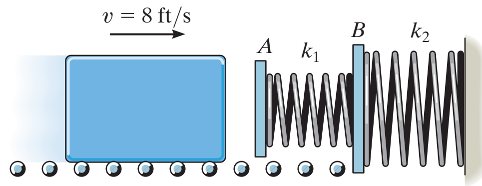
$$0 + (44.145 + 10) = \frac{1}{2}(10)(v_s)_2{}^2 + (0 + 40)$$

$$(v_s)_2 = 1.68\text{ m/s} \qquad \textbf{Ans.}$$



14–94.

The double-spring bumper is used to stop the 1500-lb steel billet in the rolling mill. Determine the maximum displacement of the plate A if the billet strikes the plate with a speed of 8 ft/s. Neglect the mass of the springs, rollers and the plates A and B. Take $k_1 = 3000$ lb/ft, $k_2 = 45\,000$ lb/ft.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{1500}{32.2}\right)(8)^2 + 0 = 0 + \frac{1}{2}(3000)s_L^2 + \frac{1}{2}(4500)s_2^2 \qquad \textbf{(1)}$$

$$F_s = 3000s_1 = 4500s_2; \qquad \textbf{(2)}$$

$$s_1 = 1.5s_2$$

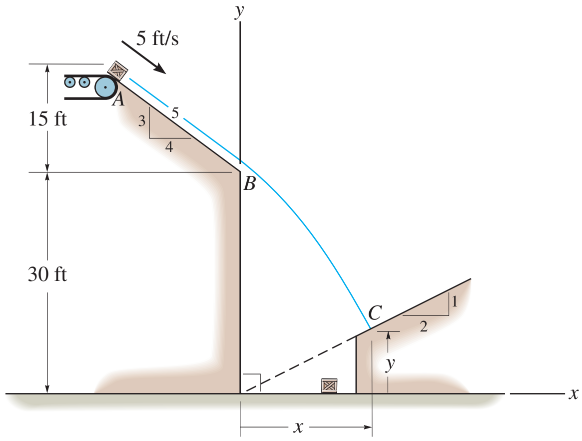
Solving Eqs. (1) and (2) yields:

$$s_2 = 0.5148 \text{ ft} \qquad s_1 = 0.7722 \text{ ft}$$

$$s_A = s_1 + s_2 = 0.7722 + 0.5148 = 1.29 \text{ ft} \qquad \textbf{Ans.}$$

14–95.

The 2-lb box has a velocity of 5 ft/s when it begins to slide down the smooth inclined surface at A. Determine the point C (x, y) where it strikes the lower incline.



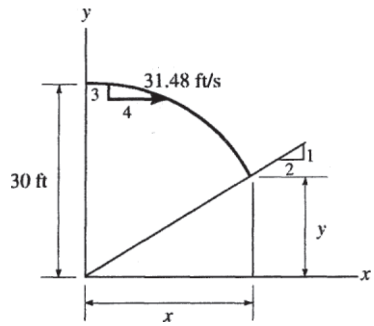
SOLUTION

Datum at A:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2 - 2(15)$$

$$v_B = 31.48 \text{ ft/s}$$



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$x = 0 + 31.48\left(\frac{4}{5}\right)t \qquad \textbf{(1)}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 30 - 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

Equation of inclined surface:

$$\frac{y}{x} = \frac{1}{2}; \qquad y = \frac{1}{2}x \qquad \textbf{(2)}$$

Thus,

$$30 - 18.888t - 16.1t^2 = 12.592t$$

$$-16.1t^2 - 31.480t + 30 = 0$$

Solving for the positive root,

$$t = 0.7014 \text{ s}$$

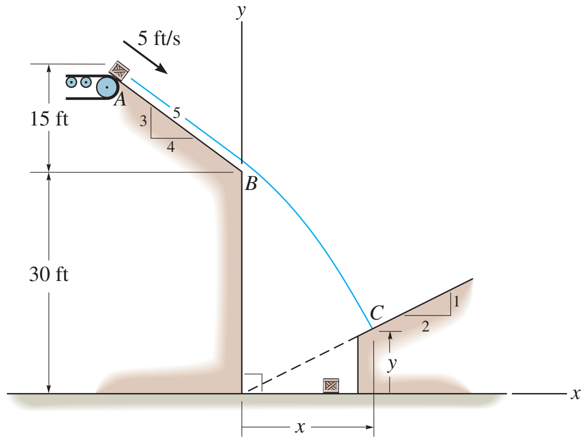
From Eqs. (1) and (2):

$$x = 31.48\left(\frac{4}{5}\right)(0.7014) = 17.66 = 17.7 \text{ ft} \qquad \textbf{Ans.}$$

$$y = \frac{1}{2}(17.664) = 8.832 = 8.83 \text{ ft} \qquad \textbf{Ans.}$$

*14–96.

The 2-lb box has a velocity of 5 ft/s when it begins to slide down the smooth inclined surface at A. Determine its speed just before hitting the surface at C and the time to travel from A to C. The coordinates of point C are $x = 17.66$ ft, and $y = 8.832$ ft.



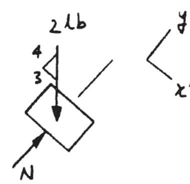
SOLUTION

Datum at A:

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_C)^2 - 2[15 + (30 - 8.832)]$$

$v_C = 48.5$ ft/s **Ans.**



$$+\searrow \Sigma F_{x'} = ma_{x'}; \qquad 2\left(\frac{3}{5}\right) = \left(\frac{2}{32.2}\right)a_{x'}$$

$$a_{x'} = 19.32 \text{ ft/s}^2$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2 - 2(15)$$

$v_B = 31.48$ ft/s

$(+\searrow)$ $\quad v_B = v_A + a_c t$

$31.48 = 5 + 19.32t_{AB}$

$t_{AB} = 1.371$ s

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$x = 0 + 31.48\left(\frac{4}{5}\right)t \qquad \textbf{(1)}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$y = 30 - 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

Equation of inclined surface:

$$\frac{y}{x} = \frac{1}{2}; \qquad y = \frac{1}{2}x \qquad \textbf{(2)}$$

Thus

$30 - 18.888t - 16.1t^2 = 12.592t$

$-16.1t^2 - 31.480t + 30 = 0$

Solving for the positive root:

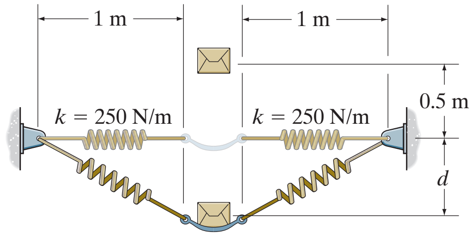
$t = 0.7014$ s

Total time is

$t = 1.371 + 0.7014 = 2.07$ s **Ans.**

14–97.

A pan of negligible mass is attached to two identical springs of stiffness $k = 250$ N/m. If a 10-kg box is dropped from a height of 0.5 m above the pan, determine the maximum vertical displacement d. Initially each spring has a tension of 50 N.



SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the box at positions (1) and (2) are $(V_g)_1 = mgh_1 = 10(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 10(9.81)[-(0.5 + d)] = -98.1(0.5 + d)$. Initially, the spring stretches $s_1 = \dfrac{50}{250} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = 1 - 0.2 = 0.8$ m and the initial elastic potential of each spring is $(V_e)_1 = (2)\dfrac{1}{2}ks_1{}^2 = 2(250/2)(0.2^2) = 10$ J. When the box is at position (2), the spring stretches $s_2 = \left(\sqrt{d^2 + 1^2} - 0.8\right)$ m. The elastic potential energy of the springs when the box is at this position is

$$(V_e)_2 = (2)\frac{1}{2}ks_2{}^2 = 2(250/2)\left[\sqrt{d^2 + 1} - 0.8\right]^2 = 250\left(d^2 - 1.6\sqrt{d^2 + 1} + 1.64\right).$$

Conservation of Energy:

$$T_1 + V_1 + T_2 + V_2$$

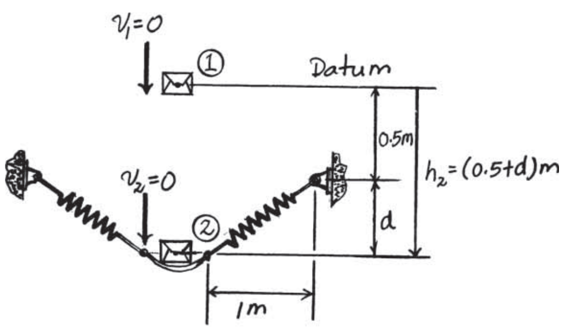
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + (V_e)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + (V_e)_2\right]$$

$$0 + (0 + 10) = 0 + \left[-98.1(0.5 + d) + 250\left(d^2 - 1.6\sqrt{d^2 + 1} + 1.64\right)\right]$$

$$250d^2 - 98.1d - 400\sqrt{d^2 + 1} + 350.95 = 0$$

Solving the above equation by trial and error,

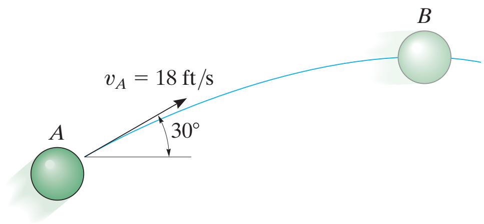
$$d = 1.34 \text{ m} \qquad \textbf{Ans.}$$



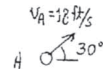
(a)

15–1.

A 2-lb ball is thrown in the direction shown with an initial speed $v_A = 18$ ft/s. Determine the time needed for it to reach its highest point B and the speed at which it is traveling at B. Use the principle of impulse and momentum for the solution.



SOLUTION



$(+\uparrow)$ $$m(v_y)_1 + \Sigma\int F\,dt = m(v_y)_2$$

$$\frac{2}{32.2}(18\sin 30^\circ) - 2(t) = 0$$

$$t = 0.2795 = 0.280\text{ s}$$ **Ans.**

$(\xrightarrow{+})$ $$m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

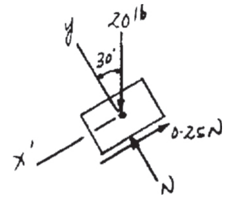
$$\frac{2}{32.2}(18\cos 30^\circ) + 0 = \frac{2}{32.2}(v_B)$$

$$v_B = 15.588 = 15.6\text{ ft/s}$$ **Ans.**

15–2.

A 20-lb block slides down a 30° inclined plane with an initial velocity of 2 ft/s. Determine the velocity of the block in 3 s if the coefficient of kinetic friction between the block and the plane is $\mu_k = 0.25$.

SOLUTION



$$(+\nwarrow) \qquad m(v_{v'}) + \Sigma \int_{t_1}^{t_2} F_{y'} dt = m(v_{y'})_2$$

$$0 + N(3) - 20 \cos 30°(3) = 0 \qquad N = 17.32 \text{ lb}$$

$$(+\swarrow) \qquad m(v_{x'})_1 + \Sigma \int_{t_1}^{t_2} F_{x'} dt = m(v_{x'})_2$$

$$\frac{20}{32.2}(2) + 20 \sin 30°(3) - 0.25(17.32)(3) = \frac{20}{32.2} v$$

$v = 29.4$ ft/s **Ans.**

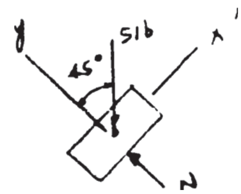
15–3.

A 5-lb block is given an initial velocity of 10 ft/s up a 45° smooth slope. Determine the time for it to travel up the slope before it stops.

SOLUTION

$$(\nearrow +) \qquad m(v_{x'})_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_{x'})_2$$

$$\frac{5}{32.2}(10) + (-5 \sin 45°)t = 0$$



$t = 0.439$ s **Ans.**

***15–4.**

The 180-lb iron worker is secured by a fall-arrest system consisting of a harness and lanyard AB, which is fixed to the beam. If the lanyard has a slack of 4 ft, determine the average impulsive force developed in the lanyard if he happens to fall 4 feet. Neglect his size in the calculation and assume the impulse takes place in 0.6 seconds.

SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 180(4) = \frac{1}{2}\left(\frac{180}{32.2}\right)v^2$$

$$v = 16.05 \text{ ft/s}$$

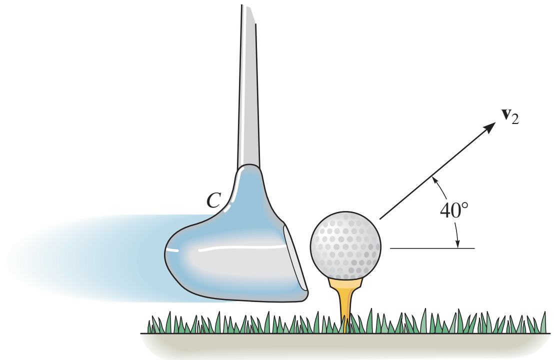
$$(+\downarrow) \qquad mv_1 + \int F\,dt = mv_2$$

$$\frac{180}{32.2}(16.05) + 180(0.6) - F(0.6) = 0$$

$$F = 329.5 \text{ lb} = 330 \text{ lb} \qquad \textbf{Ans.}$$

15–5.

A man hits the 50-g golf ball such that it leaves the tee at an angle of 40° with the horizontal and strikes the ground at the same elevation a distance of 20 m away. Determine the impulse of the club C on the ball. Neglect the impulse caused by the ball's weight while the club is striking the ball.



SOLUTION

$(\overset{+}{\rightarrow})\qquad s_x = (s_0)_x + (v_0)_x t$

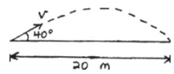
$$20 = 0 + v\cos 40°(t)$$

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$0 = 0 + v\sin 40°(t) - \frac{1}{2}(9.81)t^2$$

$$t = 1.85 \text{ s}$$

$$v = 14.115 \text{ m/s}$$



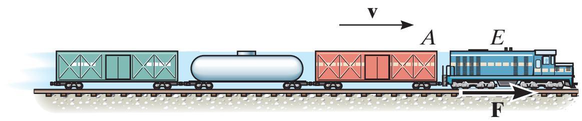
$(\overset{+}{\nearrow})\qquad mv_1 + \Sigma\int F\,dt = mv_2$

$$0 + \int F\,dt = (0.05)(14.115)$$

$\int F\,dt = 0.706 \text{ N}\cdot\text{s} \measuredangle_\theta\, 40°$ **Ans.**

15–6.

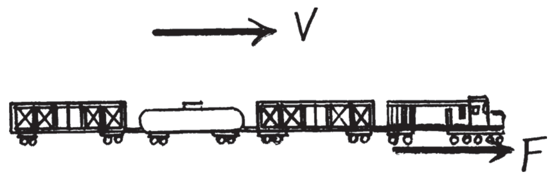
A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force T developed at the coupling between the engine E and the first car A. The wheels of the engine provide a resultant frictional tractive force **F** which gives the train forward motion, whereas the car wheels roll freely. Also, determine F acting on the engine wheels.



SOLUTION

$(v_x)_2 = 40 \text{ km/h} = 11.11 \text{ m/s}$

Entire train:

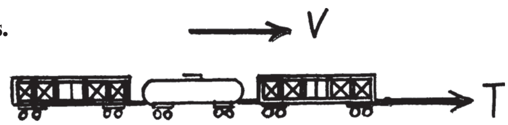


$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + F(80) = [50 + 3(30)]\left(10^3\right)(11.11)$$

$$F = 19.4 \text{ kN}$$ **Ans.**

Three cars:

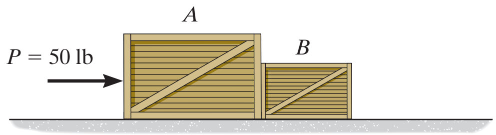


$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + T(80) = 3(30)\left(10^3\right)(11.11) \qquad T = 12.5 \text{ kN}$$ **Ans.**

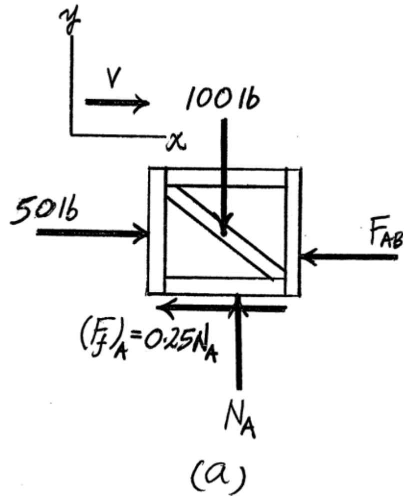
15–7.

Crates A and B weigh 100 lb and 50 lb, respectively. If they start from rest, determine their speed when $t = 5$ s. Also, find the force exerted by crate A on crate B during the motion. The coefficient of kinetic friction between the crates and the ground is $\mu_k = 0.25$.



SOLUTION

Free-Body Diagram: The free-body diagram of crates A and B are shown in Figs. a and b, respectively. The frictional force acting on each crate is $(F_f)_A = \mu_k N_A = 0.25N_A$ and $(F_f)_B = \mu_k N_B = 0.25N_B$.



Principle of Impulse and Momentum: Referring to Fig. a,

$(+\uparrow)$ $$m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$\frac{100}{32.2}(0) + N_A(5) - 100(5) = \frac{100}{32.2}(0)$$

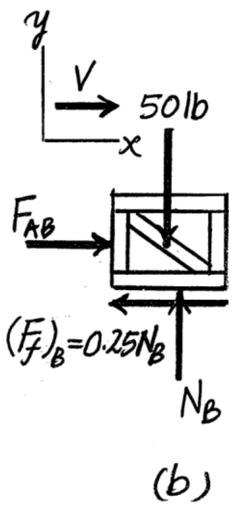
$$N_A = 100 \text{ lb}$$

$(\overset{+}{\rightarrow})$ $$m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$\frac{100}{32.2}(0) + 50(5) - 0.25(100)(5) - F_{AB}(5) = \frac{100}{32.2}v$$

$$v = 40.25 - 1.61F_{AB} \qquad \textbf{(1)}$$

By considering Fig. b,



$(+\uparrow)$ $$m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$\frac{50}{32.2}(0) + N_B(5) - 50(5) = \frac{50}{32.2}(0)$$

$$N_B = 50 \text{ lb}$$

$(\overset{+}{\rightarrow})$ $$m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$\frac{50}{32.2}(0) + F_{AB}(5) - 0.25(50)(5) = \frac{50}{32.2}v$$

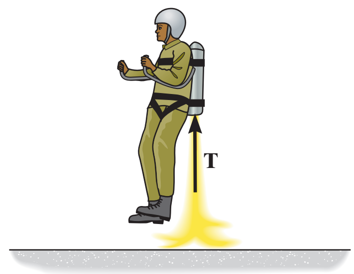
$$v = 3.22\,F_{AB} - 40.25 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$F_{AB} = 16.67 \text{ lb} = 16.7 \text{ lb} \qquad v = 13.42 \text{ ft/s} = 13.4 \text{ ft/s} \qquad \textbf{Ans.}$$

***15–8.**

If the jets exert a vertical thrust of $T = (500t^{3/2})$N, where t is in seconds, determine the man's speed when $t = 3$ s. The total mass of the man and the jet suit is 100 kg. Neglect the loss of mass due to the fuel consumed during the lift which begins from rest on the ground.



SOLUTION

Free-Body Diagram: The thrust **T** must overcome the weight of the man and jet before they move. Considering the equilibrium of the free-body diagram of the man and jet shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 500t^{3/2} - 100(9.81) = 0 \qquad t = 1.567 \text{ s}$$

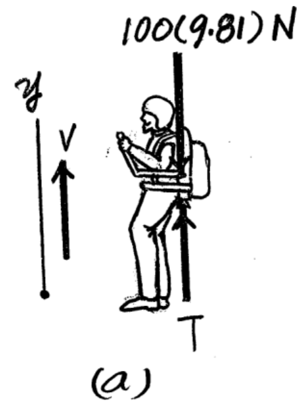
Principle of Impulse and Momentum: Only the impulse generated by thrust **T** after $t = 1.567$ s contributes to the motion. Referring to Fig. *a*,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$100(0) + \int_{1.567 \text{ s}}^{3 \text{ s}} 500t^{3/2} dt - 100(9.81)(3 - 1.567) = 100v$$

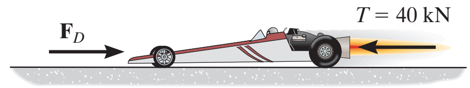
$$\left(200t^{5/2}\right)\Big|_{1.567 \text{ s}}^{3 \text{ s}} - 1405.55 = 100v$$

$$v = 11.0 \text{ m/s}$$ **Ans.**



15–9.

Under a constant thrust of $T = 40$ kN, the 1.5-Mg dragster reaches its maximum speed of 125 m/s in 8 s starting from rest. Determine the average drag resistance $\mathbf{F}_D$ during this period of time.



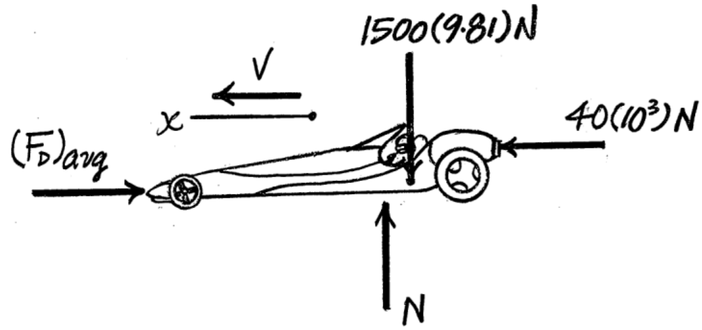
SOLUTION

Principle of Impulse and Momentum: The final speed of the dragster is $v_2 = 125$ m/s. Referring to the free-body diagram of the dragster shown in Fig. *a*,

$$(\overset{+}{\leftarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_2)_x$$

$$1500(0) + 40(10^3)(8) - (F_D)_{\text{avg}}(8) = 1500(125)$$

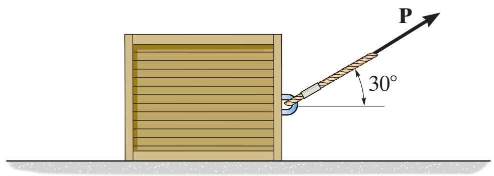
$$(F_D)_{\text{avg}} = 16\,562.5 \text{ N} = 16.6 \text{ kN} \qquad \textbf{Ans.}$$



(a)

15–10.

The 50-kg crate is pulled by the constant force **P**. If the crate starts from rest and achieves a speed of 10 m/s in 5 s, determine the magnitude of **P**. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



SOLUTION

Impulse and Momentum Diagram: The frictional force acting on the crate is $F_f = \mu_k N = 0.2N$.

Principle of Impulse and Momentum:

$(+\uparrow) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$

$$0 + N(5) + P(5)\sin 30° - 50(9.81)(5) = 0$$

$$N = 490.5 - 0.5P \qquad \textbf{(1)}$$

$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_2)_x$

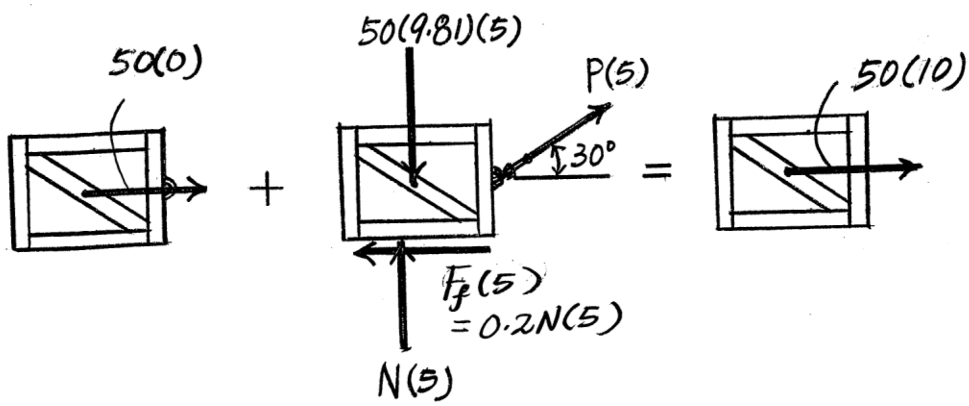
$$50(0) + P(5)\cos 30° - 0.2N(5) = 50(10)$$

$$4.3301P - N = 500 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$N = 387.97 \text{ N}$$

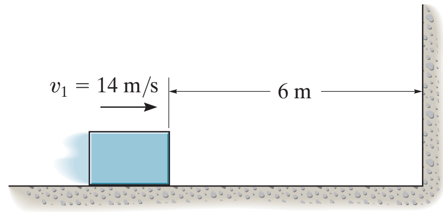
$$P = 205 \text{ N} \qquad \textbf{Ans.}$$



(a)

15–11.

When the 5-kg block is 6 m from the wall, it is sliding at $v_1 = 14$ m/s. If the coefficient of kinetic friction between the block and the horizontal plane is $\mu_k = 0.3$, determine the impulse of the wall on the block necessary to stop the block. Neglect the friction impulse acting on the block during the collision.



SOLUTION

Equation of Motion: The acceleration of the block must be obtained first before one can determine the velocity of the block before it strikes the wall.

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 5(9.81) = 5(0) \qquad N = 49.05 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -0.3(49.05) = -5a \qquad a = 2.943 \text{ m/s}^2$$

Kinematics: Applying the equation $v^2 = v_0^2 + 2a_c\,(s - s_0)$ yields

$$(\overset{+}{\rightarrow}) \qquad v^2 = 14^2 + 2(-2.943)(6 - 0) \qquad v = 12.68 \text{ m/s}$$

Principle of Linear Impulse and Momentum: Applying Eq. 15–4, we have

$$m(v_x)_1 + \Sigma \int_{t1}^{t2} F_x dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 5(12.68) - I = 5(0)$$

$$I = 63.4 \text{ N} \cdot \text{s}$$ **Ans.**

***15–12.**

For a short period of time, the frictional driving force acting on the wheels of the 2.5-Mg van is $F_D = (600t^2)$ N, where t is in seconds. If the van has a speed of 20 km/h when $t = 0$, determine its speed when $t = 5$ s.

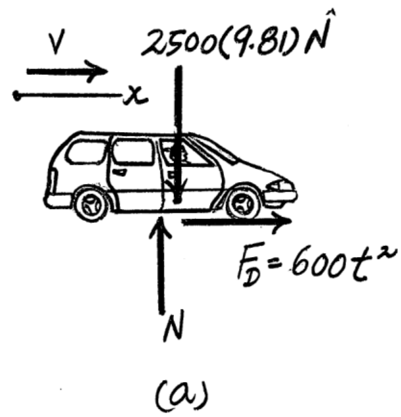
SOLUTION

Principle of Impulse and Momentum: The initial speed of the van is $v_1 = \left[20(10^3)\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 5.556$ m/s. Referring to the free-body diagram of the van shown in Fig. *a*,

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_2)_x$$

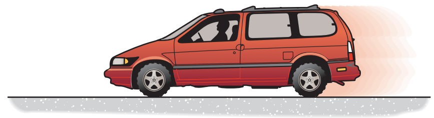
$$2500(5.556) + \int_0^{5\text{ s}} 600t^2\,dt = 2500\,v_2$$

$$v_2 = 15.6 \text{ m/s}$$ **Ans.**



15–13.

The 2.5-Mg van is traveling with a speed of 100 km/h when the brakes are applied and all four wheels lock. If the speed decreases to 40 km/h in 5 s, determine the coefficient of kinetic friction between the tires and the road.



SOLUTION

Free-Body Diagram: The free-body diagram of the van is shown in Fig. *a*. The frictional force is $F_f = \mu_k N$ since all the wheels of the van are locked and will cause the van to slide.

Principle of Impulse and Momentum: The initial and final speeds of the van are $v_1 = \left[100(10^3)\,\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 27.78\text{ m/s}$ and $v_2 = \left[40(10^3)\,\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 11.11\text{ m/s}$. Referring to Fig. *a*,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_2)_y$$

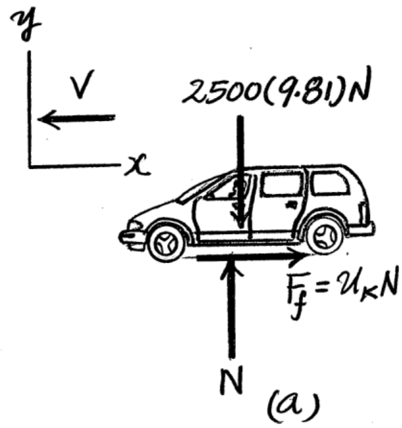
$$2500(0) + N(5) - 2500(9.81)(5) = 2500(0)$$

$$N = 24\,525\text{ N}$$

$$(\overset{+}{\leftarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_2)_x$$

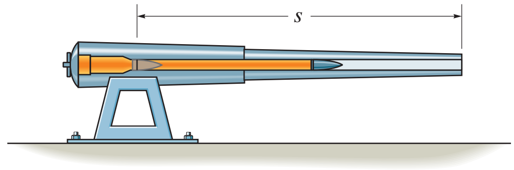
$$2500(27.78) + [-\mu_k(24525)(5)] = 2500(11.1)$$

$$\mu_k = 0.340 \qquad \textbf{Ans.}$$

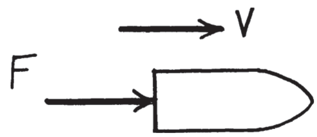


15–14.

The force acting on a projectile having a mass m as it passes horizontally through the barrel of the cannon is $F = C \sin(\pi t/t')$. Determine the projectile's velocity when $t = t'$. If the projectile reaches the end of the barrel at this instant, determine the length s.



SOLUTION



$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + \int_0^t C \sin\left(\frac{\pi t}{t'}\right) = mv$$

$$-C\left(\frac{t'}{\pi}\right)\cos\left(\frac{\pi t}{t'}\right)\Big|_0^t = mv$$

$$v = \frac{Ct'}{\pi m}\left(1 - \cos\left(\frac{\pi t}{t'}\right)\right)$$

When $t = t'$,

$$v_2 = \frac{2C\,t'}{\pi m} \qquad \text{`Ans.}$$

$$ds = v\,dt$$

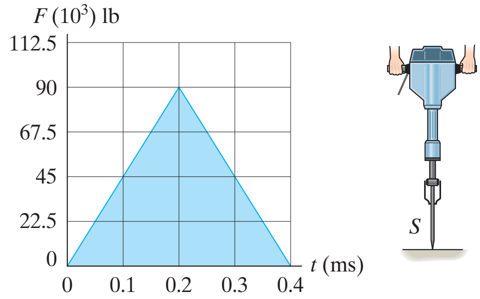
$$\int_0^s ds = \int_0^t \left(\frac{C\,t'}{\pi\, m}\right)\left(1 - \cos\left(\frac{\pi t}{t'}\right)\right) dt$$

$$s = \left(\frac{C\,t'}{\pi m}\right)\left[t - \frac{t'}{\pi}\sin\left(\frac{\pi t}{t'}\right)\right]_0^{t'}$$

$$s = \frac{Ct'^2}{\pi m} \qquad \textbf{Ans.}$$

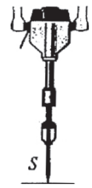
15–15.

During operation the breaker hammer develops on the concrete surface a force which is indicated in the graph. To achieve this the 2-lb spike S is fired from rest into the surface at 200 ft/s. Determine the speed of the spike just after rebounding.



SOLUTION

$(+\downarrow)$ $$mv_1 + \int F\,dt = mv_2$$

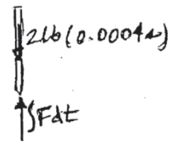


$$\frac{2}{32.2}(200) + 2(0.0004) - Area = \frac{-2}{32.2}(v)$$

$$Area = \frac{1}{2}(90)(10^3)(0.4)(10^{-3}) = 18 \text{ lb} \cdot \text{s}$$

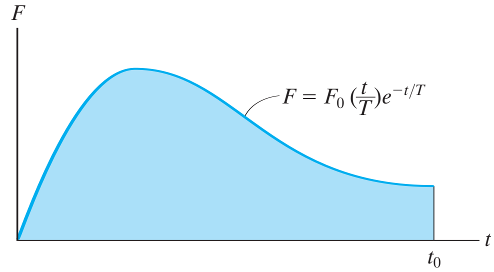
Thus,

$$v = 89.8 \text{ ft/s}$$ **Ans.**



***15–16.**

The twitch in a muscle of the arm develops a force which can be measured as a function of time as shown in the graph. If the effective contraction of the muscle lasts for a time t_0, determine the impulse developed by the muscle.



SOLUTION

$$I = \int F\,dt = \int_0^{t_0} F_0\left(\frac{t}{T}\right)e^{-t/T}dt$$

$$I = \frac{F_0}{T}\int_0^{t_0} te^{-(t/T)}dt$$

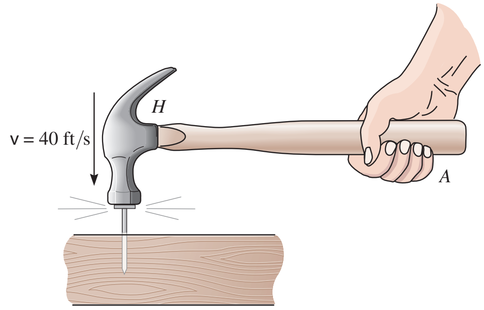
$$I = -F_0\left[Te^{-t/T}\left(\frac{t}{T}+1\right)\right]_0^{t_0}$$

$$I = -F_0\left[Te^{-t_0/T}\left(\frac{t_0}{T}+1\right)-T\right]$$

$$I = TF_0\left[1-e^{-t_0/T}\left(1+\frac{t_0}{T}\right)\right]$$ **Ans.**

15–17.

A hammer head H having a weight of 0.25 lb is moving vertically downward at 40 ft/s when it strikes the head of a nail of negligible mass and drives it into a block of wood. Find the impulse on the nail if it is assumed that the grip at A is loose, the handle has a negligible mass, and the hammer stays in contact with the nail while it comes to rest. Neglect the impulse caused by the weight of the hammer head during contact with the nail.

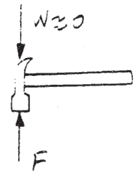


SOLUTION

$$(+\downarrow) \qquad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$$

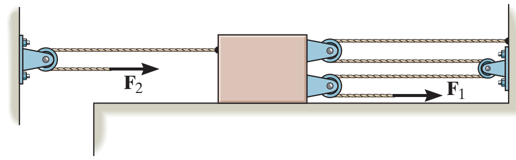
$$\left(\frac{0.25}{32.2}\right)(40) - \int F\,dt = 0$$

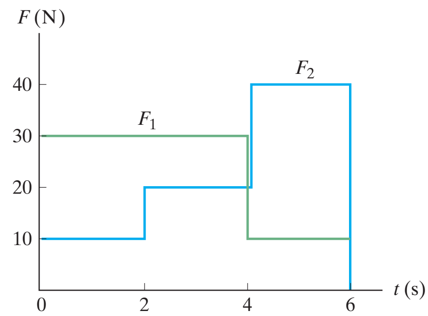
$$\int F\,dt = 0.311\ \text{lb}\cdot\text{s}$$ **Ans.**



15–18.

The 40-kg slider block is moving to the right with a speed of 1.5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.





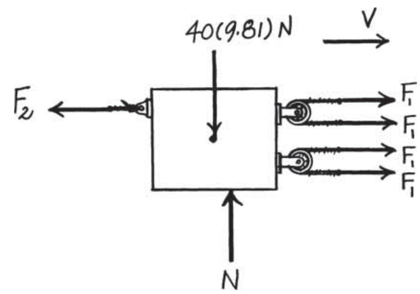
SOLUTION

The impulses acting on the block are equal to the areas under the graph.

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

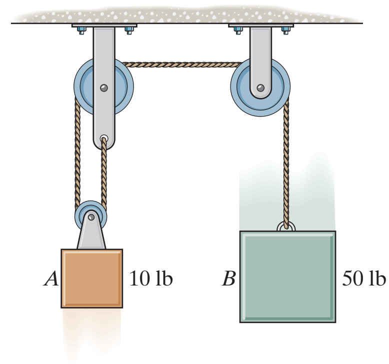
$$40(1.5) + 4[(30)4 + 10(6 - 4)] - [10(2) + 20(4 - 2) + 40(6 - 4)] = 40v_2$$

$$v_2 = 12.0 \text{ m/s} \;(\rightarrow) \qquad \textbf{Ans.}$$



15–19.

Determine the velocity of each block 2 s after the blocks are released from rest. Neglect the mass of the pulleys and cord.

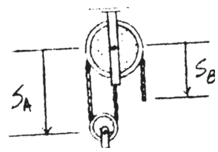


SOLUTION

Kinematics: The speed of block A and B can be related by using the position coordinate equation.

$$2s_A + s_B = l$$

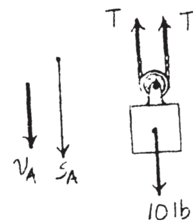
$$2v_A + v_B = 0 \qquad \textbf{(1)}$$



Principle of Linear Impulse and Momentum: Applying Eq. 15–4 to block A, we have

$$m\left(v_y\right)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m\left(v_y\right)_2$$

$$(+\uparrow) \qquad -\left(\frac{10}{32.2}\right)(0) + 2T(2) - 10(2) = -\left(\frac{10}{32.2}\right)(v_A) \qquad \textbf{(2)}$$



Applying Eq. 15–4 to block B, we have

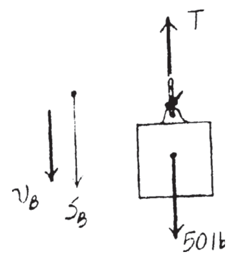
$$m\left(v_y\right)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m\left(v_y\right)_2$$

$$(+\uparrow) \qquad -\left(\frac{50}{32.2}\right)(0) + T(2) - 50(2) = -\left(\frac{50}{32.2}\right)(v_B) \qquad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3) yields

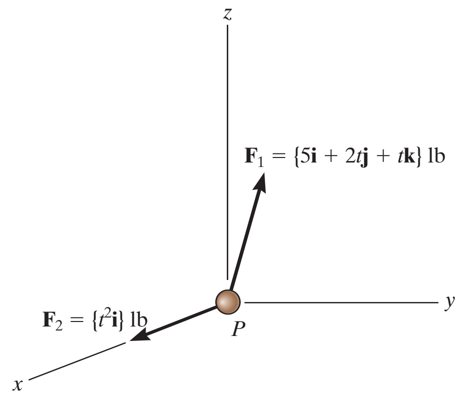
$$v_A = -27.6\ \text{ft/s} = 27.6\ \text{ft/s}\ \uparrow \qquad v_B = 55.2\ \text{ft/s}\ \downarrow \qquad \textbf{Ans.}$$

$$T = 7.143\ \text{lb}$$



***15–20.**

The particle P is acted upon by its weight of 3 lb and forces $\mathbf{F}_1$ and $\mathbf{F}_2$, where t is in seconds. If the particle orginally has a velocity of $\mathbf{v}_1 = \{3\mathbf{i} + 1\mathbf{j} + 6\mathbf{k}\}$ ft/s, determine its speed after 2 s.



SOLUTION

$$mv_1 + \sum \int_0^2 F dt = mv_2$$

Resolving into scalar components,

$$\frac{3}{32.2}(3) + \int_0^2 (5 + t^2)dt = \frac{3}{32.2}(v_x)$$

$$\frac{3}{32.2}(1) + \int_0^2 2t dt = \frac{3}{32.2}(v_y)$$

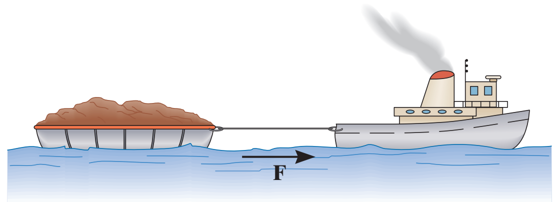
$$\frac{3}{32.2}(6) + \int_0^2 (t - 3)dt = \frac{3}{32.2}(v_z)$$

$v_x = 138.96$ ft/s $\quad v_y = 43.933$ ft/s $\quad v_z = -36.933$ ft/s

$v = \sqrt{(138.96)^2 + (43.933)^2 + (-36.933)^2} = 150$ ft/s **Ans.**

15–21.

If it takes 35 s for the 50-Mg tugboat to increase its speed uniformly to 25 km/h, starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force **F** which gives the tugboat forward motion, whereas the barge moves freely. Also, determine F acting on the tugboat. The barge has a mass of 75 Mg.

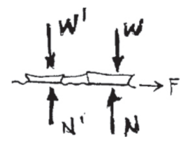


SOLUTION

$$25\left(\frac{1000}{3600}\right) = 6.944 \text{ m/s}$$

System:

$$(\overset{+}{\rightarrow}) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

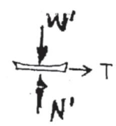


$$[0+0] + F(35) = (50 + 75)(10^3)(6.944)$$

$$F = 24.8 \text{ kN}$$ **Ans.**

Barge:

$$(\overset{+}{\rightarrow}) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$



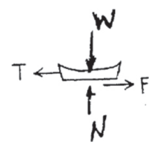
$$0 + T(35) = (75)(10^3)(6.944)$$

$$T = 14.881 = 14.9 \text{ kN}$$ **Ans.**

Also, using this result for T,

Tugboat:

$$(\overset{+}{\rightarrow}) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

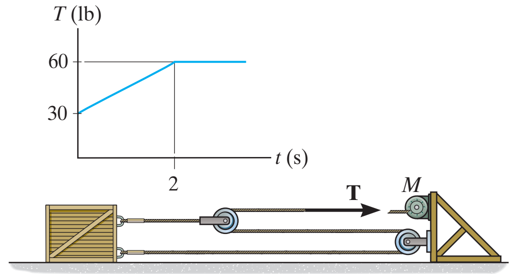


$$0 + F(35) - (14.881)(35) = (50)(10^3)(6.944)$$

$$F = 24.8 \text{ kN}$$ **Ans.**

15–22.

If the force T exerted on the cable by the motor M is indicated by the graph, determine the speed of the 500-lb crate when $t = 4$ s, starting from rest. The coefficients of static and kinetic friction are $\mu_s = 0.3$ and $\mu_k = 0.25$, respectively.



SOLUTION

Free-Body Diagram: Here, force 3**T** must overcome the friction $\mathbf{F}_f$ before the crate moves. For $0 \le t \le 2$ s, $\dfrac{T - 30}{t - 0} = \dfrac{60 - 30}{2 - 0}$ or $T = \left(15t + 30\right)$ lb. Considering the free-body diagram of the crate shown in Fig. a, where $F_f = \mu_k N = 0.3N$,

$$+\uparrow \Sigma F_y = 0; \qquad N - 500 = 0 \qquad N = 500 \text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3(15t + 30) - 0.3(500) = 0 \qquad t = 1.333 \text{ s}$$

Principle of Impulse and Momentum: Only the impulse of 3**T** after $t = 1.333$ s contributes to the motion. The impulse of **T** is equal to the area under the **T** vs. t graph. At $t = 1.333$ s, $T = 50$ lb. Thus,

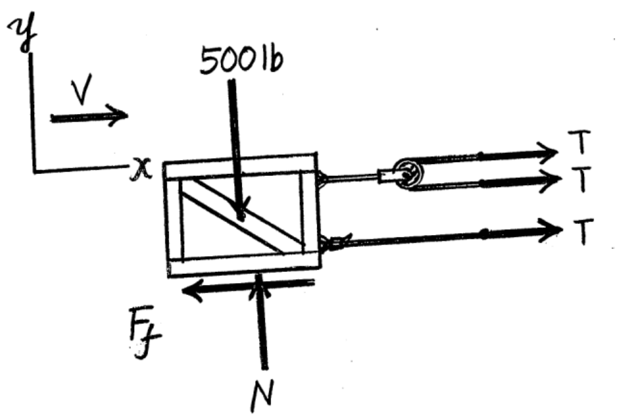
$$I = \int 3T\,dt = 3\left[\frac{1}{2}(50 + 60)(2 - 1.333) + 60(4 - 2)\right] = 470 \text{ lb} \cdot \text{s}$$

Since the crate moves, $F_f = \mu_k N = 0.25(500) = 125$ lb. Referring to Fig. a,

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_2)_x$$

$$\frac{500}{32.2}(0) + 470 - 125(4 - 1.333) = \left(\frac{500}{32.2}\right)v$$

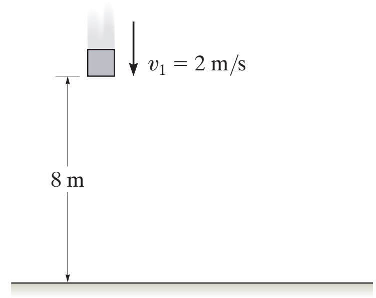
$$v = 8.80 \text{ ft/s}$$ **Ans.**



(a)

15–23.

The 5-kg block is moving downward at $v_1 = 2$ m/s when it is 8 m from the sandy surface. Determine the impulse of the sand on the block necessary to stop its motion. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.



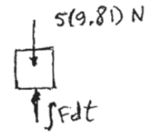
SOLUTION

Just before impact

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(5)(2)^2 + 8(5)(9.81) = \frac{1}{2}(5)(v^2)$$

$$v = 12.687 \text{ m/s}$$



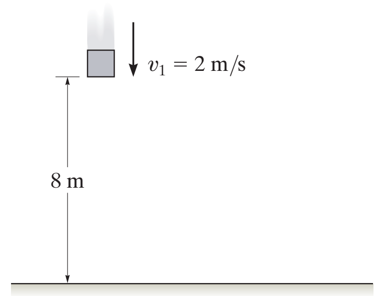
$$(+\downarrow) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

$$5(12.687) - \int F\,dt = 0$$

$$\text{I} = \int F\,dt = 63.4 \text{ N} \cdot \text{s} \qquad \textbf{Ans.}$$

***15–24.**

The 5-kg block is falling downward at $v_1 = 2$ m/s when it is 8 m from the sandy surface. Determine the average impulsive force acting on the block by the sand if the motion of the block is stopped in 0.9 s once the block strikes the sand. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.



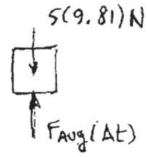
SOLUTION

Just before impact

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(5)(2)^2 + 8(5)(9.81) = \frac{1}{2}(5)(v^2)$$

$$v = 12.69 \text{ m/s}$$



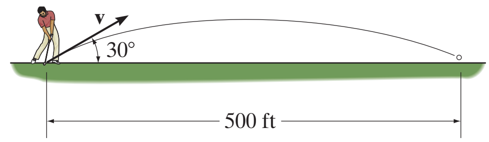
$$(+\downarrow) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

$$5(12.69) - F_{avg}(0.9) = 0$$

$$F_{avg} = 70.5 \text{ N} \qquad \textbf{Ans.}$$

15–25.

The 0.1-lb golf ball is struck by the club and then travels along the trajectory shown. Determine the average impulsive force the club imparts on the ball if the club maintains contact with the ball for 0.5 ms.



SOLUTION

Kinematics: By considering the x-motion of the golf ball, Fig. a,

$$\left(\xrightarrow{+}\right) \qquad s_x = (s_0) + (v_0)_x t$$

$$500 = 0 + v\cos 30^\circ\, t$$

$$t = \frac{500}{v\cos 30^\circ}$$

Subsequently, using the result of t and considering the y-motion of the golf ball,

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$0 = 0 + v\sin 30^\circ\left(\frac{500}{v\cos 30^\circ}\right) + \frac{1}{2}(-32.2)\left(\frac{500}{v\cos 30^\circ}\right)^2$$

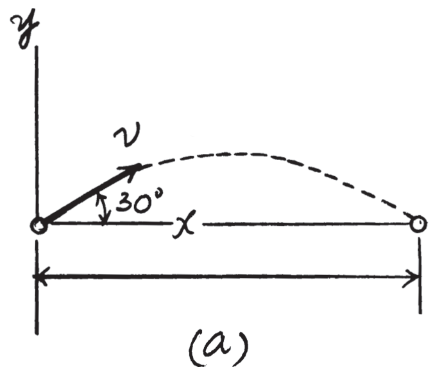
$$v = 136.35\ \text{ft/s}$$

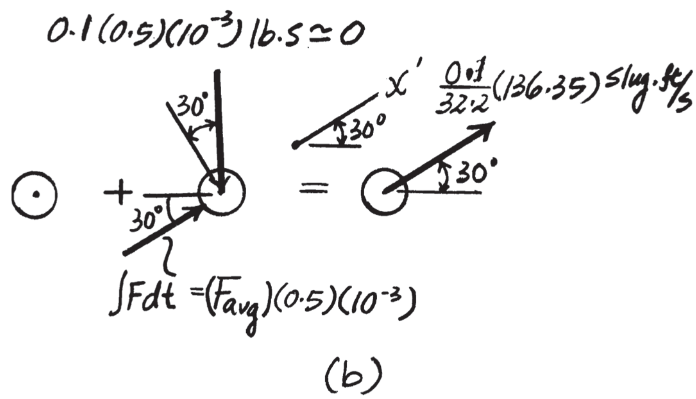
Principle of Impulse and Momentum: Here, the impulse generated by the weight of the golf ball is very small compared to that generated by the force of the impact. Hence, it can be neglected. By referring to the impulse and momentum diagram shown in Fig. b,

$$m(v_1)_{x'} + \Sigma\int_{t_1}^{t_2} F_{x'}\,dt = m(v_2)_{x'}$$

$$0 + F_{\text{avg}}(0.5)(10^{-3}) = \frac{0.1}{32.2}(136.35)$$

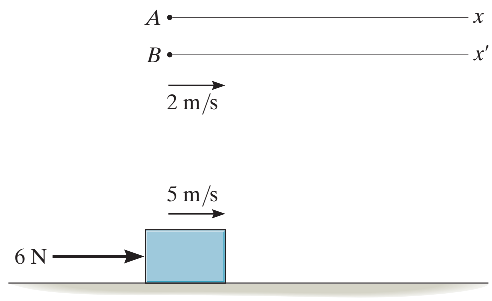
$$F_{\text{avg}} = 847\ \text{lb}$$ **Ans.**





15–26.

As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block in 4 s if it has an initial speed of 5 m/s measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis that moves at a constant velocity of 2 m/s relative to A.



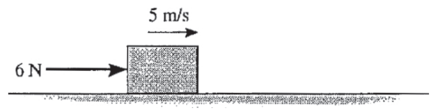
SOLUTION

Observer A:

$$(\xrightarrow{+}) \qquad m v_1 + \sum \int F\,dt = m v_2$$

$$10(5) + 6(4) = 10v$$

$$v = 7.40 \text{ m/s}$$ **Ans.**



Observer B:

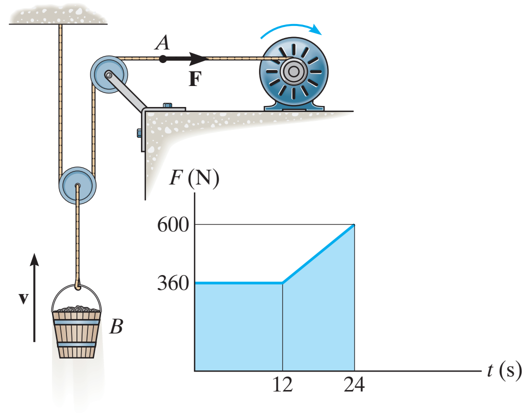
$$(\xrightarrow{+}) \qquad m v_1 + \sum \int F\,dt = m v_2$$

$$10(3) + 6(4) = 10v$$

$$v = 5.40 \text{ m/s}$$ **Ans.**

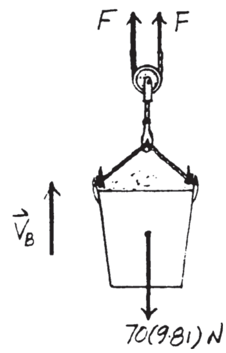
15–27.

The winch delivers a horizontal towing force **F** to its cable at A which varies as shown in the graph. Determine the speed of the 70-kg bucket when $t = 18$ s. Originally the bucket is moving upward at $v_1 = 3$ m/s.



SOLUTION

Principle of Linear Impulse and Momentum: For the time period $12\text{ s} \le t < 18\text{ s}$, $\dfrac{F - 360}{t - 12} = \dfrac{600 - 360}{24 - 12}$, $F = (20t + 120)$ N. Applying Eq. 15–4 to bucket B, we have



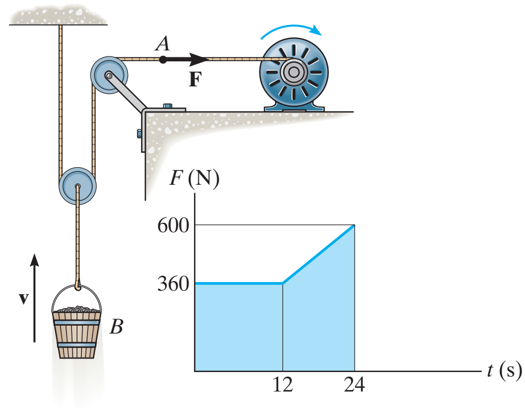
$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$(+\uparrow) \qquad 70(3) + 2\left[360(12) + \int_{12\,s}^{18\,s} (20t + 120)dt\right] - 70(9.81)(18) = 70v_2$$

$$v_2 = 21.8 \text{ m/s} \qquad \textbf{Ans.}$$

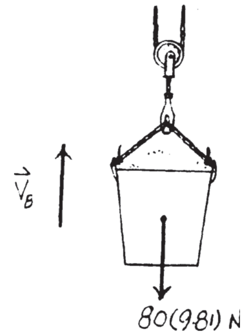
***15–28.**

The winch delivers a horizontal towing force **F** to its cable at A which varies as shown in the graph. Determine the speed of the 80-kg bucket when $t = 24$ s. Originally the bucket is released from rest.



SOLUTION

Principle of Linear Impulse and Momentum: The total impluse exerted on bucket B can be obtained by evaluating the area under the F–t graph. Thus, $I = \Sigma \int_{t_1}^{t_2} F_y\,dt = 2\left[360(12) + \frac{1}{2}(360 + 600)(24 - 12)\right] = 20160\ \text{N}\cdot\text{s}$. Applying Eq. 15–4 to the bucket B, we have



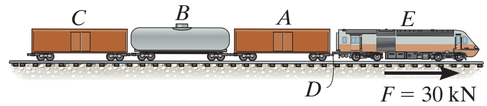
$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$(+\uparrow) \qquad 80(0) + 20160 - 80(9.81)(24) = 80v_2$$

$$v_2 = 16.6\text{m/s} \qquad \textbf{Ans.}$$

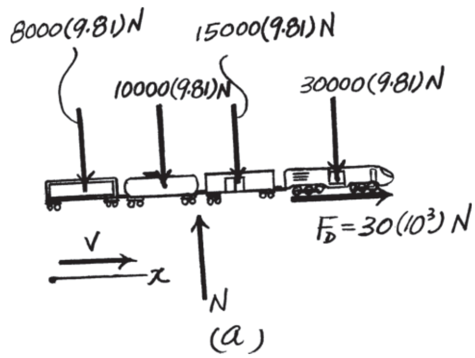
15–29.

The train consists of a 30-Mg engine E, and cars A, B, and C, which have a mass of 15 Mg, 10 Mg, and 8 Mg, respectively. If the tracks provide a traction force of $F = 30$ kN on the engine wheels, determine the speed of the train when $t = 30$ s, starting from rest. Also, find the horizontal coupling force at D between the engine E and car A. Neglect rolling resistance.



SOLUTION

Principle of Impulse and Momentum: By referring to the free-body diagram of the entire train shown in Fig. *a*, we can write



$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$63\,000(0) + 30(10^3)(30) = 63\,000v$$

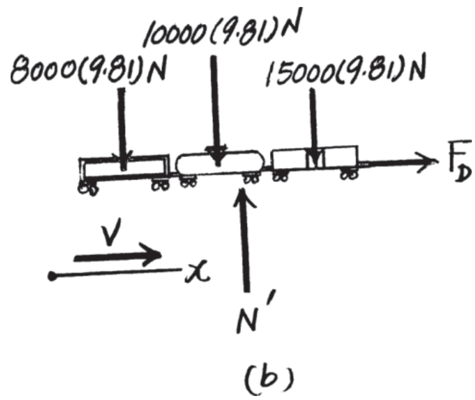
$$v = 14.29 \text{ m/s} \qquad \textbf{Ans.}$$

Using this result and referring to the free-body diagram of the train's car shown in Fig. *b*,

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_2)_x$$

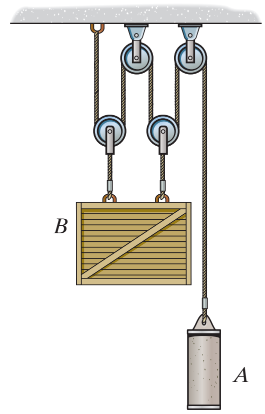
$$33000(0) + F_D(30) = 33\,000(14.29)$$

$$F_D = 15\,714.29 \text{ N} = 15.7 \text{ kN} \qquad \textbf{Ans.}$$



15–30.

The crate B and cylinder A have a mass of 200 kg and 75 kg, respectively. If the system is released from rest, determine the speed of the crate and cylinder when $t = 3$ s. Neglect the mass of the pulleys.



SOLUTION

Free-Body Diagram: The free-body diagrams of cylinder A and crate B are shown in Figs. b and c. $\mathbf{v}_A$ and $\mathbf{v}_B$ must be assumed to be directed downward so that they are consistent with the positive sense of s_A and s_B shown in Fig. a.

Principle of Impulse and Momentum: Referring to Fig. b,

$$(+\downarrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$75(0) + 75(9.81)(3) - T(3) = 75v_A$$

$$v_A = 29.43 - 0.04T \tag{1}$$

From Fig. b,

$$(+\downarrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$200(0) + 2500(9.81)(3) - 4T(3) = 200v_B$$

$$v_B = 29.43 - 0.06T \tag{2}$$

Kinematics: Expressing the length of the cable in terms of s_A and s_B and referring to Fig. a,

$$s_A + 4s_B = l \tag{3}$$

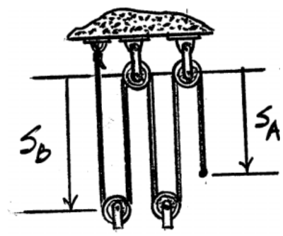
Taking the time derivative,

$$v_A + 4v_B = 0 \tag{4}$$

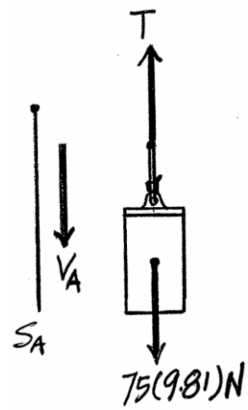
Solving Eqs. (1), (2), and (4) yields

$v_B = -2.102 \text{ m/s} = 2.10 \text{ m/s} \uparrow \qquad v_A = 8.409 \text{ m/s} = 8.41 \text{ m/s} \downarrow$ **Ans.**

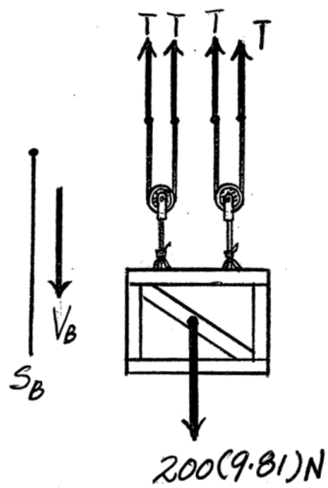
$T = 525.54$ N



(a)



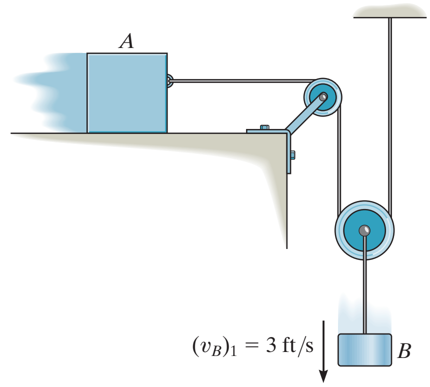
(b)



(c)

15–31.

Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity $(v_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of A when $t = 1$ s. Assume that the horizontal plane is smooth. Neglect the mass of the pulleys and cords.



SOLUTION

$s_A + 2s_B = l$

$v_A = -2v_B$

$(\overset{+}{\leftarrow}) \qquad mv_1 + \Sigma \int F\,dt = mv_2$

$$-\frac{10}{32.2}(2)(3) - T(1) = \frac{10}{32.2}(v_A)_2$$

$(+\downarrow) \qquad mv_1 + \Sigma \int F\,dt = mv_2$

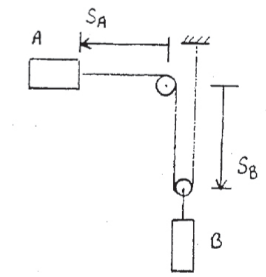
$$\frac{3}{32.2}(3) + 3(1) - 2T(1) = \frac{3}{32.2}\left(-\frac{(v_A)_2}{2}\right)$$

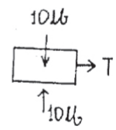
$$-32.2T - 10(v_A)_2 = 60$$

$$-64.4T + 1.5(v_A)_2 = -105.6$$

$$T = 1.40 \text{ lb}$$

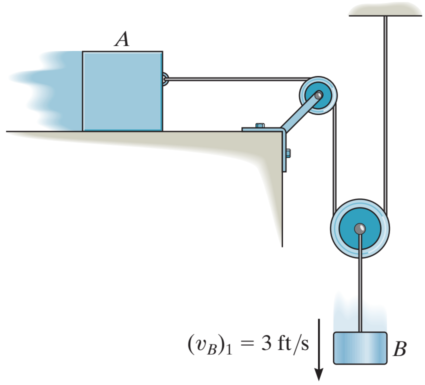
$$(v_A)_2 = -10.5 \text{ ft/s} = 10.5 \text{ ft/s} \rightarrow$$ **Ans.**



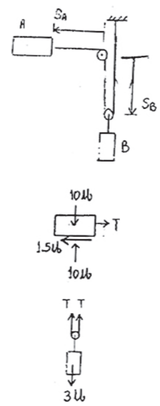


***15–32.**

Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity $(v_B)_1 = 3\text{ ft/s}$ at $t = 0$, determine the velocity of A when $t = 1\text{ s}$. The coefficient of kinetic friction between the horizontal plane and block A is $\mu_A = 0.15$.



SOLUTION



$$s_A + 2s_B = l$$

$$v_A = -2v_B$$

$$(\overset{+}{\leftarrow}) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

$$-\frac{10}{32.2}(2)(3) - T(1) + 0.15(10) = \frac{10}{32.2}(v_A)_2$$

$$(+\downarrow) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

$$\frac{3}{32.2}(3) - 3(1) - 2T(1) = \frac{3}{32.2}\left(\frac{(v_A)_2}{2}\right)$$

$$-\,32.2T - 10(v_A)_2 = 11.70$$

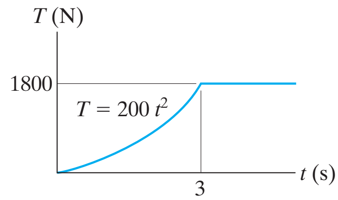
$$-\,64.4T + 1.5(v_A)_2 = -105.6$$

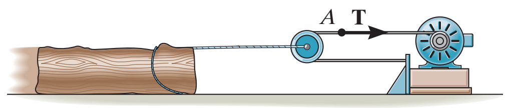
$$T = 1.50\text{ lb}$$

$$(v_A)_2 = -6.00\text{ ft/s} = 6.00\text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

15–33.

The log has a mass of 500 kg and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the log when $t = 5$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.

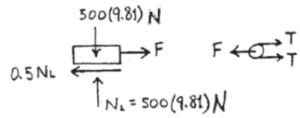




SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad F - 0.5(500)(9.81) = 0$

$$F = 2452.5 \text{ N}$$

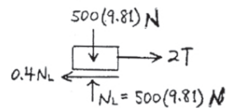


Thus,

$$2T = F$$

$$2(200t^2) = 2452.5$$

$$t = 2.476 \text{ s to start log moving}$$



$(\xrightarrow{+}) \qquad m\,v_1 + \Sigma \int F\,dt = m\,v_2$

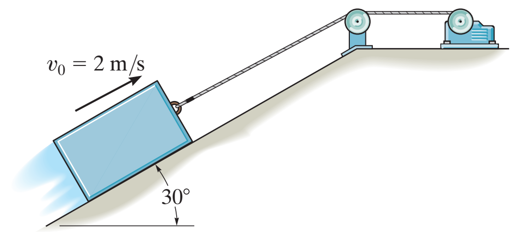
$$0 + 2\int_{2.476}^{3} 200t^2\,dt + 2(1800)(5-3) - 0.4(500)(9.81)(5-2.476) = 500v_2$$

$$400\left(\frac{t^3}{3}\right)\Bigg|_{2.476}^{3} + 2247.91 = 500v_2$$

$v_2 = 7.65$ m/s **Ans.**

15–34.

The 50-kg block is hoisted up the incline using the cable and motor arrangement shown. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.4$. If the block is initially moving up the plane at $v_0 = 2$ m/s, and at this instant ($t = 0$) the motor develops a tension in the cord of $T = (300 + 120\sqrt{t})$ N, where t is in seconds, determine the velocity of the block when $t = 2$ s.



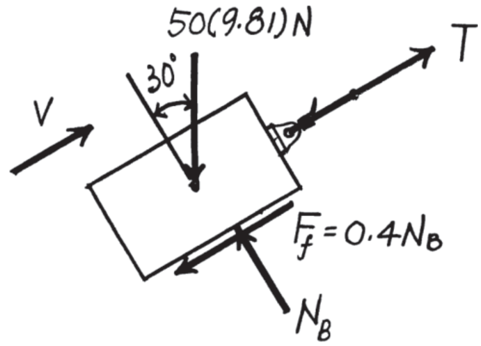
SOLUTION

$+\nwarrow \Sigma F_x = 0; \qquad N_B - 50(9.81)\cos 30° = 0 \qquad N_B = 424.79\text{ N}$

$(+\nearrow) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$

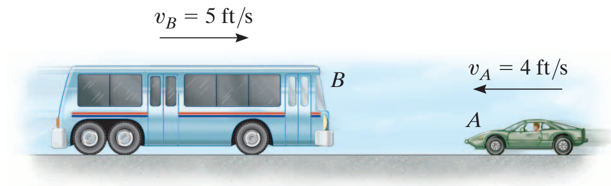
$$50(2) + \int_0^2 \left(300 + 120\sqrt{t}\right)dt - 0.4(424.79)(2) - 50(9.81)\sin 30°(2) = 50v_2$$

$v_2 = 1.92$ m/s **Ans.**



15–35.

The bus B has a weight of 15 000 lb and is traveling to the right at 5 ft/s. Meanwhile a 3000-lb car A is traveling at 4 ft/s to the left. If the vehicles crash head-on and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.



SOLUTION

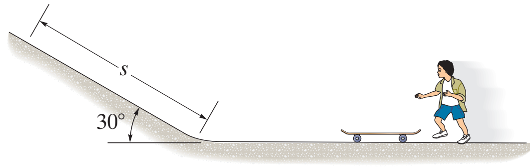
$(\xrightarrow{+})\qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v$

$$\frac{15\,000}{32.2}(5) - \frac{3000}{32.2}(4) = \frac{18\,000}{32.2}v$$

$v = 3.5$ ft/s $\rightarrow$ **Ans.**

***15–36.**

The 50-kg boy jumps on the 5-kg skateboard with a horizontal velocity of 5 m/s. Determine the distance s the boy reaches up the inclined plane before momentarily coming to rest. Neglect the skateboard's rolling resistance.



SOLUTION

Free-Body Diagram: The free-body diagram of the boy and skateboard system is shown in Fig. *a*. Here, $\mathbf{W}_b, \mathbf{W}_{sb}$, and $\mathbf{N}$ are nonimpulsive forces. The pair of impulsive forces $\mathbf{F}$ resulting from the impact during landing cancel each other out since they are internal to the system.

Conservation of Linear Momentum: Since the resultant of the impulsive force along the x axis is zero, the linear momentum of the system is conserved along the x axis.

$$(\overset{+}{\leftarrow}) \qquad m_b(v_b)_1 + m_{sb}(v_{sb})_1 = (m_b + m_{sb})v$$

$$50(5) + 5(0) = (50 + 5)v$$

$$v = 4.545 \text{ m/s}$$

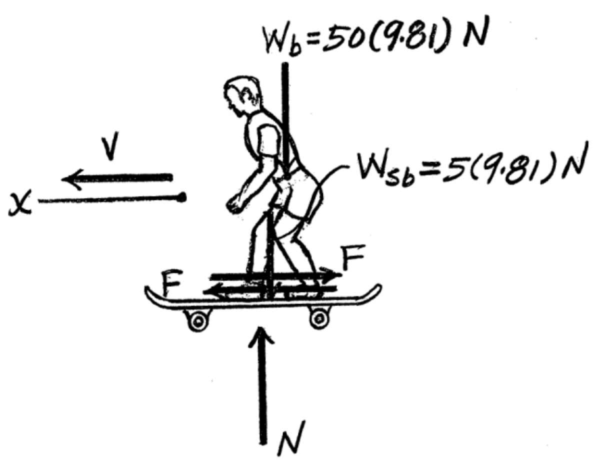
Conservation of Energy: With reference to the datum set in Fig. *b*, the gravitational potential energy of the boy and skateboard at positions A and B are $\left(V_g\right)_A = (m_b + m_{sb})gh_A = 0$ and $\left(V_g\right)_B = (m_b + m_{sb})gh_B = (50 + 5)(9.81)(s \sin 30°) = 269.775s$.

$$T_A + V_A = T_B + V_B$$

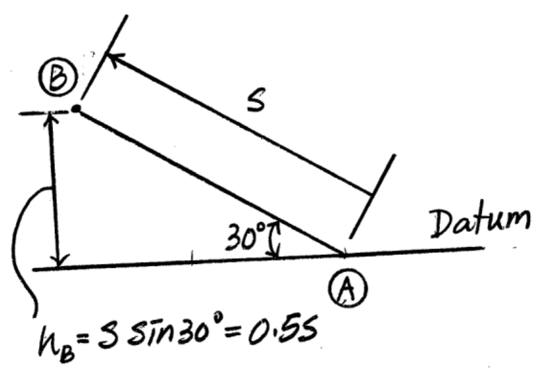
$$\frac{1}{2}(m_b + m_{sb})v_A{}^2 + \left(V_g\right)_A = \frac{1}{2}(m_b + m_{sb})v_B{}^2 + \left(V_g\right)_B$$

$$\frac{1}{2}(50 + 5)\left(4.545^2\right) + 0 = 0 + 269.775s$$

$$s = 2.11 \text{ m} \qquad \textbf{Ans.}$$



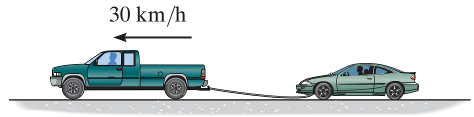
(a)



(b)

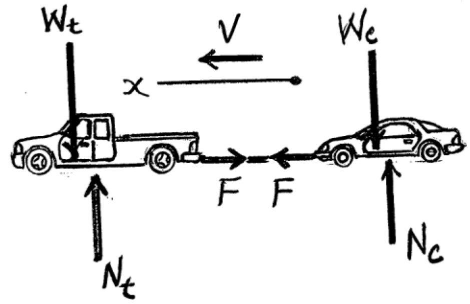
15–37.

The 2.5-Mg pickup truck is towing the 1.5-Mg car using a cable as shown. If the car is initially at rest and the truck is coasting with a velocity of 30 km/h when the cable is slack, determine the common velocity of the truck and the car just after the cable becomes taut. Also, find the loss of energy.



SOLUTION

Free-Body Diagram: The free-body diagram of the truck and car system is shown in Fig. *a*. Here, $\mathbf{W}_t$, $\mathbf{W}_C$, $\mathbf{N}_t$, and $\mathbf{N}_C$ are nonimpulsive forces. The pair of impulsive forces $\mathbf{F}$ generated at the instant the cable becomes taut are internal to the system and thus cancel each other out.



(a)

Conservation of Linear Momentum: Since the resultant of the impulsive force is zero, the linear momentum of the system is conserved along the x axis. The initial speed of the truck is $(v_t)_1 = \left[30(10^3)\,\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 8.333$ m/s.

$$(\overset{+}{\leftarrow}) \qquad m_t(v_t)_1 + m_C(v_C)_1 = (m_t + m_C)v_2$$

$$2500(8.333) + 0 = (2500 + 1500)v_2$$

$$v_2 = 5.208\text{ m/s} = 5.21\text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

Kinetic Energy: The initial and final kinetic energy of the system is

$$T_1 = \frac{1}{2}m_t(v_t)_1^2 + \frac{1}{2}m_C(v_C)_1^2$$

$$= \frac{1}{2}(2500)(8.333^2) + 0$$

$$= 86\,805.56\text{ J}$$

and

$$T_2 = (m_t + m_C)v_2^2$$

$$= \frac{1}{2}(2500 + 1500)(5.208^2)$$

$$= 54\,253.47$$

Thus, the loss of energy during the impact is

$$\Delta E = T_1 - T_2 = 86\,805.56 - 54\,253.47 = 32.55(10^3)\text{ J} = 32.6\text{ kJ} \qquad \textbf{Ans.}$$

15–38.

A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

SOLUTION

$(\xrightarrow{+})\qquad \Sigma m v_1 = \Sigma m v_2$

$$15\,000(1.5) - 12\,000(0.75) = 27\,000(v_2)$$

$$v_2 = 0.5 \text{ m/s} \qquad \textbf{Ans.}$$

$$T_1 = \frac{1}{2}(15\,000)(1.5)^2 + \frac{1}{2}(12\,000)(0.75)^2 = 20.25 \text{ kJ}$$

$$T_2 = \frac{1}{2}(27\,000)(0.5)^2 = 3.375 \text{ kJ}$$

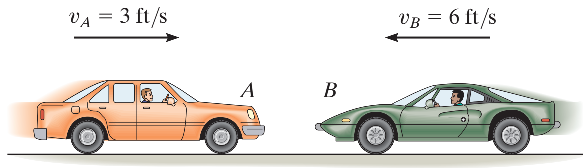
$$\Delta T = T_1 - T_2$$

$$= 20.25 - 3.375 = 16.9 \text{ kJ} \qquad \textbf{Ans.}$$

This energy is dissipated as noise, shock, and heat during the coupling.

15–39.

The car A has a weight of 4500 lb and is traveling to the right at 3 ft/s. Meanwhile a 3000-lb car B is traveling at 6 ft/s to the left. If the cars crash head-on and become entangled, determine their common velocity just after the collision. Assume that the brakes are not applied during collision.



SOLUTION

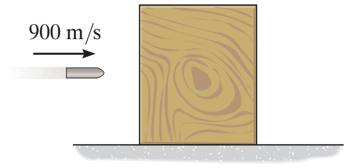
$(\overset{+}{\rightarrow})$ $\qquad m_A\,(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$

$$\frac{4500}{32.2}(3) - \frac{3000}{32.2}(6) = \frac{7500}{32.2}v_2$$

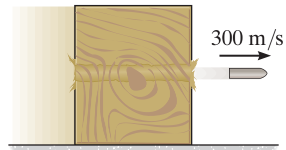
$$v_2 = -0.600 \text{ ft/s} = 0.600 \text{ ft/s} \leftarrow$$ **Ans.**

*15–40.

The 200-g projectile is fired with a velocity of 900 m/s towards the center of the 15-kg wooden block, which rests on a rough surface. If the projectile penetrates and emerges from the block with a velocity of 300 m/s, determine the velocity of the block just after the projectile emerges. How long does the block slide on the rough surface, after the projectile emerges, before it comes to rest again? The coefficient of kinetic friction between the surface and the block is $\mu_k = 0.2$.



Before



After

SOLUTION

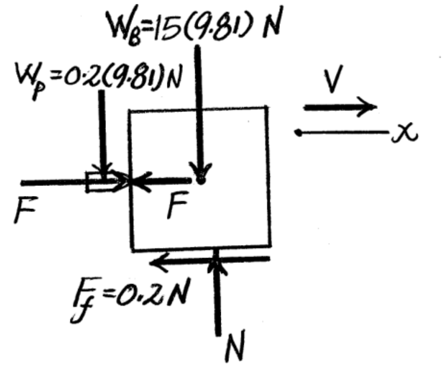
Free-Body Diagram: The free-body diagram of the projectile and block system is shown in Fig. *a*. Here, $\mathbf{W}_B$, $\mathbf{W}_P$, $\mathbf{N}$, and $\mathbf{F}_f$ are nonimpulsive forces. The pair of impulsive forces $\mathbf{F}$ resulting from the impact cancel each other out since they are internal to the system.

Conservation of Linear Momentum: Since the resultant of the impulsive force along the x axis is zero, the linear momentum of the system is conserved along the x axis.

$$(\overset{+}{\rightarrow}) \qquad m_P(v_P)_1 + m_B(v_B)_1 = m_P(v_P)_2 + m_B(v_B)_2$$

$$0.2(900) + 15(0) = 0.2(300) + 15(v_B)_2$$

$$(v_B)_2 = 8\,\text{m/s} \rightarrow \qquad \textbf{Ans.}$$



(a)

Principle of Impulse and Momentum: Using the result of $(v_B)_2$, and referring to the free-body diagram of the block shown in Fig. *b*,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_2)_y$$

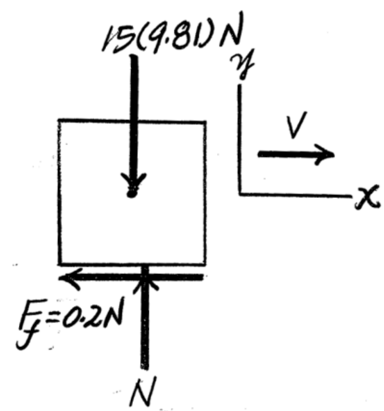
$$15(0) + N(t) - 15(9.81)(t) = 15(0)$$

$$N = 147.15\text{ N}$$

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$15(8) + [-0.2(147.15)(t)] = 15(0)$$

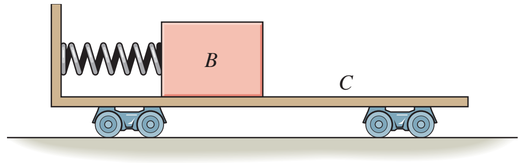
$$t = 4.077\text{ s} = 4.08\text{ s} \qquad \textbf{Ans.}$$



(b)

15–41.

The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block relative to the *ground* after the spring becomes undeformed. Neglect the mass of the cart's wheels and the spring in the calculation. Also neglect friction. Take $k = 300$ N/m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)(v_b)^2 + \frac{1}{2}(75)(v_c)^2$$

$$12 = 50\, v_b^2 + 75\, v_c^2$$

$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = 50\, v_b - 75\, v_c$$

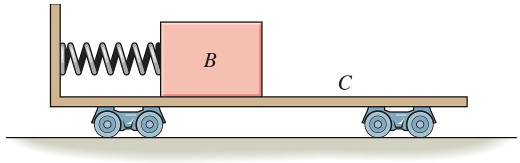
$$v_b = 1.5 v_c$$

$$v_c = 0.253 \text{ m/s} \leftarrow$$

$$v_b = 0.379 \text{ m/s} \rightarrow$$ **Ans.**

15–42.

The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block with respect to the *cart* after the spring becomes undeformed. Neglect the mass of the wheels and the spring in the calculation. Also neglect friction. Take $k = 300$ N/m.

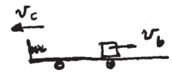


SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)(v_b)^2 + \frac{1}{2}(75)(v_c)^2$$

$$12 = \; = 50\, v_b^2 + 75\, v_c^2$$



$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = 50\, v_b - 75\, v_c$$

$$v_b = 1.5 v_c$$

$$v_c = 0.253 \text{ m/s} \leftarrow$$

$$v_b = 0.379 \text{ m/s} \rightarrow$$

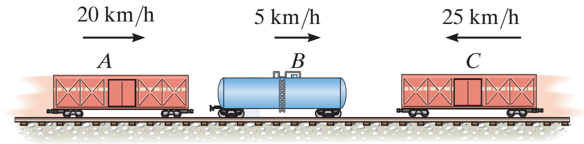
$$\mathbf{v}_b = \mathbf{v}_c + \mathbf{v}_{b/c}$$

$(\overset{+}{\rightarrow})\qquad 0.379 = -0.253 + \mathbf{v}_{b/c}$

$v_{b/c} = 0.632 \text{ m/s} \rightarrow$ **Ans.**

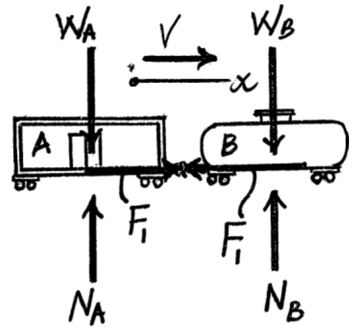
15–43.

The three freight cars A, B, and C have masses of 10 Mg, 5 Mg, and 20 Mg, respectively. They are traveling along the track with the velocities shown. Car A collides with car B first, followed by car C. If the three cars couple together after collision, determine the common velocity of the cars after the two collisions have taken place.



SOLUTION

Free-Body Diagram: The free-body diagram of the system of cars A and B when they collide is shown in Fig. a. The pair of impulsive forces $\mathbf{F}_1$ generated during the collision cancel each other since they are internal to the system. The free-body diagram of the coupled system composed of cars A and B and car C when they collide is shown in Fig. b. Again, the internal pair of impulsive forces $\mathbf{F}_2$ generated during the collision cancel each other.



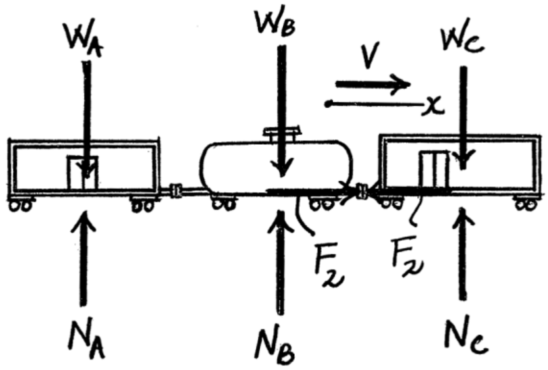
(a)

Conservation of Linear Momentum: When A collides with B, and then the coupled cars A and B collide with car C, the resultant impulsive force along the x axis is zero. Thus, the linear momentum of the system is conserved along the x axis. The initial speed of the cars A, B, and C are

$$(v_A)_1 = \left[20(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 5.556\text{ m/s}$$

$$(v_B)_1 = \left[5(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 1.389\text{ m/s},$$

and $$(v_C)_1 = \left[25(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 6.944\text{ m/s}$$



(b)

For the first case,

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$10000(5.556) + 5000(1.389) = (10000 + 5000)v_{AB}$$

$$v_{AB} = 4.167\text{ m/s} \rightarrow$$

Using the result of v_{AB} and considering the second case,

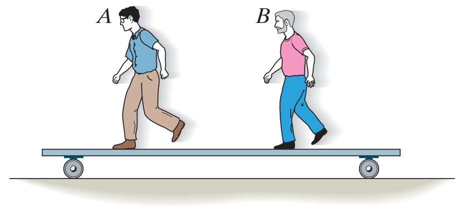
$$(\overset{+}{\rightarrow}) \qquad (m_A + m_B)v_{AB} + m_C(v_C)_1 = (m_A + m_B + m_C)v_{ABC}$$

$$(10000 + 5000)(4.167) + [-20000(6.944)] = (10000 + 5000 + 20000)v_{ABC}$$

$$v_{ABC} = -2.183\text{ m/s} = 2.18\text{ m/s} \leftarrow$$ **Ans.**

***15–44.**

Two men A and B, each having a weight of 160 lb, stand on the 200-lb cart. Each runs with a speed of 3 ft/s measured relative to the cart. Determine the final speed of the cart if (a) A runs and jumps off, then B runs and jumps off the same end, and (b) both run at the same time and jump off at the same time. Neglect the mass of the wheels and assume the jumps are made horizontally.



SOLUTION

(a) A jumps first.

$(\overset{+}{\leftarrow})\ 0 + 0 = m_A v_A - (m_C + m_B)v'_C \qquad \text{However, } v_A = -v'_C + 3$

$$0 = \frac{160}{32.2}(-v'_C + 3) - \frac{360}{32.2}v'_C$$

$$v'_C = 0.9231 \text{ ft/s} \rightarrow$$

And then B jumps

$0 + (m_C + m_B)\, v'_C = m_B\, v_B - m_C v_C \qquad \text{However, } v_B = -v_C + 3$

$$\frac{360}{32.2}(-0.9231) = \frac{160}{32.2}(-v_C + 3) - \frac{200}{32.2}v_C$$

$v_C = 2.26 \text{ ft/s} \rightarrow$ **Ans.**

(b) Both men jump at the same time

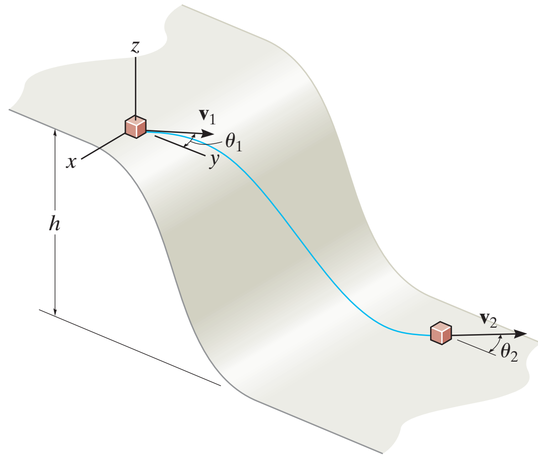
$(\overset{+}{\leftarrow})\ 0 + 0 = (m_A + m_B)v - m_C v_C \qquad \text{However, } v = -v_C + 3$

$$0 = \left(\frac{160}{32.2} + \frac{160}{32.2}\right)(-v_C + 3) - \frac{200}{32.2}v_C$$

$v_C = 1.85 \text{ ft/s} \rightarrow$ **Ans.**

15–45.

The block of mass m is traveling at v_1 in the direction θ_1 shown at the top of the smooth slope. Determine its speed v_2 and its direction θ_2 when it reaches the bottom.



SOLUTION

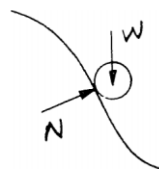
There are no impulses in the v ***direction:***

$$mv_1 \sin\theta_1 = mv_2 \sin\theta_2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv_2^2 + 0$$

$$v_2 = \sqrt{v_1^2 + 2gh}$$ **Ans.**

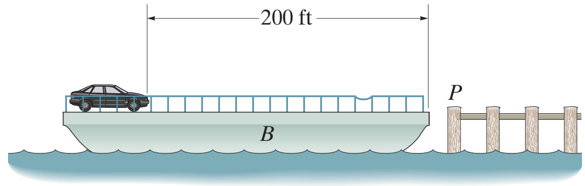


$$\sin\theta_2 = \frac{v_1 \sin\theta_1}{\sqrt{v_1^2 + 2gh}}$$

$$\theta_2 = \sin^{-1}\left(\frac{v_1 \sin\theta_1}{\sqrt{v_1^2 + 2gh}}\right)$$ **Ans.**

15–46.

The barge B weighs 30 000 lb and supports an automobile weighing 3000 lb. If the barge is not tied to the pier P and someone drives the automobile to the other side of the barge for unloading, determine how far the barge moves away from the pier. Neglect the resistance of the water.



SOLUTION

Relative Velocity: The relative velocity of the car with respect to the barge is $v_{c/b}$. Thus, the velocity of the car is

$$(\overset{+}{\rightarrow}) \qquad v_c = -v_b + v_{c/b} \qquad \textbf{(1)}$$

Conservation of Linear Momentum: If we consider the car and the barge as a system, then the impulsive force caused by the traction of the tires is *internal* to the system. Therefore, it will cancel out. As the result, the linear momentum is conserved along the x axis.

$$0 = m_c v_c + m_b v_b$$

$$(\overset{+}{\rightarrow}) \qquad 0 + 0 = \left(\frac{3000}{32.2}\right) v_c - \left(\frac{30\,000}{32.2}\right) v_b \qquad \textbf{(2)}$$

Substituting Eq. (1) into (2) yields

$$11v_b - v_{c/b} = 0 \qquad \textbf{(3)}$$

Integrating Eq. (3) becomes

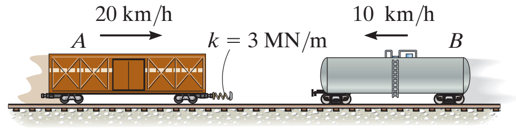
$$(\overset{+}{\rightarrow}) \qquad 11s_b - s_{c/b} = 0 \qquad \textbf{(4)}$$

Here, $s_{c/b} = 200$ ft. Then, from Eq. (4)

$$11s_b - 200 = 0 \qquad s_b = 18.2 \text{ ft} \qquad \textbf{Ans.}$$

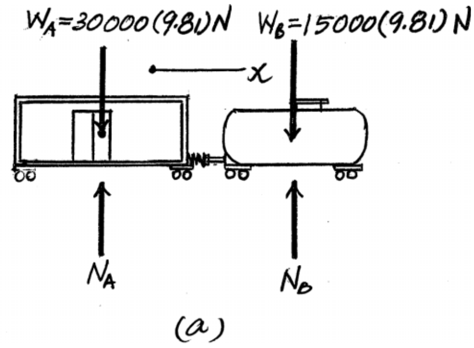
15–47.

The 30-Mg freight car A and 15-Mg freight car B are moving towards each other with the velocities shown. Determine the maximum compression of the spring mounted on car A. Neglect rolling resistance.



SOLUTION

Conservation of Linear Momentum: Referring to the free-body diagram of the freight cars A and B shown in Fig. a, notice that the linear momentum of the system is conserved along the x axis. The initial speed of freight cars A and B are $(v_A)_1 = \left[20(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 5.556\text{ m/s}$ and $(v_B)_1 = \left[10(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 2.778\text{ m/s}$. At this instant, the spring is compressed to its maximum, and no relative motion occurs between freight cars A and B and they move with a common speed.



$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$30(10^3)(5.556) + \left[-15(10^3)(2.778)\right] = \left[30(10^3) + 15(10^3)\right]v_2$$

$$v_2 = 2.778\text{ m/s} \rightarrow$$

Conservation of Energy: The initial and final elastic potential energy of the spring is $(V_e)_1 = \frac{1}{2}ks_1^2 = 0$ and $(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(3)(10^6)s_{max}^2 = 1.5(10^6)s_{max}^2$.

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$\left[\frac{1}{2}m_A(v_A)_1^2 + \frac{1}{2}m_B(v_B)_1^2\right] + (V_e)_1 = \frac{1}{2}(m_A + m_B)v_2^2 + (V_e)_2$$

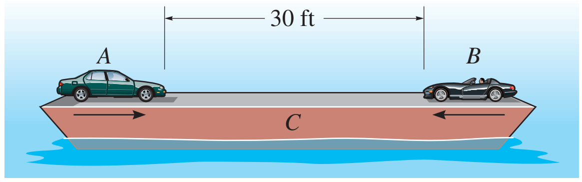
$$\frac{1}{2}(30)\left(10^3\right)(5.556^2) + \frac{1}{2}(15)\left(10^3\right)\left(2.778^2\right) + 0$$

$$= \frac{1}{2}\left[30\left(10^3\right) + 15\left(10^3\right)\right]\left(2.778^2\right) + 1.5\left(10^6\right)s_{max}^2$$

$$s_{max} = 0.4811\text{ m} = 481\text{ mm}$$ **Ans.**

***15–48.**

The barge weighs 45 000 lb and supports two automobiles A and B, which weigh 4000 lb and 3000 lb, respectively. If the automobiles start from rest and drive towards each other, accelerating at $a_A = 4\ \text{ft/s}^2$ and $a_B = 8\ \text{ft/s}^2$ until they reach a constant speed of 6 ft/s relative to the barge, determine the speed of the barge just before the automobiles collide. How much time does this take? Originally the barge is at rest. Neglect water resistance.



SOLUTION

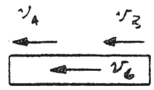
$(\xleftarrow{+})\qquad v_A = v_C + v_{A/C} = v_C - 6$

$(\xleftarrow{+})\qquad v_B = v_C + v_{B/C} = v_C + 6$

$(\xleftarrow{+})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 = m_A(v_C - 6) + m_B(v_C + 6) + m_C v_C$$

$$0 = \left(\frac{4000}{32.2}\right)(v_C - 6) + \left(\frac{3000}{32.2}\right)(v_C + 6) + \left(\frac{45\,000}{32.2}\right)v_C$$



$$v_C = 0.1154\ \text{ft/s} = 0.115\ \text{ft/s}$$ **Ans.**

For A:

$(\xrightarrow{+})\qquad v = v_0 + a_c t$

$$6 = 0 + 4t_A$$

$$t_A = 1.5\ \text{s}$$

$(\xrightarrow{+})\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$s = 0 + 0 + \frac{1}{2}(4)(1.5)^2 = 4.5\ \text{ft}$$

For B:

$(\xleftarrow{+})\qquad v = v_0 + a_c t$

$$6 = 0 + 8t_B$$

$$t_B = 0.75\ \text{s}$$

$(\xleftarrow{+})\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$s = 0 + 0 + \frac{1}{2}(8)(0.75)^2 = 2.25\ \text{ft}$$

For the remaining $(1.5 - 0.75)\ \text{s} = 0.75\ \text{s}$

$s = vt = 6(0.75) = 4.5\ \text{ft}$

Thus,

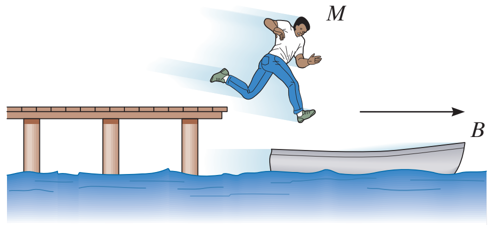
$s = 30 - 4.5 - 4.5 - 2.25 = 18.75\ \text{ft}$

$t' = \dfrac{s/2}{v} = \dfrac{18.75/2}{6} = 1.5625$

$t = 1.5 + 1.5625 = 3.06\ \text{s}$ **Ans.**

15–49.

The man M weighs 150 lb and jumps onto the boat B which has a weight of 200 lb. If he has a horizontal component of velocity *relative to the boat* of 3 ft/s, just before he enters the boat, and the boat is traveling $v_B = 2$ ft/s away from the pier when he makes the jump, determine the resulting velocity of the man and boat.



SOLUTION

$(\xrightarrow{+})$ $\quad v_M = v_B + v_{M/B}$

$$v_M = 2 + 3$$

$$v_M = 5 \text{ ft/s}$$

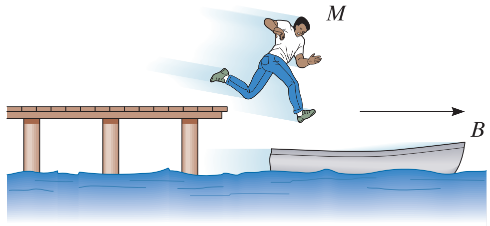
$(\xrightarrow{+})$ $\quad \Sigma m\, v_1 = \Sigma m\, v_2$

$$\frac{150}{32.2}(5) + \frac{200}{32.2}(2) = \frac{350}{32.2}(v_B)_2$$

$$(v_B)_2 = 3.29 \text{ ft/s} \qquad \textbf{Ans.}$$

15–50.

The man M weighs 150 lb and jumps onto the boat B which is originally at rest. If he has a horizontal component of velocity of 3 ft/s just before he enters the boat, determine the weight of the boat if it has a velocity of 2 ft/s once the man enters it.



SOLUTION

$(\xrightarrow{+})\qquad v_M = v_B + v_{M/B}$

$$v_M = 0 + 3$$

$$v_M = 3 \text{ ft/s}$$

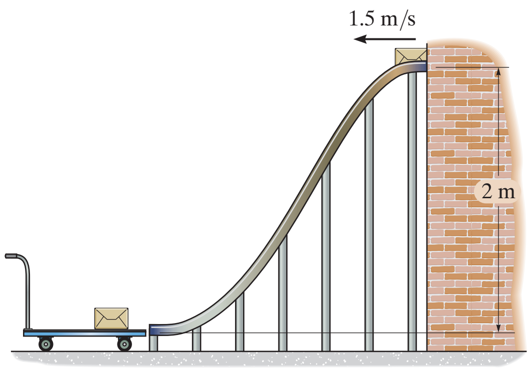
$(\xrightarrow{+})\qquad \Sigma m(v_1) = \Sigma m(v_2)$

$$\frac{150}{32.2}(3) + \frac{W_B}{32.2}(0) = \frac{(W_B + 150)}{32.2}(2)$$

$$W_B = 75 \text{ lb}$$ **Ans.**

15–51.

The 20-kg package has a speed of 1.5 m/s when it is delivered to the smooth ramp. After sliding down the ramp it lands onto a 10-kg cart as shown. Determine the speed of the cart and package after the package stops sliding on the cart.



SOLUTION

Conservation of Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the package at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 20(9.81)(2) = 392.4$ J and $\left(V_g\right)_2 = mgh_2 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m(v_P)_1^{\,2} + (V_g)_1 = \frac{1}{2}m(v_P)_2^{\,2} + (V_g)_2$$

$$\frac{1}{2}(20)(1.5^2) + 392.4 = \frac{1}{2}(20)(v_P)_2^{\,2} + 0$$

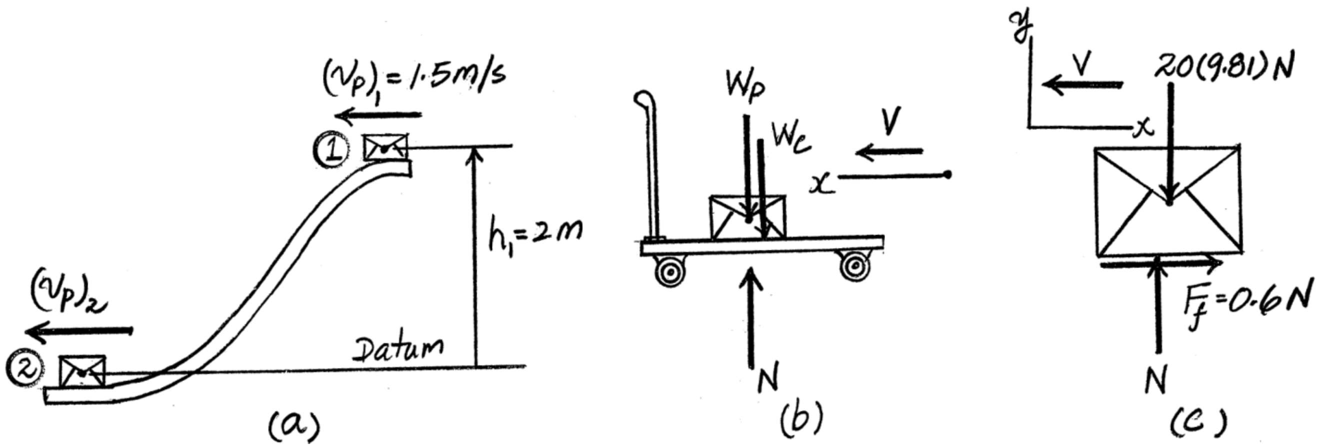
$$(v_P)_2^{\,2} = 6.441 \text{ m/s} \leftarrow$$

Conservation of Linear Momentum: Referring to the free-body diagram of the package and cart system shown in Fig. *b*, the linear momentum is conserved along the *x* axis since no impulsive force acts along it. The package stops sliding on the cart when they move with a common speed. At this instant,

$$(\overset{+}{\leftarrow}) \qquad m_P(v_P)_2 + m_C(v_C)_1 = (m_P + m_C)v$$

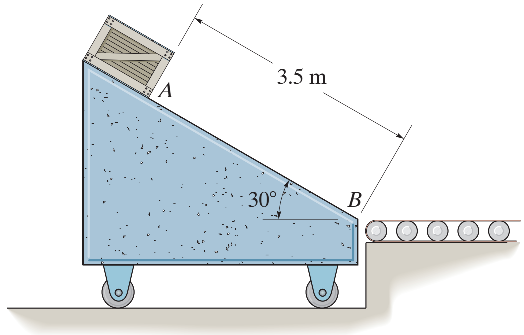
$$20(6.441) + 0 = (20 + 10)v$$

$$v = 4.29 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$



***15–52.**

The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at A and slides down 3.5 m to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?



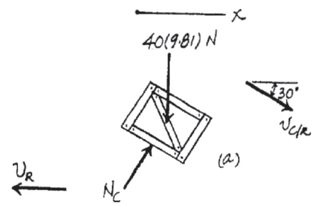
SOLUTION

Conservation of Energy: The datum is set at lowest point B. When the crate is at point A, it is $3.5 \sin 30^{\circ} = 1.75$ m *above* the datum. Its gravitational potential energy is $10(9.81)(1.75) = 171.675$ N · m. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 171.675 = \frac{1}{2}(10)v_C^2 + \frac{1}{2}(40)v_R^2$$

$$171.675 = 5v_C^2 + 20v_R^2 \tag{1}$$

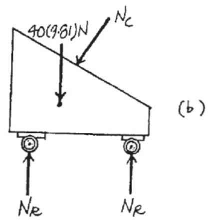


Relative Velocity: The velocity of the crate is given by

$$\mathbf{v}_C = \mathbf{v}_R + \mathbf{v}_{C/R}$$

$$= -v_R\mathbf{i} + (v_{C/R}\cos 30°\mathbf{i} - v_{C/R}\sin 30°\mathbf{j})$$

$$= (0.8660\,v_{C/R} - v_R)\mathbf{i} - 0.5\,v_{C/R}\mathbf{j} \tag{2}$$



The magnitude of v_C is

$$v_C = \sqrt{(0.8660\,v_{C/R} - v_R)^2 + (-0.5\,v_{C/R})^2}$$

$$= \sqrt{v_{C/R}^2 + v_R^2 - 1.732\,v_R v_{C/R}} \tag{3}$$

Conservation of Linear Momentum: If we consider the crate and the ramp as a system, from the FBD, one realizes that the normal reaction N_C (*impulsive force*) is *internal* to the system and will cancel each other. As the result, the linear momentum is conserved along the x axis.

$$0 = m_C(v_C)_x + m_R v_R$$

$$(\overset{+}{\rightarrow}) \qquad 0 = 10(0.8660\,v_{C/R} - v_R) + 40(-v_R)$$

$$0 = 8.660\,v_{C/R} - 50\,v_R \tag{4}$$

Solving Eqs. (1), (3), and (4) yields

$$v_R = 1.101 \text{ m/s} = 1.10 \text{ m/s} \quad v_C = 5.43 \text{ m/s} \qquad \textbf{Ans.}$$

$$v_{C/R} = 6.356 \text{ m/s}$$

From Eq. (2)

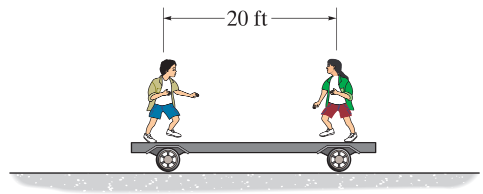
$$\mathbf{v}_C = [0.8660(6.356) - 1.101]\mathbf{i} - 0.5(6.356)\mathbf{j} = \{4.403\mathbf{i} - 3.178\mathbf{j}\} \text{ m/s}$$

Thus, the directional angle ϕ of v_C is

$$\phi = \tan^{-1}\frac{3.178}{4.403} = 35.8° \qquad \measuredangle\phi \qquad \textbf{Ans.}$$

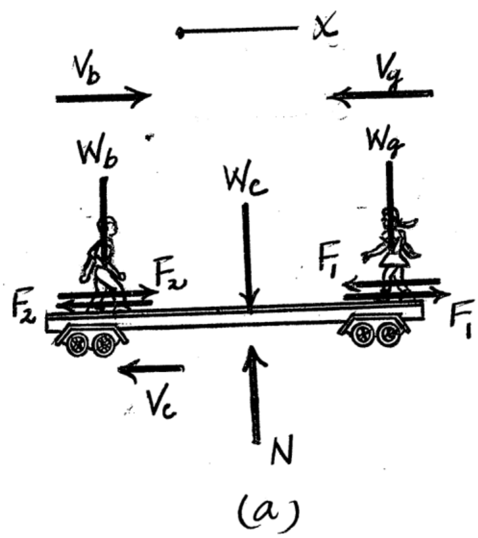
15–53.

The 80-lb boy and 60-lb girl walk towards each other with a constant speed on the 300-lb cart. If their velocities, measured relative to the cart, are 3 ft/s to the right and 2 ft/s to the left, respectively, determine the velocities of the boy and girl during the motion. Also, find the distance the cart has traveled at the instant the boy and girl meet.



SOLUTION

Conservation of Linear Momentum: From the free-body diagram of the boy, girl, and cart shown in Fig. *a*, the pairs of impulsive forces $\mathbf{F}_1$ and $\mathbf{F}_2$ generated during the walk cancel each other since they are internal to the system. Thus, the resultant of the impulsive forces along the *x* axis is zero, and the linear momentum of the system is conserved along the *x* axis.



$(\overset{+}{\rightarrow})$ $$\Sigma mv_1 = \Sigma mv_2$$

$$0 + 0 + 0 = \frac{80}{32.2}v_b - \frac{60}{32.2}(v_g) - \frac{300}{32.2}v_c$$

$$80v_b - 60v_g - 300v_c = 0 \qquad \textbf{(1)}$$

Kinematics: Applying the relative velocity equation and considering the motion of the boy,

$$\mathbf{v}_b = \mathbf{v}_c + \mathbf{v}_{b/c}$$

$(\overset{+}{\rightarrow})$ $$v_b = -v_c + 3 \qquad \textbf{(2)}$$

For the girl,

$$\mathbf{v}_g = \mathbf{v}_c + \mathbf{v}_{g/c}$$

$(\overset{+}{\rightarrow})$ $$-v_g = -v_c - 2$$

$$v_g = v_c + 2 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3), yields

$$v_b = 2.727 \text{ ft/s} = 2.73 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

$$v_g = 2.273 \text{ ft/s} = 2.27 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

$$v_c = 0.2727 \text{ ft/s} \leftarrow$$

The velocity of the girl relative to the boy can be determined from

$$\mathbf{v}_g = \mathbf{v}_b + \mathbf{v}_{g/b}$$

$(\overset{+}{\rightarrow})$ $$-2.273 = 2.727 + v_{g/b}$$

$$v_{g/b} = -5 \text{ ft/s} = 5 \text{ ft/s} \leftarrow$$

Here, $s_{g/b} = 20$ ft and $v_{g/b} = 5$ ft/s is constant. Thus,

$(\overset{+}{\leftarrow})$ $$s_{g/b} = (s_{g/b})_0 + v_{g/b}t$$

$$20 = 0 + 5t$$

$$t = 4 \text{ s}$$

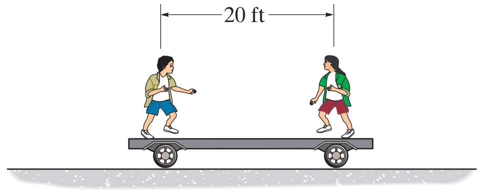
Thus, the distance the cart travels is given by

$(\overset{+}{\leftarrow})$ $$s_c = (s_c)_0 + v_ct$$

$$= 0 + 0.2727(4)$$

15–54.

The 80-lb boy and 60-lb girl walk towards each other with constant speed on the 300-lb cart. If their velocities measured relative to the cart are 3 ft/s to the right and 2 ft/s to the left, respectively, determine the velocity of the cart while they are walking.



SOLUTION

Conservation of Linear Momentum: From the free-body diagram of the body, girl, and cart shown in Fig. *a*, the pairs of impulsive forces $\mathbf{F}_1$ and $\mathbf{F}_2$ generated during the walk cancel each other since they are internal to the system. Thus, the resultant of the impulsive forces along the *x* axis is zero, and the linear momentum of the system is conserved along the *x* axis.

$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 + 0 = \frac{80}{32.2}v_b - \frac{60}{32.2}(v_g) - \frac{300}{32.2}v_c$$

$$80v_b - 60v_g - 300v_c = 0 \qquad \textbf{(1)}$$

Kinematics: Applying the relative velocity equation and considering the motion of the boy,

$$\mathbf{v}_b = \mathbf{v}_c + \mathbf{v}_{b/c}$$

$(\overset{+}{\rightarrow})\qquad v_b = -v_c + 3 \qquad \textbf{(2)}$

For the girl,

$$\mathbf{v}_g = \mathbf{v}_c + \mathbf{v}_{g/c}$$

$(\overset{+}{\rightarrow})\qquad -v_g = -v_c - 2$

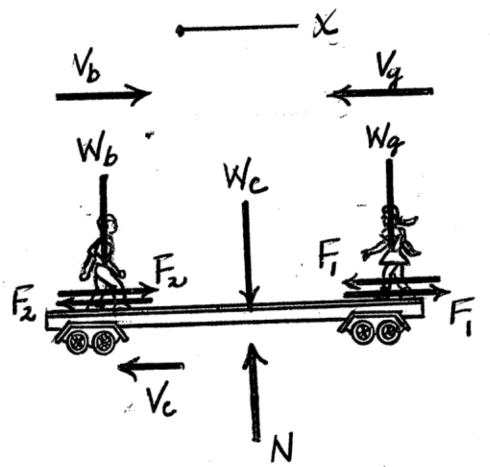
$$v_g = v_c + 2 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3), yields

$$v_b = 2.727 \text{ ft/s} \rightarrow$$

$$v_g = 2.273 \text{ ft/s} \leftarrow$$

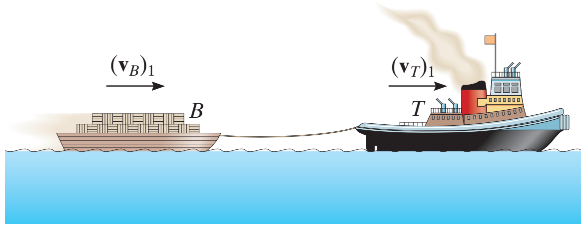
$$v_c = 0.2727 \text{ ft/s} = 0.273 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$



(a)

15–55.

A tugboat T having a mass of 19 Mg is tied to a barge B having a mass of 75 Mg. If the rope is "elastic" such that it has a stiffness $k = 600$ kN/m, determine the maximum stretch in the rope during the initial towing. Originally both the tugboat and barge are moving in the same direction with speeds $(v_T)_1 = 15$ km/h and $(v_B)_1 = 10$ km/h, respectively. Neglect the resistance of the water.

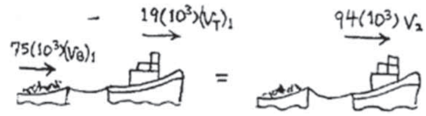


SOLUTION

$$(v_T)_1 = 15 \text{ km/h} = 4.167 \text{ m/s}$$

$$(v_B)_1 = 10 \text{ km/h} = 2.778 \text{ m/s}$$

When the rope is stretched to its maximum, both the tug and barge have a common velocity. Hence,



$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

$$19\,000(4.167) + 75\,000(2.778) = (19\,000 + 75\,000)v_2$$

$$v_2 = 3.059 \text{ m/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = \frac{1}{2}(19\,000)(4.167)^2 + \frac{1}{2}(75\,000)(2.778)^2 = 454.282 \text{ kJ}$$

$$T_2 = \frac{1}{2}(19\,000 + 75\,000)(3.059)^2 = 439.661 \text{ kJ}$$

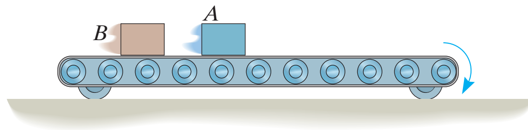
Hence,

$$454.282(10^3) + 0 = 439.661(10^3) + \frac{1}{2}(600)(10^3)x^2$$

$$x = 0.221 \text{ m}$$ **Ans.**

***15–56.**

Two boxes A and B, each having a weight of 160 lb, sit on the 500-lb conveyor which is free to roll on the ground. If the belt starts from rest and begins to run with a speed of 3 ft/s, determine the final speed of the conveyor if (a) the boxes are not stacked and A falls off then B falls off, and (b) A is stacked on top of B and both fall off together.



SOLUTION

a) Let v_b be the velocity of A and B.

$$(\overset{+}{\rightarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$0 = \left(\frac{320}{32.2}\right)(v_b) - \left(\frac{500}{32.2}\right)(v_c)$$

$$(\overset{+}{\rightarrow}) \qquad v_b = v_c + v_{b/c}$$

$$v_b = -v_c + 3$$

Thus, $v_b = 1.83 \text{ ft/s} \rightarrow \qquad v_c = 1.17 \text{ ft/s} \leftarrow$

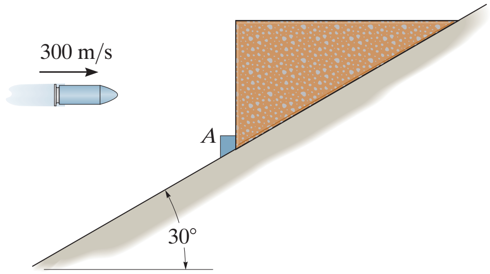
When a box falls off, it exerts no impulse on the conveyor, and so does not alter the momentum of the conveyor. Thus,

a) $v_c = 1.17 \text{ ft/s} \leftarrow$ **Ans.**

b) $v_c = 1.17 \text{ ft/s} \leftarrow$ **Ans.**

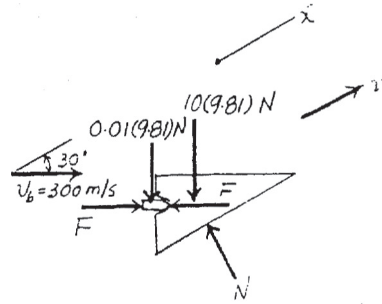
15–57.

The 10-kg block is held at rest on the smooth inclined plane by the stop block at A. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.



SOLUTION

Conservation of Linear Momentum: If we consider the block and the bullet as a system, then from the FBD, the *impulsive* force F caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the x' axis.



$$m_b(v_b)_{x'} = (m_b + m_B)\, v_{x'}$$

$$0.01(300 \cos 30°) = (0.01 + 10)\, v$$

$$v = 0.2595 \text{ m/s}$$

Conservation of Energy: The datum is set at the blocks initial position. When the block and the embedded bullet is at their highest point they are h *above* the datum. Their gravitational potential energy is $(10 + 0.01)(9.81)h = 98.1981h$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(10 + 0.01)\left(0.2595^2\right) = 0 + 98.1981h$$

$$h = 0.003433 \text{ m} = 3.43 \text{ mm}$$

$$d = 3.43 \,/\, \sin 30° = 6.87 \text{ mm} \qquad \textbf{Ans.}$$

15–58.

A ball having a mass of 200 g is released from rest at a height of 400 mm above a very large fixed metal surface. If the ball rebounds to a height of 325 mm above the surface, determine the coefficient of restitution between the ball and the surface.

SOLUTION

Before impact

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0.2(9.81)(0.4) = \frac{1}{2}(0.2)v_1^2 + 0$$

$$v_1 = 2.801 \text{ m/s}$$

After the impact

$$\frac{1}{2}(0.2)v_2^2 = 0 + 0.2(9.81)(0.325)$$

$$v_2 = 2.525 \text{ m/s}$$

Coefficient of restitution:

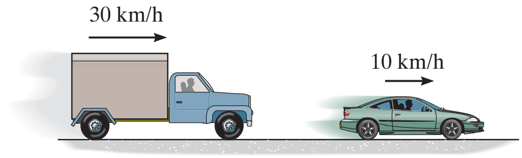
$$(+\downarrow) \qquad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$= \frac{0 - (-2.525)}{2.801 - 0}$$

$$= 0.901 \qquad \textbf{Ans.}$$

15–59.

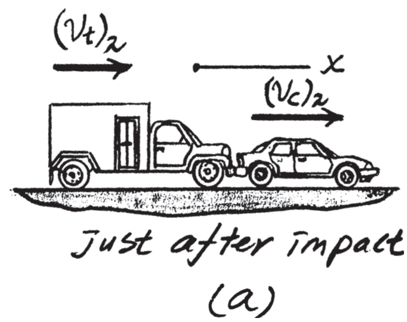
The 5-Mg truck and 2-Mg car are traveling with the free-rolling velocities shown just before they collide. After the collision, the car moves with a velocity of 15 km/h to the right *relative* to the truck. Determine the coefficient of restitution between the truck and car and the loss of energy due to the collision.



SOLUTION

Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact).

The initial speeds of the truck and car are $(v_t)_1 = \left[30\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 8.333\text{ m/s}$ and $(v_c)_1 = \left[10\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\,h}{3600\text{ s}}\right) = 2.778\text{ m/s}$.



By referring to Fig. *a*,

$$(\overset{+}{\rightarrow}) \qquad m_t(v_t)_1 + m_c(v_c)_1 = m_t(v_t)_2 + m_c(v_c)_2$$

$$5000(8.333) + 2000(2.778) = 5000(v_t)_2 + 2000(v_c)_2$$

$$5(v_t)_2 + 2(v_c)_2 = 47.22 \qquad \textbf{(1)}$$

Coefficient of Restitution: Here, $(v_{c/t}) = \left[15\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 4.167\text{ m/s} \rightarrow$. Applying the relative velocity equation,

$$(\mathbf{v}_c)_2 = (\mathbf{v}_t)_2 + (\mathbf{v}_{c/t})_2$$

$$(\overset{+}{\rightarrow}) \qquad (v_c)_2 = (v_t)_2 + 4.167$$

$$(v_c)_2 - (v_t)_2 = 4.167 \qquad \textbf{(2)}$$

Applying the coefficient of restitution equation,

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_c)_2 - (v_t)_2}{(v_t)_1 - (v_c)_1}$$

$$e = \frac{(v_c)_2 - (v_t)_2}{8.333 - 2.778} \qquad \textbf{(3)}$$

15–59. continued

Substituting Eq. (2) into Eq. (3),

$$e = \frac{4.167}{8.333 - 2.778} = 0.75 \qquad \textbf{Ans.}$$

Solving Eqs. (1) and (2) yields

$$(v_t)_2 = 5.556 \text{ m/s}$$

$$(v_c)_2 = 9.722 \text{ m/s}$$

Kinetic Energy: The kinetic energy of the system just before and just after the collision are

$$T_1 = \frac{1}{2} m_t (v_t)_1{}^2 + \frac{1}{2} m_c (v_c)_1{}^2$$

$$= \frac{1}{2}(5000)(8.333^2) + \frac{1}{2}(2000)(2.778^2)$$

$$= 181.33\left(10^3\right) \text{ J}$$

$$T_2 = \frac{1}{2} m_t (v_t)_2{}^2 + \frac{1}{2} m_c (v_c)_2{}^2$$

$$= \frac{1}{2}(5000)(5.556^2) + \frac{1}{2}(2000)(9.722^2)$$

$$= 171.68\left(10^3\right) \text{J}$$

Thus,

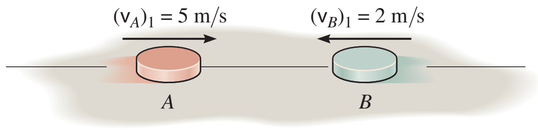
$$\Delta E = T_1 - T_2 = 181.33\left(10^3\right) - 171.68\left(10^3\right)$$

$$= 9.645\left(10^3\right) \text{ J}$$

$$= 9.65 \text{ kJ} \qquad \textbf{Ans.}$$

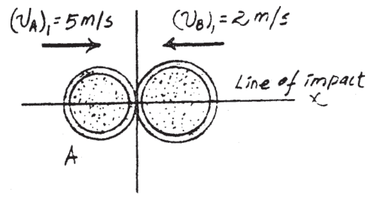
***15–60.**

Disk A has a mass of 2 kg and is sliding forward on the *smooth* surface with a velocity $(v_A)_1 = 5$ m/s when it strikes the 4-kg disk B, which is sliding towards A at $(v_B)_1 = 2$ m/s, with direct central impact. If the coefficient of restitution between the disks is $e = 0.4$, compute the velocities of A and B just after collision.



SOLUTION

Conservation of Momentum :



$$m_A\,(v_A)_1 + m_B\,(v_B)_1 = m_A\,(v_A)_2 + m_B\,(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(5) + 4(-2) = 2(v_A)_2 + 4(v_B)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution :

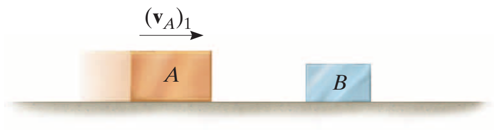
$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.4 = \frac{(v_B)_2 - (v_A)_2}{5 - (-2)} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_A)_2 = -1.53 \text{ m/s} = 1.53 \text{ m/s} \leftarrow \qquad (v_B)_2 = 1.27\text{m/s} \rightarrow \qquad \textbf{Ans.}$$

15–61.



Block A has a mass of 3 kg and is sliding on a rough horizontal surface with a velocity $(v_A)_1 = 2$ m/s when it makes a direct collision with block B, which has a mass of 2 kg and is originally at rest. If the collision is perfectly elastic $(e = 1)$, determine the velocity of each block just after collision and the distance between the blocks when they stop sliding. The coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.3$.

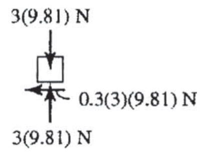
SOLUTION

$(\xrightarrow{+})\qquad \sum mv_1 = \sum mv_2$

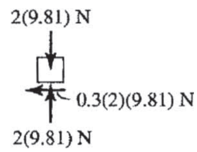
$$3(2) + 0 = 3(v_A)_2 + 2(v_B)_2$$

$(\xrightarrow{+})\qquad e = \dfrac{(v_B)_2 - (v_A)_2)}{(v_A)_1 - (v_B)_1}$

$$1 = \frac{(v_B)_2 - (v_A)_2}{2 - 0}$$



Solving



$$(v_A)_2 = 0.400 \text{ m/s} \rightarrow$$ **Ans.**

$$(v_B)_2 = 2.40 \text{ m/s} \rightarrow$$ **Ans.**

Block A:

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(3)(0.400)^2 - 3(9.81)(0.3)d_A = 0$$

$$d_A = 0.0272 \text{ m}$$

Block B:

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(2)(2.40)^2 - 2(9.81)(0.3)d_B = 0$$

$$d_B = 0.9786 \text{ m}$$

$$d = d_B - d_A = 0.951 \text{ m}$$ **Ans.**

15–62.

If two disks A and B have the same mass and are subjected to direct central impact such that the collision is perfectly elastic ($e = 1$), prove that the kinetic energy before collision equals the kinetic energy after collision. The surface upon which they slide is smooth.

SOLUTION

$(\xrightarrow{+})$ $$\Sigma m\, v_1 = \Sigma m\, v_2$$

$$m_A\,(v_A)_1 + m_B\,(v_B)_1 = m_A\,(v_A)_2 + m_B\,(v_B)_2$$

$$m_A\big[(v_A)_1 - (v_A)_2\big] = m_B\big[(v_B)_2 - (v_B)_1\big] \qquad \textbf{(1)}$$

$(\xrightarrow{+})$ $$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = 1$$

$$(v_B)_2 - (v_A)_2 = (v_A)_1 - (v_B)_1 \qquad \textbf{(2)}$$

Combining Eqs. (1) and (2):

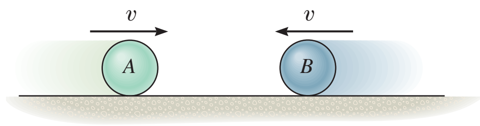
$$m_A\big[(v_A)_1 - (v_A)_2\big]\big[(v_A)_1 + (v_A)_2\big] = m_B\big[(v_B)_2 - (v_B)_1\big]\big[(v_B)_2 + (v_B)_1\big]$$

Expand and multiply by $\frac{1}{2}$:

$$\frac{1}{2}m_A\,(v_A)_1^2 + \frac{1}{2}m_B\,(v_B)_1^2 = \frac{1}{2}m_A(v_A)_2^2 + \frac{1}{2}m_B\,(v_B)_2^2 \qquad \textbf{Q.E.D.}$$

15–63.

Each ball has a mass m and the coefficient of restitution between the balls is e. If they are moving towards one another with a velocity v, determine their speeds after collision. Also, determine their common velocity when they reach the state of maximum deformation. Neglect the size of each ball.



SOLUTION

$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

$$mv - mv = mv_A + mv_B$$

$$v_A = -v_B$$

$(\overset{+}{\rightarrow})\qquad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = \dfrac{v_B - v_A}{v - (-v)}$

$$2ve = 2v_B$$

$v_B = ve \rightarrow$ **Ans.**

$v_A = -ve = ve \leftarrow$ **Ans.**

At maximum deformation $v_A = v_B = v'$.

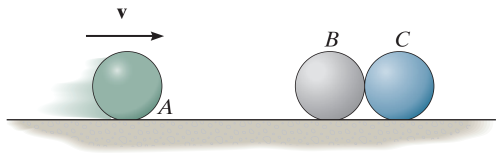
$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

$$mv - mv = (2m)v'$$

$v' = 0$ **Ans.**

***15–64.**

The three balls each have a mass m. If A has a speed v just before a direct collision with B, determine the speed of C after collision. The coefficient of restitution between each pair of balls e. Neglect the size of each ball.



SOLUTION

Conservation of Momentum: When ball A strikes ball B, we have

$$m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2$$

$$(\xrightarrow{+}) \qquad mv + 0 = m(v_A)_2 + m(v_B)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\xrightarrow{+}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{v - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_A)_2 = \frac{v(1 - e)}{2} \qquad (v_B)_2 = \frac{v(1 + e)}{2}$$

Conservation of Momentum: When ball B strikes ball C, we have

$$m_B (v_B)_2 + m_C (v_C)_1 = m_B (v_B)_3 + m_C (v_C)_2$$

$$(\xrightarrow{+}) \qquad m\left[\frac{v(1 + e)}{2}\right] + 0 = m(v_B)_3 + m(v_C)_2 \qquad \textbf{(3)}$$

Coefficient of Restitution:

$$e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}$$

$$(\xrightarrow{+}) \qquad e = \frac{(v_C)_2 - (v_B)_3}{\frac{v(1 + e)}{2} - 0} \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

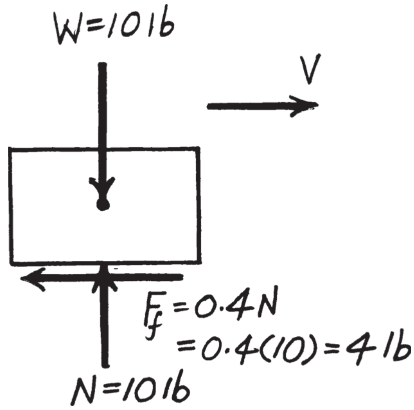
$$(v_C)_2 = \frac{v(1 + e)^2}{4} \qquad \textbf{Ans.}$$

$$(v_B)_3 = \frac{v(1 - e^2)}{4}$$

15–65.

A 1-lb ball A is traveling horizontally at 20 ft/s when it strikes a 10-lb block B that is at rest. If the coefficient of restitution between A and B is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the time for the block B to stop sliding.

SOLUTION



$(\xrightarrow{+})$ $\qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{1}{32.2}\right)(20) + 0 = \left(\frac{1}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$(\xrightarrow{+})$ $\qquad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$

Thus,

$(v_B)_2 = 2.909 \text{ ft/s} \rightarrow$

$(v_A)_2 = -9.091 \text{ ft/s} = 9.091 \text{ ft/s} \leftarrow$

Block B:

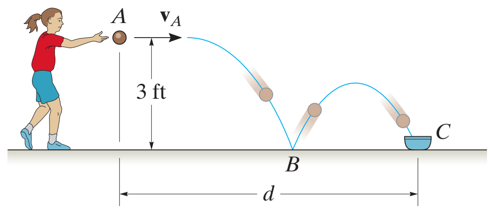
$(\xrightarrow{+})$ $\qquad m\,v_1 + \Sigma \int F\,dt = m\,v_2$

$$\left(\frac{10}{32.2}\right)(2.909) - 4t = 0$$

$t = 0.226 \text{ s}$ **Ans.**

15–66.

If the girl throws the ball with a horizontal velocity of 8 ft/s, determine the distance d so that the ball bounces once on the smooth surface and then lands in the cup at C. Take $e = 0.8$.



SOLUTION

$(+\downarrow)$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$(v_1)_y^2 = 0 + 2(32.2)(3)$$

$$(v_1)_y = 13.90 \downarrow$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$3 = 0 + 0 + \frac{1}{2}(32.2)(t_{AB})^2$$

$$t_{AB} = 0.43167 \text{ s}$$

$(+\downarrow)$ $\quad e = \dfrac{(v_2)_y}{(v_1)_y}$

$$0.8 = \frac{(v_2)_y}{13.90}$$

$$(v_2)_y = 11.1197 \uparrow$$

$(+\downarrow)$ $\quad v = v_0 + a_c t$

$$11.1197 = -11.1197 + 32.2(t_{BC})$$

$$t_{BC} = 0.6907 \text{ s}$$

Total time is $t_{AC} = 1.1224$ s

Since the x component of momentum is conserved

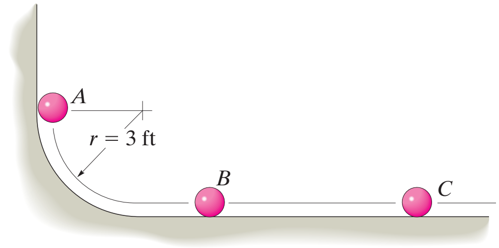
$$d = v_A(t_{AC})$$

$$d = 8(1.1224)$$

$$d = 8.98 \text{ ft}$$ **Ans.**

15–67.

The three balls each weigh 0.5 lb and have a coefficient of restitution of $e = 0.85$. If ball A is released from rest and strikes ball B and then ball B strikes ball C, determine the velocity of each ball after the second collision has occurred. The balls slide without friction.



SOLUTION

Ball A:

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0.5)(3) = \frac{1}{2}\left(\frac{0.5}{32.2}\right)(v_A)_1^2 + 0$$

$$(v_A)_1 = 13.90 \text{ ft/s}$$

Balls A and B:

$$(\stackrel{+}{\rightarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$\left(\frac{0.5}{32.2}\right)(13.90) + 0 = \left(\frac{0.5}{32.2}\right)(v_A)_2 + \left(\frac{0.5}{32.2}\right)(v_B)_2$$

$$(\stackrel{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2)}{(v_A)_1 - (v_B)_1}$$

$$0.85 = \frac{(v_B)_2 - (v_A)_2}{13.90 - 0}$$

Solving:

$$(v_A)_2 = 1.04 \text{ ft/s} \qquad \textbf{Ans.}$$

$$(v_B)_2 = 12.86 \text{ ft/s}$$

Balls B and C:

$$(\stackrel{+}{\rightarrow}) \qquad \Sigma m v_2 = \Sigma m v_3$$

$$\left(\frac{0.5}{32.2}\right)(12.86) + 0 = \left(\frac{0.5}{32.2}\right)(v_B)_3 + \left(\frac{0.5}{32.2}\right)(v_C)_3$$

$$(\stackrel{+}{\rightarrow}) \qquad e = \frac{(v_C)_3 - (v_B)_3}{(v_B)_2 - (v_C)_2}$$

$$0.85 = \frac{(v_C)_3 - (v_B)_3}{12.86 - 0}$$

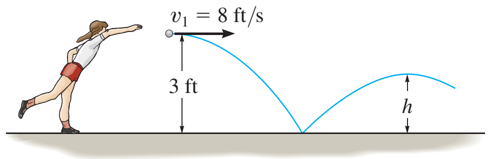
Solving:

$$(v_B)_3 = 0.964 \text{ ft/s} \qquad \textbf{Ans.}$$

$$(v_C)_3 = 11.9 \text{ ft/s} \qquad \textbf{Ans.}$$

***15–68.**

The girl throws the ball with a horizontal velocity of $v_1 = 8$ ft/s. If the coefficient of restitution between the ball and the ground is $e = 0.8$, determine (a) the velocity of the ball just after it rebounds from the ground and (b) the maximum height to which the ball rises after the first bounce.



SOLUTION

Kinematics: By considering the vertical motion of the falling ball, we have

$$(+\downarrow) \qquad (v_1)_y^2 = (v_0)_y^2 + 2a_c\left[s_y - (s_0)_y\right]$$

$$(v_1)_y^2 = 0^2 + 2(32.2)(3 - 0)$$

$$(v_1)_y = 13.90 \text{ ft/s}$$

Coefficient of Restitution (y):

$$e = \frac{\left(v_g\right)_2 - (v_2)_y}{(v_1)_y - \left(v_g\right)_1}$$

$$(+\uparrow) \qquad 0.8 = \frac{0 - (v_2)_y}{-13.90 - 0}$$

$$(v_2)_y = 11.12 \text{ ft/s}$$

Conservation of "x" Momentum: The momentum is conserved along the x axis.

$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 = m(v_x)_2\,; \qquad (v_x)_2 = 8 \text{ ft/s} \rightarrow$$

The magnitude and the direction of the rebounding velocity for the ball is

$$v_2 = \sqrt{(v_x)_2^2 + \left(v_y\right)_2^2} = \sqrt{8^2 + 11.12^2} = 13.7 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{11.12}{8}\right) = 54.3° \qquad \textbf{Ans.}$$

Kinematics: By considering the vertical motion of tfie ball after it rebounds from the ground, we have

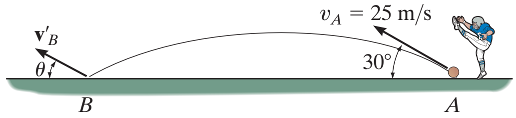
$$(+\uparrow) \qquad (v)_y^2 = (v_2)_y^2 + 2a_c\left[s_y - (s_2)_y\right]$$

$$0 = 11.12^2 + 2(-32.2)(h - 0)$$

$$h = 1.92 \text{ ft} \qquad \textbf{Ans.}$$

15–69.

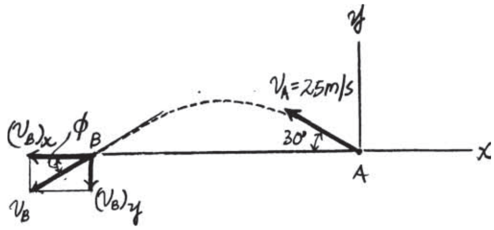
A 300-g ball is kicked with a velocity of $v_A = 25$ m/s at point A as shown. If the coefficient of restitution between the ball and the field is $e = 0.4$, determine the magnitude and direction θ of the velocity of the rebounding ball at B.



SOLUTION

Kinematics: The parabolic trajectory of the football is shown in Fig. *a*. Due to the symmetrical properties of the trajectory, $v_B = v_A = 25$ m/s and $\phi = 30°$.

Conservation of Linear Momentum: Since no impulsive force acts on the football along the x axis, the linear momentum of the football is conserved along the x axis.



(a)

$$\left(\overset{+}{\leftarrow}\right) \qquad m\left(v_B\right)_x = m\left(v'_B\right)_x$$

$$0.3\left(25\cos 30°\right) = 0.3\left(v'_B\right)_x$$

$$\left(v'_B\right)_x = 21.65 \text{ m/s} \leftarrow$$

Coefficient of Restitution: Since the ground does not move during the impact, the coefficient of restitution can be written as

$$(+\uparrow) \qquad e = \frac{0 - \left(v'_B\right)_y}{\left(v_B\right)_y - 0}$$

$$0.4 = \frac{-\left(v'_B\right)_y}{-25\sin 30°}$$

$$\left(v'_B\right)_y = 5 \text{ m/s} \uparrow$$

Thus, the magnitude of $\mathbf{v}'_B$ is

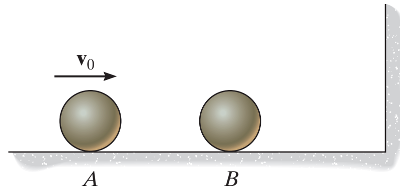
$$v'_B = \sqrt{\left(v'_B\right)_x + \left(v'_B\right)_y} = \sqrt{21.65^2 + 5^2} = 22.2 \text{ m/s} \qquad \textbf{Ans.}$$

and the angle of $\mathbf{v}'_B$ is

$$\theta = \tan^{-1}\left[\frac{\left(v'_B\right)_y}{\left(v'_B\right)_x}\right] = \tan^{-1}\left(\frac{5}{21.65}\right) = 13.0° \qquad \textbf{Ans.}$$

15–70.

Two smooth spheres A and B each have a mass m. If A is given a velocity of v_0, while sphere B is at rest, determine the velocity of B just after it strikes the wall. The coefficient of restitution for any collision is e.



SOLUTION

Impact: The first impact occurs when sphere A strikes sphere B. When this occurs, the linear momentum of the system is conserved along the x axis (line of impact). Referring to Fig. a,

$$(\overset{+}{\rightarrow}) \qquad m_A v_A + m_B v_B = m_A(v_A)_1 + m_B(v_B)_1$$

$$m v_0 + 0 = m(v_A)_1 + m(v_B)_1$$

$$(v_A)_1 + (v_B)_1 = v_0 \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_1 - (v_A)_1}{v_A - v_B}$$

$$e = \frac{(v_B)_1 - (v_A)_1}{v_0 - 0}$$

$$(v_B)_1 - (v_A)_1 = e v_0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

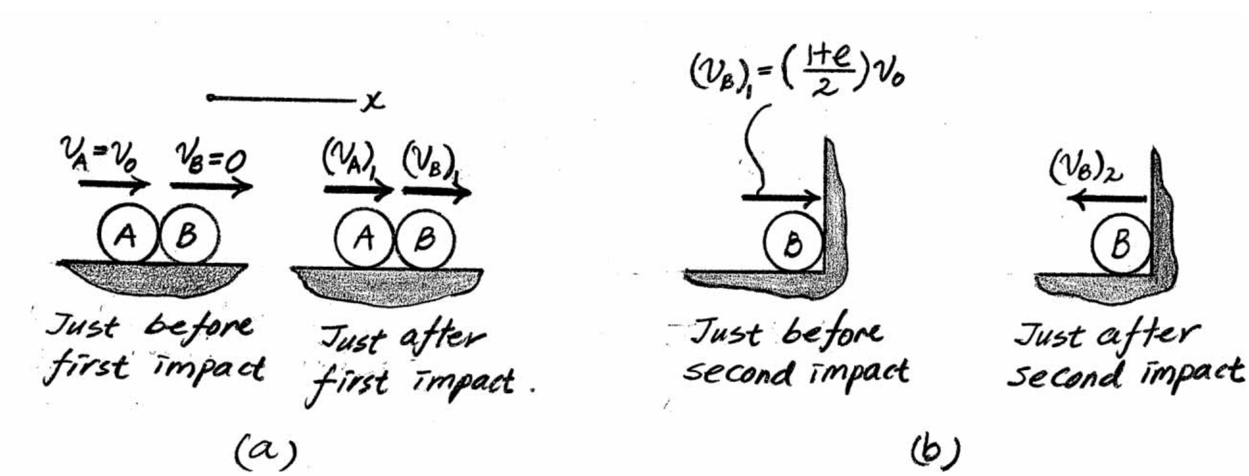
$$(v_B)_1 = \left(\frac{1+e}{2}\right)v_0 \rightarrow \qquad (v_A)_1 = \left(\frac{1-e}{2}\right)v_0 \rightarrow$$

The second impact occurs when sphere B strikes the wall, Fig. b. Since the wall does not move during the impact, the coefficient of restitution can be written as

$$(\overset{+}{\rightarrow}) \qquad e = \frac{0 - \left[-(v_B)_2\right]}{(v_B)_1 - 0}$$

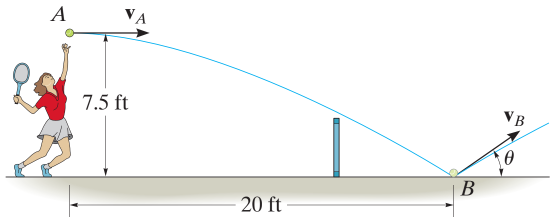
$$e = \frac{0 + (v_B)_2}{\left[\dfrac{1+e}{2}\right]v_0 - 0}$$

$$(v_B)_2 = \frac{e(1+e)}{2}v_0 \qquad \textbf{Ans.}$$



15–71.

It was observed that a tennis ball when served horizontally 7.5 ft above the ground strikes the smooth ground at B 20 ft away. Determine the initial velocity $\mathbf{v}_A$ of the ball and the velocity $\mathbf{v}_B$ (and θ) of the ball just after it strikes the court at B. Take $e = 0.7$.



SOLUTION

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$$20 = 0 + v_A t$$

$(+\downarrow)\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$7.5 = 0 + 0 + \frac{1}{2}(32.2)t^2$$

$$t = 0.682524$$

$$v_A = 29.303 = 29.3 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_{Bx1} = 29.303 \text{ ft/s}$$

$(+\downarrow)\quad v = v_0 + a_c t$

$$v_{By1} = 0 + 32.2(0.68252) = 21.977 \text{ ft/s}$$

$(\overset{+}{\rightarrow})\quad mv_1 = mv_2$

$$v_{B2x} = v_{B1x} = 29.303 \text{ ft/s} \rightarrow$$

$$e = \frac{v_{By2}}{v_{By1}}$$

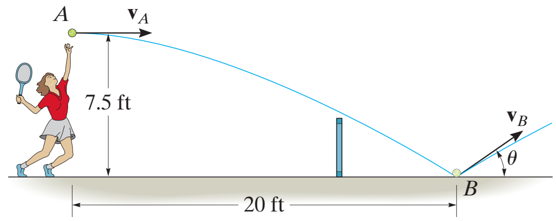
$$0.7 = \frac{v_{By2}}{21.977}, \qquad v_{By2} = 15.384 \text{ ft/s} \uparrow$$

$$v_{B2} = \sqrt{(29.303)^2 + (15.384)^2} = 33.1 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{15.384}{29.303} = 27.7^\circ \;\measuredangle\theta \qquad \textbf{Ans.}$$

*15–72.

The tennis ball is struck with a horizontal velocity $\mathbf{v}_A$, strikes the smooth ground at B, and bounces upward at $\theta = 30°$. Determine the initial velocity $\mathbf{v}_A$, the final velocity $\mathbf{v}_B$, and the coefficient of restitution between the ball and the ground.



SOLUTION

$(+\downarrow)$ $\quad v^2 = v_0^2 + 2\,a_c(s - s_0)$

$$(v_{By})_1^2 = 0 + 2(32.2)(7.5 - 0)$$

$$v_{By1} = 21.9773 \text{ m/s}$$

$(+\downarrow)$ $\quad v = v_0 + a_c t$

$$21.9773 = 0 + 32.2\,t$$

$$t = 0.68252 \text{ s}$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t$

$$20 = 0 + v_A\,(0.68252)$$

$$v_A = 29.303 = 29.3 \text{ ft/s} \qquad \textbf{Ans.}$$

$(\xrightarrow{+})$ $\quad mv_1 = mv_2$

$$v_{Bx2} = v_{Bx1} = v_A = 29.303$$

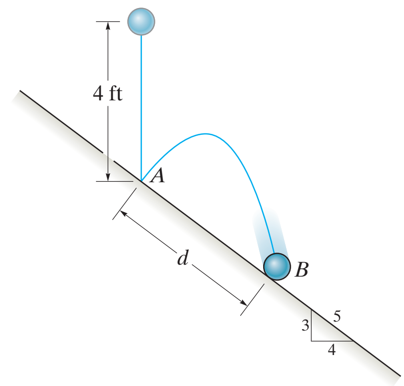
$$v_{B_2} = 29.303/\cos 30° = 33.8 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_{By2} = 29.303 \tan 30° = 16.918 \text{ ft/s}$$

$$e = \frac{v_{By2}}{v_{By1}} = \frac{16.918}{21.9773} = 0.770 \qquad \textbf{Ans.}$$

15–73.

The 1 lb ball is dropped from rest and falls a distance of 4 ft before striking the smooth plane at A. If $e = 0.8$, determine the distance d to where it again strikes the plane at B.

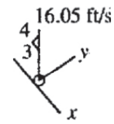


SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(m)(v_A)_1^2 - m(32.2)(4)$$

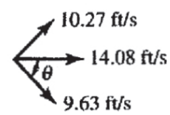
$$(v_A)_1 = \sqrt{2(32.2)(4)} = 16.05 \text{ ft/s}$$



$$\searrow + \ (v_A)_{2x} = \frac{3}{5}(16.05) = 9.63 \text{ ft/s}$$

$$\nearrow + \ (v_A)_{2y} = 0.8\left(\frac{4}{5}\right)(16.05) = 10.27 \text{ ft/s}$$

$$(v_A)_2 = \sqrt{(9.63)^2 + (10.27)^2} = 14.08 \text{ ft/s}$$



$$\theta = \tan^{-1}\left(\frac{10.27}{9.63}\right) = 46.85^\circ$$

$$\phi = 46.85^\circ - \tan^{-1}\left(\frac{3}{4}\right) = 9.977^\circ$$

$$(\overset{+}{\rightarrow}) \ s = s_0 + v_0 t$$

$$d\left(\frac{4}{5}\right) = 0 + 14.08 \cos 9.977^\circ \ (t)$$

$$(+\downarrow) \ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

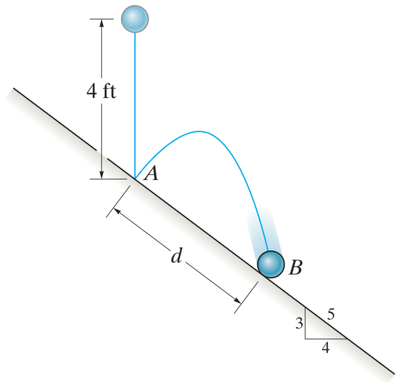
$$d\left(\frac{3}{5}\right) = 0 - 14.08 \sin 9.977^\circ \ (t) + \frac{1}{2}(32.2)t^2$$

$$t = 0.798 \text{ s}$$

$$d = 13.8 \text{ ft}$$ **Ans.**

15–74.

The 1 lb ball is dropped from rest and falls a distance of 4 ft before striking the smooth plane at A. If it rebounds and in $t = 0.5$ s again strikes the plane at B, determine the coefficient of restitution e between the ball and the plane. Also, what is the distance d?

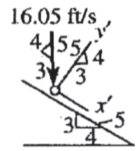


SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(m)(v_A)_1^2 - m(32.2)(4)$$

$$(v_A)_1 = \sqrt{2(32.2)(4)} = 16.05 \text{ ft/s}$$



$$+\searrow \qquad (v_A)_{2x'} = \frac{3}{5}(16.05) = 9.63 \text{ ft/s}$$

$$\nearrow + \qquad (v_A)_{2y'} = e\left(\frac{4}{5}\right)(16.05) = 12.84e \text{ ft/s}$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$$

$$\frac{4}{5}(d) = 0 + v_{A2x}(0.5)$$

$$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$\frac{3}{5}(d) = 0 - v_{A2y}(0.5) + \frac{1}{2}(32.2)(0.5)^2$$

$$(\overset{+}{\rightarrow}) \qquad 0.5\left[9.63\left(\frac{4}{5}\right) + 12.84e\left(\frac{3}{5}\right)\right] = \frac{4}{5}d$$

$$(+\uparrow) \qquad 0.5\left[-9.63\left(\frac{3}{5}\right) + 12.84e\left(\frac{4}{5}\right)\right] = 4.025 - \frac{3}{5}d$$

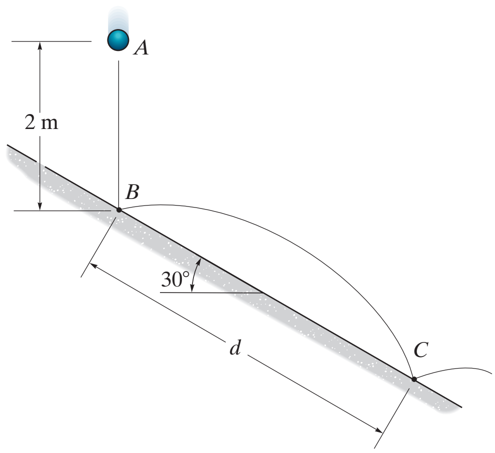
Solving,

$$e = 0.502 \qquad \textbf{Ans.}$$

$$d = 7.23 \text{ ft} \qquad \textbf{Ans.}$$

15–75.

The 1-kg ball is dropped from rest at point A, 2 m above the smooth plane. If the coefficient of restitution between the ball and the plane is $e = 0.6$, determine the distance d where the ball again strikes the plane.



SOLUTION

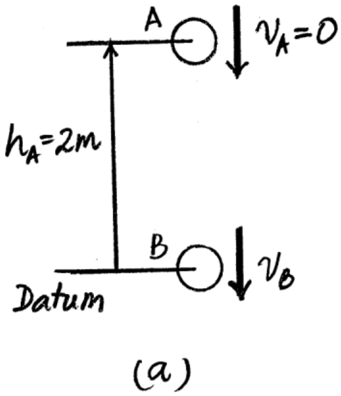
Conservation of Energy: By considering the ball's fall from position (1) to position (2) as shown in Fig. a,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m_A v_A^2 + (V_g)_A = \frac{1}{2}m_B v_B^2 + (V_g)_B$$

$$0 + 1(9.81)(2) = \frac{1}{2}(1)v_B^2 + 0$$

$$v_B = 6.264 \text{ m/s} \downarrow$$



Conservation of Linear Momentum: Since no impulsive force acts on the ball along the inclined plane (x' axis) during the impact, linear momentum of the ball is conserved along the x' axis. Referring to Fig. b,

$$m_B(v_B)_{x'} = m_B(v'_B)_{x'}$$

$$1(6.264)\sin 30° = 1(v'_B)\cos\theta$$

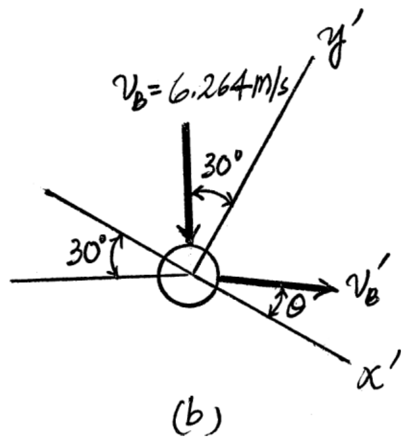
$$v'_B \cos\theta = 3.1321 \qquad \textbf{(1)}$$

Coefficient of Restitution: Since the inclined plane does not move during the impact,

$$e = \frac{0 - (v'_B)_{y'}}{(v'_B)_{y'} - 0}$$

$$0.6 = \frac{0 - v'_B \sin\theta}{-6.264\cos 30° - 0}$$

$$v'_B \sin\theta = 3.2550 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$\theta = 46.10° \qquad v'_B = 4.517 \text{ m/s}$$

Kinematics: By considering the x and y motion of the ball after the impact, Fig. c,

$(\overset{+}{\rightarrow})$ $$s_x = (s_0)_x + (v'_B)_x t$$

$$d\cos 30° = 0 + 4.517\cos 16.10° t$$

$$t = 0.1995d \qquad \textbf{(3)}$$

$(+\uparrow)$ $$s_y = (s_0)_y + (v'_B)_y t + \frac{1}{2}a_y t^2$$

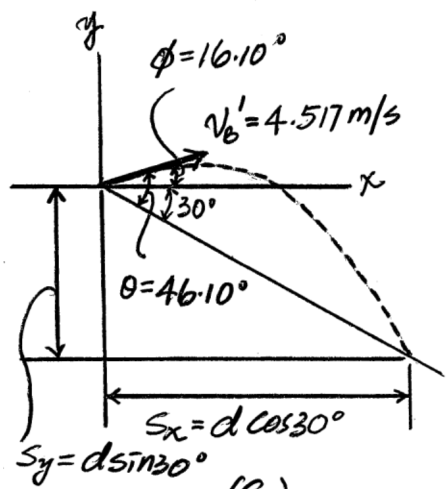
$$-d\sin 30° = 0 + 4.517\sin 16.10° t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 1.2528t - 0.5d = 0 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

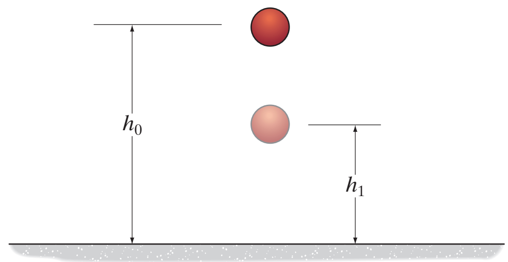
$$d = 3.84 \text{ m} \qquad \textbf{Ans.}$$

$$t = 0.7663 \text{ s}$$



***15–76.**

A ball of mass m is dropped vertically from a height h_0 above the ground. If it rebounds to a height of h_1, determine the coefficient of restitution between the ball and the ground.



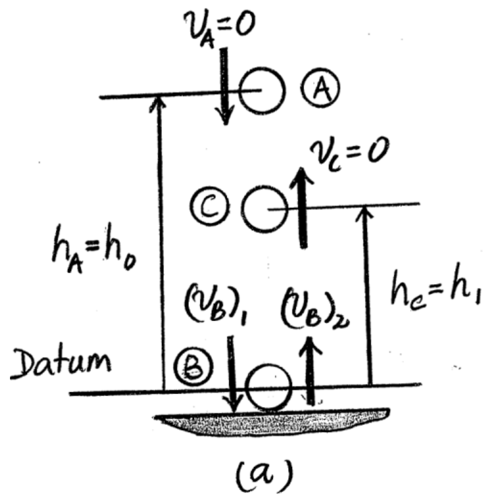
SOLUTION

Conservation of Energy: First, consider the ball's fall from position A to position B. Referring to Fig. a,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^{\,2} + (V_g)_A = \frac{1}{2}mv_B^{\,2} + (V_g)_B$$

$$0 + mg(h_0) = \frac{1}{2}m(v_B)_1^{\,2} + 0$$



Subsequently, the ball's return from position B to position C will be considered.

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^{\,2} + (V_g)_B = \frac{1}{2}mv_C^{\,2} + (V_g)_C$$

$$\frac{1}{2}m(v_B)_2^{\,2} + 0 = 0 + mgh_1$$

$$(v_B)_2 = \sqrt{2gh_1}\uparrow$$

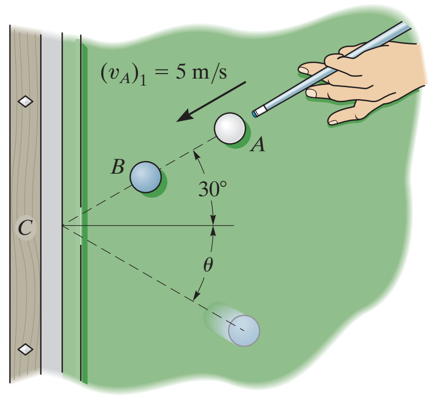
Coefficient of Restitution: Since the ground does not move,

$$(+\uparrow) \qquad e = -\frac{(v_B)_2}{(v_B)_1}$$

$$e = -\frac{\sqrt{2gh_1}}{-\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}}$$ **Ans.**

15–77.

The cue ball A is given an initial velocity $(v_A)_1 = 5$ m/s. If it makes a direct collision with ball B ($e = 0.8$), determine the velocity of B and the angle θ just after it rebounds from the cushion at C ($e' = 0.6$). Each ball has a mass of 0.4 kg. Neglect the size of each ball.



SOLUTION

Conservation of Momentum: When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$0.4(5) + 0 = 0.4(v_A)_2 + 0.4(v_B)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.8 = \frac{(v_B)_2 - (v_A)_2}{5 - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$(v_A)_2 = 0.500$ m/s $\quad (v_B)_2 = 4.50$ m/s

Conservation of "y" Momentum: When ball B strikes the cushion at C, we have

$$m_B(v_{B_y})_2 = m_B(v_{B_y})_3$$

$$(+\downarrow) \qquad 0.4(4.50 \sin 30°) = 0.4(v_B)_3 \sin\theta$$

$$(v_B)_3 \sin\theta = 2.25 \qquad \textbf{(3)}$$

Coefficient of Restitution (x):

$$e = \frac{(v_C)_2 - (v_{B_x})_3}{(v_{B_x})_2 - (v_C)_1}$$

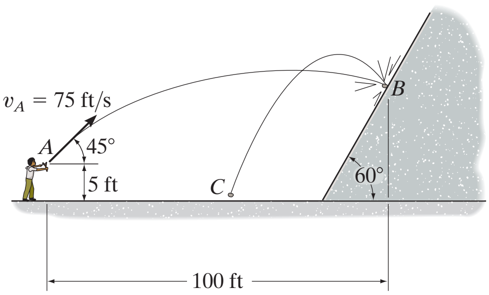
$$(\overset{+}{\leftarrow}) \qquad 0.6 = \frac{0 - [-(v_B)_3 \ \cos\theta]}{4.50 \cos 30° - 0} \qquad \textbf{(4)}$$

Solving Eqs. (1) and (2) yields

$(v_B)_3 = 3.24$ m/s $\quad \theta = 43.9°$ **Ans.**

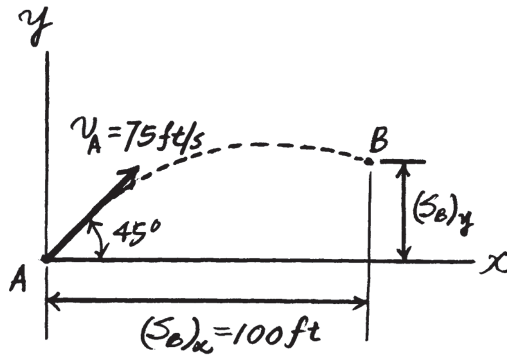
15–78.

Using a slingshot, the boy fires the 0.2-lb marble at the concrete wall, striking it at B. If the coefficient of restitution between the marble and the wall is $e = 0.5$, determine the speed of the marble after it rebounds from the wall.



SOLUTION

Kinematics: By considering the x and y motion of the marble from A to B, Fig. *a*,



(a)

$(\xrightarrow{+})$ $$\left(s_B\right)_x = \left(s_A\right)_x + \left(v_A\right)_x t$$

$$100 = 0 + 75 \cos 45^\circ\, t$$

$$t = 1.886 \text{ s}$$

and

$(+\uparrow)$ $$\left(s_B\right)_y = \left(s_A\right)_y + \left(v_A\right)_y t + \frac{1}{2} a_y t^2$$

$$\left(s_B\right)_y = 0 + 75 \sin 45^\circ(1.886) + \frac{1}{2}(-32.2)(1.886^2)$$

$$= 42.76 \text{ ft}$$

and

$(+\uparrow)$ $$\left(v_B\right)_y = \left(v_A\right)_y + a_y t$$

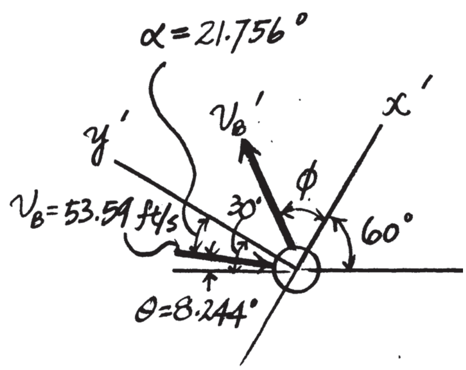
$$\left(v_B\right)_y = 75 \sin 45^\circ + (-32.2)(1.886) = -7.684 \text{ ft/s} = 7.684 \text{ ft/s} \downarrow$$

Since $\left(v_B\right)_x = \left(v_A\right)_x = 75 \cos 45^\circ = 53.03$ ft/s, the magnitude of $\mathbf{v}_B$ is

$$v_B = \sqrt{\left(v_B\right)_x{}^2 + \left(v_B\right)_y{}^2} = \sqrt{53.03^2 + 7.684^2} = 53.59 \text{ ft/s}$$

and the direction angle of $\mathbf{v}_B$ is

$$\theta = \tan^{-1}\left[\frac{\left(v_B\right)_y}{\left(v_B\right)_x}\right] = \tan^{-1}\left(\frac{7.684}{53.03}\right) = 8.244^\circ$$



(b)

Conservation of Linear Momentum: Since no impulsive force acts on the marble along the inclined surface of the concrete wall (x' axis) during the impact, the linear momentum of the marble is conserved along the x' axis. Referring to Fig. *b*,

$(+\nearrow)$ $$m_B\left(v_B'\right)_{x'} = m_B\left(v_B'\right)_{x'}$$

$$\frac{0.2}{32.2}\left(53.59 \sin 21.756^\circ\right) = \frac{0.2}{32.2}\left(v_B' \cos \phi\right)$$

$$v_B' \cos \phi = 19.862 \qquad \textbf{(1)}$$

15–78. continued

Coefficient of Restitution: Since the concrete wall does not move during the impact, the coefficient of restitution can be written as

$$(+\nwarrow) \qquad e = \frac{0 - \left(v'_B\right)_{y'}}{\left(v'_B\right)_{y'} - 0}$$

$$0.5 = \frac{-v'_B \sin\phi}{-53.59 \cos 21.756^\circ}$$

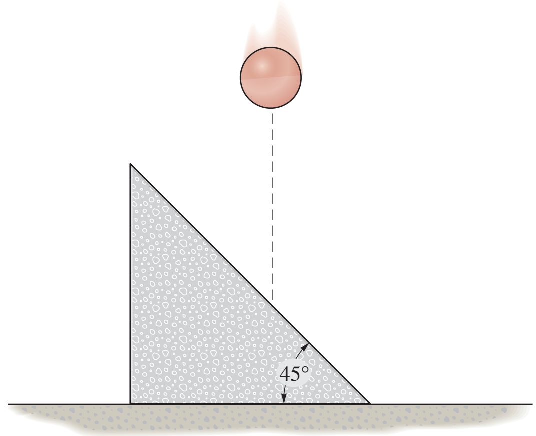
$$v'_B \sin\phi = 24.885 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v'_B = 31.8 \text{ ft/s} \qquad \textbf{Ans.}$$

15–79.

The sphere of mass m falls and strikes the triangular block with a vertical velocity v. If the block rests on a smooth surface and has a mass $3\,m$, determine its velocity just after the collision. The coefficient of restitution is e.



SOLUTION

Conservation of "x′" Momentum:

$$m\,(v)_1 = m\,(v)_2$$

$$(\searrow +) \qquad m(\,v \sin 45°) = m(v_{sx'})_2$$

$$(v_{sx'})_2 = \frac{\sqrt{2}}{2}\,v$$

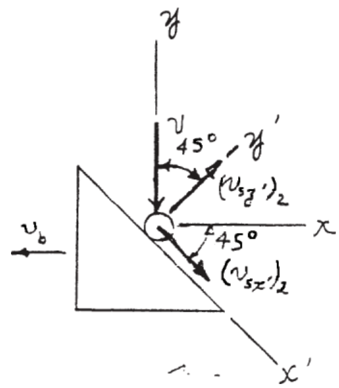
Coefficient of Restitution (y′):

$$e = \frac{(v_b)_2 - \left(v_{s_{y'}}\right)_2}{\left(v_{s_{y'}}\right)_1 - (v_b)_1}$$

$$(+\swarrow) \qquad e = \frac{v_b \cos 45° - \left[-\left(v_{sy'}\right)_2\right]}{v \cos 45° - 0}$$

$$\left(v_{sy'}\right)_2 = \frac{\sqrt{2}}{2}(ev - v_b)$$

Conservation of "x" Momentum:



$$0 = m_s\,(v_s)_x + m_b\,v_b$$

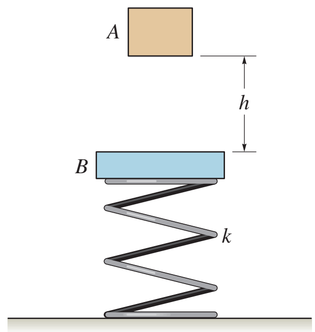
$$(\overset{+}{\leftarrow}) \qquad 0 + 0 = 3mv_b - m\left(v_{sy'}\right)_2 \cos 45° - m(v_{sx'})_2 \cos 45°$$

$$3v_b - \frac{\sqrt{2}}{2}(ev - v_b)\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\,v\,\frac{\sqrt{2}}{2} = 0$$

$$v_b = \left(\frac{1 + e}{7}\right)v \qquad \textbf{Ans.}$$

***15–80.**

Block A, having a mass m, is released from rest, falls a distance h and strikes the plate B having a mass $2m$. If the coefficient of restitution between A and B is e, determine the velocity of the plate just after collision. The spring has a stiffness k.



SOLUTION

Just before impact, the velocity of A is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}mv_A^2 - mgh$$

$$v_A = \sqrt{2gh}$$

$(+\downarrow)$ $$e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gh}}$$

$$e\sqrt{2gh} = (v_B)_2 - (v_A)_2 \qquad \textbf{(1)}$$

$(+\downarrow)$ $$\Sigma mv_1 = \Sigma mv_2$$

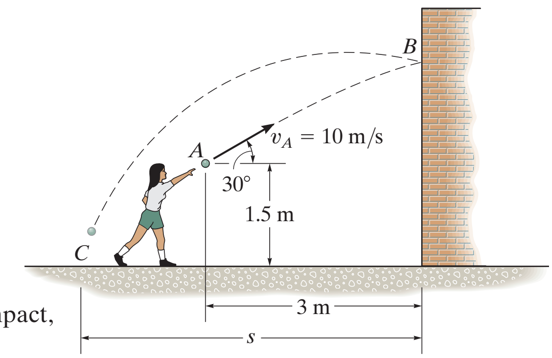
$$m(v_A) + 0 = m(v_A)_2 + 2m(v_B)_2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) for $(v_B)_2$ yields;

$$(v_B)_2 = \frac{1}{3}\sqrt{2gh}(1 + e) \qquad \textbf{Ans.}$$

15–81.

The girl throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10\text{ m/s}$. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if the coefficient of restitution $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



SOLUTION

Kinematics: By considering the horizontal motion of the ball before the impact, we have

$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + v_x t$$

$$3 = 0 + 10 \cos 30° t \qquad t = 0.3464 \text{ s}$$

By considering the vertical motion of the ball before the impact, we have

$$(+\uparrow) \qquad v_y = (v_0)_y + (a_c)_y t$$

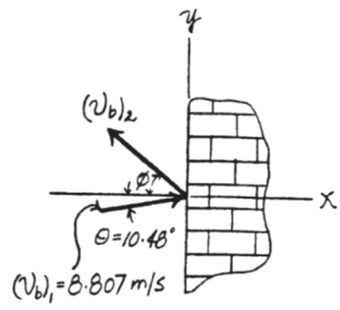
$$= 10 \sin 30° + (-9.81)(0.3464)$$

$$= 1.602 \text{ m/s}$$

The vertical position of point B above the ground is given by

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$(s_B)_y = 1.5 + 10 \sin 30°(0.3464) + \frac{1}{2}(-9.81)\left(0.3464^2\right) = 2.643 \text{ m}$$



Thus, the magnitude of the velocity and its directional angle are

$$(v_b)_1 = \sqrt{(10 \cos 30°)^2 + 1.602^2} = 8.807 \text{ m/s} = 8.81 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1} \frac{1.602}{10 \cos 30°} = 10.48° = 10.5° \qquad \textbf{Ans.}$$

Conservation of "y" Momentum: When the ball strikes the wall with a speed of $(v_b)_1 = 8.807$ m/s, it rebounds with a speed of $(v_b)_2$.

$$m_b \left(v_{b_y}\right)_1 = m_b \left(v_{b_y}\right)_2$$

$$(\xleftarrow{+}) \qquad m_b (1.602) = m_b \left[(v_b)_2 \sin \phi\right]$$

$$(v_b)_2 \sin \phi = 1.602 \qquad \textbf{(1)}$$

Coefficient of Restitution (x):

$$e = \frac{(v_w)_2 - \left(v_{b_x}\right)_2}{\left(v_{b_x}\right)_1 - (v_w)_1}$$

$$(\xrightarrow{+}) \qquad 0.5 = \frac{0 - \left[-(v_b)_2 \cos \phi\right]}{10 \cos 30° - 0} \qquad \textbf{(2)}$$

15–81. continued

Solving Eqs. (1) and (2) yields

$$\phi = 20.30° = 20.3° \qquad (v_b)_2 = 4.617 \text{ m/s} = 4.62 \text{ m/s} \qquad \textbf{Ans.}$$

Kinematics: By considering the vertical motion of the ball after the impact, we have

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$-2.643 = 0 + 4.617 \sin 20.30° t_1 + \frac{1}{2}(-9.81)t_1^2$$

$$t_1 = 0.9153 \text{ s}$$

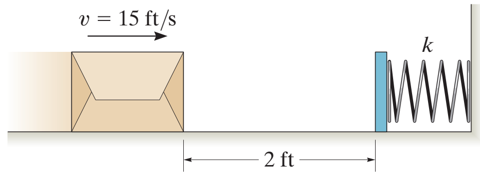
By considering the horizontal motion of the ball after the impact, we have

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$s = 0 + 4.617 \cos 20.30°(0.9153) = 3.96 \text{ m} \qquad \textbf{Ans.}$$

15–82.

The 20-lb box slides on the surface for which $\mu_k = 0.3$. The box has a velocity $v = 15$ ft/s when it is 2 ft from the plate. If it strikes the smooth plate, which has a weight of 10 lb and is held in position by an unstretched spring of stiffness $k = 400$ lb/ft, determine the maximum compression imparted to the spring. Take $e = 0.8$ between the box and the plate. Assume that the plate slides smoothly.

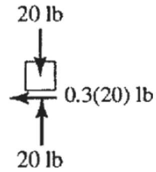


SOLUTION

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{20}{32.2}\right)(15)^2 - (0.3)(20)(2) = \frac{1}{2}\left(\frac{20}{32.2}\right)(v_2)^2$$

$$v_2 = 13.65 \text{ ft/s}$$



$$(\overset{+}{\rightarrow}) \qquad \sum mv_1 = \sum mv_2$$

$$\left(\frac{20}{32.2}\right)(13.65) = \left(\frac{20}{32.2}\right)v_A + \frac{10}{32.2}v_B$$

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{v_P - v_A}{13.65}$$

Solving,

$$v_P = 16.38 \text{ ft/s}, \quad v_A = 5.46 \text{ ft/s}$$

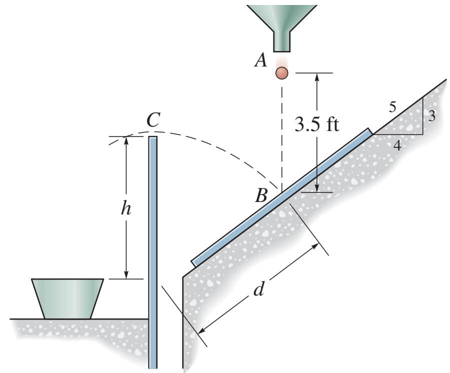
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(16.38)^2 + 0 = 0 + \frac{1}{2}(400)(s)^2$$

$$s = 0.456 \text{ ft}$$ **Ans.**

15–83.

Before a cranberry can make it to your dinner plate, it must pass a bouncing test which rates its quality. If cranberries having an $e \geq 0.8$ are to be accepted, determine the dimensions d and h for the barrier so that when a cranberry falls from rest at A it strikes the plate at B and bounces over the barrier at C.



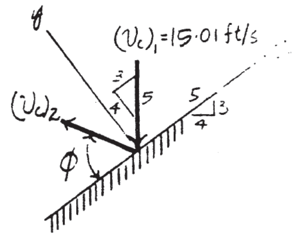
SOLUTION

Conservation of Energy: The datum is set at point B. When the cranberry falls from a height of 3.5 ft *above* the datum, its initial gravitational potential energy is $W(3.5) = 3.5\,W$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 3.5W = \frac{1}{2}\left(\frac{W}{32.2}\right)(v_c)_1^2 + 0$$

$$(v_c)_1 = 15.01 \text{ ft/s}$$

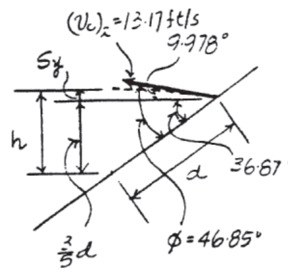


Conservation of "x′" Momentum: When the cranberry strikes the plate with a speed of $(v_c)_1 = 15.01$ ft/s, it rebounds with a speed of $(v_c)_2$.

$$m_c\left(v_{c_{x'}}\right)_1 = m_c\left(v_{c_{x'}}\right)_2$$

$$(+\swarrow) \qquad m_c\,(15.01)\left(\frac{3}{5}\right) = m_c\left[(v_c)_2 \cos\phi\right]$$

$$(v_c)_2 \cos\phi = 9.008 \qquad \textbf{(1)}$$



Coefficient of Restitution (y′):

$$e = \frac{(v_P)_2 - \left(v_{c_{y'}}\right)_2}{\left(v_{c_{y'}}\right)_1 - (v_P)_1}$$

$$(\nwarrow +) \qquad 0.8 = \frac{0 - (v_c)_2 \sin\phi}{-15.01\left(\frac{4}{5}\right) - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\phi = 46.85° \qquad (v_c)_2 = 13.17 \text{ ft/s}$$

Kinematics: By considering the vertical motion of the cranberry after the impact, we have

$$(+\uparrow) \qquad v_y = (v_0)_y + a_c t$$

$$0 = 13.17 \sin 9.978° + (-32.2)t \qquad t = 0.07087 \text{ s}$$

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y\, t^2$$

$$= 0 + 13.17 \sin 9.978°\,(0.07087) + \frac{1}{2}(-32.2)\left(0.07087^2\right)$$

$$= 0.080864 \text{ ft}$$

15–83. continued

By considering the horizontal motion of the cranberry after the impact, we have

$$\left(\overset{+}{\leftarrow}\right) \qquad s_x = (s_0)_x + v_x t$$

$$\frac{4}{5}d = 0 + 13.17 \cos 9.978° \,(0.07087)$$

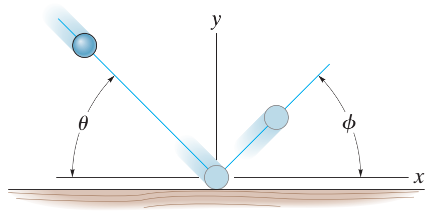
$$d = 1.149 \text{ ft} = 1.15 \text{ ft} \qquad \textbf{Ans.}$$

Thus,

$$h = s_y + \frac{3}{5}d = 0.080864 + \frac{3}{5}(1.149) = 0.770 \text{ ft} \qquad \textbf{Ans.}$$

***15–84.**

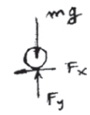
A ball is thrown onto a rough floor at an angle θ. If it rebounds at an angle ϕ and the coefficient of kinetic friction is μ, determine the coefficient of restitution e. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x\,\Delta t = \mu F_y\,\Delta t$ or $F_x = \mu F_y$.



SOLUTION

$$(+\downarrow) \qquad e = \frac{0 - [-v_2 \sin\phi]}{v_1 \sin\theta - 0} \qquad e = \frac{v_2 \sin\phi}{v_1 \sin\theta} \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \int_{t_1}^{t_2} F_x\,dx = m(v_x)_2$$



$$mv_1 \cos\theta - F_x\,\Delta t = mv_2 \cos\phi$$

$$F_x = \frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} \qquad \textbf{(2)}$$

$$(+\downarrow) \qquad m(v_y)_1 + \int_{t_1}^{t_2} F_y\,dx = m(v_y)_2$$

$$mv_1 \sin\theta - F_y\,\Delta t = -mv_2 \sin\phi$$

$$F_y = \frac{mv_1 \sin\theta + mv_2 \sin\phi}{\Delta t} \qquad \textbf{(3)}$$

Since $F_x = \mu F_y$, from Eqs. (2) and (3)

$$\frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} = \frac{\mu(mv_1 \sin\theta + mv_2 \sin\phi)}{\Delta t}$$

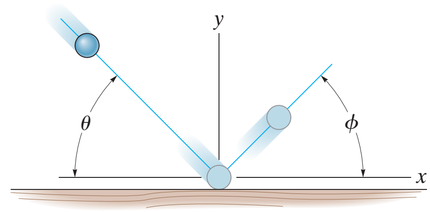
$$\frac{v_2}{v_1} = \frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi} \qquad \textbf{(4)}$$

Substituting Eq. (4) into (1) yields:

$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu\sin\theta}{\mu\sin\phi + \cos\phi}\right) \qquad \textbf{Ans.}$$

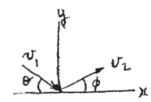
15–85.

A ball is thrown onto a rough floor at an angle of $\theta = 45°$. If it rebounds at the same angle $\phi = 45°$, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is $e = 0.6$. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.



SOLUTION

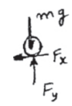
$$(+\downarrow) \qquad e = \frac{0 - [-v_2 \sin \phi]}{v_1 \sin \theta - 0} \qquad e = \frac{v_2 \sin \phi}{v_1 \sin \theta} \qquad \textbf{(1)}$$



$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \int_{t_1}^{t_2} F_x dx = m(v_x)_2$$

$$mv_1 \cos \theta - F_x \Delta t = mv_2 \cos \phi$$

$$F_x = \frac{mv_1 \cos \theta - mv_2 \cos \phi}{\Delta t} \qquad \textbf{(2)}$$



$$(+\uparrow) \qquad m(v_y)_1 + \int_{t_1}^{t_2} \mathrm{F}_y dx = m(v_y)_2$$

$$mv_1 \sin \theta - F_y \Delta t = -mv_2 \sin \phi$$

$$F_y = \frac{mv_1 \sin \theta + mv_2 \sin \phi}{\Delta t} \qquad \textbf{(3)}$$

Since $F_x = \mu F_y$, from Eqs. (2) and (3)

$$\frac{mv_1 \cos \theta - mv_2 \cos \phi}{\Delta t} = \frac{\mu(mv_1 \sin \theta + mv_2 \sin \phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi} \qquad \textbf{(4)}$$

Substituting Eq. (4) into (1) yields:

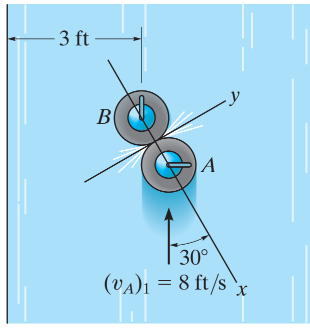
$$e = \frac{\sin \phi}{\sin \theta}\left(\frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi}\right)$$

$$0.6 = \frac{\sin 45°}{\sin 45°}\left(\frac{\cos 45° - \mu \sin 45°}{\mu \sin 45° + \cos 45°}\right)$$

$$0.6 = \frac{1 - \mu}{1 + \mu} \qquad \mu = 0.25 \qquad \textbf{Ans.}$$

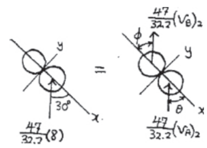
15–86.

The "stone" A used in the sport of curling slides over the ice track and strikes another "stone" B as shown. If each "stone" is smooth and has a weight of 47 lb, and the coefficient of restitution between the "stones" is $e = 0.8$, determine their speeds just after collision. Initially A has a velocity of 8 ft/s and B is at rest. Neglect friction.



SOLUTION

Line of impact (x-axis):



$$\Sigma m v_1 = \Sigma m v_2$$

$$(+\nwarrow) \qquad 0 + \frac{47}{32.2}(8)\cos 30^\circ = \frac{47}{32.2}(v_B)_{2x} + \frac{47}{32.2}(v_A)_{2x}$$

$$(+\nwarrow) \qquad e = 0.8 = \frac{(v_B)_{2x} - (v_A)_{2x}}{8\cos 30^\circ - 0}$$

Solving:

$(v_A)_{2x} = 0.6928$ ft/s

$(v_B)_{2x} = 6.235$ ft/s

Plane of impact (y-axis):

Stone A:

$$mv_1 = mv_2$$

$$(\nearrow +) \qquad \frac{47}{32.2}(8)\sin 30^\circ = \frac{47}{32.2}(v_A)_{2y}$$

$$(v_A)_{2y} = 4$$

Stone B:

$$mv_1 = mv_2$$

$$(\nearrow +) \qquad 0 = \frac{47}{32.2}(v_B)_{2y}$$

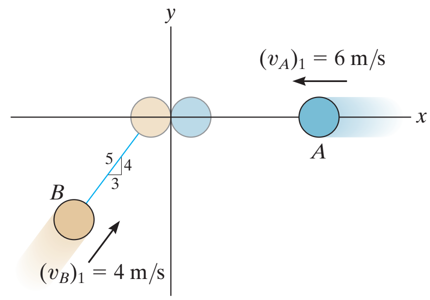
$$(v_B)_{2y} = 0$$

$$(v_A)_2 = \sqrt{(0.6928)^2 + (4)^2} = 4.06 \text{ ft/s} \qquad \textbf{Ans.}$$

$$(v_B)_2 = \sqrt{(0)^2 + (6.235)^2} = 6.235 = 6.24 \text{ ft/s} \qquad \textbf{Ans.}$$

15–87.

Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is $e = 0.75$.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = 0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$

$$0.75 = \frac{(v_A)_{2x} - (v_B)_{2x}}{4\left(\frac{3}{5}\right) - (-6)}$$

$$(v_A)_{2x} = 1.35 \text{ m/s} \rightarrow$$

$$(v_B)_{2x} = 4.95 \text{ m/s} \leftarrow$$

$(+\uparrow)$ $\quad m v_1 = m v_2$

$$0.5\left(\frac{4}{5}\right)(4) = 0.5(v_B)_{2y}$$

$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

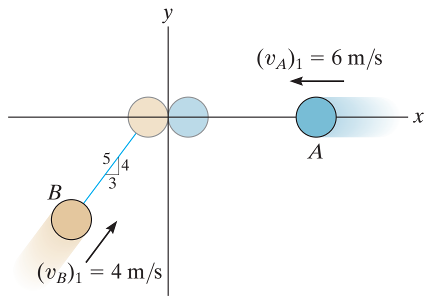
$$v_A = 1.35 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$

$$v_B = \sqrt{(4.59)^2 + (3.20)^2} = 5.89 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{3.20}{4.95} = 32.9^\circ \qquad \textbf{Ans.}$$

***15–88.**

Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision B travels along a line, 30° counterclockwise from the y axis.



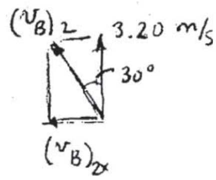
SOLUTION

$\Sigma m v_1 = \Sigma m v_2$

$$(\overset{+}{\rightarrow}) \qquad 0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = -0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$$-3.60 = -(v_B)_{2x} + (v_A)_{2x}$$

$$(+\uparrow) \qquad 0.5(4)\left(\frac{4}{5}\right) = 0.5(v_B)_{2y}$$



$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

$$(v_B)_{2x} = 3.20 \tan 30° = 1.8475 \text{ m/s} \leftarrow$$

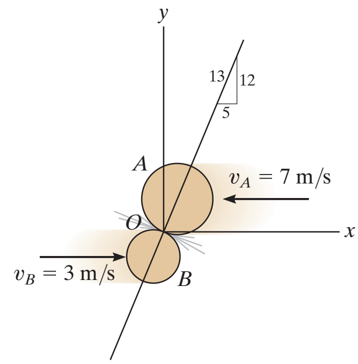
$$(v_A)_{2x} = -1.752 \text{ m/s} = 1.752 \text{ m/s} \leftarrow$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$e = \frac{-1.752-(-1.8475)}{4(\frac{3}{5})-(-6)} = 0.0113$$ **Ans.**

15–89.

Two smooth disks A and B have the initial velocities shown just before they collide at O. If they have masses $m_A = 8$ kg and $m_B = 6$ kg, determine their speeds just after impact. The coefficient of restitution is $e = 0.5$.



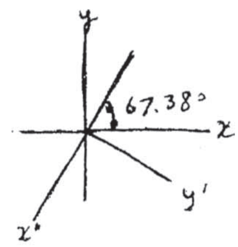
SOLUTION

$+\swarrow \Sigma m v_1 = \Sigma m v_2$

$$-6(3 \cos 67.38°) + 8(7 \cos 67.38°) = 6(v_B)_{x'} + 8(v_A)_{x'}$$

$$(+\swarrow) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.5 = \frac{(v_B)_{x'} - (v_A)_{x'}}{7 \cos 67.38° + 3 \cos 67.38°}$$



Solving,

$$(v_B)_{x'} = 2.14 \text{ m/s}$$

$$(v_A)_{x'} = 0.220 \text{ m/s}$$

$$(v_B)_{y'} = 3 \sin 67.38° = 2.769 \text{ m/s}$$

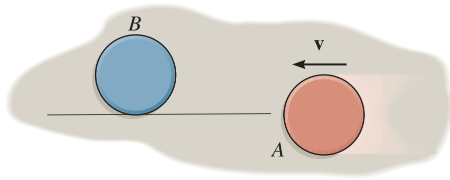
$$(v_A)_{y'} = -7 \sin 67.38° = -6.462 \text{ m/s}$$

$$v_B = \sqrt{(2.14)^2 + (2.769)^2} = 3.50 \text{ m/s} \qquad \textbf{Ans.}$$

$$v_A = \sqrt{(0.220)^2 + (6.462)^2} = 6.47 \text{ m/s} \qquad \textbf{Ans.}$$

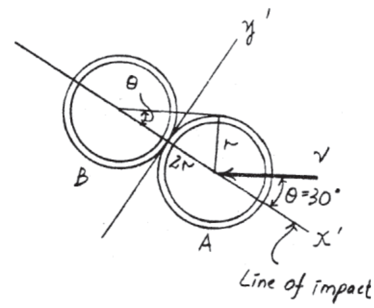
15–90.

If disk A is sliding along the tangent to disk B and strikes B with a velocity $\mathbf{v}$, determine the velocity of B after the collision and compute the loss of kinetic energy during the collision. Neglect friction. Disk B is originally at rest. The coefficient of restitution is e, and each disk has the same size and mass m.



SOLUTION

Impact: This problem involves *oblique impact* where the *line of impact* lies along x' axis (line jointing the mass center of the two impact bodies). From the geometry $\theta = \sin^{-1}\left(\dfrac{r}{2r}\right) = 30°$. The x' and y' components of velocity for disk A just before impact are



$$\left(v_{A_{x'}}\right)_1 = -v\cos 30° = -0.8660v \qquad \left(v_{A_{y'}}\right)_1 = -v\sin 30° = -0.5\,v$$

Conservation of "x′" Momentum:

$$m_A\left(v_{A_{x'}}\right)_1 + m_B\left(v_{B_{x'}}\right)_1 = m_A\left(v_{A_{x'}}\right)_2 + m_B\left(v_{B_{x'}}\right)_2$$

$$(\searrow +) \qquad m(-0.8660v) + 0 = m\left(v_{A_x}\right)_2 + m\left(v_{B_{x'}}\right)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution (x′):

$$e = \frac{\left(v_{B_{x'}}\right)_2 - \left(v_{A_{x'}}\right)_2}{\left(v_{A_{x'}}\right)_1 - \left(v_{B_{x'}}\right)_1}$$

$$e = \frac{\left(v_{B_{x'}}\right)_2 - \left(v_{A_{x'}}\right)_2}{-0.8660v - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\left(v_{B_{x'}}\right)_2 = -\frac{\sqrt{3}}{4}(1+e)\,v \qquad \left(v_{A_{x'}}\right)_2 = \frac{\sqrt{3}}{4}(e-1)\,v$$

Conservation of "y′" Momentum: The momentum is conserved along y' axis for both disks A and B.

$$(+\nearrow) \qquad m_B\left(v_{B_{y'}}\right)_1 = m_B\left(v_{B_{y'}}\right)_2; \qquad \left(v_{B_{y'}}\right)_2 = 0$$

$$(+\nearrow) \qquad m_A\left(v_{A_{y'}}\right)_1 = m_A\left(v_{A_{y'}}\right)_2; \qquad \left(v_{A_{y'}}\right)_2 = -0.5\,v$$

Thus, the magnitude the velocity for disk B just after the impact is

$$(v_B)_2 = \sqrt{(v_{B_{x'}})_2^2 + (v_{B_{y'}})_2^2}$$

$$= \sqrt{\left(-\frac{\sqrt{3}}{4}(1+e)\,v\right)^2 + 0} = \frac{\sqrt{3}}{4}(1+e)\,v \qquad \textbf{Ans.}$$

and directed toward **negative x' axis**. **Ans.**

15–90. continued

The magnitude of the velocity for disk A just after the impact is

$$(v_A)_2 = \sqrt{(v_{A_x})_2^2 + (v_{A_y})_2^2}$$

$$= \sqrt{\left[\frac{\sqrt{3}}{4}(e-1)\,v\right]^2 + (-0.5\,v)^2}$$

$$= \sqrt{\frac{v^2}{16}(3e^2 - 6e + 7)}$$

Loss of Kinetic Energy: Kinetic energy of the system before the impact is

$$U_k = \frac{1}{2}mv^2$$

Kinetic energy of the system after the impact is

$$U_k{}' = \frac{1}{2}m\left[\sqrt{\frac{v^2}{16}(3e^2 - 6e + 7)}\right]^2 + \frac{1}{2}m\left[\frac{\sqrt{3}}{4}(1+e)\,v\right]^2$$

$$= \frac{mv^2}{32}\left(6e^2 + 10\right)$$

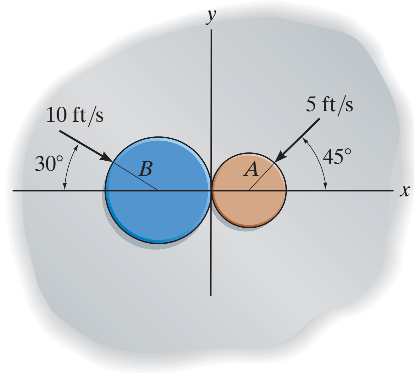
Thus, the kinetic energy loss is

$$\Delta U_k = U_k - U_k{}' = \frac{1}{2}mv^2 - \frac{mv^2}{32}(6e^2 + 10)$$

$$= \frac{3mv^2}{16}(1 - e^2) \qquad \textbf{Ans.}$$

15–91.

Two disks A and B weigh 2 lb and 5 lb, respectively. If they are sliding on the smooth horizontal plane with the velocities shown, determine their velocities just after impact. The coefficient of restitution between the disks is $e = 0.6$.



SOLUTION

Conservation of Linear Momentum: By referring to the impulse and momentum of the system of disks shown in Fig. *a*, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

$$-\frac{2}{32.2}(5)\cos 45° + \frac{5}{32.2}(10)\cos 30° = \frac{2}{32.2}v'_A\cos\theta_A + \frac{5}{32.2}v'_B\cos\theta_B$$

$$2v'_A\cos\theta_A + 5v'_B\cos\theta_B = 36.23 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum a of disks A and B are conserved along the t axis (tangent to the plane of impact). Thus,

$$(+\downarrow) \qquad m_A(v_A)_t = m_A(v'_A)_t$$

$$\frac{2}{32.2}(5)\sin 45° = \frac{2}{32.2}v'_A\sin\theta_A$$

$$v'_A\sin\theta_A = 3.5355 \qquad \textbf{(2)}$$

and

$$(+\downarrow) \qquad m_B(v_B)_t = m_B(v'_B)_t$$

$$\frac{2}{32.2}(10)\sin 30° = = \frac{2}{32.2}v'_B\sin\theta_B$$

$$v'_B\sin\theta_B = 5 \qquad \textbf{(3)}$$

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v'_A)_n - (v'_B)_n}{(v_B)_n - (v_A)_n}$$

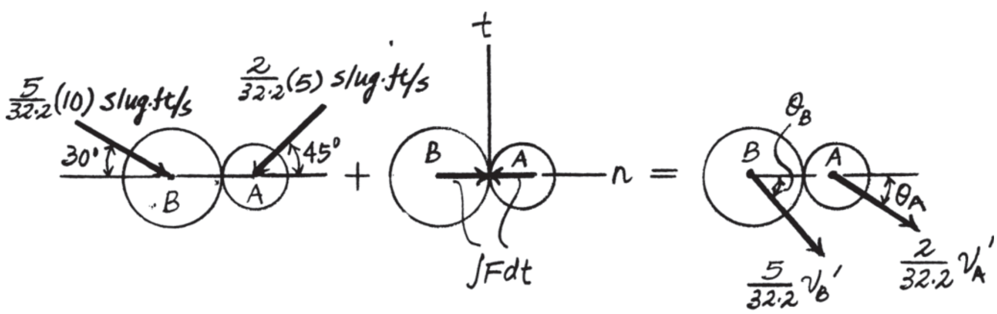
$$0.6 = \frac{v'_A\cos\theta_A - v'_B\cos\theta_B}{10\cos 30° - (-5\cos 45°)}$$

$$v'_A\cos\theta_A - v'_B\cos\theta_B = 7.317 \qquad \textbf{(4)}$$

Solving Eqs. (1), (2), (3), and (4), yields

$v'_A = 11.0$ ft/s $\qquad \theta_A = 18.8°$ **Ans.**

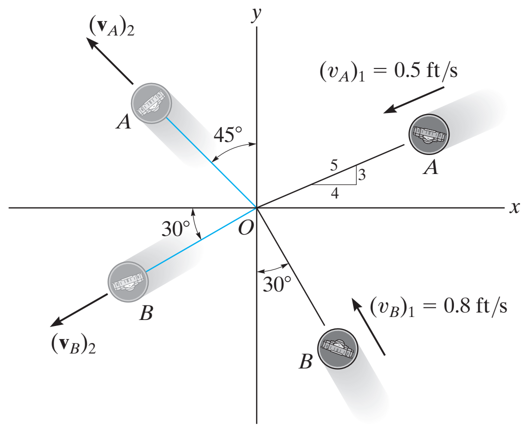
$v'_B = 5.88$ ft/s $\qquad \theta_B = 58.3°$ **Ans.**



(a)

***15–92.**

Two smooth coins A and B, each having the same mass, slide on a smooth surface with the motion shown. Determine the speed of each coin after collision if they move off along the blue paths. *Hint:* Since the line of impact has not been defined, apply the conservation of momentum along the x and y axes, respectively.



SOLUTION

$$\Sigma m v_1 = m v_2$$

$$(\xrightarrow{+}) \qquad -m(0.8)\sin 30° - m(0.5)\left(\frac{4}{5}\right) = -m(v_A)_2 \sin 45° - m(v_B)_2 \cos 30°$$

$$0.8 = 0.707(v_A)_2 + 0.866(v_B)_2$$

$$(+\uparrow) \qquad m(0.8)\cos 30° - m(0.5)\left(\frac{3}{5}\right) = m(v_A)_2 \cos 45° - m(v_B)_2 \sin 30°$$

$$-0.3928 = -0.707(v_A)_2 + 0.5(v_B)_2$$

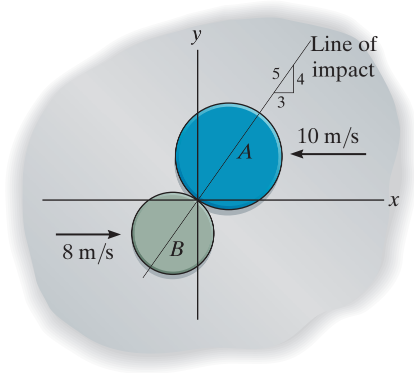
Solving,

$$(v_B)_2 = 0.298 \text{ ft/s} \qquad \textbf{Ans.}$$

$$(v_A)_2 = 0.766 \text{ ft/s} \qquad \textbf{Ans.}$$

15–93.

Disks A and B have a mass of 15 kg and 10 kg, respectively. If they are sliding on a smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.8$.



SOLUTION

Conservation of Linear Momentum: By referring to the impulse and momentum of the system of disks shown in Fig. *a*, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$+\nearrow \; m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v_A'\right)_n + m_B\left(v_B'\right)_n$$

$$15(10)\left(\frac{3}{5}\right) - 10(8)\left(\frac{3}{5}\right) = 15v_A' \cos\phi_A + 10v_B' \cos\phi_B$$

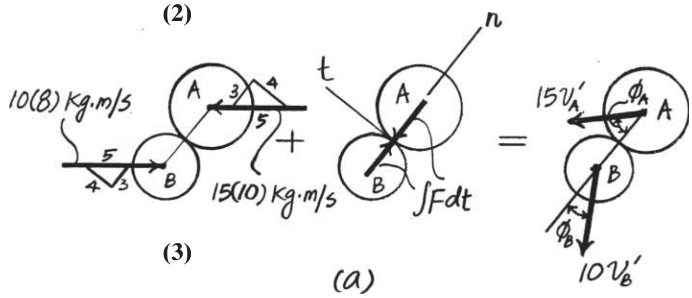
$$15v_A' \cos\phi_A + 10v_B' \cos\phi_B = 42 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis (tangent to? plane of impact). Thus,

$$+\nwarrow \; m_A\left(v_A\right)_t = m_A\left(v_A'\right)_t$$

$$15(10)\left(\frac{4}{5}\right) = 15v_A' \sin\phi_A$$

$$v_A' \sin\phi_A = 8 \qquad \textbf{(2)}$$



(a)

and

$$+\nwarrow \; m_B\left(v_B\right)_t = m_B\left(v_B'\right)_t$$

$$10(8)\left(\frac{4}{5}\right) = 10\,v_B' \sin\phi_B$$

$$v_B' \sin\phi_B = 6.4 \qquad \textbf{(3)}$$

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$+\nearrow \; e = \frac{(v_B')_n - (v_A')_n}{(v_A)_n - (v_B)_n}$$

$$0.8 = \frac{v_B' \cos\phi_B - v_A' \cos\phi_A}{10\left(\frac{3}{5}\right) - \left[-8\left(\frac{3}{5}\right)\right]}$$

$$v_B' \cos\phi_B - v_A' \cos\phi_A = 8.64 \qquad \textbf{(4)}$$

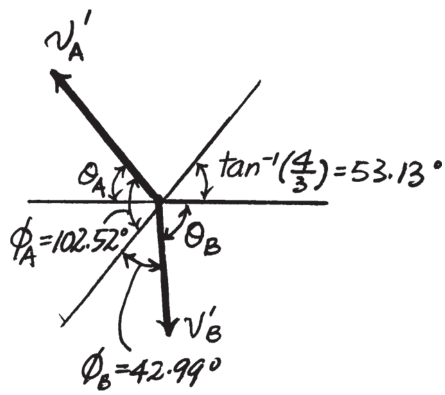
Solving Eqs. (1), (2), (3), and (4), yeilds

$$v_A' = 8.19 \text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_A = 102.52°$$

$$v_B' = 9.38 \text{ m/s} \qquad \textbf{Ans.}$$

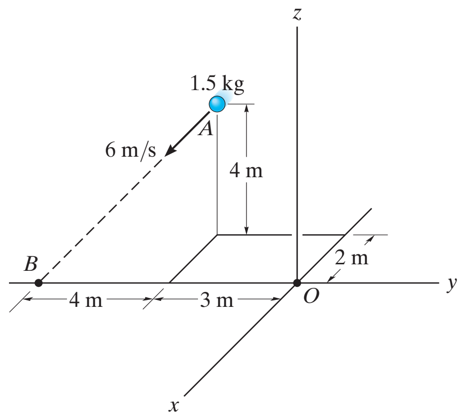
$$\phi_B = 42.99°$$



(b)

15–94.

Determine the angular momentum $\mathbf{H}_O$ of the particle about point O.



SOLUTION

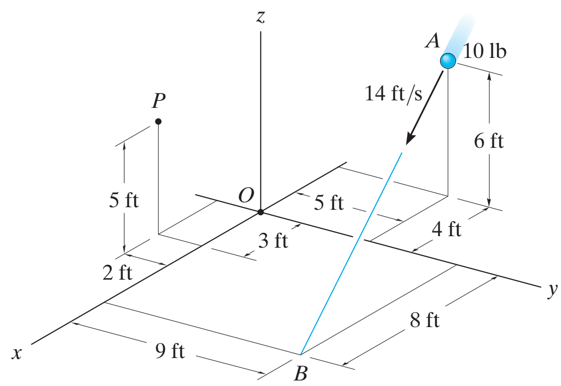
$\mathbf{r}_{OB} = \{-7\mathbf{j}\}\text{ m} \qquad \mathbf{v}_A = 6\left(\dfrac{2\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}}{\sqrt{(2^2 + (-4)^2 + (-4)^2}}\right) = \{2\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}\}\text{ m/s}$

$\mathbf{H}_O = \mathbf{r}_{OB} \times m\mathbf{v}_A$

$$\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -7 & 0 \\ 1.5(2) & 1.5(-4) & 1.5(-4) \end{vmatrix} = \{42\mathbf{i} + 21\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

15–95.

Determine the angular momentum $\mathbf{H}_O$ of the particle about point O.



SOLUTION

$\mathrm{r}_{OB} = \{8\mathbf{i} + 9\mathbf{j}\}\text{ft} \qquad \mathbf{v}_A = 14\left(\dfrac{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}}{\sqrt{12^2 + 4^2 + (-6)^2}}\right) = \{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}\}\ \text{ft/s}$

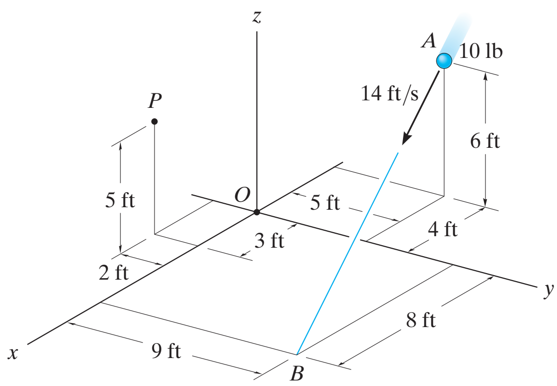
$\mathrm{H}_O = \mathrm{r}_{OB} \times m v_A$

$$\mathrm{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 9 & 0 \\ \left(\dfrac{10}{32.2}\right)(12) & \left(\dfrac{10}{32.2}\right)(4) & \left(\dfrac{10}{32.2}\right)(-6) \end{vmatrix}$$

$$= \{-16.8\mathbf{i} + 14.9\mathbf{j} - 23.6\mathbf{k}\}\ \text{slug} \quad \text{ft}^2/\text{s}$$ **Ans.**

***15–96.**

Determine the angular momentum $\mathbf{H}_P$ of the particle about point P.



SOLUTION

$$\mathbf{r}_{PB} = \{5\mathbf{i} + 11\mathbf{j} - 5\mathbf{k}\}\text{ ft}$$

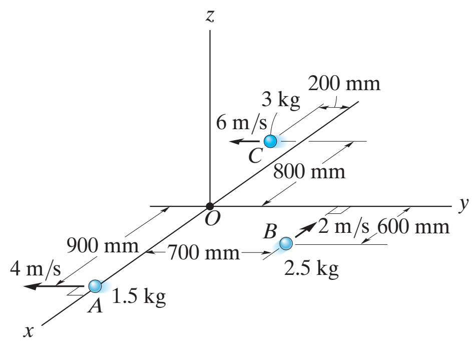
$$\mathbf{v}_A = 14\left(\frac{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}}{\sqrt{12)^2 + (4)^2 + (-6)^2}}\right) = \{12\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}\}\text{ ft/s}$$

$$\mathrm{H}_P = \mathbf{r}_{PB} \times m\mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 11 & -5 \\ \left(\frac{10}{32.2}\right)(12) & \left(\frac{10}{32.2}\right)(4) & \left(\frac{10}{32.2}\right)(-6) \end{vmatrix}$$

$$= \{-14.3\mathbf{i} - 9.32\mathbf{j} - 34.8\mathbf{k}\}\text{ slug}\cdot\text{ft}^2/\text{s} \qquad \textbf{Ans.}$$

15–97.

Determine the total angular momentum $\mathbf{H}_O$ for the system of three particles about point O. All the particles are moving in the x–y plane.



SOLUTION

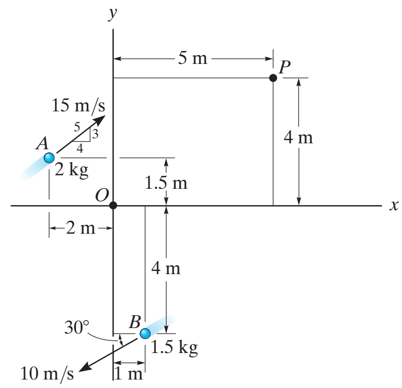
$$\mathrm{H}_O = \Sigma \mathbf{r} \times mv$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 0 \\ 0 & -1.5(4) & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0.7 & 0 \\ -2.5(2) & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.8 & -0.2 & 0 \\ 0 & 3(-6) & 0 \end{vmatrix}$$

$$= \{12.5\mathbf{k}\}\ \mathrm{kg \cdot m^2/s}$$ **Ans.**

15–98.

Determine the angular momentum $\mathbf{H}_O$ of each of the two particles about point O. Use a scalar solution.



SOLUTION

$$\circlearrowleft + (H_A)_O = -2(15)\left(\frac{4}{5}\right)(1.5) - 2(15)\left(\frac{3}{5}\right)(2)$$

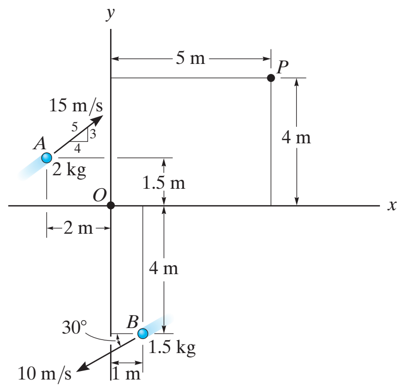
$$= -72.0 \text{ kg}\cdot\text{m}^2/\text{s} = 72.0 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \qquad \textbf{Ans.}$$

$$\circlearrowleft + (H_B)_O = -1.5(10)(\cos 30°)(4) - 1.5(10)(\sin 30°)(1)$$

$$= -59.5 \text{ kg}\cdot\text{m}^2/\text{s} = 59.5 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \qquad \textbf{Ans.}$$

15–99.

Determine the angular momentum $\mathbf{H}_P$ of each of the two particles about point P. Use a scalar solution.



SOLUTION

$$\circlearrowleft +(H_A)_P = 2(15)\left(\frac{4}{5}\right)(2.5) - 2(15)\left(\frac{3}{5}\right)(7)$$

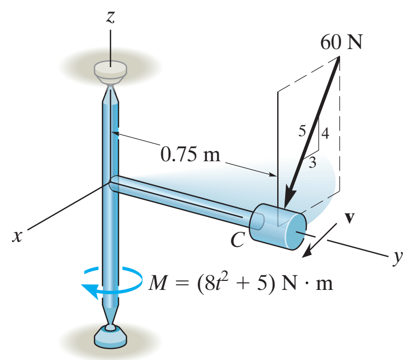
$$= -66.0 \text{ kg}\cdot\text{m}^2/\text{s} = 66.0 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \qquad \textbf{Ans.}$$

$$\circlearrowleft +(H_B)_P = -1.5(10)(\cos 30^\circ)(8) + 1.5(10)(\sin 30^\circ)(4)$$

$$= -73.9 \text{ kg}\cdot\text{m}^2/\text{s} \circlearrowright \qquad \textbf{Ans.}$$

***15–100.**

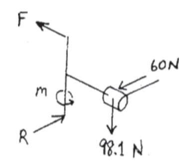
The small cylinder C has a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the frame is subjected to a couple $M = (8t^2 + 5)\ \text{N}\cdot\text{m}$, where t is in seconds, and the cylinder is subjected to a force of 60 N, which is always directed as shown, determine the speed of the cylinder when $t = 2$ s. The cylinder has a speed $v_0 = 2$ m/s when $t = 0$.



SOLUTION

$$(H_z)_1 + \Sigma\int M_z\,dt = (H_z)_2$$

$$(10)(2)(0.75) + 60(2)\left(\frac{3}{5}\right)(0.75) + \int_0^2 (8t^2 + 5)dt = 10v(0.75)$$

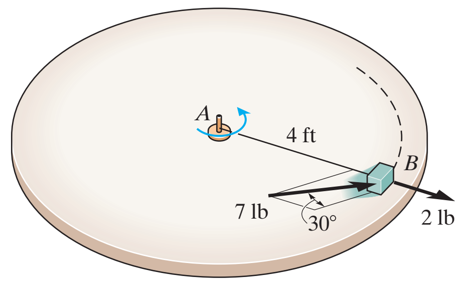


$$69 + \left[\frac{8}{3}t^3 + 5t\right]_0^2 = 7.5v$$

$v = 13.4$ m/s **Ans.**

15–101.

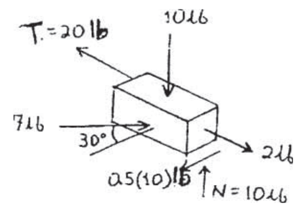
The 10-lb block rests on a surface for which $\mu_k = 0.5$. It is acted upon by a radial force of 2 lb and a horizontal force of 7 lb, always directed at 30° from the tangent to the path as shown. If the block is initially moving in a circular path with a speed $v_1 = 2$ ft/s at the instant the forces are applied, determine the time required before the tension in cord AB becomes 20 lb. Neglect the size of the block for the calculation.



SOLUTION

$\Sigma F_n = ma_n;$

$$20 - 7\sin 30^\circ - 2 = \frac{10}{32.2}\left(\frac{v^2}{4}\right)$$



$v = 13.67$ ft/s

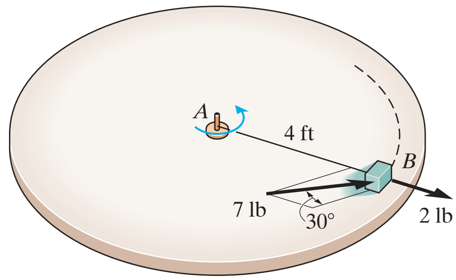
$$(H_A)_t + \Sigma\int M_A\,dt = (H_A)_2$$

$$\left(\frac{10}{32.2}\right)(2)(4) + (7\cos 30^\circ)(4)(t) - 0.5(10)(4)\,t = \frac{10}{32.2}(13.67)(4)$$

$t = 3.41$ s **Ans.**

15–102.

The 10-lb block is originally at rest on the smooth surface. It is acted upon by a radial force of 2 lb and a horizontal force of 7 lb, always directed at 30° from the tangent to the path as shown. Determine the time required to break the cord, which requires a tension $T = 30$ lb. What is the speed of the block when this occurs? Neglect the size of the block for the calculation.



SOLUTION

$\Sigma F_n = ma_n;$

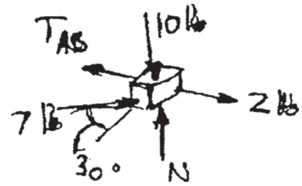
$$30 - 7\sin 30° - 2 = \frac{10}{32.2}\left(\frac{v^2}{4}\right)$$

$v = 17.764$ ft/s

$$(H_A)_1 + \Sigma \int M_A\,dt = (H_A)_2$$

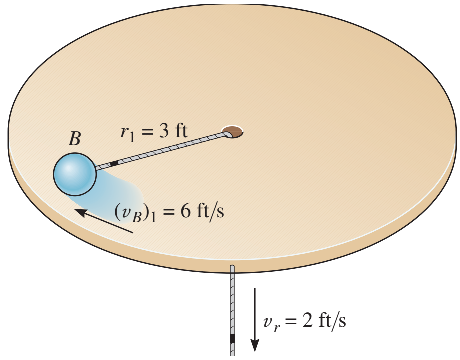
$$0 + (7\cos 30°)(4)(t) = \frac{10}{32.2}(17.764)(4)$$

$t = 0.910$ s **Ans.**



15–103.

A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine the ball's speed at the instant $r_2 = 2$ ft. How much work has to be done to pull down the cord? Neglect friction and the size of the ball.



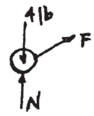
SOLUTION

$$H_1 = H_2$$

$$\frac{4}{32.2}(6)(3) = \frac{4}{32.2}v_\theta(2)$$

$$v_\theta = 9 \text{ ft/s}$$

$$v_2 = \sqrt{9^2 + 2^2} = 9.22 \text{ ft/s} \qquad \textbf{Ans.}$$



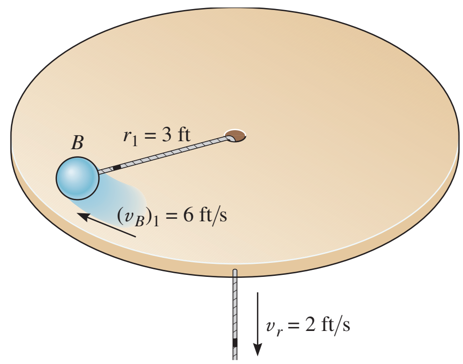
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4}{32.2}\right)(6)^2 + \Sigma U_{1-2} = \frac{1}{2}\left(\frac{4}{32.2}\right)(9.22)^2$$

$$\Sigma U_{1-2} = 3.04 \text{ ft} \cdot \text{lb} \qquad \textbf{Ans.}$$

***15–104.**

A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine how much time is required for the ball to reach a speed of 12 ft/s. How far r_2 is the ball from the hole when this occurs? Neglect friction and the size of the ball.



SOLUTION

$$v = \sqrt{(v_\theta)^2 + (2)^2}$$

$$12 = \sqrt{(v_\theta)^2 + (2)^2}$$

$$v_\theta = 11.832 \text{ ft/s}$$

$$H_1 = H_2$$

$$\frac{4}{32.2}(6)(3) = \frac{4}{32.2}(11.832)(r_2)$$

$$r_2 = 1.5213 = 1.52 \text{ ft} \qquad \textbf{Ans.}$$

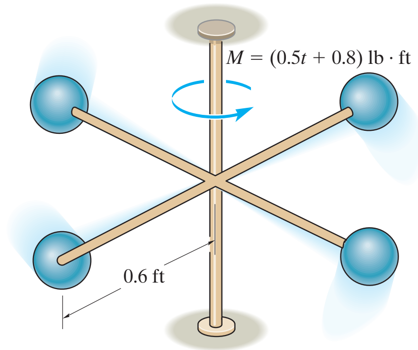
$$\Delta r = v_r t$$

$$(3 - 1.5213) = 2t$$

$$t = 0.739 \text{ s} \qquad \textbf{Ans.}$$

15–105.

The four 5-lb spheres are rigidly attached to the crossbar frame having a negligible weight. If a couple moment $M = (0.5t + 0.8)$ lb · ft, where t is in seconds, is applied asshown, determine the speed of each of the spheres in 4 seconds starting from rest. Neglect the size of the spheres.



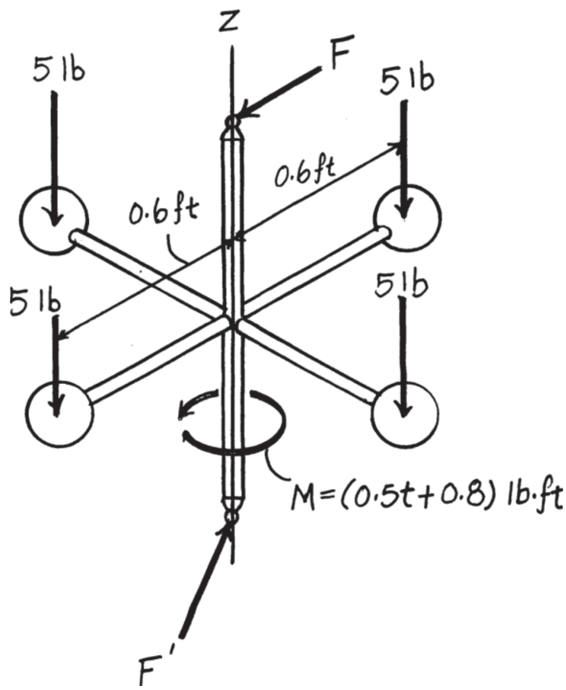
SOLUTION

$$(H_z)_1 + \Sigma \int M_z\, dt = (H_z)_2$$

$$0 + \int_0^4 (0.5\,t + 0.8)\;dt = 4\left[\left(\frac{5}{32.2}\right)(0.6\,v_2)\right]$$

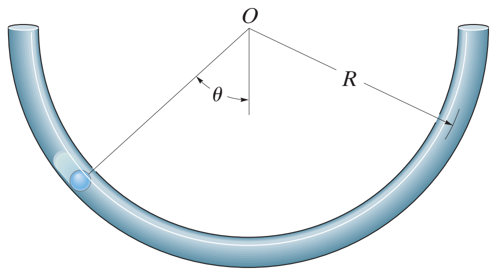
$$7.2 = 0.37267\,v_2$$

$$v_2 = 19.3\text{ ft/s}$$ **Ans.**



15–106.

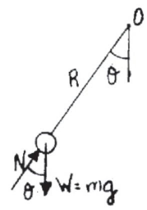
A small particle having a mass m is placed inside the semicircular tube. The particle is placed at the position shown and released. Apply the principle of angular momentum about point O $(\Sigma M_O = \dot{H}_O)$, and show that the motion of the particle is governed by the differential equation $\ddot{\theta} + (g/R)\sin\theta = 0$.



SOLUTION

$$\circlearrowleft + \Sigma M_O = \frac{dH_O}{dt}; \qquad -Rmg\sin\theta = \frac{d}{dt}(mvR)$$

$$g\sin\theta = -\frac{dv}{dt} = -\frac{d^2s}{dt^2}$$



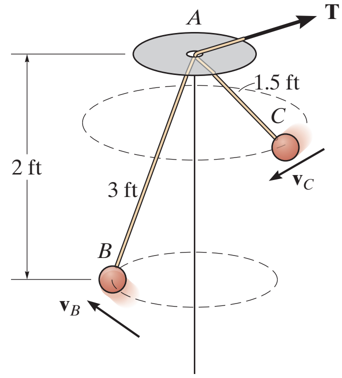
But, $s = R\theta$

Thus, $g\sin\theta = -R\ddot{\theta}$

or, $\ddot{\theta} + \left(\frac{g}{R}\right)\sin\theta = 0$ **Q.E.D.**

15–107.

The ball B has a weight of 5 lb and is originally rotating in a circle. As shown, the cord AB has a length of 3 ft and passes through the hole A, which is 2 ft above the plane of motion. If 1.5 ft of cord is pulled through the hole, determine the speed of the ball when it moves in a circular path at C.



SOLUTION

Equation of Motion: When the ball is travelling around the first circular path, $\theta = \sin^{-1}\frac{2}{3} = 41.81°$ and $r_1 = 3\cos 41.81° = 2.236$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad T_1\left(\frac{2}{3}\right) - 5 = 0 \qquad T_1 = 7.50 \text{ lb}$$

$$\Sigma F_n = ma_n; \qquad 7.50\cos 41.81° = \frac{5}{32.2}\left(\frac{v_t^2}{2.236}\right)$$

$$v_1 = 8.972 \text{ ft/s}$$

When the ball is traveling around *the* second circular path, $r_2 = 1.5\cos\phi$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad T_2 \sin\phi - 5 = 0 \qquad \textbf{(1)}$$

$$\Sigma F_n = ma_n; \qquad T_2\cos\phi = \frac{5}{32.2}\left(\frac{v_2^2}{1.5\cos\phi}\right) \qquad \textbf{(2)}$$

Conservation of Angular Momentum: Since no force acts on the ball along the tangent of the circular, path the angular momentum is conserved about z axis. Applying Eq. 15–23, we have

$$(\mathbf{H}_o)_1 = (\mathbf{H}_o)_2$$

$$r_1 m v_1 = r_2 m v_2$$

$$2.236\left(\frac{5}{32.2}\right)(8.972) = 1.5\cos\phi\left(\frac{5}{32.2}\right)v_2 \qquad \textbf{(3)}$$

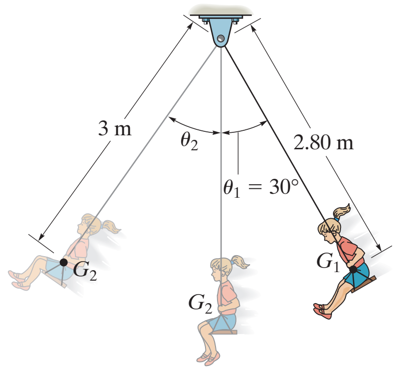
Solving Eqs. (1), (2) and (3) yields

$$\phi = 13.8678° \qquad T_2 = 20.85 \text{ lb}$$

$$v_2 = 13.8 \text{ ft/s} \qquad \textbf{Ans.}$$

***15–108.**

A child having a mass of 50 kg holds her legs up as shown as she swings downward from rest at $\theta_1 = 30^\circ$. Her center of mass is located at point G_1. When she is at the bottom position $\theta = 0^\circ$, she *suddenly* lets her legs come down, shifting her center of mass to position G_2. Determine her speed in the upswing due to this sudden movement and the angle θ_2 to which she swings before momentarily coming to rest. Treat the child's body as a particle.



SOLUTION

First before $\theta = 30^\circ$;

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2.80(1 - \cos 30^\circ)(50)(9.81) = \frac{1}{2}(50)(v_1)^2 + 0$$

$$v_1 = 2.713 \text{ m/s}$$

$$H_1 = H_2$$

$$50(2.713)(2.80) = 50(v_2)(3)$$

$$v_2 = 2.532 = 2.53 \text{ m/s}$$ **Ans.**

Just after $\theta = 0^\circ$;

$$T_2 + V_2 = T_3 + V_3$$

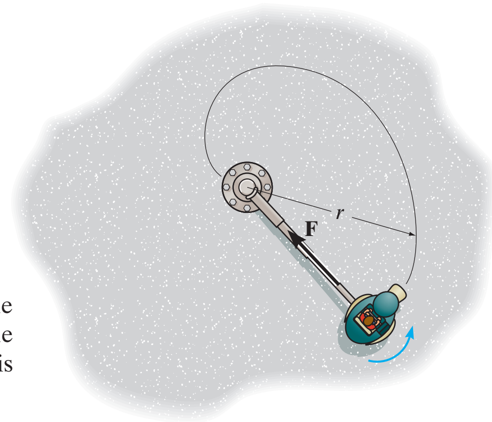
$$\frac{1}{2}(50)(2.532)^2 + 0 = 0 + 50(9.81)(3)(1 - \cos \theta_2)$$

$$0.1089 = 1 - \cos \theta_2$$

$$\theta_2 = 27.0^\circ$$ **Ans.**

15–109.

The 150-lb car of an amusement park ride is connected to a rotating telescopic boom. When $r = 15$ ft, the car is moving on a horizontal circular path with a speed of 30 ft/s. If the boom is shortened at a rate of 3 ft/s, determine the speed of the car when $r = 10$ ft. Also, find the work done by the axial force $\mathbf{F}$ along the boom. Neglect the size of the car and the mass of the boom.



SOLUTION

Conservation of Angular Momentum: By referring to Fig. *a*, we notice that the angular momentum of the car is conserved about an axis perpendicular to the page passing through point O, since no angular impulse acts on the car about this axis. Thus,

$$\left(H_O\right)_1 = \left(H_O\right)_2$$

$$r_1 m v_1 = r_2 m\left(v_2\right)_\theta$$

$$\left(v_2\right)_\theta = \frac{r_1 v_1}{r_2} = \frac{15(30)}{10} = 45 \text{ ft/s}$$

Thus, the magnitude of $\mathbf{v}_2$ is

$$v_2 = \sqrt{\left(v_2\right)_r{}^2 - \left(v_2\right)_\theta{}^2} = \sqrt{3^2 + 45^2} = 45.10 \text{ ft/s} = 45.1 \text{ ft/s} \qquad \textbf{Ans.}$$

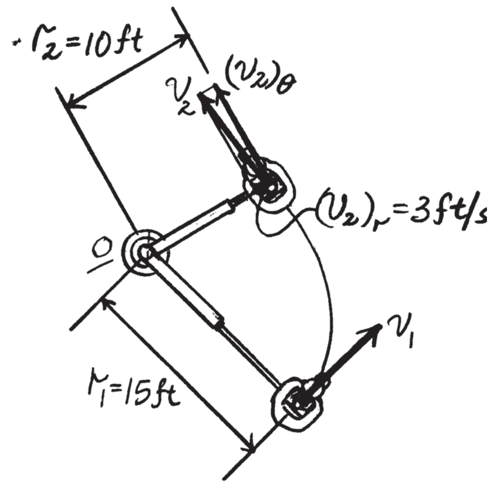
Principle of Work and Energy: Using the result of v_2,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2} m v_1{}^2 + U_F = \frac{1}{2} m v_2{}^2$$

$$\frac{1}{2}\left(\frac{150}{32.2}\right)(30^2) + U_F = \frac{1}{2}\left(\frac{150}{32.2}\right)(45.10^2)$$

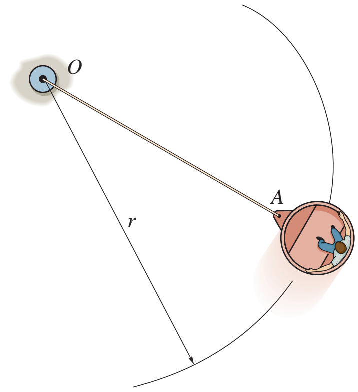
$$U_F = 2641 \text{ ft} \cdot \text{lb} \qquad \textbf{Ans.}$$



(a)

15–110.

An amusement park ride consists of a car which is attached to the cable OA. The car rotates in a horizontal circular path and is brought to a speed $v_1 = 4$ ft/s when $r = 12$ ft. The cable is then pulled in at the constant rate of 0.5 ft/s. Determine the speed of the car in 3 s.



SOLUTION

Conservation of Angular Momentum: Cable OA is shorten by $\Delta r = 0.5(3) = 1.50$ ft. Thus, at this instant $r_2 = 12 - 1.50 = 10.5$ ft. Since no force acts on the car along the tangent of the moving path, the angular momentum is conserved about point O. Applying Eq. 15–23, we have

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

$$r_1\, m v_1 = r_2\, m v'$$

$$12(m)(4) = 10.5(m)\, v'$$

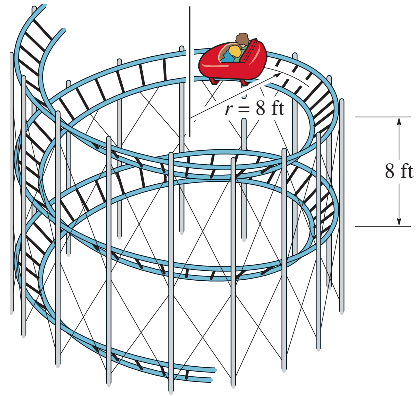
$$v' = 4.571 \text{ ft/s}$$

The speed of car after 3 s is

$$v_2 = \sqrt{0.5^2 + 4.571^2} = 4.60 \text{ ft/s}$$ **Ans.**

15–111.

The 800-lb roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 8 ft for every one revolution, determine the speed of the car in $t = 4$ s. Also, how far has the car descended in this time? Neglect friction and the size of the car.



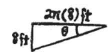
SOLUTION

$$\theta = \tan^{-1}\left(\frac{8}{2\pi(8)}\right) = 9.043^\circ$$

$\Sigma F_y = 0; \qquad N - 800 \cos 9.043^\circ = 0$

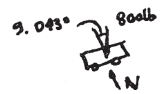
$$N = 790.1 \text{ lb}$$

$$H_1 + \int M\, dt = H_2$$



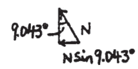
$$0 + \int_0^4 8(790.1 \sin 9.043^\circ)\, dt = \frac{800}{32.2}(8)v_t$$

$$v_t = 20.0 \text{ ft/s}$$



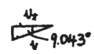
$$v = \frac{20}{\cos 9.043^\circ} = 20.2 \text{ ft/s}$$ **Ans.**

$$T_1 + \Sigma U_{1-2} = T_2$$



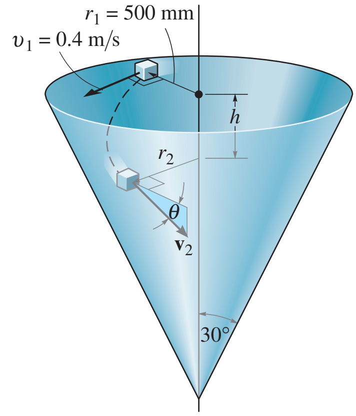
$$0 + 800h = \frac{1}{2}\left(\frac{800}{32.2}\right)(20.2)^2$$

$$h = 6.36 \text{ ft}$$ **Ans.**



***15–112.**

A small block having a mass of 0.1 kg is given a horizontal velocity of $v_1 = 0.4$ m/s when $r_1 = 500$ mm. It slides along the smooth conical surface. Determine the distance h it must descend for it to reach a speed of $v_2 = 2$ m/s. Also, what is the angle of descent θ, that is, the angle measured from the horizontal to the tangent of the path?



