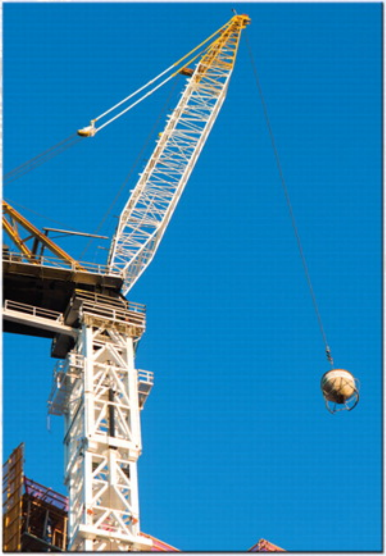
Engineering Mechanics

DYNAMICS

INSTRUCTOR SOLUTIONS MANUAL

Fourteenth Edition



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12–1.

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)\ \text{m/s}^2$, where t is in seconds. What is the particle's velocity when $t = 6$ s, and what is its position when $t = 11$ s?

SOLUTION

$a = 2t - 6$

$dv = a\,dt$

$$\int_0^v dv = \int_0^t (2t - 6)\,dt$$

$v = t^2 - 6t$

$ds = v\,dt$

$$\int_0^s ds = \int_0^t (t^2 - 6t)\,dt$$

$$s = \frac{t^3}{3} - 3t^2$$

When $t = 6\ s$,

$v = 0$ **Ans.**

When $t = 11\ s$,

$s = 80.7$ m **Ans.**

Ans:
$s = 80.7$ m

12–2.

If a particle has an initial velocity of $v_0 = 12\ \text{ft/s}$ to the right, at $s_0 = 0$, determine its position when $t = 10\ \text{s}$, if $a = 2\ \text{ft/s}^2$ to the left.

SOLUTION

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$= 0 + 12(10) + \frac{1}{2}(-2)(10)^2$$

$$= 20\ \text{ft}$$ **Ans.**

Ans:
$s = 20\ \text{ft}$

12–3.

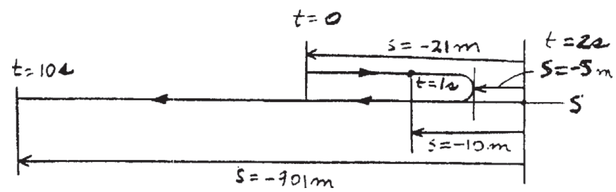
A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

SOLUTION

$v = 12 - 3t^2$ **(1)**

$a = \dfrac{dv}{dt} = -6t\big|_{t=4} = -24 \text{ m/s}^2$ **Ans.**

$\int_{-10}^{s} ds = \int_{1}^{t} v\,dt = \int_{1}^{t} (12 - 3t^2)\,dt$



$s + 10 = 12t - t^3 - 11$

$s = 12t - t^3 - 21$

$s\big|_{t=0} = -21$

$s\big|_{t=10} = -901$

$\Delta s = -901 - (-21) = -880$ m **Ans.**

From Eq. (1):

$v = 0$ when $t = 2s$

$s\big|_{t=2} = 12(2) - (2)^3 - 21 = -5$

$s_T = (21 - 5) + (901 - 5) = 912$ m **Ans.**

Ans:
$a = -24 \text{ m/s}^2$
$\Delta s = -880$ m
$s_T = 912$ m

***12–4.**

A particle travels along a straight line with a constant acceleration. When $s = 4$ ft, $v = 3$ ft/s and when $s = 10$ ft, $v = 8$ ft/s. Determine the velocity as a function of position.

SOLUTION

Velocity: To determine the constant acceleration a_c, set $s_0 = 4$ ft, $v_0 = 3$ ft/s, $s = 10$ ft and $v = 8$ ft/s and apply Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$8^2 = 3^2 + 2a_c\,(10 - 4)$$

$$a_c = 4.583 \text{ ft/s}^2$$

Using the result $a_c = 4.583$ ft/s^2, the velocity function can be obtained by applying Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v^2 = 3^2 + 2(4.583)\,(s - 4)$$

$$v = \left(\sqrt{9.17s - 27.7}\right) \text{ft/s}$$ **Ans.**

Ans:
$v = \left(\sqrt{9.17s - 27.7}\right)$ ft/s

12–5.

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

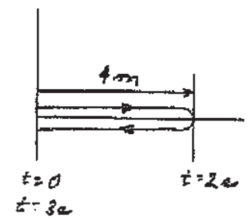
SOLUTION

$v = 6t - 3t^2$

$a = \dfrac{dv}{dt} = 6 - 6t$

At $t = 3$ s

$a = -12 \text{ m/s}^2$ **Ans.**



$ds = v\,dt$

$$\int_0^s ds = \int_0^t (6t - 3t^2)dt$$

$s = 3t^2 - t^3$

At $t = 3$ s

$s = 0$ **Ans.**

Since $v = 0 = 6t - 3t^2$, when $t = 0$ and $t = 2$ s.

when $t = 2$ s, $\quad s = 3(2)^2 - (2)^3 = 4$ m

$s_T = 4 + 4 = 8$ m **Ans.**

$(v_{sp})_{\text{avg}} = \dfrac{s_T}{t} = \dfrac{8}{3} = 2.67$ m/s **Ans.**

Ans:
$s_T = 8$ m
$v_{\text{avg}} = 2.67$ m/s

12–6.

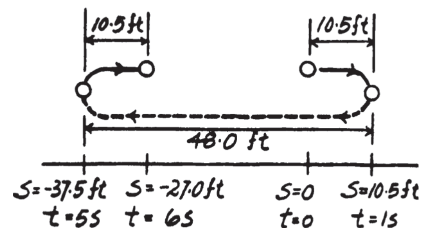
The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

SOLUTION

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft} \qquad \textbf{Ans.}$$

Total DistanceTraveled: The velocity of the particle can be determined by applying Eq. 12–1.



$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$$t = 1 \text{ s} \qquad \text{and} \qquad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

$$s|_{t=0\text{ s}} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s|_{t=1\text{ s}} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$

$$s|_{t=5\text{ s}} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{\text{tot}} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$s|_{t=6\text{ s}} = -27.0$ ft
$s_{\text{tot}} = 69.0$ ft

12–7.

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

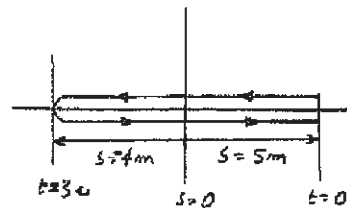
SOLUTION

$s = t^2 - 6t + 5$

$v = \dfrac{ds}{dt} = 2t - 6$

$a = \dfrac{dv}{dt} = 2$

$v = 0$ when $t = 3$



$s|_{t=0} = 5$

$s|_{t=3} = -4$

$s|_{t=6} = 5$

$v_{\text{avg}} = \dfrac{\Delta s}{\Delta t} = \dfrac{0}{6} = 0$ **Ans.**

$(v_{sp})_{\text{avg}} = \dfrac{s_T}{\Delta t} = \dfrac{9 + 9}{6} = 3 \text{ m/s}$ **Ans.**

$a|_{t=6} = 2 \text{ m/s}^2$ **Ans.**

Ans:
$v_{\text{avg}} = 0$
$(v_{\text{sp}})_{\text{avg}} = 3 \text{ m/s}$
$a|_{t=6\text{ s}} = 2 \text{ m/s}^2$

***12–8.**

A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

SOLUTION

$s = 10t^2 + 20$

(a) $s|_{1\text{ s}} = 10(1)^2 + 20 = 30 \text{ mm}$

$s|_{5\text{ s}} = 10(5)^2 + 20 = 270 \text{ mm}$

$\Delta s = 270 - 30 = 240 \text{ mm}$ **Ans.**

(b) $\Delta t = 5 - 1 = 4 \text{ s}$

$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{240}{4} = 60 \text{ mm/s}$ **Ans.**

(c) $a = \dfrac{d^2 s}{dt^2} = 20 \text{ mm/s}^2$ (for all t) **Ans.**

Ans:
$\Delta s = 240$ mm
$v_{avg} = 60$ mm/s
$a = 20$ mm/s^2

12–9.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 1\ \text{m}$ and $v = 2\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6\ \text{s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$a = 2t - 1$

$dv = a\,dt$

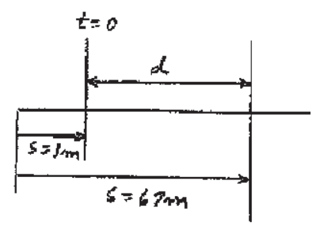
$$\int_2^v dv = \int_0^t (2t - 1)dt$$

$v = t^2 - t + 2$

$dx = v\,dt$

$$\int_t^s ds = \int_0^t (t^2 - t + 2)dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$



When $t = 6$ s

$v = 32\ \text{m/s}$ **Ans.**

$s = 67\ \text{m}$ **Ans.**

Since $v \neq 0$ for $0 \leq t \leq 6$ s, then

$d = 67 - 1 = 66\ \text{m}$ **Ans.**

Ans:
$v = 32\ \text{m/s}$
$s = 67\ \text{m}$
$d = 66\ \text{m}$

12–10.

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s^2, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

SOLUTION

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

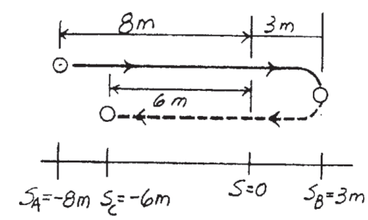
$v = 1.29$ m/s **Ans.**

Ans:
$v = 1.29$ m/s

12–11.

A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

SOLUTION



Average Velocity: The displacement from A to C is $\Delta s = s_C - S_A = -6 - (-8) = 2$ m.

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{2}{4+5} = 0.222 \text{ m/s} \qquad \textbf{Ans.}$$

Average Speed: The distances traveled from A to B and B to C are $s_{A \to B} = 8 + 3 = 11.0$ m and $s_{B \to C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{\text{Tot}} = s_{A \to B} + s_{B \to C} = 11.0 + 9.00 = 20.0$ m.

$$(v_{sp})_{\text{avg}} = \frac{s_{\text{Tot}}}{\Delta t} = \frac{20.0}{4+5} = 2.22 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{\text{avg}} = 0.222$ m/s
$(v_{\text{sp}})_{\text{avg}} = 2.22$ m/s

***12–12.**

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

SOLUTION

$v = v_1 + a_c t$

$120 = 70 + 6000(t)$

$t = 8.33(10^{-3})\text{ hr} = 30\text{ s}$ **Ans.**

$v^2 = v_1^2 + 2\,a_c(s - s_1)$

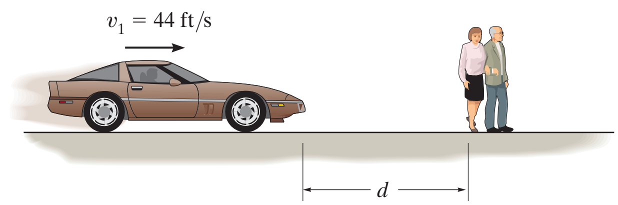
$(120)^2 = 70^2 + 2(6000)(s - 0)$

$s = 0.792\text{ km} = 792\text{ m}$ **Ans.**

Ans:
$t = 30\text{ s}$
$s = 792\text{ m}$

12–13.

Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral*: If you must drink, please don't drive!



SOLUTION

Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 33.0)$$

$$d = 517 \text{ ft} \qquad \textbf{Ans.}$$

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 132)$$

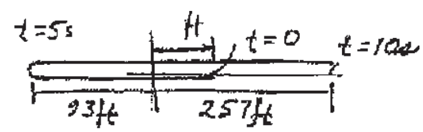
$$d = 616 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
Normal: $d = 517$ ft
drunk: $d = 616$ ft

12–14.

The position of a particle along a straight-line path is defined by $s = (t^3 - 6t^2 - 15t + 7)$ ft, where t is in seconds. Determine the total distance traveled when $t = 10$ s. What are the particle's average velocity, average speed, and the instantaneous velocity and acceleration at this time?

SOLUTION



$s = t^3 - 6t^2 - 15t + 7$

$v = \dfrac{ds}{dt} = 3t^2 - 12t - 15$

When $t = 10$ s,

$v = 165$ ft/s **Ans.**

$a = \dfrac{dv}{dt} = 6t - 12$

When $t = 10$ s,

$a = 48$ ft/s^2 **Ans.**

When $v = 0$,

$0 = 3t^2 - 12t - 15$

The positive root is

$t = 5$ s

When $t = 0$, $\quad s = 7$ ft

When $t = 5$ s, $\quad s = -93$ ft

When $t = 10$ s, $\quad s = 257$ ft

Total distance traveled

$s_T = 7 + 93 + 93 + 257 = 450$ ft **Ans.**

$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{257 - 7}{10 - 0} = 25.0$ ft/s **Ans.**

$(v_{sp})_{avg} = \dfrac{s_T}{\Delta t} = \dfrac{450}{10} = 45.0$ ft/s **Ans.**

Ans:
$v = 165$ ft/s
$a = 48$ ft/s^2
$s_T = 450$ ft
$v_{avg} = 25.0$ ft/s
$(v_{sp})_{avg} = 45.0$ ft/s

12–15.

A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

SOLUTION

$$a = \frac{d\nu}{dt} = -\mathrm{k}\nu^3$$

$$\int_{\nu 0}^{\nu} \nu^{-3}\, d\nu = \int_0^t -k\, dt$$

$$-\frac{1}{2}(\nu^{-2} - \nu_0^{-2}) = -kt$$

$$\nu = \left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{-\frac{1}{2}}$$ **Ans.**

$$ds = \nu\, dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \left.\frac{2\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}}}{2k}\right|_0^t$$

$$s = \frac{1}{k}\left[\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{\nu_0}\right]$$ Ans.

Ans:

$$v = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}$$

$$s = \frac{1}{k}\left[\left(2kt + \frac{1}{v_0^2}\right)^{1/2} - \frac{1}{v_0}\right]$$

***12–16.**

A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})$ m/s^2, where v is in m/s. Determine how far it travels before it stops. How much time does this take?

SOLUTION

Distance Traveled: The distance traveled by the particle can be determined by applying Eq. 12–3.

$$ds = \frac{vdv}{a}$$

$$\int_0^s ds = \int_{6\text{ m/s}}^{v} \frac{v}{-1.5v^{\frac{1}{2}}}\,dv$$

$$s = \int_{6\text{ m/s}}^{v} -0.6667\, v^{\frac{1}{2}}\,dv$$

$$= \left(-0.4444v^{\frac{3}{2}} + 6.532\right)\text{ m}$$

When $v = 0$, $\quad s = -0.4444\left(0^{\frac{3}{2}}\right) + 6.532 = 6.53\text{ m}$ **Ans.**

Time: The time required for the particle to stop can be determined by applying Eq. 12–2.

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = -\int_{6\text{ m/s}}^{v} \frac{dv}{1.5v^{\frac{1}{2}}}$$

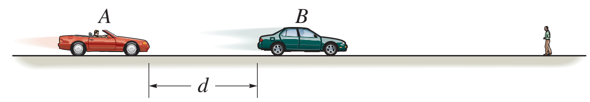
$$t = -1.333\left(v^{\frac{1}{2}}\right)\Big|_{6\text{ m/s}}^{v} = \left(3.266 - 1.333v^{\frac{1}{2}}\right)\text{s}$$

When $v = 0$, $\quad t = 3.266 - 1.333\left(0^{\frac{1}{2}}\right) = 3.27\text{ s}$ **Ans.**

Ans:
$s = 6.53$ m
$t = 3.27$ s

12–17.

Car B is traveling a distance d ahead of car A. Both cars are traveling at 60 ft/s when the driver of B suddenly applies the brakes, causing his car to decelerate at 12 ft/s^2. It takes the driver of car A 0.75 s to react (this is the normal reaction time for drivers). When he applies his brakes, he decelerates at 15 ft/s^2. Determine the minimum distance d be tween the cars so as to avoid a collision.



SOLUTION

For B:

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v_B = 60 - 12t$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_B = d + 60t - \frac{1}{2}(12)t^2 \qquad \textbf{(1)}$$

For A:

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v_A = 60 - 15(t - 0.75), \quad [t > 0.75]$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 60(0.75) + 60(t - 0.75) - \frac{1}{2}(15)(t - 0.75)^2, \quad [t > 0.74] \qquad \textbf{(2)}$$

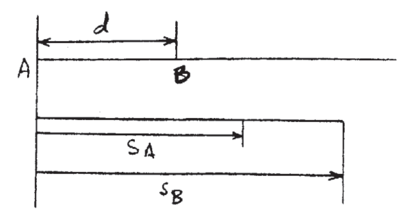
Require $v_A = v_B$ the moment of closest approach.

$60 - 12t = 60 - 15(t - 0.75)$

$t = 3.75$ s

Worst case without collision would occur when $s_A = s_B$.

At $t = 3.75$ s, from Eqs. (1) and (2):



$60(0.75) + 60(3.75 - 0.75) - 7.5(3.75 - 0.75)^2 = d + 60(3.75) - 6(3.75)^2$

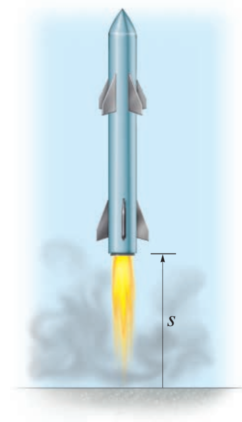
$157.5 = d + 140.625$

$d = 16.9$ ft **Ans.**

Ans:
$d = 16.9$ ft

12–18.

The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$$a\, ds = v\, dv$$

$$\int_0^s (6 + 0.02\, s)\, ds = \int_0^v v\, dv$$

$$6\, s + 0.01\, s^2 = \frac{1}{2} v^2$$

$$v = \sqrt{12\, s + 0.02\, s^2}$$

$$ds = v\, dt$$

$$\int_0^{100} \frac{ds}{\sqrt{12\, s + 0.02\, s^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{0.02}} \ln \left[\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}} \right]_0^{100} = t$$

$t = 5.62$ s **Ans.**

Ans:
$t = 5.62$ s

12–19.

A train starts from rest at station A and accelerates at $0.5\ \text{m/s}^2$ for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at $1\ \text{m/s}^2$ until it is brought to rest at station B. Determine the distance between the stations.

SOLUTION

Kinematics: For stage (1) motion, $v_0 = 0$, $s_0 = 0$, $t = 60$ s, and $a_c = 0.5\ \text{m/s}^2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(60^2) = 900\ \text{m}$$

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v_1 = 0 + 0.5(60) = 30\ \text{m/s}$$

For stage (2) motion, $v_0 = 30\ \text{m/s}$, $s_0 = 900$ m, $a_c = 0$ and $t = 15(60) = 900$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_2 = 900 + 30(900) + 0 = 27\,900\ \text{m}$$

For stage (3) motion, $v_0 = 30\ \text{m/s}$, $v = 0$, $s_0 = 27\,900$ m and $a_c = -1\ \text{m/s}^2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$0 = 30 + (-1)t$$

$$t = 30\ \text{s}$$

$$\overset{+}{\rightarrow} \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_3 = 27\,900 + 30(30) + \frac{1}{2}(-1)(30^2)$$

$$= 28\,350\ \text{m} = 28.4\ \text{km}$$ **Ans.**

Ans:
$s = 28.4$ km

***12–20.**

The velocity of a particle traveling along a straight line is $v = (3t^2 - 6t)$ ft/s, where t is in seconds. If $s = 4$ ft when $t = 0$, determine the position of the particle when $t = 4$ s. What is the total distance traveled during the time interval $t = 0$ to $t = 4$ s? Also, what is the acceleration when $t = 2$ s?

SOLUTION

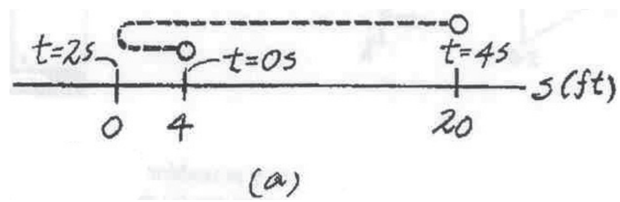
Position: The position of the particle can be determined by integrating the kinematic equation $ds = v\,dt$ using the initial condition $s = 4$ ft when $t = 0$ s. Thus,

$$(\xrightarrow{+}) \qquad ds = v\,dt$$

$$\int_{4\text{ ft}}^{s} ds = \int_0^t \left(3t^2 - 6t\right)dt$$

$$s\Big|_{4\text{ ft}}^{s} = (t^3 - 3t^2)\Big|_0^t$$

$$s = \left(t^3 - 3t^2 + 4\right)\text{ ft}$$



When $t = 4$ s,

$$s|_{4\text{ s}} = 4^3 - 3(4^2) + 4 = 20\text{ ft} \qquad \textbf{Ans.}$$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$v = 3t^2 - 6t = 0$$

$$t(3t - 6) = 0$$

$$t = 0 \text{ and } t = 2\text{ s}$$

The position of the particle at $t = 0$ and 2 s is

$$s|_{0\text{ s}} = 0 - 3\left(0^2\right) + 4 = 4\text{ ft}$$

$$s|_{2\text{ s}} = 2^3 - 3\left(2^2\right) + 4 = 0$$

Using the above result, the path of the particle shown in Fig. *a* is plotted. From this figure,

$$s_{\text{Tot}} = 4 + 20 = 24\text{ ft} \qquad \textbf{Ans.}$$

Acceleration:

$$(\xrightarrow{+}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 6t)$$

$$a = (6t - 6)\text{ ft/s}^2$$

When $t = 2$ s,

$$a|_{t=2\text{ s}} = 6(2) - 6 = 6\text{ ft/s}^2 \rightarrow \qquad \textbf{Ans.}$$

Ans:
$s_{\text{Tot}} = 24$ ft
$a|_{t=2\text{ s}} = 6\text{ ft}/s^2 \rightarrow$

12–21.

A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.

SOLUTION

$v = 60(1 - e^{-t})$

$$\int_0^s ds = \int v\,dt = \int_0^3 60(1 - e^{-t})dt$$

$s = 60(t + e^{-t})\big|_0^3$

$s = 123$ ft **Ans.**

$$a = \frac{dv}{dt} = 60(e^{-t})$$

At $t = 3$ s

$a = 60e^{-3} = 2.99$ ft/s^2 **Ans.**

Ans:
$s = 123$ ft
$a = 2.99$ ft/s^2

12–22.

A sandbag is dropped from a balloon which is ascending vertically at a constant speed of 6 m/s. If the bag is released with the same upward velocity of 6 m/s when $t = 0$ and hits the ground when $t = 8$ s, determine the speed of the bag as it hits the ground and the altitude of the balloon at this instant.

SOLUTION

$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$h = 0 + (-6)(8) + \frac{1}{2}(9.81)(8)^2$$

$$= 265.92 \text{ m}$$

During $t = 8$ s, the balloon rises

$$h' = vt = 6(8) = 48 \text{ m}$$

Altitude $= h + h' = 265.92 + 48 = 314$ m **Ans.**

$(+\downarrow) \qquad v = v_0 + a_c t$

$v = -6 + 9.81(8) = 72.5$ m/s **Ans.**

Ans:
$h = 314$ m
$v = 72.5$ m/s

12–23.

A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)\ \mathrm{m/s^2}$, where v is in meters per second. If $v = 20\ \mathrm{m/s}$ when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

SOLUTION

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^{v} \frac{dv}{v} = \int_{0}^{t} -2\,dt$$

$$\ln \frac{v}{20} = -2t$$

$$v = \left(20e^{-2t}\right)\ \mathrm{m/s} \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = \left(-40e^{-2t}\right)\ \mathrm{m/s^2} \qquad \textbf{Ans.}$$

$$\int_{0}^{s} ds = v\,dt = \int_{0}^{t} (20e^{-2t})dt$$

$$s = -10e^{-2t}\big|_0^t = -10\left(e^{-2t} - 1\right)$$

$$s = 10\left(1 - e^{-2t}\right)\ \mathrm{m} \qquad \textbf{Ans.}$$

Ans:
$v = \left(20e^{-2t}\right)\ \mathrm{m/s}$
$a = \left(-40e^{-2t}\right)\ \mathrm{m/s^2}$
$s = 10\left(1 - e^{-2t}\right)\ \mathrm{m}$

***12–24.**

The acceleration of a particle traveling along a straight line is $a = \frac{1}{4}s^{1/2}$ m/s^2, where s is in meters. If $v = 0$, $s = 1$ m when $t = 0$, determine the particle's velocity at $s = 2$ m.

SOLUTION

Velocity:

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_1^s \frac{1}{4}s^{1/2}ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left.\frac{1}{6}s^{3/2}\right|_1^s$$

$$v = \frac{1}{\sqrt{3}}(s^{3/2} - 1)^{1/2}\text{ m/s}$$

When $s = 2$ m, $v = 0.781$ m/s. **Ans.**

Ans:
$v = 0.781$ m/s

12–25.

If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s^2, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{(1)}$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \qquad \textbf{Ans.}$$

b) If $t \rightarrow \infty$, $\dfrac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. (1)

$$v_{max} = 100 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
(a) $v = 45.5$ m/s
(b) $v_{max} = 100$ m/s

12–26.

The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION

$a = 2t - 9$

$$\int_{10}^{v} dv = \int_{0}^{t} (2t - 9)\, dt$$

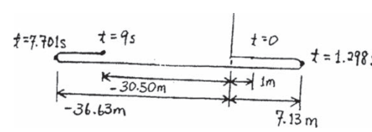
$$v - 10 = t^2 - 9\,t$$

$$v = t^2 - 9\,t + 10$$

$$\int_{1}^{s} ds = \int_{0}^{t} (t^2 - 9t + 10)\, dt$$

$$s - 1 = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t$$

$$s = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t + 1$$



Note when $v = t^2 - 9\,t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298$ s, $\quad s = 7.13$ m

When $t = 7.701$ s, $\quad s = -36.63$ m

When $t = 9$ s, $\quad s = -30.50$ m

(a) $\quad s = -30.5$ m **Ans.**

(b) $\quad s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$\quad s_{Tot} = 56.0$ m **Ans.**

(c) $\quad v = 10$ m/s **Ans.**

Ans:
(a) $s = -30.5$ m
(b) $s_{\text{Tot}} = 56.0$ m
(c) $v = 10$ m/s

12–27.

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2{}_f)(v^2{}_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

SOLUTION

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^{2'}} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f + v}{v_f - v}\right)\Bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v}{v_f - v}\right)$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v_f/2}{v_f - v_f/2}\right)$$

$$t = 0.549\left(\frac{v_f}{g}\right)$$ **Ans.**

Ans:

$$t = 0.549\left(\frac{v_f}{g}\right)$$

***12–28.**

Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)\text{ ft/s}^2$ and $a_B = (12t^2 - 8)\text{ ft/s}^2$, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

SOLUTION

Velocity: The velocity of particles A and B can be determined using Eq. 12-2.

$$dv_A = a_A dt$$

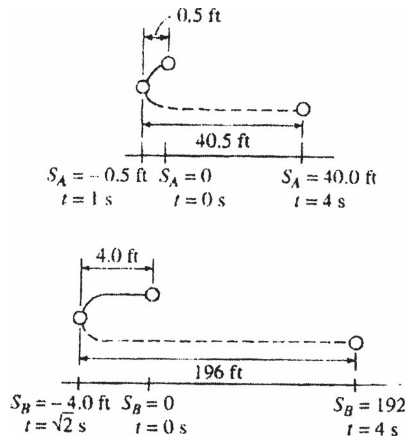
$$\int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$

$$v_A = 3t^2 - 3t$$

$$dv_B = a_B dt$$

$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$

$$v_B = 4t^3 - 8t$$



The times when particle A stops are

$3t^2 - 3t = 0 \qquad t = 0$ s and $= 1$ s

The times when particle B stops are

$4t^3 - 8t = 0 \quad t = 0$ s and $t = \sqrt{2}$ s

Position: The position of particles A and B can be determined using Eq. 12-1.

$$ds_A = v_A dt$$

$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t)dt$$

$$s_A = t^3 - \frac{3}{2}t^2$$

$$ds_B = v_B dt$$

$$\int_0^{s_B} ds_B = \int_0^t (4t^3 - 8t)dt$$

$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1$ s and 4 s are

$$s_A|_{t=1\text{ s}} = 1^3 - \frac{3}{2}(1^2) = -0.500\text{ ft}$$

$$s_A|_{t=4\text{ s}} = 4^3 - \frac{3}{2}(4^2) = 40.0\text{ ft}$$

Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0\text{ ft}$$ **Ans.**

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4\text{ ft}$$

$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192\text{ ft}$$

Particle B has traveled

$$d_B = 2(4) + 192 = 200\text{ ft}$$ **Ans.**

At $t = 4$ s the distance beween A and B is

$$\Delta s_{AB} = 192 - 40 = 152\text{ ft}$$ **Ans.**

Ans:
$d_A = 41.0$ ft
$d_B = 200$ ft
$\Delta s_{AB} = 152$ ft

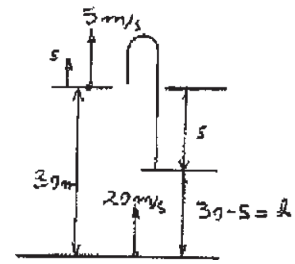
12–29.

A ball A is thrown vertically upward from the top of a 30-m-high building with an initial velocity of 5 m/s. At the same instant another ball B is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

SOLUTION

Origin at roof:

Ball A:



$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-s = 0 + 5t - \frac{1}{2}(9.81)t^2$$

Ball B:

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-s = -30 + 20t - \frac{1}{2}(9.81)t^2$$

Solving,

$t = 2$ s **Ans.**

$s = 9.62$ m

Distance from ground,

$d = (30 - 9.62) = 20.4$ m **Ans.**

Also, origin at ground,

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_A = 30 + 5t + \frac{1}{2}(-9.81)t^2$$

$$s_B = 0 + 20t + \frac{1}{2}(-9.81)t^2$$

Require

$$s_A = s_B$$

$$30 + 5t + \frac{1}{2}(-9.81)t^2 = 20t + \frac{1}{2}(-9.81)t^2$$

$t = 2$ s **Ans.**

$s_B = 20.4$ m **Ans.**

Ans:
$h = 20.4$ m
$t = 2$ s

12–30.

A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

SOLUTION

Velocity: $v_0 = 27$ m/s at $t_0 = 0$ s. Applying Eq. 12–2, we have

$$(+\downarrow) \qquad dv = a\,dt$$

$$\int_{27}^{v} dv = \int_0^t -6t\,dt$$

$$v = \left(27 - 3t^2\right)\ \text{m/s} \qquad \textbf{(1)}$$

At $v = 0$, from Eq. (1)

$$0 = 27 - 3t^2 \qquad t = 3.00\ \text{s}$$

Distance Traveled: $s_0 = 0$ m at $t_0 = 0$ s. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$$(+\downarrow) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right) dt$$

$$s = \left(27t - t^3\right)\ \text{m} \qquad \textbf{(2)}$$

At $t = 3.00$ s, from Eq. (2)

$$s = 27(3.00) - 3.00^3 = 54.0\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$s = 54.0$ m

12–31.

The velocity of a particle traveling along a straight line is $v = v_0 - ks$, where k is constant. If $s = 0$ when $t = 0$, determine the position and acceleration of the particle as a function of time.

SOLUTION

Position:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{v_0 - ks}$$

$$t\Big|_0^t = -\frac{1}{k}\ln\left(v_0 - ks\right)\Big|_0^s$$

$$t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - ks}\right)$$

$$e^{kt} = \frac{v_0}{v_0 - ks}$$

$$s = \frac{v_0}{k}\left(1 - e^{-kt}\right)$$ **Ans.**

Velocity:

$$v = \frac{ds}{dt} = \frac{d}{dt}\left[\frac{v_0}{k}\left(1 - e^{-kt}\right)\right]$$

$$v = v_0 e^{-kt}$$

Acceleration:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(v_0 e^{-kt}\right)$$

$$a = -kv_0 e^{-kt}$$ **Ans.**

Ans:

$s = \frac{v_0}{k}\left(1 - e^{-kt}\right)$

$a = -kv_0 e^{-kt}$

***12–32.**

Ball A is thrown vertically upwards with a velocity of v_0. Ball B is thrown upwards from the same point with the same velocity t seconds later. Determine the elapsed time $t < 2v_0/g$ from the instant ball A is thrown to when the balls pass each other, and find the velocity of each ball at this instant.

SOLUTION

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = v_0$, $(s_A)_0 = 0$, $s_A = h, t_A = t'$, and $(a_c)_A = -g$.

$$(+\uparrow) \qquad s_A = (s_A)_0 + (v_A)_0 t_A + \frac{1}{2}(a_c)_A t_A{}^2$$

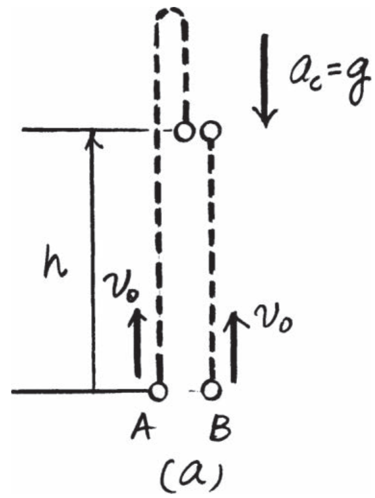
$$h = 0 + v_0 t' + \frac{1}{2}(-g)(t')^2$$

$$h = v_0 t' - \frac{g}{2}t'^2 \qquad \textbf{(1)}$$

$$(+\uparrow) \qquad v_A = (v_A)_0 + (a_c)_A t_A$$

$$v_A = v_0 + (-g)(t')$$

$$v_A = v_0 - gt' \qquad \textbf{(2)}$$



The motion of ball B requires $(v_B)_0 = v_0$, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - t$, and $(a_c)_B = -g$.

$$(+\uparrow) \qquad s_B = (s_B)_0 + (v_B)_0 t_B + \frac{1}{2}(a_c)_B t_B{}^2$$

$$h = 0 + v_0(t' - t) + \frac{1}{2}(-g)(t' - t)^2$$

$$h = v_0(t' - t) - \frac{g}{2}(t' - t)^2 \qquad \textbf{(3)}$$

$$(+\uparrow) \qquad v_B = (v_B)_0 + (a_c)_B t_B$$

$$v_B = v_0 + (-g)(t' - t)$$

$$v_B = v_0 - g(t' - t) \qquad \textbf{(4)}$$

Solving Eqs. (1) and (3),

$$v_0 t' - \frac{g}{2}t'^2 = v_0(t' - t) - \frac{g}{2}(t' - t)^2$$

$$t' = \frac{2v_0 + gt}{2g} \qquad \textbf{Ans.}$$

Substituting this result into Eqs. (2) and (4),

$$v_A = v_0 - g\left(\frac{2v_0 + gt}{2g}\right)$$

$$= -\frac{1}{2}gt = \frac{1}{2}gt\downarrow \qquad \textbf{Ans.}$$

$$v_B = v_0 - g\left(\frac{2v_0 + gt}{2g} - t\right)$$

$$= \frac{1}{2}gt\uparrow \qquad \textbf{Ans.}$$

Ans:

$$t' = \frac{2v_0 + gt}{2g}$$

$$v_A = \frac{1}{2}gt\downarrow$$

$$v_B = \frac{1}{2}gt\uparrow$$

12–33.

As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81\text{ m/s}^2$ and $R = 6356\text{ km}$, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

SOLUTION

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0R^2\int_0^\infty \frac{dy}{(R + y)^2}$$

$$\left.\frac{v^2}{2}\right|_v^0 = \left.\frac{g_0R^2}{R + y}\right|_0^\infty$$

$$v = \sqrt{2g_0R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167\text{ m/s} = 11.2\text{ km/s}$$ **Ans.**

Ans:
$v = 11.2\text{ km/s}$

12–34.

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–36), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–36.

SOLUTION

From Prob. 12–36,

$$(+\uparrow) \qquad a = -g_0 \frac{R^2}{(R+y)^2}$$

Since $a\,dy = v\,dv$

then

$$-g_0 R^2 \int_{y_0}^{y} \frac{dy}{(R+y)^2} = \int_0^v v\,dv$$

$$g_0 R^2 \left[\frac{1}{R+y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R+y} - \frac{1}{R+y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R+y)(R+y_0)}} \qquad \textbf{Ans.}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356+500)(10^6)}}$$

$$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow \qquad \textbf{Ans.}$$

Ans:

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R+y)(R+y_0)}}$$

$v_{imp} = 3.02$ km/s

12–35.

A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s². After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v–t graph for the motion.

SOLUTION

Total Distance Traveled: The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12–5 with $s_0 = 0$ and $v_0 = 0$.

$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(t')^2 = 0.25(t')^2$$

The velocity at time t can be obtained by applying Eq. 12–4 with $v_0 = 0$.

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t = 0 + 0.5t = 0.5t \qquad \textbf{(1)}$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is traveling at a constant velocity of $v = 0.5t'$ (Eq. (1)). Thus, the distance for this part of motion is

$$(\xrightarrow{+}) \qquad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{\text{Tot}} = 2000$, then

$$s_{\text{Tot}} = s_1 + s_2$$

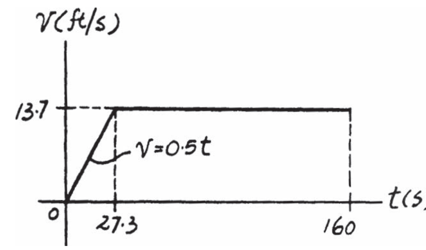
$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

$$0.25(t')^2 - 80t' + 2000 = 0$$

Choose a root that is less than 160 s, then

$$t' = 27.34 \text{ s} = 27.3 \text{ s} \qquad \textbf{Ans.}$$

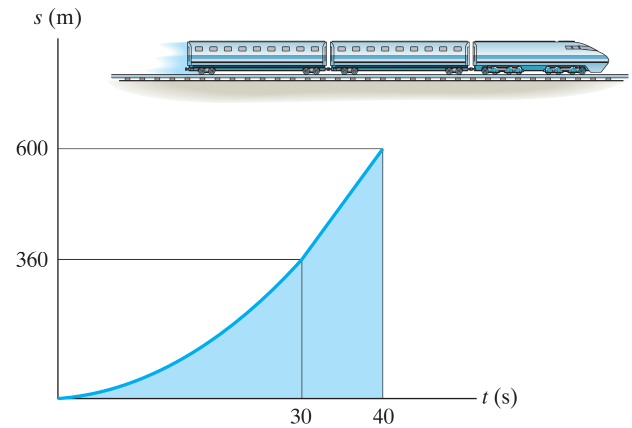
v – t Graph: The equation for the velocity is given by Eq. (1). When $t = t' = 27.34$ s, $v = 0.5(27.34) = 13.7$ ft/s.



Ans:
$t' = 27.3$ s.
When $t = 27.3$ s, $v = 13.7$ ft/s.

***12–36.**

The s–t graph for a train has been experimentally determined. From the data, construct the v–t and a–t graphs for the motion; $0 \le t \le 40$ s. For $0 \le t \le 30$ s, the curve is $s = (0.4t^2)$ m, and then it becomes straight for $t \ge 30$ s.



SOLUTION

$0 \le t \le 30$:

$$s = 0.4t^2$$

$$v = \frac{ds}{dt} = 0.8t$$

$$a = \frac{dv}{dt} = 0.8$$

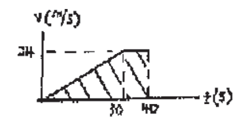
$30 \le t \le 40$:

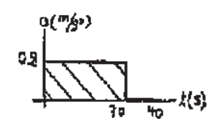
$$s - 360 = \left(\frac{600 - 360}{40 - 30}\right)(t - 30)$$

$$s = 24(t - 30) + 360$$

$$v = \frac{ds}{dt} = 24$$

$$a = \frac{dv}{dt} = 0$$





Ans:

$s = 0.4t^2$

$v = \dfrac{ds}{dt} = 0.8t$

$a = \dfrac{dv}{dt} = 0.8$

$s = 24(t - 30) + 360$

$v = \dfrac{ds}{dt} = 24$

$a = \dfrac{dv}{dt} = 0$

12–37.

Two rockets start from rest at the same elevation. Rocket A accelerates vertically at 20 m/s^2 for 12 s and then maintains a constant speed. Rocket B accelerates at 15 m/s^2 until reaching a constant speed of 150 m/s. Construct the a–t, v–t, and s–t graphs for each rocket until $t = 20$ s. What is the distance between the rockets when $t = 20$ s?

SOLUTION

For rocket A

For $t < 12$ s

$$+\uparrow \; v_A = (v_A)_0 + a_A t$$

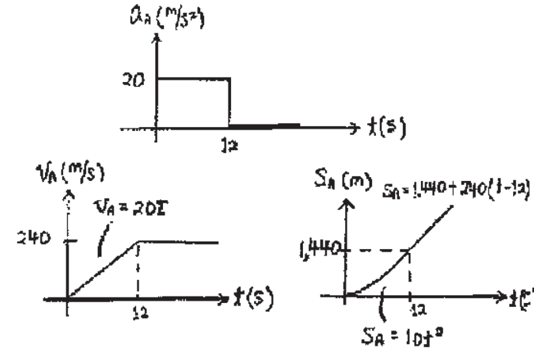
$$v_A = 0 + 20\,t$$

$$v_A = 20\,t$$

$$+\uparrow \; s_A = (s_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(20)\,t^2$$

$$s_A = 10\,t^2$$



When $t = 12$ s, $\quad v_A = 240$ m/s

$s_A = 1440$ m

For $t > 12$ s

$v_A = 240$ m/s

$s_A = 1440 + 240(t - 12)$

For rocket B

For $t < 10$ s

$$+\uparrow \; v_B = (v_B)_0 + a_B t$$

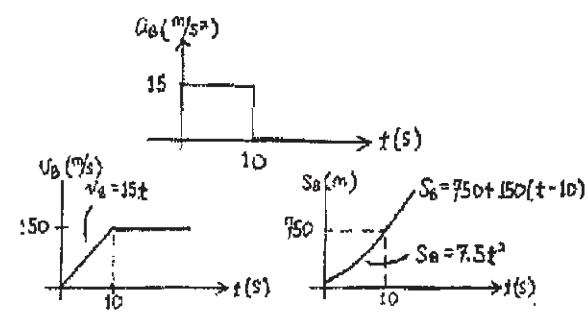
$$v_B = 0 + 15\,t$$

$$v_B = 15\,t$$

$$+\uparrow \; s_B = (s_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(15)\,t^2$$

$$s_B = 7.5\,t^2$$



When $t = 10$ s, $\quad v_B = 150$ m/s

$s_B = 750$ m

For $t > 10$ s

$v_B = 150$ m/s

$s_B = 750 + 150(t - 10)$

When $t = 20$ s, $\quad s_A = 3360$ m, $\quad s_B = 2250$ m

$\Delta s = 1110 \text{ m} = 1.11 \text{ km}$ **Ans.**

Ans:
$\Delta s = 1.11$ km

12–38.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \le t \le 4$ s.

SOLUTION

a–t Graph:

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$
$$a = (2t - 4)\,\text{m/s}^2$$

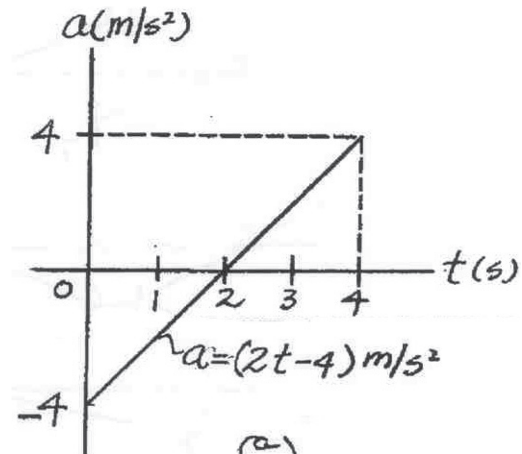
Thus,

$$a|_{t=0} = 2(0) - 4 = -4\ \text{m/s}^2$$

$$a|_{t=2} = 0$$

$$a|_{t=4\,\text{s}} = 2(4) - 4 = 4\ \text{m/s}^2$$

The $a-t$ graph is shown in Fig. *a*.



(a)

v–t Graph: The slope of the $v-t$ graph is zero when $a = \dfrac{dv}{dt} = 0$. Thus,

$$a = 2t - 4 = 0 \qquad t = 2\ \text{s}$$

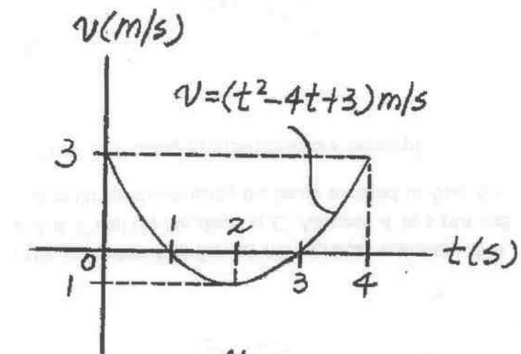
The velocity of the particle at $t = 0$ s, 2 s, and 4 s are

$$v|_{t=0\,\text{s}} = 0^2 - 4(0) + 3 = 3\ \text{m/s}$$

$$v|_{t=2\,\text{s}} = 2^2 - 4(2) + 3 = -1\ \text{m/s}$$

$$v|_{t=4\,\text{s}} = 4^2 - 4(4) + 3 = 3\ \text{m/s}$$

The $v-t$ graph is shown in Fig. *b*.



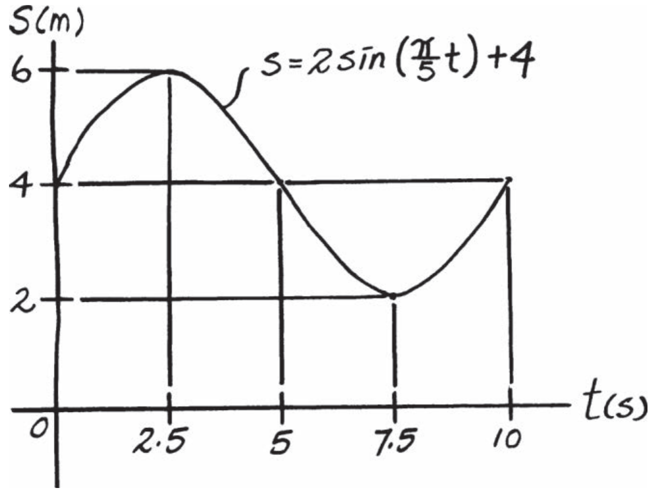
(b)

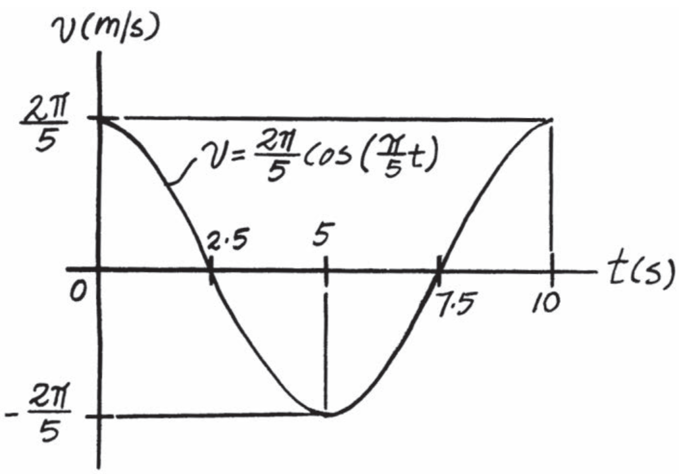
Ans:
$a|_{t=0} = -4\ \text{m/s}^2$
$a|_{t=2\,\text{s}} = 0$
$a|_{t=4\,\text{s}} = 4\ \text{m/s}^2$
$v|_{t=0} = 3\ \text{m/s}$
$v|_{t=2\,\text{s}} = -1\ \text{m/s}$
$v|_{t=4\,\text{s}} = 3\ \text{m/s}$

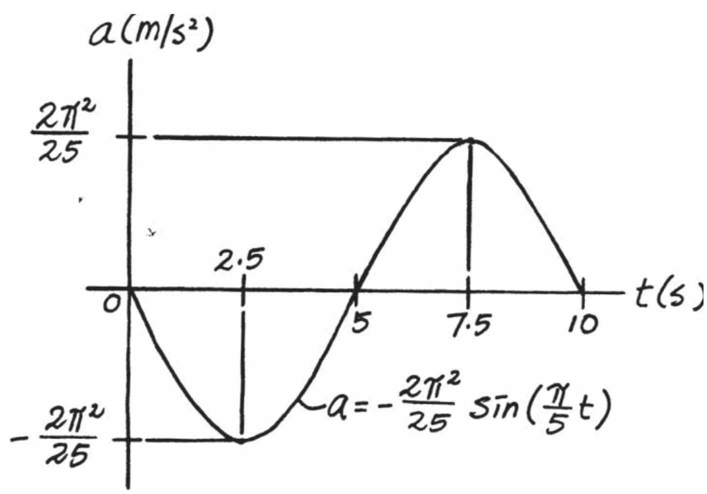
12–39.

If the position of a particle is defined by $s = [2 \sin (\pi/5)t + 4]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \leq t \leq 10$ s.

SOLUTION







Ans:

$$s = 2 \sin\left(\frac{\pi}{5}t\right) + 4$$

$$v = \frac{2\pi}{5} \cos\left(\frac{\pi}{5}t\right)$$

$$a = -\frac{2\pi^2}{25} \sin\left(\frac{\pi}{5}t\right)$$

***12–40.**

An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s^2 until it reaches a constant speed of 220 mi/h. Draw the *s–t*, *v–t*, and *a–t* graphs that describe the motion.

SOLUTION

$v_1 = 0$

$$v_2 = 162\frac{\text{mi}}{\text{h}}\,\frac{(1\text{h})\ 5280\ \text{ft}}{(3600\ \text{s})(1\ \text{mi})} = 237.6\ \text{ft/s}$$

$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$

$(237.6)^2 = 0^2 + 2(a_c)(5000 - 0)$

$a_c = 5.64538\ \text{ft/s}^2$

$v_2 = v_1 + a_c t$

$237.6 = 0 + 5.64538\,t$

$t = 42.09 = 42.1\ \text{s}$

$$v_3 = 220\,\frac{\text{mi}}{\text{h}}\,\frac{(1\text{h})\ 5280\ \text{ft}}{(3600\ \text{s})(1\ \text{mi})} = 322.67\ \text{ft/s}$$

$v_3^2 = v_2^2 + 2a_c(s_3 - s_2)$

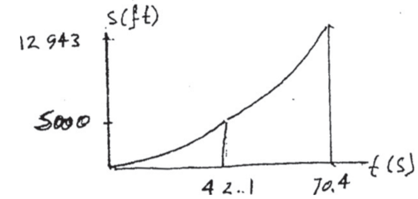
$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$

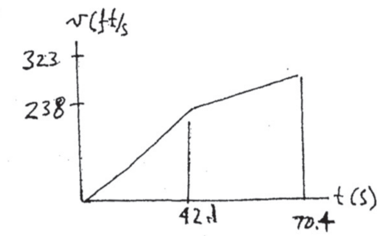
$s = 12\,943.34\ \text{ft}$

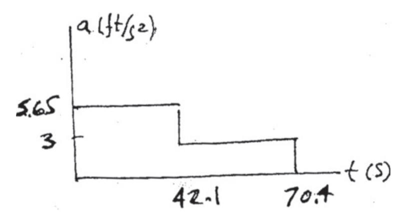
$v_3 = v_2 + a_c t$

$322.67 = 237.6 + 3\,t$

$t = 28.4\ \text{s}$



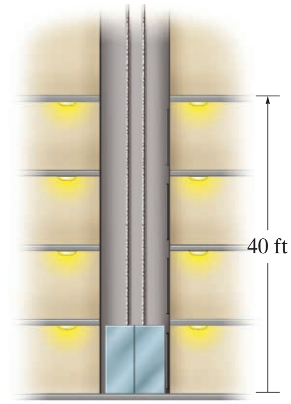




Ans:
$s = 12\,943.34\ \text{ft}$
$v_3 = v_2 + a_c t$
$t = 28.4\ \text{s}$

12–41.

The elevator starts from rest at the first floor of the building. It can accelerate at 5 ft/s^2 and then decelerate at 2 ft/s^2. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the a–t, v–t, and s–t graphs for the motion.



SOLUTION

$+\uparrow\; v_2 = v_1 + a_c t_1$

$v_{max} = 0 + 5\,t_1$

$+\uparrow\; v_3 = v_2 + a_c\, t$

$0 = v_{max} - 2\,t_2$

Thus

$t_1 = 0.4\,t_2$

$+\uparrow\; s_2 = s_1 + v_1 t_1 + \frac{1}{2} a_c t_1^2$

$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5\,t_1^2$

$+\uparrow\; 40 - h = 0 + v_{max} t_2 - \frac{1}{2}(2)\,t_2^2$

$+\uparrow\; v^2 = v_1^2 + 2\,a_c(s - s_1)$

$v_{max}^2 = 0 + 2(5)(h - 0)$

$v_{max}^2 = 10h$

$0 = v_{max}^2 + 2(-2)(40 - h)$

$v_{max}^2 = 160 - 4h$

Thus,

$10\,h = 160 - 4h$

$h = 11.429$ ft

$v_{max} = 10.69$ ft/s

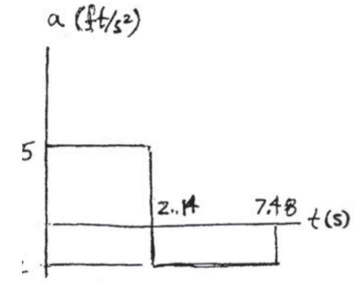
$t_1 = 2.138$ s

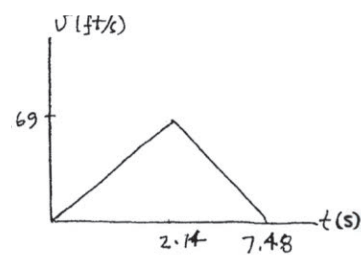
$t_2 = 5.345$ s

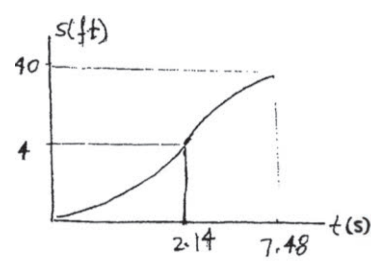
$t = t_1 + t_2 = 7.48$ s **Ans.**

When $t = 2.145$, $v = v_{max} = 10.7$ ft/s

and $h = 11.4$ ft.



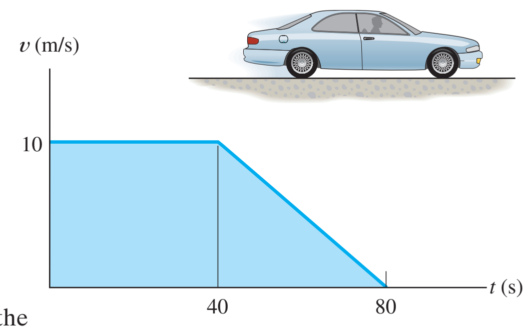




Ans:
$t = 7.48$ s. When $t = 2.14$ s,
$v = v_{max} = 10.7$ ft/s
$h = 11.4$ ft

12–42.

The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops ($t = 80$ s). Construct the a–t graph.



SOLUTION

Distance Traveled: The total distance traveled can be obtained by computing the area under the $v - t$ graph.

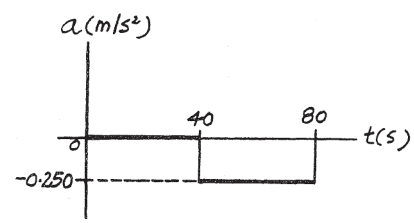
$$s = 10(40) + \frac{1}{2}(10)(80 - 40) = 600 \text{ m} \qquad \textbf{Ans.}$$

a – t Graph: The acceleration in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval $0 \text{ s} \leq t < 40$ s,

$$a = \frac{dv}{dt} = 0$$

For time interval $40 \text{ s} < t \leq 80$ s, $\frac{v - 10}{t - 40} = \frac{0 - 10}{80 - 40}$, $v = \left(-\frac{1}{4}t + 20\right)$ m/s.

$$a = \frac{dv}{dt} = -\frac{1}{4} = -0.250 \text{ m/s}^2$$



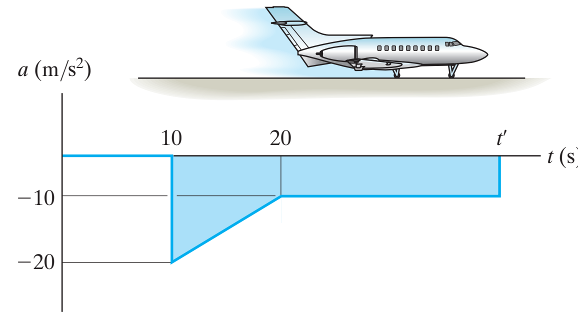
For $0 \leq t < 40$ s, $a = 0$.

For $40 \text{ s} < t \leq 80$, $a = -0.250 \text{ m/s}^2$.

Ans:
$s = 600$ m. For $0 \leq t < 40$ s, $a = 0$. For $40 \text{ s} < t \leq 80$ s, $a = -0.250 \text{ m/s}^2$

12–43.

The motion of a jet plane just after landing on a runway is described by the a–t graph. Determine the time t' when the jet plane stops. Construct the v–t and s–t graphs for the motion. Here $s = 0$, and $v = 300$ ft/s when $t = 0$.



SOLUTION

***v–t* Graph.** The v–t function can be determined by integrating $dv = a\,dt$. For $0 \le t < 10$ s, $a = 0$. Using the initial condition $v = 300$ ft/s at $t = 0$,

$$\int_{300\text{ ft/s}}^{v} dv = \int_0^t 0\,dt$$

$$v - 300 = 0$$

$$v = 300 \text{ ft/s}$$ **Ans.**

For $10\text{ s} < t < 20\text{ s}$, $\dfrac{a - (-20)}{t - 10} = \dfrac{-10 - (-20)}{20 - 10}$, $a = (t - 30)\text{ ft/s}^2$. Using the initial condition $v = 300$ ft/s at $t = 10$ s,

$$\int_{300\text{ ft/s}}^{v} dv = \int_{10\text{ s}}^{t} (t - 30)\,dt$$

$$v - 300 = \left(\frac{1}{2}t^2 - 30t\right)\Bigg|_{10\text{ s}}^{t}$$

$$v = \left\{\frac{1}{2}t^2 - 30t + 550\right\}\text{ ft/s}$$ **Ans.**

At $t = 20$ s,

$$v\Big|_{t=20\text{ s}} = \frac{1}{2}(20^2) - 30(20) + 550 = 150\text{ ft/s}$$

For $20\text{ s} < t < t'$, $a = -10$ ft/s. Using the initial condition $v = 150$ ft/s at $t = 20$ s,

$$\int_{150\text{ ft/s}}^{v} dv = \int_{20\text{ s}}^{t} -10\,dt$$

$$v - 150 = (-10t)\Big|_{20\text{ s}}^{t}$$

$$v = (-10t + 350)\text{ ft/s}$$ **Ans.**

It is required that at $t = t'$, $v = 0$. Thus

$$0 = -10\,t' + 350$$

$$t' = 35\text{ s}$$ **Ans.**

Using these results, the v–t graph shown in Fig. *a* can be plotted ***s-t* Graph**. The s–t function can be determined by integrating $ds = v\,dt$. For $0 \le t < 10$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 300\,dt$$

$$s = \{300\,t\}\text{ ft}$$ **Ans.**

At $= 10$ s,

$$s\big|_{t=10\text{ s}} = 300(10) = 3000\text{ ft}$$

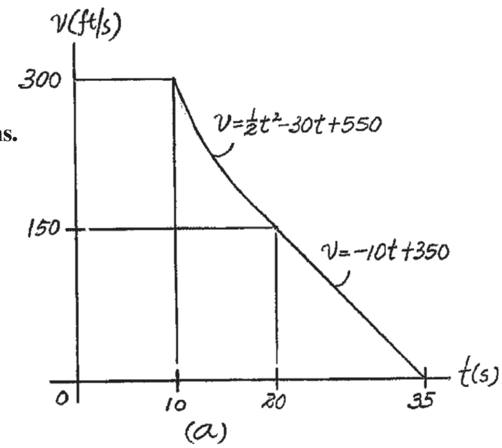
12–43. Continued

For 10 s < t < 20 s, the initial condition is $s = 3000$ ft at $t = 10$ s.

$$\int_{3000\text{ ft}}^{s} ds = \int_{10\text{ s}}^{t} \left(\frac{1}{2}t^2 - 30t + 550\right)dt$$

$$s - 3000 = \left(\frac{1}{6}t^3 - 15t^2 + 550t\right)\Big|_{10\text{ s}}^{t}$$

$$s = \left\{\frac{1}{6}t^3 - 15t^2 + 550t - 1167\right\}\text{ ft} \qquad \textbf{Ans.}$$



At $t = 20$ s,

$$s = \frac{1}{6}(20^3) - 15(20^2) + 550(20) - 1167 = 5167\text{ ft}$$

For 20 s $< t \leq$ 35 s, the initial condition is $s = 5167$ ft at $t = 20$ s.

$$\int_{5167\text{ ft}}^{s} ds = \int_{20\text{ s}}^{t} (-10t + 350)\,dt$$

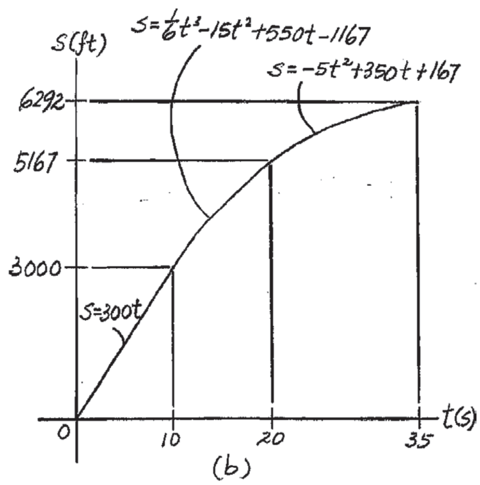
$$s - 5167 = (-5t^2 + 350t)\Big|_{20\text{ s}}^{t}$$

$$s = \{-5t^2 + 350t + 167\}\text{ ft} \qquad \textbf{Ans.}$$

At $t = 35$ s,

$$s\Big|_{t = 35\text{ s}} = -5(35^2) + 350(35) + 167 = 6292\text{ ft}$$

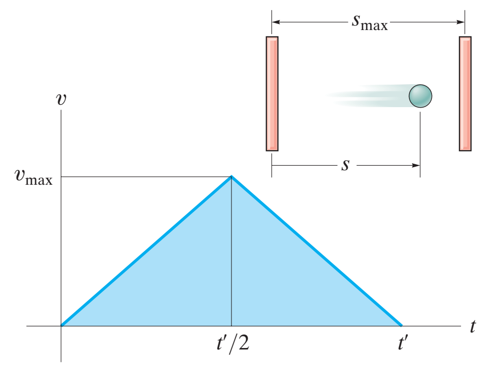
using these results, the s-t graph shown in Fig. b can be plotted.



Ans:
$t' = 35$ s
For $0 \leq t < 10$ s,
$s = \{300t\}$ ft
$v = 300$ ft/s
For 10 s $< t <$ 20 s,
$s = \left\{\frac{1}{6}t^3 - 15t^2 + 550t - 1167\right\}$ ft
$v = \left\{\frac{1}{2}t^2 - 30t + 550\right\}$ ft/s
For 20 s $< t \leq$ 35 s,
$s = \{-5t^2 + 350t + 167\}$ ft
$v = (-10t + 350)$ ft/s

***12–44.**

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of 4 m/s^2. If the plates are spaced 200 mm apart, determine the maximum velocity v_{max} and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.



SOLUTION

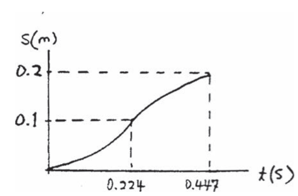
$a_c = 4\text{ m/s}^2$

$\frac{s}{2} = 100\text{ mm} = 0.1\text{ m}$

$v^2 = v_0^2 + 2\,a_c(s - s_0)$

$v_{max}^2 = 0 + 2(4)(0.1 - 0)$

$v_{max} = 0.89442\text{ m/s} \qquad = 0.894\text{ m/s}$ **Ans.**



$v = v_0 + a_c\,t'$

$0.89442 = 0 + 4(\frac{t'}{2})$

$t' = 0.44721\text{ s} \qquad = 0.447\text{ s}$ **Ans.**

$s = s_0 + v_0\,t + \frac{1}{2}\,a_c\,t^2$

$s = 0 + 0 + \frac{1}{2}(4)(t)^2$

$s = 2\,t^2$

When $t = \frac{0.44721}{2} = 0.2236 = 0.224$ s,

$s = 0.1$ m

$\int_{0.894}^{v} ds = -\int_{0.2235}^{t} 4\,dt$

$v = -4\,t + 1.788$

$\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t + 1.788)\,dt$

$s = -2\,t^2 + 1.788\,t - 0.2$

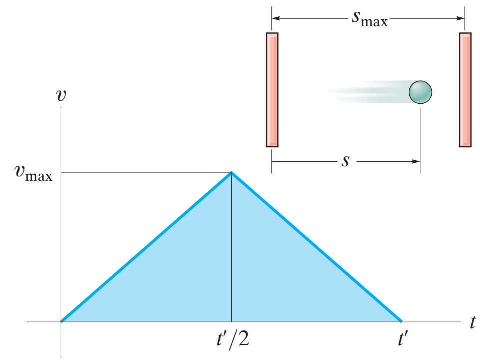
When $t = 0.447$ s,

$s = 0.2$ m

Ans:
$t' = 0.447$ s
$s = 0.2$ m

12–45.

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



SOLUTION

For $0 < t < 0.1$ s,

$$v = 100\,t$$

$$a = \frac{dv}{dt} = 100$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 100\,t\,dt$$

$$s = 50\,t^2$$

When $t = 0.1$ s,

$$s = 0.5 \text{ m}$$

For $0.1 \text{ s} < t < 0.2$ s,

$$v = -100\,t + 20$$

$$a = \frac{dv}{dt} = -100$$

$$ds = v\,dt$$

$$\int_{0.5}^s ds = \int_{0.1}^t (-100t + 20)dt$$

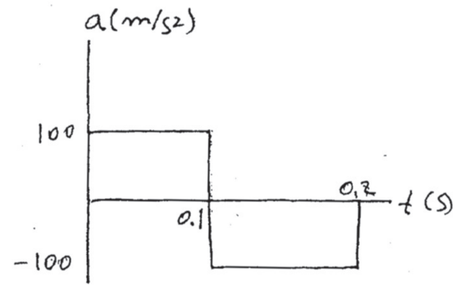
$$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$$

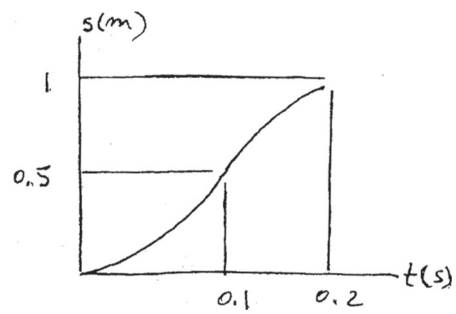
$$s = -50\,t^2 + 20\,t - 1$$

When $t = 0.2$ s,

$$s = 1 \text{ m}$$

When $t = 0.1$ s, $s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When t = 0.2 s, $s = 1$ m.

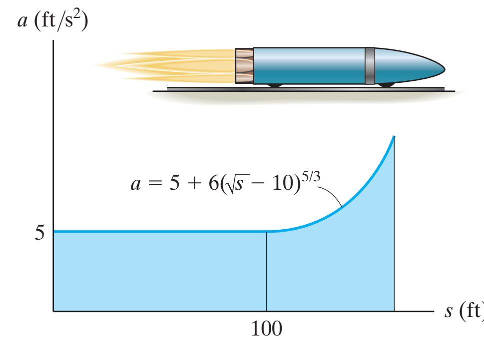




Ans:
When $t = 0.1$ s,
$s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When $t = 0.2$ s, $s = 1$ m.

12–46.

The a–s graph for a rocket moving along a straight track has been experimentally determined. If the rocket starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



SOLUTION

$0 \le s < 100$

$$\int_0^v v\,dv = \int_0^s 5\,ds$$

$$\frac{1}{2}v^2 = 5\,\mathrm{s}$$

$$v = \sqrt{10\,\mathrm{s}}$$

At $s = 75$ ft, $\quad v = \sqrt{750} = 27.4$ ft/s **Ans.**

At $s = 100$ ft, $\quad v = 31.623$

$$v\,dv = a\,ds$$

$$\int_{31.623}^{v} v\,dv = \int_{100}^{125} \left[5 + 6\left(\sqrt{s} - 10\right)^{5/3}\right] ds$$

$$\frac{1}{2}v^2\Big|_{31.623}^{v} = 201.0324$$

$v = 37.4$ ft/s **Ans.**

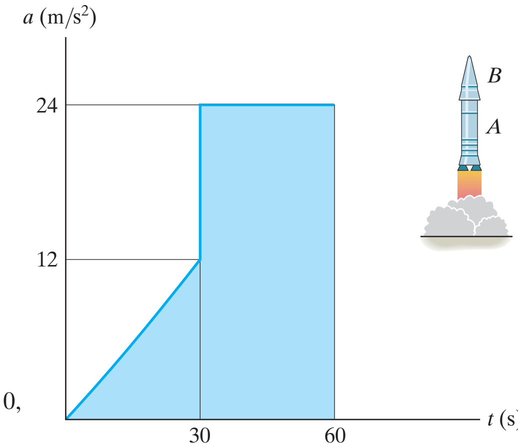
Ans:

$$v\Big|_{s=75\text{ ft}} = 27.4\text{ ft/s}$$

$$v\Big|_{s=125\text{ ft}} = 37.4\text{ ft/s}$$

12–47.

A two-stage rocket is fired vertically from rest at $s = 0$ with the acceleration as shown. After 30 s the first stage, A, burns out and the second stage, B, ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \le t \le 60$ s.



SOLUTION

***v*–*t* Graph.** The $v-t$ function can be determined by integrating $dv = a\,dt$.

For $0 \le t < 30$ s, $a = \dfrac{12}{30}t = \left(\dfrac{2}{5}t\right)$ m/s^2. Using the initial condition $v = 0$ at $t = 0$,

$$\int_0^v dv = \int_0^t \frac{2}{5}t\,dt$$

$$v = \left\{\frac{1}{5}t^2\right\}\text{ m/s} \qquad \textbf{Ans.}$$

At $t = 30$ s,

$$v\Big|_{t=30\text{ s}} = \frac{1}{5}(30^2) = 180\text{ m/s}$$

For $30 < t \le 60$ s, $a = 24$ m/s^2. Using the initial condition $v = 180$ m/s at $t = 30$ s,

$$\int_{180\text{ m/s}}^{v} dv = \int_{30\text{ s}}^{t} 24\,dt$$

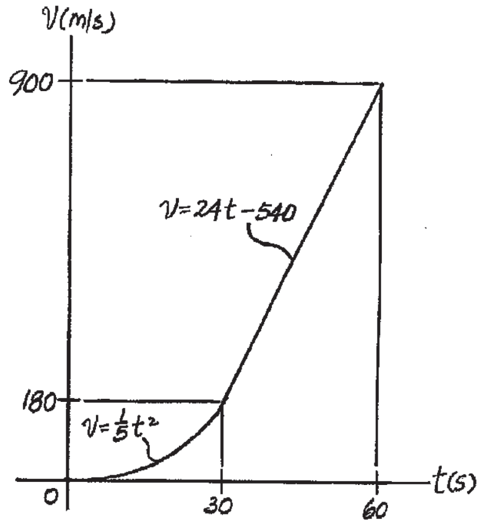
$$v - 180 = 24\,t\Big|_{30\text{ s}}^{t}$$

$$v = \{24t - 540\}\text{ m/s} \qquad \textbf{Ans.}$$

At $t = 60$ s,

$$v\Big|_{t=60\text{ s}} = 24(60) - 540 = 900\text{ m/s}$$

Using these results, $v-t$ graph shown in Fig. a can be plotted.



(a)

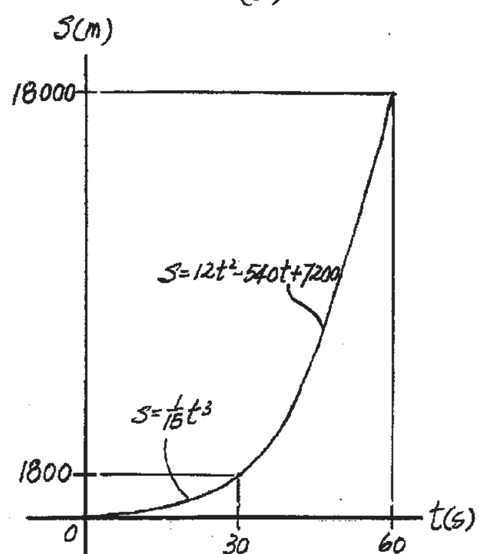
***s*–*t* Graph.** The $s-t$ function can be determined by integrating $ds = v\,dt$. For $0 \le t < 30$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t \frac{1}{5}t^2 dt$$

$$s = \left\{\frac{1}{15}t^3\right\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 30$ s,

$$s\Big|_{t=30\text{ s}} = \frac{1}{15}(30^3) = 1800\text{ m}$$



(b)

12–47. Continued

For 30 s $< t \leq$ 60 s, the initial condition is $s = 1800$ m at $t = 30$ s.

$$\int_{1800\text{ m}}^{s} ds = \int_{30\text{ s}}^{t} (24t - 540)dt$$

$$s - 1800 = (12t^2 - 540t)\Big|_{30\text{ s}}^{t}$$

$$s = \{12t^2 - 540t + 7200\}\text{ m}$$

At $t = 60$ s,

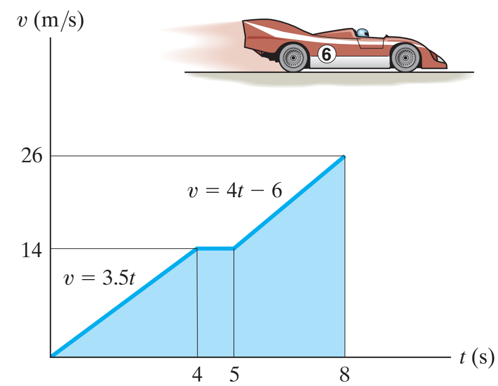
$$s\Big|_{t = 60\text{ s}} = 12(60^2) - 540(60) + 7200 = 18000\text{ m}$$

Using these results, the $s-t$ graph in Fig. b can be plotted.

Ans:
For $0 \leq t < 30$ s,
$v = \left\{\frac{1}{5}t^2\right\}$ m/s
$s = \left\{\frac{1}{15}t^3\right\}$ m
For $30 \leq t \leq 60$ s,
$v = \{24t - 540\}$ m/s
$s = \{12t^2 - 540t + 7200\}$ m

***12–48.**

The race car starts from rest and travels along a straight road until it reaches a speed of 26 m/s in 8 s as shown on the v–t graph. The flat part of the graph is caused by shifting gears. Draw the a–t graph and determine the maximum acceleration of the car.



SOLUTION

For $0 \le t < 4$ s

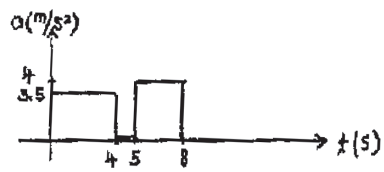
$$a = \frac{\Delta v}{\Delta t} = \frac{14}{4} = 3.5 \text{ m/s}^2$$

For $4 \text{ s} \le t < 5$ s

$$a = \frac{\Delta v}{\Delta t} = 0$$

For $5 \text{ s} \le t < 8$ s

$$a = \frac{\Delta v}{\Delta t} = \frac{26 - 14}{8 - 5} = 4 \text{ m/s}^2$$

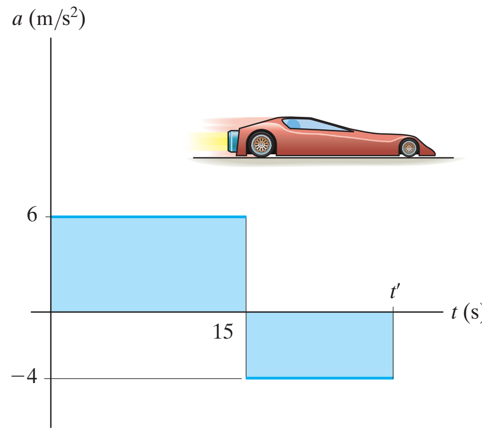


$a_{max} = 4.00 \text{ m/s}^2$ **Ans.**

Ans:
$a_{max} = 4.00 \text{ m/s}^2$

12–49.

The jet car is originally traveling at a velocity of 10 m/s when it is subjected to the acceleration shown. Determine the car's maximum velocity and the time t' when it stops. When $t = 0, s = 0$.



SOLUTION

***v–t* Function.** The $v-t$ function can be determined by integrating $dv = a\,dt$. For $0 \le t < 15$ s, $a = 6\ \text{m/s}^2$. Using the initial condition $v = 10$ m/s at $t = 0$,

$$\int_{10\text{ m/s}}^{v} dv = \int_0^t 6dt$$

$$v - 10 = 6t$$

$$v = \{6t + 10\}\ \text{m/s}$$

The maximum velocity occurs when $t = 15$ s. Then

$$v_{\max} = 6(15) + 10 = 100\ \text{m/s} \qquad \textbf{Ans.}$$

For $15\text{ s} < t \le t'$, $a = -4$ m/s, Using the initial condition $v = 100$ m/s at $t = 15$ s,

$$\int_{100\text{ m/s}}^{v} dv = \int_{15\text{ s}}^{t} -4dt$$

$$v - 100 = (-4t)\Big|_{15\text{ s}}^{t}$$

$$v = \{-4t + 160\}\ \text{m/s}$$

It is required that $v = 0$ at $t = t'$. Then

$$0 = -4t' + 160 \qquad t' = 40\text{ s} \qquad \textbf{Ans.}$$

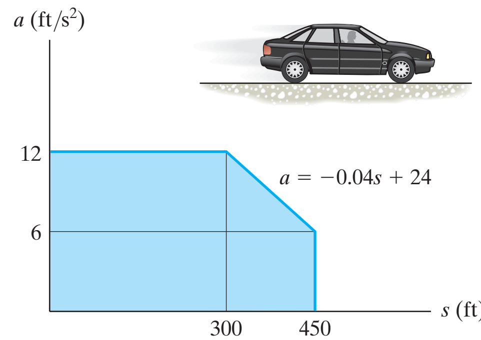
Ans:

$v_{\max} = 100$ m/s

$t' = 40$ s

12–50.

The car starts from rest at $s = 0$ and is subjected to an acceleration shown by the a–s graph. Draw the v–s graph and determine the time needed to travel 200 ft.



SOLUTION

For $s < 300$ ft

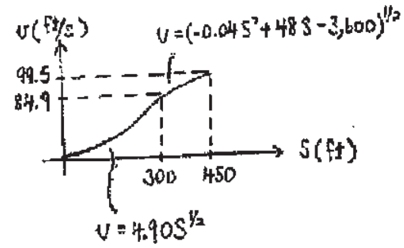
$a\,ds = v\,dv$

$$\int_0^s 12\,ds = \int_0^v v\,dv$$

$$12\text{ s} = \frac{1}{2}v^2$$

$$v = 4.90\text{ s}^{1/2}$$

At $s = 300$ ft, $\quad v = 84.85$ ft/s



For 300 ft $< s < 450$ ft

$a\,ds = v\,dv$

$$\int_{300}^{s} (24 - 0.04\,s)\,ds = \int_{84.85}^{v} v\,dv$$

$$24\,s - 0.02\,s^2 - 5400 = 0.5\,v^2 - 3600$$

$$v = (-0.04\,s^2 + 48\,s - 3600)^{1/2}$$

At $s = 450$ ft, $\quad v = 99.5$ ft/s

$$v = 4.90\text{ s}^{1/2}$$

$$\frac{ds}{dt} = 4.90\text{ s}^{1/2}$$

$$\int_0^{200} s^{-1/2}\,ds = \int_0^t 4.90\,dt$$

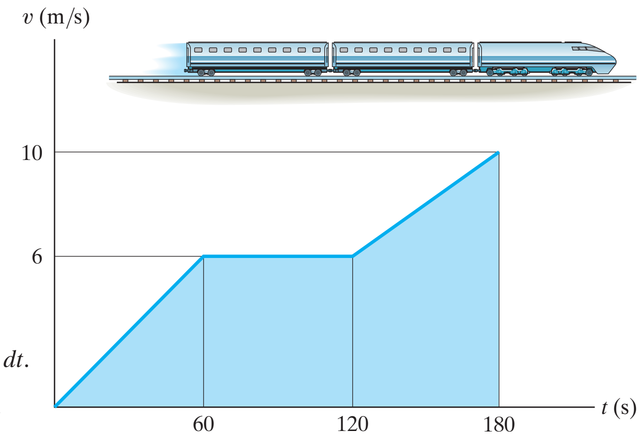
$$2\,s^{1/2}\Big|_0^{200} = 4.90\,t$$

$t = 5.77$ s **Ans.**

Ans:
For $0 \le s < 300$ ft,
$v = \{4.90\,s^{1/2}\}$ m/s.
For 300 ft $< s \le 450$ ft,
$v = \{(-0.04s^2 + 48s - 3600)^{1/2}\}$ m/s.
$s = 200$ ft when $t = 5.77$ s.

12–51.

The v–t graph for a train has been experimentally determined. From the data, construct the s–t and a–t graphs for the motion for $0 \leq t \leq 180$ s. When $t = 0$, $s = 0$.



SOLUTION

***s–t* Graph.** The s–t function can be determined by integrating $ds = v\,dt$. For $0 \leq t < 60$ s, $v = \frac{6}{60}t = \left(\frac{1}{10}t\right)$ m/s. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t \left(\frac{1}{10}t\right)dt$$

$$s = \left\{\frac{1}{20}t^2\right\} \text{ m} \qquad \textbf{Ans.}$$

When $t = 60$ s,

$$s|_{t-60\text{ s}} = \frac{1}{20}(60^2) = 180 \text{ m}$$

For $60 \text{ s} < t < 120$ s, $v = 6$ m/s. Using the initial condition $s = 180$ m at $t = 60$ s,

$$\int_{180\text{ m}}^s ds = \int_{60\text{ s}}^t 6\,dt$$

$$s - 180 = 6t\Big|_{60\text{ s}}^{t}$$

$$s = \{6t - 180\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 120$ s,

$$s|_{t-120\text{ s}} = 6(120) - 180 = 540 \text{ m}$$

For $120 \text{ s} < t \leq 180$ s, $\frac{v-6}{t-120} = \frac{10-6}{180-120}$; $v = \left\{\frac{1}{15}t - 2\right\}$ m/s. Using the initial condition $s = 540$ m at $t = 120$ s,

$$\int_{540\text{ m}}^s ds = \int_{120\text{ s}}^t \left(\frac{1}{15}\text{t} - 2\right)dt$$

$$s - 540 = \left(\frac{1}{30}t^2 - 2t\right)\Big|_{120\text{ s}}^{t}$$

$$s = \left\{\frac{1}{30}t^2 - 2t + 300\right\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 180$ s,

$$s|_{t=180\text{ s}} = \frac{1}{30}(180^2) - 2(180) + 300 = 1020 \text{ m}$$

Using these results, s–t graph shown in Fig. a can be plotted.

12–51. Continued

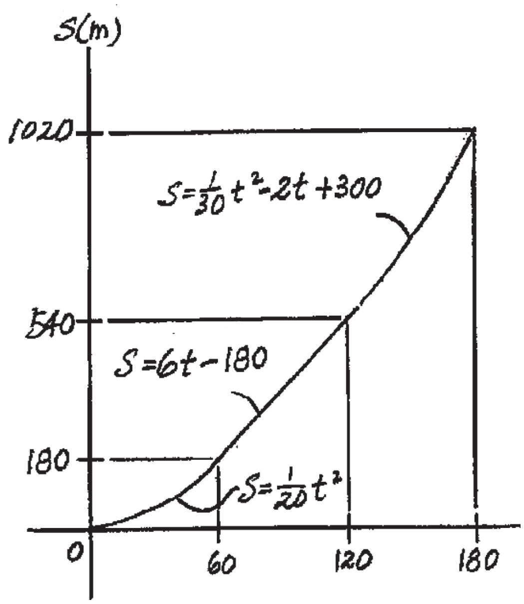
***a–t* Graph.** The *a–t* function can be determined using $a = \dfrac{dv}{dt}$.

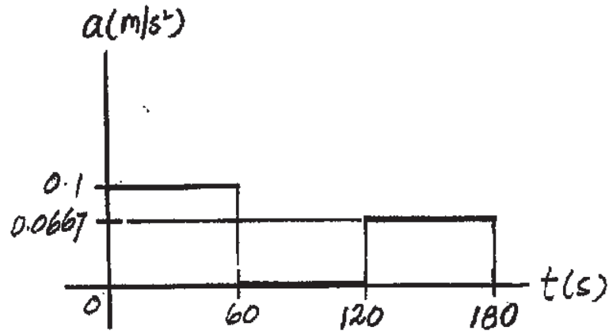
For $0 \le t < 60$ s, $\quad a = \dfrac{d\left(\frac{1}{10}t\right)}{dt} = 0.1\ \text{m/s}^2$ **Ans.**

For $60\text{ s} < t < 120$ s, $\quad a = \dfrac{d(6)}{dt} = 0$ **Ans.**

For $120\text{ s} < t \le 180$ s, $\quad a = \dfrac{d\left(\frac{1}{15}t - 2\right)}{dt} = 0.0667\ \text{m/s}^2$ **Ans.**

Using these results, *a–t* graph shown in Fig. *b* can be plotted.

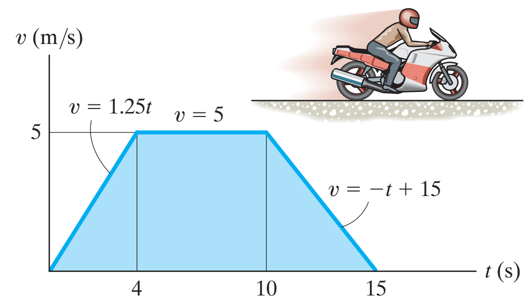




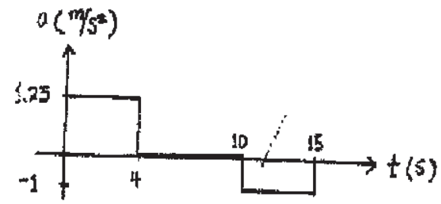
Ans:
For $0 \le t < 60$ s,
$s = \left\{\dfrac{1}{20}t^2\right\}$ m,
$a = 0.1\ \text{m/s}^2$.
For $60\text{ s} < t < 120$ s,
$s = \{6t - 180\}$ m,
$a = 0$. For $120\text{ s} < t \le 180$ s,
$s = \left\{\dfrac{1}{30}t^2 - 2t + 300\right\}$ m,
$a = 0.0667\ \text{m/s}^2$.

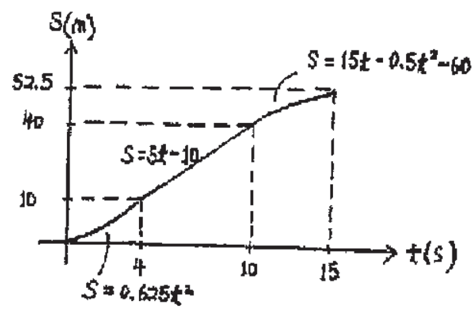
***12–52.**

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the total distance the motorcycle travels until it stops when $t = 15$ s. Also plot the a–t and s–t graphs.



SOLUTION





For $t < 4$ s

$$a = \frac{dv}{dt} = 1.25$$

$$\int_0^s ds = \int_0^t 1.25\, t\, dt$$

$$s = 0.625\, t^2$$

When $t = 4$ s, $\quad s = 10$ m

For $4\text{ s} < t < 10$ s

$$a = \frac{dv}{dt} = 0$$

$$\int_{10}^s ds = \int_4^t 5\, dt$$

$$s = 5\, t - 10$$

When $t = 10$ s, $\quad s = 40$ m

For $10\text{ s} < t < 15$ s

$$a = \frac{dv}{dt} = -1$$

$$\int_{40}^s ds = \int_{10}^t (15 - t)\, dt$$

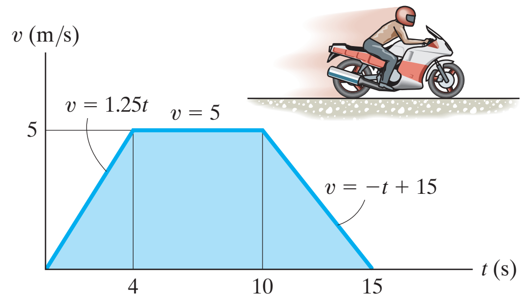
$$s = 15\, t - 0.5\, t^2 - 60$$

When $t = 15$ s, $\quad s = 52.5$ m **Ans.**

Ans:
When $t = 15$ s, $\quad s = 52.5$ m

12–53.

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



SOLUTION

At $t = 8$ s

$$a = \frac{dv}{dt} = 0$$ **Ans.**

$$\Delta s = \int v\,dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (8 - 4)(5) = 30$$

$s = 30$ m **Ans.**

At $t = 12$ s

$$a = \frac{dv}{dt} = \frac{-5}{5} = -1 \text{ m/s}^2$$ **Ans.**

$$\Delta s = \int v\,dt$$

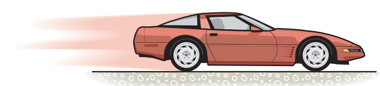
$$s - 0 = \frac{1}{2}(4)(5) + (10 - 4)(5) + \frac{1}{2}(15 - 10)(5) - \frac{1}{2}\left(\frac{3}{5}\right)(5)\left(\frac{3}{5}\right)(5)$$

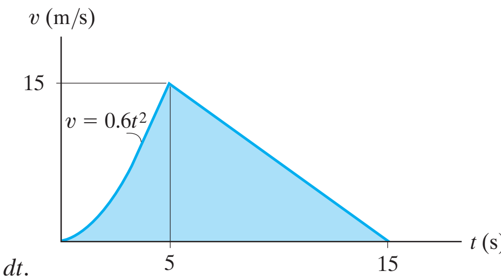
$s = 48$ m **Ans.**

Ans:
At $t = 8$ s,
$a = 0$ and $s = 30$ m.
At $t = 12$ s,
$a = -1 \text{ m/s}^2$
and $s = 48$ m.

12–54.

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t and a–t graphs. Also determine the average speed and the distance traveled for the 15-s time interval. When $t = 0$, $s = 0$.





SOLUTION

***s–t* Graph.** The s–t function can be determined by integrating $ds = v\,dt$. For $0 \le t < 5$ s, $v = 0.6t^2$. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t 0.6t^2 dt$$

$$s = \{0.2t^3\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 5$ s,

$$s|_{t=5\text{ s}} = 0.2(5^3) = 25 \text{ m}$$

For $5 \text{ s} < t \le 15$ s, $\dfrac{v - 15}{t - 5} = \dfrac{0 - 15}{15 - 5}$; $v = \dfrac{1}{2}(45 - 3t)$. Using the initial condition $s = 25$ m at $t = 5$ s,

$$\int_{25\text{ m}}^s ds = \int_{5\text{ s}}^t \frac{1}{2}(45 - 3t)dt$$

$$s - 25 = \frac{45}{2}t - \frac{3}{4}t^2 - 93.75$$

$$s = \left\{\frac{1}{4}(90t - 3t^2 - 275)\right\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 15$ s,

$$s = \frac{1}{4}\left[90(15) - 3(15^2) - 275\right] = 100 \text{ m} \qquad \textbf{Ans.}$$

Thus the average speed is

$$v_{\text{avg}} = \frac{s_T}{t} = \frac{100 \text{ m}}{15 \text{ s}} = 6.67 \text{ m/s} \qquad \textbf{Ans.}$$

using these results, the s–t graph shown in Fig. a can be plotted.

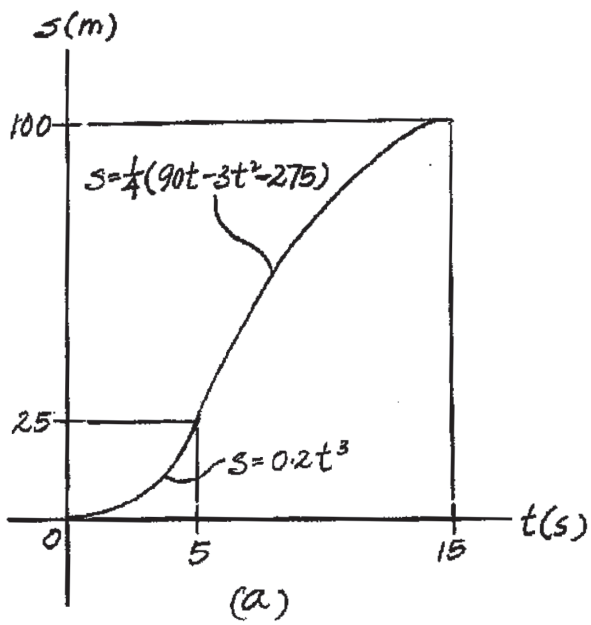
12–54. Continued

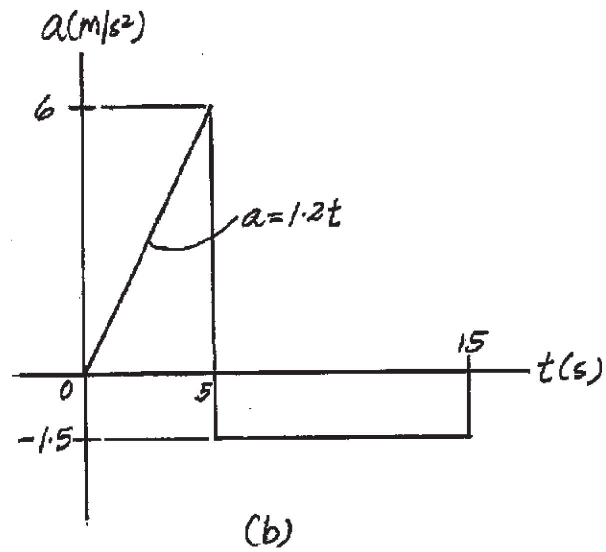
***a–t* Graph.** The *a–t* function can be determined using $a = \frac{dv}{dt}$.

For $0 \le t < 5$ s, $\quad a = \dfrac{d(0.6\,t^2)}{dt} = \{1.2\,t\}\ \text{m/s}^2$ **Ans.**

At $t = 5$ s, $\quad a = 1.2(5) = 6\ \text{m/s}^2$ **Ans.**

For $5\ \text{s} < t \le 15$ s, $\quad a = \dfrac{d\left[\frac{1}{2}(45 - 3t)\right]}{dt} = -1.5\ \text{m/s}^2$ **Ans.**

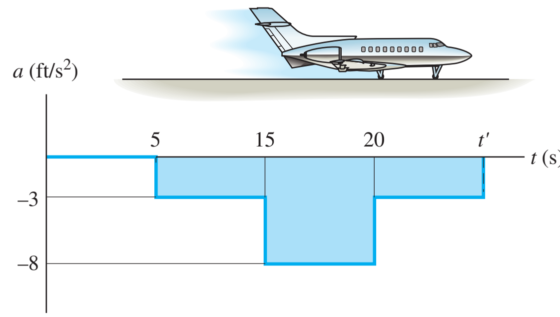




Ans:
For $0 \le t < 5$ s,
$s = \{0.2t^3\}$ m
$a = \{1.2t\}\ \text{m/s}^2$
For $5\ \text{s} < t \le 15$ s,
$s = \left\{\frac{1}{4}(90t - 3t^2 - 275)\right\}$ m
$a = -1.5\ \text{m/s}^2$
At $t = 15$ s,
$s = 100$ m
$v_{\text{avg}} = 6.67$ m/s

12–55.

An airplane lands on the straight runway, originally traveling at 110 ft/s when $s = 0$. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane and construct the s–t graph for the motion.



SOLUTION

$v_0 = 110$ ft/s

$\Delta v = \int a\, dt$

$0 - 110 = -3(15 - 5) - 8(20 - 15) - 3(t' - 20)$

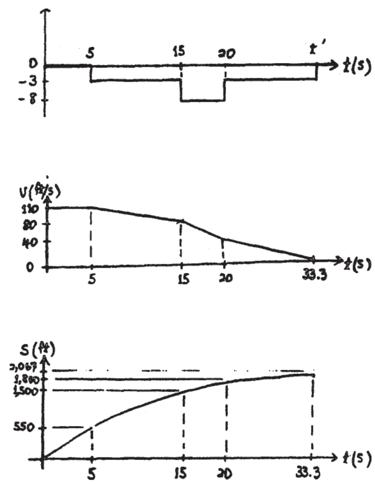
$t' = 33.3$ s **Ans.**

$s\big|_{t = 5\text{s}} = 550$ ft

$s\big|_{t = 15\text{s}} = 1500$ ft

$s\big|_{t = 20\text{s}} = 1800$ ft

$s\big|_{t = 33.3\text{s}} = 2067$ ft

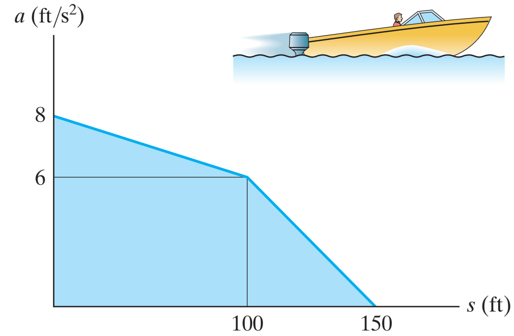


Ans:

$t' = 33.3$ s
$s\big|_{t=5\text{ s}} = 550$ ft
$s\big|_{t=15\text{ s}} = 1500$ ft
$s\big|_{t=20\text{ s}} = 1800$ ft
$s\big|_{t=33.3\text{ s}} = 2067$ ft

***12–56.**

Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the a–s graph. Determine the boat's speed when $s = 50$ ft, 100 ft, and 150 ft.



SOLUTION

***v–s* Function.** The $v-s$ function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 100$ ft, $\dfrac{a-8}{s-0} = \dfrac{6-8}{100-0}$, $a = \left\{-\dfrac{1}{50}s + 8\right\}$ ft/s^2. Using the initial condition $v-0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(-\frac{1}{50}s + 8\right) ds$$

$$\frac{v^2}{2}\Big|_0^v = \left(-\frac{1}{100}s^2 + 8\,s\right)\Big|_0^s$$

$$\frac{v^2}{2} = 8s - \frac{1}{100}s^2$$

$$v = \left\{\sqrt{\frac{1}{50}(800\,s - s^2)}\right\} \text{ ft/s}$$

At $s = 50$ ft,

$$v|_{s=50\text{ ft}} = \sqrt{\frac{1}{50}[800\,(50) - 50^2]} = 27.39 \text{ ft/s} = 27.4 \text{ ft/s} \qquad \textbf{Ans.}$$

At $s = 100$ ft,

$$v|_{s=100\text{ ft}} = \sqrt{\frac{1}{50}[800\,(100) - 100^2]} = 37.42 \text{ ft/s} = 37.4 \text{ ft/s} \qquad \textbf{Ans.}$$

For 100 ft $< s \le 150$ ft, $\dfrac{a-0}{s-150} = \dfrac{6-0}{100-150}$; $a = \left\{-\dfrac{3}{25}s + 18\right\}$ ft/s^2. Using the initial condition $v = 37.42$ ft/s at $s = 100$ ft,

$$\int_{37.42\text{ ft/s}}^v v\,dv = \int_{100\text{ ft}}^s \left(-\frac{3}{25}s + 18\right) ds$$

$$\frac{v^2}{2}\Big|_{37.42\text{ ft/s}}^v = \left(-\frac{3}{50}s^2 + 18s\right)\Big|_{100\text{ ft}}^s$$

$$v = \left\{\frac{1}{5}\sqrt{-3s^2 + 900s - 25000}\right\} \text{ ft/s}$$

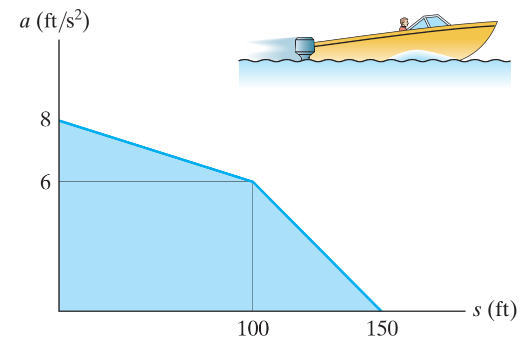
At $s = 150$ ft

$$v|_{s=150\text{ ft}} = \frac{1}{5}\sqrt{-3(150^2) + 900\,(150) - 25000} = 41.23 \text{ ft/s} = 41.2 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v|_{s=50\text{ ft}} = 27.4$ ft/s
$v|_{s=100\text{ ft}} = 37.4$ ft/s
$v|_{s=150\text{ ft}} = 41.2$ ft/s

12–57.

Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the a–s graph. Construct the v–s graph.



SOLUTION

v–s Graph. The $v-s$ function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 100$ ft, $\dfrac{a-8}{s-0} = \dfrac{6-8}{100-0}$, $a = \left\{-\dfrac{1}{50}s + 8\right\}$ ft/s^2 using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(-\frac{1}{50}s + 8\right) ds$$

$$\left.\frac{v^2}{2}\right|_0^v = \left.\left(-\frac{1}{100}s^2 + 8s\right)\right|_0^s$$

$$\frac{v^2}{2} = 8s - \frac{1}{100}s^2$$

$$v = \left\{\sqrt{\frac{1}{50}(800s - s^2)}\right\} \text{ ft/s}$$

At $s = 25$ ft, 50 ft, 75 ft and 100 ft

$$v|_{s=25\text{ ft}} = \sqrt{\frac{1}{50}[800(25) - 25^2]} = 19.69 \text{ ft/s}$$

$$v|_{s=50\text{ ft}} = \sqrt{\frac{1}{50}[800(50) - 50^2]} = 27.39 \text{ ft/s}$$

$$v|_{s=75\text{ ft}} = \sqrt{\frac{1}{50}[800(75) - 75^2]} = 32.98 \text{ ft/s}$$

$$v|_{s=100\text{ ft}} = \sqrt{\frac{1}{50}[800(100) - 100^2]} = 37.42 \text{ ft/s}$$

For 100 ft $< s \le 150$ ft, $\dfrac{a-0}{s-150} = \dfrac{6-0}{100-150}$; $a = \left\{-\dfrac{3}{25}s + 18\right\}$ ft/s^2 using the initial condition $v = 37.42$ ft/s at $s = 100$ ft,

$$\int_{37.42\text{ ft/s}}^v v\,dv = \int_{100\text{ ft}}^s \left(-\frac{3}{25}s + 18\right) ds$$

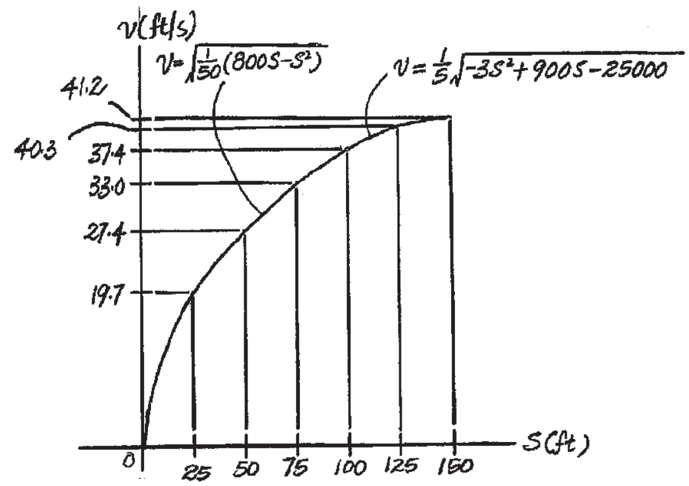
$$\left.\frac{v^2}{2}\right|_{37.42\text{ ft/s}}^v = \left.\left(-\frac{3}{50}s^2 + 18s\right)\right|_{100\text{ ft}}^s$$

$$v = \left\{\frac{1}{5}\sqrt{-3s^2 + 900s - 25000}\right\} \text{ ft/s}$$

At $s = 125$ ft and $s = 150$ ft

$$v|_{s=125\text{ ft}} = \frac{1}{5}\sqrt{-3(125^2) + 900(125) - 25000} = 40.31 \text{ ft/s}$$

$$v|_{s=150\text{ ft}} = \frac{1}{5}\sqrt{-3(150^2) + 900(150) - 25000} = 41.23 \text{ ft/s}$$



Ans:

For $0 \le s < 100$ ft,

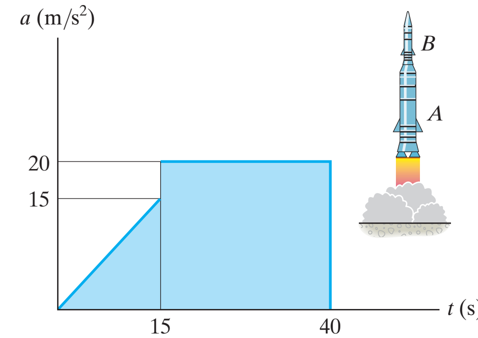
$$v = \left\{\sqrt{\frac{1}{50}(800s - s^2)}\right\} \text{ ft/s}$$

For 100 ft $< s \le 150$ ft,

$$v = \left\{\frac{1}{5}\sqrt{-3s^2 + 900s - 25000}\right\} \text{ ft/s}$$

12–58.

A two-stage rocket is fired vertically from rest with the acceleration shown. After 15 s the first stage A burns out and the second stage B ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \leq t \leq 40$ s.



SOLUTION

For $0 \leq t < 15$

$a = t$

$$\int_0^v dv = \int_0^t t\,dt$$

$$v = \frac{1}{2}t^2$$

$v = 112.5$ when $t = 15$ s

$$\int_0^s ds = \int_0^t \frac{1}{2}t^2\,dt$$

$$s = \frac{1}{6}t^3$$

$s = 562.5$ when $t = 15$ s

For $15 < t < 40$

$a = 20$

$$\int_{112.5}^{v} dv = \int_{1.5}^{t} 20\,dt$$

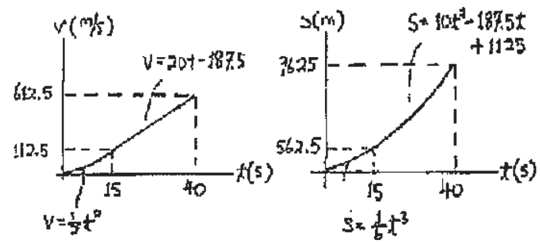
$v = 20t - 187.5$

$v = 612.5$ when $t = 40$ s

$$\int_{562.5}^{s} ds = \int_{15}^{t} (20\,t - 187.5)\,dt$$

$s = 10\,t^2 - 187.5\,t + 1125$

$s = 9625$ when $t = 40$ s



Ans:

For $0 \leq t < 15$ s,

$$v = \left\{\frac{1}{2}t^2\right\} \text{m/s}$$

$$s = \left\{\frac{1}{6}t^3\right\} \text{m}$$

For $15 \text{ s} < t \leq 40$ s,

$v = \{20t - 187.5 \text{ m/s}\}$

$s = \{10t^2 - 187.5t + 1125\}$ m

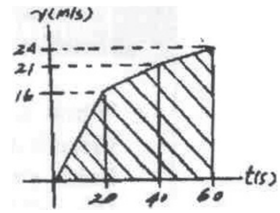
12–59.

The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

Plot the $v-t$ graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

SOLUTION



The total distance traveled is equal to the area under the graph.

$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s_T = 980$ m

***12–60.**

A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v–t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

SOLUTION

For package:

$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s_2 - s_0)$

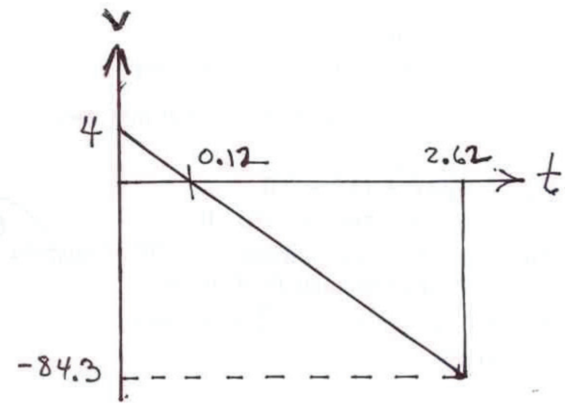
$$v^2 = (4)^2 + 2(-32.2)(0 - 100)$$

$$v = 80.35\ \text{ft/s}\ \downarrow$$

$(+\uparrow)\qquad v = v_0 + a_c t$

$$-80.35 = 4 + (-32.2)t$$

$$t = 2.620\ \text{s}$$



For elevator:

$(+\uparrow)\qquad s_2 = s_0 + vt$

$$s = 100 + 4(2.620)$$

$$s = 110\ \text{ft}$$ **Ans.**

Ans:
$s = 110$ ft

12–61.

Two cars start from rest side by side and travel along a straight road. Car A accelerates at $4\ \text{m/s}^2$ for 10 s and then maintains a constant speed. Car B accelerates at $5\ \text{m/s}^2$ until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?

SOLUTION

Car A:

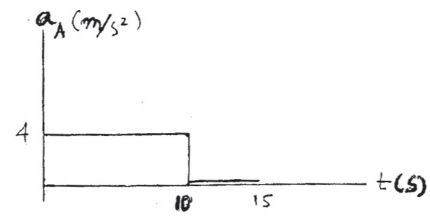
$v = v_0 + a_c t$

$v_A = 0 + 4t$

At $t = 10$ s, $\quad v_A = 40$ m/s

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$

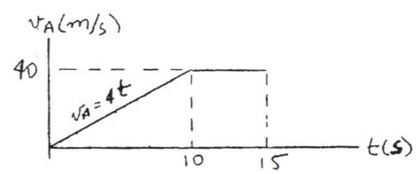


At $t = 10$ s, $\quad s_A = 200$ m

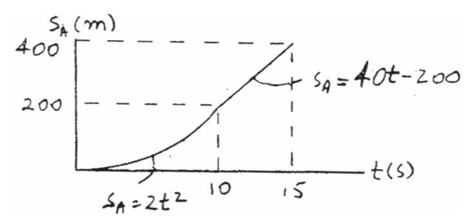
$t > 10$ s, $\quad ds = v\, dt$

$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\, dt$$

$$s_A = 40t - 200$$



At $t = 15$ s, $\quad s_A = 400$ m



Car B:

$$v = v_0 + a_c t$$

$$v_B = 0 + 5t$$

When $v_B = 25$ m/s, $\quad t = \frac{25}{5} = 5$ s

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$$

When $t = 10$ s, $v_A = (v_A)_{max} = 40$ m/s and $s_A = 200$ m.

When $t = 5$ s, $s_B = 62.5$ m.

When $t = 15$ s, $s_A = 400$ m and $s_B = 312.5$ m.

12–61. Continued

At $t = 5$ s, $\quad s_B = 62.5$ m

$t > 5$ s, $\quad ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_5^t 25\,dt$$

$$s_B - 62.5 = 25t - 125$$

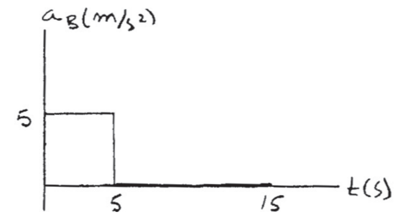
$$s_B = 25t - 62.5$$

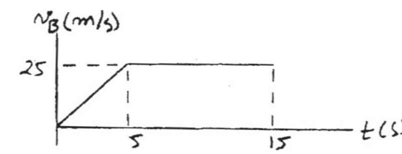
When $t = 15$ s, $\quad s_B = 312.5$

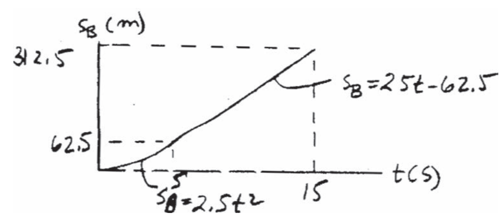
Distance between the cars is

$\Delta s = s_A - s_B = 400 - 312.5 = 87.5$ m **Ans.**

Car A is ahead of car B.





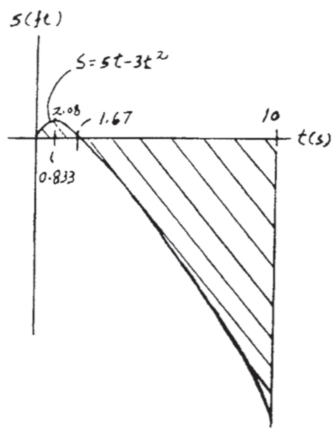


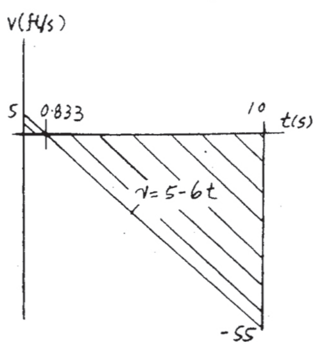
Ans:
When $t = 5$ s,
$s_B = 62.5$ m.
When $t = 10$ s,
$v_A = (v_A)_{\max} = 40$ m/s and
$s_A = 200$ m.
When $t = 15$ s,
$s_A = 400$ m and $s_B = 312.5$ m.
$\Delta s = s_A - s_B = 87.5$ m

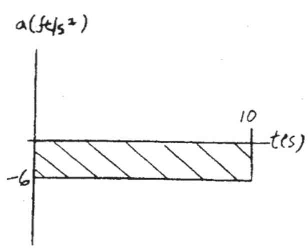
12–62.

If the position of a particle is defined as $s = (5t - 3t^2)$ ft, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \leq t \leq 10$ s.

SOLUTION



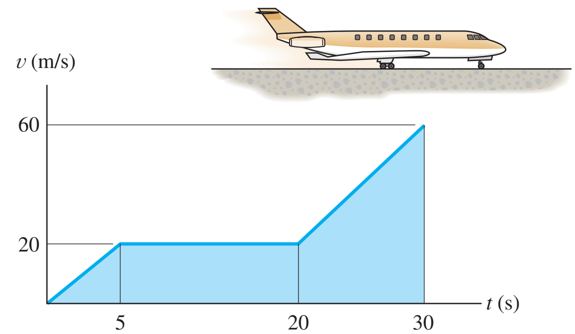




Ans:
$v = \{5 - 6t\}$ ft/s
$a = -6$ ft/s^2

12–63.

From experimental data, the motion of a jet plane while traveling along a runway is defined by the $v-t$ graph. Construct the $s-t$ and $a-t$ graphs for the motion. When $t=0, s=0$.



SOLUTION

***s − t* Graph:** The position in terms of time t can be obtained by applying $v=\frac{ds}{dt}$. For time interval $\mathbf{0\ s \le t < 5\ s}$, $v=\frac{20}{5}t=(4t)$ m/s.

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 4t\,dt$$

$$s = \left(2t^2\right)\ \text{m}$$

When $t=5$ s, $\quad s = 2\left(5^2\right) = 50$ m,

For time interval $\mathbf{5\ s < t < 20\ s}$,

$$ds = v\,dt$$

$$\int_{50\text{ m}}^s ds = \int_{5\text{ a}}^t 20\,dt$$

$$s = (20t - 50)\ \text{m}$$

When $t=20$ s, $\quad s = 20(20) - 50 = 350$ m

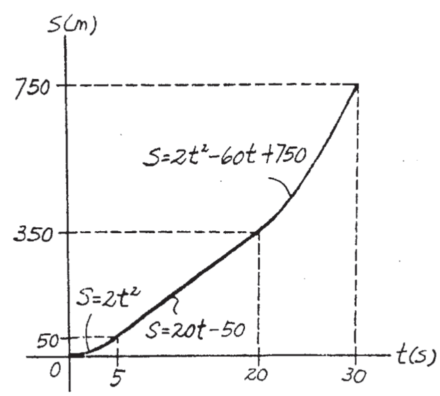
For time interval $\mathbf{20\ s < t \le 30\ s}$, $\frac{v-20}{t-20}=\frac{60-20}{30-20}$, $v=(4t-60)$ m/s.

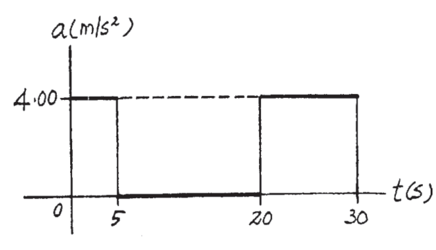
$$ds = v\,dt$$

$$\int_{350\text{ m}}^s ds = \int_{20\text{ a}}^t (4t-60)\,dt$$

$$s = \left(2t^2 - 60t + 750\right)\ \text{m}$$

When $t=30$ s, $\quad s = 2\left(30^2\right) - 60(30) + 750 = 750$ m



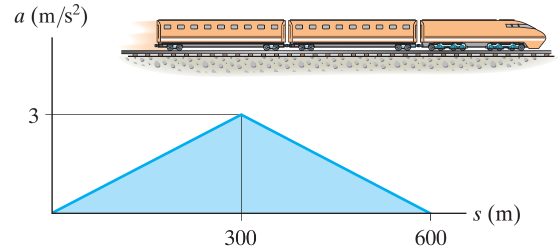


***a − t* Graph:** The acceleration function in terms of time t can be obtained by applying $a=\frac{dv}{dt}$. For time interval $\mathbf{0\ s \le t < 5\ s}$, $\mathbf{5\ s < t < 20\ s}$ and $\mathbf{20\ s < t \le 30\ s}$, $a=\frac{dv}{dt}=4.00\ \text{m/s}^2$, $a=\frac{dv}{dt}=0$ and $a=\frac{dv}{dt}=4.00\ \text{m/s}^2$, respectively.

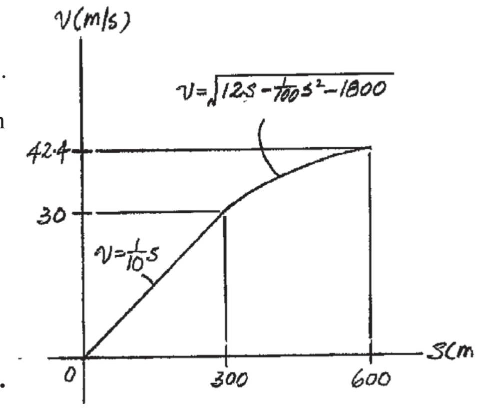
Ans:
For $0 \le t < 5$ s,
$s = \left\{2t^2\right\}$ m and $a = 4\ \text{m/s}^2$.
For $5\ \text{s} < t < 20$ s,
$s = \{20t - 50\}$ m and $a = 0$.
For $20\ \text{s} < t \le 30$ s,
$s = \left\{2t^2 - 60t + 750\right\}$ m
and $a = 4\ \text{m/s}^2$.

***12–64.**

The motion of a train is described by the a–s graph shown. Draw the v–s graph if $v = 0$ at $s = 0$.



SOLUTION



***v*–*s* Graph.** The $v-s$ function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 300$ m, $a = \left(\frac{3}{300}\right)s = \left(\frac{1}{100}s\right)$ m/s^2. Using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(\frac{1}{100}s\right)ds$$

$$\frac{v^2}{2} = \frac{1}{200}s^2$$

$$v = \left\{\frac{1}{10}s\right\}\text{ m/s} \qquad \textbf{Ans.}$$

At $s = 300$ m,

$$v|_{s=300\text{ m}} = \frac{1}{10}(300) = 30\text{ m/s}$$

For $300\text{ m} < s \le 600$ m, $\frac{a-3}{s-300} = \frac{0-3}{600-300}$; $a = \left\{-\frac{1}{100}s + 6\right\}$ m/s^2, using the initial condition $v = 30$ m/s at $s = 300$ m,

$$\int_{30\text{ m/s}}^{v} v\,dv = \int_{300\text{ m}}^{s}\left(-\frac{1}{100}s + 6\right)ds$$

$$\left.\frac{v^2}{2}\right|_{30\text{ m/s}}^{v} = \left.\left(-\frac{1}{200}s^2 + 6s\right)\right|_{300\text{ m}}^{s}$$

$$\frac{v^2}{2} - 450 = 6s - \frac{1}{200}s^2 - 1350$$

$$v = \left\{\sqrt{12s - \frac{1}{100}s^2 - 1800}\right\}\text{ m/s} \qquad \textbf{Ans.}$$

At $s = 600$ m,

$$v = \sqrt{12(600) - \frac{1}{100}(600^2) - 1800} = 42.43\text{ m/s}$$

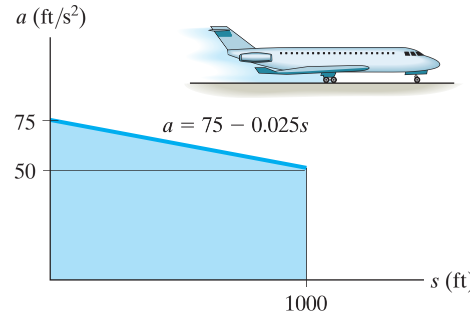
Using the these results, the $v-s$ graph shown in Fig. a can be plotted.

Ans:

$$v = \left\{\frac{1}{10}s\right\}\text{ m/s}$$

12–65.

The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled 1000 ft. Also, how much time is required for it to travel 1000 ft?



SOLUTION

***v–s* Function.** Here, $\dfrac{a-75}{s-0} = \dfrac{50-75}{1000-0}$; $a = \{75 - 0.025s\}\ \text{ft/s}^2$. The function $v_{(s)}$ can be determined by integrating $v\,dv = a\,ds$. Using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s (75 - 0.025\,s)\,ds$$

$$\frac{v^2}{2} = 75\,s - 0.0125\,s^2$$

$$v = \left\{\sqrt{150\,s - 0.025\,s^2}\right\}\ \text{ft/s}$$

At $s = 1000$ ft,

$$v = \sqrt{150\,(1000) - 0.025(1000^2)}$$

$$= 353.55\ \text{ft/s} = 354\ \text{ft/s}$$ **Ans.**

Time. t as a function of s can be determined by integrating $dt = \dfrac{ds}{v}$. Using the initial condition $s = 0$ at $t = 0$;

$$\int_0^t dt = \int_0^s \frac{ds}{\sqrt{150\,s - 0.025\,s^2}}$$

$$t = \left[-\frac{1}{\sqrt{0.025}}\sin^{-1}\left(\frac{150 - 0.05\,s}{150}\right)\right]\Bigg|_0^s$$

$$t = \frac{1}{\sqrt{0.025}}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{150 - 0.05\,s}{150}\right)\right]$$

At $s = 1000$ ft,

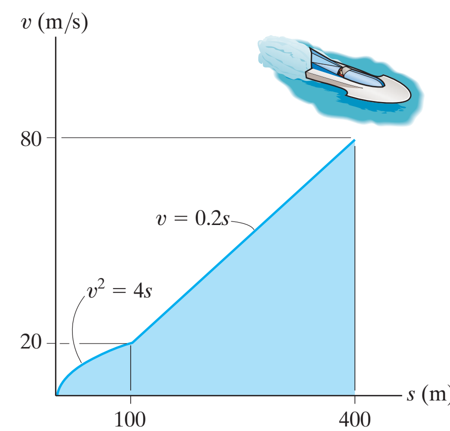
$$t = \frac{1}{\sqrt{0.025}}\left\{\frac{\pi}{2} - \sin^{-1}\left[\frac{150 - 0.05(1000)}{150}\right]\right\}$$

$$= 5.319\ \text{s} = 5.32\ \text{s}$$ **Ans.**

Ans:
$v = 354$ ft/s
$t = 5.32$ s

12–66.

The boat travels along a straight line with the speed described by the graph. Construct the s–t and a–s graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.



SOLUTION

***s–t* Graph:** For $0 \le s < 100$ m, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{2s^{1/2}}$$

$$t = s^{1/2}$$

$$s = \left(t^2\right) \text{ m}$$

When $s = 100$ m,

$$100 = t^2 \qquad t = 10 \text{ s}$$

For $100 \text{ m} < s \le 400$ m, the initial condition is $s = 100$ m when $t = 10$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_{10\text{ s}}^t dt = \int_{100\text{ m}}^s \frac{ds}{0.2s}$$

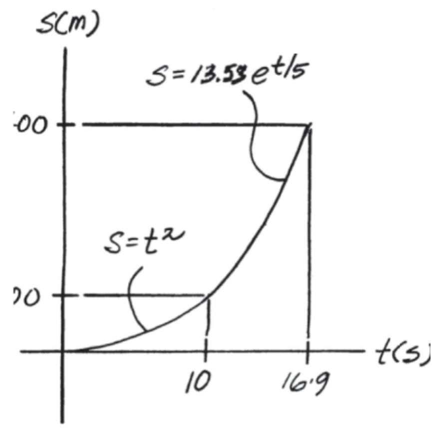
$$t - 10 = 5\ln\frac{s}{100}$$

$$\frac{t}{5} - 2 = \ln\frac{s}{100}$$

$$e^{t/5-2} = \frac{s}{100}$$

$$\frac{e^{t/5}}{e^2} = \frac{s}{100}$$

$$s = \left(13.53e^{t/5}\right) \text{ m}$$



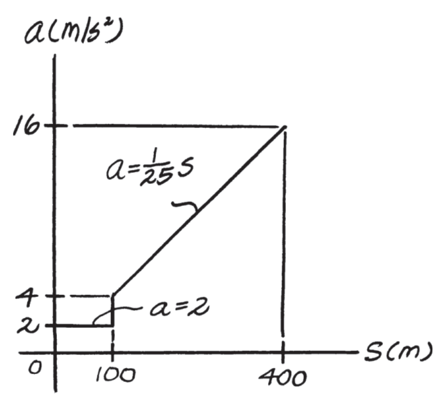
(a)

When $s = 400$ m,

$$400 = 13.53e^{t/5}$$

$$t = 16.93 \text{ s} = 16.9 \text{ s} \qquad \textbf{Ans.}$$

The s–t graph is shown in Fig. a.



(b)

***a–s* Graph:** For $0 \text{ m} \le s < 100$ m,

$$a = v\frac{dv}{ds} = \left(2s^{1/2}\right)\left(s^{-1/2}\right) = 2 \text{ m/s}^2$$

For $100 \text{ m} < s \le 400$ m,

$$a = v\frac{dv}{ds} = (0.2s)(0.2) = 0.04s$$

When $s = 100$ m and 400 m,

$$a\big|_{s=100\text{ m}} = 0.04(100) = 4 \text{ m/s}^2$$

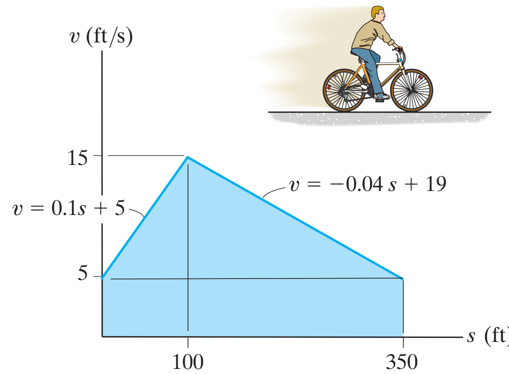
$$a\big|_{s=400\text{ m}} = 0.04(400) = 16 \text{ m/s}^2$$

The a–s graph is shown in Fig. b.

Ans:
When $s = 100$ m,
$t = 10$ s.
When $s = 400$ m,
$t = 16.9$ s.
$a\big|_{s=100\text{ m}} = 4 \text{ m/s}^2$
$a\big|_{s=400\text{ m}} = 16 \text{ m/s}^2$

12–67.

The $v-s$ graph of a cyclist traveling along a straight road is shown. Construct the $a-s$ graph.



SOLUTION

a–s Graph: For $0 \le s < 100$ ft,

$$(\overset{+}{\rightarrow}) \quad a = v\frac{dv}{ds} = (0.1s + 5)(0.1) = (0.01s + 0.5)\text{ ft/s}^2$$

Thus at $s = 0$ and 100 ft

$$a|_{s=0} = 0.01(0) + 0.5 = 0.5\text{ ft/s}^2$$

$$a|_{s=100\text{ ft}} = 0.01(100) + 0.5 = 1.5\text{ ft/s}^2$$

For 100 ft $< s \le 350$ ft,

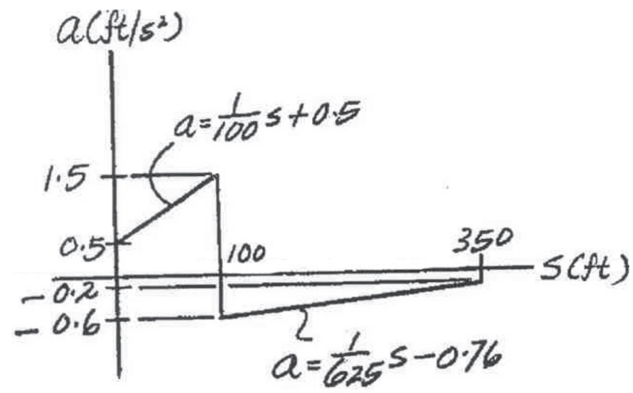
$$(\overset{+}{\rightarrow}) \quad a = v\frac{dv}{ds} = (-0.04s + 19)(-0.04) = (0.0016s - 0.76)\text{ ft/s}^2$$

Thus at $s = 100$ ft and 350 ft

$$a|_{s=100\text{ ft}} = 0.0016(100) - 0.76 = -0.6\text{ ft/s}^2$$

$$a|_{s=350\text{ ft}} = 0.0016(350) - 0.76 = -0.2\text{ ft/s}^2$$

The $a-s$ graph is shown in Fig. a.



(a)

Thus at $s = 0$ and 100 ft

$$a|_{s=0} = 0.01(0) + 0.5 = 0.5\text{ ft/s}^2$$

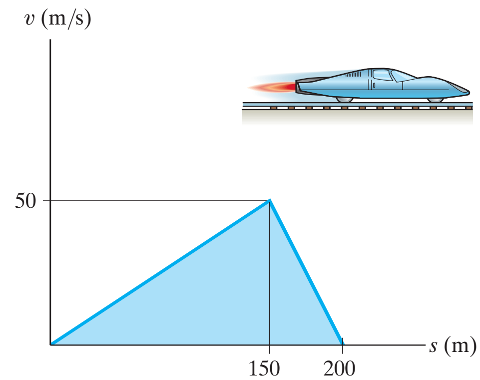
$$a|_{s=100\text{ ft}} = 0.01(100) + 0.5 = 1.5\text{ ft/s}^2$$

At $s = 100$ ft, a changes from $a_{\max} = 1.5\text{ ft/s}^2$ to $a_{\min} = -0.6\text{ ft/s}^2$.

Ans:
At $s = 100$ s,
a changes from $a_{\max} = 1.5\text{ ft/s}^2$
to $a_{\min} = -0.6\text{ ft/s}^2$.

***12–68.**

The v–s graph for a test vehicle is shown. Determine its acceleration when $s = 100$ m and when $s = 175$ m.



SOLUTION

$0 \leq s \leq 150\text{m}$: $\quad v = \frac{1}{3}s,$

$$dv = \frac{1}{3}ds$$

$$v\,dv = a\,ds$$

$$\frac{1}{3}s\left(\frac{1}{3}ds\right) = a\,ds$$

$$a = \frac{1}{9}s$$

At $s = 100$ m, $\quad a = \frac{1}{9}(100) = 11.1 \text{ m/s}^2$ **Ans.**

$150 \leq s \leq 200$ m; $\quad v = 200 - s,$

$$dv = -\,ds$$

$$v\,dv = a\,ds$$

$$(200 - s)(-\,ds) = a\,ds$$

$$a = s - 200$$

At $s = 175$ m, $\quad a = 175 - 200 = -25 \text{ m/s}^2$ **Ans.**

Ans:
At $s = 100$ s, $\quad a = 11.1 \text{ m/s}^2$
At $s = 175$ m, $\quad a = -25 \text{ m/s}^2$

12–69.

If the velocity of a particle is defined as $\mathbf{v}(t) = \{0.8t^2\mathbf{i} + 12t^{1/2}\mathbf{j} + 5\mathbf{k}\}$ m/s, determine the magnitude and coordinate direction angles α, β, γ of the particle's acceleration when $t = 2$ s.

SOLUTION

$$\mathbf{v}(t) = 0.8t^2\,\mathbf{i} + 12t^{1/2}\,\mathbf{j} + 5\mathbf{k}$$

$$\mathbf{a} = \frac{dv}{dt} = 1.6\mathbf{i} + 6t^{1/2}\mathbf{j}$$

When $t = 2$ s, $\quad \mathbf{a} = 3.2\mathbf{i} + 4.243\mathbf{j}$

$$a = \sqrt{(3.2)^2 + (4.243)^2} = 5.31 \text{ m/s}^2$$ **Ans.**

$$u_o = \frac{\mathbf{a}}{a} = 0.6022\mathbf{i} + 0.7984\mathbf{j}$$

$$\alpha = \cos^{-1}(0.6022) = 53.0^\circ$$ **Ans.**

$$\beta = \cos^{-1}(0.7984) = 37.0^\circ$$ **Ans.**

$$\gamma = \cos^{-1}(0) = 90.0^\circ$$ **Ans.**

Ans:
$a = 5.31$ m/s^2
$\alpha = 53.0^\circ$
$\beta = 37.0^\circ$
$\gamma = 90.0^\circ$

12–70.

The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

SOLUTION

Position: The position $\mathbf{r}$ of the particle can be determined by integrating the kinematic equation $d\mathbf{r} = \boldsymbol{v}dt$ using the initial condition $\mathbf{r} = \mathbf{0}$ at $t = 0$ as the integration limit. Thus,

$$d\mathbf{r} = \boldsymbol{v}dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left[3\mathbf{i} + (6 - 2t)\mathbf{j}\right]dt$$

$$\mathbf{r} = \left[3t\mathbf{i} + \left(6t - t^2\right)\mathbf{j}\right]\text{m}$$

When $t = 1$ s and 3 s,

$$r\big|_{t=1\text{ s}} = 3(1)\mathbf{i} + \left[6(1) - 1^2\right]\mathbf{j} = [3\mathbf{i} + 5\mathbf{j}]\text{ m/s}$$

$$r\big|_{t=3\text{ s}} = 3(3)\mathbf{i} + \left[6(3) - 3^2\right]\mathbf{j} = [9\mathbf{i} + 9\mathbf{j}]\text{ m/s}$$

Thus, the displacement of the particle is

$$\Delta\mathbf{r} = \mathbf{r}\big|_{t=3\text{ s}} - \mathbf{r}\big|_{t=1\text{ s}}$$

$$= (9\mathbf{i} + 9\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})$$

$$= \{6\mathbf{i} + 4\mathbf{j}\}\text{ m} \qquad \textbf{Ans.}$$

Ans:
$\Delta\mathbf{r} = \{6\mathbf{i} + 4\mathbf{j}\}$ m

12–71.

A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s^2. Determine the particle's position (x, y, z) at $t = 1$ s.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$dv = \mathrm{a}dt$$

$$\int_0^v dv = \int_0^t (6t\mathbf{i} + 12t^2\mathbf{k})\,dt$$

$$v = \{3t^2\mathbf{i} + 4t^3\mathbf{k}\}\text{ ft/s}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$dr = \mathrm{v}dt$$

$$\int_{\mathrm{r}_1}^{\mathrm{r}} dr = \int_0^t (3t^2\mathbf{i} + 4t^3\mathbf{k})\,dt$$

$$\mathrm{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

$$\mathrm{r} = \{(t^3 + 3)\,\mathbf{i} + 2\mathbf{j} + (t^4 + 5)\mathbf{k}\}\text{ ft}$$

When $t = 1\,s$, $\mathrm{r} = (1^3 + 3)\mathbf{i} + 2\mathbf{j} + (1^4 + 5)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}$ ft.

The coordinates of the particle are

(4 ft, 2 ft, 6 ft) **Ans.**

Ans:
(4 ft, 2 ft, 6 ft)

***12–72.**

The velocity of a particle is given by $\boldsymbol{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

SOLUTION

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\boldsymbol{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12\left(2^2\right)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \boldsymbol{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right]\ \text{m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\}\ \text{m}.$$

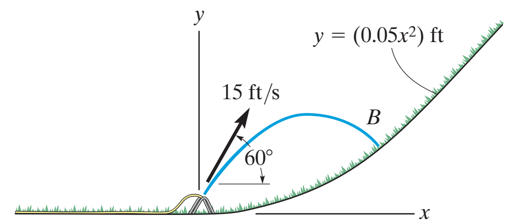
Thus, the coordinate of the particle is

$$(42.7,\ 16.0,\ 14.0)\ \text{m} \qquad \textbf{Ans.}$$

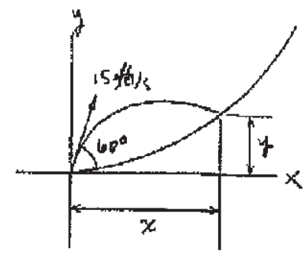
Ans:
(42.7, 16.0, 14.0) m

12–73.

The water sprinkler, positioned at the base of a hill, releases a stream of water with a velocity of 15 ft/s as shown. Determine the point *B(x, y)* where the water strikes the ground on the hill. Assume that the hill is defined by the equation $y = (0.05x^2)$ ft and neglect the size of the sprinkler.



SOLUTION



$v_x = 15 \cos 60° = 7.5 \text{ ft/s} \qquad v_y = 15 \sin 60° = 12.99 \text{ ft/s}$

$(\xrightarrow{+}) \quad s = v_0 t$

$x = 7.5t$

$(+\uparrow) \quad s = s_o + v_o t + \frac{1}{2} a_c t^2$

$y = 0 + 12.99t + \frac{1}{2}(-32.2)t^2$

$y = 1.732x - 0.286x^2$

Since $y = 0.05x^2$,

$0.05x^2 = 1.732x - 0.286x^2$

$x(0.336x - 1.732) = 0$

$x = 5.15 \text{ ft}$ **Ans.**

$y = 0.05(5.15)^2 = 1.33 \text{ ft}$ **Ans.**

Also,

$(\xrightarrow{+}) \quad s = v_0 t$

$x = 15 \cos 60° t$

$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$y = 0 + 15 \sin 60° t + \frac{1}{2}(-32.2)t^2$

Since $y = 0.05x^2$

$12.99t - 16.1t^2 = 2.8125t^2 \quad t = 0.6869 \text{ s}$

So that,

$x = 15 \cos 60° (0.6868) = 5.15 \text{ ft}$ **Ans.**

$y = 0.05(5.15)^2 = 1.33 \text{ ft}$ **Ans.**

Ans:
(5.15 ft, 1.33 ft)

12–74.

A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}\ \text{ft/s}^2$. Determine the particle's position (x, y, z) when $t = 2$ s.

SOLUTION

$$\mathbf{a} = 6t\mathbf{i} + 12t^2\mathbf{k}$$

$$\int_0^v d\mathbf{v} = \int_0^t (6t\mathbf{i} + 12t^2\mathbf{k})\, dt$$

$$\mathbf{v} = 3t^2\mathbf{i} + 4t^3\mathbf{k}$$

$$\int_{\mathrm{r}_0}^{\mathrm{r}} d\mathbf{r} = \int_0^t (3t^2\mathbf{i} + 4t^3\mathbf{k})\, dt$$

$$\mathbf{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

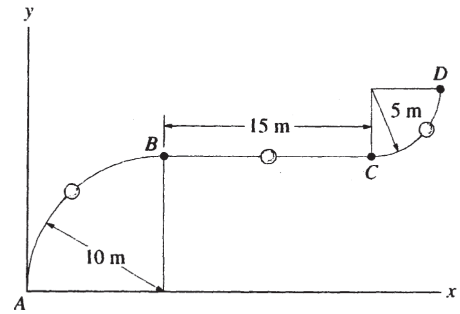
When $t = 2$ s

$\mathbf{r} = \{11\mathbf{i} + 2\mathbf{j} + 21\mathbf{k}\}$ ft **Ans.**

Ans:
$\mathbf{r} = \{11\mathbf{i} + 2\mathbf{j} + 21\mathbf{k}\}$ ft

12–75.

A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



SOLUTION

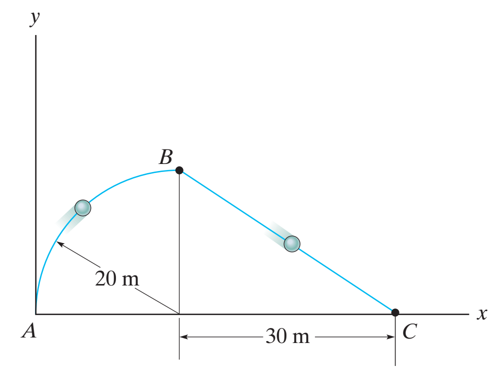
$$s_T = \frac{1}{4}(2\pi)(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sp} = \frac{s_T}{t_t} = \frac{38.56}{2 + 4 + 3} = 4.28 \text{ m/s}$$ **Ans.**

Ans:
$(v_{\text{sp}})_{\text{avg}} = 4.28 \text{ m/s}$

***12–76.**

A particle travels along the curve from A to B in 5 s. It takes 8 s for it to go from B to C and then 10 s to go from C to A. Determine its average speed when it goes around the closed path.



SOLUTION

The total distance traveled is

$$S_{\text{Tot}} = S_{AB} + S_{BC} + S_{CA}$$

$$= 20\left(\frac{\pi}{2}\right) + \sqrt{20^2 + 30^2} + (30 + 20)$$

$$= 117.47 \text{ m}$$

The total time taken is

$$t_{\text{Tot}} = t_{AB} + t_{BC} + t_{CA}$$

$$= 5 + 8 + 10$$

$$= 23 \text{ s}$$

Thus, the average speed is

$$(v_{sp})_{avg} = \frac{S_{\text{Tot}}}{t_{\text{Tot}}} = \frac{117.47 \text{ m}}{23 \text{ s}} = 5.107 \text{ m/s} = 5.11 \text{ m/s}$$ **Ans.**

Ans:
$(v_{sp})_{avg} = 5.11 \text{ m/s}$

12–77.

The position of a crate sliding down a ramp is given by $x = (0.25t^3)$ m, $y = (1.5t^2)$ m, $z = (6 - 0.75t^{5/2})$ m, where t is in seconds. Determine the magnitude of the crate's velocity and acceleration when $t = 2$ s.

SOLUTION

Velocity: By taking the time derivative of x, y, and z, we obtain the x, y, and z components of the crate's velocity.

$$v_x = \dot{x} = \frac{d}{dt}\left(0.25t^3\right) = \left(0.75t^2\right) \text{ m/s}$$

$$v_y = \dot{y} = \frac{d}{dt}\left(1.5t^2\right) = \left(3t\right) \text{ m/s}$$

$$v_z = \dot{z} = \frac{d}{dt}\left(6 - 0.75t^{5/2}\right) = \left(-1.875t^{3/2}\right) \text{ m/s}$$

When $t = 2$ s,

$$v_x = 0.75\left(2^2\right) = 3 \text{ m/s} \qquad v_y = 3(2) = 6 \text{ m/s} \qquad v_z = -1.875\left(2\right)^{3/2} = -5.303 \text{ m/s}$$

Thus, the magnitude of the crate's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2 + v_z{}^2} = \sqrt{3^2 + 6^2 + (-5.303)^2} = 8.551 \text{ ft/s} = 8.55 \text{ ft} \qquad \textbf{Ans.}$$

Acceleration: The x, y, and z components of the crate's acceleration can be obtained by taking the time derivative of the results of v_x, v_y, and v_z, respectively.

$$a_x = \dot{v}_x = \frac{d}{dt}\left(0.75t^2\right) = (1.5t) \text{ m/s}^2$$

$$a_y = \dot{v}_y = \frac{d}{dt}(3t) = 3 \text{ m/s}^2$$

$$a_z = \dot{v}_z = \frac{d}{dt}\left(-1.875t^{3/2}\right) = \left(-2.815t^{1/2}\right) \text{ m/s}^2$$

When $t = 2$ s,

$$a_x = 1.5(2) = 3 \text{ m/s}^2 \qquad a_y = 3 \text{ m/s}^2 \qquad a_z = -2.8125\left(2^{1/2}\right) = -3.977 \text{ m/s}^2$$

Thus, the magnitude of the crate's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} = \sqrt{3^2 + 3^2 + (-3.977)^2} = 5.815 \text{ m/s}^2 = 5.82 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 8.55$ ft/s
$a = 5.82$ m/s^2

12–78.

A rocket is fired from rest at $x = 0$ and travels along a parabolic trajectory described by $y^2 = [120(10^3)x]$ m. If the x component of acceleration is $a_x = \left(\frac{1}{4}t^2\right)$ m/s^2, where t is in seconds, determine the magnitude of the rocket's velocity and acceleration when $t = 10$ s.

SOLUTION

Position: The parameter equation of x can be determined by integrating a_x twice with respect to t.

$$\int dv_x = \int a_x dt$$

$$\int_0^{v_x} dv_x = \int_0^t \frac{1}{4}t^2 dt$$

$$v_x = \left(\frac{1}{12}t^3\right) \text{ m/s}$$

$$\int dx = \int v_x dt$$

$$\int_0^x dx = \int_0^t \frac{1}{12}t^3 dt$$

$$x = \left(\frac{1}{48}t^4\right) \text{ m}$$

Substituting the result of x into the equation of the path,

$$y^2 = 120\left(10^3\right)\left(\frac{1}{48}t^4\right)$$

$$y = \left(50t^2\right) \text{ m}$$

Velocity:

$$v_y = \dot{y} = \frac{d}{dt}\left(50t^2\right) = \left(100t\right) \text{ m/s}$$

When $t = 10$ s,

$$v_x = \frac{1}{12}\left(10^3\right) = 83.33 \text{ m/s} \qquad v_y = 100(10) = 1000 \text{ m/s}$$

Thus, the magnitude of the rocket's velocity is

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}} = \sqrt{83.33^2 + 1000^2} = 1003 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_y = \dot{v}_y = \frac{d}{dt}(100t) = 100 \text{ m/s}^2$$

When $t = 10$ s,

$$a_x = \frac{1}{4}\left(10^2\right) = 25 \text{ m/s}^2$$

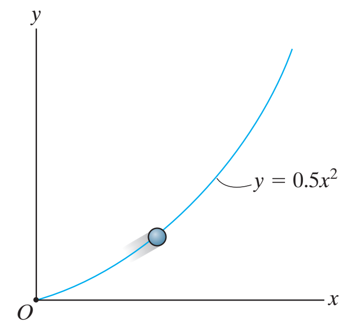
Thus, the magnitude of the rocket's acceleration is

$$a = \sqrt{a_x^{\,2} + a_y^{\,2}} = \sqrt{25^2 + 100^2} = 103 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 1003$ m/s
$a = 103$ m/s^2

12–79.