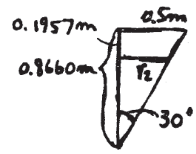
SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(0.1)(0.4)^2 + 0.1(9.81)(h) = \frac{1}{2}(0.1)(2)^2$$

$$h = 0.1957 \text{ m} = 196 \text{ mm}$$ **Ans.**

From similar triangles



$$r_2 = \frac{(0.8660 - 0.1957)}{0.8660}(0.5) = 0.3870 \text{ m}$$

$$(H_0)_1 = (H_0)_2$$

$$0.5(0.1)(0.4) = 0.3870(0.1)(v_2')$$

$$v_2' = 0.5168 \text{ m/s}$$

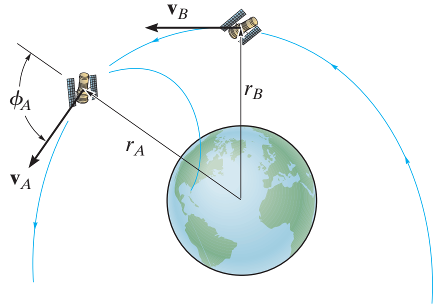
$$v_2 = \cos\theta = v_2'$$

$$2\cos\theta = 0.5168$$

$$\theta = 75.0°$$ **Ans.**

15–113.

An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10\text{ km/s}$ when the distance from the center of the earth is $r_A = 15$ Mm. If the launch angle at this position is $\phi_A = 70°$, determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy.



SOLUTION

$$(H_O)_1 = (H_O)_2$$

$$m_s\,(v_A \sin\phi_A)r_A = m_s\,(v_B)r_B$$

$$700[10(10^3)\sin 70°](15)(10^6) = 700(v_B)(r_B) \qquad \textbf{(1)}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m_s\,(v_A)^2 - \frac{GM_e m_s}{r_A} = \frac{1}{2}m_s\,(v_B)^2 - \frac{GM_e m_s}{r_B}$$

$$\frac{1}{2}(700)[10(10^3)]^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} = \frac{1}{2}(700)(v_B)^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B} \qquad \textbf{(2)}$$

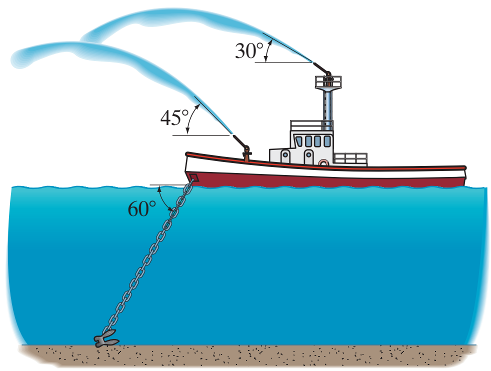
Solving,

$v_B = 10.2\text{ km/s}$ **Ans.**

$r_B = 13.8\text{ Mm}$ **Ans.**

15–114.

The fire boat discharges two streams of seawater, each at a flow of $0.25\ \text{m}^3/\text{s}$ and with a nozzle velocity of 50 m/s. Determine the tension developed in the anchor chain, needed to secure the boat. The density of seawater is $\rho_{sw} = 1020\ \text{kg/m}^3$.



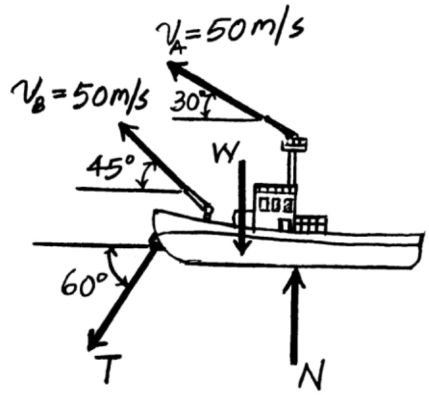
SOLUTION

Steady Flow Equation: Here, the mass flow rate of the sea water at nozzles A and B are $\dfrac{dm_A}{dt} = \dfrac{dm_B}{dt} = \rho sw\, Q = 1020(0.25) = 225\ \text{kg/s}$. Since the sea water is collected from the larger reservoir (the sea), the velocity of the sea water entering the control volume can be considered zero. By referring to the free-body diagram of the control volume (the boat),

$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm_A}{dt}\left(v_A\right)_x + \frac{dm_B}{dt}\left(v_B\right)_x;$$

$$T \cos 60° = 225(50 \cos 30°) + 225(50 \cos 45°)$$

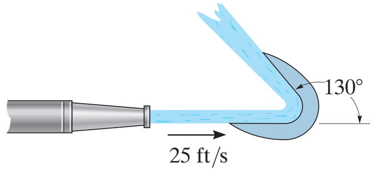
$$T = 40\,114.87\ \text{N} = 40.1\ \text{kN}$$ **Ans.**



(a)

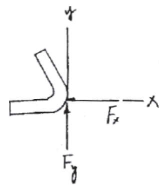
15–115.

A jet of water having a cross-sectional area of 4 in^2 strikes the fixed blade with a speed of 25 ft/s. Determine the horizontal and vertical components of force which the blade exerts on the water, $\gamma_w = 62.4 \text{ lb/ft}^3$.



SOLUTION

$$Q = Av = \left(\frac{4}{144}\right)(25) = 0.6944 \text{ ft}^3\text{/s}$$



$$\frac{dm}{dt} = \rho Q = \left(\frac{62.4}{32.2}\right)(0.6944) = 1.3458 \text{ slug/s}$$

$v_{Ax} = 25 \text{ ft/s} \qquad v_{Ay} = 0$

$v_{Bx} = -25 \cos 50° \text{ft/s} \qquad v_{By} = 25 \sin 50° \text{ft/s}$

$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_A); \qquad -F_x = 1.3458[-25 \cos 50° - 25]$$

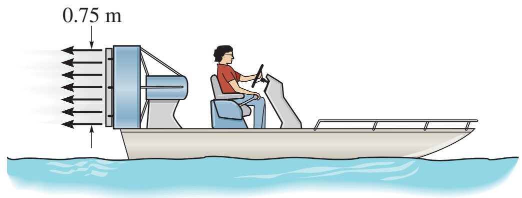
$$F_x = 55.3 \text{ lb}$$ **Ans.**

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{\text{By}} - v_{\text{Ay}}); \qquad F_y = 1.3458(25 \sin 50° - 0)$$

$$F_y = 25.8 \text{ lb}$$ **Ans.**

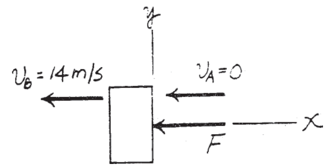
***15–116.**

The 200-kg boat is powered by the fan which develops a slipstream having a diameter of 0.75 m. If the fan ejects air with a speed of 14 m/s, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$ and that the entering air is essentially at rest. Neglect the drag resistance of the water.



SOLUTION

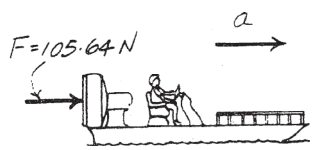
Equations of Steady Flow: Initially, the boat is at rest hence $v_B = v_{a/b} = 14\ \text{m/s}$. Then, $Q = v_B A = 14\left[\frac{\pi}{4}\left(0.75^2\right)\right] = 6.185\ \text{m}^3/\text{s}$ and $\frac{dm}{dt} = \rho_a Q = 1.22(6.185) = 7.546\ \text{kg/s}$. Applying Eq. 15–26, we have



$$\Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x}); \qquad -F = 7.546(-14 - 0) \qquad F = 105.64\ \text{N}$$

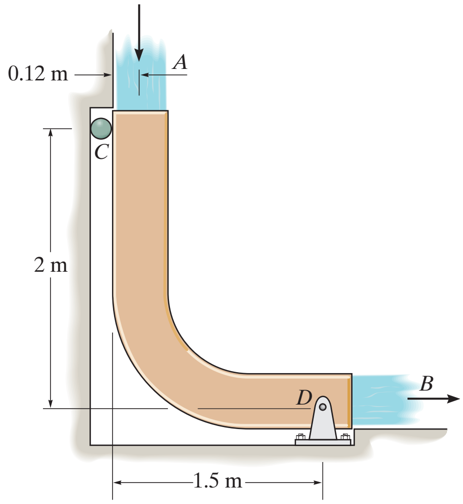
Equation of Motion :

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 105.64 = 200a \qquad a = 0.528\ \text{m/s}^2 \qquad \textbf{Ans.}$$



15–117.

The chute is used to divert the flow of water, $Q = 0.6\ \mathrm{m^3/s}$. If the water has a cross-sectional area of $0.05\ \mathrm{m^2}$, determine the force components at the pin D and roller C necessary for equilibrium. Neglect the weight of the chute and weight of the water on the chute, $\rho_w = 1\ \mathrm{Mg/m^3}$.



SOLUTION

Equations of Steady Flow: Here, the flow rate $Q = 0.6\ \mathrm{m^2/s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.6}{0.05} = 12.0\ \mathrm{m/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = 1000\,(0.6) = 600\ \mathrm{kg/s}$. Applying Eqs. 15–26 and 15–28, we have

$$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}\left(d_{DB}\, v_B - d_{DA}\, v_A\right);$$

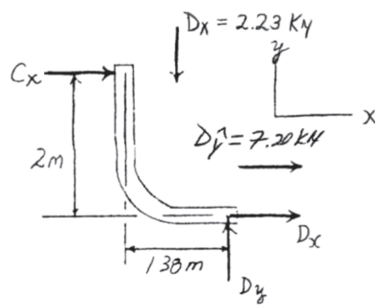
$$-C_x\,(2) = 600\,[0 - 1.38(12.0)] \qquad C_x = 4968\ \mathrm{N} = 4.97\ \mathrm{kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}\left(v_{B_x} - v_{A_x}\right);$$

$$D_x + 4968 = 600\,(12.0 - 0) \qquad D_x = 2232\mathrm{N} = 2.23\ \mathrm{kN} \qquad \textbf{Ans.}$$

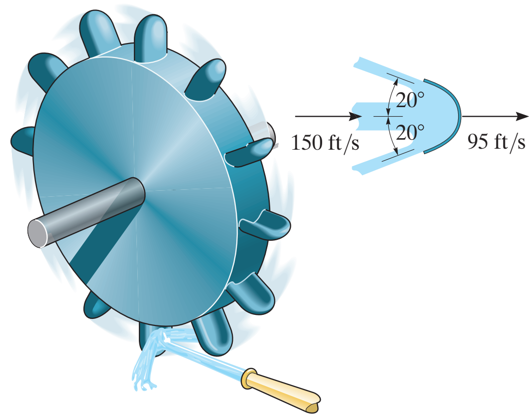
$$+\uparrow \Sigma F_y = \Sigma \frac{dm}{dt}\left(v_{\mathrm{out}_y} - v_{\mathrm{in}_y}\right);$$

$$D_y = 600[0 - (-12.0)] \qquad D_y = 7200\ \mathrm{N} = 7.20\ \mathrm{kN} \qquad \textbf{Ans.}$$



15–118.

The buckets on the *Pelton wheel* are subjected to a 2-in-diameter jet of water, which has a velocity of 150 ft/s. If each bucket is traveling at 95 ft/s when the water strikes it, determine the power developed by the bucket. $\gamma_w = 62.4\ \text{lb/ft}^3$.

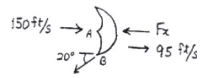


SOLUTION

$v_A = 150 - 95 = 55\ \text{ft/s} \rightarrow$

$(\overset{+}{\rightarrow})(v_B)_x = -55\cos 20° + 95 = 43.317\ \text{ft/s} \rightarrow$

$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$



$$F_x = \left(\frac{62.4}{32.2}\right)(\pi)\left(\frac{1}{12}\right)^2[(55)(55\cos 20° - (-55)]$$

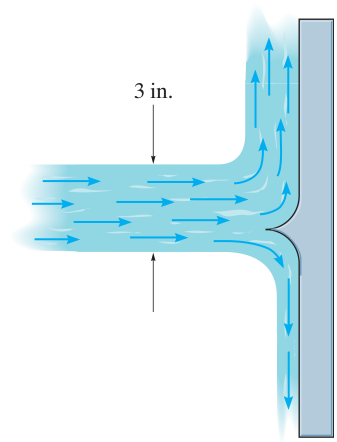
$F_x = 248.07\ \text{lb}$

$P = 248.07(95) = 23{,}567\ \text{ft} \cdot \text{lb/s}$

$P = 42.8\ \text{hp}$ **Ans.**

15–119.

The blade divides the jet of water having a diameter of 3 in. If one-fourth of the water flows downward while the other three-fourths flows upwards, and the total flow is $Q = 0.5\ \text{ft}^3/\text{s}$, determine the horizontal and vertical components of force exerted on the blade by the jet, $\gamma_w = 62.4\ \text{lb/ft}^3$.



SOLUTION

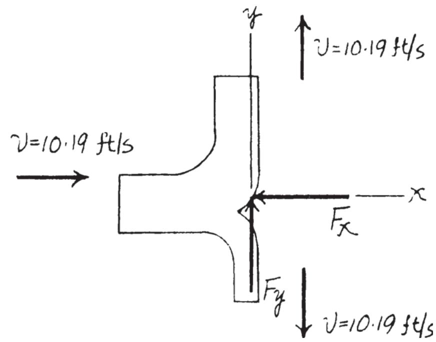
Equations of Steady Flow: Here, the flow rate $Q = 0.5\ \text{ft}^2/\text{s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.5}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2} = 10.19\ \text{ft/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = \dfrac{62.4}{32.2}(0.5) = 0.9689\ \text{slug/s}$.

Applying Eq. 15–25 we have

$$\Sigma F_x = \Sigma \frac{dm}{dt}\left(v_{\text{out}_s} - v_{\text{in}_s}\right); \quad -F_x = 0 - 0.9689\,(10.19) \qquad F_x = 9.87\ \text{lb} \qquad \textbf{Ans.}$$

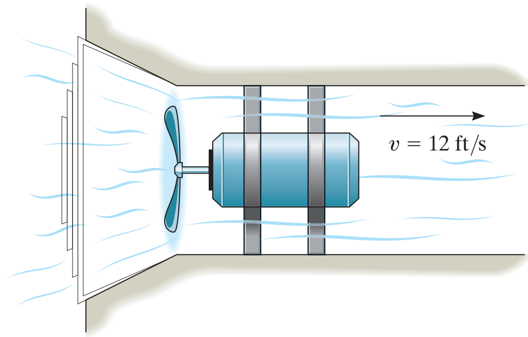
$$\Sigma F_y = \Sigma \frac{dm}{dt}\left(v_{\text{out}_y} - v_{\text{in}_y}\right); \quad F_y = \frac{3}{4}(0.9689)(10.19) + \frac{1}{4}(0.9689)(-10.19)$$

$$F_y = 4.93\ \text{lb} \qquad \textbf{Ans.}$$



***15–120.**

The fan draws air through a vent with a speed of 12 ft/s. If the cross-sectional area of the vent is 2 ft^2, determine the horizontal thrust on the blade. The specific weight of the air is $\gamma_a = 0.076\ lb/ft^3$.

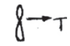


SOLUTION

$$\frac{dm}{dt} = \rho v A$$

$$= \frac{0.076}{32.2}(12)(2)$$

$$= 0.05665\ \text{slug/s}$$

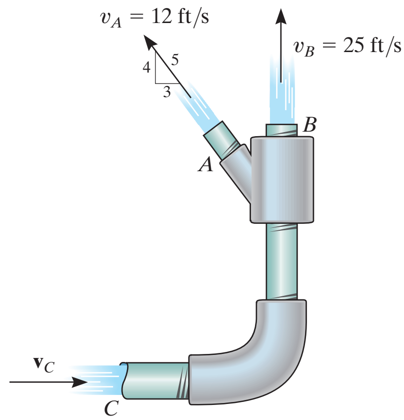


$$\Sigma F = \frac{dm}{dt}(v_B - v_A)$$

$$T = 0.05665(12 - 0) = 0.680\ \text{lb}$$ **Ans.**

15–121.

The gauge pressure of water at C is 40 lb/in^2. If water flows out of the pipe at A and B with velocities $v_A = 12$ ft/s and $v_B = 25$ ft/s, determine the horizontal and vertical components of force exerted on the elbow necessary tohold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 0.75 in. at C, and at A and B the diameter is 0.5 in. $\gamma_w = 62.4$ lb/ft^3.



SOLUTION

$$\frac{dm_A}{dt} = \frac{62.4}{32.2}(12)(\pi)\left(\frac{0.25}{12}\right)^2 = 0.03171 \text{ slug/s}$$

$$\frac{dm_B}{dt} = \frac{62.4}{32.2}(25)(\pi)\left(\frac{0.25}{12}\right)^2 = 0.06606 \text{ slug/s}$$

$$\frac{dm_C}{dt} = 0.03171 + 0.06606 = 0.09777 \text{ slug/s}$$

$$v_C A_C = v_A A_A + v_B A_B$$

$$v_C(\pi)\left(\frac{0.375}{12}\right)^2 = 12(\pi)\left(\frac{0.25}{12}\right)^2 + 25(\pi)\left(\frac{0.25}{12}\right)^2$$

$$v_C = 16.44 \text{ ft/s}$$

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm_B}{dt} v_{B_x} + \frac{dm_A}{dt} v_{A_x} - \frac{dm_C}{dt} v_{C_x}$$

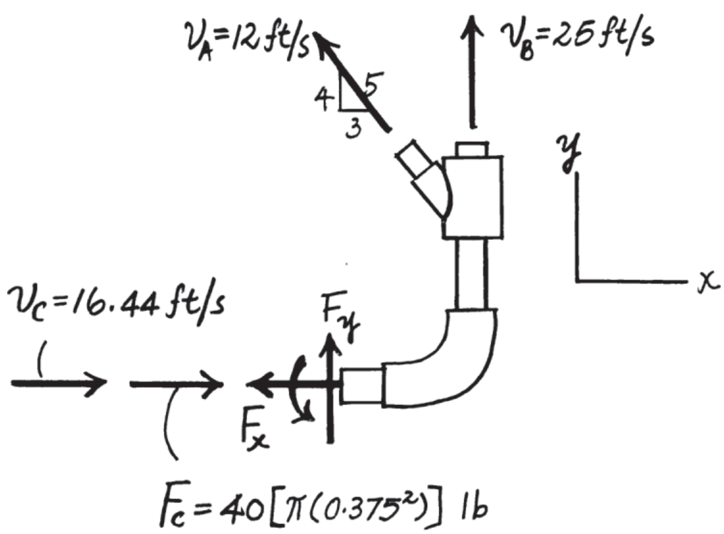
$$40(\pi)(0.375)^2 - F_x = 0 - 0.03171(12)\left(\frac{3}{5}\right) - 0.09777(16.44)$$

$F_x = 19.5$ lb **Ans.**

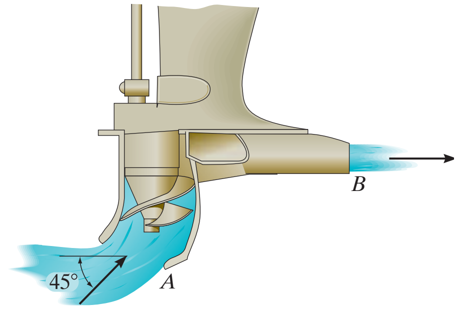
$$+\uparrow \Sigma F_y = \frac{dm_B}{dt} v_{B_y} + \frac{dm_A}{dt} v_{A_y} - \frac{dm_C}{dt} v_{C_y}$$

$$F_y = 0.06606(25) + 0.03171\left(\frac{4}{5}\right)(12) - 0$$

$F_y = 1.9559 = 1.96$ lb **Ans.**



15–122.



A speedboat is powered by the jet drive shown. Seawater is drawn into the pump housing at the rate of 20 ft^3/s through a 6-in.-diameter intake A. An impeller accelerates the water flow and forces it out horizontally through a 4-in.- diameter nozzle B. Determine the horizontal and vertical components of thrust exerted on the speedboat. The specific weight of seawater is $\gamma_{sw} = 64.3\ \text{lb/ft}^3$.

SOLUTION

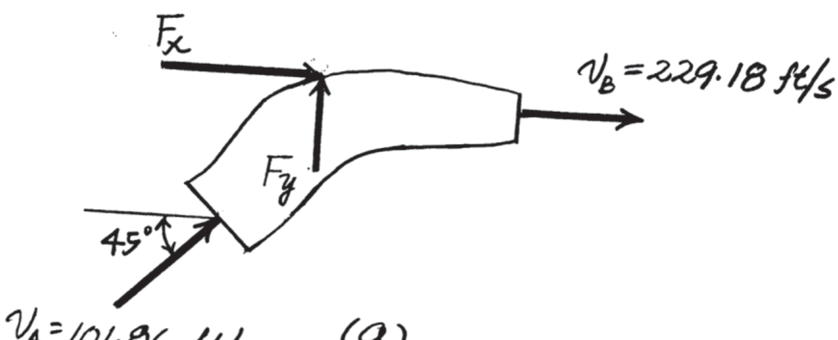
Steady Flow Equation: The speed of the sea water at the hull bottom A and B are $v_A = \dfrac{Q}{A_A} = \dfrac{20}{\frac{\pi}{4}\left(\frac{6}{12}\right)^2} = 101.86\ \text{ft/s}$ and $v_B = \dfrac{Q}{A_B} = \dfrac{20}{\frac{\pi}{4}\left(\frac{4}{12}\right)^2} = 229.18\ \text{ft/s}$ and the mass flow rate at the hull bottom A and nozle B are the same, i.e., $\dfrac{dm_A}{dt} = \dfrac{dm_B}{dt} = \dfrac{dm}{dt} = \rho_{sw} Q = \left(\dfrac{64.3}{32.2}\right)(20) = 39.94\ \text{slug/s}$. By referring to the free-body diagram of the control volume shown in Fig. a,

$$\left(\overset{+}{\rightarrow}\right)\Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \qquad F_x = 39.94(229.18 - 101.86\cos 45^\circ)$$

$$= 6276.55\ \text{lb} = 6.28\ \text{kip} \qquad \textbf{Ans.}$$

$$(+\uparrow)\Sigma F_y = \frac{dm}{dt}\left[(v_B)_y - (v_A)_y\right]; \qquad F_y = 39.94(101.86\sin 45^\circ - 0)$$

$$= 2876.53\ \text{lb} = 2.28\ \text{kip} \qquad \textbf{Ans.}$$



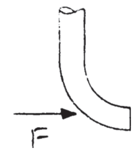
(a)

15–123.

A plow located on the front of a locomotive scoops up snow at the rate of $10\ \text{ft}^3/\text{s}$ and stores it in the train. If the locomotive is traveling at a constant speed of 12 ft/s, determine the resistance to motion caused by the shoveling. The specific weight of snow is $\gamma_s = 6\ \text{lb/ft}^3$.

SOLUTION

$$\Sigma F_x = m\frac{dv}{dt} + v_{D/t}\frac{dm_t}{dt}$$

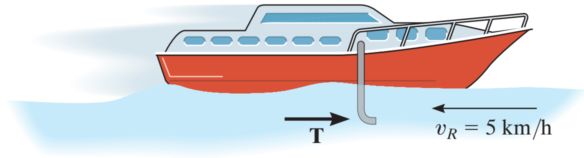


$$F = 0 + (12 - 0)\left(\frac{10(6)}{32.2}\right)$$

$$F = 22.4\ \text{lb}$$ **Ans.**

***15–124.**

The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured *relative* to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust T on the tube that is required to overcome the resistance due to the water collection and yet maintain the constant speed of the boat. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

$$\frac{dm}{dt} = \frac{40}{80} = 0.5\ \text{kg/s}$$

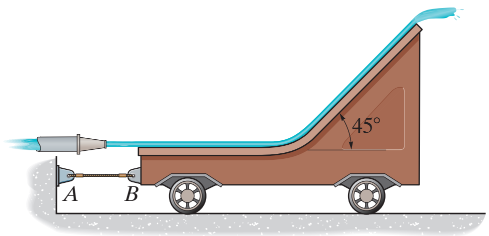
$$v_{D/t} = (70)\left(\frac{1000}{3600}\right) = 19.444\ \text{m/s}$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$T = 0 + 19.444(0.5) = 9.72\ \text{N}$$ **Ans.**

15–125.

Water is discharged from a nozzle with a velocity of 12 m/s and strikes the blade mounted on the 20-kg cart. Determine the tension developed in the cord, needed to hold the cart stationary, and the normal reaction of the wheels on the cart. The nozzle has a diameter of 50 mm and the density of water is $\rho_w = 1000\ \text{kg/mg}^3$.



SOLUTION

Steady Flow Equation: Here, the mass flow rate at sections A and B of the control volume is $\dfrac{dm}{dt} = \rho_W Q = \rho_W A v = 1000\left[\dfrac{\pi}{4}(0.05^2)\right](12) = 7.5\pi\ \text{kg/s}$

Referring to the free-body diagram of the control volume shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}[(v_B)_x - (v_A)_x]; \qquad -F_x = 7.5\pi(12\cos 45° - 12)$$

$$F_x = 82.81\ \text{N}$$

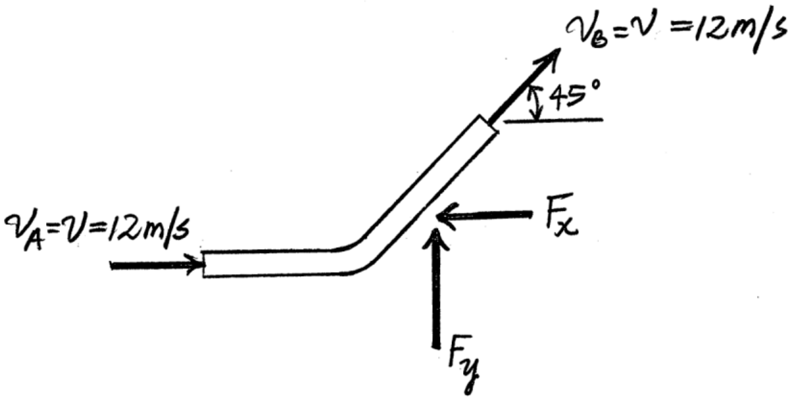
$$+\uparrow \Sigma F_y = \frac{dm}{dt}[(v_B)_y - (v_A)_y]; \qquad F_y = 7.5\pi(12\sin 45° - 0)$$

$$F_y = 199.93\ \text{N}$$

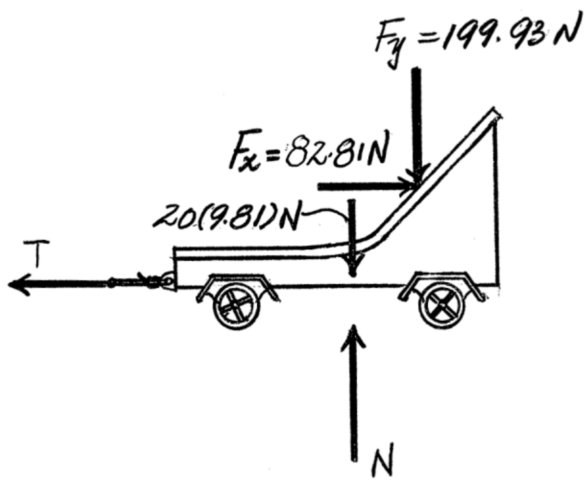
Equilibrium: Using the results of $\mathbf{F}_x$ and $\mathbf{F}_y$ and referring to the free-body diagram of the cart shown in Fig. b,

$\xrightarrow{+} \Sigma F_x = 0; \qquad 82.81 - T = 0 \qquad T = 82.8\ \text{N}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad N - 20(9.81) - 199.93 = 0 \qquad N = 396\ \text{N}$ **Ans.**

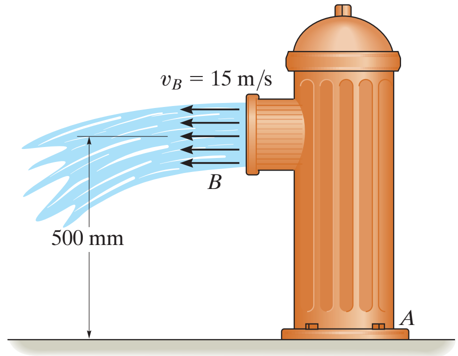


(a)

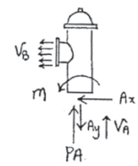


15–126.

Water is flowing from the 150-mm-diameter fire hydrant with a velocity $v_B = 15$ m/s. Determine the horizontal and vertical components of force and the moment developed at the base joint A, if the static (gauge) pressure at A is 50 kPa. The diameter of the fire hydrant at A is 200 mm. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION



$$\frac{dm}{dt} = \rho v_A A_B = 1000(15)(\pi)(0.075)^2$$

$$\frac{dm}{dt} = 265.07 \text{ kg/s}$$

$$v_A = \left(\frac{dm}{dt}\right)\frac{1}{\rho A_A} = \frac{265.07}{1000(\pi)(0.1)^2}$$

$$v_A = 8.4375 \text{ m/s}$$

$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$

$A_x = 265.07(15-0) = 3.98$ kN **Ans.**

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay})$$

$$-A_y + 50(10^3)(\pi)(0.1)^2 = 265.07(0-8.4375)$$

$A_y = 3.81$ kN **Ans.**

$$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}(d_{AB}\, v_B - d_{AA}\, v_A)$$

$$M = 265.07(0.5(15) - 0)$$

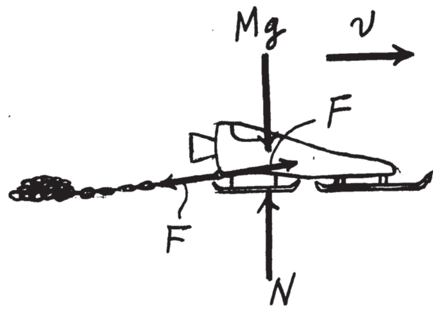
$M = 1.99\ \text{kN}\cdot\text{m}$ **Ans.**

15–127.

A coil of heavy open chain is used to reduce the stopping distance of a sled that has a mass M and travels at a speed of v_0. Determine the required mass per unit length of the chain needed to slow down the sled to $(1/2)v_0$ within a distance $x = s$ if the sled is hooked to the chain at $x = 0$. Neglect friction between the chain and the ground.

SOLUTION

Observing the free-body diagram of the system shown in Fig. *a*, notice that the pair of forces **F**, which develop due to the change in momentum of the chain, cancel each other since they are internal to the system. Here, $v_{D/s} = v$ since the chain on the ground is at rest. The rate at which the system gains mass is $\dfrac{dm_s}{dt} = m'v$ and the mass of the system is $m = m'x + M$. Referring to Fig. *a*,



(a)

$$\left(\overset{+}{\rightarrow}\right)\Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad 0 = \left(m'x + M\right)\frac{dv}{dt} + v\left(m'v\right)$$

$$0 = \left(m'x + M\right)\frac{dv}{dt} + m'v^2 \tag{1}$$

Since $\dfrac{dx}{dt} = v$ or $dt = \dfrac{dx}{v}$,

$$\left(m'x + M\right)v\frac{dv}{dx} + m'v^2 = 0$$

$$\frac{dv}{v} = -\left(\frac{m'}{m'x + M}\right)dx \tag{2}$$

Integrating using the limit $v = v_0$ at $x = 0$ and $v = \dfrac{1}{2}v_0$ at $x = s$,

$$\int_{v_0}^{\frac{1}{2}v_0}\frac{dv}{v} = -\int_0^s\left(\frac{m'}{m'x + M}\right)dx$$

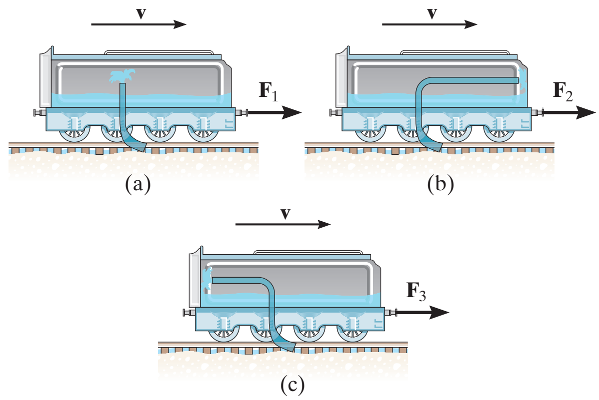
$$\text{1n } v\Big|_{v_0}^{\frac{1}{2}v_0} = -\text{1n}\left(m'x + M\right)\Big|_0^s$$

$$\frac{1}{2} = \frac{M}{m's + M}$$

$$m' = \frac{M}{s} \qquad \textbf{Ans.}$$

***15–128.**

The car is used to scoop up water that is lying in a trough at the tracks. Determine the force needed to pull the car forward at constant velocity **v** for each of the three cases. The scoop has a cross-sectional area A and the density of water is ρ_w.



SOLUTION

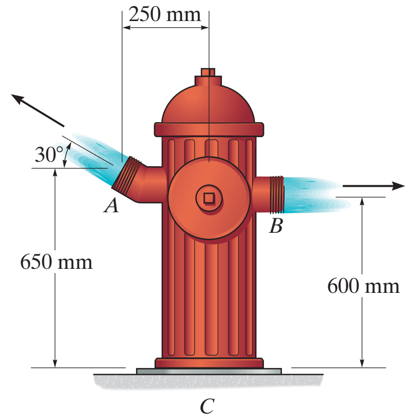
The system consists of the car and the scoop. In all cases

$$\Sigma F_t = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

$$F = 0 - v(\rho)(A)\,v$$

$$F = v^2\,\rho\,A \qquad \textbf{Ans.}$$

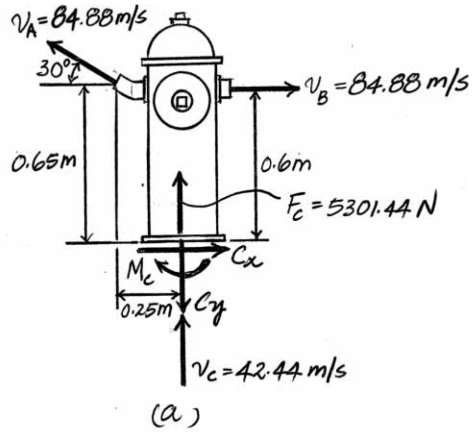
15–129.



The water flow enters below the hydrant at C at the rate of $0.75\ \text{m}^3/\text{s}$. It is then divided equally between the two outlets at A and B. If the gauge pressure at C is 300 kPa, determine the horizontal and vertical force reactions and the moment reaction on the fixed support at C. The diameter of the two outlets at A and B is 75 mm, and the diameter of the inlet pipe at C is 150 mm. The density of water is $\rho_w = 1000\ \text{kg/m}^3$. Neglect the mass of the contained water and the hydrant.

SOLUTION

Free-Body Diagram: The free-body diagram of the control volume is shown in Fig. a. The force exerted on section A due to the water pressure is $F_C = p_C A_C = 300(10^3)\left[\frac{\pi}{4}(0.15^2)\right] = 5301.44$ N. The mass flow rate at sections A, B, and C, are $\frac{dm_A}{dt} = \frac{dm_B}{dt} = \rho_W\left(\frac{Q}{2}\right) = 1000\left(\frac{0.75}{2}\right) = 375$ kg/s and $\frac{dm_C}{dt} = \rho_W Q = 1000(0.75) = 750$ kg/s.



(a)

The speed of the water at sections A, B, and C are

$$v_A = v_B = \frac{Q/2}{A_A} = \frac{0.75/2}{\frac{\pi}{4}(0.075^2)} = 84.88\ \text{m/s} \qquad v_C = \frac{Q}{A_C} = \frac{0.75}{\frac{\pi}{4}(0.15^2)} = 42.44\ \text{m/s}.$$

Steady Flow Equation: Writing the force steady flow equations along the x and y axes,

$$\xrightarrow{+} \Sigma F_x = \frac{dm_A}{dt}(v_A)_x + \frac{dm_B}{dt}(v_B)_x - \frac{dm_C}{dt}(v_C)_x;$$

$$C_x = -375(84.88\cos 30°) + 375(84.88) - 0$$

$$C_x = 4264.54\ \text{N} = 4.26\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \frac{dm_A}{dt}(v_A)_y + \frac{dm_B}{dt}(v_B)_y - \frac{dm_C}{dt}(v_C)_y;$$

$$-C_y + 5301.44 = 375(84.88\sin 30°) + 0 - 750(42.44)$$

$$Cy = 21\,216.93\ \text{N} = 2.12\ \text{kN} \qquad \textbf{Ans.}$$

Writing the steady flow equation about point C,

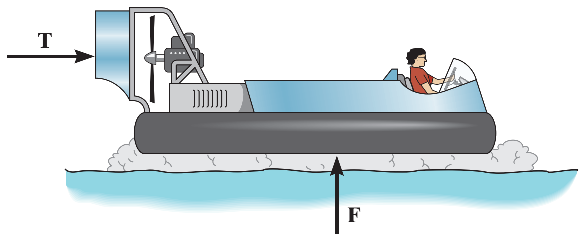
$$+\Sigma M_C = \frac{dm_A}{dt}dv_A + \frac{dm_B}{dt}dv_B - \frac{dm_C}{dt}dv_C;$$

$$-M_C = 375(0.65)(84.88\cos 30°) - 375(0.25)(84.88\sin 30°) + [-375(0.6)(84.88)] - 0$$

$$M_C = 5159.28\ \text{N}\cdot\text{m} = 5.16\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

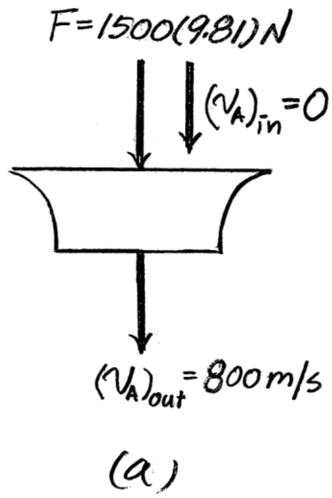
15–130.

The mini hovercraft is designed so that air is drawn in at a constant rate of 20 m^3/s by the-fan blade and channeled to provide a vertical thrust **F**, just sufficient to lift the hovercraft off the water, while the remaining air is channeled to produce a horizontal thrust **T** on the hovercraft. If the air is discharged horizontally at 200 m/s and vertically at 800 m/s, determine the thrust **T** produced. The hovercraft and its passenger have a total mass of 1.5 Mg, and the density of the air is $\rho_a = 1.20\ \text{kg/m}^3$.



SOLUTION

Steady Flow Equation: The free-body diagram of the control volume A is shown in Fig. a. The mass flow rate through the control volume is $\dfrac{dm_A}{dt} = \rho_a Q_A = 1.20Q_A$. Since the air intakes from a large reservoir (atmosphere), the velocity of the air entering the control volume can be considered zero; i.e., $(v_A)_{\text{in}} = 0$. Also, the force acting on the control volume is required to equal the weight of the hovercraft. Thus, $F = 1500(9.81)$ N.



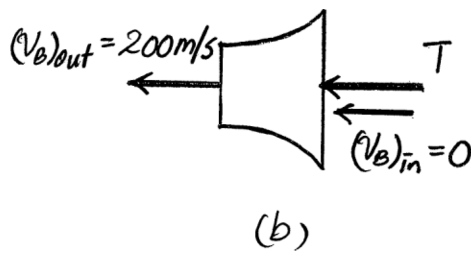
$$(+\downarrow)\Sigma F_y = \frac{dm_A}{dt}\left[(v_A)_{\text{out}} - (v_A)_{\text{in}}\right];\quad 1500(9.81) = 1.20Q_A\,(800 - 0)$$

$$Q_A = 15.328\ \text{m}^3/\text{s}$$

The flow rate through the control volume B, Fig. b, is $Q_B = 20 - Q_A = 20 - 15.328 = 4.672\ \text{m}^3/\text{s}$. Thus, the mass flow rate of this control volume is $\dfrac{dm_B}{dt} = \rho_a Q_B = 1.20(4.672) = 5.606$ kg/s. Again, the intake velocity of the control volume B is equal to zero; i.e., $(v_B)_{\text{in}} = 0$. Referring to the free-body diagram of this control volume, Fig. b,

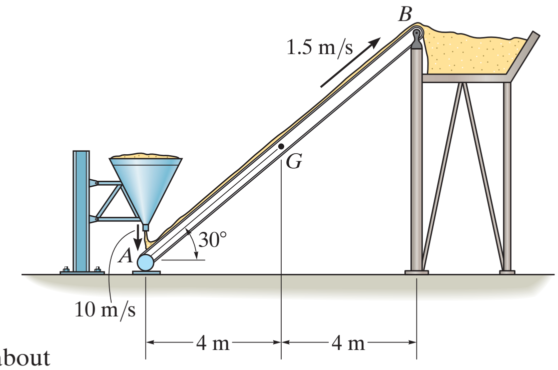
$$(\overset{+}{\leftarrow})\Sigma F_x = \frac{dm_B}{dt}\left[(v_B)_{\text{out}} - (v_B)_{\text{in}}\right];\quad T = 5.606(200 - 0) = 1121.25\ \text{N} = 1.12\ \text{kN}$$

Ans.



15–131.

Sand is discharged from the silo at A at a rate of 50 kg/s with a vertical velocity of 10 m/s onto the conveyor belt, which is moving with a constant velocity of 1.5 m/s. If the conveyor system and the sand on it have a total mass of 750 kg and center of mass at point G, determine the horizontal and vertical components of reaction at the pin support B roller support A. Neglect the thickness of the conveyor.



SOLUTION

Steady Flow Equation: The moment steady flow equation will be written about point B to eliminate $\mathbf{B}_x$ and $\mathbf{B}_y$. Referring to the free-body diagram of the control volume shown in Fig. a,

$$+\Sigma M_B = \frac{dm}{dt}(dv_B - dv_A); \qquad 750(9.81)(4) - A_y(8) = 50[0 - 8(5)]$$

$$A_y = 4178.5 \text{ N} = 4.18 \text{ kN} \qquad \textbf{Ans.}$$

Writing the force steady flow equation along the x and y axes,

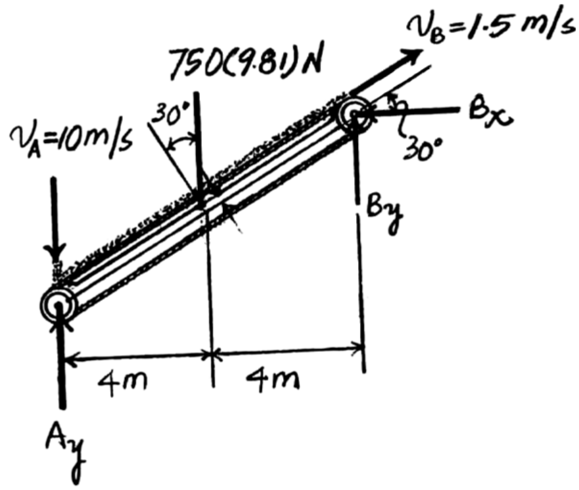
$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}[(v_B)_x - (v_A)_x]; \qquad -B_x = 50(1.5 \cos 30° - 0)$$

$$B_x = |-64.95 \text{ N}| = 65.0 \text{ N} \rightarrow \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}[(v_B)_y - (v_A)_y]; \qquad B_y + 4178.5 - 750(9.81)$$

$$= 50[1.5 \sin 30° - (-10)]$$

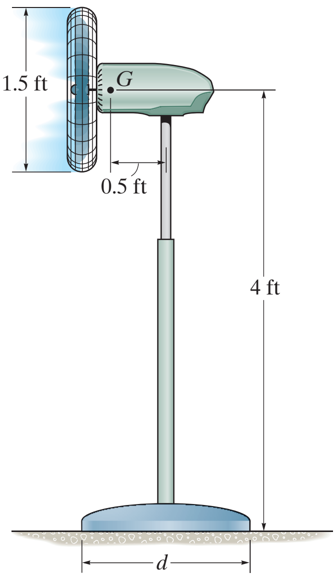
$$B_y = 3716.25 \text{ N} = 3.72 \text{ kN} \uparrow \qquad \textbf{Ans.}$$



(a)

***15–132.**

The fan blows air at 6000 ft^3/min. If the fan has a weight of 30 lb and a center of gravity at G, determine the smallest diameter d of its base so that it will not tip over. The specific weight of air is $\gamma = 0.076\ \text{lb/ft}^3$.



SOLUTION

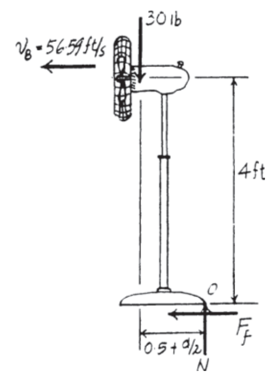
Equations of Steady Flow: Here $Q = \left(\dfrac{6000\ \text{ft}^3}{\text{min}}\right) \times \left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 100\ \text{ft}^3/\text{s}$. Then,

$v = \dfrac{Q}{A} = \dfrac{100}{\frac{\pi}{4}(1.5^2)} = 56.59\ \text{ft/s}$. Also, $\dfrac{dm}{dt} = \rho_a Q = \dfrac{0.076}{32.2}(100) = 0.2360\ \text{slug/s}$.

Applying Eq. 15–26 we have

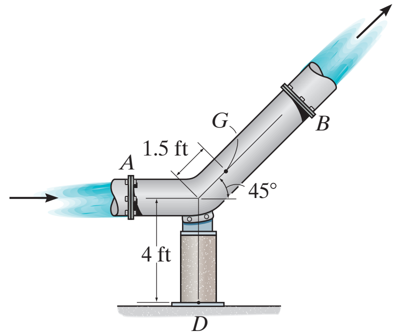
$$\circlearrowleft +\Sigma M_O = \frac{dm}{dt}(d_{OB}\,v_B - d_{OA}\,v_A); \quad 30\left(0.5 + \frac{d}{2}\right) = 0.2360\,[4(56.59) - 0]$$

$$d = 2.56\ \text{ft} \qquad \textbf{Ans.}$$



15–133.

The bend is connected to the pipe at flanges A and B as shown. If the diameter of the pipe is 1 ft and it carries a discharge of 50 ft^3/s, determine the horizontal and vertical components of force reaction and the moment reaction exerted at the fixed base D of the support. The total weight of the bend and the water within it is 500 lb, with a mass center at point G. The gauge pressure of the water at the flanges at A and B are 15 psi and 12 psi, respectively. Assume that no force is transferred to the flanges at A and B. The specific weight of water is $\gamma_w = 62.4 \text{ lb/ft}^3$.



SOLUTION

Free-Body Diagram: The free-body of the control volume is shown in Fig. *a*. The force exerted on sections A and B due to the water pressure is $F_A = P_A A_A = 15\left[\frac{\pi}{4}\left(12^2\right)\right] = 1696.46 \text{ lb}$ and $F_B = P_B A_B = 12\left[\frac{\pi}{4}\left(12^2\right)\right] = 1357.17 \text{ lb}$. The speed of the water at, sections A and B are $v_A = v_B = \dfrac{Q}{A} = \dfrac{50}{\frac{\pi}{4}\left(1^2\right)} = 63.66 \text{ ft/s}$. Also, the mass flow rate at these two sections are $\dfrac{dm_A}{dt} = \dfrac{dm_B}{dt} = \rho_W Q = \left(\dfrac{62.4}{32.2}\right)(50) = 96.894 \text{ slug/s}$.

Steady Flow Equation: The moment steady flow equation will be wr itten about point D to eliminate D_x and D_y.

$$\circlearrowleft +\Sigma M_D = \frac{dm_B}{dt} dv_B - \frac{dm_A}{dt} dv_A;$$

$$M_D + 1357.17 \cos 45°(4) - 500\left(1.5 \cos 45°\right) - 1696.46(4)$$

$$= -96.894(4)\left(63.66 \cos 45°\right) - [-96.894(4)(63.66)]$$

$$M_D = 10\,704.35 \text{ lb} \cdot \text{ft} = 10.7 \text{ kip} \cdot \text{ft}$$ **Ans.**

15–133. continued

Writing the force steady flow equation along the x and y axes,

$$(+\uparrow)\Sigma F_y = \frac{dm}{dt}\left[(v_B)_y - (v_A)_y\right];$$

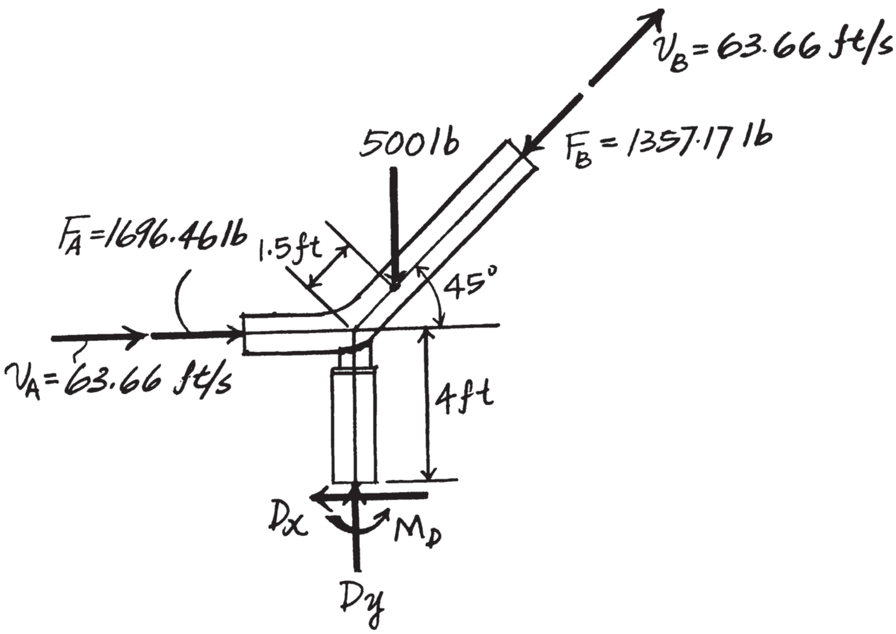
$$D_y - 500 - 1357.17 \sin 45° = 96.894(63.66 \sin 45° - 0)$$

$$D_y = 5821.44 \text{ lb} = 5.82 \text{ kip}$$ **Ans.**

$$\left(\xrightarrow{+}\right)\Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right];$$

$$1696.46 - 1357.17 \cos 45° - D_x = 96.894[63.66 \cos 45° - 63.66]$$

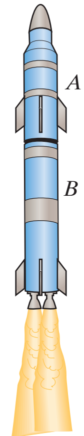
$$D_x = 2543.51 \text{ lb} = 2.54 \text{ kip}$$ **Ans.**



(a)

15–134.

Each of the two stages A and B of the rocket has a mass of 2 Mg when their fuel tanks are empty. They each carry 500 kg of fuel and are capable of consuming it at a rate of 50 kg/sand eject it with a constant velocity of 2500 m/s, measured with respect to the rocket. The rocket is launched vertically from rest by first igniting stage B. Then stage A is ignited immediately after all the fuel in B is consumed and A has separated from B. Determine the maximum velocity of stage A. Neglect drag resistance and the variation of the rocket's weight with altitude.



SOLUTION

The mass of the rocket at any instant t is $m = (M + m_0) - qt$. Thus, its weight at the same instant is $W = mg = [(M + m_0) - qt]g$.

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \quad -[(M + m_0) - qt]g = [(M + m_0) - qt]\frac{dv}{dt} - v_{D/e}q$$

$$\frac{dv}{dt} = \frac{v_{D/e}q}{(M + m_0) - qt} - g$$

During the first stage, $M = 4000$ kg, $m_0 = 1000$ kg, $q = 50$ kg/s, and $v_{D/e} = 2500$ m/s. Thus,

$$\frac{dv}{dt} = \frac{2500(50)}{(4000 + 1000) - 50t} - 9.81$$

$$\frac{dv}{dt} = \left(\frac{2500}{100 - t} - 9.81\right) \text{m/s}^2$$

The time that it take to complete the first stage is equal to the time for all the fuel in the rocket to be consumed, i.e., $t = \frac{500}{50} = 10$ s. Integrating,

$$\int_0^{v_1} dv = \int_0^{10\text{ s}} \left(\frac{2500}{100 - t} - 9.81\right)dt$$

$$v_1 = \left[-2500 \ln(100 - t) - 9.81t\right]\Big|_0^{10\text{ s}}$$

$$= 165.30 \text{ m/s}$$

During the second stage of launching, $M = 2000$ kg, $m_0 = 500$ kg, $q = 50$ kg/s, and $v_{D/e} = 2500$ m/s. Thus, Eq. (1) becomes

$$\frac{dv}{dt} = \frac{2500(50)}{(2000 + 500) - 50t} - 9.81$$

$$\frac{dv}{dt} = \left(\frac{2500}{50 - t} - 9.81\right) \text{m/s}^2$$

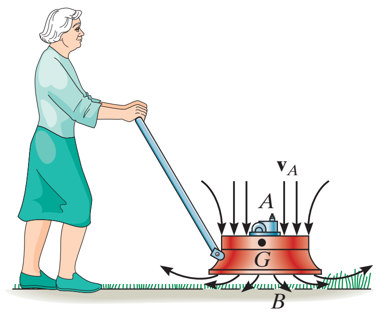
The maximum velocity of rocket A occurs when it has consumed all the fuel. Thus, the time taken is given by $t = \frac{500}{50} = 10$ s. Integrating with the initial condition $v = v_1 = 165.30$ m/s when $t = 0$ s,

$$\int_{165.30\text{ m/s}}^{v_{\max}} dv = \int_0^{10\text{ s}} \left(\frac{2500}{50 - t} - 9.81\right)dt$$

$$v_{\max} - 165.30 = \left[-2500 \ln(50 - t) - 9.81t\right]\Big|_0^{10\text{ s}}$$

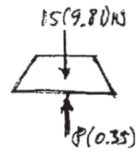
15–135.

A power lawn mower hovers very close over the ground. This is done by drawing air in at a speed of 6 m/s through an intake unit A, which has a cross-sectional area of $A_A = 0.25\ \text{m}^2$, and then discharging it at the ground, B, where the cross-sectional area is $A_B = 0.35\ \text{m}^2$. If air at A is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has a mass of 15 kg with center of mass at G. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$.



SOLUTION

$$\frac{dm}{dt} = \rho\, A_A v_A = 1.22(0.25)(6) = 1.83\ \text{kg/s}$$



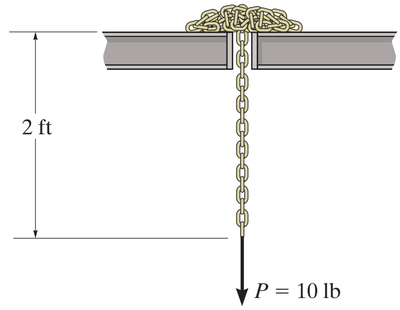
$$+\uparrow \Sigma F_y = \frac{dm}{dt}((v_B)_y - (v_A)_y)$$

$$P = (0.35) - 15(9.81) = 1.83(0 - (-6))$$

$$P = 452\ \text{Pa}$$ **Ans.**

***15–136.**

The 12-ft-long open-link chain has 2 ft of its end suspended from the hole as shown. If a force of $P = 10$ lb is applied to its end and the chain is released from rest, determine the velocity of the chain at the instant the entire chain has been extended. The chain has a weight of 2 lb/ft.



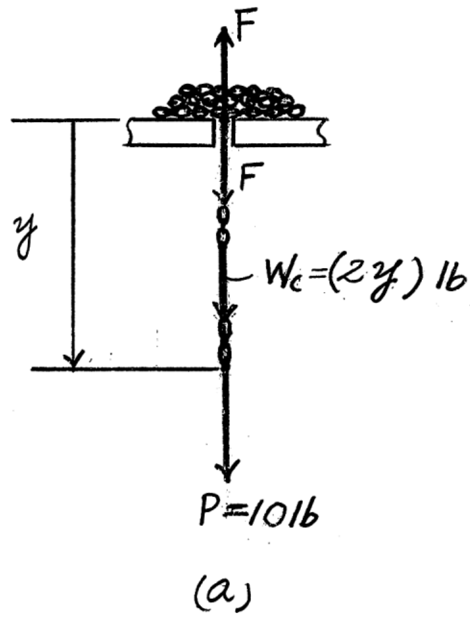
SOLUTION

From the free-body diagram of the system shown in Fig. *a*, **F** cancels since it is internal to the system. Here, $v_{D/s} = v$ since the chain on the horizontal surface is at rest. The rate at which the chain gains mass is $\frac{dm_s}{dt} = \left(\frac{2}{32.2}v\right)$ slug/s, and the mass of the chain is $m = \left(\frac{2y}{32.2}\right)$ slug.

Referring to Fig. *a*,

$$+\downarrow \Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad 10 + 2y = \left(\frac{2y}{32.2}\right)\frac{dv}{dt} + v\left(\frac{2}{32.2}v\right)$$

$$2y\frac{dv}{dt} + 2v^2 = 322 + 64.4y$$



Since $\frac{dy}{dt} = v$ or $dt = \frac{dy}{v}$, then

$$2yv\frac{dv}{dy} + 2v^2 = 322 + 64.4y$$

Multiplying by $y\,dy$,

$$\left(2vy^2\frac{dv}{dy} + 2yv^2\right)dy = \left(322y + 64.4y^2\right)dy$$

Since $\frac{d\left(v^2y^2\right)}{dy} = 2vy^2\frac{dv}{dy} + 2yv^2$, then

$$d\left(v^2y^2\right) = \left(322y + 64.4y^2\right)dy$$

Integrating,

$$v^2y^2 = 161y^2 + 21.467y^3 + C$$

Substituting the initial condition $v = 0$ at $y = 2$ ft,

$$0 = 161\left(2^2\right) + 21.467\left(2^3\right) + C \qquad C = -815.73\,\frac{\text{ft}^4}{\text{s}^2}$$

Thus,

$$v^2y^2 = 161y^2 + 21.467y^3 - 815.73$$

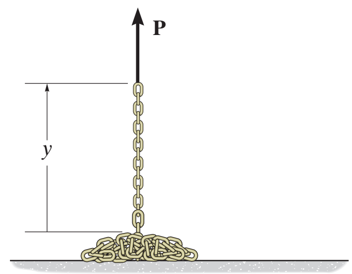
At the instant the entire chain is in motion $y = 12$ ft.

$$v^2\left(12^2\right) = 161\left(12^2\right) + 21.467\left(12^3\right) - 815.73$$

$$v = 20.3 \text{ ft/s}$$ **Ans.**

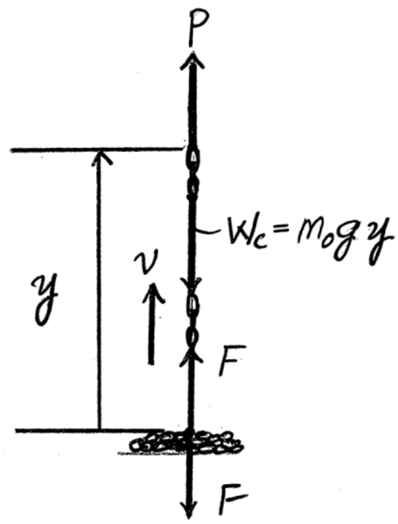
15–137.

A chain of mass m_0 per unit length is loosely coiled on the floor. If one end is subjected to a constant force **P** when $y = 0$, determine the velocity of the chain as a function of y.



SOLUTION

From the free-body diagram of the system shown in Fig. a, F cancels since it is internal to the system. Here, $v_{D/s} = v$ since the chain on the horizontal surface is at rest. The rate at which the chain gains mass is $\dfrac{dm_s}{dt} = m_0 v$, and the mass of the system is $m = m_0 y$. Referring to Fig. a,



(a)

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad P - m_0 g y = m_0 y \frac{dv}{dt} + v(m_0 v)$$

$$P - m_0 g y = m_0 y \frac{dv}{dt} + m_0 v^2$$

$$y\frac{dv}{dt} + v^2 = \frac{P}{m_0} - gy$$

Since $\dfrac{dy}{dt} = v$ or $dt = \dfrac{dy}{v}$,

$$vy\frac{dv}{dy} + v^2 = \frac{P}{m_0} - gy$$

Multiplying by $2y\,dy$,

$$\left(2vy^2\frac{dv}{dy} + 2v^2 y\right)dy = \left(\frac{2P}{m_0}y - 2gy^2\right)dy \qquad \textbf{(1)}$$

Since $\dfrac{d\left(v^2y^2\right)}{dy} = 2vy^2\dfrac{dv}{dy} + 2yv^2$, then Eq. (1) can be written as

$$d\left(v^2y^2\right) = \left(\frac{2P}{m_0}y - 2gy^2\right)dy$$

Integrating,

$$\int d\left(v^2y^2\right) = \int\left(\frac{2P}{m_0}y - 2gy^2\right)dy$$

$$v^2y^2 = \frac{P}{m_0}y^2 - \frac{2}{3}gy^3 + C$$

Substituting $v = 0$ at $y = 0$,

$$0 = 0 - 0 + C \qquad C = 0$$

Thus,

$$v^2y^2 = \frac{P}{m_0}y^2 - \frac{2}{3}gy^3$$

$$v = \sqrt{\frac{P}{m_0} - \frac{2}{3}gy} \qquad \textbf{Ans.}$$

15–138.

The second stage of a two-stage rocket weighs 2000 lb (empty) and is launched from the first stage with a velocity of 3000 mi/h. The fuel in the second stage weighs 1000 lb. If it is consumed at the rate of 50 lb/s and ejected with a relative velocity of 8000 ft/s, determine the acceleration of the second stage just after the engine is fired. What is the rocket's acceleration just before all the fuel is consumed? Neglect the effect of gravitation.

SOLUTION

Initially,

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\left(\frac{dm_e}{dt}\right)$$

$$0 = \frac{3000}{32.2}a - 8000\left(\frac{50}{32.2}\right)$$

$$a = 133 \text{ ft/s}^2$$ **Ans.**

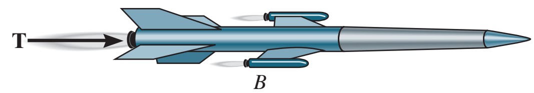
Finally,

$$0 = \frac{2000}{32.2}a - 8000\left(\frac{50}{32.2}\right)$$

$$a = 200 \text{ ft/s}^2$$ **Ans.**

15–139.

The missile weighs 40 000 lb. The constant thrust provided by the turbojet engine is $T = 15\,000$ lb. Additional thrust is provided by *two* rocket boosters B. The propellant in each booster is burned at a constant rate of 150 lb/s, with a relative exhaust velocity of 3000 ft/s. If the mass of the propellant lost by the turbojet engine can be neglected, determine the velocity of the missile after the 4-s burn time of the boosters. The initial velocity of the missile is 300 mi/h.



SOLUTION

$$\stackrel{+}{\rightarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

At a time t, $m = m_0 - ct$, where $c = \dfrac{dm_e}{dt}$.

$$T = (m_0 - ct)\frac{dv}{dt} - v_{D/e}c$$

$$\int_{v_0}^{v} dv = \int_0^t \left(\frac{T + cv_{D/e}}{m_0 - ct}\right)dt$$

$$v = \left(\frac{T + cv_{D/e}}{c}\right)\ln\left(\frac{m_0}{m_0 - ct}\right) + v_0 \qquad \textbf{(1)}$$

Here, $m_0 = \dfrac{40\,000}{32.2} = 1242.24$ slug, $c = 2\left(\dfrac{150}{32.2}\right) = 9.3168$ slug/s, $v_{D/e} = 3000$ ft/s,

$t = 4$ s, $v_0 = \dfrac{300(5280)}{3600} = 440$ ft/s.

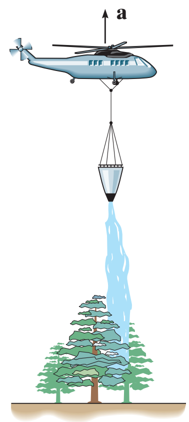
Substitute the numerical values into Eq. (1):

$$v_{max} = \left(\frac{15\,000 + 9.3168(3000)}{9.3168}\right)\ln\left(\frac{1242.24}{1242.24 - 9.3168(4)}\right) + 440$$

$v_{max} = 580$ ft/s **Ans.**

***15–140.**

The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to the helicopter, determine the initial upward acceleration the helicopter experiences as the water is being released.



SOLUTION

$$+\uparrow \Sigma F_t = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

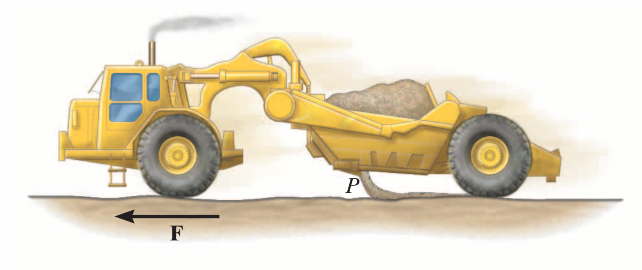
Initially, the bucket is full of water, hence $m = 10(10^3) + 0.5(10^3) = 10.5(10^3)$ kg

$0 = 10.5(10^3)a - (10)(50)$

$a = 0.0476 \text{ m/s}^2$ **Ans.**

15–141.

The earthmover initially carries 10 m^3 of sand having a density of 1520 kg/m^3. The sand is unloaded horizontally through a 2.5 m^2 dumping port P at a rate of 900 kg/s measured relative to the port. Determine the resultant tractive force **F** at its front wheels if the acceleration of the earthmover is 0.1 m/s^2 when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.



SOLUTION

When half the sand remains,

$$m = 30\,000 + \frac{1}{2}(10)(1520) = 37\,600 \text{ kg}$$

$$\frac{dm}{dt} = 900 \text{ kg/s} = \rho\, v_{D/e}\, A$$

$$900 = 1520(v_{D/e})(2.5)$$

$$v_{D/e} = 0.237 \text{ m/s}$$

$$a = \frac{dv}{dt} = 0.1$$

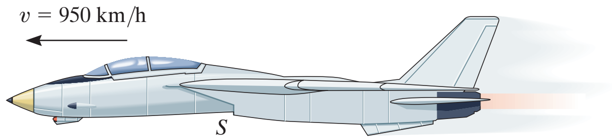
$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - \frac{dm}{dt}v$$

$$F = 37\,600(0.1) - 900(0.237)$$

$$F = 3.55 \text{ kN}$$ **Ans.**

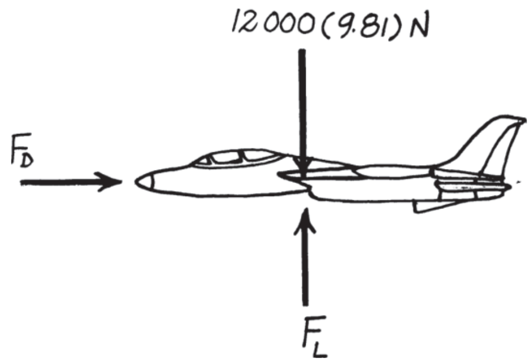
15–142.

The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops S at the rate of 50 m^3/s. If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m^3. *Hint:* Since mass both enters and exits the plane, Eqs. 15–28 and 15–29 must be combined to yield $\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$.



SOLUTION

$$\Sigma F_s = m\frac{dv}{dt} - \frac{dm_e}{dt}(v_{D/E}) + \frac{dm_i}{dt}(v_{D/i}) \qquad \textbf{(1)}$$



$v = 950 \text{ km/h} = 0.2639 \text{ km/s}, \qquad \frac{dv}{dt} = 0$

$v_{D/E} = 0.45 \text{ km/s}$

$v_{D/t} = 0.2639 \text{ km/s}$

$\frac{dm_t}{dt} = 50(1.22) = 61.0 \text{ kg/s}$

$\frac{dm_e}{dt} = 0.4 + 61.0 = 61.4 \text{ kg/s}$

Forces T and R are incorporated into Eq. (1) as the last two terms in the equation.

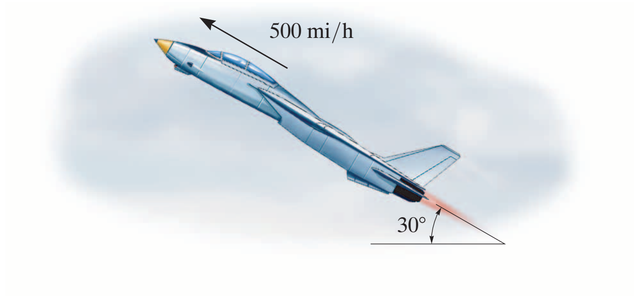
$(\overset{+}{\leftarrow}) \; -F_D = 0 - (0.45)(61.4) + (0.2639)(61)$

$F_D = 11.5 \text{ kN}$ **Ans.**

15–143.

The jet is traveling at a speed of 500 mi/h, 30° with the horizontal. If the fuel is being spent at 3 lb/s, and the engine takes in air at 400 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 32 800 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.7v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. *Hint:* Since mass both enters and exits the plane, Eqs. 15-28 and 15-29 must be combined to yield

$$\Sigma F_x = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}.$$

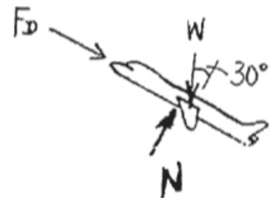


SOLUTION

$$\frac{dm_i}{dt} = \frac{400}{32.2} = 12.42 \text{ slug/s}$$

$$\frac{dm_e}{dt} = \frac{403}{32.2} = 12.52 \text{ slug/s}$$

$$v = v_{D/i} = 500 \text{ mi/h} = 733.3 \text{ ft/s}$$



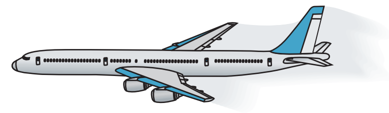
$$\nwarrow + \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$-(15\,000)\sin 30° - 0.7(733.3)^2 = \frac{15\,000}{32.2}\frac{dv}{dt} - 32\,800(12.52) + 733.3(12.42)$$

$$a = \frac{dv}{dt} = 37.5 \text{ ft/s}^2$$ **Ans.**

***15–144.**

A four-engine commercial jumbo jet is cruising at a constant speed of 800 km/h in level flight when all four engines are in operation. Each of the engines is capable of discharging combustion gases with a velocity of 775 m/s relative to the plane. If during a test two of the engines, one on each side of the plane, are shut off, determine the new cruising speed of the jet. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. Neglect the loss of mass due to fuel consumption.



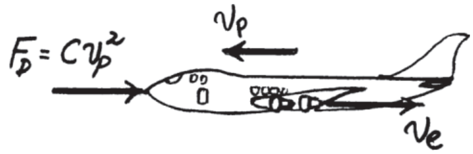
SOLUTION

Steady Flow Equation: Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is

$$(\overset{+}{\rightarrow}) \qquad v_e + v_p + v_{e/p}$$

When the four engines are in operation, the airplane has a constant speed of $v_p = \left[800(10^3)\ \frac{\text{m}}{\text{h}}\right]\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 222.22$ m/s. Thus,

$$(\overset{+}{\rightarrow}) \qquad v_e = -222.22 + 775 = 552.78\ \text{m/s} \rightarrow$$



(a)

Referring to the free-body diagram of the airplane shown in Fig. *a*,

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \qquad C(222.22^2) = 4\,\frac{dm}{dt}\,(552.78 - 0)$$

$$C = 0.044775\,\frac{dm}{dt}$$

When only two engines are in operation, the exit speed of the air is

$$(\overset{+}{\rightarrow}) \qquad v_e = -v_p + 775$$

Using the result for C,

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \quad \left(0.044775\,\frac{dm}{dt}\right)(v_p{}^2) = 2\frac{dm}{dt}\left[-v_p + 775) - 0\right]$$

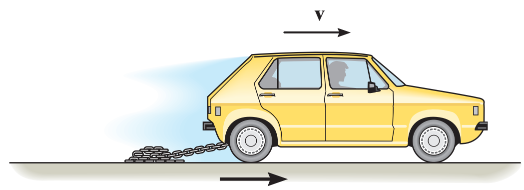
$$0.044775v_p{}^2 + 2v_p - 1550 = 0$$

Solving for the positive root,

$v_p = 165.06$ m/s $= 594$ km/h **Ans.**

15–145.

The car has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force F that must be supplied by the rear wheels of the car, necessary to maintain a constant speed v while the chain is being drawn out.



SOLUTION

$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time t, $m = m_0 + ct$, where $c = \dfrac{dm_i}{dt} = \dfrac{m'dx}{dt} = m'v$.

Here, $v_{D/i} = v$, $\quad \dfrac{dv}{dt} = 0$.

$$F = (m_0 - m'v\,)(0) + v(m'v) = m'v^2 \qquad \textbf{Ans.}$$

15–146.

A rocket has an empty weight of 500 lb and carries 300 lb of fuel. If the fuel is burned at the rate of 1.5 lb/s and ejected with a velocity of 4400 ft/s relative to the rocket, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.

SOLUTION

$$+\uparrow \Sigma F_s = \frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

At a time t, $m = m_0 - ct$, where $c = \dfrac{dm_e}{dt}$. In space the weight of the rocket is zero.

$$0 = (m_0 - ct)\frac{dv}{dt} - v_{D/e}\,c$$

$$\int_0^v dv = \int_0^t \left(\frac{cv_{D/e}}{m_0 - ct}\right)dt$$

$$v = v_{D/e} \ln\left(\frac{m_0}{m_0 - ct}\right) \qquad \textbf{(1)}$$

The maximum speed occurs when all the fuel is consumed, that is, when $t = \frac{300}{15} = 20$ s.

Here, $m_0 = \frac{500 + 300}{32.2} = 24.8447$ slug, $c = \frac{15}{32.2} = 0.4658$ slug/s, $v_{D/e} = 4400$ ft/s.

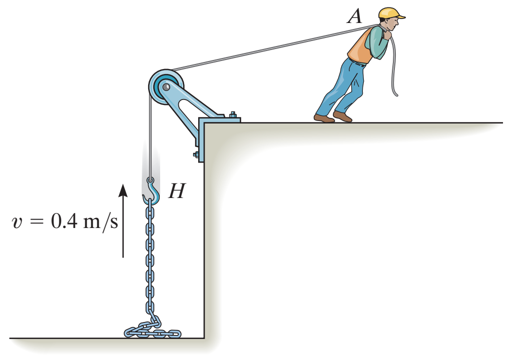
Substitute the numerical into Eq. (1):

$$v_{max} = 4400\ 1n\left(\frac{24.8447}{24.8447 - (0.4658(20))}\right)$$

$$v_{max} = 2068 \text{ ft/s} \qquad \textbf{Ans.}$$

15–147.

Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed $v = 0.4\text{ m/s}$. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.

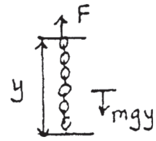


SOLUTION

$$\frac{dv}{dt} = 0, \qquad y = vt$$

$$m_i = my = mvt$$

$$\frac{dm_i}{dt} = mv$$



$$+\uparrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\left(\frac{dm_i}{dt}\right)$$

$$F - mgvt = 0 + v(mv)$$

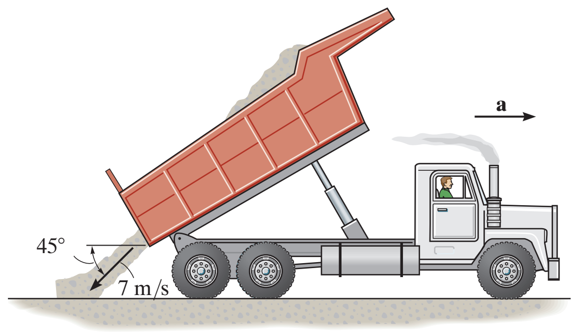
$$F = m(gvt + v^2)$$

$$= 2[9.81(0.4)t + (0.4)^2]$$

$$F = (7.85t + 0.320)\text{ N} \qquad \textbf{Ans.}$$

***15–148.**

The truck has a mass of 50 Mg when empty. When it is unloading 5 m^3 of sand at a constant rate of 0.8 m^3/s, the sand flows out the back at a speed of 7 m/s, measured relative to the truck, in the direction shown. If the truck is free to roll, determine its initial acceleration just as the load begins to empty. Neglect the mass of the wheels and any frictional resistance to motion. The density of sand is $\rho_s = 1520\ \text{kg/m}^3$.



SOLUTION

A System That Loses Mass: Initially, the total mass of the truck is $m = 50(10^3) + 5(1520) = 57.6(10^3)$ kg and $\dfrac{dm_e}{dt} = 0.8(1520) = 1216$ kg/s.

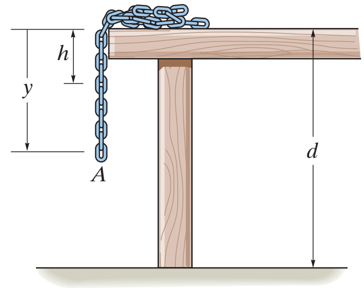
Applying Eq. 15–29, we have

$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad 0 = 57.6(10^3)a - (0.8\cos 45°)(1216)$$

$$a = 0.104\ \text{m/s}^2 \qquad \textbf{Ans.}$$

15–149.

The chain has a total length $L < d$ and a mass per unit length of m'. If a portion h of the chain is suspended over the table and released, determine the velocity of its end A as a function of its position y. Neglect friction.



SOLUTION

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/e}\frac{dm_e}{dt}$$

$$m'gy = m'y\frac{dv}{dt} + v(m'v)$$

$$m'gy = m'\left(y\frac{dv}{dt} + v^2\right)$$

Since $dt = \dfrac{dy}{v}$, we have

$$gy = vy\frac{dv}{dy} + v^2$$

Multiply by $2y$ and integrate:

$$\int 2gy^2\,dy = \int\left(2vy^2\frac{dv}{dy} + 2yv^2\right)dy$$

$$\frac{2}{3}g^3y^3 + C = v^2y^2$$

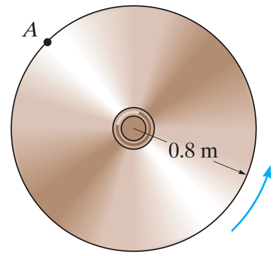
when $v = 0$, $y = h$, so that $C = -\dfrac{2}{3}gh^3$

Thus, $v^2 = \dfrac{2}{3}g\left(\dfrac{y^3 - h^3}{y^2}\right)$

$$v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)}$$ **Ans.**

16–1.

The angular velocity of the disk is defined by $\omega = (5t^2 + 2)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and acceleration of point A on the disk when $t = 0.5$ s.



SOLUTION

$$\omega = (5\,t^2 + 2)\ \text{rad/s}$$

$$\alpha = \frac{d\omega}{dt} = 10\,t$$

$$t = 0.5\ \text{s}$$

$$\omega = 3.25\ \text{rad/s}$$

$$\alpha = 5\ \text{rad/s}^2$$

$$v_A = \omega r = 3.25(0.8) = 2.60\ \text{m/s}$$ **Ans.**

$$a_z = \alpha r = 5(0.8) = 4\ \text{m/s}^2$$

$$a_n = \omega^2 r = (3.25)^2(0.8) = 8.45\ \text{m/s}^2$$

$$a_A = \sqrt{(4)^2 + (8.45)^2} = 9.35\ \text{m/s}^2$$ **Ans.**

16–2.

A flywheel has its angular speed increased uniformly from 15 rad/s to 60 rad/s in 80 s. If the diameter of the wheel is 2 ft, determine the magnitudes of the normal and tangential components of acceleration of a point on the rim of the wheel when $t = 80$ s, and the total distance the point travels during the time period.

SOLUTION

$$\omega = \omega_0 + \alpha_c t$$

$$60 = 15 + \alpha_c(80)$$

$$\alpha_c = 0.5625 \text{ rad/s}^2$$

$$a_t = \alpha r = (0.5625)(1) = 0.562 \text{ ft/s}^2$$ **Ans.**

$$a_n = \omega^2 r = (60)^2(1) = 3600 \text{ ft/s}^2$$ **Ans.**

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

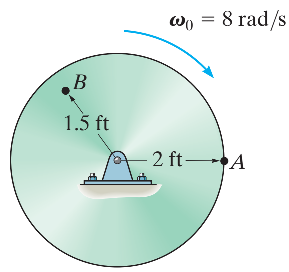
$$(60)^2 = (15)^2 + 2(0.5625)(\theta - 0)$$

$$\theta = 3000 \text{ rad}$$

$$s = \theta r = 3000(1) = 3000 \text{ ft}$$ **Ans.**

16–3.

The disk is originally rotating at $\omega_0 = 8\ \text{rad/s}$. If it is subjected to a constant angular acceleration of $\alpha = 6\ \text{rad/s}^2$, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant $t = 0.5\ \text{s}$.



SOLUTION

$\omega = \omega_0 + \alpha_c t$

$\omega = 8 + 6(0.5) = 11\ \text{rad/s}$

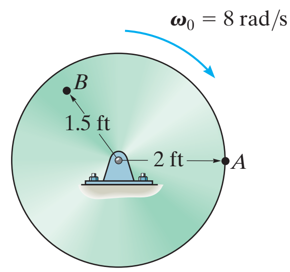
$v = r\omega; \qquad v_A = 2(11) = 22\ \text{ft/s}$ **Ans.**

$a_t = r\alpha; \qquad (a_A)_t = 2(6) = 12.0\ \text{ft/s}^2$ **Ans.**

$a_n = \omega^2 r; \qquad (a_A)_n = (11)^2(2) = 242\ \text{ft/s}^2$ **Ans.**

***16–4.**

The disk is originally rotating at $\omega_0 = 8$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 6$ rad/s^2, determine the magnitudes of the velocity and the n and t components of acceleration of point B just after the wheel undergoes 2 revolutions.



SOLUTION

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = (8)^2 + 2(6)[2(2\pi) - 0]$$

$$\omega = 14.66 \text{ rad/s}$$

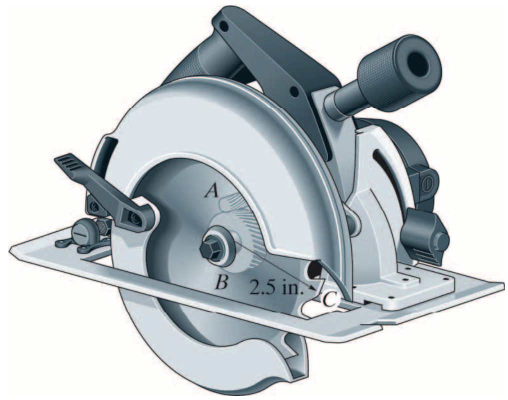
$$v_B = \omega r = 14.66(1.5) = 22.0 \text{ ft/s}$$ **Ans.**

$$(a_B)_t = \alpha r = 6(1.5) = 9.00 \text{ ft/s}^2$$ **Ans.**

$$(a_B)_n = \omega^2 r = (14.66)^2(1.5) = 322 \text{ ft/s}^2$$ **Ans.**

16–5.

Initially the motor on the circular saw turns its drive shaft at $\omega = (20t^{2/3})$ rad/s, where t is in seconds. If the radii of gears A and B are 0.25 in. and 1 in., respectively, determine the magnitudes of the velocity and acceleration of a tooth C on the saw blade after the drive shaft rotates $\theta = 5$ rad starting from rest.



SOLUTION

$$\omega = 20\,t^{2/3}$$

$$\alpha = \frac{d\omega}{dt} = \frac{40}{3}t^{-1/3}$$

$$d\theta = \omega\,dt$$

$$\int_0^\theta d\theta = \int_0^t 20\,t^{2/3}\,dt$$

$$\theta = 20\left(\frac{3}{5}\right)t^{5/3}$$

When $\theta = 5$ rad

$$t = 0.59139\text{ s}$$

$$\alpha = 15.885\text{ rad/s}^2$$

$$\omega = 14.091\text{ rad/s}$$

$$\omega_A\,r_A = \omega_B\,r_B$$

$$14.091(0.25) = \omega_B(1)$$

$$\omega_B = 3.523\text{ rad/s}$$

$$v_C = \omega_B\,r = 3.523(2.5) = 8.81\text{ in./s}$$ **Ans.**

$$\alpha_A\,r_A = \alpha_B\,r_B$$

$$15.885(0.25) = \alpha_B(1)$$

$$\alpha_B = 3.9712\text{ rad/s}^2$$

$$(a_C)_t = \alpha_B\,r = 3.9712(2.5) = 9.928\text{ in./s}^2$$

$$(a_C)_n = \omega_B^2 r = (3.523)^2(2.5) = 31.025\text{ in./s}^2$$

$$a_C = \sqrt{(9.928)^2 + (31.025)^2}$$

$$= 32.6\text{ in./s}^2$$ **Ans.**

16–6.

A wheel has an initial clockwise angular velocity of 10 rad/s and a constant angular acceleration of 3 rad/s^2. Determine the number of revolutions it must undergo to acquire a clockwise angular velocity of 15 rad/s. What time is required?

SOLUTION

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$(15)^2 = (10)^2 + 2(3)(\theta - 0)$$

$$\theta = 20.83 \text{ rad} = 20.83\left(\frac{1}{2\pi}\right) = 3.32 \text{ rev.} \qquad \textbf{Ans.}$$

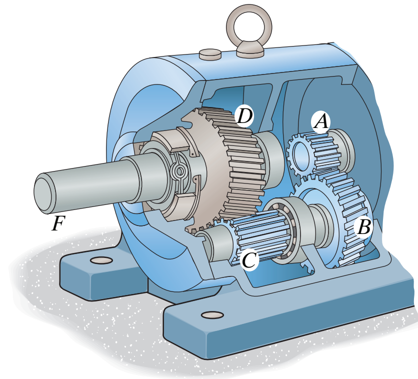
$$\omega = \omega_0 + \alpha_c t$$

$$15 = 10 + 3t$$

$$t = 1.67 \text{ s} \qquad \textbf{Ans.}$$

16–7.

If gear A rotates with a constant angular acceleration of $\alpha_A = 90\ \text{rad/s}^2$, starting from rest, determine the time required for gear D to attain an angular velocity of 600 rpm. Also, find the number of revolutions of gear D to attain this angular velocity. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



SOLUTION

Gear B is in mesh with gear A. Thus,

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{15}{50}\right)(90) = 27\ \text{rad/s}^2$$

Since gears C and B share the same shaft, $\alpha_C = \alpha_B = 27\ \text{rad/s}^2$. Also, gear D is in mesh with gear C. Thus,

$$\alpha_D r_D = \alpha_C r_C$$

$$\alpha_D = \left(\frac{r_C}{r_D}\right)\alpha_C = \left(\frac{25}{75}\right)(27) = 9\ \text{rad/s}^2$$

The final angular velocity of gear D is $\omega_D = \left(\frac{600\ \text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 20\pi\ \text{rad/s}$. Applying the constant acceleration equation,

$$\omega_D = (\omega_D)_0 + \alpha_D t$$

$$20\pi = 0 + 9t$$

$$t = 6.98\ \text{s} \qquad \textbf{Ans.}$$

and

$$\omega_D^{\,2} = (\omega_D)_0^{\,2} + 2\alpha_D[\theta_D - (\theta_D)_0]$$

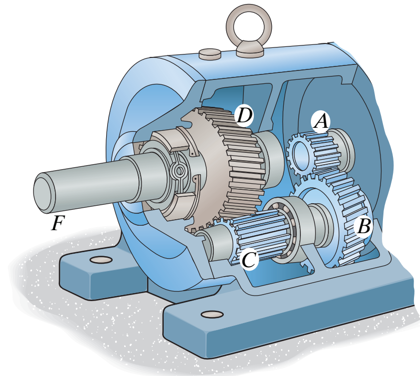
$$(20\pi)^2 = 0^2 + 2(9)(\theta_D - 0)$$

$$\theta_D = (219.32\ \text{rad})\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)$$

$$= 34.9\ \text{rev} \qquad \textbf{Ans.}$$

***16–8.**

If gear A rotates with an angular velocity of $\omega_A = (\theta_A + 1)$ rad/s, where θ_A is the angular displacement of gear A, measured in radians, determine the angular acceleration of gear D when $\theta_A = 3$ rad, starting from rest. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



SOLUTION

Motion of Gear A:

$$\alpha_A d\theta_A = \omega_A d\omega_A$$

$$\alpha_A d\theta_A = (\theta_A + 1)\, d(\theta_A + 1)$$

$$\alpha_A d\theta_A = (\theta_A + 1)\; d\theta_A$$

$$\alpha_A = (\theta_A + 1)$$

At $\theta_A = 3$ rad,

$$\alpha_A = 3 + 1 = 4 \text{ rad/s}^2$$

Motion of Gear D: Gear A is in mesh with gear B. Thus,

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{15}{50}\right)(4) = 1.20 \text{ rad/s}^2$$

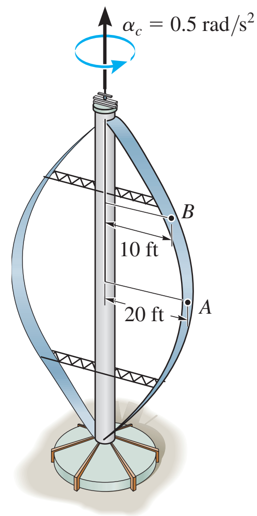
Since gears C and B share the same shaft $\alpha_C = \alpha_B = 1.20 \text{ rad/s}^2$. Also, gear D is in mesh with gear C. Thus,

$$\alpha_D r_D = \alpha_C r_C$$

$$\alpha_D = \left(\frac{r_C}{r_D}\right)\alpha_C = \left(\frac{25}{75}\right)(1.20) = 0.4 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

16–9.

The vertical-axis windmill consists of two blades that have a parabolic shape. If the blades are originally at rest and begin to turn with a constant angular acceleration of $\alpha_c = 0.5\ \text{rad/s}^2$, determine the magnitude of the velocity and acceleration of points A and B on the blade after the blade has rotated through two revolutions.



SOLUTION

Angular Motion: The angular velocity of the blade after the blade has rotated $2(2\pi) = 4\pi$ rad can be obtained by applying Eq. 16–7.

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = 0^2 + 2(0.5)(4\pi - 0)$$

$$\omega = 3.545\ \text{rad/s}$$

Motion of A and B: The magnitude of the velocity of point A and B on the blade can be determined using Eq. 16–8.

$$v_A = \omega r_A = 3.545(20) = 70.9\ \text{ft/s} \qquad \textbf{Ans.}$$

$$v_B = \omega r_B = 3.545(10) = 35.4\ \text{ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of point A and B can be determined using Eqs. 16–11 and 16–12 respectively.

$$(a_t)_A = \alpha r_A = 0.5(20) = 10.0\ \text{ft/s}^2$$

$$(a_n)_A = \omega^2 r_A = \left(3.545^2\right)(20) = 251.33\ \text{ft/s}^2$$

$$(a_t)_B = \alpha r_B = 0.5(10) = 5.00\ \text{ft/s}^2$$

$$(a_n)_B = \omega^2 r_B = \left(3.545^2\right)(10) = 125.66\ \text{ft/s}^2$$

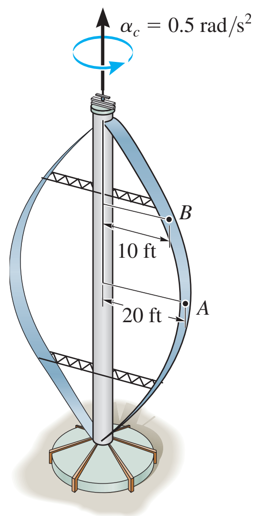
The magnitude of the acceleration of points A and B are

$$(a)_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{10.0^2 + 251.33^2} = 252\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$(a)_B = \ = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{5.00^2 + 125.66^2} = 126\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

16–10.

The vertical-axis windmill consists of two blades that have a parabolic shape. If the blades are originally at rest and begin to turn with a constant angular acceleration of $\alpha_c = 0.5\ \text{rad/s}^2$, determine the magnitude of the velocity and acceleration of points A and B on the blade when $t = 4$ s.



SOLUTION

Angular Motion: The angular velocity of the blade at $t = 4$ s can be obtained by applying Eq. 16–5.

$$\omega = \omega_0 + \alpha_c t = 0 + 0.5(4) = 2.00\ \text{rad/s}$$

Motion of A and B: The magnitude of the velocity of points A and B on the blade can be determined using Eq. 16–8.

$$v_A = \omega r_A = 2.00(20) = 40.0\ \text{ft/s} \qquad \textbf{Ans.}$$

$$v_B = \omega r_B = 2.00(10) = 20.0\ \text{ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of points A and B can be determined using Eqs. 16–11 and 16–12 respectively.

$$(a_t)_A = \alpha r_A = 0.5(20) = 10.0\ \text{ft/s}^2$$

$$(a_n)_A = \omega^2 r_A = \left(2.00^2\right)(20) = 80.0\ \text{ft/s}^2$$

$$(a_t)_B = \alpha r_B = 0.5(10) = 5.00\ \text{ft/s}^2$$

$$(a_n)_B = \omega^2 r_B = \left(2.00^2\right)(10) = 40.0\ \text{ft/s}^2$$

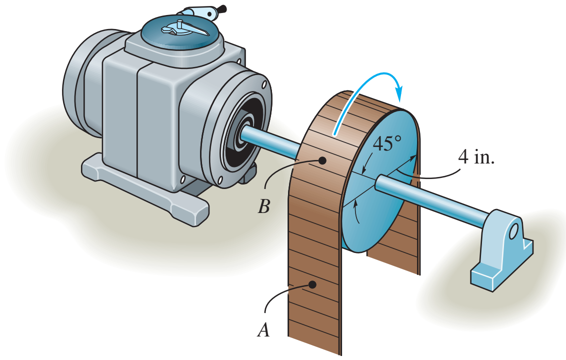
The magnitude of the acceleration of points A and B are

$$(a)_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{10.0^2 + 80.0^2} = 80.6\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$(a)_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{5.00^2 + 40.0^2} = 40.3\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

16–11.

If the angular velocity of the drum is increased uniformly from $6\ \text{rad/s}$ when $t = 0$ to $12\ \text{rad/s}$ when $t = 5\ \text{s}$, determine the magnitudes of the velocity and acceleration of points A and B on the belt when $t = 1$ s. At this instant the points are located as shown.



SOLUTION

$$\omega = \omega_0 + \alpha_c t$$

$$12 = 6 + \alpha(5) \qquad \alpha = 1.2\ \text{rad/s}^2$$

At $t = 1$ s,

$$\omega = 6 + 1.2(1) = 7.2\ \text{rad/s}$$

$$v_A = v_B = \omega\, r = 7.2\left(\frac{4}{12}\right) = 2.4\ \text{ft/s} \qquad \textbf{Ans.}$$

$$a_A = \alpha r = 1.2\left(\frac{4}{12}\right) = 0.4\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

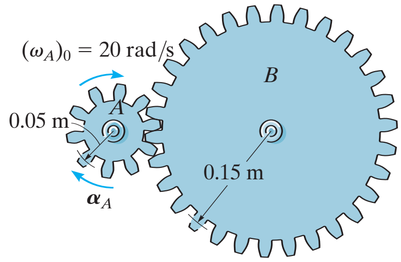
$$(a_B)_t = \alpha r = 1.2\left(\frac{4}{12}\right) = 0.4\ \text{ft/s}^2$$

$$(a_B)_n = \omega^2 r = (7.2)^2\left(\frac{4}{12}\right) = 17.28\ \text{ft/s}^2$$

$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{0.4^2 + 17.28^2} = 17.3\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

***16–12.**

A motor gives gear A an angular acceleration of $\alpha_A = (0.25\theta^3 + 0.5)\ \text{rad/s}^2$, where θ is in radians. If this gear is initially turning at $(\omega_A)_0 = 20\ \text{rad/s}$, determine the angular velocity of gear B after A undergoes an angular displacement of 10 rev.



SOLUTION

$$\alpha_A = 0.25\,\theta^3 + 0.5$$

$$\alpha\,d\theta = \omega\,d\omega$$

$$\int_0^{20\pi} (0.25\,\theta^3 + 0.5)d\theta_A = \int_{20}^{\omega_A} \omega_A\,d\omega_A$$

$$(0.0625\,\theta^4 + 0.5\,\theta)\Big|_0^{20\pi} = \frac{1}{2}(\omega_A)^2\Big|_{20}^{\omega_A}$$

$$\omega_A = 1395.94\ \text{rad/s}$$

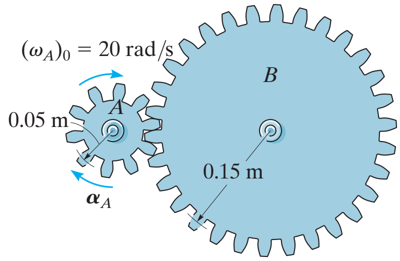
$$\omega_A\,r_A = \omega_B\,r_B$$

$$1395.94(0.05) = \omega_B\,(0.15)$$

$$\omega_B = 465\ \text{rad/s} \qquad \textbf{Ans.}$$

16–13.

A motor gives gear A an angular acceleration of $\alpha_A = (4t^3)\ \text{rad/s}^2$, where t is in seconds. If this gear is initially turning at $(\omega_A)_0 = 20\ \text{rad/s}$, determine the angular velocity of gear B when $t = 2$ s.



SOLUTION

$$\alpha_A = 4\,t^3$$

$$d\omega = \alpha\, dt$$

$$\int_{20}^{\omega_A} d\omega_A = \int_0^t \alpha_A\, dt = \int_0^t 4\,t^3 dt$$

$$\omega_A = t^4 + 20$$

When $t = 2$ s,

$$\omega_A = 36\ \text{rad/s}$$

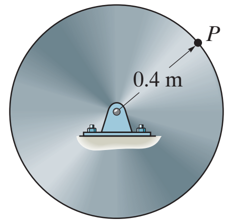
$$\omega_A\, r_A = \omega_B\, r_B$$

$$36(0.05) = \omega_B\,(0.15)$$

$$\omega_B = 12\ \text{rad/s} \qquad \textbf{Ans.}$$

16–14.

The disk starts from rest and is given an angular acceleration $\alpha = (2t^2)\ \text{rad/s}^2$, where t is in seconds. Determine the angular velocity of the disk and its angular displacement when $t = 4$ s.



SOLUTION

$$\alpha = \frac{d\omega}{dt} = 2t^2$$

$$\int_0^\omega d\omega = \int_0^t 2t^2 dt$$

$$\omega = \frac{2}{3}t^3\Big|_0^t$$

$$\omega = \frac{2}{3}t^3$$

When $t = 4$ s,

$$\omega = \frac{2}{3}(4)^3 = 42.7\ \text{rad/s}$$ **Ans.**

$$\int_0^\theta d\theta = \int_0^t \frac{2}{3}t^3 dt$$

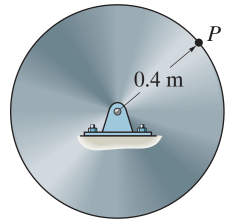
$$\theta = \frac{1}{6}t^4$$

When $t = 4$ s,

$$\theta = \frac{1}{6}(4)^4 = 42.7\ \text{rad}$$ **Ans.**

16–15.

The disk starts from rest and is given an angular acceleration $\alpha = (5t^{1/2})$ rad/s^2, where t is in seconds. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $t = 2$ s.



SOLUTION

Motion of the Disk: Here, when $t = 0, \omega = 0$.

$$d\omega = adt$$

$$\int_0^\omega d\omega = \int_0^t 5t^{\frac{1}{2}}dt$$

$$\omega\Big|_0^\omega = \frac{10}{3}t^{\frac{3}{2}}\Big|_0^t$$

$$\omega = \left\{\frac{10}{3}t^{\frac{3}{2}}\right\} \text{ rad/s}$$

When $t = 2$ s,

$$\omega = \frac{10}{3}\left(2^{\frac{3}{2}}\right) = 9.428 \text{ rad/s}$$

When $t = 2$ s,

$$\alpha = 5\left(2^{\frac{1}{2}}\right) = 7.071 \text{ rad/s}^2$$

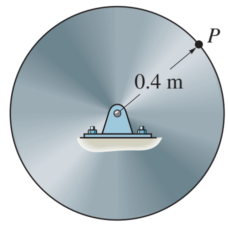
Motion of point P: The tangential and normal components of the acceleration of point P when $t = 2$ s are

$$a_t = \alpha r = 7.071(0.4) = 2.83 \text{ m/s}^2$$ **Ans.**

$$a_n = \omega^2 r = 9.428^2(0.4) = 35.6 \text{ m/s}^2$$ **Ans.**

***16–16.**

The disk starts at $\omega_0 = 1$ rad/s when $\theta = 0$, and is given an angular acceleration $\alpha = (0.3\theta)$ rad/s^2, where θ is in radians. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $\theta = 1$ rev.



SOLUTION

$$\alpha = 0.3\theta$$

$$\int_1^\omega \omega d\omega = \int_0^\theta 0.3\theta d\theta$$

$$\frac{1}{2}\omega^2\Big|_1^\omega = 0.15\theta^2\Big|_0^\theta$$

$$\frac{\omega^2}{2} - 0.5 = 0.15\theta^2$$

$$\omega = \sqrt{0.3\theta^2 + 1}$$

At $\theta = 1 \text{ rev} = 2\pi$ rad

$$\omega = \sqrt{0.3(2\pi)^2 + 1}$$

$$\omega = 3.584 \text{ rad/s}$$

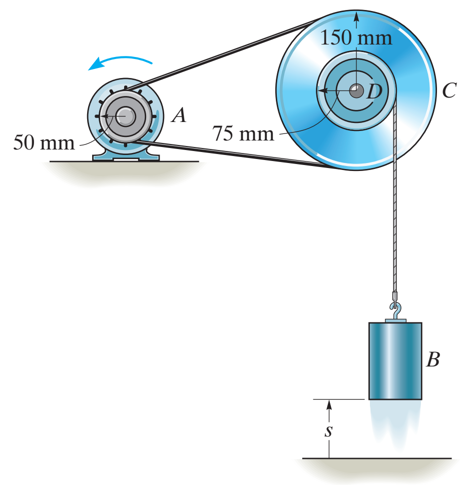
$$a_t = \alpha r = 0.3(2\pi) \text{ rad/s}^2(0.4 \text{ m}) = 0.7540 \text{ m/s}^2$$ **Ans.**

$$a_n = \omega^2 r = (3.584 \text{ rad/s})^2(0.4 \text{ m}) = 5.137 \text{ m/s}^2$$ **Ans.**

$$a_p = \sqrt{(0.7540)^2 + (5.137)^2} = 5.19 \text{ m/s}^2$$

16–17.

Starting at $(\omega_A)_0 = 3$ rad/s, when $\theta = 0$, $s = 0$, pulley A is given an angular acceleration $\alpha = (0.6\theta)$ rad/s², where θ is in radians. Determine the speed of block B when it has risen $s = 0.5$ m. The pulley has an inner hub D which is fixed to C and turns with it.



SOLUTION

$$\alpha_a = 0.6\theta_A$$

$$\theta_C = \frac{0.5}{0.075} = 6.667 \text{ rad}$$

$$\theta_A(0.05) = (6.667)(0.15)$$

$$\theta_A = 20 \text{ rad}$$

$$\alpha d\theta = \omega d\omega$$

$$\int_0^{20} 0.6\theta_A d\theta_A = \int_3^{\omega_A} \omega_A d\omega_A$$

$$0.3\theta_A^2\Big|_0^{20} = \frac{1}{2}\omega_A^2\Big|_3^{\omega_A}$$

$$120 = \frac{1}{2}\omega_A^2 - 4.5$$

$$\omega_A = 15.780 \text{ rad/s}$$

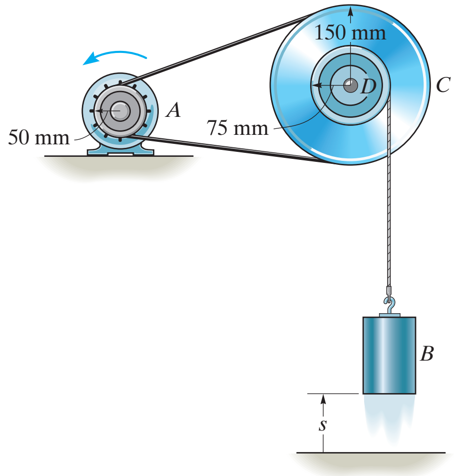
$$15.780(0.05) = \omega_C(0.15)$$

$$\omega_C = 5.260 \text{ rad/s}$$

$$v_B = 5.260(0.075) = 0.394 \text{ m/s}$$ **Ans.**

16–18.

Starting from rest when $s = 0$, pulley A is given a constant angular acceleration $\alpha_c = 6\ \text{rad/s}^2$. Determine the speed of block B when it has risen $s = 6$ m. The pulley has an inner hub D which is fixed to C and turns with it.



SOLUTION

$$\alpha_A\, r_A = \alpha_C\, r_C$$

$$6(50) = \alpha_C(150)$$

$$\alpha_C = 2\ \text{rad/s}^2$$

$$a_B = \alpha_C\, r_B = 2(0.075) = 0.15\ \text{m/s}^2$$

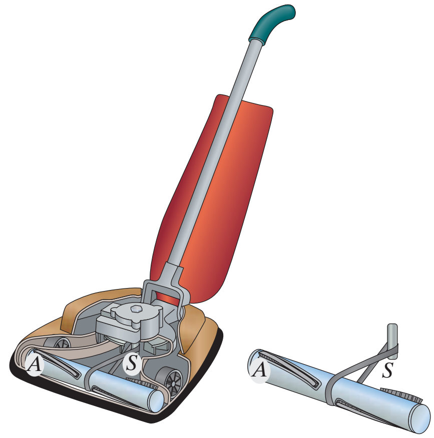
$$(+\uparrow) \qquad v^2 = v_0^2 + 2\, a_c\,(s - s_0)$$

$$v^2 = 0 + 2(0.15)(6 - 0)$$

$$v = 1.34\ \text{m/s} \qquad \textbf{Ans.}$$

16–19.

The vacuum cleaner's armature shaft S rotates with an angular acceleration of $\alpha = 4\omega^{3/4}$ rad/s², where ω is in rad/s. Determine the brush's angular velocity when $t = 4$ s, starting from $\omega_0 = 1$ rad/s, at $\theta = 0$. The radii of the shaft and the brush are 0.25 in. and 1 in., respectively. Neglect the thickness of the drive belt.



SOLUTION

Motion of the Shaft: The angular velocity of the shaft can be determined from

$$\int dt = \int \frac{d\omega_S}{\alpha_S}$$

$$\int_0^t dt = \int_1^{\omega_s} \frac{d\omega_S}{4\omega_S{}^{3/4}}$$

$$t\Big|_0^t = \omega_S{}^{1/4}\Big|_1^{\omega_s}$$

$$t = \omega_S{}^{1/4} - 1$$

$$\omega_S = (t + 1)^4$$

When $t = 4$ s

$$\omega_s = 5^4 = 625 \text{ rad/s}$$

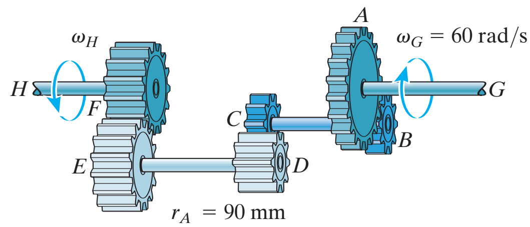
Motion of the Beater Brush: Since the brush is connected to the shaft by a non-slip belt, then

$$\omega_B r_B = \omega_s r_s$$

$$\omega_B = \left(\frac{r_s}{r_B}\right)\omega_s = \left(\frac{0.25}{1}\right)(625) = 156 \text{ rad/s} \qquad \textbf{Ans.}$$

***16–20.**

The operation of "reverse" for a three-speed automotive transmission is illustrated schematically in the figure. If the crank shaft G is turning with an angular speed of 60 rad/s, determine the angular speed of the drive shaft H. Each of the gears rotates about a fixed axis. Note that gears A and B, C and D, E and F are in mesh. The radii of each of these gears are reported in the figure.



$r_A = 90$ mm
$r_B = r_C = 30$ mm
$r_D = 50$ mm
$r_E = 70$ mm
$r_F = 60$ mm

SOLUTION

$$60(90) = \omega_{BC}(30)$$

$$\omega_{BC} = 180 \text{ rad/s}$$

$$180(30) = 50(\omega_{DE})$$

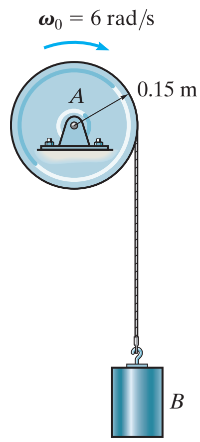
$$\omega_{DE} = 108 \text{ rad/s}$$

$$108(70) = (60)\,(\omega_H)$$

$$\omega_H = 126 \text{ rad/s}$$ **Ans.**

16–21.

A motor gives disk A an angular acceleration of $\alpha_A = (0.6t^2 + 0.75)\ \text{rad/s}^2$, where t is in seconds. If the initial angular velocity of the disk is $\omega_0 = 6\ \text{rad/s}$, determine the magnitudes of the velocity and acceleration of block B when $t = 2$ s.



SOLUTION

$$d\omega = \alpha\, dt$$

$$\int_6^{\omega} d\omega = \int_0^2 (0.6\, t^2 + 0.75)\, dt$$

$$\omega - 6 = (0.2\, t^3 + 0.75\, t)\Big|_0^2$$

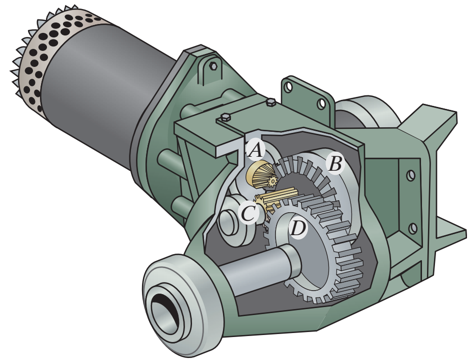
$$\omega = 9.10\ \text{rad/s}$$

$$v_B = \omega r = 9.10(0.15) = 1.37\ \text{m/s}$$ **Ans.**

$$a_B = a_t = \alpha r = [0.6(2)^2 + 0.75](0.15) = 0.472\ \text{m/s}^2$$ **Ans.**

16–22.

For a short time the motor turns gear A with an angular acceleration of $\alpha_A = (30t^{1/2})$ rad/s^2, where t is in seconds. Determine the angular velocity of gear D when $t = 5$ s, starting from rest. Gear A is initially at rest. The radii of gears A, B, C, and D are $r_A = 25$ mm, $r_B = 100$ mm, $r_C = 40$ mm, and $r_D = 100$ mm, respectively.



SOLUTION

Motion of the Gear A: The angular velocity of gear A can be determined from

$$\int d\omega_A = \int \alpha dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t 30t^{1/2} dt$$

$$\omega_A\Big|_0^{\omega_A} = 20t^{3/2}\Big|_0^t$$

$$\omega_A = \left(20t^{3/2}\right) \text{ rad/s}$$

When $t = 5$ s

$$\omega_A = 20\left(5^{3/2}\right) = 223.61 \text{ rad/s}$$

Motion of Gears B, C, and D: Gears B and C which are mounted on the same axle will have the same angular velocity. Since gear B is in mesh with gear A, then

$$\omega_B r_B = \omega_A r_A$$

$$\omega_C = \omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{25}{100}\right)(223.61) = 55.90 \text{ rad/s}$$

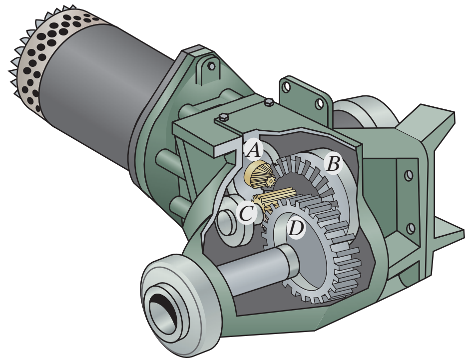
Also, gear D is in mesh with gear C. Then

$$\omega_D r_D = \omega_C r_C$$

$$\omega_D = \left(\frac{r_C}{r_D}\right)\omega_C = \left(\frac{40}{100}\right)(55.90) = 22.4 \text{ rad/s}$$ **Ans.**

16–23.

The motor turns gear A so that its angular velocity increases uniformly from zero to 3000 rev/min after the shaft turns 200 rev. Determine the angular velocity of gear D when $t = 3$ s. The radii of gears A, B, C, and D are $r_A = 25$ mm, $r_B = 100$ mm, $r_C = 40$ mm, and $r_D = 100$ mm, respectively.



SOLUTION

Motion of Wheel A: Here, $\omega_A = \left(3000\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{rev}}\right) = 100\pi\text{ rad/s}$

when $\theta_A = (200\text{ rev})\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) = 400\pi$ rad. Since the angular acceleration of gear A is constant, it can be determined from

$$\omega_A{}^2 = (\omega_A)_0{}^2 + 2\alpha_A\left[\theta_A - (\theta_A)_0\right]$$

$$(100\pi)^2 = 0^2 + 2\alpha_A(400\pi - 0)$$

$$\alpha_A = 39.27\text{ rad/s}^2$$

Thus, the angular velocity of gear A when $t = 3$ s is

$$\omega_A = (\omega_A)_0 + \alpha_A t$$

$$= 0 + 39.27(3)$$

$$= 117.81\text{ rad/s}$$

Motion of Gears B, C, and D: Gears B and C which are mounted on the same axle will have the same angular velocity. Since gear B is in mesh with gear A, then

$$\omega_B r_B = \omega_B r_A$$

$$\omega_C = \omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{25}{100}\right)(117.81) = 29.45\text{ rad/s}$$

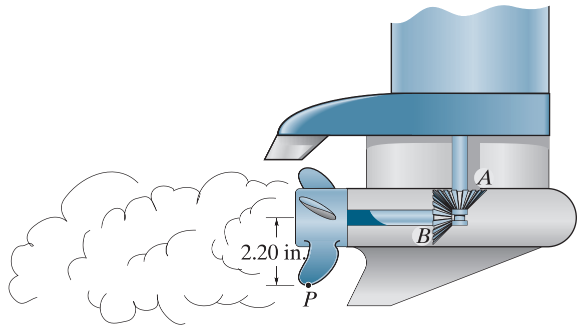
Also, gear D is in mesh with gear C. Then

$$\omega_D r_D = \omega_C r_C$$

$$\omega_D = \left(\frac{r_C}{r_D}\right)\omega_C = \left(\frac{40}{100}\right)(29.45) = 11.8\text{ rad/s}$$ **Ans.**

***16–24.**

The gear A on the drive shaft of the outboard motor has a radius $r_A = 0.5$ in. and the meshed pinion gear B on the propeller shaft has a radius $r_B = 1.2$ in. Determine the angular velocity of the propeller in $t = 1.5$ s, if the drive shaft rotates with an angular acceleration $\alpha = (400t^3)$ rad/s^2, where t is in seconds. The propeller is originally at rest and the motor frame does not move.



SOLUTION

Angular Motion: The angular velocity of gear A at $t = 1.5$ s must be determined first. Applying Eq. 16–2, we have

$$d\omega = \alpha dt$$

$$\int_0^{\omega_A} d\omega = \int_0^{1.5\,s} 400t^3\, dt$$

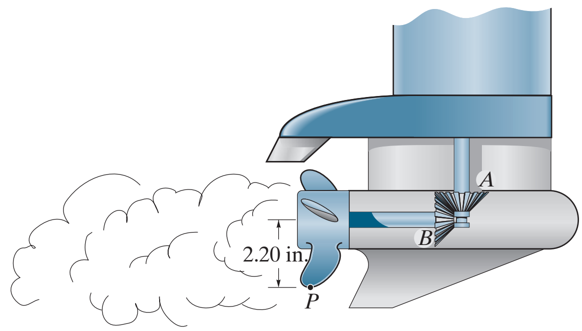
$$\omega_A = 100t^4\big|_0^{1.5\,s} = 506.25 \text{ rad/s}$$

However, $\omega_A r_A = \omega_B r_B$ where ω_B is the angular velocity of propeller. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{0.5}{1.2}\right)(506.25) = 211 \text{ rad/s} \qquad \textbf{Ans.}$$

16–25.

For the outboard motor in Prob. 16–24, determine the magnitude of the velocity and acceleration of point P located on the tip of the propeller at the instant $t = 0.75$ s.



SOLUTION

Angular Motion: The angular velocity of gear A at $t = 0.75$ s must be determined first. Applying Eq. 16–2, we have

$$d\omega = \alpha dt$$

$$\int_0^{\omega_A} d\omega = \int_0^{0.75\,s} 400t^3\,dt$$

$$\omega_A = 100t^4\Big|_0^{0.75\,s} = 31.64 \text{ rad/s}$$

The angular acceleration of gear A at $t = 0.75$ s is given by

$$\alpha_A = 400\left(0.75^3\right) = 168.75 \text{ rad/s}^2$$

However, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$ where ω_B and α_B are the angular velocity and acceleration of propeller. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{0.5}{1.2}\right)(31.64) = 13.18 \text{ rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{0.5}{1.2}\right)(168.75) = 70.31 \text{ rad/s}^2$$

Motion of P: The magnitude of the velocity of point P can be determined using Eq. 16–8.

$$v_P = \omega_B r_P = 13.18\left(\frac{2.20}{12}\right) = 2.42 \text{ ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of point P can be determined using Eqs. 16–11 and 16–12, respectively.

$$a_r = \alpha_B r_P = 70.31\left(\frac{2.20}{12}\right) = 12.89 \text{ ft/s}^2$$

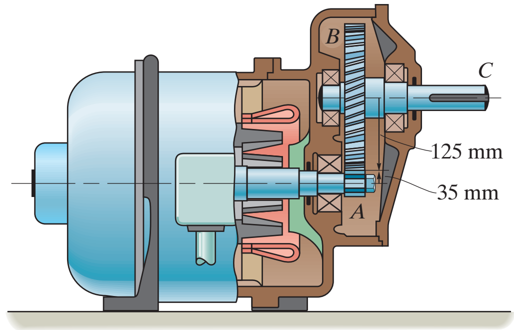
$$a_n = \omega_B^2 r_P = \left(13.18^2\right)\left(\frac{2.20}{12}\right) = 31.86 \text{ ft/s}^2$$

The magnitude of the acceleration of point P is

$$a_P = \sqrt{a_r^2 + a_n^2} = \sqrt{12.89^2 + 31.86^2} = 34.4 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

16–26.

The pinion gear A on the motor shaft is given a constant angular acceleration $\alpha = 3\ \text{rad/s}^2$. If the gears A and B have the dimensions shown, determine the angular velocity and angular displacement of the output shaft C, when $t = 2$ s starting from rest. The shaft is fixed to B and turns with it.



SOLUTION

$$\omega = \omega_0 + \alpha_c t$$

$$\omega_A = 0 + 3(2) = 6\ \text{rad/s}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$\theta_A = 0 + 0 + \frac{1}{2}(3)(2)^2$$

$$\theta_A = 6\ \text{rad}$$

$$\omega_A r_A = \omega_B r_B$$

$$6(35) = \omega_B(125)$$

$$\omega_C = \omega_B = 1.68\ \text{rad/s}$$ **Ans.**

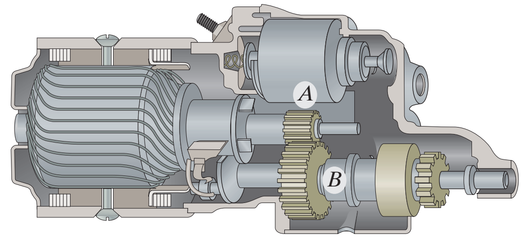
$$\theta_A r_A = \theta_B r_B$$

$$6(35) = \theta_B(125)$$

$$\theta_C = \theta_B = 1.68\ \text{rad}$$ **Ans.**

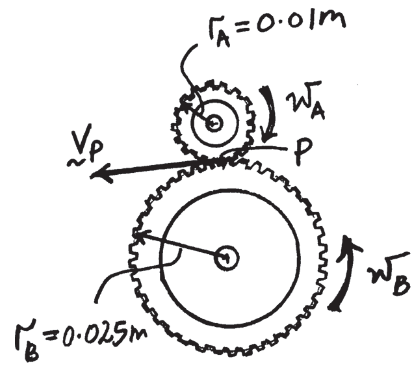
16–27.

For a short time, gear A of the automobile starter rotates with an angular acceleration of $\alpha_A = (450t^2 + 60)$ rad/s^2, where t is in seconds. Determine the angular velocity and angular displacement of gear B when $t = 2$ s, starting from rest. The radii of gears A and B are 10 mm and 25 mm, respectively.



SOLUTION

Motion of Gear A: Applying the kinematic equation of variable angular acceleration,



(a)

$$\int d\omega_A = \int \alpha_A dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t \left(450t^2 + 60\right)dt$$

$$\omega_A\Big|_0^{\omega_A} = 150t^3 + 60t\Big|_0^t$$

$$\omega_A = \left(150t^3 + 60t\right) \text{ rad/s}$$

When $t = 2$ s,

$$\omega_A = 150(2)^3 + 60(2) = 1320 \text{ rad/s}$$

$$\int d\theta_A = \int \omega_A\, dt$$

$$\int_0^{\theta_A} d\theta_A = \int_0^t \left(150t^3 + 60t\right)dt$$

$$\theta_A\Big|_0^{\theta_A} = 37.5t^4 + 30t^2\Big|_0^t$$

$$\theta_A = \left(37.5t^4 + 30t^2\right) \text{ rad}$$

When $t = 2$ s

$$\theta_A = 37.5(2)^4 + 30(2)^2 = 720 \text{ rad}$$

Motion of Gear B: Since gear B is meshed with gear A, Fig. a, then

$$v_p = \omega_A r_A = \omega_B r_B$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right)$$

$$= (1320)\left(\frac{0.01}{0.025}\right)$$

$$= 528 \text{ rad/s} \qquad \textbf{Ans.}$$

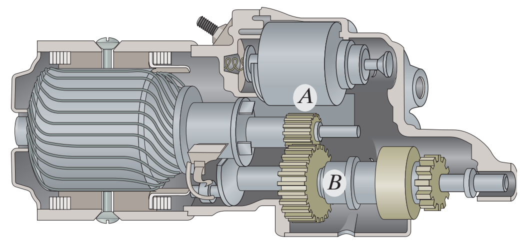
$$\theta_B = \theta_A\left(\frac{r_A}{r_B}\right)$$

$$= 720\left(\frac{0.01}{0.025}\right)$$

$$= 288 \text{ rad} \qquad \textbf{Ans.}$$

***16–28.**

For a short time, gear A of the automobile starter rotates with an angular acceleration of $\alpha_A = (50\omega^{1/2})$ rad/s^2, where ω is in rad/s. Determine the angular velocity of gear B when $t = 1$ s. Orginally $(\omega_A)_0 = 1$ rad/s when $t = 0$. The radii of gears A and B are 10 mm and 25 mm, respectively.



SOLUTION

Motion of Gear A: We have

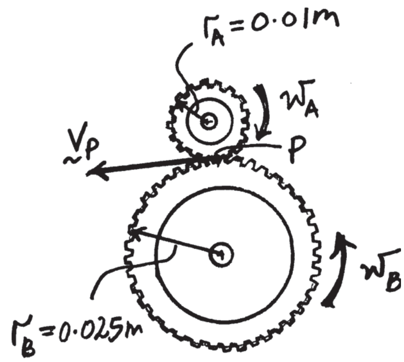
$$\int dt = \int \frac{d\omega_A}{\alpha_A}$$

$$\int_0^t dt = \int_1^{\omega_A} \frac{d\omega_A}{50\omega_A{}^{1/2}}$$

$$t\Big|_0^t = \frac{1}{25}\omega_A{}^{1/2}\Big|_1^{\omega_A}$$

$$t = \frac{1}{25}\omega_A{}^{1/2} - \frac{1}{25}$$

$$\omega_A = (25t + 1)^2$$



(a)

When $t = 1$ s, $\omega_A = 676$ rad/s

Motion of Gear B: Since gear B is meshed with gear A, Fig. a, then

$$v_p = \omega_A r_A = \omega_B r_B$$

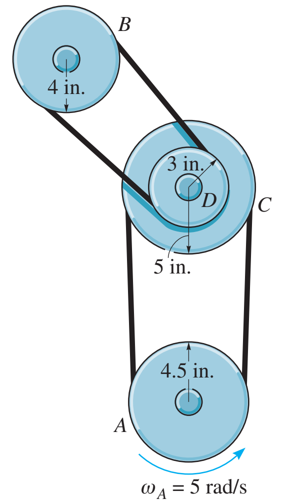
$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right)$$

$$= 676\left(\frac{0.01}{0.025}\right)$$

$$= 270 \text{ rad/s} \qquad \textbf{Ans.}$$

16–29.

A mill in a textile plant uses the belt-and-pulley arrangement shown to transmit power. When $t = 0$ an electric motor is turning pulley A with an angular velocity of $\omega_A = 5\text{ rad/s}$. If this pulley is subjected to a constant angular acceleration 2 rad/s^2, determine the angular velocity of pulley B after B turns 6 revolutions. The hub at D is rigidly *connected* to pulley C and turns with it.



SOLUTION

When $\theta_B = 6$ rev;

$$4(6) = 3\,\theta_C$$

$$\theta_C = 8\text{ rev}$$

$$8(5) = 4.5(\theta_A)$$

$$\theta_A = 8.889\text{ rev}$$

$$(\omega_A)_2^2 = (\omega_A)_1^2 + 2\alpha_C[(\theta_A)_2 - (\theta_A)_1]$$

$$(\omega_A)_2^2 = (5)^2 + 2(2)[(8.889)(2\pi) - 0]$$

$$(\omega_A)_2 = 15.76\text{ rad/s}$$

$$15.76(4.5) = 5\omega_C$$

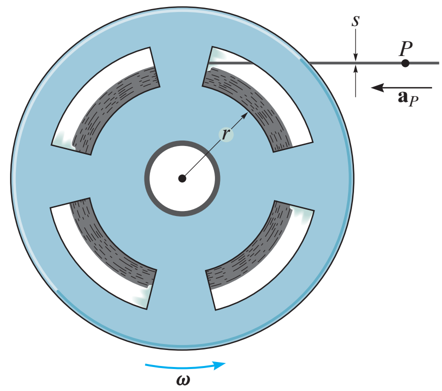
$$\omega_C = 14.18\text{ rad/s}$$

$$14.18(3) = 4(\omega_B)_2$$

$$(\omega_B)_2 = 10.6\text{ rad/s} \qquad \textbf{Ans.}$$

16–30.

A tape having a thickness s wraps around the wheel which is turning at a constant rate $\boldsymbol{\omega}$. Assuming the unwrapped portion of tape remains horizontal, determine the acceleration of point P of the unwrapped tape when the radius of the wrapped tape is r. *Hint*: Since $v_P = \omega r$, take the time derivative and note that $dr/dt = \omega(s/2\pi)$.



SOLUTION

$$v_P = \omega r$$

$$a = \frac{dv_P}{dt} = \frac{d\omega}{dt} r + \omega \frac{dr}{dt}$$

Since $\dfrac{d\omega}{dt} = 0$,

$$a = \omega\left(\frac{dr}{dt}\right)$$

In one revolution r is increased by s, so that

$$\frac{2\pi}{\theta} = \frac{s}{\Delta r}$$

Hence,

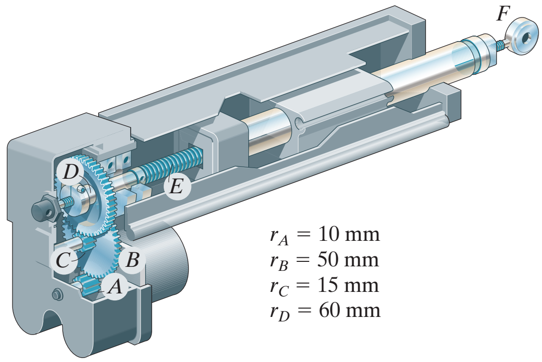
$$\Delta r = \frac{s}{2\pi}\theta$$

$$\frac{dr}{dt} = \frac{s}{2\pi}\omega$$

$$a = \frac{s}{2\pi}\omega^2$$ **Ans.**

16–31.

Due to the screw at E, the actuator provides linear motion to the arm at F when the motor turns the gear at A. If the gears have the radii listed in the figure, and the screw at E has a pitch $p = 2$ mm, determine the speed at F when the motor turns A at $\omega_A = 20$ rad/s. *Hint*: The screw pitch indicates the amount of advance of the screw for each full revolution.



SOLUTION

$$\omega_A r_A = \omega_B r_B$$

$$\omega_C r_C = \omega_D r_D$$

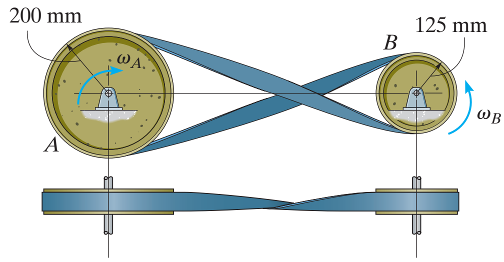
Thus,

$$\omega_D = \frac{r_A}{r_B}\frac{r_C}{r_D}\omega_A = \frac{10}{50}\frac{15}{60}20 = 1 \text{ rad/s}$$

$$v_F = \frac{1 \text{ rad / s}}{2\pi \text{ rad}}\frac{1 \text{ rev}}{}(2 \text{ mm}) = 0.318 \text{ mm /s}$$ **Ans.**

***16–32.**

The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If A has a constant angular acceleration of $\alpha_A = 30\ \text{rad/s}^2$, determine the tangential and normal components of acceleration of a point located at the rim of B when $t = 3$ s, starting from rest.



SOLUTION

Motion of Wheel A: Since the angular acceleration of wheel A is constant, its angular velocity can be determined from

$$\omega_A = (\omega_A)_0 + \alpha_C t$$

$$= 0 + 30(3) = 90\ \text{rad/s}$$

Motion of Wheel B: Since wheels A and B are connected by a nonslip belt, then

$$\omega_B r_B = \omega_A r_A$$

$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{200}{125}\right)(90) = 144\ \text{rad/s}$$

and

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{200}{125}\right)(30) = 48\ \text{rad/s}^2$$

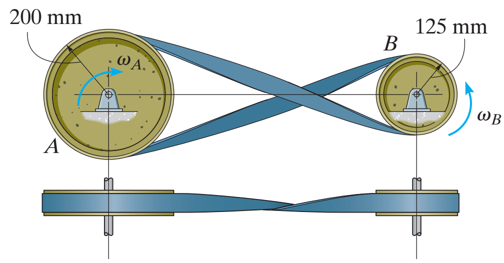
Thus, the tangential and normal components of the acceleration of point P located at the rim of wheel B are

$$(a_p)_t = \alpha_B r_B = 48(0.125) = 6\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$(a_p)_n = \omega_B{}^2 r_B = (144^2)(0.125) = 2592\ \text{m/s}^2 \qquad \textbf{Ans.}$$

16–33.

The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If the angular displacement of A is $\theta_A = (5t^3 + 10t^2)$ rad, where t is in seconds, determine the angular velocity and angular acceleration of B when $t = 3$ s.



SOLUTION

Motion of Wheel A: The angular velocity and angular acceleration of wheel A can be determined from

$$\omega_A = \frac{d\theta_A}{dt} = \left(15t^2 + 20t\right) \text{ rad/s}$$

and

$$\alpha_A = \frac{d\omega_A}{dt} = \left(30t + 20\right) \text{ rad/s}$$

When $t = 3$ s,

$$\omega_A = 15\left(3^2\right) + 20(3) = 195 \text{ rad/s}$$

$$\alpha_A = 30(3) + 20 = 110 \text{ rad/s}$$

Motion of Wheel B: Since wheels A and B are connected by a nonslip belt, then

$$\omega_B r_B = \omega_A r_A$$

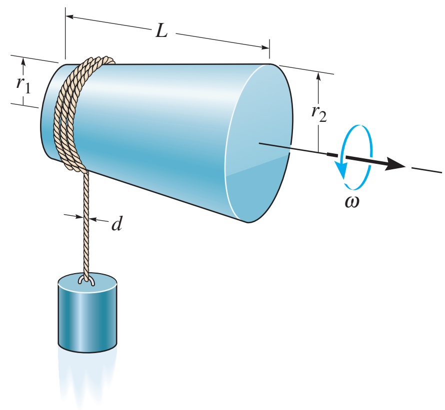
$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{200}{125}\right)(195) = 312 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{200}{125}\right)(110) = 176 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

16–34.

The rope of diameter d is wrapped around the tapered drum which has the dimensions shown. If the drum is rotating at a constant rate of ω, determine the upward acceleration of the the block. Neglect the small horizontal displacement of the block.



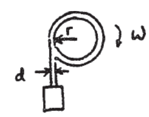
SOLUTION

$$v = \omega r$$

$$a = \frac{d(\omega r)}{dt}$$

$$= \frac{d\omega}{dt} r + \omega \frac{dr}{dt}$$

$$= \omega\left(\frac{dr}{dt}\right)$$



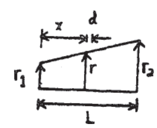
$$r = r_1 + \left(\frac{r_2 - r_1}{L}\right)x$$

$$dr = \left(\frac{r_2 - r_1}{L}\right)dx$$

But $dx = \dfrac{d\theta}{2\pi} \cdot d$

Thus $\dfrac{dr}{dt} = \dfrac{1}{2\pi}\left(\dfrac{r_2 - r_1}{L}\right)d\left(\dfrac{d\theta}{dt}\right)$

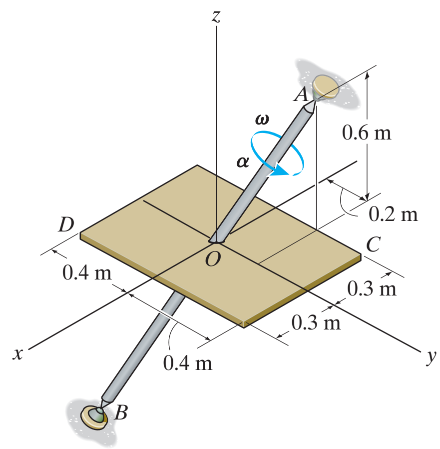
$$= \frac{1}{2\pi}\left(\frac{r_2 - r_1}{L}\right)d\omega$$



Thus, $a = \dfrac{\omega^2}{2\pi}\left(\dfrac{r_2 - r_1}{L}\right)d$ **Ans.**

16–35.

If the shaft and plate rotates with a constant angular velocity of $\omega = 14$ rad/s, determine the velocity and acceleration of point C located on the corner of the plate at the instant shown. Express the result in Cartesian vector form.



SOLUTION

We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

Since ω is constant

$$\alpha = 0$$

For convenience, $\mathbf{r}_C = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point C can be determined from

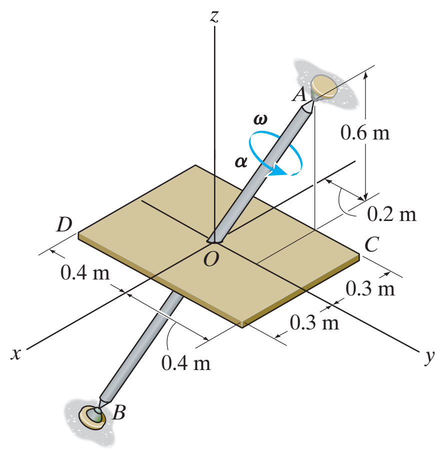
$$\begin{aligned}\mathbf{v}_C &= \omega \times \mathbf{r}_C \\ &= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j}) \\ &= [-4.8\mathbf{i} - 3.6\mathbf{j} - 1.2\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}\end{aligned}$$

and

$$\begin{aligned}\mathbf{a}_C &= \alpha \times \mathbf{r}_C + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_c) \\ &= 0 + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})] \\ &= [38.4\mathbf{i} - 64.8\mathbf{j} + 40.8\mathbf{k}]\text{m/s}^2 \qquad \textbf{Ans.}\end{aligned}$$

***16–36.**

At the instant shown, the shaft and plate rotates with an angular velocity of $\omega = 14$ rad/s and angular acceleration of $\alpha = 7$ rad/s^2. Determine the velocity and acceleration of point D located on the corner of the plate at this instant. Express the result in Cartesian vector form.



SOLUTION

We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω and α is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

$$\alpha = \alpha\mathbf{u}_{OA} = 7\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}] \text{ rad/s}$$

For convenience, $\mathbf{r}_D = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point D can be determined from

$$\mathbf{v}_D = \omega \times r_D$$

$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (0.3\mathbf{i} - 0.4\mathbf{j})$$

$$= [4.8\mathbf{i} + 3.6\mathbf{j} + 1.2\mathbf{k}]\text{m/s}$$ **Ans.**

and

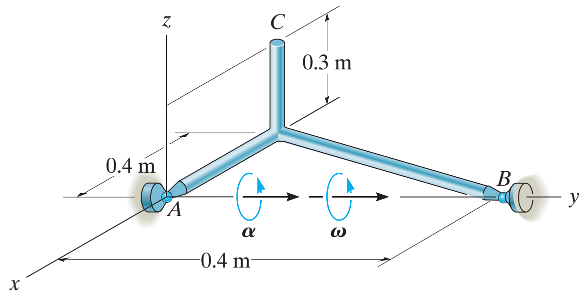
$$\mathbf{a}_D = \alpha \times \mathbf{r}_D - \omega^2 \mathbf{r}_D$$

$$= (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j}) + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$

$$= [-36.0\mathbf{i} + 66.6\mathbf{j} - 40.2\mathbf{k}]\text{m/s}^2$$ **Ans.**

16–37.

The rod assembly is supported by ball-and-socket joints at A and B. At the instant shown it is rotating about the y axis with an angular velocity $\omega = 5$ rad/s and has an angular acceleration $\alpha = 8$ rad/s^2. Determine the magnitudes of the velocity and acceleration of point C at this instant. Solve the problem using Cartesian vectors and Eqs. 16–9 and 16–13.



SOLUTION

$$v_C = \boldsymbol{\omega} \times \mathbf{r}$$

$$v_C = 5\mathbf{j} \times (-0.4\mathbf{i} + 0.3\mathbf{k}) = \{1.5\mathbf{i} + 2\mathbf{k}\}\text{ m/s}$$

$$v_C = \sqrt{1.5^2 + 2^2} = 2.50\text{ m/s} \qquad \textbf{Ans.}$$

$$\mathrm{a}_C = \mathrm{a} \times \mathbf{r} - \omega^2\mathbf{r}$$

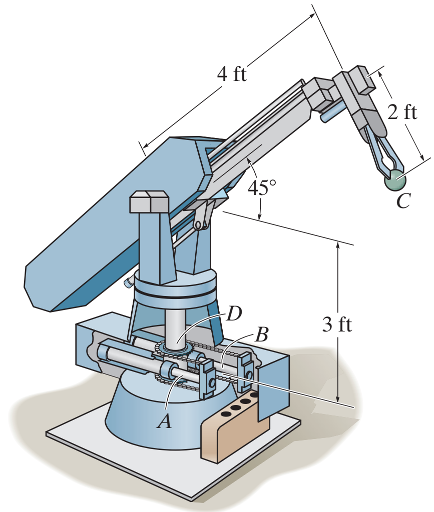
$$= 8\mathbf{j} \times (-0.4\mathbf{i} + 0.3\mathbf{k}) - 5^2(-0.4\mathbf{i} + 0.3\mathbf{k})$$

$$= \{12.4\mathbf{i} - 4.3\mathbf{k}\}\text{ m/s}^2$$

$$a_C = \sqrt{12.4^2 + (-4.3)^2} = 13.1\text{ m/s}^2 \qquad \textbf{Ans.}$$

16–38.

Rotation of the robotic arm occurs due to linear movement of the hydraulic cylinders A and B. If this motion causes the gear at D to rotate clockwise at 5 rad/s, determine the magnitude of velocity and acceleration of the part C held by the grips of the arm.



SOLUTION

Motion of Part C: Since the shaft that turns the robot's arm is attached to gear D, then the angular velocity of the robot's arm $\omega_R = \omega_D = 5.00$ rad/s. The distance of part C from the rotating shaft is $r_C = 4 \cos 45° + 2 \sin 45° = 4.243$ ft. The magnitude of the velocity of part C can be determined using Eq. 16–8.

$$v_C = \omega_R r_C = 5.00(4.243) = 21.2 \text{ ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of part C can be determined using Eqs. 16–11 and 16–12 respectively.

$$a_t = \alpha r_C = 0$$

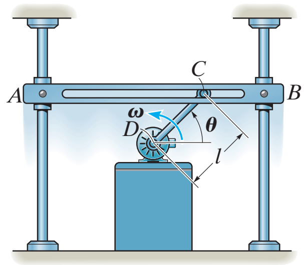
$$a_n = \omega_R^2 r_C = \left(5.00^2\right)(4.243) = 106.07 \text{ ft/s}^2$$

The magnitude of the acceleration of point C is

$$a_C = \sqrt{a_t^2 + a_n^2} = \sqrt{0^2 + 106.07^2} = 106 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

16–39.

The bar DC rotates uniformly about the shaft at D with a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity and acceleration of the bar AB, which is confined by the guides to move vertically.



SOLUTION

$$y = l \sin\theta$$

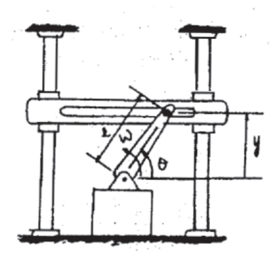
$$\dot{y} = v_y = l \cos\theta\dot{\theta}$$

$$\ddot{y} = a_y = l(\cos\theta\ddot{\theta} - \sin\theta\dot{\theta})$$

Here $v_y = v_{AB}$, $a_y = a_{AB}$, and $\dot{\theta} = \omega$, $\ddot{\theta} = \alpha = 0$.

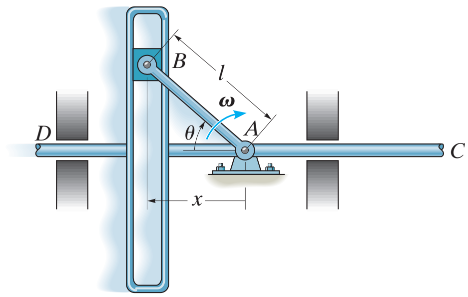
$$v_{AB} = l\cos\theta(\omega) = \omega l \cos\theta \qquad \textbf{Ans.}$$

$$a_{AB} = l\left[\cos\theta(0) - \sin\theta(\omega)^2\right] = -\omega^2 l \sin\theta \qquad \textbf{Ans.}$$



***16–40.**

The mechanism is used to convert the constant circular motion ω of rod AB into translating motion of rod CD and the attached vertical slot. Determine the velocity and acceleration of CD for any angle θ of AB .

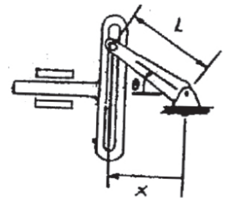


SOLUTION

$$x = l\cos\theta$$

$$\dot{x} = v_x = -l\sin\theta\dot{\theta}$$

$$\dot{x} = a_x = -l(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)$$



Here $v_x = v_{CD}, a_x = a_{CD}$, and $\dot{\theta} = \omega, \ddot{\theta} = \alpha = 0$.

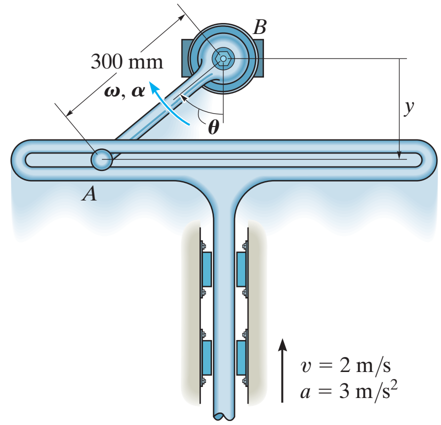
$$v_{CD} = -l\sin\theta(\omega) = -\omega l\sin\theta \qquad \textbf{Ans.}$$

$$a_{CD} = -l\left[\sin\theta(0) + \cos\theta(\omega)^2\right] = -\omega^2 l\cos\theta \qquad \textbf{Ans.}$$

Negative signs indicate that both v_{CD} and a_{CD} are directed opposite to positive x.

16–41.

At the instant $\theta = 50°$, the slotted guide is moving upward with an acceleration of $3\ \text{m/s}^2$ and a velocity of $2\ \text{m/s}$. Determine the angular acceleration and angular velocity of link AB at this instant. *Note:* The upward motion of the guide is in the negative y direction.



SOLUTION

$$y = 0.3 \cos \theta$$

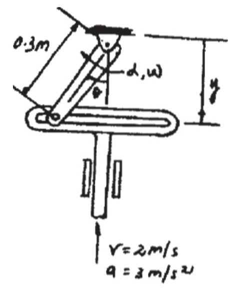
$$\dot{y} = v_y = -0.3 \sin \theta \dot{\theta}$$

$$\ddot{y} = a_y = -0.3\left(\sin \theta \ddot{\theta} + \cos \theta \dot{\theta}^2\right)$$

Here $v_y = -2\ \text{m/s}$, $a_y = -3\ \text{m/s}^2$, and $\dot{\theta} = \omega, \dot{\theta} = \omega, \ddot{\theta} = \alpha, \theta = 50°$.

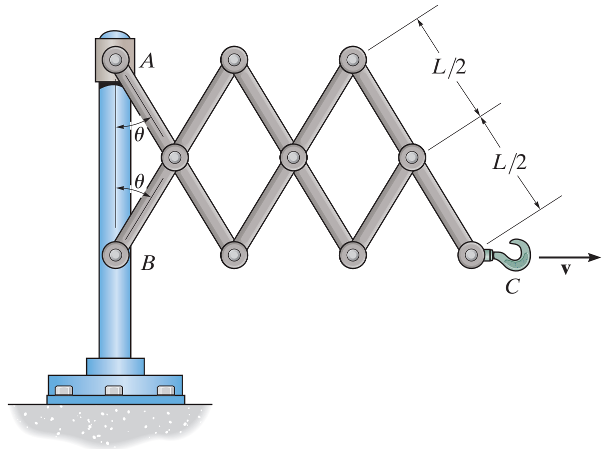
$$-2 = -0.3 \sin 50°(\omega) \qquad \omega = 8.70\ \text{rad/s} \qquad \textbf{Ans.}$$

$$-3 = -0.3[\sin 50°(\alpha) + \cos 50°(8.70)^2] \qquad \alpha = -50.5\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



16–42.

The mechanism shown is known as a Nuremberg scissors. If the hook at C moves with a constant velocity of $\mathbf{v}$, determine the velocity and acceleration of collar A as a function of θ. The collar slides freely along the vertical guide.



SOLUTION

$$x = 3L\sin\theta$$

$$v = \dot{x} = 3L\cos\theta\,\dot{\theta}$$

$$y = L\cos\theta$$

$$\dot{y} = -L\sin\theta\,\dot{\theta}$$

$$\frac{\dot{y}}{v} = -\frac{L\sin\theta\,\dot{\theta}}{3L\cos\theta\,\dot{\theta}}$$

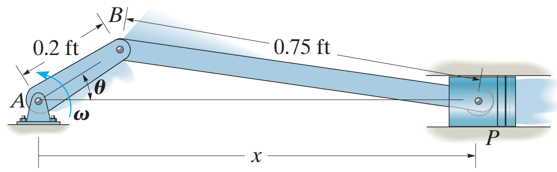
$$\dot{y} = (v\tan\theta)/3 \downarrow$$ **Ans.**

$$\ddot{y} = \frac{v}{3}(\sec^2\theta\,\dot{\theta}) = \frac{v}{3}\left(\frac{1}{\cos^2\theta}\right)\left(\frac{v}{3L\cos\theta}\right)$$

$$\ddot{y} = \frac{v^2}{9L\cos^3\theta} \downarrow$$ **Ans.**

16–43.

The crankshaft AB is rotating at a constant angular velocity of $\omega = 150$ rad/s. Determine the velocity of the piston P at the instant $\theta = 30°$.



SOLUTION

$$x = 0.2\cos\theta + \sqrt{(0.75)^2 - (0-2\sin\theta)^2}$$

$$\dot{x} = -0.2\sin\theta\ddot{\theta} + \frac{1}{2}\left[(0.75)^2 - (0.2\sin\theta)^2\right]^{-\frac{1}{2}}(-2)(0.2\sin\theta)(0.2\cos\theta)\ddot{\theta}$$

$$v_P = -0.2\omega\sin\theta - \left(\frac{1}{2}\right)\frac{(0.2)^2\omega\sin 2\theta}{\sqrt{(0.75)^2 - (0.2\sin\theta)^2}}$$

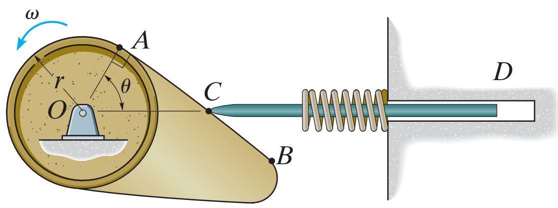
At $\theta = 30°, \omega = 150$ rad/s

$$v_P = -0.2(150)\sin 30° - \left(\frac{1}{2}\right)\frac{(0.2)^2(150)\sin 60°}{\sqrt{(0.75)^2 - (0.2\sin 30°)^2}}$$

$$v_P = -18.5\text{ ft/s} = 18.5\text{ ft/s} \leftarrow$$ **Ans.**

***16–44.**

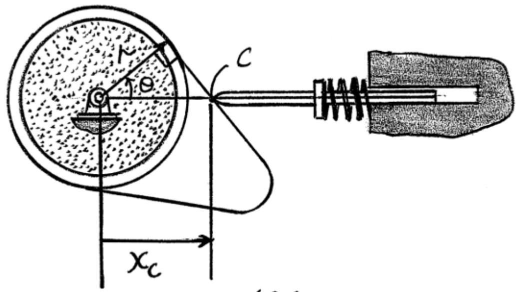
Determine the velocity and acceleration of the follower rod CD as a function of θ when the contact between the cam and follower is along the straight region AB on the face of the cam. The cam rotates with a constant counterclockwise angular velocity $\boldsymbol{\omega}$.



SOLUTION

Position Coordinate: From the geometry shown in Fig. a,

$$x_C = \frac{r}{\cos\theta} = r\sec\theta$$



(a)

Time Derivative: Taking the time derivative,

$$v_{CD} = \dot{x}_C = r\sec\theta\tan\theta\dot{\theta} \qquad \textbf{(1)}$$

Here, $\dot{\theta} = +\omega$ since ω acts in the positive rotational sense of θ. Thus, Eq. (1) gives

$$v_{CD} = r\omega\sec\theta\tan\theta \rightarrow \qquad \textbf{Ans.}$$

The time derivative of Eq. (1) gives

$$a_{CD} = \ddot{x}_C = r\{\sec\theta\tan\theta\ddot{\theta} + \dot{\theta}[\sec\theta(\sec^2\theta\dot{\theta}) + \tan\theta(\sec\theta\tan\theta\dot{\theta})]\}$$

$$a_{CD} = r[\sec\theta\tan\theta\ddot{\theta} + (\sec^3\theta + \sec\theta\tan^2\theta)\dot{\theta}^2]$$

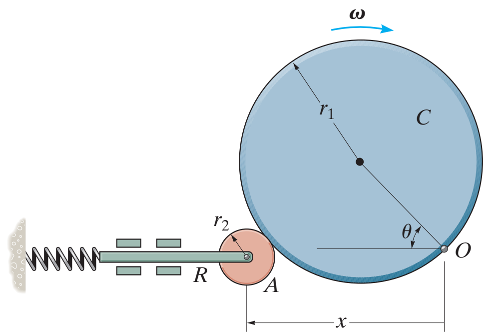
Since $\dot{\theta} = \omega$ is constant, $\ddot{\theta} = \alpha = 0$. Then,

$$a_{CD} = r[\sec\theta\tan\theta(0) + (\sec^3\theta + \sec\theta\tan^2\theta)\omega^2]$$

$$= r\omega^2\left(\sec^3\theta + \sec\theta\tan^2\theta\right) \rightarrow \qquad \textbf{Ans.}$$

16–45.

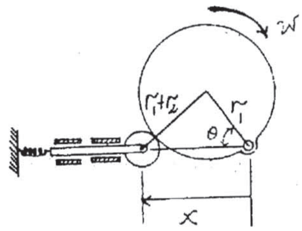
Determine the velocity of rod R for any angle θ of the cam C if the cam rotates with a constant angular velocity $\boldsymbol{\omega}$. The pin connection at O does not cause an interference with the motion of A on C.



SOLUTION

Position Coordinate Equation: Using law of cosine.

$$(r_1 + r_2)^2 = x^2 + r_1^2 - 2r_1 x \cos\theta \qquad \textbf{(1)}$$



Time Derivatives: Taking the time derivative of Eq. (1). we have

$$0 = 2x\frac{dx}{dt} - 2r_1\left(-x\sin\theta\frac{d\theta}{dt} + \cos\theta\frac{dx}{dt}\right) \qquad \textbf{(2)}$$

However $v = \dfrac{dx}{dt}$ and $\omega = \dfrac{d\theta}{dt}$. From Eq.(2),

$$0 = xv - r_1(v\cos\theta - x\omega\sin\theta)$$

$$v = \frac{r_1 x\omega\sin\theta}{r_1\cos\theta - x} \qquad \textbf{(3)}$$

However, the positive root of Eq.(1) is

$$x = r_1\cos\theta + \sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1r_2}$$

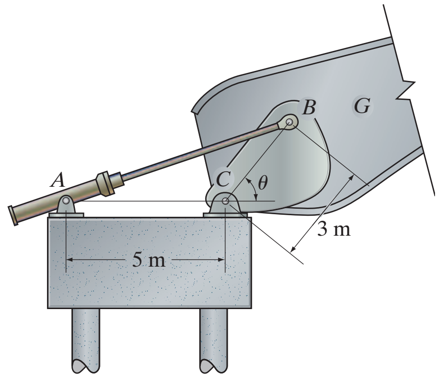
Substitute into Eq.(3), we have

$$v = -\left(\frac{r_1^2\omega\sin 2\theta}{2\sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1r_2}} + r_1\omega\sin\theta\right) \qquad \textbf{Ans.}$$

Note: Negative sign indicates that v is directed in the opposite direction to that of positive x.

16–46.

The bridge girder G of a bascule bridge is raised and lowered using the drive mechanism shown. If the hydraulic cylinder AB shortens at a constant rate of 0.15 m/s, determine the angular velocity of the bridge girder at the instant $\theta = 60°$.



SOLUTION

Position Coordinates: Applying the law of cosines to the geometry shown in Fig. a,

$$s^2 = 3^2 + 5^2 - 2(3)(5)\cos\left(180°-\theta\right)$$

$$s^2 = 34 - 30\cos\left(180°-\theta\right)$$

However, $\cos\left(180°-\theta\right) = -\cos\theta$. Thus,

$$s^2 = 34 + 30\cos\theta$$

Time Derivatives: Taking the time derivative,

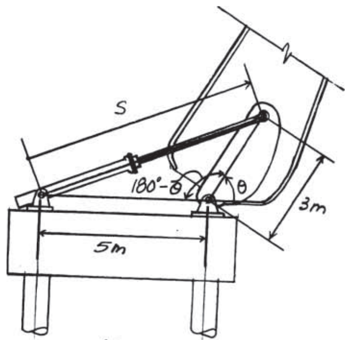
$$2s\dot{s} = 0 + 30\left(-\sin\theta\dot{\theta}\right)$$

$$s\dot{s} = -15\sin\theta\dot{\theta} \qquad \textbf{(1)}$$

When $\theta = 60°$, $s = \sqrt{34+30\cos 60°} = 7$ m. Also, $\dot{s} = -0.15$ m/s since $\dot{s}$ is directed towards the negative sense of s. Thus, Eq. (1) gives

$$7\left(-0.15\right) = -15\sin 60°\dot{\theta}$$

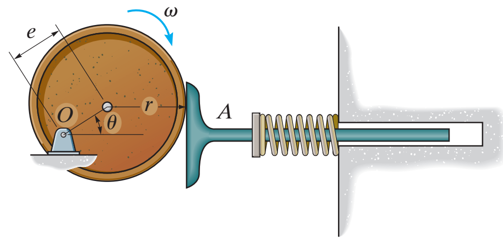
$$\omega = \dot{\theta} = 0.0808 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)

16–47.

The circular cam of radius r is rotating clockwise with a constant angular velocity $\boldsymbol{\omega}$ about the pin at O, which is at an eccentric distance e from the center of the cam. Determine the velocity and acceleration of the follower rod A as a function of θ.



SOLUTION

Position Coordinates: From the geometry shown in Fig. a,

$$x_A = e\cos\theta + r \qquad \textbf{(1)}$$

Time Derivatives: Taking the time derivative,

$$v_A = \dot{x}_A = -e\sin\theta\dot{\theta} \qquad \textbf{(2)}$$

Since ω acts in the negative rotational sense of θ, then $\dot{\theta} = -\omega$. Thus, Eq. (2) gives

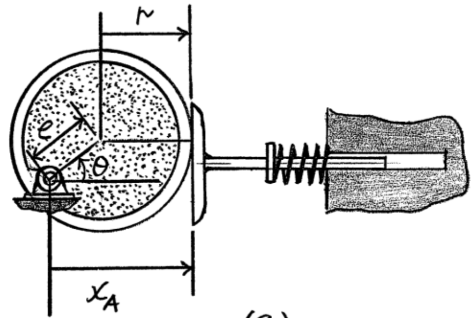
$$v_A = -e\sin\theta(-\omega) = e\omega\sin\theta \rightarrow \qquad \textbf{Ans.}$$

Taking the time derivative of Eq. (2) gives

$$a_A = \ddot{x}_A = -e\left(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right) \qquad \textbf{(3)}$$

Since ω is constant, $\ddot{\theta} = \alpha = 0$. Then Eq. (3) gives

$$\begin{aligned} a_A &= -e[\sin\theta(0) + \cos\theta(\omega^2)] \\ &= -e\omega^2\cos\theta \\ &= e\omega^2\cos\theta \leftarrow \qquad \textbf{Ans.} \end{aligned}$$

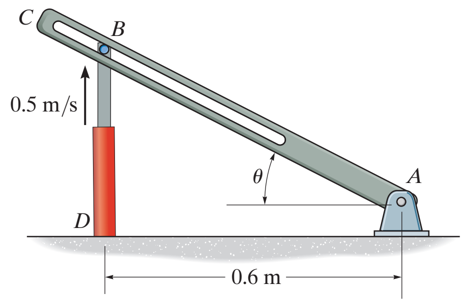


(a)

The negative sign indicates that $\mathbf{a}_A$ acts towards the negative sense of x_A.

***16–48.**

Peg B mounted on hydraulic cylinder BD slides freely along the slot in link AC. If the hydraulic cylinder extends at a constant rate of 0.5 m/s, determine the angular velocity and angular acceleration of the link at the instant $\theta = 45°$.



SOLUTION

Position Coordinate: From the geometry shown in Fig. a,

$$y_C = 0.6\ \tan\theta \text{ m}$$

Time Derivatives: Taking the time derivative,

$$v_C = \dot{y}_C = (0.6 \sec^2\theta\dot{\theta}) \text{ m/s} \qquad \textbf{(1)}$$

Here, $v_C = 0.5$ m/s since v_C acts in the positive sense of y_C. When $\theta = 45°$, Eq. (1) gives

$$0.5 = \left(0.6 \sec^2 45°\right)\dot{\theta}$$

$$\omega_{AB} = \dot{\theta} = 0.4167 \text{ rad/s} = 0.417 \text{ rad/s} \qquad \textbf{Ans.}$$

The time derivative of Eq. (1) gives

$$a_C = \ddot{y}_C = 0.6\left(\sec^2\theta\ddot{\theta} + 2\sec\theta\sec\theta\tan\theta\dot{\theta}^2\right)$$

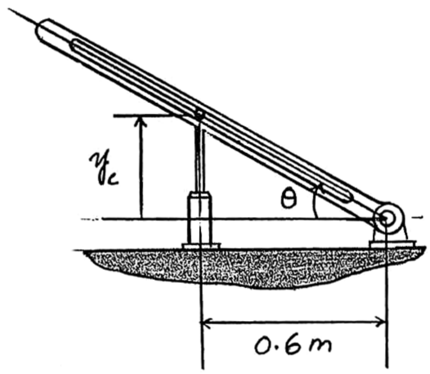
$$a_C = 0.6\sec^2\theta\left(\ddot{\theta} + 2\tan\theta\dot{\theta}^2\right) \qquad \textbf{(2)}$$

Since v_C is constant, $a_C = 0$. Thus, Eq. (2) gives

$$0 = \ddot{\theta} + 2\tan 45°\left(0.4167^2\right)$$

$$\alpha_{AB} = \ddot{\theta} = -0.3472 \text{ rad/s}^2 = 0.347 \text{ rad/s}^2 ↺ \qquad \textbf{Ans.}$$

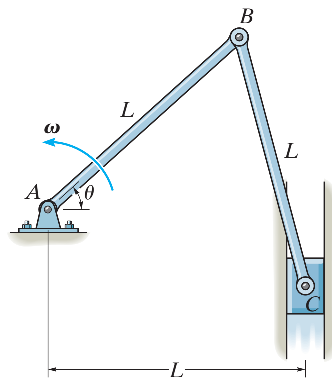
The negative sign indicates that α_{AB} acts counterclockwise.



(a)

16–49.

Bar AB rotates uniformly about the fixed pin A with a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity and acceleration of block C, at the instant $\theta = 60°$.



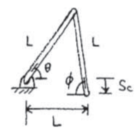
SOLUTION

$$L\cos\theta + L\cos\phi = L$$

$$\cos\theta + \cos\phi = 1$$

$$\sin\theta\,\dot{\theta}] + \sin\phi\,\dot{\phi} = 0 \qquad \textbf{(1)}$$

$$\cos\theta(\dot{\theta})^2 + \sin\theta\ddot{\theta} + \sin\phi\,\ddot{\phi} + \cos\phi\ (\dot{\phi})^2 = 0 \qquad \textbf{(2)}$$



When $\theta = 60°, \phi = 60°$,

thus, $\dot{\theta} = -\dot{\phi} = \omega$ (from Eq. (1))

$\ddot{\theta} = 0$

$\ddot{\phi} = -1.155\omega^2$ (from Eq.(2))

Also, $s_C = L\sin\phi - L\sin\theta$

$$v_C = L\cos\phi\,\dot{\phi} - L\cos\theta\,\dot{\theta}$$

$$a_C = -L\sin\phi\,(\dot{\phi})^2 + L\cos\phi\,(\ddot{\phi}) - L\cos\theta(\ddot{\theta}) + L\sin\theta(\dot{\theta})^2$$

At $\theta = 60°, \phi = 60°$

$$s_C = 0$$

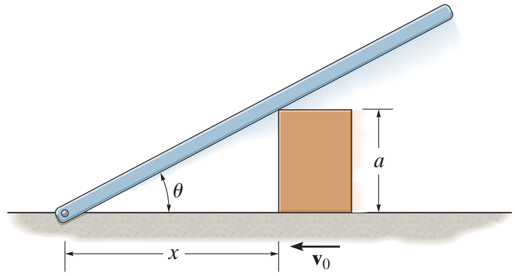
$$v_C = L(\cos 60°)(-\omega) - L\cos 60°(\omega) = -L\omega = L\omega\uparrow \qquad \textbf{Ans.}$$

$$a_C = -L\sin 60°(-\omega)^2 + L\cos 60°(-1.155\omega^2) + 0 + L\sin 60°(\omega)^2$$

$$a_C = -0.577\,L\omega^2 = 0.577\,L\omega^2\uparrow \qquad \textbf{Ans.}$$

16–50.

The block moves to the left with a constant velocity $\mathbf{v}_0$. Determine the angular velocity and angular acceleration of the bar as a function of θ.



SOLUTION

Position Coordinate Equation: From the geometry,

$$x = \frac{a}{\tan\theta} = a\cot\theta \qquad \textbf{(1)}$$

Time Derivatives: Taking the time derivative of Eq. (1), we have

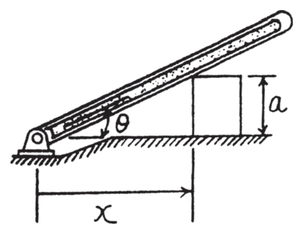
$$\frac{dx}{dt} = -a\csc^2\theta\frac{d\theta}{dt} \qquad \textbf{(2)}$$

Since v_0 is directed toward negative x, then $\frac{dx}{dt} = -v_0$. Also, $\frac{d\theta}{dt} = \omega$.

From Eq.(2),

$$-v_0 = -a\csc^2\theta(\omega)$$

$$\omega = \frac{v_0}{a\csc^2\theta} = \frac{v_0}{a}\sin^2\theta \qquad \textbf{Ans.}$$



Here, $\alpha = \frac{d\omega}{dt}$. Then from the above expression

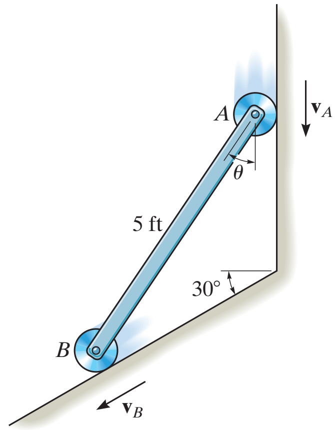
$$\alpha = \frac{v_0}{a}(2\sin\theta\cos\theta)\frac{d\theta}{dt} \qquad \textbf{(3)}$$

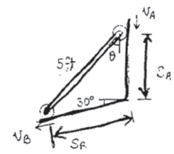
However, $2\sin\theta\cos\theta = \sin 2\theta$ and $\omega = \frac{d\theta}{dt} = \frac{v_0}{a}\sin^2\theta$. Substitute these values into Eq.(3) yields

$$\alpha = \frac{v_0}{a}\sin 2\theta\left(\frac{v_0}{a}\sin^2\theta\right) = \left(\frac{v_0}{a}\right)^2\sin 2\theta\sin^2\theta \qquad \textbf{Ans.}$$

16–51.

The bar is confined to move along the vertical and inclined planes. If the velocity of the roller at A is $v_A = 6$ ft/s when $\theta = 45°$, determine the bar's angular velocity and the velocity of roller B at this instant.





SOLUTION

$$s_B \cos 30° = 5 \sin \theta$$

$$s_B = 5.774 \sin \theta$$

$$\dot{s}_B = 5.774 \cos \theta \, \dot{\theta} \qquad \textbf{(1)}$$

$$5 \cos \theta = s_A + s_B \sin 30°$$

$$-5 \sin \theta \, \dot{\theta} = \dot{s}_A + \dot{s}_B \sin 30° \qquad \textbf{(2)}$$

Combine Eqs.(1) and (2):

$$-5 \sin \theta \, \dot{\theta} = -6 + 5.774 \cos \theta \, (\dot{\theta})(\sin 30°)$$

$$-3.536\dot{\theta} = -6 + 2.041\dot{\theta}$$

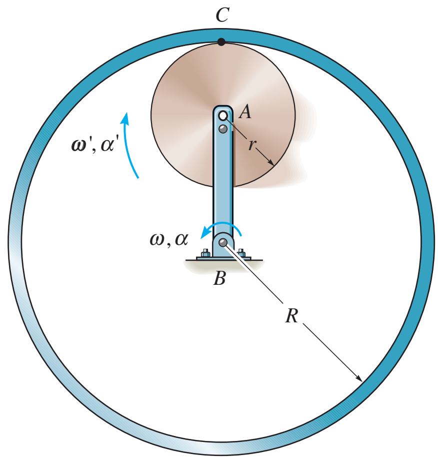
$$\omega = \dot{\theta} = 1.08 \text{ rad/s} \qquad \textbf{Ans.}$$

From Eq.(1):

$$v_B = \dot{s}_B = 5.774 \cos 45°(1.076) = 4.39 \text{ ft/s} \qquad \textbf{Ans.}$$

***16–52.**

Arm AB has an angular velocity of $\boldsymbol{\omega}$ and an angular acceleration of $\boldsymbol{\alpha}$. If no slipping occurs between the disk and the fixed curved surface, determine the angular velocity and angular acceleration of the disk.



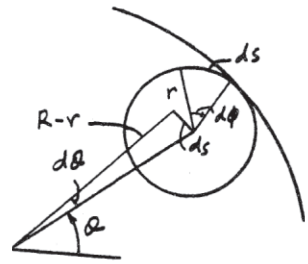
SOLUTION

$$ds = (R - r)\,d\theta = -\,r\,d\phi$$

$$(R - r)\left(\frac{d\theta}{dt}\right) = -r\left(\frac{d\phi}{dt}\right)$$

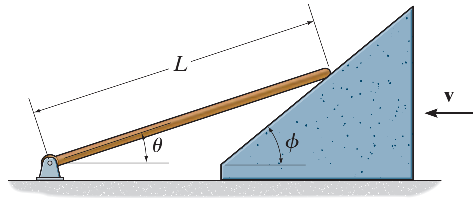
$$\omega' = -\frac{(R - r)\,\omega}{r}$$ **Ans.**

$$\alpha' = -\frac{(R - r)\,\alpha}{r}$$ **Ans.**



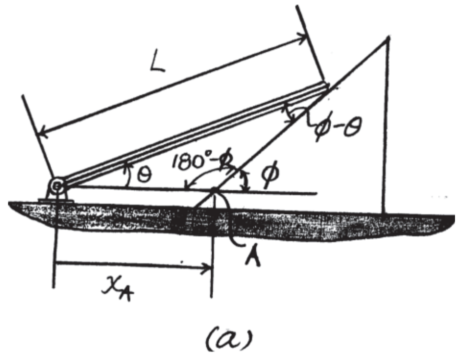
16–53.

If the wedge moves to the left with a constant velocity $\mathbf{v}$, determine the angular velocity of the rod as a function of θ.



SOLUTION

Position Coordinates: Applying the law of sines to the geometry shown in Fig. *a*,



$$\frac{x_A}{\sin(\phi - \theta)} = \frac{L}{\sin\left(180^\circ - \phi\right)}$$

$$x_A = \frac{L\sin(\phi - \theta)}{\sin\left(180^\circ - \phi\right)}$$

However, $\sin\left(180^\circ - \phi\right) = \sin\phi$. Therefore,

$$x_A = \frac{L\sin(\phi - \theta)}{\sin\phi}$$

Time Derivative: Taking the time derivative,

$$\dot{x}_A = \frac{L\cos(\phi - \theta)(-\dot{\theta})}{\sin\phi}$$

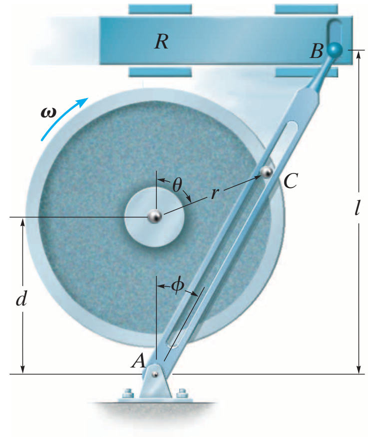
$$v_A = \dot{x}_A = -\frac{L\cos(\phi - \theta)\dot{\theta}}{\sin\phi} \qquad \textbf{(1)}$$

Since point A is on the wedge, its velocity is $v_A = -v$. The negative sign indicates that $\mathbf{v}_A$ is directed towards the negative sense of x_A. Thus, Eq. (1) gives

$$\dot{\theta} = \frac{v\sin\phi}{L\cos(\phi - \theta)} \qquad \textbf{Ans.}$$

16–54.

The slotted yoke is pinned at A while end B is used to move the ram R horizontally. If the disk rotates with a constant angular velocity $\boldsymbol{\omega}$, determine the velocity and acceleration of the ram. The crank pin C is fixed to the disk and turns with it.

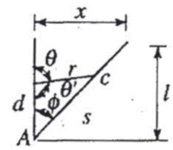


SOLUTION

$$x = l \tan \phi \tag{1}$$

However $\dfrac{r}{\sin \phi} = \dfrac{s}{\sin (180° - \theta)} = \dfrac{s}{\sin \theta}$ $\qquad \sin \phi = \dfrac{r}{s} \sin \theta$

$$d = s \cos \phi - r \cos \theta \qquad \cos \phi = \frac{d + r \cos \theta}{s}$$



From Eq. (1) $x = l\left(\dfrac{\sin \phi}{\cos \phi}\right) = l\left(\dfrac{\dfrac{r}{s}\sin \theta}{\dfrac{d + r\cos\theta}{s}}\right) = \dfrac{lr \sin \theta}{d + r \cos \theta}$

$$\dot{x} = v = \frac{(d + r\cos\theta)(lr\cos\theta\dot{\theta}) - (lr\sin\theta)(-r\sin\theta\dot{\theta})}{(d + r\cos\theta)^2} \text{ Where } \dot{\theta} = \omega$$

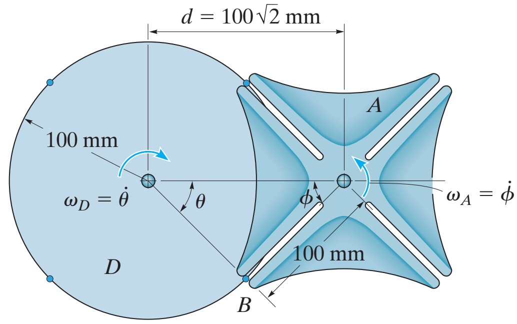
$$= \frac{lr(r + d\cos\theta)}{(d + r\cos\theta)^2}\omega \qquad \textbf{Ans.}$$

$$\ddot{x} = a = lr\omega\left[\frac{(d + r\cos\theta)^2(-d\sin\theta\dot{\theta}) - (r + d\cos\theta)(2)(d + r\cos\theta)(-r\sin\theta\dot{\theta})}{(d + r\cos\theta)^4}\right]$$

$$= \frac{lr\sin\theta(2r^2 - d^2 + rd\cos\theta)}{(d + r\cos\theta)^3}\omega^2 \qquad \textbf{Ans.}$$

16–55.

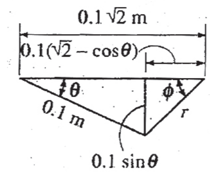
The Geneva wheel A provides intermittent rotary motion ω_A for continuous motion $\omega_D = 2$ rad/s of disk D. By choosing $d = 100\sqrt{2}$ mm, the wheel has zero angular velocity at the instant pin B enters or leaves one of the four slots. Determine the magnitude of the angular velocity $\boldsymbol{\omega}_A$ of the Geneva wheel at any angle θ for which pin B is in contact with the slot.



SOLUTION

$$\tan\phi = \frac{0.1\sin\theta}{0.1\left(\sqrt{2} - \cos\theta\right)} = \frac{\sin\theta}{\sqrt{2} - \cos\theta}$$

$$\sec^2\phi\,\dot{\phi} = \frac{\left(\sqrt{2} - \cos\theta\right)(\cos\theta\dot{\theta}) - \sin\theta(\sin\theta\dot{\theta})}{\left(\sqrt{2} - \cos\theta\right)^2} = \frac{\sqrt{2}\cos\theta - 1}{\left(\sqrt{2} - \cos\theta\right)^2}\dot{\theta} \qquad \textbf{(1)}$$



From the geometry:

$$r^2 = (0.1\sin\theta)^2 + [0.1\left(\sqrt{2} - \cos\theta\right)]^2 = 0.01\left(3 - 2\sqrt{2}\cos\theta\right)$$

$$\sec^2\phi = \frac{r^2}{[0.1\left(\sqrt{2} - \cos\theta\right)]^2} = \frac{0.01\left(3 - 2\sqrt{2}\cos\theta\right)}{[0.1\left(\sqrt{2} - \cos\theta\right)]^2} = \frac{(3 - 2\sqrt{2}\cos\theta)}{\left(\sqrt{2} - \cos\theta\right)^2}$$

From Eq. (1)

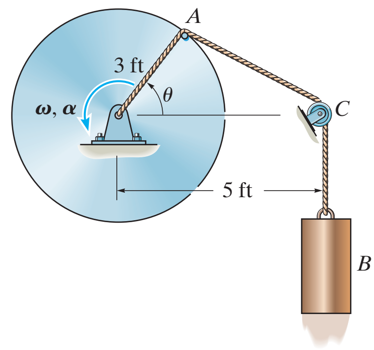
$$\frac{\left(3 - 2\sqrt{2}\cos\theta\right)}{\left(\sqrt{2} - \cos\theta\right)^2}\dot{\phi} = \frac{\sqrt{2}\cos\theta - 1}{\left(\sqrt{2} - \cos\theta\right)^2}\dot{\theta}$$

$$\dot{\phi} = \frac{\sqrt{2}\cos\theta - 1}{3 - 2\sqrt{2}\cos\theta}\dot{\theta} \quad \textit{Here } \phi = \omega_A \textit{ and } \dot{\theta} = \omega_D = 2\text{ rad/s}$$

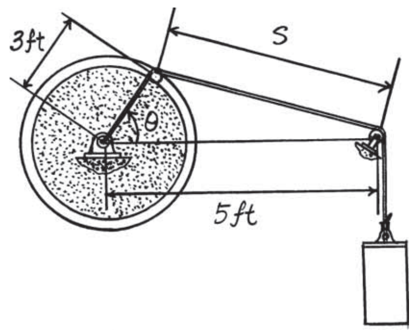
$$\omega_A = 2\left(\frac{\sqrt{2}\cos\theta - 1}{3 - 2\sqrt{2}\cos\theta}\right) \qquad \textbf{Ans.}$$

***16–56.**

At the instant shown, the disk is rotating with an angular velocity of $\boldsymbol{\omega}$ and has an angular acceleration of $\boldsymbol{\alpha}$. Determine the velocity and acceleration of cylinder B at this instant. Neglect the size of the pulley at C.



SOLUTION



$$s = \sqrt{3^2 + 5^2 - 2(3)(5)\cos\theta}$$

$$v_B = \dot{s} = \frac{1}{2}(34 - 30\cos\theta)^{-\frac{1}{2}}(30\sin\theta)\dot{\theta}$$

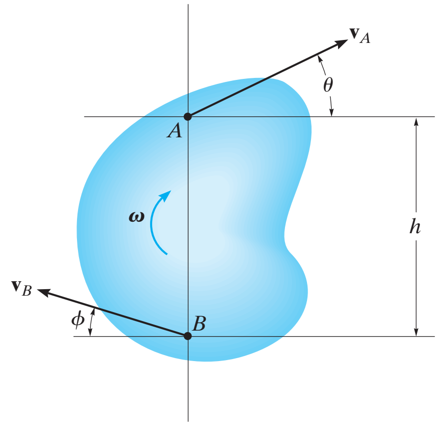
$$v_B = \frac{15\,\omega\sin\theta}{(34 - 30\cos\theta)^{\frac{1}{2}}}$$ **Ans.**

$$a_B = \ddot{s} = \frac{15\,\omega\cos\theta\dot{\theta} + 15\dot{\omega}\sin\theta}{\sqrt{34 - 30\cos\theta}} + \frac{\left(-\frac{1}{2}\right)(15\omega\sin\theta)\left(30\sin\theta\dot{\theta}\right)}{(34 - 30\cos\theta)^{\frac{3}{2}}}$$

$$= \frac{15\,(\omega^2\cos\theta + \alpha\sin\theta)}{(34 - 30\cos\theta)^{\frac{1}{2}}} - \frac{225\,\omega^2\sin^2\theta}{(34 - 30\cos\theta)^{\frac{3}{2}}}$$ **Ans.**

16–57.

If h and θ are known, and the speed of A and B is $v_A = v_B = v$, determine the angular velocity $\boldsymbol{\omega}$ of the body and the direction ϕ of $\mathbf{v}_B$.



SOLUTION

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$$

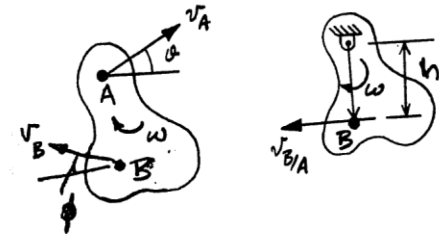
$$-v\cos\phi\mathbf{i} + v\sin\phi\mathbf{j} = v\cos\theta\mathbf{i} + v\sin\theta\mathbf{j} + (-\omega\mathbf{k}) \times (-h\mathbf{j})$$

$(\xrightarrow{+})$ $\quad -v\cos\phi = v\cos\theta - \omega h$ **(1)**

$(+\uparrow)$ $\quad v\sin\phi = v\sin\theta$ **(2)**

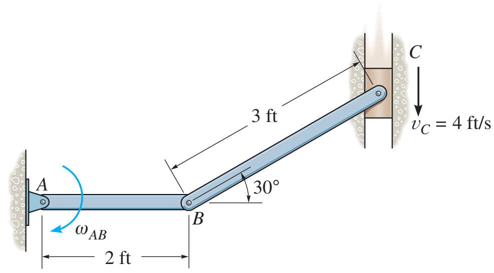
From Eq. (2), $\quad \phi = \theta$ **Ans.**

From Eq. (1), $\quad \omega = \dfrac{2v}{h}\cos\theta$ **Ans.**



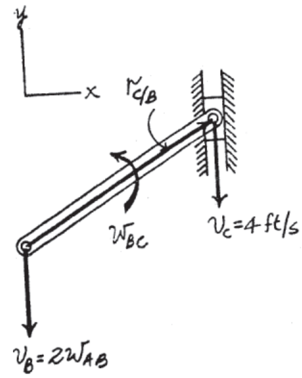
16–58.

If the block at C is moving downward at 4 ft/s, determine the angular velocity of bar AB at the instant shown.



SOLUTION

***Kinematic Diagram*:** Since link AB is rotating about fixed point A, then v_B is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 2\omega_{AB}$. At the instant shown, v_B is directed towards the *negative y* axis. Also, block C is moving downward vertically due to the constraint of the guide. Then v_c is directed toward *negative* y axis.



***Velocity Equation*:** Here, $\mathbf{r}_{C/A} = \{3\cos 30°\mathbf{i} + 3\sin 30°\mathbf{j}\}$ ft $= \{2.598\mathbf{i} + 1.50\mathbf{j}\}$ ft. Applying Eq. 16–16, we have

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$-4\mathbf{j} = -2\omega_{AB}\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (2.598\mathbf{i} + 1.50\mathbf{j})$$

$$-4\mathbf{j} = -1.50\omega_{BC}\mathbf{i} + (2.598\omega_{BC} - 2\omega_{AB})\mathbf{j}$$

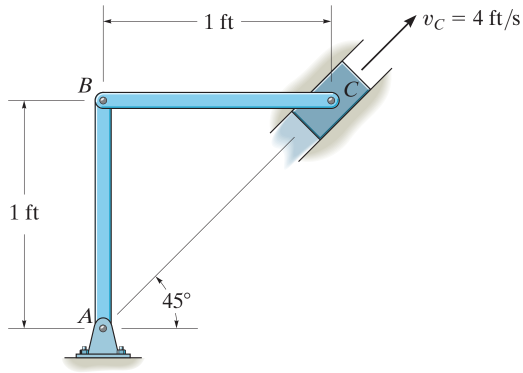
Equating **i** and **j** components gives

$$0 = -1.50\omega_{BC} \qquad \omega_{BC} = 0$$

$$-4 = 2.598(0) - 2\omega_{AB} \qquad \omega_{AB} = 2.00 \text{ rad/s} \qquad \textbf{Ans.}$$

16–59.

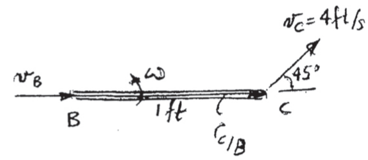
The velocity of the slider block C is 4 ft/s up the inclined groove. Determine the angular velocity of links AB and BC and the velocity of point B at the instant shown.



SOLUTION

For link BC

$$\mathbf{v}_C = \{-4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j}\}\text{ ft/s} \qquad \mathbf{v}_B = -v_B\mathbf{i} \qquad \omega = \omega_{BC}\mathbf{k}$$



$$\mathbf{r}_{B/C} = \{1\mathbf{i}\}\text{ ft}$$

$$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$$

$$-4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j} = -v_B\mathbf{i} + (\omega_{BC}\mathbf{k}) \times (1\mathbf{i})$$

$$-4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j} = -v_B\mathbf{i} + \omega_{BC}\mathbf{j}$$

Equating the **i** and **j** components yields:

$$-4\cos 45° = -v_B \qquad v_B = 2.83\text{ ft/s}$$ **Ans.**

$$-4\cos 45° = \omega_{BC} \qquad \omega_{BC} = 2.83\text{ rad/s}$$ **Ans.**

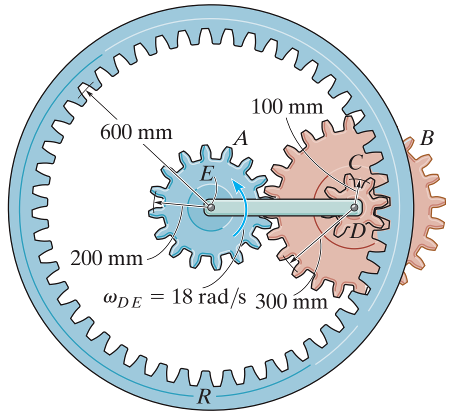
For link AB: Link AB rotates about the fixed point A. Hence

$$\mathbf{v}_B = \omega_{AB} r_{AB}$$

$$2.83 = \omega_{AB}(1) \qquad \omega_{AB} = 2.83\text{ rad/s}$$ **Ans.**

***16–60.**

The epicyclic gear train consists of the sun gear A which is in mesh with the planet gear B. This gear has an inner hub C which is fixed to B and in mesh with the fixed ring gear R. If the connecting link DE pinned to B and C is rotating at $\omega_{DE} = 18$ rad/s about the pin at E, determine the angular velocities of the planet and sun gears.



SOLUTION

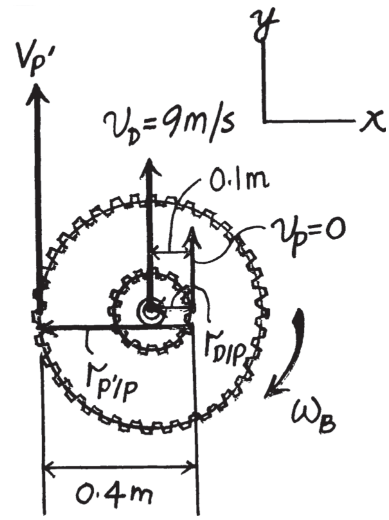
$v_D = r_{DE}\,\omega_{DE} = (0.5)(18) = 9 \text{ m/s} \uparrow$

The velocity of the contact point P with the ring is zero.

$\mathbf{v}_D = \mathbf{v}_P + \omega \times \mathbf{r}_{D/P}$

$9\mathbf{j} = 0 + (-\omega_B\,\mathbf{k}) \times (-0.1\mathbf{i})$

$\omega_B = 90 \text{ rad/s} \;\circlearrowright$ **Ans.**



Let P' be the contact point between A and B.

$\mathbf{v}_{P'} = \mathbf{v}_P + \omega \times \mathbf{r}_{P'/P}$

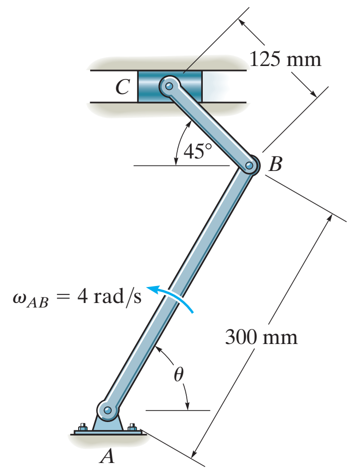
$v_{P'}\,\mathbf{j} = \mathbf{0} + (-90\mathbf{k}) \times (-0.4\mathbf{i})$

$v_{P'} = 36 \text{ m/s} \uparrow$

$\omega_A = \dfrac{v_{P'}}{r_A} = \dfrac{36}{0.2} = 180 \text{ rad/s} \;\circlearrowleft$ **Ans.**

16–61.

The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the velocity of the slider block C at the instant $\theta = 60°$, if link AB is rotating at 4 rad/s.



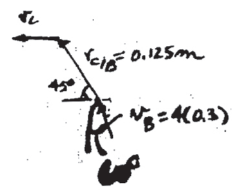
SOLUTION

$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$-v_C\mathbf{i} = -4(0.3)\sin 30°\mathbf{i} + 4(0.3)\cos 30°\mathbf{j} + \omega\mathbf{k} \times (-0.125\cos 45°\mathbf{i} + 0.125\sin 45°\mathbf{j})$

$-v_C = -1.0392 - 0.008839\omega$

$0 = 0.6 - 0.08839\omega$



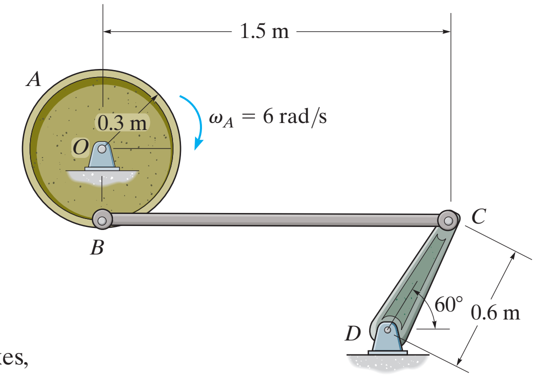
Solving,

$$\omega = 6.79 \text{ rad/s}$$

$$v_C = 1.64 \text{ m/s}$$ **Ans.**

16–62.

If the flywheel is rotating with an angular velocity of $\omega_A = 6$ rad/s, determine the angular velocity of rod BC at the instant shown.



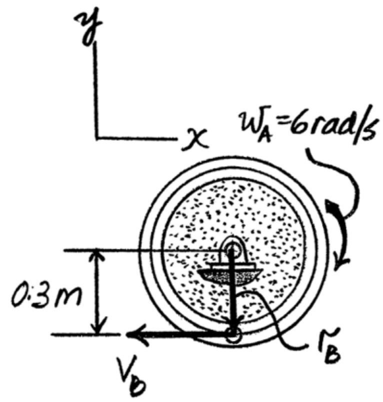
SOLUTION

Rotation About a Fixed Axis: Flywheel A and rod CD rotate about fixed axes, Figs. a and b. Thus, the velocity of points B and C can be determined from

$$v_B = \omega_A \times \mathbf{r}_B = (-6\mathbf{k}) \times (-0.3\mathbf{j}) = [-1.8\mathbf{i}] \text{ m/s}$$

$$v_C = \omega_{CD} \times \mathbf{r}_C = (\omega_{CD}\mathbf{k}) \times (0.6 \cos 60° \, \mathbf{i} + 0.6 \sin 60° \, \mathbf{j})$$

$$= -0.5196\omega_{CD}\mathbf{i} + 0.3\omega_{CD}\mathbf{j}$$



(a)

General Plane Motion: By referring to the kinematic diagram of link BC shown in Fig. c and applying the relative velocity equation, we have

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$-1.8\mathbf{i} = -0.5196\omega_{CD}\mathbf{i} + 0.3\omega_{CD}\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (-1.5\mathbf{i})$$

$$-1.8\mathbf{i} = -0.5196\omega_{CD}\mathbf{i} + (0.3\omega_{CD} - 1.5\omega_{BC})\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components

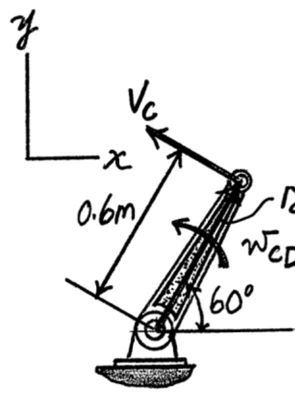
$$-1.8 = -0.5196\omega_{CD}$$

$$0 = 0.3\omega_{CD} - 1.5\omega_{BC}$$

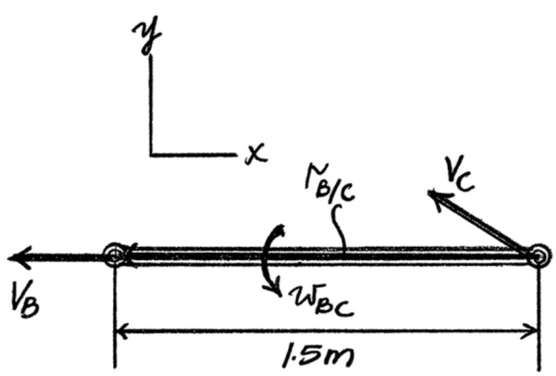
Solving,

$$\omega_{CD} = 3.46 \text{ rad/s}$$

$$\omega_{BC} = 0.693 \text{ rad/s}$$ **Ans.**



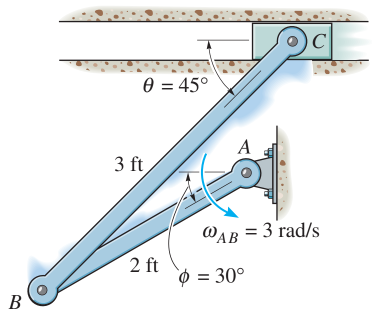
(b)



(c)

16–63.

If the angular velocity of link AB is $\omega_{AB} = 3$ rad/s, determine the velocity of the block at C and the angular velocity of the connecting link CB at the instant $\theta = 45°$ and $\phi = 30°$.



SOLUTION

$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

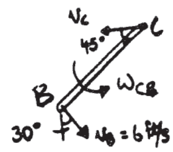
$$\begin{bmatrix} v_C \\ \leftarrow \end{bmatrix} = \begin{bmatrix} 6 \\ 30°\searrow \end{bmatrix} + \begin{bmatrix} \omega_{CB}(3) \\ 45°\nwarrow \end{bmatrix}$$

$(\overset{+}{\rightarrow})\qquad -v_C = 6 \sin 30° - \omega_{CB}(3)\cos 45°$

$(+\uparrow)\qquad 0 = -6\cos 30° + \omega_{CB}(3)\sin 45°$

$\omega_{CB} = 2.45$ rad/s $\circlearrowright$ **Ans.**

$v_C = 2.20$ ft/s $\leftarrow$ **Ans.**



Also,

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$

$-v_C\mathbf{i} = (6\sin 30°\mathbf{i} - 6\cos 30°\mathbf{j}) + (\omega_{CB}\mathbf{k}) \times (3\cos 45°\mathbf{i} + 3\sin 45°\mathbf{j})$

$\left(\overset{+}{\rightarrow}\right)\qquad -v_C = 3 - 2.12\omega_{CB}$

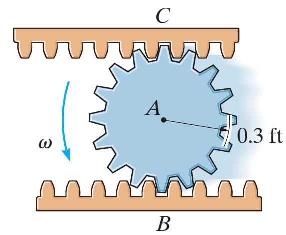
$(+\uparrow)\qquad 0 = -5.196 + 2.12\omega_{CB}$

$\omega_{CB} = 2.45$ rad/s $\circlearrowright$ **Ans.**

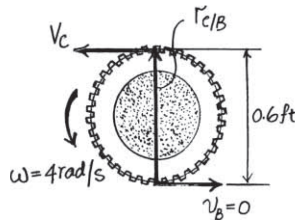
$v_C = 2.20$ ft/s $\leftarrow$ **Ans.**

***16–64.**

Pinion gear A rolls on the fixed gear rack B with an angular velocity $\omega = 4$ rad/s. Determine the velocity of the gear rack C.



SOLUTION



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$(\overset{+}{\leftarrow})$ $$v_C = 0 + 4(0.6)$$

$$v_C = 2.40 \text{ ft/s}$$ **Ans.**

Also:

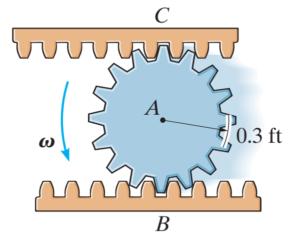
$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

$$-v_C\,\mathbf{i} = 0 + (4\mathbf{k}) \times (0.6\mathbf{j})$$

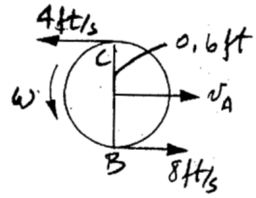
$$v_C = 2.40 \text{ ft/s}$$ **Ans.**

16–65.

The pinion gear rolls on the gear racks. If B is moving to the right at 8 ft/s and C is moving to the left at 4 ft/s, determine the angular velocity of the pinion gear and the velocity of its center A.



SOLUTION



$$v_C = v_B + v_{C/B}$$

$(\overset{+}{\rightarrow})\qquad -4 = 8 - 0.6(\omega)$

$$\omega = 20 \text{ rad/s} \qquad \textbf{Ans.}$$

$$v_A = v_B + v_{A/B}$$

$(\overset{+}{\rightarrow})\qquad v_A = 8 - 20(0.3)$

$$v_A = 2 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

Also:

$$v_C = v_B + \omega \times \mathbf{r}_{C/B}$$

$$-4\mathbf{i} = 8\mathbf{i} + (\omega\mathbf{k}) \times (0.6\mathbf{j})$$

$$-4 = 8 - 0.6\omega$$

$$\omega = 20 \text{ rad/s} \qquad \textbf{Ans.}$$

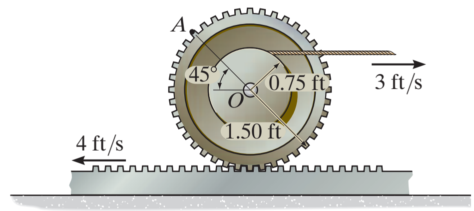
$$v_A = v_B + \omega \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{i} = 8\mathbf{i} + 20\mathbf{k} \times (0.3\mathbf{j})$$

$$v_A = 2 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

16–66.

Determine the angular velocity of the gear and the velocity of its center O at the instant shown.



SOLUTION

General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of the gear shown in Fig. a,

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{B/C}$$

$$3\mathbf{i} = -4\mathbf{i} + \left(-\omega\mathbf{k}\right) \times \left(2.25\mathbf{j}\right)$$

$$3\mathbf{i} = \left(2.25\omega - 4\right)\mathbf{i}$$

Equating the **i** components yields

$$3 = 2.25\omega - 4 \qquad \textbf{(1)}$$

$$\omega = 3.111 \text{ rad/s} \qquad \textbf{Ans. (2)}$$

For points O and C,

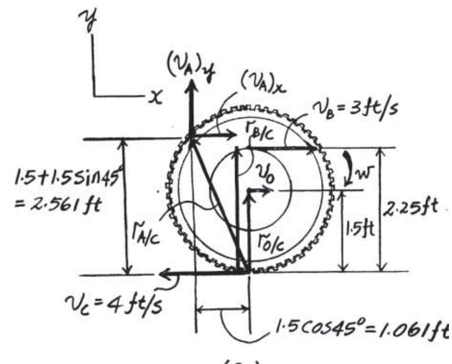
$$\mathbf{v}_O = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{O/C}$$

$$= -4\mathbf{i} + \left(-3.111\mathbf{k}\right) \times \left(1.5\mathbf{j}\right)$$

$$= [0.6667\mathbf{i}] \text{ ft/s}$$

Thus,

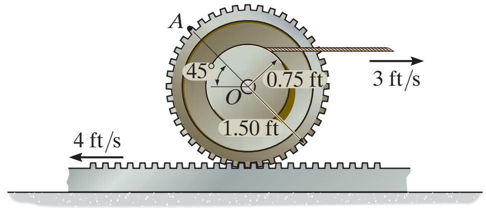
$$v_O = 0.667 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$



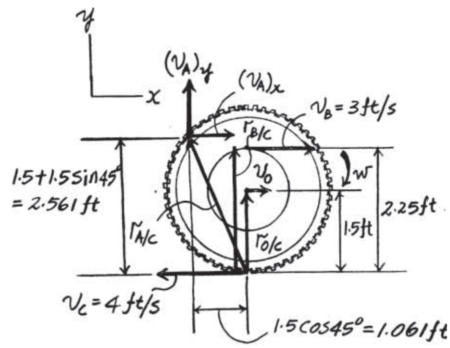
(a)

16–67.

Determine the velocity of point A on the rim of the gear at the instant shown.



SOLUTION



(a)

General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of the gear shown in Fig. *a*,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times \mathbf{r}_{B/C}$$

$$3\mathbf{i} = -4\mathbf{i} + (-\omega\mathbf{k}) \times (2.25\mathbf{j})$$

$$3\mathbf{i} = (2.25\omega - 4)\mathbf{i}$$

Equating the $\mathbf{i}$ components yields

$$3 = 2.25\omega - 4 \qquad \textbf{(1)}$$

$$\omega = 3.111 \text{ rad/s} \qquad \textbf{(2)}$$

For points A and C,

$$\mathbf{v}_A = \mathbf{v}_C + \omega \times \mathbf{r}_{A/C}$$

$$(v_A)_x\,\mathbf{i} + (v_A)_y\,\mathbf{j} = -4\mathbf{i} + (-3.111\mathbf{k}) \times (-1.061\mathbf{i} + 2.561\mathbf{j})$$

$$(v_A)_x\,\mathbf{i} + (v_A)_y\,\mathbf{j} = 3.9665\mathbf{i} + 3.2998\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields

$$(v_A)_x = 3.9665 \text{ ft/s} \qquad (v_A)_y = 3.2998 \text{ ft/s}$$

Thus, the magnitude of v_A is

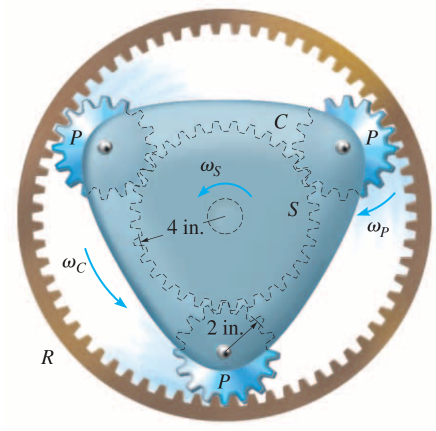
$$v_A = \sqrt{(v_A)_x{}^2 + (v_A)_y{}^2} = \sqrt{3.9665^2 + 3.2998^2} = 5.16 \text{ ft/s} \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(v_A)_y}{(v_A)_x}\right] = \tan^{-1}\left(\frac{3.2998}{3.9665}\right) = 39.8° \qquad \textbf{Ans.}$$

***16–68.**

Part of an automatic transmission consists of a *fixed* ring gear R, three equal planet gears P, the sun gear S, and the planet carrier C, which is shaded. If the sun gear is rotating at $\omega_S = 6$ rad/s, determine the angular velocity ω_C of the *planet carrier*. Note that C is pin connected to the center of each of the planet gears.



SOLUTION

$$\mathbf{v}_D = \mathbf{v}_A + \mathbf{v}_{D/A}$$

$$\underset{\swarrow}{24} = 0 + \underset{\swarrow}{4(\omega_P)}$$

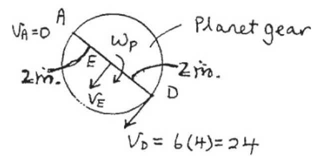
$$\omega_P = 6 \text{ rad/s}$$

$$\mathbf{v}_E = \mathbf{v}_A + \mathbf{v}_{E/A}$$

$$\underset{\swarrow}{v_E} = 0 + \underset{\swarrow}{6(2)}$$

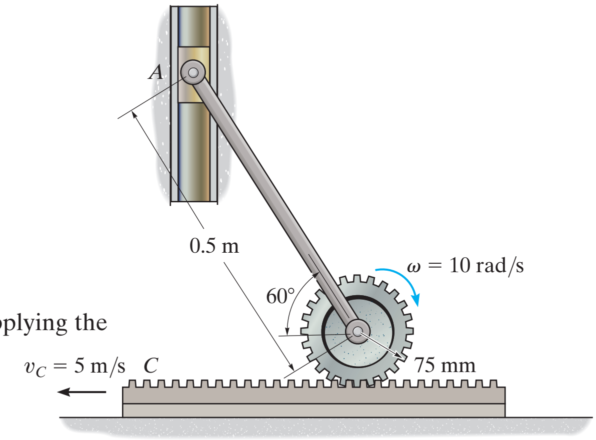
$$v_E = 12 \text{ in./s}$$

$$\omega_C = \frac{12}{6} = 2 \text{ rad/s}$$ **Ans.**



16–69.

If the gear rotates with an angular velocity of $\omega = 10$ rad/s and the gear rack moves at $v_C = 5$ m/s, determine the velocity of the slider block A at the instant shown.



SOLUTION

General Plane Motion: Referring to the diagram shown in Fig. *a* and applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times \mathbf{r}_{B/C}$$

$$= -5\mathbf{i} + (-10\mathbf{k}) \times (0.075\mathbf{j})$$

$$= [-4.25\mathbf{i}] \text{ m/s}$$

Then, applying the relative velocity equation to link AB shown in Fig. *b*,

$$\mathbf{v}_A = \mathbf{v}_B + \omega_{AB} \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{j} = -4.25\mathbf{i} + (-\omega_{AB}\,\mathbf{k}) \times (-0.5\cos 60°\,\mathbf{i} + 0.5\sin 60°\,\mathbf{j})$$

$$v_A\mathbf{j} = (0.4330\omega_{AB} - 4.25)\mathbf{i} + 0.25\omega_{AB}\mathbf{j}$$

Equating the **i** and **j** components, yields

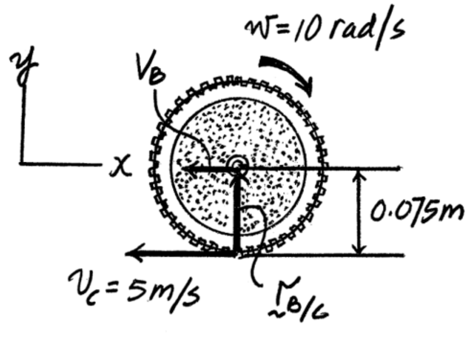
$$0 = 0.4330\omega_{AB} - 4.25 \quad \textbf{(1)}$$

$$v_A = 0.25\omega_{AB} \quad \textbf{(2)}$$

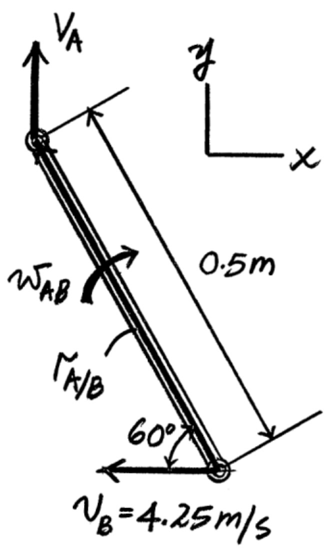
Solving Eqs. (1) and (2) yields

$$\omega_{AB} = 9.815 \text{ rad/s}$$

$$v_A = 2.45 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



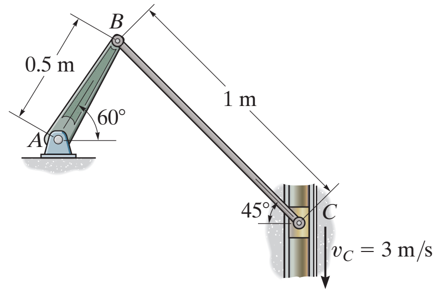
(a)



(b)

16–70.

If the slider block C is moving at $v_C = 3$ m/s, determine the angular velocity of BC and the crank AB at the instant shown.



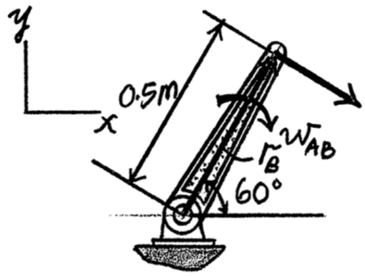
SOLUTION

Rotation About a Fixed Axis: Referring to Fig. a,

$$v_B = \omega_{AB} \times \mathbf{r}_B$$

$$= (-\omega_{AB}\,\mathbf{k}) \times (0.5 \cos 60°\,\mathbf{i} + 0.5 \sin 60°\,\mathbf{j})$$

$$= 0.4330\omega_{AB}\mathbf{i} - 0.25\omega_{AB}\mathbf{j}$$



(a)

General Plane Motion: Applying the relative velocity equation and referring to the kinematic diagram of link BC shown in Fig. b,

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$0.4330\omega_{AB}\mathbf{i} - 0.25\omega_{AB}\mathbf{j} = -3\mathbf{j} + (-\omega_{BC}\mathbf{k}) \times (-1 \cos 45°\,\mathbf{i} + 1 \sin 45°\,\mathbf{j})$$

$$0.4330\omega_{AB}\mathbf{i} - 0.25\omega_{AB}\mathbf{j} = 0.7071\omega_{BC}\mathbf{i} + (0.7071\omega_{BC} - 3)\mathbf{j}$$

Equating the **i** and **j** components yields,

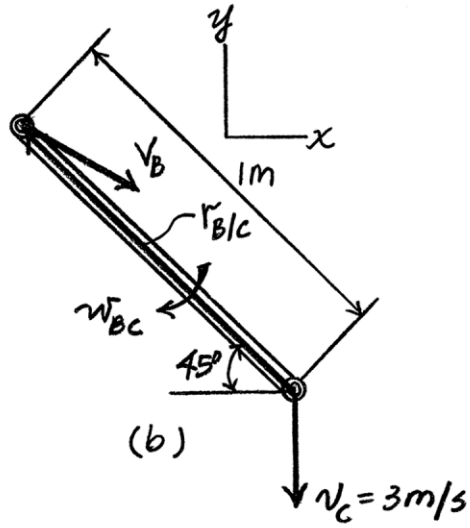
$$0.4330\omega_{AB} = 0.7071\omega_{BC}$$

$$-0.25\omega_{AB} = 0.7071\omega_{BC} - 3$$

Solving,

$$\omega_{BC} = 2.69 \text{ rad/s} \qquad \textbf{Ans.}$$

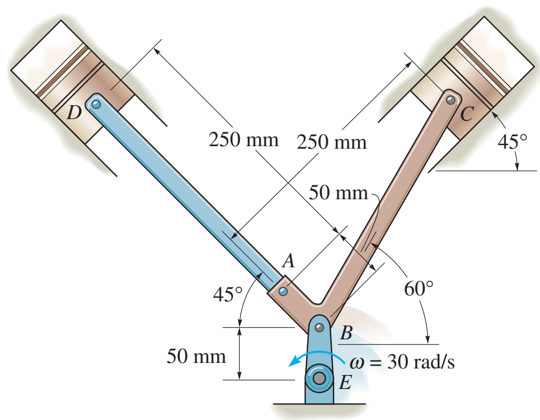
$$\omega_{AB} = 4.39 \text{ rad/s} \qquad \textbf{Ans.}$$



(b)

16–71.

The two-cylinder engine is designed so that the pistons are connected to the crankshaft BE using a master rod ABC and articulated rod AD. If the crankshaft is rotating at $\omega = 30$ rad/s, determine the velocities of the pistons C and D at the instant shown.



SOLUTION

$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

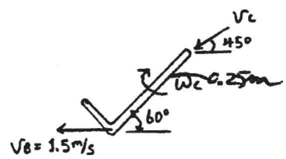
$$\underset{45^\circ\swarrow}{v_C} = \underset{\leftarrow}{1.5} + \underset{\nwarrow 30^\circ}{(0.25)\,\omega}$$

$(\overset{+}{\leftarrow})\qquad v_C \cos 45^\circ = 1.5 - \omega_c(0.25)(\cos 30^\circ)$

$(+\downarrow)\qquad v_C \sin 45^\circ = 0 + \omega_c(0.25)\,(\sin 30^\circ)$

$v_C = 0.776$ m/s **Ans.**

$\omega_C = 4.39$ rad/s



$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

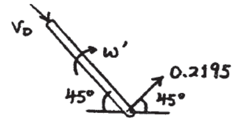
$$\underset{\rightarrow}{v_A} = \underset{\leftarrow}{1.5} + \underset{45^\circ\nearrow}{[0.05(4.39) = 0.2195]}$$

$\mathbf{v}_D = \mathbf{v}_A + \mathbf{v}_{D/A}$

$$\underset{45^\circ\searrow}{\mathbf{v}_D} = \underset{45^\circ\swarrow}{1.5} + \underset{\nearrow}{0.2195} + \underset{\nearrow}{\omega'(0.25)}$$

$(\searrow+)\qquad v_D = -1.5 \sin 45^\circ$

$v_D = 1.06$ m/s $\nwarrow$ **Ans.**



Also:

$\mathbf{v}_B = \omega \times \mathbf{r}_{B/E}$

$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$

$-v_C \cos 45^\circ\mathbf{i} - v_C \sin\ 45^\circ\mathbf{j} = (30\mathbf{k}) \times (0.05\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (0.25 \cos\ 60^\circ\mathbf{i} + 0.25 \sin 60^\circ\mathbf{j})$

$-v_C \cos 45^\circ = -1.5 - \omega_{BC}(0.2165)$

$-v_C \sin 45^\circ = 0.125\omega_{BC}$

$v_C = 0.776$ m/s **Ans.**

$\omega_{BC} = 4.39$ rad/s

$\mathbf{v}_A = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{A/B}$

$\mathbf{v}_D = \mathbf{v}_A + \omega_{AD} \times \mathbf{r}_{D/A}$

$$v_D \cos 45^\circ\mathbf{i} - v_D \sin 45^\circ\mathbf{j} = (30\mathbf{k}) \times (0.05\mathbf{j}) + (-4.39\mathbf{k}) \times (0.05 \cos 45^\circ\mathbf{i} + 0.05 \sin 45^\circ\mathbf{j}) + (\omega_{AD}\mathbf{k}) \times (-0.25 \cos 45^\circ\mathbf{i} + 0.25 \sin 45^\circ\mathbf{j})$$

$v_D \cos 45^\circ = -1.5 + 0.1552 - \omega_{AD}(0.1768)$

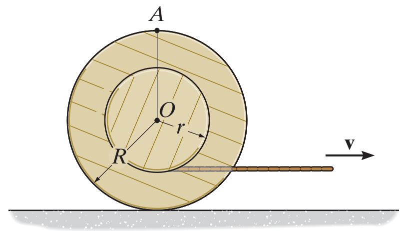
$-v_D \sin 45^\circ = 0.1552 - 0.1768\omega_{AD}$

$\omega_{AD} = 3.36$ rad/s

$v_D = 1.06$ m/s **Ans.**

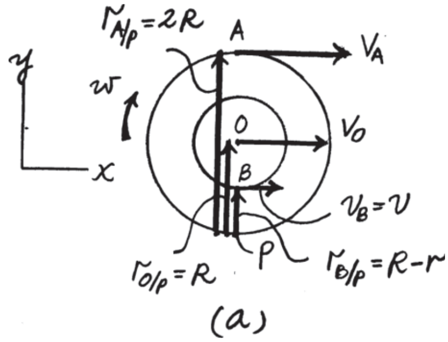
***16–72.**

Determine the velocity of the center O of the spool when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.



SOLUTION

Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. a.



General Plane Motion: Applying the relative velocity equation and referring to Fig. a,

$$\mathbf{v}_B = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the $\mathbf{i}$ components, yields

$$v = \omega(R - r) \qquad \omega = \frac{v}{R - r}$$

Using this result,

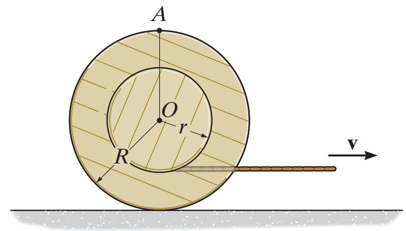
$$\mathbf{v}_O = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{O/P}$$

$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times R\mathbf{j}$$

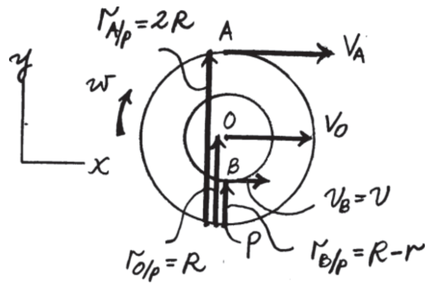
$$\mathbf{v}_O = \left(\frac{R}{R - r}\right)v \rightarrow$$ **Ans.**

16–73.

Determine the velocity of point A on the outer rim of the spool at the instant shown when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.



SOLUTION



(a)

Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. *a*.

General Plane Motion: Applying the relative velocity equation and referring to Fig. *a*,

$$\mathbf{v}_B = \mathbf{v}_P + \omega \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the $\mathbf{i}$ components, yields

$$v = \omega(R - r) \qquad \omega = \frac{v}{R - r}$$

Using this result,

$$\mathbf{v}_A = \mathbf{v}_P + \omega \times \mathbf{r}_{A/P}$$

$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times 2R\mathbf{j}$$

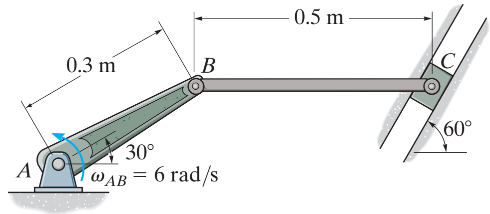
$$= \left[\left(\frac{2R}{R - r}\right)v\right]\mathbf{i}$$

Thus,

$$v_A = \left(\frac{2R}{R - r}\right)v \rightarrow$$ **Ans.**

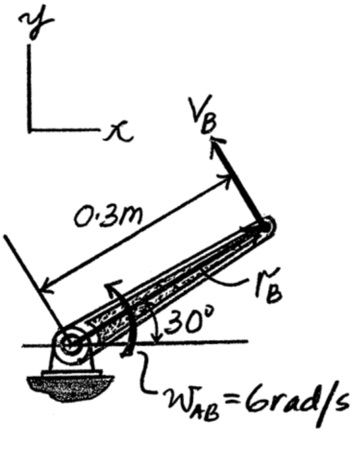
16–74.

If crank AB rotates with a constant angular velocity of $\omega_{AB} = 6$ rad/s, determine the angular velocity of rod BC and the velocity of the slider block at the instant shown. The rod is in a horizontal position.



SOLUTION

Rotation About a Fixed Axis: Referring to Fig. *a*,



(a)

$$v_B = \omega_{AB} \times r_B$$

$$= (6\mathbf{k}) \times (0.3 \cos 30° \,\mathbf{i} + 0.3 \sin 30° \,\mathbf{j})$$

$$= [-0.9\mathbf{i} + 1.559\mathbf{j}]$$

General Plane Motion: Applying the relative velocity equation to the kinematic diagram of link BC shown in Fig. *b*,

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$(-0.9\mathbf{i} + 1.559\mathbf{j}) = (-v_C \cos 60° \,\mathbf{i} - v_C \sin 60° \,\mathbf{j}) + (-\omega_{BC}\mathbf{k}) \times (-0.5\mathbf{i})$$

$$-0.9\mathbf{i} + 1.559\mathbf{j} = -0.5v_C\mathbf{i} + (0.5\omega_{BC} - 0.8660v_C)\mathbf{j}$$

Equating the **i** and **j** components yields

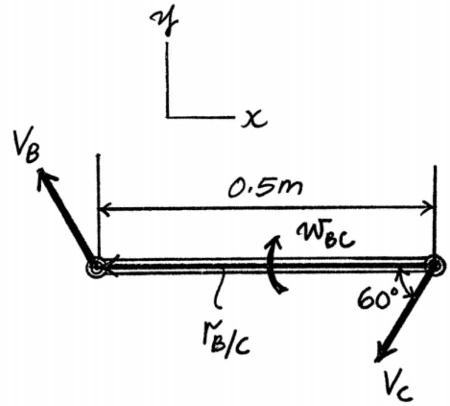
$$-0.9 = -0.5v_C \qquad \textbf{(1)}$$

$$1.559 = 0.5\omega_{BC} - 0.8660v_C \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_C = 1.80 \text{ m/s} \qquad \textbf{Ans.}$$

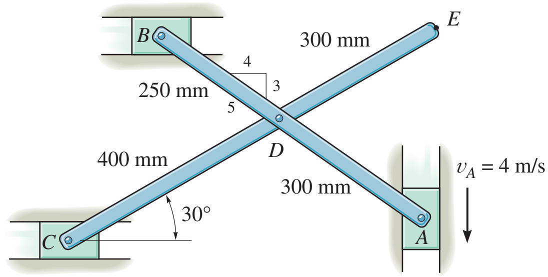
$$\omega_{BC} = 6.24 \text{ rad/s} \qquad \textbf{Ans.}$$



(b)

16–75.

If the slider block A is moving downward at $v_A = 4$ m/s, determine the velocity of block B at the instant shown.



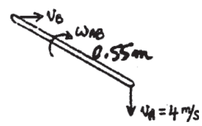
SOLUTION

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$\underset{\rightarrow}{v_B} = 4\downarrow + \omega_{AB}(0.55)$

$(\overset{+}{\rightarrow}) \qquad v_B = 0 + \omega_{AB}(0.55)(\frac{3}{5})$

$(+\uparrow) \qquad 0 = -4 + \omega_{AB}(0.55)(\frac{4}{5})$



Solving,

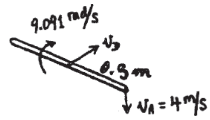
$\omega_{AB} = 9.091$ rad/s

$v_B = 3.00$ m/s **Ans.**

Also:

$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$

$v_B\mathbf{i} = -4\mathbf{j} + (-\omega_{AB}\mathbf{k}) \times \{\frac{-4}{5}(0.55)\mathbf{i} + \frac{3}{5}(0.55)\mathbf{j}\}$

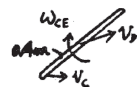


$v_B = \omega_{AB}\,(0.33)$

$0 = -4 + 0.44\omega_{AB}$

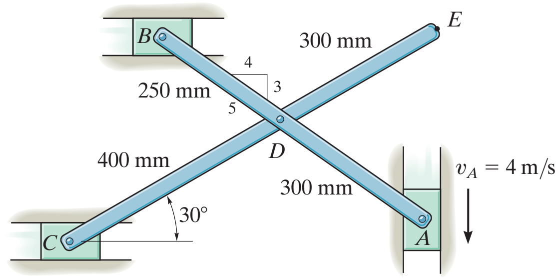
$\omega_{AB} = 9.091$ rad/s

$v_B = 3.00$ m/s **Ans.**



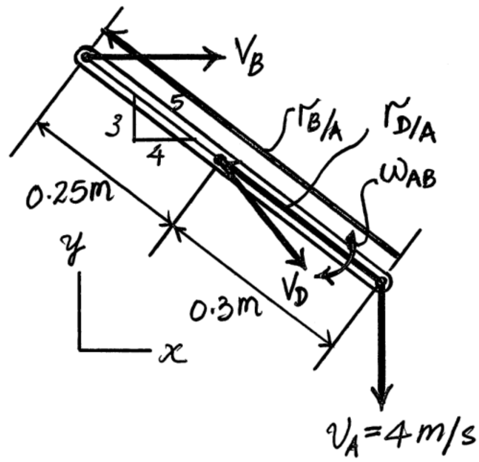
***16–76.**

If the slider block A is moving downward at $v_A = 4$ m/s, determine the velocity of block C at the instant shown.



SOLUTION

General Plane Motion: Applying the relative velocity equation by referring to the kinematic diagram of link AB shown in Fig. a,



(a)

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{i} = -4\mathbf{j} + (-\omega_{AB}\mathbf{k}) \times \left[-0.55\left(\frac{4}{5}\right)\mathbf{i} + 0.55\left(\frac{3}{5}\right)\mathbf{j}\right]$$

$$v_B\mathbf{i} = 0.33\omega_{AB}\,\mathbf{i} + (0.44\omega_{AB} - 4)\,\mathbf{j}$$

Equating **j** component,

$$0 = 0.44\omega_{AB} - 4 \qquad \omega_{AB} = 9.091 \text{ rad/s} \circlearrowright$$

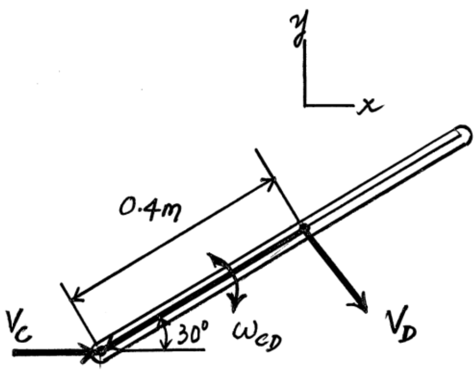
Using the result of ω_{AB},

$$\mathbf{v}_D = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{D/A}$$

$$= -4\mathbf{j} + (-9.091\mathbf{k}) \times \left[-0.3\left(\frac{4}{5}\right)\mathbf{i} + 0.3\left(\frac{3}{5}\right)\mathbf{j}\right]$$

$$= \{1.636\mathbf{i} - 1.818\mathbf{j}\} \text{ m/s}$$

Using the result of $\mathbf{v}_D$ to consider the motion of link CDE, Fig. b,



(b)

$$\mathbf{v}_C = \mathbf{v}_D + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D}$$

$$v_C\mathbf{i} = (1.636\mathbf{i} - 1.818\mathbf{j}) + (-\omega_{CD}\mathbf{k}) \times (-0.4\cos 30°\,\mathbf{i} - 0.4\sin 30°\,\mathbf{j})$$

$$v_C\mathbf{i} = (1.636 - 0.2\omega_{CD})\mathbf{i} + (0.3464\omega_{CD} - 1.818)\mathbf{j}$$

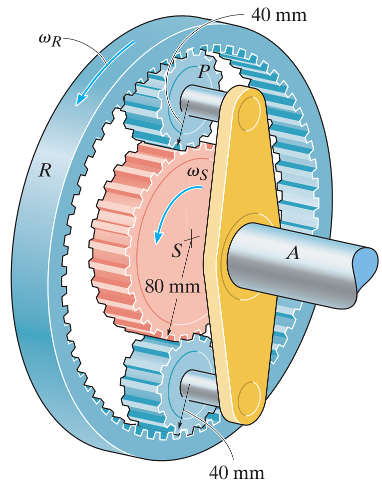
Equating **j** and **i** components,

$$0 = 0.3464\omega_{CD} - 1.818 \qquad \omega_{CD} = 5.249 \text{ rad/s} \circlearrowright$$

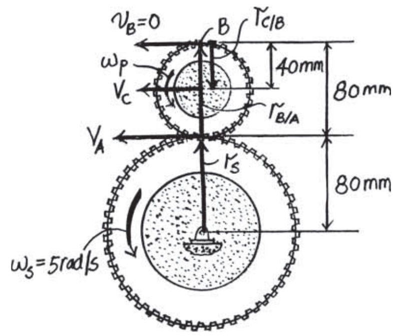
$$v_C = 1.636 - 0.2(5.249) = 0.587 \text{ m/s} \rightarrow$$ **Ans.**

16–77.

The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear R is held fixed, $\omega_R = 0$, and the sun gear S is rotating at $\omega_S = 5$ rad/s. Determine the angular velocity of each of the planet gears P and shaft A.



SOLUTION



$v_A = 5(80) = 400$ mm/s $\leftarrow$

$v_B = 0$

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$0 = -400\mathbf{i} + (\omega_p\,\mathbf{k}) \times (80\mathbf{j})$

$0 = -400\mathbf{i} - 80\omega_p\,\mathbf{i}$

$\omega_P = -5 \text{ rad/s} = 5 \text{ rad/s}$ **Ans.**

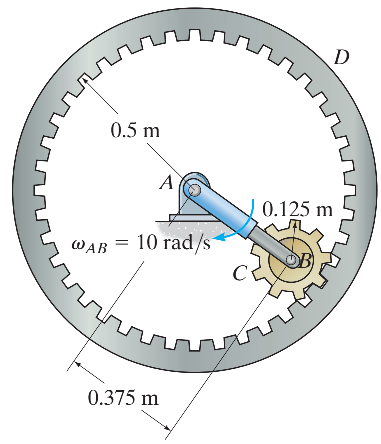
$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$\mathbf{v}_C = 0 + (-5\mathbf{k}) \times (-40\mathbf{j}) = -200\mathbf{i}$

$\omega_A = \dfrac{200}{120} = 1.67 \text{ rad/s}$ **Ans.**

16–78.