The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



SOLUTION

Position: The x position of the particle can be obtained by applying the $v_x = \dfrac{dx}{dt}$.

$$dx = v_x dt$$

$$\int_0^x dx = \int_0^t 5t dt$$

$$x = \left(2.50t^2\right) \text{ ft}$$

Thus, $y = 0.5\left(2.50t^2\right)^2 = \left(3.125t^4\right)$ ft. At $t = 1$ s, $x = 2.5\left(1^2\right) = 2.50$ ft and $y = 3.125\left(1^4\right) = 3.125$ ft. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50 - 0)^2 + (3.125 - 0)^2} = 4.00 \text{ ft} \qquad \textbf{Ans.}$$

Acceleration: Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x} \qquad \textbf{(1)}$$

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. (1) becomes

$$a_y = v_x^2 + xa_x \qquad \textbf{(2)}$$

When $t = 1$ s, $v_x = 5(1) = 5$ ft/s $a_x = \dfrac{dv_x}{dt} = 5$ ft/s^2, and $x = 2.50$ ft. Then, from Eq. (2)

$$a_y = 5^2 + 2.50(5) = 37.5 \text{ ft/s}^2$$

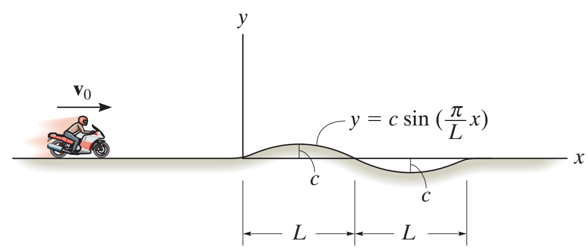
Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$d = 4.00$ ft
$a = 37.8$ ft/s^2

***12–80.**

The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



SOLUTION

$$y = c\sin\left(\frac{\pi}{L}x\right)$$

$$\dot{y} = \frac{\pi}{L}c\left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c\,v_x\left(\cos\frac{\pi}{L}x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

$$v_0^2 = v_x^2\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]$$

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}} \qquad \textbf{Ans.}$$

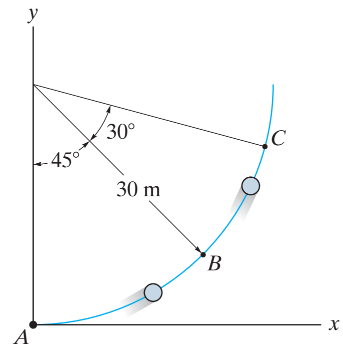
Ans:

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2\cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

12–81.

A particle travels along the circular path from A to B in 1 s. If it takes 3 s for it to go from A to C, determine its *average velocity* when it goes from B to C.



SOLUTION

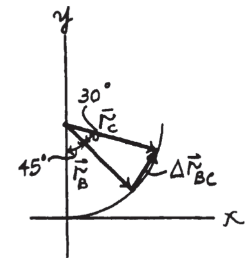
Position: The coordinates for points B and C are [30 sin 45°, 30 − 30 cos 45°] and [30 sin 75°, 30 − 30 cos 75°]. Thus,

$$\mathbf{r}_B = (30 \sin 45° - 0)\mathbf{i} + [(30 - 30 \cos 45°) - 30]\mathbf{j}$$

$$= \{21.21\mathbf{i} - 21.21\mathbf{j}\} \text{ m}$$

$$\mathbf{r}_C = (30 \sin 75° - 0)\mathbf{i} + [(30 - 30 \cos 75°) - 30]\mathbf{j}$$

$$= \{28.98\mathbf{i} - 7.765\mathbf{j}\} \text{ m}$$



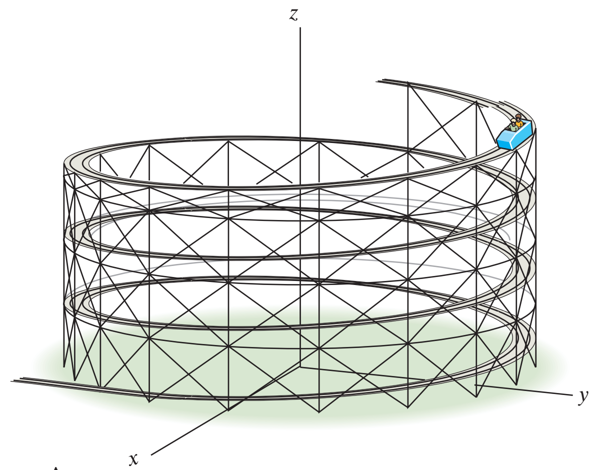
Average Velocity: The displacement from point B to C is $\Delta\mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B = (28.98\mathbf{i} - 7.765\mathbf{j}) - (21.21\mathbf{i} - 21.21\mathbf{j}) = \{7.765\mathbf{i} + 13.45\mathbf{j}\}$ m.

$$(\mathbf{v}_{BC})_{\text{avg}} = \frac{\Delta\mathbf{r}_{BC}}{\Delta t} = \frac{7.765\mathbf{i} + 13.45\mathbf{j}}{3 - 1} = \{3.88\mathbf{i} + 6.72\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$(\mathbf{v}_{BC})_{\text{avg}} = \{3.88\mathbf{i} + 6.72\mathbf{j}\}$ m/s

12–82.

The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c, h, and b are constants. Determine the magnitudes of its velocity and acceleration.



SOLUTION

$x = c \sin kt \qquad \dot{x} = \text{ck} \cos kt \qquad \ddot{x} = -ck^2 \sin kt$

$y = c \cos kt \qquad \dot{y} = -\text{ck} \sin kt \qquad \ddot{y} = -ck^2 \cos kt$

$z = h - bt \qquad \dot{z} = -b \qquad \ddot{z} = 0$

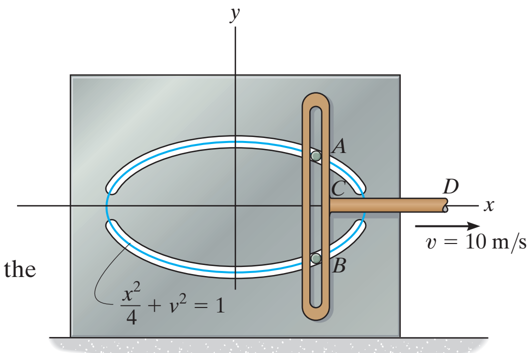
$v = \sqrt{(ck \cos kt)^2 + (-ck \sin kt)^2 + (-b)^2} = \sqrt{c^2k^2 + b^2}$ **Ans.**

$a = \sqrt{(-ck^2 \sin kt)^2 + (-ck^2 \cos kt)^2 + 0} = ck^2$ **Ans.**

Ans:
$v = \sqrt{c^2 k^2 + b^2}$
$a = ck^2$

12–83.

Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg A when $x = 1\,\text{m}$.



SOLUTION

Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\frac{x^2}{4} + y^2 = 1$$

$$\frac{1}{4}(2x\dot{x}) + 2y\dot{y} = 0$$

$$\frac{1}{2}x\dot{x} + 2y\dot{y} = 0$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \qquad \textbf{(1)}$$

At $x = 1$ m,

$$\frac{(1)^2}{4} + y^2 = 1 \qquad\qquad y = \frac{\sqrt{3}}{2}\text{ m}$$

Here, $v_x = 10$ m/s and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \qquad\qquad v_y = -2.887\text{ m/s} = 2.887\text{ m/s}\downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{10^2 + 2.887^2} = 10.4\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) = 0$$

$$\frac{1}{2}\left(\dot{x}^2 + x\ddot{x}\right) + 2\left(\dot{y}^2 + y\ddot{y}\right) = 0$$

or

$$\frac{1}{2}\left(v_x{}^2 + xa_x\right) + 2\left(v_y{}^2 + ya_y\right) = 0 \qquad \textbf{(2)}$$

Since v_x is constant, $a_x = 0$. When $x = 1$ m, $y = \frac{\sqrt{3}}{2}$ m, $v_x = 10$ m/s, and $v_y = -2.887$ m/s. Substituting these values into Eq. (2),

$$\frac{1}{2}\left(10^2 + 0\right) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] = 0$$

$$a_y = -38.49\text{ m/s}^2 = 38.49\text{ m/s}^2\downarrow$$

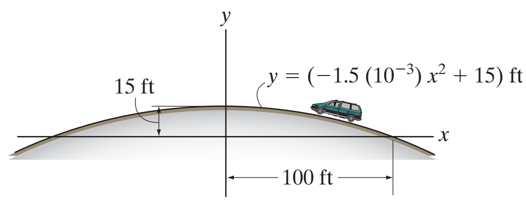
Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{0^2 + (-38.49)^2} = 38.5\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 10.4$ m/s
$a = 38.5\text{ m/s}^2$

***12–84.**

The van travels over the hill described by $y = (-1.5(10^{-3})\,x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when $x = 50$ ft.



SOLUTION

Velocity: The x and y components of the van's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = -1.5\left(10^{-3}\right)x^2 + 15$$

$$\dot{y} = -3\left(10^{-3}\right)x\dot{x}$$

or

$$v_y = -3\left(10^{-3}\right)xv_x$$

When $x = 50$ ft,

$$v_y = -3\left(10^{-3}\right)(50)v_x = -0.15v_x \qquad \textbf{(1)}$$

The magnitude of the van's velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \textbf{(2)}$$

Substituting $v = 75$ ft/s and Eq. (1) into Eq. (2),

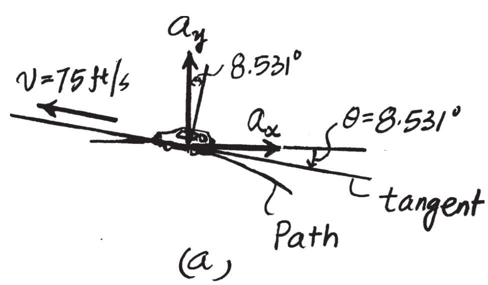
$$75 = \sqrt{{v_x}^2 + (-0.15v_x)^2}$$

$$v_x = 74.2 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Substituting the result of v_x into Eq. (1), we obtain

$$v_y = -0.15(-74.17) = 11.12 \text{ ft/s} = 11.1 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the van's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.



$$\ddot{y} = -3\left(10^{-3}\right)(\dot{x}\dot{x} + x\ddot{x})$$

or

$$a_y = -3\left(10^{-3}\right)\left({v_x}^2 + xa_x\right)$$

When $x = 50$ ft, $v_x = -74.17$ ft/s. Thus,

$$a_y = -3\left(10^{-3}\right)\left[(-74.17)^2 + 50a_x\right]$$

$$a_y = -(16.504 + 0.15a_x) \qquad \textbf{(3)}$$

Since the van travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at $x = 50$ ft is $\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\Big|_{x=50\text{ ft}} = \tan^{-1}\left[-3\left(10^{-3}\right)x\right]\Big|_{x=50\text{ ft}} = \tan^{-1}(-0.15) = -8.531°$.
Thus, from the diagram shown in Fig. *a*,

$$a_x \cos 8.531° - a_y \sin 8.531° = 0 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

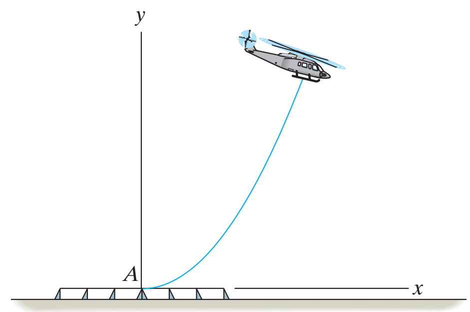
$$a_x = -2.42 \text{ ft/s} = 2.42 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$a_y = -16.1 \text{ ft/s} = 16.1 \text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

Ans:
$v_x = 74.2$ ft/s $\leftarrow$
$v_y = 11.1$ ft/s $\uparrow$
$a_x = 2.42$ ft/s^2 $\leftarrow$
$a_y = 16.1$ ft/s^2 $\downarrow$

12–85.

The flight path of the helicopter as it takes off from A is defined by the parametric equations $x = (2t^2)$ m and $y = (0.04t^3)$ m, where t is the time in seconds. Determine the distance the helicopter is from point A and the magnitudes of its velocity and acceleration when $t = 10$ s.



SOLUTION

$x = 2t^2 \qquad y = 0.04t^3$

At $t = 10$ s, $\qquad x = 200$ m $\qquad y = 40$ m

$d = \sqrt{(200)^2 + (40)^2} = 204 \text{ m}$ **Ans.**

$$v_x = \frac{dx}{dt} = 4t$$

$$a_x = \frac{dv_x}{dt} = 4$$

$$v_y = \frac{dy}{dt} = 0.12t^2$$

$$a_y = \frac{dv_y}{dt} = 0.24t$$

At $t = 10$ s,

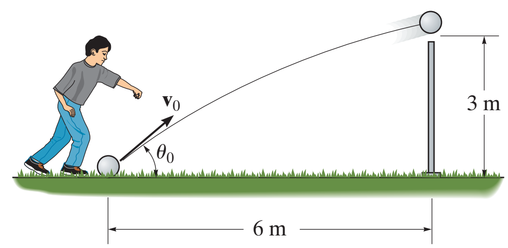
$v = \sqrt{(40)^2 + (12)^2} = 41.8 \text{ m/s}$ **Ans.**

$a = \sqrt{(4)^2 + (2.4)^2} = 4.66 \text{ m/s}^2$ **Ans.**

Ans:
$d = 204$ m
$v = 41.8$ m/s
$a = 4.66$ m/s^2

12–86.

Determine the minimum initial velocity v_0 and the corresponding angle θ_0 at which the ball must be kicked in order for it to just cross over the 3-m high fence.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set so that its origin coincides with the ball's initial position.

x-Motion: Here, $(v_0)_x = v_0 \cos\theta$, $x_0 = 0$, and $x = 6$ m. Thus,

$$\left(\xrightarrow{+}\right) \qquad x = x_0 + (v_0)x^t$$

$$6 = 0 + (v_0 \cos\theta)t$$

$$t = \frac{6}{v_0 \cos\theta} \qquad \textbf{(1)}$$

y-Motion: Here, $(v_0)_x = v_0 \sin\theta$, $a_y = -g = -9.81 \text{ m/s}^2$, and $y_0 = 0$. Thus,

$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$3 = 0 + v_0 (\sin\theta)\, t + \frac{1}{2}(-9.81)t^2$$

$$3 = v_0 (\sin\theta)\, t - 4.905t^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$v_0 = \sqrt{\frac{58.86}{\sin 2\theta - \cos^2\theta}} \qquad \textbf{(3)}$$

From Eq. (3), we notice that v_0 is minimum when $f(\theta) = \sin 2\theta - \cos^2\theta$ is maximum. This requires $\dfrac{df(\theta)}{d\theta} = 0$

$$\frac{df(\theta)}{d\theta} = 2\cos 2\theta + \sin 2\theta = 0$$

$$\tan 2\theta = -2$$

$$2\theta = 116.57°$$

$$\theta = 58.28° = 58.3° \qquad \textbf{Ans.}$$

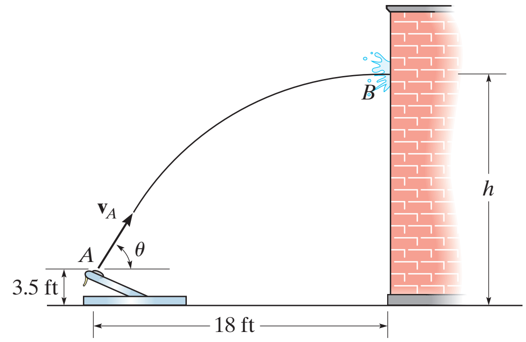
Substituting the result of θ into Eq. (2), we have

$$(v_0)_{min} = \sqrt{\frac{58.86}{\sin 116.57° - \cos^2 58.28°}} = 9.76 \text{ m/s} \qquad \textbf{Ans.}$$

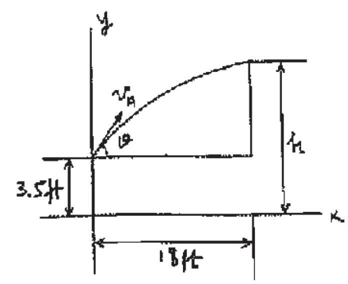
Ans:
$\theta = 58.3°$
$(v_0)_{\text{min}} = 9.76 \text{ m/s}$

12–87.

The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B, determine the velocity $\mathbf{v}_A$ at which it was launched, the angle of release θ, and the height h.



SOLUTION



$$(\overset{+}{\rightarrow}) \quad s = v_0 t$$

$$18 = v_A \cos\theta(1.5) \qquad \textbf{(1)}$$

$$(+\uparrow) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = (v_A \sin\theta)^2 + 2(-32.2)(h - 3.5)$$

$$(+\uparrow) \quad v = v_0 + a_c t$$

$$0 = v_A \sin\theta - 32.2(1.5) \qquad \textbf{(2)}$$

To solve, first divide Eq. (2) by Eq. (1) to get θ. Then

$$\theta = 76.0^\circ \qquad \textbf{Ans.}$$

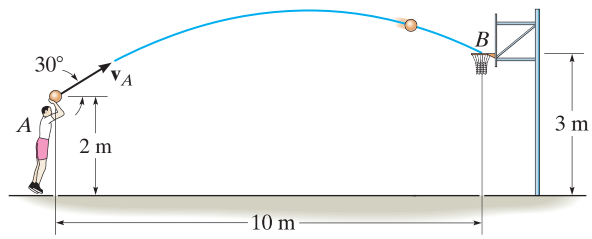
$$v_A = 49.8 \text{ ft/s} \qquad \textbf{Ans.}$$

$$h = 39.7 \text{ ft} \qquad \textbf{Ans.}$$

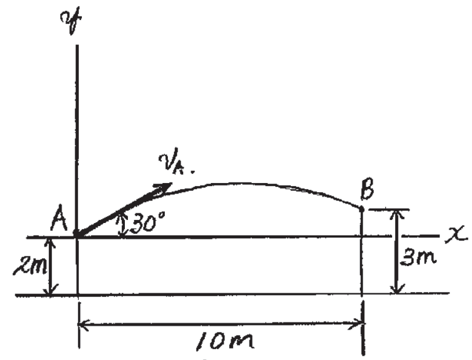
Ans:
$\theta = 76.0^\circ$
$v_A = 49.8 \text{ ft/s}$
$h = 39.7 \text{ ft}$

***12–88.**

Neglecting the size of the ball, determine the magnitude v_A of the basketball's initial velocity and its velocity when it passes through the basket.



SOLUTION



(a)

Coordinate System. The origin of the x-y coordinate system will be set to coinside with point A as shown in Fig. a

Horizontal Motion. Here $(v_A)_x = v_A \cos 30° \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 10\text{ m} \rightarrow$.

$$\left(\xrightarrow{+}\right)\quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$10 = 0 + v_A \cos 30°\, t$$

$$t = \frac{10}{v_A \cos 30°} \qquad \textbf{(1)}$$

Also,

$$\left(\xrightarrow{+}\right)\quad (v_B)_x = (v_A)_x = v_A \cos 30° \qquad \textbf{(2)}$$

Vertical Motion. Here, $(v_A)_y = v_A \sin 30° \uparrow$, $(s_A)_y = 0$, $(s_B)_y = 3 - 2 = 1\text{ m}\uparrow$ and $a_y = 9.81\text{ m/s}^2 \downarrow$

$$(+\uparrow)\quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$1 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 0.5\, v_A\, t + 1 = 0 \qquad \textbf{(3)}$$

Also

$$(+\uparrow)\quad (v_B)_y = (v_A)_y + a_y t$$

$$(v_B)_y = v_A \sin 30° + (-9.81)t$$

$$(v_B)_y = 0.5\, v_A - 9.81t \qquad \textbf{(4)}$$

Solving Eq. (1) and (3)

$$v_A = 11.705\text{ m/s} = 11.7\text{ m/s} \qquad \textbf{Ans.}$$

$$t = 0.9865\text{ s}$$

Substitute these results into Eq. (2) and (4)

$$(v_B)_x = 11.705 \cos 30° = 10.14\text{ m/s} \rightarrow$$

$$(v_B)_y = 0.5(11.705) - 9.81(0.9865) = -3.825\text{ m/s} = 3.825\text{ m/s} \downarrow$$

Thus, the magnitude of $\mathbf{v}_B$ is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{10.14^2 + 3.825^2} = 10.83\text{ m/s} = 10.8\text{ m/s} \qquad \textbf{Ans.}$$

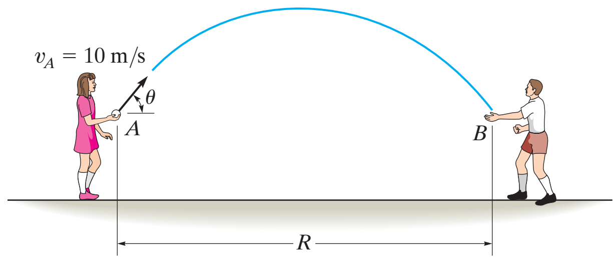
And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{3.825}{10.14}\right) = 20.67° = 20.7° \qquad \textbf{Ans.}$$

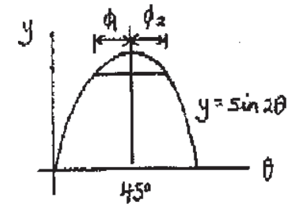
Ans:
$v_A = 11.7\text{ m/s}$
$v_B = 10.8\text{ m/s}$
$\theta = 20.7°$ ⦪

12–89.

The girl at A can throw a ball at $v_A = 10\text{ m/s}$. Calculate the maximum possible range $R = R_{max}$ and the associated angle θ at which it should be thrown. Assume the ball is caught at B at the same elevation from which it is thrown.



SOLUTION



$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$$R = 0 + (10 \cos \theta)t$$

$(+\uparrow)\quad v = v_0 + a_c t$

$$-10 \sin \theta = 10 \sin \theta - 9.81t$$

$$t = \frac{20}{9.81} \sin \theta$$

Thus, $R = \dfrac{200}{9.81} \sin \theta \cos \theta$

$$R = \frac{100}{9.81} \sin 2\theta \qquad \textbf{(1)}$$

Require,

$$\frac{dR}{d\theta} = 0$$

$$\frac{100}{9.81} \cos 2\theta(2) = 0$$

$$\cos 2\theta = 0$$

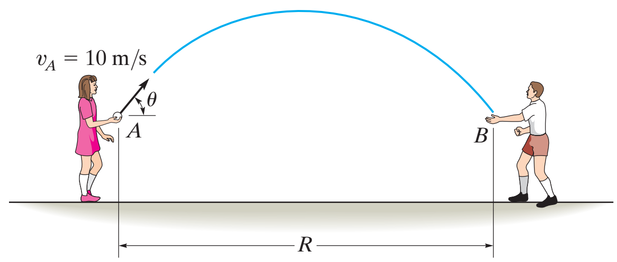
$$\theta = 45^\circ \qquad \textbf{Ans.}$$

$$R = \frac{100}{9.81}(\sin 90^\circ) = 10.2\text{ m} \qquad \textbf{Ans.}$$

Ans:
$R_{max} = 10.2\text{ m}$
$\theta = 45^\circ$

12–90.

Show that the girl at A can throw the ball to the boy at B by launching it at equal angles measured up or down from a 45° inclination. If $v_A = 10$ m/s, determine the range R if this value is 15°, i.e., $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$. Assume the ball is caught at the same elevation from which it is thrown.



SOLUTION

$(\xrightarrow{+})\quad s = s_0 + v_0 t$

$$R = 0 + (10\cos\theta)t$$

$(+\uparrow)\quad v = v_0 + a_c t$

$$-10\sin\theta = 10\sin\theta - 9.81t$$

$$t = \frac{20}{9.81}\sin\theta$$

Thus, $\quad R = \dfrac{200}{9.81}\sin\theta\cos\theta$

$$R = \frac{100}{9.81}\sin 2\theta \qquad \textbf{(1)}$$

Since the function $y = \sin 2\theta$ is symmetric with respect to $\theta = 45°$ as indicated, Eq. (1) will be satisfied if $|\phi_1| = |\phi_2|$

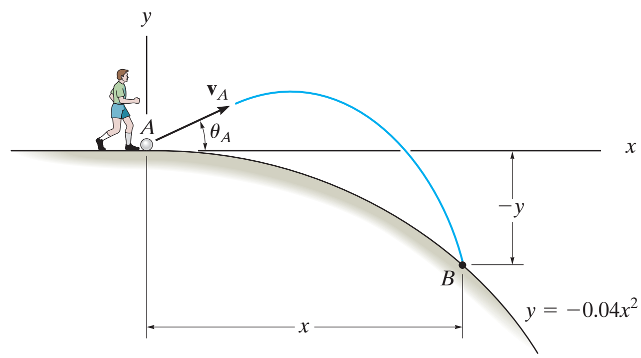
Choosing $\phi = 15°$ or $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$, and substituting into Eq. (1) yields

$R = 8.83$ m **Ans.**

Ans:
$R = 8.83$ m

12–91.

The ball at A is kicked with a speed $v_A = 80$ ft/s and at an angle $\theta_A = 30°$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.



SOLUTION

$(v_A)_x = 80 \cos 30° = 69.28$ ft/s

$(v_A)_y = 80 \sin 30° = 40$ ft/s

$(\xrightarrow{+})\; s = s_0 + v_0 t$

$x = 0 + 69.28t$ **(1)**

$(+\uparrow)\; s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$-y = 0 + 40t + \frac{1}{2}(-32.2)t^2$ **(2)**

$y = -0.04x^2$

From Eqs. (1) and (2):

$-y = 0.5774x - 0.003354x^2$

$0.04x^2 = 0.5774x - 0.003354x^2$

$0.04335x^2 = 0.5774x$

$x = 13.3$ ft **Ans.**

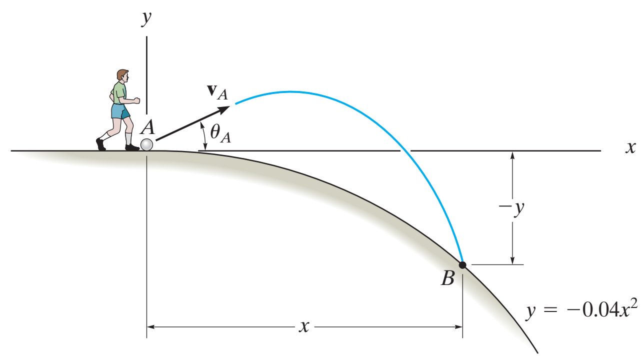
Thus

$y = -0.04\,(13.3)^2 = -7.09$ ft **Ans.**

Ans:
(13.3 ft, −7.09 ft)

***12–92.**

The ball at A is kicked such that $\theta_A = 30°$. If it strikes the ground at B having coordinates $x = 15$ ft, $y = -9$ ft, determine the speed at which it is kicked and the speed at which it strikes the ground.



SOLUTION

$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$$15 = 0 + v_A \cos 30°\, t$$

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-9 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-32.2)t^2$$

$v_A = 16.5$ ft/s **Ans.**

$t = 1.047$ s

$(\overset{+}{\rightarrow})$ $(v_B)_x = 16.54 \cos 30° = 14.32$ ft/s

$(+\uparrow)$ $v = v_0 + a_c t$

$(v_B)_y = 16.54 \sin 30° + (-32.2)(1.047)$

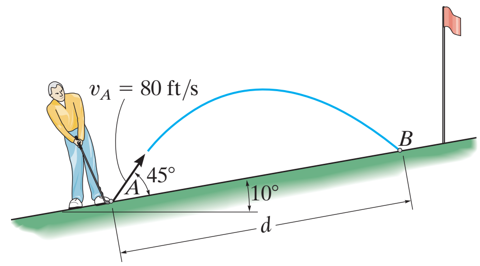
$= -25.45$ ft/s

$v_B = \sqrt{(14.32)^2 + (-25.45)^2} = 29.2$ ft/s **Ans.**

Ans:
$v_A = 16.5$ ft/s
$t = 1.047$ s
$v_B = 29.2$ ft/s

12–93.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



SOLUTION

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$d \cos 10° = 0 + 80 \cos 55°\ t$

$(+\uparrow)\ s = s_0 + v_0\, t + \frac{1}{2} a_c t^2$

$d \sin 10° = 0 + 80 \sin 55°\ t - \frac{1}{2}(32.2)(t^2)$

Solving

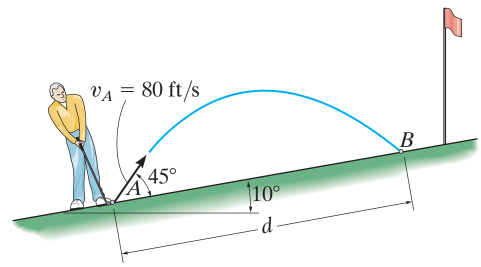
$t = 3.568$ s

$d = 166$ ft **Ans.**

Ans:
$d = 166$ ft

12–94.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the speed at which it strikes the ground at B and the time of flight from A to B.



SOLUTION

$(v_A)_x = 80 \cos 55° = 44.886$

$(v_A)_y = 80 \sin 55° = 65.532$

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$d \cos 10° = 0 + 45.886t$

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$d \sin 10° = 0 + 65.532\,(t) + \frac{1}{2}(-32.2)\left(t^2\right)$

$d = 166$ ft

$t = 3.568 = 3.57$ s **Ans.**

$(v_B)_x = (v_A)_x = 45.886$

$(+\uparrow)\quad v = v_0 + a_c t$

$(v_B)_y = 65.532 - 32.2(3.568)$

$(v_B)_y = -49.357$

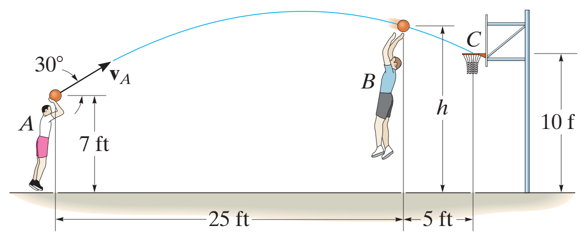
$v_B = \sqrt{(45.886)^2 + (-49.357)^2}$

$v_B = 67.4$ ft/s **Ans.**

Ans:
$t = 3.57$ s
$v_B = 67.4$ ft/s

12–95.

The basketball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$30 = 0 + v_A \cos 30° \, t_{AC}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$10 = 7 + v_A \sin 30° \, t_{AC} - \frac{1}{2}(32.2)(t_{AC}^2)$$

Solving

$v_A = 36.73 = 36.7 \text{ ft/s}$ **Ans.**

$t_{AC} = 0.943 \text{ s}$

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$25 = 0 + 36.73 \cos 30° \, t_{AB}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$h = 7 + 36.73 \sin 30° \, t_{AB} - \frac{1}{2}(32.2)(t_{AB}^2)$$

Solving

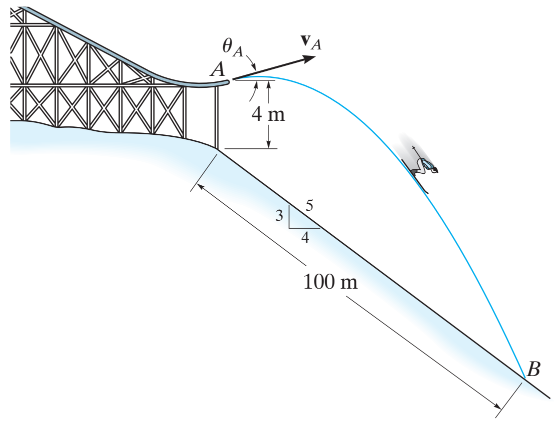
$t_{AB} = 0.786 \text{ s}$

$h = 11.5 \text{ ft}$ **Ans.**

Ans:
$v_A = 36.7 \text{ ft/s}$
$h = 11.5 \text{ ft}$

***12–96.**

It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the time of flight t_{AB}.



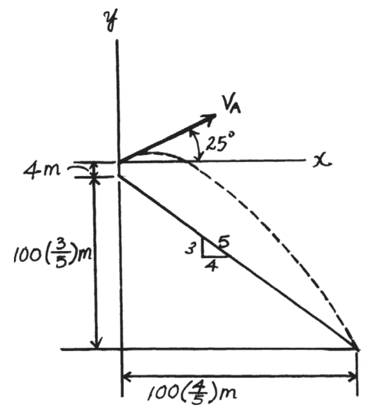
SOLUTION

$(\overset{+}{\rightarrow})\qquad s = v_0 t$

$$100\left(\frac{4}{5}\right) = v_A \cos 25° t_{AB}$$

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-4 - 100\left(\frac{3}{5}\right) = 0 + v_A \sin 25° t_{AB} + \frac{1}{2}(-9.81)t_{AB}^2$$



Solving,

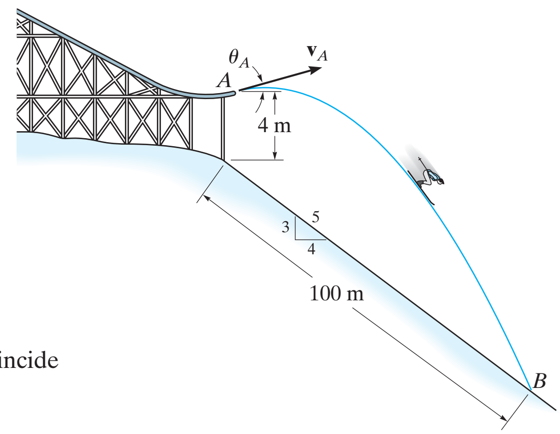
$v_A = 19.4 \text{ m/s}$ **Ans.**

$t_{AB} = 4.54 \text{ s}$ **Ans.**

Ans:
$v_A = 19.4 \text{ m/s}$
$t_{AB} = 4.54 \text{ s}$

12–97.

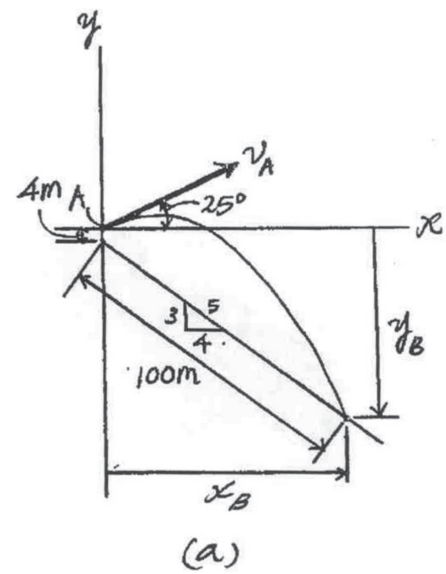
It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the speed at which he strikes the ground.



SOLUTION

Coordinate System: $x-y$ coordinate system will be set with its origin to coincide with point A as shown in Fig. a.

x-motion: Here, $x_A = 0,\ x_B = 100\left(\frac{4}{5}\right) = 80$ m and $(v_A)_x = v_A \cos 25°$.



$$\left(\xrightarrow{+}\right) \qquad x_B = x_A + (v_A)_x t$$

$$80 = 0 + (v_A \cos 25°)t$$

$$t = \frac{80}{v_A \cos 25°} \qquad \textbf{(1)}$$

y-motion: Here, $y_A = 0, y_B = -[4 + 100\left(\frac{3}{5}\right)] = -64$ m and $(v_A)_y = v_A \sin 25°$ and $a_y = -g = -9.81 \text{ m/s}^2$.

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-64 = 0 + v_A \sin 25°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - v_A \sin 25°\, t = 64 \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2) yieldS

$$4.905\left(\frac{80}{v_A \cos 25°}\right)^2 = v_A \sin 25°\left(\frac{80}{v_A \cos 25°}\right) = 64$$

$$\left(\frac{80}{v_A \cos 25°}\right)^2 = 20.65$$

$$\frac{80}{v_A \cos 25°} = 4.545$$

$$v_A = 19.42 \text{ m/s} = 19.4 \text{ m/s} \qquad \textbf{Ans.}$$

Substitute this result into Eq. (1),

$$t = \frac{80}{19.42 \cos 25°} = 4.54465$$

12–97. Continued

Using this result,

$$(+\uparrow) \qquad (v_B)_y = (v_A)_y + a_y t$$

$$= 19.42 \sin 25° + (-9.81)(4.5446)$$

$$= -36.37 \text{ m/s} = 36.37 \text{ m/s} \downarrow$$

And

$$(\overset{+}{\rightarrow}) \qquad (v_B)_x = (v_A)_x = v_A \cos 25° = 19.42 \cos 25° = 17.60 \text{ m/s} \rightarrow$$

Thus,

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2}$$

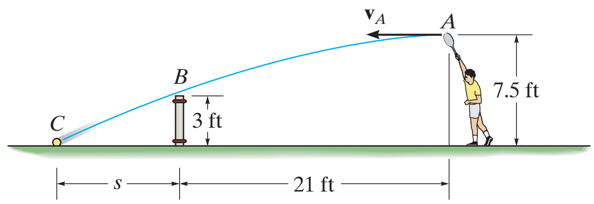
$$= \sqrt{36.37^2 + 17.60^2}$$

$$= 40.4 \text{ m/s}$$ **Ans.**

Ans:
$v_A = 19.4 \text{ m/s}$
$v_B = 40.4 \text{ m/s}$

12–98.

Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B. Also, find the distance s where the ball strikes the ground.



SOLUTION

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 0$. For the ball to travel from A to B, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 3$ ft, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$3 = 7.5 + 0 + \frac{1}{2}(-32.2)t_1^2$$

$$t_1 = 0.5287 \text{ s}$$

For the ball to travel from A to C, the initial and final vertical positions are $(s_0)_y = 7.5$ ft and $s_y = 0$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 7.5 + 0 + \frac{1}{2}(-32.2)t_2^2$$

$$t_2 = 0.6825 \text{ s}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A$. For the ball to travel from A to B, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 21$ ft, respectively. The time is $t = t_1 = 0.5287$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$21 = 0 + v_A(0.5287)$$

$$v_A = 39.72 \text{ ft/s} = 39.7 \text{ ft/s} \qquad \textbf{Ans.}$$

For the ball to travel from A to C, the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (21 + s)$ ft, respectively. The time is $t = t_2 = 0.6825$ s.

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

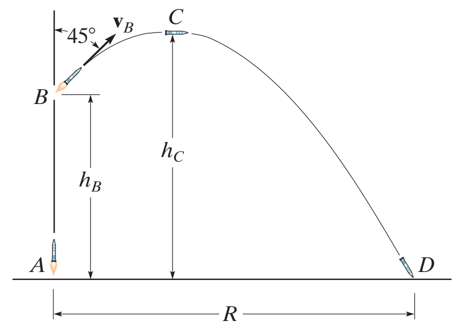
$$21 + s = 0 + 39.72(0.6825)$$

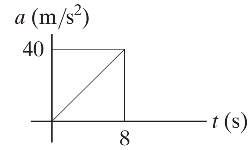
$$s = 6.11 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$v_A = 39.7$ ft/s
$s = 6.11$ ft

12–99.

The missile at A takes off from rest and rises vertically to B, where its fuel runs out in 8 s. If the acceleration varies with time as shown, determine the missile's height h_B and speed v_B. If by internal controls the missile is then suddenly pointed 45° as shown, and allowed to travel in free flight, determine the maximum height attained, h_C, and the range R to where it crashes at D.





SOLUTION

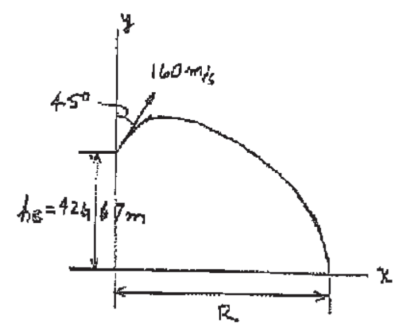
$$a = \frac{40}{8}t = 5t$$

$$dv = a\,dt$$

$$\int_0^v dv = \int_0^t 5t\,dt$$

$$v = 2.5t^2$$

When $t = 8$ s, $\quad v_B = 2.5(8)^2 = 160$ m/s **Ans.**



$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 2.5t^2\,dt$$

$$x = \frac{2.5}{3}t^3$$

$$h_B = \frac{2.5}{3}(8)^3 = 426.67 = 427 \text{ m}$$ **Ans.**

$$(v_B)_x = 160 \sin 45° = 113.14 \text{ m/s}$$

$$(v_B)_y = 160 \cos 45° = 113.14 \text{ m/s}$$

$$(+\uparrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = (113.14)^2 + 2(-9.81)(s_c - 426.67)$$

$$h_c = 1079.1 \text{ m} = 1.08 \text{ km}$$ **Ans.**

$$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$$

$$R = 0 + 113.14t$$

$$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$0 = 426.67 + 113.14t + \frac{1}{2}(-9.81)t^2$$

Solving for the positive root, $t = 26.36$ s

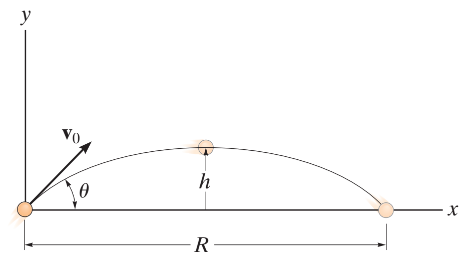
Then,

$$R = 113.14\,(26.36) = 2983.0 = 2.98 \text{ km}$$ **Ans.**

Ans:
$v_B = 160$ m/s
$h_B = 427$ m
$h_C = 1.08$ km
$R = 2.98$ km

***12–100.**

The projectile is launched with a velocity $\mathbf{v}_0$. Determine the range R, the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0. The acceleration due to gravity is g.



SOLUTION

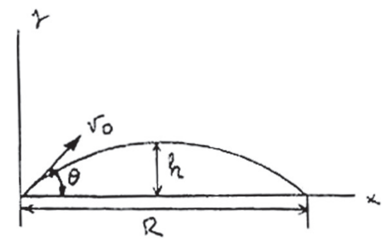
$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$$R = 0 + (v_0 \cos\theta)t$$

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$0 = 0 + (v_0 \sin\theta)\, t + \frac{1}{2}(-g)t^2$$

$$0 = v_0 \sin\theta - \frac{1}{2}(g)\left(\frac{R}{v_0 \cos\theta}\right)$$



$$R = \frac{v_0^2}{g} \sin 2\theta \qquad \textbf{Ans.}$$

$$t = \frac{R}{v_0 \cos\theta} = \frac{v_0^2\,(2 \sin\theta \cos\theta)}{v_0\, g \cos\theta}$$

$$= \frac{2v_0}{g} \sin\theta \qquad \textbf{Ans.}$$

$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (v_0 \sin\theta)^2 + 2(-g)(h - 0)$$

$$h = \frac{v_0^2}{2g} \sin^2\theta \qquad \textbf{Ans.}$$

Ans:

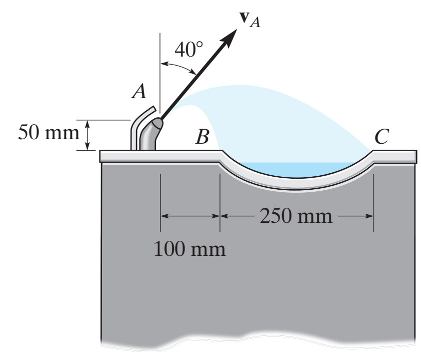
$R = \frac{v_0}{g} \sin 2\theta$

$t = \frac{2v_0}{g} \sin\theta$

$h = \frac{v_0^2}{2g} \sin^2\theta$

12–101.

The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at *B* and *C*.



SOLUTION

Horizontal Motion:

$$(\xrightarrow{+}) \qquad s = v_0 t$$

$$R = v_A \sin 40°\, t \quad t = \frac{R}{v_A \sin 40°} \qquad \textbf{(1)}$$

Vertical Motion:

$$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-0.05 = 0 + v_A \cos 40° t + \frac{1}{2}(-9.81)t^2 \qquad \textbf{(2)}$$

Substituting Eq.(1) into (2) yields:

$$-0.05 = v_A \cos 40° \left(\frac{R}{v_A \sin 40°}\right) + \frac{1}{2}(-9.81)\left(\frac{R}{v_A \sin 40°}\right)^2$$

$$v_A = \sqrt{\frac{4.905R^2}{\sin 40° \,(R \cos 40° + 0.05 \sin 40°)}}$$

At point B, $\quad R = 0.1$ m.

$$v_{\text{min}} = v_A = \sqrt{\frac{4.905\,(0.1)^2}{\sin 40°\,(0.1 \cos 40° + 0.05 \sin 40°)}} = 0.838 \text{ m/s} \qquad \textbf{Ans.}$$

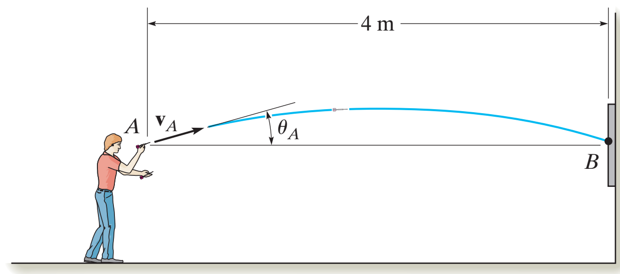
At point C, $\quad R = 0.35$ m.

$$v_{\text{max}} = v_A = \sqrt{\frac{4.905\,(0.35)^2}{\sin 40°\,(0.35 \cos 40° + 0.05 \sin 40°)}} = 1.76 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{\text{min}} = 0.838$ m/s
$v_{\text{max}} = 1.76$ m/s

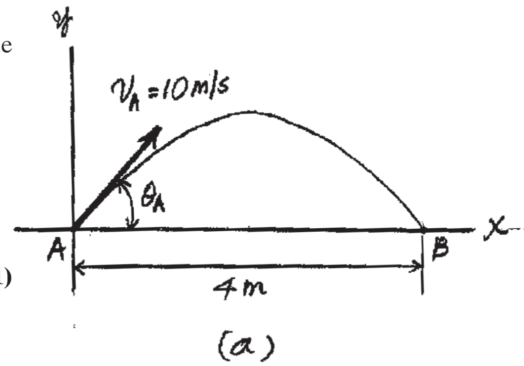
12–102.

If the dart is thrown with a speed of 10 m/s, determine the shortest possible time before it strikes the target. Also, what is the corresponding angle θ_A at which it should be thrown, and what is the velocity of the dart when it strikes the target?



SOLUTION

Coordinate System. The origin of the x-y coordinate system will be set to coincide with point A as shown in Fig. a.



Horizontal Motion. Here, $(v_A)_x = 10 \cos \theta_A \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 4 \text{ m} \rightarrow$.

$$(\overset{+}{\rightarrow}) \quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$4 = 0 + 10 \cos \theta_A t$$

$$t = \frac{4}{10 \cos \theta_A} \qquad \textbf{(1)}$$

Also,

$$(\overset{+}{\rightarrow}) \quad (v_B)_x = (v_A)_x = 10 \cos \theta_A \rightarrow \qquad \textbf{(2)}$$

Vertical Motion. Here, $(v_A)_y = 10 \sin \theta_A \uparrow$, $(s_A)_y = (s_B)_y = 0$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$

$$(+\uparrow) \quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$0 = 0 + (10 \sin \theta_A)\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - (10 \sin \theta_A)\, t = 0$$

$$t\,(4.905t - 10 \sin \theta_A) = 0$$

Since $t \neq 0$, then

$$4.905t - 10 \sin \theta_A = 0 \qquad \textbf{(3)}$$

Also

$$(+\uparrow) \quad (v_B)_y^2 = (v_A)_y^2 + 2\, a_y\, [(s_B)_y - (s_A)_y]$$

$$(v_B)_y^2 = (10 \sin \theta_A)^2 + 2\,(-9.81)\,(0 - 0)$$

$$(v_B)_y = -10 \sin \theta_A = 10 \sin \theta_A \downarrow \qquad \textbf{(4)}$$

Substitute Eq. (1) into (3)

$$4.905\left(\frac{4}{10 \cos \theta_A}\right) - 10 \sin \theta_A = 0$$

$$1.962 - 10 \sin \theta_A \cos \theta_A = 0$$

Using the trigonometry identity $\sin 2\theta_A = 2 \sin \theta_A \cos \theta_A$, this equation becomes

$$1.962 - 5 \sin 2\theta_A = 0$$

$$\sin 2\theta_A = 0.3924$$

$$2\theta_A = 23.10° \text{ and } 2\theta_A = 156.90°$$

$$\theta_A = 11.55° \text{ and } \theta_A = 78.45°$$

12–102. Continued

Since the shorter time is required, Eq. (1) indicates that smaller θ_A must be choosen. Thus

$$\theta_A = 11.55^\circ = 11.6^\circ \qquad \textbf{Ans.}$$

and

$$t = \frac{4}{10\cos 11.55^\circ} = 0.4083 \text{ s} = 0.408 \text{ s} \qquad \textbf{Ans.}$$

Substitute the result of θ_A into Eq. (2) and (4)

$$(v_B)_x = 10\cos 11.55^\circ = 9.7974 \text{ m/s} \rightarrow$$

$$(v_B)_y = 10\sin 11.55^\circ = 2.0026 \text{ m/s} \downarrow$$

Thus, the magnitude of v_B is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{9.7974^2 + 2.0026^2} = 10 \text{ m/s} \qquad \textbf{Ans.}$$

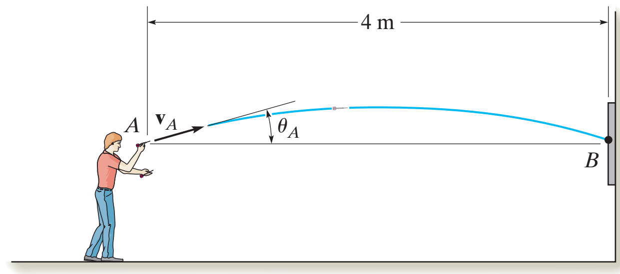
And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{2.0026}{9.7974}\right) = 11.55^\circ = 11.6^\circ \qquad \textbf{Ans.}$$

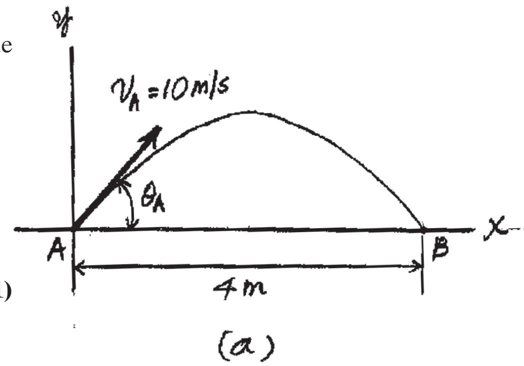
Ans:
$\theta_A = 11.6^\circ$
$t = 0.408$ s
$\theta_B = 11.6^\circ$

12–103.

If the dart is thrown with a speed of 10 m/s, determine the longest possible time when it strikes the target. Also, what is the corresponding angle θ_A at which it should be thrown, and what is the velocity of the dart when it strikes the target?



SOLUTION



Coordinate System. The origin of the x-y coordinate system will be set to coincide with point A as shown in Fig. a.

Horizontal Motion. Here, $(v_A)_x = 10\cos\theta_A \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 4\text{ m} \rightarrow$.

$$(\overset{+}{\rightarrow})\quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$4 = 0 + 10\cos\theta_A\, t$$

$$t = \frac{4}{10\cos\theta_A} \qquad \textbf{(1)}$$

Also,

$$(\overset{+}{\rightarrow})\quad (v_B)_x = (v_A)_x = 10\cos\theta_A \rightarrow \qquad \textbf{(2)}$$

Vertical Motion. Here, $(v_A)_y = 10\sin\theta_A \uparrow$, $(s_A)_y = (s_B)_y = 0$ and $a_y = -9.81\text{ m/s}^2 \downarrow$.

$$(+\uparrow)\quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2}a_y t^2$$

$$0 = 0 + (10\sin\theta_A)\,t + \frac{1}{2}(-9.81)\,t^2$$

$$4.905t^2 - (10\sin\theta_A)\,t = 0$$

$$t\,(4.905t - 10\sin\theta_A) = 0$$

Since $t \neq 0$, then

$$4.905\,t - 10\sin\theta_A = 0 \qquad \textbf{(3)}$$

Also,

$$(v_B)_y^2 = (v_A)_y^2 + 2\,a_y\,[(s_B)_y - (s_A)_y]$$

$$(v_B)_y^2 = (10\sin\theta_A)^2 + 2\,(-9.81)\,(0 - 0)$$

$$(v_B)_y = -10\sin\theta_A = 10\sin\theta_A \downarrow \qquad \textbf{(4)}$$

Substitute Eq. (1) into (3)

$$4.905\left(\frac{4}{10\cos\theta_A}\right) - 10\sin\theta_A = 0$$

$$1.962 - 10\sin\theta_A\cos\theta_A = 0$$

Using the trigonometry identity $\sin 2\theta_A = 2\sin\theta_A\cos\theta_A$, this equation becomes

$$1.962 - 5\sin 2\theta_A = 0$$

$$\sin 2\theta_A = 0.3924$$

$$2\theta_A = 23.10^\circ \text{ and } 2\theta_A = 156.90^\circ$$

$$\theta_A = 11.55^\circ \text{ and } \theta_A = 78.44^\circ$$

12–103. Continued

Since the longer time is required, Eq. (1) indicates that larger θ_A must be choosen. Thus,

$$\theta_A = 78.44° = 78.4° \qquad \textbf{Ans.}$$

and

$$t = \frac{4}{10 \cos 78.44°} = 1.9974 \text{ s} = 2.00 \text{ s} \qquad \textbf{Ans.}$$

Substitute the result of θ_A into Eq. (2) and (4)

$$(v_B)_x = 10 \cos 78.44° = 2.0026 \text{ m/s} \rightarrow$$

$$(v_B)_y = 10 \sin 78.44° = 9.7974 \text{ m/s} \downarrow$$

Thus, the magnitude of v_B is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{2.0026^2 + 9.7974^2} = 10 \text{ m/s} \qquad \textbf{Ans.}$$

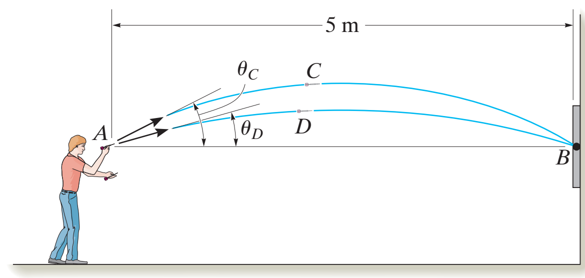
And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{9.7974}{2.0026}\right) = 78.44° = 78.4° \qquad \textbf{Ans.}$$

Ans:
$\theta_A = 78.4°$
$t = 2.00 \text{ s}$
$\theta_B = 78.4°$

***12–104.**

The man at A wishes to throw two darts at the target at B so that they arrive at the *same time*. If each dart is thrown with a speed of 10 m/s, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at θ_C ($>\theta_D$), then the second dart is thrown at θ_D.



SOLUTION

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$5 = 0 + (10\cos\theta)\,t \qquad$ **(1)**

$(+\uparrow)\qquad v = v_0 + a_c t$

$-10\sin\theta = 10\sin\theta - 9.81\,t$

$$t = \frac{2(10\sin\theta)}{9.81} = 2.039\sin\theta$$

From Eq. (1),

$5 = 20.39\sin\theta\cos\theta$

Since $\quad \sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2\theta = 0.4905$

The two roots are $\theta_D = 14.7°$ **Ans.**

$\theta_C = 75.3°$ **Ans.**

From Eq. (1): $t_D = 0.517$ s

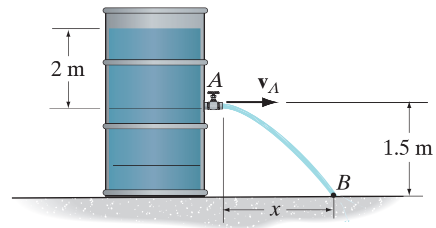
$t_C = 1.97$ s

So that $\Delta t = t_C - t_D = 1.45$ s **Ans.**

Ans:
$\theta_D = 14.7°$
$\theta_C = 75.3°$
$\Delta t = t_C - t_D = 1.45$ s

12–105.

The velocity of the water jet discharging from the orifice can be obtained from $v = \sqrt{2gh}$, where $h = 2\text{ m}$ is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach point B and the horizontal distance x where it hits the surface.



SOLUTION

Coordinate System: The x–y coordinate system will be set so that its origin coincides with point A. The speed of the water that the jet discharges from A is

$$v_A = \sqrt{2(9.81)(2)} = 6.264 \text{ m/s}$$

***x*-Motion:** Here, $(v_A)_x = v_A = 6.264$ m/s, $x_A = 0$, $x_B = x$, and $t = t_A$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$x = 0 + 6.264 t_A \qquad \textbf{(1)}$$

***y*-Motion:** Here, $(v_A)_y = 0$, $a_y = -g = -9.81 \text{ m/s}^2$, $y_A = 0$ m, $y_B = -1.5$ m, and $t = t_A$. Thus,

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-1.5 = 0 + 0 + \frac{1}{2}(-9.81) t_A{}^2$$

$$t_A = 0.553 \text{ s} \qquad \textbf{Ans.}$$

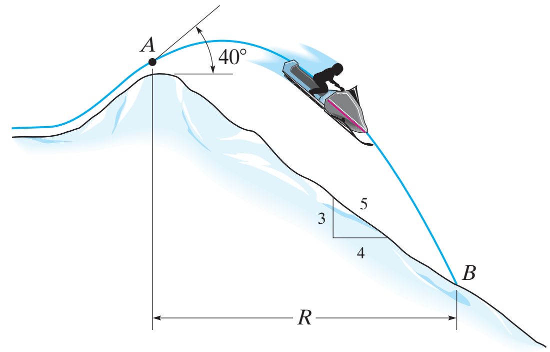
Thus,

$$x = 0 + 6.264(0.553) = 3.46 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$t_A = 0.553$ s
$x = 3.46$ m

12–106.

The snowmobile is traveling at 10 m/s when it leaves the embankment at A. Determine the time of flight from A to B and the range R of the trajectory.



SOLUTION

$(\xrightarrow{+})$ $\quad s_B = s_A + v_A t$

$$R = 0 + 10 \cos 40^\circ\, t$$

$(+\uparrow)$ $\quad s_B = s_A + v_A t + \frac{1}{2} a_c t^2$

$$-R\left(\frac{3}{4}\right) = 0 + 10 \sin 40^\circ t - \frac{1}{2}(9.81)\, t^2$$

Solving:

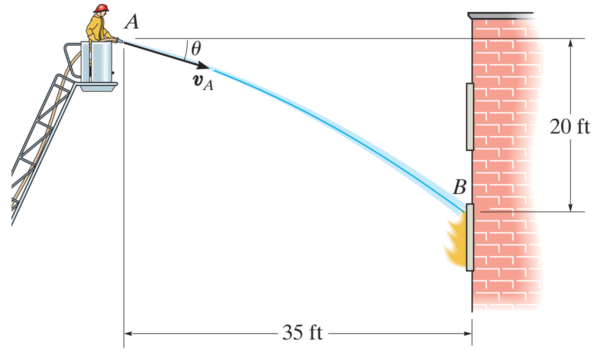
$R = 19.0$ m **Ans.**

$t = 2.48$ s **Ans.**

Ans:
$R = 19.0$ m
$t = 2.48$ s

12–107.

The fireman wishes to direct the flow of water from his hose to the fire at B. Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at $v_A = 80$ ft/s.



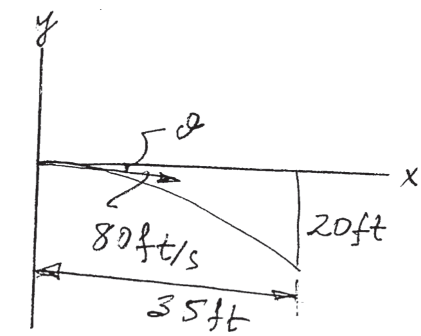
SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$35 = 0 + (80)(\cos\theta)t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-20 = 0 - 80(\sin\theta)t + \frac{1}{2}(-32.2)t^2$$



Thus,

$$20 = 80 \sin\theta \frac{0.4375}{\cos\theta} t + 16.1\left(\frac{0.1914}{\cos^2\theta}\right)$$

$$20\cos^2\theta = 17.5 \sin 2\theta + 3.0816$$

Solving,

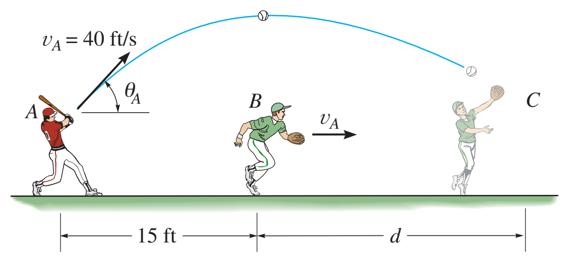
$\theta_1 = 24.9°$ (below the horizontal) **Ans.**

$\theta_2 = 85.2°$ (above the horizontal) **Ans.**

Ans:
$\theta_1 = 24.9°$ ↘
$\theta_2 = 85.2°$ ↗

***12–108.**

The baseball player A hits the baseball at $v_A = 40$ ft/s and $\theta_A = 60°$ from the horizontal. When the ball is directly overhead of player B he begins to run under it. Determine the constant speed at which B must run and the distance d in order to make the catch at the same elevation at which the ball was hit.



SOLUTION

Vertical Motion: The vertical component of initial velocity for the football is $(v_0)_y = 40 \sin 60° = 34.64$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = 0$, respectively.

$$(+\uparrow) \quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$0 = 0 + 34.64t + \frac{1}{2}(-32.2)\,t^2$$

$$t = 2.152 \text{ s}$$

Horizontal Motion: The horizontal component of velocity for the baseball is $(v_0)_x = 40 \cos 60° = 20.0$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = R$, respectively.

$$(\xrightarrow{+}) \quad s_x = (s_0)_x + (v_0)_x t$$

$$R = 0 + 20.0(2.152) = 43.03 \text{ ft}$$

The distance for which player B must travel in order to catch the baseball is

$d = R - 15 = 43.03 - 15 = 28.0$ ft **Ans.**

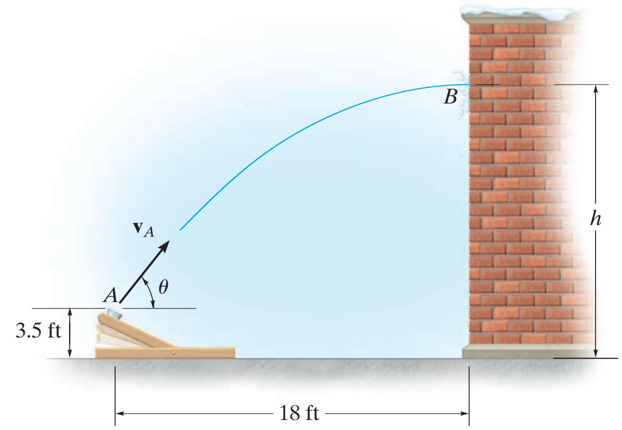
Player B is required to run at a same speed as the horizontal component of velocity of the baseball in order to catch it.

$v_B = 40 \cos 60° = 20.0$ ft/s **Ans.**

Ans:
$d = 28.0$ ft
$v_B = 20.0$ ft/s

12–109.

The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B, determine the velocity $\mathbf{v}_A$ at which it was launched, the angle of release θ, and the height h.



SOLUTION

$(\xrightarrow{+})$ $\quad s = v_0 t$

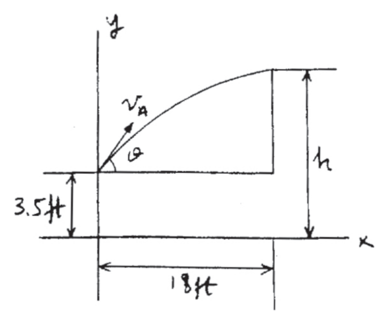
$$18 = v_A \cos\theta(1.5) \qquad \textbf{(1)}$$

$(+\uparrow)$ $\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (v_A \sin\theta)^2 + 2(-32.2)(h - 3.5)$$

$(+\uparrow)$ $\quad v = v_0 + a_c t$

$$0 = v_A \sin\theta - 32.2(1.5) \qquad \textbf{(2)}$$



To solve, first divide Eq. (2) by Eq. (1), to get θ. Then

$$\theta = 76.0^\circ \qquad \textbf{Ans.}$$

$$v_A = 49.8 \text{ ft/s} \qquad \textbf{Ans.}$$

$$h = 39.7 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$\theta = 76.0^\circ$
$v_A = 49.8 \text{ ft/s}$
$h = 39.7$ ft

12–110.

An automobile is traveling on a curve having a radius of 800 ft. If the acceleration of the automobile is $5\ \text{ft/s}^2$, determine the constant speed at which the automobile is traveling.

SOLUTION

Acceleration: Since the automobile is traveling at a constant speed, $a_t = 0$. Thus, $a_n = a = 5\ \text{ft/s}^2$. Applying Eq. 12–20, $a_n = \dfrac{v^2}{\rho}$, we have

$$v = \sqrt{\rho a_n} = \sqrt{800(5)} = 63.2\ \text{ft/s} \qquad \textbf{Ans.}$$

Ans:
$v = 63.2\ \text{ft/s}$

12–111.

Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

SOLUTION

Acceleration: Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$7.5 = \frac{v^2}{200}$$

$$v = 38.7 \text{ m/s}$$ **Ans.**

Ans:
$v = 38.7 \text{ m/s}$

***12–112.**

A boat has an initial speed of 16 ft/s. If it then increases its speed along a circular path of radius $\rho = 80$ ft at the rate of $\dot{v} = (1.5s)$ ft/s, where s is in feet, determine the time needed for the boat to travel $s = 50$ ft.

SOLUTION

$a_t = 1.5s$

$$\int_0^s 1.5s\,ds = \int_{16}^v v\,dv$$

$$0.75\,s^2 = 0.5\,v^2 - 128$$

$$v = \frac{ds}{dt} = \sqrt{256 + 1.5\,s^2}$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 170.7}} = \int_0^t 1.225\,dt$$

$$\ln\left(s + \sqrt{s^2 + 170.7}\right)\Big|_0^s = 1.225t$$

$$\ln\left(s + \sqrt{s^2 + 170.7}\right) - 2.570 = 1.225t$$

At $s = 50$ ft,

$t = 1.68$ s **Ans.**

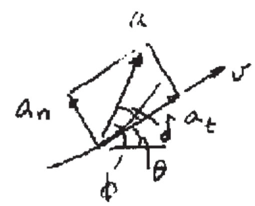
Ans:
$t = 1.68$ s

12–113.

The position of a particle is defined by $\mathbf{r} = \{4(t - \sin t)\mathbf{i} + (2t^2 - 3)\mathbf{j}\}$ m, where t is in seconds and the argument for the sine is in radians. Determine the speed of the particle and its normal and tangential components of acceleration when $t = 1$ s.

SOLUTION

$\mathbf{r} = 4(t - \sin t)\,\mathbf{i} + (2\,t^2 - 3)\mathbf{j}$



$\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 4(1 - \cos t)\mathbf{i} + (4\,t)\mathbf{j}$

$\mathbf{v}|_{t=1} = 1.83879\mathbf{i} + 4\mathbf{j}$

$v = \sqrt{(1.83879)^2 + (4)^2} = 4.40\ \text{m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{4}{1.83879}\right) = 65.312°$ ∠θ

$\mathbf{a} = 4\sin \mathbf{r}\mathbf{i} + 4\mathbf{j}$

$\mathbf{a}|_{t=1} = 3.3659\mathbf{i} + 4\mathbf{j}$

$a = \sqrt{(3.3659)^2 + (4)^2} = 5.22773\ \text{m/s}^2$

$\phi = \tan^{-1}\left(\dfrac{4}{3.3659}\right) = 49.920°$ ∠θ

$\delta = \theta - \phi = 15.392°$

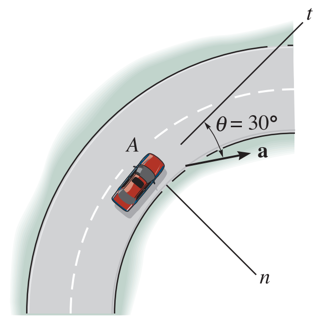
$a_2 = 5.22773\cos 15.392° = 5.04\ \text{m/s}^2$ **Ans.**

$a_n = 5.22773\sin 15.392° = 1.39\ \text{m/s}^2$ **Ans.**

Ans:
$v = 4.40\ \text{m/s}$
$a_t = 5.04\ \text{m/s}^2$
$a_n = 1.39\ \text{m/s}^2$

12–114.

The automobile has a speed of 80 ft/s at point A and an acceleration $\mathbf{a}$ having a magnitude of $10\ \text{ft/s}^2$, acting in the direction shown. Determine the radius of curvature of the path at point A and the tangential component of acceleration.



SOLUTION

Acceleration: The tangential acceleration is

$$a_t = a\cos 30° = 10\cos 30° = 8.66\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

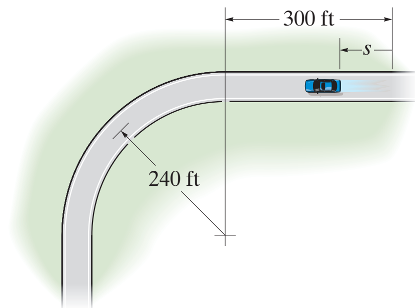
and the normal acceleration is $a_n = a\sin 30° = 10\sin 30° = 5.00\ \text{ft/s}^2$. Applying Eq. 12–20, $a_n = \dfrac{v^2}{\rho}$, we have

$$\rho = \frac{v^2}{a_n} = \frac{80^2}{5.00} = 1280\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$a_t = 8.66\ \text{ft/s}^2$
$\rho = 1280\ \text{ft}$

12–115.

The automobile is originally at rest at $s = 0$. If its speed is increased by $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration when $t = 18$ s.



SOLUTION

$a_t = 0.05t^2$

$$\int_0^v dv = \int_0^t 0.05\, t^2\, dt$$

$v = 0.0167\, t^3$

$$\int_0^s ds = \int_0^t 0.0167\, t^3\, dt$$

$s = 4.167(10^{-3})\, t^4$

When $t = 18$ s, $\quad s = 437.4$ ft

Therefore the car is on a curved path.

$v = 0.0167(18^3) = 97.2$ ft/s **Ans.**

$$a_n = \frac{(97.2)^2}{240} = 39.37 \text{ ft/s}^2$$

$a_t = 0.05(18^2) = 16.2$ ft/s^2

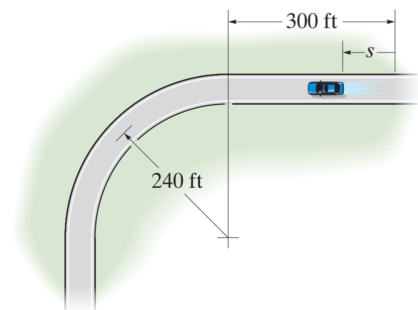
$a = \sqrt{(39.37)^2 + (16.2)^2}$

$a = 42.6$ ft/s^2 **Ans.**

Ans:
$v = 97.2$ ft/s
$a = 42.6$ ft/s^2

***12–116.**

The automobile is originally at rest $s = 0$. If it then starts to increase its speed at $\dot{v} = (0.05t^2)$ ft/s^2, where t is in seconds, determine the magnitudes of its velocity and acceleration at $s = 550$ ft.



SOLUTION

The car is on the curved path.

$a_t = 0.05\,t^2$

$$\int_0^v dv = \int_0^t 0.05\,t^2\,dt$$

$v = 0.0167\,t^3$

$$\int_0^s ds = \int_0^t 0.0167\,t^3\,dt$$

$s = 4.167(10^{-3})\,t^4$

$550 = 4.167(10^{-3})\,t^4$

$t = 19.06$ s

So that

$v = 0.0167(19.06)^3 = 115.4$

$v = 115$ ft/s **Ans.**

$$a_n = \frac{(115.4)^2}{240} = 55.51 \text{ ft/s}^2$$

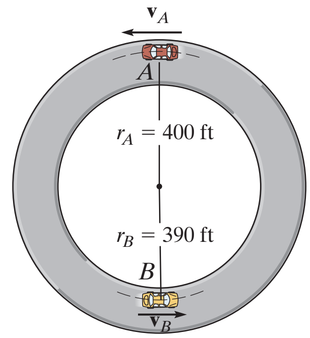
$a_t = 0.05(19.06)^2 = 18.17$ ft/s^2

$a = \sqrt{(55.51)^2 + (18.17)^2} = 58.4$ ft/s^2 **Ans.**

Ans:
$v = 115$ ft/s
$a = 58.4$ ft/s^2

12–117.

The two cars A and B travel along the circular path at constant speeds $v_A = 80$ ft/s and $v_B = 100$ ft/s, respectively. If they are at the positions shown when $t = 0$, determine the time when the cars are side by side, and the time when they are 90° apart.



SOLUTION

a) Referring to Fig. a, when cars A and B are side by side, the relation between their angular displacements is

$$\theta_B = \theta_A + \pi \qquad \textbf{(1)}$$

Here, $s_A = v_A t = 80\,t$ and $s_B = v_B t = 100\,t$. Apply the formula $s = r\theta$ or $\theta = \dfrac{s}{r}$. Then

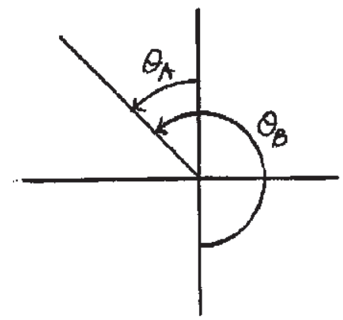
$$\theta_B = \frac{s_B}{r_B} = \frac{100\,t}{390} = \frac{10}{39}t$$

$$\theta_A = \frac{s_A}{r_A} = \frac{80\,t}{400} = \frac{1}{5}t$$

Substitute these results into Eq. (1)

$$\frac{10}{39}t = \frac{1}{5}t + \pi$$

$$t = 55.69\text{ s} = 55.7\text{ s} \qquad \textbf{Ans.}$$



(a)

(b) Referring to Fig. a, when cars A and B are 90° apart, the relation between their angular displacements is

$$\theta_B + \frac{\pi}{2} = \theta_A + \pi$$

$$\theta_B = \theta_A + \frac{\pi}{2} \qquad \textbf{(2)}$$

Here, $s_A = v_A t = 80\,t$ and $s_B = v_B t = 100\,t$. Applying the formula $s = r\theta$ or $\theta = \dfrac{s}{r}$. Then

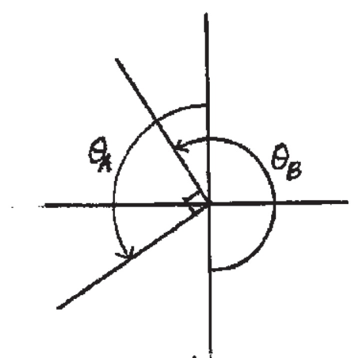
$$\theta_B = \frac{s_B}{r_B} = \frac{100\,t}{390} = \frac{10}{39}t$$

$$\theta_A = \frac{s_A}{r_A} = \frac{80\,t}{400} = \frac{1}{5}t$$

Substitute these results into Eq. (2)

$$\frac{10}{39}t = \frac{1}{5}t + \frac{\pi}{2}$$

$$t = 27.84\text{ s} = 27.8\text{ s} \qquad \textbf{Ans.}$$

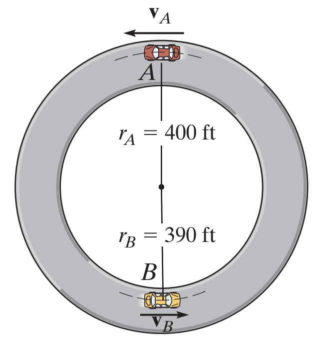


(b)

Ans:
When cars A and B are side by side, $t = 55.7$ s.
When cars A and B are 90° apart, $t = 27.8$ s.

12–118.

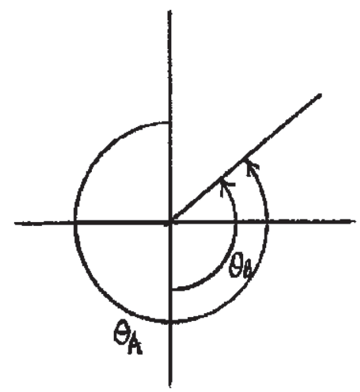
Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 60 ft/s and is increasing its speed at the rate of 15 ft/s^2 until it travels for a distance of 100π ft, after which it maintains a constant speed. Car B has a speed of 120 ft/s and is decreasing its speed at 15 ft/s^2 until it travels a distance of 65π ft, after which it maintains a constant speed. Determine the time when they come side by side.



SOLUTION

Referring to Fig. a, when cars A and B are side by side, the relation between their angular displacements is

$$\theta_A = \theta_B + \pi \qquad \textbf{(1)}$$



(a)

The constant speed achieved by cars A and B can be determined from

$$(v_A)_c^2 = (v_A)_0^2 + 2(a_A)_t\,[s_A - (s_0)_A]$$

$$(v_A)_c^2 = 60^2 + 2(15)\,(100\pi - 0)$$

$$(v_A)_c = 114.13 \text{ ft/s}$$

$$(v_B)_c^2 = (v_B)_0^2 + 2(a_B)_t\,[s_B - (s_0)_B]$$

$$(v_B)_c^2 = 120^2 + 2\,(-15)\,(65\pi - 0)$$

$$(v_B)_c = 90.96 \text{ ft/s}$$

The time taken to achieve these constant speeds can be determined from

$$(v_A)_c = (v_A)_0 + (a_A)_t\,(t_A)_1$$

$$114.13 = 60 + 15(t_A)_1$$

$$(t_A)_1 = 3.6084 \text{ s}$$

$$(v_B)_c = (v_B)_0 + (a_B)_t\,(t_B)_1$$

$$90.96 = 120 + (-15)\,(t_B)_1$$

$$(t_B)_1 = 1.9359 \text{ s}$$

Let t be the time taken for cars A and B to be side by side. Then, the times at which cars A and B travel with constant speed are $(t_A)_2 = t - (t_A)_1 = t - 3.6084$ and $(t_B)_2 = t - (t_B)_1 = t - 1.9359$. Here, $(s_A)_1 = 100\pi$, $(s_A)_2 = (v_A)_c\,(t_A)_2 = 114.13\,(t - 3.6084)$, $(s_B)_1 = 65\pi$ and $(s_B)_2 = (v_B)_c\,(t_B)_2 = 90.96\,(t - 1.9359)$.

Using, the formula $s = r\theta$ or $\theta = \dfrac{s}{r}$,

$$\theta_A = (\theta_A)_1 + (\theta_A)_2 = \frac{(s_A)_1}{r_A} + \frac{(s_A)_2}{r_A} = \frac{100\pi}{400} + \frac{114.13\,(t - 3.6084)}{400}$$

$$= 0.2853\,t - 0.24414$$

$$\theta_B = (\theta_B)_1 + (\theta_B)_2 = \frac{(s_B)_1}{r_B} + \frac{(s_B)_2}{r_B} = \frac{65\,\pi}{390} + \frac{90.96\,(t - 1.9359)}{390}$$

$$= 0.2332\,t + 0.07207$$

Substitute these results into Eq. (1),

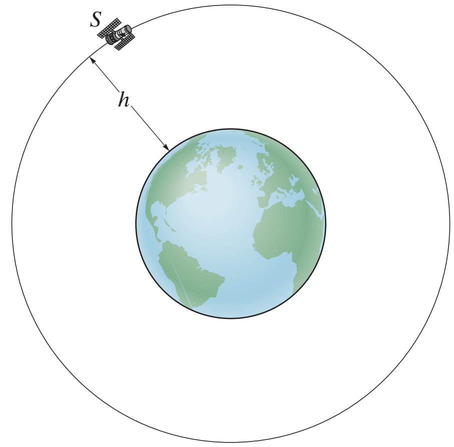
$$0.2853\,t - 0.24414 = 0.2332\,t + 0.07207 + \pi$$

$$t = 66.39 \text{ s} = 66.4 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 66.4$ s

12–119.

The satellite S travels around the earth in a circular path with a constant speed of 20 Mm/h. If the acceleration is 2.5 m/s^2, determine the altitude h. Assume the earth's diameter to be 12 713 km.



SOLUTION

$$v = 20 \text{ Mm/h} = \frac{20(10^6)}{3600} = 5.56(10^3) \text{ m/s}$$

Since $a_t = \dfrac{dv}{dt} = 0$, then,

$$a = a_n = 2.5 = \frac{v^2}{\rho}$$

$$\rho = \frac{(5.56(10^3))^2}{2.5} = 12.35(10^6) \text{ m}$$

The radius of the earth is

$$\frac{12\,713(10^3)}{2} = 6.36(10^6) \text{ m}$$

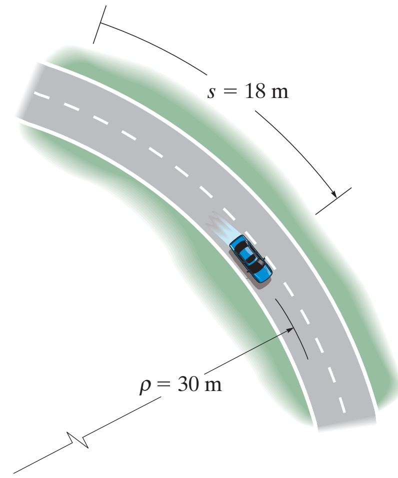
Hence,

$$h = 12.35(10^6) - 6.36(10^6) = 5.99(10^6) \text{ m} = 5.99 \text{ Mm}$$ **Ans.**

Ans:
$h = 5.99$ Mm

***12–120.**

The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18$ m starting from rest. Neglect the size of the car.



SOLUTION

$$\int_0^v dv = \int_0^t 0.5e^t\,dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving,

$$t = 3.7064\ \text{s}$$

$$v = 0.5(e^{3.7064} - 1) = 19.85\ \text{m/s} = 19.9\ \text{m/s} \qquad \textbf{Ans.}$$

$$a_t = \dot{v} = 0.5e^t\big|_{t=3.7064\ \text{s}} = 20.35\ \text{m/s}^2$$

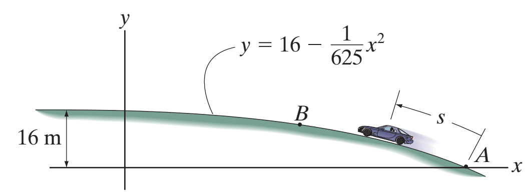
$$a_n = \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 19.9\ \text{m/s}$
$a = 24.2\ \text{m/s}^2$

12–121.

The car passes point A with a speed of 25 m/s after which its speed is defined by $v = (25 - 0.15s)$ m/s. Determine the magnitude of the car's acceleration when it reaches point B, where $s = 51.5$ m and $x = 50$ m.



SOLUTION

Velocity: The speed of the car at B is

$$v_B = \left[25 - 0.15(51.5)\right] = 17.28 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=50\text{ m}} = 324.58 \text{ m}$$

Acceleration:

$$a_n = \frac{v_B^{\,2}}{\rho} = \frac{17.28^2}{324.58} = 0.9194 \text{ m/s}^2$$

$$a_t = v\frac{dv}{ds} = (25 - 0.15s)(-0.15) = (0.225s - 3.75) \text{ m/s}^2$$

When the car is at B $(s = 51.5 \text{ m})$

$$a_t = \left[0.225(51.5) - 3.75\right] = -2.591 \text{ m/s}^2$$

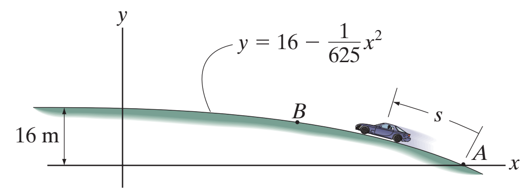
Thus, the magnitude of the car's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-2.591)^2 + 0.9194^2} = 2.75 \text{ m/s}^2$$ **Ans.**

Ans:
$a = 2.75 \text{ m/s}^2$

12–122.

If the car passes point A with a speed of 20 m/s and begins to increase its speed at a constant rate of $a_t = 0.5\ \text{m/s}^2$, determine the magnitude of the car's acceleration when $s = 101.68$ m and $x = 0$.



SOLUTION

Velocity: The speed of the car at C is

$$v_C{}^2 = v_A{}^2 + 2a_t(s_C - s_A)$$

$$v_C{}^2 = 20^2 + 2(0.5)(100 - 0)$$

$$v_C = 22.361\ \text{m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$

$$\frac{dy}{dx} = -3.2\left(10^{-3}\right)x$$

$$\frac{d^2y}{dx^2} = -3.2\left(10^{-3}\right)$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(-3.2\left(10^{-3}\right)x\right)^2\right]^{3/2}}{\left|-3.2\left(10^{-3}\right)\right|}\right|_{x=0} = 312.5\ \text{m}$$

Acceleration:

$$a_t = \dot{v} = 0.5\ \text{m/s}$$

$$a_n = \frac{v_C{}^2}{\rho} = \frac{22.361^2}{312.5} = 1.60\ \text{m/s}^2$$

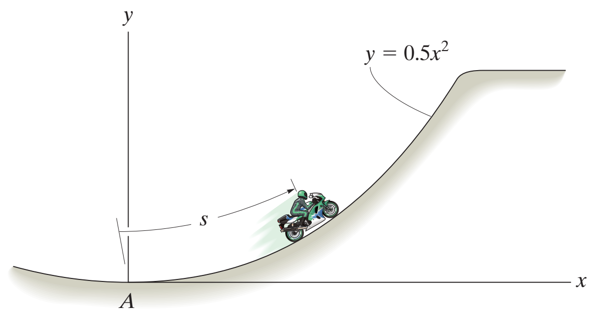
The magnitude of the car's acceleration at C is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} = 1.68\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 1.68\ \text{m/s}^2$

12–123.

The motorcycle is traveling at 1 m/s when it is at A. If the speed is then increased at $\dot{v} = 0.1\ \text{m/s}^2$, determine its speed and acceleration at the instant $t = 5$ s.



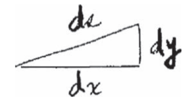
SOLUTION

$a_t = \nu = 0.1$

$$s = s_0 + \nu_0\, t + \frac{1}{2}a_c t^2$$

$$s = 0 + 1(5) + \frac{1}{2}(0.1)(5)^2 = 6.25\ \text{m}$$

$$\int_0^{6.25} ds = \int_0^{x} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$



$y = 0.5x^2$

$$\frac{dy}{dx} = x$$

$$\frac{d^2y}{dx^2} = 1$$

$$6.25 = \int_0^x \sqrt{1 + x^2}\, dx$$

$$6.25 = \frac{1}{2}\left[x\sqrt{1 + x^2} + 1\text{n}\left(x + \sqrt{1 + x^2}\right)\right]_0^x$$

$$x\sqrt{1 + x^2} + 1\text{n}\left(x + \sqrt{1 + x^2}\right) = 12.5$$

Solving,

$x = 3.184$ m

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + x^2]^{\frac{3}{2}}}{|1|}\Bigg|_{x=3.184} = 37.17\ \text{m}$$

$\nu = \nu_0 + a_c t$

$= 1 + 0.1(5) = 1.5\ \text{m/s}$ **Ans.**

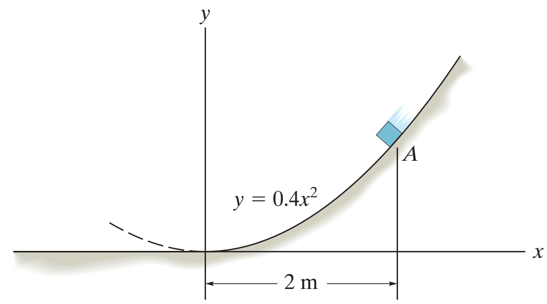
$$a_n = \frac{\nu^2}{\rho} = \frac{(1.5)^2}{37.17} = 0.0605\ \text{m/s}^2$$

$a = \sqrt{(0.1)^2 + (0.0605)^2} = 0.117\ \text{m/s}^2$ **Ans.**

Ans:
$v = 1.5\ \text{m/s}$
$a = 0.117\ \text{m/s}^2$

***12–124.**

The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at A ($x_A = 2$ m, $y_A = 1.6$ m), the speed is $v = 8$ m/s and the increase in speed is $dv/dt = 4$ m/s^2. Determine the magnitude of the acceleration of the box at this instant.



SOLUTION

$$y = 0.4\,x^2$$

$$\left.\frac{dy}{dx}\right|_{x=2\text{ m}} = 0.8x\Big|_{x=2\text{ m}} = 1.6$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2\text{ m}} = 0.8$$

$$\rho = \left.\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=2\text{ m}} = \frac{\left[1 + (1.6)^2\right]^{3/2}}{|0.8|} = 8.396\text{ m}$$

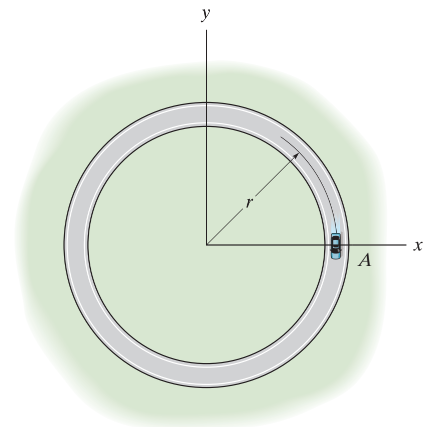
$$a_n = \frac{{v_B}^2}{\rho} = \frac{8^2}{8.396} = 7.622\text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4)^2 + (7.622)^2} = 8.61\text{ m/s}^2$$ **Ans.**

Ans:
$a = 8.61$ m/s^2

12–125.

The car travels around the circular track having a radius of $r = 300$ m such that when it is at point A it has a velocity of 5 m/s, which is increasing at the rate of $\dot{v} = (0.06t)$ m/s^2, where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled one-third the way around the track.



SOLUTION

$a_t = v = 0.06t$

$dv = a_t\, dt$

$$\int_s^v dv = \int_0^t 0.06t\, dt$$

$v = 0.03t^2 + 5$

$ds = v\, dt$

$$\int_0^s ds = \int_0^t (0.03t^2 + 5)\, dt$$

$s = 0.01t^3 + 5t$

$$s = \frac{1}{3}(2\pi(300)) = 628.3185$$

$0.01t^3 + 5t - 628.3185 = 0$

Solve for the positive root,

$t = 35.58$ s

$v = 0.03(35.58)^2 + 5 = 42.978$ m/s $= 43.0$ m/s **Ans.**

$$a_n = \frac{v^2}{\rho} = \frac{(42.978)^2}{300} = 6.157 \text{ m/s}^2$$

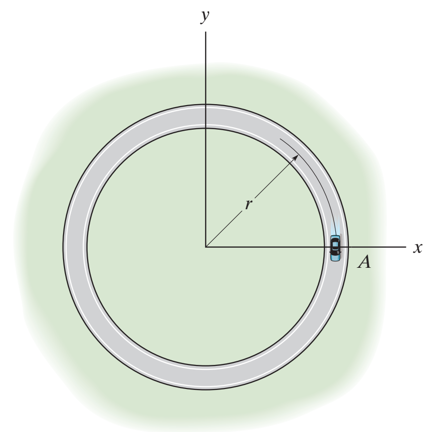
$a_t = 0.06(35.58) = 2.135$ m/s^2

$a = \sqrt{(6.157)^2 + (2.135)^2} = 6.52$ m/s^2 **Ans.**

Ans:
$v = 43.0$ m/s
$a = 6.52$ m/s^2

12–126.

The car travels around the portion of a circular track having a radius of $r = 500$ ft such that when it is at point A it has a velocity of 2 ft/s, which is increasing at the rate of $\dot{v} = (0.002t)$ ft/s^2, where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled three-fourths the way around the track.



SOLUTION

$a_t = 0.002\text{ s}$

$a_t\, ds = v\, dv$

$$\int_0^s 0.002s\, ds = \int_2^v v\, dv$$

$$0.001s^2 = \frac{1}{2}v^2 - \frac{1}{2}(2)^2$$

$$v^2 = 0.002s^2 + 4$$

$$s = \frac{3}{4}[2\pi(500)] = 2356.194 \text{ ft}$$

$$v^2 = 0.002(2356.194)^2 + 4$$

$$v = 105.39 \text{ ft/s} = 105 \text{ ft/s}$$ **Ans.**

$$a_n = \frac{v^2}{\rho} = \frac{(105.39)^2}{500} = 22.21 \text{ ft/s}^2$$

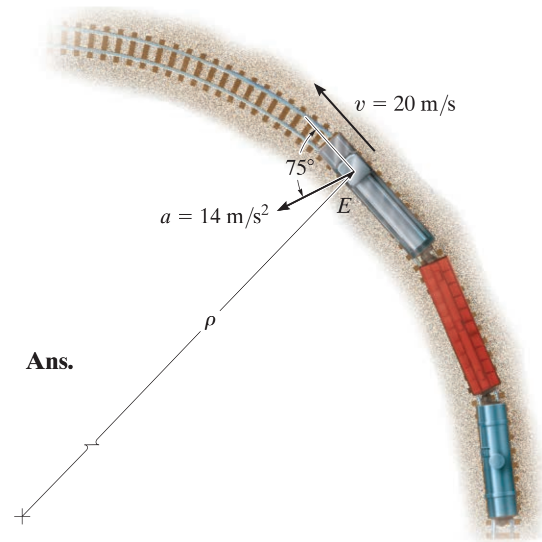
$$a_t = 0.002(2356.194) = 4.712 \text{ ft/s}^2$$

$$a = \sqrt{(22.21)^2 + (4.712)^2} = 22.7 \text{ ft/s}^2$$ **Ans.**

Ans:
$v = 105$ ft/s
$a = 22.7$ ft/s^2

12–127.

At a given instant the train engine at E has a speed of 20 m/s and an acceleration of 14 m/s^2 acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.



SOLUTION

$a_t = 14 \cos 75° = 3.62 \text{ m/s}^2$ **Ans.**

$a_n = 14 \sin 75°$

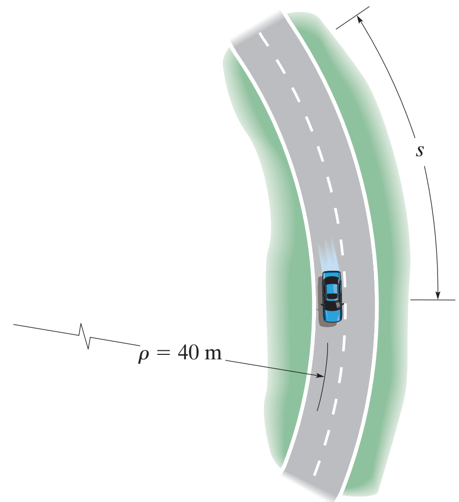
$$a_n = \frac{(20)^2}{\rho}$$

$\rho = 29.6$ m **Ans.**

Ans:
$a_t = 3.62 \text{ m/s}^2$
$\rho = 29.6$ m

***12–128.**

The car has an initial speed $v_0 = 20$ m/s. If it increases its speed along the circular track at $s = 0$, $a_t = (0.8s)$ m/s^2, where s is in meters, determine the time needed for the car to travel $s = 25$ m.



SOLUTION

The distance traveled by the car along the circular track can be determined by integrating $v\,dv = a_t\,ds$. Using the initial condition $v = 20$ m/s at $s = 0$,

$$\int_{20\text{ m/s}}^{v} v\,dv = \int_0^{s} 0.8\,s\,ds$$

$$\left.\frac{v^2}{2}\right|_{20\text{ m/s}}^{v} = 0.4\,s^2$$

$$v = \left\{\sqrt{0.8\,(s^2 + 500)}\right\}\text{ m/s}$$

The time can be determined by integrating $dt = \dfrac{ds}{v}$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^{t} dt = \int_0^{25\text{ m}} \frac{ds}{\sqrt{0.8(s^2 + 500)}}$$

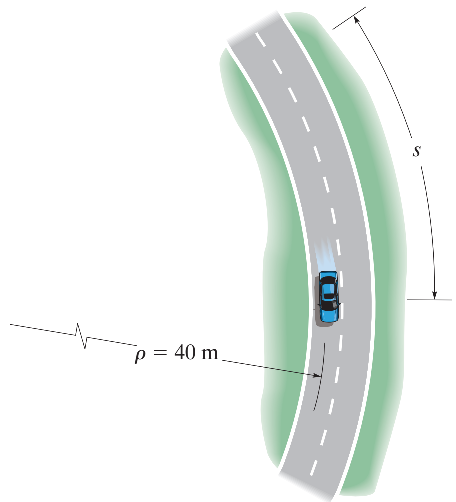
$$t = \frac{1}{\sqrt{0.8}}\left[\ln\left(s + \sqrt{s^2 + 500}\right)\right]\Big|_0^{25\text{ m}}$$

$$= 1.076\text{ s} = 1.08\text{ s}$$ **Ans.**

Ans:
$t = 1.08$ s

12–129.

The car starts from rest at $s = 0$ and increases its speed at $a_t = 4\ \text{m/s}^2$. Determine the time when the magnitude of acceleration becomes $20\ \text{m/s}^2$. At what position s does this occur?



SOLUTION

Acceleration. The normal component of the acceleration can be determined from

$$\theta_r = \frac{v^2}{\rho}; \qquad a_r = \frac{v^2}{40}$$

From the magnitude of the acceleration

$$a = \sqrt{a_t^2 + a_n^2}; \quad 20 = \sqrt{4^2 + \left(\frac{v^2}{40}\right)^2} \qquad v = 28.00\ \text{m/s}$$

Velocity. Since the car has a constant tangential accelaration of $a_t = 4\ \text{m/s}^2$,

$$v = v_0 + a_t t; \quad 28.00 = 0 + 4t$$

$t = 6.999\ \text{s} = 7.00\ \text{s}$ **Ans.**

$$v^2 = v_0^2 + 2a_t s; \ \ 28.00^2 = 0^2 + 2(4)\ s$$

$s = 97.98\ \text{m} = 98.0\ \text{m}$ **Ans.**

Ans:
$t = 7.00\ \text{s}$
$s = 98.0\ \text{m}$

12–130.

A boat is traveling along a circular curve having a radius of 100 ft. If its speed at $t = 0$ is 15 ft/s and is increasing at $\dot{v} = (0.8t)$ ft/s^2, determine the magnitude of its acceleration at the instant $t = 5$ s.

SOLUTION

$$\int_{15}^{v} dv = \int_{0}^{5} 0.8t\,dt$$

$$v = 25 \text{ ft/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{100} = 6.25 \text{ ft/s}^2$$

At $t = 5$ s, $\quad a_t = \dot{v} = 0.8(5) = 4 \text{ ft/s}^2$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{4^2 + 6.25^2} = 7.42 \text{ ft/s}^2$$ **Ans.**

Ans:
$a = 7.42$ ft/s^2

12–131.

A boat is traveling along a circular path having a radius of 20 m. Determine the magnitude of the boat's acceleration when the speed is $v = 5$ m/s and the rate of increase in the speed is $\dot{v} = 2\text{ m/s}^2$.

SOLUTION

$a_t = 2\text{ m/s}^2$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{20} = 1.25\text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.25^2} = 2.36\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 2.36\text{ m/s}^2$

***12–132.**

Starting from rest, a bicyclist travels around a horizontal circular path, $\rho = 10$ m, at a speed of $v = (0.09t^2 + 0.1t)$ m/s, where t is in seconds. Determine the magnitudes of his velocity and acceleration when he has traveled $s = 3$ m.

SOLUTION

$$\int_0^s ds = \int_0^t (0.09t^2 + 0.1t)dt$$

$$s = 0.03t^3 + 0.05t^2$$

When $s = 3$ m, $\quad 3 = 0.03t^3 + 0.05t^2$

Solving,

$t = 4.147$ s

$$v = \frac{ds}{dt} = 0.09t^2 + 0.1t$$

$$v = 0.09(4.147)^2 + 0.1(4.147) = 1.96 \text{ m/s}$$ **Ans.**

$$a_t = \frac{dv}{dt} = 0.18t + 0.1\bigg|_{t=4.147\text{ s}} = 0.8465 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{1.96^2}{10} = 0.3852 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8465)^2 + (0.3852)^2} = 0.930 \text{ m/s}^2$$ **Ans.**

Ans:
$v = 1.96$ m/s
$a = 0.930$ m/s^2

12–133.

A particle travels around a circular path having a radius of 50 m. If it is initially traveling with a speed of 10 m/s and its speed then increases at a rate of $\dot{v} = (0.05\,v)$ m/s^2, determine the magnitude of the particle's acceleraton four seconds later.

SOLUTION

Velocity: Using the initial condition $v = 10$ m/s at $t = 0$ s,

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_{10\text{ m/s}}^{v} \frac{dv}{0.05v}$$

$$t = 20\ln\frac{v}{10}$$

$$v = (10e^{t/20})\text{ m/s}$$

When $t = 4$ s,

$$v = 10e^{4/20} = 12.214\text{ m/s}$$

Acceleration: When $v = 12.214$ m/s ($t = 4$ s),

$$a_t = 0.05(12.214) = 0.6107\text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(12.214)^2}{50} = 2.984\text{ m/s}^2$$

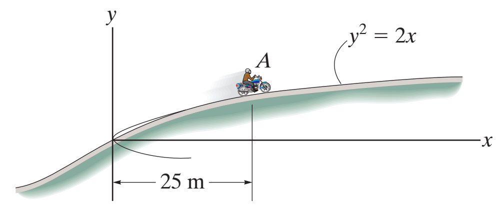
Thus, the magnitude of the particle's acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6107^2 + 2.984^2} = 3.05\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 3.05\text{ m/s}^2$

12–134.

The motorcycle is traveling at a constant speed of 60 km/h. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = \sqrt{2}x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{2}x^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{2}x^{-3/2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{2}\sqrt{2}x^{-1/2}\right)^2\right]^{3/2}}{\left|-\frac{1}{4}\sqrt{2}x^{-3/2}\right|}\right|_{x=25\text{ m}} = 364.21\text{ m}$$

Acceleration: The speed of the motorcycle at a is

$$v = \left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 16.67\text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{364.21} = 0.7627\text{ m/s}^2$$

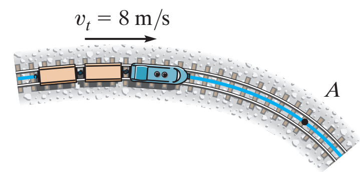
Since the motorcycle travels with a constant speed, $a_t = 0$. Thus, the magnitude of the motorcycle's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0^2 + 0.7627^2} = 0.763\text{ m/s}^2$$ **Ans.**

Ans:
$a = 0.763\text{ m/s}^2$

12–135.

When $t = 0$, the train has a speed of 8 m/s, which is increasing at 0.5 m/s^2. Determine the magnitude of the acceleration of the engine when it reaches point A, at $t = 20$ s. Here the radius of curvature of the tracks is $\rho_A = 400$ m.



SOLUTION

Velocity. The velocity of the train along the track can be determined by integrating $dv = a_t\,dt$ with initial condition $v = 8$ m/s at $t = 0$.

$$\int_{8\text{ m/s}}^{v} dv = \int_0^t 0.5\,dt$$

$$v - 8 = 0.5\,t$$

$$v = \{0.5\,t + 8\}\text{ m/s}$$

At $t = 20$ s,

$$v|_{t=20\,s} = 0.5(20) + 8 = 18\text{ m/s}$$

Acceleration. Here, the tangential component is $a_t = 0.5\text{ m/s}^2$. The normal component can be determined from

$$a_n = \frac{v^2}{\rho} = \frac{18^2}{400} = 0.81\text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2}$$

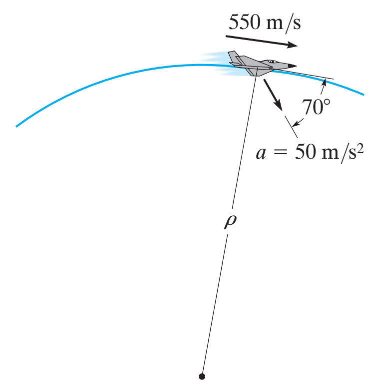
$$= \sqrt{0.5^2 + 0.81^2}$$

$$= 0.9519\text{ m/s}^2 = 0.952\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.952\text{ m/s}^2$

***12–136.**

At a given instant the jet plane has a speed of 550 m/s and an acceleration of 50 m/s^2 acting in the direction shown. Determine the rate of increase in the plane's speed, and also the radius of curvature ρ of the path.



SOLUTION

Acceleration. With respect to the n–t coordinate established as shown in Fig. a, the tangential and normal components of the acceleration are

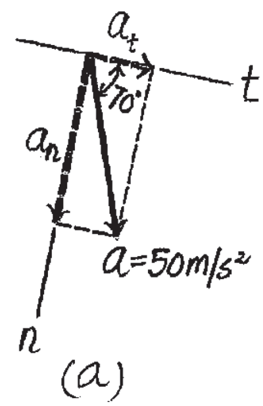
$$a_t = 50 \cos 70^\circ = 17.10 \text{ m/s}^2 = 17.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = 50 \sin 70^\circ = 46.98 \text{ m/s}^2$$

However,

$$a_n = \frac{v^2}{\rho}; \qquad 46.98 = \frac{550^2}{\rho}$$

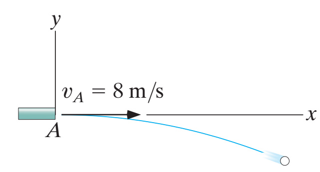
$$\rho = 6438.28 \text{ m} = 6.44 \text{ km} \qquad \textbf{Ans.}$$



Ans:
$a_t = 17.1 \text{ m/s}^2$
$a_n = 46.98 \text{ m/s}^2$
$\rho = 6.44$ km

12–137.

The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



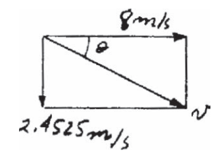
SOLUTION

$v_x = 8$ m/s

$(\xrightarrow{+})\qquad s = v_0 t$

$x = 8t$

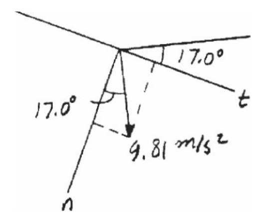
$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$



$$y = 0 + 0 + \frac{1}{2}(-9.81)t^2$$

$$y = -4.905t^2$$

$$y = -4.905\left(\frac{x}{8}\right)^2$$



$y = -0.0766x^2$ (Parabola) **Ans.**

$v = v_0 + a_c t$

$v_y = 0 - 9.81t$

When $t = 0.25$ s,

$v_y = -2.4525$ m/s

$v = \sqrt{(8)^2 + (2.4525)^2} = 8.37$ m/s **Ans.**

$\theta = \tan^{-1}\left(\frac{2.4525}{8}\right) = 17.04°$

$a_x = 0 \quad a_y = 9.81\ \text{m/s}^2$

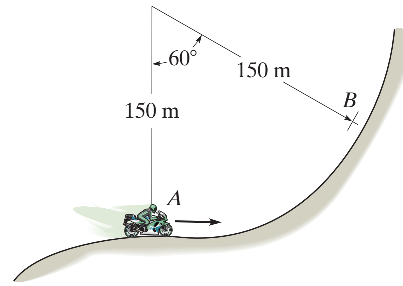
$a_n = 9.81 \cos 17.04° = 9.38\ \text{m/s}^2$ **Ans.**

$a_t = 9.81 \sin 17.04° = 2.88\ \text{m/s}^2$ **Ans.**

Ans:
$y = -0.0766x^2$
$v = 8.37$ m/s
$a_n = 9.38\ \text{m/s}^2$
$a_t = 2.88\ \text{m/s}^2$

12–138.

The motorcycle is traveling at 40 m/s when it is at A. If the speed is then decreased at $\dot{v} = -(0.05\, s)$ m/s^2, where s is in meters measured from A, determine its speed and acceleration when it reaches B.



SOLUTION

Velocity. The velocity of the motorcycle along the circular track can be determined by integrating $v dv = a_d s$ with the initial condition $v = 40$ m/s at $s = 0$. Here, $a_t = -0.05s$.

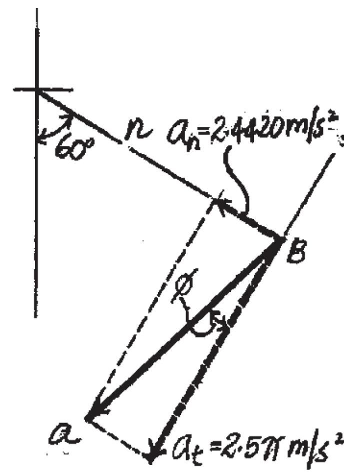
$$\int_{40\text{ m/s}}^{v} v dv = \int_0^s -0.05\, s ds$$

$$\left.\frac{v^2}{2}\right|_{40\text{ m/s}}^{v} = -0.025\, s^2 \Big|_0^s$$

$$v = \left\{\sqrt{1600 - 0.05\, s^2}\right\} \text{ m/s}$$

At B, $s = r\theta = 150\left(\dfrac{\pi}{3}\right) = 50\pi$ m. Thus

$$v_B = v|_{s=50\pi\text{m}} = \sqrt{1600 - 0.05(50\pi)^2} = 19.14 \text{ m/s} = 19.1 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

Acceleration. At B, the tangential and normal components are

$$a_t = 0.05(50\pi) = 2.5\pi \text{ m/s}^2$$

$$a_n = \frac{v_B^2}{\rho} = \frac{19.14^2}{150} = 2.4420 \text{ m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(2.5\pi)^2 + 2.4420^2} = 8.2249 \text{ m/s}^2 = 8.22 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle ϕ measured from the negative t-axis, Fig. a.

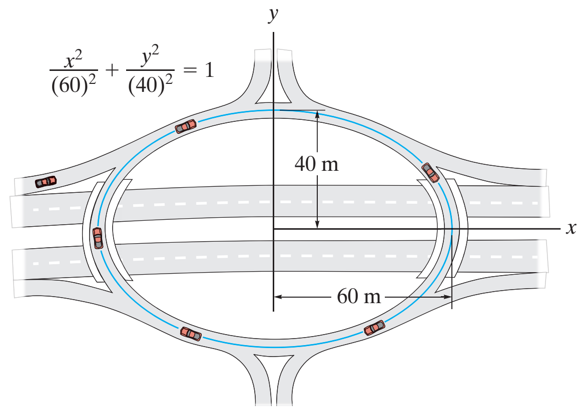
$$\phi = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{2.4420}{2.5\pi}\right)$$

$$= 17.27° = 17.3° \qquad \textbf{Ans.}$$

Ans:
$v_B = 19.1$ m/s
$a = 8.22$ m/s^2
$\phi = 17.3°$
up from negative$-t$ axis

12–139.

Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the minimum acceleration experienced by the passengers.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)^2$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{b^4}{a^2y^2}\right)\left(\frac{x^2}{a^2}\right)$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \frac{b^4}{a^2y^2}\left(1 - \frac{y^2}{b^2}\right)$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \frac{b^4}{a^2y^2} + \frac{b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^2}{a^2y^3}\right|}$$

At $x = 0$, $y = h$,

$$\rho = \frac{a^2}{b}$$

12–139. Continued

Thus

$a_t = 0$

$$a_{\min} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{a^2}{b}} = \frac{v^2 b}{a^2}$$

Set $a = 60$ m, $b = 40$ m,

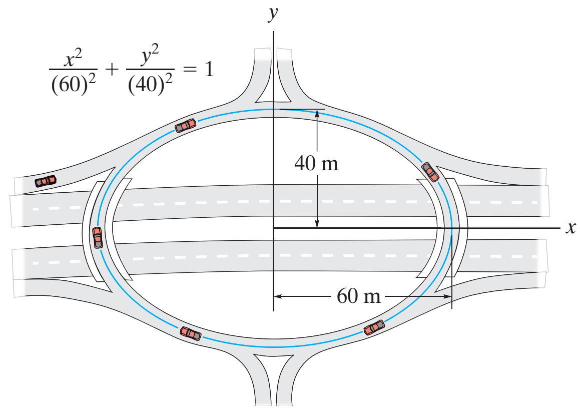
$$v = \frac{60(10)^3}{3600} = 16.67 \text{ m/s}$$

$$a_{\min} = \frac{(16.67)^2(40)}{(60)^2} = 3.09 \text{ m/s}^2$$ **Ans.**

Ans:
$a_{\min} = 3.09 \text{ m/s}^2$

***12–140.**

Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the maximum acceleration experienced by the passengers.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)^2$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^4}{a^2y^3}\right|}$$

At $x = a$, $y = 0$,

$$\rho = \frac{b^2}{a}$$

Then

$$a_t = 0$$

$$a_{\max} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{b^2}{n}} = \frac{v^2a}{b^3}$$

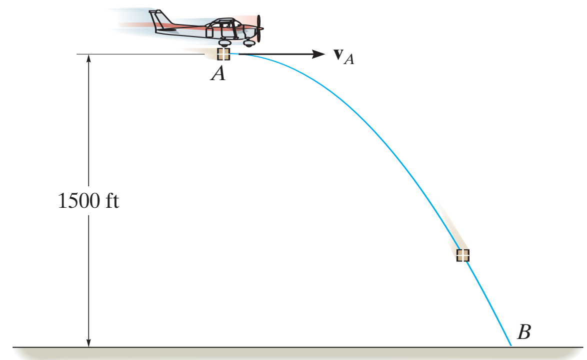
Set $a = 60$ m, $b = 40$ m, $v = \frac{60(10^3)}{3600} = 16.67$ m/s

$$a_{\max} = \frac{(16.67)^2(60)}{(40)^2} = 10.4 \text{ m/s}^2$$ **Ans.**

Ans:
$a_{\max} = 10.4 \text{ m/s}^2$

12–141.

A package is dropped from the plane which is flying with a constant horizontal velocity of $v_A = 150$ ft/s. Determine the normal and tangential components of acceleration and the radius of curvature of the path of motion (a) at the moment the package is released at A, where it has a horizontal velocity of $v_A = 150$ ft/s, and (b) *just before* it strikes the ground at B.



SOLUTION

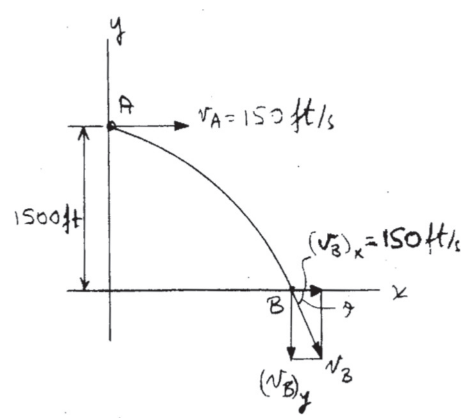
Initially (Point A):

$(a_n)_A = g = 32.2 \text{ ft/s}^2$ **Ans.**

$(a_t)_A = 0$ **Ans.**

$$(a_n)_A = \frac{v_A^2}{\rho_A}; \quad 32.2 = \frac{(150)^2}{\rho_A}$$

$\rho_A = 698.8$ ft **Ans.**



$(v_B)_x = (v_A)_x = 150$ ft/s

$(+\downarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$$(v_B)_y^2 = 0 + 2(32.2)(1500 - 0)$$

$$(v_B)_y = 310.8 \text{ ft/s}$$

$$v_B = \sqrt{(150)^2 + (310.8)^2} = 345.1 \text{ ft/s}$$

$$\theta = \tan^{-1}\left(\frac{v_{B_y}}{v_{B_x}}\right) = \tan^{-1}\left(\frac{310.8}{150}\right) = 64.23°$$

$(a_n)_B = g\cos\theta = 32.2\cos 64.24° = 14.0 \text{ ft/s}^2$ **Ans.**

$(a_t)_B = g\sin\theta = 32.2\sin 64.24° = 29.0 \text{ ft/s}^2$ **Ans.**

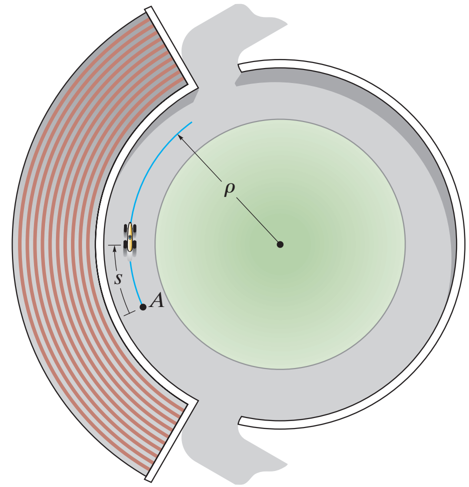
$$(a_n)_B = \frac{v_B^2}{\rho_B}; \quad 14.0 = \frac{(345.1)^2}{\rho_B}$$

$\rho_B = 8509.8 \text{ ft} = 8.51(10^3) \text{ ft}$ **Ans.**

Ans:
$(a_n)_A = g = 32.2 \text{ ft/s}^2$
$(a_t)_A = 0$
$\rho_A = 699$ ft
$(a_n)_B = 14.0 \text{ ft/s}^2$
$(a_t)_B = 29.0 \text{ ft/s}^2$
$\rho_B = 8.51(10^3)$ ft

12–142.

The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



SOLUTION

$$a_t = 0.4s = \frac{v\,dv}{ds}$$

$$a\,ds = v\,dv$$

$$\int_0^s 0.4s\,ds = \int_{15}^v v\,dv$$

$$\frac{0.4s^2}{2}\bigg|_0^s = \frac{v^2}{2}\bigg|_{15}^v$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$$

$$1\text{n}\,(s + \sqrt{s^2 + 562.5})\bigg|_0^s = 0.632\,456t$$

$$1\text{n}\,(s + \sqrt{s^2 + 562.5}) - 3.166\,196 = 0.632\,456t$$

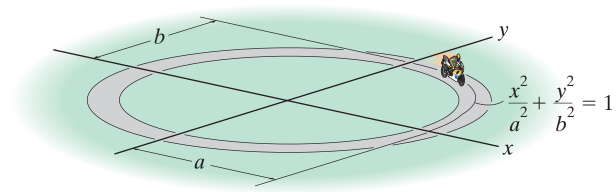
At $s = 20$ m,

$t = 1.21$ s **Ans.**

Ans:
$t = 1.21$ s

12–143.

The motorcycle travels along the elliptical track at a constant speed v. Determine its greatest acceleration if $a > b$.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{b^2x}{a^2y}\right)^2\right]^{3/2}}{\frac{-b^4}{a^2y^3}}$$

At $x = a$, $y = 0$,

$$\rho = \frac{b^2}{a}$$

Then

$$a_t = 0$$

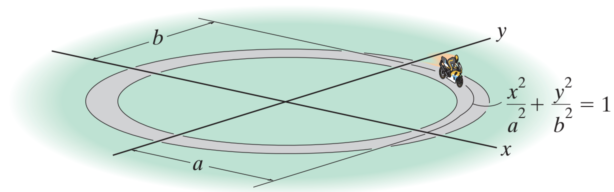
$$a_{max} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{b^2}{a}} = \frac{v^2a}{b^2}$$ **Ans.**

Ans:

$$a_{max} = \frac{v^2a}{b^2}$$

***12–144.**

The motorcycle travels along the elliptical track at a constant speed v. Determine its smallest acceleration if $a > b$.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{b^2x}{a^2y}\right)^2\right]^{3/2}}{\frac{-b^4}{a^2y^3}}$$

At $x = 0, y = b$,

$$|\rho| = \frac{a^2}{b}$$

Thus

$$a_t = 0$$

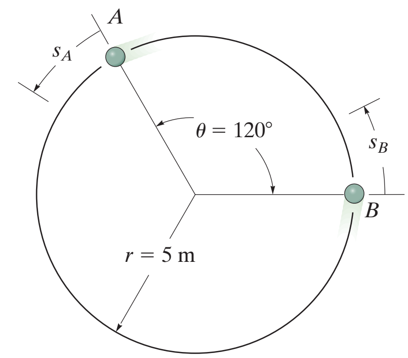
$$a_{\min} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{a^3}{b}} = \frac{v^2b}{a^2} \qquad \textbf{Ans.}$$

Ans:

$$a_{\min} = \frac{v^2b}{a^2}$$

12–145.

Particles A and B are traveling counter-clockwise around a circular track at a constant speed of 8 m/s. If at the instant shown the speed of A begins to increase by $(a_t)_A = (0.4s_A)$ m/s^2, where s_A is in meters, determine the distance measured counterclockwise along the track from B to A when $t = 1$ s. What is the magnitude of the acceleration of each particle at this instant?



SOLUTION

Distance Traveled: Initially the distance between the particles is

$$d_0 = \rho d\theta = 5\left(\frac{120°}{180°}\right)\pi = 10.47 \text{ m}$$

When $t = 1$ s, B travels a distance of

$$d_B = 8(1) = 8 \text{ m}$$

The distance traveled by particle A is determined as follows:

$$v dv = a ds$$

$$\int_{8 \text{ m/s}}^{v} v dv = \int_0^s 0.4 \, s ds$$

$$v = 0.6325\sqrt{s^2 + 160} \qquad \textbf{(1)}$$

$$dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{0.6325\sqrt{s^2 + 160}}$$

$$1 = \frac{1}{0.6325}\left(\text{In}\left[\frac{\sqrt{s^2 + 160} + s}{\sqrt{160}}\right]\right)$$

$$s = 8.544 \text{ m}$$

Thus the distance between the two cyclists after $t = 1$ s is

$$d = 10.47 + 8.544 - 8 = 11.0 \text{ m} \qquad \textbf{Ans.}$$

Acceleration:

For A, when $t = 1$ s,

$$\left(a_t\right)_A = \dot{v}_A = 0.4\left(8.544\right) = 3.4176 \text{ m/s}^2$$

$$v_A = 0.6325\sqrt{8.544^2 + 160} = 9.655 \text{ m/s}$$

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{9.655^2}{5} = 18.64 \text{ m/s}^2$$

The magnitude of the A's acceleration is

$$a_A = \sqrt{3.4176^2 + 18.64^2} = 19.0 \text{ m/s}^2 \qquad \textbf{Ans.}$$

For B, when $t = 1$ s,

$$\left(a_t\right)_B = \dot{v}_A = 0$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{8^2}{5} = 12.80 \text{ m/s}^2$$

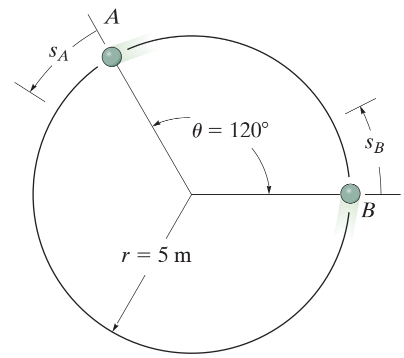
The magnitude of the B's acceleration is

$$a_B = \sqrt{0^2 + 12.80^2} = 12.8 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$d = 11.0$ m
$a_A = 19.0$ m/s^2
$a_B = 12.8$ m/s^2

12–146.

Particles A and B are traveling around a circular track at a speed of 8 m/s at the instant shown. If the speed of B is increasing by $(a_t)_B = 4\ \text{m/s}^2$, and at the same instant A has an increase in speed of $(a_t)_A = 0.8t\ \text{m/s}^2$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?



SOLUTION

Distance Traveled: Initially the distance between the two particles is $d_0 = \rho\theta = 5\left(\frac{120°}{180°}\pi\right) = 10.47$ m. Since particle B travels with a constant acceleration, distance can be obtained by applying equation

$$s_B = (s_0)_B + (v_0)_B t + \frac{1}{2}a_c t^2$$

$$s_B = 0 + 8t + \frac{1}{2}(4)\,t^2 = \left(8t + 2t^2\right)\text{ m} \qquad \textbf{[1]}$$

The distance traveled by particle A can be obtained as follows.

$$dv_A = a_A\,dt$$

$$\int_{8\text{ m/s}}^{v_A} dv_A = \int_0^t 0.8\,t\,dt$$

$$v_A = \left(0.4t^2 + 8\right)\ \text{m/s} \qquad \textbf{[2]}$$

$$ds_A = v_A\,dt$$

$$\int_0^{s_A} ds_A = \int_0^t \left(0.4t^2 + 8\right)dt$$

$$s_A = 0.1333t^3 + 8t$$

In order for the collision to occur

$$s_A + d_0 = s_B$$

$$0.1333t^3 + 8t + 10.47 = 8t + 2t^2$$

Solving by trial and error $\quad t = 2.5074\text{ s} = 2.51\text{ s}$ **Ans.**

Note: If particle A strikes B then, $s_A = 5\left(\frac{240°}{180°}\pi\right) + s_B$. This equation will result in $t = 14.6\text{ s} > 2.51$ s.

Acceleration: The tangential acceleration for particle A and B when $t = 2.5074$ are $(a_t)_A = 0.8t = 0.8\,(2.5074) = 2.006\ \text{m/s}^2$ and $(a_t)_B = 4\ \text{m/s}^2$, respectively. When $t = 2.5074$ s, from Eq. [1], $v_A = 0.4\left(2.5074^2\right) + 8 = 10.51$ m/s and $v_B = (v_0)_B + a_c t = 8 + 4(2.5074) = 18.03$ m/s. To determine the normal acceleration, apply Eq. 12–20.

$$(a_n)_A = \frac{v_A^2}{\rho} = \frac{10.51^2}{5} = 22.11\ \text{m/s}^2$$

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{18.03^2}{5} = 65.01\ \text{m/s}^2$$

The magnitude of the acceleration for particles A and B just before collision are

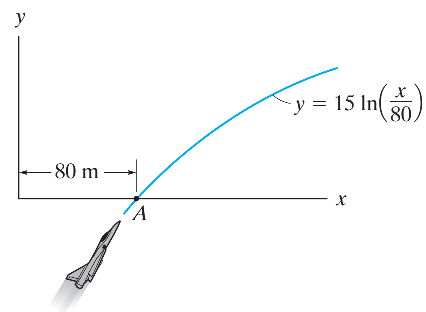
$$a_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{2.006^2 + 22.11^2} = 22.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$a_B = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{4^2 + 65.01^2} = 65.1\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$t = 2.51$ s
$a_A = 22.2\ \text{m/s}^2$
$a_B = 65.1\ \text{m/s}^2$

12–147.

The jet plane is traveling with a speed of 120 m/s which is decreasing at 40 m/s^2 when it reaches point A. Determine the magnitude of its acceleration when it is at this point. Also, specify the direction of flight, measured from the x axis.



SOLUTION

$$y = 15 \ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x}\bigg|_{x = 80\text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}\bigg|_{x = 80\text{ m}} = -0.002344$$

$$\rho\bigg|_{x = 80\text{ m}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x = 80\text{ m}}$$

$$= \frac{[1 + (0.1875)^2]^{3/2}}{|-0.002344|} = 449.4\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(120)^2}{449.4} = 32.04\text{ m/s}^2$$

$$a_n = -40\text{ m/s}^2$$

$$a = \sqrt{(-40)^2 + (32.04)^2} = 51.3\text{ m/s}^2$$ **Ans.**

Since

$$\frac{dy}{dx} = \tan\theta = 0.1875$$

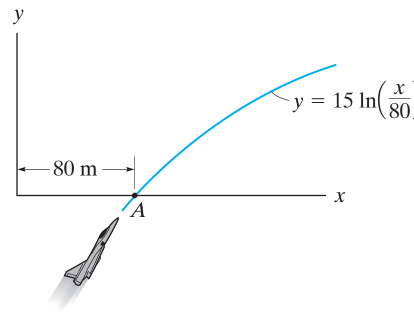
$$\theta = 10.6°$$ **Ans.**

Ans:

$\theta = 10.6°$

***12–148.**

The jet plane is traveling with a constant speed of 110 m/s along the curved path. Determine the magnitude of the acceleration of the plane at the instant it reaches point $A(y = 0)$.



SOLUTION

$$y = 15 \ln\left(\frac{x}{80}\right)$$

$$\frac{dy}{dx} = \frac{15}{x}\bigg|_{x = 80\text{ m}} = 0.1875$$

$$\frac{d^2y}{dx^2} = -\frac{15}{x^2}\bigg|_{x = 80\text{ m}} = -0.002344$$

$$\rho\bigg|_{x = 80\text{ m}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x = 80\text{ m}}$$

$$= \frac{\left[1 + (0.1875)^2\right]^{3/2}}{|-0.002344|} = 449.4\text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(110)^2}{449.4} = 26.9\text{ m/s}^2$$

Since the plane travels with a constant speed, $a_t = 0$. Hence

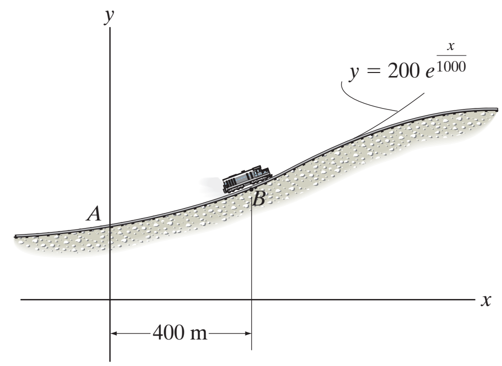
$$a = a_n = 26.9\text{ m/s}^2$$ **Ans.**

Ans:

$a = 26.9\text{ m/s}^2$

12–149.

The train passes point B with a speed of 20 m/s which is decreasing at $a_t = -0.5\ \text{m/s}^2$. Determine the magnitude of acceleration of the train at this point.



SOLUTION

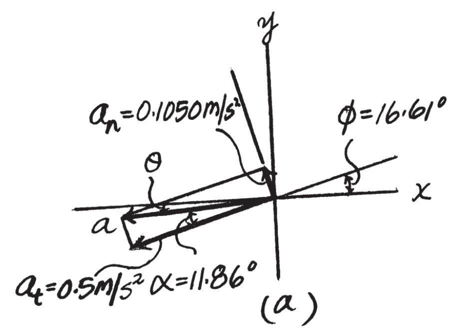
Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 200\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2\left(\frac{1}{1000}\right)e^{\frac{x}{1000}} = 0.2\left(10^{-3}\right)e^{\frac{x}{1000}}.$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(0.2\,e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2\left(10^{-3}\right)e^{\frac{x}{1000}}\right|}\right|_{x=400\text{ m}} = 3808.96\text{ m}$$



Acceleration:

$$a_t = \dot{v} = -0.5\ \text{m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{20^2}{3808.96} = 0.1050\ \text{m/s}^2$$

The magnitude of the train's acceleration at B is

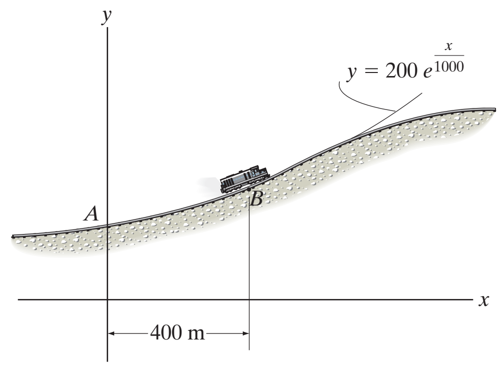
$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-0.5)^2 + 0.1050^2} = 0.511\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$a = 0.511\ \text{m/s}^2$

12–150.

The train passes point A with a speed of 30 m/s and begins to decrease its speed at a constant rate of $a_t = -0.25 \text{ m/s}^2$. Determine the magnitude of the acceleration of the train when it reaches point B, where $s_{AB} = 412$ m.



SOLUTION

Velocity: The speed of the train at B can be determined from

$$v_B{}^2 = v_A{}^2 + 2a_t(s_B - s_A)$$

$$v_B{}^2 = 30^2 + 2(-0.25)(412 - 0)$$

$$v_B = 26.34 \text{ m/s}$$

Radius of Curvature:

$$y = 200e^{\frac{x}{1000}}$$

$$\frac{dy}{dx} = 0.2e^{\frac{x}{1000}}$$

$$\frac{d^2y}{dx^2} = 0.2\left(10^{-3}\right)e^{\frac{x}{1000}}$$

$$\rho = \frac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\left|\dfrac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(0.2e^{\frac{x}{1000}}\right)^2\right]^{3/2}}{\left|0.2\left(10^{-3}\right)e^{\frac{x}{1000}}\right|}\right|_{x=400\text{ m}} = 3808.96 \text{ m}$$

Acceleration:

$$a_t = \dot{v} = -0.25 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{26.34^2}{3808.96} = 0.1822 \text{ m/s}^2$$

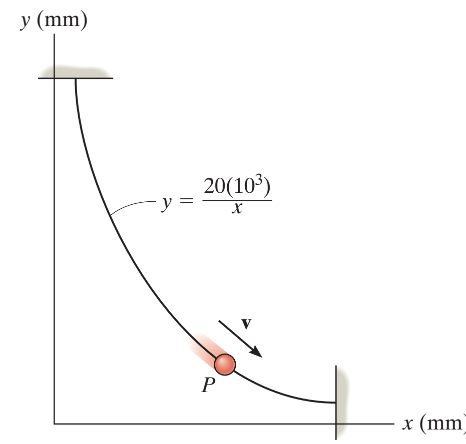
The magnitude of the train's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{\left(-0.5\right)^2 + 0.1822^2} = 0.309 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.309 \text{ m/s}^2$

12–151.

The particle travels with a constant speed of 300 mm/s along the curve. Determine the particle's acceleration when it is located at point (200 mm, 100 mm) and sketch this vector on the curve.



SOLUTION

$v = 300$ mm/s

$$a_t = \frac{dv}{dt} = 0$$

$$y = \frac{20(10^3)}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=200} = -\frac{20(10^3)}{x^2} = -0.5$$

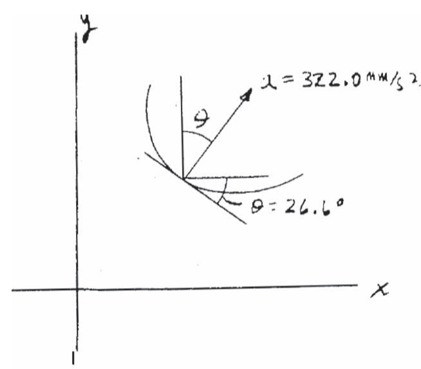
$$\left.\frac{d^2y}{dx^2}\right|_{x=200} = \frac{40(10^3)}{x^3} = 5(10^{-3})$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + (-0.5)^2]^{\frac{3}{2}}}{\left|5(10^{-3})\right|} = 279.5 \text{ mm}$$

$$a_n = \frac{v^2}{\rho} = \frac{(300)^2}{279.5} = 322 \text{ mm/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$= \sqrt{(0)^2 + (322)^2} = 322 \text{ mm/s}^2 \qquad \textbf{Ans.}$$



Since $\frac{dy}{dx} = -0.5$,

$$\theta = \tan^{-1}(-0.5) = 26.6° \qquad \textbf{Ans.}$$

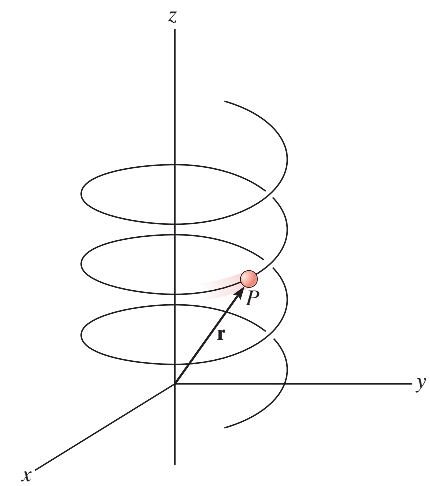
Ans:

$a = 322$ mm/s^2

$\theta = 26.6°$

***12–152.**

A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = \{2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + (2t)\mathbf{k}\}$ m, where t is in seconds and the arguments for the sine and cosine are given in radians. When $t = 8$ s, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v}_P$ and acceleration $\mathbf{a}_P$ of the particle in terms of their $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components. The binormal is parallel to $\mathbf{v}_P \times \mathbf{a}_P$. Why?



SOLUTION

$$\mathbf{r}_P = 2\cos(0.1t)\mathbf{i} + 1.5\sin(0.1t)\mathbf{j} + 2t\mathbf{k}$$

$$\mathbf{v}_P = \dot{\mathbf{r}} = -0.2\sin(0.1t)\mathbf{i} + 0.15\cos(0.1t)\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = \ddot{\mathbf{r}} = -0.02\cos(0.1t)\mathbf{i} - 0.015\sin(0.1t)\mathbf{j}$$

When $t = 8$ s,

$$\mathbf{v}_P = -0.2\sin(0.8\text{ rad})\mathbf{i} + 0.15\cos(0.8\text{ rad})\mathbf{j} + 2\mathbf{k} = -0.143\,47\mathbf{i} + 0.104\,51\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{a}_P = -0.02\cos(0.8\text{ rad})\mathbf{i} - 0.015\sin(0.8\text{ rad})\mathbf{j} = -0.013\,934\mathbf{i} - 0.010\,76\mathbf{j}$$

Since the binormal vector is perpendicular to the plane containing the n–t axis, and $\mathbf{a}_p$ and $\mathbf{v}_p$ are in this plane, then by the definition of the cross product,

$$\mathbf{b} = \mathbf{v}_P \times \mathbf{a}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.14\,347 & 0.104\,51 & 2 \\ -0.013\,934 & -0.010\,76 & 0 \end{vmatrix} = 0.021\,52\mathbf{i} - 0.027\,868\mathbf{j} + 0.003\mathbf{k}$$

$$b = \sqrt{(0.02152)^2 + (-0.027868)^2 + (0.003)^2} = 0.035\,338$$

$$\mathbf{u}_b = 0.608\,99\mathbf{i} - 0.788\,62\mathbf{j} + 0.085\mathbf{k}$$

$\alpha = \cos^{-1}(0.608\,99) = 52.5°$ **Ans.**

$\beta = \cos^{-1}(-0.788\,62) = 142°$ **Ans.**

$\gamma = \cos^{-1}(0.085) = 85.1°$ **Ans.**

Note: The direction of the binormal axis may also be specified by the unit vector $\mathbf{u}_{b'} = -\mathbf{u}_b$, which is obtained from $\mathbf{b}' = \mathbf{a}_p \times \mathbf{v}_p$.

For this case, $\alpha = 128°$, $\beta = 37.9°$, $\gamma = 94.9°$ **Ans.**

Ans:

$\alpha = 52.5°$
$\beta = 142°$
$\gamma = 85.1°$
$\alpha = 128°, \beta = 37.9°, \gamma = 94.9°$

12–153.

The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m, where t is in seconds. Determine the normal and tangential components of the particle's velocity and acceleration when $t = 2$ s.

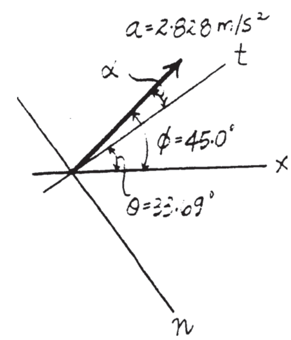
SOLUTION

Velocity: Here, $\mathbf{r} = \left\{\left(2t + t^2\right)\mathbf{i} + t^2\mathbf{j}\right\}$ m. To determine the velocity **v**, apply Eq. 12–7.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \{(2 + 2t)\,\mathbf{i} + 2t\mathbf{j}\}\text{ m/s}$$

When $t = 2$ s, $\mathbf{v} = [2 + 2(2)]\mathbf{i} + 2(2)\mathbf{j} = \{6\mathbf{i} + 4\mathbf{j}\}$ m/s. Then $v = \sqrt{6^2 + 4^2}$ $= 7.21$ m/s. Since the velocity is always directed tangent to the path,

$$v_n = 0 \qquad \text{and} \qquad v_t = 7.21\text{ m/s} \qquad \textbf{Ans.}$$



The velocity **v** makes an angle $\theta = \tan^{-1}\dfrac{4}{6} = 33.69°$ with the x axis.

Acceleration: To determine the acceleration **a**, apply Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{2\mathbf{i} + 2\mathbf{j}\}\text{ m/s}^2$$

Then

$$a = \sqrt{2^2 + 2^2} = 2.828\text{ m/s}^2$$

The acceleration **a** makes an angle $\phi = \tan^{-1}\dfrac{2}{2} = 45.0°$ with the x axis. From the figure, $\alpha = 45° - 33.69 = 11.31°$. Therefore,

$$a_n = a\sin\alpha = 2.828\sin 11.31° = 0.555\text{ m/s}^2 \qquad \textbf{Ans.}$$

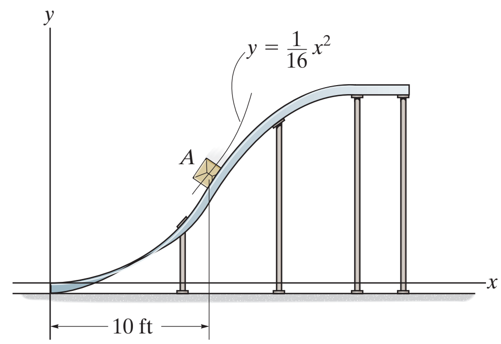
$$a_t = a\cos\alpha = 2.828\cos 11.31° = 2.77\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$v_n = 0$
$v_t = 7.21$ m/s
$a_n = 0.555$ m/s^2
$a_t = 2.77$ m/s^2

12–154.

If the speed of the crate at A is 15 ft/s, which is increasing at a rate $\dot{v} = 3\text{ ft/s}^2$, determine the magnitude of the acceleration of the crate at this instant.



SOLUTION

Radius of Curvature:

$$y = \frac{1}{16}x^2$$

$$\frac{dy}{dx} = \frac{1}{8}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{8}$$

Thus,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{8}x\right)^2\right]^{3/2}}{\left|\frac{1}{8}\right|}\right|_{x=10\text{ ft}} = 32.82\text{ ft}$$

Acceleration:

$$a_t = \dot{v} = 3\text{ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{15^2}{32.82} = 6.856\text{ ft/s}^2$$

The magnitude of the crate's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 6.856^2} = 7.48\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:

$a = 7.48\text{ ft/s}^2$

12–155.

A particle is moving along a circular path having a radius of 4 in. such that its position as a function of time is given by $\theta = \cos 2t$, where θ is in radians and t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = 30°$.

SOLUTION

When $\theta = \frac{\pi}{6}$ rad, $\quad \frac{\pi}{6} = \cos 2t \quad t = 0.5099 \text{ s}$

$$\dot{\theta} = \frac{d\theta}{dt} = -2\sin 2t\bigg|_{t=0.5099\text{ s}} = -1.7039 \text{ rad/s}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = -4\cos 2t\bigg|_{t=0.5099\text{ s}} = -2.0944 \text{ rad/s}^2$$

$$r = 4 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4(-1.7039)^2 = -11.6135 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4(-2.0944) + 0 = -8.3776 \text{ in./s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-11.6135)^2 + (-8.3776)^2} = 14.3 \text{ in./s}^2$$ **Ans.**

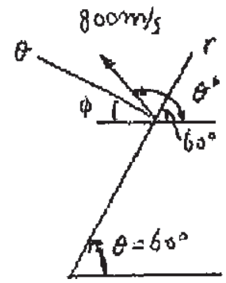
Ans:

$a = 14.3 \text{ in./s}^2$

***12–156.**

For a short time a rocket travels up and to the right at a constant speed of 800 m/s along the parabolic path $y = 600 - 35x^2$. Determine the radial and transverse components of velocity of the rocket at the instant $\theta = 60°$, where θ is measured counterclockwise from the x axis.

SOLUTION



$y = 600 - 35x^2$

$\dot{y} = -70x\dot{x}$

$\dfrac{dy}{dx} = -70x$

$\tan 60° = \dfrac{y}{x}$

$y = 1.732051x$

$1.732051x = 600 - 35x^2$

$x^2 + 0.049487x - 17.142857 = 0$

Solving for the positive root,

$x = 4.1157 \text{ m}$

$\tan \theta' = \dfrac{dy}{dx} = -288.1$

$\theta' = 89.8011°$

$\phi = 180° - 89.8011° - 60° = 30.1989°$

$v_r = 800 \cos 30.1989° = 691 \text{ m/s}$ **Ans.**

$v_\theta = 800 \sin 30.1989° = 402 \text{ m/s}$ **Ans.**

Ans:

$v_r = 691 \text{ m/s}$

$v_\theta = 402 \text{ m/s}$

12–157.

A particle moves along a path defined by polar coordinates $r = (2e^t)$ ft and $\theta = (8t^2)$ rad, where t is in seconds. Determine the components of its velocity and acceleration when $t = 1$ s.

SOLUTION

When $t = 1$ s,

$r = 2e^t = 5.4366$

$\dot{r} = 2e^t = 5.4366$

$\ddot{r} = 2e^t = 5.4366$

$\theta = 8t^2$

$\dot{\theta} = 16t = 16$

$\ddot{\theta} = 16$

$v_r = r = 5.44 \text{ ft/s}$ **Ans.**

$v_\theta = r\dot{\theta} = 5.4366(16) = 87.0 \text{ ft/s}$ **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 5.4366 - 5.4366(16)^2 = -1386 \text{ ft/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5.4366(16) + 2(5.4366)(16) = 261 \text{ ft/s}^2$ **Ans.**

Ans:

$v_r = 5.44 \text{ ft/s}$

$v_\theta = 87.0 \text{ ft/s}$

$a_r = -1386 \text{ ft/s}^2$

$a_\theta = 261 \text{ ft/s}^2$

12–158.

An airplane is flying in a straight line with a velocity of 200 mi/h and an acceleration of 3 mi/h^2. If the propeller has a diameter of 6 ft and is rotating at an angular rate of 120 rad/s, determine the magnitudes of velocity and acceleration of a particle located on the tip of the propeller.

SOLUTION

$$v_{Pl} = \left(\frac{200\text{ mi}}{\text{h}}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 293.3\text{ ft/s}$$

$$a_{Pl} = \left(\frac{3\text{ mi}}{\text{h}^2}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2 = 0.001\,22\ \text{ ft/s}^2$$

$$v_{Pr} = 120(3) = 360\text{ ft/s}$$

$$v = \sqrt{v_{Pl}^2 + v_{Pr}^2} = \sqrt{(293.3)^2 + (360)^2} = 464\text{ ft/s}$$ **Ans.**

$$a_{Pr} = \frac{v_{Pr}^2}{\rho} = \frac{(360)^2}{3} = 43\,200\text{ ft/s}^2$$

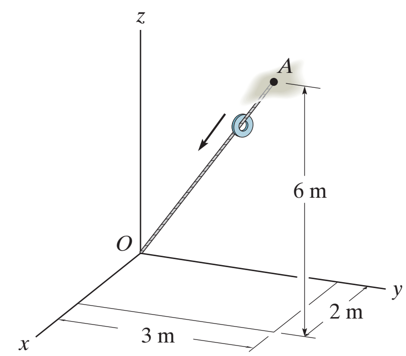
$$a = \sqrt{a_{Pl}^2 + a_{Pr}^2} = \sqrt{(0.001\,22)^2 + (43\,200)^2} = 43.2(10^3)\ \text{ ft/s}^2$$ **Ans.**

Ans:

$v = 464$ ft/s
$a = 43.2(10^3)$ ft/s^2

12–159.