If the ring gear D rotates counterclockwise with an angular velocity of $\omega_D = 5$ rad/s while link AB rotates clockwise with an angular velocity of $\omega_{AB} = 10$ rad/s, determine the angular velocity of gear C.

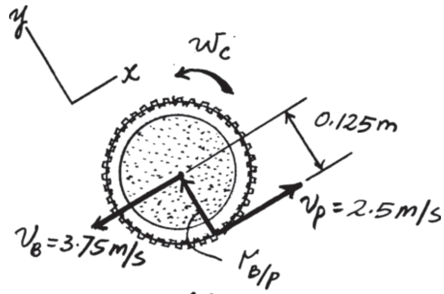


SOLUTION

Rotation About a Fixed Axis: Since link AB and gear D rotate about a fixed axis, Fig. a, the velocity of the center B and the contact point of gears D and C is

$$v_B = \omega_{AB}\, r_B = 10(0.375) = 3.75 \text{ m/s}$$

$$v_P = \omega_D\, r_P = 5(0.5) = 2.5 \text{ m/s}$$



(a)

General Plane Motion: Applying the relative velocity equation and referring to the kinematic diagram of gear C shown in Fig. b,

$$\mathbf{v}_B = \mathbf{v}_P + \omega_C \times r_{B/P}$$

$$-3.75\mathbf{i} = 2.5\mathbf{i} + (\omega_C\,\mathbf{k}) \times (0.125\mathbf{j})$$

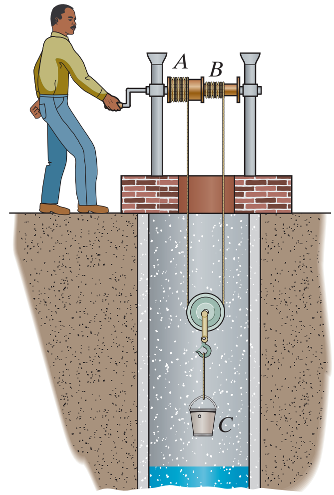
$$-3.75\mathbf{i} = (2.5 - 0.125\omega_C)\mathbf{i}$$

Thus,

$$-3.75 = 2.5 - 0.125\omega_C$$

$$\omega_C = 50 \text{ rad/s} \qquad \textbf{Ans.}$$

16–79.



The differential drum operates in such a manner that the rope is unwound from the small drum B and wound up on the large drum A. If the radii of the large and small drums are R and r, respectively, and for the pulley it is $(R + r)/2$, determine the speed at which the bucket C rises if the man rotates the handle with a constant angular velocity of ω. Neglect the thickness of the rope.

SOLUTION

Rotation About a Fixed Axis: Since the datum rotates about a fixed axis, Fig. *a*, we obtain $v_B = \omega r$ and $v_A = \omega R$.

General Plane Motion: Applying the relative velocity equation and referring to the kinematic diagram of pulley shown in Fig. *b*,

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_P \times \mathbf{r}_{A/B}$$

$$\omega R\mathbf{j} = -\omega r\mathbf{j} + (-\omega_P\mathbf{k}) \times (-2r_P\mathbf{i})$$

$$\omega R\mathbf{j} = (-\omega r + 2\omega_P r_P)\mathbf{j}$$

Equating the **j** components, yields

$$\omega R = -\omega r + 2\omega_P r_P$$

$$\omega_P = \frac{\omega(R + r)}{2r_P}$$

Since $r_P = (R + r)/2$, then

$$\omega_P = \frac{\omega(R + r)}{(R + r)} = \omega$$

Using this result, we have

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_P \times \mathbf{r}_{C/B}$$

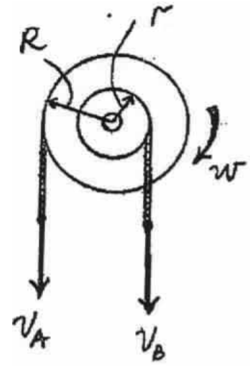
$$= -\omega r\mathbf{j} + (-\omega\mathbf{k}) \times (-r_P\mathbf{i})$$

$$= \left[-\omega r + \frac{\omega(R + r)}{2}\right]\mathbf{j}$$

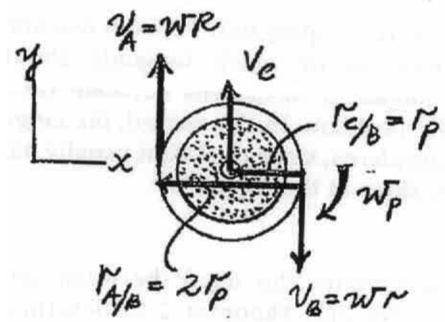
$$= \left[\frac{\omega}{2}(R - r)\right]\mathbf{j}$$

Thus,

$$v_C = \frac{\omega}{2}(R - r) \uparrow$$ **Ans.**



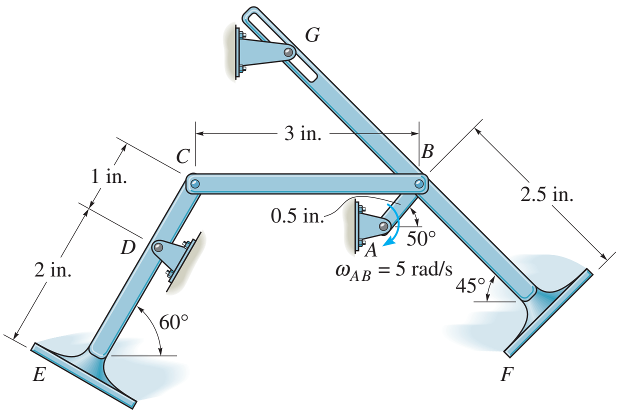
(a)



(b)

***16–80.**

Mechanical toy animals often use a walking mechanism as shown idealized in the figure. If the driving crank AB is propelled by a spring motor such that $\omega_{AB} = 5$ rad/s, determine the velocity of the rear foot E at the instant shown. Although not part of this problem, the upper end of the foreleg has a slotted guide which is constrained by the fixed pin at G.



SOLUTION

$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

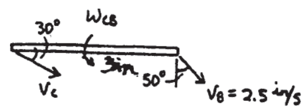
$$\underset{\measuredangle 30^\circ}{v_C} = \underset{\measuredangle 50^\circ}{2.5} + \underset{\downarrow}{3\omega}$$

$(\overset{+}{\rightarrow})\qquad v_C \cos 30^\circ = 2.5 \sin 50^\circ + 0$

$v_C = 2.21$ in./s

$$\omega_{EC} = \frac{2.21}{1} = 2.21 \text{ rad/s}$$

$v_E = (2.21)(2) = 4.42$ in./s **Ans.**



Also:

$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A}$

$\mathbf{v}_C = \omega_{EC} \times \mathbf{r}_{C/D}$

$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$(\omega_{EC}\mathbf{k}) \times (\cos 60^\circ\mathbf{i} + \sin 60^\circ\mathbf{j}) = (-5\mathbf{k}) \times (0.5 \cos 50^\circ\mathbf{i} + 0.5 \sin 50^\circ\mathbf{j}) + (\omega\mathbf{k}) \times (-3\mathbf{i})$

$-0.866\omega_{EC} = 1.915$

$0.5\omega_{EC} = -1.607 - 3\omega$

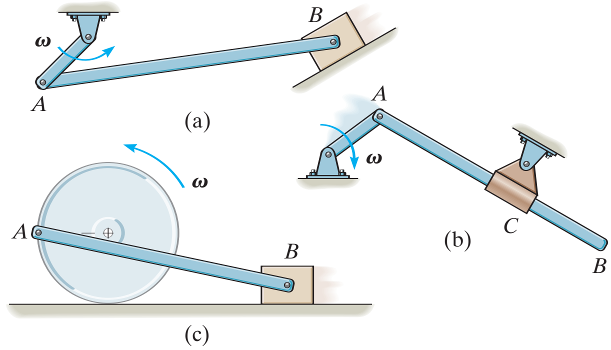
$\omega_{EC} = -2.21$ rad/s

$\omega = -0.167$ rad/s

$v_E = 2(2.21) = 4.42$ in./s **Ans.**

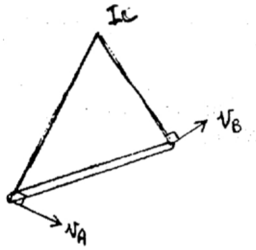
16–81.

In each case show graphically how to locate the instantaneous center of zero velocity of link AB. Assume the geometry is known.

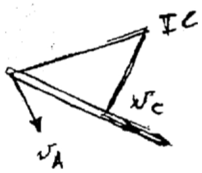


SOLUTION

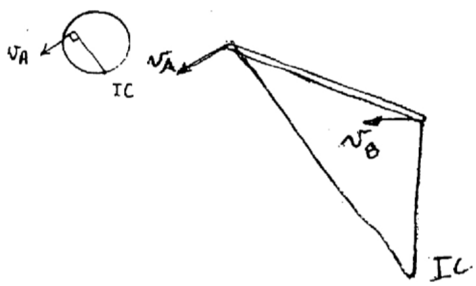
a)



b)

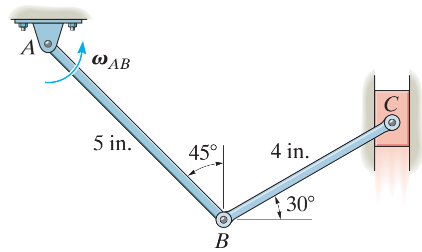


c)



16–82.

Determine the angular velocity of link AB at the instant shown if block C is moving upward at 12 in./s.



SOLUTION

$$\frac{4}{\sin 45^\circ} = \frac{r_{IC-B}}{\sin 30^\circ} = \frac{r_{IC-C}}{\sin 105^\circ}$$

$r_{IC-C} = 5.464$ in.

$r_{IC-B} = 2.828$ in.

$v_C = \omega_{BC}(r_{IC-C})$

$12 = \omega_{BC}(5.464)$

$\omega_{BC} = 2.1962$ rad/s

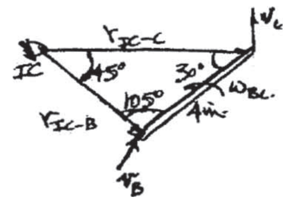
$v_B = \omega_{BC}(r_{IC-B})$

$\quad = 2.1962(2.828) = 6.211$ in./s

$v_B = \omega_{AB}\, r_{AB}$

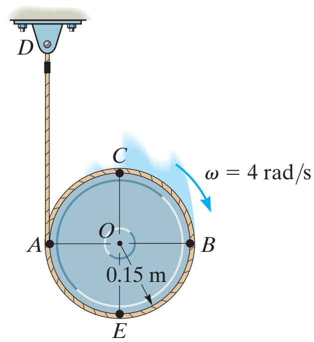
$6.211 = \omega_{AB}(5)$

$\omega_{AB} = 1.24$ rad/s **Ans.**



16–83.

At the instant shown, the disk is rotating at $\omega = 4$ rad/s. Determine the velocities of points A, B, and C.



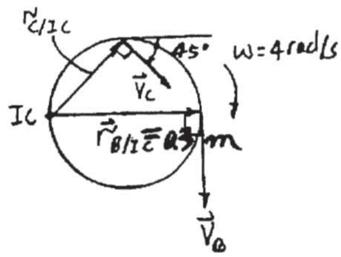
SOLUTION

The instantaneous center is located at point A. Hence, $v_A = 0$ **Ans.**

$$r_{C/IC} = \sqrt{0.15^2 + 0.15^2} = 0.2121 \text{ m} \qquad r_{B/IC} = 0.3 \text{ m}$$

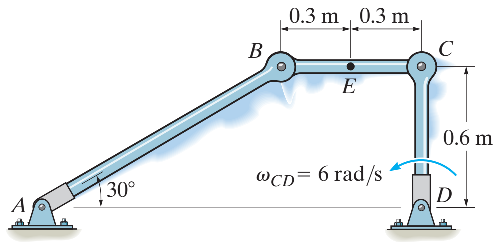
$$v_B = \omega\, r_{B/IC} = 4(0.3) = 1.2 \text{ m/s}$$ **Ans.**

$$v_C = \omega\, r_{C/IC} = 4(0.2121) = 0.849 \text{ m/s} \quad \searrow 45^\circ$$ **Ans.**



***16–84.**

If link CD has an angular velocity of $\omega_{CD} = 6$ rad/s, determine the velocity of point B on link BC and the angular velocity of link AB at the instant shown.

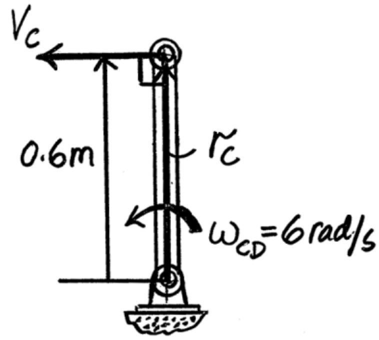


SOLUTION

Rotation About Fixed Axis: Referring to Fig. *a* and *b*,

$$v_C = \omega_{CD} r_C = 6(0.6) = 3.60 \text{ m/s} \leftarrow$$

$$v_B = \omega_{AB} r_B = \omega_{AB}(1.2) \; 60° \measuredangle \qquad \textbf{(1)}$$



(a)

General Plane Motion: The location of IC for link BC is indicated in Fig. *c*. From the geometry of this figure,

$$r_{C/IC} = 0.6 \tan 30° = 0.3464 \text{ m}$$

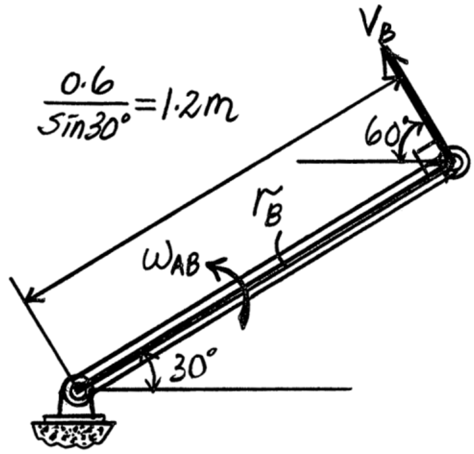
$$r_{B/IC} = \frac{0.6}{\cos 30°} = 0.6928 \text{ m}$$

Thus, the angular velocity of link BC can be determined from

$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{3.60}{0.3464} = 10.39 \text{ rad/s}$$

Then

$$v_B = \omega_{BC} r_{B/IC} = 10.39\,(0.6928) = 7.20 \text{ m/s} \; 60° \measuredangle \qquad \textbf{Ans.}$$

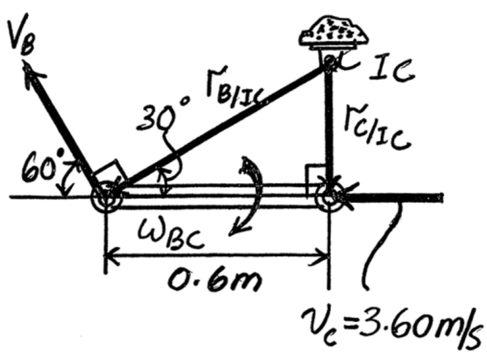


(b)

Substitute this result into Eq. (1),

$$7.20 = \omega_{AB}(1.2)$$

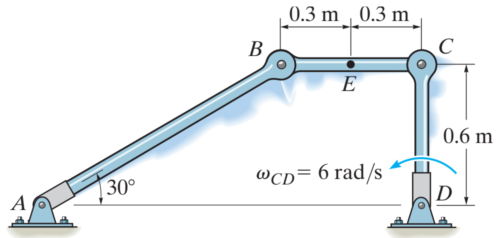
$$\omega_{AB} = 6 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$



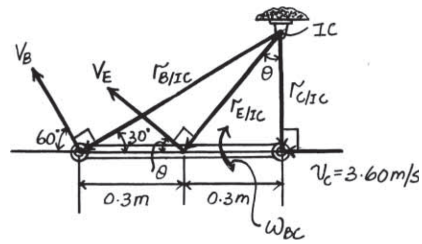
(c)

16–85.

If link CD has an angular velocity of $\omega_{CD} = 6$ rad/s, determine the velocity of point E on link BC and the angular velocity of link AB at the instant shown.



SOLUTION



$$v_C = \omega_{CD}\,(r_{CD}) = (6)(0.6) = 3.60 \text{ m/s}$$

$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{3.60}{0.6 \tan 30^\circ} = 10.39 \text{ rad/s}$$

$$v_B = \omega_{BC}\, r_{B/IC} = (10.39)\left(\frac{0.6}{\cos 30^\circ}\right) = 7.20 \text{ m/s}$$

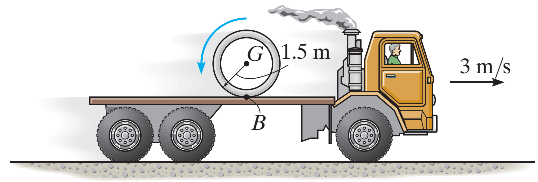
$$\omega_{AB} = \frac{v_B}{r_{AB}} = \frac{7.20}{\left(\dfrac{0.6}{\sin 30^\circ}\right)} = 6 \text{ rad/s} \circlearrowright$$ **Ans.**

$$v_E = \omega_{BC}\, r_{E/IC} = 10.39\sqrt{(0.6 \tan 30^\circ)^2 + (0.3)^2} = 4.76 \text{ m/s}$$ **Ans.**

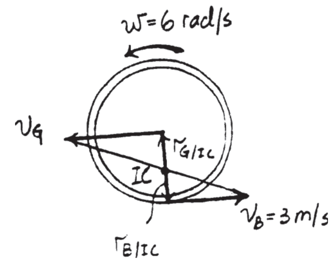
$$\theta = \tan^{-1}\left(\frac{0.3}{0.6 \tan 30^\circ}\right) = 40.9^\circ$$ **Ans.**

16–86.

At the instant shown, the truck travels to the right at 3 m/s, while the pipe rolls counterclockwise at $\omega = 6$ rad/s without slipping at B. Determine the velocity of the pipe's center G.



SOLUTION



Kinematic Diagram: Since the pipe rolls without slipping, then the velocity of point B must be the same as that of the truck, i.e; $v_B = 3$ m/s.

Instantaneous Center: $r_{B/IC}$ must be determined first in order to locate the the instantaneous center of zero velocity of the pipe.

$$v_B = \omega r_{B/IC}$$

$$3 = 6(r_{B/IC})$$

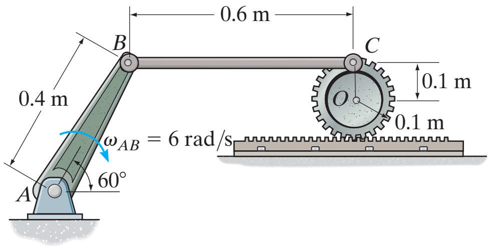
$$r_{B/IC} = 0.5 \text{ m}$$

Thus, $r_{G/IC} = 1.5 - r_{B/IC} = 1.5 - 0.5 = 1.00$ m. Then

$$v_G = \omega r_{G/IC} = 6(1.00) = 6.00 \text{ m/s} \leftarrow$$ **Ans.**

16–87.

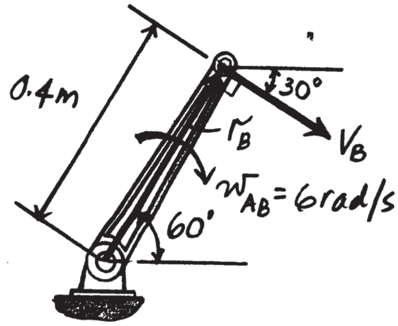
If crank AB is rotating with an angular velocity of $\omega_{AB} = 6$ rad/s, determine the velocity of the center O of the gear at the instant shown.



SOLUTION

Rotation About a Fixed Axis: Referring to Fig. *a*,

$$v_B = \omega_{AB}\, r_B = 6(0.4) = 2.4 \text{ m/s}$$



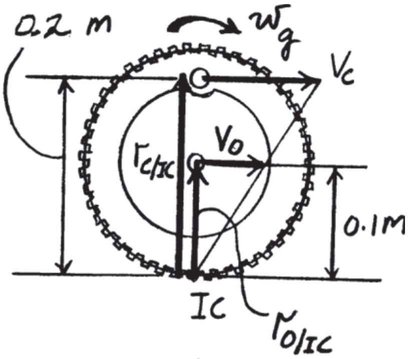
(a)

General Plane Motion: Since the gear rack is stationary, the *IC* of the gear is located at the contact point between the gear and the rack, Fig. *b*. Thus, $\mathbf{v}_O$ and $\mathbf{v}_C$ can be related using the similar triangles shown in Fig. *b*,

$$\omega_g = \frac{v_C}{r_{C/IC}} = \frac{v_O}{r_{O/IC}}$$

$$\frac{v_C}{0.2} = \frac{v_O}{0.1}$$

$$v_C = 2v_O$$



(b)

The location of the *IC* for rod *BC* is indicated in Fig. *c*. From the geometry shown,

$$r_{B/IC} = \frac{0.6}{\cos 60^\circ} = 1.2 \text{ m}$$

$$r_{C/IC} = 0.6 \tan 60^\circ = 1.039 \text{ m}$$

Thus, the angular velocity of rod *BC* can be determined from

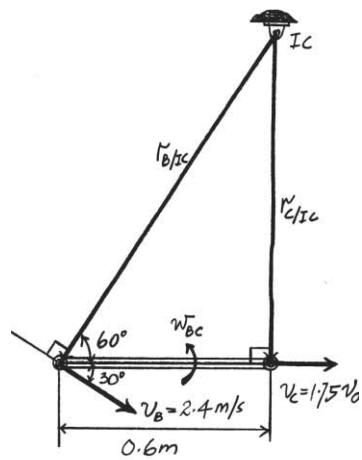
$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{2.4}{1.2} = 2 \text{ rad/s}$$

Then,

$$v_C = \omega_{BC}\, r_{C/IC}$$

$$2v_O = 2(1.039)$$

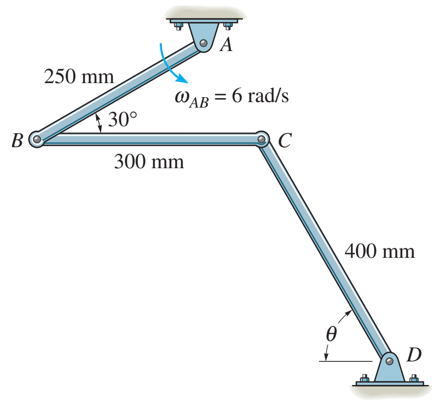
$$v_O = 1.04 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



(c)

***16–88.**

If link AB is rotating at $\omega_{AB} = 6$ rad/s, determine the angular velocities of links BC and CD at the instant $\theta = 60^\circ$.



SOLUTION

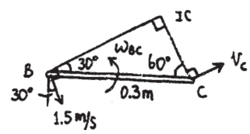
$$r_{IC-B} = 0.3\cos 30^\circ = 0.2598 \text{ m}$$

$$r_{IC-C} = 0.3\cos 60^\circ = 0.1500 \text{ m}$$

$$\omega_{BC} = \frac{1.5}{0.2598} = 5.774 = 5.77 \text{ rad/s}$$ **Ans.**

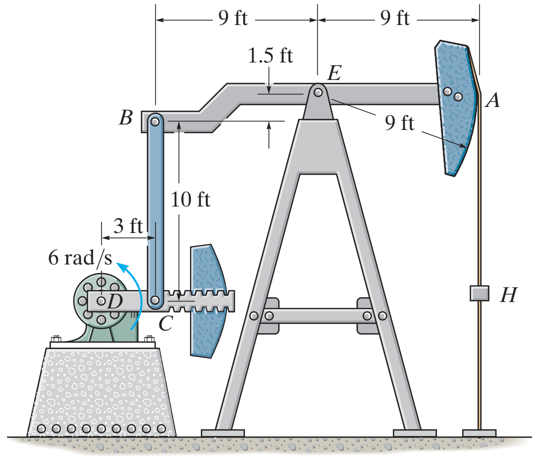
$$v_C = 5.774(0.15) = 0.8661 \text{ m/s}$$

$$\omega_{CD} = \frac{0.8661}{0.4} = 2.17 \text{ rad/s}$$ **Ans.**

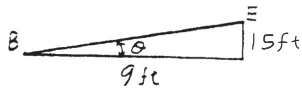


16–89.

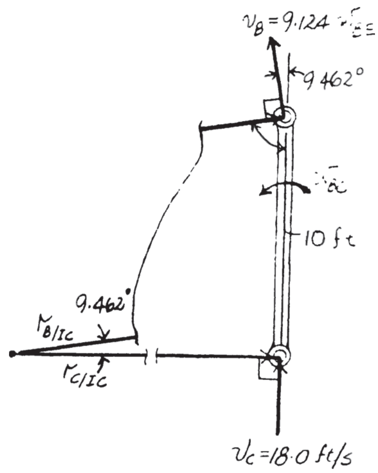
The oil pumping unit consists of a walking beam AB, connecting rod BC, and crank CD. If the crank rotates at a constant rate of 6 rad/s, determine the speed of the rod hanger H at the instant shown. *Hint:* Point B follows a circular path about point E and therefore the velocity of B is *not* vertical.



SOLUTION



Kinematic Diagram: From the geometry, $\theta = \tan^{-1}\left(\dfrac{1.5}{9}\right) = 9.462°$ and $r_{BE} = \sqrt{9^2 + 1.5^2} = 9.124$ ft. Since crank CD and beam BE are rotating about fixed points D and E, then $\mathbf{v}_C$ and $\mathbf{v}_B$ are always directed perpendicular to crank CD and beam BE, respectively. The magnitude of $\mathbf{v}_C$ and $\mathbf{v}_B$ are $v_C = \omega_{CD} r_{CD} = 6(3) = 18.0$ ft/s and $v_B = \omega_{BE} r_{BE} = 9.124\omega_{BE}$. At the instant shown, $\mathbf{v}_C$ is directed vertically while $\mathbf{v}_B$ is directed with an angle 9.462° with the vertical.



Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. From the geometry

$$r_{B/IC} = \frac{10}{\sin 9.462°} = 60.83 \text{ ft}$$

$$r_{C/IC} = \frac{10}{\tan 9.462°} = 60.0 \text{ ft}$$

The angular velocity of link BC is given by

$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{18.0}{60.0} = 0.300 \text{ rad/s}$$

Thus, the angular velocity of beam BE is given by

$$v_B = \omega_{BC} r_{B/IC}$$

$$9.124\omega_{BE} = 0.300(60.83)$$

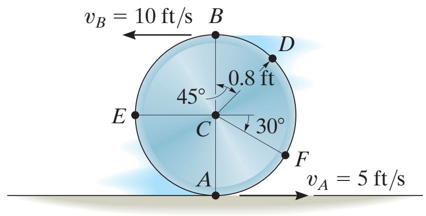
$$\omega_{BE} = 2.00 \text{ rad/s}$$

The speed of rod hanger H is given by

$$v_H = \omega_{BE} r_{EA} = 2.00(9) = 18.0 \text{ ft/s}$$ **Ans.**

16–90.

Due to slipping, points A and B on the rim of the disk have the velocities shown. Determine the velocities of the center point C and point D at this instant.

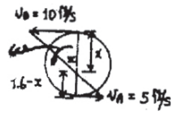


SOLUTION

$$\frac{1.6 - x}{5} = \frac{x}{10}$$

$$5x = 16 - 10x$$

$$x = 1.06667 \text{ ft}$$

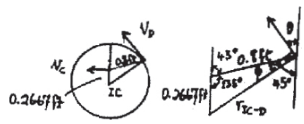


$$\omega = \frac{10}{1.06667} = 9.375 \text{ rad/s}$$

$$r_{IC-D} = \sqrt{(0.2667)^2 + (0.8)^2 - 2(0.2667)(0.8)\cos 135^\circ} = 1.006 \text{ ft}$$

$$\frac{\sin\phi}{0.2667} = \frac{\sin 135^\circ}{1.006}$$

$$\phi = 10.80^\circ$$



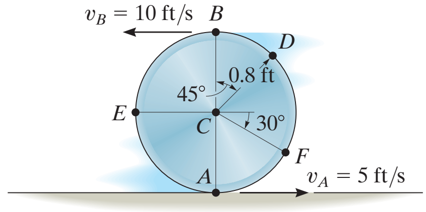
$$v_C = 0.2667(9.375) = 2.50 \text{ ft/s}$$ **Ans.**

$$v_D = 1.006(9.375) = 9.43 \text{ ft/s}$$ **Ans.**

$$\theta = 45^\circ + 10.80^\circ = 55.8^\circ$$

16–91.

Due to slipping, points A and B on the rim of the disk have the velocities shown. Determine the velocities of the center point C and point E at this instant.



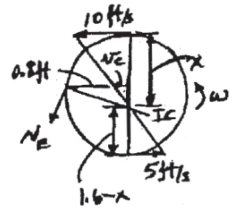
SOLUTION

$$\frac{1.6 - x}{5} = \frac{x}{10}$$

$$5x = 16 - 10x$$

$$x = 1.06667 \text{ ft}$$

$$\omega = \frac{10}{1.06667} = 9.375 \text{ rad/s}$$



$$v_C = \omega(r_{IC-C})$$

$$= 9.375(1.06667 - 0.8)$$

$$= 2.50 \text{ ft/s}$$ **Ans.**

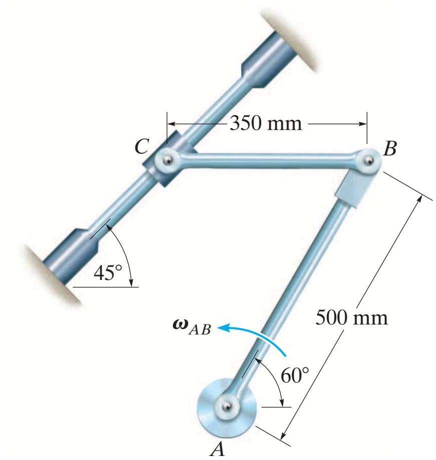
$$v_E = \omega(r_{IC-E})$$

$$= 9.375\sqrt{(0.8)^2 + (0.26667)^2}$$

$$= 7.91 \text{ ft/s}$$ **Ans.**

***16–92.**

Knowing that the angular velocity of link AB is $\omega_{AB} = 4$ rad/s, determine the velocity of the collar at C and the angular velocity of link CB at the instant shown. Link CB is horizontal at this instant.



SOLUTION

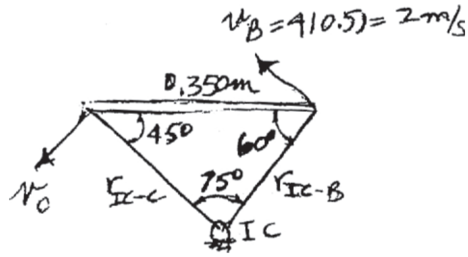
$$\frac{0.350}{\sin 75^\circ} = \frac{r_{IC-B}}{\sin 45^\circ} = \frac{r_{IC-C}}{\sin 60^\circ}$$

$$r_{IC-B} = 0.2562 \text{ m}$$

$$r_{IC-C} = 0.3138 \text{ m}$$

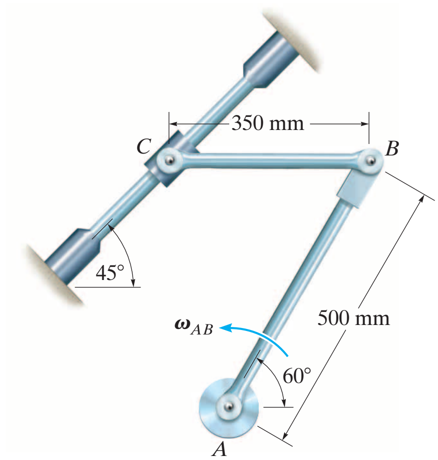
$$\omega_{CB} = \frac{2}{0.2562} = 7.8059 = 7.81 \text{ rad/s}$$ **Ans.**

$$v_C = 7.8059(0.3138) = 2.45 \text{ m/s}$$ **Ans.**



16–93.

If the collar at C is moving downward to the left at $v_C = 8$ m/s, determine the angular velocity of link AB at the instant shown.



SOLUTION

$$\frac{0.350}{\sin 75^\circ} = \frac{r_{IC-B}}{\sin 45^\circ} = \frac{r_{IC-C}}{\sin 60^\circ}$$

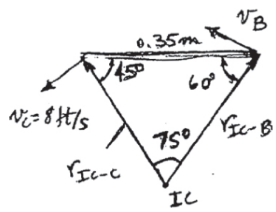
$$r_{IC-B} = 0.2562 \text{ m}$$

$$r_{IC-C} = 0.3138 \text{ m}$$

$$\omega_{CB} = \frac{8}{0.3138} = 25.494 \text{ rad/s}$$

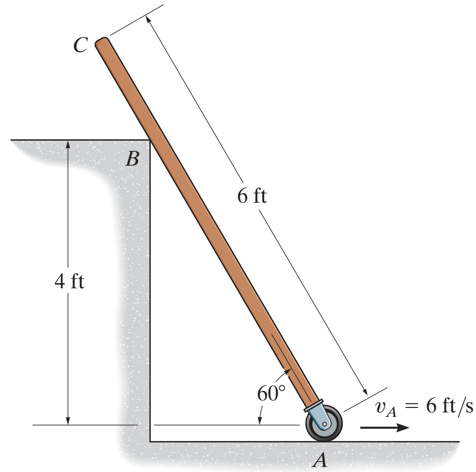
$$v_B = 25.494(0.2562) = 6.5315 \text{ m/s}$$

$$\omega_{AB} = \frac{6.5315}{0.5} = 13.1 \text{ rad/s}$$ **Ans.**



16–94.

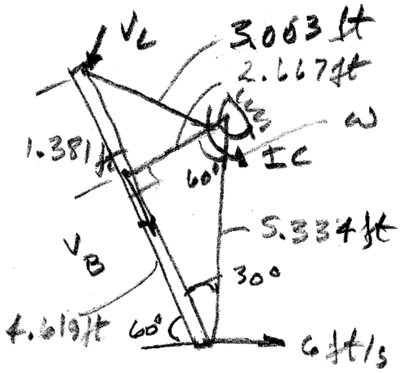
If the roller is given a velocity of $v_A = 6$ ft/s to the right, determine the angular velocity of the rod and the velocity of C at the instant shown.

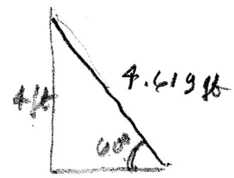


SOLUTION

$$\omega = \frac{6 \text{ ft/s}}{5.334 \text{ ft}} = 1.125 \text{ rad/s} = 1.12 \text{ rad/s} \qquad \textbf{Ans.}$$

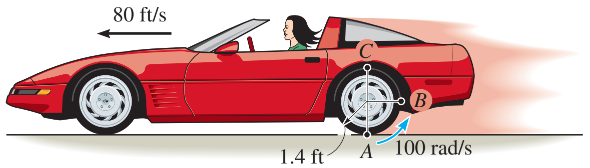
$$v_C = (1.125 \text{ rad/s})(3.003 \text{ ft}) = 3.38 \text{ ft/s} \qquad \textbf{Ans.}$$



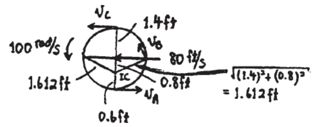


16–95.

As the car travels forward at 80 ft/s on a wet road, due to slipping, the rear wheels have an angular velocity $\omega = 100$ rad/s. Determine the speeds of points A, B, and C caused by the motion.



SOLUTION



$$r = \frac{80}{100} = 0.8 \text{ ft}$$

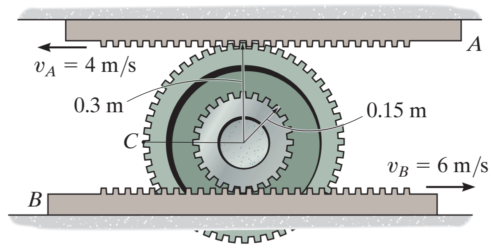
$$v_A = 0.6(100) = 60.0 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

$$v_C = 2.2(100) = 220 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

$$v_B = 1.612(100) = 161 \text{ ft/s} \quad \measuredangle 60.3^\circ \qquad \textbf{Ans.}$$

***16–96.**

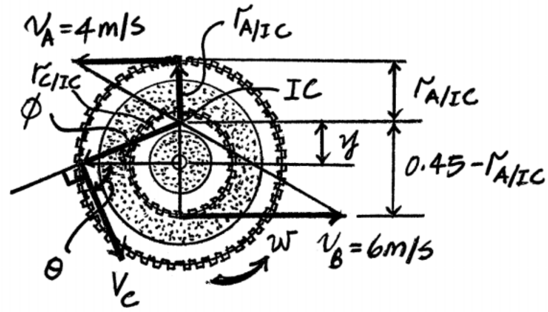
Determine the angular velocity of the double-tooth gear and the velocity of point C on the gear.



SOLUTION

General Plane Motion: The location of the IC can be found using the similar triangles shown in Fig. a.

$$\frac{r_{A/IC}}{4} = \frac{0.45 - r_{A/IC}}{6} \qquad r_{A/IC} = 0.18 \text{ m}$$



(a)

Then,

$$y = 0.3 - r_{A/IC} = 0.3 - 0.18 = 0.12 \text{ m}$$

and

$$r_{C/IC} = \sqrt{0.3^2 + 0.12^2} = 0.3231 \text{ m}$$

$$\phi = \tan^{-1}\left(\frac{0.12}{0.3}\right) = 21.80°$$

Thus, the angular velocity of the gear can be determined from

$$\omega = \frac{v_A}{r_{A/IC}} = \frac{4}{0.18} = 22.22 \text{ rad/s} = 22.2 \text{ rad/s} \qquad \textbf{Ans.}$$

Then

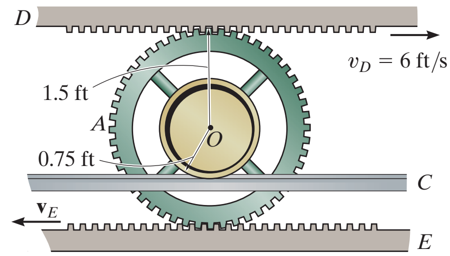
$$v_C = \omega r_{C/IC} = 22.2(0.3231) = 7.18 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is

$$\phi = 90° - \phi = 90° - 21.80° = 68.2° \qquad \textbf{Ans.}$$

16–97.

The wheel is rigidly attached to gear A, which is in mesh with gear racks D and E. If D has a velocity of $v_D = 6$ f t/s to the right and the wheel rolls on track C without slipping, determine the velocity of gear rack E.



SOLUTION

General Plane Motion: Since the wheel rolls without slipping on track C, the IC is located there, Fig. a. Here,

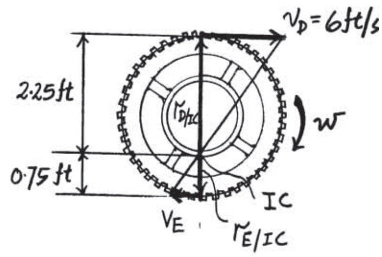
$$r_{D/IC} = 2.25 \text{ ft} \qquad r_{E/IC} = 0.75 \text{ ft}$$

Thus, the angular velocity of the gear can be determined from

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{6}{2.25} = 2.667 \text{ rad/s}$$

Then,

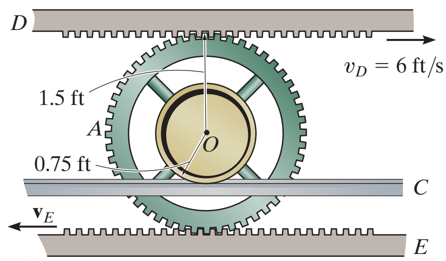
$$v_E = \omega r_{E/IC} = 2.667(0.75) = 2 \text{ ft/s} \leftarrow$$ **Ans.**



(a)

16–98.

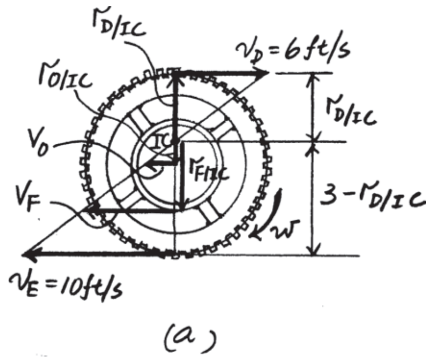
The wheel is rigidly attached to gear A, which is in mesh with gear racks D and E. If the racks have a velocity of $v_D = 6\text{ ft/s}$ and $v_E = 10\text{ ft/s}$, show that it is necessary for the wheel to slip on the fixed track C. Also find the angular velocity of the gear and the velocity of its center O.



SOLUTION

General Plane Motion: The location of the IC can be found using the similar triangles shown in Fig. a,

$$\frac{r_{D/IC}}{6} = \frac{3 - r_{D/IC}}{10} \qquad r_{D/IC} = 1.125\text{ ft}$$



(a)

Thus,

$$r_{O/IC} = 1.5 - r_{D/IC} = 1.5 - 1.125 = 0.375\text{ft}$$

$$r_{F/IC} = 2.25 - r_{D/IC} = 2.25 - 1.125 = 1.125\text{ft}$$

Thus, the angular velocity of the gear is

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{6}{1.125} = 5.333\text{ rad/s} = 5.33\text{ rad/s} \qquad \textbf{Ans.}$$

The velocity of the contact point F between the wheel and the track is

$$v_F = \omega r_{F/IC} = 5.333(1.125) = 6\text{ ft/s} \leftarrow$$

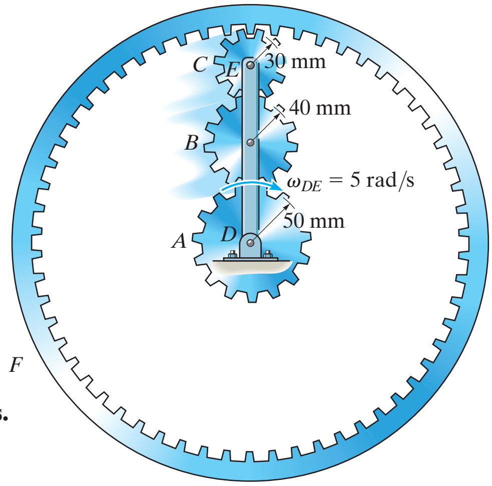
Since $v_F \neq 0$, **the wheel slips on the track** **(Q.E.D.)**

The velocity of center O of the gear is

$$v_O = \omega r_{O/IC} = 5.333(0.375) = 2\text{ft/s} \leftarrow \qquad \textbf{Ans.}$$

16–99.

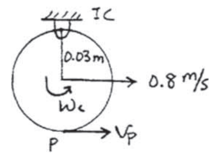
The epicyclic gear train is driven by the rotating link DE, which has an angular velocity $\omega_{DE} = 5$ rad/s. If the ring gear F is fixed, determine the angular velocities of gears A, B, and C.



SOLUTION

$$v_E = 0. \qquad = 0.8 \text{ m/s}$$

$$\omega_C = \frac{0}{0.} \qquad 26.7 \text{ rad/s}$$ **Ans.**



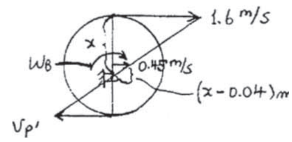
$$v_P = (0. \qquad 6.7) = 1.6 \text{ m/s}$$

$$\frac{1.6}{x} = \frac{\quad}{x}$$

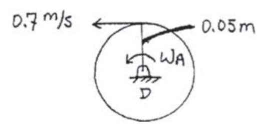
$$x = 0.05$$

$$\omega_B = \frac{\quad}{0} \quad ; = 28.75 \text{ rad/s}$$ **Ans.**

$$v_{P'} = 28.75(0.08 - 0.05565) = 0.700 \text{ m/s} \leftarrow$$

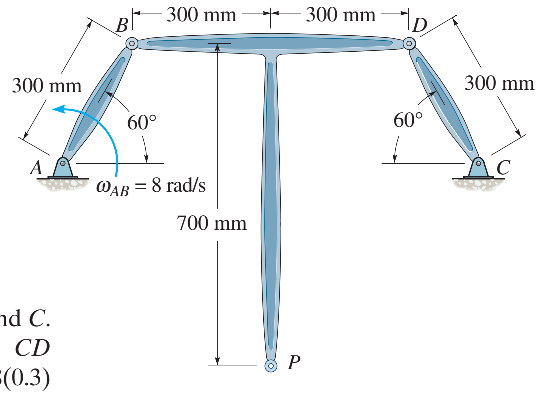


$$\omega_A = \frac{0.700}{0.05} = 14.0 \text{ rad/s}$$ **Ans.**



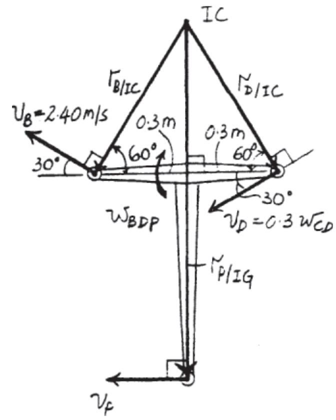
***16–100.**

The similar links AB and CD rotate about the fixed pins at A and C. If AB has an angular velocity $\omega_{AB} = 8\text{ rad/s}$, determine the angular velocity of BDP and the velocity of point P.



SOLUTION

Kinematic Diagram: Since link AB and CD is rotating about fixed points A and C. then $\mathbf{v}_B$ and $\mathbf{v}_D$ are always directed perpendicular to link AB and CD respectively. The magnitude of $\mathbf{v}_B$ and $\mathbf{v}_D$ are $v_B = \omega_{AB} r_{AB} = 8(0.3) = 2.40\text{ m/s}$ and $v_D = \omega_{CD} r_{CD} = 0.3\omega_{CD}$. At the instant shown. $\mathbf{v}_B$ and $\mathbf{v}_D$ are directed at 30° with the horizontal.



Instantaneous Center: The instantaneous center of zero velocity of link BDP at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_D$. From the geometry.

$$r_{B/IC} = \frac{0.3}{\cos 60^\circ} = 0.600\text{ m}$$

$$r_{P/IC} = 0.3 \tan 60^\circ + 0.7 = 1.220\text{ m}$$

The angular velocity of link BDP is given by

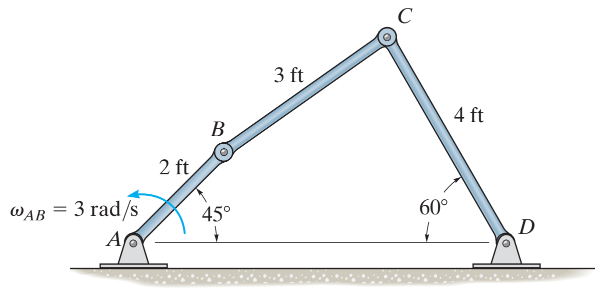
$$\omega_{BDP} = \frac{v_B}{r_{B/IC}} = \frac{2.40}{0.600} = 4.00\text{ rad/s} \qquad \textbf{Ans.}$$

Thus, the velocity of point P is given by

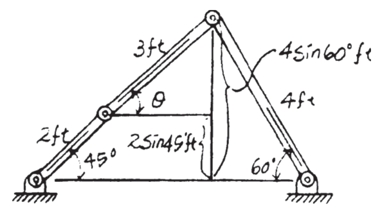
$$v_P = \omega_{BDP} r_{P/IC} = 4.00(1.220) = 4.88\text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

16–101.

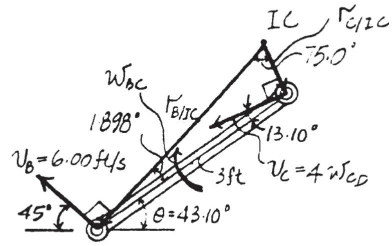
If rod AB is rotating with an angular velocity $\omega_{AB} = 3$ rad/s, determine the angular velocity of rod BC at the instant shown.



SOLUTION



Kinematic Diagram: From the geometry, $\theta = \sin^{-1}\left(\dfrac{4\sin 60^\circ - 2\sin 45^\circ}{3}\right) = 43.10^\circ$. Since links AB and CD is rotating about fixed points A and D, then $\mathbf{v}_B$ and $\mathbf{v}_C$ are always directed perpendicular to links AB and CD, respectively. The magnitude of $\mathbf{v}_B$ and $\mathbf{v}_C$ are $v_B = \omega_{AB}\, r_{AB} = 3(2) = 6.00$ ft/s and $v_C = \omega_{CD} r_{CD} = 4\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ is directed at an angle of 45° while $\mathbf{v}_C$ is directed at 30°



Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 103.1^\circ} = \frac{3}{\sin 75^\circ} \qquad r_{B/IC} = 3.025 \text{ ft}$$

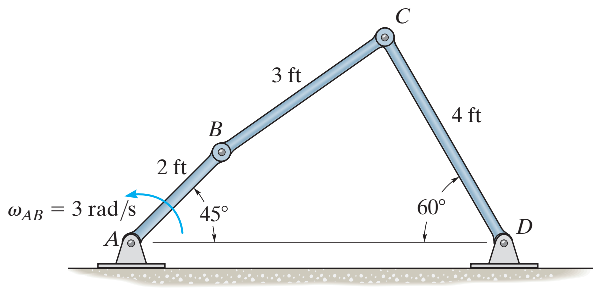
$$\frac{r_{C/IC}}{\sin 1.898^\circ} = \frac{3}{\sin 75^\circ} \qquad r_{C/IC} = 0.1029 \text{ ft}$$

The angular velocity of link BC is given by

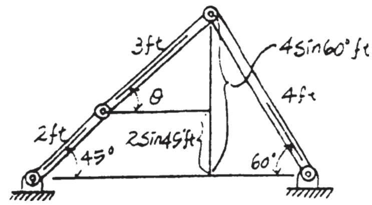
$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{6.00}{3.025} = 1.983 \text{ rad/s} = 1.98 \text{ rad/s} \qquad \textbf{Ans.}$$

16–102.

If rod AB is rotating with an angular velocity $\omega_{AB} = 3$ rad/s, determine the angular velocity of rod CD at the instant shown.

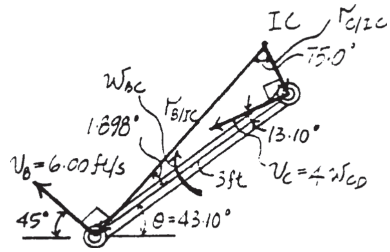


SOLUTION



Kinematic Diagram: From the geometry. $\theta = \sin^{-1}\left(\dfrac{4\sin 60° - 2\sin 45°}{3}\right) = 43.10°$. Since links AB and CD is rotating about fixed points A and D, then $\mathbf{v}_B$ and $\mathbf{v}_C$ are always directed perpendicular to links AB and CD, respectively. The magnitude of $\mathbf{v}_B$ and $\mathbf{v}_C$ are $v_B = \omega_{AB} r_{AB} = 3(2) = 6.00$ ft/s and $v_C = \omega_{CD} r_{CD} = 4\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ is directed at an angle of 45° while $\mathbf{v}_C$ is directed at 30°.

Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have



$$\frac{r_{B/IC}}{\sin 103.1°} = \frac{3}{\sin 75°} \qquad r_{B/IC} = 3.025 \text{ ft}$$

$$\frac{r_{C/IC}}{\sin 1.898°} = \frac{3}{\sin 75°} \qquad r_{C/IC} = 0.1029 \text{ ft}$$

The angular velocity of link BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{6.00}{3.025} = 1.983 \text{ rad/s}$$

Thus, the angular velocity of link CD is given by

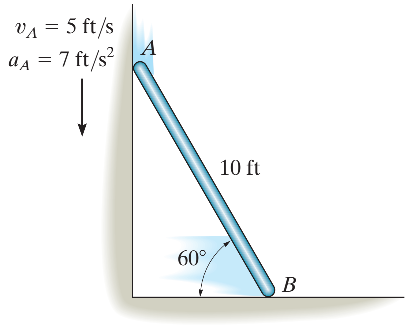
$$v_C = \omega_{BC} r_{C/IC}$$

$$4\omega_{CD} = 1.983(0.1029)$$

$$\omega_{CD} = 0.0510 \text{ rad/s} \qquad \textbf{Ans.}$$

16–103.

At a given instant the top end A of the bar has the velocity and acceleration shown. Determine the acceleration of the bottom B and the bar's angular acceleration at this instant.



SOLUTION

$$\omega = \frac{5}{5} = 1.00 \text{ rad/s}$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

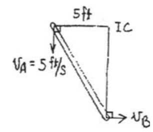
$$\underset{\rightarrow}{a_B} = \underset{\downarrow}{7} + \underset{\nwarrow 30^\circ}{10} + \underset{\measuredangle 30^\circ}{\alpha(10)}$$

$(\overset{+}{\rightarrow})\qquad a_B = 0 - 10\sin 30^\circ + \alpha(10)\cos 30^\circ$

$(+\uparrow)\qquad 0 = -7 + 10\cos 30^\circ + \alpha(10)\sin 30^\circ$

$\alpha = -0.3321 \text{ rad/s}^2 = 0.332 \text{ rad/s}^2$ ↻ **Ans.**

$a_B = -7.875 \text{ ft/s}^2 = 7.88 \text{ ft/s}^2 \leftarrow$ **Ans.**



Also:

$\mathbf{a}_B = \mathbf{a}_A - \omega^2\mathbf{r}_{B/A} + \alpha \times \mathbf{r}_{B/A}$

$a_B\mathbf{i} = -7\mathbf{j} - (1)^2(10\cos 60^\circ\mathbf{i} - \sin 60^\circ\mathbf{j}) + (\alpha\mathbf{k}) \times (10\cos 60^\circ\mathbf{i} - 10\sin 60^\circ\mathbf{j})$

$\overset{+}{\rightarrow}\qquad a_B = -10\cos 60^\circ + \alpha(10\sin 60^\circ)$

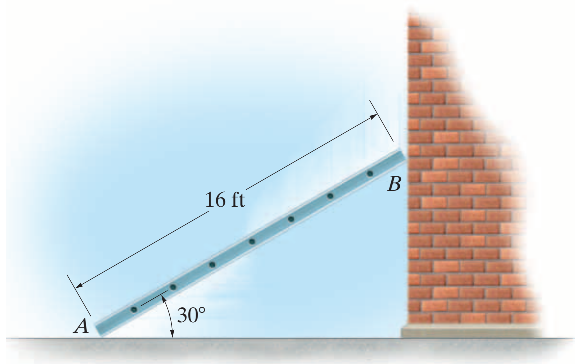
$(+\uparrow)\qquad 0 = -7 + 10\sin 60^\circ + \alpha(10\cos 60^\circ)$

$\alpha = -0.3321 \text{ rad/s}^2 = 0.332 \text{ rad/s}^2$ ↻ **Ans.**

$a_B = -7.875 \text{ ft/s}^2 = 7.88 \text{ ft/s}^2 \leftarrow$ **Ans.**

***16–104.**

At a given instant the bottom A of the ladder has an acceleration $a_A = 4\ \text{ft/s}^2$ and velocity $v_A = 6\ \text{ft/s}$, both acting to the left. Determine the acceleration of the top of the ladder, B, and the ladder's angular acceleration at this same instant.



SOLUTION

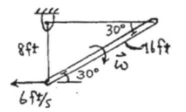
$$\omega = \frac{6}{8} = 0.75\ \text{rad/s}$$

$$a_B = a_A + (a_{B/A})_n + (a_{B/A})_t$$

$$\underset{\downarrow}{a_B} = \underset{\leftarrow}{4} + \underset{30^\circ \swarrow}{(0.75)^2(16)} + \underset{30^\circ \searrow}{\alpha(16)}$$

$$(\overset{+}{\leftarrow}) \qquad 0 = 4 + (0.75)^2(16)\cos 30^\circ - \alpha(16)\sin 30^\circ$$

$$(+\downarrow) \qquad a_B = 0 + (0.75)^2(16)\sin 30^\circ + \alpha(16)\cos 30^\circ$$

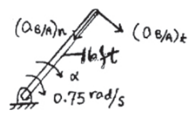


Solving,

$$\alpha = 1.47\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

$$a_B = 24.9\ \text{ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

Also:



$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$-a_B\mathbf{j} = -\mathbf{4i} + (\alpha\text{k}) \times (16\cos 30^\circ\mathbf{i} + 16\sin 30^\circ\mathbf{j}) - (0.75)^2(16\cos 30^\circ\mathbf{i} + 16\sin 30^\circ\mathbf{j})$$

$$0 = -4 - 8\alpha - 7.794$$

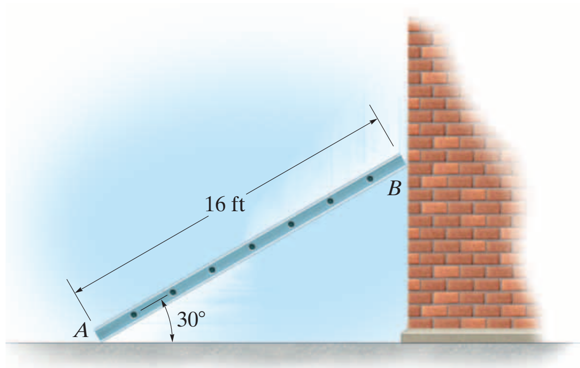
$$-a_B = 13.856\alpha - 4.5$$

$$\alpha = 1.47\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

$$a_B = 24.9\ \text{ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

16–105.

At a given instant the top B of the ladder has an acceleration $a_B = 2\ \text{ft/s}^2$ and a velocity of $v_B = 4\ \text{ft/s}$, both acting downward. Determine the acceleration of the bottom A of the ladder, and the ladder's angular acceleration at this instant.



SOLUTION

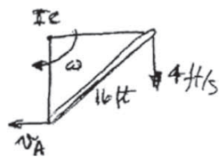
$$\omega = \frac{4}{16 \cos 30^\circ} = 0.288675\ \text{rad/s}$$

$$\mathbf{a}_A = \mathbf{a}_B + \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$$

$$-a_A\mathbf{i} = -2\mathbf{j} + (\alpha\mathbf{k}) \times (-16 \cos 30^\circ\mathbf{i} - 16 \sin 30^\circ\mathbf{j}) - (0.288675)^2(-16 \cos 30^\circ\mathbf{i} - 16 \sin 30^\circ\mathbf{j})$$

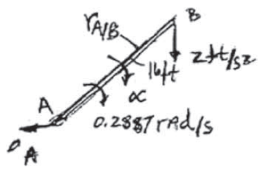
$$-a_A = 8\alpha + 1.1547$$

$$0 = -2 - 13.856\,\alpha + 0.6667$$



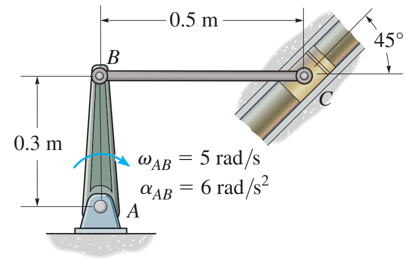
$$\alpha = -0.0962\ \text{rad/s}^2 = 0.0962\ \text{rad/s}^2 \circlearrowright$$ **Ans.**

$$a_A = -0.385\ \text{ft/s}^2 = 0.385\ \text{ft/s}^2 \rightarrow$$ **Ans.**



16–106.

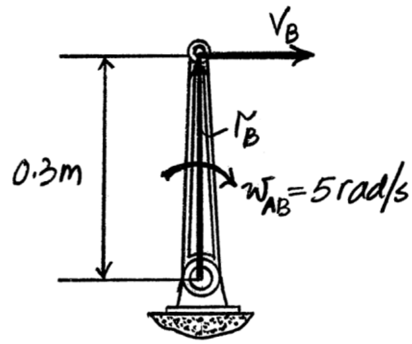
Crank AB is rotating with an angular velocity of $\omega_{AB} = 5\ \text{rad/s}$ and an angular acceleration of $\alpha_{AB} = 6\ \text{rad/s}^2$. Determine the angular acceleration of BC and the acceleration of the slider block C at the instant shown.



SOLUTION

Angular Velocity: Since crank AB rotates about a fixed axis, Fig. a,

$$v_B = \omega_{AB}\, r_B = 5(0.3) = 1.5\ \text{m/s} \rightarrow$$



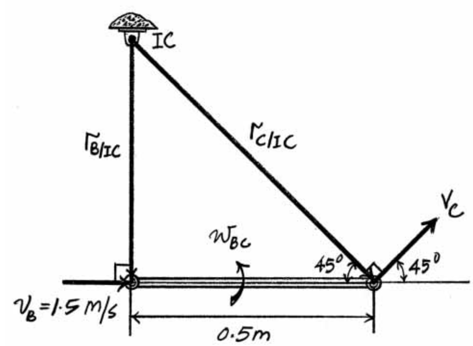
(a)

The location of the IC for link BC is indicated in Fig. b. From the geometry of this figure,

$$r_{B/IC} = 0.5 \tan 45° = 0.5\ \text{m}$$

Then,

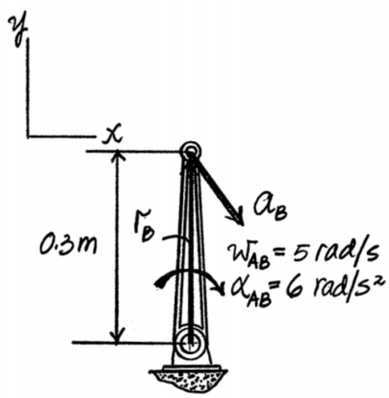
$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.5}{0.5} = 3\ \text{rad/s}$$



(b)

Acceleration and Angular Acceleration: Since crank AB rotates about a fixed axis, Fig. c,

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_B - \omega_{AB}^2 \mathbf{r}_B$$
$$= (-6\mathbf{k}) \times (0.3\mathbf{j}) - 5^2(0.3\mathbf{j})$$
$$= [1.8\mathbf{i} - 7.5\mathbf{j}]\ \text{m/s}^2$$



(c)

Using this result and applying the relative acceleration equation by referring to Fig. d,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$
$$a_C \cos 45°\mathbf{i} + a_C \sin 45°\mathbf{j} = (1.8\mathbf{i} - 7.5\mathbf{j}) + (\alpha_{BC}\mathbf{k}) \times (0.5\mathbf{i}) - 3^2(0.5\mathbf{i})$$
$$0.7071 a_C\mathbf{i} + 0.7071 a_C\mathbf{j} = -2.7\mathbf{i} + (0.5\alpha_{BC} - 7.5)\mathbf{j}$$

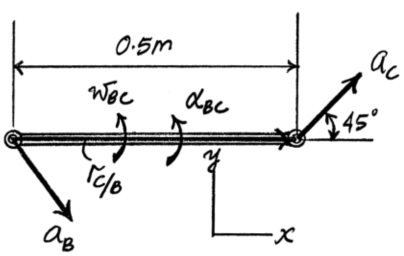
Equating the **i** and **j** components,

$$0.7071 a_C = -2.7 \qquad \textbf{(1)}$$
$$0.7071 a_C = 0.5\alpha_{BC} - 7.5 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$a_C = -3.818\ \text{m/s}^2 = 3.82\ \text{m/s}^2 \swarrow \qquad \textbf{Ans.}$$
$$\alpha_{BC} = 9.60\ \text{rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

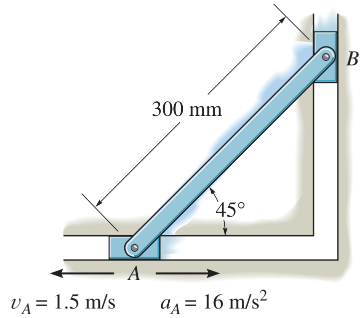
The negative sign indicates that $\mathbf{a}_C$ acts in the opposite sense to that shown in Fig. c.



(d)

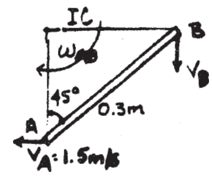
16–107.

At a given instant, the slider block A has the velocity and deceleration shown. Determine the acceleration of block B and the angular acceleration of the link at this instant.



SOLUTION

$$\omega_{AB} = \frac{v_B}{r_{A/IC}} = \frac{1.5}{0.3 \cos 45^\circ} = 7.07 \text{ rad/s}$$



$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$-a_B \mathbf{j} = 16\mathbf{i} + (\alpha\mathbf{k}) \times (0.3 \cos 45^\circ \mathbf{i} + 0.3 \sin 45^\circ \mathbf{j}) - (7.07)^2 (0.3 \cos 45^\circ \mathbf{i} + 0.3 \sin 45^\circ \mathbf{j})$$

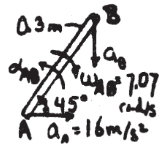
$$\left(\xrightarrow{+}\right) \quad 0 = 16 - \alpha(0.3) \sin 45^\circ - (7.07)^2 (0.3) \cos 45^\circ$$

$$(+\downarrow) \quad a_B = 0 - \alpha(0.3) \cos 45^\circ + (7.07)^2 (0.3) \sin 45^\circ$$

Solving:

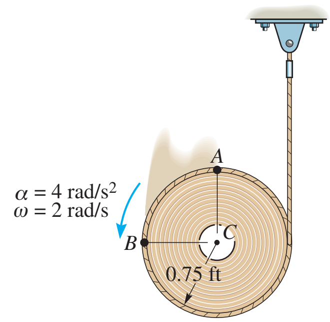
$\alpha_{A/B} = 25\ 4 \text{ rad/s}^2$ ↺ **Ans.**

$a_B = 5.21 \text{ m/s}^2 \quad \downarrow$ **Ans.**



***16–108.**

As the cord unravels from the cylinder, the cylinder has an angular acceleration of $\alpha = 4\ \text{rad/s}^2$ and an angular velocity of $\omega = 2\ \text{rad/s}$ at the instant shown. Determine the accelerations of points A and B at this instant.



SOLUTION

$$a_C = 4(0.75) = 3\ \text{ft/s}^2 \quad \downarrow$$

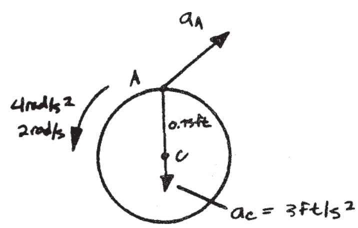
$$\mathbf{a}_A = \mathbf{a}_C + \alpha \times \mathbf{r}_{A/C} - \omega^2 \mathbf{r}_{A/C}$$

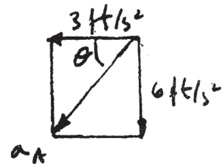
$$\mathbf{a}_A = -3\mathbf{j} + 4\mathbf{k} \times (0.75\mathbf{j}) - (2)^2\,(0.75\mathbf{j})$$

$$\mathbf{a}_A = \{-3\mathbf{i} - 6\mathbf{j}\}\ \text{ft/s}^2$$

$$a_A = \sqrt{(-3)^2 + (-6)^2} = 6.71\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{6}{3}\right) = 63.4^\circ \qquad \textbf{Ans.}$$





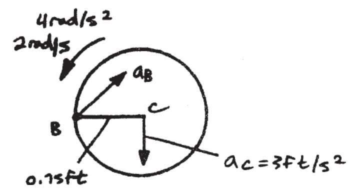
$$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

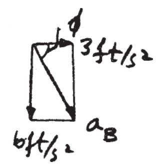
$$\mathbf{a}_B = -3\mathbf{j} + 4\mathbf{k} \times (-0.75\mathbf{i}) - (2)^2(-0.75\mathbf{i})$$

$$\mathbf{a}_B = \{3\mathbf{i} - 6\mathbf{j}\}\ \text{ft/s}^2$$

$$a_B = \sqrt{(3)^2 + (-6)^2} = 6.71\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

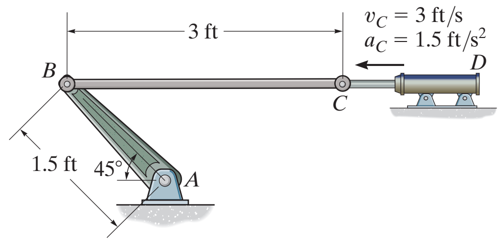
$$\phi = \tan^{-1}\left(\frac{6}{3}\right) = 63.4^\circ \qquad \textbf{Ans.}$$





16–109.

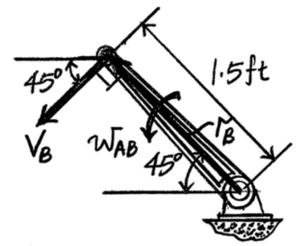
The hydraulic cylinder is extending with a velocity of $v_C = 3\text{ ft/s}$ and an acceleration of $a_C = 1.5\text{ ft/s}^2$. Determine the angular acceleration of links BC and AB at the instant shown.



SOLUTION

Angular Velocity: Since link AB rotates about a fixed axis, Fig. a, then

$$v_B = \omega_{AB} r_B = \omega_{AB}(1.5)$$



(a)

The location of the IC for link BC is indicated in Fig. b. From the geometry of this figure,

$$r_{C/IC} = 3\tan 45° = 3\text{ ft} \qquad r_{B/IC} = \frac{3}{\cos 45°} = 4.243\text{ ft}$$

Then

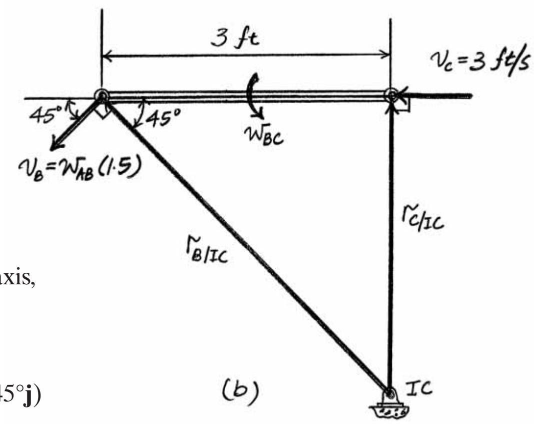
$$\omega_{BC} = \frac{v_C}{r_{C/IC}} = \frac{3}{3} = 1\text{ rad/s}$$

and

$$v_B = \omega_{BC} r_{B/IC}$$

$$\omega_{AB}(1.5) = (1)(4.243)$$

$$\omega_{AB} = 2.828\text{ rad/s}$$



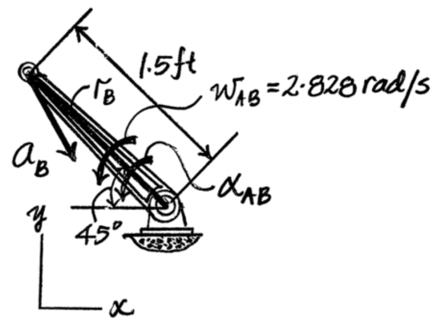
(b)

Acceleration and Angular Acceleration: Since crank AB rotates about a fixed axis, Fig. c, then

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_B - \omega_{AB}^2 \mathbf{r}_B$$

$$= (\alpha_{AB}\mathbf{k}) \times (-1.5\cos 45°\mathbf{i} + 1.5\sin 45°\mathbf{j}) - 2.828^2(-1.5\cos 45°\mathbf{i} + 1.5\sin 45°\mathbf{j})$$

$$= (8.485 - 1.061\alpha_{AB})\mathbf{i} - (8.485 + 1.061\alpha_{AB})\mathbf{j}$$



(c)

Using this result and applying the relative acceleration equation by referring to Fig. d,

$$\mathbf{a}_B = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C}$$

$$(8.485 - 1.061\alpha_{AB})\mathbf{i} - (8.485 + 1.061\alpha_{AB})\mathbf{j} = -1.5\mathbf{i} + (\alpha_{BC}\mathbf{k}) \times (-3\mathbf{i}) - 1^2(-3\mathbf{i})$$

$$(8.485 - 1.061\alpha_{AB})\mathbf{i} - (8.485 + 1.061\alpha_{AB})\mathbf{j} = 1.5\mathbf{i} - 3\alpha_{BC}\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components, yields

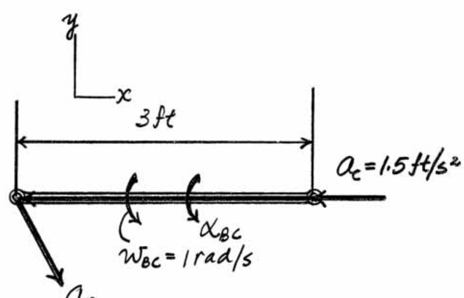
$$8.485 - 1.061\alpha_{AB} = 1.5 \qquad \textbf{(1)}$$

$$-(8.485 + 1.061\alpha_{AB}) = -3\alpha_{BC} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

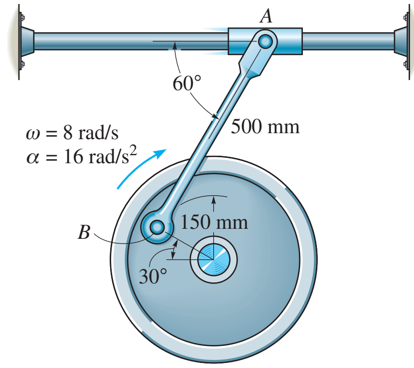
$$\alpha_{AB} = 6.59\text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$\alpha_{BC} = 5.16\text{ rad/s}^2 \qquad \textbf{Ans.}$$



16–110.

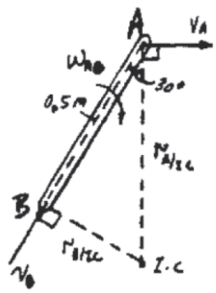
At a given instant the wheel is rotating with the angular motions shown. Determine the acceleration of the collar at A at this instant.



SOLUTION

Using instantaneous center method:

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{8(0.15)}{0.5\tan 30^\circ} = 4.157\text{ rad/s}$$



(a)

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$\underset{\leftarrow}{a_A} = \underset{\measuredangle 60^\circ}{2.4} + \underset{\measuredangle 30^\circ}{9.6} + \underset{60^\circ\measuredangle}{(4.157)^2(0.5)} + \underset{\measuredangle 60^\circ}{\alpha(0.5)}$$

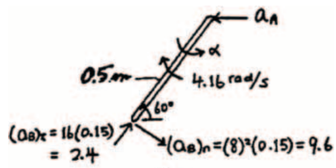
$(\overset{+}{\rightarrow})\quad -a_A = 2.4\cos 60^\circ + 9.6\cos 30^\circ - 8.65\cos 60^\circ - \alpha(0.5)\sin 60^\circ$

$(+\uparrow)\quad 0 = 2.4\sin 60^\circ - 9.6\sin 30^\circ - 8.65\sin 60^\circ + \alpha(0.5)\cos 60^\circ$

$\alpha = 40.8\text{ rad/s}^2 \circlearrowleft$

$a_A = 12.5\text{ m/s}^2 \leftarrow$ **Ans.**

Also:



(b)

$$\mathbf{a}_A = \mathbf{a}_B + \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$$

$$a_A\mathbf{i} = (8)^2(0.15)(\cos 30^\circ)\mathbf{i} - (8)^2(0.15)\sin 30^\circ\mathbf{j} + (16)(0.15)\sin 30^\circ\mathbf{i} + (16)(0.15)\cos 30^\circ\mathbf{j}$$
$$+ (\alpha\mathbf{k}) \times (0.5\cos 60^\circ\mathbf{i} + 0.5\sin 60^\circ\mathbf{j}) - (4.157^2)(0.5\cos 60^\circ\mathbf{i} + 0.5\sin 60^\circ\mathbf{j})$$

$a_A = 8.314 + 1.200 - 0.433\alpha - 4.326$

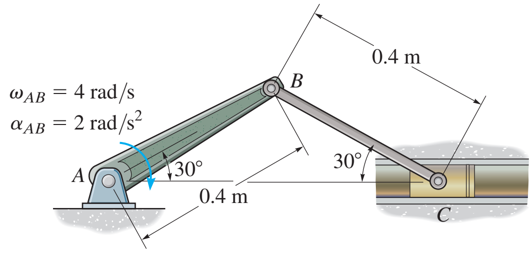
$0 = -4.800 + 2.0785 + 0.25\alpha - 7.4935$

$\alpha = 40.8\text{ rad/s}^2 \circlearrowleft$

$a_A = 12.5\text{ m/s}^2 \leftarrow$ **Ans.**

16–111.

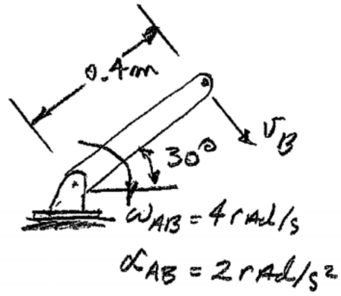
Crank AB rotates with the angular velocity and angular acceleration shown. Determine the acceleration of the slider block C at the instant shown.



SOLUTION

Angular Velocity: Since crank AB rotates about a fixed axis, Fig. a,

$$v_B = \omega_{AB} r_B = 4(0.4) = 1.6 \text{ m/s}$$



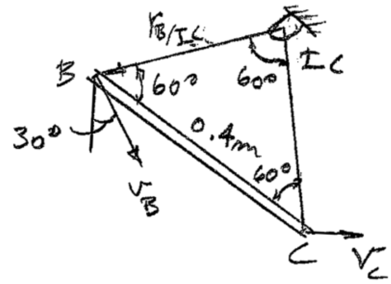
(a)

The location of the IC for link BC is indicated in Fig. b. From the geometry of this figure,

$$r_{B/IC} = 0.4 \text{ m}$$

Then

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.6}{0.4} = 4 \text{ rad/s}$$



(b)

Acceleration and Angular Acceleration: Since crank AB rotates about a fixed axis, Fig. a

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_B - \omega_{AB}^2 \mathbf{r}_B$$

$$= (-2\mathbf{k}) \times (0.4 \cos 30°\mathbf{i} + 0.4 \sin 30°\mathbf{j}) - 4^2(0.4 \cos 30°\mathbf{i} + 0.4 \sin 30°\mathbf{j})$$

$$= [-5.143\mathbf{i} - 3.893\mathbf{j}] \text{ m/s}^2$$

Using this result and applying the relative acceleration equation by referring to Fig. c,

$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

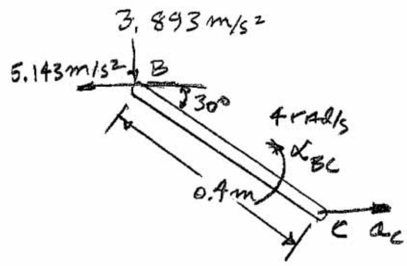
$$a_C\mathbf{i} = (-5.143\mathbf{i} - 3.893\mathbf{j}) + (\alpha_{BC}\mathbf{k}) \times (0.4 \cos 30°\mathbf{i} - 0.4 \sin 30°\mathbf{j}) - 4^2(0.4 \cos 30°\mathbf{i} - 0.4 \sin 30°\mathbf{j})$$

$$a_C\mathbf{i} = (0.2\alpha_{BC} - 10.69)\mathbf{i} + (0.3464\alpha_{BC} - 0.6928)\mathbf{j}$$

Equating the **i** and **j** components, yields

$$a_C = 0.2\alpha_{BC} - 10.69 \quad \textbf{(1)}$$

$$0 = 0.3464\alpha_{BC} - 0.6928 \quad \textbf{(2)}$$



(c)

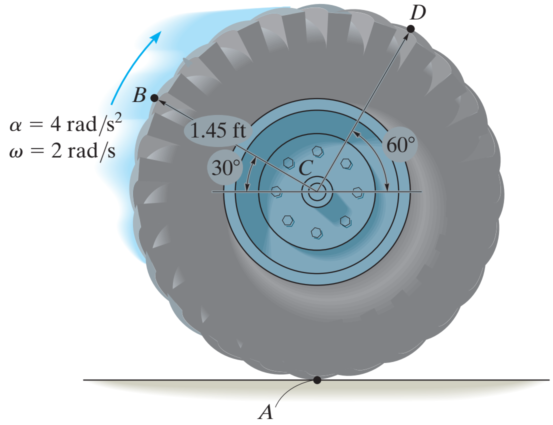
Solving Eqs. (1) and (2),

$$\alpha_{BC} = 2 \text{ rad/s}^2$$

$$a_C = -10.29 \text{ m/s}^2 = 10.3 \text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

***16–112.**

The wheel is moving to the right such that it has an angular velocity $\omega = 2$ rad/s and angular acceleration $\alpha = 4\ \text{rad/s}^2$ at the instant shown. If it does not slip at A, determine the acceleration of point B.



SOLUTION

Since no slipping

$$a_C = \alpha r = 4(1.45) = 5.80\ \text{ft/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

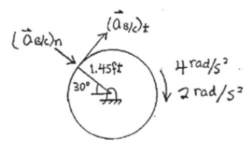
$$\mathbf{a}_B = \underset{\rightarrow}{5.80} + \underset{30^\circ}{(2)^2(1.45)} + \underset{30^\circ}{4(1.45)}$$

$$(\overset{+}{\rightarrow}) \qquad (a_B)_x = 5.80 + 5.02 + 2.9 = 13.72$$

$$(+\uparrow) \qquad (a_B)_y = 0 - 2.9 + 5.02 = 2.12$$

$$a_B = \sqrt{(13.72)^2 + (2.12)^2} = 13.9\ \text{ft/s}^2$$

$$\theta = \tan^{-1}\left(\frac{2.123}{13.72}\right) = 8.80^\circ \ \angle\theta \qquad \textbf{Ans.}$$



Also:

$$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

$$\mathbf{a}_B = 5.80\mathbf{i} + (-4\mathbf{k}) \times (-1.45 \cos 30^\circ \mathbf{i} + 1.45 \sin 30^\circ \mathbf{j}) - (2)^2(-1.45 \cos 30^\circ \mathbf{i} + 1.45 \sin 30^\circ \mathbf{j})$$

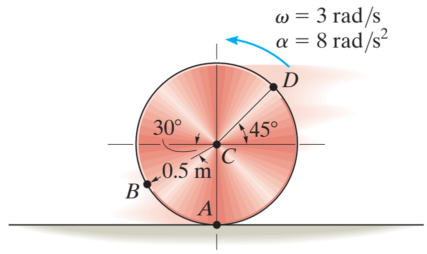
$$\mathbf{a}_B = \{13.72\mathbf{i} + 2.123\mathbf{j}\}\ \text{ft/s}^2$$

$$a_B = \sqrt{(13.72)^2 + (2.123)^2} = 13.9\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{2.123}{13.72}\right) = 8.80^\circ \ \angle\theta \qquad \textbf{Ans.}$$

16–113.

The disk is moving to the left such that it has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point B.

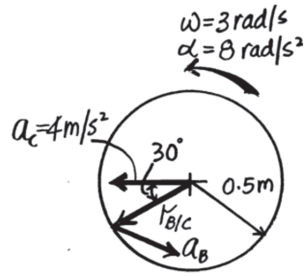


SOLUTION

$$a_C = 0.5(8) = 4\ \text{m/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$\mathbf{a}_B = \begin{bmatrix} 4 \\ \leftarrow \end{bmatrix} + \begin{bmatrix} (3)^2(0.5) \\ \nearrow 30^\circ \end{bmatrix} + \begin{bmatrix} (0.5)(8) \\ \searrow 30^\circ \end{bmatrix}$$



$(\xrightarrow{+})$ $\quad (a_B)_x = -4 + 4.5\cos 30^\circ + 4\sin 30^\circ = 1.897\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_B)_y = 0 + 4.5\sin 30^\circ - 4\cos 30^\circ = -1.214\ \text{m/s}^2$

$a_B = \sqrt{(1.897)^2 + (-1.214)^2} = 2.25\ \text{m/s}^2$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{1.214}{1.897}\right) = 32.6^\circ \searrow$ **Ans.**

Also,

$$\mathbf{a}_B = \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

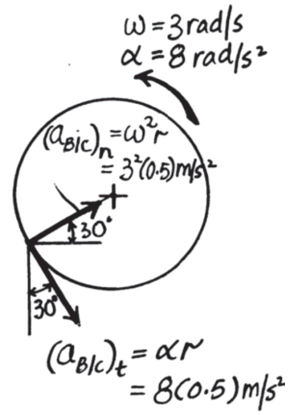
$$(a_B)_x\,\mathbf{i} + (a_B)_y\,\mathbf{j} = -4\mathbf{i} + (8\mathbf{k}) \times (-0.5\cos 30^\circ\mathbf{i} - 0.5\sin 30^\circ\mathbf{j}) - (3)^2(-0.5\cos 30^\circ\mathbf{i} - 0.5\sin 30^\circ\mathbf{j})$$

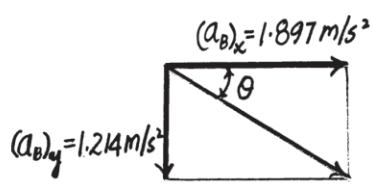
$(\xrightarrow{+})$ $\quad (a_B)_x = -4 + 8(0.5\sin 30^\circ) + (3)^2(0.5\cos 30^\circ) = 1.897\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_B)_y = 0 - 8(0.5\cos 30^\circ) + (3)^2(0.5\sin 30^\circ) = -1.214\ \text{m/s}^2$

$\theta = \tan^{-1}\left(\dfrac{1.214}{1.897}\right) = 32.6^\circ \searrow$ **Ans.**

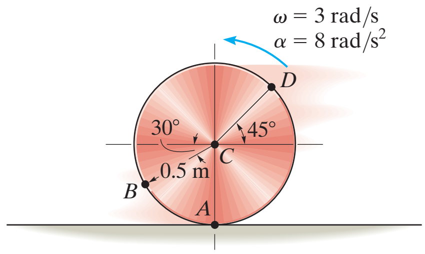
$a_B = \sqrt{(1.897)^2 + (-1.214)^2} = 2.25\ \text{m/s}^2$ **Ans.**



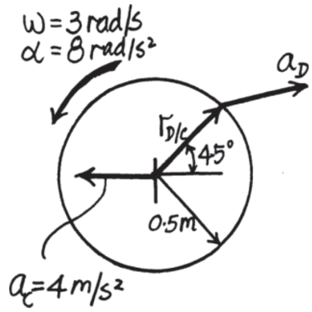


16–114.

The disk is moving to the left such that it has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point D.



SOLUTION



$$a_C = 0.5(8) = 4\ \text{m/s}^2$$

$$\mathbf{a}_D = \mathbf{a}_C + \mathbf{a}_{D/C}$$

$$\mathbf{a}_D = \begin{bmatrix} 4 \\ \leftarrow \end{bmatrix} + \begin{bmatrix} (3)^2(0.5) \\ \swarrow 45^\circ \end{bmatrix} + \begin{bmatrix} 8(0.5) \\ 45^\circ \nwarrow \end{bmatrix}$$

$(\overset{+}{\rightarrow})$ $\quad (a_D)_x = -4 - 4.5 \sin 45^\circ - 4 \cos 45^\circ = -10.01\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_D)_y = 0 - 4.5 \cos 45^\circ + 4 \sin 45^\circ = -0.3536\ \text{m/s}^2$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02^\circ \qquad \textbf{Ans.}$$

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10.0\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Also,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha \times \mathbf{r}_{D/C} - \omega^2\,\mathbf{r}_{D/C}$$

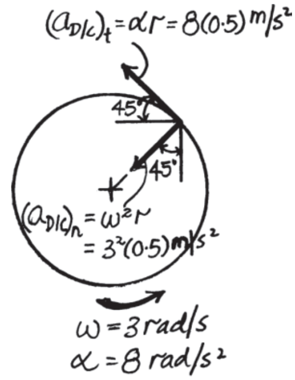
$$(a_D)_x\,\mathbf{i} + (a_D)_y\,\mathbf{j} = -4\mathbf{i} + (8\mathbf{k}) \times (0.5 \cos 45^\circ\mathbf{i} + 0.5 \sin 45^\circ\mathbf{j}) - (3)^2\,(0.5 \cos 45^\circ\mathbf{i} + 0.5 \sin 45^\circ\mathbf{j})$$

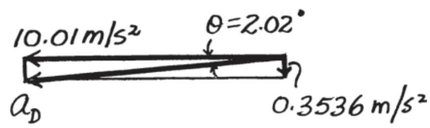
$(\overset{+}{\rightarrow})$ $\quad (a_D)_x = -4 - 8(0.5 \sin 45^\circ) - (3)^2(0.5 \cos 45^\circ) = -10.01\ \text{m/s}^2$

$(+\uparrow)$ $\quad (a_D)_y = +8(0.5 \cos 45^\circ) - (3)^2\,(0.5 \sin 45^\circ) = -0.3536\ \text{m/s}^2$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02^\circ \qquad \textbf{Ans.}$$

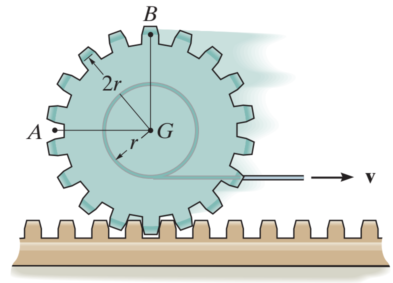
$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10.0\ \text{m/s}^2 \qquad \textbf{Ans.}$$





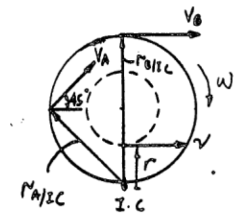
16–115.

A cord is wrapped around the inner spool of the gear. If it is pulled with a constant velocity $\mathbf{v}$, determine the velocities and accelerations of points A and B. The gear rolls on the fixed gear rack.



SOLUTION

Velocity analysis:

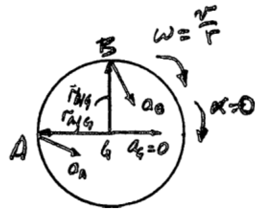


$$\omega = \frac{v}{r}$$

$$v_B = \omega r_{B/IC} = \frac{v}{r}(4r) = 4v \rightarrow$$ **Ans.**

$$v_A = \omega\, r_{A/IC} = \frac{v}{r}\left(\sqrt{(2r)^2 + (2r)^2}\right) = 2\sqrt{2}v \qquad \measuredangle 45^\circ$$ **Ans.**

Acceleration equation: From Example 16–3, Since $a_G = 0, \alpha = 0$



$\mathbf{r}_{B/G} = 2r\,\mathbf{j} \qquad \mathbf{r}_{A/G} = -2r\,\mathbf{i}$

$$\mathbf{a}_B = \mathbf{a}_G + \alpha \times \mathbf{r}_{B/G} - \omega^2 \mathbf{r}_{B/G}$$

$$= 0 + 0 - \left(\frac{v}{r}\right)^2 (2r\mathbf{j}) = -\frac{2v^2}{r}\mathbf{j}$$

$$a_B = \frac{2v^2}{r} \downarrow$$ **Ans.**

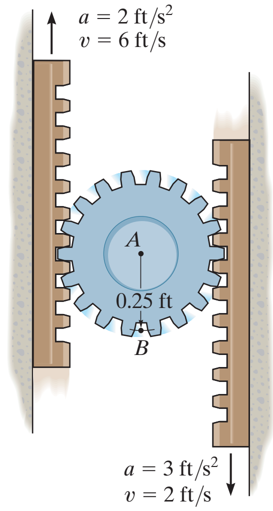
$$\mathbf{a}_A = \mathbf{a}_G + \alpha \times \mathbf{r}_{A/G} - \omega^2 \mathbf{r}_{A/G}$$

$$= 0 + 0 - \left(\frac{v}{r}\right)^2 (-2r\mathbf{i}) = \frac{2v^2}{r}\mathbf{i}$$

$$a_A = \frac{2v^2}{r} \rightarrow$$ **Ans.**

***16–116.**

At a given instant, the gear racks have the velocities and accelerations shown. Determine the acceleration of point A.

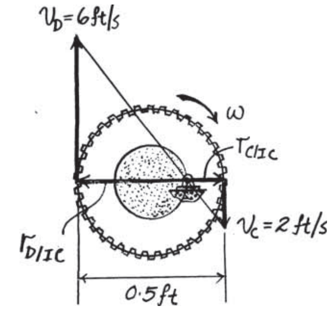


SOLUTION

Velocity Analysis: The angular velocity of the gear can be obtained by using the method of instantaneous center of zero velocity. From similar triangles,

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{v_C}{r_{C/IC}}$$

$$\frac{6}{r_{D/IC}} = \frac{2}{r_{C/IC}} \qquad \textbf{(1)}$$



Where

$$r_{D/IC} + r_{C/IC} = 0.5 \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2) yields

$$r_{D/IC} = 0.375 \text{ ft} \qquad r_{C/IC} = 0.125 \text{ ft}$$

Thus,

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{6}{0.375} = 16.0 \text{ rad/s}$$

Acceleration Equation: The angular acceleration of the gear can be obtained by analyzing the angular motion of points C and D. Applying Eq. 16–18 with $\mathbf{r}_{D/C} = \{-0.5\mathbf{i}\}$ ft, we have

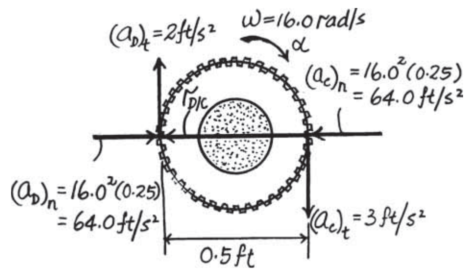
$$\mathbf{a}_D = \mathbf{a}_C + \alpha \times \mathbf{r}_{D/C} - \omega^2\,\mathbf{r}_{D/C}$$

$$(a_D)_n\mathbf{i} + 2\mathbf{j} = -(a_C)_n\mathbf{i} - 3\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.5\mathbf{i}) - 16.0^2\,(-0.5\mathbf{i})$$

$$(a_D)_n\mathbf{i} + 2\mathbf{j} = -(a_C)_n\mathbf{i} + (0.5\alpha - 3)\mathbf{j} + 128\mathbf{i}$$

Equating the **j** components, we have

$$2 = 0.5\,\alpha - 3 \qquad \alpha = 10.0 \text{ rad/s}^2$$



The acceleration of point A can be obtained by analyzing the angular motion of points A and C. Applying Eq. 16–18 with $\mathbf{r}_{A/C} = \{-0.25\mathbf{i}\}$ ft, we have

$$\mathbf{a}_A = \mathbf{a}_C + \alpha \times \mathbf{r}_{A/C} - \omega^2\,\mathbf{r}_{A/C}$$

$$a_A\mathbf{j} = -(a_C)_n\mathbf{i} - 3\mathbf{j} + (-10.0\mathbf{k}) \times (-0.25\mathbf{i}) - 16.0^2\,(-0.25\mathbf{i})$$

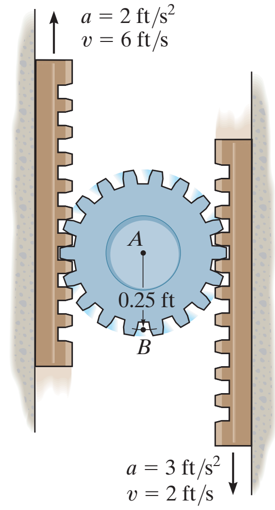
Equating the **i** and **j** components, we have

$$a_A = 0.500 \text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

$$(a_C)_n = 64 \text{ m/s}^2 \leftarrow$$

16–117.

At a given instant, the gear racks have the velocities and accelerations shown. Determine the acceleration of point B.



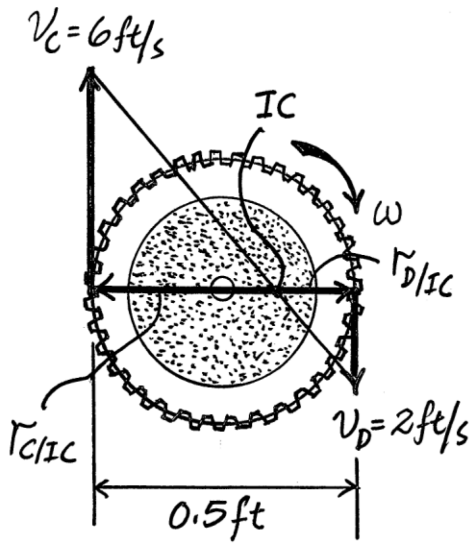
SOLUTION

Angular Velocity: The method of IC will be used. The location of IC for the gear is indicated in Fig. *a*. using the similar triangle,

$$\frac{2}{r_{D/IC}} = \frac{6}{0.5 - r_{D/IC}} \qquad r_{D/IC} = 0.125 \text{ ft}$$

Thus,

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{2}{0.125 \text{ ft}} = 16 \text{ rad/s} \circlearrowright$$



(a)

Acceleration and Angular Acceleration: Applying the relative acceleration equation for points C and D by referring to Fig. *b*,

$$\mathbf{a}_C = \mathbf{a}_D + \boldsymbol{\alpha} \times \mathbf{r}_{C/D} - \omega^2 \mathbf{r}_{C/D}$$

$$(a_C)_n\mathbf{i} + 2\mathbf{j} = -(a_D)_n\mathbf{i} - 3\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.5\mathbf{i}) - 16^2(-0.5\mathbf{i})$$

$$(a_C)_n\mathbf{i} + 2\mathbf{j} = [128 - (a_D)_n]\mathbf{i} + (0.5\alpha - 3)\mathbf{j}$$

Equating **j** component,

$$2 = 0.5\alpha - 3$$

Using this result, the relative acceleration equation applied to points A and C, Fig. *b*, gives

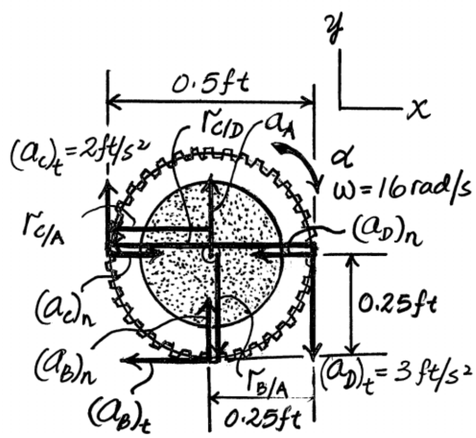
$$\mathbf{a}_C = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{C/A} - \omega^2 \mathbf{r}_{C/A}$$

$$(a_C)_n\mathbf{i} + 2\mathbf{j} = a_A\mathbf{j} + (-10\mathbf{k}) \times (-0.25\mathbf{i}) - 16^2(-0.25\mathbf{i})$$

$$(a_C)_n\mathbf{i} + 2\mathbf{j} = 64\mathbf{i} + (a_A + 2.5)\mathbf{j}$$

Equating **j** component,

$$2 = a_A + 2.5 \qquad a_A = -0.5 \text{ ft/s}^2 = 0.5 \text{ ft/s}^2 \downarrow$$



(b)

Using this result to apply the relative acceleration equation to points A and B,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$-(a_B)_t\mathbf{i} + (a_B)_n\mathbf{j} = -0.5\mathbf{j} + (-10\mathbf{k}) \times (-0.25\mathbf{j}) - 16^2(-0.25\mathbf{j})$$

$$-(a_B)_t\mathbf{i} + (a_B)_n\mathbf{j} = -2.5\mathbf{i} + 63.5\mathbf{j}$$

Equating **i** and **j** components,

$$(a_B)_t = 2.50 \text{ ft/s}^2 \qquad (a_B)_n = 63.5 \text{ ft/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is

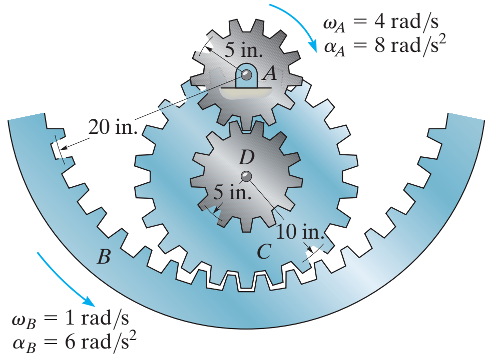
$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{2.50^2 + 63.5^2} = 63.55 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_B)_n}{(a_B)_t}\right] = \tan^{-1}\left(\frac{63.5}{2.50}\right) = 87.7° \ \theta \qquad \textbf{Ans.}$$

16–118.

At a given instant gears A and B have the angular motions shown. Determine the angular acceleration of gear C and the acceleration of its center point D at this instant. Note that the inner hub of gear C is in mesh with gear A and its outer rim is in mesh with gear B.

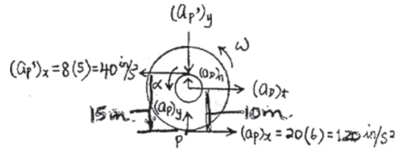


SOLUTION

$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/P'}$

$(\xrightarrow{+})\qquad 120 = -40 + \alpha(15)$

$\alpha = 10.67 \text{ rad/s}^2 \circlearrowleft$ **Ans.**



$\mathbf{a}_P = \mathbf{a}_D + \mathbf{a}_{P/D}$

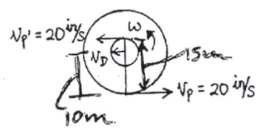
$(\xrightarrow{+})\qquad 120 = (a_D)_t + (10.67)(10)$

$(a_D)_t = 13.3 \text{ in./s}^2 \rightarrow$

$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/P'}$

$(\xrightarrow{+})\qquad 20 = -20 + \omega(15)$

$\omega = 2.667$ rad/s



$\mathbf{v}_D = \mathbf{v}_P + \mathbf{v}_{D/P}$

$(\xleftarrow{+})\qquad v_D = -20 + 10(2.667)$

$v_D = 6.67$ in./s

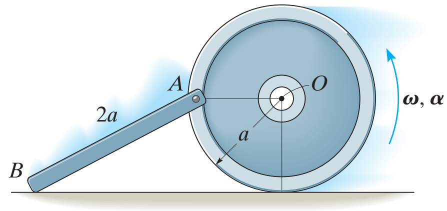
$$(a_D)_n = \frac{(6.67)^2}{10} = 4.44 \text{ in./s}^2 \uparrow$$

$$\theta = \tan^{-1}\left(\frac{4.44}{13.3}\right) = 18.4^\circ$$

$$a_D = \sqrt{(4.44)^2 + (13.3)^2} = 14.1 \text{ in./s}^2$$ **Ans.**

16–119.

The wheel rolls without slipping such that at the instant shown it has an angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$. Determine the velocity and acceleration of point B on the rod at this instant.



SOLUTION

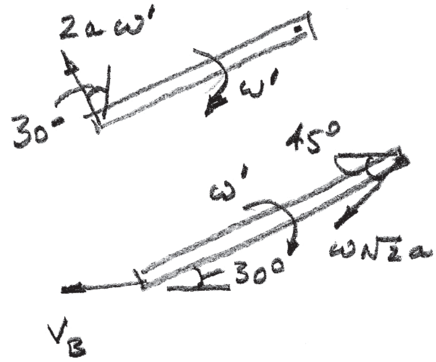
$\bar{v}_B = \bar{v}_A + \bar{v}_{B/A\,(Pin)}$

$\overset{+}{\leftarrow}\quad v_B = \dfrac{1}{\sqrt{2}}\left(\omega\sqrt{2}a\right) + 2a\omega'\left(\dfrac{1}{2}\right)$

$+\uparrow\quad O = -\dfrac{1}{\sqrt{2}}\left(\omega\sqrt{2}a\right) + 2a\omega'\left(\dfrac{\sqrt{3}}{2}\right)$

$\omega' = \dfrac{\omega}{\sqrt{3}}$

$v_B = 1.58\,\omega a$ **Ans.**



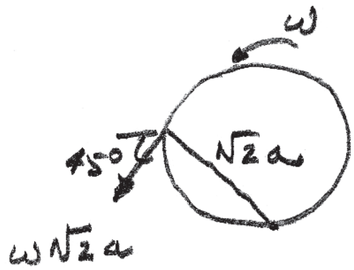
$\bar{a}_A = \bar{a}_O + \bar{a}_{A/O\,(Pin)}$

$(a_A)_x + (a_A)_y = \alpha a + \alpha(a) + \omega^2 a$

$\leftarrow \quad \downarrow \quad \leftarrow \quad \downarrow \quad \rightarrow$

$(a_A)_x = \alpha a - \omega^2 a$

$(a_A)_y = \alpha a$



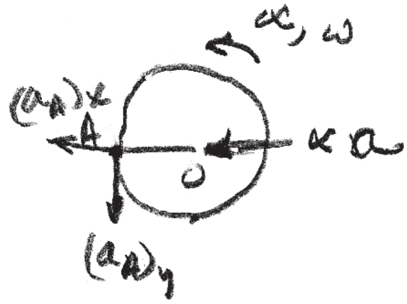
$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A\,(Pin)}$

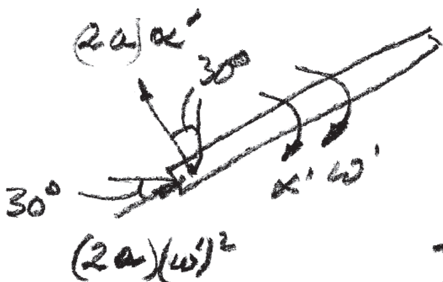
$a_B = \alpha a - \omega^2 a + 2a(\alpha')\left(\dfrac{1}{2}\right) - 2a\left(\dfrac{\omega}{\sqrt{3}}\right)^2\dfrac{\sqrt{3}}{2}$

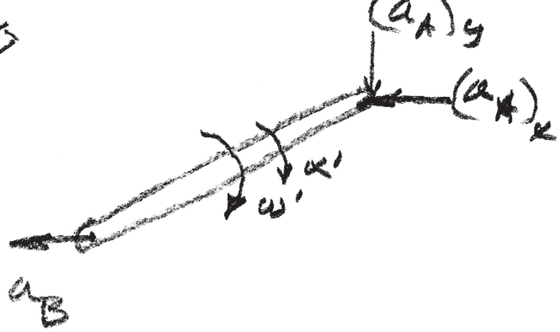
$O = -\alpha a + 2a\alpha'\left(\dfrac{2}{\sqrt{3}}\right) + 2a\left(\dfrac{\omega}{\sqrt{3}}\right)^2\left(\dfrac{1}{2}\right)$

$\alpha' = 0.577\alpha - 0.1925\omega^2$

$a_B = 1.58\alpha a - 1.77\omega^2 a$ **Ans.**

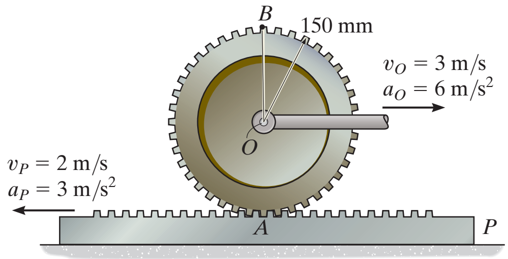






***16–120.**

The center O of the gear and the gear rack P move with the velocities and accelerations shown. Determine the angular acceleration of the gear and the acceleration of point B located at the rim of the gear at the instant shown.



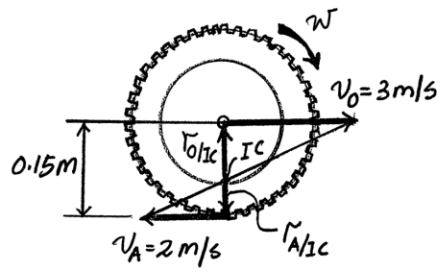
SOLUTION

Angular Velocity: The location of the IC is indicated in Fig. a. Using similar triangles,

$$\frac{3}{r_{O/IC}} = \frac{2}{0.15 - r_{O/IC}} \qquad r_{O/IC} = 0.09 \text{ m}$$

Thus,

$$\omega = \frac{v_O}{r_{O/IC}} = \frac{3}{0.09} = 33.33 \text{ rad/s}$$



(a)

Acceleration and Angular Acceleration: Applying the relative acceleration equation to points O and A and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_O + \alpha \times \mathbf{r}_{A/O} - \omega^2 r_{A/O}$$

$$-3\mathbf{i} + (a_A)_n\mathbf{j} = 6\mathbf{i} + (-\alpha\mathbf{k}) \times (-0.15\mathbf{j}) - 33.33^2(-0.15\mathbf{j})$$

$$-3\mathbf{i} + (a_A)_n\mathbf{j} = (6 - 0.15\alpha)\mathbf{i} + 166.67\mathbf{j}$$

Equating the **i** components,

$$-3 = 6 - 0.15\alpha$$

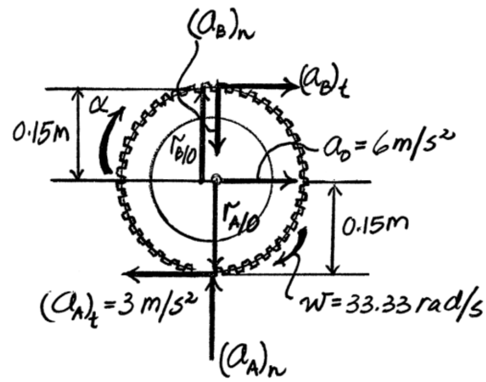
$$\alpha = 60 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Using this result, the relative acceleration equation is applied to points O and B, Fig. b, which gives

$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 r_{B/O}$$

$$(a_B)_t\mathbf{i} - (a_B)_n\mathbf{j} = 6\mathbf{i} + (-60\mathbf{k}) \times (0.15\mathbf{j}) - 33.33^2(0.15\mathbf{j})$$

$$(a_B)_t\mathbf{i} - (a_B)_n\mathbf{j} = 15\mathbf{i} - 166.67\mathbf{j}$$



(b)

Equating the **i** and **j** components,

$$(a_B)_t = 15 \text{ m/s}^2 \qquad (a_B)_n = 166.67 \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is

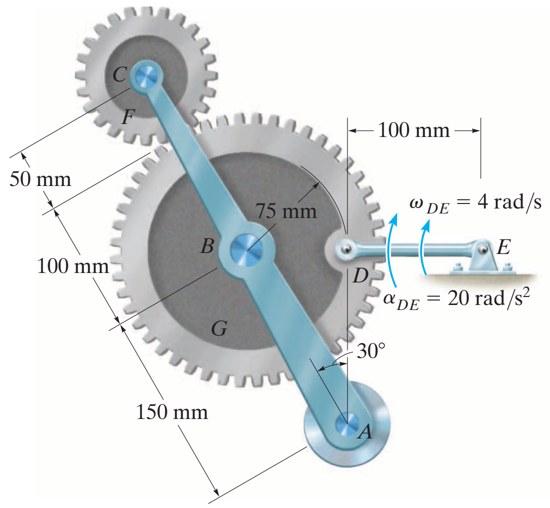
$$a_B = \sqrt{(a_B)_t^{\,2} + (a_B)_n^{\,2}} = \sqrt{15^2 + 166.67^2} = 167 \text{ m/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_B)_n}{(a_B)_t}\right] = \tan^{-1}\left(\frac{166.67}{15}\right) = 84.9° \measuredangle$$

16–121.

The tied crank and gear mechanism gives rocking motion to crank AC, necessary for the operation of a printing press. If link DE has the angular motion shown, determine the respective angular velocities of gear F and crank AC at this instant, and the angular acceleration of crank AC.



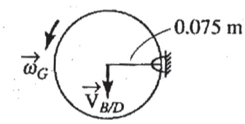
SOLUTION

Velocity analysis:

$$v_D = \omega_{DE} r_{D/E} = 4(0.1) = 0.4 \text{ m/s} \uparrow$$

$$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D}$$

$$\underset{\measuredangle 30^\circ}{v_B} = \underset{\uparrow}{0.4} + \underset{\downarrow}{(\omega_G)(0.075)}$$



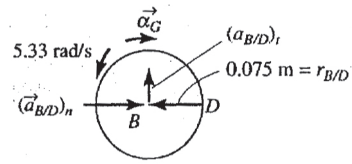
$(\xrightarrow{+})$ $\quad v_B \cos 30^\circ = 0, \qquad v_B = 0$

$(+\uparrow)$ $\quad \omega_G = 5.33 \text{ rad/s}$

Since $v_B = 0, \qquad v_C = 0, \qquad \omega_{AC} = 0$ **Ans.**

$$\omega_F r_F = \omega_G r_G$$

$$\omega_F = 5.33\left(\frac{100}{50}\right) = 10.7 \text{ rad/s}$$ **Ans.**



Acceleration analysis:

$$(a_D)_n = (4)^2(0.1) = 1.6 \text{ m/s}^2 \rightarrow$$

$$(a_D)_t = (20)(0.1) = 2 \text{ m/s}^2 \uparrow$$

$$(\mathbf{a}_B)_n + (\mathbf{a}_B)_t = (\mathbf{a}_D)_n + (\mathbf{a}_D)_t + (\mathbf{a}_{B/D})_n + (\mathbf{a}_{B/D})_t$$

$$0 + \underset{\measuredangle 30^\circ}{(a_B)_t} = \underset{\rightarrow}{1.6} + \underset{\uparrow}{2} + \underset{\rightarrow}{(5.33)^2(0.075)} + \underset{\uparrow}{\alpha_G(0.075)}$$

$(+\uparrow)$ $\quad (a_B)_t \sin 30^\circ = 0 + 2 + 0 + \alpha_G(0.075)$

$(\xrightarrow{+})$ $\quad (a_B)_t \cos 30^\circ = 1.6 + 0 + (5.33)^2(0.075) + 0$

Solving,

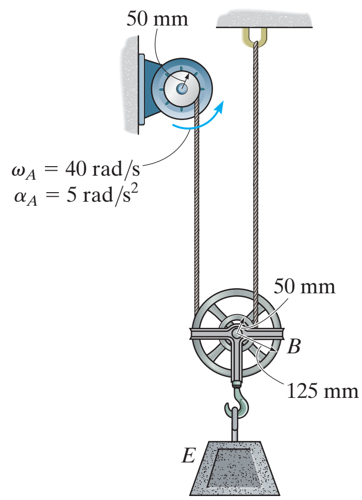
$$(a_B)_t = 4.31 \text{ m/s}^2, \qquad \alpha_G = 2.052 \text{ rad/s}^2 \circlearrowright$$

Hence,

$$\alpha_{AC} = \frac{(a_B)_t}{r_{B/A}} = \frac{4.31}{0.15} = 28.7 \text{ rad/s}^2 \circlearrowright$$ **Ans.**

16–122.

Pulley A rotates with the angular velocity and angular acceleration shown. Determine the angular acceleration of pulley B at the instant shown.



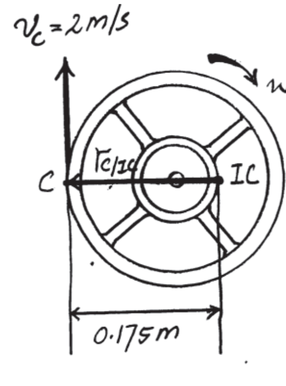
SOLUTION

Angular Velocity: Since pulley A rotates about a fixed axis,

$$v_C = \omega_A r_A = 40(0.05) = 2 \text{ m/s} \uparrow$$

The location of the IC is indicated in Fig. a. Thus,

$$\omega_B = \frac{v_C}{r_{C/IC}} = \frac{2}{0.175} = 11.43 \text{ rad/s}$$



(a)

Acceleration and Angular Acceleration: For pulley A,

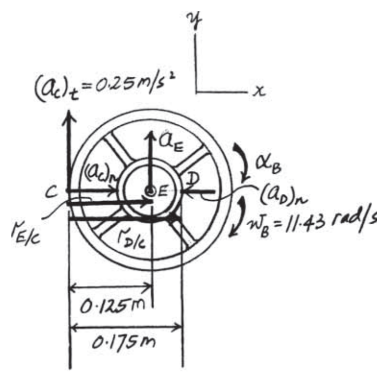
$$(a_C)_t = \alpha_A r_A = 5(0.05) = 0.25 \text{ m/s}^2 \uparrow$$

Using this result and applying the relative acceleration equation to points C and D by referring to Fig. b,

$$\mathbf{a}_D = \mathbf{a}_C + \alpha_B \times \mathbf{r}_{D/C} - \omega_B{}^2 r_{D/C}$$

$$(a_D)_n\,\mathbf{i} = (a_C)_n\,\mathbf{i} + 0.25\mathbf{j} + (-\alpha_B\mathbf{k}) \times (0.175\mathbf{i}) - 11.43^2(0.175\mathbf{i})$$

$$(a_D)_n\,\mathbf{i} = [(a_C)_n - 22.86]\mathbf{i} + (0.25 - 0.175\alpha_B)\mathbf{j}$$



(b)

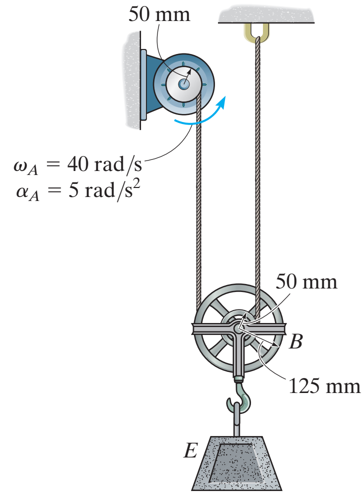
Equating the $\mathbf{j}$ components,

$$0 = 0.25 - 0.175\alpha_B$$

$$\alpha_B = 1.43 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

16–123.

Pulley A rotates with the angular velocity and angular acceleration shown. Determine the acceleration of block E at the instant shown.



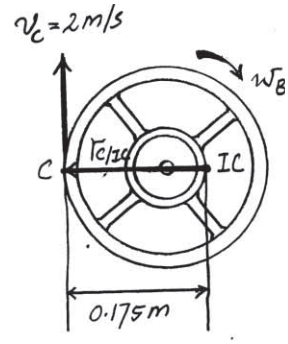
SOLUTION

Angular Velocity: Since pulley A rotates about a fixed axis,

$$v_C = \omega_A r_A = 40(0.05) = 2 \text{ m/s} \uparrow$$

The location of the IC is indicated in Fig. *a*. Thus,

$$\omega_B = \frac{v_C}{r_{C/IC}} = \frac{2}{0.175} = 11.43 \text{ rad/s}$$



(a)

Acceleration and Angular Acceleration: For pulley A,

$$(a_C)_t = \alpha_A r_A = 5(0.05) = 0.25 \text{ m/s}^2 \uparrow$$

Using this result and applying the relative acceleration equation to points C and D by referring to Fig. *b*,

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_B \times \mathbf{r}_{D/C} - \omega_B{}^2 \mathbf{r}_{D/C}$$

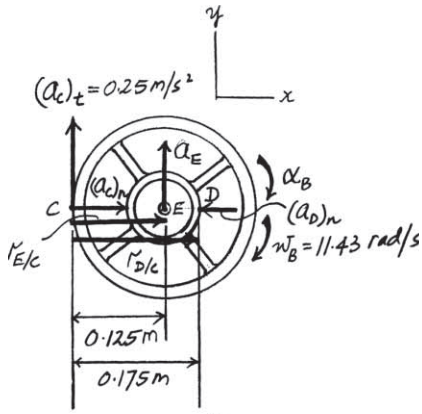
$$(a_D)_n\,\mathbf{i} = (a_C)_n\,\mathbf{i} + 0.25\mathbf{j} + (-\alpha_B\mathbf{k}) \times (0.175\mathbf{i}) - 11.43^2(0.175\mathbf{i})$$

$$(a_D)_n\,\mathbf{i} = [(a_C)_n - 22.86]\mathbf{i} + (0.25 - 0.175\alpha_B)\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$0 = 0.25 - 0.175\alpha_B$$

$$\alpha_B = 1.429 \text{ rad/s} = 1.43 \text{ rad/s}^2$$



(b)

Using this result, the relative acceleration equation applied to points C and E, Fig. *b*, gives

$$\mathbf{a}_E = \mathbf{a}_C + \boldsymbol{\alpha}_B \times \mathbf{r}_{E/C} - \omega_B{}^2 \mathbf{r}_{E/C}$$

$$a_E\,\mathbf{j} = [(a_C)_n\,\mathbf{i} + 0.25\mathbf{j}] + (-1.429\mathbf{k}) \times (0.125\mathbf{i}) - 11.43^2(0.125\mathbf{i})$$

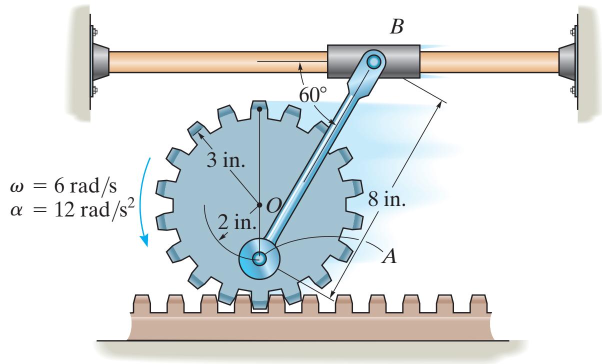
$$a_E\,\mathbf{j} = [(a_C)_n - 16.33]\mathbf{i} + 0.0714\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$a_E = 0.0714 \text{ m/s}^2 \uparrow \qquad \textbf{Ans.}$$

***16–124.**

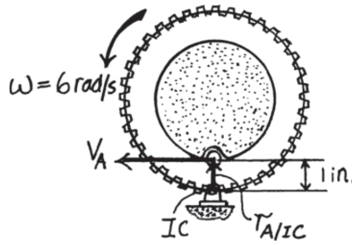
At a given instant, the gear has the angular motion shown. Determine the accelerations of points A and B on the link and the link's angular acceleration at this instant.



SOLUTION

For the gear

$$v_A = \omega r_{A/IC} = 6(1) = 6 \text{ in./s}$$



$$\mathbf{a}_O = -12(3)\mathbf{i} = \{-36\mathbf{i}\} \text{ in./s}^2 \qquad \mathbf{r}_{A/O} = \{-2\mathbf{j}\} \text{ in.} \qquad \alpha = \{12\mathbf{k}\} \text{ rad/s}^2$$

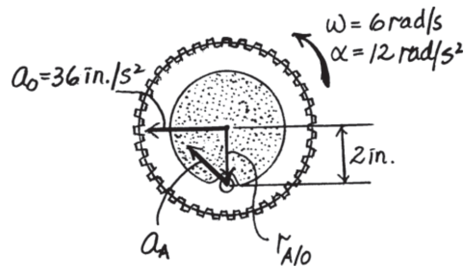
$$\mathbf{a}_A = \mathbf{a}_0 + \alpha \times \mathbf{r}_{A/O} - \omega^2 \mathbf{r}_{A/O}$$

$$= -36\mathbf{i} + (12\mathbf{k}) \times (-2\mathbf{j}) - (6)^2(-2\mathbf{j})$$

$$= \{-12\mathbf{i} + 72\mathbf{j}\} \text{ in./s}^2$$

$$a_A = \sqrt{(-12)^2 + 72^2} = 73.0 \text{ in./s}^2 \qquad \textbf{Ans.}$$

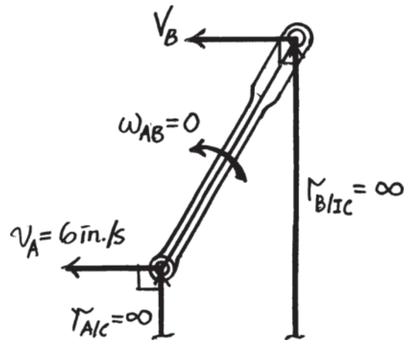
$$\theta = \tan^{-1}\left(\frac{72}{12}\right) = 80.5° \qquad \textbf{Ans.}$$



For link AB

The IC is at ∞, so $\omega_{AB} = 0$, i.e.,

$$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{6}{\infty} = 0$$



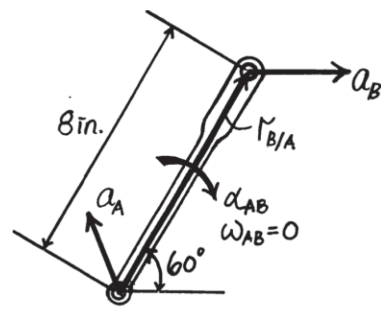
$$\mathbf{a}_B = a_B\mathbf{i} \qquad \alpha_{AB} = -\alpha_{AB}\mathbf{k} \qquad \mathbf{r}_{B/A} = \{8\cos 60°\mathbf{i} + 8\sin 60°\mathbf{j}\} \text{ in.}$$

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$a_B\mathbf{i} = (-12\mathbf{i} + 72\mathbf{j}) + (-\alpha_{AB}\mathbf{k}) \times (8\cos 60°\mathbf{i} + 8\sin 60°\mathbf{j}) - \mathbf{0}$$

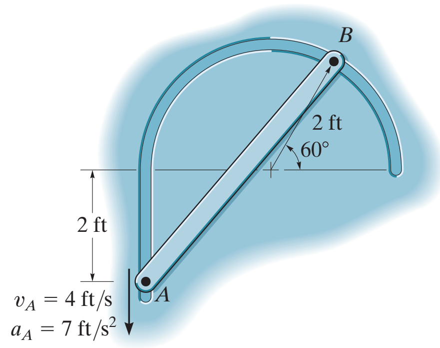
$$(\xrightarrow{+}) \qquad a_B = -12 + 8\sin 60°(18) = 113 \text{ in./s}^2 \rightarrow \qquad \textbf{Ans.}$$

$$(+\uparrow) \qquad 0 = 72 - 8\cos 60°\alpha_{AB} \qquad \alpha_{AB} = 18 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



16–125.

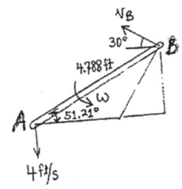
The ends of the bar AB are confined to move along the paths shown. At a given instant, A has a velocity of $v_A = 4\text{ ft/s}$ and an acceleration of $a_A = 7\text{ ft/s}^2$. Determine the angular velocity and angular acceleration of AB at this instant.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$$\underset{30^\circ \nwarrow}{v_B} = \underset{\downarrow}{4} + \underset{\nwarrow 51.21^\circ}{\omega(4.788)}$$



$(\overset{+}{\rightarrow})\qquad -v_B\cos 30^\circ = 0 - \omega(4.788)\sin 51.21^\circ$

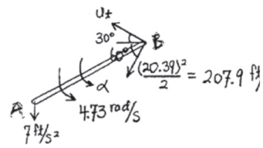
$(+\uparrow)\qquad v_B\sin 30^\circ = -4 + \omega(4.788)\cos 51.21^\circ$

$v_B = 20.39\text{ ft/s}\qquad 30^\circ \nwarrow$

$\omega = 4.73\text{ rad/s} \circlearrowleft$ **Ans.**

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$$\underset{30^\circ \nwarrow}{a_t} + \underset{60^\circ \swarrow}{207.9} = \underset{\downarrow}{7} + \underset{\swarrow 51.21^\circ}{107.2} + \underset{\nwarrow 51.21^\circ}{4.788(\alpha)}$$



$(\overset{+}{\leftarrow})\qquad a_t\cos 30^\circ + 207.9\cos 60^\circ = 0 + 107.2\cos 51.21^\circ + 4.788\alpha(\sin 51.21^\circ)$

$(+\uparrow)\qquad a_t\sin 30^\circ - 207.9\sin 60^\circ = -7 - 107.2\sin 51.21^\circ + 4.788\alpha(\cos 51.21^\circ)$

$a_t(0.866) - 3.732\alpha = -36.78$

$a_t(0.5) - 3\alpha = 89.49$

$a_t = -607\text{ ft/s}^2$

$\alpha = -131\text{ rad/s}^2 = 131\text{ rad/s}^2 \circlearrowright$ **Ans.**

Also:

$\mathbf{v}_B = \mathbf{v}_A = \omega \times \mathbf{r}_{B/A}$

$-v_B\cos 30^\circ\mathbf{i} + v_B\sin 30^\circ\mathbf{j} = -4\mathbf{j} + (\omega\mathbf{k}) \times (3\mathbf{i} + 3.732\mathbf{j})$

$-v_B\cos 30^\circ = -\omega(3.732)$

$v_B\sin 30^\circ = -4 + \omega(3)$

$\omega = 4.73\text{ rad/s} \circlearrowleft$ **Ans.**

$v_B = 20.39\text{ ft/s}$

$\mathbf{a}_B = \mathbf{a}_A - \omega^2\mathbf{r}_{B/A} + \alpha \times \mathbf{r}_{B/A}$

$(-a_t\cos 30^\circ\mathbf{i} + a_t\sin 30^\circ\mathbf{j}) + (-207.9\cos 60^\circ\mathbf{i} - 207.9\sin 60^\circ\mathbf{j}) = -7\mathbf{j} - (4.732)^2(3\mathbf{i} + 3.732\mathbf{j})$

$\qquad +(\alpha\mathbf{k}) \times (3\mathbf{i} + 3.732\mathbf{j})$

$-a_t\cos 30^\circ - 207.9\cos 60^\circ = -(4.732)^2(3) - \alpha(3.732)$

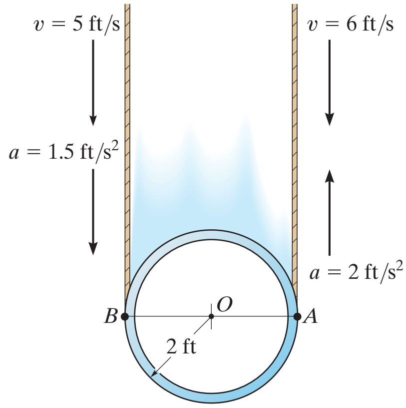
$a_t\sin 30^\circ - 207.9\sin 60^\circ = -7 - (4.732)^2(3.732) + \alpha(3)$

$a_t = -607\text{ ft/s}^2$

$\alpha = -131\text{ rad/s}^2 = 131\text{ rad/s}^2 \circlearrowright$ **Ans.**

16–126.

At a given instant, the cables supporting the pipe have the motions shown. Determine the angular velocity and angular acceleration of the pipe and the velocity and acceleration of point B located on the pipe.



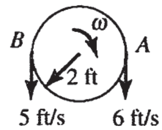
SOLUTION

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$(+\downarrow) \quad 5 = 6 - \omega\,(4)$$

$\omega = 0.25 \text{ rad/s} \circlearrowright$ **Ans.**

$v_B = 5.00 \text{ ft/s} \downarrow$ **Ans.**

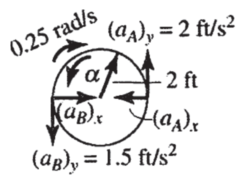


$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\underset{\downarrow}{1.5} + \underset{\rightarrow}{(a_B)_x} = \underset{\uparrow}{2} + \underset{\leftarrow}{(a_A)_x} + \underset{\downarrow}{(\alpha)(4)} + \underset{\rightarrow}{(0.25)^2\,(4)}$$

$$(+\downarrow) \quad 1.5 = -2 + \alpha(4)$$

$\alpha = 0.875 \text{ rad/s}^2 \circlearrowleft$ **Ans.**



$$\mathbf{a}_B = \mathbf{a}_O + \mathbf{a}_{B/O}$$

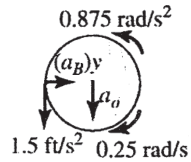
$$\underset{\downarrow}{1.5} + \underset{\rightarrow}{(a_B)_x} = \underset{\downarrow}{\alpha_O} + \underset{\downarrow}{0.875(2)} + \underset{\rightarrow}{(0.25)^2(2)}$$

$$(\overset{+}{\rightarrow}) \quad (a_B)_x = (0.25)^2(2) = 0.125 \text{ ft/s}^2$$

$a_B = \sqrt{(1.5)^2 + (0.125)^2} = 1.51 \text{ ft/s}^2$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{1.5}{0.125}\right) = 85.2^\circ \;\; \measuredangle\theta$ **Ans.**

Also:



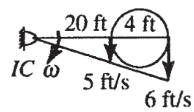
$\omega = \dfrac{5}{20} = 0.25 \text{ rad/s}$ **Ans.**

$v_B = 5.00 \text{ ft/s} \downarrow$ **Ans.**

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$-1.5\mathbf{j} + (a_B)_x\mathbf{i} = 2\mathbf{j} - (a_A)_x\mathbf{i} + (\alpha\mathbf{k}) \times (-4\mathbf{i}) - (0.25)^2(-4\mathbf{i})$$

$$-1.5 = 2 - 4\alpha$$



$\alpha = 0.875 \text{ rad/s}^2 \circlearrowleft$ **Ans.**

$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$-1.5\mathbf{j} + (a_B)_x\mathbf{i} = -a_O\mathbf{j} + (\alpha\mathbf{k}) \times (-2\mathbf{i}) - (0.25)^2(-2\mathbf{i})$$

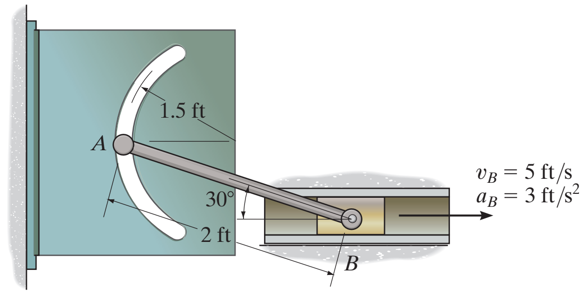
$$(a_B)_x = (0.25)^2(2) = 0.125$$

$a_B = \sqrt{(1.5)^2 + (0.125)^2} = 1.51 \text{ ft/s}^2$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{1.5}{0.125}\right) = 85.2^\circ \;\; \measuredangle\theta$ **Ans.**

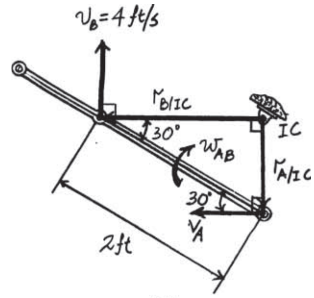
16–127.

The slider block moves with a velocity of $v_B = 5$ ft/s and an acceleration of $a_B = 3$ ft/s^2. Determine the angular acceleration of rod AB at the instant shown.



SOLUTION

Angular Velocity: The velocity of point A is directed along the tangent of the circular slot. Thus, the location of the IC for rod AB is indicated in Fig. *a*. From the geometry of this figure,



(a)

$$r_{B/IC} = 2 \sin 30^\circ = 1 \text{ ft} \qquad r_{A/IC} = 2 \cos 30^\circ = 1.732 \text{ ft}$$

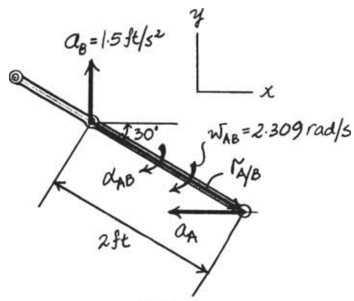
Thus,

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{5}{1} = 5 \text{ rad/s}$$

Then

$$v_A = \omega_{AB}\, r_{A/IC} = 5(1.732) = 8.660 \text{ ft/s}$$

Acceleration and Angular Acceleration: Since point A travels along the circular slot, the normal component of its acceleration has a magnitude of $(a_A)_n = \dfrac{v_A{}^2}{\rho} = \dfrac{8.660^2}{1.5} = 50$ ft/s^2 and is directed towards the center of the circular slot. The tangential component is directed along the tangent of the slot. Applying the relative acceleration equation and referring to Fig. *b*,



(b)

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\, \mathbf{r}_{A/B}$$

$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = 3\mathbf{i} + (\alpha_{AB}\,\mathbf{k}) \times (-2 \cos 30^\circ \mathbf{i} + 2 \sin 30^\circ \mathbf{j}) - 5^2(-2 \cos 30^\circ\, \mathbf{i} + 2 \sin 30^\circ \mathbf{j})$$

$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = (46.30 - \alpha_{AB})\mathbf{i} + (1.732\alpha_{AB} + 25)\mathbf{j}$$

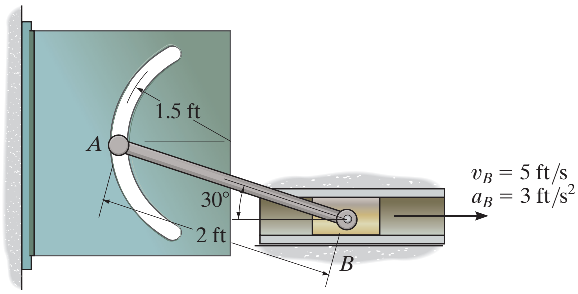
Equating the **i** components,

$$50 = 46.30 - \alpha_{AB}$$

$$\alpha_{AB} = -3.70 \text{ rad/s}^2 = 3.70 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

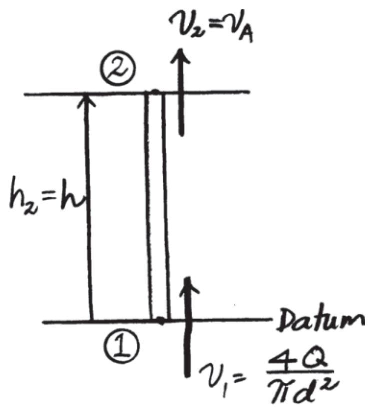
***16–128.**

The slider block moves with a velocity of $v_B = 5$ ft/s and an acceleration of $a_B = 3$ ft/s^2. Determine the acceleration of A at the instant shown.



SOLUTION

Angualr Velocity: The velocity of point A is directed along the tangent of the circular slot. Thus, the location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure,



(a)

$$r_{B/IC} = 2\sin 30° = 1\text{ ft} \qquad r_{A/IC} = 2\cos 30° = 1.732\text{ ft}$$

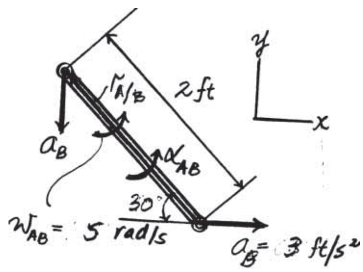
Thus,

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{5}{1} = 5\text{ rad/s}$$

Then

$$v_A = \omega_{AB}\, r_{A/IC} = 5(1.732) = 8.660\text{ ft/s}$$

Acceleration and Angular Acceleration: Since point A travels along the circular slot, the normal component of its acceleration has a magnitude of $(a_A)_n = \frac{v_A{}^2}{\rho} = \frac{8.660^2}{1.5} = 50$ ft/s^2 and is directed towards the center of the circular slot. The tangential component is directed along the tangent of the slot. Applying the relative acceleration equation and referring to Fig. b,



(b)

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\,\mathbf{r}_{A/B}$$

$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = 3\mathbf{i} + (\alpha_{AB}\,\mathbf{k}) \times (-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}) - 5^2(-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j})$$

$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = (46.30 - \alpha_{AB})\mathbf{i} - (1.732\alpha_{AB} + 25)\mathbf{j}$$

Equating the **i** and **j** components,

$$50 = 46.30 - \alpha_{AB}$$

$$-(a_A)_t = -(1.732\alpha_{AB} + 25)$$

Solving,

$$\alpha_{AB} = -3.70\text{ rad/s}^2$$

$$(a_A)_t = 18.59\text{ ft/s}^2 \downarrow$$

Thus, the magnitude of $\mathbf{a}_A$ is

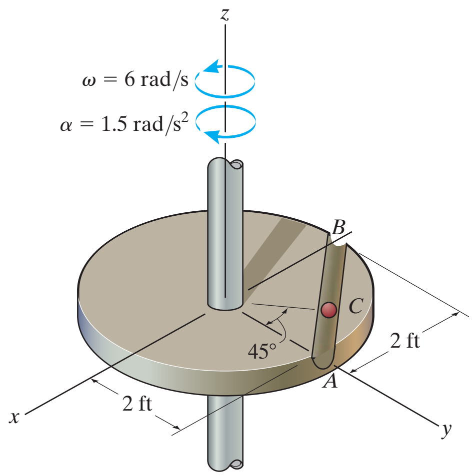
$$a_A = \sqrt{(a_A)_t{}^2 + (a_A)_n{}^2} = \sqrt{18.59^2 + 50^2} = 53.3\text{ft/s}^2 \qquad \textbf{Ans.}$$

and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_A)_t}{(a_A)_n}\right] = \tan^{-1}\left(\frac{18.59}{50}\right) = 20.4° \qquad \textbf{Ans.}$$

16–129.

Ball C moves along the slot from A to B with a speed of 3 ft/s, which is increasing at 1.5 ft/s^2, both measured relative to the circular plate. At this same instant the plate rotates with the angular velocity and angular deceleration shown. Determine the velocity and acceleration of the ball at this instant.



SOLUTION

Reference Frames: The xyz rotating reference frame is attached to the plate and coincides with the fixed reference frame XYZ at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0}$ $\qquad$ $\omega = [6\mathbf{k}]$ rad/s $\qquad$ $\dot{\omega} = \alpha = [-1.5\mathbf{k}]$ rad/s^2

For the motion of ball C with respect to the xyz frame,

$$(\mathbf{v}_{\text{rel}})_{xyz} = (-3\sin 45°\mathbf{i} - 3\cos 45°\mathbf{j})\text{ ft/s} = [-2.121\mathbf{i} - 2.121\mathbf{j}]\text{ ft/s}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = (-1.5\sin 45°\mathbf{i} - 1.5\cos 45°\mathbf{j})\text{ ft/s}^2 = [-1.061\mathbf{i} - 1.061\mathbf{j}]\text{ ft/s}^2$$

From the geometry shown in Fig. b, $r_{C/O} = 2\cos 45° = 1.414$ ft. Thus,

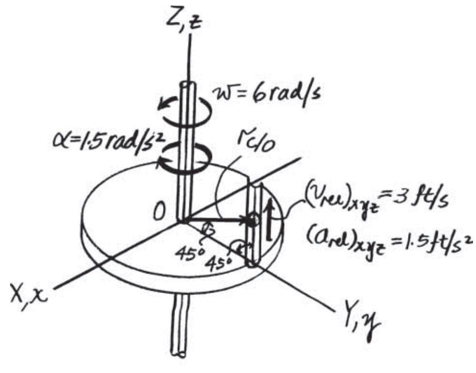
$$\mathbf{r}_{C/O} = (-1.414\sin 45°\mathbf{i} + 1.414\cos 45°\mathbf{j})\text{ft} = [-1\mathbf{i} + 1\mathbf{j}]\text{ ft}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_C = \mathbf{v}_O + \omega \times \mathbf{r}_{C/O} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (6\mathbf{k}) \times (-1\mathbf{i} + 1\mathbf{j}) + (-2.121\mathbf{i} - 2.121\mathbf{j})$$

$$= [-8.12\mathbf{i} - 8.12\mathbf{j}]\text{ ft/s}$$ **Ans.**



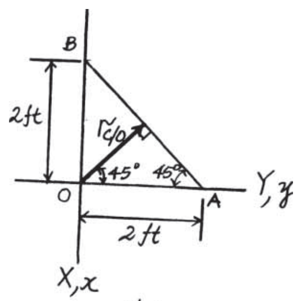
(a)

Acceleration: Applying the relative acceleration equation, we have

$$\mathbf{a}_C = \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{C/O} + \omega \times (\omega \times \mathbf{r}_{C/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (a_{\text{rel}})_{xyz}$$

$$= 0 + (1.5\mathbf{k}) \times (-1\mathbf{i} + 1\mathbf{j}) + (6\mathbf{k}) \times [(6\mathbf{k}) \times (-1\mathbf{i} + 1\mathbf{j})] + 2(6\mathbf{k}) \times (-2.121\mathbf{i} - 2.121\mathbf{j}) + (-1.061\mathbf{i} - 1.061\mathbf{j})$$

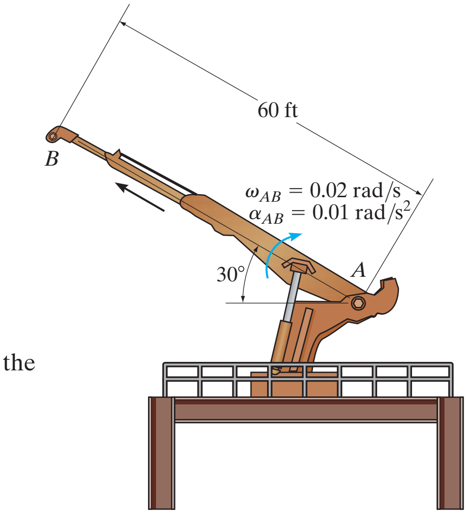
$$= [61.9\mathbf{i} - 61.0\mathbf{j}]\text{ft/s}^2$$ **Ans.**



(b)

16–130.

The crane's telescopic boom rotates with the angular velocity and angular acceleration shown. At the same instant, the boom is extending with a constant speed of 0.5 ft/s, measured relative to the boom. Determine the magnitudes of the velocity and acceleration of point B at this instant.



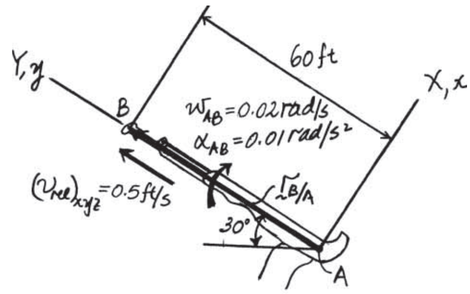
SOLUTION

Reference Frames: The xyz rotating reference frame is attached to boom AB and coincides with the XY fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xy frame with respect to the XY frame is

$\mathbf{v}_A = \mathbf{a}_A = \mathbf{0}$ $\qquad \omega_{AB} = [-0.02\mathbf{k}]$ rad/s $\qquad \dot{\omega}_{AB} = \alpha = [-0.01\mathbf{k}]$ rad/s^2

For the motion of point B with respect to the xyz frame, we have

$\mathbf{r}_{B/A} = [60\mathbf{j}]$ ft $\qquad (\mathbf{v}_{\text{rel}})_{xyz} = [0.5\mathbf{j}]$ ft/s $\qquad (\mathbf{a}_{\text{rel}})_{xyz} = \mathbf{0}$



(a)

Velocity: Applying the relative velocity equation,

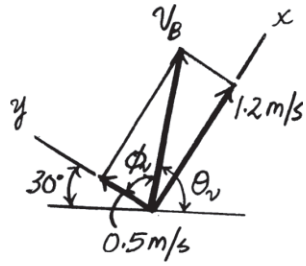
$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-0.02\mathbf{k}) \times (60\mathbf{j}) + 0.5\mathbf{j}$$

$$= [1.2\mathbf{i} + 0.5\mathbf{j}] \text{ ft / s}$$

Thus, the magnitude of $\mathbf{v}_B$, Fig. b, is

$$v_B = \sqrt{1.2^2 + 0.5^2} = 1.30 \text{ ft/s} \qquad \textbf{Ans.}$$



(b)

Acceleration: Applying the relative acceleration equation,

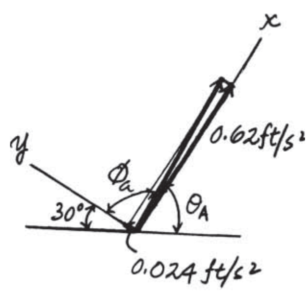
$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-0.01\mathbf{k}) \times (60\mathbf{j}) + (-0.02\mathbf{k}) \times [(-0.02\mathbf{k}) \times (60\mathbf{j})] + 2(-0.02\mathbf{k}) \times (0.5\mathbf{j}) + \mathbf{0}$$

$$= [0.62\mathbf{i} - 0.024\,\mathbf{j}] \text{ ft/s}^2$$

Thus, the magnitude of a_B, Fig. c, is

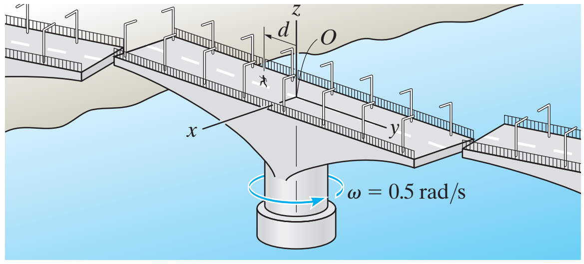
$$a_B = \sqrt{0.62^2 + (-0.024)^2} = 0.6204 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



(c)

16–131.

While the swing bridge is closing with a constant rotation of 0.5 rad/s, a man runs along the roadway at a constant speed of 5 ft/s relative to the roadway. Determine his velocity and acceleration at the instant $d = 15$ ft.



SOLUTION

$\Omega = \{0.5\mathbf{k}\}$ rad/s

$\dot{\Omega} = \mathbf{0}$

$\mathbf{r}_{m/o} = \{-15\,\mathbf{j}\}$ ft

$(v_{m/o})_{xyz} = \{-5\mathbf{j}\}$ ft/s

$(a_{m/o})_{xyz} = \mathbf{0}$

$v_m = \mathbf{v}_o + \Omega \times \mathbf{r}_{m/o} + (\mathbf{v}_{m/o})_{xyz}$

$v_m = 0 + (0.5\mathbf{k}) \times (-15\mathbf{j}) - 5\mathbf{j}$

$\mathbf{v}_m = \{7.5\mathbf{i} - 5\mathbf{j}\}$ ft/s **Ans.**

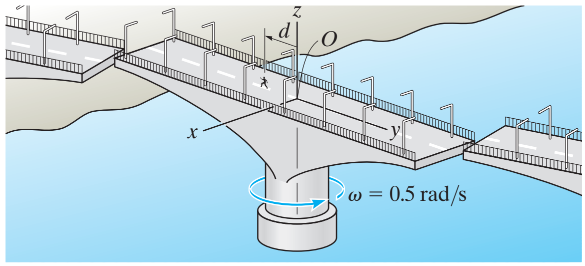
$\mathbf{a}_m = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{m/O} + \Omega \times (\Omega \times \mathbf{r}_{m/O}) + 2\Omega \times (\mathbf{v}_{m/O})_{xyz} + (\mathbf{a}_{m/O})_{xyz}$

$\mathbf{a}_m = \mathbf{0} + \mathbf{0} + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (-15\mathbf{j})] + 2(0.5\mathbf{k}) \times (-5\mathbf{j}) + \mathbf{0}$

$\mathbf{a}_m = \{5\mathbf{i} + 3.75\mathbf{j}\}$ ft/s^2 **Ans.**

***16–132.**

While the swing bridge is closing with a constant rotation of 0.5 rad/s, a man runs along the roadway such that when $d = 10$ ft he is running outward from the center at 5 ft/s with an acceleration of 2 ft/s^2, both measured relative to the roadway. Determine his velocity and acceleration at this instant.



SOLUTION

$\Omega = \{0.5\mathbf{k}\}$ rad/s

$\dot{\Omega} = \mathbf{0}$

$\mathbf{r}_{m/o} = \{-10\,\mathbf{j}\}$ ft

$(v_{m/O})_{xyz} = \{-5\mathbf{j}\}$ ft/s

$(a_{m/O})_{xyz} = \{-2\mathbf{j}\}$ ft/s^2

$v_m = \mathbf{v}_o + \Omega \times \mathbf{r}_{m/o} + (\mathbf{v}_{m/o})_{xyz}$

$v_m = 0 + (0.5\mathbf{k}) \times (-10\mathbf{j}) - 5\mathbf{j}$

$\mathbf{v}_m = \{5\mathbf{i} - 5\mathbf{j}\}$ ft/s **Ans.**

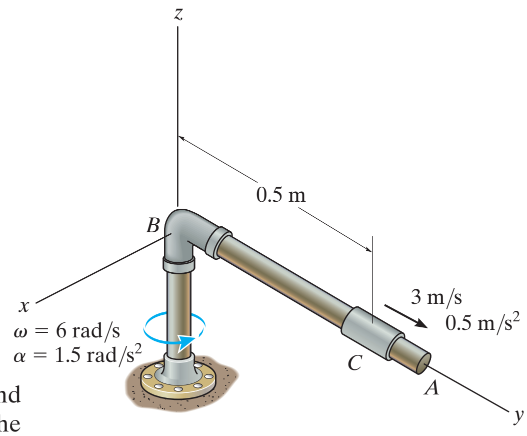
$\mathbf{a}_m = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{m/O} + \Omega \times (\Omega \times \mathbf{r}_{m/O}) + 2\Omega \times (\mathbf{v}_{m/O})_{xyz} + (\mathbf{a}_{m/O})_{xyz}$

$\mathbf{a}_m = \mathbf{0} + \mathbf{0} + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (-10\mathbf{j})] + 2(0.5\mathbf{k}) \times (-5\mathbf{j}) - 2\mathbf{j}$

$\mathbf{a}_m = \{5\mathbf{i} + 0.5\mathbf{j}\}$ ft/s^2 **Ans.**

16–133.

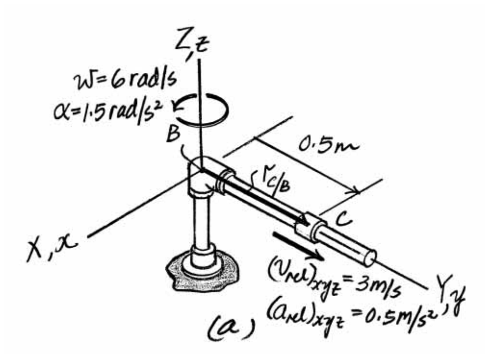
Collar C moves along rod BA with a velocity of 3 m/s and an acceleration of 0.5 m/s^2, both directed from B towards A and measured relative to the rod. At the same instant, rod AB rotates with the angular velocity and angular acceleration shown. Determine the collar's velocity and acceleration at this instant.



SOLUTION

Reference Frames: The xyz rotating reference frame is attached to rod AB and coincides with the XYZ reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$\mathbf{v}_B = \mathbf{a}_B = 0 \qquad \omega_B = \omega = [6\mathbf{k}]\text{ rad/s} \qquad \dot{\omega}_{AB} = \alpha = [1.5\mathbf{k}]\text{ rad/s}^2$$



For the motion of collar C with respect to the xyz frame, we have

$$\mathbf{r}_{C/B} = [0.5\mathbf{j}]\text{ m} \qquad (\mathbf{v}_{\text{rel}})_{xyz} = [3\mathbf{j}]\text{ m/s} \qquad (\mathbf{a}_{\text{rel}})_{xyz} = [0.5\mathbf{j}]\text{ m/s}^2$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{AB} \times \mathbf{r}_{C/B} + (v_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (6\mathbf{k}) \times (0.5\mathbf{j}) + 3\mathbf{j}$$

$$= [-3\mathbf{i} + 3\mathbf{j}]\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

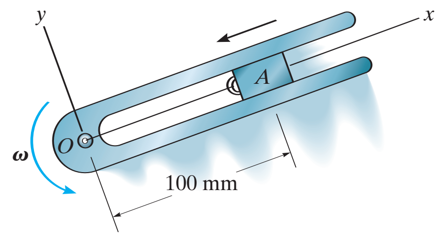
$$a_C = \mathbf{a}_B + \dot{\omega}_{AB} \times \mathbf{r}_{C/B} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{C/B}) + 2\omega_{AB} \times (v_{\text{rel}})_{xyz} + (a_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (1.5\mathbf{k}) \times (0.5\mathbf{j}) + (6\mathbf{k}) \times (6\mathbf{k} \times 0.5\mathbf{j}) + 2(6\mathbf{k}) \times (3\mathbf{j}) + (0.5\mathbf{j})$$

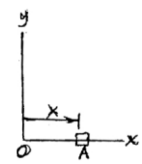
$$= [-36.75\mathbf{i} - 17.5\mathbf{j}]\text{ m/s}^2 \qquad \textbf{Ans.}$$

16–134.

Block A, which is attached to a cord, moves along the slot of a horizontal forked rod. At the instant shown, the cord is pulled down through the hole at O with an acceleration of $4\ \text{m/s}^2$ and its velocity is $2\ \text{m/s}$. Determine the acceleration of the block at this instant. The rod rotates about O with a constant angular velocity $\omega = 4\ \text{rad/s}$.



SOLUTION



Motion of moving reference.

$\mathbf{v}_O = \mathbf{0}$

$\mathbf{a}_O = \mathbf{0}$

$\Omega = 4\mathbf{k}$

$\dot{\Omega} = \mathbf{0}$

Motion of A with respect to moving reference.

$\mathbf{r}_{A/O} = 0.1\mathbf{i}$

$\mathbf{v}_{A/O} = -2\mathbf{i}$

$\mathbf{a}_{A/O} = -4\mathbf{i}$

Thus,

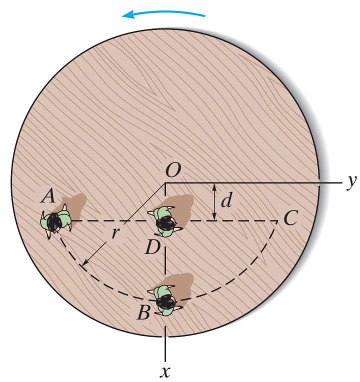
$$\mathbf{a}_A = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{A/O} + \Omega \times (\Omega \times \mathbf{r}_{A/O}) + 2\Omega \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$$

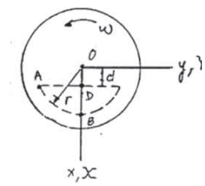
$$= \mathbf{0} + \mathbf{0} + (4\mathbf{k}) \times (4\mathbf{k} \times 0.1\mathbf{i}) + 2(4\mathbf{k} \times (-2\mathbf{i})) - 4\mathbf{i}$$

$\mathbf{a}_A = \{-5.60\mathbf{i} - 16\mathbf{j}\}\ \text{m/s}^2$ **Ans.**

16–135.

A girl stands at A on a platform which is rotating with a constant angular velocity $\omega = 0.5$ rad/s. If she walks at a constant speed of $v = 0.75$ m/s measured relative to the platform, determine her acceleration (a) when she reaches point D in going along the path ADC, $d = 1$ m; and (b) when she reaches point B if she follows the path ABC, $r = 3$ m.





SOLUTION

(a)

$$\mathbf{a}_D = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{D/O} + \Omega \times (\Omega \times \mathbf{r}_{D/O}) + 2\Omega \times (\mathbf{v}_{D/O})_{xyz} + (\mathbf{a}_{D/O})_{xyz} \qquad \textbf{(1)}$$

Motion of moving reference	*Motion of D with respect to moving reference*
$\mathbf{a}_O = 0$	$\mathbf{r}_{D/O} = \{1\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{D/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\dot{\Omega} = 0$	$(\mathbf{a}_{D/O})_{xyz} = \mathbf{0}$

Substitute the data into Eq.(1):

$$\mathbf{a}_B = 0 + (0) \times (1\mathbf{i}) + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (1\mathbf{i})] + 2(0.5\mathbf{k}) \times (0.75\mathbf{j}) + \mathbf{0}$$

$$= \{-1\mathbf{i}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

(b)

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \qquad \textbf{(2)}$$

Motion of moving reference	*Motion of B with respect to moving reference*
$\mathbf{a}_o = \mathbf{0}$	$\mathbf{r}_{B/O} = \{3\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{B/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\Omega = \mathbf{0}$	$(\mathbf{a}_{B/O})_{xyz} = -(a_{B/O})_n\,\mathbf{i} + (a_{B/O})_t\,\mathbf{j}$
	$= -\left(\frac{0.75^2}{3}\right)\mathbf{i}$
	$= \{-0.1875\mathbf{i}\}$ m/s

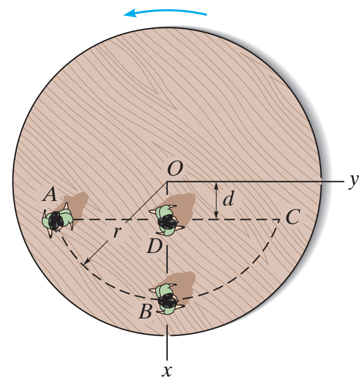
Substitute the data into Eq.(2):

$$\mathbf{a}_B = \mathbf{0} + (\mathbf{0}) \times (3\mathbf{i}) + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (3\mathbf{i})] + 2(0.5\mathbf{k}) \times (0.75\mathbf{j}) + (-0.1875\mathbf{i})$$

$$= \{-1.69\mathbf{i}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

***16–136.**

A girl stands at A on a platform which is rotating with an angular acceleration $\alpha = 0.2\ \text{rad/s}^2$ and at the instant shown has an angular velocity $\omega = 0.5\ \text{rad/s}$. If she walks at a constant speed $v = 0.75\ \text{m/s}$ measured relative to the platform, determine her acceleration (a) when she reaches point D in going along the path ADC, $d = 1$ m; and (b) when she reaches point B if she follows the path ABC, $r = 3$ m.



SOLUTION

(a)

$$\mathbf{a}_D = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{D/O} + \Omega \times (\Omega \times \mathbf{r}_{D/O}) + 2\Omega \times (\mathbf{v}_{D/O})_{xyz} + (\mathbf{a}_{D/O})_{xyz} \qquad \textbf{(1)}$$

Motion of moving reference	*Motion of D with respect to moving reference*
$\mathbf{a}_O = 0$	$\mathbf{r}_{D/O} = \{1\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{D/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\dot{\Omega} = \{0.2\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{D/O})_{xyz} = \mathbf{0}$

Substitute the data into Eq.(1):

$$\mathbf{a}_B = \mathbf{0} + (0.2\mathbf{k}) \times (1\mathbf{i}) + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (1\mathbf{i})] + 2(0.5\mathbf{k}) \times (0.75\mathbf{j}) + \mathbf{0}$$

$$= \{-1\mathbf{i} + 0.2\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

(b)

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \qquad \textbf{(2)}$$

Motion of moving reference	*Motion of B with respect to moving reference*
$\mathbf{a}_O = \mathbf{0}$	$\mathbf{r}_{B/O} = \{3\mathbf{i}\}$ m
$\Omega = \{0.5\mathbf{k}\}$ rad/s	$(\mathbf{v}_{B/O})_{xyz} = \{0.75\mathbf{j}\}$ m/s
$\dot{\Omega} = \{0.2\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{B/O})_{xyz} = -(a_{B/O})_n\,\mathbf{i} + (a_{B/O})_t\,\mathbf{j}$
	$= -\left(\frac{0.75^2}{3}\right)\mathbf{i}$
	$= \{-0.1875\mathbf{i}\}$ m/s

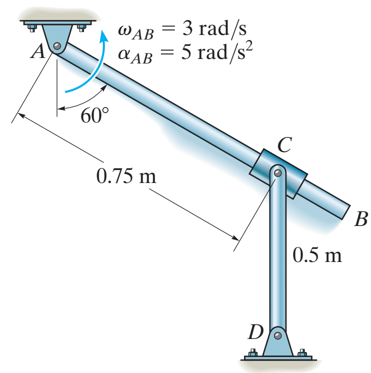
Substitute the data into Eq.(2):

$$\mathbf{a}_B = 0 + (0.2\mathbf{k}) \times (3\mathbf{i}) + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (3\mathbf{i})] + 2(0.5\mathbf{k}) \times (0.75\mathbf{j}) + (-0.1875\mathbf{i})$$

$$= \{-1.69\mathbf{i} + 0.6\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

16–137.

At the instant shown, rod AB has an angular velocity $\omega_{AB} = 3$ rad/s and an angular acceleration $\alpha_{AB} = 5$ rad/s^2. Determine the angular velocity and angular acceleration of rod CD at this instant. The collar at C is pin-connected to CD and slides over AB.



SOLUTION

$$\mathbf{r}_{C/A} = (0.75\sin 60°)\mathbf{i} - (0.75\cos 60°)\mathbf{j}$$

$$\mathbf{r}_{C/A} = \{0.6495\mathbf{i} - 0.375\mathbf{j}\} \text{ m}$$

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D}$$

$$= (\omega_{CD}\mathbf{k}) \times (0.5\mathbf{j})$$

$$= \{-0.5\omega_{CD}\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2\mathbf{r}_{CD}$$

$$= (\alpha_{CD}\mathbf{k}) \times (0.5\mathbf{j}) - \omega_{CD}^2(0.5\mathbf{j})$$

$$a_C = \{-0.5\alpha_{CD}\mathbf{i} - \omega_{CD}^2(0.5)\mathbf{j}\} \text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$-0.5\omega_{CD}\mathbf{i} = 0 + (3\mathbf{k}) \times (0.6495\mathbf{i} - 0.375\mathbf{j}) + v_{C/A}\sin 60°\mathbf{i} - v_{C/A}\cos 60°\mathbf{j}$$

$$-0.5\omega_{CD} = 1.125 + 0.866v_{C/A}$$

$$0 = 1.9485 - 0.5v_{C/A}$$

$$v_{C/A} = 3.897 \text{ m/s}$$

$$\omega_{CD} = -9.00 \text{ rad/s} = 9.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$\mathbf{a}_C = 0 + (5\mathbf{k}) \times (0.6495\mathbf{i} - 0.375\mathbf{j}) + (3\mathbf{k}) \times [(3\mathbf{k}) \times (0.6495\mathbf{i} - 0.375\mathbf{j})]$$

$$+ 2(3\mathbf{k}) \times [3.897(0.866)\mathbf{i} - 0.5(3.897)\mathbf{j}] + 0.866a_{C/A}\mathbf{i} - 0.5a_{C/A}\mathbf{j}$$

$$0.5\alpha_{CD}\mathbf{i} - (-9.00)^2(0.5)\mathbf{j} = 0 + 1.875\mathbf{i} + 3.2475\mathbf{j} - 5.8455\mathbf{i} + 3.375\mathbf{j} + 11.6910\mathbf{i}$$

$$+ 20.2488\mathbf{j} + 0.866a_{C/A}\mathbf{i} - 0.5a_{C/A}\mathbf{j}$$

$$0.5\alpha_{CD} = 7.7205 + 0.866a_{C/A}$$

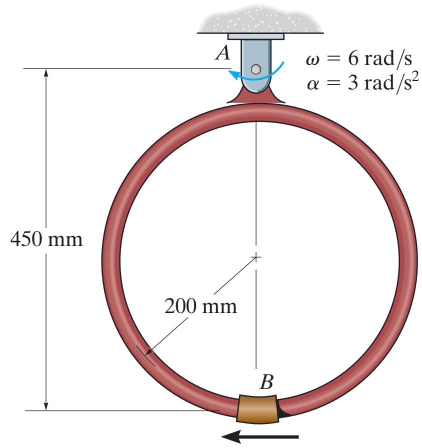
$$-40.5 = 26.8713 - 0.5a_{C/A}$$

$$a_{C/A} = 134.7 \text{ m/s}^2$$

$$\alpha_{CD} = 249 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

16–138.

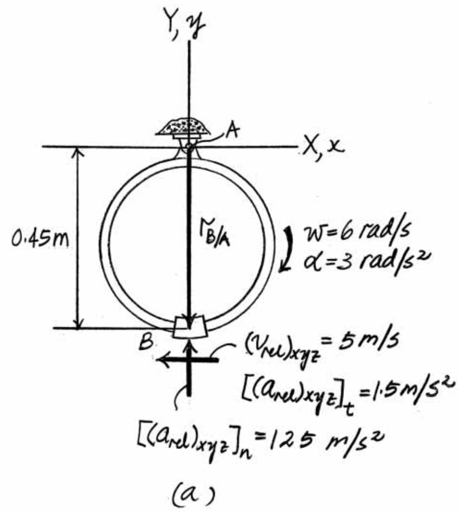
Collar B moves to the left with a speed of 5 m/s, which is increasing at a constant rate of 1.5 m/s^2, relative to the hoop, while the hoop rotates with the angular velocity and angular acceleration shown. Determine the magnitudes of the velocity and acceleration of the collar at this instant.



SOLUTION

Reference Frames: The xyz rotating reference frame is attached to the hoop and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$v_A = a_A = \mathbf{0} \qquad \omega = [-6\mathbf{k}]\text{ rad/s} \qquad \dot{\omega} = \alpha = [-3\mathbf{k}]\text{ rad/s}^2$$



For the motion of collar B with respect to the xyz frame,

$$\mathbf{r}_{B/A} = [-0.45\mathbf{j}]\text{ m}$$

$$(v_{\text{rel}})_{xyz} = [-5\mathbf{i}]\text{ m/s}$$

The normal components of $(\mathbf{a}_{\text{rel}})_{xyz}$ is $[(a_{\text{rel}})_{xyz}]_n = \dfrac{(v_{\text{rel}})_{xyz}{}^2}{\rho} = \dfrac{5^2}{0.2} = 125\text{ m/s}^2$. Thus,

$$(\mathbf{a}_{\text{rel}})_{xyz} = [-1.5\mathbf{i} + 125\mathbf{j}]\text{ m/s}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-6\mathbf{k}) \times (-0.45\mathbf{j}) + (-5\mathbf{i})$$

$$= [-7.7\mathbf{i}]\text{ m/s}$$

Thus,

$$v_B = 7.7\text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega} \times \mathbf{r}_{B/A} + \omega \times (\omega \times \mathbf{r}_{B/A}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

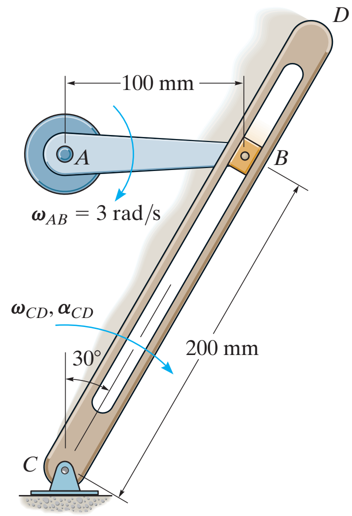
$$= \mathbf{0} + (-3\mathbf{k}) \times (-0.45\mathbf{j}) + (-6\mathbf{k}) \times [(-6\mathbf{k}) \times (-0.45\mathbf{j})] + 2(-6\mathbf{k}) \times (-5\mathbf{i}) + (-1.5\mathbf{i} + 125\mathbf{j})$$

$$= [-2.85\mathbf{i} + 201.2\mathbf{j}]\text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is therefore

$$a_B = \sqrt{2.85^2 + 201.2^2} = 201\text{ m/s}^2 \qquad \textbf{Ans.}$$

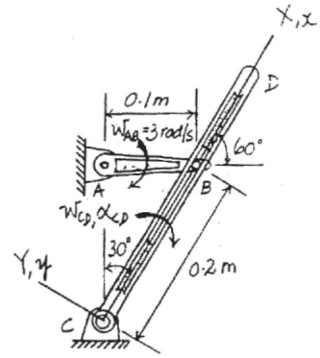
16–139.



Block B of the mechanism is confined to move within the slot member CD. If AB is rotating at a constant rate of $\omega_{AB} = 3$ rad/s, determine the angular velocity and angular acceleration of member CD at the instant shown.

SOLUTION

Coordinate Axes: The origin of both the fixed and moving frames of reference are located at point C. The x, y, z moving frame is attached to and rotates with rod CD since peg B slides along the slot in member CD.



Kinematic Equation: Applying Eqs. 16–24 and 16–27, we have

$$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz} \qquad \textbf{(1)}$$

$$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz} \qquad \textbf{(2)}$$

Motion of moving reference	*Motion of C with respect to moving reference*
$\mathbf{v}_C = \mathbf{0}$	$\mathbf{r}_{B/C} = \{0.2\mathbf{i}\}$ m
$\mathbf{a}_C = \mathbf{0}$	$(\mathbf{v}_{B/C})_{xyz} = (v_{B/C})_{xyz}\,\mathbf{i}$
$\Omega = -\omega_{CD}\,\mathbf{k}$	$(\mathbf{a}_{B/C})_{xyz} = (a_{B/C})_{xyz}\,\mathbf{i}$
$\dot{\Omega} = -\alpha_{CD}\,\mathbf{k}$	

The velocity and acceleration of peg B can be determined using Eqs. 16–9 and 16–14 with $\mathbf{r}_{B/A} = \{0.1\cos 60°\mathbf{i} - 0.1\sin 60°\mathbf{j}\}$ m $= \{0.05\mathbf{i} - 0.08660\mathbf{j}\}$ m.

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = -3\mathbf{k} \times (0.05\mathbf{i} - 0.08660\mathbf{j})$$
$$= \{-0.2598\mathbf{i} - 0.150\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = 0 - 3^2(0.05\mathbf{i} - 0.08660\mathbf{j})$$
$$= \{-0.450\mathbf{i} + 0.7794\mathbf{j}\} \text{ m/s}$$

Substitute the above data into Eq.(1) yields

$$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$$
$$-0.2598\mathbf{i} - 0.150\mathbf{j} = 0 + (-\omega_{CD}\mathbf{k}) \times 0.2\mathbf{i} + (v_{B/C})_{xyz}\,\mathbf{i}$$
$$-0.2598\mathbf{i} - 0.150\mathbf{j} = (v_{B/C})_{xyz}\mathbf{i} - 0.2\omega_{CD}\mathbf{j}$$

Equating **i** and **j** components, we have

$$(v_{B/C})_{xyz} = -0.2598 \text{ m/s}$$
$$\omega_{CD} = 0.750 \text{ rad/s} \qquad \textbf{Ans.}$$

Substitute the above data into Eq.(2) yields

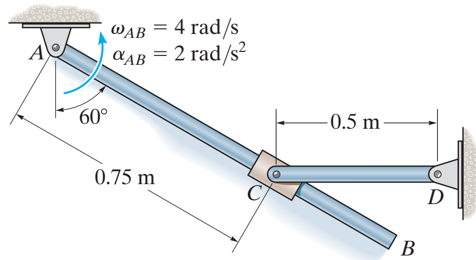
$$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$$
$$-0.450\mathbf{i} + 0.7794\mathbf{j} = 0 + (-\alpha_{CD}\mathbf{k}) \times 0.2\mathbf{i} + (-0.750\mathbf{k}) \times [(-0.750\mathbf{k}) \times 0.2\mathbf{i}]$$
$$+ 2(-0.750\mathbf{k}) \times (-0.2598\mathbf{i}) + (a_{B/C})_{xyz}\,\mathbf{i}$$
$$-0.450\mathbf{i} + 0.7794\mathbf{j} = \left[(a_{B/C})_{xyz} - 0.1125\right]\mathbf{i} + (0.3897 - 0.2\alpha_{CD})\mathbf{j}$$

Equating **i** and **j** components, we have

$$(a_{B/C})_{xyz} = -0.3375 \text{ m/s}^2$$

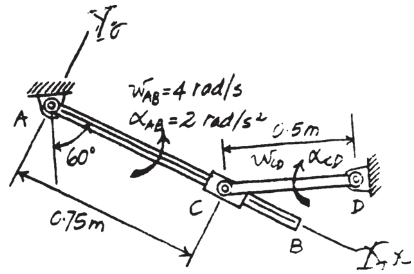
***16–140.**

At the instant shown rod AB has an angular velocity $\omega_{AB} = 4$ rad/s and an angular acceleration $\alpha_{AB} = 2$ rad/s^2. Determine the angular velocity and angular acceleration of rod CD at this instant. The collar at C is pin connected to CD and slides freely along AB.



SOLUTION

Coordinate Axes: The origin of both the fixed and moving frames of reference are located at point A. The x, y, z moving frame is attached to and rotate with rod AB since collar C slides along rod AB.



Kinematic Equation: Applying Eqs. 16–24 and 16–27, we have

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \tag{1}$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz} \tag{2}$$

Motion of moving reference

$\mathbf{v}_A = \mathbf{0}$

$\mathbf{a}_A = \mathbf{0}$

$\Omega = 4\mathbf{k}$ rad/s

$\dot{\Omega} = 2\mathbf{k}$ rad/s^2

Motion of C with respect to moving reference

$r_{C/A} = \{0.75\mathbf{i}\}$m

$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{xyz}\,\mathbf{i}$

The velocity and acceleration of collar C can be determined using Eqs. 16–9 and 16–14 with $\mathbf{r}_{C/D} = \{-0.5\cos 30°\mathbf{i} - 0.5\sin 30°\mathbf{j}\}$m $= \{-0.4330\mathbf{i} - 0.250\mathbf{j}\}$ m.

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = -\omega_{CD}\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j})$$

$$= -0.250\omega_{CD}\mathbf{i} + 0.4330\omega_{CD}\mathbf{j}$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$$

$$= -\alpha_{CD}\,\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j}) - \omega_{CD}^2(-0.4330\mathbf{i} - 0.250\mathbf{j})$$

$$= \left(0.4330\omega_{CD}^2 - 0.250\,\alpha_{CD}\right)\mathbf{i} + \left(0.4330\alpha_{CD} + 0.250\omega_{CD}^2\right)\mathbf{j}$$

Substitute the above data into Eq.(1) yields

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$-0.250\,\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\mathbf{j} = \mathbf{0} + 4\mathbf{k} \times 0.75\mathbf{i} + (v_{C/A})_{xyz}\,\mathbf{i}$$

$$-0.250\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\,\mathbf{j} = (v_{C/A})_{xyz}\,\mathbf{i} + 3.00\mathbf{j}$$

Equating **i** and **j** components and solve, we have

$$(v_{C/A})_{xyz} = -1.732 \text{ m/s}$$

$$\omega_{CD} = 6.928 \text{ rad/s} = 6.93 \text{ rad/s} \qquad \textbf{Ans.}$$

16–140. continued

Substitute the above data into Eq.(2) yields

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (v_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$\left[0.4330\left(6.928^2\right) - 0.250\,\alpha_{CD}\right]\mathbf{i} + \left[0.4330\alpha_{CD} + 0.250\left(6.928^2\right)\right]\mathbf{j}$$

$$= \mathbf{0} + 2\mathbf{k} \times 0.75\mathbf{i} + 4\mathbf{k} \times (4\mathbf{k} \times 0.75\mathbf{i}) + \mathbf{2}\,(4\mathbf{k}) \times (-1.732\mathbf{i}) + (a_{C/A})_{xyz}\,\mathbf{i}$$

$$(20.78 - 0.250\alpha_{CD})\mathbf{i} + (0.4330\,\alpha_{CD} + 12)\mathbf{j} = \left[(a_{C/A})_{xyz} - 12.0\right]\mathbf{i} - 12.36\mathbf{j}$$

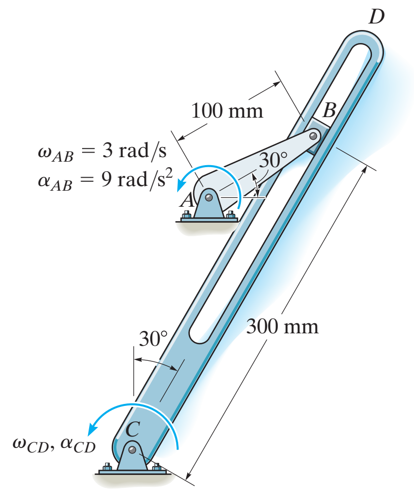
Equating **i** and **j** components, we have

$$(a_{C/A})_{xyz} = 46.85\ \mathrm{m/s^2}$$

$$\alpha_{CD} = -56.2\ \mathrm{rad/s^2} = 56.2\ \mathrm{rad/s^2}\ \circlearrowright \qquad \textbf{Ans.}$$

16–141.

The "quick-return" mechanism consists of a crank AB, slider block B, and slotted link CD. If the crank has the angular motion shown, determine the angular motion of the slotted link at this instant.



SOLUTION

$v_B = 3(0.1) = 0.3$ m/s

$(a_B)_t = 9(0.1) = 0.9$ m/s^2

$(a_B)_n = (3)^2\,(0.1) = 0.9$ m/s^2

$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$

$0.3 \cos 60°\mathbf{i} + 0.3 \sin 60°\mathbf{j} = \mathbf{0} + (\omega_{CD}\mathbf{k}) \times (0.3\mathbf{i}) + v_{B/C}\,\mathbf{i}$

$v_{B/C} = 0.15$ m/s

$\omega_{CD} = 0.866$ rad/s $\circlearrowleft$ **Ans.**

$\mathbf{a}_B = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{B/C} + \Omega \times (\Omega \times \mathbf{r}_{B/C}) + 2\Omega \times (\mathbf{v}_{B/C})_{xyz} + (\mathbf{a}_{B/C})_{xyz}$

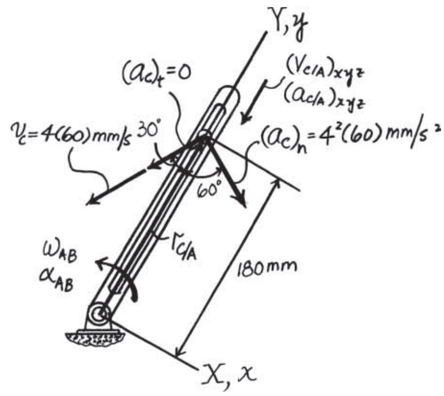
$0.9 \cos 60°\mathbf{i} - 0.9 \cos 30°\mathbf{i} + 0.9 \sin 60°\mathbf{j} + 0.9 \sin 30°\mathbf{j} = \mathbf{0} + (\alpha_{CD}\,\mathbf{k}) \times (0.3\mathbf{i})$

$+(0.866\mathbf{k}) \times (0.866\mathbf{k} \times 0.3\mathbf{i}) + 2(0.866\mathbf{k} \times 0.15\mathbf{i}) + a_{B/C}\,\mathbf{i}$

$-0.3294\mathbf{i} + 1.2294\mathbf{j} = 0.3\alpha_{CD}\,\mathbf{j} - 0.225\mathbf{i} + 0.2598\mathbf{j} + a_{B/C}\,\mathbf{i}$

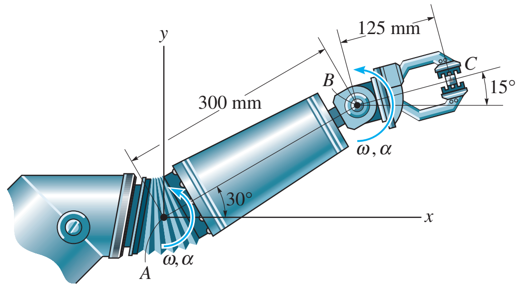
$a_{B/C} = -0.104$ m/s^2

$\alpha_{CD} = 3.23$ rad/s^2 $\circlearrowleft$ **Ans.**



16–142.

At the instant shown, the robotic arm AB is rotating counter clockwise at $\omega = 5$ rad/s and has an angular acceleration $\alpha = 2$ rad/s^2. Simultaneously, the grip BC is rotating counterclockwise at $\omega' = 6$ rad/s and $\alpha' = 2$ rad/s2, both measured relative to a fixed reference. Determine the velocity and acceleration of the object heldat the grip C.



SOLUTION

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} \quad \textbf{(1)}$$

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz} \quad \textbf{(2)}$$

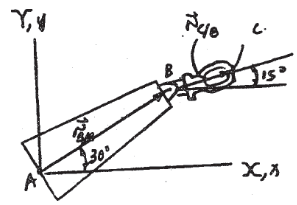
Motion of moving reference

Motion of C with respect to moving reference

$$\mathbf{r}_{C/B} = \{0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j}\} \text{ m}$$

$\Omega = \{6\mathbf{k}\}$ rad/s $\qquad (\mathbf{v}_{C/B})_{xyz} = 0$

$\dot{\Omega} = \{2\mathbf{k}\}$ rad/s^2 $\qquad (\mathbf{a}_{C/B})_{xyz} = 0$



Motion of *B*:

$$\mathbf{v}_B = \omega \times \mathbf{r}_{B/A}$$
$$= (5\mathbf{k}) \times (0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j})$$
$$= \{-0.75\mathbf{i} + 1.2990\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_B = \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$
$$= (2\mathbf{k}) \times (0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j}) - (5)^2(0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j})$$
$$= \{-6.7952\mathbf{i} - 3.2304\mathbf{j}\} \text{ m/s}^2$$

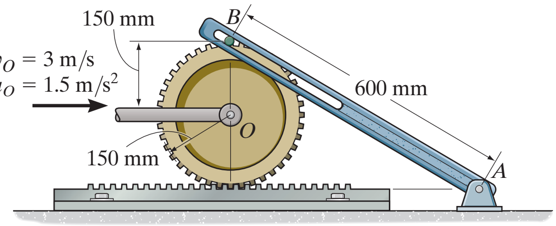
Substitute the data into Eqs. (1) and (2) yields:

$$\mathbf{v}_C = (-0.75\mathbf{i} + 1.2990\mathbf{j}) + (6\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j}) + 0$$
$$= \{-0.944\mathbf{i} + 2.02\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = (-6.79527\mathbf{i} - 3.2304\mathbf{j}) + (2\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j})$$
$$+ (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j})] + 0 + 0$$
$$= \{-11.2\mathbf{i} - 4.15\mathbf{j}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

16–143.

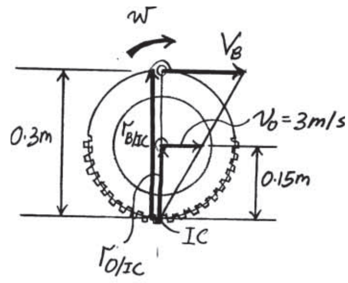
Peg B on the gear slides freely along the slot in link AB. If the gear's center O moves with the velocity and acceleration shown, determine the angular velocity and angular acceleration of the link at this instant.



SOLUTION

Gear Motion: The IC of the gear is located at the point where the gear and the gear rack mesh, Fig. a. Thus,

$$\omega = \frac{v_O}{r_{O/IC}} = \frac{3}{0.15} = 20 \text{ rad/s}$$



(a)

Then,

$$v_B = \omega r_{B/IC} = 20(0.3) = 6 \text{ m/s} \rightarrow$$

Since the gear rolls on the gear rack, $\alpha = \dfrac{a_O}{r} = \dfrac{1.5}{0.15} = 10$ rad/s. By referring to Fig. b,

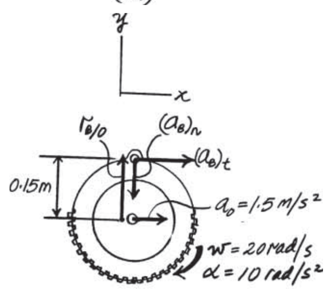
$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$(a_B)_t \mathbf{i} - (a_B)_n \mathbf{j} = 1.5\mathbf{i} + (-10\mathbf{k}) \times 0.15\mathbf{j} - 20^2(0.15\mathbf{j})$$

$$(a_B)_t \mathbf{i} - (a_B)_n \mathbf{j} = 3\mathbf{i} - 60\mathbf{j}$$

Thus,

$$(a_B)_t = 3 \text{ m/s}^2 \qquad\qquad (a_B)_n = 60 \text{ m/s}^2$$



(b)

Reference Frame: The $x'y'z'$ rotating reference frame is attached to link AB and coincides with the XYZ fixed reference frame, Figs. c and d. Thus, $\mathbf{v}_B$ and $\mathbf{a}_B$ with respect to the XYZ frame is

$$\mathbf{v}_B = [6 \sin 30°\mathbf{i} - 6 \cos 30° \mathbf{j}] = [3\mathbf{i} - 5.196\mathbf{j}] \text{ m/s}$$

$$\mathbf{a}_B = (3 \sin 30° - 60 \cos 30°)\mathbf{i} + (-3 \cos 30° - 60 \sin 30°)\mathbf{j}$$

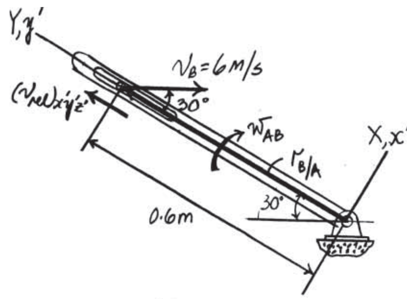
$$= [-50.46\mathbf{i} - 32.60\mathbf{j}] \text{ m/s}^2$$

For motion of the $x'y'z'$ frame with reference to the XYZ reference frame,

$$\mathbf{v}_A = \mathbf{a}_A = \mathbf{0} \qquad \boldsymbol{\omega}_{AB} = -\omega_{AB}\mathbf{k} \qquad \dot{\boldsymbol{\omega}}_{AB} = -\alpha_{AB}\,\mathbf{k}$$

For the motion of point B with respect to the $x'y'z'$ frame is

$$\mathbf{r}_{B/A} = [0.6\mathbf{j}]\text{m} \qquad (\mathbf{v}_{\text{rel}})_{x'y'z'} = (v_{\text{rel}})_{x'y'z'}\,\mathbf{j} \qquad (\mathbf{a}_{\text{rel}})_{x'y'z'} = (a_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$



(C)

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{x'y'z'}$$

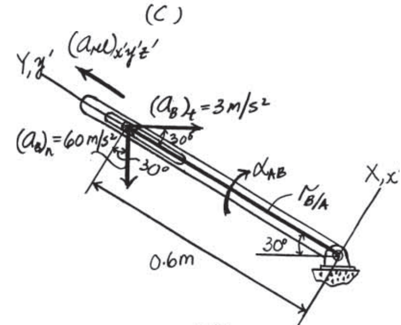
$$3\mathbf{i} - 5.196\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (v_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$

$$3\mathbf{i} - 5.196\mathbf{j} = 0.6\omega_{AB}\,\mathbf{i} + (v_{\text{rel}})_{x'y'z'}\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields

$$3 = 0.6\omega_{AB} \qquad\qquad \omega_{AB} = 5 \text{ rad/s} \qquad\qquad \textbf{Ans.}$$

$$(v_{\text{rel}})_{x'y'z'} = -5.196 \text{ m/s}$$



(d)

Acceleration: Applying the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\omega}}_{AB} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\omega}_{AB} \times (\mathbf{v}_{\text{rel}})_{x'y'z'} + (\mathbf{a}_{\text{rel}})_{x'y'z'}$$

$$-50.46\mathbf{i} - 32.60\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (-5\mathbf{k}) \times [(-5\mathbf{k}) \times (0.6\mathbf{j})] + 2(-5\mathbf{k}) \times (-5.196\mathbf{j}) + (a_{\text{rel}})_{x'y'z'}\mathbf{j}$$

$$-50.46\mathbf{i} - 32.60\mathbf{j} = (0.6\alpha_{AB} - 51.96)\mathbf{i} + \left[(a_{\text{rel}})_{x'y'z'} - 15\right]\mathbf{j}$$

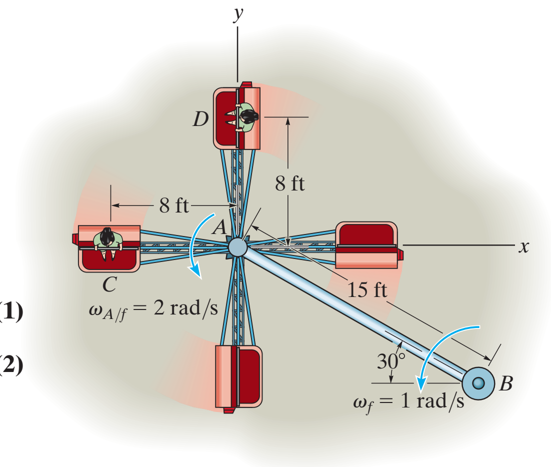
Equating the $\mathbf{i}$ components,

$$-50.46 = 0.6\alpha_{AB} - 51.96$$

$$\alpha_{AB} = 2.5 \text{ rad/s}^2 \qquad\qquad \textbf{Ans.}$$

*16–144.

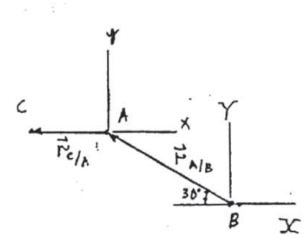
The cars on the amusement-park ride rotate around the axle at A with a constant angular velocity $\omega_{A/f} = 2$ rad/s, measured relative to the frame AB. At the same time the frame rotates around the main axle support at B with a constant angular velocity $\omega_f = 1$ rad/s. Determine the velocity and acceleration of the passenger at C at the instant shown.



SOLUTION

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \qquad \textbf{(1)}$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz} \qquad \textbf{(2)}$$



Motion of moving refernce	*Motion of C with respect to moving reference*
$\Omega = \{3\mathbf{k}\}$ rad/s	$\mathbf{r}_{C/A} = \{-8\mathbf{i}\}$ ft
$\dot{\Omega} = \mathbf{0}$	$(\mathbf{v}_{C/A})_{xyz} = \mathbf{0}$
	$(\mathbf{a}_{C/A})_{xyz} = \mathbf{0}$

Motion of A:

$$\mathbf{v}_A = \omega \times \mathbf{r}_{A/B}$$

$$= (1\mathbf{k}) \times (-15\cos 30°\mathbf{i} + 15\sin 30°\mathbf{j})$$

$$= \{-7.5\mathbf{i} - 12.99\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B}$$

$$= \mathbf{0} - (1)^2(-15\cos 30°\mathbf{i} + 15\sin 30°\mathbf{j})$$

$$= \{12.99\mathbf{i} - 7.5\mathbf{j}\} \text{ ft/s}^2$$

Substitute the data into Eqs.(1) and (2) yields:

$$\mathbf{v}_C = (-7.5\mathbf{i} - 12.99\mathbf{j}) + (3\mathbf{k}) \times (-8\mathbf{i}) + \mathbf{0}$$

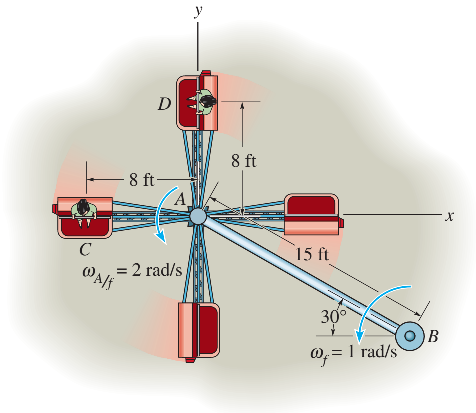
$$= \{-7.5\mathbf{i} - 37.0\mathbf{j}\} \text{ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = (12.99\mathbf{i} - 7.5\mathbf{j}) + \mathbf{0} + (3\mathbf{k}) \times [(3\mathbf{k}) \times (-8\mathbf{i}) + \mathbf{0} + \mathbf{0}]$$

$$= \{85.0\mathbf{i} - 7.5\mathbf{j}\}\text{ft/s}^2 \qquad \textbf{Ans.}$$

16–145.

The cars on the amusement-park ride rotate around the axle at A with a constant angular velocity $\omega_{A/f} = 2$ rad/s, measured relative to the frame AB. At the same time the frame rotates around the main axle support at B with aconstant angular velocity $\omega_f = 1$ rad/s. Determine the velocity and acceleration of the passenger at D at the instant shown.



SOLUTION

$$\mathbf{v}_D = \mathbf{v}_A + \Omega \times \mathbf{r}_{D/A} + (\mathbf{v}_{D/A})_{xyz} \qquad \textbf{(1)}$$

$$\mathbf{a}_D = \mathbf{a}_A + \dot{\omega} \times \mathbf{r}_{D/A} + \Omega \times (\Omega \times \mathbf{r}_{D/A}) + 2\Omega \times (\mathbf{v}_{D/A})_{xyz} + (\mathbf{a}_{D/A})_{xyz} \qquad \textbf{(2)}$$

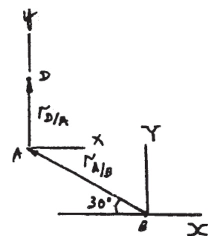
Motion of moving reference	***Motion of D with respect to moving reference***
	$\mathbf{r}_{D/A} = \{8\mathbf{j}\}$ ft
$\Omega = \{3\mathbf{k}\}$ rad/s	$(\mathbf{v}_{D/A})_{xyz} = 0$
$\dot{\Omega} = 0$	$(\mathbf{a}_{D/A})_{xyz} = \mathbf{0}$

Motion of A:

$$\mathbf{v}_A = \omega \times \mathbf{r}_{A/B}$$

$$= (1\mathbf{k}) \times (-15\cos 30°\mathbf{i} + 15\sin 30°\mathbf{j})$$

$$= \{-7.5\mathbf{i} - 12.99\mathbf{j}\} \text{ ft/s}$$



$$\mathbf{a}_A = \alpha \times \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B}$$

$$= \mathbf{0} - (1)^2(-15\cos 30°\mathbf{i} + 15\sin 30°\mathbf{j})$$

$$= \{12.99\mathbf{i} - 7.5\mathbf{j}\} \text{ ft/s}^2$$

Substitute the data into Eqs. (1) and (2) yields:

$$\mathbf{v}_D = (-7.5\mathbf{i} - 12.99\mathbf{j}) + (3\mathbf{k}) \times (8\mathbf{j}) + 0$$

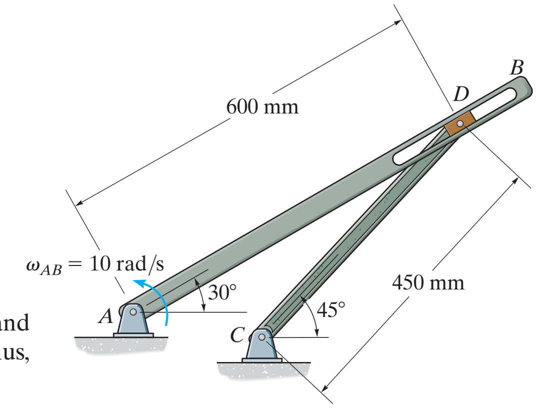
$$= \{-31.5\mathbf{i} - 13.0\mathbf{j}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_D = (12.99\mathbf{i} - 7.5\mathbf{j}) + \mathbf{0} + (3\mathbf{k}) \times [(3\mathbf{k}) \times (8\mathbf{j})] + \mathbf{0} + \mathbf{0}$$

$$= \{13.0\mathbf{i} - 79.5\mathbf{j}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

16–146.

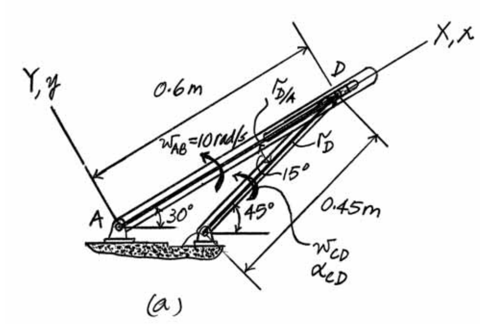
If the slotted arm AB rotates about the pin A with a constant angular velocity of $\omega_{AB} = 10$ rad/s, determine the angular velocity of link CD at the instant shown.



SOLUTION

Reference Frame: The xyz rotating reference frame is attached to link AB and coincides with the XYZ fixed reference frame at the instant considered, Fig. *a*. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$\mathbf{v}_A = \mathbf{0} \qquad \boldsymbol{\omega}_{AB} = [10\mathbf{k}] \text{ rad/s} \qquad \dot{\boldsymbol{\omega}}_{AB} = \mathbf{0}$$



(a)

For the motion of point D relative to the xyz frame, we have

$$\mathbf{r}_{D/A} = [0.6\mathbf{i}] \text{ m} \qquad (\mathbf{v}_{\text{rel}})_{xyz} = (v_{\text{rel}})_{xyz}\mathbf{i}$$

Since link CD rotates about a fixed axis, $\mathbf{v}_D$ can be determined from

$$\mathbf{v}_D = \boldsymbol{\omega}_{CD} \times \mathbf{r}_D$$

$$= (\omega_{CD}\mathbf{k}) \times (0.45 \cos 15°\,\mathbf{i} + 0.45 \sin 15°\,\mathbf{j})$$

$$= -0.1165\omega_{CD}\,\mathbf{i} + 0.4347\omega_{CD}\mathbf{j}$$

Velocity: Applying the relative velocity equation, we have

$$\mathbf{v}_D = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{D/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-0.1165\omega_{CD}\mathbf{i} + 0.4347\omega_{CD}\mathbf{j} = \mathbf{0} + (10\mathbf{k}) \times (0.6\mathbf{i}) + (v_{\text{rel}})_{xyz}\,\mathbf{i}$$

$$-0.1165\omega_{CD}\mathbf{i} + 0.4347\omega_{CD}\mathbf{j} = (v_{\text{rel}})_{xyz}\mathbf{i} + 6\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components

$$-0.1165\omega_{CD} = (v_{\text{rel}})_{xyz}$$

$$0.4347\omega_{CD} = 6$$

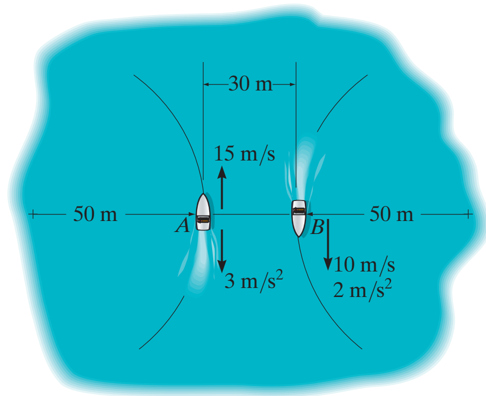
Solving,

$$\omega_{CD} = 13.80 \text{ rad/s} = 13.8 \text{ rad/s}$$ **Ans.**

$$(v_{\text{rel}})_{xyz} = -1.608 \text{ m/s}$$

16–147.

At the instant shown, boat A travels with a speed of 15 m/s, which is decreasing at 3 m/s^2, while boat B travels with a speed of 10 m/s, which is increasing at 2 m/s^2. Determine the velocity and acceleration of boat A with respect to boat B at this instant.

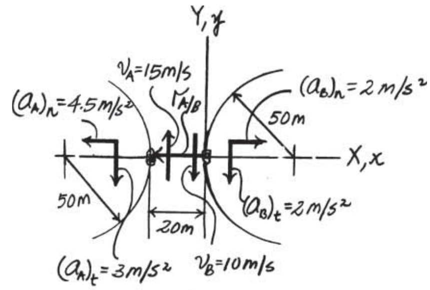


SOLUTION

Reference Frame: The xyz rotating reference frame is attached to boat B and coincides with the XYZ fixed reference frame at the instant considered, Fig. *a*. Since boats A and B move along the circular paths, their normal components of acceleration are $(a_A)_n = \dfrac{v_A{}^2}{\rho} = \dfrac{15^2}{50} = 4.5\text{ m/s}^2$ and $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{10^2}{50} = 2\text{ m/s}^2$. Thus, the motion of boats A and B with respect to the XYZ frame are

$$\mathbf{v}_A = [15\mathbf{j}]\text{ m/s} \qquad \mathbf{v}_B = [-10\mathbf{j}]\text{ m/s}$$

$$\mathbf{a}_A = [-4.5\mathbf{i} - 3\mathbf{j}]\text{ m/s}^2 \qquad \mathbf{a}_B = [2\mathbf{i} - 2\mathbf{j}]\text{ m/s}^2$$



(a)

Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_B}{\rho} = \frac{10}{50} = 0.2\text{ rad/s} \qquad \boldsymbol{\omega} = [0.2\mathbf{k}]\text{ rad/s}$$

$$\dot{\omega} = \frac{(a_B)_t}{\rho} = \frac{2}{50} = 0.04\text{ rad/s}^2 \qquad \dot{\boldsymbol{\omega}} = [0.04\mathbf{k}]\text{ rad/s}^2$$

And the position of boat A with respect to B is

$$\mathbf{r}_{A/B} = [-20\mathbf{i}]\text{ m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$15\mathbf{j} = -10\mathbf{j} + (0.2\mathbf{k}) \times (-20\mathbf{i}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$15\mathbf{j} = -14\mathbf{j} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [29\mathbf{j}]\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) + 2\boldsymbol{\omega} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

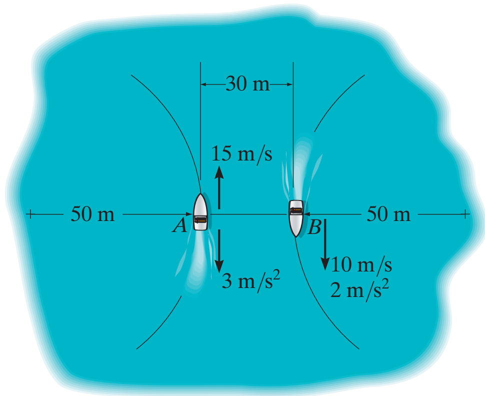
$$(-4.5\mathbf{i} - 3\mathbf{j}) = (2\mathbf{i} - 2\mathbf{j}) + (0.04\mathbf{k}) \times (-20\mathbf{i}) + (0.2\mathbf{k}) \times \left[(0.2\mathbf{k}) \times (-20\mathbf{i})\right] + 2(0.2\mathbf{k}) \times (29\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-4.5\mathbf{i} - 3\mathbf{j} = -8.8\mathbf{i} - 2.8\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [4.3\mathbf{i} - 0.2\mathbf{j}]\text{ m/s}^2 \qquad \textbf{Ans.}$$

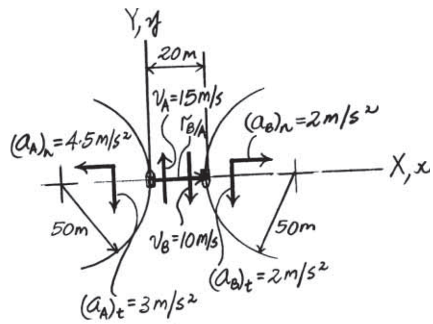
***16–148.**

At the instant shown, boat A travels with a speed of 15 m/s, which is decreasing at $3\,\text{m/s}^2$, while boat B travels with a speed of 10 m/s, which is increasing at $2\,\text{m/s}^2$. Determine the velocity and acceleration of boat B with respect to boat A at this instant.



SOLUTION

Reference Frame: The xyz rotating reference frame is attached to boat A and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since boats A and B move along the circular paths, their normal components of acceleration are $(a_A)_n = \dfrac{v_A{}^2}{\rho} = \dfrac{15^2}{50} = 4.5\ \text{m/s}^2$ and $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{10^2}{50} = 2\ \text{m/s}^2$. Thus, the motion of boats A and B with respect to the XYZ frame are



(a)

$$\mathbf{v}_A = [15\mathbf{j}]\ \text{m/s} \qquad \mathbf{v}_B = [-10\mathbf{j}]\ \text{m/s}$$

$$\mathbf{a}_A = [-4.5\mathbf{i} - 3\mathbf{j}]\ \text{m/s}^2 \qquad \mathbf{a}_B = [2\mathbf{i} - 2\mathbf{j}]\ \text{m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_A}{\rho} = \frac{15}{50} = 0.3\ \text{rad/s} \qquad \omega = [0.3\mathbf{k}]\ \text{rad/s}$$

$$\dot{\omega} = \frac{(a_A)_t}{\rho} = \frac{3}{50} = 0.06\ \text{rad/s}^2 \qquad \dot{\omega} = [-0.06\mathbf{k}]\ \text{rad/s}^2$$

And the position of boat B with respect to boat A is

$$\mathbf{r}_{B/A} = [20\mathbf{i}]\ \text{m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-10\mathbf{j} = 15\mathbf{j} + (0.3\mathbf{k}) \times (20\mathbf{i}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$-10\mathbf{j} = 21\mathbf{j} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [-31\mathbf{j}]\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega} \times \mathbf{r}_{B/A} + \omega(\omega \times \mathbf{r}_{B/A}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

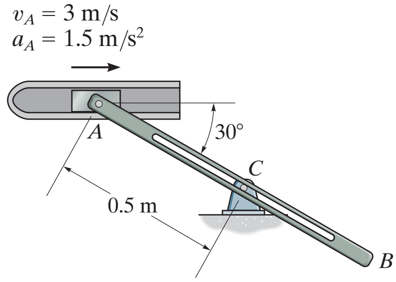
$$(2\mathbf{i} - 2\mathbf{j}) = (-4.5\mathbf{i} - 3\mathbf{j}) + (-0.06\mathbf{k}) \times (20\mathbf{i}) + (0.3\mathbf{k}) \times \left[(0.3\mathbf{k}) \times (20\mathbf{i})\right] + 2(0.3\mathbf{k}) \times (-31\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$2\mathbf{i} - 2\mathbf{j} = 12.3\mathbf{i} - 4.2\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [-10.3\mathbf{i} + 2.2\mathbf{j}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

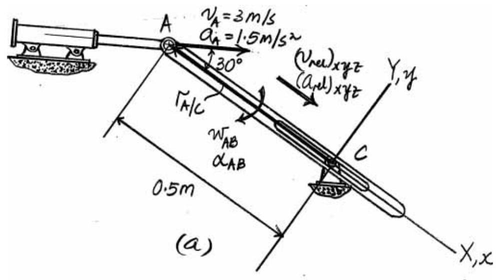
16–149.

If the piston is moving with a velocity of $v_A = 3\text{ m/s}$ and acceleration of $a_A = 1.5\text{ m/s}^2$, determine the angular velocity and angular acceleration of the slotted link at the instant shown. Link AB slides freely along its slot on the fixed peg C.



SOLUTION

Reference Frame: The xyz reference frame centered at C rotates with link AB and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is



$$\mathbf{v}_C = \mathbf{a}_C = \mathbf{0} \qquad \boldsymbol{\omega}_{AB} = -\omega_{AB}\mathbf{k} \qquad \boldsymbol{\alpha}_{AB} = -\alpha_{AB}\mathbf{k}$$

The motion of point A with respect to the xyz frame is

$$\mathbf{r}_{A/C} = [-0.5\mathbf{i}]\text{ m} \qquad (\mathbf{v}_{\text{rel}})_{xyz} = (v_{\text{rel}})_{xyz}\mathbf{i} \qquad (\mathbf{a}_{\text{rel}})_{xyz} = (a_{\text{rel}})_{xyz}\mathbf{i}$$

The motion of point A with respect to the XYZ frame is

$$\mathbf{v}_A = 3\cos 30°\mathbf{i} + 3\sin 30°\mathbf{j} = [2.598\mathbf{i} + 1.5\mathbf{j}]\text{ m/s}$$

$$a_A = 1.5\cos 30°\mathbf{i} + 1.5\sin 30°\mathbf{j} = [1.299\mathbf{i} + 0.75\mathbf{j}]\text{ m/s}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$2.598\mathbf{i} + 1.5\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (-0.5\mathbf{i}) + (v_{\text{rel}})_{xyz}\mathbf{i}$$

$$2.598\mathbf{i} + 1.5\mathbf{j} = (v_{\text{rel}})_{xyz}\mathbf{i} + 0.5\omega_{AB}\mathbf{j}$$

Equating the **i** and **j** components,

$$(v_{\text{rel}})_{xyz} = 2.598\text{ m/s}$$

$$0.5\omega_{AB} = 1.5 \qquad \omega_{AB} = 3\text{ rad/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\boldsymbol{\omega}}_{AB} \times \mathbf{r}_{A/C} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times r_{A/C}) + 2\boldsymbol{\omega}_{AB} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$1.299\mathbf{i} + 0.75\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (-0.5\mathbf{i}) + (-3\mathbf{k}) \times [(-3\mathbf{k}) \times (-0.5\mathbf{i})] + 2(-3\mathbf{k}) \times (2.598\mathbf{i}) + (a_{\text{rel}})_{xyz}\mathbf{i}$$

$$1.299\mathbf{i} + 0.75\mathbf{j} = \left[4.5 + (a_{\text{rel}})_{xyz}\right]\mathbf{i} + (0.5\alpha_{AB} - 15.59)\mathbf{j}$$

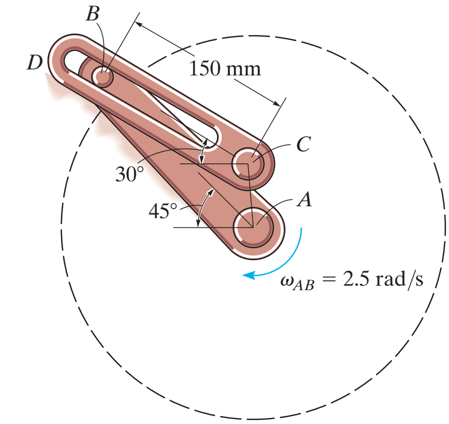
Equating the **j** components,

$$0.75 = 0.5\alpha_{AB} - 15.59$$

$$\alpha_{AB} = 32.68\text{ rad/s}^2 = 32.7\text{ rad/s}^2 \qquad \textbf{Ans.}$$

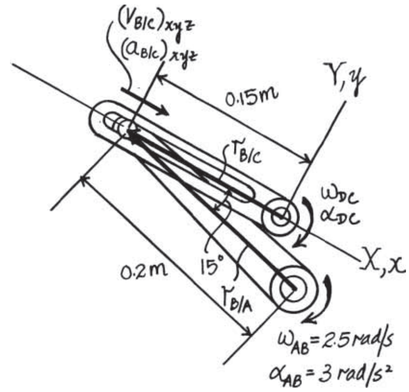
16–150.

The two-link mechanism serves to amplify angular motion. Link AB has a pin at B which is confined to move within the slot of link CD. If at the instant shown, AB (input) has an angular velocity of $\omega_{AB} = 2.5$ rad/s, determine the angular velocity of CD (output) at this instant.



SOLUTION

$$\frac{r_{BA}}{\sin 120^\circ} = \frac{0.15 \text{ m}}{\sin 45^\circ}$$



$\mathbf{r}_{BA} = 0.1837$ m

$\mathbf{v}_C = \mathbf{0}$

$\mathbf{a}_C = \mathbf{0}$

$\Omega = -\omega_{DC}\mathbf{k}$

$\dot{\Omega} = -\alpha_{DC}\mathbf{k}$

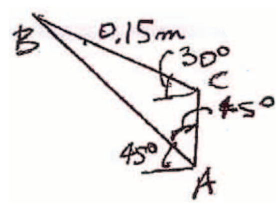
$\mathbf{r}_{B/C} = \{-0.15\,\mathbf{i}\}$ m

$(\mathbf{v}_{B/C})_{xyz} = (v_{B/C})_{xyz}\mathbf{i}$

$(\mathbf{a}_{B/C})_{xyz} = (a_{B/C})_{xyz}\mathbf{i}$

$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = (-2.5\mathbf{k}) \times (-0.1837 \cos 15^\circ\mathbf{i} + 0.1837 \sin 15^\circ\mathbf{j})$

$\quad = \{0.1189\mathbf{i} + 0.4436\mathbf{j}\}$ m/s



$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$

$0.1189\mathbf{i} + 0.4436\mathbf{j} = \mathbf{0} + (-\omega_{DC}\mathbf{k}) \times (-0.15\mathbf{i}) + (v_{B/C})_{xyz}\mathbf{i}$

$0.1189\mathbf{i} + 0.4436\mathbf{j} = (v_{B/C})_{xyz}\mathbf{i} + 0.15\omega_{DC}\mathbf{j}$

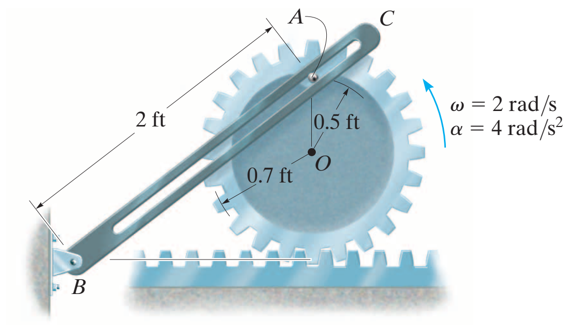
Solving:

$$(v_{B/C})_{xyz} = 0.1189 \text{ m/s}$$

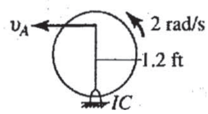
$$\omega_{DC} = 2.96 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

16–151.

The gear has the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link BC at this instant. The peg at A is fixed to the gear.



SOLUTION

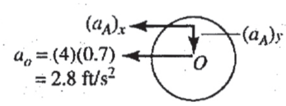


$v_A = (1.2)(2) = 2.4 \text{ ft/s} \leftarrow$

$a_O = 4(0.7) = 2.8 \text{ ft/s}^2$

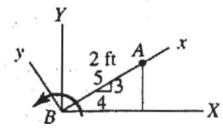
$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O}$

$\mathbf{a}_A = \underset{\leftarrow}{2.8} + \underset{\leftarrow}{4(0.5)} + \underset{\downarrow}{(2)^2(0.5)}$



$\mathbf{a}_A = \underset{\leftarrow}{4.8} + \underset{\downarrow}{2}$

$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$



$-2.4\mathbf{i} = 0 + (\Omega\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}) + v_{A/B}\left(\frac{4}{5}\right)\mathbf{i} + v_{A/B}\left(\frac{3}{5}\right)\mathbf{j}$

$-2.4\mathbf{i} = 1.6\Omega\mathbf{j} - 1.2\Omega\mathbf{i} + 0.8v_{A/B}\mathbf{i} + 0.6v_{A/B}\mathbf{j}$

$-2.4 = -1.2\Omega + 0.8v_{A/B}$

$0 = 1.6\Omega + 0.6v_{A/B}$

Solving,

$\omega_{BC} = \Omega = 0.720 \text{ rad/s} \circlearrowleft$ **Ans.**

$v_{A/B} = -1.92 \text{ ft/s}$

$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$

$-4.8\mathbf{i} - 2\mathbf{j} = 0 + (\dot{\Omega}\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}) + (0.72\mathbf{k}) \times (0.72\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}))$

$+2(0.72\mathbf{k}) \times [-(0.8)(1.92)\mathbf{i} - 0.6(1.92)\mathbf{j}] + 0.8a_{B/A}\mathbf{i} + 0.6a_{B/A}\mathbf{j}$

$-4.8\mathbf{i} - 2\mathbf{j} = 1.6\dot{\Omega}\mathbf{j} - 1.2\dot{\Omega}\mathbf{i} - 0.8294\mathbf{i} - 0.6221\mathbf{j} - 2.2118\mathbf{j} + 1.6589\mathbf{i} + 0.8a_{B/A}\mathbf{i} + 0.6a_{B/A}.$

$-4.8 = -1.2\dot{\Omega} - 0.8294 + 1.6589 + 0.8a_{B/A}$

$-2 = 1.6\dot{\Omega} - 0.6221 - 2.2118 + 0.6a_{B/A}$

$-4.6913 = -\dot{\Omega} + 0.667a_{B/A}$

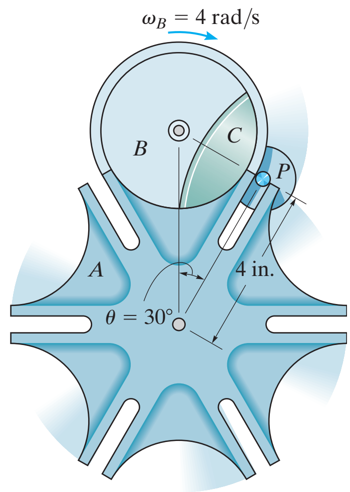
$0.5212 = \dot{\Omega} + 0.357a_{B/A}$

$\alpha_{BC} = \dot{\Omega} = 2.02 \text{ rad/s}^2 \circlearrowleft$ **Ans.**

$a_{B/A} = -4.00 \text{ ft/s}^2$

***16–152.**

The Geneva mechanism is used in a packaging system to convert constant angular motion into intermittent angular motion. The star wheel A makes one sixth of a revolution for each full revolution of the driving wheel B and the attached guide C. To do this, pin P, which is attached to B, slides into one of the radial slots of A, thereby turning wheel A, and then exits the slot. If B has a constant angular velocity of $\omega_B = 4$ rad/s, determine $\boldsymbol{\omega}_A$ and $\boldsymbol{\alpha}_A$ of wheel A at the instant shown.



SOLUTION

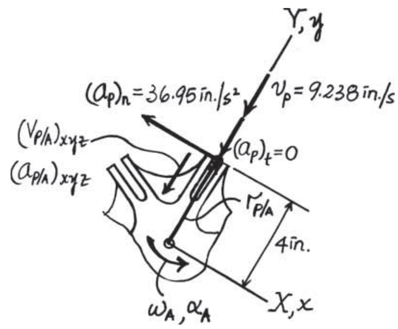
The circular path of motion of P has a radius of

$$r_P = 4 \tan 30° = 2.309 \text{ in.}$$

Thus,

$$\mathbf{v}_P = -4(2.309)\mathbf{j} = -9.238\mathbf{j}$$

$$\mathbf{a}_P = -(4)^2(2.309)\mathbf{i} = -36.95\mathbf{i}$$



Thus,

$$\mathbf{v}_P = \mathbf{v}_A + \Omega \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$

$$-9.238\mathbf{j} = \mathbf{0} + (\omega_A\,\mathbf{k}) \times (4\mathbf{j}) - v_{P/A}\,\mathbf{j}$$

Solving,

$$\omega_A = 0 \qquad \textbf{Ans.}$$

$$v_{P/A} = 9.238 \text{ in./s}$$

$$\mathbf{a}_P = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{P/A} + \Omega \times (\Omega \times \mathbf{r}_{P/A}) + 2\Omega \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$

$$-36.95\mathbf{i} = \mathbf{0} + (\alpha_A\mathbf{k}) \times (4\mathbf{j}) + \mathbf{0} + \mathbf{0} - a_{P/A}\,\mathbf{j}$$

Solving,

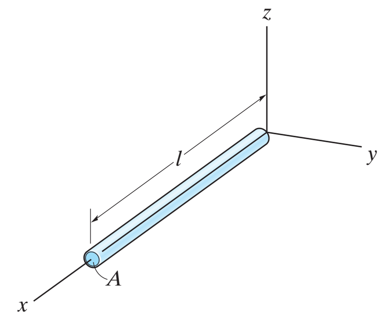
$$-36.95 = -4\alpha_A$$

$$\alpha_A = 9.24 \text{ rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

$$a_{P/A} = 0$$

17–1.

Determine the moment of inertia I_y for the slender rod. The rod's density ρ and cross-sectional area A are constant. Express the result in terms of the rod's total mass m.



SOLUTION

$$I_y = \int_M x^2\, dm$$

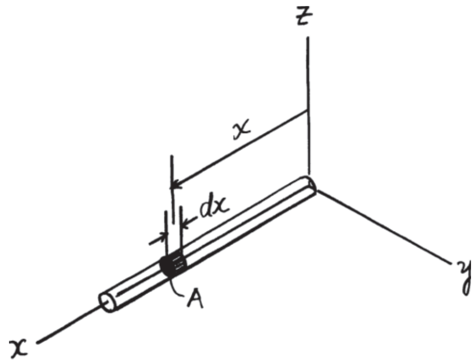
$$= \int_0^l x^2 (\rho\, A\, dx)$$

$$= \frac{1}{3} \rho\, A\, l^3$$

$$m = \rho\, A\, l$$

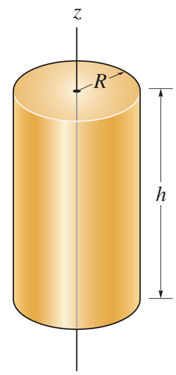
Thus,

$$I_y = \frac{1}{3} m\, l^2$$ **Ans.**



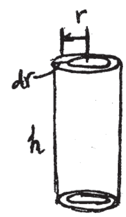
17–2.

The solid cylinder has an outer radius R, height h, and is made from a material having a density that varies from its center as $\rho = k + ar^2$, where k and a are constants. Determine the mass of the cylinder and its moment of inertia about the z axis.



SOLUTION

Consider a shell element of radius r and mass



$$dm = \rho\, dV = \rho(2\pi\, r\, dr)h$$

$$m = \int_0^R (k + ar^2)(2\pi\, r\, dr)h$$

$$m = 2\pi\, h\left(\frac{kR^2}{2} + \frac{aR^4}{4}\right)$$

$$m = \pi\, h\, R^2\left(k + \frac{aR^2}{2}\right) \qquad \textbf{Ans.}$$

$$dI = r^2\, dm = r^2(\rho)(2\pi\, r\, dr)h$$

$$I_z = \int_0^R r^2(k + ar^2)(2\pi\, r\, dr)\, h$$

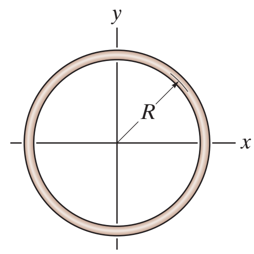
$$I_z = 2\pi h \int_0^R (k\, r^3 + a\, r^5)\, dr$$

$$I_z = 2\pi h\left[\frac{k\, R^4}{4} + \frac{aR^6}{6}\right]$$

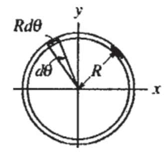
$$I_z = \frac{\pi\, h\, R^4}{2}\left[k + \frac{2\, aR^2}{3}\right] \qquad \textbf{Ans.}$$

17–3.

Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.



SOLUTION



$$I_z = \int_0^{2\pi} \rho\, A(R\, d\theta)R^2 = 2\pi\, \rho\, A\, R^3$$

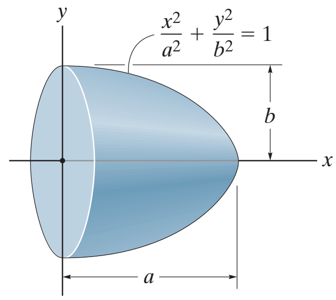
$$m = \int_0^{2\pi} \rho\, A\, R\, d\theta = 2\pi\, \rho\, A\, R$$

Thus,

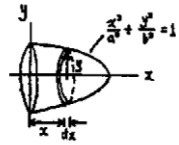
$$I_z = m R^2$$ **Ans.**

***17–4.**

Determine the moment of inertia of the semiellipsoid with respect to the x axis and express the result in terms of the mass m of the semiellipsoid. The material has a constant density ρ.



SOLUTION



$$dI_x = \frac{y^2 dm}{2}$$

$$m = \int_v \rho dV$$

$$= \int_0^a \rho\pi b^2\left(1 - \frac{x^2}{a^2}\right)dx$$

$$= \frac{2}{3}\rho\pi a b^2$$

$$I_x = \frac{1}{2}\rho\pi \int_0^a b^4\left(1 - \frac{x^2}{a^2}\right)^2 dx$$

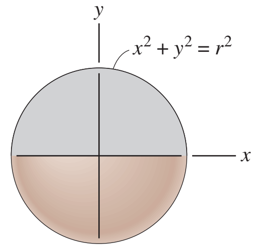
$$= \frac{4}{15}\rho\pi a b^4$$

Thus,

$$I_x = \frac{2}{5}mb^2$$ **Ans.**

17–5.

The sphere is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the sphere. The material has a constant density ρ.



SOLUTION

$$dI_x = \frac{y^2\,dm}{2}$$

$$dm = \rho\,dV = \rho(\pi y^2\,dx) = \rho\,\pi(r^2 - x^2)\,dx$$

$$dI_x = \frac{1}{2}\rho\,\pi(r^2 - x^2)^2\ \ dx$$

$$I_x = \int_{-r}^{r}\frac{1}{2}\rho\,\pi(r^2 - x^2)^2\,dx$$

$$= \frac{8}{15}\pi\rho\,r^5$$

$$m = \int_{-r}^{r}\rho\,\pi(r^2 - x^2)\,dx$$

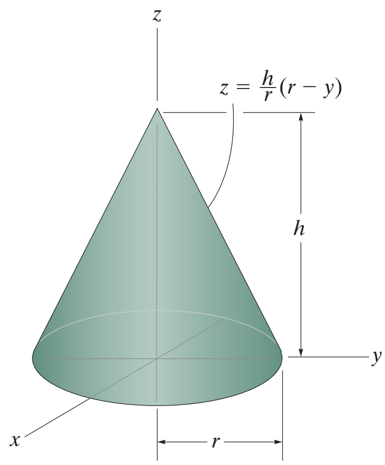
$$= \frac{4}{3}\rho\,\pi\,r^3$$

Thus,

$$I_x = \frac{2}{5}m\,r^2 \qquad \textbf{Ans.}$$

17–6.

Determine the mass moment of inertia I_z of the cone formed by revolving the shaded area around the z axis. The density of the material is ρ. Express the result in terms of the mass m of the cone.