The small washer is sliding down the cord OA. When it is at the midpoint, its speed is 28 m/s and its acceleration is 7 m/s^2. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.



SOLUTION

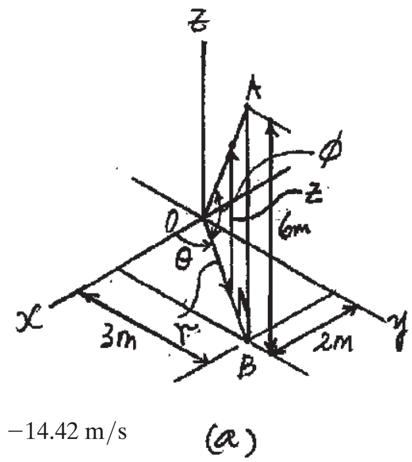
The position of the washer can be defined using the cylindrical coordinate system (r, θ and z) as shown in Fig. a. Since θ is constant, there will be no transverse component for **v** and **a**. The velocity and acceleration expressed as Cartesian vectors are

$$\mathbf{v} = v\left(-\frac{\mathbf{r}_{AO}}{r_{AO}}\right) = 28\left[\frac{(0-2)\mathbf{i} + (0-3)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-2)^2 + (0-3)^2 + (0-6)^2}}\right] = \{-8\mathbf{i} - 12\mathbf{j} - 24\mathbf{k}\}\text{ m/s}$$

$$\mathbf{a} = a\left(-\frac{\mathbf{r}_{AO}}{r_{AO}}\right) = 7\left[\frac{(0-2)\mathbf{i} + (0-3)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(0-2)^2 + (0-3)^2 + (0-6)^2}}\right] = \{-2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}\}\text{ m}^2/\text{s}$$

$$\mathbf{u}_r = \frac{\mathbf{r}_{OB}}{r_{OB}} = \frac{2\mathbf{i} + 3\mathbf{j}}{\sqrt{2^2 + 3^2}} = \frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}$$

$$\mathbf{u}_z = \mathbf{k}$$



Using vector dot product

$$v_r = \mathbf{v}\cdot\mathbf{u}_r = (-8\,\mathbf{i} - 12\,\mathbf{j} - 24\,\mathbf{k})\cdot\left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = -8\left(\frac{2}{\sqrt{13}}\right) + \left[-12\left(\frac{3}{\sqrt{13}}\right)\right] = -14.42\text{ m/s}$$

$$v_z = \mathbf{v}\cdot\mathbf{u}_z = (-8\,\mathbf{i} - 12\,\mathbf{j} - 24\,\mathbf{k})\cdot(\mathbf{k}) = -24.0\text{ m/s}$$

$$a_r = \mathbf{a}\cdot\mathbf{u}_r = (-2\,\mathbf{i} - 3\,\mathbf{j} - 6\,\mathbf{k})\cdot\left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = -2\left(\frac{2}{\sqrt{13}}\right) + \left[-3\left(\frac{3}{\sqrt{13}}\right)\right] = -3.606\text{ m/s}^2$$

$$a_z = a\cdot\mathbf{u}_z = (-2\,\mathbf{i} - 3\,\mathbf{j} - 6\,\mathbf{k})\cdot\mathbf{k} = -6.00\text{ m/s}^2$$

Thus, in vector form

$\mathbf{v} = \{-14.2\,\mathbf{u}_r - 24.0\,\mathbf{u}_z\}$ m/s **Ans.**

$a = \{-3.61\,\mathbf{u}_r - 6.00\,\mathbf{u}_z\}$ m/s^2 **Ans.**

These components can also be determined using trigonometry by first obtain angle ϕ shown in Fig. a.

$$OA = \sqrt{2^2 + 3^2 + 6^2} = 7\text{ m} \qquad OB = \sqrt{2^2 + 3^2} = \sqrt{13}$$

Thus,

$\sin\phi = \dfrac{6}{7}$ and $\cos\phi = \dfrac{\sqrt{13}}{7}$. Then

$$v_r = -v\cos\phi = -28\left(\frac{\sqrt{13}}{7}\right) = -14.42\text{ m/s}$$

$$v_z = -v\sin\phi = -28\left(\frac{6}{7}\right) = -24.0\text{ m/s}$$

$$a_r = -a\cos\phi = -7\left(\frac{\sqrt{13}}{7}\right) = -3.606\text{ m/s}^2$$

$$a_z = -a\sin\phi = -7\left(\frac{6}{7}\right) = -6.00\text{ m/s}^2$$

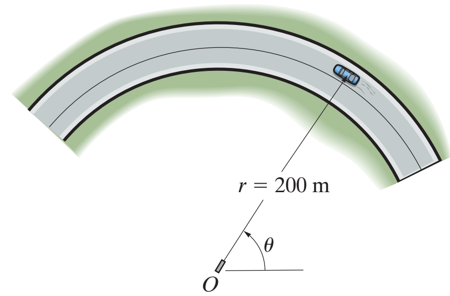
Ans:

$\mathbf{v} = \{-14.2\mathbf{u}_r - 24.0\mathbf{u}_z\}$ m/s

$\mathbf{a} = \{-3.61\mathbf{u}_r - 6.00\mathbf{u}_z\}$ m/s^2

***12–160.**

A radar gun at O rotates with the angular velocity of $\dot{\theta} = 0.1$ rad/s and angular acceleration of $\ddot{\theta} = 0.025$ rad/s^2, at the instant $\theta = 45°$, as it follows the motion of the car traveling along the circular road having a radius of $r = 200$ m. Determine the magnitudes of velocity and acceleration of the car at this instant.



SOLUTION

Time Derivatives: Since r is constant,

$$\dot{r} = \ddot{r} = 0$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 200(0.1) = 20 \text{ m/s}$$

Thus, the magnitude of the car's velocity is

$$v = \sqrt{v_r{}^2 + v_\theta{}^2} = \sqrt{0^2 + 20^2} = 20 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \dot{r} - r\dot{\theta}^2 = 0 - 200(0.1^2) = -2 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 200(0.025) + 0 = 5 \text{ m/s}^2$$

Thus, the magnitude of the car's acceleration is

$$a = \sqrt{a_r{}^2 + a_\theta{}^2} = \sqrt{(-2)^2 + 5^2} = 5.39 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:

$v = 20$ m/s
$a = 5.39$ m/s^2

12–161.

If a particle moves along a path such that $r = (2\cos t)$ ft and $\theta = (t/2)$ rad, where t is in seconds, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$r = 2\cos t \quad \dot{r} = -2\sin t \qquad \ddot{r} = -2\cos t$

$\theta = \dfrac{t}{2} \qquad \dot{\theta} = \dfrac{1}{2} \qquad \ddot{\theta} = 0$

$v_r = \dot{r} = -2\sin t$ **Ans.**

$v_\theta = r\dot{\theta} = (2\cos t)\left(\dfrac{1}{2}\right) = \cos t$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = -2\cos t - (2\cos t)\left(\dfrac{1}{2}\right)^2 = -\dfrac{5}{2}\cos t$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\cos t(0) + 2(-2\sin t)\left(\dfrac{1}{2}\right) = -2\sin t$ **Ans.**

Ans:

$v_r = -2\sin t$

$v_\theta = \cos t$

$a_r = -\dfrac{5}{2}\cos t$

$a_\theta = -2\sin t$

12–162.

If a particle moves along a path such that $r = (e^{at})$ m and $\theta = t$, where t is in seconds, plot the path $r = f(\theta)$, and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$r = e^{at} \qquad \dot{r} = ae^{at} \qquad \ddot{r} = a^2e^{et}$

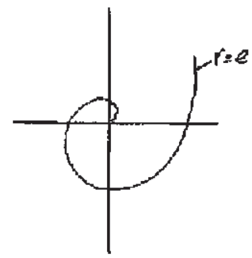
$\theta = t \qquad \dot{\theta} = 1 \qquad \ddot{\theta} = 0$

$v_r = \dot{r} = ae^{at}$ **Ans.**

$v_a = r\dot{\theta} = e^{at}(1) = e^{at}$ **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = a^2e^{at} - e^{at}(1)^2 = e^{at}(a^2 - 1)$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = e^{at}(0) + 2(ae^{at})(1) = 2ae^{at}$ **Ans.**



Ans:

$v_r = ae^{at}$

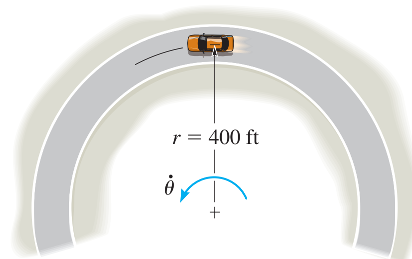
$v_\theta = e^{at}$

$a_r = e^{at}(a^2 - 1)$

$a_\theta = 2ae^{at}$

12–163.

The car travels along the circular curve having a radius $r = 400$ ft. At the instant shown, its angular rate of rotation is $\dot{\theta} = 0.025$ rad/s, which is decreasing at the rate $\ddot{\theta} = -0.008$ rad/s^2. Determine the radial and transverse components of the car's velocity and acceleration at this instant and sketch these components on the curve.



SOLUTION

$r = 400 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

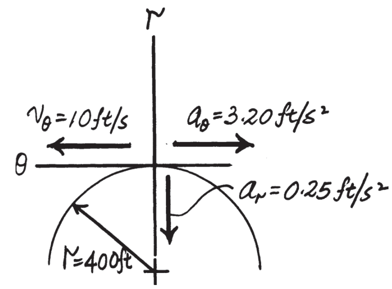
$\dot{\theta} = 0.025 \qquad \theta = -0.008$

$v_r = \dot{r} = 0$ **Ans.**

$v_\theta = r\dot{\theta} = 400(0.025) = 10$ ft/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.025)^2 = -0.25$ ft/s^2 **Ans.**

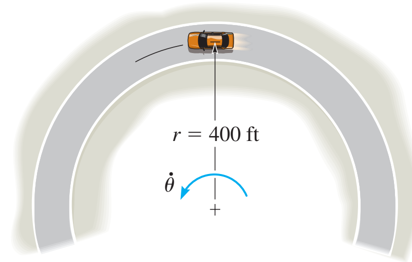
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(-0.008) + 0 = -3.20$ ft/s^2 **Ans.**



Ans:
$v_r = 0$
$v_\theta = 10$ ft/s
$a_r = -0.25$ ft/s^2
$a_\theta = -3.20$ ft/s^2

***12–164.**

The car travels along the circular curve of radius $r = 400$ ft with a constant speed of $v = 30$ ft/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the car's acceleration.



SOLUTION

$r = 400 \text{ ft} \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 400\left(\dot{\theta}\right)$

$$v = \sqrt{(0)^2 + \left(400\,\dot{\theta}\right)^2} = 30$$

$\dot{\theta} = 0.075 \text{ rad/s}$ **Ans.**

$\ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.075)^2 = -2.25 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 400(0) + 2(0)(0.075) = 0$

$a = \sqrt{(-2.25)^2 + (0)^2} = 2.25 \text{ ft/s}^2$ **Ans.**

Ans:

$\dot{\theta} = 0.075 \text{ rad/s}$
$a = 2.25 \text{ ft/s}^2$

12–165.

The time rate of change of acceleration is referred to as the *jerk*, which is often used as a means of measuring passenger discomfort. Calculate this vector, $\dot{\mathbf{a}}$, in terms of its cylindrical components, using Eq. 12–32.

SOLUTION

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{u}_\theta + \ddot{z}\,\mathbf{u}_z$$

$$\dot{\mathbf{a}} = \left(\dddot{r} - \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(\ddot{r} - r\dot{\theta}^2\right)\dot{\mathbf{u}}_r + \left(\dot{r}\ddot{\theta} + r\dddot{\theta} + 2\ddot{r}\dot{\theta} + 2\dot{r}\ddot{\theta}\right)\mathbf{u}_\theta + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\dot{\mathbf{u}}_\theta + \dddot{z}\,\mathbf{u}_z + \ddot{z}\dot{\mathbf{u}}_z$$

But, $\mathbf{u}_r = \dot{\theta}\mathbf{u}_\theta \quad \dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad \dot{\mathbf{u}}_z = 0$

Substituting and combining terms yields

$$\dot{\mathbf{a}} = \left(\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta}\right)\mathbf{u}_r + \left(3\dot{r}\ddot{\theta} + r\dddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3\right)\mathbf{u}_\theta + \left(\dddot{z}\right)\mathbf{u}_z \qquad \textbf{Ans.}$$

Ans:

$\dot{\mathbf{a}} = (\dddot{r} - 3\dot{r}\dot{\theta}^2 - 3r\dot{\theta}\ddot{\theta})\mathbf{u}_r$
$+ (3\dot{r}\ddot{\theta} + \dot{r}\ddot{\theta} + 3\ddot{r}\dot{\theta} - r\dot{\theta}^3)\mathbf{u}_\theta + (\dddot{z})\mathbf{u}_z$

12–166.

A particle is moving along a circular path having a radius of 6 in. such that its position as a function of time is given by $\theta = \sin 3t$, where θ is in radians, the argument for the sine are in radians, and t is in seconds. Determine the acceleration of the particle at $\theta = 30°$. The particle starts from rest at $\theta = 0°$.

SOLUTION

$r = 6$ in., $\quad \dot{r} = 0, \quad \ddot{r} = 0$

$\theta = \sin 3t$

$\dot{\theta} = 3 \cos 3t$

$\ddot{\theta} = -9 \sin 3t$

At $\theta = 30°$,

$\frac{30°}{180°}\pi = \sin 3t$

$t = 10.525$ s

Thus,

$\dot{\theta} = 2.5559$ rad/s

$\ddot{\theta} = -4.7124$ rad/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 6(2.5559)^2 = -39.196$

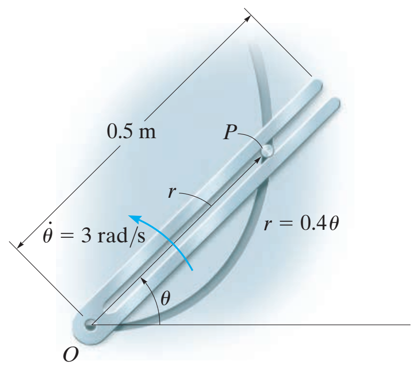
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 6(-4.7124) + 0 = -28.274$

$a = \sqrt{(-39.196)^2 + (-28.274)^2} = 48.3$ in./s^2 **Ans.**

Ans:
$a = 48.3$ in./s^2

12–167.

The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \pi/3$ rad.



SOLUTION

$\dot{\theta} = 3$ rad/s $\quad r = 0.4\,\theta$

$$\dot{r} = 0.4\,\dot{\theta}$$

$$\ddot{r} = 0.4\,\ddot{\theta}$$

At $\theta = \dfrac{\pi}{3}$, $\quad r = 0.4189$

$$\dot{r} = 0.4(3) = 1.20$$

$$\ddot{r} = 0.4(0) = 0$$

$v = \dot{r} = 1.20$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 0.4189(3) = 1.26$ m/s **Ans.**

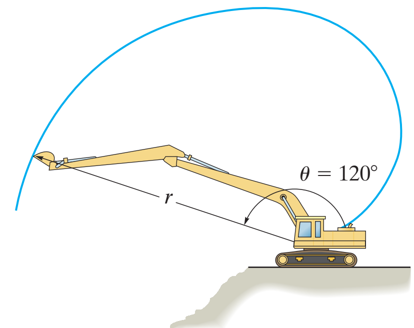
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.4189(3)^2 = -3.77$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20$ m/s^2 **Ans.**

Ans:
$v_r = 1.20$ m/s
$v_\theta = 1.26$ m/s
$a_r = -3.77$ m/s^2
$a_\theta = 7.20$ m/s^2

***12–168.**

For a short time the bucket of the backhoe traces the path of the cardioid $r = 25(1 - \cos\theta)$ ft. Determine the magnitudes of the velocity and acceleration of the bucket when $\theta = 120°$ if the boom is rotating with an angular velocity of $\dot{\theta} = 2$ rad/s and an angular acceleration of $\ddot{\theta} = 0.2$ rad/s^2 at the instant shown.



SOLUTION

$r = 25(1 - \cos\theta) = 25(1 - \cos 120°) = 37.5$ ft

$\dot{r} = 25\sin\theta\dot{\theta} = 25\sin 120°(2) = 43.30$ ft/s

$\ddot{r} = 25[\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta}] = 25[\cos 120°(2)^2 + \sin 120°(0.2)] = -45.67$ ft/s^2

$v_r = \dot{r} = 43.30$ ft/s

$v_\theta = r\dot{\theta} = 37.5(2) = 75$ ft/s

$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{43.30^2 + 75^2} = 86.6$ ft/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = -45.67 - 37.5(2)^2 = -195.67$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 37.5(0.2) + 2(43.30)(2) = 180.71$ ft/s^2

$a = \sqrt{a_s^2 + a_\theta^2} = \sqrt{(-195.67)^2 + 180.71^2} = 266$ ft/s^2 **Ans.**

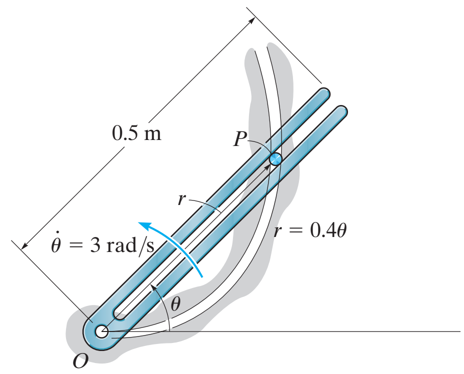
Ans:

$v = 86.6$ ft/s

$a = 266$ ft/s^2

12–169.

The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$ rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\,\theta)$ m, where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5$ m.



SOLUTION

$r = 0.4\,\theta$

$\dot{r} = 0.4\,\dot{\theta}$

$\ddot{r} = 0.4\,\ddot{\theta}$

$\dot{\theta} = 3$

$\ddot{\theta} = 0$

At $r = 0.5$ m,

$$\theta = \frac{0.5}{0.4} = 1.25 \text{ rad}$$

$\dot{r} = 1.20$

$\ddot{r} = 0$

$v_r = \dot{r} = 1.20 \text{ m/s}$ **Ans.**

$v_\theta = r\,\dot{\theta} = 0.5(3) = 1.50 \text{ m/s}$ **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 0.5(3)^2 = -4.50 \text{ m/s}^2$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1.20)(3) = 7.20 \text{ m/s}^2$ **Ans.**

Ans:
$v_r = 1.20 \text{ m/s}$
$v_\theta = 1.50 \text{ m/s}$
$a_r = -4.50 \text{ m/s}^2$
$a_\theta = 7.20 \text{ m/s}^2$

12–170.

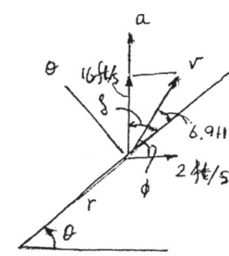
A particle moves in the x–y plane such that its position is defined by $\mathbf{r} = \{2t\mathbf{i} + 4t^2\mathbf{j}\}$ ft, where t is in seconds. Determine the radial and transverse components of the particle's velocity and acceleration when $t = 2$ s.

SOLUTION

$\mathbf{r} = 2t\mathbf{i} + 4\,t^2\mathbf{j}|_{t=2} = 4\mathbf{i} + 16\mathbf{j}$

$\mathbf{v} = 2\mathbf{i} + 8\,t\mathbf{j}|_{t=2} = 2\mathbf{i} + 16\mathbf{j}$

$\mathbf{a} = 8\mathbf{j}$



$\theta = \tan^{-1}\left(\dfrac{16}{4}\right) = 75.964°$

$v = \sqrt{(2)^2 + (16)^2} = 16.1245$ ft/s

$\phi = \tan^{-1}\left(\dfrac{16}{2}\right) = 82.875°$

$a = 8$ ft/s^2

$\phi - \theta = 6.9112°$

$v_r = 16.1245 \cos 6.9112° = 16.0$ ft/s **Ans.**

$v_\theta = 16.1245 \sin 6.9112° = 1.94$ ft/s **Ans.**

$\delta = 90° - \theta = 14.036°$

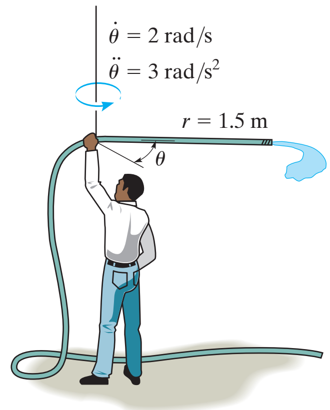
$a_r = 8 \cos 14.036° = 7.76$ ft/s^2 **Ans.**

$a_\theta = 8 \sin 14.036° = 1.94$ ft/s^2 **Ans.**

Ans:
$v_r = 16.0$ ft/s
$v_\theta = 1.94$ ft/s
$a_r = 7.76$ ft/s^2
$a_\theta = 1.94$ ft/s^2

12–171.

At the instant shown, the man is twirling a hose over his head with an angular velocity $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 3$ rad/s^2. If it is assumed that the hose lies in a horizontal plane, and water is flowing through it at a constant rate of 3 m/s, determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r = 1.5$ m.



SOLUTION

$r = 1.5$

$\dot{r} = 3$

$\ddot{r} = 0$

$\dot{\theta} = 2$

$\ddot{\theta} = 3$

$v_r = \dot{r} = 3$

$v_\theta = r\dot{\theta} = 1.5(2) = 3$

$v = \sqrt{(3)^2 + (3)^2} = 4.24$ m/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(2)^2 = 6$

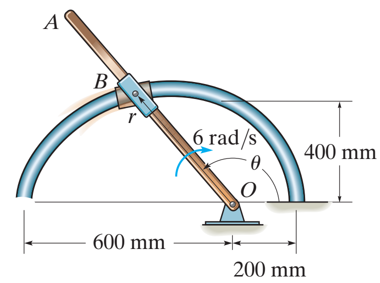
$a_\theta = r\,\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.5(3) + 2(3)(2) = 16.5$

$a = \sqrt{(6)^2 + (16.5)^2} = 17.6$ m/s^2 **Ans.**

Ans:
$v = 4.24$ m/s
$a = 17.6$ m/s^2

***12–172.**

The rod OA rotates clockwise with a constant angular velocity of 6 rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 200(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 150°$.



SOLUTION

Velocity. Using the chain rule, the first and second time derivatives of r can be determined.

$$r = 200(2 - \cos\theta)$$

$$\dot{r} = 200(\sin\theta)\,\dot{\theta} = \{200(\sin\theta)\,\dot{\theta}\}\ \text{mm/s}$$

$$\ddot{r} = \{200[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}]\}\ \text{mm/s}^2$$

The radial and transverse components of the velocity are

$$v_r = \dot{r} = \{200(\sin\theta)\dot{\theta}\}\ \text{mm/s}$$

$$v_\theta = r\dot{\theta} = \{200(2 - \cos\theta)\dot{\theta}\}\ \text{mm/s}$$

Since $\dot{\theta}$ is in the opposite sense to that of positive θ, $\dot{\theta} = -6$ rad/s. Thus, at $\theta = 150°$,

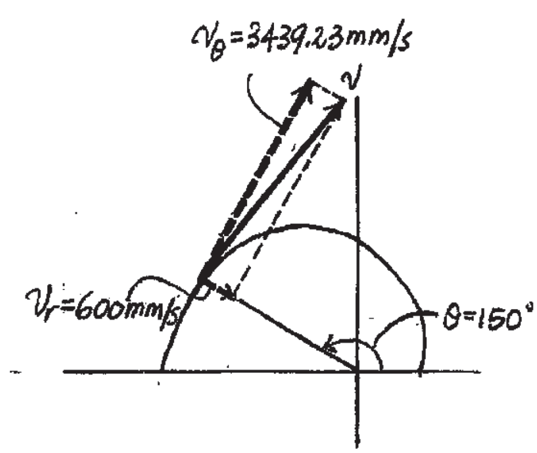
$$v_r = 200(\sin 150°)(-6) = -600\ \text{mm/s}$$

$$v_\theta = 200(2 - \cos 150°)(-6) = -3439.23\ \text{mm/s}$$

Thus, the magnitude of the velocity is

$$v = \sqrt{v_r^2 + v_\theta^2}\quad \sqrt{(-600)^2 + (-3439.23)^2} = 3491\ \text{mm/s} = 3.49\ \text{m/s} \qquad \textbf{Ans.}$$

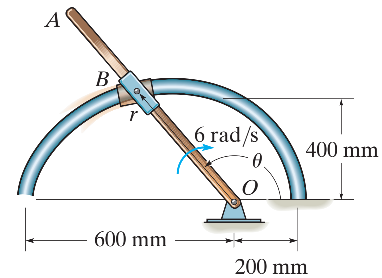
These components are shown in Fig. *a*



Ans:
$v = 3.49$ m/s

12–173.

Determine the magnitude of the acceleration of the slider blocks in Prob. 12–172 when $\theta = 150°$.



SOLUTION

Acceleration. Using the chain rule, the first and second time derivatives of r can be determined

$$r = 200(2 - \cos\theta)$$

$$\dot{r} = 200(\sin\theta)\dot{\theta} = \{200(\sin\theta)\dot{\theta}\}\ \text{mm/s}$$

$$\ddot{r} = \{200[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}]\}\ \text{mm/s}^2$$

Here, since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$. Since $\dot{\theta}$ is in the opposite sense to that of positive θ, $\dot{\theta} = -6$ rad/s. Thus, at $\theta = 150°$

$$r = 200(2 - \cos 150°) = 573.21\ \text{mm}$$

$$\dot{r} = 200(\sin 150°)(-6) = -600\ \text{mm/s}$$

$$\ddot{r} = 200\left[(\cos 150°)(-6)^2 + \sin 150°(0)\right] = -6235.38\ \text{mm/s}^2$$

The radial and transverse components of the acceleration are

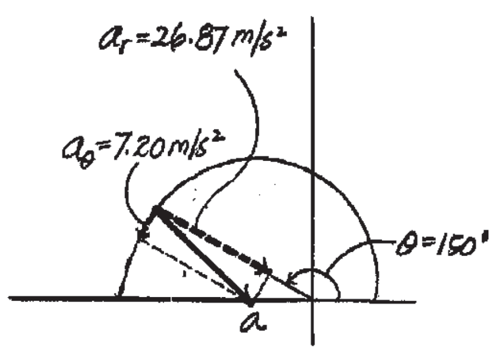
$$a_r = \ddot{r} - r\dot{\theta}^2 = -6235.38 - 573.21\,(-6)^2 = -26870.77\ \text{mm/s}^2 = -26.87\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 573.21(0) + 2(-600)(-6) = 7200\ \text{mm/s}^2 = 7.20\ \text{m/s}^2$$

Thus, the magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-26.87)^2 + 7.20^2} = 27.82\ \text{m/s}^2 = 27.8\ \text{m/s}^2 \qquad \textbf{Ans.}$$

These components are shown in Fig. *a*.

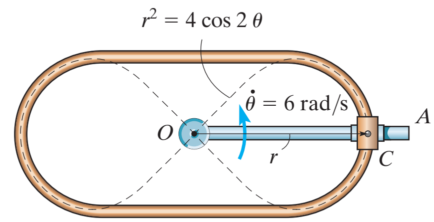


(a)

Ans:
$a = 27.8\ \text{m/s}^2$

12–174.

A double collar C is pin connected together such that one collar slides over a fixed rod and the other slides over a rotating rod. If the geometry of the fixed rod for a short distance can be defined by a lemniscate, $r^2 = (4 \cos 2\theta)$ ft^2, determine the collar's radial and transverse components of velocity and acceleration at the instant $\theta = 0°$ as shown. Rod OA is rotating at a constant rate of $\dot{\theta} = 6$ rad/s.



SOLUTION

$r^2 = 4 \cos 2\theta$

$r\dot{r} = -4 \sin 2\theta\, \dot{\theta}$

$r\ddot{r} = \dot{r}^2 = -4 \sin 2\theta\, \ddot{\theta} - 8 \cos 2\theta\, \dot{\theta}^2$

when $\theta = 0, \dot{\theta} = 6, \ddot{\theta} = 0$

$r = 2, \dot{r} = 0, \ddot{r} = -144$

$v_r = \dot{r} = 0$ **Ans.**

$v_\theta = r\dot{\theta} = 2(6) = 12$ ft/s **Ans.**

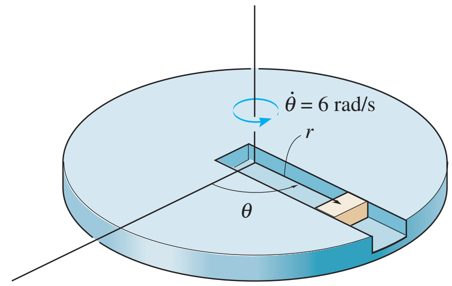
$a_r = \ddot{r} - r\dot{\theta}^2 = -144 - 2(6)^2 = -216$ ft/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2(0) + 2(0)(6) = 0$ **Ans.**

Ans:
$v_r = 0$
$v_\theta = 12$ ft/s
$a_r = -216$ ft/s^2
$a_\theta = 0$

12–175.

A block moves outward along the slot in the platform with a speed of $\dot{r} = (4t)$ m/s, where t is in seconds. The platform rotates at a constant rate of 6 rad/s. If the block starts from rest at the center, determine the magnitudes of its velocity and acceleration when $t = 1$ s.



SOLUTION

$\dot{r} = 4t|_{t=1} = 4 \qquad \ddot{r} = 4$

$\dot{\theta} = 6 \qquad \ddot{\theta} = 0$

$$\int_0^1 dr = \int_0^1 4t\,dt$$

$r = 2t^2\Big]_0^1 = 2\text{ m}$

$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} = \sqrt{(4)^2 + [2(6)]^2} = 12.6\text{ m/s}$ **Ans.**

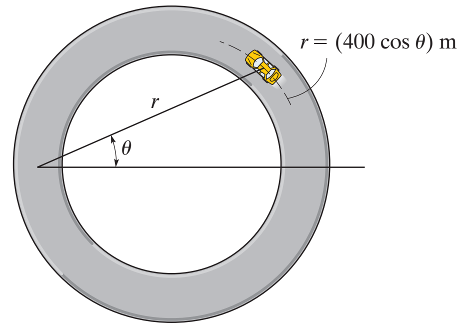
$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2} = \sqrt{[4 - 2(6)^2]^2 + [0 + 2(4)(6)]^2}$ **Ans.**

$= 83.2\text{ m/s}^2$

Ans:
$v = 12.6\text{ m/s}$
$a = 83.2\text{ m/s}^2$

***12–176.**

The car travels around the circular track with a constant speed of 20 m/s. Determine the car's radial and transverse components of velocity and acceleration at the instant $\theta = \pi/4$ rad.



SOLUTION

$v = 20$ m/s

$\theta = \dfrac{\pi}{4} = 45°$

$r = 400 \cos \theta$

$\dot{r} = -400 \sin \theta\, \dot{\theta}$

$\ddot{r} = -400(\cos \theta (\dot{\theta})^2 + \sin \theta\, \ddot{\theta})$

$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$0 = \dot{r}\ddot{r} + r\dot{\theta}(\dot{r}\dot{\theta} + r\ddot{\theta})$

Thus

$r = 282.84$

$(20)^2 = [-400 \sin 45°\, \dot{\theta}]^2 + [282.84\, \dot{\theta}]^2$

$\dot{\theta} = 0.05$

$\dot{r} = -14.14$

$0 = -14.14[-400(\cos 45°)(0.05)^2 + \sin 45°\, \ddot{\theta}] + 282.84(0.05)[-14.14(0.05) + 282.84\ddot{\theta}]$

$\ddot{\theta} = 0$

$\ddot{r} = -0.707$

$v_r = \dot{r} = -14.1$ m/s **Ans.**

$v_\theta = r\dot{\theta} = 282.84(0.05) = 14.1$ m/s **Ans.**

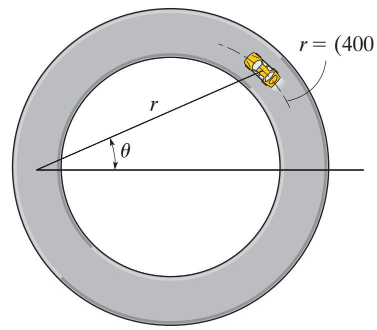
$a_r = \ddot{r} - r(\dot{\theta})^2 = -0.707 - 282.84(0.05)^2 = -1.41$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \dot{\theta} + 2(-14.14)(0.05) = -1.41$ m/s^2 **Ans.**

Ans:
$v_r = -14.1$ m/s
$v_\theta = 14.1$ m/s
$a_r = -1.41$ m/s^2
$a_\theta = -1.41$ m/s^2

12–177.

The car travels around the circular track such that its transverse component is $\theta = (0.006t^2)$ rad, where t is in seconds. Determine the car's radial and transverse components of velocity and acceleration at the instant $t = 4$ s.



SOLUTION

$\theta = 0.006\, t^2\big|_{t=4} = 0.096 \text{ rad} = 5.50°$

$\dot{\theta} = 0.012\, t\big|_{t=4} = 0.048 \text{ rad/s}$

$\ddot{\theta} = 0.012 \text{ rad/s}^2$

$r = 400 \cos\theta$

$\dot{r} = -400 \sin\theta\, \dot{\theta}$

$\ddot{r} = -400(\cos\theta\, (\dot{\theta})^2 + \sin\theta\, \ddot{\theta})$

At $\theta = 0.096$ rad

$r = 398.158$ m

$\dot{r} = -1.84037$ m/s

$\ddot{r} = -1.377449 \text{ m/s}^2$

$v_r = \dot{r} = -1.84$ m/s **Ans.**

$v_\theta = r\,\dot{\theta} = 398.158(0.048) = 19.1$ m/s **Ans.**

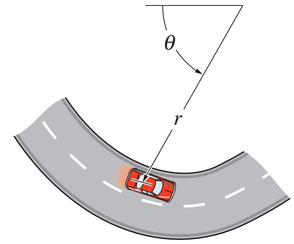
$a_r = \ddot{r} - r\,(\dot{\theta})^2 = -1.377449 - 398.158(0.048)^2 = -2.29 \text{ m/s}^2$ **Ans.**

$a_\theta = r\,\ddot{\theta} = 2\dot{r}\,\dot{\theta} = 398.158\,(0.012) + 2(-1.84037)(0.048) = 4.60 \text{ m/s}^2$ **Ans.**

Ans:
$v_r = -1.84$ m/s
$v_\theta = 19.1$ m/s
$a_r = -2.29 \text{ m/s}^2$
$a_\theta = 4.60 \text{ m/s}^2$

12–178.

The car travels along a road which for a short distance is defined by $r = (200/\theta)$ ft, where θ is in radians. If it maintains a constant speed of $v = 35$ ft/s, determine the radial and transverse components of its velocity when $\theta = \pi/3$ rad.



SOLUTION

$$r = \frac{200}{\theta}\bigg|_{\theta=\pi/3\text{ rad}} = \frac{600}{\pi}\text{ ft}$$

$$\dot{r} = -\frac{200}{\theta^2}\dot{\theta}\bigg|_{\theta=\pi/3\text{ rad}} = -\frac{1800}{\pi^2}\dot{\theta}$$

$$v_r = \dot{r} = -\frac{1800}{\pi^2}\dot{\theta} \qquad v_\theta = r\dot{\theta} = \frac{600}{\pi}\dot{\theta}$$

$$v^2 = v_r^2 + v_\theta^2$$

$$35^2 = \left(-\frac{1800}{\pi^2}\dot{\theta}\right)^2 + \left(\frac{600}{\pi}\dot{\theta}\right)^2$$

$$\dot{\theta} = 0.1325\text{ rad/s}$$

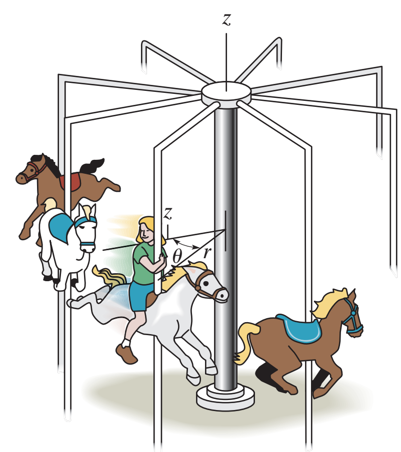
$$v_r = -\frac{1800}{\pi^2}(0.1325) = -24.2\text{ ft/s}$$ **Ans.**

$$v_\theta = \frac{600}{\pi}(0.1325) = 25.3\text{ ft/s}$$ **Ans.**

Ans:
$v_r = -24.2$ ft/s
$v_\theta = 25.3$ ft/s

12–179.

A horse on the merry-go-round moves according to the equations $r = 8$ ft, $\theta = (0.6t)$ rad, and $z = (1.5 \sin \theta)$ ft, where t is in seconds. Determine the cylindrical components of the velocity and acceleration of the horse when $t = 4$ s.



SOLUTION

$r = 8 \qquad \theta = 0.6\,t$

$\dot{r} = 0 \qquad \dot{\theta} = 0.6$

$\ddot{r} = 0 \qquad \ddot{\theta} = 0$

$z = 1.5 \sin \theta$

$\dot{z} = 1.5 \cos \theta\, \dot{\theta}$

$\ddot{z} = -1.5 \sin \theta\, (\dot{\theta})^2 + 1.5 \cos \theta\, \ddot{\theta}$

At $t = 4$ s

$\theta = 2.4$

$\dot{z} = -0.6637$

$\ddot{z} = -0.3648$

$v_r = 0$ **Ans.**

$v_\theta = 4.80$ ft/s **Ans.**

$v_z = -0.664$ ft/s **Ans.**

$a_r = 0 - 8(0.6)^2 = -2.88$ ft/s^2 **Ans.**

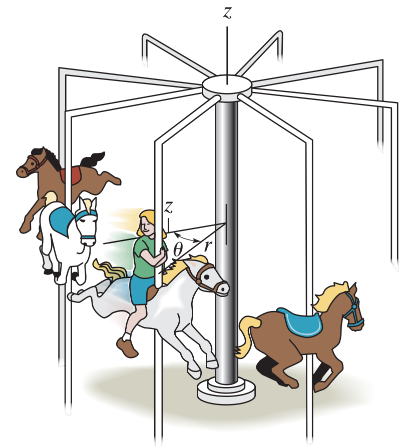
$a_\theta = 0 + 0 = 0$ **Ans.**

$a_z = -0.365$ ft/s^2 **Ans.**

Ans:
$v_r = 0$
$v_\theta = 4.80$ ft/s
$v_z = -0.664$ ft/s
$a_r = -2.88$ ft/s^2
$a_\theta = 0$
$a_z = -0.365$ ft/s^2

***12–180.**

A horse on the merry-go-round moves according to the equations $r = 8$ ft, $\dot{\theta} = 2$ rad/s and $z = (1.5 \sin \theta)$ ft, where t is in seconds. Determine the maximum and minimum magnitudes of the velocity and acceleration of the horse during the motion.



SOLUTION

$r = 8$

$\dot{r} = 0 \qquad \dot{\theta} = 2$

$\ddot{r} = 0 \qquad \ddot{\theta} = 0$

$z = 1.5 \sin \theta$

$\dot{z} = 1.5 \cos \theta\, \dot{\theta}$

$\ddot{z} = -1.5 \sin \theta\, (\dot{\theta})^2 + 1.5 \cos \theta\, \ddot{\theta}$

$v_r = \dot{r} = 0$

$v_\theta = r\dot{\theta} = 8(2) = 16$ ft/s

$(v_z)_{\text{max}} = \dot{z} = 1.5(\cos 0°)(2) = 3$ ft/s

$(v_z)_{\text{min}} = \dot{z} = 1.5(\cos 90°)(2) = 0$

$v_{\text{max}} = \sqrt{(16)^2 + (3)^2} = 16.3$ ft/s **Ans.**

$v_{\text{min}} = \sqrt{(16)^2 + (0)^2} = 16$ ft/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 8(2)^2 = -32$ ft/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$

$(a_z)_{\text{max}} = \ddot{z} = -1.5(\sin 90°)(2)^2 = -6$

$(a_z)_{\text{min}} = \ddot{z} = -1.5(\sin 0°)(2)^2 = 0$

$a_{\text{max}} = \sqrt{(-32)^2 + (0)^2 + (-6)^2} = 32.6$ ft/s^2 **Ans.**

$a_{\text{min}} = \sqrt{(-32)^2 + (0)^2 + (0)^2} = 32$ ft/s^2 **Ans.**

Ans:

$v_{\text{max}} = 16.3$ ft/s

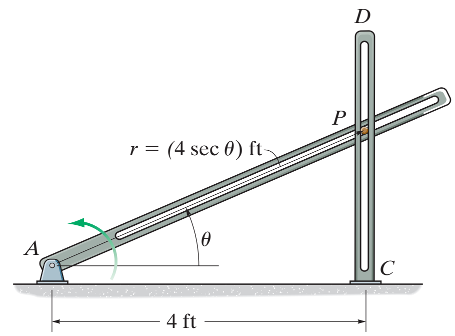
$v_{\text{min}} = 16$ ft/s

$a_{\text{max}} = 32.6$ ft/s^2

$a_{\text{min}} = 32$ ft/s^2

12–181.

If the slotted arm AB rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 2$ rad/s, determine the magnitudes of the velocity and acceleration of peg P at $\theta = 30°$. The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB.



SOLUTION

Time Derivatives:

$r = 4\sec\theta$

$\dot{r} = (4\sec\theta(\tan\theta)\dot{\theta})$ ft/s $\qquad \dot{\theta} = 2$ rad/s

$\ddot{r} = 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}(\sec\theta(\sec^2\theta)\dot{\theta} + \tan\theta\sec\theta(\tan\theta)\dot{\theta})] \qquad \ddot{\theta} = 0$

$= 4[\sec\theta(\tan\theta)\dot{\theta} + \dot{\theta}^2(\sec 3\theta + \tan^2\theta\sec\theta)]$ ft/s^2

When $\theta = 30°$,

$$r|_{\theta=30°} = 4\sec 30° = 4.619\text{ ft}$$

$$\dot{r}|_{\theta=30°} = (4\sec 30°\tan 30°)(2) = 5.333\text{ ft/s}$$

$$\ddot{r}|_{\theta=30°} = 4[0 + 2^2(\sec^3 30° + \tan^2 30°\sec 30°)] = 30.79\text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.333\text{ ft/s} \qquad v_\theta = r\dot{\theta} = 4.619(2) = 9.238\text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.333^2 + 9.238^2} = 10.7\text{ ft/s}$$ **Ans.**

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 30.79 - 4.619(2^2) = 12.32\text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(5.333)(2) = 21.23\text{ ft/s}^2$$

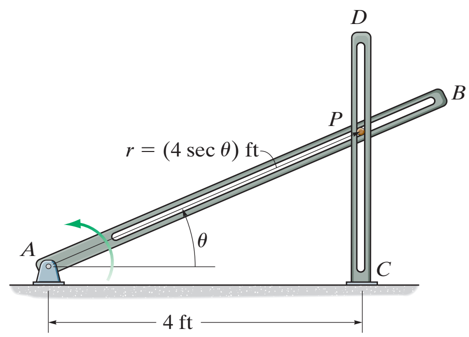
Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{12.32^2 + 21.23^2} = 24.6\text{ ft/s}^2$$ **Ans.**

Ans:
$v = 10.7$ ft/s
$a = 24.6$ ft/s^2

12–182.

The peg is constrained to move in the slots of the fixed bar CD and rotating bar AB. When $\theta = 30°$, the angular velocity and angular acceleration of arm AB are $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 3$ rad/s^2, respectively. Determine the magnitudes of the velocity and acceleration of the peg P at this instant.



SOLUTION

Time Derivatives:

$$r = 4\sec\theta$$

$$\dot{r} = (4\sec\theta(\tan\theta)\dot{\theta})\text{ ft/s} \qquad \dot{\theta} = 2\text{ rad/s}$$

$$\ddot{r} = 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}(\sec\theta\sec^2\theta\dot{\theta} + \tan\theta\sec\theta(\tan\theta)\dot{\theta})] \qquad \ddot{\theta} = 3\text{ rad/s}^2$$

$$= 4[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}^2(\sec^3\theta° + \tan^2\theta°\sec\theta°)]\text{ ft/s}^2$$

When $\theta = 30°$,

$$r|_{\theta=30°} = 4\sec 30° = 4.619\text{ ft}$$

$$\dot{r}|_{\theta=30°} = (4\sec 30°\tan 30°)(2) = 5.333\text{ ft/s}$$

$$\ddot{r}|_{\theta=30°} = 4[(\sec 30°\tan 30°)(3) + 2^2(\sec^3 30° + \tan^2 30°\sec 30°)] = 38.79\text{ ft/s}^2$$

Velocity:

$$v_r = \dot{r} = 5.333\text{ ft/s} \qquad v_\theta = r\dot{\theta} = 4.619(2) = 9.238\text{ ft/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{5.333^2 + 9.238^2} = 10.7\text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 38.79 - 4.619(2^2) = 20.32\text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.619(3) + 2(5.333)(2) = 35.19\text{ ft/s}^2$$

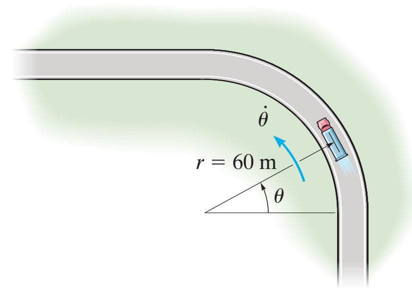
Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{20.32^2 + 35.19^2} = 40.6\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 10.7$ ft/s
$a = 40.6$ ft/s^2

12–183.

A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a constant speed $v = 20$ m/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the truck's acceleration.



SOLUTION

$r = 60$

$\dot{r} = 0$

$\ddot{r} = 0$

$v = 20$

$v_r = \dot{r} = 0$

$v_\theta = r\,\dot{\theta} = 60\,\dot{\theta}$

$v = \sqrt{(v_r)^2 + (v_\theta)^2}$

$20 = 60\,\dot{\theta}$

$\dot{\theta} = 0.333$ rad/s **Ans.**

$a_r = \ddot{r} - r(\dot{\theta})^2$

$= 0 - 60(0.333)^2$

$= -6.67$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 60\ddot{\theta}$

Since

$v = r\dot{\theta}$

$\dot{v} = \dot{r}\dot{\theta} + r\ddot{\theta}$

$0 = 0 + 60\,\ddot{\theta}$

$\ddot{\theta} = 0$

Thus,

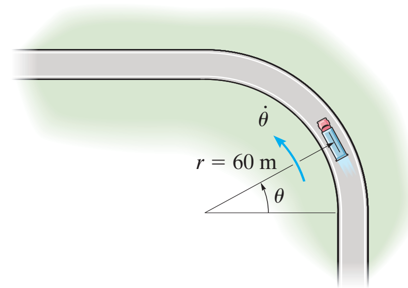
$a_\theta = 0$

$a = \left|a_r\right| = 6.67$ m/s^2 **Ans.**

Ans:
$\dot{\theta} = 0.333$ rad/s
$a = 6.67$ m/s^2

***12–184.**

A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a speed of 20 m/s which is increasing at $3\ \text{m/s}^2$. Determine the truck's radial and transverse components of acceleration.



SOLUTION

$r = 60$

$a_t = 3\ \text{m/s}^2$

$$a_n = \frac{v^2}{r} = \frac{(20)^2}{60} = 6.67\ \text{m/s}^2$$

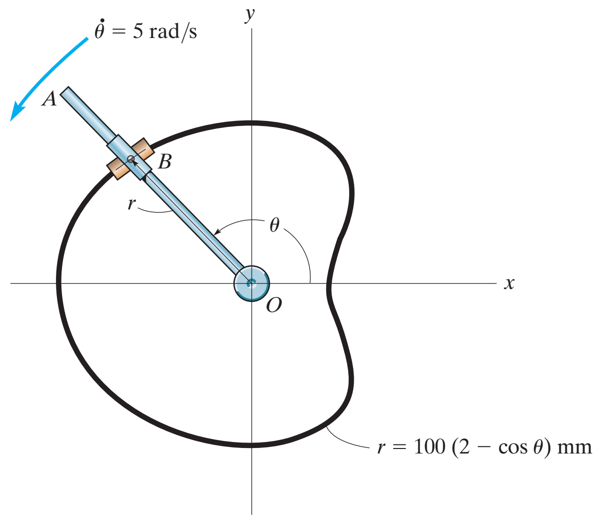
$a_r = -a_n = -6.67\ \text{m/s}^2$ **Ans.**

$a_\theta = a_t = 3\ \text{m/s}^2$ **Ans.**

Ans:
$a_r = -6.67\ \text{m/s}^2$
$a_\theta = 3\ \text{m/s}^2$

12–185.

The rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 100(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 120°$.



SOLUTION

$\dot{\theta} = 5$

$r = 100(2 - \cos\theta)$

$\dot{r} = 100 \sin\theta\dot{\theta} = 500 \sin\theta$

$\ddot{r} = 500 \cos\theta\dot{\theta} = 2500 \cos\theta$

At $\theta = 120°$,

$v_r = \dot{r} = 500 \sin 120° = 433.013$

$v_\theta = r\dot{\theta} = 100\,(2 - \cos 120°)(5) = 1250$

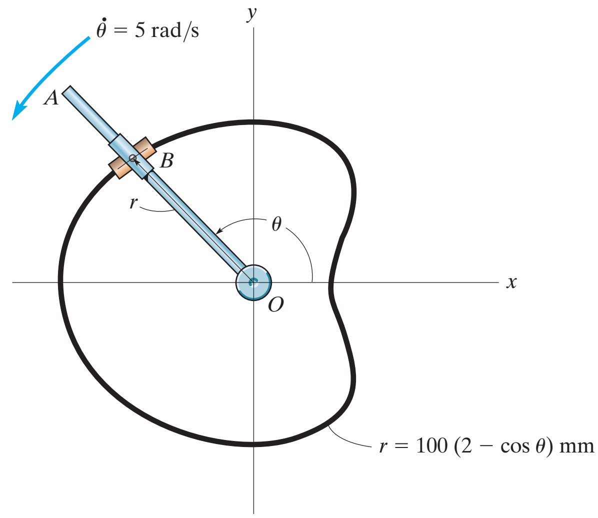
$v = \sqrt{(433.013)^2 + (1250)^2} = 1322.9 \text{ mm/s} = 1.32 \text{ m/s}$ **Ans.**

Ans:

$v = 1.32$ m/s

12–186.

Determine the magnitude of the acceleration of the slider blocks in Prob. 12–185 when $\theta = 120°$.



SOLUTION

$\dot{\theta} = 5$

$\dot{\theta} = 0$

$r = 100(2 - \cos\theta)$

$\dot{r} = 100 \sin\theta\dot{\theta} = 500 \sin\theta$

$\ddot{r} = 500 \cos\theta\dot{\theta} = 2500 \cos\theta$

$a_r = \ddot{r} - r\dot{\theta}^2 = 2500 \cos\theta - 100(2 - \cos\theta)(5)^2 = 5000(\cos 120° - 1) = -7500 \text{ mm/s}^2$

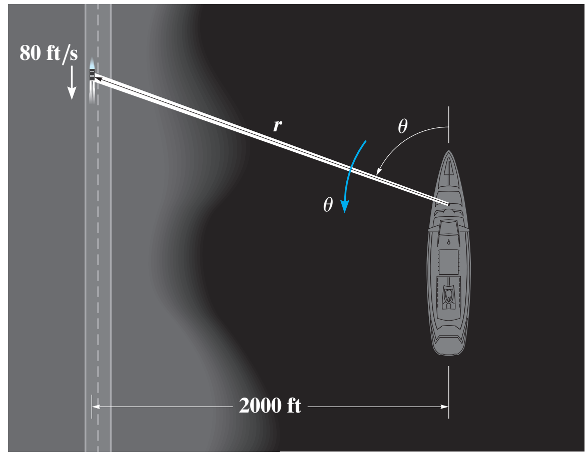
$a_s = r\dot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(500 \sin\theta)(5) = 5000 \sin 120° = 4330.1 \text{ mm/s}^2$

$a = \sqrt{(-7500)^2 + (4330.1)^2} = 8660.3 \text{ mm/s}^2 = 8.66 \text{ m/s}^2$ **Ans.**

Ans:
$a = 8.66 \text{ m/s}^2$

12–187.

The searchlight on the boat anchored 2000 ft from shore is turned on the automobile, which is traveling along the straight road at a constant speed of 80 ft/s. Determine the angular rate of rotation of the light when the automobile is $r = 3000$ ft from the boat.



SOLUTION

$r = 2000 \csc\theta$

$\dot{r} = -2000 \csc\theta \operatorname{ctn}\theta\, \dot{\theta}$

At $r = 3000$ ft, $\quad \theta = 41.8103°$

$\dot{r} = -3354.102\,\dot{\theta}$

$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

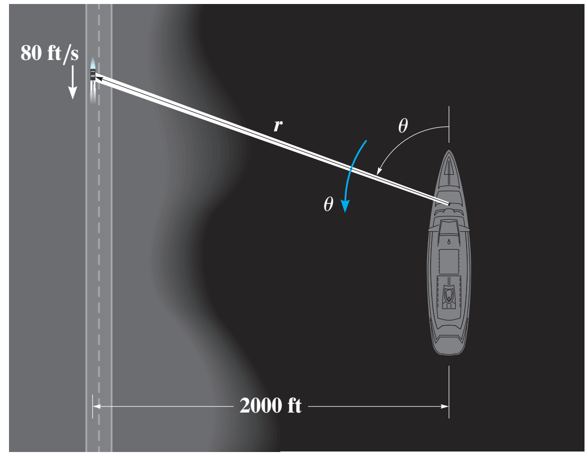
$(80)^2 = [(-3354.102)^2 + (3000)^2](\dot{\theta})^2$

$\dot{\theta} = 0.0177778 = 0.0178$ rad/s **Ans.**

Ans:
$\dot{\theta} = 0.0178$ rad/s

***12–188.**

If the car in Prob. 12–187 is accelerating at 15 ft/s^2 and has a velocity of 80 ft/s at the instant $r = 3000$ ft, determine the required angular acceleration $\ddot{\theta}$ of the light at this instant.



SOLUTION

$r = 2000 \csc \theta$

$\dot{r} = -2000 \csc\theta \operatorname{ctn}\theta\, \dot{\theta}$

At $r = 3000$ ft, $\theta = 41.8103°$

$\dot{r} = -3354.102\, \dot{\theta}$

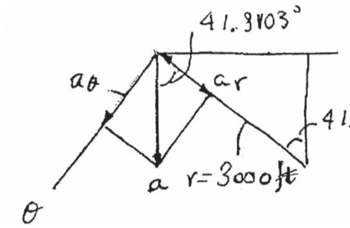
$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta}$

$a_\theta = 3000\,\ddot{\theta} + 2(-3354.102)(0.0177778)^2$

Since $a_\theta = 15 \sin 41.8103° = 10$ m/s

Then,

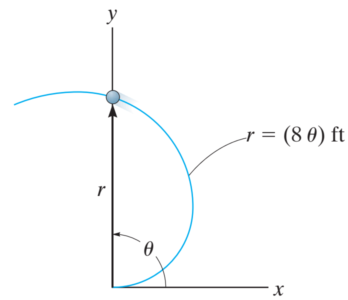
$\ddot{\theta} = 0.00404 \text{ rad/s}^2$ **Ans.**



Ans:
$\ddot{\theta} = 0.00404 \text{ rad/s}^2$

12–189.

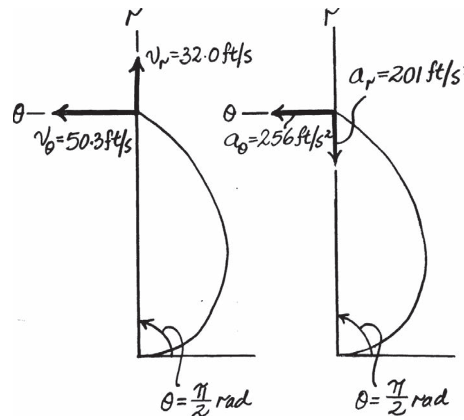
A particle moves along an Archimedean spiral $r = (8\theta)$ ft, where θ is given in radians. If $\dot{\theta} = 4$ rad/s (constant), determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \pi/2$ rad. Sketch the curve and show the components on the curve.



SOLUTION

Time Derivatives: Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi \text{ ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0 \text{ ft/s} \qquad \ddot{r} = 8\ddot{\theta} = 0$$



Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 32.0 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3 \text{ ft/s} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

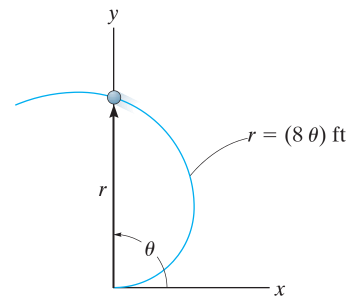
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 4\pi\left(4^2\right) = -201 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(32.0)(4) = 256 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_r = -201 \text{ ft/s}^2$
$a_\theta = 256 \text{ ft/s}^2$

12–190.

Solve Prob. 12–189 if the particle has an angular acceleration $\ddot{\theta} = 5\ \text{rad/s}^2$ when $\dot{\theta} = 4\ \text{rad/s}$ at $\theta = \pi/2$ rad.



SOLUTION

Time Derivatives: Here,

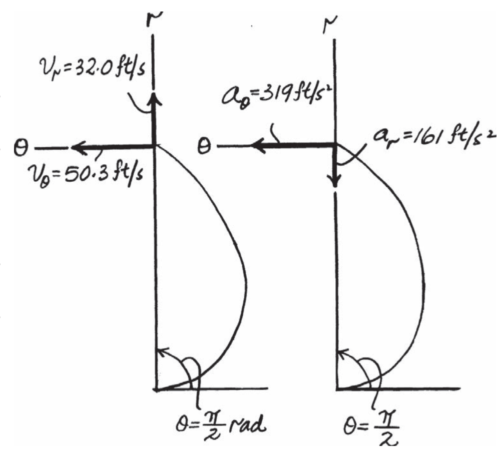
$$r = 8\theta = 8\left(\frac{\pi}{2}\right) = 4\pi\ \text{ft} \qquad \dot{r} = 8\dot{\theta} = 8(4) = 32.0\ \text{ft/s}$$

$$\ddot{r} = 8\ddot{\theta} = 8(5) = 40\ \text{ft/s}^2$$

Velocity: Applying Eq. 12–25, we have

$$v_r = \dot{r} = 32.0\ \text{ft/s} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = 4\pi(4) = 50.3\ \text{ft/s} \qquad \textbf{Ans.}$$



Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 40 - 4\pi\left(4^2\right) = -161\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4\pi(5) + 2(32.0)(4) = 319\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Ans:

$v_r = 32.0\ \text{ft/s}$

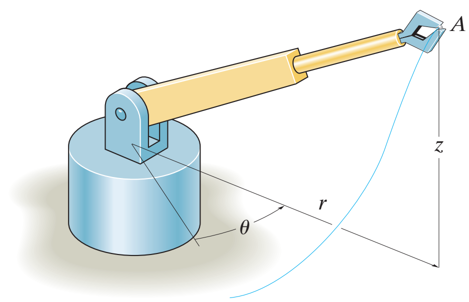
$v_\theta = 50.3\ \text{ft/s}$

$a_r = -161\ \text{ft/s}^2$

$a_\theta = 319\ \text{ft/s}^2$

12–191.

The arm of the robot moves so that $r = 3$ ft is constant, and its grip A moves along the path $z = (3 \sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t \qquad r = 3 \qquad z = 3 \sin 2t$

$\dot{\theta} = 0.5 \qquad \dot{r} = 0 \qquad \dot{z} = 6 \cos 2t$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = -12 \sin 2t$

At $t = 3$ s,

$z = -0.8382$

$\dot{z} = 5.761$

$\ddot{z} = 3.353$

$v_r = 0$

$v_\theta = 3(0.5) = 1.5$

$v_z = 5.761$

$v = \sqrt{(0)^2 + (1.5)^2 + (5.761)^2} = 5.95 \text{ ft/s}$ **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 0 = 0$

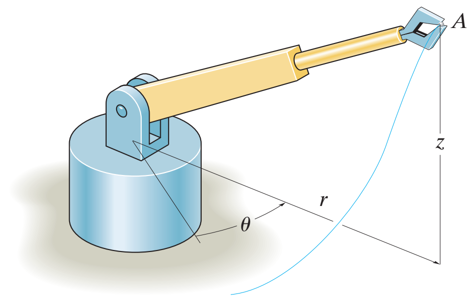
$a_z = 3.353$

$a = \sqrt{(-0.75)^2 + (0)^2 + (3.353)^2} = 3.44 \text{ ft/s}^2$ **Ans.**

Ans:
$v = 5.95 \text{ ft/s}$
$a = 3.44 \text{ ft/s}^2$

***12–192.**

For a short time the arm of the robot is extending at a constant rate such that $\dot{r} = 1.5$ ft/s when $r = 3$ ft, $z = (4t^2)$ ft, and $\theta = 0.5t$ rad, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the grip A when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t$ rad $\qquad r = 3$ ft $\qquad z = 4\,t^2$ ft

$\dot{\theta} = 0.5$ rad/s $\qquad \dot{r} = 1.5$ ft/s $\qquad \dot{z} = 8\,t$ ft/s

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$ ft/s^2

At $t = 3$ s,

$\theta = 1.5 \qquad r = 3 \qquad z = 36$

$\dot{\theta} = 0.5 \qquad \dot{r} = 1.5 \qquad \dot{z} = 24$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = 8$

$v_r = 1.5$

$v_\theta = 3(0.5) = 1.5$

$v_z = 24$

$v = \sqrt{(1.5)^2 + (1.5)^2 + (24)^2} = 24.1$ ft/s **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 2(1.5)(0.5) = 1.5$

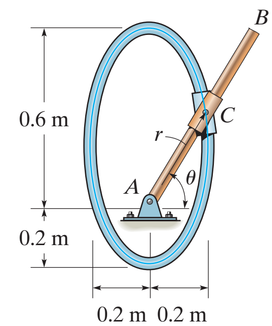
$a_z = 8$

$a = \sqrt{(-0.75)^2 + (1.5)^2 + (8)^2} = 8.17$ ft/s^2 **Ans.**

Ans:
$v = 24.1$ ft/s
$a = 8.17$ ft/s^2

12–193.

The double collar C is pin connected together such that one collar slides over the fixed rod and the other slides over the rotating rod AB. If the angular velocity of AB is given as $\dot{\theta} = (e^{0.5\,t^2})$ rad/s, where t is in seconds, and the path defined by the fixed rod is $r = |(0.4\sin\theta + 0.2)|$ m, determine the radial and transverse components of the collar's velocity and acceleration when $t = 1$ s. When $t = 0$, $\theta = 0$. Use Simpson's rule with $n = 50$ to determine θ at $t = 1$ s.



SOLUTION

$\dot{\theta} = e^{0.5\,t^2}\big|_{t=1} = 1.649$ rad/s

$\ddot{\theta} = e^{0.5\,t^2}\,t\big|_{t=1} = 1.649$ rad/s^2

$$\theta = \int_0^1 e^{0.5\,t^2}dt = 1.195 \text{ rad} = 68.47^\circ$$

$r = 0.4\sin\theta + 0.2$

$\dot{r} = 0.4\cos\theta\,\dot{\theta}$

$\ddot{r} = -0.4\sin\theta\,\dot{\theta}^2 + 0.4\cos\theta\,\ddot{\theta}$

At $t = 1$ s,

$r = 0.5721$

$\dot{r} = 0.2421$

$\ddot{r} = -0.7697$

$v_r = \dot{r} = 0.242$ m/s **Ans.**

$v_\theta = r\,\dot{\theta} = 0.5721(1.649) = 0.943$ m/s **Ans.**

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.7697 - 0.5721(1.649)^2$

$a_r = -2.33$ m/s^2 **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

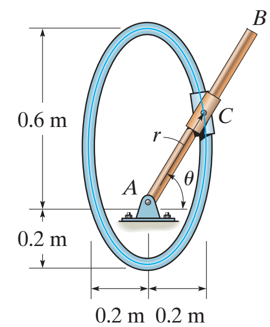
$= 0.5721(1.649) + 2(0.2421)(1.649)$

$a_\theta = 1.74$ m/s^2 **Ans.**

Ans:
$v_r = 0.242$ m/s
$v_\theta = 0.943$ m/s
$a_r = -2.33$ m/s^2
$a_\theta = 1.74$ m/s^2

12–194.

The double collar C is pin connected together such that one collar slides over the fixed rod and the other slides over the rotating rod AB. If the mechanism is to be designed so that the largest speed given to the collar is 6 m/s, determine the required constant angular velocity $\dot{\theta}$ of rod AB. The path defined by the fixed rod is $r = (0.4 \sin\theta + 0.2)$ m.



SOLUTION

$r = 0.4 \sin\theta + 0.2$

$\dot{r} = 0.4 \cos\theta\, \dot{\theta}$

$v_r = \dot{r} = 0.4 \cos\theta\, \dot{\theta}$

$v_\theta = r\,\dot{\theta} = (0.4 \sin\theta + 0.2)\,\dot{\theta}$

$v^2 = v_r^2 + v_\theta^3$

$(6)^2 = [(0.4 \cos\theta)^2 + (0.4 \sin\theta + 0.2)^2](\dot{\theta})^2$

$36 = [0.2 + 0.16 \sin\theta](\dot{\theta})^2$

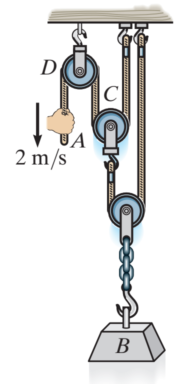
The greatest speed occurs when $\theta = 90°$.

$\dot{\theta} = 10.0$ rad/s **Ans.**

Ans:
$\dot{\theta} = 10.0$ rad/s

12–195.

If the end of the cable at A is pulled down with a speed of 2 m/s, determine the speed at which block B rises.



SOLUTION

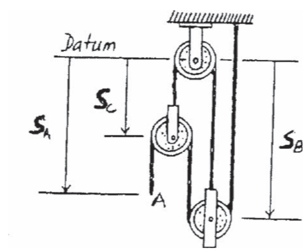
Position-Coordinate Equation: Datum is established at fixed pulley D. The position of point A, block B and pulley C with respect to datum are s_A, s_B, and s_C respectively. Since the system consists of two cords, two position-coordinate equations can be derived.

$$(s_A - s_C) + (s_B - s_C) + s_B = l_1 \qquad \textbf{(1)}$$

$$s_B + s_C = l_2 \qquad \textbf{(2)}$$

Eliminating s_C from Eqs. (1) and (2) yields

$$s_A + 4s_B = l_1 = 2l_2$$



Time Derivative: Taking the time derivative of the above equation yields

$$v_A + 4v_B = 0 \qquad \textbf{(3)}$$

Since $v_A = 2$ m/s, from Eq. (3)

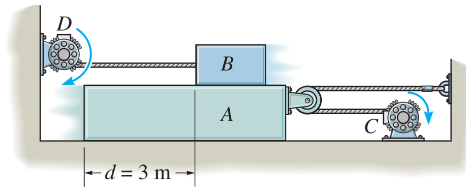
$(+\downarrow)$ $$2 + 4v_B = 0$$

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

Ans:
$v_B = 0.5$ m/s

***12–196.**

The motor at C pulls in the cable with an acceleration $a_C = (3t^2)\ \text{m/s}^2$, where t is in seconds. The motor at D draws in its cable at $a_D = 5\ \text{m/s}^2$. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the velocities of blocks A and B when this occurs.



SOLUTION

For A:

$s_A + (s_A - s_C) = l$

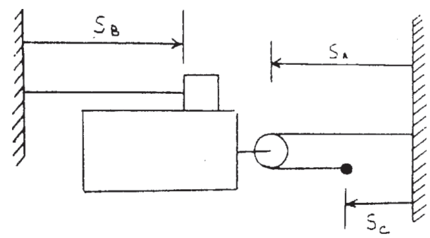
$2v_A = v_C$

$2a_A = a_C = -3t^2$

$a_A = -1.5t^2 = 1.5t^2 \quad \rightarrow$

$v_A = 0.5t^3 \quad \rightarrow$

$s_A = 0.125t^4 \quad \rightarrow$



For B:

$a_B = 5\ \text{m/s}^2 \quad \leftarrow$

$v_B = 5t \quad \leftarrow$

$s_B = 2.5t^2 \quad \leftarrow$

Require $s_A + s_B = d$

$0.125t^4 + 2.5t^2 = 3$

Set $u = t^2 \qquad 0.125u^2 + 2.5u = 3$

The positive root is $u = 1.1355$. Thus,

$t = 1.0656 = 1.07$ s **Ans.**

$v_A = 0.5(1.0656)^3 = 0.6050$

$v_B = 5(1.0656) = 5.3281$ m/s

$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

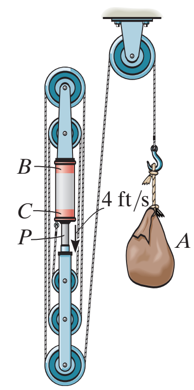
$0.6050\mathbf{i} = -5.3281\mathbf{i} + v_{A/B}\,\mathbf{i}$

$v_{A/B} = 5.93$ m/s $\quad \rightarrow$ **Ans.**

Ans:
$t = 1.07$ s
$v_{A/B} = 5.93$ ms/s $\rightarrow$

12–197.

The pulley arrangement shown is designed for hoisting materials. If *BC remains fixed* while the plunger *P* is pushed downward with a speed of 4 ft/s, determine the speed of the load at *A*.



SOLUTION

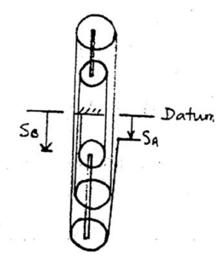
$5\,s_B + (s_B - s_A) = l$

$6\,s_B - s_A = l$

$6\,v_B - v_A = 0$

$6(4) = v_A$

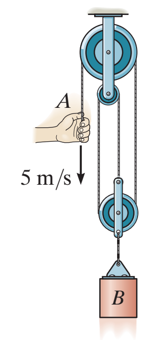
$v_A = 24\text{ ft/s}$ **Ans.**



Ans:
$v = 24\text{ ft/s}$

12–198.

If the end of the cable at A is pulled down with a speed of 5 m/s, determine the speed at which block B rises.



SOLUTION

Position Coordinate. The positions of pulley B and point A are specified by position coordinates s_B and s_A, respectively, as shown in Fig. a. This is a single-cord pulley system. Thus,

$$s_B + 2(s_B - a) + s_A = l$$

$$3s_B + s_A = l + 2a \qquad \textbf{(1)}$$

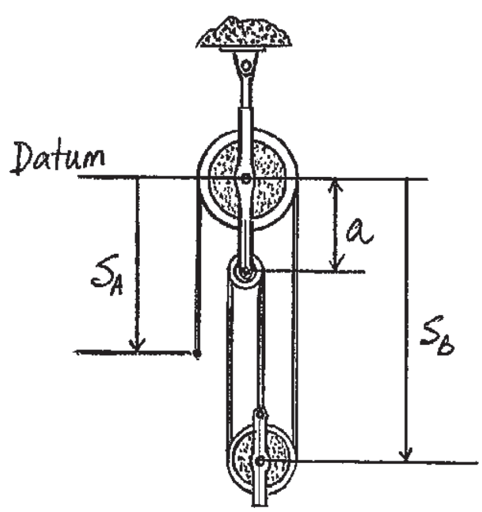
Time Derivative. Taking the time derivative of Eq. (1),

$$3v_B + v_A = 0 \qquad \textbf{(2)}$$

Here $v_A = +5$ m/s, since it is directed toward the positive sense of s_A. Thus,

$$3v_B + 5 = 0 \qquad v_B = -1.667 \text{ m/s} = 1.67 \text{ m/s}\uparrow \qquad \textbf{Ans.}$$

The negative sign indicates that v_B is directed toward the negative sense of s_B.

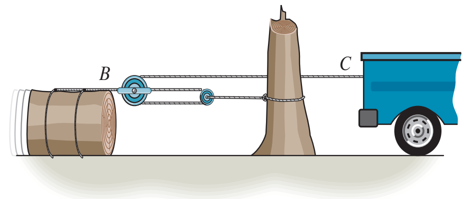


(a)

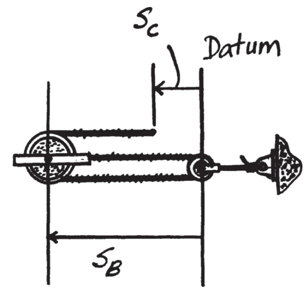
Ans:
$v_B = 1.67$ m/s

12–199.

Determine the displacement of the log if the truck at C pulls the cable 4 ft to the right.



SOLUTION



$2s_B + (s_B - s_C) = l$

$3s_B - s_C = l$

$3\Delta s_B - \Delta s_C = 0$

Since $\Delta s_C = -4$, then

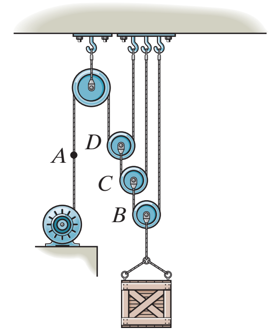
$3\Delta s_B = -4$

$\Delta s_B = -1.33 \text{ ft} = 1.33 \text{ ft} \rightarrow$ **Ans.**

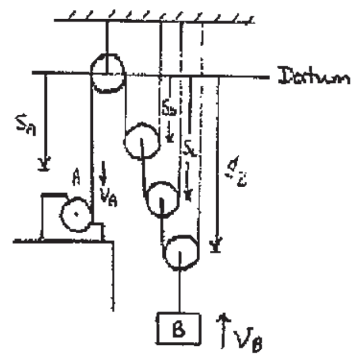
Ans:
$\Delta s_B = 1.33 \text{ ft} \rightarrow$

***12–200.**

Determine the constant speed at which the cable at A must be drawn in by the motor in order to hoist the load 6 m in 1.5 s.



SOLUTION



$$v_B = \frac{6}{1.5} = 4 \text{ m/s} \uparrow$$

$$s_B + (s_B - s_C) = l_1$$

$$s_C + (s_C - s_D) = l_2$$

$$s_A + 2\,s_D = l_3$$

Thus,

$$2s_B - s_C = l_1$$

$$2s_C - s_D = l_2$$

$$s_A + 2s_D = l_3$$

$$2v_A = v_C$$

$$2v_C = v_D$$

$$v_A = -2v_D$$

$$2(2v_B) = v_D$$

$$v_A = -2(4v_B)$$

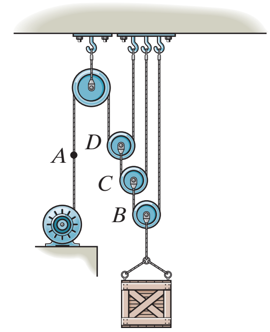
$$v_A = -8v_B$$

$$v_A = -8(-4) = 32 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

Ans:
$v_A = 32 \text{ m/s}\downarrow$

12–201.

Starting from rest, the cable can be wound onto the drum of the motor at a rate of $v_A = (3t^2)$ m/s, where t is in seconds. Determine the time needed to lift the load 7 m.



SOLUTION

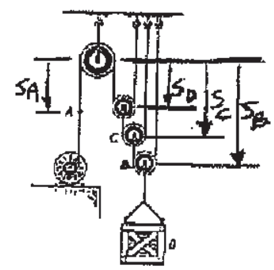
$$v_B = \frac{6}{1.5} = 4 \text{ m/s} \uparrow$$

$$s_B + (s_B - s_C) = l_1$$

$$s_C + (s_C - s_D) = l_2$$

$$s_A + 2s_D) = l_3$$

Thus,



$$2s_B - s_C = l_1$$

$$2s_C - s_D = l_2$$

$$s_A + 2s_D = l_3$$

$$2v_B = v_C$$

$$2v_C = v_D$$

$$v_A = -2v_D$$

$$v_A = -8v_B$$

$$3\,t^2 = -8v_B$$

$$v_B = \frac{-3}{8}\,t^2$$

$$s_B = \int_0^t \frac{-3}{8}\,t^2\,dt$$

$$s_B = \frac{-1}{8}\,t^3$$

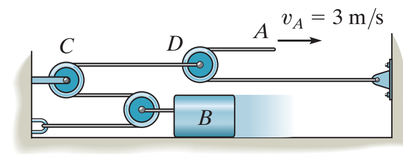
$$-7 = \frac{-1}{8}\,t^3$$

$t = 3.83$ s **Ans.**

Ans:
$t = 3.83$ s

12–202.

If the end A of the cable is moving at $v_A = 3$ m/s, determine the speed of block B.



SOLUTION

Position Coordinates. The positions of pulley B, D and point A are specified by position coordinates s_B, s_D and s_A respectively as shown in Fig. a. The pulley system consists of two cords which give

$$2\,s_B + s_D = l_1 \qquad \textbf{(1)}$$

and

$$(s_A - s_D) + (b - s_D) = l_2$$

$$s_A - 2\,s_D = l_2 - b \qquad \textbf{(2)}$$

Time Derivative. Taking the time derivatives of Eqs. (1) and (2), we get

$$2v_B + v_D = 0 \qquad \textbf{(3)}$$

$$v_A - 2v_D = 0 \qquad \textbf{(4)}$$

Eliminate v_0 from Eqs. (3) and (4),

$$v_A + 4v_B = 0 \qquad \textbf{(5)}$$

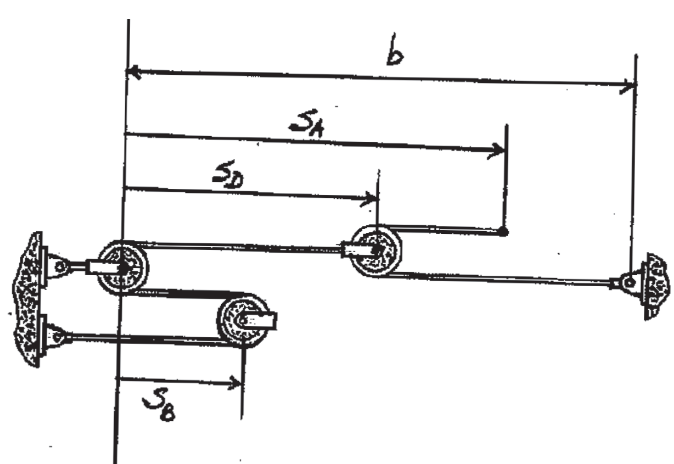
Here $v_A = +3$ m/s since it is directed toward the positive sense of s_A.

Thus

$$3 + 4v_B = 0$$

$$v_B = -0.75 \text{ m/s} = 0.75 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_D$ is directed toward the negative sense of $\mathbf{s}_B$.



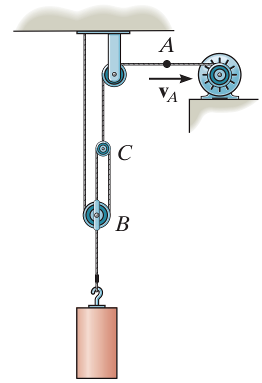
(a)

Ans:

$v_B = 0.75$ m/s

12–203.

Determine the time needed for the load at B to attain a speed of 10 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 3 m/s^2.



SOLUTION

Position Coordinates. The position of pulleys B, C and point A are specified by position coordinates s_B, s_C and s_A respectively as shown in Fig. a. The pulley system consists of two cords which gives

$$s_B + 2(s_B - s_C) = l_1$$

$$3s_B - 2s_C = l_1 \qquad \textbf{(1)}$$

And

$$s_C + s_A = l_2 \qquad \textbf{(2)}$$

Time Derivative. Taking the time derivative twice of Eqs. (1) and (2),

$$3a_B - 2a_C = 0 \qquad \textbf{(3)}$$

And

$$a_C + a_A = 0 \qquad \textbf{(4)}$$

Eliminate a_C from Eqs. (3) and (4)

$$3a_B + 2a_A = 0$$

Here, $a_A = +3\text{ m/s}^2$ since it is directed toward the positive sense of s_A. Thus,

$$3a_B + 2(3) = 0 \qquad a_B = -2\text{ m/s}^2 = 2\text{ m/s}^2 \uparrow$$

The negative sign indicates that $\mathbf{a}_B$ is directed toward the negative sense of s_B. Applying kinematic equation of constant acceleration,

$$+\uparrow \qquad v_B = (v_B)_0 + a_B t$$

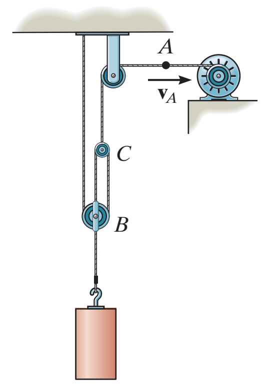
$$10 = 0 + 2t$$

$$t = 5.00\text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 5.00$ s

***12–204.**

The cable at A is being drawn toward the motor at $v_A = 8$ m/s. Determine the velocity of the block.



SOLUTION

Position Coordinates. The position of pulleys B, C and point A are specified by position coordinates s_B, s_C and s_A respectively as shown in Fig. a. The pulley system consists of two cords which give

$$s_B + 2(s_B - s_C) = l_1$$

$$3s_B - 2s_C = l_1 \qquad \textbf{(1)}$$

And

$$s_C + s_A = l_2 \qquad \textbf{(2)}$$

Time Derivative. Taking the time derivatives of Eqs. (1) and (2), we get

$$3v_B - 2v_C = 0 \qquad \textbf{(3)}$$

And

$$v_C + v_A = 0 \qquad \textbf{(4)}$$

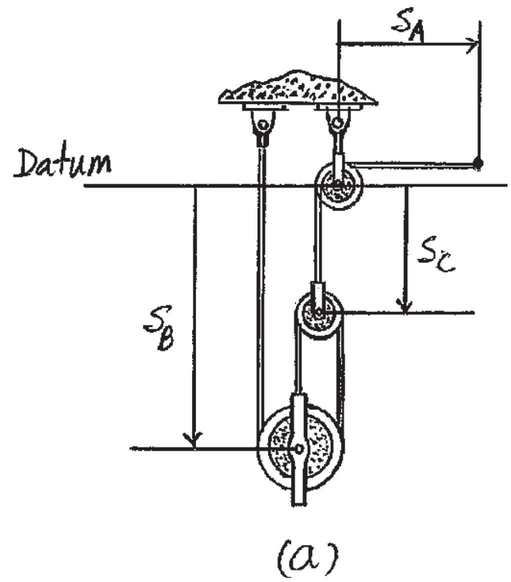
Eliminate v_C from Eqs. (3) and (4),

$$3v_B + 2v_A = 0$$

Here $v_A = +8$ m/s since it is directed toward the positive sense of s_A. Thus,

$$3v_B + 2(8) = 0 \qquad v_B = -5.33 \text{ m/s} = 5.33 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_B$ is directed toward the negative sense of s_B.

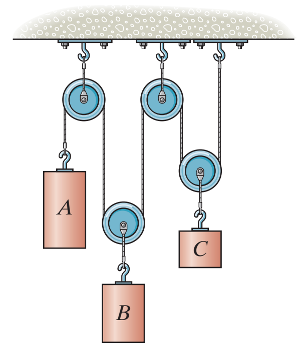


(a)

Ans:
$v_B = 5.33 \text{ m/s} \uparrow$

12–205.

If block A of the pulley system is moving downward at 6 ft/s while block C is moving down at 18 ft/s, determine the relative velocity of block B with respect to C.



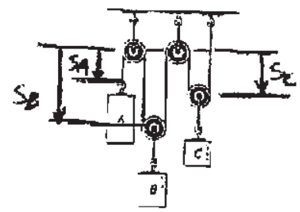
SOLUTION

$s_A + 2s_B + 2s_C = l$

$v_A + 2v_B + 2v_C = 0$

$6 + 2v_B + 2(18) = 0$

$v_B = -21 \text{ ft/s} = 21 \text{ ft/s} \uparrow$



$+\downarrow v_B = v_C + v_{B/C}$

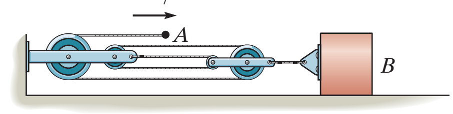
$-21 = 18 + v_{B/C}$

$v_{B/C} = -39 \text{ ft/s} = 39 \text{ ft/s} \uparrow$ **Ans.**

Ans:
$v_{B/C} = 39 \text{ ft/s} \uparrow$

12–206.

Determine the speed of the block at B.



SOLUTION

Position Coordinate. The positions of pulley B and point A are specified by position coordinates s_B and s_A respectively as shown in Fig. *a*. This is a single cord pulley system. Thus,

$$s_B + 2(s_B - a - b) + (s_B - a) + s_A = l$$

$$4s_B + s_A = l + 3a + 2b \qquad \textbf{(1)}$$

Time Derivative. Taking the time derivative of Eq. (1),

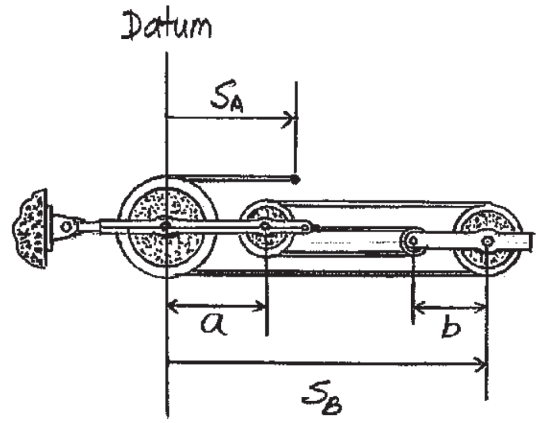
$$4v_B + v_A = 0 \qquad \textbf{(2)}$$

Here, $v_A = +6$ m/s since it is directed toward the positive sense of s_A. Thus,

$$4v_B + 6 = 0$$

$$v_B = -1.50 \text{ m/s} = 1.50 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{v}_B$ is directed towards negative sense of s_B.

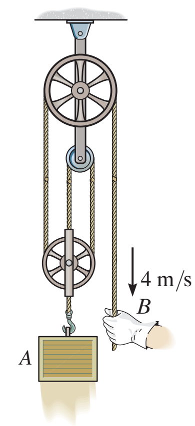


(a)

Ans:
$v_B = 1.50$ m/s

12–207.

Determine the speed of block A if the end of the rope is pulled down with a speed of 4 m/s.



SOLUTION

Position Coordinates: By referring to Fig. a, the length of the cord written in terms of the position coordinates s_A and s_B is

$$s_B + s_A + 2(s_A - a) = l$$

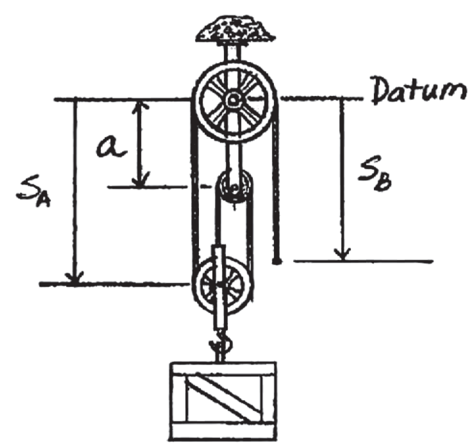
$$s_B + 3s_A = l + 2a$$

Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad v_B + 3v_A = 0$$

Here, $v_B = 4$ m/s. Thus,

$$4 + 3v_A = 0 \qquad v_A = -133 \text{ m/s} = 1.33 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

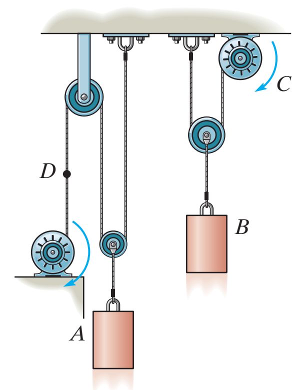


(a)

Ans:
$v_A = 1.33$ m/s

***12–208.**

The motor draws in the cable at C with a constant velocity of $v_C = 4$ m/s. The motor draws in the cable at D with a constant acceleration of $a_D = 8\ \text{m/s}^2$. If $v_D = 0$ when $t = 0$, determine (a) the time needed for block A to rise 3 m, and (b) the relative velocity of block A with respect to block B when this occurs.



SOLUTION

(a) $a_D = 8\ \text{m/s}^2$

$v_D = 8\,t$

$s_D = 4\,t^2$

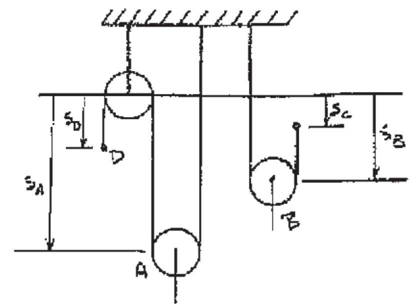
$s_D + 2s_A = l$

$\Delta s_D = -2\Delta s_A$ **(1)**

$\Delta s_A = -2\,t^2$

$-3 = -2\,t^2$

$t = 1.2247 = 1.22\ \text{s}$ **Ans.**



(b) $v_A = s_A = -4\,t = -4(1.2247) = -4.90\ \text{m/s} = 4.90\ \text{m/s}\uparrow$

$s_B + (s_B - s_C) = l$

$2v_B = v_C = -4$

$v_B = -2\ \text{m/s} = 2\ \text{m/s}\uparrow$

$(+\downarrow)\quad \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

$-4.90 = -2 + v_{A/B}$

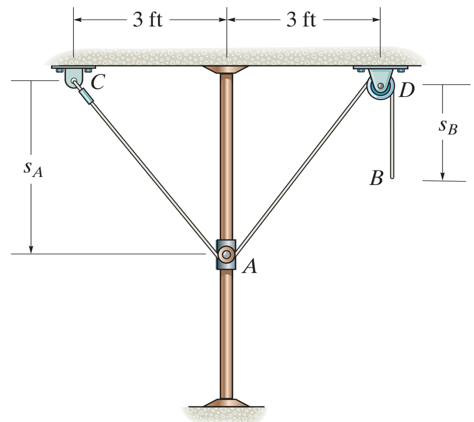
$v_{A/B} = -2.90\ \text{m/s} = 2.90\ \text{m/s}\uparrow$ **Ans.**

Ans:

$v_{A/B} = 2.90\ \text{m/s}\uparrow$

12–209.

The cord is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. Determine the velocity and acceleration of the end of the cord at B if at the instant $s_A = 4$ ft the collar is moving upwards at 5 ft/s, which is decreasing at 2 ft/s^2.



SOLUTION

$$2\sqrt{s_A^2 + 3^2} + s_B = l$$

$$2\left(\frac{1}{2}\right)\left(s_A^2 + 9\right)^{-\frac{1}{2}}\left(2s_A\,\dot{s}_A\right) + \dot{s}_B = 0$$

$$\dot{s}_B = -\frac{2s_A\dot{s}_A}{\left(s_A^2 + 9\right)^{\frac{1}{2}}}$$

$$\ddot{s}_B = -2\dot{s}_A^2\left(s_A^2 + 9\right)^{-\frac{1}{2}} - \left(2s_A\,\ddot{s}_A\right)\left(s_A^2 + 9\right)^{-\frac{1}{2}} - \left(2s_A\,\dot{s}_A\right)\left[\left(-\frac{1}{2}\right)\left(s_A^2 + 9\right)^{-\frac{3}{2}}\left(2s_A\,\dot{s}_A\right)\right]$$

$$\ddot{s}_B = -\frac{2\left(\dot{s}_A + s_A\,\ddot{s}_A\right)}{\left(s_A^2 + 9\right)^{\frac{1}{2}}} + \frac{2\left(s_A\,\dot{s}_A\right)^2}{\left(s_A^2 + 9\right)^{\frac{3}{2}}}$$

At $s_A = 4$ ft,

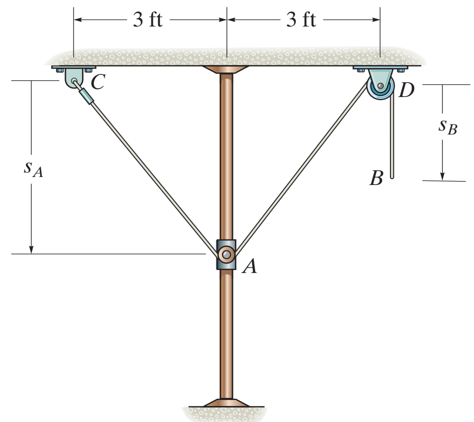
$$v_B = \dot{s}_B = -\frac{2(4)(-5)}{(4^2 + 9)^{\frac{1}{2}}} = 8 \text{ ft/s} \downarrow$$ **Ans.**

$$a_B = \ddot{s}_B = -\frac{2\left[(-5)^2 + (4)(2)\right]}{(4^2 + 9)^{\frac{1}{2}}} + \frac{2[(4)(-5)]^2}{(4^2 + 9)^{\frac{3}{2}}} = -6.80 \text{ ft/s}^2 = 6.80 \text{ ft/s}^2 \uparrow$$ **Ans.**

Ans:
$v_B = 8$ ft/s$\downarrow$
$a_B = 6.80$ ft/s$^2 \uparrow$

12–210.

The 16-ft-long cord is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. When $s_B = 6$ ft, the end of the cord at B is pulled downwards with a velocity of 4 ft/s and is given an acceleration of 3 ft/s^2. Determine the velocity and acceleration of the collar at this instant.



SOLUTION

$$2\sqrt{s_A^2 + 3^2} + s_B = l$$

$$2\left(\frac{1}{2}\right)(s_A^2 + 9)^{-\frac{1}{2}}\left(2s_A\dot{s}_A\right) + \dot{s}_B = 0$$

$$\dot{s}_B = -\frac{2s_A\,\dot{s}_A}{(s_A^2 + 9)^{\frac{1}{2}}}$$

$$\ddot{s}_B = -2\dot{s}_A^2(s_A^2 + 9)^{-\frac{1}{2}} - \left(2s_A\ddot{s}_A\right)(s_A^2 + 9)^{-\frac{1}{2}} - \left(2s_A\dot{s}_A\right)\left[\left(-\frac{1}{2}\right)(s_A^2 + 9)^{-\frac{3}{2}}\left(2s_A\dot{s}_A\right)\right]$$

$$\ddot{s}_B = -\frac{2(\dot{s}_A^2 + s_A\,\ddot{s}_A)}{(s_A^2 + 9)^{\frac{1}{2}}} + \frac{2(s_A\,\dot{s}_A)^2}{(s_A^2 + 9)^{\frac{3}{2}}}$$

At $s_B = 6$ ft, $\dot{s}_B = 4$ ft/s, $\ddot{s}_B = 3$ ft/s^2

$$2\sqrt{s_A^2 + 3^2} + 6 = 16$$

$$s_A = 4 \text{ ft}$$

$$4 = -\frac{2(4)(\dot{s}_A)}{(4^2 + 9)^{\frac{1}{2}}}$$

$v_A = \dot{s}_A = -2.5 \text{ ft/s} = 2.5 \text{ ft/s}\uparrow$ **Ans.**

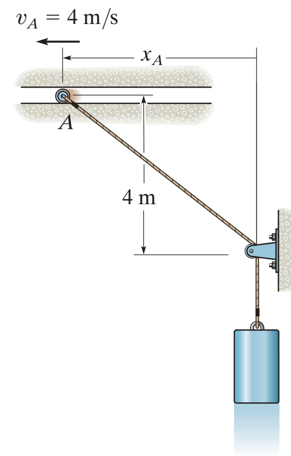
$$3 = -\frac{2\left[(-2.5)^2 + 4(\ddot{s}_A)\right]}{(4^2 + 9)^{\frac{1}{2}}} + \frac{2[4(-2.5)]^2}{(4^2 + 9)^{\frac{3}{2}}}$$

$a_A = \ddot{s}_A = -2.4375 = 2.44 \text{ ft/s}^2\uparrow$ **Ans.**

Ans:
$v_A = 2.5 \text{ ft/s}\uparrow$
$a_A = 2.44 \text{ ft/s}^2\uparrow$

12–211.

The roller at A is moving with a velocity of $v_A = 4\text{ m/s}$ and has an acceleration of $a_A = 2\text{ m/s}^2$ when $x_A = 3\text{ m}$. Determine the velocity and acceleration of block B at this instant.



SOLUTION

Position Coordinates. The position of roller A and block B are specified by position coordinates x_A and y_B respectively as shown in Fig. a. We can relate these two position coordinates by considering the length of the cable, which is constant

$$\sqrt{x_A^2 + 4^2} + y_B = l$$

$$y_B = l - \sqrt{x_A^2 + 16} \qquad \textbf{(1)}$$

Velocity. Taking the time derivative of Eq. (1) using the chain rule,

$$\frac{dy_B}{dt} = 0 - \frac{1}{2}\left(x_A^2 + 16\right)^{-\frac{1}{2}}(2x_A)\frac{dx_A}{dt}$$

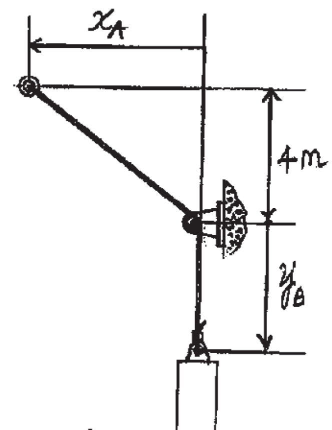
$$\frac{dy_B}{dt} = -\frac{x_A}{\sqrt{x_A^2 + 16}}\frac{dx_A}{dt}$$

However, $\dfrac{dy_B}{dt} = v_B$ and $\dfrac{dx_A}{dt} = v_A$. Then

$$v_B = -\frac{x_A}{\sqrt{x_A^2 + 16}}v_A \qquad \textbf{(2)}$$

At $x_A = 3\text{ m}$, $v_A = +4\text{ m/s}$ since $\mathbf{v}_A$ is directed toward the positive sense of x_A. Then Eq. (2) give

$$v_B = -\frac{3}{\sqrt{3^2 + 16}}(4) = -2.40\text{ m/s} = 2.40\text{ m/s}\uparrow \qquad \textbf{Ans.}$$



(a)

The negative sign indicates that $\mathbf{v}_B$ is directed toward the negative sense of y_B.

Acceleration. Taking the time derivative of Eq. (2),

$$\frac{dv_B}{dt} = -\left[x_A\left(-\frac{1}{2}\right)(x_A^2 + 16)^{-3/2}(2x_A)\frac{dx_A}{dt} + (x_A^2 + 16)^{-1/2}\frac{dx_A}{dt}\right]$$

$$v_A - x_A(x_A^2 + 16)^{-1/2}\frac{dv_A}{dt}$$

However, $\dfrac{dv_B}{dt} = a_B$, $\dfrac{dv_A}{dt} = a_A$ and $\dfrac{dx_A}{dt} = v_A$. Then

$$a_B = \frac{x_A^2 v_A^2}{\left(x_A^2 + 16\right)^{3/2}} - \frac{v_A^2}{\left(x_A^2 + 16\right)^{1/2}} - \frac{x_A a_A}{\left(x_A^2 + 16\right)^{1/2}}$$

$$a_B = -\frac{16\,v_A^2 + a_A x_A\left(x_A^2 + 16\right)}{\left(x_A^2 + 16\right)^{3/2}}$$

At $x_A = 3\text{ m}$, $v_A = +4\text{ m/s}$, $a_A = +2\text{ m/s}^2$ since $\mathbf{v}_A$ and a_A are directed toward the positive sense of x_A.

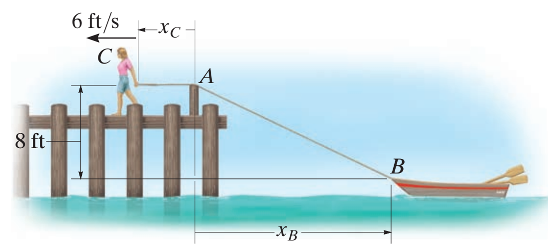
$$a_B = -\frac{16\left(4^2\right) + 2(3)\left(3^2 + 16\right)}{\left(3^2 + 16\right)^{3/2}} = -3.248\text{ m/s}^2 = 3.25\text{ m/s}^2\uparrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{a}_B$ is directed toward the negative sense of y_B.

Ans:
$v_B = 2.40\text{ m/s}\uparrow$
$a_B = 3.25\text{ m/s}^2\uparrow$

***12–212.**

The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at a constant speed of 6 ft/s. Determine how fast the boat approaches the pier at the instant the rope length AB is 50 ft.



SOLUTION

The length l of cord is

$$\sqrt{(8)^2 + x_B^2} + x_C = l$$

Taking the time derivative:

$$\frac{1}{2}[(8)^2 + x_B^2]^{-1/2}\, 2\, x_B \dot{x}_B + \dot{x}_C = 0 \qquad \textbf{(1)}$$

$$\dot{x}_C = 6 \text{ ft/s}$$

When $AB = 50$ ft,

$$x_B = \sqrt{(50)^2 - (8)^2} = 49.356 \text{ ft}$$

From Eq. (1)

$$\frac{1}{2}[(8)^2 + (49.356)^2]^{-1/2}\, 2(49.356)(\dot{x}_B) + 6 = 0$$

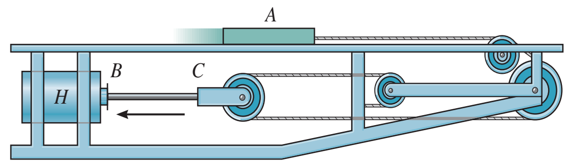
$$\dot{x}_B = -6.0783 = 6.08 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Ans:

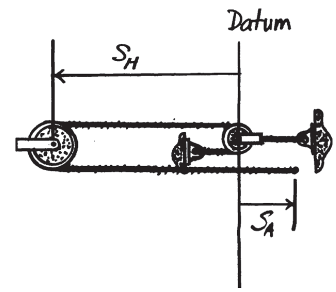
$\dot{x}_B = 6.08 \text{ ft/s} \leftarrow$

12–213.

If the hydraulic cylinder H draws in rod BC at 2 ft/s, determine the speed of slider A.



SOLUTION



$2s_H + s_A = l$

$2v_H = -v_A$

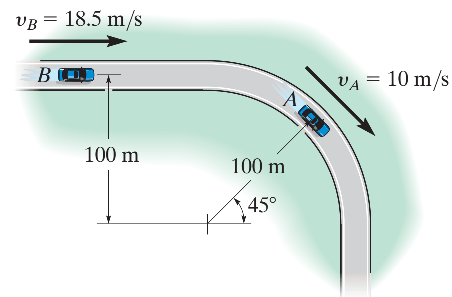
$2(2) = -v_A$

$v_A = -4 \text{ ft/s} = 4 \text{ ft/s} \leftarrow$ **Ans.**

Ans:
$v_A = 4 \text{ ft/s}$

12–214.

At the instant shown, the car at A is traveling at 10 m/s around the curve while increasing its speed at 5 m/s^2. The car at B is traveling at 18.5 m/s along the straightaway and increasing its speed at 2 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



SOLUTION

$$v_A = 10 \cos 45°\mathbf{i} - 10 \sin 45°\mathbf{j} = \{7.071\mathbf{i} - 7.071\mathbf{j}\} \text{ m/s}$$

$$v_B = \{18.5\mathbf{i}\} \text{ m/s}$$

$$v_{A/B} = v_A - v_B$$

$$= (7.071\mathbf{i} - 7.071\mathbf{j}) - 18.5\mathbf{i} = \{-11.429\mathbf{i} - 7.071\mathbf{j}\} \text{ m/s}$$

$$v_{A/B} = \sqrt{(-11.429)^2 + (-7.071)^2} = 13.4 \text{ m/s}$$ **Ans.**

$$\theta = \tan^{-1}\frac{7.071}{11.429} = 31.7°$$ **Ans.**

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{10^2}{100} = 1 \text{ m/s}^2 \qquad (a_A)_t = 5 \text{ m/s}^2$$

$$\mathbf{a}_A = (5 \cos 45° - 1 \cos 45°)\mathbf{i} + (-1 \sin 45° - 5 \sin 45°)\mathbf{j}$$

$$= \{2.828\mathbf{i} - 4.243\mathbf{j}\} \text{ m/s}^2$$

$$\mathbf{a}_B = \{2\mathbf{i}\} \text{ m/s}^2$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{u}_B$$

$$= (2.828\mathbf{i} - 4.243\mathbf{j}) - 2\mathbf{i} = \{0.828\mathbf{i} - 4.24\mathbf{j}\} \text{ m/s}^2$$

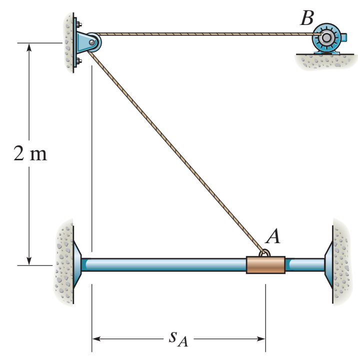
$$a_{A/B} = \sqrt{0.828^2 + (-4.243)^2} = 4.32 \text{ m/s}^2$$ **Ans.**

$$\theta = \tan^{-1}\frac{4.243}{0.828} = 79.0°$$ **Ans.**

Ans:
$v_{A/B} = 13.4$ m/s
$\theta_v = 31.7°$
$a_{A/B} = 4.32$ m/s^2
$\theta_a = 79.0°$

12–215.

The motor draws in the cord at B with an acceleration of $a_B = 2\ \text{m/s}^2$. When $s_A = 1.5$ m, $v_B = 6$ m/s. Determine the velocity and acceleration of the collar at this instant.



SOLUTION

Position Coordinates. The position of collar A and point B are specified by s_A and s_C respectively as shown in Fig. a. We can relate these two position coordinates by considering the length of the cable, which is constant.

$$s_B + \sqrt{s_A^2 + 2^2} = l$$

$$s_B = l - \sqrt{s_A^2 + 4} \qquad \textbf{(1)}$$

Velocity. Taking the time derivative of Eq. (1),

$$\frac{ds_B}{dt} = 0 - \frac{1}{2}\left(s_A^2 + 4\right)^{-1/2}\left(2s_A\frac{ds_A}{dt}\right)$$

$$\frac{ds_B}{dt} = -\frac{s_A}{\sqrt{s_A^2 + 4}}\frac{ds_A}{dt}$$

However, $\dfrac{ds_B}{dt} = v_B$ and $\dfrac{ds_A}{dt} = v_A$. Then this equation becomes

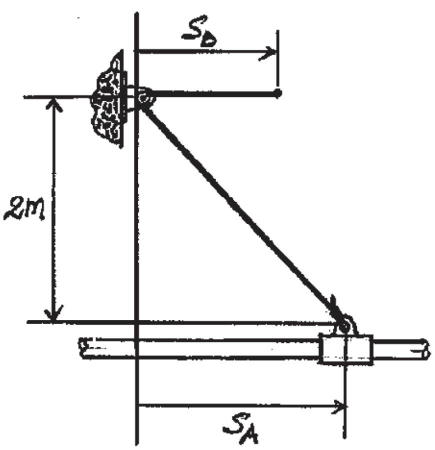
$$v_B = -\frac{s_A}{\sqrt{s_A^2 + 4}}v_A \qquad \textbf{(2)}$$

At the instant $s_A = 1.5$ m, $v_B = +6$ m/s. v_B is positive since it is directed toward the positive sense of s_B.

$$6 = -\frac{1.5}{\sqrt{1.5^2 + 4}}v_A$$

$$v_A = -10.0\ \text{m/s} = 10.0\ \text{m/s} \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that v_A is directed toward the negative sense of s_A.



(a)

Acceleration. Taking the time derivative of Eq. (2),

$$\frac{dv_B}{dt} = -\left[s_A\left(-\frac{1}{2}\right)\left(s_A^2 + 4\right)^{-3/2}\left(2s_A\frac{ds_A}{dt}\right) + \left(s_A^2 + 4\right)^{-1/2}\frac{ds_A}{dt}\right]$$

$$v_A - s_A\left(s_A^2 + 4\right)^{-1/2}\frac{dv_A}{dt}$$

However, $\dfrac{dv_B}{dt} = a_B$, $\dfrac{dv_A}{dt} = a_A$ and $\dfrac{ds_A}{dt} = v_A$. Then

$$a_B = \frac{s_A^2 v_A^2}{\left(s_A^2 + 4\right)^{3/2}} - \frac{v_A^2}{\left(s_A^2 + 4\right)^{1/2}} - \frac{a_A s_A}{\left(s_A^2 + 4\right)^{1/2}}$$

$$a_B = -\frac{4v_A^2 + a_A s_A\left(s_A^2 + 4\right)}{\left(s_A^2 + 4\right)^{3/2}}$$

At the instant $s_A = 1.5$ m, $a_B = +2\ \text{m/s}^2$. $\mathbf{a}_B$ is positive since it is directed toward the positive sense of s_B. Also, $v_A = -10.0$ m/s. Then

$$2 = -\left[\frac{4(-10.0)^2 + a_A(1.5)(1.5^2 + 4)}{(1.5^2 + 4)^{3/2}}\right]$$

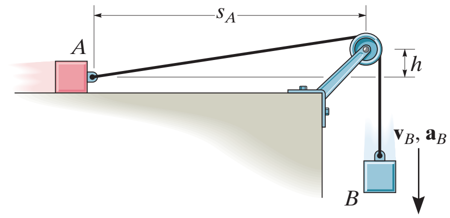
$$a_A = -46.0\ \text{m/s}^2 = 46.0\ \text{m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{a}_A$ is directed toward the negative sense of s_A.

Ans:
$v_A = 10.0\ \text{m/s} \leftarrow$
$a_A = 46.0\ \text{m/s}^2 \leftarrow$

***12–216.**

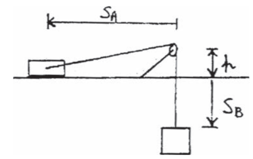
If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.



SOLUTION

$$l = s_B + \sqrt{s_B^2 + h^2}$$

$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2} 2 s_A \dot{s}_A$$



$$v_A = \dot{s}_A = \frac{-\dot{s}_B(s_A^2 + h^2)^{1/2}}{s_A}$$

$$v_A = -v_B \left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} \qquad \textbf{Ans.}$$

$$a_A = \dot{v}_A = -\dot{v}_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2} - v_B\left(\frac{1}{2}\right)\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{-1/2}(h^2)(-2)(s_A)^{-3}\dot{s}_A$$

$$a_A = -a_B\left(1+\left(\frac{h}{s_B}\right)^2\right)^{1/2} + \frac{v_A v_B h^2}{s_A^3}\left(1+\left(\frac{h}{s_A}\right)^2\right)^{-1/2} \qquad \textbf{Ans.}$$

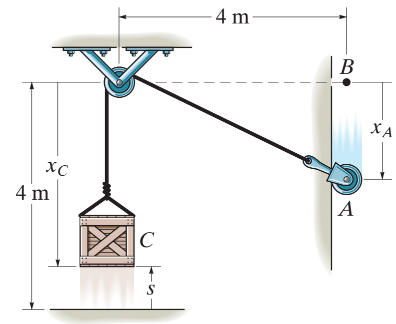
Ans:

$$v_A = -v_B\left(1+\left(\frac{h}{s_A}\right)^2\right)^{1/2}$$

$$a_A = -a_B\left(1+\left(\frac{h}{s_B}\right)^2\right)^{1/2} + \frac{v_A v_B h^2}{s_A^3}\left(1+\left(\frac{h}{S_A}\right)^2\right)^{-1/2}$$

12–217.

The crate C is being lifted by moving the roller at A downward with a constant speed of $v_A = 2\text{ m/s}$ along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1\text{ m}$. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.

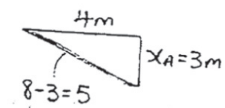


SOLUTION

$$x_C + \sqrt{x_A^2 + (4)^2} = l$$

$$\dot{x}_C + \frac{1}{2}(x_A^2 + 16)^{-1/2}(2x_A)(\dot{x}_A) = 0$$

$$\ddot{x}_C - \frac{1}{2}(x_A^2 + 16)^{-3/2}(2x_A^2)(\dot{x}_A^2) + (x_A^2 + 16)^{-1/2}(\dot{x}_A)^2 + (x_A^2 + 16)^{-1/2}(x_A)(\ddot{x}_A) = 0$$



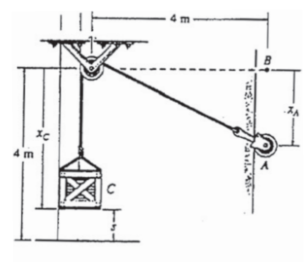
$l = 8\text{ m}$, and when $s = 1\text{ m}$,

$x_C = 3\text{ m}$

$x_A = 3\text{ m}$

$v_A = \dot{x}_A = 2\text{ m/s}$

$a_A = \ddot{x}_A = 0$



Thus,

$v_C + [(3)^2 + 16]^{-1/2}(3)(2) = 0$

$v_C = -1.2\text{ m/s} = 1.2\text{ m/s}\uparrow$ **Ans.**

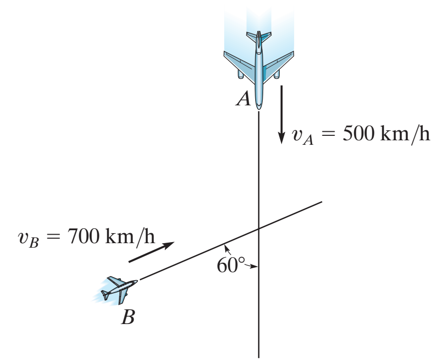
$a_C - [(3)^2 + 16]^{-3/2}(3)^2(2)^2 + [(3)^2 + 16]^{-1/2}(2)^2 + 0 = 0$

$a_C = -0.512\text{ m/s}^2 = 0.512\text{ m/s}^2\uparrow$ **Ans.**

Ans:
$v_C = 1.2\text{ m/s}\uparrow$
$a_C = 0.512\text{ m/s}^2\uparrow$

12–218.

Two planes, A and B, are flying at the same altitude. If their velocities are $v_A = 500$ km/h and $v_B = 700$ km/h such that the angle between their straight-line courses is $\theta = 60°$, determine the velocity of plane B with respect to plane A.



SOLUTION

Relative Velocity. Express $\mathbf{v}_A$ and $\mathbf{v}_B$ in Cartesian vector form,

$$\mathbf{v}_A = \{-500\,\mathbf{j}\}\text{ km/h}$$

$$\mathbf{v}_B = \{700 \sin 60°\mathbf{i} + 700 \cos 60°\mathbf{j}\}\text{ km/h} = \left\{350\sqrt{3}\mathbf{i} + 350\mathbf{j}\right\}\text{ km/h}$$

Applying the relative velocity equation.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$350\sqrt{3}\mathbf{i} + 350\mathbf{j} = -500\mathbf{j} + \mathbf{v}_{B/A}$$

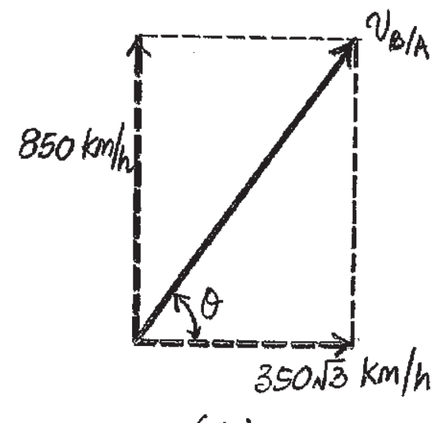
$$\mathbf{v}_{B/A} = \left\{350\sqrt{3}\mathbf{i} + 850\mathbf{j}\right\}\text{km/h}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(350\sqrt{3})^2 + 850^2} = 1044.03\text{ km/h} = 1044\text{ km/h} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. a.

$$\theta = \tan^{-1}\left(\frac{850}{350\sqrt{3}}\right) = 54.50° = 54.5° \measuredangle \qquad \textbf{Ans.}$$



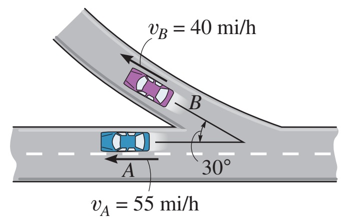
(a)

Ans:

$v_{B/A} = 1044$ km/h
$\theta = 54.5° \measuredangle$

12–219.

At the instant shown, cars A and B are traveling at speeds of 55 mi/h and 40 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A. Car B moves along a curve having a radius of curvature of 0.5 mi.



SOLUTION

$$v_B = -40\cos 30°\mathbf{i} + 40\sin 30°\mathbf{j} = \{-34.64\mathbf{i} + 20\mathbf{j}\}\text{ mi/h}$$

$$v_A = \{-55\mathbf{i}\}\text{ mi/h}$$

$$v_{B/A} = v_B - v_A$$

$$= (-34.64\mathbf{i} + 20\mathbf{j}) - (-55\mathbf{i}) = \{20.36\mathbf{i} + 20\mathbf{j}\}\text{ mi/h}$$

$$v_{B/A} = \sqrt{20.36^2 + 20^2} = 28.5\text{ mi/h} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{20}{20.36} = 44.5° \measuredangle \qquad \textbf{Ans.}$$

$$(a_B)_n = \frac{v_A^2}{\rho} = \frac{40^2}{0.5} = 3200\text{ mi/h}^2 \qquad (a_B)_t = 1200\text{ mi/h}^2$$

$$\mathbf{a}_B = (3200\cos 60° - 1200\cos 30°)\mathbf{i} + (3200\sin 60° + 1200\sin 30°)\mathbf{j}$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\quad\text{mi/h}^2$$

$$\mathbf{a}_A = 0$$

$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$

$$= \{560.77\mathbf{i} + 3371.28\mathbf{j}\} - 0 = \{560.77\mathbf{i} + 3371.28\mathbf{j}\}\quad\text{mi/h}^2$$

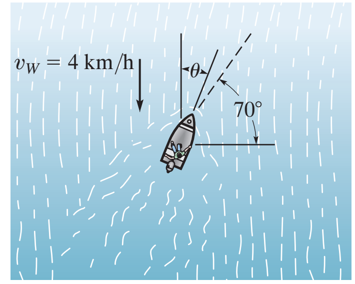
$$a_{B/A} = \sqrt{(560.77)^2 + (3371.28)^2} = 3418\text{ mi/h}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{3371.28}{560.77} = 80.6° \measuredangle \qquad \textbf{Ans.}$$

Ans:
$v_{B/A} = 28.5$ mi/h
$\theta_v = 44.5° \measuredangle$
$a_{B/A} = 3418$ mi/h^2
$\theta_a = 80.6° \measuredangle$

***12–220.**

The boat can travel with a speed of 16 km/h in still water. The point of destination is located along the dashed line. If the water is moving at 4 km/h, determine the bearing angle θ at which the boat must travel to stay on course.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_W + \mathbf{v}_{B/W}$

$v_B \cos 70°\mathbf{i} + v_B \sin 70°\mathbf{j} = -4\mathbf{j} + 16 \sin \theta\mathbf{i} + 16 \cos \theta\mathbf{j}$

$(\overset{+}{\rightarrow})$ $\quad v_B \cos 70° = 0 + 16 \sin \theta$

$(+\uparrow)$ $\quad v_B \sin 70° = -4 + 16 \cos \theta$

$2.748 \sin \theta - \cos \theta + 0.25 = 0$

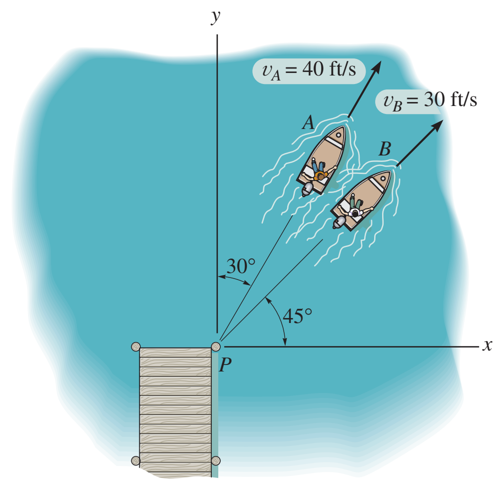
Solving,

$\theta = 15.1°$ **Ans.**

Ans:
$\theta = 15.1°$

12–221.

Two boats leave the pier P at the same time and travel in the directions shown. If $v_A = 40$ ft/s and $v_B = 30$ ft/s, determine the velocity of boat A relative to boat B. How long after leaving the pier will the boats be 1500 ft apart?



SOLUTION

Relative Velocity:

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$40 \sin 30°\mathbf{i} + 40 \cos 30°\mathbf{j} = 30 \cos 45°\mathbf{i} + 30 \sin 45°\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-1.213\mathbf{i} + 13.43\mathbf{j}\} \text{ ft/s}$$

Thus, the magnitude of the relative velocity $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-1.213)^2 + 13.43^2} = 13.48 \text{ ft/s} = 13.5 \text{ ft/s} \qquad \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\frac{13.43}{1.213} = 84.8° \qquad \textbf{Ans.}$$

One can obtained the time t required for boats A and B to be 1500 ft apart by noting that boat B is at rest and boat A travels at the relative speed $v_{A/B} = 13.48$ ft/s for a distance of 1500 ft. Thus

$$t = \frac{1500}{v_{A/B}} = \frac{1500}{13.48} = 111.26 \text{ s} = 1.85 \text{ min} \qquad \textbf{Ans.}$$

Ans:

$v_B = 13.5$ ft/s
$\theta = 84.8°$
$t = 1.85$ min

12–222.

A car is traveling north along a straight road at 50 km/h. An instrument in the car indicates that the wind is coming from the east. If the car's speed is 80 km/h, the instrument indicates that the wind is coming from the northeast. Determine the speed and direction of the wind.

SOLUTION

Solution I

Vector Analysis: For the first case, the velocity of the car and the velocity of the wind relative to the car expressed in Cartesian vector form are $\mathbf{v}_c = [50\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_1\,\mathbf{i}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 50\mathbf{j} + (v_{w/c})_1\,\mathbf{i}$$

$$\mathbf{v}_w = (v_{w/c})_1\mathbf{i} + 50\mathbf{j} \qquad \textbf{(1)}$$

For the second case, $v_C = [80\mathbf{j}]$ km/h and $\mathbf{v}_{W/C} = (v_{W/C})_2 \cos 45°\mathbf{i} + (v_{W/C})_2 \sin 45°\,\mathbf{j}$. Applying the relative velocity equation, we have

$$\mathbf{v}_w = \mathbf{v}_c + \mathbf{v}_{w/c}$$

$$\mathbf{v}_w = 80\mathbf{j} + (v_{w/c})_2 \cos 45°\mathbf{i} + (v_{w/c})_2 \sin 45°\,\mathbf{j}$$

$$\mathbf{v}_w = (v_{w/c})_2 \cos 45°\,\mathbf{i} + \left[80 + (v_{w/c})_2 \sin 45°\right]\mathbf{j} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) and then the **i** and **j** components,

$$(v_{w/c})_1 = (v_{w/c})_2 \cos 45° \qquad \textbf{(3)}$$

$$50 = 80 + (v_{w/c})_2 \sin 45° \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$(v_{w/c})_2 = -42.43 \text{ km/h} \qquad (v_{w/c})_1 = -30 \text{ km/h}$$

Substituting the result of $(v_{w/c})_1$ into Eq. (1),

$$\mathbf{v}_w = [-30\mathbf{i} + 50\mathbf{j}] \text{ km/h}$$

Thus, the magnitude of $\mathbf{v}_W$ is

$$v_w = \sqrt{(-30)^2 + 50^2} = 58.3 \text{ km/h} \qquad \textbf{Ans.}$$

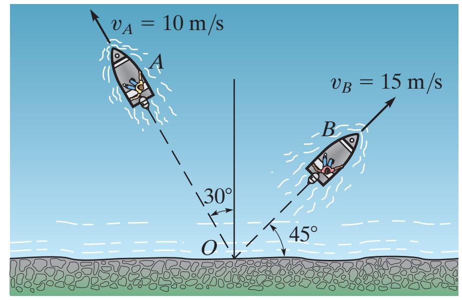
and the directional angle θ that $\mathbf{v}_W$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{50}{30}\right) = 59.0° \qquad \textbf{Ans.}$$

Ans:
$v_w = 58.3$ km/h
$\theta = 59.0°$

12–223.

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 10$ m/s and $v_B = 15$ m/s, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 600 m apart?



SOLUTION

Relative Velocity. The velocity triangle shown in Fig. *a* is drawn based on the relative velocity equation $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$. Using the cosine law,

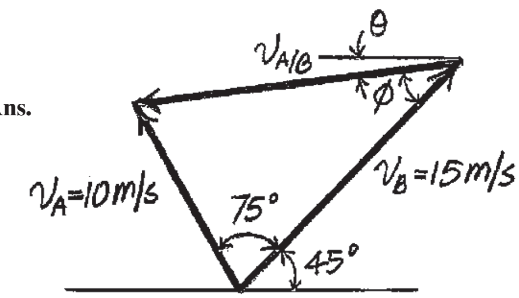
$$v_{A/B} = \sqrt{10^2 + 15^2 - 2(10)(15)\cos 75^\circ} = 15.73 \text{ m/s} = 15.7 \text{ m/s} \qquad \textbf{Ans.}$$

Then, the sine law gives

$$\frac{\sin\phi}{10} = \frac{\sin 75^\circ}{15.73} \qquad \phi = 37.89^\circ$$

The direction of $\mathbf{v}_{A/B}$ is defined by

$$\theta = 45^\circ - \phi = 45^\circ - 37.89^\circ = 7.11^\circ \qquad \textbf{Ans.}$$



(a)

Alternatively, we can express $\mathbf{v}_A$ and $\mathbf{v}_B$ in Cartesian vector form

$$\mathbf{v}_A = \{-10\sin 30^\circ\mathbf{i} + 10\cos 30^\circ\mathbf{j}\} \text{ m/s} = \{-5.00\mathbf{i} + 5\sqrt{3}\mathbf{j}\} \text{ m/s}$$

$$\mathbf{v}_B = \{15\cos 45^\circ\mathbf{i} + 15\sin 45^\circ\mathbf{j}\} \text{ m/s} = \{7.5\sqrt{2}\mathbf{i} + 7.5\sqrt{2}\mathbf{j}\} \text{ m/s.}$$

Applying the relative velocity equation

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-500\mathbf{i} + 5\sqrt{3}\mathbf{j} = 7.5\sqrt{2}\mathbf{i} + 7.5\sqrt{2}\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-15.61\mathbf{i} - 1.946\mathbf{j}\} \text{ m/s}$$

Thus the magnitude of $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-15.61)^2 + (-1.946)^2} = 15.73 \text{ m/s} = 15.7 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. *b*,

$$\theta = \tan^{-1}\left(\frac{1.946}{15.61}\right) = 7.1088^\circ = 7.11^\circ \qquad \textbf{Ans.}$$

(b)

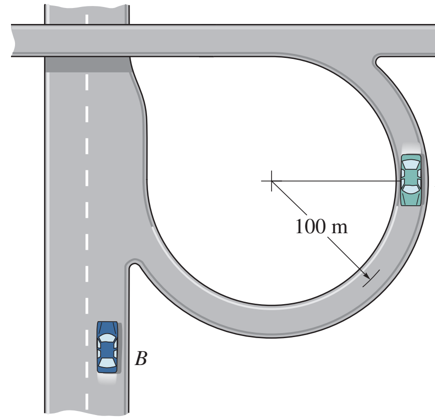
Here $s_{A/B} = 600$ m. Thus

$$t = \frac{s_{A/B}}{v_{A/B}} = \frac{600}{15.73} = 38.15 \text{ s} = 38.1 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_{A/B} = 15.7$ m/s
$\theta = 7.11^\circ$
$t = 38.1$ s

***12–224.**

At the instant shown, car A has a speed of 20 km/h, which is being increased at the rate of 300 km/h^2 as the car enters an expressway. At the same instant, car B is decelerating at 250 km/h^2 while traveling forward at 100 km/h. Determine the velocity and acceleration of A with respect to B.



SOLUTION

$\mathbf{v}_A = \{-20\mathbf{j}\}$ km/h $\qquad \mathbf{v}_B = \{100\mathbf{j}\}$ km/h

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$= (-20\mathbf{j} - 100\mathbf{j}) = \{-120\mathbf{j}\} \text{ km/h}$$

$v_{A/B} = 120$ km/h $\downarrow$ **Ans.**

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{20^2}{0.1} = 4000 \text{ km/h}^2 \qquad (a_A)_t = 300 \text{ km/h}^2$$

$$\mathbf{a}_A = -4000\mathbf{i} + (-300\mathbf{j})$$

$$= \{-4000\mathbf{i} - 300\mathbf{j}\} \text{ km/h}^2$$

$$\mathbf{a}_B = \{-250\mathbf{j}\} \text{ km/h}^2$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B$$

$$= (-4000\mathbf{i} - 300\mathbf{j}) - (-250\mathbf{j}) = \{-4000\mathbf{i} - 50\mathbf{j}\} \text{ km/h}^2$$

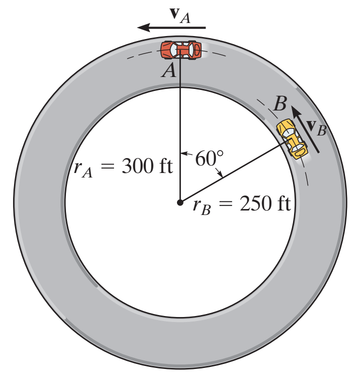
$a_{A/B} = \sqrt{(-4000)^2 + (-50)^2} = 4000$ km/h^2 **Ans.**

$\theta = \tan^{-1}\dfrac{50}{4000} = 0.716°$ **Ans.**

Ans:
$v_{A/B} = 120$ km/h $\downarrow$
$a_{A/B} = 4000$ km/h^2
$\theta = 0.716°$

12–225.

Cars A and B are traveling around the circular race track. At the instant shown, A has a speed of 90 ft/s and is increasing its speed at the rate of 15 ft/s^2, whereas B has a speed of 105 ft/s and is decreasing its speed at 25 ft/s^2. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



SOLUTION

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-90\mathbf{i} = -105 \sin 30°\,\mathbf{i} + 105 \cos 30°\,\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-37.5\mathbf{i} - 90.93\mathbf{j}\} \text{ ft/s}$$

$$v_{A/B} = \sqrt{(-37.5)^2 + (-90.93)^2} = 98.4 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{90.93}{37.5}\right) = 67.6° \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

$$-15\mathbf{i} - \frac{(90)^2}{300}\mathbf{j} = 25 \cos 60°\mathbf{i} - 25 \sin 60°\mathbf{j} - 44.1 \sin 60°\mathbf{i} - 44.1 \cos 60°\mathbf{j} + \mathbf{a}_{A/B}$$

$$\mathbf{a}_{A/B} = \{10.69\mathbf{i} + 16.70\mathbf{j}\} \text{ ft/s}^2$$

$$a_{A/B} = \sqrt{(10.69)^2 + (16.70)^2} = 19.8 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

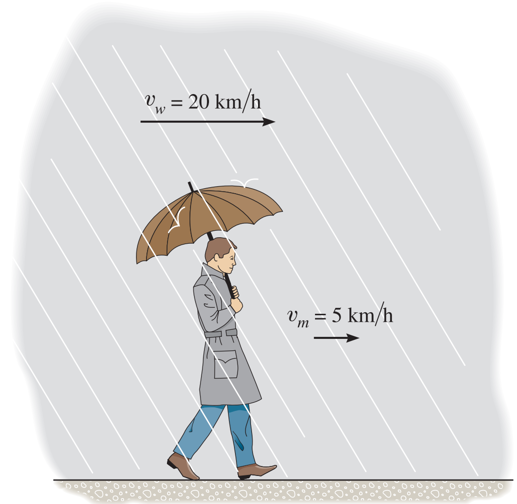
$$\theta = \tan^{-1}\left(\frac{16.70}{10.69}\right) = 57.4° \qquad \textbf{Ans.}$$

Ans:

$v_{A/B} = 98.4$ ft/s
$\theta_v = 67.6°$
$a_{A/B} = 19.8$ ft/s^2
$\theta_a = 57.4°$

12–226.

A man walks at 5 km/h in the direction of a 20-km/h wind. If raindrops fall vertically at 7 km/h in *still air*, determine the direction in which the drops appear to fall with respect to the man.



SOLUTION

Relative Velocity: The velocity of the rain must be determined first. Applying Eq. 12–34 gives

$$\mathbf{v}_r = \mathbf{v}_w + \mathbf{v}_{r/w} = 20\,\mathbf{i} + (-7\,\mathbf{j}) = \{20\,\mathbf{i} - 7\,\mathbf{j}\}\ \text{km/h}$$

Thus, the relatives velocity of the rain with respect to the man is

$$\mathbf{v}_r = \mathbf{v}_m + \mathbf{v}_{r/m}$$

$$20\,\mathbf{i} - 7\,\mathbf{j} = 5\,\mathbf{i} + \mathbf{v}_{r/m}$$

$$\mathbf{v}_{r/m} = \{15\,\mathbf{i} - 7\,\mathbf{j}\}\ \text{km/h}$$

The magnitude of the relative velocity $\mathbf{v}_{r/m}$ is given by

$$v_{r/m} = \sqrt{15^2 + (-7)^2} = 16.6\ \text{km/h}$$ **Ans.**

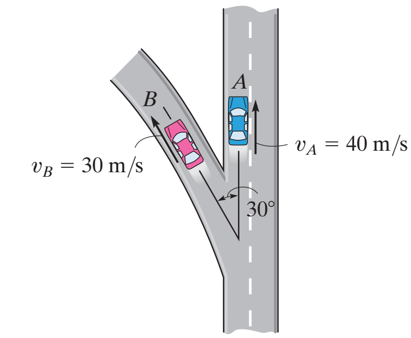
And its direction is given by

$$\theta = \tan^{-1}\frac{7}{15} = 25.0^\circ$$ **Ans.**

Ans:
$v_{r/m} = 16.6$ km/h
$\theta = 25.0^\circ$

12–227.

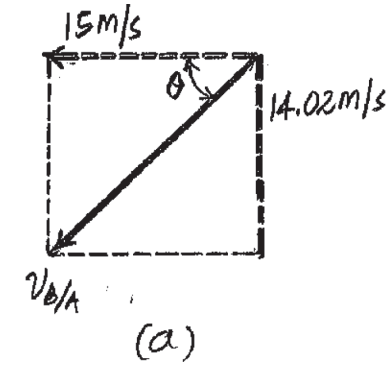
At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s, respectively. If B is increasing its velocity by 2 m/s^2, while A maintains a constant velocity, determine the velocity and acceleration of B with respect to A. The radius of curvature at B is $\rho_B = 200$ m.



SOLUTION

Relative velocity. Express $\mathbf{v}_A$ and $\mathbf{v}_B$ as Cartesian vectors.

$$\mathbf{v}_A = \{40\mathbf{j}\}\ \text{m/s} \quad \mathbf{v}_B = \{-30 \sin 30°\mathbf{i} + 30 \cos 30°\mathbf{j}\}\ \text{m/s} = \{-15\mathbf{i} + 15\sqrt{3}\mathbf{j}\}\ \text{m/s}$$



Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-15\mathbf{i} + 15\sqrt{3}\mathbf{j} = 40\mathbf{j} + \mathbf{v}_{B/A}$$

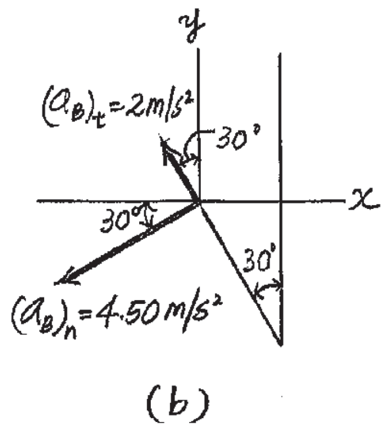
$$\mathbf{v}_{B/A} = \{-15\mathbf{i} - 14.02\mathbf{j}\}\ \text{m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(-15)^2 + (-14.02)^2} = 20.53\ \text{m/s} = 20.5\ \text{m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. *a*

$$\theta = \tan^{-1}\left(\frac{14.02}{15}\right) = 43.06° = 43.1° \measuredangle \qquad \textbf{Ans.}$$



Relative Acceleration. Here, $(a_B)_t = 2\ \text{m/s}^2$ and $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{30^2}{200} = 4.50\ \text{m/s}^2$ and their directions are shown in Fig. *b*. Then, express $\mathbf{a}_B$ as a Cartesian vector,

$$\mathbf{a}_B = (-2 \sin 30° - 4.50 \cos 30°)\mathbf{i} + (2 \cos 30° - 4.50 \sin 30°)\mathbf{j}$$

$$= \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\}\ \text{m/s}^2$$

Applying the relative acceleration equation with $\mathbf{a}_A = \mathbf{0}$,

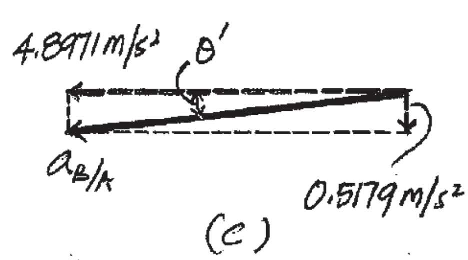
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$-4.8971\mathbf{i} - 0.5179\mathbf{j} = \mathbf{0} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{-4.8971\mathbf{i} - 0.5179\mathbf{j}\}\ \text{m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/A}$ is

$$\mathbf{a}_{B/A} = \sqrt{(-4.8971)^2 + (-0.5179)^2} = 4.9244\ \text{m/s}^2 = 4.92\ \text{m/s}^2 \qquad \textbf{Ans.}$$



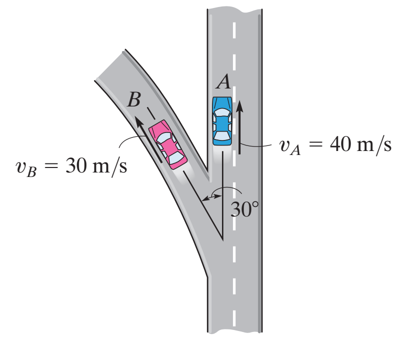
And its direction is defined by angle θ', Fig. *c*,

$$\theta' = \tan^{-1}\left(\frac{0.5179}{4.8971}\right) = 6.038° = 6.04° \measuredangle \qquad \textbf{Ans.}$$

Ans:
$v_{B/A} = 20.5$ m/s
$\theta_v = 43.1° \measuredangle$
$a_{B/A} = 4.92$ m/s^2
$\theta_a = 6.04° \measuredangle$

***12–228.**

At the instant shown, cars A and B are traveling at velocities of 40 m/s and 30 m/s, respectively. If A is increasing its speed at 4 m/s^2, whereas the speed of B is decreasing at 3 m/s^2, determine the velocity and acceleration of B with respect to A. The radius of curvature at B is $\rho_B = 200$ m.



SOLUTION

Relative velocity. Express v_A and v_B as Cartesian vector.

$\mathbf{v}_A = \{40\mathbf{j}\}$ m/s $\quad \mathbf{v}_B = \{-30 \sin 30°\mathbf{i} + 30 \cos 30°\mathbf{j}\}$ m/s $= \{-15\mathbf{i} + 15\sqrt{3}\mathbf{j}\}$ m/s

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$-15\mathbf{i} + 15\sqrt{3}\mathbf{j} = 40\mathbf{j} + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \{-15\mathbf{i} - 14.02\} \text{ m/s}$$

Thus the magnitude of $\mathbf{v}_{B/A}$ is

$$v_{B/A} = \sqrt{(-15)^2 + (-14.02)^2} = 20.53 \text{ m/s} = 20.5 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig a.

$$\theta = \tan^{-1}\left(\frac{14.02}{15}\right) = 43.06° = 43.1° \qquad \textbf{Ans.}$$

Relative Acceleration. Here $(a_B)_t = 3$ m/s^2 and $(a_B)_n = \dfrac{v_B^2}{\rho} = \dfrac{30^2}{200} = 4.5$ m/s^2 and their directions are shown in Fig. b. Then express $\mathbf{a}_B$ as a Cartesian vector,

$$\mathbf{a}_B = (3 \sin 30° - 4.50 \cos 30°)\mathbf{i} + (-3 \cos 30° - 4.50 \sin 30°)\mathbf{j}$$

$$= \{-2.3971\mathbf{i} - 4.8481\mathbf{j}\} \text{ m/s}^2$$

Applying the relative acceleration equation with $\mathbf{a}_A = \{4\mathbf{j}\}$ m/s^2,

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$-2.3971\mathbf{i} - 4.8481\mathbf{j} = 4\mathbf{j} + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{B/A} = \{-2.3971\mathbf{i} - 8.8481\mathbf{j}\} \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_{B/A}$ is

$$\mathbf{a}_{B/A} = \sqrt{(-2.3971)^2 + (-8.8481)^2} = 9.167 \text{ m/s}^2 = 9.17 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by angle θ', Fig. c

$$\theta' = \tan^{-1}\left(\frac{8.8481}{2.3971}\right) = 74.84° = 74.8° \qquad \textbf{Ans.}$$

Ans:

$v_{B/A} = 20.5$ m/s

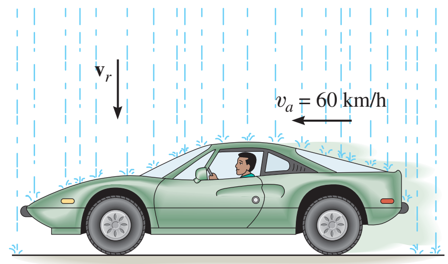
$\theta = 43.1°$

$\mathbf{a}_{B/A} = 9.17$ m/s^2

$\theta' = 74.8°$

12–229.

A passenger in an automobile observes that raindrops make an angle of 30° with the horizontal as the auto travels forward with a speed of 60 km/h. Compute the terminal (constant) velocity $\mathbf{v}_r$ of the rain if it is assumed to fall vertically.



SOLUTION

$v_r = v_a + v_{r/a}$

$-v_r\mathbf{j} = -60\mathbf{i} + v_{r/a}\cos 30°\mathbf{i} - v_{r/a}\sin 30°\mathbf{j}$

$(\overset{+}{\rightarrow})\qquad 0 = -60 + v_{r/a}\cos 30°$

$(+\uparrow)\qquad -v_r = 0 - v_{r/a}\sin 30°$

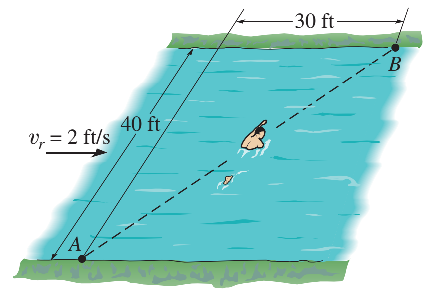
$v_{r/a} = 69.3$ km/h

$v_r = 34.6$ km/h **Ans.**

Ans:
$v_r = 34.6$ km/h$\downarrow$

12–230.

A man can swim at 4 ft/s in still water. He wishes to cross the 40-ft-wide river to point B, 30 ft downstream. If the river flows with a velocity of 2 ft/s, determine the speed of the man and the time needed to make the crossing. *Note*: While in the water he must not direct himself toward point B to reach this point. Why?

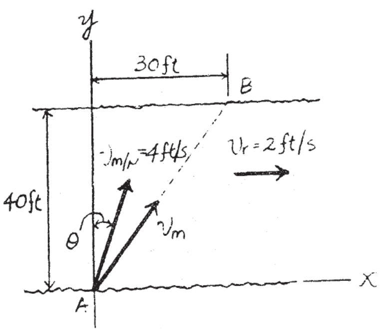


SOLUTION

Relative Velocity:

$$v_m = v_r + v_{m/r}$$

$$\frac{3}{5}v_m\mathbf{i} + \frac{4}{5}v_m\mathbf{j} = 2\mathbf{i} + 4\sin\theta\mathbf{i} + 4\cos\theta\mathbf{j}$$



Equating the i and j components, we have

$$\frac{3}{5}v_m = 2 + 4\sin\theta \qquad \textbf{(1)}$$

$$\frac{4}{5}v_m = 4\cos\theta \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\theta = 13.29°$$

$$v_m = 4.866 \text{ ft/s} = 4.87 \text{ ft/s} \qquad \textbf{Ans.}$$

Thus, the time t required by the boat to travel from points A to B is

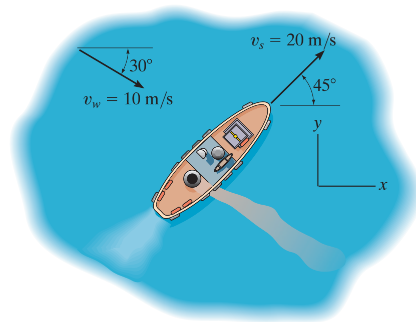
$$t = \frac{s_{AB}}{v_b} = \frac{\sqrt{40^2 + 30^2}}{4.866} = 10.3 \text{ s} \qquad \textbf{Ans.}$$

In order for the man to reached point B, the man has to direct himself at an angle $\theta = 13.3°$ with y axis.

Ans:
$v_m = 4.87$ ft/s
$t = 10.3$ s

12–231.

The ship travels at a constant speed of $v_s = 20$ m/s and the wind is blowing at a speed of $v_w = 10$ m/s, as shown. Determine the magnitude and direction of the horizontal component of velocity of the smoke coming from the smoke stack as it appears to a passenger on the ship.



SOLUTION

Solution I

Vector Analysis: The velocity of the smoke as observed from the ship is equal to the velocity of the wind relative to the ship. Here, the velocity of the ship and wind expressed in Cartesian vector form are $\mathbf{v}_s = [20\cos 45°\,\mathbf{i} + 20\sin 45°\,\mathbf{j}]$ m/s $= [14.14\mathbf{i} + 14.14\mathbf{j}]$ m/s and $\mathbf{v}_w = [10\cos 30°\,\mathbf{i} - 10\sin 30°\,\mathbf{j}] = [8.660\mathbf{i} - 5\mathbf{j}]$ m/s. Applying the relative velocity equation,

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$

$$8.660\mathbf{i} - 5\mathbf{j} = 14.14\mathbf{i} + 14.14\mathbf{j} + \mathbf{v}_{w/s}$$

$$\mathbf{v}_{w/s} = [-5.482\mathbf{i} - 19.14\mathbf{j}] \text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{w/s}$ is given by

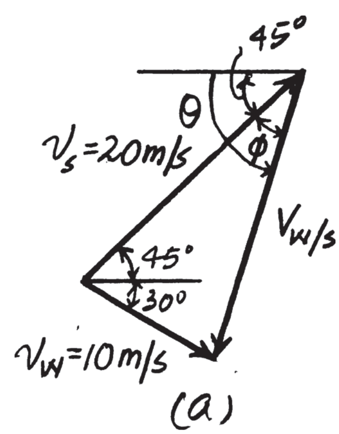
$$v_w = \sqrt{(-5.482)^2 + (-19.14)^2} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

and the direction angle θ that $\mathbf{v}_{w/s}$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{19.14}{5.482}\right) = 74.0° \;\measuredangle \qquad \textbf{Ans.}$$

Solution II

Scalar Analysis: Applying the law of cosines by referring to the velocity diagram shown in Fig. *a*,



$$v_{w/s} = \sqrt{20^2 + 10^2 - 2(20)(10)\cos 75°}$$

$$= 19.91 \text{ m/s} = 19.9 \text{ m/s} \qquad \textbf{Ans.}$$

Using the result of $v_{w/s}$ and applying the law of sines,

$$\frac{\sin\phi}{10} = \frac{\sin 75°}{19.91} \qquad \phi = 29.02°$$

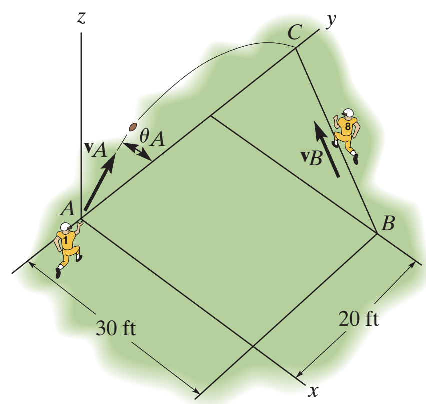
Thus,

$$\theta = 45° + \phi = 74.0° \;\measuredangle \qquad \textbf{Ans.}$$

Ans:
$v_{w/s} = 19.9$ m/s
$\theta = 74.0° \;\measuredangle$

***12–232.**

The football player at A throws the ball in the y–z plane at a speed $v_A = 50$ ft/s and an angle $\theta_A = 60°$ with the horizontal. At the instant the ball is thrown, the player is at B and is running with constant speed along the line BC in order to catch it. Determine this speed, v_B, so that he makes the catch at the same elevation from which the ball was thrown.



SOLUTION

$(\xrightarrow{+})\quad s = s_0 + v_0\,t$

$d_{AC} = 0 + (50\cos 60°)\,t$

$(+\uparrow)\,v = v_0 + a_c\,t$

$-50\sin 60° = 50\sin 60° - 32.2\,t$

$t = 2.690\text{ s}$

$d_{AC} = 67.24\text{ ft}$

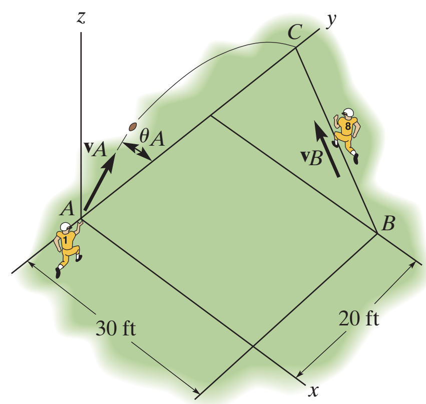
$d_{BC} = \sqrt{(30)^2 + (67.24 - 20)^2} = 55.96\text{ ft}$

$v_B = \dfrac{55.96}{2.690} = 20.8\text{ ft/s}$ **Ans.**

Ans:
$v_B = 20.8\text{ ft/s}$

12–233.

The football player at A throws the ball in the y–z plane with a speed $v_A = 50$ ft/s and an angle $\theta_A = 60°$ with the horizontal. At the instant the ball is thrown, the player is at B and is running at a constant speed of $v_B = 23$ ft/s along the line BC. Determine if he can reach point C, which has the same elevation as A, before the ball gets there.



SOLUTION

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$d_{AC} = 0 + (50 \cos 60°)\, t$

$(+\uparrow)\quad v = v_0 + a_c t$

$-50 \sin 60° = 50 \sin 60° - 32.2\, t$

$t = 2.690$ s

$d_{AC} = 67.24$ ft

$d_{BC} = \sqrt{(30)^2 + (67.24 - 20)^2} = 55.96$ ft

$$v_B = \frac{d_{BC}}{t} = \frac{55.96}{(2.690)} = 20.8 \text{ ft/s}$$

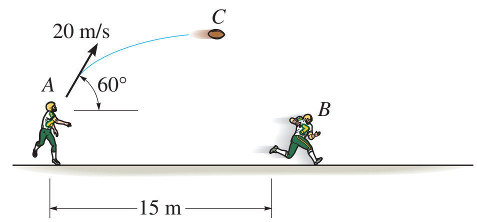
Since $v_B = 20.8 \text{ ft/s} < (v_B)_{\max} = 23 \text{ ft/s}$

Yes, he can catch the ball. **Ans.**

Ans:
Yes, he can catch the ball.

12–234.

At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.



SOLUTION

Ball:

$(\xrightarrow{+})\ s = s_0 + v_0 t$

$s_C = 0 + 20 \cos 60^\circ\, t$

$(+\uparrow)\quad v = v_0 + a_c t$

$-20 \sin 60^\circ = 20 \sin 60^\circ - 9.81\, t$

$t = 3.53$ s

$s_C = 35.31$ m

Player B:

$(\xrightarrow{+})\ s_B = s_0 + \nu_B t$

Require,

$35.31 = 15 + v_B(3.53)$

$v_B = 5.75$ m/s **Ans.**

At the time of the catch

$(v_C)_x = 20 \cos 60^\circ = 10 \text{ m/s} \rightarrow$

$(v_C)_y = 20 \sin 60^\circ = 17.32 \text{ m/s} \downarrow$

$v_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$10\mathbf{i} - 17.32\mathbf{j} = 5.751\mathbf{i} + (v_{C/B})_x \mathbf{i} + (v_{C/B})_y \mathbf{j}$

$(\xrightarrow{+})\quad 10 = 5.75 + (v_{C/B})_x$

$(+\uparrow)\quad -17.32 = (v_{C/B})_y$

$(v_{C/B})_x = 4.25 \text{ m/s} \rightarrow$

$(v_{C/B})_y = 17.32 \text{ m/s} \downarrow$

$v_{C/B} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{17.32}{4.25}\right) = 76.2^\circ$ ⦪ **Ans.**

$a_C = \mathbf{a}_B + \mathbf{a}_{C/B}$

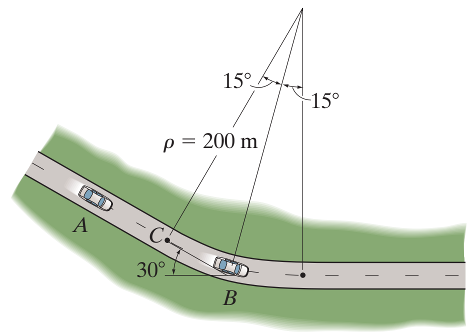
$-9.81\,\mathbf{j} = 0 + \mathbf{a}_{C/B}$

$a_{C/B} = 9.81 \text{ m/s}^2 \downarrow$ **Ans.**

Ans:
$v_B = 5.75$ m/s
$v_{C/B} = 17.8$ m/s
$\theta = 76.2^\circ$ ⦪
$a_{C/B} = 9.81 \text{ m/s}^2 \downarrow$

12–235.

At the instant shown, car A travels along the straight portion of the road with a speed of 25 m/s. At this same instant car B travels along the circular portion of the road with a speed of 15 m/s. Determine the velocity of car B relative to car A.

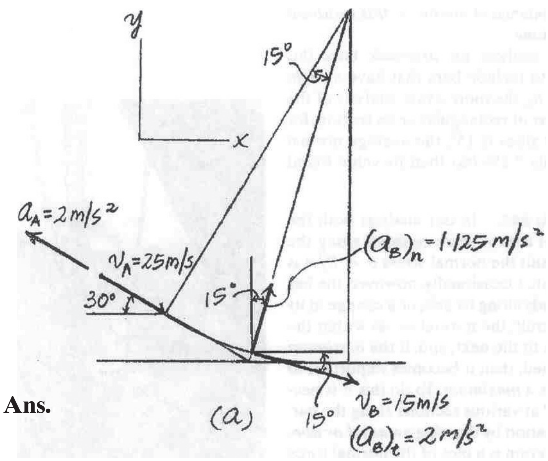


SOLUTION

Velocity: Referring to Fig. a, the velocity of cars A and B expressed in Cartesian vector form are

$$\mathbf{v}_A = [25\cos 30^\circ\,\mathbf{i} - 25\sin 30^\circ\,\mathbf{j}]\text{ m/s} = [21.65\mathbf{i} - 12.5\mathbf{j}]\text{ m/s}$$

$$\mathbf{v}_B = [15\cos 15^\circ\,\mathbf{i} - 15\sin 15^\circ\,\mathbf{j}]\text{ m/s} = [14.49\mathbf{i} - 3.882\mathbf{j}]\text{ m/s}$$



Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$14.49\mathbf{i} - 3.882\mathbf{j} = 21.65\mathbf{i} - 12.5\mathbf{j} + \mathbf{v}_{B/A}$$

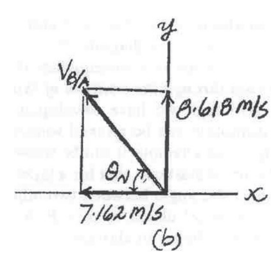
$$\mathbf{v}_{B/A} = [-7.162\mathbf{i} + 8.618\mathbf{j}]\text{ m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/A}$ is given by

$$v_{B/A} = \sqrt{(-7.162)^2 + 8.618^2} = 11.2\text{ m/s} \qquad \textbf{Ans.}$$

The direction angle θ_v of $\mathbf{v}_{B/A}$ measured down from the negative x axis, Fig. b is

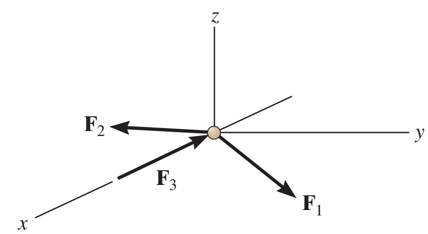
$$\theta_v = \tan^{-1}\left(\frac{8.618}{7.162}\right) = 50.3^\circ \qquad \textbf{Ans.}$$



Ans:
$v_{B/A} = 11.2\text{ m/s}$
$\theta = 50.3^\circ$

13–1.

The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{-2t\mathbf{i}\}$ lb, where t is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.



SOLUTION

$$\Sigma\mathbf{F} = m\mathbf{a}; \quad (2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}) + (t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}) - 2t\mathbf{i} - 6\mathbf{k} = \left(\frac{6}{32.2}\right)(\mathbf{a}_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

Equating components:

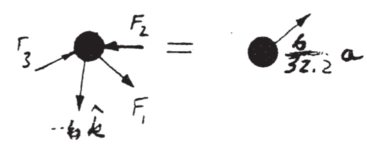
$$\left(\frac{6}{32.2}\right)a_x = t^2 - 2t + 2 \quad \left(\frac{6}{32.2}\right)a_y = -4t + 6 \quad \left(\frac{6}{32.2}\right)a_z = -2t - 7$$

Since $dv = a\,dt$, integrating from $\nu = 0, t = 0$, yields

$$\left(\frac{6}{32.2}\right)v_x = \frac{t^3}{3} - t^2 + 2t \qquad \left(\frac{6}{32.2}\right)v_y = -2t^2 + 6t \qquad \left(\frac{6}{32.2}\right)v_z = -t^2 - 7t$$

Since $ds = v\,dt$, integrating from $s = 0, t = 0$ yields

$$\left(\frac{6}{32.2}\right)s_x = \frac{t^4}{12} - \frac{t^3}{3} + t^2 \quad \left(\frac{6}{32.2}\right)s_y = -\frac{2t^3}{3} + 3t^2 \quad \left(\frac{6}{32.2}\right)s_z = -\frac{t^3}{3} - \frac{7t^2}{2}$$



When $t = 2$ s then, $s_x = 14.31$ ft, $s_y = 35.78$ ft $s_z = -89.44$ ft

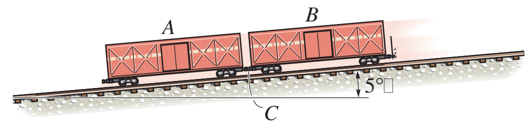
Thus,

$s = \sqrt{(14.31)^2 + (35.78)^2 + (-89.44)^2} = 97.4$ ft **Ans.**

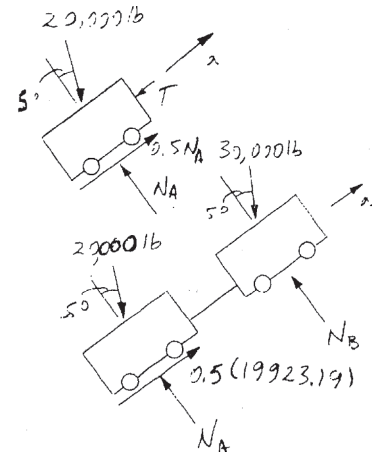
Ans:
$s = 97.4$ ft

13–2.

The two boxcars A and B have a weight of 20 000 lb and 30 000 lb, respectively. If they are freely coasting down the incline when the brakes are applied to all the wheels of car A, determine the force in the coupling C between the two cars. The coefficient of kinetic friction between the wheels of A and the tracks is $\mu_k = 0.5$. The wheels of car B are free to roll. Neglect their mass in the calculation. *Suggestion:* Solve the problem by representing single resultant normal forces acting on A and B, respectively.



SOLUTION



Car A:

$+\nwarrow \Sigma F_y = 0;$ $\quad N_A - 20\,000 \cos 5° = 0 \quad N_A = 19\,923.89 \text{ lb}$

$$+\nearrow \Sigma F_x = ma_x; \quad 0.5(19\,923.89) - T - 20\,000 \sin 5° = \left(\frac{20\,000}{32.2}\right)a \qquad \textbf{(1)}$$

Both cars:

$$+\nearrow \Sigma F_x = ma_x; \quad 0.5(19\,923.89) - 50\,000 \sin 5° = \left(\frac{50\,000}{32.2}\right)a$$

Solving,

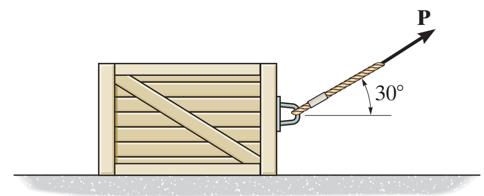
$a = 3.61 \text{ ft/s}^2$

$T = 5.98 \text{ kip}$ **Ans.**

Ans:
$T = 5.98$ kip

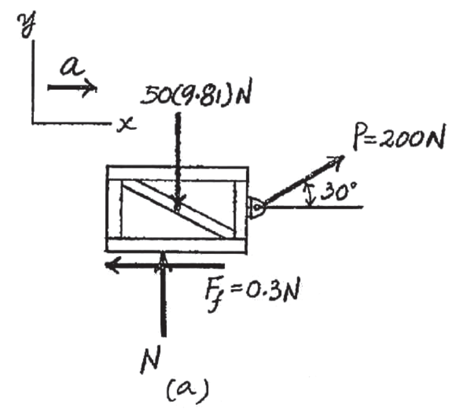
13–3.

If the coefficient of kinetic friction between the 50-kg crate and the ground is $\mu_k = 0.3$, determine the distance the crate travels and its velocity when $t = 3$ s. The crate starts from rest, and $P = 200$ N.



SOLUTION

Free-Body Diagram: The kinetic friction $F_f = \mu_k N$ is directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.



Equations of Motion: Here, $a_y = 0$. Thus,

$$+\uparrow \Sigma F_y = 0; \qquad N - 50(9.81) + 200 \sin 30^\circ = 0$$

$$N = 390.5 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \quad 200 \cos 30^\circ - 0.3(390.5) = 50a$$

$$a = 1.121 \text{ m/s}^2$$

Kinematics: Since the acceleration **a** of the crate is constant,

$$\left(\xrightarrow{+}\right) \qquad v = v_0 + a_c t$$

$$v = 0 + 1.121(3) = 3.36 \text{ m/s} \qquad \textbf{Ans.}$$

and

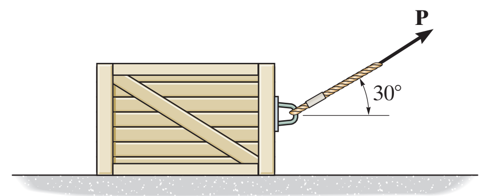
$$\left(\xrightarrow{+}\right) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(1.121)\left(3^2\right) = 5.04 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$v = 3.36$ m/s
$s = 5.04$ m

***13–4.**

If the 50-kg crate starts from rest and achieves a velocity of $v = 4\text{ m/s}$ when it travels a distance of 5 m to the right, determine the magnitude of force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$.



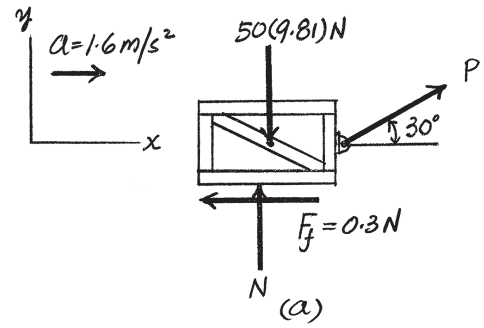
SOLUTION

Kinematics: The acceleration **a** of the crate will be determined first since its motion is known.

$(\overset{+}{\rightarrow})\qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$

$$4^2 = 0^2 + 2a(5 - 0)$$

$$a = 1.60\text{ m/s}^2 \rightarrow$$



Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.3N$ is required to be directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.

Equations of Motion:

$+\uparrow \Sigma F_y = ma_y;\qquad N + P\sin 30° - 50(9.81) = 50(0)$

$$N = 490.5 - 0.5P$$

Using the results of **N** and **a**,

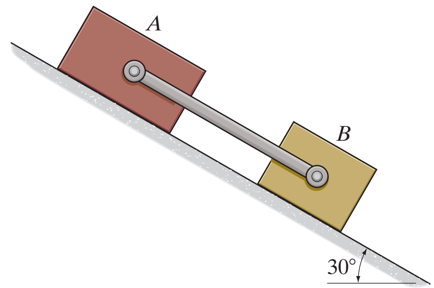
$\overset{+}{\rightarrow} \Sigma F_x = ma_x;\qquad P\cos 30° - 0.3(490.5 - 0.5P) = 50(1.60)$

$$P = 224\text{ N}$$ **Ans.**

Ans:
$P = 224\text{ N}$

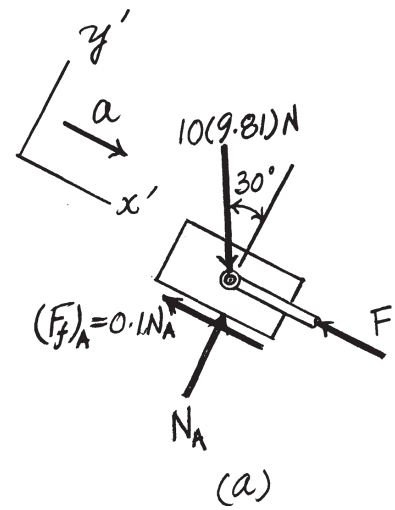
13–5.

If blocks A and B of mass 10 kg and 6 kg, respectively, are placed on the inclined plane and released, determine the force developed in the link. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.



SOLUTION

Free-Body Diagram: Here, the kinetic friction $(F_f)_A = \mu_A N_A = 0.1N_A$ and $(F_f)_B = \mu_B N_B = 0.3N_B$ are required to act up the plane to oppose the motion of the blocks which are down the plane. Since the blocks are connected, they have a common acceleration **a**.



Equations of Motion: By referring to Figs. (a) and (b),

$+\nearrow \Sigma F_{y'} = ma_{y'}$; $\quad N_A - 10(9.81)\cos 30° = 10(0)$

$N_A = 84.96 \text{ N}$

$\searrow + \Sigma F_{x'} = ma_{x'}$; $\quad 10(9.81)\sin 30° - 0.1(84.96) - F = 10a$

$$40.55 - F = 10a \qquad \textbf{(1)}$$

and

$+\nearrow \Sigma F_{y'} = ma_{y'}$; $\quad N_B - 6(9.81)\cos 30° = 6(0)$

$N_B = 50.97 \text{ N}$

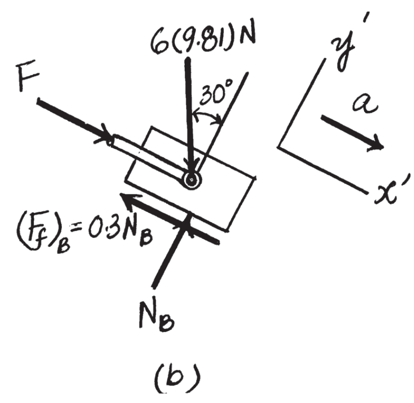
$\searrow + \Sigma F_{x'} = ma_{x'}$; $\quad F + 6(9.81)\sin 30° - 0.3(50.97) = 6a$

$$F + 14.14 = 6a \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$a = 3.42 \text{ m/s}^2$

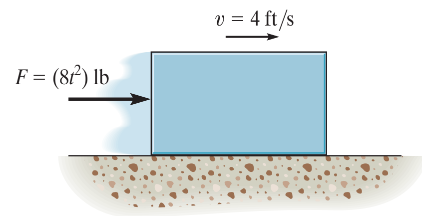
$F = 6.37 \text{ N}$ **Ans.**



Ans:
$F = 6.37 \text{ N}$

13–6.

The 10-lb block has a speed of 4 ft/s when the force of $F = (8t^2)$ lb is applied. Determine the velocity of the block when $t = 2$ s. The coefficient of kinetic friction at the surface is $\mu_k = 0.2$.



SOLUTION

Equations of Motion. Here the friction is $F_f = \mu_k N = 0.2N$. Referring to the FBD of the block shown in Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 10 = \frac{10}{32.2}(0) \qquad N = 10 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 8t^2 - 0.2(10) = \frac{10}{32.2}a$$

$$a = 3.22(8t^2 - 2) \text{ ft/s}^2$$

Kinematics. The velocity of the block as a function of t can be determined by integrating $dv = a\,dt$ using the initial condition $v = 4$ ft/s at $t = 0$.

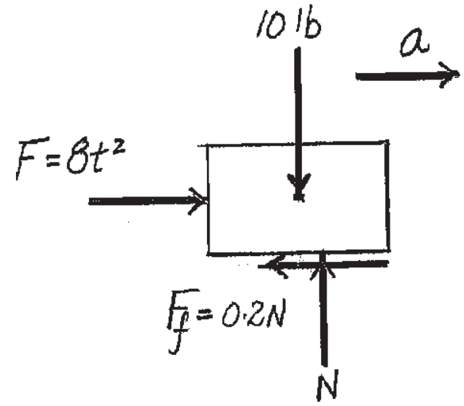
$$\int_{4 \text{ ft/s}}^{v} dv = \int_0^t 3.22\,(8t^2 - 2)dt$$

$$v - 4 = 3.22\left(\frac{8}{3}t^3 - 2t\right)$$

$$v = \{8.5867t^3 - 6.44t + 4\} \text{ ft/s}$$

When $t = 2$ s,

$$v = 8.5867(2^3) - 6.44(2) + 4$$
$$= 59.81 \text{ ft/s}$$
$$= 59.8 \text{ ft/s} \qquad \textbf{Ans.}$$

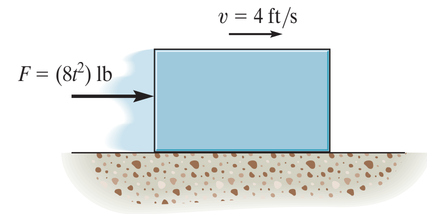


(a)

Ans:
$v = 59.8$ ft/s

13–7.

The 10-lb block has a speed of 4 ft/s when the force of $F = (8t^2)$ lb is applied. Determine the velocity of the block when it moves $s = 30$ ft. The coefficient of kinetic friction at the surface is $\mu_s = 0.2$.



SOLUTION

Equations of Motion. Here the friction is $F_f = \mu_k N = 0.2N$. Referring to the FBD of the block shown in Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 10 = \frac{10}{32.2}(0) \qquad N = 10 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 8t^2 - 0.2(10) = \frac{10}{32.2}a$$

$$a = 3.22(8t^2 - 2) \text{ ft/s}^2$$

Kinematics. The velocity of the block as a function of t can be determined by integrating $dv = adt$ using the initial condition $v = 4$ ft/s at $t = 0$.

$$\int_{4 \text{ ft/s}}^{v} dv = \int_0^t 3.22(8t^2 - 2)dt$$

$$v - 4 = 3.22\left(\frac{8}{3}t^3 - 2t\right)$$

$$v = \{8.5867t^3 - 6.44t + 4\} \text{ ft/s}$$

The displacement as a function of t can be determined by integrating $ds = vdt$ using the initial condition $s = 0$ at $t = 0$

$$\int_0^s ds = \int_0^t (8.5867t^3 - 6.44t + 4)dt$$

$$s = \{2.1467t^4 - 3.22t^2 + 4t\} \text{ ft}$$

At $s = 30$ ft,

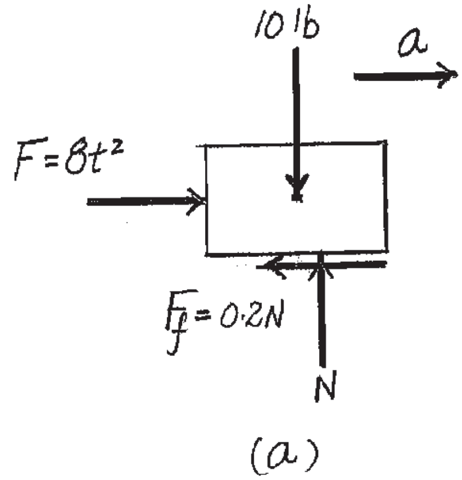
$$30 = 2.1467t^4 - 3.22t^2 + 4t$$

Solved by numerically,

$$t = 2.0089 \text{ s}$$

Thus, at $s = 30$ ft,

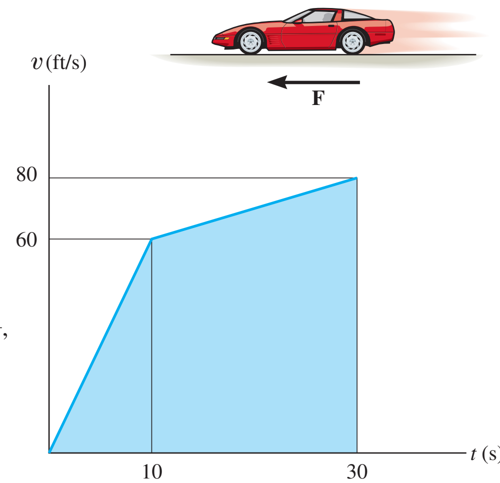
$$v = 8.5867(2.0089^3) - 6.44(2.0089) + 4$$
$$= 60.67 \text{ ft/s}$$
$$= 60.7 \text{ ft/s}$$ **Ans.**



Ans:
$v = 60.7$ ft/s

***13–8.**

The speed of the 3500-lb sports car is plotted over the 30-s time period. Plot the variation of the traction force **F** needed to cause the motion.



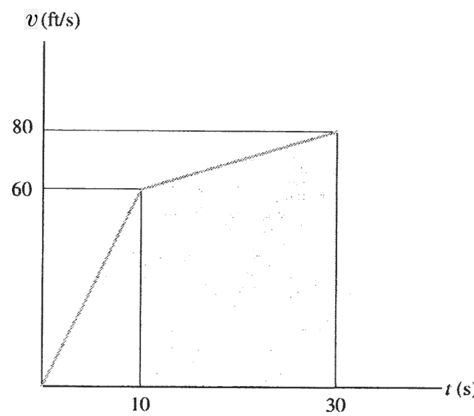
SOLUTION

Kinematics: For $\mathbf{0} \leq \mathbf{t} < \mathbf{10}$ **s**. $v = \frac{60}{10}t = \{6t\}$ ft/s. Applying equation $a = \frac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 6 \text{ ft/s}^2$$

For $\mathbf{10} < t \leq \mathbf{30}$ **s**, $\frac{v - 60}{t - 10} = \frac{80 - 60}{30 - 10}$, $v = \{t + 50\}$ ft/s. Applying equation $a = \frac{dv}{dt}$, we have

$$a = \frac{dv}{dt} = 1 \text{ ft/s}^2$$



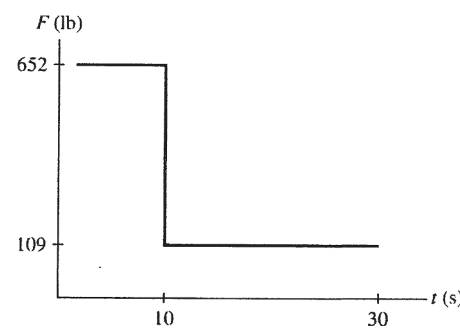
Equation of Motion:

For $\mathbf{0} \leq \mathbf{t} < \mathbf{10}$ **s**

$$\overset{+}{\leftarrow} \sum F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(6) = 652 \text{ lb}$$ **Ans.**

For $\mathbf{10} < \boldsymbol{t} \leq \mathbf{30}$ **s**

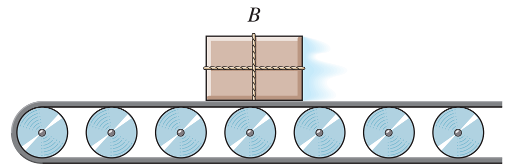
$$\overset{+}{\leftarrow} \sum F_x = ma_x; \quad F = \left(\frac{3500}{32.2}\right)(1) = 109 \text{ lb}$$ **Ans.**



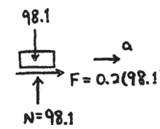
Ans:
$\overset{+}{\leftarrow} \Sigma F_x = ma_x; F = 652$ lb
$\overset{+}{\leftarrow} \Sigma F_x = ma_x; F = 109$ lb

13–9.

The conveyor belt is moving at 4 m/s. If the coefficient of static friction between the conveyor and the 10-kg package B is $\mu_s = 0.2$, determine the shortest time the belt can stop so that the package does not slide on the belt.



SOLUTION



$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0.2(98.1) = 10\,a$

$a = 1.962 \text{ m/s}^2$

$(\xrightarrow{+})\, v = v_0 + a_c t$

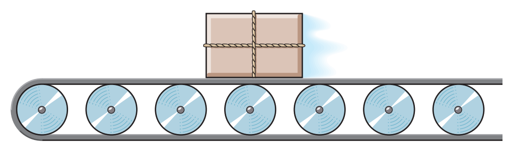
$4 = 0 + 1.962\,t$

$t = 2.04 \text{ s}$ **Ans.**

Ans:
$t = 2.04 \text{ s}$

13–10.

The conveyor belt is designed to transport packages of various weights. Each 10-kg package has a coefficient of kinetic friction $\mu_k = 0.15$. If the speed of the conveyor is 5 m/s, and then it suddenly stops, determine the distance the package will slide on the belt before coming to rest.



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 0.15\, m(9.81) = ma$

$a = 1.4715 \text{ m/s}^2$

$(\xrightarrow{+})\ v^2 = v_0^2 + 2a_c(s - s_0)$

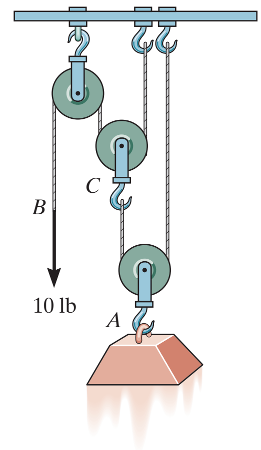
$0 = (5)^2 + 2(-1.4715)(s - 0)$

$s = 8.49 \text{ m}$ **Ans.**

Ans:
$s = 8.49 \text{ m}$

13–11.

Determine the time needed to pull the cord at B down 4 ft starting from rest when a force of 10 lb is applied to the cord. Block A weighs 20 lb. Neglect the mass of the pulleys and cords.



SOLUTION

$+\uparrow \Sigma F_y = ma_y; \qquad 40 - 20 = \dfrac{20}{32.2} a_A$ **Ans.**

$$a_A = 32.2 \text{ ft/s}^2$$

$s_B + 2s_C = l; \qquad a_B = -2a_C$

$2s_A - s_C = l'; \qquad 2a_A = a_C$

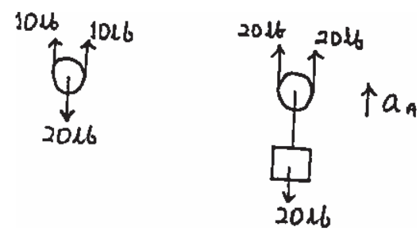
$a_B = -4a_A$

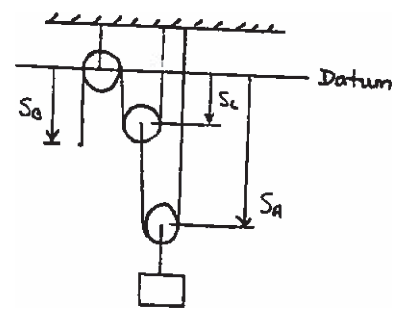
$a_B = 128.8 \text{ ft/s}^2$

$(+\downarrow) \quad s = s_0 + v_0 t + \dfrac{1}{2} a_c t^2$

$$4 = 0 + 0 + \frac{1}{2}(128.8)\, t^2$$

$t = 0.249$ s **Ans.**

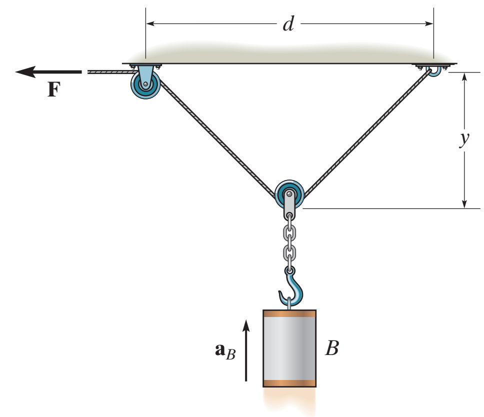




Ans:
$t = 0.249$ s

***13–12.**

Cylinder B has a mass m and is hoisted using the cord and pulley system shown. Determine the magnitude of force **F** as a function of the block's vertical position y so that when **F** is applied the block rises with a constant acceleration $\mathbf{a}_B$. Neglect the mass of the cord and pulleys.

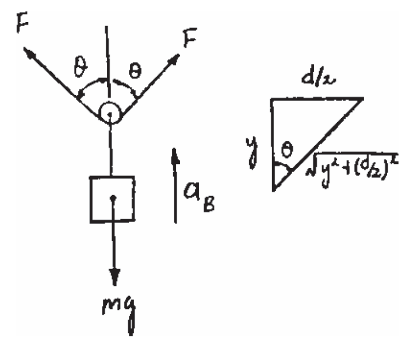


SOLUTION

$+\uparrow \Sigma F_y = ma_y;\qquad 2F\cos\theta - mg = ma_B \qquad \text{where } \cos\theta = \dfrac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}$

$$2F\left(\frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}\right) - mg = ma_B$$

$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y} \qquad \textbf{Ans.}$$

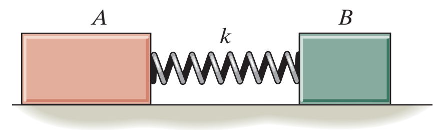


Ans:

$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y}$$

13–13.

Block A has a weight of 8 lb and block B has a weight of 6 lb. They rest on a surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. If the spring has a stiffness of $k = 20\text{ lb/ft}$, and it is compressed 0.2 ft, determine the acceleration of each block just after they are released.

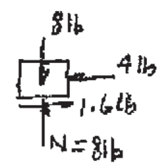


SOLUTION

Block A:

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad 4 - 1.6 = \frac{8}{32.2} a_A$$

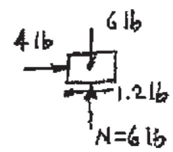
$$a_A = 9.66\text{ ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$



Block B:

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 4 - 12 = \frac{6}{32.2} a_B$$

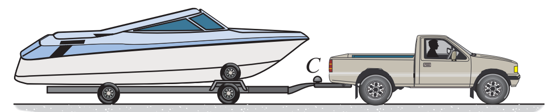
$$a_B = 15.0\text{ ft/s}^2 \rightarrow \qquad \textbf{Ans.}$$



Ans:
$a_A = 9.66\text{ ft/s}^2 \leftarrow$
$a_B = 15.0\text{ ft/s}^2 \rightarrow$

13–14.

The 2-Mg truck is traveling at 15 m/s when the brakes on all its wheels are applied, causing it to skid for a distance of 10 m before coming to rest. Determine the constant horizontal force developed in the coupling C, and the frictional force developed between the tires of the truck and the road during this time. The total mass of the boat and trailer is 1 Mg.



SOLUTION

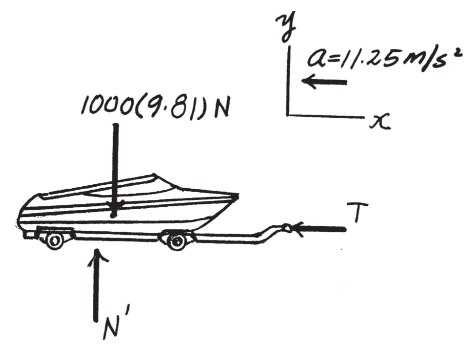
Kinematics: Since the motion of the truck and trailer is known, their common acceleration **a** will be determined first.

$$\left(\xrightarrow{+}\right) \qquad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$0 = 15^2 + 2a(10 - 0)$$

$$a = -11.25\ \text{m/s}^2 = 11.25\ \text{m/s}^2 \leftarrow$$

Free-Body Diagram: The free-body diagram of the truck and trailer are shown in Figs. (a) and (b), respectively. Here, **F** representes the frictional force developed when the truck skids, while the force developed in coupling C is represented by **T**.



(a)

Equations of Motion: Using the result of **a** and referrning to Fig. (a),

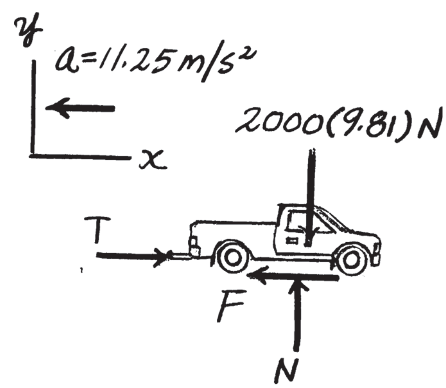
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -T = 1000(-11.25)$$

$$T = 11\,250\ \text{N} = 11.25\ \text{kN} \qquad \textbf{Ans.}$$

Using the results of **a** and **T** and referring to Fig. (b),

$$+\uparrow \Sigma F_x = ma_x; \qquad 11\,250 - F = 2000(-11.25)$$

$$F = 33\,750\ \text{N} = 33.75\ \text{kN} \qquad \textbf{Ans.}$$

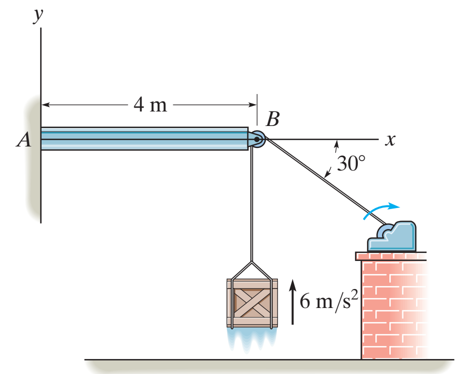


(b)

Ans:
$T = 11.25$ kN
$F = 33.75$ kN

13–15.

The motor lifts the 50-kg crate with an acceleration of 6 m/s^2. Determine the components of force reaction and the couple moment at the fixed support A.



SOLUTION

Equation of Motion. Referring to the FBD of the crate shown in Fig. *a*,

$+\uparrow \Sigma F_y = ma_y; \qquad T - 50(9.81) = 50(6) \qquad T = 790.5 \text{ N}$

Equations of Equilibrium. Since the pulley is smooth, the tension is constant throughout entire cable. Referring to the FBD of the pulley shown in Fig. *b*,

$\xrightarrow{+} \Sigma F_x = 0; \qquad 790.5 \cos 30° - B_x = 0 \qquad B_x = 684.59 \text{ N}$

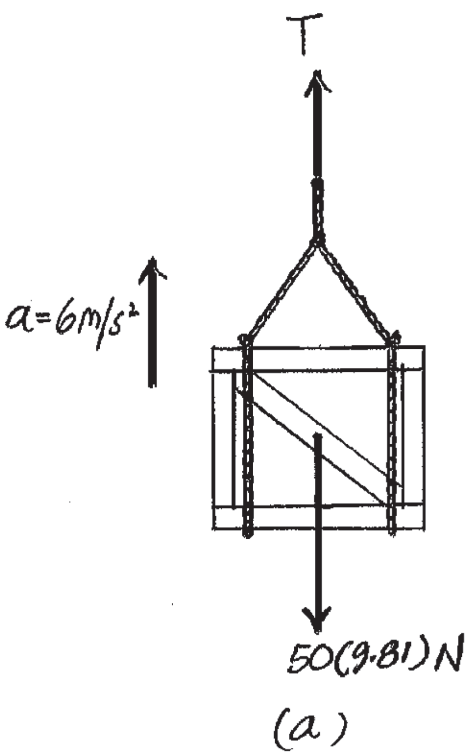
$+\uparrow \Sigma F_y = 0; \qquad B_y - 790.5 - 790.5 \sin 30° = 0 \qquad B_y = 1185.75 \text{ N}$

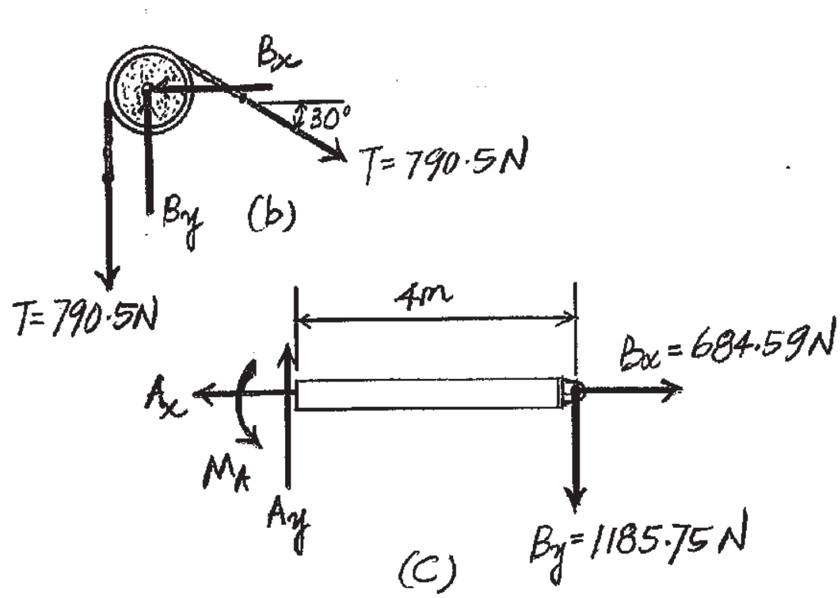
Consider the FBD of the cantilever beam shown in Fig. *c*,

$\xrightarrow{+} \Sigma F_x = 0; \qquad 684.59 - A_x = 0 \qquad A_x = 684.59 \text{ N} = 685 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y - 1185.75 = 0 \qquad A_y = 1185.75 \text{ N} = 1.19 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0; \quad M_A - 1185.75(4) = 0 \qquad M_A = 4743 \text{ N} \cdot \text{m} = 4.74 \text{ kN} \cdot \text{m}$ **Ans.**

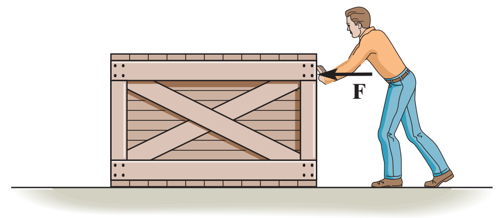




Ans:
$A_x = 685$ N
$A_y = 1.19$ kN
$M_A = 4.74$ kN · m

***13–16.**

The 75-kg man pushes on the 150-kg crate with a horizontal force **F**. If the coefficients of static and kinetic friction between the crate and the surface are $\mu_s = 0.3$ and $\mu_k = 0.2$, and the coefficient of static friction between the man's shoes and the surface is $\mu_s = 0.8$, show that the man is able to move the crate. What is the greatest acceleration the man can give the crate?



SOLUTION

Equation of Equilibrium. Assuming that the crate is on the verge of sliding $(F_f)_C = \mu_s N_C = 0.3N_C$. Referring to the FBD of the crate shown in Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad N_C - 150(9.81) = 0 \qquad N_C = 1471.5 \text{ N}$

$\underset{\rightarrow}{+} \Sigma F_x = 0; \qquad 0.3(1471.5) - F = 0 \qquad F = 441.45 \text{ N}$

Referring to the FBD of the man, Fig. *b*,

$+\uparrow \Sigma F_y = 0; \qquad N_m - 75(9.81) = 0 \qquad N_B = 735.75 \text{ N}$

$\underset{\rightarrow}{+} \Sigma F_x = 0; \qquad 441.45 - (F_f)_m = 0 \qquad (F_f)_m = 441.45 \text{ N}$

Since $(F_f)_m < \mu'_s N_m = 0.8(735.75) = 588.6$ N, **the man is able to move the crate**.

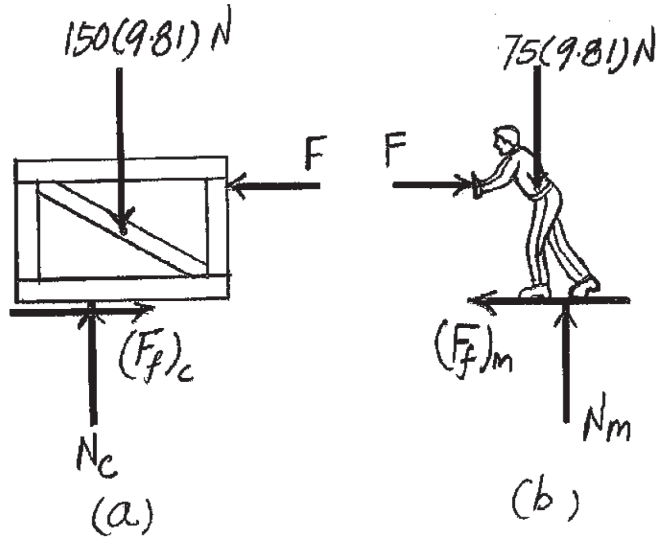
Equation of Motion. The greatest acceleration of the crate can be produced when the man is on the verge of slipping. Thus, $(F_f)_m = \mu'_s N_m = 0.8(735.75) = 588.6$ N.

$\underset{\rightarrow}{+} \Sigma F_x = 0; \qquad F - 588.6 = 0 \qquad F = 588.6 \text{ N}$

Since the crate slides, $(F_f)_C = \mu_k N_C = 0.2(1471.5) = 294.3$ N. Thus,

$\underset{\leftarrow}{+} \Sigma F_x = ma_x; \qquad 588.6 - 294.3 = 150\,a$

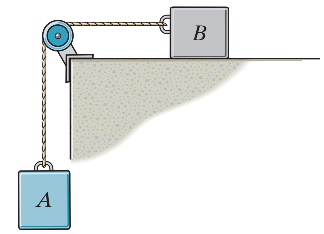
$$a = 1.962 \text{ m/s}^2 = 1.96 \text{ m/s}^2 \qquad \textbf{Ans.}$$



Ans:
$a = 1.96 \text{ m/s}^2$

13–17.

Determine the acceleration of the blocks when the system is released. The coefficient of kinetic friction is μ_k, and the mass of each block is m. Neglect the mass of the pulleys and cord.



SOLUTION

Free Body Diagram. Since the pulley is smooth, the tension is constant throughout the entire cord. Since block B is required to slide, $F_f = \mu_k N$. Also, blocks A and B are attached together with inextensible cord, so $a_A = a_B = a$. The FBDs of blocks A and B are shown in Figs. a and b, respectively.

Equations of Motion. For block A, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad T - mg = m(-a) \qquad \textbf{(1)}$$

For block B, Fig. b,

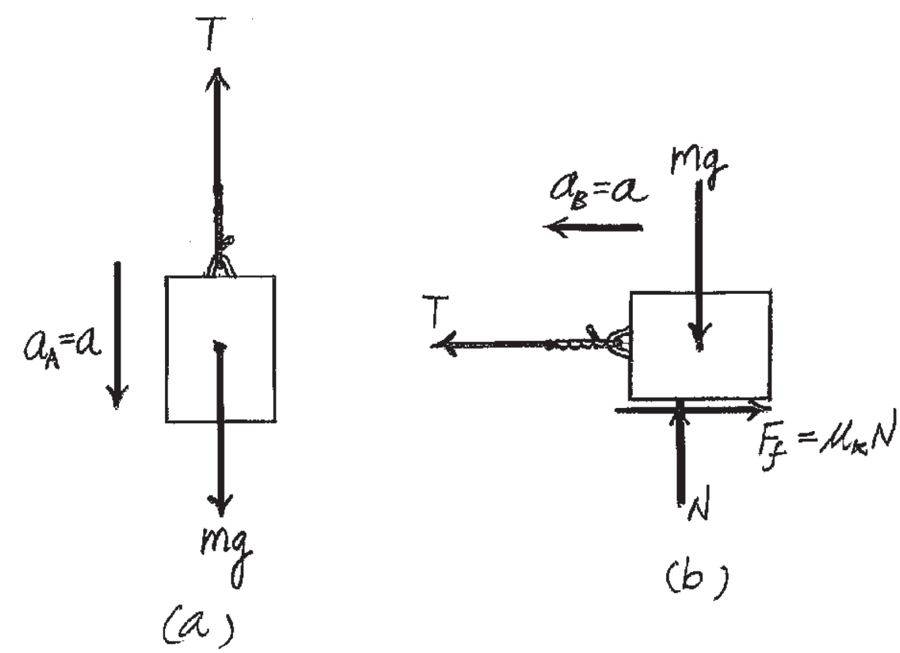
$$+\uparrow \Sigma F_y = ma_y; \qquad N - mg = m(0) \qquad N = mg$$

$$(\overset{+}{\leftarrow})\, \Sigma F_x = ma_x; \quad T - \mu_k mg = ma \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$a = \frac{1}{2}(1 - \mu_k)\, g \qquad \textbf{Ans.}$$

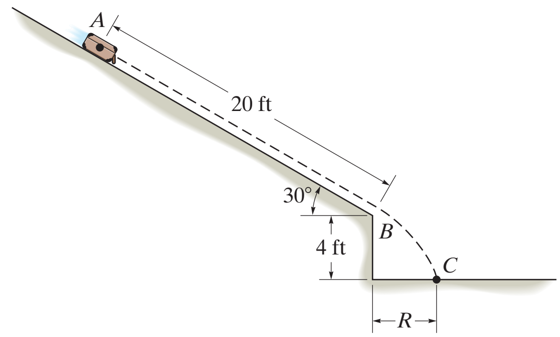
$$T = \frac{1}{2}(1 + \mu_k)\, mg$$



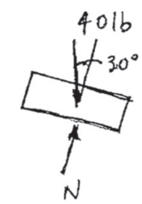
Ans:
$a = \frac{1}{2}(1 - \mu_k)\, g$

13–18.

A 40-lb suitcase slides from rest 20 ft down the smooth ramp. Determine the point where it strikes the ground at C. How long does it take to go from A to C?



SOLUTION



$+\searrow \Sigma F_x = m\,a_x; \qquad 40 \sin 30° = \dfrac{40}{32.2}a$

$a = 16.1 \text{ ft/s}^2$

$(+\searrow) v^2 = v_0^2 + 2\,a_c(s - s_0);$

$v_B^2 = 0 + 2(16.1)(20)$

$v_B = 25.38 \text{ ft/s}$

$(+\searrow)\ v = v_0 + a_c\,t;$

$25.38 = 0 + 16.1\,t_{AB}$

$t_{AB} = 1.576 \text{ s}$

$(\overset{+}{\rightarrow}) s_x = (s_x)_0 + (v_x)_0\,t$

$R = 0 + 25.38 \cos 30°(t_{BC})$

$(+\downarrow)\ s_y = (s_y)_0 + (v_y)_0\,t + \dfrac{1}{2}a_c t^2$

$4 = 0 + 25.38 \sin 30°\ t_{BC} + \dfrac{1}{2}(32.2)(t_{BC})^2$

$t_{BC} = 0.2413 \text{ s}$

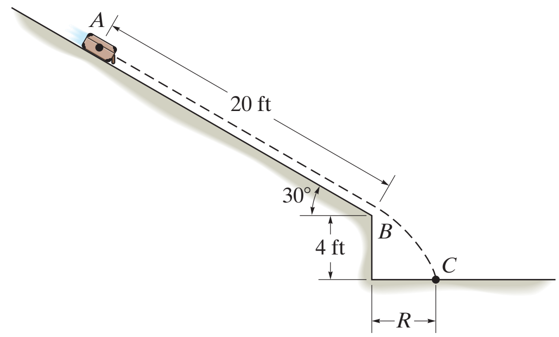
$R = 5.30 \text{ ft}$ **Ans.**

Total time $= t_{AB} + t_{BC} = 1.82 \text{ s}$ **Ans.**

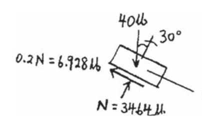
Ans:
$R = 5.30 \text{ ft}$
$t_{AC} = 1.82 \text{ s}$

13–19.

Solve Prob. 13–18 if the suitcase has an initial velocity down the ramp of $v_A = 10$ ft/s and the coefficient of kinetic friction along AB is $\mu_k = 0.2$.



SOLUTION



$+\searrow \Sigma F_x = ma_x;\qquad 40 \sin 30° - 6.928 = \dfrac{40}{32.2} a$

$a = 10.52 \text{ ft/s}^2$

$(+\searrow)\ v^2 = v_0^2 + 2\, a_c(s - s_0);$

$v_B^2 = (10)^2 + 2(10.52)(20)$

$v_B = 22.82$ ft/s

$(+\searrow)\ v = v_0 + a_c\, t;$

$22.82 = 10 + 10.52\, t_{AB}$

$t_{AB} = 1.219$ s

$(\overset{+}{\rightarrow})\ s_x = (s_x)_0 + (v_x)_0\, t$

$R = 0 + 22.82 \cos 30° \,(t_{BC})$

$(+\downarrow)\ s_y = (s_y)_0 + (v_y)_0\, t + \dfrac{1}{2} a_c\, t^2$

$4 = 0 + 22.82 \sin 30°\, t_{BC} + \dfrac{1}{2}(32.2)(t_{BC})^2$

$t_{BC} = 0.2572$ s

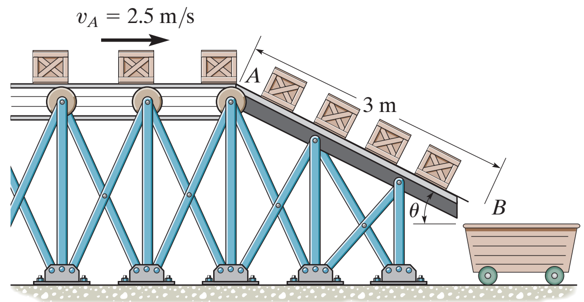
$R = 5.08$ ft **Ans.**

Total time $= t_{AB} + t_{BC} = 1.48$ s **Ans.**

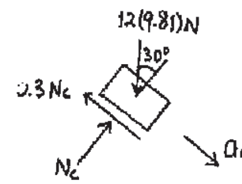
Ans:
$R = 5.08$ ft
$t_{AC} = 1.48$ s

***13–20.**

The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5\ \text{m/s}$, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs. Take $\theta = 30°$.



SOLUTION



$\nearrow + \Sigma F_y = ma_y;\qquad N_C - 12(9.81)\cos 30° = 0$

$N_C = 101.95\ \text{N}$

$+\searrow \Sigma F_x = ma_x;\qquad 12(9.81)\sin 30° - 0.3(101.95) = 12\,a_C$

$a_C = 2.356\ \text{m/s}^2$

$(+\searrow)\qquad v_B^2 = v_A^2 + 2\,a_C(s_B - s_A)$

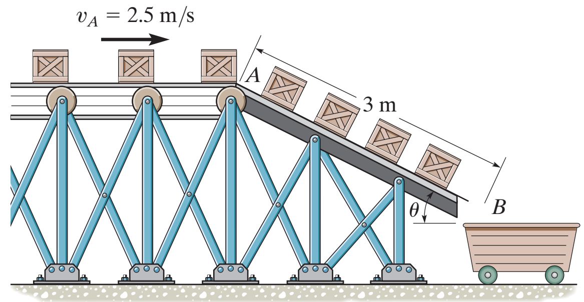
$v_B^2 = (2.5)^2 + 2(2.356)(3 - 0)$

$v_B = 4.5152 = 4.52\ \text{m/s}$ **Ans.**

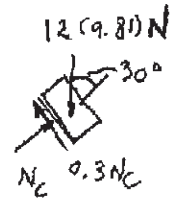
Ans:
$v_B = 4.52\ \text{m/s}$

13–21.

The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5$ m/s, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the smallest incline θ of the ramp so that the crates will slide off and fall into the cart.



SOLUTION



$(+\searrow)\quad v_B^2 = v_A^2 + 2a_C(s_B - s_A)$

$$0 = (2.5)^2 + 2(a_C)(3 - 0)$$

$$a_C = 1.0417$$

$\nearrow + \Sigma F_y = ma_y;\qquad N_C - 12(9.81)\cos\theta = 0$

$$N_C = 117.72\cos\theta$$

$+\searrow \Sigma F_x = ma_x;\qquad 12(9.81)\sin\theta - 0.3(N_C) = 12\,(1.0417)$

$$117.72\sin\theta - 35.316\cos\theta - 12.5 = 0$$

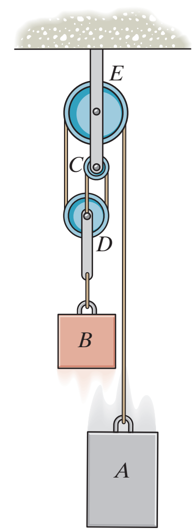
Solving,

$\theta = 22.6°$ **Ans.**

Ans:
$\theta = 22.6°$

13–22.

The 50-kg block A is released from rest. Determine the velocity of the 15-kg block B in 2 s.



SOLUTION

Kinematics. As shown in Fig. a, the position of block B and point A are specified by s_B and s_A respectively. Here the pulley system has only one cable which gives

$$s_A + s_B + 2(s_B - a) = l$$

$$s_A + 3s_B = l + 2a \qquad \textbf{(1)}$$

Taking the time derivative of Eq. (1) twice,

$$a_A + 3a_B = 0 \qquad \textbf{(2)}$$

Equations of Motion. The FBD of blocks B and A are shown in Fig. b and c. To be consistent to those in Eq. (2), $\mathbf{a}_A$ and $\mathbf{a}_B$ are assumed to be directed towards the positive sense of their respective position coordinates s_A and s_B. For block B,

$$+\uparrow \Sigma F_y = ma_y; \qquad 3T - 15(9.81) = 15(-a_B) \qquad \textbf{(3)}$$

For block A,

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 50(9.81) = 50(-a_A) \qquad \textbf{(4)}$$

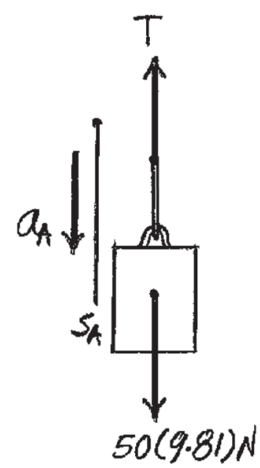
Solving Eqs. (2), (3) and (4),

$$a_B = -2.848 \text{ m/s}^2 = 2.848 \text{ m/s}^2 \uparrow \qquad a_A = 8.554 \text{ m/s}^2 \qquad T = 63.29 \text{ N}$$

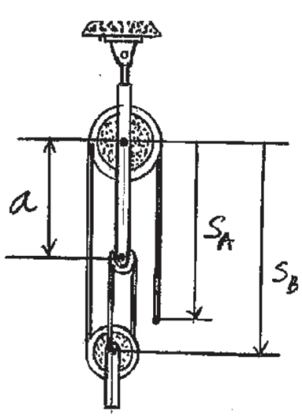
The negative sign indicates that $\mathbf{a}_B$ acts in the sense opposite to that shown in FBD. The velocity of block B can be determined using

$$+\uparrow \quad v_B = (v_A)_0 + a_B t; \qquad v_B = 0 + 2.848(2)$$

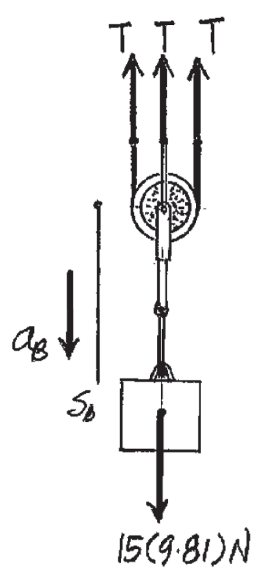
$$v_B = 5.696 \text{ m/s} = 5.70 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(c)



(a)

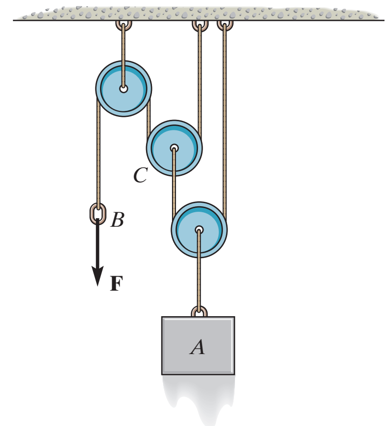


(b)

Ans:
$v_B = 5.70 \text{ m/s} \uparrow$

13–23.

If the supplied force $F = 150$ N, determine the velocity of the 50-kg block A when it has risen 3 m, starting from rest.



SOLUTION

Equations of Motion. Since the pulleys are smooth, the tension is constant throughout each entire cable. Referring to the FBD of pulley C, Fig. a, of which its mass is negligible.

$+\uparrow \Sigma F_y = 0; \qquad 150 + 150 - T = 0 \qquad T = 300 \text{ N}$

Subsequently, considered the FBD of block A shown in Fig. b,

$+\uparrow \Sigma F_y = ma_y; \qquad 300 + 300 - 50(9.81) = 50a$

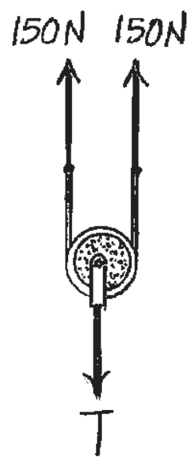
$$a = 2.19 \text{ m/s}^2 \uparrow$$

Kinematics. Using the result of **a**,

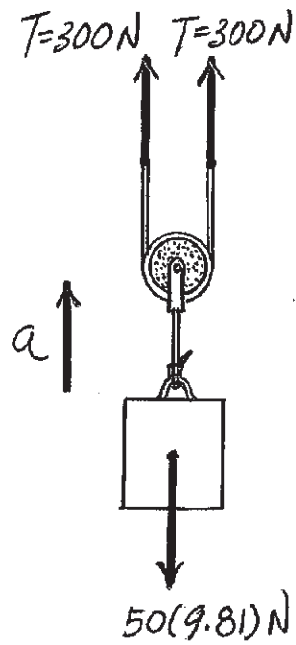
$(+\uparrow) \quad v^2 = v_0^2 + 2a_c s;$

$$v^2 = 0^2 + 2(2.19)(3)$$

$$v = 3.6249 \text{ m/s} = 3.62 \text{ m/s}$$ **Ans.**



(a)

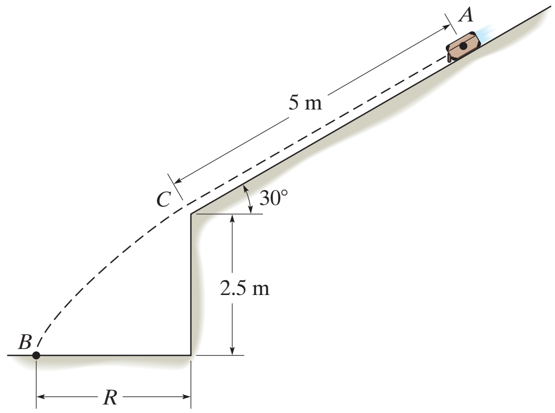


(b)

Ans:
$v = 3.62 \text{ m/s} \uparrow$

***13–24.**

A 60-kg suitcase slides from rest 5 m down the smooth ramp. Determine the distance R where it strikes the ground at B. How long does it take to go from A to B?



SOLUTION

Equation of Motion. Referring to the FBD of the suitcase shown in Fig. *a*

$+\swarrow \Sigma F_{x'} = ma_{x'}; \qquad 60(9.81) \sin 30° = 60a \qquad a = 4.905 \text{ m/s}^2$

Kinematics. From A to C, the suitcase moves along the inclined plane (straight line).

$(+\swarrow)\ v^2 = v_0^2 + 2a_c s; \qquad v^2 = 0^2 + 2(4.905)(5)$

$$v = 7.0036 \text{ m/s}$$

$(+\swarrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2; \qquad 5 = 0 + 0 + \frac{1}{2}(4.905)t_{AC}^2$

$$t_{AC} = 1.4278 \text{ s}$$

From C to B, the suitcase undergoes projectile motion. Referring to x–y coordinate system with origin at C, Fig. *b*, the vertical motion gives

$(+\downarrow) \quad s_y = (s_0)_y + v_y t + \frac{1}{2} a_y t^2;$

$$2.5 = 0 + 7.0036 \sin 30°\, t_{CB} + \frac{1}{2}(9.81)t_{CB}^2$$

$$4.905\, t_{CB}^2 + 3.5018\, t_{CB} - 2.5 = 0$$

Solve for positive root,

$$t_{CB} = 0.4412 \text{ s}$$

Then, the horizontal motion gives

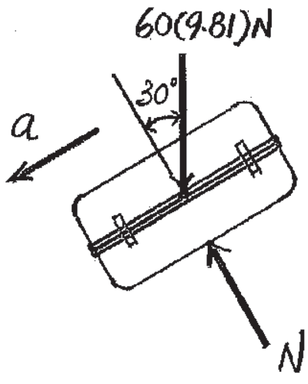
$(\overset{+}{\leftarrow}) \quad s_x = (s_0)_x + v_x t;$

$$R = 0 + 7.0036 \cos 30° (0.4412)$$

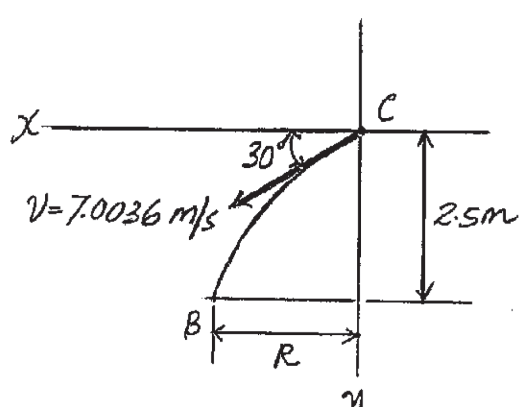
$$= 2.676 \text{ m} = 2.68 \text{ m} \qquad \textbf{Ans.}$$

The time taken from A to B is

$$t_{AB} = t_{AC} + t_{CB} = 1.4278 + 0.4412 = 1.869 \text{ s} = 1.87 \text{ s} \qquad \textbf{Ans.}$$



(a)

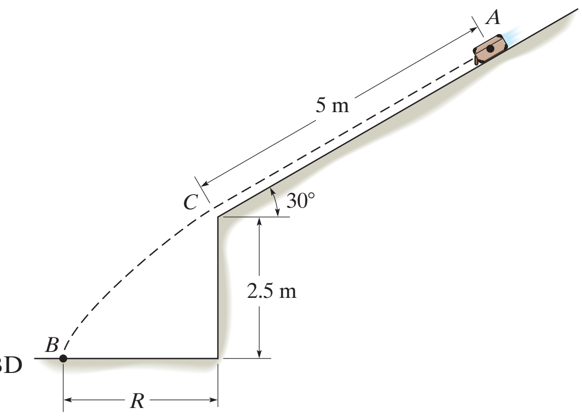


(b)

Ans:
$(\overset{+}{\leftarrow}) \quad s_x = 2.68$ m
$t_{AB} = 1.87$ s

13–25.

Solve Prob. 13–24 if the suitcase has an initial velocity down the ramp of $v_A = 2\text{ m/s}$, and the coefficient of kinetic friction along AC is $\mu_k = 0.2$.



SOLUTION

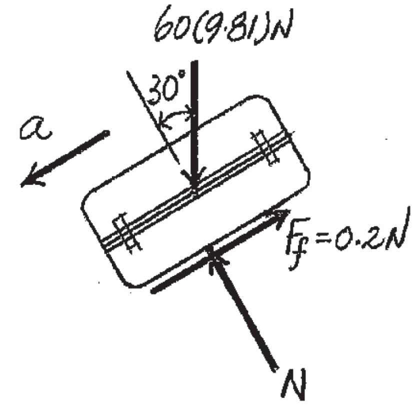
Equations of Motion. The friction is $F_f = \mu_k N = 0.2N$. Referring to the FBD of the suitcase shown in Fig. a

$$\nwarrow^{+} \Sigma F_{y'} = ma_{y'}; \qquad N - 60(9.81)\cos 30^\circ = 60(0)$$

$$N = 509.74\text{ N}$$

$$+\swarrow \Sigma F_{x'} = ma_{x'}; \qquad 60(9.81)\sin 30^\circ - 0.2(509.74) = 60\,a$$

$$a = 3.2059\text{ m/s}^2 \swarrow$$



(a)

Kinematics. From A to C, the suitcase moves along the inclined plane (straight line).

$$(+\swarrow) \qquad v^2 = v_0^2 + 2a_c s; \qquad v^2 = 2^2 + 2(3.2059)(5)$$

$$v = 6.0049\text{ m/s} \swarrow$$

$$(+\swarrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2; \qquad 5 = 0 + 2t_{AC} + \frac{1}{2}(3.2059)t_{AC}^2$$

$$1.6029\,t_{AC}^2 + 2t_{AC} - 5 = 0$$

Solve for positive root,

$$t_{AC} = 1.2492\text{ s}$$

From C to B, the suitcase undergoes projectile motion. Referring to x–y coordinate system with origin at C, Fig. b, the vertical motion gives

$$(+\downarrow) \qquad s_y = (s_0)_y + v_y t + \frac{1}{2}a_y t^2;$$

$$2.5 = 0 + 6.0049\sin 30^\circ\,t_{CB} + \frac{1}{2}(9.81)t_{CB}^2$$

$$4.905\,t_{CB}^2 + 3.0024\,t_{CB} - 2.5 = 0$$

Solve for positive root,

$$t_{CB} = 0.4707\text{ s}$$

Then, the horizontal motion gives

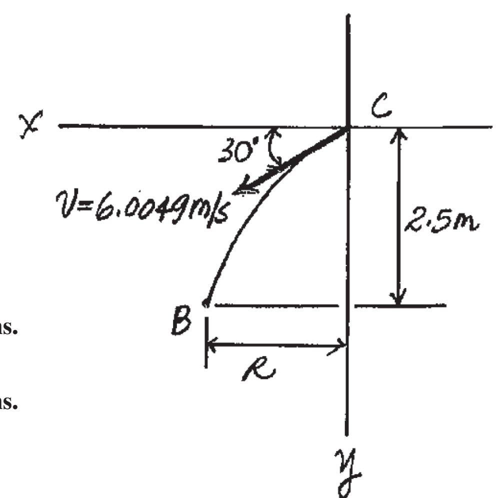
$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t;$$

$$R = 0 + 6.0049\cos 30^\circ\,(0.4707)$$

$$= 2.448\text{ m} = 2.45\text{ m} \qquad \textbf{Ans.}$$

The time taken from A to B is

$$t_{AB} = t_{AC} + t_{CB} = 1.2492 + 0.4707 = 1.7199\text{ s} = 1.72\text{ s} \qquad \textbf{Ans.}$$

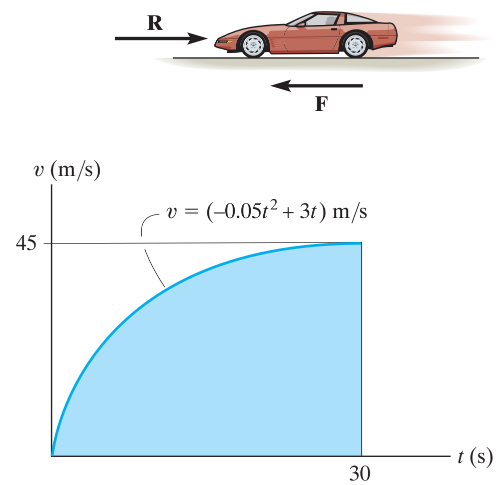


(b)

Ans:
$R = 2.45\text{ m}$
$t_{AB} = 1.72\text{ s}$

13–26.

The 1.5 Mg sports car has a tractive force of $F = 4.5$ kN. If it produces the velocity described by v-t graph shown, plot the air resistance R versus t for this time period.



SOLUTION

Kinematic. For the v–t graph, the acceleration of the car as a function of t is

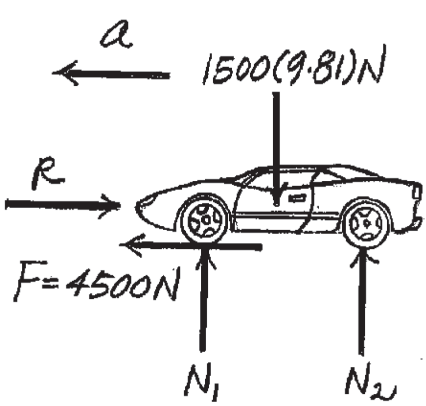
$$a = \frac{dv}{dt} = \{-0.1t + 3\}\,\text{m/s}^2$$

Equation of Motion. Referring to the FBD of the car shown in Fig. a,

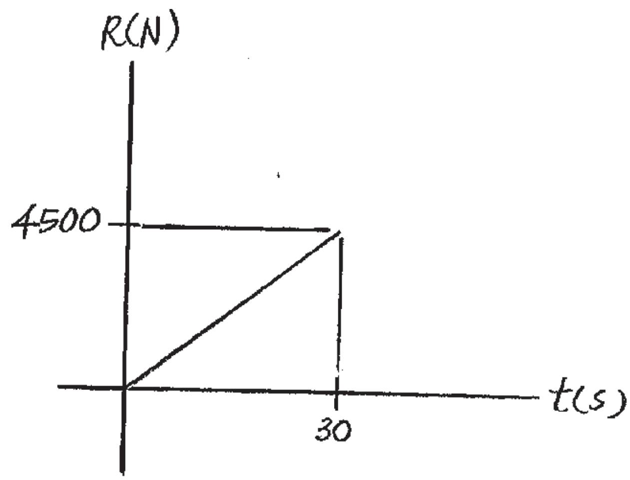
$$(\overset{+}{\leftarrow})\,\Sigma F_x = ma_x; \qquad 4500 - R = 1500(-0.1t + 3)$$

$$R = \{150t\}\,\text{N}$$

The plot of R vs t is shown in Fig. b



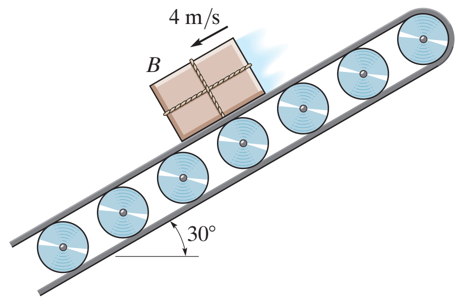
(a)



Ans:
$R = \{150t\}$ N

13–27.

The conveyor belt is moving downward at 4 m/s. If the coefficient of static friction between the conveyor and the 15-kg package B is $\mu_s = 0.8$, determine the shortest time the belt can stop so that the package does not slide on the belt.



SOLUTION

Equations of Motion. It is required that the package is on the verge to slide. Thus, $F_f = \mu_s N = 0.8N$. Referring to the FBD of the package shown in Fig. a,

$+\nwarrow \Sigma F_{y'} = ma_{y'};\quad N - 15(9.81)\cos 30° = 15(0)\quad N = 127.44\text{ N}$

$+\nearrow \Sigma F_{x'} = ma_{x'};\quad 0.8(127.44) - 15(9.81)\sin 30° = 15\,a$

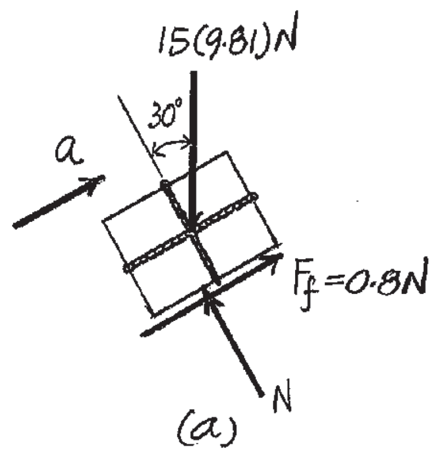
$$a = 1.8916\text{ m/s}^2 \nearrow$$

Kinematic. Since the package is required to stop, $v = 0$. Here $v_0 = 4$ m/s.

$(+\swarrow)\quad v = v_0 + a_0 t;$

$$0 = 4 + (-1.8916)\,t$$

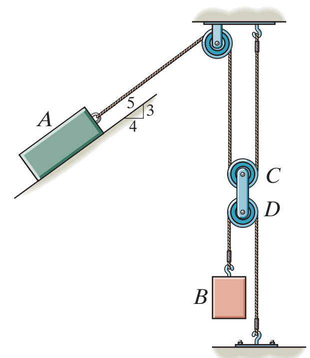
$$t = 2.1146\text{ s} = 2.11\text{ s}$$ **Ans.**



Ans:
$t = 2.11$ s

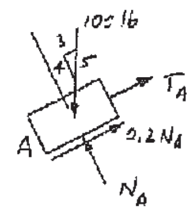
***13–28.**

At the instant shown the 100-lb block A is moving down the plane at 5 ft/s while being attached to the 50-lb block B. If the coefficient of kinetic friction between the block and the incline is $\mu_k = 0.2$, determine the acceleration of A and the distance A slides before it stops. Neglect the mass of the pulleys and cables.



SOLUTION

Block A:



$$+\swarrow \Sigma F_x = ma_x; \qquad -T_A - 0.2N_A + 100\left(\frac{3}{5}\right) = \left(\frac{100}{32.2}\right)a_A$$

$$+\nwarrow \Sigma F_y = ma_y; \qquad N_A - 100\left(\frac{4}{5}\right) = 0$$

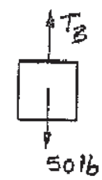
Thus,

$$T_A - 44 = -3.1056a_A \qquad \textbf{(1)}$$

Block B:

$$+\uparrow \Sigma F_y = ma_y; \qquad T_B - 50 = \left(\frac{50}{32.2}\right)a_B$$

$$T_B - 50 = 1.553a_B \qquad \textbf{(2)}$$

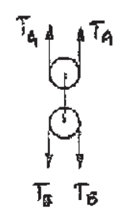


Pulleys at C and D:

$$+\uparrow \Sigma F_y = 0; \qquad 2T_A - 2T_B = 0$$

$$T_A = T_B \qquad \textbf{(3)}$$

Kinematics:



$s_A + 2s_C = l$

$s_D + (s_D - s_B) = l'$

$s_C + d + s_D = d'$

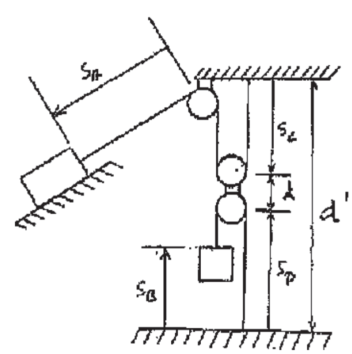
Thus,

$a_A = -2a_C$

$2a_D = a_B$

$a_C = -a_{D'}$

so that $a_A = a_B$ **(4)**



Solving Eqs. (1)–(4):

$a_A = a_B = -1.288 \text{ ft/s}^2$

$T_A = T_B = 48.0 \text{ lb}$

Thus,

$a_A = 1.29 \text{ ft/s}^2$ **Ans.**

$$(+\swarrow) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

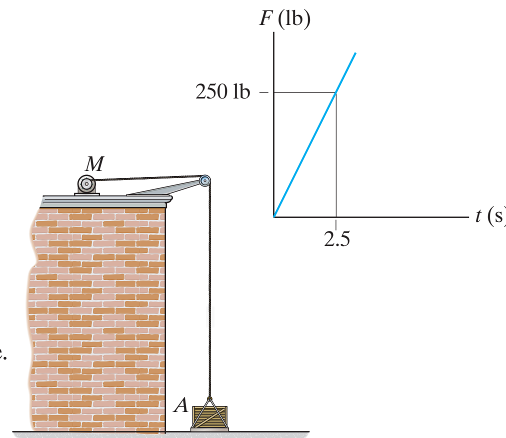
$$0 = (5)^2 + 2(-1.288)(s - 0)$$

$s = 9.70 \text{ ft}$ **Ans.**

Ans:
$a_A = 1.29 \text{ ft/s}^2$
$s = 9.70 \text{ ft}$

13–29.

The force exerted by the motor on the cable is shown in the graph. Determine the velocity of the 200-lb crate when $t = 2.5$ s.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. *a*.

Equilibrium: For the crate to move, force **F** must overcome the weight of the crate. Thus, the time required to move the crate is given by

$$+\uparrow \Sigma F_y = 0; \qquad 100t - 200 = 0 \qquad t = 2\text{ s}$$

Equation of Motion: For $2\text{ s} < t < 2.5\text{ s}$, $F = \dfrac{250}{2.5}t = (100t)$ lb. By referring to Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad 100t - 200 = \frac{200}{32.2}a$$

$$a = (16.1t - 32.2)\text{ ft/s}^2$$

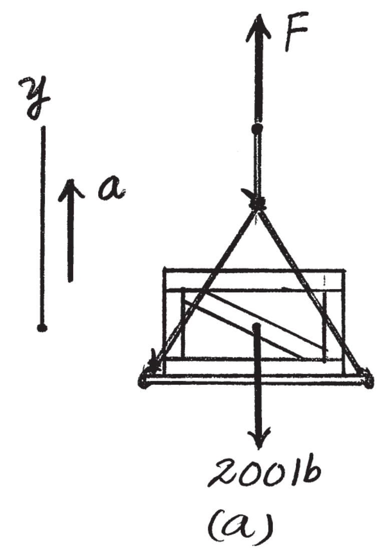
Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = a\,dt$. For $2\text{ s} \le t < 2.5\text{ s}$, $v = 0$ at $t = 2$ s will be used as the lower integration limit. Thus,

$$(+\uparrow) \qquad \int dv = \int a\,dt$$

$$\int_0^v dv = \int_{2\text{ s}}^t (16.1t - 32.2)dt$$

$$v = \left(8.05t^2 - 32.2t\right)\Big|_{2\text{ s}}^t$$

$$= \left(8.05t^2 - 32.2t + 32.2\right)\text{ft/s}$$



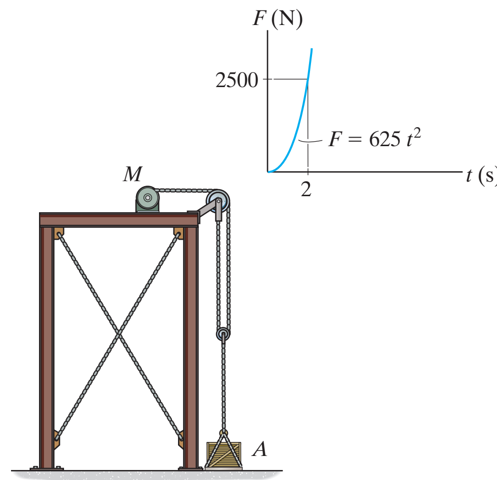
When $t = 2.5$ s,

$$v = 8.05(2.5^2) - 32.2(2.5) + 32.2 = 2.01\text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v = 2.01$ ft/s

13–30.

The force of the motor M on the cable is shown in the graph. Determine the velocity of the 400-kg crate A when $t = 2$ s.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. a.

Equilibrium: For the crate to move, force 2**F** must overcome its weight. Thus, the time required to move the crate is given by

$$+\uparrow \Sigma F_y = 0; \qquad 2(625t^2) - 400(9.81) = 0$$

$$t = 1.772 \text{ s}$$

Equations of Motion: $F = \left(625t^2\right)$ N. By referring to Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 2\left(625t^2\right) - 400(9.81) = 400a$$

$$a = (3.125t^2 - 9.81) \text{ m/s}^2$$

Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = a\,dt$. For $1.772 \text{ s} \le t < 2 \text{ s}$, $v = 0$ at $t = 1.772$ s will be used as the lower integration limit. Thus,

$$(+\uparrow) \qquad \int dv = \int a\,dt$$

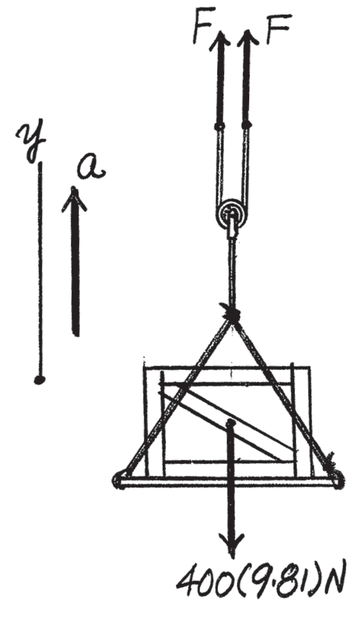
$$\int_0^v dv = \int_{1.772 \text{ s}}^t \left(3.125t^2 - 9.81\right)dt$$

$$v = \left(1.0417t^3 - 9.81t\right)\Big|_{1.772 \text{ s}}^t$$

$$= \left(1.0417t^3 - 9.81t + 11.587\right) \text{ m/s}$$

When $t = 2$ s,

$$v = 1.0417(2^3) - 9.81(2) + 11.587 = 0.301 \text{ m/s} \qquad \textbf{Ans.}$$

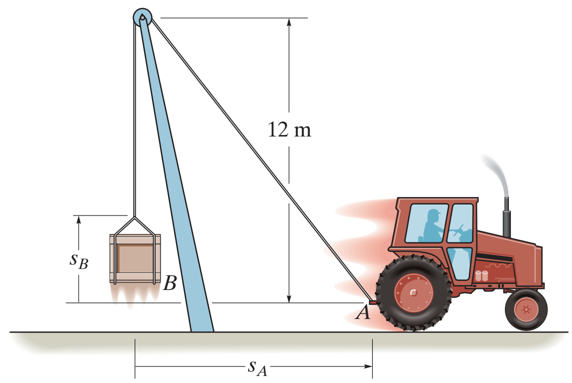


(a)

Ans:
$v = 0.301$ m/s

13–31.

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.



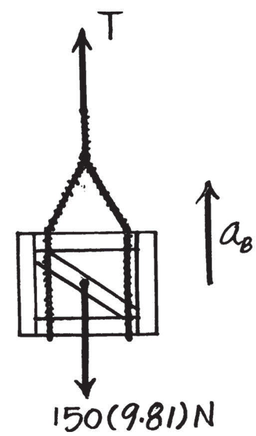
SOLUTION

$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-s_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$



$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$

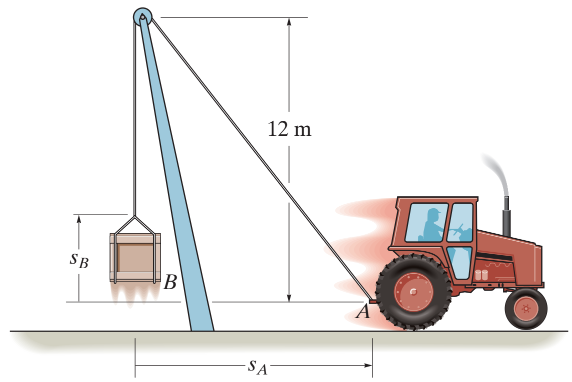
$+\uparrow \Sigma F_y = ma_y; \qquad T - 150(9.81) = 150(1.0487)$

$T = 1.63$ kN **Ans.**

Ans:
$T = 1.63$ kN

***13–32.**

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



SOLUTION

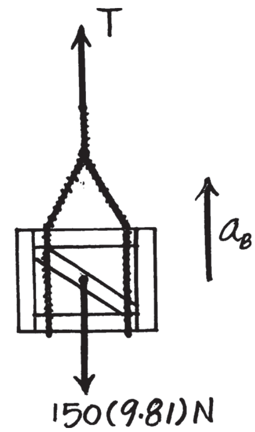
$$12 = s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \frac{1}{2}\left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(2s_A\dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A\dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A\ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2\,\dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A\ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{((5)^2 + 144)^{\frac{3}{2}}} - \frac{(4)^2 + (5)(3)}{((5)^2 + 144)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$$



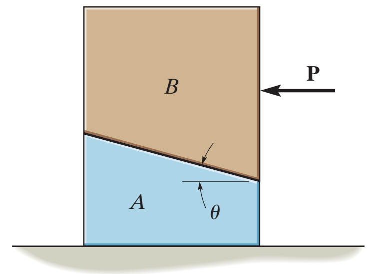
$+\uparrow \Sigma F_y = ma_y; \qquad T - 150(9.81) = 150(2.2025)$

$T = 1.80$ kN **Ans.**

Ans:
$T = 1.80$ kN

13–33.

Block A and B each have a mass m. Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that it will not slide on A. Also, what is the corresponding acceleration? The coefficient of static friction between A and B is μ_s. Neglect any friction between A and the horizontal surface.



SOLUTION

Equations of Motion. Since block B is required to be on the verge to slide on A, $F_f = \mu_s N_B$. Referring to the FBD of block B shown in Fig. a,

$+\uparrow \Sigma F_y = ma_y; \qquad N_B \cos\theta - \mu_s N_B \sin\theta - mg = m(0)$

$$N_B = \frac{mg}{\cos\theta - \mu_s \sin\theta} \qquad \textbf{(1)}$$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - N_B \sin\theta - \mu_s N_B \cos\theta = ma$

$$P - N_B(\sin\theta + \mu_s\cos\theta) = ma \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2),

$$P - \left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)mg = ma \qquad \textbf{(3)}$$

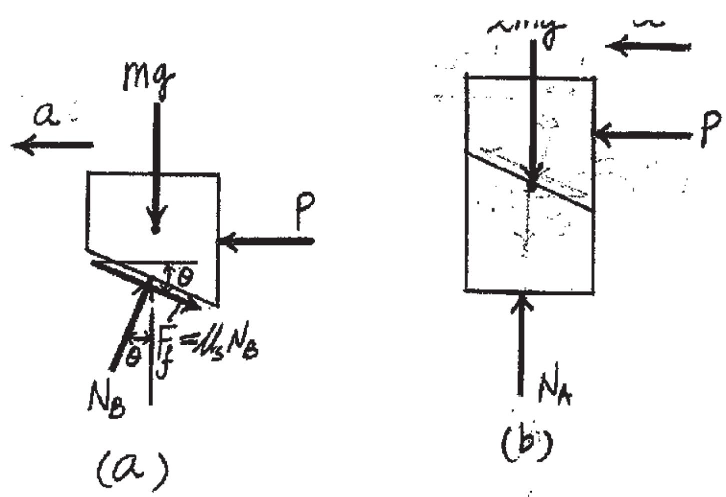
Referring to the FBD of blocks A and B shown in Fig. b

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P = 2\,ma \qquad \textbf{(4)}$

Solving Eqs. (2) into (3),

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) \qquad \textbf{Ans.}$$

$$a = \left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)g \qquad \textbf{Ans.}$$



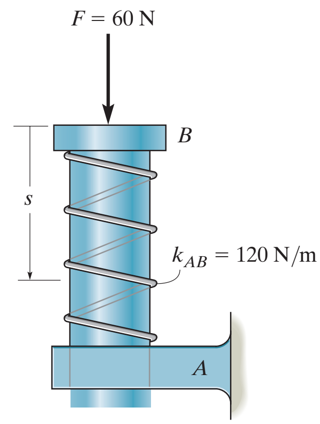
Ans:

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$a = \left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)g$$

13–34.

The 4-kg smooth cylinder is supported by the spring having a stiffness of $k_{AB} = 120$ N/m. Determine the velocity of the cylinder when it moves downward $s = 0.2$ m from its equilibrium position, which is caused by the application of the force $F = 60$ N.



SOLUTION

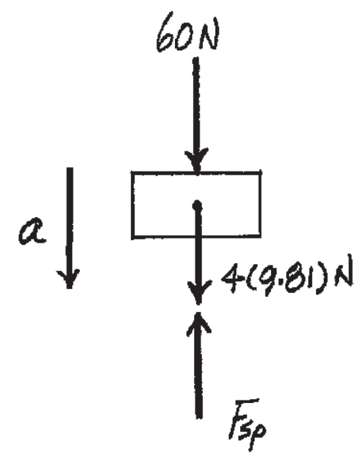
Equation of Motion. At the equilibrium position, realizing that $F_{sp} = kx_0 = 120x_0$ the compression of the spring can be determined from

$$+\uparrow \Sigma F_y = 0; \qquad 120x_0 - 4(9.81) = 0 \qquad x_0 = 0.327 \text{ m}$$

Thus, when 60 N force is applied, the compression of the spring is $x = s + x_0 = s + 0.327$. Thus, $F_{sp} = kx = 120(s + 0.327)$. Then, referring to the FBD of the collar shown in Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad 120(s + 0.327) - 60 - 4(9.81) = 4(-a)$$

$$a = \{15 - 30\,s\} \text{ m/s}^2$$



(a)

Kinematics. Using the result of **a** and integrate $\int v dv = a ds$ with the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v dv = \int_0^s (15 - 30\,s) ds$$

$$\frac{v^2}{2} = 15\,s - 15\,s^2$$

$$v = \left\{\sqrt{30(s - s^2)}\right\} \text{ m/s}$$

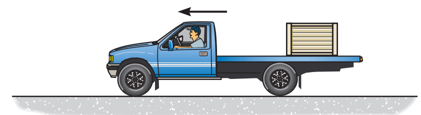
At $s = 0.2$ m,

$$v = \sqrt{30(0.2 - 0.2^2)} = 2.191 \text{ m/s} = 2.19 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 2.19$ m/s

13–35.

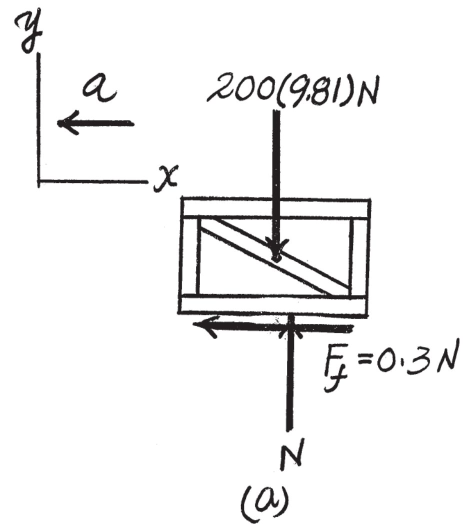
The coefficient of static friction between the 200-kg crate and the flat bed of the truck is $\mu_s = 0.3$. Determine the shortest time for the truck to reach a speed of 60 km/h, starting from rest with constant acceleration, so that the crate does not slip.



SOLUTION

Free-Body Diagram: When the crate accelerates with the truck, the frictional force F_f develops. Since the crate is required to be on the verge of slipping, $F_f = \mu_s N = 0.3N$.

Equations of Motion: Here, $a_y = 0$. By referring to Fig. *a*,



$$+\uparrow \Sigma F_y = ma_y; \qquad N - 200(9.81) = 200(0)$$

$$N = 1962 \text{ N}$$

$$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad -0.3(1962) = 200(-a)$$

$$a = 2.943 \text{ m/s}^2 \leftarrow$$

Kinematics: The final velocity of the truck is $v = \left(60\frac{\text{km}}{\text{h}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 16.67$ m/s. Since the acceleration of the truck is constant,

$$(\overset{+}{\leftarrow}) \qquad v = v_0 + a_c t$$

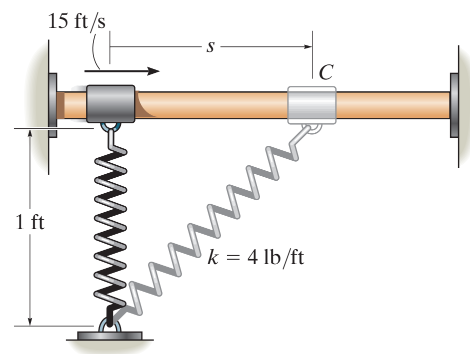
$$16.67 = 0 + 2.943t$$

$$t = 5.66 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 5.66$ s

***13–36.**

The 2-lb collar C fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



SOLUTION

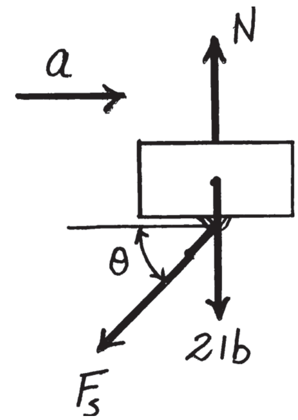
$F_s = kx; \qquad F_s = 4\left(\sqrt{1 + s^2} - 1\right)$

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -4\left(\sqrt{1 + s^2} - 1\right)\left(\dfrac{s}{\sqrt{1 + s^2}}\right) = \left(\dfrac{2}{32.2}\right)\left(v\dfrac{dv}{ds}\right)$

$$-\int_0^1 \left(4s\,ds - \frac{4s\,ds}{\sqrt{1 + s^2}}\right) = \int_{15}^{v} \left(\frac{2}{32.2}\right) v\,dv$$

$$-\left[2s^2 - 4\sqrt{1 + s^2}\right]_0^1 = \frac{1}{32.2}\left(v^2 - 15^2\right)$$

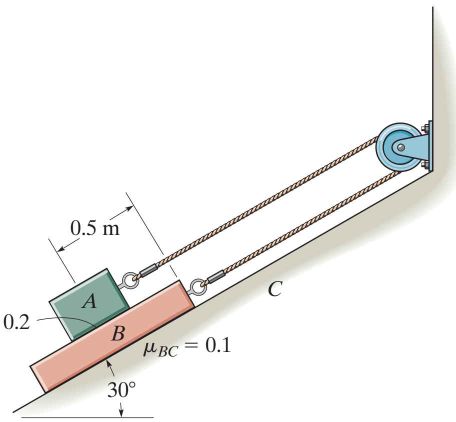
$v = 14.6$ ft/s **Ans.**



Ans:
$v = 14.6$ ft/s

13–37.

The 10-kg block A rests on the 50-kg plate B in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide 0.5 m *on the plate* when the system is released from rest.



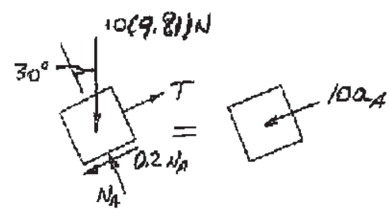
SOLUTION

Block A:

$+\nwarrow \Sigma F_y = ma_y;\quad N_A - 10(9.81)\cos 30° = 0 \qquad N_A = 84.96\text{ N}$

$+\swarrow \Sigma F_x = ma_x;\quad -T + 0.2(84.96) + 10(9.81)\sin 30° = 10a_A$

$T - 66.04 = -10a_A \qquad \textbf{(1)}$



Block B:

$+\nwarrow \Sigma F_y = ma_y;\quad N_B - 84.96 - 50(9.81)\cos 30° = 0$

$N_B = 509.7\text{ N}$

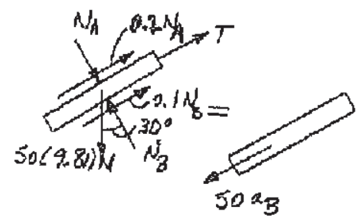
$+\swarrow \Sigma F_x = ma_x;\quad -0.2(84.96) - 0.1(509.7) - T + 50(9.81\sin 30°) = 50a_B$

$177.28 - T = 50a_B \qquad \textbf{(2)}$

$s_A + s_B = l$

$\Delta s_A = -\Delta s_B$

$a_A = -a_B \qquad \textbf{(3)}$



Solving Eqs. (1) – (3):

$a_B = 1.854\text{ m/s}^2$

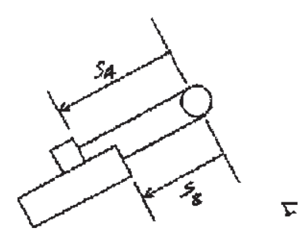
$a_A = -1.854\text{ m/s}^2 \qquad T = 84.58\text{ N}$

In order to slide 0.5 m along the plate the block must move 0.25 m. Thus,

$(+\swarrow)\quad s_B = s_A + s_{B/A}$

$-\Delta s_A = \Delta s_A + 0.5$

$\Delta s_A = -0.25\text{ m}$



$(+\swarrow)\quad s_A = s_0 + v_0 t + \frac{1}{2}a_A t^2$

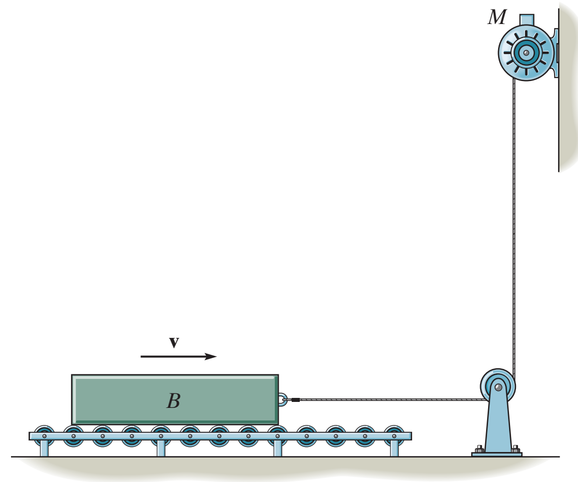
$$-0.25 = 0 + 0 + \frac{1}{2}(-1.854)t^2$$

$t = 0.519\text{ s}$ **Ans.**

Ans:
$t = 0.519\text{ s}$

13–38.

The 300-kg bar B, originally at rest, is being towed over a series of small rollers. Determine the force in the cable when $t = 5$ s, if the motor M is drawing in the cable for a short time at a rate of $v = (0.4t^2)$ m/s, where t is in seconds $(0 \leq t \leq 6\text{ s})$. How far does the bar move in 5 s? Neglect the mass of the cable, pulley, and the rollers.



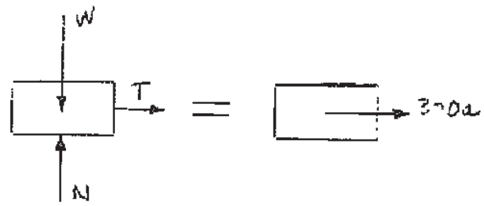
SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad T = 300a$

$v = 0.4t^2$

$$a = \frac{dv}{dt} = 0.8t$$

When $t = 5$ s, $a = 4\text{ m/s}^2$



$T = 300(4) = 1200\text{ N} = 1.20\text{ kN}$ **Ans.**

$ds = v\,dt$

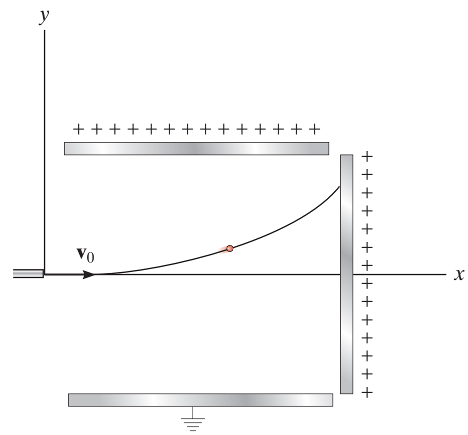
$$\int_0^s ds = \int_0^5 0.4t^2\,ds$$

$$s = \left(\frac{0.4}{3}\right)(5)^3 = 16.7\text{ m}$$ **Ans.**

Ans:
$s = 16.7$ m

13–39.

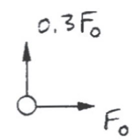
An electron of mass m is discharged with an initial horizontal velocity of $\mathbf{v}_0$. If it is subjected to two fields of force for which $F_x = F_0$ and $F_y = 0.3F_0$, where F_0 is constant, determine the equation of the path, and the speed of the electron at any time t.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 = ma_x$$

$$+\uparrow \Sigma F_y = ma_y; \qquad 0.3\,F_0 = ma_y$$



Thus,

$$\int_{v_0}^{v_x} dv_x = \int_0^t \frac{F_0}{m}\,dt$$

$$v_x = \frac{F_0}{m}t + v_0$$

$$\int_0^{v_y} dv_y = \int_0^t \frac{0.3F_0}{m}\,dt \qquad v_y = \frac{0.3F_0}{m}t$$

$$v = \sqrt{\left(\frac{F_0}{m}t + v_0\right)^2 + \left(\frac{0.3F_0}{m}t\right)^2}$$

$$= \frac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0tmv_0 + m^2v_0^2} \qquad \textbf{Ans.}$$

$$\int_0^x dx = \int_0^t \left(\frac{F_0}{m}t + v_0\right)dt$$

$$x = \frac{F_0t^2}{2m} + v_0\,t$$

$$\int_0^y dy = \int_0^t \frac{0.3F_0}{m}\,t\,dt$$

$$y = \frac{0.3F_0t^2}{2m}$$

$$t = \left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$

$$x = \frac{F_0}{2m}\left(\frac{2m}{0.3F_0}\right)y + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$

$$x = \frac{y}{0.3} + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}} \qquad \textbf{Ans.}$$

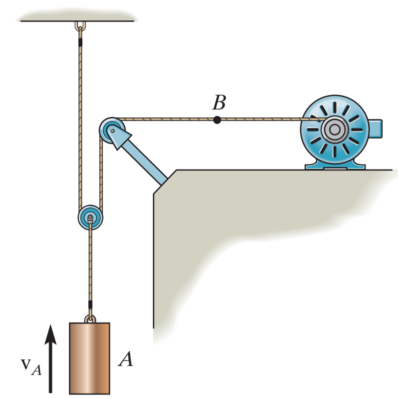
Ans:

$$v = \frac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0tmv_0 + m^2v_0^2}$$

$$x = \frac{y}{0.3} + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{1/2}$$

***13–40.**

The 400-lb cylinder at A is hoisted using the motor and the pulley system shown. If the speed of point B on the cable is increased at a constant rate from zero to $v_B = 10\text{ ft/s}$ in $t = 5\text{ s}$, determine the tension in the cable at B to cause the motion.



SOLUTION

$2s_A + s_B = l$

$2a_A = -a_B$

$(\overset{+}{\rightarrow})\quad v = v_0 + a_B t$

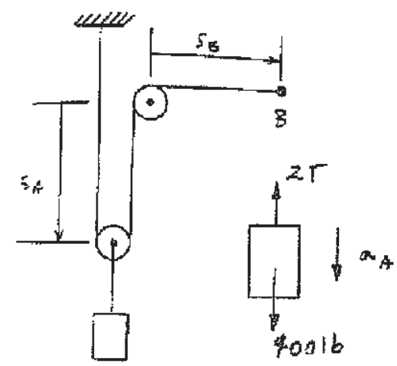
$10 = 0 + a_B(5)$

$a_B = 2\text{ ft/s}^2$

$a_A = -1\text{ ft/s}^2$

$+\downarrow \Sigma F_y = ma_y;\qquad 400 - 2T = \left(\dfrac{400}{32.2}\right)(-1)$

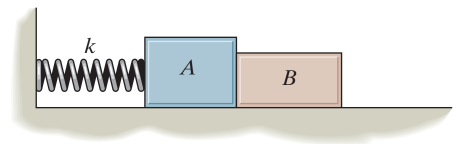
Thus, $\quad T = 206\text{ lb}$ **Ans.**



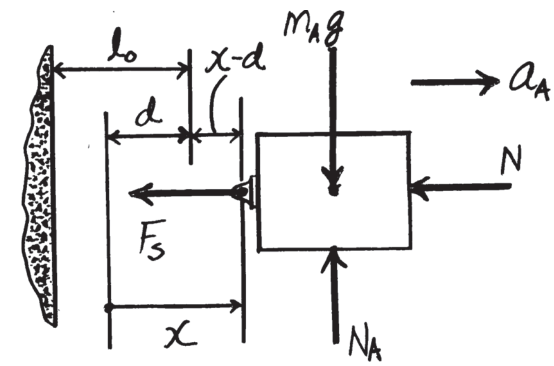
Ans:
$T = 206\text{ lb}$

13–41.

Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?



SOLUTION



Block A:

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad -k(x-d) - N = m_A a_A$

Block B:

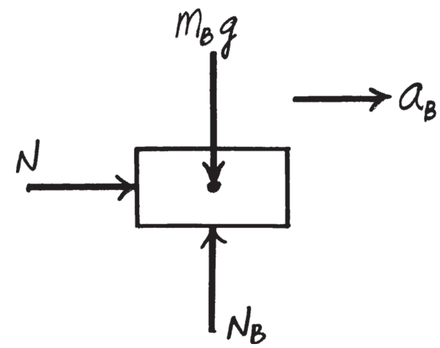
$\xrightarrow{+} \Sigma F_x = ma_x;\qquad N = m_B a_B$

Since $a_A = a_B = a$,

$-k(x-d) - m_B a = m_A a$

$$a = \frac{k(d-x)}{(m_A + m_B)} \qquad N = \frac{km_B(d-x)}{(m_A+m_B)}$$

$N = 0$ when $d - x = 0$, or $x = d$ **Ans.**



$v\,dv = a\,dx$

$$\int_0^v v\,dv = \int_0^d \frac{k(d-x)}{(m_A+m_B)}dx$$

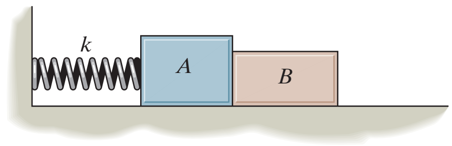
$$\frac{1}{2}v^2 = \frac{k}{(m_A+m_B)}\left[(d)x - \frac{1}{2}x^2\right]_0^d = \frac{1}{2}\frac{kd^2}{(m_A+m_B)}$$

$$v = \sqrt{\frac{kd^2}{(m_A+m_B)}}$$ **Ans.**

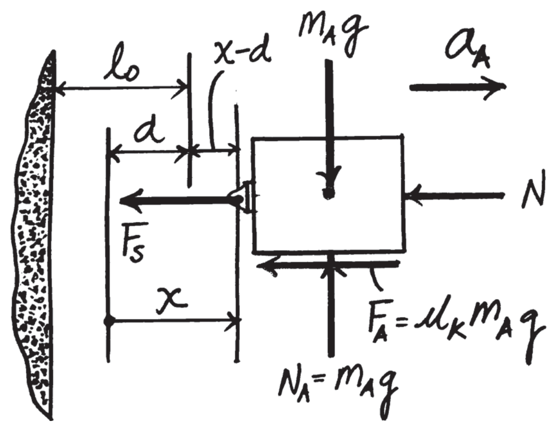
Ans:
$x = d$
$v = \sqrt{\dfrac{kd^2}{m_A + m_B}}$

13–42.

Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?



SOLUTION



Block A:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -k(x-d) - N - \mu_k m_A g = m_A a_A$$

Block B:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad N - \mu_k m_B g = m_B a_B$$

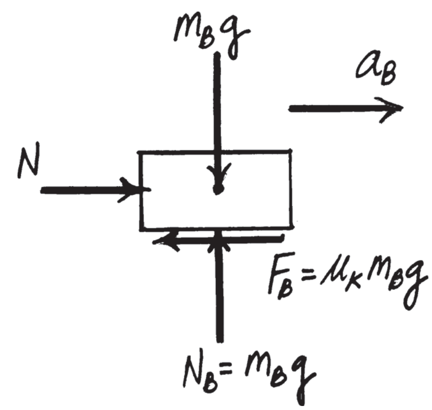
Since $a_A = a_B = a$,

$$a = \frac{k(d-x) - \mu_k g(m_A + m_B)}{(m_A + m_B)} = \frac{k(d-x)}{(m_A + m_B)} - \mu_k g$$

$$N = \frac{k m_B (d-x)}{(m_A + m_B)}$$

$N = 0$, then $x = d$ for separation. **Ans.**

At the moment of separation:



$$v\,dv = a\,dx$$

$$\int_0^v v\,dv = \int_0^d \left[\frac{k(d-x)}{(m_A + m_B)} - \mu_k g\right] dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2 - \mu_k g\,x\right]_0^d$$

$$v = \sqrt{\frac{kd^2 - 2\mu_k g(m_A + m_B)d}{(m_A + m_B)}}$$

Require $v > 0$, so that

$$kd^2 - 2\mu_k g(m_A + m_B)d > 0$$

Thus,

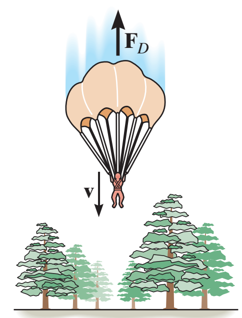
$$kd > 2\mu_k g(m_A + m_B)$$

$$d > \frac{2\mu_k g}{k}(m_A + m_B)$$ **Q.E.D.**

Ans:
$x = d$ for separation.

13–43.

A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.



SOLUTION

$+\downarrow \Sigma F_z = ma_z;$ $\qquad mg - kv^2 = m\dfrac{dv}{dt}$

$$m\int_0^v \frac{m\,dv}{(mg - kv^2)} = \int_0^t dt$$

$$\frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k} - v^2} = t$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right)\ln\left[\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}\right]_0^v = t$$

$$\frac{k}{m}t\left(2\sqrt{\frac{mg}{k}}\right) = \ln\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$e^{2t\sqrt{\frac{mg}{k}}} = \frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$\sqrt{\frac{mg}{k}}e^{2t\sqrt{\frac{mg}{k}}} - v\,e^{2t\sqrt{\frac{mg}{k}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{mg}{k}}} - 1}{e^{2t\sqrt{\frac{mg}{k}}} + 1}\right] \qquad \textbf{Ans.}$$

When $t \rightarrow \infty$ $\qquad v_t = \sqrt{\dfrac{mg}{k}}$ $\qquad$ **Ans.**

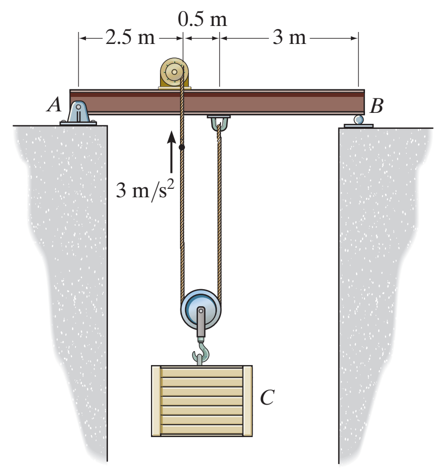
Ans:

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{mg/k}} - 1}{e^{2t\sqrt{mg/k}} + 1}\right]$$

$$v_t = \sqrt{\frac{mg}{k}}$$

***13–44.**

If the motor draws in the cable with an acceleration of $3\,\text{m/s}^2$, determine the reactions at the supports A and B. The beam has a uniform mass of 30 kg/m, and the crate has a mass of 200 kg. Neglect the mass of the motor and pulleys.



SOLUTION

$S_c + (S_c - S_p)$

$2v_c = v_p$

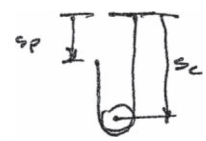
$2a_c = a_p$

$2a_c = 3\ \text{m/s}^2$

$a_c = 1.5\ \text{m/s}^2$

$+\uparrow \Sigma F_y = ma_y \qquad 2T - 1962 = 200(1.5)$

$T = 1{,}131\ \text{N}$



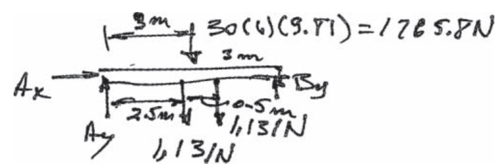
$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(6) - (1765.8 + 1{,}131)3 - (1{,}131)(2.5) = 0$

$B_y = 1{,}919.65\ \text{N} = 1.92\ \text{kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y - 1765.8 - (2)(1{,}131) + 1919.65 = 0$

$A_y = 2108.15\ \text{N} = 2.11\ \text{kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**



13–45.

If the force exerted on cable AB by the motor is $F = (100t^{3/2})$ N, where t is in seconds, determine the 50-kg crate's velocity when $t = 5$ s. The coefficients of static and kinetic friction between the crate and the ground are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. Initially the crate is at rest.

SOLUTION

Free-Body Diagram: The frictional force $\mathbf{F}_f$ is required to act to the left to oppose the motion of the crate which is to the right.

Equations of Motion: Here, $a_y = 0$. Thus,

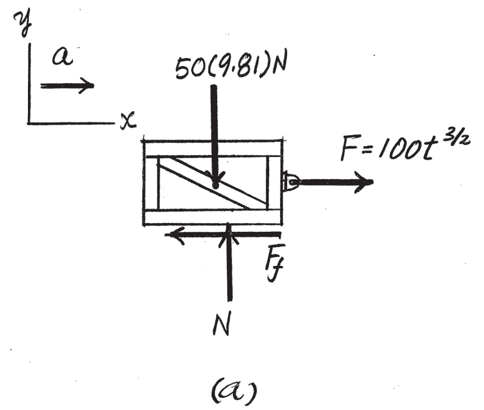
$$+\uparrow \Sigma F_y = ma_y; \qquad N - 50(9.81) = 50(0)$$

$$N = 490.5 \text{ N}$$

Realizing that $F_f = \mu_k N = 0.3(490.5) = 147.15$ N,

$$+\uparrow \Sigma F_x = ma_x; \qquad 100t^{3/2} - 147.15 = 50a$$

$$a = \left(2t^{3/2} - 2.943\right) \text{ m/s}$$



Equilibrium: For the crate to move, force $\mathbf{F}$ must overcome the static friction of $F_f = \mu_s N = 0.4(490.5) = 196.2$ N. Thus, the time required to cause the crate to be on the verge of moving can be obtained from.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 100t^{3/2} - 196.2 = 0$$

$$t = 1.567 \text{ s}$$

Kinematics: Using the result of **a** and integrating the kinematic equation $dv = a\,dt$ with the initial condition $v = 0$ at $t = 1.567$ as the lower integration limit,

$$(\overset{+}{\rightarrow}) \qquad \int dv = \int a dt$$

$$\int_0^v dv = \int_{1.567 \text{ s}}^t \left(2t^{3/2} - 2.943\right)dt$$

$$v = \left(0.8t^{5/2} - 2.943t\right)\Big|_{1.567 \text{ s}}^t$$

$$v = \left(0.8t^{5/2} - 2.943t + 2.152\right) \text{ m/s}$$

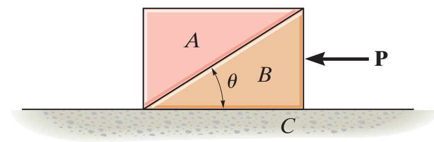
When $t = 5$ s,

$$v = 0.8(5)^{5/2} - 2.943(5) + 2.152 = 32.16 \text{ ft/s} = 32.2 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v = 32.2$ ft/s

13–46.

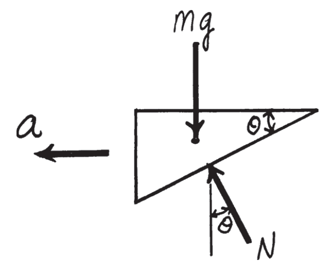
Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not move relative to B. All surfaces are smooth.



SOLUTION

Require

$a_A = a_B = a$



Block A:

$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad N\sin\theta = ma$

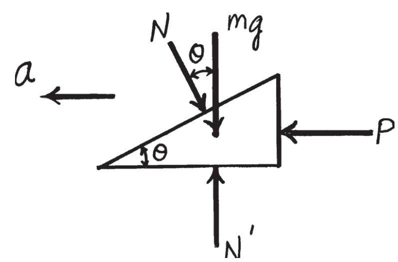
$a = g\tan\theta$

Block B:

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - N\sin\theta = ma$

$P - mg\tan\theta = mg\tan\theta$

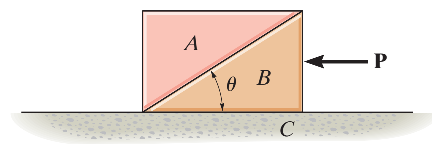
$P = 2mg\tan\theta$ **Ans.**



Ans:
$P = 2mg\tan\theta$

13–47.

Blocks A and B each have a mass m. Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that A will not slip on B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.

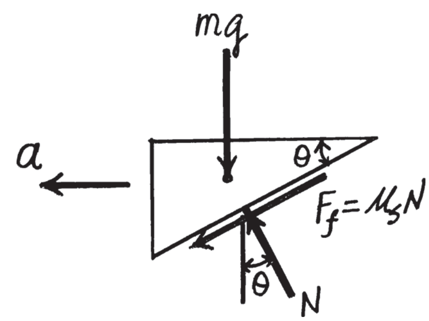


SOLUTION

Require

$a_A = a_B = a$

Block A:



$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - \mu_s N\sin\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad N\sin\theta + \mu_s N\cos\theta = ma$

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

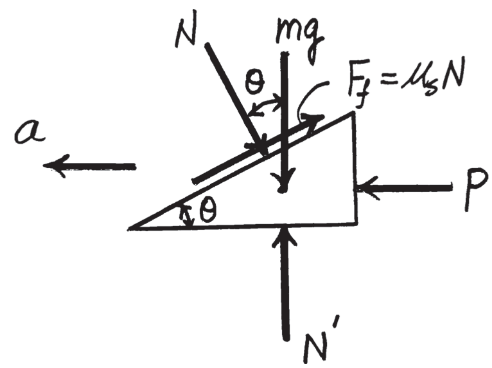
$$a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

Block B:

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - \mu_s N\cos\theta - N\sin\theta = ma$

$$P - mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) \qquad \textbf{Ans.}$$

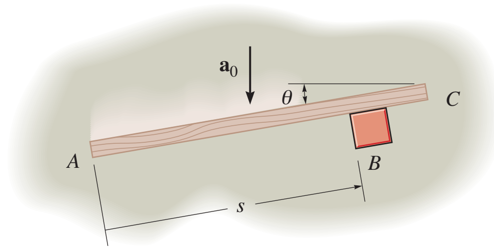


Ans:

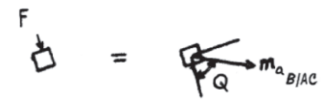
$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

***13–48.**

The smooth block B of negligible size has a mass m and rests on the horizontal plane. If the board AC pushes on the block at an angle θ with a constant acceleration $\mathbf{a}_0$, determine the velocity of the block along the board and the distance s the block moves along the board as a function of time t. The block starts from rest when $s = 0$, $t = 0$.



SOLUTION



$\nearrow + \Sigma F_x = m\,a_x;\qquad 0 = m\,a_B \sin\phi$

$$\mathbf{a}_B = \mathbf{a}_{AC} + \mathbf{a}_{B/AC}$$

$$\mathbf{a}_B = \mathbf{a_0} + \mathbf{a}_{B/AC}$$

$\nearrow +\qquad a_B \sin\phi = -a_0 \sin\theta + a_{B/AC}$

Thus,

$$0 = m(-a_0 \sin\theta + a_{B/AC})$$

$$a_{B/AC} = a_0 \sin\theta$$

$$\int_0^{v_{B/AC}} dv_{B/AC} = \int_0^t a_0 \sin\theta\, dt$$

$$v_{B/AC} = a_0 \sin\theta\, t \qquad \textbf{Ans.}$$

$$s_{B/AC} = s = \int_0^t a_0 \sin\theta\, t\, dt$$

$$s = \frac{1}{2} a_0 \sin\theta\, t^2 \qquad \textbf{Ans.}$$

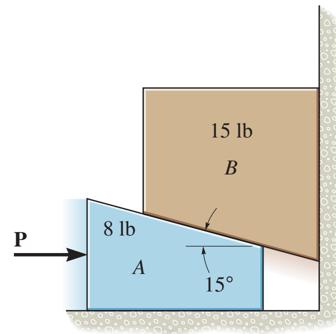
Ans:

$v_{B/AC} = a_0 \sin\theta\, t$

$s = \frac{1}{2} a_0 \sin\theta\, t^2$

13–49.

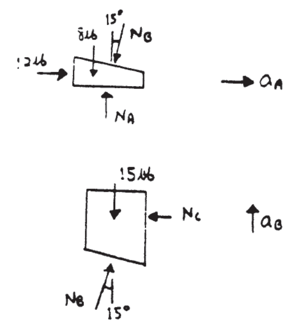
If a horizontal force $P = 12$ lb is applied to block A determine the acceleration of block B. Neglect friction.



SOLUTION

Block A:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 12 - N_B \sin 15^\circ = \left(\frac{8}{32.2}\right)a_A \qquad \textbf{(1)}$$



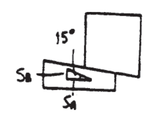
Block B:

$$+\uparrow \Sigma F_y = ma_y; \qquad N_B \cos 15^\circ - 15 = \left(\frac{15}{32.2}\right)a_B \qquad \textbf{(2)}$$

$$s_B = s_A \tan 15^\circ$$

$$a_B = a_A \tan 15^\circ \qquad \textbf{(3)}$$

Solving Eqs. (1)–(3)



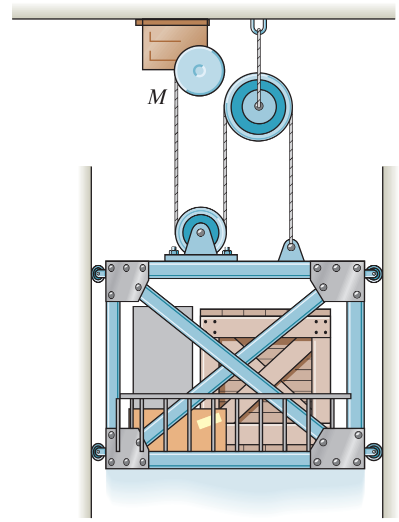
$$a_A = 28.3 \text{ ft/s}^2 \qquad N_B = 19.2 \text{ lb}$$

$$a_B = 7.59 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_B = 7.59 \text{ ft/s}^2$

13–50.

A freight elevator, including its load, has a mass of 1 Mg. It is prevented from rotating due to the track and wheels mounted along its sides. If the motor M develops a constant tension $T = 4$ kN in its attached cable, determine the velocity of the elevator when it has moved upward 6 m starting from rest. Neglect the mass of the pulleys and cables.



SOLUTION

Equation of Motion. Referring to the FBD of the freight elevator shown in Fig. a,

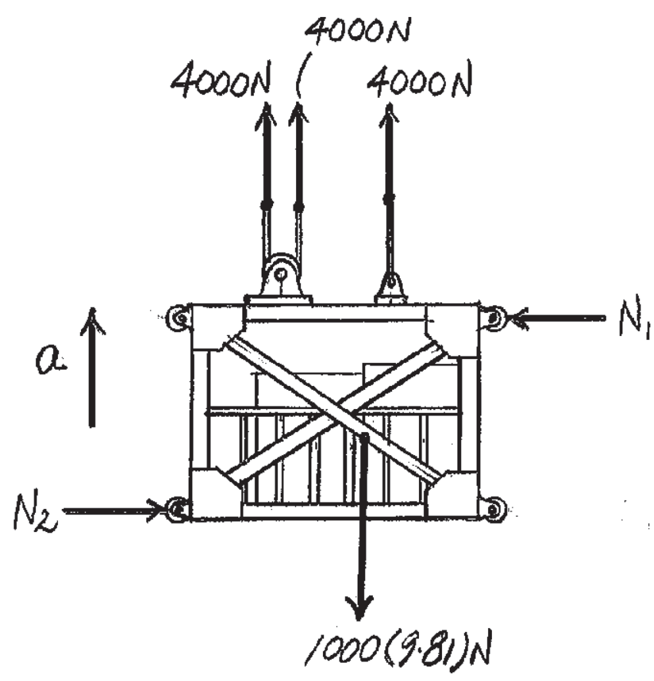
$$+\uparrow \Sigma F_y = ma_y; \qquad 3(4000) - 1000(9.81) = 1000a$$

$$a = 2.19 \text{ m/s}^2 \uparrow$$

Kinematics. Using the result of **a**,

$$(+\uparrow) \qquad v^2 = v_0^2 + 2as; \qquad v^2 = 0^2 + 2(2.19)(6)$$

$$v = 5.126 \text{ m/s} = 5.13 \text{ m/s} \qquad \textbf{Ans.}$$

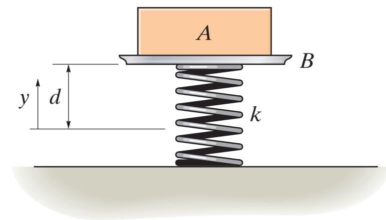


(a)

Ans:
$v = 5.13$ m/s

13–51.

The block A has a mass m_A and rests on the pan B, which has a mass m_B. Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.

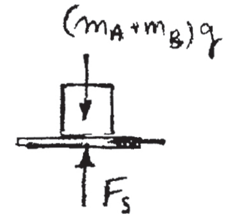


SOLUTION

For Equilibrium

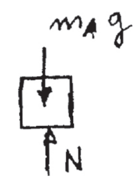
$$+\uparrow \Sigma F_y = ma_y; \qquad F_s = (m_A + m_B)g$$

$$y_{eq} = \frac{F_s}{k} = \frac{(m_A + m_B)g}{k}$$



Block:

$$+\uparrow \Sigma F_y = ma_y; \qquad -m_A g + N = m_A a$$



Block and pan

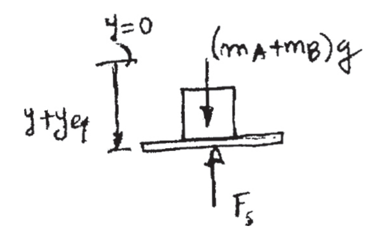
$$+\uparrow \Sigma F_y = ma_y; \qquad -(m_A + m_B)g + k(y_{eq} + y) = (m_A + m_B)a$$

Thus,

$$-(m_A + m_B)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = (m_A + m_B)\left(\frac{-m_A g + N}{m_A}\right)$$

Require $y = d, N = 0$

$$kd = -(m_A + m_B)g$$



Since d is downward,

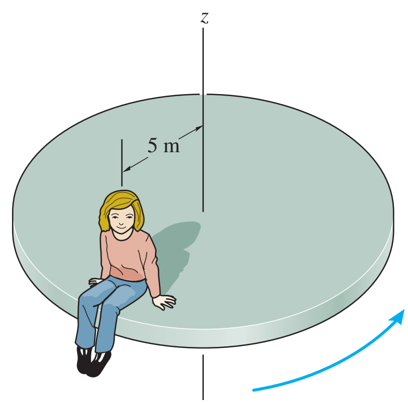
$$d = \frac{(m_A + m_B)g}{k} \qquad \textbf{Ans.}$$

Ans:

$$d = \frac{(m_A + m_B)g}{k}$$

***13–52.**

A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.



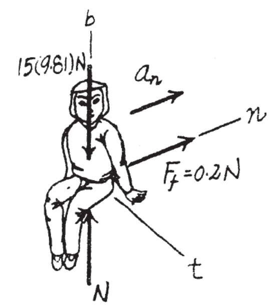
SOLUTION

Equation of Motion: Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15\text{ N}$$

$$\Sigma F_n = ma_n; \qquad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$$

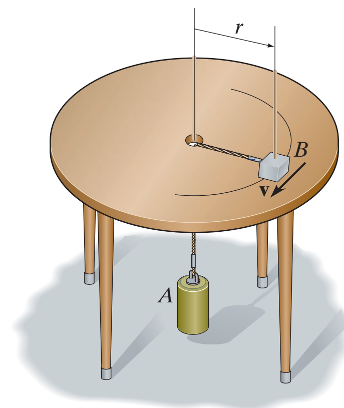
$$v = 3.13\text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 3.13\text{ m/s}$

13–53.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block is given a speed of $v = 10\,\text{m/s}$, determine the radius r of the circular path along which it travels.



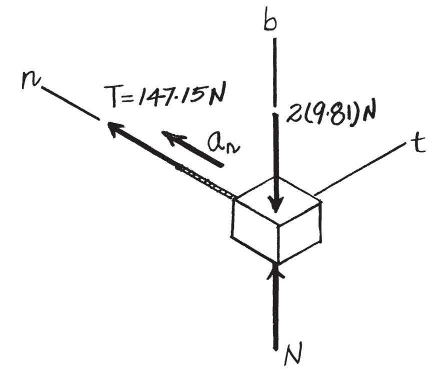
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{ N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{10^2}{r}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{10^2}{r}\right)$$

$$r = 1.36\text{ m}$$ **Ans.**

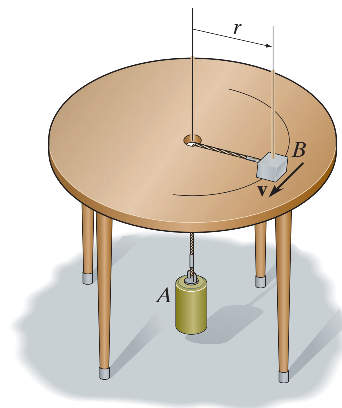


(a)

Ans:
$r = 1.36\text{ m}$

13–54.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5\,\text{m}$, determine the speed of the block.



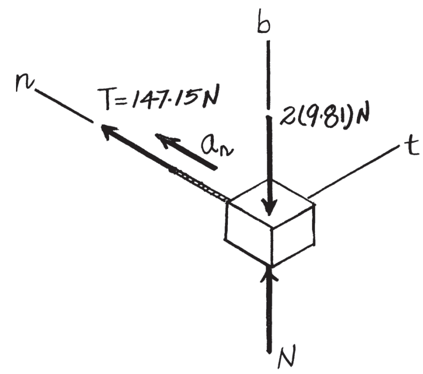
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{ N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{r} = \dfrac{v^2}{1.5}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{v^2}{1.5}\right)$$

$$v = 10.5\text{ m/s} \qquad \textbf{Ans.}$$

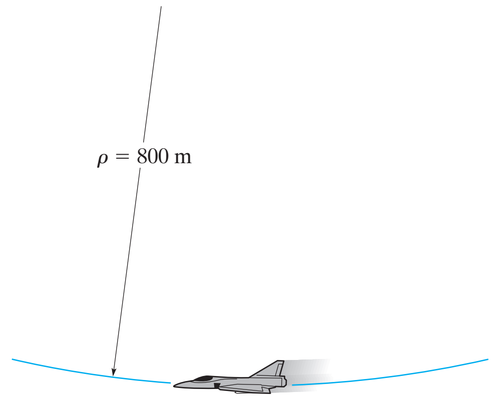


(a)

Ans:
$v = 10.5\text{ m/s}$

13–55.

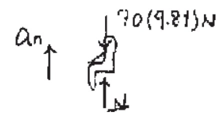
Determine the maximum constant speed at which the pilot can travel around the vertical curve having a radius of curvature $\rho = 800$ m, so that he experiences a maximum acceleration $a_n = 8g = 78.5\ \text{m/s}^2$. If he has a mass of 70 kg, determine the normal force he exerts on the seat of the airplane when the plane is traveling at this speed and is at its lowest point.



SOLUTION

$$a_n = \frac{v^2}{\rho}; \qquad 78.5 = \frac{v^2}{800}$$

$v = 251$ m/s **Ans.**



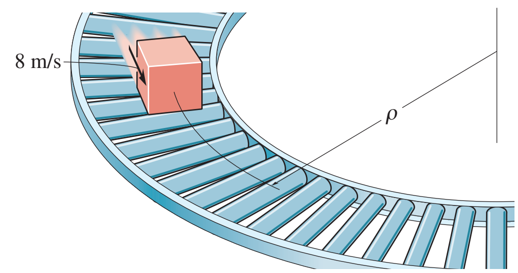
$+\uparrow \Sigma F_n = ma_n; \qquad N - 70(9.81) = 70(78.5)$

$N = 6.18$ kN **Ans.**

Ans:
$N = 6.18$ kN

***13–56.**

Cartons having a mass of 5 kg are required to move along the assembly line at a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.



SOLUTION

$+\uparrow \Sigma F_b = m\, a_b; \qquad N - W = 0$

$$N = W$$

$$F_x = 0.7W$$

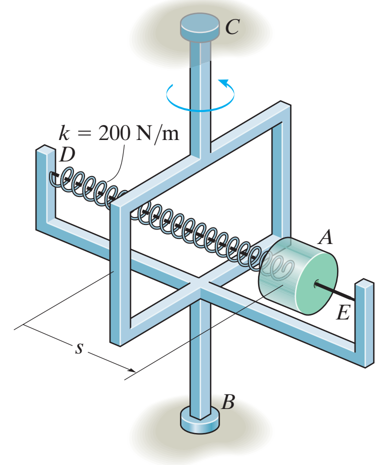
$\overset{+}{\leftarrow} \Sigma F_n = m a_n; \qquad 0.7W = \dfrac{W}{9.81}\left(\dfrac{8^2}{\rho}\right)$

$$\rho = 9.32 \text{ m}$$ **Ans.**

Ans:
$\rho = 9.32$ m

13–57.

The collar A, having a mass of 0.75 kg, is attached to a spring having a stiffness of $k = 200$ N/m. When rod BC rotates about the vertical axis, the collar slides outward along the smooth rod DE. If the spring is unstretched when $s = 0$, determine the constant speed of the collar in order that $s = 100$ mm. Also, what is the normal force of the rod on the collar? Neglect the size of the collar.



SOLUTION

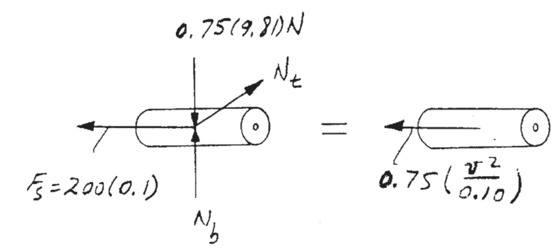
$\Sigma F_b = 0;$ $\quad N_b - 0.75(9.81) = 0 \quad N_b = 7.36$

$\Sigma F_n = ma_n;$ $\quad 200(0.1) = 0.75\left(\dfrac{v^2}{0.10}\right)$

$\Sigma F_t = ma_t;$ $\quad N_t = 0$

$v = 1.63$ m/s **Ans.**

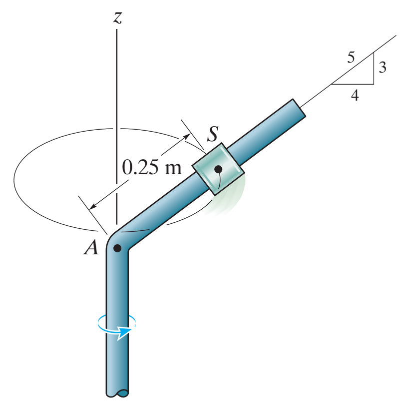
$N = \sqrt{(7.36)^2 + (0)} = 7.36$ N **Ans.**



Ans:
$v = 1.63$ m/s
$N = 7.36$ N

13–58.

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.



SOLUTION

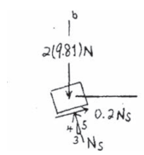
$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $$N_s\left(\frac{3}{5}\right) - 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

$+\uparrow \Sigma F_b = m\,a_b;$ $$N_s\left(\frac{4}{5}\right) + 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$$N_s = 21.3 \text{ N}$$

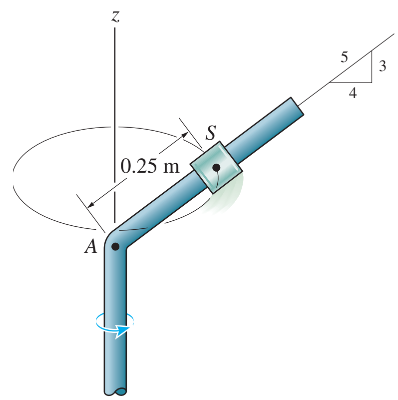
$$v = 0.969 \text{ m/s}$$ **Ans.**



Ans:
$v = 0.969$ m/s

13–59.

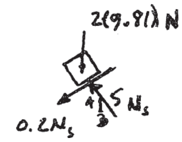
The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.



SOLUTION

$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $\quad N_s\left(\frac{3}{5}\right) + 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$



$+\uparrow \Sigma F_b = m\,a_b;$ $\quad N_s\left(\frac{4}{5}\right) - 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$

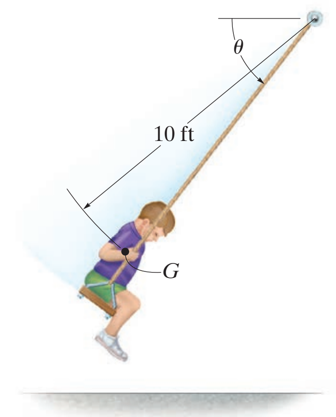
$$N_s = 28.85 \text{ N}$$

$$v = 1.48 \text{ m/s}$$ **Ans.**

Ans:
$v = 1.48$ m/s

***13–60.**

At the instant $\theta = 60°$, the boy's center of mass G has a downward speed $v_G = 15\text{ ft/s}$. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



SOLUTION

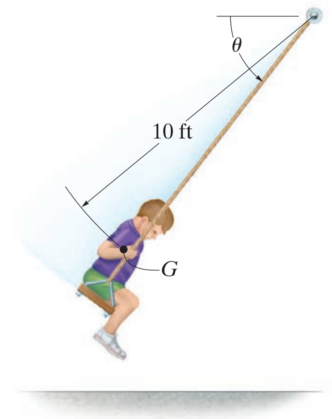
$+\searrow \Sigma F_t = ma_t;\qquad 60\cos 60° = \dfrac{60}{32.2}a_t \qquad a_t = 16.1\text{ ft/s}^2$ **Ans.**

$\nearrow + \Sigma F_n = ma_n;\qquad 2T - 60\sin 60° = \dfrac{60}{32.2}\left(\dfrac{15^2}{10}\right) \qquad T = 46.9\text{ lb}$ **Ans.**

Ans:
$a_t = 16.1\text{ ft/s}^2$
$T = 46.9\text{ lb}$

13–61.

At the instant $\theta = 60^\circ$, the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = 90^\circ$. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.



SOLUTION

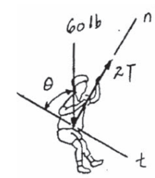
$+\searrow\Sigma_t = ma_t; \qquad 60\cos\theta = \dfrac{60}{32.2}a_t \qquad a_t = 32.2\cos\theta$

$\nearrow + \Sigma F_n = ma_n; \qquad 2T - 60\sin\theta = \dfrac{60}{32.2}\left(\dfrac{v^2}{10}\right)$ **(1)**

$v\,dv = a\,ds \qquad$ however $ds = 10d\theta$

$$\int_0^v v\,dv = \int_{60^\circ}^{90^\circ} 322\cos\theta\,d\theta$$

$v = 9.289\text{ ft/s}$ **Ans.**



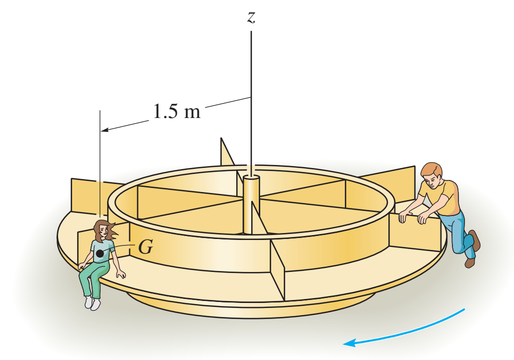
From Eq. (1)

$2T - 60\sin 90^\circ = \dfrac{60}{32.2}\left(\dfrac{9.289^2}{10}\right) \qquad T = 38.0\text{ lb}$ **Ans.**

Ans:
$v = 9.29\text{ ft/s}$
$T = 38.0\text{ lb}$

13–62.

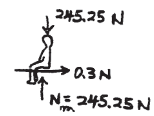
A girl having a mass of 25 kg sits at the edge of the merry-go-round so her center of mass G is at a distance of 1.5 m from the axis of rotation. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which she can have before she begins to slip off the merry-go-round. The coefficient of static friction between the girl and the merry-go-round is $\mu_s = 0.3$.



SOLUTION

$\xrightarrow{+} \Sigma F_n = ma_n; \qquad 0.3(245.25) = 25\left(\frac{v^2}{1.5}\right)$

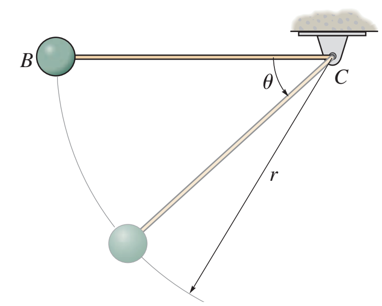
$v = 2.10 \text{ m/s}$ **Ans.**



Ans:
$v = 2.10 \text{ m/s}$

13–63.

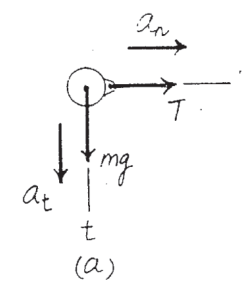
The pendulum bob B has a weight of 5 lb and is released from rest in the position shown, $\theta = 0°$. Determine the tension in string BC just after the bob is released, $\theta = 0°$, and also at the instant the bob reaches point D, $\theta = 45°$. Take $r = 3$ ft.