SOLUTION

Differential Element: The mass of the disk element shown shaded in Fig. a is $dm = \rho\, dV = \rho\pi r^2 dz$. Here, $r = y = r_o - \dfrac{r_o}{h}z$. Thus, $dm = \rho\pi\left(r_o - \dfrac{r_o}{h}z\right)^2 dz$. The mass moment of inertia of this element about the z axis is

$$dI_z = \frac{1}{2}dmr^2 = \frac{1}{2}(\rho\pi r^2 dz)r^2 = \frac{1}{2}\rho\pi r^4 dz = \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

Mass: The mass of the cone can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^h \rho\pi\left(r_o - \frac{r_o}{h}z\right)^2 dz$$

$$= \rho\pi\left[\frac{1}{3}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{3}\rho\pi r_o^2 h$$

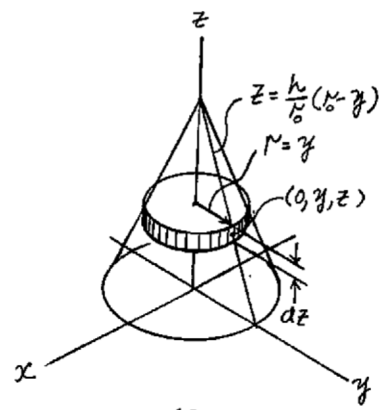
Mass Moment of Inertia: Integrating dI_z, we obtain

$$I_z = \int dI_z = \int_0^h \frac{1}{2}\rho\pi\left(r_o - \frac{r_o}{h}z\right)^4 dz$$

$$= \frac{1}{2}\rho\pi\left[\frac{1}{5}\left(r_o - \frac{r_o}{h}z\right)^3\left(-\frac{h}{r_o}\right)\right]\Bigg|_0^h = \frac{1}{10}\rho\pi r_o^4 h$$

From the result of the mass, we obtain $\rho\pi r_o^2 h = 3m$. Thus, I_z can be written as

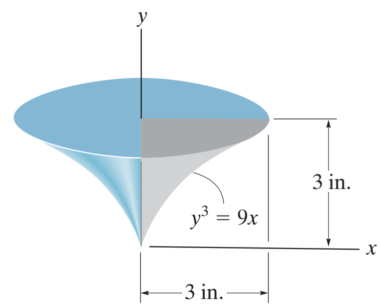
$$I_z = \frac{1}{10}\left(\rho\pi r_o^2 h\right)r_o^2 = \frac{1}{10}(3m)r_o^2 = \frac{3}{10}mr_o^2 \qquad \textbf{Ans.}$$



(a)

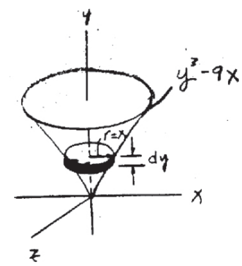
17–7.

The solid is formed by revolving the shaded area around the y axis. Determine the radius of gyration k_y. The specific weight of the material is $\gamma = 380\ \text{lb/ft}^3$.



SOLUTION

The moment of inertia of the solid : The mass of the disk element $dm = \rho\pi x^2\,dy = \frac{1}{81}\rho\pi y^6\,dy$.



$$dI_y = \frac{1}{2}dmx^2$$

$$= \frac{1}{2}\left(\rho\pi x^2\,dy\right)x^2$$

$$= \frac{1}{2}\rho\pi x^4\,dy = \frac{1}{2(9^4)}\rho\pi y^{12}\,dy$$

$$I_y = \int dI_y = \frac{1}{2(9^4)}\rho\pi\int_0^3 y^{12}\,dy$$

$$= 29.632\rho$$

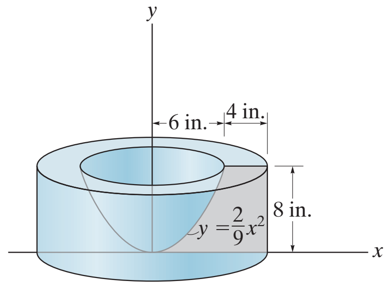
The mass of the solid:

$$m = \int_m dm = \frac{1}{81}\rho\pi\int_0^3 y^6\,dy = 12.117\rho$$

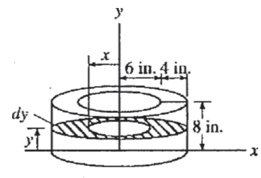
$$k_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{29.632\rho}{12.117\rho}} = 1.56\text{ in.}$$ **Ans.**

***17–8.**

The concrete shape is formed by rotating the shaded area about the y axis. Determine the moment of inertia I_y. The specific weight of concrete is $\gamma = 150\ \text{lb/ft}^3$.



SOLUTION



$$d\,I_y = \frac{1}{2}(dm)(10)^2 - \frac{1}{2}(dm)x^2$$

$$= \frac{1}{2}[\pi\rho(10)^2\,dy](10)^2 - \frac{1}{2}\pi\rho x^2\,dy x^2$$

$$I_y = \frac{1}{2}\pi\rho\left[\int_0^8 (10)^4\,dy - \int_0^8 \left(\frac{9}{2}\right)^2 y^2 dy\right]$$

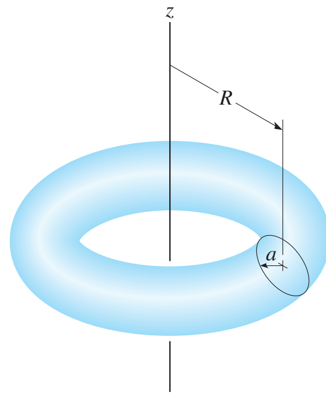
$$= \frac{\frac{1}{2}\pi(150)}{32.2(12)^3}\left[(10)^4(8) - \left(\frac{9}{2}\right)^2\left(\frac{1}{3}\right)(8)^3\right]$$

$$= 324.1\ \text{slug}\cdot\text{in}^2$$

$$I_y = 2.25\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans.}$$

17–9.

Determine the moment of inertia I_z of the torus. The mass of the torus is m and the density ρ is constant. *Suggestion:* Use a shell element.



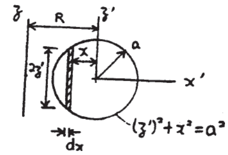
SOLUTION

$$dm = 2\pi(R - x)(2z'\rho\, dx)$$

$$dI_z = (R - x)^2 dm$$

$$= 4\pi\rho[(R^3 - 3R^2 x + 3Rx^2 - x^3)\sqrt{a^2 - x^2}\, dx]$$

$$I_z = 4\pi\rho[R^3 \int_{-a}^{a} \sqrt{a^2 - x^2}\, dx - 3R^2 \int_{-a}^{a} x^3\sqrt{a^2 - x^2}\, dx + 3R \int_{-a}^{a} \sqrt{a^2 - x^2}\, dx - \int_{-a}^{a} x^3\sqrt{a^2 - x^2}$$



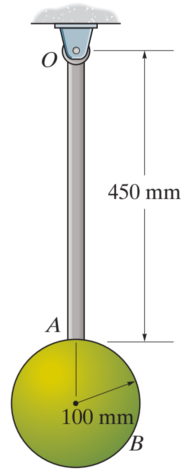
$$= 2\pi^2 \rho R a^2 (R^2 + \frac{3}{4}a^2)$$

Since $m = \rho V = 2\pi R \rho \pi a^2$

$$I_z = m(R^2 + \frac{3}{4}a^2) \qquad \textbf{Ans.}$$

17–10.

Determine the mass moment of inertia of the pendulum about an axis perpendicular to the page and passing through point O. The slender rod has a mass of 10 kg and the sphere has a mass of 15 kg.



SOLUTION

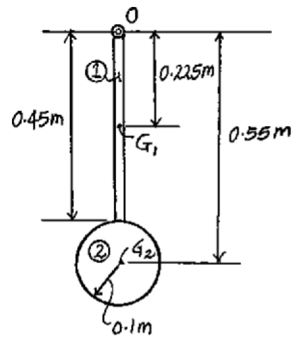
Composite Parts: The pendulum can be subdivided into two segments as shown in Fig. *a*. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Moment of Inertia: The moment of inertia of the slender rod segment (1) and the sphere segment (2) about the axis passing through their center of mass can be computed from $(I_G)_1 = \frac{1}{12}ml^2$ and $(I_G)_2 = \frac{2}{5}mr^2$. The mass moment of inertia of each segment about an axis passing through point O can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

$$= \left[\frac{1}{12}(10)(0.45^2) + 10(0.225^2)\right] + \left[\frac{2}{5}(15)(0.1^2) + 15(0.55^2)\right]$$

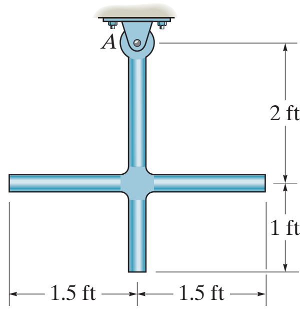
$$= 5.27 \text{ kg} \cdot \text{m}^2$$ **Ans.**



(a)

17–11.

The slender rods have a weight of 3 lb/ft. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point A.

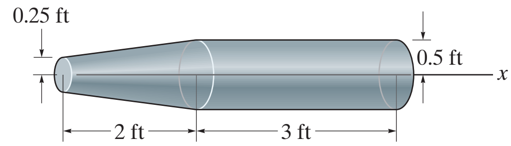


SOLUTION

$$I_A = \frac{1}{3}\left[\frac{3(3)}{32.2}\right](3)^2 + \frac{1}{12}\left[\frac{3(3)}{32.2}\right](3)^2 + \left[\frac{3(3)}{32.2}\right](2)^2 = 2.17 \text{ slug}\cdot\text{ft}^2$$ **Ans.**

***17–12.**

Determine the moment of inertia of the solid steel assembly about the x axis. Steel has a specific weight of $\gamma_{st} = 490 \text{ lb/ft}^3$.



SOLUTION

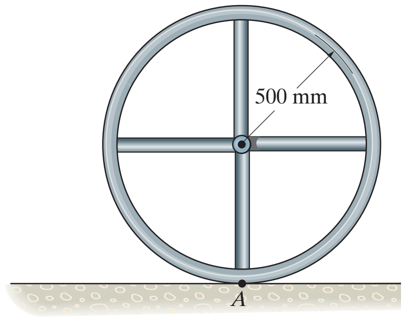
$$I_x = \frac{1}{2} m_1 (0.5)^2 + \frac{3}{10} m_2 (0.5)^2 - \frac{3}{10} m_3 (0.25)^2$$

$$= \left[\frac{1}{2}\pi(0.5)^2(3)(0.5)^2 + \frac{3}{10}\left(\frac{1}{3}\right)\pi(0.5)^2(4)(0.5)^2 - \frac{3}{10}\left(\frac{1}{2}\right)\pi(0.25)^2(2)(0.25)^2\right]\left(\frac{490}{32.2}\right)$$

$$= 5.64 \text{ slug} \cdot \text{ft}^2$$ **Ans.**

17–13.

The wheel consists of a thin ring having a mass of 10 kg and four spokes made from slender rods, each having a mass of 2 kg. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point A.



SOLUTION

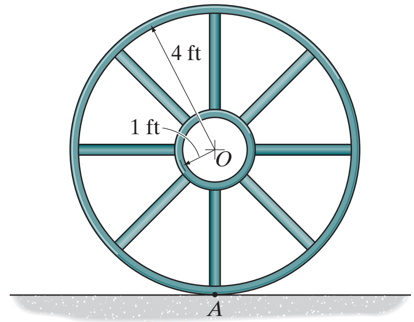
$$I_A = I_o + md^3$$

$$= \left[2\left[\frac{1}{12}(4)(1)^2\right] + 10(0.5)^2\right] + 18(0.5)^2$$

$$= 7.67 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

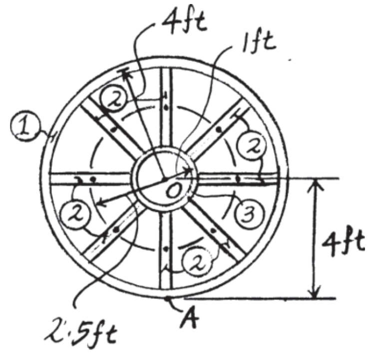
17–14.

If the large ring, small ring and each of the spokes weigh 100 lb, 15 lb, and 20 lb, respectively, determine the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A.



SOLUTION

Composite Parts: The wheel can be subdivided into the segments shown in Fig. *a*. The spokes which have a length of $(4 - 1) = 3$ ft and a center of mass located at a distance of $\left(1 + \frac{3}{2}\right)$ ft $= 2.5$ ft from point O can be grouped as segment (2).



(a)

Mass Moment of Inertia: First, we will compute the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point O.

$$I_O = \left(\frac{100}{32.2}\right)(4^2) + 8\left[\frac{1}{12}\left(\frac{20}{32.2}\right)(3^2) + \left(\frac{20}{32.2}\right)(2.5^2)\right] + \left(\frac{15}{32.2}\right)(1^2)$$

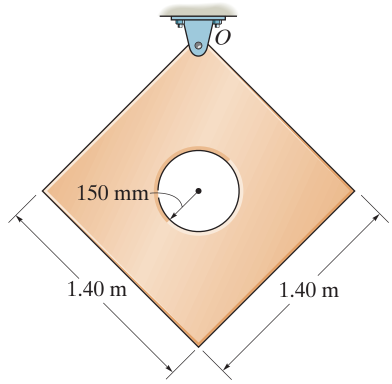
$$= 84.94 \text{ slug}\cdot\text{ft}^2$$

The mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A can be found using the parallel-axis theorem $I_A = I_O + md^2$, where $m = \frac{100}{32.2} + 8\left(\frac{20}{32.2}\right) + \frac{15}{32.2} = 8.5404$ slug and $d = 4$ ft. Thus,

$$I_A = 84.94 + 8.5404(4^2) = 221.58 \text{ slug}\cdot\text{ft}^2 = 222 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans.}$$

17–15.

Determine the moment of inertia about an axis perpendicular to the page and passing through the pin at O. The thin plate has a hole in its center. Its thickness is 50 mm, and the material has a density $\rho = 50\ \text{kg/m}^3$.



SOLUTION

$$I_G = \frac{1}{12}\left[50(1.4)(1.4)(0.05)\right]\left[(1.4)^2 + (1.4)^2\right] - \frac{1}{2}\left[50(\pi)(0.15)^2(0.05)\right](0.15)^2$$

$$= 1.5987\ \text{kg}\cdot\text{m}^2$$

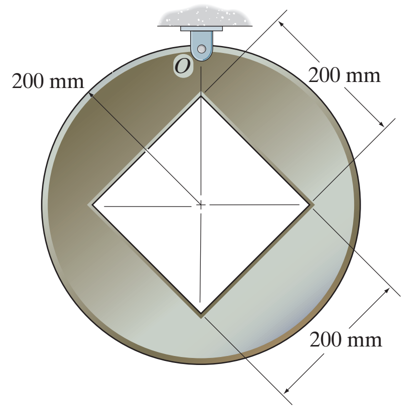
$$I_O = I_G + md^2$$

$$m = 50(1.4)(1.4)(0.05) - 50(\pi)(0.15)^2(0.05) = 4.7233\ \text{kg}$$

$$I_O = 1.5987 + 4.7233(1.4\sin 45^\circ)^2 = 6.23\ \text{kg}\cdot\text{m}^2$$ **Ans.**

***17–16.**

Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \text{kg/m}^2$.



SOLUTION

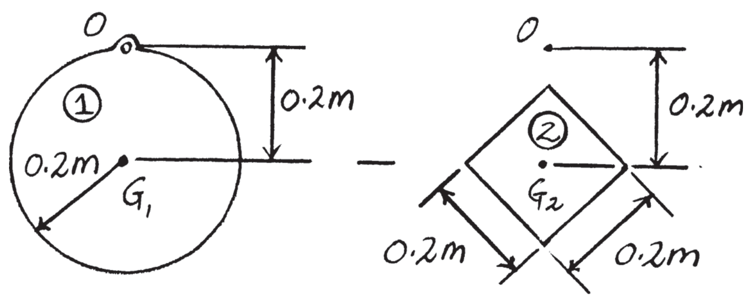
Composite Parts: The plate can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered as a negative part. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Mass Moment of Inertia: The moment of inertia of segments (1) and (2) are computed as $m_1 = \pi(0.2^2)(20) = 0.8\pi$ kg and $m_2 = (0.2)(0.2)(20) = 0.8$ kg. The moment of inertia of the plate about an axis perpendicular to the page and passing through point O for each segment can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

$$= \left[\frac{1}{2}(0.8\pi)(0.2^2) + 0.8\pi(0.2^2)\right] - \left[\frac{1}{12}(0.8)(0.2^2 + 0.2^2) + 0.8(0.2^2)\right]$$

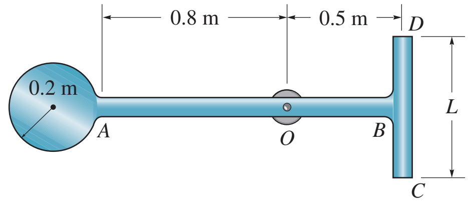
$$= 0.113\ \text{kg}\cdot\text{m}^2$$ **Ans.**



(a)

17–17.

The assembly consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass of 2 kg/m. Determine the length L of DC so that the center of mass is at the bearing O. What is the moment of inertia of the assembly about an axis perpendicular to the page and passing through O?



SOLUTION

Measured from the right side,

$$y = \frac{6(1.5) + 2(1.3)(0.65)}{6 + 1.3(2) + L(2)} = 0.5$$

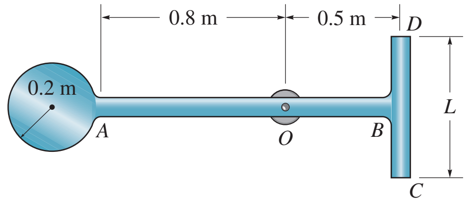
$L = 6.39$ m **Ans.**

$$I_O = \frac{1}{2}(6)(0.2)^2 + 6(1)^2 + \frac{1}{12}(2)(1.3)(1.3)^2 + 2(1.3)(0.15)^2 + \frac{1}{12}(2)(6.39)(6.39)^2 + 2(6.39)(0.5)^2$$

$I_O = 53.2 \text{ kg} \cdot \text{m}^2$ **Ans.**

17–18.

The assembly consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass of 2 kg/m. If $L = 0.75$ m, determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through O.



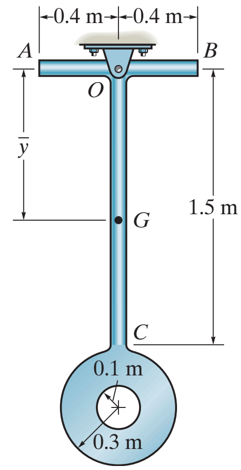
SOLUTION

$$I_O = \frac{1}{2}(6)(0.2)^2 + 6(1)^2 + \frac{1}{12}(2)(1.3)(1.3)^2 + 2(1.3)(0.15)^2 + \frac{1}{12}(2)(0.75)(0.75)^2 + 2(0.75)(0.5)^2$$

$I_O = 6.99\ \text{kg}\cdot\text{m}^2$ **Ans.**

17–19.

The pendulum consists of two slender rods AB and OC which have a mass of 3 kg/m. The thin plate has a mass of 12 kg/m^2. Determine the location $\bar{y}$ of the center of mass G of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



SOLUTION

$$\bar{y} = \frac{1.5(3)(0.75) + \pi(0.3)^2(12)(1.8) - \pi(0.1)^2(12)(1.8)}{1.5(3) + \pi(0.3)^2(12) - \pi(0.1)^2(12) + 0.8(3)}$$

$$= 0.8878 \text{ m} = 0.888 \text{ m} \qquad \textbf{Ans.}$$

$$I_G = \frac{1}{12}(0.8)(3)(0.8)^2 + 0.8(3)(0.8878)^2$$

$$+\frac{1}{12}(1.5)(3)(1.5)^2 + 1.5(3)(0.75 - 0.8878)^2$$

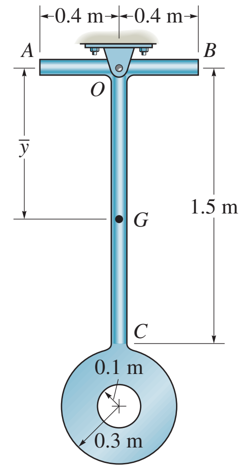
$$+\frac{1}{2}[\pi(0.3)^2(12)(0.3)^2 + [\pi(0.3)^2(12)](1.8 - 0.8878)^2$$

$$-\frac{1}{2}[\pi(0.1)^2(12)(0.1)^2 - [\pi(0.1)^2(12)](1.8 - 0.8878)^2$$

$$I_G = 5.61 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

***17–20.**

The pendulum consists of two slender rods AB and OC which have a mass of 3 kg/m. The thin plate has a mass of 12 kg/m^2. Determine the moment of inertia of the pendulum about an axis perpendicular to the page and passing through the pin at O.



SOLUTION

$$I_o = \frac{1}{12}[3(0.8)](0.8)^2 + \frac{1}{3}[3(1.5)](1.5)^2 + \frac{1}{2}[12(\pi)(0.3)^2](0.3)^2$$

$$+ [12(\pi)(0.3)^2](1.8)^2 - \frac{1}{2}[12(\pi)(0.1)^2](0.1)^2 - [12(\pi)(0.1)^2](1.8)^2$$

$$= 13.43 = 13.4 \text{ kg} \cdot \text{m}^2$$ **Ans.**

Also, from the solution to Prob. 17–16,

$$m = 3(0.8 + 1.5) + 12[\pi(0.3)^2 - \pi(0.1)^2] = 9.916 \text{ kg}$$

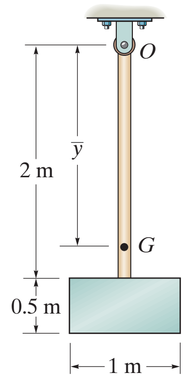
$$I_o = I_G + m\,d^2$$

$$= 5.61 + 9.916(0.8878)^2$$

$$= 13.4 \text{ kg} \cdot \text{m}^2$$ **Ans.**

17–21.

The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



SOLUTION

$$\bar{y} = \frac{\Sigma\, \bar{y}m}{\Sigma\, m} = \frac{1(3) + 2.25(5)}{3 + 5} = 1.781 \text{ m} = 1.78 \text{ m} \qquad \textbf{Ans.}$$

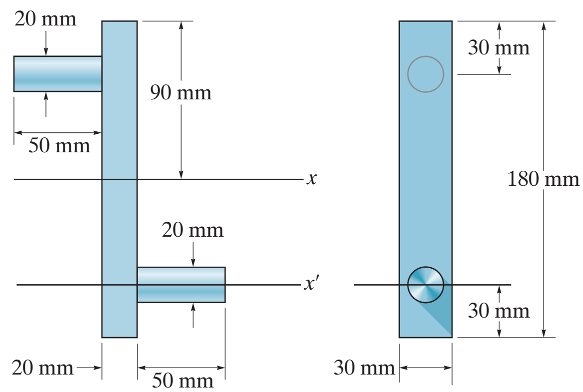
$$I_G = \Sigma \bar{I}_G + md^2$$

$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

$$= 4.45 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

17–22.

Determine the moment of inertia of the overhung crank about the x axis. The material is steel having a destiny of $\rho = 7.85\ \text{Mg/m}^3$.



SOLUTION

$$m_c = 7.85(10^3)\big((0.05)\pi(0.01)^2\big) = 0.1233\ \text{kg}$$

$$m_\rho = 7.85(10^3)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_x = 2\left[\frac{1}{2}(0.1233)(0.01)^2 + (0.1233)(0.06)^2\right]$$

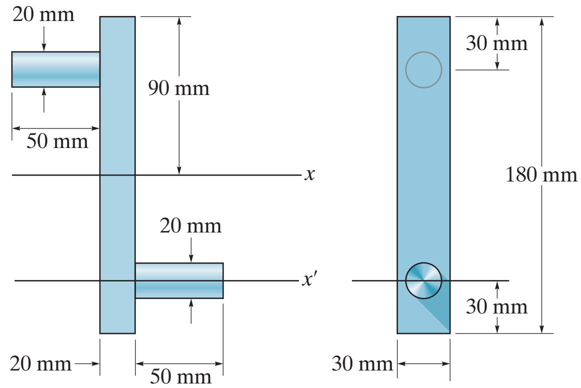
$$+ \left[\frac{1}{12}(0.8478)\big((0.03)^2 + (0.180)^2\big)\right]$$

$$= 0.00325\ \text{kg}\cdot\text{m}^2 = 3.25\ \text{g}\cdot\text{m}^2$$

Ans.

17–23.

Determine the moment of inertia of the overhung crank about the x' axis. The material is steel having a destiny of $\rho = 7.85\ \mathrm{Mg/m^3}$.



SOLUTION

$$m_c = 7.85\left(10^3\right)\left((0.05)\pi(0.01)^2\right) = 0.1233\ \mathrm{kg}$$

$$m_p = 7.85\left(10^3\right)\left((0.03)(0.180)(0.02)\right) = 0.8478\ \mathrm{kg}$$

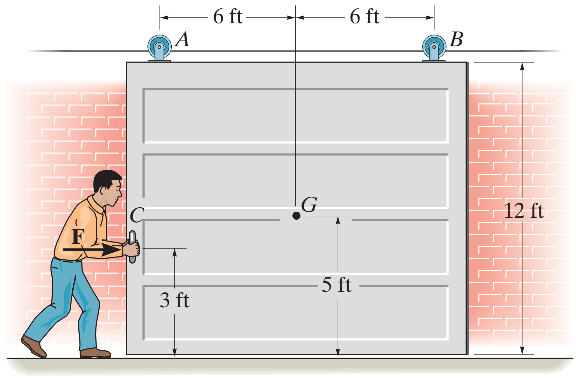
$$I_x = \left[\frac{1}{2}(0.1233)(0.01)^2\right] + \left[\frac{1}{2}(0.1233)(0.02)^2 + (0.1233)(0.120)^2\right]$$

$$+ \left[\frac{1}{12}(0.8478)\left((0.03)^2 + (0.180)^2\right) + (0.8478)(0.06)^2\right]$$

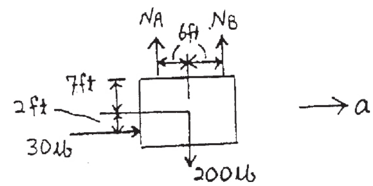
$$= 0.00719\ \mathrm{kg \cdot m^2} = 7.19\ \mathrm{g \cdot m^2}$$ **Ans.**

***17–24.**

The door has a weight of 200 lb and a center of gravity at G. Determine how far the door moves in 2 s, starting from rest, if a man pushes on it at C with a horizontal force $F = 30$ lb. Also, find the vertical reactions at the rollers A and B.



SOLUTION



$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad 30 = \left(\frac{200}{32.2}\right)a_G$$

$$a_G = 4.83 \text{ ft/s}^2$$

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad N_B(12) - 200(6) + 30(9) = \left(\frac{200}{32.2}\right)(4.83)(7)$$

$$N_B = 95.0 \text{ lb}$$ **Ans.**

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + 95.0 - 200 = 0$$

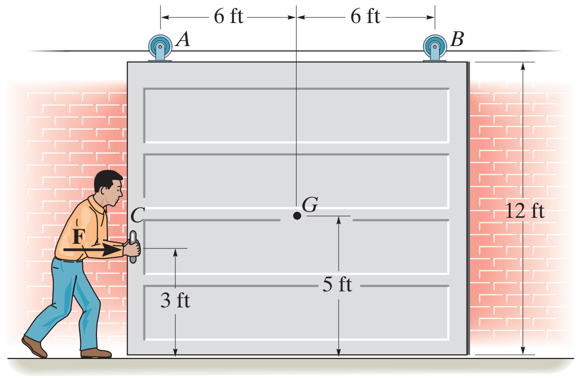
$$N_A = 105 \text{ lb}$$ **Ans.**

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_G t^2$$

$$s = 0 + 0 + \frac{1}{2}(4.83)(2)^2 = 9.66 \text{ ft}$$ **Ans.**

17–25.

The door has a weight of 200 lb and a center of gravity at G. Determine the constant force F that must be applied to the door to push it open 12 ft to the right in 5 s, starting from rest. Also, find the vertical reactions at the rollers A and B.

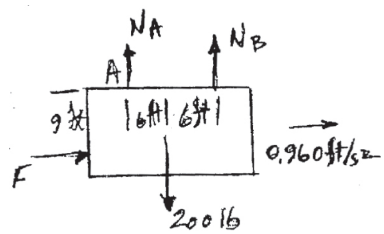


SOLUTION

$$(\xrightarrow{+})\, s = s_0 + v_0 t + \frac{1}{2} a_G t^2$$

$$12 = 0 + 0 + \frac{1}{2} a_G (5)^2$$

$$a_c = 0.960 \text{ ft/s}^2$$



$\xrightarrow{+} \Sigma F_x = m(a_G)_x\,;$ $\qquad F = \dfrac{200}{32.2}(0.960)$

$F = 5.9627 \text{ lb} = 5.96 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A\,;$ $\qquad N_B(12) - 200(6) + 5.9627(9) = \dfrac{200}{32.2}(0.960)(7)$

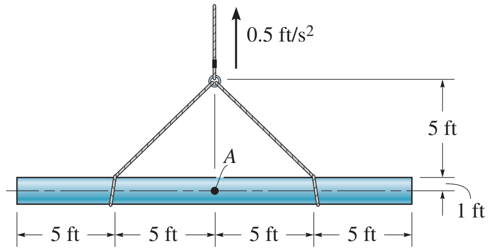
$N_B = 99.0 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y\,;$ $\qquad N_A + 99.0 - 200 = 0$

$N_A = 101 \text{ lb}$ **Ans.**

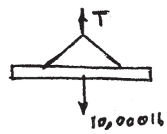
17–26.

The uniform pipe has a weight of 500 lb/ft and diameter of 2 ft. If it is hoisted as shown with an acceleration of 0.5 ft/s^2, determine the internal moment at the center A of the pipe due to the lift.



SOLUTION

Pipe:



$$+\uparrow \Sigma F_y = m\,a_y; \qquad T - 10\,000 = \frac{10\,000}{32.2}(0.5)$$

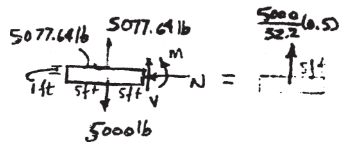
$$T = 10\,155.27 \text{ lb}$$

Cables:

$$+\uparrow \Sigma F_y = 0; \qquad 10\,155.27 - 2P\cos 45^\circ = 0$$

$$P = 7\,180.867 \text{ lb}$$

$$P_x = P_y = 7\,180.867\left(\frac{1}{\sqrt{2}}\right) = 5\,077.64 \text{ lb}$$

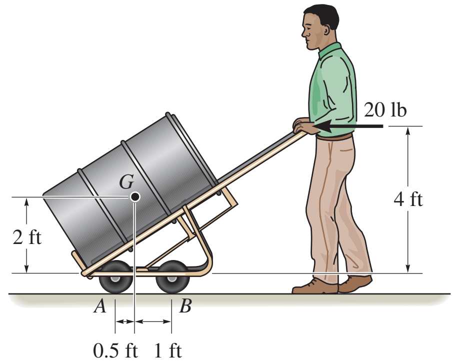


$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad M_A + 5000(5) - 5077.64(5) - 5077.64(1) = \frac{-5000}{32.2}(0.5)(5)$$

$$M_A = 5077.6 \text{ lb} \cdot \text{ft} = 5.08(10^3) \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

17–27.

The drum truck supports the 600-lb drum that has a center of gravity at G. If the operator pushes it forward with a horizontal force of 20 lb, determine the acceleration of the truck and the normal reactions at each of the four wheels. Neglect the mass of the wheels.

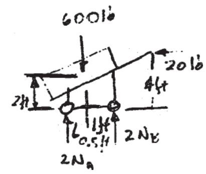


SOLUTION

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 20 = \left(\frac{600}{32.2}\right) a_G$

$a_G = 1.0733 \text{ ft/s}^2 = 1.07 \text{ ft/s}^2$ **Ans.**

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 20(4) - 600(0.5) + 2N_B(1.5) = \frac{600}{32.2}(1.0733)(2)$



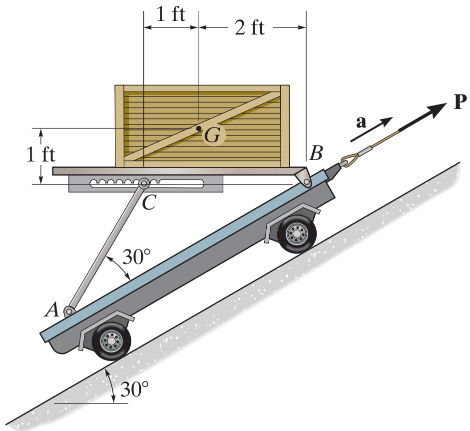
$N_B = 86.7 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2N_A + 2(86.7) - 600 = 0$

$N_A = 213 \text{ lb}$ **Ans.**

***17–28.**

If the cart is given a constant acceleration of $a = 6\ \text{ft/s}^2$ up the inclined plane, determine the force developed in rod AC and the horizontal and vertical components of force at pin B. The crate has a weight of 150 lb with center of gravity at G, and it is secured on the platform, so that it does not slide. Neglect the platform's weight.



SOLUTION

Equations of Motion: $\mathbf{F}_{AC}$ can be obtained directly by writing the moment equation of motion about B,

$$+\Sigma M_B = \Sigma(M_k)_B;$$

$$150(2) - F_{AC}\sin 60°(3) = -\left(\frac{150}{32.2}\right)(6)\cos 30°(1) - \left(\frac{150}{32.2}\right)(6)\sin 30°(2)$$

$$F_{AC} = 135.54\ \text{lb} = 136\ \text{lb} \qquad \textbf{Ans.}$$

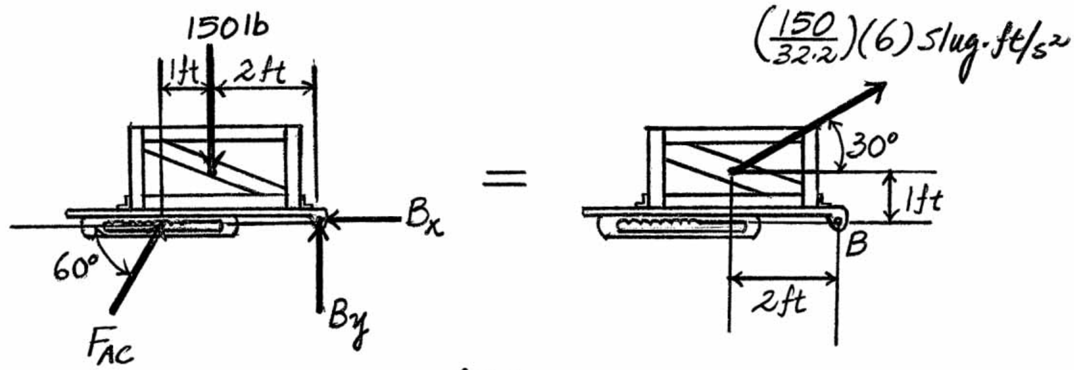
Using this result and writing the force equations of motion along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad 135.54\cos 60° - B_x = \left(\frac{150}{32.2}\right)(6)\cos 30°$$

$$B_x = 43.57\ \text{lb} = 43.6\ \text{lb} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad B_y + 135.54\sin 60° - 150 = \frac{150}{32.2}(6)\sin 30°$$

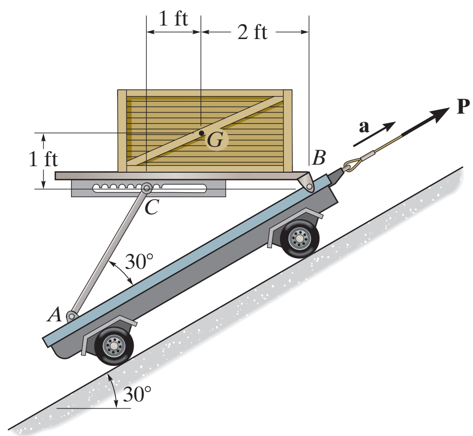
$$B_y = 46.59\ \text{lb} = 46.6\ \text{lb} \qquad \textbf{Ans.}$$



(a)

17–29.

If the strut AC can withstand a maximum compression force of 150 lb before it fails, determine the cart's maximum permissible acceleration. The crate has a weight of 150 lb with center of gravity at G, and it is secured on the platform, so that it does not slide. Neglect the platform's weight.



SOLUTION

Equations of Motion: $\mathbf{F}_{AC}$ in terms of $\mathbf{a}$ can be obtained directly by writing the moment equation of motion about B.

$$+\Sigma M_B = \Sigma (M_k)_B;$$

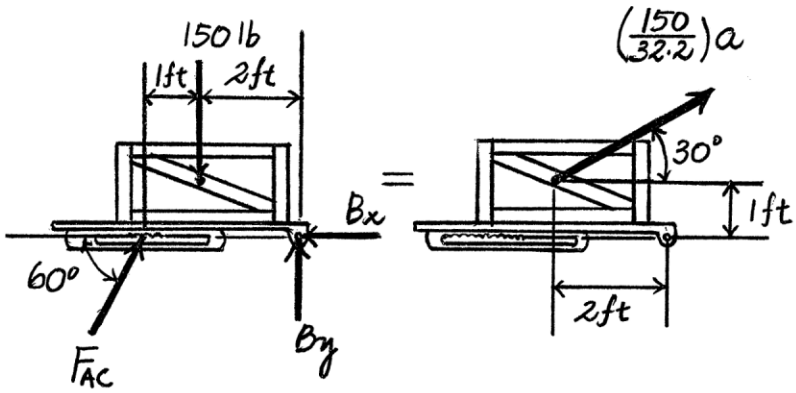
$$150(2) - F_{AC}\sin 60°(3) = -\left(\frac{150}{32.2}\right)a\cos 30°(1) - \left(\frac{150}{32.2}\right)a\sin 30°(2)$$

$$F_{AC} = (3.346a + 115.47)\text{ lb}$$

Assuming AC is about to fail,

$$F_{AC} = 150 = 3.346a + 115.47$$

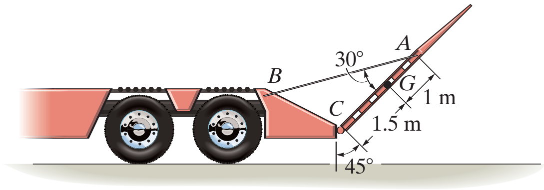
$$a = 10.3\text{ ft/s}^2$$ **Ans.**



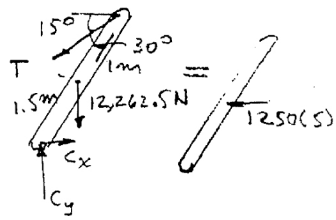
(a)

17–30.

The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at G. If it is supported by the cable AB and hinge at C, determine the tension in the cable when the truck begins to accelerate at 5 m/s^2. Also, what are the horizontal and vertical components of reaction at the hinge C?



SOLUTION



$\circlearrowleft + \Sigma M_C = \Sigma (M_k)_C; \qquad T \sin 30°(2.5) - 12\,262.5(1.5 \cos 45°) = 1250(5)(1.5 \sin 45°)$

$T = 15\,708.4 \text{ N} = 15.7 \text{ kN}$ **Ans.**

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad -C_x + 15\,708.4 \cos 15° = 1250(5)$

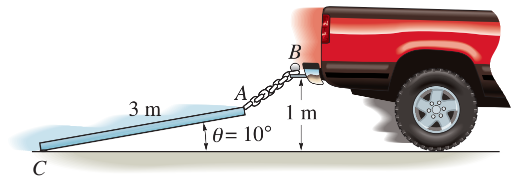
$C_x = 8.92 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad C_y - 12\,262.5 - 15\,708.4 \sin 15° = 0$

$C_y = 16.3 \text{ kN}$ **Ans.**

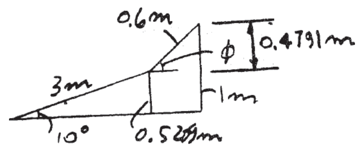
17–31.

The pipe has a length of 3 m and a mass of 500 kg. It is attached to the back of the truck using a 0.6-m-long chain AB. If the coefficient of kinetic friction at C is $\mu_k = 0.4$, determine the acceleration of the truck if the angle $\theta = 10^\circ$ with the road as shown.



SOLUTION

$$\phi = \sin^{-1}\left(\frac{0.4791}{0.6}\right) = 52.98^\circ$$



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;$ $\quad T\cos 52.98^\circ - 0.4N_C = 500a_G$

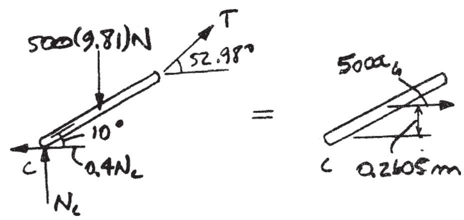
$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_C - 500(9.81) + T\sin 52.98^\circ = 0$

$\circlearrowleft +\Sigma M_C = \Sigma(M_k)_C;$ $\quad -500(9.81)(1.5\cos 10^\circ) + T\sin(52.98^\circ - 10^\circ)(3) = -500a_G(0.2605)$

$T = 3.39\text{ kN}$

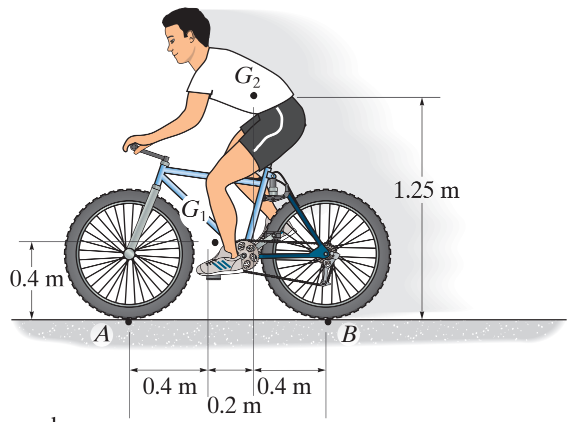
$N_C = 2.19\text{ kN}$

$a_G = 2.33\text{ m/s}^2$ **Ans.**



***17–32.**

The mountain bike has a mass of 40 kg with center of mass at point G_1, while the rider has a mass of 60 kg with center of mass at point G_2. Determine the maximum deceleration when the brake is applied to the front wheel, without causing the rear wheel B to leave the road. Assume that the front wheel does not slip. Neglect the mass of all the wheels.

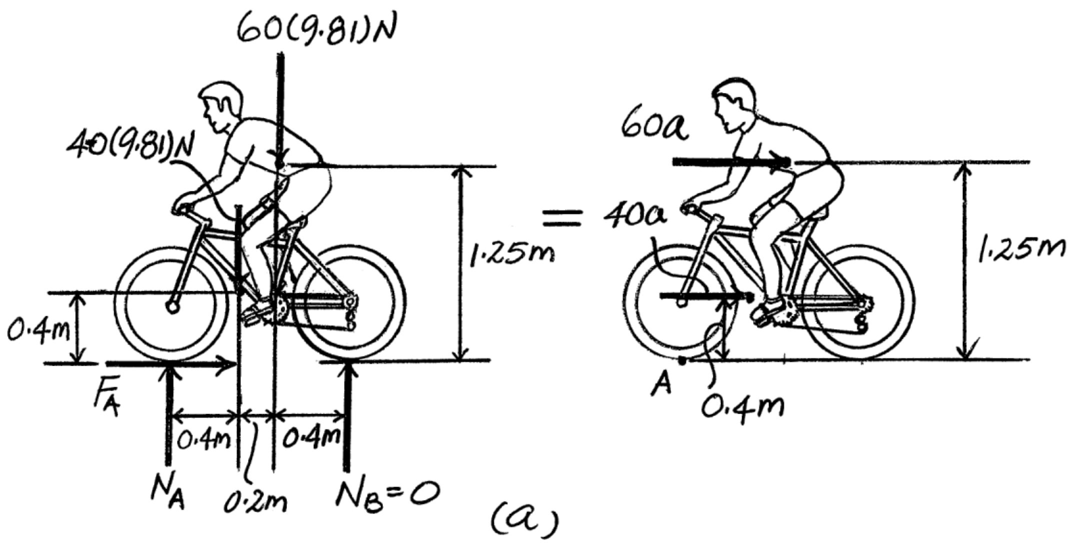


SOLUTION

Equations of Motion: Since the rear wheel B is required to just leave the road, $N_B = 0$. Thus, the acceleration **a** of the bike can be obtained directly by writing the moment equation of motion about point A.

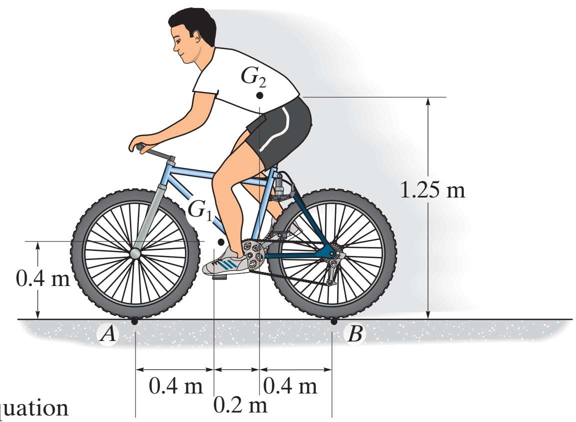
$$+\Sigma M_A = (M_k)_A; \quad -40(9.81)(0.4) - 60(9.81)(0.6) = -40a(0.4) - 60a(1.25)$$

$$a = 5.606 \text{ m/s}^2 = 5.61 \text{ m/s}^2 \qquad \textbf{Ans.}$$



17–33.

The mountain bike has a mass of 40 kg with center of mass at point G_1, while the rider has a mass of 60 kg with center of mass at point G_2. When the brake is applied to the front wheel, it causes the bike to decelerate at a constant rate of 3 m/s^2. Determine the normal reaction the road exerts on the front and rear wheels. Assume that the rear wheel is free to roll. Neglect the mass of all the wheels.



SOLUTION

Equations of Motion: $\mathbf{N}_B$ can be obtained directly by writing the moment equation of motion about point A.

$$+\Sigma M_A = (M_k)_A;$$

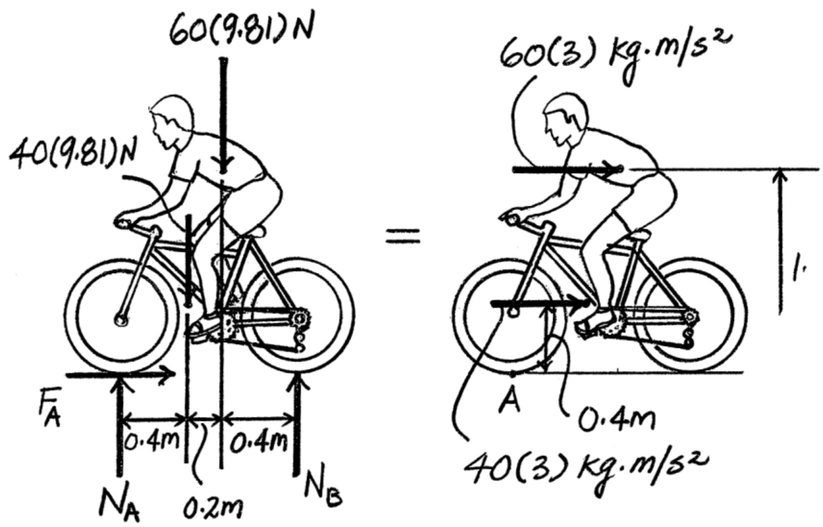
$$N_B(1) - 40(9.81)(0.4) - 60(9.81)(0.6) = -60(3)(1.25) - 40(3)(0.4)$$

$$N_B = 237.12 \text{ N} = 237 \text{ N} \qquad \textbf{Ans.}$$

Using this result and writing the force equations of motion along the y axis,

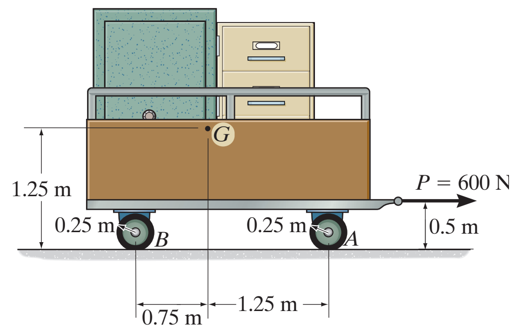
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + 237.12 - 40(9.81) - 60(9.81) = 0$$

$$N_A = 743.88 \text{ N} = 744 \text{ N} \qquad \textbf{Ans.}$$



17–34.

The trailer with its load has a mass of 150 kg and a center of mass at G. If it is subjected to a horizontal force of $P = 600$ N, determine the trailer's acceleration and the normal force on the pair of wheels at A and at B. The wheels are free to roll and have negligible mass.



SOLUTION

Equations of Motion: Writing the force equation of motion along the x axis,

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 600 = 150a \qquad a = 4 \text{ m/s}^2 \rightarrow \qquad \textbf{Ans.}$$

Using this result to write the moment equation about point A,

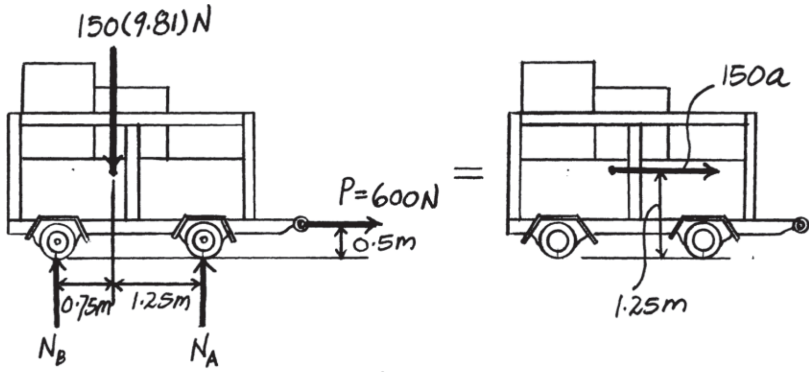
$$\circlearrowleft +\Sigma M_A = (M_k)_A; \qquad 150(9.81)(1.25) - 600(0.5) - N_B(2) = -150(4)(1.25)$$

$$N_B = 1144.69 \text{ N} = 1.14 \text{ kN} \qquad \textbf{Ans.}$$

Using this result to write the force equation of motion along the y axis,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 1144.69 - 150(9.81) = 150(0)$$

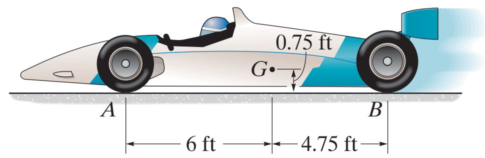
$$N_A = 326.81 \text{ N} = 327 \text{ N} \qquad \textbf{Ans.}$$



(a)

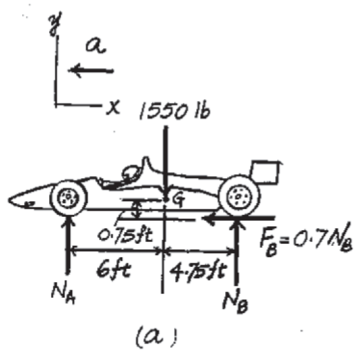
17–35.

At the start of a race, the rear drive wheels B of the 1550-lb car slip on the track. Determine the car's acceleration and the normal reaction the track exerts on the front pair of wheels A and rear pair of wheels B. The coefficient of kinetic friction is μ_k=0.7, and the mass center of the car is at G. The front wheels are free to roll. Neglect the mass of all the wheels.



SOLUTION

Equations of Motion: Since the rear wheels B are required to slip, the frictional force developed is $F_B = \mu_s N_B = 0.7N_B$.



$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad 0.7N_B = \frac{1550}{32.2}a \tag{1}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 1550 = 0 \tag{2}$$

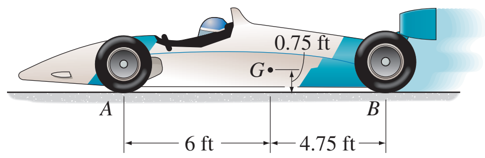
$$\circlearrowleft + \Sigma M_G = 0; \qquad N_B(4.75) - 0.7N_B(0.75) - N_A(6) = 0 \tag{3}$$

Solving Eqs. (1), (2), and (3) yields

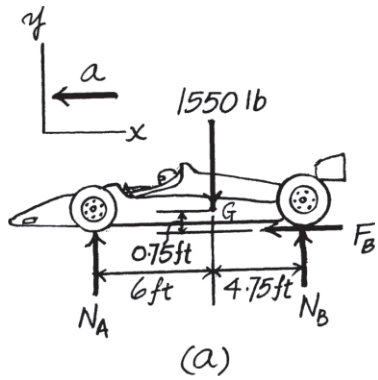
$N_A = 640.46 \text{ lb} = 640 \text{ lb} \qquad N_B = 909.54 \text{ lb} = 910 \text{ lb} \qquad a = 13.2 \text{ ft/s}^2$ **Ans.**

***17–36.**

Determine the maximum acceleration that can be achieved by the car without having the front wheels A leave the track or the rear drive wheels B slip on the track. The coefficient of static friction is $\mu_s = 0.9$. The car's mass center is at G, and the front wheels are free to roll. Neglect the mass of all the wheels.



SOLUTION



Equations of Motion:

$$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x; \qquad F_B = \frac{1550}{32.2}a \tag{1}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \quad N_A + N_B - 1550 = 0 \tag{2}$$

$$\circlearrowleft +\Sigma M_G = 0; \qquad N_B(4.75) - F_B(0.75) - N_A(6) = 0 \tag{3}$$

If we assume that the front wheels are about to leave the track, $N_A = 0$. Substituting this value into Eqs. (2) and (3) and solving Eqs. (1), (2), (3),

$$N_B = 1550 \text{ lb} \qquad F_B = 9816.67 \text{ lb} \qquad a = 203.93 \text{ ft/s}^2$$

Since $F_B > (F_B)_{\max} = \mu_s N_B = 0.9(1550) \text{ lb} = 1395 \text{ lb}$, the rear wheels will slip. Thus, the solution must be reworked so that the rear wheels are about to slip.

$$F_B = \mu_s N_B = 0.9N_B \tag{4}$$

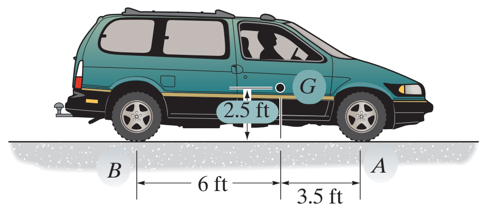
Solving Eqs. (1), (2), (3), and (4) yields

$$N_A = 626.92 \text{ lb} \qquad N_B = 923.08 \text{ lb}$$

$$a = 17.26 \text{ ft/s}^2 = 17.3 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

17–37.

If the 4500-lb van has front-wheel drive, and the coefficient of static friction between the front wheels A and the road is $\mu_s = 0.8$, determine the normal reactions on the pairs of front and rear wheels when the van has maximum acceleration. Also, find this maximum acceleration. The rear wheels are free to roll. Neglect the mass of the wheels.



SOLUTION

Equations of Motion: The maximum acceleration occurs when the front wheels are about to slip. Thus, $F_A = \mu_s N_A = 0.8 N_A$. Referring to the free-body diagram of the van shown in Fig. a, we have

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 0.8N_A = \left(\frac{4500}{32.2}\right) a_{\max} \qquad \textbf{(1)}$$

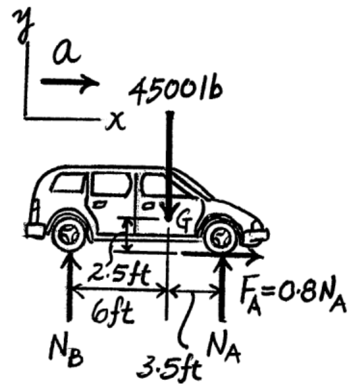
$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 4500 = 0 \qquad \textbf{(2)}$$

$$+\Sigma M_G = 0; \quad N_A(3.5) + 0.8N_A(2.5) - N_B(6) = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

$N_A = 2347.82 \text{ lb} = 2.35 \text{ kip} \qquad N_B = 2152.171 \text{ lb} = 2.15 \text{ kip}$ **Ans.**

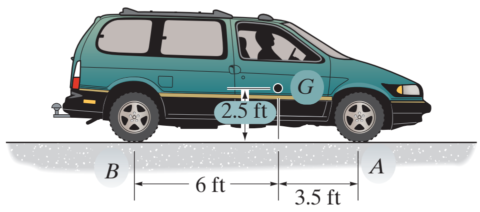
$a_{\max} = 13.44 \text{ ft/s}^2 = 13.4 \text{ ft/s}^2$ **Ans.**



(a)

17–38.

If the 4500-lb van has rear-wheel drive, and the coefficient of static friction between the front wheels B and the road is $\mu_s = 0.8$, determine the normal reactions on the pairs of front and rear wheels when the van has maximum acceleration. The front wheels are free to roll. Neglect the mass of the wheels.



SOLUTION

Equations of Motion: The maximum acceleration occurs when the rear wheels are about to slip. Thus, $F_B = \mu_s N_B = 0.8N_B$. Referring to Fig. a,

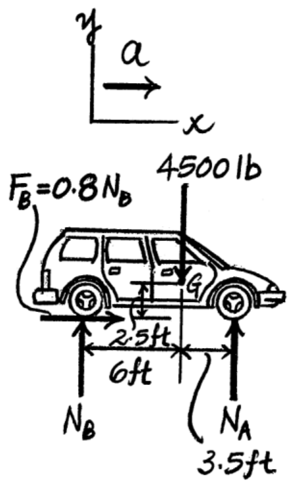
$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \quad 0.8N_B = \left(\frac{4500}{32.2}\right)a_{\max} \tag{1}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \quad N_A + N_B - 4500 = \left(\frac{4500}{32.2}\right)(0) \tag{2}$$

$$+\Sigma M_G = 0; \quad N_A(3.5) + 0.8N_B(2.5) - N_B(6) = 0 \tag{3}$$

Solving Eqs. (1), (2), and (3) yields

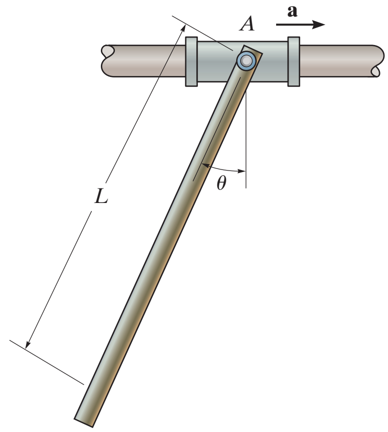
$N_A = 2.40$ kip $\qquad N_B = 2.10$ kip $\qquad a_{\max} = 12.02\ \text{ft/s}^2 = 12.0\ \text{ft/s}^2$ **Ans.**



(a)

17–39.

The uniform bar of mass m is pin connected to the collar, which slides along the smooth horizontal rod. If the collar is given a constant acceleration of $\mathbf{a}$, determine the bar's inclination angle θ. Neglect the collar's mass.

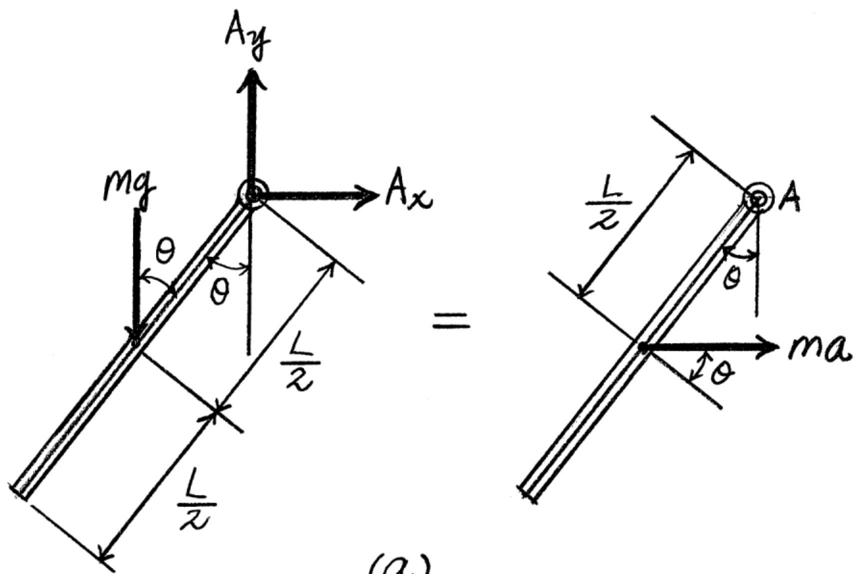


SOLUTION

Equations of Motion: Writing the moment equation of motion about point A,

$$+\Sigma M_A = (M_k)_A; \qquad mg \sin\theta\left(\frac{L}{2}\right) = ma\cos\theta\left(\frac{L}{2}\right)$$

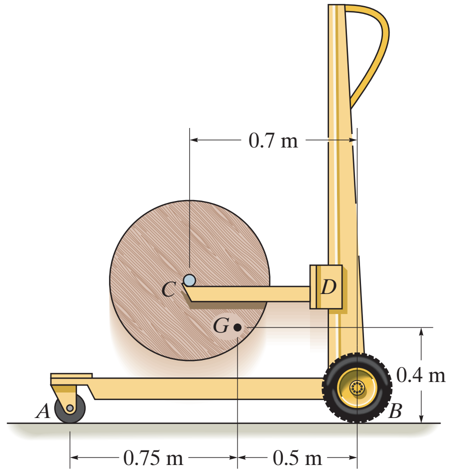
$$\theta = \tan^{-1}\left(\frac{a}{g}\right) \qquad \textbf{Ans.}$$



(a)

***17–40.**

The lift truck has a mass of 70 kg and mass center at G. If it lifts the 120-kg spool with an acceleration of $3\ \text{m/s}^2$, determine the reactions on each of the four wheels. The loading is symmetric. Neglect the mass of the movable arm CD.



SOLUTION

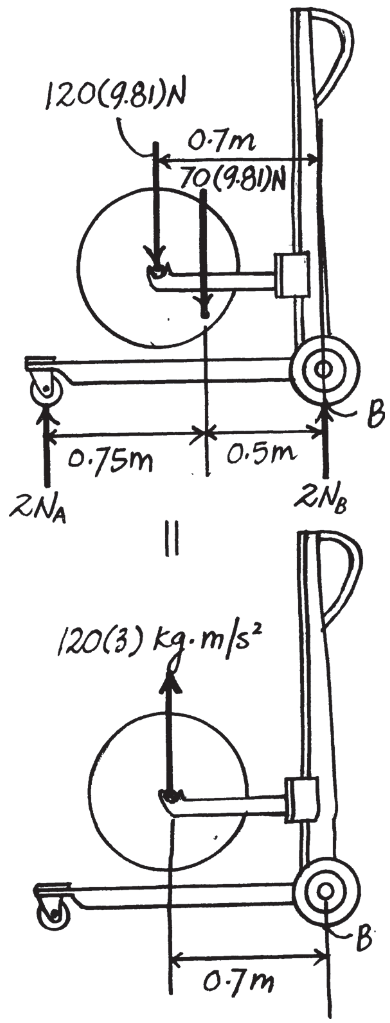
$\zeta + \Sigma M_B = \Sigma (M_k)_B; \qquad 70(9.81)(0.5) + 120(9.81)(0.7) - 2N_A(1.25)$

$= -120(3)(0.7)$

$N_A = 567.76\ \text{N} = 568\ \text{N}$ **Ans.**

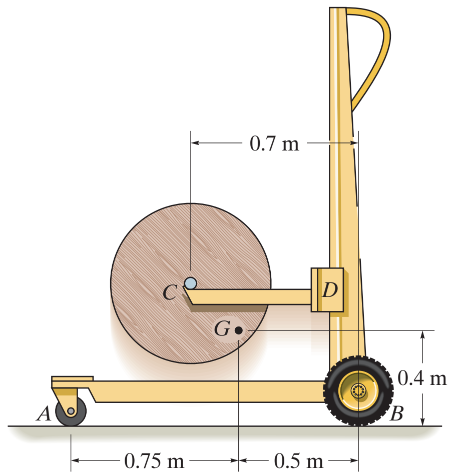
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2(567.76) + 2N_B - 120(9.81) - 70(9.81) = 120(3)$

$N_B = 544\ \text{N}$ **Ans.**



17–41.

The lift truck has a mass of 70 kg and mass center at G. Determine the largest upward acceleration of the 120-kg spool so that no reaction on the wheels exceeds 600 N.



SOLUTION

Assume $N_A = 600$ N.

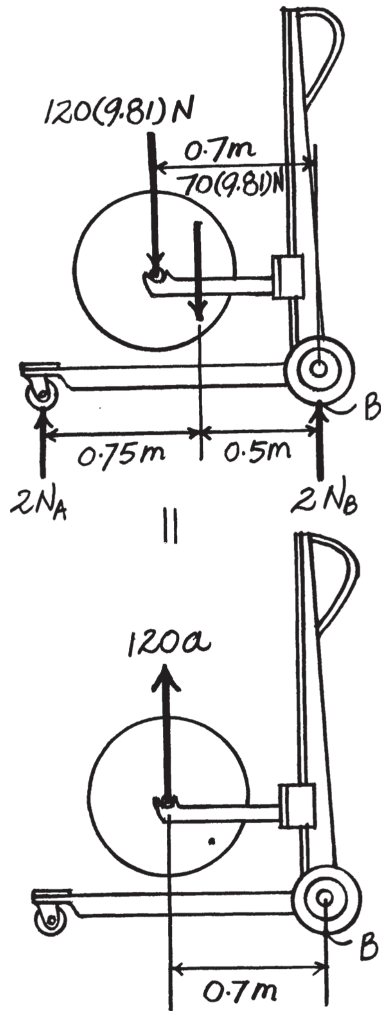
$$\circlearrowleft + \Sigma M_B = \Sigma (M_k)_B; \qquad 70(9.81)(0.5) + 120(9.81)(0.7) - 2(600)(1.25) = -120a(0.7)$$

$$a = 3.960 \text{ m/s}^2$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 2(600) + 2N_B - 120(9.81) - 70(9.81) = 120(3.960)$$

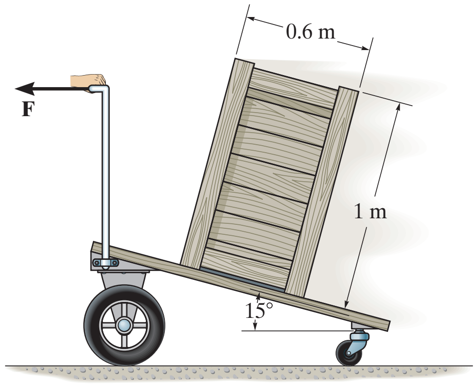
$$N_B = 570 \text{ N} < 600 \text{ N} \qquad \textbf{OK}$$

Thus $a = 3.96$ m/s^2 **Ans.**



17–42.

The uniform crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine the smallest acceleration that will cause the crate either to tip or slip relative to the cart. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.



SOLUTION

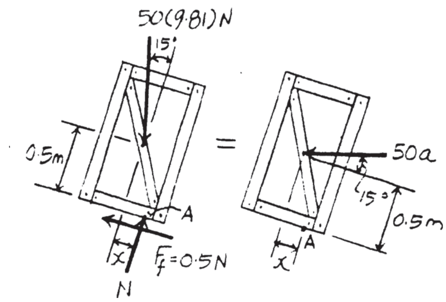
Equations of Motion: Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

$\zeta + \Sigma M_A = \Sigma (M_k)_A; \qquad 50(9.81)\cos 15°(x) - 50(9.81)\sin 15°(0.5)$

$$= 50a\cos 15°(0.5) + 50a\sin 15°(x) \qquad \textbf{(1)}$$

$+\nearrow \Sigma F_{y'} = m(a_G)_{y'}; \qquad N - 50(9.81)\cos 15° = -50a\sin 15° \qquad \textbf{(2)}$

$\searrow + \Sigma F_{x'} = m(a_G)_{x'}; \qquad 50(9.81)\sin 15° - 0.5N = -50a\cos 15° \qquad \textbf{(3)}$



Solving Eqs. (1), (2), and (3) yields

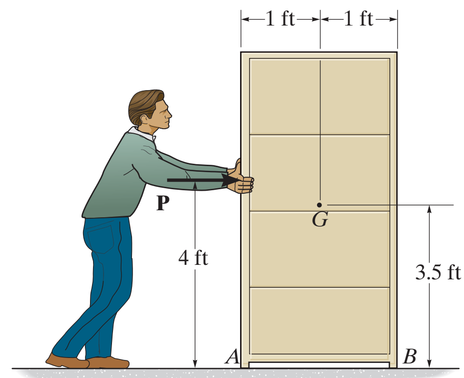
$$N = 447.81\text{ N} \qquad x = 0.250\text{ m}$$

$$a = 2.01\text{ m/s}^2 \qquad \textbf{Ans.}$$

Since $x < 0.3$ m, then crate will not tip. Thus, **the crate slips**. **Ans.**

17–43.

Determine the acceleration of the 150-lb cabinet and the normal reaction under the legs A and B if $P = 35$ lb. The coefficients of static and kinetic friction between the cabinet and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively. The cabinet's center of gravity is located at G.



SOLUTION

Equations of Equilibrium: The free-body diagram of the cabinet under the static condition is shown in Fig. *a*, where **P** is the unknown minimum force needed to move the cabinet. We will assume that the cabinet slides before it tips. Then, $F_A = \mu_s N_A = 0.2N_A$ and $F_B = \mu_s N_B = 0.2N_B$.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad P - 0.2N_A - 0.2N_B = 0 \qquad \textbf{(1)}$$

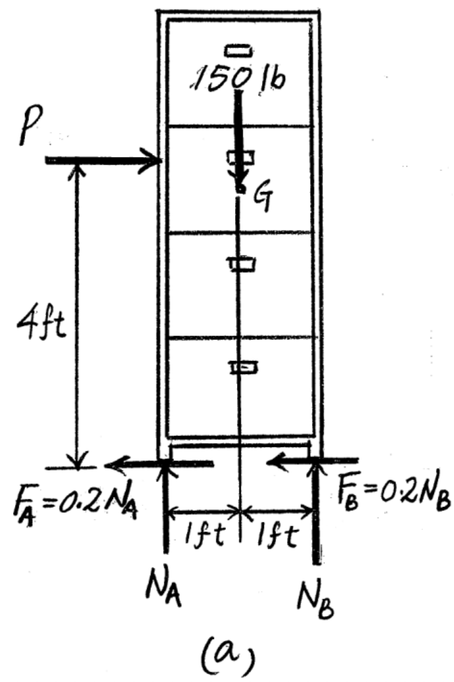
$$+\uparrow\Sigma F_y = 0; \qquad N_A + N_B - 150 = 0 \qquad \textbf{(2)}$$

$$+\Sigma M_A = 0; \qquad N_B(2) - 150(1) - P(4) = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

$$P = 30 \text{ lb} \qquad N_A = 15 \text{ lb} \qquad N_B = 135 \text{ lb}$$

Since $P < 35$ lb and $\mathbf{N}_A$ is positive, the cabinet will slide.



Equations of Motion: Since the cabinet is in motion, $F_A = \mu_k N_A = 0.15N_A$ and $F_B = \mu_k N_B = 0.15N_B$. Referring to the free-body diagram of the cabinet shown in Fig. *b*,

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad 35 - 0.15N_A - 0.15N_B = \left(\frac{150}{32.2}\right)a \qquad \textbf{(4)}$$

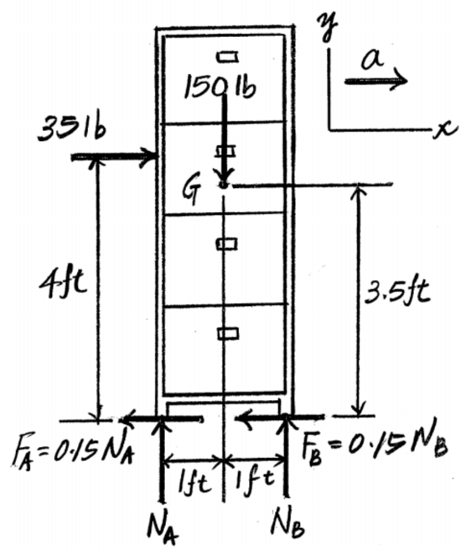
$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad N_A + N_B - 150 = 0 \qquad \textbf{(5)}$$

$$+\Sigma M_G = 0; \quad N_B(1) - 0.15N_B(3.5) - 0.15N_A(3.5) - N_A(1) - 35(0.5) = 0 \quad \textbf{(6)}$$

Solving Eqs. (4), (5), and (6) yields

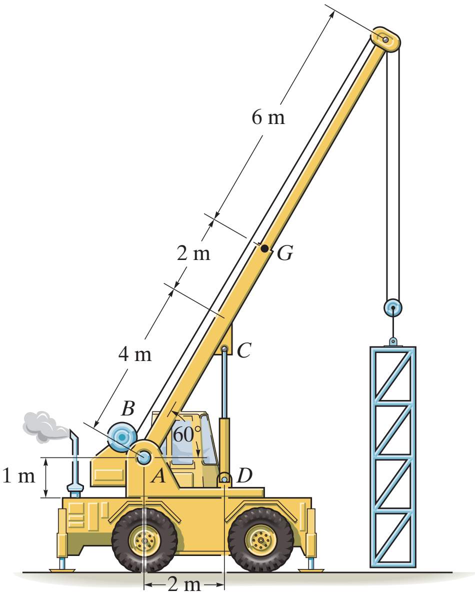
$$a = 2.68 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$N_A = 26.9 \text{ lb} \qquad N_B = 123 \text{ lb} \qquad \textbf{Ans.}$$



***17–44.**

The assembly has a mass of 8 Mg and is hoisted using the boom and pulley system. If the winch at B draws in the cable with an acceleration of $2\ \text{m/s}^2$, determine the compressive force in the hydraulic cylinder needed to support the boom. The boom has a mass of 2 Mg and mass center at G.



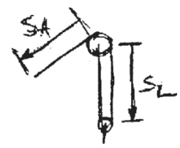
SOLUTION

$$s_B + 2s_L = l$$

$$a_B = -2a_L$$

$$2 = -2a_L$$

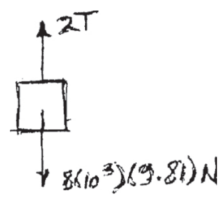
$$a_L = -1\ \text{m/s}^2$$



Assembly:

$$+\uparrow \Sigma F_y = m\,a_y; \qquad 2T - 8(10^3)(9.81) = 8(10^3)(1)$$

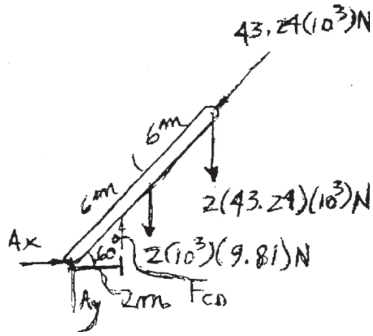
$$T = 43.24\ \text{kN}$$



Boom:

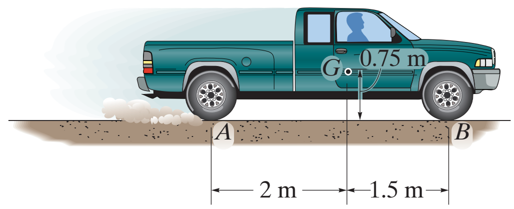
$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CD}(2) - 2(10^3)(9.81)(6\cos 60°) - 2(43.24)(10^3)(12\cos 60°) = 0$$

$$F_{CD} = 289\ \text{kN} \qquad \textbf{Ans.}$$



17–45.

The 2-Mg truck achieves a speed of 15 m/s with a constant acceleration after it has traveled a distance of 100 m, starting from rest. Determine the normal force exerted on each pair of front wheels B and rear driving wheels A. Also, find the traction force on the pair of wheels at A. The front wheels are free to roll. Neglect the mass of the wheels.



SOLUTION

Kinematics: The acceleration of the truck can be determined from

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$15^2 = 0 + 2a(100 - 0)$$

$$a = 1.125 \text{ m/s}^2$$

Equations of Motion: $\mathbf{N}_B$ can be obtained directly by writing the moment equation of motion about point A.

$+\Sigma M_A = (M_k)_A; \qquad N_B(3.5) - 2000(9.81)(2) = -2000(1.125)(0.75)$

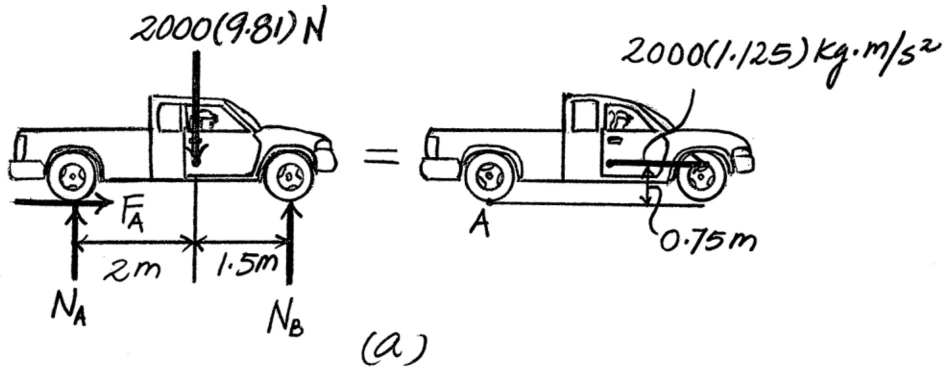
$N_B = 10\,729.29 \text{ N} = 10.7 \text{ kN}$ **Ans.**

Using this result and writing the force equations of motion along the x and y axes,

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_A = 2000(1.125) = 2250 \text{ N} = 2.25 \text{ kN}$ **Ans.**

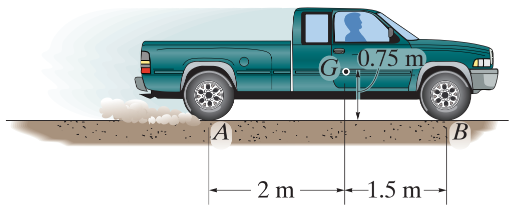
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + 10\,729.29 - 2000(9.81) = 0$

$N_A = 8890.71 \text{ N} = 8.89 \text{ kN}$ **Ans.**



17–46.

Determine the shortest time possible for the rear-wheel drive, 2-Mg truck to achieve a speed of 16 m/s with a constant acceleration starting from rest. The coefficient of static friction between the wheels and the road surface is $\mu_s = 0.8$. The front wheels are free to roll. Neglect the mass of the wheels.



SOLUTION

Equations of Motion: The maximum acceleration of the truck occurs when its rear wheels are on the verge of slipping. Thus, $F_A = \mu_s N_A = 0.8N_A$. Referring to the free-body diagram of the truck shown in Fig. *a*, we can write

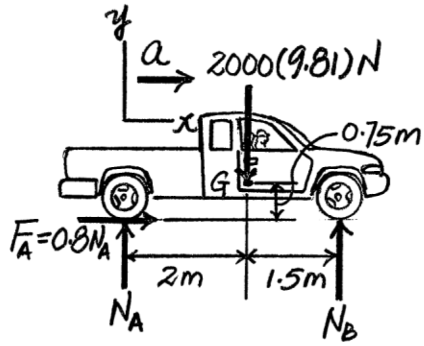
$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 0.8N_A = 2000a \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + N_B - 2000(9.81) = 0 \qquad \textbf{(2)}$$

$$+\Sigma M_G = 0; \quad N_B(1.5) + 0.8N_A(0.75) - N_A(2) = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

$N_A = 10\,148.28$ N $\qquad N_B = 9471.72$ N $\qquad a = 4.059$ m/s^2



(a)

Kinematics: Since the acceleration of the truck is constant, we can apply

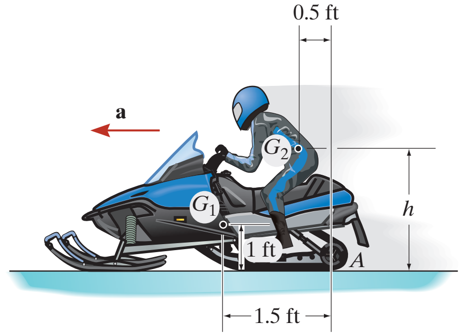
$$(\xrightarrow{+}) \qquad v = v_0 + at$$

$$16 = 0 + 4.059t$$

$$t = 3.94 \text{ s} \qquad \textbf{Ans.}$$

17–47.

The snowmobile has a weight of 250 lb, centered at G_1, while the rider has a weight of 150 lb, centered at G_2. If the acceleration is $a = 20\ \text{ft/s}^2$, determine the maximum height h of G_2 of the rider so that the snowmobile's front skid does not lift off the ground. Also, what are the traction (horizontal) force and normal reaction under the rear tracks at A?



SOLUTION

Equations of Motion: Since the front skid is required to be on the verge of lift off, $N_B = 0$. Writing the moment equation about point A and referring to Fig. a,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 250(1.5) + 150(0.5) = \frac{150}{32.2}(20)(h_{\text{max}}) + \frac{250}{32.2}(20)(1)$$

$$h_{\text{max}} = 3.163\ \text{ft} = 3.16\ \text{ft} \qquad \textbf{Ans.}$$

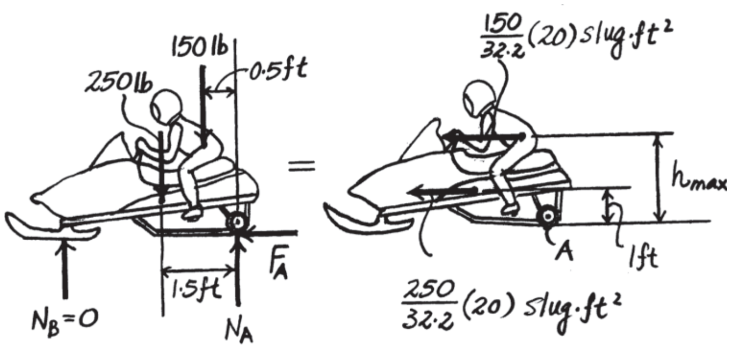
Writing the force equations of motion along the x and y axes,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_A = \frac{150}{32.2}(20) + \frac{250}{32.2}(20)$$

$$F_A = 248.45\ \text{lb} = 248\ \text{lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 250 - 150 = 0$$

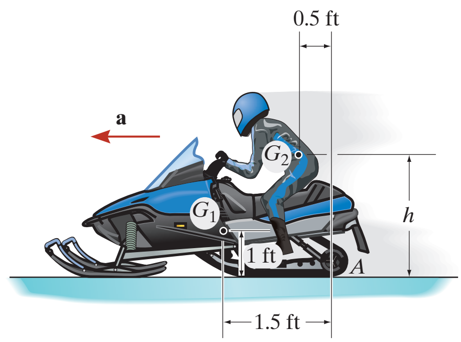
$$N_A = 400\ \text{lb} \qquad \textbf{Ans.}$$



(a)

***17–48.**

The snowmobile has a weight of 250 lb, centered at G_1, while the rider has a weight of 150 lb, centered at G_2. If $h = 3$ ft, determine the snowmobile's maximum permissible acceleration **a** so that its front skid does not lift off the ground. Also, find the traction (horizontal) force and the normal reaction under the rear tracks at A.



SOLUTION

Equations of Motion: Since the front skid is required to be on the verge of lift off, $N_B = 0$. Writing the moment equation about point A and referring to Fig. a,

$$\circlearrowleft + \Sigma M_A = (\mathcal{M}_k)_A; \qquad 250(1.5) + 150(0.5) = \left(\frac{150}{32.2}a_{\max}\right)(3) + \left(\frac{250}{32.2}a_{\max}\right)(1)$$

$$a_{\max} = 20.7 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

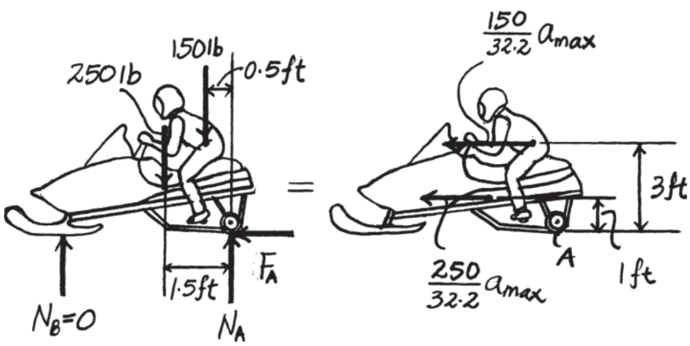
Writing the force equations of motion along the x and y axes and using this result, we have

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_A = \frac{150}{32.2}(20.7) + \frac{250}{32.2}(20.7)$$

$$F_A = 257.14 \text{ lb} = 257 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 150 - 250 = 0$$

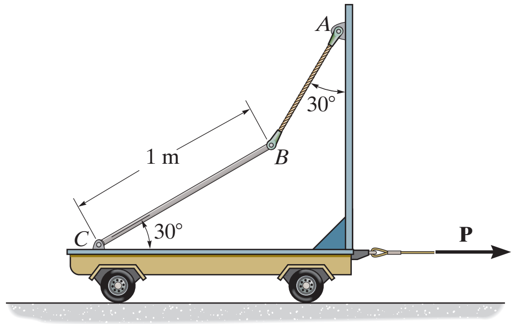
$$N_A = 400 \text{ lb} \qquad \textbf{Ans.}$$



(a)

17–49.

If the cart's mass is 30 kg and it is subjected to a horizontal force of $P = 90$ N, determine the tension in cord AB and the horizontal and vertical components of reaction on end C of the uniform 15-kg rod BC.



SOLUTION

Equations of Motion: The acceleration **a** of the cart and the rod can be determined by considering the free-body diagram of the cart and rod system shown in Fig. *a*.

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 90 = (15 + 30)a \qquad a = 2\ \text{m/s}^2$$

The force in the cord can be obtained directly by writing the moment equation of motion about point C by referring to Fig. *b*.

$$+\Sigma M_C = (M_k)_C; \qquad F_{AB} \sin 30°(1) - 15(9.81)\cos 30°(0.5) = -15(2)\sin 30°(0.5)$$

$$F_{AB} = 112.44\ \text{N} = 112\ \text{N} \qquad \textbf{Ans.}$$

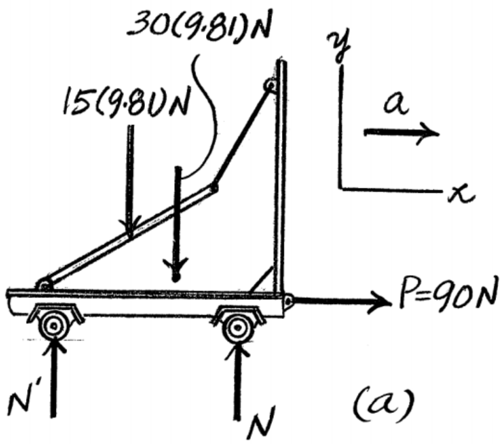
Using this result and applying the force equations of motion along the x and y axes,

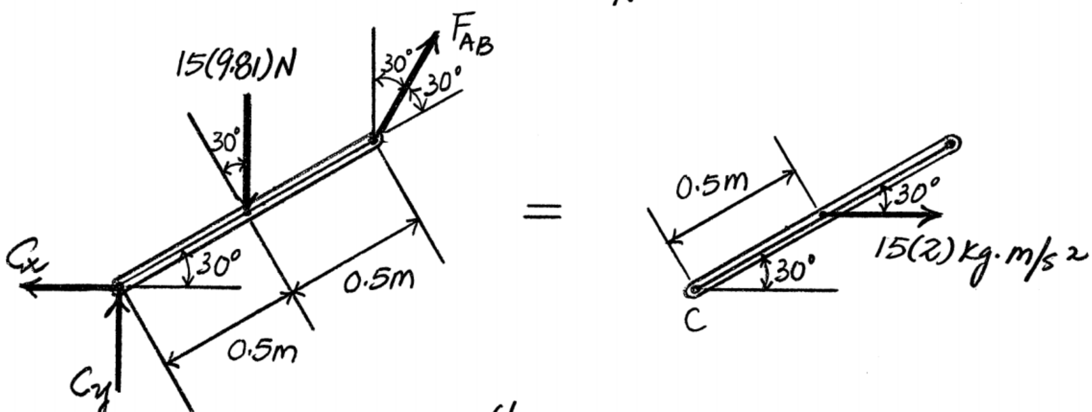
$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad -C_x + 112.44 \sin 30° = 15(2)$$

$$C_x = 26.22\ \text{N} = 26.2\ \text{N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad C_y + 112.44 \cos 30° - 15(9.81) = 0$$

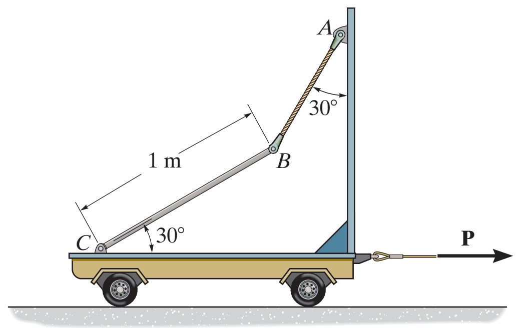
$$C_y = 49.78\ \text{N} = 49.8\ \text{N} \qquad \textbf{Ans.}$$





17–50.

If the cart's mass is 30 kg, determine the horizontal force P that should be applied to the cart so that the cord AB just becomes slack. The uniform rod BC has a mass of 15 kg.



SOLUTION

Equations of Motion: Since cord AB is required to be on the verge of becoming slack, $F_{AB} = 0$. The corresponding acceleration **a** of the rod can be obtained directly by writing the moment equation of motion about point C. By referring to Fig. a.

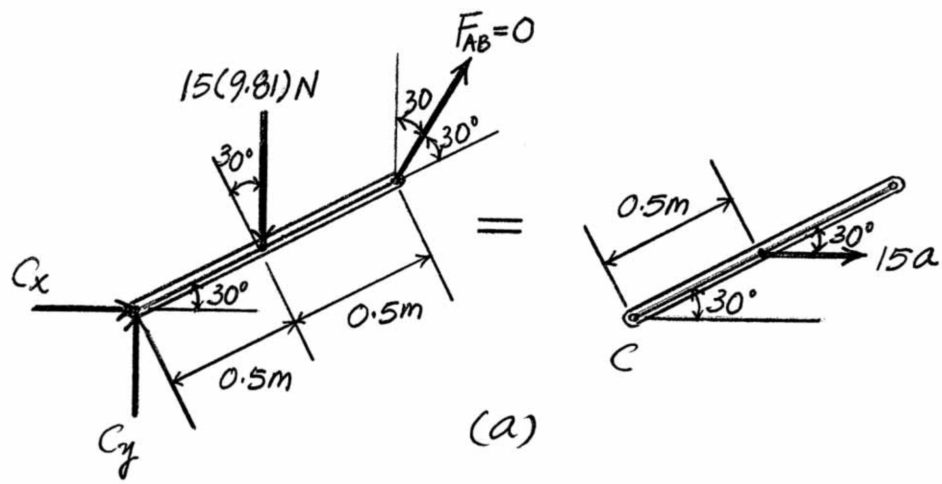
$$+\Sigma M_C = \Sigma(M_C)_A; \qquad -15(9.81)\cos 30°(0.5) = -15a\sin 30°(0.5)$$

$$a = 16.99\ \text{m/s}^2$$

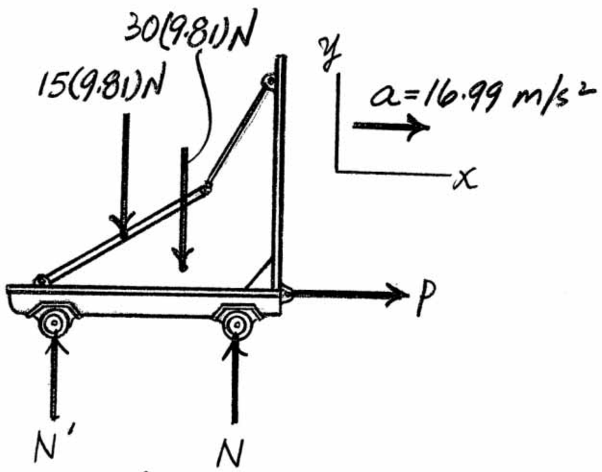
Using this result and writing the force equation of motion along the x axis and referring to the free-body diagram of the cart and rod system shown in Fig. b,

$$(\overset{+}{\rightarrow})\Sigma F_x = m(a_G)_x; \qquad P = (30 + 15)(16.99)$$

$$= 764.61\ \text{N} = 765\ \text{N}$$ **Ans.**



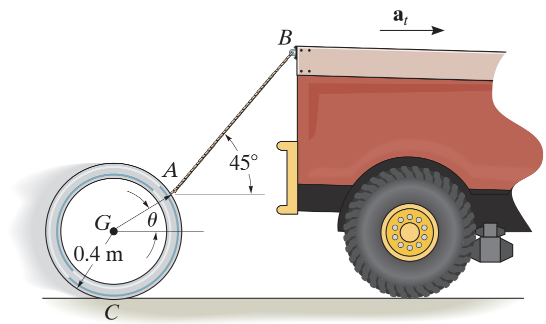
(a)



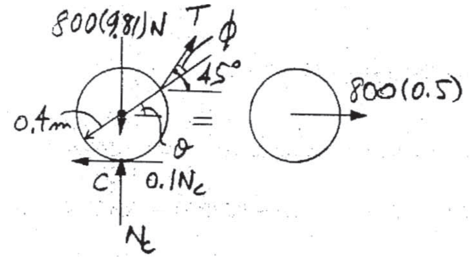
(b)

17–51.

The pipe has a mass of 800 kg and is being towed behind the truck. If the acceleration of the truck is $a_t = 0.5\ \text{m/s}^2$, determine the angle θ and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



SOLUTION



$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -0.1N_C + T\cos 45° = 800(0.5)$

$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 800(9.81) + T\sin 45° = 0$

$\circlearrowleft + \Sigma M_G = 0; \qquad -0.1N_C(0.4) + T\sin\phi(0.4) = 0$

$N_C = 6770.9\ \text{N}$

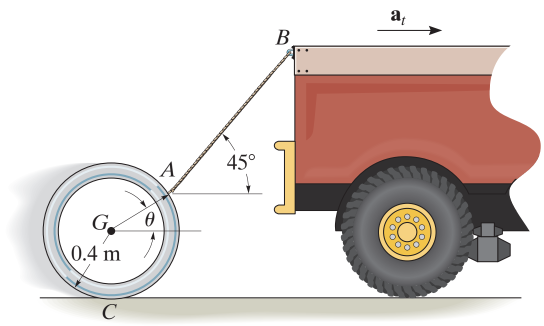
$T = 1523.24\ \text{N} = 1.52\ \text{kN}$ **Ans.**

$\sin\phi = \dfrac{0.1(6770.9)}{1523.24} \qquad \phi = 26.39°$

$\theta = 45° - \phi = 18.6°$ **Ans.**

***17–52.**

The pipe has a mass of 800 kg and is being towed behind a truck. If the angle $\theta = 30°$, determine the acceleration of the truck and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad T \cos 45° - 0.1 N_C = 800a$

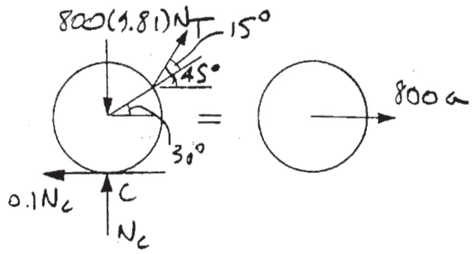
$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 800(9.81) + T \sin 45° = 0$

$\circlearrowleft + \Sigma M_G = 0; \qquad T \sin 15°(0.4) - 0.1 N_C(0.4) = 0$

$N_C = 6161 \text{ N}$

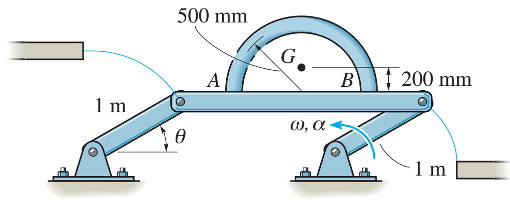
$T = 2382 \text{ N} = 2.38 \text{ kN}$ **Ans.**

$a = 1.33 \text{ m/s}^2$ **Ans.**

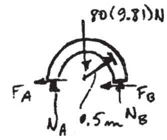


17–53.

The arched pipe has a mass of 80 kg and rests on the surface of the platform. As it is hoisted from one level to the next, $\alpha = 0.25\ \text{rad/s}^2$ and $\omega = 0.5\ \text{rad/s}$ at the instant $\theta = 30°$. If it does not slip, determine the normal reactions of the arch on the platform at this instant.



SOLUTION

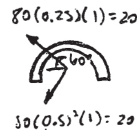


$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + N_B - 80(9.81) = 20\sin 60° - 20\cos 60°$$

$$N_A + N_B = 792.12$$

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad N_B(1) - 80(9.81)(0.5) = 20\cos 60°(0.2) + 20\sin 60°(0.5)$$

$$- 20\cos 60°(0.5) + 20\sin 60°(0.2)$$

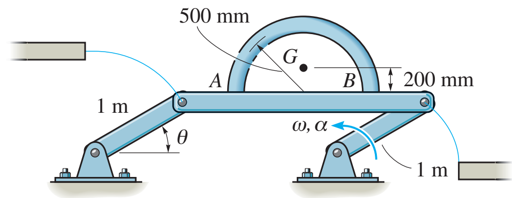


$$N_B = 402\ \text{N} \qquad \textbf{Ans.}$$

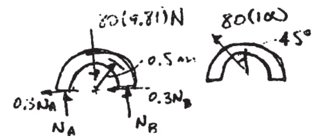
$$N_A = 391\ \text{N} \qquad \textbf{Ans.}$$

17–54.

The arched pipe has a mass of 80 kg and rests on the surface of the platform for which the coefficient of static friction is $\mu_s = 0.3$. Determine the greatest angular acceleration α of the platform, starting from rest when $\theta = 45°$, without causing the pipe to slip on the platform.



SOLUTION



$a_G = (a_G)_t = (1)(\alpha)$

$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A; \qquad N_B(1) - 80(9.81)(0.5) = 80(1\alpha)(\sin 45°)(0.2) + 80(1\alpha)(\cos 45°)(0.5)$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0.3N_A + 0.3N_B = 80(1\alpha)\sin 45°$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + N_B - 80(9.81) = 80(1\alpha)\cos 45°$

Solving,

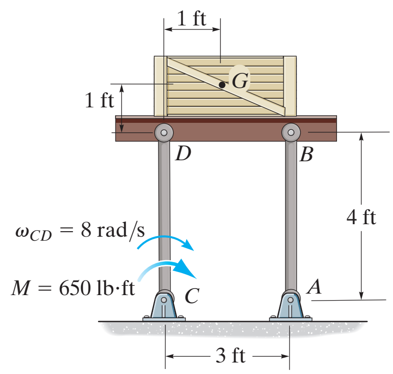
$\alpha = 5.95 \text{ rad/s}^2$ **Ans.**

$N_A = 494 \text{ N}$

$N_B = 628 \text{ N}$

17–55.

At the instant shown, link CD rotates with an angular velocity of $\omega_{CD} = 8$ rad/s. If link CD is subjected to a couple moment of $M = 650$ lb · ft, determine the force developed in link AB and the angular acceleration of the links at this instant. Neglect the weight of the links and the platform. The crate weighs 150 lb and is fully secured on the platform.



SOLUTION

Equilibrium: Since the mass of link CD can be neglected, $\mathbf{D}_t$ can be obtained directly by writing the moment equation of equilibrium about point C using the free-body diagram of link CD, Fig. *a*,

$$\circlearrowleft + \Sigma M_C = 0; \qquad D_t(4) - 650 = 0 \qquad D_t = 162.5 \text{ lb}$$

Equations of Motion: Since the crate undergoes curvilinear translation, $(a_G)_n = \omega^2 r_G = 8^2(4) = 256 \text{ ft/s}^2$ and $(a_G)_t = \alpha r_G = \alpha(4)$. Referring to the free-body diagram of the crate, Fig. *b*, we have

$$\Sigma F_t = m(a_G)_t; \qquad 162.5 = \frac{150}{32.2}\left[\alpha(4)\right] \qquad \alpha = 8.72 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

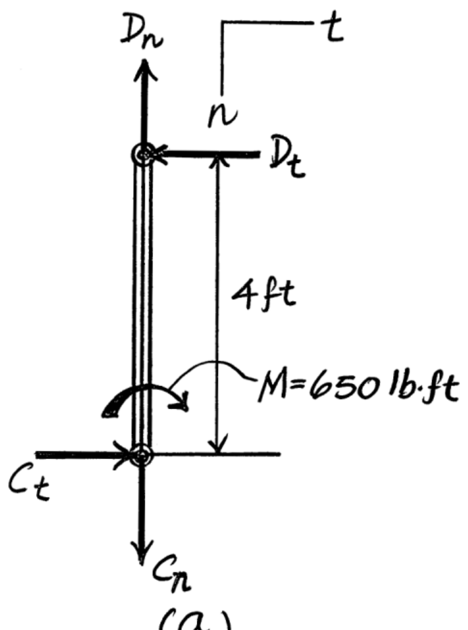
$$\Sigma F_n = m(a_G)_n; \qquad D_n + F_{AB} + 150 = \frac{150}{32.2}(256) \qquad \textbf{(1)}$$

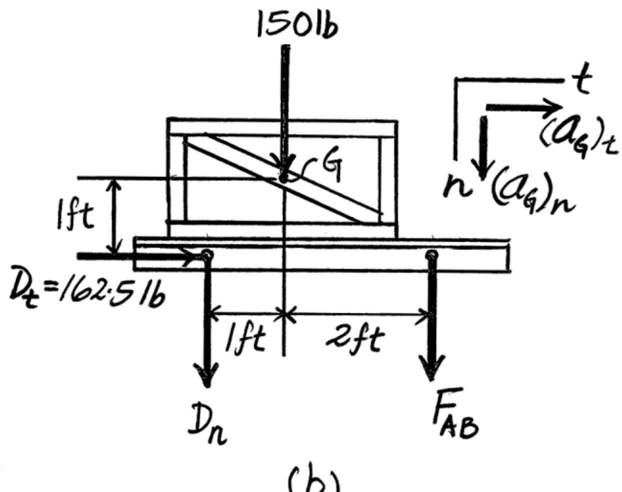
$$\circlearrowleft \Sigma M_G = 0; \qquad D_n(1) - F_{AB}(2) + 162.5(1) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), we obtain

$$F_{AB} = 402 \text{ lb} \qquad \textbf{Ans.}$$

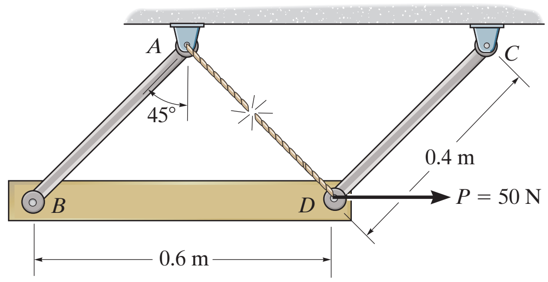
$$D_n = 641 \text{ lb}$$





***17–56.**

Determine the force developed in the links and the acceleration of the bar's mass center immediately after the cord fails. Neglect the mass of links AB and CD. The uniform bar has a mass of 20 kg.



SOLUTION

Equations of Motion: Since the bar is still at rest at the instant the cord fails, $v_G = 0$. Thus, $(a_G)_n = \dfrac{v_G^2}{r} = 0$. Referring to the free-body diagram of the bar, Fig. a,

$$\Sigma F_n = m(a_G)_n; \qquad T_{AB} + T_{CD} - 20(9.81)\cos 45° + 50\cos 45° = 0$$

$$\Sigma F_t = m(a_G)_t; \qquad 20(9.81)\sin 45° + 50\sin 45° = 20(a_G)_t$$

$$+\Sigma M_G = 0; \qquad T_{CD}\cos 45°(0.3) - T_{AB}\cos 45°(0.3) = 0$$

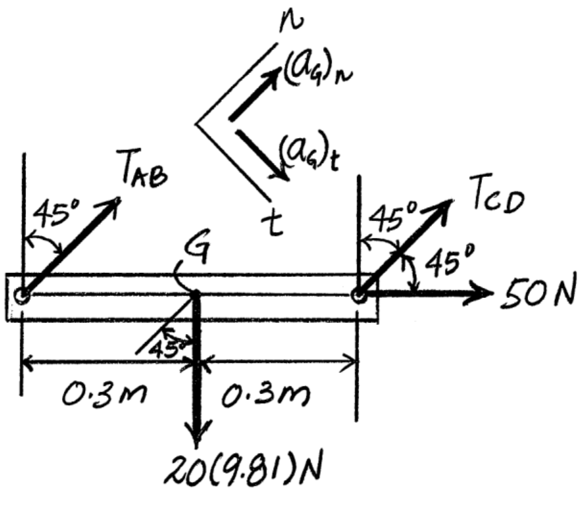
Solving,

$$T_{AB} = T_{CD} = 51.68 \text{ N} = 51.7 \text{ N} \qquad \textbf{Ans.}$$

$$(a_G)_t = 8.704 \text{ m/s}^2$$

Since $(a_G)_n = 0$, then

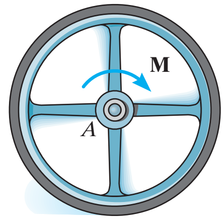
$$a_G = (a_G)_t = 8.70 \text{ m/s}^2 \searrow \qquad \textbf{Ans.}$$



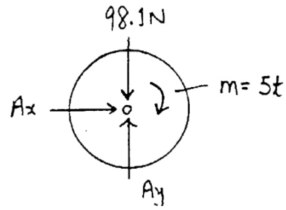
(a)

17–57.

The 10-kg wheel has a radius of gyration $k_A = 200$ mm. If the wheel is subjected to a moment $M = (5t)$ N·m, where t is in seconds, determine its angular velocity when $t = 3$ s starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad A_x = 0$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 10(9.81) = 0$

$\circlearrowright + \Sigma M_A = I_a\,\alpha; \qquad 5t = 10(0.2)^2\alpha$

$$\alpha = \frac{d\omega}{dt} = 12.5t$$

$$\omega = \int_0^3 12.5t\,dt = \frac{12.5}{2}(3)^2$$

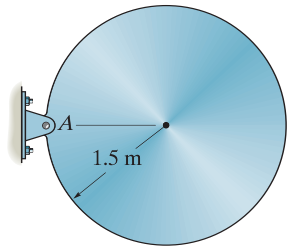
$$\omega = 56.2 \text{ rad/s}$$ **Ans.**

$$A_x = 0$$ **Ans.**

$$A_y = 98.1 \text{ N}$$ **Ans.**

17–58.

The 80-kg disk is supported by a pin at A. If it is released from rest from the position shown, determine the initial horizontal and vertical components of reaction at the pin.



SOLUTION

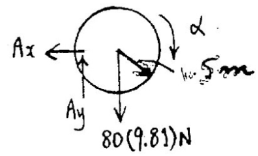
$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 80(9.81) = -80(1.5)(\alpha)$

$\circlearrowright + \Sigma M_A = I_A \alpha; \qquad 80(9.81)(1.5) = [\frac{3}{2}(80)(1.5)^2]\alpha$

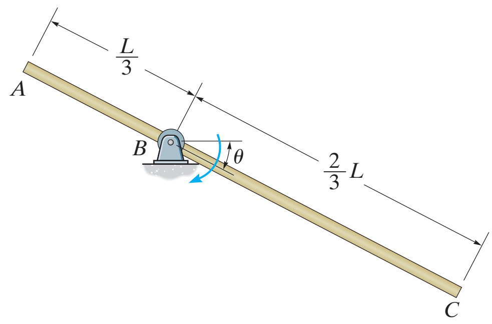
$\alpha = 4.36 \text{ rad/s}^2$

$A_y = 262 \text{ N}$ **Ans.**



17–59.

The uniform slender rod has a mass m. If it is released from rest when $\theta = 0°$, determine the magnitude of the reactive force exerted on it by pin B when $\theta = 90°$.



SOLUTION

Equations of Motion: Since the rod rotates about a fixed axis passing through point B, $(a_G)_t = \alpha r_G = \alpha\left(\frac{L}{6}\right)$ and $(a_G)_n = \omega^2 r_G = \omega^2\left(\frac{L}{6}\right)$. The mass moment of inertia of the rod about its G is $I_G = \frac{1}{12}mL^2$. Writing the moment equation of motion about point B,

$$+\Sigma M_B = \Sigma(M_k)_B; \quad -mg\cos\theta\left(\frac{L}{6}\right) = -m\left[\alpha\left(\frac{L}{6}\right)\right]\left(\frac{L}{6}\right) - \left(\frac{1}{12}mL^2\right)\alpha$$

$$\alpha = \frac{3g}{2L}\cos\theta$$

This equation can also be obtained by applying $\Sigma M_B = I_B\alpha$, where $I_B = \frac{1}{12}mL^2 + m\left(\frac{L}{6}\right)^2 = \frac{1}{9}mL^2$. Thus,

$$+\Sigma M_B = I_B\alpha; \quad -mg\cos\theta\left(\frac{L}{6}\right) = -\left(\frac{1}{9}mL^2\right)\alpha$$

$$\alpha = \frac{3g}{2L}\cos\theta$$

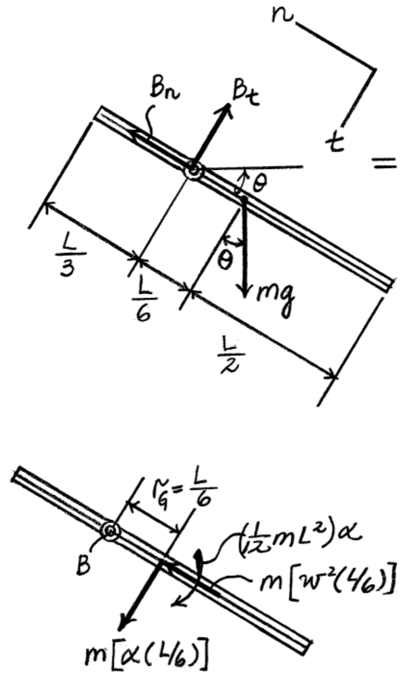
Using this result and writing the force equation of motion along the n and t axes,

$$\Sigma F_t = m(a_G)_t; \quad mg\cos\theta - B_t = m\left[\left(\frac{3g}{2L}\cos\theta\right)\left(\frac{L}{6}\right)\right]$$

$$B_t = \frac{3}{4}mg\cos\theta \quad \textbf{(1)}$$

$$\Sigma F_n = m(a_G)_n; \quad B_n - mg\sin\theta = m\left[\omega^2\left(\frac{L}{6}\right)\right]$$

$$B_n = \frac{1}{6}m\omega^2 L + mg\sin\theta \quad \textbf{(2)}$$



(a)

Kinematics: The angular velocity of the rod can be determined by integrating

$$\int \omega\, d\omega = \int \alpha\, d\theta$$

$$\int_0^\omega \omega\, d\omega = \int_0^\theta \frac{3g}{2L}\cos\theta\, d\theta$$

$$\omega = \sqrt{\frac{3g}{L}\sin\theta}$$

When $\theta = 90°$, $\omega = \sqrt{\frac{3g}{L}}$. Substituting this result and $\theta = 90°$ into Eqs. (1) and (2),

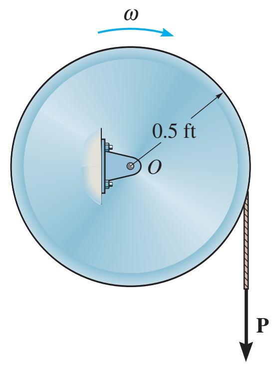
$$B_t = \frac{3}{4}mg\cos 90° = 0$$

$$B_n = \frac{1}{6}m\left(\frac{3g}{L}\right)(L) + mg\sin 90° = \frac{3}{2}mg$$

$$F_A = \sqrt{A_t^2 + A_n^2} = \sqrt{0^2 + \left(\frac{3}{2}mg\right)^2} = \frac{3}{2}mg \quad \textbf{Ans.}$$

***17–60.**

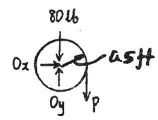
The drum has a weight of 80 lb and a radius of gyration $k_O = 0.4$ ft. If the cable, which is wrapped around the drum, is subjected to a vertical force $P = 15$ lb, determine the time needed to increase the drum's angular velocity from $\omega_1 = 5$ rad/s to $\omega_2 = 25$ rad/s. Neglect the mass of the cable.



SOLUTION

$\circlearrowright + \Sigma M_O = I_O \alpha;$ $\quad 15(0.5) = [\frac{80}{32.2}(0.4)^2]\alpha$ **(1)**

$\alpha = 18.87 \text{ rad/s}^2$



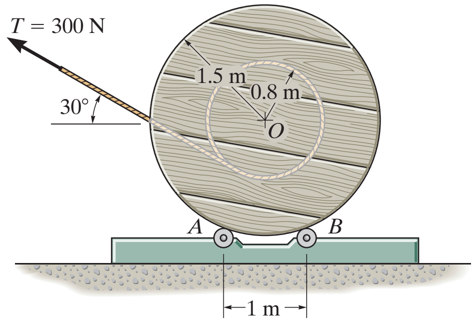
$(\circlearrowright +)\omega = \omega_0 + \alpha t$

$$25 = 5 + 18.87\,t$$

$$t = 1.06 \text{ s}$$ **Ans.**

17–61.

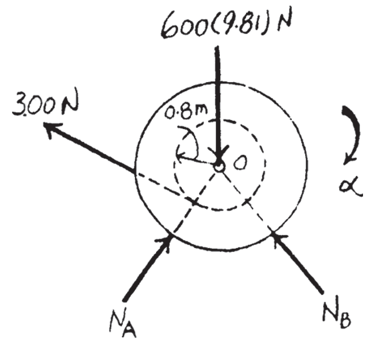
Cable is unwound from a spool supported on small rollers at A and B by exerting a force of $T = 300$ N on the cable in the direction shown. Compute the time needed to unravel 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and a centroidal radius of gyration of $k_O = 1.2$ m. For the calculation, neglect the mass of the cable being unwound and the mass of the rollers at A and B. The rollers turn with no friction.



SOLUTION

Equations of Motion: The mass moment of inertia of the spool about point O is given by $I_O = mk_O^2 = 600\left(1.2^2\right) = 864 \text{ kg} \cdot \text{m}^2$. Applying Eq. 17–16, we have

$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad -300(0.8) = -864\alpha \qquad \alpha = 0.2778 \text{ rad/s}^2$$



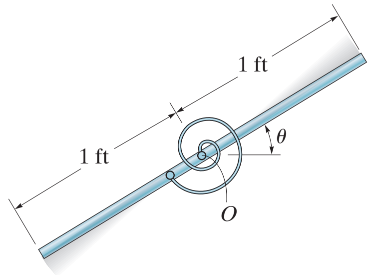
Kinematics: Here, the angular displacement $\theta = \dfrac{s}{r} = \dfrac{5}{0.8} = 6.25$ rad. Applying equation $\theta = \theta_0 + \omega_0\,t + \dfrac{1}{2}\alpha t^2$, we have

$$(\circlearrowright +) \qquad 6.25 = 0 + 0 + \frac{1}{2}(0.2778)t^2$$

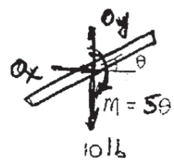
$$t = 6.71 \text{ s} \qquad \textbf{Ans.}$$

17–62.

The 10-lb bar is pinned at its center O and connected to a torsional spring. The spring has a stiffness $k = 5 \text{ lb} \cdot \text{ft/rad}$, so that the torque developed is $M = (5\theta) \text{ lb} \cdot \text{ft}$, where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$.



SOLUTION



$$\circlearrowleft + \Sigma M_O = I_O\alpha; \quad -5\theta = [\frac{1}{12}(\frac{10}{32.2})(2)^2]\alpha$$

$$-48.3\,\theta = \alpha$$

$$\alpha\, d\theta = \omega\, d\omega$$

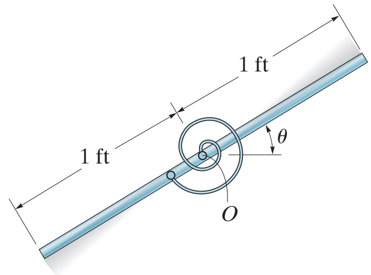
$$-\int_{\frac{\pi}{2}}^{O} 48.3\,\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$$

$$\frac{48.3}{2}(\frac{\pi}{2})^2 = \frac{1}{2}\omega^2$$

$$\omega = 10.9 \text{ rad/s}$$ **Ans.**

17–63.

The 10-lb bar is pinned at its center O and connected to a torsional spring. The spring has a stiffness $k = 5\ \text{lb}\cdot\text{ft/rad}$, so that the torque developed is $M = (5\theta)\ \text{lb}\cdot\text{ft}$, where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 45°$.



SOLUTION

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 5\theta = [\frac{1}{12}(\frac{10}{32.2})(2)^2]\alpha$$

$$\alpha = -48.3\theta$$

$$\alpha\, d\theta = \omega\, d\omega$$

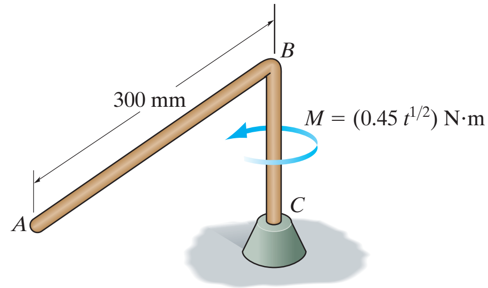
$$-\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 48.3\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$$

$$-24.15\left((\frac{\pi}{4})^2 - (\frac{\pi}{2})^2\right) = \frac{1}{2}\omega^2$$

$$\omega = 9.45\ \text{rad/s}$$ **Ans.**

***17–64.**

If shaft BC is subjected to a torque of $M = (0.45t^{1/2})\,\text{N}\cdot\text{m}$, where t is in seconds, determine the angular velocity of the 3-kg rod AB when $t = 4$ s, starting from rest. Neglect the mass of shaft BC.



SOLUTION

Equations of Motion: The mass moment of inertia of the rod about the z axis is $I_z = I_G + md^2 = \frac{1}{12}(3)\left(0.3^2\right) + 3\left(0.15^2\right) = 0.09\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about the z axis,

$$\Sigma M_z = I_z\alpha; \qquad 0.45t^{1/2} = 0.09\alpha \qquad \alpha = 5t^{1/2}\ \text{rad/s}^2$$

Kinematics: The angular velocity of the rod can be obtained by integration.

$$\int d\omega = \int \alpha\, dt$$

$$\int_0^\omega d\omega = \int_0^t 5t^{1/2}dt$$

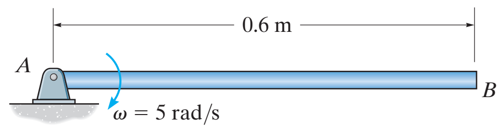
$$\omega = \left(3.333t^{3/2}\right)\text{rad/s}$$

When $t = 4$ s,

$$\omega = 3.333\left(4^{3/2}\right) = 26.7\ \text{rad/s} \qquad \textbf{Ans.}$$

17–65.

Determine the vertical and horizontal components of reaction at the pin support A and the angular acceleration of the 12-kg rod at the instant shown, when the rod has an angular velocity of $\omega = 5$ rad/s.



SOLUTION

Equations of Motion: Since the rod rotates about a fixed axis passing through point A, $(a_G)_t = \alpha r_G = \alpha(0.3)$ and $(a_G)_n = \omega^2 r_G = \left(5^2\right)(0.3) = 7.5\text{ m/s}^2$. The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}mL^2 = \frac{1}{12}(12)\left(0.6^2\right) = 0.36\text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A and referring to Fig. a,

$$+\Sigma M_A = \Sigma(M_k)_A;\quad -12(9.81)(0.3) = -0.36\alpha - 12[\alpha(0.3)](0.3)$$

$$\alpha = 24.525\text{ rad/s}^2 = 24.5\text{ rad/s}^2 \qquad \textbf{Ans.}$$

This result can also be obtained by applying $\Sigma M_A = I_A\alpha$, where $I_A = \frac{1}{3}ml^2 = \frac{1}{3}(12)(0.6^2) = 1.44\text{ kg}\cdot\text{m}^2$. Thus,

$$+\Sigma M_A = I_A\alpha;\qquad -12(9.81)(0.3) = -1.44\alpha$$

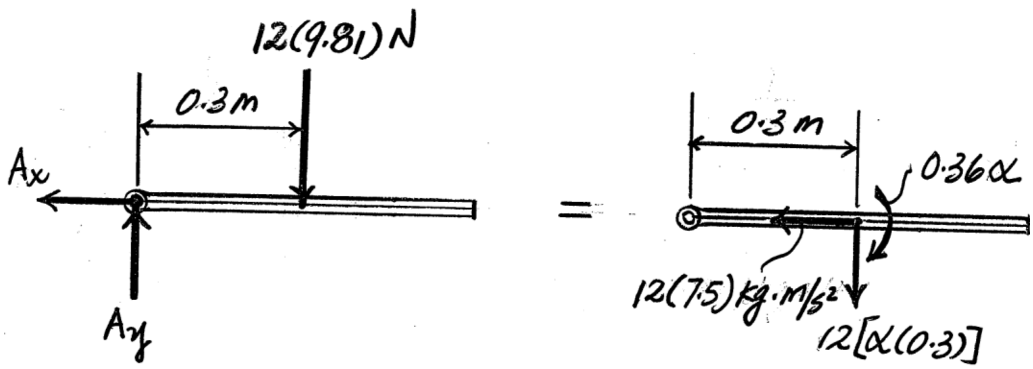
$$\alpha = 24.525\text{ rad/s}^2 = 24.5\text{ rad/s}^2 \qquad \textbf{Ans.}$$

Using this result to write the force equations of motion along the x and y axes, we have

$$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x;\quad A_x = 12(7.5) = 90\text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = m(a_G)_y;\quad A_y = 12(9.81) = -12[24.525(0.3)]$$

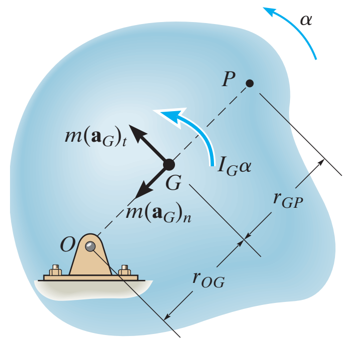
$$A_y = 29.43\text{ N} = 29.4\text{ N} \qquad \textbf{Ans.}$$



(a)

17–66.

The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis passing through O is shown in the figure. Show that $I_G\boldsymbol{\alpha}$ may be eliminated by moving the vectors $m(\mathbf{a}_G)_t$ and $m(\mathbf{a}_G)_n$ to point P, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass G of the body. Here k_G represents the radius of gyration of the body about an axis passing through G. The point P is called the *center of percussion* of the body.



SOLUTION

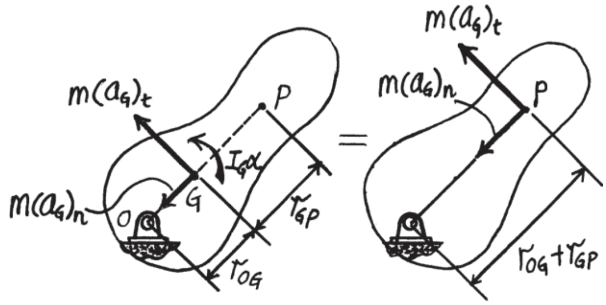
$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + \left(mk_G^2\right)\alpha$$

However,

$$k_G^2 = r_{OG}\, r_{GP} \text{ and } \alpha = \frac{(a_G)_t}{r_{OG}}$$

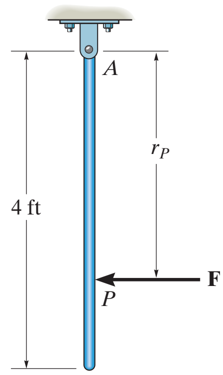
$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + (mr_{OG}\, r_{GP})\left[\frac{(a_G)_t}{r_{OG}}\right]$$

$$= m(a_G)_t(r_{OG} + r_{GP}) \qquad \textbf{Q.E.D.}$$



17–67.

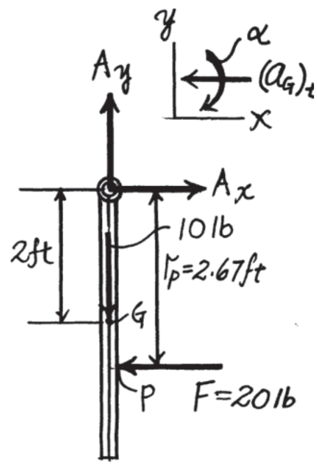
Determine the position r_P of the center of percussion P of the 10-lb slender bar. (See Prob. 17–66.) What is the horizontal component of force that the pin at A exerts on the bar when it is struck at P with a force of $F = 20$ lb?



SOLUTION

Using the result of Prob 17–66,

$$r_{GP} = \frac{k_G^2}{r_{AG}} = \frac{\left[\sqrt{\frac{1}{12}\left(\frac{ml^2}{m}\right)}\right]^2}{\frac{l}{2}} = \frac{1}{6}l$$



Thus,

$$r_P = \frac{1}{6}l + \frac{1}{2}l = \frac{2}{3}l = \frac{2}{3}(4) = 2.67 \text{ ft} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad 20(2.667) = \left[\frac{1}{3}\left(\frac{10}{32.2}\right)(4)^2\right]\alpha$$

$$\alpha = 32.2 \text{ rad/s}^2$$

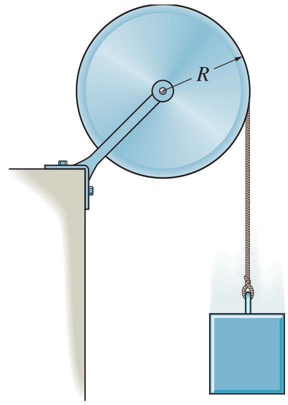
$$(a_G)_t = 2(32.2) = 64.4 \text{ ft/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad -A_x + 20 = \left(\frac{10}{32.2}\right)(64.4)$$

$$A_x = 0 \qquad \textbf{Ans.}$$

***17–68.**

The disk has a mass M and a radius R. If a block of mass m is attached to the cord, determine the angular acceleration of the disk when the block is released from rest. Also,what is the velocity of the block after it falls a distance $2R$ starting from rest?



SOLUTION

$\circlearrowleft +\Sigma M_O = \Sigma(\mathrm{M}_k)_O; \qquad mgR = \frac{1}{2}MR^2(\alpha) + m(\alpha R)R$

$$\alpha = \frac{2mg}{R(M + 2m)} \qquad \textbf{Ans.}$$

$$a = \alpha R$$

$$v^2 = v_0^2 + 2\,a(s - s_0)$$

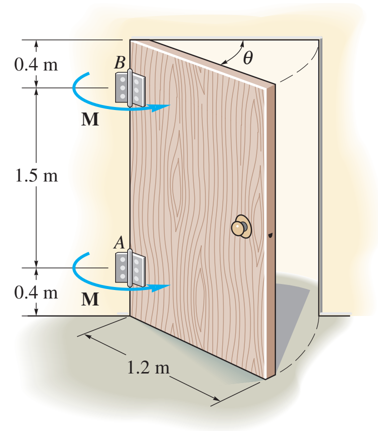
$$v^2 = 0 + 2\left(\frac{2mgR}{R(M + 2m)}\right)(2R - 0)$$

FBD $\neq$ KD

$$v = \sqrt{\frac{8mgR}{(M + 2m)}} \qquad \textbf{Ans.}$$

17–69.

The door will close automatically using torsional springs mounted on the hinges. Each spring has a stiffness $k = 50\ \text{N}\cdot\text{m/rad}$ so that the torque on each hinge is $M = (50\theta)\ \text{N}\cdot\text{m}$, where θ is measured in radians. If the door is released from rest when it is open at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$. For the calculation, treat the door as a thin plate having a mass of 70 kg.



SOLUTION

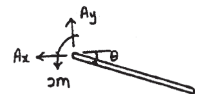
$$I_{AB} = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}(70)(1.2)^2 + 70(0.6)^2 = 33.6\ \text{kg}\cdot\text{m}^2$$

$$\Sigma M_{AB} = I_{AB}\,\alpha; \qquad 2(50\theta) = -33.6(\alpha) \qquad \alpha = -2.97620\theta$$

$$\omega d\omega = \alpha d\theta$$

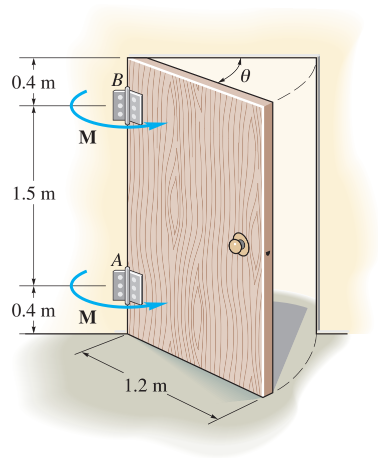
$$\int_0^{\omega} \omega d\omega = -\int_{\frac{\pi}{2}}^{0} 2.97620\theta d\theta$$

$$\omega = 2.71\ \text{rad/s}$$ **Ans.**



17–70.

The door will close automatically using torsional springs mounted on the hinges. If the torque on each hinge is $M = k\theta$, where θ is measured in radians, determine the required torsional stiffness k so that the door will close $(\theta = 0°)$ with an angular velocity $\omega = 2$ rad/s when it is released from rest at $\theta = 90°$. For the calculation, treat the door as a thin plate having a mass of 70 kg.



SOLUTION

$$\Sigma M_A = I_A\alpha; \qquad 2M = -\left[\frac{1}{12}(70)(1.2)^2 + 70(0.6)^2\right](\alpha)$$

$$M = -16.8\alpha$$

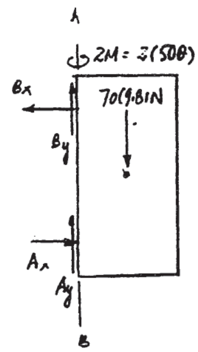
$$k\theta = -16.8\alpha$$

$$\alpha\, d\theta = \omega\, d\omega$$

$$-k\int_{\frac{\pi}{2}}^{0} \theta d\theta = 16.8\int_0^2 \omega\, d\omega$$

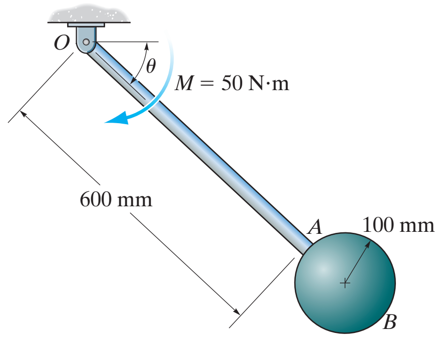
$$\frac{k}{2}\left(\frac{\pi}{2}\right)^2 = \frac{16.8}{2}(2)^2$$

$$k = 27.2\ \text{N}\cdot\text{m/rad}$$ **Ans.**



17–71.

The pendulum consists of a 10-kg uniform slender rod and a 15-kg sphere. If the pendulum is subjected to a torque of $M = 50\ \text{N}\cdot\text{m}$, and has an angular velocity of 3 rad/s when $\theta = 45°$, determine the magnitude of the reactive force pin O exerts on the pendulum at this instant.



SOLUTION

Equations of Motion: Since the pendulum rotates about a fixed axis passing through point O, $[(a_G)_{OA}]_t = \alpha(r_G)_{OA} = \alpha(0.3)$, $[(a_G)_B]_t = \alpha(r_G)_B = \alpha(0.7)$, $[(a_G)_{OA}]_n = \omega^2(r_G)_{OA} = (3^2)(0.3) = 2.7\ \text{m/s}^2$, and $[(a_G)_B]_n = \omega^2(r_G)_B = (3^2)(0.7) = 6.3\ \text{m/s}^2$. The mass moment of inertia of the rod and sphere about their respective mass centers are $(I_G)_{OA} = \dfrac{1}{12}ml^2 = \dfrac{1}{12}(10)(0.6^2) = 0.3\ \text{kg}\cdot\text{m}^2$ and $(I_G)_B = \dfrac{2}{5}mr^2 = \dfrac{2}{5}(15)(0.1^2) = 0.06\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point O, we have

$$+\Sigma M_O = \Sigma(M_k)_O; \quad -10(9.81)\cos 45°(0.3) - 15(9.81)\cos 45°(0.7) - 50 =$$

$$-10[\alpha(0.3)](0.3) - 0.3\alpha - 15[\alpha(0.7)](0.7) - 0.06\alpha$$

$$\alpha = 16.68\ \text{rad/s}^2$$

This result can also be obtained by applying $\Sigma M_O = I_O\alpha$, where $I_O = \Sigma I_G + md^2 = \dfrac{1}{12}(10)(0.6^2) + 10(0.3^2) + \dfrac{2}{5}(15)(0.1^2) + 15(0.7^2) = 8.61\ \text{kg}\cdot\text{m}^2$. Thus,

$$+\Sigma M_O = I_O\alpha; \quad -10(9.81)\cos 45°(0.3) - 15(9.81)\cos 45°(0.7) - 50 = -8.61\alpha$$

$$\alpha = 16.68\ \text{rad/s}^2$$

Using this result to write the force equations of motion along the n and t axes,

$$\Sigma F_t = m(a_G)_t; \quad 10(9.81)\cos 45° + 15(9.81)\cos 45° + O_t$$

$$= 10[16.68(0.3)] + 15[16.68(0.7)]$$

$$O_t = 51.81\ \text{N}$$

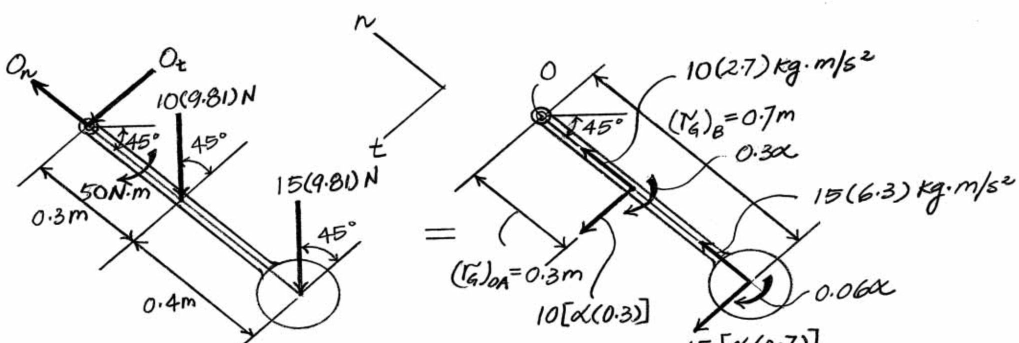
$$\Sigma F_n = m(a_G)_n; \quad O_n - 10(9.81)\sin 45° - 15(9.81)\sin 45° = 10(2.7) + 15(6.3)$$

$$O_n = 294.92\ \text{N}$$

Thus,

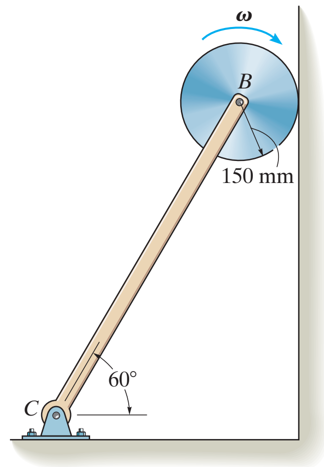
$$F_O = \sqrt{O_t^2 + O_n^2} = \sqrt{51.81^2 + 294.92^2}$$

$$= 299.43\ \text{N} = 299\ \text{N} \qquad \textbf{Ans.}$$



***17–72.**

The disk has a mass of 20 kg and is originally spinning at the end of the strut with an angular velocity of $\omega = 60$ rad/s. If it is then placed against the wall, where the coefficient of kinetic friction is $\mu_k = 0.3$, determine the time required for the motion to stop. What is the force in strut BC during this time?



SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad F_{CB} \sin 30° - N_A = 0$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad F_{CB} \cos 30° - 20(9.81) + 0.3N_A = 0$

$\circlearrowleft + \Sigma M_B = I_B\,\alpha; \qquad 0.3N_A(0.15) = \left[\frac{1}{2}(20)(0.15)^2\right]\alpha$

$$N_A = 96.6 \text{ N}$$

$$F_{CB} = 193 \text{ N} \qquad \textbf{Ans.}$$

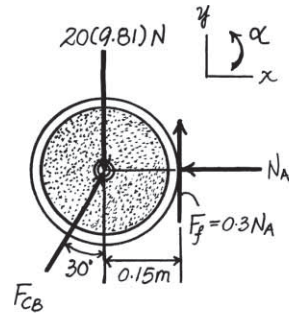
$$\alpha = 19.3 \text{ rad/s}^2$$

$\circlearrowleft +$

$$\omega = \omega_0 + \alpha_c t$$

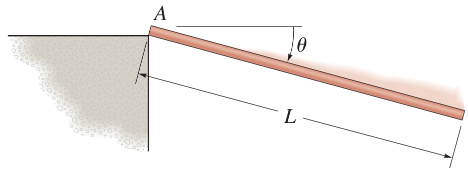
$$0 = 60 + (-19.3)\,t$$

$$t = 3.11 \text{ s} \qquad \textbf{Ans.}$$



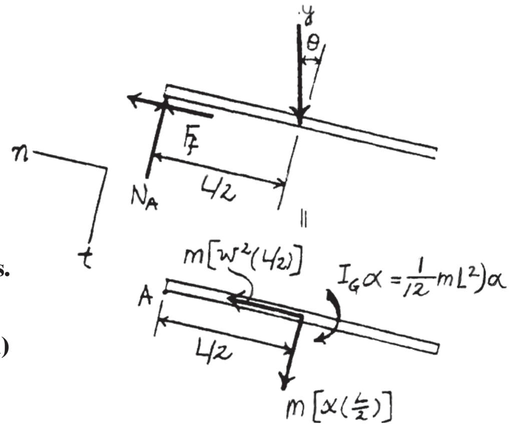
17–73.

The slender rod of length L and mass m is released from rest when $\theta = 0°$. Determine as a function of θ the normal and the frictional forces which are exerted by the ledge on the rod at A as it falls downward. At what angle θ does the rod begin to slip if the coefficient of static friction at A is μ?



SOLUTION

Equations of Motion: The mass moment inertia of the rod about its mass center is given by $I_G = \frac{1}{12}mL^2$. At the instant shown, the normal component of acceleration of the mass center for the rod is $(a_G)_n = \omega^2 r_G = \omega^2\left(\frac{L}{2}\right)$. The tangential component of acceleration of the mass center for the rod is $(a_G)_t = \alpha r_s = \alpha\left(\frac{L}{2}\right)$.



$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_O; \qquad -mg\cos\theta\left(\frac{L}{2}\right) = -\left(\frac{1}{12}mL^2\right)\alpha - m\left[\alpha\left(\frac{L}{2}\right)\right]\left(\frac{L}{2}\right)$$

$$\alpha = \frac{3g}{2L}\cos\theta$$

$$+\swarrow \Sigma F_t = m(a_G)_t; \qquad mg\cos\theta - N_A = m\left[\frac{3g}{2L}\cos\theta\left(\frac{L}{2}\right)\right]$$

$$N_A = \frac{mg}{4}\cos\theta \qquad \textbf{Ans.}$$

$$\nwarrow + \Sigma F_n = m(a_G)_n; \qquad F_f - mg\sin\theta = m\left[\omega^2\left(\frac{L}{2}\right)\right] \qquad \textbf{(1)}$$

Kinematics: Applying equation $\omega\,d\omega = a\,d\theta$, we have

$$\int_0^{\omega} \omega\,d\omega = \int_{0°}^{\theta} \frac{3g}{2L}\cos\theta\,d\theta$$

$$\omega^2 = \frac{3g}{L}\sin\theta$$

Substitute $\omega^2 = \frac{3g}{L}\sin\theta$ into Eq. (1) gives

$$F_f = \frac{5mg}{2}\sin\theta \qquad \textbf{Ans.}$$

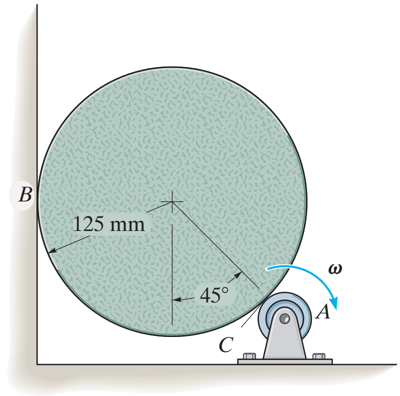
If the rod is on the verge of slipping at A, $F_f = \mu N_A$. Substitute the data obtained above, we have

$$\frac{5mg}{2}\sin\theta = \mu\left(\frac{mg}{4}\cos\theta\right)$$

$$\theta = \tan^{-1}\left(\frac{\mu}{10}\right) \qquad \textbf{Ans.}$$

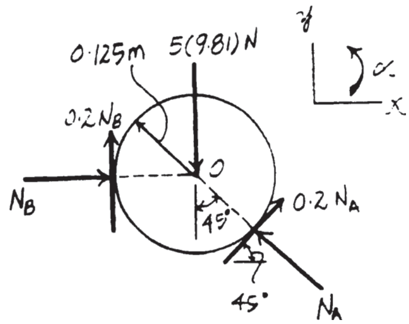
17–74.

The 5-kg cylinder is initially at rest when it is placed in contact with the wall B and the rotor at A. If the rotor always maintains a constant clockwise angular velocity $\omega = 6$ rad/s, determine the initial angular acceleration of the cylinder. The coefficient of kinetic friction at the contacting surfaces B and C is $\mu_k = 0.2$.



SOLUTION

Equations of Motion: The mass moment of inertia of the cylinder about point O is given by $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(5)(0.125^2) = 0.0390625\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have



$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad N_B + 0.2N_A\cos 45° - N_A\sin 45° = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad 0.2N_B + 0.2N_A\sin 45° + N_A\cos 45° - 5(9.81) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 0.2N_A(0.125) - 0.2N_B(0.125) = 0.0390625\alpha \qquad \textbf{(3)}$$

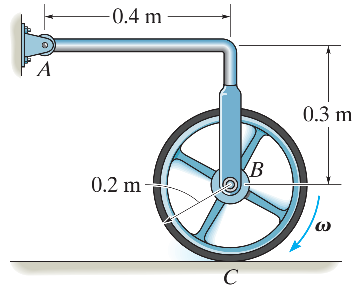
Solving Eqs. (1), (2), and (3) yields;

$$N_A = 51.01\text{ N} \qquad N_B = 28.85\text{ N}$$

$$\alpha = 14.2\text{ rad/s}^2 \qquad \textbf{Ans.}$$

17–75.

The wheel has a mass of 25 kg and a radius of gyration $k_B = 0.15$ m. It is originally spinning at $\omega_1 = 40$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at A exerts on AB during this time? Neglect the mass of AB.



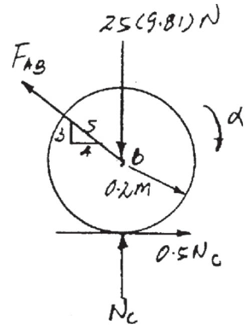
SOLUTION

$I_B = mk_B^2 = 25(0.15)^2 = 0.5625\ \text{kg}\cdot\text{m}^2$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad \left(\frac{3}{5}\right) F_{AB} + N_C - 25(9.81) = 0$ **(1)**

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;\qquad 0.5N_C - \left(\frac{4}{5}\right)F_{AB} = 0$ **(2)**

$\circlearrowleft + \Sigma M_B = I_B\alpha;\qquad 0.5N_C(0.2) = 0.5625(-\alpha)$ **(3)**



Solvings Eqs. (1), (2) and (3) yields:

$F_{AB} = 111.48\ \text{N} \qquad N_C = 178.4\ \text{N}$

$\alpha = -31.71\ \text{rad/s}^2$

$A_x = \frac{4}{5}F_{AB} = 0.8(111.48) = 89.2\ \text{N}$ **Ans.**

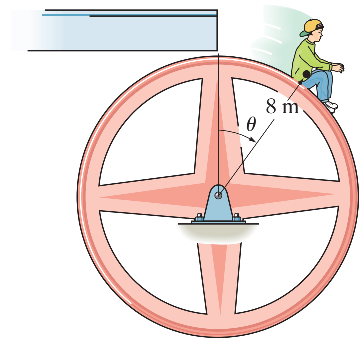
$A_y = \frac{3}{5}F_{AB} = 0.6(111.48) = 66.9\ \text{N}$ **Ans.**

$\omega = \omega_0 + \alpha_c t$

$0 = 40 + (-31.71)\,t$

$t = 1.26\ \text{s}$ **Ans.**

***■17–76.**



A 40-kg boy sits on top of the large wheel which has a mass of 400 kg and a radius of gyration $k_G = 5.5$ m. If the boy essentially starts from rest at $\theta = 0°$, and the wheel begins to rotate freely, determine the angle at which the boy begins to slip. The coefficient of static friction between the wheel and the boy is $\mu_s = 0.5$. Neglect the size of the boy in the calculation.

SOLUTION

$\circlearrowright + \Sigma M_O = \Sigma(M_k)_O; \qquad 392.4(8\sin\theta) = 400(5.5)^2\,\alpha + 40(8)(\alpha)(8)$

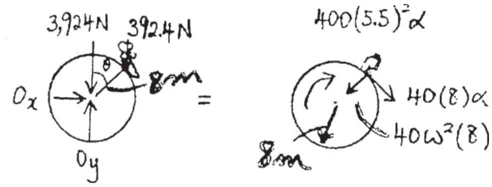
$$0.2141\sin\theta = \alpha$$

$$\alpha\, d\theta = \omega\, d\omega$$

$$\int_0^\theta 0.2141\sin\theta\, d\theta = \int_0^\omega \omega d\omega$$

$$-0.2141\cos\theta\Big|_0^\theta = \frac{1}{2}\omega^2$$

$$\omega^2 = 0.4283(1 - \cos\theta)$$



$+\swarrow \Sigma F_{y'} = m(a_G)_{y'}; \qquad 392.4\cos\theta - N = 40(\omega^2)(8)$

$+\searrow \Sigma F_{x'} = m(a_G)_{x'}; \qquad 392.4\sin\theta - 0.5\,N = 40(8)(\alpha)$

$$N = 392.4\cos\theta - 137.05(1 - \cos\theta) = 529.45\cos\theta - 137.05$$

$$392.4\sin\theta - 0.5(529.45\cos\theta - 137.05) = 320(0.2141\sin\theta)$$

$$323.89\sin\theta - 264.73\cos\theta + 68.52 = 0$$

$$-\sin\theta + 0.8173\cos\theta = 0.2116$$

Solve by trial and error

$$\theta = 29.8° \qquad \textbf{Ans.}$$

Note: The boy will loose contact with the wheel when $N = 0$, i.e.

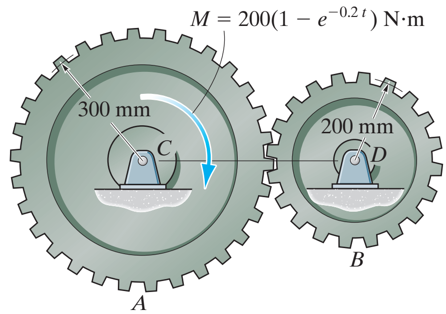
$$N = 529.45\cos\theta - 137.05 = 0$$

$$\theta = 75.0° > 29.8°$$

Hence slipping occurs first.

17–77.

Gears A and B have a mass of 50 kg and 15 kg, respectively. Their radii of gyration about their respective centers of mass are $k_C = 250\text{ mm}$ and $k_D = 150\text{ mm}$. If a torque of $M = 200(1 - e^{-0.2t})\text{ N}\cdot\text{m}$, where t is in seconds, is applied to gear A, determine the angular velocity of both gears when $t = 3\text{ s}$, starting from rest.



SOLUTION

Equations of Motion: Since gear B is in mesh with gear A, $\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{0.3}{0.2}\right)\alpha_A = 1.5\alpha_A$. The mass moment of inertia of gears A and B about their respective centers are $I_C = m_A k_C^{\,2} = 50(0.25^2) = 3.125\text{ kg}\cdot\text{m}^2$ and $I_D = m_B k_D^{\,2} = 15(0.15^2) = 0.3375\text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about the gears' center using the free-body diagrams of gears A and B, Figs. a and b,

$$\circlearrowleft + \Sigma M_C = I_C\alpha_A; \qquad F(0.3) - 200(1 - e^{-0.2t}) = -3.125\alpha_A \qquad \textbf{(1)}$$

and

$$\circlearrowleft + \Sigma M_D = I_D\alpha_B; \qquad F(0.2) = 0.3375(1.5\alpha_A) \qquad \textbf{(2)}$$

Eliminating F from Eqs. (1) and (2) yields

$$\alpha_A = 51.49(1 - e^{-0.2t})\text{ rad/s}^2$$

Kinematics: The angular velocity of gear A can be determined by integration.

$$\int d\omega_A = \int \alpha_A\,dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t 51.49(1 - e^{-0.2t})dt$$

$$\omega_A = 51.49(t + 5e^{-0.2t} - 5)\text{ rad/s}$$

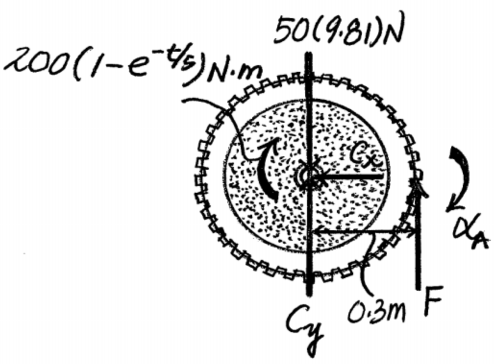
When $t = 3\text{ s}$,

$$\omega_A = 51.49(3 + 5e^{-0.2(3)} - 5) = 38.31\text{ rad/s} = 38.3\text{ rad/s} \qquad \textbf{Ans.}$$

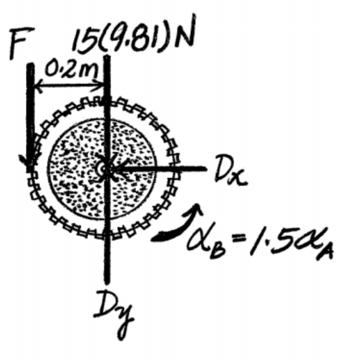
Then

$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{0.3}{0.2}\right)(38.31)$$

$$= 57.47\text{ rad/s} = 57.5\text{ rad/s} \qquad \textbf{Ans.}$$

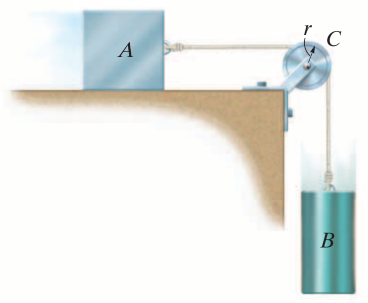


(a)



(b)

17–78.

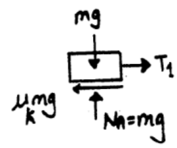


Block A has a mass m and rests on a surface having a coefficient of kinetic friction μ_k. The cord attached to A passes over a pulley at C and is attached to a block B having a mass $2m$. If B is released, determine the acceleration of A. Assume that the cord does not slip over the pulley. The pulley can be approximated as a thin disk of radius r and mass $\frac{1}{4}m$. Neglect the mass of the cord.

SOLUTION

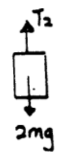
Block A:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad T_1 - \mu_k mg = ma \qquad (1)$$



Block B:

$$+\downarrow \Sigma F_y = ma_y; \qquad 2mg - T_2 = 2ma \qquad (2)$$

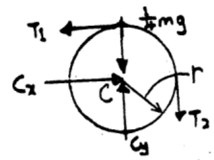


Pulley C:

$$\circlearrowleft + \Sigma M_C = I_G\alpha; \qquad T_2 r - T_1 r = \left[\frac{1}{2}\left(\frac{1}{4}m\right)r^2\right]\left(\frac{a}{r}\right)$$

$$T_2 - T_1 = \frac{1}{8}ma \qquad (3)$$

Substituting Eqs. (1) and (2) into (3),

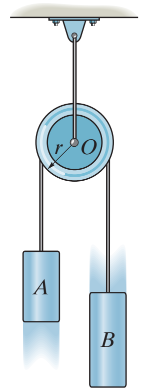


$$2mg - 2ma - (ma + \mu_k mg) = \frac{1}{8}ma \qquad (2 - \mu_k)g = \frac{25}{8}a$$

$$2mg - \mu_k mg = \frac{1}{8}ma + 3ma \qquad a = \frac{8}{25}(2 - \mu_k)g \qquad \textbf{Ans.}$$

17–79.

The two blocks A and B have a mass of 5 kg and 10 kg, respectively. If the pulley can be treated as a disk of mass 3 kg and radius 0.15 m, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.



SOLUTION

Kinematics: Since the pulley rotates about a fixed axis passes through point O, its angular acceleration is

$$\alpha = \frac{a}{r} = \frac{a}{0.15} = 6.6667a$$

The mass moment of inertia of the pulley about point O is

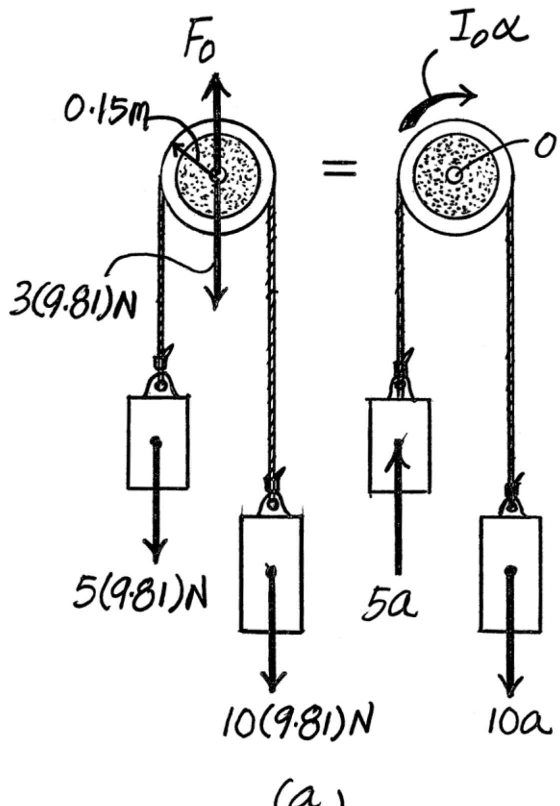
$$I_o = \frac{1}{2}Mr^2 = \frac{1}{2}(3)(0.15^2) = 0.03375\ \text{kg}\cdot\text{m}^2$$

Equation of Motion: Write the moment equation of motion about point O by referring to the free-body and kinetic diagram of the system shown in Fig. a,

$$\circlearrowleft + \Sigma M_o = \Sigma(M_k)_o; \qquad 5(9.81)(0.15) - 10(9.81)(0.15)$$

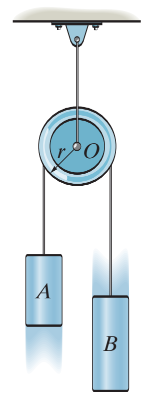
$$= -0.03375(6.6667a) - 5a(0.15) - 10a(0.15)$$

$$a = 2.973\ \text{m/s}^2 = 2.97\ \text{m/s}^2$$ **Ans.**



***17–80.**

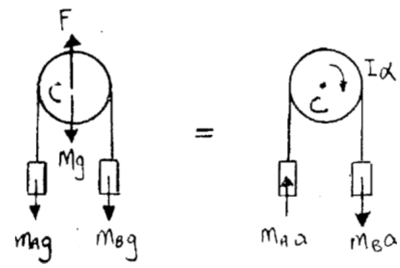
The two blocks A and B have a mass m_A and m_B, respectively, where $m_B > m_A$. If the pulley can be treated as a disk of mass M, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.



SOLUTION

$$a = \alpha r$$

$$\circlearrowleft +\Sigma M_C = \Sigma (M_k)_C; \qquad m_B\, g(r) - m_A\, g(r) = \left(\frac{1}{2}Mr^2\right)\alpha + m_B\, r^2\alpha + m_A r^2 \alpha$$

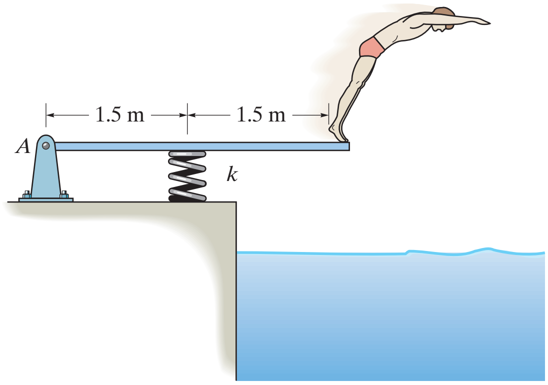


$$\alpha = \frac{g(m_B - m_A)}{r\left(\frac{1}{2}M + m_B + m_A\right)}$$

$$a = \frac{g(m_B - m_A)}{\left(\frac{1}{2}M + m_B + m_A\right)} \qquad \textbf{Ans.}$$

17–81.

Determine the angular acceleration of the 25-kg diving board and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount of 200 mm, $\omega = 0$, and the board is horizontal. Take $k = 7$ kN/m.



SOLUTION

$$\circlearrowleft + \sum M_A = I_A\alpha; \qquad 1.5(1400 - 245.25) = \left[\frac{1}{3}(25)(3)^2\right]\alpha$$

$$+\uparrow \sum F_t = m(a_G)_t; \qquad 1400 - 245.25 - A_y = 25(1.5\alpha)$$

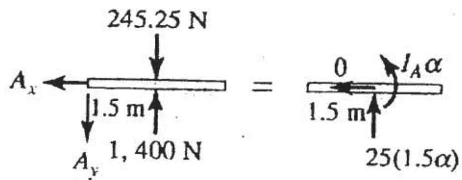
$$\overset{+}{\leftarrow} \sum F_n = m(a_G)_n; \qquad A_x = 0$$

Solving,

$A_x = 0$ **Ans.**

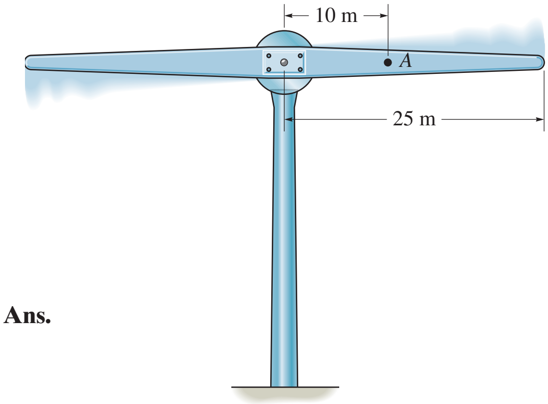
$A_y = 289$ N **Ans.**

$\alpha = 23.1$ rad/s^2 **Ans.**



17–82.

The lightweight turbine consists of a rotor which is powered from a torque applied at its center. At the instant the rotor is horizontal it has an angular velocity of 15 rad/s and a clockwise angular acceleration of 8 rad/s^2. Determine the internal normal force, shear force, and moment at a section through A. Assume the rotor is a 50-m-long slender rod, having a mass of 3 kg/m.



SOLUTION

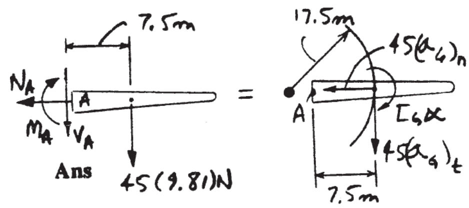
$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n; \qquad N_A = 45(15)^2(17.5) = 177\text{kN}$ **Ans.**

$+\downarrow \Sigma F_t = m(a_G)_t; \qquad V_A + 45(9.81) = 45(8)(17.5)$

$V_A = 5.86 \text{ kN}$ **Ans.**

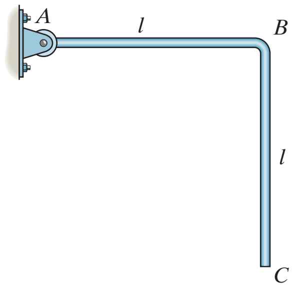
$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad M_A + 45(9.81)(7.5) = \left[\frac{1}{12}(45)(15)^2\right](8) + [45(8)(17.5)](7.5)$

$M_A = 50.7 \text{ kN}\cdot\text{m}$ **Ans.**



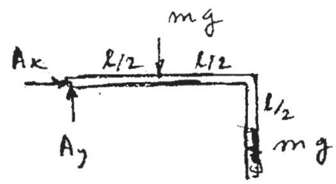
17–83.

The two-bar assembly is released from rest in the position shown. Determine the initial bending moment at the fixed joint B. Each bar has a mass m and length l.



SOLUTION

Assembly:



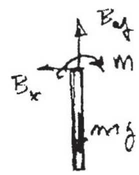
$$I_A = \frac{1}{3}ml^2 + \frac{1}{12}(m)(l)^2 + m(l^2 + (\frac{l}{2})^2)$$

$$= 1.667\ ml^2$$

$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad mg(\frac{l}{2}) + mg(l) = (1.667ml^2)\alpha$$

$$\alpha = \frac{0.9\ g}{l}$$

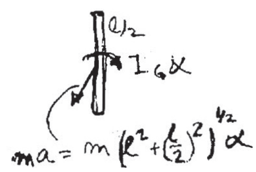
Segment BC:



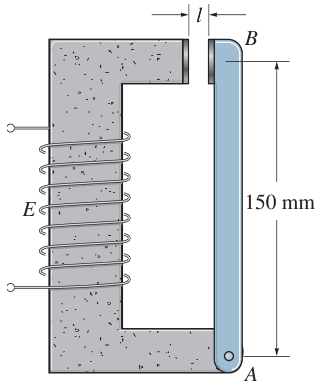
$$\circlearrowleft + \Sigma M_B = \Sigma(M_k)_B; \qquad M = \left[\frac{1}{12}ml^2\right]\alpha + m(l^2 + (\frac{l}{2})^2)^{1/2}\alpha(\frac{l/2}{l^2 + (\frac{l}{2})^2})(\frac{l}{2})$$

$$M = \frac{1}{3}ml^2\,\alpha = \frac{1}{3}ml^2(\frac{0.9g}{l})$$

$$M = 0.3gml \qquad \textbf{Ans.}$$



***17–84.**

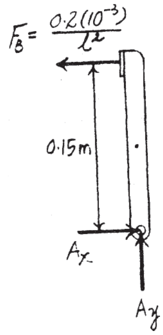


The armature (slender rod) AB has a mass of 0.2 kg and can pivot about the pin at A. Movement is controlled by the electromagnet E, which exerts a horizontal attractive force on the armature at B of $F_B = (0.2(10^{-3})l^{-2})$ N, where l in meters is the gap between the armature and the magnet at any instant. If the armature lies in the horizontal plane, and is originally at rest, determine the speed of the contact at B the instant $l = 0.01$ m. Originally $l = 0.02$ m.

SOLUTION

Equation of Motion: The mass moment of inertia of the armature about point A is given by $I_A = I_G + mr_G^2 = \frac{1}{12}(0.2)\left(0.15^2\right) + 0.2\left(0.075^2\right) = 1.50\left(10^{-3}\right)\text{kg}\cdot\text{m}^2$

Applying Eq. 17–16, we have



$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad \frac{0.2(10^{-3})}{l^2}(0.15) = 1.50\left(10^{-3}\right)\alpha$$

$$\alpha = \frac{0.02}{l^2}$$

Kinematic: From the geometry, $l = 0.02 - 0.15\theta$. Then $dl = -0.15\,d\theta$ or $d\theta = -\frac{dl}{0.15}$. Also, $\omega = \frac{v}{0.15}$ hence $d\omega = \frac{dv}{0.15}$. Substitute into equation $\omega d\omega = \alpha d\theta$, we have

$$\frac{v}{0.15}\left(\frac{dv}{0.15}\right) = \alpha\left(-\frac{dl}{0.15}\right)$$

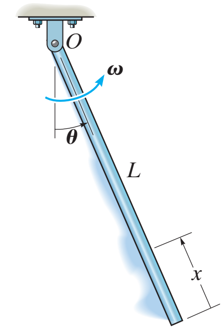
$$vdv = -0.15\,\alpha dl$$

$$\int_0^v vdv = \int_{0.02\text{ m}}^{0.01\text{ m}} -0.15\left(\frac{0.02}{l^2}\right)dl$$

$$v = 0.548\text{ m/s}$$ **Ans.**

17–85.

The bar has a weight per length of w. If it is rotating in the vertical plane at a constant rate $\boldsymbol{\omega}$ about point O, determine the internal normal force, shear force, and moment as a function of x and θ.

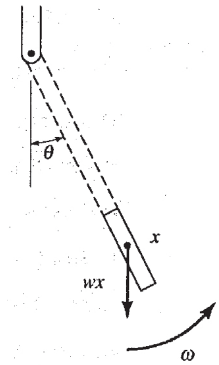


SOLUTION

$$a = \omega^2\left(L - \frac{x}{z}\right)\overset{\theta}{\nwarrow}$$

Forces:

$$\frac{wx}{g}\omega^2\left(L - \frac{x}{z}\right)\overset{\theta}{\nwarrow} = N\overset{\theta}{\nwarrow} + S\measuredangle\theta + wx\downarrow \qquad \textbf{(1)}$$



Moments:

$$I\alpha = M - S\left(\frac{x}{2}\right)$$

$$O = M - \frac{1}{2}Sx \qquad \textbf{(2)}$$

Solving (1) and (2),

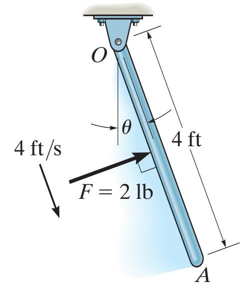
$$N = wx\left[\frac{\omega^2}{g}\left(L - \frac{x}{2}\right) + \cos\theta\right] \qquad \textbf{Ans.}$$

$$S = wx\sin\theta \qquad \textbf{Ans.}$$

$$M = \frac{1}{2}wx^2\sin\theta \qquad \textbf{Ans.}$$

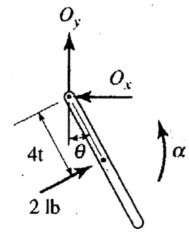
17–86.

A force $F = 2$ lb is applied perpendicular to the axis of the 5-lb rod and moves from O to A at a constant rate of 4 ft/s. If the rod is at rest when $\theta = 0°$ and **F** is at O when $t = 0$, determine the rod's angular velocity at the instant the force is at A. Through what angle has the rod rotated when this occurs? The rod rotates in the *horizontal plane*.



SOLUTION

$$I_O = \frac{1}{3}mR^2 = \frac{1}{3}\left(\frac{5}{32.2}\right)(4)^2 = 0.8282 \text{ slug}\cdot\text{ft}^2$$



$$\circlearrowleft + \sum M_O = I_O\alpha; \quad 2(4t) = 0.8282(\alpha)$$

$$\alpha = 9.66t$$

$$d\omega = \alpha dt$$

$$\int_0^\omega d\omega \doteq \int_0^t 9.66t\, dt$$

$$\omega = 4.83t^2$$

When $t = 1$ s,

$$\omega = 4.83(1)^2 = 4.83 \text{ rad/s} \qquad \textbf{Ans.}$$

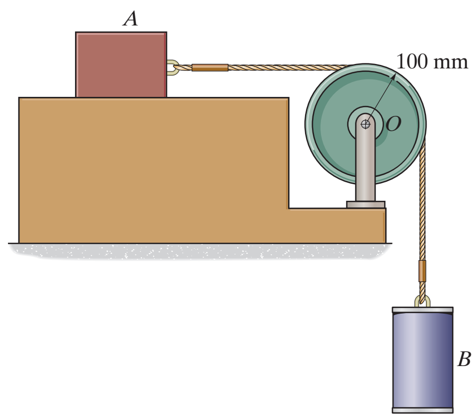
$$d\theta = \omega\, dt$$

$$\int_0^\theta d\theta = \int_0^t 4.83t^2\, dt$$

$$\theta = 1.61 \text{ rad} = 92.2° \qquad \textbf{Ans.}$$

17–87.

The 15-kg block A and 20-kg cylinder B are connected by a light cord that passes over a 5-kg pulley (disk). If the system is released from rest, determine the cylinder's velocity after its has traveled downwards 2 m. Neglect friction between the plane and the block, and assume the cord does not slip over the pulley.



SOLUTION

Equations of Motion: Since the pulley rotates about a fixed axis passing through point O, its angular acceleration is $\alpha = \frac{a}{r} = \frac{a}{0.1} = 10a$. The mass moment of inertia of the pulley about point O is $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(5)(0.1^2) = 0.025\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of equilibrium about point O and realizing that the moment of 15(9.81) N and N_A cancel out, we have

$$+\Sigma M_O = \Sigma(M_k)_O; \quad -20(9.81)(0.1) = -15a(0.1) - 20a(0.1) - 0.025(10a)$$

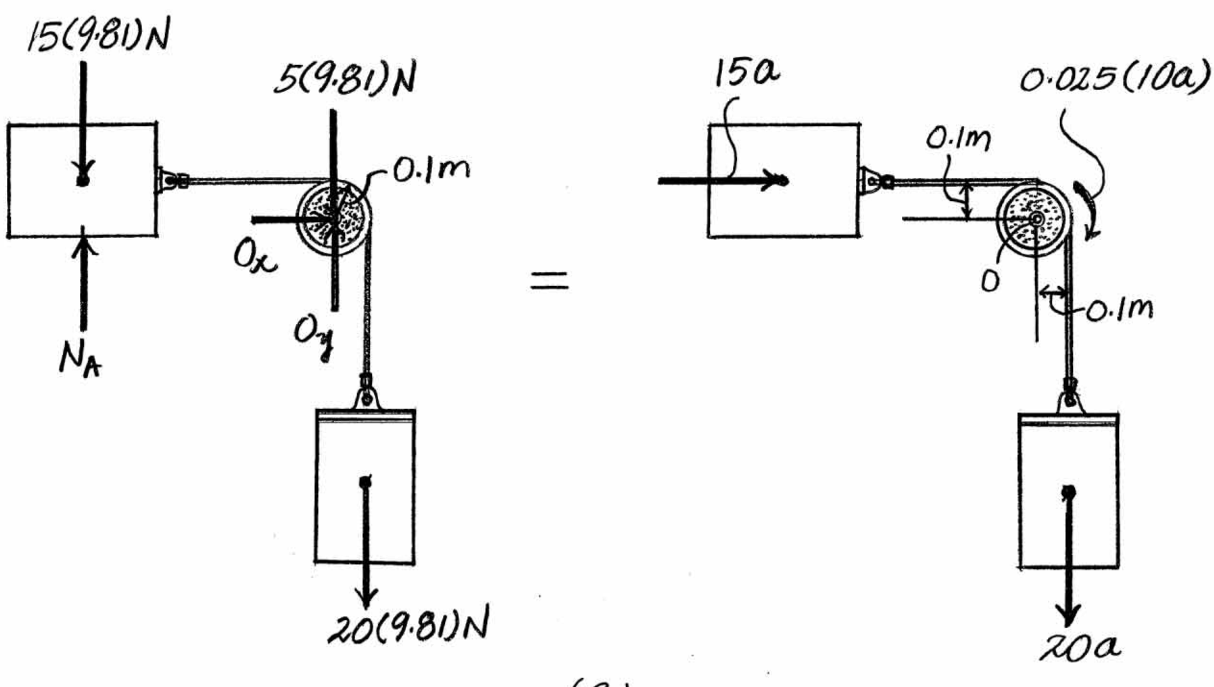
$$a = 5.232\ \text{m/s}^2$$

Kinematics: Since the angular acceleration is constant,

$$(+\downarrow) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$v^2 = 0^2 + 2(5.232)(2 - 0)$$

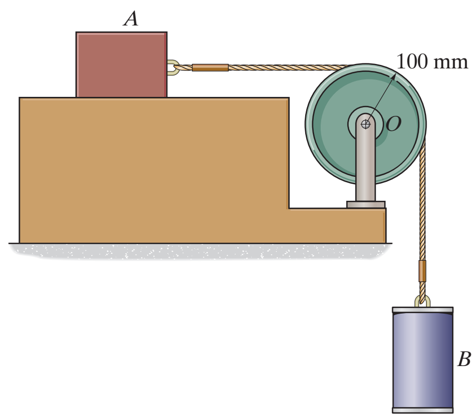
$$v = 4.57\ \text{m/s} \downarrow \qquad \textbf{Ans.}$$



(a)

*17–88.

The 15-kg block A and 20-kg cylinder B are connected by a light cord that passes over a 5-kg pulley (disk). If the system is released from rest, determine the cylinder's velocity after its has traveled downwards 2 m. The coefficient of kinetic friction between the block and the horizontal plane is $\mu_k = 0.3$. Assume the cord does not slip over the pulley.



SOLUTION

Equations of Motion: Since the block is in motion, $F_A = \mu_k N_A = 0.3N_A$. Referring to the free-body diagram of block A shown in Fig. a,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 15(9.81) = 0 \qquad N_A = 147.15 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad T_1 - 0.3(147.15) = 15a \tag{1}$$

Referring to the free-body diagram of the cylinder, Fig. b,

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad T_2 - 20(9.81) = -20a \tag{2}$$

Since the pulley rotates about a fixed axis passing through point O, $\alpha = \dfrac{a}{r} = \dfrac{a}{0.1} = 10a$. The mass moment of inertia of the pulley about O is $I_O = \dfrac{1}{2}mr^2 = \dfrac{1}{2}(5)(0.1^2) = 0.025 \text{ kg}\cdot\text{m}^2$. Writing the moment equation of motion about point O using the free-body diagram of the pulley shown in Fig. c,

$$+\Sigma M_O = I_O\alpha; \qquad T_1(0.1) - T_2(0.1) = -0.025(10a) \tag{3}$$

Solving Eqs. (1) through (3) yields

$$a = 4.0548 \text{ m/s}^2 \qquad T_2 = 115.104 \text{ N} \qquad T_1 = 104.967 \text{ N}$$

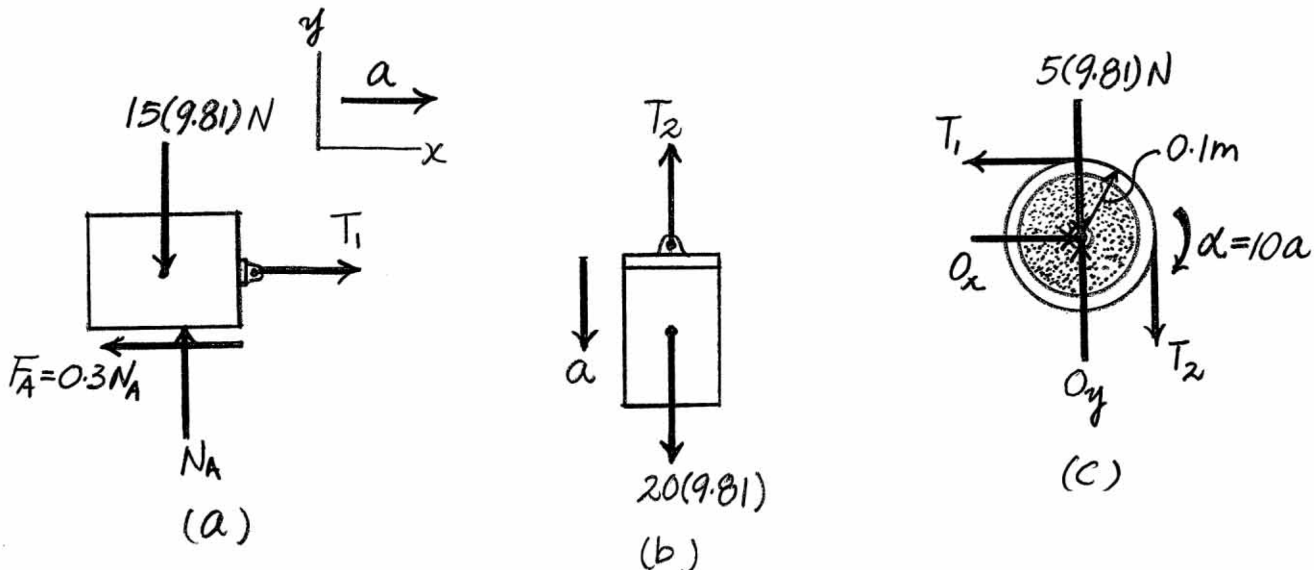
Kinematics: Since the acceleration is constant,

$$(+\downarrow) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$v^2 = 0^2 + 2(4.0548)(2 - 0)$$

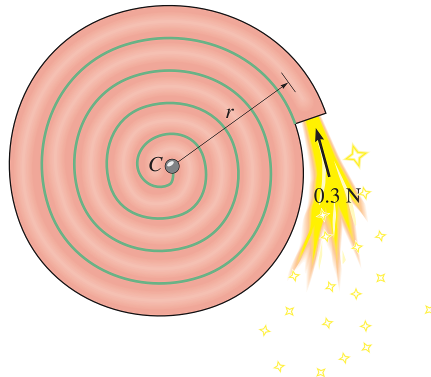
$$v = 4.027 \text{ m/s} = 4.03 \text{ m/s} \downarrow$$

Ans.



17–89.

The “Catherine wheel” is a firework that consists of a coiled tube of powder which is pinned at its center. If the powder burns at a constant rate of 20 g/s such as that the exhaust gases always exert a force having a constant magnitude of 0.3 N, directed tangent to the wheel, determine the angular velocity of the wheel when 75% of the mass is burned off. Initially, the wheel is at rest and has a mass of 100 g and a radius of r = 75 mm. For the calculation, consider the wheel to always be a thin disk.

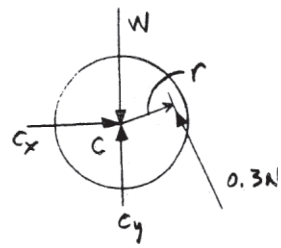


SOLUTION

Mass of wheel when 75% of the powder is burned = 0.025 kg

$$\text{Time to burn off 75 \%} = \frac{0.075\text{ kg}}{0.02\text{ kg/s}} = 3.75\text{ s}$$

$$m(t) = 0.1 - 0.02t$$



Mass of disk per unit area is

$$\rho_0 = \frac{m}{A} = \frac{0.1\text{ kg}}{\pi(0.075\text{ m})^2} = 5.6588\text{ kg/m}^2$$

At any time t,

$$5.6588 = \frac{0.1 - 0.02t}{\pi r^2}$$

$$r(t) = \sqrt{\frac{0.1 - 0.02t}{\pi(5.6588)}}$$

$$+\Sigma M_C = I_C\alpha; \qquad 0.3r = \frac{1}{2}mr^2\alpha$$

$$\alpha = \frac{0.6}{mr} = \frac{0.6}{(0.1 - 0.02t)\sqrt{\dfrac{0.1 - 0.02t}{\pi(5.6588)}}}$$

$$\alpha = 0.6\left(\sqrt{\pi(5.6588)}\right)[0.1 - 0.02t]^{-\frac{3}{2}}$$

$$\alpha = 2.530[0.1 - 0.02t]^{-\frac{3}{2}}$$

$$d\omega = \alpha\, dt$$

$$\int_0^\omega d\omega = 2.530\int_0^t [0.1 - 0.02t]^{-\frac{3}{2}}\, dt$$

$$\omega = 253\left[(0.1 - 0.02t)^{-\frac{1}{2}} - 3.162\right]$$

For $t = 3.75$ s,

$$\omega = 800\text{ rad/s} \qquad \textbf{Ans.}$$

17–90.

If the disk in Fig. 17–21*a rolls without slipping*, show that when moments are summed about the instantaneous center of zero velocity, *IC*, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where I_{IC} represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

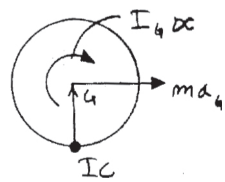
SOLUTION

$\zeta + \Sigma M_{IC} = \Sigma (M_K)_{IC}; \qquad \Sigma M_{IC} = I_G \alpha + (m a_G) r$

Since there is no slipping, $a_G = \alpha r$

Thus, $\Sigma M_{IC} = \left(I_G + m r^2\right)\alpha$

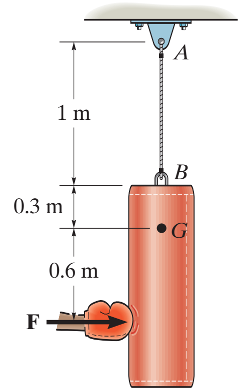
By the parallel–axis thoerem, the term in parenthesis represents I_{IC}. Thus,



$\Sigma M_{IC} = I_{IC}\alpha$ **Q.E.D.**

17–91.

The 20-kg punching bag has a radius of gyration about its center of mass G of $k_G = 0.4$ m. If it is initially at rest and is subjected to a horizontal force $F = 30$ N, determine the initial angular acceleration of the bag and the tension in the supporting cable AB.



SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 30 = 20(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T - 196.2 = 20(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad 30(0.6) = 20(0.4)^2 \alpha$

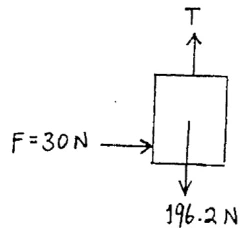
$\alpha = 5.62 \text{ rad/s}^2$ **Ans.**

$(a_G)_x = 1.5 \text{ m/s}^2$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

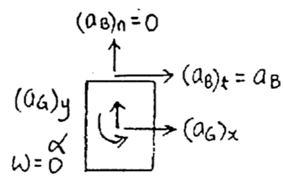
$a_B \mathbf{i} = (a_G)_y \mathbf{j} + (a_G)_x \mathbf{i} - \alpha(0.3)\mathbf{i}$

$(+\uparrow) \qquad (a_G)_y = 0$



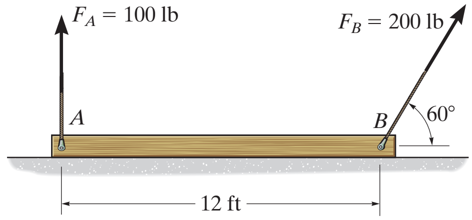
Thus,

$T = 196 \text{ N}$ **Ans.**



*17–92.

The uniform 150-lb beam is initially at rest when the forces are applied to the cables. Determine the magnitude of the acceleration of the mass center and the angular acceleration of the beam at this instant.



SOLUTION

Equations of Motion: The mass moment of inertia of the beam about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{150}{32.2}\right)(12^2) = 55.90 \text{ slug}\cdot\text{ft}^2$.

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 200 \cos 60° = \frac{150}{32.2}(a_G)_x$$

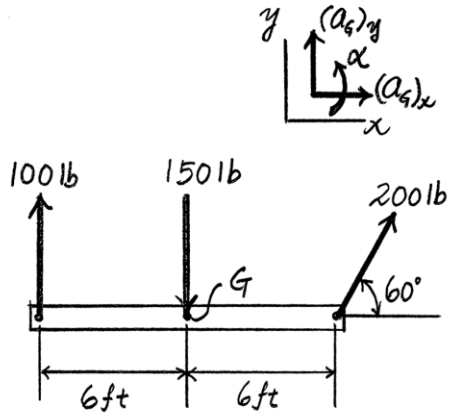
$$(a_G)_x = 21.47 \text{ ft/s}^2$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad 100 + 200 \sin 60° - 150 = \frac{150}{32.2}(a_G)_y$$

$$(a_G)_y = 26.45 \text{ ft/s}^2$$

$$+\Sigma M_G = I_G\alpha; \quad 200 \sin 60°(6) - 100(6) = 55.90\alpha$$

$$\alpha = 7.857 \text{ rad/s}^2 = 7.86 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



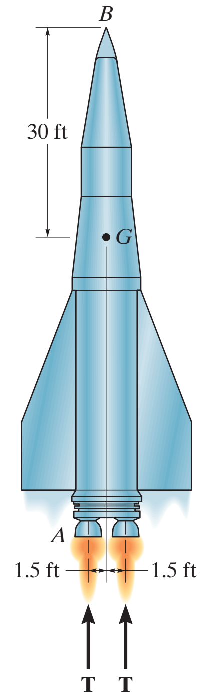
(a)

Thus, the magnitude of $\mathbf{a}_G$ is

$$a_G = \sqrt{(a_G)_x^{\,2} + (a_G)_y^{\,2}} = \sqrt{21.47^2 + 26.45^2} = 34.1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

17–93.

The rocket has a weight of 20 000 lb, mass center at G, and radius of gyration about the mass center of $k_G = 21$ ft when it is fired. Each of its two engines provides a thrust $T = 50\,000$ lb. At a given instant, engine A suddenly fails to operate. Determine the angular acceleration of the rocket and the acceleration of its nose B.



SOLUTION

$\circlearrowleft +\Sigma M_G = I_G\alpha;\qquad 50\,000(1.5) = \dfrac{20\,000}{32.2}(21)^2\alpha$

$$\alpha = 0.2738 \text{ rad/s}^2 = 0.274 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad 50\,000 - 20\,000 = \dfrac{20\,000}{32.2}a_G$

$$a_G = 48.3 \text{ ft/s}^2$$

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$$

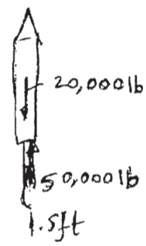
Since $\omega = 0$

$$\mathbf{a}_B = 48.3\mathbf{j} - 0.2738(30)\mathbf{i}$$

$$= 48.34\mathbf{j} - 8.214\mathbf{i}$$

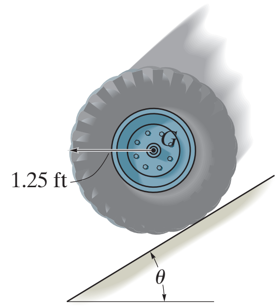
$$a_B = \sqrt{(48.3)^2 + (8.214)^2} = 49.0 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{48.3}{8.214} = 80.3^\circ \;\; \theta \measuredangle \qquad \textbf{Ans.}$$



17–94.

The wheel has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, determine the wheel's angular acceleration as it rolls down the incline. Set $\theta = 12°$.

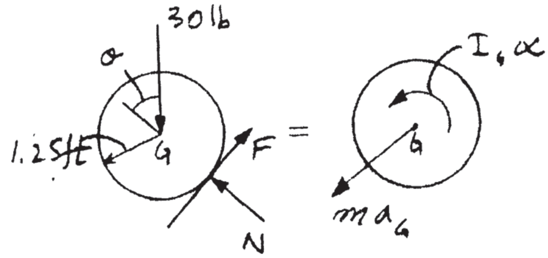


SOLUTION

$+\swarrow \Sigma F_x = m(a_G)_x; \qquad 30 \sin 12° - F = \left(\frac{30}{32.2}\right) a_G$

$+\nwarrow \Sigma F_y = m(a_G)_y; \qquad N - 30 \cos 12° = 0$

$\circlearrowleft + \Sigma M_G = I_G\,\alpha; \qquad F(1.25) = \left[\left(\frac{30}{32.2}\right)(0.6)^2\right]\alpha$



Assume the wheel does not slip.

$$a_G = (1.25)\alpha$$

Solving:

$F = 1.17$ lb

$N = 29.34$ lb

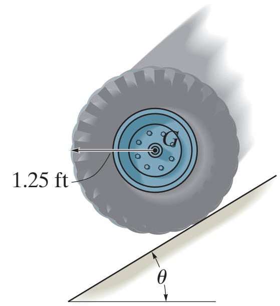
$a_G = 5.44 \text{ ft/s}^2$

$\alpha = 4.35 \text{ rad/s}^2$ **Ans.**

$F_{max} = 0.2(29.34) = 5.87 \text{ lb} > 1.17 \text{ lb}$ **OK**

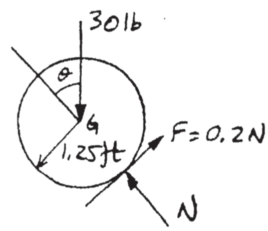
17–95.

The wheel has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, determine the maximum angle θ of the inclined plane so that the wheel rolls without slipping.



SOLUTION

Since wheel is on the verge of slipping:



$$+\swarrow \Sigma F_x = m(a_G)_x; \qquad 30 \sin\theta - 0.2N = \left(\frac{30}{32.2}\right)(1.25\alpha) \qquad \textbf{(1)}$$

$$+\nwarrow \Sigma F_y = m(a_G)_y; \qquad N - 30\cos\theta = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_C = I_G\,\alpha; \qquad 0.2N(1.25) = \left[\left(\frac{30}{32.2}\right)(0.6)^2\right]\alpha \qquad \textbf{(3)}$$

Substituting Eqs.(2) and (3) into Eq. (1),

$$30\sin\theta - 6\cos\theta = 26.042\cos\theta$$

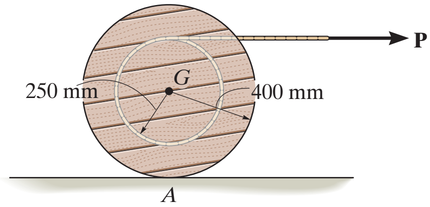
$$30\sin\theta = 32.042\cos\theta$$

$$\tan\theta = 1.068$$

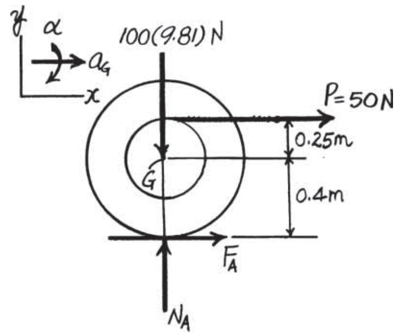
$$\theta = 46.9^\circ \qquad \textbf{Ans.}$$

***17–96.**

The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 50 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G\alpha;\qquad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\alpha$

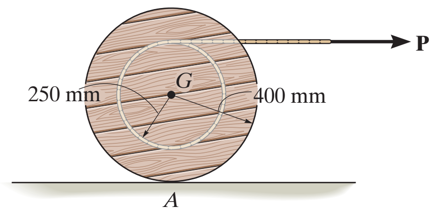
$\alpha = 1.30 \text{ rad/s}^2$ **Ans.**

$a_G = 0.520 \text{ m/s}^2 \qquad N_A = 981 \text{ N} \qquad F_A = 2.00 \text{ N}$

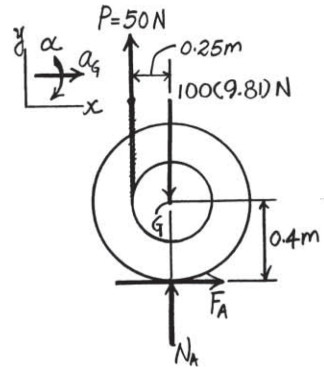
Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 2.00 \text{ N}$ **OK**

17–97.

Solve Prob. 17–96 if the cord and force $P = 50\text{ N}$ are directed vertically upwards.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)x;\quad F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_A + 50 - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G\alpha;\qquad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\,\alpha$

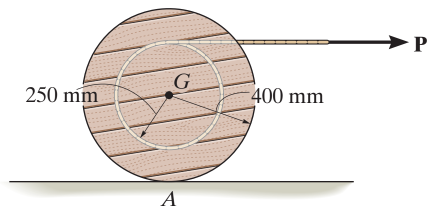
$\alpha = 0.500\text{ rad/s}^2$ **Ans.**

$a_G = 0.2\text{ m/s}^2 \qquad N_A = 931\text{ N} \qquad F_A = 20\text{ N}$

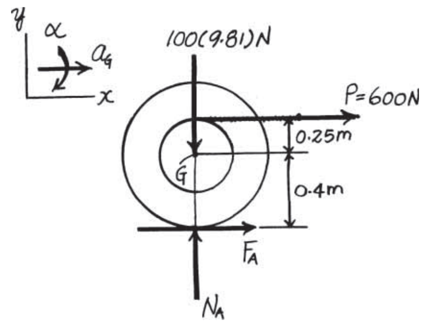
Since $(F_A)_{\max} = 0.2(931) = 186.2\text{ N} > 20\text{ N}$ **OK**

17–98.

The spool has a mass of 100 kg and a radius of gyration $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 600$ N.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 600 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G\,\alpha;\qquad 600(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\alpha$

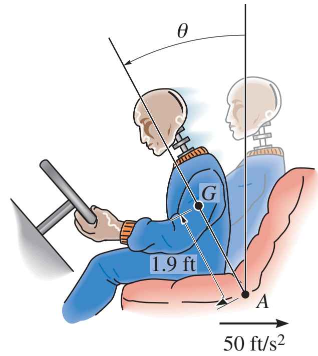
$\alpha = 15.6 \text{ rad/s}^2$ **Ans.**

$a_G = 6.24 \text{ m/s}^2 \qquad N_A = 981 \text{ N} \qquad F_A = 24.0 \text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 24.0 \text{ N}$ **OK**

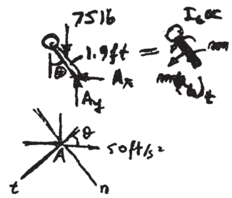
17–99.

The upper body of the crash dummy has a mass of 75 lb, a center of gravity at G, and a radius of gyration about G of $k_G = 0.7$ ft. By means of the seat belt this body segment is assumed to be pin-connected to the seat of the car at A. If a crash causes the car to decelerate at 50 ft/s^2, determine the angular velocity of the body when it has rotated to $\theta = 30°$.



SOLUTION

$$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A; \qquad 75(1.9\sin\theta) = \left[\left(\frac{75}{32.2}\right)(0.7)^2\right]\alpha + \left[\frac{75}{32.2}(a_G)_t\right](1.9)$$



$$+\swarrow \mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_n + (\mathbf{a}_{G/A})_t$$

$$(a_G)_t = -50\cos\theta + 0 + (\alpha)(1.9)$$

$$142.5\sin\theta = 1.1413\alpha - 221.273\cos\theta + 8.4084\alpha$$

$$142.5\sin\theta + 221.273\cos\theta = 9.5497\alpha$$

$$\omega\, d\omega = \alpha\, d\theta$$

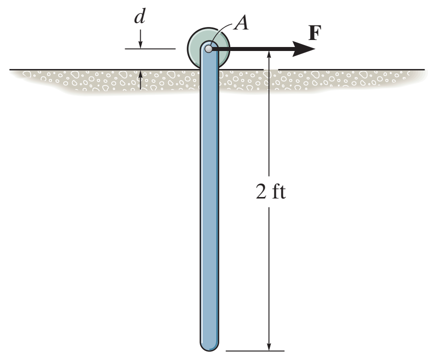
$$\int_0^{\omega} \omega\, d\omega = \int_0^{30°} (14.922\sin\theta + 23.17\cos\theta)d\theta$$

$$\frac{1}{2}\omega^2 = -14.922(\cos 30° - \cos 0°) + 23.17(\sin 30° - \sin 0°)$$

$$\omega = 5.21 \text{ rad/s}$$ **Ans.**

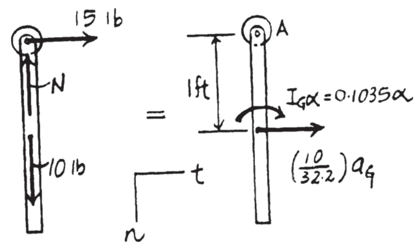
***17–100.**

A uniform rod having a weight of 10 lb is pin supported at A from a roller which rides on a horizontal track. If the rod is originally at rest, and a horizontal force of $F = 15$ lb is applied to the roller, determine the acceleration of the roller. Neglect the mass of the roller and its size d in the computations.



SOLUTION

Equations of Motion: The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035 \text{ slug} \cdot \text{ft}^2$. At the instant force $\mathbf{F}$ is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17–16, we have

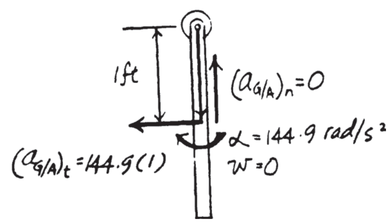


$$\Sigma F_t = m(a_G)_t; \qquad 15 = \left(\frac{10}{32.2}\right)a_G \quad a_G = 48.3 \text{ ft/s}^2$$

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 0 = \left(\frac{10}{32.2}\right)(48.3)(1) - 0.1035\,\alpha$$

$$\alpha = 144.9 \text{ rad/s}^2$$

Kinematics: Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of roller A can be obtain by analyzing the motion of points A and G. Applying Eq. 16–17, we have



$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

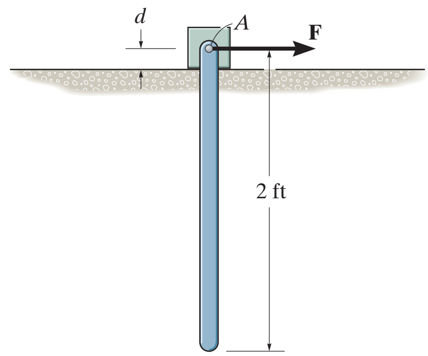
$$\left[\underset{\rightarrow}{48.3}\right] = \left[\underset{\rightarrow}{a_A}\right] + \left[\underset{\leftarrow}{144.9(1)}\right] + [0]$$

$$(\overset{+}{\rightarrow}) \qquad 48.3 = a_A - 144.9$$

$$a_A = 193 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

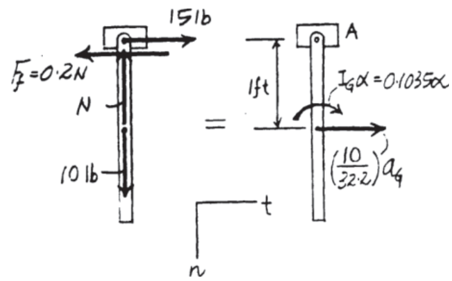
17–101.

Solve Prob. 17–100 assuming that the roller at A is replaced by a slider block having a negligible mass. The coefficient of kinetic friction between the block and the track is $\mu_k = 0.2$. Neglect the dimension d and the size of the block in the computations.



SOLUTION

Equations of Motion: The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035 \text{ slug} \cdot \text{ft}^2$. At the instant force **F** is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17–16, we have



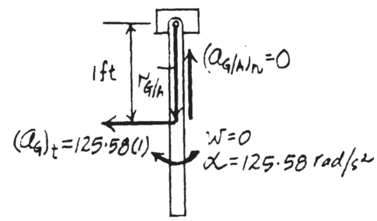
$$\Sigma F_n = m(a_G)_n; \qquad 10 - N = 0 \qquad N = 10.0 \text{ lb}$$

$$\Sigma F_t = m(a_G)_t; \qquad 15 - 0.2(10.0) = \left(\frac{10}{32.2}\right)a_G \qquad a_G = 41.86 \text{ ft/s}^2$$

$$\zeta + \Sigma M_A = \Sigma(M_k)_A; \quad 0 = \left(\frac{10}{32.2}\right)(41.86)(1) - 0.1035\alpha$$

$$\alpha = 125.58 \text{ rad/s}^2$$

Kinematics: Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of block A can be obtain by analyzing the motion of points A and G. Applying Eq. 16–17, we have



$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

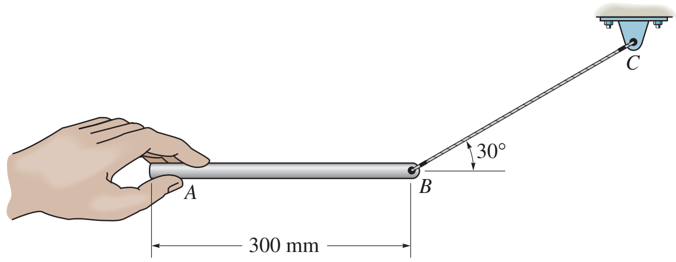
$$\left[\underset{\rightarrow}{41.86}\right] = \left[\underset{\rightarrow}{a_A}\right] + \left[\underset{\leftarrow}{125.58(1)}\right] + [0]$$

$$(\overset{+}{\rightarrow}) \qquad 41.86 = a_A - 125.58$$

$$a_A = 167 \text{ ft/s}^2$$ **Ans.**

17–102.

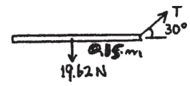
The 2-kg slender bar is supported by cord BC and then released from rest at A. Determine the initial angular acceleration of the bar and the tension in the cord.



SOLUTION

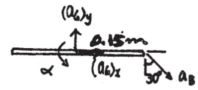
$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad T \cos 30° = 2(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T \sin 30° - 19.62 = 2(a_G)_y$



$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad T \sin 30°(0.15) = [\frac{1}{12}(2)(0.3)^2]\alpha$

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$$



$$a_B \sin 30°\mathbf{i} - a_B \cos 30°\mathbf{j} = (a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} + \alpha(0.15)\mathbf{j}$$

$(\xrightarrow{+}) \qquad (a_B) \sin 30° = (a_G)_x$

$(+\uparrow) \qquad (a_B) \cos 30° = -(a_G)_y - \alpha(0.15)$

Thus,

$$1.7321(a_G)_x = -(a_G)_y - 0.15\alpha$$

$T = 5.61\text{ N}$ **Ans.**

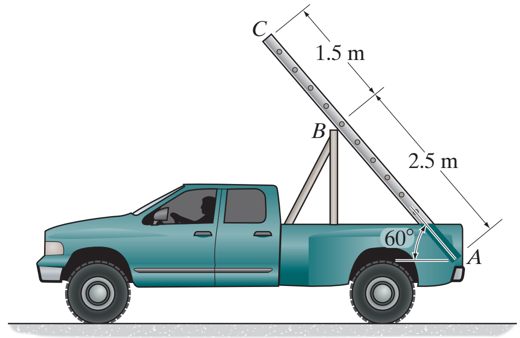
$(a_G)_x = 2.43\text{ m/s}^2$

$(a_G)_y = -8.41\text{ m/s}^2$

$\alpha = 28.0\text{ rad/s}^2$ **Ans.**

17–103.

If the truck accelerates at a constant rate of $6\,\text{m/s}^2$, starting from rest, determine the initial angular acceleration of the 20-kg ladder. The ladder can be considered as a uniform slender rod. The support at B is smooth.



SOLUTION

Equations of Motion: We must first show that the ladder will rotate when the acceleration of the truck is $6\ \text{m/s}^2$. This can be done by determining the minimum acceleration of the truck that will cause the ladder to lose contact at B, $N_B = 0$. Writing the moment equation of motion about point A using Fig. a,

$$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A; \qquad 20(9.81)\cos 60°(2) = 20a_{\min}\,(2\sin 60°)$$

$$a_{\min} = 5.664\ \text{m/s}^2$$

Since $a_{\min} < 6\ \text{m/s}^2$, the ladder will in the fact rotate. The mass moment of inertia about its mass center is $I_G = \dfrac{1}{12}ml^2 = \dfrac{1}{12}(20)\left(4^2\right) = 26.67\ \text{kg}\cdot\text{m}^2$. Referring to Fig. b,

$$\circlearrowleft +\Sigma M_A = \Sigma(M_k)_A; \qquad 20(9.81)\cos 60°(2) = -20(a_G)_x(2\sin 60°)$$

$$- 20(a_G)_y(2\cos 60°) - 26.67\alpha \qquad \textbf{(1)}$$

Kinematics: The acceleration of A is equal to that of the truck. Thus, $a_A = 6\ \text{m/s}^2 \leftarrow$. Applying the relative acceleration equation and referring to Fig. c,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -6\mathbf{i} + (-\alpha\mathbf{k}) \times (-2\cos 60°\,\mathbf{i} + 2\sin 60°\,\mathbf{j}) - \mathbf{0}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = (2\sin 60°\,\alpha - 6)\mathbf{i} + \alpha\mathbf{j}$$

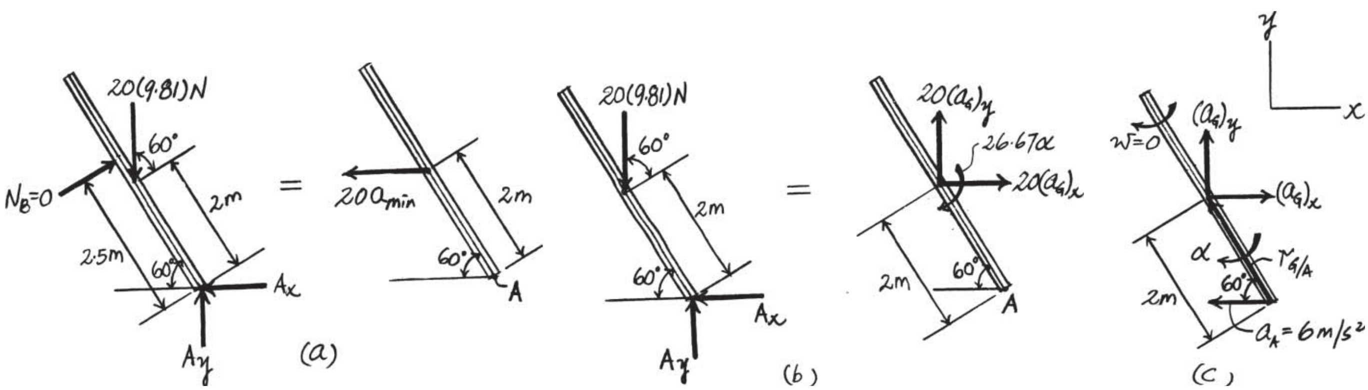
Equating the **i** and **j** components,

$$(a_G)_x = 2\sin 60°\,\alpha - 6 \qquad \textbf{(2)}$$

$$(a_G)_y = \alpha \qquad \textbf{(3)}$$

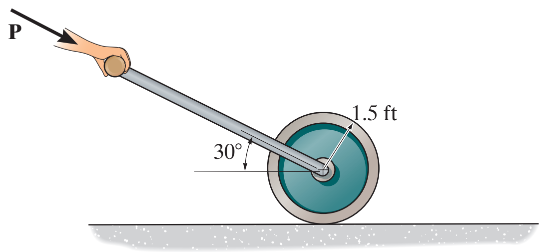
Substituting Eqs. (2) and (3) into Eq. (1),

$$\alpha = 0.1092\ \text{rad/s}^2 = 0.109\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



***17–104.**

If $P = 30$ lb, determine the angular acceleration of the 50-lb roller. Assume the roller to be a uniform cylinder and that no slipping occurs.



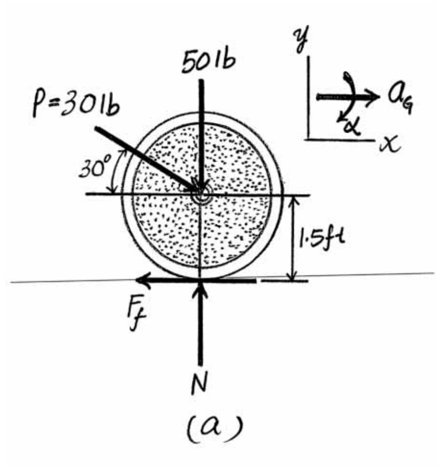
SOLUTION

Equations of Motion: The mass moment of inertia of the roller about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{50}{32.2}\right)\left(1.5^2\right) = 1.7469 \text{ slug} \cdot \text{ft}^2$. We have

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 30 \cos 30° - F_f = \frac{50}{32.2} a_G \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - 50 - 30 \sin 30° = 0 \qquad N = 65 \text{ lb}$$

$$+\Sigma M_G = I_G \alpha; \qquad F_f(1.5) = 1.7469\alpha \qquad \textbf{(2)}$$



Since the roller rolls without slipping,

$$a_G = \alpha r = \alpha(1.5) \qquad \textbf{(3)}$$

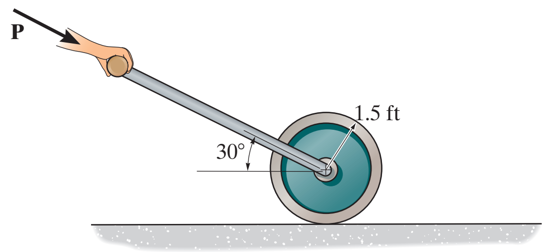
Solving Eqs. (1) through (3) yields

$$\alpha = 7.436 \text{ rad/s}^2 = 7.44 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$F_f = 8.660 \text{ lb} \qquad a_G = 11.15 \text{ ft/s}^2$$

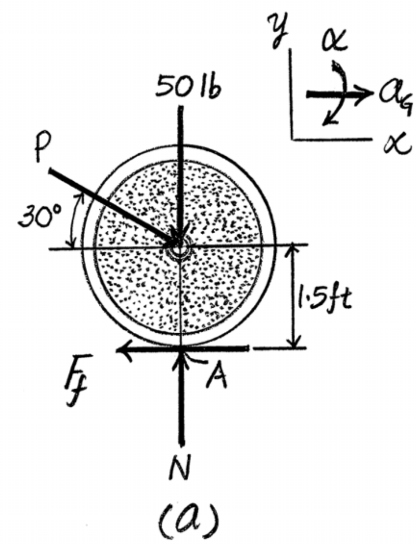
17–105.

If the coefficient of static friction between the 50-lb roller and the ground is $\mu_s = 0.25$, determine the maximum force P that can be applied to the handle, so that roller rolls on the ground without slipping. Also, find the angular acceleration of the roller. Assume the roller to be a uniform cylinder.



SOLUTION

Equations of Motion: The mass moment of inertia of the roller about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{50}{32.2}\right)\left(1.5^2\right) = 1.7469 \text{ slug}\cdot\text{ft}^2$. We have



$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \quad P\cos 30° - F_f = \frac{50}{32.2}a_G \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = m(a_G)_y; \quad N - P\sin 30° - 50 = 0 \qquad \textbf{(2)}$$

$$+\Sigma M_G = I_G\alpha; \qquad F_f(1.5) = 1.7469\alpha \qquad \textbf{(3)}$$

Since the roller is required to be on the verge of slipping,

$$a_G = \alpha r = \alpha(1.5) \qquad \textbf{(4)}$$

$$F_f = \mu_s N = 0.25N \qquad \textbf{(5)}$$

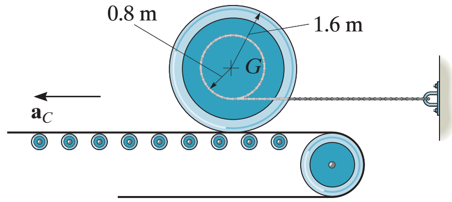
Solving Eqs. (1) through (5) yields

$$\alpha = 18.93 \text{ rad/s}^2 = 18.9 \text{ rad/s}^2 \qquad P = 76.37 \text{ lb} = 76.4 \text{ lb} \qquad \textbf{Ans.}$$

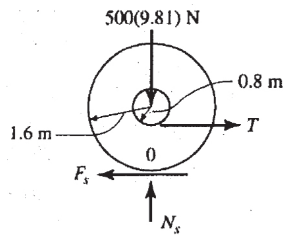
$$N = 88.18 \text{ lb} \qquad a_G = 28.39 \text{ ft/s}^2 \qquad F_f = 22.05 \text{ lb}$$

17–106.

The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.4$. If the conveyor accelerates at $a_C = 1\ \text{m/s}^2$, determine the initial tension in the wire and the angular acceleration of the spool. The spool is originally at rest.



SOLUTION



$\xrightarrow{+} \sum F_x = m(a_G)_x; \qquad -F_s + T = 500a_G$

$+\uparrow \sum F_y = m(a_G)_y; \qquad N_s - 500(9.81) = 0$

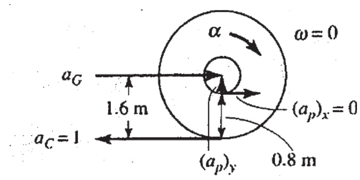
$\circlearrowright + \sum M_G = I_G\alpha; \qquad F_s(1.6) - T(0.8) = 500(1.30)^2\alpha$

$$\mathbf{a}_p = \mathbf{a}_G + \mathbf{a}_{p/G}$$

$$(a_p)_y\mathbf{j} = a_G\,\mathbf{i} - 0.8\alpha\mathbf{i}$$

$$\alpha_G = 0.8\alpha$$

$$N_s = 4905\text{ N}$$



Assume no slipping

$$\alpha = \frac{ac}{0.8} = \frac{1}{0.8} = 1.25\text{ rad/s} \qquad \textbf{Ans.}$$

$$a_G = 0.8(1.25) = 1\ \text{m/s}^2$$

$$T = 2.32\text{ kN} \qquad \textbf{Ans.}$$

$$F_s = 1.82\text{ kN}$$

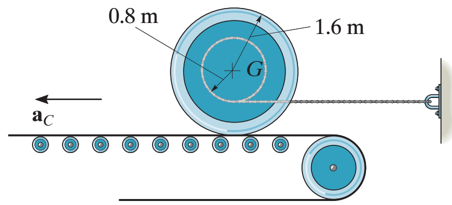
Since

$$(F_s)_{\max} = 0.5(4.905) = 2.45 > 1.82$$

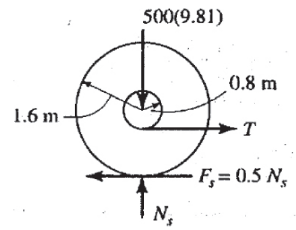
(No slipping occurs)

17–107.

The spool has a mass of 500 kg and a radius of gyration $k_G = 1.30$ m. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s = 0.5$. Determine the greatest acceleration a_C of the conveyor so that the spool will not slip. Also, what are the initial tension in the wire and the angular acceleration of the spool? The spool is originally at rest.



SOLUTION



$\overset{+}{\rightarrow}\sum F_x = m(a_G)_x; \qquad T - 0.5N_S = 500a_G$

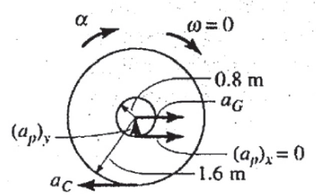
$+\uparrow\sum F_y = m(a_G)_y; \qquad N_s - 500(9.81) = 0$

$\circlearrowleft + \sum M_G = I_G\alpha, \qquad 0.5N_s(1.6) - T(0.8) = 500(1.30)^2\alpha$

$a_p = a_C + a_{p/G}$

$(a_p)_y\mathbf{j} = a_G\mathbf{i} - 0.8\alpha\mathbf{i}$

$a_G = 0.8\alpha$



Solving;

$N_s = 4905$ N

$T = 3.13$ kN **Ans.**

$\alpha = 1.684$ rad/s **Ans.**

$a_G = 1.347$ m/s^2

Since no slipping

$\mathbf{a}_C = \mathbf{a}_G + \mathbf{a}_{C/G}$

$ac = 1.347\mathbf{i} - (1.684)(1.6)\mathbf{i}$

$a_C = 1.35$ m/s^2 **Ans.**

Also,

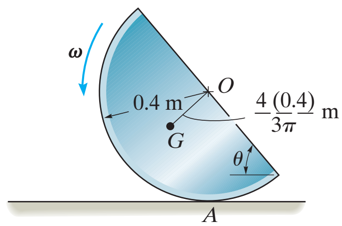
$\circlearrowleft + \sum M_{IC} = I_{IC}\alpha; \qquad 0.5N_s(0.8) = [500(1.30)^2 + 500(0.8)^2]\alpha$

Since $N_S = 4905$ N

$\alpha = 1.684$ rad/s

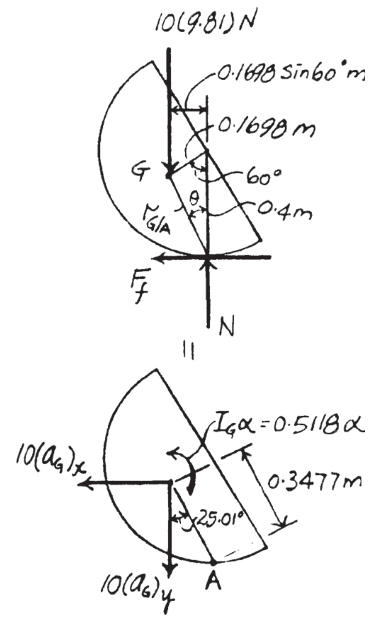
***17–108.**

The semicircular disk having a mass of 10 kg is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. If the coefficient of static friction at A is $\mu_s = 0.5$, determine if the disk slips at this instant.



SOLUTION

Equations of Motion: The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)\left(0.4^2\right) - 10\,(0.1698^2) = 0.5118\ \text{kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)\,(0.4)\cos 60°} = 0.3477$ m Also, using law of sines, $\dfrac{\sin\theta}{0.1698} = \dfrac{\sin 60°}{0.3477}$, $\theta = 25.01°$. Applying Eq. 17–16, we have



$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 10(9.81)(0.1698\sin 60°) = 0.5118\alpha$$
$$+ 10(a_G)_x \cos 25.01°(0.3477)$$
$$+ 10(a_G)_y \sin 25.01°(0.3477) \qquad \textbf{(1)}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_f = 10(a_G)_x \qquad \textbf{(2)}$$

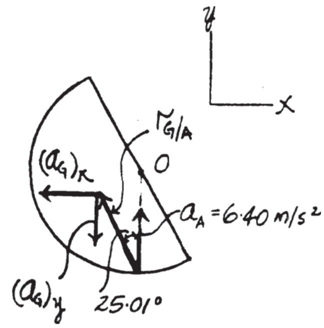
$$+\uparrow F_y = m(a_G)_y; \qquad N - 10(9.81) = -10(a_G)_y \qquad \textbf{(3)}$$

Kinematics: Assume that the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477\sin 25.01°\mathbf{i} + 0.3477\cos 25.01°\mathbf{j}\}$ m $= \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16–18, we have

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k} \times (-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = (2.3523 - 0.3151\,\alpha)\,\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$



Equating **i** and **j** components, we have

$$(a_G)_x = 0.3151\alpha - 2.3523 \qquad \textbf{(4)}$$

$$(a_G)_y = 0.1470\alpha - 1.3581 \qquad \textbf{(5)}$$

Solving Eqs. (1), (2), (3), (4), and (5) yields:

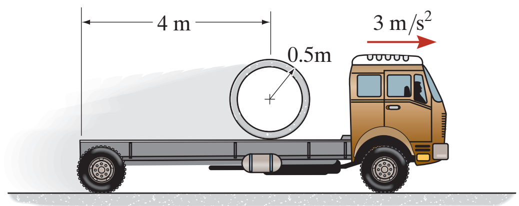
$$\alpha = 13.85\ \text{rad/s}^2 \qquad (a_G)_x = 2.012\ \text{m/s}^2 \qquad (a_G)_y = 0.6779\ \text{m/s}^2$$

$$F_f = 20.12\ \text{N} \qquad N = 91.32\ \text{N}$$

Since $F_f < (F_f)_{max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular **disk does not slip**. **Ans.**

17–109.

The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of $3\,\text{m/s}^2$, determine the culvert's angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.



SOLUTION

Equations of Motion: The mass moment of inertia of the culvert about its mass center is $I_G = mr^2 = 500\left(0.5^2\right) = 125\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A using Fig. *a*,

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \quad 0 = 125\alpha - 500a_G(0.5) \qquad \textbf{(1)}$$

Kinematics: Since the culvert does not slip at A, $(a_A)_t = 3\ \text{m/s}^2$. Applying the relative acceleration equation and referring to Fig. *b*,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 r_{G/A}$$

$$a_G\mathbf{i} - 3\mathbf{i} + (a_A)_n\,\mathbf{j} + (\alpha\mathbf{k} \times 0.5\mathbf{j}) - \omega^2(0.5\mathbf{j})$$

$$a_G\mathbf{i} = (3 - 0.5\alpha)\mathbf{i} + \left[(a_A)_n - 0.5\omega^2\right]\mathbf{j}$$

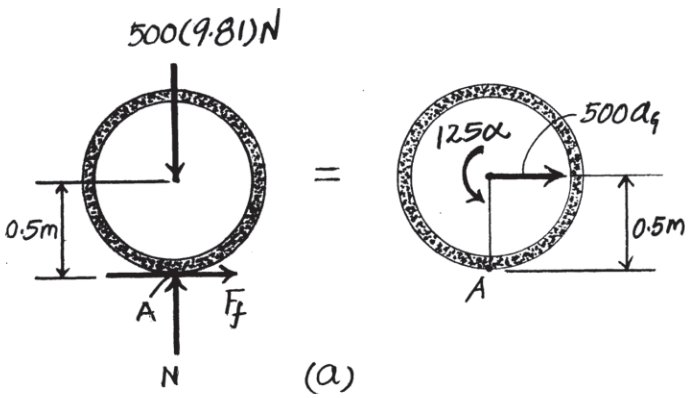
Equating the **i** components,

$$a_G = 3 - 0.5\alpha \qquad \textbf{(2)}$$

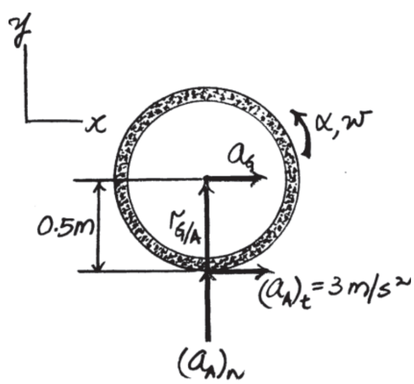
Solving Eqs. (1) and (2) yields

$$a_G = 1.5\ \text{m/s}^2 \rightarrow$$

$$\alpha = 3\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



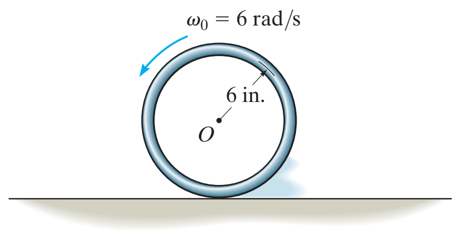
(a)



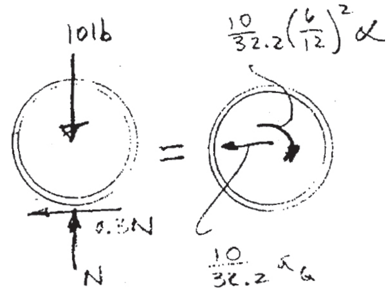
(b)

17–110.

The 10-lb hoop or thin ring is given an initial angular velocity of 6 rad/s when it is placed on the surface. If the coefficient of kinetic friction between the hoop and the surface is $\mu_k = 0.3$, determine the distance the hoop moves before it stops slipping.



SOLUTION



$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N - 10 = 0 \qquad N = 10 \text{ lb}$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad 0.3(10) = \left(\frac{10}{32.2}\right)a_G \qquad a_G = 9.66 \text{ ft/s}^2$

$\circlearrowleft + \Sigma M_G = I_G\alpha;\qquad 0.3(10)\left(\frac{6}{12}\right) = \left(\frac{10}{32.2}\right)\left(\frac{6}{12}\right)^2\alpha \qquad \alpha = 19.32 \text{ rad/s}^2$

When slipping ceases, $v_G = \omega r = 0.5\omega$ **(1)**

$$(\circlearrowleft +) \qquad \omega = \omega_0 + \alpha t$$

$$\omega = 6 + (-19.32)t \qquad \textbf{(2)}$$

$$(\overset{+}{\leftarrow}) \qquad v_G = (v_G)_0 + a_G t$$

$$v_G = 0 + 9.66t \qquad \textbf{(3)}$$

Solving Eqs. (1) to (3) yields:

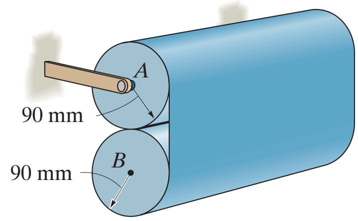
$$t = 0.1553 \text{ s} \qquad v_G = 1.5 \text{ ft/s} \qquad \omega = 3 \text{ rad/s}$$

$$(\overset{+}{\leftarrow}) \qquad s = s_0 + (v_G)_0 t + \tfrac{1}{2}a_G t^2$$

$$= 0 + 0 + \tfrac{1}{2}(9.66)(0.1553)^2$$

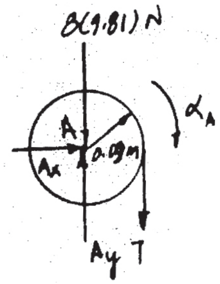
$$= 0.116 \text{ ft} = 1.40 \text{ in.}$$ **Ans.**

17–111.



A long strip of paper is wrapped into two rolls, each having a mass of 8 kg. Roll A is pin supported about its center whereas roll B is not centrally supported. If B is brought into contact with A and released from rest, determine the initial tension in the paper between the rolls and the angular acceleration of each roll. For the calculation, assume the rolls to be approximated by cylinders.

SOLUTION



For roll A.

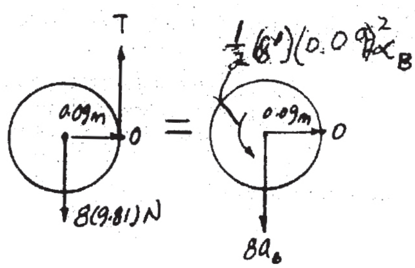
$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad T(0.09) = \tfrac{1}{2}(8)(0.09)^2\alpha_A \qquad \textbf{(1)}$$

For roll B

$$\circlearrowright + \Sigma M_O = \Sigma (M_k)_O; \qquad 8(9.81)(0.09) = \tfrac{1}{2}(8)(0.09)^2\alpha_B + 8a_B(0.09) \qquad \textbf{(2)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y\,; \qquad T - 8(9.81) = -8a_B \qquad \textbf{(3)}$$

Kinematics:



$$\mathbf{a}_B = \mathbf{a}_O + (\mathbf{a}_{B/O})_t + (\mathbf{a}_{B/O})_n$$

$$\begin{bmatrix} a_B \\ \downarrow \end{bmatrix} = \begin{bmatrix} a_O \\ \downarrow \end{bmatrix} + \begin{bmatrix} \alpha_B\,(0.09) \\ \downarrow \end{bmatrix} + [0]$$

$$(+\downarrow) \qquad a_B = a_O + 0.09\alpha_B \qquad \textbf{(4)}$$

also,

$$(+\downarrow) \qquad a_O = \alpha_A\,(0.09) \qquad \textbf{(5)}$$

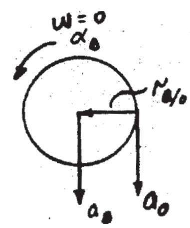
Solving Eqs. (1)–(5) yields:

$\alpha_A = 43.6\ \text{rad/s}^2$ **Ans.**

$\alpha_B = 43.6\ \text{rad/s}^2$ **Ans.**

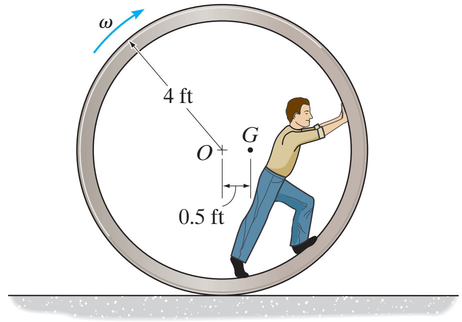
$T = 15.7\ \text{N}$ **Ans.**

$a_B = 7.85\ \text{m/s}^2 \qquad a_O = 3.92\ \text{m/s}^2$



***17–112.**

The circular concrete culvert rolls with an angular velocity of $\omega = 0.5$ rad/s when the man is at the position shown. At this instant the center of gravity of the culvert and the man is located at point G, and the radius of gyration about G is $k_G = 3.5$ ft. Determine the angular acceleration of the culvert. The combined weight of the culvert and the man is 500 lb. Assume that the culvert rolls without slipping, and the man does not move within the culvert.



SOLUTIONS

Equations of Motion: The mass moment of inertia of the system about its mass center is $I_G = mk_G^2 = \frac{500}{32.2}(3.5^2) = 190.22\ \text{slug}\cdot\text{ft}^2$. Writing the moment equation of motion about point A, Fig. a,

$$+\ \Sigma M_A = \Sigma(M_k)_A;\ -500(0.5) = -\frac{500}{32.2}(a_G)_x(4) - \frac{500}{32.2}(a_G)_y(0.5) - 190.22\alpha \quad \textbf{(1)}$$

Kinematics: Since the culvert rolls without slipping,

$$a_0 = \alpha r = \alpha(4) \rightarrow$$

Applying the relative acceleration equation and referrring to Fig. b,

$$a_G = a_O + \alpha \times r_{G/O} - \omega^2 \mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = 4\alpha\mathbf{i} + (-\alpha\mathbf{k}) \times (0.5\mathbf{i}) - (0.5^2)(0.5\mathbf{i})$$

$$(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = (4\alpha - 0.125)\mathbf{i} - 0.5\alpha\mathbf{j}$$

Equation the $\mathbf{i}$ and $\mathbf{j}$ components,

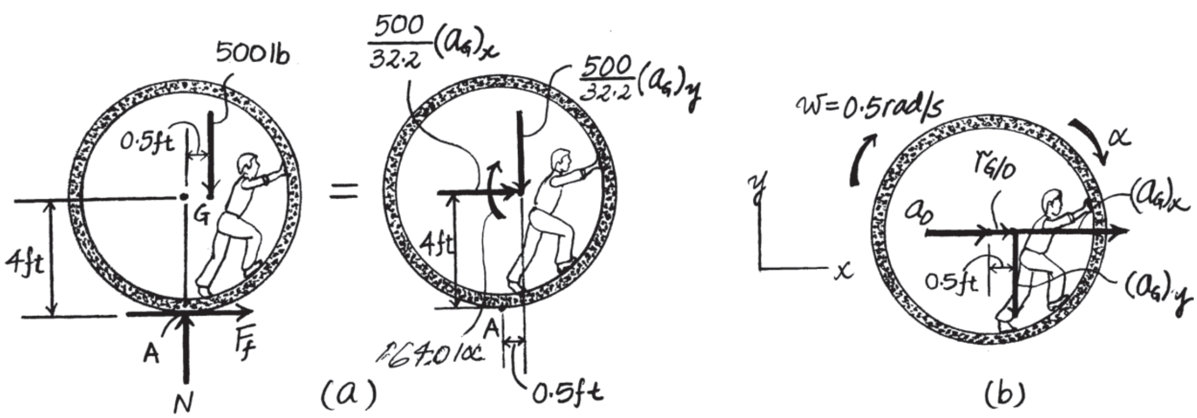
$$(a_G)_x = 4\alpha - 0.125 \quad \textbf{(2)}$$

$$(a_G)_y = 0.5\alpha \quad \textbf{(3)}$$

Subtituting Eqs. (2) and (3) into Eq. (1),

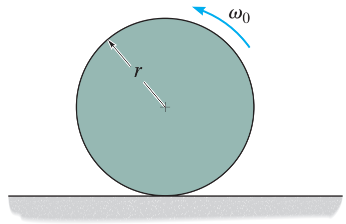
$$-500(0.5) = -\frac{500}{32.2}(4\alpha - 0.125)(4) - \frac{500}{32.2}(0.5\alpha)(0.5) - 190.22\alpha$$

$$\alpha = 0.582\ \text{rad/s}^2 \quad \textbf{Ans.}$$



17–113.

The uniform disk of mass m is rotating with an angular velocity of ω_0 when it is placed on the floor. Determine the initial angular acceleration of the disk and the acceleration of its mass center. The coefficient of kinetic friction between the disk and the floor is μ_k.



SOLUTION

Equations of Motion. Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$. We have

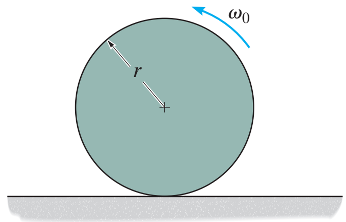
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - mg = 0 \qquad N = mg$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad \mu_k(mg) = ma_G \qquad a_G = \mu_k g \leftarrow \qquad \textbf{Ans.}$$

$$+\Sigma M_G = I_G\alpha; \qquad -\mu_k(mg)r = \left(\frac{1}{2}mr^2\right)\alpha \qquad \alpha = \frac{2\mu_k g}{r} \qquad \textbf{Ans.}$$

17–114.

The uniform disk of mass m is rotating with an angular velocity of ω_0 when it is placed on the floor. Determine the time before it starts to roll without slipping. What is the angular velocity of the disk at this instant? The coefficient of kinetic friction between the disk and the floor is μ_k.



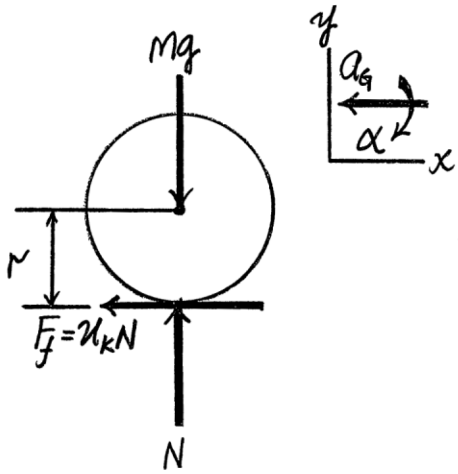
SOLUTION

Equations of Motion: Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$.

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N - mg = 0 \qquad N = mg$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad \mu_k(mg) = ma_G \qquad a_G = \mu_k g$

$+\Sigma M_G = I_G\alpha;\qquad -\mu_k(mg)r = -\left(\frac{1}{2}mr^2\right)\alpha \qquad \alpha = \dfrac{2\mu_k g}{r}$



(a)

Kinematics: At the instant when the disk rolls without slipping, $v_G = \omega r$. Thus,

$(\overset{+}{\leftarrow})$

$$v_G = (v_G)_0 + a_G t$$

$$\omega r = 0 + \mu_k g t$$

$$t = \frac{\omega r}{\mu_k g} \qquad (1)$$

and

$$\omega = \omega_0 + \alpha t$$

$(\circlearrowleft +)$

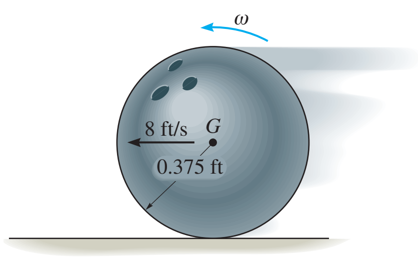
$$\omega = \omega_0 + \left(-\frac{2\mu_k g}{r}\right)t \qquad (2)$$

Solving Eqs. (1) and (2) yields

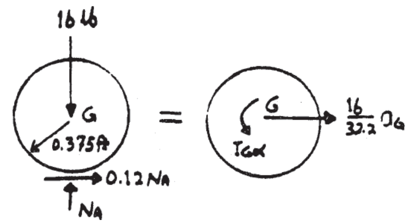
$$\omega = \frac{1}{3}\omega_0 \qquad t = \frac{\omega_0 r}{3\mu_k g} \qquad \textbf{Ans.}$$

17–115.

The 16-lb bowling ball is cast horizontally onto a lane such that initially $\omega = 0$ and its mass center has a velocity $v = 8$ ft/s. If the coefficient of kinetic friction between the lane and the ball is $\mu_k = 0.12$, determine the distance the ball travels before it rolls without slipping. For the calculation, neglect the finger holes in the ball and assume the ball has a uniform density.



SOLUTION



$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.12N_A = \frac{16}{32.2}a_G$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 16 = 0$$

$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad 0.12N_A(0.375) = \left[\frac{2}{5}\left(\frac{16}{32.2}\right)(0.375)^2\right]\alpha$$

Solving,

$$N_A = 16 \text{ lb}; \qquad a_G = 3.864 \text{ ft/s}^2; \qquad \alpha = 25.76 \text{ rad/s}^2$$

When the ball rolls without slipping $v = \omega(0.375)$,

$$(\circlearrowleft +) \qquad \omega = \omega_0 + \alpha_c t$$

$$\frac{v}{0.375} = 0 + 25.76t$$

$$v = 9.660t$$

$$(\xleftarrow{+}) \qquad v = v_0 + a_c t$$

$$9.660t = 8 - 3.864t$$

$$t = 0.592 \text{ s}$$

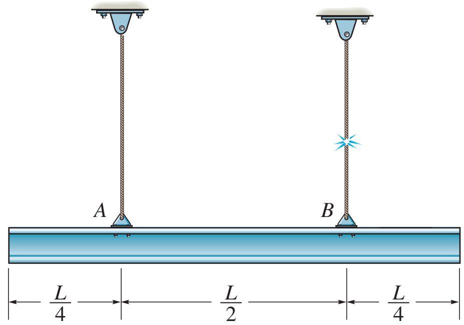
$$(\xleftarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s = 0 + 8(0.592) - \frac{1}{2}(3.864)(0.592)^2$$

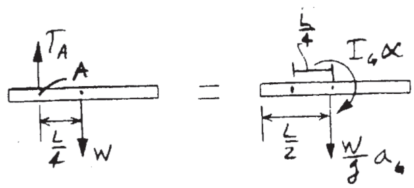
$$s = 4.06 \text{ ft} \qquad \textbf{Ans.}$$

***17–116.**

The uniform beam has a weight W. If it is originally at rest while being supported at A and B by cables, determine the tension in cable A if cable B suddenly fails. Assume the beam is a slender rod.



SOLUTION



$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_A - W = -\frac{W}{g}a_G$$

$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad W\left(\frac{L}{4}\right) = \left[\frac{1}{12}\left(\frac{W}{g}\right)L^2\right]\alpha + \frac{W}{g}\left(\frac{L}{4}\right)\alpha\left(\frac{L}{4}\right)$$

$$1 = \frac{1}{g}\left(\frac{L}{4} + \frac{L}{3}\right)\alpha$$

Since $a_G = \alpha\left(\frac{L}{4}\right)$.

$$\alpha = \frac{12}{7}\left(\frac{g}{L}\right)$$

$$T_A = W - \frac{W}{g}(\alpha)\left(\frac{L}{4}\right) = W - \frac{W}{g}\left(\frac{12}{7}\right)\left(\frac{g}{L}\right)\left(\frac{L}{4}\right)$$

$$T_A = \frac{4}{7}W$$ **Ans.**

Also,

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_A - W = -\frac{W}{g}a_G$$

$$\circlearrowleft + \Sigma M_G = I_G\,\alpha; \qquad T_A\left(\frac{L}{4}\right) = \left[\frac{1}{12}\left(\frac{W}{g}\right)L^2\right]\alpha$$

Since $a_G = \frac{L}{4}\alpha$

$$T_A = \frac{1}{3}\left(\frac{W}{g}\right)L\alpha$$

$$\frac{1}{3}\left(\frac{W}{g}\right)L\alpha - W = -\frac{W}{g}\left(\frac{L}{4}\right)\alpha$$

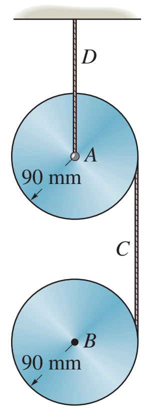
$$\alpha = \frac{12}{7}\left(\frac{g}{L}\right)$$

$$T_A = \frac{1}{3}\left(\frac{W}{g}\right)L\left(\frac{12}{7}\right)\left(\frac{g}{L}\right)$$

$$T_A = \frac{4}{7}W$$ **Ans.**

17–117.

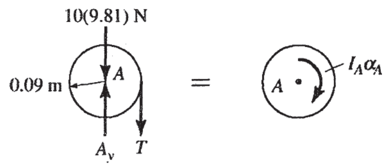
A cord C is wrapped around each of the two 10-kg disks. If they are released from rest, determine the tension in the fixed cord D. Neglect the mass of the cord.



SOLUTION

For A:

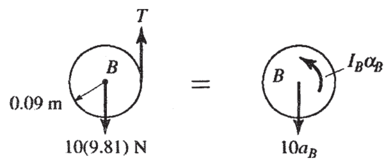
$$\circlearrowright + \sum M_A = I_A \alpha_A; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_A \qquad \textbf{(1)}$$



For B:

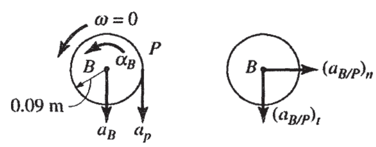
$$\circlearrowleft + \sum M_B = I_B \alpha_B; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_B \qquad \textbf{(2)}$$

$$+\downarrow \sum F_y = m(a_B)_y; \qquad 10(9.81) - T = 10a_B \qquad \textbf{(3)}$$



$$a_B = a_P + (a_{B/P})_t + (a_{B/P})_n$$

$$(+\downarrow) a_B = 0.09\alpha_A + 0.09\alpha_B + 0 \qquad \textbf{(4)}$$



Solving.

$$a_B = 7.85 \text{ m/s}^2$$

$$\alpha_A = 43.6 \text{ rad/s}^2$$

$$\alpha_B = 43.6 \text{ rad/s}^2$$

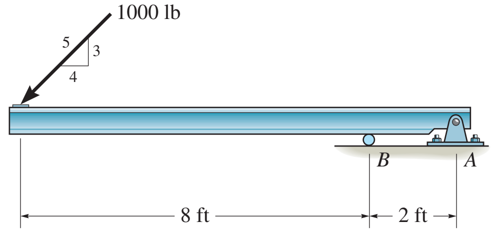
$$T = 19.6 \text{ N} \qquad \textbf{Ans.}$$

$$A_y = 10(9.81) + 19.62$$

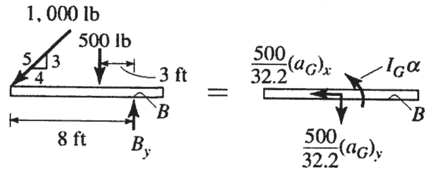
$$= 118 \text{ N}$$

17–118.

The 500-lb beam is supported at A and B when it is subjected to a force of 1000 lb as shown. If the pin support at A suddenly fails, determine the beam's initial angular acceleration and the force of the roller support on the beam. For the calculation, assume that the beam is a slender rod so that its thickness can be neglected.



SOLUTION



$$\overset{+}{\leftarrow}\sum F_x = m(a_G)_x; \qquad 1000\left(\frac{4}{5}\right) = \frac{500}{32.2}(a_G)_x$$

$$+\downarrow\sum F_y = m(a_G)_y; \qquad 1000\left(\frac{3}{5}\right) + 500 - B_y = \frac{500}{32.2}(a_G)_y$$

$$\circlearrowleft + \sum M_B = \sum(M_k)_B; \quad 500(3) + 1000\left(\frac{3}{5}\right)(8) = \frac{500}{32.2}(a_G)_y(3) + \left[\frac{1}{12}\left(\frac{500}{32.2}\right)(10)^2\right]\alpha$$

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$$

$$-a_B\mathbf{i} = -(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} + \alpha(3)\mathbf{j}$$

$$(+\downarrow) \quad (a_G)_y = \alpha(3)$$

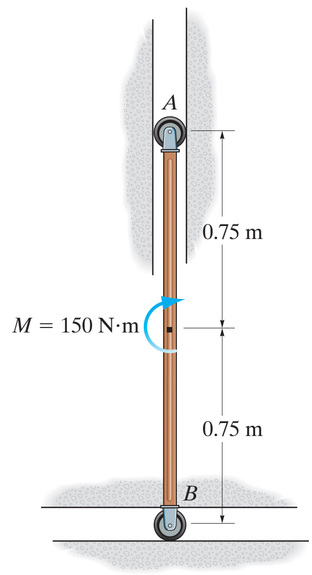
$\alpha = 23.4 \text{ rad/s}^2$ **Ans.**

$B_y = 9.62 \text{ lb}$ **Ans.**

$B_y > 0$ means that the beam stays in contact with the roller support.

17–119.

The 30-kg uniform slender rod AB rests in the position shown when the couple moment of $M = 150\ \text{N}\cdot\text{m}$ is applied. Determine the initial angular acceleration of the rod. Neglect the mass of the rollers.



SOLUTION

Equations of Motion: Here, the mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(30)(1.5^2) = 5.625\ \text{kg}\cdot\text{m}^2$. Writing the moment equations of motion about the intersection point A of the lines of action of $\mathbf{N}_A$ and $\mathbf{N}_B$ and using, Fig. a,

$$+\Sigma M_A = \Sigma(M_k)_A; \qquad -150 = 30(a_G)_x(0.75) - 5.625\alpha$$

$$5.625\alpha - 22.5(a_G)_x = 150 \qquad \textbf{(1)}$$

Kinematics: Applying the relative acceleration equation to points A and G, Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -a_A\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.75\mathbf{j}) - \mathbf{0}$$

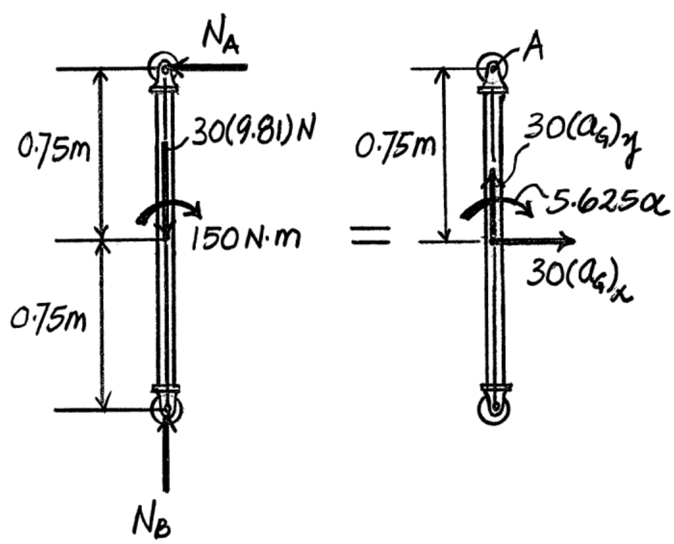
$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -0.75\alpha\mathbf{i} - a_A\mathbf{j}$$

Equating the $\mathbf{i}$ components,

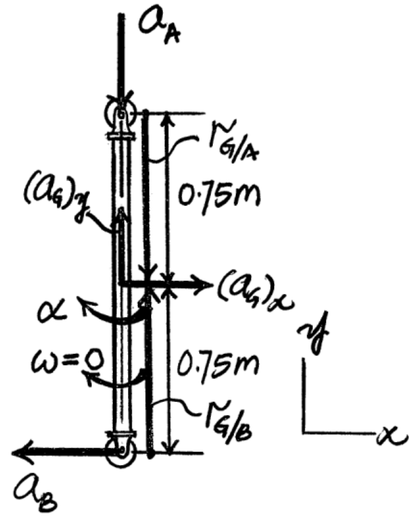
$$(a_G)_x = -0.75\alpha \qquad \textbf{(2)}$$

Substituting Eq. (2) into Eq. (1),

$$\alpha = 6.667\ \text{rad/s}^2 = 6.67\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



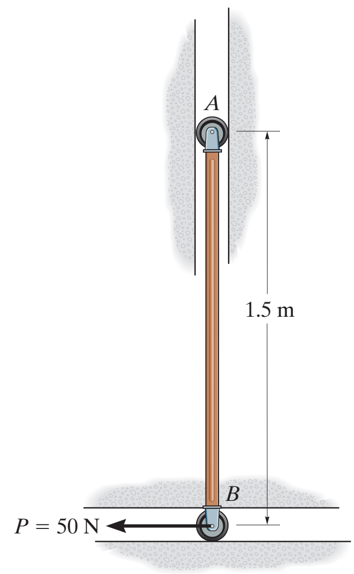
(a)



(b)

***17–120.**

The 30-kg slender rod AB rests in the position shown when the horizontal force $P = 50\text{ N}$ is applied. Determine the initial angular acceleration of the rod. Neglect the mass of the rollers.



SOLUTION

Equations of Motion: Here, the mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(30)(1.5^2) = 5.625\text{ kg}\cdot\text{m}^2$. Writing the moment equations of motion about the intersection point A of the lines of action of $\mathbf{N}_A$ and $\mathbf{N}_B$ and using, Fig. a,

$$+\Sigma M_A = \Sigma(M_k)_A; \qquad -50(0.15) = 30(a_G)_x(0.75) - 5.625\alpha$$

$$5.625\alpha - 22.5(a_G)_x = 75 \qquad \textbf{(1)}$$

Kinematics: Applying the relative acceleration equation to points A and G, Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -a_A\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.75\mathbf{j}) - \mathbf{0}$$

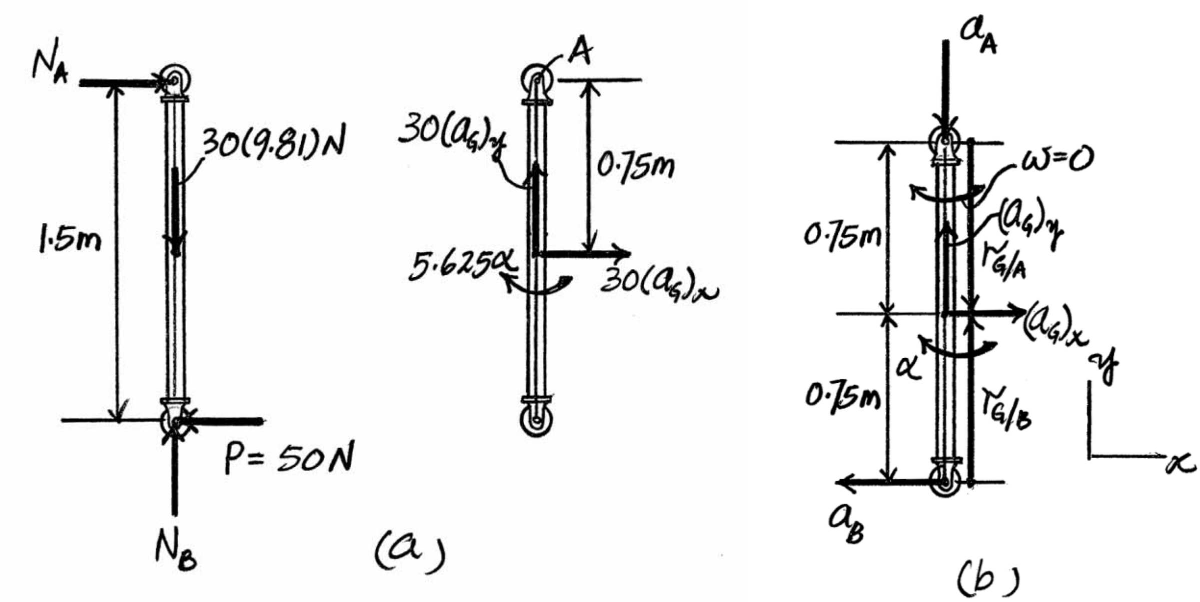
$$(a_G)_x\mathbf{i} + (a_G)_y\mathbf{j} = -0.75\alpha\mathbf{i} - a_A\mathbf{j}$$

Equating the **i** components,

$$(a_G)_x = -0.75\alpha \qquad \textbf{(2)}$$

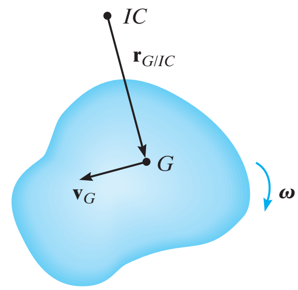
Substituting Eq. (2) into Eq. (1),

$$\alpha = 3.333\text{ rad/s}^2 = 3.33\text{ rad/s}^2 \qquad \textbf{Ans.}$$



18–1.

At a given instant the body of mass m has an angular velocity $\boldsymbol{\omega}$ and its mass center has a velocity $\mathbf{v}_G$. Show that its kinetic energy can be represented as $T = \frac{1}{2}I_{IC}\omega^2$, where I_{IC} is the moment of inertia of the body determined about the instantaneous axis of zero velocity, located a distance $r_{G/IC}$ from the mass center as shown.



SOLUTION

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\,\omega^2 \qquad \text{where } v_G = \omega r_{G/IC}$$

$$= \frac{1}{2}m(\omega r_{G/IC})^2 + \frac{1}{2}I_G\,\omega^2$$

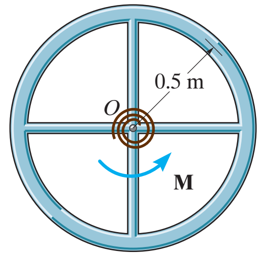
$$= \frac{1}{2}\left(mr_{G/IC}^2 + I_G\right)\omega^2 \qquad \text{However } mr_{G/IC}^2 + I_G = I_{IC}$$

$$= \frac{1}{2}I_{IC}\,\omega^2$$

Q.E.D.

18–2.

The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2\ \mathrm{N \cdot m/rad}$, and the wheel is rotated until the torque $M = 25\ \mathrm{N \cdot m}$ is developed, determine the maximum angular velocity of the wheel if it is released from rest.



SOLUTION

Kinetic Energy and Work: The mass moment of inertia of the wheel about point O is

$$I_O = m_R r^2 + 2\left(\frac{1}{12} m_r l^2\right)$$

$$= 5(0.5^2) + 2\left[\frac{1}{12}(2)(1^2)\right]$$

$$= 1.5833\ \mathrm{kg \cdot m^2}$$

Thus, the kinetic energy of the wheel is

$$T = \frac{1}{2} I_O \omega^2 = \frac{1}{2}(1.5833)\,\omega^2 = 0.79167\,\omega^2$$

Since the wheel is released from rest, $T_1 = 0$. The torque developed is $M = k\theta = 2\theta$. Here, the angle of rotation needed to develop a torque of $M = 25\ \mathrm{N \cdot m}$ is

$$2\theta = 25 \qquad \theta = 12.5\ \mathrm{rad}$$

The wheel achieves its maximum angular velocity when the spacing is unwound that is when the wheel has rotated $\theta = 12.5$ rad. Thus, the work done by $\underset{\infty}{\mathbf{M}}$ is

$$U_M = \int M d\theta = \int_0^{12.5\ \mathrm{rad}} 2\theta\, d\theta$$

$$= \theta^2 \Big|_0^{12.5\ \mathrm{rad}} = 156.25\ \mathrm{J}$$

Principle of Work and Energy:

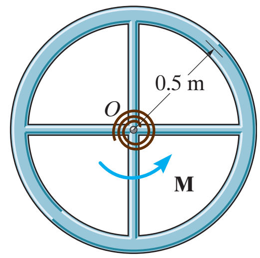
$$T_1 + \Sigma u_{1-2} = T_2$$

$$0 + 156.25 = 0.79167\,\omega^2$$

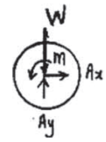
$$\omega = 14.0\ \mathrm{rad/s}$$ **Ans.**

18–3.

The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2\ \mathrm{N\cdot m/rad}$, so that the torque on the center of the wheel is $M = (2\theta)\ \mathrm{N\cdot m}$, where θ is in radians, determine the maximum angular velocity of the wheel if it is rotated two revolutions and then released from rest.



SOLUTION



$$I_o = 2\left[\frac{1}{12}(2)(1)^2\right] + 5(0.5)^2 = 1.583$$

$$T_1 + \Sigma U_{1-2} = T_2$$

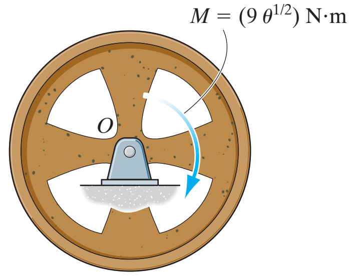
$$0 + \int_0^{4\pi} 2\theta\, d\theta = \frac{1}{2}(1.583)\,\omega^2$$

$$(4\pi)^2 = 0.7917\omega^2$$

$\omega = 14.1$ rad/s **Ans.**

***18–4.**

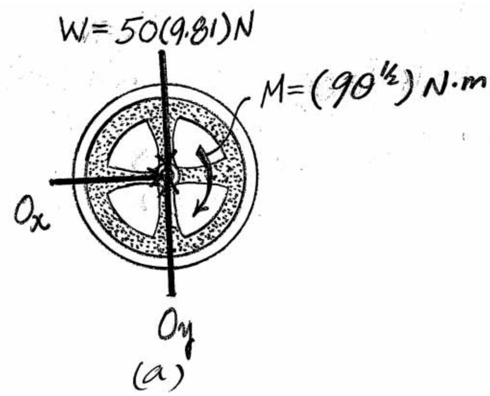
The 50-kg flywheel has a radius of gyration of $k_0 = 200$ mm about its center of mass. If it is subjected to a torque of $M = (9\theta^{1/2})$ N · m, where θ is in radians, determine its angular velocity when it has rotated 5 revolutions, starting from rest.



SOLUTION

Kinetic Energy and Work: The mass moment inertia of the flywheel about its mass center is $I_O = mk_O^2 = 50(0.2^2) = 2$ kg · m^2. Thus, the kinetic energy of the flywheel is

$$T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}(2)\omega^2 = \omega^2$$



Since the wheel is initially at rest, $T_1 = 0$. Referring to Fig. *a*, **W**, $\mathbf{O}_x$, and $\mathbf{O}_y$ do no work while **M** does positive work. When the wheel rotates $\theta = (5 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 10\pi$, the work done by M is

$$U_M = \int M d\theta = \int_0^{10\pi} 9\theta^{1/2} d\theta$$

$$= 6\theta^{3/2}\Big|_0^{10\pi}$$

$$= 1056.52 \text{ J}$$

Principle of Work and Energy:

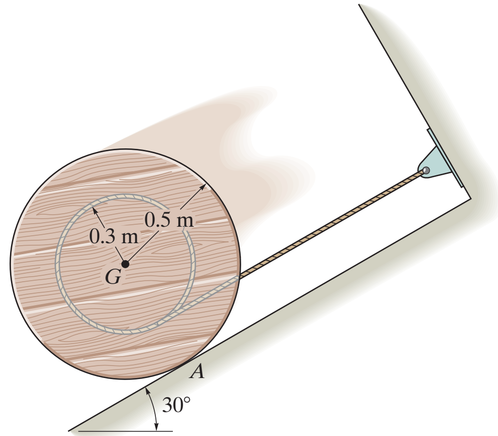
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 1056.52 = \omega^2$$

$$\omega = 32.5 \text{ rad/s} \qquad \textbf{Ans.}$$

18–5.

The spool has a mass of 60 kg and a radius of gyration $k_G = 0.3$ m. If it is released from rest, determine how far its center descends down the smooth plane before it attains an angular velocity of $\omega = 6$ rad/s. Neglect friction and the mass of the cord which is wound around the central core.

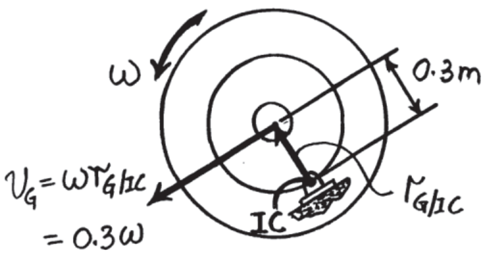


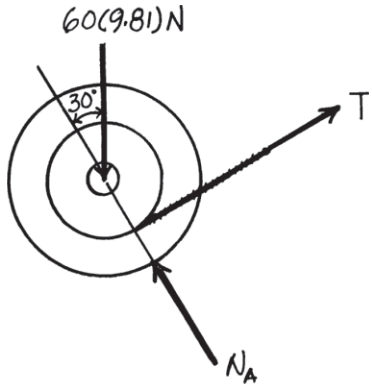
SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 60(9.81)\sin 30°(s) = \frac{1}{2}\left[60(0.3)^2\right](6)^2 + \frac{1}{2}(60)\left[0.3(6)\right]^2$$

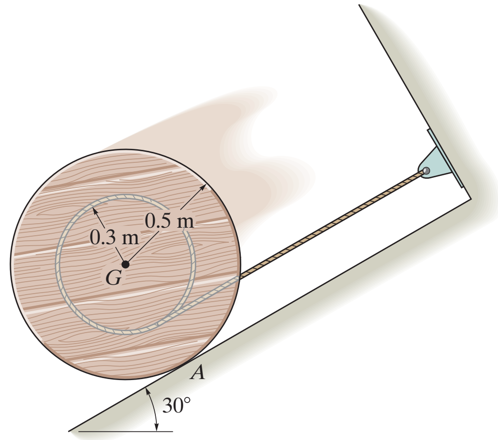
$$s = 0.661 \text{ m}$$ **Ans.**





18–6.

Solve Prob. 18–5 if the coefficient of kinetic friction between the spool and plane at A is $\mu_k = 0.2$.



SOLUTION

$$\frac{s_G}{0.3} = \frac{s_A}{(0.5 - 0.3)}$$

$$s_A = 0.6667 s_G$$

$$+\nwarrow \Sigma F_y = 0; \qquad N_A - 60(9.81)\cos 30^\circ = 0$$

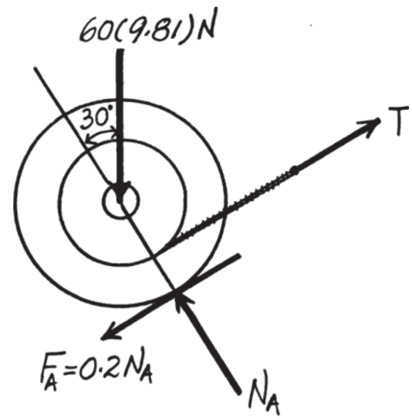
$$N_A = 509.7 \text{ N}$$

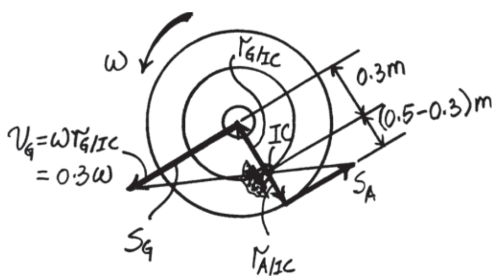
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 60(9.81)\sin 30^\circ(s_G) - 0.2(509.7)(0.6667 s_G) = \frac{1}{2}\left[60(0.3)^2\right](6)^2$$

$$+ \frac{1}{2}(60)\left[(0.3)(6)\right]^2$$

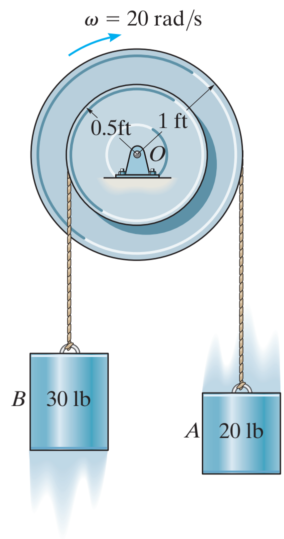
$$s_G = 0.859 \text{ m} \qquad \textbf{Ans.}$$





18–7.

The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a radius of gyration about its center of $k_O = 0.6$ ft. If it rotates with an angular velocity of 20 rad/s clockwise, determine the kinetic energy of the system. Assume that neither cable slips on the pulley.



SOLUTION

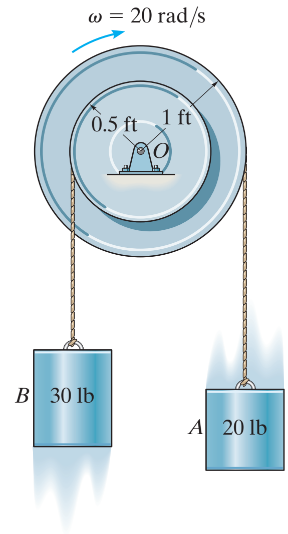
$$T = \frac{1}{2} I_O \omega_O^2 + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$T = \frac{1}{2}\left(\frac{50}{32.2}(0.6)^2\right)(20)^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)\left[(20)(1)\right]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)\left[(20)(0.5)\right]^2$$

$$= 283 \text{ ft} \cdot \text{lb}$$ **Ans.**

*18–8.

The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a centroidal radius of gyration of $k_O = 0.6$ ft and is turning with an angular velocity of 20 rad/s clockwise. Determine the angular velocity of the pulley at the instant the 20-lb weight moves 2 ft downward.



SOLUTION

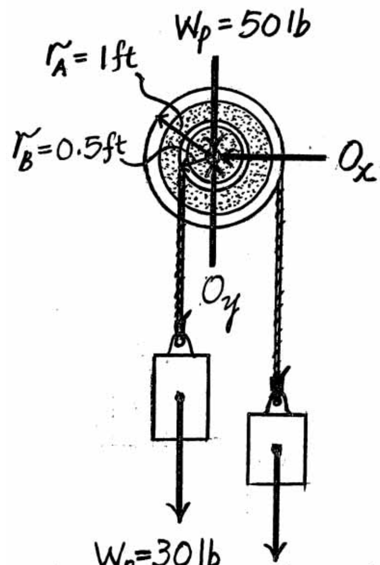
Kinetic Energy and Work: Since the pulley rotates about a fixed axis, $v_A = \omega r_A = \omega(1)$ and $v_B = \omega r_B = \omega(0.5)$. The mass moment of inertia of the pulley about point O is $I_O = mk_O{}^2 = \left(\frac{50}{32.2}\right)(0.6^2) = 0.5590\ \text{slug}\cdot\text{ft}^2$. Thus, the kinetic energy of the system is

$$T = \frac{1}{2}I_O\omega^2 + \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B{}^2$$

$$= \frac{1}{2}(0.5590)\omega^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)[\omega(1)]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)[\omega(0.5)]^2$$

$$= 0.7065\omega^2$$

Thus, $T_1 = 0.7065(20^2) = 282.61$ ft·lb. Referring to the FBD of the system shown in Fig. *a*, we notice that $\mathbf{O}_x$, $\mathbf{O}_y$, and $\mathbf{W}_p$ do no work while $\mathbf{W}_A$ does positive work and $\mathbf{W}_B$ does negative work. When A moves 2 ft downward, the pulley rotates



(a)

$$\theta = \frac{S_A}{r_A} = \frac{S_B}{r_B}$$

$$\frac{2}{1} = \frac{S_B}{0.5}$$

$$S_B = 2(0.5) = 1\text{ ft}\uparrow$$

Thus, the work of $\mathbf{W}_A$ and $\mathbf{W}_B$ are

$$U_{W_A} = W_AS_A = 20(2) = 40\text{ ft}\cdot\text{lb}$$

$$U_{W_B} = -W_BS_B = -30(1) = -30\text{ ft}\cdot\text{lb}$$

Principle of Work and Energy:

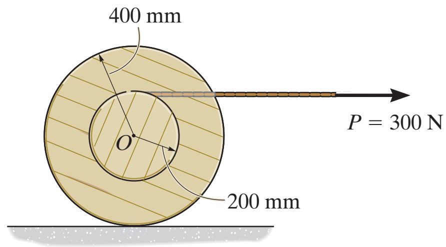
$$T_1 + U_{1-2} = T_2$$

$$282.61 + [40 + (-30)] = 0.7065\,\omega^2$$

$$\omega = 20.4\text{ rad/s}$$ **Ans.**

18–9.

If the cable is subjected to force of $P = 300$ N, and the spool starts from rest, determine its angular velocity after its center of mass O has moved 1.5 m. The mass of the spool is 100 kg and its radius of gyration about its center of mass is $k_O = 275$ mm. Assume that the spool rolls without slipping.



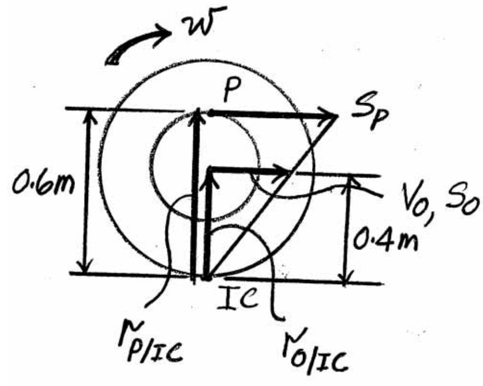
SOLUTION

Kinetic Energy and Work: Referring to Fig. a, we have

$$v_O = \omega r_{O/IC} = \omega(0.4)$$

The mass moment of inertia of the spool about its mass center is $I_O = mk_O{}^2 = 100(0.275^2) = 7.5625\ \text{kg} \cdot \text{m}^2$. Thus, the kinetic energy of the spool is

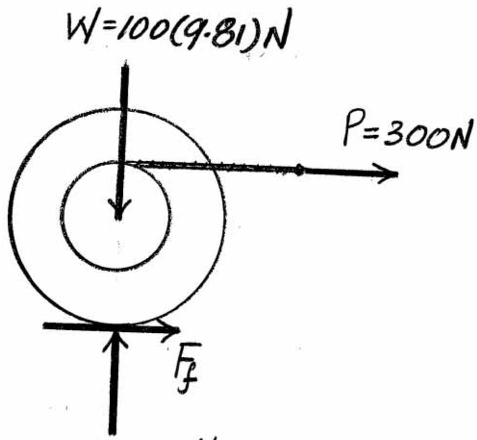
$$\begin{aligned} T &= \frac{1}{2} mv_O{}^2 + \frac{1}{2} I_O\omega^2 \\ &= \frac{1}{2}(100)[\omega(0.4)]^2 + \frac{1}{2}(7.5625)\omega^2 \\ &= 11.78125\omega^2 \end{aligned}$$



(a)

Since the spool is initially at rest, $T_1 = 0$. Referring to Fig. b, **W**, **N**, and $\mathbf{F}_f$ do no work **P** does positive work. When the center O of the spool moves to the right by $S_O = 1.5$ m, **P** displaces $s_P = \dfrac{r_{P/IC}}{r_{O/IC}} s_O = \left(\dfrac{0.6}{0.4}\right)(1.5) = 2.25$ m. Thus, the work done by **P** is

$$U_P = Ps_p = 300(2.25) = 675\ \text{J}$$



(b)

Principle of Work and Energy:

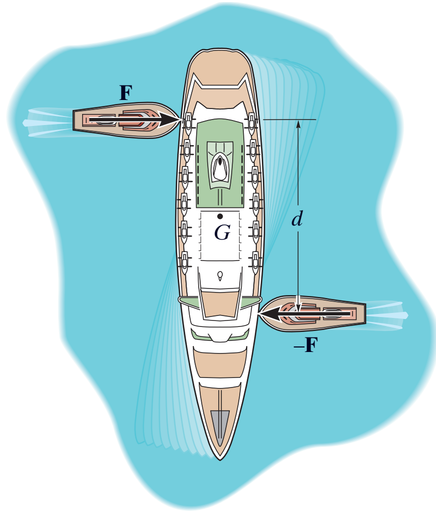
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 675 = 11.78125\omega^2$$

$$\omega = 7.57\ \text{rad/s} \qquad \textbf{Ans.}$$

18–10.

The two tugboats each exert a constant force **F** on the ship. These forces are always directed perpendicular to the ship's centerline. If the ship has a mass m and a radius of gyration about its center of mass G of k_G, determine the angular velocity of the ship after it turns 90°. The ship is originally at rest.



SOLUTION

Principle of Work and Energy: The two tugboats create a couple moment of $M = Fd$ to rotate the ship through an angular displacement of $\theta = \dfrac{\pi}{2}$ rad. The mass moment of inertia about its mass center is $I_G = mk_G^2$. Applying Eq. 18–14, we have

$$T_1 + \sum U_{1-2} = T_2$$

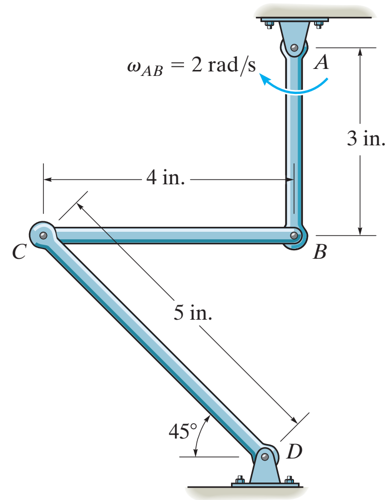
$$0 + M\theta = \frac{1}{2} I_G \omega^2$$

$$0 + Fd\left(\frac{\pi}{2}\right) = \frac{1}{2}\left(mk_G^2\right)\omega^2$$

$$\omega = \frac{1}{k_G}\sqrt{\frac{\pi F d}{m}} \qquad \textbf{Ans.}$$

18–11.

At the instant shown, link AB has an angular velocity $\omega_{AB} = 2$ rad/s. If each link is considered as a uniform slender bar with a weight of 0.5 lb/in., determine the total kinetic energy of the system.



SOLUTION

$$\omega_{BC} = \frac{6}{4} = 1.5 \text{ rad/s}$$

$$v_C = 1.5(4\sqrt{2}) = 8.4853 \text{ in./s}$$

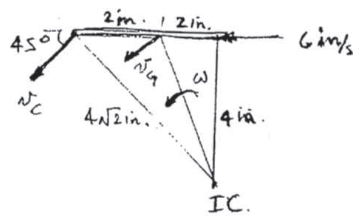
$$r_{IC-G} = \sqrt{(2)^2 + (4)^2} = 4.472$$

$$v_G = 1.5(4.472) = 6.7082 \text{ in./s}$$

$$\omega_{DC} = \frac{8.4853}{5} = 1.697 \text{ rad/s}$$

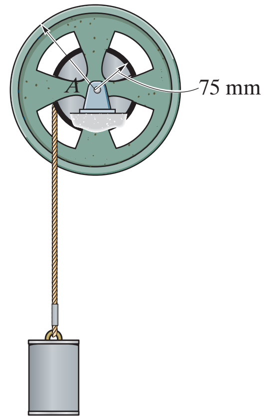
$$T = \frac{1}{2}\left[\frac{1}{3}\left(\frac{3(0.5)}{32.2}\right)\left(\frac{3}{12}\right)^2\right](2)^2 + \frac{1}{2}\left[\frac{4(0.5)}{32.2}\right]\left(\frac{6.7082}{12}\right)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{4(0.5)}{32.2}\right)\left(\frac{4}{12}\right)^2\right](1.5)^2$$

$$+ \frac{1}{2}\left[\frac{1}{3}\left(\frac{5(0.5)}{32.2}\right)\left(\frac{5}{12}\right)^2\right](1.697)^2 = 0.0188 \text{ ft} \cdot \text{lb} \qquad \textbf{Ans.}$$



***18–12.**

Determine the velocity of the 50-kg cylinder after it has descended a distance of 2 m. Initially, the system is at rest. The reel has a mass of 25 kg and a radius of gyration about its center of mass A of $k_A = 125$ mm.

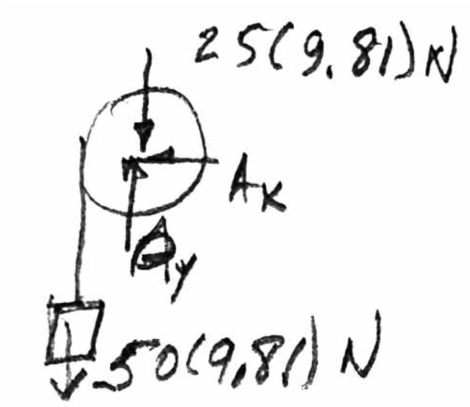


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

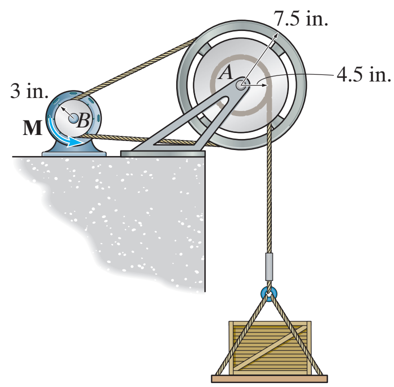
$$0 + 50(9.81)(2) = \frac{1}{2}[(25)(0.125)^2]\left(\frac{v}{0.075}\right)^2 + \frac{1}{2}(50)\,v^2$$

$$v = 4.05 \text{ m/s}$$ **Ans.**



18–13.

The wheel and the attached reel have a combined weight of 50 lb and a radius of gyration about their center of $k_A = 6$ in. If pulley B attached to the motor is subjected to a torque of $M = 40(2 - e^{-0.1\theta})$ lb · ft, where θ is in radians, determine the velocity of the 200-lb crate after it has moved upwards a distance of 5 ft, starting from rest. Neglect the mass of pulley B.



SOLUTION

Kinetic Energy and Work: Since the wheel rotates about a fixed axis $v_C = \omega r_C = \omega(0.375)$. The mass moment of inertia of A about its mass center is $I_A = mk_A{}^2 = \left(\frac{50}{32.2}\right)(0.5^2) = 0.3882 \text{ slug} \cdot \text{ft}^2$. Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2} I_A \omega^2 + \frac{1}{2} m_C v_C{}^2$$

$$= \frac{1}{2}(0.3882)\omega^2 + \frac{1}{2}\left(\frac{200}{32.2}\right)\left[\omega(0.375)\right]^2$$

$$= 0.6308\omega^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{W}_A$ do no work, $\mathbf{M}$ does positive work, and $\mathbf{W}_C$ does negative work. When crate C moves 5 ft upward, wheel A rotates through an angle of $\theta_A = \frac{s_C}{r} = \frac{5}{0.375} = 13.333$ rad. Then, pulley B rotates through an angle of $\theta_B = \frac{r_A}{r_B}\theta_A = \left(\frac{0.625}{0.25}\right)(13.333) = 33.33$ rad. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}_C$ is

$$U_M = \int M d\theta_B = \int_0^{33.33 \text{ rad}} 40\left(2 - e^{-0.1\theta}\right)d\theta$$

$$= \left[40\left(2\theta + 10e^{-0.1\theta}\right)\right]\Bigg|_0^{33.33 \text{ rad}}$$

$$= 2280.93 \text{ ft} \cdot \text{lb}$$

$$U_{W_C} = -W_C s_C = -200(5) = -1000 \text{ ft} \cdot \text{lb}$$

Principle of Work and Energy:

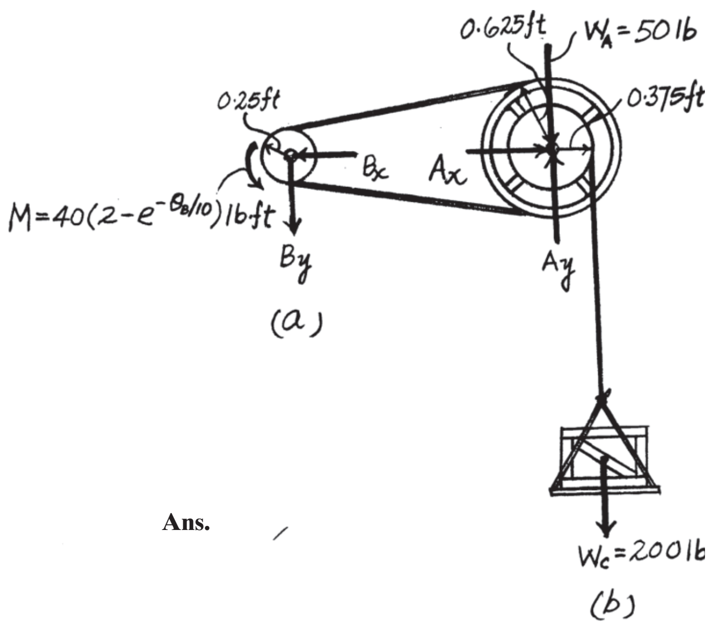
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [2280.93 - 1000] = 0.6308\omega^2$$

$$\omega = 45.06 \text{ rad/s}$$

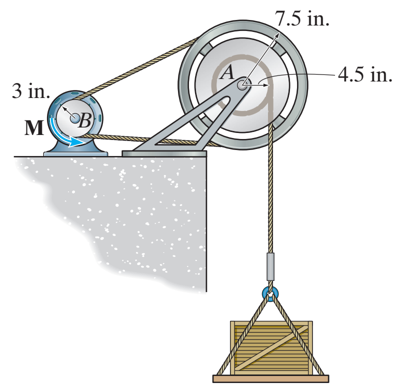
Thus,

$$v_C = 45.06(0.375) = 16.9 \text{ ft/s} \uparrow$$ **Ans.**



18–14.

The wheel and the attached reel have a combined weight of 50 lb and a radius of gyration about their center of $k_A = 6$ in. If pulley B that is attached to the motor is subjected to a torque of $M = 50\text{ lb}\cdot\text{ft}$, determine the velocity of the 200-lb crate after the pulley has turned 5 revolutions. Neglect the mass of the pulley.



SOLUTION

Kinetic Energy and Work: Since the wheel at A rotates about a fixed axis, $v_C = \omega r_C = \omega(0.375)$. The mass moment of inertia of wheel A about its mass center is $I_A = mk_A{}^2 = \left(\frac{50}{32.2}\right)(0.5^2) = 0.3882\text{ slug}\cdot\text{ft}^2$. Thus, the kinetic energy of the system is

$$T = T_A + T_C$$
$$= \frac{1}{2}I_A\,\omega^2 + \frac{1}{2}m_C\,v_C{}^2$$
$$= \frac{1}{2}(0.3882)\omega^2 + \frac{1}{2}\left(\frac{200}{32.2}\right)\left[\omega(0.375)\right]^2$$
$$= 0.6308\omega^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{W}_A$ do no work, $\mathbf{M}$ does positive work, and $\mathbf{W}_C$ does negative work. When pulley B rotates $\theta_B = (5\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 10\pi\text{ rad}$, the wheel rotates through an angle of $\theta_A = \frac{r_B}{r_A}\theta_B = \left(\frac{0.25}{0.625}\right)(10\pi) = 4\pi$. Thus, the crate displaces upwards through a distance of $s_C = r_C\,\theta_A = 0.375(4\pi) = 1.5\pi$ ft. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}_C$ is

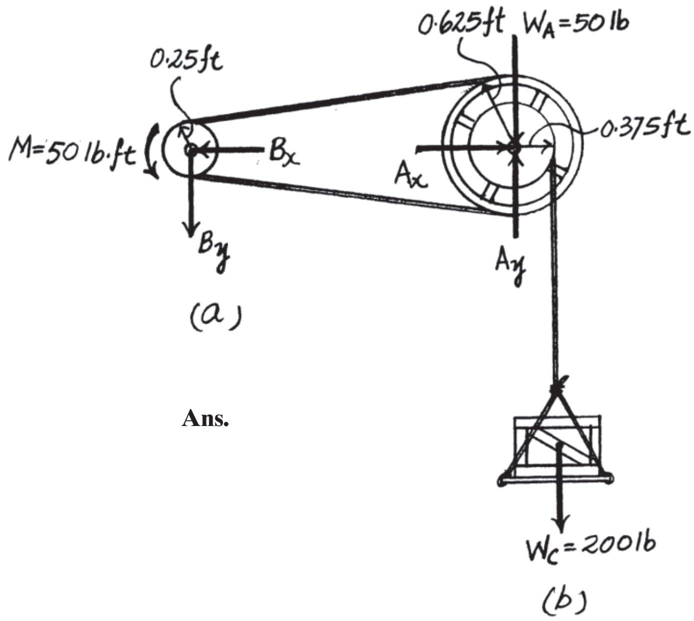
$$U_M = M\theta_B = 50(10\pi) = 500\pi\text{ ft}\cdot\text{lb}$$
$$U_{W_C} = -W_C\,s_C = -200(1.5\pi) = -300\pi\text{ ft}\cdot\text{lb}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$
$$0 + [500\pi - 300\pi] = 0.6308\omega^2$$
$$\omega = 31.56\text{ rad/s}$$

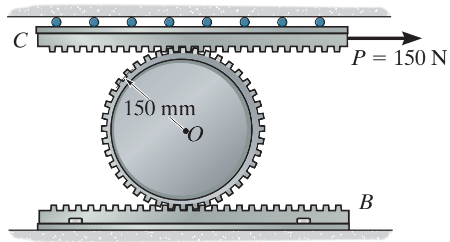
Thus,

$$v_C = 31.56(0.375) = 11.8\text{ ft/s}\uparrow$$ **Ans.**



18–15.

The 50-kg gear has a radius of gyration of 125 mm about its center of mass O. If gear rack B is stationary, while the 25-kg gear rack C is subjected to a horizontal force of $P = 150$ N, determine the speed of C after the gear's center O has moved to the right a distance of 0.3 m, starting from rest.



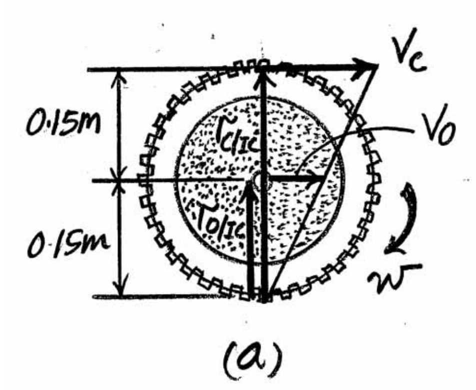
SOLUTION

Kinetic Energy and Work: Referring to Fig. *a*,

$$\omega = \frac{v_C}{r_{C/IC}} = \frac{v_C}{0.3} = 3.333v_C$$

Then,

$$v_O = \omega r_{O/IC} = (3.333v_C)(0.15) = 0.5v_C$$



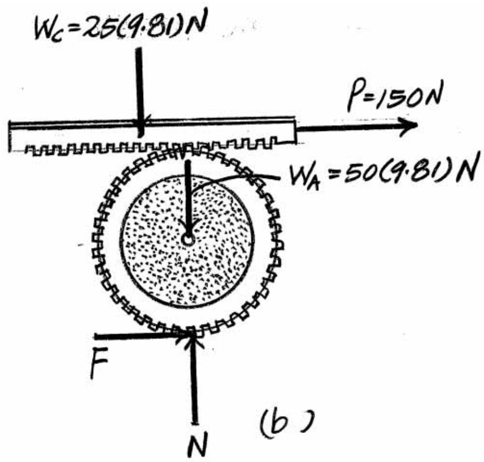
The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2 = 50(0.125^2) = 0.78125 \text{ kg} \cdot \text{m}^2$. Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \left[\frac{1}{2}m_A v_O{}^2 + \frac{1}{2}I_O\omega^2\right] + \frac{1}{2}m_C v_C{}^2$$

$$= \left[\frac{1}{2}(50)(0.5v_C)^2 + \frac{1}{2}(0.78125)(3.333v_C)^2\right] + \frac{1}{2}(25)\,v_C{}^2$$

$$= 23.090v_C{}^2$$



Since the system is initially at rest, $T_1 = 0$. Referring to Fig. *b*, $\mathbf{W}_C$, $\mathbf{W}_A$, $\mathbf{F}$, and $\mathbf{N}$ do no work, while $\mathbf{P}$ does positive work. When the center O of the gear travels to the right through a distance of $s_O = 0.3$ m, P displaces horizontally through a distance of $s_C = \dfrac{r_{C/IC}}{r_{O/IC}}s_O = \left(\dfrac{0.3}{0.15}\right)(0.3) = 0.6$ m. Thus, the work done by $\mathbf{P}$ is

$$U_P = Ps_D = 150(0.6) = 90 \text{ J}$$

Principle of Work and Energy:

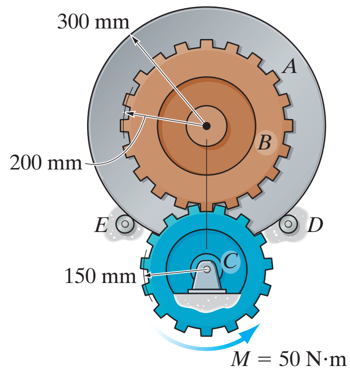
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 90 = 23.090v_C{}^2$$

$$v_C = 1.97 \text{ m/s}$$

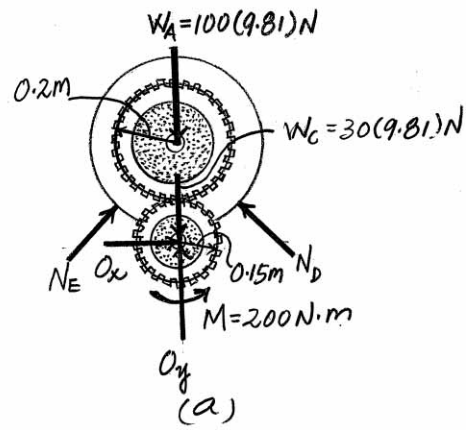
***18–16.**

Gear B is rigidly attached to drum A and is supported by two small rollers at E and D. Gear B is in mesh with gear C and is subjected to a torque of $M = 50\ \text{N} \cdot \text{m}$. Determine the angular velocity of the drum after C has rotated 10 revolutions, starting from rest. Gear B and the drum have 100 kg and a radius of gyration about their rotating axis of 250 mm. Gear C has a mass of 30 kg and a radius of gyration about its rotating axis of 125 mm.



SOLUTION

Kinetic Energy and Work: Since gear B is in mesh with gear C and both gears rotate about fixed axes, $\omega_C = \left(\dfrac{r_B}{r_C}\right)\omega_A = \left(\dfrac{0.2}{0.15}\right)\omega_A = 1.333\omega_A$. The mass moment of the drum and gear C about their rotating axes are $I_A = m_A k^2 = 100(0.25^2) = 6.25\ \text{kg} \cdot \text{m}^2$ and $I_C = m_C k^2 = 30(0.125^2) = 0.46875\ \text{kg} \cdot \text{m}^2$. Thus, the kinetic energy of the system is



$$T = T_A + T_C$$

$$= \frac{1}{2} I_A {\omega_A}^2 + \frac{1}{2} I_C {\omega_C}^2$$

$$= \frac{1}{2}(6.25){\omega_A}^2 + \frac{1}{2}(0.46875)(1.333\omega_A)^2$$

$$= 3.5417{\omega_A}^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. a, $\mathbf{M}$ does positive work. When the gear C rotates $\theta = (10\ \text{rev})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 20\pi$, the work done by $\mathbf{M}$ is

$$U_M = 50(20\pi) = 1000\pi\ \text{J}$$

Principle of Work and Energy:

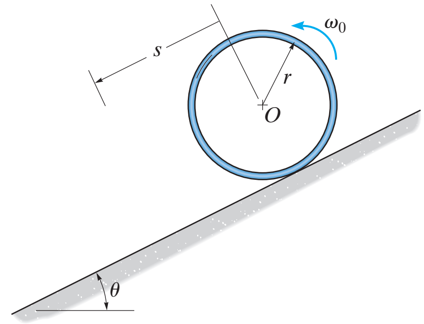
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 1000\pi = 3.5417{\omega_A}^2$$

$$\omega_A = 29.8\ \text{rad/s} \qquad \textbf{Ans.}$$

18–17.

The center O of the thin ring of mass m is given an angular velocity of ω_0. If the ring rolls without slipping, determine its angular velocity after it has traveled a distance of s down the plane. Neglect its thickness.

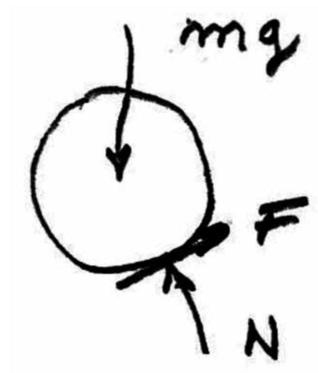


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

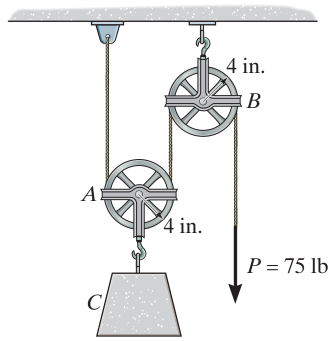
$$\frac{1}{2}(mr^2 + mr^2)\omega_0{}^2 + mg(s \sin \theta) = \frac{1}{2}(mr^2 + mr^2)\omega^2$$

$$\omega = \sqrt{\omega_0{}^2 + \frac{g}{r^2} s \sin \theta}$$ **Ans.**



18–18.

If the end of the cord is subjected to a force of $P = 75$ lb, determine the speed of the 100-lb block C after P has moved a distance of 4 ft, starting from rest. Pulleys A and B are identical, each of which has a weight of 10 lb and a radius of gyration of $k = 3$ in. about its center of mass.

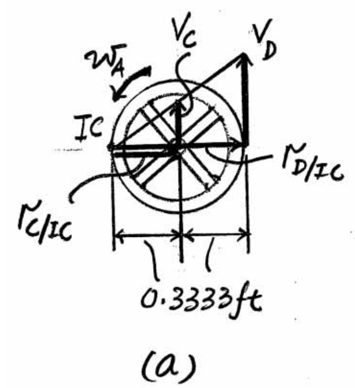


SOLUTION

Kinetic Energy and Work: Referring to Fig. a, we have

$$v_D = \omega_A r_{D/IC} = \omega_A(0.6667)$$

$$(v_G)_A = v_C = \omega_A r_{C/IC} = \omega_A(0.3333)$$



Since pulley B rotates about a fixed axis, its angular velocity is

$$\omega_B = \frac{v_D}{r_B} = \frac{\omega_A(0.6667)}{0.3333} = 2\omega_A$$

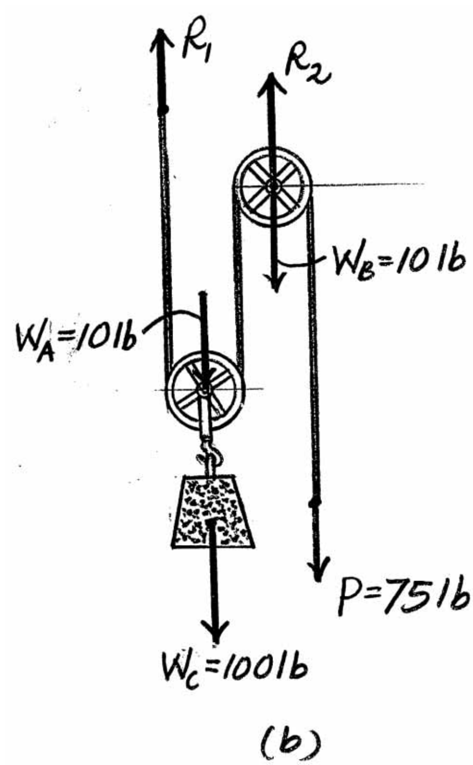
The mass moment of inertia of pulleys A and B about their resperctive mass centers are $(I_A)_G = (I_B)_G = mk^2 = \left(\frac{10}{32.2}\right)\left(\frac{3}{12}\right)^2 = 0.01941 \text{ slug}\cdot\text{ft}^2$. Thus, the kinetic enegry of the system is

$$T = T_A + T_B + T_C$$

$$= \left[\frac{1}{2}m_A(v_G)_A^2 + \frac{1}{2}(I_G)_A\omega_A^2\right] + \frac{1}{2}(I_G)_B\omega_B^2 + \frac{1}{2}m_C v_C^2$$

$$= \left[\frac{1}{2}\left(\frac{10}{32.2}\right)[\omega_A(0.3333)]^2 + \frac{1}{2}(0.01941)\omega_A^2\right] + \frac{1}{2}(0.01941)(2\omega_A)^2 + \frac{1}{2}\left(\frac{100}{32.2}\right)[\omega_A(0.3333)]^2$$

$$= 0.2383\omega_A^2$$



Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{W}_B$ do no work, $\mathbf{P}$ does positive work, and $\mathbf{W}_A$ and $\mathbf{W}_C$ do negative work. When P moves $s_D = 4$ ft downward, the center of the pulley moves upward through a distance of $s_C = \frac{r_{C/IC}}{r_{D/IC}} s_D = \frac{0.3333}{0.6667}(4) = 2$ ft. Thus, the work done by $\mathbf{W}_A$, $\mathbf{W}_C$, and $\mathbf{P}$ is

$$U_{W_A} = -W_A s_C = -10(2) = -20 \text{ ft}\cdot\text{lb}$$

$$U_{W_C} = -W_C s_C = -100(2) = -200 \text{ ft}\cdot\text{lb}$$

$$U_P = P s_D = 75(4) = 300 \text{ ft}\cdot\text{lb}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [-20 + (-200) + 300] = 0.2383\omega_A^2$$

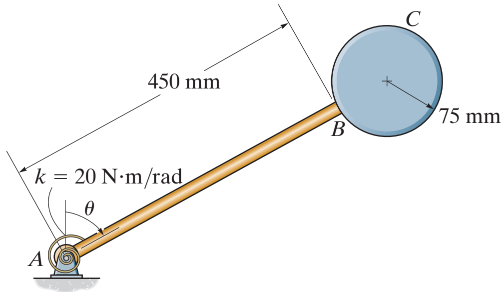
$$\omega_A = 18.32 \text{ rad/s}$$

Thus,

$$v_C = 18.32(0.3333) = 6.11 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

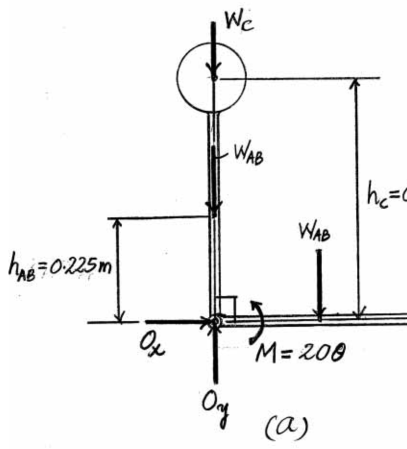
18–19.

When $\theta = 0°$, the assembly is held at rest, and the torsional spring is untwisted. If the assembly is released and falls downward, determine its angular velocity at the instant $\theta = 90°$. Rod AB has a mass of 6 kg, and disk C has a mass of 9 kg.



SOLUTION

Kinetic Energy and Work: Since the rod rotates about a fixed axis, $(v_G)_{AB} = \omega r_{G_{AB}} = \omega(0.225)$ and $(v_G)_C = \omega r_{GC} = \omega(0.525)$. The mass moment of the rod and the disk about their respective mass centers are $(I_{AB})_G = \frac{1}{12}ml^2 = \frac{1}{12}(6)(0.45^2)$ $= 0.10125\ \text{kg}\cdot\text{m}^2$ and $(I_C)_G = \frac{1}{2}mr^2 = \frac{1}{2}(9)(0.075^2) = 0.0253125\ \ \text{kg}\cdot\text{m}^2$. Thus, the kinetic energy of the pendulum is



$$T = \Sigma\frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\omega^2$$

$$= \left[\frac{1}{2}(6)[\omega(0.225)]^2 + \frac{1}{2}(0.10125)\omega^2\right] + \left[\frac{1}{2}(9)[\omega(0.525)]^2 + \frac{1}{2}(0.0253125)\omega^2\right]$$

$$= 1.4555\omega^2$$

This result can also be obtained by applying $T = \frac{1}{2}I_O\omega^2$, where $I_O = \left[\frac{1}{12}(6)(0.45^2) + 6(0.225^2)\right] + \left[\frac{1}{2}(9)(0.075^2) + 9(0.525^2)\right] = 2.9109\ \text{kg}\cdot\text{m}^2$. Thus,

$$T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}(2.9109)\omega^2 = 1.4555\omega^2$$

Since the pendulum is initially at rest, $T_1 = 0$. Referring to Fig. a, $\mathbf{O}_x$ and $\mathbf{O}_y$ do no work, $\mathbf{W}_C$ and $\mathbf{W}_{AB}$ do positive work, and $\mathbf{M}$ does negative work. When $\theta = 90°$, $\mathbf{W}_{AB}$ and $\mathbf{W}_C$ displace vertically through distances of $h_{AB} = 0.225$ m and $h_C = 0.525$ m. Thus, the work done by $\mathbf{W}_{AB}$, $\mathbf{W}_C$, and $\mathbf{M}$ is

$$U_{W_{AB}} = W_{AB}h_{AB} = 6(9.81)(0.225) = 13.24\ \text{J}$$

$$U_{W_C} = W_Ch_C = 9(9.81)(0.525) = 46.35\ \text{J}$$

$$U_M = -\int M\,d\theta = -\int_0^{\pi/2} 20\theta\,d\theta = -24.67\ \text{J}$$

Principle of Work and Energy:

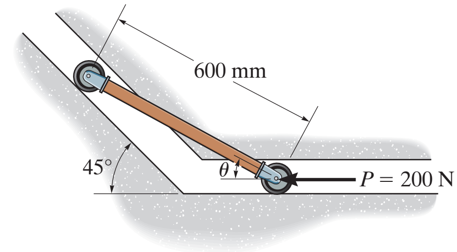
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [13.24 + 46.35 + (-24.67)] = 1.4555\omega^2$$

$$\omega = 4.90\ \text{rad/s} \qquad \textbf{Ans.}$$

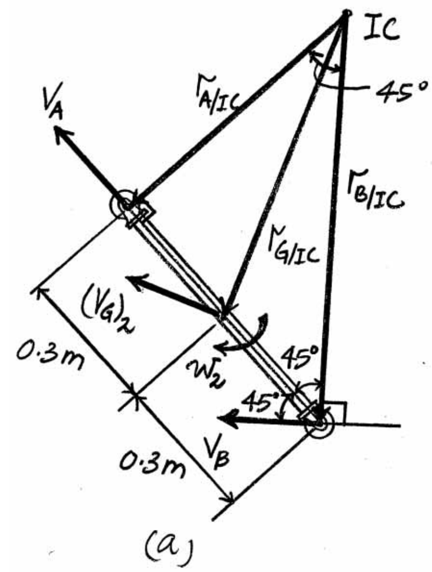
***18–20.**

If $P = 200$ N and the 15-kg uniform slender rod starts from rest at $\theta = 0°$, determine the rod's angular velocity at the instant just before $\theta = 45°$.



SOLUTION

Kinetic Energy and Work: Referring to Fig. *a*,



$$r_{A/IC} = 0.6 \tan 45° = 0.6 \text{ m}$$

Then

$$r_{G/IC} = \sqrt{0.3^2 + 0.6^2} = 0.6708 \text{ m}$$

Thus,

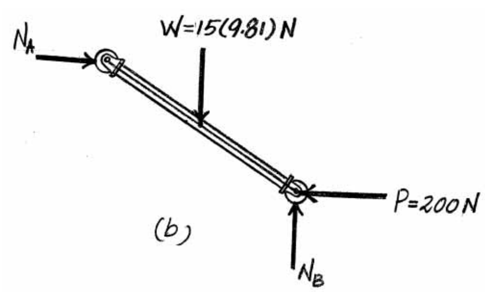
$$(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(0.6708)$$

The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(15)(0.6^2) = 0.45 \text{ kg}\cdot\text{m}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2^{\,2} + \frac{1}{2}I_G\,\omega_2^{\,2}$$

$$= \frac{1}{2}(15)[w_2(0.6708)]^2 + \frac{1}{2}(0.45)\,\omega_2^{\,2}$$

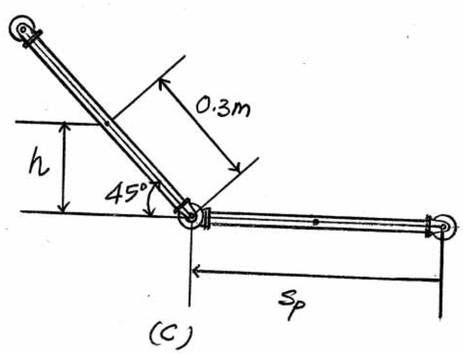
$$= 3.6\omega_2^{\,2}$$



Since the rod is initially at rest, $T_1 = 0$. Referring to Fig. *b*, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while **P** does positive work and **W** does negative work. When $\theta = 45°$, **P** displaces through a horizontal distance $s_P = 0.6$ m and W displaces vertically upwards through a distance of $h = 0.3 \sin 45°$, Fig. *c*. Thus, the work done by **P** and **W** is

$$U_P = Ps_P = 200(0.6) = 120 \text{ J}$$

$$U_W = -Wh = -15(9.81)(0.3 \sin 45°) = -31.22 \text{ J}$$



Principle of Work and Energy:

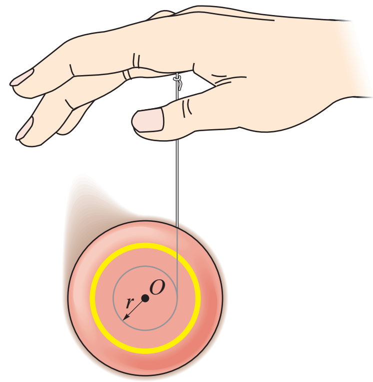
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [120 - 31.22] = 3.6\omega_2^{\,2}$$

$$\omega_2 = 4.97 \text{ rad/s}$$ **Ans.**

18–21.

A yo-yo has a weight of 0.3 lb and a radius of gyration $k_O = 0.06$ ft. If it is released from rest, determine how far it must descend in order to attain an angular velocity $\omega = 70$ rad/s. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is $r = 0.02$ ft.



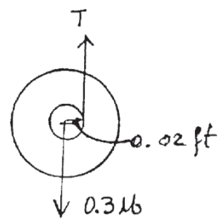
SOLUTION

$v_G = (0.02)70 = 1.40$ ft/s

$T_1 + \Sigma U_{1-2} = T_2$

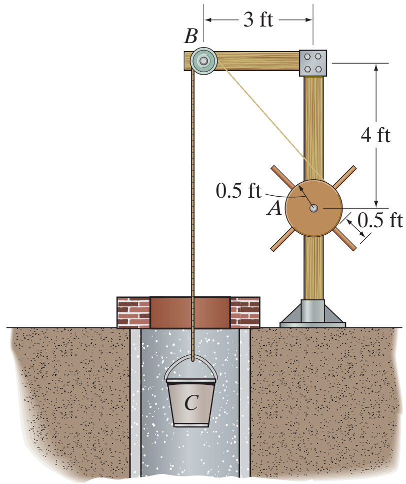
$$0 + (0.3)(s) = \frac{1}{2}\left(\frac{0.3}{32.2}\right)(1.40)^2 + \frac{1}{2}\left[(0.06)^2\left(\frac{0.3}{32.2}\right)\right](70)^2$$

$s = 0.304$ ft **Ans.**



18–22.

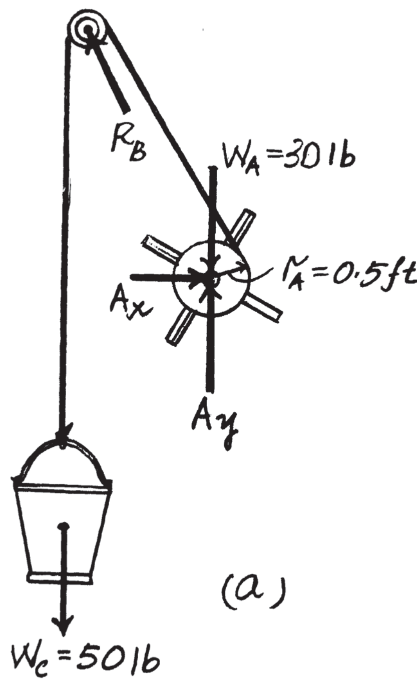
If the 50-lb bucket is released from rest, determine its velocity after it has fallen a distance of 10 ft. The windlass A can be considered as a 30-lb cylinder, while the spokes are slender rods, each having a weight of 2 lb. Neglect the pulley's weight.



SOLUTION

Kinetic Energy and Work: Since the windlass rotates about a fixed axis, $v_C = \omega_A r_A$ or $\omega_A = \dfrac{v_C}{r_A} = \dfrac{v_C}{0.5} = 2v_C$. The mass moment of inertia of the windlass about its mass center is

$$I_A = \frac{1}{2}\left(\frac{30}{32.2}\right)(0.5^2) + 4\left[\frac{1}{12}\left(\frac{2}{32.2}\right)(0.5^2) + \frac{2}{32.2}(0.75^2)\right] = 0.2614 \text{ slug}\cdot\text{ft}^2$$



Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2}I_A\omega^2 + \frac{1}{2}m_C v_C{}^2$$

$$= \frac{1}{2}(0.2614)(2v_C)^2 + \frac{1}{2}\left(\frac{50}{32.2}\right)v_C{}^2$$

$$= 1.2992v_C{}^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. a, $\mathbf{W}_A$, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{R}_B$ do no work, while $\mathbf{W}_C$ does positive work. Thus, the work done by $\mathbf{W}_C$, when it displaces vertically downward through a distance of $s_C = 10$ ft, is

$$U_{W_C} = W_C s_C = 50(10) = 500 \text{ ft}\cdot\text{lb}$$

Principle of Work and Energy:

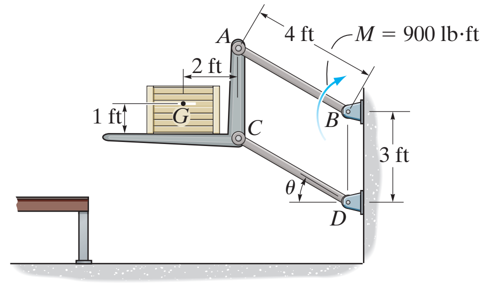
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 500 = 1.2992v_C{}^2$$

$$v_C = 19.6 \text{ ft/s} \qquad \textbf{Ans.}$$

18–23.

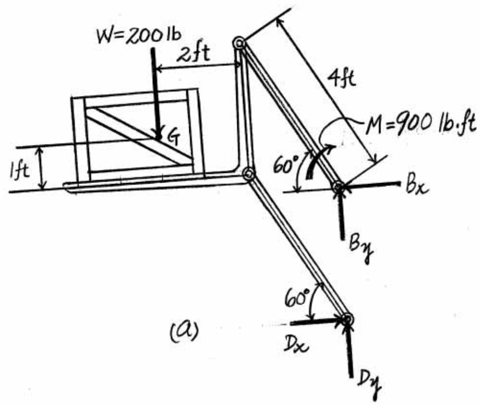
The combined weight of the load and the platform is 200 lb, with the center of gravity located at G. If a couple moment of $M = 900\ \text{lb}\cdot\text{ft}$ is applied to link AB, determine the angular velocity of links AB and CD at the instant $\theta = 60°$. The system is at rest when $\theta = 0°$. Neglect the weight of the links.



SOLUTION

Kinetic Energy and Work: Since the weight of the links are negligible and the crate and platform undergo curvilinear translation, the kinetic energy of the system is

$$T = \frac{1}{2}mv_G^2 = \frac{1}{2}\left(\frac{200}{32.2}\right)v_G^2 = 3.1056v_G^2$$



Since the system is initially at rest, $T_1 = 0$. Referring to Fig. a, $\mathbf{B}_x$, $\mathbf{B}_y$, $\mathbf{D}_x$, and $\mathbf{D}_y$ do no work while $\mathbf{M}$ does positive work and $\mathbf{W}$ does negative work. When $\theta = 60°$, $\mathbf{W}$ displaces upward through a distance of $h = 4\sin 60°\ \text{ft} = 3.464\ \text{ft}$. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}$ is

$$U_M = M\theta = 900\left(\frac{\pi}{3}\right) = 300\pi\ \text{ft}\cdot\text{lb}$$

$$U_W = -Wh = -200(3.464) = -692.82\ \text{ft}\cdot\text{lb}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [300\pi - 692.82] = 3.1056v_G^2$$

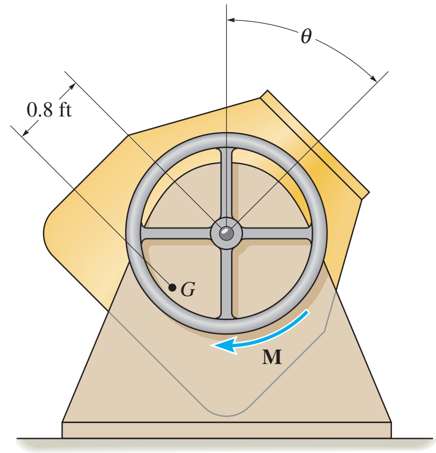
$$v_G = 8.966\ \text{ft/s}$$

Thus,

$$\omega_{AB} = \omega_{CD} = \frac{v_G}{\rho} = \frac{8.966}{4} = 2.24\ \text{rad/s} \qquad \textbf{Ans.}$$

***18–24.**

The tub of the mixer has a weight of 70 lb and a radius of gyration $k_G = 1.3$ ft about its center of gravity. If a constant torque $M = 60\text{ lb}\cdot\text{ft}$ is applied to the dumping wheel, determine the angular velocity of the tub when it has rotated $\theta = 90°$. Originally the tub is at rest when $\theta = 0°$.

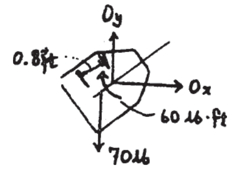


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

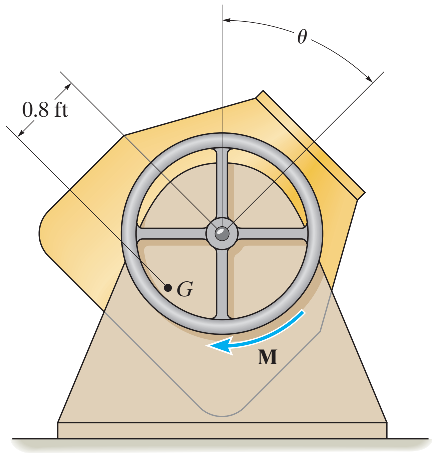
$$0 + 60\left(\frac{\pi}{2}\right) - 70(0.8) = \frac{1}{2}\left[\left(\frac{70}{32.2}\right)(1.3)^2\right](\omega)^2 + \frac{1}{2}\left[\frac{70}{32.2}\right](0.8\omega)^2$$

$\omega = 3.89$ rad/s **Ans.**



18–25.

The tub of the mixer has a weight of 70 lb and a radius of gyration $k_G = 1.3$ ft about its center of gravity. If a constant torque $M = 60\ \text{lb}\cdot\text{ft}$ is applied to the tub, determine its angular velocity when it has rotated $\theta = 45°$. Originally the tub is at rest when $\theta = 0°$.



SOLUTION

Kinetic Energy and Work: The mass moment of inertia of the tub about point O is

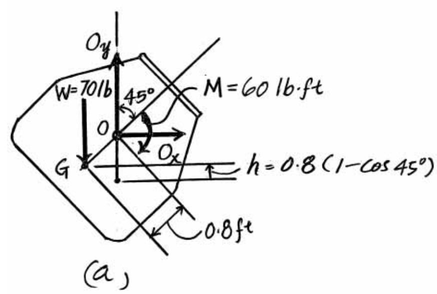
$$I_O = mk_G^2 + mr_G^2$$

$$= \frac{70}{32.2}(1.3^2) + \frac{70}{32.2}(0.8^2)$$

$$= 5.0652\ \text{slug}\cdot\text{ft}^2$$

Thus, kinetic energy of the tub is

$$T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}(5.0652)\omega^2 = 2.5326\,\omega^2$$



Initially, the tub is at rest. Thus, $T_1 = 0$. Referring to the FBD of the tub, Fig. *a*, we notice that $\mathbf{O}_x$ and $\mathbf{O}_y$ do no work while $\mathbf{M}$ does positive work and $\mathbf{W}$ does negative work. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}$ are

$$U_M = M\theta = 60\left(\frac{\pi}{4}\right) = 15\pi\ \text{ft}\cdot\text{lb}$$

$$U_W = -Wh = -70[0.8(1 - \cos 45°)] = -16.40\ \text{ft}\cdot\text{lb}$$

Principle of Work and Energy:

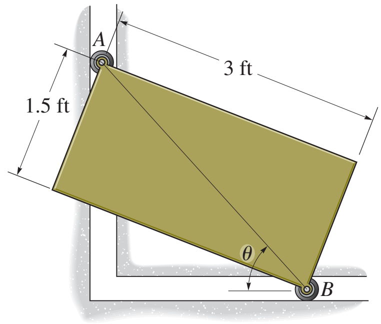
$$T_1 + U_{1-2} = T_2$$

$$0 + [15\pi + (-16.40)] = 2.5326\,\omega^2$$

$$\omega = 3.48\ \text{rad/s} \qquad \textbf{Ans.}$$

18–26.

Two wheels of negligible weight are mounted at corners A and B of the rectangular 75-lb plate. If the plate is released from rest at $\theta = 90°$, determine its angular velocity at the instant just before $\theta = 0°$.



SOLUTION

Kinetic Energy and Work: Referring Fig. a,

$$(v_G)_2 = \omega r_{A/IC} = \omega\left(\sqrt{0.75^2 + 1.5^2}\right) = 1.677\omega^2$$

The mass moment of inertia of the plate about its mass center is $I_G = \frac{1}{12}m(a^2 + b^2) = \frac{1}{12}\left(\frac{75}{32.2}\right)(1.5^2 + 3^2) = 2.1836\text{ slug}\cdot\text{ft}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}\omega_2{}^2$$

$$= \frac{1}{2}\left(\frac{75}{32.2}\right)(1.677\omega_2)^2 + \frac{1}{2}I_G(2.1836)\omega_2{}^2$$

$$= 4.3672\omega_2{}^2$$

Since the plate is initially at rest, $T_1 = 0$. Referring to Fig. b, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while $\mathbf{W}$ does positive work. When $\theta = 0°$, $\mathbf{W}$ displaces vertically through a distance of $h = \sqrt{0.75^2 + 1.5^2} = 1.677$ ft, Fig. c. Thus, the work done by $\mathbf{W}$ is

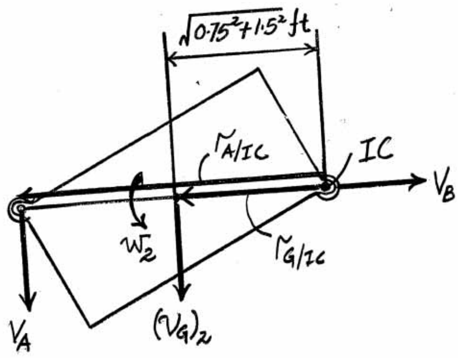
$$U_W = Wh = 75(1.677) = 125.78\text{ ft}\cdot\text{lb}$$

Principle of Work and Energy:

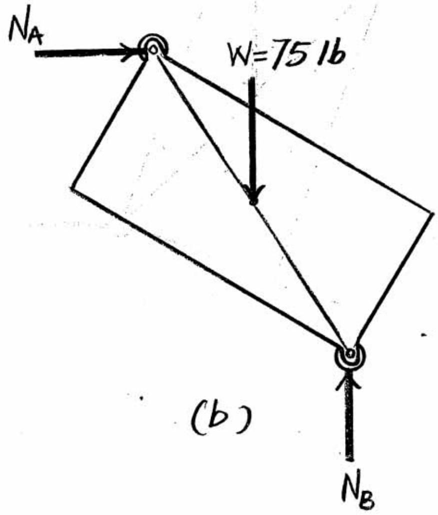
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 125.78 = 4.3672\omega_2{}^2$$

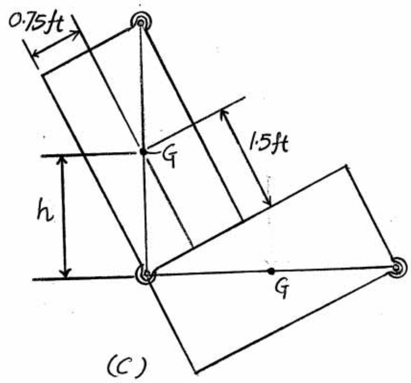
$$\omega_2 = 5.37\text{ rad/s}$$ **Ans.**



(a)



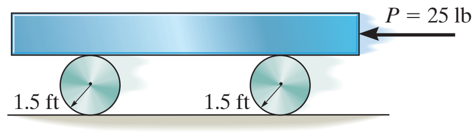
(b)



(c)

18–27.

The 100-lb block is transported a short distance by using two cylindrical rollers, each having a weight of 35 lb. If a horizontal force $P = 25$ lb is applied to the block, determine the block's speed after it has been displaced 2 ft to the left. Originally the block is at rest. No slipping occurs.

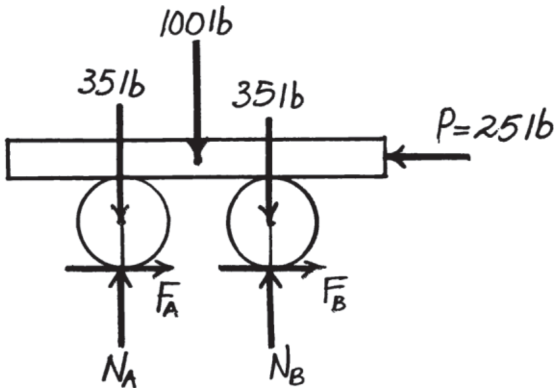


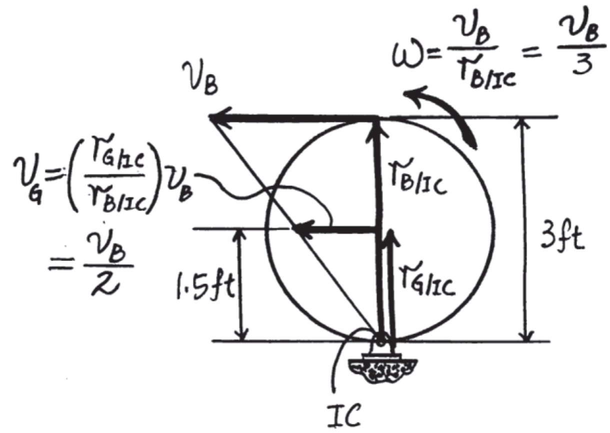
SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 25(2) = \frac{1}{2}\left(\frac{100}{32.2}\right)(v_B)^2 + 2\left[\frac{1}{2}\left(\frac{35}{32.2}\right)\left(\frac{v_B}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{2}\left(\frac{35}{32.2}\right)(1.5)^2\right)\left(\frac{v_B}{3}\right)^2\right]$$

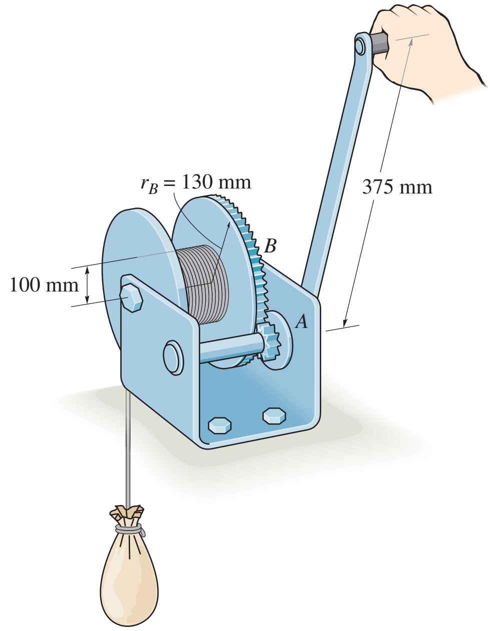
$$v_B = 5.05 \text{ ft/s}$$ **Ans.**





***18–28.**

The hand winch is used to lift the 50-kg load. Determine the work required to rotate the handle five revolutions. The gear at A has a radius of 20 mm.



SOLUTION

$$20(\theta_A) = \theta_B(130)$$

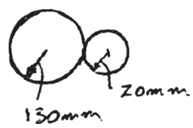
When $\theta_A = 5$ rev. $= 10\,\pi$

$$\theta_B = 4.8332 \text{ rad}$$

Thus load moves up

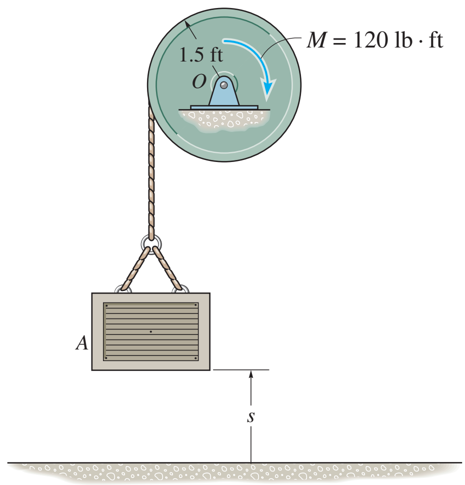
$$s = 4.8332(0.1 \text{ m}) = 0.48332 \text{ m}$$

$$U = 50(9.81)(0.48332) = 237 \text{ J}$$ **Ans.**



18–29.

A motor supplies a constant torque or twist of $M = 120 \text{ lb} \cdot \text{ft}$ to the drum. If the drum has a weight of 30 lb and a radius of gyration of $k_O = 0.8$ ft, determine the speed of the 15-lb crate A after it rises $s = 4$ ft starting from rest. Neglect the mass of the cord.



SOLUTION

Free Body Diagram: The weight of the crate does *negative* work since it acts in the opposite direction to that of its displacement s_w. Also, the couple moment $\mathbf{M}$ does positive work as it acts in the same direction of its angular displacement θ. The reactions O_x, O_y and the weight of the drum do no work since point O does not displace.

Kinematic: Since the drum rotates about point O, the angular velocity of the drum and the speed of the crate can be related by $\omega_D = \dfrac{v_A}{r_D} = \dfrac{v_A}{1.5} = 0.6667\, v_A$. When the crate rises $s = 4$ ft, the angular displacement of the drum is given by $\theta = \dfrac{s}{r_D} = \dfrac{4}{1.5} = 2.667$ rad.

Principle of Work and Energy: The mass moment of inertia of the drum about point O is $I_O = mk_O^2 = \left(\dfrac{30}{32.2}\right)(0.8^2) = 0.5963 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 18–13, we have

$$T_1 + \sum U_{1-2} = T_2$$

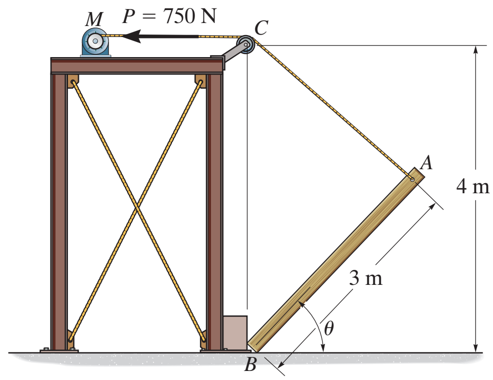
$$0 + M\theta - W_C s_C = \frac{1}{2} I_O \omega^2 + \frac{1}{2} m_C v_C^2$$

$$0 + 120(2.667) - 15(4) = \frac{1}{2}(0.5963)(0.6667 v_A)^2 + \frac{1}{2}\left(\frac{15}{32.2}\right) v_A^2$$

$$v_A = 26.7 \text{ ft/s}$$ **Ans.**

18–30.

Motor M exerts a constant force of $P = 750$ N on the rope. If the 100-kg post is at rest when $\theta = 0°$, determine the angular velocity of the post at the instant $\theta = 60°$. Neglect the mass of the pulley and its size, and consider the post as a slender rod.



SOLUTION

Kinetic Energy and Work: Since the post rotates about a fixed axis, $v_G = \omega r_G = \omega(1.5)$. The mass moment of inertia of the post about its mass center is $I_G = \frac{1}{12}(100)(3^2) = 75\ \text{kg}\cdot\text{m}^2$. Thus, the kinetic energy of the post is

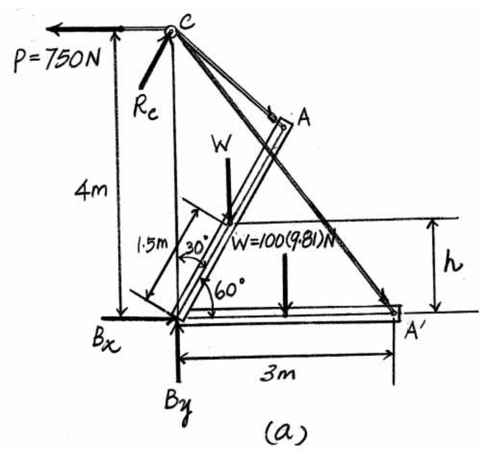
$$T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\omega^2$$

$$= \frac{1}{2}(100)[\omega(1.5)]^2 + \frac{1}{2}(75)\omega^2$$

$$= 150\omega^2$$

This result can also be obtained by applying $T = \frac{1}{2}I_B\omega^2$, where $I_B = \frac{1}{12}(100)(3^2) + 100(1.5^2) = 300\ \text{kg}\cdot\text{m}^2$. Thus,

$$T = \frac{1}{2}I_B\omega^2 = \frac{1}{2}(300)\omega^2 = 150\omega^2$$



Since the post is initially at rest, $T_1 = 0$. Referring to Fig. a, $\mathbf{B}_x$, $\mathbf{B}_y$, and $\mathbf{R}_C$ do no work, while $\mathbf{P}$ does positive work and $\mathbf{W}$ does negative work. When $\theta = 60°$, $\mathbf{P}$ displaces $s_P = A'C - AC$, where $AC = \sqrt{4^2 + 3^2 - 2(4)(3)\cos 30°} = 2.053$ m and $A'C = \sqrt{4^2 + 3^2} = 5$ m. Thus, $s_P = 5 - 2.053 = 2.947$ m. Also, $\mathbf{W}$ displaces vertically upwards through a distance of $h = 1.5\sin 60° = 1.299$ m. Thus, the work done by $\mathbf{P}$ and $\mathbf{W}$ is

$$U_P = Ps_P = 750(2.947) = 2210.14\ \text{J}$$

$$U_W = -Wh = -100(9.81)(1.299) = -1274.36\ \text{J}$$

Principle of Work and Energy:

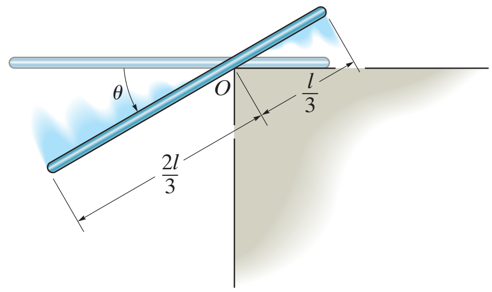
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [2210.14 - 1274.36] = 150\omega^2$$

$$\omega = 2.50\ \text{rad/s}$$ **Ans.**

18–31.

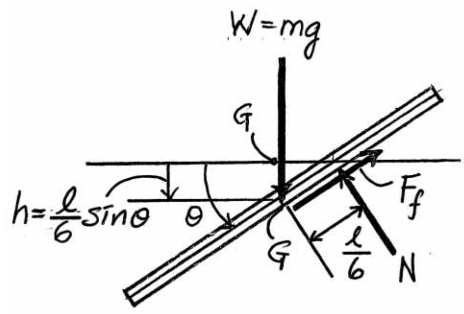
The uniform bar has a mass m and length l. If it is released from rest when $\theta = 0°$, determine its angular velocity as a function of the angle θ before it slips.



SOLUTION

Kinetic Energy and Work: Before the bar slips, the bar rotates about the fixed axis passing through point O. The mass moment of inertia of the bar about this axis is $I_O = \frac{1}{12}ml^2 + m\left(\frac{l}{6}\right)^2 = \frac{1}{9}ml^2$. Thus, the kinetic energy of the bar is

$$T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}\left(\frac{1}{9}ml^2\right)\omega^2 = \frac{1}{18}ml^2\omega^2$$



(a)

Initially, the bar is at rest. Thus, $T_1 = 0$. Referring to the FBD of the bar, Fig. a, we notice that **N** and $\mathbf{F}_f$ do no work while **W** does positive work which is given by

$$U_W = Wh = mg\left(\frac{l}{6}\sin\theta\right) = \frac{mgl}{6}\sin\theta$$

Principle of Work and Energy:

$$T_1 + U_{1-2} = T_2$$

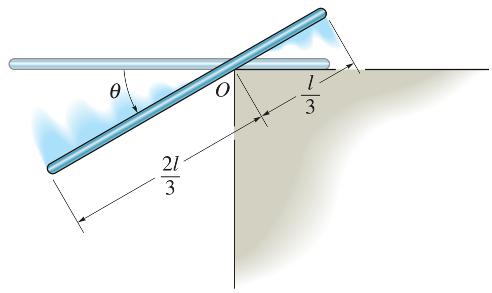
$$0 + \frac{mgl}{6}\sin\theta = \frac{1}{18}ml^2\omega^2$$

$$\omega^2 = \frac{3g}{l}\sin\theta$$

$$\omega = \sqrt{\frac{3g}{l}\sin\theta} \qquad \textbf{Ans.}$$

***18–32.**

The uniform bar has a mass m and length l. If it is released from rest when $\theta = 0°$, determine the angle θ at which it first begins to slip. The coefficient of static friction at O is $\mu_s = 0.3$.

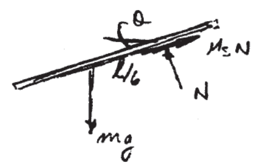


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + m\,g\left(\frac{l}{6}\sin\theta\right) = \frac{1}{2}\left[\frac{1}{12}m\,l^2 + m\left(\frac{l}{6}\right)^2\right]\omega^2$$

$$\omega = \sqrt{\frac{3\,g\sin\theta}{l}}$$



$\circlearrowleft + \Sigma M_O = I_O\,\alpha;$ $$m\,g\cos\theta\left(\frac{l}{6}\right) = \left[\frac{1}{12}m\,l^2 + m\left(\frac{l}{6}\right)^2\right]\alpha$$

$$\alpha = \frac{3\,g\cos\theta}{2\,l}$$

$+ \Sigma F_n = m(a_G)_n;$ $$\mu_s N - m\,g\sin\theta = m\left(\frac{3\,g\sin\theta}{l}\right)\left(\frac{l}{6}\right)$$

$$\mu_s N = 1.5\,m\,g\sin\theta$$

$+\Sigma F_t = m(a_G)_t;$ $$-N + m\,g\cos\theta = m\left(\frac{3\,g\cos\theta}{2\,l}\right)\left(\frac{l}{6}\right)$$

$$N = 0.75\,m\,g\cos\theta$$

Thus,

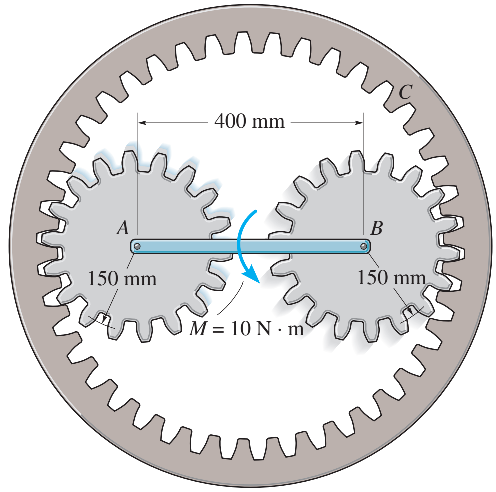
$$\mu_s = \frac{1.5}{0.75}\tan\theta$$

$$0.3 = 2\tan\theta$$

$$\theta = 8.53°$$ **Ans.**

18–33.

The two 2-kg gears A and B are attached to the ends of a 3-kg slender bar. The gears roll within the fixed ring gear C, which lies in the horizontal plane. If a 10-N·m torque is applied to the center of the bar as shown, determine the number of revolutions the bar must rotate starting from rest in order for it to have an angular velocity of $\omega_{AB} = 20$ rad/s. For the calculation, assume the gears can be approximated by thin disks. What is the result if the gears lie in the vertical plane?



SOLUTION

Energy equation (where G refers to the center of one of the two gears):

$$M\theta = T_2$$

$$10\theta = 2\left(\frac{1}{2} I_G \omega_{\text{gear}}^2\right) + 2\left(\frac{1}{2} m_{\text{gear}}\right)(0.200\omega_{AB})^2 + \frac{1}{2} I_{AB}\omega_{AB}^2$$

Using $m_{\text{gear}} = 2$ kg, $I_G = \frac{1}{2}(2)(0.150)^2 = 0.0225$ kg $\cdot$ m^2,

$I_{AB} = \frac{1}{12}(3)(0.400)^2 = 0.0400$ kg $\cdot$ m^2 and $\omega_{\text{gear}} = \frac{200}{150}\omega_{AB}$,

$$10\theta = 0.0225\left(\frac{200}{150}\right)^2 \omega_{AB}^2 + 2(0.200)^2\omega_{AB}^2 + 0.0200\omega_{AB}^2$$

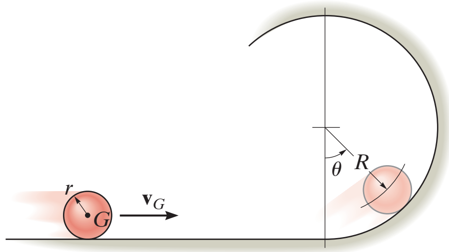
When $\omega_{AB} = 20$ rad/s,

$$\theta = 5.60 \text{ rad}$$

$$= 0.891 \text{ rev, regardless of orientation} \qquad \textbf{Ans.}$$

18–34.

A ball of mass m and radius r is cast onto the horizontal surface such that it rolls without slipping. Determine its angular velocity at the instant $\theta = 90°$, if it has an initial speed of v_G as shown.



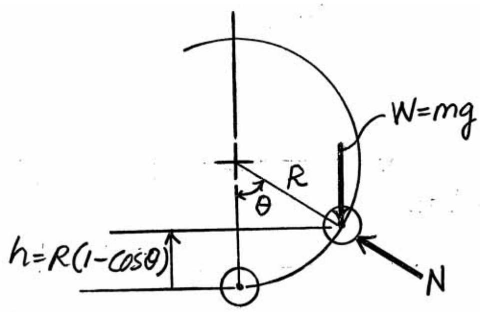
SOLUTION

Kinetic Energy and Work: Since the ball rolls without slipping, $v_G = \omega r$ or $\omega = \dfrac{v_G}{r}$.

The mass moment of inertia of the ball about its mass cener is $I_G = \dfrac{2}{5}mr^2$. Thus, the kinetic energy of the ball is

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$
$$= \frac{1}{2}mv_G^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_G}{r}\right)^2$$
$$= \frac{7}{10}mv_G^2$$

Initially, the ball has a speed of v_G. Thus, $T_1 = \dfrac{7}{10}mv_G^2$. Referring to the FBD of the ball, Fig. a, we notice that $\mathbf{N}$ does no work while $\mathbf{W}$ does negative work. When $\theta = 90°$, $h = R(1 - \cos 90°) = R$. Thus,



(a)

$$U_W = -Wh = -mgR$$

Principle of Work and Energy:

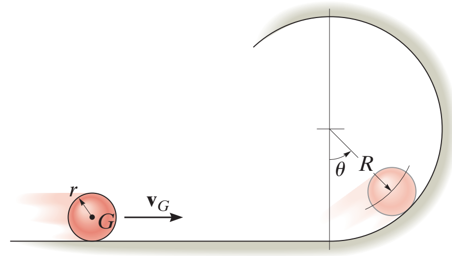
$$T_1 + U_{1-2} = T_2$$
$$\frac{7}{10}mv_G^2 + (-mgR) = \frac{7}{10}m(v_G)_2^2$$
$$(v_G)_2 = \sqrt{\frac{1}{7}(7v_G^2 - 10gR)}$$

so that

$$\omega_2 = \frac{(v_G)_2}{r} = \sqrt{v_G^2 - \frac{10}{7}gR}/r$$ **Ans.**

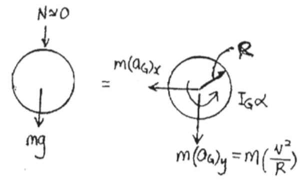
18–35.

A ball of mass m and radius r is cast onto the horizontal surface such that it rolls without slipping. Determine the minimum speed v_G of its mass center G so that it rolls completely around the loop of radius $R + r$ without leaving the track.



SOLUTION

$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad mg = m\left(\frac{v^2}{R}\right)$$



$$v^2 = gR$$

$$T_1 + \Sigma U_{1-2} = T_2$$

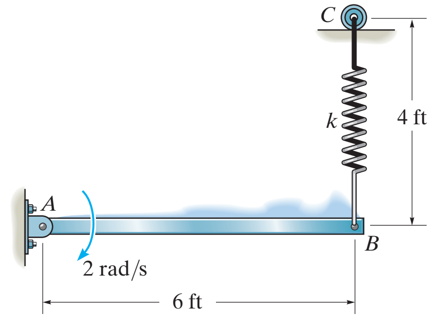
$$\frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_G^2}{r^2}\right) + \frac{1}{2}mv_G^2 - mg(2R) = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{gR}{r^2}\right) + \frac{1}{2}m(gR)$$

$$\frac{1}{5}v_G^2 + \frac{1}{2}v_G^2 = 2gR + \frac{1}{5}gR + \frac{1}{2}gR$$

$$v_G = 3\sqrt{\frac{3}{7}gR}$$ **Ans.**

***18–36.**

At the instant shown, the 50-lb bar rotates clockwise at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at C. If the spring has an unstretched length of 2 ft and a stiffness of $k = 6$ lb/ft, determine the angular velocity of the bar the instant it has rotated 30° clockwise.



SOLUTION

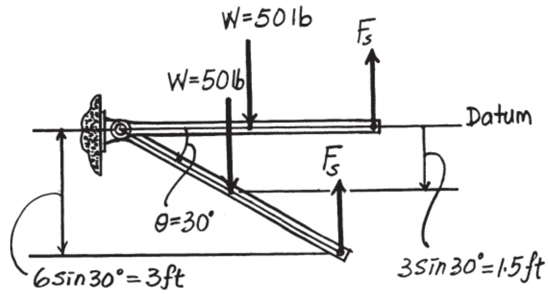
Datum through A.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(6)(4-2)^2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right]\omega^2$$

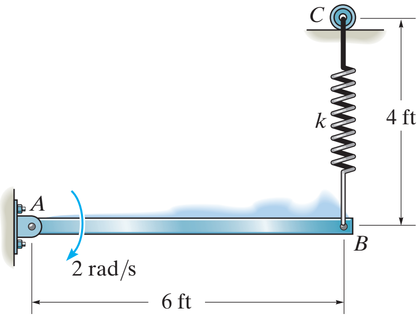
$$+\frac{1}{2}(6)(7-2)^2 - 50(1.5)$$

$$\omega = 2.30 \text{ rad/s}$$ **Ans.**



18–37.

At the instant shown, the 50-lb bar rotates clockwise at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at C. If the spring has an unstretched length of 2 ft and a stiffness of $k = 12$ lb/ft, determine the angle θ, measured from the horizontal, to which the bar rotates before it momentarily stops.



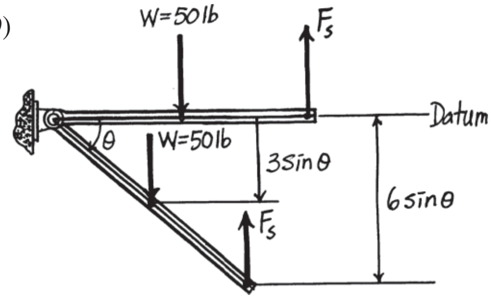
SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(12)(4-2)^2 = 0 + \frac{1}{2}(12)(4 + 6\sin\theta - 2)^2 - 50(3\sin\theta)$$

$$61.2671 = 24(1 + 3\sin\theta)^2 - 150\sin\theta$$

$$37.2671 = -6\sin\theta + 216\sin^2\theta$$



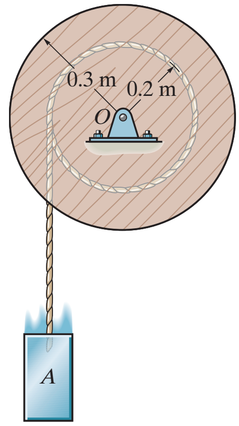
Set $x = \sin\theta$, and solve the quadratic equation for the positive root:

$$\sin\theta = 0.4295$$

$$\theta = 25.4^\circ \qquad \textbf{Ans.}$$

18–38.

The spool has a mass of 50 kg and a radius of gyration $k_O = 0.280$ m. If the 20-kg block A is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 5$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



SOLUTION

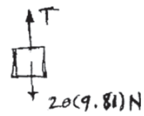
$v_A = 0.2\omega = 0.2(5) = 1\text{ m/s}$

System:

$T_1 + V_1 = T_2 + V_2$

$$[0 + 0] + 0 = \frac{1}{2}(20)(1)^2 + \frac{1}{2}[50(0.280)^2](5)^2 - 20(9.81)\,s$$

$s = 0.30071\text{ m} = 0.301\text{ m}$ **Ans.**



Block:

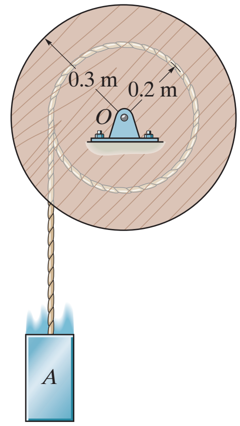
$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 20(9.81)(0.30071) - T(0.30071) = \frac{1}{2}(20)(1)^2$$

$T = 163\text{ N}$ **Ans.**

18–39.

The spool has a mass of 50 kg and a radius of gyration $k_O = 0.280$ m. If the 20-kg block A is released from rest, determine the velocity of the block when it descends 0.5 m.



SOLUTION

Potential Energy: With reference to the datum established in Fig. *a*, the gravitational potential energy of block A at position 1 and 2 are

$$V_1 = (V_g)_1 = W_A y_1 = 20(9.81)(0) = 0$$

$$V_2 = (V_g)_2 = -W_A y_2 = -20(9.81)(0.5) = -98.1 \text{ J}$$

Kinetic Energy: Since the spool rotates about a fixed axis, $\omega = \frac{v_A}{r_A} = \frac{v_A}{0.2} = 5v_A$. Here, the mass moment of inertia about the fixed axis passes through point O is $I_O = mk_O^2 = 50(0.280)^2 = 3.92 \text{ kg} \cdot \text{m}^2$. Thus, the kinetic energy of the system is

$$T = \frac{1}{2}I_O\omega^2 + \frac{1}{2}m_A v_A^2$$

$$= \frac{1}{2}(3.92)(5v_A)^2 + \frac{1}{2}(20)v_A^2 = 59v_A^2$$

Since the system is at rest initially, $T_1 = 0$

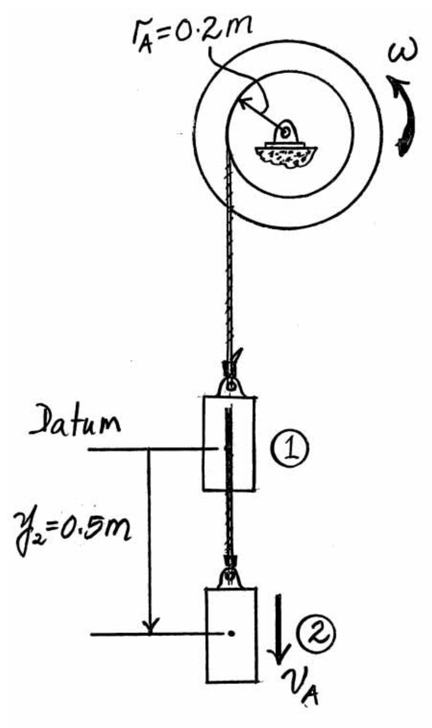
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 59v_A^2 + (-98.1)$$

$$v_A = 1.289 \text{ m/s}$$

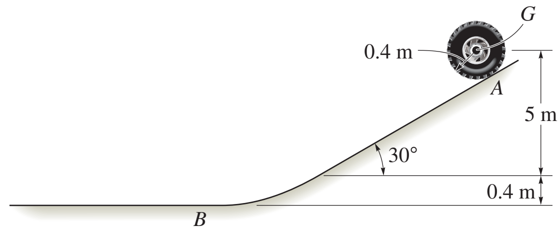
$$= 1.29 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

***18–40.**

An automobile tire has a mass of 7 kg and radius of gyration $k_G = 0.3$ m. If it is released from rest at A on the incline, determine its angular velocity when it reaches the horizontal plane. The tire rolls without slipping.



SOLUTION

$$v_G = 0.4\omega$$

Datum at lowest point.

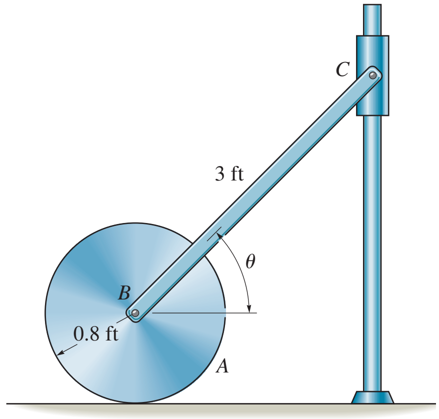
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 7(9.81)(5) = \frac{1}{2}(7)(0.4\omega)^2 + \frac{1}{2}[7\,(0.3)^2]\omega^2 + 0$$

$$\omega = 19.8 \text{ rad/s}$$ **Ans.**

18–41.

The system consists of a 20-lb disk A, 4-lb slender rod BC, and a 1-lb smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e., $\theta = 0°$. The system is released from rest when $\theta = 45°$.

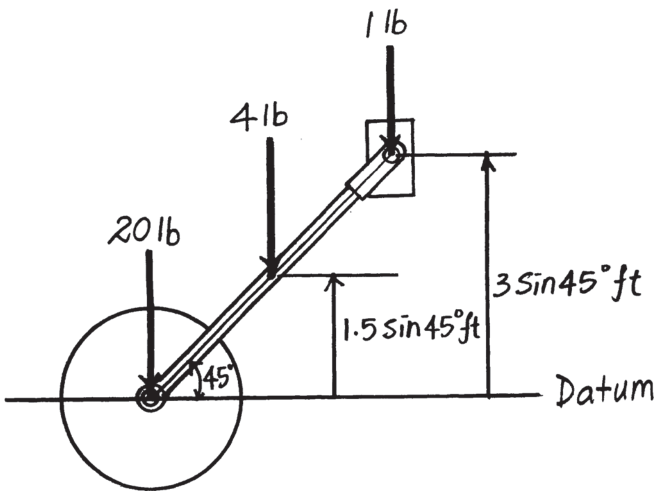


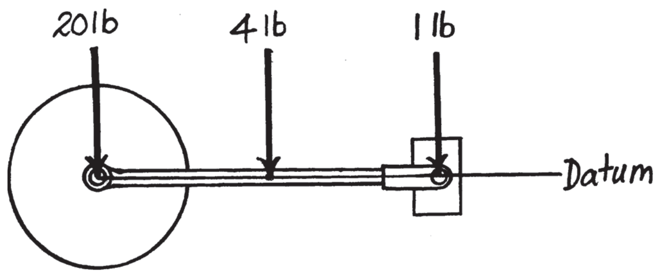
SOLUTION

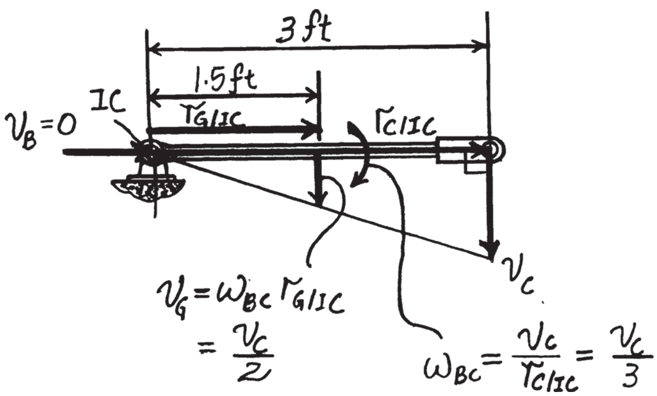
$$T_1 + V_1 = T_2 + V_2$$

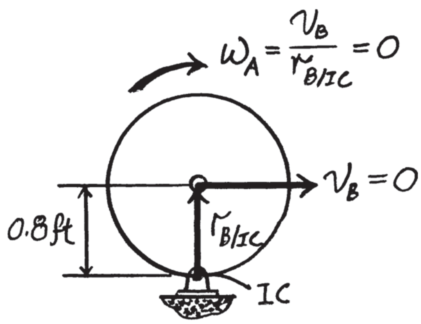
$$0 + 4(1.5\sin 45°) + 1(3\sin 45°) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{4}{32.2}\right)(3)^2\right]\left(\frac{v_C}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{32.2}\right)(v_C)^2 + 0$$

$$v_C = 13.3 \text{ ft/s} \qquad \textbf{Ans.}$$



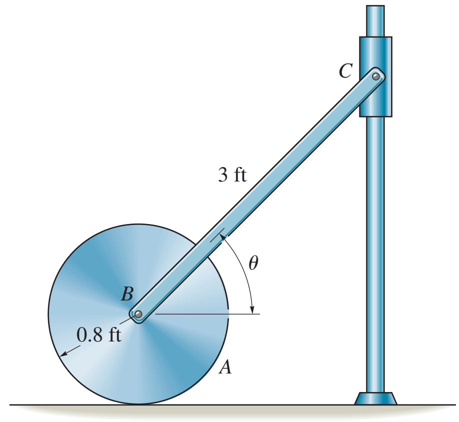






18–42.

The system consists of a 20-lb disk A, 4-lb slender rod BC, and a 1-lb smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant $\theta = 30°$. The system is released from rest when $\theta = 45°$.



SOLUTION

$$v_B = 0.8\omega_A$$

$$\omega_{BC} = \frac{v_B}{1.5} = \frac{v_C}{2.598} = \frac{v_G}{1.5}$$

Thus,

$$v_B = v_G = 1.5\omega_{BC} \qquad v_C = 2.598\omega_{BC}$$

$$\omega_A = 1.875\,\omega_{BC}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 4(1.5\sin 45°) + 1(3\sin 45°)$$

$$= \frac{1}{2}\left[\frac{1}{2}\left(\frac{20}{32.2}\right)(0.8)^2\right](1.875\omega_{BC})^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)(1.5\,\omega_{BC})^2$$

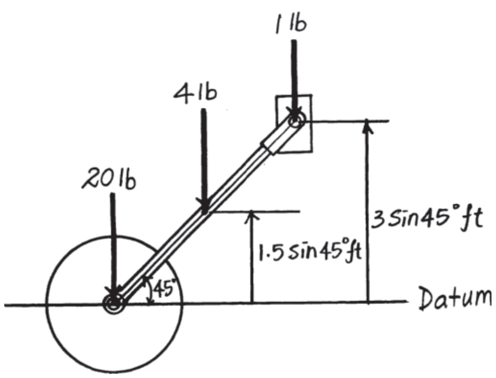
$$+\frac{1}{2}\left[\frac{1}{12}\left(\frac{4}{32.2}\right)(3)^2\right]\omega_{BC}^2 + \frac{1}{2}\left(\frac{4}{32.2}\right)(1.5\omega_{BC})^2$$

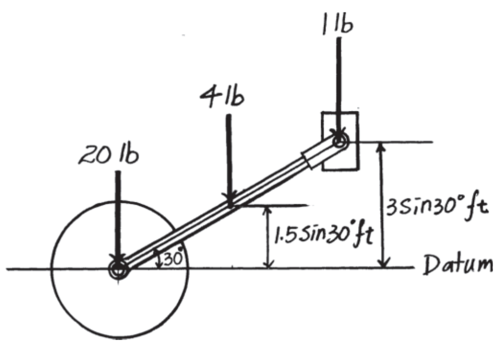
$$+\frac{1}{2}\left(\frac{1}{32.2}\right)(2.598\omega_{BC})^2 + 4(1.5\sin 30°) + 1(3\sin 30°)$$

$$\omega_{BC} = 1.180\text{ rad/s}$$

Thus,

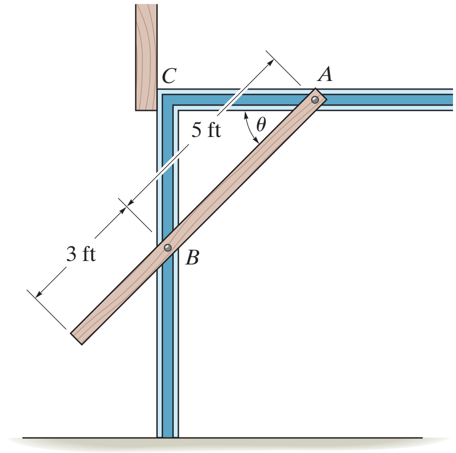
$$v_C = 2.598(1.180) = 3.07\text{ ft/s}$$ **Ans.**





18–43.

The door is made from one piece, whose ends move along the horizontal and vertical tracks. If the door is in the open position, $\theta = 0^\circ$, and then released, determine the speed at which its end A strikes the stop at C. Assume the door is a 180-lb thin plate having a width of 10 ft.



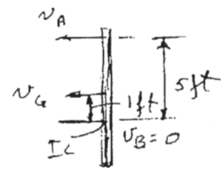
SOLUTION

$T_1 + V_1 = T_2 + V_2$

$$0 + 0 = \frac{1}{2}\left[\frac{1}{12}\left(\frac{180}{32.2}\right)(8)^2\right]\omega^2 + \frac{1}{2}\left(\frac{180}{32.2}\right)(1\omega)^2 - 180(4)$$

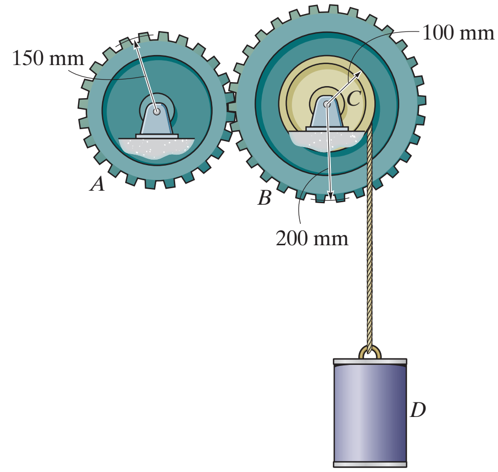
$\omega = 6.3776 \text{ rad/s}$

$v_C = \omega(5) = 6.3776(5) = 31.9 \text{ m/s}$ **Ans.**



***18–44.**

Determine the speed of the 50-kg cylinder after it has descended a distance of 2 m, starting from rest. Gear A has a mass of 10 kg and a radius of gyration of 125 mm about its center of mass. Gear B and drum C have a combined mass of 30 kg and a radius of gyration about their center of mass of 150 mm.



SOLUTION

Potential Energy: With reference to the datum shown in Fig. a, the gravitational potential energy of block D at position (1) and (2) is

$$V_1 = (V_g)_1 = W_D(y_D)_1 = 50(9.81)(0) = 0$$

$$V_2 = (V_g)_2 = -W_D(y_D)_2 = -50(9.81)(2) = -981 \text{ J}$$

Kinetic Energy: Since gear B rotates about a fixed axis, $\omega_B = \dfrac{v_D}{r_D} = \dfrac{v_D}{0.1} = 10v_D$.

Also, since gear A is in mesh with gear B, $\omega_A = \left(\dfrac{r_B}{r_A}\right)\omega_B = \left(\dfrac{0.2}{0.15}\right)(10v_D) = 13.33v_D$.

The mass moment of inertia of gears A and B about their mass centers are $I_A = m_A k_A{}^2 = 10(0.125^2) = 0.15625 \text{ kg}\cdot\text{m}^2$ and $I_B = m_B k_B{}^2 = 30(0.15^2) = 0.675 \text{ kg}\cdot\text{m}^2$. Thus, the kinetic energy of the system is

$$\begin{aligned} T &= \frac{1}{2}I_A\omega_A{}^2 + \frac{1}{2}I_B\omega_B{}^2 + \frac{1}{2}m_D v_D{}^2 \\ &= \frac{1}{2}(0.15625)(13.33v_D)^2 + \frac{1}{2}(0.675)(10v_D)^2 + \frac{1}{2}(50)v_D{}^2 \\ &= 72.639v_D{}^2 \end{aligned}$$

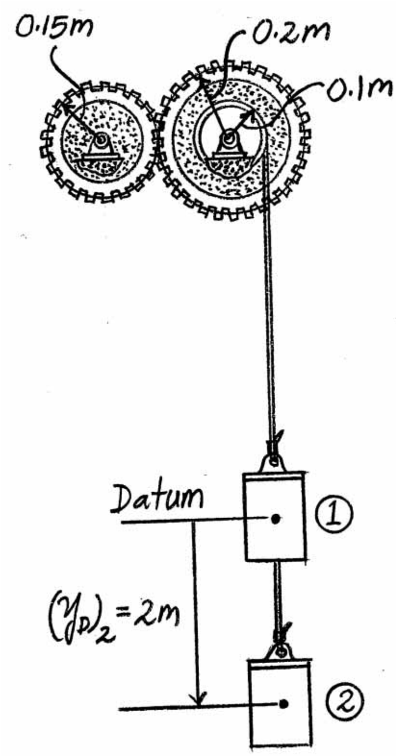
Since the system is initially at rest, $T_1 = 0$.

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 72.639v_D{}^2 - 981$$

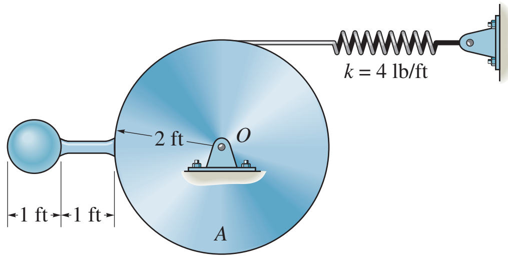
$$v_D = 3.67 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

18–45.

The disk A is pinned at O and weighs 15 lb. A 1-ft rod weighing 2 lb and a 1-ft-diameter sphere weighing 10 lb are welded to the disk, as shown. If the spring is orginally stretched 1 ft and the sphere is released from the position shown, determine the angular velocity of the disk when it has rotated 90°.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0 + 0] + \frac{1}{2}(4)(1)^2 = \frac{1}{2}[\frac{1}{2}(\frac{15}{32.2})(2)^2]\omega^2 + \frac{1}{2}[\frac{1}{12}(\frac{2}{32.2})(1)^2]\omega^2 + \frac{1}{2}(\frac{2}{32.2})(\nu_G)_R^2$$

$$+ \frac{1}{2}[\frac{2}{5}(\frac{10}{32.2})(0.5)^2]\omega^2 + \frac{1}{2}(\frac{10}{32.2})(\nu_G)_s^2 - 2(2.5) - 10(3.5) + \frac{1}{2}(4)(1 + 2(\frac{\pi}{2}))^2$$

Since

$$(\nu_G)_S = 3.5\omega$$

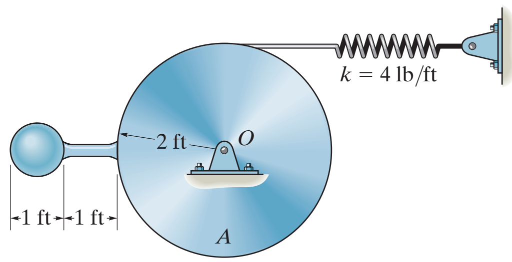
$$(\nu_G)_R = 2.5\omega$$

Substituting and solving, yields

$$\omega = 1.73 \text{ rad/s} \qquad \textbf{Ans.}$$

18–46.

The disk A is pinned at O and weighs 15 lb. A 1-ft rod weighing 2 lb and a 1-ft-diameter sphere weighing 10 lb are welded to the disk, as shown. If the spring is originally stretched 1 ft and the sphere is released from the position shown, determine the angular velocity of the disk when it has rotated 45°.



SOLUTION

Potential Energy: From the geometry shown in Fig. a, we obtain $(y_{G1})_2 = 2.5 \sin 45° \text{ ft} = 1.7678 \text{ ft}$ and $(y_{G2})_2 = 3.5 \sin 45° = 2.4749 \text{ ft}$. With reference to the datum set in Fig. a, the initial and final gravitational potential energy of the system is

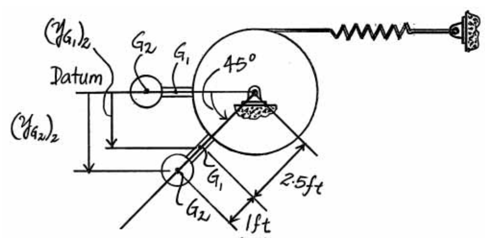
$$(V_g)_1 = W_1(y_{G1})_1 + W_2(y_{G2})_1 = 2(0) + 10(0) = 0$$

$$(V_g)_2 = -W_1(y_{G1})_2 - W_2(y_{G2})_2 = -2(1.7678) - 10(2.4749)$$
$$= -28.284 \text{ ft} \cdot \text{lb}$$

The initial and final stretch of the spring is $s_1 = 1$ ft and $s_2 = 1 + \dfrac{\pi}{4}(2) = 2.5708$ ft. Thus the initial and final elastic potential energy of the system are

$$(V_e)_1 = \frac{1}{2} k s_1^2 = \frac{1}{2}(4)(1^2) = 2 \text{ ft} \cdot \text{lb}$$

$$(V_e)_2 = \frac{1}{2} k s_2^2 = \frac{1}{2}(4)(2.5708)^2 = 13.218 \text{ ft} \cdot \text{lb}$$



(a)

Kinetic Energy: The mass moment of inertia of the disk assembly about the fixed axis passing through point O is

$$I_O = \frac{2}{5}\left(\frac{10}{32.2}\right)(0.5^2) + \left(\frac{10}{32.2}\right)(3.5^2) + \frac{1}{12}\left(\frac{2}{32.2}\right)(1^2) + \left(\frac{2}{32.2}\right)(2.5^2)$$
$$+ \frac{1}{2}\left(\frac{15}{32.2}\right)(2^2)$$
$$= 5.1605 \text{ slug} \cdot \text{ft}^2$$

Thus, the kinetic energy of the system is

$$T = \frac{1}{2} I_O \omega^2 = \frac{1}{2}(5.1605)\omega^2 = 2.5802\omega^2$$

Since the system is at rest initially, $T_1 = 0$

Conservation of Energy:

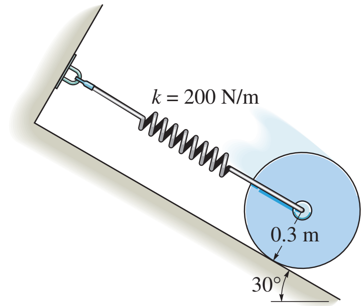
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0 + 2) = 2.5802\omega^2 + (-28.284 + 13.218)$$

$$\omega = 2.572 \text{ rad/s} = 2.57 \text{ rad/s}$$ **Ans.**

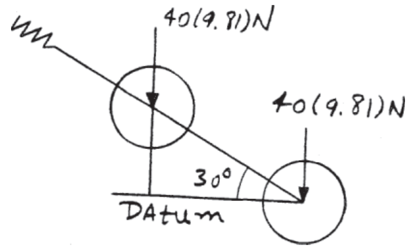
18–47.

At the instant the spring becomes undeformed, the center of the 40-kg disk has a speed of 4 m/s. From this point determine the distance d the disk moves down the plane before momentarily stopping. The disk rolls without slipping.



SOLUTION

Datum at lowest point.



$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{2}(40)(0.3)^2\right]\left(\frac{4}{0.3}\right)^2 + \frac{1}{2}(40)(4)^2 + 40(9.81)d \sin 30^\circ = 0 + \frac{1}{2}(200)d^2$$

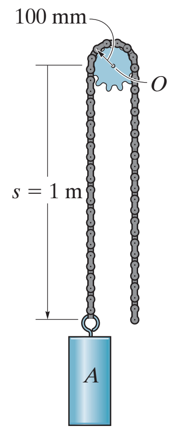
$$100d^2 - 196.2d - 480 = 0$$

Solving for the positive root

$$d = 3.38 \text{ m} \qquad \textbf{Ans.}$$

***18–48.**

A chain that has a negligible mass is draped over the sprocket which has a mass of 2 kg and a radius of gyration of $k_O = 50$ mm. If the 4-kg block A is released from rest from the position $s = 1$ m, determine the angular velocity of the sprocket at the instant $s = 2$ m.

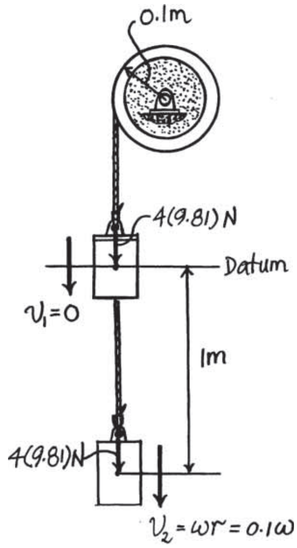


SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

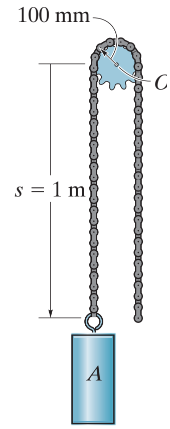
$$0 + 0 + 0 = \frac{1}{2}(4)(0.1\,\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2 - 4(9.81)(1)$$

$$\omega = 41.8 \text{ rad/s}$$ **Ans.**



18–49.

Solve Prob. 18–48 if the chain has a mass of 0.8 kg/m. For the calculation neglect the portion of the chain that wraps over the sprocket.



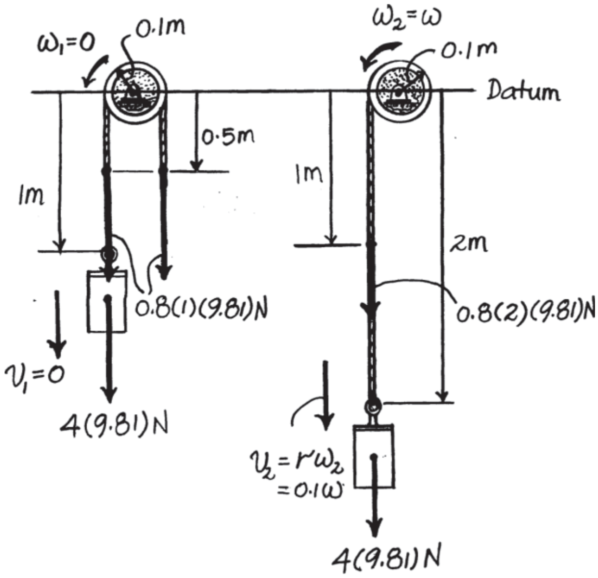
SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 - 4(9.81)(1) - 2\left[0.8(1)(9.81)(0.5)\right] = \frac{1}{2}(4)(0.1\,\omega)^2 + \frac{1}{2}\left[2(0.05)^2\right]\omega^2$$

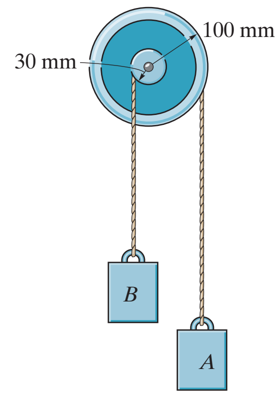
$$+ \frac{1}{2}(0.8)(2)(0.1\,\omega)^2 - 4(9.81)(2) - 0.8(2)(9.81)(1)$$

$$\omega = 39.3 \text{ rad/s}$$ **Ans.**



18–50.

The compound disk pulley consists of a hub and attached outer rim. If it has a mass of 3 kg and a radius of gyration $k_G = 45$ mm, determine the speed of block A after A descends 0.2 m from rest. Blocks A and B each have a mass of 2 kg. Neglect the mass of the cords.

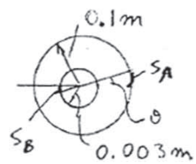


SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0 + 0] + [0 + 0] = \frac{1}{2}[3(0.045)^2]\omega^2 + \frac{1}{2}(2)(0.03\omega)^2 + \frac{1}{2}(2)(0.1\omega)^2 - 2(9.81)s_A + 2(9.81)s_B$$

$$\theta = \frac{s_B}{0.03} = \frac{s_A}{0.1}$$



$$s_B = 0.3\, s_A$$

Set $s_A = 0.2$ m, $s_B = 0.06$ m

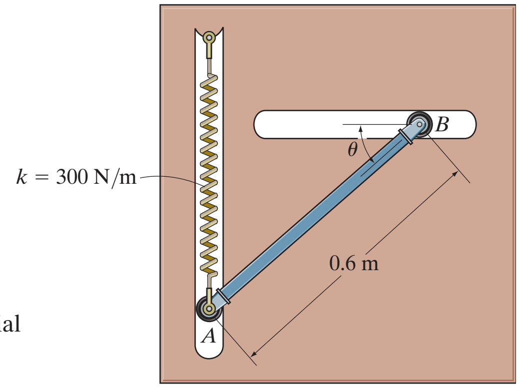
Substituting and solving yields,

$$\omega = 14.04 \text{ rad/s}$$

$$v_A = 0.1(14.04) = 1.40 \text{ m/s}$$ **Ans.**

18–51.

A spring having a stiffness of $k = 300\ \text{N/m}$ is attached to the end of the 15-kg rod, and it is unstretched when $\theta = 0°$. If the rod is released from rest when $\theta = 0°$, determine its angular velocity at the instant $\theta = 30°$. The motion is in the vertical plane.



SOLUTION

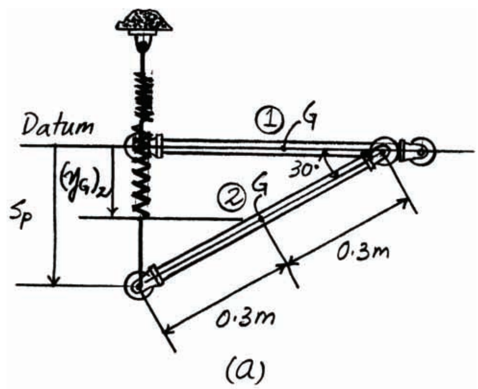
Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the rod at positions (1) and (2) is

$$\left(V_g\right)_1 = W(y_G)_1 = 15(9.81)(0) = 0$$

$$\left(V_g\right)_2 = -W(y_G)_2 = -15(9.81)(0.3\sin 30°) = -22.0725\ \text{J}$$

Since the spring is initially unstretched, $(V_e)_1 = 0$. When $\theta = 30°$, the stretch of the spring is $s_P = 0.6\sin 30° = 0.3\ \text{m}$. Thus, the final elastic potential energy of the spring is

$$\left(V_e\right)_2 = \frac{1}{2}ks_P{}^2 = \frac{1}{2}(300)\left(0.3^2\right) = 13.5\ \text{J}$$

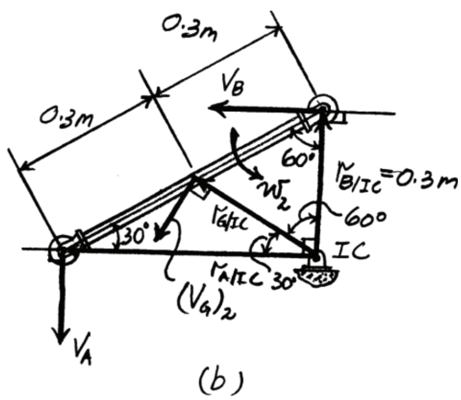


Thus,

$$V_1 = (V_g)_1 + (V_e)_1 = 0 + 0 = 0$$

$$V_2 = (V_g)_2 + (V_e)_2 = -22.0725 + 13.5 = -8.5725\ \text{J}$$

Kinetic Energy: Since the rod is initially at rest, $T_1 = 0$. From the geometry shown in Fig. *b*, $r_{G/IC} = 0.3$ m. Thus, $(V_G)_2 = \omega_2 r_{G/IC} = \omega_2\,(0.3)$. The mass moment of inertia of the rod about its mass center is $I_G = \dfrac{1}{12}ml^2 = \dfrac{1}{12}(15)\left(0.6^2\right) = 0.45\ \text{kg}\cdot\text{m}^2$. Thus, the final kinetic energy of the rod is



$$T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2$$

$$= \frac{1}{2}(15)\left[\omega_2\left(0.3\right)\right]^2 + \frac{1}{2}\left(0.45\right)\omega_2{}^2$$

$$= 0.9\omega_2{}^2$$

Conservation of Energy:

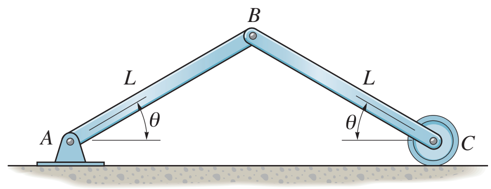
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0.9\omega_2{}^2 - 8.5725$$

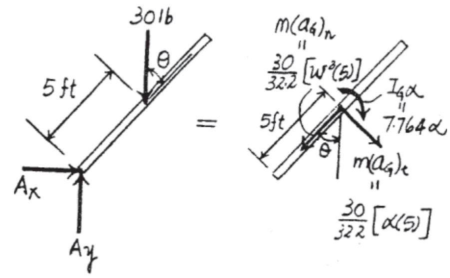
$$\omega_2 = 3.09\ \text{rad/s} \qquad \textbf{Ans.}$$

***18–52.**

The two bars are released from rest at the position θ. Determine their angular velocities at the instant they become horizontal. Neglect the mass of the roller at C. Each bar has a mass m and length L.



SOLUTION



Potential Energy: Datum is set at point A. When links AB and BC is at their initial position, their center of gravity is located $\frac{L}{2}\sin\theta$ *above* the datum. Their gravitational potential energy at this position is $mg\left(\frac{L}{2}\sin\theta\right)$. Thus, the initial and final potential energies are

$$V_1 = 2\left(\frac{mgL}{2}\sin\theta\right) = mgL\sin\theta \qquad V_2 = 0$$

Kinetic Energy: When links AB and BC are in the horizontal position, then $v_B = \omega_{AB}L$ which is directed vertically downward since link AB is rotating about fixed point A. Link BC is subjected to general plane motion and its instantaneous center of zero velocity is located at point C. Thus, $v_B = \omega_{BC}r_{B/IC}$ or $\omega_{AB}L = \omega_{BC}L$, hence $\omega_{AB} = \omega_{BC} = \omega$. The mass moment inertia for link AB and BC about point A and C is $(I_{AB})_A = (I_{BC})_C = \frac{1}{2}mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{3}mL^2$. Since links AB and CD are at rest initially, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}(I_{AB})_A\,\omega_{AB}^2 + \frac{1}{2}(I_{BC})_C\,\omega_{BC}^2$$

$$= \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 + \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

$$= \frac{1}{3}mL^2\,\omega^2$$

Conservation of Energy: Applying Eq. 18–18, we have

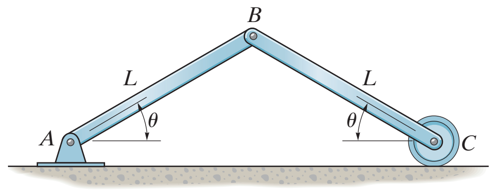
$$T_1 + V_1 = T_2 + V_2$$

$$0 + mgL\sin\theta = \frac{1}{3}mL^2\,\omega^2 + 0$$

$$\omega_{AB} = \omega_{BC} = \omega = \sqrt{\frac{3g}{L}\sin\theta} \qquad \textbf{Ans.}$$

18–53.

The two bars are released from rest at the position $\theta = 90°$. Determine their angular velocities at the instant they become horizontal. Neglect the mass of the roller at C. Each bar has a mass m and length L.



SOLUTION

Potential Energy: With reference to the datum established in Fig. *a*, the gravitational potential energy of the system at position (1) and (2) are

$$(V_1)_g = W_{AB}(y_{G_{AB}})_1 + W_{BC}(y_{G_{BC}})_1 = mg\left(\frac{L}{2}\right) + mg\left(\frac{L}{2}\right) = mg\,L$$

$$(V_2)_g = W_{AB}(y_{G_{AB}})_2 + W_{BC}(y_{G_{BC}})_2 = 0$$

Kinetic Energy: Since the system is at rest initially, $T_1 = 0$. Referring to the kinematic diagram of the system at position (2) shown in Fig. *b*,

$$v_B = \omega_{AB}\,r_{AB} = \omega_{BC}\,r_{G_{BC/IC}}; \qquad \omega_{AB}(L) = \omega_{BC}(L)$$
$$\omega_{AB} = \omega_{BC}$$

Also,

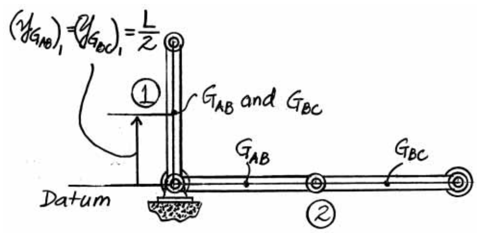
$$v_{G_{BC}} = \omega_{BC}\,r_{G_{BC/IC}} = \omega_{BC}\left(\frac{L}{2}\right)$$

The mass moment of inertia of bar AB about the fixed axis passing through A is $I_A = \frac{1}{3}mL^2$ and the mass moment of inertia of bar BC about its mass center is $I_{G_{BC}} = \frac{1}{12}mL^2$. Thus, the kinetic energy of the system is

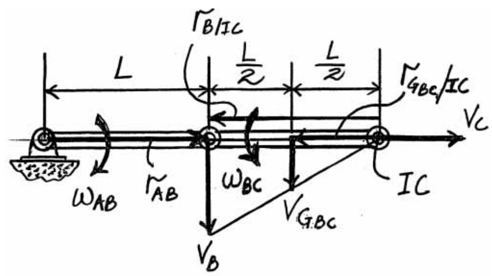
$$T = \frac{1}{2}I_A\omega_{AB}{}^2 + \frac{1}{2}I_{G_{BC}}\omega_{BC}{}^2 + \frac{1}{2}m_{BC}\,v_{G_{BC}}{}^2$$

$$= \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_{BC}{}^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_{BC}{}^2 + \frac{1}{2}m\left[\omega_{BC}\left(\frac{L}{2}\right)\right]^2$$

$$= \frac{1}{3}mL^2\,\omega_{BC}^2$$



(a)



(b)

Conservation of Energy:

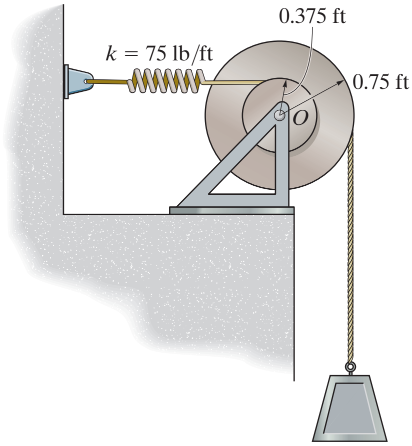
$$T_1 + V_1 = T_2 + V_2$$

$$0 + mg\,L = \frac{1}{3}mL^2\,\omega_{BC}{}^2$$

$$\omega_{AB} = \omega_{BC} = \sqrt{\frac{3g}{L}} \qquad \textbf{Ans.}$$

18–54.

If the 250-lb block is released from rest when the spring is unstretched, determine the velocity of the block after it has descended 5 ft. The drum has a weight of 50 lb and a radius of gyration of $k_O = 0.5$ ft about its center of mass O.



SOLUTION

Potential Energy: With reference to the datum shown in Fig. *a*, the gravitational potential energy of the system when the block is at position 1 and 2 is

$$(V_g)_1 = W(y_G)_1 = 250(0) = 0$$

$$(V_g)_2 = -W(y_G)_2 = -250(5) = -1250 \text{ ft} \cdot \text{lb}$$

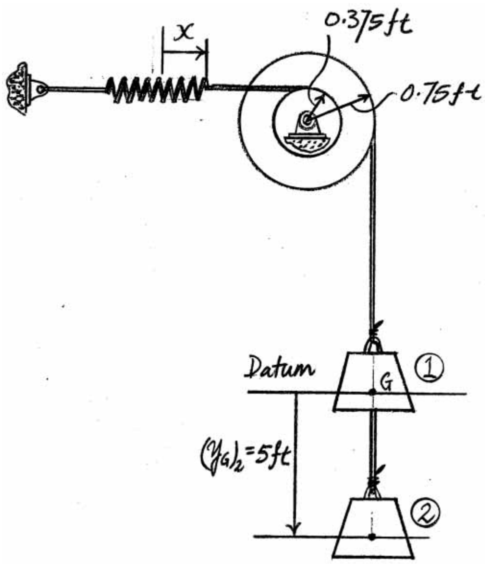
When the block descends $s_b = 5$ ft, the drum rotates through an angle of $\theta = \frac{s_b}{r_b} = \frac{5}{0.75} = 6.667$ rad. Thus, the stretch of the spring is $x = s + s_0 = r_{sp}\theta + 0 = 0.375(6.667) = 2.5$ ft. The elastic potential energy of the spring is

$$(V_e)_2 = \frac{1}{2}kx^2 = \frac{1}{2}(75)(2.5^2) = 234.375 \text{ ft} \cdot \text{lb}$$

Since the spring is initially unstretched, $(V_e)_1 = 0$. Thus,

$$V_1 = (V_g)_1 + (V_e)_1 = 0$$

$$V_2 = (V_g)_2 + (V_e)_2 = -1250 + 234.375 = -1015.625 \text{ ft} \cdot \text{lb}$$



(a)

Kinetic Energy: Since the drum rotates about a fixed axis passing through point O, $\omega = \frac{v_b}{r_b} = \frac{v_b}{0.75} = 1.333v_b$. The mass moment of inertia of the drum about its mass center is $I_O = mk_O^2 = \frac{50}{32.2}\left(0.5^2\right) = 0.3882 \text{ slug} \cdot \text{ft}^2$.

$$\begin{aligned} T &= \frac{1}{2}I_O\omega^2 + \frac{1}{2}m_b v_b^2 \\ &= \frac{1}{2}(0.3882)(1.333v_b)^2 + \frac{1}{2}\left(\frac{250}{32.2}\right)v_b^2 \\ &= 4.2271v_b^2 \end{aligned}$$

Since the system is initially at rest, $T_1 = 0$.

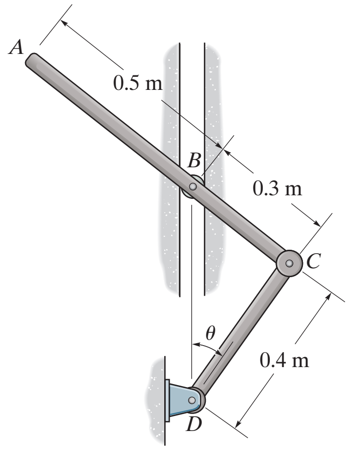
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 4.2271v_b^2 - 1015.625$$

$$v_b = 15.5 \text{ ft/s} \quad \downarrow \qquad \textbf{Ans.}$$

18–55.

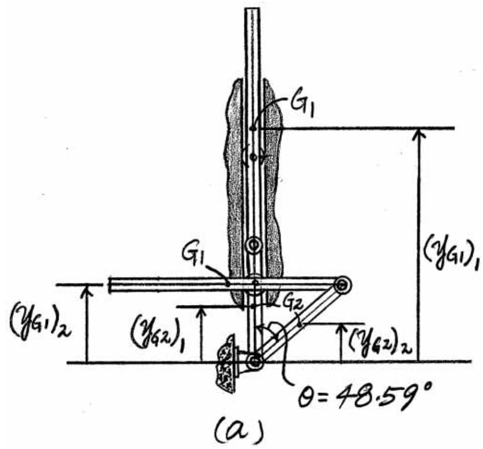
The 6-kg rod ABC is connected to the 3-kg rod CD. If the system is released from rest when $\theta = 0°$, determine the angular velocity of rod ABC at the instant it becomes horizontal.



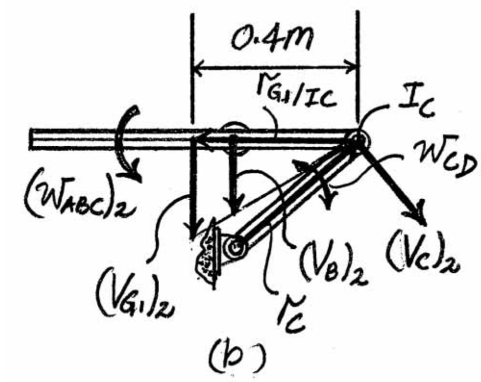
SOLUTION

Potential Energy: When rod ABC is in the horizontal position, Fig. a, $\theta = \sin^{-1}\left(\dfrac{0.3}{0.4}\right) = 48.59°$. With reference to the datum in Fig. a, the initial and final gravitational potential energy of the system is

$$V_1 = (V_g)_1 = W_1(y_{G1})_1 + W_2(y_{G2})_1$$
$$= 6(9.81)(0.8) + 3(9.81)(0.2) = 52.974 \text{ J}$$
$$V_2 = (V_g)_2 = W_1(y_{G1})_2 + W_2(y_{G2})_2$$
$$= 6(9.81)(0.4 \cos 48.59°) + 3(9.81)(0.2 \cos 48.59°) = 19.466 \text{ J}$$



Kinetic Energy: Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $(v_{G1})_2 = (\omega_{ABC})_2\, r_{G1/IC} = (\omega_{ABC})_2(0.4)$. Since point C is at the $IC(v_C)_2 = 0$. Then, $\omega_{CD} = \dfrac{(v_C)_2}{r_C} = \dfrac{0}{0.4} = 0$. The mass moment of inertia of rod ABC about its mass center is $I_{G1} = \dfrac{1}{12}(6)(0.8^2) = 0.32 \text{ kg} \cdot \text{m}^2$. Thus, the final kinetic energy of the system is



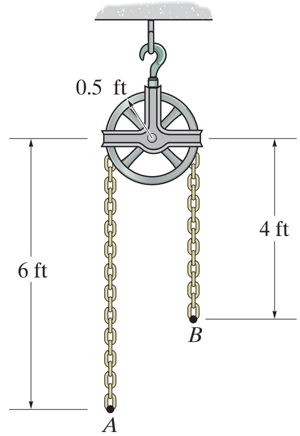
$$T_2 = \frac{1}{2} m_1 (v_{G1})_2{}^2 + \frac{1}{2} I_{G1} (\omega_{ABC})_2{}^2$$
$$= \frac{1}{2}(6)\left[(\omega_{ABC})_2(0.4)\right]^2 + \frac{1}{2}(0.32)(\omega_{ABC})_2{}^2$$
$$= 0.64\omega_{ABC}{}^2$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 52.974 = 0.64\omega_{ABC}{}^2 + 19.466$$
$$(\omega_{ABC})_2 = 7.24 \text{ rad/s} \qquad \textbf{Ans.}$$

***18–56.**

If the chain is released from rest from the position shown, determine the angular velocity of the pulley after the end B has risen 2 ft. The pulley has a weight of 50 lb and a radius of gyration of 0.375 ft about its axis. The chain weighs 6 lb/ft.



SOLUTION

Potential Energy: $(y_{G1})_1 = 2$ ft, $(y_{G2})_1 = 3$ ft, $(y_{G1})_2 = 1$ ft, and $(y_{G2})_2 = 4$ ft. With reference to the datum in Fig. *a*, the gravitational potential energy of the chain at position (1) and (2) is

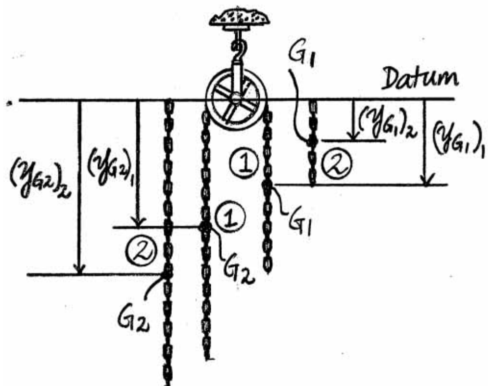
$$V_1 = (V_g)_1 = W_1(y_{G1})_1 - W_2(y_{G2})_1$$
$$= -6(4)(2) - 6(6)(3) = -156 \text{ ft}\cdot\text{lb}$$
$$V_2 = (V_g)_2 = -W_1(y_{G1})_2 + W_2(y_{G2})_2$$
$$= -6(2)(1) - 6(8)(4) = -204 \text{ ft}\cdot\text{lb}$$

Kinetic Energy: Since the system is initially at rest, $T_1 = 0$. The pulley rotates about a fixed axis, thus, $(V_{G1})_2 = (V_{G2})_2 = \omega_2 r = \omega_2(0.5)$. The mass moment of inertia of the pulley about its axis is $I_O = mk_O{}^2 = \dfrac{50}{32.2}(0.375^2) = 0.2184 \text{ slug}\cdot\text{ft}^2$. Thus, the final kinetic energy of the system is

$$T = \frac{1}{2}I_O\omega_2{}^2 + \frac{1}{2}m_1(V_{G1})_2{}^2 + \frac{1}{2}m_2\,(V_{G2})_2{}^2$$
$$= \frac{1}{2}(0.2184)\omega_2{}^2 + \frac{1}{2}\left[\frac{6(2)}{32.2}\right][\omega_2(0.5)]^2 + \frac{1}{2}\left[\frac{6(8)}{32.2}\right][\omega_2(0.5)]^2 + \frac{1}{2}\left[\frac{6(0.5)(\pi)}{32.2}\right][\omega_2(0.5)]^2$$
$$= 0.3787\omega_2{}^2$$

Conservation of Energy:

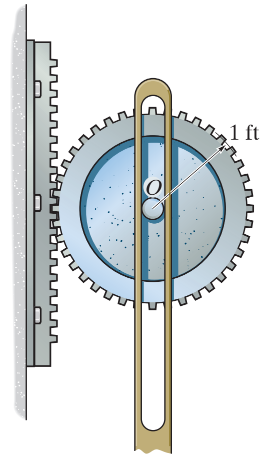
$$T_1 + V_1 = T_2 + V_2$$
$$0 + (-156) = 0.3787\omega_2^2 = (-204)$$
$$\omega_2 = 11.3 \text{ rad/s}$$ **Ans.**



(a)

18–57.

If the gear is released from rest, determine its angular velocity after its center of gravity O has descended a distance of 4 ft. The gear has a weight of 100 lb and a radius of gyration about its center of gravity of $k = 0.75$ ft.



SOLUTION

Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the gear at position 1 and 2 is

$$V_1 = (V_g)_1 = W(y_O)_1 = 100(0) = 0$$

$$V_2 = (V_g)_2 = -W_1(y_O)_2 = -100(4) = -400 \text{ ft} \cdot \text{lb}$$

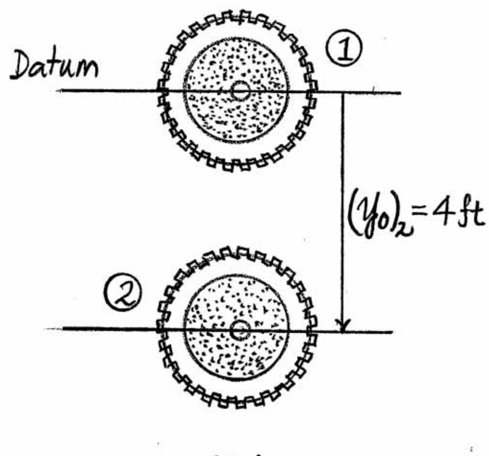
Kinetic Energy: Referring to Fig. *b*, we obtain $v_O = \omega r_{O/IC} = \omega(1)$. The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2 = \frac{100}{32.2}(0.75^2) = 1.7469 \text{ kg} \cdot \text{m}^2$.

Thus,

$$T = \frac{1}{2} m v_O{}^2 + \frac{1}{2} I_O \omega^2$$

$$= \frac{1}{2}\left(\frac{100}{32.2}\right)[\omega(1)]^2 + \frac{1}{2}(1.7469)\omega^2$$

$$= 2.4262\omega^2$$



(a)

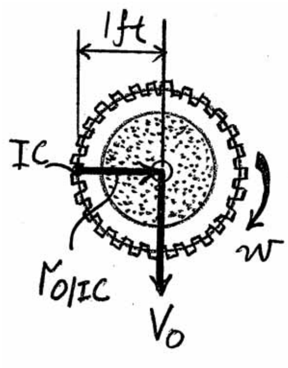
Since the gear is initially at rest, $T_1 = 0$.

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 2.4262\omega^2 - 400$$

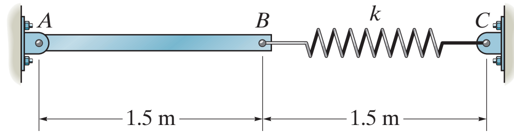
$$\omega = 12.8 \text{ rad/s}$$ **Ans.**



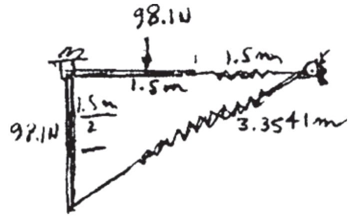
(b)

18–58.

When the slender 10-kg bar AB is horizontal it is at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated clockwise 90°.



SOLUTION



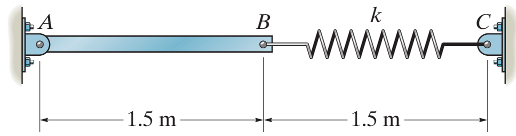
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \frac{1}{2}(k)(3.3541 - 1.5)^2 - 98.1\left(\frac{1.5}{2}\right)$$

$$k = 42.8 \text{ N/m} \qquad \textbf{Ans.}$$

18–59.

When the slender 10-kg bar AB is horizontal it is at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated clockwise 45°.



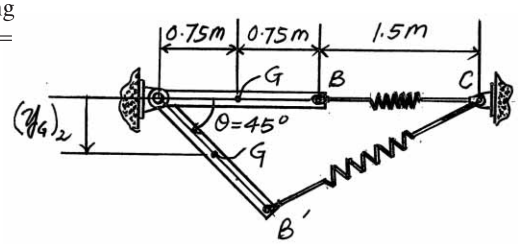
SOLUTION

Potential Energy: From the geometry shown in Fig. a, we obtain $(y_G)_2 = 0.75 \sin 45° = 0.5303$ m and $CB' = \sqrt{3^2 + 1.5^2 - 2(3)(1.5)\cos 45°} = 2.2104$. With reference to the datum established in Fig. a, the initial and final gravitational potential energy of the system is

$$(V_g)_1 = W_{AB}(y_G)_1 = 0$$

$$(V_g)_2 = -W_{AB}(y_G)_2 = -10(9.81)(0.5303) = -52.025 \text{ J}$$

Initially, the spring is unstretched. Thus, $(V_e)_1 = 0$. At the final position, the spring stretches $S = CB' - CB = 2.2104 - 1.5 = 0.7104$ m. Then $(V_e)_1 = 0$ and $(V_e)_2 = \frac{1}{2}ks^2 = \frac{1}{2}k\,(0.7104^2) = 0.2524k$.



(a)

$$V_1 = (V_e)_1 + (V_g)_1 = 0$$

$$V_2 = (V_e)_2 + (V_g)_2 = 0.2524k - 52.025$$

Kinetic Energy: Since the bar is at rest initially and stops momentarily at the final position, $T_1 = T_2 = 0$.

***Conservation of Energy*:**

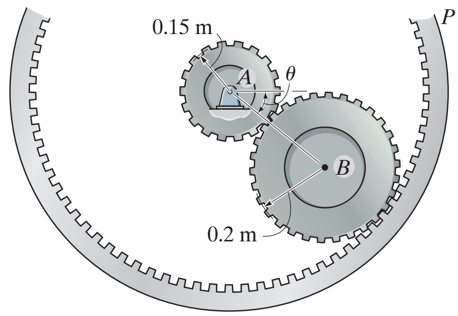
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + 0.2524k - 52.025$$

$$k = 206.15 \text{ N/m} = 206 \text{ N/m}$$ **Ans.**

***18–60.**

If the 40-kg gear B is released from rest at $\theta = 0°$, determine the angular velocity of the 20-kg gear A at the instant $\theta = 90°$. The radii of gyration of gears A and B about their respective centers of mass are $k_A = 125$ mm and $k_B = 175$ mm. The outer gear ring P is fixed.



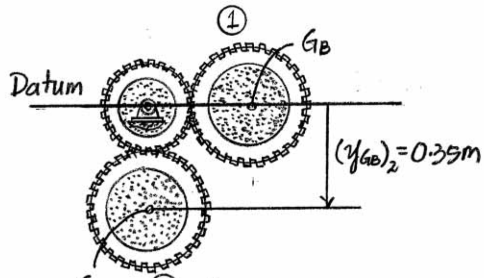
SOLUTION

Potential Energy: With reference to the datum in Fig. a, the gravitational potential energy of gear B at positions (1) and (2) is

$$V_1 = (V_g)_1 = W_B(y_{GB})_1 = 40(9.81)(0) = 0$$

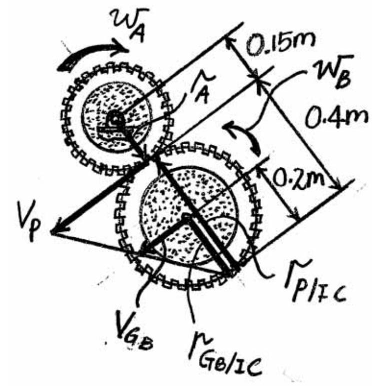
$$V_2 = (V_g)_2 = -W_B(y_{GB})_2 = -40(9.81)(0.35) = -137.34 \text{ J}$$

Kinetic Energy: Referring to Fig. b, $v_P = \omega_A r_A = \omega_A(0.15)$. Then, $\omega_B = \dfrac{v_P}{r_{P/IC}} = \dfrac{\omega_A(0.15)}{0.4} = 0.375\omega_A$. Subsequently, $v_{GB} = \omega_B r_{GB/IC} = (0.375\omega_A)(0.2) = 0.075\omega_A$. The mass moments of inertia of gears A and B about their mass centers are $I_A = m_A k_A^2 = 20(0.125^2) = 0.3125 \text{ kg}\cdot\text{m}^2$ and $I_B = m_B k_B^2 = 40(0.175^2) = 1.225 \text{ kg}\cdot\text{m}^2$. Thus, the kinetic energy of the system is



(a)

$$\begin{aligned} T &= T_A + T_B \\ &= \frac{1}{2} I_A \omega_A^2 + \left[\frac{1}{2} m_B v_{GB}^2 + \frac{1}{2} I_B \omega_B^2\right] \\ &= \frac{1}{2}(0.3125)\omega_A^2 + \left[\frac{1}{2}(40)(0.075\omega_A)^2 + \frac{1}{2}(1.225)(0.375\omega_A)^2\right] \\ &= 0.3549\omega_A^2 \end{aligned}$$



(b)

Since the system is initially at rest, $T_1 = 0$.

Conservation of Energy:

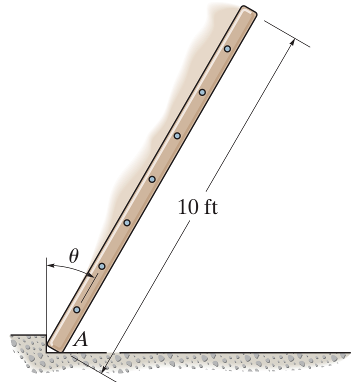
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0.3549\omega_A^2 - 137.34$$

$$\omega_A = 19.7 \text{ rad/s} \qquad \textbf{Ans.}$$

18–61.

A uniform ladder having a weight of 30 lb is released from rest when it is in the vertical position. If it is allowed to fall freely, determine the angle θ at which the bottom end A starts to slide to the right of A. For the calculation, assume the ladder to be a slender rod and neglect friction at A.



SOLUTION

Potential Energy: Datum is set at point A. When the ladder is at its initial and final position, its center of gravity is located 5 ft and $(5\cos\theta)$ ft *above* the datum. Its initial and final gravitational potential energy are $30(5) = 150\ \text{ft}\cdot\text{lb}$ and $30(5\cos\theta) = 150\cos\theta\ \text{ft}\cdot\text{lb}$, respectively. Thus, the initial and final potential energy are

$$V_1 = 150\ \text{ft}\cdot\text{lb} \qquad V_2 = 150\cos\theta\ \text{ft}\cdot\text{lb}$$

Kinetic Energy: The mass moment inertia of the ladder about point A is $I_A = \frac{1}{12}\left(\frac{30}{32.2}\right)(10^2) + \left(\frac{30}{32.2}\right)(5^2) = 31.06\ \text{slug}\cdot\text{ft}^2$. Since the ladder is initially at rest, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}I_A\omega^2 = \frac{1}{2}(31.06)\omega^2 = 15.53\omega^2$$

Conservation of Energy: Applying Eq. 18–18, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 150 = 15.53\omega^2 + 150\cos\theta$$

$$\omega^2 = 9.66(1 - \cos\theta)$$

Equation of Motion: The mass moment inertia of the ladder about its mass center is $I_G = \frac{1}{12}\left(\frac{30}{32.2}\right)(10^2) = 7.764\ \text{slug}\cdot\text{ft}^2$. Applying Eq. 17–16, we have

$$+\ \Sigma M_A = \Sigma(M_k)_A; \qquad -30\sin\theta(5) = -7.764\alpha - \left(\frac{30}{32.2}\right)[\alpha(5)](5)$$

$$\alpha = 4.83\sin\theta$$

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad A_x = -\frac{30}{32.2}[9.66(1 - \cos\theta)(5)]\sin\theta + \frac{30}{32.2}[4.83\sin\theta(5)]\cos\theta$$

$$A_x = -\frac{30}{32.2}(48.3\sin\theta - 48.3\sin\theta\cos\theta - 24.15\sin\theta\cos\theta)$$

$$= 45.0\sin\theta\,(1 - 1.5\cos\theta) = 0$$

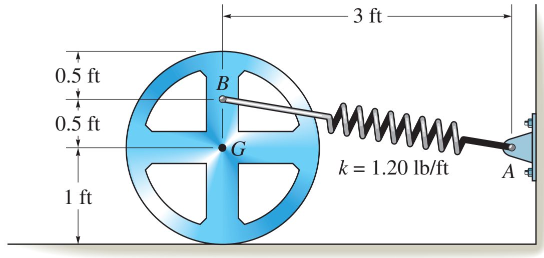
If the ladder begins to slide, then $A_x = 0$. Thus, for $\theta > 0$,

$$45.0\sin\theta\,(1 - 1.5\cos\theta) = 0$$

$$\theta = 48.2^\circ$$ **Ans.**

18–62.

The 50-lb wheel has a radius of gyration about its center of gravity G of $k_G = 0.7$ ft. If it rolls without slipping, determine its angular velocity when it has rotated clockwise 90° from the position shown. The spring AB has a stiffness $k = 1.20$ lb/ft and an unstretched length of 0.5 ft. The wheel is released from rest.



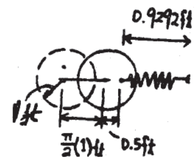
SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(1.20)[\sqrt{(3)^2 + (0.5)^2} - 0.5]^2 = \frac{1}{2}[\frac{50}{32.2}(0.7)^2]\omega^2 + \frac{1}{2}(\frac{50}{32.2})(1\omega)^2$$

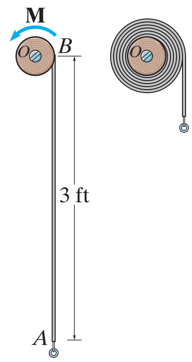
$$+ \frac{1}{2}(1.20)(0.9292 - 0.5)^2$$

$$\omega = 1.80 \text{ rad/s} \qquad \textbf{Ans.}$$



18–63.

The uniform window shade AB has a total weight of 0.4 lb. When it is released, it winds up around the spring-loaded core O. Motion is caused by a spring within the core, which is coiled so that it exerts a torque $M = 0.3(10^{-3})\theta$ lb · ft, where θ is in radians, on the core. If the shade is released from rest, determine the angular velocity of the core at the instant the shade is completely rolled up, i.e., after 12 revolutions. When this occurs, the spring becomes uncoiled and the radius of gyration of the shade about the axle at O is $k_O = 0.9$ in. *Note:* The elastic potential energy of the torsional spring is $V_e = \frac{1}{2}k\theta^2$, where $M = k\theta$ and $k = 0.3(10^{-3})$ lb · ft/rad.

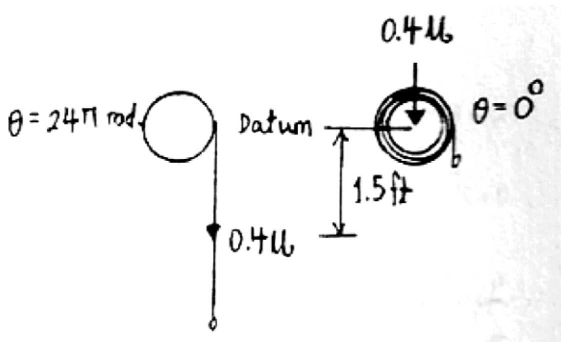


SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

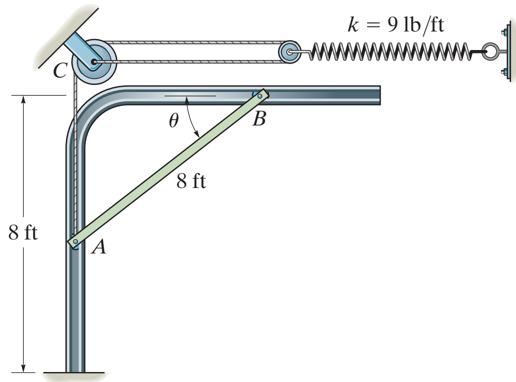
$$0 - (0.4)(1.5) + \frac{1}{2}(0.3)(10^{-3})(24\pi)^2 = \frac{1}{2}\left(\frac{0.4}{32.2}\right)\left(\frac{0.9}{12}\right)^2\omega^2$$

$\omega = 85.1$ rad/s **Ans.**



***18–64.**

The motion of the uniform 80-lb garage door is guided at its ends by the track. Determine the required initial stretch in the spring when the door is open, $\theta = 0°$, so that when it falls freely it comes to rest when it just reaches the fully closed position, $\theta = 90°$. Assume the door can be treated as a thin plate, and there is a spring and pulley system on each of the two sides of the door.



SOLUTION

$$s_A + 2s_s = l$$

$$\Delta s_A = -2\Delta s_s$$

$$8 \text{ ft} = -2\Delta s_s$$

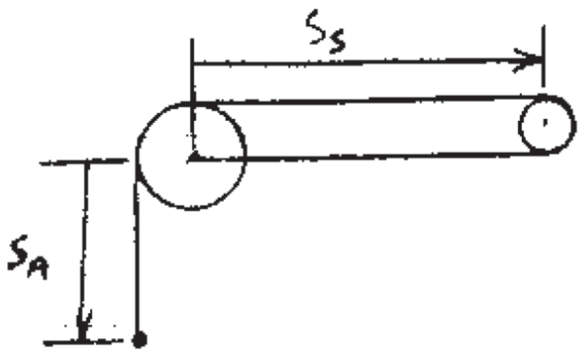
$$\Delta s_s = -4 \text{ ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(9)s^2\right] = 0 - 80(4) + 2\left[\frac{1}{2}(9)(4 + s)^2\right]$$

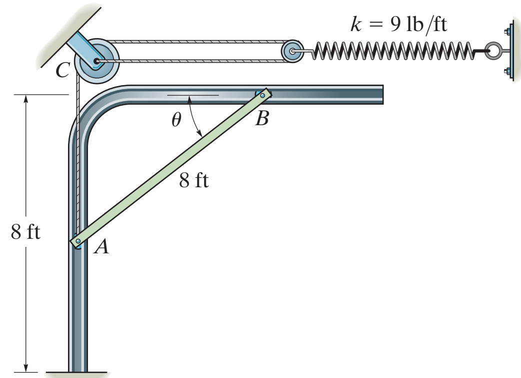
$$9s^2 = -320 + 9(16 + 8s + s^2)$$

$$s = 2.44 \text{ ft} \qquad \textbf{Ans.}$$



18–65.

The motion of the uniform 80-lb garage door is guided at its ends by the track. If it is released from rest at $\theta = 0°$, determine the door's angular velocity at the instant $\theta = 30°$. The spring is originally stretched 1 ft when the door is held open, $\theta = 0°$. Assume the door can be treated as a thin plate, and there is a spring and pulley system on each of the two sides of the door.



SOLUTION

$$v_G = 4\omega$$

$$s_A + 2s_s = l$$

$$\Delta s_A = -2\Delta s_s$$

$$4 \text{ ft} = -2\Delta s_s$$

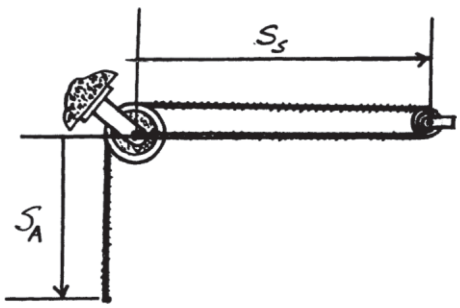
$$\Delta s_s = -2 \text{ ft}$$

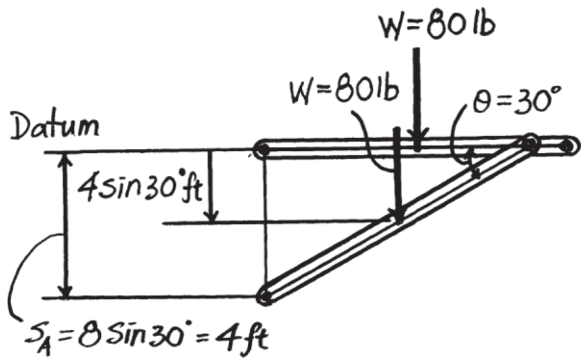
$$T_1 + V_1 = T_2 + V_2$$

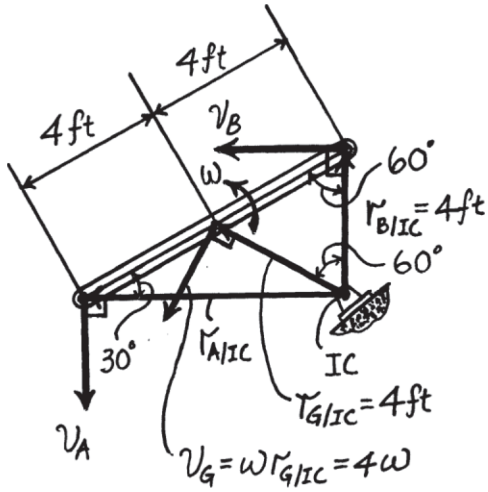
$$0 + 2\left[\frac{1}{2}(9)(1)^2\right] = \frac{1}{2}\left(\frac{80}{32.2}\right)(4\omega)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{80}{32.2}\right)(8)^2\right]\omega^2 - 80(4 \sin 30°)$$

$$+ 2\left[\frac{1}{2}(9)(2+1)^2\right]$$

$$\omega = 1.82 \text{ rad/s} \qquad \textbf{Ans.}$$

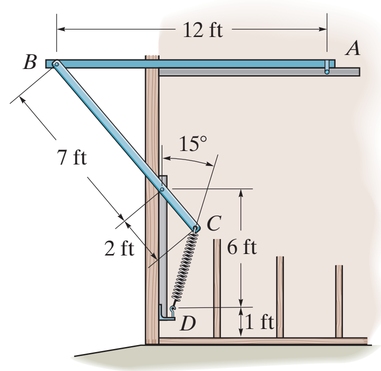






18–66.

The end A of the garage door AB travels along the horizontal track, and the end of member BC is attached to a spring at C. If the spring is originally unstretched, determine the stiffness k so that when the door falls downward from rest in the position shown, it will have zero angular velocity the moment it closes, i.e., when it and BC become vertical. Neglect the mass of member BC and assume the door is a thin plate having a weight of 200 lb and a width and height of 12 ft. There is a similar connection and spring on the other side of the door.



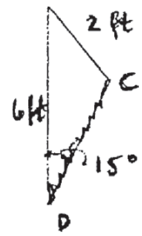
SOLUTION

$$(2)^2 = (6)^2 + (CD)^2 - 2(6)(CD)\cos 15^\circ$$

$$CD^2 - 11.591CD + 32 = 0$$

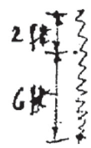
Selecting the smaller root:

$$CD = 4.5352 \text{ ft}$$



$$T_1 + V_1 = T_2 + V_2$$

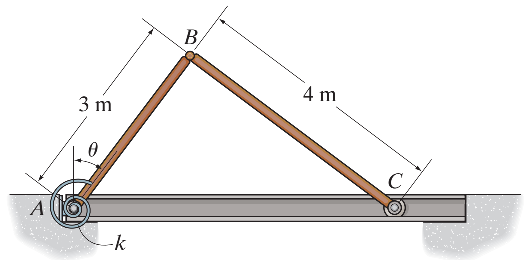
$$0 + 0 = 0 + 2\left[\frac{1}{2}(k)(8 - 4.5352)^2\right] - 200(6)$$



$$k = 100 \text{ lb/ft}$$ **Ans.**

18–67.

Determine the stiffness k of the torsional spring at A, so that if the bars are released from rest when $\theta = 0°$, bar AB has an angular velocity of 0.5 rad/s at the closed position, $\theta = 90°$. The spring is uncoiled when $\theta = 0°$. The bars have a mass per unit length of 10 kg/m.



SOLUTION

Potential Energy: With reference to the datum in Fig. a, the gravitational potential energy of the system at its open and closed positions is

$$\left(V_g\right)_1 = W_{AB}\,(y_{G1})_1 + W_{BC}\,(y_{G2})_1$$

$$= 10(3)(9.81)(1.5) + 10(4)(9.81)(1.5) = 1030.5\text{ J}$$

$$\left(V_g\right)_2 = W_{AB}\,(y_{G1})_2 + W_{BC}\,(y_{G2})_2$$

$$= 10(3)(9.81)(0) + 10(4)(9.81)(0) = 0$$

Since the spring is initially uncoiled, $(V_e)_1 = 0$. When the panels are in the closed position, the coiled angle of the spring is $\theta = \dfrac{\pi}{2}$ rad. Thus,

$$(V_e)_2 = \frac{1}{2}k\theta^2 = \frac{1}{2}k\left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{8}k$$

And so,

$$V_1 = \left(V_g\right)_1 + (V_e)_1 = 1030.5 + 0 = 1030.5\text{ J}$$

$$V_2 = \left(V_g\right)_2 + (V_e)_2 = 0 + \frac{\pi^2}{8}k = \frac{\pi^2}{8}k$$

Kinetic Energy: Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $(v_B)_2 = (\omega_{AB})_2 r_B = 0.5(3) = 1.5$ m/s. Then, $(\omega_{BC})_2 = \dfrac{(v_B)_2}{r_{B/IC}} = \dfrac{1.5}{4} = 0.375$ rad/s. Subsequently, $(v_G)_2 = (\omega_{BC})_2 r_{G2/IC} = 0.375(2) = 0.75$ m/s. The mass moments of inertia of AB about point A and BC about its mass center are

$$(I_{AB})_A = \frac{1}{3}ml^2 = \frac{1}{3}[10(3)]\left(3^2\right) = 90\text{ kg}\cdot\text{m}^2$$

and

$$(I_{BC})_{G2} = \frac{1}{12}ml^2 = \frac{1}{12}[10(4)]\left(4^2\right) = 53.33\text{ kg}\cdot\text{m}^2$$

Thus,

$$T_2 = \frac{1}{2}(I_{AB})_A\,(\omega_{AB})_2{}^2 + \left[\frac{1}{2}m_{BC}(v_{G2})^2 + \frac{1}{2}(I_{BC})_{G2}\,(\omega_{BC})_2{}^2\right]$$

$$= \frac{1}{2}(90)\left(0.5^2\right) + \left[\frac{1}{2}[10(4)]\left(0.75^2\right) + \frac{1}{2}(53.33)\left(0.375^2\right)\right]$$

$$= 26.25\text{ J}$$

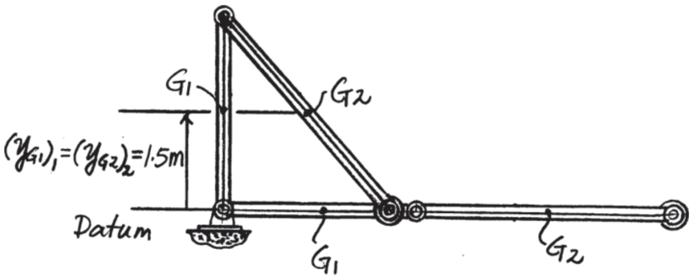
18–67. continued

Conservation of Energy:

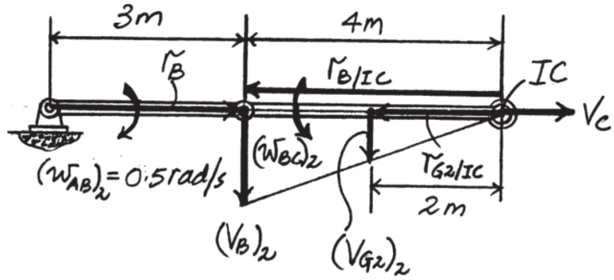
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 1030.5 = 26.25 + \frac{\pi^2}{8}k$$

$$k = 814 \text{ N} \cdot \text{m/rad} \qquad \textbf{Ans.}$$



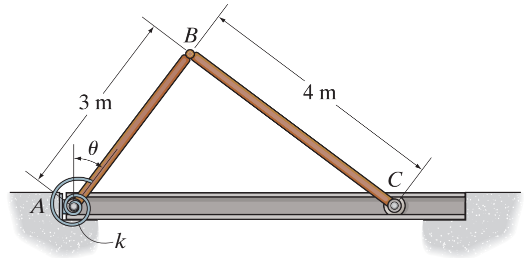
(a)



(b)

***18–68.**

The torsional spring at A has a stiffness of $k = 900\ \text{N} \cdot \text{m/rad}$ and is uncoiled when $\theta = 0°$. Determine the angular velocity of the bars, AB and BC, when $\theta = 0°$, if they are released from rest at the closed position, $\theta = 90°$. The bars have a mass per unit length of $10\ \text{kg/m}$.



SOLUTION

Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the system at its open and closed positions is

$$\left(V_g\right)_1 = W_{AB}\,(y_{G1})_1 + W_{BC}\,(y_{G2})_1$$

$$= 10(3)(9.81)(0) + 10(4)(9.81)(0) = 0$$

$$\left(V_g\right)_2 = W_{AB}\,(y_{G1})_2 + W_{BC}\,(y_{G2})_2$$

$$= 10(3)(9.81)(1.5) + 10(4)(9.81)(1.5) = 1030.05\ \text{J}$$

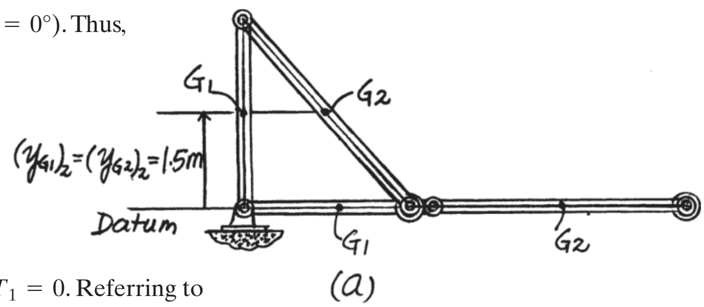
When the panel is in the closed position, the coiled angle of the spring is $\theta = \dfrac{\pi}{2}$ rad.

Thus,

$$(V_e)_1 = \frac{1}{2}k\theta^2 = \frac{1}{2}(900)\left(\frac{\pi}{2}\right)^2 = 112.5\pi^2\ \text{J}$$

The spring is uncoiled when the panel is in the open position ($\theta = 0°$). Thus,

$$(V_e)_2 = 0$$



And so,

$$V_1 = \left(V_g\right)_1 + (V_e)_1 = 0 + 112.5\pi^2 = 112.5\pi^2\ \text{J}$$

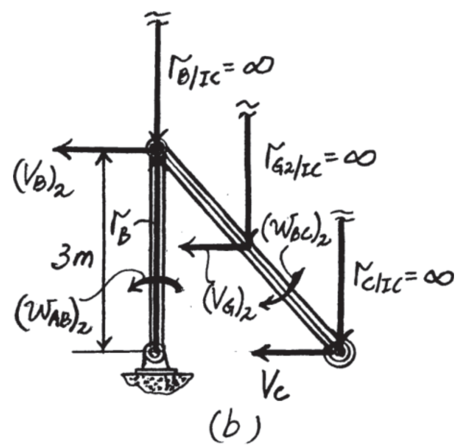
$$V_2 = \left(V_g\right)_2 + (V_e)_2 = 1030.05 + 0 = 1030.05\ \text{J}$$

Kinetic Energy: Since the panel is at rest in the closed position, $T_1 = 0$. Referring to Fig. *b*, the *IC* for *BC* is located at infinity. Thus,

$$(\omega_{BC})_2 = 0 \qquad \textbf{Ans.}$$

Then,

$$(v_G)_2 = (v_B)_2 = (\omega_{AB})_2\, r_B = (\omega_{AB})_2(3)$$



The mass moments of inertia of AB about point A and BC about its mass center are

$$(I_{AB})_A = \frac{1}{3}ml^2 = \frac{1}{3}[10(3)]\left(3^2\right) = 90\ \text{kg} \cdot \text{m}^2$$

and

$$(I_{BC})_{G2} = \frac{1}{12}ml^2 = \frac{1}{12}[10(4)]\left(4^2\right) = 53.33\ \text{kg} \cdot \text{m}^2$$

Thus,

$$T_2 = \frac{1}{2}(I_{AB})_A(\omega_{AB})_2{}^2 + \frac{1}{2}m_{BC}(v_{G2})^2$$

$$= \frac{1}{2}(90)(\omega_{AB})_2{}^2 + \frac{1}{2}[10(4)]\left[(\omega_{AB})_2(3)\right]^2$$

$$= 225(\omega_{AB})_2{}^2$$

18–68. continued

Conservation of Energy:

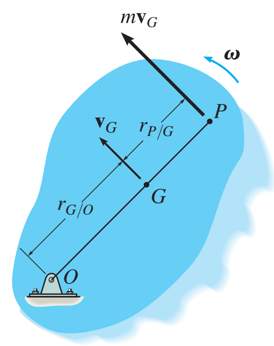
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 112.5\pi^2 = 225(\omega_{AB})_2{}^2 + 1030.05$$

$$(\omega_{AB})_2 = 0.597 \text{ rad/s}$$ **Ans.**

19–1.

The rigid body (slab) has a mass m and rotates with an angular velocity $\boldsymbol{\omega}$ about an axis passing through the fixed point O. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude mv_G and acting through point P, called the *center of percussion,* which lies at a distance $r_{P/G} = k_G^2/r_{G/O}$ from the mass center G. Here k_G is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through G.



SOLUTION

$$H_O = (r_{G/O} + r_{P/G})\, mv_G = r_{G/O}\,(mv_G) + I_G\omega, \qquad \text{where} \quad I_G = mk_G^2$$

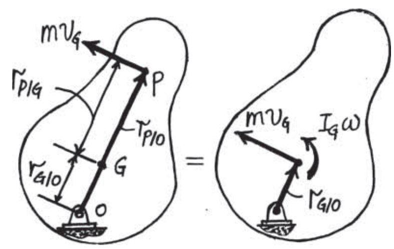
$$r_{G/O}\,(mv_G) + r_{P/G}\,(mv_G) = r_{G/O}\,(mv_G) + (mk_G^2)\,\omega$$

$$r_{P/G} = \frac{k_G^2}{v_G/\omega}$$

However, $\quad v_G = \omega r_{G/O} \quad$ or $\quad r_{G/O} = \dfrac{v_G}{\omega}$

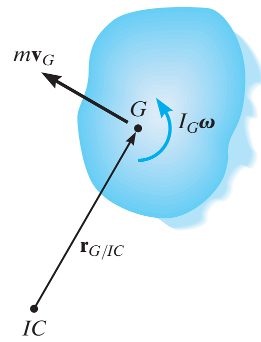
$$r_{P/G} = \frac{k_G^2}{r_{G/O}}$$

Q.E.D.



19–2.

At a given instant, the body has a linear momentum $\mathbf{L} = m\mathbf{v}_G$ and an angular momentum $\mathbf{H}_G = I_G\boldsymbol{\omega}$ computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity IC can be expressed as $\mathbf{H}_{IC} = I_{IC}\boldsymbol{\omega}$, where I_{IC} represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the IC is located at a distance $r_{G/IC}$ away from the mass center G.



SOLUTION

$$H_{IC} = r_{G/IC}\,(mv_G) + I_G\,\omega, \qquad \text{where} \quad v_G = \omega r_{G/IC}$$

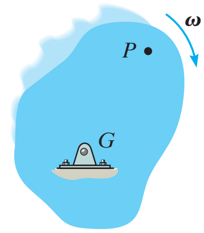
$$= r_{G/IC}\,(m\omega r_{G/IC}) + I_G\,\omega$$

$$= (I_G + mr^2_{G/IC})\,\omega$$

$$= I_{IC}\,\omega \qquad \textbf{Q.E.D.}$$

19–3.

Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center G, the angular momentum is the same when computed about any other point P.



SOLUTION

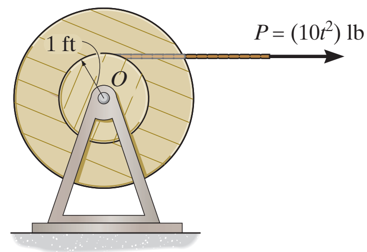
Since $v_G = 0$, the linear momentum $L = mv_G = 0$. Hence the angular momentum about any point P is

$$H_P = I_G\,\omega$$

Since ω is a free vector, so is $\mathbf{H}_P$. **Q.E.D.**

***19–4.**

The cable is subjected to a force of $P = (10t^2)$ lb, where t is in seconds. Determine the angular velocity of the spool 3 s after **P** is applied, starting from rest. The spool has a weight of 150 lb and a radius of gyration of 1.25 ft about its center of gravity.



SOLUTION

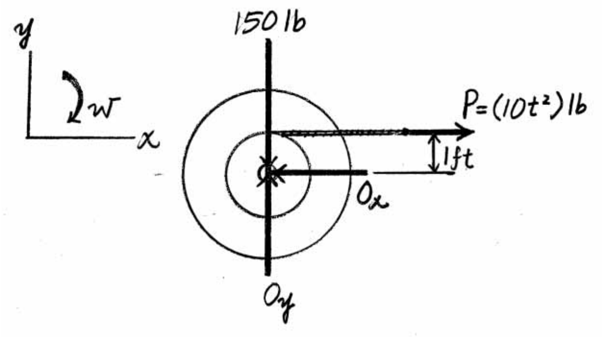
Principle of Angular Impulse and Momentum: The mass moment of inertia of the spool about its mass center is $I_O = mk_O{}^2 = \dfrac{150}{32.2}(1.25)^2 = 7.279 \text{ slug} \cdot \text{ft}^2$.

$$I_O\omega_1 + \Sigma \int_{t_1}^{t_2} M_O dt = I_O\omega_2$$

$$0 - \int_0^{3\text{ s}} 10t^2(1)dt = 7.279\omega_2$$

$$\left.\frac{10t^3}{3}\right|_0^{3\text{ s}} = 7.279\omega_2$$

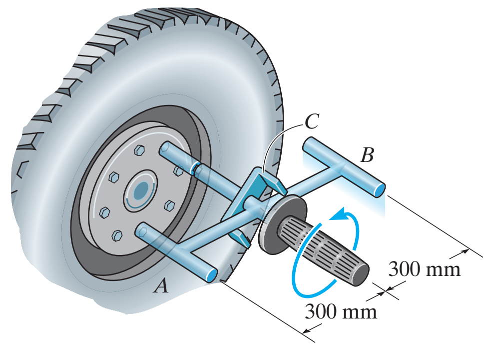
$$\omega_2 = 12.4 \text{ rad/s}$$ **Ans.**



(a)

19–5.

The impact wrench consists of a slender 1-kg rod AB which is 580 mm long, and cylindrical end weights at A and B that each have a diameter of 20 mm and a mass of 1 kg. This assembly is free to rotate about the handle and socket, which are attached to the lug nut on the wheel of a car. If the rod AB is given an angular velocity of 4 rad/s and it strikes the bracket C on the handle without rebounding, determine the angular impulse imparted to the lug nut.



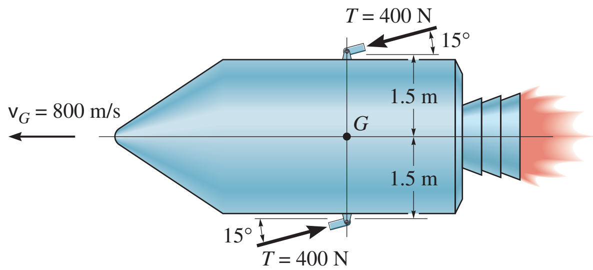
SOLUTION

$$I_{\text{axle}} = \frac{1}{12}(1)(0.6 - 0.02)^2 + 2\left[\frac{1}{2}(1)(0.01)^2 + 1(0.3)^2\right] = 0.2081 \text{ kg}\cdot\text{m}^2$$

$$\int M dt = I_{\text{axle}}\,\omega = 0.2081(4) = 0.833 \text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

19–6.

The space capsule has a mass of 1200 kg and a moment of inertia $I_G = 900 \text{ kg} \cdot \text{m}^2$ about an axis passing through G and directed perpendicular to the page. If it is traveling forward with a speed $v_G = 800 \text{ m/s}$ and executes a turn by means of two jets, which provide a constant thrust of 400 N for 0.3 s, determine the capsule's angular velocity just after the jets are turned off.

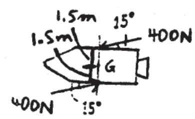


SOLUTION

$\circlearrowleft +$ $$(H_G)_1 + \Sigma \int M_G dt = (H_G)_2$$

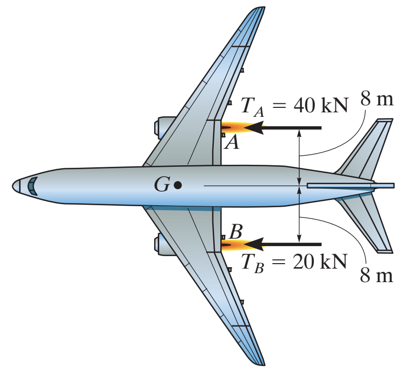
$$0 + 2[400 \cos 15°(0.3)(1.5)] = 900\omega_2$$

$$\omega_2 = 0.386 \text{ rad/s}$$ **Ans.**



19–7.

The airplane is traveling in a straight line with a speed of 300 km/h, when the engines A and B produce a thrust of $T_A = 40$ kN and $T_B = 20$ kN, respectively. Determine the angular velocity of the airplane in $t = 5$ s. The plane has a mass of 200 Mg, its center of mass is located at G, and its radius of gyration about G is $k_G = 15$ m.



SOLUTION

Principle of Angular Impulse and Momentum: The mass moment of inertia of the airplane about its mass center is $I_G = mk_G{}^2 = 200\left(10^3\right)\left(15^2\right) = 45\left(10^6\right)\ \text{kg}\cdot\text{m}^2$. Applying the angular impulse and momentum equation about point G,

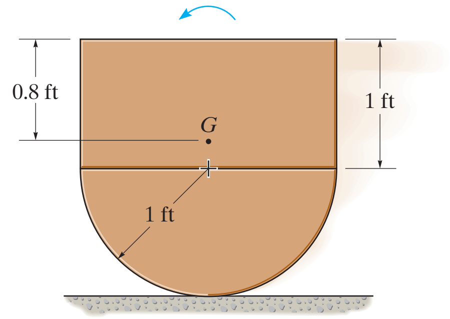
$$I_z\omega_1 + \Sigma\int_{t_1}^{t_2} M_G dt = I_G\omega_2$$

$$0 + 40\left(10^3\right)(5)(8) - 20\left(10^3\right)(5)(8) = 45\left(10^6\right)\omega$$

$$\omega = 0.0178 \text{ rad/s}$$ **Ans.**

***19–8.**

The assembly weighs 10 lb and has a radius of gyration $k_G = 0.6$ ft about its center of mass G. The kinetic energy of the assembly is 31 ft · lb when it is in the position shown. If it is rolling counterclockwise on the surface without slipping, determine its linear momentum at this instant.



SOLUTION

$$I_G = (0.6)^2\left(\frac{10}{32.2}\right) = 0.1118 \text{ slug} \cdot \text{ft}^2$$

$$T = \frac{1}{2}\left(\frac{10}{32.2}\right)v_G^2 + \frac{1}{2}(0.1118)\,\omega^2 = 31 \qquad \textbf{(1)}$$

$$v_G = 1.2\,\omega$$

Substitute into Eq. (1),

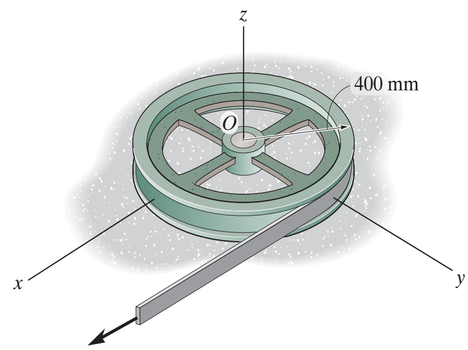
$$\omega = 10.53 \text{ rad/s}$$

$$v_G = 10.53(1.2) = 12.64 \text{ ft/s}$$

$$L = mv_G = \frac{10}{32.2}(12.64) = 3.92 \text{ slug} \cdot \text{ft/s} \qquad \textbf{Ans.}$$

19–9.

The wheel having a mass of 100 kg and a radius of gyration about the z axis of $k_z = 300$ mm, rests on the smooth horizontal plane. If the belt is subjected to a force of $P = 200$ N, determine the angular velocity of the wheel and the speed of its center of mass O, three seconds after the force is applied.



SOLUTION

Principle of Angular Impulse and Momentum: The mass moment of inertia of the wheel about the z axis is $I_z = mk_z^2 = 100(0.3^2) = 9\ \text{kg}\cdot\text{m}^2$. Applying the linear and angular impulse and momentum equations using the free-body diagram of the wheel shown in Fig. a,

$$\overset{+}{\leftarrow} \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + 200(3) = 100(v_O)_2$$

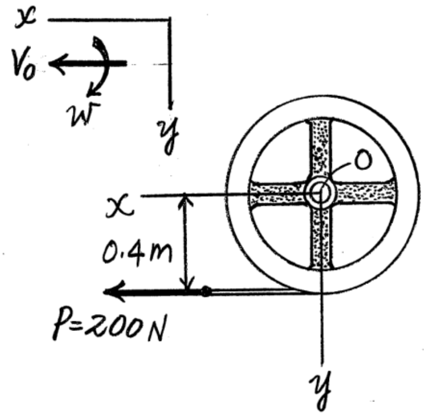
$$(v_O)_2 = 6\ \text{m/s} \qquad \textbf{Ans.}$$

and

$$I_z\omega_1 + \Sigma\int_{t_1}^{t_2} M_z\,dt = I_z\omega_2$$

$$0 - [200(0.4)(3)] = -9\omega_2$$

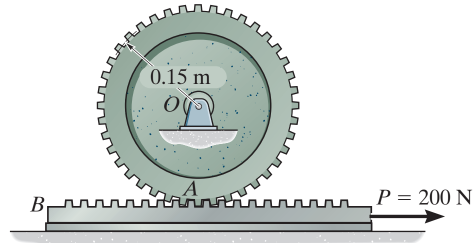
$$\omega_2 = 26.7\ \text{rad/s} \qquad \textbf{Ans.}$$



(a)

19–10.

The 30-kg gear A has a radius of gyration about its center of mass O of $k_O = 125$ mm. If the 20-kg gear rack B is subjected to a force of $P = 200$ N, determine the time required for the gear to obtain an angular velocity of 20 rad/s, starting from rest. The contact surface between the gear rack and the horizontal plane is smooth.

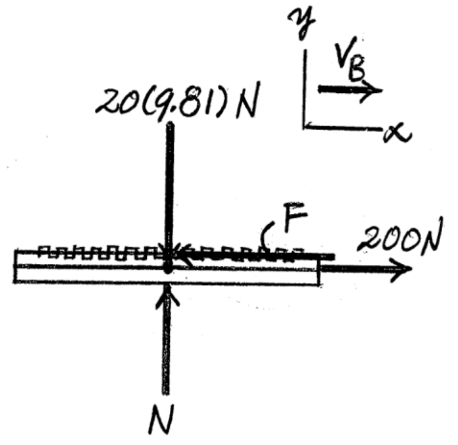


SOLUTION

Kinematics: Since the gear rotates about the fixed axis, the final velocity of the gear rack is required to be

$$(v_B)_2 = \omega_2 r_B = 20(0.15) = 3 \text{ m/s} \rightarrow$$

Principle of Impulse and Momentum: Applying the linear impulse and momentum equation along the x axis using the free-body diagram of the gear rack shown in Fig. a,



(a)

$$(\overset{+}{\rightarrow}) \qquad m(v_B)_1 + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_B)_2$$

$$0 + 200(t) - F(t) = 20(3)$$

$$F(t) = 200t - 60 \qquad \textbf{(1)}$$

The mass moment of inertia of the gear about its mass center is $I_O = mk_O^2 = 30(0.125^2) = 0.46875 \text{ kg}\cdot\text{m}^2$. Writing the angular impulse and momentum equation about point O using the free-body diagram of the gear shown in Fig. b,

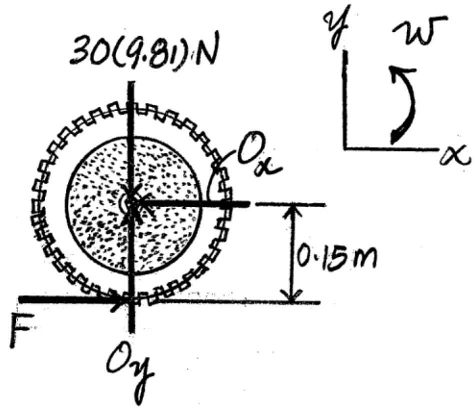
$$I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O dt = I_O\omega_2$$

$$0 + F(t)(0.15) = 0.46875(20)$$

$$F(t) = 62.5 \qquad \textbf{(2)}$$

Substituting Eq. (2) into Eq. (1) yields

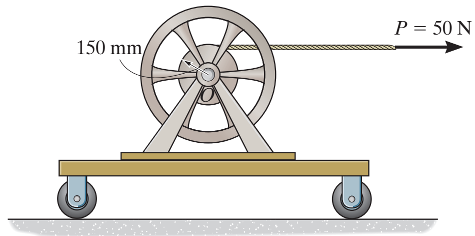
$$t = 0.6125 \text{ s} \qquad \textbf{Ans.}$$



(b)

19–11.

The 30-kg reel is mounted on the 20-kg cart. If the cable wrapped around the inner hub of the reel is subjected to a force of $P = 50$ N, determine the velocity of the cart and the angular velocity of the reel when $t = 4$ s. The radius of gyration of the reel about its center of mass O is $k_O = 250$ mm. Neglect the size of the small wheels.



SOLUTION

Principle of Impulse and Momentum: The mass moment of inertia of the reel about its mass center is $I_O = mk_O{}^2 = 30(0.250^2) = 1.875\ \text{kg}\cdot\text{m}^2$. Referring to Fig. *a*,

$$m\big[(v_O)_1\big]_x + \Sigma\int_{t_1}^{t_2} F_x dt = m\big[(v_O)_2\big]_x$$

$$(\overset{+}{\rightarrow}) \qquad 0 + 50(4) - O_x(4) = 30v$$

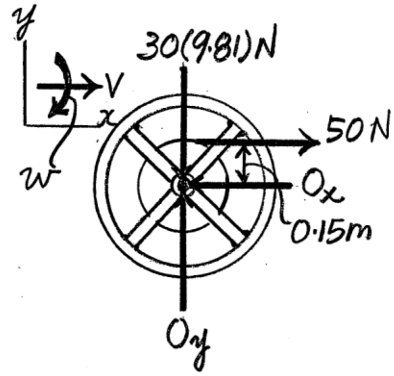
$$O_x = 50 - 7.5v \qquad \textbf{(1)}$$

and

$$I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O dt = I_O\omega_2$$

$$0 + 50(4)(0.15) = 1.875\omega$$

$$\omega = 16\ \text{rad/s} \qquad \textbf{Ans.}$$



(a)

Referring to Fig. *b*,

$$\overset{+}{\rightarrow} \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

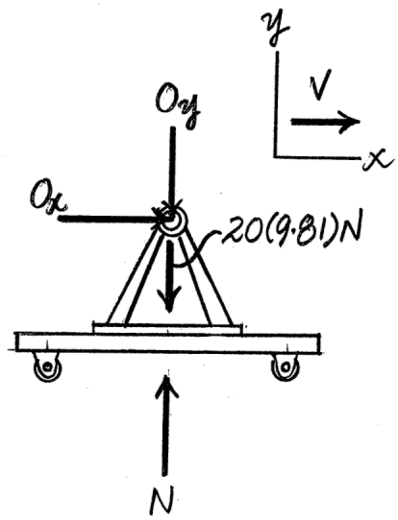
$$0 + O_x(4) = 20v$$

$$O_x = 5v \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v = 4\ \text{m/s} \qquad \textbf{Ans.}$$

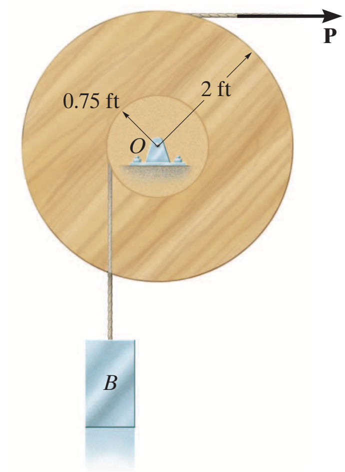
$$O_x = 20\ \text{N}$$



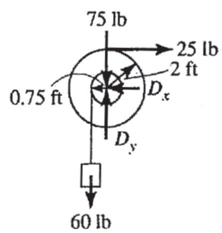
(b)

***19–12.**

The spool has a weight of 75 lb and a radius of gyration $k_O = 1.20$ ft. If the block B weighs 60 lb, and a force $P = 25$ lb is applied to the cord, determine the speed of the block in 5 s starting from rest. Neglect the mass of the cord.



SOLUTION



$\circlearrowleft +$ $$(H_O)_1 + \sum \int M_O\, dt = (H_O)_2$$

$$0 - 60(0.75)(5) + 25(2)(5) = \frac{75}{32.2}(1.20)^2\omega$$

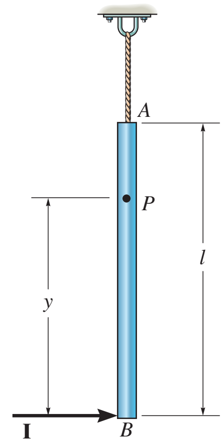
$$+ \left[\frac{60}{32.2}(0.75\omega)\right](0.75)$$

$$\omega = 5.679 \text{ rad/s}$$

$$v_B = \omega r = (5.679)(0.75) = 4.26 \text{ ft/s}$$ **Ans.**

19–13.

The slender rod has a mass m and is suspended at its end A by a cord. If the rod receives a horizontal blow giving it an impulse $\mathbf{I}$ at its bottom B, determine the location y of the point P about which the rod appears to rotate during the impact.



SOLUTION

Principle of Impulse and Momentum:

$$(\circlearrowleft +) \qquad I_G\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\,\omega_2$$

$$0 + I\left(\frac{l}{2}\right) = \left[\frac{1}{12}ml^2\right]\omega \qquad I = \frac{1}{6}ml\omega$$

$$(\overset{+}{\rightarrow}) \qquad m(v_{Ax})_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_{Ax})_2$$

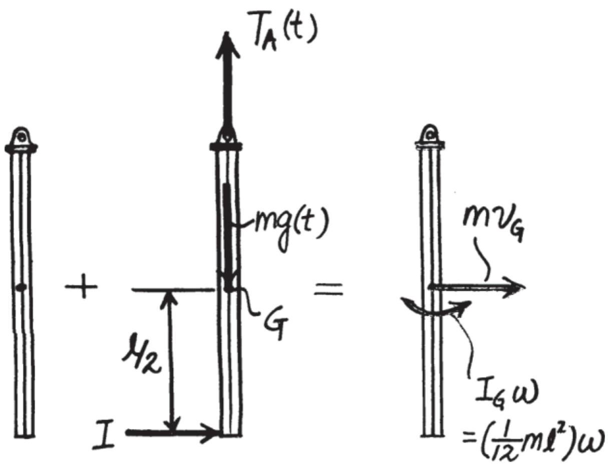
$$0 + \frac{1}{6}ml\omega = mv_G \qquad v_G = \frac{l}{6}\,\omega$$

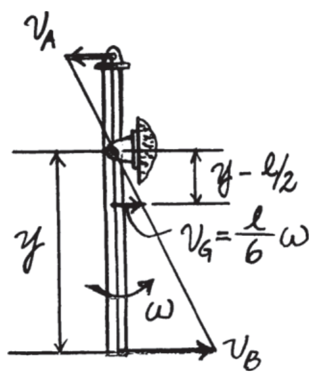
Kinematics: Point P is the IC.

$$v_B = \omega\,y$$

Using similar triangles,

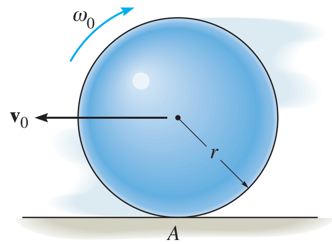
$$\frac{\omega\,y}{y} = \frac{\frac{l}{6}\omega}{y - \frac{l}{2}} \qquad y = \frac{2}{3}l \qquad \textbf{Ans.}$$



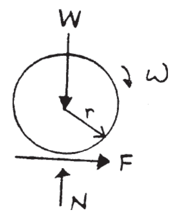


19–14.

If the ball has a weight W and radius r and is thrown onto a *rough surface* with a velocity $\mathbf{v}_0$ parallel to the surface, determine the amount of backspin, $\boldsymbol{\omega}_0$, it must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of friction at A for the calculation.



SOLUTION



$$(\overset{+}{\leftarrow}) \qquad m(v_{Gx})_1 + \Sigma \int F_x \, dt = m(v_{Gx})_2$$

$$\frac{W}{g} v_0 - F_t = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft(+) \qquad (H_G)_1 + \Sigma \int M_G \, dt = (H_G)_2$$

$$-\frac{2}{5}\left(\frac{W}{g} r^2\right)\omega_0 + Ft(r) = 0 \qquad \textbf{(2)}$$

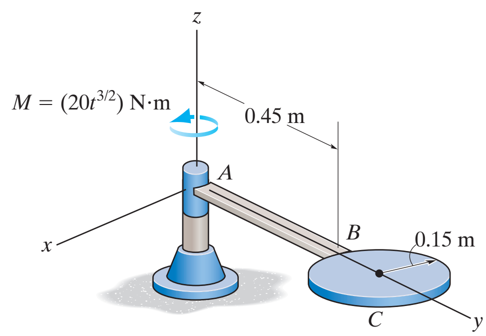
Eliminate Ft between Eqs. (1) and (2):

$$\frac{2}{5}\left(\frac{W}{g} r^2\right)\omega_0 = \left[\frac{W}{g}\left(\frac{v_0}{t}\right)\right] t(r)$$

$$\omega_0 = 2.5\left(\frac{v_0}{r}\right) \qquad \textbf{Ans.}$$

19–15.

The assembly shown consists of a 10-kg rod AB and a 20-kg circular disk C. If it is subjected to a torque of $M = (20t^{3/2})\ \text{N}\cdot\text{m}$, where t is it in seconds, determine its angular velocity when $t = 3$ s. When $t = 0$ the assembly is rotating at $\omega_1 = \{-6\mathbf{k}\}$ rad/s.



SOLUTION

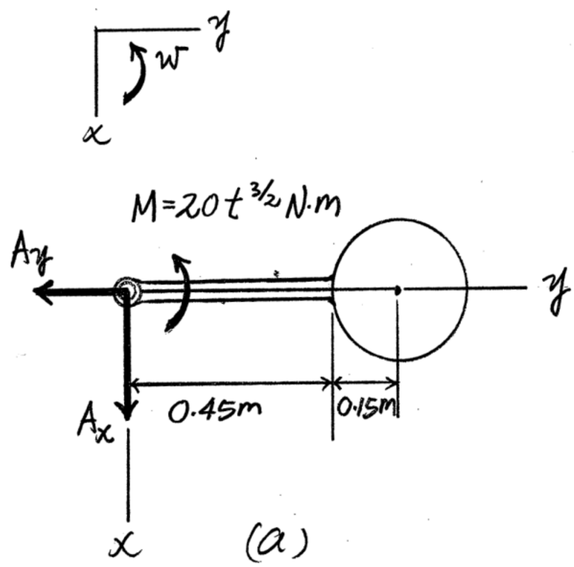
Principle of Angular Impulse and Momentum: The mass moment of inertia of the assembly about the z axis is $I_z = \frac{1}{3}(10)\left(0.45^2\right) + \left[\frac{1}{2}(20)\left(0.15^2\right) + 20\left(0.6^2\right)\right] = 8.10\ \text{kg}\cdot\text{m}^2$. Using the free-body diagram of the assembly shown in Fig. a,

$$I_z\omega_1 + \Sigma\int_{t_1}^{t_2} M_z dt = I_z\omega_2$$

$$8.10(-6) + \int_0^{3\text{ s}} 20t^{3/2}dt = 8.10\omega_2$$

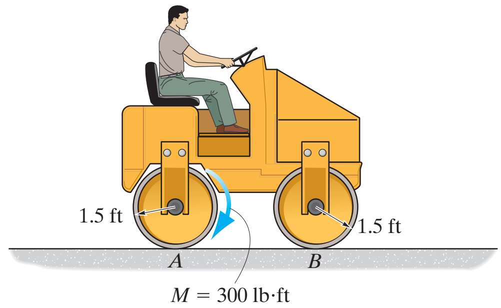
$$-48.6 + 8t^{5/2}\Big|_0^{3\text{ s}} = 8.10\omega_2$$

$$\omega_2 = 9.40\ \text{rad/s}$$ **Ans.**



***19–16.**

The frame of a tandem drum roller has a weight of 4000 lb excluding the two rollers. Each roller has a weight of 1500 lb and a radius of gyration about its axle of 1.25 ft. If a torque of $M = 300 \text{ lb} \cdot \text{ft}$ is supplied to the rear roller A, determine the speed of the drum roller 10 s later, starting from rest.



SOLUTION

Principle of Impulse and Momentum: The mass moments of inertia of the rollers about their mass centers are $I_C = I_D = \frac{1500}{32.2}\left(1.25^2\right) = 72.787 \text{ slug} \cdot \text{ft}^2$. Since the rollers roll without slipping, $\omega = \frac{v}{r} = \frac{v}{1.5} = 0.6667v$. Using the free-body diagrams of the rear roller and front roller, Figs. a and b, and the momentum diagram of the rollers, Fig. c,

$$(H_A)_1 + \Sigma \int_{t_1}^{t_2} M_A dt = (H_A)_2$$

$$0 + 300(10) - C_x(10)(1.5) = \frac{1500}{32.2}v(1.5) + 72.787(0.6667v)$$

$$C_x = 200 - 7.893v \qquad \textbf{(1)}$$

and

$$(H_B)_1 + \Sigma \int_{t_1}^{t_2} M_B dt = (H_B)_2$$

$$0 + D_x(10)(1.5) = \frac{1500}{32.2}v(1.5) + 72.787(0.6667v)$$

$$D_x = 7.893v \qquad \textbf{(2)}$$

Referring to the free-body diagram of the frame shown in Fig. d,

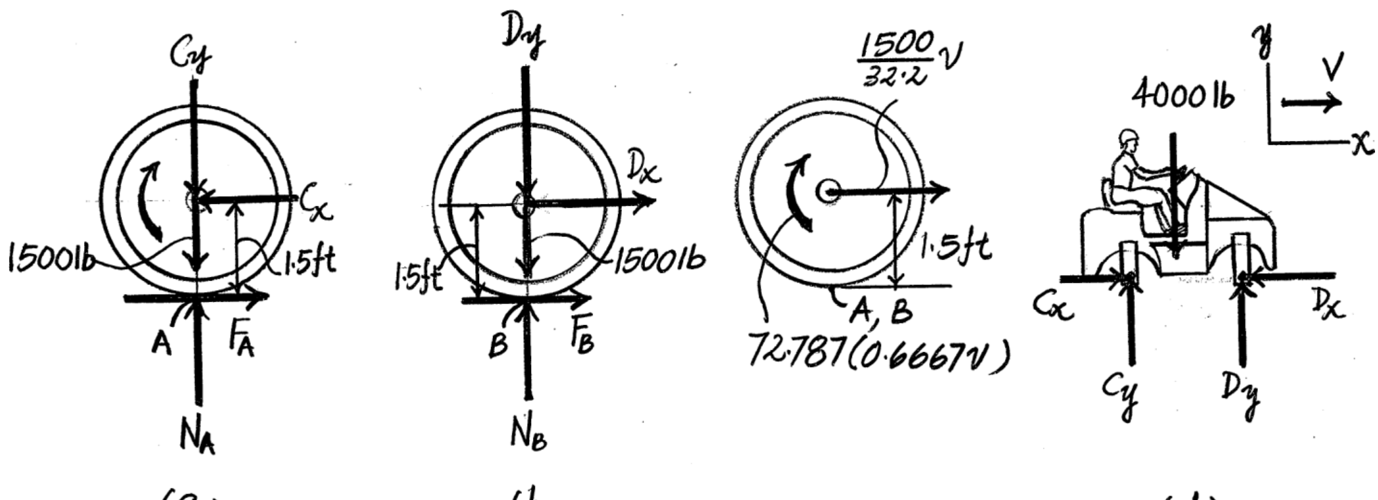
$$\stackrel{+}{\rightarrow} \qquad m\left[(v_G)_x\right]_1 + \Sigma \int_{t_1}^{t_2} F_x dt = m\left[(v_G)_x\right]_2$$

$$0 + C_x(10) - D_x(10) = \frac{4000}{32.2}v \qquad \textbf{(3)}$$

Substituting Eqs. (1) and (2) into Eq. (3),

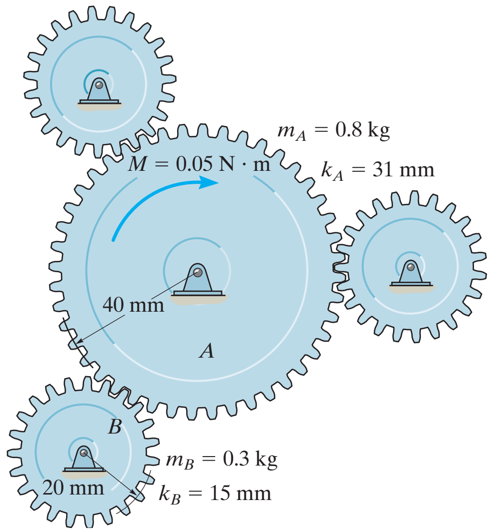
$$(200 - 7.893v)(10) - 7.893v(10) = \frac{4000}{32.2}v$$

$$v = 7.09 \text{ ft/s} \qquad \textbf{Ans.}$$



19–17.