SOLUTION

Equation of Motion: Since the bob is just being release, $v = 0$. Applying Eq. 13–8 to FBD(a), we have

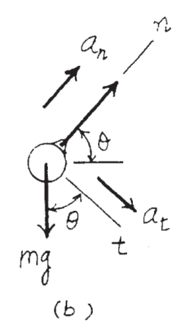
$$\Sigma F_n = ma_n; \qquad T = \frac{5}{32.2}\left(\frac{0^2}{3}\right) = 0 \qquad \textbf{Ans.}$$



Applying Eq. 13–8 to FBD(b), we have

$$\Sigma F_t = ma_t; \qquad 5\cos\theta = \frac{5}{32.2}a_t \qquad a_t = 32.2\cos\theta$$

$$\Sigma F_n = ma_n; \qquad T - 5\sin\theta = \frac{5}{32.2}\left(\frac{v^2}{3}\right) \qquad [1]$$



Kinematics: The speed of the bob at the instant when $\theta = 45°$ can be determined using $v\,dv = a_t\,ds$. However, $ds = 3d\theta$, then $v\,dv = 3a_t\,d\theta$.

$$\int_0^v v\,dv = 3(32.2)\int_0^{45°}\cos\theta\,d\theta$$

$$v^2 = 136.61 \text{ ft}^2/\text{s}^2$$

Substitute $\theta = 45°$ and $v^2 = 136.61\text{ ft}^2/\text{s}^2$ into Eq. [1] yields

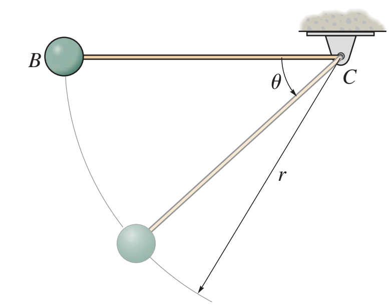
$$T - 5\sin 45° = \frac{5}{32.2}\left(\frac{136.61}{3}\right)$$

$$T = 10.6 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$T = 0$
$T = 10.6$ lb

***13–64.**

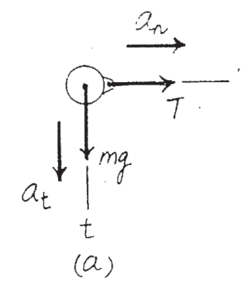
The pendulum bob B has a mass m and is released from rest when $\theta = 0°$. Determine the tension in string BC immediately afterwards, and also at the instant the bob reaches the arbitrary position θ.



SOLUTION

Equation of Motion: Since the bob is just being release, $v = 0$. Applying Eq. 13–8 to FBD(a), we have

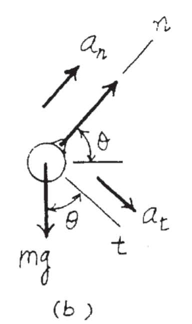
$$\Sigma F_n = ma_n; \qquad T = m\left(\frac{0^2}{r}\right) = 0 \qquad \textbf{Ans.}$$



Applying Eq. 13–8 to FBD(b), we have

$$\Sigma F_t = ma_t; \qquad mg\cos\theta = ma_t \qquad a_t = g\cos\theta$$

$$\Sigma F_n = ma_n; \qquad T - mg\sin\theta = m\left(\frac{v^2}{r}\right) \qquad \textbf{[1]}$$



Kinematics: The speed of the bob at the arbitrary position θ can be detemined using $v\,dv = a_t\,ds$. However, $ds = r\,d\theta$, then $v\,dv = a_t\,r\,d\theta$.

$$\int_0^v v\,dv = gr\int_0^\theta \cos\theta\,d\theta$$

$$v^2 = 2gr\sin\theta$$

Substitute $v^2 = 2gr\sin\theta$ into Eq. [1] yields

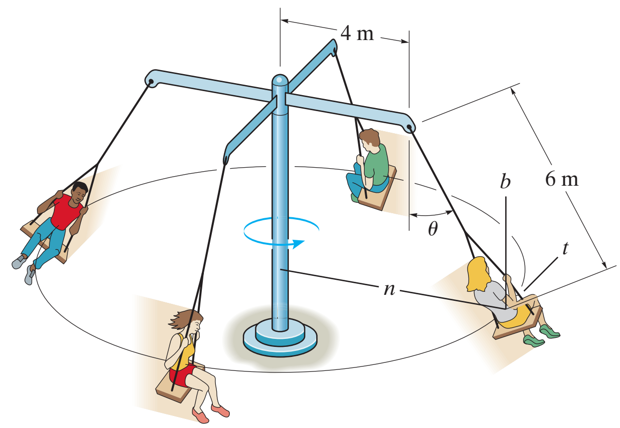
$$T - mg\sin\theta = m\left(\frac{2gr\sin\theta}{r}\right)$$

$$T = 3mg\sin\theta \qquad \textbf{Ans.}$$

Ans:
$T = 0$
$T = 3mg\sin\theta$

13–65.

Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30°$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the n, t, and b directions which the chair exerts on a 50-kg passenger during the motion?

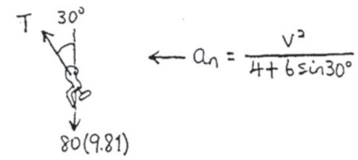


SOLUTION

$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $\qquad T \sin 30° = 80\left(\dfrac{v^2}{4 + 6 \sin 30°}\right)$

$+\uparrow \Sigma F_b = 0;$ $\qquad T \cos 30° - 80(9.81) = 0$

$T = 906.2 \text{ N}$

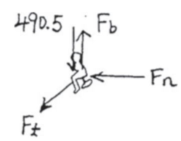


$v = 6.30 \text{ m/s}$ **Ans.**

$\Sigma F_n = m\,a_n;$ $\qquad F_n = 50\left(\dfrac{(6.30)^2}{7}\right) = 283 \text{ N}$ **Ans.**

$\Sigma F_t = m\,a_t;$ $\qquad F_t = 0$ **Ans.**

$\Sigma F_b = m\,a_b;$ $\qquad F_b - 490.5 = 0$

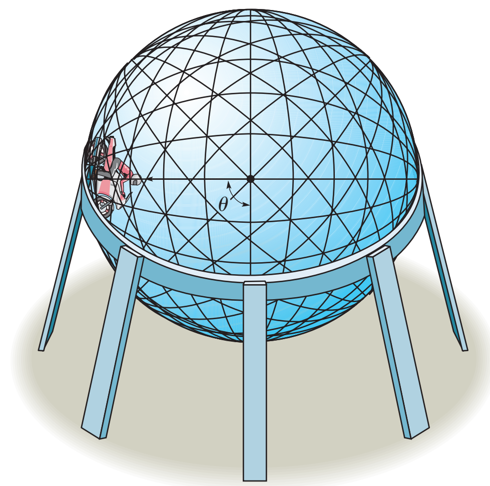


$F_b = 490 \text{ N}$ **Ans.**

Ans:
$v = 6.30 \text{ m/s}$
$F_n = 283 \text{ N}$
$F_t = 0$
$F_b = 490 \text{ N}$

13–66.

A motorcyclist in a circus rides his motorcycle within the confines of the hollow sphere. If the coefficient of static friction between the wheels of the motorcycle and the sphere is $\mu_s = 0.4$, determine the minimum speed at which he must travel if he is to ride along the wall when $\theta = 90°$. The mass of the motorcycle and rider is 250 kg, and the radius of curvature to the center of gravity is $\rho = 20$ ft. Neglect the size of the motorcycle for the calculation.



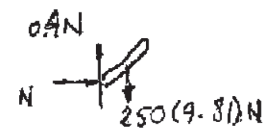
SOLUTION

$\xrightarrow{+} \Sigma F_n = ma_n;\qquad N = 250\left(\frac{v^2}{20}\right)$

$+\uparrow \Sigma F_b = ma_b;\qquad 0.4\,N - 250(9.81) = 0$

Solving,

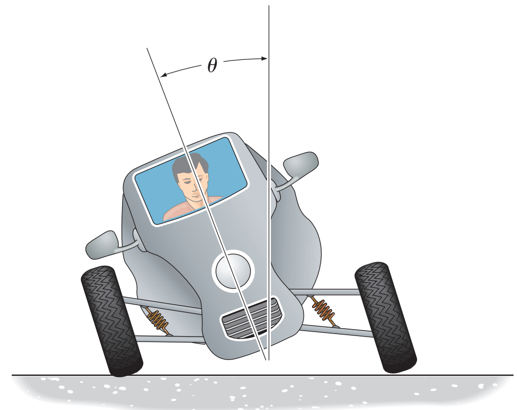
$v = 22.1 \text{ m/s}$ **Ans.**



Ans:
$v = 22.1 \text{ m/s}$

13–67.

The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.



SOLUTION

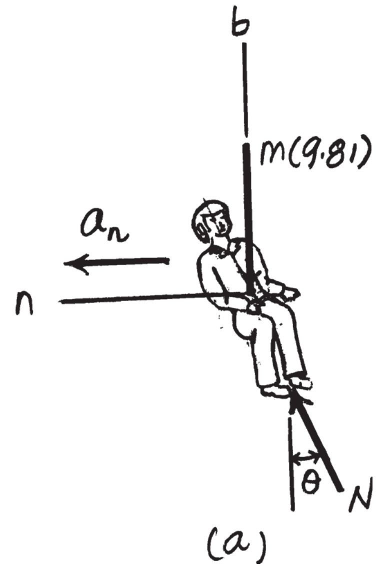
Free-Body Diagram: The free-body diagram of the passenger is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: The speed of the passenger is $v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22$ m/s. Thus, the normal component of the passenger's acceleration is given by $a_n = \frac{v^2}{\rho} = \frac{22.22^2}{100} = 4.938\text{ m/s}^2$. By referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad N\cos\theta - m(9.81) = 0 \qquad N = \frac{9.81m}{\cos\theta}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad \frac{9.81m}{\cos\theta}\sin\theta = m(4.938)$$

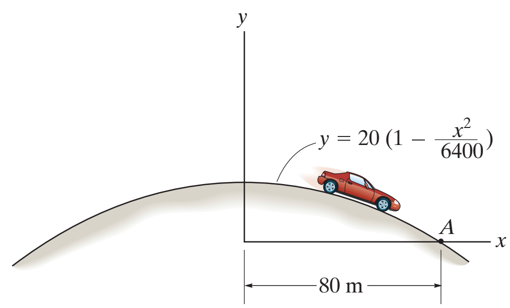
$$\theta = 26.7^\circ$$ **Ans.**



Ans:
$\theta = 26.7^\circ$

***13–68.**

The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.



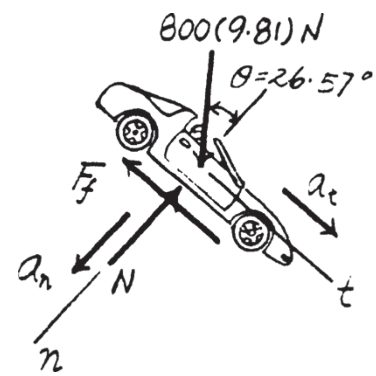
SOLUTION

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$

and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$



Equations of Motion: Here, $a_t = 0$. Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57^\circ - F_f = 800(0)$$

$$F_f = 3509.73\text{ N} = 3.51\text{ kN} \qquad \textbf{Ans.}$$

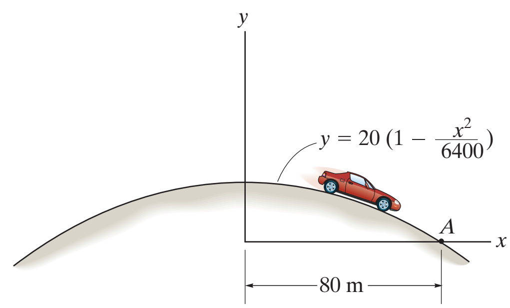
$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$$

$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_f = 3.51$ kN
$N = 6.73$ kN

13–69.

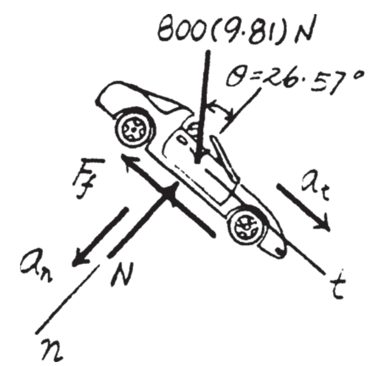
The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.



SOLUTION

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$



and the radius of curvature at point A is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (-0.00625x)^2\right]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$

Equation of Motion: Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57^\circ - F_f = 800(3)$$

$$F_f = 1109.73\text{ N} = 1.11\text{ kN} \qquad \textbf{Ans.}$$

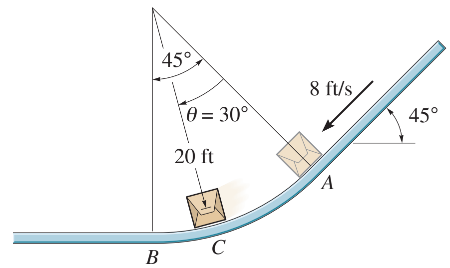
$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$$

$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

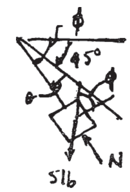
Ans:
$F_f = 1.11$ kN
$N = 6.73$ kN

13–70.

The package has a weight of 5 lb and slides down the chute. When it reaches the curved portion AB, it is traveling at 8 ft/s $(\theta = 0°)$. If the chute is smooth, determine the speed of the package when it reaches the intermediate point C $(\theta = 30°)$ and when it reaches the horizontal plane $(\theta = 45°)$. Also, find the normal force on the package at C.



SOLUTION



$+\swarrow \Sigma F_t = ma_t; \qquad 5 \cos \phi = \dfrac{5}{32.2} a_t$

$$a_t = 32.2 \cos \phi$$

$+\nwarrow \Sigma F_n = ma_n; \qquad N - 5 \sin \phi = \dfrac{5}{32.2} \left(\dfrac{v^2}{20}\right)$

$$v\,dv = a_t\,ds$$

$$\int_g^v v\,dv = \int_{45°}^{\phi} 32.2 \cos \phi \,(20\,d\phi)$$

$$\frac{1}{2}v^2 - \frac{1}{2}(8)^2 = 644(\sin \phi - \sin 45°)$$

At $\phi = 45° + 30° = 75°$,

$$v_C = 19.933 \text{ ft/s} = 19.9 \text{ ft/s} \qquad \textbf{Ans.}$$

$$N_C = 7.91 \text{ lb} \qquad \textbf{Ans.}$$

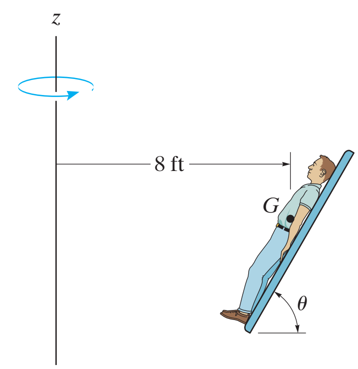
At $\phi = 45° + 45° = 90°$

$$v_B = 21.0 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v_C = 19.9$ ft/s
$N_C = 7.91$ lb
$v_B = 21.0$ ft/s

13–71.

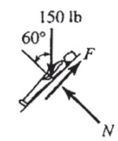
The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60°$.



SOLUTION

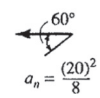
$+\nwarrow \sum F_y = m(a_n)_y;\qquad N - 150\cos 60° = \dfrac{150}{32.2}\left(\dfrac{20^2}{8}\right)\sin\ 60°$

$N = 277$ lb **Ans.**



$+\swarrow \sum F_x = m(a_n)_x;\qquad -F + 150\sin\ 60° = \dfrac{150}{32.2}\left(\dfrac{20^2}{8}\right)\cos 60°$

$F = 13.4$ lb **Ans.**



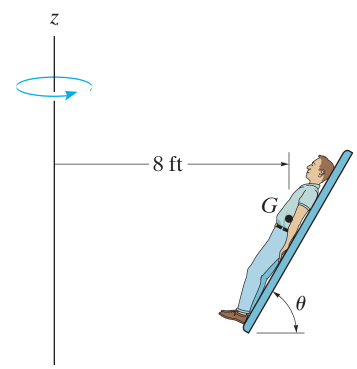
Note: No slipping occurs

Since $\mu_s N = 138.4 \text{ lb} > 13.4$ lb

Ans:
$N = 277$ lb
$F = 13.4$ lb

***13–72.**

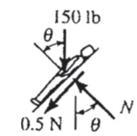
The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. If he rotates about the z axis with a constant speed $v = 30$ ft/s, determine the smallest angle θ of the cushion at which he will begin to slip off.



SOLUTION

$\overset{+}{\leftarrow} \Sigma F_n = m a_n; \qquad 0.5N\cos\theta + N\sin\theta = \dfrac{150}{32.2}\left(\dfrac{(30)^2}{8}\right)$

$+\uparrow \Sigma F_b = 0; \qquad -150 + N\cos\theta - 0.5\,N\sin\theta = 0$



$$N = \frac{150}{\cos\theta - 0.5\sin\theta}$$

$$\frac{(0.5\cos\theta + \sin\theta)150}{(\cos\theta - 0.5\sin\theta)} = \frac{150}{32.2}\left(\frac{(30)^2}{8}\right)$$

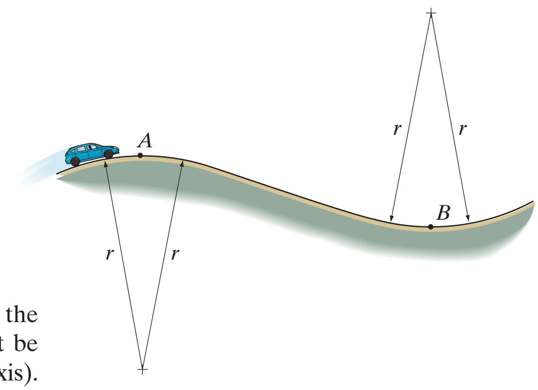
$$0.5\cos\theta + \sin\theta = 3.49378\cos\theta - 1.74689\sin\theta$$

$\theta = 47.5°$ **Ans.**

Ans:
$\theta = 47.5°$

13–73.

Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?



SOLUTION

Free-Body Diagram: The free-body diagram of the car at the top and bottom of the vertical curved road are shown in Figs. (a) and (b), respectively. Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the vertical curved road (positive n axis).

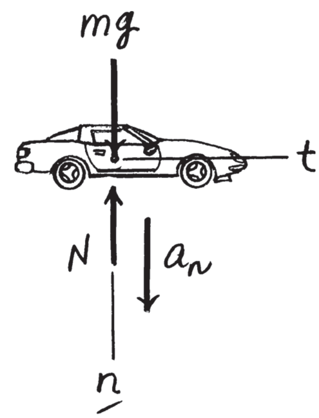
Equations of Motion: When the car is on top of the vertical curved road, it is required that its tires are about to lose contact with the road surface. Thus, $N = 0$. Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{r}$ and referring to Fig. (a),

$$+\downarrow \Sigma F_n = ma_n; \qquad mg = m\left(\frac{v^2}{r}\right) \qquad v = \sqrt{gr} \qquad \textbf{Ans.}$$

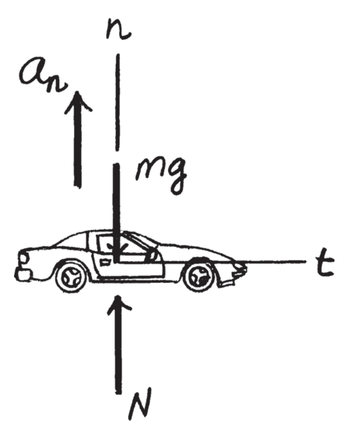
Using the result of v, the normal component of car acceleration is $a_n = \dfrac{v^2}{\rho} = \dfrac{gr}{r} = g$ when it is at the lowest point on the road. By referring to Fig. (b),

$$+\uparrow \Sigma F_n = ma_n; \qquad N - mg = mg$$

$$N = 2mg \qquad \textbf{Ans.}$$



(a)

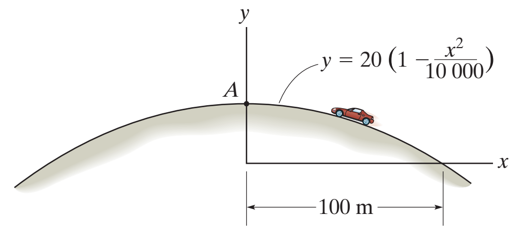


(b)

Ans:
$v = \sqrt{gr}$
$N = 2mg$

13–74.

Determine the maximum constant speed at which the 2-Mg car can travel over the crest of the hill at A without leaving the surface of the road. Neglect the size of the car in the calculation.



SOLUTION

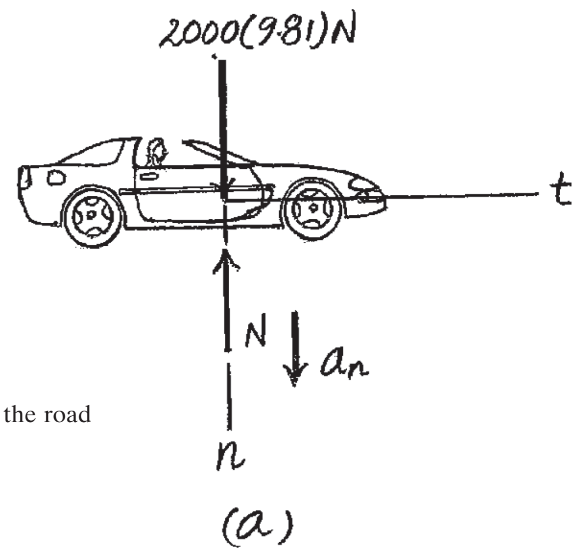
Geometry. The radius of curvature of the road at A must be determined first. Here

$$\frac{dy}{dx} = 20\left(-\frac{2x}{10000}\right) = -0.004x$$

$$\frac{d^2y}{dx^2} = -0.004$$

At point A, $x = 0$. Thus, $\left.\frac{dy}{dx}\right|_{x=0} = 0$. Then

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{(1 + 0^2)^{3/2}}{0.004} = 250 \text{ m}$$



Equation of Motion. Since the car is required to be on the verge to leave the road surface, $N = 0$.

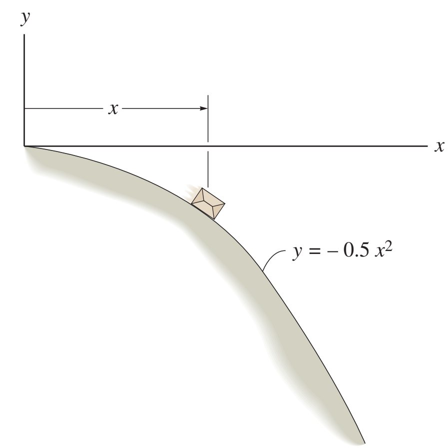
$$\Sigma F_n = ma_n; \qquad 2000(9.81) = 2000\left(\frac{v^2}{250}\right)$$

$$v = 49.52 \text{ m/s} = 49.5 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 49.5$ m/s

13–75.

The box has a mass m and slides down the smooth chute having the shape of a parabola. If it has an initial velocity of v_0 at the origin, determine its velocity as a function of x. Also, what is the normal force on the box, and the tangential acceleration as a function of x?



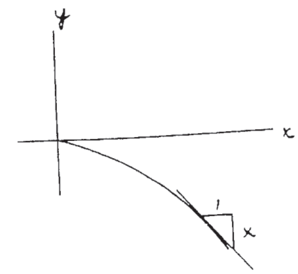
SOLUTION

$$x = -\frac{1}{2}x^2$$

$$\frac{dy}{dx} = -x$$

$$\frac{d^2y}{dx^2} = -1$$

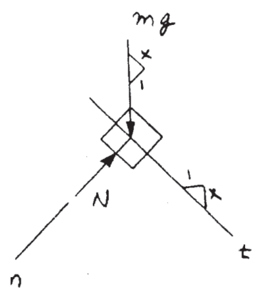
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + x^2\right]^{\frac{3}{2}}}{|-1|} = \left(1 + x^2\right)^{\frac{3}{2}}$$



$+\swarrow \Sigma F_n = ma_n\,;\qquad mg\left(\frac{1}{\sqrt{1 + x^2}}\right) - N = m\left(\frac{v^2}{(1 + x^2)^{\frac{3}{2}}}\right)$ **(1)**

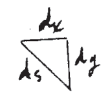
$+\searrow \Sigma F_t = ma_t\,;\qquad mg\left(\frac{x}{\sqrt{1 + x^2}}\right) = ma_t$

$$a_t = g\left(\frac{x}{\sqrt{1 + x^2}}\right)$$ **Ans.**



$$v\,dv = a_t\,ds = g\left(\frac{x}{\sqrt{1 + x^2}}\right)ds$$

$$ds = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx = \left(1 + x^2\right)^{\frac{1}{2}} dx$$



$$\int_{v_0}^{v} v\,dv = \int_0^x gx\,dx$$

$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = g\left(\frac{x^2}{2}\right)$$

$$v = \sqrt{v_0^2 + gx^2}$$ **Ans.**

From Eq. (1):

$$N = \frac{m}{\sqrt{1 + x^2}}\left[g - \frac{(v_0^2 + gx^2)}{(1 + x^2)}\right]$$ **Ans.**

Ans:

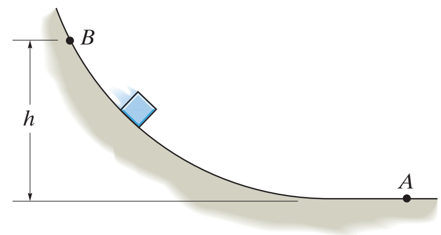
$a_t = g\left(\frac{x}{\sqrt{1 + x^2}}\right)$

$v = \sqrt{v_0^2 + gx^2}$

$N = \frac{m}{\sqrt{1 + x^2}}\left[g - \frac{v_0^2 + gx^2}{1 + x^2}\right]$

***13–76.**

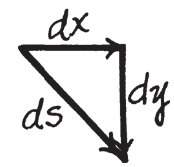
Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



SOLUTION

$+\searrow \Sigma F_t = ma_t; \qquad mg \sin\theta = ma_t \qquad a_t = g\sin\theta$

$v\,dv = a_t\,ds = g\sin\theta\,ds \qquad$ However $\quad dy = ds\sin\theta$

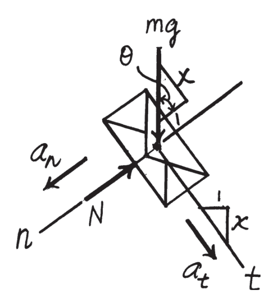


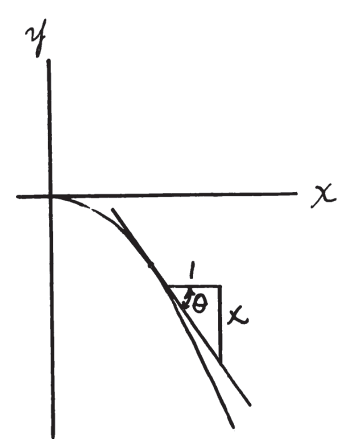
$$\int_0^v v\,dv = \int_0^h g\,dy$$

$$\frac{v^2}{2} = gh$$

$$v = \sqrt{2gh}$$

Q.E.D.

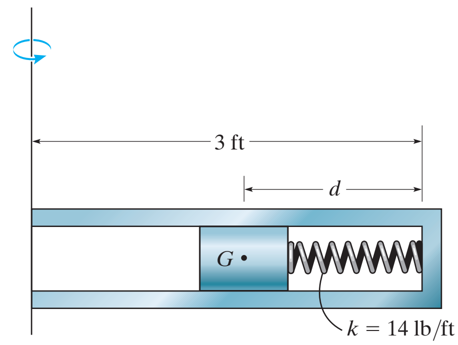




Ans:
$v = \sqrt{2gh}$

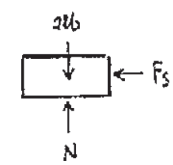
12–77.

The cylindrical plug has a weight of 2 lb and it is free to move within the confines of the smooth pipe. The spring has a stiffness $k = 14$ lb/ft and when no motion occurs the distance $d = 0.5$ ft. Determine the force of the spring on the plug when the plug is at rest with respect to the pipe. The plug is traveling with a constant speed of 15 ft/s, which is caused by the rotation of the pipe about the vertical axis.



SOLUTION

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\quad F_s = \dfrac{2}{32.2}\left[\dfrac{(15)^2}{3 - d}\right]$



$F_s = ks;$ $\quad F_s = 14(0.5 - d)$

Thus, $\quad 14(0.5 - d) = \dfrac{2}{32.2}\left[\dfrac{(15)^2}{3 - d}\right]$

$$(0.5 - d)(3 - d) = 0.9982$$

$$1.5 - 3.5d + d^2 = 0.9982$$

$$d^2 - 3.5d + 0.5018 = 0$$

Choosing the root < 0.5 ft

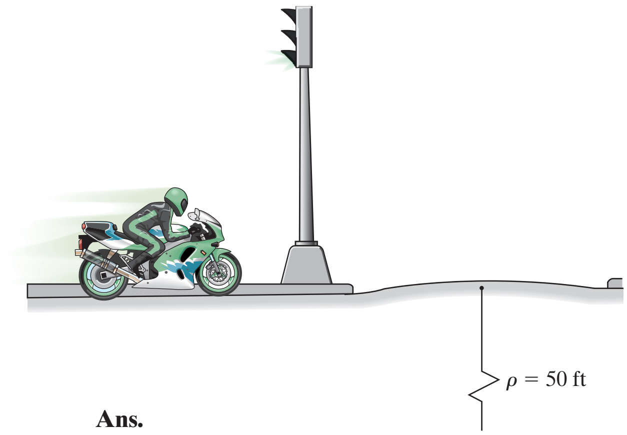
$$d = 0.1498 \text{ ft}$$

$$F_s = 14(0.5 - 0.1498) = 4.90 \text{ lb}$$ **Ans.**

Ans:
$F_s = 4.90$ lb

13–78.

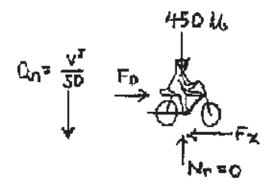
When crossing an intersection, a motorcyclist encounters the slight bump or crown caused by the intersecting road. If the crest of the bump has a radius of curvature $\rho = 50$ ft, determine the maximum constant speed at which he can travel without leaving the surface of the road. Neglect the size of the motorcycle and rider in the calculation. The rider and his motorcycle have a total weight of 450 lb.



SOLUTION

$$+\downarrow \Sigma F_n = ma_n; \qquad 450 - 0 = \frac{450}{32.2}\left(\frac{v^2}{50}\right)$$

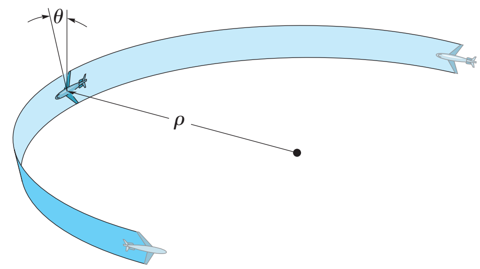
$$v = 40.1 \text{ ft/s}$$ **Ans.**



Ans:
$v = 40.1$ ft/s

13–79.

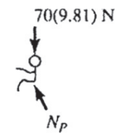
The airplane, traveling at a constant speed of 50 m/s, is executing a horizontal turn. If the plane is banked at $\theta = 15°$, when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has a mass of 70 kg.



SOLUTION

$+\uparrow \sum F_b = ma_b; \qquad N_P \sin 15° - 70(9.81) = 0$

$N_P = 2.65 \text{ kN}$ **Ans.**



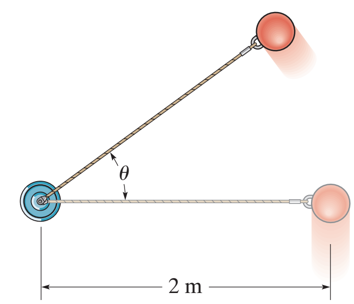
$\overset{+}{\leftarrow} \sum F_n = ma_n; \qquad N_P \cos 15° = 70\left(\frac{50^2}{\rho}\right)$

$\rho = 68.3 \text{ m}$ **Ans.**

Ans:
$N_P = 2.65 \text{ kN}$
$\rho = 68.3 \text{ m}$

***13–80.**

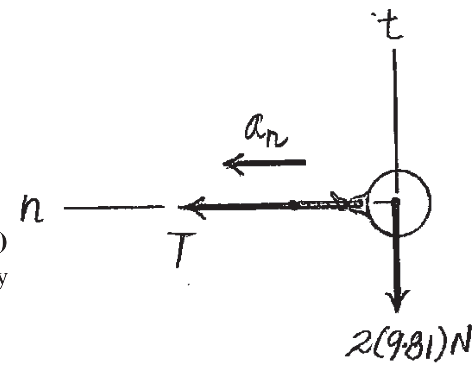
The 2-kg pendulum bob moves in the vertical plane with a velocity of 8 m/s when $\theta = 0°$. Determine the initial tension in the cord and also at the instant the bob reaches $\theta = 30°$. Neglect the size of the bob.



SOLUTION

Equations of Motion. Referring to the FBD of the bob at position $\theta = 0°$, Fig. *a*,

$$\Sigma F_n = ma_n; \qquad T = 2\left(\frac{8^2}{2}\right) = 64.0 \text{ N} \qquad \textbf{Ans.}$$



(a)

For the bob at an arbitrary position θ, the FBD is shown in Fig. *b*.

$$\Sigma F_t = ma_t; \qquad -2(9.81)\cos\theta = 2a_t$$

$$a_t = -9.81\cos\theta$$

$$\Sigma F_n = ma_n; \qquad T + 2(9.81)\sin\theta = 2\left(\frac{v^2}{2}\right)$$

$$T = v^2 - 19.62\sin\theta \qquad \textbf{(1)}$$

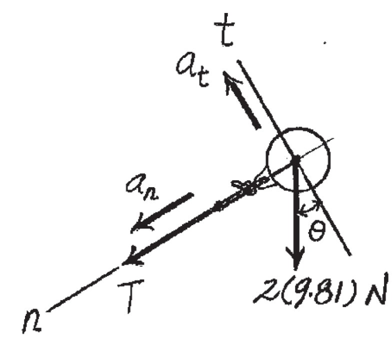
Kinematics. The velocity of the bob at the position $\theta = 30°$ can be determined by integrating $v\,dv = a_t\,ds$. However, $ds = r\,d\theta = 2\,d\theta$.
Then,

$$\int_{8\text{ m/s}}^{v} v\,dv = \int_{0°}^{30°} -9.81\cos\theta(z\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_{8\text{ m/s}}^{v} = -19.62\sin\theta\Big|_{0°}^{30°}$$

$$\frac{v^2}{2} - \frac{8^2}{2} = -19.62(\sin 30° - 0)$$

$$v^2 = 44.38 \text{ m}^2/\text{s}^2$$



(b)

Substitute this result and $\theta = 30°$ into Eq. (1),

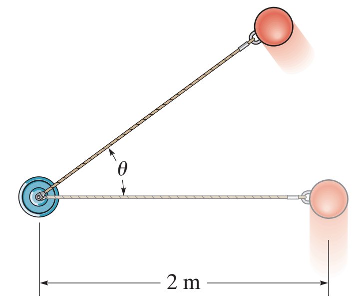
$$T = 44.38 - 19.62\sin 30°$$

$$= 34.57 \text{ N} = 34.6 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$T = 64.0$ N
$T = 34.6$ N

13–81.

The 2-kg pendulum bob moves in the vertical plane with a velocity of 6 m/s when $\theta = 0°$. Determine the angle θ where the tension in the cord becomes zero.



SOLUTION

Equation of Motion. The FBD of the bob at an arbitrary position θ is shown in Fig. a. Here, it is required that $T = 0$.

$$\Sigma F_t = ma_t; \qquad -2(9.81)\cos\theta = 2a_t$$

$$a_t = -9.81\cos\theta$$

$$\Sigma F_n = ma_n; \qquad 2(9.81)\sin\theta = 2\left(\frac{v^2}{2}\right)$$

$$v^2 = 19.62\sin\theta \qquad \textbf{(1)}$$

Kinematics. The velocity of the bob at an arbitrary position θ can be determined by integrating $vdv = a_t ds$. However, $ds = rd\theta = 2d\theta$.

Then

$$\int_{6\text{ m/s}}^{v} vdv = \int_{0°}^{\theta} -9.81\cos\theta(2d\theta)$$

$$\left.\frac{v^2}{2}\right|_{6\text{ m/s}}^{v} = -19.62\sin\theta\Big|_{0°}^{\theta}$$

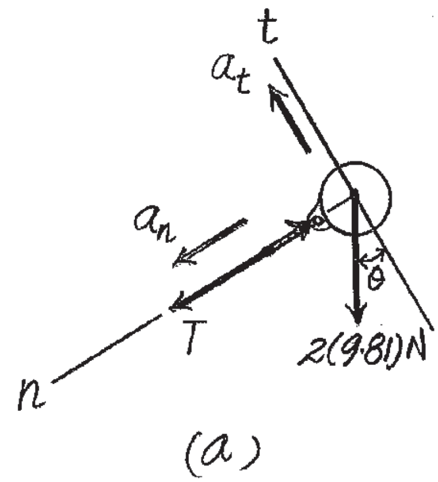
$$v^2 = 36 - 39.24\sin\theta \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2)

$$19.62\sin\theta = 36 - 39.24\sin\theta$$

$$58.86\sin\theta = 36$$

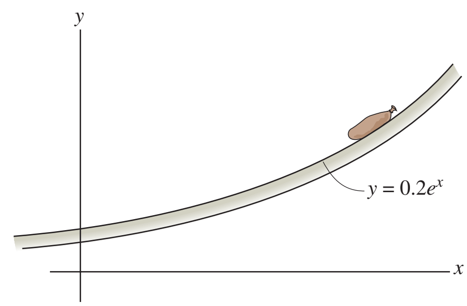
$$\theta = 37.71° = 37.7° \qquad \textbf{Ans.}$$



Ans:
$\theta = 37.7°$

13–82.

The 8-kg sack slides down the smooth ramp. If it has a speed of 1.5 m/s when $y = 0.2$ m, determine the normal reaction the ramp exerts on the sack and the rate of increase in the speed of the sack at this instant.



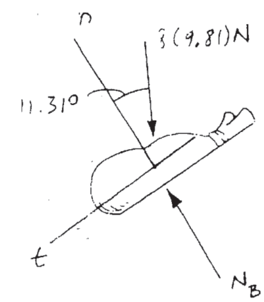
SOLUTION

$y = 0.2 \qquad x = 0$

$y = 0.2e^x$

$$\frac{dy}{dx} = 0.2e^x \bigg|_{x=0} = 0.2$$

$$\frac{d^2y}{dx^2} = 0.2e^x \bigg|_{x=0} = 0.2$$



$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (0.2)^2\right]^{\frac{3}{2}}}{|0.2|} = 5.303$$

$\theta = \tan^{-1}(0.2) = 11.31°$

$+\nwarrow \Sigma F_n = ma_n; \qquad N_B - 8(9.81)\cos 11.31° = 8\left(\frac{(1.5)^2}{5.303}\right)$

$N_B = 80.4 \text{ N}$ **Ans.**

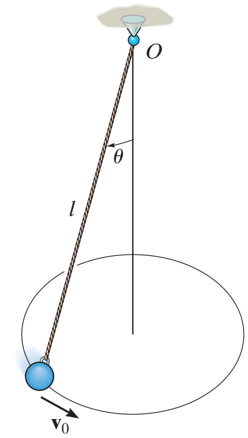
$+\swarrow \Sigma F_t = ma_t; \qquad 8(9.81)\sin 11.31° = 8a_t$

$a_t = 1.92 \text{ m/s}^2$ **Ans.**

Ans:
$N_B = 80.4$ N
$a_t = 1.92 \text{ m/s}^2$

13–83.

The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan\theta\sin\theta = v_0^2/gl$. Neglect air resistance and the size of the ball.



SOLUTION

$$\xrightarrow{+} \Sigma F_n = ma_n; \qquad T\sin\theta = m\left(\frac{v_0^2}{r}\right)$$

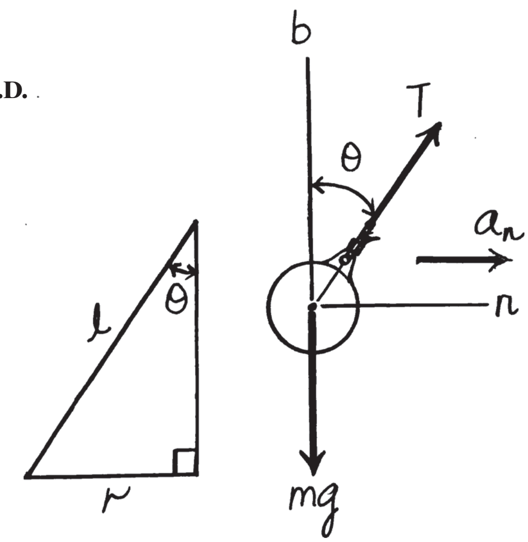
$$+\uparrow \Sigma F_b = 0; \qquad T\cos\theta - mg = 0$$

$$\text{Since } r = l\sin\theta \qquad T = \frac{mv_0^2}{l\sin^2\theta}$$

$$\left(\frac{mv_0^2}{l}\right)\left(\frac{\cos\theta}{\sin^2\theta}\right) = mg$$

$$\tan\theta\sin\theta = \frac{v_0^2}{gl}$$

Q.E.D.

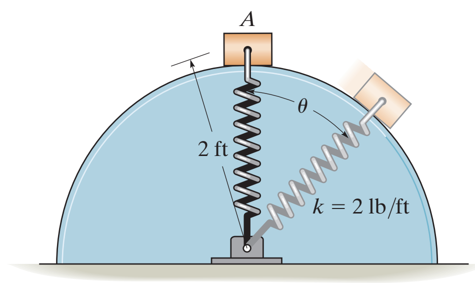


Ans:

$$\tan\theta\sin\theta = \frac{v_0^2}{gl}$$

***13–84.**

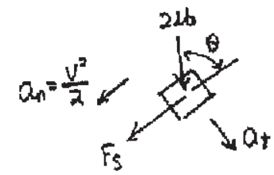
The 2-lb block is released from rest at A and slides down along the smooth cylindrical surface. If the attached spring has a stiffness $k = 2$ lb/ft, determine its unstretched length so that it does not allow the block to leave the surface until $\theta = 60°$.



SOLUTION

$+\swarrow \Sigma F_n = ma_n;\quad F_s + 2\cos\theta = \frac{2}{32.2}\left(\frac{v^2}{2}\right)$ **(1)**

$+\searrow \Sigma F_t = ma_t;\quad 2\sin\theta = \frac{2}{32.2}a_t$



$a_t = 32.2\sin\theta$

$v\,dv = a_t\,ds;\quad \int_0^v v\,dv = \int_0^\theta 32.2(\sin\theta)2d\theta$

$\frac{1}{2}v^2 = 64.4(-\cos\theta + 1)$

When $\theta = 60°$

$v^2 = 64.4$

From Eq. (1)

$F_s + 2\cos 60° = \frac{2}{32.2}\left(\frac{64.4}{2}\right)$

$F_s = 1$ lb

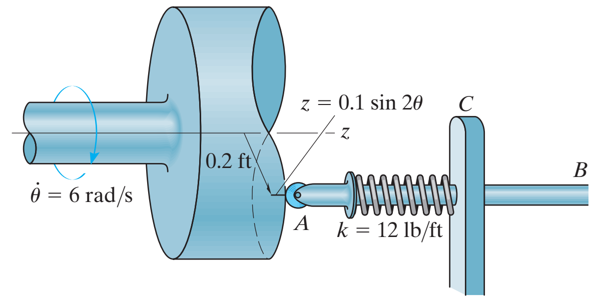
$F_s = ks;\quad 1 = 2s;\quad s = 0.5$ ft

$l_0 = l - s = 2 - 0.5 = 1.5$ ft **Ans.**

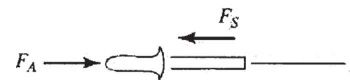
Ans:
$l_0 = 1.5$ ft

13–85.

The spring-held follower AB has a weight of 0.75 lb and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.2$ ft and $z = (0.1 \sin \theta)$ ft. If the cam is rotating at a constant rate of 6 rad/s, determine the force at the end A of the follower when $\theta = 90°$. In this position the spring is compressed 0.4 ft. Neglect friction at the bearing C.



SOLUTION



$z = 0.1 \sin 2\theta$

$\dot{z} = 0.2 \cos 2\theta\dot{\theta}$

$\ddot{z} = -0.4 \sin 2\theta\dot{\theta}^2 + 0.2 \cos 2\theta\ddot{\theta}$

$\dot{\theta} = 6$ rad/s

$\ddot{\theta} = 0$

$\ddot{z} = -14.4 \sin 2\theta$

$\sum F_z = ma_z; \qquad F_A - 12(z + 0.3) = m\ddot{z}$

$$F_A - 12(0.1 \sin 2\theta + 0.3) = \frac{0.75}{32.2}(-14.4 \sin 2\theta)$$

For $\theta = 45°$,

$$F_A - 12(0.4) = \frac{0.75}{32.2}(-14.4)$$

$F_A = 4.46$ lb **Ans.**

Ans:
$F_A = 4.46$ lb

13–86.

Determine the magnitude of the resultant force acting on a 5-kg particle at the instant $t = 2$ s, if the particle is moving along a horizontal path defined by the equations $r = (2t + 10)$ m and $\theta = (1.5t^2 - 6t)$ rad, where t is in seconds.

SOLUTION

$r = 2t + 10|_{t=2\text{ s}} = 14$

$\dot{r} = 2$

$\ddot{r} = 0$

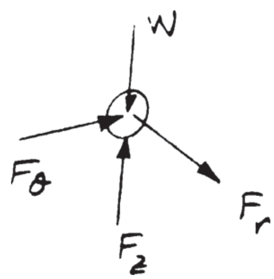
$\theta = 1.5t^2 - 6t$

$\dot{\theta} = 3t - 6|_{t=2\,s} = 0$

$\ddot{\theta} = 3$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0 = 0$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 14(3) + 0 = 42$



Hence,

$\Sigma F_r = ma_r; \qquad F_r = 5(0) = 0$

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 5(42) = 210\text{ N}$

$F = \sqrt{(F_r)^2 + (F_\theta)^2} = 210\text{ N}$ **Ans.**

Ans:
$F = 210\text{ N}$

13–87.

The path of motion of a 5-lb particle in the horizontal plane is described in terms of polar coordinates as $r = (2t + 1)$ ft and $\theta = (0.5t^2 - t)$ rad, where t is in seconds. Determine the magnitude of the unbalanced force acting on the particle when $t = 2$ s.

SOLUTION

$r = 2t + 1|_{t=2\,s} = 5\text{ ft} \qquad \dot{r} = 2\text{ ft/s} \qquad \ddot{r} = 0$

$\theta = 0.5t^2 - t|_{t=2\,s} = 0\text{ rad} \qquad \dot{\theta} = t - 1|_{t=2\,s} = 1\text{ rad/s} \qquad \ddot{\theta} = 1\text{ rad/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 5(1)^2 = -5\text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 5(1) + 2(2)(1) = 9\text{ ft/s}^2$

$\Sigma F_r = ma_r; \qquad F_r = \dfrac{5}{32.2}(-5) = -0.7764\text{ lb}$

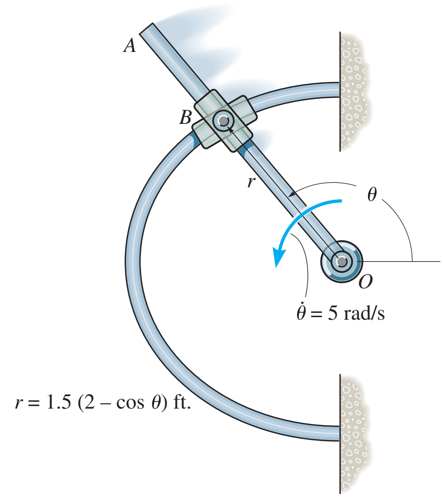
$\Sigma F_\theta = ma_\theta; \qquad F_\theta = \dfrac{5}{32.2}(9) = 1.398\text{ lb}$

$F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(-0.7764)^2 + (1.398)^2} = 1.60\text{ lb}$ **Ans.**

Ans:
$F = 1.60$ lb

***13–88.**

Rod OA rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other slides over the *horizontal* curved rod, of which the shape is described by the equation $r = 1.5(2 - \cos\theta)$ ft. If both collars weigh 0.75 lb, determine the normal force which the curved rod exerts on one collar at the instant $\theta = 120^\circ$. Neglect friction.



SOLUTION

Kinematic: Here, $\dot{\theta} = 5$ rad/s and $\ddot{\theta} = 0$. Taking the required time derivatives at $\theta = 120^\circ$, we have

$$r = 1.5(2 - \cos\theta)|_{\theta=120^\circ} = 3.75 \text{ ft}$$

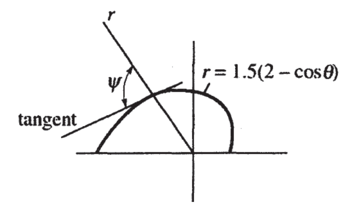
$$\dot{r} = 1.5 \sin\theta\dot{\theta}|_{\theta=120^\circ} = 6.495 \text{ ft/s}$$

$$\ddot{r} = 1.5(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)|_{\theta=120^\circ} = -18.75 \text{ ft/s}^2$$

Applying Eqs. 12–29, we have

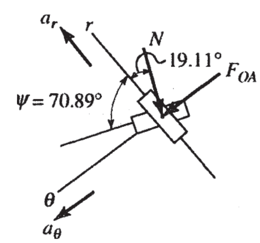
$$a_r = \ddot{r} - r\dot{\theta}^2 = -18.75 - 3.75(5^2) = -112.5 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.75(0) + 2(6.495)(5) = 64.952 \text{ ft/s}^2$$



Equation of Motion: The angle ψ must be obtained first.

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{1.5(2 - \cos\theta)}{1.5\sin\theta}\bigg|_{\theta=120^\circ} = 2.8867 \qquad \psi = 70.89^\circ$$



Applying Eq. 13–9, we have

$$\sum F_r = ma_r; \qquad -N\cos 19.11^\circ = \frac{0.75}{32.2}(-112.5)$$

$$N = 2.773 \text{ lb} = 2.77 \text{ lb} \qquad \textbf{Ans.}$$

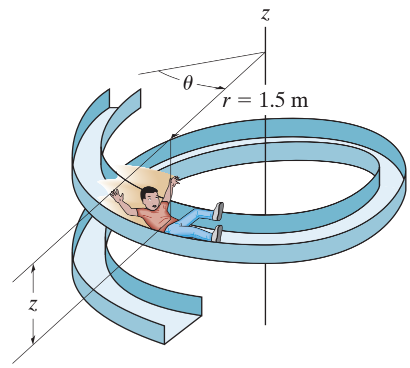
$$\sum F_\theta = ma_\theta; \qquad F_{OA} + 2.773\sin 19.11^\circ = \frac{0.75}{32.2}(64.952)$$

$$F_{OA} = 0.605 \text{ lb}$$

Ans:
$N = 2.77$ lb

13–89.

The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.

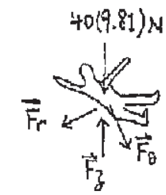


SOLUTION

$r = 1.5 \qquad \theta = 0.7t \qquad z = -0.5t$

$\dot{r} = \ddot{r} = 0 \qquad \dot{\theta} = 0.7 \qquad \dot{z} = -0.5$

$\ddot{\theta} = 0 \qquad \ddot{z} = 0$



$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = 0$

$\Sigma F_r = ma_r; \qquad F_r = 40(-0.735) = -29.4 \text{ N}$ **Ans.**

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 0$ **Ans.**

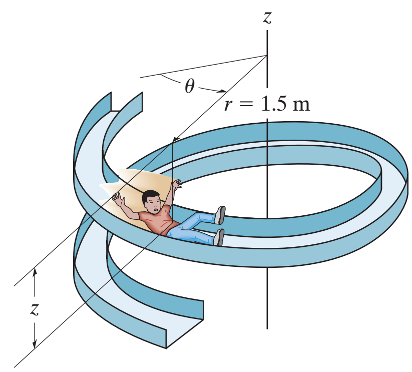
$\Sigma F_z = ma_z; \qquad F_z - 40(9.81) = 0$

$F_z = 392 \text{ N}$ **Ans.**

Ans:
$F_r = -29.4$ N
$F_\theta = 0$
$F_z = 392$ N

13–90.

The 40-kg boy is sliding down the smooth spiral slide such that $z = -2$ m/s and his speed is 2 m/s. Determine the r, θ, z components of force the slide exerts on him at this instant. Neglect the size of the boy.



SOLUTION

$r = 1.5$ m

$\dot{r} = 0$

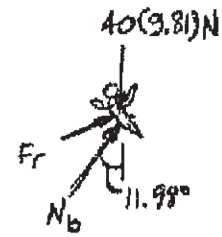
$\ddot{r} = 0$

$v_\theta = 2 \cos 11.98° = 1.9564$ m/s

$v_z = -2 \sin 11.98° = -0.41517$ m/s

$v_\theta = r\dot{\theta}; \qquad 1.9564 = 1.5\,\dot{\theta}$

$\dot{\theta} = 1.3043$ rad/s



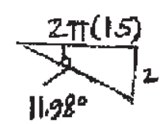
$\Sigma F_r = ma_r; \qquad -F_r = 40(0 - 1.5(1.3043)^2)$

$F_r = 102$ N **Ans.**

$\Sigma F_\theta = ma_\theta; \qquad N_b \sin 11.98° = 40(a_\theta)$

$\Sigma F_z = ma_z; \qquad -N_b \cos 11.98° + 40(9.81) = 40a_z$

Require $\tan 11.98° = \dfrac{a_z}{a_\theta}, \qquad a_\theta = 4.7123a_z$



Thus,

$a_z = 0.423$ m/s^2

$a_\theta = 1.99$ m/s^2

$N_b = 383.85$ N

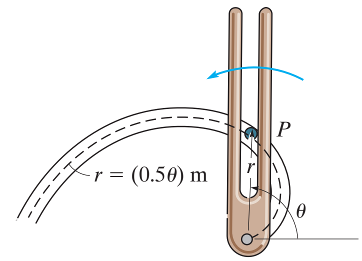
$N_z = 383.85 \cos 11.98° = 375$ N **Ans.**

$N_\theta = 383.85 \sin 11.98° = 79.7$ N **Ans.**

Ans:
$F_r = 102$ N
$F_z = 375$ N
$F_\theta = 79.7$ N

13–91.

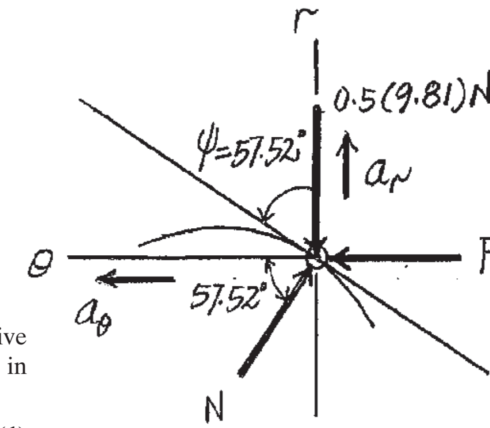
Using a forked rod, a 0.5-kg smooth peg P is forced to move along the *vertical slotted* path $r = (0.5\theta)$ m, where θ is in radians. If the angular position of the arm is $\theta = (\frac{\pi}{8}t^2)$ rad, where t is in seconds, determine the force of the rod on the peg and the normal force of the slot on the peg at the instant $t = 2$ s. The peg is in contact with only *one edge* of the rod and slot at any instant.



SOLUTION

Equation of Motion. Here, $r = 0.5\theta$. Then $\dfrac{dr}{d\theta} = 0.5$. The angle ψ between the extended radial line and the tangent can be determined from

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.5\theta}{0.5} = \theta$$



At the instant $t = 25$, $\theta = \dfrac{\pi}{8}(2^2) = \dfrac{\pi}{2}$ rad

$$\tan\psi = \frac{\pi}{2} \qquad \psi = 57.52^\circ$$

The positive sign indicates that ψ is measured from extended radial line in positive sense of θ (counter clockwise) to the tangent. Then the FBD of the peg shown in Fig. *a* can be drawn.

$\Sigma F_r = ma_r;\quad N\sin 57.52^\circ - 0.5(9.81) = 0.5a_r$ **(1)**

$\Sigma F_\theta = ma_\theta;\quad F - N\cos 57.52^\circ = 0.5a_\theta$ **(2)**

Kinematics. Using the chain rule, the first and second derivatives of r and θ with respect to t are

$$r = 0.5\theta = 0.5\left(\frac{\pi}{8}t^2\right) = \frac{\pi}{16}t^2 \qquad \theta = \frac{\pi}{8}t^2$$

$$\dot{r} = \frac{\pi}{8}t \qquad \dot{\theta} = \frac{\pi}{4}t$$

$$\ddot{r} = \frac{\pi}{8} \qquad \ddot{\theta} = \frac{\pi}{4}$$

When $t = 2$ s,

$$r = \frac{\pi}{16}(2^2) = \frac{\pi}{4}\text{ m} \qquad \theta = \frac{\pi}{8}(2^2) = \frac{\pi}{2}\text{ rad}$$

$$\dot{r} = \frac{\pi}{8}(2) = \frac{\pi}{4}\text{ m/s} \qquad \dot{\theta} = \frac{\pi}{4}(2) = \frac{\pi}{2}\text{ rad/s}$$

$$\ddot{r} = \frac{\pi}{8}\text{ m/s}^2 \qquad \ddot{\theta} = \frac{\pi}{4}\text{ rad/s}^2$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{\pi}{8} - \frac{\pi}{4}\left(\frac{\pi}{2}\right)^2 = -1.5452\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{\pi}{4}\left(\frac{\pi}{4}\right) + 2\left(\frac{\pi}{4}\right)\left(\frac{\pi}{2}\right) = 3.0843\text{ m/s}^2$$

Substitute these results in Eqs. (1) and (2)

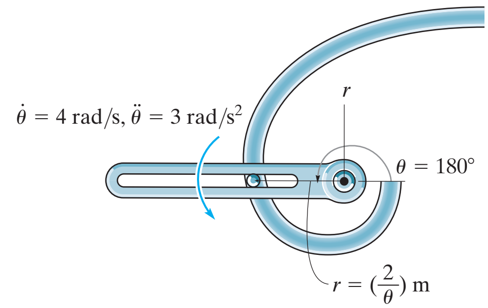
$N = 4.8987\text{ N} = 4.90\text{ N}$ **Ans.**

$F = 4.173\text{ N} = 4.17\text{ N}$ **Ans.**

Ans:
$N = 4.90$ N
$F = 4.17$ N

***13–92.**

The arm is rotating at a rate of $\dot{\theta} = 4\text{ rad/s}$ when $\ddot{\theta} = 3\text{ rad/s}^2$ and $\theta = 180°$. Determine the force it must exert on the 0.5-kg smooth cylinder if it is confined to move along the slotted path. Motion occurs in the horizontal plane.



SOLUTION

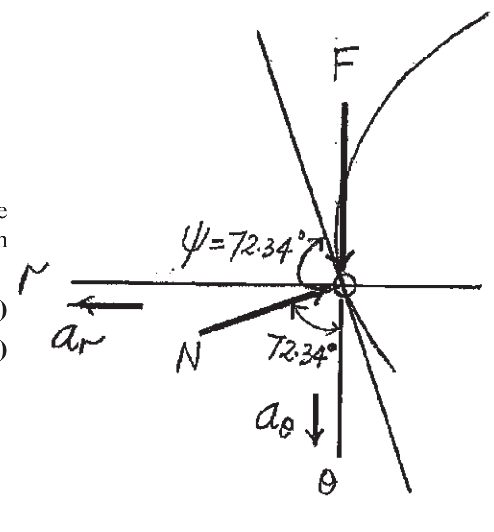
Equation of Motion. Here, $r = \dfrac{2}{\theta}$. Then $\dfrac{dr}{d\theta} = -\dfrac{2}{\theta^2}$. The angle ψ between the extended radial line and the tangent can be determined from

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2/\theta}{-2/\theta^2} = -\theta$$

At $\theta = 180° = \pi$ rad,

$$\tan\psi = -\pi \qquad \psi = -72.34°$$

The negative sign indicates that ψ is measured from extended radial line in the negative sense of θ (clockwise) to the tangent. Then, the FBD of the peg shown in Fig. *a* can be drawn.



(a)

$$\Sigma F_r = ma_r; \qquad -N\sin 72.34° = 0.5a_r \qquad \textbf{(1)}$$

$$\Sigma F_\theta = ma_\theta; \qquad F - N\cos 72.34° = 0.5a_\theta \qquad \textbf{(2)}$$

Kinematics. Using the chain rule, the first and second time derivatives of r are

$$r = 2\theta^{-1}$$

$$\dot{r} = -2\theta^{-2}\dot{\theta} = -\left(\frac{2}{\theta^2}\right)\dot{\theta}$$

$$\ddot{r} = -2\left(-2\theta^{-3}\dot{\theta}^2 + \theta^{-2}\ddot{\theta}\right) = \frac{2}{\theta^3}\left(2\dot{\theta}^2 - \theta\ddot{\theta}\right)$$

When $\theta = 180° = \pi$ rad, $\dot{\theta} = 4\text{ rad/s}$ and $\ddot{\theta} = 3\text{ rad/s}^2$. Thus

$$r = \frac{2}{\pi}\text{ m} = 0.6366\text{ m}$$

$$\dot{r} = -\left(\frac{2}{\pi^2}\right)(4) = -0.8106\text{ m/s}$$

$$\ddot{r} = \frac{2}{\pi^3}\left[2(4^2) - \pi(3)\right] = 1.4562\text{ m/s}^2$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 1.4562 - 0.6366(4^2) = -8.7297\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6366(3) + 2(-0.8106)(4) = -4.5747\text{ m/s}^2$$

Substitute these result into Eqs. (1) and (2),

$$N = 4.5807\text{ N}$$

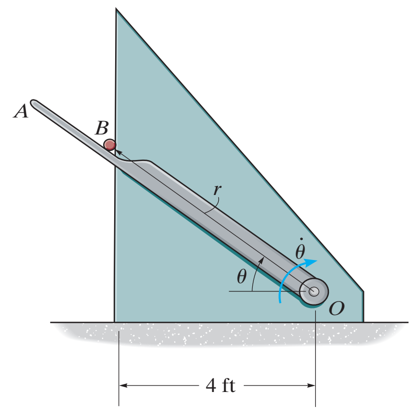
$$F = -0.8980\text{ N} = -0.898\text{ N} \qquad \textbf{Ans.}$$

The negative sign indicates that **F** acts in the sense opposite to that shown in the FBD.

Ans:
$F = -0.898\text{ N}$

13–93.

If arm OA rotates with a constant clockwise angular velocity of $\dot{\theta} = 1.5$ rad/s. determine the force arm OA exerts on the smooth 4-lb cylinder B when $\theta = 45°$.



SOLUTION

Kinematics: Since the motion of cylinder B is known, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ will be determined first. Here, $\frac{4}{r} = \cos\theta$ or $r = 4\sec\theta$ ft. The value of r and its time derivatives at the instant $\theta = 45°$ are

$$r = 4\sec\theta\big|_{\theta=45°} = 4\sec 45° = 5.657 \text{ ft}$$

$$\dot{r} = 4\sec\theta(\tan\theta)\dot{\theta}\big|_{\theta=45°} = 4\sec 45°\tan 45°(1.5) = 8.485 \text{ ft/s}$$

$$\ddot{r} = 4\left[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}\left(\sec\theta\ \sec^2\theta\dot{\theta} + \tan\theta\sec\theta\tan\theta\dot{\theta}\right)\right]$$

$$= 4\left[\sec\theta(\tan\theta)\ddot{\theta} + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan^2\theta\dot{\theta}^2\right]\Big|_{\theta=45°}$$

$$= 4\left[\sec 45°\tan 45°(0) + \sec^3 45°(1.5)^2 + \sec 45°\tan^2 45°(1.5)^2\right]$$

$$= 38.18 \text{ ft/s}^2$$

Using the above time derivatives,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 38.18 - 5.657\left(1.5^2\right) = 25.46 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} - 2\dot{r}\dot{\theta} = 5.657(0) + 2(8.485)(1.5) = 25.46 \text{ ft/s}^2$$

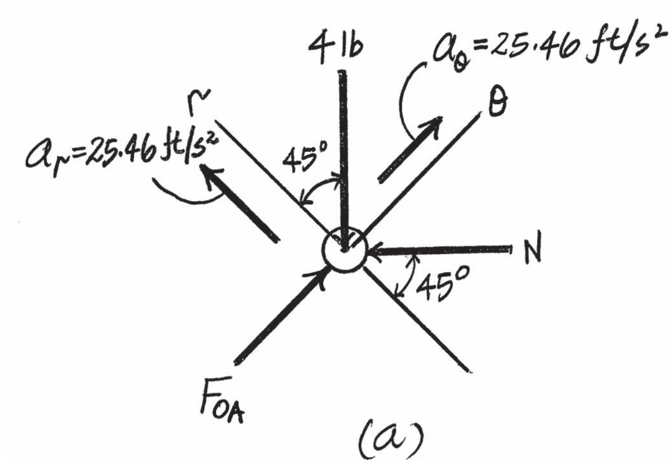
Equations of Motion: By referring to the free-body diagram of the cylinder shown in Fig. a,

$\Sigma F_r = ma_r;$ $\qquad N\cos 45° - 4\cos 45° = \frac{4}{32.2}(25.46)$

$N = 8.472$ lb

$\Sigma F_\theta = ma_\theta;$ $\qquad F_{OA} - 8.472\sin 45° - 4\sin 45° = \frac{4}{32.2}(25.46)$

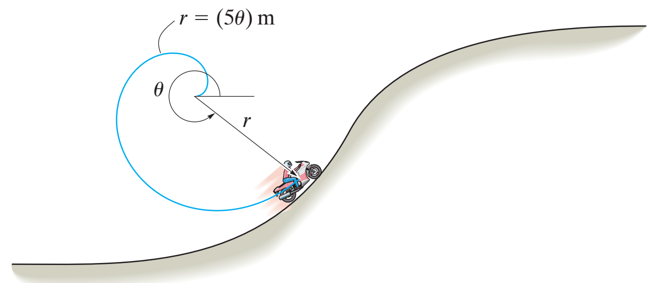
$F_{OA} = 12.0$ lb **Ans.**



Ans:
$F_{OA} = 12.0$ lb

13–94.

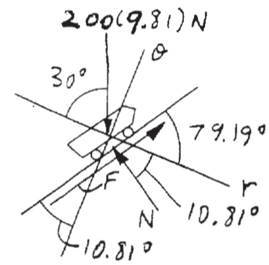
Determine the normal and frictional driving forces that the partial spiral track exerts on the 200-kg motorcycle at the instant $\theta = \frac{5}{3}\pi$ rad, $\dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8$ rad/s^2. Neglect the size of the motorcycle.



SOLUTION

$$\theta = \left(\frac{5}{3}\pi\right) = 300^\circ \qquad \dot{\theta} = 0.4 \qquad \ddot{\theta} = 0.8$$

$$r = 5\theta = 5\left(\frac{5}{3}\pi\right) = 26.18$$



$$\dot{r} = 5\dot{\theta} = 5(0.4) = 2$$

$$\ddot{r} = 5\ddot{\theta} = 5(0.8) = 4$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4 - 26.18(0.4)^2 = -0.1888$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 26.18(0.8) + 2(2)(0.4) = 22.54$$

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{5\left(\frac{5}{3}\pi\right)}{5} = 5.236 \qquad \psi = 79.19^\circ$$

$+\searrow\Sigma F_r = ma_r;$ $\quad F\sin 10.81^\circ - N\cos 10.81^\circ + 200(9.81)\cos 30^\circ = 200(-0.1888)$

$+\nearrow\Sigma F_\theta = ma_\theta;$ $\quad F\cos 10.81^\circ - 200(9.81)\sin 30^\circ + N\sin 10.81^\circ = 200(22.54)$

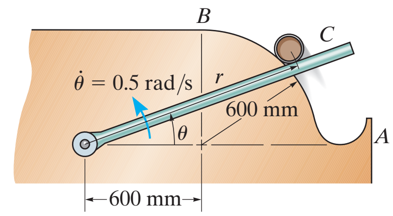
$F = 5.07$ kN **Ans.**

$N = 2.74$ kN **Ans.**

Ans:
$F = 5.07$ kN
$N = 2.74$ kN

13–95.

A smooth can C, having a mass of 3 kg, is lifted from a feed at A to a ramp at B by a rotating rod. If the rod maintains a constant angular velocity of $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the can at the instant $\theta = 30°$. Neglect the effects of friction in the calculation and the size of the can so that $r = (1.2\cos\theta)$ m. The ramp from A to B is circular, having a radius of 600 mm.



SOLUTION

$$r = 2(0.6\cos\theta) = 1.2\cos\theta$$

$$\dot{r} = -1.2\sin\theta\dot{\theta}$$

$$\ddot{r} = -1.2\cos\theta\dot{\theta}^2 - 1.2\sin\theta\ddot{\theta}$$

At $\theta = 30°$, $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$

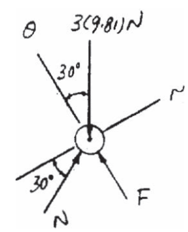
$$r = 1.2\cos 30° = 1.0392\text{ m}$$

$$\dot{r} = -1.2\sin 30°(0.5) = -0.3\text{ m/s}$$

$$\ddot{r} = -1.2\cos 30°(0.5)^2 - 1.2\sin 30°(0) = -0.2598\text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.2598 - 1.0392(0.5)^2 = -0.5196\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.0392(0) + 2(-0.3)(0.5) = -0.3\text{ m/s}^2$$



$+\nearrow\Sigma F_r = ma_r;\qquad N\cos 30° - 3(9.81)\sin 30° = 3(-0.5196)\qquad N = 15.19\text{ N}$

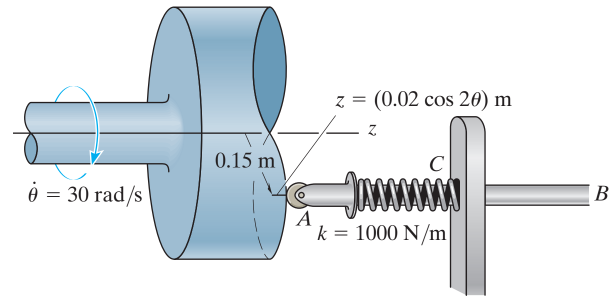
$+\nwarrow\Sigma F_\theta = ma_\theta;\qquad F + 15.19\sin 30° - 3(9.81)\cos 30° = 3(-0.3)$

$$F = 17.0\text{ N}$$ **Ans.**

Ans:
$F = 17.0$ N

***13–96.**

The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.15$ m and $z = (0.02 \cos 2\theta)$ m. If the cam is rotating at a constant rate of 30 rad/s, determine the force component F_z at the end A of the follower when $\theta = 30°$. The spring is uncompressed when $\theta = 90°$. Neglect friction at the bearing C.



SOLUTION

Kinematics. Using the chain rule, the first and second time derivatives of z are

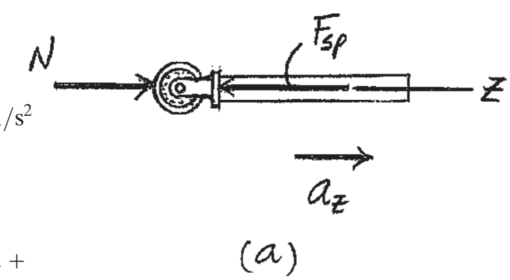
$$z = (0.02 \cos 2\theta) \text{ m}$$

$$\dot{z} = 0.02[-\sin 2\theta(2\dot{\theta})] = [-0.04(\sin 2\theta)\dot{\theta}] \text{ m/s}$$

$$\ddot{z} = -0.04[\cos 2\theta(2\dot{\theta})\dot{\theta} + (\sin 2\theta)\ddot{\theta}] = [-0.04(2 \cos 2\theta(\dot{\theta})^2 + \sin 2\theta(\ddot{\theta}))] \text{ m/s}^2$$

Here, $\dot{\theta} = 30$ rad/s and $\ddot{\theta} = 0$. Then

$$\ddot{z} = -0.04[2 \cos 2\theta(30^2) + \sin 2\theta(0)] = (-72 \cos 2\theta) \text{ m/s}^2$$



Equation of Motion. When $\theta = 30°$, the spring compresses $x = 0.02 + 0.02 \cos 2(30°) = 0.03$ m. Thus, $F_{sp} = kx = 1000(0.03) = 30$ N. Also, at this position $a_z = \ddot{z} = -72 \cos 2(30°) = -36.0 \text{ m/s}^2$. Referring to the FBD of the follower, Fig. a,

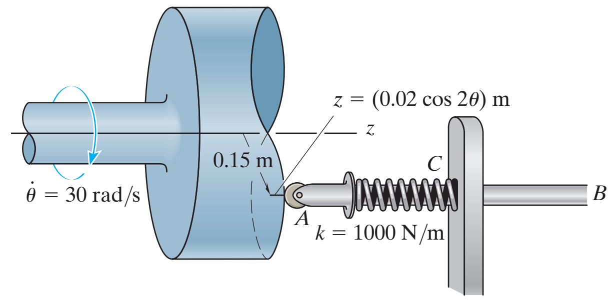
$$\Sigma F_z = ma_z; \qquad N - 30 = 0.5(-36.0)$$

$$N = 12.0 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$N = 12.0$ N

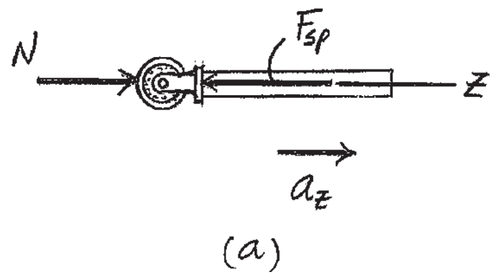
13–97.

The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.15$ m and $z = (0.02 \cos 2\theta)$ m. If the cam is rotating at a constant rate of 30 rad/s, determine the maximum and minimum force components F_z the follower exerts on the cam if the spring is uncompressed when $\theta = 90°$.



SOLUTION

Kinematics. Using the chain rule, the first and second time derivatives of z are



$$z = (0.02 \cos 2\theta) \text{ m}$$

$$\dot{z} = 0.02[-\sin 2\theta(2\dot{\theta})] = (-0.04 \sin 2\theta\dot{\theta}) \text{ m/s}$$

$$\ddot{z} = -0.04[\cos 2\theta(2\dot{\theta})\dot{\theta} + \sin 2\theta\ddot{\theta}] = [-0.04(2 \cos 2\theta(\dot{\theta})^2 + \sin 2\theta(\ddot{\theta}))] \text{ m/s}^2$$

Here $\dot{\theta} = 30$ rad/s and $\ddot{\theta} = 0$. Then,

$$\ddot{z} = -0.04[2 \cos 2\theta(30^2) + \sin 2\theta(0)] = (-72 \cos 2\theta) \text{ m/s}^2$$

Equation of Motion. At any arbitrary θ, the spring compresses $x = 0.02(1 + \cos 2\theta)$. Thus, $F_{sp} = kx = 1000[0.02(1 + \cos 2\theta)] = 20\,(1 + \cos 2\theta)$. Referring to the FBD of the follower, Fig. *a*,

$$\Sigma F_z = ma_z; \qquad N - 20(1 + \cos 2\theta) = 0.5(-72 \cos 2\theta)$$

$$N = (20 - 16 \cos 2\theta) \text{ N}$$

N is maximum when $\cos 2\theta = -1$. Then

$$(N)_{max} = 36.0 \text{ N} \qquad \textbf{Ans.}$$

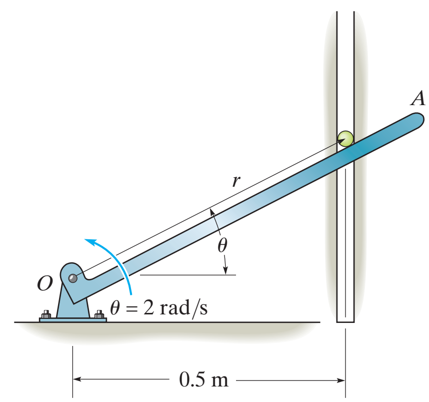
N is minimum when $\cos 2\theta = 1$. Then

$$(N)_{min} = 4.00 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$(N)_{max} = 36.0$ N
$(N)_{min} = 4.00$ N

13–98.

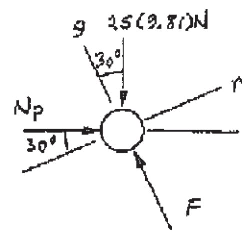
The particle has a mass of 0.5 kg and is confined to move along the smooth vertical slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



SOLUTION

$r = \dfrac{0.5}{\cos\theta} = 0.5\sec\theta, \qquad \dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$

$\ddot{r} = 0.5\sec\theta\tan\theta\ddot{\theta} + 0.5\sec^3\theta\dot{\theta}^2 + 0.5\sec\theta\tan^2\theta\dot{\theta}^2$



At $\theta = 30°$.

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 0.5774$ m

$\dot{r} = 0.6667$ m/s

$\ddot{r} = 3.8490$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 3.8490 - 0.5774(2)^2 = 1.5396$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(0.6667)(2) = 2.667$ m/s^2

$+\nearrow\Sigma F_r = ma_r; \qquad N_P\cos 30° - 0.5(9.81)\sin 30° = 0.5(1.5396)$

$N_P = 3.7208 = 3.72$ N **Ans.**

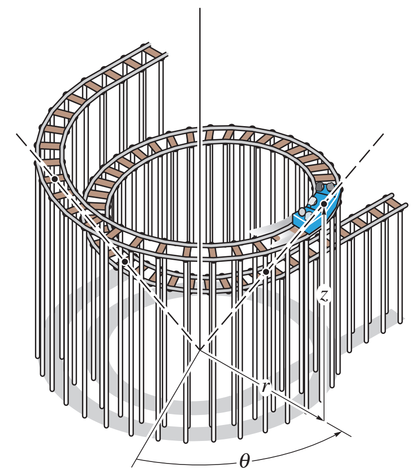
$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 3.7208\sin 30° - 0.5(9.81)\cos 30° = 0.5(2.667)$

$F = 7.44$ N **Ans.**

Ans:
$N_s = 3.72$ N
$F_r = 7.44$ N

13–99.

A car of a roller coaster travels along a track which for a short distance is defined by a conical spiral, $r = \frac{3}{4}z$, $\theta = -1.5z$, where r and z are in meters and θ in radians. If the angular motion $\dot{\theta} = 1\text{ rad/s}$ is always maintained, determine the r, θ, z components of reaction exerted on the car by the track at the instant $z = 6\text{ m}$. The car and passengers have a total mass of 200 kg.



SOLUTION

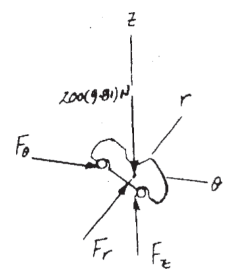
$$r = 0.75z \qquad \dot{r} = 0.75\dot{z} \qquad \ddot{r} = 0.75\ddot{z}$$

$$\theta = -1.5z \qquad \dot{\theta} = -1.5\dot{z} \qquad \ddot{\theta} = -1.5\ddot{z}$$

$$\dot{\theta} = 1 = -1.5\dot{z} \qquad \dot{z} = -0.6667\text{ m/s} \qquad \ddot{z} = 0$$

At $z = 6$ m,

$$r = 0.75(6) = 4.5\text{ m} \qquad \dot{r} = 0.75(-0.6667) = -0.5\text{ m/s} \qquad \ddot{r} = 0.75(0) = 0 \qquad \ddot{\theta} = 0$$



$$a_t = \ddot{r} - r\dot{\theta}^2 = 0 - 4.5(1)^2 = -4.5\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.5(0) + 2(-0.5)(1) = -1\text{ m/s}^2$$

$$a_z = \ddot{z} = 0$$

$\Sigma F_t = ma_r;$ $\quad F_r = 200(-4.5)$ $\quad F_r = -900\text{ N}$ **Ans.**

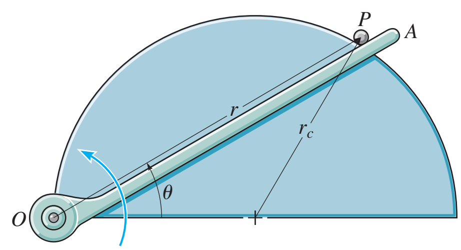
$\Sigma F_\theta = ma_\theta;$ $\quad F_\theta = 200(-1)$ $\quad F_\theta = -200\text{N}$ **Ans.**

$\Sigma F_z = ma_z;$ $\quad F_z - 200(9.81) = 0$ $\quad F_z = 1962\text{ N} = 1.96\text{ kN}$ **Ans.**

Ans:
$F_r = -900\text{ N}$
$F_\theta = -200\text{ N}$
$F_z = 1.96\text{ kN}$

***13–100.**

The 0.5-lb ball is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4$ rad/s and an angular acceleration $\ddot{\theta} = 0.8\ \text{rad/s}^2$ at the instant $\theta = 30°$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_c = 0.4$ ft.

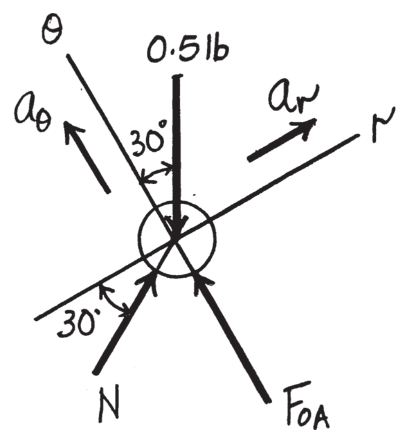


SOLUTION

$r = 2(0.4)\cos\theta = 0.8\cos\theta$

$\dot{r} = -0.8\sin\theta\dot{\theta}$

$\ddot{r} = -0.8\cos\theta\dot{\theta}^2 - 0.8\sin\theta\ddot{\theta}$



At $\theta = 30°$, $\dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8\ \text{rad/s}^2$

$r = 0.8\cos 30° = 0.6928$ ft

$\dot{r} = -0.8\sin 30°(0.4) = -0.16$ ft/s

$\ddot{r} = -0.8\cos 30°(0.4)^2 - 0.8\sin 30°(0.8) = -0.4309\ \text{ft/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.4309 - 0.6928(0.4)^2 = -0.5417\ \text{ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6928(0.8) + 2(-0.16)(0.4) = 0.4263\ \text{ft/s}^2$

$+\nearrow\Sigma F_r = ma_r;\quad N\cos 30° - 0.5\sin 30° = \dfrac{0.5}{32.2}(-0.5417)\quad N = 0.2790$ lb

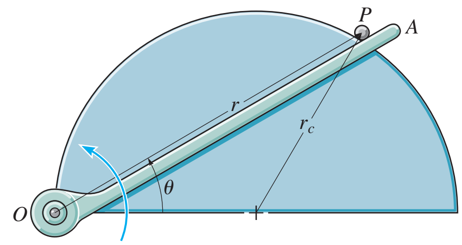
$\nwarrow+\Sigma F_\theta = ma_\theta;\quad F_{OA} + 0.2790\sin 30° - 0.5\cos 30° = \dfrac{0.5}{32.2}(0.4263)$

$F_{OA} = 0.300$ lb **Ans.**

Ans:
$F_{OA} = 0.300$ lb

13–101.

The ball of mass m is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has a constant angular velocity $\dot{\theta}_0$, determine the angle $\theta \leq 45°$ at which the ball starts to leave the surface of the semicylinder. Neglect friction and the size of the ball.



SOLUTION

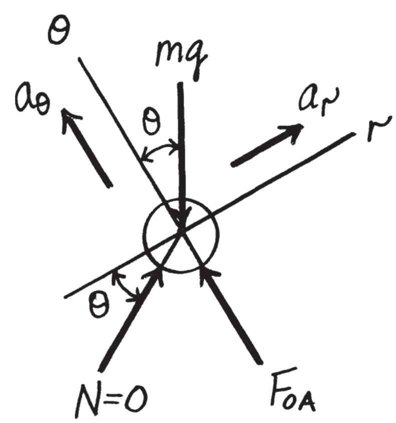
$r = 2r_c \cos\theta$

$\dot{r} = -2r_c \sin\theta\dot{\theta}$

$\ddot{r} = -2r_c \cos\theta\dot{\theta}^2 - 2r_c \sin\theta\ddot{\theta}$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2r_c \cos\theta\dot{\theta}_0^2 - 2r_c \cos\theta\dot{\theta}_0^2 = -4r_c \cos\theta\dot{\theta}_0^2$$



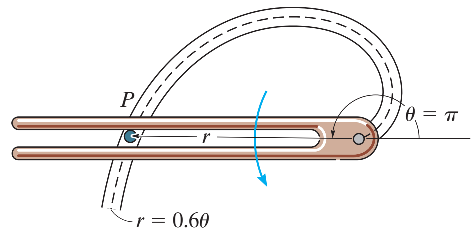
$+\nearrow\Sigma F_r = ma_r;\qquad -mg\sin\theta = m(-4r_c\cos\theta\dot{\theta}_0^2)$

$$\tan\theta = \frac{4r_c\,\dot{\theta}_0^2}{g} \qquad \theta = \tan^{-1}\left(\frac{4r_c\,\dot{\theta}_0^2}{g}\right) \qquad \textbf{Ans.}$$

Ans:
$\theta = \tan^{-1}\left(\frac{4r_c\dot{\theta}_0^2}{g}\right)$

13–102.

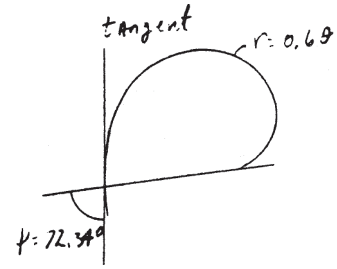
Using a forked rod, a smooth cylinder P, having a mass of 0.4 kg, is forced to move along the *vertical slotted* path $r = (0.6\theta)$ m, where θ is in radians. If the cylinder has a constant speed of $v_C = 2$ m/s, determine the force of the rod and the normal force of the slot on the cylinder at the instant $\theta = \pi$ rad. Assume the cylinder is in contact with only *one edge* of the rod and slot at any instant. *Hint:* To obtain the time derivatives necessary to compute the cylinder's acceleration components a_r and a_θ, take the first and second time derivatives of $r = 0.6\theta$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$, to determine $\ddot{\theta}$.



SOLUTION

$r = 0.6\theta \qquad \dot{r} = 0.6\dot{\theta} \qquad \ddot{r} = 0.6\ddot{\theta}$

$v_r = \dot{r} = 0.6\dot{\theta} \qquad v_\theta = r\dot{\theta} = 0.6\theta\dot{\theta}$

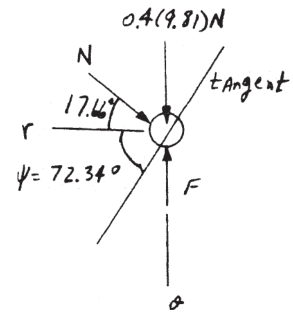


$v^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

$2^2 = \left(0.6\dot{\theta}\right)^2 + \left(0.6\theta\dot{\theta}\right)^2 \qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1 + \theta^2}}$

$0 = 0.72\dot{\theta}\ddot{\theta} + 0.36\left(2\theta\dot{\theta}^3 + 2\theta^2\dot{\theta}\ddot{\theta}\right) \qquad \ddot{\theta} = -\dfrac{\theta\dot{\theta}^2}{1 + \theta^2}$

At $\theta = \pi$ rad, $\qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1 + \pi^2}} = 1.011 \text{ rad/s}$



$\ddot{\theta} = -\dfrac{(\pi)(1.011)^2}{1 + \pi^2} = -0.2954 \text{ rad/s}^2$

$r = 0.6(\pi) = 0.6\,\pi \text{ m} \qquad \dot{r} = 0.6(1.011) = 0.6066 \text{ m/s}$

$\ddot{r} = 0.6(-0.2954) = -0.1772 \text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1772 - 0.6\pi(1.011)^2 = -2.104 \text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6\pi(-0.2954) + 2(0.6066)(1.011) = 0.6698 \text{ m/s}^2$

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.6\theta}{0.6} = \theta = \pi \qquad \psi = 72.34°$

$\overset{+}{\leftarrow} \Sigma F_r = ma_r; \qquad -N\cos 17.66° = 0.4(-2.104) \qquad N = 0.883 \text{ N}$ **Ans.**

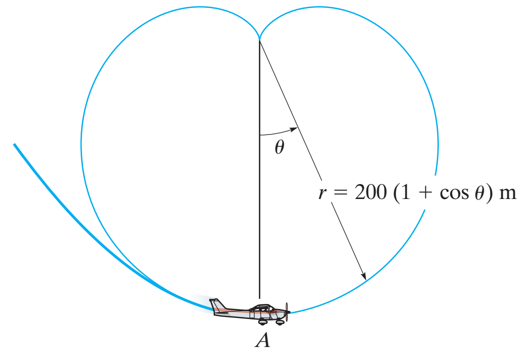
$+\downarrow \Sigma F_\theta = ma_\theta; \qquad -F + 0.4(9.81) + 0.883\sin 17.66° = 0.4(0.6698)$

$F = 3.92 \text{ N}$ **Ans.**

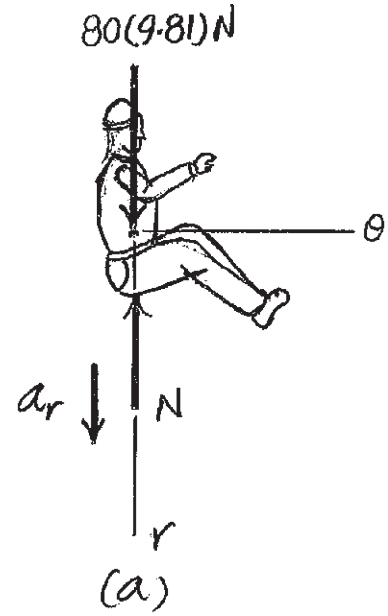
Ans:
$N = 0.883$ N
$F = 3.92$ N

13–103.

The pilot of the airplane executes a vertical loop which in part follows the path of a cardioid, $r = 200(1 + \cos\theta)$ m, where θ is in radians. If his speed at A is a constant $v_p = 85$ m/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He has a mass of 80 kg. *Hint:* To determine the time derivatives necessary to calculate the acceleration components a_r and a_θ, take the first and second time derivatives of $r = 200(1 + \cos\theta)$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$.



SOLUTION



Kinematic. Using the chain rule, the first and second time derivatives of r are

$$r = 200(1 + \cos\theta)$$

$$\dot{r} = 200(-\sin\theta)(\dot{\theta}) = -200(\sin\theta)\dot{\theta}$$

$$\ddot{r} = -200[(\cos\theta)(\dot{\theta})^2 + (\sin\theta)(\ddot{\theta})]$$

When $\theta = 0°$,

$$r = 200(1 + \cos 0°) = 400 \text{ m}$$

$$\dot{r} = -200(\sin 0°)\,\dot{\theta} = 0$$

$$\ddot{r} = -200\left[(\cos 0°)(\dot{\theta})^2 + (\sin 0°)(\ddot{\theta})\right] = -200\dot{\theta}^2$$

Using Eq. 12–26

$$v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$$

$$v^2 = \dot{r}^2 + (r\dot{\theta})^2$$

$$85^2 = 0^2 + (400\dot{\theta})^2$$

$$\dot{\theta} = 0.2125 \text{ rad/s}$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = -200\left(0.2125^2\right) - 400\left(0.2125^2\right) = -27.09 \text{ m/s}^2$$

Equation of Motion. Referring to the FBD of the pilot, Fig. *a*,

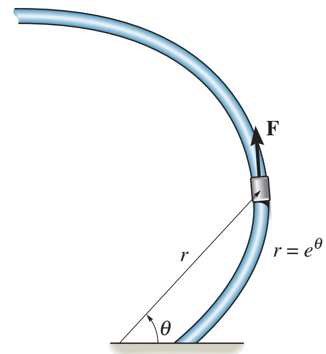
$\downarrow + \Sigma F_r = ma_r;\qquad 80(9.81) - N = 80(-27.09)$

$$N = 2952.3 \text{ N} = 2.95 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$N = 2.95$ N

***13–104.**

The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 45°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



SOLUTION

$r = e^{\theta}$

$\dot{r} = e^{\theta}\,\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 45°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 2.1933$

$\dot{r} = 4.38656$

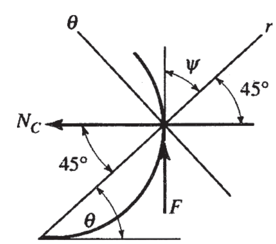
$\ddot{r} = 8.7731$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 8.7731 - 2.1933(2)^2 = 0$

$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(4.38656)(2) = 17.5462 \text{ m/s}^2$

$$\tan\psi = \frac{r}{\left(\dfrac{dr}{d\dot{\theta}}\right)} = e^{\theta}/e^{\theta} = 1$$

$\psi = \theta = 45°$



$\nearrow + \sum F_r = ma_r;\qquad -N_C\cos 45° + F\cos 45° = 2(0)$

$+\nwarrow \sum F_{\theta} = ma_{\theta};\qquad F\sin 45° + N_C\sin 45° = 2(17.5462)$

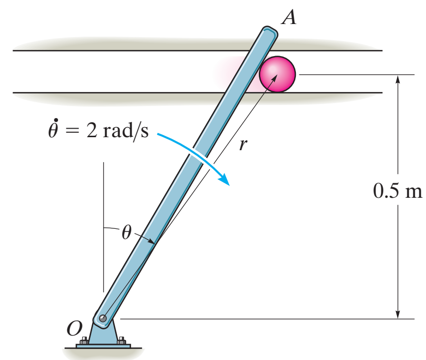
$N = 24.8$ N **Ans.**

$F = 24.8$ N **Ans.**

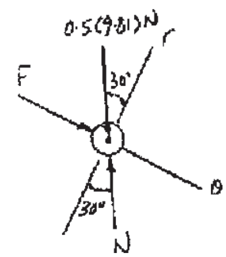
Ans:
$N = 24.8$ N
$F = 24.8$ N

13–105.

The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



SOLUTION



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta})\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}]$$

When $\theta = 30°, \dot{\theta} = 2$ rad/s and $\ddot{\theta} = 0$

$$r = 0.5\sec 30° = 0.5774 \text{ m}$$

$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667 \text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(0)\right]$$

$$= 3.849 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667 \text{ m/s}^2$$

$\nearrow + \Sigma F_r = ma_r;\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(1.540)$

$N = 5.79$ N **Ans.**

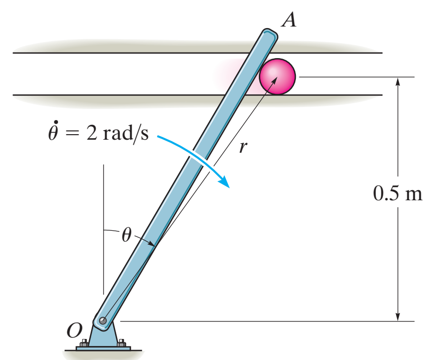
$+\searrow \Sigma F_\theta = ma_\theta;\quad F + 0.5(9.81)\sin 30° - 5.79\sin 30° = 0.5(2.667)$

$F = 1.78$ N **Ans.**

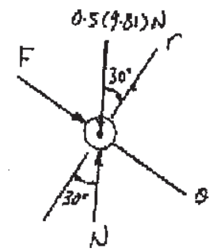
Ans:
$F_r = 1.78$ N
$N_s = 5.79$ N

13–106.

Solve Prob. 13–105 if the arm has an angular acceleration of $\ddot{\theta} = 3\text{ rad/s}^2$ when $\dot{\theta} = 2\text{ rad/s}$ at $\theta = 30°$.



SOLUTION



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta})\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30°$, $\dot{\theta} = 2\text{ rad/s}$ and $\ddot{\theta} = 3\text{ rad/s}^2$

$$r = 0.5\sec 30° = 0.5774\text{ m}$$

$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667\text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(3)\right]$$

$$= 4.849\text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4.849 - 0.5774(2)^2 = 2.5396\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(3) + 2(0.6667)(2) = 4.3987\text{ m/s}^2$$

$\nearrow + \Sigma F_r = ma_r;\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(2.5396)$

$N = 6.3712 = 6.37\text{ N}$ **Ans.**

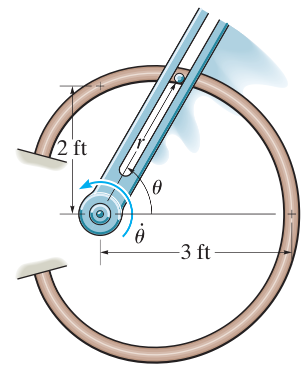
$+\searrow \Sigma F_\theta = ma_\theta;\quad F + 0.5(9.81)\sin 30° - 6.3712\sin 30° = 0.5(4.3987)$

$F = 2.93\text{ N}$ **Ans.**

Ans:
$F_r = 2.93\text{ N}$
$N_s = 6.37\text{ N}$

13–107.

The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where t is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.



SOLUTION

$r = 2 + \cos\theta \qquad \theta = 0.5t^2$

$\dot{r} = -\sin\theta\theta \qquad \dot{\theta} = t$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$

At $t = 1$ s, $\theta = 0.5$ rad, $\theta = 1$ rad/s, and $\ddot{\theta} = 1$ rad/s^2

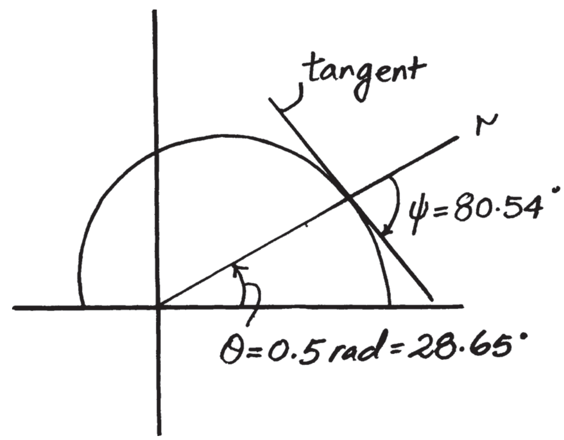
$r = 2 + \cos 0.5 = 2.8776$ ft

$\dot{r} = -\sin 0.5(1) = -0.4974 \text{ ft/s}^2$

$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357 \text{ ft/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -1.375 - 2.8776(1)^2 = -4.2346 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1) = 1.9187 \text{ ft/s}^2$

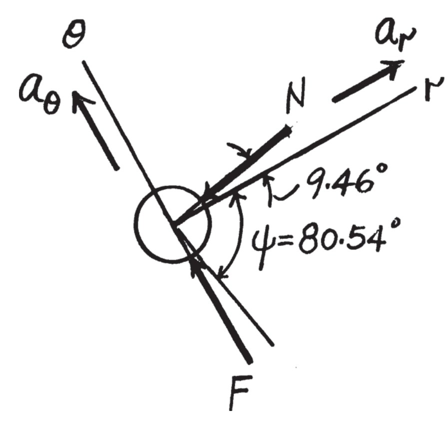


$$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{2 + \cos\theta}{-\sin\theta}\right|_{\theta=0.5\text{ rad}} = -6.002 \qquad \psi = -80.54°$$

$+\nearrow\Sigma F_r = ma_r; \qquad -N\cos 9.46° = \dfrac{2}{32.2}(-4.2346) \qquad N = 0.2666 \text{ lb}$

$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 0.2666\sin 9.46° = \dfrac{2}{32.2}(1.9187)$

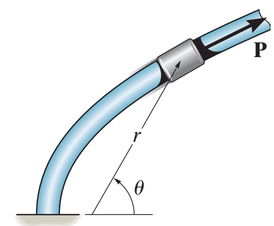
$F = 0.163$ lb **Ans.**



Ans:
$F = 0.163$ lb

***13–108.**

The collar, which has a weight of 3 lb, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = 4/(1 - \cos\theta)$, where θ is in radians and r is in feet. If the collar's angular rate is constant and equals $\dot{\theta} = 4$ rad/s, determine the tangential retarding force P needed to cause the motion and the normal force that the collar exerts on the rod at the instant $\theta = 90°$.

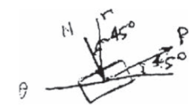


SOLUTION

$$r = \frac{4}{1 - \cos\theta}$$

$$\dot{r} = \frac{-4\sin\theta\,\dot{\theta}}{(1 - \cos\theta)^2}$$

$$\ddot{r} = \frac{-4\sin\theta\,\ddot{\theta}}{(1 - \cos\theta)^2} + \frac{-4\cos\theta\,(\dot{\theta})^2}{(1 - \cos\theta)^2} + \frac{8\sin^2\theta\,\dot{\theta}^2}{(1 - \cos\theta)^3}$$



At $\theta = 90°$, $\quad \dot{\theta} = 4, \quad \ddot{\theta} = 0$

$$r = 4$$

$$\dot{r} = -16$$

$$\ddot{r} = 128$$

$$a_r = \ddot{r} - r(\theta)^2 = 128 - 4(4)^2 = 64$$

$$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta} = 0 + 2(-16)(4) = -128$$

$$r = \frac{4}{1 - \cos\theta}$$

$$\frac{dr}{d\theta} = \frac{-4\sin\theta}{(1 - \cos\theta)^2}$$

$$\tan\psi = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \left.\frac{\frac{4}{1 - \cos\theta)}}{\frac{-4\sin\theta}{(1 - \cos\theta)^2}}\right|_{\theta = 90^\circ} = \frac{4}{-4} = -1$$

$$\psi = -45° = 135°$$

$$+\uparrow \Sigma F_r = m\,a_r; \qquad P\sin 45° - N\cos 45° = \frac{3}{32.2}(64)$$

$$\overset{+}{\leftarrow} \Sigma F_\theta = ma_\theta; \qquad -P\cos 45° - N\sin 45° = \frac{3}{32.2}(-128)$$

Solving,

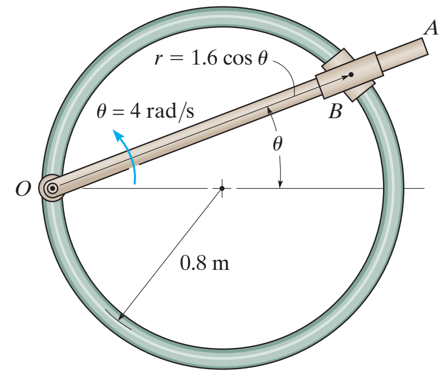
$$P = 12.6 \text{ lb} \qquad \textbf{Ans.}$$

$$N = 4.22 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$P = 12.6$ lb
$N = 4.22$ lb

13–109.

Rod OA rotates counterclockwise at a constant angular rate $\dot{\theta} = 4\text{ rad/s}$. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other collar slides over the circular rod described by the equation $r = (1.6\cos\theta)$ m. If *both* collars have a mass of 0.5 kg, determine the force which the circular rod exerts on one of the collars and the force that OA exerts on the other collar at the instant $\theta = 45°$. Motion is in the horizontal plane.



SOLUTION

$r = 1.6\cos\theta$

$\dot{r} = -1.6\sin\theta\dot{\theta}$

$\ddot{r} = -1.6\cos\theta\dot{\theta}^2 - 1.6\sin\theta\ddot{\theta}$

At $\theta = 45°, \dot{\theta} = 4\text{ rad/s}$ and $\ddot{\theta} = 0$

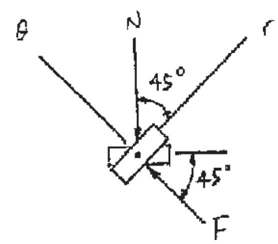
$r = 1.6\cos 45° = 1.1314\text{ m}$

$\dot{r} = -1.6\sin 45°(4) = -4.5255\text{ m/s}$

$\ddot{r} = -1.6\cos 45°(4)^2 - 1.6\sin 45°(0) = -18.1019\text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -18.1019 - 1.1314(4)^2 = -36.20\text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.1314(0) + 2(-4.5255)(4) = -36.20\text{ m/s}^2$



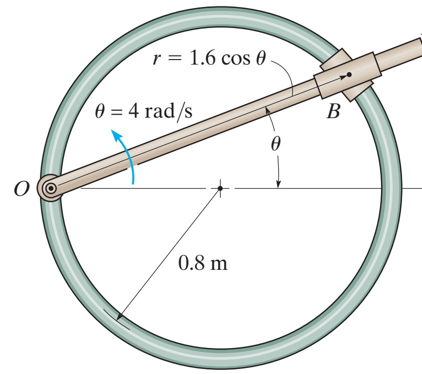
$\nearrow+\Sigma F_r = ma_r;$ $\quad -N_C\cos 45° = 0.5(-36.20)$ $\quad N_C = 25.6\text{ N}$ **Ans.**

$+\nwarrow\Sigma F_\theta = ma_\theta;$ $\quad F_{OA} - 25.6\sin 45° = 0.5(-36.20)$ $\quad F_{OA} = 0$ **Ans.**

Ans:
$F_r = 25.6\text{ N}$
$F_{OA} = 0$

13–110.

Solve Prob. 13–109 if motion is in the vertical plane.



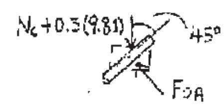
SOLUTION

$r = 1.6 \cos\theta$

$\dot{r} = -1.6 \sin\theta\dot{\theta}$

$\ddot{r} = -1.6 \cos\theta\dot{\theta}^2 - 1.6 \sin\theta\ddot{\theta}$

At $\theta = 45^\circ$, $\dot{\theta} = 4$ rad/s and $\ddot{\theta} = 0$



$r = 1.6 \cos 45^\circ = 1.1314$ m

$\dot{r} = -1.6 \sin 45^\circ(4) = -4.5255$ m/s

$\ddot{r} = -1.6 \cos 45^\circ(4)^2 - 1.6 \sin 45^\circ(0) = -18.1019$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -18.1019 - 1.1314(4)^2 = -36.20$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.1314(0) + 2(-4.5255)(4) = -36.20$ m/s^2

$+\swarrow \Sigma F_r = ma_r$; $\quad -N_C \cos 45^\circ - 4.905 \cos 45^\circ = 0.5(-36.204)$

$+\nwarrow \Sigma F_\theta = ma_\theta$; $\quad F_{OA} - N_C \sin 45^\circ - 4.905 \sin 45^\circ = 0.5(-36.204)$

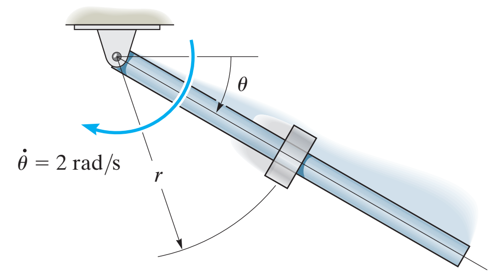
$N_C = 20.7$ N **Ans.**

$F_{OA} = 0$ **Ans.**

Ans:
$F_r = 20.7$ N
$F_{OA} = 0$

13–111.

A 0.2-kg spool slides down along a smooth rod. If the rod has a constant angular rate of rotation $\dot{\theta} = 2$ rad/s in the vertical plane, show that the equations of motion for the spool are $\ddot{r} - 4r - 9.81 \sin\theta = 0$ and $0.8\dot{r} + N_s - 1.962\cos\theta = 0$, where N_s is the magnitude of the normal force of the rod on the spool. Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-2t} + C_2 e^{2t} - (9.81/8)\sin 2t$. If r, $\dot{r}$, and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 to determine r at the instant $\theta = \pi/4$ rad.

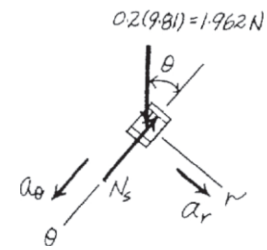


SOLUTION

Kinematic: Here, $\dot{\theta}. = 2$ rad/s and $\ddot{\theta} = 0$. Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - r(2^2) = \ddot{r} - 4r$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(0) + 2\dot{r}(2) = 4\dot{r}$$



Equation of Motion: Applying Eq. 13–9, we have

$\Sigma F_r = ma_r;$ $\quad 1.962\sin\theta = 0.2(\ddot{r} - 4r)$

$$\ddot{r} - 4r - 9.81\sin\theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(1)}$$

$\Sigma F_\theta = ma_\theta;$ $\quad 1.962\cos\theta - N_s = 0.2(4\dot{r})$

$$0.8\dot{r} + N_s - 1.962\cos\theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(2)}$$

Since $\theta. = 2$ rad/s, then $\int_0^\theta \dot{\theta} = \int_0^1 2dt, \theta = 2t$. The solution of the differential equation (Eq.(1)) is given by

$$r = C_1\, e^{-2t} + C_2\, e^{2t} - \frac{9.81}{8}\sin 2t \qquad \textbf{(3)}$$

Thus,

$$\dot{r} = -2\,C_1 e^{-2t} + 2C_2\, e^{2t} - \frac{9.81}{4}\cos 2t \qquad \textbf{(4)}$$

At $t = 0, r = 0$. From Eq.(3) $0 = C_1(1) + C_2(1) - 0$ **(5)**

At $t = 0, \dot{r} = 0$. From Eq.(4) $0 = -2\,C_1(1) + 2C_2(1) - \frac{9.81}{4}$ **(6)**

Solving Eqs. (5) and (6) yields

$$C_1 = -\frac{9.81}{16} \qquad C_2 = \frac{9.81}{16}$$

Thus,

$$r = -\frac{9.81}{16}e^{-2t} + \frac{9.81}{16}e^{2t} - \frac{9.81}{8}\sin 2t$$

$$= \frac{9.81}{8}\left(\frac{-e^{-2t} + e^{2t}}{2} - \sin 2t\right)$$

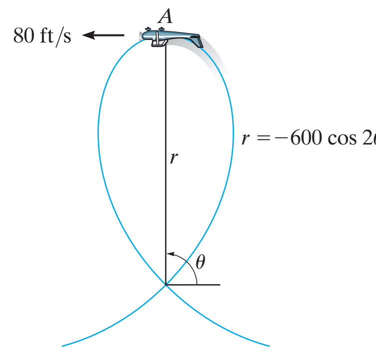
$$= \frac{9.81}{8}(\sin\text{h}\, 2t - \sin 2t)$$

At $\theta = 2t = \frac{\pi}{4}$, $\quad r = \frac{9.81}{8}\left(\sin\text{h}\frac{\pi}{4} - \sin\frac{\pi}{4}3\right) = 0.198$ m **Ans.**

Ans:
$r = 0.198$ m

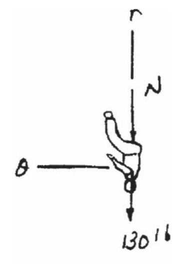
***13–112.**

The pilot of an airplane executes a vertical loop which in part follows the path of a "four-leaved rose," $r = (-600 \cos 2\theta)$ ft, where θ is in radians. If his speed at A is a constant $v_P = 80$ ft/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He weighs 130 lb. *Hint:* To determine the time derivatives necessary to compute the acceleration components a_r and a_θ, take the first and second time derivatives of $r = 400(1 + \cos\theta)$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$, to determine $\ddot{\theta}$.



SOLUTION

$r = -600 \cos 2\theta \qquad \dot{r} = 1200 \sin 2\theta\dot{\theta} \qquad \ddot{r} = 1200\left(2 \cos 2\theta\dot{\theta}^2 + \sin 2\theta\ddot{\theta}\right)$



At $\theta = 90°$

$r = -600 \cos 180° = 600 \text{ ft} \qquad \dot{r} = 1200 \sin 180°\dot{\theta} = 0$

$\ddot{r} = 1200\left(2 \cos 180°\dot{\theta}^2 + \sin 180°\ddot{\theta}\right) = -2400\dot{\theta}^2$

$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = 600\dot{\theta}$

$v_p^2 = v_r^2 + v_\theta^2$

$80^2 = 0^2 + \left(600\,\dot{\theta}\right)^2 \qquad \dot{\theta} = 0.1333 \text{ rad/s}$

$\ddot{r} = -2400(0.1333)^2 = -42.67 \text{ ft/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -42.67 - 600(0.1333)^2 = -53.33 \text{ ft/s}^2$

$+\uparrow \Sigma F_r = ma_r; \qquad -N - 130 = \dfrac{130}{32.2}(-53.33) \qquad N = 85.3 \text{ lb}$ **Ans.**

Ans:
$N = 85.3$ lb

13–113.

The earth has an orbit with eccentricity $e = 0.0167$ around the sun. Knowing that the earth's minimum distance from the sun is $146(10^6)$ km, find the speed at which the earth travels when it is at this distance. Determine the equation in polar coordinates which describes the earth's orbit about the sun.

SOLUTION

$$e = \frac{Ch^2}{GM_S} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0\, v_0^2}\right) \text{ and } h = r_0\, v_0$$

$$e = \frac{1}{GM_S\, r_0}\left(1 - \frac{GM_S}{r_0\, v_0^2}\right)(r_0 v_0)^2 \qquad e = \left(\frac{r_0\, v_0^2}{GM_S} - 1\right) \qquad \frac{r_0 v_0^2}{GM_S} = e + 1$$

$$v_0 = \sqrt{\frac{GM_S\,(e+1)}{r_0}}$$

$$= \sqrt{\frac{66.73(10^{-12})(1.99)(10^{30})(0.0167 + 1)}{146(10^9)}} = 30409 \text{ m/s} = 30.4 \text{ km/s} \qquad \textbf{Ans.}$$

$$\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0\, v_0^2}\right)\cos\theta + \frac{GM_S}{r_0^2 v_0^2}$$

$$\frac{1}{r} = \frac{1}{146(10^9)}\left(1 - \frac{66.73(10^{-12})(1.99)(10^{30})}{151.3(10^9)(30409)^2}\right)\cos\theta + \frac{66.73(10^{-12})(1.99)(10^{30})}{\left[146(10^9)\right]^2 (30409)^2}$$

$$\frac{1}{r} = 0.348(10^{-12})\cos\theta + 6.74(10^{-12}) \qquad \textbf{Ans.}$$

Ans:
$v_o = 30.4$ km/s
$\frac{1}{r} = 0.348\left(10^{-12}\right)\cos\theta + 6.74\left(10^{-12}\right)$

13–114.

A communications satellite is in a circular orbit above the earth such that it always remains directly over a point on the earth's surface. As a result, the period of the satellite must equal the rotation of the earth, which is approximately 24 hours. Determine the satellite's altitude h above the earth's surface and its orbital speed.

SOLUTION

The period of the satellite around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$T = \frac{2\pi r_0}{v_s}$$

$$24(3600) = \frac{2\pi\left[h + 6.378(10^6)\right]}{v_s}$$

$$v_s = \frac{2\pi\left[h + 6.378(10^6)\right]}{86.4(10^3)} \qquad \textbf{(1)}$$

The velocity of the satellite orbiting around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$v_S = \sqrt{\frac{GM_e}{r_0}}$$

$$v_S = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{h + 6.378(10^6)}} \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2),

$h = 35.87(10^6)\text{ m} = 35.9\text{ Mm}$ $\qquad$ $v_S = 3072.32\text{ m/s} = 3.07\text{ km/s}$ **Ans.**

Ans:
$h = 35.9\text{ mm}$
$v_s = 3.07\text{ km/s}$

13–115.

The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

SOLUTION

For a 800-km orbit

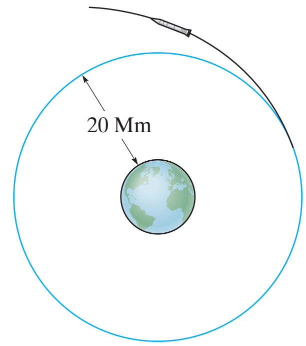
$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800 + 6378)(10^3)}}$$

$$= 7453.6 \text{ m/s} = 7.45 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v_0 = 7.45 \text{ km/s}$

***13–116.**

The rocket is in circular orbit about the earth at an altitude of 20 Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



SOLUTION

The speed of the rocket in circular orbit is

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})\left[5.976(10^{24})\right]}{20(10^6) + 6378(10^3)}} = 3888.17 \text{ m/s}$$

To escape the earth's gravitational field, the rocket must enter the parabolic trajectory, which require its speed to be

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]}{20(10^6) + 6378(10^3)}} = 5498.70 \text{ m/s}$$

The required increment in speed is

$$\begin{aligned}\Delta v = v_e - v_c &= 5498.70 - 3888.17 \\ &= 1610.53 \text{ m/s} \\ &= 1.61(10^3) \text{ m/s}\end{aligned}$$ **Ans.**

Ans:
$\Delta v = 1.61(10^3)$ m/s

13–117.

Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

SOLUTION

From Eq. 13–19,

$$\frac{1}{r} = C\cos\theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$,

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C, from Eqs. 13–28 and 13–29,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13–31,

$$T = \frac{\pi}{h}(2a)(b)$$

Thus,

$$b^2 = \frac{T^2h^2}{4\pi^2a^2}$$

$$\frac{4\pi^2a^3}{T^2h^2} = \frac{GM_s}{h^2}$$

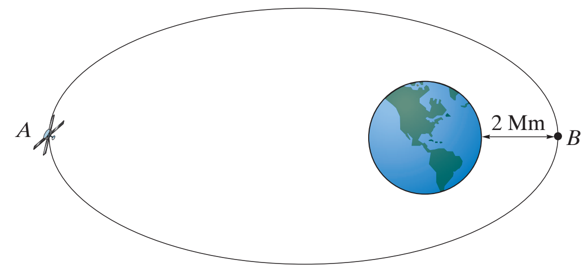
$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3 \qquad \textbf{Q.E.D.}$$

Ans:

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3$$

13–118.

The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.



SOLUTION

$$e = \frac{Ch^2}{GM_e}$$

where $C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)$ and $h = r_0\, v_0$.

$$e = \frac{1}{GM_e\, r_0}\left(1 - \frac{GM_e}{r_0\, v_0^2}\right)(r_0\, v_0)^2$$

$$e = \left(\frac{r_0\, v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0\, v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e\,(e+1)}{r_0}}$$

where $r_0 = r_p = 2(10^6) + 6378(10^3) = 8.378(10^6)$ m.

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.25 + 1)}{8.378(10^6)}} = 7713 \text{ m/s} = 7.71 \text{ km/s} \qquad \textbf{Ans.}$$

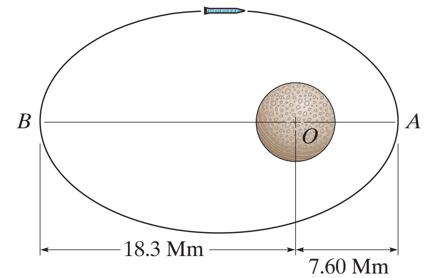
$$r_a = \frac{r_0}{\frac{2GM_e}{r_0\, v_0} - 1} = \frac{8.378(10^6)}{\frac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)(7713)^2} - 1} = 13.96(10^6) \text{ m}$$

$$v_A = \frac{r_p}{r_a} v_B = \frac{8.378(10^6)}{13.96(10^6)}(7713) = 4628 \text{ m/s} = 4.63 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 7.71$ km/s
$v_A = 4.63$ km/s

13–119.

The rocket is traveling in free flight along the elliptical orbit. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the orbit shown, determine the rocket's speed when it is at A and at B.



SOLUTION

Applying Eq. 13–27,

$$r_a = \frac{r_p}{\left(2GM/r_p v_p^2\right) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM\, r_a}{r_p(r_p + r_a)}}$$

The elliptical orbit has $r_p = 7.60(10^6)$ m, $r_a = 18.3(10^6)$ m and $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73\left(10^{-12}\right)\right]\left[0.6(5.976)\left(10^{24}\right)\right]\left[18.3\left(10^6\right)\right]}{7.60\left(10^6\right)\left[7.60\left(10^6\right) + 18.3\left(10^6\right)\right]}}$$

$$= 6669.99 \text{ m/s} = 6.67\left(10^3\right) \text{ m/s} \qquad \textbf{Ans.}$$

In this case,

$$h = r_p v_A = r_a v_B$$

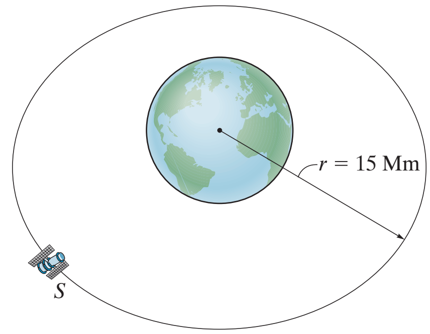
$$7.60\left(10^6\right)(6669.99) = 18.3\left(10^6\right)v_B$$

$$v_B = 2770.05 \text{ m/s} = 2.77\left(10^3\right) \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 6.67\left(10^3\right)$ m/s
$v_B = 2.77\left(10^3\right)$ m/s

***13–120.**

Determine the constant speed of satellite S so that it circles the earth with an orbit of radius $r = 15$ Mm. *Hint:* Use Eq. 13–1.



SOLUTION

$$F = G\frac{m_s m_e}{r^2} \qquad \text{Also} \qquad F = m_s\left(\frac{v_s^2}{r}\right) \qquad \text{Hence}$$

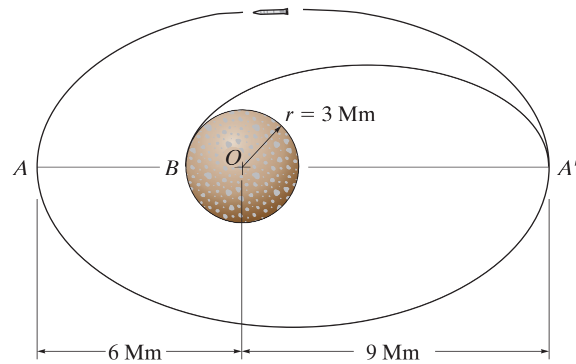
$$m_s\left(\frac{v_0^2}{r}\right) = G\frac{m_s m_e}{r^2}$$

$$v = \sqrt{G\frac{m_e}{r}} = \sqrt{66.73(10^{-12})\left(\frac{5.976(10^{24})}{15(10^6)}\right)} = 5156 \text{ m/s} = 5.16 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v = 5.16$ km/s

13–121.

The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.



SOLUTION

Central-Force Motion: Use $r_a = \dfrac{r_0}{(2\,GM/r_0\,v_0^2)-1}$, with $r_0 = r_p = 6\left(10^6\right)$ m and $M = 0.70M_e$, we have

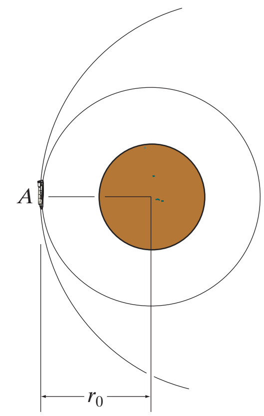
$$9\left(10^6\right) = \frac{6(10)^6}{\left(\dfrac{2(66.73)\,(10^{-12})\,(0.7)\,[5.976(10^{24})]}{6(10^6)v_P^2}\right) - 1}$$

$$v_A = 7471.89 \text{ m/s} = 7.47 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 7.47$ km/s

13–122.

The Viking Explorer approaches the planet Mars on a parabolic trajectory as shown. When it reaches point A its velocity is 10 Mm/h. Determine r_0 and the required velocity at A so that it can then maintain a circular orbit as shown. The mass of Mars is 0.1074 times the mass of the earth.



SOLUTION

When the Viking explorer approaches point A on a parabolic trajectory, its velocity at point A is given by

$$v_A = \sqrt{\frac{2GM_M}{r_0}}$$

$$\left[10(10^6)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \sqrt{\frac{2(66.73)(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{r_0}}$$

$$r_0 = 11.101(10^6)\text{ m} = 11.1\text{ Mm}$$ **Ans.**

When the explorer travels along a circular orbit of $r_0 = 11.101(10^6)$ m, its velocity is

$$v_{A'} = \sqrt{\frac{GM_r}{r_0}} = \sqrt{\frac{66.73(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{11.101(10^6)}}$$

$$= 1964.19\text{ m/s}$$

Thus, the required sudden decrease in the explorer's velocity is

$$\Delta v_A = v_A - v_{A'}$$

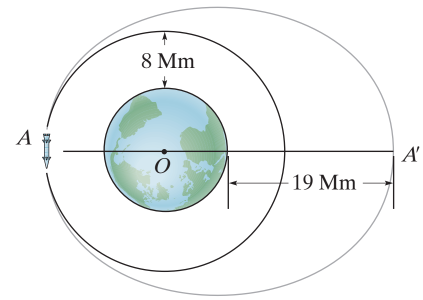
$$= 10(10^6)\left(\frac{1}{3600}\right) - 1964.19$$

$$= 814\text{ m/s}$$ **Ans.**

Ans:
$r_0 = 11.1$ Mm
$\Delta v_A = 814$ m/s

13–123.

The rocket is initially in free-flight circular orbit around the earth. Determine the speed of the rocket at A. What change in the speed at A is required so that it can move in an elliptical orbit to reach point A'?



SOLUTION

The required speed to remain in circular orbit containing point A of which $r_0 = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m can be determined from

$$(v_A)_C = \sqrt{\frac{GM_e}{r_0}}$$

$$= \sqrt{\frac{[66.73(10^{-12})][5.976(10^{24})]}{14.378(10^6)}}$$

$$= 5266.43 \text{ m/s} = 5.27(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

To more from A to A', the rocket has to follow the elliptical orbit with $r_p = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m and $r_a = 19(10^6) + 6378(10^3) = 25.378(10^6)$ m. The required speed at A to do so can be determined using Eq. 13–27.

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here, $v_p = (v_A)_e$. Then

$$(v_A)_e = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][25.378(10^6)]}{14.378(10^6)[14.378(10^6) + 25.378(10^6)]}}$$

$$= 5950.58 \text{ m/s}$$

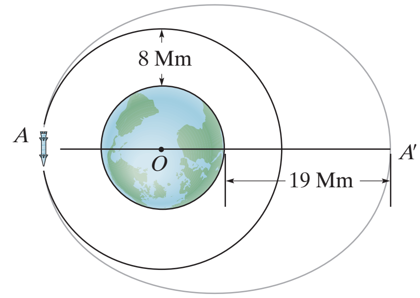
Thus, the required change in speed is

$$\Delta v = (v_A)_e - (v_A)_c = 5950.58 - 5266.43 = 684.14 \text{ m/s} = 684 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$(v_A)_C = 5.27(10^3)$ m/s
$\Delta v = 684$ m/s

***13–124.**

The rocket is in free-flight circular orbit around the earth. Determine the time needed for the rocket to travel from the inner orbit at A to the outer orbit at A'.



SOLUTION

To move from A to A', the rocket has to follow the elliptical orbit with $r_p = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m and $r_a = 19(10^6) + 6378(10^3) = 25.378(10^6)$ m. The required speed at A to do so can be determined using Eq. 13–27.

$$r_a = \frac{r_p}{(2GM_e/r_P v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here, $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]\left[25.378(10^6)\right]}{14.378(10^6)\left[14.378(10^6) + 25.378(10^6)\right]}} = 5950.58 \text{ m/s}$$

Then

$$h = v_A r_p = 5950.58\left[14.378(10^6)\right] = 85.5573(10^9) \text{ m}^2/\text{s}$$

The period of this elliptical orbit can be determined using Eq. 13–31.

$$\begin{aligned} T &= \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a} \\ &= \frac{\pi}{85.5573(10^9)}\left[14.378(10^6) + 25.378(10^6)\right]\sqrt{\left[14.378(10^6)\right]\left[25.378(10^6)\right]} \\ &= 27.885(10^3) \text{ s} \end{aligned}$$

Thus, the time required to travel from A to A' is

$$t = \frac{T}{2} = \frac{27.885(10^3)}{2} = 13.94(10^3) \text{ s} = 3.87 \text{ h} \qquad \textbf{Ans.}$$

Ans:
$t = 3.87$ h

13–125.

A satellite is launched with an initial velocity $v_0 = 2500 \text{ mi/h}$ parallel to the surface of the earth. Determine the required altitude (or range of altitudes) above the earth's surface for launching if the free-flight trajectory is to be (a) circular, (b) parabolic, (c) elliptical, and (d) hyperbolic. Take $G = 34.4(10^{-9})(\text{lb}\cdot\text{ft}^2)/\text{slug}^2$, $M_e = 409(10^{21})$ slug, the earth's radius $r_e = 3960$ mi, and $1 \text{ mi} = 5280 \text{ ft}$.

SOLUTION

$$v_0 = 2500 \text{ mi/h} = 3.67(10^3) \text{ ft/s}$$

(a) $$e = \frac{C^2 h}{GM_e} = 0 \qquad \text{or } C = 0$$

$$1 = \frac{GM_e}{r_0\, v_0^2}$$

$$GM_e = 34.4(10^{-9})(409)(10^{21})$$

$$= 14.07(10^{15})$$

$$r_0 = \frac{GM_e}{v_0^2} = \frac{14.07(10^{15})}{[3.67(10^{13})]^2} = 1.046(10^9) \text{ ft}$$

$$r = \frac{1.047(10^9)}{5280} - 3960 = 194(10^3) \text{ mi}$$ **Ans.**

(b) $$e = \frac{C^2 h}{GM_e} = 1$$

$$\frac{1}{GM_e}(r_0^2 v_0^2)\left(\frac{1}{r_0}\right)\left(1 - \frac{GM_e}{r_0\, v_0^2}\right) = 1$$

$$r_0 = \frac{2GM_e}{v_0^2} = \frac{2(14.07)(10^{15})}{[3.67(10^3)]^2} = 2.09(10^9) \text{ ft} = 396(10^3) \text{ mi}$$

$$r = 396(10^3) - 3960 = 392(10^3) \text{ mi}$$ **Ans.**

(c) $$e < 1$$

$$194(10^3) \text{ mi} < r < 392(10^3) \text{ mi}$$ **Ans.**

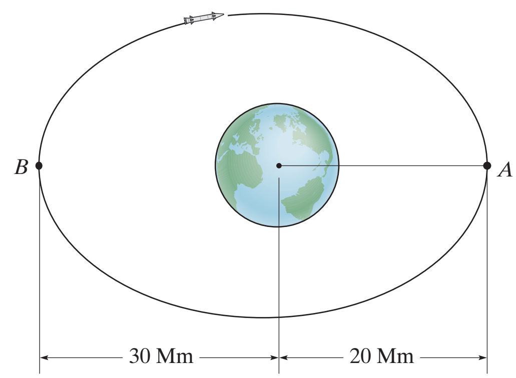
(d) $$e > 1$$

$$r > 392(10^3) \text{ mi}$$ **Ans.**

Ans:
(a) $r = 194\,(10^3)$ mi
(b) $r = 392\,(10^3)$ mi
(c) $194\,(10^3) \text{ mi} < r < 392\,(10^3)$ mi
(d) $r > 392\,(10^3)$ mi

13–126.

The rocket is traveling around the earth in free flight along the elliptical orbit. If the rocket has the orbit shown, determine the speed of the rocket when it is at A and at B.



SOLUTION

Here $r_p = 20(10^6)$ m and $r_a = 30(10^6)$ m. Applying Eq. 13–27,

$$r_a = \frac{r_p}{\left(2GM_e/r_p v_p^2\right) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_P = \sqrt{\frac{2GM_e r_a}{r_p\left(r_p + r_a\right)}}$$

Here $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73\left(10^{-12}\right)\right]\left[5.976\left(10^{24}\right)\right]\left[30\left(10^6\right)\right]}{20\left(10^6\right)\left[20\left(10^6\right) + 30\left(10^6\right)\right]}}$$

$$= 4891.49 \text{ m/s} = 4.89\left(10^3\right) \text{ m/s} \qquad \textbf{Ans.}$$

For the same orbit h is constant. Thus,

$$h = r_p v_p = r_a v_a$$

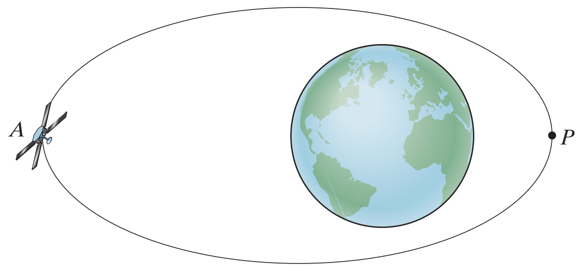
$$\left[20\left(10^6\right)\right](4891.49) = \left[30\left(10^6\right)\right] v_B$$

$$v_B = 3261.00 \text{ m/s} = 3.26\left(10^3\right) \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 4.89\left(10^3\right)$ m/s
$v_B = 3.26\left(10^3\right)$ m/s

13–127.

An elliptical path of a satellite has an eccentricity $e = 0.130$. If it has a speed of 15 Mm/h when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?



SOLUTION

$$e = 0.130$$

$$v_p = v_0 = 15 \text{ Mm/h} = 4.167 \text{ km/s}$$

$$e = \frac{Ch^2}{GM_e} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_e}\right)$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1$$

$$r_0 = \frac{(e+1)GM_e}{v_0^2}$$

$$= \frac{1.130(66.73)(10^{-12})(5.976)(10^{24})}{\left[4.167(10^3)\right]^2}$$

$$= 25.96 \text{ Mm}$$

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e+1}$$

$$r_A = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e+1}\right) - 1}$$

$$r_A = \frac{r_0(e+1)}{1-e}$$

$$= \frac{25.96(10^6)(1.130)}{0.870}$$

$$= 33.71(10^6) \text{ m} = 33.7 \text{ Mm}$$

$$v_A = \frac{v_0 r_0}{r_A}$$

$$= \frac{15(25.96)(10^6)}{33.71(10^6)}$$

$$= 11.5 \text{ Mm/h}$$ **Ans.**

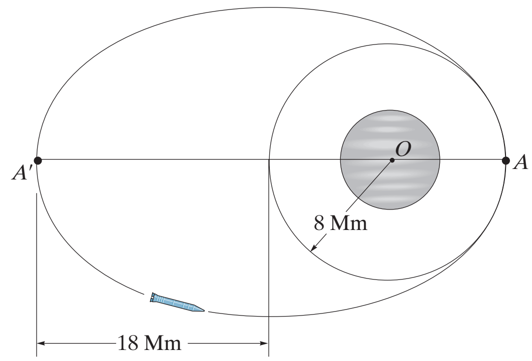
$$d = 33.71(10^6) - 6.378(10^6)$$

$$= 27.3 \text{ Mm}$$ **Ans.**

Ans:
$v_A = 11.5$ Mm/h
$d = 27.3$ Mm

***13–128.**

A rocket is in free-flight elliptical orbit around the planet Venus. Knowing that the periapsis and apoapsis of the orbit are 8 Mm and 26 Mm, respectively, determine (a) the speed of the rocket at point A', (b) the required speed it must attain at A just after braking so that it undergoes an 8-Mm free-flight circular orbit around Venus, and (c) the periods of both the circular and elliptical orbits. The mass of Venus is 0.816 times the mass of the earth.



SOLUTION

a)

$$M_\nu = 0.816(5.976(10^{24})) = 4.876(10^{24})$$

$$OA' = \frac{OA}{\left(\dfrac{2GM\nu}{OA\,\nu_A^2} - 1\right)}$$

$$26(10)^6 = \frac{8(10^6)}{\left(\dfrac{2(66.73)(10^{-12})4.876(10^{24})}{8(10^6)\nu_A^2} - 1\right)}$$

$$\frac{81.35(10^6)}{\nu_A^2} = 1.307$$

$$\nu_A = 7887.3 \text{ m/s} = 7.89 \text{ km/s}$$

$$\nu_A = \frac{OA\,\nu_A}{OA'} = \frac{8(10^6)(7887.3)}{26(10^6)} = 2426.9 \text{ m/s} = 2.43 \text{ m/s} \qquad \textbf{Ans.}$$

b)

$$\nu_{A''} = \sqrt{\frac{GM_\nu}{OA'}} = \sqrt{\frac{66.73(10^{-12})4.876(10^{24})}{8(10^6)}}$$

$$\nu_{A''} = 6377.7 \text{ m/s} = 6.38 \text{ km/s} \qquad \textbf{Ans.}$$

c)

Circular orbit:

$$T_c = \frac{2\pi OA}{\nu_{A''}} = \frac{2\pi 8(10^6)}{6377.7} = 7881.41 \text{ s} = 2.19 \text{ h} \qquad \textbf{Ans.}$$

Elliptic orbit:

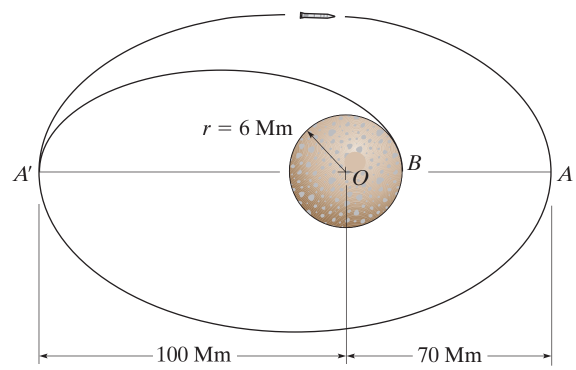
$$T_e = \frac{\pi}{OA\nu_A}(OA + OA')\sqrt{(OA)(OA')} = \frac{\pi}{8(10^6)(7886.8)}(8 + 26)\left(10^6\right)\left(\sqrt{(8)(26)}\right)\left(10^6\right)$$

$$T_e = 24414.2 \text{ s} = 6.78 \text{ h} \qquad \textbf{Ans.}$$

Ans:
$v_A = 2.43$ m/s
$v_{A''} = 6.38$ km/s
$T_c = 2.19$ h
$T_e = 6.78$ h

13–129.

The rocket is traveling in a free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the orbit shown, determine the rocket's velocity when it is at point A.



SOLUTION

Applying Eq. 13–27,

$$r_a = \frac{r_p}{\left(2GM/r_p v_p^2\right) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GMr_a}{r_p\left(r_p + r_a\right)}}$$

The rocket is traveling around the elliptical orbit with $r_p = 70\left(10^6\right)$ m, $r_a = 100\left(10^6\right)$ m and $v_p = v_A$. Then

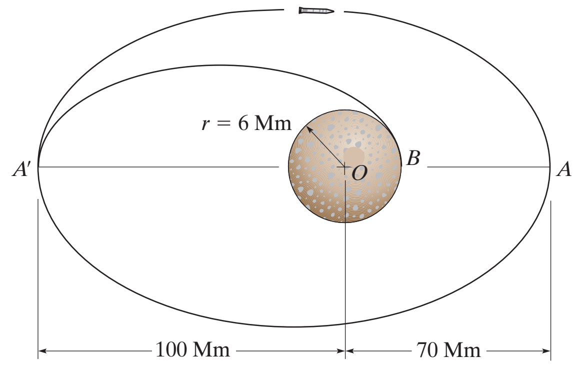
$$v_A = \sqrt{\frac{2\left[66.73\left(10^{-12}\right)\right]\left[0.6(5.976)\left(10^{24}\right)\right]\left[100\left(10^6\right)\right]}{70\left(10^6\right)\left[70\left(10^6\right) + 100\left(10^6\right)\right]}}$$

$$= 2005.32 \text{ m/s} = 2.01\left(10^3\right) \text{ m/s}$$ **Ans.**

Ans:
$v_A = 2.01\left(10^3\right)$ m/s

13–130.

If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that the landing occurs at B. How long does it take for the rocket to land, going from A' to B? The planet has no atmosphere, and its mass is 0.6 times that of the earth.



SOLUTION

Applying Eq. 13–27,

$$r_a = \frac{r_p}{\left(2GM/r_p v_p^2\right) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GMr_a}{r_p(r_p + r_a)}}$$

To land on B, the rocket has to follow the elliptical orbit $A'B$ with $r_p = 6(10^6)$, $r_a = 100(10^6)$ m and $v_p = v_B$.

$$v_B = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[0.6(5.976)(10^{24})\right]\left[100(10^6)\right]}{6(10^6)\left[6(10^6) + 100(10^6)\right]}} = 8674.17 \text{ m/s}$$

In this case

$$h = r_p v_B = r_a v_{A'}$$

$$6(10^6)(8674.17) = 100(10^6)v_{A'}$$

$$v_{A'} = 520.45 \text{ m/s} = 521 \text{ m/s} \qquad \textbf{Ans.}$$

The period of the elliptical orbit can be determined using Eq. 13–31.

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{6(10^6)(8674.17)}\left[6(10^6) + 100(10^6)\right]\sqrt{\left[6(10^6)\right]\left[100(10^6)\right]}$$

$$= 156.73(10^3) \text{ s}$$

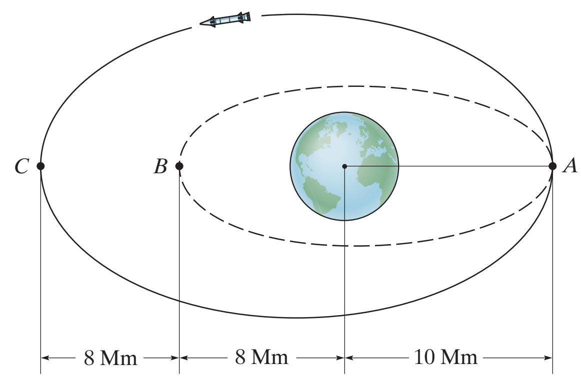
Thus, the time required to travel from A' to B is

$$t = \frac{T}{2} = 78.365(10^3) \text{ s} = 21.8 \text{ h} \qquad \textbf{Ans.}$$

Ans:
$v_{A'} = 521$ m/s
$t = 21.8$ h

13–131.

The rocket is traveling around the earth in free flight along an elliptical orbit AC. If the rocket has the orbit shown, determine the rocket's velocity when it is at point A.



SOLUTION

For orbit AC, $r_p = 10\left(10^6\right)$ m and $r_a = 16\left(10^6\right)$ m. Applying Eq. 13–27

$$r_a = \frac{r_p}{\left(2GM_e/r_p v_p^2\right) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p\left(r_p + r_a\right)}}$$

Here $v_p = v_A$. Then

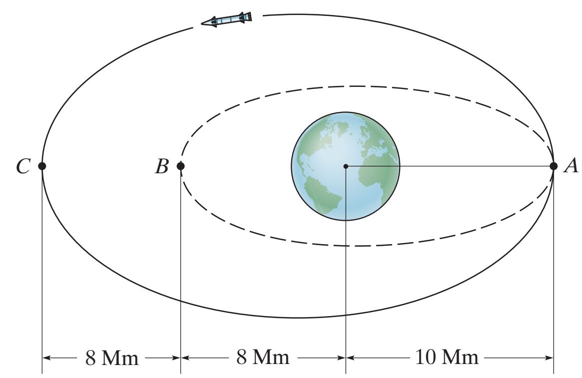
$$v_A = \sqrt{\frac{2\left[66.73\left(10^{-12}\right)\right]\left[5.976\left(10^{24}\right)\right]\left[16\left(10^6\right)\right]}{10\left(10^6\right)\left[10\left(10^6\right) + 16\left(10^6\right)\right]}}$$

$$= 7005.74 \text{ m/s} = 7.01\left(10^3\right) \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 7.01\left(10^3\right)$ m/s

***13–132.**

The rocket is traveling around the earth in free flight along the elliptical orbit AC. Determine its change in speed when it reaches A so that it travels along the elliptical orbit AB.



SOLUTION

Applying Eq. 13–27,

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_a v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

For orbit AC, $r_p = 10(10^6)$ m, $r_a = 16(10^6)$ m and $v_p = (v_A)_{AC}$. Then

$$(v_A)_{AC} = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][16(10^6)]}{10(10^6)[10(10^6) + 16(10^6)]}} = 7005.74 \text{ m/s}$$

For orbit AB, $r_p = 8(10^6)$ m, $r_a = 10(10^6)$ m and $v_p = v_B$. Then

$$v_B = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][10(10^6)]}{8(10^6)[8(10^6) + 10(10^6)]}} = 7442.17 \text{ m/s}$$

Since h is constant at any position of the orbit,

$$h = r_p v_p = r_a v_a$$

$$8(10^6)(7442.17) = 10(10^6)(v_A)_{AB}$$

$$(v_A)_{AB} = 5953.74 \text{ m/s}$$

Thus, the required change in speed is

$$\Delta v = (v_A)_{AB} - (v_A)_{AC} = 5953.74 - 7005.74$$

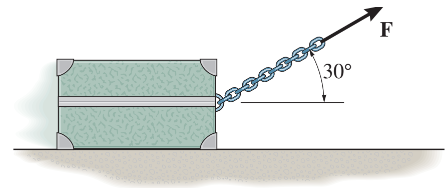
$$= -1052.01 \text{ m/s} = -1.05 \text{ km/s} \qquad \textbf{Ans.}$$

The negative sign indicates that the speed must be decreased.

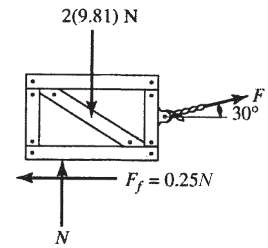
Ans:
$\Delta v = -1.05$ km/s

14–1.

The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



SOLUTION



Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25N$. Applying Eq. 13–7, we have

$$+\uparrow \sum F_y = ma_y; \qquad N + 100 \sin 30^\circ - 20(9.81) = 20(0)$$

$$N = 146.2 \text{ N}$$

Principle of Work and Energy: The horizontal component of force F which acts in the direction of displacement does *positive* work, whereas the friction force $F_f = 0.25(146.2) = 36.55$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force F and the weight of the crate do not displace hence do no work. Applying Eq.14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(20)(8^2) + \int_{15\text{ m}}^{25\text{ m}} 100 \cos 30^\circ \, ds$$

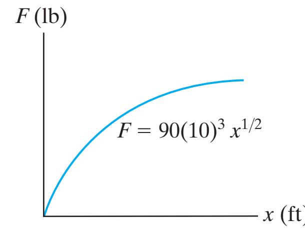
$$- \int_{15\text{ m}}^{25\text{ m}} 36.55 \, ds = \frac{1}{2}(20)v^2$$

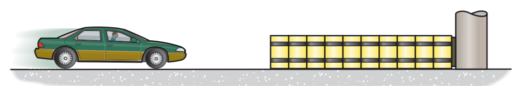
$v = 10.7$ m/s **Ans.**

Ans:
$v = 10.7$ m/s

14–2.

For protection, the barrel barrier is placed in front of the bridge pier. If the relation between the force and deflection of the barrier is $F = (90(10^3)x^{1/2})$ lb, where x is in ft, determine the car's maximum penetration in the barrier. The car has a weight of 4000 lb and it is traveling with a speed of 75 ft/s just before it hits the barrier.