A motor transmits a torque of $M = 0.05\ \text{N}\cdot\text{m}$ to the center of gear A. Determine the angular velocity of each of the three (equal) smaller gears in 2 s starting from rest. The smaller gears (B) are pinned at their centers, and the masses and centroidal radii of gyration of the gears are given in the figure.



SOLUTION

Gear A:

$(\circlearrowright +)$ $$(H_A)_1 + \Sigma \int M_A\, dt = (H_A)_2$$

$$0 - 3(F)(2)(0.04) + 0.05(2) = [0.8(0.031)^2]\omega_A$$

Gear B:

$(\circlearrowleft +)$ $$(H_B)_1 + \Sigma \int M_B\, dt = (H_B)_2$$

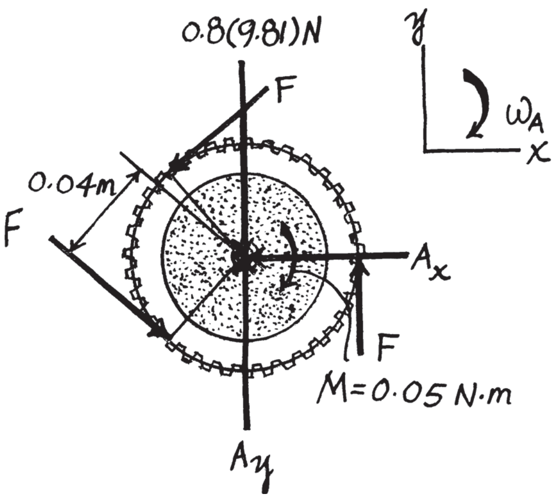
$$0 + (F)(2)(0.02) = [0.3(0.015)^2]\omega_B$$

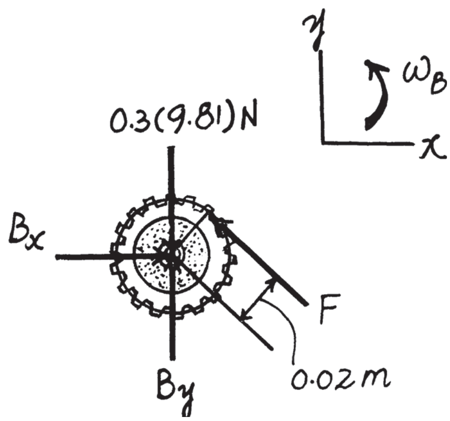
Since $0.04\omega_A = 0.02\omega_B$, or $\omega_B = 2\omega_A$, then solving,

$F = 0.214$ N

$\omega_A = 63.3$ rad/s

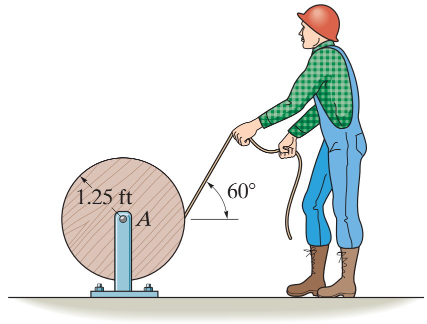
$\omega_B = 127$ rad/s **Ans.**





19–18.

The man pulls the rope off the reel with a constant force of 8 lb in the direction shown. If the reel has a weight of 250 lb and radius of gyration $k_G = 0.8$ ft about the trunnion (pin) at A, determine the angular velocity of the reel in 3 s starting from rest. Neglect friction and the weight of rope that is removed.



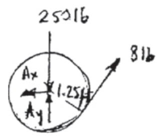
SOLUTION

$$\zeta+ \qquad (H_A)_1 + \Sigma \int M_A \, dt = (H_A)_2$$

$$0 + 8(1.25)(3) = \left[\frac{250}{32.2}(0.8)^2\right]\omega$$

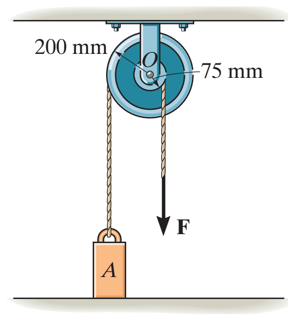
$$\omega = 6.04 \text{ rad/s}$$

Ans.



19–19.

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration $k_O = 110\text{ mm}$. If the block at A has a mass of 40 kg, determine the speed of the block in 3 s after a constant force $F = 2\text{ kN}$ is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest. Neglect the mass of the rope.



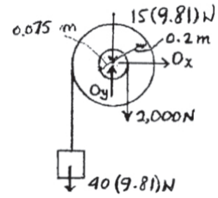
SOLUTION

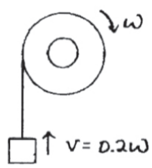
$$(\circlearrowleft +) \qquad (H_O)_1 + \Sigma \int M_O\, dt = (H_O)_2$$

$$0 + 2000(0.075)(3) - 40(9.81)(0.2)(3) = 15(0.110)^2\omega + 40(0.2\omega)\,(0.2)$$

$$\omega = 120.4 \text{ rad/s}$$

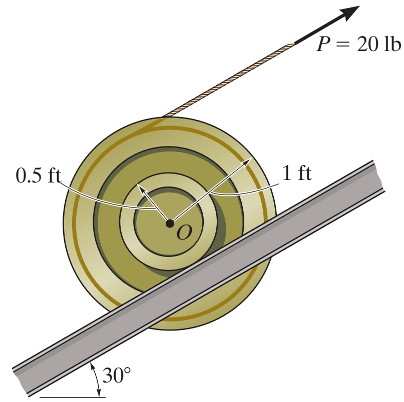
$$v_A = 0.2(120.4) = 24.1 \text{ m/s} \qquad \textbf{Ans.}$$





***19–20.**

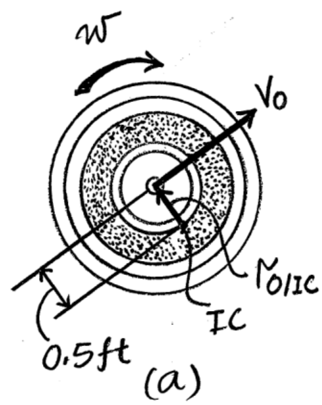
The cable is subjected to a force of $P = 20$ lb, and the spool rolls up the rail without slipping. Determine the angular velocity of the spool in 5 s, starting from rest. The spool has a weight of 100 lb and a radius of gyration about its center of gravity O of $k_O = 0.75$ ft.



SOLUTION

Kinematics: Referring to Fig. *a*,

$$v_O = \omega r_{O/IC} = \omega(0.5)$$

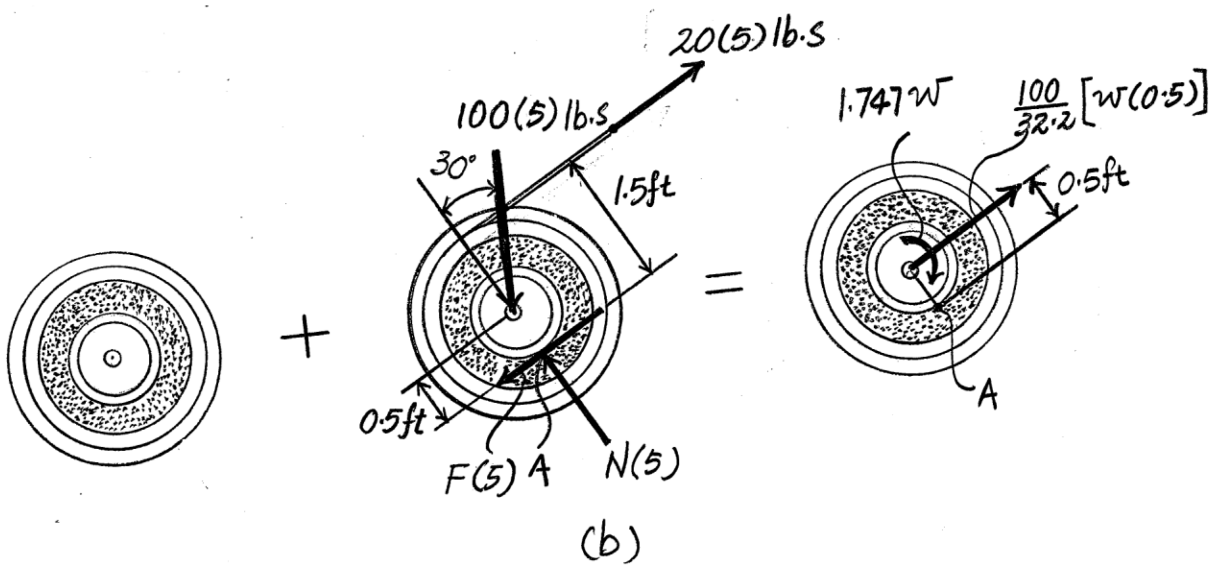


Principle of Angular Impulse and Momentum: The mass moments of inertia of the spool about its mass center is $I_O = mk_O^2 = \dfrac{100}{32.2}(0.75^2) = 1.747 \text{ slug} \cdot \text{ft}^2$. Writing the angular impulse and momentum equation about point A shown in Fig. *b*,

$$(H_A)_1 + \Sigma \int_{t_1}^{t_2} M_A dt = (H_A)_2$$

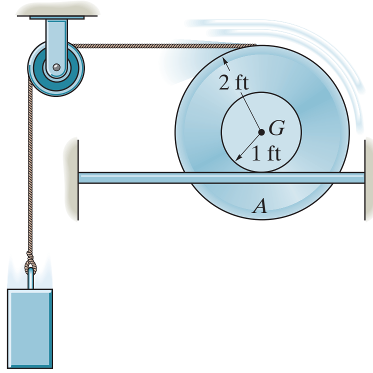
$$0 + 100(5)\sin 30°(0.5) - 20(5)(1.5) = -1.747\omega - \frac{100}{32.2}[\omega(0.5)](0.5)$$

$$\omega = 9.91 \text{ rad/s}$$ **Ans.**



19–21.

The inner hub of the wheel rests on the horizontal track. If it does not slip at A, determine the speed of the 10-lb block in 2 s after the block is released from rest. The wheel has a weight of 30 lb and a radius of gyration $k_G = 1.30$ ft. Neglect the mass of the pulley and cord.

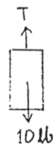


SOLUTION

Spool,

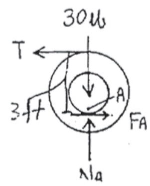
$$(\circlearrowleft +) \qquad (H_A)_1 + \Sigma \int M_A \, dt = (H_A)_2$$

$$0 + T(3)(2) = \left[\frac{30}{32.2}(1.3)^2 + \frac{30}{32.2}(1)^2\right]\left(\frac{v_B}{3}\right)$$



Block,

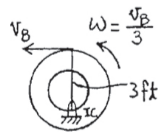
$$(+\downarrow) \qquad m(v_y)_1 + \Sigma \int F_y \, dt = m(v_y)_2$$



$$0 + 10(2) - T(2) = \frac{10}{32.2} v_B$$

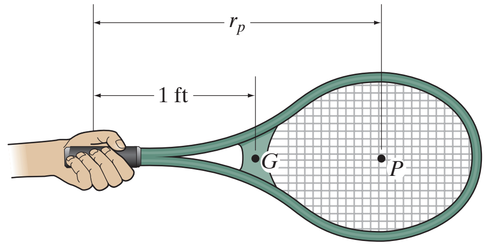
$$v_B = 34.0 \text{ ft/s} \qquad \textbf{Ans.}$$

$$T = 4.73 \text{ lb}$$



19–22.

The 1.25-lb tennis racket has a center of gravity at G and a radius of gyration about G of $k_G = 0.625$ ft. Determine the position P where the ball must be hit so that 'no sting' is felt by the hand holding the racket, i.e., the horizontal force exerted by the racket on the hand is zero.



SOLUTION

Principle of Impulse and Momentum: Here, we will assume that the tennis racket is initially at rest and rotates about point A with an angular velocity of ω immediately after it is hit by the ball, which exerts an impulse of $\int F\,dt$ on the racket, Fig. a. The mass moment of inertia of the racket about its mass center is $I_G = \left(\frac{1.25}{32.2}\right)(0.625^2) = 0.01516\ \text{slug}\cdot\text{ft}^2$. Since the racket about point A, $(v_G) = \omega r_G = \omega(1)$. Referring to Fig. b,

$$\overset{+}{\leftarrow} \qquad m(v_G)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_G)_2$$

$$0 + \int F\,dt = \left(\frac{1.25}{32.2}\right)[\omega(1)]$$

$$\int F\,dt = 0.03882\omega \qquad \textbf{(1)}$$

and

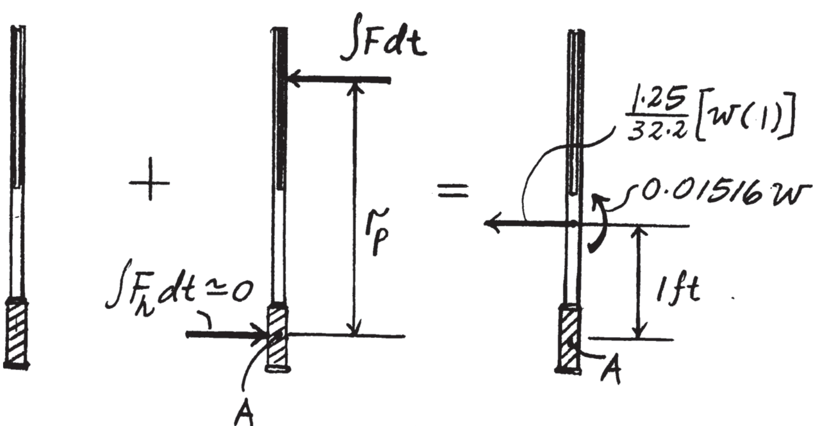
$$\circlearrowleft + \qquad (H_A)_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = (H_A)_2$$

$$0 + \left(\int F\,dt\right)r_P = 0.01516\omega + \frac{1.25}{32.2}[\omega(1)](1)$$

$$\int F\,dt = \frac{0.05398\omega}{r_P} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) yields

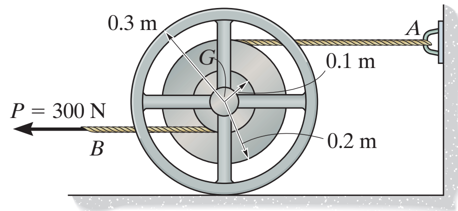
$$0.03882\omega = \frac{0.05398\omega}{r_P} \qquad r_P = 1.39\ \text{ft}$$ **Ans.**



(a)

19–23.

The 100-kg reel has a radius of gyration about its center of mass G of $k_G = 200$ mm. If the cable B is subjected to a force of $P = 300$ N, determine the time required for the reel to obtain an angular velocity of 20 rad/s. The coefficient of kinetic friction between the reel and the plane is $\mu_k = 0.15$.



SOLUTION

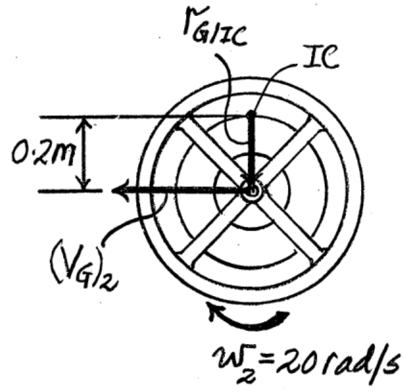
Kinematics: Referring to Fig. *a*, the final velocity of the center O of the spool is

$$(v_G)_2 = \omega_2 r_{G/IC} = 20(0.2) = 4 \text{ m/s} \leftarrow$$

Principle of Impulse and Momentum: The mass moment of inertia of the spool about its mass center is $I_G = mk_G^2 = 100(0.2^2) = 4 \text{ kg}\cdot\text{m}^2$. Applying the linear impulse and momentum equation along the y axis,

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$$

$$0 + N(t) - 100(9.81)(t) = 0 \qquad N = 981 \text{ N}$$



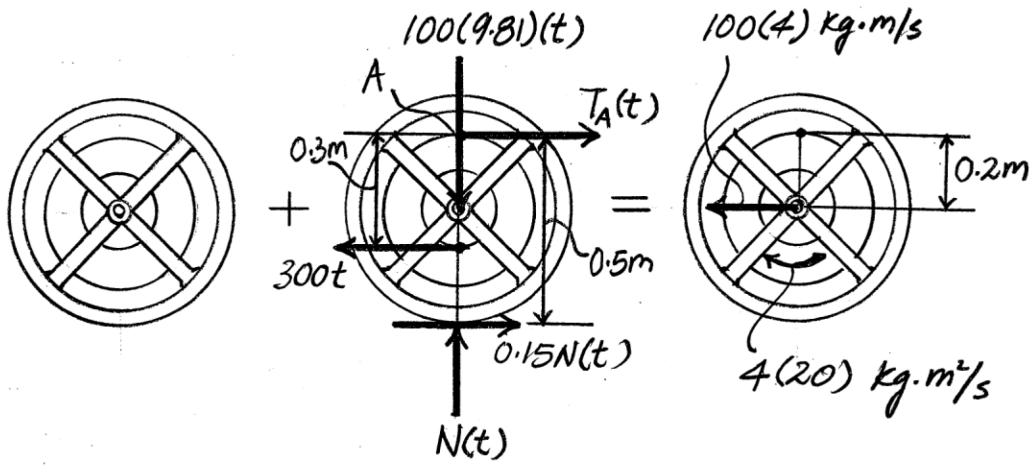
(a)

Using this result to write the angular impulse and momentum equation about the IC,

$$\circlearrowleft + \qquad (H_{IC})_1 + \Sigma\int_{t_1}^{t_2} M_{IC}\,dt = (H_{IC})_2$$

$$0 + 0.15(981)(t)(0.5) - 300t(0.3) = -100(4)(0.2) - 4(20)$$

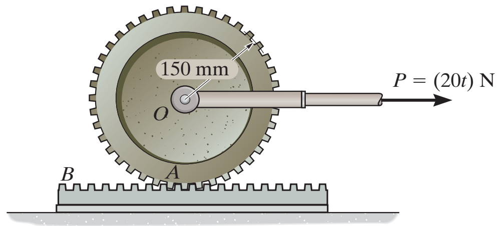
$$t = 9.74 \text{ s}$$ **Ans.**



(b)

***19–24.**

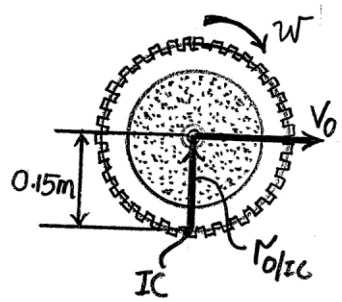
The 30-kg gear is subjected to a force of $P = (20t)$ N, where t is in seconds. Determine the angular velocity of the gear at $t = 4$ s, starting from rest. Gear rack B is fixed to the horizontal plane, and the gear's radius of gyration about its mass center O is $k_O = 125$ mm.



SOLUTION

Kinematics: Referring to Fig. a,

$$v_O = \omega r_{O/IC} = \omega(0.15)$$



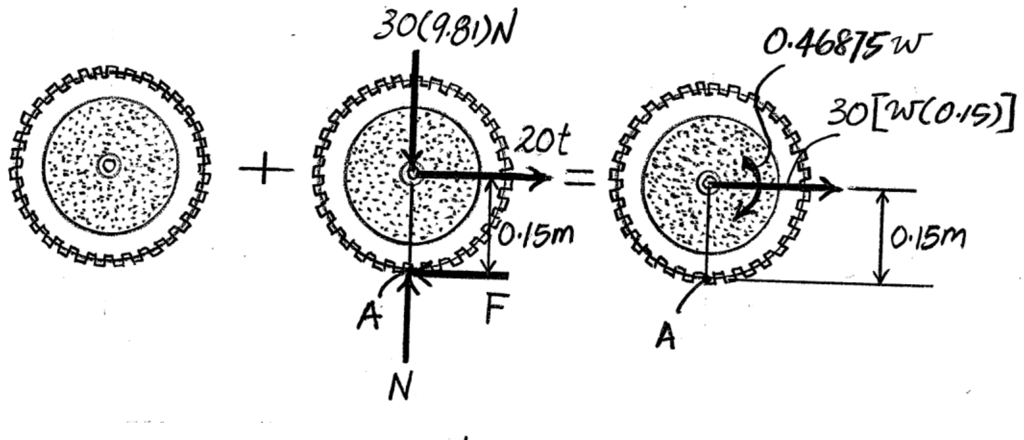
Principle of Angular Impulse and Momentum: The mass moment of inertia of the gear about its mass center is $I_O = mk_O^2 = 30(0.125^2) = 0.46875\ \text{kg}\cdot\text{m}^2$. Writing the angular impulse and momentum equation about point A shown in Fig. b,

$$(H_A)_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = (H_A)_2$$

$$0 + \int_0^{4\text{ s}} 20t(0.15)dt = 0.46875\omega + 30\,[\omega(0.15)]\,(0.15)$$

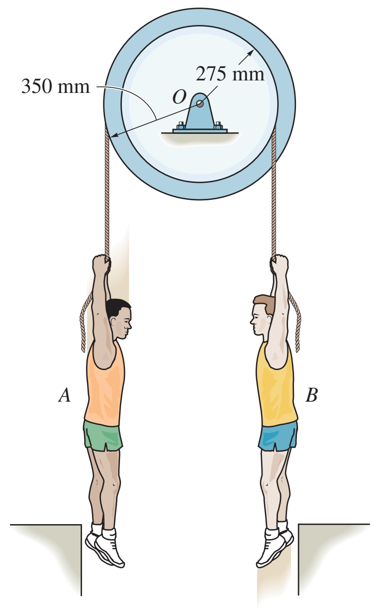
$$1.5t^2\Big|_0^{4\text{ s}} = 1.14375\omega$$

$$\omega = 21.0 \text{ rad/s}$$ **Ans.**



19–25.

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 30 kg and a radius of gyration $k_O = 250$ mm. If two men A and B grab the suspended ropes and step off the ledges at the same time, determine their speeds in 4 s starting from rest. The men A and B have a mass of 60 kg and 70 kg, respectively. Assume they do not move relative to the rope during the motion. Neglect the mass of the rope.



SOLUTION

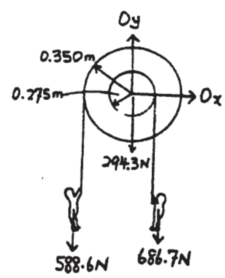
$$\zeta+ \qquad (H_O)_1 + \Sigma \int M_O dt = (H_O)_2$$

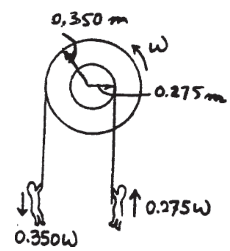
$$0 + 588.6(0.350)(4) - 686.7(0.275)(4) = 30(0.25)^2\omega + 60(0.35\omega)(0.35) + 70(0.275\omega)(0.275)$$

$$\omega = 4.73 \text{ rad/s}$$

$$v_A = 0.35(4.73) = 1.66 \text{ m/s} \qquad \textbf{Ans.}$$

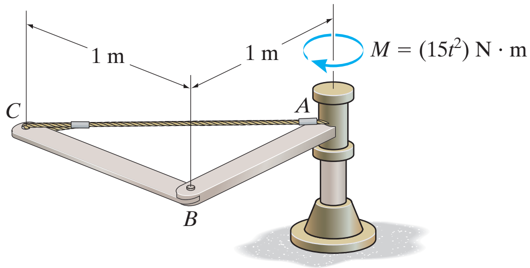
$$v_B = 0.275(4.73) = 1.30 \text{ m/s} \qquad \textbf{Ans.}$$





19–26.

If the shaft is subjected to a torque of $M = (15t^2)\ \text{N}\cdot\text{m}$, where t is in seconds, determine the angular velocity of the assembly when $t = 3$ s, starting from rest. Rods AB and BC each have a mass of 9 kg.



SOLUTION

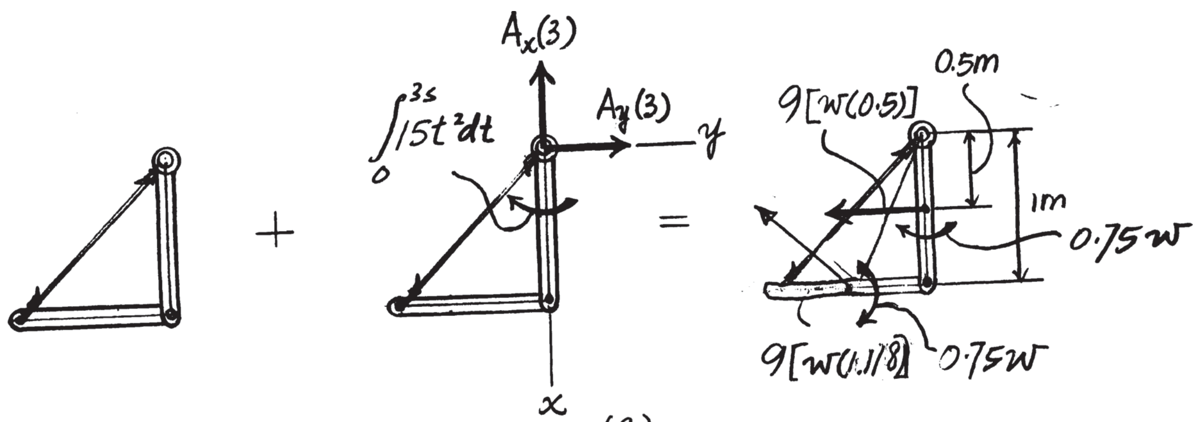
Principle of Impulse and Momentum: The mass moment of inertia of the rods about their mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(9)\left(1^2\right) = 0.75\ \text{kg}\cdot\text{m}^2$. Since the assembly rotates about the fixed axis, $(v_G)_{AB} = \omega(r_G)_{AB} = \omega(0.5)$ and $(v_G)_{BC} = \omega(r_G)_{BC} = \omega\left(\sqrt{1^2 + (0.5)^2}\right) = \omega(1.118)$. Referring to Fig. *a*,

$$\circlearrowleft + \qquad (H_z)_1 + \Sigma\int_{t_1}^{t_2} M_z\,dt = (H_z)_2$$

$$0 + \int_0^{3\,\text{s}} 15t^2 dt = 9\left[\omega(0.5)\right](0.5) + 0.75\omega + 9\left[\omega(1.118)\right](1.118) + 0.75\omega$$

$$5t^3\Big|_0^{3\,\text{s}} = 15\omega$$

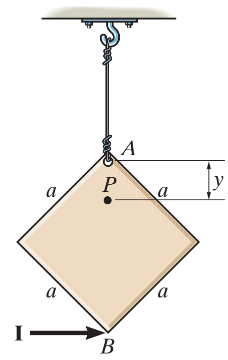
$$\omega = 9\ \text{rad/s} \qquad \textbf{Ans.}$$



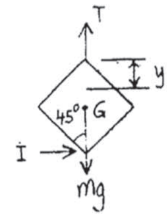
(a)

19–27.

The square plate has a mass m and is suspended at its corner A by a cord. If it receives a horizontal impulse $\mathbf{I}$ at corner B, determine the location y of the point P about which the plate appears to rotate during the impact.



SOLUTION



$(\circlearrowleft +)\qquad (H_G)_1 + \Sigma \int M_G\, dt = (H_G)_2$

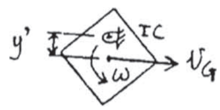
$$0 + I\left(\frac{a}{\sqrt{2}}\right) = \frac{m}{12}(a^2 + a^2)\,\omega$$

$(\overset{+}{\rightarrow})\qquad m(v_{Gx})_1 + \Sigma \int F_x dt = m(v_{Gx})_2$

$$0 + I = m v_G$$

$$\omega = \frac{6I}{\sqrt{2}am}$$

$$v_G = \frac{I}{m}$$

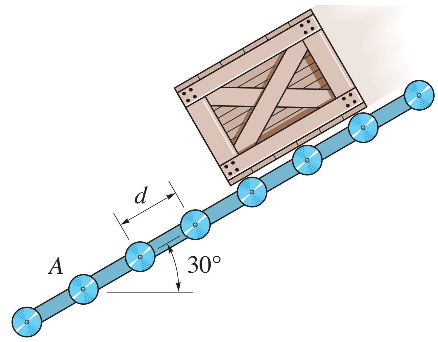


$$y' = \frac{v_G}{\omega} = \frac{\frac{I}{m}}{\frac{6I}{\sqrt{2}\,am}} = \frac{\sqrt{2}a}{6}$$

$$y = \frac{3\sqrt{2}}{6}a - \frac{\sqrt{2}}{6}a = \frac{\sqrt{2}}{3}a \qquad \textbf{Ans.}$$

***19–28.**

The crate has a mass m_c. Determine the constant speed v_0 it acquires as it moves down the conveyor. The rollers each have a radius of r, mass m, and are spaced d apart. Note that friction causes each roller to rotate when the crate comes in contact with it.

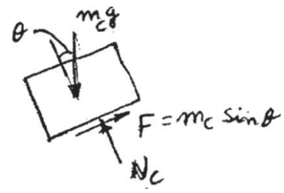


SOLUTION

The number of rollers per unit length is $1/d$.

Thus in one second, $\frac{v_0}{d}$ rollers are contacted.

If a roller is brought to full angular speed of $\omega = \frac{v_0}{r}$ in t_0 seconds, then the moment of inertia that is effected is



$$I' = I\left(\frac{v_0}{d}\right)(t_0) = \left(\frac{1}{2}m\,r^2\right)\left(\frac{v_0}{d}\right)t_0$$

Since the frictional impluse is

$F = m_c \sin\theta$ then

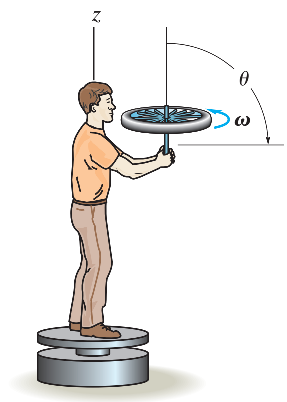
$$\circlearrowleft + (H_G)_1 + \Sigma \int M_G\,dt = (H_G)_2$$

$$0 + (m_c \sin\theta)\,r\,t_0 = \left[\left(\frac{1}{2}m\,r^2\right)\left(\frac{v_0}{d}\right)t_0\right]\left(\frac{v_0}{r}\right)$$

$$v_0 = \sqrt{(2\,g\sin\theta\,d)\left(\frac{m_c}{m}\right)}$$ **Ans.**

19–29.

A man has a moment of inertia I_z about the z axis. He is originally at rest and standing on a small platform which can turn freely. If he is handed a wheel which is rotating at $\boldsymbol{\omega}$ and has a moment of inertia I about its spinning axis, determine his angular velocity if (a) he holds the wheel upright as shown, (b) turns the wheel out, $\theta = 90°$, and (c) turns the wheel downward, $\theta = 180°$. Neglect the effect of holding the wheel a distance d away from the z axis.



SOLUTION

a)

$$\sum(H_z)_1 = \Sigma(H)_2; \qquad 0 + I\omega = I_z\omega_M + I\omega \qquad \omega_M = 0$$ **Ans.**

b)

$$\sum(H_z)_1 = \Sigma(H)_2; \qquad 0 + I\omega = I_z\omega_M + 0 \qquad \omega_M = \frac{I}{I_z}\omega$$ **Ans.**

c)

$$\sum(H_z)_1 = \Sigma(H)_2; \qquad 0 + I\omega = I_z\omega_M - I\omega \qquad \omega_M = \frac{2I}{I_y}\omega$$ **Ans.**

19–30.

Two wheels A and B have masses m_A and m_B, and radii of gyration about their central vertical axes of k_A and k_B, respectively. If they are freely rotating in the same direction at $\boldsymbol{\omega}_A$ and $\boldsymbol{\omega}_B$ about the same vertical axis, determine their common angular velocity after they are brought into contact and slipping between them stops.

SOLUTION

$$(\Sigma\text{Syst. Angular Momentum})_1 = (\Sigma\text{Syst. Angular Momentum})_2$$

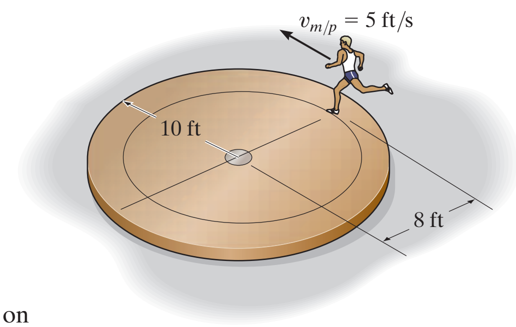
$$(m_A k_A^2)\,\omega_A + (m_B k_B^2)\,\omega_B = (m_A k_A^2)\,\omega'_A + (m_B k_B^2)\omega'_B$$

Set $\omega'_A = \omega'_B = \omega$. then

$$\omega = \frac{m_A k_A^2\,\omega_A + m_B k_B^2\,\omega_B}{m_A k_A^2 + m_B k_B^2} \qquad \textbf{Ans.}$$

19–31.

A 150-lb man leaps off the circular platform with a velocity of $v_{m/p} = 5$ ft/s, relative to the platform. Determine the angular velocity of the platform afterwards. Initially the man and platform are at rest. The platform weighs 300 lb and can be treated as a uniform circular disk.

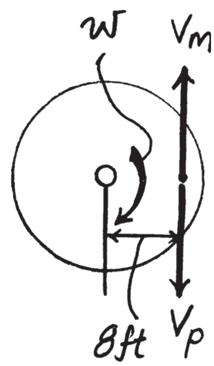


SOLUTION

Kinematics: Since the platform rotates about a fixed axis, the speed of point P on the platform to which the man leaps is $v_P = \omega r = \omega(8)$. Applying the relative velocity equation,

$$v_m = v_P + v_{m/P}$$

$$(+\uparrow) \qquad v_m = -\omega(8) + 5 \qquad \textbf{(1)}$$



Conservation of Angular Momentum: As shown in Fig. b, the impulse $\int F dt$ generated during the leap is internal to the system. Thus, angular momentum of the system is conserved about the axis perpendicular to the page passing through point O. The mass moment of inertia of the platform about this axis is

$$I_O = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{300}{32.2}\right)(10^2) = 465.84 \text{ slug} \cdot \text{ft}^2$$

Then

$$(H_O)_1 = (H_O)_2$$

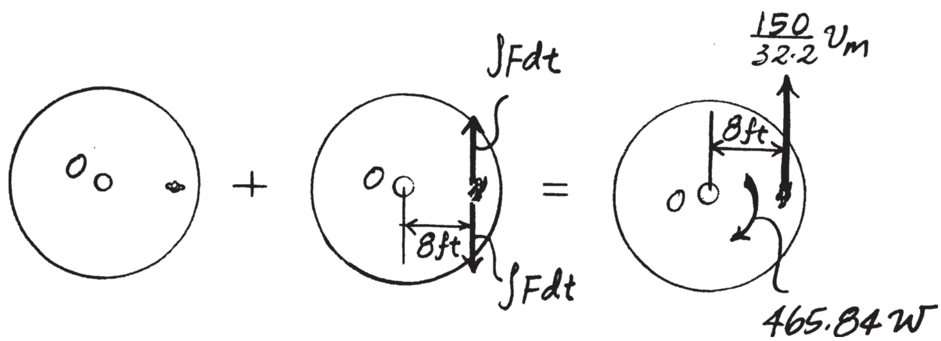
$$0 = \left(\frac{150}{32.2} v_m\right)(8) - 465.84\omega$$

$$v_m = 12.5\omega \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

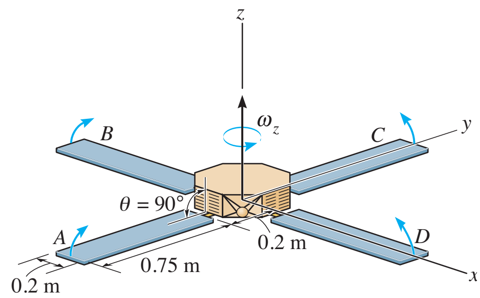
$$\omega = 0.244 \text{ rad/s} \qquad \textbf{Ans.}$$

$$v_m = 3.05 \text{ ft/s}$$



***19–32.**

The space satellite has a mass of 125 kg and a moment of inertia $I_z = 0.940\ \text{kg}\cdot\text{m}^2$, excluding the four solar panels A, B, C, and D. Each solar panel has a mass of 20 kg and can be approximated as a thin plate. If the satellite is originally spinning about the z axis at a constant rate $\omega_z = 0.5$ rad/s when $\theta = 90°$, determine the rate of spin if all the panels are raised and reach the upward position, $\theta = 0°$, at the same instant



SOLUTION

$$\circlearrowleft + \qquad H_1 = H_2$$

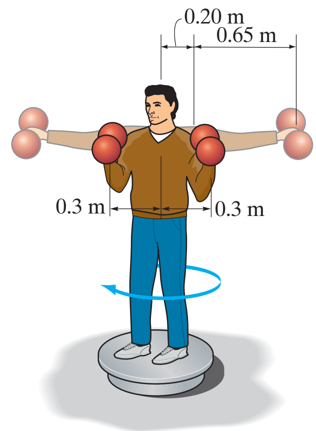
$$2\left[\frac{1}{2}(4)(0.15)^2\right](5) = 2\left[\frac{1}{2}(4)(0.15)^2\right]\omega$$

$$+\ 2[4(0.75\omega)(0.75)] + \left[\frac{1}{12}(2)(1.50)^2\right]\omega$$

$$\omega = 0.0906\ \text{rad/s} \qquad \textbf{Ans.}$$

19–33.

The 80-kg man is holding two dumbbells while standing on a turntable of negligible mass, which turns freely about a vertical axis. When his arms are fully extended, the turntable is rotating with an angular velocity of 0.5 rev/s. Determine the angular velocity of the man when he retracts his arms to the position shown. When his arms are fully extended, approximate each arm as a uniform 6-kg rod having a length of 650 mm, and his body as a 68-kg solid cylinder of 400-mm diameter. With his arms in the retracted position, assume the man as an 80-kg solid cylinder of 450-mm diameter. Each dumbbell consists of two 5-kg spheres of negligible size.



SOLUTION

Conservation of Angular Momentum: Since no external angular impulse acts on the system during the motion, angular momentum about the axis of rotation (z axis) is conserved. The mass moment of inertia of the system when the arms are in the fully extended position is

$$(I_z)_1 = 2\left[10(0.85^2)\right] + 2\left[\frac{1}{12}(6)(0.65^2) + 6(0.525^2)\right] + \frac{1}{2}(68)(0.2^2)$$

$$= 19.54\ \mathrm{kg \cdot m^2}$$

And the mass moment of inertia of the system when the arms are in the restracted position is

$$(I_z)_2 = 2\left[10(0.3^2)\right] + \frac{1}{2}(80)(0.225^2)$$

$$= 3.825\ \mathrm{kg \cdot m^2}$$

Thus,

$$(H_z)_1 = (H_z)_2$$

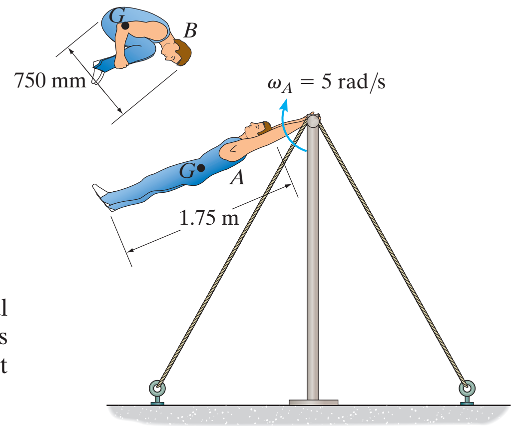
$$(I_z)_1\omega_1 = (I_z)_2\omega_2$$

$$19.54(0.5) = 3.825\omega_2$$

$$\omega_2 = 2.55\ \mathrm{rev/s} \qquad \textbf{Ans.}$$

19–34.

The 75-kg gymnast lets go of the horizontal bar in a fully stretched position A, rotating with an angular velocity of $\omega_A = 3$ rad/s. Estimate his angular velocity when he assumes a tucked position B. Assume the gymnast at positions A and B as a uniform slender rod and a uniform circular disk, respectively.



SOLUTION

Conservation of Angular Momentum: Other than the weight, there is no external impulse during the motion. Thus, the angular momentum of the gymnast is conserved about his mass cener G. The mass moments of inertia of the gymnast at the fully stretched and tucked positions are $(I_A)_G = \frac{1}{12}ml^2 = \frac{1}{12}(75)(1.75^2) = 19.14\ \text{kg}\cdot\text{m}^2$ and $(I_B)_G = \frac{1}{12}mr^2 = \frac{1}{2}(75)(0.375^2) = 5.273\ \text{kg}\cdot\text{m}^2$. Thus,

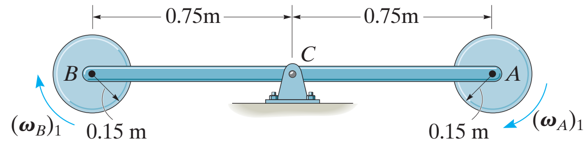
$$(H_A)_G = (H_B)_G$$

$$19.14(3) = 5.273\omega_B$$

$$\omega_B = 10.9\ \text{rad/s}$$ **Ans.**

19–35.

The 2-kg rod ACB supports the two 4-kg disks at its ends. If both disks are given a clockwise angular velocity $(\omega_A)_1 = (\omega_B)_1 = 5$ rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after both disks have stopped spinning relative to the rod due to frictional resistance at the pins A and B. Motion is in the *horizontal plane*. Neglect friction at pin C.



SOLUTION

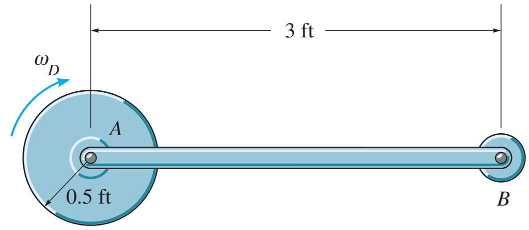
$$\circlearrowright + \qquad H_1 = H_2$$

$$2\left[\frac{1}{2}(4)(0.15)^2\right](5) = 2\left[\frac{1}{2}(4)(0.15)^2\right]\omega + 2[4(0.75\,\omega)(0.75)] + \left[\frac{1}{12}(2)(1.50)^2\right]\omega$$

$\omega = 0.0906$ rad/s **Ans.**

***19–36.**

The 5-lb rod AB supports the 3-lb disk at its end. If the disk is given an angular velocity $\omega_D = 8$ rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after the disk has stopped spinning relative to the rod due to frictional resistance at the bearing A. Motion is in the *horizontal plane*. Neglect friction at the fixed bearing B.



SOLUTION

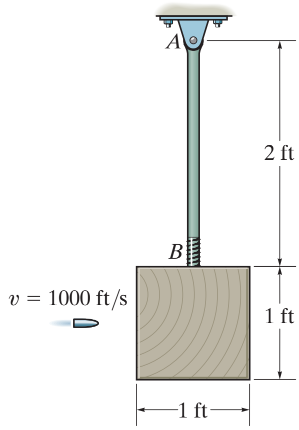
$$\Sigma(H_B)_1 = \Sigma(H_B)_2$$

$$\left[\frac{1}{2}\left(\frac{3}{32.2}\right)(0.5)^2\right](8) + 0 = \left[\frac{1}{3}\left(\frac{5}{32.2}\right)(3)^2\right]\omega + \left[\frac{1}{2}\left(\frac{3}{32.2}\right)(0.5)^2\right]\omega + \left(\frac{3}{32.2}\right)(3\omega)(3)$$

$\omega = 0.0708$ rad/s **Ans.**

19–37.

The pendulum consists of a 5-lb slender rod AB and a 10-lb wooden block. A projectile weighing 0.2 lb is fired into the center of the block with a velocity of 1000 ft/s. If the pendulum is initially at rest, and the projectile embeds itself into the block, determine the angular velocity of the pendulum just after the impact.



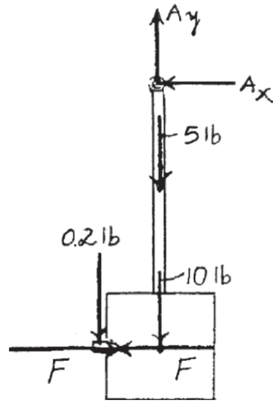
SOLUTION

Mass Moment of Inertia: The mass moment inertia of the pendulum and the embeded bullet about point A is

$$(I_2)_2 = \frac{1}{12}\left(\frac{5}{32.2}\right)(2^2) + \frac{5}{32.2}(1^2) + \frac{1}{12}\left(\frac{10}{32.2}\right)(1^2 + 1^2) + \frac{10}{32.2}(2.5^2) + \frac{0.2}{32.2}(2.5^2)$$

$$= 2.239 \text{ slug} \cdot \text{ft}^2$$

Conservation of Angular Momentum: Since force F due to the impact is *internal* to the system consisting of the pendulum and the bullet, it will cancel out. Thus, angular momentum is conserved about point A. Applying Eq. 19–17, we have



$$(H_A)_1 = (H_A)_2$$

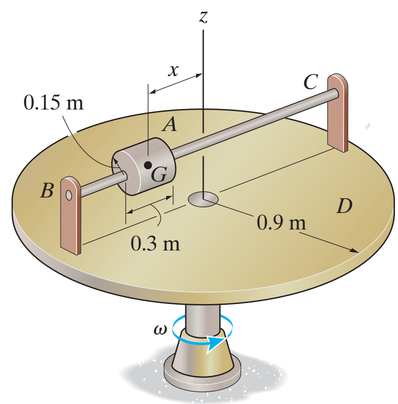
$$(m_b\, v_b)\,(r_b) = (I_A)_2\, \omega_2$$

$$\left(\frac{0.2}{32.2}\right)(1000)(2.5) = 2.239\omega_2$$

$$\omega_2 = 6.94 \text{ rad/s} \qquad \textbf{Ans.}$$

19–38.

The 20-kg cylinder A is free to slide along rod BC. When the cylinder is at $x = 0$, the 50-kg circular disk D is rotating with an angular velocity of 5 rad/s. If the cylinder is given a slight push, determine the angular velocity of the disk when the cylinder strikes B at $x = 600$ mm. Neglect the mass of the brackets and the smooth rod.



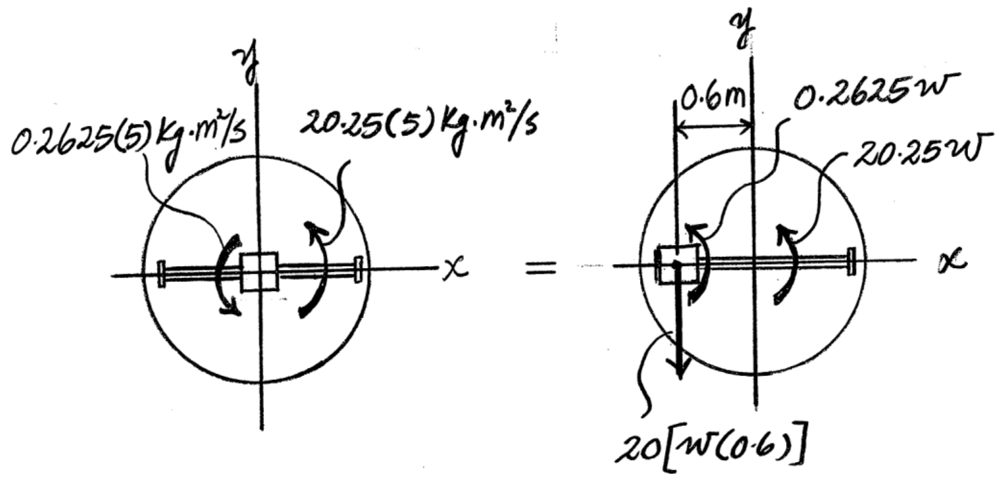
SOLUTION

Conservation of Angular Momentum: Since no external angular impulse acts on the system during the motion, angular momentum is conserved about the z axis. The mass moments of inertia of the cylinder and the disk about their centers of mass are $(I_A)_G = \frac{1}{12}m(3r^2 + h^2) = \frac{1}{12}(20)[3(0.15^2) + 0.3^2] = 0.2625\ \text{kg}\cdot\text{m}^2$ and $(I_D)_G = \frac{1}{2}mr^2 = \frac{1}{2}(50)(0.9^2) = 20.25\ \text{kg}\cdot\text{m}^2$. Since the disk rotates about a fixed z axis, $(v_G)_A = \omega(r_G)_A = \omega(0.6)$. Referring to Fig. a,

$$(H_z)_1 = (H_z)_2$$

$$0.2625(5) + 20.25(5) = 20.25\omega + 0.2625\omega + 20[\omega(0.6)](0.6)$$

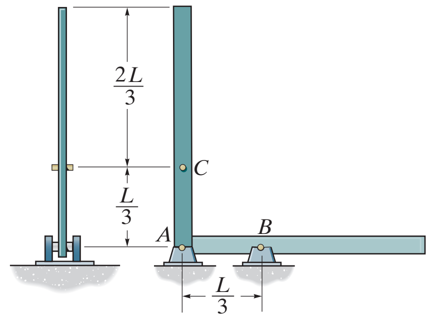
$$\omega = 3.70 \text{ rad/s}$$ **Ans.**



(a)

19–39.

The slender bar of mass m pivots at support A when it is released from rest in the vertical position. When it falls and rotates 90°, pin C will strike support B, and pin at A will leave its support. Determine the angular velocity of the bar immediately after the impact. Assume the pin at B will not rebound.



SOLUTION

Conservation of Energy: With reference to the datum in Fig. a, $V_1 = (V_g)_1 = W(y_G)_1 = mg\left(\frac{L}{2}\right)$ and $V_2 = (V_g)_2 = W(y_G)_2 = 0$. Since the rod rotates about point A, $T_2 = \frac{1}{2} I_A \omega_2{}^2 = \frac{1}{2}\left[\frac{1}{3} mL^2\right]\omega_2{}^2 = \frac{1}{6} mL^2\omega_2{}^2$. Since the rod is initially at rest, $T_1 = 0$. Then,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + mg\left(\frac{L}{2}\right) = \frac{1}{6} mL^2\omega_2{}^2 + 0$$

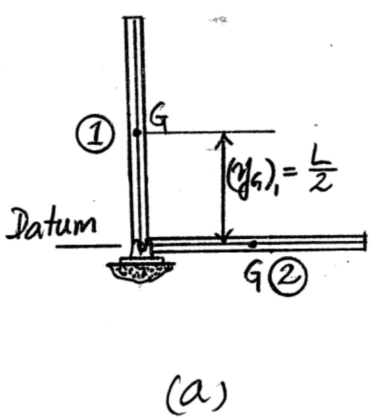
$$\omega_2 = \sqrt{\frac{3g}{L}}$$

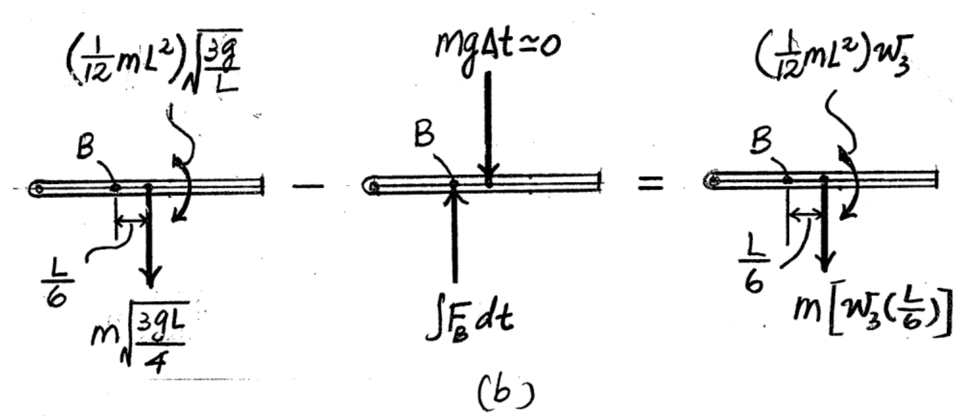
Conservation of Angular Momentum: Since the rod rotates about point A just before the impact, $(v_G)_2 = \omega_2 r_{AG} = \sqrt{\frac{3g}{L}}\left(\frac{L}{2}\right) = \sqrt{\frac{3gL}{4}}$. Also, the rod rotates about B immediately after the impact, $(v_G)_3 = \omega_3 r_{BG} = \omega_3\left(\frac{L}{6}\right)$. Angular momentum is conserved about point B. Thus,

$$(H_B)_2 = (H_B)_3$$

$$m\sqrt{\frac{3gL}{4}}\left(\frac{L}{6}\right) + \left(\frac{1}{12} mL^2\right)\sqrt{\frac{3g}{L}} = \left(\frac{1}{12} mL^2\right)\omega_3 + m\left[\omega_3\left(\frac{L}{6}\right)\right]\left(\frac{L}{6}\right)$$

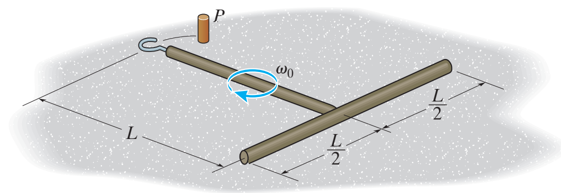
$$\omega_3 = \frac{3}{2}\sqrt{\frac{3g}{L}} \qquad \textbf{Ans.}$$





***19–40.**

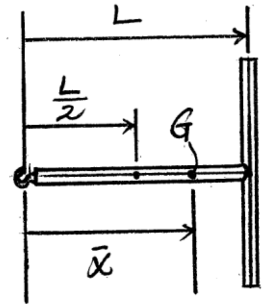
The uniform rod assembly rotates with an angular velocity of ω_0 on the smooth horizontal plane just before the hook strikes the peg P without rebound. Determine the angular velocity of the assembly immediately after the impact. Each rod has a mass of m.



SOLUTION

Center of Mass: Referring to Fig. a,

$$\bar{x} = \frac{\Sigma x_c m}{\Sigma m} = \frac{\left(\frac{L}{2}\right)(m) + L(m)}{2m} = \frac{3}{4}L$$



(a)

Thus, the mass moment of the assembly about its mass center G is

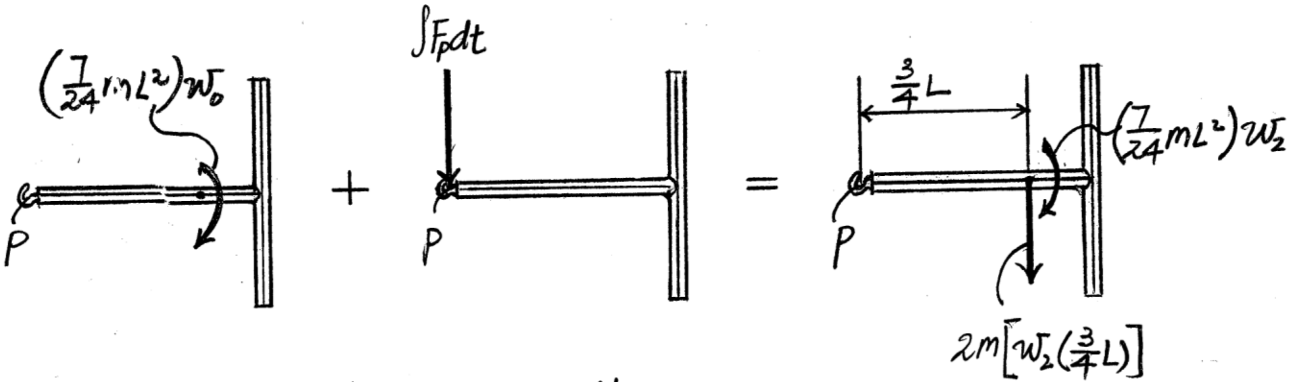
$$I_G = \left[\frac{1}{12}mL^2 + m\left(\frac{L}{4}\right)^2\right] + \left[\frac{1}{12}mL^2 + m\left(\frac{L}{4}\right)^2\right] = \frac{7}{24}mL^2$$

Conservation of Angular Momentum: Referring to Fig. b, angular momentum is conserved about P since the sum of the angular impulses about this point is zero. Here, immediately after the impact, $(v_G)_2 = \omega_2 r_{PG} = \omega_2\left(\frac{3}{4}L\right)$. Thus,

$$(H_P)_1 = (H_P)_2$$

$$\left(\frac{7}{24}mL^2\right)\omega_0 = \left(\frac{7}{24}mL^2\right)\omega_2 + 2m\left[\omega_2\left(\frac{3}{4}L\right)\right]\left(\frac{3}{4}L\right)$$

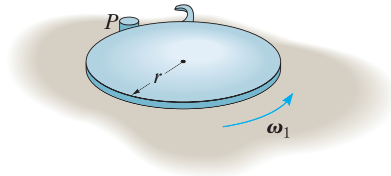
$$\omega_2 = \frac{7}{34}\omega_0 \qquad \textbf{Ans.}$$



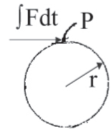
(b)

19–41.

A thin disk of mass m has an angular velocity $\boldsymbol{\omega}_1$ while rotating on a smooth surface. Determine its new angular velocity just after the hook at its edge strikes the peg P and the disk starts to rotate about P without rebounding.



SOLUTION



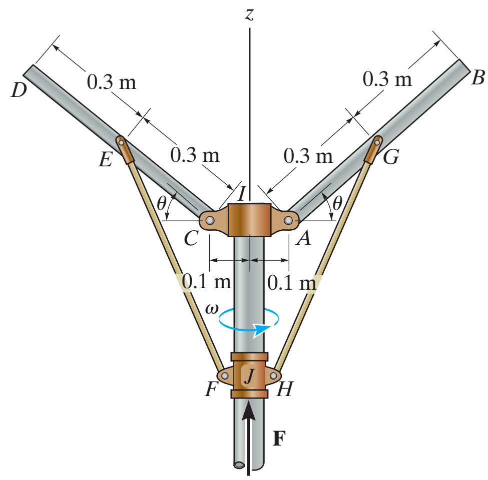
$$H_1 = H_2$$

$$\left(\frac{1}{2}mr^2\right)\omega_1 = \left[\frac{1}{2}mr^2 + mr^2\right]\omega_2$$

$$\omega_2 = \frac{1}{3}\omega_1 \qquad \textbf{Ans.}$$

19–42.

The vertical shaft is rotating with an angular velocity of 3 rad/s when $\theta = 0°$. If a force **F** is applied to the collar so that $\theta = 90°$, determine the angular velocity of the shaft. Also, find the work done by force **F**. Neglect the mass of rods GH and EF and the collars I and J. The rods AB and CD each have a mass of 10 kg.



SOLUTION

Conservation of Angular Momentum: Referring to the free-body diagram of the assembly shown in Fig. a, the sum of the angular impulses about the z axis is zero. Thus, the angular momentum of the system is conserved about the axis. The mass moments of inertia of the rods about the z axis when $\theta = 0°$ and $90°$ are

$$(I_z)_1 = 2\left[\frac{1}{12}(10)(0.6^2) + 10(0.3 + 0.1)^2\right] = 3.8\ \text{kg}\cdot\text{m}^2$$

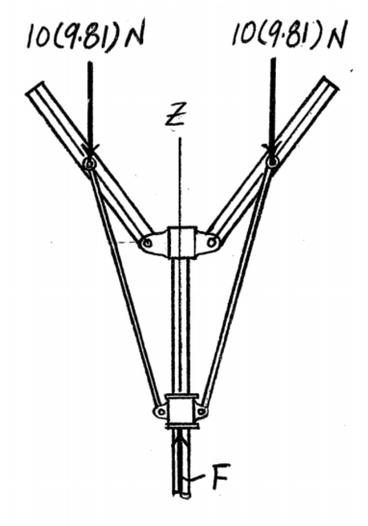
$$(I_z)_2 = 2\left[10(0.1^2)\right] = 0.2\ \text{kg}\cdot\text{m}^2$$

Thus,

$$(H_z)_1 = (H_z)_2$$

$$3.8(3) = 0.2\omega_2$$

$$\omega_2 = 57\ \text{rad/s} \qquad \textbf{Ans.}$$



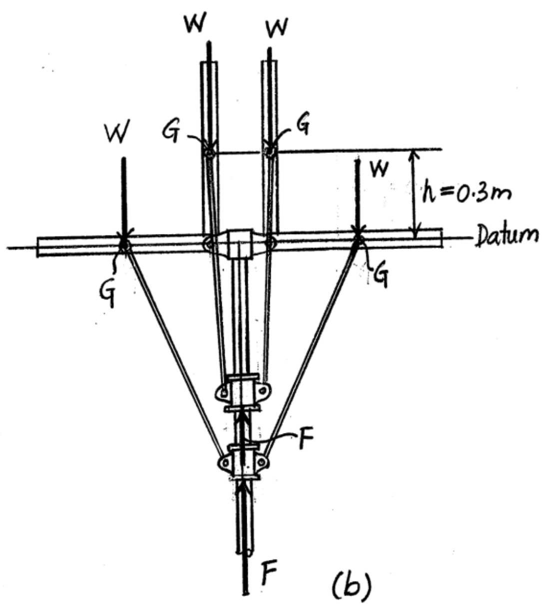
(a)

Principle of Work and Energy: As shown on the free-body diagram of the assembly, Fig. b, **W** does negative work, while **F** does positive work. The work of **W** is $U_W = -Wh = -10(9.81)(0.3) = -29.43$ J. The initial and final kinetic energy of the assembly is $T_1 = \frac{1}{2}(I_z)_1\omega_1{}^2 = \frac{1}{2}(3.8)(3^2) = 17.1$ J and $T_2 = \frac{1}{2}(I_z)_2\omega_2{}^2 = \frac{1}{2}(0.2)(57^2) = 324.9$ J. Thus,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$17.1 + 2(-29.43) + U_F = 324.9$$

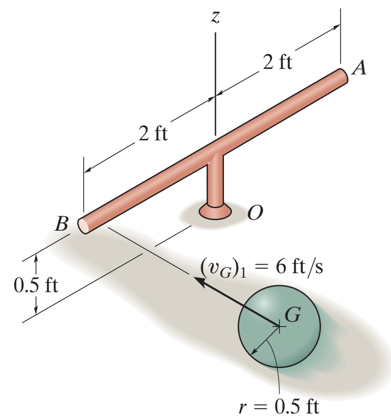
$$U_F = 367\ \text{J} \qquad \textbf{Ans.}$$



(b)

19–43.

The mass center of the 3-lb ball has a velocity of $(v_G)_1 = 6\text{ ft/s}$ when it strikes the end of the smooth 5-lb slender bar which is at rest. Determine the angular velocity of the bar about the z axis just after impact if $e = 0.8$.



SOLUTION

Conservation of Angular Momentum: Since force F due to the impact is *internal* to the system consisting of the slender bar and the ball, it will cancel out. Thus, angular momentum is conserved about the z axis. The mass moment of inertia of the slender bar about the z axis is $I_z = \frac{1}{12}\left(\frac{5}{32.2}\right)(4^2) = 0.2070\text{ slug}\cdot\text{ft}^2$. Here, $\omega_2 = \frac{(v_B)_2}{2}$. Applying Eq. 19–17, we have

$$(H_z)_1 = (H_z)_2$$

$$\left[m_b\,(v_G)_1\right](r_b) = I_z\,\omega_2 + \left[m_b\,(v_G)_2\right](r_b)$$

$$\left(\frac{3}{32.2}\right)(6)(2) = 0.2070\left[\frac{(v_B)_2}{2}\right] + \left(\frac{3}{32.2}\right)(v_G)_2(2) \qquad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{(v_B)_2 - (v_G)_2}{(v_G)_1 - (v_B)_1}$$

$$0.8 = \frac{(v_B)_2 - (v_G)_2}{6 - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

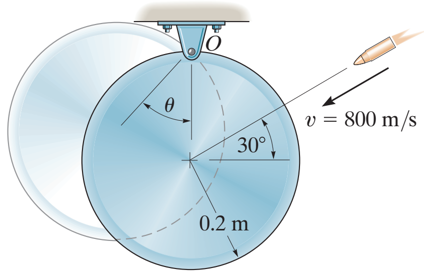
$$(v_G)_2 = 2.143\text{ ft/s} \qquad (v_B)_2 = 6.943\text{ ft/s}$$

Thus, the angular velocity of the slender rod is given by

$$\omega_2 = \frac{(v_B)_2}{2} = \frac{6.943}{2} = 3.47\text{ rad/s} \qquad \textbf{Ans.}$$

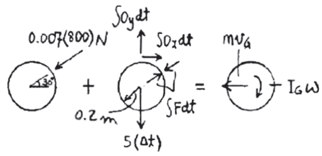
***19–44.**

A 7-g bullet having a velocity of 800 m/s is fired into the edge of the 5-kg disk as shown. Determine the angular velocity of the disk just after the bullet becomes embedded in it. Also, calculate how far θ the disk will swing until it stops. The disk is originally at rest.



SOLUTION

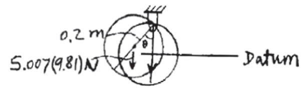
$$\zeta+ \qquad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$$



$$0.007(800)\cos 30^\circ(0.2) + 0 = \frac{1}{2}(5.007)(0.2)^2\omega + 5.007(0.2\omega)(0.2)$$

$\omega = 3.23$ rad/s **Ans.**

$$T_1 + V_1 = T_2 + V_2$$

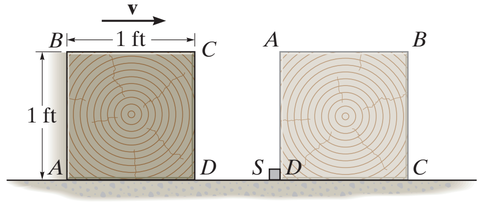


$$\frac{1}{2}(5.007)[3.23(0.2)]^2 + \frac{1}{2}\left[\frac{1}{2}(5.007)(0.2)^2\right](3.23)^2 + 0 = 0 + 0.2(1 - \cos\theta)(5.007)(9.81)$$

$\theta = 32.8^\circ$ **Ans.**

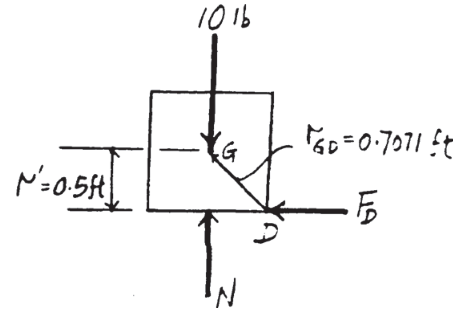
19–45.

The 10-lb block slides on the smooth surface when the corner D hits a stop block S. Determine the minimum velocity $\mathbf{v}$ the block should have which would allow it to tip over on its side and land in the position shown. Neglect the size of S. *Hint:* During impact consider the weight of the block to be nonimpulsive.



SOLUTION

Conservation of Energy: If the block tips over about point D, it must at least achieve the dash position shown. Datum is set at point D. When the block is at its initial and final position, its center of gravity is located 0.5 ft and 0.7071 ft *above* the datum. Its initial and final potential energy are $10(0.5) = 5.00\text{ ft}\cdot\text{lb}$ and $10(0.7071) = 7.071\text{ ft}\cdot\text{lb}$. The mass moment of inertia of the block about point D is



$$I_D = \frac{1}{12}\left(\frac{10}{32.2}\right)\left(1^2 + 1^2\right) + \left(\frac{10}{32.2}\right)\left(\sqrt{0.5^2 + 0.5^2}\right)^2 = 0.2070\text{ slug}\cdot\text{ft}^2$$

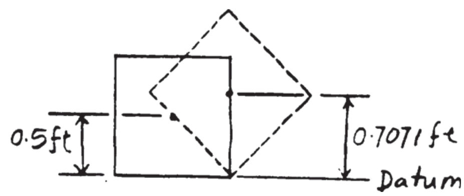
The initial kinetic energy of the block (after the impact) is $\frac{1}{2}I_D\,\omega_2^2 = \frac{1}{2}(0.2070)\,\omega_2^2$. Applying Eq. 18–18, we have

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(0.2070)\,\omega_2^2 + 5.00 = 0 + 7.071$$

$$\omega_2 = 4.472\text{ rad/s}$$

Conservation of Angular Momentum: Since the weight of the block and the normal reaction N are *nonimpulsive* forces, the angular momentum is conserves about point D. Applying Eq. 19–17, we have



$$(H_D)_1 = (H_D)_2$$

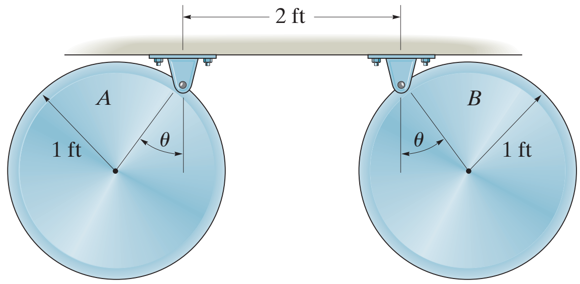
$$(mv_G)(r') = I_D\,\omega_2$$

$$\left[\left(\frac{10}{32.2}\right)v\right](0.5) = 0.2070(4.472)$$

$$v = 5.96\text{ ft/s} \qquad \textbf{Ans.}$$

19–46.

The two disks each weigh 10 lb. If they are released from rest when $\theta = 30^\circ$, determine θ after they collide and rebound from each other. The coefficient of restitution is $e = 0.75$. When $\theta = 0^\circ$, the disks hang so that they just touch one another.



SOLUTION

$$I_c = \frac{1}{2}\left(\frac{10}{32.2}\right)(1)^2 + \frac{10}{32.2}(1)^2 = 0.46584 \text{ slug}\cdot\text{ft}^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 10(1 - \cos 30^\circ) = \frac{1}{2}(0.46584)\omega_1^2 + 0$$

$$\omega_1 = 2.398 \text{ rad/s}$$

Coefficient of restitution:

$$e = \frac{(v_D)_{B_2} - (v_D)_{A_2}}{(v_D)_{A_1} - (v_D)_{B_1}} = 0.75 = \frac{\omega_2 - (-\ \omega_2)}{2.398 - (-\ 2.398)} \qquad \textbf{(1)}$$

Where, v_D is the speed of point D on disk A or B. Note that $(v_D)_B = -(v_D)_A$ and $(v_D)_A = r\omega = (v_D)_B$.

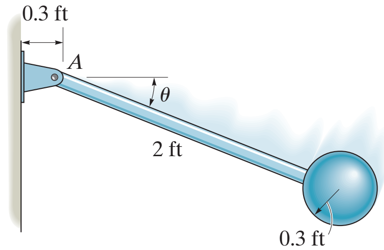
Solving Eq.(1); $\omega_2 = 1.799$ rad/s

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(0.46584)(1.799)^2 + 0 = 0 + 10(1 - \cos\theta)$$

$$\theta = 22.4^\circ \qquad \textbf{Ans.}$$

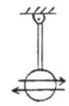
19–47.



The pendulum consists of a 10-lb solid ball and 4-lb rod. If it is released from rest when $\theta_1 = 0°$, determine the angle θ_2 after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest. Take $e = 0.6$.

SOLUTION

$$I_A = \frac{1}{3}\left(\frac{4}{32.2}\right)(2)^2 + \frac{2}{5}\left(\frac{10}{32.2}\right)(0.3)^2 + \frac{10}{32.2}(2.3)^2 = 1.8197 \text{ slug} \cdot \text{ft}^2$$



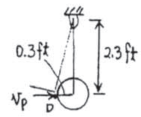
Just before impact:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}[1.8197]\omega^2 - 4(1) - 10(2.3)$$

$$\omega = 5.4475 \text{ rad/s}$$

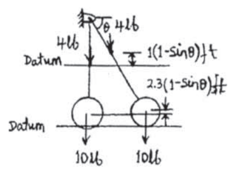
$$v_P = 2.3(5.4475) = 12.53 \text{ ft/s}$$



Since wall does not move

$$e = 0.6 = \frac{v_P'}{12.529}$$

$$v_P' = 7.518 \text{ ft/s}$$



$$\omega' = \frac{7.518}{2.3} = 3.2685 \text{ rad/s}$$

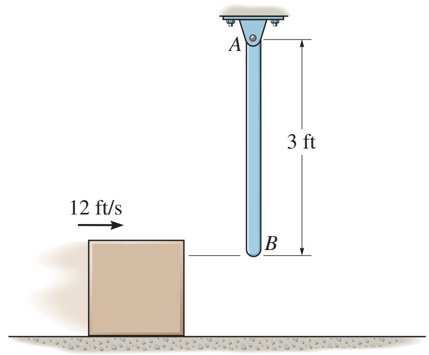
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(1.8197)(3.2685)^2 = 4(1)(1 - \sin\theta_2) + 10(2.3)(1 - \sin\theta_2)$$

$$\theta_2 = 39.8° \qquad \textbf{Ans.}$$

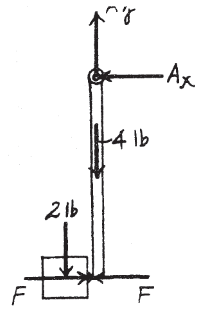
***19–48.**

The 4-lb rod AB is hanging in the vertical position. A 2-lb block, sliding on a smooth horizontal surface with a velocity of 12 ft/s, strikes the rod at its end B. Determine the velocity of the block immediately after the collision. The coefficient of restitution between the block and the rod at B is $e = 0.8$.



SOLUTION

Conservation of Angular Momentum: Since force F due to the impact is *internal* to the system consisting of the slender rod and the block, it will cancel out. Thus, angular momentum is conserved about point A. The mass moment of inertia of the slender rod about point A is $I_A = \frac{1}{12}\left(\frac{4}{32.2}\right)(3^2) + \frac{4}{32.2}(1.5^2) = 0.3727 \text{ slug} \cdot \text{ft}^2$. Here, $\omega_2 = \frac{(v_B)_2}{3}$. Applying Eq. 19–17, we have



$$(H_A)_1 = (H_A)_2$$

$$[m_b(v_b)_1](r_b) = I_A\omega_2 + [m_b(v_b)_2](r_b)$$

$$\left(\frac{2}{32.2}\right)(12)(3) = 0.3727\left[\frac{(v_B)_2}{3}\right] + \left(\frac{2}{32.2}\right)(v_b)_2(3) \qquad \textbf{[1]}$$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{(v_B)_2 - (v_b)_2}{(v_b)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.8 = \frac{(v_B)_2 - (v_b)_2}{12 - 0} \qquad \textbf{[2]}$$

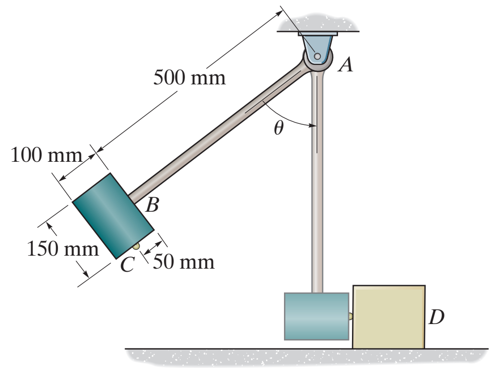
Solving Eqs. [1] and [2] yields

$$(v_b)_2 = 3.36 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

$$(v_B)_2 = 12.96 \text{ ft/s} \rightarrow$$

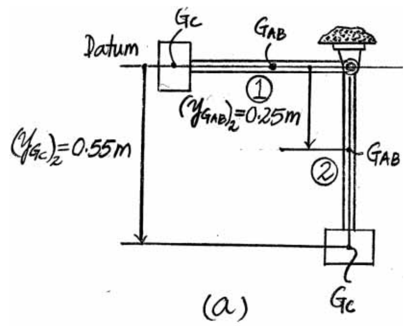
19–49.

The hammer consists of a 10-kg solid cylinder C and 6-kg uniform slender rod AB. If the hammer is released from rest when $\theta = 90^\circ$ and strikes the 30-kg block D when $\theta = 0^\circ$, determine the velocity of block D and the angular velocity of the hammer immediately after the impact. The coefficient of restitution between the hammer and the block is $e = 0.6$.



SOLUTION

Conservation of Energy: With reference to the datum in Fig. a, $V_1 = (V_g)_1 = W_{AB}(y_{GAB})_1 + W_C(y_{GC})_1 = 0$ and $V_2 = (V_g)_2 = -W_{AB}(y_{GAB})_2 - W_C(y_{GC})_2 = -6(9.81)(0.25) - 10(9.81)(0.55) = -68.67$ J. Initially, $T_1 = 0$. Since the hammer rotates about the fixed axis, $(v_{GAB})_2 = \omega_2 r_{GAB} = \omega_2(0.25)$ and $(v_{GC})_2 = \omega_2 r_{GC} = \omega_2(0.55)$. The mass moment of inertia of rod AB and cylinder C about their mass centers is $I_{GAB} = \frac{1}{12}ml^2 = \frac{1}{12}(6)(0.5^2) = 0.125\ \text{kg}\cdot\text{m}^2$ and $I_C = \frac{1}{12}m(3r^2 + h^2) = \frac{1}{12}(10)\left[3(0.05^2) + 0.15^2\right] = 0.025\ \text{kg}\cdot\text{m}^2$. Thus,



(a)

$$T_2 = \frac{1}{2}I_{GAB}\omega_2^2 + \frac{1}{2}m_{AB}(v_{GAB})_2^2 + \frac{1}{2}I_{GC}\,\omega_2^2 + \frac{1}{2}m_C(v_{GC})_2^2$$

$$= \frac{1}{2}(0.125)\omega_2^2 + \frac{1}{2}(6)\left[\omega_2(0.25)\right]^2 + \frac{1}{2}(0.025)\omega_2^2 + \frac{1}{2}(10)\left[\omega_2(0.55)\right]^2$$

$$= 1.775\,\omega_2^2$$

Then,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 1.775\omega_2^2 + (-68.67)$$

$$\omega_2 = 6.220\ \text{rad/s}$$

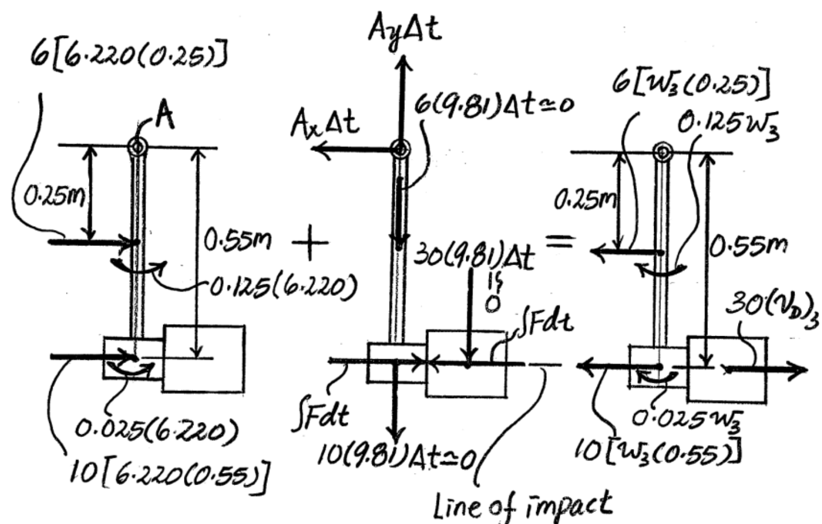
Conservation of Angular Momentum: The angular momentum of the system is conserved point A. Then,

$$(H_A)_1 = (H_A)_2$$

$$0.125(6.220) + 6[6.220(0.25)](0.25) + 0.025(6.220) + 10[6.220(0.55)](0.55)$$

$$= 30v_D(0.55) - 0.125\omega_3 - 6[\omega_3(0.25)](0.25) - 0.025\omega_3 - 10[\omega_3(0.55)](0.55)$$

$$16.5v_D - 3.55\omega_3 = 22.08 \qquad \textbf{(1)}$$



19–49. continued

Coefficient of Restitution: Referring to Fig. *c*, the components of the velocity of the impact point P just before and just after impact along the line of impact are $\left[(v_P)_x\right]_2 = (v_{GC})_2 = \omega_2 r_{GC} = 6.220(0.55) = 3.421\text{ m/s} \rightarrow$ and $\left[(v_P)_x\right]_3 = (v_{GC})_3 = \omega_3 r_{GC} = \omega_3\,(0.55) \leftarrow$. Thus,

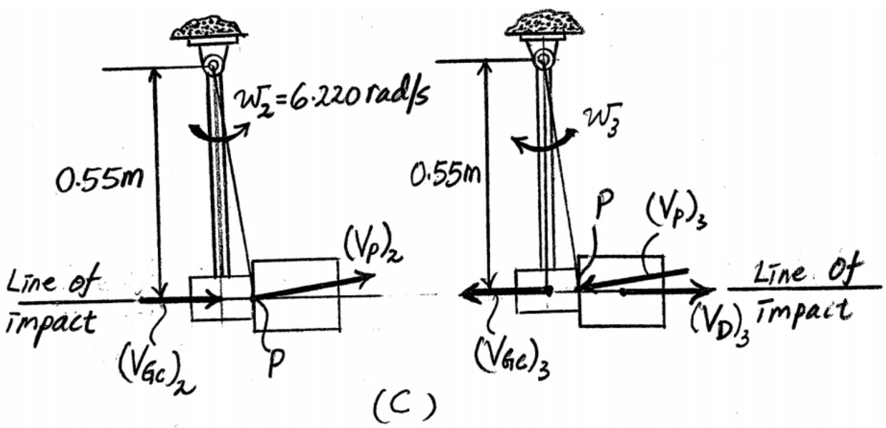
$$\overset{+}{\rightarrow} \qquad e = \frac{(v_D)_3 - \left[(v_P)_x\right]_3}{\left[(v_P)_x\right]_2 - (v_D)_2}$$

$$0.6 = \frac{(v_D)_3 - \left[-\omega_3(0.55)\right]}{3.421 - 0}$$

$$(v_D)_3 + 0.55\omega_3 = 2.053 \qquad \textbf{(2)}$$

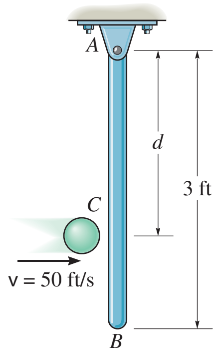
Solving Eqs. (1) and (2),

$$(v_D)_3 = 1.54\text{ m/s} \qquad \omega_3 = 0.934\text{ rad/s} \qquad \textbf{Ans.}$$



19–50.

The 6-lb slender rod AB is originally at rest, suspended in the vertical position. A 1-lb ball is thrown at the rod with a velocity $v = 50$ ft/s and strikes the rod at C. Determine the angular velocity of the rod just after the impact. Take $e = 0.7$ and $d = 2$ ft.



SOLUTION

$\circlearrowleft + (H_A)_1 = (H_A)_2$

$$\left(\frac{1}{32.2}\right)(50)(2) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(3)^2\right]\omega_2 + \frac{1}{32.2}(v_{BL})(2)$$

$$e = 0.7 = \frac{v_C - v_{BL}}{50 - 0}$$

$$v_C = 2\omega_2$$

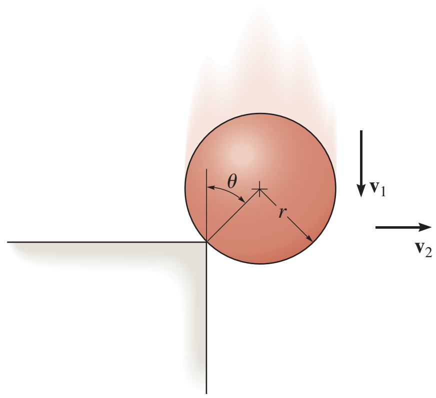
Thus,

$$\omega_2 = 7.73 \text{ rad/s}$$ **Ans.**

$$v_{BL} = -19.5 \text{ ft/s}$$

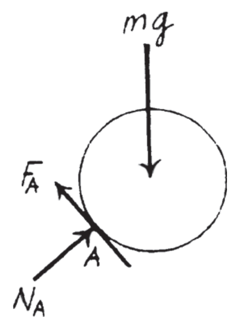
19–51.

The solid ball of mass m is dropped with a velocity $\mathbf{v}_1$ onto the edge of the rough step. If it rebounds horizontally off the step with a velocity $\mathbf{v}_2$, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is e.



SOLUTION

Conservation of Angular Momentum: Since the weight of the solid ball is a *nonimpulsive force*, then angular momentum is conserved about point A. The mass moment of inertia of the solid ball about its mass center is $I_G = \frac{2}{5}mr^2$. Here, $\omega_2 = \frac{v_2 \cos\theta}{r}$. Applying Eq. 19–17, we have



$$(H_A)_1 = (H_A)_2$$

$$\left[m_b\,(v_b)_1\right](r') = I_G\,\omega_2 + \left[m_b\,(v_b)_2\right](r'')$$

$$(mv_1)(r\sin\theta) = \left(\frac{2}{5}mr^2\right)\left(\frac{v_2\cos\theta}{r}\right) + (mv_2)(r\cos\theta)$$

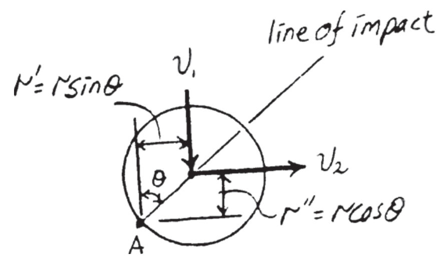
$$\frac{v_2}{v_1} = \frac{5}{7}\tan\theta \qquad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{0 - (v_b)_2}{(v_b)_1 - 0}$$

$$e = \frac{-(v_2\sin\theta)}{-v_1\cos\theta}$$

$$\frac{v_2}{v_1} = \frac{e\cos\theta}{\sin\theta} \qquad \textbf{(2)}$$



Equating Eqs. (1) and (2) yields

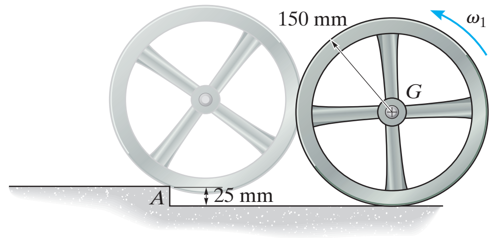
$$\frac{5}{7}\tan\theta = \frac{e\cos\theta}{\sin\theta}$$

$$\tan^2\theta = \frac{7}{5}e$$

$$\theta = \tan^{-1}\left(\sqrt{\frac{7}{5}e}\right) \qquad \textbf{Ans.}$$

***19–52.**

The wheel has a mass of 50 kg and a radius of gyration of 125 mm about its center of mass G. Determine the minimum value of the angular velocity ω_1 of the wheel, so that it strikes the step at A without rebounding and then rolls over it without slipping.



SOLUTION

Conservation of Angular Momentum: Referring to Fig. *a*, the sum of the angular impulses about point A is zero. Thus, angular momentum of the wheel is conserved about this point. Since the wheel rolls without slipping, $(v_G)_1 = \omega_1 r = \omega_1(0.15)$ and $(v_G)_2 = \omega_2 r = \omega_2(0.15)$. The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2 = 50(0.125^2) = 0.78125\ \text{kg}\cdot\text{m}^2$. Thus,

$$(H_A)_1 = (H_A)_2$$

$$50[\omega_1(0.15)](0.125) + 0.78125\omega_1 = 50[\omega_2(0.15)](0.15) + 0.78125\omega_2$$

$$\omega_1 = 1.109\omega_2 \qquad \textbf{(1)}$$

Conservation of Energy: With reference to the datum in Fig. *a*, $V_2 = (V_g)_2 = W(y_G)_2 = 0$ and $V_3 = (V_g)_3 = W(y_G)_3 = 50(9.81)(0.025) = 12.2625$ J. Since the wheel is required to be at rest in the final position, $T_3 = 0$. The initial kinetic energy of the wheel is $T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2 = \frac{1}{2}(50)[\omega_2(0.15)]^2 + \frac{1}{2}(0.78125)(\omega_2{}^2) = 0.953125\omega_2{}^2$. Then

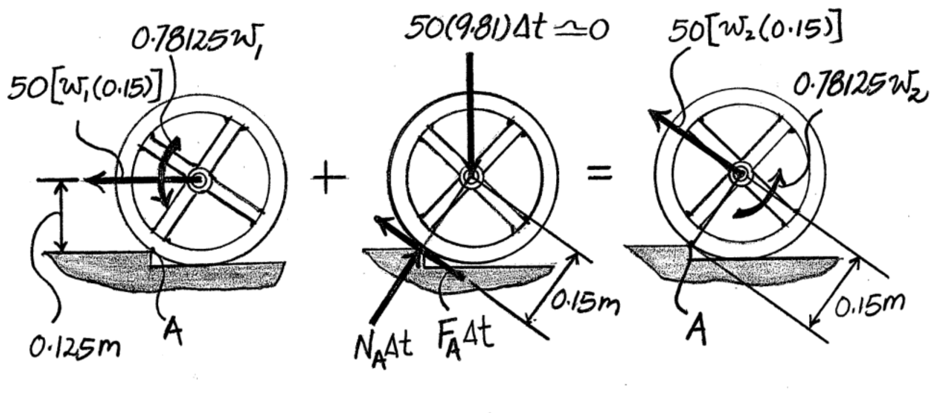
$$T_2 + V_2 = T_3 + V_3$$

$$0.953125\omega_2{}^2 + 0 = 0 + 12.2625$$

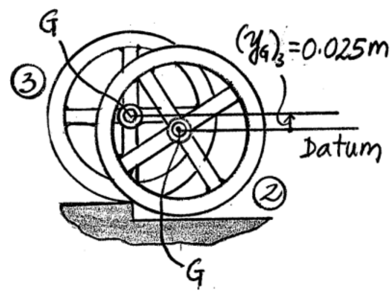
$$\omega_2 = 3.587\ \text{rad/s}$$

Substituting this result into Eq. (1), we obtain

$$\omega_1 = 3.98\ \text{rad/s} \qquad \textbf{Ans.}$$



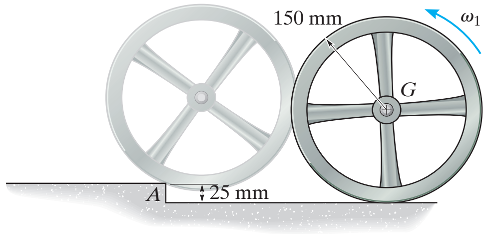
(a)



(b)

19–53.

The wheel has a mass of 50 kg and a radius of gyration of 125 mm about its center of mass G. If it rolls without slipping with an angular velocity of $\omega_1 = 5$ rad/s before it strikes the step at A, determine its angular velocity after it rolls over the step. The wheel does not loose contact with the step when it strikes it.



SOLUTION

Conservation of Angular Momentum: Referring to Fig. a, the sum of the angular impulses about point A is zero. Thus, angular momentum of the wheel is conserved about this point. Since the wheel rolls without slipping, $(v_G)_1 = \omega_1 r = (5)(0.15) = 0.75$ m/s and $\omega_2 = \omega_2 r = \omega_2(0.15)$. The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2 = 50(0.125^2) = 0.78125\ \text{kg}\cdot\text{m}^2$. Thus,

$$(H_A)_1 = (H_A)_2$$

$$50(0.75)(0.125) + 0.78125(5) = 50[\omega_2(0.15)](0.15) + 0.78125\omega_2$$

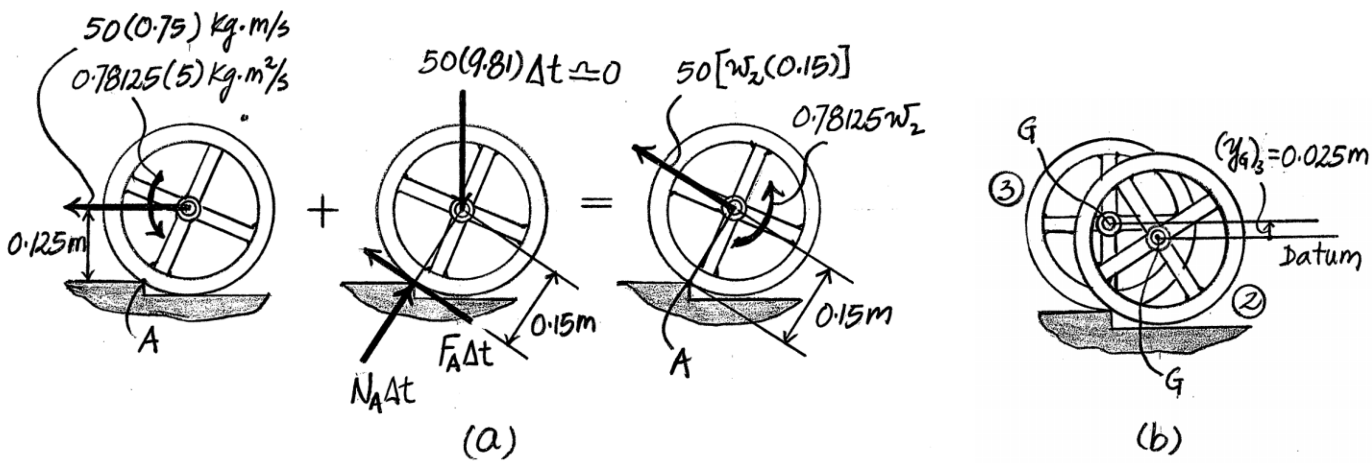
$$\omega_2 = 4.508 \text{ rad/s} \qquad \textbf{(1)}$$

Conservation of Energy: With reference to the datum in Fig. a, $V_2 = (V_g)_2 = W(y_G)_2 = 0$ and $V_3 = (V_g)_3 = W(y_G)_3 = 50(9.81)(0.025) = 12.2625$ J. The initial kinetic energy of the wheel is $T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}(50)[\omega(0.15)]^2 + \frac{1}{2}(0.78125)\omega^2 = 0.953125\omega^2$. Thus, $T_2 = 0.953125\omega_2{}^2 = 0.953125(4.508^2) = 19.37$ J and $T_3 = 0.953125\omega_3{}^2$.

$$T_2 + V_2 = T_3 + V_3$$

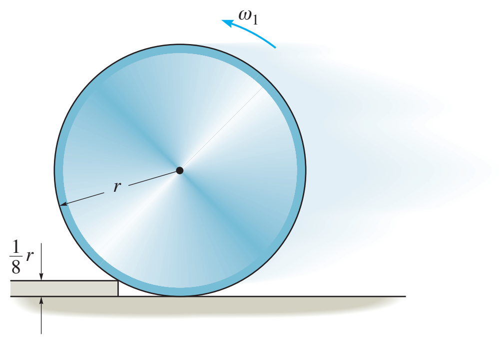
$$19.37 + 0 = 0.953125\omega_3{}^2 + 12.2625$$

$$\omega_3 = 2.73 \text{ rad/s} \qquad \textbf{Ans.}$$



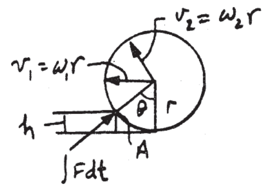
19–54.

The disk has a mass m and radius r. If it strikes the step without rebounding, determine the largest angular velocity the disk can have and not lose contact with the step.



SOLUTION

$$(H_A)_1 = \frac{1}{2}mr^2(\omega_1) + m(\omega_1 r)(r - h)$$



$$(H_A)_2 = \frac{1}{2}mr^2(\omega_2) + m(\omega_2 r)(r)$$

$$(H_A)_1 = (H_A)_2$$

$$\left[\frac{1}{2}m\,r^2 + m\,r\,(r - h)\right]\omega_1 = \frac{3}{2}m\,r^2\omega_2$$

$$\left(\frac{3}{2}r - h\right)\omega_1 = \frac{3}{2}r\,\omega_2$$

$\swarrow + \Sigma F_n = m\,a_n;\qquad W\cos\theta - F = m(\omega_2^2 r)$

$$F = mg\left(\frac{r-h}{r}\right) - m(\omega_2^2 r)$$

$$F = mg\left(\frac{r-h}{r}\right) - m\,r\left(\frac{2}{3}\right)^2\left(\frac{\frac{3}{2}r - h}{r}\right)^2\omega_1^2$$

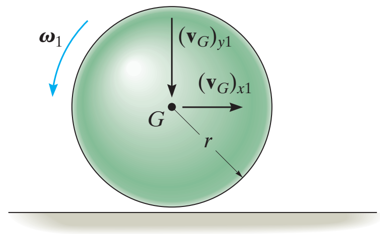
Set $h = \frac{1}{8}r$; also note that for maximum ω_1 F will approach zero. Thus

$$mg\left(\frac{r - \frac{1}{8}r}{r}\right) - mr\left(\frac{2}{3}\right)^2\left(\frac{\frac{3}{2}r - \frac{r}{8}}{r}\right)^2\omega_1^2$$

$\omega_1 = 1.02\sqrt{\dfrac{g}{r}}$ **Ans.**

19–55.

A solid ball with a mass m is thrown on the ground such that at the instant of contact it has an angular velocity $\boldsymbol{\omega}_1$ and velocity components $(\mathbf{v}_G)_{x1}$ and $(\mathbf{v}_G)_{y1}$ as shown. If the ground is rough so no slipping occurs, determine the components of the velocity of its mass center just after impact. The coefficient of restitution is e.



SOLUTION

Coefficient of Restitution (y direction):

$$(+\downarrow) \qquad e = \frac{0 - (v_G)_{y2}}{(v_G)_{y1} - 0} \qquad (v_G)_{y2} = -e(v_G)_{y1} = e(v_G)_{y1} \uparrow \qquad \textbf{Ans.}$$

Conservation of angular momentum about point on the ground:

$$(\circlearrowleft +) \qquad (H_A)_1 = (H_A)_2$$

$$-\frac{2}{5}mr^2\omega_1 + m(v_G)_{x1}r = \frac{2}{5}mr^2\omega_2 + m(v_G)_{x2}r$$

Since no slipping, $(v_G)_{x2} = \omega_2 r$ then,

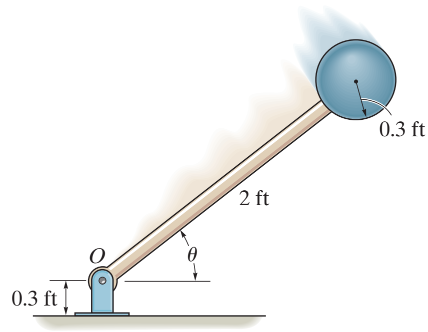
$$\omega_2 = \frac{5\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right)}{7r}$$

Therefore

$$(v_G)_{x2} = \frac{5}{7}\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right) \qquad \textbf{Ans.}$$

***19–56.**

The pendulum consists of a 10-lb sphere and 4-lb rod. If it is released from rest when $\theta = 90^\circ$, determine the angle θ of rebound after the sphere strikes the floor. Take $e = 0.8$.



SOLUTION

$$I_A = \frac{1}{3}\left(\frac{4}{32.2}\right)(2)^2 + \frac{2}{5}\left(\frac{10}{32.2}\right)(0.3)^2 + \left(\frac{10}{32.2}\right)(2.3)^2 = 1.8197 \text{ slug}\cdot\text{ft}^2$$

Just before impact:

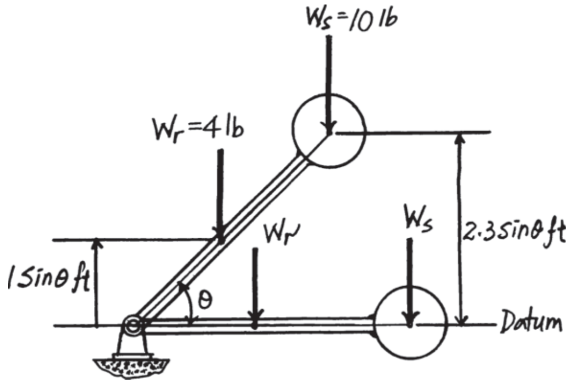
Datum through O.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 4(1) + 10(2.3) = \frac{1}{2}(1.8197)\omega^2 + 0$$

$$\omega_2 = 5.4475 \text{ rad/s}$$

$$v = 2.3(5.4475) = 12.529 \text{ ft/s}$$



Since the floor does not move,

$$(+\uparrow) \qquad e = 0.8 = \frac{(v_P) - 0}{0 - (-12.529)}$$

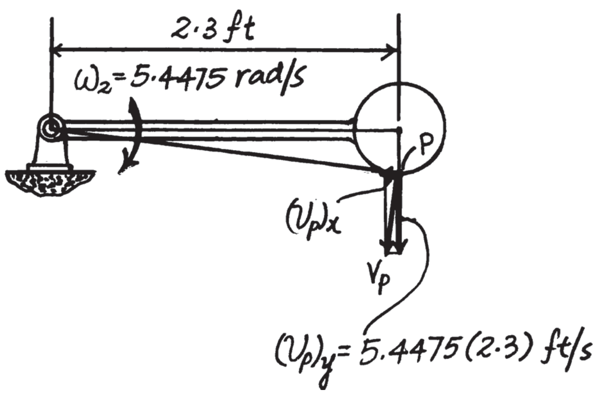
$$(v_P)_3 = 10.023 \text{ ft/s}$$

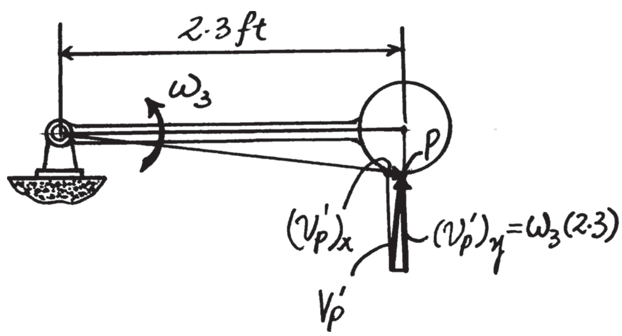
$$\omega_3 = \frac{10.023}{2.3} = 4.358 \text{ rad/s}$$

$$T_3 + V_3 = T_4 + V_4$$

$$\frac{1}{2}(1.8197)(4.358)^2 + 0 = 4(1 \sin\theta_1) + 10(2.3 \sin\theta_1)$$

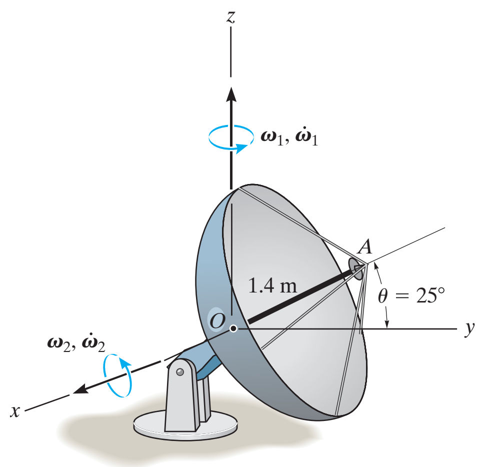
$$\theta_1 = 39.8^\circ$$ **Ans.**





20–1.

At a given instant, the satellite dish has an angular motion $\omega_1 = 6$ rad/s and $\dot{\omega}_1 = 3$ rad/s^2 about the z axis. At this same instant $\theta = 25°$, the angular motion about the x axis is $\omega_2 = 2$ rad/s, and $\dot{\omega}_2 = 1.5$ rad/s^2. Determine the velocity and acceleration of the signal horn A at this instant.



SOLUTION

Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the satellite at this instant (with reference to X, Y, Z) can be expressed in terms of **i, j, k** components.

$$\omega = \omega_1 + \omega_2 = \{2\mathbf{i} + 6\mathbf{k}\}\ \text{rad/s}$$

Angular Acceleration: The angular acceleration α will be determined by investigating separately the time rate of change of *each angular velocity component* with respect to the fixed XYZ frame. ω_2 is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_1 = \{6\mathbf{k}\}$ rad/s. Applying Eq. 20–6 with $(\dot{\omega}_2)_{xyz} = \{1.5\mathbf{i}\}$ rad/s^2. we have

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5\mathbf{i} + 6\mathbf{k} \times 2\mathbf{i} = \{1.5\mathbf{i} + 12\mathbf{j}\}\ \text{rad/s}^2$$

Since ω_1 is always directed along the Z axis ($\Omega = \mathbf{0}$), then

$$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + \mathbf{0} \times \omega_1 = \{3\mathbf{k}\}\ \text{rad/s}^2$$

Thus, the angular acceleration of the satellite is

$$\alpha = \dot{\omega}_1 + \dot{\omega}_2 = \{1.5\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}\}\ \text{rad/s}^2$$

Velocity and Acceleration: Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{1.4 \cos 25°\mathbf{j} + 1.4 \sin 25°\mathbf{k}\}$ m $= \{1.2688\mathbf{j} + 0.5917\mathbf{k}\}$ m, we have

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (2\mathbf{i} + 6\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})$$

$$= \{-7.61\mathbf{i} - 1.18\mathbf{j} + 2.54\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

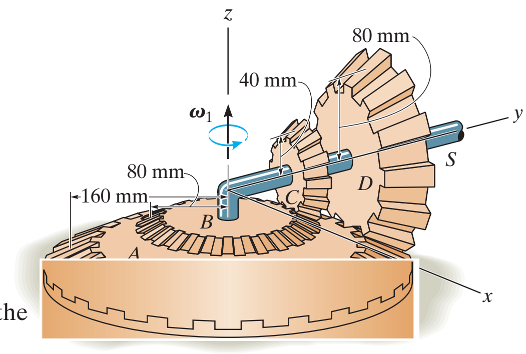
$$= (1.3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})$$

$$+ (2\mathbf{i} + 6\mathbf{k}) \times [(2\mathbf{i} + 6\mathbf{k}) \times (1.2688\mathbf{j} + 0.5917\mathbf{k})]$$

$$= \{10.4\mathbf{i} - 51.6\mathbf{j} - 0.463\mathbf{k}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

20–2.

Gears A and B are fixed, while gears C and D are free to rotate about the shaft S. If the shaft turns about the z axis at a constant rate of $\omega_1 = 4$ rad/s, determine the angular velocity and angular acceleration of gear C.



SOLUTION

The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity IA.

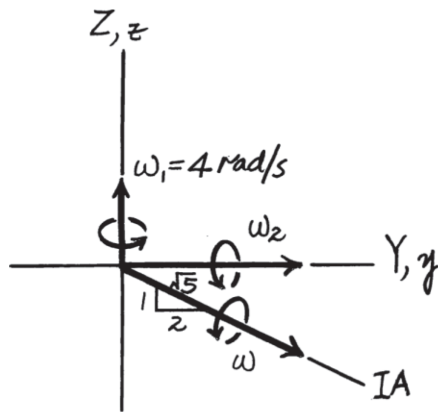
$$\omega = \omega_1 + \omega_2$$

$$\frac{2}{\sqrt{5}}\omega\mathbf{j} - \frac{1}{\sqrt{5}}\omega\mathbf{k} = 4\mathbf{k} + \omega_2\mathbf{j}$$

Equating $\mathbf{j}$ and $\mathbf{k}$ components

$$-\frac{1}{\sqrt{5}}\omega = 4 \qquad \omega = -8.944 \text{ rad/s}$$

$$\omega_2 = \frac{2}{\sqrt{5}}(-8.944) = -8.0 \text{ rad/s}$$



Hence $\quad \omega = \dfrac{2}{\sqrt{5}}(-8.944)\mathbf{j} - \dfrac{1}{\sqrt{5}}(-8.944)\mathbf{k} = \{-8.0\mathbf{j} + 4.0\mathbf{k}\}$ rad/s **Ans.**

For ω_2, $\Omega = \omega_1 = \{4\mathbf{k}\}$ rad/s.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2$$
$$= 0 + (4\mathbf{k}) \times (-8\mathbf{j})$$
$$= \{32\mathbf{i}\} \text{ rad/s}^2$$

For ω_1, $\Omega = \mathbf{0}$.

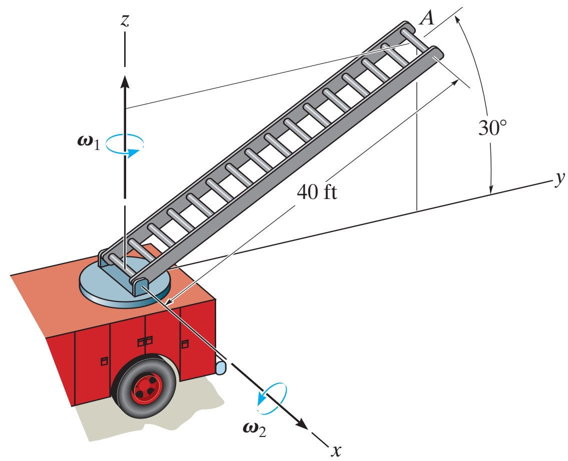
$$(\dot{\omega}_1)_{XYZ} = (\dot{\omega}_1)_{xyz} + \Omega \times \omega_1 = 0 + 0 = 0$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = \mathbf{0} + (32\mathbf{i}) = \{32\mathbf{i}\} \text{ rad/s}^2$$ **Ans.**

20–3.

The ladder of the fire truck rotates around the z axis with an angular velocity $\omega_1 = 0.15$ rad/s, which is increasing at 0.8 rad/s^2. At the same instant it is rotating upward at a constant rate $\omega_2 = 0.6$ rad/s. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.



SOLUTION

$$\omega = \omega_1 + \omega_2 = 0.15\mathbf{k} + 0.6\mathbf{i} = \{0.6\mathbf{i} + 0.15\mathbf{k}\}\text{ rad/s}$$

Angular acceleration: For ω_1, $\omega = \omega_1 = \{0.15\mathbf{k}\}$ rad/s.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \omega \times \omega_2$$

$$= 0 + (0.15\mathbf{k}) \times (0.6\mathbf{i}) = \{0.09\mathbf{j}\}\text{ rad/s}^2$$

For ω_1, $\Omega = 0$.

$$(\dot{\omega}_1)_{XYZ} = (\dot{\omega}_1)_{xyz} + \omega \times \omega_1 = (0.8\mathbf{k}) + 0 = \{0.8\mathbf{k}\}\text{ rad/s}^2$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = 0.8\mathbf{k} + 0.09\mathbf{j} = \{0.09\mathbf{j} + 0.8\mathbf{k}\}\text{ rad/s}^2$$

$$\mathbf{r}_A = 40\cos 30^\circ\mathbf{j} + 40\sin 30^\circ\mathbf{k} = \{34.641\mathbf{j} + 20\mathbf{k}\}\text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$

$$= (0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$$

$$= \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}\text{ ft/s} \qquad \textbf{Ans.}$$

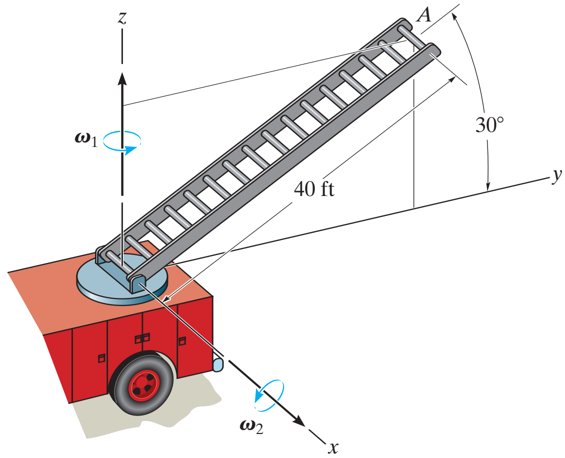
$$\mathbf{a}_A = \alpha \times \mathbf{r} + \omega \times \mathbf{v}_A$$

$$= (0.09\mathbf{j} + 0.8\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k}) + (0.6\mathbf{i} + 0.15\mathbf{k}) \times (-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k})$$

$$= \{-24.1\mathbf{i} - 13.3\mathbf{j} - 7.20\mathbf{k}\}\text{ ft/s}^2 \qquad \textbf{Ans.}$$

***20–4.**

The ladder of the fire truck rotates around the z axis with an angular velocity of $\omega_1 = 0.15$ rad/s, which is increasing at 0.2 rad/s^2. At the same instant it is rotating upwards at $\omega_2 = 0.6$ rad/s while increasing at 0.4 rad/s^2. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.



SOLUTION

$$\mathbf{r}_{A/O} = 40\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k}$$

$$\mathbf{r}_{A/O} = \{34.641\mathbf{j} + 20\mathbf{k}\}\text{ ft}$$

$$\Omega = \omega_1\,\mathbf{k} + \omega_2\,\mathbf{i} = \{0.6\mathbf{i} + 0.15\mathbf{k}\}\text{ rad/s}$$

$$\dot{\omega} = \dot{\omega}_1\,\mathbf{k} + \dot{\omega}_2\,\mathbf{i} + \omega_1\,\mathbf{k} \times \omega_2\,\mathbf{i}$$

$$\dot{\Omega} = 0.2\mathbf{k} + 0.4\mathbf{i} + 0.15\mathbf{k} \times 0.6\mathbf{i} = \{0.4\mathbf{i} + 0.09\mathbf{j} + 0.2\mathbf{k}\}\text{ rad/s}^2$$

$$\mathbf{v}_A = \Omega \times \mathbf{r}_{A/O} = (0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$$

$$\mathbf{v}_A = \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}\text{ ft/s}$$ **Ans.**

$$\mathbf{a}_A = \Omega \times (\Omega \times \mathbf{r}_{A/O}) + \dot{\omega} \times \mathbf{r}_{A/O}$$

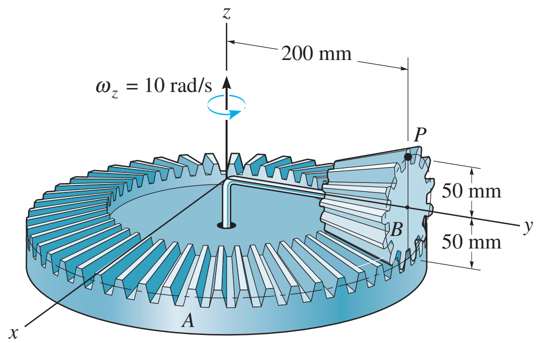
$$\mathbf{a}_A = (0.6\mathbf{i} + 0.15\mathbf{k}) \times [(0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})]$$

$$+ (0.4\mathbf{i} + 0.09\mathbf{j} + 0.2\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$$

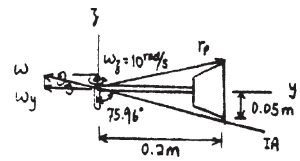
$$\mathbf{a}_A = \{-3.33\mathbf{i} - 21.3\mathbf{j} + 6.66\mathbf{k}\}\text{ ft/s}^2$$ **Ans.**

20–5.

Gear B is connected to the rotating shaft, while the plate gear A is fixed. If the shaft is turning at a constant rate of $\omega_z = 10$ rad/s about the z axis, determine the magnitudes of the angular velocity and the angular acceleration of gear B. Also, determine the magnitudes of the velocity and acceleration of point P.



SOLUTION



$\omega_z = 10$ rad/s

$\omega_y = -10 \tan 75.96° = -40$ rad/s

$\omega_x = 0$

$\boldsymbol{\omega} = \{-40\mathbf{j} + 10\mathbf{k}\}$ rad/s

$\omega = \sqrt{(-40)^2 + (10)^2} = 41.2$ rad/s **Ans.**

$\mathbf{r}_P = \{0.2\mathbf{j} + 0.05\mathbf{k}\}$ m

$\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{r}_P = (-40\mathbf{j} + 10\mathbf{k}) \times (0.2\mathbf{j} + 0.05\mathbf{k})$

$\mathbf{v}_P = \{-4\mathbf{i}\}$ m/s

$v_P = 4.00$ m/s **Ans.**

Let $\Omega = \omega_z$,

$$\dot{\boldsymbol{\omega}} = \left(\dot{\boldsymbol{\omega}}\right)_{xyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega}$$

$$= \mathbf{0} + (10\mathbf{k}) \times (-40\mathbf{j} + 10\mathbf{k}) = \{400\mathbf{i}\} \text{ rad/s}^2$$

$\alpha = \dot{\omega} = 400$ rad/s^2 **Ans.**

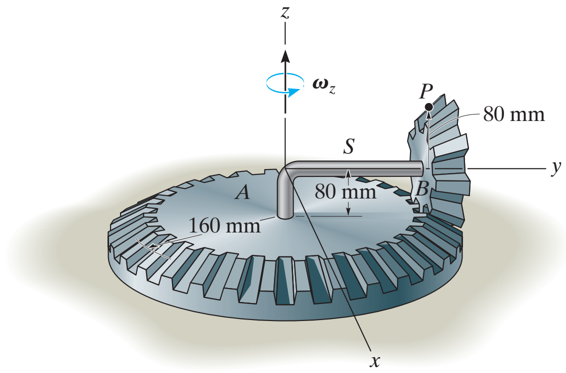
$\mathbf{a}_P = \boldsymbol{\alpha} \times \mathbf{r}_P + \boldsymbol{\omega} \times \mathbf{v}_P = (400\mathbf{i}) \times (0.2\mathbf{j} + 0.05\mathbf{k}) + (-40\mathbf{j} + 10\mathbf{k}) \times (-4\mathbf{i})$

$= \{-60\mathbf{j} - 80\mathbf{k}\}$ m/s^2

$a_P = \sqrt{(-60)^2 + (-80)^2} = 100$ m/s^2 **Ans.**

20–6.

Gear A is fixed while gear B is free to rotate on the shaft S. If the shaft is turning about the z axis at $\omega_z = 5$ rad/s, while increasing at 2 rad/s^2, determine the velocity and acceleration of point P at the instant shown. The face of gear B lies in a vertical plane.



SOLUTION

$$\Omega = \{5\mathbf{k} - 10\mathbf{j}\}\ \text{rad/s}$$

$$\dot{\Omega} = \{50\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\}\ \text{rad/s}^2$$

$$\mathbf{v}_P = \Omega \times \mathbf{r}_P$$

$$\mathbf{v}_P \quad (5\mathbf{k} - 10\mathbf{j} \times (160\mathbf{j} + 80\mathbf{k})$$

$$\mathbf{v}_P = \{-1600\mathbf{i}\}\ \text{mm/s}$$

$$= \{-1.60\mathbf{i}\}\ \text{m/s}$$ **Ans.**

$$\mathbf{a}_P = \Omega \times \mathbf{v}_P + \dot{\Omega} \times \mathbf{r}_P$$

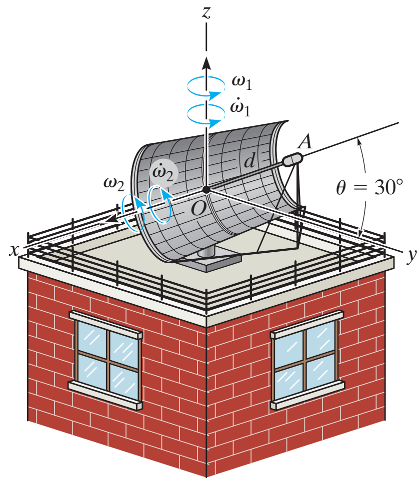
$$\mathbf{a}_P = \{50\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\} \times (160\mathbf{j} + 80\mathbf{k}) + (-10\mathbf{j} + 5\mathbf{k}) \times (-1600\mathbf{i})$$

$$\mathbf{a}_P = \{-640\mathbf{i} - 12000\mathbf{j} - 8000\mathbf{k}\}\ \text{mm/s}^2$$

$$\mathbf{a}_P = \{-0.640\mathbf{i} - 12.0\mathbf{j} - 8.00\mathbf{k}\}\ \text{m/s}^2$$ **Ans.**

20–7.

At a given instant, the antenna has an angular motion $\omega_1 = 3 \text{ rad/s}$ and $\dot{\omega}_1 = 2 \text{ rad/s}^2$ about the z axis. At this same instant $\theta = 30^\circ$, the angular motion about the x axis is $\omega_2 = 1.5 \text{ rad/s}$, and $\dot{\omega}_2 = 4 \text{ rad/s}^2$. Determine the velocity and acceleration of the signal horn A at this instant. The distance from O to A is $d = 3$ ft.



SOLUTION

$$\mathbf{r}_A = 3\cos 30°\mathbf{j} + 3\sin 30°\mathbf{k} = \{2.598\mathbf{j} + 1.5\mathbf{k}\} \text{ ft}$$

$$\Omega = \omega_1 + \omega_2 = 3\mathbf{k} + 1.5\mathbf{i}$$

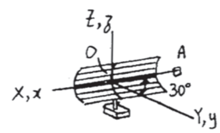
$$\mathbf{v}_A = \Omega \times \mathbf{r}_A$$

$$\mathbf{v}_A = (3\mathbf{k} + 1.5\mathbf{i}) \times (2.598\mathbf{j} + 1.5\mathbf{k})$$

$$= -7.794\mathbf{i} + 3.897\mathbf{k} - 2.25\mathbf{j}$$

$$= \{-7.79\mathbf{i} - 2.25\mathbf{j} + 3.90\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\dot{\Omega} = \dot{\omega}_1 + \dot{\omega}_2$$



$$= (2\mathbf{k} + 0) + (4\mathbf{i} + 3\mathbf{k} \times 1.5\mathbf{i})$$

$$= 4\mathbf{i} + 4.5\mathbf{j} + 2\mathbf{k}$$

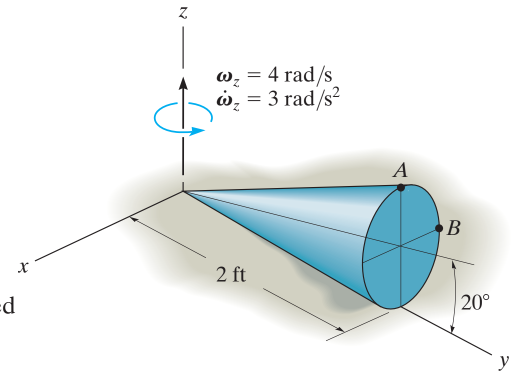
$$\mathbf{a}_A = \dot{\omega} \times r_A + \Omega \times \mathbf{v}_A$$

$$\mathbf{a}_A = (4\mathbf{i} + 4.5\mathbf{j} + 2\mathbf{k}) \times (2.598\mathbf{j} + 1.5\mathbf{k}) + (3\mathbf{k} + 1.5\mathbf{i}) \times (-7.794\mathbf{i} - 2.25\mathbf{j} + 3.879\mathbf{k})$$

$$\mathbf{a}_A = \{8.30\mathbf{i} - 35.2\mathbf{j} + 7.02\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

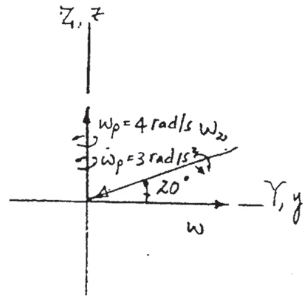
***20–8.**

The cone rolls without slipping such that at the instant shown $\omega_z = 4\text{ rad/s}$ and $\dot{\omega}_z = 3\text{ rad/s}^2$. Determine the velocity and acceleration of point A at this instant.



SOLUTION

Angular velocity: The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity (y axis).



$$\omega = \omega_1 + \omega_2$$

$$\omega\,\mathbf{j} = 4\mathbf{k} + (\omega_2 \cos 20°\mathbf{j} + \omega_2 \sin 20°\mathbf{k})$$

$$\omega\,\mathbf{j} = \omega_2 \cos 20°\mathbf{j} + (\mathbf{4} + \omega_2 \sin 20°)\,\mathbf{k}$$

Equating $\mathbf{j}$ and $\mathbf{k}$ components:

$$4 + \omega_2 \sin 20° = 0 \qquad \omega_2 = -11.70\text{ rad/s}$$

$$\omega = -11.70\cos 20° = -10.99\text{ rad/s}$$

Hence, $\omega = \{-10.99\mathbf{j}\}\text{ rad/s}$

$$\omega_2 = -11.70\cos 20°\mathbf{j} + (-11.70\sin 20°)\,\mathbf{k} = \{-10.99\mathbf{j} - 4\mathbf{k}\}\text{ rad/s}$$

Angular acceleration:

$$(\dot{\omega}_1)_{xyz} = \{3\mathbf{k}\}\text{ rad/s}^2$$

$$(\dot{\omega}_2)_{xyz} = \left(-\frac{3}{\sin 20°}\right)\cos 20°\mathbf{j} - 3\mathbf{k} = \{-8.2424\mathbf{j} - 3\mathbf{k}\}\text{ rad/s}^2$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

$$= \left[(\dot{\omega}_1)_{xyz} + \Omega \times \omega_1\right] + \left[(\dot{\omega}_2)_{xyz} + \Omega \times \omega_2\right]$$

$\Omega = \omega_1 = \{4\mathbf{k}\}\text{ rad/s}$ then

$$\dot{\omega} = [3\mathbf{k} + \mathbf{0}] + [(-8.2424\mathbf{j} - 3\mathbf{k}) + 4\mathbf{k} \times (-10.99\mathbf{j} - 4\mathbf{k})]$$

$$= \{43.9596\mathbf{i} - 8.2424\mathbf{j}\}\text{ rad/s}$$

$$\mathbf{r}_A = 2\cos 40°\mathbf{j} + 2\sin 40°\mathbf{k} = \{1.5321\mathbf{j} + 1.2856\mathbf{k}\}\text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$

$$= (-10.99\mathbf{j}) \times (1.5321\mathbf{j} + 1.2856\mathbf{k})$$

$$= \{-14.1\mathbf{i}\}\text{ ft/s} \qquad \textbf{Ans.}$$

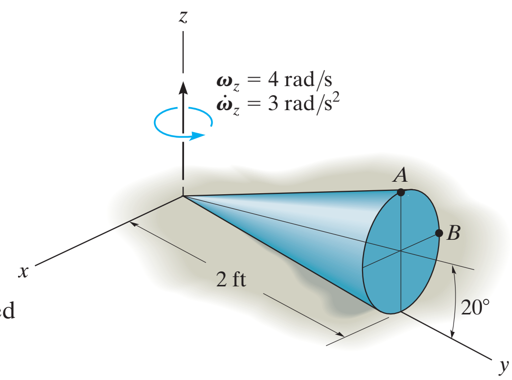
$$\mathbf{a}_A = \mathbf{a} \times \mathbf{r}_A + \omega \times \mathbf{v}_A$$

$$= (43.9596\mathbf{i} - 8.2424\mathbf{j}) \times (1.5321\mathbf{j} + 1.2856\mathbf{k}) + (-10.99\mathbf{j}) \times (-14.1\mathbf{i})$$

$$= \{-10.6\mathbf{i} - 56.5\mathbf{j} - 87.9\mathbf{k}\}\text{ ft/s}^2 \qquad \textbf{Ans.}$$

20–9.

The cone rolls without slipping such that at the instant shown $\omega_z = 4\text{ rad/s}$ and $\dot{\omega}_z = 3\text{ rad/s}^2$. Determine the velocity and acceleration of point B at this instant.



SOLUTION

Angular velocity: The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity (y axis).

$$\omega = \omega_1 + \omega_2$$

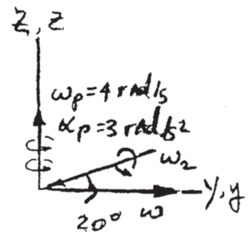
$$\omega\,\mathbf{j} = 4\mathbf{k} + (\omega_2 \cos 20^\circ\mathbf{j} + \omega_2 \sin 20^\circ\mathbf{k})$$

$$\omega\,\mathbf{j} = \omega_2 \cos 20^\circ\mathbf{j} + (\mathbf{4} + \omega_2 \sin 20^\circ)\,\mathbf{k}$$

Equating $\mathbf{j}$ and $\mathbf{k}$ components:

$$4 + \omega_2 \sin 20^\circ = 0 \qquad \omega_2 = -11.70\text{ rad/s}$$

$$\omega = -11.70\cos 20^\circ = -10.99\text{ rad/s}$$



Hence, $\omega = \{-10.99\mathbf{j}\}\text{ rad/s}$

$$\omega_2 = -11.70\cos 20^\circ\mathbf{j} + (-11.70\sin 20^\circ)\,\mathbf{k} = \{-10.99\mathbf{j} - 4\mathbf{k}\}\text{ rad/s}$$

Angular acceleration:

$$(\dot{\omega}_1)_{xyz} = \{3\mathbf{k}\}\text{ rad/s}^2$$

$$(\dot{\omega}_2)_{xyz} = \left(-\frac{3}{\sin 20^\circ}\right)\cos 20^\circ\mathbf{j} - 3\mathbf{k} = \{-8.2424\mathbf{j} - 3\mathbf{k}\}\text{ rad/s}^2$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

$$= \left[(\dot{\omega}_1)_{xyz} + \Omega \times \omega_1\right] + \left[(\dot{\omega}_2)_{xyz} + \Omega \times \omega_2\right]$$

$$\Omega = \omega_1 = \{4\mathbf{k}\}\text{ rad/s then}$$

$$\dot{\omega} = [3\mathbf{k} + \mathbf{0}] + [(-8.2424\mathbf{j} - 3\mathbf{k}) + 4\mathbf{k} \times (-10.99\mathbf{j} - 4\mathbf{k})]$$

$$= \{43.9596\mathbf{i} - 8.2424\mathbf{j}\}\text{ rad/s}$$

$$\mathbf{r}_B = 2\sin 20^\circ\mathbf{i} + 2\cos 20^\circ\mathbf{j} + 2\sin 20^\circ\cos 20^\circ\mathbf{k}$$

$$= -0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k}$$

$$\mathbf{v}_B = \omega \times \mathbf{r}_B$$

$$= (-10.99\mathbf{j}) \times (-0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k})$$

$$= -7.0642\mathbf{i} - 7.5176\mathbf{k}$$

$$= \{-7.06\mathbf{i} - 7.52\mathbf{k}\}\text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \alpha \times \mathbf{r}_B + \omega \times \mathbf{v}_B$$

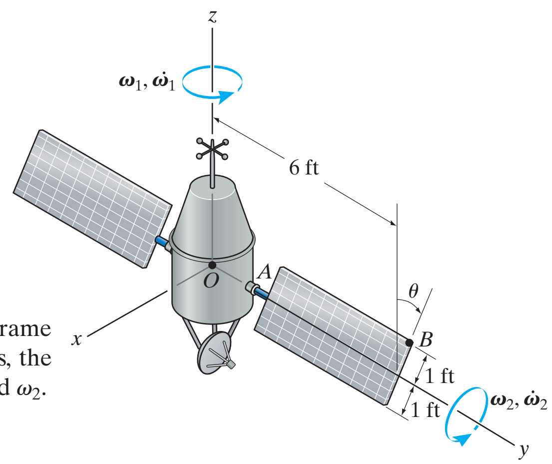
$$= (43.9596\mathbf{i} - 8.2424\mathbf{j}) \times (-0.68404\mathbf{i} + 1.8794\mathbf{j} + 0.64279\mathbf{k})$$

$$+ (-10.99\mathbf{j}) \times (-7.0642\mathbf{i} - 7.5176\mathbf{k})$$

$$= \{77.3\mathbf{i} - 28.3\mathbf{j} - 0.657\mathbf{k}\}\text{ ft/s}^2 \qquad \textbf{Ans.}$$

20–10.

At the instant when $\theta = 90°$, the satellite's body is rotating with an angular velocity of $\omega_1 = 15$ rad/s and angular acceleration of $\dot{\omega}_1 = 3$ rad/s^2. Simultaneously, the solar panels rotate with an angular velocity of $\omega_2 = 6$ rad/s and angular acceleration of $\dot{\omega}_2 = 1.5$ rad/s^2. Determine the velocity and acceleration of point B on the solar panel at this instant.



SOLUTION

Here, the solar panel rotates about a fixed point O. The XYZ fixed reference frame is set to coincide with the xyz rotating frame at the instant considered. Thus, the angular velocity of the solar panel can be obtained by vector addition of ω_1 and ω_2.

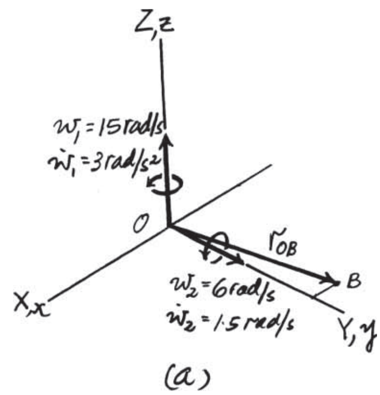
$$\omega = \omega_1 + \omega_2 = [6\mathbf{j} + 15\mathbf{k}] \text{ rad/s}$$

The angular acceleration of the solar panel can be determined from

$$\alpha = \dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

If we set the xyz frame to have an angular velocity of $\Omega = \omega_1 = [15\mathbf{k}]$ rad/s, then the direction of ω_2 will remain constant with respect to the xyz frame, which is along the y axis. Thus,

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5\mathbf{j} + (15\mathbf{k} \times 6\mathbf{j}) = [-90\mathbf{i} + 1.5\mathbf{j}] \text{ rad/s}^2$$



Since ω_1 is always directed along the Z axis when $\Omega = \omega_1$, then

$$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + \omega_1 \times \omega_1 = [3\mathbf{k}] \text{ rad/s}^2$$

Thus,

$$\alpha = 3\mathbf{k} + (-90\mathbf{i} + 1.5\mathbf{j})$$

$$= [-90\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}] \text{ rad/s}^2$$

When $\theta = 90°$, $\mathbf{r}_{OB} = [-1\mathbf{i} + 6\mathbf{j}]$ ft. Thus,

$$\mathbf{v}_B = \omega \times r_{OB} = (6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})$$

$$= [-90\mathbf{i} - 15\mathbf{j} + 6\mathbf{k}] \text{ ft/s} \qquad \textbf{Ans.}$$

and

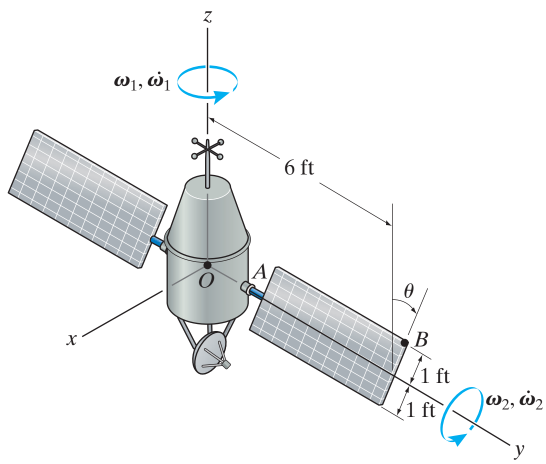
$$\mathbf{a}_B = \alpha \times \mathbf{r}_{OB} + \omega \times (\omega \times \mathbf{r}_{OB})$$

$$= (-90\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j}) + (6\mathbf{j} + 15\mathbf{k}) \times [(6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})]$$

$$= [243\mathbf{i} - 1353\mathbf{j} + 1.5\mathbf{k}] \text{ ft/s}^2 \qquad \textbf{Ans.}$$

20–11.

At the instant when $\theta = 90^\circ$, the satellite's body travels in the x direction with a velocity of $\mathbf{v}_O = \{500\mathbf{i}\}$ m/s and acceleration of $\mathbf{a}_O = \{50\mathbf{i}\}$ m/s^2. Simultaneously, the body also rotates with an angular velocity of $\omega_1 = 15$ rad/s and angular acceleration of $\dot{\omega}_1 = 3$ rad/s^2. At the same time, the solar panels rotate with an angular velocity of $\omega_2 = 6$ rad/s and angular acceleration of $\dot{\omega}_2 = 1.5$ rad/s^2 Determine the velocity and acceleration of point B on the solar panel.



SOLUTION

The XYZ translating reference frame is set to coincide with the xyz rotating frame at the instant considered. Thus, the angular velocity of the solar panel at this instant can be obtained by vector addition of ω_1 and ω_2.

$$\omega = \omega_1 + \omega_2 = [6\mathbf{j} + 15\mathbf{k}] \text{ rad/s}$$

The angular acceleration of the solar panel can be determined from

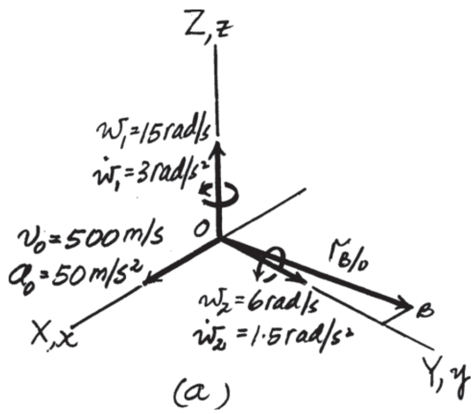
$$\alpha = \dot{\omega} = \omega_1 + \omega_2$$

If we set the xyz frame to have an angular velocity of $\Omega = \omega_1 = [15\mathbf{k}]$ rad/s, then the direction of ω_2 will remain constant with respect to the xyz frame, which is along the y axis. Thus,

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \omega_1 \times \omega_2 = 1.5\mathbf{j} + (15\mathbf{k} \times 6\mathbf{j}) = [-90\mathbf{i} + 15\mathbf{j}] \text{ rad/s}^2$$

Since ω_1 is always directed along the Z axis when $\Omega = \omega_1$, then

$$\dot{\omega}_1 = (\dot{\omega}_1)_{xyz} + \omega_1 \times \omega_1 = [3\mathbf{k}] \text{ rad/s}^2$$



(a)

Thus,

$$\alpha = 3\mathbf{k} + (-90\mathbf{i} + 1.5\mathbf{j})$$

$$= [-90\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}] \text{ rad/s}^2$$

When $\theta = 90^\circ$, $\mathbf{r}_{B/O} = [-1\mathbf{i} + 6\mathbf{j}]$ ft. Since the satellite undergoes general motion, then

$$\mathbf{v}_B = \mathbf{v}_O + \omega \times r_{B/O} = (500\mathbf{i}) + (6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})$$

$$= [410\mathbf{i} - 15\mathbf{j} + 6\mathbf{k}] \text{ ft/s}$$ **Ans.**

and

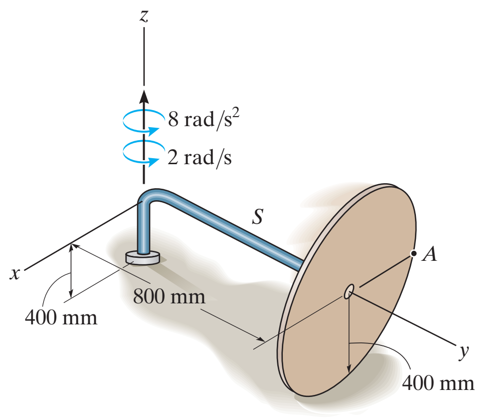
$$\mathbf{a}_B = \mathbf{a}_O + \alpha \times \mathbf{r}_{B/O} + \omega \times (\omega \times \mathbf{r}_{B/O})$$

$$= 50\mathbf{i} + (-90\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j}) + (6\mathbf{j} + 15\mathbf{k}) \times [(6\mathbf{j} + 15\mathbf{k}) \times (-1\mathbf{i} + 6\mathbf{j})]$$

$$= [293\mathbf{i} - 1353\mathbf{j} + 1.5\mathbf{k}] \text{ ft/s}^2$$ **Ans.**

***20–12.**

The disk B is free to rotate on the shaft S. If the shaft is turning about the z axis at $\omega_z = 2$ rad/s, while increasing at 8 rad/s^2, determine the velocity and acceleration of point A at the instant shown.



SOLUTION

Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident Thus, the angular velocity of the disk at this instant (with reference to X, Y, Z) can be expressed in terms of **i**, **j**, **k** components. The velocity of the center of the disk is $v = \omega_z(0.8) = 2(0.8) = 1.60$ m/s. Since the disk rolls without slipping, its spinning anngular velocity is given by $\omega_s = \frac{v}{r} = \frac{1.60}{0.4} = 4$ rad/s and is directed towards – j. Thus, $\omega_s = \{-4\mathbf{j}\}$ rad/s.

$$\omega = \omega_z + \omega_z = \{-4\mathbf{j} + 2\mathbf{k}\} \text{ rad/s}$$

Angular Acceleration: The angular acceleration α will be determined by investigating separately the time rate of change of *each angular velocity component* with respect to the fixed XYZ frame, ω_s is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_z = \{2\mathbf{k}\}$ rad/s. The tangential acceleration of the center of the disk is $a = \dot{\omega}_z(0.8) = 8(0.8) = 6.40$ m/s^2. Since the disk rolls without slipping, its spinning anngular acceleration is given by $\dot{\omega}_s = \frac{a}{r} = \frac{6.40}{0.4} = 16$ rad/s^2 and directed towards – j. Thus, $(\dot{\omega}_s)_{xyz} = \{-16\mathbf{j}\}$ rad/s^2. Applying Eq. 20–6, we have

$$\dot{\omega}_s = (\dot{\omega}_s)_{xyz} + \omega_z \times \omega_s = -16\mathbf{j} + 2\mathbf{k} \times (-4\mathbf{j}) = \{8\mathbf{i} - 16\mathbf{j}\} \text{ rad/s}^2$$

Since ω_z is always directed along Z axis ($\Omega = \mathbf{0}$), then

$$\dot{\omega}_z = (\dot{\omega}_z)_{xyz} + \mathbf{0} \times \omega_z = \{8\mathbf{k}\} \text{ rad/s}^2$$

Thus, the angular acceleration of the disk is

$$\alpha = \dot{\omega}_z + \dot{\omega}_z = \{8\mathbf{i} - 16\mathbf{j} + 8\mathbf{k}\} \text{ rad/s}^2$$

Velocity and Acceleration: Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{-0.4\mathbf{i} + 0.8\mathbf{j}\}$ m, we have

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-4\mathbf{j} + 2\mathbf{k}) \times (-0.4\mathbf{i} + 0.8\mathbf{j})$$

$$= \{-1.60\mathbf{i} - 0.800\mathbf{j} - 1.60\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$a_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

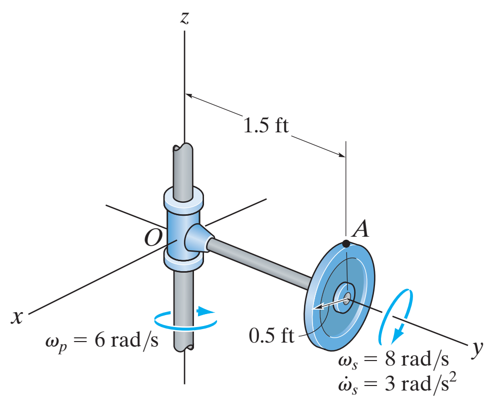
$$= (8\mathbf{i} - 16\mathbf{j} + 8\mathbf{k}) \times (-0.4\mathbf{i} + 0.8\mathbf{j})$$

$$+ (-4\mathbf{j} + 2\mathbf{k}) \times [(-4\mathbf{j} + 2\mathbf{k}) \times (-0.4\mathbf{i} + 0.8\mathbf{j})]$$

$$= \{1.6\mathbf{i} - 6.40\mathbf{j} - 6.40\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

20–13.

The disk spins about the arm with an angular velocity of $\omega_s = 8\ \text{rad/s}$, which is increasing at a constant rate of $\dot{\omega}_s = 3\ \text{rad/s}^2$ at the instant shown. If the shaft rotates with a constant angular velocity of $\omega_p = 6\ \text{rad/s}$, determine the velocity and acceleration of point A located on the rim of the disk at this instant.



SOLUTION

The XYZ fixed reference frame is set to coincide with the rotating xyz reference frame at the instant considered. Thus, the angular velocity of the disk at this instant can be obtained by vector addition of ω_s and ω_p.

$$\omega = \omega_s + \omega_p = [-8\mathbf{j} - 6\mathbf{k}]\ \text{rad/s}$$

The angular acceleration of the disk is determined from

$$\alpha = \dot{\omega} = \dot{\omega}_s + \dot{\omega}_p$$

If we set the xyz rotating frame to have an angular velocity of $\Omega = \omega_p = [-6\mathbf{k}]\ \text{rad/s}$, the direction of ω_s will remain unchanged with respect to the xyz rotating frame which is along the y axis. Thus,

$$\dot{\omega}_s = (\dot{\omega}_s)_{xyz} + \omega_p \times \omega_s = -3\mathbf{j} + (-6\mathbf{k}) \times (-8\mathbf{j}) = [-48\mathbf{i} - 3\mathbf{j}]\ \text{rad/s}^2$$

Since ω_p is always directed along the Z axis where $\Omega = \omega_p$ and since it has a constant magnitude, $\left(\dot{\omega}_p\right)_{xyz} = 0$. Thus,

$$\dot{\omega}_p = \left(\dot{\omega}_p\right)_{xyz} + \omega_p \times \omega_p = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

Thus,

$$\alpha = [-48\mathbf{i} - 3\mathbf{j}] + 0 = [-48\mathbf{i} - 3\mathbf{j}]\ \text{rad/s}^2$$

Here, $\mathbf{r}_A = [1.5\mathbf{j} + 0.5\mathbf{k}]$ ft, so that

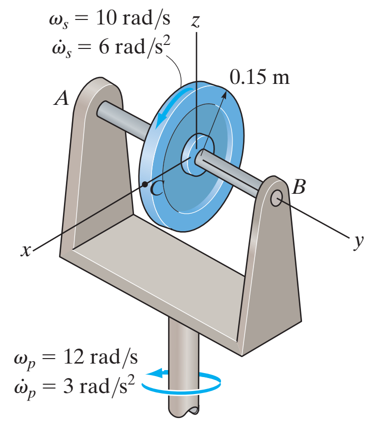
$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-8\mathbf{j} - 6\mathbf{k}) \times (1.5\mathbf{j} + 0.5\mathbf{k}) = [5\mathbf{i}]\ \text{ft/s} \qquad \textbf{Ans.}$$

and

$$\begin{aligned}\mathbf{a}_A &= \alpha \times \mathbf{r}_A + \omega \times (\omega \times r_A)\\ &= (-48\mathbf{i} - 3\mathbf{j}) \times (1.5\mathbf{j} + 0.5\mathbf{k}) + (-8\mathbf{j} - 6\mathbf{k}) \times [(-8\mathbf{j} - 6\mathbf{k}) \times (1.5\mathbf{j} + 0.5\mathbf{k})]\\ &= [-1.5\mathbf{i} - 6\mathbf{j} - 32\mathbf{k}]\ \text{ft/s}^2 \qquad \textbf{Ans.}\end{aligned}$$

20–14.

The wheel is spinning about shaft AB with an angular velocity of $\omega_s = 10$ rad/s, which is increasing at a constant rate of $\dot{\omega}_2 = 6$ rad/s^2, while the frame precesses about the z axis with an angular velocity of $\omega_p = 12$ rad/s, which is increasing at a constant rate of $\dot{\omega}_p = 3$ rad/s^2. Determine the velocity and acceleration of point C located on the rim of the wheel at this instant.



SOLUTION

The XYZ fixed reference frame is set to coincide with the rotating xyz reference frame at the instant considered. Thus, the angular velocity of the wheel at this instant can be obtained by vector addition of ω_s and ω_p.

$$\omega = \omega_s + \omega_p = [10\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

The angular acceleration of the disk is determined from

$$\alpha = \dot{\omega} = \dot{\omega}_s + \dot{\omega}_p$$

If we set the xyz rotating frame to have an angular velocity of $\Omega = \omega_p = [12\mathbf{k}]$ rad/s, the direction of ω_s will remain unchanged with respect to the xyz rotating frame which is along the y axis. Thus,

$$\dot{\omega}_s = (\dot{\omega}_s)_{xyz} + \omega_p \times \omega_s = 6\mathbf{j} + (12\mathbf{k}) \times (10\mathbf{j}) = [-120\mathbf{i} + 6\mathbf{j}] \text{ rad/s}^2$$

Since ω_p is always directed along the Z axis where $\Omega = \omega_p$, then

$$\dot{\omega}_p = \left(\dot{\omega}_p\right)_{xyz} + \omega_p \times \omega_p = 3\mathbf{k} + \mathbf{0} = [3\mathbf{k}] \text{ rad/s}^2$$

Thus, $\alpha = (-120\mathbf{i} + 6\mathbf{j}) + 3\mathbf{k} = [-120\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}]$ rad/s^2

Here, $\mathbf{r}_C = [0.15\mathbf{i}]$ m, so that

$$v_C = \omega \times \mathbf{r}_C = (10\mathbf{j} + 12\mathbf{k}) \times (0.15\mathbf{i}) = [1.8\mathbf{j} - 1.5\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

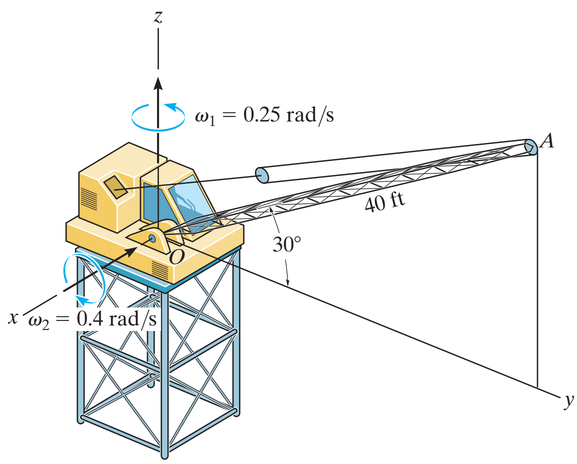
$$a_C = \alpha \times \mathbf{r}_C + \omega \times (\omega \times \mathbf{r}_C)$$

$$= (-120\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) \times (0.15\mathbf{i}) + (10\mathbf{j} + 12\mathbf{k}) \times [(10\mathbf{j} + 12\mathbf{k}) \times (0.15\mathbf{i})]$$

$$= [-36.6\mathbf{i} + 0.45\mathbf{j} - 0.9\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

20–15.

At the instant shown, the tower crane rotates about the z axis with an angular velocity $\omega_1 = 0.25$ rad/s, which is increasing at 0.6 rad/s^2. The boom OA rotates downward with an angular velocity $\omega_2 = 0.4$ rad/s, which is increasing at 0.8 rad/s^2. Determine the velocity and acceleration of point A located at the end of the boom at this instant.



SOLUTION

$\omega = \omega_1 + \omega_2 = \{-0.4\,\mathbf{i} + 0.25\mathbf{k}\}$ rad/s

$\Omega = \{0.25\,\mathbf{k}\}$ rad/s

$\dot{\omega} = (\dot{\omega})_{xyz} + \Omega \times \omega = (-0.8\,\mathbf{i} + 0.6\mathbf{k}) + (0.25\mathbf{k}) \times (-0.4\,\mathbf{i} + 0.25\mathbf{k})$

$= \{-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}\}$ rad/s^2

$\mathbf{r}_A = 40\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k} = \{34.64\mathbf{j} + 20\mathbf{k}\}$ ft

$\mathbf{v}_A = \omega \times \mathbf{r}_A = (1 - 0.4\,\mathbf{i} + 0.25\,\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$

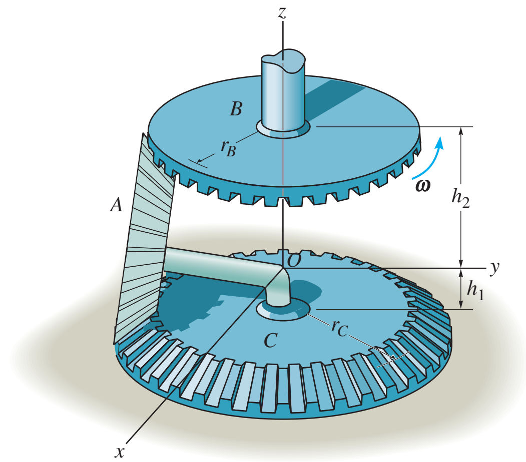
$\mathbf{v}_A = \{-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k}\}$ ft/s **Ans.**

$\mathbf{a}_A = \alpha \cdot \mathbf{r}_A + \omega \times \mathbf{v}_A = (-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k}) + (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k})$

$\mathbf{a}_A = \{-24.8\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}$ ft/s^2 **Ans.**

***20–16.**

If the top gear B rotates at a constant rate of ω, determine the angular velocity of gear A, which is free to rotate about the shaft and rolls on the bottom fixed gear C.



SOLUTION

$$\mathbf{v}_P = \omega\mathbf{k} \times (-r_B\,\mathbf{j}) = \omega r_B\,\mathbf{i}$$

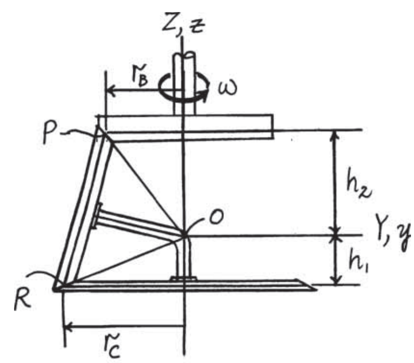
Also,

$$\mathbf{v}_P = \boldsymbol{\omega}_A \times (-r_B\,\mathbf{j} + h_2\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_{Ax} & \omega_{Ay} & \omega_{Az} \\ 0 & -r_B & h_2 \end{vmatrix}$$

$$= (\omega_{Ay}\,h_2 + \omega_{Az}\,r_B)\mathbf{i} - (\omega_{Ax}\,h_2)\mathbf{j} - \omega_{Ax}\,r_B\,\mathbf{k}$$

Thus,

$$\omega r_B = \omega_{Ay}\,h_2 + \omega_{Az}\,r_B \qquad \textbf{(1)}$$



$$0 = \omega_{Ax}\,h_2$$

$$0 = \omega_{Ax}\,r_B$$

$$\omega_{Ax} = 0$$

$$\mathbf{v}_R = \mathbf{0} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega_{Ay} & \omega_{Az} \\ 0 & -r_C & -h_1 \end{vmatrix} = (-\omega_{Ay}\,h_1 + \omega_{Az}\,r_C)\mathbf{i}$$

$$\omega_{Ay} = \omega_{Az}\left(\frac{r_C}{h_1}\right)$$

From Eq. (1)

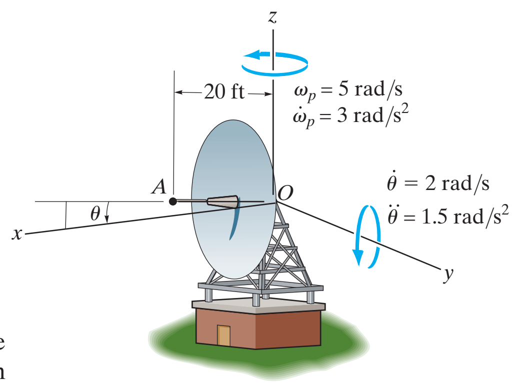
$$\omega r_B = \omega_{Az}\left[\left(\frac{r_C\,h_2}{h_1}\right) + r_B\right]$$

$$\omega_{Az} = \frac{r_B\,h_1\,\omega}{r_C\,h_2 + r_B\,h_1}; \qquad \omega_{Ay} = \left(\frac{r_C}{h_1}\right)\left(\frac{r_B\,h_1\omega}{r_C\,h_2 + r_B\,h_1}\right)$$

$$\boldsymbol{\omega}_A = \left(\frac{r_C}{h_1}\right)\left(\frac{r_B\,h_1\,\omega}{r_C\,h_2 + r_B\,h_1}\right)\mathbf{j} + \left(\frac{r_B\,h_1\omega}{r_C\,h_2 + r_B\,h_1}\right)\mathbf{k} \qquad \textbf{Ans.}$$

20–17.

When $\theta = 0°$, the radar disk rotates about the y axis with an angular velocity of $\dot{\theta} = 2$ rad/s, increasing at a constant rate of $\ddot{\theta} = 1.5$ rad/s^2. Simultaneously, the disk also precesses about the z axis with an angular velocity of $\omega_p = 5$ rad/s, increasing at a constant rate of $\dot{\omega}_P = 3$ rad/s^2. Determine the velocity and acceleration of the receiver A at this instant.



SOLUTION

The XYZ fixed reference frame is set to coincide with the rotating xyz reference frame at the instant considered. Thus, the angular velocity of the radar disk at this instant can be obtained by vector addition of $\dot{\theta}$ and ω_p.

$$\omega = \dot{\theta} + \omega_p = [2\mathbf{j} + 5\mathbf{k}] \text{ rad/s}$$

The angular acceleration of the disk is determined from

$$\dot{\omega} = \ddot{\theta} + \dot{\omega}_p$$

If we set the xyz frame to have an angular velocity of $\Omega = \omega_p = [5\mathbf{k}]$ rad/s, the direction of $\dot{\theta}$ will remain unchanged with respect to the xyz rotating frame which is along the y axis. Thus,

$$\ddot{\theta} = \left(\ddot{\theta}\right)_{xyz} + \omega_p \times \dot{\theta} = (1.5\mathbf{j}) + (5\mathbf{k} \times 2\mathbf{j}) = [-10\mathbf{i} + 1.5\mathbf{j}] \text{ rad/s}^2$$

Since ω_p is always directed along the Z axis where $\Omega = \omega_p$, then

$$\dot{\omega}_p = \left(\dot{\omega}_p\right)_{xyz} + \omega_p \times \omega_p = 3\mathbf{k} + \mathbf{0} = [3\mathbf{k}] \text{ rad/s}^2$$

Thus,

$$\alpha = -10\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k} = [-10\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}] \text{ rad/s}^2$$

When, $\theta = 0°$, $\mathbf{r}_A = 20\mathbf{i}$. Thus,

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (2\mathbf{j} + 5\mathbf{k}) \times (20\mathbf{i}) = [100\mathbf{j} - 40\mathbf{k}] \text{ ft/s} \qquad \textbf{Ans.}$$

and

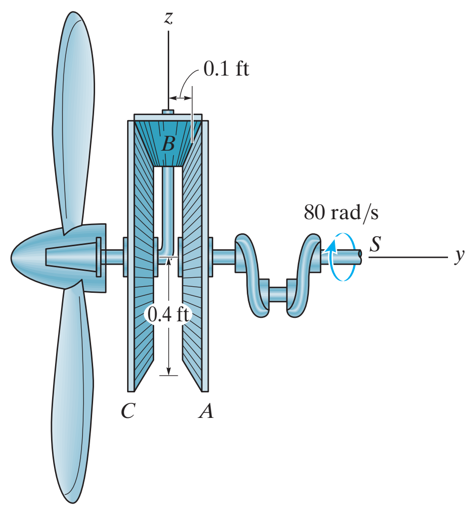
$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

$$= (-10\mathbf{i} + 1.5\mathbf{j} + 3\mathbf{k}) \times (20\mathbf{i}) + (2\mathbf{j} + 5\mathbf{k}) \times [(2\mathbf{j} + 5\mathbf{k}) \times (20\mathbf{i})]$$

$$= [-580\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}] \text{ ft/s}^2 \qquad \textbf{Ans.}$$

20–18.

Gear A is fixed to the crankshaft S, while gear C is fixed. Gear B and the propeller are free to rotate. The crankshaft is turning at 80 rad/s about its axis. Determine the magnitudes of the angular velocity of the propeller and the angular acceleration of gear B.

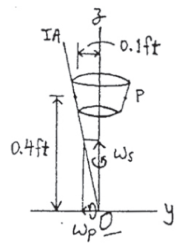


SOLUTION

Point P on gear B has a speed of

$v_P = 80(0.4) = 32$ ft/s

The IA is located along the points of contant of B and C



$$\frac{\omega_P}{0.1} = \frac{\omega_s}{0.4}$$

$$\omega_s = 4\omega_P$$

$$\omega = -\omega_P\,\mathbf{j} + \omega_s\,\mathbf{k}$$

$$= -\omega_P\mathbf{j} + 4\omega_P\mathbf{k}$$

$$\mathbf{r}_{P/O} = 0.1\mathbf{j} = 0.4\mathbf{k}$$

$$\mathbf{v}_P = -32\mathbf{i}$$

$$\mathbf{v}_P = \omega \times \mathbf{r}_{P/O}$$

$$-32\mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -\omega_P & 4\omega_P \\ 0 & 0.1 & 0.4 \end{vmatrix}$$

$$-32\mathbf{i} = -0.8\omega_P\mathbf{i}$$

$$\omega_P = 40 \text{ rad/s}$$

$\omega_P = \{-40\mathbf{j}\}$ rad/s **Ans.**

$$\omega_s = 4(40)\,\mathbf{k} = \{160\mathbf{k}\} \text{ rad/s}$$

Thus,

$$\omega = \omega_P + \omega_s$$

Let the x,y,z axes have an angular velocity of $\Omega \times \omega_P$, then

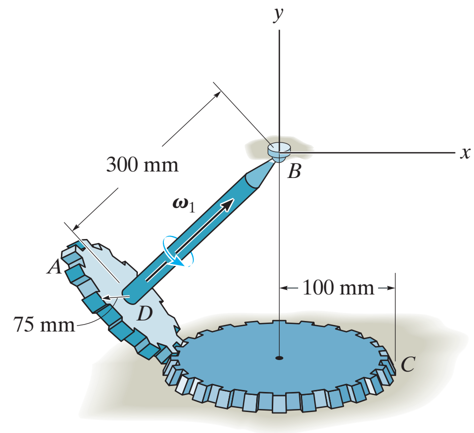
$$\alpha = \dot{\omega} = \dot{\omega}_P + \dot{\omega}_s = \mathbf{0} + \omega_P \times (\omega_s + \omega_P)$$

$$\alpha = (-40\mathbf{j}) \times (160\mathbf{k} - 40\mathbf{j})$$

$\alpha = \{-6400\mathbf{i}\}$ rad/s^2 **Ans.**

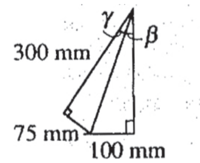
20–19.

Shaft BD is connected to a ball-and-socket joint at B, and a beveled gear A is attached to its other end. The gear is in mesh with a fixed gear C. If the shaft and gear A are *spinning* with a constant angular velocity $\omega_1 = 8\text{ rad/s}$, determine the angular velocity and angular acceleration of gear A.



SOLUTION

$$\gamma = \tan^{-1}\frac{75}{300} = 14.04^\circ \qquad \beta = \sin^{-1}\frac{100}{\sqrt{300^2 + 75^2}} = 18.87^\circ$$



The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity IA.

$$\frac{\omega}{\sin 147.09^\circ} = \frac{8}{\sin 18.87^\circ} \qquad \omega = 13.44\text{ rad/s}$$

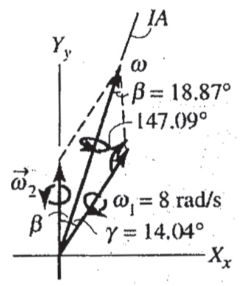
$$\omega = 13.44 \sin 18.87^\circ\,\mathbf{i} + 13.44 \cos 18.87^\circ\,\mathbf{j}$$

$$= \{4.35\mathbf{i} + 12.7\mathbf{j}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\frac{\omega_2}{\sin 14.04^\circ} = \frac{8}{\sin 18.87^\circ} \qquad \omega_2 = 6.00\text{ rad/s}$$

$$\omega_2 = \{6\mathbf{j}\}\text{ rad/s}$$

$$\omega_1 = 8 \sin 32.91^\circ\,\mathbf{i} + 8 \cos 32.91^\circ\,\mathbf{j} = \{4.3466\mathbf{i} + 6.7162\mathbf{j}\}\text{ rad/s}$$



For ω_1, $\Omega = \omega_2 = \{6\mathbf{j}\}\text{ rad/s}$

$$(\omega_1)_{xyz} = (\omega_1)_{xyz} + \Omega \times \omega_1$$

$$= \mathbf{0} + (6\mathbf{j}) \times (4.3466\mathbf{i} + 6.7162\mathbf{j})$$

$$= \{-26.08\mathbf{k}\}\text{ rad/s}^2$$

For ω_2, $\Omega = \mathbf{0}$.

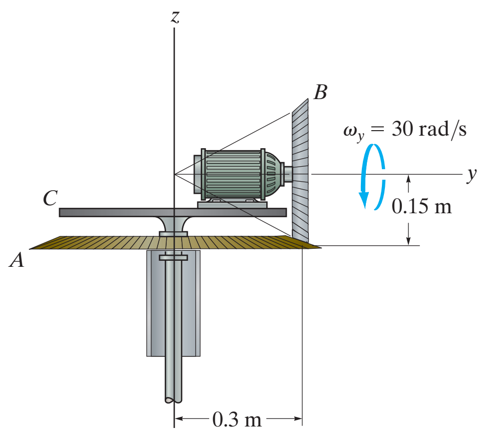
$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2 = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = \mathbf{0} + (-26.08\mathbf{k}) = \{-26.1\mathbf{k}\}\text{ rad/s}^2 \qquad \textbf{Ans.}$$

***20–20.**

Gear B is driven by a motor mounted on turntable C. If gear A is held fixed, and the motor shaft rotates with a constant angular velocity of $\omega_y = 30\text{ rad/s}$, determine the angular velocity and angular acceleration of gear B.



SOLUTION

The angular velocity ω of gear B is directed along the instantaneous axis of zero velocity, which is along the line where gears A and B mesh since gear A is held fixed. From Fig. a, the vector addition gives

$$\omega = \omega_y + \omega_z$$

$$\frac{2}{\sqrt{5}}\omega\mathbf{j} - \frac{1}{\sqrt{5}}\omega\mathbf{k} = 30\mathbf{j} - \omega_z\mathbf{k}$$

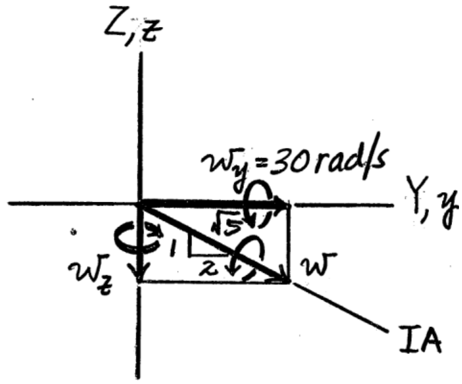
Equating the **j** and **k** components gives

$$\frac{2}{\sqrt{5}}\omega = 30 \qquad \omega = 15\sqrt{5}\text{ rad/s}$$

$$-\frac{1}{\sqrt{5}}\left(15\sqrt{5}\right) = -\omega_z \qquad \omega_z = 15\text{ rad/s}$$

Thus,

$$\omega = [30\mathbf{j} - 15\mathbf{k}]\text{ rad/s}$$ **Ans.**



(a)

Here, we will set the XYZ fixed reference frame to coincide with the xyz rotating frame at the instant considered. If the xyz frame rotates with an angular velocity of $\Omega = \omega_z = [-15\mathbf{k}]\text{ rad/s}$, then ω_y will always be directed along the y axis with respect to the xyz frame. Thus,

$$\dot{\omega}_y = \left(\dot{\omega}_y\right)_{xyz} + \omega_z \times \omega_y = 0 + (-15\mathbf{k}) \times (30\mathbf{j}) = [450\mathbf{i}]\text{ rad/s}^2$$

When $\Omega = \omega_z$, ω_z is always directed along the z axis. Therefore,

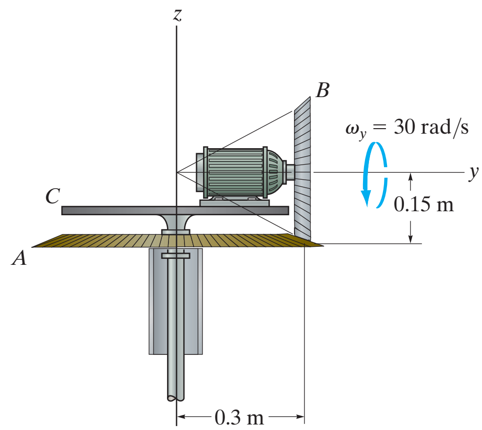
$$\dot{\omega}_z = \left(\dot{\omega}_z\right)_{xyz} + \omega_z \times \omega_z = 0 + 0 = 0$$

Thus,

$$\alpha = \dot{\omega}_y + \dot{\omega}_z = (450\mathbf{i}) + 0 = [450\mathbf{i}]\text{ rad/s}^2$$ **Ans.**

20–21.

Gear B is driven by a motor mounted on turntable C. If gear A and the motor shaft rotate with constant angular speeds of $\omega_A = \{10\mathbf{k}\}$ rad/s and $\omega_y = \{30\mathbf{j}\}$ rad/s, respectively, determine the angular velocity and angular acceleration of gear B.



SOLUTION

If the angular velocity of the turn-table is ω_z, then the angular velocity of gear B is

$$\omega = \omega_y + \omega_z = [30\mathbf{j} + \omega_z\mathbf{k}] \text{ rad/s}$$

Since gear A rotates about the fixed axis (z axis), the velocity of the contact point **P** between gears A and B is

$$\mathbf{v}_p = \omega_A \times r_A = (10\mathbf{k}) \times (0.3\mathbf{j}) = [-3\mathbf{i}] \text{ m/s}$$

Since gear B rotates about a fixed point O, the origin of the xyz frame, then $\mathbf{r}_{OP} = [0.3\mathbf{j} - 0.15\mathbf{k}]$ m.

$$\mathbf{v}_p = \omega \times \mathbf{r}_{OP}$$

$$-3\mathbf{i} = (30\mathbf{j} + \omega_z\mathbf{k}) \times (0.3\mathbf{j} - 0.15\mathbf{k})$$

$$-3\mathbf{i} = -(4.5 + 0.3\omega_z)\mathbf{i}$$

Thus,

$$-3 = -(4.5 + 0.3\omega_z)$$

$$\omega_z = -5 \text{ rad/s}$$

Then,

$$\omega = [30\mathbf{j} - 5\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

Here, we will set the XYZ fixed reference frame to conincide with the xyz rotating frame at the instant considered. If the xyz frame rotates with an angular velocity of $\Omega = \omega_z = [-5\mathbf{k}]$ rad/s, then ω_y will always be directed along the y axis with respect to the xyz frame. Thus,

$$\dot{\omega}_y = \left(\dot{\omega}_y\right)_{xyz} + \omega_z \times \omega_y = \mathbf{0} + (-5\mathbf{k}) \times (30\mathbf{j}) = [150\mathbf{i}] \text{ rad/s}^2$$

When $\Omega = \omega_z, \omega_z$ is always directed along the z axis. Therefore,

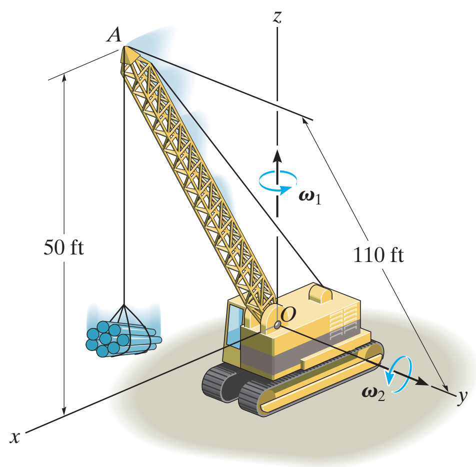
$$\dot{\omega}_z = \left(\dot{\omega}_z\right)_{xyz} + \omega_z \times \omega_z = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

Thus,

$$\alpha = \dot{\omega}_y + \dot{\omega}_z = (150\mathbf{i} + 0) = [150\mathbf{i}] \text{ rad/s}^2 \qquad \textbf{Ans.}$$

20–22.

The crane boom OA rotates about the z axis with a constant angular velocity of $\omega_1 = 0.15$ rad/s, while it is rotating downward with a constant angular velocity $\omega_2 = 0.2$ rad/s. Determine the velocity and acceleration of point A located at the end of the boom at the instant shown.



SOLUTION

$$\omega = \omega_1 + \omega_2 = \{0.2\mathbf{j} + 0.15\mathbf{k}\}\ \text{rad/s}$$

Let the x, y, z axes rotate at $\Omega = \omega_1$, then

$$\dot{\omega} = (\dot{\omega})_{xyz} + \omega_1 \times \omega_2$$

$$\dot{\omega} = \mathbf{0} + 0.15\mathbf{k} \times 0.2\mathbf{j} = \{-0.03\mathbf{i}\}\ \text{rad/s}^2$$

$$\mathbf{r}_A = \left[\sqrt{(110)^2 - (50)^2}\right]\mathbf{i} + 50\mathbf{k} = \{97.98\mathbf{i} + 50\mathbf{k}\}\ \text{ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 97.98 & 0 & 50 \end{vmatrix}$$

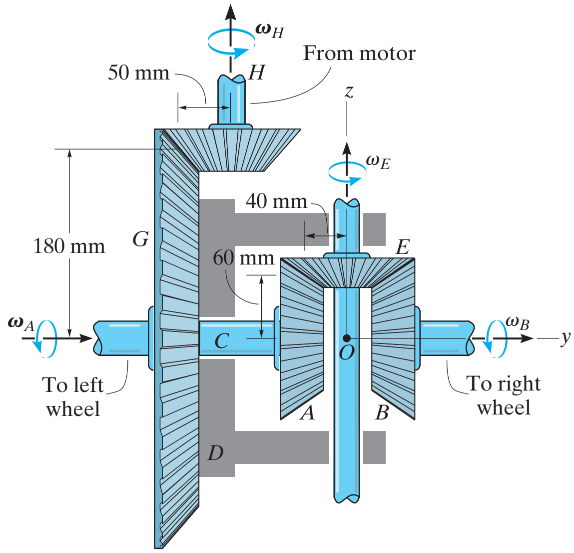
$$\mathbf{v}_A = \{10\mathbf{i} + 14.7\mathbf{j} - 19.6\mathbf{k}\}\ \text{ft/s}$$ **Ans.**

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.03 & 0 & 0 \\ 97.98 & 0 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 10 & 14.7 & -19.6 \end{vmatrix}$$

$$\mathbf{a}_A = \{-6.12\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\ \text{ft/s}^2$$ **Ans.**

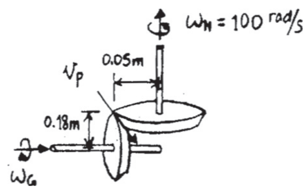
20–23.

The differential of an automobile allows the two rear wheels to rotate at different speeds when the automobile travels along a curve. For operation, the rear axles are attached to the wheels at one end and have beveled gears A and B on their other ends. The differential case D is placed over the left axle but can rotate about C independent of the axle. The case supports a pinion gear E on a shaft, which meshes with gears A and B. Finally, a ring gear G is *fixed* to the differential case so that the case rotates with the ring gear when the latter is driven by the drive pinion H. This gear, like the differential case, is free to rotate about the left wheel axle. If the drive pinion is turning at $\omega_H = 100$ rad/s and the pinion gear E is spinning about its shaft at $\omega_E = 30$ rad/s, determine the angular velocity, ω_A and ω_B, of each axle.



SOLUTION

$$v_P = \omega_H r_H = 100(50) = 5000 \text{ mm/s}$$



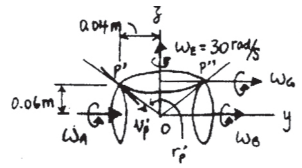
$$\omega_G = \frac{5000}{180} = 27.78 \text{ rad/s}$$

Point O is a fixed point of rotation for gears A, E, and B.

$$\Omega = \omega_G + \omega_E = \{27.78\mathbf{j} + 30\mathbf{k}\} \text{ rad/s}$$

$$\mathbf{v}_{P'} = \Omega \times \mathbf{r}_{P'} = (27.78\mathbf{j} + 30\mathbf{k}) \times (-40\mathbf{j} + 60\mathbf{k}) = \{2866.7\mathbf{i}\} \text{ mm/s}$$

$$\omega_A = \frac{2866.7}{60} = 47.8 \text{ rad/s}$$ **Ans.**

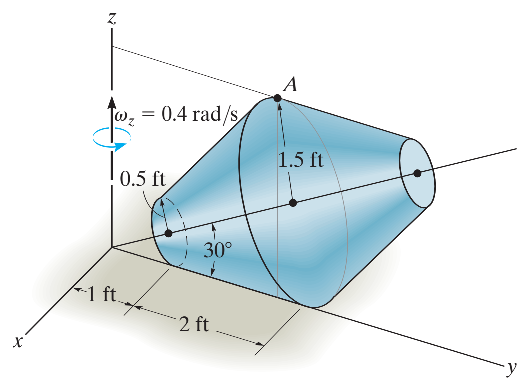


$$\mathbf{v}_{P''} = \Omega \times \mathbf{r}_{P''} = (27.78\mathbf{j} + 30\mathbf{k}) \times (40\mathbf{j} + 60\mathbf{k}) = \{466.7\mathbf{i}\} \text{ mm/s}$$

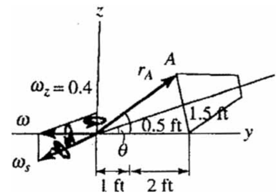
$$\omega_B = \frac{466.7}{60} = 7.78 \text{ rad/s}$$ **Ans.**

***20–24.**

The truncated double cone rotates about the z axis at a constant rate $\omega_z = 0.4$ rad/s without slipping on the horizontal plane. Determine the velocity and acceleration of point A on the cone.



SOLUTION



$$\theta = \sin^{-1}\left(\frac{0.5}{1}\right) = 30^\circ$$

$$\omega_s = \frac{0.4}{\sin 30^\circ} = 0.8 \text{ rad/s}$$

$$\omega = 0.8 \cos 30^\circ = 0.6928 \text{ rad/s}$$

$$\omega = \{-0.6928\mathbf{j}\} \text{ rad/s}$$

$$\Omega = 0.4\mathbf{k}$$

$$\dot{\omega} = (\dot{\omega})_{xyz} + \Omega \times \omega \qquad \textbf{(1)}$$

$$= 0 + (0.4\mathbf{k}) \times (-0.6928\mathbf{j})$$

$$\dot{\omega} = \{0.2771\mathbf{i}\} \text{ rad/s}^2$$

$$\mathbf{r}_A = (3 - 3\sin 30^\circ)\mathbf{j} + 3\cos 30^\circ\,\mathbf{k}$$

$$= (1.5\mathbf{j} + 2.598\mathbf{k}) \text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$

$$= (-0.6928\mathbf{j}) \times (1.5\mathbf{j} + 2.598\mathbf{k})$$

$$\mathbf{v}_A = \{-1.80\mathbf{i}\} \text{ ft/s} \qquad \textbf{Ans.}$$

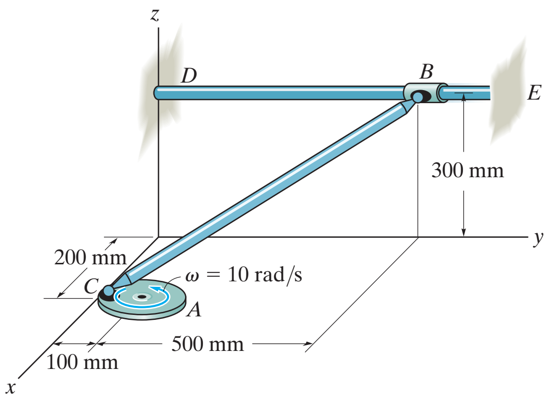
$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A$$

$$= (0.2771\mathbf{i}) \times (1.5\mathbf{j} + 2.598\mathbf{k}) + (-0.6928\mathbf{j}) \times (-1.80\mathbf{i})$$

$$\mathbf{a}_A = (-0.720\mathbf{j} - 0.831\mathbf{k}) \text{ ft/s}^2 \qquad \textbf{Ans.}$$

20–25.

Disk A rotates at a constant angular velocity of 10 rad/s. If rod BC is joined to the disk and a collar by ball-and-socket joints, determine the velocity of collar B at the instant shown. Also, what is the rod's angular velocity $\boldsymbol{\omega}_{BC}$ if it is directed perpendicular to the axis of the rod?



SOLUTION

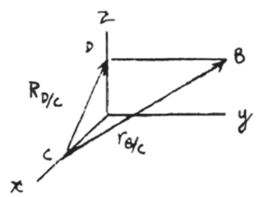
$\mathbf{v}_C = \{1\mathbf{i}\}$ m/s $\qquad \mathbf{v}_B = -v_B\mathbf{j} \qquad \omega_{BC} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$r_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$-\mathbf{v}_B = 1\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.2 & 0.6 & 0.3 \end{vmatrix}$$

Equating **i**, **j**, and **k** components



$$1 - 0.3\omega_y - 0.6\omega_z = 0 \qquad \textbf{(1)}$$

$$0.3\omega_x + 0.2\omega_z = v_B \qquad \textbf{(2)}$$

$$0.6\omega_x + 0.2\omega_y = 0 \qquad \textbf{(3)}$$

Since ω_{BC} is perpendicular to the axis of the rod,

$$\omega_{BC} \cdot \mathbf{r}_{B/C} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k})\cdot(-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}) = 0$$

$$-0.2\omega_x + 0.6\omega_y + 0.3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$\omega_x = 0.204$ rad/s $\qquad \omega_y = -0.612$ rad/s $\qquad \omega_z = 1.36$ rad/s $\qquad v_B = 0.333$ m/s

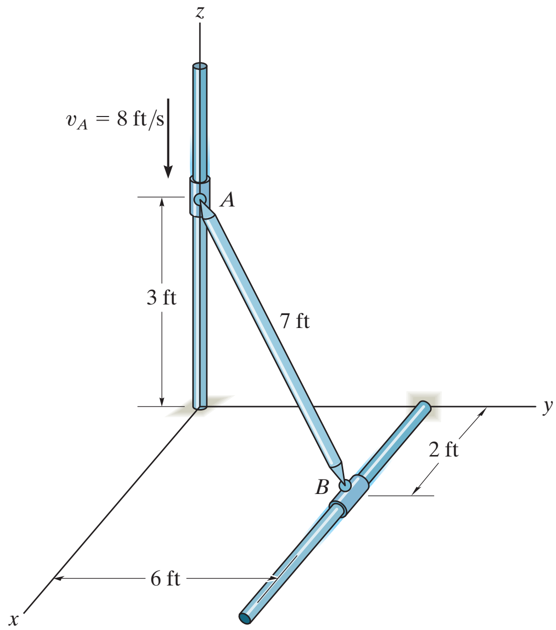
Then

$$\omega_{BC} = \{0.204\mathbf{i} - 0.612\mathbf{j} + 1.36\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-0.333\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

20–26.

If the rod is attached with ball-and-socket joints to smooth collars A and B at its end points, determine the speed of B at the instant shown if A is moving downward at a constant speed of $v_A = 8$ ft/s. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.



SOLUTION

$\mathbf{v}_A = \{-8\mathbf{k}\}$ ft/s

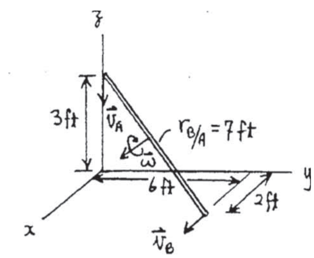
$\mathbf{v}_B = v_B\mathbf{i}$

$\mathbf{r}_{B/A} = \{2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}$ ft

$\omega = \{\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}\}$ rad/s

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$$v_B\mathbf{i} = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 2 & 6 & -3 \end{vmatrix}$$



Expanding and equating components yields:

$$v_B = -3\omega_y - 6\omega_z \quad \textbf{(1)}$$

$$0 = 3\omega_x + 2\omega_z \quad \textbf{(2)}$$

$$0 = -8 + 6\omega_x - 2\omega_y \quad \textbf{(3)}$$

Also, $\omega \cdot \mathbf{r}_{B/A} = 0$

$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$

$$2\omega_x + 6\omega_y - 3\omega_z = 0 \quad \textbf{(4)}$$

Solving Eqs. (1)–(4) yields

$\omega_x = 0.9796$ rad/s

$\omega_y = -1.061$ rad/s

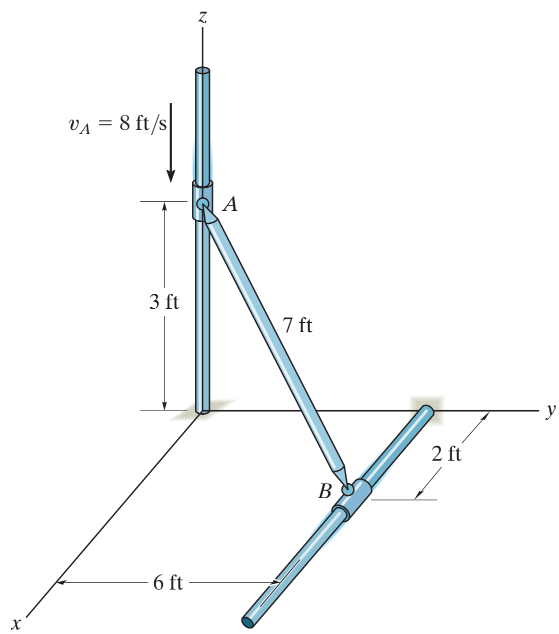
$\omega_z = -1.469$ rad/s

$\omega = \{0.980\mathbf{i} - 1.06\mathbf{j} - 1.47\mathbf{k}\}$ rad/s **Ans.**

$v_B = 12.0$ ft/s **Ans.**

20–27.

If the collar at A is moving downward with an acceleration $\mathbf{a}_A = \{-5\mathbf{k}\}\ \text{ft/s}^2$, at the instant its speed is $v_A = 8\ \text{ft/s}$, determine the acceleration of the collar at B at this instant.



SOLUTION

$\mathbf{a}_B = a_B\,\mathbf{i}, \qquad \mathbf{a}_A = -5\mathbf{k}$

From Prob. 20–26,

$$\omega_{AB} = 0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \omega_{AB} \times \mathbf{v}_{B/A} + \alpha_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A = 12\mathbf{i} + 8\mathbf{k}$$

$$\mathbf{a}_{B/A} = \{0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}\} \times (12\mathbf{i} + 8\mathbf{k}) + \{\alpha_x\,\mathbf{i} + \alpha_x\mathbf{j} + \alpha_z\,\mathbf{k}\} \times (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$$

$$a_B\,\mathbf{i} = -5\mathbf{k} + \{0.9796\mathbf{i} - 1.0612\mathbf{j} - 1.4694\mathbf{k}\} \times (12\mathbf{i} + 8\mathbf{k})$$

$$+ \{(-3\alpha_y - 6\alpha_z)\mathbf{i} + (2\alpha_z + 3\alpha_x)\mathbf{j} + (6\alpha_x - 2\alpha_y)\mathbf{k}\,\}$$

$$3\alpha_y + 6\alpha_z + a_B = -8.4897$$

$$-3\alpha_x - 2\alpha_z = -25.4696$$

$$-6\alpha_x + 2\alpha_y = 7.7344$$

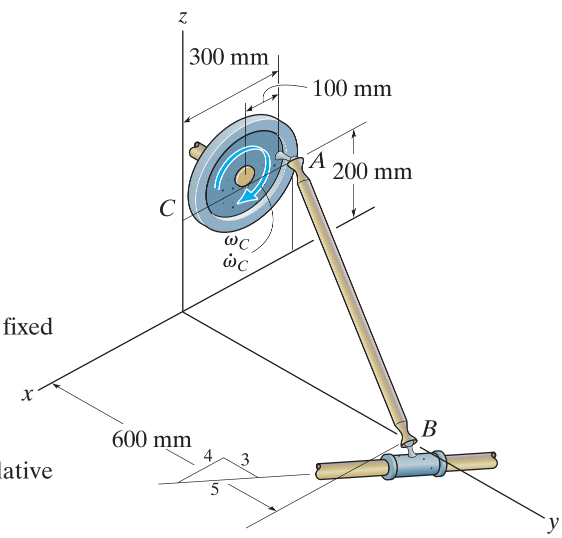
Solving these equations

$$\mathbf{a}_B = -96.5$$

$$\mathbf{a}_B = \{-96.5\mathbf{i}\}\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

***20–28.**

If wheel C rotates with a constant angular velocity of $\omega_C = 10\ \text{rad/s}$, determine the velocity of the collar at B when rod AB is in the position shown.



SOLUTION

Here, $\mathbf{r}_{CA} = [-0.1\mathbf{i}]$ m and $\omega_C = [-10\mathbf{j}]$ rad/s. Since wheel C rotates about a fixed axis, then

$$\mathbf{v}_A = \omega_C \times \mathbf{r}_{CA} = (-10\mathbf{j}) \times (-0.1\mathbf{i}) = [-1\mathbf{k}]\ \text{m/s}$$

Since rod AB undergoes general motion, $\mathbf{v}_A$ and $\mathbf{v}_B$ can be related using the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

Assume $\mathbf{v}_B = -\frac{4}{5}v_B\mathbf{i} + \frac{3}{5}v_B\mathbf{j}$ and $\omega_{AB} = \left[(\omega_{AB})_x\,\mathbf{i} + (\omega_{AB})_y\,\mathbf{j} + (\omega_{AB})_z\,\mathbf{k}\right]$. Also, $\mathbf{r}_{B/A} = [0.3\mathbf{i} + 0.6\mathbf{j} - 0.2\mathbf{k}]$ m. Thus,

$$-\frac{4}{5}v_B\mathbf{i} + \frac{3}{5}v_B\mathbf{j} = (-1\mathbf{k}) + \left[(\omega_{AB})_x\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\mathbf{k}\right] \times (0.3\mathbf{i} + 0.6\mathbf{j} - 0.2\mathbf{k})$$

$$-\frac{4}{5}v_B\mathbf{i} + \frac{3}{5}v_B\mathbf{j} = \left[-0.2(\omega_{AB})_y - 0.6(\omega_{AB})_z\right]\mathbf{i} + \left[0.2(\omega_{AB})_x + 0.3(\omega_{AB})_z\right]\mathbf{j} + \left[0.6(\omega_{AB})_x - 0.3(\omega_{AB})_y - 1\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$-\frac{4}{5}v_B = -0.2(\omega_{AB})_y - 0.6(\omega_{AB})_z \qquad \textbf{(1)}$$

$$\frac{3}{5}v_B = 0.2(\omega_{AB})_x + 0.3(\omega_{AB})_z \qquad \textbf{(2)}$$

$$0 = 0.6(\omega_{AB})_x - 0.3(\omega_{AB})_y - 1 \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\omega_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left[(\omega_{AB})_x\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\mathbf{k}\right] \cdot (0.3\mathbf{i} + 0.6\mathbf{j} - 0.2\mathbf{k}) = 0$$

$$0.3(\omega_{AB})_x + 0.6(\omega_{AB})_y - 0.2(\omega_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

$$(\omega_{AB})_x = 1.633\ \text{rad/s} \qquad (\omega_{AB})_y = -0.06803\ \text{rad/s} \qquad (\omega_{AB})_z = 2.245\ \text{rad/s}$$

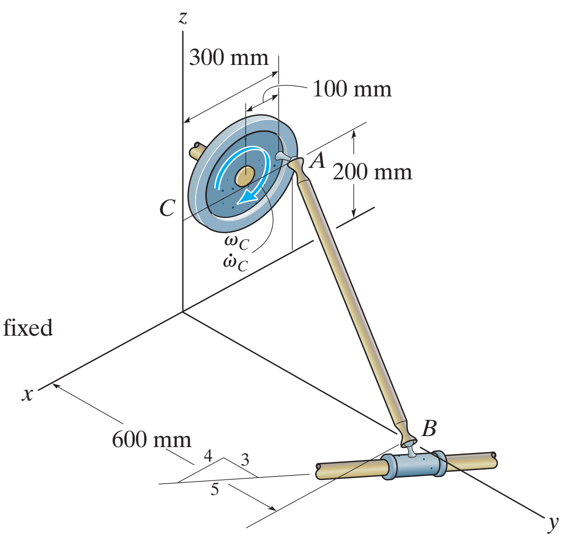
$$v_B = 1.667\ \text{m/s}$$

Then,

$$\mathbf{v}_B = -\frac{4}{5}(1.667)\mathbf{i} + \frac{3}{5}(1.667)\mathbf{j} = [-1.33\mathbf{i} + 1\mathbf{j}]\ \text{m/s} \qquad \textbf{Ans.}$$

20–29.

At the instant rod AB is in the position shown wheel C rotates with an angular velocity of $\omega_C = 10$ rad/s and has an angular acceleration of $\alpha_C = 1.5$ rad/s^2. Determine the acceleration of collar B at this instant.



SOLUTION

Here, $\mathbf{r}_{CA} = [-0.1\mathbf{i}]$ m and $\alpha_C = [-1.5\mathbf{j}]$ rad/s^2. Since wheel C rotates about a fixed axis, then

$$\mathbf{a}_A = \alpha_C \times \mathbf{r}_{CA} + \omega_C \times (\omega_C \times \mathbf{r}_{CA})$$
$$= (-1.5\mathbf{j}) \times (-0.1\mathbf{i}) + (-10\mathbf{j}) \times [(-10\mathbf{j}) \times (-0.1\mathbf{i})]$$
$$= [10\mathbf{i} - 0.15\mathbf{k}] \text{ m/s}^2$$

For general motion, $\mathbf{a}_A$ and $\mathbf{a}_B$ can be related using the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A})$$

Assume $\mathbf{a}_B = -\frac{4}{5}a_B\mathbf{i} + \frac{3}{5}a_B\mathbf{j}$ and $\alpha_{AB} = (\alpha_{AB})_x\mathbf{i} + (\alpha_{AB})_y\mathbf{j} + (\alpha_{AB})_z\mathbf{k}$. Also, $\mathbf{r}_{B/A} = [0.3\mathbf{i} + 0.6\mathbf{j} - 0.2\mathbf{k}]$ m. Using the result of Prob. 20–28, $\omega_{AB} = [1.633\mathbf{i} - 0.06803\mathbf{j} + 2.245\mathbf{k}]$ rad/s. Thus,

$$-\frac{4}{5}a_B\mathbf{i} + \frac{3}{5}a_B\mathbf{j} = (10\mathbf{i} - 0.15\mathbf{k}) + \left[(\alpha_{AB})_x\mathbf{i} + (\alpha_{AB})_y\mathbf{j} + (\alpha_{AB})_z\mathbf{k}\right] \times (0.3\mathbf{i} + 0.6\mathbf{j} - 0.2\mathbf{k})$$
$$+ (1.633\mathbf{i} - 0.06803\mathbf{j} + 2.245\mathbf{k}) \times [(1.633\mathbf{i} - 0.06803\mathbf{j} + 2.245\mathbf{k}) \times (0.3\mathbf{i} + 0.6\mathbf{j} - 0.2\mathbf{k})]$$

$$-\frac{4}{5}a_B\mathbf{i} + \frac{3}{5}a_B\mathbf{j} = \left[7.6871 - 0.2(\alpha_{AB})_y - 0.6(\alpha_{AB})_z\right]\mathbf{i} + \left[0.2(\alpha_{AB})_x + 0.3(\alpha_{AB})_z - 4.6259\right]\mathbf{j}$$
$$+ \left[0.6(\alpha_{AB})_x - 0.3(\alpha_{AB})_y + 1.3920\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$-\frac{4}{5}a_B = 7.6871 - 0.2(\alpha_{AB})_y - 0.6(\alpha_{AB})_z \qquad \textbf{(1)}$$

$$\frac{3}{5}a_B = 0.2(\alpha_{AB})_x + 0.3(\alpha_{AB})_z - 4.6259 \qquad \textbf{(2)}$$

$$0 = 0.6(\alpha_{AB})_x - 0.3(\alpha_{AB})_y + 1.3920 \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\alpha_{AB} \cdot \mathbf{r}_{B/A} = 0$$
$$\left[(\alpha_{AB})_x\mathbf{i} + (\alpha_{AB})_y\mathbf{j} + (\alpha_{AB})_z\mathbf{k}\right] \cdot (0.3\mathbf{i} + 0.6\mathbf{j} - 0.2\mathbf{k}) = 0$$
$$0.3(\alpha_{AB})_x + 0.6(\alpha_{AB})_y - 0.2(\alpha_{AB})_z = 0 \qquad \textbf{(4)}$$

20–29. continued

Solving Eqs. (1) through (4),

$$(\alpha_{AB})_x = -1.342 \text{ rad/s}^2 \qquad (\alpha_{AB})_y = 1.955 \text{ rad/s}^2 \qquad (\alpha_{AB})_z = 3.851 \text{ rad/s}^2$$

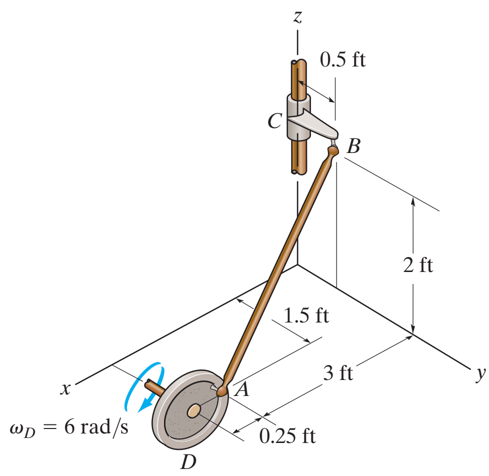
$$a_B = -6.231 \text{ m/s}^2$$

Then,

$$\mathbf{a}_B = -\frac{4}{5}(-6.231)\mathbf{i} + \frac{3}{5}(-6.231)\mathbf{j} = [4.99\mathbf{i} - 3.74\mathbf{j}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

20–30.

If wheel D rotates with an angular velocity of $\omega_D = 6$ rad/s, determine the angular velocity of the follower link BC at the instant shown. The link rotates about the z axis at $z = 2$ ft.



SOLUTION

Here, $\mathbf{r}_D = [-0.25\mathbf{i}]$ ft and $\omega_D = [-6\mathbf{j}]$ rad/s. Since wheel D rotates about a fixed axis, then

$$\mathbf{v}_A = \omega_D \times \mathbf{r}_D = (-6\mathbf{j}) \times (-0.25\mathbf{i}) = [-1.5\mathbf{k}] \text{ ft/s}$$

Also, link BC rotates about a fixed axis. Assume $\omega_{BC} = \omega_{BC}\,\mathbf{k}$ and $\mathbf{r}_{CB} = [0.5\mathbf{j}]$ ft. Thus,

$$\mathbf{v}_B = \omega_{BC} \times \mathbf{r}_{CB} = (\omega_{BC}\mathbf{k}) \times (0.5\mathbf{j}) = -0.5\omega_{BC}\,\mathbf{i}$$

Since rod AB undergoes general motion, $\mathbf{v}_A$ and $\mathbf{v}_B$ can be related using the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

Here, $\omega_{AB} = \left[(\omega_{AB})_x\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\mathbf{k}\right]$ and $\mathbf{r}_{B/A} = [-3\mathbf{i} - 1\mathbf{j} + 2\mathbf{k}]$ ft. Thus,

$$-0.5\omega_{BC}\mathbf{i} = (-1.5\mathbf{k}) + \left[(\omega_{AB})_x\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\mathbf{k}\right] \times (-3\mathbf{i} - 1\mathbf{j} + 2\mathbf{k})$$

$$-0.5\omega_{BC}\mathbf{i} = \left[2(\omega_{AB})_y + (\omega_{AB})_z\right]\mathbf{i} - \left[2(\omega_{AB})_x + 3(\omega_{AB})_z\right]\mathbf{j} + \left[3(\omega_{AB})_y - (\omega_{AB})_x - 1.5\right]\mathbf{k}$$

Equating the **i**, **j**, and **k** components

$$-0.5\omega_{BC} = 2(\omega_{AB})_y + (\omega_{AB})_z \qquad \textbf{(1)}$$

$$0 = -\left[2(\omega_{AB})_x + 3(\omega_{AB})_z\right] \qquad \textbf{(2)}$$

$$0 = 3(\omega_{AB})_y - (\omega_{AB})_x - 1.5 \qquad \textbf{(3)}$$

The fourth equation can be obtained from the dot product of

$$\omega_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left[(\omega_{AB})_x\mathbf{i} + (\omega_{AB})_y\mathbf{j} + (\omega_{AB})_z\mathbf{k}\right] \cdot (-3\mathbf{i} - 1\mathbf{j} + 2\mathbf{k}) = 0$$

$$-3(\omega_{AB})_x - (\omega_{AB})_y + 2(\omega_{AB})_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

$$(\omega_{AB})_x = -0.1071 \text{ rad/s} \qquad (\omega_{AB})_y = 0.4643 \text{ rad/s} \qquad (\omega_{AB})_z = 0.07143 \text{ rad/s}$$

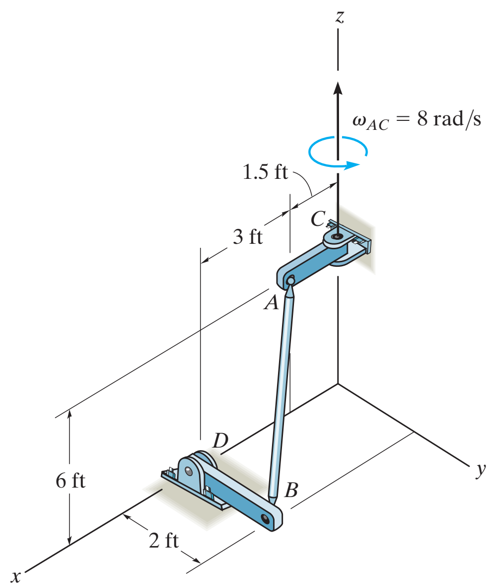
$$\omega_{BC} = -2 \text{ rad/s}$$

Then,

$$\omega_{BC} = [-2\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

20–31.

Rod AB is attached to the rotating arm using ball-and-socket joints. If AC is rotating with a constant angular velocity of 8 rad/s about the pin at C, determine the angular velocity of link BD at the instant shown.



SOLUTION

$\mathbf{v}_A = 8(1.5)\mathbf{j} = \{12\mathbf{j}\}$ ft/s

$\mathbf{v}_B = -v_B\mathbf{k}$

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$-v_B\mathbf{k} = 12\mathbf{j} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$

$$0 = -6\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$0 = 12 + 6\omega_x + 3\omega_z \qquad \textbf{(2)}$$

$-v_B = 2\omega_x - 3\omega_y$

Thus,

$$-v_B = 2\left(-\frac{12}{2} - \frac{3}{6}\omega_z\right) - 3\left(\frac{-\omega_z}{3}\right)$$

$-v_B = -4 - \omega_z + \omega_z$

$v_B = 4$ ft/s

$\omega_{BD} = -(\frac{4}{2})\mathbf{r}$

$\omega_{BD} = \{-2.00\mathbf{i}\}$ rad/s **Ans.**

Also, assuming ω is perpendicular to $\mathbf{r}_{B/A}$,

$$\omega \cdot \mathbf{r}_{B/A} = 0$$

$$3\omega_x + 2\omega_y - 6\omega_z = 0 \qquad \textbf{(3)}$$

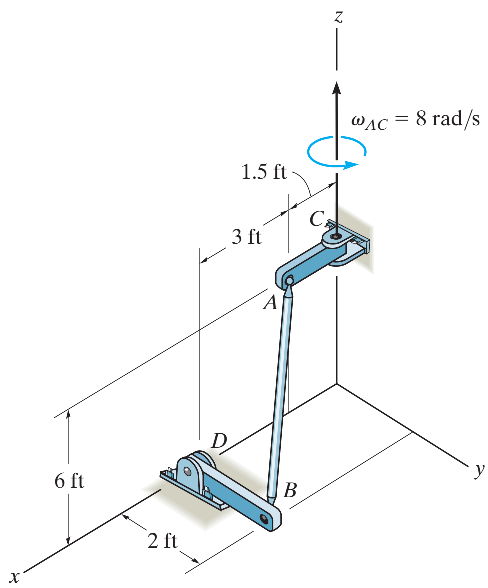
Solving Eqs. (1), (2), and (3),

$$\omega = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$= \{-1.633\mathbf{i} + 0.2449\mathbf{j} - 0.7347\mathbf{k}\} \text{ rad/s}$$

***20–32.**

Rod AB is attached to the rotating arm using ball and-socket joints. If AC is rotating about point C with an angular velocity of 8 rad/s and has an angular acceleration of 6 rad/s^2 at the instant shown, determine the angular velocity and angular acceleration of link BD at this instant.



SOLUTION

See Prob. 20–31.

$\omega_{BD} = \{-2.00\mathbf{i}\}$ rad/s **Ans.**

$\omega = \{-1.633\mathbf{i} + 0.2449\mathbf{j} - 0.7347\mathbf{k}\}$ rad/s

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$-4\mathbf{k} = 12\mathbf{j} + \mathbf{v}_{B/A}$

$\mathbf{v}_{B/A} = \{-12\mathbf{j} - 4\mathbf{k}\}$ ft/s

$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} + \omega \times (\mathbf{v}_{B/A})$

$$-(2)^2(2)\mathbf{j} + (a_B)_z\mathbf{k} = -96\mathbf{i} + 9\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 3 & 2 & -6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.633 & 0.2449 & -0.7347 \\ 0 & -12 & -4 \end{vmatrix}$$

$0 = -96 + \alpha_y(-6) - 2\alpha_z - 9.796$

$-8 = 9 + 6\alpha_x + 3\alpha_z - 6.5308$

$(a_B)_z = \alpha_x(2) - \alpha_y(3) + 19.592$

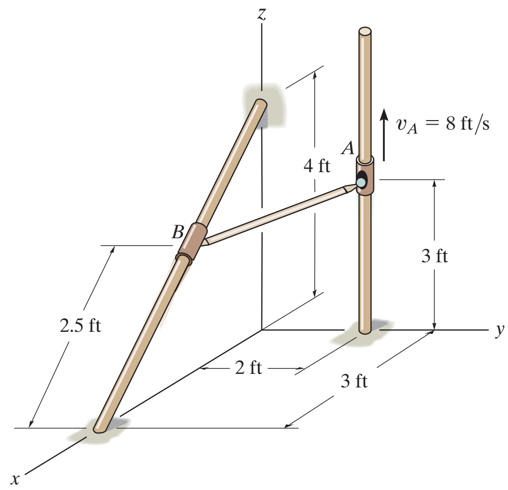
$(a_B)_z = 69.00$ ft/s^2

$$\alpha_{BD} = \frac{69.00}{2} = 34.5 \text{ rad/s}^2$$

$\alpha_{BD} = \{34.5\mathbf{i}\}$ rad/s^2 **Ans.**

20–33.

Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A moves upward with a velocity of $\mathbf{v}_A = \{8\mathbf{k}\}$ ft/s, determine the angular velocity of the rod and the speed of collar B at the instant shown. Assume that the rod's angular velocity is directed perpendicular to the rod.



SOLUTION

$$\mathbf{v}_A = \{8\mathbf{k}\}\text{ ft/s} \qquad \mathbf{v}_B = -\frac{3}{5}v_B\mathbf{i} + \frac{4}{5}v_B\mathbf{k} \qquad \omega_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\mathbf{r}_{B/A} = \{1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}\text{ ft}$$

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$-\frac{3}{5}v_B\,\mathbf{i} + \frac{4}{5}v_B\mathbf{k} = 8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1.5 & -2 & -1 \end{vmatrix}$$

Equating **i**, **j**, and **k**

$$-\omega_y + 2\omega_z = -\frac{3}{5}v_B \qquad \textbf{(1)}$$

$$\omega_x + 1.5\omega_z = 0 \qquad \textbf{(2)}$$

$$\omega - 2\omega_x - 1.5\omega_x = \frac{4}{5}v_B \qquad \textbf{(3)}$$

Since ω_{AB} is perpendicular to the axis of the rod,

$$\omega_{AB}\cdot\mathbf{r}_{B/A} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k})\cdot(1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}) = 0$$

$$1.5\omega_x - 2\omega_y - \omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs.(1) to (4) yields:

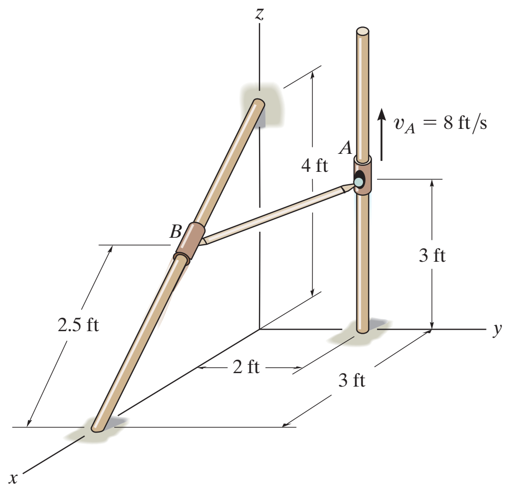
$$\omega_x = 1.1684\text{ rad/s} \qquad \omega_y = 1.2657\text{rad/s} \qquad \omega_z = -0.7789\text{ rad/s}$$

$$v_B = 4.71\text{ ft/s} \qquad \textbf{Ans.}$$

Then $\omega_{AB} = \{1.17\mathbf{i} + 1.27\mathbf{j} - 0.779\mathbf{k}\}$ rad/s **Ans.**

20–34.

Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A moves upward with an acceleration of $\mathbf{a}_A = \{4\mathbf{k}\}\,\text{ft/s}^2$, determine the angular acceleration of rod AB and the magnitude of acceleration of collar B. Assume that the rod's angular acceleration is directed perpendicular to the rod.



SOLUTION

From Prob. 20–33

$$\boldsymbol{\omega}_{AB} = \{1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k}\}\ \text{rad/s}$$

$$\mathbf{r}_{B/A} = \{1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}\ \text{ft}$$

$$\boldsymbol{\alpha}_{AB} = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}$$

$$\mathbf{a}_A = \{4\mathbf{k}\}\ \text{ft/s}^2 \qquad \mathbf{a}_B = -\frac{3}{5}a_B\,\mathbf{i} + \frac{4}{5}a_B\,\mathbf{k}$$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A})$$

$$-\frac{3}{5}a_B\,\mathbf{i} + \frac{4}{5}a_B\,\mathbf{k} = 4\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\,\mathbf{k}) \times (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k})$$

$$+(1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k})$$

$$\times\left[(1.1684\mathbf{i} + 1.2657\mathbf{j} - 0.7789\mathbf{k}) \times (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k})\right]$$

Equating **i**, **j**, and **k** components

$$-\alpha_y + 2\alpha_z - 5.3607 = -\frac{3}{5}a_B \qquad \textbf{(1)}$$

$$\alpha_x + 1.5\alpha_z + 7.1479 = 0 \qquad \textbf{(2)}$$

$$7.5737 - 2\alpha_x - 1.5\alpha_y = \frac{4}{5}a \qquad \textbf{(3)}$$

Since α_{AB} is perpendicular to the axis of the rod,

$$\boldsymbol{\alpha}_{AB} \cdot \mathbf{r}_{B/A} = (\alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k}) \cdot (1.5\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}) = 0$$

$$1.5\alpha_x - 2\alpha_y - \alpha_z = 0 \qquad \textbf{(4)}$$

Solving Eqs.(1) to (4) yields:

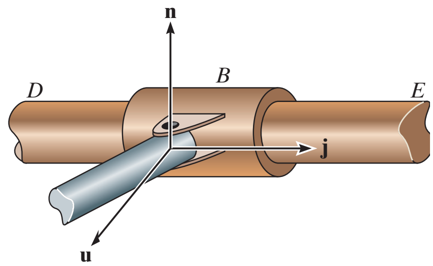
$$\alpha_x = -2.7794\ \text{rad/s}^2 \qquad \alpha_y = -0.6285\text{rad/s}^2 \qquad \alpha_z = -2.91213\ \text{rad/s}^2$$

$$a_B = 17.6\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Then $\boldsymbol{\alpha}_{AB} = \{-2.78\mathbf{i} - 0.628\mathbf{j} - 2.91\mathbf{k}\}\ \text{rad/s}^2$ **Ans.**

20–35.

Solve Prob. 20–25 if the connection at B consists of a pin as shown in the figure below, rather than a ball-and-socket joint. *Hint:* The constraint allows rotation of the rod both about bar DE (**j** direction) and about the axis of the pin (**n** direction). Since there is no rotational component in the **u** direction, i.e., perpendicular to **n** and **j** where $\mathbf{u} = \mathbf{j} \times \mathbf{n}$, an additional equation for solution can be obtained from $\boldsymbol{\omega} \cdot \mathbf{u} = 0$. The vector **n** is in the same direction as $\mathbf{r}_{B/C} \times \mathbf{r}_{D/C}$.



SOLUTION

$\mathbf{v}_C = \{1\mathbf{i}\}\ \text{m/s} \qquad \mathbf{v}_B = -v_B\mathbf{j} \qquad \omega_{BC} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$\mathbf{r}_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\ \text{m}$$

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$-v_B\mathbf{j} = 1\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.2 & 0.6 & 0.3 \end{vmatrix}$$

Equating **i**, **j**, and **k** components

$$1 + 0.3\omega_x - 0.6\omega_z = 0 \qquad \textbf{(1)}$$

$$0.3\omega_x + 0.2\omega_z = v_B \qquad \textbf{(2)}$$

$$0.6\omega_x + 0.2\omega_z = 0 \qquad \textbf{(3)}$$

Also,

$$\mathbf{r}_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\ \text{m}$$

$$\mathbf{r}_{D/C} = \{-0.2\mathbf{i} + 0.3\mathbf{k}\}\ \text{m}$$

$$\mathbf{r}_{B/C} \times \mathbf{r}_{D/C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.2 & 0.6 & 0.3 \\ -0.2 & 0 & 0.3 \end{vmatrix} = \{0.18\mathbf{i} + 0.12\mathbf{k}\}\ \text{m}^2$$

$$\mathbf{n} = \frac{0.18\mathbf{i} + 0.12\mathbf{k}}{\sqrt{0.18^2 + 0.12^2}} = 0.8321\mathbf{i} + 0.5547\mathbf{k}$$

$$\mathbf{u} = \mathbf{j} \times \mathbf{n} = \mathbf{j} \times (0.8321\mathbf{i} + 0.5547\mathbf{k}) = 0.5547\mathbf{i} - 0.8321\mathbf{k}$$

$$\omega_{BC} \cdot \mathbf{u} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k}) \cdot (0.5547\mathbf{i} - 0.8321\mathbf{k}) = 0$$

$$0.5547\omega_x - 0.8321\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$\omega_x = 0.769\ \text{rad/s} \qquad \omega_y = -2.31\ \text{rad/s} \qquad \omega_z = 0.513\ \text{rad/s} \qquad v_B = 0.333\ \text{m/s}$

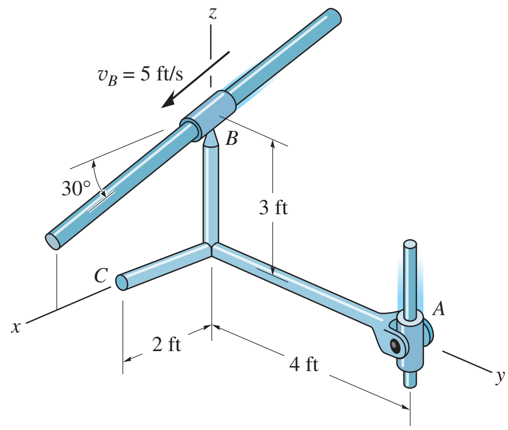
Then

$$\omega_{BC} = \{0.769\mathbf{i} - 2.31\mathbf{j} + 0.513\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-0.333\mathbf{j}\}\ \text{m/s} \qquad \textbf{Ans.}$$

***20–36.**

The rod assembly is supported at B by a ball-and-socket joint and at A by a clevis. If the collar at B moves in the x–z plane with a speed $v_B = 5$ ft/s, determine the velocity of points A and C on the rod assembly at the instant shown. *Hint:* See Prob. 20–35.



SOLUTION

$\mathbf{v}_B = \{5\cos 30°\mathbf{i} - 5\sin 30°\mathbf{k}\}$ ft/s $\qquad \mathbf{v}_A = -v_A\mathbf{k} \qquad \boldsymbol{\omega}_{ABC} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$\mathbf{r}_{A/B} = \{4\mathbf{j} - 3\mathbf{k}\}$ ft $\qquad \mathbf{r}_{C/B} = \{2\mathbf{i} - 3\mathbf{k}\}$ ft

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{ABC} \times \mathbf{r}_{A/B}$$

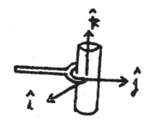
$$-v_A\mathbf{k} = 5\cos 30°\mathbf{i} - 5\sin 30°\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & 4 & -3 \end{vmatrix}$$

Equating **i**, **j**, and **k** components

$$5\cos 30° - 3\omega_y - 4\omega_z = 0 \qquad \textbf{(1)}$$

$$3\omega_x = 0 \qquad \textbf{(2)}$$

$$4\omega_x - 5\sin 30° = -v_A \qquad \textbf{(3)}$$



Also, since there is no rotation about the y axis

$$\boldsymbol{\omega}_{ABC} \cdot \mathbf{j} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (\mathbf{j}) = 0$$

$$\omega_y = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$\omega_x = \omega_y = 0 \qquad \omega_z = 1.083$ rad/s $\qquad v_A = 2.5$ ft/s$\downarrow$

Then $\qquad \boldsymbol{\omega}_{ABC} = \{1.083\mathbf{k}\}$ rad/s

$$\mathbf{v}_A = \{-2.50\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{ABC} \times \mathbf{r}_{C/B}$$

$$= 5\cos 30°\mathbf{i} - 5\sin 30°\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1.083 \\ 2 & 0 & -3 \end{vmatrix}$$

$$= \{4.33\mathbf{i} + 2.17\mathbf{j} - 2.50\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans.}$$

20–37.

Solve Example 20–5 such that the x, y, z axes move with curvilinear translation, $\Omega = \mathbf{0}$ in which case the collar appears to have both an angular velocity $\Omega_{xyz} = \omega_1 + \omega_2$ and radial motion.

SOLUTION

Relative to XYZ, let xyz have

$$\Omega = 0 \qquad \dot{\Omega} = 0$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\}\ \text{m}$$

$$\mathbf{v}_B = \{2\mathbf{j}\}\ \text{m/s}$$

$$\mathbf{a}_B = \{0.75\mathbf{j} + 8\mathbf{k}\}\ \text{m/s}^2$$

Relative to xyz, let $x'\ y'\ z'$ be coincident with xyz and be fixed to BD. Then

$$\Omega_{xyz} = \omega_1 + \omega_2 = \{4\mathbf{i} + 5\mathbf{k}\}\ \text{rad/s} \qquad \dot{\omega}_{xyz} = \dot{\omega}_1 + \dot{\omega}_2 = \{1.5\mathbf{i} - 6\mathbf{k}\}\ \text{rad/s}^2$$

$$(\mathbf{r}_{C/B})_{xyz} = \{0.2\mathbf{j}\}\ \text{m}$$

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\mathbf{r}_{C/B})_{xyz}$$

$$= 3\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})$$

$$= \{-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k}\}\ \text{m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = (\ddot{\mathbf{r}}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{x'y'z'}\right]$$

$$+\left[(\dot{\omega}_1 + \dot{\omega}_2) \times (\mathbf{r}_{C/B})_{xyz}\right] + \left[(\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right]$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[2\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times 3\mathbf{j}\right] + \left[(1.5\mathbf{i} - 6\mathbf{k}) \times 0.2\mathbf{j}\right] + \left[(4\mathbf{i} + 5\mathbf{k}) \times (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})\right]$$

$$= \{-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k}\}\ \text{m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + 0 + (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})$$

$$= \{-1.00\mathbf{i} + 5.00\mathbf{j} + 0.800\mathbf{k}\}\ \text{m/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \mathbf{0} + \mathbf{0} + \mathbf{0} + (-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k})$$

$$= \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\ \text{m/s}^2$$ **Ans.**

20–38.

Solve Example 20–5 by fixing x, y, z axes to rod BD so that $\mathbf{\Omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$. In this case the collar appears only to move radially outward along BD; hence $\mathbf{\Omega}_{xyz} = \mathbf{0}$.

SOLUTION

Relative to XYZ, let $x'\ y'\ z'$ be concident with XYZ and have $\Omega' = \omega_1$ and $\dot{\Omega}' = \dot{\omega}_1$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \{4\mathbf{i} + 5\mathbf{k}\}\ \text{rad/s}$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \left[\left(\dot{\omega}_1\right)_{x'y'z'} + \omega_1 \times \omega_1\right] + \left[\left(\omega_2\right)_{x'y'z'} + \omega_1 \times \omega_2\right]$$

$$= (1.5\mathbf{i} + \mathbf{0}) + \left[-6\mathbf{k} + (4\mathbf{i}) \times (5\mathbf{k})\right] = \{1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}\}\ \text{rad/s}^2$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\}\ \text{m}$$

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = \left(\dot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \mathbf{r}_B = \mathbf{0} + (4\mathbf{i}) \times (-0.5\mathbf{k}) = \{2\mathbf{j}\}\ \text{m/s}$$

$$\mathbf{a}_B = \dot{\mathbf{r}}_B = \left[\left(\ddot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \left(\dot{\mathbf{r}}_\mathbf{B}\right)_{x'y'z'}\right] + \dot{\omega}_1 \times r_B + \omega_1 \times \dot{\mathbf{r}}_B$$

$$= \mathbf{0} + \mathbf{0} + \left[(1.5\mathbf{i}) \times (-0.5\mathbf{k})\right] + (4\mathbf{i} \times 2\mathbf{j}) = \{0.75\mathbf{j} + 8\mathbf{k}\}\ \text{m/s}^2$$

Relative to $x'y'z'$, let xyz have

$$\Omega_{x'y'z'} = \mathbf{0}; \qquad \dot{\Omega}_{x'y'z'} = \mathbf{0};$$

$$\left(r_{C/B}\right)_{xyz} = \{0.2\mathbf{j}\}\ \text{m}$$

$$(\mathbf{v}_{C/B})_{xyz} = \{3\mathbf{j}\}\ \text{m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \{2\mathbf{j}\}\ \text{m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 3\mathbf{j}$$

$$= \{-1\mathbf{i} + 5\mathbf{j} + 0.8\mathbf{k}\}\text{m/s} \qquad \textbf{Ans.}$$

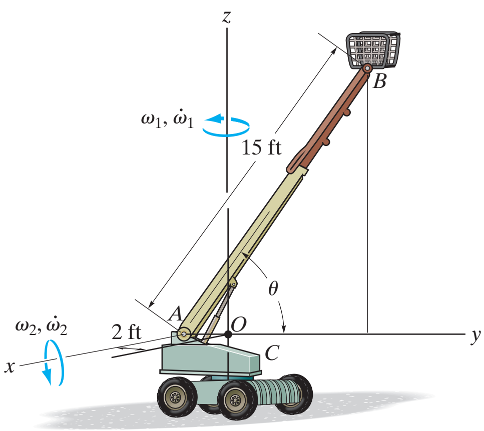
$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \left[(1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}) \times (0.2\mathbf{j})\right] + (4\mathbf{i} + 5\mathbf{k}) \times \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 2\left[(4\mathbf{i} + 5\mathbf{k}) \times (3\mathbf{j})\right] + 2\mathbf{j}$$

$$\mathbf{a}_C = \{-28.2\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

20–39.

At the instant $\theta = 60°$, the telescopic boom AB of the construction lift is rotating with a constant angular velocity about the z axis of $\omega_1 = 0.5$ rad/s and about the pin at A with a constant angular speed of $\omega_2 = 0.25$ rad/s. Simultaneously, the boom is extending with a velocity of 1.5 ft/s, and it has an acceleration of 0.5 ft/s^2, both measured relative to the construction lift. Determine the velocity and acceleration of point B located at the end of the boom at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

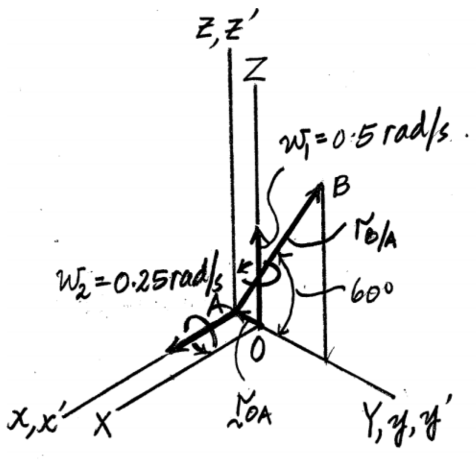
$$\Omega = \omega_1 = \{0.5\mathbf{k}\}\ \text{rad/s} \qquad\qquad \dot{\Omega} = \dot{\omega}_1 = 0$$

Since point A rotates about a fixed axis (Z axis), its motion can determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (0.5\mathbf{k}) \times (-2\mathbf{j}) = \{1\mathbf{i}\}\ \text{ft/s}$$

and

$$\begin{aligned} a_A &= \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA}) \\ &= 0 + (0.5\mathbf{k}) \times (0.5\mathbf{k}) \times (-2\mathbf{j}) \\ &= \{0.5\mathbf{j}\}\ \text{ft/s}^2 \end{aligned}$$



(a)

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity of $\Omega' = \omega_2 = \{0.25\mathbf{i}\}$ rad/s, the direction of $\mathbf{r}_{B/A}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $\mathbf{r}_{B/A}$,

$$\begin{aligned} (\mathbf{v}_{B/A})_{xyz} &= (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times \mathbf{r}_{B/A}\right] \\ &= (1.5\cos 60°\mathbf{j} + 1.5\sin 60°\mathbf{k}) + 0.25\mathbf{i} \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k}) \\ &= \{-2.4976\mathbf{j} + 3.1740\mathbf{k}\}\ \text{ft/s} \end{aligned}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega} = \dot{\omega}_2 = 0$. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$\begin{aligned} (\mathbf{a}_{B/A})_{xyz} &= (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times \mathbf{r}_{B/A} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz} \\ &= \left[(0.5\cos 60°\mathbf{j} + 0.5\sin 60°\mathbf{k}) + 0.25\mathbf{i} \times (1.5\cos 60°\mathbf{j} + 1.5\sin 60°\mathbf{k})\right] + 0.25\mathbf{i} \times (-2.4976\mathbf{j} + 3.1740\mathbf{k}) \\ &= \{-0.8683\mathbf{j} - 0.003886\mathbf{k}\}\ \text{ft/s}^2 \end{aligned}$$

20–39. continued

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (1\mathbf{i}) + (0.5\mathbf{k}) \times (15\cos 60^\circ \mathbf{j} + 15\sin 60^\circ \mathbf{k}) + (-2.4976\mathbf{j} + 3.1740\mathbf{k})$$

$$= \{-2.75\mathbf{i} - 2.50\mathbf{j} + 3.17\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

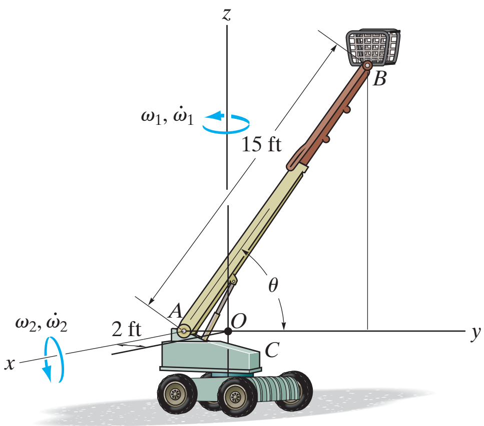
$$= 0.5\mathbf{j} + 0 + 0.5\mathbf{k} \times \left[(0.5\mathbf{k}) \times (15\cos 60^\circ \mathbf{j} + 15\sin 60^\circ \mathbf{k})\right]$$

$$+ 2(0.5\mathbf{k}) \times (-2.4976\mathbf{j} + 3.1740\mathbf{k}) + (-0.8683\mathbf{j} - 0.003886\mathbf{k})$$

$$= \{2.50\mathbf{i} - 2.24\mathbf{j} - 0.00389\mathbf{k}\}\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

***20–40.**

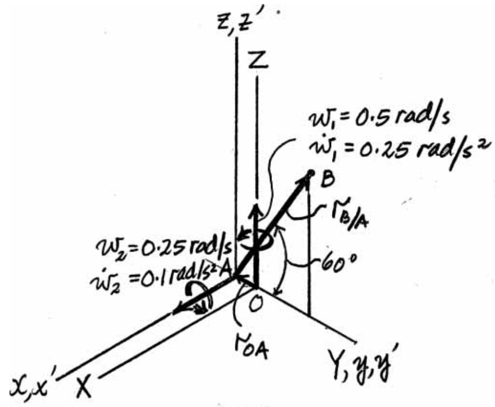
At the instant $\theta = 60°$, the construction lift is rotating about the z axis with an angular velocity of $\omega_1 = 0.5$ rad/s and an angular acceleration of $\dot{\omega}_1 = 0.25$ rad/s^2 while the telescopic boom AB rotates about the pin at A with an angular velocity of $\omega_2 = 0.25$ rad/s and angular acceleration of $\dot{\omega}_2 = 0.1$ rad/s^2. Simultaneously, the boom is extending with a velocity of 1.5 ft/s, and it has an acceleration of 0.5 ft/s^2, both measured relative to the frame. Determine the velocity and acceleration of point B located at the end of the boom at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = \{0.5\mathbf{k}\} \text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \{0.25\mathbf{k}\} \text{ rad/s}^2$$



(a)

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (0.5\mathbf{k}) \times (-2\mathbf{j}) = \{1\mathbf{i}\} \text{ ft/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= (0.25\mathbf{k}) \times (-2\mathbf{j}) + (0\ .5\mathbf{k}) \times [0.5\mathbf{k} \times (-2\mathbf{j})]$$

$$= \{0.5\mathbf{i} + 0.5\mathbf{j}\} \text{ ft/s}^2$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity of $\Omega' = \omega_2 = [0.25\mathbf{i}]$ rad/s, the direction of $\mathbf{r}_{B/A}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $\mathbf{r}_{B/A}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times \mathbf{r}_{B/A}\right]$$

$$= (1.5 \cos 60°\mathbf{j} + 1.5 \sin 60°\mathbf{k}) + [0.25\mathbf{i} \times (15 \cos 60°\mathbf{j} + 15 \sin 60°\mathbf{k})]$$

$$= \{-2.4976\mathbf{j} + 3.1740\mathbf{k}\} \text{ ft/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega} = \dot{\omega}_2 = [0.1\mathbf{i}]$ rad/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$(\mathbf{a}_{B/A})_{xyz} = (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times \mathbf{r}_{B/A}$$

$$+ \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= (0.5 \cos\ 60°\mathbf{j} + 0.5 \sin 60°\mathbf{k}) + (0.25\mathbf{i}) \times (1.5 \cos 60°\mathbf{j} + 1.5 \sin 60°\mathbf{k})$$

$$+ (0.1\mathbf{i}) \times (15 \cos 60°\mathbf{j} + 15 \sin 60°\mathbf{k}) + (0.25\mathbf{i}) \times (-2.4976\mathbf{j} + 3.1740\mathbf{k})$$

$$= \{-2.1673\mathbf{j} + 0.7461\mathbf{k}\} \text{ ft/s}^2$$

20–40. continued

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= [1\mathbf{i}] + (0.5\mathbf{k}) \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k}) + (-2.4976\mathbf{j} + 3.1740\mathbf{k})$$

$$= \{-2.75\mathbf{i} - 2.50\mathbf{j} + 3.17\mathbf{k}\}\ \text{ft/s}$$ **Ans.**

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (0.5\mathbf{i} + 0.5\mathbf{j}) + (0.25\mathbf{k}) \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k}) + (0.5\mathbf{k})$$

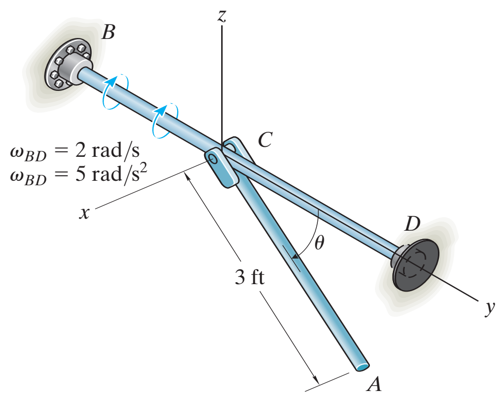
$$\times [(0.5\mathbf{k}) \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k})] + 2(0.5\mathbf{k})$$

$$\times (-2.4976\mathbf{j} + 3.1740\mathbf{k}) + (-2.1673\mathbf{j} + 0.7461\mathbf{k})$$

$$= \{1.12\mathbf{i} - 3.54\mathbf{j} + 0.746\mathbf{k}\}\ \text{ft/s}^2$$ **Ans.**

20–41.