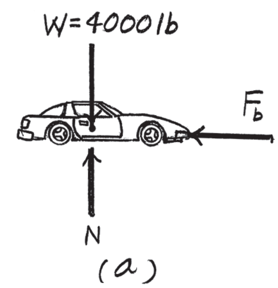
SOLUTION

Principle of Work and Energy: The speed of the car just before it crashes into the barrier is $v_1 = 75$ ft/s. The maximum penetration occurs when the car is brought to a stop, i.e., $v_2 = 0$. Referring to the free-body diagram of the car, Fig. *a*, **W** and **N** do no work; however, $\mathbf{F}_b$ does negative work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(75^2) + \left[-\int_0^{x_{max}} 90(10^3)x^{1/2}dx\right] = 0$$

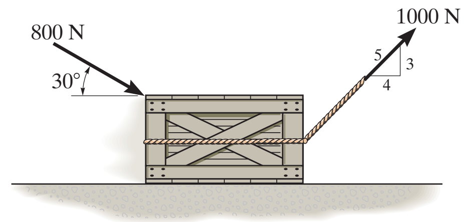
$$x_{max} = 3.24 \text{ ft} \qquad \textbf{Ans.}$$



Ans:
$x_{max} = 3.24$ ft

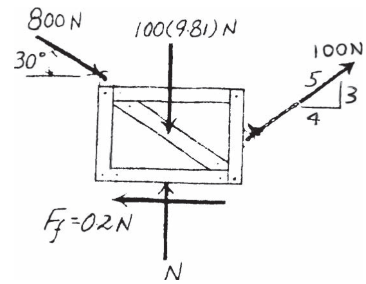
14–3.

The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



SOLUTION

Equations of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.2N$. Applying Eq. 13–7, we have



$$+\uparrow \Sigma F_y = ma_y; \qquad N + 1000\left(\frac{3}{5}\right) - 800 \sin 30° - 100(9.81) = 100(0)$$

$$N = 781 \text{ N}$$

Principle of Work and Energy: The horizontal components of force 800 N and 1000 N which act in the direction of displacement do *positive* work, whereas the friction force $F_f = 0.2(781) = 156.2$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of 800 N and 1000 N force and the weight of the crate do not displace, hence they do no work. Since the crate is originally at rest, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

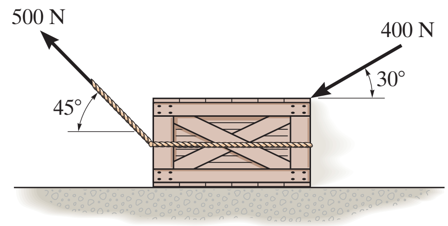
$$0 + 800 \cos 30°(s) + 1000\left(\frac{4}{5}\right)s - 156.2s = \frac{1}{2}(100)\left(6^2\right)$$

$$s = 1.35\text{m} \qquad \textbf{Ans.}$$

Ans:
$s = 1.35$ m

***14–4.**

The 100-kg crate is subjected to the forces shown. If it is originally at rest, determine the distance it slides in order to attain a speed of $v = 8\text{ m/s}$. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



SOLUTION

Work. Consider the force equilibrium along the y axis by referring to the FBD of the crate, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N + 500 \sin 45° - 100(9.81) - 400 \sin 30° = 0$$

$$N = 827.45\text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.2(827.45) = 165.49$ N. Here, F_1 and F_2 do positive work whereas F_f does negative work. W and N do no work

$$U_{F_1} = 400 \cos 30°\text{ s} = 346.41\text{ s}$$

$$U_{F_2} = 500 \cos 45°\text{ s} = 353.55\text{ s}$$

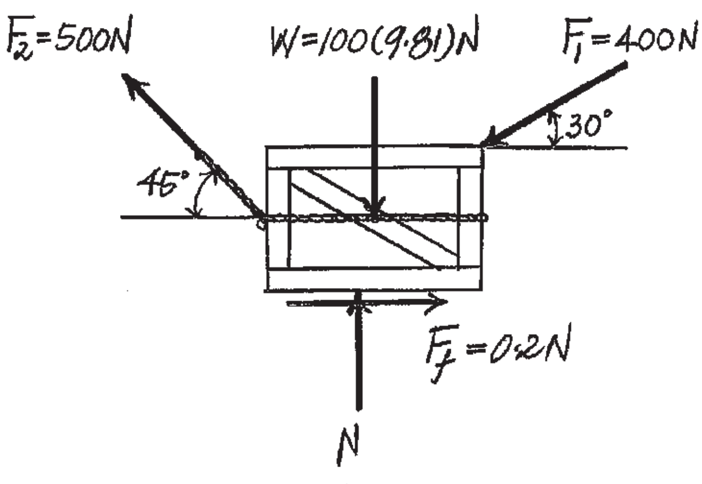
$$U_{F_f} = -165.49\text{ s}$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 346.41\text{ s} + 353.55\text{ s} + (-165.49\text{ s}) = \frac{1}{2}(100)(8^2)$$

$$s = 5.987\text{ m} = 5.99\text{ m} \qquad \textbf{Ans.}$$

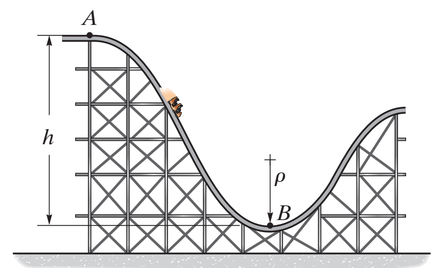


(a)

Ans:
$s = 5.99$ m

14–5.

Determine the required height h of the roller coaster so that when it is essentially at rest at the crest of the hill A it will reach a speed of 100 km/h when it comes to the bottom B. Also, what should be the minimum radius of curvature ρ for the track at B so that the passengers do not experience a normal force greater than $4mg = (39.24m)$ N? Neglect the size of the car and passenger.



SOLUTION

$$100 \text{ km/h} = \frac{100(10^3)}{3600} = 27.778 \text{ m/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + m(9.81)h = \frac{1}{2}m(27.778)^2$$

$$h = 39.3 \text{ m} \qquad \textbf{Ans.}$$

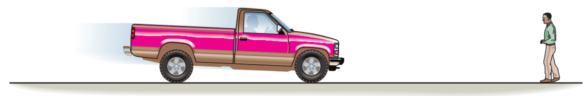
$$+\uparrow \Sigma F_n = ma_n; \qquad 39.24\, m - mg = m\left(\frac{(27.778)^2}{\rho}\right)$$

$$\rho = 26.2 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$h = 39.3$ m
$\rho = 26.2$ m

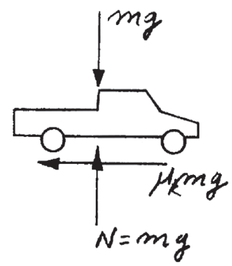
14–6.

When the driver applies the brakes of a light truck traveling 40 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?



SOLUTION

$$40 \text{ km/h} = \frac{40(10^3)}{3600} = 11.11 \text{ m/s} \qquad 80 \text{ km/h} = 22.22 \text{ m/s}$$



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m(11.11)^2 - \mu_k\, mg(3) = 0$$

$$\mu_k g = 20.576$$

$$T_1 + \Sigma U_{1-2} = T_2$$

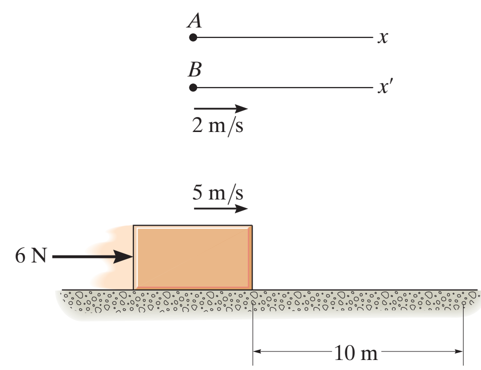
$$\frac{1}{2}m(22.22)^2 - (20.576)m(d) = 0$$

$$d = 12 \text{ m}$$ **Ans.**

Ans:
$d = 12$ m

14–7.

As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of 5 m/s and travels 10 m, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of 2 m/s relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.



SOLUTION

Observer A:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10)(5)^2 + 6(10) = \frac{1}{2}(10)v_2^2$$

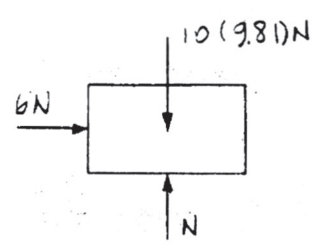
$v_2 = 6.08$ m/s **Ans.**

Observer B:

$$F = ma$$

$$6 = 10a \qquad a = 0.6 \text{ m/s}^2$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$



$$10 = 0 + 5t + \frac{1}{2}(0.6)t^2$$

$$t^2 + 16.67t - 33.33 = 0$$

$$t = 1.805 \text{ s}$$

At $v = 2$ m/s, $s' = 2(1.805) = 3.609$ m

Block moves $10 - 3.609 = 6.391$ m

Thus

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10)(3)^2 + 6(6.391) = \frac{1}{2}(10)v_2^2$$

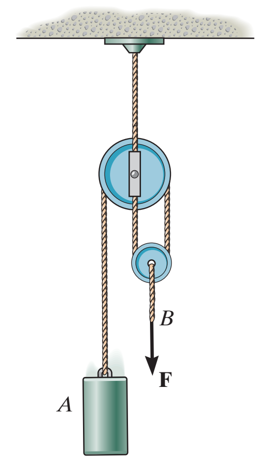
$v_2 = 4.08$ m/s **Ans.**

Note that this result is 2 m/s less than that observed by A.

Ans:
Observer A: $v_2 = 6.08$ m/s
Observer B: $v_2 = 4.08$ m/s

***14–8.**

A force of $F = 250$ N is applied to the end at B. Determine the speed of the 10-kg block when it has moved 1.5 m, starting from rest.



SOLUTION

Work. with reference to the datum set in Fig. *a*,

$$S_W + 2s_F = l$$

$$\delta S_W + 2\delta s_F = 0 \qquad \textbf{(1)}$$

Assuming that the block moves upward 1.5 m, then $\delta S_W = -1.5$ m since it is directed in the negative sense of S_W. Substituted this value into Eq. (1),

$$-1.5 + 2\delta s_F = 0 \qquad \delta s_F = 0.75 \text{ m}$$

Thus,

$$U_F = F\delta S_F = 250(0.75) = 187.5 \text{ J}$$

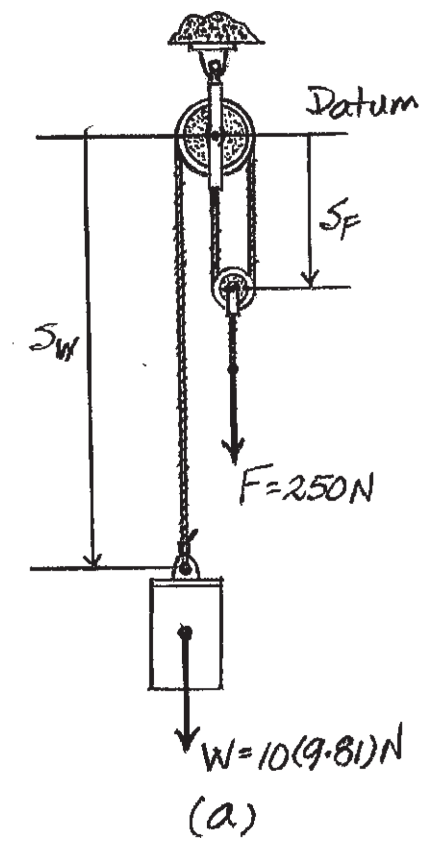
$$U_W = -W\delta S_W = -10(9.81)(1.5) = -147.15 \text{ J}$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + U_{1-2} = T_2$$

$$0 + 187.5 + (-147.15) = \frac{1}{2}(10)v^2$$

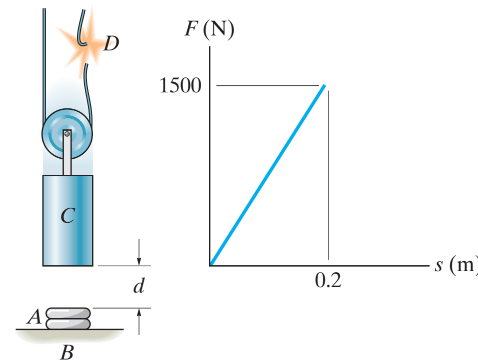
$$v = 2.841 \text{ m/s} = 2.84 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 2.84$ m/s

14–9.

The "air spring" A is used to protect the support B and prevent damage to the conveyor-belt tensioning weight C in the event of a belt failure D. The force developed by the air spring as a function of its deflection is shown by the graph. If the block has a mass of 20 kg and is suspended a height $d = 0.4$ m above the top of the spring, determine the maximum deformation of the spring in the event the conveyor belt fails. Neglect the mass of the pulley and belt.



SOLUTION

Work. Referring to the FBD of the tensioning weight, Fig. a, W does positive work whereas force F does negative work. Here the weight displaces downward $S_W = 0.4 + x_{max}$ where x_{max} is the maximum compression of the air spring. Thus

$$U_W = 20(9.81)(0.4 + x_{max}) = 196.2(0.4 + x_{max})$$

The work of F is equal to the area under the F-S graph shown shaded in Fig. b, Here $\dfrac{F}{x_{max}} = \dfrac{1500}{0.2}$; $F = 7500x_{max}$. Thus

$$U_F = -\frac{1}{2}(7500\,x_{max})(x_{max}) = -3750x_{max}^2$$

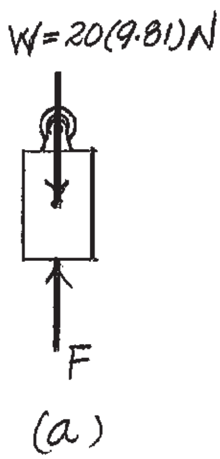
Principle of Work And Energy. Since the block is at rest initially and is required to stop momentarily when the spring is compressed to the maximum, $T_1 = T_2 = 0$. Applying Eq. 14–7,

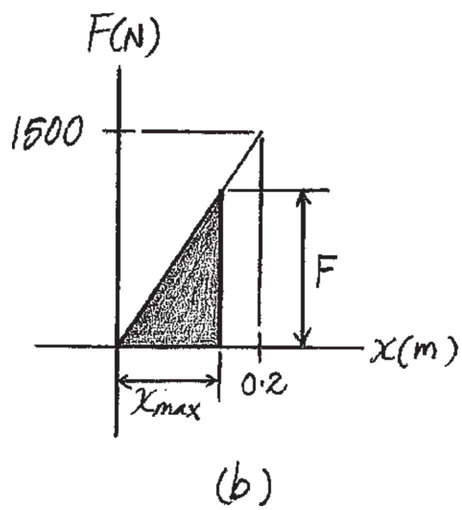
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 196.2(0.4 + x_{max}) + (-3750x_{max}^2) = 0$$

$$3750x_{max}^2 - 196.2x_{max} - 78.48 = 0$$

$$x_{max} = 0.1732 \text{ m} = 0.173 \text{ m} < 0.2 \text{ m} \quad \textbf{(O.K!)} \qquad \textbf{Ans.}$$

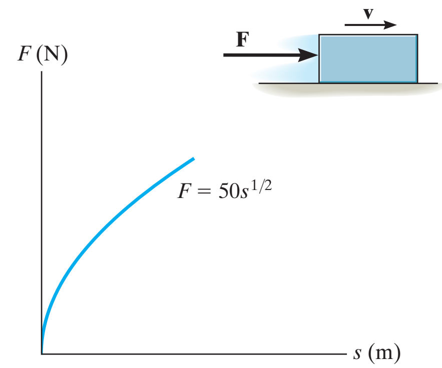




Ans:
$x_{max} = 0.173$ m

14–10.

The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block must slide before its velocity becomes 15 m/s. When $s = 0$ the block is moving to the right at $v = 6$ m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



SOLUTION

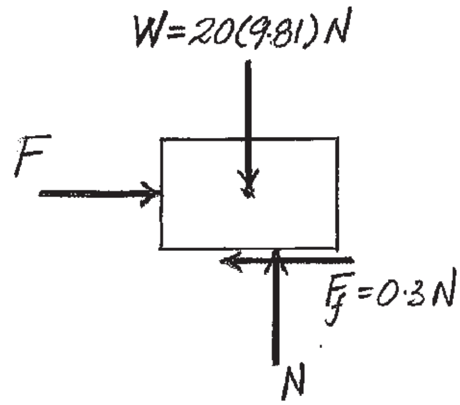
Work. Consider the force equilibrium along y axis, by referring to the FBD of the block, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N - 20(9.81) = 0 \qquad N = 196.2 \text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.3(196.2) = 58.86$ N. Here, force F does positive work whereas friction F_f does negative work. The weight W and normal reaction N do no work.

$$U_F = \int F ds = \int_0^s 50 s^{\frac{1}{2}}\, ds = \frac{100}{3} s^{\frac{3}{2}}$$

$$U_{F_f} = -58.86\, s$$



(a)

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(6^2) + \frac{100}{3} s^{\frac{3}{2}} + (-58.86s) = \frac{1}{2}(20)(15^2)$$

$$\frac{100}{3} s^{\frac{3}{2}} - 58.86s - 1890 = 0$$

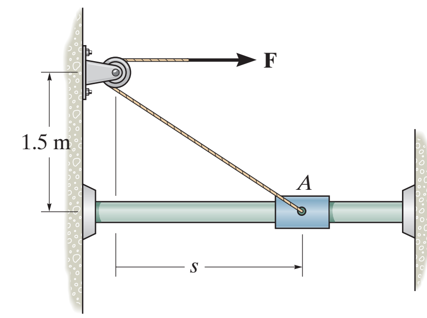
Solving numerically,

$$s = 20.52 \text{ m} = 20.5 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s = 20.5$ m

14–11.

The force of $F = 50$ N is applied to the cord when $s = 2$ m. If the 6-kg collar is orginally at rest, determine its velocity at $s = 0$. Neglect friction.



SOLUTION

Work. Referring to the FBD of the collar, Fig. a, we notice that force F does positive work but W and N do no work. Here, the displacement of F is $s = \sqrt{2^2 + 1.5^2} - 1.5 = 1.00$ m

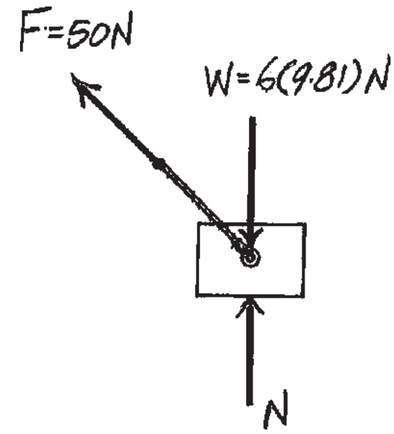
$$U_F = 50(1.00) = 50.0 \text{ J}$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 50 = \frac{1}{2}(6)v^2$$

$$v = 4.082 \text{ m/s} = 4.08 \text{ m/s} \qquad \textbf{Ans.}$$

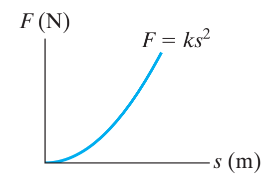


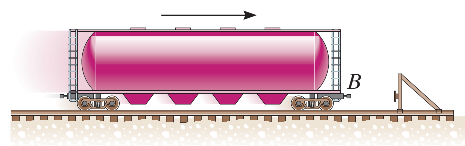
(a)

Ans:
$v = 4.08$ m/s

***14–12.**

Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.



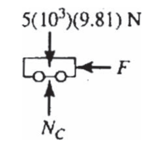


SOLUTION

$$\frac{1}{2}(5000)(4)^2 - \int_0^{0.2} ks^2\, ds = 0$$

$$40\,000 - k\frac{(0.2)^3}{3} = 0$$

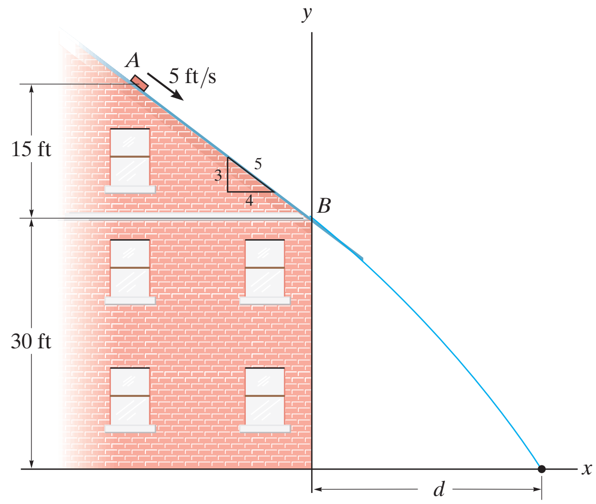
$$k = 15.0\ \text{MN/m}^2$$ **Ans.**



Ans:
$k = 15.0\ \text{MN/m}^2$

14–13.

The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the brick just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.



SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

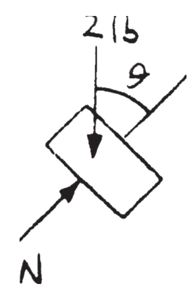
$v_B = 31.48 \text{ ft/s} = 31.5 \text{ ft/s}$ **Ans.**

$$\left(\xrightarrow{+}\right) \qquad s = s_0 + v_0 t$$

$$d = 0 + 31.48\left(\frac{4}{5}\right)t$$

$$(+\downarrow) \qquad s = s_0 + v_0 t - \frac{1}{2}a_c t^2$$

$$30 = 0 + 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(32.2)t^2$$



$$16.1t^2 + 18.888t - 30 = 0$$

Solving for the positive root,

$$t = 0.89916 \text{ s}$$

$d = 31.48\left(\frac{4}{5}\right)(0.89916) = 22.6 \text{ ft}$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_C$$

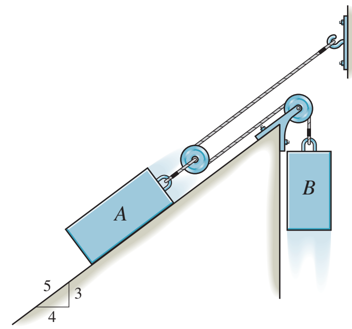
$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2$$

$v_C = 54.1 \text{ ft/s}$ **Ans.**

Ans:
$v_B = 31.5$ ft/s
$d = 22.6$ ft
$v_C = 54.1$ ft/s

14–14.

Block A has a weight of 60 lb and block B has a weight of 10 lb. Determine the speed of block A after it moves 5 ft down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.

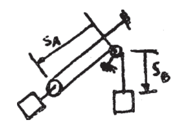


SOLUTION

$$2\,s_A + s_B = l$$

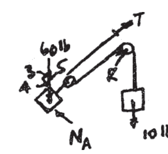
$$2\,\Delta s_A + \Delta s_B = 0$$

$$2v_A + v_B = 0$$



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 60\left(\frac{3}{5}\right)(5) - 10(10) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(2v_A)^2$$

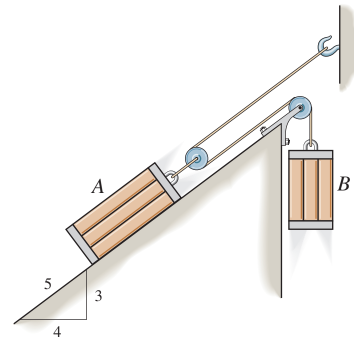


$v_A = 7.18\text{ ft/s}$ **Ans.**

Ans:
$v_A = 7.18\text{ ft/s}$

14–15.

The two blocks A and B have weights $W_A = 60$ lb and $W_B = 10$ lb. If the kinetic coefficient of friction between the incline and block A is $\mu_k = 0.2$, determine the speed of A after it moves 3 ft down the plane starting from rest. Neglect the mass of the cord and pulleys.



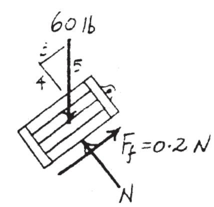
SOLUTION

Kinematics: The speed of the block A and B can be related by using position coordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

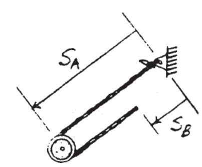
$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2(3) = 6\text{ ft}$$

$$2v_A - v_B = 0 \tag{1}$$



Equation of Motion: Applying Eq. 13–7, we have

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 60\left(\frac{4}{5}\right) = \frac{60}{32.2}(0) \qquad N = 48.0\text{ lb}$$



Principle of Work and Energy: By considering the whole system, W_A which acts in the direction of the displacement does *positive* work. W_B and the friction force $F_f = \mu_k N = 0.2(48.0) = 9.60$ lb does *negative* work since they act in the opposite direction to that of displacement Here, W_A is being displaced vertically (downward) $\frac{3}{5}\Delta s_A$ and W_B is being displaced vertically (upward) Δs_B. Since blocks A and B are at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_A\left(\frac{3}{5}\Delta s_A\right) - F_f\Delta s_A - W_B\Delta s_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B\ v_B^2$$

$$60\left[\frac{3}{5}(3)\right] - 9.60(3) - 10(6) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$1236.48 = 60v_A^2 + 10v_B^2 \tag{2}$$

Eqs. (1) and (2) yields

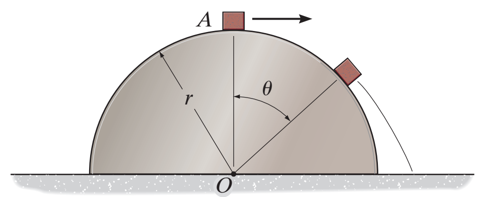
$$v_A = 3.52\text{ ft/s} \qquad \textbf{Ans.}$$

$$v_B = 7.033\text{ ft/s}$$

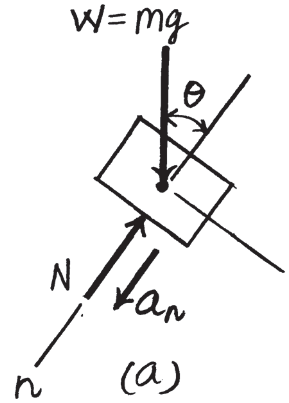
Ans:
$v_A = 3.52\text{ ft/s}$

***14–16.**

A small box of mass m is given a speed of $v = \sqrt{\frac{1}{4}gr}$ at the top of the smooth half cylinder. Determine the angle θ at which the box leaves the cylinder.



SOLUTION



Principle of Work and Energy: By referring to the free-body diagram of the block, Fig. a, notice that **N** does no work, while **W** does positive work since it displaces downward though a distance of $h = r - r\cos\theta$.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m\left(\frac{1}{4}gr\right) + mg(r - r\cos\theta) = \frac{1}{2}mv^2$$

$$v^2 = gr\left(\frac{9}{4} - 2\cos\theta\right) \qquad (1)$$

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{gr\left(\frac{9}{4} - 2\cos\theta\right)}{r} = g\left(\dfrac{9}{4} - 2\cos\theta\right)$. By referring to Fig. a,

$$\Sigma F_n = ma_n; \qquad mg\cos\theta - N = m\left[g\left(\frac{9}{4} - 2\cos\theta\right)\right]$$

$$N = mg\left(3\cos\theta - \frac{9}{4}\right)$$

It is required that the block leave the track. Thus, $N = 0$.

$$0 = mg\left(3\cos\theta - \frac{9}{4}\right)$$

Since $mg \neq 0$,

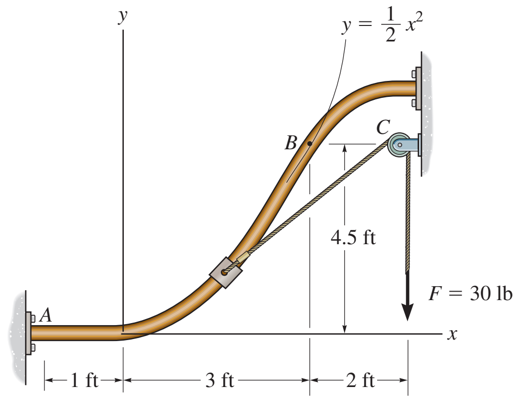
$$3\cos\theta - \frac{9}{4} = 0$$

$$\theta = 41.41^\circ = 41.4^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 41.4^\circ$

14–17.

If the cord is subjected to a constant force of $F = 30$ lb and the smooth 10-lb collar starts from rest at A, determine its speed when it passes point B. Neglect the size of pulley C.



SOLUTION

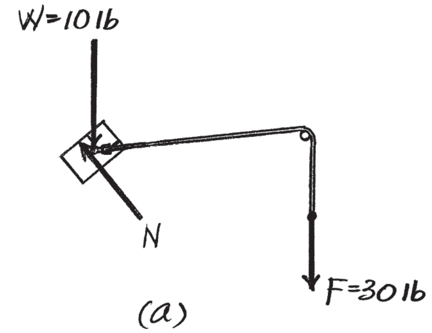
Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: By referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces upward through a distance $h = 4.5$ ft, while force **F** displaces a distance of $s = AC - BC = \sqrt{6^2 + 4.5^2} - 2 = 5.5$ ft. The work of **F** is positive, whereas **W** does negative work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 30(5.5) + [-10(4.5)] = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

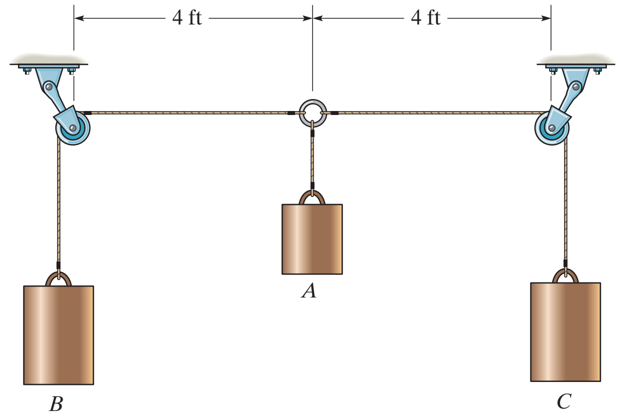
$$v_B = 27.8 \text{ ft/s} \qquad \textbf{Ans.}$$



Ans:
$v_B = 27.8$ ft/s

14–18.

When the 12-lb block A is released from rest it lifts the two 15-lb weights B and C. Determine the maximum distance A will fall before its motion is momentarily stopped. Neglect the weight of the cord and the size of the pulleys.



SOLUTION

Consider the entire system:

$$t = \sqrt{y^2 + 4^2}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$(0 + 0 + 0) + 12y - 2(15)\left(\sqrt{y^2 + 4^2} - 4\right) = (0 + 0 + 0)$$

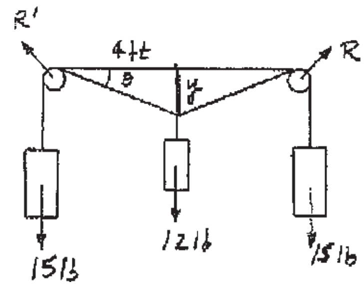
$$0.4y = \sqrt{y^2 + 16} - 4$$

$$(0.4y + 4)^2 = y^2 + 16$$

$$-0.84y^2 + 3.20y + 16 = 16$$

$$-0.84y + 3.20 = 0$$

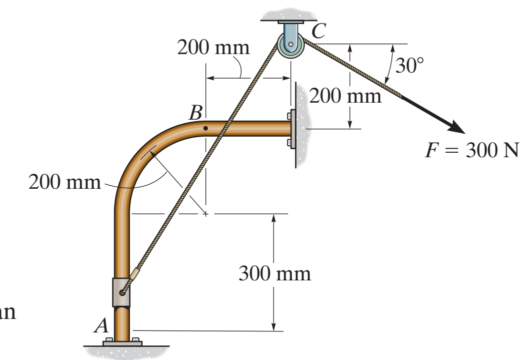
$$y = 3.81 \text{ ft} \qquad \textbf{Ans.}$$



Ans:
$y = 3.81$ ft

14–19.

If the cord is subjected to a constant force of $F = 300$ N and the 15-kg smooth collar starts from rest at A, determine the velocity of the collar when it reaches point B. Neglect the size of the pulley.



SOLUTION

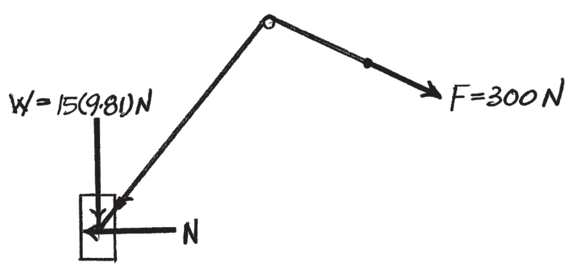
Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: Referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces vertically upward a distance $h = (0.3 + 0.2)\text{ m} = 0.5\text{ m}$, while force F displaces a distance of $s = AC - BC = \sqrt{0.7^2 + 0.4^2} - \sqrt{0.2^2 + 0.2^2} = 0.5234\text{ m}$. Here, the work of **F** is positive, whereas **W** does negative work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 300(0.5234) + [-15(9.81)(0.5)] = \frac{1}{2}(15)v_B^2$$

$$v_B = 3.335\text{ m/s} = 3.34\text{ m/s} \qquad \textbf{Ans.}$$

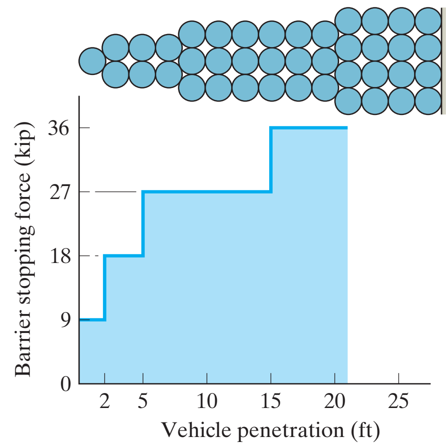


(a)

Ans:
$v_B = 3.34$ m/s

***14–20.**

The crash cushion for a highway barrier consists of a nest of barrels filled with an impact-absorbing material. The barrier stopping force is measured versus the vehicle penetration into the barrier. Determine the distance a car having a weight of 4000 lb will penetrate the barrier if it is originally traveling at 55 ft/s when it strikes the first barrel.



SOLUTION

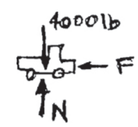
$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(55)^2 - Area = 0$$

$Area = 187.89 \text{ kip} \cdot \text{ft}$

$2(9) + (5 - 2)(18) + x(27) = 187.89$

$x = 4.29 \text{ ft} < (15 - 5) \text{ ft}$ **(O.K!)**



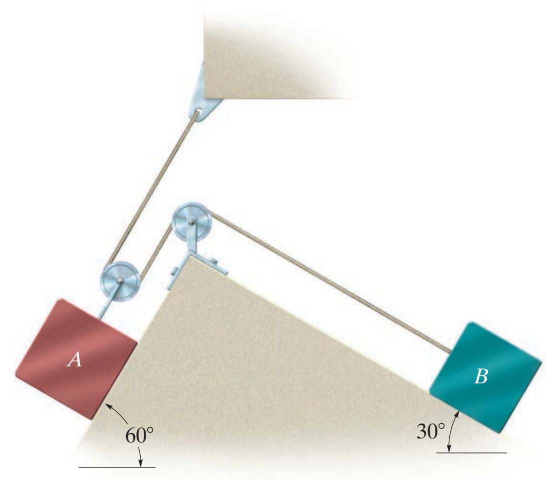
Thus

$s = 5 \text{ ft} + 4.29 \text{ ft} = 9.29 \text{ ft}$ **Ans.**

Ans:
$s = 9.29$ ft

14–21.

Determine the velocity of the 60-lb block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



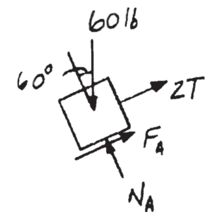
SOLUTION

Block A:

$+\nwarrow \Sigma F_y = ma_y; \qquad N_A - 60 \cos 60° = 0$

$N_A = 30 \text{ lb}$

$F_A = 0.1(30) = 3 \text{ lb}$



Block B:

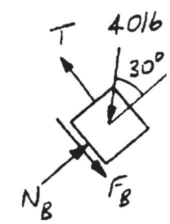
$+\nearrow \Sigma F_y = ma_y; \qquad N_B - 40 \cos 30° = 0$

$N_B = 34.64 \text{ lb}$

$F_B = 0.1(34.64) = 3.464 \text{ lb}$

Use the system of both blocks. N_A, N_B, T, and R do no work.

$T_1 + \Sigma U_{1-2} = T_2$



$$(0 + 0) + 60 \sin 60°|\Delta s_A| - 40 \sin 30°|\Delta s_B| - 3|\Delta s_A| - 3.464|\Delta s_B| = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)v_B^2$$

$2s_A + s_B = l$

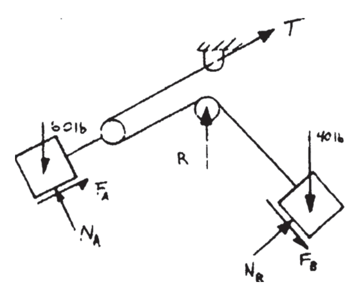
$2\Delta s_A = -\Delta s_B$

When $|\Delta s_B| = 2$ ft, $|\Delta s_A| = 1$ ft

Also,

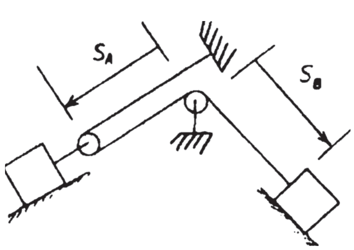
$2v_A = -v_B$

Substituting and solving,



$v_A = 0.771 \text{ ft/s}$ **Ans.**

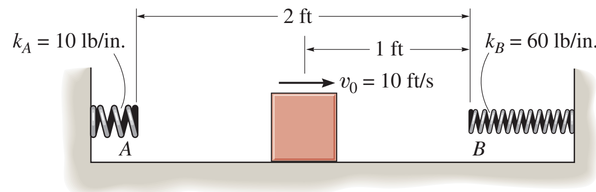
$v_B = -1.54 \text{ ft/s}$



Ans:
$v_A = 0.771 \text{ ft/s}$

14–22.

The 25-lb block has an initial speed of $v_0 = 10$ ft/s when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.



SOLUTION

Principle of Work and Energy: Here, the friction force $F_f = \mu_k N = 0.4(25) = 10.0$ lb. Since the friction force is always opposite the motion, it does negative work. When the block strikes spring B and stops momentarily, the spring force does *negative* work since it acts in the opposite direction to that of displacement. Applying Eq. 14–7, we have

$$T_l + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{25}{32.2}\right)(10)^2 - 10(1 + s_1) - \frac{1}{2}(60)s_1^2 = 0$$

$$s_1 = 0.8275 \text{ ft}$$

Assume the block bounces back and stops without striking spring A. The spring force does *positive* work since it acts in the direction of displacement. Applying Eq. 14–7, we have

$$T_2 + \sum U_{2-3} = T_3$$

$$0 + \frac{1}{2}(60)(0.8275^2) - 10(0.8275 + s_2) = 0$$

$$s_2 = 1.227 \text{ ft}$$

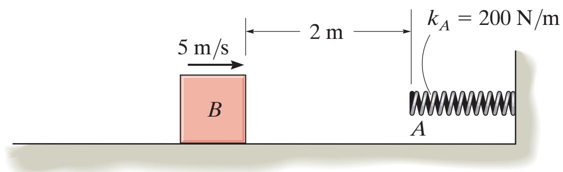
Since $s_2 = 1.227 \text{ ft} < 2$ ft, the block stops before it strikes spring A. Therefore, the above assumption was correct. Thus, the total distance traveled by the block before it stops is

$$s_{\text{Tot}} = 2s_1 + s_2 + 1 = 2(0.8275) + 1.227 + 1 = 3.88 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$s_{\text{Tot}} = 3.88$ ft

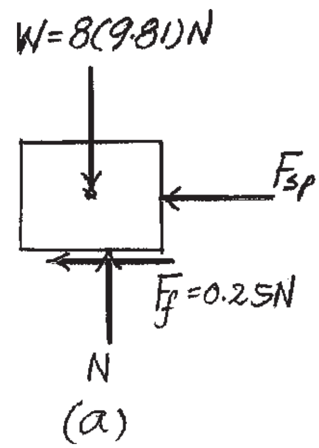
14–23.

The 8-kg block is moving with an initial speed of 5 m/s. If the coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$, determine the compression in the spring when the block momentarily stops.



SOLUTION

Work. Consider the force equilibrium along y axis by referring to the FBD of the block, Fig. *a*



$$+\uparrow \Sigma F_y = 0; \qquad N - 8(9.81) = 0 \qquad N = 78.48 \text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.25(78.48) = 19.62$ N and $F_{sp} = kx = 200\,x$. Here, the spring force F_{sp} and F_f both do negative work. The weight W and normal reaction N do no work.

$$U_{F_{sp}} = -\int_0^x 200\,x\,dx = -100\,x^2$$

$$U_{F_f} = -19.62(x + 2)$$

Principle of Work And Energy. It is required that the block stopped momentarily, $T_2 = 0$. Applying Eq. 14–7

$$T_1 + \Sigma\,U_{1-2} = T_2$$

$$\frac{1}{2}(8)(5^2) + (-100x^2) + [-19.62(x + 2)] = 0$$

$$100x^2 + 19.62x - 60.76 = 0$$

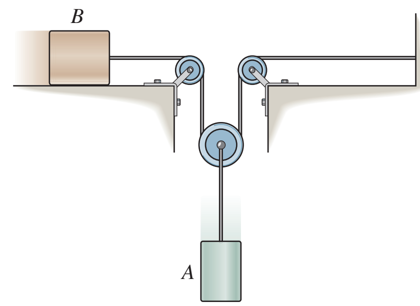
Solved for positive root,

$$x = 0.6875 \text{ m} = 0.688 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$x = 0.688$ m

***14–24.**

At a given instant the 10-lb block A is moving downward with a speed of 6 ft/s. Determine its speed 2 s later. Block B has a weight of 4 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of the cord and pulleys.



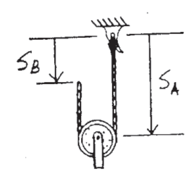
SOLUTION

Kinematics: The speed of the block A and B can be related by using the position coordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

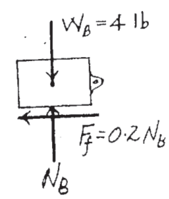
$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A \qquad \textbf{[1]}$$

$$v_B = 2v_A \qquad \textbf{[2]}$$



Equation of Motion:

$$+\Sigma F_{y'} = ma_{y'}; \qquad N_B - 4 = \frac{4}{32.2}(0) \qquad N_B = 4.00\text{ lb}$$



Principle of Work and Energy: By considering the whole system, W_A, which acts in the direction of the displacement, does *positive* work. The friction force $F_f = \mu_k N_B = 0.2(4.00) = 0.800$ lb does *negative* work since it acts in the opposite direction to that of displacement. Here, W_A is being displaced vertically (downward) Δs_A. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}m_A\left(v_A^2\right)_0 + \frac{1}{2}m_B\left(v_B^2\right)_0 + W_A\Delta s_A - F_f\Delta s_B$$

$$= \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 \qquad \textbf{[3]}$$

From Eq. [1], $(v_B)_0 = 2(v_A)_0 = 2(6) = 12$ ft/s. Also, $\Delta s_A = \left[\frac{(v_A)_0 + v_A}{2}\right](2) = (v_A)_0 + v_A = 6 + v_A$ and $\Delta s_B = 2\Delta s_A = 12 + 2v_A$ (Eq. [2]). Substituting these values into Eq. [3] yields

$$\frac{1}{2}\left(\frac{10}{32.2}\right)\left(6^2\right) + \frac{1}{2}\left(\frac{4}{32.2}\right)\left(12^2\right) + 10(6 + v_A) - 0.800(12 + 2v_A)$$

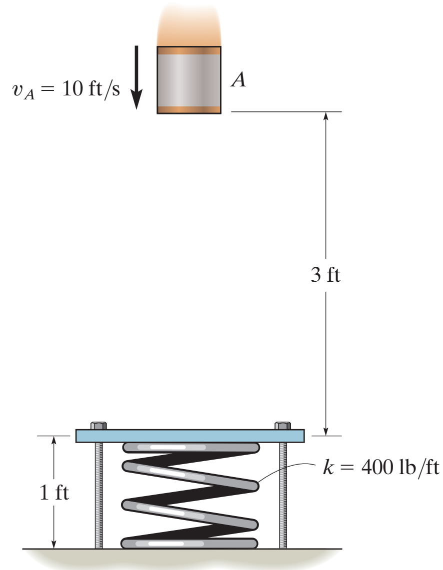
$$= \frac{1}{2}\left(\frac{10}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{4}{32.2}\right)\left(4v_A^2\right)$$

$$v_A = 26.8\text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 26.8$ ft/s

14–25.

The 5-lb cylinder is falling from A with a speed $v_A = 10$ ft/s onto the platform. Determine the maximum displacement of the platform, caused by the collision. The spring has an unstretched length of 1.75 ft and is originally kept in compression by the 1-ft long cables attached to the platform. Neglect the mass of the platform and spring and any energy lost during the collision.



SOLUTION

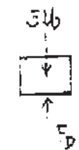
$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}\left(\frac{5}{32.2}\right)(10^2) + 5(3 + s) - \left[\frac{1}{2}(400)(0.75 + s)^2 - \frac{1}{2}(400)(0.75)^2\right] = 0$$

$200\,s^2 + 295\,s - 22.76 = 0$

$s = 0.0735 \text{ ft} < 1 \text{ ft}$ **(O.K!)**

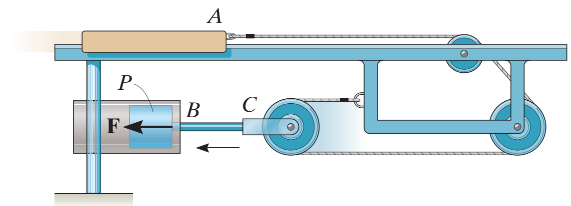
$s = 0.0735$ ft **Ans.**



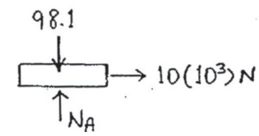
Ans:
$s = 0.0735$ ft

14–26.

The catapulting mechanism is used to propel the 10-kg slider A to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies a constant force $F = 20$ kN to rod BC such that it moves it 0.2 m, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.



SOLUTION

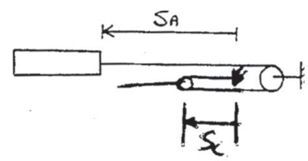


$$2\,s_C + s_A = l$$

$$2\,\Delta\,s_C + \Delta\,s_A = 0$$

$$2(0.2) = -\Delta\,s_A$$

$$-0.4 = \Delta\,s_A$$



$$T_1 + \Sigma U_{1-2} = T_2$$

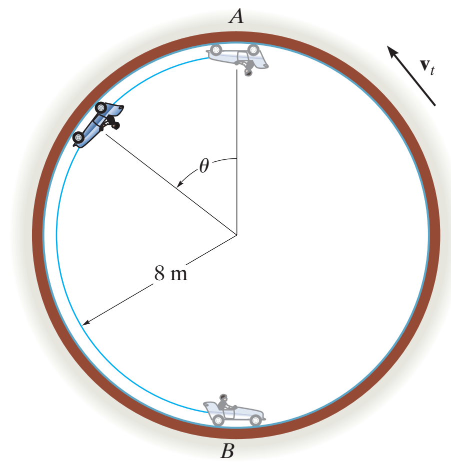
$$0 + (10\,000)(0.4) = \frac{1}{2}(10)(v_A)^2$$

$$v_A = 28.3 \text{ m/s}$$ **Ans.**

Ans:
$v_A = 28.3$ m/s

14–27.

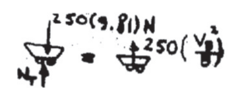
The "flying car" is a ride at an amusement park which consists of a car having wheels that roll along a track mounted inside a rotating drum. By design the car cannot fall off the track, however motion of the car is developed by applying the car's brake, thereby gripping the car to the track and allowing it to move with a constant speed of the track, $v_t = 3$ m/s. If the rider applies the brake when going from B to A and then releases it at the top of the drum, A, so that the car coasts freely down along the track to B ($\theta = \pi$ rad), determine the speed of the car at B and the normal reaction which the drum exerts on the car at B. Neglect friction during the motion from A to B. The rider and car have a total mass of 250 kg and the center of mass of the car and rider moves along a circular path having a radius of 8 m.



SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}(250)(3)^2 + 250(9.81)(16) = \frac{1}{2}(250)(v_B)^2$$



$v_B = 17.97 = 18.0$ m/s **Ans.**

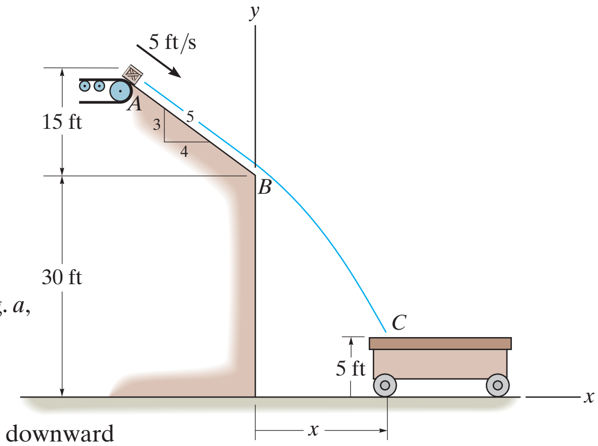
$$+\uparrow \Sigma F_n = ma_n \qquad N_B - 250(9.81) = 250\left(\frac{(17.97)^2}{8}\right)$$

$N_B = 12.5$ kN **Ans.**

Ans:
$v_B = 18.0$ m/s
$N_B = 12.5$ kN

***14–28.**

The 10-lb box falls off the conveyor belt at 5-ft/s. If the coefficient of kinetic friction along AB is $\mu_k = 0.2$, determine the distance x when the box falls into the cart.



SOLUTION

Work. Consider the force equilibrium along the y axis by referring to Fig. a,

$$+\uparrow \Sigma F_{y'} = 0; \qquad N - 10\left(\frac{4}{5}\right) = 0 \qquad N = 8.00 \text{ lb}$$

Thus, $F_f = \mu_k N = 0.2(8.00) = 1.60$ lb. To reach B, W displaces vertically downward 15 ft and the box slides 25 ft down the inclined plane.

$$U_w = 10(15) = 150 \text{ ft} \cdot \text{lb}$$

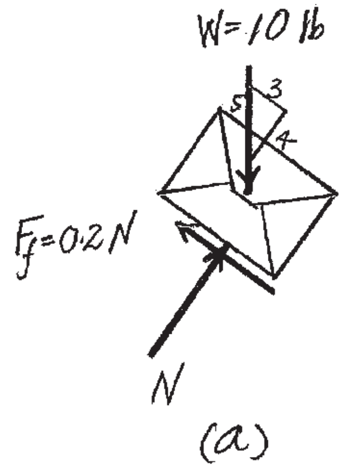
$$U_{F_f} = -1.60(25) = -40 \text{ ft} \cdot \text{lb}$$

Principle of Work And Energy. Applying Eq. 14–7

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(5^2) + 150 + (-40) = \frac{1}{2}\left(\frac{10}{32.2}\right) v_B^2$$

$$v_B = 27.08 \text{ ft/s}$$



Kinematics. Consider the vertical motion with reference to the x-y coordinate system,

$$(+\uparrow)\ (S_C)_y = (S_B)_y + (v_B)_y t + \frac{1}{2} a_y t^2;$$

$$5 = 30 - 27.08\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

$$16.1t^2 + 16.25t - 25 = 0$$

Solve for positive root,

$$t = 0.8398 \text{ s}$$

Then, the horizontal motion gives

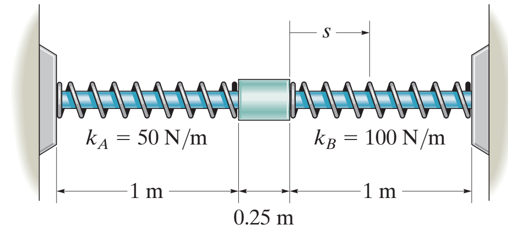
$$\overset{+}{\rightarrow} \quad (S_c)_x = (S_B)_x + (v_B)_x t;$$

$$x = 0 + 27.08\left(\frac{4}{5}\right)(0.8398) = 18.19 \text{ ft} = 18.2 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$x = 18.2$ ft

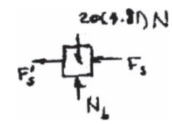
14–29.

The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



SOLUTION

$T_1 + \Sigma U_{1-2} = T_2$



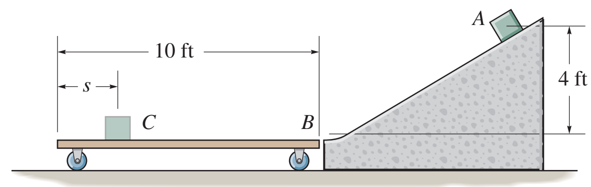
$$\frac{1}{2}(20)(2)^2 - \frac{1}{2}(50)(s)^2 - \frac{1}{2}(100)(s)^2 = 0$$

$s = 0.730$ m **Ans.**

Ans:
$s = 0.730$ m

14–30.

The 30-lb box A is released from rest and slides down along the smooth ramp and onto the surface of a cart. If the cart is *prevented from moving*, determine the distance s from the end of the cart to where the box stops. The coefficient of kinetic friction between the cart and the box is $\mu_k = 0.6$.



SOLUTION

Principle of Work and Energy: W_A which acts in the direction of the vertical displacement does *positive* work when the block displaces 4 ft vertically. The friction force $F_f = \mu_k N = 0.6(30) = 18.0$ lb does *negative* work since it acts in the opposite direction to that of displacement Since the block is at rest initially and is required to stop, $T_A = T_C = 0$. Applying Eq. 14–7, we have

$$T_A + \sum U_{A-C} = T_C$$

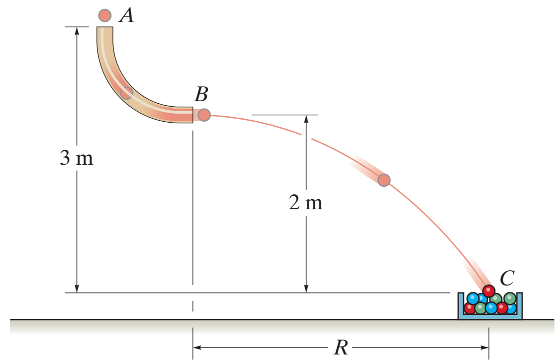
$$0 + 30(4) - 18.0s' = 0 \qquad s' = 6.667 \text{ ft}$$

Thus, $$s = 10 - s' = 3.33 \text{ ft}$$ **Ans.**

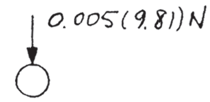
Ans:
$s = 3.33$ ft

14–31.

Marbles having a mass of 5 g are dropped from rest at A through the smooth glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.



SOLUTION



$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + [0.005(9.81)(3 - 2)] = \frac{1}{2}(0.005)v_B^2$$

$$v_B = 4.429 \text{ m/s}$$

$$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$2 = 0 + 0 = \frac{1}{2}(9.81)t^2$$

$$t = 0.6386 \text{ s}$$

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$$

$$R = 0 + 4.429(0.6386) = 2.83 \text{ m}$$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_1$$

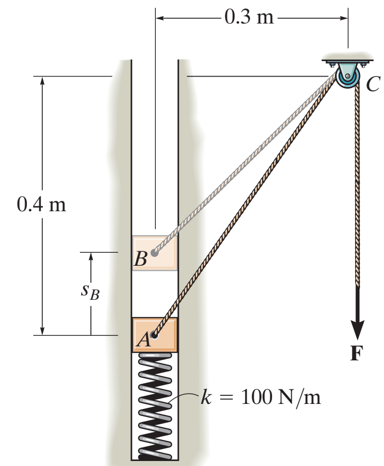
$$0 + [0.005(9.81)(3) = \frac{1}{2}(0.005)v_C^2$$

$$v_C = 7.67 \text{ m/s}$$ **Ans.**

Ans:
$R = 2.83$ m
$v_C = 7.67$ m/s

***14–32.**

The block has a mass of 0.8 kg and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the size and mass of the pulley. *Hint:* The work of **F** can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\,\Delta l$.



SOLUTION

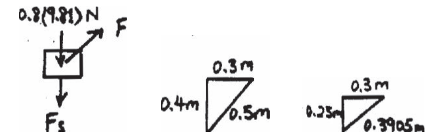
$$l_{AC} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5 \text{ m}$$

$$l_{BC} = \sqrt{(0.4 - 0.15)^2 + (0.3)^2} = 0.3905 \text{ m}$$

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + F(0.5 - 0.3905) - \frac{1}{2}(100)(0.15)^2 - (0.8)(9.81)(0.15) = \frac{1}{2}(0.8)(2.5)^2$$

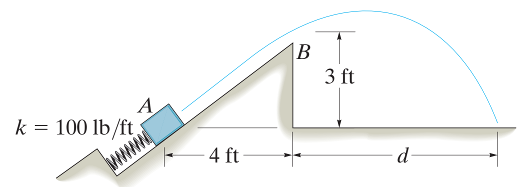
$$F = 43.9 \text{ N}$$ **Ans.**



Ans:
$F = 43.9$ N

14–33.

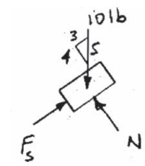
The 10-lb block is pressed against the spring so as to compress it 2 ft when it is at A. If the plane is smooth, determine the distance d, measured from the wall, to where the block strikes the ground. Neglect the size of the block.



SOLUTION

$T_A + \Sigma U_{A-B} = T_B$

$$0 + \frac{1}{2}(100)(2)^2 - (10)(3) = \ = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

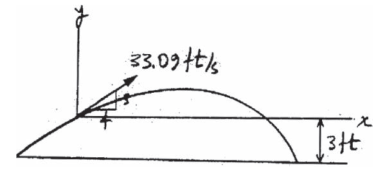


$v_B = 33.09$ ft/s

$(\xrightarrow{+})$ $\quad s = s_0 + v_0 t$

$$d = 0 + 33.09\left(\frac{4}{5}\right)t$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$



$$-3 = 0 + (33.09)\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

$$16.1t^2 - 19.853t - 3 = 0$$

Solving for the positive root,

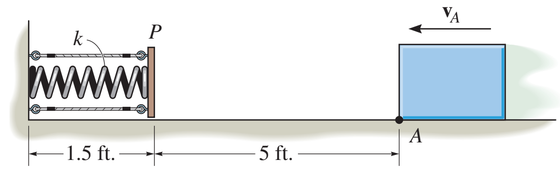
$t = 1.369$ s

$d = 33.09\left(\frac{4}{5}\right)(1.369) = 36.2$ ft **Ans.**

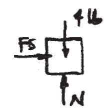
Ans:
$d = 36.2$ ft

14–34.

The spring bumper is used to arrest the motion of the 4-lb block, which is sliding toward it at $v = 9$ ft/s. As shown, the spring is confined by the plate P and wall using cables so that its length is 1.5 ft. If the stiffness of the spring is $k = 50$ lb/ft, determine the required unstretched length of the spring so that the plate is not displaced more than 0.2 ft after the block collides into it. Neglect friction, the mass of the plate and spring, and the energy loss between the plate and block during the collision.



SOLUTION



$T_1 + \Sigma U_{1-2} = T_2$

$$\frac{1}{2}\left(\frac{4}{32.2}\right)(9)^2 - \left[\frac{1}{2}(50)(s - 1.3)^2 - \frac{1}{2}(50)(s - 1.5)^2\right] = 0$$

$0.20124 = s^2 - 2.60\,s + 1.69 - (s^2 - 3.0\,s + 2.25)$

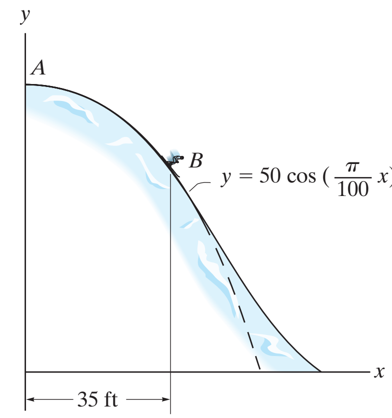
$0.20124 = 0.4\,s - 0.560$

$s = 1.90$ ft **Ans.**

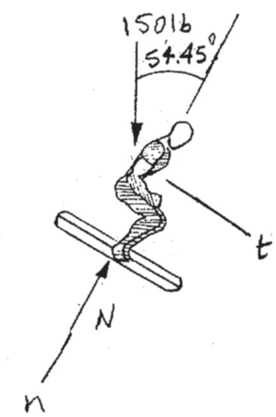
Ans:
$s = 1.90$ ft

14–35.

When the 150-lb skier is at point A he has a speed of 5 ft/s. Determine his speed when he reaches point B on the smooth slope. For this distance the slope follows the cosine curve shown. Also, what is the normal force on his skis at B and his rate of increase in speed? Neglect friction and air resistance.



SOLUTION



$$y = 50\cos\left(\frac{\pi}{100}\right)x\bigg|_{x=35} = 22.70\text{ ft}$$

$$\frac{dy}{dx} = \tan\theta = -50\left(\frac{\pi}{100}\right)\sin\left(\frac{\pi}{100}\right)x = -\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{100}\right)x\bigg|_{x=35} = -1.3996$$

$$\theta = -54.45^\circ$$

$$\frac{d^2y}{dx^2} = -\left(\frac{\pi^2}{200}\right)\cos\left(\frac{\pi}{100}\right)x\bigg|_{x=35} = -0.02240$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(-1.3996)^2\right]^{\frac{3}{2}}}{|-0.02240|} = 227.179$$

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{150}{32.2}\right)(5)^2 + 150(50 - 22.70) = \frac{1}{2}\left(\frac{150}{32.2}\right)v_B^2$$

$v_B = 42.227\text{ ft/s} = 42.2\text{ ft/s}$ **Ans.**

$+\swarrow\Sigma F_n = ma_n;\qquad -N + 150\cos 54.45^\circ = \left(\frac{150}{32.2}\right)\left(\frac{(42.227)^2}{227.179}\right)$

$N = 50.6\text{ lb}$ **Ans.**

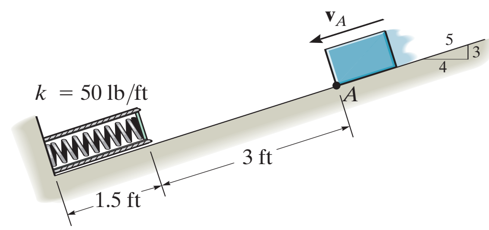
$+\searrow\Sigma F_t = ma_t;\qquad 150\sin 54.45^\circ = \left(\frac{150}{32.2}\right)a_t$

$a_t = 26.2\text{ ft/s}^2$ **Ans.**

Ans:
$v_B = 42.2\text{ ft/s}$
$N = 50.6\text{ lb}$
$a_t = 26.2\text{ ft/s}^2$

***14–36.**

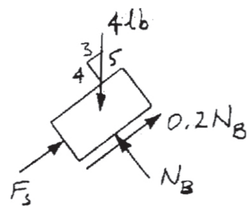
The spring has a stiffness $k = 50$ lb/ft and an *unstretched length* of 2 ft. As shown, it is confined by the plate and wall using cables so that its length is 1.5 ft. A 4-lb block is given a speed v_A when it is at A, and it slides down the incline having a coefficient of kinetic friction $\mu_k = 0.2$. If it strikes the plate and pushes it forward 0.25 ft before stopping, determine its speed at A. Neglect the mass of the plate and spring.



SOLUTION

$$+\nwarrow \Sigma F_y = 0; \qquad N_B - 4\left(\frac{4}{5}\right) = 0$$

$$N_B = 3.20 \text{ lb}$$



$$T_1 + \Sigma U_{1-2} = T_2$$

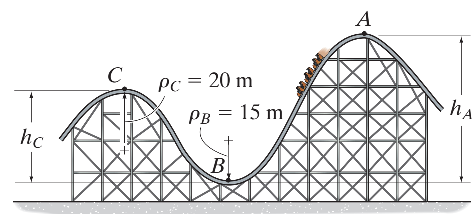
$$\frac{1}{2}\left(\frac{4}{32.2}\right)v_A^2 + (3 + 0.25)\left(\frac{3}{5}\right)(4) - 0.2(3.20)(3 + 0.25) - \left[\frac{1}{2}(50)(0.75)^2 - \frac{1}{2}(50)(0.5)^2\right] = 0$$

$v_A = 5.80$ ft/s **Ans.**

Ans:
$v_A = 5.80$ ft/s

14–37.

If the track is to be designed so that the passengers of the roller coaster do not experience a normal force equal to zero or more than 4 times their weight, determine the limiting heights h_A and h_C so that this does not occur. The roller coaster starts from rest at position A. Neglect friction.



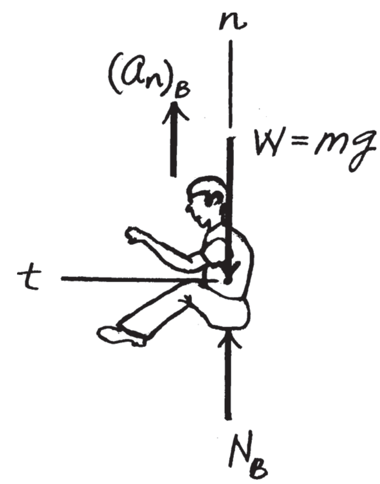
SOLUTION

Free-Body Diagram: The free-body diagram of the passenger at positions B and C are shown in Figs. a and b, respectively.

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. The requirement at position B is that $N_B = 4mg$. By referring to Fig. a,

$$+\uparrow \Sigma F_n = ma_n; \qquad 4mg - mg = m\left(\frac{v_B^{\,2}}{15}\right)$$

$$v_B^{\,2} = 45g$$



(a)

At position C, N_C is required to be zero. By referring to Fig. b,

$$+\downarrow \Sigma F_n = ma_n; \qquad mg - 0 = m\left(\frac{v_C^{\,2}}{20}\right)$$

$$v_C^{\,2} = 20g$$

Principle of Work and Energy: The normal reaction **N** does no work since it always acts perpendicular to the motion. When the rollercoaster moves from position A to B, **W** displaces vertically downward $h = h_A$ and does positive work.

We have

$$T_A + \Sigma U_{A\text{–}B} = T_B$$

$$0 + mgh_A = \frac{1}{2}m(45g)$$

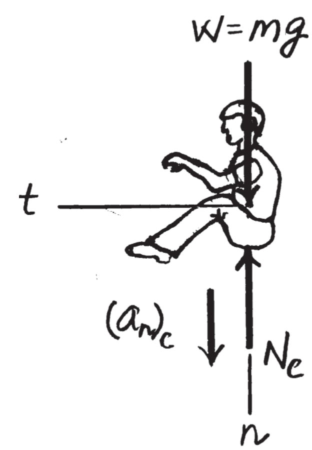
$$h_A = 22.5 \text{ m} \qquad \textbf{Ans.}$$

When the rollercoaster moves from position A to C, **W** displaces vertically downward $h = h_A - h_C = (22.5 - h_C)$ m.

$$T_A + \Sigma U_{A\text{–}B} = T_B$$

$$0 + mg(22.5 - h_C) = \frac{1}{2}m(20g)$$

$$h_C = 12.5 \text{ m} \qquad \textbf{Ans.}$$

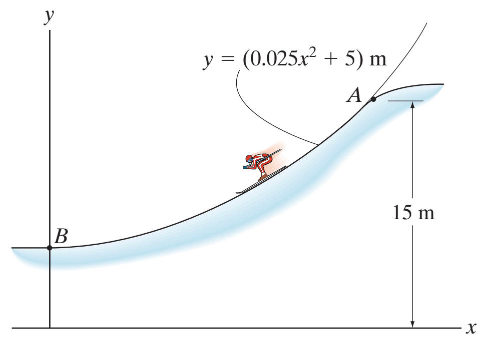


(b)

Ans:
$h_A = 22.5$ m
$h_C = 12.5$ m

14–38.

If the 60-kg skier passes point A with a speed of $5\,\text{m/s}$, determine his speed when he reaches point B. Also find the normal force exerted on him by the slope at this point. Neglect friction.



SOLUTION

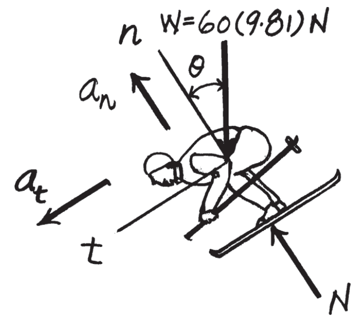
Free-Body Diagram: The free-body diagram of the skier at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: By referring to Fig. *a*, we notice that **N** does no work since it always acts perpendicular to the motion. When the skier slides down the track from A to B, **W** displaces vertically downward $h = y_A - y_B = 15 - \left[0.025\left(0^2\right) + 5\right] = 10\text{ m}$ and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}(60)(5^2) + \left[60(9.81)(10)\right] = \frac{1}{2}(60)v_B{}^2$$

$$v_B = 14.87\text{ m/s} = 14.9\text{ m/s} \qquad \textbf{Ans.}$$



(a)

$dy/dx = 0.05x$

$d^2y/dx^2 = 0.05$

$$\rho = \frac{[1 + 0]^{3/2}}{0.5} = 20\text{ m}$$

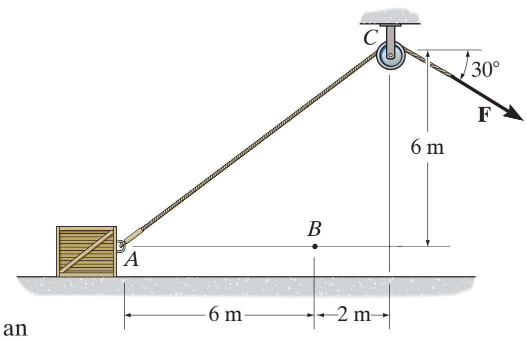
$$+\uparrow \Sigma F_n = ma_n; \qquad N - 60(9.81) = 60\left(\frac{(14.87)^2}{20}\right)$$

$$N = 1.25\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$v_B = 14.9\text{ m/s}$
$N = 1.25\text{ kN}$

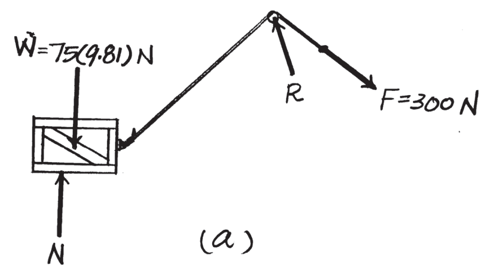
14–39.

If the 75-kg crate starts from rest at A, determine its speed when it reaches point B. The cable is subjected to a constant force of $F = 300$ N. Neglect friction and the size of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675$ m. Here, the work of **F** is positive.

$$T_1 + \Sigma U_{1-2} = T_2$$

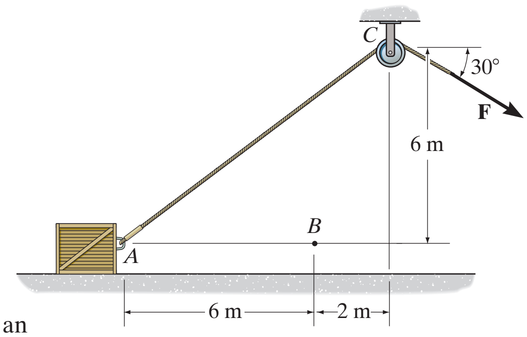
$$0 + 300(3.675) = \frac{1}{2}(75)v_B{}^2$$

$$v_B = 5.42 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 5.42$ m/s

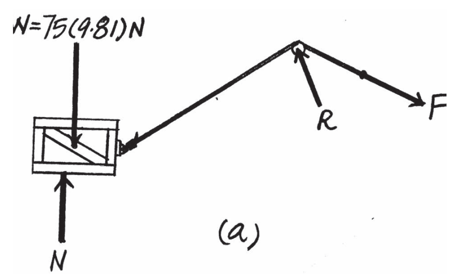
***14–40.**

If the 75-kg crate starts from rest at A, and its speed is $6\,\mathrm{m/s}$ when it passes point B, determine the constant force $\mathbf{F}$ exerted on the cable. Neglect friction and the size of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675\text{ m}$. Here, the work of **F** is positive.

$$T_1 + \Sigma U_{1-2} = T_2$$

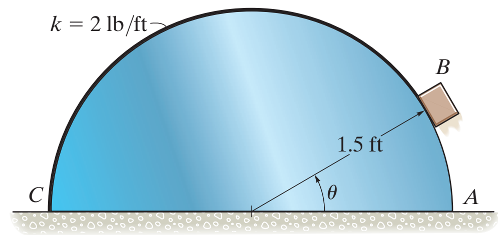
$$0 + F(3.675) = \frac{1}{2}(75)(6^2)$$

$$F = 367\text{ N}$$ **Ans.**

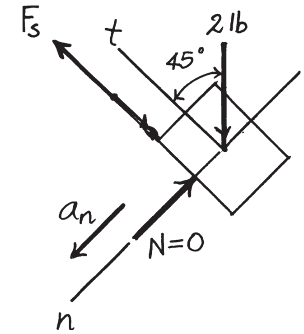
Ans:
$F = 367\text{ N}$

14–41.

A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at A ($\theta = 0°$), determine the unstretched length of the cord so that the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.



SOLUTION



$+\swarrow \Sigma F_n = ma_n; \qquad 2 \sin 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right)$

$$v = 5.844 \text{ ft/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

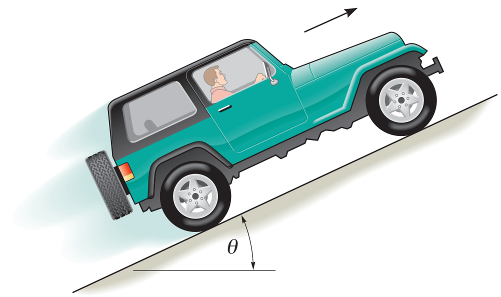
$$0 + \frac{1}{2}(2)\left[\pi(1.5) - l_0\right]^2 - \frac{1}{2}(2)\left[\frac{3\pi}{4}(1.5) - l_0\right]^2 - 2(1.5 \sin 45°) = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2$$

$l_0 = 2.77$ ft **Ans.**

Ans:
$l_0 = 2.77$ ft

14–42.

The jeep has a weight of 2500 lb and an engine which transmits a power of 100 hp to *all* the wheels. Assuming the wheels do not slip on the ground, determine the angle θ of the largest incline the jeep can climb at a constant speed $v = 30$ ft/s.

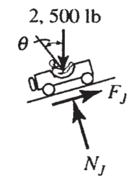


SOLUTION

$P = F_J v$

$100(550) = 2500 \sin \theta(30)$

$\theta = 47.2^\circ$ **Ans.**



Ans:
$\theta = 47.2^\circ$

14–43.

Determine the power input for a motor necessary to lift 300 lb at a constant rate of 5 ft/s. The efficiency of the motor is $\epsilon = 0.65$.

SOLUTION

Power: The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 300(5) = 1500 \text{ ft} \cdot \text{lb/s}$$

Using Eq. 14–11, the required power input for the motor to provide the above power output is

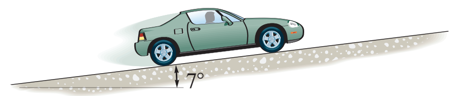
$$\text{power input} = \frac{\text{power output}}{\epsilon}$$

$$= \frac{1500}{0.65} = 2307.7 \text{ ft} \cdot \text{lb/s} = 4.20 \text{ hp} \qquad \textbf{Ans.}$$

Ans:
$P_i = 4.20$ hp

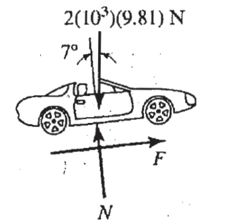
***14–44.**

An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100$ km/h. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\epsilon = 0.65$.



SOLUTION

Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.



$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 2(10^3)(9.81)\sin 7^\circ = 2(10^3)(0)$$

$$F = 2391.08 \text{ N}$$

Power: Here, the speed of the car is $v = \left[\dfrac{100(10^3)\text{ m}}{\text{h}}\right] \times \left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 27.78$ m/s. The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F}\cdot\mathbf{v} = 2391.08(27.78) = 66.418(10^3)\text{ W} = 66.418\text{ kW}$$

Using Eq. 14–11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{66.418}{0.65} = 102\text{ kW} \qquad \textbf{Ans.}$$

Ans:
power input = 102 kW

14–45.

The Milkin Aircraft Co. manufactures a turbojet engine that is placed in a plane having a weight of 13000 lb. If the engine develops a constant thrust of 5200 lb, determine the power output of the plane when it is just ready to take off with a speed of 600 mi/h.

SOLUTION

At 600 ms/h.

$$P = 5200(600)\left(\frac{88\text{ ft/s}}{60\text{ m/h}}\right)\frac{1}{550} = 8.32\,(10^3)\text{ hp} \qquad \textbf{Ans.}$$

Ans:
$P = 8.32\,(10^3)$ hp

14–46.

To dramatize the loss of energy in an automobile, consider a car having a weight of 5000 lb that is traveling at 35 mi/h. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy. (1 mi = 5280 ft.)

SOLUTION

Energy: Here, the speed of the car is $v = \left(\frac{35\text{ mi}}{\text{h}}\right) \times \left(\frac{5280\text{ ft}}{1\text{ mi}}\right) \times \left(\frac{1\text{ h}}{3600\text{ s}}\right) =$ 51.33 ft/s. Thus, the kinetic energy of the car is

$$U = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{5000}{32.2}\right)\left(51.33^2\right) = 204.59\left(10^3\right)\text{ ft}\cdot\text{lb}$$

The power of the bulb is $P_{bulb} = 100\text{ W} \times \left(\frac{1\text{ hp}}{746\text{ W}}\right) \times \left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) =$ 73.73 ft · lb/s. Thus,

$$t = \frac{U}{P_{bulb}} = \frac{204.59(10^3)}{73.73} = 2774.98\text{ s} = 46.2\text{ min} \qquad \textbf{Ans.}$$

Ans:
$t = 46.2$ min

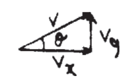
14–47.

The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.

SOLUTION

Step height: 0.125 m

The number of steps: $\dfrac{4}{0.125} = 32$



Total load: $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \dfrac{4}{2} = 2$ m, then

$$U = 47\,088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$v_y = v \sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW} \qquad \textbf{Ans.}$$

Also,

$$P = \mathbf{F} \cdot \mathbf{v} = 47\,088(0.2683) = 12.6 \text{ kW} \qquad \textbf{Ans.}$$

Ans:
$P = 12.6$ kW

***14–48.**

The man having the weight of 150 lb is able to run up a 15-ft-high flight of stairs in 4 s. Determine the power generated. How long would a 100-W light bulb have to burn to expend the same amount of energy? *Conclusion:* Please turn off the lights when they are not in use!

SOLUTION

Power: The work done by the man is

$$U = Wh = 150(15) = 2250\ \text{ft} \cdot \text{lb}$$

Thus, the power generated by the man is given by

$$P_{man} = \frac{U}{t} = \frac{2250}{4} = 562.5\ \text{ft} \cdot \text{lb/s} = 1.02\ \text{hp} \qquad \textbf{Ans.}$$

The power of the bulb is $P_{bulb} = 100\ \text{W} \times \left(\frac{1\ \text{hp}}{746\ \text{W}}\right) \times \left(\frac{550\ \text{ft} \cdot \text{lb/s}}{1\ \text{hp}}\right)$

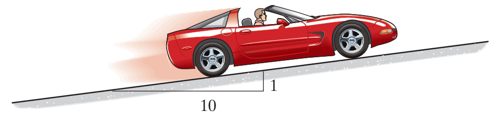
$= 73.73\ \text{ft} \cdot \text{lb/s}$. Thus,

$$t = \frac{U}{P_{bulb}} = \frac{2250}{73.73} = 30.5\ \text{s} \qquad \textbf{Ans.}$$

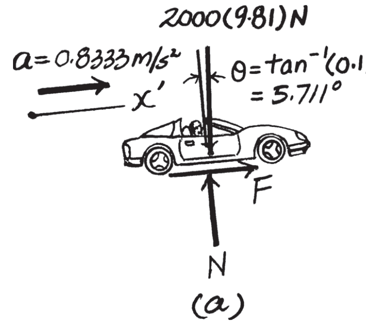
Ans:
$P_{man} = 1.02$ hp
$t = 30.5$ s

14–49.

The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\epsilon = 0.8$. Also, find the average power supplied by the engine.



SOLUTION



Kinematics: The constant acceleration of the car can be determined from

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$25 = 0 + a_c(30)$$

$$a_c = 0.8333\ \text{m/s}^2$$

Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,

$$\Sigma F_{x'} = ma_{x'}; \qquad F - 2000(9.81)\sin 5.711^\circ = 2000(0.8333)$$

$$F = 3618.93\text{N}$$

Power: The maximum power output of the motor can be determined from

$$(P_{\text{out}})_{\max} = \mathbf{F}\cdot\mathbf{v}_{\max} = 3618.93(25) = 90\,473.24\ \text{W}$$

Thus, the maximum power input is given by

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{90473.24}{0.8} = 113\,091.55\ \text{W} = 113\ \text{kW} \qquad \textbf{Ans.}$$

The average power output can be determined from

$$(P_{\text{out}})_{\text{avg}} = \mathbf{F}\cdot\mathbf{v}_{\text{avg}} = 3618.93\left(\frac{25}{2}\right) = 45\,236.62\ \text{W}$$

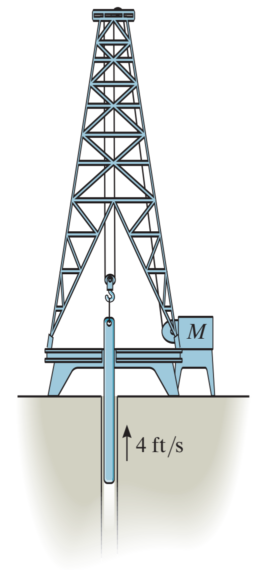
Thus,

$$(P_{\text{in}})_{\text{avg}} = \frac{(P_{\text{out}})_{\text{avg}}}{\varepsilon} = \frac{45236.62}{0.8} = 56\,545.78\ \text{W} = 56.5\ \text{kW} \qquad \textbf{Ans.}$$

Ans:
$P_{\max} = 113$ kW
$P_{\text{avg}} = 56.5$ kW

14–50.

Determine the power output of the draw-works motor M necessary to lift the 600-lb drill pipe upward with a constant speed of 4 ft/s. The cable is tied to the top of the oil rig, wraps around the lower pulley, then around the top pulley, and then to the motor.



SOLUTION

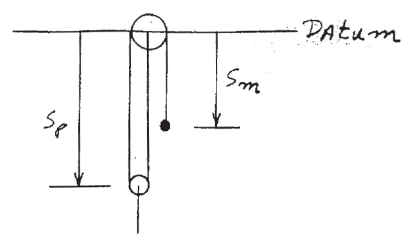
$$2s_P + s_M = l$$

$$2v_P = -v_M$$

$$2(-4) = -v_M$$

$$v_M = 8 \text{ ft/s}$$

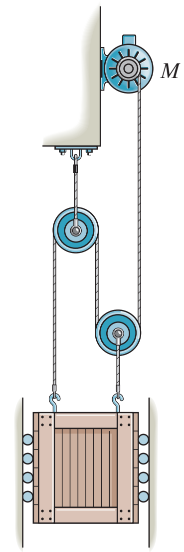
$$P_o = Fv = \left(\frac{600}{2}\right)(8) = 2400 \text{ ft}\cdot\text{lb/s} = 4.36 \text{ hp}$$ **Ans.**



Ans:
$P_o = 4.36$ hp

14–51.

The 1000-lb elevator is hoisted by the pulley system and motor M. If the motor exerts a constant force of 500 lb on the cable, determine the power that must be supplied to the motor at the instant the load has been hoisted $s = 15$ ft starting from rest. The motor has an efficiency of $\varepsilon = 0.65$.



SOLUTION

Equation of Motion. Referring to the FBD of the elevator, Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad 3(500) - 1000 = \frac{1000}{32.2}a$$

$$a = 16.1 \text{ ft/s}^2$$

When $S = 15ft$,

$$+\uparrow\ v^2 = v_0^2 + 2a_c(S - S_0); \quad v^2 = 0^2 + 2(16.1)(15)$$

$$v = 21.98 \text{ ft/s}$$

Power. Applying Eq. 14–9, the power output is

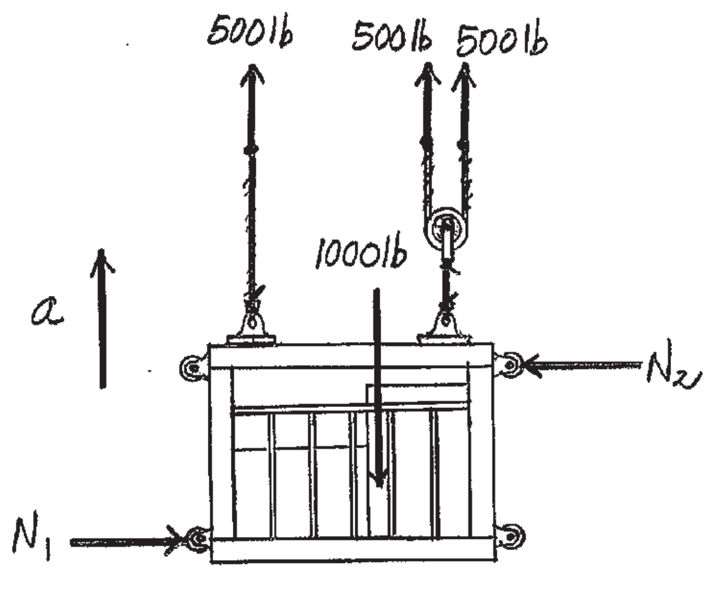
$$P_{out} = F \cdot V = 3(500)(21.98) = 32.97\left(10^3\right) \text{ lb} \cdot \text{ft/s}$$

The power input can be determined using Eq. 14–9

$$\Sigma = \frac{P_{out}}{P_{in}}; \qquad 0.65 = \frac{32.97\left(10^3\right)}{P_{in}}$$

$$P_{in} = \left[50.72\left(10^3\right) \text{ lb} \cdot \text{ft/s}\right]\left(\frac{1 \text{ hp}}{550 \text{ lb} \cdot \text{ft/s}}\right)$$

$$= 92.21 \text{ hp} = 92.2 \text{ hp}$$ **Ans.**

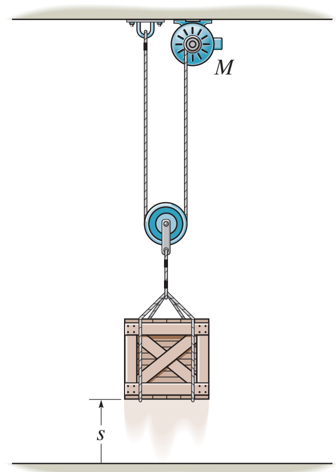


(a)

Ans:
$P = 92.2$ hp

***14–52.**

The 50-lb crate is given a speed of 10 ft/s in $t = 4$ s starting from rest. If the acceleration is constant, determine the power that must be supplied to the motor when $t = 2$ s. The motor has an efficiency $\varepsilon = 0.65$. Neglect the mass of the pulley and cable.



SOLUTION

$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 50 = \dfrac{50}{32.2}a$

$(+\uparrow)\ v = v_0 + a_c t$

$10 = 0 + a(4)$

$a = 2.5 \text{ ft/s}^2$

$T = 26.94 \text{ lb}$

$In\ t = 2\ s$

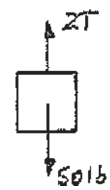
$(+\uparrow)\ v = v_0 + a_c l$

$v = 0 + 2.5(2) = 5 \text{ ft/s}$

$s_C + (s_C - s_P) = l$

$2\,v_C = v_P$

$2(5) = v_P = 10 \text{ ft/s}$



$P_0 = 26.94(10) = 269.4$

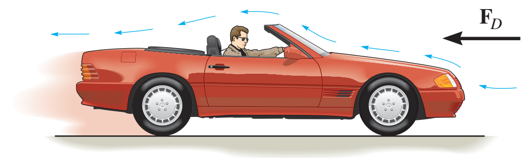
$P_1 = \dfrac{269.4}{0.65} = 414.5 \text{ ft} \cdot \text{lb/s}$

$P_1 = 0.754 \text{ hp}$ **Ans.**

Ans:
$P_1 = 0.754 \text{ hp}$

14–53.

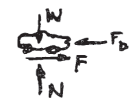
The sports car has a mass of 2.3 Mg, and while it is traveling at 28 m/s the driver causes it to accelerate at 5 m/s^2. If the drag resistance on the car due to the wind is $F_D = (0.3v^2)$ N, where v is the velocity in m/s, determine the power supplied to the engine at this instant. The engine has a running efficiency of $\epsilon = 0.68$.



SOLUTION

$\xrightarrow{+} \Sigma F_x = m\,a_x;\qquad F - 0.3v^2 = 2.3(10^3)(5)$

$$F = 0.3v^2 + 11.5(10^3)$$



At $v = 28$ m/s

$$F = 11\,735.2\text{ N}$$

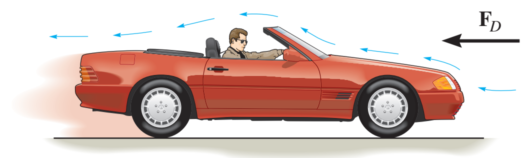
$$P_O = (11\,735.2)(28) = 328.59\text{ kW}$$

$$P_i = \frac{P_O}{e} = \frac{328.59}{0.68} = 438\text{ kW}$$ **Ans.**

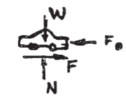
Ans:
$P_i = 483$ kW

14–54.

The sports car has a mass of 2.3 Mg and accelerates at 6 m/s^2, starting from rest. If the drag resistance on the car due to the wind is $F_D = (10v)$ N, where v is the velocity in m/s, determine the power supplied to the engine when $t = 5$ s. The engine has a running efficiency of $\epsilon = 0.68$.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m\,a_x; \qquad F - 10v = 2.3(10^3)(6)$

$$F = 13.8(10^3) + 10\,v$$

$(\xrightarrow{+})\; v = v_0 + a_c\,t$

$$v = 0 + 6(5) = 30 \text{ m/s}$$

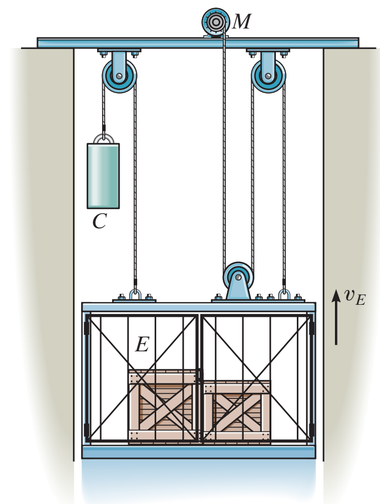
$$P_O = \mathbf{F} \cdot \mathbf{v} = [13.8(10^3) + 10(30)](30) = 423.0 \text{ kW}$$

$$P_i = \frac{P_O}{e} = \frac{423.0}{0.68} = 622 \text{ kW}$$ **Ans.**

Ans:
$P_i = 622$ kW

14–55.

The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\epsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.



SOLUTION

Elevator:

Since $a = 0$,

$+\uparrow \Sigma F_y = 0; \qquad 60(9.81) + 3T - 400(9.81) = 0$

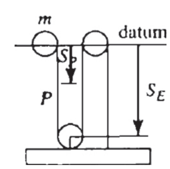
$$T = 1111.8 \text{ N}$$

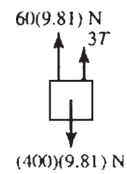
$2s_E + (s_E - s_P) = l$

$3v_E = v_P$

Since $v_E = -4$ m/s, $\qquad v_P = -12$ m/s

$$P_i = \frac{\mathbf{F} \cdot \mathbf{v}_P}{e} = \frac{(1111.8)(12)}{0.6} = 22.2 \text{ kW}$$ **Ans.**

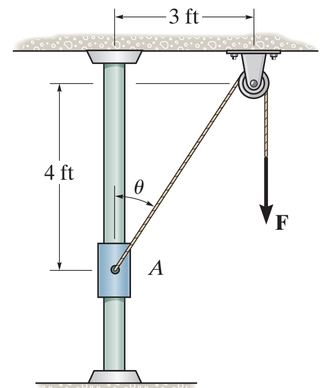




Ans:
$P_i = 22.2$ kW

***14–56.**

The 10-lb collar starts from rest at A and is lifted by applying a constant vertical force of $F = 25$ lb to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = 60°$.



SOLUTION

Work of **F**

$U_{1-2} = 25(5 - 3.464) = 38.40\ \text{lb} \cdot \text{ft}$

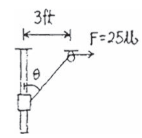
$T_1 + \Sigma U_{1-2} = T_2 s$

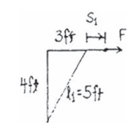
$$0 + 38.40 - 10(4 - 1.732) = \frac{1}{2}\left(\frac{10}{32.2}\right)v^2$$

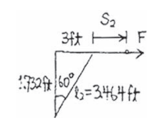
$v = 10.06\ \text{ft/s}$

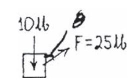
$P = \mathbf{F} \cdot \mathbf{v} = 25 \cos 60°(10.06) = 125.76\ \text{ft} \cdot \text{lb/s}$

$P = 0.229\ \text{hp}$ **Ans.**





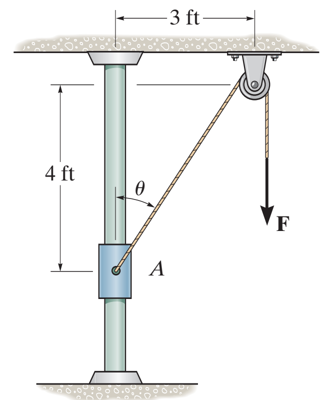




Ans:
$P = 0.229$ hp

14–57.

The 10-lb collar starts from rest at A and is lifted with a constant speed of 2 ft/s along the smooth rod. Determine the power developed by the force **F** at the instant shown.



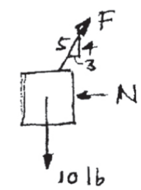
SOLUTION

$+\uparrow\Sigma F_y = m\,a_y;\qquad F\left(\frac{4}{5}\right) - 10 = 0$

$F = 12.5\text{ lb}$

$P = \mathbf{F}\cdot\mathbf{v} = 12.5\left(\frac{4}{5}\right)(2) = 20\text{ lb}\cdot\text{ft/s}$

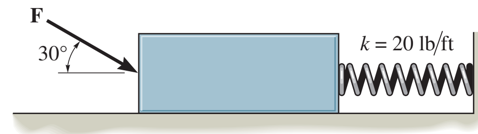
$= 0.0364\text{ hp}$ **Ans.**



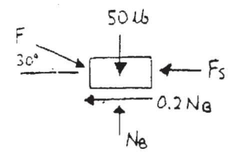
Ans:
$P = 0.0364\text{ hp}$

14–58.

The 50-lb block rests on the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force $F = (40 + s^2)$ lb, where s is in ft, acts on the block in the direction shown. If the spring is originally unstretched ($s = 0$) and the block is at rest, determine the power developed by the force the instant the block has moved $s = 1.5$ ft.



SOLUTION



$+\uparrow \Sigma F_y = 0; \qquad N_B - (40 + s^2)\sin 30° - 50 = 0$

$N_B = 70 + 0.5s^2$

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + \int_0^{1.5} (40 + s^2)\cos 30°\, ds - \frac{1}{2}(20)(1.5)^2 - 0.2\int_0^{1.5} (70 + 0.5s^2)\,ds = \frac{1}{2}\left(\frac{50}{32.2}\right)v_2^2$$

$0 + 52.936 - 22.5 - 21.1125 = 0.7764v_2^2$

$v_2 = 3.465$ ft/s

When $s = 1.5$ ft,

$F = 40 + (1.5)^2 = 42.25$ lb

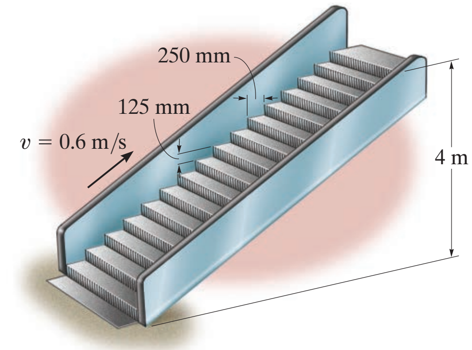
$P = \mathbf{F} \cdot \mathbf{v} = (42.25 \cos 30°)(3.465)$

$P = 126.79$ ft·lb/s $= 0.231$ hp **Ans.**

Ans:
$P = 0.231$ hp

14–59.

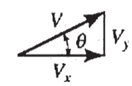
The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.



SOLUTION

Step height: 0.125 m

The number of steps: $\dfrac{4}{0.125} = 32$



Total load: $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \dfrac{4}{2} = 2$ m, then

$$U = 47\,088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$v_y = v\sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW}$$ **Ans.**

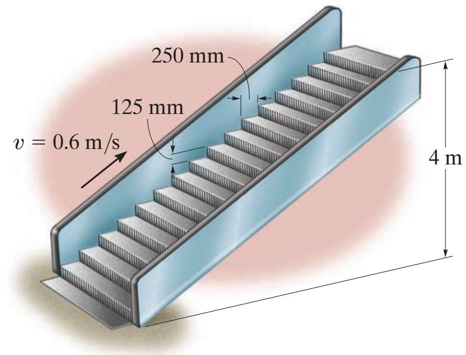
Also,

$$P = \mathbf{F}\cdot\mathbf{v} = 47\,088(0.2683) = 12.6 \text{ kW}$$ **Ans.**

Ans:
$P = 12.6$ kW

***14–60.**

If the escalator in Prob. 14–47 is not moving, determine the constant speed at which a man having a mass of 80 kg must walk up the steps to generate 100 W of power—the same amount that is needed to power a standard light bulb.



SOLUTION

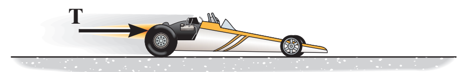
$$P = \frac{U_{1-2}}{t} = \frac{(80)(9.81)(4)}{t} = 100 \qquad t = 31.4 \text{ s}$$

$$v = \frac{s}{t} = \frac{\sqrt{(32(0.25))^2 + 4^2}}{31.4} = 0.285 \text{ m/s} \qquad \textbf{Ans.}$$

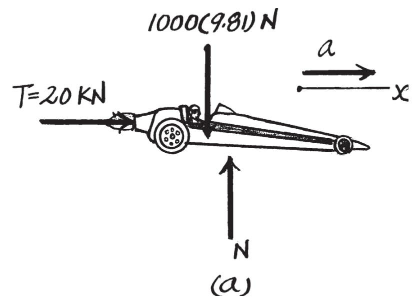
Ans:
$v = 0.285$ m/s

14–61.

If the jet on the dragster supplies a constant thrust of $T = 20$ kN, determine the power generated by the jet as a function of time. Neglect drag and rolling resistance, and the loss of fuel. The dragster has a mass of 1 Mg and starts from rest.



SOLUTION



Equations of Motion: By referring to the free-body diagram of the dragster shown in Fig. a,

$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 20(10^3) = 1000(a)\qquad a = 20\text{ m/s}^2$

Kinematics: The velocity of the dragster can be determined from

$$\left(\xrightarrow{+}\right)\qquad v = v_0 + a_c t$$

$$v = 0 + 20t = (20t)\text{ m/s}$$

Power:

$$P = \mathbf{F}\cdot\mathbf{v} = 20(10^3)(20t)$$

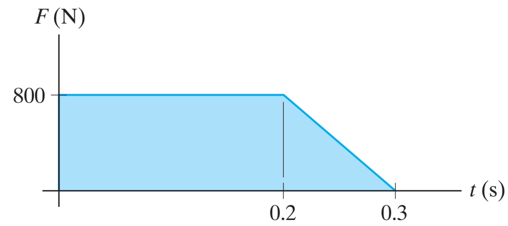
$$= \left[400(10^3)t\right]\text{ W} \qquad \textbf{Ans.}$$

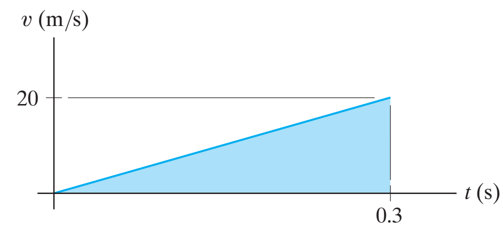
Ans:

$P = \left\{400(10^3)t\right\}\text{ W}$

14–62.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3$ s.





SOLUTION

For $0 \leq t \leq 0.2$

$F = 800$ N

$v = \frac{20}{0.3}t = 66.67t$

$\boldsymbol{P} = \mathbf{F} \cdot \mathbf{v} = 53.3t$ kW **Ans.**

For $0.2 \leq t \leq 0.3$

$F = 2400 - 8000t$

$v = 66.67t$

$P = \mathbf{F} \cdot \mathbf{v} = (160t - 533t^2)$ kW **Ans.**

$$U = \int_0^{0.3} P\,dt$$

$$U = \int_0^{0.2} 53.3t\,dt + \int_{0.2}^{0.3} (160t - 533t^2)\,dt$$

$$= \frac{53.3}{2}(0.2)^2 + \frac{160}{2}[(0.3)^2 - (0.2)^2] - \frac{533}{3}[(0.3)^3 - (0.2)^3]$$

$= 1.69$ kJ **Ans.**

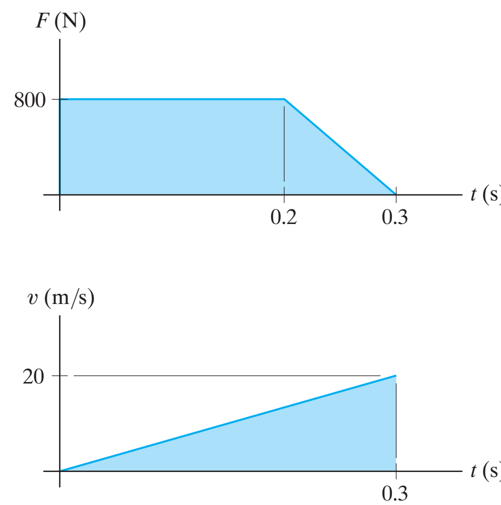
Ans:

$P = \left\{160\,t - 533t^2\right\}$ kW

$U = 1.69$ kJ

14–63.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the 0.3-second time period.



SOLUTION

See solution to Prob. 14–62.

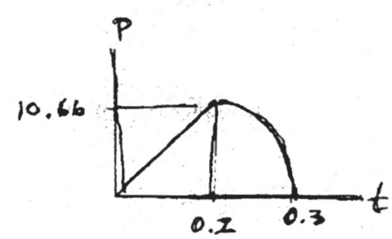
$P = 160\,t - 533\,t^2$

$$\frac{dP}{dt} = 160 - 1066.6\,t = 0$$

$t = 0.15 \text{ s} < 0.2 \text{ s}$

Thus maximum occurs at $t = 0.2$ s

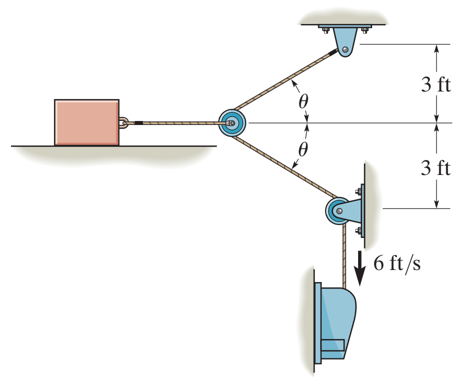
$P_{max} = 53.3(0.2) = 10.7$ kW **Ans.**



Ans:
$P_{\max} = 10.7$ kW

***14–64.**

The block has a weight of 80 lb and rests on the floor for which $\mu_k = 0.4$. If the motor draws in the cable at a constant rate of 6 ft/s, determine the output of the motor at the instant $\theta = 30°$. Neglect the mass of the cable and pulleys.

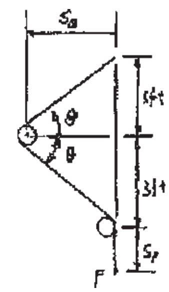


SOLUTION

$$2\left(\sqrt{s_B^2 + 3^2}\right) + S_P = 1 \qquad \textbf{(1)}$$

Time derivative of Eq. (1) yields:

$$\frac{2S_B\dot{s}_B}{\sqrt{s_B^2 + 0}} + \dot{s}_P = 0 \quad \text{Where } \dot{s}_B = v_B \text{ and } \dot{s}_P = v_P \qquad \textbf{(2)}$$



$$\frac{2s_B v_B}{\sqrt{s_B^2 + 9}} + v_P = 0 \quad v_B = \frac{\sqrt{s_B^2 + 9}}{2s_B}v_p \qquad \textbf{(3)}$$

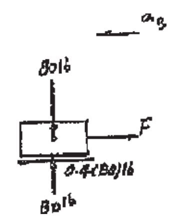
Time derivative of Eq. (2) yields:

$$\frac{1}{(s_B^2 + 9)^{3/2}}[2(s_B^2 + 9)s_B^2 - 2s_B^2 s_B^2 + 2s_B(s_B^2 + 9)\ddot{s}_B] + \ddot{s}_B = 0$$

where $\ddot{s}_p = a_P = 0$ and $\ddot{s}_B = a_B$

$$2(s_B^2 + 9)v_B^2 - 2s_B^2 v_{B2} + 2s_B(s_B^2 + 9)a_B = 0$$

$$v_B = \frac{s_B^2 v_B^2 - v_B^2(s_B^2) + 9}{s_B(s_B^2 + 9)} \qquad \textbf{(4)}$$



At $\theta = 30°$, $\quad s_B = \dfrac{3}{\tan 30°} = 5.196$ ft

From Eq. (3) $\quad v_B = -\dfrac{\sqrt{5.196^2 + 9}}{2(5.196)}(6) = -3.464$ ft/s

From Eq. (4) $\quad a_B = \dfrac{5.196^2(-3.464)^2 - (-3.464^2)(5.196^2 + 9)}{5.196\,(5.196^2 + 9)} = -0.5773 \text{ ft/s}^2$

$\overset{+}{\rightarrow} \Sigma F_x = ma; \quad p - 0.4(80) = \dfrac{80}{32.2}(-0.5773) \qquad p = 30.57$ lb

$\mathbf{F_0} = \theta \cdot v = 30.57(3.464) = 105.9$ ft·lb/s $= 0.193$ hp **Ans.**

Also,

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad -F + 2T\cos 30° = 0$

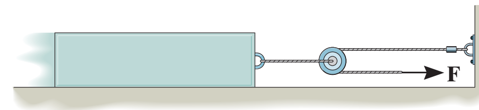
$$T = \frac{30.57}{2\cos 30°} = 17.65 \text{ lb}$$

$\mathbf{F_0} = \mathbf{T} \cdot v_p = 17.65(6) = 105.9$ ft · lb/s $= 0.193$ hp **Ans.**

Ans:
$\mathbf{F_0} = 0.193$ hp
$\mathbf{F_0} = 0.193$ hp

14–65.

The block has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. If a force $F = (60t^2)$ N, where t is in seconds, is applied to the cable, determine the power developed by the force when $t = 5$ s. *Hint:* First determine the time needed for the force to cause motion.

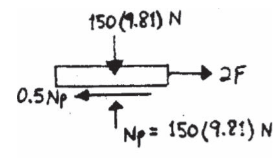


SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F - 0.5(150)(9.81) = 0$

$$F = 367.875 = 60t^2$$

$$t = 2.476 \text{ s}$$

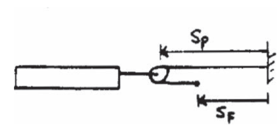


$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 2(60t^2) - 0.4(150)(9.81) = 150a_p$

$$a_p = 0.8t^2 - 3.924$$

$dv = a\,dt$

$$\int_0^v dv = \int_{2.476}^{5} \left(0.8t^2 - 3.924\right) dt$$



$$v = \left(\frac{0.8}{3}\right)t^3 - 3.924t\Big|_{2.476}^{5} = 19.38 \text{ m/s}$$

$s_P + (s_P - s_F) = l$

$2v_P = v_F$

$v_F = 2(19.38) = 38.76$ m/s

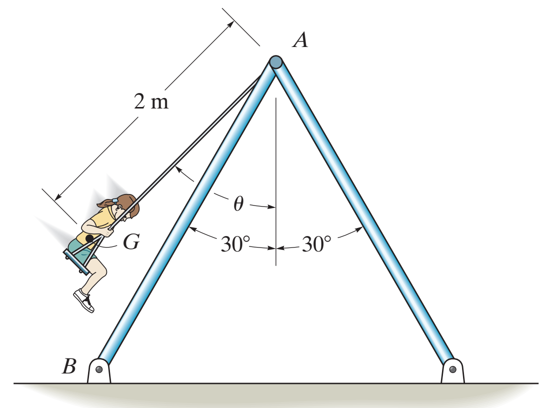
$F = 60(5)^2 = 1500$ N

$P = \mathbf{F} \cdot \mathbf{v} = 1500(38.76) = 58.1$ kW **Ans.**

Ans:
$P = 58.1$ kW

14–66.

The girl has a mass of 40 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 60°$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.



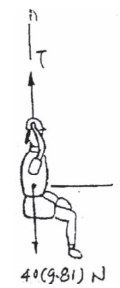
SOLUTION

The maximum tension in the cable occurs when $\theta = 0°$.

$$T_1 + V_1 = T_2 + V_2$$

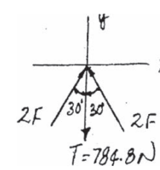
$$0 + 40(9.81)(-2\cos 60°) = \frac{1}{2}(40)v^2 + 40(9.81)(-2)$$

$$v = 4.429 \text{ m/s}$$



$$+\uparrow \Sigma F_n = ma_n; \qquad T - 40(9.81) = (40)\left(\frac{4.429^2}{2}\right) \qquad T = 784.8 \text{ N}$$

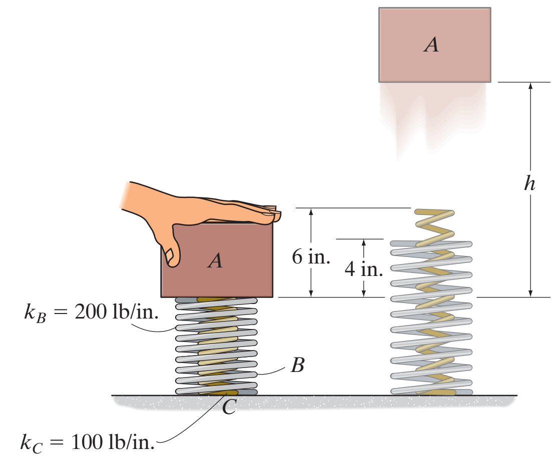
$$+\uparrow \Sigma F_y = 0; \qquad 2(2F)\cos 30° - 784.8 = 0 \qquad F = 227 \text{ N}$$ **Ans.**



Ans:
$F = 227$ N

14–67.

The 30-lb block A is placed on top of two nested springs B and C and then pushed down to the position shown. If it is then released, determine the maximum height h to which it will rise.



SOLUTION

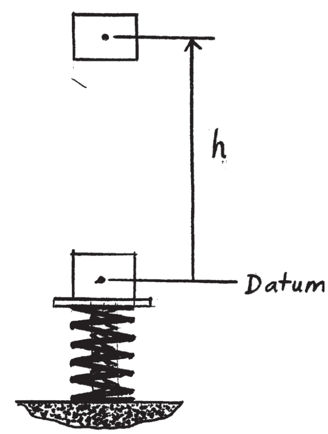
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + 0 + \frac{1}{2}(200)(4)^2 + \frac{1}{2}(100)(6)^2 = 0 + h(30) + 0$$

$$h = 113 \text{ in.}$$ **Ans.**

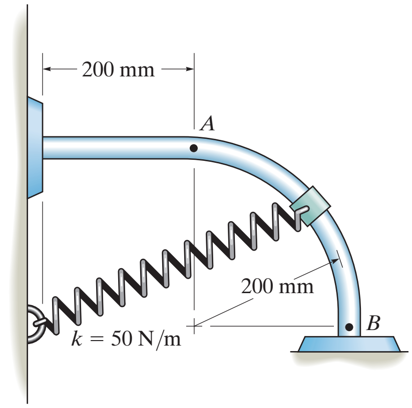


(a)

Ans:
$h = 133$ in.

***14–68.**

The 5-kg collar has a velocity of 5 m/s to the right when it is at A. It then travels down along the smooth guide. Determine the speed of the collar when it reaches point B, which is located just before the end of the curved portion of the rod. The spring has an unstretched length of 100 mm and B is located just before the end of the curved portion of the rod.



SOLUTION

Potential Energy. With reference to the datum set through B the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 5(9.81)(0.2) = 9.81 \text{ J}$$

$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{0.2^2 + 0.2^2} - 0.1 = 0.1828 \text{ m}$ and $x_B = 0.4 - 0.1 = 0.3 \text{ m}$ respectively. Thus, the elastic potential energies in the spring at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(50)\left(0.1828^2\right) = 0.8358 \text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(50)\left(0.3^2\right) = 2.25 \text{ J}$$

Conservation of Energy.

$$T_A + V_A = T_B + V_B$$

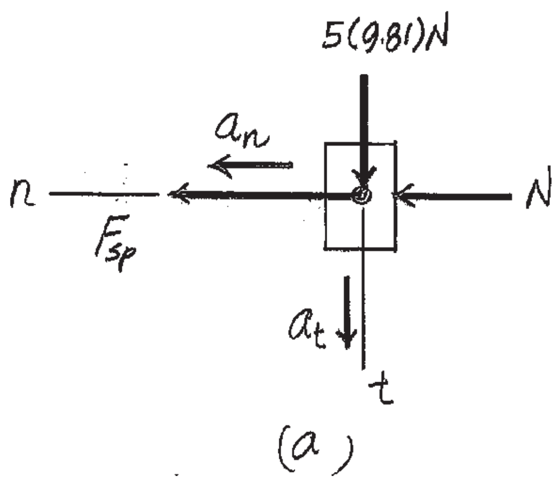
$$\frac{1}{2}(5)\left(5^2\right) + 9.81 + 0.8358 = \frac{1}{2}(5)v_B^2 + 0 + 2.25$$

$$v_B = 5.325 \text{ m/s} = 5.33 \text{ m/s} \qquad \textbf{Ans.}$$

Equation of Motion. At B, $F_{sp} = kx_B = 50(0.3) = 15 \text{ N}$. Referring to the FBD of the collar, Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 15 = 5\left(\frac{5.325^2}{0.2}\right)$$

$$N = 693.95 \text{ N} = 694 \text{ N} \qquad \textbf{Ans.}$$

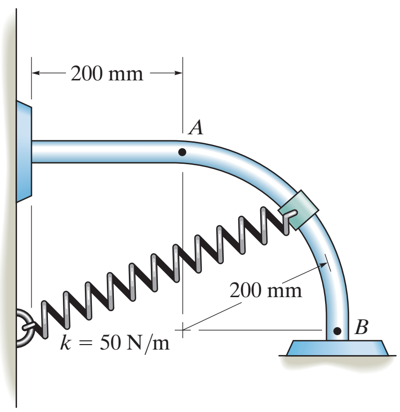


(a)

Ans:
$v_B = 5.33 \text{ m/s}$
$N = 694 \text{ N}$

14–69.

The 5-kg collar has a velocity of 5 m/s to the right when it is at A. It then travels along the smooth guide. Determine its speed when its center reaches point B and the normal force it exerts on the rod at this point. The spring has an unstretched length of 100 mm and B is located just before the end of the curved portion of the rod.



SOLUTION

Potential Energy. With reference to the datum set through B the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 5(9.81)(0.2) = 9.81\text{ J}$$

$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{0.2^2 + 0.2^2} - 0.1 = 0.1828\text{ m}$ and $x_B = 0.4 - 0.1 = 0.3\text{ m}$ respectively. Thus, the elastic potential energies in the spring at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(50)\left(0.1828^2\right) = 0.8358\text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(50)\left(0.3^2\right) = 2.25\text{ J}$$

Conservation of Energy.

$$T_A + V_A = T_B + V_B$$

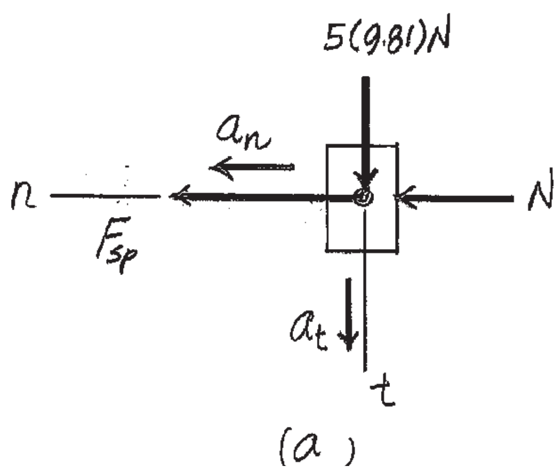
$$\frac{1}{2}(5)\left(5^2\right) + 9.81 + 0.8358 = \frac{1}{2}(5)v_B^2 + 0 + 2.25$$

$$v_B = 5.325\text{ m/s} = 5.33\text{ m/s}$$ **Ans.**

Equation of Motion. At B, $F_{sp} = kx_B = 50(0.3) = 15\text{ N}$. Referring to the FBD of the collar, Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 15 = 5\left(\frac{5.325^2}{0.2}\right)$$

$$N = 693.95\text{ N} = 694\text{ N}$$ **Ans.**

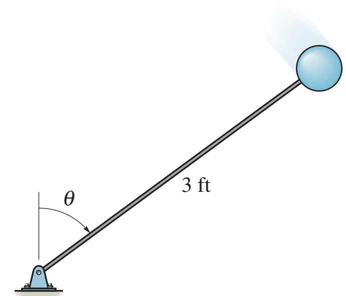


(a)

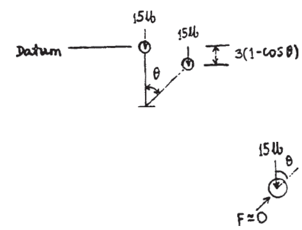
Ans:
$N = 694\text{ N}$

14–70.

The ball has a weight of 15 lb and is fixed to a rod having a negligible mass. If it is released from rest when $\theta = 0°$, determine the angle θ at which the compressive force in the rod becomes zero.



SOLUTION



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{15}{32.2}\right)v^2 - 15(3)(1 - \cos\theta)$$

$$v^2 = 193.2(1 - \cos\theta)$$

$+\swarrow\Sigma F_n = ma_n;$ $$15\cos\theta = \frac{15}{32.2}\left[\frac{193.2(1 - \cos\theta)}{3}\right]$$

$$\cos\theta = 2 - 2\cos\theta$$

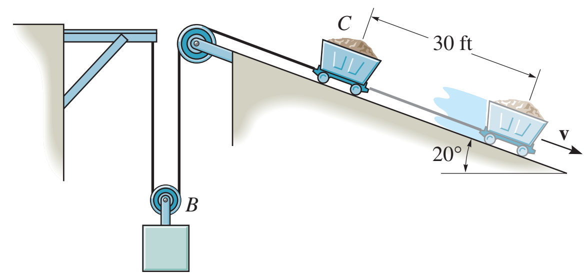
$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\theta = 48.2°$$ **Ans.**

Ans:
$\theta = 48.2°$

14–71.

The car C and its contents have a weight of 600 lb, whereas block B has a weight of 200 lb. If the car is released from rest, determine its speed when it travels 30 ft down the 20° incline. *Suggestion:* To measure the gravitational potential energy, establish separate datums at the initial elevations of B and C.



SOLUTION

$$2s_B + s_C = l$$

$$2\Delta s_B = -\Delta s_C$$

$$\Delta s_B = -\frac{30}{2} = -15 \text{ ft}$$

$$2v_B = -v_C$$

Establish two datums at the initial elevations of the car and the block, respectively.

$$T_1 + V_1 = T_2 + V_2$$

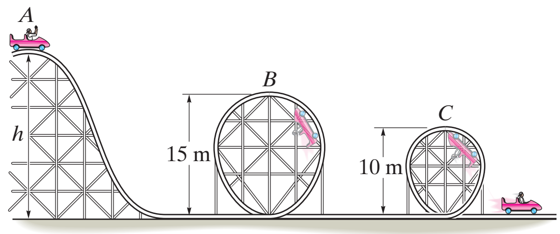
$$0 + 0 = \frac{1}{2}\left(\frac{600}{32.2}\right)(v_C)^2 + \frac{1}{2}\left(\frac{200}{32.2}\right)\left(\frac{-v_C}{2}\right)^2 + 200(15) - 600 \sin 20°(30)$$

$v_C = 17.7$ ft/s **Ans.**

Ans:
$v_C = 17.7$ ft/s

***14–72.**

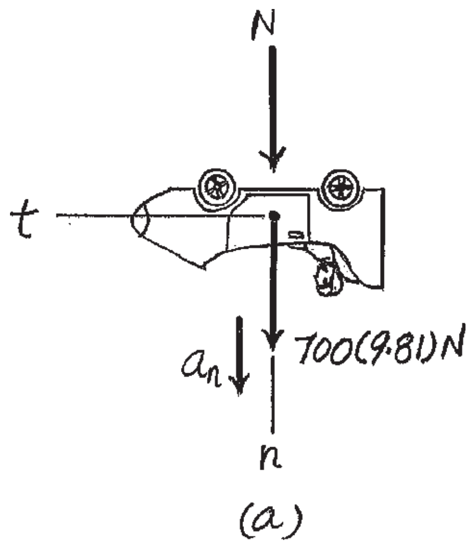
The roller coaster car has a mass of 700 kg, including its passenger. If it starts from the top of the hill A with a speed $v_A = 3\text{ m/s}$, determine the minimum height h of the hill crest so that the car travels around the inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take $\rho_B = 7.5$ m and $\rho_C = 5$ m.



SOLUTION

Equation of Motion. Referring to the FBD of the roller-coaster car shown in Fig. *a*,

$$\Sigma F_n = ma_n; \qquad N + 700(9.81) = 700\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$



When the roller-coaster car is about to leave the loop at B and C, $N = 0$. At B and C, $\rho_B = 7.5\ m$ and $\rho_C = 5\ m$. Then Eq. (1) gives

$$0 + 700(9.81) = 700\left(\frac{v_B^2}{7.5}\right) \qquad v_B^2 = 73.575\text{ m}^2/\text{s}^2$$

and

$$0 + 700(9.81) = 700\left(\frac{v_C^2}{5}\right) \qquad v_C^2 = 49.05\text{ m}^2/\text{s}^2$$

Judging from the above results, the coster car will not leave the loop at C if it safely passes through B. Thus

$$N_B = 0 \qquad \textbf{Ans.}$$

Conservation of Energy. The datum will be set at the ground level. With $v_B^2 = 73.575\text{ m}^2/\text{s}^2$,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(700)\left(3^2\right) + 700(9.81)h = \frac{1}{2}(700)(73.575) + 700(9.81)(15)$$

$$h = 18.29\text{ m} = 18.3\text{ m} \qquad \textbf{Ans.}$$

And from B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}(700)(73.575) + 700(9.81)(15) = \frac{1}{2}(700)v_c^2 + 700(9.81)(10)$$

$$v_c^2 = 171.675\text{ m}^2/\text{s}^2 > 49.05\text{ m}^2/\text{s}^2 \qquad \textbf{(O.K!)}$$

Substitute this result into Eq. 1 with $\rho_C = 5$ m,

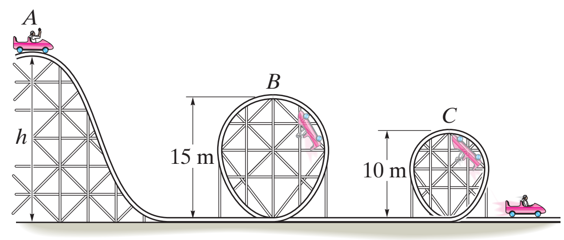
$$N_c + 700(9.81) = 700\left(\frac{171.675}{5}\right)$$

$$N_c = 17.17\left(10^3\right)\text{ N} = 17.2\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$N_B = 0$
$h = 18.3$ m
$N_c = 17.2$ kN

14–73.

The roller coaster car has a mass of 700 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill crest so that the car travels around both inside the loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take $\rho_B = 7.5$ m and $\rho_C = 5$ m.



SOLUTION

Equation of Motion. Referring to the FBD of the roller-coaster car shown in Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 700(9.81) = 700\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$

When the roller-coaster car is about to leave the loop at B and C, $N = 0$. At B and C, $\rho_B = 7.5$ m and $\rho_C = 5$ m. Then Eq. (1) gives

$$0 + 700(9.81) = 700\left(\frac{v_B^2}{7.5}\right) \qquad v_B^2 = 73.575\ \text{m}^2/\text{s}^2$$

and

$$0 + 700(9.81) = 700\left(\frac{v_C^2}{5}\right) \qquad v_C^2 = 49.05\ \text{m}^2/\text{s}^2$$

Judging from the above result the coaster car will not leave the loop at C provided it passes through B safely. Thus

$$N_B = 0 \qquad \textbf{Ans.}$$

Conservation of Energy. The datum will be set at the ground level. Applying Eq. 14– from A to B with $v_B^2 = 73.575\ \text{m}^2/\text{s}^2$,

$$T_A + V_A = T_B + V_B$$

$$0 + 700(9.81)h = \frac{1}{2}(700)(73.575) + 700(9.81)(15)$$

$$h = 18.75\ \text{m} \qquad \textbf{Ans.}$$

And from B to C,

$$T_B + V_B = T_C + V_C$$

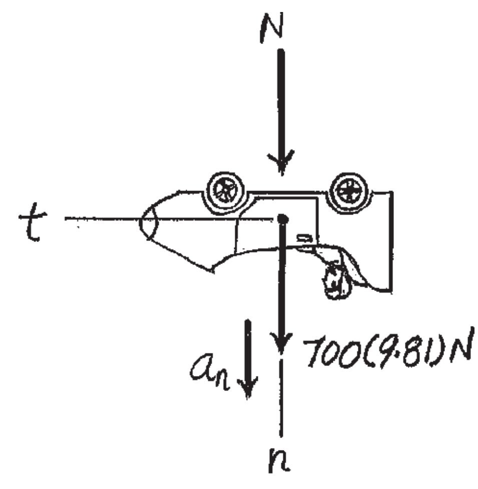
$$\frac{1}{2}(700)(73.575) + 700(9.81)(15) = \frac{1}{2}(700)v_C^2 + 700(9.81)(10)$$

$$v_C^2 = 171.675\ \text{m}^2/\text{s}^2 > 49.05\ \text{m}^2/\text{s}^2 \qquad \textbf{(O.K!)}$$

Substitute this result into Eq. 1 with $\rho_C = 5$ m,

$$N_C + 700(9.81) = 700\left(\frac{171.675}{5}\right)$$

$$N_c = 17.17(10^3)N = 17.2\ \text{kN} \qquad \textbf{Ans.}$$

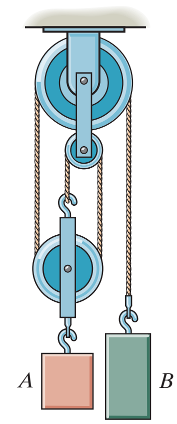


(a)

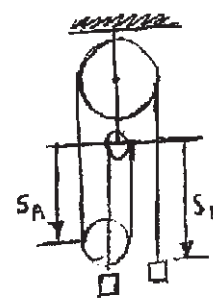
Ans:
$N_B = 0$
$h = 18.75$ m
$N_C = 17.2$ kN

14–74.

The assembly consists of two blocks A and B which have a mass of 20 kg and 30 kg, respectively. Determine the speed of each block when B descends 1.5 m. The blocks are released from rest. Neglect the mass of the pulleys and cords.



SOLUTION



$$3s_A + s_B = l$$

$$3\Delta s_A = -\Delta s_B$$

$$3v_A = -v_B$$

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + (0 + 0) = \frac{1}{2}(20)(v_A)^2 + \frac{1}{2}(30)(-3v_A)^2 + 20(9.81)\left(\frac{1.5}{3}\right) - 30(9.81)(1.5)$$

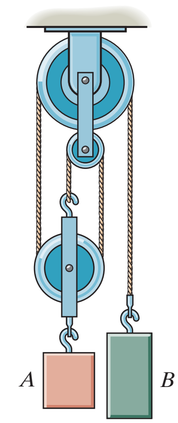
$v_A = 1.54\text{ m/s}$ **Ans.**

$v_B = 4.62\text{ m/s}$ **Ans.**

Ans:
$v_A = 1.54\text{ m/s}$
$v_B = 4.62\text{ m/s}$

14–75.

The assembly consists of two blocks A and B, which have a mass of 20 kg and 30 kg, respectively. Determine the distance B must descend in order for A to achieve a speed of 3 m/s starting from rest.



SOLUTION

$3s_A + s_B = l$

$3\Delta s_A = -\Delta s_B$

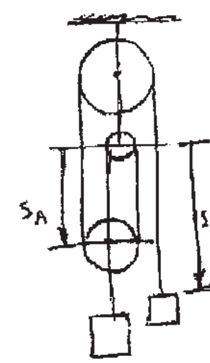
$3v_A = -v_B$

$v_B = -9\text{ m/s}$

$T_1 + V_1 = T_2 + V_2$

$$(0 + 0) + (0 + 0) = \frac{1}{2}(20)(3)^2 + \frac{1}{2}(30)(-9)^2 + 20(9.81)\left(\frac{s_B}{3}\right) - 30(9.81)(s_B)$$

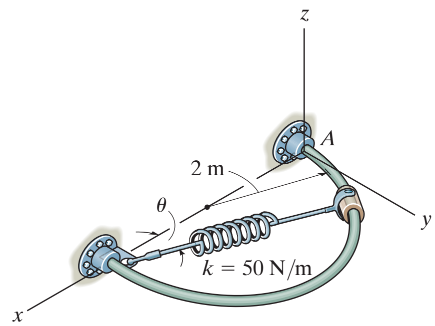
$s_B = 5.70\text{ m}$ **Ans.**



Ans:
$s_B = 5.70\text{ m}$

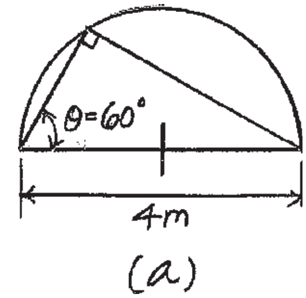
***14–76.**

The spring has a stiffness $k = 50\ \text{N/m}$ and an unstretched length of 0.3 m. If it is attached to the 2-kg smooth collar and the collar is released from rest at A ($\theta = 0°$), determine the speed of the collar when $\theta = 60°$. The motion occurs in the horizontal plane. Neglect the size of the collar.



SOLUTION

Potential Energy. Since the motion occurs in the horizontal plane, there will be no change in gravitational potential energy when $\theta = 0°$, the spring stretches $x_1 = 4 - 0.3 = 3.7\ \text{m}$. Referring to the geometry shown in Fig. *a*, the spring stretches $x_2 = 4 \cos 60° - 0.3 = 1.7\ \text{m}$. Thus, the elastic potential energies in the spring when $\theta = 0°$ and $60°$ are



$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(50)(3.7^2) = 342.25\ \text{J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(50)(1.7^2) = 72.25\ \text{J}$$

Conservation of Energy. Since the collar is released from rest when $\theta = 0°$, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

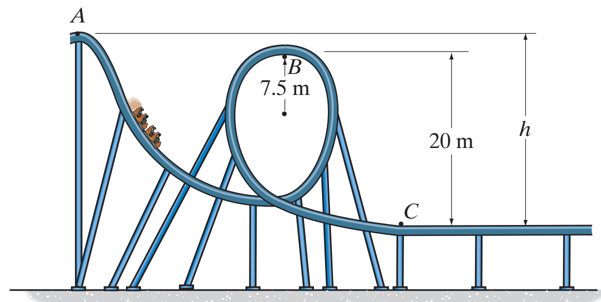
$$0 + 342.25 = \frac{1}{2}(2)v^2 + 72.25$$

$$v = 16.43\ \text{m/s} = 16.4\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 16.4\ \text{m/s}$

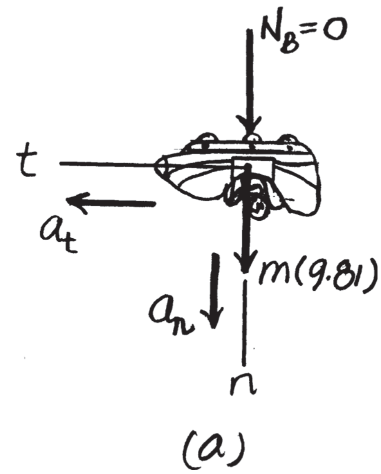
14–77.

The roller coaster car having a mass m is released from rest at point A. If the track is to be designed so that the car does not leave it at B, determine the required height h. Also, find the speed of the car when it reaches point C. Neglect friction.



SOLUTION

Equation of Motion: Since it is required that the roller coaster car is about to leave the track at B, $N_B = 0$. Here, $a_n = \dfrac{v_B^{\,2}}{\rho_B} = \dfrac{v_B^{\,2}}{7.5}$. By referring to the free-body diagram of the roller coaster car shown in Fig. a,



$$\Sigma F_n = ma_n; \qquad m(9.81) = m\left(\frac{v_B^{\,2}}{7.5}\right) \qquad v_B^{\,2} = 73.575\ \text{m}^2/\text{s}^2$$

Potential Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the rollercoaster car at positions A, B, and C are $\left(V_g\right)_A = mgh_A = m(9.81)h = 9.81mh$, $\left(V_g\right)_B = mgh_B = m(9.81)(20) = 196.2$ m, and $\left(V_g\right)_C = mgh_C = m(9.81)(0) = 0$.

Conservation of Energy: Using the result of $v_B^{\,2}$ and considering the motion of the car from position A to B,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^{\,2} + \left(V_g\right)_A = \frac{1}{2}mv_B^{\,2} + \left(V_g\right)_B$$

$$0 + 9.81mh = \frac{1}{2}m(73.575) + 196.2m$$

$$h = 23.75\ \text{m} \qquad \textbf{Ans.}$$

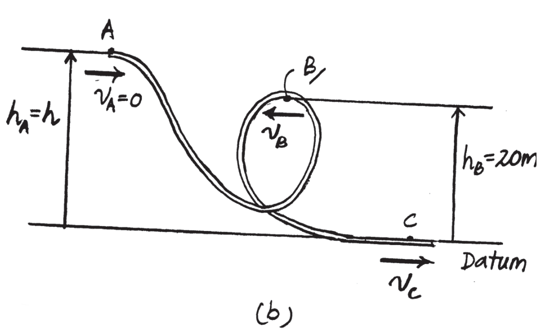
Also, considering the motion of the car from position B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^{\,2} + \left(V_g\right)_B = \frac{1}{2}mv_C^{\,2} + \left(V_g\right)_C$$

$$\frac{1}{2}m(73.575) + 196.2m = \frac{1}{2}mv_C^{\,2} + 0$$

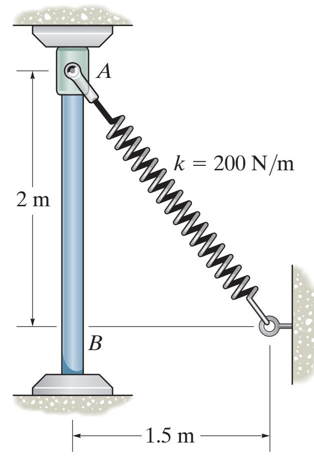
$$v_C = 21.6\ \text{m/s} \qquad \textbf{Ans.}$$



Ans:
$h = 23.75$ m
$v_C = 21.6$ m/s

14–78.

The spring has a stiffness $k = 200$ N/m and an unstretched length of 0.5 m. If it is attached to the 3-kg smooth collar and the collar is released from rest at A, determine the speed of the collar when it reaches B. Neglect the size of the collar.



SOLUTION

Potential Energy. With reference to the datum set through B, the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 3(9.81)(2) = 58.86 \text{ J}$$
$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{1.5^2 + 2^2} - 0.5 = 2.00$ m and $x_B = 1.5 - 0.5 = 1.00$ m. Thus, the elastic potential energies in the spring when the collar is at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(200)\left(2.00^2\right) = 400 \text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(200)\left(1.00^2\right) = 100 \text{ J}$$

Conservation of Energy. Since the collar is released from rest at A, $T_A = 0$.

$$T_A + V_A = T_B + V_B$$

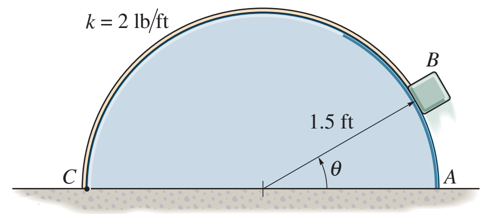
$$0 + 58.86 + 400 = \frac{1}{2}(3)v_B^2 + 0 + 100$$

$$v_B = 15.47 \text{ m/s} = 15.5 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 15.5$ m/s

14–79.

A 2-lb block rests on the smooth semicylindrical surface at A. An elastic cord having a stiffness of $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at C. If the block is released from rest at A, determine the longest unstretched length of the cord so the block begins to leave the cylinder at the instant $\theta = 45°$. Neglect the size of the block.



SOLUTION

Equation of Motion: It is required that $N = 0$. Applying Eq. 13–8, we have

$$\Sigma F_n = ma_n; \qquad 2\cos 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right) \qquad v^2 = 34.15\ \text{m}^2/\text{s}^2$$

Potential Energy: Datum is set at the base of cylinder. When the block moves to a position $1.5 \sin 45° = 1.061$ ft *above* the datum, its gravitational potential energy at this position is $2(1.061) = 2.121$ ft · lb. The initial and final elastic potential energy are $\frac{1}{2}(2)\,[\pi(1.5) - l]^2$ and $\frac{1}{2}(2)\,[0.75\pi(1.5) - l]^2$, respectively.

Conservation of Energy:

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

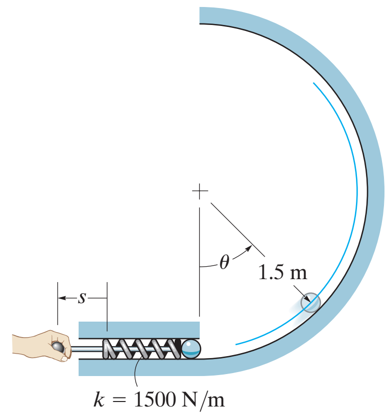
$$0 + \frac{1}{2}(2)\,[\pi(1.5) - l]^2 = \frac{1}{2}\left(\frac{2}{32.2}\right)(34.15) + 2.121 + \frac{1}{2}(2)[0.75\pi(1.5) - l]^2$$

$$l = 2.77\ \text{ft}$$ **Ans.**

Ans:
$l = 2.77$ ft

***14–80.**

When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 100$ mm and released, determine the speed of the 0.3-kg ball and the normal reaction of the circular track on the ball when $\theta = 60°$. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



SOLUTION

Potential Energy. With reference to the datum set through the center of the circular track, the gravitational potential energies of the ball when $\theta = 0°$ and $\theta = 60°$ are

$$(V_g)_1 = -mgh_1 = -0.3(9.81)(1.5) = -4.4145 \text{ J}$$

$$(V_g)_2 = -mgh_2 = -0.3(9.81)(1.5 \cos 60°) = -2.20725 \text{ J}$$

When $\theta = 0°$, the spring compress $x_1 = 0.1$ m and is unstretched when $\theta = 60°$. Thus, the elastic potential energies in the spring when $\theta = 0°$ and $60°$ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1500)\left(0.1^2\right) = 7.50 \text{ J}$$

$$(V_e)^2 = 0$$

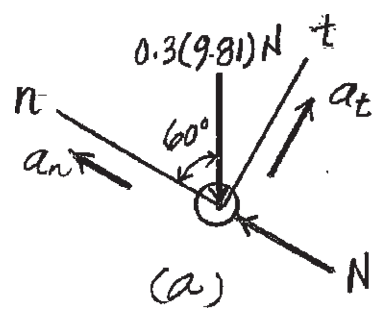
Conservation of Energy. Since the ball starts from rest, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-4.4145) + 7.50 = \frac{1}{2}(0.3)v^2 + (-2.20725) + 0$$

$$v^2 = 35.285 \text{ m}^2/\text{s}^2$$

$$v = 5.94 \text{ m/s} \qquad \textbf{Ans.}$$



Equation of Motion. Referring to the FBD of the ball, Fig. *a*,

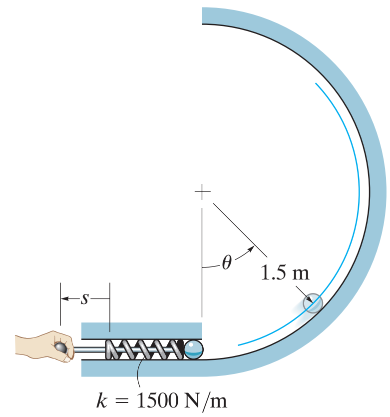
$$\Sigma F_n = ma_n; \qquad N - 0.3(9.81)\cos 60° = 0.3\left(\frac{35.285}{1.5}\right)$$

$$N = 8.5285 \text{ N} = 8.53 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$v = 5.94$ m/s
$N = 8.53$ N

14–81.

When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 100$ mm and released, determine the maximum angle θ the ball will travel without leaving the circular track. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



SOLUTION

Equation of Motion. It is required that the ball leaves the track, and this will occur provided $\theta > 90°$. When this happens, $N = 0$. Referring to the FBD of the ball, Fig. *a*

$$\Sigma F_n = ma_n; \qquad 0.3(9.81)\sin(\theta - 90°) = 0.3\left(\frac{v^2}{1.5}\right)$$

$$v^2 = 14.715 \sin(\theta - 90°) \qquad \textbf{(1)}$$

Potential Energy. With reference to the datum set through the center of the circular track Fig. *b*, the gravitational potential Energies of the ball when $\theta = 0°$ and θ are

$$(V_g)_1 = -mgh_1 = -0.3(9.81)(1.5) = -4.4145 \text{ J}$$

$$(V_g)_2 = mgh_2 = 0.3(9.81)[1.5\sin(\theta - 90°)]$$

$$= 4.4145\sin(\theta - 90°)$$

When $\theta = 0°$, the spring compresses $x_1 = 0.1$ m and is unstretched when the ball is at θ for max height. Thus, the elastic potential energies in the spring when $\theta = 0°$ and θ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1500)(0.1^2) = 7.50 \text{ J}$$

$$(V_e)_2 = 0$$

Conservation of Energy. Since the ball starts from rest, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-4.4145) + 7.50 = \frac{1}{2}(0.3)v^2 + 4.4145\sin(\theta - 90°) + 0$$

$$v^2 = 20.57 - 29.43\sin(\theta - 90°) \qquad \textbf{(2)}$$

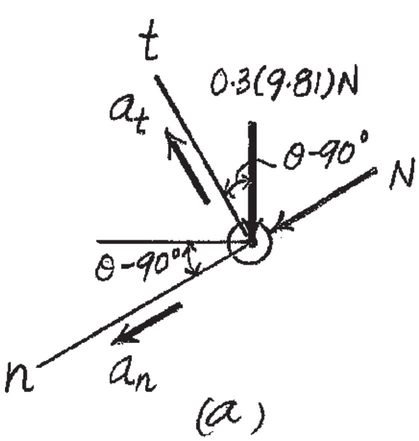
Equating Eqs. (1) and (2),

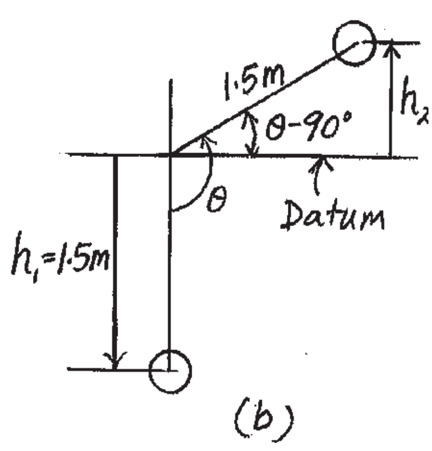
$$14.715\sin(\theta - 90°) = 20.57 - 29.43\sin(\theta - 90°)$$

$$\sin(\theta - 90°) = 0.4660$$

$$\theta - 90° = 27.77°$$

$$\theta = 117.77° = 118° \qquad \textbf{Ans.}$$

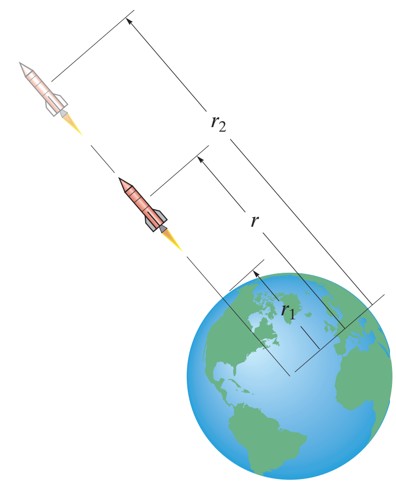




Ans:
$\theta = 118°$

14–82.

If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_e m/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_e m/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \to \infty$. Also, prove that F is a conservative force.



SOLUTION

The work is computed by moving F from position r_1 to a farther position r_2.

$$V_g = -U = -\int F\,dr$$

$$= -G\,M_e\,m \int_{r1}^{r2} \frac{dr}{r^2}$$

$$= -G\,M_e\,m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

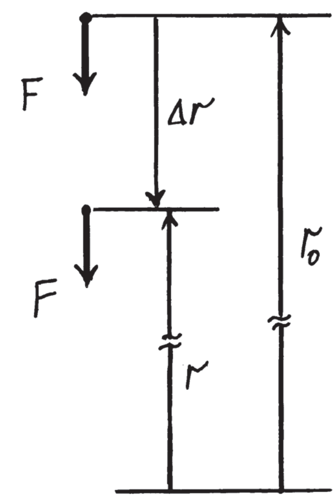
As $r_1 \to \infty$, let $r_2 = r_1$, $F_2 = F_1$, then

$$V_g \to \frac{-G\,M_e\,m}{r}$$

To be conservative, require

$$F = -\nabla V_g = -\frac{\partial}{\partial r}\left(-\frac{G\,M_e\,m}{r}\right)$$

$$= \frac{-G\,M_e\,m}{r^2}$$ **Q.E.D.**

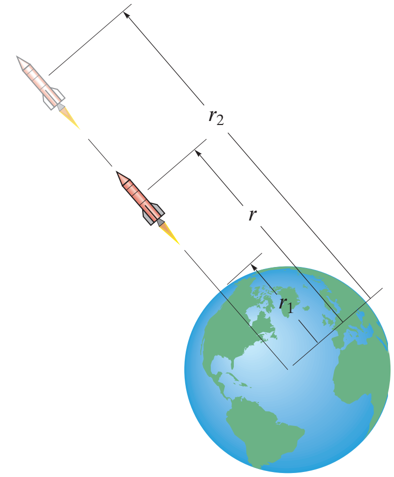


Ans:

$$F = \frac{-G\,M_e\,m}{r^2}$$

14–83.