At a given instant, rod BD is rotating about the y axis with an angular velocity $\omega_{BD} = 2\ \text{rad/s}$ and an angular acceleration $\dot{\omega}_{BD} = 5\ \text{rad/s}^2$. Also, when $\theta = 60^\circ$ link AC is rotating downward such that $\dot{\theta} = 2\ \text{rad/s}$ and $\ddot{\theta} = 8\ \text{rad/s}^2$. Determine the velocity and acceleration of point A on the link at this instant.



SOLUTION

$$\Omega = -2\mathbf{i} - 2\mathbf{j}$$

$$\mathbf{r}_{A/C} = 3\cos 60^\circ\mathbf{j} - 3\sin 60^\circ\mathbf{k} = 1.5\mathbf{j} - 2.5980762\mathbf{k}$$

$$\dot{\Omega} = -5\mathbf{j} - 8\mathbf{i} + (-2\mathbf{j}) \times (-2\mathbf{i}) = \{-8\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}\}\ \text{rad/s}^2$$

$$\mathbf{v}_A = \mathbf{v}_C + \Omega \times \mathbf{r}_{A/C} + (\mathbf{v}_{A/C})_{xyz}$$

$$\mathbf{v}_A = (-2\mathbf{i} - 2\mathbf{j}) \times (1.5\mathbf{j} - 2.5980762\mathbf{k}) + \mathbf{0}$$

$$\mathbf{v}_A = 5.19615\mathbf{i} - 5.19615\mathbf{j} - 3\mathbf{k} + \mathbf{0}$$

$$\mathbf{v}_A = \{5.20\mathbf{i} - 5.20\mathbf{j} - 3.00\mathbf{k}\}\ \text{ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\Omega} \times \mathbf{r}_{A/C} + \Omega \times (\Omega \times \mathbf{r}_{A/C})$$

$$+ 2(\Omega \times (\mathbf{v}_{A/C})_{xyz}) + (a_{A/C})_{xyz}$$

$$= \mathbf{0} + (-8\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}) \times (1.5\mathbf{j} - 2.5980762\mathbf{k})$$

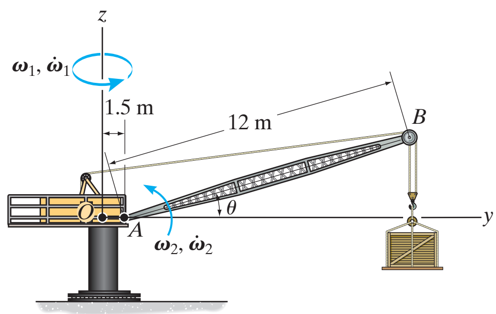
$$+ (-2\mathbf{i} - 2\mathbf{j}) \times (5.19615\mathbf{i} - 5.19615\mathbf{j} - 3\mathbf{k}) + \mathbf{0} + \mathbf{0}$$

$$\mathbf{a}_A = 24.9904\mathbf{i} - 26.7846\mathbf{j} + 8.7846\mathbf{k}$$

$$\mathbf{a}_A = \{25\mathbf{i} - 26.8\mathbf{j} + 8.78\mathbf{k}\}\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

20–42.

At the instant $\theta = 30°$, the frame of the crane and the boom AB rotate with a constant angular velocity of $\omega_1 = 1.5$ rad/s and $\omega_2 = 0.5$ rad/s, respectively. Determine the velocity and acceleration of point B at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. The angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [1.5\mathbf{k}] \text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$$

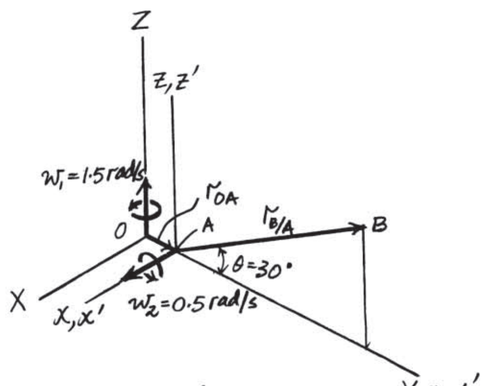
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (1.5\mathbf{k}) \times (1.5\mathbf{j}) = [-2.25\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= \mathbf{0} + (1.5\mathbf{k}) \times [(1.5\mathbf{k}) \times (1.5\mathbf{j})]$$

$$= [-3.375\mathbf{j}] \text{ m/s}^2$$



(a)

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [0.5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (0.5\mathbf{i}) \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30° \mathbf{k})$$

$$= [-3\mathbf{j} + 5.196\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{A/B})_{xyz}$,

$$(\mathbf{a}_{A/B})_{xyz} = (\ddot{\mathbf{r}}_{A/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{A/B})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{A/B})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{A/B})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{A/B})_{xyz}$$

$$= [0 + 0] + 0 + (0.5\mathbf{i}) \times (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-2.598\mathbf{j} - 1.5\mathbf{k}] \text{ m/s}^2$$

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (-2.25\mathbf{i}) + 1.5\mathbf{k} \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30° \,\mathbf{k}) + (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

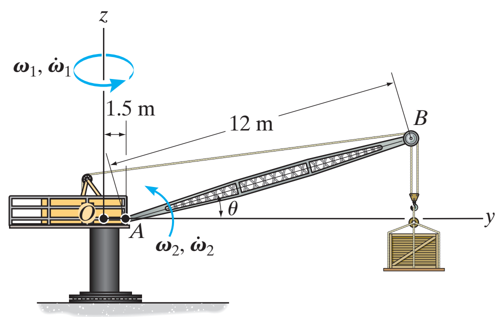
$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{AB})_{xyz} + (\mathbf{a}_{AB})_{xyz}$$

$$= (-3.375\mathbf{j}) + 0 + 1.5\mathbf{k} \times \left[(1.5\mathbf{k}) \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30° \,\mathbf{k})\right] + 2(1.5\mathbf{k}) \times (-3\mathbf{j} + 5.196\mathbf{k}) + (-2.598\mathbf{j} - 1.5\mathbf{k})$$

$$= [9\mathbf{i} - 29.4\mathbf{j} - 1.5\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

20–43.

At the instant $\theta = 30°$, the frame of the crane is rotating with an angular velocity of $\omega_1 = 1.5$ rad/s and angular acceleration of $\dot{\omega}_1 = 0.5$ rad/s^2, while the boom AB rotates with an angular velocity of $\omega_2 = 0.5$ rad/s and angular acceleration of $\dot{\omega}_2 = 0.25$ rad/s^2. Determine the velocity and acceleration of point B at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [1.5\mathbf{k}]\ \text{rad/s} \qquad \dot{\Omega} = [0.5\mathbf{k}]\ \text{rad/s}^2$$

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (1.5\mathbf{k}) \times (1.5\mathbf{j}) = [-2.25\mathbf{i}]\ \text{m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= (0.5\mathbf{k}) \times (1.5\mathbf{j}) + (1.5\mathbf{k}) \times \left[(1.5\mathbf{k}) \times (1.5\mathbf{j})\right]$$

$$= [-0.75\mathbf{i} - 3.375\mathbf{j}]\ \text{m/s}^2$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [0.5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (0.5\mathbf{i}) \times (12 \cos 30°\,\mathbf{j} + 12 \sin 30°\,\mathbf{k})$$

$$= [-3\mathbf{j} + 5.196\mathbf{k}]\ \text{m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = [0.25\mathbf{i}]$ m/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$(\mathbf{a}_{B/A}) = (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\Omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= [0 + 0] + (0.25\mathbf{i}) \times (12 \cos 30°\,\mathbf{j} + 12 \sin 30°\mathbf{k}) + 0.5\mathbf{i} \times (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-4.098\mathbf{j} + 1.098\mathbf{k}]\ \text{m/s}^2$$

20–43. continued

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (-2.25\mathbf{i}) + 1.5\mathbf{k} \times (12 \cos 30^\circ\, \mathbf{j} + 12 \sin 30^\circ \mathbf{k}) + (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

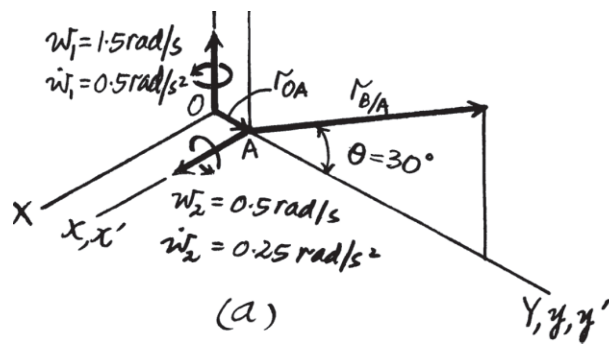
and

$$\mathbf{a}_B = \mathbf{a}_A = \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (-0.75\mathbf{i} - 3.375\mathbf{j}) + 0.5\mathbf{k} \times (12 \cos 30^\circ \mathbf{j} + 12 \sin 30^\circ\, \mathbf{k}) + (1.5\mathbf{k}) \times \left[(1.5\mathbf{k}) \times (12 \cos 30^\circ\, \mathbf{j} + 12 \sin 30^\circ\, \mathbf{k})\right]$$

$$+2(1.5\mathbf{k}) \times (-3\mathbf{j} + 5.196\mathbf{k}) + (-4.098\mathbf{j} + 1.098\mathbf{k})$$

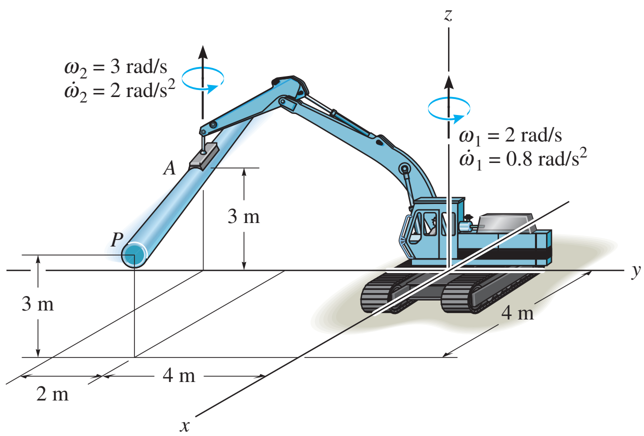
$$= [3.05\mathbf{i} - 30.9\mathbf{j} + 1.10\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$



(a)

***20–44.**

At the instant shown, the boom is rotating about the z axis with an angular velocity $\omega_1 = 2$ rad/s and angular acceleration $\dot{\omega}_1 = 0.8$ rad/s^2. At this same instant the swivel is rotating at $\omega_2 = 3$ rad/s when $\dot{\omega}_2 = 2$ rad/s^2, both measured relative to the boom. Determine the velocity and acceleration of point P on the pipe at this instant.



SOLUTION

Relative to XYZ, let xyz have

$\Omega = \{2\mathbf{k}\}$ rad/s $\qquad \Omega = \{0.8\mathbf{k}\}$ rad/s (Ω does not change direction relative to XYZ.)

$\mathbf{r}_A = \{-6\mathbf{j} + 3\mathbf{k}\}$ m ($\mathbf{r}_A$ changes direction relative to XYZ.)

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{xyz} + \Omega \times \mathbf{r}_A = \mathbf{0} + (2\mathbf{k}) \times (-6\mathbf{j} + 3\mathbf{k}) = \{12\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = [(\ddot{\mathbf{r}}_A)_{xyz} + \Omega \times (\dot{\mathbf{r}}_A)_{xyz}] + \dot{\Omega} \times \mathbf{r}_A + \Omega \times \dot{\mathbf{r}}_A$$

$$= \mathbf{0} + \mathbf{0} + (0.8\mathbf{k}) \times (-6\mathbf{j} + 3\mathbf{k}) + (2\mathbf{k}) \times (12\mathbf{i})$$

$$= \{4.8\mathbf{i} + 24\mathbf{j}\} \text{ m/s}^2$$

Relative to xyz, let x', y', z' have the origin at A and

$\Omega_{xyz} = \{3\mathbf{k}\}$ rad/s $\qquad \dot{\Omega}_{xyz} = \{2\mathbf{k}\}$ rad/s^2 (Ω_{xyz} does not change direction relative to xyz.)

$(\mathbf{r}_{P/A})_{xyz} = \{4\mathbf{i} + 2\mathbf{j}\}$ m ($(\mathbf{r}_{P/A})_{xyz}$ changes direction relative to xyz.)

$$(\mathbf{v}_{P/A})_{xyz} = (\dot{\mathbf{r}}_{P/A})_{xyz} = [(\dot{\mathbf{r}}_{P/A})_{x'y'z'} + \Omega_{xyz} \times (\mathbf{r}_{P/A})_{xyz}]$$

$$= \mathbf{0} + (3\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})$$

$$= \{-6\mathbf{i} + 12\mathbf{j}\} \text{ m/s}$$

$$(\mathbf{a}_{P/A})_{xyz} = (\ddot{\mathbf{r}}_{P/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{P/A})_{x'y'z'} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{P/A})_{x'y'z'}\right] + \left[\dot{\Omega}_{xyz} \times (\mathbf{r}_{P/A})_{xyz}\right] + \left[\Omega_{xyz} \times (\dot{\mathbf{r}}_{P/A})_{xyz}\right]$$

$$= \mathbf{0} + \mathbf{0} + [(2\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})] + [(3\mathbf{k}) \times (-6\mathbf{i} + 12\mathbf{j})]$$

$$= \{-40\mathbf{i} - 10\mathbf{j}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_A + \Omega \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$

$$= 12\mathbf{i} + [(2\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})] + (-6\mathbf{i} + 12\mathbf{j})$$

$$= \{2\mathbf{i} + 20\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_P = \mathbf{a}_A + \Omega \times \mathbf{r}_{P/A} + \Omega \times (\Omega \times \mathbf{r}_{P/A}) + 2\Omega \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$

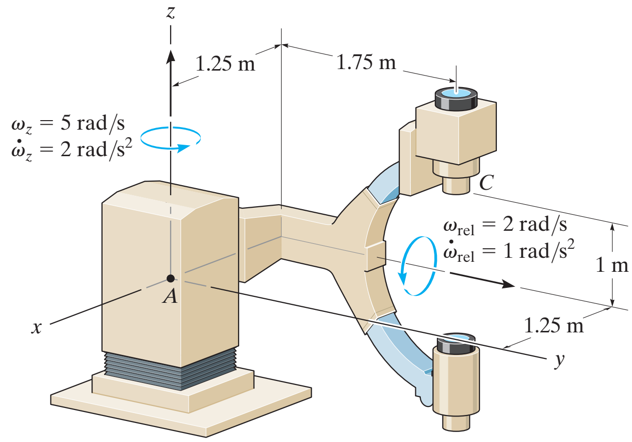
$$= (4.8\mathbf{i} + 24\mathbf{j}) + \left[(0.8\mathbf{k}) \times (4\mathbf{i} + 2\mathbf{j})\right] + 2\mathbf{k} \times \left[2\mathbf{k} \times (4\mathbf{i} + 2\mathbf{j})\right]$$

$$+ \left[2(2\mathbf{k}) \times (-6\mathbf{i} + 12\mathbf{j})\right] + (-40\mathbf{i} - 10\mathbf{j})$$

$$= \{-101\mathbf{i} - 14.8\mathbf{j}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

20–45.

During the instant shown the frame of the X-ray camera is rotating about the vertical axis at $\omega_z = 5\ \text{rad/s}$ and $\dot{\omega}_z = 2\ \text{rad/s}^2$. Relative to the frame the arm is rotating at $\omega_{\text{rel}} = 2\ \text{rad/s}$ and $\dot{\omega}_{\text{rel}} = 1\ \text{rad/s}^2$. Determine the velocity and acceleration of the center of the camera C at this instant.

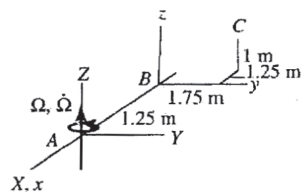


SOLUTION

$\Omega = \{5\mathbf{k}\}\ \text{rad/s}$

$\dot{\Omega} = \{2\mathbf{k}\}\ \text{rad/s}^2$

$\mathbf{r}_B = \{-1.25\mathbf{i}\}\ \text{m}$



$\omega_B = \mathbf{0} + 5\mathbf{k} \times (-1.25\mathbf{i}) = -6.25\mathbf{j}$

$\mathbf{a}_B = \mathbf{0} + 2\mathbf{k} \times (-1.25\mathbf{i}) + \mathbf{0} + 5\mathbf{k} \times (-6.25\mathbf{j})$

$= 31.25\mathbf{i} - 2.5\mathbf{j}$

$\Omega_{xyz} = \{2\mathbf{j}\}\ \text{rad/s}$

$\dot{\Omega}_{xyz} = \{1\mathbf{j}\}\ \text{rad/s}^2$

$\mathbf{r}_{C/B} = \{1.75\mathbf{j} + 1\mathbf{k}\}\ \text{m}$

$(\mathbf{v}_{C/B})_{xyz} = \dot{\mathbf{r}}_{C/B} = \mathbf{0} + (2\mathbf{j}) \times (1.75\mathbf{j} + 1\mathbf{k}) = 2\mathbf{i}$

$(\mathbf{a}_{C/B})_{xyz} = \ddot{\mathbf{r}}_{C/B} = \mathbf{0} + (1\mathbf{j}) \times (1.75\mathbf{j} + 1\mathbf{k}) + \mathbf{0} + (2\mathbf{j}) \times (2\mathbf{i})$

$= 1\mathbf{i} - 4\mathbf{k}$

$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/A})_{xyz}$

$\mathbf{v}_C = -6.25\mathbf{j} + 5\mathbf{k} \times (1.75\mathbf{j} + 1\mathbf{k}) + 2\mathbf{i}$

$\mathbf{v}_C = \{-6.75\mathbf{i} - 6.25\mathbf{j}\}\ \text{m/s}$ **Ans.**

$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2(\Omega \times (\mathbf{v}_{C/B})_{xyz}) + (a_{C/B})_{xyz}$

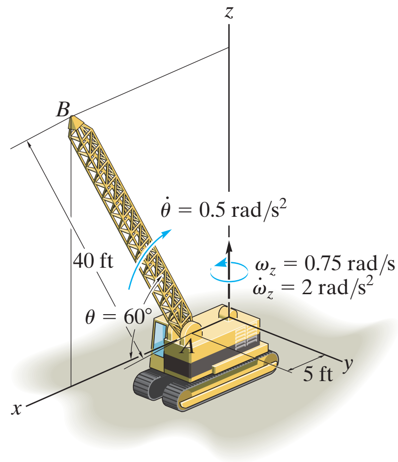
$= (31.25\mathbf{i} - 2.5\mathbf{j}) + (2\mathbf{k}) \times (1.75\mathbf{j} + 1\mathbf{k}) + 5\mathbf{k} \times [(5\mathbf{k}) \times (1.75\mathbf{j} + 1\mathbf{k})]$

$+ 2(5\mathbf{k}) \times (2\mathbf{i}) + (1\mathbf{i} - 4\mathbf{k})$

$\mathbf{a}_C = \{28.75\mathbf{i} - 26.25\mathbf{j} - 4\mathbf{k}\}\ \text{m/s}^2$ **Ans.**

20–46.

The boom AB of the crane is rotating about the z axis with an angular velocity $\omega_z = 0.75$ rad/s, which is increasing at $\dot{\omega}_z = 2$ rad/s^2. At the same instant, $\theta = 60°$ and the boom is rotating upward at a constant rate $\dot{\theta} = 0.5$ rad/s^2. Determine the velocity and acceleration of the tip B of the boom at this instant.



SOLUTION

Motion of moving reference:

$$\Omega = \omega_z = \{0.75\mathbf{k}\}\text{ rad/s}$$

$$\dot{\Omega} = (\dot{\omega})_{xyz} = \{2\mathbf{k}\}\text{ rad/s}^2$$

$$\mathbf{r}_O = \{5\mathbf{i}\}\text{ ft}$$

$$\mathbf{v}_O = \dot{\mathbf{r}}_O = (\dot{\mathbf{r}}_O)_{xyz} + \Omega \times \mathbf{r}_O$$

$$= \mathbf{0} + (0.75\mathbf{k}) \times (5\mathbf{i})$$

$$= \{3.75\mathbf{j}\}\text{ ft/s}$$

$$\mathbf{a}_O = \ddot{\mathbf{r}}_O = [(\ddot{\mathbf{r}}_O)_{xyz} + \Omega \times (\dot{\mathbf{r}}_O)_{xyz}] + \dot{\Omega} \times \mathbf{r}_O + \Omega \times \dot{\mathbf{r}}_O$$

$$= \mathbf{0} + \mathbf{0} + (2\mathbf{k}) \times (5\mathbf{i}) + (0.75\mathbf{k}) \times (3.75\mathbf{j})$$

$$= \{-\,2.8125\mathbf{i} + 10\mathbf{j}\}\text{ ft/s}^2$$

Motion of B with respect to moving reference:

$$\Omega_{B/O} = \{-\,0.5\mathbf{j}\}\text{ rad/s}$$

$$\dot{\Omega}_{B/O} = 0$$

$$\mathbf{r}_{B/O} = 40\cos 60°\mathbf{i} + 40\sin 60°\mathbf{k} = \{20\mathbf{i} + 34.64\mathbf{k}\}\text{ ft}$$

$$(v_{B/O})_{xyz} = \dot{\mathbf{r}}_{B/O} = (\dot{\mathbf{r}}_{B/O})_{xyz} + \Omega_{B/O} \times \mathbf{r}_{B/O}$$

$$= \mathbf{0} + (-\,0.5\mathbf{j}) \times (20\mathbf{i} + 34.64\mathbf{k})$$

$$= \{-\,17.32\mathbf{i} + 10\mathbf{k}\}\text{ ft/s}$$

$$(\mathbf{a}_{B/O})_{xyz} = \ddot{\mathbf{r}}_{B/O} = [(\ddot{\mathbf{r}}_{B/O})_{xyz} + \Omega_{B/O} \times (\dot{\mathbf{r}}_{B/O})_{xyz}] + \dot{\Omega}_{B/O} \times \mathbf{r}_{B/O} + \Omega_{B/O} \times \dot{\mathbf{r}}_{B/O}$$

$$= \mathbf{0} + \mathbf{0} + \mathbf{0} + (-\,0.5\mathbf{j}) \times (-\,17.32\mathbf{i} + 10\mathbf{k})$$

$$= \{-\,5\mathbf{i} - 8.66\mathbf{k}\}\text{ ft/s}^2$$

$$\mathbf{v}_B = \mathbf{v}_O + \Omega \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz}$$

$$= (3.75\mathbf{j}) + (0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (-17.32\mathbf{i} + 10\mathbf{k})$$

$$\mathbf{v}_B = \{-17.3\mathbf{i} + 18.8\mathbf{j} + 10.0\mathbf{k}\}\ \text{ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz}$$

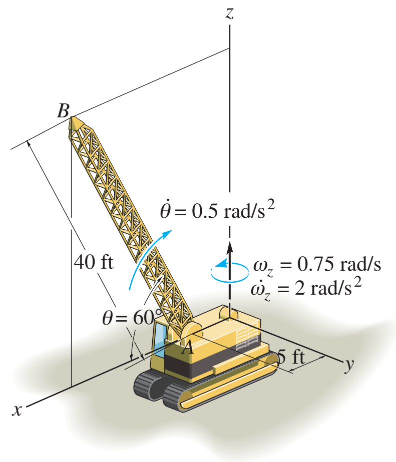
$$= (-2.8125\mathbf{i} + 10\mathbf{j}) + (2\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (0.75\mathbf{k}) \times [(0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k})]$$

$$+ 2(0.75\mathbf{k}) \times (-17.32\mathbf{i} + 10\mathbf{k}) + (-5\mathbf{i} - 8.66\mathbf{j})$$

$$\mathbf{a}_B = \{-19.1\mathbf{i} + 24.0\mathbf{j} - 8.66\mathbf{k}\}\text{ ft/s}^{\mathbf{2}} \qquad \textbf{Ans.}$$

20–47.

The boom AB of the crane is rotating about the z axis with an angular velocity of $\omega_z = 0.75$ rad/s, which is increasing at $\dot{\omega}_z = 2$ rad/s^2. At the same instant, $\theta = 60°$ and the boom is rotating upward at $\dot{\theta} = 0.5$ rad/s^2, which is increasing at $\ddot{\theta} = 0.75$ rad/s^2. Determine the velocity and acceleration of the tip B of the boom at this instant.



SOLUTION

Motion of moving reference:

$$\Omega = \omega_z = \{0.75\mathbf{k}\} \text{ rad/s}$$

$$\dot{\Omega} = (\dot{\omega})_{xyz} = \{2\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_O = \{5\mathbf{i}\} \text{ ft}$$

$$\mathbf{v}_O = \dot{\mathbf{r}}_O = (\dot{\mathbf{r}}_O)_{xyz} + \Omega \times \mathbf{r}_O$$

$$= \mathbf{0} + (0.75\mathbf{k}) \times (5\mathbf{i})$$

$$= \{3.75\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{a}_O = \ddot{\mathbf{r}}_\mathbf{O} = [(\ddot{\mathbf{r}}_O)_{xyz} + \Omega \times (\mathbf{r}_O)_{xyz}] + \dot{\omega} \times \mathbf{r}_O + \Omega \times \dot{\mathbf{r}}_O$$

$$= \mathbf{0} + \mathbf{0} + (2\mathbf{k}) \times (5\mathbf{i}) + (0.75\mathbf{k}) \times (3.75\mathbf{j})$$

$$= \{-2.8125\mathbf{i} + 10\mathbf{j}\} \text{ ft/s}^2$$

Motion of B with respect to moving reference:

$$\Omega_{B/O} = \{-0.5\mathbf{j}\} \text{ rad/s}$$

$$\dot{\Omega}_{B/O} = \{-0.75\mathbf{j}\} \text{ rad/s}^2$$

$$\mathbf{r}_{B/O} = 40 \cos 60°\mathbf{i} + 40 \sin 60°\mathbf{k} = \{20\mathbf{i} + 34.64\mathbf{k}\} \text{ ft}$$

$$(\mathbf{v}_{B/O})_{xyz} = \dot{\mathbf{r}}_{B/O} = (\dot{\mathbf{r}}_{B/O})_{xyz} + \Omega_{B/O} \times \mathbf{r}_{B/O}$$

$$= \mathbf{0} + (-0.5\mathbf{j}) \times (20\mathbf{i} + 34.64\mathbf{k})$$

$$= \{-17.32\mathbf{i} + 10\mathbf{k}\} \text{ ft/s}$$

$$(\mathbf{a}_{B/O})_{xyz} = \ddot{\mathbf{r}}_{B/O} = [(\ddot{\mathbf{r}}_{B/O})_{xyz} + \Omega_{B/O} \times (\dot{\mathbf{r}}_{B/O})_{xyz}] + \dot{\Omega}_{B/O} \times \mathbf{r}_{B/O} + \Omega_{B/O} \times \dot{\mathbf{r}}_{B/O}$$

$$= \mathbf{0} + \mathbf{0} + (-0.75\mathbf{j}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (-0.5\mathbf{j}) \times (-17.32\mathbf{i} + 10\mathbf{k})$$

$$= \{-30.98\mathbf{i} + 6.34\mathbf{k}\} \text{ ft/s}^2$$

$$\mathbf{v}_B = \mathbf{v}_O + \Omega \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz}$$

$$= (3.75\mathbf{j}) + (0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (-17.32\mathbf{i} + 10\mathbf{k})$$

$$\mathbf{v}_B = \{-17.3\mathbf{i} + 18.8\mathbf{j} + 10.0\mathbf{k}\} \text{ ft/s}$$ **Ans.**

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz}$$

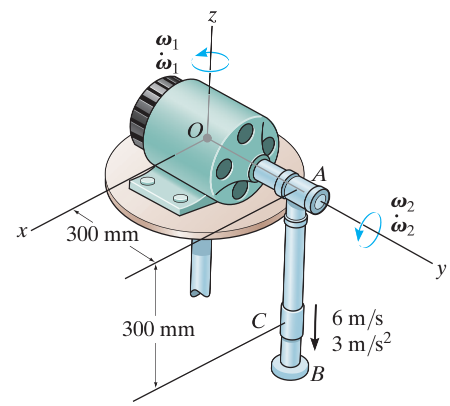
$$= (-2.8125\mathbf{i} + 10\mathbf{j}) + (2\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k}) + (0.75\mathbf{k}) \times [(0.75\mathbf{k}) \times (20\mathbf{i} + 34.64\mathbf{k})]$$

$$+ 2(0.75\mathbf{k}) \times (-17.32\mathbf{i} + 10\mathbf{k}) + (-30.98\mathbf{i} + 6.34\mathbf{j})$$

$$\mathbf{a}_B = \{-45.0\mathbf{i} + 24.0\mathbf{j} + 6.34\mathbf{k}\} \text{ ft/s}^2$$ **Ans.**

*20–48.

At the instant shown, the motor rotates about the z axis with an angular velocity of $\omega_1 = 3$ rad/s and angular acceleration of $\dot{\omega}_1 = 1.5$ rad/s^2. Simultaneously, shaft OA rotates with an angular velocity of $\omega_2 = 6$ rad/s and angular acceleration of $\dot{\omega}_2 = 3$ rad/s^2, and collar C slides along rod AB with a velocity and acceleration of 6 m/s and 3 m/s^2. Determine the velocity and acceleration of collar C at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [3\mathbf{k}] \text{ rad/s} \qquad \dot{\omega} = [1.5\mathbf{k}] \text{ rad/s}^2$$

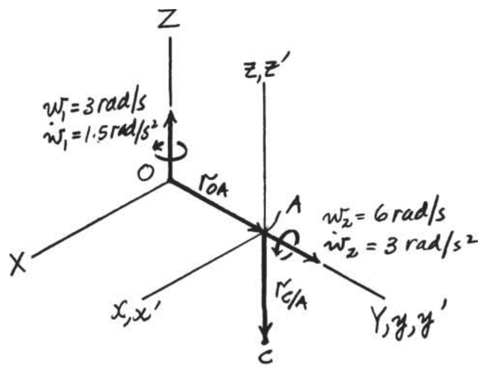
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (3\mathbf{k}) \times (0.3\mathbf{j}) = [-0.9\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega \times \mathbf{r}_{OA})$$

$$= (1.5\mathbf{k}) \times (0.3\mathbf{j}) + (3\mathbf{k}) \times (3\mathbf{k} \times 0.3\mathbf{j})$$

$$= [-0.45\mathbf{i} - 2.7\mathbf{j}] \text{ m/s}^2$$



(a)

In order to determine the motion of point C relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $'x\,'y\,'z$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [6\mathbf{j}]$ rad/s, the direction of $\mathbf{r}_{C/A}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative $\left(\dot{\mathbf{r}}_{C/A}\right)_{xyz}$,

$$(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} = \left[(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{C/A})_{xyz}\right]$$

$$= (-6\mathbf{k}) + 6\mathbf{j} \times (-0.3\mathbf{k})$$

$$= [-1.8\mathbf{i} - 6\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = [3\mathbf{j}]$ rad/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{C/A})_{xyz}$,

$$(\mathbf{a}_{C/A})_{xyz} = (\ddot{\mathbf{r}}_{C/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{x'y'z'}\right] + \dot{\omega}_2 \times \mathbf{r}_{C/A} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{xyz}$$

$$= [(-3\mathbf{k}) + 6\mathbf{j} \times (-6\mathbf{k})] + (3\mathbf{j}) \times (-0.3\mathbf{k}) + 6\mathbf{j} \times (-1.8\mathbf{i} - 6\mathbf{k})$$

$$= [-72.9\mathbf{i} + 7.8\mathbf{k}] \text{ m/s}$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$= (-0.9\mathbf{i}) + 3\mathbf{k} \times (-0.3\mathbf{k}) + (-1.8\mathbf{i} - 6\mathbf{k})$$

$$= [-2.7\mathbf{i} - 6\mathbf{k}] \text{ m/s}$$ **Ans.**

and

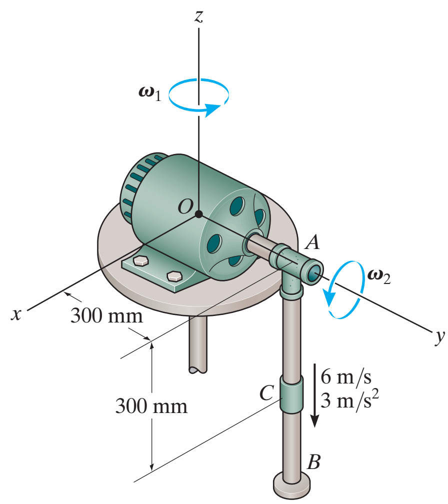
$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$= (-0.45\mathbf{i} - 2.7\mathbf{j}) + 1.5\mathbf{k} \times (-0.3\mathbf{k}) + (3\mathbf{k}) \times [(3\mathbf{k}) \times (-0.3\mathbf{k})] + 2(3\mathbf{k}) \times (-1.8\mathbf{i} - 6\mathbf{k}) + (-72.9\mathbf{i} + 7.8\mathbf{k})$$

$$= [-73.35\mathbf{i} - 13.5\mathbf{j} + 7.8\mathbf{k}] \text{ m/s}$$ **Ans.**

20–49.

The motor rotates about the z axis with a constant angular velocity of $\omega_1 = 3$ rad/s. Simultaneously, shaft OA rotates with a constant angular velocity of $\omega_2 = 6$ rad/s. Also, collar C slides along rod AB with a velocity and acceleration of 6 m/s and 3 m/s^2. Determine the velocity and acceleration of collar C at the instant shown.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [3\mathbf{k}] \text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$$

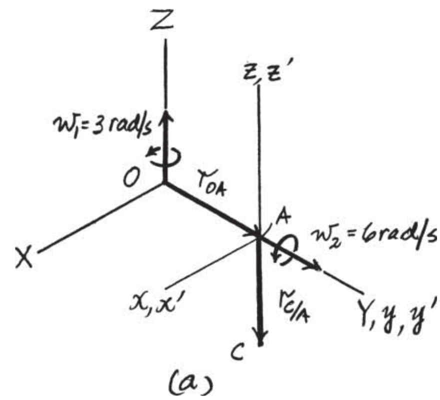
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (3\mathbf{k}) \times (0.3\mathbf{j}) = [-0.9\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= \mathbf{0} + (3\mathbf{k}) \times \left[(3\mathbf{k}) \times (0.3\mathbf{j})\right]$$

$$= [-2.7\mathbf{j}] \text{ m/s}^2$$



In order to determine the motion of the point C relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [6\mathbf{j}]$ rad/s, the direction of $\mathbf{r}_{C/A}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative $(\dot{\mathbf{r}}_{C/A})_{xyz}$,

$$(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} = \left[(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{C/A})_{xyz}\right]$$

$$= (-6\mathbf{k}) + 6\mathbf{j} \times (-0.3\mathbf{k})$$

$$= [-1.8\mathbf{i} - 6\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{C/A})_{xyz}$,

$$(\mathbf{a}_{C/A})_{xyz} = (\ddot{\mathbf{r}}_{C/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{x'y'z'}\right] + \dot{\omega}_2 \times \mathbf{r}_{C/A} + \omega_2 \times (\dot{\mathbf{r}}_{C/A})_{xyz}$$

$$= [(-3\mathbf{k}) + 6\mathbf{j} \times (-6\mathbf{k})] + \mathbf{0} + [6\mathbf{i} \times (-1.8\mathbf{j} - 6\mathbf{k})]$$

$$= [-72\mathbf{i} + 7.8\mathbf{k}] \text{ m/s}^2$$

Thus,

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$= (-0.9\mathbf{i}) + 3\mathbf{k} \times (-0.3\mathbf{k}) + (-1.8\mathbf{i} - 6\mathbf{k})$$

$$= [-2.7\mathbf{i} - 6\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

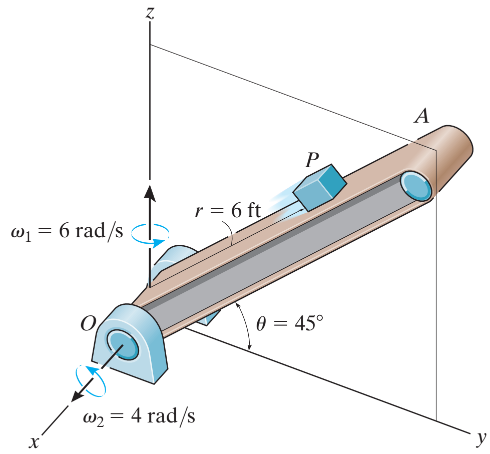
$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$= (-2.7\mathbf{j}) + \mathbf{0} + 3\mathbf{k} \times [(3\mathbf{k}) \times (-0.3\mathbf{k})] + 2(3\mathbf{k}) \times (-1.8\mathbf{i} - 6\mathbf{k}) + (-72\mathbf{i} + 7.8\mathbf{k})$$

$$= [-72\mathbf{i} - 13.5\mathbf{j} + 7.8\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

20–50.

At the instant shown, the arm OA of the conveyor belt is rotating about the z axis with a constant angular velocity $\omega_1 = 6\ \text{rad/s}$, while at the same instant the arm is rotating upward at a constant rate $\omega_2 = 4\ \text{rad/s}$. If the conveyor is running at a constant rate $\dot{r} = 5\ \text{ft/s}$, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



SOLUTION

$$\Omega = \omega_1 = \{6\mathbf{k}\}\ \text{rad/s}$$

$$\dot{\Omega} = \mathbf{0}$$

$$\mathbf{r}_O = \mathbf{v}_O = \mathbf{a}_O = \mathbf{0}$$

$$\Omega_{P/O} = \{4\mathbf{i}\}\ \text{rad/s}$$

$$\dot{\Omega}_{P/O} = \mathbf{0}$$

$$\mathbf{r}_{P/O} = \{4.243\mathbf{j} + 4.243\mathbf{k}\}\ \text{ft}$$

$$(\mathbf{v}_{P/O})_{xyz} = (\dot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times \mathbf{r}_{P/O}$$

$$= (5\cos 45°\mathbf{j} + 5\sin 45°\mathbf{k}) + (4\mathbf{i}) \times (4.243\mathbf{j} + 4.243\mathbf{k})$$

$$= \{-13.44\mathbf{j} + 20.51\mathbf{k}\}\ \text{ft/s}$$

$$(\mathbf{a}_{P/O}) = (\ddot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times (\dot{\mathbf{r}}_{P/O})_{xyz} + \dot{\Omega}_{P/O} \times \mathbf{r}_{P/O} + \Omega_{P/O} \times \dot{\mathbf{r}}_{P/O}$$

$$= \mathbf{0} + (4\mathbf{i}) \times (3.536\mathbf{j} + 3.536\mathbf{k}) + \mathbf{0} + (4\mathbf{i}) \times (-13.44\mathbf{j} + 20.51\mathbf{k})$$

$$= \{-96.18\mathbf{j} - 39.60\mathbf{k}\}\ \text{ft/s}^2$$

$$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{r}_{P/O})_{xyz}$$

$$= \mathbf{0} + (6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k}) + (-13.44\mathbf{j} + 20.51\mathbf{k})$$

$$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}\ \text{ft/s} \qquad \textbf{Ans.}$$

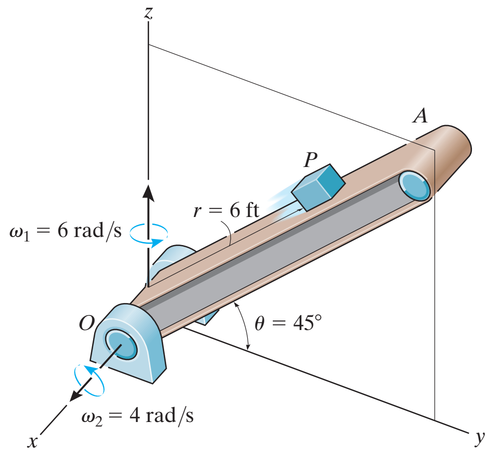
$$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times (\Omega \times \mathbf{r}_{P/O}) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (6\mathbf{k}) \times [(6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k})] + 2(6\mathbf{k}) \times (-13.44\mathbf{j} + 20.51\mathbf{k}) + (-96.18\mathbf{j} - 39.60\mathbf{k})$$

$$\mathbf{a}_P = \{161\mathbf{i} - 249\mathbf{j} - 39.6\mathbf{k}\}\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

20–51.

At the instant shown, the arm OA of the conveyor belt is rotating about the z axis with a constant angular velocity $\omega_1 = 6\text{ rad/s}$, while at the same instant the arm is rotating upward at a constant rate $\omega_2 = 4\text{ rad/s}$. If the conveyor is running at a rate $\dot{r} = 5\text{ ft/s}$, which is increasing at $\ddot{r} = 8\text{ ft/s}^2$, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



SOLUTION

$\Omega = \omega_1 = \{6\mathbf{k}\}\text{ rad/s}$

$\dot{\Omega} = \mathbf{0}$

$\mathbf{r}_O = \mathbf{v}_O = \mathbf{a}_O = \mathbf{0}$

$\Omega_{P/O} = \{4\mathbf{i}\}\text{ rad/s}$

$\dot{\Omega}_{P/O} = \mathbf{0}$

$\mathbf{r}_{P/O} = \{4.243\mathbf{j} + 4.243\mathbf{k}\}\text{ ft}$

$$(\mathbf{v}_{P/O})_{xyz} = (\dot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times \mathbf{r}_{P/O}$$

$$= (5\cos 45°\mathbf{j} + 5\sin 45°\mathbf{k}) + (4\mathbf{i}) \times (4.243\mathbf{j} + 4.243\mathbf{k})$$

$$= \{-13.44\mathbf{j} + 20.51\mathbf{k}\}\text{ ft/s}$$

$$(\mathbf{a}_{P/O})_{xyz} = 8\cos 45\mathbf{j} + 8\sin 45°\mathbf{k} - 96.18\mathbf{j} - 39.60\mathbf{k}$$

$$= \{-90.52\mathbf{j} - 33.945\mathbf{k}\}\text{ ft/s}^2$$

$$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz}$$

$$= \mathbf{0} + (6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k}) + (-13.44\mathbf{j} + 20.51\mathbf{k})$$

$$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}\text{ ft/s}$$ **Ans.**

$$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times (\Omega \times \mathbf{r}_{P/O}) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$$

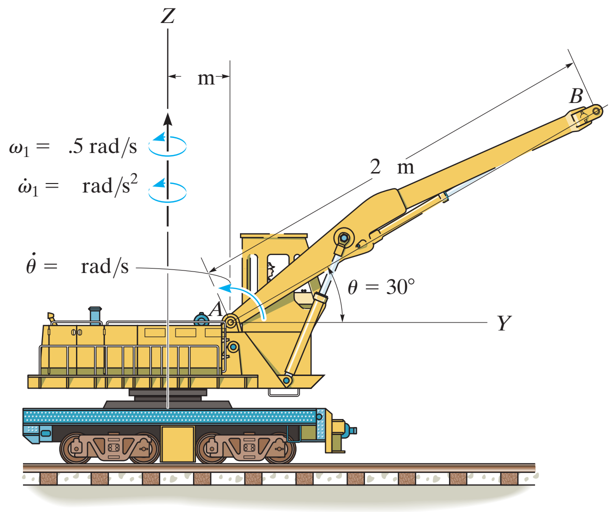
$$= \mathbf{0} + \mathbf{0} + (6\mathbf{k}) \times [(6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k})] + 2(6\mathbf{k}) \times (-13.44\mathbf{j} + 20.51\mathbf{k}) + (-90.52\mathbf{j} - 33.945\mathbf{k})$$

$$= -152.75\mathbf{j} + 161.23\mathbf{i} - 90.52\mathbf{j} - 33.945\mathbf{k}$$

$$\mathbf{a}_P = \{161\mathbf{i} - 243\mathbf{j} - 33.9\mathbf{k}\}\text{ ft/s}^2$$ **Ans.**

***20–52.**

The boom AB of the locomotive crane is rotating about the Z axis with an angular velocity $\omega_1 = 0.5$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. At this same instant, $\theta = 30°$ and the boom is rotating upward at a constant rate of $\dot{\theta} = 3$ rad/s. Determine the velocity and acceleration of the tip B of the boom at this instant.



SOLUTION

$\Omega = \{0.5\mathbf{k}\}$ rad/s $\qquad \dot{\Omega} = \{3\mathbf{k}\}$ rad/s^2 $\qquad \mathbf{r}_A = \{3\mathbf{j}\}$ m

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{xyz} + \Omega \times \mathbf{r}_A = \mathbf{0} + (0.5\mathbf{k}) \times (3\mathbf{j}) = \{-1.5\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = [(\ddot{\mathbf{r}}_A)_{xyz} + \Omega \times (\dot{\mathbf{r}}_A)_{xyz}] + \dot{\Omega} \times \mathbf{r}_A + \Omega \times \dot{\mathbf{r}}_A$$

$$= \mathbf{0} + \mathbf{0} + (3\mathbf{k}) \times (3\mathbf{j}) + (0.5\mathbf{k}) \times (-1.5\mathbf{i})$$

$$= \{-9\mathbf{i} - 0.75\mathbf{j}\} \text{ m/s}^2$$

$\Omega_{xyz} = \{3\text{i}\}$ rad/s $\qquad \dot{\Omega}_{xyz} = 0$

$$\mathbf{r}_{B/A} = 20 \cos 30°\mathbf{j} + 20 \sin 30° = \{17.23\mathbf{j} + 10\mathbf{k}\} \text{ m}$$

$$(\mathbf{v}_{B/A})_{xyz} = \dot{\mathbf{r}}_{B/A} = (\dot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times r_{B/A}$$

$$= 0 + (3\mathbf{i}) \times (17.32\mathbf{j} + 10\mathbf{k})$$

$$= \{-30\mathbf{j} + 51.96\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{B/A})_{xyz} = \ddot{\mathbf{r}}_{B/A} = [(\ddot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{B/A})_{xyz}] + [\dot{\omega}_{xyz} \times \mathbf{r}_{B/A}] + [\Omega_{xyz} \times \dot{\mathbf{r}}_{B/A}]$$

$$(\mathbf{a}_{B/A})_{xyz} = \mathbf{0} + \mathbf{0} + \mathbf{0} + [(3\mathbf{i}) \times (-30\mathbf{j} + 51.9\mathbf{k})] - \{-155.8\mathbf{j} - 90\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= -1.5\mathbf{i} + [(0.5\mathbf{k}) \times (17.32\mathbf{j} + 10\mathbf{k})] + (-30\mathbf{j} + 51.96\mathbf{k})$$

$$= \{-10.2\mathbf{i} - 30\mathbf{j} + 52.0\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

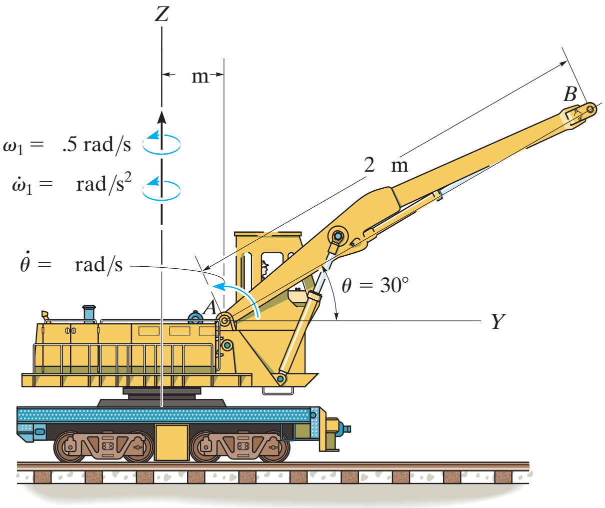
$$= (-9\text{i} - 0.75\mathbf{j}) + [(3\mathbf{k}) \times (17.32\mathbf{j} + 10\mathbf{k})] + 0.5\mathbf{k} \times [0.5\mathbf{k} \times (17.32\mathbf{j} + 10\mathbf{k})]$$

$$+ [2(0.5\mathbf{k}) \times (-30\mathbf{j} + 51.96\mathbf{k})] + (-155.88\mathbf{j} - 90\mathbf{k})$$

$$= \{-31.0\mathbf{i} - 161\mathbf{j} - 90\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

20–53.

The locomotive crane is traveling to the right at 2 m/s and has an acceleration of 1.5 m/s^2, while the boom is rotating about the Z axis with an angular velocity $\omega_1 = 0.5$ rad/s, which is increasing at $\dot{\omega}_1 = 3$ rad/s^2. At this same instant, $\theta = 30^\circ$ and the boom is rotating upward at a constant rate $\dot{\theta} = 3$ rad/s. Determine the velocity and acceleration of the tip B of the boom at this instant.



SOLUTION

$\Omega = \{0.5\mathbf{k}\}$ rad/s $\qquad \dot{\Omega} = \{3\mathbf{k}\}$ rad/s^2 $\qquad \mathbf{r}_A = \{3\mathbf{j}\}$ m

$$\mathbf{v}_A = \dot{\mathbf{r}}_A = (\dot{\mathbf{r}}_A)_{xyz} + \Omega \times \mathbf{r}_A = 2\mathbf{j} + (0.5\mathbf{k}) \times (3\mathbf{j}) = \{-1.5\mathbf{i} + 2\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_A = \ddot{\mathbf{r}}_A = [(\ddot{\mathbf{r}}_A)_{xyz} + \Omega \times (\dot{\mathbf{r}}_A)_{xyz}] + \dot{\Omega} \times \mathbf{r}_A + \Omega \times \dot{\mathbf{r}}_A$$

$$= 1.5\mathbf{j} + (0.5\mathbf{k}) \times (2\mathbf{j}) + (3\mathbf{k}) \times (3\mathbf{j}) + (0.5\mathbf{k}) \times (-1.5\mathbf{i} + 2\mathbf{j})$$

$$= \{-11\mathbf{i} + 0.75\mathbf{j}\} \text{ m/s}^2$$

$\Omega_{xyz} = \{3\text{i}\}$ rad/s $\qquad \dot{\Omega}_{xyz} = 0$

$$\mathbf{r}_{B/A} = 20 \cos 30^\circ \mathbf{j} + 20 \sin 30^\circ = \{17.32\mathbf{j} + 10\mathbf{k}\} \text{ m}$$

$$(\mathbf{v}_{B/A})_{xyz} = \dot{\mathbf{r}}_{B/A} = (\dot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{B/A}$$

$$= \mathbf{0} + (3\mathbf{i}) \times (17.32\mathbf{j} + 10\mathbf{k})$$

$$= \{-30\mathbf{j} + 51.96\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{B/A})_{xyz} = \ddot{\mathbf{r}}_{B/A} = [(\ddot{\mathbf{r}}_{B/A})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{B/A})_{xyz}] + [\dot{\Omega}_{xyz} \times \mathbf{r}_{B/A}] + [\Omega_{xyz} \times \dot{\mathbf{r}}_{B/A}]$$

$$(\mathbf{a}_{B/A})_{xyz} = \mathbf{0} + \mathbf{0} + \mathbf{0} + [(3\mathbf{i}) \times (-30\mathbf{j} + 51.96\mathbf{k})] = \{-155.88\mathbf{j} - 90\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= -1.5\mathbf{i} + 2\mathbf{j} + [(0.5\mathbf{k}) \times (17.32\mathbf{j} + 10\mathbf{k})] + (-30\mathbf{j} + 51.96\mathbf{k})$$

$$= \{-10.2\mathbf{i} - 28\mathbf{j} + 52.0\mathbf{k}\} \text{ m/s}$$ **Ans.**

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

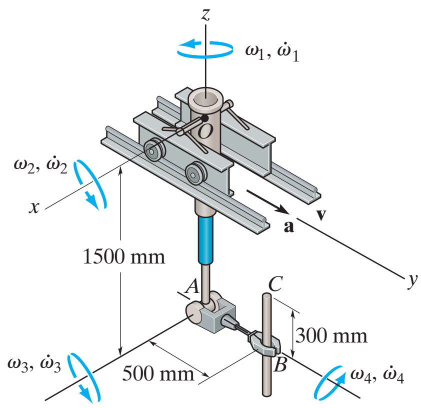
$$= (-11\mathbf{i} + 0.75\mathbf{j}) + [(3\mathbf{k}) \times (17.32\mathbf{j} + 10\mathbf{k})] + 0.5\mathbf{k} \times [0.5\mathbf{k} \times (17.32\mathbf{j} + 10\mathbf{k})]$$

$$+ [2(0.5\mathbf{k}) \times (-30\mathbf{j} + 51.96\mathbf{k})] + (-155.88\mathbf{j} - 90\mathbf{k})$$

$$= \{-33.0\mathbf{i} - 159\mathbf{i} - 90\mathbf{k}\} \text{ m/s}^2$$ **Ans.**

20–54.

The robot shown has four degrees of rotational freedom, namely, arm OA rotates about the x and z axes, arm AB rotates about the x axis, and CB rotates about the y axis. At the instant shown, $\omega_2 = 1.5$ rad/s, $\dot{\omega}_2 = 1$ rad/s^2, $\omega_3 = 3$ rad/s, $\dot{\omega}_3 = 0.5$ rad/s^2, $\omega_4 = 6$ rad/s, $\dot{\omega}_4 = 3$ rad/s^2, and $\omega_1 = \dot{\omega}_1 = 0$. If the robot does not translate, i.e., $\mathbf{v} = \mathbf{a} = \mathbf{0}$, determine the velocity and acceleration of point C at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_2 = [1.5\mathbf{i}] \text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_2 = [1\mathbf{i}] \text{ rad/s}^2$$

Here, arm OA rotates about a fixed axis (X axis), the motion of point A can be determined from

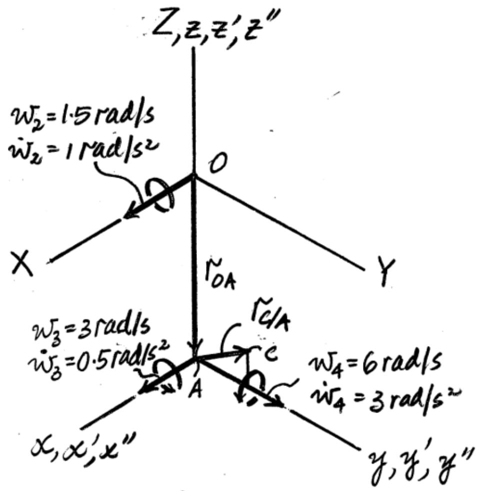
$$\mathbf{v}_A = \omega_2 \times \mathbf{r}_{OA} = (1.5\mathbf{i}) \times (-1.5\mathbf{k}) = [2.25\mathbf{j}] \text{ m/s}$$

and

$$a_A = \dot{\omega}_2 \times \mathbf{r}_{A/O} + \omega_2 \times (\omega_2 \times \mathbf{r}_{OA})$$

$$= (1\mathbf{i}) \times (-1.5\mathbf{k}) + (1.5\mathbf{i}) \times [(1.5\mathbf{i}) \times (-1.5\mathbf{k})]$$

$$= [1.5\mathbf{j} + 3.375\mathbf{k}] \text{ m/s}^2$$



(a)

In order to determine the motion of point C relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity of $\Omega'_{xyz} = \omega_3 + \omega_4 = [3\mathbf{i} + 6\mathbf{j}]$ rad/s, the direction of $\mathbf{r}_{C/A}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $\mathbf{r}_{C/A}$,

$$(\mathbf{v}_{C/A})_{xyz} = (\dot{\mathbf{r}}_{C/A})_{xyz} = [(\dot{\mathbf{r}}_{C/A})_{x'y'z'} + \Omega'_{xyz} \times \mathbf{r}_{C/A}]$$

$$= 0 + (3\mathbf{i} + 6\mathbf{j}) \times (0.5\mathbf{j} + 0.3\mathbf{k})$$

$$= \{1.8\mathbf{i} - 0.9\mathbf{j} + 1.5\mathbf{k}\} \text{ m/s}$$

Here, the direction of Ω' changes with respect to the xyz frame. To account for this change a third $x''y''z''$ rotating frame is set to coincide with the xyz frame, Fig. a. If the angular velocity of $x''y''z''$ is $\Omega''_{xyz} = \omega_3 = \{3\mathbf{i}\}$ rad/s, then the direction of ω_4 will remain unchanged with respect to the $x''y''z''$ frame. Thus,

$$(\dot{\omega}_4)_{xyz} = (\dot{\omega}_4)_{x''y''z''} + \omega_3 \times \omega_4 = 3\mathbf{j} + (3\mathbf{i} \times 6\mathbf{j}) = \{3\mathbf{j} + 18\mathbf{k}\} \text{ rad/s}^2$$

Since ω_3 has a constant direction with respect to the xyz frame when $\Omega'' = \omega_3$, then

$$(\dot{\omega}_3)_{xyz} = (\dot{\omega}_3)_{x''y''z''} + \omega_3 \times \omega_3 = \{0.5\mathbf{i}\} \text{ rad/s}^2$$

Thus,

$$(\dot{\Omega}')_{xyz} = (\dot{\omega}_4)_{xyz} + (\dot{\omega}_3)_{xyz} = (3\mathbf{j} + 18\mathbf{k}) + (0.5\mathbf{i}) = \{0.5\mathbf{i} + 3\mathbf{j} + 18\mathbf{k}\} \text{ rad/s}^2$$

Taking the time derivate of $(\dot{\mathbf{r}}_{C/A})_{xyz}$,

$$(\mathbf{a}_{C/A})_{xyz} = (\ddot{\mathbf{r}}_{C/A})_{xyz} = [(\ddot{\mathbf{r}}_{C/A})_{x'y'z'} + \Omega'_{xyz} \times (\dot{r}_{C/A})_{x'y'z'}]$$
$$+ \dot{\Omega}'_{xyz} \times \mathbf{r}_{C/A} + \Omega'_{xyz} \times (\dot{\mathbf{r}}_{C/A})_{xyz}$$

$$= [0 + 0] + (0.5\mathbf{i} + 3\mathbf{j} + 18\mathbf{k}) \times (0.5\mathbf{j} + 0.3\mathbf{k}) + (3\mathbf{i} + 6\mathbf{j}) \times (1.8\mathbf{i} - 0.9\mathbf{j} + 1.5\mathbf{k})$$

$$= \{0.9\mathbf{i} - 4.65\mathbf{j} - 13.25\mathbf{k}\} \text{ m/s}^2$$

20–54. continued

Thus,

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$= (2.25\mathbf{j}) + (1.5\mathbf{i}) \times (0.5\mathbf{j} + 0.3\mathbf{k}) + (1.8\mathbf{i} - 0.9\mathbf{j} + 1.5\mathbf{k})$$

$$= \{1.8\mathbf{i} + 0.9\mathbf{j} + 2.25\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$= (1.5\mathbf{j} + 3.375\mathbf{k}) + 1\mathbf{i} \times (0.5\mathbf{j} + 0.3\mathbf{k}) + (1.5\mathbf{i}) \times [(1.5\mathbf{i}) \times (0.5\mathbf{j}) + 0.3\mathbf{k})]$$

$$+ 2(1.5\mathbf{i}) \times (1.8\mathbf{i} - 0.9\mathbf{j} + 1.5\mathbf{k}) + (0.9\mathbf{i} - 4.65\mathbf{j} - 13.25\mathbf{k})$$

$$= \{0.9\mathbf{i} - 9.075\mathbf{j} - 12.75\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

21–1.

Show that the sum of the moments of inertia of a body, $I_{xx} + I_{yy} + I_{zz}$, is independent of the orientation of the x, y, z axes and thus depends only on the location of its origin.

SOLUTION

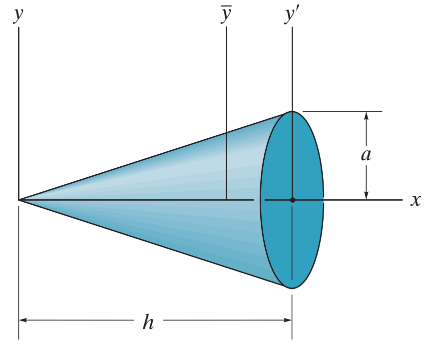
$$I_{xx} + I_{yy} + I_{zz} = \int_m (y^2 + z^2)dm + \int_m (x^2 + z^2)dm + \int_m (x^2 + y^2)dm$$

$$= 2\int_m (x^2 + y^2 + z^2)dm$$

However, $x^2 + y^2 + z^2 = r^2$, where r is the distance from the origin O to dm. Since $|r|$ is constant, it does not depend on the orientation of the x, y, z axis. Consequently, $I_{xx} + I_{yy} + I_{zz}$ is also indepenent of the orientation of the x, y, z axis. **Q.E.D.**

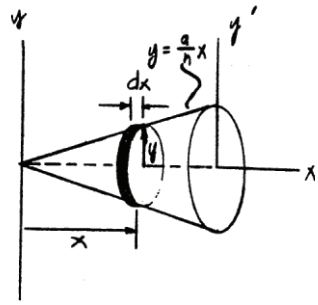
21–2.

Determine the moment of inertia of the cone with respect to a vertical $\bar{y}$ axis passing through the cone's center of mass. What is the moment of inertia about a parallel axis y' that passes through the diameter of the base of the cone? The cone has a mass m.



SOLUTION

The mass of the differential element is $dm = \rho dV = \rho(\pi y^2)\,dx = \dfrac{\rho\pi a^2}{h^2}x^2 dx$.



$$dI_y = \frac{1}{4}dmy^2 + dmx^2$$

$$= \frac{1}{4}\left[\frac{\rho\pi a^2}{h^2}x^2\,dx\right]\left(\frac{a}{h}x\right)^2 + \left(\frac{\rho\pi a^2}{h^2}x^2\right)x^2\,dx$$

$$= \frac{\rho\,\pi a^2}{4h^4}(4h^2 + a^2)\,x^4\,dx$$

$$I_y = \int dI_y = \frac{\rho\pi a^2}{4h^4}(4h^2 + a^2)\int_0^h x^4 dx = \frac{\rho\,\pi a^2 h}{20}(4h^2 + a^2)$$

However,

$$m = \int_m dm = \frac{\rho\pi a^2}{h^2}\int_0^h x^2\,dx = \frac{\rho\,\pi a^2 h}{3}$$

Hence,

$$I_y = \frac{3m}{20}(4h^2 + a^2)$$

Using the parallel axis theorem:

$$I_y = I_{\bar{y}} + md^2$$

$$\frac{3m}{20}(4h^2 + a^2) = I_{\bar{y}} + m\left(\frac{3h}{4}\right)^2$$

$$I_{\bar{y}} = \frac{3m}{80}(h^2 + 4a^2) \qquad \textbf{Ans.}$$

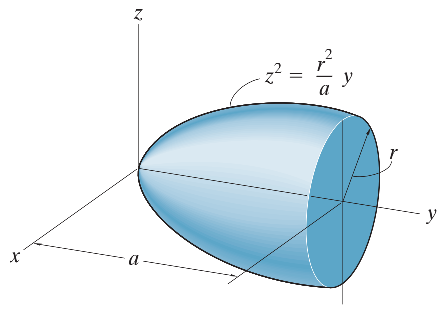
$$I_{y'} = I_{\bar{y}} + md^2$$

$$= \frac{3m}{80}(h^2 + 4a^2) + m\left(\frac{h}{4}\right)^2$$

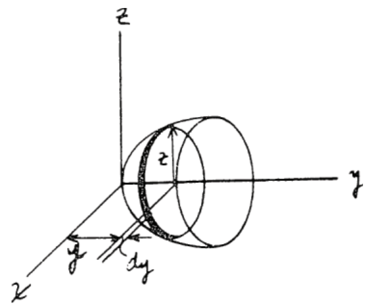
$$= \frac{m}{20}(2h^2 + 3a^2) \qquad \textbf{Ans.}$$

21–3.

Determine the moments of inertia I_x and I_y of the paraboloid of revolution. The mass of the paraboloid is m.



SOLUTION



$$m = \rho\int_0^a \pi z^2\,dy = \rho\pi\int_0^a\left(\frac{r^2}{a}\right)\bar{y}\,dy = \rho\pi\left(\frac{r^2}{2}\right)a$$

$$I_y = \int_m \frac{1}{2}dm\,z^2 = \frac{1}{2}\rho\pi\int_0^a z^4dy = \frac{1}{2}\rho\pi\left(\frac{r^4}{a^2}\right)\int_0^a y^2\,dy = \rho\pi\left(\frac{r^4}{6}\right)a$$

Thus,

$$I_y = \frac{1}{3}mr^2 \qquad \textbf{Ans.}$$

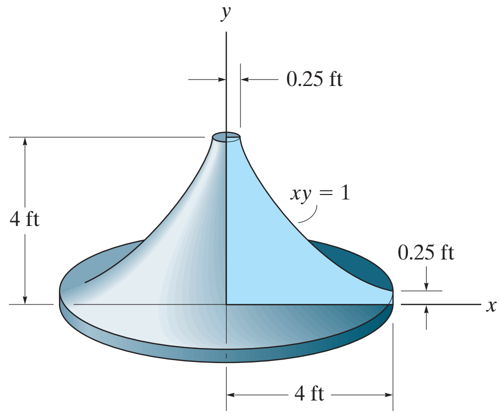
$$I_x = \int_m\left(\frac{1}{4}dm\,z^2 + dm\,y^2\right) = \frac{1}{4}\rho\pi\int_0^a z^4\,dy + \rho\int_0^a \pi z^2\,y^2\,dy$$

$$= \frac{1}{4}\rho\pi\left(\frac{r^4}{a^2}\right)\int_0^a y^2\,dy + \rho\pi\left(\frac{r^2}{a}\right)\int_0^a y^3dy = \frac{\rho\pi r^4 a}{12} + \frac{\rho\pi r^2 a^3}{4} = \frac{1}{6}mr^2 + \frac{1}{2}ma^2$$

$$I_x = \frac{m}{6}(r^2 + 3a^2) \qquad \textbf{Ans.}$$

***21–4.**

Determine the radii of gyration k_x and k_y for the solid formed by revolving the shaded area about the y axis. The density of the material is ρ.



SOLUTION

For k_y: The mass of the differential element is $dm = \rho dV = \rho(\pi x^2)\,dy = \rho\pi \dfrac{dy}{y^2}$.

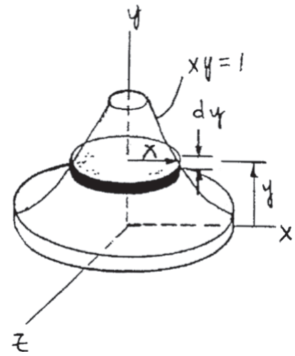
$$dI_y = \tfrac{1}{2}dmx^2 = \tfrac{1}{2}\left[\rho\pi\tfrac{dy}{y^2}\right]\left(\tfrac{1}{y^2}\right) = \tfrac{1}{2}\rho\pi\tfrac{dy}{y^4}$$

$$I_y = \int dI_y = \tfrac{1}{2}\rho\pi\int_{0.25}^{4}\tfrac{dy}{y^4} + \frac{1}{2}\left[\rho(\pi)(4)^2(0.25)\right](4)^2$$

$$= 134.03\rho$$

However, $$m = \int_m dm = \rho\pi\int_{0.25}^{4}\tfrac{dy}{y^2} + \rho\left[\pi(4)^2(0.25)\right] = 24.35\rho$$

Hence, $$k_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{134.03\rho}{24.35\rho}} = 2.35\text{ ft}$$ **Ans.**



For k_x: $0.25\text{ ft} < \text{y} \leq 4\text{ ft}$

$$dI'_x = \frac{1}{4}dmx^2 + dmy^2$$

$$= \frac{1}{4}\left[\rho\pi\tfrac{dy}{y}\right]\left(\frac{\mathrm{I}}{\mathrm{y}^2}\right) + \left(\rho\pi\tfrac{dy}{y}\right)y^2$$

$$= \rho\pi\left(\tfrac{1}{4y^4} + 1\right)dy$$

$$I'_x = \int dI'_x = \rho\pi\int_{0.25}^{4}\left(\tfrac{1}{4y^4} + 1\right)dy = 28.53\rho$$

$$I''_x = \tfrac{1}{4}\left[\rho\pi(4)^2(0.25)\right](4)^2 + \left[\rho\pi(4)^2(0.25)\right](0.125)^2$$

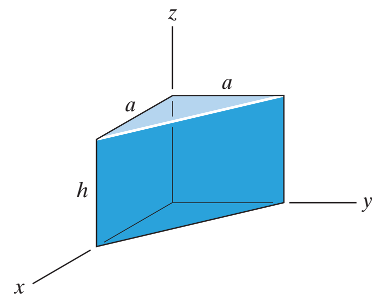
$$= 50.46\rho$$

$$I_x = I'_x + I''_x = 28.53\rho + 50.46\rho = 78.99\rho$$

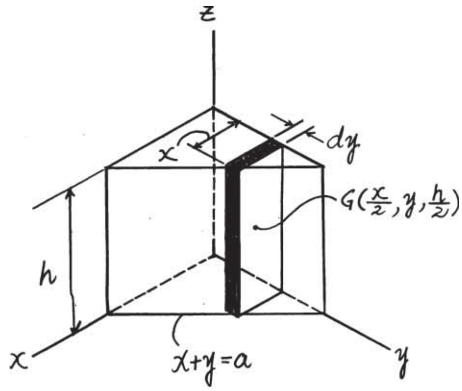
Hence, $$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{78.99\rho}{24.35\rho}} = 1.80\text{ ft}$$ **Ans.**

21–5.

Determine by direct integration the product of inertia I_{yz} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the total mass m of the prism.



SOLUTION



The mass of the differential element is $dm = \rho dV = \rho hxdy = \rho h(a - y)dy$.

$$m = \int_m dm = \rho h \int_0^a (a - y)dy = \frac{\rho a^2 h}{2}$$

Using the parallel axis theorem:

$$dI_{yz} = (dI_{y'z'})_G + dm y_G z_G$$

$$= 0 + (\rho hxdy)(y)\left(\frac{h}{2}\right)$$

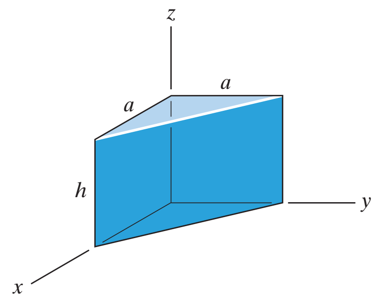
$$= \frac{\rho h^2}{2} xydy$$

$$= \frac{\rho h^2}{2}(ay - y^2)\,dy$$

$$I_{yz} = \frac{\rho h^2}{2}\int_0^a (ay - y^2)\,dy = \frac{\rho a^3 h^2}{12} = \frac{1}{6}\left(\frac{\rho a^2 h}{2}\right)(ah) = \frac{m}{6}ah \qquad \textbf{Ans.}$$

21–6.

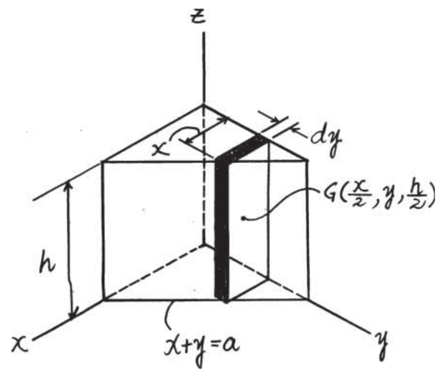
Determine by direct integration the product of inertia I_{xy} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the total mass m of the prism.



SOLUTION

The mass of the differential element is $dm = \rho dV = \rho hxdy = \rho h(a - y)dy$.

$$m = \int_m dm = \rho h \int_0^a (a - y)dy = \frac{\rho a^2 h}{2}$$



Using the parallel axis theorem:

$$dI_{xy} = (dI_{x'y'})_G + dm x_G y_G$$

$$= 0 + (\rho hxdy)\left(\frac{x}{2}\right)(y)$$

$$= \frac{\rho h^2}{2} x^2 y dy$$

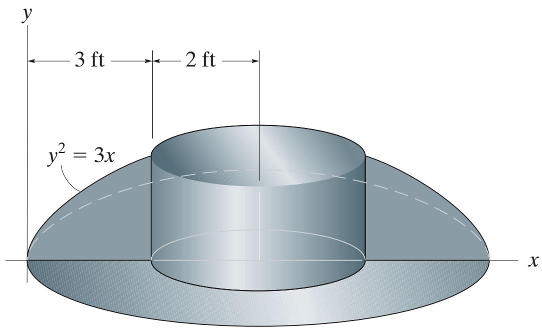
$$= \frac{\rho h^2}{2}(y^3 - 2ay^2 + a^2 y)\, dy$$

$$I_{xy} = \frac{\rho h}{2}\int_0^a (y^3 - 2ay^2 + a^2 y)\, dy$$

$$= \frac{\rho a^4 h}{24} = \frac{1}{12}\left(\frac{\rho a^2 h}{2}\right)a^2 = \frac{m}{12}a^2 \qquad \textbf{Ans.}$$

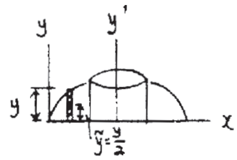
21–7.

Determine the product of inertia I_{xy} of the object formed by revolving the shaded area about the line $x = 5$ ft. Express the result in terms of the density of the material, ρ.



SOLUTION

$$\int_0^3 dm = \rho 2\pi \int_0^3 (5 - x)y\,dx = \rho 2\pi \int_0^3 (5 - x)\sqrt{3x}\,dx = 38.4\rho\pi$$



$$\int_0^3 \tilde{y}\,dm = \rho 2\pi \int_0^3 \frac{y}{2}(5 - x)y\,dx$$

$$= \rho\pi \int_0^3 (5 - x)(3x)\,dx$$

$$= 40.5\rho\pi$$

Thus, $\bar{y} = \dfrac{40.5\rho\pi}{38.4\rho\pi} = 1.055$ ft

The solid is symmetric about y, thus

$$I_{xy'} = 0$$

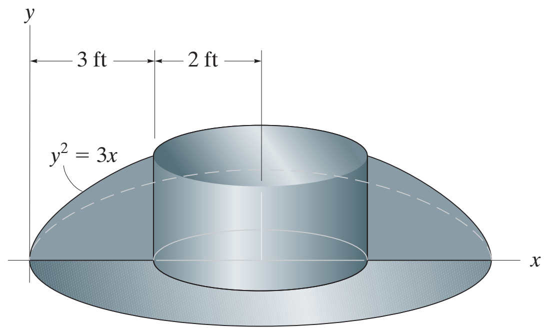
$$I_{xy} = I_{xy'} + \bar{x}\bar{y}m$$

$$= 0 + 5(1.055)(38.4\rho\pi)$$

$$I_{xy} = 636\rho$$ **Ans.**

***21–8.**

Determine the moment of inertia I_y of the object formed by revolving the shaded area about the line $x = 5$ ft. Express the result in terms of the density of the material, ρ.



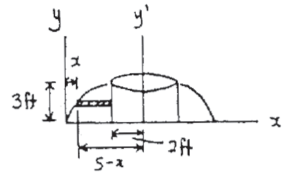
SOLUTION

$$I_{y'} = \int_0^3 \frac{1}{2}\, dm\, r^2 - \frac{1}{2}(m')(2)^2$$

$$\int_0^3 \frac{1}{2} dm\, r^2 = \frac{1}{2}\int_0^3 \rho\,\pi(5 - x)^4\, dy$$

$$= \frac{1}{2}\rho\,\pi \int_0^3 \left(5 - \frac{y^2}{3}\right)^4 dy$$

$$= 490.29\,\rho\,\pi$$



$$m' = \rho\,\pi\,(2)^2(3) = 12\,\rho\,\pi$$

$$I_{y'} = 490.29\,\rho\,\pi - \frac{1}{2}(12\,\rho\,\pi)(2)^2 = 466.29\,\rho\,\pi$$

Mass of body;

$$m = \int_0^3 \rho\,\pi\,(5 - x)^2\, dy - m'$$

$$= \int_0^3 \rho\,\pi\,(5 - \frac{y^2}{3})^2\, dy - 12\,\rho\,\pi$$

$$= 38.4\,\rho\,\pi$$

$$I_y = 466.29\,\rho\,\pi + (38.4\,\rho\,\pi)(5)^2$$

$$= 1426.29\,\rho\,\pi$$

$$I_y = 4.48(10^3)\,\rho$$ **Ans.**

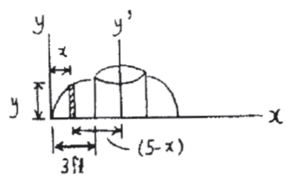
Also,

$$I_{y'} = \int_0^3 r^2\, dm$$

$$= \int_0^3 (5 - x)^2\,\rho(2\pi)(5 - x)y\, dx$$

$$= 2\,\rho\,\pi \int_0^3 (5 - x)^3(3x)^{1/2}\, dx$$

$$= 466.29\,\rho\,\pi$$



$$m = \int_0^3 dm$$

$$= 2\,\rho\,\pi \int_0^3 (5 - x)y\, dx$$

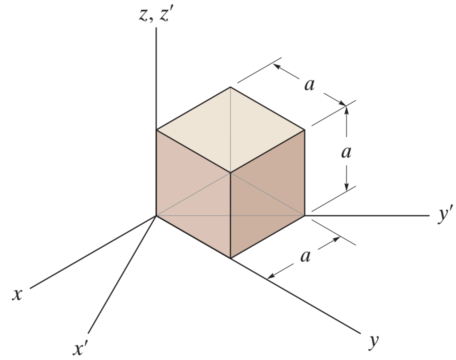
$$= 2\,\rho\,\pi \int_0^3 (5 - x)(3x)^{1/2}\, dx$$

$$= 38.4\,\rho\,\pi$$

$$I_y = 466.29\,\rho\,\pi + 38.4\,\rho\,\pi(5)^2 = 4.48(10^3)\rho$$ **Ans.**

21–9.

Determine the elements of the inertia tensor for the cube with respect to the x, y, z coordinate system. The mass of the cube is m.



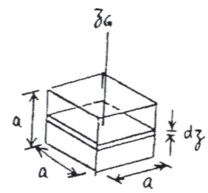
SOLUTION

$$dI_{zG} = \frac{1}{12}(dm)(a^2 + a^2)$$

$$I_{zG} = \frac{1}{6}a^2 \int_0^a \rho(a^2)\, dz$$

$$= \frac{1}{6}a^2 (\rho a^3)$$

$$= \frac{1}{6}ma^2$$

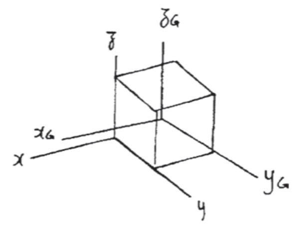


$$I_x = I_y = I_z = \frac{1}{6}ma^2 + m[(\frac{a}{2})^2 + (\frac{a}{2})^2] = \frac{2}{3}ma^2$$

$$I_{xy} = 0 + m(-\frac{a}{2})(\frac{a}{2}) = -\frac{1}{4}ma^2$$

$$I_{yz} = 0 + m(\frac{a}{2})(\frac{a}{2}) = \frac{1}{4}ma^2$$

$$I_{xz} = 0 + m(-\frac{a}{2})(\frac{a}{2}) = -\frac{1}{4}ma^2$$



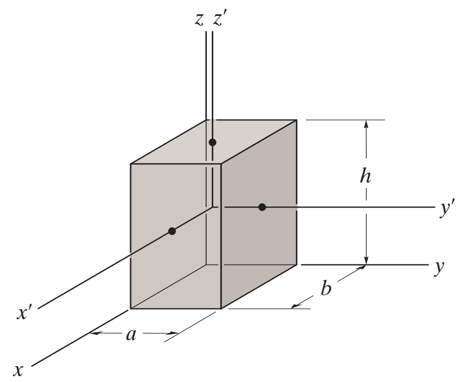
Changing the signs of the produts of inertia as represented by the equation in the text, we have

$$\begin{pmatrix} \frac{2}{3}ma^2 & \frac{1}{4}ma^2 & \frac{1}{4}ma^2 \\ \frac{1}{4}ma^2 & \frac{2}{3}ma^2 & -\frac{1}{4}ma^2 \\ \frac{1}{4}ma^2 & -\frac{1}{4}ma^2 & \frac{2}{3}ma^2 \end{pmatrix}$$

Ans.

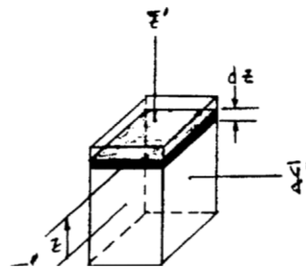
21–10.

Determine the mass moment of inertia of the homogeneous block with respect to its centroidal x' axis. The mass of the block is m.



SOLUTION

The mass of the differential element is $dm = \rho dV = \rho abdz$.



$$dI_{x'} = \frac{1}{12}dma^2 + dmz^2$$

$$= \frac{1}{12}(\rho abdz)a^2 + (\rho abdz)z^2$$

$$= \frac{\rho ab}{12}(a^2 + 12z^2)\,dz$$

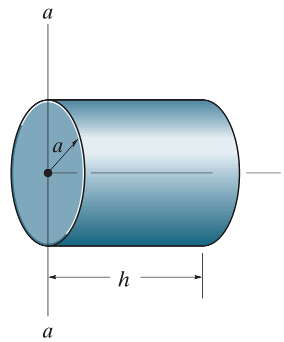
$$I_{x'} = \int dI_{x'} = \frac{\rho ab}{12}\int_{-\frac{h}{2}}^{\frac{h}{2}}(a^2 + 12z^2)\,dz = \frac{\rho abh}{12}(a^2 + h^2)$$

However, $$m = \int_m dm = \rho ab\int_{-\frac{h}{2}}^{\frac{h}{2}} dz = \rho abh$$

Hence, $$I_{x'} = \frac{m}{12}(a^2 + h^2)$$ **Ans.**

21–11.

Determine the moment of inertia of the cylinder with respect to the a–a axis of the cylinder. The cylinder has a mass m.



SOLUTION

The mass of the differential element is $dm = \rho dV = \rho\left(\pi a^2\right) dy$.

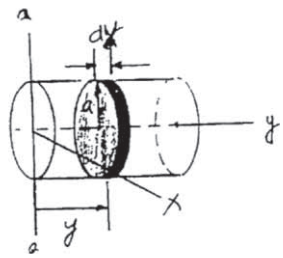
$$dI_{aa} = \tfrac{1}{4} dma^2 + dm\left(y^2\right)$$

$$= \tfrac{1}{4}\left[\rho\left(\pi a^2\right) dy\right]a^2 + \left[\rho\left(\pi a^2\right) dy\right]y^2$$

$$= \left(\tfrac{1}{4}\rho\pi a^4 + \rho\pi a^2 y^2\right) dy$$

$$I_{aa} = \int dI_{aa} = \int_0^h \left(\tfrac{1}{4}\rho\pi a^4 + \rho\pi a^2 y^2\right) dy$$

$$= \frac{\rho\pi a^2 h}{12}(3a^2 + 4h^2)$$

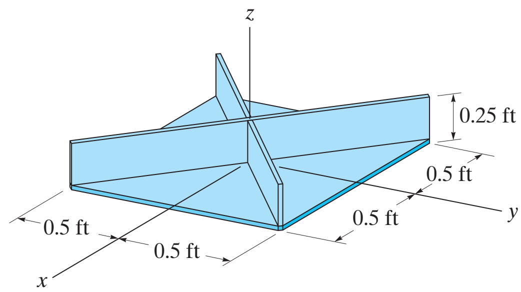


However, $$m = \int_m dm = \int_0^h \rho\left(\pi a^2\right) dy = \rho\pi a^2 h$$

Hence, $$I_{aa} = \frac{m}{12}\left(3a^2 + 4h^2\right)$$ **Ans.**

***21–12.**

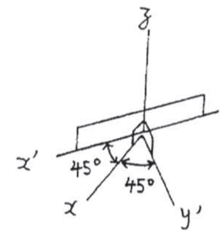
Determine the moment of inertia I_x of the composite plate assembly. The plates have a specific weight of 6 lb/ft^2.



SOLUTION

Horizontial plate:

$$I_{xx} = \frac{1}{12}\left(\frac{6(1)(1)}{32.2}\right)(1)^2 = 0.0155$$



Vertical plates:

$$l_{xx'} = 0.707, \qquad l_{xy'} = 0.707, \qquad l_{xz'} = 0$$

$$I_{x'x'} = \frac{1}{3}\left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)\left(\frac{1}{4}\right)^2 = 0.001372$$

$$I_{y'y'} = \left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)\left(\frac{1}{12}\right)\left[\left(\frac{1}{4}\right)^2 + (1\sqrt{2})^2\right] + \left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)\left(\frac{1}{8}\right)^2$$

$$= 0.01235$$

Using Eq. 21–5,

$$I_{xx} = (0.707)^2(0.001372) + (0.707)^2(0.01235)$$

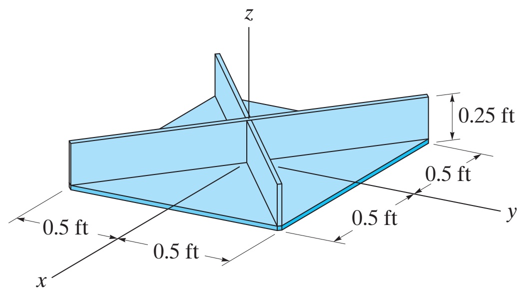
$$= 0.00686$$

Thus,

$$I_{xx} = 0.0155 + 2(0.00686) = 0.0292\ \text{slug} \cdot \text{ft}^2 \qquad \textbf{Ans.}$$

21–13.

Determine the product of inertia I_{yz} of the composite plate assembly. The plates have a weight of 6 lb/ft^2.



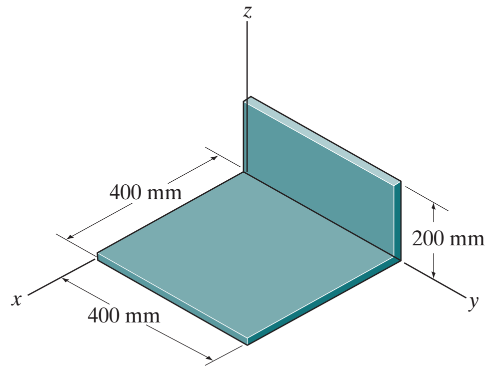
SOLUTION

Due to symmetry,

$I_{yz} = 0$ **Ans.**

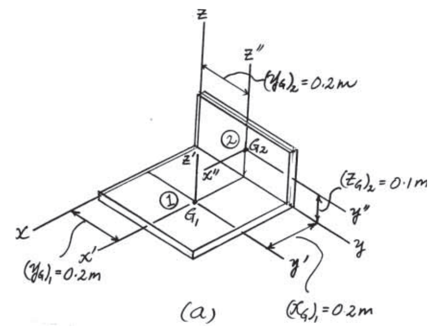
21–14.

Determine the products of inertia I_{xy}, I_{yz}, and I_{xz}, of the thin plate. The material has a density per unit area of $50\ \text{kg/m}^2$.



SOLUTION

The masses of segments (1) and (2) shown in Fig. *a* are $m_1 = 50(0.4)(0.4) = 8\ \text{kg}$ and $m_2 = 50(0.4)(0.2) = 4\ \text{kg}$. Due to symmetry $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{x'z'} = 0$ for segment (1) and $\bar{I}_{x''y''} = \bar{I}_{y''z''} = \bar{I}_{x''z''} = 0$ for segment (2).



$$I_{xy} = \Sigma \bar{I}_{x'y'} + m x_G y_G$$

$$= \left[0 + 8(0.2)(0.2)\right] + \left[0 + 4(0)(0.2)\right]$$

$$= 0.32\ \text{kg} \cdot \text{m}^2$$ **Ans.**

$$I_{yz} = \Sigma \bar{I}_{y'z'} + m y_G z_G$$

$$= \left[0 + 8(0.2)(0)\right] + \left[0 + 4(0.2)(0.1)\right]$$

$$= 0.08\ \text{kg} \cdot \text{m}^2$$ **Ans.**

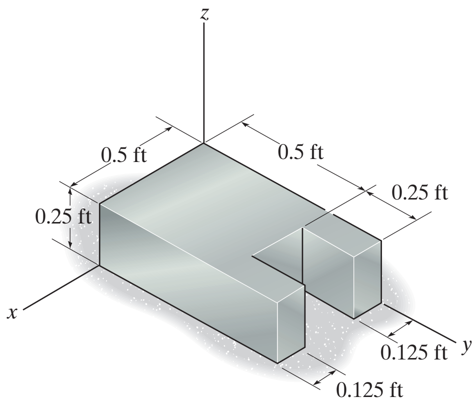
$$I_{xz} = \Sigma \bar{I}_{x'z'} + m x_G z_G$$

$$= \left[0 + 8(0.2)(0)\right] + \left[0 + 4(0)(0.1)\right]$$

$$= 0$$ **Ans.**

21–15.

Determine the products of inertia I_{xy}, I_{yz}, and I_{xz} of the solid. The material is steel, which has a specific weight of $490\ \text{lb/ft}^3$.



SOLUTION

Consider the block to be made of a large one, 1, minus the slot 2. Thus, $m_1 = \dfrac{\gamma}{g}V_1 = \dfrac{490}{32.2}(0.5)(0.75)(0.25) = 1.4266$ slug and $m_2 = \dfrac{\gamma}{g}V_2 = \dfrac{490}{32.2}(0.25)(0.25)(0.25) = 0.2378$ slug. Due to symmetry $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{x'z'} = 0$ and $\bar{I}_{x''y''} = \bar{I}_{y''z''} = \bar{I}_{x''z''} = 0$.

Since the slot 2 is a hole, it should be considered as a negative segment. Thus

$$I_{xy} = \Sigma(\bar{I}_{x'y'} + m x_G y_G)$$

$$= [0 + 1.4266(0.25)(0.375)] - [0 + 0.2378(0.25)(0.625)]$$

$$= 0.0966\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans.}$$

$$I_{yz} = \Sigma(\bar{I}_{y'z'} + m y_G z_G)$$

$$= [0 + 1.4266(0.375)(0.125)] - [0 + 0.2378(0.625)(0.125)]$$

$$= 0.0483\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans.}$$

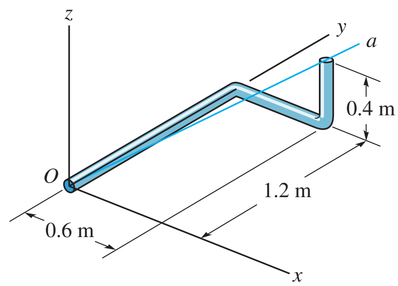
$$I_{xz} = \Sigma(\bar{I}_{x'z'} + m x_G z_G)$$

$$= [0 + 1.4266(0.25)(0.125)] - [0 + 0.2378(0.25)(0.125)]$$

$$= 0.0372\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans.}$$

***21–16.**

The bent rod has a mass of 4 kg/m. Determine the moment of inertia of the rod about the Oa axis.



SOLUTION

$$I_{xy} = [4(1.2)](0)(0.6) + [4(0.6)](0.3)(1.2) + [4(0.4)](0.6)(1.2) = 2.016 \text{ kg}\cdot\text{m}^2$$

$$I_{yz} = [4(1.2)](0.6)(0) + [4(0.6)](1.2)(0) + [4(0.4)](1.2)(0.2) = 0.384 \text{ kg}\cdot\text{m}^2$$

$$I_{zx} = [4(1.2)](0)(0) + [4(0.6)](0)(0.3) + [4(0.4)](0.2)(0.6) = 0.192 \text{ kg}\cdot\text{m}^2$$

$$I_x = \frac{1}{3}[4(1.2)](1.2)^2 + [4(0.6)](1.2)^2 + \left[\frac{1}{12}[4(0.4)](0.4)^2 + [4(0.4)](1.2^2 + 0.2^2)\right]$$

$$= 8.1493 \text{ kg}\cdot\text{m}^2$$

$$I_y = 0 + \frac{1}{3}[4(0.6)](0.6)^2 + \left[\frac{1}{12}[4(0.4)](0.4)^2 + [4(0.4)]\left(0.6^2 + 0.2^2\right)\right]$$

$$= 0.9493 \text{ kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{3}[4(1.2)](1.2)^2 + \left[\frac{1}{12}[4(0.6)](0.6)^2 + [4(0.6)](0.3^2 + 1.2^2)\right] + [4(0.4)](1.2^2 + 0.6^2)$$

$$= 8.9280 \text{ kg}\cdot\text{m}^2$$

$$vu\frac{0.6\mathbf{i} + 1.2\mathbf{j} + 0.4\mathbf{k}}{\sqrt{0.6^2 + 1.2^2 + 0.4^2}} = \frac{3}{7}\mathbf{i} + \frac{6}{6}\mathbf{j} + \frac{2}{7}\mathbf{k}$$

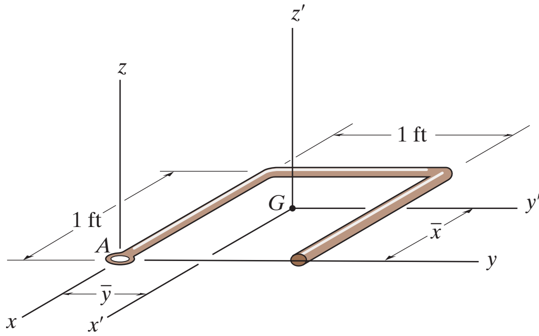
$$I_{Oa} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy}u_x u_y - 2I_{yz}u_y u_z - 2I_{zx}u_z u_x$$

$$= 8.1493\left(\frac{3}{7}\right)^2 + 0.9493\left(\frac{6}{7}\right)^2 + 8.9280\left(\frac{2}{7}\right)^2 - 2(2.016)\left(\left(\frac{3}{7}\right)\left(\frac{6}{7}\right) - 290.384\right)\left(\frac{6}{7}\right)\left(\frac{2}{7}\right) - 2(0.192)\left(\frac{2}{7}\right)\left(\frac{3}{7}\right)$$

$$= 1.21 \text{ kg}\cdot\text{m}^2$$ **Ans.**

21–17.

The bent rod has a weight of 1.5 lb/ft. Locate the center of gravity $G(\bar{x}, \bar{y})$ and determine the principal moments of inertia $I_{x'}$, $I_{y'}$, and $I_{z'}$ of the rod with respect to the x', y', z' axes.



SOLUTION

Due to symmetry

$\bar{y} = 0.5$ ft **Ans.**

$$\bar{x} = \frac{\Sigma \bar{x} W}{\Sigma w} = \frac{(-1)(1.5)(1) + 2\left[(-0.5)(1.5)(1)\right]}{3\left[1.5(1)\right]} = -0.667 \text{ ft}$$ **Ans.**

$$I_{x'} = 2\left[\left(\frac{1.5}{32.2}\right)(0.5)^2\right] + \frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2$$

$$= 0.0272 \text{ slug} \cdot \text{ft}^2$$ **Ans.**

$$I_{y'} = 2\left[\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.667 - 0.5)^2\right] + \left(\frac{1.5}{32.2}\right)(1 - 0.667)_2$$

$$= 0.0155 \text{ slug} \cdot \text{ft}^2$$ **Ans.**

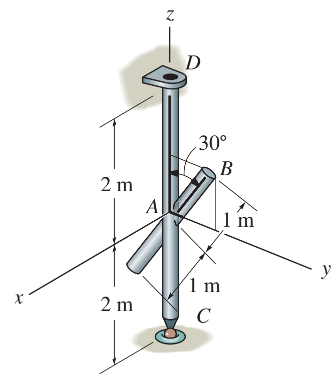
$$I_{z'} = 2\left[\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.5^2 + 0.1667^2)\right]$$

$$+ \frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.3333)^2$$

$$= 0.0427 \text{ slug} \cdot \text{ft}^2$$ **Ans.**

21–18.

Determine the moments of inertia about the x, y, z axes of the rod assembly. The rods have a mass of 0.75 kg/m.



SOLUTION

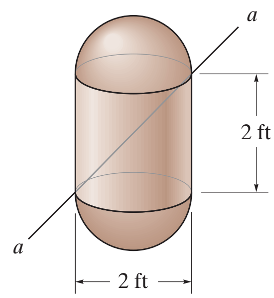
$$I_x = \frac{1}{12}\left[0.75(4)\right](4)^2 + \frac{1}{12}\left[0.75(2)\right](2)^2 = 4.50\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_y = \frac{1}{12}\left[0.75(4)\right](4)^2 + \frac{1}{12}\left[0.75(2)\right](2\cos 30^\circ)^2 = 4.38\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

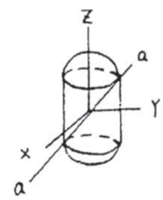
$$I_z = 0 + \frac{1}{12}\left[0.75(2)\right](2\sin 30^\circ)^2 = 0.125\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

21–19.

Determine the moment of inertia of the composite body about the *aa* axis. The cylinder weighs 20 lb, and each hemisphere weighs 10 lb.



SOLUTION



$u_{az} = 0.707$

$u_{ax} = 0$

$u_{ay} = 0.707$

$$I_{zz} = \frac{1}{2}\left(\frac{20}{32.2}\right)(1)^2 + 2\left[\frac{2}{5}\left(\frac{10}{32.2}\right)(1)^2\right]$$

$$= 0.5590 \text{ slug} \cdot \text{ft}^2$$

$$I_{xx} = I_{yy} = \frac{1}{12}\left(\frac{20}{32.2}\right)[3(1)^2 + (2)^2] + 2\left[0.259\left(\frac{10}{32.2}\right)(1)^2 + \frac{10}{32.2}\left(\frac{11}{8}\right)^2\right]$$

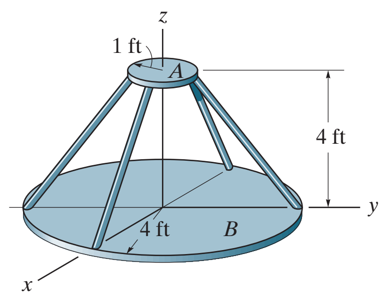
$$I_{xx} = I_{yy} = 1.6975 \text{ slug} \cdot \text{ft}^2$$

$$I_{aa} = 0 + (0.707)^2(1.6975) + (0.707)^2(0.559)$$

$I_{aa} = 1.13 \text{ slug} \cdot \text{ft}^2$ **Ans.**

***21–20.**

The assembly consists of a 15-lb plate A, 40-lb plate B, and four 7-lb rods. Determine the moments of inertia of the assembly with respect to the principal x, y, z axes.



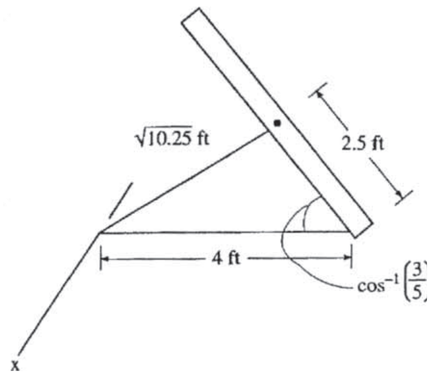
SOLUTION

Due to symmetry

$$\bar{y} = \bar{x} = 0$$

$$\bar{z} = \frac{\Sigma \bar{z} w}{\Sigma w} = \frac{4(15) + 0(40) + 2(28)}{15 + 40}$$

$$= 1.3976 \text{ ft}$$



$$I_z = (I_z)_{\text{upper}} + (I_z)_{\text{lower}} + (I_z)_{\text{rods}}$$

$$= \frac{1}{2}\left(\frac{15}{32.2}\right)(1)^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)(4)^2$$

$$+ 4\left[\frac{1}{12}\left(\frac{7}{32.2}\right)(3)^2 + \left(\frac{7}{32.2}\right)(2.5)^2\right]$$

$I_z = 16.3 \text{ slug} \cdot \text{ft}^2$ **Ans.**

$$I_x = (I_x)_{\text{upper}} + (I_x)_{\text{lower}} + (I_x)_{\text{rods}-1} + (I_x)_{\text{rods}-2}$$

$$= \left[\frac{1}{4}\left(\frac{15}{32.2}\right)(1)^2 + \left(\frac{15}{32.2}\right)(4)^2\right] + \frac{1}{4}\left(\frac{40}{32.2}\right)(4)^2$$

$$+ 2\left[\frac{1}{3}\left(\frac{7}{32.2}\right)(4)^2\right] + 2\left[\frac{1}{12}\left(\frac{7}{32.2}\right)(5)^2 + \left(\frac{7}{32.2}\right)(10.25)\right]$$

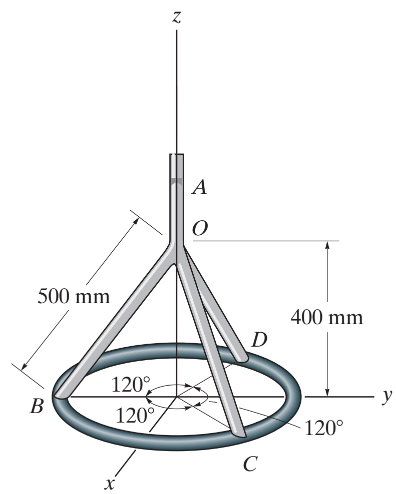
$I_x = 20.2 \text{ slug} \cdot \text{ft}^2$ **Ans.**

And by symmetry,

$I_y = 20.2 \text{ slug} \cdot \text{ft}^2$ **Ans.**

21–21.

Determine the moment of inertia of the rod-and-thin-ring assembly about the z axis. The rods and ring have a mass per unit length of 2 kg/m.



SOLUTION

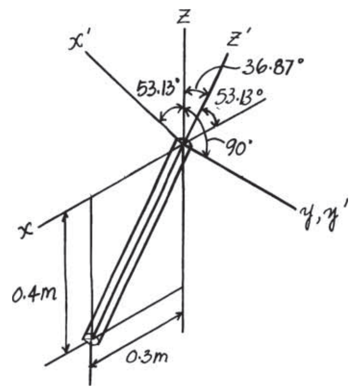
For the rod,

$$u_{x'} = 0.6, \qquad u_{y'} = 0, \qquad u_{z'} = 0.8$$

$$I_x = I_y = \frac{1}{3}[(0.5)(2)](0.5)^2 = 0.08333 \text{ kg}\cdot\text{m}^2$$

$$I_{x'} = 0$$

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$



From Eq. 21–5,

$$I_z = 0.08333(0.6)^2 + 0 + 0 - 0 - 0 - 0$$

$$I_z = 0.03 \text{ kg}\cdot\text{m}^2$$

For the ring,

The radius is $r = 0.3$ m

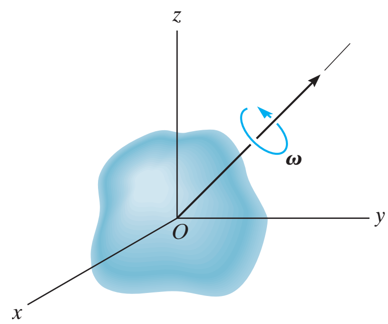
Thus,

$$I_z = mR^2 = [2\,(2\pi)(0.3)](0.3)^2 = 0.3393 \text{ kg}\cdot\text{m}^2$$

Thus the moment of inertia of the assembly is

$$I_z = 3(0.03) + 0.339 = 0.429 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

21–22.



If a body contains *no planes of symmetry,* the principal moments of inertia can be determined mathematically. To show how this is done, consider the rigid body which is spinning with an angular velocity $\boldsymbol{\omega}$, directed along one of its principal axes of inertia. If the principal moment of inertia about this axis is I, the angular momentum can be expressed as $\mathbf{H} = I\boldsymbol{\omega} = I\omega_x\mathbf{i} + I\omega_y\mathbf{j} + I\omega_z\mathbf{k}$. The components of $\mathbf{H}$ may also be expressed by Eqs. 21–10, where the inertia tensor is assumed to be known. Equate the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components of both expressions for $\mathbf{H}$ and consider ω_x, ω_y, and ω_z to be unknown. The solution of these three equations is obtained provided the determinant of the coefficients is zero. Show that this determinant, when expanded, yields the cubic equation

$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2$$
$$+ (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2)I$$
$$- (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I_{yz}^2$$
$$- I_{yy}I_{zx}^2 - I_{zz}I_{xy}^2) = 0$$

The three positive roots of I, obtained from the solution of this equation, represent the principal moments of inertia I_x, I_y, and I_z.

SOLUTION

$$\mathbf{H} = I\omega = I\omega_x\,\mathbf{i} + I\omega_y\,\mathbf{j} + I\omega_z\,\mathbf{k}$$

Equating the **i**, **j**, **k** components to the scalar equations (Eq. 21–10) yields

$$(I_{xx} - I)\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z = 0$$

$$-I_{xx}\,\omega_x + (I_{xy} - I)\,\omega_y - I_{yz}\,\omega_z = 0$$

$$-I_{zx}\,\omega_z - I_{zy}\,\omega_y + (I_{zz} - I)\,\omega_z = 0$$

Solution for ω_x, ω_y, and ω_z requires

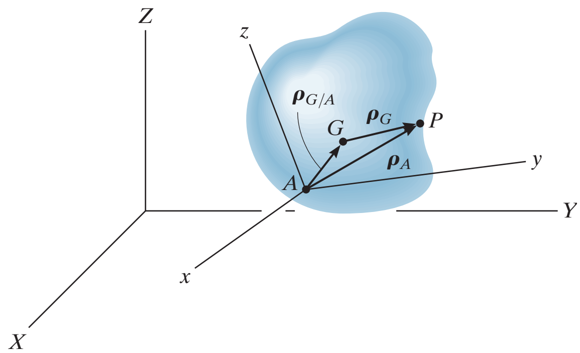
$$\begin{vmatrix} (I_{xx} - I) & -I_{xy} & -I_{xz} \\ -I_{yx} & (I_{yy} - I) & -I_{yz} \\ -I_{zx} & -I_{zy} & (I_{zz} - I) \end{vmatrix} = 0$$

Expanding

$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + \left(I_{xx}\,I_{yy} + I_{yy}\,I_{zz} + I_{zz}\,I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2\right)I$$
$$- \left(I_{xx}\,I_{yy}\,I_{zz} - 2I_{xy}\,I_{yz}\,I_{zx} - I_{xx}\,I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}\,I_{xy}^2\right) = 0 \textbf{ Q.E.D.}$$

21–23.

Show that if the angular momentum of a body is determined with respect to an arbitrary point A, then $\mathbf{H}_A$ can be expressed by Eq. 21–9. This requires substituting $\boldsymbol{\rho}_A = \boldsymbol{\rho}_G + \boldsymbol{\rho}_{G/A}$ into Eq. 21–6 and expanding, noting that $\int \boldsymbol{\rho}_G \, dm = \mathbf{0}$ by definition of the mass center and $\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega} \times \boldsymbol{\rho}_{G/A}$.



SOLUTION

$$\mathbf{H}_A = \left(\int_m \rho_A \, dm\right) \times \mathbf{v}_A + \int_m \rho_A \times (\omega \times \rho_A) dm$$

$$= \left(\int_m (\rho_G + \rho_{G/A}) \, dm\right) \times \mathbf{v}_A + \int_m (\rho_G + \rho_{G/A}) \times \left[\omega \times \rho_G + \rho_{G/A})\right] dm$$

$$= \left(\int_m \rho_G \, dm\right) \times \mathbf{v}_A + (\rho_{G/A} \times \mathbf{v}_A) \int_m dm + \int_m \rho_G \times (\omega \times \rho_G) \, dm$$

$$+ \left(\int_m \rho_G dm\right) \times (\omega \times \rho_{G/A}) + \rho_{G/A} \times \left(\omega \times \int_m \rho_G \, dm\right) + \rho_{G/A} \times (\omega \times \rho_{G/A}) \int_m dm$$

Since $\int_m \rho_G \, dm = 0$ and from Eq. 21–8 $\mathbf{H}_G = \int_m \rho_G \times (\omega \times \rho_G) dm$

$$\mathbf{H}_A = (\rho_{G/A} \times \mathbf{v}_A) m + \mathbf{H}_G + \rho_{G/A} \times (\omega \times \rho_{G/A}) m$$

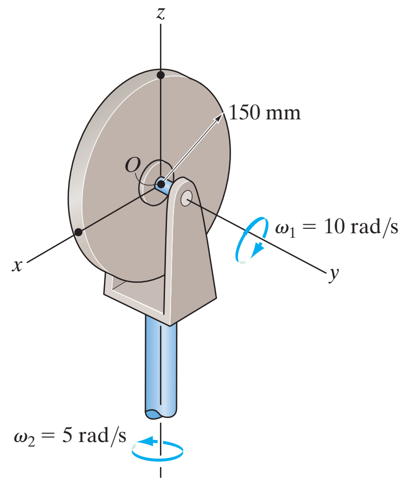
$$= \rho_{G/A} \times (\mathbf{v}_A + (\omega \times \rho_{G/A})) m + \mathbf{H}_G$$

$$= (\rho_{G/A} \times m\mathbf{v}_G) + \mathbf{H}_G$$

Q.E.D.

***21–24.**

The 15-kg circular disk spins about its axle with a constant angular velocity of $\omega_1 = 10$ rad/s. Simultaneously, the yoke is rotating with a constant angular velocity of $\omega_2 = 5$ rad/s. Determine the angular momentum of the disk about its center of mass O, and its kinetic energy.



SOLUTION

The mass moments of inertia of the disk about the x, y, and z axes are

$$I_x = I_z = \frac{1}{4}mr^2 = \frac{1}{4}(15)(0.15^2) = 0.084375\ \text{kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{2}mr^2 = \frac{1}{2}(15)(0.15^2) = 0.16875\ \text{kg}\cdot\text{m}^2$$

Due to symmetry,

$$I_{xy} = I_{yz} = I_{xz} = 0$$

Here, the angular velocity of the disk can be determined from the vector addition of ω_1 and ω_2. Thus,

$$\omega = \omega_1 + \omega_2 = [-10\mathbf{j} + 5\mathbf{k}]\ \text{rad/s}$$

so that

$$\omega_x = 0 \qquad \omega_y = -10\ \text{rad/s} \qquad \omega_z = 5\ \text{rad/s}$$

Since the disk rotates about a fixed point O, we can apply

$$H_x = I_x\omega_x = 0.084375(0) = 0$$

$$H_y = I_y\omega_y = 0.16875(-10) = -1.6875\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$H_z = I_z\omega_z = 0.084375(5) = 0.421875\ \text{kg}\cdot\text{m}^2/\text{s}$$

Thus,

$$H_O = [-1.69\mathbf{j} + 0.422\mathbf{k}]\ \text{kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

The kinetic energy of the disk can be determined from

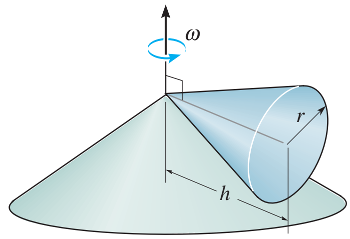
$$T = \frac{1}{2}I_x{\omega_x}^2 + \frac{1}{2}I_y{\omega_y}^2 + \frac{1}{2}I_z{\omega_z}^2$$

$$= \frac{1}{2}(0.084375)(0^2) + \frac{1}{2}(0.16875)(-10)^2 + \frac{1}{2}(0.084375)(5^2)$$

$$= 9.49\ \mathbf{J}$$ **Ans.**

21–25.

The cone has a mass m and rolls without slipping on the conical surface so that it has an angular velocity about the vertical axis of $\boldsymbol{\omega}$. Determine the kinetic energy of the cone due to this motion.

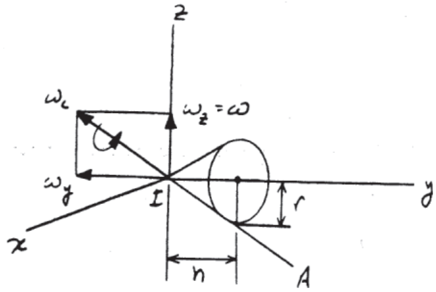


SOLUTION

$$\frac{\omega_z}{r} = \frac{\omega_y}{h}$$

$$\omega_y = \left(\frac{h}{r}\right)\omega_z = \left(\frac{h}{r}\right)\omega$$

$$I_z = \left(\frac{3}{80}\right)m\,(4r^2 + h^2) + m\left(\frac{3}{4}h\right)^2 = \left(\frac{3}{20}\right)mr^2 + \left(\frac{3}{5}\right)mh^2 = \left(\frac{3}{20}\right)m(r^2 + 4h^2)$$



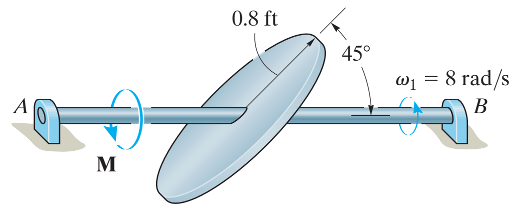
$$T = \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2$$

$$= 0 + \frac{1}{2}\left(\frac{3}{10}mr^2\right)\left(\frac{h}{r}\omega\right)^2 + \frac{1}{2}\left[\left(\frac{3}{20}\right)m(r^2 + 4h^2)\right]\omega^2 = \frac{m\omega^2}{20}\left[3h^2 + \frac{3}{2}r^2 + 6h^2\right]$$

$$T = \frac{9mh^2}{20}\left[1 + \frac{r^2}{6h^2}\right]\omega^2 \qquad \textbf{Ans.}$$

21–26.

The circular disk has a weight of 15 lb and is mounted on the shaft AB at an angle of 45° with the horizontal. Determine the angular velocity of the shaft when $t = 3$ s if a constant torque $M = 2\text{ lb}\cdot\text{ft}$ is applied to the shaft. The shaft is originally spinning at $\omega_1 = 8$ rad/s when the torque is applied.



SOLUTION

Due to symmetry

$$I_{xy} = I_{yz} = I_{zx} = 0$$

$$I_y = I_z = \frac{1}{2}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.07453\text{ slug}\cdot\text{ft}^2$$

$$I_x = \frac{1}{2}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.1491\text{ slug}\cdot\text{ft}^2$$

For x' axis

$$u_x = \cos 45° = 0.7071 \qquad u_y = \cos 45° = 0.7071$$

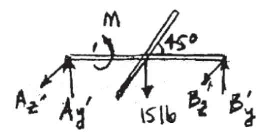
$$u_z = \cos 90° = 0$$

$$I_{z'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

$$= 0.1491(0.7071)^2 + 0.07453(0.7071)^2 + 0 - 0 - 0 - 0$$

$$= 0.1118\text{ slug}\cdot\text{ft}^2$$

Principle of impulse and momentum:



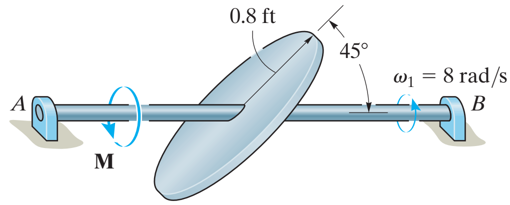
$$(H_{x'})_1 + \Sigma\int M_{x'}\,dt = (H_{x'})_2$$

$$0.1118(8) + 2(3) = 0.1118\,\omega_2$$

$\omega_2 = 61.7$ rad/s **Ans.**

21–27.

The circular disk has a weight of 15 lb and is mounted on the shaft AB at an angle of 45° with the horizontal. Determine the angular velocity of the shaft when $t = 2$ s if a torque $M = (4e^{0.1t})$ lb · ft, where t is in seconds, is applied to the shaft. The shaft is originally spinning at $\omega_1 = 8$ rad/s when the torque is applied.



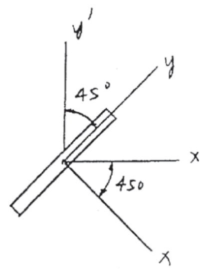
SOLUTION

Due to symmetry

$I_{xy} = I_{yz} = I_{zx} = 0$

$I_y = I_z = \frac{1}{4}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.07453 \text{ slug} \cdot \text{ft}^2$

$I_x = \frac{1}{2}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.1491 \text{ slug} \cdot \text{ft}^2$



For x' axis

$u_x = \cos 45° = 0.7071 \qquad u_y = \cos 45° = 0.7071$

$u_z = \cos 90° = 0$

$$I_{z'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

$$= 0.1491(0.7071)^2 + 0.07453(0.7071)^2 + 0 - 0 - 0 - 0$$

$$= 0.1118 \text{ slug} \cdot \text{ft}^2$$

Principle of impulse and momentum:

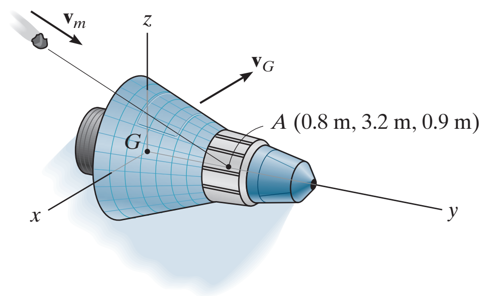
$$(H_{x'})_1 + \Sigma \int M_{x'}\, dt = (H_{x'})_2$$

$$0.1118(8) + \int_0^2 4e^{0.1\,t}\, dt = 0.1118\omega_2$$

$\omega_2 = 87.2$ rad/s **Ans.**

***21–28.**

The space capsule has a mass of 5 Mg and the radii of gyration are $k_x = k_z = 1.30$ m and $k_y = 0.45$ m. If it travels with a velocity $\mathbf{v}_G = \{400\mathbf{j} + 200\mathbf{k}\}$ m/s, compute its angular velocity just after it is struck by a meteoroid having a mass of 0.80 kg and a velocity $\mathbf{v}_m = \{-300\mathbf{i} + 200\mathbf{j} - 150\mathbf{k}\}$ m/s. Assume that the meteoroid embeds itself into the capsule at point A and that the capsule initially has no angular velocity.



SOLUTION

Conservation of Angular Momentum: The angular momentum is conserved about the center of mass of the space capsule G. Neglect the mass of the meteroid after the impact.

$$(H_G)_1 = (H_G)_2$$

$$\mathbf{r}_{GA} \times m_m \mathbf{v}_m = I_G \,\omega$$

$$(0.8\mathbf{i} + 3.2\mathbf{j} + 0.9\mathbf{k}) \times 0.8(-300\mathbf{i} + 200\mathbf{j} - 150\mathbf{k})$$
$$= 5000\left(1.30^2\right)\omega_x\,\mathbf{i} + 5000\left(0.45^2\right)\omega_y\,\mathbf{j} + 5000\left(1.30^2\right)\omega_z\,\mathbf{k}$$

$$-528\mathbf{i} - 120\mathbf{j} + 896\mathbf{k} = 8450\omega_x\,\mathbf{i} + 1012.5\omega_y\,\mathbf{j} + 8450\,\omega_z\,\mathbf{k}$$

Equating **i**, **j** and **k** components, we have

$$-528 = 8450\omega_x \qquad \omega_x = -0.06249 \text{ rad/s}$$

$$-120 = 1012.5\omega_y \qquad \omega_y = -0.11852 \text{ rad/s}$$

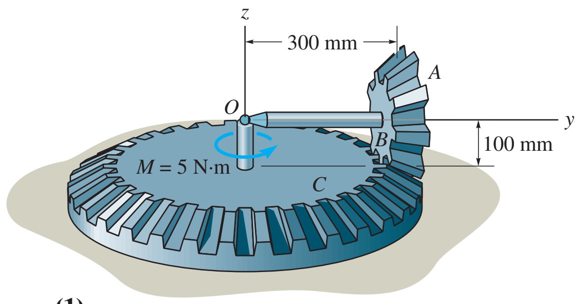
$$896 = 8450\omega_z \qquad \omega_z = 0.1060 \text{ rad/s}$$

Thus,

$$\omega = \{-0.0625\mathbf{i} - 0.119\mathbf{j} + 0.106\mathbf{k}\} \text{ rad/s} \qquad \textbf{Ans.}$$

21–29.

The 2-kg gear A rolls on the fixed plate gear C. Determine the angular velocity of rod OB about the z axis after it rotates one revolution about the z axis, starting from rest. The rod is acted upon by the constant moment $M = 5\ \text{N} \cdot \text{m}$. Neglect the mass of rod OB. Assume that gear A is a uniform disk having a radius of 100 mm.



SOLUTION

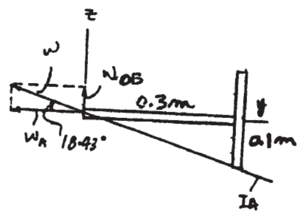
$$\omega_{OB} = \omega_A \tan 18.43° = 0.3333\omega_A \qquad \textbf{(1)}$$

$$I_y = \tfrac{1}{2}(2)(0.1)^2 = 0.01\ \text{kg} \cdot \text{m}^2$$

$$I_z = \tfrac{1}{4}(2)(0.1)^2 + 2(0.3)^2 = 0.185\ \text{kg} \cdot \text{m}^2$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 5(2\pi)\tfrac{1}{2}(0.01)\omega_A^2 + \tfrac{1}{2}(0.185)\omega_{OB}^2 \qquad \textbf{(2)}$$



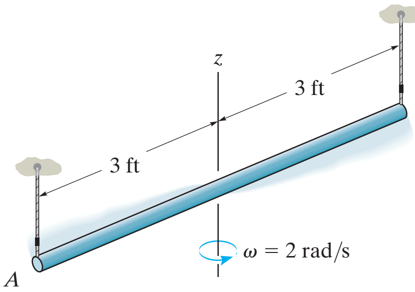
Solving Eqs. (1) and (2) yields:

$$\omega_{OB} = 15.1\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\omega_A = 45.35\ \text{rad/s}$$

21–30.

The rod weighs 3 lb/ft and is suspended from parallel cords at A and B. If the rod has an angular velocity of 2 rad/s about the z axis at the instant shown, determine how high the center of the rod rises at the instant the rod momentarily stops swinging.



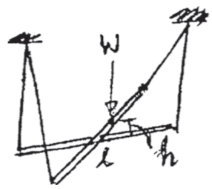
SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{12}\frac{W}{g}l^2\right]\omega^2 + 0 = 0 + Wh$$

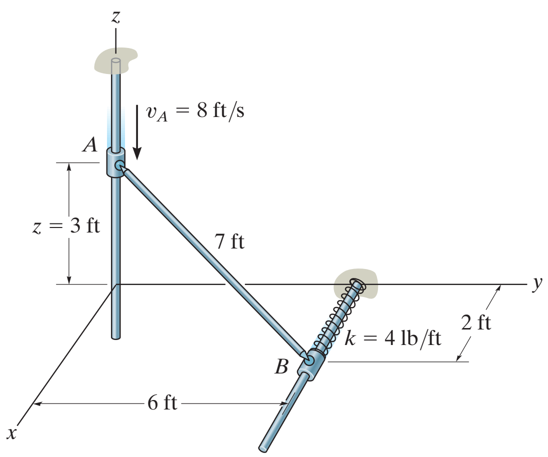
$$h = \frac{1}{24}\frac{l^2\omega^2}{g} = \frac{1}{24}\frac{(6)^2(2)^2}{(32.2)}$$

$$h = 0.1863 \text{ ft} = 2.24 \text{ in.}$$ **Ans.**

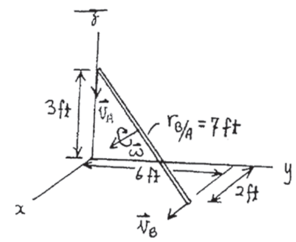


21–31.

Rod AB has a weight of 6 lb and is attached to two smooth collars at its ends by ball-and-socket joints. If collar A is moving downward with a speed of 8 ft/s when $z = 3$ ft, determine the speed of A at the instant $z = 0$. The spring has an unstretched length of 2 ft. Neglect the mass of the collars. Assume the angular velocity of rod AB is perpendicular to its axis.



SOLUTION



$\mathbf{v}_A = \{-8\mathbf{k}\}$ ft/s

$\mathbf{v}_B = v_B\,\mathbf{i}$

$\mathbf{r}_{B/A} = \{2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}$ ft

$\omega = \{\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}\}$ rad/s

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$$v_B\,\mathbf{i} = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 2 & 6 & -3 \end{vmatrix}$$

expanding and equating components yields,

$$v_B = -3\,\omega_y - 6\omega_z \qquad \textbf{(1)}$$

$$0 = 3\omega_x + 2\omega_z \qquad \textbf{(2)}$$

$$0 = -8 + 6\omega_x - 2\omega_z \qquad \textbf{(3)}$$

Also, $\omega \cdot \mathbf{r}_{B/A} = 0$

$(\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \cdot (2\mathrm{i} + 6\mathbf{j} - 3\mathbf{k}) = 0$

$$2\omega_x + 6\omega_y - 3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1)–(4) yields;

$\omega_x = 0.9796$ rad/s

$\omega_y = -1.061$ rad/s

$\omega_z = -1.469$ rad/s

Thus,

$$\omega = \sqrt{(0.9796)^2 + (-1.061)^2 + (-1.469)^2} = 2.06 \text{ rad/s}$$

$\mathbf{v}_G = \mathbf{v}_A + \omega \times \mathbf{r}_{G/A}$

where $\mathbf{r}_{G/A} = \dfrac{1}{2}\mathbf{r}_{B/A} = \dfrac{1}{2}(2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$ft

$$\mathbf{v}_G = -8\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9796 & -1.061 & -1.469 \\ 1 & 3 & -1.5 \end{vmatrix}$$

$\mathbf{v}_G = \{5.9985\mathbf{i} - 4\mathbf{k}\}$ ft/s

$$v_G = \sqrt{(5.9985)^2 + (-4)^2} = 7.2097 \text{ ft/s}$$

21–31. continued

Hence, since ω is directed perpendicular to the axis of the rod,

$$T_1 = \frac{1}{2}I\omega^2 + \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}[\frac{1}{12}(\frac{6}{32.2})(7)^2](2.06)^2 + \frac{1}{2}(\frac{6}{32.2})(7.2097)^2$$

$$= 6.46 \text{ lb} \cdot \text{ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$6.46 + 1.5(6) = T_2 + \frac{1}{2}(4)(3.6056 - 2)^2$$

$$T_2 = 10.304 \text{ lb} \cdot \text{ft}$$

$$\mathbf{v}_A = \mathbf{v}_B + \omega \times \mathbf{r}_{A/B}$$

$$-v_A\mathbf{k} = v_B\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 3.6056 & 6 & 0 \end{vmatrix}$$

$$0 = v_B - 6\omega_z$$

$$0 = \omega_z(0.36056)$$

$$-v_A = 6\omega_x - 3.6056\,\omega_y$$

$$\omega \cdot \mathbf{r}_{A/B} = 0$$

$$3.6056\,\omega_x + 6\omega_y = 0$$

Solving,

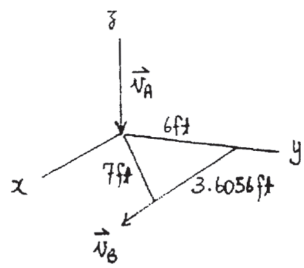
$$\omega_z = 0$$

$$v_B = 0 \text{ (location of } IC)$$

$$v_A = 13.590\omega_y$$

$$\omega_x = -1.664\,\omega_y$$

$$\mathbf{v}_G = \mathbf{v}_B + \omega \times \mathbf{r}_{G/B}$$



$$\mathbf{v}_G = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.664 & 1 & 0 \\ -1.803 & -3 & 0 \end{vmatrix}\omega_y = \{6.795\,\omega_y\,\mathbf{k}\} \text{ ft/s}$$

$$T_2 = \frac{1}{2}(\frac{6}{32.2})(6.795\omega_y)^2 + \frac{1}{2}[\frac{1}{12}(\frac{6}{32.2})(7)^2][(-1.664\omega_y)^2 + \omega_y^2]$$

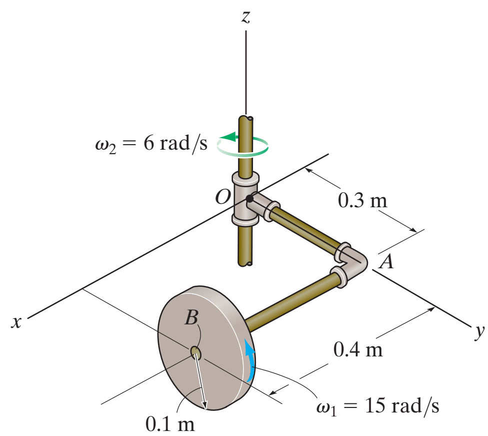
$$= 10.304$$

$$\omega_y = -1.34 \text{ rad/s}$$

$v_A = 13.590(-1.34) = 18.2 \text{ ft/s} \downarrow$ **Ans.**

***21–32.**

The 5-kg circular disk spins about AB with a constant angular velocity of $\omega_1 = 15$ rad/s. Simultaneously, the shaft to which arm OAB is rigidly attached, rotates with a constant angular velocity of $\omega_2 = 6$ rad/s. Determine the angular momentum of the disk about point O, and its kinetic energy.

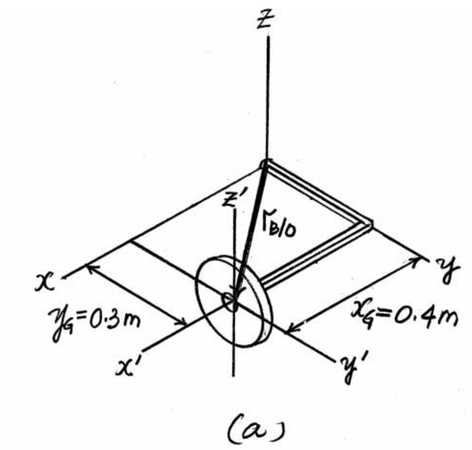


SOLUTION

The mass moments of inertia of the disk about the x', y', and z' axes are

$$I_{x'} = \frac{1}{2}mr^2 = \frac{1}{2}(5)(0.1^2) = 0.025 \text{ kg}\cdot\text{m}^2$$

$$I_{y'} = I_{z'} = \frac{1}{4}mr^2 = \frac{1}{4}(5)(0.1^2) = 0.0125 \text{ kg}\cdot\text{m}^2$$



Due to symmetry, the products of inertia of the disk with respect to its centroidal planes are equal to zero.

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

Here, the angular velocity of the disk can be determined from the vector addition of ω_1 and ω_2. Thus,

$$\omega = \omega_1 + \omega_2 = [15\mathbf{i} + 6\mathbf{k}] \text{ rad/s}$$

The angular momentum of the disk about its mass center B can be obtained by applying

$$(H_B)_x = I_{x'}\omega_x = 0.025(15) = 0.375 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(H_B)_y = I_{y'}\omega_y = 0.0125(0) = 0$$

$$(H_B)_z = I_{z'}\omega_z = 0.0125(6) = 0.075 \text{ kg}\cdot\text{m}^2/\text{s}$$

Thus,

$$\mathbf{H}_B = [0.375\mathbf{i} + 0.075\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s}$$

Since the mass center B rotates about the z axis with a constant angular velocity of $\omega_2 = [6\mathbf{k}]$ rad/s, its velocity is

$$\mathbf{v}_B = \omega_2 \times r_{B/O} = (6\mathbf{k}) \times (0.4\mathbf{i} + 0.3\mathbf{j}) = [-1.8\mathbf{i} + 2.4\mathbf{j}] \text{ m/s}$$

Since the disk does not rotate about a fixed point O, its angular momentum must be determined from

$$\mathbf{H}_O = r_{B/O} \times m\mathbf{v}_B + \mathbf{H}_B$$

$$= (0.4\mathbf{i} + 0.3\mathbf{j}) \times 5(-1.8\mathbf{i} + 2.4\mathbf{j}) + (0.375\mathbf{i} + 0.075\mathbf{k})$$

$$= [0.375\mathbf{i} + 7.575\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

The kinetic energy of the disk is therefore

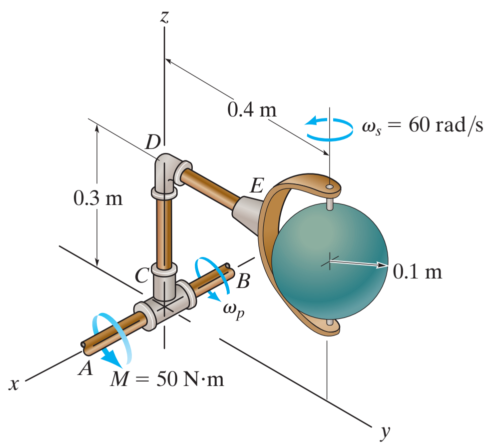
$$T = \frac{1}{2}\omega\cdot\mathbf{H}_O$$

$$= \frac{1}{2}(15\mathbf{i} + 6\mathbf{k})\cdot(0.375\mathbf{i} + 7.575\mathbf{k})$$

$$= 25.5 \text{ J}$$ **Ans.**

21–33.

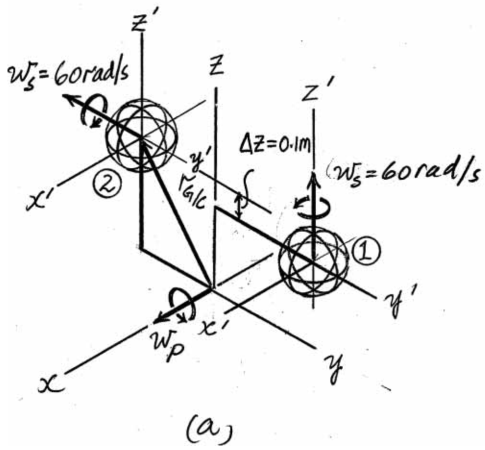
The 20-kg sphere rotates about the axle with a constant angular velocity of $\omega_s = 60$ rad/s. If shaft AB is subjected to a torque of $M = 50$ N·m, causing it to rotate, determine the value of ω_p after the shaft has turned 90° from the position shown. Initially, $\omega_p = 0$. Neglect the mass of arm CDE.



SOLUTION

The mass moments of inertia of the sphere about the x', y', and z' axes are

$$I_{x'} = I_{y'} = I_{z'} = \frac{2}{5}mr^2 = \frac{2}{5}(20)(0.1^2) = 0.08\ \text{kg}\cdot\text{m}^2$$



When the sphere is at position 1, Fig. a, $\omega_p = 0$. Thus, the velocity of its mass center is zero and its angular velocity is $\omega_1 = [60\mathbf{k}]$ rad/s. Thus, its kinetic energy at this position is

$$T = \frac{1}{2}m(v_G)_1^2 + \frac{1}{2}I_{x'}(\omega_1)_{x'}^2 + \frac{1}{2}I_{y'}(\omega_1)_{y'}^2 + \frac{1}{2}I_{z'}(\omega_1)_{z'}^2$$

$$= 0 + 0 + 0 + \frac{1}{2}(0.08)(60^2)$$

$$= 144\ \text{J}$$

When the sphere is at position 2, Fig. a, $\omega_p = \omega_p\mathbf{i}$. Then the velocity of its mass center is $(\mathbf{v}_G)_2 = \omega_p \times \mathbf{v}_{G/C} = (\omega_p\mathbf{i}) \times (-0.3\mathbf{j} + 0.4\mathbf{k}) = -0.4\omega_p\mathbf{j} - 0.3\omega_p\mathbf{k}$. Then $(v_G)_2^2 = (-0.4\omega_p)^2 + (-0.3\omega_p)^2 = 0.25\omega_p^2$. Also, its angular velocity at this position is $\omega_2 = \omega_p\mathbf{i} - 60\mathbf{j}$. Thus, its kinetic energy at this position is

$$T = \frac{1}{2}m(v_G)_2^2 + \frac{1}{2}I_{x'}(\omega_2)_{x'}^2 + \frac{1}{2}I_{y'}(\omega_2)_{y'}^2 + \frac{1}{2}I_{z'}(\omega_2)_{z'}^2$$

$$= \frac{1}{2}(20)\left(0.25\omega_p^2\right) + \frac{1}{2}(0.08)\left(\omega_p^2\right) + \frac{1}{2}(0.08)(-60)^2$$

$$= 2.54\omega_p^2 + 144$$

When the sphere moves from position 1 to position 2, its center of gravity raises vertically $\Delta z = 0.1$ m. Thus, its weight $\mathbf{W}$ does negative work.

$$U_W = -W\Delta z = -20(9.81)(0.1) = -19.62\ \text{J}$$

Here, the couple moment M does positive work.

$$U_W = M\theta = 50\left(\frac{\pi}{2}\right) = 25\pi\text{J}$$

Applying the principle of work and energy,

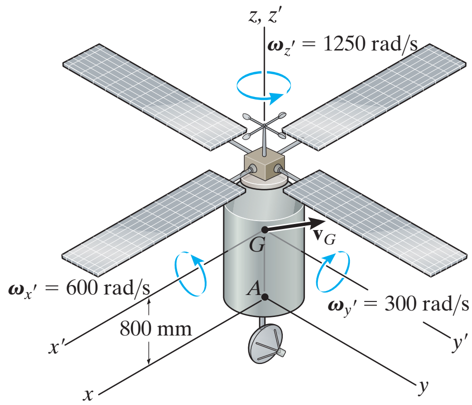
$$T_1 + \Sigma U_{1-2} = T_2$$

$$144 + 25\pi + (-19.62) = 2.54\omega_p^2 + 144$$

$$\omega_p = 4.82\ \text{rad/s}$$ **Ans.**

21–34.

The 200-kg satellite has its center of mass at point G. Its radii of gyration about the z', x', y' axes are $k_{z'} = 300$ mm, $k_{x'} = k_{y'} = 500$ mm, respectively. At the instant shown, the satellite rotates about the x', y', and z' axes with the angular velocity shown, and its center of mass G has a velocity of $\mathbf{v}_G = \{-250\mathbf{i} + 200\mathbf{j} + 120\mathbf{k}\}$ m/s. Determine the angular momentum of the satellite about point A at this instant.



SOLUTION

The mass moments of inertia of the satellite about the x', y', and z' axes are

$$I_{x'} = I_{y'} = 200\left(0.5^2\right) = 50 \text{ kg}\cdot\text{m}^2$$

$$I_{z'} = 200\left(0.3^2\right) = 18 \text{ kg}\cdot\text{m}^2$$

Due to symmetry, the products of inertia of the satellite with respect to the x', y', and z' coordinate system are equal to zero.

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

The angular velocity of the satellite is

$$\omega = [600\mathbf{i} + 300\mathbf{j} + 1250\mathbf{k}] \text{ rad/s}$$

Thus,

$$\omega_{x'} = 600 \text{ rad/s} \qquad \omega_{y'} = -300 \text{ rad/s} \qquad \omega_{z'} = 1250 \text{ rad/s}$$

Then, the components of the angular momentum of the satellite about its mass center G are

$$(H_G)_{x'} = I_{x'}\omega_{x'} = 50(600) = 30\,000 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(H_G)_{y'} = I_{y'}\omega_{y'} = 50(-300) = -15\,000 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(H_G)_{z'} = I_{z'}\omega_{z'} = 18(1250) = 22\,500 \text{ kg}\cdot\text{m}^2/\text{s}$$

Thus,

$$\mathbf{H}_G = [30\,000\mathbf{i} - 15\,000\mathbf{j} + 22\,500\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s}$$

The angular momentum of the satellite about point A can be determined from

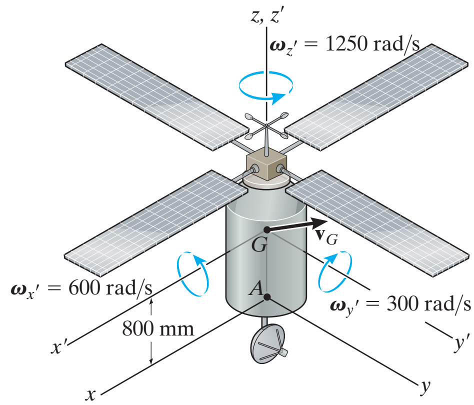
$$\mathbf{H}_A = \mathbf{r}_{G/A} \times m\mathbf{v}_G + \mathbf{H}_G$$

$$= (0.8\mathbf{k}) \times 200(-250\mathbf{i} + 200\mathbf{j} + 120\mathbf{k}) + (30\,000\mathbf{i} - 15\,000\mathbf{j} + 22\,500\mathbf{k})$$

$$= [-2000\mathbf{i} - 55\,000\mathbf{j} + 22\,500\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

21–35.

The 200-kg satellite has its center of mass at point G. Its radii of gyration about the z', x', y' axes are $k_{z'} = 300$ mm, $k_{x'} = k_{y'} = 500$ mm, respectively. At the instant shown, the satellite rotates about the x', y', and z' axes with the angular velocity shown, and its center of mass G has a velocity of $\mathbf{v}_G = \{-250\mathbf{i} + 200\mathbf{j} + 120\mathbf{k}\}$ m/s. Determine the kinetic energy of the satellite at this instant.



SOLUTION

The mass moments of inertia of the satellite about the x', y', and z' axes are

$$I_{x'} = I_{y'} = 200\left(0.5^2\right) = 50\ \text{kg}\cdot\text{m}^2$$

$$I_{z'} = 200\left(0.3^2\right) = 18\ \text{kg}\cdot\text{m}^2$$

Due to symmetry, the products of inertia of the satellite with respect to the x', y', and z, coordinate system are equal to zero.

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

The angular velocity of the satellite is

$$\omega = [600\mathbf{i} - 300\mathbf{j} + 1250\mathbf{k}]\ \text{rad/s}$$

Thus,

$$\omega_{x'} = 600\ \text{rad/s} \qquad \omega_{y'} = -300\ \text{rad/s} \qquad \omega_{z'} = 1250\ \text{rad/s}$$

Since $v_G{}^2 = (-250)^2 + 200^2 + 120^2 = 116\,900\ \text{m}^2/\text{s}^2$, the kinetic energy of the satellite can be determined from

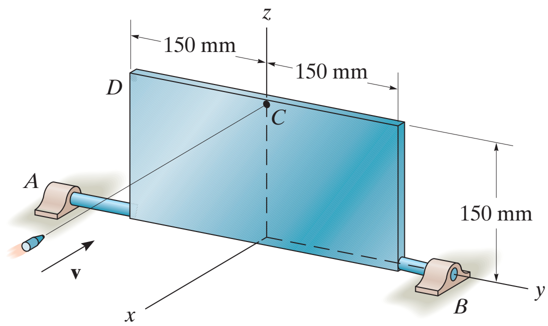
$$T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_{x'}\omega_{x'}{}^2 + \frac{1}{2}I_{y'}\omega_{y'}{}^2 + \frac{1}{2}I_{z'}\omega_{z'}{}^2$$

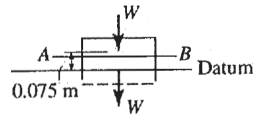
$$= \frac{1}{2}(200)(116\,900) + \frac{1}{2}(50)\left(600^2\right) + \frac{1}{2}(50)(-300)^2 + \frac{1}{2}(18)\left(1250^2\right)$$

$$= 37.0025\left(10^6\right)\text{J} = 37.0\ \text{MJ}$$ **Ans.**

***21–36.**

The 15-kg rectangular plate is free to rotate about the y axis because of the bearing supports at A and B. When the plate is balanced in the vertical plane, a 3-g bullet is fired into it, perpendicular to its surface, with a velocity $\mathbf{v} = \{-2000\mathbf{i}\}$ m/s. Compute the angular velocity of the plate at the instant it has rotated 180°. If the bullet strikes corner D with the same velocity $\mathbf{v}$, instead of at C, does the angular velocity remain the same? Why or why not?





SOLUTION

Consider the projectile and plate as an entire system.

Angular momentum is conserved about the AB axis.

$$(\mathbf{H}_{AB})_1 = -(0.003)(2000)(0.15)\mathbf{j} = \{-0.9\mathbf{j}\}$$

$$(\mathbf{H}_{AB})_1 = (\mathbf{H}_{AB})_2$$

$$-0.9\mathbf{j} = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

Equating components,

$$\omega_x = 0$$

$$\omega_z = 0$$

$$\omega_y = \frac{-0.9}{\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right]} = -8 \text{ rad/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right](8)^2 + 15(9.81)(0.15)$$

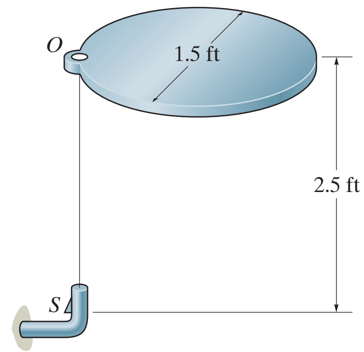
$$= \frac{1}{2}\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right]\omega_{AB}^2$$

$\omega_{AB} = 21.4$ rad/s **Ans.**

If the projectile strikes the plate at D, the angular velocity is the same, only the impulsive reactions at the bearing supports A and B will be different.

21–37.

The circular plate has a weight of 19 lb and a diameter of 1.5 ft. If it is released from rest and falls horizontally 2.5 ft onto the hook at S, which provides a permanent connection, determine the velocity of the mass center of the plate just after the connection with the hook is made.



SOLUTION

Conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 19(2.5) = \tfrac{1}{2}\left(\tfrac{19}{32.2}\right)(v_G)_2^2 + 0$$

$$(v_G)_2 = 12.69 \text{ ft/s}$$

Conservation of momentum about point O:

$$(\mathbf{H}_O)_1 = \left[-\left(\tfrac{19}{32.2}\right)(12.69)(0.75)\right]\mathbf{i} = \{-5.6153\mathbf{i}\} \text{ slug}\cdot\text{ft}^2/\text{s}$$

$$I_x = \left[\tfrac{1}{4}\left(\tfrac{19}{32.2}\right)(0.75)^2 + \left(\tfrac{19}{32.2}\right)(0.75)^2\right] = 0.4149 \text{ slug}\cdot\text{ft}^2$$

$$I_y = \frac{1}{4}\left(\frac{19}{32.2}\right)(0.75)^2 = 0.08298 \text{ slug}\cdot\text{ft}^2$$

$$I_z = \left[\tfrac{1}{2}\left(\tfrac{19}{32.2}\right)(0.75)^2 + \left(\tfrac{19}{32.2}\right)(0.75)^2\right] = 0.4979 \text{ slug}\cdot\text{ft}^2$$

$$(\mathbf{H}_O)_3 = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$= 0.4149\omega_x\mathbf{i} + 0.8298\omega_y\mathbf{j} + 0.4979\omega_z\mathbf{k}$$

$$(\mathbf{H}_O)_2 = (\mathbf{H}_O)_3$$

$$-5.6153\mathbf{i} = 0.4149\omega_x\mathbf{i} + 0.08298\omega_y\mathbf{j} + 0.4979\omega_z\mathbf{k}$$

Equating **i**, **j** and **k** components

$-5.6153\mathbf{i} = 0.4149\omega_x \qquad \omega_x = -13.54 \text{ rad/s}$

$0 = 0.89298\omega_y \qquad \omega_y = 0$

$0 = 0.4979\omega_z \qquad \omega_z = 0$

Hence $\qquad \omega = \{-13.54\mathbf{i}\} \text{ rad/s}$

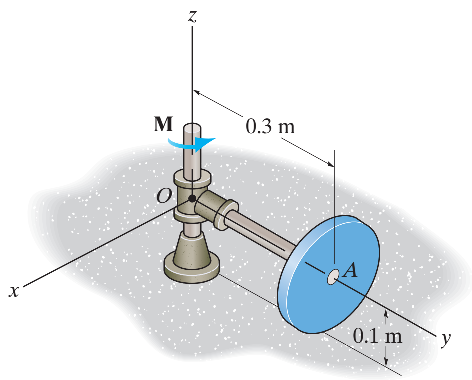
$$\mathbf{v}_G = \omega \times \mathbf{r}_{G/O}$$

$$= (-13.54\mathbf{i}) \times (0.75\mathbf{j})$$

$$= \{-10.2\mathbf{k}\} \text{ ft/s}$$ **Ans.**

21–38.

The 10-kg disk rolls on the horizontal plane without slipping. Determine the magnitude of its angular momentum when it is spinning about the y axis at 2 rad/s.



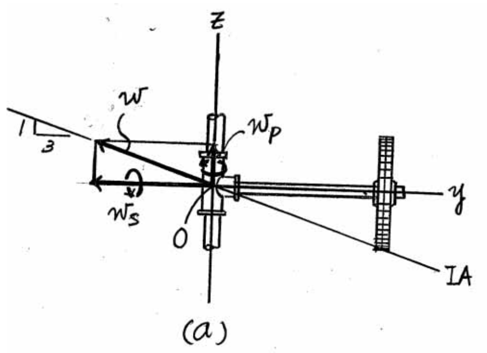
SOLUTION

Here, the disk rotates about the fixed point O and its angular velocity is

$$\omega = -\omega_S\,\mathbf{j} + \omega_P\,\mathbf{k}$$

Since the disk rolls without slipping on the horizontal plane, the instantaneous axis of zero velocity (IA) is shown in Fig. a. Thus,

$$\frac{\omega_P}{\omega_S} = \frac{1}{3} \quad \omega_P = \frac{1}{3}\omega_S$$



Then,

$$\omega = -\omega_S\,\mathbf{j} + \frac{1}{3}\omega_S\,\mathbf{k}$$

Thus, $\omega_x = 0$, $\omega_y = -\omega_S$, and $\omega_z = \dfrac{1}{3}\omega_S$.

The mass moments of inertia of the disk about the x, y, and z axes are

$$I_x = I_z = \frac{1}{4}(10)(0.1^2) + 10(0.3^2) = 0.925 \text{ kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{2}(10)(0.1^2) = 0.05 \text{ kg}\cdot\text{m}^2$$

Due to symmetry,

$$I_{xy} = I_{yz} = I_{xz} = 0$$

Thus, the components of angular momentum about point O are

$$H_x = I_{x'}\omega_x = 0.925(0) = 0$$

$$H_y = I_{y'}\omega_y = 0.05(-\omega_S) = -0.05\omega_S$$

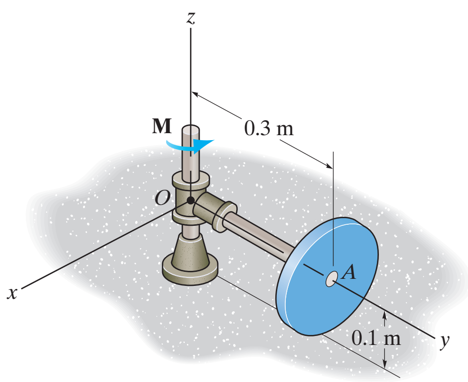
$$H_z = I_{z'}\omega_z = 0.925\left(\frac{1}{3}\omega_S\right) = 0.3083\omega_S$$

At $\omega_S = 2$ rad/s,

$$H = \sqrt{0 + [(0.05(2)]^2 + [0.3083(2)]^2}$$
$$= 0.625 \text{ kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

21–39.

If arm OA is subjected to a torque of $M = 5\text{ N}\cdot\text{m}$, determine the spin angular velocity of the 10-kg disk after the arm has turned 2 rev, starting from rest. The disk rolls on the horizontal plane without slipping. Neglect the mass of the arm.

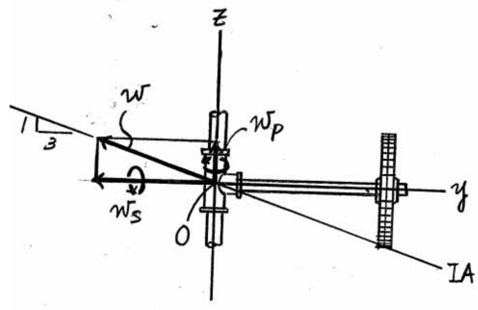


SOLUTION

Here, the disk rotates about the fixed point O and its angular velocity is

$$\omega = -\omega_S\mathbf{j} + \omega_P\mathbf{k}$$

Since the disk rolls without slipping on the horizontal plane, the instantaneous axis of zero velocity (IA) is shown in Fig. a. Thus,



(a)

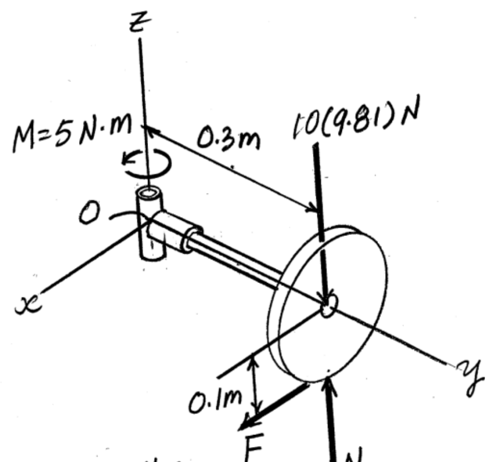
$$\frac{\omega_P}{\omega_S} = \frac{1}{3} \qquad \omega_P = \frac{1}{3}\omega_S$$

Then,

$$\omega = -\omega_S\mathbf{j} + \frac{1}{3}\omega_S\mathbf{k}$$

So that, $\omega_x = 0$, $\omega_y = -\omega_S$, and $\omega_z = \frac{1}{3}\omega_S$.

The mass moment of inertia of the disk about the x, y, and z axes are



(b)

$$I_x = I_z = \frac{1}{4}(10)(0.1^2) + 10(0.3^2) = 0.925\text{ kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{2}(10)(0.1^2) = 0.05\text{ kg}\cdot\text{m}^2$$

Due to symmetry,

$$I_{xy} = I_{yz} = I_{xz} = 0$$

Thus, the kinetic energy of the disk can be determined from

$$\begin{aligned} T &= \frac{1}{2}I_x\omega_x^{\,2} + \frac{1}{2}I_y\omega_y^{\,2} + \frac{1}{2}I_z\omega_z^{\,2} \\ &= \frac{1}{2}(0.925)(0^2) + \frac{1}{2}(0.05)(-\omega_S)^2 + \frac{1}{2}(0.925)\left(\frac{1}{3}\omega_S\right)^2 \\ &= 0.07639\omega_S^{\,2} \end{aligned}$$

Referring to the free-body diagram of the disk shown in Fig. b, $W = 10(9.81)$ N, and **N** do no work.

The frictional force **F** also does no work since the disk rolls without slipping. Only the couple moment does work.

$$U_M = M\theta = 5[2(2\pi)] = 20\pi\text{ J}$$

Applying the principle of work and energy,

$$\begin{aligned} T_1 &= \Sigma U_{1-2} = T_2 \\ 0 + 20\pi &= 0.07639\omega_S^{\,2} \\ \omega_S &= 28.68\text{ rad/s} = 28.7\text{ rad/s} \end{aligned}$$

Ans.

***21–40.**

Derive the scalar form of the rotational equation of motion about the x axis if $\mathbf{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *not constant* with respect to time.

SOLUTION

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\,\mathbf{i} + H_y\,\mathbf{j} + H_z\,\mathbf{k})$$

$$= \left(\dot{H}_x\,\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\,\mathbf{k}\right)_{xyz} + \Omega \times (H_x\,\mathbf{i} + H_y\mathbf{j} + H_z\,\mathbf{k})$$

Substitute $\Omega = \Omega_x\,\mathbf{i} + \Omega_y\,\mathbf{j} + \Omega_z\,\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z\,H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x\,H_z + \Omega_z H_x\right)\mathbf{j}$$
$$+ \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y\,H_x + \Omega_x\,H_y\right)\mathbf{k}$$

Subsitute H_x, H_y and H_z using Eq. 21–10. For the $\mathbf{i}$ component

$$\Sigma M_x = \frac{d}{dt}(I_x\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\omega_x)$$
$$+ \Omega_y\,(I_z\,\omega_z - I_{zx}\,\omega_x - I_{zy}\,\omega_y) \qquad \textbf{Ans.}$$

One can obtain y and z components in a similar manner.

21–41.

Derive the scalar form of the rotational equation of motion about the x axis if $\boldsymbol{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *constant* with respect to time.

SOLUTION

In general

$$\mathbf{M} = \frac{d}{dt}(H_x \mathbf{i} + H_y \mathbf{j} + H_z \mathbf{k})$$

$$= \left(\dot{H}_x \mathbf{i} + \dot{H}_y \mathbf{j} + \dot{H}_z \mathbf{k}\right)_{xyz} + \Omega \times (H_x \mathbf{i} + H_y \mathbf{j} + H_z \mathbf{k})$$

Substitute $\Omega = \Omega_x \mathbf{i} + \Omega_y \mathbf{j} + \Omega_z \mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} + \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x, H_y and H_z using Eq. 21–10. For the $\mathbf{i}$ component

$$\Sigma M_x = \frac{d}{dt}(I_x \omega_x - I_{xy}\omega_y - I_{xz}\omega_z) - \Omega_z (I_y \omega_y - I_{yz}\omega_z - I_{yx}\omega_x) + \Omega_y (I_z \omega_z - I_{zx}\omega_x - I_{zy}\omega_y)$$

For constant inertia, expanding the time derivative of the above equation yields

$$\Sigma M_x = (I_x \dot{\omega}_x - I_{xy}\dot{\omega}_y - I_{xz}\dot{\omega}_z) - \Omega_z (I_y \omega_y - I_{yz}\omega_z - I_{yx}\omega_x) + \Omega_y (I_z \omega_z - I_{zx}\omega_x - I_{zy}\omega_y) \qquad \textbf{Ans.}$$

One can obtain y and z components in a similar manner.

21–42.

Derive the Euler equations of motion for $\boldsymbol{\Omega} \neq \boldsymbol{\omega}$, i.e., Eqs. 21–26.

SOLUTION

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\,\mathbf{i} + H_y\,\mathbf{j} + H_z\,\mathbf{k})$$

$$= \left(\dot{H}_x\,\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\,\mathbf{k}\right)_{xyz} + \Omega \times (H_x\,\mathbf{i} + H_y\mathbf{j} + H_z\,\mathbf{k})$$

Substitute $\Omega = \Omega_x\,\mathbf{i} + \Omega_y\,\mathbf{j} + \Omega_z\,\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z\,H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x\,H_z + \Omega_z H_x\right)\mathbf{j}$$
$$+ \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y\,H_x + \Omega_x\,H_y\right)\mathbf{k}$$

Substitute H_x, H_y and H_z using Eq. 21–10. For the $\mathbf{i}$ component

$$\Sigma M_x = \frac{d}{dt}(I_x\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\omega_x)$$
$$+ \Omega_y\,(I_z\,\omega_z - I_{zx}\,\omega_x - I_{zy}\,\omega_y)$$

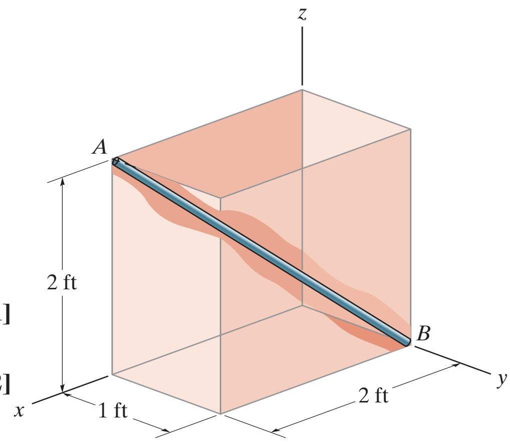
Set $I_{xy} = I_{yz} = I_{zx} = 0$ and require I_x, I_y, I_z to be constant. This yields

$$\Sigma M_x = I_x\,\dot{\omega}_x - I_y\,\Omega_z\,\omega_y + I_z\,\Omega_y\omega_z \qquad \textbf{Ans.}$$

One can obtain y and z components in a similar manner.

21–43.

The 4-lb bar rests along the smooth corners of an open box. At the instant shown, the box has a velocity $\mathbf{v} = \{3\mathbf{j}\}$ ft/s and an acceleration $\mathbf{a} = \{-6\mathbf{j}\}$ ft/s^2. Determine the x, y, z components of force which the corners exert on the bar.

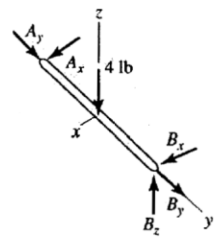


SOLUTION

$$\Sigma F_x = m(a_G)_x\,; \qquad A_x + B_x = 0 \qquad \textbf{[1]}$$

$$\Sigma F_y = m(a_G)_y\,; \qquad A_y + B_y = \left(\frac{4}{32.2}\right)(-6) \qquad \textbf{[2]}$$

$$\Sigma F_z = m(a_G)_z\,; \qquad B_z - 4 = 0 \qquad B_z = 4 \text{ lb} \qquad \textbf{Ans.}$$



Applying Eq. 21–25 with $\omega_x = \omega_y = \omega_z = 0$ $\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$

$$\Sigma(M_G)_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z; \qquad B_y(1) - A_y(1) + 4(0.5) = 0 \qquad \textbf{[3]}$$

$$\Sigma(M_G)_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x; \qquad A_x(1) - B_x(1) + 4(1) = 0 \qquad \textbf{[4]}$$

Solving Eqs. [1] to [4] yields:

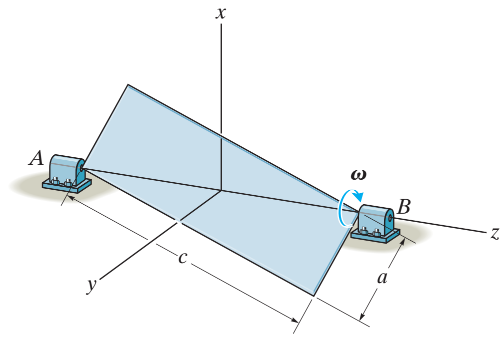
$A_x = -2.00$ lb $\quad A_y = 0.627$ lb $\quad B_x = 2.00$ lb $\quad B_y = -1.37$ lb **Ans.**

$$\Sigma(M_G)_z = I_z\dot{\omega}_z - (I_x - I_y)\,\omega_x\omega_y;$$

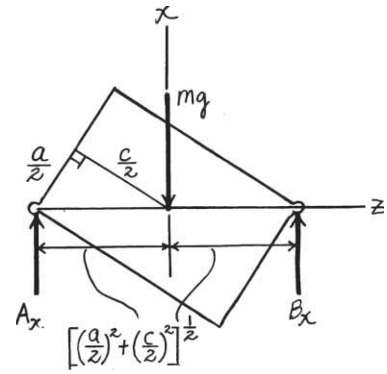
$$(-2.00)(0.5) - (2.00)(0.5) - (-1.37)(1) + (0.627)(1) = 0 \qquad \textbf{(O.K!)}$$

***21–44.**

The uniform rectangular plate has a mass of $m = 2$ kg and is given a rotation of $\omega = 4$ rad/s about its bearings at A and B. If $a = 0.2$ m and $c = 0.3$ m, determine the vertical reactions at the instant shown. Use the x, y, z axes shown and note that $I_{zx} = -\left(\dfrac{mac}{12}\right)\left(\dfrac{c^2 - a^2}{c^2 + a^2}\right)$.



SOLUTION



$$\omega_x = 0, \qquad \omega_y = 0, \qquad \omega_z = -4$$

$$\dot{\omega}_x = 0, \qquad \dot{\omega}_y = 0, \qquad \dot{\omega}_z = 0$$

$$\Sigma M_y = I_{yy}\,\dot{\omega}_y - (I_{zz} - I_{xx})\omega_z\,\omega_x - I_{yz}\left(\dot{\omega}_z - \omega_x\,\omega_y\right)$$

$$- I_{zx}\left(\omega_z^2 - \omega_x^2\right) - I_{xy}\left(\dot{\omega}_x + \omega_y\,\omega_z\right)$$

$$B_x\left[\left(\frac{a}{2}\right)^2 + \left(\frac{c}{2}\right)^2\right]^{\frac{1}{2}} - A_x\left[\left(\frac{a}{2}\right)^2 + \left(\frac{c}{2}\right)^2\right]^{\frac{1}{2}} = -I_{zx}\,(\omega)^2$$

$$B_x - A_x = \left(\frac{mac}{6}\right)\left(\frac{c^2 - a^2}{\left[a^2 + c^2\right]^{\frac{3}{2}}}\right)\omega^2$$

$$\Sigma F_x = m(a_G)_x\,; \qquad A_x + B_x - mg = 0$$

Substitute the data,

$$B_x - A_x = \frac{2(0.2)(0.3)}{6}\left[\frac{(0.3)^2 - (0.2)^2}{\left[(0.3)^2 + (0.2)^2\right]^{\frac{3}{2}}}\right](-4)^2 = 0.34135$$

$$A_x + B_x = 2(9.81)$$

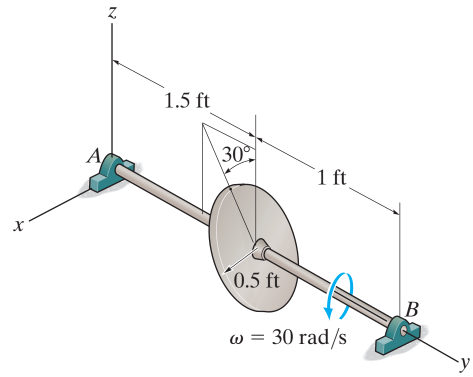
Solving:

$$A_x = 9.64\ \text{N} \qquad \textbf{Ans.}$$

$$B_x = 9.98\ \text{N} \qquad \textbf{Ans.}$$

21–45.

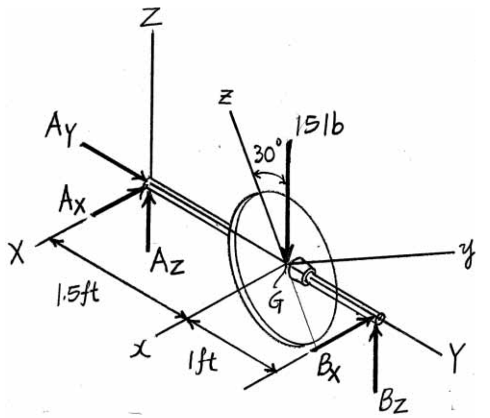
If the shaft AB is rotating with a constant angular velocity of $\omega = 30$ rad/s, determine the X, Y, Z components of reaction at the thrust bearing A and journal bearing B at the instant shown. The disk has a weight of 15 lb. Neglect the weight of the shaft AB.



SOLUTION

The rotating xyz frame is set with its origin at the plate's mass center, Fig. a. This frame will be fixed to the disk so that its angular velocity is $\Omega = \omega$ and the x, y, and z axes will always be the principle axes of inertia of the disk. Referring to Fig. b,

$$\omega = [30\cos 30°\mathbf{j} - 30\sin 30°\mathbf{k}]\text{ rad/s} = [25.98\mathbf{j} - 15\mathbf{k}]\text{ rad/s}$$



(a)

Thus,

$$\omega_x = 0 \qquad \omega_y = 25.98\text{ rad/s} \qquad \omega_z = -15\text{ rad/s}$$

Since ω is always directed towards the $+Y$ axis and has a constant magnitude, $\dot{\omega} = 0$. Also, since $\Omega = \omega$, $\left(\dot{\omega}_{xyz}\right) = \dot{\omega} = 0$. Thus,

$$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$$

The mass moments of intertia of the disk about the x, y, z axes are

$$I_x = I_z = \frac{1}{4}\left(\frac{15}{32.2}\right)\left(0.5^2\right) = 0.02911\text{ slug}\cdot\text{ft}^2$$

$$I_y = \frac{1}{2}\left(\frac{15}{32.2}\right)\left(0.5^2\right) = 0.05823\text{ slug}\cdot\text{ft}^2$$

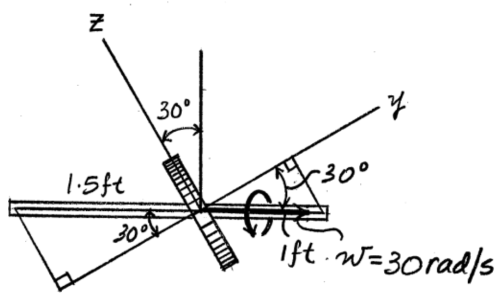
Applying the equations of motion,

$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z;\ B_Z(1) - A_Z(1.5) = 0 - (0.05823 - 0.02911)(25.98)(-15)$

$$B_Z - 1.5A_Z = 11.35 \qquad \textbf{(1)}$$

$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x;\ B_X(1\sin 30°) - A_X(1.5\sin 30°) = 0 - 0$

$$B_X - 1.5A_X = 0 \qquad \textbf{(2)}$$



(b)

$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y;\ B_X(1\cos 30°) - A_X(1.5\cos 30°) = 0 - 0$

$$B_X - 1.5A_X = 0$$

$\Sigma F_X = m(a_G)_X;\qquad A_X + B_X = 0 \qquad$ **(3)**

$\Sigma F_Y = m(a_G)_Y;\qquad A_Y = 0 \qquad$ **Ans.**

$\Sigma F_Z = m(a_G)_Z;\qquad A_Z + B_Z - 15 = 0 \qquad$ **(4)**

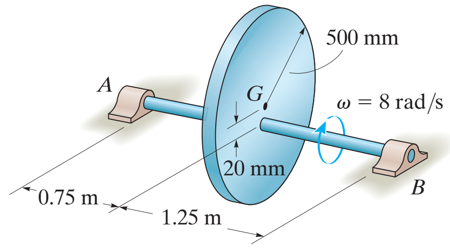
Solving Eqs. (1) through (4),

$A_Z = 1.461$ lb $\qquad B_Z = 13.54$ lb $= 13.5$ lb $\qquad$ **Ans.**

$A_X = B_X = 0 \qquad$ **Ans.**

21–46.

The 40-kg flywheel (disk) is mounted 20 mm off its true center at G. If the shaft is rotating at a constant speed $\omega = 8$ rad/s, determine the maximum reactions exerted on the journal bearings at A and B.

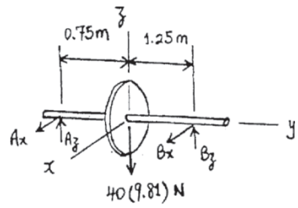


SOLUTION

$$\omega_x = 0$$

$$\omega_y = -8 \text{ rad/s}$$

$$\omega_z = 0$$



$$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\, \omega_y\, \omega_z;$$

$$B_z(1.25) - A_z(0.75) = 0 - 0$$

$$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\, \omega_x\, \omega_y;$$

$$-B_x(1.25) + A_x(0.75) = 0 - 0$$

$$\Sigma F_x = ma_x; \qquad A_x + B_x = 0$$

$$\Sigma F_z = ma_z; \qquad A_z + B_z - 40(9.81) = 40(8)^2(0.020)$$

Solving,

$$A_x = 0$$

$$B_x = 0$$

$$A_z = 277 \text{ N}$$

$$B_z = 166 \text{ N}$$

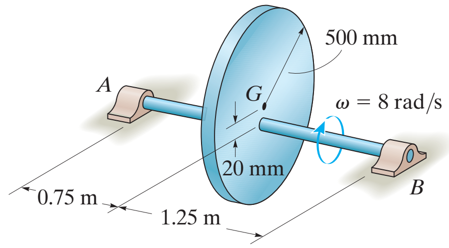
Thus,

$$F_A = 277 \text{ N} \qquad \textbf{Ans.}$$

$$F_B = 166 \text{ N} \qquad \textbf{Ans.}$$

21–47.

The 40-kg flywheel (disk) is mounted 20 mm off its true center at G. If the shaft is rotating at a constant speed $\omega = 8$ rad/s, determine the minimum reactions exerted on the journal bearings at A and B during the motion.

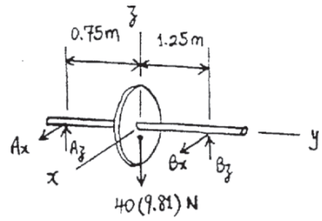


SOLUTION

$$\omega_x = 0$$

$$\omega_y = -8 \text{ rad/s}$$

$$\omega_z = 0$$



$$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\,\omega_y\,\omega_z;$$

$$B_z(1.25) - A_z(0.75) = 0 - 0$$

$$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\,\omega_x\,\omega_y;$$

$$-B_x(1.25) + A_x(0.75) = 0 - 0$$

$$\Sigma F_x = ma_x; \qquad A_x + B_x = 0$$

$$\Sigma F_z = ma_z; \qquad A_z + B_z - 40(9.81) = -40(8)^2(0.020)$$

Solving,

$$A_x = 0$$

$$B_x = 0$$

$$A_z = 213.25 \text{ N}$$

$$B_z = 127.95 \text{ N}$$

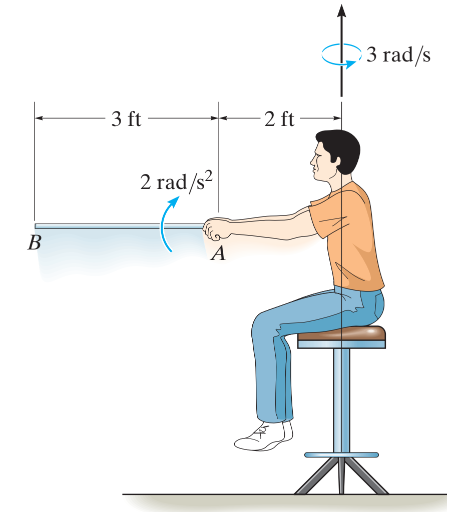
Thus,

$$F_A = 213 \text{ N} \qquad \textbf{Ans.}$$

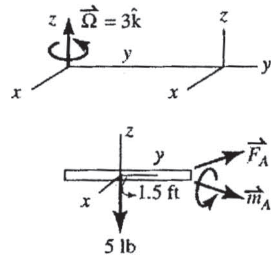
$$F_B = 128 \text{ N} \qquad \textbf{Ans.}$$

***21–48.**

The man sits on a swivel chair which is rotating with a constant angular velocity of 3 rad/s. He holds the uniform 5-lb rod AB horizontal. He suddenly gives it an angular acceleration of $2\ \text{rad/s}^2$, measured relative to him, as shown. Determine the required force and moment components at the grip, A, necessary to do this. Establish axes at the rod's center of mass G, with $+z$ upward, and $+y$ directed along the axis of the rod towards A.



SOLUTION



$$I_x = I_z = \frac{1}{12}\left(\frac{5}{32.2}\right)(3)^2 = 0.1165\ \text{ft}^4$$

$$I_y = 0$$

$$\Omega = \omega = 3\mathbf{k}$$

$$\omega_x = \omega_y = 0$$

$$\omega_z = 3\ \text{rad/s}$$

$$\dot{\Omega} = (\dot{\omega}_{xyz}) + \Omega \times \omega = -2\mathbf{i} + 0$$

$$\dot{\omega}_x = -2\ \text{rad/s}^2$$

$$\dot{\omega}_y = \dot{\omega}_z = 0$$

$$(a_G)_x = 0$$

$$(a_G)_y = (3.5)(3)^2 = 31.5\ \text{ft/s}^2$$

$$(a_G)_z = 2(1.5) = 3\ \text{ft/s}^2$$

$\Sigma F_x = m(a_G)_x; \qquad A_x = 0$ **Ans.**

$\Sigma F_y = m(a_G)_y; \qquad A_y = \dfrac{5}{32.2}(31.5) = 4.89\ \text{lb}$ **Ans.**

$\Sigma F_z = m(a_G)_z; \qquad -5 + A_z = \dfrac{5}{32.2}(3)$

$A_z = 5.47\ \text{lb}$ **Ans.**

$$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\,\omega_z;$$

$$M_x + 5.47(1.5) = 0.1165(-2) - 0$$

$M_x = -8.43\ \text{lb}\cdot\text{ft}$ **Ans.**

$$\Sigma M_y = I_y\dot{\omega}_x - (I_z - I_x)\omega_z\,\omega_x;$$

$$0 + M_y = 0 - 0$$

$M_y = 0$ **Ans.**

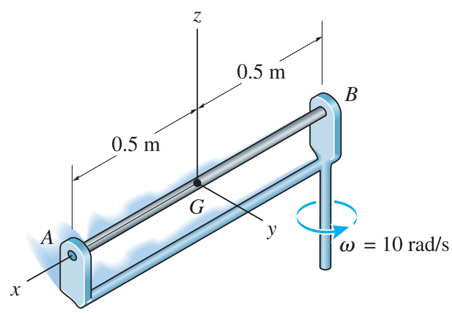
$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\,\omega_y;$$

$$M_z = 0 - 0$$

$M_z = 0$ **Ans.**

21–49.

The 5-kg rod AB is supported by a rotating arm. The support at A is a journal bearing, which develops reactions normal to the rod. The support at B is a thrust bearing, which develops reactions both normal to the rod and along the axis of the rod. Neglecting friction, determine the x, y, z components of reaction at these supports when the frame rotates with a constant angular velocity of $\omega = 10$ rad/s.



SOLUTION

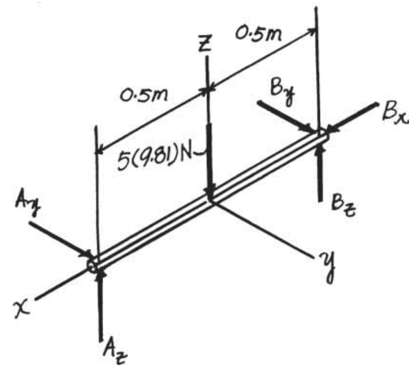
$$I_y = I_z = \frac{1}{12}(5)(1)^2 = 0.4167 \text{ kg}\cdot\text{m}^2 \qquad I_x = 0$$

Applying Eq. 21–25 with $\omega_x = \omega_y = 0 \;\; \omega_z = 10 \text{ rad/s} \;\; \dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$

$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z; \qquad 0 = 0$

$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x; \qquad B_z(0.5) - A_z(0.5) = 0$ **(1)**

$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y; \qquad A_y(0.5) - B_y(0.5) = 0$ **(2)**



Also,

$\Sigma F_x = m(a_G)_x; \qquad B_x = -5(10)^2(0.5) \qquad B_x = -250\text{N}$ **Ans.**

$\Sigma F_y = m(a_G)_y; \qquad A_y + B_y = 0$ **(3)**

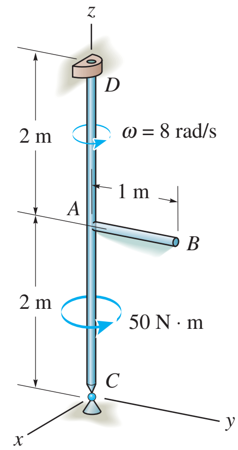
$\Sigma F_z = m(a_G)_z; \qquad A_z + B_z - 5(9.81) = 0$ **(4)**

Solving Eqs. (1) to (4) yields:

$$A_y = B_y = 0 \qquad A_z = B_z = 24.5 \text{ N}$$ **Ans.**

21–50.

The rod assembly is supported by a ball-and-socket joint at C and a journal bearing at D, which develops only x and y force reactions. The rods have a mass of 0.75 kg/m. Determine the angular acceleration of the rods and the components of reaction at the supports at the instant $\omega = 8$ rad/s as shown.



SOLUTION

$\Omega = \omega = 8\mathbf{k}$

$\omega_x = \omega_y = 0, \qquad \omega_z = 8$ rad/s

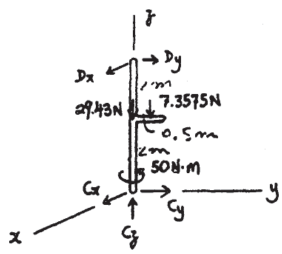
$\dot{\omega}_x = \dot{\omega}_y = 0, \qquad \dot{\omega}_z = \dot{\omega}_z$

$I_{xz} = I_{xy} = 0$

$I_{yz} = 0.75(1)(2)(0.5) = 0.75 \text{ kg} \cdot \text{m}^2$

$I_{zz} = \frac{1}{3}(0.75)(1)(1)^2 = 0.25 \text{ kg} \cdot \text{m}^2$

Eqs. 21–24 become



$\Sigma M_x = I_{yz}\omega_z^2$

$\Sigma M_y = -I_{yz}\dot{\omega}_z$

$\Sigma M_z = I_{zz}\dot{\omega}_z$

Thus,

$-D_y(4) - 7.3575(0.5) = 0.75(8)^2$

$D_y = -12.9$ N **Ans.**

$D_x(4) = -0.75\,\omega_z$

$50 = 0.25\,\dot{\omega}_z$

$\dot{\omega}_z = 200 \text{ rad/s}^2$ **Ans.**

$D_x = -37.5$ N **Ans.**

$\Sigma F_x = m(a_G)_x; \qquad C_x - 37.5 = -1(0.75)(200)(0.5)$

$C_x = -37.5$ N **Ans.**

$\Sigma F_y = m(a_G)_y; \qquad C_y - 12.9 = -(1)(0.75)(8)^2(0.5)$

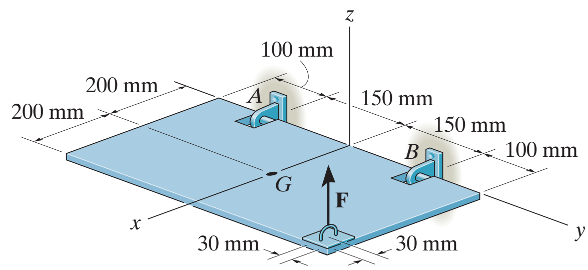
$C_y = -11.1$ N **Ans.**

$\Sigma F_z = m(a_G)_z; \qquad C_z - 7.3575 - 29.43 = 0$

$C_z = 36.8$ N **Ans.**

21–51.

The uniform hatch door, having a mass of 15 kg and a mass center at G, is supported in the horizontal plane by bearings at A and B. If a vertical force $F = 300$ N is applied to the door as shown, determine the components of reaction at the bearings and the angular acceleration of the door. The bearing at A will resist a component of force in the y direction, whereas the bearing at B will not. For the calculation, assume the door to be a thin plate and neglect the size of each bearing. The door is originally at rest.

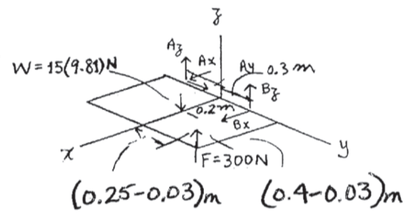


SOLUTION

$$\omega_x = \omega_y = \omega_z = 0$$

$$\dot{\omega}_x = \dot{\omega}_z = 0$$

Eqs. 21–25 reduce to



$\Sigma M_x = 0; \qquad 300(0.25 - 0.03) + B_z(0.15) - A_z(0.15) = 0$

$B_z - A_z = -440$ **(1)**

$\Sigma M_y = I_y\dot{\omega}_y; \qquad 15(9.81)(0.2) - (300)(0.4 - 0.03) = [\frac{1}{12}(15)(0.4)^2 + 15(0.2)^2]\dot{\omega}_y$

$\dot{\omega}_y = -102 \text{ rad/s}^2$ **Ans.**

$\Sigma M_z = 0; \qquad -B_x(0.15) + A_x(0.15) = 0$

$\Sigma F_x = m(a_G)_x; \qquad -A_x + B_x = 0$

$A_x = B_x = 0$ **Ans.**

$\Sigma F_y = m(a_G)_y; \qquad A_y = 0$ **Ans.**

$\Sigma F_z = m(a_G)_z; \qquad 300 - 15(9.81) + B_z + A_z = 15(101.96)(0.2)$

$B_z + A_z = 153.03$ **(2)**

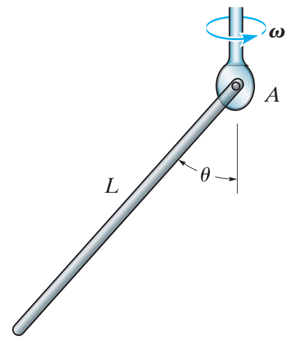
Solving Eqs. (1) and (2) yields

$$A_z = 297 \text{ N}$$ **Ans.**

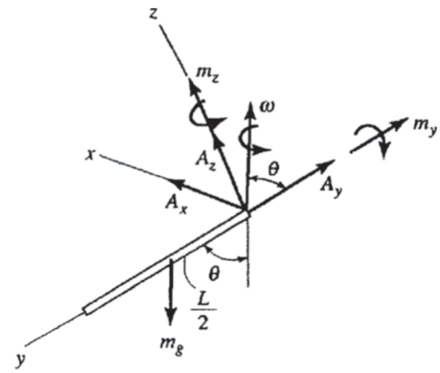
$$B_z = -143 \text{ N}$$ **Ans.**

***21–52.**

The conical pendulum consists of a bar of mass m and length L that is supported by the pin at its end A. If the pin is subjected to a rotation $\boldsymbol{\omega}$, determine the angle θ that the bar makes with the vertical as it rotates.



SOLUTION



$$I_x = I_z = \frac{1}{3}mL^2, \qquad I_y = 0$$

$$\omega_x = 0, \qquad \omega_y = -\omega\cos\theta, \qquad \omega_z = \omega\sin\theta$$

$$\dot{\omega}_x = 0, \qquad \dot{\omega}_y = 0, \qquad \dot{\omega}_z = 0$$

$$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z$$

$$-mg\left(\frac{L}{2}\sin\theta\right) = 0 - \left(0 - \frac{1}{3}mL^2\right)(-\omega\cos\theta)(\omega\sin\theta)$$

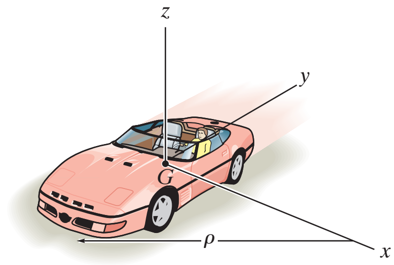
$$\frac{g}{2} = \frac{1}{3}L\omega^2\cos\theta$$

$$\cos\theta = \frac{3g}{2L\omega^2}$$

$$\theta = \cos^{-1}\left(\frac{3g}{2L\omega^2}\right)$$ **Ans.**

21–53.

The car travels around the curved road of radius ρ such that its mass center has a constant speed v_G. Write the equations of rotational motion with respect to the x, y, z axes. Assume that the car's six moments and products of inertia with respect to these axes are known.



SOLUTION

Applying Eq. 21–24 with $\omega_x = 0, \quad \omega_y = 0, \quad \omega_z = \dfrac{v_G}{\rho},$

$$\omega_x = \omega_y = \omega_z = 0$$

$$\Sigma M_x = -I_{yz}\left[0 - \left(\frac{v_G}{\rho}\right)^2\right] = \frac{I_{yz}}{\rho^2}v_G^2 \qquad \textbf{Ans.}$$

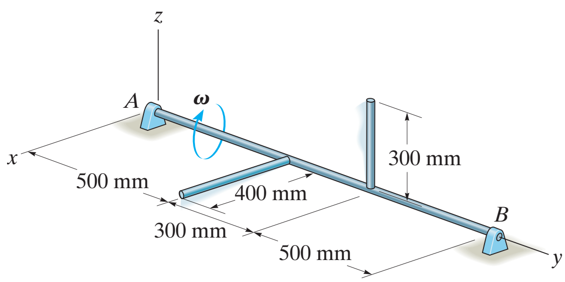
$$\Sigma M_y = -I_{zx}\left[\left(\frac{v_G}{\rho}\right)^2 - 0\right] = -\frac{I_{zx}}{\rho^2}v_G^2 \qquad \textbf{Ans.}$$

$$\Sigma M_z = 0 \qquad \textbf{Ans.}$$

Note: This result indicates the normal reactions of the tires on the ground are not all necessarily equal. Instead, they depend upon the speed of the car, radius of curvature, and the products of inertia I_{yz} and I_{zx}. (See Example 13–6.)

21–54.

The rod assembly is supported by journal bearings at A and B, which develops only x and y force reactions on the shaft. If the shaft AB is rotating in the direction shown at $\boldsymbol{\omega} = \{-5\mathbf{j}\}$ rad/s, determine the reactions at the bearings when the assembly is in the position shown. Also, what is the shaft's angular acceleration? The mass of each rod is 1.5 kg/m.

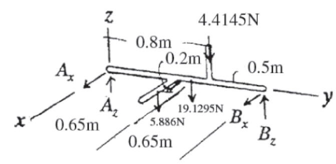


SOLUTION

$$\omega_x = \omega_z = 0, \qquad \omega_y = -5 \text{ rad/s}$$

$$\dot{\omega}_x = \dot{\omega}_z = 0$$

Eqs. 21–24 become



$$\Sigma M_x = -I_{xy}\dot{\omega}_y - I_{yz}\omega_y^2$$

$$\Sigma M_y = I_{yy}\,\dot{\omega}_y$$

$$\Sigma M_z = I_{xy}\,\omega_y^2 - I_{yz}\,\dot{\omega}_y$$

$$I_{yy} = \frac{1}{3}(0.4)(1.5)(0.4)^2 + \frac{1}{3}(0.3)(1.5)(0.3)^2 = 0.0455 \text{ kg} \cdot \text{m}^2$$

$$I_{yz} = [0 + (1.5)(0.3)(0.15)(0.8)] = 0.0540 \text{ kg} \cdot \text{m}^2$$

$$I_{xy} = [0 + (1.5)(0.4)(0.2)(0.5)] = 0.0600 \text{ kg} \cdot \text{m}^2$$

Thus,

$$-5.886(0.5) - 19.1295(0.65) - 4.4145(0.8) + B_z(1.3) = -0.0600\,\dot{\omega}_y - 0.0540(-5)^2$$

$$5.886(0.2) = 0.0455\,\dot{\omega}_y$$

$$-B_x\,(1.3) = 0.0600(-5)^2 - (0.0540)\,\dot{\omega}_y$$

$\dot{\omega}_y = 25.9 \text{ rad/s}^2$ **Ans.**

$B_x = -0.0791 \text{ N}$ **Ans.**

$B_z = 12.3 \text{ N}$ **Ans.**

$\Sigma F_x = m(a_G)_x; \qquad A_x - 0.0791 = -0.4(1.5)(5)^2(0.2) + 0.3(1.5)(25.9)(0.15)$

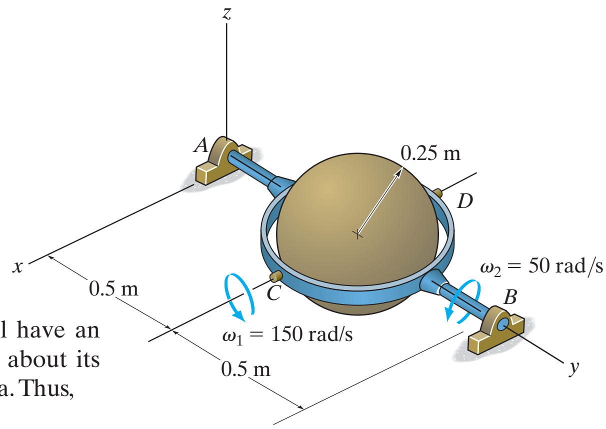
$A_x = -1.17 \text{ N}$ **Ans.**

$\Sigma F_z = m(a_G)_z; \qquad A_z + 12.31 - 5.886 - 19.1295 - 4.4145 = -0.4(1.5)(25.9)(0.2) - 0.3(1.5)(5)^2(0.15)$

$A_z = 12.3 \text{ N}$ **Ans.**

21–55.

The 20-kg sphere is rotating with a constant angular speed of $\omega_1 = 150$ rad/s about axle CD, which is mounted on the circular ring. The ring rotates about shaft AB with a constant angular speed of $\omega_2 = 50$ rad/s. If shaft AB is supported by a thrust bearing at A and a journal bearing at B, determine the X, Y, Z components of reaction at these bearings at the instant shown. Neglect the mass of the ring and shaft.



SOLUTION

The rotating xyz frame is established as shown in Fig. a. This frame will have an angular velocity of $\Omega = \omega_2 = [50\mathbf{j}]$ rad/s. Since the sphere is symmetric about its spinning axis, the x, y, and z axes will remain as the principal axes of inertia. Thus,

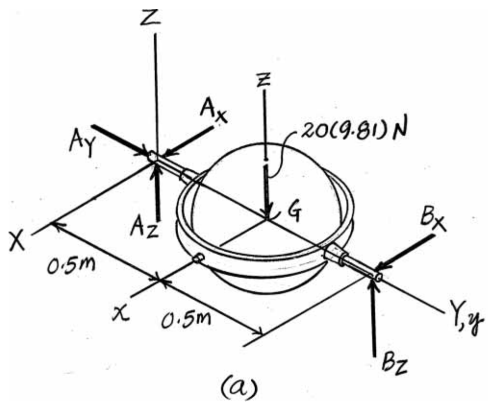
$$I_x = I_y = I_z = \frac{2}{5}mr^2 = \frac{2}{5}(20)(0.25)^2 = 0.5\ \text{kg} \cdot \text{m}^2$$

The angular velocity of the sphere is $\omega = \omega_1 + \omega_2 = [150\mathbf{i} + 50\mathbf{j}]$ rad/s. Thus,

$$\omega_x = 150\ \text{rad/s} \qquad \omega_y = 50\ \text{rad/s} \qquad \omega_z = 0$$

Since the directions of ω_1 and ω_2 do not change with respect to the xyz frame and their magnitudes are constant, $\dot{\omega}_{xyz} = \mathbf{0}$. Thus,

$$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$$



(a)

Applying the equations of motion and referring to the free-body diagram shown in Fig. a,

$\Sigma M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z;\qquad B_Z(0.5) - A_Z(0.5) = 0 - 0 + 0$

$$B_Z - A_Z = 0 \tag{1}$$

$\Sigma M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x;\qquad 0 = 0$

$\Sigma M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y;\qquad A_X(0.5) - B_X(0.5) = 0 - 0.5(50)(150) + 0$

$$A_X - B_X = -7500 \tag{2}$$

Since the mass center G does not move, $\mathbf{a}_G = \mathbf{0}$. Thus,

$\Sigma F_X = m(a_G)_X;\qquad A_X + B_X = 0 \qquad$ **(3)**

$\Sigma F_Y = m(a_G)_Y;\qquad A_Y = 0 \qquad$ **Ans.**

$\Sigma F_Z = m(a_G)_Z;\qquad A_Z + B_Z - 20(9.81) = 0 \qquad$ **(4)**

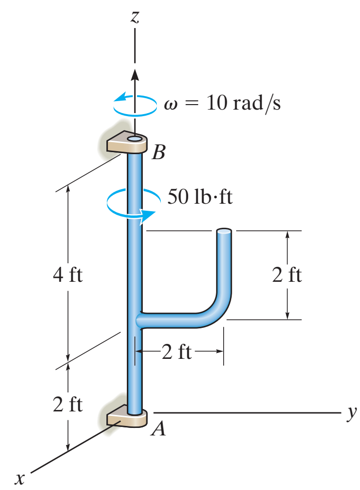
Solving Eqs. (1) through (4),

$$A_Z = B_Z = 98.1\ \text{N}$$

$A_X = -3750\ \text{N} = -3.75\ \text{kN} \qquad B_X = 3750\ \text{N} = 3.75\ \text{kN} \qquad$ **Ans.**

***21–56.**

The rod assembly has a weight of 5 lb/ft. It is supported at B by a smooth journal bearing, which develops x and y force reactions, and at A by a smooth thrust bearing, which develops x, y, and z force reactions. If a 50-lb·ft torque is applied along rod AB, determine the components of reaction at the bearings when the assembly has an angular velocity $\omega = 10$ rad/s at the instant shown.



SOLUTION

$$I_y = \frac{1}{3}\left[\frac{6(5)}{32.2}\right](6)^2 + \frac{1}{12}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](3)^2 + \left[\frac{2(5)}{32.2}\right](2)^2$$

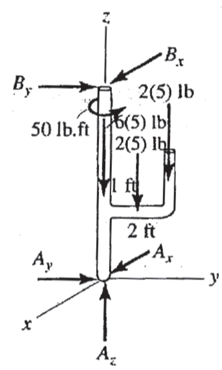
$$= 15.3209 \text{ slug} \cdot \text{ft}^2$$

$$I_x = \frac{1}{3}\left[\frac{6(5)}{32.2}\right](6)^2 + \frac{1}{12}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](2^2 + 3^2) + \frac{1}{12}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](1^2 + 2^2)$$

$$I_x = 16.9772 \text{ slug} \cdot \text{ft}^2$$

$$I_z = \frac{1}{3}\left[\frac{2(5)}{32.2}\right](2)^2 + \left[\frac{2(5)}{32.2}\right](2)^2 = 1.6563 \text{ slug} \cdot \text{ft}^2$$

$$I_{yz} = \left[\frac{2(5)}{32.2}\right](1)(2) + \left[\frac{2(5)}{32.2}\right](2)(3) = 2.4845 \text{ slug} \cdot \text{ft}^2 \qquad I_{xy} = I_{zx} = 0$$



Applying Eq. 21-24 with $\omega_x = \omega_y = 0$, $\omega_z = 10$ rad/s, $\dot{\omega}_x = \dot{\omega}_y = 0$

$$-B_y(6) - 2(5)(1) - 2(5)(2) = 0 - 0 - 0 - 2.4845(0 - 10^2) - 0$$

$$B_y = -46.4 \qquad \textbf{Ans.}$$

$$B_x(6) = 0 - 0 - 2.4845\dot{\omega}_z - 0 - 0 \qquad \textbf{(1)}$$

$$50 = 1.6563\dot{\omega}_z \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$\dot{\omega}_z = 30.19 \text{ rad/s}^2$$

$$B_x = -12.5 \text{ lb} \qquad \textbf{Ans.}$$

$$\Sigma F_x = m(a_G)_x; \qquad A_x + (-12.50) = -\left[\frac{2(5)}{32.2}\right](1)(30.19) - \left[\frac{2(5)}{32.2}\right](2)(30.19)$$

$$A_x = -15.6 \text{ lb} \qquad \textbf{Ans.}$$

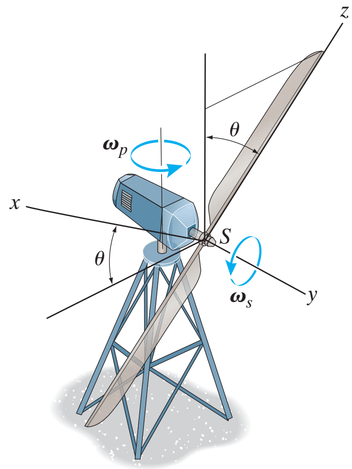
$$\Sigma F_y = m(a_G)_y; \qquad A_y + (-46.41) = -\left[\frac{2(5)}{32.2}\right](1)(10)^2 - \left[\frac{2(5)}{32.2}\right](2)(10)^2$$

$$A_y = -46.8 \text{ lb} \qquad \textbf{Ans.}$$

$$\Sigma F_z = m(a_G)_z; \qquad A_z - 2(5) - 2(5) - 6(5) = 0 \quad A_z = 50 \text{ lb} \qquad \textbf{Ans.}$$

21–57.

The blades of a wind turbine spin about the shaft S with a constant angular speed of ω_s, while the frame precesses about the vertical axis with a constant angular speed of ω_p. Determine the x, y, and z components of moment that the shaft exerts on the blades as a function of θ. Consider each blade as a slender rod of mass m and length l.



SOLUTION

The rotating xyz frame shown in Fig. a will be attached to the blade so that it rotates with an angular velocity of $\Omega = \omega$, where $\omega = \omega_s + \omega_p$. Referring to Fig. b $\omega_p = \omega_p \sin\theta\mathbf{i} + \omega_p\cos\theta\mathbf{k}$. Thus, $\omega = \omega_p\sin\theta\mathbf{i} + \omega_s\mathbf{j} + \omega_p\cos\theta\mathbf{k}$. Then

$$\omega_x = \omega_p\sin\theta \qquad \omega_y = \omega_s \quad \omega_z = \omega_p\cos\theta$$

The angular acceleration of the blade $\dot{\omega}$ with respect to the XYZ frame can be obtained by setting another $x'y'z'$ frame having an angular velocity of $\Omega' = \omega_p = \omega_p\sin\theta\mathbf{i} + \omega_p\cos\theta\mathbf{k}$. Thus,

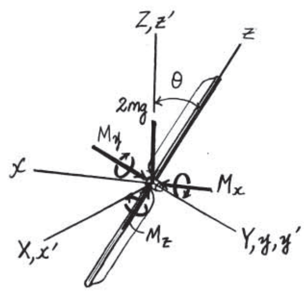
$$\begin{aligned}
\dot{\omega} &= \left(\dot{\omega}_{x'y'z'}\right) + \Omega' \times \omega \\
&= (\dot{\omega}_1)_{x'y'z'} + (\dot{\omega}_2)_{x'y'z'} + \Omega' \times \omega_S + \Omega' \times \omega_P \\
&= \mathbf{0} + \mathbf{0} + \left(\omega_p\sin\theta\mathbf{i} + \omega_p\cos\theta\mathbf{k}\right) \times (\omega_s\mathbf{j}) + \mathbf{0} \\
&= -\omega_s\omega_p\cos\theta\mathbf{i} + \omega_s\omega_p\sin\theta\mathbf{k}
\end{aligned}$$

Since $\Omega = \omega$, $\dot{\omega}_{x'y'z'} = \dot{\omega}$. Thus,

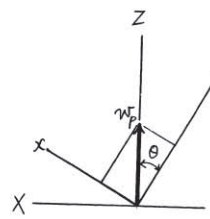
$$\dot{\omega}_x = -\omega_s\omega_p\cos\theta \qquad \dot{\omega}_y = 0 \qquad \dot{\omega}_z = \omega_s\omega_p\sin\theta$$

Also, the x, y, and z axes will remain as principle axes of inertia for the blade. Thus,

$$I_x = I_y = \frac{1}{12}(2m)(2l)^2 = \frac{2}{3}ml^2 \qquad I_z = 0$$



(a)



(b)

Applying the moment equations of motion and referring to the free-body diagram shown in Fig. a,

$$\Sigma M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z; \qquad M_x = \frac{2}{3}ml^2(-\omega_s\omega_p\cos\theta) - \left(\frac{2}{3}ml^2 - 0\right)(\omega_s)(\omega_p\cos\theta)$$

$$= -\frac{4}{3}ml^2\,\omega_s\omega_p\cos\theta \qquad \textbf{Ans.}$$

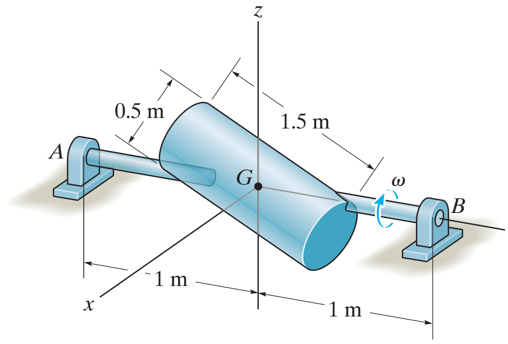
$$\Sigma M_y = I_y\dot{\omega}_y - \left(I_z - I_x\right)\omega_z\omega_x; \qquad M_y = 0 - \left(0 - \frac{2}{3}ml^2\right)(\omega_p\cos\theta)(\omega_p\sin\theta)$$

$$= \frac{1}{3}ml^2\,\omega_p^{\,2}\sin 2\theta \qquad \textbf{Ans.}$$

$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y; \qquad M_z = 0 - 0 = 0 \qquad \textbf{Ans.}$$

21–58.

The cylinder has a mass of 30 kg and is mounted on an axle that is supported by bearings at A and B. If the axle is turning at $\boldsymbol{\omega} = \{-40\mathbf{j}\}$ rad/s, determine the vertical components of force acting at the bearings at this instant.



SOLUTION

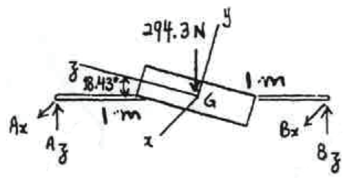
$\omega_x = 0$

$\omega_y = -40 \sin 18.43° = -12.65 \text{ rad/s}$

$\omega_z = 40 \cos 18.43° = 37.95 \text{ rad/s}$

$\dot{\omega}_x = 0, \qquad \dot{\omega}_y = 0, \qquad \dot{\omega}_z = 0$

$(a_G)_x = (a_G)_y = (a_G)_z = 0$



$$I_x = I_y = \frac{1}{12}(30)[3(0.25)^2 + (1.5)^2] = 6.09375 \text{ kg} \cdot \text{m}^2$$

$$I_z = \frac{1}{2}(30)(0.25)^2 = 0.9375 \text{ kg} \cdot \text{m}^2$$

Using the first of Eqs. 21–25,

$$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z$$

$$B_z(1) - A_z(1) = 0 - (6.09375 - 0.9375)(-12.65)(37.95)$$

$$B_z - A_z = 2475$$

Also, summing forces in the vertical direction,

$$\Sigma F_z = m(a_G)_z; \qquad A_z + B_z - 294.3 = 0$$

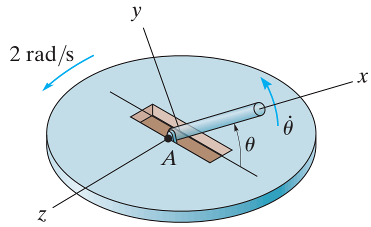
Solving,

$A_z = -1.09 \text{ kN}$ **Ans.**

$B_z = 1.38 \text{ kN}$ **Ans.**

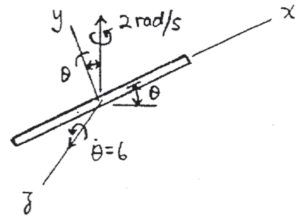
21–59.

The *thin rod* has a mass of 0.8 kg and a total length of 150 mm. It is rotating about its midpoint at a constant rate while the table to which its axle A is fastened is rotating at 2 rad/s. Determine the x, y, z moment components which the axle exerts on the rod when the rod is in any position θ.



SOLUTION

The x,y,z *axes* are fixed as shown.



$$\omega_x = 2 \sin \theta$$

$$\omega_y = 2 \cos \theta$$

$$\omega_z = \dot{\theta} = 6$$

$$\dot{\omega}_x = 2\dot{\theta} \cos \theta = 12 \cos \theta$$

$$\dot{\omega}_y = -2\dot{\theta} \sin \theta = -12 \sin \theta$$

$$\dot{\omega}_z = 0$$

$$I_x = 0$$

$$I_y = I_z = \frac{1}{12}(0.8)(0.15)^2 = 1.5(10^{-3})$$

Using Eqs. 21–25:

$$\Sigma M_x = 0 - 0 = 0 \qquad \textbf{Ans.}$$

$$\Sigma M_y = 1.5(10^{-3})(-12 \sin \theta) - [1.5(10^{-3}) - 0](6)(2 \sin \theta)$$

$$\Sigma M_y = (-0.036 \sin \theta)\ \text{N} \cdot \text{m} \qquad \textbf{Ans.}$$

$$\Sigma M_z = 0 - [0 - 1.5(10^{-3})](2 \sin \theta)(2 \cos \theta)$$

$$\Sigma M_z = 0.006 \sin \theta \cos \theta = (0.003 \sin 2\theta)\ \text{N} \cdot \text{m} \qquad \textbf{Ans.}$$

***21–60.**

Show that the angular velocity of a body, in terms of Euler angles ϕ, θ, and ψ, can be expressed as $\omega = (\dot{\phi} \sin\theta \sin\psi + \dot{\theta}\cos\psi)\mathbf{i} + (\dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi)\mathbf{j} + (\dot{\phi}\cos\theta + \dot{\psi})\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are directed along the x, y, z axes as shown in Fig. 21–15*d*.

SOLUTION

From Fig. 21–15*b*. due to rotation ϕ, the x, y, z components of $\dot{\phi}$ are simply $\dot{\phi}$ along z axis.

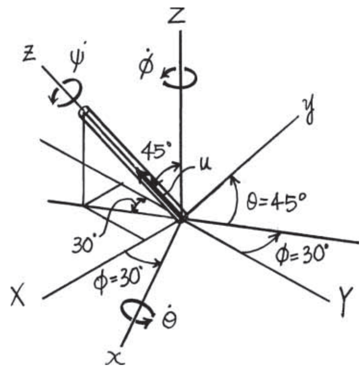
From Fig 21–15*c*, due to rotation θ, the x, y, z components of $\dot{\phi}$ and $\dot{\theta}$ are $\dot{\phi}\sin\theta$ in the y direction, $\dot{\phi}\cos\theta$ in the z direction, and $\dot{\theta}$ in the x direction.

Lastly, rotation ψ. Fig. 21–15*d*, produces the final components which yields

$$\omega = \left(\dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi\right)\mathbf{i} + \left(\dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi\right)\mathbf{j} + \left(\dot{\phi}\cos\theta + \dot{\psi}\right)\mathbf{k} \quad \textbf{Q.E.D.}$$

21–61.

A thin rod is initially coincident with the Z axis when it is given three rotations defined by the Euler angles $\phi = 30°$, $\theta = 45°$, and $\psi = 60°$. If these rotations are given in the order stated, determine the coordinate direction angles α, β, γ of the axis of the rod with respect to the X, Y, and Z axes. Are these directions the same for any order of the rotations? Why?



SOLUTION

$$\mathbf{u} = (1 \sin 45°) \sin 30° \, \mathbf{i} - (1 \sin 45°) \cos 30° \mathbf{j} + 1 \cos 45° \, \mathbf{k}$$

$$\mathbf{u} = 0.3536\mathbf{i} - 0.6124\mathbf{j} + 0.7071\mathbf{k}$$

$$\alpha = \cos^{-1} 0.3536 = 69.3°$$ **Ans.**

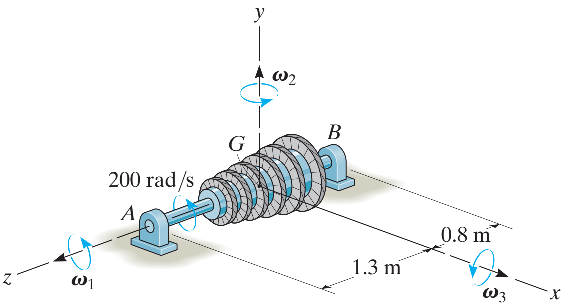
$$\beta = \cos^{-1}(-0.6124) = 128°$$ **Ans.**

$$\gamma = \cos^{-1}(0.7071) = 45°$$ **Ans.**

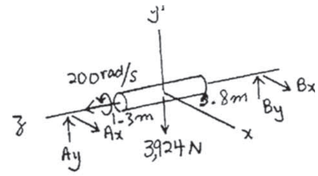
No, the orientation of the rod will not be the same for any order of rotation, because finite rotations are not vectors.

21–62.

The turbine on a ship has a mass of 400 kg and is mounted on bearings A and B as shown. Its center of mass is at G, its radius of gyration is $k_z = 0.3$ m, and $k_x = k_y = 0.5$ m. If it is spinning at 200 rad/s, determine the vertical reactions at the bearings when the ship undergoes each of the following motions: (a) rolling, $\omega_1 = 0.2$ rad/s, (b) turning, $\omega_2 = 0.8$ rad/s, (c) pitching, $\omega_3 = 1.4$ rad/s.



SOLUTION



a) $\omega_1 = 0.2 + 200 = 200.2$ rad/s

$\Sigma F_y = m(a_G)_y; \qquad A_y + B_y - 3924 = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z;$

$B_y(0.8) - A_y(1.3) = 0 - 0$

Thus,

$A_y = 1.49$ kN **Ans.**

$B_y = 2.43$ kN **Ans.**

b) $\Omega = 0.8\mathbf{j}$

$\omega = 0.8\mathbf{j} + 200\mathbf{k}$

$\dot{\omega} = 0 + 0.8\mathbf{j} \text{ X } (0.8\mathbf{j} + 200\mathbf{k}) = 160\mathbf{i}$

$\Sigma F_y = m(a_G)_y; \qquad A_y + B_y - 3924 = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z;$

$B_y(0.8) - A_y(1.3) = 400(0.5)^2(160) - [400(0.5)^2 - 400(0.3)^2](0.8)(200)$

$B_y(0.8) - A_y(1.3) = 5760$

Thus,

$A_y = -1.24$ kN **Ans.**

$B_y = 5.17$ kN **Ans.**

c) $\Omega = 1.4\mathbf{i}$

$\omega = 1.4\mathbf{i} + 200\mathbf{k}$

$\dot{\omega} = 1.4\mathbf{i} \times (1.4\mathbf{i} + 200\mathbf{k}) = -280\mathbf{j}$

$\Sigma F_y = m(a_G)_y; \qquad A_y + B_y - 3924 = 0$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z;$

$B_y(0.8) - A_y(1.3) = 0 - 0$

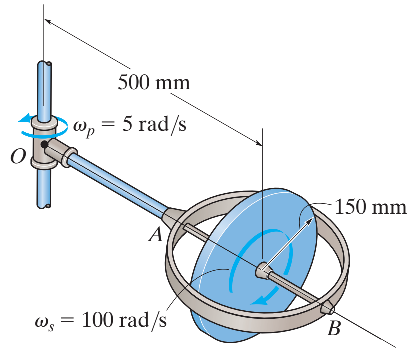
Thus,

$A_y = 1.49$ kN **Ans.**

$B_y = 2.43$ kN **Ans.**

21–63.

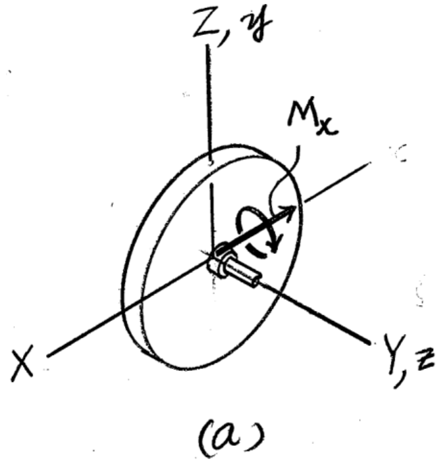
The 10-kg disk spins about axle AB at a constant rate of $\omega_s = 100$ rad/s. If the supporting arm precesses about the vertical axis at a constant rate of $\omega_p = 5$ rad/s, determine the internal moment at O caused only by the gyroscopic action.



SOLUTION

Here, $\theta = 90°$, $\dot{\phi} = \omega_p = 5$ rad/s, and $\dot{\psi} = -\omega_s = -100$ rad/s are constants. This is the special case of precession.

$I_z = \frac{1}{2}(10)(0.15^2) = 0.1125\ \text{kg}\cdot\text{m}^2$, $\Omega_z = \dot{\phi} = 5$ rad/s, and $\omega_z = \dot{\psi} = -100$ rad/s.

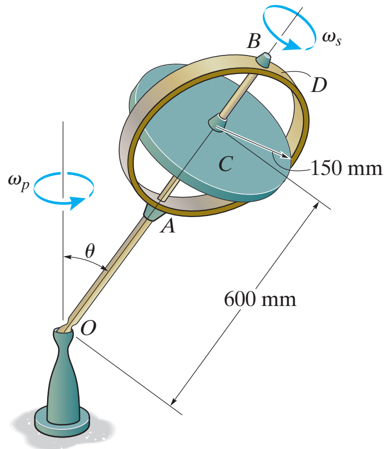


Thus,

$\Sigma M_x = I_z \Omega_y \omega_z;$ $\qquad M_x = 0.1125(5)(-100) = -56.25\ \text{N}\cdot\text{m}$ **Ans.**

***21–64.**

The 10-kg disk spins about axle AB at a constant rate of $\omega_s = 250$ rad/s, and $\theta = 30°$. Determine the rate of precession of arm OA. Neglect the mass of arm OA, axle AB, and the circular ring D.



SOLUTION

Since $\theta = 30°$, $\dot{\psi} = \omega_s = 250$ rad/s, and $\dot{\phi} = \omega_p$ are constant, the disk undergoes steady precession. $I_z = \frac{1}{2}(10)(0.15^2) = 0.1125\ \text{kg} \cdot \text{m}^2$ and $I = I_x = I_y = \frac{1}{4}(10)(0.15^2) + 10(0.6^2) = 3.65625\ \text{kg} \cdot \text{m}^2$. Thus,

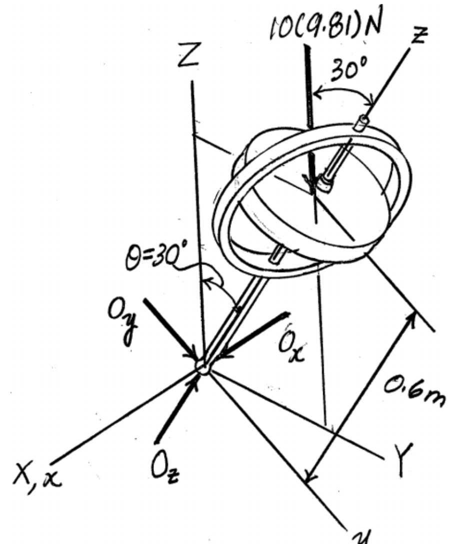
$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta \cos\theta + I_z \dot{\phi} \sin\theta(\dot{\phi}\cos\theta + \dot{\psi})$$

$$-10(9.81)\sin 30°(0.6) = -3.65625\omega_p^2 \sin 30° \cos 30° + 0.1125\omega_p \sin 30°(\omega_p \cos 30° + 250)$$

$$1.5345\omega_p^2 - 14.0625\omega_p - 29.43 = 0$$

Solving,

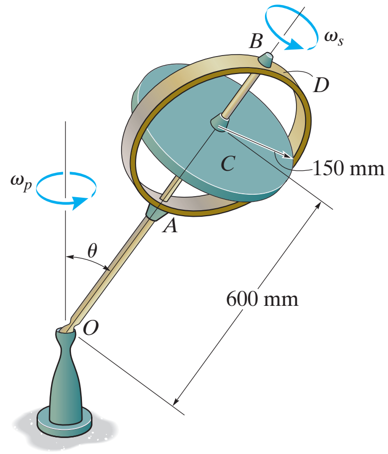
$$\omega_p = 10.9 \text{ rad/s or } -1.76 \text{ rad/s}$$ **Ans.**



(a)

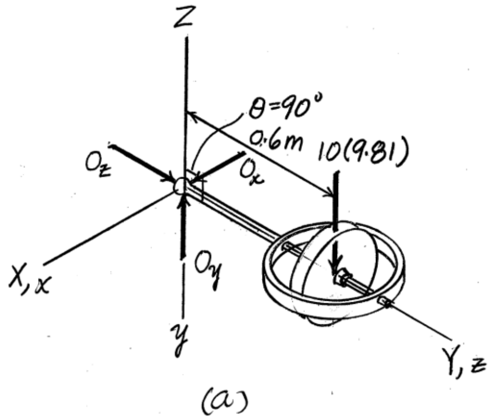
21–65.

When OA precesses at a constant rate of $\omega_p = 5$ rad/s, when $\theta = 90°$, determine the required spin of the 10-kg disk C. Neglect the mass of arm OA, axle AB, and the circular ring D.



SOLUTION

Here, $\theta = 90°$, $\dot{\psi} = \omega_s$, and $\dot{\phi} = \omega_p = 5$ rad/s are constant. Thus, this is a special case of steady precession.



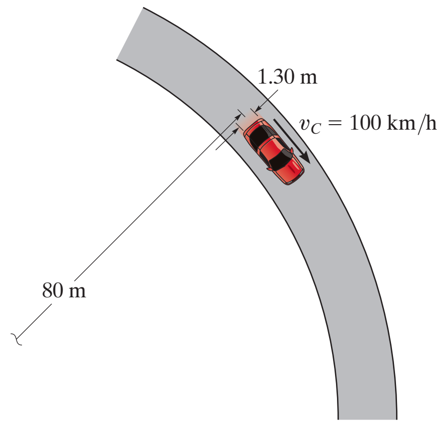
$I_z = \frac{1}{2}(10)(0.15^2) = 0.1125\ \text{kg}\cdot\text{m}^2$, $\Omega_y = -\dot{\phi} = -5$ rad/s, and $\omega_z = \dot{\psi} = \omega_s$. Then,

$\Sigma M_x = I_z \Omega_y \omega_z; \quad -10(9.81)(0.6) = 0.1125(-5)(\omega_s)$

$$\omega_s = 104.64\ \text{rad/s} = 105\ \text{rad/s} \qquad \textbf{Ans.}$$

21–66.

The car travels at a constant speed of $v_C = 100\text{ km/h}$ around the horizontal curve having a radius of 80 m. If each wheel has a mass of 16 kg, a radius of gyration $k_G = 300\text{ mm}$ about its spinning axis, and a radius of 400 mm, determine the difference between the normal forces of the rear wheels, caused by the gyroscopic effect. The distance between the wheels is 1.30 m.



SOLUTION

$$I = 2[16(0.3)^2] = 2.88\text{ kg}\cdot\text{m}^2$$

$$\omega_s = \frac{100(1000)}{3600(0.4)} = 69.44\text{ rad/s}$$

$$\omega_p = \frac{100(1000)}{80(3600)} = 0.347\text{ rad/s}$$

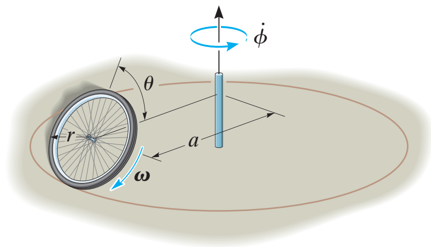
$$M = I\,\omega_s\,\omega_p$$

$$\Delta F(1.30) = 2.88(69.44)(0.347)$$

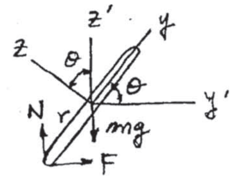
$$\Delta F = 53.4\text{ N}$$ **Ans.**

21–67.

A wheel of mass m and radius r rolls with constant spin $\boldsymbol{\omega}$ about a circular path having a radius a. If the angle of inclination is θ, determine the rate of precession. Treat the wheel as a thin ring. No slipping occurs.



SOLUTION



Since no slipping occurs,

$$(r)\dot{\psi} = (a + r\cos\theta)\dot{\phi}$$

or

$$\dot{\psi} = \left(\frac{a + r\cos\theta}{r}\right)\dot{\phi} \qquad \textbf{(1)}$$

Also,

$$\omega = \dot{\phi} + \dot{\psi}$$

$$\Sigma F_{y'} = m(a_G)_{y'}; \qquad F = m(a\,\dot{\phi}^2) \qquad \textbf{(2)}$$

$$\Sigma F_{z'} = m(a_G)_{z'}; \qquad N - mg = 0 \qquad \textbf{(3)}$$

$$I_x = I_y = \frac{mr^2}{2}, \qquad I_z = mr^2$$

$$\omega = \dot{\phi}\sin\theta\,\mathbf{j} + (-\dot{\psi} + \dot{\phi}\cos\theta)\mathbf{k}$$

Thus,

$$\omega_x = 0, \qquad \omega_y = \dot{\phi}\sin\theta, \qquad \omega_{z'} = -\dot{\psi} + \dot{\phi}\cos\theta$$

$$\dot{\omega} = \dot{\phi} \times \dot{\psi} = -\dot{\phi}\dot{\psi}\sin\theta$$

$$\dot{\omega}_x = -\dot{\phi}\dot{\psi}\sin\theta, \qquad \dot{\omega}_y = \dot{\omega}_z = 0$$

Applying

$$\Sigma M_x = I_x\,\dot{\omega}_x + (I_z - I_y)\omega_z\omega_y$$

$$F\,r\sin\theta - N\,r\cos\theta = \frac{m\,r^2}{2}(-\dot{\phi}\dot{\psi}\sin\theta) + \left(m\,r^2 - \frac{mr^2}{2}\right)(-\dot{\psi} + \dot{\phi}\cos\theta)(\dot{\phi}\sin\theta)$$

Using Eqs. (1), (2) and (3), and eliminating $\dot{\psi}$, we have

$$m\,a\,\dot{\phi}^2\,r\sin\theta - m\,g\,r\cos\theta = \frac{m\,r^2}{2}(-\dot{\phi})\sin\theta\left(\frac{a + r\cos\theta}{r}\right)\dot{\phi} + \frac{mr^2}{2}\left(\frac{-\dot{\phi}\,a}{r}\right)\dot{\phi}\sin\theta$$

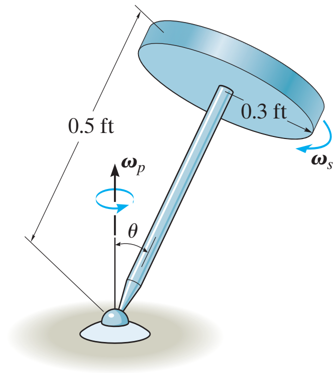
$$m\,a\,\dot{\phi}^2\sin\theta\,r - m\,g\,r\cos\theta = \frac{m\,r^2}{2}\left(\frac{-\dot{\phi}^2 a}{r}\right)\sin\theta - \frac{m\,r^2}{2}(\dot{\phi}^2\sin\theta\cos\theta)$$

$$2\,g\cos\theta = a\,\dot{\phi}^2\sin\theta + r\,\dot{\phi}^2\sin\theta\cos\theta$$

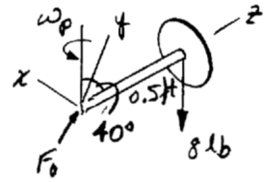
$$\dot{\phi} = \left(\frac{2\,g\cot\theta}{a + r\cos\theta}\right)^{1/2} \qquad \textbf{Ans.}$$

***21–68.**

The top consists of a thin disk that has a weight of 8 lb and a radius of 0.3 ft. The rod has a negligible mass and a length of 0.5 ft. If the top is spinning with an angular velocity $\omega_s = 300$ rad/s, determine the steady-state precessional angular velocity ω_p of the rod when $\theta = 40°$.



SOLUTION



$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta\left(\phi\cos\theta + \dot{\psi}\right)$$

$$8(0.5\sin 40°) = -\left[\frac{1}{4}\left(\frac{8}{32.2}\right)(0.3)^2 + \left(\frac{8}{32.2}\right)(0.5)^2\right]\omega_p^2 \sin 40°\cos 40°$$

$$+\left[\frac{1}{2}\left(\frac{8}{32.2}\right)(0.3)^2\right]\omega_p \sin 40°\,(\omega_p\cos 40° + 300)$$

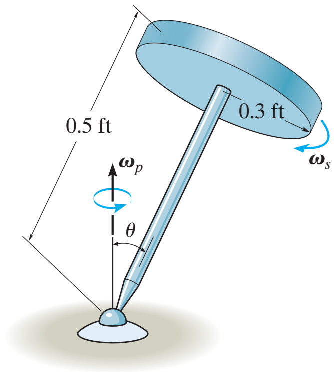
$$0.02783\omega_p^2 - 2.1559\omega_p + 2.571 = 0$$

$\omega_p = 1.21$ rad/s **Ans.** (Low precession)

$\omega_p = 76.3$ rad/s **Ans.** (High precession)

21–69.

Solve Prob. 21–68 when $\theta = 90°$.

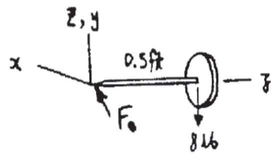


SOLUTION

$$\Sigma M_x = I_z \Omega_y \, \omega_z$$

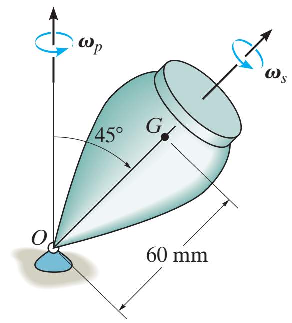
$$8(0.5) = \left[\frac{1}{2}\left(\frac{8}{32.2}\right)(0.3)^2\right]\omega_p\,(300)$$

$$\omega_p = 1.19 \text{ rad/s}$$ **Ans.**



21–70.

The top has a mass of 90 g, a center of mass at G, and a radius of gyration $k = 18$ mm about its axis of symmetry. About any transverse axis acting through point O the radius of gyration is $k_t = 35$ mm. If the top is connected to a ball-and-socket joint at O and the precession is $\omega_p = 0.5$ rad/s, determine the spin $\boldsymbol{\omega}_s$.



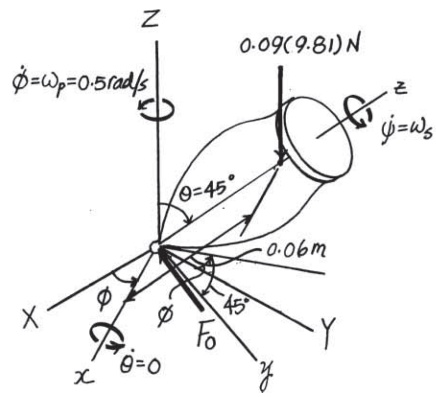
SOLUTION

$$\omega_p = 0.5 \text{ rad/s}$$

$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta \cos\theta + I_z \dot{\phi} \sin\theta\left(\dot{\phi}\cos\theta + \dot{\psi}\right)$$

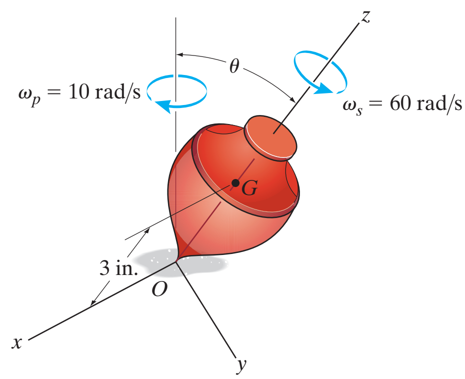
$$0.090(9.81)(0.06)\sin 45^\circ = -0.090(0.035)^2\,(0.5)^2\,(0.7071)^2 + 0.090(0.018)^2(0.5)(0.7071)\left[0.5(0.7071) + \dot{\psi}\right]$$

$$\omega_s = \psi = 3.63\left(10^3\right) \text{ rad/s}$$ **Ans.**



21–71.

The 1-lb top has a center of gravity at point G. If it spins about its axis of symmetry and precesses about the vertical axis at constant rates of $\omega_s = 60$ rad/s and $\omega_p = 10$ rad/s, respectively, determine the steady state angle θ. The radius of gyration of the top about the z axis is $k_z = 1$ in., and about the x and y axes it is $k_x = k_y = 4$ in.

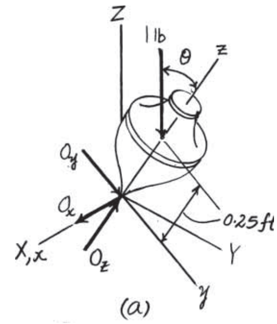


SOLUTION

Since $\dot{\psi} = \omega_s = 60$ rad/s and $\dot{\phi} = \omega_p = -10$ rad/s and θ are constant, the top undergoes steady precession.

$$I_z = \left(\frac{1}{32.2}\right)\left(\frac{1}{12}\right)^2 = 215.67\left(10^{-6}\right) \text{ slug} \cdot \text{ft}^2 \quad \text{and} \quad I = I_x = I_y = \left(\frac{1}{32.2}\right)\left(\frac{4}{12}\right)^2$$

$$= 3.4507\left(10^{-3}\right) \text{ slug} \cdot \text{ft}^2.$$



Thus,

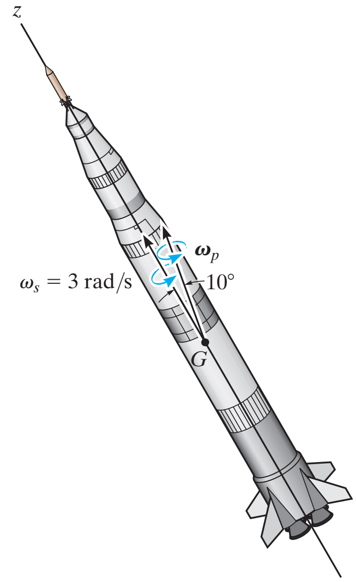
$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta\left(\dot{\phi}\cos + \dot{\psi}\right)$$

$$-1\sin\theta(0.25) = -3.4507\left(10^{-3}\right)(-10)^2\sin\theta\cos\theta + 215.67\left(10^{-6}\right)(-10)\sin\theta[(-10)\cos\theta + 60]$$

$$\theta = 68.1^\circ \qquad \textbf{Ans.}$$

***21–72.**

While the rocket is in free flight, it has a spin of 3 rad/s and precesses about an axis measured 10° from the axis of spin. If the ratio of the axial to transverse moments of inertia of the rocket is 1/15, computed about axes which pass through the mass center G, determine the angle which the resultant angular velocity makes with the spin axis. Construct the body and space cones used to describe the motion. Is the precession regular or retrograde?



SOLUTION

Determine the angle β from the result of Prob. 21-75

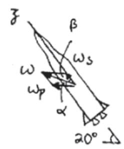
$$\tan\theta = \frac{I}{I_z}\tan\beta$$

$$\tan 10^\circ = \frac{15}{1}\tan\beta$$

$$\beta = 0.673^\circ \qquad \textbf{Ans.}$$

Thus,

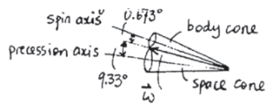
$$\alpha = 10^\circ - 0.673^\circ = 9.33^\circ$$



Hence,

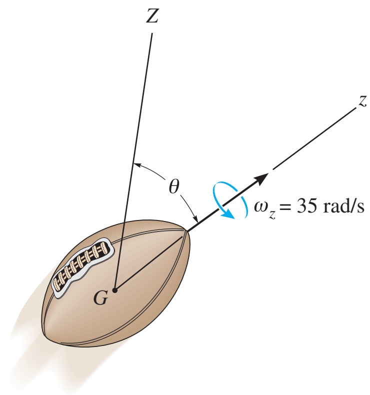
Regular Precession **Ans.**

Since $I_z < I$.



21–73.

The 0.2-kg football is thrown with a spin $\omega_z = 35$ rad/s. If the angle θ is measured as 60°, determine the precession about the Z axis. The radius of gyration about the spin axis is $k_z = 0.05$ m, and about a transverse axis it is $k_t = 0.1$ m.



SOLUTION

Gyroscopic Motion: Here, the spinning angular velocity $\psi = \omega_s = 35$ rad/s. The moment inertia of the football about the z axis is $I_z = 0.2(0.05^2) = 0.5(10^{-3})\ \text{kg}\cdot\text{m}^2$ and the moment inertia of the football about its transverse axis is $I = 0.2(0.1^2) = 2(10^{-3})\ \text{kg}\cdot\text{m}^2$. Applying the third of Eq. 21–36 with $\theta = 60°$, we have

$$\dot{\psi} = \frac{I - I_z}{I\,I_z} H_G \cos\theta$$

$$35 = \left[\frac{2(10^{-3}) - 0.5(10^{-3})}{2(10^{-3})0.5(10^{-3})}\right] H_G \cos 60°$$

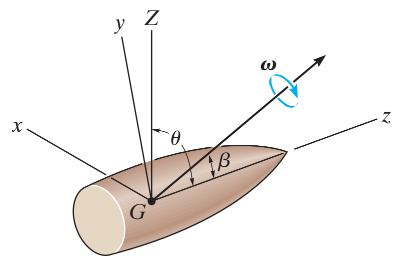
$$H_G = 0.04667\ \text{kg}\cdot\text{m}^2/\text{s}$$

Applying the second of Eq. 21–36, we have

$$\dot{\phi} = \frac{H_G}{I} = \frac{004667}{2(10^{-3})} = 23.3\ \text{rad/s} \qquad \textbf{Ans.}$$

21–74.

The projectile shown is subjected to torque-free motion. The transverse and axial moments of inertia are I and I_z, respectively. If θ represents the angle between the precessional axis Z and the axis of symmetry z, and β is the angle between the angular velocity $\boldsymbol{\omega}$ and the z axis, show that β and θ are related by the equation $\tan\theta = (I/I_z)\tan\beta$.



SOLUTION

From Eq. 21–34 $\omega_y = \dfrac{H_G \sin\theta}{I}$ and $\omega_z = \dfrac{H_G \cos\theta}{I_z}$ Hence $\dfrac{\omega_y}{\omega_z} = \dfrac{I_z}{I}\tan\theta$

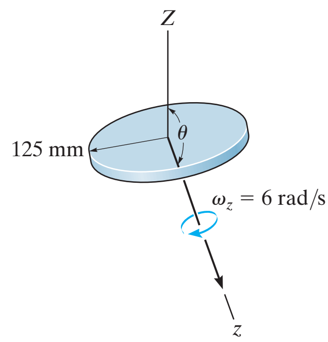
However, $\omega_y = \omega \sin\beta$ and $\omega_z = \omega\cos\beta$

$$\frac{\omega_y}{\omega_z} = \tan\beta = \frac{I_z}{I}\tan\theta$$

$$\tan\theta = \frac{I}{I_z}\tan\beta \qquad \textbf{Q.E.D.}$$

21–75.

The 4-kg disk is thrown with a spin $\omega_z = 6$ rad/s. If the angle θ is measured as 160°, determine the precession about the Z axis.



SOLUTION

$$I = \frac{1}{4}(4)(0.125)^2 = 0.015625 \text{ kg}\cdot\text{m}^2 \qquad I_z = \frac{1}{2}(4)(0.125)^2 = 0.03125 \text{ kg}\cdot\text{m}^2$$

Applying Eq. 21–36 with $\theta = 160°$ and $\dot{\psi} = 6$ rad/s

$$\dot{\psi} = \frac{I - I_z}{II_z} H_O \cos\theta$$

$$6 = \frac{0.015625 - 0.03125}{0.015625(0.03125)} H_O \cos 160°$$

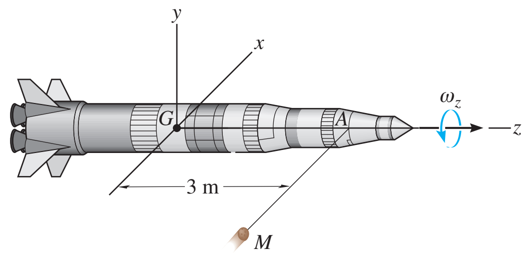
$$H_G = 0.1995 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$\phi = \frac{H_G}{I} = \frac{0.1995}{0.015625} = 12.8 \text{ rad/s}$$ **Ans.**

Note that this is a case of retrograde precession since $I_z > I$.

***21–76.**

The rocket has a mass of 4 Mg and radii of gyration $k_z = 0.85$ m and $k_y = 2.3$ m. It is initially spinning about the z axis at $\omega_z = 0.05$ rad/s when a meteoroid M strikes it at A and creates an impulse $\mathbf{I} = \{300\mathbf{i}\}$ N·s. Determine the axis of precession after the impact.



SOLUTION

The impulse creates an angular momentum about the y axis of

$$H_y = 300(3) = 900 \text{ kg}\cdot\text{m}^2/\text{s}$$

Since

$$\omega_z = 0.05 \text{ rad/s}$$

then

$$\mathrm{H}_G = 900\mathbf{j} + [4000(0.85)^2](0.05)\mathbf{k} = 900\mathbf{j} + 144.5\mathbf{k}$$

The axis of precession is defined by $\mathbf{H}_G$.

$$\mathrm{u}_{H_G} = \frac{900\mathbf{j} + 144.5\mathbf{k}}{911.53} = 0.9874\mathbf{j} + 0.159\mathbf{k}$$

Thus,

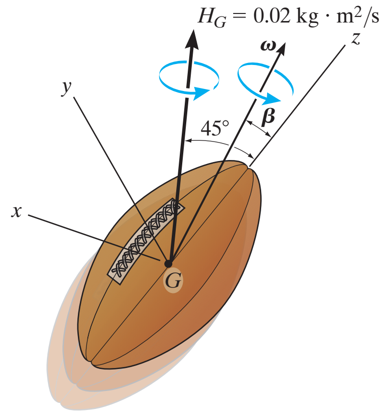
$$\alpha = \cos^{-1}(0) = 90° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}(0.9874) = 9.12° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}(0.159) = 80.9° \qquad \textbf{Ans.}$$

21–77.

The football has a mass of 450 g and radii of gyration about its axis of symmetry (z axis) and its transverse axes (x or y axis) of $k_z = 30$mm and $k_x = k_y = 50$mm, respectively. If the football has an angular momentum of $H_G = 0.02\text{kg}\cdot\text{m}^2/\text{s}$, determine its precession ϕ and spin ψ. Also, find the angle β that the angular velocity vector makes with the z axis.



SOLUTION

Since the weight is the only force acting on the football, it undergoes torque-free motion. $I_z = 0.45\left(0.03^2\right) = 0.405\left(10^{-3}\right)\text{ kg}\cdot\text{m}^2$, $I = I_x = I_y = 0.45\left(0.05^2\right) = 1.125\left(10^{-3}\right)\text{ kg}\cdot\text{m}^2$, and $\theta = 45°$.

Thus,

$$\dot{\phi} = \frac{H_G}{I} = \frac{0.02}{1.125\left(10^{-3}\right)} = 17.78\text{ rad/s} = 17.8\text{ rad/s} \qquad \textbf{Ans.}$$

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta = \frac{1.125\left(10^{-3}\right) - 0.405\left(10^{-3}\right)}{1.125\left(10^{-3}\right)(0.405)\left(10^{-3}\right)}(0.02)\cos 45°$$

$$= 22.35\text{ rad/s} = 22.3\text{ rad/s} \qquad \textbf{Ans.}$$

Also,

$$\omega_y = \frac{H_G \sin\theta}{I} = \frac{0.02\sin 45°}{1.125\left(10^{-3}\right)} = 12.57\text{ rad/s}$$

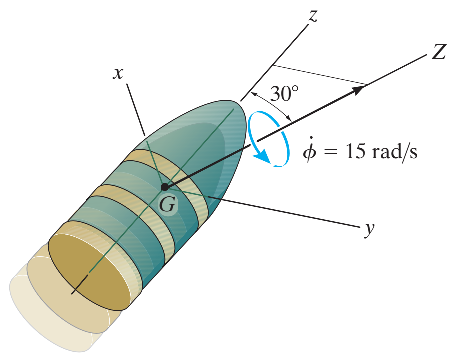
$$\omega_z = \frac{H_G \cos\theta}{I_z} = \frac{0.02\cos 45°}{0.405\left(10^{-3}\right)} = 34.92\text{ rad/s}$$

Thus,

$$\beta = \tan^{-1}\left(\frac{\omega_y}{\omega_z}\right) = \tan^{-1}\left(\frac{12.57}{34.92}\right) = 19.8° \qquad \textbf{Ans.}$$

21–78.

The projectile precesses about the Z axis at a constant rate of $\dot{\phi} = 15\text{ rad/s}$ when it leaves the barrel of a gun. Determine its spin $\dot{\psi}$ and the magnitude of its angular momentum $\mathbf{H}_G$. The projectile has a mass of 1.5 kg and radii of gyration about its axis of symmetry (z axis) and about its transverse axes (x and y axes) of $k_z = 65\text{ mm}$ and $k_x = k_y = 125\text{ mm}$, respectively.



SOLUTION

Since the only force that acts on the projectile is its own weight, the projectile undergoes torque-free motion. $I_z = 1.5\left(0.065^2\right) = 6.3375\left(10^{-3}\right)\text{ kg}\cdot\text{m}^2$, $I = I_x = I_y = 1.5\left(0.125^2\right) = 0.0234375\text{ kg}\cdot\text{m}^2$, and $\theta = 30°$. Thus,

$$\dot{\phi} = \frac{H_G}{I}; \quad H_G = I\dot{\phi} = 0.0234375(15) = 0.352\text{ kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

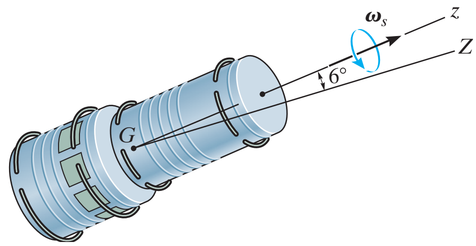
$$\dot{\psi} = \frac{I - I_z}{I_z}\dot{\phi}\cos\theta$$

$$= \frac{0.0234375 - 6.3375\left(10^{-3}\right)}{6.3375\left(10^{-3}\right)}(15)\cos 30°$$

$$= 35.1\text{ rad/s} \qquad \textbf{Ans.}$$

21–79.

The space capsule has a mass of 3.2 Mg, and about axes passing through the mass center G the axial and transverse radii of gyration are $k_z = 0.90$ m and $k_t = 1.85$ m, respectively. If it spins at $\omega_s = 0.8$ rev/s, determine its angular momentum. Precession occurs about the Z axis.



SOLUTION

Gyroscopic Motion: Here, the spinning angular velocity $\psi = \omega_s = 0.8(2\pi) = 1.6\pi$ rad/s. The moment of inertia of the satelite about the z axis is $I_z = 3200\left(0.9^2\right) = 2592\ \text{kg}\cdot\text{m}^2$ and the moment of inertia of the satelite about its transverse axis is $I = 3200\left(1.85^2\right) = 10\,952\ \text{kg}\cdot\text{m}^2$. Applying the third of Eq. 21–36 with $\theta = 6°$, we have

$$\dot{\psi} = \frac{I - I_z}{II_z} H_G \cos\theta$$

$$1.6\pi = \left[\frac{10\,952 - 2592}{10\,952(2592)}\right] H_G \cos 6°$$

$$H_G = 17.16\left(10^3\right)\ \text{kg}\cdot\text{m}^2/\text{s} = 17.2\ \text{Mg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

22–1.

A spring has a stiffness of 600 N/m. If a 4-kg block is attached to the spring, pushed 50 mm above its equilibrium position, and released from rest, determine the equation which describes the block's motion. Assume that positive displacement is measured downward.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{4}} = 12.25 \text{ rad/s}$$

$$v = 0, \qquad x = -0.05 \text{ m at } t = 0$$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$-0.05 = 0 + B$$

$$B = -0.05$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$0 = A(12.25) - 0$$

$$A = 0$$

Thus, $x = -0.05 \cos (12.2t)$ m **Ans.**

22–2.

When a 2-kg block is suspended from a spring, the spring is stretched a distance of 40 mm. Determine the frequency and the period of vibration for a 0.5-kg block attached to the same spring.

SOLUTION

$$k = \frac{F}{y} = \frac{2(9.81)}{0.040} = 490.5 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{490.5}{0.5}} = 31.321$$

$$f = \frac{\omega_n}{2\pi} = \frac{31.321}{2\pi} = 4.985 = 4.98 \text{ Hz}$$ **Ans.**

$$\tau = \frac{1}{f} = \frac{1}{4.985} = 0.201 \text{ s}$$ **Ans.**

22–3.

A spring is stretched 200 mm by a 15-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 0.75 m/s, determine the equation which describes the motion. What is the phase angle? Assume that positive displacement is downward.

SOLUTION

$$k = \frac{F}{y} = \frac{15(9.81)}{0.2} = 735.75\ \text{N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{735.75}{15}} = 7.00$$

$$y = A \sin \omega_n t + B \cos \omega_n t$$

$$y = 0.1\ \text{m when } t = 0,$$

$$0.1 = 0 + B; \qquad B = 0.1$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$v = 0.75\ \text{m/s when } t = 0,$$

$$0.75 = A(7.00)$$

$$A = 0.107$$

$$y = 0.107 \sin (7.00t) + 0.100 \cos (7.00t) \qquad \textbf{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{0.100}{0.107}\right) = 43.0^\circ \qquad \textbf{Ans.}$$

***22–4.**

When a 20-lb weight is suspended from a spring, the spring is stretched a distance of 4 in. Determine the natural frequency and the period of vibration for a 10-lb weight attached to the same spring.

SOLUTION

$$k = \frac{20}{\frac{4}{12}} = 60 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{60}{\frac{10}{32.2}}} = 13.90 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\tau = \frac{2\pi}{\omega_n} = 0.452 \text{ s} \qquad \textbf{Ans.}$$

22–5.

When a 3-kg block is suspended from a spring, the spring is stretched a distance of 60 mm. Determine the natural frequency and the period of vibration for a 0.2-kg block attached to the same spring.

SOLUTION

$$k = \frac{F}{\Delta x} = \frac{3(9.81)}{0.060} = 490.5 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{490.5}{0.2}} = 49.52 = 49.5 \text{ rad/s} \qquad \textbf{Ans.}$$

$$f = \frac{\omega_n}{2\pi} = \frac{49.52}{2\pi} = 7.88 \text{ Hz} \qquad \textbf{Ans.}$$

$$\tau = \frac{1}{f} = \frac{1}{7.88} = 0.127 \text{ s}$$

22–6.

An 8-kg block is suspended from a spring having a stiffness $k = 80$ N/m. If the block is given an upward velocity of 0.4 m/s when it is 90 mm above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that positive displacement is measured downward.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{80}{8}} = 3.162 \text{ rad/s}$$

$$v = -0.4 \text{ m/s}, \qquad x = -0.09 \text{ m at } t = 0$$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$-0.09 = 0 + B$$

$$B = -0.09$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$-0.4 = A(3.162) - 0$$

$$A = -0.126$$

Thus, $\quad x = -0.126 \sin (3.16t) - 0.09 \cos (3.16t)$ m **Ans.**

$C = \sqrt{A^2 + B^2} = \sqrt{(-0.126)^2 + (-0.09)} = 0.155$ m **Ans.**

22–7.

A 2-lb weight is suspended from a spring having a stiffness $k = 2\ \text{lb/in.}$ If the weight is pushed 1 in. upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What is the amplitude and the natural frequency of the vibration?

SOLUTION

$$k = 2(12) = 24\ \text{lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{24}{\frac{2}{32.2}}} = 19.66 = 19.7\ \text{rad/s} \qquad \textbf{Ans.}$$

$$y = -\frac{1}{12}, \qquad v = 0 \text{ at } t = 0$$

From Eqs. 22–3 and 22–4,

$$-\frac{1}{12} = 0 + B$$

$$B = -0.0833$$

$$0 = A\omega_n + 0$$

$$A = 0$$

$$C = \sqrt{A^2 + B^2} = 0.0833\ \text{ft} = 1\ \text{in.} \qquad \textbf{Ans.}$$

Position equation,

$$y = (0.0833 \cos 19.7t)\ \text{ft} \qquad \textbf{Ans.}$$

***22–8.**

A 6-lb weight is suspended from a spring having a stiffness $k = 3\text{ lb/in.}$ If the weight is given an upward velocity of 20 ft/s when it is 2 in. above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the weight, measured from the equilibrium position. Assume positive displacement is downward.

SOLUTION

$$k = 3(12) = 36\text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{36}{\frac{6}{32.2}}} = 13.90\text{ rad/s}$$

$$t = 0, \qquad v = -20\text{ ft/s}, \qquad y = -\frac{1}{6}\text{ ft}$$

From Eq. 22–3,

$$-\frac{1}{6} = 0 + B$$

$$B = -0.167$$

From Eq. 22–4,

$$-20 = A(13.90) + 0$$

$$A = -1.44$$

Thus,

$$y = [-1.44 \sin(13.9t) - 0.167 \cos(13.9t)]\text{ ft} \qquad \textbf{Ans.}$$

From Eq. 22–10,

$$C = \sqrt{A^2 + B^2} = \sqrt{(1.44)^2 + (-0.167)^2} = 1.45\text{ ft} \qquad \textbf{Ans.}$$

22–9.

A 3-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is pushed 50 mm upward from its equilibrium position and then released from rest, determine the equation that describes the motion. What are the amplitude and the frequency of the vibration? Assume that positive displacement is downward.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{3}} = 8.165 \qquad \textbf{Ans.}$$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$x = -0.05 \text{ m when } t = 0,$$

$$-0.05 = 0 + B; \qquad B = -0.05$$

$$v = Ap \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$v = 0 \text{ when } t = 0,$$

$$0 = A(8.165) - 0; \qquad A = 0$$

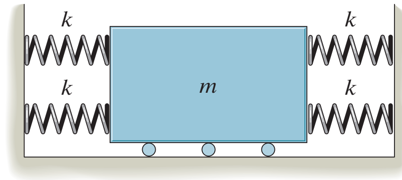
Hence,

$$x = -0.05 \cos (8.16t) \qquad \textbf{Ans.}$$

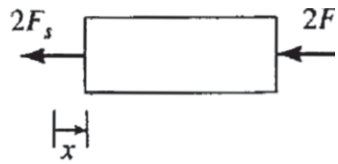
$$C = \sqrt{A^2 + B^2} = \sqrt{(0)^2 + (-0.05)} = 0.05 \text{ m} = 50 \text{ mm} \qquad \textbf{Ans.}$$

22–10.

Determine the frequency of vibration for the block. The springs are originally compressed Δ.



SOLUTION



$$\xrightarrow{+} \Sigma_x = ma_x; \qquad -4kx = m\ddot{x}$$

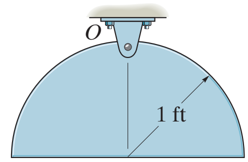
$$\ddot{x} + \frac{4k}{m}x = 0$$

$$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$$

$$f = \frac{1}{\pi}\sqrt{\frac{k}{m}}$$ **Ans.**

22–11.

The semicircular disk weighs 20 lb. Determine the natural period of vibration if it is displaced a small amount and released.



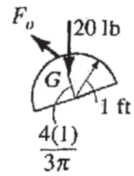
SOLUTION

Moment of Inertia about O:

$I_A = I_G + md^2$ where A is the center of the semicircle.

$$\frac{1}{2}\left(\frac{20}{32.2}\right)(1)^2 = I_G + \left(\frac{20}{32.2}\right)\left[\frac{4(1)}{3\pi}\right]^2$$

$$I_G = 0.1987 \text{ slug} \cdot \text{ft}^2$$



$$I_O = I_G + md^2$$

$$= 0.1987 + \left(\frac{20}{32.2}\right)\left[1 - \frac{4(1)}{3\pi}\right]^2 = 0.4045 \text{ slug} \cdot \text{ft}^2$$

Equation of Motion:

$$\Sigma M_O = I_{Oxs}\alpha; \qquad 20\left[1 - \frac{4(1)}{3\pi}\right]\sin\theta = -0.4045\ddot{\theta}$$

$$\ddot{\theta} + 28.462 \sin\theta = 0$$

However, for small rotation $\theta = \theta$. Hence

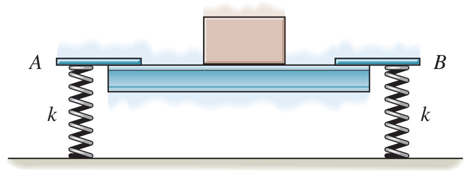
$$\ddot{\theta} + 28.462\theta = 0$$

From the above differential equation, $\omega_n = \sqrt{28.462} = 5.335$.

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{5.335} = 1.18 \text{ s}$$ **Ans.**

***22–12.**

The uniform beam is supported at its ends by two springs A and B, each having the same stiffness k. When nothing is supported on the beam, it has a period of vertical vibration of 0.83 s. If a 50-kg mass is placed at its center, the period of vertical vibration is 1.52 s. Compute the stiffness of each spring and the mass of the beam.



SOLUTION

$$\tau = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{\tau^2}{(2\pi)^2} = \frac{m}{k}$$

$$\frac{(0.83)^2}{(2\pi)^2} = \frac{m_B}{2k} \qquad \textbf{(1)}$$

$$\frac{(1.52)^2}{(2\pi)^2} = \frac{m_B + 50}{2k} \qquad \textbf{(2)}$$

Eqs. (1) and (2) become

$$m_B = 0.03490k$$

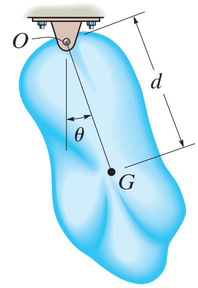
$$m_B + 50 = 0.1170k$$

$$m_B = 21.2 \text{ kg} \qquad \textbf{Ans.}$$

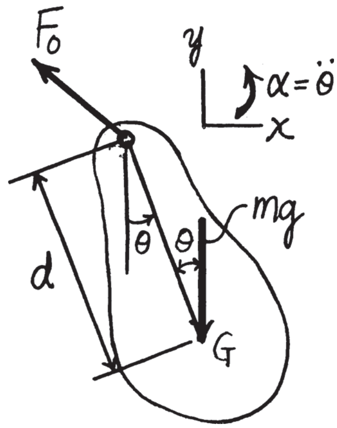
$$k = 609 \text{ N/m} \qquad \textbf{Ans.}$$

22–13.

The body of arbitrary shape has a mass m, mass center at G, and a radius of gyration about G of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration.



SOLUTION



$\circlearrowleft +\Sigma M_O = I_O\,\alpha; \qquad -mgd\sin\theta = \left[mk_G^2 + md^2\right]\ddot{\theta}$

$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\sin\theta = 0$$

However, for small rotation $\sin\theta \approx \theta$. Hence

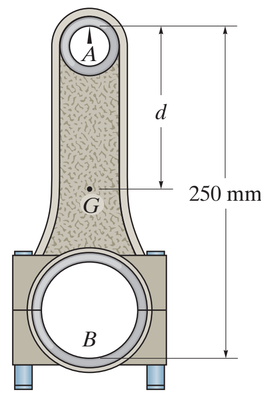
$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\theta = 0$$

From the above differential equation, $\omega_n = \sqrt{\dfrac{gd}{k_G^2 + d^2}}$.

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\dfrac{gd}{k_G^2 + d^2}}} = 2\pi\sqrt{\frac{k_G^2 + d^2}{gd}}$$ **Ans.**

22–14.

The connecting rod is supported by a knife edge at A and the period of vibration is measured as $\tau_A = 3.38$ s. It is then removed and rotated 180° so that it is supported by the knife edge at B. In this case the perod of vibration is measured as $\tau_B = 3.96$ s. Determine the location d of the center of gravity G, and compute the radius of gyration k_G.



SOLUTION

Free-body Diagram: In general, when an object of arbitary shape having a mass m is pinned at O and is displaced by an angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum monent about point O to eliminate O_x and O_y.

$$\circlearrowleft + \Sigma M_O = I_O \alpha; \qquad -mg \sin\theta(l) = I_O\,\alpha \tag{1}$$

Kinematics: Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta = \theta$ if θ is small, then substitute these values into Eq. (1), we have

$$-mgl\theta = I_O\,\ddot{\theta} \qquad \text{or} \qquad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \tag{2}$$

From Eq. (2), $\omega_n{}^2 = \dfrac{mgl}{I_O}$, thus, $\omega_n = \sqrt{\dfrac{mgl}{I_O}}$. Applying Eq. 22–12, we have

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_O}{mgl}} \tag{3}$$

When the rod is rotating about A, $\tau = \tau_A = 3.38$ s and $l = d$. Substitute these values into Eq. (3), we have

$$3.38 = 2\pi\sqrt{\frac{I_A}{mgd}} \qquad I_A = 0.2894mgd$$

When the rod is rotating about B, $\tau = \tau_B = 3.96$ s and $l = 0.25 - d$. Substitute these values into Eq. (3), we have

$$3.96 = 2\pi\sqrt{\frac{I_B}{mg\,(0.25 - d)}} \qquad I_B = 0.3972mg\,(0.25 - d)$$

However, the mass moment inertia of the rod about its mass center is

$$I_G = I_A - md^2 = I_B - m(0.25 - d)^2$$

Then,

$$0.2894mgd - md^2 = 0.3972mg\,(0.25 - d) - m\,(0.25 - d)^2$$

$$d = 0.1462 \text{ m} = 146 \text{ mm} \qquad \textbf{Ans.}$$

Thus, the mass moment inertia of the rod about its mass center is

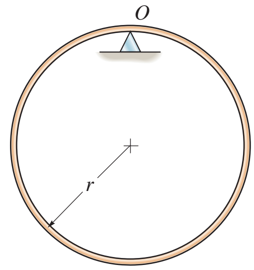
$$I_G = I_A - md^2 = 0.2894m\,(9.81)(0.1462) - m\left(0.1462^2\right) = 0.3937\,m$$

The radius of gyration is

$$k_G = \sqrt{\frac{I_G}{m}} = \sqrt{\frac{0.3937m}{m}} = 0.627 \text{ m} \qquad \textbf{Ans.}$$

22–15.

The thin hoop of mass m is supported by a knife-edge. Determine the natural period of vibration for small amplitudes of swing.



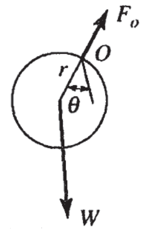
SOLUTION

$$I_O = mr^2 + mr^2 = 2mr^2$$

$$\zeta + \sum M_O = I_O\alpha; \qquad -mgr\theta = (2mr^2)\ddot{\theta}$$

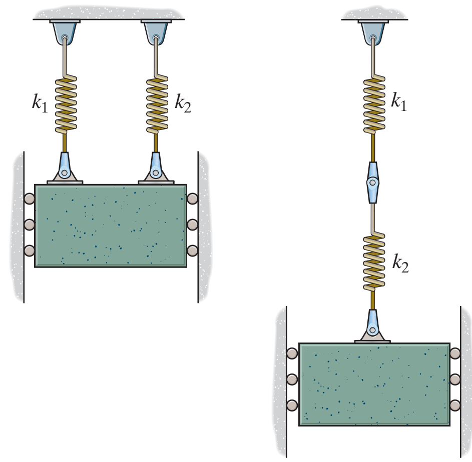
$$\ddot{\theta} + \left(\frac{g}{2r}\right)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2r}{g}} \qquad \textbf{Ans.}$$



***22–16.**

A block of mass m is suspended from two springs having a stiffness of k_1 and k_2, arranged a) parallel to each other, and b) as a series. Determine the equivalent stiffness of a single spring with the same oscillation characteristics and the period of oscillation for each case.



(a) (b)

SOLUTION

(a) When the springs are arranged in parallel, the equivalent spring stiffness is

$$k_{eq} = k_1 + k_2 \qquad \textbf{Ans.}$$

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$$

Thus, the period of oscillation of the system is

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\dfrac{k_1 + k_2}{m}}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}} \qquad \textbf{Ans.}$$

(b) When the springs are arranged in a series, the equivalent stiffness of the system can be determined by equating the stretch of both spring systems subjected to the same load F.

$$\frac{F}{k_1} + \frac{F}{k_2} = \frac{F}{k_{eq}}$$

$$\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k_{eq}}$$

$$\frac{k_2 + k_1}{k_1 k_2} = \frac{1}{k_{eq}}$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} \qquad \textbf{Ans.}$$

The natural frequency of the system is

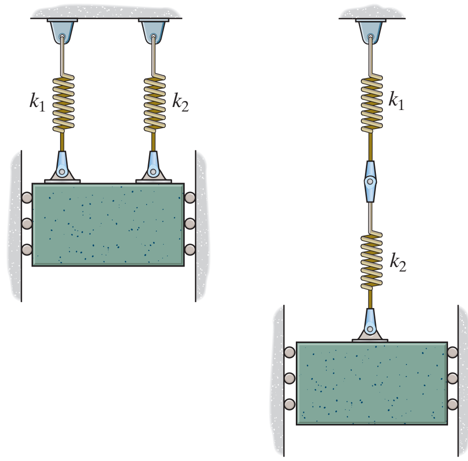
$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{\left(\dfrac{k_1 k_2}{k_2 + k_1}\right)}{m}}$$

Thus, the period of oscillation of the system is

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\dfrac{\left(\dfrac{k_1 k_2}{k_2 + k_1}\right)}{m}}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}} \qquad \textbf{Ans.}$$

22–17.

The 15-kg block is suspended from two springs having a different stiffness and arranged a) parallel to each other, and b) as a series. If the natural periods of oscillation of the parallel system and series system are observed to be 0.5 s and 1.5 s, respectively, determine the spring stiffnesses k_1 and k_2.



(a) (b)

SOLUTION

The equivalent spring stiffness of the spring system arranged in parallel is $(k_{eq})_P = k_1 + k_2$ and the equivalent stiffness of the spring system arranged in a series can be determined by equating the stretch of the system to a single equivalent spring when they are subjected to the same load.

$$\frac{F}{k_1} + \frac{F}{k_2} = \frac{F}{(k_{eq})_S}$$

$$\frac{k_2 + k_1}{k_1 k_2} = \frac{1}{(k_{eq})_S}$$

$$(k_{eq})_S = \frac{k_1 k_2}{k_1 + k_2}$$

Thus the natural frequencies of the parallel and series spring system are

$$(\omega_n)_P = \sqrt{\frac{(k_{eq})_P}{m}} = \sqrt{\frac{k_1 + k_2}{15}}$$

$$(\omega_n)_S = \sqrt{\frac{(k_{eq})_S}{m}} = \sqrt{\frac{\left(\frac{k_1 k_2}{k_1 + k_2}\right)}{15}} = \sqrt{\frac{k_1 k_2}{15(k_1 + k_2)}}$$

Thus, the natural periods of oscillation are

$$\tau_P = \frac{2\pi}{(\omega_n)_P} = 2\pi\sqrt{\frac{15}{k_1 + k_2}} = 0.5 \qquad \textbf{(1)}$$

$$\tau_S = \frac{2\pi}{(\omega_n)_S} = 2\pi\sqrt{\frac{15(k_1 + k_2)}{k_1 k_2}} = 1.5 \qquad \textbf{(2)}$$

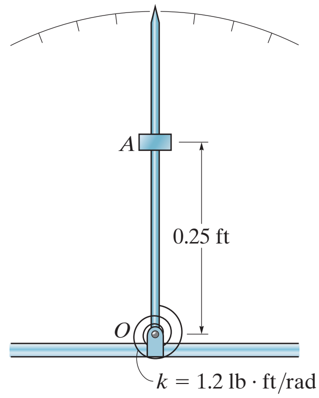
Solving Eqs. (1) and (2),

$$k_1 = 2067 \text{ N/m or } 302 \text{ N/m} \qquad \textbf{Ans.}$$

$$k_2 = 302 \text{ N/m or } 2067 \text{ N/m} \qquad \textbf{Ans.}$$

22–18.

The pointer on a metronome supports a 0.4-lb slider A, which is positioned at a fixed distance from the pivot O of the pointer. When the pointer is displaced, a torsional spring at O exerts a restoring torque on the pointer having a magnitude $M = (1.2\theta)$ lb $\cdot$ ft, where θ represents the angle of displacement from the vertical, measured in radians. Determine the natural period of vibration when the pointer is displaced a small amount θ and released. Neglect the mass of the pointer.



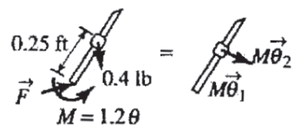
SOLUTION

$$I_O = \frac{0.4}{32.2}(0.25)^2 = 0.7764(10^{-3})\text{ slug}\cdot\text{ft}^2$$

$$\circlearrowleft + \sum M_O = I_O\,\alpha; \qquad -1.2\theta + 0.4(0.25)\sin\theta = 0.7764(10^{-3})\ddot{\theta}$$

For small θ, $\sin\theta = \theta$

So that



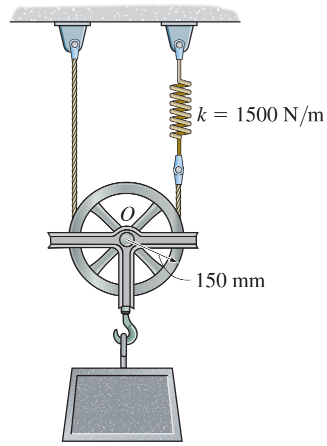
$$\ddot{\theta} + 1417.5\theta = 0$$

$$\omega_n = \sqrt{1417.5} = 37.64\text{ rad/s}$$

$$\tau = \frac{2\pi}{37.64} = 0.167\text{ s}$$ **Ans.**

22–19.

The 50-kg block is suspended from the 10-kg pulley that has a radius of gyration about its center of mass of 125 mm. If the block is given a small vertical displacement and then released, determine the natural frequency of oscillation.



SOLUTION

Equation of Motion: When the system is in the equilibrium position, the moment equation of equilibrium written about the IC using the free-body diagram of the system shown in Fig. a gives

$$\circlearrowleft + \Sigma M_{IC} = 0; \qquad \left(F_{sp}\right)_{st}(0.3) - 10(9.81)(0.15) - 50(9.81)(0.15) = 0$$

$$\left(F_{sp}\right)_{st} = 294.3 \text{ N}$$

Thus, the initial stretch of the spring is $s_0 = \dfrac{\left(F_{sp}\right)_{st}}{k} = \dfrac{294.3}{1500} = 0.1962$ m. Referring the pulley shown in Fig. a, the spring stretches further $s_A = r_{A/IC}\theta = 0.3\theta$ when the pulley rotates through a small angle θ. Thus, $F_{sp} = k(s_0 + s_1) = 1500(0.1962 + 0.3\theta) = 294.3 + 450\theta$. Also, $a_G = \ddot{\theta} r_{G/IC} = \ddot{\theta}(0.15)$. The mass moment of inertia of the pulley about its mass center is $I_G = mk_G^2 = 10(0.125^2) = 0.15625 \text{ kg}\cdot\text{m}^2$. Referring to the free-body and kinetic diagrams of the pulley shown in Fig. b,

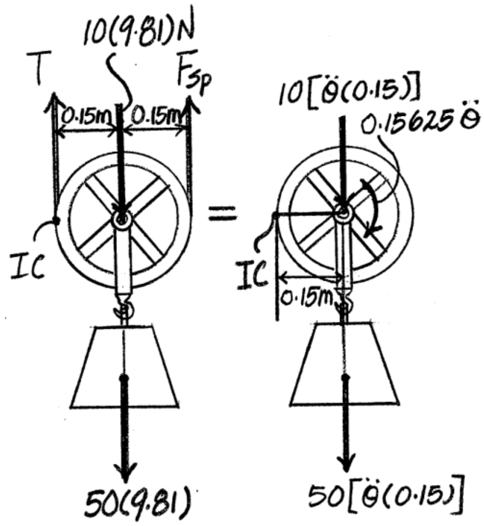
$$\Sigma M_{IC} = \Sigma(M_k)_{IC}; \qquad 10(9.81)(0.15) + 50(9.81)(0.15) - (294.3 + 450\theta)(0.3)$$

$$= 10\left[\ddot{\theta}(0.15)\right](0.15) + 50\left[\ddot{\theta}(0.15)\right](0.15) + 0.15625\ddot{\theta}$$

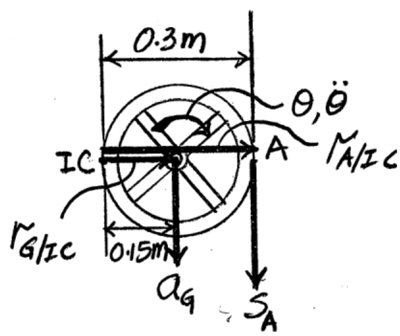
$$\ddot{\theta} + 89.63\theta = 0$$

Comparing this equation to that of the standard form, the natural frequency of the system is

$$\omega_n = \sqrt{89.63} \text{ rad/s} = 9.47 \text{ rad/s}$$ **Ans.**



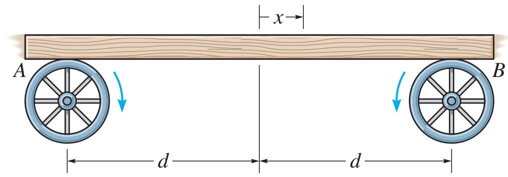
(a)



(b)

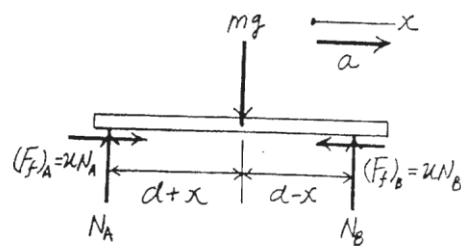
***22–20.**

A uniform board is supported on two wheels which rotate in opposite directions at a constant angular speed. If the coefficient of kinetic friction between the wheels and board is μ, determine the frequency of vibration of the board if it is displaced slightly, a distance x from the midpoint between the wheels, and released.



SOLUTION

Freebody Diagram: When the board is being displaced x to the right, the *restoring force* is due to the unbalance friction force at A and B $\left[(F_f)_B > (F_f)_A\right]$.



Equation of Motion:

$\circlearrowleft + \Sigma M_A = \Sigma(M_A)_k;$ $$N_B(2d) - mg(d + x) = 0$$

$$N_B = \frac{mg(d + x)}{2d}$$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $$N_A + \frac{mg(d + x)}{2d} - mg = 0$$

$$N_A = \frac{mg(d - x)}{2d}$$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;$ $$\mu\left[\frac{mg(d - x)}{2d}\right] - \mu\left[\frac{mg(d + x)}{2d}\right] = ma$$

$$a + \frac{\mu g}{d}x = 0 \qquad \textbf{(1)}$$

Kinematics: Since $a = \dfrac{d^2x}{dt^2} = \ddot{x}$, then substitute this value into Eq.(1), we have

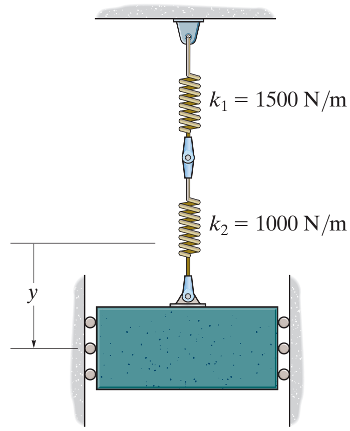
$$\ddot{x} + \frac{\mu g}{d}x = 0 \qquad \textbf{(2)}$$

From Eq.(2), $\omega_n{}^2 = \dfrac{\mu g}{d}$, thus, $\omega_n = \sqrt{\dfrac{\mu g}{d}}$. Applying Eq. 22–4, we have

$$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\mu g}{d}} \qquad \textbf{Ans.}$$

22–21.

If the 20-kg block is given a downward velocity of 6 m/s at its equilibrium position, determine the equation that describes the amplitude of the block's oscillation.



SOLUTION

The equivalent stiffness of the springs in a series can be obtained by equating the stretch of the spring system to an equivalent single spring when they are subjected to the same load. Thus

$$\frac{F}{k_1} + \frac{F}{k_2} = \frac{F}{k_{eq}}$$

$$\frac{k_2 + k_1}{k_1 k_2} = \frac{1}{k_{eq}}$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{1500(1000)}{1500 + 1000} = 600 \text{ N/m}$$

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{600}{20}} = 5.4772 \text{ rad/s}$$

The equation that describes the oscillation of the system becomes

$$y = C \sin(5.4772t + \phi) \text{ m} \qquad \textbf{(1)}$$

Since $y = 0$ when $t = 0$, Eq. (1) gives

$$0 = C \sin \phi$$

Since $C \neq 0$, $\sin\phi = 0$. Then $\phi = 0$. Thus, Eq. (1) becomes

$$y = C \sin(5.4772t) \text{ m} \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2),

$$\dot{y} = v = 5.4772C \cos(5.4772t) \text{ m/s} \qquad \textbf{(3)}$$

Here, $v = 6$ m/s when $t = 0$. Thus, Eq. (3) gives

$$6 = 5.4772C \cos 0$$

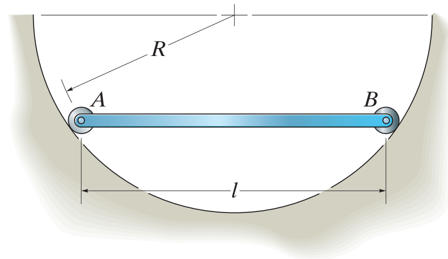
$$C = 1.095 \text{ m} = 1.10 \text{ m}$$

Then

$$y = 1.10 \sin(5.48t) \text{ m} \qquad \textbf{Ans.}$$

22–22.

The bar has a length l and mass m. It is supported at its ends by rollers of negligible mass. If it is given a small displacement and released, determine the natural frequency of vibration.



SOLUTION

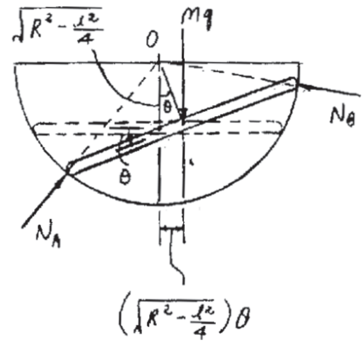
Moment of inertia about point O:

$$I_O = \frac{1}{12}ml^2 + m\left(\sqrt{R^2 - \frac{l^2}{4}}\right)^2 = m\left(R^2 - \frac{1}{6}l^2\right)$$

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad mg\left(\sqrt{R^2 - \frac{l^2}{4}}\right)\theta = -m\left(R^2 - \frac{1}{6}l^2\right)\ddot{\theta}$$

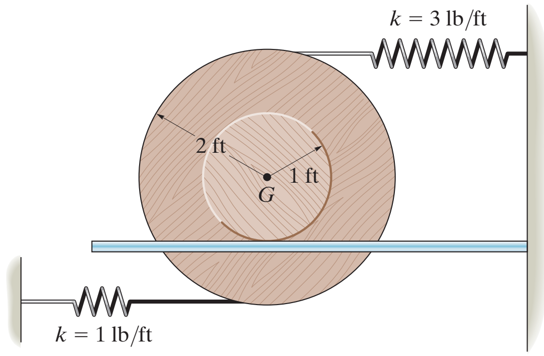
$$\ddot{\theta} + \frac{3g(4R^2 - l^2)^{\frac{1}{2}}}{6R^2 - l^2}\theta = 0$$

From the above differential equation, $\omega_n = \sqrt{\dfrac{3g(4R^2 - l^2)^{\frac{1}{2}}}{6R^2 - l^2}}$. **Ans.**



22–23.

The 50-lb spool is attached to two springs. If the spool is displaced a small amount and released, determine the natural period of vibration. The radius of gyration of the spool is $k_G = 1.5$ ft. The spool rolls without slipping.



SOLUTION

$$I_{IC} = \frac{50}{32.2}(1.5)^2 + \frac{50}{32.2}(1)^2 = 5.047 \text{ slug} \cdot \text{ft}^2$$

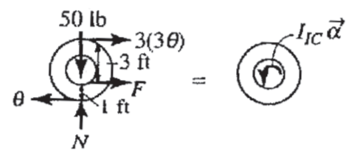
$$\circlearrowleft + \sum M_{IC} = I_{IC}\alpha; \qquad -3(3\theta)(3) - 1(\theta) = 5.047\ddot{\theta}$$

$$\ddot{\theta} + 5.5483\theta = 0$$

$$\omega_n = \sqrt{5.5483}$$

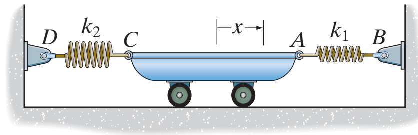
$$\tau = \frac{2\pi}{\sqrt{5.5483}} = 2.67 \text{ s}$$

Ans.



***22–24.**

The cart has a mass of m and is attached to two springs, each having a stiffness of $k_1 = k_2 = k$, unstretched length of l_0, and a stretched length of l when the cart is in the equilibrium position. If the cart is displaced a distance of $x = x_0$ such that both springs remain in tension $(x_0 < l - l_0)$, determine the natural frequency of oscillation.



SOLUTION

Equation of Motion: When the cart is displaced x to the right, the stretch of springs AB and CD are $s_{AB} = (l - l_0) - x_0$ and $s_{AC} = (l - l_0) + x$. Thus, $F_{AB} = ks_{AB} = k[(l - l_0) - x]$ and $F_{AC} = ks_{AC} = k[(l - l_0) + x]$. Referring to the free-body diagram of the cart shown in Fig. a,

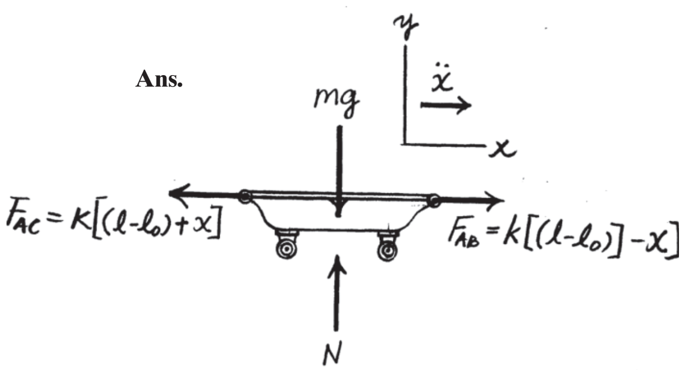
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad k[(l - l_0) - x] - k[(l - l_0) + x] = m\overline{x}$$

$$-2kx = m\overline{x}$$

$$\overline{x} + \frac{2k}{m}x = 0$$

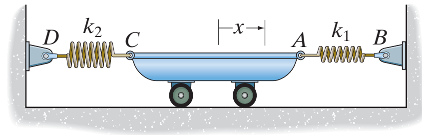
Simple Harmonic Motion: Comparing this equation with that of the standard form, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{2k}{m}} \qquad \textbf{Ans.}$$



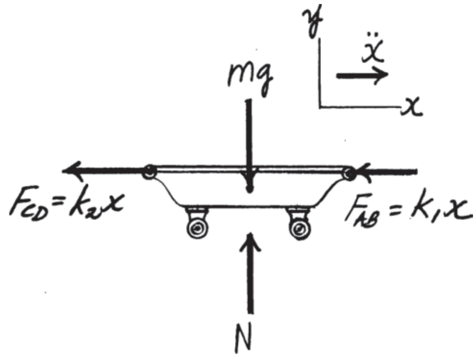
22–25.

The cart has a mass of m and is attached to two springs, each having a stiffness of k_1 and k_2, respectively. If both springs are unstretched when the cart is in the equilibrium position shown, determine the natural frequency of oscillation.



SOLUTION

Equation of Motion: When the cart is displaced x to the right, spring CD stretches $s_{CD} = x$ and spring AB compresses $s_{AB} = x$. Thus, $F_{CD} = k_2 s_{CD} = k_2 x$ and $F_{AB} = k_1 s_{AB} = k_1 x$. Referring to the free-body diagram of the cart shown in Fig. *a*,



$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -k_1 x - k_2 x = m\ddot{x}$

$$-(k_1 + k_2)x = m\ddot{x}$$

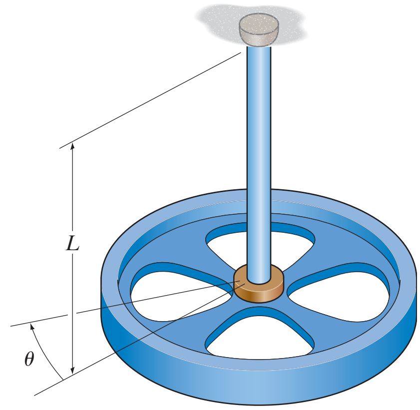
$$\ddot{x} + \left(\frac{k_1 + k_2}{m}\right)x = 0$$

Simple Harmonic Motion: Comparing this equation with that of the standard equation, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_1 + k_2}{m}}$$ **Ans.**

22–26.

A flywheel of mass m, which has a radius of gyration about its center of mass of k_O, is suspended from a circular shaft that has a torsional resistance of $M = C\theta$. If the flywheel is given a small angular displacement of θ and released, determine the natural period of oscillation.

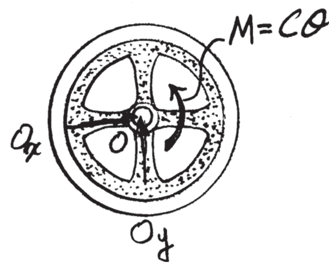


SOLUTION

Equation of Motion: The mass moment of inertia of the wheel about point O is $I_O = mk_O{}^2$. Referring to Fig. a,

$$\circlearrowleft + \qquad \Sigma M_O = I_O\,\alpha; \qquad -C\theta = mk_O{}^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{C}{mk_O{}^2}\theta = 0$$



Comparing this equation to the standard equation, the natural circular frequency of the wheel is

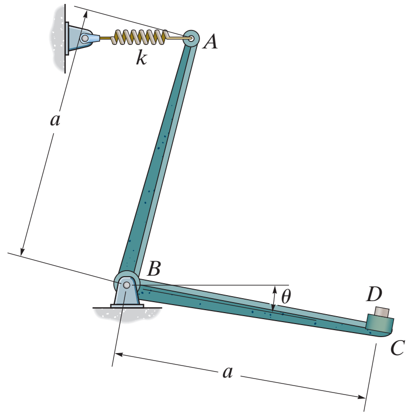
$$\omega_n = \sqrt{\frac{C}{mk_O{}^2}} = \frac{1}{k_O}\sqrt{\frac{C}{m}}$$

Thus, the natural period of the oscillation is

$$\tau = \frac{2\pi}{\omega_n} = 2\pi k_O\sqrt{\frac{m}{C}} \qquad \textbf{Ans.}$$

22–27.

If a block D of negligible size and of mass m is attached at C, and the bell crank of mass M is given a small angular displacement of θ, the natural period of oscillation is τ_1. When D is removed, the natural period of oscillation is τ_2. Determine the bell crank's radius of gyration about its center of mass, pin B, and the spring's stiffness k. The spring is unstrectched at $\theta = 0°$, and the motion occurs in the *horizontal plane*.



SOLUTION

Equation of Motion: When the bell crank rotates through a small angle θ, the spring stretches $s = a\theta$. Thus, the force in the spring is $F_{sp} = ks = k(a\theta)$. The mass moment of inertia of the bell crank about its mass center B is $I_B = Mk_B^2$. Referring to the free-body diagram of the bell crank shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_B = I_B\alpha; \qquad -k(a\theta)\cos\theta(a) = Mk_B^2\ddot{\theta}_B$$

$$\ddot{\theta} + \frac{ka^2}{Mk_B^2}(\cos\theta)\theta = 0 \tag{1}$$

Since θ is very small, $\cos\theta \cong 1$. Then Eq.(1) becomes

$$\ddot{\theta} + \frac{ka^2}{Mk_B^2}\theta = 0$$

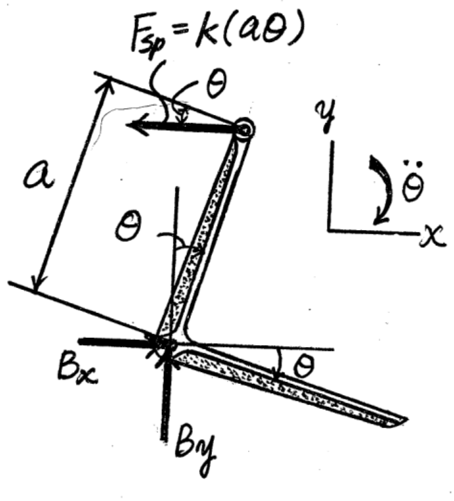
Since the bell crank rotates about point B, $a_C = \alpha r_{BC} = \ddot{\theta}(a)$. Referring to the free-body diagram shown in Fig. *b*,

$$\circlearrowleft + \Sigma M_B = \Sigma(M_k)_B; \qquad -k(a\theta)\cos\theta(a) = Mk_B^2\ddot{\theta} + m\left[\ddot{\theta}(a)\right](a)$$

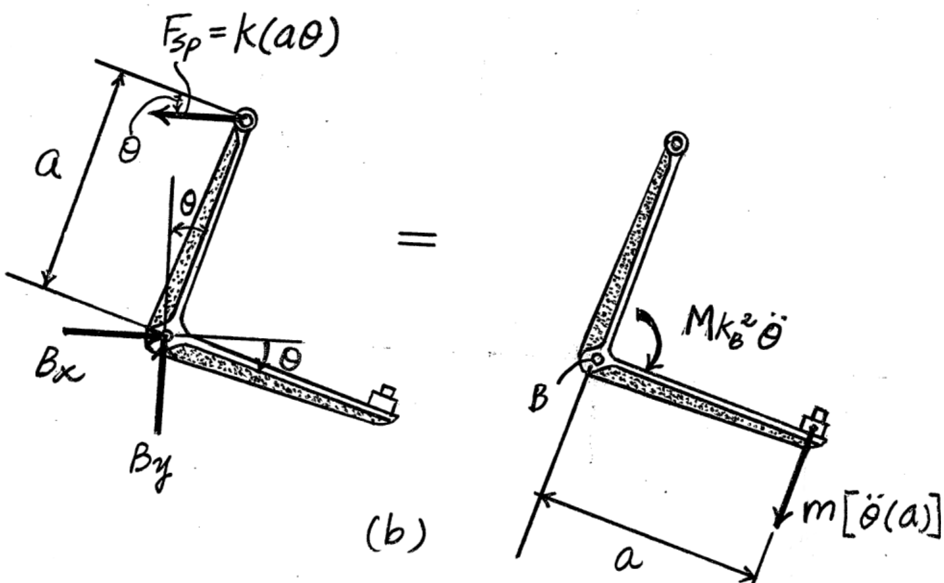
$$\ddot{\theta} + \frac{ka^2}{Mk_B^2 + ma^2}(\cos\theta)\theta = 0 \tag{2}$$

Again, $\cos\theta \cong 1$, since θ is very small. Thus, Eq. (2) becomes

$$\ddot{\theta} + \frac{ka^2}{Mk_B^2 + ma^2}\theta = 0$$



(a)



(b)

22–27. continued

Thus, the natural frequencies of the two oscillations are

$$(\omega_n)_2 = \sqrt{\frac{ka^2}{Mk_B^{\,2}}}$$

$$(\omega_n)_1 = \sqrt{\frac{ka^2}{Mk_B^{\,2} + ma^2}}$$

The natural periods of the two oscillations are

$$\tau_2 = \frac{2\pi}{(\omega_n)_2} = 2\pi\sqrt{\frac{Mk_B^{\,2}}{ka^2}}$$

$$\tau_1 = \frac{2\pi}{(\omega_n)_1} = 2\pi\sqrt{\frac{Mk_B^{\,2} + ma^2}{ka^2}}$$

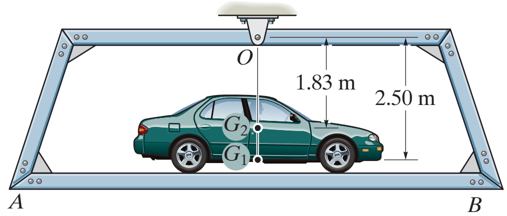
Solving,

$$k_B = a\sqrt{\frac{m}{M}\left(\frac{\tau_2^{\,2}}{\tau_1^{\,2} - \tau_2^{\,2}}\right)} \qquad \textbf{Ans.}$$

$$k = \frac{4\pi^2}{\tau_1^{\,2} - \tau_2^{\,2}}\,m \qquad \textbf{Ans.}$$

***22–28.**

The platform AB when empty has a mass of 400 kg, center of mass at G_1, and natural period of oscillation $\tau_1 = 2.38$ s. If a car, having a mass of 1.2 Mg and center of mass at G_2, is placed on the platform, the natural period of oscillation becomes $\tau_2 = 3.16$ s. Determine the moment of inertia of the car about an axis passing through G_2.



SOLUTION

Free-body Diagram: When an object arbitrary shape having a mass m is pinned at O and being displaced by an angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum moment about point O to eliminate O_x and O_y.

$$\circlearrowleft + \Sigma M_O = I_O\alpha: \qquad -mg\sin\theta(l) = I_O\alpha \qquad \textbf{(1)}$$

Kinematics: Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta = \theta$ if θ is small, then substituting these values into Eq. (1), we have

$$-mgl\theta = I_O\ddot{\theta} \qquad \text{or} \qquad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \qquad \textbf{(2)}$$

From Eq. (2), $\omega_n^2 = \dfrac{mgl}{I_O}$, thus, $\omega_n = \sqrt{\dfrac{mgl}{I_O}}$, Applying Eq. 22–12, we have

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_O}{mgl}} \qquad \textbf{(3)}$$

When the platform is empty, $\tau = \tau_1 = 2.38$ s, $m = 400$ kg and $l = 2.50$ m. Substituting these values into Eq. (3), we have

$$2.38 = 2\pi\sqrt{\frac{(I_O)_p}{400(9.81)(2.50)}} \quad (I_O)_p = 1407.55 \text{ kg} \cdot \text{m}^2$$

When the car is on the platform, $\tau = \tau_2 = 3.16$ s, $m = 400 \text{ kg} + 1200 \text{ kg} = 1600$ kg. $l = \dfrac{2.50(400) + 1.83(1200)}{1600} = 1.9975$ m and $I_O = (I_O)_C + (I_O)_p = (I_O)_C + 1407.55$. Substituting these values into Eq. (3), we have

$$3.16 = 2\pi\sqrt{\frac{(I_O)_C + 1407.55}{1600(9.81)(1.9975)}} \quad (I_O)_C = 6522.76 \text{ kg} \cdot \text{m}^2$$

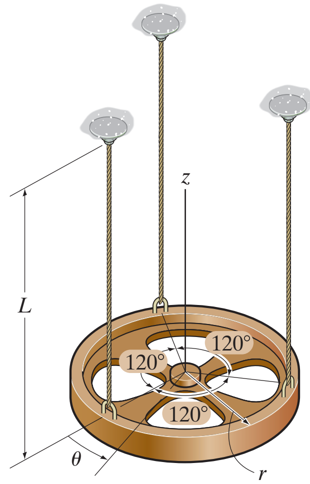
Thus, the mass moment inertia of the car about its mass center is

$$(I_G)_C = (I_O)_C - m_C d^2$$

$$= 6522.76 - 1200(1.83^2) = 2.50(10^3) \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

22–29.

A wheel of mass m is suspended from three equal-length cords. When it is given a small angular displacement of θ about the z axis and released, it is observed that the period of oscillation is τ. Determine the radius of gyration of the wheel about the z axis.



SOLUTION

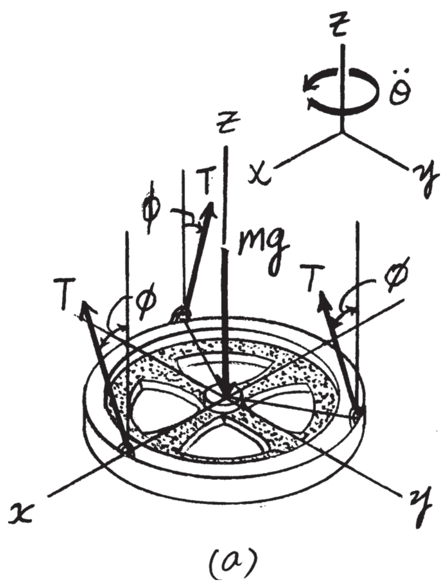
Equation of Motion: Due to symmetry, the force in each cord is the same. The mass moment of inertia of the wheel about is z axis is $I_z = mk_z^2$. Referring to the free-body diagram of the wheel shown in Fig. *a*,

$$+\uparrow \Sigma F_z = ma_z; \qquad 3T\cos\phi - mg = 0 \qquad T = \frac{mg}{3\cos\phi}$$

Then,

$$\uparrow + \Sigma M_z = I_z\alpha; \qquad -3\left(\frac{mg}{3\cos\phi}\right)\sin\phi(\mathrm{r}) = mk_z^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{gr}{k_z^2}\tan\phi = 0 \qquad \textbf{(1)}$$



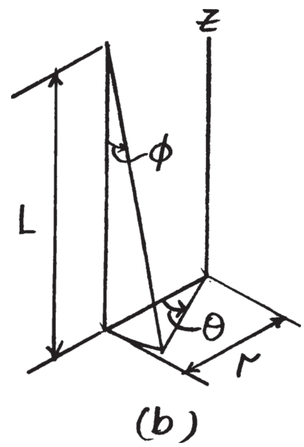
Since θ is very small, from the geometry of Fig. *b*,

$$r\theta = L\phi$$

$$\phi = \frac{r}{L}\theta$$

Substituting this result into Eq. (1)

$$\ddot{\theta} + \frac{gr}{k_z^2}\tan\left(\frac{r}{L}\theta\right) = 0 \qquad \textbf{(2)}$$



Since θ is very small, $\tan\left(\frac{r}{L}\theta\right) \cong \frac{r}{L}\theta$. Thus,

$$\ddot{\theta} + \frac{gr^2}{kz^2L}\theta = 0$$

Comparing this equation to that of the standard form, the natural circular frequency of the wheel is

$$\omega_n = \sqrt{\frac{gr^2}{k_z^2L}} = \frac{r}{k_z}\sqrt{\frac{g}{L}}$$

Thus, the natural period of oscillation is

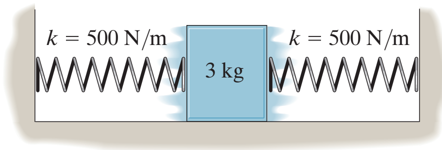
$$\tau = \frac{2\pi}{\omega_n}$$

$$\tau = 2\pi\left(\frac{k_z}{r}\sqrt{\frac{L}{g}}\right)$$

$$k_z = \frac{\tau r}{2\pi}\sqrt{\frac{g}{L}} \qquad \textbf{Ans.}$$

22–30.

Determine the differential equation of motion of the 3-kg block when it is displaced slightly and released. The surface is smooth and the springs are originally unstretched.



SOLUTION

$$T + V = \text{const.}$$

$$T = \frac{1}{2}(3)\dot{x}^2$$

$$V = \frac{1}{2}(500)x^2 + \frac{1}{2}(500)x^2$$

$$T + V = 1.5\dot{x}^2 + 500x^2$$

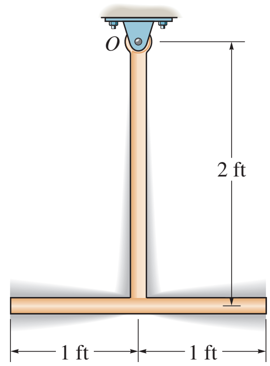
$$1.5(2\dot{x})\,\ddot{x} + 1000x\dot{x} = 0$$

$$3\ddot{x} + 1000x = 0$$

$$\ddot{x} + 333x = 0 \qquad \textbf{Ans.}$$

22–31.

Determine the natural period of vibration of the pendulum. Consider the two rods to be slender, each having a weight of 8 lb/ft.

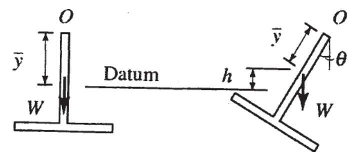


SOLUTION

$$\bar{y} = \frac{1(8)(2) + 2(8)(2)}{8(2) + 8(2)} = 1.5 \text{ ft}$$

$$I_O = \frac{1}{32.2}\left[\frac{1}{12}(2)(8)(2)^2 + 2(8)(1)^2\right]$$

$$+ \frac{1}{32.2}\left[\frac{1}{12}(2)(8)(2)^2 + 2(8)(2)^2\right] = 2.8157 \text{ slug}\cdot\text{ft}^2$$



$$h = \bar{y}\,(1 - \cos\theta)$$

$$T + V = \text{const}$$

$$T = \frac{1}{2}(2.8157)(\dot{\theta})^2 = 1.4079\,\dot{\theta}^2$$

$$V = 8(4)(1.5)(1 - \cos\theta) = 48(1 - \cos\theta)$$

$$T + V = 1.4079\dot{\theta}^2 + 48(1 - \cos\theta)$$

$$1.4079\,(2\dot{\theta})\ddot{\theta} + 48(\sin\theta)\dot{\theta} = 0$$

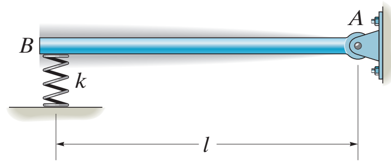
For small θ, $\sin\theta = \theta$, then

$$\ddot{\theta} + 17.047\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{17.047}} = 1.52 \text{ s}$$ **Ans.**

***22–32.**

The uniform rod of mass m is supported by a pin at A and a spring at B. If the end B is given a small downward displacement and released, determine the natural period of vibration.



SOLUTION

$$T = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\dot{\theta}^2$$

$$V = \frac{1}{2}k(y_{eq} + y_2)^2 - mgy_1$$

$$= \frac{1}{2}k(l\theta_{eq} + l\theta)^2 - mg\left(\frac{1}{2}\right)\theta$$

$$T + V = \frac{1}{6}ml^2\dot{\theta}^2 + \frac{1}{2}k(l\theta_{eq} + l\theta)^2 - mg\left(\frac{l\theta}{2}\right)$$

Time derivative

$$0 = \frac{1}{3}ml^2\dot{\theta}\ddot{\theta} + kl(\theta_{eq} + \theta)\dot{\theta} - mgl\frac{\dot{\theta}}{2}$$

For equilibrium

$$k(l\theta_{eq}) = mgl/2, \theta_{eq} = \frac{mg}{2k}$$

Thus,

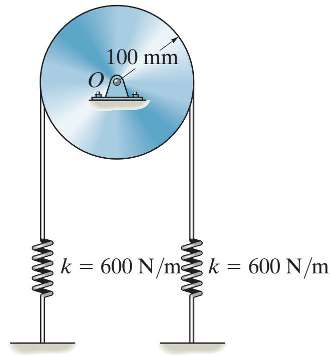
$$0 = \frac{1}{3}ml\ddot{\theta} + k\theta$$

$$\ddot{\theta} + (3k/m)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{3k}}$$ **Ans.**

22–33.

The 7-kg disk is pin-connected at its midpoint. Determine the natural period of vibration of the disk if the springs have sufficient tension in them to prevent the cord from slipping on the disk as it oscillates. *Hint:* Assume that the initial stretch in each spring is δ_O. This term will cancel out after taking the time derivative of the energy equation.

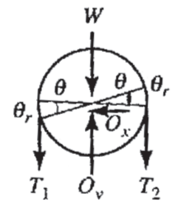


SOLUTION

$$E = T + V$$

$$= \frac{1}{2}k(\theta r + \delta_O)^2 + \frac{1}{2}k(\theta r - \delta_O)^2 + \frac{1}{2}I_O(\dot{\theta})^2$$

$$E = k(\theta r + \delta_O)\dot{\theta} r + k(\theta r - \delta_O)\dot{\theta} r + I_O\dot{\theta}\ddot{\theta} = 0$$



Thus,

$$\ddot{\theta} + \frac{2kr^2}{I_O}\theta = 0$$

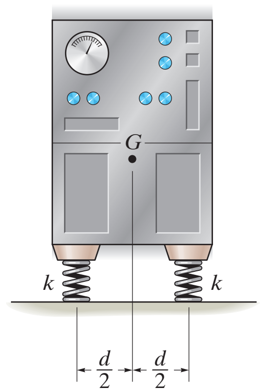
$$\omega_n = \sqrt{\frac{2kr^2}{I_O}}$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_O}{2kr^2}}$$

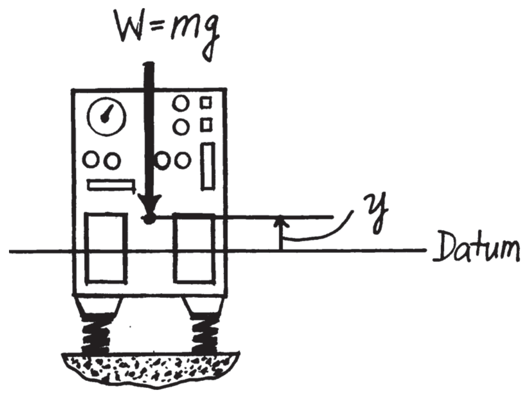
$$\tau = 2\pi\sqrt{\frac{\frac{1}{2}(7)(0.1)^2}{2(600)(0.1)^2}} = 0.339 \text{ s}$$ **Ans.**

22–34.

The machine has a mass m and is uniformly supported by *four* springs, each having a stiffness k. Determine the natural period of vertical vibration.



SOLUTION



$$T + V = \text{const.}$$

$$T = \frac{1}{2}m(\dot{y})^2$$

$$V = m\,g\,y + \frac{1}{2}(4k)(\Delta s - y)^2$$

$$T + V = \frac{1}{2}m(\dot{y})^2 + m\,g\,y + \frac{1}{2}(4k)(\Delta s - y)^2$$

$$m\,\dot{y}\,\ddot{y} + m\,g\,\dot{y} - 4k(\Delta s - y)\dot{y} = 0$$

$$m\,\ddot{y} + m\,g + 4ky - 4k\Delta s = 0$$

Since $\Delta s = \dfrac{mg}{4k}$

Then

$$m\ddot{y} + 4ky = 0$$

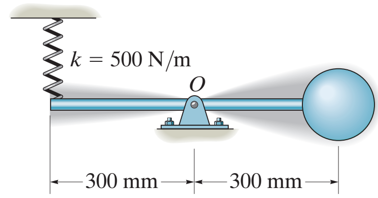
$$y + \frac{4k}{m}y = 0$$

$$\omega_n = \sqrt{\frac{4k}{m}}$$

$$\tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{m}{k}} \qquad \textbf{Ans.}$$

22–35.

Determine the natural period of vibration of the 3-kg sphere. Neglect the mass of the rod and the size of the sphere.

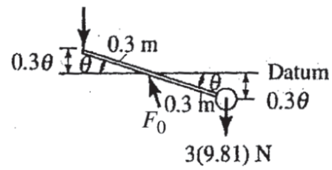


SOLUTION

$$E = T + V$$

$$= \frac{1}{2}(3)(0.3\dot{\theta})^2 + \frac{1}{2}(500)(\delta_{st} + 0.3\theta)^2 - 3(9.81)(0.3\theta)$$

$$E = \dot{\theta}[(3(0.3)^2\ddot{\theta} + 500(\delta_{st} + 0.3\theta)(0.3) - 3(9.81)(0.3)] = 0$$



By statics,

$$T(0.3) = 3(9.81)(0.3)$$

$$T = 3(9.81)\text{ N}$$

$$\delta_{st} = \frac{3(9.81)}{500}$$

Thus,

$$3(0.3)^2\ddot{\theta} + 500(0.3)^2\theta = 0$$

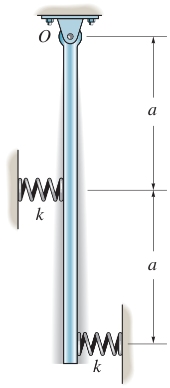
$$\ddot{\theta} + 166.67\theta = 0$$

$$\omega_n = \sqrt{166.67} = 12.91\text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{12.91} = 0.487\text{ s}$$ **Ans.**

***22–36.**

The slender rod has a mass m and is pinned at its end O. When it is vertical, the springs are unstretched. Determine the natural period of vibration.



SOLUTION

$$T + V = \frac{1}{2}\left[\frac{1}{3}m(2a)^2\right]\dot{\theta}^2 + \frac{1}{2}k(2\theta a)^2 + \frac{1}{2}k(\theta a)^2 + mga(1 - \cos\theta)$$

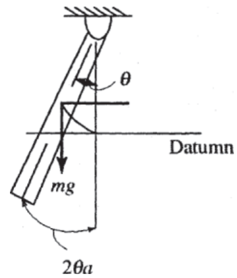
$$0 = \frac{4}{3}ma^2\dot{\theta}\,\ddot{\theta} + 4ka^2\theta\dot{\theta} + ka^2\theta\dot{\theta} + mga\sin\theta\dot{\theta}$$

$$\sin\theta = \theta$$

$$\frac{4}{3}ma^2\ddot{\theta} + 5ka^2\theta + mga\theta = 0$$

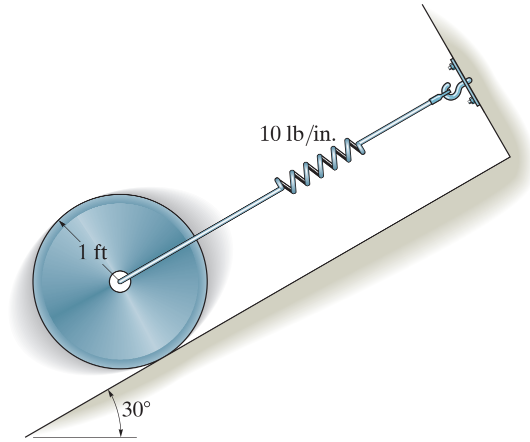
$$\ddot{\theta} + \left(\frac{15ka + 3mg}{4ma}\right)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{4\pi}{\sqrt{3}}\left(\frac{ma}{5ka + mg^2}\right)^{\frac{1}{2}}$$ **Ans.**



22–37.

Determine the natural frequency of vibration of the 20-lb disk. Assume the disk does not slip on the inclined surface.



SOLUTION

θ is the displacement of the disk.

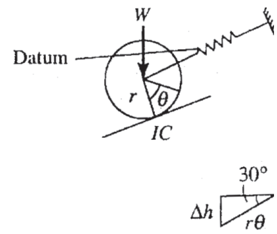
The disk rolls a distance $s = r\theta$

$$\Delta k = r\theta \sin 30^\circ$$

$$E = T + V$$

$$= \frac{1}{2}I_{IC}(\dot{\theta})^2 + \frac{1}{2}k[\delta_{st} + \theta r]^2 - W(r\theta \sin 30^\circ)$$

$$E = \dot{\theta}(I_{IC}\ddot{\theta} + k\delta_{st}r + k\theta r^2 - Wr\sin 30^\circ) = 0$$



Since $k\delta_{st} = Wr\sin 30^\circ$

$$\ddot{\theta} + \frac{kr^2}{I_{IC}}\theta = 0$$

$$I_{IC} = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$$

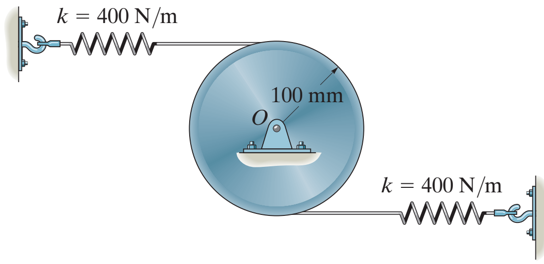
Thus,

$$\omega_n = \sqrt{\frac{kr^2}{I_{IC}}}$$

$$= \sqrt{\frac{10(12)(1)^2}{\frac{3}{2}\left(\frac{20}{32.2}\right)(1)^2}} = 11.3 \text{ rad/s}$$ **Ans.**

22–38.

If the disk has a mass of 8 kg, determine the natural frequency of vibration. The springs are originally unstretched.

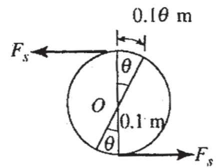


SOLUTION

$$I_O = \frac{1}{2}(8)(0.1)^2 = 0.04$$

$$T_{max} = V_{max}$$

$$\frac{1}{2}I_O(\omega_n\theta_{max})^2 = 2\left[\frac{1}{2}k(r\theta_{max})^2\right]$$



Thus,

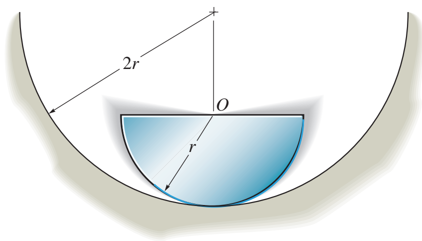
$$\omega_n = \sqrt{\frac{2kr^2}{I_O}}$$

$$= \sqrt{\frac{2(400)(0.1)^2}{0.04}} = 14.1 \text{ rad/s}$$

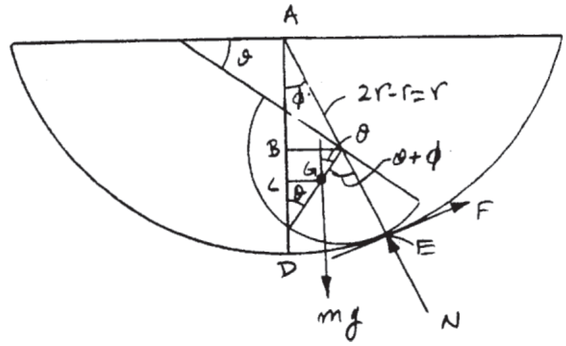
Ans.

22–39.

The semicircular disk has a mass m and radius r, and it rolls without slipping in the semicircular trough. Determine the natural period of vibration of the disk if it is displaced slightly and released. *Hint:* $I_O = \frac{1}{2}mr^2$.



SOLUTION



$$AB = (2r - r)\cos\phi = r\cos\phi, \qquad BC = \frac{4r}{3\pi}\cos\theta$$

$$AC = r\cos\phi + \frac{4r}{3\pi}\cos\theta, \qquad DE = 2r\phi = r(\theta + \phi)$$

$$\phi = \theta$$

$$AC = r\left(1 + \frac{4}{3\pi}\right)\cos\theta$$

Thus, the change in elevation of G is

$$h = 2r - \left(r - \frac{4r}{3\pi}\right) - AC = r\left(1 + \frac{4}{3\pi}\right)(1 - \cos\theta)$$

Since no slipping occurs,

$$v_G = \dot{\theta}\left(r - \frac{4r}{3\pi}\right)$$

$$I_G = I_O - m\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{1}{2} - \left(\frac{4}{3\pi}\right)^2\right)mr^2$$

$$T = \frac{1}{2}m\dot{\theta}^2 r^2\left(1 - \frac{4}{3\pi}\right)^2 + \frac{1}{2}\left(\frac{1}{2} - \left(\frac{4}{3\pi}\right)^2\right)mr^2\dot{\theta}^2 = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\dot{\theta}^2$$

$$T + V = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\dot{\theta}^2 + mgr\left(1 + \frac{4}{3\pi}\right)(1 - \cos\theta)$$

$$0 = mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\dot{\theta}\,\ddot{\theta} + mgr\left(1 + \frac{4}{3\pi}\right)\sin\theta\dot{\theta}$$

$$\sin\theta \approx \theta$$

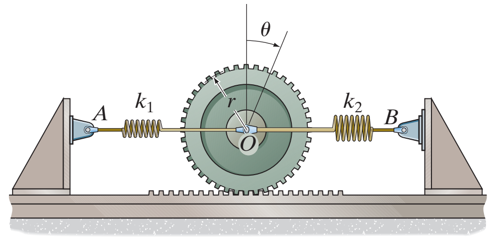
$$\ddot{\theta} + \frac{g\left(1 + \frac{4}{3\pi}\right)}{r\left(\frac{3}{2} - \frac{8}{3\pi}\right)}\theta = 0$$

$$\omega_n = 1.479\sqrt{\frac{g}{r}}$$

$$\tau = \frac{2\pi}{\omega_n} = 4.25\sqrt{\frac{r}{g}} \qquad \textbf{Ans.}$$

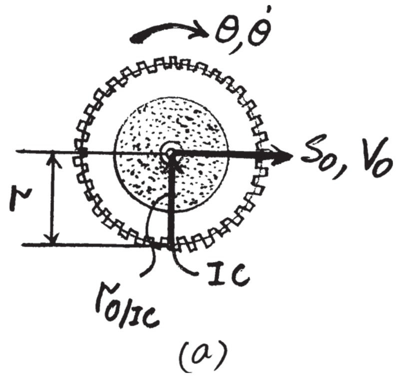
***22–40.**

The gear of mass m has a radius of gyration about its center of mass O of k_O. The springs have stiffnesses of k_1 and k_2, respectively, and both springs are unstretched when the gear is in an equilibrium position. If the gear is given a small angular displacement of θ and released, determine its natural period of oscillation.



SOLUTION

Potential and Kinetic Energy: Since the gear rolls on the gear rack, springs AO and BO stretch and compress $s_O = r_{O/IC}\theta = r\theta$. When the gear rotates a small angle θ, Fig. a, the elastic potential energy of the system is



$$V = V_e = \frac{1}{2}k_1 s_O{}^2 + \frac{1}{2}k_2 s_O{}^2$$

$$= \frac{1}{2}k_1\,(r\theta)^2 + \frac{1}{2}k_2\,(r\theta)^2$$

$$= \frac{1}{2}r^2(k_1 + k_2)\theta^2$$

Also, from Fig. a, $v_O = \dot{\theta}r_{O/IC} = \dot{\theta}r$. The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2$.

Thus, the kinetic energy of the system is

$$T = \frac{1}{2}mv_O{}^2 + \frac{1}{2}I_O\,\omega^2$$

$$= \frac{1}{2}m(\dot{\theta}r)^2 + \frac{1}{2}\left(mk_O{}^2\right)\dot{\theta}^2$$

$$= \frac{1}{2}m(r^2 + k_O{}^2)\dot{\theta}^2$$

The energy function of the system is therefore

$$T + V = \text{constant}$$

$$\frac{1}{2}m\left(r^2 + k_O{}^2\right)\dot{\theta}^2 + \frac{1}{2}r^2(k_1 + k_2)\theta^2 = \text{constant}$$

Taking the time derivative of this equation,

$$m\left(r^2 + k_O{}^2\right)\dot{\theta}\,\ddot{\theta} + r^2(k_1 + k_2)\theta\dot{\theta} = 0$$

$$\dot{\theta}\left[m\left(r^2 + k_O{}^2\right)\ddot{\theta} + r^2(k_1 + k_2)\theta\right] = 0$$

Since $\dot{\theta}$ is not always equal to zero, then

$$m\left(r^2 + k_O{}^2\right)\ddot{\theta} + r^2(k_1 + k_2)\theta = 0$$

$$\ddot{\theta} + \frac{r^2(k_1 + k_2)}{m\left(r^2 + k_O{}^2\right)}\theta = 0$$

***22–40. continued**

Comparing this equation to that of the standard form, the natural circular frequency of the system is

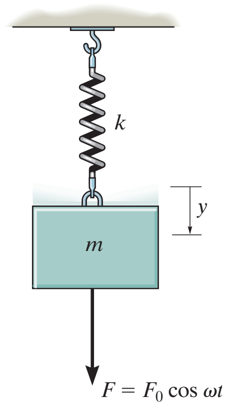
$$\omega_n = \sqrt{\frac{r^2(k_1 + k_2)}{m\left(r^2 + k_O{}^2\right)}}$$

Thus, the natural period of the oscillation is

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m\left(r^2 + k_O{}^2\right)}{r^2(k_1 + k_2)}} \qquad \textbf{Ans.}$$

22–41.

If the block is subjected to the periodic force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{y} + (k/m)y = (F_0/m)\cos \omega t$, where y is measured from the equilibrium position of the block. What is the general solution of this equation?



SOLUTION

$$+\downarrow \Sigma F_y = ma_y; \qquad F_0 \cos \omega t + W - k\delta_{st} - ky = m\ddot{y}$$

Since $W = k\delta_{st}$,

$$\ddot{y} + \left(\frac{k}{m}\right)y = \frac{F_0}{m}\cos \omega t \qquad \textbf{(1) (Q.E.D.)}$$

$$y_c = A \sin \omega_n y + B \cos \omega_n y \qquad \text{(complementary solution)}$$

$$y_p = C \cos \omega t \qquad \text{(particular solution)}$$

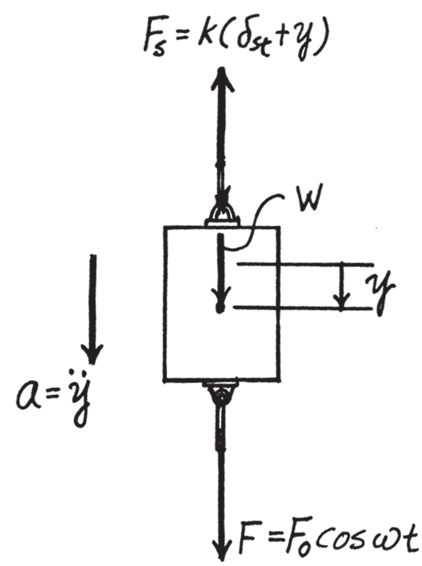
Substitute y_p into Eq. (1).

$$C\left(-\omega^2 + \frac{k}{m}\right)\cos \omega t = \frac{F_0}{m}\cos \omega t$$

$$C = \frac{\frac{F_0}{m}}{\left(\frac{k}{m} - \omega^2\right)}$$

$$y = y_c + y_p$$

$$y = A \sin \omega_n + B \cos \omega_n + \left(\frac{F_0}{(k - m\omega^2)}\right)\cos \omega t \qquad \textbf{Ans.}$$



22–42.

The block shown in Fig. 22–16 has a mass of 20 kg, and the spring has a stiffness $k = 600\ \text{N/m}$. When the block is displaced and released, two successive amplitudes are measured as $x_1 = 150$ mm and $x_2 = 87$ mm. Determine the coefficient of viscous damping, c.

SOLUTION

Assuming that the system is underdamped.

$$x_1 = De^{-\left(\frac{c}{2m}\right)t_1} \qquad \textbf{(1)}$$

$$x_2 = De^{-\left(\frac{c}{2m}\right)t_2} \qquad \textbf{(2)}$$

Divide Eq. (1) by Eq. (2) $\dfrac{x_1}{x_2} = \dfrac{e^{-\left(\frac{c}{2m}\right)t_1}}{e^{-\left(\frac{c}{2m}\right)t_2}}$

$$\ln\left(\frac{x_1}{x_2}\right) = \left(\frac{c}{2m}\right)(t_2 - t_1) \qquad \textbf{(3)}$$

However, $t_2 - t_1 = \tau_c = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{\omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2}}$ and $\omega_n = \dfrac{C_c}{2m}$

$$t_2 - t_1 = \frac{4m\pi}{C_c\sqrt{1 - \left(\frac{c}{C_c}\right)^2}} \qquad \textbf{(4)}$$

Substitute Eq. (4) into Eq. (3) yields:

$$\ln\left(\frac{x_1}{x_2}\right) = \left(\frac{c}{2m}\right)\frac{4m\pi}{C_c\sqrt{1 - \left(\frac{C}{C_c}\right)^2}}$$

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{C}{C_c}\right)}{\sqrt{1 - \left(\frac{C}{C_c}\right)^2}} \qquad \textbf{(5)}$$

From Eq. (5)

$$x_1 = 0.15\ \text{m} \quad x_2 = 0.087\text{m} \quad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{20}} = 5.477\ \text{rad/s}$$

$$C_c = 2m\omega_n = 2(20)(5.477) = 219.09\ \text{N}\cdot\text{s/m}$$

$$\ln\left(\frac{0.15}{0.087}\right) = \frac{2\pi\left(\frac{c}{219.09}\right)}{\sqrt{1 - \left(\frac{c}{219.09}\right)^2}}$$

$$c = 18.9\ \text{N}\cdot\text{s/m} \qquad \textbf{Ans.}$$

Since $C < C_c$, the system is underdamped. Therefore, the assumption is OK!

22–43.

A 4-lb weight is attached to a spring having a stiffness $k = 10\ \text{lb/ft}$. The weight is drawn downward a distance of 4 in. and released from rest. If the support moves with a vertical displacement $\delta = (0.5 \sin 4t)$ in., where t is in seconds, determine the equation which describes the position of the weight as a function of time.

SOLUTION

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega_0 t$$

$$v = \dot{y} = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t + \frac{\delta_0 \omega_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \cos \omega_0 t$$

The initial condition when $t = 0$, $y = y_0$, and $v = v_0$ is

$$y_0 = 0 + B + 0 \qquad B = y_0$$

$$v_0 = A\omega_n - 0 + \frac{\delta_0\, \omega_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \qquad A = \frac{v_0}{\omega_n} - \frac{\delta_0 \omega_0}{\omega_n - \frac{\omega_0^{\,2}}{\omega_n}}$$

Thus,

$$y = \left(\frac{v_0}{\omega_n} - \frac{\delta_0\, \omega_0}{\omega_n - \frac{\omega_0^{\,2}}{\omega_n}}\right) \sin \omega_n t + y_0 \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega_0 t$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10}{4/32.2}} = 8.972$$

$$\frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} = \frac{0.5/12}{1 - \left(\frac{4}{8.972}\right)^2} = 0.0520$$

$$\frac{v_0}{\omega_n} - \frac{\delta_0\, \omega_0}{\omega_n - \frac{\omega_0^{\,2}}{\omega_n}} = 0 - \frac{(0.5/12)4}{8.972 - \frac{4^2}{8.972}} = -0.0232$$

$$y = (-0.0232 \sin 8.97t + 0.333 \cos 8.97t + 0.0520 \sin 4t)\ \text{ft} \qquad \textbf{Ans.}$$

***22–44.**

A 4-kg block is suspended from a spring that has a stiffness of $k = 600$ N/m. The block is drawn downward 50 mm from the equilibrium position and released from rest when $t = 0$. If the support moves with an impressed displacement of $\delta = (10 \sin 4t)$ mm, where t is in seconds, determine the equation that describes the vertical motion of the block. Assume positive displacement is downward.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{4}} = 12.25$$

The general solution is defined by Eq. 22–23 with $k\delta_0$ substituted for F_0.

$$y = A \sin \omega_n t + B \cos \omega_n t + \left(\frac{\delta_0}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]} \right) \sin \omega t$$

$\delta = (0.01 \sin 4t)$m, hence $\delta_0 = 0.01, \omega = 4$, so that

$y = A \sin 12.25t + B \cos 12.25t + 0.0112 \sin 4t$

$y = 0.05$ when $t = 0$

$0.05 = 0 + B + 0; \qquad B = 0.05$ m

$\dot{y} = A(12.25) \cos 12.25t - B(12.25) \sin 12.25t + 0.0112(4) \cos 4t$

$v = y = 0$ when $t = 0$

$0 = A(12.25) - 0 + 0.0112(4); \qquad A = -0.00366$ m

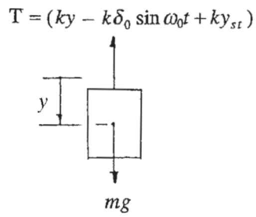
Expressing the result in mm, we have

$y = (-3.66 \sin 12.25t + 50 \cos 12.25t + 11.2 \sin 4t)$ mm **Ans.**

22–45.

Use a block-and-spring model like that shown in Fig. 22–14*a*, but suspended from a vertical position and subjected to a periodic support displacement $\delta = \delta_0 \sin \omega_0 t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.

SOLUTION



$+\uparrow \Sigma F_x = ma_x; \qquad k(y - \delta_0 \sin \omega_0 t + y_{st}) - mg = -m\ddot{y}$

$$m\ddot{y} + ky + ky_{st} - mg = k\delta_0 \sin \omega_0 t$$

However, from equilibrium

$ky_{st} - mg = 0$, therefore

$$\ddot{y} + \frac{k}{m}y = \frac{k\delta_0}{m}\sin \omega\, t \qquad \text{where } \omega_n = \sqrt{\frac{k}{m}}$$

$$\ddot{y} + \omega_n^2\, y = \frac{k\delta_0}{m}\sin \omega\, t \qquad \textbf{Ans. (1)}$$

The general solution of the above differential equation is of the form of $y = y_c + y_p$, where

$$y_c = A \sin \omega_n t + B \cos \omega_n t$$

$$y_p = C \sin \omega_0 t \qquad \textbf{(2)}$$

$$\ddot{y}_p = -C\omega_0^2 \sin \omega_0 t \qquad \textbf{(3)}$$

Substitute Eqs. (2) and (3) into (1) yields:

$$-C\omega^2 \sin \omega_0 t + \omega_n^2(C \sin \omega_0 t) = \frac{k\delta_0}{m} \sin \omega_0 t$$

$$C = \frac{\frac{k\delta_0}{m}}{\omega_n^2 - \omega_0^2} = \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}$$

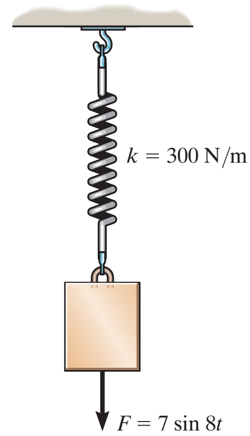
The general solution is therefore

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega\, t \qquad \textbf{Ans.}$$

The constants A and B can be found from the initial conditions.

22–46.

A 5-kg block is suspended from a spring having a stiffness of 300 N/m. If the block is acted upon by a vertical force $F = (7 \sin 8t)$ N, where t is in seconds, determine the equation which describes the motion of the block when it is pulled down 100 mm from the equilibrium position and released from rest at $t = 0$. Assume that positive displacement is downward.



SOLUTION

The general solution is defined by:

$$y = A \sin \omega_n t + B \cos \omega_n t + \left(\frac{\frac{F_0}{k}}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \right) \sin \omega_0 t$$

Since

$$F = 7 \sin 8t, \qquad F_0 = 7 \text{ N}, \qquad \omega_0 = 8 \text{ rad/s}, \qquad k = 300 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{5}} = 7.746 \text{ rad/s}$$

Thus,

$$y = A \sin 7.746t + B \cos 7.746t + \left(\frac{\frac{7}{300}}{1 - \left(\frac{8}{7.746}\right)^2} \right) \sin 8t$$

$y = 0.1$ m when $t = 0$,

$$0.1 = 0 + B - 0; \qquad B = 0.1 \text{ m}$$

$$\dot{y} = A(7.746) \cos 7.746t - B(7.746) \sin 7.746t - (0.35)(8) \cos 8t$$

$y = \dot{y} = 0$ when $t = 0$,

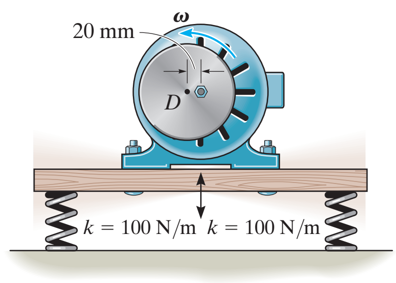
$$\dot{y} = A(7.746) - 2.8 = 0; \qquad A = 0.361$$

Expressing the results in mm, we have

$$y = (361 \sin 7.75t + 100 \cos 7.75t - 350 \sin 8t) \text{ mm}$$ **Ans.**

22–47.

The electric motor has a mass of 50 kg and is supported by *four springs,* each spring having a stiffness of 100 N/m. If the motor turns a disk D which is mounted eccentrically, 20 mm from the disk's center, determine the angular velocity ω at which resonance occurs. Assume that the motor only vibrates in the vertical direction.



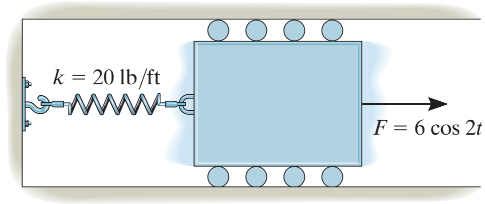
SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4(100)}{50}} = 2.83 \text{ rad/s}$$

$$\omega_n = \omega = 2.83 \text{ rad/s} \qquad \textbf{Ans.}$$

***22–48.**

The 20-lb block is attached to a spring having a stiffness of 20 lb/ft. A force $F = (6 \cos 2t)$ lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.



SOLUTION

$$C = \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{20}{\frac{20}{32.2}}} = 5.6745 \text{ rad/s}$$

$$C = \frac{\frac{6}{20}}{1 - \left(\frac{2}{5.6745}\right)^2} = 0.343 \text{ ft}$$

$$x_p = C \cos 2t$$

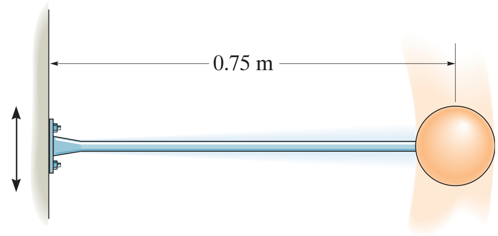
$$\dot{x}_p = -C(2) \sin 2t$$

Maximum velocity is

$$v_{max} = C(2) = 0.343(2) = 0.685 \text{ ft/s} \qquad \textbf{Ans.}$$

22–49.

The light elastic rod supports a 4-kg sphere. When an 18-N vertical force is applied to the sphere, the rod deflects 14 mm. If the wall oscillates with harmonic frequency of 2 Hz and has an amplitude of 15 mm, determine the amplitude of vibration for the sphere.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{18}{0.014} = 1285.71 \text{ N/m}$$

$$\omega_0 = 2 \text{ Hz} = 2(2\pi) = 12.57 \text{ rad/s}$$

$$\delta_0 = 0.015 \text{ m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1285.71}{4}} = 17.93$$

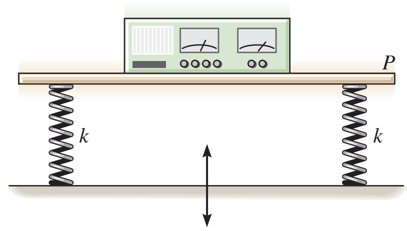
Using Eq. 22–22, the amplitude is

$$(x_p)_{max} = \left| \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \right| = \left| \frac{0.015}{1 - \left(\frac{12.57}{17.93}\right)^2} \right|$$

$$(x_p)_{max} = 0.0295 \text{ m} = 29.5 \text{ mm}$$ **Ans.**

22–50.

The instrument is centered uniformly on a platform P, which in turn is supported by *four* springs, each spring having a stiffness $k = 130$ N/m. If the floor is subjected to a vibration $\omega_0 = 7$ Hz, having a vertical displacement amplitude $\delta_0 = 0.17$ ft, determine the vertical displacement amplitude of the platform and instrument. The instrument and the platform have a total weight of 18 lb.



SOLUTION

$$k = 4(130) = 520 \text{ lb/ft}$$

$$\delta_0 = 0.17 \text{ ft}$$

$$\omega_0 = 7 \text{ Hz} = 7(2\pi) = 43.98 \text{ rad/s}$$

Using Eq. 22–22, the amplitude is

$$(x_p)_{max} = \left|\frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}\right|$$

Since $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{520}{\frac{18}{32.2}}} = 30.50 \text{ rad/s}$

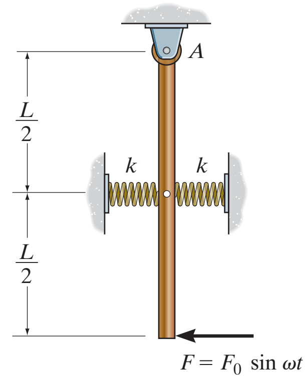
Then,

$$(x_p)_{max} = \left|\frac{0.17}{1 - \left(\frac{43.98}{30.50}\right)^2}\right| = 0.157 \text{ ft}$$

$$(x_p)_{max} = 1.89 \text{ in.}$$ **Ans.**

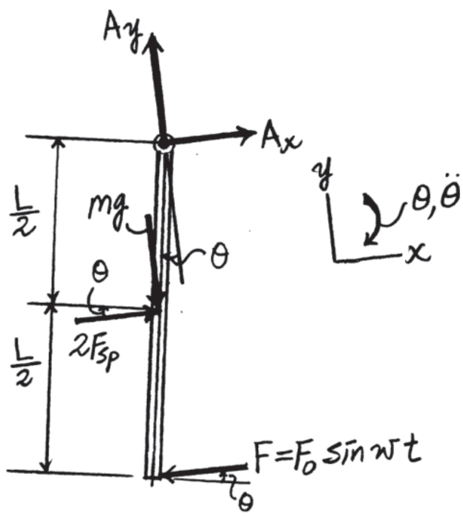
22–51.

The uniform rod has a mass of m. If it is acted upon by a periodic force of $F = F_0 \sin \omega t$, determine the amplitude of the steady-state vibration.



SOLUTION

Equation of Motion: When the rod rotates through a small angle θ, the springs compress and stretch $s = r_{AG}\theta = \dfrac{L}{2}\theta$. Thus, the force in each spring is $F_{sp} = ks = \dfrac{kL}{2}\theta$. The mass moment of inertia of the rod about point A is $I_A = \dfrac{1}{3}mL^2$. Referring to the free-body diagram of the rod shown in Fig. a,



(a)

$$+\Sigma M_A = I_A\alpha; \qquad F_O \sin \omega t \cos \theta(L) - mg \sin \theta\left(\frac{L}{2}\right) - 2\left(\frac{kL}{2}\theta\right)\cos \theta\left(\frac{L}{2}\right)$$

$$= \frac{1}{3}mL^2\dot{\theta}$$

Since θ is small, $\sin \theta \cong 0$ and $\cos \theta \cong 1$. Thus, this equation becomes

$$\frac{1}{3}mL\ddot{\theta} + \frac{1}{2}(mg + kL)\theta = F_O \sin \omega t$$

$$\ddot{\theta} + \frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right)\theta = \frac{3F_O}{mL}\sin \omega t \qquad \textbf{(1)}$$

The particular solution of this differential equation is assumed to be in the form of

$$\theta_p = C \sin \omega t \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2) twice,

$$\ddot{\theta}_p = -C\omega^2 \sin \omega t \qquad \textbf{(3)}$$

Substituting Eqs. (2) and (3) into Eq. (1),

$$-C\omega^2 \sin \omega t + \frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right)(C \sin \omega t) = \frac{3F_O}{mL}\sin \omega t$$

$$C\left[\frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right) - \omega^2\right]\sin \omega t = \frac{3F_O}{mL}\sin \omega t$$

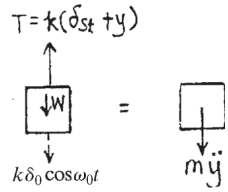
$$C = \frac{3F_O/\,mL}{\dfrac{3}{2}\left(\dfrac{g}{L} + \dfrac{k}{m}\right) - \omega^2}$$

$$C = \frac{3F_O}{\dfrac{3}{2}(mg + Lk) - mL\omega^2} \qquad \textbf{Ans.}$$

***22–52.**

Use a block-and-spring model like that shown in Fig. 22–14*a* but suspended from a vertical position and subjected to a periodic support displacement of $\delta = \delta_0 \cos \omega_0 t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.

SOLUTION



$+\downarrow \Sigma F_y = ma_y; \quad k\delta_0 \cos \omega_0 t + W - k\delta_{st} - ky = m\ddot{y}$

Since $W = k\delta_{st}$,

$$\ddot{y} + \frac{k}{m} y = \frac{k\delta_0}{m} \cos \omega_0 t \qquad \textbf{(1)}$$

$y_C = A \sin \omega_n y + B \cos \omega_n y$ (General sol.)

$y_P = C \cos \omega_0 t$ (Particular sol.)

Substitute y_p into Eq. (1)

$$C(-\omega_0^2 + \frac{k}{m}) \cos \omega_0 t = \frac{k\delta_0}{m} \cos \omega_0 t$$

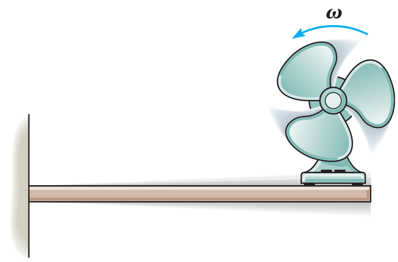
$$C = \frac{\frac{k\delta_0}{m}}{\left(\frac{k}{m} - \omega_0^2\right)}$$

Thus, $y = y_C + y_P$

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\frac{k\delta_0}{m}}{\left(\frac{k}{m} - \omega_0^2\right)} \cos \omega_0 t \qquad \textbf{Ans.}$$

22–53.

The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the angular velocity of the fan blade at which resonance will occur. *Hint:* See the first part of Example 22.8.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

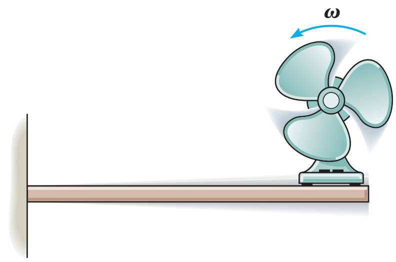
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

Resonance occurs when

$$\omega = \omega_n = 14.0 \text{ rad/s}$$ **Ans.**

22–54.

In Prob. 22–53, determine the amplitude of steady-state vibration of the fan if its angular velocity is 10 rad/s.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\,\omega^2 = 3.5(0.1)(10)^2 = 35 \text{ N}$$

Using Eq. 22–22, the amplitude is

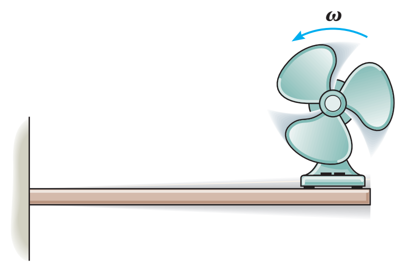
$$(x_p)_{\max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2} \right|$$

$$(x_p)_{\max} = \left| \frac{\frac{35}{4905}}{1 - \left(\frac{10}{14.01}\right)^2} \right| = 0.0146 \text{ m}$$

$$(x_p)_{\max} = 14.6 \text{ mm}$$ **Ans.**

22–55.

What will be the amplitude of steady-state vibration of the fan in Prob. 22–53 if the angular velocity of the fan blade is 18 rad/s? *Hint:* See the first part of Example 22.8.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\omega^2 = 3.5(0.1)(18)^2 = 113.4 \text{ N}$$

Using Eq. 22–22, the amplitude is

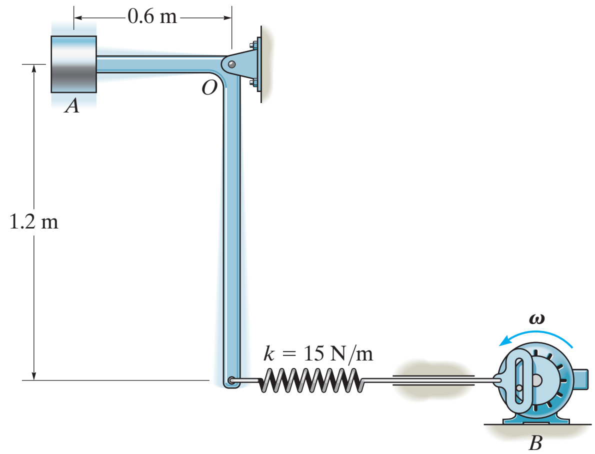
$$(x_p)_{\max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2} \right|$$

$$(x_p)_{\max} = \left| \frac{\frac{113.4}{4905}}{1 - \left(\frac{18}{14.01}\right)^2} \right| = 0.0355 \text{ m}$$

$$(x_p)_{\max} = 35.5 \text{ mm}$$ **Ans.**

***22–56.**

The small block at A has a mass of 4 kg and is mounted on the bent rod having negligible mass. If the rotor at B causes a harmonic movement $\delta_B = (0.1 \cos 15t)$ m, where t is in seconds, determine the steady-state amplitude of vibration of the block.



SOLUTION

$+\Sigma M_O = I_O\,\alpha; \qquad 4(9.81)(0.6) - F_s(1.2) = 4(0.6)^2\ddot{\theta}$

$$F_s = kx = 15(x + x_{st} - 0.1 \cos 15t)$$

$$x_{st} = \frac{4(9.81)(0.6)}{1.2(15)}$$

Thus,

$$-15(x - 0.1 \cos 15t)(1.2) = 4(0.6)^2\ddot{\theta}$$

$$x = 1.2\theta$$

$$\theta + 15\theta = 1.25 \cos 15t$$

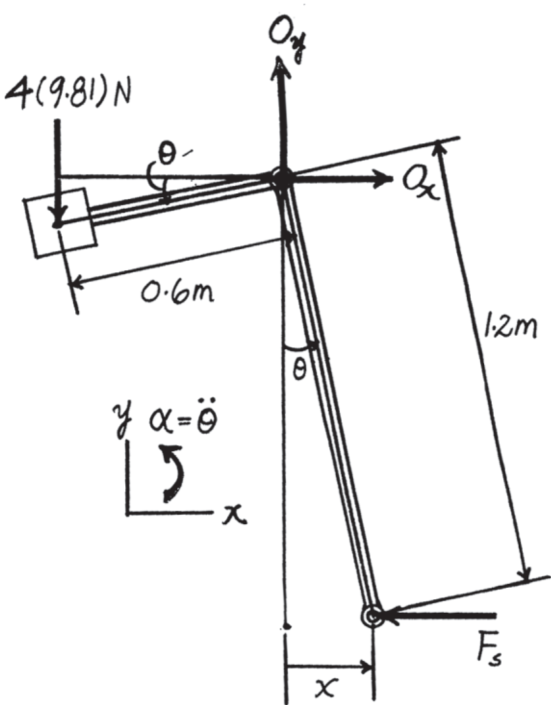
Set $x_p = C \cos 15t$

$$-C(15)^2 \cos 15t + 15(C \cos 15t) = 1.25 \cos 15t$$

$$C = \frac{1.25}{15 - (15)^2} = -0.00595 \text{ m}$$

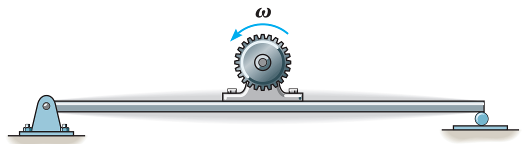
$$\theta_{\max} = C = 0.00595 \text{ rad}$$

$$y_{\max} = (0.6 \text{ m})(0.00595 \text{ rad}) = 0.00357 \text{ rad}$$ **Ans.**



22–57.

The electric motor turns an eccentric flywheel which is equivalent to an unbalanced 0.25-lb weight located 10 in. from the axis of rotation. If the static deflection of the beam is 1 in. due to the weight of the motor, determine the angular velocity of the flywheel at which resonance will occur. The motor weights 150 lb. Neglect the mass of the beam.



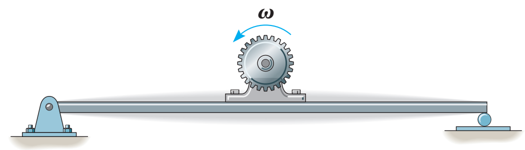
SOLUTION

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.66$$

Resonance occurs when $\qquad \omega_0 = \omega_n = 19.7 \text{ rad/s}$ **Ans.**

22–58.

What will be the amplitude of steady-state vibration of the motor in Prob. 22–57 if the angular velocity of the flywheel is 20 rad/s?



SOLUTION

The constant value F_O of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_O = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)(20)^2 = 2.588 \text{ lb}$$

Hence $F = 2.588 \sin 20t$

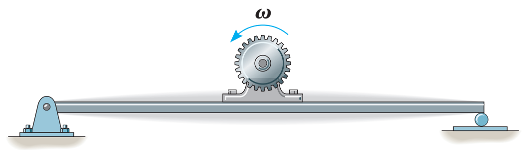
$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

From Eq. 22–21, the amplitude of the steady state motion is

$$C = \left|\frac{F_0/k}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}\right| = \left|\frac{2.588/1800}{1 - \left(\frac{20}{19.657}\right)^2}\right| = 0.04085 \text{ ft} = 0.490 \text{ in.} \qquad \textbf{Ans.}$$

22–59.

Determine the angular velocity of the flywheel in Prob. 22–57 which will produce an amplitude of vibration of 0.25 in.



SOLUTION

The constant value F_O of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_O = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)\omega^2 = 0.006470\omega^2$$

$$F = 0.006470\omega^2 \sin \omega t$$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

From Eq. 22.21, the amplitude of the steady-state motion is

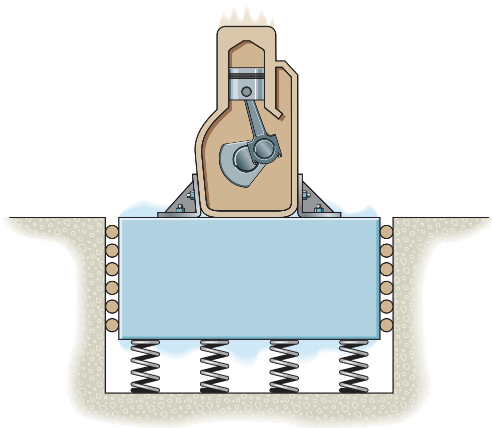
$$C = \left|\frac{F_0/k}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}\right|$$

$$\frac{0.25}{12} = \left|\frac{0.006470\left(\dfrac{\omega^2}{1800}\right)}{1 - \left(\dfrac{\omega}{19.657}\right)^2}\right|$$

$$\omega = 19.7 \text{ rad/s}$$ **Ans.**

***22–60.**

The engine is mounted on a foundation block which is spring-supported. Describe the steady-state vibration of the system if the block and engine have a total weight of 1500 lb and the engine, when running, creates an impressed force $F = (50 \sin 2t)$ lb, where t is in seconds. Assume that the system vibrates only in the vertical direction, with the positive displacement measured downward, and that the total stiffness of the springs can be represented as $k = 2000$ lb/ft.



SOLUTION

The steady-state vibration is defined by Eq. 22–22.

$$x_p = \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega_0 t$$

Since $F = 50 \sin 2t$

Then $F_0 = 50$ lb, $\omega_0 = 0$ rad/s

$$k = 2000 \text{ lb/ft}$$

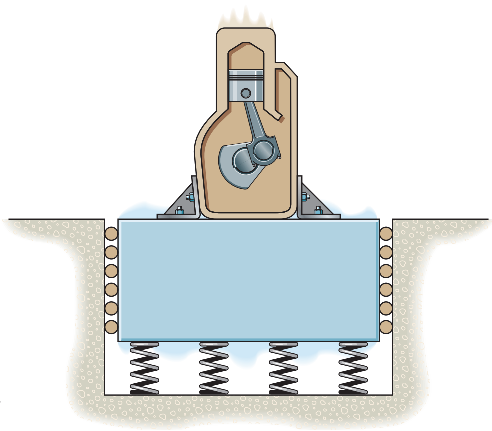
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2000}{\frac{1500}{32.2}}} = 6.55 \text{ rad/s}$$

$$\text{Hence, } x_p = \frac{\frac{50}{2000}}{1 - \left(\frac{2}{6.55}\right)^2} \sin 2t$$

$$x_p = (0.0276 \sin 2t) \text{ ft}$$ **Ans.**

22–61.

Determine the rotational speed ω of the engine in Prob. 22–60 which will cause resonance.



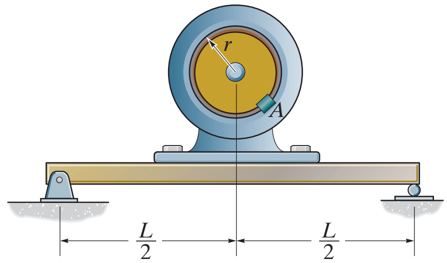
SOLUTION

Resonance occurs when

$$\omega = \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{\frac{2000}{1500}}{32.2}} = 6.55\ \text{rad/s} \qquad \textbf{Ans.}$$

22–62.

The motor of mass M is supported by a simply supported beam of negligible mass. If block A of mass m is clipped onto the rotor, which is turning at constant angular velocity of ω, determine the amplitude of the steady-state vibration. *Hint:* When the beam is subjected to a concentrated force of P at its mid-span, it deflects $\delta = PL^3/48EI$ at this point. Here E is Young's modulus of elasticity, a property of the material, and I is the moment of inertia of the beam's cross-sectional area.



SOLUTION

In this case, $P = k_{eq}\delta$. Then, $k_{eq} = \dfrac{P}{\delta} = \dfrac{P}{PL^3/48EI} = \dfrac{48EI}{L^3}$. Thus, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{\frac{48EI}{L^3}}{M}} = \sqrt{\frac{48EI}{ML^3}}$$

Here, $F_O = ma_n = m(\omega^2 r)$. Thus,

$$Y = \frac{F_O/k_{eq}}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}$$

$$Y = \frac{\dfrac{m(\omega^2 r)}{48EI/L^3}}{1 - \dfrac{\omega^2}{48EI/ML^3}}$$

$$Y = \frac{mr\omega^2 L^3}{48EI - M\omega^2 L^3}$$ **Ans.**

22–63.

A block having a mass of 0.8 kg is suspended from a spring having a stiffness of 120 N/m. If a dashpot provides a damping force of 2.5 N when the speed of the block is 0.2 m s, determine the period of free vibration.

SOLUTION

$$F = cv \qquad c = \frac{F}{v} = \frac{2.5}{0.2} = 12.5 \text{ N}\cdot\text{s}/\text{m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{120}{0.8}} = 12.247 \text{ rad/s}$$

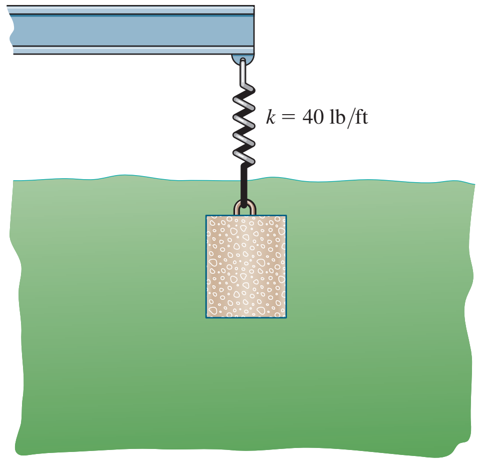
$$C_c = 2m\omega_n = 2(0.8)(12.247) = 19.60 \text{ N}\cdot\text{s/m}$$

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 12.247\sqrt{1 - \left(\frac{12.5}{19.6}\right)^2} = 9.432 \text{ rad/s}$$

$$\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{9.432} = 0.666 \text{ s} \qquad \textbf{Ans.}$$

***22–64.**

The block, having a weight of 15 lb, is immersed in a liquid such that the damping force acting on the block has a magnitude of $F = (0.8|v|)$ lb, where v is the velocity of the block in ft/s. If the block is pulled down 0.8 ft and released from rest, determine the position of the block as a function of time. The spring has a stiffness of $k = 40$ lb/ft. Consider positive displacement to be downward.



SOLUTION

Viscous Damped Free Vibration: Here $c = 0.8\ \text{lb}\cdot\text{s/ft}$, $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{40}{15/32.2}} = 9.266$ rad/s and $c_c = 2m\omega_n = 2\left(\dfrac{15}{32.2}\right)(9.266) = 8.633\ \text{lb}\cdot\text{s/ft}$. Since $c < c_c$, the system is underdamped and the solution of the differential equation is in the form of

$$y = D\left[e^{-(c/2m)t}\sin(\omega_d t + \phi)\right] \qquad \textbf{(1)}$$

Taking the time derivative of Eq. (1), we have

$$v = \dot{y} = D\left[-\left(\frac{c}{2m}\right)e^{-(c/2m)t}\sin(\omega_d t + \phi) + \omega_d e^{-(c/2m)t}\cos(\omega_d t + \phi)\right]$$

$$= De^{-(c/2m)t}\left[-\left(\frac{c}{2m}\right)\sin(\omega_d t + \phi) + \omega_d\cos(\omega_d t + \phi)\right] \qquad \textbf{(2)}$$

Applying the initial condition $v = 0$ at $t = 0$ to Eq. (2), we have

$$0 = De^{-0}\left[-\left(\frac{c}{2m}\right)\sin(0 + \phi) + \omega_d\cos(0 + \phi)\right]$$

$$0 = D\left[-\left(\frac{c}{2m}\right)\sin\phi + \omega_d\cos\phi\right] \qquad \textbf{(3)}$$

Here, $\dfrac{c}{2m} = \dfrac{0.8}{2(15/32.2)} = 0.8587$ and $\omega_d = \omega_n\sqrt{1 - \left(\dfrac{c}{c_c}\right)^2} = 9.266\sqrt{1 - \left(\dfrac{0.8}{8.633}\right)^2}$ $= 9.227$ rad/s. Substituting these values into Eq. (3) yields

$$0 = D[-0.8587\sin\phi + 9.227\cos\phi] \qquad \textbf{(4)}$$

Applying the initial condition $y = 0.8$ ft at $t = 0$ to Eq. (1), we have

$$0.8 = D\left[e^{-0}\sin(0 + \phi)\right]$$

$$0.8 = D\sin\phi \qquad \textbf{(5)}$$

Solving Eqs. (4) and (5) yields

$$\phi = 84.68° = 1.50\ \text{rad} \qquad D = 0.8035\ \text{ft}$$

Substituting these values into Eq. (1) yields

$$y = 0.803\left[e^{-0.8597}\sin(9.23t + 1.48)\right] \qquad \textbf{Ans.}$$

22–65.

A 7-lb block is suspended from a spring having a stiffness of $k = 75$ lb/ft. The support to which the spring is attached is given simple harmonic motion which can be expressed as $\delta = (0.15 \sin 2t)$ ft, where t is in seconds. If the damping factor is $c/c_c = 0.8$, determine the phase angle ϕ of forced vibration.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$$\delta = 0.15 \sin 2t$$

$$\delta_0 = 0.15, \omega = 2$$

$$\phi' = \tan^{-1}\left(\frac{2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right) = \tan^{-1}\left(\frac{2(0.8)\left(\frac{2}{18.57}\right)}{1 - \left(\frac{2}{18.57}\right)^2}\right)$$

$$\phi' = 9.89^\circ$$ **Ans.**

22–66.

Determine the magnification factor of the block, spring, and dashpot combination in Prob. 22–65.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$$\delta = 0.15 \sin 2t$$

$$\delta_0 = 0.15, \quad \omega = 2$$

$$\text{MF} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_n}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}} = \frac{1}{\sqrt{\left[1 - \left(\frac{2}{18.57}\right)^2\right]^2 + \left[2(0.8)\left(\frac{2}{18.57}\right)\right]^2}}$$

$$\text{MF} = 0.997$$ **Ans.**

22–67.

A block having a mass of 7 kg is suspended from a spring that has a stiffness $k = 600$ N/m. If the block is given an upward velocity of 0.6 m/s from its equilibrium position at $t = 0$, determine its position as a function of time. Assume that positive displacement of the block is downward and that motion takes place in a medium which furnishes a damping force $F = (50|v|)$ N, where v is the velocity of the block in m/s.

SOLUTION

$$c = 50 \text{ N s/m} \quad k = 600 \text{ N/m} \quad m = 7 \text{ kg}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{7}} = 9.258 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2(7)(9.258) = 129.6 \text{ N} \cdot \text{s/m}$$

Since $c < c_z$, the system is underdamped,

$$\omega_d = \omega_n \sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 9.258 \sqrt{1 - \left(\frac{50}{129.6}\right)^2} = 8.542 \text{ rad/s}$$

$$\frac{c}{2m} = \frac{50}{2(7)} = 3.751$$

From Eq. 22-32

$$y = D\left[e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$$

$$v = \dot{y} = D\left[e^{-\left(\frac{c}{2m}\right)t} \omega_d \cos(\omega_d t + \phi) + \left(-\frac{c}{2m}\right) e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$$

$$v = De^{-\left(\frac{c}{2m}\right)t}\left[\omega_d \cos(\omega_d t + \phi) - \frac{c}{2m} \sin(\omega_d t + \phi)\right]$$

Applying the initial condition at $t = 0$, $y = 0$ and $v = -0.6$ m/s.

$$0 = D[e^{-0} \sin(0 + \phi)] \qquad \text{since} \quad D \neq 0$$

$$\sin\phi = 0 \qquad \phi = 0°$$

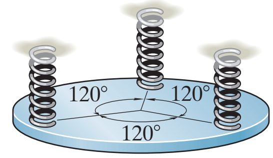
$$-0.6 = De^{-0}[8.542 \cos 0° - 0]$$

$$D = -0.0702 \text{ m}$$

$$y = [-0.0702 e^{-3.57t} \sin(8.540)] \text{ m} \qquad \textbf{Ans.}$$

***22–68.**

The 4-kg circular disk is attached to three springs, each spring having a stiffness $k = 180\ \text{N/m}$. If the disk is immersed in a fluid and given a downward velocity of $0.3\ \text{m/s}$ at the equilibrium position, determine the equation which describes the motion. Consider positive displacement to be measured downward, and that fluid resistance acting on the disk furnishes a damping force having a magnitude $F = (60|v|)\ \text{N}$, where v is the velocity of the block in m/s.



SOLUTION

$$k = 540\ \text{N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{540}{4}} = 11.62\ \text{rad/s}$$

$$c_c = 2m\omega_n = 2(4)(11.62) = 92.95$$

$$F = 60v, \text{ so that } c = 60$$

Since $c < c_c$, system is underdamped.

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2}$$

$$= 11.62\sqrt{1 - \left(\frac{60}{92.95}\right)^2}$$

$$= 8.87\ \text{rad/s}$$

$$y = A[e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)] \qquad \textbf{(1)}$$

$$y = 0, v = 0.3 \text{ at } t = 0$$

$$0 = A\sin\phi$$

$$A \neq 0 \text{ (trivial solution) so that } \phi = 0$$

$$v = y = A[-\frac{c}{2m}e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi) + e^{-\left(\frac{c}{2m}\right)t}\cos(\omega_d t + \phi)(\omega_d)]$$

Since $\phi = 0$

$$0.3 = A[0 + 1(8.87)]$$

$$A = 0.0338$$

Substituting into Eq. (1)

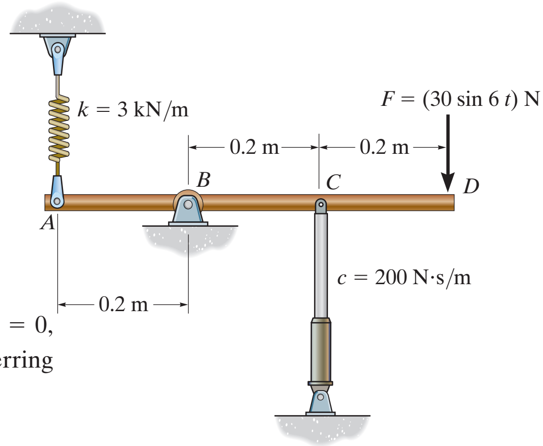
$$y = 0.0338[e^{-\left(\frac{60}{2(4)}\right)t}\sin(8.87)t]$$

Expressing the result in mm

$$y = [33.8e^{-7.5t}\sin(8.87t)]\ \text{mm} \qquad \textbf{Ans.}$$

22–69.

If the 12-kg rod is subjected to a periodic force of $F = (30 \sin 6t)$ N, where t is in seconds, determine the steady-state vibration amplitude θ_{max} of the rod about the pin B. Assume θ is small.

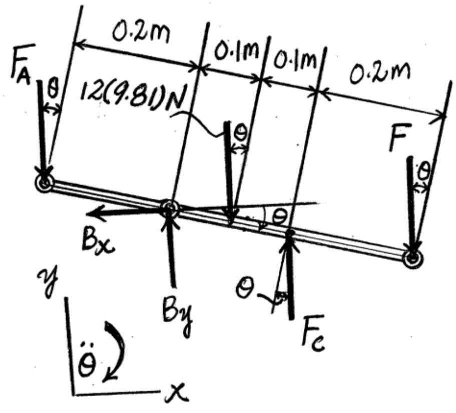


SOLUTION

Equation of Motion: When the rod is in equilibrium, $\theta = 0°$, $F = 0$, $F_c = c\dot{y}_c = 0$, and $\ddot{\theta} = 0$. Writing the moment equation of equilibrium about point B by referring to the free-body diagram of the rod, Fig. a,

$$+\Sigma M_B = 0; \qquad F_A(0.2) - 12(9.81)(0.1) = 0 \qquad F_A = 58.86 \text{ N}$$

Thus, the initial compression of the spring is $s_O = \dfrac{F_A}{k} = \dfrac{58.86}{3000} = 0.01962$ m. When the rod rotates about point B through a small angle θ, the spring compresses further by $s_1 = 0.2\theta$. Thus, the force in the spring is $F_A = k(s_O + s_1) = 3000(0.01962 + 0.2\theta) = 58.85 + 600\theta$. Also, the velocity of point C on the rod is $v_c = \dot{y}_c = 0.2\dot{\theta}$. Thus, $F_c = c\dot{y}_c = 200(0.2\dot{\theta}) = 40\dot{\theta}$. The mass moment of inertia of the rod about B is $I_B = \dfrac{1}{12}(12)(0.6)^2 + 12(0.1)^2 = 0.48 \text{ kg}\cdot\text{m}^2$. Again, referring to Fig. a,



(a)

$$+\Sigma M_B = I_B\alpha; \quad (58.86 + 600\theta)\cos\theta(0.2) + 40\dot{\theta}\cos\theta(0.2) - (30\sin 6t)\cos\theta(0.4)$$
$$- 12(9.81)\cos\theta(0.1) = -0.48\ddot{\theta}$$

$$\ddot{\theta} + 16.67\cos\theta\dot{\theta} + 250(\cos\theta)\theta = 25\sin 6t\cos\theta$$

Since θ is small, $\cos\theta \cong 1$. Thus, this equation becomes

$$\ddot{\theta} + 16.67\dot{\theta} + 250\theta = 25\sin 6t$$

Comparing this equation to that of the standard form,

$$\frac{k_{eq}}{m} = 250 \qquad k_{eq} = 250(12) = 3000 \text{ N/m}$$

$$\frac{c_{eq}}{m} = 16.667 \qquad c_{eq} = 16.667(12) = 200 \text{ N}\cdot\text{s/m}$$

$$\frac{F_O}{m} = 25 \qquad F_O = 25(12) = 300 \text{ N}$$

$$\omega_n = \sqrt{3000/12} = \sqrt{250}$$

Thus,

$$c_c = 2m\omega_n = 2(12)\sqrt{250} = 379.47 \text{ N}\cdot\text{s/m}$$

Then,

$$\frac{c_{eq}}{c_c} = \frac{200}{379.47} = 0.5270$$

$$\theta_{max} = \frac{300/3000}{\sqrt{\left[1 - \left(\dfrac{6}{\sqrt{250}}\right)^2\right]^2 + \left[\dfrac{2(0.5270)(6)}{\sqrt{250}}\right]^2}}$$

22–70.

The damping factor, c/c_c, may be determined experimentally by measuring the successive amplitudes of vibrating motion of a system. If two of these maximum displacements can be approximated by x_1 and x_2, as shown in Fig. 22–17, show that the ratio $\ln x_1/x_2 = 2\pi(c/c_c)/\sqrt{1 - (c/c_e)^2}$. The quantity $\ln x_1/x_2$ is called the *logarithmic decrement*.

SOLUTION

Using Eq. 22–32,

$$x = D\left[e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$$

The maximum displacement is

$$x_{max} = De^{-\left(\frac{c}{2m}\right)t}$$

At $t = t_1$, and $t = t_2$

$$x_1 = De^{-\left(\frac{c}{2m}\right)t_1}$$

$$x_2 = De^{-\left(\frac{c}{2m}\right)t2}$$

Hence,

$$\frac{x_1}{x_2} = \frac{De^{-\left(\frac{c}{2m}\right)t1}}{De^{-\left(\frac{c}{2m}\right)t_2} = e^{-\left(\frac{c}{2m}\right)(t_1 - t_2)}}$$

Since $\omega_d t_2 - \omega_d t_1 = 2\pi$

then $t_2 - t_1 = \dfrac{2\pi}{\omega_d}$

so that $\ln\left(\dfrac{x_1}{x_2}\right) = \dfrac{c\pi}{m\omega_d}$

Using Eq. 22–33, $c_c = 2m\omega_n$

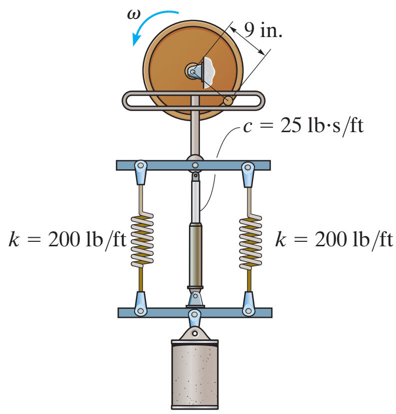
$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = \frac{c_c}{2m}\sqrt{1 - \left(\frac{c}{c_r}\right)^2}$$

So that,

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{c}{c_c}\right)}{\sqrt{1 - \left(\frac{c}{c_c}\right)^2}} \qquad \textbf{Q.E.D.}$$

22–71.

If the amplitude of the 50-lb cylinder's steady-vibration is 6 in., determine the wheel's angular velocity ω.



SOLUTION

In this case, $Y = \dfrac{6}{12} = 0.5$ ft, $\delta_O = \dfrac{9}{12} = 0.75$ ft, and $k_{eq} = 2k = 2(200) = 400$ lb/ft. Then

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{400}{(50/32.2)}} = 16.05 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2\left(\frac{50}{32.2}\right)(16.05) = 49.84 \text{ lb}\cdot\text{s/ft}$$

$$\frac{c}{c_c} = \frac{25}{49.84} = 0.5016$$

$$Y = \frac{\delta_O}{\sqrt{\left[1 - \left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left(\dfrac{2(c/c_c)\omega}{\omega_n}\right)^2}}$$

$$0.5 = \frac{0.75}{\sqrt{\left[1 - \left(\dfrac{\omega}{16.05}\right)^2\right]^2 + \left(\dfrac{2(0.5016)\omega}{16.05}\right)^2}}$$

$$15.07(10^{-6})\omega^4 - 3.858(10^{-3})\omega^2 - 1.25 = 0$$

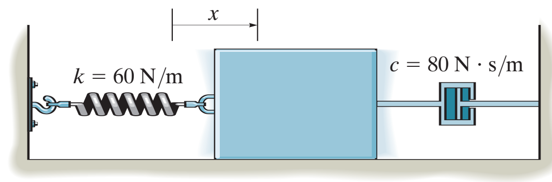
Solving for the positive root of this equation,

$$\omega^2 = 443.16$$

$$\omega = 21.1 \text{ rad/s} \qquad \textbf{Ans.}$$

***22–72.**

The 10-kg block-spring-damper system is continually damped. If the block is displaced to $x = 50$ mm and released from rest, determine the time required for it to return to the position $x = 2$ mm.



SOLUTION

$$m = 10\text{ kg}, \qquad c = 80, \qquad k = 60$$

$$c_c = 2\sqrt{km} = 2\sqrt{600} = 49.0 < c \quad \text{(Overdamped)}$$

$$x = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

$$\lambda_{1,2} = \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} = \frac{-80}{20} \pm \sqrt{\left(\frac{80}{20}\right)^2 - \frac{60}{10}}$$

$$\lambda_1 = -0.8377, \qquad \lambda_2 = -7.1623$$

At $t = 0$, $x = 0.05, \dot{x} = 0$

$$0.05 = A + B$$

$$A = 0.05 - B$$

$$\dot{x} = A\lambda_1 e^{\lambda_1 t} + B\lambda_2 e^{\lambda_2 t}$$

$$0 = A\lambda_1 + B\lambda_2$$

$$0 = (0.05 - B)\lambda_1 + B\lambda_2$$

$$B = \frac{0.05\lambda_1}{\lambda_1 - \lambda_2} = -6.6228(10^{-3})$$

$$A = 0.056623$$

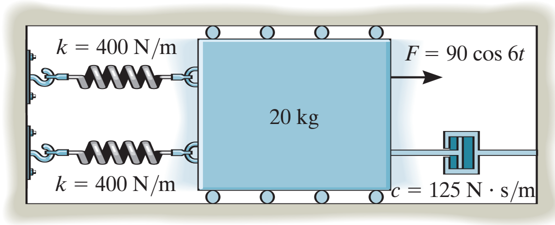
$$x = 0.056623e^{-0.8377t} - 6.6228(10^{-3})e^{-7.1623t}$$

Set $x = 0.0002$ m and solve,

$$t = 3.99\text{ s}$$ **Ans.**

22–73.

The 20-kg block is subjected to the action of the harmonic force $F = (90 \cos 6t)$ N, where t is in seconds. Write the equation which describes the steady-state motion.



SOLUTION

$$F = 90 \cos 6t$$

$$F_0 = 90 \text{ N}, \qquad \omega_0 = 6 \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{20}} = 6.32 \text{ rad/s}$$

From Eq. 22–29,

$$c_c = 2m\omega_n = 2(20)(6.32) = 253.0$$

Using Eqs. 22–39,

$$x = C' \cos(\omega_0 t - \phi)$$

$$C' = \frac{\frac{F_n}{k}}{\sqrt{\left[1 - \left(\frac{\omega_0}{\omega_r}\right)^2\right]^2 + \left[2\left(\frac{C}{C_r}\right)\left(\frac{\omega_0}{\omega_n}\right)\right]^2}}$$

$$= \frac{\frac{90}{800}}{\sqrt{[1 - \left(\frac{6}{6.32}\right)^2]^2 + [2\left(\frac{125}{253.0}\right)\left(\frac{6}{6.32}\right)]^2}}$$

$$= 0.119$$

$$\phi = \tan^{-1}\left[\frac{\frac{c\omega_0}{k}}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{\frac{125(6)}{800}}{1 - \left(\frac{6}{6.32}\right)^2}\right]$$

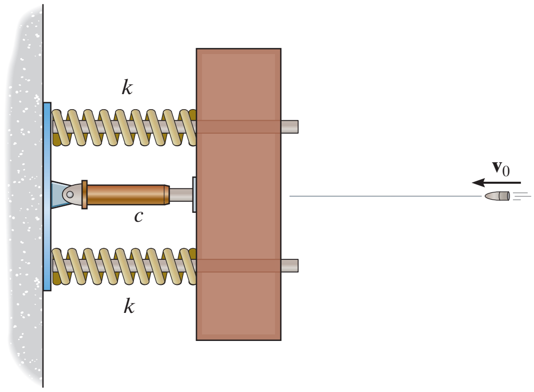
$$\phi = 83.9°$$

Thus,

$$x = 0.119 \cos(6t - 83.9°) \text{ m}$$ **Ans.**

22–74.

A bullet of mass m has a velocity of $\mathbf{v}_0$ just before it strikes the target of mass M. If the bullet embeds in the target, and the vibration is to be critically damped, determine the dashpot's critical damping coefficient, and the springs' maximum compression. The target is free to move along the two horizontal guides that are "nested" in the springs.



SOLUTION

Since the springs are arranged in parallel, the equivalent stiffness of the single spring system is $k_{eq} = 2k$. Also, when the bullet becomes embedded in the target, $m_T = m + M$. Thus, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m_T}} = \sqrt{\frac{2k}{m + M}}$$

When the system is critically damped

$$c = c_c = 2m_T\omega_n = 2(m + M)\sqrt{\frac{2k}{m + M}} = \sqrt{8(m + M)k} \qquad \textbf{Ans.}$$

The equation that describes the critically dampened system is

$$x = (A + Bt)e^{-\omega_n t}$$

When $t = 0, x = 0$. Thus,

$$A = 0$$

Then,

$$x = Bte^{-\omega_n t} \qquad \textbf{(1)}$$

Taking the time derivative,

$$v = \dot{x} = Be^{-\omega_n t} - B\omega_n te^{-\omega_n t}$$

$$v = Be^{-\omega_n t}(1 - \omega_n t) \qquad \textbf{(2)}$$

Since linear momentum is conserved along the horizontal during the impact, then

$(\overset{+}{\leftarrow})$ $$mv_0 = (m + M)v$$

$$v = \left(\frac{m}{m + M}\right)v_0$$

Here, when $t = 0$, $v = \left(\frac{m}{m + M}\right)v_0$. Thus, Eq. (2) gives

$$B = \left(\frac{m}{m + M}\right)v_0$$

And Eqs. (1) and (2) become

$$x = \left[\left(\frac{m}{m + M}\right)v_0\right]te^{-\omega_n t} \qquad \textbf{(3)}$$

$$v = \left[\left(\frac{m}{m + M}\right)v_0\right]e^{-\omega_n t}(1 - \omega_n t) \qquad \textbf{(4)}$$

22–74. continued

The maximum compression of the spring occurs when the block stops. Thus, Eq. (4) gives

$$0 = \left[\left(\frac{m}{m + M}\right)v_0\right](1 - \omega_n t)$$

Since $\left(\frac{m}{m + M}\right)v_0 \neq 0$, then

$$1 - \omega_n t = 0$$

$$t = \frac{1}{\omega_n} = \sqrt{\frac{m + M}{2k}}$$

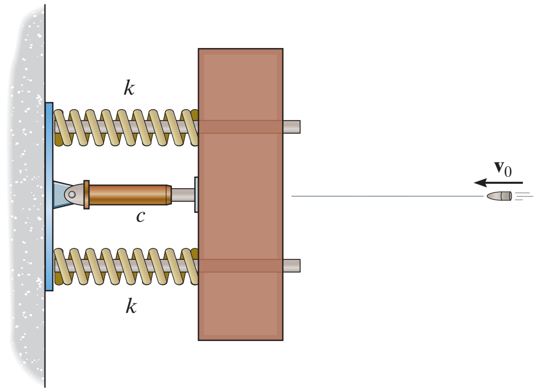
Substituting this result into Eq. (3)

$$x_{\max} = \left[\left(\frac{m}{m + M}\right)v_0\right]\left(\sqrt{\frac{m + M}{2k}}\right)e^{-1}$$

$$= \left[\frac{m}{e}\sqrt{\frac{1}{2k(m + M)}}\right]v_0 \qquad \textbf{Ans.}$$

22–75.

A bullet of mass m has a velocity $\mathbf{v}_0$ just before it strikes the target of mass M. If the bullet embeds in the target, and the dashpot's damping coefficient is $0 < c \ll c_c$, determine the springs' maximum compression. The target is free to move along the two horizontal guides that are "nested" in the springs.



SOLUTION

Since the springs are arranged in parallel, the equivalent stiffness of the single spring system is $k_{eq} = 2k$. Also, when the bullet becomes embedded in the target, $m_T = m + M$. Thus, the natural circular frequency of the system

$$\omega_n = \sqrt{\frac{k_{eq}}{m_T}} = \sqrt{\frac{2k}{m + M}}$$

The equation that describes the underdamped system is

$$x = Ce^{-(c/2m_T)t} \sin(\omega_d t + \phi) \qquad \textbf{(1)}$$

When $t = 0$, $x = 0$. Thus, Eq. (1) gives

$$0 = C \sin \phi$$

Since $C \neq 0$, $\sin \phi = 0$. Then $\phi = 0$. Thus, Eq. (1) becomes

$$x = Ce^{-(c/2m_T)t} \sin \omega_d t \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2),

$$v = \dot{x} = C\left[\omega_d e^{-(c/2m_T)t} \cos \omega_d t - \frac{c}{2m_T} e^{-(c/2m_T)t} \sin \omega_d t\right]$$

$$v = Ce^{-(c/2m_T)t}\left[\omega_d \cos \omega_d t - \frac{c}{2m_T} \sin \omega_d t\right] \qquad \textbf{(3)}$$

Since linear momentum is conserved along the horizontal during the impact, then

$$(\overset{+}{\leftarrow}) \qquad mv_0 = (m + M)v$$

$$v = \left(\frac{m}{m + M}\right)v_0$$

When $t = 0$, $v = \left(\frac{m}{m + M}\right)v_0$. Thus, Eq. (3) gives

$$\left(\frac{m}{m + M}\right)v_0 = C\omega_d \qquad C = \left(\frac{m}{m + M}\right)\frac{v_0}{\omega_d}$$

And Eqs. (2) becomes

$$x = \left[\left(\frac{m}{m + M}\right)\frac{v_0}{\omega_d}\right]e^{-(c/2m_T)t} \sin \omega_d t \qquad \textbf{(4)}$$

22–75. continued

The maximum compression of the spring occurs when

$$\sin \omega_d t = 1$$

$$\omega_d t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2\omega_d}$$

Substituting this result into Eq. (4),

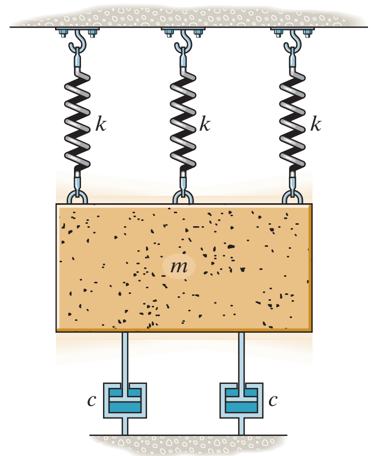
$$x_{\max} = \left[\left(\frac{m}{m + M}\right)\frac{v_0}{\omega_d}\right]e^{-[c/2(m + M)]\left(\frac{\pi}{2\omega_d}\right)}$$

However, $\omega_d = \sqrt{\dfrac{k_{eq}}{m_T} - \left(\dfrac{c}{2m_T}\right)^2} = \sqrt{\dfrac{2k}{m + M} - \dfrac{c^2}{4(m + M)^2}} = \dfrac{1}{2(m + M)}$ $\sqrt{8k(m + M) - c^2}$. Substituting this result into Eq. (5),

$$x_{\max} = \frac{2mv_0}{\sqrt{8k(m + M) - c^2}}e^{-\left[\frac{\pi c}{2\sqrt{8k(m + M) - c^2)}}\right]}$$

***22–76.**

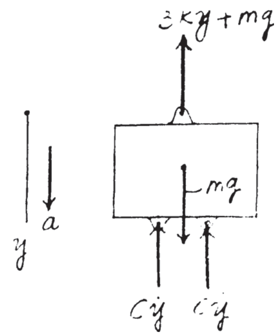
Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs? Take $k = 100$ N/m, $c = 200$ N·s/m, $m = 25$ kg.



SOLUTION

Free-body Diagram: When the block is being displaced by an amount y vertically downward, the *restoring force* is developed by the three springs attached the block.

Equation of Motion:



$+\uparrow \Sigma F_x = 0;\qquad 3ky + mg + 2c\dot{y} - mg = -m\ddot{y}$

$$m\ddot{y} + 2c\dot{y} + 3ky = 0 \qquad \textbf{(1)}$$

Here, $m = 25$ kg, $c = 200$ N·s/m and $k = 100$ N/m. Substituting these values into Eq. (1) yields

$$25\ddot{y} + 400\dot{y} + 300y = 0$$

$$\ddot{y} + 16\dot{y} + 12y = 0 \qquad \textbf{Ans.}$$

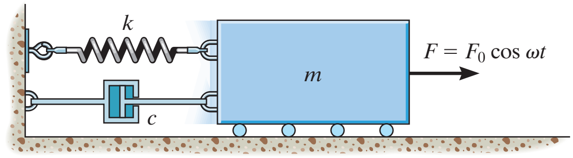
Comparing the above differential equation with Eq. 22–27, we have $m = 1$kg, $c = 16$ N·s/m and $k = 12$ N/m. Thus, $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{12}{1}} = 3.464$ rad/s

$$c_c = 2m\omega_n = 2(1)(3.464) = 6.928 \text{ N}\cdot\text{s/m}$$

Since $c > c_c$, the system will not vibrate. Therefore it is **overdamped.** **Ans.**

22–77.

Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.



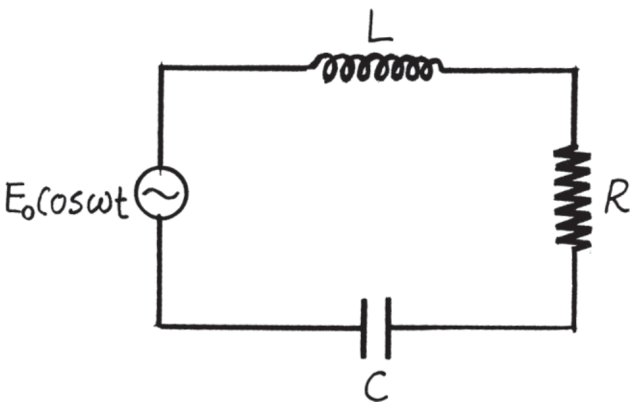
SOLUTION

For the block,

$$mx + cx + kx = F_0 \cos \omega t$$

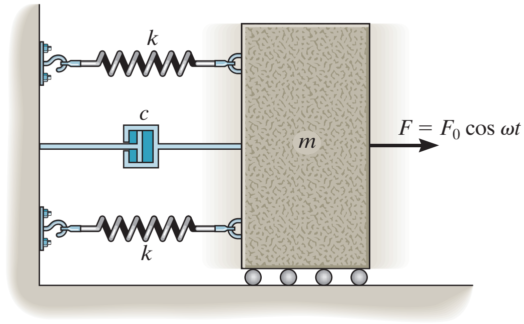
Using Table 22–1,

$$Lq + Rq + \left(\frac{1}{C}\right)q = E_0 \cos \omega t \qquad \textbf{Ans.}$$



22–78.

Draw the electrical circuit that is equivalent to the mechanical system shown. What is the differential equation which describes the charge q in the circuit?



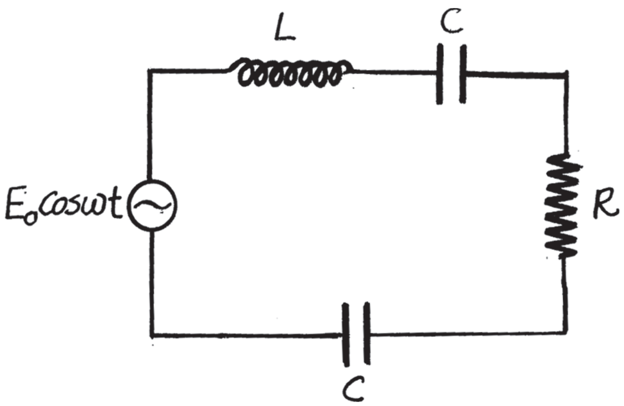
SOLUTION

Electrical Circuit Analogs: The differential equation that deseribes the motion of the given mechanical system is

$$m\ddot{x} + c\dot{x} + 2kx = F_0 \cos \omega t$$

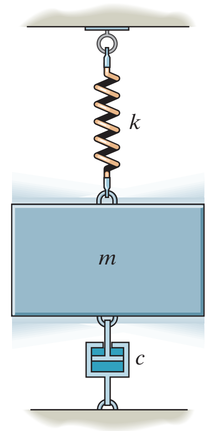
From Table 22–1 of the text, the differential equation of the analog electrical circuit is

$$L\ddot{q} + R\dot{q} + \left(\frac{2}{C}\right)q = E_0 \cos \omega t \qquad \textbf{Ans.}$$



22–79.

Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.



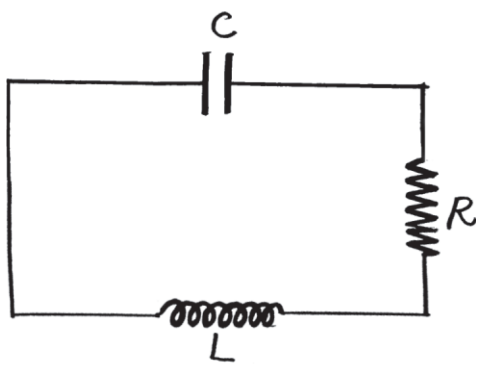
SOLUTION

For the block

$$m\ddot{y} + c\dot{y} + ky = 0$$

Using Table 22–1

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0 \qquad \textbf{Ans.}$$



R1–1.

An automobile is traveling with a *constant speed* along a horizontal circular curve that has a radius $\rho = 750$ ft. If the magnitude of acceleration is $a = 8\ \text{ft/s}^2$, determine the speed at which the automobile is traveling.

SOLUTION

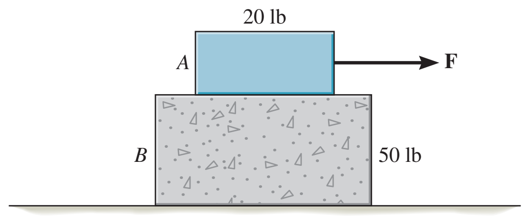
$$a = a_n = 8 = \frac{v^2}{\rho}$$

$$8 = \frac{v^2}{750}$$

$v = 77.5$ ft/s **Ans.**

R1–2.

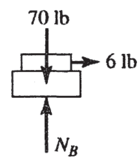
Block B rests on a smooth surface. If the coefficients of friction between A and B are $\mu_s = 0.4$ and $\mu_k = 0.3$, determine the acceleration of each block if (a) $F = 6$ lb, and (b) $F = 50$ lb.



SOLUTION

a) The maximum friction force between blocks A and B is

$$F_{\max} = 0.4(20) = 8 \text{ lb} > 6 \text{ lb}$$

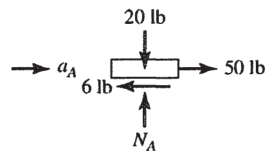


Thus, both blocks move together.

$$\xrightarrow{+} \sum F_x = ma_x; \qquad 6 = \frac{70}{32.2}a$$

$$a_B = a_A = a = 2.76 \text{ ft/s}^2$$ **Ans.**

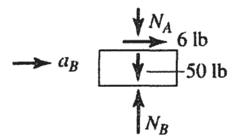
b) In this case 8 lb $<$ $F = 50$ lb



Block A:

$$\xrightarrow{+} \sum F_x = ma_x; \qquad 20(0.3) = \frac{20}{32.2}a_A$$

$$a_A = 70.8 \text{ ft/s}^2$$ **Ans.**



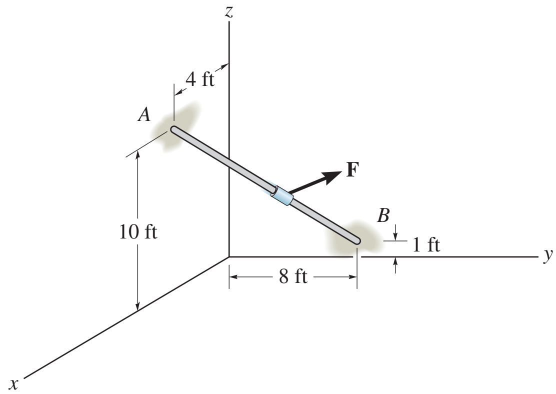
Block B:

$$\xrightarrow{+} \sum F_x = ma_x; \qquad 20(0.3) = \frac{50}{32.2}a_B$$

$$a_B = 3.86 \text{ ft/s}^2$$ **Ans.**

R1–3.

The small 2-lb collar starting from rest at A slides down along the smooth rod. During the motion, the collar is acted upon by a force $\mathbf{F} = \{10\mathbf{i} + 6y\mathbf{j} + 2z\mathbf{k}\}$ lb, where x, y, z are in feet. Determine the collar's speed when it strikes the wall at B.



SOLUTION

$$r_{AB} = r_B - r_A = -4\mathbf{i} + 8\mathbf{j} - 9\mathbf{k}$$

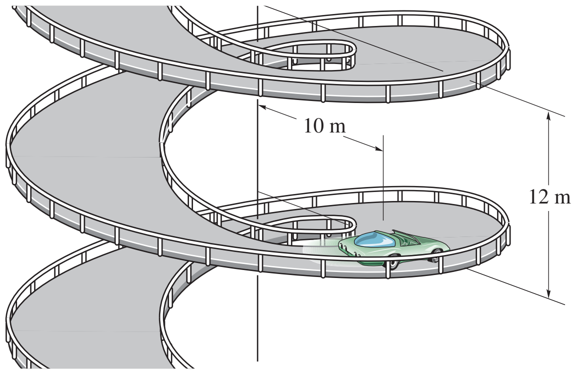
$$T_1 + \Sigma \int F ds = T_2$$

$$0 + 2(10-1) + \int_4^0 10dx + \int_0^8 6y\,dy + \int_{10}^1 2z\,dz = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

$$v_B = 47.8 \text{ ft/s}$$ **Ans.**

***R1–4.**

The automobile travels from a parking deck down along a cylindrical spiral ramp at a constant speed of $v = 1.5$ m/s. If the ramp descends a distance of 12 m for every full revolution, $\theta = 2\pi$ rad, determine the magnitude of the car's acceleration as it moves along the ramp, $r = 10$ m. *Hint:* For part of the solution, note that the tangent to the ramp at any point is at an angle of $\phi = \tan^{-1}(12/[2\pi(10)]) = 10.81^\circ$ from the horizontal. Use this to determine the velocity components v_θ and v_z, which in turn are used to determine $\dot{\theta}$ and $\dot{z}$.



SOLUTION

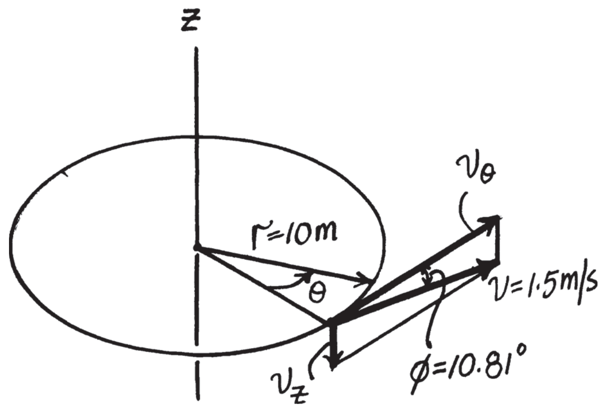
$$\phi = \tan^{-1}\left(\frac{12}{2\pi(10)}\right) = 10.81^\circ$$

$v = 1.5$ m/s

$v_r = 0$

$v_\theta = 1.5 \cos 10.81^\circ = 1.473$ m/s

$v_z = -1.5 \sin 10.81^\circ = -0.2814$ m/s



Since

$r = 10 \qquad \dot{r} = 0 \qquad r = 0$

$v_\theta = r\,\dot{\theta} = 1.473 \qquad \theta = \dfrac{1.473}{10} = 0.1473$

Since $\theta = 0$

$a_r = r - \dot{r}\theta^2 = 0 - 10(0.1473)^2 = -0.217$

$a_\theta = \dot{r}\ddot{\theta} + 2\,r\,\theta = 10(0) + 2(0)(0.1473) = 0$

$a_z = \ddot{z} = 0$

$a = \sqrt{(-0.217)^2 + (0)^2 + (0)^2} = 0.217 \text{ m/s}^2$ **Ans.**

R1–5.

A rifle has a mass of 2.5 kg. If it is loosely gripped and a 1.5-g bullet is fired from it with a horizontal muzzle velocity of 1400 m/s, determine the recoil velocity of the rifle just after firing.

SOLUTION

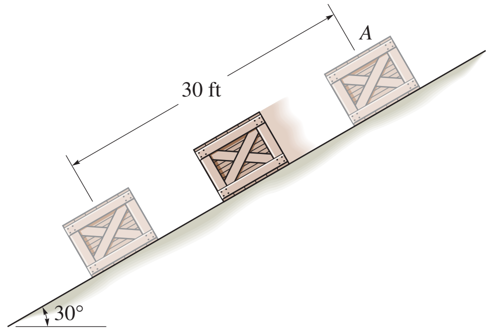
$\xrightarrow{+}$ $$\Sigma m v_1 = \Sigma m v_2$$

$$0 + 0 = 0.0015(1400) - 2.5(v_R)_2$$

$$(v_R)_2 = 0.840 \text{ m/s}$$ **Ans.**

R1–6.

If a 150-lb crate is released from rest at A, determine its speed after it slides 30 ft down the plane. The coefficient of kinetic friction between the crate and plane is $\mu_k = 0.3$.



SOLUTION

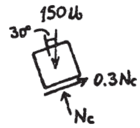
$+\nwarrow \Sigma F_y = 0; \qquad N_C - 150 \cos 30° = 0$

$$N_C = 129.9 \text{ lb}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

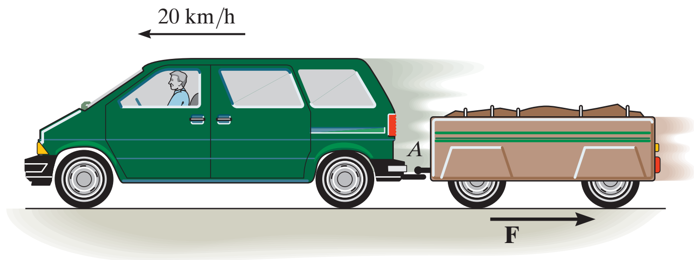
$$0 + 150 \sin 30°(30) - (0.3)129.9(30) = \frac{1}{2}\left(\frac{150}{32.2}\right)v_2^2$$

$v_2 = 21.5$ ft/s **Ans.**



R1–7.

The van is traveling at 20 km/h when the coupling of the trailer at A fails. If the trailer has a mass of 250 kg and coasts 45 m before coming to rest, determine the constant horizontal force F created by rolling friction which causes the trailer to stop.



SOLUTION

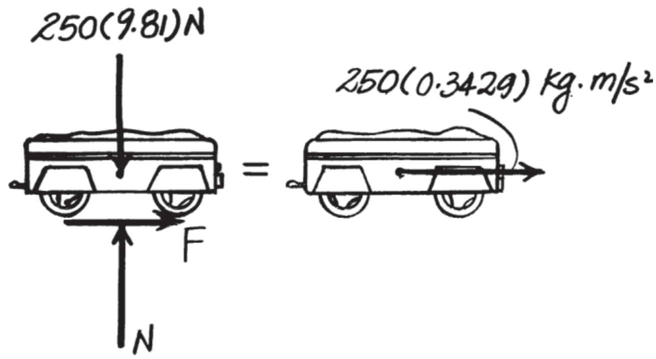
$$20 \text{ km/h} = \frac{20(10^3)}{3600} = 5.556 \text{ m/s}$$

$$(\overset{+}{\leftarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = 5.556^2 + 2(a)(45 - 0)$$

$$a = -0.3429 \text{ m/s}^2 = 0.3429 \text{ m/s}^2 \rightarrow$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F = 250(0.3429) = 85.7 \text{ N}$$ **Ans.**



***R1–8.**

The position of a particle along a straight line is given by $s = (t^3 - 9t^2 + 15t)$ ft, where t is in seconds. Determine its maximum acceleration and maximum velocity during the time interval $0 \leq t \leq 10$ s.

SOLUTION

$s = t^3 - 9t^2 + 15t$

$$v = \frac{ds}{dt} = 3t^2 - 18t + 15$$

$$a = \frac{dv}{dt} = 6t - 18$$

a_{max} occurs at $t = 10$ s,

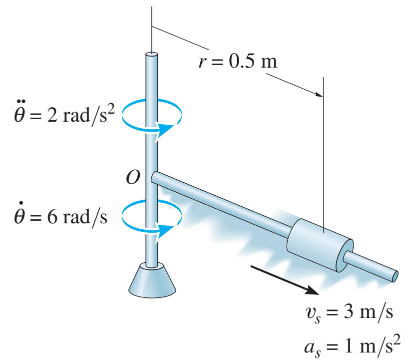
$a_{max} = 6(10) - 18 = 42 \text{ ft/s}^2$ **Ans.**

v_{max} occurs when $t = 10$ s

$v_{max} = 3(10)^2 - 18(10) + 15 = 135 \text{ ft/s}$ **Ans.**

R1–9.

The spool, which has a mass of 4 kg, slides along the rotating rod. At the instant shown, the angular rate of rotation of the rod is $\dot{\theta} = 6 \text{ rad/s}$ and this rotation is increasing at $\ddot{\theta} = 2 \text{ rad/s}^2$. At this same instant, the spool has a velocity of 3 m/s and an acceleration of 1 m/s^2, both measured relative to the rod and directed away from the center O when $r = 0.5$ m. Determine the radial frictional force and the normal force, both exerted by the rod on the spool at this instant.



SOLUTION

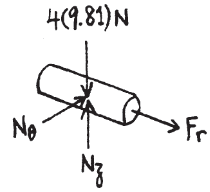
$r = 0.5 \text{ m}$

$\dot{r} = 3 \text{ m/s} \qquad \dot{\theta} = 6 \text{ rad/s}$

$\ddot{r} = 1 \text{ m/s}^2 \qquad \ddot{\theta} = 2 \text{ rad/s}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 1 - 0.5(6)^2 = -17$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(2) + 2(3)(6) = 37$



$\Sigma F_r = ma_r; \qquad F_r = 4(-17) = -68 \text{ N}$

$\Sigma F_\theta = ma_\theta; \qquad N_\theta = 4(37) = 148 \text{ N}$

$\Sigma F_z = ma_z; \qquad N_z - 4(9.81) = 0$

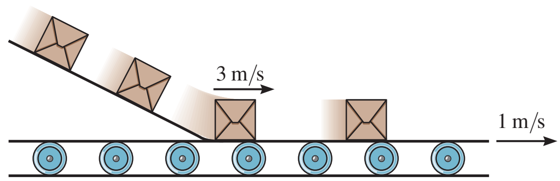
$N_z = 39.24 \text{ N}$

$F_r = -68 \text{ N}$ **Ans.**

$N = \sqrt{(148)^2 + (39.24)^2} = 153 \text{ N}$ **Ans.**

R1–10.

Packages having a mass of 6 kg slide down a smooth chute and land horizontally with a speed of 3 m/s on the surface of a conveyor belt. If the coefficient of kinetic friction between the belt and a package is $\mu_k = 0.2$, determine the time needed to bring the package to rest on the belt if the belt is moving in the same direction as the package with a speed $v = 1$ m/s.

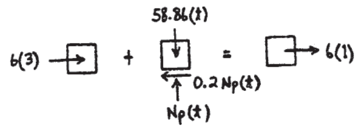


SOLUTION

$(+\uparrow) \qquad m(v_1)_y + \Sigma \int F_y \, dt = m(v_2)_y$

$$0 + N_p(t) - 58.86(t) = 0$$

$$N_p = 58.86 \text{ N}$$



$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma \int F_x \, dt = m(v_2)x$

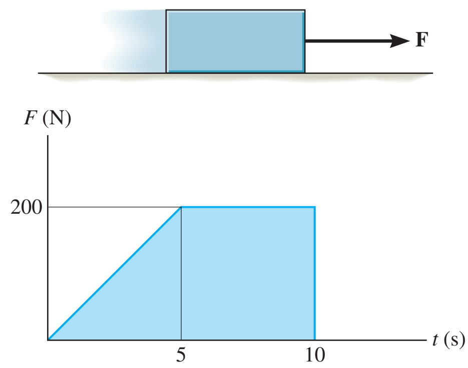
$$6(3) - 0.2(58.86)(t) = 6(1)$$

$$t = 1.02 \text{ s}$$

Ans.

R1–11.

A 20-kg block is originally at rest on a horizontal surface for which the coefficient of static friction is $\mu_s = 0.6$ and the coefficient of kinetic friction is $\mu_k = 0.5$. If a horizontal force F is applied such that it varies with time as shown, determine the speed of the block in 10 s. *Hint:* First determine the time needed to overcome friction and start the block moving.



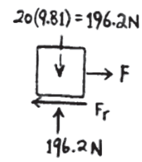
SOLUTION

The crate starts moving when

$$F = F_r = 0.6(196.2) = 117.72\ \text{N}$$

From the graph since

$$F = \frac{200}{5}t, \qquad 0 \le t \le 5\ \text{s}$$



The time needed for the crate to start moving is

$$t = \frac{5}{200}(117.72) = 2.943\ \text{s}$$

Hence, the impulse due to F is equal to the area under the curve from $2.943\ \text{s} \le t \le 10\ \text{s}$

$$\overset{+}{\rightarrow} \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + \int_{2.943}^{5} \frac{200}{5} t\, dt + \int_{5}^{10} 200\, dt - (0.5)196.2(10 - 2.943) = 20v_2$$

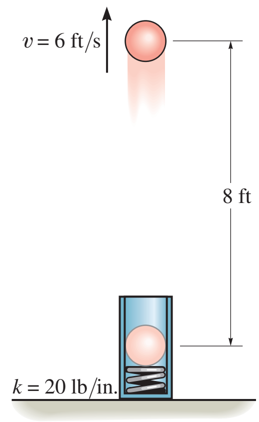
$$40\left(\frac{1}{2}t^2\right)\Big|_{2.943}^{5} + 200(10 - 5) - 692.292 = 20v_2$$

$$634.483 = 20v_2$$

$$v_2 = 31.7\ \text{m/s} \qquad \textbf{Ans.}$$

***R1–12.**

The 6-lb ball is fired from a tube by a spring having a stiffness $k = 20\ \text{lb/in.}$ Determine how far the spring must be compressed to fire the ball from the compressed position to a height of 8 ft, at which point it has a velocity of 6 ft/s.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(20)(12)(x^2) = \frac{1}{2}\left(\frac{6}{32.2}\right)(6)^2 + 8(6)$$

$$x = 0.654\ \text{ft} = 7.85\ \text{in.}$$ **Ans.**

R1–13.

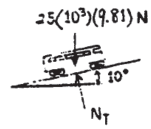
A train car, having a mass of 25 Mg, travels up a 10° incline with a constant speed of 80 km/h. Determine the power required to overcome the force of gravity.

SOLUTION

$v = 80 \text{ km/h} = 22.22 \text{ m/s}$

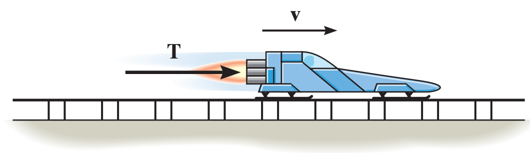
$P = \mathbf{F} \cdot \mathbf{v} = 25(10^3)(9.81)(22.22)(\sin 10°)$

$P = 946 \text{ kW}$ **Ans.**



R1–14.

The rocket sled has a mass of 4 Mg and travels from rest along the smooth horizontal track such that it maintains a constant power output of 450 kW. Neglect the loss of fuel mass and air resistance, and determine how far it must travel to reach a speed of $v = 60$ m/s.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = m\,a_x; \qquad F = m\,a = m\left(\frac{v\,dv}{ds}\right)$$

$$P = Fv = m\left(\frac{v^2\,dv}{ds}\right)$$

$$\int P\,ds = m\int v^2\,dv$$

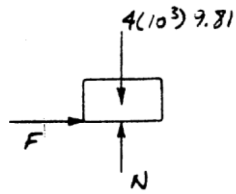
$$P\int_0^s ds = m\int_0^v v^2\,dv$$

$$Ps = \frac{m\,v3}{3}$$

$$s = \frac{m\,v^3}{3\,P}$$

$$s = \frac{4\,(10^3)(60)^3}{3(450)(10^3)} = 640 \text{ m}$$

Ans.



■**R1–15.**

A projectile, initially at the origin, moves vertically downward along a straight-line path through a fluid medium such that its velocity is defined as $v = 3(8e^{-t} + t)^{1/2}$ m/s, where t is in seconds. Plot the position s of the projectile during the first 2 s. Use the Runge-Kutta method to evaluate s with incremental values of $h = 0.25$ s.

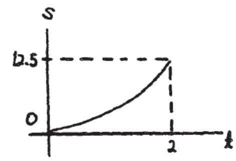
SOLUTION

$v = 3\,(8\,e^{-1} + t)^{1/2}$

$s_0 = 0$ at $t = 0$

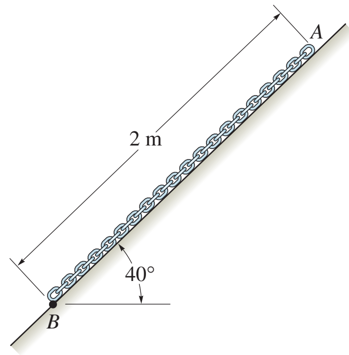
Using the Runge–Kutta method:

s	t
0	0
2.01	0.25
3.83	0.50
5.49	0.75
7.03	1.00
8.48	1.25
9.87	1.50
11.2	1.75
12.5	2.00



***R1–16.**

The chain has a mass of 3 kg/m. If the coefficient of kinetic friction between the chain and the plane is $\mu_k = 0.2$, determine the velocity at which the end A will pass point B when the chain is released from rest.



SOLUTION

$+\nwarrow\Sigma F_y = ma_y;\qquad -2(3)(9.81)\cos 40° + N_C = 0$

$N_C = 45.09\text{ N}$

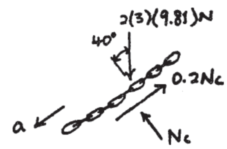
$+\swarrow\Sigma F_x = ma_x;\qquad 2(3)(9.81)\sin 40° - 0.2(45.09) = 2(3)\,a$

$a = 4.80\text{ m/s}^2$

$+\swarrow\quad v_2^2 = v_1^2 + 2as$

$v_2^2 = 0 + 2(4.80)(2)$

$v_2 = 4.38\text{ m/s}$ **Ans.**



Also,

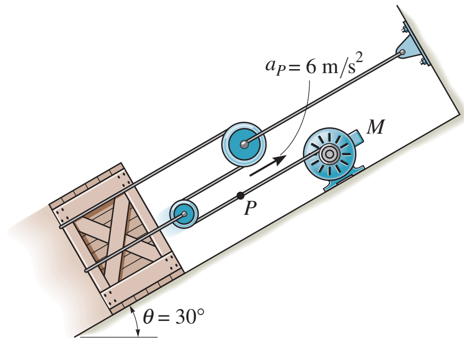
$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 2(3)(9.81)(2\sin 40°) - 0.2(45.09)(2) = \frac{1}{2}(2)(3)(v^2)$$

$v = 4.38\text{ m/s}$ **Ans.**

R1–17.

The motor M pulls in its attached rope with an acceleration $a_p = 6\ \text{m/s}^2$. Determine the towing force exerted by M on the rope in order to move the 50-kg crate up the inclined plane. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$. Neglect the mass of the pulleys and rope.

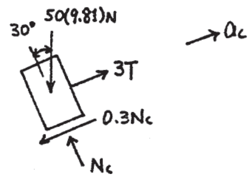


SOLUTION

$\nwarrow + \Sigma F_y = ma_y;\qquad N_C - 50(9.81)\cos 30° = 0$

$N_C = 424.79$

$\nearrow + \Sigma F_x = ma_x;\qquad 3T - 0.3(424.79) - 50(9.81)\sin 30° = 50a_C \qquad \textbf{(1)}$

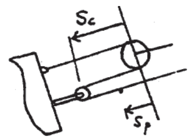


Kinematics, $2s_C + (s_C - s_p) = l$

Taking two time derivatives, yields

$$3a_C = a_p$$

Thus, $a_C = \dfrac{6}{3} = 2$

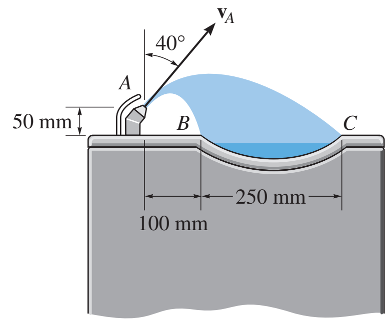


Substituting into Eq. (1) and solving,

$$T = 158\ \text{N} \qquad \textbf{Ans.}$$

R1–18.

The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and C.



SOLUTION

$$\left(\xrightarrow{+}\right) \quad s_x = v_x t$$

$$R = v_A \sin 40° t \qquad t = \frac{R}{v_A \sin 40°} \tag{1}$$

$$(+\uparrow) \quad s_y = (s_y)_0 + v_y t + \tfrac{1}{2} a_c t^2$$

$$-0.05 = 0 + v_A \cos 40° t + \tfrac{1}{2}(-9.81)t^2 \tag{2}$$

Substituting Eq. (1) into (2) yields:

$$-0.05 = v_A \cos 40°\left(\frac{R}{v_A \sin 40°}\right) + \frac{1}{2}(-9.81)\left(\frac{R}{v_A \sin 40°}\right)^2$$

$$v_A = \sqrt{\frac{4.905 \sin 40° R^2}{\sin^2 40°(R \cos 40° + 0.05 \sin 40°)}}$$

At point B, $R = 0.1$ m.

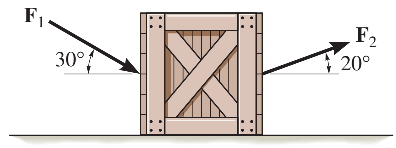
$$v_A = \sqrt{\frac{4.905 \sin 40°(0.1)^2}{\sin^2 40°(0.1 \cos 40° + 0.05 \sin 40°)}} = 0.838 \text{ m/s}$$ **Ans.**

At point C, $R = 0.35$ m.

$$v_A = \sqrt{\frac{4.905 \sin 40°(0.35)^2}{\sin^2 40°(0.35 \cos 40° + 0.05 \sin 40°)}} = 1.76 \text{ m/s}$$ **Ans.**

R1–19.

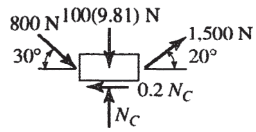
The 100-kg crate is subjected to the action of two forces, $F_1 = 800$ N and $F_2 = 1.5$ kN, as shown. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



SOLUTION

$$+\uparrow \sum F_y = 0; \qquad N_C - 800 \sin 30° - 100(9.81) + 1500 \sin 20° = 0$$

$$N_C = 867.97 \text{ N}$$



$$T_1 + \sum U_{1-2} = T_2$$

$$0 + 800 \cos 30°(s) - 0.2(867.97)(s) + 1500 \cos 20°(s) = \frac{1}{2}(100)(6)^2$$

$$s(1928.7) = 1800$$

$$s = 0.933 \text{ m} \qquad \textbf{Ans.}$$

***R1–20.**

If a particle has an initial velocity $v_0 = 12$ ft/s to the right, and a constant acceleration of 2 ft/s^2 to the left, determine the particle's displacement in 10 s. Originally $s_0 = 0$.

SOLUTION

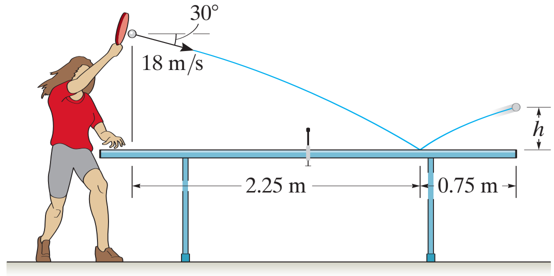
$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 12(10) + \frac{1}{2}(-2)(10)^2$$

$$s = 20.0 \text{ ft}$$ **Ans.**

R1–21.

The ping-pong ball has a mass of 2 g. If it is struck with the velocity shown, determine how high h it rises above the end of the smooth table after the rebound. Take $e = 0.8$.



SOLUTION

$(\xrightarrow{+})\qquad s = s_0 + v_0 t$

$$2.25 = 0 + 18\cos 30° t$$

$$t = 0.14434 \text{ s}$$

$(v_x)_1 = (v_x)_2 = 18 \cos 30° = 15.5885 \text{ m/s}$

$(+\downarrow)\quad v = v_0 + a_c t$

$(v_y)_1 = 18 \sin 30° + 9.81(0.14434)$

$(v_y)_1 = 10.4160 \text{ m/s}$

$(+\uparrow)\quad e = 0.8 = \dfrac{(v_y)_2}{10.4160}$

$(v_y)_2 = 8.3328 \text{ m/s}$

$(\xrightarrow{+})\qquad s = s_0 + v_0 t$

$0.75 = 0 + 15.5885t$

$t = 0.048112 \text{ s}$

$(+\uparrow)\quad s = s_0 + v_0 t + \dfrac{1}{2} a_c t^2$

$h = 0 + 8.3328(0.048112) - \dfrac{1}{2}(9.81)(0.048112)^2$

$h = 0.390 \text{ m}$ **Ans.**

R1–22.

A sports car can accelerate at $6\ \text{m/s}^2$ and decelerate at $8\ \text{m/s}^2$. If the maximum speed it can attain is 60 m/s, determine the shortest time it takes to travel 900 m starting from rest and then stopping when $s = 900$ m.

SOLUTION

Time to accelerate to 60 m/s,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$60 = 0 + 6t$$

$$t = 10\ \text{s}$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(6)(10^2)$$

$$s = 300\ \text{m}$$

Time to decelerate to a stop,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$0 = 60 - 8t$$

$$t = 7.5\ \text{s}$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 60(7.5) - \frac{1}{2}(8)(7.5^2)$$

$$s = 225\ \text{m}$$

Time to travel at 60 m/s,

$$900 - 300 - 225 = 375\ \text{m}$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$$

$$375 = 0 + 60t$$

$$t = 6.25\ \text{s}$$

Total time $t = 10 + 7.5 + 6.25 = 23.8\ \text{s}$ **Ans.**

R1–23.

A 2-kg particle rests on a smooth horizontal plane and is acted upon by forces $F_x = 0$ and $F_y = 3\text{ N}$. If $x = 0$, $y = 0$, $v_x = 6\text{ m/s}$, and $v_y = 2\text{ m/s}$ when $t = 0$, determine the equation $y = f(x)$ which describes the path.

SOLUTION

$$+\uparrow \Sigma F_y = ma_y; \qquad 3 = 2a_y \qquad a_y = 1.5\text{ m/s}^2 \qquad \textbf{(1)}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0 = 2a_x \qquad a_x = 0 \qquad \textbf{(2)}$$

$$a_y = \frac{dv_y}{dt} = 1.5$$

$$\int_2^{v_y} dv_y = 1.5\int_0^t dt$$

$$v_y = \frac{dy}{dt} = 1.5t + 2$$

$$\int_0^y dy = \int_0^t (1.5t + 2)\,dt$$

$$y = 0.75t^2 + 2t \qquad \textbf{(3)}$$

$$a_x = \frac{dv_x}{dt} = 0$$

$$\int_6^{v_x} dv_x = \int_0^t 0dt$$

$$v_x = \frac{dx}{dt} = 6$$

$$\int_0^x dx = \int_0^t 6dt$$

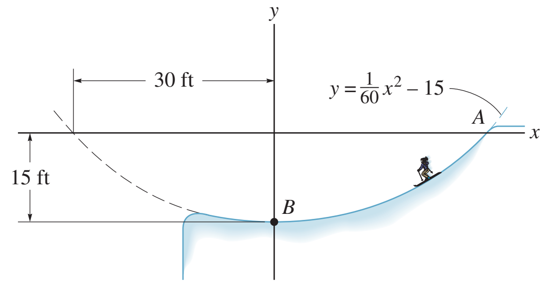
$$x = 6t \qquad \textbf{(4)}$$

Eliminating t from Eq. (3) and (4) yields:

$y = 0.0208x^2 + 0.333x$ (**Parabola**) **Ans.**

***R1–24.**

A skier starts from rest at A (30 ft, 0) and descends the smooth slope, which may be approximated by a parabola. If she has a weight of 120 lb, determine the normal force she exerts on the ground at the instant she arrives at point B.



SOLUTION

$$T_A + V_A = T_B + V_B$$

$$0 + (120)(15) = \frac{1}{2}\left(\frac{120}{32.2}\right)v_B^2 + 0$$

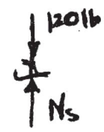
$$v_B = 31.08 \text{ ft/s}$$

$$y = \frac{1}{60}x^2 - 15$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \left.\frac{1}{30}x\right|_{x=0} = 0$$

$$\frac{d^2y}{dx^2} = \frac{1}{30}$$

$$\rho = \left|\frac{[1(\frac{dy}{dx})^2]^{3/2}}{\frac{d^2y}{dx^2}}\right| = \frac{(1+0)^{3/2}}{\frac{1}{30}} = 30 \text{ ft}$$

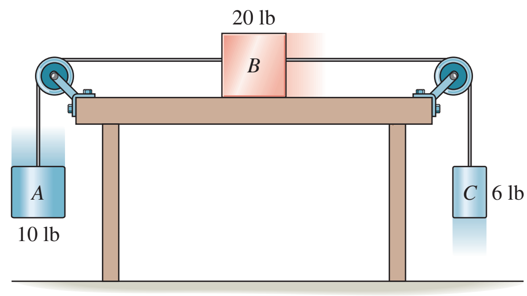


$$+\uparrow\Sigma F_n = ma_n; \qquad N_s - 120 = \frac{120}{32.2}\left[\frac{(31.08)^2}{30}\right]$$

$$N_s = 240 \text{ lb}$$ **Ans.**

R1–25.

The 20-lb block B rests on the surface of a table for which the coefficient of kinetic friction is $\mu_k = 0.1$. Determine the speed of the 10-lb block A after it has moved downward 2 ft from rest. Neglect the mass of the pulleys and cords.

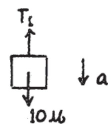


SOLUTION

Block A:

$$+\downarrow \Sigma F_y = ma_y; \qquad -T_1 + 10 = \frac{10}{32.2}a \qquad \textbf{(1)}$$

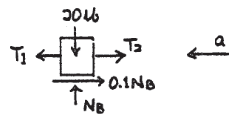
Block B:



$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad -T_2 + T_1 - 0.1N_B = \frac{20}{32.2}a \qquad \textbf{(2)}$$

$$+\uparrow \Sigma F_y = ma_y; \qquad N_B - 20 = 0 \qquad \textbf{(3)}$$

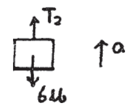
Block C:



$$+\uparrow \Sigma F_y = ma_y; \qquad T_2 - 6 = \frac{6}{32.2}a \qquad \textbf{(4)}$$

Solving Eqs. (1)–(4) for a,

$a = 1.79 \text{ ft/s}^2$



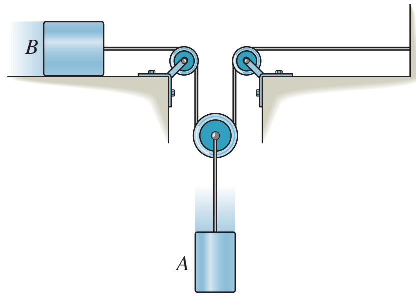
$(+\downarrow) v^2 = v_0^2 + 2a_c(s - s_0)$

$v^2 = 0 + 2(1.79)(2 - 0)$

$v = 2.68 \text{ ft/s}$ **Ans.**

R1–26.

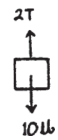
At a given instant the 10-lb block A is moving downward with a speed of 6 ft/s. Determine its speed 2 s later. Block B has a weight of 4 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of the pulleys and cord.



SOLUTION

Block A:

$$+\downarrow \Sigma F_y = ma_y; \qquad 10 - 2T = \frac{10}{32.2} a_A$$



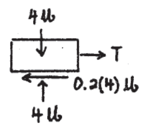
Block B:

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad -T + 0.2(4) = \frac{4}{32.2} a_B$$

Kinematics:

$$2s_A + s_B = l$$

$$2a_A = -a_B$$

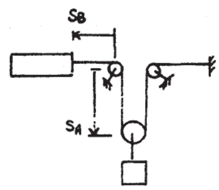


Solving,

$$T = 3.38 \text{ lb}$$

$$a_A = 10.40 \text{ ft/s}^2$$

$$a_B = -20.81 \text{ ft/s}^2$$



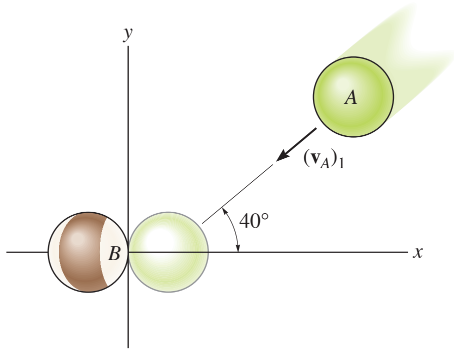
$(+\downarrow)$ $\qquad v_A = (v_A)_0 + a_A t$

$v_A = 6 + 10.40(2)$

$v_A = 26.8 \text{ ft/s} \downarrow$ **Ans.**

R1–27.

Two smooth billiard balls A and B have an equal mass of $m = 200$ g. If A strikes B with a velocity of $(v_A)_1 = 2$ m/s as shown, determine their final velocities just after collision. Ball B is originally at rest and the coefficient of restitution is $e = 0.75$.



SOLUTION

$(v_A)_{x_1} = -2 \cos 40° = -1.532$ m/s

$(v_A)_{y_1} = -2 \sin 40° = -1.285$ m/s

$(\overset{+}{\rightarrow})$ $\quad m_A(v_A)_{x_1} + m_B(v_B)_{x_1} = m_A(v_A)_{x_2} + m_B(v_B)_{x_2}$

$$-2(1.532) + 0 = 0.2(v_A)_{x_2} + 0.2(v_B)_{x_2} \tag{1}$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_{rel})_2}{(v_{rel})_1}$

$$0.75 = \frac{(v_A)_{x_2} - (v_B)_{x_2}}{1.532} \tag{2}$$

Solving Eqs. (1) and (2)

$(v_A)_{x_2} = -0.1915$ m/s

$(v_B)_{x_2} = -1.3405$ m/s

For A:

$(+\downarrow)$ $\quad m_A(v_A)_{y_1} = m_A(v_A)_{y_2}$

$(v_A)_{y_2} = 1.285$ m/s

For B:

$(+\uparrow)$ $\quad m_B(v_B)_{y_1} = m_B(v_B)_{y_2}$

$(v_B)_{y_2} = 0$

Hence $(v_B)_2 = (v_B)_{x_2} = 1.34 \text{ m/s} \leftarrow$ **Ans.**

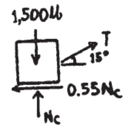
$(v_A)_2 = \sqrt{(-0.1915)^2 + (1.285)^2} = 1.30$ m/s **Ans.**

$(\theta_A)_2 = \tan^{-1}\left(\dfrac{0.1915}{1.285}\right) = 8.47°$ $\measuredangle$ **Ans.**

***R1–28.**

A crate has a weight of 1500 lb. If it is pulled along the ground at a constant speed for a distance of 20 ft, and the towing cable makes an angle of 15° with the horizontal, determine the tension in the cable and the work done by the towing force. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.55$.

SOLUTION



$+\uparrow\Sigma F_y = 0;\quad N_C - 1500 + T\sin 15^\circ = 0$

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad T\cos 15^\circ - 0.55N_C = 0$

$T = 744.4 \text{ lb} = 744 \text{ lb}$ **Ans.**

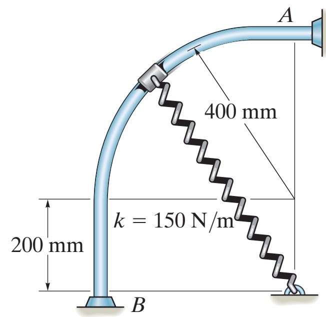
$N_C = 1307.3 \text{ lb}$

$U_{1-2} = (744.4\cos 15^\circ)(20) = 14\,380.7 \text{ ft}\cdot\text{lb}$

$U_{1-2} = 14.4 \text{ ft}\cdot\text{kip}$ **Ans.**

R1–29.

The collar of negligible size has a mass of 0.25 kg and is attached to a spring having an unstretched length of 100 mm. If the collar is released from rest at A and travels along the smooth guide, determine its speed just before it strikes B.

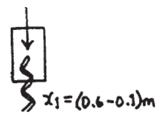


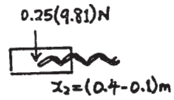
SOLUTION

$$T_A + V_A = T_B + V_B$$

$$0 + (0.25)(9.81)(0.6) + \frac{1}{2}(150)(0.6 - 0.1)^2 = \frac{1}{2}(0.25)(v_B)^2 + \frac{1}{2}(150)(0.4 - 0.1)^2$$

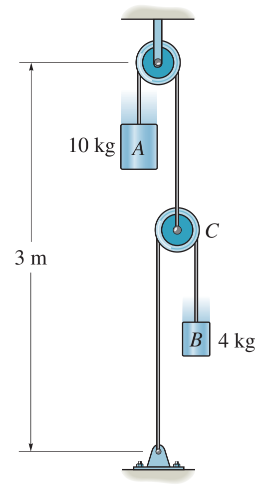
$v_B = 10.4$ m/s **Ans.**





R1–30.

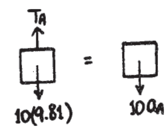
Determine the tension developed in the two cords and the acceleration of each block. Neglect the mass of the pulleys and cords. *Hint:* Since the system consists of *two* cords, relate the motion of block A to C, and of block B to C. Then, by elimination, relate the motion of A to B.



SOLUTION

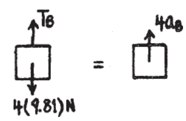
Block A:

$+\downarrow \Sigma F_y = ma_y; \qquad 10(9.81) - T_A = 10a_A$ **(1)**



Block B:

$+\uparrow \Sigma F_y = ma_y; \qquad T_B - 4(9.81) = 4a_B$ **(2)**

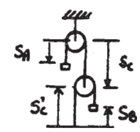


Pulley C:

$+\uparrow \Sigma F_y = 0; \qquad T_A - 2T_B = 0$ **(3)**

Kinematics:

$$s_A + s_C = l$$



Taking the two time derivatives:

$$a_A = -a_C$$

Also,

$$s_C{}' + (s_C{}' - s_B) = l'$$

So that $2a_C{}' = a_B$

Since $a_C{}' = -a_C$

$$a_B = 2\,a_A \qquad \textbf{(4)}$$

Solving Eqs.(1)–(4),

$a_A = 0.755 \text{ m/s}^2$ **Ans.**

$a_B = 1.51 \text{ m/s}^2$ **Ans.**

$T_A = 90.6 \text{ N}$ **Ans.**

$T_B = 45.3 \text{ N}$ **Ans.**

R1–31.

The baggage truck A has a mass of 800 kg and is used to pull each of the 300-kg cars. Determine the tension in the couplings at B and C if the tractive force $\mathbf{F}$ on the truck is $F = 480$ N. What is the speed of the truck when $t = 2$ s, starting from the rest? The car wheels are free to roll. Neglect the mass of the wheels.

SOLUTION

$\xrightarrow{+} \Sigma F_x = m\,a_x; \qquad 480 = [800 + 2(300)]a$

$a = 0.3429 \text{ m/s}^2$

$(\xrightarrow{+}) \qquad v = v_0 + a_c t$

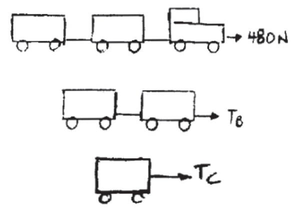
$v = 0 + 0.3429(2) = 0.686 \text{ m/s}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = m\,a_x; \qquad T_B = 2(300)(0.3429)$

$T_B = 205.71 = 206 \text{ N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = m\,a_x; \qquad T_C = (300)(0.3429)$

$T_C = 102.86 = 103 \text{ N}$ **Ans.**



***R1–32.**

The baggage truck A has a mass of 800 kg and is used to pull each of the 300-kg cars. If the tractive force $\mathbf{F}$ on the truck is $F = 480$ N, determine the initial acceleration of the truck. What is the acceleration of the truck if the coupling at C suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.

SOLUTION

$\xrightarrow{+} \Sigma F_x = m\,a_x;$ $\quad 480 = [800 + 2(300)]a$

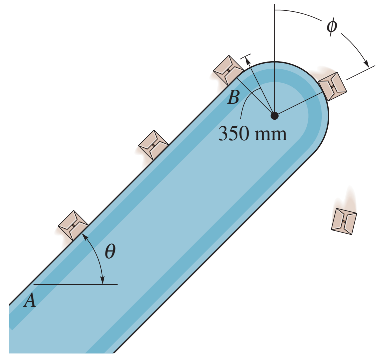
$a = 0.3429 = 0.343 \text{ m/s}^2$ **Ans.**

$\xrightarrow{+} \Sigma F_x = m\,a_x;$ $\quad 480 = (800 + 300)a$

$a = 0.436 \text{ m/s}^2$ **Ans.**

■**R1–33.**

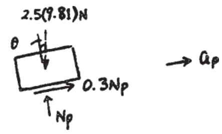
Packages having a mass of 2.5 kg ride on the surface of the conveyor belt. If the belt starts from rest and with constant acceleration increases to a speed of 0.75 m/s in 2 s, determine the maximum angle of tilt, θ, so that none of the packages slip on the inclined surface AB of the belt. The coefficient of static friction between the belt and each package is $\mu_s = 0.3$. At what angle ϕ do the packages first begin to slip off the surface of the belt if the belt is moving at a constant speed of 0.75 m/s?



SOLUTION

$+\Sigma F_y = ma_y;\qquad N_P - 2.5(9.81)\cos\theta = 0\qquad$ **(1)**

$+\Sigma F_x = ma_x;\qquad 0.3N_P - 2.5(9.81)\sin\theta = 2.5a_P\qquad$ **(2)**



Kinematics:

$$v = v_0 + a_c t$$

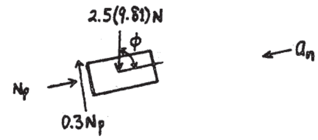
$$0.75 = 0 + a_P(2)$$

$$a_P = 0.375\ \text{m/s}^2$$

Combining Eqs. (1) and (2), using a_p,

$$0.3\cos\theta - \sin\theta = 0.0382$$

$$\theta = 14.6^\circ \qquad \textbf{Ans.}$$



$\nwarrow + \Sigma F_n = ma_n;\qquad 2.5(9.81)\cos\phi - N_P = 2.5\left[\dfrac{(0.75)^2}{0.350}\right]$

$\nearrow + \Sigma F_t = ma_t;\qquad 2.5(9.81)\sin\phi - 0.3N_P = 0$

Combining these equations,

$$\cos\phi - 3.33\sin\phi = 0.164$$

$$\phi = 14.0^\circ \qquad \textbf{Ans.}$$

R1–34.

A particle travels in a straight line such that for a short time $2\text{ s} \leq t \leq 6\text{ s}$ its motion is described by $v = (4/a)$ ft/s where a is in ft/s^2. If $v = 6$ ft/s when $t = 2$ s, determine the particle's acceleration when $t = 3$ s.

SOLUTION

$$a = \frac{dv}{dt} = \frac{4}{v}$$

$$\int_6^v v\,dv = \int_2^t 4\,dt$$

$$\frac{1}{2}v^2 - 18 = 4t - 8$$

$$v^2 = 8t + 20$$

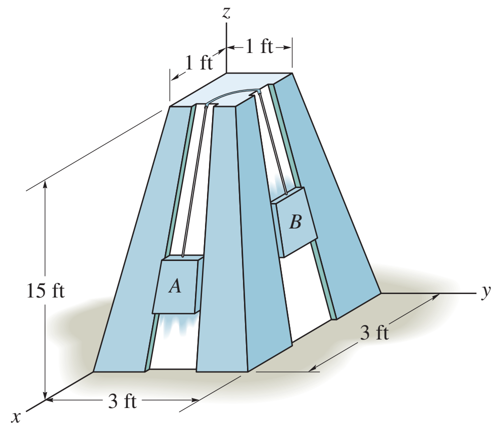
At $t = 3$ s, choosing the positive root

$$v = 6.63\text{ ft/s}$$

$$a = \frac{4}{6.63} = 0.603\text{ ft/s}^2 \qquad \textbf{Ans.}$$

R1–35.

The blocks A and B weigh 10 and 30 lb, respectively. They are connected together by a light cord and ride in the frictionless grooves. Determine the speed of each block after block A moves 6 ft up along the plane. The blocks are released from rest.



SOLUTION

$$\frac{6}{z} = \frac{\sqrt{15^2 + 2^2}}{15}$$

$$z = 5.95 \text{ ft}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{10}{32.2}\right)v_2^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)v_2^2 + 10(5.95) - 30(5.95)$$

$$v_2 = 13.8 \text{ ft/s}$$ **Ans.**

***R1–36.**

A motorcycle starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 50 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, a car located 6000 ft down the road is traveling toward the motocycle at a constant speed of 30 ft/s. Determine the time and the distance traveled by the motorcycle when they pass each other.

SOLUTION

Motorcycle:

$$\left(\xrightarrow{+}\right) \qquad v = v_0 + a_c t'$$

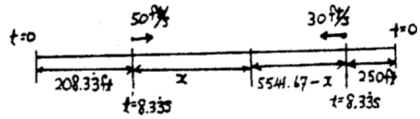
$$50 = 0 + 6t$$

$$t' = 8.33 \text{ s}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$(50)^2 = 0 + 2(6)(s' - 0)$$

$$s' = 208.33 \text{ ft}$$



In $t' = 8.33$ s car travels

$$s'' = v_0 t' = 30(8.33) = 250 \text{ ft}$$

Distance between motorcycle and car:

$$s = v_0 t; \qquad 6000 - 250 - 208.33 = 5541.67 \text{ ft}$$

When passing occurs for motorcycle:

$$s = v_0 t; \qquad x = 50(t'')$$

For car:

$$s = v_0 t; \qquad 5541.67 - x = 30(t'')$$

Solving,

$$x = 3463.54 \text{ ft}$$

$$t' = 69.27 \text{ s}$$

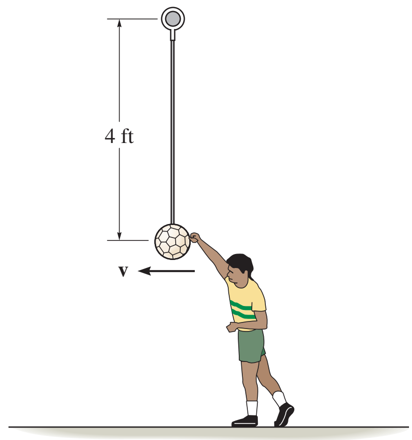
Thus, for the motorcycle,

$$t = 69.27 + 8.33 = 77.6 \text{ s} \qquad \textbf{Ans.}$$

$$s_m = 208.33 + 3463.54 = 3.67(10)^3 \text{ ft} \qquad \textbf{Ans.}$$

R1–37.

The 5-lb ball, attached to the cord, is struck by the boy. Determine the smallest speed he must impart to the ball so that it will swing around in a vertical circle, without causing the cord to become slack.



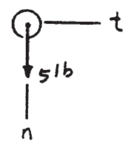
SOLUTION

$$+\downarrow \Sigma F_n = ma_n; \qquad 5 = \frac{5}{32.2}\left(\frac{v_2^2}{4}\right) \qquad v_2^2 = 128.8 \text{ ft}^2/\text{s}^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{5}{32.2}\right)v^2 + 0 = \frac{1}{2}\left(\frac{5}{32.2}\right)(128.8) + 5(8)$$

$$v = 25.4 \text{ ft/s}$$ **Ans.**



R1–38.

A projectile, initially at the origin, moves along a straight-line path through a fluid medium such that its velocity is $v = 1800(1 - e^{-0.3t})$ mm/s, where t is in seconds. Determine the displacement of the projectile during the first 3 s.

SOLUTION

$$v = \frac{ds}{dt} = 1800(1 - e^{-0.3t})$$

$$\int_0^s ds = \int_0^t 1800(1 - e^{-0.3t})\,dt$$

$$s = 1800\left(t + \frac{1}{0.3}e^{-0.3t}\right) - 6000$$

Thus, in $t = 3$ s

$$s = 1800\left(3 + \frac{1}{0.3}e^{-0.3(3)}\right) - 6000$$

$$s = 1839.4 \text{ mm} = 1.84 \text{ m}$$ **Ans.**

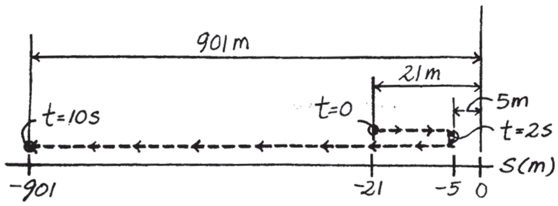
R1–39.

A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

SOLUTION

$v = 12 - 3t^2$ **(1)**

$a = \dfrac{dv}{dt} = -6t\Big|_{t=4} = -24\ \text{m/s}^2$ **Ans.**



$\displaystyle\int_{-10}^{s} ds = \int_{1}^{t} v\,dt = \int_{1}^{t} \left(12 - 3t^2\right)dt$

$s + 10 = 12t - t^3 - 11$

$s = 12t - t^3 - 21$

$s|_{t=0} = -21$

$s|_{t=10} = -901$

$\Delta s = -901 - (-21) = -880$ m **Ans.**

From Eq. (1):

$v = 0$ when $t = 2s$

$s|_{t=2} = 12(2) - (2)^3 - 21 = -5$

$s_T = (21 - 5) + (901 - 5) = 912$ m **Ans.**

***R1–40.**

A particle is moving along a circular path of 2-m radius such that its position as a function of time is given by $\theta = (5t^2)$ rad, where t is in seconds. Determine the magnitude of the particle's acceleration when $\theta = 30°$. The particle starts from rest when $\theta = 0°$.

SOLUTION

$r = 2\text{ m} \qquad \theta = 5t^2$

$\dot{r} = 0 \qquad \dot{\theta} = 10t$

$\ddot{r} = 0 \qquad \ddot{\theta} = 10$

$$\mathrm{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta$$

$$= [0 - 2(10t)^2]\mathbf{u}_r + [2(10) + 0]\mathbf{u}_\theta$$

$$= \{-200t^2\mathbf{u}_r + 20\mathbf{u}_\theta\}\text{ m/s}^2$$

When $\theta = 30° = 30\left(\dfrac{\pi}{180}\right) = 0.524$ rad

Then,

$$0.524 = 5t^2$$

$$t = 0.324\text{ s}$$

Hence,

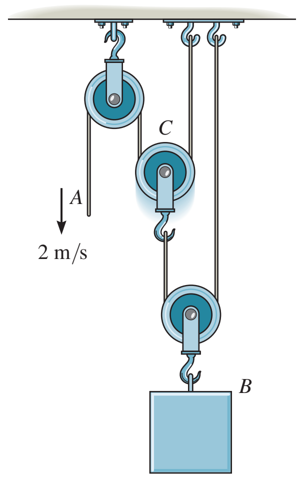
$$\mathrm{a} = [-200(0.324)^2]\mathbf{u}_r + 20\mathbf{u}_\theta$$

$$= \{-20.9\mathbf{u}_r + 20\mathbf{u}_\theta\}\text{ m/s}^2$$

$$a = \sqrt{(-20.9)^2 + (20)^2} = 29.0\text{ m/s}^2$$ **Ans.**

R1–41.

If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



SOLUTION

Two cords:

$$s_A + 2s_C = l$$

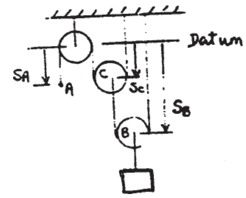
$$s_B + (s_B - s_C) = l'$$

Thus, $v_A = -2v_C$

$2v_B = v_C$

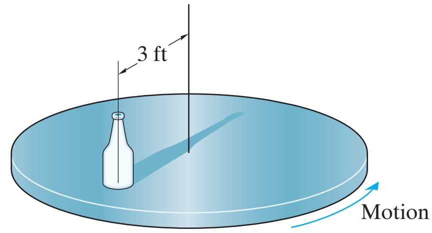
$4v_B = -v_A$

$$v_B = \frac{-2}{4} = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow$$ **Ans.**



R1–42.

The bottle rests at a distance of 3 ft from the center of the horizontal platform. If the coefficient of static friction between the bottle and the platform is $\mu_s = 0.3$, determine the maximum speed that the bottle can attain before slipping. Assume the angular motion of the platform is slowly increasing.

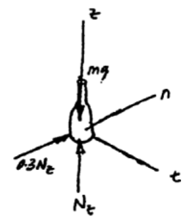


SOLUTION

$$\Sigma F_z = ma_z; \qquad N_z - mg = 0 \qquad N_z = mg$$

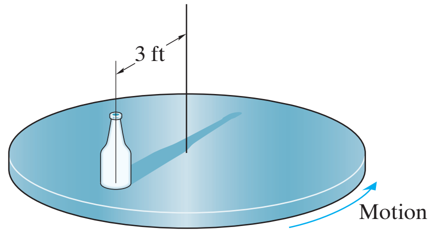
$$\Sigma F_x = ma_n; \qquad 0.3(mg) = m\left(\frac{v^2}{r}\right)$$

$$v = \sqrt{0.3gr} = \sqrt{0.3(32.2)(3)} = 5.38 \text{ ft/s} \qquad \textbf{Ans.}$$

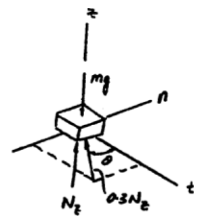


R1–43.

Work Prob. R1–42 assuming that the platform starts rotating from rest so that the speed of the bottle is increased at 2 ft/s^2.



SOLUTION



$\Sigma F_z = ma_z;$ $\qquad N_z - mg = 0 \qquad N_z = mg$

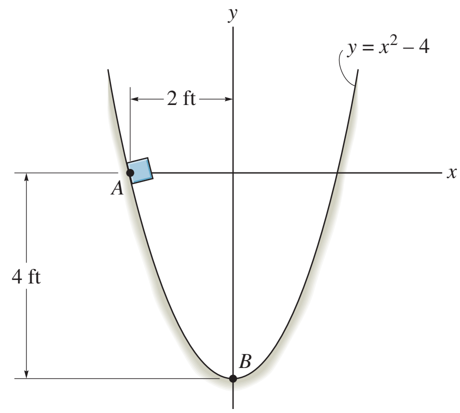
$\Sigma F_t = ma_t;$ $\qquad -0.3(mg)\cos\theta = -m(2) \qquad \theta = 78.05^\circ$

$\Sigma F_n = ma_n;$ $\qquad 0.3(mg)\sin 78.05^\circ = -m\left(\dfrac{v^2}{3}\right)$

$v = 5.32 \text{ ft/s}$ **Ans.**

***R1–44.**

A 3-lb block, initially at rest at point A, slides along the smooth parabolic surface. Determine the normal force acting on the block when it reaches B. Neglect the size of the block.



SOLUTION

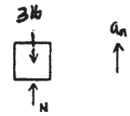
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 3(4) = \frac{1}{2}\left(\frac{3}{32.2}\right)v_2^2 + 0$$

$$v_2 = 16.05 \text{ ft/s}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{[1 + (2x)^2]^{3/2}}{2}$$

At $x = 0$, $\rho = 0.5$ ft



$$+\uparrow \Sigma F_n = ma_n; \qquad N - 3 = \frac{3}{32.2}\left[\frac{(16.05)^2}{0.5}\right]$$

$N = 51.0$ lb **Ans.**

R1–45.

A car starts from rest and moves along a straight line with an acceleration of $a = (3s^{-1/3})\ \mathrm{m/s^2}$, where s is in meters. Determine the car's acceleration when $t = 4$ s.

SOLUTION

$$a = 3s^{-\frac{1}{3}}$$

$$a\,ds = \nu\,d\nu$$

$$\int_0^s 3s^{-\frac{1}{3}}\,ds = \int_0^v \nu\,d\nu$$

$$\frac{3}{2}(3)s^{\frac{2}{3}} = \frac{1}{2}\nu^2$$

$$\nu = 3s^{\frac{1}{3}}$$

$$\frac{ds}{dt} = 3s^{\frac{1}{3}}$$

$$\int_0^s s^{-\frac{1}{3}}\,ds = \int_0^t 3\,dt$$

$$\frac{3}{2}s^{\frac{2}{3}} = 3t$$

$$s = (2t)^{\frac{3}{2}}$$

$$s\big|_{t=4} = (2(4))^{\frac{3}{2}} = 22.62 = 22.6\ \mathrm{m}$$

$$a\big|_{t=4} = 3(22.62)^{-\frac{1}{3}} = 1.06\ \mathrm{m/s^2}$$ **Ans.**

R1–46.

A particle travels along a curve defined by the equation $s = (t^3 - 3t^2 + 2t)$ m. where t is in seconds. Draw the $s-t$, $v-t$, and $a-t$ graphs for the particle for $0 \le t \le 3$ s.

SOLUTION

$$s = t^3 - 3t^2 + 2t$$

$$v = \frac{ds}{dt} = 3t^2 - 6t + 2 \qquad \textbf{Ans.}$$

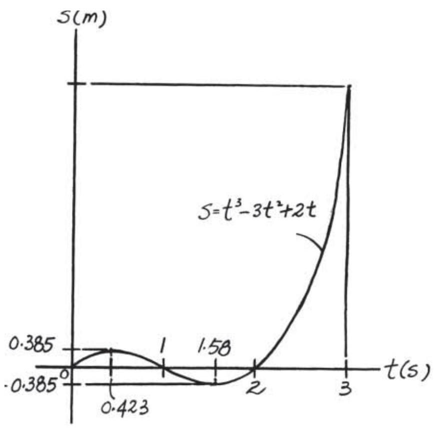
$$a = \frac{dv}{dt} = 6t - 6 \qquad \textbf{Ans.}$$

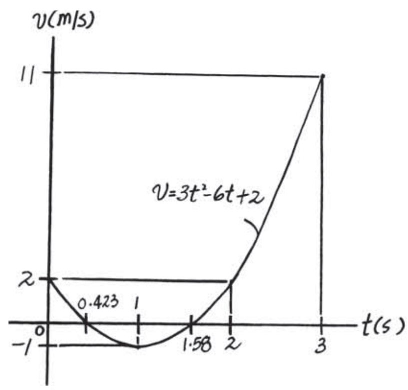
$v = 0$ at $0 = 3t^2 - 6t + 2$

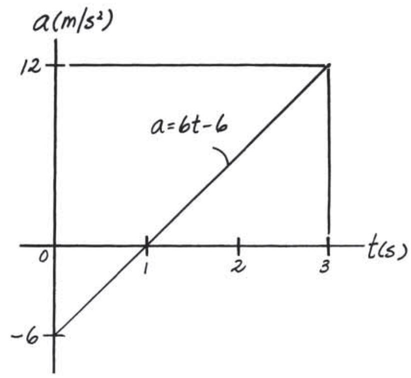
$t = 1.577$ s, and $t = 0.4226$ s,

$s|_{t=1.577} = -0.386$ m

$s|_{t=0.4226} = 0.385$ m

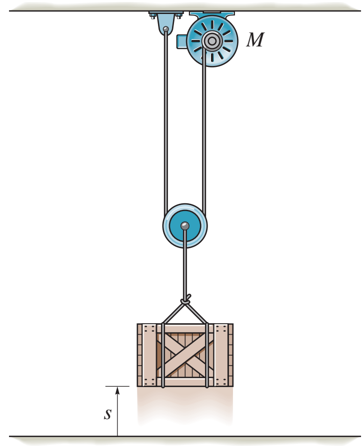






R1–47.

The crate, having a weight of 50 lb, is hoisted by the pulley system and motor M. If the crate starts from rest and, by constant acceleration, attains a speed of 12 ft/s after rising 10 ft, determine the power that must be supplied to the motor at the instant $s = 10$ ft. The motor has an efficiency $\epsilon = 0.74$.



SOLUTION

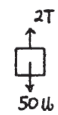
$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$$(12)^2 = 0 + 2a_c(10 - 0)$$

$$a_c = 7.20 \text{ ft/s}^2$$

$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 50 = \dfrac{50}{32.2}(7.20)$

$$T = 30.6 \text{ lb}$$



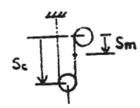
$s_C + (s_C - s_M) = l$

$v_M = 2v_C$

$v_M = 2(12) = 24$ ft/s

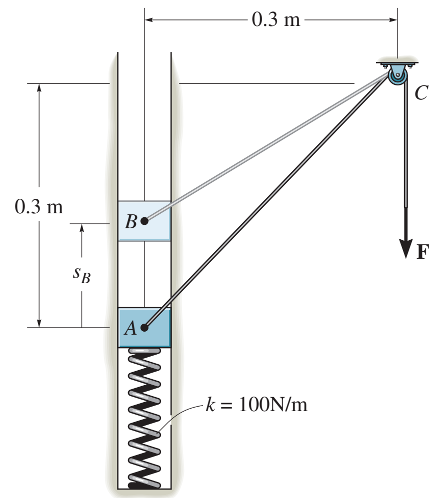
$P_0 = \mathbf{T} \cdot \mathbf{v} = 30.6(24) = 734.2 \text{ lb} \cdot \text{ft/s}$

$P_i = \dfrac{734.2}{0.74} = 992.1 \text{ lb} \cdot \text{ft/s} = 1.80 \text{ hp}$ **Ans.**



***R1–48.**

The block has a mass of 0.5 kg and moves within the smooth vertical slot. If the block starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5\ \text{m/s}$ when it reaches B; $s_B = 0.15$ m. Neglect the mass of the cord and pulley.



SOLUTION

The work done by F depends upon the difference in the cord length AC–BC.

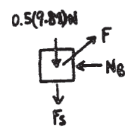
$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + F[\sqrt{(0.3)^2 + (0.3)^2} - \sqrt{(0.3)^2 + (0.3 - 0.15)^2}] - 0.5(9.81)(0.15)$$

$$-\frac{1}{2}(100)(0.15)^2 = \frac{1}{2}(0.5)(2.5)^2$$

$$F(0.0889) = 3.423$$

$$F = 38.5\ \text{N} \qquad \textbf{Ans.}$$



R1–49.

A ball having a mass of 200 g is released from rest at a height of 400 mm above a very large fixed metal surface. If the ball rebounds to a height of 325 mm above the surface, determine the coefficient of restitution between the ball and the surface.

SOLUTION

Just before impact

$$T_1 + T_2 = T_2 + V_2$$

$$0 + 0.2(9.81)(0.4) = \frac{1}{2}(0.2)(v_2)^2 + 0$$

$$v_2 = 2.80 \text{ m/s}$$

Just after impact

$$T_3 + V_3 = T_4 + V_4$$

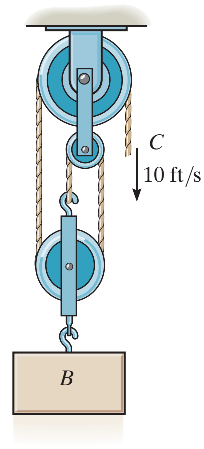
$$\frac{1}{2}(0.2)(v_3)^2 + 0 = 0 + 0.2(9.81)(0.325)$$

$$v_3 = 2.53 \text{ m/s}$$

$$e = \frac{(v_{rel})_2}{(v_{rel})_1} = \frac{2.53}{2.80} = 0.901 \qquad \textbf{Ans.}$$

R1–50.

Determine the speed of block B if the end of the cable at C is pulled downward with a speed of 10 ft/s. What is the relative velocity of the block with respect to C?



SOLUTION

$3s_B + s_C = l$

$3v_B = -v_C$

$3v_B = -(10)$

$v_B = -3.33 \text{ ft/s} = 3.33 \text{ ft/s} \uparrow$ **Ans.**

$(+\downarrow) \quad v_B = v_C + v_{B/C}$

$-3.33 = 10 + v_{B/C}$

$v_{B/C} = -13.3 \text{ ft/s} = 13.3 \text{ ft/s} \uparrow$ **Ans.**