A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.



SOLUTION

$$F = G\frac{M_e m}{r^2}$$

$$F_{1-2} = \int F\,dr = GM_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

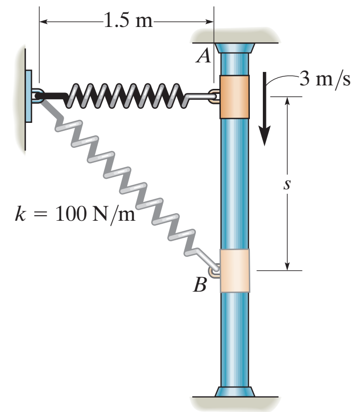
$$= GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \textbf{Ans.}$$

Ans:

$$F = GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

***14–84.**

The 4-kg smooth collar has a speed of 3 m/s when it is at $s = 0$. Determine the maximum distance s it travels before it stops momentarily. The spring has an unstretched length of 1 m.



SOLUTION

Potential Energy. With reference to the datum set through A the gravitational potential energies of the collar at A and B are

$$(V_g)_A = 0 \qquad (V_g)_B = -mgh_B = -4(9.81)\, S_{max} = -39.24\, S_{max}$$

At A and B, the spring stretches $x_A = 1.5 - 1 = 0.5$ m and $x_B = \sqrt{S_{max}^2 + 1.5^2} - 1$. Thus, the elastic potential Energies in the spring when the collar is at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(100)\left(0.5^2\right) = 12.5 \text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(100)\left(\sqrt{S_{max}^2 + 1.5^2} - 1\right)^2 = 50\left(S_{max}^2 - 2\sqrt{S_{max}^2 + 1.5^2} + 3.25\right)$$

Conservation of Energy. Since the collar is required to stop momentarily at B, $T_B = 0$.

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(4)\left(3^2\right) + 0 + 12.5 = 0 + (-39.24\, S_{max}) + 50\left(S_{max}^2 - 2\sqrt{S_{max}^2 + 1.5^2} + 3.25\right)$$

$$50\, S_{max}^2 - 100\sqrt{S_{max}^2 + 1.5^2} - 39.24\, S_{max} + 132 = 0$$

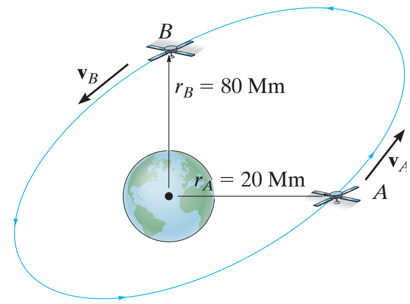
Solving numerically,

$$S_{max} = 1.9554 \text{ m} = 1.96 \text{ m}$$ **Ans.**

Ans:
$S_{max} = 1.96$ m

14–85.

A 60-kg satellite travels in free flight along an elliptical orbit such that at A, where $r_A = 20$ Mm, it has a speed $v_A = 40$ Mm/h. What is the speed of the satellite when it reaches point B, where $r_B = 80$ Mm? *Hint:* See Prob. 14–82, where $M_e = 5.976(10^{24})$ kg and $G = 66.73(10^{-12})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$.



SOLUTION

$$v_A = 40\ \text{Mm/h} = 11\,111.1\ \text{m/s}$$

Since $V = -\dfrac{GM_e m}{r}$

$$T_1 + V_1 = T_2 + V_2$$

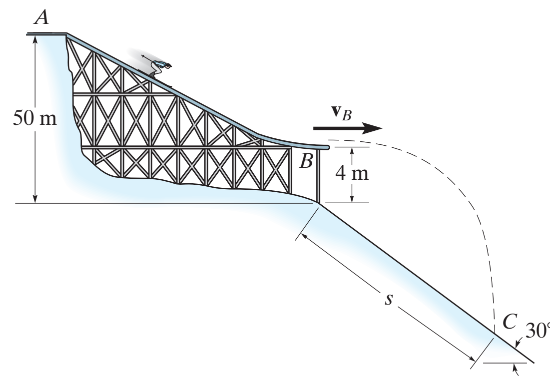
$$\frac{1}{2}(60)(11\,111.1)^2 - \frac{66.73(10)^{-12}(5.976)(10)^{23}(60)}{20(10)^6} = \frac{1}{2}(60)v_B^2 - \frac{66.73(10)^{-12}(5.976)(10)^{24}(60)}{80(10)^6}$$

$$v_B = 9672\ \text{m/s} = 34.8\ \text{Mm/h}$$ **Ans.**

Ans:
$v_B = 34.8$ Mm/h

14–86.

The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, compute the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



SOLUTION

$$T_A + V_A = T_B + V_B$$

$$0 + 70(9.81)(46) = \frac{1}{2}(70)v^2 + 0$$

$$v = 30.04 \text{ m/s} = 30.0 \text{ m/s}$$ **Ans.**

$$(+\downarrow)\ s_y = (s_y)_0 + (v_0)_y t + \frac{1}{2}a_c t^2$$

$$4 + s \sin 30^\circ = 0 + 0 + \frac{1}{2}(9.81)t^2 \qquad \textbf{(1)}$$

$$(\overset{+}{\leftarrow})\, s_x = v_x\, t$$

$$s \cos 30^\circ = 30.04t \qquad \textbf{(2)}$$

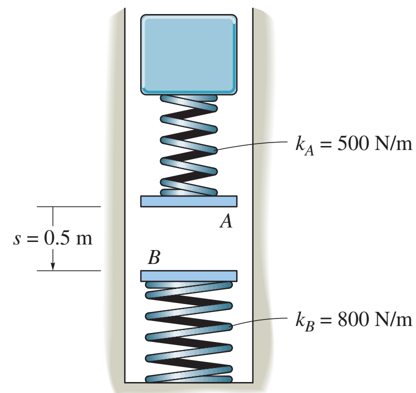
$$s = 130 \text{ m}$$ **Ans.**

$$t = 3.75 \text{ s}$$

Ans:
$s = 130$ m

14–87.

The block has a mass of 20 kg and is released from rest when $s = 0.5$ m. If the mass of the bumpers A and B can be neglected, determine the maximum deformation of each spring due to the collision.



SOLUTION

Datum at initial position:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \frac{1}{2}(500)s_A^2 + \frac{1}{2}(800)s_B^2 + 20(9.81)\left[-(s_A + s_B) - 0.5\right] \quad \textbf{(1)}$$

Also, $F_s = 500s_A = 800s_B \qquad s_A = 1.6s_B$ **(2)**

Solving Eqs. (1) and (2) yields:

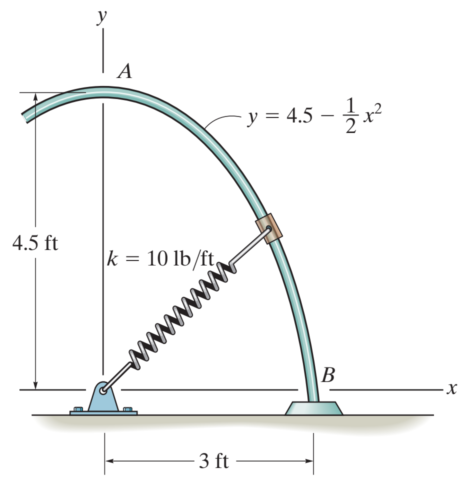
$s_B = 0.638$ m **Ans.**

$s_A = 1.02$ m **Ans.**

Ans:
$s_B = 0.638$ m
$s_A = 1.02$ m

***14–88.**

The 2-lb collar has a speed of 5 ft/s at A. The attached spring has an unstretched length of 2 ft and a stiffness of $k = 10$ lb/ft. If the collar moves over the smooth rod, determine its speed when it reaches point B, the normal force of the rod on the collar, and the rate of decrease in its speed.



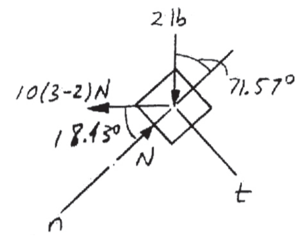
SOLUTION

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + \frac{1}{2}(10)(4.5 - 2)^2 + 2(4.5) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_B)^2 + \frac{1}{2}(10)(3 - 2)^2 + 0$$

$$v_B = 34.060 \text{ ft/s} = 34.1 \text{ ft/s} \qquad \textbf{Ans.}$$



$$y = 4.5 - \frac{1}{2}x^2$$

$$\frac{dy}{dx} = \tan\theta = -x\bigg|_{x=3} = -3$$

$$\theta = -71.57^\circ \qquad \frac{d^2y}{dx^2} = -1$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + (-3)^2]^{\frac{3}{2}}}{|-1|} = 31.623 \text{ ft}$$

$$+\swarrow\Sigma F_n = ma_n; \qquad -N + 10\cos 18.43^\circ + 2\cos 71.57^\circ = \left(\frac{2}{32.2}\right)\left(\frac{(34.060)^2}{31.623}\right)$$

$$N = 7.84 \text{ lb} \qquad \textbf{Ans.}$$

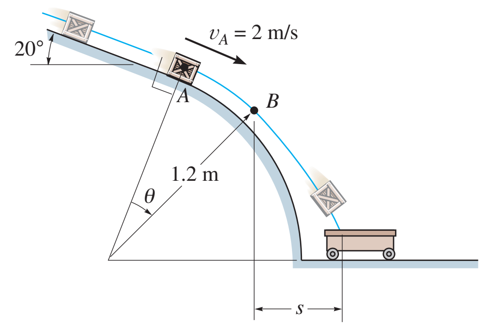
$$+\searrow\Sigma F_t = ma_t; \qquad 2\sin 71.57^\circ - 10\sin 18.43^\circ = \left(\frac{2}{32.2}\right)a_t$$

$$a_t = -20.4 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_B = 34.1$ ft/s
$N = 7.84$ lb
$a_t = -20.4$ ft/s^2

14–89.

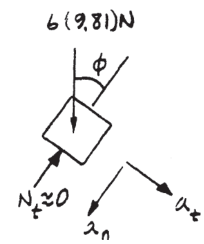
When the 6-kg box reaches point A it has a speed of $v_A = 2$ m/s. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.



SOLUTION

At point B:

$$+\swarrow \Sigma F_n = ma_n; \qquad 6(9.81)\cos\phi = 6\left(\frac{v_B^2}{1.2}\right) \qquad \textbf{(1)}$$



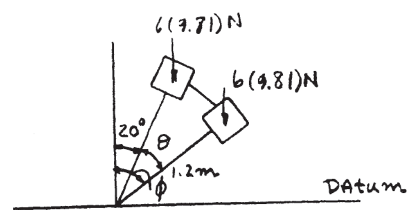
Datum at bottom of curve:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(6)(2)^2 + 6(9.81)(1.2\cos 20^\circ) = \frac{1}{2}(6)(v_B)^2 + 6(9.81)(1.2\cos\phi)$$

$$13.062 = 0.5v_B^2 + 11.772\cos\phi \qquad \textbf{(2)}$$

Substitute Eq. (1) into Eq. (2), and solving for v_B,



$$v_B = 2.951 \text{ m/s}$$

Thus, $\quad \phi = \cos^{-1}\left(\frac{(2.951)^2}{1.2(9.81)}\right) = 42.29^\circ$

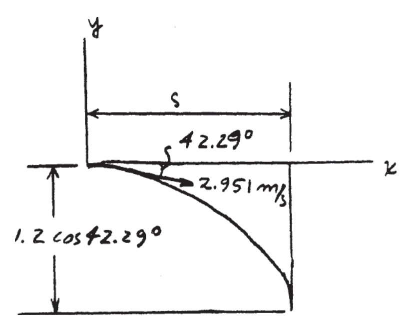
$$\theta = \phi - 20^\circ = 22.3^\circ \qquad \textbf{Ans.}$$

$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$-1.2\cos 42.29^\circ = 0 - 2.951(\sin 42.29^\circ)t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 + 1.9857t - 0.8877 = 0$$

Solving for the positive root: $t = 0.2687$ s



$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t$

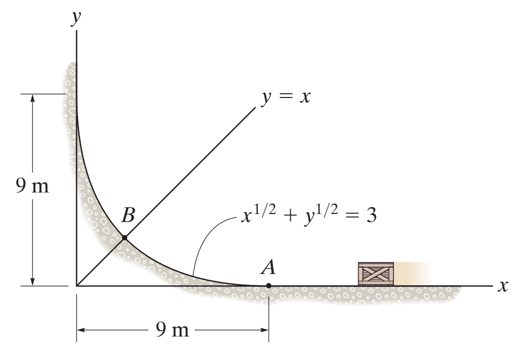
$$s = 0 + (2.951\cos 42.29^\circ)(0.2687)$$

$$s = 0.587 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$\theta = 22.3^\circ$
$s = 0.587$ m

14–90.

When the 5-kg box reaches point A it has a speed $v_A = 10\text{ m/s}$. Determine the normal force the box exerts on the surface when it reaches point B. Neglect friction and the size of the box.



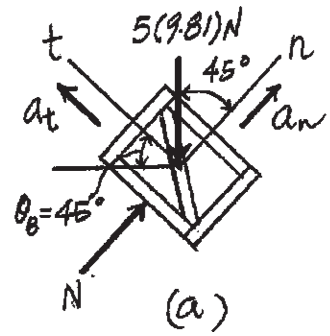
SOLUTION

Conservation of Energy. At point B, $y = x$

$$x^{\frac{1}{2}} + x^{\frac{1}{2}} = 3$$

$$x = \frac{9}{4}\text{ m}$$

Then $y = \dfrac{9}{4}$ m. With reference to the datum set to coincide with the x axis, the gravitational potential energies of the box at points A and B are



$$(V_g)_A = 0 \qquad (V_g)_B = mgh_B = 5(9.81)\left(\frac{9}{4}\right) = 110.3625\text{ J}$$

Applying the energy equation,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(5)\left(10^2\right) + 0 = \frac{1}{2}(5)v_B^2 + 110.3625$$

$$v_B^2 = 55.855\text{ m}^2/\text{s}^2$$

Equation of Motion. Here, $y = \left(3 - x^{\frac{1}{2}}\right)^2$. Then, $\dfrac{dy}{dx} = 2\left(3 - x^{\frac{1}{2}}\right)\left(-\dfrac{1}{2}x^{-\frac{1}{2}}\right)$ $= \dfrac{x^{\frac{1}{2}} - 3}{x^{\frac{1}{2}}} = 1 - \dfrac{3}{x^{\frac{1}{2}}}$ and $\dfrac{d^2y}{dx^2} = \dfrac{3}{2}x^{-\frac{3}{2}} = \dfrac{3}{2x^{\frac{3}{2}}}$. At point B, $x = \dfrac{9}{4}$ m. Thus,

$$\tan\theta_B = \left.\frac{dy}{dx}\right|_{x=\frac{9}{4}\text{ m}} = 1 - \frac{3}{\left(\frac{9}{4}\right)^{\frac{1}{2}}} = -1 \qquad \theta_B = -45° = 45°$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{9}{4}\text{ m}} = \frac{3}{2\left(\frac{9}{4}\right)^{\frac{3}{2}}} = 0.4444$$

The radius of curvature at B is

$$P_B = \frac{\left[1 + (dy/dx)^2\right]^{\frac{3}{2}}}{\left|d^2y/dx^2\right|} = \frac{\left[1 + (-1)^2\right]^{\frac{3}{2}}}{0.4444} = 6.3640\text{ m}$$

Referring to the FBD of the box, Fig. *a*

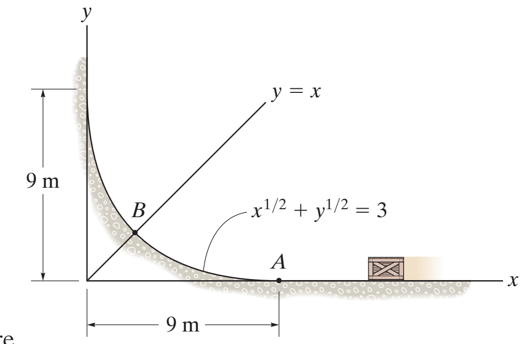
$$\Sigma F_n = ma_n; \qquad N - 5(9.81)\cos 45° = 5\left(\frac{55.855}{6.3640}\right)$$

$$N = 78.57\text{ N} = 78.6\text{ N} \qquad \textbf{Ans.}$$

Ans:
$N = 78.6\text{ N}$

14–91.

When the 5-kg box reaches point A it has a speed $v_A = 10\text{ m/s}$. Determine how high the box reaches up the surface before it comes to a stop. Also, what is the resultant normal force on the surface at this point and the acceleration? Neglect friction and the size of the box.



SOLUTION

Conservation of Energy. With reference to the datum set coincide with x axis, the gravitational potential energy of the box at A and C (at maximum height) are

$$(V_g)_A = 0 \qquad (V_g)_C = mgh_c = 5(9.81)(y) = 49.05y$$

It is required that the box stop at C. Thus, $T_c = 0$

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}(5)\left(10^2\right) + 0 = 0 + 49.05y$$

$$y = 5.0968\text{ m} = 5.10\text{ m} \qquad \textbf{Ans.}$$

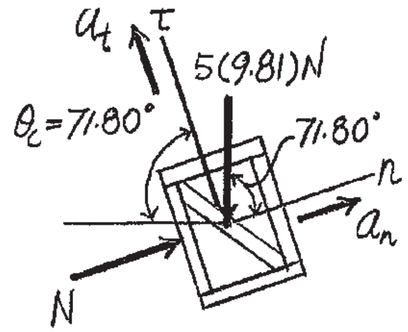
Then,

$$x^{\frac{1}{2}} + 5.0968^{\frac{1}{2}} = 3 \qquad x = 0.5511\text{ m}$$

Equation of Motion. Here, $y = \left(3 - x^{\frac{1}{2}}\right)^2$. Then, $\dfrac{dy}{dx} = 2\left(3 - x^{\frac{1}{2}}\right)\left(-\dfrac{1}{2}x^{\frac{1}{2}}\right)$ $= \dfrac{x^{\frac{1}{2}} - 3}{x^{\frac{1}{2}}} = 1 - \dfrac{3}{x^{\frac{1}{2}}}$ and $\dfrac{d^2y}{dx^2} = \dfrac{3}{2}x^{\frac{-3}{2}} = \dfrac{3}{2x^{\frac{3}{2}}}$ At point C, $x = 0.5511$ m.

Thus

$$\tan\theta_c = \left.\frac{dy}{dx}\right|_{x=0.5511\text{ m}} = 1 - \frac{3}{0.5511^{\frac{1}{2}}} = -3.0410 \qquad \theta_C = -71.80° = 71.80°$$



(a)

Referring to the FBD of the box, Fig. a,

$$\Sigma F_n = ma_n; \qquad N - 5(9.81)\cos 71.80° = 5\left(\frac{0^2}{\rho_C}\right)$$

$$N = 15.32\text{ N} = 15.3\text{ N} \qquad \textbf{Ans.}$$

$$\Sigma F_t = ma_t; \qquad -5(9.81)\sin 71.80° = 5a_t$$

$$a_t = -9.3191\text{ m/s}^2 = 9.32\text{ m/s}^2 \searrow$$

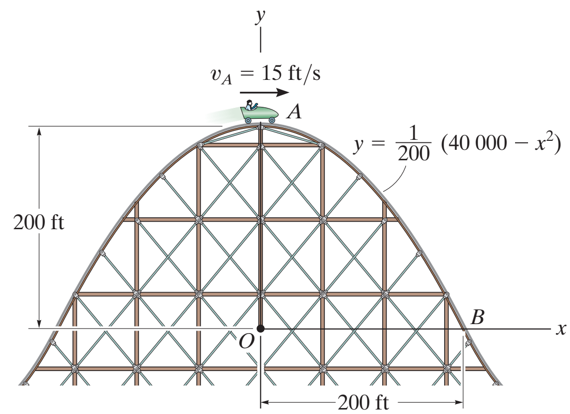
Since $a_n = 0$, Then

$$a = a_t = 9.32\text{ m/s}^2 \searrow \qquad \textbf{Ans.}$$

Ans:
$y = 5.10$ m
$N = 15.3$ N
$a = 9.32\text{ m/s}^2 \searrow$

***14–92.**

The roller-coaster car has a speed of 15 ft/s when it is at the crest of a vertical parabolic track. Determine the car's velocity and the normal force it exerts on the track when it reaches point B. Neglect friction and the mass of the wheels. The total weight of the car and the passengers is 350 lb.

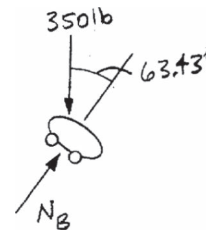


SOLUTION

$$y = \frac{1}{200}(40\,000 - x^2)$$

$$\frac{dy}{dx} = -\frac{1}{100}x\bigg|_{x=200} = -2, \qquad \theta = \tan^{-1}(-2) = -63.43^\circ$$

$$\frac{d^2y}{dx^2} = -\frac{1}{100}$$



Datum at A:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{350}{32.2}\right)(15)^2 + 0 = \frac{1}{2}\left(\frac{350}{32.2}\right)(v_B)^2 - 350(200)$$

$v_B = 114.48 = 114$ ft/s **Ans.**

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + (-2)^2]^{\frac{3}{2}}}{\left|-\frac{1}{100}\right|} = 1118.0 \text{ ft}$$

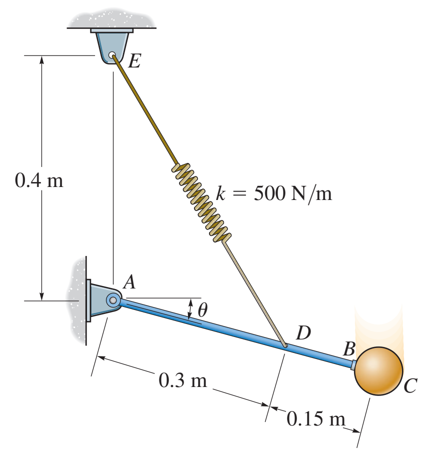
$+\swarrow\Sigma F_n = ma_n; \qquad 350\cos 63.43^\circ - N_B = \left(\frac{350}{32.2}\right)\frac{(114.48)^2}{1118.0}$

$N_B = 29.1$ lb **Ans.**

Ans:
$v_B = 114$ ft/s
$N_B = 29.1$ lb

14–93.

The 10-kg sphere C is released from rest when $\theta = 0°$ and the tension in the spring is 100 N. Determine the speed of the sphere at the instant $\theta = 90°$. Neglect the mass of rod AB and the size of the sphere.



SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the sphere at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 10(9.81)(0.45) = 44.145$ J and $\left(V_g\right)_2 = mgh_2 = 10(9.81)(0) = 0$. When the sphere is at position (1), the spring stretches $s_1 = \dfrac{100}{500} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = \sqrt{0.3^2 + 0.4^2} - 0.2 = 0.3$ m, and the elastic potential energy of the spring is $\left(V_e\right)_1 = \frac{1}{2}ks_1^2 = \frac{1}{2}(500)(0.2^2) = 10$ J. When the sphere is at position (2), the spring stretches $s_2 = 0.7 - 0.3 = 0.4$ m, and the elastic potential energy of the spring is $\left(V_e\right)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(500)(0.4^2) = 40$ J.

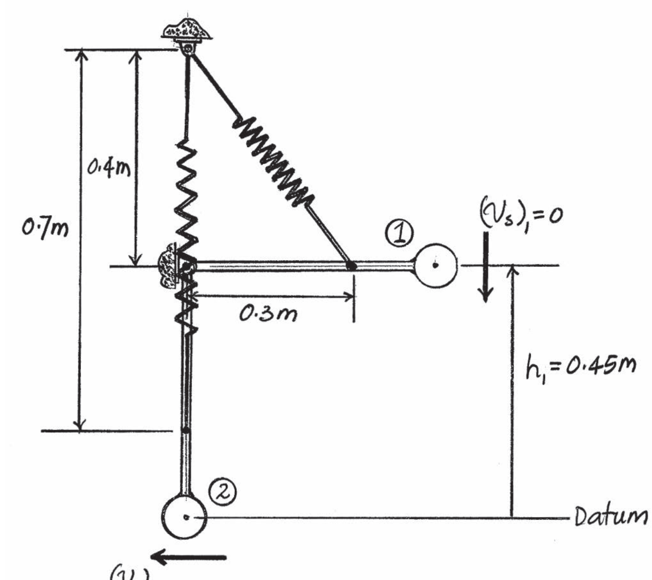
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_s\left(v_s\right)_1^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}m_s\left(v_s\right)_2^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + (44.145 + 10) = \frac{1}{2}(10)(v_s)_2^2 + (0 + 40)$$

$$(v_s)_2 = 1.68 \text{ m/s} \qquad \textbf{Ans.}$$

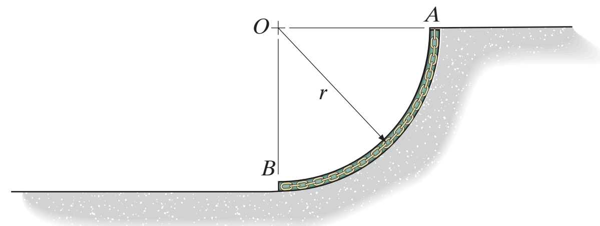


(a)

Ans:
$v = 1.68$ m/s

14–94.

A quarter-circular tube AB of mean radius r contains a smooth chain that has a mass per unit length of m_0. If the chain is released from rest from the position shown, determine its speed when it emerges completely from the tube.



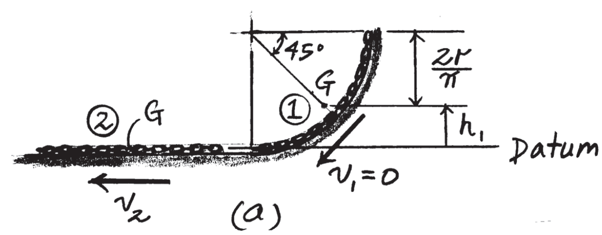
SOLUTION

Potential Energy: The location of the center of gravity G of the chain at positions (1) and (2) are shown in Fig. *a*. The mass of the chain is $m = m_0\left(\frac{\pi}{2}r\right) = \frac{\pi}{2}m_0 r$. Thus, the center of mass is at $h_1 = r - \frac{2r}{\pi} = \left(\frac{\pi - 2}{\pi}\right)r$. With reference to the datum set in Fig. *a* the gravitational potential energy of the chain at positions (1) and (2) are

$$\left(V_g\right)_1 = mgh_1 = \left(\frac{\pi}{2}m_0 rg\right)\left(\frac{\pi - 2}{\pi}\right)r = \left(\frac{\pi - 2}{2}\right)m_0 r^2 g$$

and

$$\left(V_g\right)_2 = mgh_2 = 0$$



Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1{}^2 + \left(V_g\right)_1 = \frac{1}{2}mv_2{}^2 + \left(V_g\right)_2$$

$$0 + \left(\frac{\pi - 2}{2}\right)m_0 r^2 g = \frac{1}{2}\left(\frac{\pi}{2}m_0 r\right)v_2{}^2 + 0$$

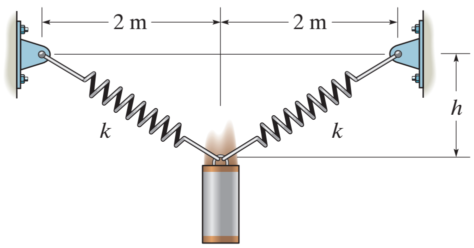
$$v_2 = \sqrt{\frac{2}{\pi}(\pi - 2)gr} \qquad \textbf{Ans.}$$

Ans:

$v_2 = \sqrt{\frac{2}{\pi}(\pi - 2)gr}$

14–95.

The cylinder has a mass of 20 kg and is released from rest when $h = 0$. Determine its speed when $h = 3$ m. Each spring has a stiffness $k = 40$ N/m and an unstretched length of 2 m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

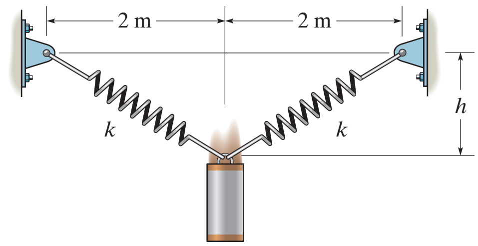
$$0 + 0 = 0 + 2\left[\frac{1}{2}(40)\left(\sqrt{3^2 + 2^2} - 2^2\right)\right] - 20(9.81)(3) + \frac{1}{2}(20)v^2$$

$v = 6.97$ m/s **Ans.**

Ans:
$v = 6.97$ m/s

***14–96.**

If the 20-kg cylinder is released from rest at $h = 0$, determine the required stiffness k of each spring so that its motion is arrested or stops when $h = 0.5$ m. Each spring has an unstretched length of 1 m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}k(2-1)^2\right] = 0 - 20(9.81)(0.5) + 2\left[\frac{1}{2}k\left(\sqrt{(2)^2 + (0.5)^2} - 1\right)^2\right]$$

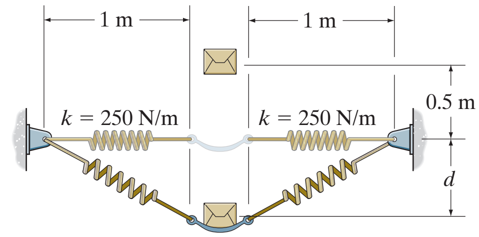
$$k = -98.1 + 1.12689\,k$$

$$k = 773 \text{ N/m}$$ **Ans.**

Ans:
$k = 773$ N/m

14–97.

A pan of negligible mass is attached to two identical springs of stiffness $k = 250$ N/m. If a 10-kg box is dropped from a height of 0.5 m above the pan, determine the maximum vertical displacement d. Initially each spring has a tension of 50 N.



SOLUTION

Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the box at positions (1) and (2) are $(V_g)_1 = mgh_1 = 10(9.81)(0) = 0$ and $(V_g)_2 = mgh_2 = 10(9.81)[-(0.5 + d)] = -98.1(0.5 + d)$. Initially, the spring stretches $s_1 = \dfrac{50}{250} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = 1 - 0.2 = 0.8$ m and the initial elastic potential of each spring is $(V_e)_1 = (2)\dfrac{1}{2}ks_1{}^2 = 2(250/2)(0.2^2) = 10$ J. When the box is at position (2), the spring stretches $s_2 = \left(\sqrt{d^2 + 1^2} - 0.8\right)$ m. The elastic potential energy of the springs when the box is at this position is

$$(V_e)_2 = (2)\frac{1}{2}ks_2{}^2 = 2(250/2)\left[\sqrt{d^2+1} - 0.8\right]^2 = 250\left(d^2 - 1.6\sqrt{d^2+1} + 1.64\right).$$

Conservation of Energy:

$$T_1 + V_1 + T_2 + V_2$$

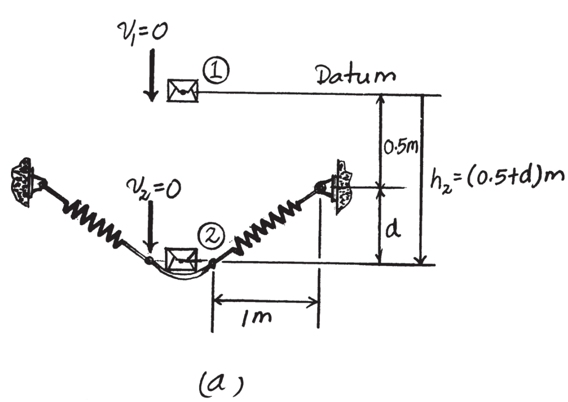
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + \left(0 + 10\right) = 0 + \left[-98.1\left(0.5 + d\right) + 250\left(d^2 - 1.6\sqrt{d^2+1} + 1.64\right)\right]$$

$$250d^2 - 98.1d - 400\sqrt{d^2+1} + 350.95 = 0$$

Solving the above equation by trial and error,

$$d = 1.34 \text{ m} \qquad \textbf{Ans.}$$

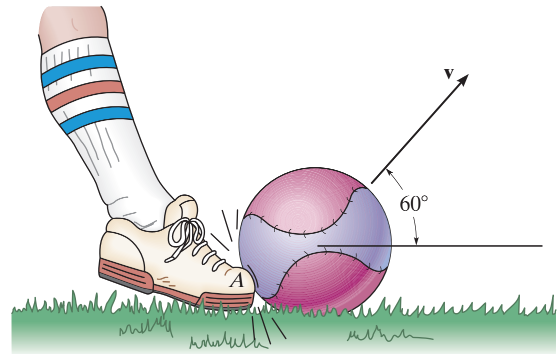


(a)

Ans:
$d = 1.34$ m

15–1.

A man kicks the 150-g ball such that it leaves the ground at an angle of 60° and strikes the ground at the same elevation a distance of 12 m away. Determine the impulse of his foot on the ball at A. Neglect the impulse caused by the ball's weight while it's being kicked.



SOLUTION

Kinematics. Consider the vertical motion of the ball where

$(s_0)_y = s_y = 0$, $(v_0)_y = v \sin 60° \uparrow$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$,

$$(+\uparrow) \quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_y t^2; \quad 0 = 0 + v \sin 60° t + \frac{1}{2}(-9.81)t^2$$

$$t(v \sin 60° - 4.905t) = 0$$

Since $t \neq 0$, then

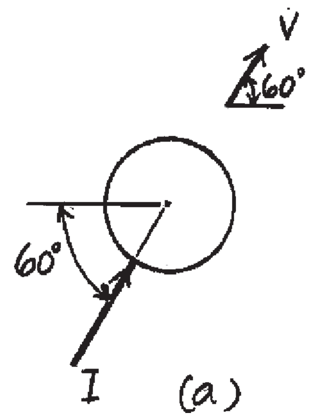
$$v \sin 60° - 4.905t = 0$$

$$t = 0.1766\, v \qquad \textbf{(1)}$$

Then, consider the horizontal motion where $(v_0)_x = v \cos 60°$, and $(s_0)_x = 0$,

$$(\overset{+}{\rightarrow}) \quad s_x = (s_0)_x + (v_0)_x t; \quad 12 = 0 + v \cos 60° t$$

$$t = \frac{24}{v} \qquad \textbf{(2)}$$



(a)

Equating Eqs. (1) and (2)

$$0.1766\, v = \frac{24}{v}$$

$$v = 11.66 \text{ m/s}$$

Principle of Impulse and Momentum.

$$(+\nearrow) \quad mv_1 + \Sigma\int_{t_1}^{t_2} F\,dt = mv_2$$

$$0 + I = 0.15\,(11.66)$$

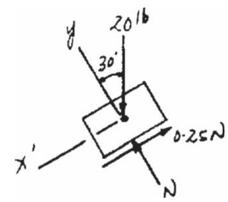
$$I = 1.749 \text{ N}\cdot\text{s} = 1.75 \text{ N}\cdot\text{s} \qquad \textbf{Ans.}$$

Ans:
$v = 1.75 \text{ N}\cdot\text{s}$

15–2.

A 20-lb block slides down a 30° inclined plane with an initial velocity of 2 ft/s. Determine the velocity of the block in 3 s if the coefficient of kinetic friction between the block and the plane is $\mu_k = 0.25$.

SOLUTION



$$(+\nwarrow) \qquad m(v_{v'}) + \Sigma\int_{t_1}^{t_2} F_{y'}dt = m(v_{y'})_2$$

$$0 + N(3) - 20\cos 30°(3) = 0 \qquad N = 17.32 \text{ lb}$$

$$(+\swarrow) \qquad m(v_{x'})_1 + \Sigma\int_{t_1}^{t_2} F_{x'}dt = m(v_{x'})_2$$

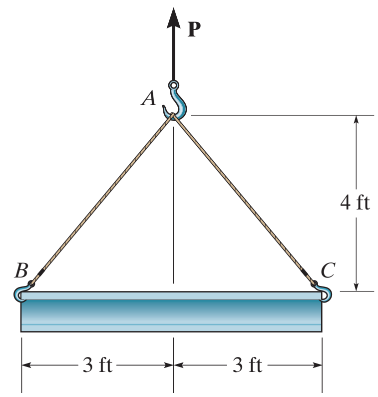
$$\frac{20}{32.2}(2) + 20\sin 30°(3) - 0.25(17.32)(3) = \frac{20}{32.2}v$$

$v = 29.4$ ft/s **Ans.**

Ans:
$v = 29.4$ ft/s

15–3.

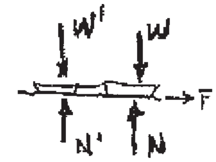
The uniform beam has a weight of 5000 lb. Determine the average tension in each of the two cables AB and AC if the beam is given an upward speed of 8 ft/s in 1.5 s starting from rest. Neglect the mass of the cables.



SOLUTION

$$25\left(\frac{1000}{3600}\right) = 6.944 \text{ m/s}$$

System:

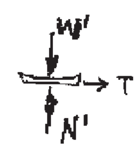


$$\left(\xrightarrow{+}\right) \quad mv_1 + \Sigma \int F\,dt = mv_2$$

$$[0 + 0] + F(35) = (50 + 75)(10^3)(6.944)$$

$$F = 24.8 \text{ kN} \qquad \textbf{Ans.}$$

Barge:



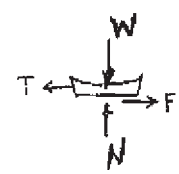
$$\left(\xrightarrow{+}\right) \quad mv_1 + \Sigma \int F\,dt = mv_2$$

$$0 + T(35) = (75)(10^3)(6.944)$$

$$T = 14.881 = 14.9 \text{ kN} \qquad \textbf{Ans.}$$

Also, using this result for T,

Tugboat:



$$\left(\xrightarrow{+}\right) \quad mv_1 + \Sigma \int F\,dt = mv_2$$

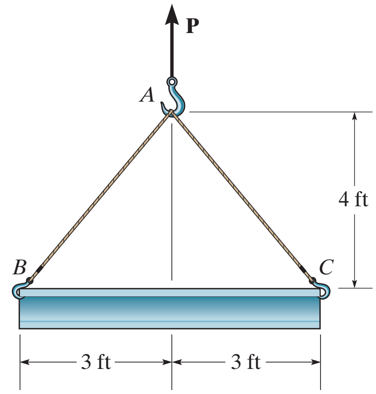
$$0 + F(35) - (14.881)(35) = (50)(10^3)(6.944)$$

$$F = 24.8 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F = 24.8$ kN

***15–4.**

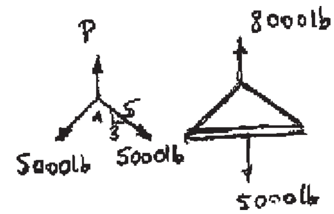
Each of the cables can sustain a maximum tension of 5000 lb. If the uniform beam has a weight of 5000 lb, determine the shortest time possible to lift the beam with a speed of 10 ft/s starting from rest.



SOLUTION

$$+\uparrow \Sigma F_y = 0; \quad P_{max} - 2\left(\frac{4}{5}\right)(5000) = 0$$

$$P_{max} = 8000 \text{ lb}$$



$$(+\uparrow) \quad mv_1 + \Sigma\int F\,dt = mv_2$$

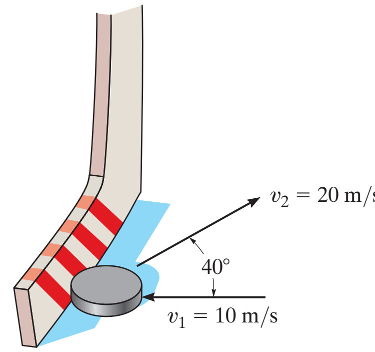
$$0 + 8000(t) - 5000(t) = \frac{5000}{32.2}(10)$$

$$t = 0.518 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 0.518$ s

15–5.

A hockey puck is traveling to the left with a velocity of $v_1 = 10$ m/s when it is struck by a hockey stick and given a velocity of $v_2 = 20$ m/s as shown. Determine the magnitude of the net impulse exerted by the hockey stick on the puck. The puck has a mass of 0.2 kg.



SOLUTION

$$v_1 = \{-10\mathbf{i}\} \text{ m/s}$$

$$v_2 = \{20 \cos 40°\mathbf{i} + 20 \sin 40°\mathbf{j}\} \text{ m/s}$$

$$\mathbf{I} = m\Delta v = (0.\,2)\,\{[20 \cos 40° - (-10)]\mathbf{i} + 20 \sin 40°\mathbf{j}\}$$

$$= \{5.0642\mathbf{i} + 2.5712\mathbf{j}\} \text{ kg} \cdot \text{m/s}$$

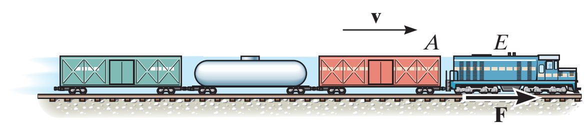
$$\mathbf{I} = \sqrt{(5.0642)^2 + (2.5712)^2}$$

$$= 5.6795 = 5.68 \text{ kg} \cdot \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$I = 5.68 \text{ N} \cdot \text{s}$

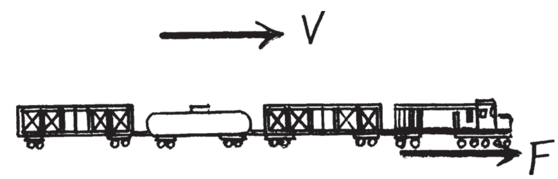
15–6.

A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force T developed at the coupling between the engine E and the first car A. The wheels of the engine provide a resultant frictional tractive force $\mathbf{F}$ which gives the train forward motion, whereas the car wheels roll freely. Also, determine F acting on the engine wheels.



SOLUTION

$(v_x)_2 = 40 \text{ km/h} = 11.11 \text{ m/s}$

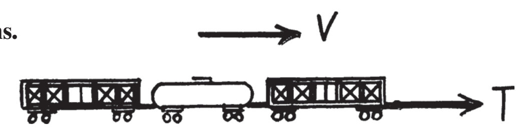


Entire train:

$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$$

$$0 + F(80) = [50 + 3(30)]\left(10^3\right)(11.11)$$

$$F = 19.4 \text{ kN}$$ **Ans.**



Three cars:

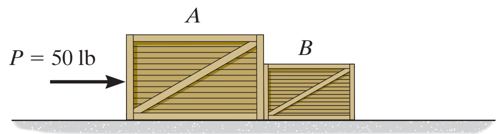
$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$$

$$0 + T(80) = 3(30)\left(10^3\right)(11.11) \qquad T = 12.5 \text{ kN}$$ **Ans.**

Ans:
$F = 19.4$ kN
$T = 12.5$ kN

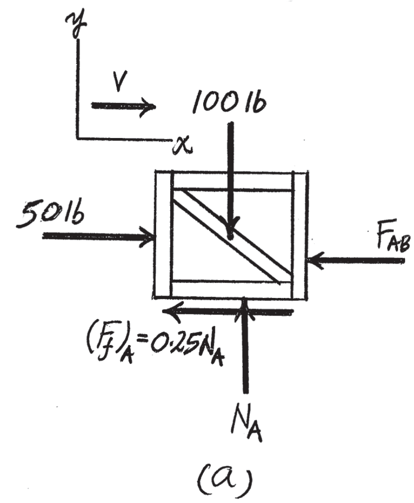
15–7.

Crates A and B weigh 100 lb and 50 lb, respectively. If they start from rest, determine their speed when $t = 5$ s. Also, find the force exerted by crate A on crate B during the motion. The coefficient of kinetic friction between the crates and the ground is $\mu_k = 0.25$.



SOLUTION

Free-Body Diagram: The free-body diagram of crates A and B are shown in Figs. a and b, respectively. The frictional force acting on each crate is $(F_f)_A = \mu_k N_A = 0.25N_A$ and $(F_f)_B = \mu_k N_B = 0.25N_B$.



Principle of Impulse and Momentum: Referring to Fig. a,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$\frac{100}{32.2}(0) + N_A(5) - 100(5) = \frac{100}{32.2}(0)$$

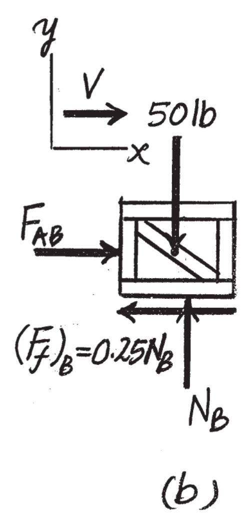
$$N_A = 100 \text{ lb}$$

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$\frac{100}{32.2}(0) + 50(5) - 0.25(100)(5) - F_{AB}(5) = \frac{100}{32.2}v$$

$$v = 40.25 - 1.61F_{AB} \qquad \textbf{(1)}$$

By considering Fig. b,



$$(+\uparrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$\frac{50}{32.2}(0) + N_B(5) - 50(5) = \frac{50}{32.2}(0)$$

$$N_B = 50 \text{ lb}$$

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$\frac{50}{32.2}(0) + F_{AB}(5) - 0.25(50)(5) = \frac{50}{32.2}v$$

$$v = 3.22F_{AB} - 40.25 \qquad \textbf{(2)}$$

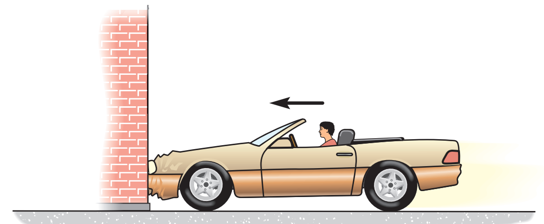
Solving Eqs. (1) and (2) yields

$$F_{AB} = 16.67 \text{ lb} = 16.7 \text{ lb} \qquad v = 13.42 \text{ ft/s} = 13.4 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$F_{AB} = 16.7$ lb
$v = 13.4$ ft/s

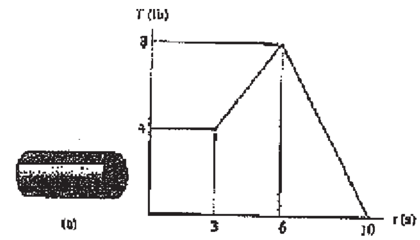
***15–8.**

The automobile has a weight of 2700 lb and is traveling forward at 4 ft/s when it crashes into the wall. If the impact occurs in 0.06 s, determine the average impulsive force acting on the car. Assume the brakes are *not applied*. If the coefficient of kinetic friction between the wheels and the pavement is $\mu_k = 0.3$, calculate the impulsive force on the wall if the brakes *were applied* during the crash. The brakes are applied to all four wheels so that all the wheels slip.



SOLUTION

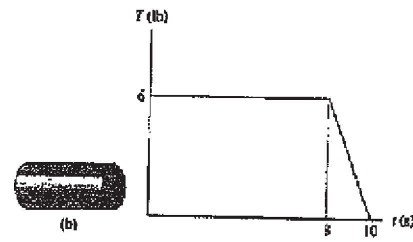
Impulse is area under curve for hole cavity.



$$I = \int F\,dt = 4(3) + \frac{1}{2}(8 + 4)(6 - 3) + \frac{1}{2}(8)(10 - 6)$$

$$= 46 \text{ lb} \cdot \text{s}$$ **Ans.**

For starred cavity:



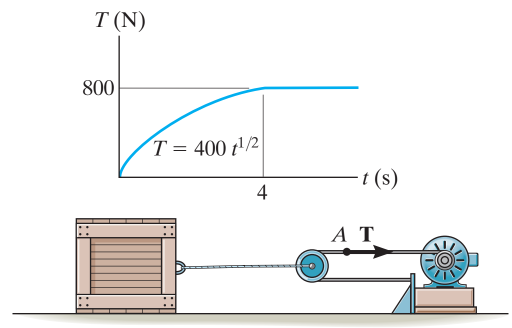
$$I = \int F\,dt = 6(8) + \frac{1}{2}(6)(10 - 8)$$

$$= 54 \text{ lb} \cdot \text{s}$$ **Ans.**

Ans:
$I = 46$ lb • s
$I = 54$ lb • s

15–9.

The 200-kg crate rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force $\boldsymbol{T}$ to its cable at A which varies as shown in the graph. Determine the speed of the crate when $t = 4$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the crate.



SOLUTION

Equilibrium. The time required to move the crate can be determined by considering the equilibrium of the crate. Since the crate is required to be on the verge of sliding, $F_f = \mu_s N = 0.5$ N. Referring to the FBD of the crate, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad N - 200(9.81) = 0 \quad N = 1962 \text{ N}$$

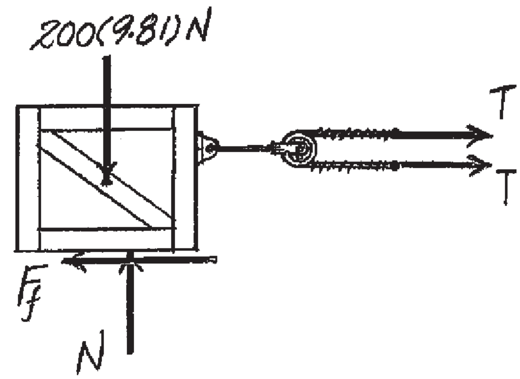
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 2\left(400t^{\frac{1}{2}}\right) - 0.5(1962) = 0 \quad t = 1.5037 \text{ s}$$

Principle of Impulse and Momentum. Since the crate is sliding, $F_f = \mu_k N = 0.4(1962) = 784.8$ N. Referring to the FBD of the crate, Fig. *a*

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$0 + 2\int_{1.5037 \text{ s}}^{4 \text{ s}} 400t^{\frac{1}{2}}\, dt - 784.8(4 - 1.5037) = 200v$$

$$v = 6.621 \text{ m/s} = 6.62 \text{ m/s} \qquad \textbf{Ans.}$$

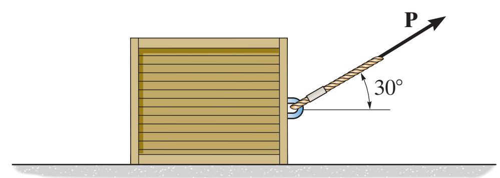


(a)

Ans:
$v = 6.62$ m/s

15–10.

The 50-kg crate is pulled by the constant force **P**. If the crate starts from rest and achieves a speed of 10 m/s in 5 s, determine the magnitude of **P**. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



SOLUTION

Impulse and Momentum Diagram: The frictional force acting on the crate is $F_f = \mu_k N = 0.2N$.

Principle of Impulse and Momentum:

$(+\uparrow) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$

$$0 + N(5) + P(5)\sin 30^\circ - 50(9.81)(5) = 0$$

$$N = 490.5 - 0.5P \qquad \textbf{(1)}$$

$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_2)_x$

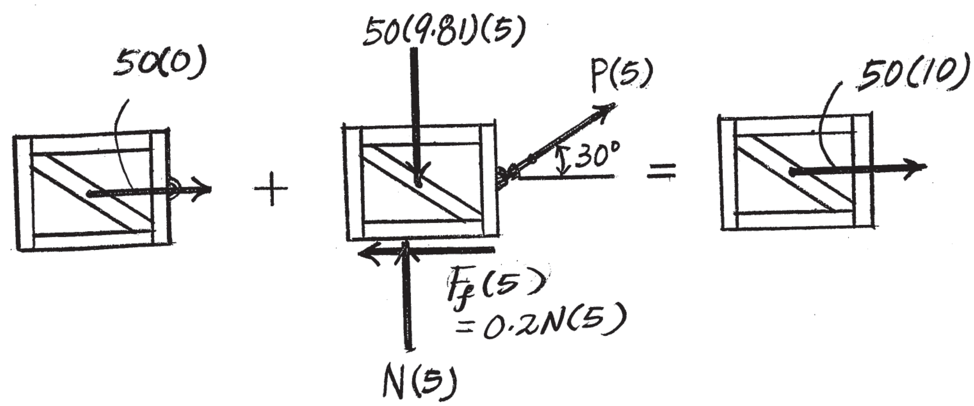
$$50(0) + P(5)\cos 30^\circ - 0.2N(5) = 50(10)$$

$$4.3301P - N = 500 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$N = 387.97 \text{ N}$$

$$P = 205 \text{ N} \qquad \textbf{Ans.}$$

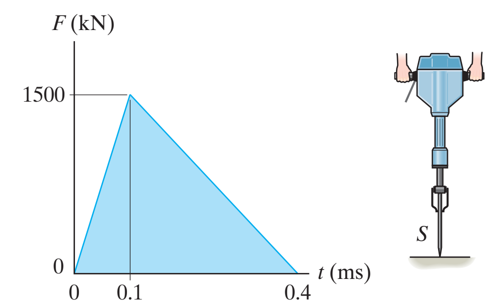


(a)

Ans:
$P = 205$ N

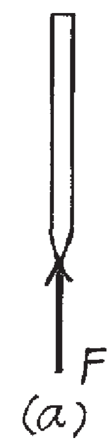
15–11.

During operation the jack hammer strikes the concrete surface with a force which is indicated in the graph. To achieve this the 2-kg spike S is fired into the surface at 90 m/s. Determine the speed of the spike just after rebounding.



SOLUTION

Principle of Impulse and Momentum. The impulse of the force $\mathbf{F}$ is equal to the area under the F–t graph. Referring to the FBD of the spike, Fig. *a*



$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$2(-90) + \frac{1}{2}\left[0.4\left(10^{-3}\right)\right]\left[1500\left(10^{3}\right)\right] = 2v$$

$$v = 60.0 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

Ans:
$v = 60.0$ m/s

***15–12.**

For a short period of time, the frictional driving force acting on the wheels of the 2.5-Mg van is $F_D = (600t^2)$ N, where t is in seconds. If the van has a speed of 20 km/h when $t = 0$, determine its speed when $t = 5$ s.

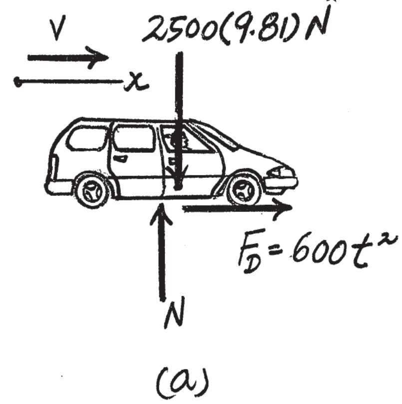
SOLUTION

Principle of Impulse and Momentum: The initial speed of the van is $v_1 = \left[20(10^3)\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 5.556$ m/s. Referring to the free-body diagram of the van shown in Fig. *a*,

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$2500(5.556) + \int_0^{5\text{ s}} 600t^2 dt = 2500\, v_2$$

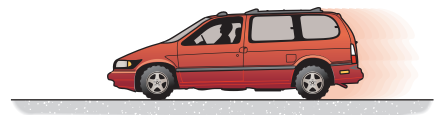
$$v_2 = 15.6 \text{ m/s}$$ **Ans.**



Ans:
$v_2 = 15.6$ m/s

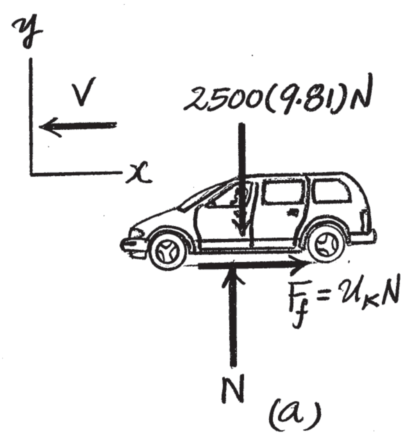
15–13.

The 2.5-Mg van is traveling with a speed of 100 km/h when the brakes are applied and all four wheels lock. If the speed decreases to 40 km/h in 5 s, determine the coefficient of kinetic friction between the tires and the road.



SOLUTION

Free-Body Diagram: The free-body diagram of the van is shown in Fig. *a*. The frictional force is $F_f = \mu_k N$ since all the wheels of the van are locked and will cause the van to slide.



Principle of Impulse and Momentum: The initial and final speeds of the van are $v_1 = \left[100(10^3)\,\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 27.78\text{ m/s}$ and $v_2 = \left[40(10^3)\,\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 11.11\text{ m/s}$. Referring to Fig. *a*,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_2)_y$$

$$2500(0) + N(5) - 2500(9.81)(5) = 2500(0)$$

$$N = 24\,525\text{ N}$$

$$(\overset{+}{\leftarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_2)_x$$

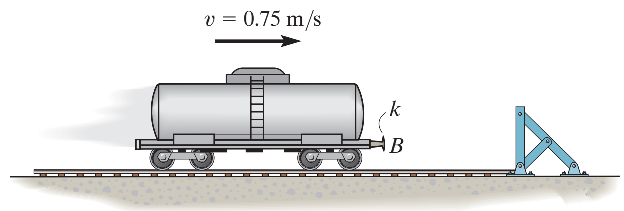
$$2500(27.78) + [-\mu_k(24525)(5)] = 2500(11.1)$$

$$\mu_k = 0.340 \qquad \textbf{Ans.}$$

Ans:
$\mu_k = 0.340$

15–14.

A tankcar has a mass of 20 Mg and is freely rolling to the right with a speed of 0.75 m/s. If it strikes the barrier, determine the horizontal impulse needed to stop the car if the spring in the bumper B has a stiffness (a) $k \to \infty$ (bumper is rigid), and (b) $k = 15$ kN/m.



SOLUTION

a) b) $\quad (\overset{+}{\rightarrow}) \quad mv_1 + \Sigma \int F\,dt = mv_2$

$$20(10^3)(0.75) - \int F\,dt = 0$$

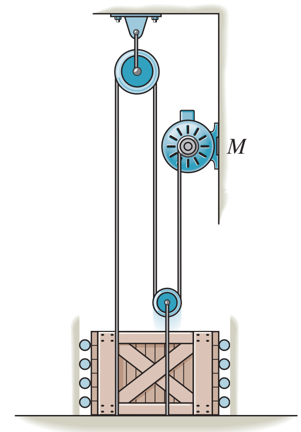
$$\int F\,dt = 15 \text{ kN} \cdot \text{s} \qquad \textbf{Ans.}$$

The impulse is the same for both cases. For the spring having a stiffness $k = 15$ kN/m, the impulse is applied over a longer period of time than for $k \to \infty$.

Ans:
$I = 15$ kN · s in both cases.

15–15.

The motor, M, pulls on the cable with a force $F = (10t^2 + 300)$ N, where t is in seconds. If the 100 kg crate is originally at rest at $t = 0$, determine its speed when $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



SOLUTION

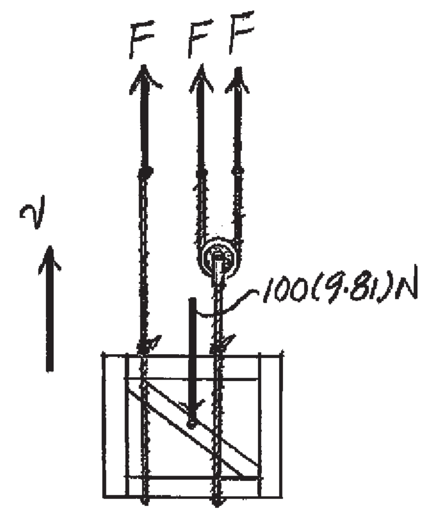
Principle of Impulse and Momentum. The crate will only move when $3(10t^2 + 300) = 100(9.81)$. Thus, this instant is $t = 1.6432$ s. Referring to the FBD of the crate, Fig. *a*,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$0 + \int_{1.6432\text{ s}}^{4\text{ s}} 3(10t^2 + 300)\,dt - 100(9.81)(4 - 1.6432) = 100v$$

$$3\left(\frac{10t^3}{3} + 300t\right)\Bigg|_{1.6432\text{ s}}^{4\text{ s}} - 2312.05 = 100v$$

$$v = 4.047\text{ m/s} = 4.05\text{ m/s}\uparrow \qquad \textbf{Ans.}$$

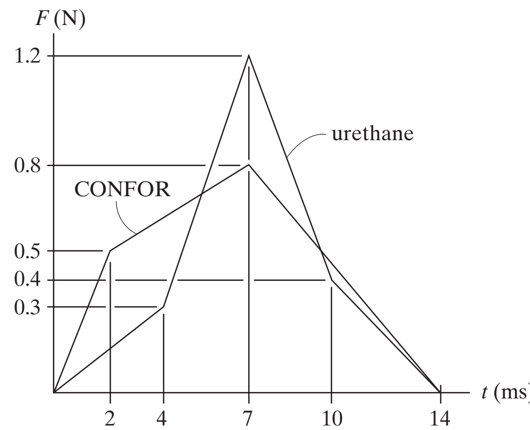


(a)

Ans:
$v = 4.05$ m/s

***15–16.**

The choice of a seating material for moving vehicles depends upon its ability to resist shock and vibration. From the data shown in the graphs, determine the impulses created by a falling weight onto a sample of urethane foam and CONFOR foam.



SOLUTION

CONFOR foam:

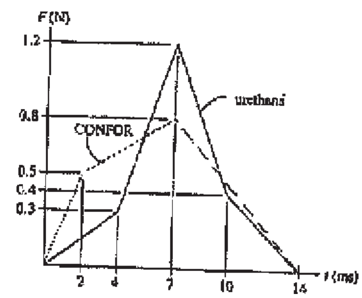
$$I_c = \int F\,dt = \left[\frac{1}{2}(2)(0.5) + \frac{1}{2}(0.5 + 0.8)(7 - 2) + \frac{1}{2}(0.8)(14 - 7)\right](10^{-3})$$

$$= 6.55\ \text{N}\cdot\text{ms} \qquad \textbf{Ans.}$$

Urethane foam:

$$I_v = \int F\,dt = \left[\frac{1}{2}(4)(0.3) + \frac{1}{2}(1.2 + 0.3)(7 - 4) + \frac{1}{2}(1.2 + 0.4)(10 - 7) + \frac{1}{2}(14 - 10)(0.4)\right](10^{-3})$$

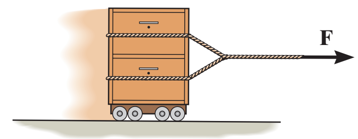
$$= 6.05\ \text{N}\cdot\text{ms} \qquad \textbf{Ans.}$$

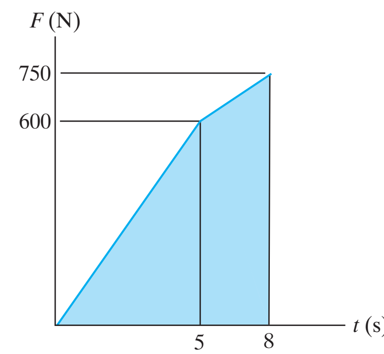


Ans:
$I_c = 6.55\ \text{N}\cdot\text{ms}$
$I_v = 6.05\ \text{N}\cdot\text{ms}$

15–17.

The towing force acting on the 400-kg safe varies as shown on the graph. Determine its speed, starting from rest, when $t = 8$ s. How far has it traveled during this time?





SOLUTION

Principle of Impulse and Momentum. The FBD of the safe is shown in Fig. *a*. For $0 \leq t < 5$ s, $F = \dfrac{600}{5}t = 120t$.

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + \int_0^t 120t\,dt = 400v$$

$$v = \{0.15t^2\}\ \text{m/s}$$

At $t = 5$ s,

$$v = 0.15(5^2) = 3.75\ \text{m/s}$$

For $5\text{ s} < t \leq 8$ s, $\dfrac{F - 600}{t - 5} = \dfrac{750 - 600}{8 - 5}$, $F = 50t + 350$. Here,

$(v_x)_1 = 3.75$ m/s and $t_1 = 5$ s.

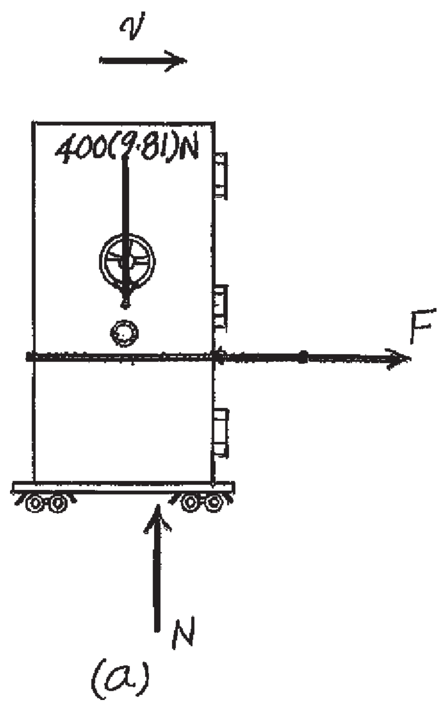
$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$400(3.75) + \int_{5\text{ s}}^{t} (50t + 350)\,dt = 400v$$

$$v = \{0.0625t^2 + 0.875t - 2.1875\}\ \text{m/s}$$

At $t = 8$ s,

$$v = 0.0625(8^2) + 0.875(8) - 2.1875 = 8.8125\ \text{m/s} = 8.81\ \text{m/s} \quad \textbf{Ans.}$$



15–17. Continued

Kinematics. The displacement of the safe can be determined by integrating $ds = v\,dt$. For $0 \leq t < 5$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 0.15t^2\,dt$$

$$s = \{0.05t^3\}\text{ m}$$

At $t = 5$ s,

$$s = 0.05\left(5^3\right) = 6.25\text{ m}$$

For $5\text{ s} < t \leq 8$ s, the initial condition is $s = 6.25$ m at $t = 5$ s.

$$\int_{6.25\text{ m}}^s ds = \int_{5\text{ s}}^t \left(0.0625t^2 + 0.875\,t - 2.1875\right)dt$$

$$s - 6.25 = \left(0.02083t^3 + 0.4375t^2 - 2.1875t\right)\Big|_{5\text{ s}}^{t}$$

$$s = \{0.02083t^3 + 0.4375t^2 - 2.1875t + 3.6458\}\text{ m}$$

At $t = 8$ s,

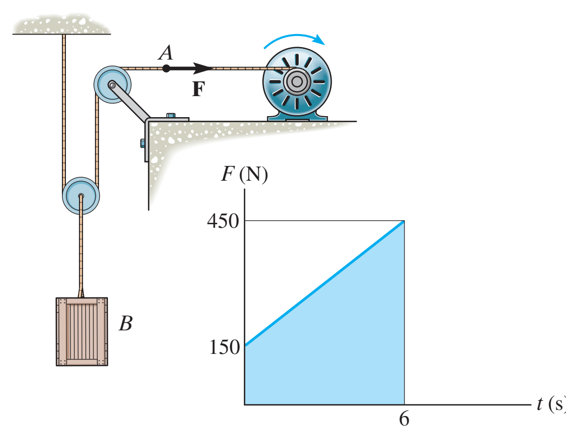
$$s = 0.02083\left(8^3\right) + 0.4375\left(8^2\right) - 2.1875(8) + 3.6458$$

$$= 24.8125\text{ m} = 24.8\text{ m} \qquad \textbf{Ans.}$$

Ans:
$v = 8.81$ m/s
$s = 24.8$ m

15–18.

The motor exerts a force $\boldsymbol{F}$ on the 40-kg crate as shown in the graph. Determine the speed of the crate when $t = 3$ s and when $t = 6$ s. When $t = 0$, the crate is moving downward at 10 m/s.



SOLUTION

Principle of Impulse and Momentum. The impulse of force $\boldsymbol{F}$ is equal to the area under the F–t graph. At $t = 3$ s, $\dfrac{F - 150}{3 - 0} = \dfrac{450 - 150}{6 - 0}$ $F = 300$ N. Referring to the FBD of the crate, Fig. *a*

$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$40(-10) + 2\left[\frac{1}{2}(150 + 300)(3)\right] - 40(9.81)(3) = 40v$$

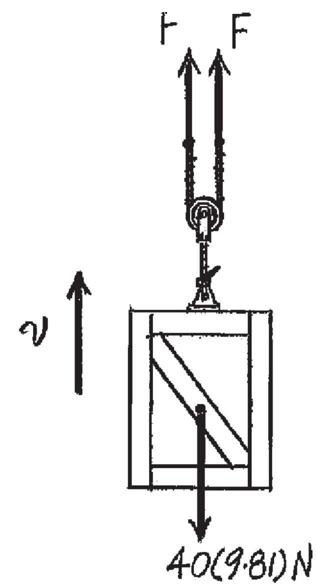
$$v = -5.68 \text{ m/s} = 5.68 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

At $t = 6$ s,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$40(-10) + 2\left[\frac{1}{2}(450 + 150)(6)\right] - 40(9.81)(6) = 40v$$

$$v = 21.14 \text{ m/s} = 21.1 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

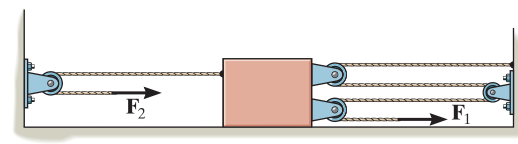


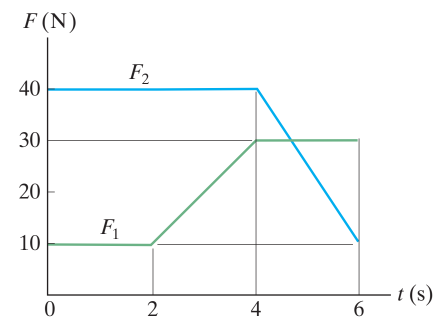
(a)

Ans:
$v|_{t=3\text{ s}} = 5.68 \text{ m/s} \downarrow$
$v|_{t=6\text{ s}} = 21.1 \text{ m/s} \uparrow$

15–19.

The 30-kg slider block is moving to the left with a speed of 5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.





SOLUTION

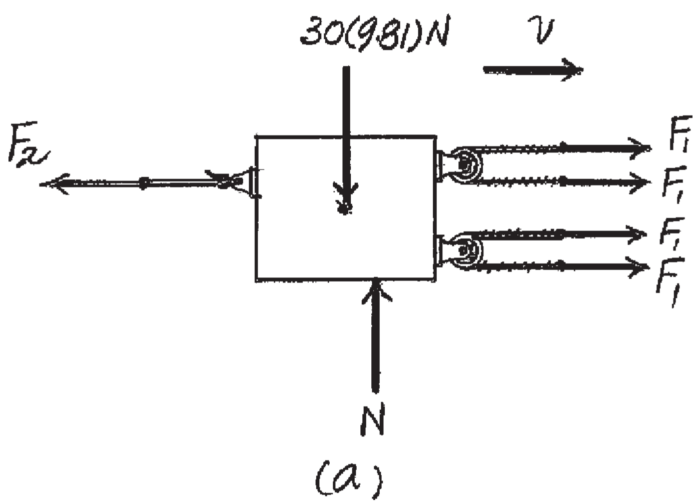
Principle of Impulse and Momentum. The impulses produced by $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal to the area under the respective F–t graph. Referring to the FBD of the block Fig. a,

$$(\overset{+}{\rightarrow})\quad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dx = m(v_x)_2$$

$$-30(5) + 4\left[10(2) + \frac{1}{2}(10+30)(4-2) + 30(6-4)\right]$$

$$+\left[-40(4) - \frac{1}{2}(10+40)(6-4)\right] = 30v$$

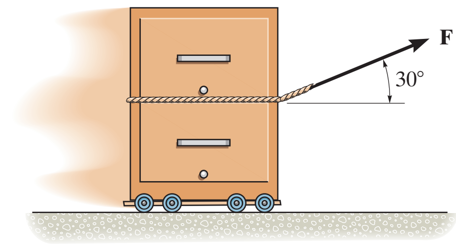
$$v = 4.00 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



Ans:
$v = 4.00$ m/s

***15–20.**

The 200-lb cabinet is subjected to the force $F = 20(t + 1)$ lb where t is in seconds. If the cabinet is initially moving to the left with a velocity of 20 ft/s, determine its speed when $t = 5$ s. Neglect the size of the rollers.



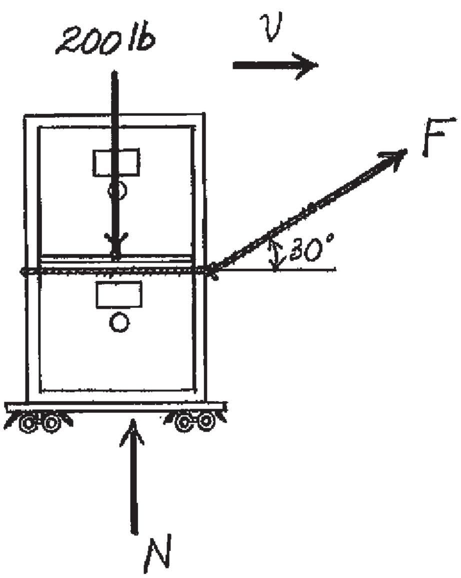
SOLUTION

Principle of Impulse and Momentum. Referring to the FBD of the cabinet, Fig. *a*

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$\frac{200}{32.2}(-20) + 20\cos 30^\circ \int_0^{5\text{ s}} (t + 1)\, dt = \frac{200}{32.2} v$$

$$v = 28.80 \text{ ft/s} = 28.8 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

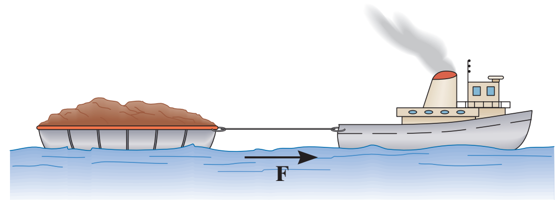


(a)

Ans:
$v = 28.8$ ft/s $\rightarrow$

15–21.

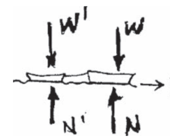
If it takes 35 s for the 50-Mg tugboat to increase its speed uniformly to 25 km/h, starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force **F** which gives the tugboat forward motion, whereas the barge moves freely. Also, determine F acting on the tugboat. The barge has a mass of 75 Mg.



SOLUTION

$$25\left(\frac{1000}{3600}\right) = 6.944 \text{ m/s}$$

System:



$$(\overset{+}{\rightarrow}) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

$$[0+0] + F(35) = (50 + 75)(10^3)(6.944)$$

$$F = 24.8 \text{ kN}$$ **Ans.**

Barge:

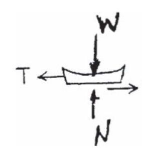
$$(\overset{+}{\rightarrow}) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

$$0 + T(35) = (75)(10^3)(6.944)$$

$$T = 14.881 = 14.9 \text{ kN}$$ **Ans.**

Also, using this result for T,

Tugboat:



$$(\overset{+}{\rightarrow}) \qquad mv_1 + \Sigma\int F\,dt = mv_2$$

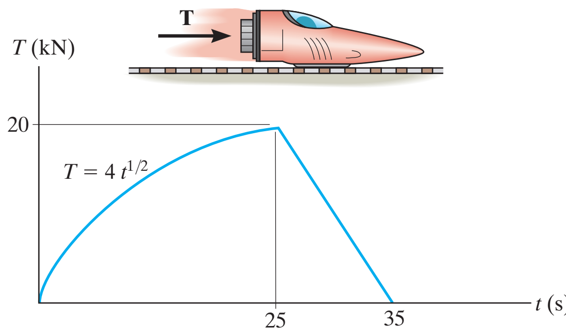
$$0 + F(35) - (14.881)(35) = (50)(10^3)(6.944)$$

$$F = 24.8 \text{ kN}$$ **Ans.**

Ans:
$T = 14.9$ kN
$F = 24.8$ kN

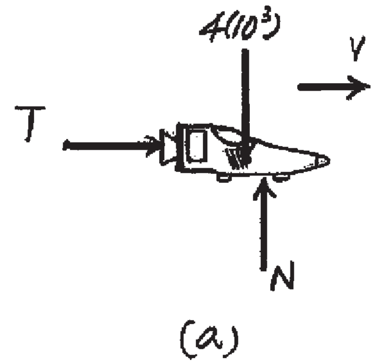
15–22.

The thrust on the 4-Mg rocket sled is shown in the graph. Determine the sleds maximum velocity and the distance the sled travels when $t = 35$ s. Neglect friction.



SOLUTION

Principle of Impulse And Momentum. The FBD of the rocket sled is shown in Fig. *a*. For $0 \leq t < 25$ s,



$$(\xrightarrow{+}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$0 + \int_0^t 4(10^3)t^{\frac{1}{2}} dt = 4(10^3)\, v$$

$$4(10^3)\left(\frac{2}{3}t^{\frac{3}{2}}\right)\Big|_0^t = 4(10^3)\, v$$

$$v = \left\{\frac{2}{3}t^{\frac{3}{2}}\right\} \text{ m/s}$$

At $t = 25$ s,

$$v = \frac{2}{3}(25)^{\frac{3}{2}} = 83.33 \text{ m/s}$$

For $25 \text{ s} < t < 35$ s, $\dfrac{T - 0}{t - 35} = \dfrac{20(10^3) - 0}{25 - 35}$ or $T = 2(10^3)(35 - t)$.

Here, $(v_x)_1 = 83.33$ m/s and $t_1 = 25$ s.

$$(\xrightarrow{+}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$4(10^3)(83.33) + \int_{25\text{ s}}^t 2(10^3)(35 - t)dt = 4(10^3)\, v$$

$$v = \{-0.25t^2 + 17.5t - 197.9167\} \text{ m/s}$$

The maximum velocity occurs at $t = 35$ s, Thus,

$$v_{max} = -0.25(35^2) + 17.5(35) - 197.9167$$

$$= 108.33 \text{ m/s} = 108 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{max} = 108$ m/s

15–22. Continued

Kinematics. The displacement of the sled can be determined by integrating $ds = vdt$. For $0 \leq t < 25$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t \frac{2}{3} t^{\frac{3}{2}} dt$$

$$s\Big|_0^s = \frac{2}{3}\left(\frac{2}{5}\right) t^{\frac{5}{2}}\Big|_0^t$$

$$s = \left\{\frac{4}{15} t^{\frac{5}{2}}\right\} \text{ m}$$

At $t = 25$ s,

$$s = \frac{4}{15}(25)^{\frac{5}{2}} = 833.33 \text{ m}$$

For $25 < t \leq 35$ s, the initial condition is $s = 833.33$ at $t = 25$ s.

$$\int_{833.33 \text{ m}}^{S} ds = \int_{25 \text{ s}}^{t} (-0.25t^2 + 17.5t - 197.9167)\, dt$$

$$s\Big|_{833.33 \text{ m}}^{S} = \left(-0.08333t^3 + 8.75t^2 - 197.9167t\right)\Big|_{25 \text{ s}}^{t}$$

$$s = \{-0.08333t^3 + 8.75t^2 - 197.9167t + 1614.58\} \text{ m}$$

At $t = 35$ s,

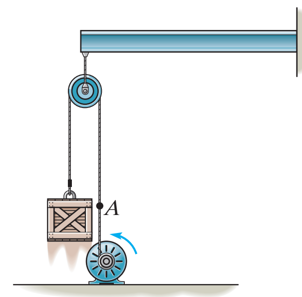
$$s = -0.08333(35^3) + 8.75(35^2) - 197.9167(35) + 1614.58$$

$$= 1833.33 \text{ m} = 1833 \text{ m}$$ **Ans.**

Ans:
$s = 1.83$ km

15–23.

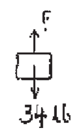
The motor pulls on the cable at A with a force $F = (30 + t^2)$ lb, where t is in seconds. If the 34-lb crate is originally on the ground at $t = 0$, determine its speed in $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



SOLUTION

$30 + t^2 = 34$

$t = 2$ s for crate to start moving



$$(+\uparrow) \quad mv_1 + \Sigma \int F\,dt = mv_2$$

$$0 + \int_2^4 (30 + t^2)dt - 34(4 - 2) = \frac{34}{32.2}v_2$$

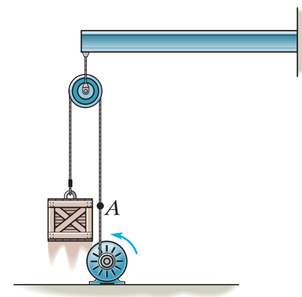
$$\left[30t + \frac{1}{3}t^3\right]_2^4 - 68 = \frac{34}{32.2}v_2$$

$$v_2 = 10.1 \text{ ft/s}$$ **Ans.**

Ans:
$v = 10.1$ ft/s

***15–24.**

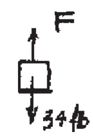
The motor pulls on the cable at A with a force $F = (e^{2t})$ lb, where t is in seconds. If the 34-lb crate is originally at rest on the ground at $t = 0$, determine the crate's velocity when $t = 2$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



SOLUTION

$F = e^{2t} = 34$

$t = 1.7632$ s for crate to start moving



$$(+\uparrow) \quad mv_1 + \Sigma \int F dt = mv_2$$

$$0 \cdot \int_{1.7632}^{2} e^{2t} dt - 34(2 - 1.7632) = \frac{34}{32.2} v_2$$

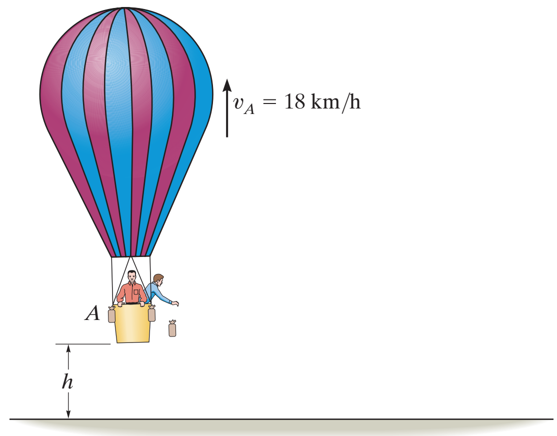
$$\frac{1}{2} e^{2t} \int_{1.7632}^{2} - 8.0519 = 1.0559\, v_2$$

$v_2 = 2.13$ m/s **Ans.**

Ans:
$v_2 = 2.13$ m/s

15–25.

The balloon has a total mass of 400 kg including the passengers and ballast. The balloon is rising at a constant velocity of 18 km/h when $h = 10$ m. If the man drops the 40-kg sand bag, determine the velocity of the balloon when the bag strikes the ground. Neglect air resistance.



SOLUTION

Kinematic. When the sand bag is dropped, it will have an upward velocity of $v_0 = \left(18 \dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1000 \text{ m}}{1 \text{ km}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right) = 5 \text{ m/s} \uparrow$. When the sand bag strikes the ground $s = 10 \text{ m} \downarrow$. The time taken for the sand bag to strike the ground can be determined from

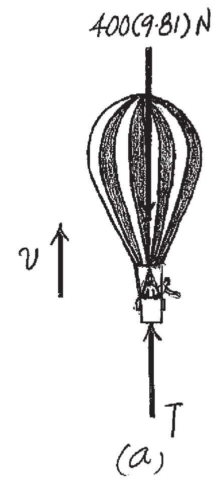
$$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2;$$

$$-10 = 0 + 5t + \frac{1}{2}(-9.81t^2)$$

$$4.905t^2 - 5t - 10 = 0$$

Solve for the positive root,

$$t = 2.0258 \text{ s}$$



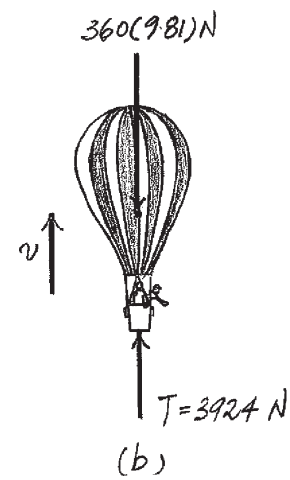
Principle of Impulse and Momentum. The FBD of the ballon when the ballon is rising with the constant velocity of 5 m/s is shown in Fig. *a*

$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$400(5) + T(t) - 400(9.81)t = 400(5)$$

$$T = 3924 \text{ N}$$

When the sand bag is dropped, the thrust $T = 3924$ N is still maintained as shown in the FBD, Fig. *b*.



$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

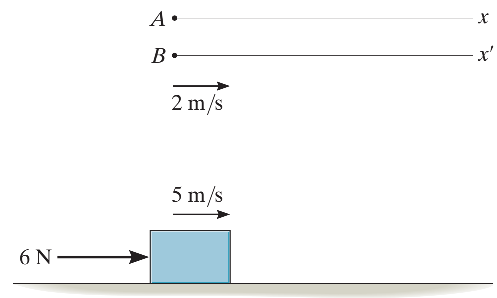
$$360(5) + 3924(2.0258) - 360(9.81)(2.0258) = 360v$$

$$v = 7.208 \text{ m/s} = 7.21 \text{ m/s} \uparrow$$ **Ans.**

Ans:
$v = 7.21 \text{ m/s} \uparrow$

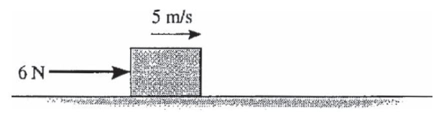
15–26.

As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which slides along the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block in 4 s if it has an initial speed of 5 m/s measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis that moves at a constant velocity of 2 m/s relative to A.



SOLUTION

Observer A:



$$(\overset{+}{\rightarrow}) \qquad m\,v_1 + \sum \int F\,dt = m\,v_2$$

$$10(5) + 6(4) = 10v$$

$$v = 7.40 \text{ m/s}$$ **Ans.**

Observer B:

$$(\overset{+}{\rightarrow}) \qquad m\,v_1 + \sum \int F\,dt = m\,v_2$$

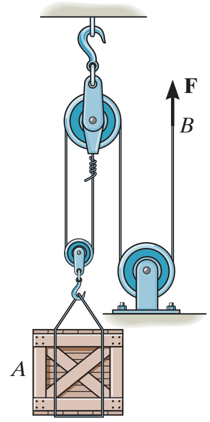
$$10(3) + 6(4) = 10v$$

$$v = 5.40 \text{ m/s}$$ **Ans.**

Ans:
Observer A: $v = 7.40$ m/s
Observer B: $v = 5.40$ m/s

15–27.

The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where t is in seconds. Determine the speed of the crate when $t = 3$ s, starting from rest.



SOLUTION

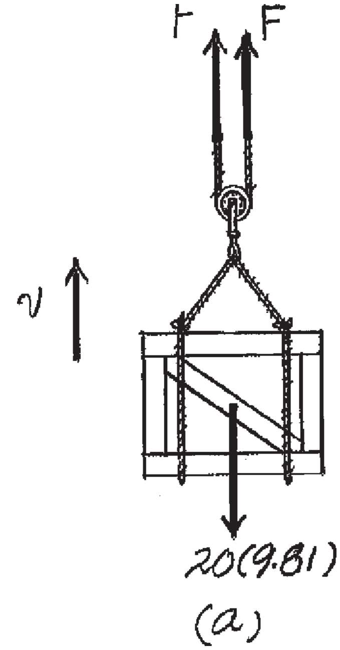
Principle of Impulse and Momentum. At $t = 0$, $F = 100$ N. Since at this instant, $2F = 200\text{ N} > W = 20(9.81) = 196.2$ N, the crate will move the instant force **F** is applied. Referring to the FBD of the crate, Fig. a,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + 2\int_0^{3\text{ s}} (100 + 5t^2)dt - 20(9.81)(3) = 20v$$

$$2\left(100t + \frac{5}{3}t^3\right)\Big|_0^{3\text{ s}} - 588.6 = 20v$$

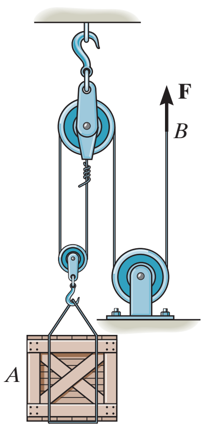
$$v = 5.07 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 5.07$ m/s

***15–28.**

The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where t is in seconds. Determine how high the crate has moved upward when $t = 3$ s, starting from rest.



SOLUTION

Principle of Impulse and Momentum. At $t = 0$, $F = 100$ N. Since at this instant, $2F = 200\text{ N} > W = 20(9.81) = 196.2$ N, the crate will move the instant force F is applied. Referring to the FBD of the crate, Fig. *a*

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + 2\int_0^t (100 + 5t^2)dt - 20(9.81)t = 20v$$

$$2\left(100t + \frac{5}{3}t^3\right)\Big|_0^t - 196.2t = 20v$$

$$v = \{0.1667t^3 + 0.19t\}\text{ m/s}$$

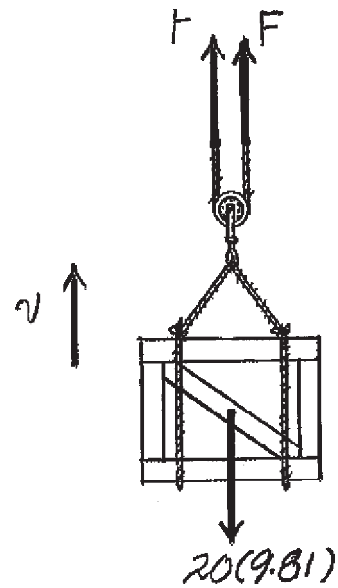
Kinematics. The displacement of the crate can be determined by integrating $ds = v\,dt$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t (0.1667t^3 + 0.19t)\,dt$$

$$s = \{0.04167t^4 + 0.095t^2\}\text{ m}$$

At $t = 3$ s,

$$s = 0.04167(3^4) + 0.095(3^2) = 4.23\text{ m} \qquad \textbf{Ans.}$$

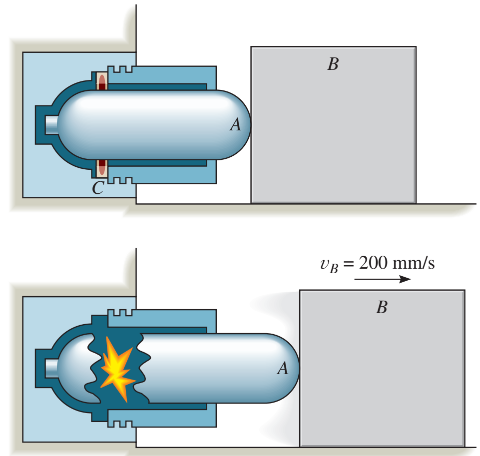


(a)

Ans:
$s = 4.23$ m

15–29.

In case of emergency, the gas actuator is used to move a 75-kg block B by exploding a charge C near a pressurized cylinder of negligible mass. As a result of the explosion, the cylinder fractures and the released gas forces the front part of the cylinder, A, to move B forward, giving it a speed of 200 mm/s in 0.4 s. If the coefficient of kinetic friction between B and the floor is $\mu_k = 0.5$, determine the impulse that the actuator imparts to B.



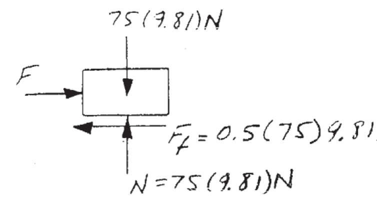
SOLUTION

Principle of Linear Impulse and Momentum: In order for the package to rest on top of the belt, it has to travel at the same speed as the belt. Applying Eq. 15–4, we have

$$m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$(+\uparrow)$ $$6(0) + Nt - 6(9.81)\,t = 6(0)$$

$$N = 58.86\text{ N}$$



$$m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$(\xrightarrow{+})$ $$6(3) + [-0.2(58.86)t] = 6(1)$$

$$t = 1.02\text{ s}$$ **Ans.**

$(\xrightarrow{+})$ $$m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

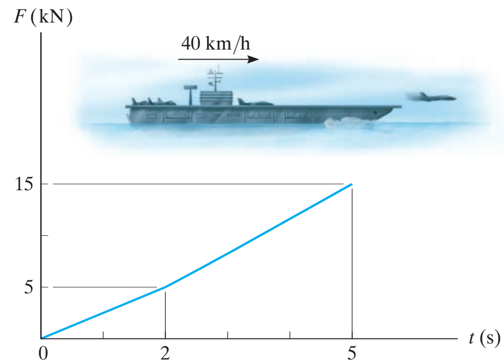
$$0 + \int F\,dt - (0.5)(9.81)(75)(0.4) = 75(0.2)$$

$$\int F\,dt = 162\text{ N}\cdot\text{s}$$ **Ans.**

Ans:
$t = 1.02$ s
$I = 162$ N·s

15–30.

A jet plane having a mass of 7 Mg takes off from an aircraft carrier such that the engine thrust varies as shown by the graph. If the carrier is traveling forward with a speed of 40 km/h, determine the plane's airspeed after 5 s.



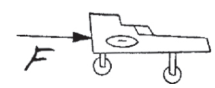
SOLUTION

The impulse exerted on the plane is equal to the area under the graph.

$v_1 = 40 \text{ km/h} = 11.11 \text{ m/s}$

$$(\xrightarrow{+}) \qquad m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$$

$$(7)(10^3)(11.11) - \frac{1}{2}(2)(5)(10^3) + \frac{1}{2}(15 + 5)(5 - 2)(10^3) = 7(10^3)_{v_2}$$

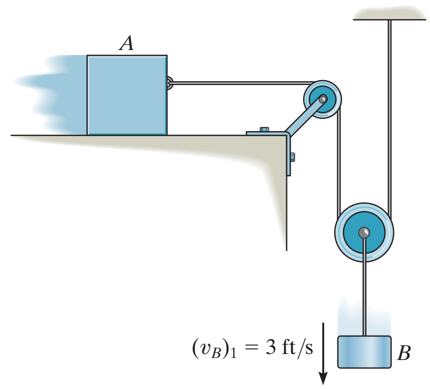


$v_2 = 16.1 \text{ m/s}$ **Ans.**

Ans:
$v = 16.1 \text{ m/s}$

15–31.

Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity $(v_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of A when $t = 1$ s. Assume that the horizontal plane is smooth. Neglect the mass of the pulleys and cords.



SOLUTION

$s_A + 2s_B = l$

$v_A = -2v_B$

$(\overset{+}{\leftarrow})$ $\quad mv_1 + \Sigma \int F\,dt = mv_2$

$$-\frac{10}{32.2}(2)(3) - T(1) = \frac{10}{32.2}(v_A)_2$$

$(+\downarrow)$ $\quad mv_1 + \Sigma \int F\,dt = mv_2$

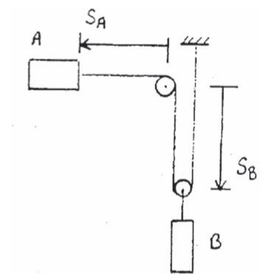
$$\frac{3}{32.2}(3) + 3(1) - 2T(1) = \frac{3}{32.2}\left(-\frac{(v_A)_2}{2}\right)$$

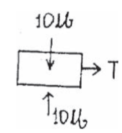
$-\,32.2T - 10(v_A)_2 = 60$

$-\,64.4T + 1.5(v_A)_2 = -105.6$

$T = 1.40$ lb

$(v_A)_2 = -10.5$ ft/s $= 10.5$ ft/s $\rightarrow$ **Ans.**

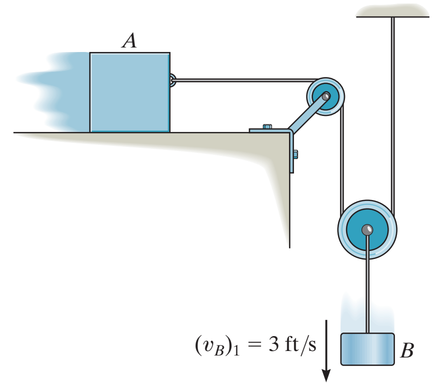




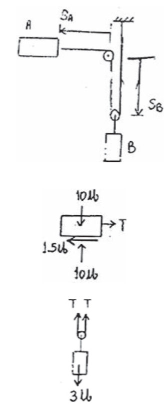
Ans:
$(v_A)_2 = 10.5$ ft/s $\rightarrow$

***15–32.**

Block A weighs 10 lb and block B weighs 3 lb. If B is moving downward with a velocity $(v_B)_1 = 3$ ft/s at $t = 0$, determine the velocity of A when $t = 1$ s. The coefficient of kinetic friction between the horizontal plane and block A is $\mu_A = 0.15$.



SOLUTION



$s_A + 2s_B = l$

$v_A = -2v_B$

$(\overset{+}{\leftarrow})\qquad mv_1 + \Sigma\int F\,dt = mv_2$

$$-\frac{10}{32.2}(2)(3) - T(1) + 0.15(10) = \frac{10}{32.2}(v_A)_2$$

$(+\downarrow)\qquad mv_1 + \Sigma\int F\,dt = mv_2$

$$\frac{3}{32.2}(3) - 3(1) - 2T(1) = \frac{3}{32.2}\left(\frac{(v_A)_2}{2}\right)$$

$$-32.2T - 10(v_A)_2 = 11.70$$

$$-64.4T + 1.5(v_A)_2 = -105.6$$

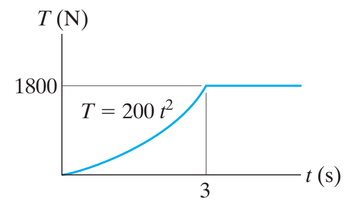
$$T = 1.50 \text{ lb}$$

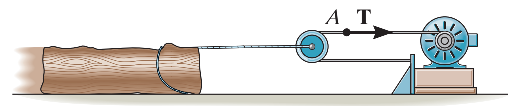
$$(v_A)_2 = -6.00 \text{ ft/s} = 6.00 \text{ ft/s} \rightarrow$$ **Ans.**

Ans:
$(v_A)_2 = 6.00$ ft/s $\rightarrow$

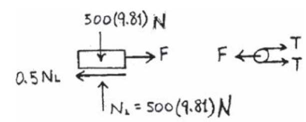
15–33.

The log has a mass of 500 kg and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the log when $t = 5$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.





SOLUTION



$\xrightarrow{+}\Sigma F_x = 0;\qquad F - 0.5(500)(9.81) = 0$

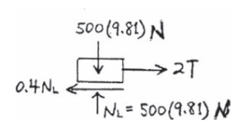
$F = 2452.5\text{ N}$

Thus,

$2T = F$

$2(200t^2) = 2452.5$

$t = 2.476$ s to start log moving



$(\xrightarrow{+})\qquad m\,v_1 + \Sigma\int F\,dt = m\,v_2$

$$0 + 2\int_{2.476}^{3} 200t^2\,dt + 2(1800)(5-3) - 0.4(500)(9.81)(5-2.476) = 500v_2$$

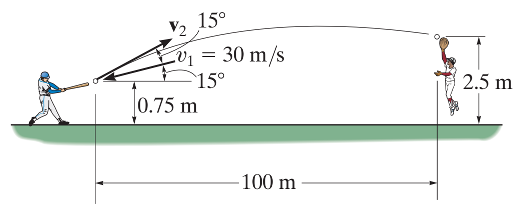
$$400\left(\frac{t^3}{3}\right)\Bigg|_{2.476}^{3} + 2247.91 = 500v_2$$

$v_2 = 7.65\text{ m/s}$ **Ans.**

Ans:
$v = 7.65\text{ m/s}$

15–34.

The 0.15-kg baseball has a speed of $v = 30$ m/s just before it is struck by the bat. It then travels along the trajectory shown before the outfielder catches it. Determine the magnitude of the average impulsive force imparted to the ball if it is in contact with the bat for 0.75 ms.



SOLUTION

$$(\overset{+}{\rightarrow}) \qquad m_A\,(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

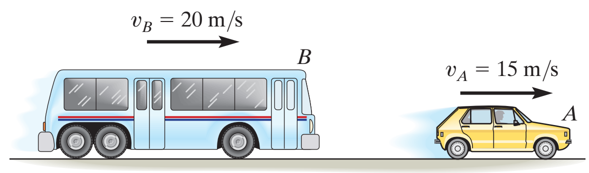
$$\frac{4500}{32.2}(3) - \frac{3000}{32.2}(6) = \frac{7500}{32.2}v_2$$

$$v_2 = -0.600 \text{ ft/s} = 0.600 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Ans:
$v = 0.6$ ft/s $\leftarrow$

15–35.

The 5-Mg bus B is traveling to the right at 20 m/s. Meanwhile a 2-Mg car A is traveling at 15 m/s to the right. If the vehicles crash and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.



SOLUTION

Conservation of Linear Momentum.

$$(\xrightarrow{+}) \qquad m_A v_A + m_B v_B = (m_A + m_B)v$$

$$\left[5(10^3)\right](20) + \left[2(10^3)\right](15) = \left[5(10^3) + 2(10^3)\right]v$$

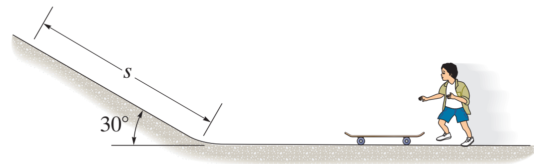
$$v = 18.57 \text{ m/s} = 18.6 \text{ m/s} \rightarrow$$ **Ans.**

Ans:

$v = 18.6 \text{ m/s} \rightarrow$

***15–36.**

The 50-kg boy jumps on the 5-kg skateboard with a horizontal velocity of 5 m/s. Determine the distance s the boy reaches up the inclined plane before momentarily coming to rest. Neglect the skateboard's rolling resistance.



SOLUTION

Free-Body Diagram: The free-body diagram of the boy and skateboard system is shown in Fig. *a*. Here, $\mathbf{W}_b$, $\mathbf{W}_{sb}$, and $\mathbf{N}$ are nonimpulsive forces. The pair of impulsive forces $\mathbf{F}$ resulting from the impact during landing cancel each other out since they are internal to the system.

Conservation of Linear Momentum: Since the resultant of the impulsive force along the x axis is zero, the linear momentum of the system is conserved along the x axis.

$$(\overset{+}{\leftarrow}) \qquad m_b(v_b)_1 + m_{sb}(v_{sb})_1 = (m_b + m_{sb})v$$

$$50(5) + 5(0) = (50 + 5)v$$

$$v = 4.545 \text{ m/s}$$

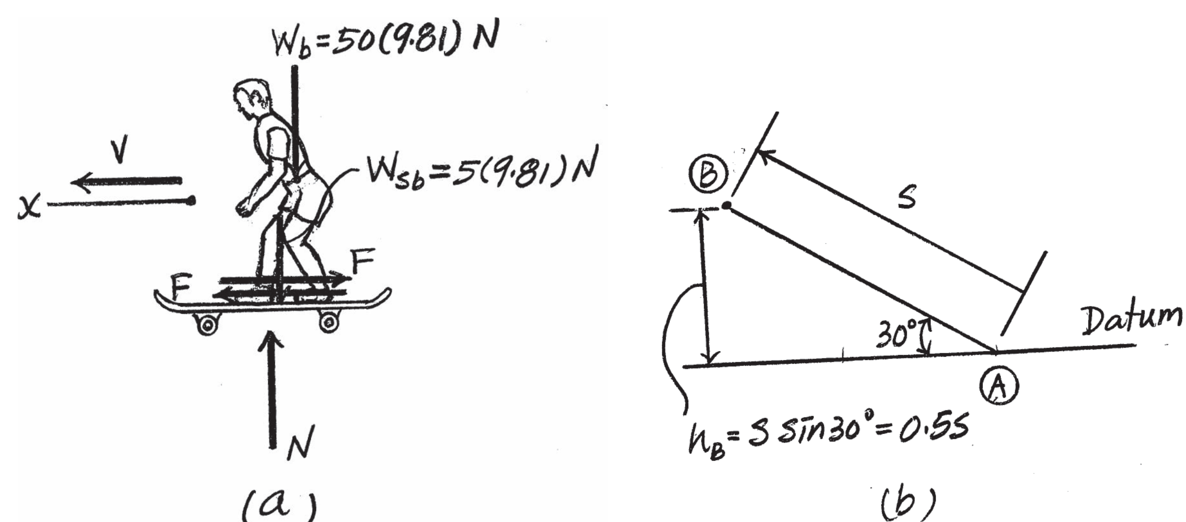
Conservation of Energy: With reference to the datum set in Fig. *b*, the gravitational potential energy of the boy and skateboard at positions A and B are $\left(V_g\right)_A = (m_b + m_{sb})gh_A = 0$ and $\left(V_g\right)_B = (m_b + m_{sb})gh_B = (50 + 5)(9.81)(s \sin 30°) = 269.775s$.

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(m_b + m_{sb})v_A{}^2 + \left(V_g\right)_A = \frac{1}{2}(m_b + m_{sb})v_B{}^2 + \left(V_g\right)_B$$

$$\frac{1}{2}(50 + 5)\left(4.545^2\right) + 0 = 0 + 269.775s$$

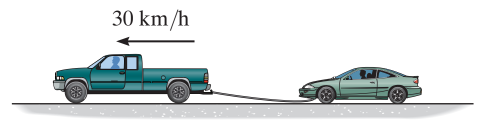
$$s = 2.11 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$s = 2.11$ m

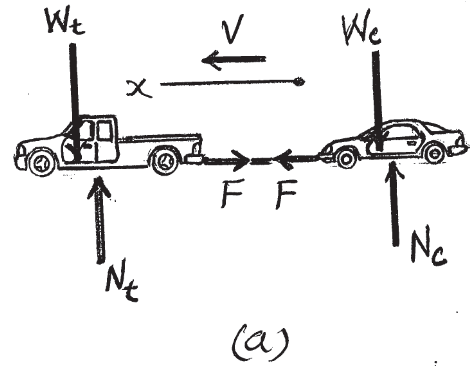
15–37.

The 2.5-Mg pickup truck is towing the 1.5-Mg car using a cable as shown. If the car is initially at rest and the truck is coasting with a velocity of 30 km/h when the cable is slack, determine the common velocity of the truck and the car just after the cable becomes taut. Also, find the loss of energy.



SOLUTION

Free-Body Diagram: The free-body diagram of the truck and car system is shown in Fig. *a*. Here, $\mathbf{W}_t$, $\mathbf{W}_C$, $\mathbf{N}_t$, and $\mathbf{N}_C$ are nonimpulsive forces. The pair of impulsive forces $\mathbf{F}$ generated at the instant the cable becomes taut are internal to the system and thus cancel each other out.



Conservation of Linear Momentum: Since the resultant of the impulsive force is zero, the linear momentum of the system is conserved along the x axis. The initial speed of the truck is $(v_t)_1 = \left[30(10^3)\dfrac{\text{m}}{\text{h}}\right]\left[\dfrac{1\text{ h}}{3600\text{ s}}\right] = 8.333$ m/s.

$$(\overset{+}{\leftarrow}) \qquad m_t(v_t)_1 + m_C(v_C)_1 = (m_t + m_C)v_2$$

$$2500(8.333) + 0 = (2500 + 1500)v_2$$

$$v_2 = 5.208 \text{ m/s} = 5.21 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

Kinetic Energy: The initial and final kinetic energy of the system is

$$T_1 = \frac{1}{2}m_t(v_t)_1^2 + \frac{1}{2}m_C(v_C)_1^2$$

$$= \frac{1}{2}(2500)(8.333^2) + 0$$

$$= 86\,805.56 \text{ J}$$

and

$$T_2 = (m_t + m_C)v_2^2$$

$$= \frac{1}{2}(2500 + 1500)(5.208^2)$$

$$= 54\,253.47$$

Thus, the loss of energy during the impact is

$$\Delta T = T_1 - T_2 = 86\,805.56 - 54\,253.47 = 32.55(10^3)\text{ J} = 32.6 \text{ kJ} \qquad \textbf{Ans.}$$

Ans:
$v = 5.21$ m/s $\leftarrow$
$\Delta T = -32.6$ kJ

15–38.

A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

SOLUTION

$(\overset{+}{\rightarrow})\quad \Sigma m v_1 = \Sigma m v_2$

$$15\,000(1.5) - 12\,000(0.75) = 27\,000(v_2)$$

$$v_2 = 0.5 \text{ m/s}$$ **Ans.**

$$T_1 = \frac{1}{2}(15\,000)(1.5)^2 + \frac{1}{2}(12\,000)(0.75)^2 = 20.25 \text{ kJ}$$

$$T_2 = \frac{1}{2}(27\,000)(0.5)^2 = 3.375 \text{ kJ}$$

$$\Delta T = T_2 - T_1$$

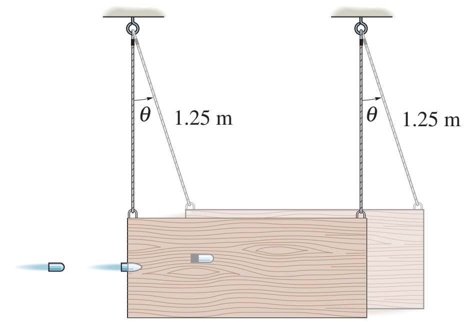
$$= 3.375 - 20.25 = -16.9 \text{ kJ}$$ **Ans.**

This energy is dissipated as noise, shock, and heat during the coupling.

Ans:
$v = 0.5$ m/s
$\Delta T = -16.9$ kJ

15–39.

A ballistic pendulum consists of a 4-kg wooden block originally at rest, $\theta = 0°$. When a 2-g bullet strikes and becomes embedded in it, it is observed that the block swings upward to a maximum angle of $\theta = 6°$. Estimate the speed of the bullet.



SOLUTION

Just after impact:

Datum at lowest point.

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(4 + 0.002)(v_B)_2^2 + 0 = 0 + (4 + 0.002)(9.81)(1.25)(1 - \cos 6°)$$

$$(v_B)_2 = 0.3665 \text{ m/s}$$

For the system of bullet and block:

$$(\overset{+}{\rightarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

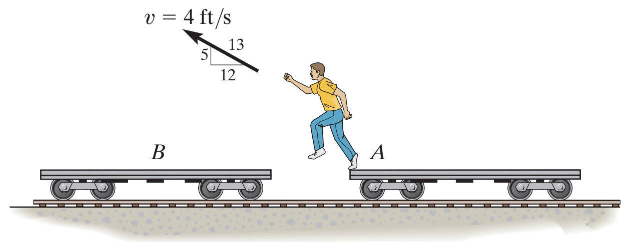
$$0.002(v_B)_1 = (4 + 0.002)(0.3665)$$

$$(v_B)_1 = 733 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 733$ m/s

***15–40.**

The boy jumps off the flat car at A with a velocity of $v = 4$ ft/s relative to the car as shown. If he lands on the second flat car B, determine the final speed of both cars after the motion. Each car has a weight of 80 lb. The boy's weight is 60 lb. Both cars are originally at rest. Neglect the mass of the car's wheels.



SOLUTION

$(\overset{+}{\leftarrow})\qquad \Sigma m(v_1) = \Sigma m(v_2)$

$$0 + 0 = -\frac{80}{32.2}v_A + \frac{60}{32.2}(v_b)_x$$

$$v_A = 0.75(v_b)_x$$

$$v_b = v_A + v_{b/A}$$

$(\overset{+}{\leftarrow})\qquad (v_b)_x = -v_A + 4\left(\frac{12}{13}\right)$

$$(v_b)_x = 2.110 \text{ ft/s}$$

$v_A = 1.58 \text{ ft/s} \rightarrow$ **Ans.**

$(\overset{+}{\leftarrow})\qquad \Sigma m(v_1) = \Sigma m(v_2)$

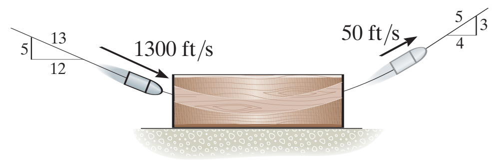
$$\frac{60}{32.2}(2.110) = \left(\frac{80}{32.2} + \frac{60}{32.2}\right)v$$

$v = 0.904 \text{ ft/s}$ **Ans.**

Ans:
$v_A = 1.58 \text{ ft/s} \rightarrow$
$v = 0.904 \text{ ft/s}$

15–41.

A 0.03-lb bullet traveling at 1300 ft/s strikes the 10-lb wooden block and exits the other side at 50 ft/s as shown. Determine the speed of the block just after the bullet exits the block, and also determine how far the block slides before it stops. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$.

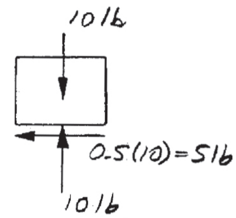


SOLUTION

$(\xrightarrow{+})\qquad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{0.03}{32.2}\right)(1300)\left(\frac{12}{13}\right) + 0 = \left(\frac{10}{32.2}\right)v_B + \left(\frac{0.03}{32.2}\right)(50)\left(\frac{4}{5}\right)$$

$v_B = 3.48\text{ ft/s}$ **Ans.**



$T_1 + \Sigma U_{1-2} = T_2$

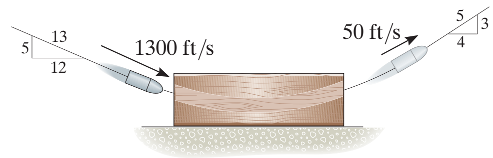
$$\frac{1}{2}\left(\frac{10}{32.2}\right)(3.48)^2 - 5(d) = 0$$

$d = 0.376\text{ ft}$ **Ans.**

Ans:
$v_B = 3.48\text{ ft/s}$
$d = 0.376\text{ ft}$

15–42.

A 0.03-lb bullet traveling at 1300 ft/s strikes the 10-lb wooden block and exits the other side at 50 ft/s as shown. Determine the speed of the block just after the bullet exits the block. Also, determine the average normal force on the block if the bullet passes through it in 1 ms, and the time the block slides before it stops. The coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$.

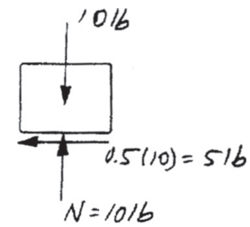


SOLUTION

$(\xrightarrow{+})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{0.03}{32.2}\right)(1300)\left(\frac{12}{13}\right) + 0 = \left(\frac{10}{32.2}\right)v_B + \left(\frac{0.03}{32.2}\right)(50)\left(\frac{4}{5}\right)$$

$v_B = 3.48 \text{ ft/s}$ **Ans.**



$(+\uparrow)$ $\quad mv_1 + \Sigma \int F\,dt = mv_2$

$$-\left(\frac{0.03}{32.2}\right)(1300)\left(\frac{5}{13}\right) - 10(1)\left(10^{-3}\right) + N(1)\left(10^{-3}\right) = \left(\frac{0.03}{32.2}\right)(50)\left(\frac{3}{5}\right)$$

$N = 504 \text{ lb}$ **Ans.**

$(\xrightarrow{+})$ $\quad mv_1 + \Sigma \int F\,dt = mv_2$

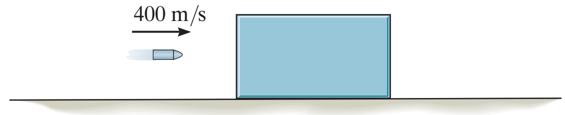
$$\left(\frac{10}{32.2}\right)(3.48) - 5(t) = 0$$

$t = 0.216 \text{ s}$ **Ans.**

Ans:
$v_B = 3.48 \text{ ft/s}$
$N_{avg} = 504 \text{ lb}$
$t = 0.216 \text{ s}$

15–43.

The 20-g bullet is traveling at 400 m/s when it becomes embedded in the 2-kg stationary block. Determine the distance the block will slide before it stops. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.



SOLUTION

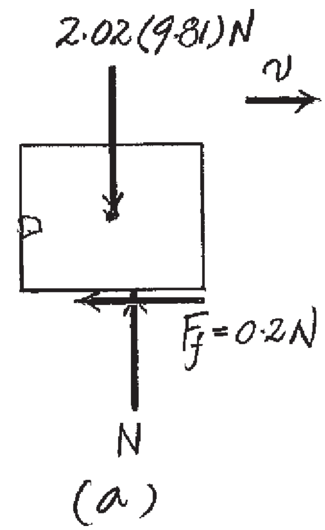
Conservation of Momentum.

$$(\xrightarrow{+}) \qquad m_b v_b + m_B v_B = (m_b + m_B)v$$

$$0.02(400) + 0 = (0.02 + 2)v$$

$$v = 3.9604 \text{ m/s}$$

Principle of Impulse and Momentum. Here, friction $F_f = \mu_k N = 0.2$ N. Referring to the FBD of the blocks, Fig. *a*,



$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + N(t) - 2.02(9.81)(t) = 0$$

$$N = 19.8162 \text{ N}$$

$$(\xrightarrow{+}) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$2.02(3.9604) + [-0.2(19.8162)t] = 2.02v$$

$$v = \{3.9604 - 1.962t\} \text{ m/s}$$

Thus, the stopping time can be determined from

$$0 = 3.9604 - 1.962t$$

$$t = 2.0186 \text{ s}$$

Kinematics. The displacement of the block can be determined by integrating $ds = v\,dt$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t (3.9604 - 1.962t)\,dt$$

$$s = \{3.9604t - 0.981t^2\} \text{ m}$$

The block stopped at $t = 2.0186$ s. Thus

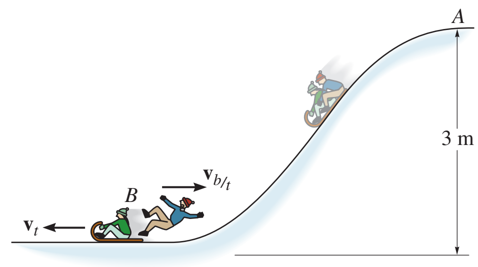
$$s = 3.9604(2.0186) - 0.981(2.0186^2)$$

$$= 3.9971 \text{ m} = 4.00 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s = 4.00$ m

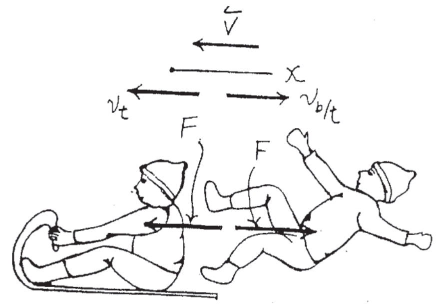
***15–44.**

A toboggan having a mass of 10 kg starts from rest at A and carries a girl and boy having a mass of 40 kg and 45 kg, respectively. When the toboggan reaches the bottom of the slope at B, the boy is pushed off from the back with a horizontal velocity of $v_{b/t} = 2\text{ m/s}$, measured relative to the toboggan. Determine the velocity of the toboggan afterwards. Neglect friction in the calculation.



SOLUTION

Conservation of Energy: The datum is set at the lowest point B. When the toboggan and its rider is at A, their position is 3 m *above* the datum and their gravitational potential energy is $(10 + 40 + 45)(9.81)(3) = 2795.85\text{ N}\cdot\text{m}$. Applying Eq. 14–21, we have



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2795.85 = \frac{1}{2}(10 + 40 + 45)\,v_B^2 + 0$$

$$v_B = 7.672\text{ m/s}$$

Relative Velocity: The relative velocity of the falling boy with respect to the toboggan is $v_{b/t} = 2\text{ m/s}$. Thus, the velocity of the boy falling off the toboggan is

$$v_b = v_t + v_{b/t}$$

$$(\overset{+}{\leftarrow}) \qquad v_b = v_t - 2 \qquad \textbf{[1]}$$

Conservation of Linear Momentum: If we consider the tobbogan and the riders as a system, then the impulsive force caused by the push is *internal* to the system. Therefore, it will cancel out. As the result, the linear momentum is conserved along the x axis.

$$m_T v_B = m_b v_b + \left(m_t + m_g\right) v_t$$

$$(\overset{+}{\leftarrow}) \qquad (10 + 40 + 45)(7.672) = 45v_b + (10 + 40)\,v_t \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

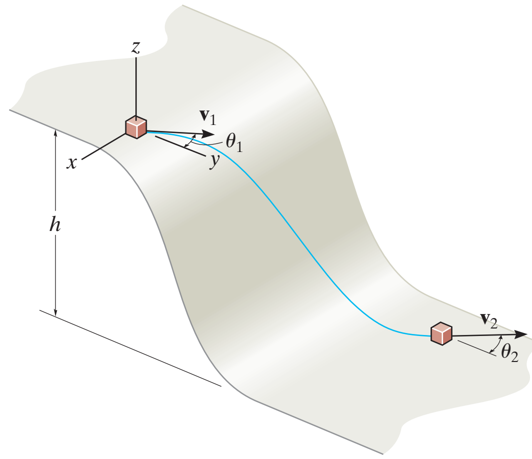
$$v_t = 8.62\text{ m/s} \qquad \textbf{Ans.}$$

$$v_b = 6.619\text{ m/s}$$

Ans:
$v_t = 8.62\text{ m/s}$

15–45.

The block of mass m is traveling at v_1 in the direction θ_1 shown at the top of the smooth slope. Determine its speed v_2 and its direction θ_2 when it reaches the bottom.



SOLUTION

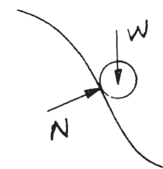
There are no impulses in the v direction:

$$mv_1 \sin\theta_1 = mv_2 \sin\theta_2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv_2^2 + 0$$

$$v_2 = \sqrt{v_1^2 + 2gh} \qquad \textbf{Ans.}$$



$$\sin\theta_2 = \frac{v_1 \sin\theta_1}{\sqrt{v_1^2 + 2gh}}$$

$$\theta_2 = \sin^{-1}\left(\frac{v_1 \sin\theta_1}{\sqrt{v_1^2 + 2gh}}\right) \qquad \textbf{Ans.}$$

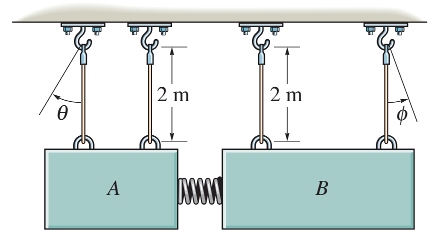
Ans:

$v_2 = \sqrt{v_1^2 + 2gh}$

$\theta_2 = \sin^{-1}\left(\dfrac{v_1 \sin\theta}{\sqrt{v_1^2 + 2gh}}\right)$

15–46.

The two blocks A and B each have a mass of 5 kg and are suspended from parallel cords. A spring, having a stiffness of $k = 60\ \mathrm{N/m}$, is attached to B and is compressed 0.3 m against A and B as shown. Determine the maximum angles θ and ϕ of the cords when the blocks are released from rest and the spring becomes unstretched.



SOLUTION

$(\xrightarrow{+})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = -5v_A + 5v_B$$

$$v_A = v_B = v$$

Just before the blocks begin to rise:

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(60)(0.3)^2 = \frac{1}{2}(5)(v)^2 + \frac{1}{2}(5)(v)^2 + 0$$

$$v = 0.7348\ \mathrm{m/s}$$

For A or B:

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

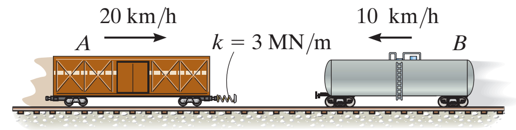
$$\frac{1}{2}(5)(0.7348)^2 + 0 = 0 + 5(9.81)(2)(1 - \cos\ \theta)$$

$\theta = \phi = 9.52°$ **Ans.**

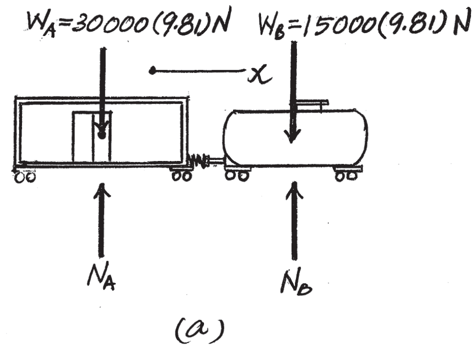
Ans:
$\theta = \phi = 9.52°$

15–47.

The 30-Mg freight car A and 15-Mg freight car B are moving towards each other with the velocities shown. Determine the maximum compression of the spring mounted on car A. Neglect rolling resistance.



SOLUTION



Conservation of Linear Momentum: Referring to the free-body diagram of the freight cars A and B shown in Fig. a, notice that the linear momentum of the system is conserved along the x axis. The initial speed of freight cars A and B are $(v_A)_1 = \left[20(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 5.556\text{ m/s}$ and $(v_B)_1 = \left[10(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 2.778\text{ m/s}$. At this instant, the spring is compressed to its maximum, and no relative motion occurs between freight cars A and B and they move with a common speed.

$$(\overset{+}{\rightarrow})\qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$30(10^3)(5.556) + \left[-15(10^3)(2.778)\right] = \left[30(10^3) + 15(10^3)\right]v_2$$

$$v_2 = 2.778\text{ m/s}\rightarrow$$

Conservation of Energy: The initial and final elastic potential energy of the spring is $(V_e)_1 = \frac{1}{2}ks_1^2 = 0$ and $(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(3)(10^6)s_{\max}^2 = 1.5(10^6)s_{\max}^2$.

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$\left[\frac{1}{2}m_A(v_A)_1^2 + \frac{1}{2}m_B(v_B)_1^2\right] + (V_e)_1 = \frac{1}{2}(m_A + m_B)v_2^2 + (V_e)_2$$

$$\frac{1}{2}(30)\left(10^3\right)(5.556^2) + \frac{1}{2}(15)\left(10^3\right)\left(2.778^2\right) + 0$$

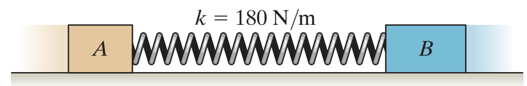
$$= \frac{1}{2}\left[30\left(10^3\right) + 15\left(10^3\right)\right]\left(2.778^2\right) + 1.5\left(10^6\right)s_{\max}^2$$

$$s_{\max} = 0.4811\text{ m} = 481\text{ mm}\qquad\textbf{Ans.}$$

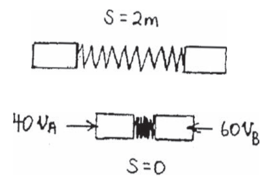
Ans:
$s_{\max} = 481$ mm

***15–48.**

Blocks A and B have masses of 40 kg and 60 kg, respectively. They are placed on a smooth surface and the spring connected between them is stretched 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.



SOLUTION



$(\xrightarrow{+})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = 40\, v_A - 60\, v_B$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(180)(2)^2 = \frac{1}{2}(40)(v_A)^2 + \frac{1}{2}(60)(v_B)^2$$

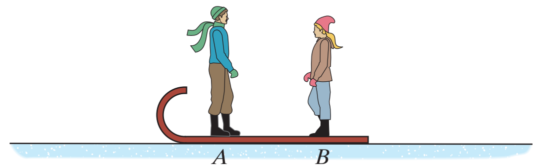
$v_A = 3.29$ m/s **Ans.**

$v_B = 2.19$ m/s **Ans.**

Ans:
$v_A = 3.29$ m/s
$v_B = 2.19$ m/s

15–49.

A boy A having a weight of 80 lb and a girl B having a weight of 65 lb stand motionless at the ends of the toboggan, which has a weight of 20 lb. If they exchange positions, A going to B and then B going to A's original position, determine the final position of the toboggan just after the motion. Neglect friction between the toboggan and the snow.



SOLUTION

A goes to B,

$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 = m_A v_A - (m_t + m_B) v_B$$

$$0 = m_A s_A - (m_t + m_B) s_B$$

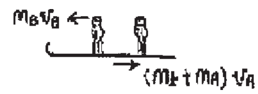
Assume B moves x to the left, then A moves $(4 - x)$ to the right

$$0 = m_A(4 - x) - (m_t + m_B)x$$

$$x = \frac{4m_A}{m_A + m_B + m_t}$$

$$= \frac{4(80)}{80 + 65 + 20} = 1.939 \text{ ft} \leftarrow$$

B goes to other end.



$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 = -m_B v_B + (m_t + m_A) v_A$$

$$0 = -m_B s_B + (m_t + m_A) s_A$$

Assume B moves x' to the right, then A moves $(4 - x')$ to the left

$$0 = -m_B(4 - x') + (m_t + m_A)x'$$

$$x' = \frac{4m_B}{m_A + m_B + m_t}$$

$$= \frac{4(65)}{80 + 65 + 20} = 1.576 \text{ ft} \rightarrow$$

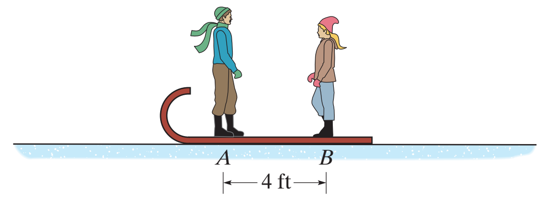
Net movement of sled is

$$x = 1.939 - 1.576 = 0.364 \text{ ft} \leftarrow$$ **Ans.**

Ans:
$x = 0.364 \text{ ft} \leftarrow$

15–50.

A boy A having a weight of 80 lb and a girl B having a weight of 65 lb stand motionless at the ends of the toboggan, which has a weight of 20 lb. If A walks to B and stops, and both walk back together to the original position of A, determine the final position of the toboggan just after the motion stops. Neglect friction between the toboggan and the snow.



SOLUTION

A goes to B,

$(\xrightarrow{+})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 = m_A v_A - (m_t + m_B) v_B$$

$$0 = m_A s_A - (m_t + m_B) s_B$$

Assume B moves x to the left, then A moves $(4 - x)$ to the right

$$0 = m_A(4 - x) - (m_t + m_B)x$$

$$x = \frac{4 m_A}{m_A + m_B + m_t}$$

$$= \frac{4(80)}{80 + 65 + 20} = 1.939 \text{ ft} \leftarrow$$

A and B go to other end.

$(\xrightarrow{+})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 = -m_B v - m_A v + m_t v_t$$

$$0 = -m_B s - m_A s + m_t s_t$$

Assume the toboggan moves x' to the right, then A and B move $(4 - x')$ to the left

$$0 = -m_B(4 - x') - m_A(4 - x') + m_t x'$$

$$x' = \frac{4(m_B + m_A)}{m_A + m_B + m_t}$$

$$= \frac{4(65 + 80)}{80 + 65 + 20} = 3.515 \text{ ft} \rightarrow$$

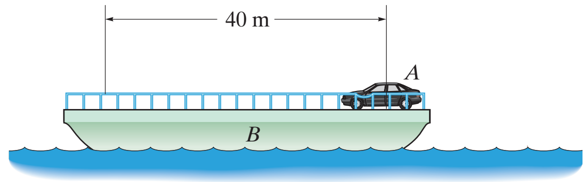
Net movement of sled is

$(\xrightarrow{+})\qquad x = 3.515 - 1.939 = 1.58 \text{ ft} \rightarrow$ **Ans.**

Ans:
$x = 1.58 \text{ ft} \rightarrow$

15–51.

The 10-Mg barge B supports a 2-Mg automobile A. If someone drives the automobile to the other side of the barge, determine how far the barge moves. Neglect the resistance of the water.



SOLUTION

Conservation of Momentum. Assuming that $\mathbf{V}_B$ is to the left,

$$(\overset{+}{\leftarrow}) \qquad m_A v_A + m_B v_B = 0$$

$$2(10^3)v_A + 10(10^3)v_B = 0$$

$$2\,v_A + 10\,v_B = 0$$

Integrate this equation,

$$2\,s_A + 10\,s_B = 0 \qquad \textbf{(1)}$$

Kinematics. Here, $s_{A/B} = 40 \text{ m} \leftarrow$, using the relative displacement equation by assuming that $\mathbf{s}_B$ is to the left,

$$(\overset{+}{\leftarrow}) \qquad \mathbf{s}_A = \mathbf{s}_B + \mathbf{s}_{A/B}$$

$$\mathbf{s}_A = \mathbf{s}_B + 40 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2),

$$s_B = -6.6667 \text{ m} = 6.67 \text{ m} \rightarrow \qquad \textbf{Ans.}$$

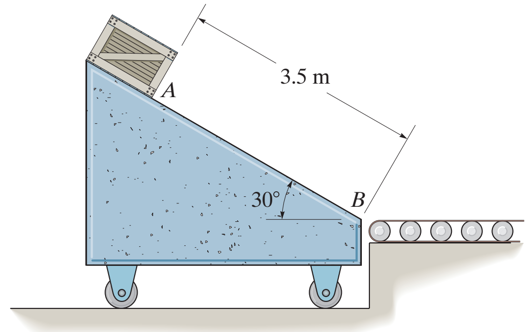
$$s_A = 33.33 \text{ m} \leftarrow$$

The negative sign indicates that $\mathbf{s}_B$ is directed to the right which is opposite to that of the assumed.

Ans:
$s_B = 6.67 \text{ m} \rightarrow$

***15–52.**

The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at A and slides down 3.5 m to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?



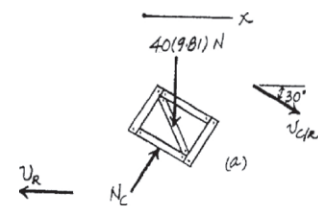
SOLUTION

Conservation of Energy: The datum is set at lowest point B. When the crate is at point A, it is $3.5 \sin 30^\circ = 1.75$ m *above* the datum. Its gravitational potential energy is $10(9.81)(1.75) = 171.675\ \text{N}\cdot\text{m}$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 171.675 = \frac{1}{2}(10)v_C^2 + \frac{1}{2}(40)v_R^2$$

$$171.675 = 5v_C^2 + 20v_R^2 \qquad \textbf{(1)}$$

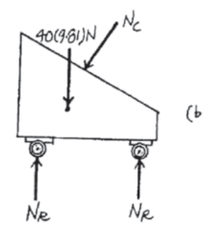


Relative Velocity: The velocity of the crate is given by

$$\mathbf{v}_C = \mathbf{v}_R + \mathbf{v}_{C/R}$$

$$= -v_R\mathbf{i} + (v_{C/R}\cos 30^\circ\mathbf{i} - v_{C/R}\sin 30^\circ\mathbf{j})$$

$$= (0.8660\,v_{C/R} - v_R)\mathbf{i} - 0.5\,v_{C/R}\mathbf{j} \qquad \textbf{(2)}$$



The magnitude of v_C is

$$v_C = \sqrt{(0.8660\,v_{C/R} - v_R)^2 + (-0.5\,v_{C/R})^2}$$

$$= \sqrt{v_{C/R}^2 + v_R^2 - 1.732\,v_R v_{C/R}} \qquad \textbf{(3)}$$

Conservation of Linear Momentum: If we consider the crate and the ramp as a system, from the FBD, one realizes that the normal reaction N_C (*impulsive force*) is *internal* to the system and will cancel each other. As the result, the linear momentum is conserved along the x axis.

$$0 = m_C(v_C)_x + m_R v_R$$

$$(\overset{+}{\rightarrow}) \qquad 0 = 10(0.8660\,v_{C/R} - v_R) + 40(-v_R)$$

$$0 = 8.660\,v_{C/R} - 50\,v_R \qquad \textbf{(4)}$$

Solving Eqs. (1), (3), and (4) yields

$$v_R = 1.101\ \text{m/s} = 1.10\ \text{m/s} \quad v_C = 5.43\ \text{m/s} \qquad \textbf{Ans.}$$

$$v_{C/R} = 6.356\ \text{m/s}$$

From Eq. (2)

$$\mathbf{v}_C = [0.8660(6.356) - 1.101]\mathbf{i} - 0.5(6.356)\mathbf{j} = \{4.403\mathbf{i} - 3.178\mathbf{j}\}\ \text{m/s}$$

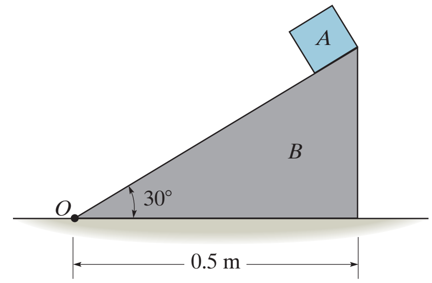
Thus, the directional angle ϕ of v_C is

$$\phi = \tan^{-1}\frac{3.178}{4.403} = 35.8^\circ \quad \phi \qquad \textbf{Ans.}$$

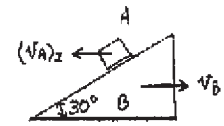
Ans:
$v_{C/R} = 6.356$ m/s
$\phi = 35.8^\circ$

15–53.

Block A has a mass of 5 kg and is placed on the smooth triangular block B having a mass of 30 kg. If the system is released from rest, determine the distance B moves from point O when A reaches the bottom. Neglect the size of block A.



SOLUTION



$(\xrightarrow{+})$ $\Sigma m v_1 = \Sigma m v_2$

$$0 = 30 v_B - 5(v_A)_x$$

$$(v_A)_x = 6 v_B$$

$$v_B = v_A + v_{B/A}$$

$(\xrightarrow{+})$ $v_B = -(v_A)_x + (v_{B/A})_x$

$$v_B = -6 v_B + (v_{B/A})_x$$

$$(v_{B/A})_x = 7 v_B$$

Integrate

$$(s_{B/A})_x = 7 s_B$$

$$(s_{B/A})_x = 0.5 \text{ m}$$

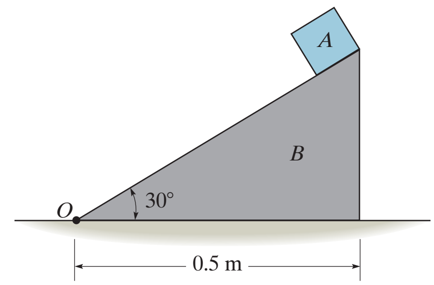
Thus,

$$s_B = \frac{0.5}{7} = 0.0714 \text{ m} = 71.4 \text{ mm} \rightarrow$$ **Ans.**

Ans:
$s_B = 71.4 \text{ mm} \rightarrow$

15–54.

Solve Prob. 15–53 if the coefficient of kinetic friction between A and B is $\mu_k = 0.3$. Neglect friction between block B and the horizontal plane.



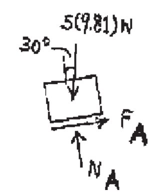
SOLUTION

$+\nwarrow\Sigma F_y = 0; \qquad N_A - 5(9.81)\cos 30° = 0$

$N_A = 42.4785 \text{ N}$

$\nearrow + \Sigma F_x = 0; \qquad F_A - 5(9.81)\sin 30° = 0$

$F_A = 24.525 \text{ N}$



$F_{max} = \mu N_A = 0.3(42.4785) = 12.74 \text{ N} < 24.525 \text{ N}$

Block indeed slides.

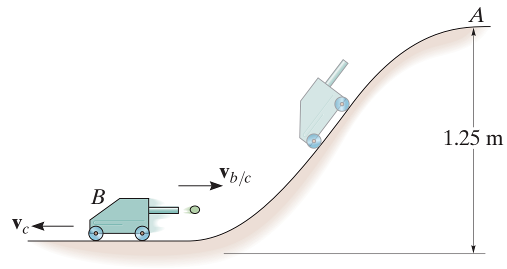
Solution is the same as in Prob. 15–53. Since F_A is internal to the system.

$s_B = 71.4 \text{ mm} \rightarrow$ **Ans.**

Ans:
$s_B = 71.4 \text{ mm} \rightarrow$

15–55.

The cart has a mass of 3 kg and rolls freely down the slope. When it reaches the bottom, a spring loaded gun fires a 0.5-kg ball out the back with a horizontal velocity of $v_{b/c} = 0.6$ m/s, measured relative to the cart. Determine the final velocity of the cart.



SOLUTION

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$0 + (3 + 0.5)(9.81)(1.25) = \frac{1}{2}(3 + 0.5)(v_B)_2^2 + 0$$

$$v_B = 4.952 \text{ m/s}$$

$$(\overset{+}{\leftarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$(3 + 0.5)(4.952) = (3)v_c - (0.5)v_b \qquad \textbf{(1)}$$

$$(\overset{+}{\leftarrow}) \qquad v_b = v_c + v_{b/c}$$

$$-v_b = v_c - 0.6 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

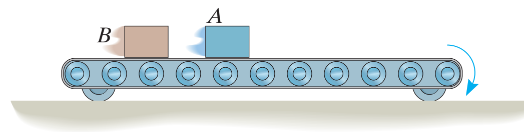
$$v_c = 5.04 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

$$v_b = -4.44 \text{ m/s} = 4.44 \text{ m/s} \leftarrow$$

Ans:
$v_c = 5.04$ m/s $\leftarrow$

***15–56.**

Two boxes A and B, each having a weight of 160 lb, sit on the 500-lb conveyor which is free to roll on the ground. If the belt starts from rest and begins to run with a speed of 3 ft/s, determine the final speed of the conveyor if (a) the boxes are not stacked and A falls off then B falls off, and (b) A is stacked on top of B and both fall off together.



SOLUTION

a) Let v_b be the velocity of A and B.

$$\left(\overset{+}{\rightarrow}\right) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$0 = \left(\frac{320}{32.2}\right)(v_b) - \left(\frac{500}{32.2}\right)(v_c)$$

$$\left(\overset{+}{\rightarrow}\right) \qquad v_b = v_c + v_{b/c}$$

$$v_b = -v_c + 3$$

Thus, $v_b = 1.83\text{ ft/s} \rightarrow \qquad v_c = 1.17\text{ ft/s} \leftarrow$

When a box falls off, it exerts no impulse on the conveyor, and so does not alter the momentum of the conveyor. Thus,

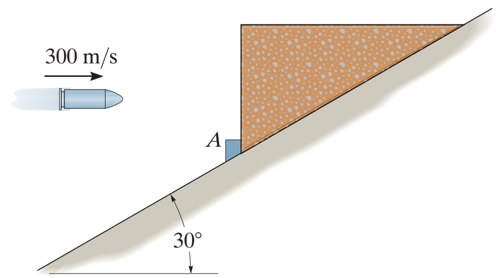
a) $v_c = 1.17\text{ ft/s} \leftarrow$ **Ans.**

b) $v_c = 1.17\text{ ft/s} \leftarrow$ **Ans.**

Ans:
a) $v_c = 1.17\text{ ft/s} \leftarrow$
b) $v_c = 1.17\text{ ft/s} \leftarrow$

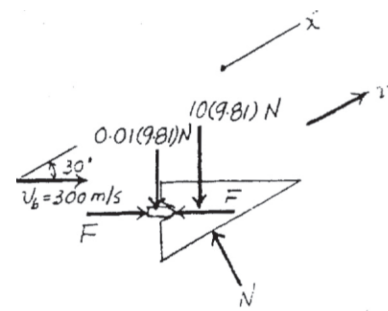
15–57.

The 10-kg block is held at rest on the smooth inclined plane by the stop block at A. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.



SOLUTION

Conservation of Linear Momentum: If we consider the block and the bullet as a system, then from the FBD, the *impulsive* force F caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the x' axis.



$$m_b(v_b)_{x'} = (m_b + m_B)\,v_{x'}$$

$$0.01(300 \cos 30°) = (0.01 + 10)\,v$$

$$v = 0.2595 \text{ m/s}$$

Conservation of Energy: The datum is set at the blocks initial position. When the block and the embedded bullet is at their highest point they are h *above* the datum. Their gravitational potential energy is $(10 + 0.01)(9.81)h = 98.1981h$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(10 + 0.01)\left(0.2595^2\right) = 0 + 98.1981h$$

$$h = 0.003433 \text{ m} = 3.43 \text{ mm}$$

$$d = 3.43 / \sin 30° = 6.87 \text{ mm}$$ **Ans.**

Ans:
$d = 6.87$ mm

15–58.

Disk A has a mass of 250 g and is sliding on a *smooth* horizontal surface with an initial velocity $(v_A)_1 = 2\text{ m/s}$. It makes a direct collision with disk B, which has a mass of 175 g and is originally at rest. If both disks are of the same size and the collision is perfectly elastic ($e = 1$), determine the velocity of each disk just after collision. Show that the kinetic energy of the disks before and after collision is the same.

SOLUTION

$(\overset{+}{\rightarrow})$ $\quad (0.250)(2) + 0 = (0.250)(v_A)_2 + (0.175)(v_B)_2$

$(\overset{+}{\rightarrow})$ $\quad e = 1 = \dfrac{(v_B)_2 - (v_A)_2}{2 - 0}$

Solving

$(v_A)_2 = 0.353\text{ m/s}$ **Ans.**

$(v_B)_2 = 2.35\text{ m/s}$ **Ans.**

$$T_1 = \frac{1}{2}(0.25)(2)^2 = 0.5\text{ J}$$

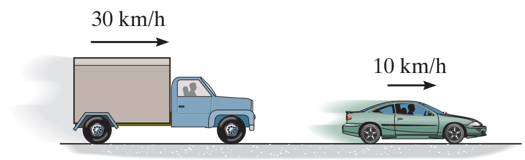
$$T_2 = \frac{1}{2}(0.25)(0.353)^2 + \frac{1}{2}(0.175)(2.35)^2 = 0.5\text{ J}$$

$T_1 = T_2$ **QED**

Ans:
$(v_A)_2 = 0.353\text{ m/s}$
$(v_B)_2 = 2.35\text{ m/s}$

15–59.

The 5-Mg truck and 2-Mg car are traveling with the free-rolling velocities shown just before they collide. After the collision, the car moves with a velocity of 15 km/h to the right *relative* to the truck. Determine the coefficient of restitution between the truck and car and the loss of energy due to the collision.

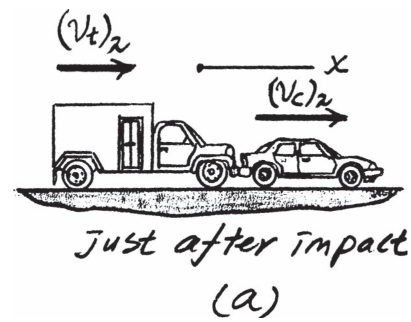


SOLUTION

Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact).

The initial speeds of the truck and car are $(v_t)_1 = \left[30\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 8.333\text{ m/s}$ and $(v_c)_1 = \left[10\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\ h}{3600\text{ s}}\right) = 2.778\text{ m/s}$.

By referring to Fig. a,



$$\left(\xrightarrow{+}\right) \qquad m_t\left(v_t\right)_1 + m_c\left(v_c\right)_1 = m_t\left(v_t\right)_2 + m_c\left(v_c\right)_2$$

$$5000(8.333) + 2000(2.778) = 5000\left(v_t\right)_2 + 2000\left(v_c\right)_2$$

$$5\left(v_t\right)_2 + 2\left(v_c\right)_2 = 47.22 \qquad \textbf{(1)}$$

Coefficient of Restitution: Here, $(v_{c/t}) = \left[15\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 4.167\text{ m/s} \rightarrow$. Applying the relative velocity equation,

$$(\mathbf{v}_c)_2 = (\mathbf{v}_t)_2 + (\mathbf{v}_{c/t})_2$$

$$\left(\xrightarrow{+}\right) \qquad (v_c)_2 = (v_t)_2 + 4.167$$

$$(v_c)_2 - (v_t)_2 = 4.167 \qquad \textbf{(2)}$$

Applying the coefficient of restitution equation,

$$\left(\xrightarrow{+}\right) \qquad e = \frac{(v_c)_2 - (v_t)_2}{(v_t)_1 - (v_c)_1}$$

$$e = \frac{(v_c)_2 - (v_t)_2}{8.333 - 2.778} \qquad \textbf{(3)}$$

15–59. Continued

Substituting Eq. (2) into Eq. (3),

$$e = \frac{4.167}{8.333 - 2.778} = 0.75 \qquad \textbf{Ans.}$$

Solving Eqs. (1) and (2) yields

$$(v_t)_2 = 5.556 \text{ m/s}$$

$$(v_c)_2 = 9.722 \text{ m/s}$$

Kinetic Energy: The kinetic energy of the system just before and just after the collision are

$$T_1 = \frac{1}{2} m_t (v_t)_1{}^2 + \frac{1}{2} m_c (v_c)_1{}^2$$

$$= \frac{1}{2}(5000)(8.333^2) + \frac{1}{2}(2000)(2.778^2)$$

$$= 181.33\left(10^3\right) \text{ J}$$

$$T_2 = \frac{1}{2} m_t (v_t)_2{}^2 + \frac{1}{2} m_c (v_c)_2{}^2$$

$$= \frac{1}{2}(5000)(5.556^2) + \frac{1}{2}(2000)(9.722^2)$$

$$= 171.68\left(10^3\right) \text{J}$$

Thus,

$$\Delta T = T_1 - T_2 = 181.33\left(10^3\right) - 171.68\left(10^3\right)$$

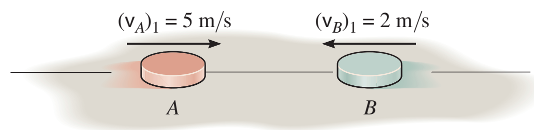
$$= 9.645\left(10^3\right) \text{ J}$$

$$= 9.65 \text{ kJ} \qquad \textbf{Ans.}$$

Ans:
$e = 0.75$
$\Delta T = -9.65 \text{ kJ}$

***15–60.**

Disk A has a mass of 2 kg and is sliding forward on the *smooth* surface with a velocity $(v_A)_1 = 5$ m/s when it strikes the 4-kg disk B, which is sliding towards A at $(v_B)_1 = 2$ m/s, with direct central impact. If the coefficient of restitution between the disks is $e = 0.4$, compute the velocities of A and B just after collision.

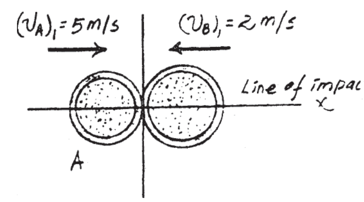


SOLUTION

Conservation of Momentum :

$$m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(5) + 4(-2) = 2(v_A)_2 + 4(v_B)_2 \qquad \textbf{(1)}$$



Coefficient of Restitution :

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.4 = \frac{(v_B)_2 - (v_A)_2}{5 - (-2)} \qquad \textbf{(2)}$$

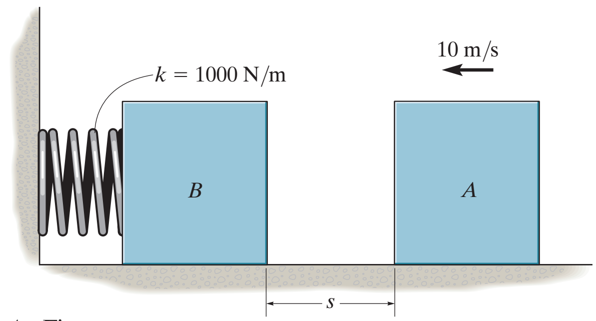
Solving Eqs. (1) and (2) yields

$$(v_A)_2 = -1.53 \text{ m/s} = 1.53 \text{ m/s} \leftarrow \qquad (v_B)_2 = 1.27 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$

Ans:
$(v_A)_2 = 1.53$ m/s $\leftarrow$
$(v_B)_2 = 1.27$ m/s $\rightarrow$

15–61.

The 15-kg block A slides on the surface for which $\mu_k = 0.3$. The block has a velocity $v = 10$ m/s when it is $s = 4$ m from the 10-kg block B. If the unstretched spring has a stiffness $k = 1000$ N/m, determine the maximum compression of the spring due to the collision. Take $e = 0.6$.



SOLUTION

Principle of Work and Energy. Referring to the FBD of block A, Fig. a, motion along the y axis gives $N_A = 15(9.81) = 147.15$ N. Thus the friction is $F_f = \mu_k N_A = 0.3(147.15) = 44.145$ N.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(15)(10^2) + (-44.145)(4) = \frac{1}{2}(15)(v_A)_1^2$$

$$(v_A)_1 = 8.7439 \text{ m/s} \leftarrow$$

Conservation of Momentum.

$$(\overset{+}{\leftarrow}) \quad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$15(8.7439) + 0 = 15(v_A)_2 + 10(v_B)_2$$

$$3(v_A)_2 + 2(v_B)_2 = 26.2317 \qquad \textbf{(1)}$$

Coefficient of Restitution.

$$(\overset{+}{\leftarrow}) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.6 = \frac{(v_B)_2 - (v_A)_2}{8.7439 - 0}$$

$$(v_B)_2 - (v_A)_2 = 5.2463 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

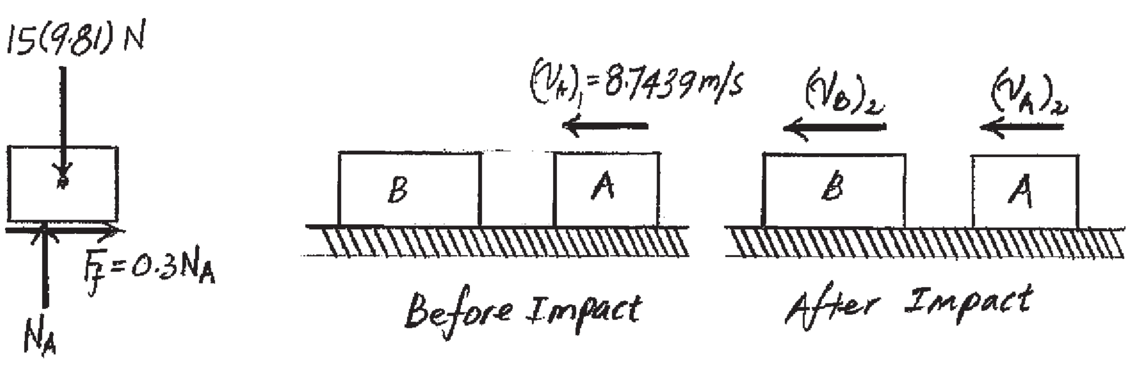
$$(v_B)_2 = 8.3942 \text{ m/s} \leftarrow \quad (v_A)_2 = 3.1478 \text{ m/s} \leftarrow$$

Conservation of Energy. When block B stops momentarily, the compression of the spring is maximum. Thus, $T_2 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(10)(8.3942^2) + 0 = 0 + \frac{1}{2}(1000)x_{max}^2$$

$$x_{max} = 0.8394 \text{ m} = 0.839 \text{ m} \qquad \textbf{Ans.}$$

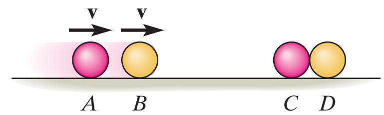


(a)

Ans:
$x_{max} = 0.839$ m

15–62.

The four smooth balls each have the same mass m. If A and B are rolling forward with velocity $\mathbf{v}$ and strike C, explain why after collision C and D each move off with velocity $\mathbf{v}$. Why doesn't D move off with velocity $2\mathbf{v}$? The collision is elastic, $e = 1$. Neglect the size of each ball.



SOLUTION

Collision will occur in the following sequence;

B strikes *C*

$(\overset{+}{\rightarrow})\qquad mv = -mv_B + mv_C$

$$v = -v_B + v_C$$

$(\overset{+}{\rightarrow})\qquad e = 1 = \dfrac{v_C + v_B}{v}$

$$v_C = v, \qquad v_B = 0$$

C strikes *D*

$(\overset{+}{\rightarrow})\qquad mv = -mv_C + mv_D$

$(\overset{+}{\rightarrow})\qquad e = 1 = \dfrac{v_D + v_C}{v}$

$v_C = 0, \qquad v_D = v$ **Ans.**

A strikes *B*

$(\overset{+}{\rightarrow})\qquad mv = -mv_A + mv_B$

$(\overset{+}{\rightarrow})\qquad e = 1 = \dfrac{v_B + v_A}{v}$

$v_B = v, \qquad v_A = 0$ **Ans.**

Finally, *B* strikes *C*

$(\overset{+}{\rightarrow})\qquad mv = -mv_B + mv_C$

$(\overset{+}{\rightarrow})\qquad e = 1 = \dfrac{v_C + v_B}{v}$

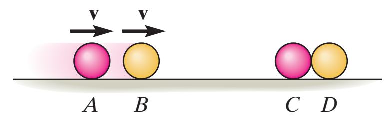
$v_C = v, \qquad v_B = 0$ **Ans.**

Note: If D rolled off with twice the velocity, its kinetic energy would be twice the energy available from the original two A and B: $\left(\frac{1}{2}mv^2 + \frac{1}{2}mv^2 \neq \frac{1}{2}(2v)^2\right)$

Ans:
$v_C = 0, v_D = v$
$v_B = v, v_A = 0$
$v_C = v, v_B = 0$

15–63.

The four balls each have the same mass m. If A and B are rolling forward with velocity $\mathbf{v}$ and strike C, determine the velocity of each ball after the first three collisions. Take $e = 0.5$ between each ball.



SOLUTION

Collision will occur in the following sequence;

B strikes C

$(\overset{+}{\rightarrow})\qquad mv = mv_B + mv_c$

$$v = v_B + v_C$$

$(\overset{+}{\rightarrow})\qquad e = 0.5 = \dfrac{v_C - v_B}{v}$

$$v_C = 0.75v \rightarrow, \qquad v_B = 0.25v \rightarrow$$

C strikes D

$(\overset{+}{\rightarrow})\qquad m(0.75v) = mv_C + mv_D$

$(\overset{+}{\rightarrow})\qquad e = 0.5 = \dfrac{v_D - v_C}{0.75v}$

$v_C = 0.1875v \rightarrow$ **Ans.**

$v_D = 0.5625v \rightarrow$ **Ans.**

A strikes B

$(\overset{+}{\rightarrow})\qquad mv + m(0.25v) = mv_A + mv_B$

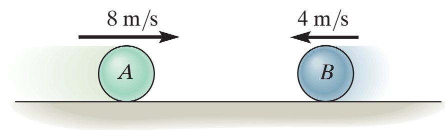
$(\overset{+}{\rightarrow})\qquad e = 0.5 = \dfrac{v_B - v_A}{(v - 0.25v)}$

$v_B = 0.8125v \rightarrow, \qquad v_A = 0.4375v \rightarrow$ **Ans.**

Ans:
$v_C = 0.1875v \rightarrow$
$v_D = 0.5625v \rightarrow$
$v_B = 0.8125v \rightarrow$
$v_A = 0.4375v \rightarrow$

***15–64.**

Ball A has a mass of 3 kg and is moving with a velocity of 8 m/s when it makes a direct collision with ball B, which has a mass of 2 kg and is moving with a velocity of 4 m/s. If $e = 0.7$, determine the velocity of each ball just after the collision. Neglect the size of the balls.



SOLUTION

Conservation of Momentum. The velocity of balls A and B before and after impact are shown in Fig. a

$$(\xrightarrow{+}) \quad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$3(8) + 2(-4) = 3v_A + 2v_B$$

$$3v_A + 2v_B = 16 \qquad \textbf{(1)}$$

Coefficient of Restitution.

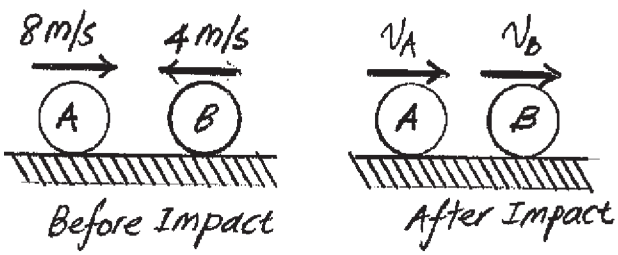
$$(\xrightarrow{+}) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \quad 0.7 = \frac{v_B - v_A}{8 - (-4)}$$

$$v_B - v_A = 8.4 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$v_B = 8.24 \text{ m/s} \rightarrow$ **Ans.**

$v_A = -0.16 \text{ m/s} = 0.160 \text{ m/s} \leftarrow$ **Ans.**



(a)

Ans:
$v_B = 8.24 \text{ m/s} \rightarrow$
$v_A = 0.160 \text{ m/s} \leftarrow$

15–65.

A 1-lb ball A is traveling horizontally at 20 ft/s when it strikes a 10-lb block B that is at rest. If the coefficient of restitution between A and B is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the time for the block B to stop sliding.

SOLUTION

$(\xrightarrow{+})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

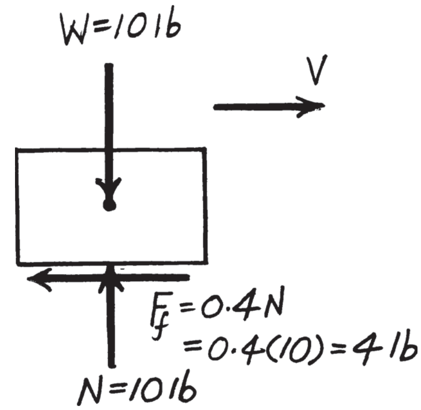
$$\left(\frac{1}{32.2}\right)(20) + 0 = \left(\frac{1}{32.2}\right)(v_A)_2 + \left(\frac{10}{32.2}\right)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$(\xrightarrow{+})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$



Thus,

$(v_B)_2 = 2.909 \text{ ft/s} \rightarrow$

$(v_A)_2 = -9.091 \text{ ft/s} = 9.091 \text{ ft/s} \leftarrow$

Block B:

$(\xrightarrow{+})$ $\quad m\,v_1 + \Sigma \int F\,dt = m\,v_2$

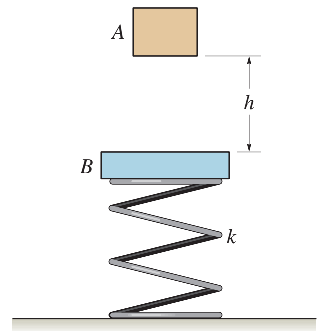
$$\left(\frac{10}{32.2}\right)(2.909) - 4t = 0$$

$t = 0.226$ s **Ans.**

Ans:
$t = 0.226$ s

15–66.

Block A, having a mass m, is released from rest, falls a distance h and strikes the plate B having a mass $2m$. If the coefficient of restitution between A and B is e, determine the velocity of the plate just after collision. The spring has a stiffness k.



SOLUTION

Just before impact, the velocity of A is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}mv_A^2 - mgh$$

$$v_A = \sqrt{2gh}$$

$(+\downarrow)$ $$e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gh}}$$

$$e\sqrt{2gh} = (v_B)_2 - (v_A)_2 \qquad \textbf{(1)}$$

$(+\downarrow)$ $$\Sigma mv_1 = \Sigma mv_2$$

$$m(v_A) + 0 = m(v_A)_2 + 2m(v_B)_2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) for $(v_B)_2$ yields;

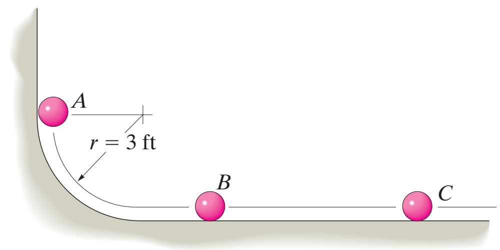
$$(v_B)_2 = \frac{1}{3}\sqrt{2gh}(1 + e) \qquad \textbf{Ans.}$$

Ans:

$$(v_B)_2 = \frac{1}{3}\sqrt{2gh}\,(1 + e)$$

15–67.

The three balls each weigh 0.5 lb and have a coefficient of restitution of $e = 0.85$. If ball A is released from rest and strikes ball B and then ball B strikes ball C, determine the velocity of each ball after the second collision has occurred. The balls slide without friction.



SOLUTION

Ball A:

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0.5)(3) = \frac{1}{2}\left(\frac{0.5}{32.2}\right)(v_A)_1^2 + 0$$

$$(v_A)_1 = 13.90 \text{ ft/s}$$

Balls A and B:

$$(\overset{+}{\rightarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$\left(\frac{0.5}{32.2}\right)(13.90) + 0 = \left(\frac{0.5}{32.2}\right)(v_A)_2 + \left(\frac{0.5}{32.2}\right)(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2)}{(v_A)_1 - (v_B)_1}$$

$$0.85 = \frac{(v_B)_2 - (v_A)_2}{13.90 - 0}$$

Solving:

$$(v_A)_2 = 1.04 \text{ ft/s} \qquad \textbf{Ans.}$$

$$(v_B)_2 = 12.86 \text{ ft/s}$$

Balls B and C:

$$(\overset{+}{\rightarrow}) \qquad \Sigma m v_2 = \Sigma m v_3$$

$$\left(\frac{0.5}{32.2}\right)(12.86) + 0 = \left(\frac{0.5}{32.2}\right)(v_B)_3 + \left(\frac{0.5}{32.2}\right)(v_C)_3$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_C)_3 - (v_B)_3}{(v_B)_2 - (v_C)_2}$$

$$0.85 = \frac{(v_C)_3 - (v_B)_3}{12.86 - 0}$$

Solving:

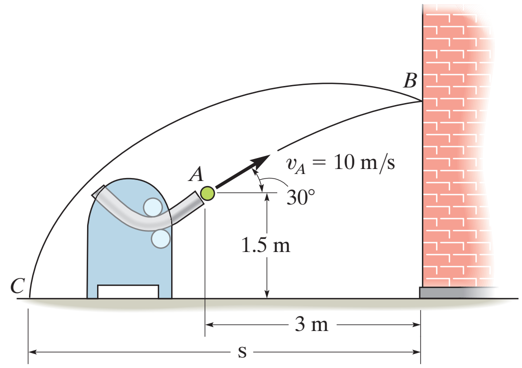
$$(v_B)_3 = 0.964 \text{ ft/s} \qquad \textbf{Ans.}$$

$$(v_C)_3 = 11.9 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$(v_A)_2 = 1.04$ ft/s
$(v_B)_3 = 0.964$ ft/s
$(v_C)_3 = 11.9$ ft/s

***15–68.**

A pitching machine throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10$ m/s as shown. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



SOLUTION

(a)

$(v_B)_{x1} = 10 \cos 30° = 8.660 \text{ m/s} \rightarrow$

$(\xrightarrow{+})\quad s = s_0 + v_0 t$

$3 = 0 + 10 \cos 30° t$

$t = 0.3464 \text{ s}$

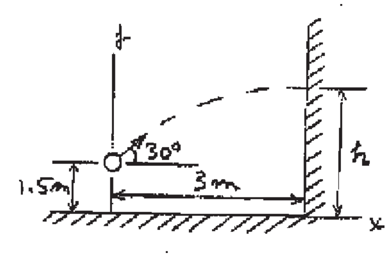
$(+\uparrow)\quad v = v_0 + a_c t$

$(v_B)_{y\,t} = 10 \sin 30° - 9.81(0.3464) = 1.602 \text{ m/s} \uparrow$

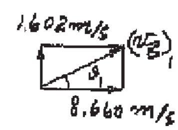
$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$h = 1.5 + 10 \sin 30°(0.3464) - \frac{1}{2}(9.81)(0.3464)^2$

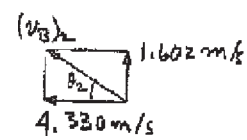
$= 2.643 \text{ m}$



$(v_B)_1 = \sqrt{(1.602)^2 + (8.660)^2} = 8.81 \text{ m/s}$ **Ans.**



$\theta_1 = \tan^{-1}\left(\frac{1.602}{8.660}\right) = 10.5°$ ⦣ **Ans.**



(b)

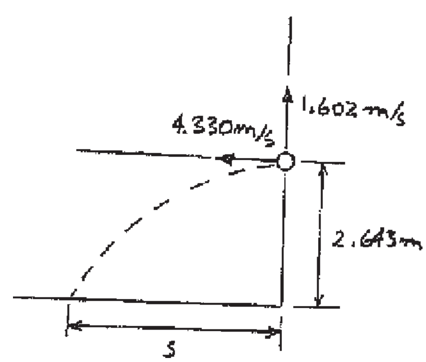
$(\xrightarrow{+})\quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.5 = \frac{(v_{Bx})_2 - 0}{0 - (8.660)}$

$(v_{Bx})_2 = 4.330 \text{ m/s} \leftarrow$

$(v_{By})_2 = (v_{By})_1 = 1.602 \text{ m/s} \uparrow$

$(v_B)_2 = \sqrt{(4.330)^2 + (1.602)^2} = 4.62 \text{ m/s}$ **Ans.**

$\theta_2 = \tan^{-1}\left(\frac{1.602}{4.330}\right) = 20.3°$ ⦥ **Ans.**



(c)

$(+\uparrow)\quad s = s_0 + v_{Bt} + \frac{1}{2} a_c t^2$

$-2.643 = 0 + 1.602(t) - \frac{1}{2}(9.81)(t)^2$

$t = 0.9153 \text{ s}$

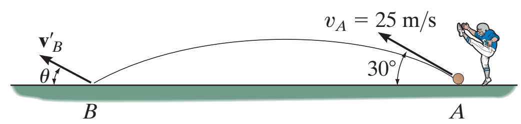
$(\xleftarrow{+})\quad s = s_0 + v_0 t$

$s = 0 + 4.330(0.9153) = 3.96 \text{ m}$ **Ans.**

Ans:
$(v_B)_1 = 8.81$ m/s
$\theta_1 = 10.5°$ ⦣
$(v_B)_2 = 4.62$ m/s
$\theta_2 = 20.3°$ ⦥
$s = 3.96$ m

15–69.

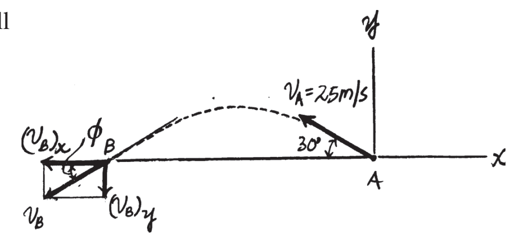
A 300-g ball is kicked with a velocity of $v_A = 25$ m/s at point A as shown. If the coefficient of restitution between the ball and the field is $e = 0.4$, determine the magnitude and direction θ of the velocity of the rebounding ball at B.



SOLUTION

Kinematics: The parabolic trajectory of the football is shown in Fig. *a*. Due to the symmetrical properties of the trajectory, $v_B = v_A = 25$ m/s and $\phi = 30°$.

Conservation of Linear Momentum: Since no impulsive force acts on the football along the x axis, the linear momentum of the football is conserved along the x axis.



(a)

$$\left(\overset{+}{\leftarrow}\right) \qquad m(v_B)_x = m(v'_B)_x$$

$$0.3(25\cos 30°) = 0.3(v'_B)_x$$

$$(v'_B)_x = 21.65 \text{ m/s} \leftarrow$$

Coefficient of Restitution: Since the ground does not move during the impact, the coefficient of restitution can be written as

$$(+\uparrow) \qquad e = \frac{0 - (v'_B)_y}{(v_B)_y - 0}$$

$$0.4 = \frac{-(v'_B)_y}{-25\sin 30°}$$

$$(v'_B)_y = 5 \text{ m/s} \uparrow$$

Thus, the magnitude of $\mathbf{v}'_B$ is

$$v'_B = \sqrt{(v'_B)_x + (v'_B)_y} = \sqrt{21.65^2 + 5^2} = 22.2 \text{ m/s} \qquad \textbf{Ans.}$$

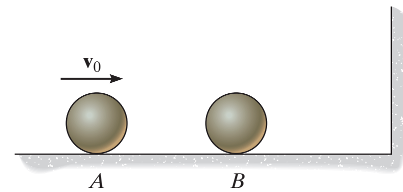
and the angle of $\mathbf{v}'_B$ is

$$\theta = \tan^{-1}\left[\frac{(v'_B)_y}{(v'_B)_x}\right] = \tan^{-1}\left(\frac{5}{21.65}\right) = 13.0° \qquad \textbf{Ans.}$$

Ans:
$v'_B = 22.2$ m/s
$\theta = 13.0°$

15–70.

Two smooth spheres A and B each have a mass m. If A is given a velocity of v_0, while sphere B is at rest, determine the velocity of B just after it strikes the wall. The coefficient of restitution for any collision is e.



SOLUTION

Impact: The first impact occurs when sphere A strikes sphere B. When this occurs, the linear momentum of the system is conserved along the x axis (line of impact). Referring to Fig. a,

$$(\overset{+}{\rightarrow}) \qquad m_A v_A + m_B v_B = m_A (v_A)_1 + m_B (v_B)_1$$

$$m v_0 + 0 = m(v_A)_1 + m(v_B)_1$$

$$(v_A)_1 + (v_B)_1 = v_0 \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_1 - (v_A)_1}{v_A - v_B}$$

$$e = \frac{(v_B)_1 - (v_A)_1}{v_0 - 0}$$

$$(v_B)_1 - (v_A)_1 = e v_0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

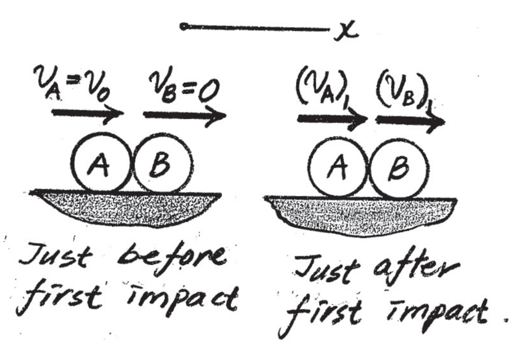
$$(v_B)_1 = \left(\frac{1+e}{2}\right) v_0 \rightarrow \qquad (v_A)_1 = \left(\frac{1-e}{2}\right) v_0 \rightarrow$$

The second impact occurs when sphere B strikes the wall, Fig. b. Since the wall does not move during the impact, the coefficient of restitution can be written as

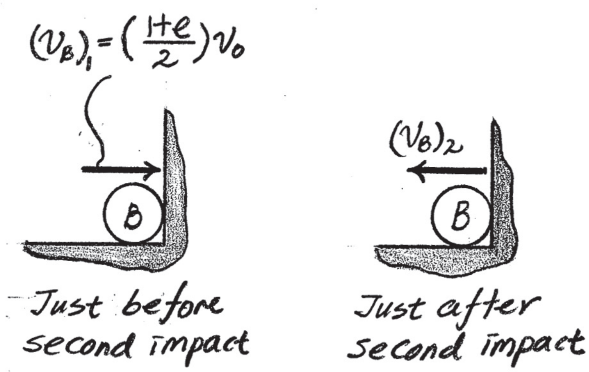
$$(\overset{+}{\rightarrow}) \qquad e = \frac{0 - \left[-(v_B)_2\right]}{(v_B)_1 - 0}$$

$$e = \frac{0 + (v_B)_2}{\left[\frac{1+e}{2}\right] v_0 - 0}$$

$$(v_B)_2 = \frac{e(1+e)}{2} v_0 \qquad \textbf{Ans.}$$



(a)



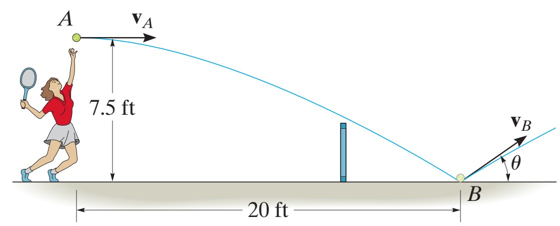
(b)

Ans:

$$(v_B)_2 = \frac{e(1+e)}{2} v_0$$

15–71.

It was observed that a tennis ball when served horizontally 7.5 ft above the ground strikes the smooth ground at B 20 ft away. Determine the initial velocity $\mathbf{v}_A$ of the ball and the velocity $\mathbf{v}_B$ (and θ) of the ball just after it strikes the court at B. Take $e = 0.7$.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$20 = 0 + v_A t$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$7.5 = 0 + 0 + \frac{1}{2}(32.2)t^2$$

$$t = 0.682524$$

$$v_A = 29.303 = 29.3 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_{Bx1} = 29.303 \text{ ft/s}$$

$(+\downarrow)$ $\quad v = v_0 + a_c t$

$$v_{By1} = 0 + 32.2(0.68252) = 21.977 \text{ ft/s}$$

$(\overset{+}{\rightarrow})$ $\quad mv_1 = mv_2$

$$v_{B2x} = v_{B1x} = 29.303 \text{ ft/s} \rightarrow$$

$$e = \frac{v_{By2}}{v_{By1}}$$

$$0.7 = \frac{v_{By2}}{21.977}, \qquad v_{By2} = 15.384 \text{ ft/s} \uparrow$$

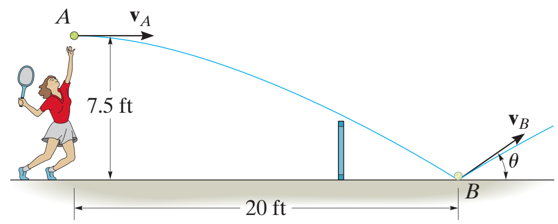
$$v_{B2} = \sqrt{(29.303)^2 + (15.384)^2} = 33.1 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{15.384}{29.303} = 27.7^\circ \measuredangle \qquad \textbf{Ans.}$$

Ans:
$v_A = 29.3$ ft/s
$v_{B2} = 33.1$ ft/s
$\theta = 27.7^\circ \measuredangle$

***15–72.**

The tennis ball is struck with a horizontal velocity $\mathbf{v}_A$, strikes the smooth ground at B, and bounces upward at $\theta = 30°$. Determine the initial velocity $\mathbf{v}_A$, the final velocity $\mathbf{v}_B$, and the coefficient of restitution between the ball and the ground.



SOLUTION

$(+\downarrow)$ $\quad v^2 = v_0^2 + 2\,a_c(s - s_0)$

$$(v_{By})_1^2 = 0 + 2(32.2)(7.5 - 0)$$

$$v_{By1} = 21.9773 \text{ m/s}$$

$(+\downarrow)$ $\quad v = v_0 + a_c t$

$$21.9773 = 0 + 32.2\,t$$

$$t = 0.68252 \text{ s}$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t$

$$20 = 0 + v_A(0.68252)$$

$$v_A = 29.303 = 29.3 \text{ ft/s} \qquad \textbf{Ans.}$$

$(\overset{+}{\rightarrow})$ $\quad mv_1 = mv_2$

$$v_{Bx2} = v_{Bx1} = v_A = 29.303$$

$$v_{B_2} = 29.303/\cos 30° = 33.8 \text{ ft/s} \qquad \textbf{Ans.}$$

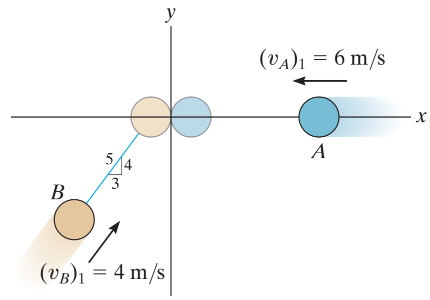
$$v_{By2} = 29.303 \tan 30° = 16.918 \text{ ft/s}$$

$$e = \frac{v_{By2}}{v_{By1}} = \frac{16.918}{21.9773} = 0.770 \qquad \textbf{Ans.}$$

Ans:
$v_A = 29.3$ ft/s
$v_{B_2} = 33.8$ ft/s
$e = 0.770$

15–73.

Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is $e = 0.75$.



SOLUTION

$(\xrightarrow{+})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = 0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$(\xrightarrow{+})$ $\quad e = \dfrac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$

$$0.75 = \frac{(v_A)_{2x} - (v_B)_{2x}}{4\left(\frac{3}{5}\right) - (-6)}$$

$$(v_A)_{2x} = 1.35 \text{ m/s} \rightarrow$$

$$(v_B)_{2x} = 4.95 \text{ m/s} \leftarrow$$

$(+\uparrow)$ $\quad m v_1 = m v_2$

$$0.5\left(\frac{4}{5}\right)(4) = 0.5(v_B)_{2y}$$

$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

$v_A = 1.35 \text{ m/s} \rightarrow$ **Ans.**

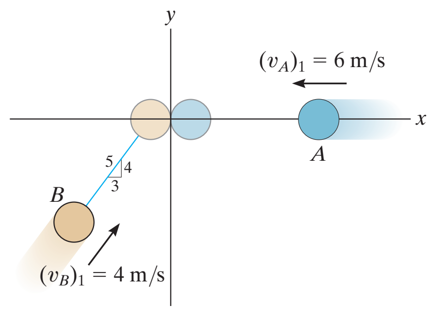
$v_B = \sqrt{(4.59)^2 + (3.20)^2} = 5.89 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\dfrac{3.20}{4.95} = 32.9$ **Ans.**

Ans:
$v_A = 1.35 \text{ m/s} \rightarrow$
$v_B = 5.89 \text{ m/s}$
$\theta = 32.9°$

15–74.

Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision B travels along a line, 30° counterclockwise from the y axis.



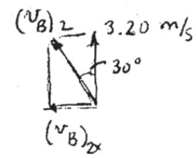
SOLUTION

$$\Sigma m v_1 = \Sigma m v_2$$

$(\overset{+}{\rightarrow})$ $\quad 0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = -0.5(v_B)_{2x} + 0.5(v_A)_{2x}$

$$-3.60 = -(v_B)_{2x} + (v_A)_{2x}$$

$(+\uparrow)$ $\quad 0.5(4)\left(\frac{4}{5}\right) = 0.5(v_B)_{2y}$



$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

$$(v_B)_{2x} = 3.20 \tan 30° = 1.8475 \text{ m/s} \leftarrow$$

$$(v_A)_{2x} = -1.752 \text{ m/s} = 1.752 \text{ m/s} \leftarrow$$

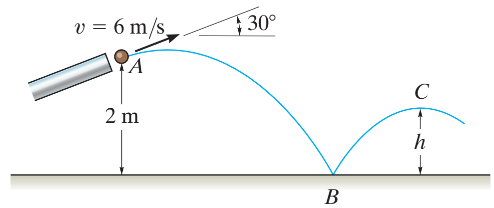
$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$

$$e = \frac{-1.752-(-1.8475)}{4\left(\frac{3}{5}\right)-(-6)} = 0.0113 \qquad \textbf{Ans.}$$

Ans:
$e = 0.0113$

15–75.

The 0.5-kg ball is fired from the tube at A with a velocity of $v = 6$ m/s. If the coefficient of restitution between the ball and the surface is $e = 0.8$, determine the height h after it bounces off the surface.



SOLUTION

Kinematics. Consider the vertical motion from A to B.

$$(+\uparrow) \quad (v_B)_y^2 = (v_A)_y^2 + 2a_y[(s_B)_y - (s_A)_y]\,;$$

$$(v_B)_y^2 = (6 \sin 30°)^2 + 2(-9.81)(-2 - 0)$$

$$(v_B)_y = 6.9455 \text{ m/s} \downarrow$$

Coefficient of Restitution. The y-component of the rebounding velocity at B is $(v'_B)_y$ and the ground does not move. Then

$$(+\uparrow) \quad e = \frac{(v_g)_2 - (v'_B)_y}{(v_B)_y - (v_g)_1}; \qquad 0.8 = \frac{0 - (v'_B)_y}{-6.9455 - 0}$$

$$(v'_B)_y = 5.5564 \text{ m/s} \uparrow$$

Kinematics. When the ball reach the maximum height h at C, $(v_c)_y = 0$.

$$(+\uparrow) \quad (v_c)_y^2 = (v'_B)_y^2 + 2a_c[(s_c)_y - (s_B)_y]\,;$$

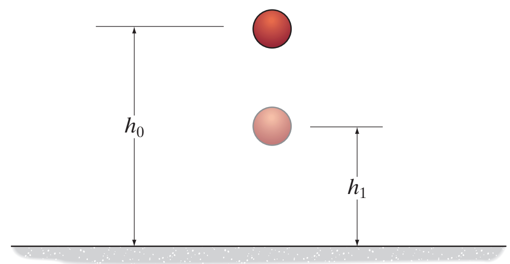
$$0^2 = 5.5564^2 + 2(-9.81)(h - 0)$$

$$h = 1.574 \text{ m} = 1.57 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$h = 1.57$ m

***15–76.**

A ball of mass m is dropped vertically from a height h_0 above the ground. If it rebounds to a height of h_1, determine the coefficient of restitution between the ball and the ground.



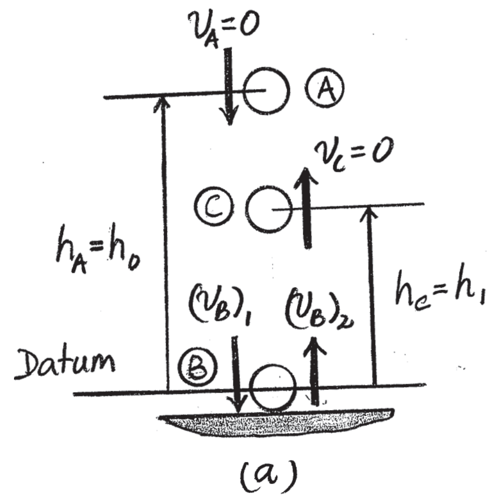
SOLUTION

Conservation of Energy: First, consider the ball's fall from position A to position B. Referring to Fig. a,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^{\,2} + (V_g)_A = \frac{1}{2}mv_B^{\,2} + (V_g)_B$$

$$0 + mg(h_0) = \frac{1}{2}m(v_B)_1^{\,2} + 0$$



Subsequently, the ball's return from position B to position C will be considered.

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^{\,2} + (V_g)_B = \frac{1}{2}mv_C^{\,2} + (V_g)_C$$

$$\frac{1}{2}m(v_B)_2^{\,2} + 0 = 0 + mgh_1$$

$$(v_B)_2 = \sqrt{2gh_1}\uparrow$$

Coefficient of Restitution: Since the ground does not move,

$$(+\uparrow) \qquad e = -\frac{(v_B)_2}{(v_B)_1}$$

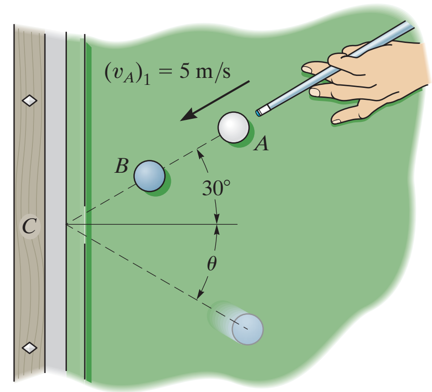
$$e = -\frac{\sqrt{2gh_1}}{-\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}} \qquad \textbf{Ans.}$$

Ans:

$$e = \sqrt{\frac{h_1}{h_0}}$$

15–77.

The cue ball A is given an initial velocity $(v_A)_1 = 5$ m/s. If it makes a direct collision with ball B ($e = 0.8$), determine the velocity of B and the angle θ just after it rebounds from the cushion at C ($e' = 0.6$). Each ball has a mass of 0.4 kg. Neglect their size.



SOLUTION

Conservation of Momentum: When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$0.4(5) + 0 = 0.4(v_A)_2 + 0.4(v_B)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.8 = \frac{(v_B)_2 - (v_A)_2}{5 - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$(v_A)_2 = 0.500$ m/s $\quad (v_B)_2 = 4.50$ m/s

Conservation of "y" Momentum: When ball B strikes the cushion at C, we have

$$m_B(v_{B_y})_2 = m_B(v_{B_y})_3$$

$$(+\downarrow) \qquad 0.4(4.50 \sin 30°) = 0.4(v_B)_3 \sin\theta$$

$$(v_B)_3 \sin\theta = 2.25 \qquad \textbf{(3)}$$

Coefficient of Restitution (x):

$$e = \frac{(v_C)_2 - (v_{B_x})_3}{(v_{B_x})_2 - (v_C)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.6 = \frac{0 - [-(v_B)_3 \cos\theta]}{4.50 \cos 30° - 0} \qquad \textbf{(4)}$$

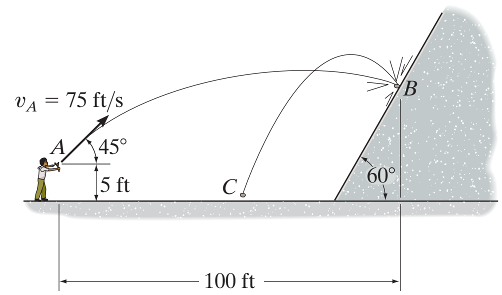
Solving Eqs. (1) and (2) yields

$(v_B)_3 = 3.24$ m/s $\quad \theta = 43.9°$ **Ans.**

Ans:
$(v_B)_3 = 3.24$ m/s
$\theta = 43.9°$

15–78.

Using a slingshot, the boy fires the 0.2-lb marble at the concrete wall, striking it at B. If the coefficient of restitution between the marble and the wall is $e = 0.5$, determine the speed of the marble after it rebounds from the wall.



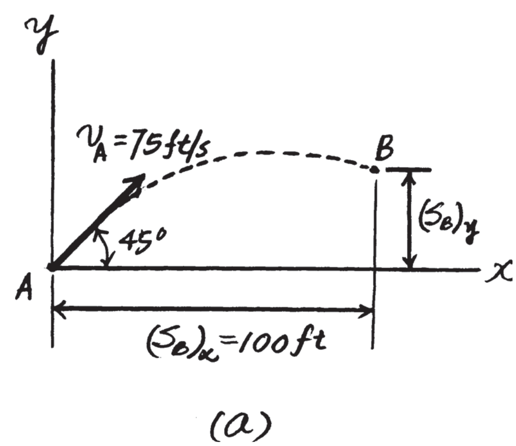
SOLUTION

Kinematics: By considering the x and y motion of the marble from A to B, Fig. *a*,

$$\left(\xrightarrow{+}\right) \qquad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$100 = 0 + 75 \cos 45° \, t$$

$$t = 1.886 \text{ s}$$



and

$$(+\uparrow) \qquad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$(s_B)_y = 0 + 75 \sin 45°(1.886) + \frac{1}{2}(-32.2)(1.886^2)$$

$$= 42.76 \text{ ft}$$

and

$$(+\uparrow) \qquad (v_B)_y = (v_A)_y + a_y t$$

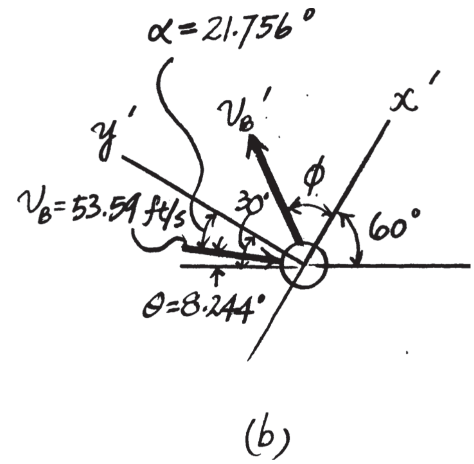
$$(v_B)_y = 75 \sin 45° + (-32.2)(1.886) = -7.684 \text{ ft/s} = 7.684 \text{ ft/s} \downarrow$$

Since $(v_B)_x = (v_A)_x = 75 \cos 45° = 53.03$ ft/s, the magnitude of $\mathbf{v}_B$ is

$$v_B = \sqrt{(v_B)_x^{\,2} + (v_B)_y^{\,2}} = \sqrt{53.03^2 + 7.684^2} = 53.59 \text{ ft/s}$$

and the direction angle of $\mathbf{v}_B$ is

$$\theta = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{7.684}{53.03}\right) = 8.244°$$



Conservation of Linear Momentum: Since no impulsive force acts on the marble along the inclined surface of the concrete wall (x' axis) during the impact, the linear momentum of the marble is conserved along the x' axis. Referring to Fig. *b*,

$$(+\nearrow) \qquad m_B(v'_B)_{x'} = m_B(v'_B)_{x'}$$

$$\frac{0.2}{32.2}(53.59 \sin 21.756°) = \frac{0.2}{32.2}(v'_B \cos \phi)$$

$$v'_B \cos \phi = 19.862 \qquad \textbf{(1)}$$

15–78. Continued

Coefficient of Restitution: Since the concrete wall does not move during the impact, the coefficient of restitution can be written as

$$(+\nwarrow) \qquad e = \frac{0-\left(v'_B\right)_{y'}}{\left(v'_B\right)_{y'} - 0}$$

$$0.5 = \frac{-v'_B \sin\phi}{-53.59 \cos 21.756°}$$

$$v'_B \sin\phi = 24.885 \qquad \textbf{(2)}$$

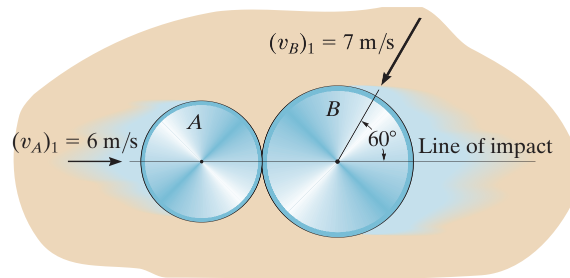
Solving Eqs. (1) and (2) yields

$$v'_B = 31.8 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v'_B = 31.8$ ft/s

15–79.

The two disks A and B have a mass of 3 kg and 5 kg, respectively. If they collide with the initial velocities shown, determine their velocities just after impact. The coefficient of restitution is $e = 0.65$.



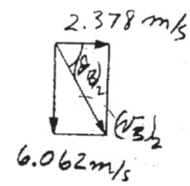
SOLUTION

$(v_{Ax}) = 6$ m/s $\qquad (v_{Ay})_1 = 0$

$(v_{Bx})_1 = -7\cos 60° = -3.5$ m/s $\qquad (v_{By})_1 = -7\cos 60° = -6.062$ m/s

$$\left(\overset{+}{\rightarrow}\right) \qquad m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$

$$3(6) - 5(3.5) = 3(v_A)_{x2} + 5(v_B)_{x2}$$



$$\left(\overset{+}{\rightarrow}\right) \qquad e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}; \qquad 0.65 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{6 - (-3.5)}$$

$(v_{Bx})_2 - (v_{Ax})_2 = 6.175$

Solving,

$(v_{Ax})_2 = -3.80$ m/s $\qquad (v_{Bx})_2 = 2.378$ m/s

$$(+\uparrow) \qquad m_A\left(v_{Ay}\right)_1 + m_A\left(v_{Ay}\right)_2$$

$$\left(v_{Ay}\right)_2 = 0$$

$$(+\uparrow) \qquad m_B\left(v_{By}\right)_1 + m_B\left(v_{Ay}\right)_2$$

$$\left(v_{By}\right)_2 = -6.062 \text{ m/s}$$

$(v_A)_2 = \sqrt{(3.80)^2 + (0)^2 = } = 3.80$ m/s $\leftarrow$ **Ans.**

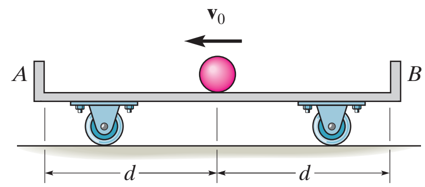
$(v_B)_2 = \sqrt{(2.378)^2 + (-6.062)^2} = 6.51$ m/s **Ans.**

$(\theta_B)_2 = \tan^{-1}\left(\frac{6.062}{2.378}\right) = 68.6°$ **Ans.**

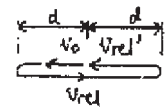
Ans:
$(v_A)_2 = 3.80$ m/s $\leftarrow$
$(v_B)_2 = 6.51$ m/s
$(\theta_B)_2 = 68.6°$

***15–80.**

A ball of negligible size and mass m is given a velocity of $\mathbf{v}_0$ on the center of the cart which has a mass M and is originally at rest. If the coefficient of restitution between the ball and walls A and B is e, determine the velocity of the ball and the cart just after the ball strikes A. Also, determine the total time needed for the ball to strike A, rebound, then strike B, and rebound and then return to the center of the cart. Neglect friction.



SOLUTION



After the first collision;

$$(\overset{+}{\leftarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$0 + m v_0 = m v_b + M v_c$$

$$(\overset{+}{\leftarrow}) \qquad e = \frac{v_c - v_b}{v_0}$$

$$m v_0 = m v_b + \frac{M}{m} v_c$$

$$e v_0 = v_c - v_b$$

$$v_0(1 + e) = \left(1 + \frac{M}{m}\right) v_c$$

$$v_c = \frac{v_0(1 + e)m}{(m + M)} \qquad \textbf{Ans.}$$

$$v_b = \frac{v_0(1 + e)m}{(m + M)} - e v_0$$

$$= v_0\left[\frac{m + me - em - eM}{m + M}\right]$$

$$= v_0\left(\frac{m - eM}{m + M}\right) \qquad \textbf{Ans.}$$

The relative velocity on the cart after the first collision is

$$e = \frac{v_{\text{ref}}}{v_0}$$

$$v_{\text{ref}} = e v_0$$

Similarly, the relative velocity after the second collision is

$$e = \frac{v_{\text{ref}}}{e v_0}$$

$$v_{ref} = e^2 v_0$$

Total time is

$$t = \frac{d}{v_0} + \frac{2d}{e v_0} + \frac{d}{e^2 v_0}$$

$$= \frac{d}{v_0}\left(1 + \frac{1}{e}\right)^2 \qquad \textbf{Ans.}$$

Ans:

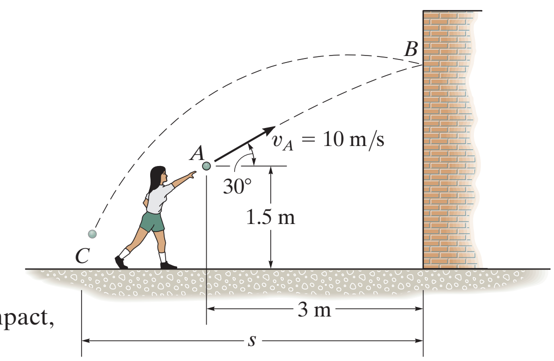
$$v_c = \frac{v_0(1 + e)m}{(m + M)}$$

$$v_b = v_0\left(\frac{m - eM}{m + M}\right)$$

$$t = \frac{d}{v_0}\left(1 + \frac{1}{e}\right)^2$$

15–81.

The girl throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10$ m/s. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if the coefficient of restitution $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



SOLUTION

Kinematics: By considering the horizontal motion of the ball before the impact, we have

$$(\xrightarrow{+}) \qquad s_x = (s_0)_x + v_x t$$

$$3 = 0 + 10 \cos 30° t \qquad t = 0.3464 \text{ s}$$

By considering the vertical motion of the ball before the impact, we have

$$(+\uparrow) \qquad v_y = (v_0)_y + (a_c)_y t$$

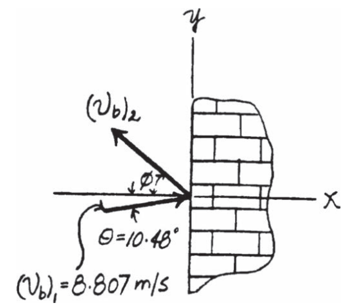
$$= 10 \sin 30° + (-9.81)(0.3464)$$

$$= 1.602 \text{ m/s}$$

The vertical position of point B above the ground is given by

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$(s_B)_y = 1.5 + 10 \sin 30°(0.3464) + \frac{1}{2}(-9.81)\left(0.3464^2\right) = 2.643 \text{ m}$$



Thus, the magnitude of the velocity and its directional angle are

$$(v_b)_1 = \sqrt{(10 \cos 30°)^2 + 1.602^2} = 8.807 \text{ m/s} = 8.81 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1} \frac{1.602}{10 \cos 30°} = 10.48° = 10.5° \qquad \textbf{Ans.}$$

Conservation of "y" Momentum: When the ball strikes the wall with a speed of $(v_b)_1 = 8.807$ m/s, it rebounds with a speed of $(v_b)_2$.

$$m_b \left(v_{b_y}\right)_1 = m_b \left(v_{b_y}\right)_2$$

$$(\xleftarrow{+}) \qquad m_b (1.602) = m_b \left[(v_b)_2 \sin \phi\right]$$

$$(v_b)_2 \sin \phi = 1.602 \qquad \textbf{(1)}$$

Coefficient of Restitution (x):

$$e = \frac{(v_w)_2 - \left(v_{b_x}\right)_2}{\left(v_{b_x}\right)_1 - (v_w)_1}$$

$$(\xrightarrow{+}) \qquad 0.5 = \frac{0 - \left[-(v_b)_2 \cos \phi\right]}{10 \cos 30° - 0} \qquad \textbf{(2)}$$

15–81. Continued

Solving Eqs. (1) and (2) yields

$$\phi = 20.30° = 20.3° \qquad (v_b)_2 = 4.617 \text{ m/s} = 4.62 \text{ m/s} \qquad \textbf{Ans.}$$

Kinematics: By considering the vertical motion of the ball after the impact, we have

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$-2.643 = 0 + 4.617 \sin 20.30° t_1 + \frac{1}{2}(-9.81)t_1^2$$

$$t_1 = 0.9153 \text{ s}$$

By considering the horizontal motion of the ball after the impact, we have

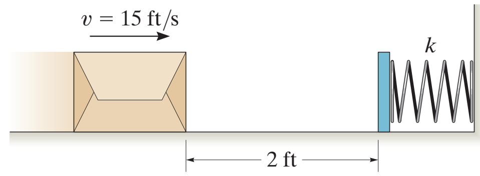
$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$s = 0 + 4.617 \cos 20.30°(0.9153) = 3.96 \text{ m} \qquad \textbf{Ans.}$$

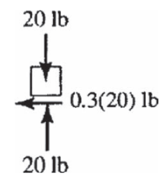
Ans:
(a) $(v_B)_1 = 8.81 \text{ m/s}, \theta = 10.5°$
(b) $(v_B)_2 = 4.62 \text{ m/s}, \phi = 20.3°$
(c) $s = 3.96$ m

15–82.

The 20-lb box slides on the surface for which $\mu_k = 0.3$. The box has a velocity $v = 15$ ft/s when it is 2 ft from the plate. If it strikes the smooth plate, which has a weight of 10 lb and is held in position by an unstretched spring of stiffness $k = 400$ lb/ft, determine the maximum compression imparted to the spring. Take $e = 0.8$ between the box and the plate. Assume that the plate slides smoothly.



SOLUTION



$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{20}{32.2}\right)(15)^2 - (0.3)(20)(2) = \frac{1}{2}\left(\frac{20}{32.2}\right)(v_2)^2$$

$$v_2 = 13.65 \text{ ft/s}$$

$$(\overset{+}{\rightarrow}) \qquad \sum mv_1 = \sum mv_2$$

$$\left(\frac{20}{32.2}\right)(13.65) = \left(\frac{20}{32.2}\right)v_A + \frac{10}{32.2}v_B$$

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.8 = \frac{v_P - v_A}{13.65}$$

Solving,

$$v_P = 16.38 \text{ ft/s}, \quad v_A = 5.46 \text{ ft/s}$$

$$T_1 + V_1 = T_2 + V_2$$

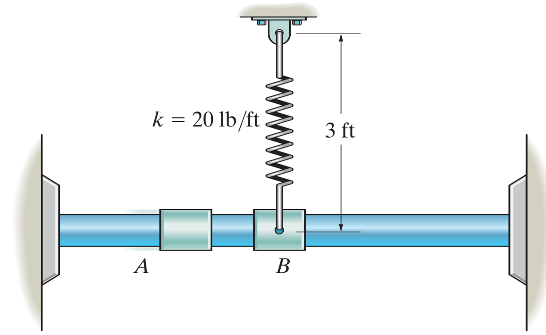
$$\frac{1}{2}\left(\frac{10}{32.2}\right)(16.38)^2 + 0 = 0 + \frac{1}{2}(400)(s)^2$$

$$s = 0.456 \text{ ft}$$ **Ans.**

Ans:
$s = 0.456$ ft

15–83.

The 10-lb collar B is at rest, and when it is in the position shown the spring is unstretched. If another 1-lb collar A strikes it so that B slides 4 ft on the smooth rod before momentarily stopping, determine the velocity of A just after impact, and the average force exerted between A and B during the impact if the impact occurs in 0.002 s. The coefficient of restitution between A and B is $e = 0.5$.



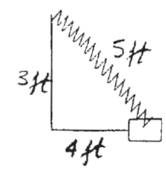
SOLUTION

Collar B after impact:

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(v_B)_2^2 + 0 = 0 + \frac{1}{2}(20)(5-3)^2$$

$$(v_B)_2 = 16.05 \text{ ft/s}$$

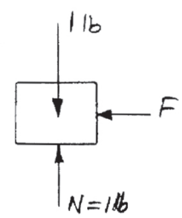


System:

$$(\overset{+}{\rightarrow}) \qquad \Sigma m_1 v_1 = \Sigma m_1 v_2$$

$$\frac{1}{32.2}(v_A)_1 + 0 = \frac{1}{32.2}(v_A)_2 + \frac{10}{32.2}(16.05)$$

$$(v_A)_1 - (v_A)_2 = 160.5$$



$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.5 = \frac{16.05 - (v_A)_2}{(v_A)_1 - 0}$$

$$0.5(v_A)_1 + (v_A)_2 = 16.05$$

Solving:

$$(v_A)_1 = 117.7 \text{ ft/s} = 118 \text{ ft/s} \rightarrow$$

$$(v_A)_2 = -42.8 \text{ ft/s} = 42.8 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Collar A:

$$(\overset{+}{\rightarrow}) \qquad mv_1 + \Sigma \int F\,dt = mv_2$$

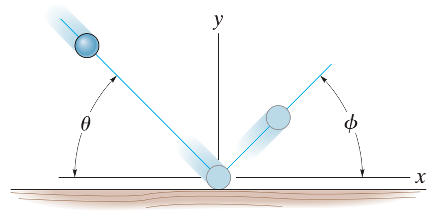
$$\left(\frac{1}{32.2}\right)(117.7) - F(0.002) = \left(\frac{1}{32.2}\right)(-42.8)$$

$$F = 2492.2 \text{ lb} = 2.49 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$(v_A)_2 = 42.8$ ft/s $\leftarrow$
$F = 2.49$ kip

***15–84.**

A ball is thrown onto a rough floor at an angle θ. If it rebounds at an angle ϕ and the coefficient of kinetic friction is μ, determine the coefficient of restitution e. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.

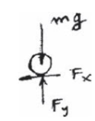


SOLUTION

$(+\downarrow)$ $$e = \frac{0 - [-v_2 \sin\phi]}{v_1 \sin\theta - 0} \qquad e = \frac{v_2 \sin\phi}{v_1 \sin\theta} \qquad \textbf{(1)}$$

$(\xrightarrow{+})$ $$m(v_x)_1 + \int_{t_1}^{t_2} F_x\, dx = m(v_x)_2$$

$$mv_1 \cos\theta - F_x \Delta t = mv_2 \cos\phi$$



$$F_x = \frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} \qquad \textbf{(2)}$$

$(+\downarrow)$ $$m(v_y)_1 + \int_{t_1}^{t_2} F_y\, dx = m(v_y)_2$$

$$mv_1 \sin\theta - F_y \Delta t = -mv_2 \sin\phi$$

$$F_y = \frac{mv_1 \sin\theta + mv_2 \sin\phi}{\Delta t} \qquad \textbf{(3)}$$

Since $F_x = \mu F_y$, from Eqs. (2) and (3)

$$\frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} = \frac{\mu(mv_1 \sin\theta + mv_2 \sin\phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi} \qquad \textbf{(4)}$$

Substituting Eq. (4) into (1) yields:

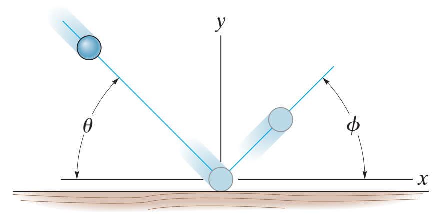
$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi}\right) \qquad \textbf{Ans.}$$

Ans:

$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi}\right)$$

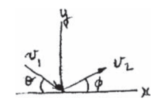
15–85.

A ball is thrown onto a rough floor at an angle of $\theta = 45°$. If it rebounds at the same angle $\phi = 45°$, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is $e = 0.6$. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x\,\Delta t = \mu F_y\,\Delta t$ or $F_x = \mu F_y$.



SOLUTION

$$(+\downarrow) \qquad e = \frac{0 - [-v_2 \sin \phi]}{v_1 \sin \theta - 0} \qquad e = \frac{v_2 \sin \phi}{v_1 \sin \theta} \qquad \textbf{(1)}$$



$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \int_{t_1}^{t_2} F_x dx = m(v_x)_2$$

$$m v_1 \cos \theta - F_x \Delta t = m v_2 \cos \phi$$

$$F_x = \frac{m v_1 \cos \theta - m v_2 \cos \phi}{\Delta t} \qquad \textbf{(2)}$$

$$(+\uparrow) \qquad m\left(v_y\right)_1 + \int_{t_1}^{t_2} F_y dx = m\left(v_y\right)_2$$

$$m v_1 \sin \theta - F_y \Delta t = -m v_2 \sin \phi$$

$$F_y = \frac{m v_1 \sin \theta + m v_2 \sin \phi}{\Delta t} \qquad \textbf{(3)}$$

Since $F_x = \mu F_y$, from Eqs. (2) and (3)

$$\frac{m v_1 \cos \theta - m v_2 \cos \phi}{\Delta t} = \frac{\mu(m v_1 \sin \theta + m v_2 \sin \phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi} \qquad \textbf{(4)}$$

Substituting Eq. (4) into (1) yields:

$$e = \frac{\sin \phi}{\sin \theta}\left(\frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi}\right)$$

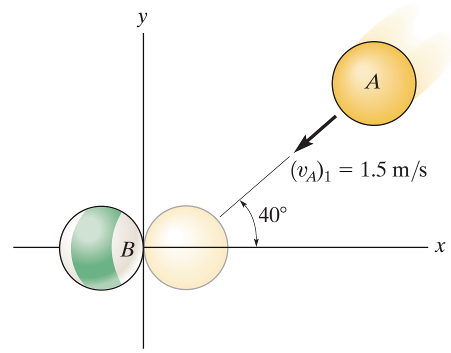
$$0.6 = \frac{\sin 45°}{\sin 45°}\left(\frac{\cos 45° - \mu \sin 45°}{\mu \sin 45° + \cos 45°}\right)$$

$$0.6 = \frac{1-\mu}{1+\mu} \qquad \mu = 0.25 \qquad \textbf{Ans.}$$

Ans:
$\mu_k = 0.25$

15–86.

Two smooth billiard balls A and B each have a mass of 200 g. If A strikes B with a velocity $(v_A)_1 = 1.5\text{ m/s}$ as shown, determine their final velocities just after collision. Ball B is originally at rest and the coefficient of restitution is $e = 0.85$. Neglect the size of each ball.



SOLUTION

$$(v_{A_x})_1 = -1.5\cos 40° = -1.1491\text{ m/s}$$

$$(v_{A_y})_1 = -1.5\sin 40° = -0.9642\text{ m/s}$$

$$(\overset{+}{\rightarrow})\qquad m_A(v_{A_x})_1 + m_B(v_{B_x})_1 = m_A(v_{A_x})_2 + m_B(v_{B_x})_2$$

$$-0.2(1.1491) + 0 = 0.2(v_{A_x})_2 + 0.2(v_{B_x})_2$$

$$(\overset{+}{\rightarrow})\qquad e = \frac{(v_{A_x})_2 - (v_{B_x})_2}{(v_{B_x})_1 - (v_{A_x})_1};\qquad 0.85 = \frac{(v_{A_x})_2 - (v_{B_x})_2}{1.1491}$$

Solving,

$$(v_{A_x})_2 = -0.08618\text{ m/s}$$

$$(v_{B_x})_2 = -1.0629\text{ m/s}$$

For A:

$$(+\downarrow)\qquad m_A(v_{A_y})_1 = m_A(v_{A_y})_2$$

$$(v_{A_y})_2 = 0.9642\text{ m/s}$$

For B:

$$(+\uparrow)\qquad m_B(v_{B_y})_1 = m_B(v_{B_y})_2$$

$$(v_{B_y})_2 = 0$$

Hence.

$$(v_B)_2 = (v_{B_x})_2 = 1.06\text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

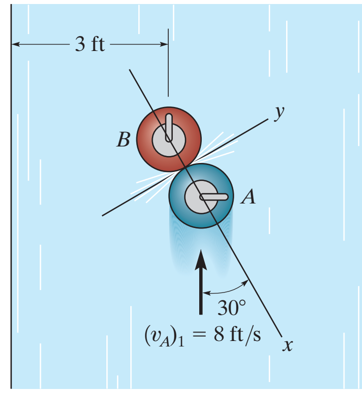
$$(v_A)_2 = \sqrt{(-0.08618)^2 + (0.9642)^2} = 0.968\text{ m/s} \qquad \textbf{Ans.}$$

$$(\theta_A)_2 = \tan^{-1}\left(\frac{0.08618}{0.9642}\right) = 5.11° \measuredangle \qquad \textbf{Ans.}$$

Ans:
$(v_B)_2 = 1.06\text{ m/s} \leftarrow$
$(v_A)_2 = 0.968\text{ m/s}$
$(\theta_A)_2 = 5.11° \measuredangle$

15–87.

The "stone" A used in the sport of curling slides over the ice track and strikes another "stone" B as shown. If each "stone" is smooth and has a weight of 47 lb, and the coefficient of restitution between the "stones" is $e = 0.8$, determine their speeds just after collision. Initially A has a velocity of 8 ft/s and B is at rest. Neglect friction.



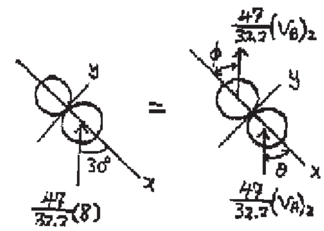
SOLUTION

Line of impact (x-axis):

$\Sigma m v_1 = \Sigma m v_2$

$$(+\nwarrow) \qquad 0 + \frac{47}{32.2}(8)\cos 30^\circ = \frac{47}{32.2}(v_B)_{2x} + \frac{47}{32.2}(v_A)_{2x}$$

$$(+\nwarrow) \qquad e = 0.8 = \frac{(v_B)_{2x} - (v_A)_{2x}}{8\cos 30^\circ - 0}$$



Solving:

$$(v_A)_{2x} = 0.6928 \text{ ft/s}$$

$$(v_B)_{2x} = 6.235 \text{ ft/s}$$

Plane of impact (y-axis):

Stone A:

$m v_1 = m v_2$

$$(\nearrow +) \qquad \frac{47}{32.2}(8)\sin 30^\circ = \frac{47}{32.2}(v_A)_{2y}$$

$$(v_A)_{2y} = 4$$

Stone B:

$m v_1 = m v_2$

$$(\nearrow +) \qquad 0 = \frac{47}{32.2}(v_B)_{2y}$$

$$(v_B)_{2y} = 0$$

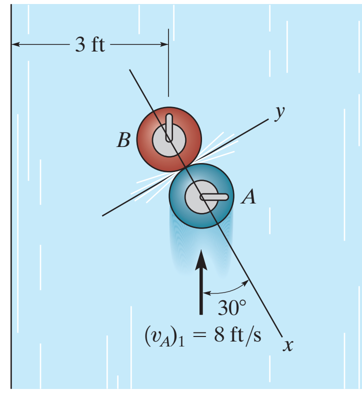
$$(v_A)_2 = \sqrt{(0.6928)^2 + (4)^2} = 4.06 \text{ ft/s} \qquad \textbf{Ans.}$$

$$(v_B)_2 = \sqrt{(0)^2 + (6.235)^2} = 6.235 = 6.24 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$(v_A)_2 = 4.06$ ft/s
$(v_B)_2 = 6.24$ ft/s

***15–88.**

The "stone" A used in the sport of curling slides over the ice track and strikes another "stone" B as shown. If each "stone" is smooth and has a weight of 47 lb, and the coefficient of restitution between the "stone" is $e = 0.8$, determine the time required just after collision for B to slide off the runway. This requires the horizontal component of displacement to be 3 ft.



SOLUTION

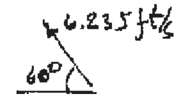
See solution to Prob. 15–87.

$$(v_B)_2 = 6.235 \text{ ft/s}$$

$$s = s_0 + v_0 t$$

$$3 = 0 + (6.235 \cos 60°)t$$

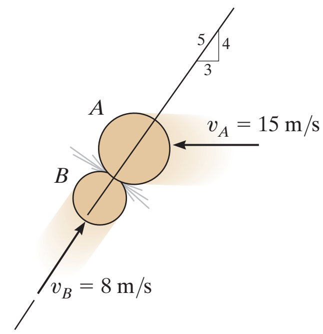
$$t = 0.962 \text{ s} \qquad \textbf{Ans.}$$



Ans:
$t = 0.962$ s

15–89.

Two smooth disks A and B have the initial velocities shown just before they collide. If they have masses $m_A = 4$ kg and $m_B = 2$ kg, determine their speeds just after impact. The coefficient of restitution is $e = 0.8$.



SOLUTION

Impact. The line of impact is along the line joining the centers of disks A and B represented by y axis in Fig. a. Thus

$$[(v_A)_1]_y = 15\left(\frac{3}{5}\right) = 9 \text{ m/s} \swarrow \qquad [(v_A)_1]_x = 15\left(\frac{4}{5}\right) = 12 \text{ m/s} \nwarrow$$

$$[(v_B)_1]_y = 8 \text{ m/s} \nearrow \qquad [(v_B)_1]_x = 0$$

Coefficient of Restitution. Along the line of impact (y axis),

$$(+\nearrow) \quad e = \frac{[(v_B)_2]_y - [(v_A)_2]_y}{[(v_A)_1]_y - [(v_B)_1]_y}; \qquad 0.8 = \frac{[(v_B)_2]_y - [(v_A)_2]_y}{-9 - 8}$$

$$[(v_A)_2]_y - [(v_B)_2]_y = 13.6 \qquad \textbf{(1)}$$

Conservation of 'y' Momentum.

$$(+\nearrow) \quad m_A[(v_A)_1]_y + m_B[(v_B)_1]_y = m_A[(v_A)_2]_y + m_B[(v_B)_2]_y$$

$$4(-9) + 2(8) = 4[(v_A)_2]_y + 2[(v_B)_2]_y$$

$$2[(v_A)_2]_y + [(v_B)_2]_y = -10 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$[(v_A)_2]_y = 1.20 \text{ m/s} \nearrow \qquad [(v_B)_2]_y = -12.4 \text{ m/s} = 12.4 \text{ m/s} \swarrow$$

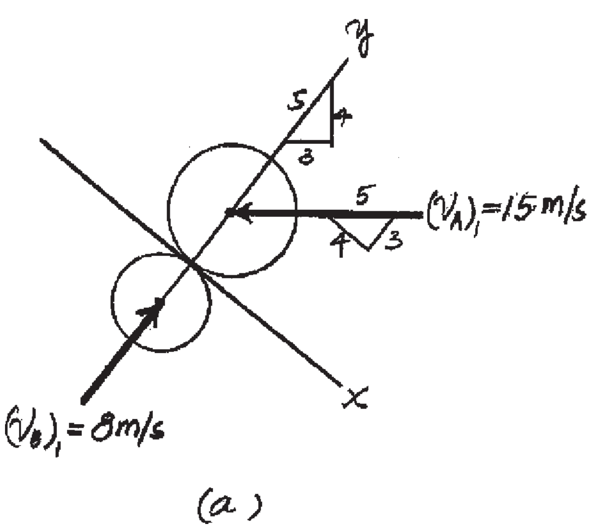
Conservation of 'x' Momentum. Since no impact occurs along the x axis, the component of velocity of each disk remain constant before and after the impact. Thus

$$[(v_A)_2]_x = [(v_A)_1]_x = 12 \text{ m/s} \nwarrow \qquad [(v_B)_2]_x = [(v_B)_1]_x = 0$$

Thus, the magnitude of the velocity of disks A and B just after the impact is

$$(v_A)_2 = \sqrt{[(v_A)_2]_x^2 + [(v_A)_2]_y^2} = \sqrt{12^2 + 1.20^2} = 12.06 \text{ m/s} = 12.1 \text{ m/s} \qquad \textbf{Ans.}$$

$$(v_B)_2 = \sqrt{[(v_B)_2]_x^2 + [(v_B)_2]_y^2} = \sqrt{0^2 + 12.4^2} = 12.4 \text{ m/s} \qquad \textbf{Ans.}$$

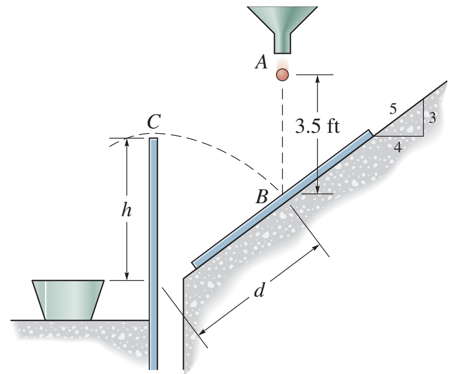


(a)

Ans:
$(v_A)_2 = 12.1$ m/s
$(v_B)_2 = 12.4$ m/s

15–90.

Before a cranberry can make it to your dinner plate, it must pass a bouncing test which rates its quality. If cranberries having an $e \geq 0.8$ are to be accepted, determine the dimensions d and h for the barrier so that when a cranberry falls from rest at A it strikes the incline at B and bounces over the barrier at C.



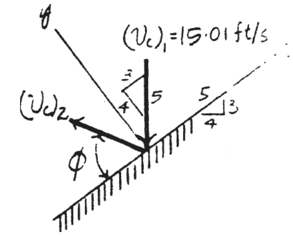
SOLUTION

Conservation of Energy: The datum is set at point B. When the cranberry falls from a height of 3.5 ft *above* the datum, its initial gravitational potential energy is $W(3.5) = 3.5\,W$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 3.5W = \frac{1}{2}\left(\frac{W}{32.2}\right)(v_c)_1^2 + 0$$

$$(v_c)_1 = 15.01 \text{ ft/s}$$

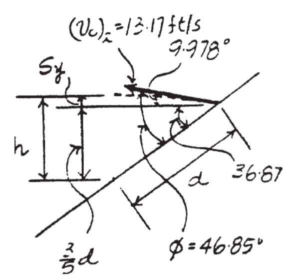


Conservation of "x′" Momentum: When the cranberry strikes the plate with a speed of $(v_c)_1 = 15.01$ ft/s, it rebounds with a speed of $(v_c)_2$.

$$m_c\left(v_{c_{x'}}\right)_1 = m_c\left(v_{c_{x'}}\right)_2$$

$$(+\swarrow) \qquad m_c\,(15.01)\left(\frac{3}{5}\right) = m_c\left[(v_c)_2 \cos \phi\right]$$

$$(v_c)_2 \cos \phi = 9.008 \qquad \textbf{(1)}$$



Coefficient of Restitution (y′):

$$e = \frac{(v_P)_2 - \left(v_{c_{y'}}\right)_2}{\left(v_{c_{y'}}\right)_1 - (v_P)_1}$$

$$(\nwarrow +) \qquad 0.8 = \frac{0 - (v_c)_2 \sin \phi}{-15.01\left(\frac{4}{5}\right) - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\phi = 46.85^\circ \qquad (v_c)_2 = 13.17 \text{ ft/s}$$

Kinematics: By considering the vertical motion of the cranberry after the impact, we have

$$(+\uparrow) \qquad v_y = (v_0)_y + a_c t$$

$$0 = 13.17 \sin 9.978^\circ + (-32.2)t \qquad t = 0.07087 \text{ s}$$

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y\, t^2$$

$$= 0 + 13.17 \sin 9.978^\circ\,(0.07087) + \frac{1}{2}(-32.2)\left(0.07087^2\right)$$

$$= 0.080864 \text{ ft}$$

15–90. Continued

By considering the horizontal motion of the cranberry after the impact, we have

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$\frac{4}{5}d = 0 + 13.17 \cos 9.978° \,(0.07087)$$

$$d = 1.149 \text{ ft} = 1.15 \text{ ft} \qquad \textbf{Ans.}$$

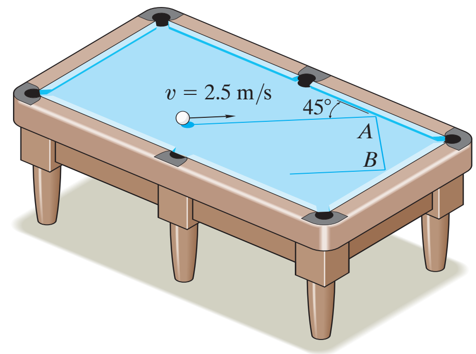
Thus,

$$h = s_y + \frac{3}{5}d = 0.080864 + \frac{3}{5}(1.149) = 0.770 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$d = 1.15$ ft
$h = 0.770$ ft

15–91.

The 200-g billiard ball is moving with a speed of 2.5 m/s when it strikes the side of the pool table at A. If the coefficient of restitution between the ball and the side of the table is $e = 0.6$, determine the speed of the ball just after striking the table twice, i.e., at A, then at B. Neglect the size of the ball.



SOLUTION

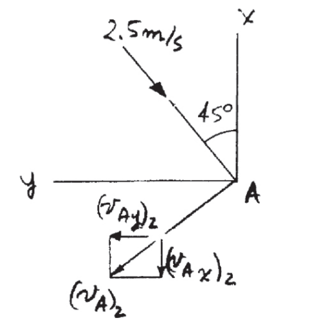
At A:

$$(v_A)_{y1} = 2.5(\sin 45°) = 1.7678 \text{ m/s} \rightarrow$$

$$e = \frac{(v_{A_y})_2}{(v_{A_y})_1}; \qquad 0.6 = \frac{(v_{A_y})_2}{1.7678}$$

$$(v_{A_y})_2 = 1.061 \text{ m/s} \leftarrow$$

$$(v_{A_x})_2 = (v_{A_x})_1 = 2.5 \cos 45° = 1.7678 \text{ m/s} \downarrow$$

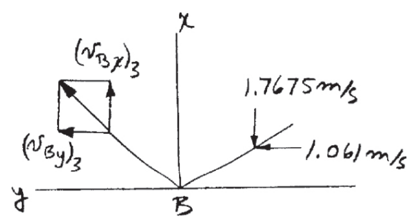


At B:

$$e = \frac{(v_{B_x})_3}{(v_{B_x})_2}; \qquad 0.6 = \frac{(v_{B_x})_3}{1.7678}$$

$$(v_{B_x})_3 = 1.061 \text{ m/s}$$

$$\left(v_{B_y}\right)_3 = \left(v_{A_y}\right)_2 = 1.061 \text{ m/s}$$



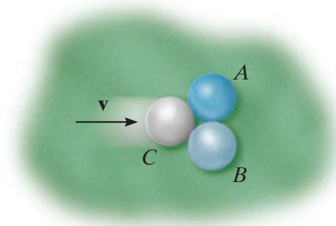
Hence,

$$(v_B)_3 = \sqrt{(1.061)^2 + (1.061)^2} = 1.50 \text{ m/s}$$ **Ans.**

Ans:
$(v_B)_3 = 1.50$ m/s

***15–92.**

The two billiard balls A and B are originally in contact with one another when a third ball C strikes each of them at the same time as shown. If ball C remains at rest after the collision, determine the coefficient of restitution. All the balls have the same mass. Neglect the size of each ball.

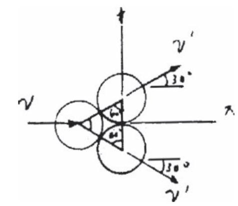


SOLUTION

Conservation of "x" momentum:

$$\left(\overset{+}{\rightarrow}\right) \qquad mv = 2mv' \cos 30^\circ$$

$$v = 2v' \cos 30^\circ \qquad \textbf{(1)}$$

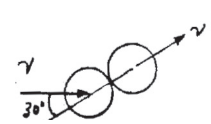


Coefficient of restitution:

$$(+\nearrow) \qquad e = \frac{v'}{v \cos 30^\circ} \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields:

$$e = \frac{v'}{2v' \cos^2 30^\circ} = \frac{2}{3} \qquad \textbf{Ans.}$$

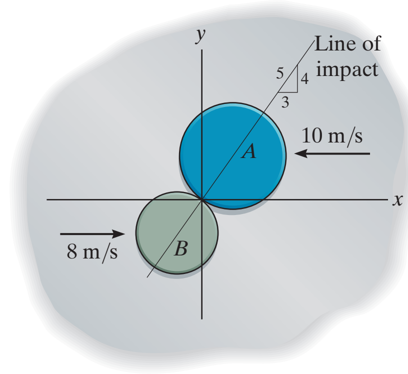


Ans:

$e = \dfrac{2}{3}$

15–93.

Disks A and B have a mass of 15 kg and 10 kg, respectively. If they are sliding on a smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.8$.



SOLUTION

Conservation of Linear Momentum: By referring to the impulse and momentum of the system of disks shown in Fig. *a*, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$+\nearrow \; m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v_A'\right)_n + m_B\left(v_B'\right)_n$$

$$15(10)\left(\frac{3}{5}\right) - 10(8)\left(\frac{3}{5}\right) = 15v_A' \cos\phi_A + 10v_B' \cos\phi_B$$

$$15v_A' \cos\phi_A + 10v_B' \cos\phi_B = 42 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis (tangent to? plane of impact). Thus,

$$+\nwarrow \; m_A\left(v_A\right)_t = m_A\left(v_A'\right)_t$$

$$15(10)\left(\frac{4}{5}\right) = 15v_A' \sin\phi_A$$

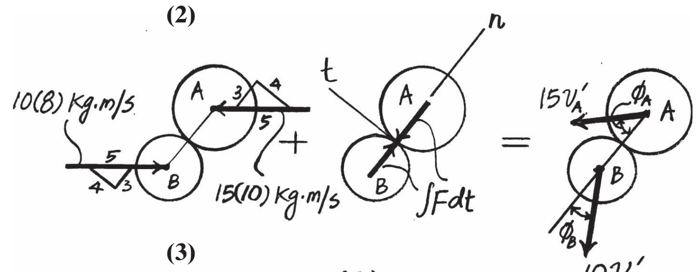
$$v_A' \sin\phi_A = 8 \qquad \textbf{(2)}$$

and

$$+\nwarrow \; m_B\left(v_B\right)_t = m_B\left(v_B'\right)_t$$

$$10(8)\left(\frac{4}{5}\right) = 10\, v_B' \sin\phi_B$$

$$v_B' \sin\phi_B = 6.4 \qquad \textbf{(3)}$$



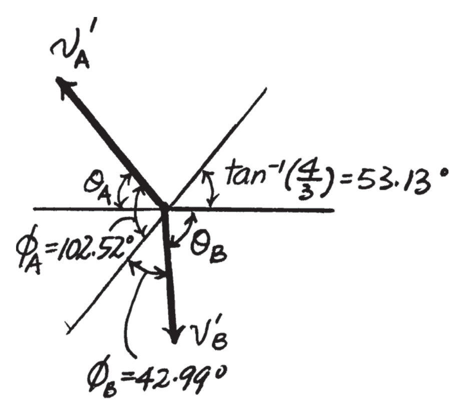
(a)

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$+\nearrow \; e = \frac{(v_B')_n - (v_A')_n}{(v_A)_n - (v_B)_n}$$

$$0.8 = \frac{v_B' \cos\phi_B - v_A' \cos\phi_A}{10\left(\frac{3}{5}\right) - \left[-8\left(\frac{3}{5}\right)\right]}$$

$$v_B' \cos\phi_B - v_A' \cos\phi_A = 8.64 \qquad \textbf{(4)}$$



(b)

Solving Eqs. (1), (2), (3), and (4), yeilds

$v_A' = 8.19$ m/s **Ans.**

$\phi_A = 102.52°$

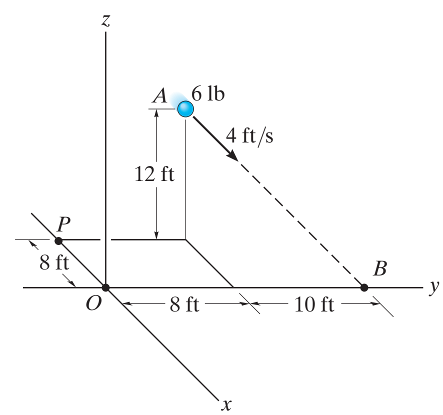
$v_B' = 9.38$ m/s **Ans.**

$\phi_B = 42.99°$

Ans:
$(v_A)_2 = 8.19$ m/s
$(v_B)_2 = 9.38$ m/s

15–94.

Determine the angular momentum $\mathbf{H}_O$ of the 6-lb particle about point O.



SOLUTION

Position and Velocity Vector. The coordinates of points A and B are $A(-8, 8, 12)$ ft and $B(0, 18, 0)$ ft. Then

$$\mathbf{r}_{OB} = \{18\mathbf{j}\}\text{ ft} \qquad \mathbf{r}_{OA} = \{-8\mathbf{i} + 8\mathbf{j} + 12\mathbf{k}\}\text{ ft}$$

$$V_A = v_A\left(\frac{\mathbf{r}_{AB}}{\mathbf{r}_{AB}}\right) = 4\left\{\frac{[0-(-8)]\mathbf{i} + (18-8)\mathbf{j} + (0-12)\mathbf{k}}{\sqrt{[0-(-8)]^2 + (18-8)^2 + (0-12)^2}}\right\}$$

$$= \left\{\frac{32}{\sqrt{308}}\mathbf{i} + \frac{40}{\sqrt{308}}\mathbf{j} - \frac{48}{\sqrt{308}}\mathbf{k}\right\}\text{ ft/s}$$

Angular Momentum about Point O.

$$\boldsymbol{H}_O = \mathbf{r}_{OB} \times m\boldsymbol{V}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 18 & 0 \\ \frac{6}{32.2}\left(\frac{32}{\sqrt{308}}\right) & \frac{6}{32.2}\left(\frac{40}{\sqrt{308}}\right) & \frac{6}{32.2}\left(-\frac{48}{\sqrt{308}}\right) \end{vmatrix}$$

$$= \{-9.1735\mathbf{i} - 6.1156\mathbf{k}\}\text{ slug}\cdot\text{ft}^2/\text{s}$$

$$= \{-9.17\mathbf{i} - 6.12\mathbf{k}\}\text{ slug}\cdot\text{ft}^2/\text{s} \qquad \textbf{Ans.}$$

Also,

$$\boldsymbol{H}_O = \mathbf{r}_{OA} \times m\boldsymbol{V}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 8 & 12 \\ \frac{6}{32.2}\left(\frac{32}{\sqrt{308}}\right) & \frac{6}{32.2}\left(\frac{40}{\sqrt{308}}\right) & \frac{6}{32.2}\left(-\frac{48}{\sqrt{308}}\right) \end{vmatrix}$$

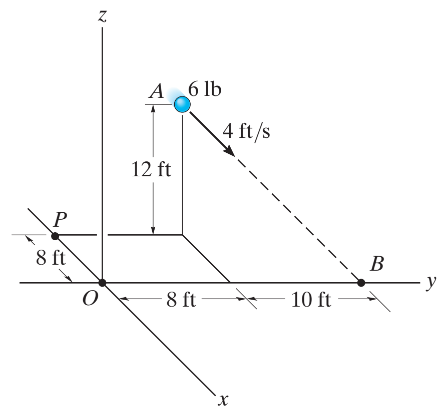
$$= \{-9.1735\mathbf{i} - 6.1156\mathbf{k}\}\text{ slug}\cdot\text{ft}^2/\text{s}$$

$$= \{-9.17\mathbf{i} - 6.12\mathbf{k}\}\text{ slug}\cdot\text{ft}^2/\text{s} \qquad \textbf{Ans.}$$

Ans:
$\{-9.17\mathbf{i} - 6.12\mathbf{k}\}\text{ slug}\cdot\text{ft}^2/\text{s}$

15–95.

Determine the angular momentum $\mathbf{H}_p$ of the 6-lb particle about point P.



SOLUTION

Position and Velocity Vector. The coordinates of points A, B and P are $A(-8, 8, 12)$ ft, $B(0, 18, 0)$ ft and $P(-8, 0, 0)$. Then

$$\boldsymbol{r}_{pB} = [0 - (-8)]\mathbf{i} + (18 - 0)\mathbf{j}] = \{8\mathbf{i} + 18\mathbf{j}\} \text{ ft}$$

$$\boldsymbol{r}_{pA} = [-8 - (-8)]\mathbf{i} + (8 - 0)\mathbf{j} + (12 - 0)\mathbf{j}] = \{8\mathbf{j} + 12\mathbf{k}\} \text{ ft}$$

$$\boldsymbol{V}_A = v_A\left(\frac{\boldsymbol{r}_{AB}}{\boldsymbol{r}_{AB}}\right) = 4\left\{\frac{[0 - (-8)]\mathbf{i} + (18 - 8)\mathbf{j} + (0 - 12)\mathbf{k}}{[0 - (-8)]^2 + (18 - 8)^2 + (0 - 12)^2}\right\}$$

$$= \left\{\frac{32}{\sqrt{308}}\mathbf{i} + \frac{40}{\sqrt{308}}\mathbf{j} - \frac{48}{\sqrt{308}}\mathbf{k}\right\} \text{ ft/s}$$

Angular Momentum about Point P.

$$\boldsymbol{H}_P = \boldsymbol{r}_{pA} \times m\boldsymbol{V}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 12 \\ \frac{6}{32.2}\left(\frac{32}{\sqrt{308}}\right) & \frac{6}{32.2}\left(\frac{40}{\sqrt{308}}\right) & \frac{6}{32.2}\left(-\frac{48}{\sqrt{308}}\right) \end{vmatrix}$$

$$= \{-9.1735\mathbf{i} + 4.0771\mathbf{j} - 2.7181\} \text{ slug} \cdot \text{ft}^2/\text{s}$$

$$= \{-9.17\mathbf{i} + 4.08\mathbf{j} - 2.72\mathbf{k}\} \text{ slug} \cdot \text{ft}^2/\text{s} \qquad \textbf{Ans.}$$

Also,

$$\boldsymbol{H}_P = \boldsymbol{r}_{pB} \times m\boldsymbol{V}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 18 & 0 \\ \frac{6}{32.2}\left(\frac{32}{\sqrt{308}}\right) & \frac{6}{32.2}\left(\frac{40}{\sqrt{308}}\right) & \frac{6}{32.2}\left(-\frac{48}{\sqrt{308}}\right) \end{vmatrix}$$

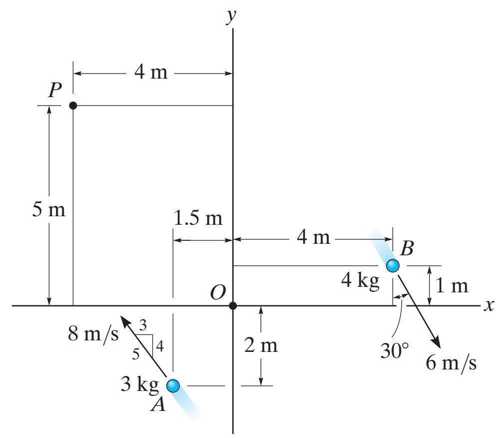
$$= \{-9.1735\mathbf{i} + 4.0771\mathbf{j} - 2.7181\mathbf{k}\} \text{ slug} \cdot \text{ft}^2/\text{s}$$

$$= \{-9.17\mathbf{i} + 4.08\mathbf{j} - 2.72\mathbf{k}\} \text{ slug} \cdot \text{ft}^2/\text{s} \qquad \textbf{Ans.}$$

Ans:
$\{-9.17\mathbf{i} + 4.08\mathbf{j} - 2.72\mathbf{k}\}$ slug $\cdot$ ft^2/s

***15–96.**

Determine the angular momentum $\mathbf{H}_o$ of each of the two particles about point O.



SOLUTION

$$\zeta + (\boldsymbol{H}_A)_O = (-1.5)\left[3(8)\left(\frac{4}{5}\right)\right] - (2)\left[3(8)\left(\frac{3}{5}\right)\right] = -57.6\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$\zeta + (\boldsymbol{H}_B)_O = (-1)[4(6\sin 30^\circ)] - (4)[4\,(6\cos 30^\circ)] = -95.14\ \text{kg}\cdot\text{m}^2/\text{s}$$

Thus

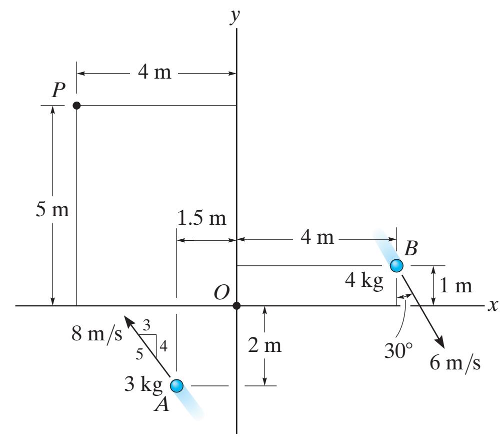
$$(\boldsymbol{H}_A)_O = \{-57.6\,\boldsymbol{k}\}\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

$$(\boldsymbol{H}_B)_O = \{-95.1\,\boldsymbol{k}\}\ \text{kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

Ans:
$(\boldsymbol{H}_A)_O = \{-57.6\,\boldsymbol{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$
$(\boldsymbol{H}_B)_O = \{-95.1\,\boldsymbol{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$

15–97.

Determine the angular momentum $\mathbf{H}_p$ of each of the two particles about point P.



SOLUTION

$$\circlearrowleft + (\boldsymbol{H}_A)_p = (2.5)\left[3(8)\left(\frac{4}{5}\right)\right] - (7)\left[3(8)\left(\frac{3}{5}\right)\right] = -52.8\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$\circlearrowleft + (\boldsymbol{H}_B)_p = (4)[4(6\sin 30°)] - 8[4\,(6\cos 30°)] = -118.28\ \text{kg}\cdot\text{m}^2/\text{s}$$

Thus,

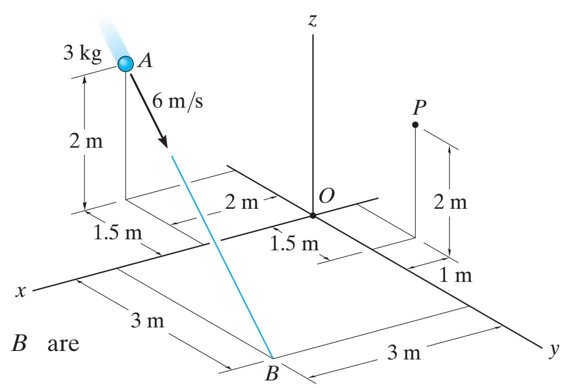
$(\boldsymbol{H}_A)_p = \{-52.8\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

$(\boldsymbol{H}_B)_p = \{-118\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

Ans:
$(H_A)_P = \{-52.8\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$
$(H_B)_P = \{-118\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$

15–98.

Determine the angular momentum $\mathbf{H}_O$ of the 3-kg particle about point O.



SOLUTION

Position and Velocity Vectors. The coordinates of points A and B are $A(2, -1.5, 2)$ m and $B(3, 3, 0)$.

$$\mathbf{r}_{OB} = \{3\mathbf{i} + 3\mathbf{j}\}\text{ m} \qquad \mathbf{r}_{OA} = \{2\mathbf{i} - 1.5\mathbf{j} + 2\mathbf{k}\}\text{ m}$$

$$V_A = v_A\left(\frac{r_{AB}}{r_{AB}}\right) = (6)\left[\frac{(3-2)\mathbf{i} + [3-(-1.5)]\mathbf{j} + (0.2)\mathbf{k}}{\sqrt{(3-2)^2 + [3-(-1.5)]^2 + (0-2)^2}}\right]$$

$$= \left\{\frac{6}{\sqrt{25.25}}\mathbf{i} + \frac{27}{\sqrt{25.25}}\mathbf{j} - \frac{12}{\sqrt{25.25}}\mathbf{k}\right\}\text{ m/s}$$

Angular Momentum about Point O. Applying Eq. 15

$$\mathbf{H}_O = \mathbf{r}_{OB} \times mV_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 0 \\ 3\left(\frac{6}{\sqrt{25.25}}\right) & 3\left(\frac{27}{\sqrt{25.25}}\right) & 3\left(-\frac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

$$= \{-21.4928\mathbf{i} + 21.4928\mathbf{j} + 37.6124\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$

$$= \{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\}\text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Also,

$$\mathbf{H}_O = \mathbf{r}_{OA} \times mV_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1.5 & 2 \\ 3\left(\frac{6}{\sqrt{25.25}}\right) & 3\left(\frac{27}{\sqrt{25.25}}\right) & 3\left(-\frac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

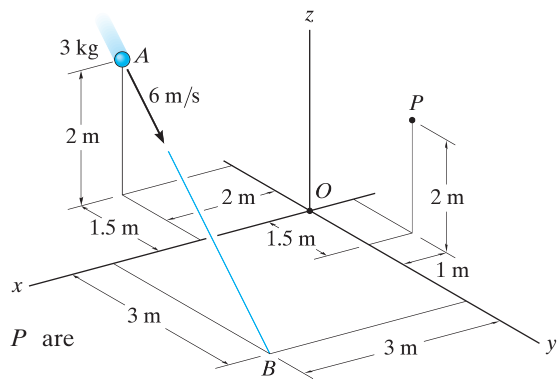
$$= \{-21.4928\mathbf{i} + 21.4928\mathbf{j} + 37.6124\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$

$$= \{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Ans:
$\{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\}\text{ kg}\cdot\text{m}^2/\text{s}$

15–99.

Determine the angular momentum $\mathbf{H}_P$ of the 3-kg particle about point P.



SOLUTION

Position and Velocity Vectors. The coordinates of points A, B and P are $A(2, -1.5, 2)$ m, $B(3, 3, 0)$ m and $P(-1, 1.5, 2)$ m.

$$\boldsymbol{r}_{PA} = [2 - (-1)]\mathbf{i} + (-1.5 - 1.5)\mathbf{j} + (2 - 2)\mathbf{k} = \{3\mathbf{i} - 3\mathbf{j}\}\text{ m}$$

$$\boldsymbol{r}_{PB} = [3 - (-1)]\mathbf{i} + (3 - 1.5)\mathbf{j} + (0 - 2)\mathbf{k} = \{4\mathbf{i} + 1.5\mathbf{j} - 2\mathbf{k}\}\text{ m}$$

$$\boldsymbol{V}_A = v_A\left(\frac{\boldsymbol{r}_{AB}}{r_{AB}}\right) = 6\left[\frac{(3-2)\mathbf{i} + [3-(-1.5)]\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(3-2)^2 + [3-(-1.5)]^2 + (0-2)^2}}\right]$$

$$= \left\{\frac{6}{\sqrt{25.25}}\mathbf{i} + \frac{27}{\sqrt{25.25}}\mathbf{j} - \frac{12}{\sqrt{25.25}}\mathbf{k}\right\}\text{ m/s}$$

Angular Momentum about Point P. Applying Eq. 15

$$\boldsymbol{H}_p = \boldsymbol{r}_{pA} \times m\boldsymbol{V}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 0 \\ 3\left(\dfrac{6}{\sqrt{25.25}}\right) & 3\left(\dfrac{27}{\sqrt{25.25}}\right) & 3\left(-\dfrac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

$$= \{21.4928\mathbf{i} + 21.4928\mathbf{j} + 59.1052\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$

$$= \{21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Also,

$$\boldsymbol{H}_p = \boldsymbol{r}_{PB} \times m\boldsymbol{V}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 1.5 & -2 \\ 3\left(\dfrac{6}{\sqrt{25.25}}\right) & 3\left(\dfrac{27}{\sqrt{25.25}}\right) & 3\left(-\dfrac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

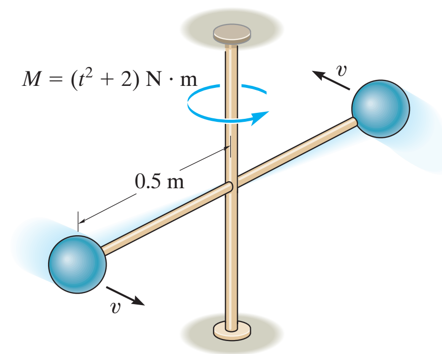
$$= \{21.4928\mathbf{i} + 21.4928\mathbf{j} + 59.1052\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$

$$= \{21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Ans:
$\{21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$

***15–100.**

Each ball has a negligible size and a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (t^2 + 2)\ \text{N}\cdot\text{m}$, where t is in seconds, determine the speed of each ball when $t = 3$ s. Each ball has a speed $v = 2$ m/s when $t = 0$.



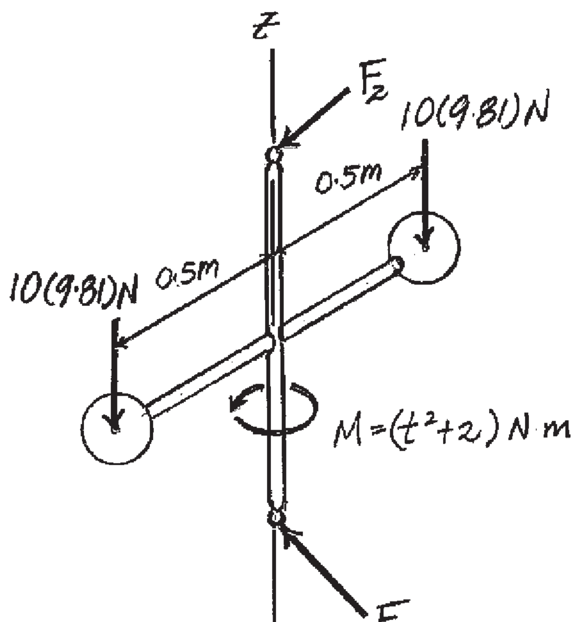
SOLUTION

Principle of Angular Impulse and Momentum. Referring to the FBD of the assembly, Fig. *a*

$$(H_Z)_1 + \Sigma \int_{t_1}^{t_2} M_Z\, dt = (H_Z)_2$$

$$2[0.5(10)(2)] + \int_0^{3\text{ s}} (t^2 + 2)dt = 2[0.5(10v)]$$

$$v = 3.50 \text{ m/s} \qquad \textbf{Ans.}$$

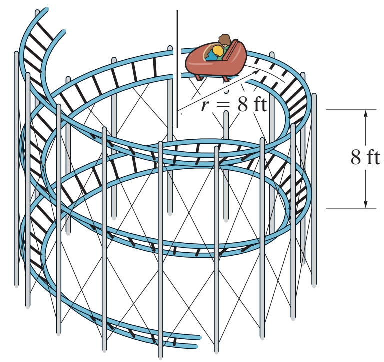


(a)

Ans:
$v = 3.50$ m/s

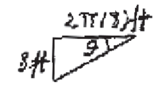
15–101.

The 800-lb roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 8 ft for every one revolution, determine the speed of the car when $t = 4$ s. Also, how far has the car descended in this time? Neglect friction and the size of the car.



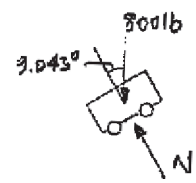
SOLUTION

$$\theta = \tan^{-1}\left(\frac{\theta}{2\pi(8)}\right) = 9.043^\circ$$



$$\Sigma F_y = 0; \qquad N - 800\cos 9.043^\circ = 0$$

$$N = 790.1 \text{ lb}$$



$$H_1 + \int M\,dt = H_2$$

$$0 + \int_0^4 8(790.1\sin 9.043^\circ)dt = \frac{800}{32.2}(8)v_t$$

$$v_t = 20.0 \text{ ft/s}$$

$$v = \frac{20.0}{\cos 9.043^\circ} = 20.2 \text{ ft/s} \qquad \textbf{Ans.}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

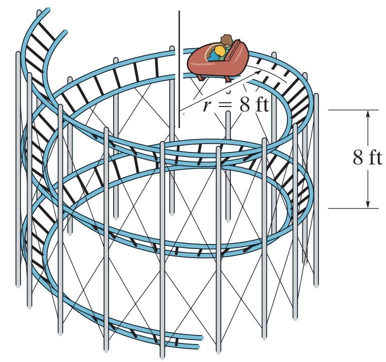
$$0 + 800h = \frac{1}{2}\left(\frac{800}{32.2}\right)(20.2)^2$$

$$h = 6.36 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$v = 20.2$ ft/s
$h = 6.36$ ft

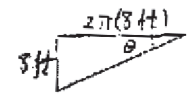
15–102.

The 800-lb roller-coaster car starts from rest on the track having the shape of a cylindrical helix. If the helix descends 8 ft for every one revolution, determine the time required for the car to attain a speed of 60 ft/s. Neglect friction and the size of the car.



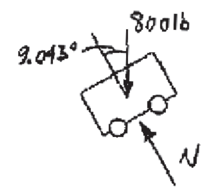
SOLUTION

$$\theta = \tan^{-1}\left(\frac{8}{2\pi(8)}\right) = 9.043^\circ$$



$\Sigma F_y = 0; \qquad N - 800\cos 9.043^\circ = 0$

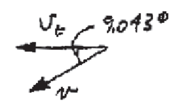
$$N = 790.1\text{ lb}$$



$$v = \frac{v_t}{\cos 9.043^\circ}$$

$$60 = \frac{v_t}{\cos 9.043^\circ}$$

$$v_t = 59.254\text{ ft/s}$$



$$H_1 + \int M\,dt = H_z$$

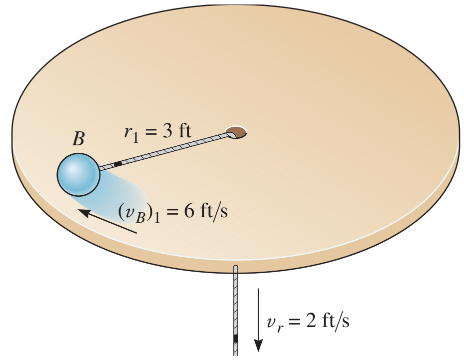
$$0 + \int_0^t 8(790.1\sin 9.043^\circ)dt = \frac{800}{32.2}(8)(59.254)$$

$t = 11.9$ s **Ans.**

Ans:
$t = 11.9$ s

15–103.

A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine the ball's speed at the instant $r_2 = 2$ ft. How much work has to be done to pull down the cord? Neglect friction and the size of the ball.



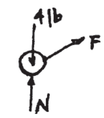
SOLUTION

$H_1 = H_2$

$$\frac{4}{32.2}(6)(3) = \frac{4}{32.2}\,v_\theta(2)$$

$v_\theta = 9$ ft/s

$v_2 = \sqrt{9^2 + 2^2} = 9.22$ ft/s **Ans.**



$T_1 + \Sigma U_{1-2} = T_2$

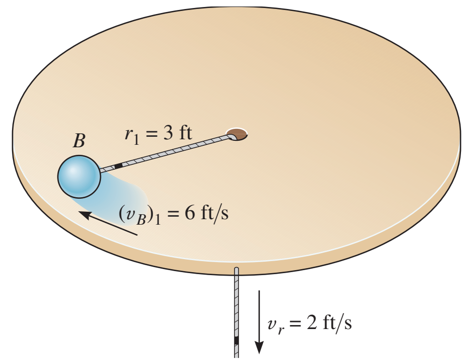
$$\frac{1}{2}\left(\frac{4}{32.2}\right)(6)^2 + \Sigma U_{1-2} = \frac{1}{2}\left(\frac{4}{32.2}\right)(9.22)^2$$

$\Sigma U_{1-2} = 3.04$ ft · lb **Ans.**

Ans:
$v_2 = 9.22$ ft/s
$\Sigma U_{1-2} = 3.04$ ft · lb

***15–104.**

A 4-lb ball B is traveling around in a circle of radius $r_1 = 3$ ft with a speed $(v_B)_1 = 6$ ft/s. If the attached cord is pulled down through the hole with a constant speed $v_r = 2$ ft/s, determine how much time is required for the ball to reach a speed of 12 ft/s. How far r_2 is the ball from the hole when this occurs? Neglect friction and the size of the ball.



SOLUTION

$$v = \sqrt{(v_\theta)^2 + (2)^2}$$

$$12 = \sqrt{(v_\theta)^2 + (2)^2}$$

$$v_\theta = 11.832 \text{ ft/s}$$

$$H_1 = H_2$$

$$\frac{4}{32.2}(6)(3) = \frac{4}{32.2}(11.832)(r_2)$$

$$r_2 = 1.5213 = 1.52 \text{ ft}$$ **Ans.**

$$\Delta r = v_r t$$

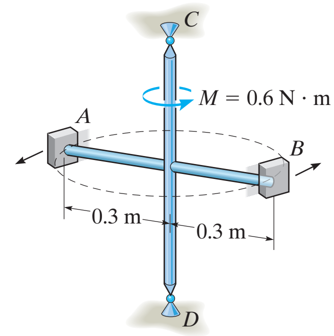
$$(3 - 1.5213) = 2t$$

$$t = 0.739 \text{ s}$$ **Ans.**

Ans:
$r_2 = 1.52$ ft
$t = 0.739$ s

15–105.

The two blocks A and B each have a mass of 400 g. The blocks are fixed to the horizontal rods, and their initial velocity along the circular path is 2 m/s. If a couple moment of $M = (0.6)\ \text{N}\cdot\text{m}$ is applied about CD of the frame, determine the speed of the blocks when $t = 3$ s. The mass of the frame is negligible, and it is free to rotate about CD. Neglect the size of the blocks.



SOLUTION

$$(H_o)_1 + \Sigma \int_{t_1}^{t_2} M_o dt = (H_o)_2$$

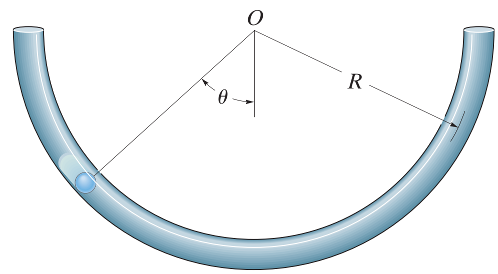
$$2[0.3(0.4)(2)] + 0.6(3) = 2[0.3(0.4)v]$$

$v = 9.50$ m/s **Ans.**

Ans:
$v = 9.50$ m/s

15–106.

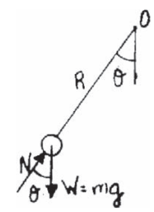
A small particle having a mass m is placed inside the semicircular tube. The particle is placed at the position shown and released. Apply the principle of angular momentum about point O $(\Sigma M_O = \dot{H}_O)$, and show that the motion of the particle is governed by the differential equation $\ddot{\theta} + (g/R)\sin\theta = 0$.



SOLUTION

$$\circlearrowleft + \Sigma M_O = \frac{dH_O}{dt}; \qquad -Rmg\sin\theta = \frac{d}{dt}(mvR)$$

$$g\sin\theta = -\frac{dv}{dt} = -\frac{d^2s}{dt^2}$$



But, $s = R\theta$

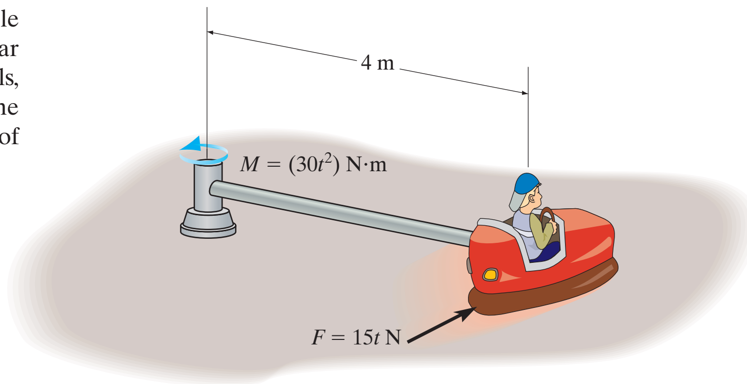
Thus, $g\sin\theta = -R\ddot{\theta}$

or, $\ddot{\theta} + \left(\frac{g}{R}\right)\sin\theta = 0$ **Q.E.D.**

Ans:
$\ddot{\theta} + \left(\frac{g}{R}\right)\sin\theta = 0$

15–107.

If the rod of negligible mass is subjected to a couple moment of $M = (30t^2)\ \text{N} \cdot \text{m}$ and the engine of the car supplies a traction force of $F = (15t)\ \text{N}$ to the wheels, where t is in seconds, determine the speed of the car at the instant $t = 5\ \text{s}$. The car starts from rest. The total mass of the car and rider is 150 kg. Neglect the size of the car.



SOLUTION

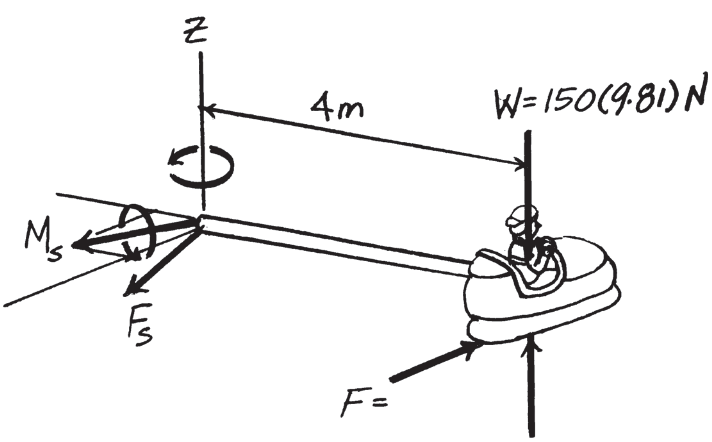
Free-Body Diagram: The free-body diagram of the system is shown in Fig. *a*. Since the moment reaction $\mathbf{M}_S$ has no component about the z axis, the force reaction $\mathbf{F}_S$ acts through the z axis, and the line of action of **W** and **N** are parallel to the z axis, they produce no angular impulse about the z axis.

Principle of Angular Impulse and Momentum:

$$(H_1)_z + \Sigma \int_{t_2}^{t_1} M_z\, dt = (H_2)_z$$

$$0 + \int_0^{5\text{ s}} 30t^2\, dt + \int_0^{5\text{ s}} 15t(4)dt = 150v(4)$$

$v = 3.33\ \text{m/s}$ **Ans.**

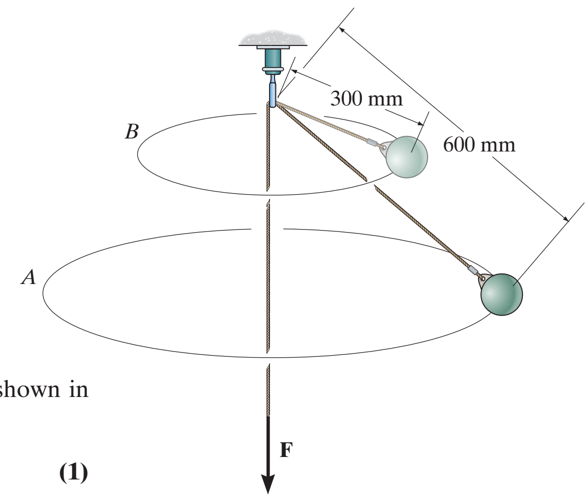


(a)

Ans:
$v = 3.33\ \text{m/s}$

***15–108.**

When the 2-kg bob is given a horizontal speed of 1.5 m/s, it begins to rotate around the horizontal circular path A. If the force $\mathbf{F}$ on the cord is increased, the bob rises and then rotates around the horizontal circular path B. Determine the speed of the bob around path B. Also, find the work done by force $\mathbf{F}$.



SOLUTION

Equations of Motion: By referring to the free-body diagram of the bob shown in Fig. *a*,

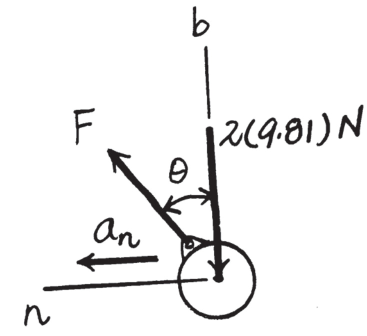
$$+\uparrow \Sigma F_b = 0; \qquad F\cos\theta - 2(9.81) = 0 \tag{1}$$

$$\leftarrow \Sigma F_n = ma_n; \qquad F\sin\theta = 2\left(\frac{v^2}{l\sin\theta}\right) \tag{2}$$

Eliminating F from Eqs. (1) and (2) yields

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v^2}{9.81l}$$

$$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v^2}{9.81l} \tag{3}$$



(a)

When $l = 0.6$ m, $v = v_1 = 5$ m/s. Using Eq. (3), we obtain

$$\frac{1-\cos^2\theta_1}{\cos\theta_1} = \frac{1.5^2}{9.81(0.6)}$$

$$\cos^2\theta_1 + 0.3823\cos\theta_1 - 1 = 0$$

Solving for the root < 1, we obtain

$$\theta_1 = 34.21^\circ$$

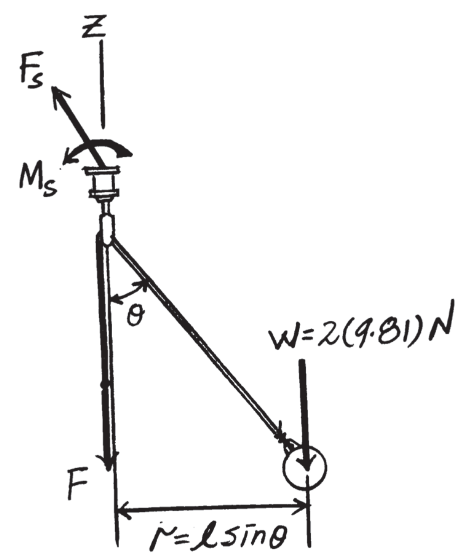
Conservation of Angular Momentum: By observing the free-body diagram of the system shown in Fig. *b*, notice that $\mathbf{W}$ and $\mathbf{F}$ are parallel to the z axis, $\mathbf{M}_S$ has no z component, and $\mathbf{F}_S$ acts through the z axis. Thus, they produce no angular impulse about the z axis. As a result, the angular momentum of the system is conserved about the z axis. When $\theta = \theta_1 = 34.21^\circ$ and $\theta = \theta_2$, $r = r_1 = 0.6\sin 34.21^\circ = 0.3373$ m and $r = r_2 = 0.3\sin\theta_2$. Thus,

$$(H_z)_1 = (H_z)_2$$

$$r_1 m v_1 = r_2 m v_2$$

$$0.3373(2)(1.5) = 0.3\sin\theta_2\,(2)v_2$$

$$v_2\sin\theta_2 = 1.6867 \tag{4}$$



(b)

***15–108. Continued**

Substituting $l = 0.3$ and $\theta = \theta_2$ $v = v_2$ into Eq. (3) yields

$$\frac{1 - \cos^2\theta_2}{\cos\theta_2} = \frac{v^2}{9.81(0.3)}$$

$$\frac{1 - \cos^2\theta_2}{\cos\theta_2} = \frac{{v_2}^2}{2.943} \qquad \textbf{(5)}$$

Eliminating v_2 from Eqs. (4) and (5),

$$\sin^3\theta_2 \tan\theta_2 - 0.9667 = 0$$

Solving the above equation by trial and error, we obtain

$$\theta_2 = 57.866°$$

Substituting the result of θ_2 into Eq. (4), we obtain

$$v_2 = 1.992 \text{ m/s} = 1.99 \text{ m/s} \qquad \textbf{Ans.}$$

Principle of Work and Energy: When θ changes from θ_1 to θ_2, **W** displaces vertically upward $h = 0.6 \cos 34.21° - 0.3 \cos 57.866° = 0.3366$ m. Thus, **W** does negatives work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m{v_1}^2 + U_F + \left(-Wh\right) = \frac{1}{2}m{v_2}^2$$

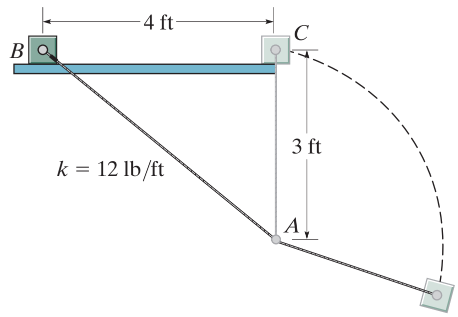
$$\frac{1}{2}(2)(1.5^2) + U_F - 2(9.81)(0.3366) = \frac{1}{2}(2)(1.992)^2$$

$$U_F = 8.32 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$v_2 = 1.99$ m/s
$U_F = 8.32$ N · m

15–109.

The elastic cord has an unstretched length $l_0 = 1.5$ ft and a stiffness $k = 12$ lb/ft. It is attached to a fixed point at A and a block at B, which has a weight of 2 lb. If the block is released from rest from the position shown, determine its speed when it reaches point C after it slides along the smooth guide. After leaving the guide, it is launched onto the smooth *horizontal* plane. Determine if the cord becomes unstretched. Also, calculate the angular momentum of the block about point A, at any instant after it passes point C.



SOLUTION

$$T_B + V_B = T_C + V_C$$

$$0 + \frac{1}{2}(12)(5 - 1.5)^2 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2 + \frac{1}{2}(12)(3 - 1.5)^2$$

$v_C = 43.95 = 44.0$ ft/s **Ans.**

There is a central force about A, and angular momentum about A is conserved.

$H_A = \frac{2}{32.2}(43.95)(3) = 8.19 \text{ slug} \cdot \text{ft}^2/\text{s}$ **Ans.**

If cord is slack $AD = 1.5$ ft

$$(H_A)_1 = (H_A)_2$$

$$8.19 = \frac{2}{32.2}(v_\theta)_D(1.5)$$

$$(v_\theta)_D = 88 \text{ ft/s}$$

But

$$T_C + V_C = T_D + V_D$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(43.95)^2 + \frac{1}{2}(12)(3 - 1.5)^2 = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_D)^2 + 0$$

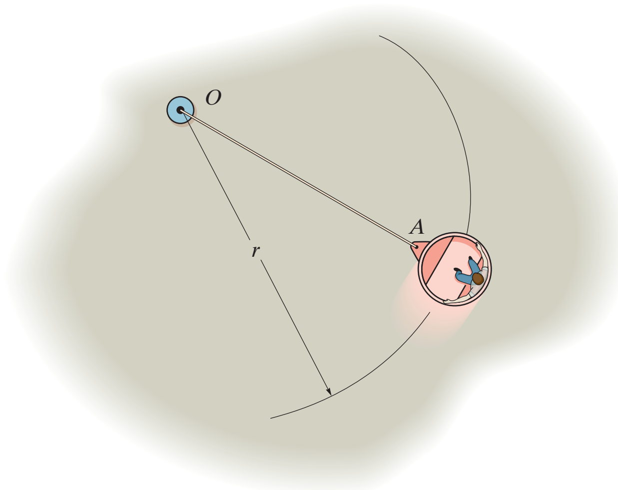
$$v_D = 48.6 \text{ ft/s}$$

Since $v_D < (v_\theta)_D$ cord will not unstretch. **Ans.**

Ans:
$v_C = 44.0$ ft/s
$H_A = 8.19 \text{ slug} \cdot \text{ft}^2/\text{s}$.
The cord will not unstretch.

15–110.

The amusement park ride consists of a 200-kg car and passenger that are traveling at 3 m/s along a circular path having a radius of 8 m. If at $t = 0$, the cable OA is pulled in toward O at 0.5 m/s, determine the speed of the car when $t = 4$ s. Also, determine the work done to pull in the cable.



SOLUTION

Conservation of Angular Momentum. At $t = 4$ s, $r_2 = 8 - 0.5(4) = 6$ m.

$$(H_0)_1 = (H_0)_2$$

$$r_1 m v_1 = r_2 m (v_2)_t$$

$$8[200(3)] = 6[200(v_2)_t]$$

$$(v_2)_t = 4.00 \text{ m/s}$$

Here, $(v_2)_t = 0.5$ m/s. Thus

$$v_2 = \sqrt{(v_2)_t^2 + (v_2)_r^2} = \sqrt{4.00^2 + 0.5^2} = 4.031 \text{ m/s} = 4.03 \text{ m/s} \qquad \textbf{Ans.}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

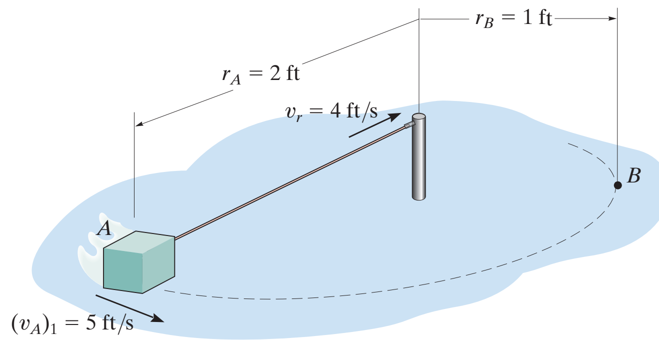
$$\frac{1}{2}(200)(3^2) + \Sigma U_{1-2} = \frac{1}{2}(200)(4.031)^2$$

$$\Sigma U_{1-2} = 725 \text{ J} \qquad \textbf{Ans.}$$

Ans:
$v_2 = 4.03$ m/s
$\Sigma U_{1-2} = 725$ J

15–111.

A box having a weight of 8 lb is moving around in a circle of radius $r_A = 2$ ft with a speed of $(v_A)_1 = 5$ ft/s while connected to the end of a rope. If the rope is pulled inward with a constant speed of $v_r = 4$ ft/s, determine the speed of the box at the instant $r_B = 1$ ft. How much work is done after pulling in the rope from A to B? Neglect friction and the size of the box.



SOLUTION

$(H_z)_A = (H_z)_B;$ $$\left(\frac{8}{32.2}\right)(2)(5) = \left(\frac{8}{32.2}\right)(1)(v_B)_{\text{tangent}}$$

$$(v_B)_{\text{tangent}} = 10 \text{ ft/s}$$

$$v_B = \sqrt{(10)^2 + (4)^2} = 10.77 = 10.8 \text{ ft/s}$$ **Ans.**

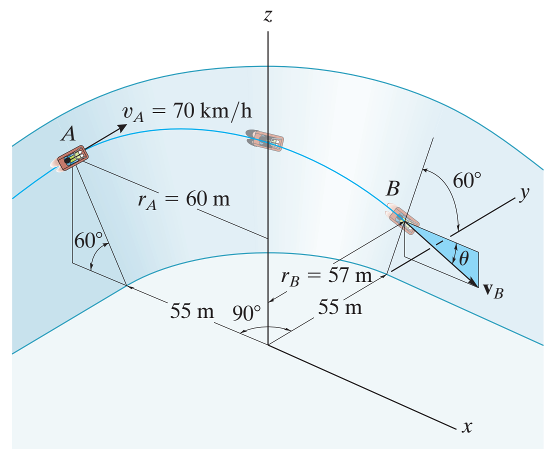
$\sum U_{AB} = T_B - T_A$ $$U_{AB} = \frac{1}{2}\left(\frac{8}{32.2}\right)(10.77)^2 - \frac{1}{2}\left(\frac{8}{32.2}\right)(5)^2$$

$$U_{AB} = 11.3 \text{ ft} \cdot \text{lb}$$ **Ans.**

Ans:
$v_B = 10.8$ ft/s
$U_{AB} = 11.3$ ft · lb

***15–112.**

A toboggan and rider, having a total mass of 150 kg, enter horizontally tangent to a 90° circular curve with a velocity of $v_A = 70$ km/h. If the track is flat and banked at an angle of 60°, determine the speed v_B and the angle θ of "descent," measured from the horizontal in a vertical x–z plane, at which the toboggan exists at B. Neglect friction in the calculation.



SOLUTION

$$v_A = 70 \text{ km/h} = 19.44 \text{ m/s}$$

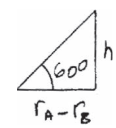
$$(H_A)_z = (H_B)_z$$

$$150(19.44)(60) = 150(v_B)\cos\theta(57) \qquad \textbf{(1)}$$

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(150)(19.44)^2 + 150(9.81)h = \frac{1}{2}(150)(v_B)^2 + 0 \qquad \textbf{(2)}$$



Since $h = (r_A - r_B)\tan 60° = (60 - 57)\tan 60° = 5.196$

Solving Eq. (1) and Eq (2):

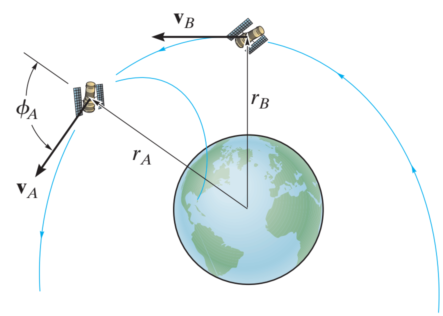
$$v_B = 21.9 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = 20.9 \qquad \textbf{Ans.}$$

Ans:
$v_B = 21.9$ m/s
$\theta = 20.9$

15–113.

An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10$ km/s when the distance from the center of the earth is $r_A = 15$ Mm. If the launch angle at this position is $\phi_A = 70°$, determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})$ kg. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy.



SOLUTION

$$(H_O)_1 = (H_O)_2$$

$$m_s\,(v_A \sin \phi_A) r_A = m_s\,(v_B) r_B$$

$$700[10(10^3)\sin 70°](15)(10^6) = 700(v_B)(r_B) \qquad \textbf{(1)}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m_s\,(v_A)^2 - \frac{GM_e m_s}{r_A} = \frac{1}{2} m_s\,(v_B)^2 - \frac{GM_e m_s}{r_B}$$

$$\frac{1}{2}(700)[10(10^3)]^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} = \frac{1}{2}(700)(v_B)^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B} \qquad \textbf{(2)}$$

Solving,

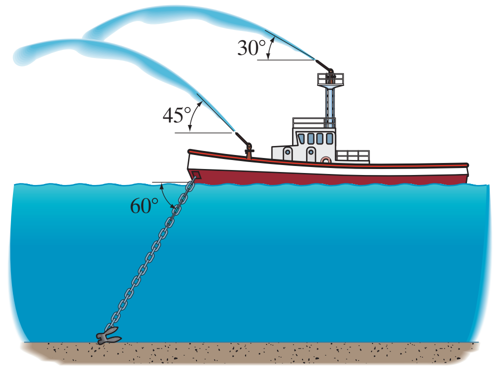
$v_B = 10.2$ km/s **Ans.**

$r_B = 13.8$ Mm **Ans.**

Ans:
$v_B = 10.2$ km/s
$r_B = 13.8$ Mm

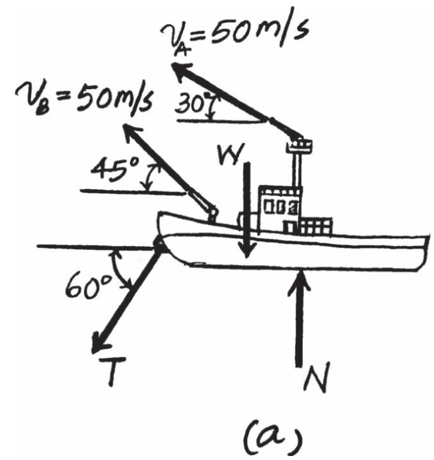
15–114.

The fire boat discharges two streams of seawater, each at a flow of $0.25\ \text{m}^3/\text{s}$ and with a nozzle velocity of 50 m/s. Determine the tension developed in the anchor chain needed to secure the boat. The density of seawater is $\rho_{sw} = 1020\ \text{kg/m}^3$.



SOLUTION

Steady Flow Equation: Here, the mass flow rate of the sea water at nozzles A and B are $\dfrac{dm_A}{dt} = \dfrac{dm_B}{dt} = \rho sw\, Q = 1020(0.25) = 225\ \text{kg/s}$. Since the sea water is collected from the larger reservoir (the sea), the velocity of the sea water entering the control volume can be considered zero. By referring to the free-body diagram of the control volume (the boat),



$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm_A}{dt}\left(v_A\right)_x + \frac{dm_B}{dt}\left(v_B\right)_x;$$

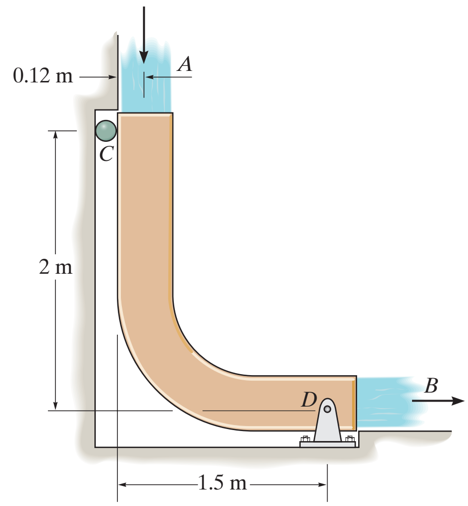
$$T \cos 60° = 225(50 \cos 30°) + 225(50 \cos 45°)$$

$$T = 40\,114.87\ \text{N} = 40.1\ \text{kN}$$ **Ans.**

Ans:
$T = 40.1\ \text{kN}$

15–115.

The chute is used to divert the flow of water, $Q = 0.6\ \text{m}^3/\text{s}$. If the water has a cross-sectional area of $0.05\ \text{m}^2$, determine the force components at the pin D and roller C necessary for equilibrium. Neglect the weight of the chute and weight of the water on the chute. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

Equations of Steady Flow: Here, the flow rate $Q = 0.6\ \text{m}^2/\text{s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.6}{0.05} = 12.0\ \text{m/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = 1000\,(0.6) = 600\ \text{kg/s}$. Applying Eqs. 15–26 and 15–28, we have

$$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}\left(d_{DB}\, v_B - d_{DA}\, v_A\right);$$

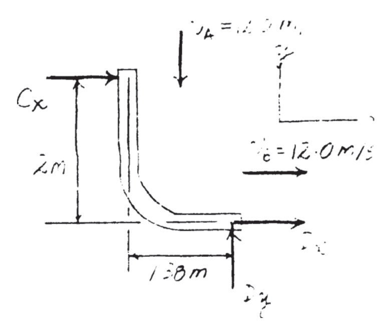
$$-C_x\,(2) = 600\,[0 - 1.38(12.0)] \qquad C_x = 4968\ \text{N} = 4.97\ \text{kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}\left(v_{B_x} - v_{A_x}\right);$$

$$D_x + 4968 = 600\,(12.0 - 0) \qquad D_x = 2232\text{N} = 2.23\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \Sigma \frac{dm}{dt}\left(v_{\text{out}_y} - v_{\text{in}_y}\right);$$

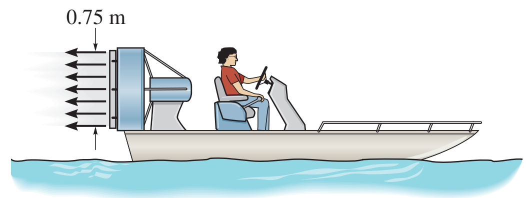
$$D_y = 600[0 - (-12.0)] \qquad D_y = 7200\ \text{N} = 7.20\ \text{kN} \qquad \textbf{Ans.}$$



Ans:
$C_x = 4.97$ kN
$D_x = 2.23$ kN
$D_y = 7.20$ kN

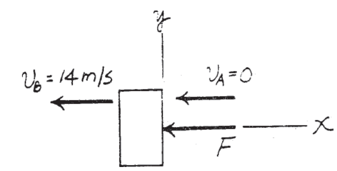
***15–116.**

The 200-kg boat is powered by the fan which develops a slipstream having a diameter of 0.75 m. If the fan ejects air with a speed of 14 m/s, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$ and that the entering air is essentially at rest. Neglect the drag resistance of the water.



SOLUTION

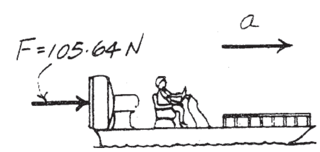
Equations of Steady Flow: Initially, the boat is at rest hence $v_B = v_{a/b} = 14\ \text{m/s}$. Then, $Q = v_B A = 14\left[\frac{\pi}{4}\left(0.75^2\right)\right] = 6.185\ \text{m}^3/\text{s}$ and $\frac{dm}{dt} = \rho_a Q = 1.22(6.185) = 7.546\ \text{kg/s}$. Applying Eq. 15–26, we have



$$\Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x}); \qquad -F = 7.546(-14 - 0) \qquad F = 105.64\ \text{N}$$

Equation of Motion :

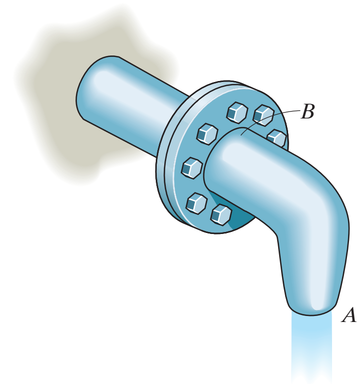
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 105.64 = 200a \qquad a = 0.528\ \text{m/s}^2$$ **Ans.**



Ans:
$a = 0.528\ \text{m/s}^2$

15–117.

The nozzle discharges water at a constant rate of 2 ft^3/s. The cross-sectional area of the nozzle at A is 4 in^2, and at B the cross-sectional area is 12 in^2. If the static gauge pressure due to the water at B is 2 lb/in^2, determine the magnitude of force which must be applied by the coupling at B to hold the nozzle in place. Neglect the weight of the nozzle and the water within it. $\gamma_w = 62.4\ \text{lb/ft}^3$.



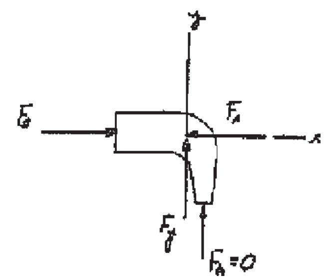
SOLUTION

$$\frac{dm}{dt} = pQ = \left(\frac{62.4}{32.2}\right)(2) = 3.876\ \text{slug/s}$$

$$(v_{Bx}) = \frac{Q}{A_B} = \frac{2}{12/144} = 24\ \text{ft/s} \qquad (v_{By}) = 0$$

$$(v_{Ay}) = \frac{Q}{A_A} = \frac{2}{4/144} = 72\ \text{ft/s} \qquad (v_{Ax}) = 0$$

$$F_B = p_B A_B = 2(12) = 24\ \text{lb}$$



Equations of steady flow:

$$\overset{+}{\rightarrow}\Sigma F_x = \frac{dm}{dt}(v_{Ax} - v_{Bx}); \qquad 24 - F_x = 3.876(0 - 24) \qquad F_x = 117.01\ \text{lb}$$

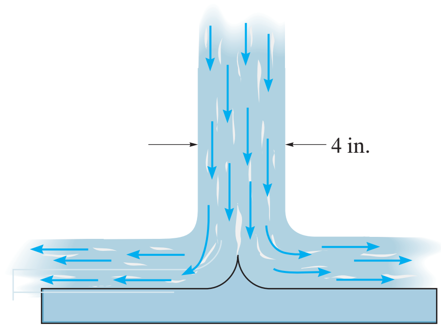
$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{Ay} - v_{By}); \qquad F_y = 3.876(72 - 0) = 279.06\ \text{lb}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{117.01^2 + 279.06^2} = 303\ \text{lb}$$ **Ans.**

Ans:
$F = 303$ lb

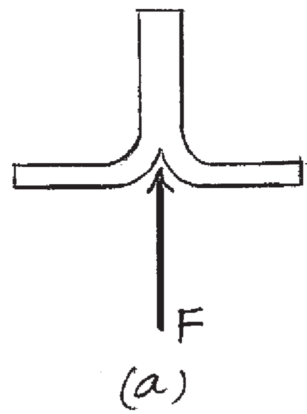
15–118.

The blade divides the jet of water having a diameter of 4 in. If one-half of the water flows to the right while the other half flows to the left, and the total flow is $Q = 1.5\ \text{ft}^3/\text{s}$, determine the vertical force exerted on the blade by the jet, $\gamma_\omega = 62.4\ \text{lb/ft}^3$.



SOLUTION

Equation of Steady Flow. Here $\dfrac{dm}{dt} = \rho_w Q = \left(\dfrac{62.4}{32.2}\right)(1.5) = 2.9068\ \text{slug/s}$. The velocity of the water jet is $v_J = \dfrac{Q}{A} = \dfrac{1.5}{\pi\left(\frac{2}{12}\right)^2} = \dfrac{54}{\pi}\ \text{ft/s}$. Referring to the FBD of the control volume shown in Fig. *a*,



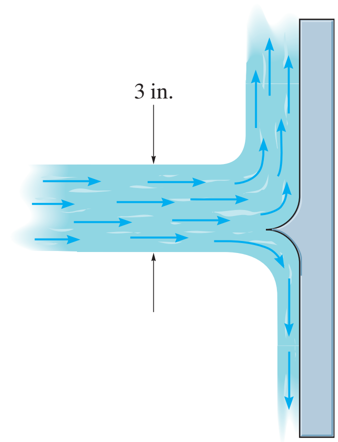
$$+\uparrow\Sigma F_y = \frac{dm}{dt}[(v_B)_y - (v_A)_y];$$

$$F = 2.9068\left[0 - \left(-\frac{54}{\pi}\right)\right] = 49.96\ \text{lb} = 50.0\ \text{lb} \qquad \textbf{Ans.}$$

Ans:
$F = 50.0\ \text{lb}$

15–119.

The blade divides the jet of water having a diameter of 3 in. If one-fourth of the water flows downward while the other three-fourths flows upwards, and the total flow is $Q = 0.5\ \text{ft}^3/\text{s}$, determine the horizontal and vertical components of force exerted on the blade by the jet, $\gamma_w = 62.4\ \text{lb/ft}^3$.



SOLUTION

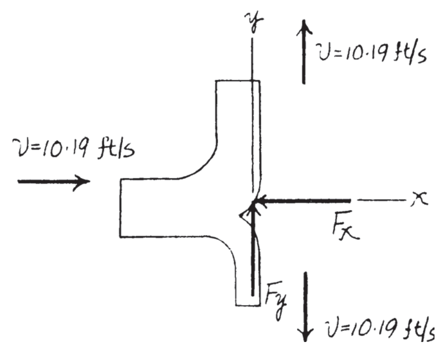
Equations of Steady Flow: Here, the flow rate $Q = 0.5\ \text{ft}^2/\text{s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.5}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2} = 10.19\ \text{ft/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = \dfrac{62.4}{32.2}(0.5) = 0.9689\ \text{slug/s}$.

Applying Eq. 15–25 we have

$$\Sigma F_x = \Sigma \frac{dm}{dt}\left(v_{\text{out}_s} - v_{\text{in}_s}\right); \quad -F_x = 0 - 0.9689\,(10.19) \qquad F_x = 9.87\ \text{lb} \qquad \textbf{Ans.}$$

$$\Sigma F_y = \Sigma \frac{dm}{dt}\left(v_{\text{out}_y} - v_{\text{in}_y}\right); \quad F_y = \frac{3}{4}(0.9689)(10.19) + \frac{1}{4}(0.9689)(-10.19)$$

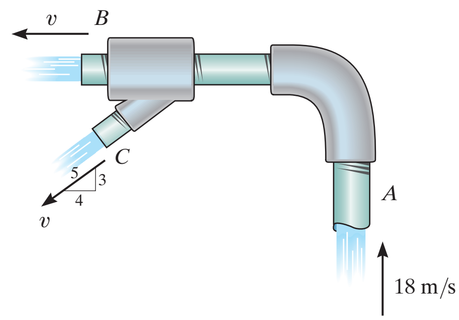
$$F_y = 4.93\ \text{lb} \qquad \textbf{Ans.}$$



Ans:
$F_x = 9.87\ \text{lb}$
$F_y = 4.93\ \text{lb}$

***15–120.**

The gauge pressure of water at A is 150.5 kPa. Water flows through the pipe at A with a velocity of 18 m/s, and out the pipe at B and C with the same velocity v. Determine the horizontal and vertical components of force exerted on the elbow necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 50 mm at A, and at B and C the diameter is 30 mm. $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

Continuity. The flow rate at B and C are the same since the pipe have the same diameter there. The flow rate at A is

$$Q_A = v_A A_A = (18)[\pi(0.025^2)] = 0.01125\pi\ \text{m}^3/\text{s}$$

Continuity negatives that

$$Q_A = Q_B + Q_C; \qquad 0.01125\pi = 2Q$$
$$Q = 0.005625\pi\ \text{m}^3/\text{s}$$

Thus,

$$v_c = v_B = \frac{Q}{A} = \frac{0.005625\pi}{\pi(0.015^2)} = 25\ \text{m/s}$$

Equation of Steady Flow. The force due to the pressure at A is $P = \rho_A A_A = (150.5)(10^3)[\pi(0.025^2)] = 94.0625\pi$ N. Here, $\dfrac{dm_A}{dt} = \rho_w Q_A = 1000(0.01125\pi) = 11.25\pi$ kg/s and $\dfrac{dm_A}{dt} = \dfrac{dM_c}{dt} = \rho_w Q = 1000(0.005625\pi) = 5.625\pi$ kg/s.

$$\overset{+}{\leftarrow}\Sigma F_x = \frac{dm_B}{dt}(v_B)_x + \frac{dm_c}{dt}(v_c)_x - \frac{dm_A}{dt}(v_A)_x;$$

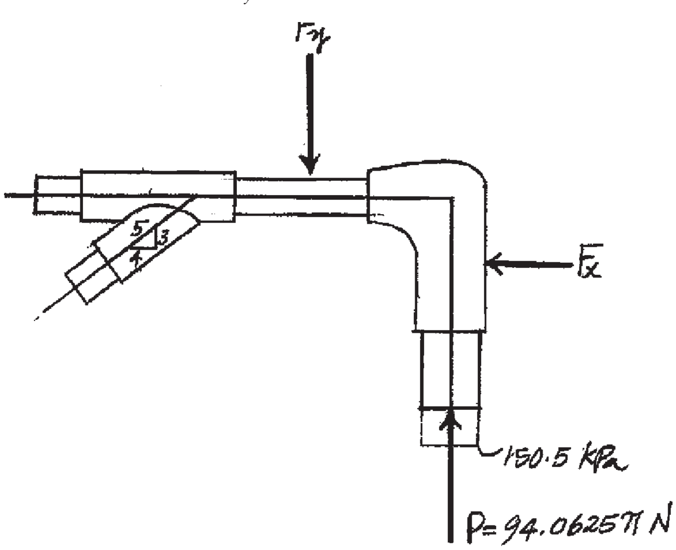
$$F_x = (5.625\pi)(25) + (5.625\pi)\left[25\left(\frac{4}{5}\right)\right] - (11.25\pi)(0)$$

$$= 795.22\ \text{N} = 795\ \text{N} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = \frac{dm_B}{dt}(v_B)_y + \frac{dm_C}{dt}(v_C)_y - \frac{dm_A}{dt}(v_A)_y;$$

$$94.0625\pi - F_y = (5.625\pi)(0) + (5.625\pi)\left[-25\left(\frac{3}{5}\right)\right] - (11.25\pi)(18)$$

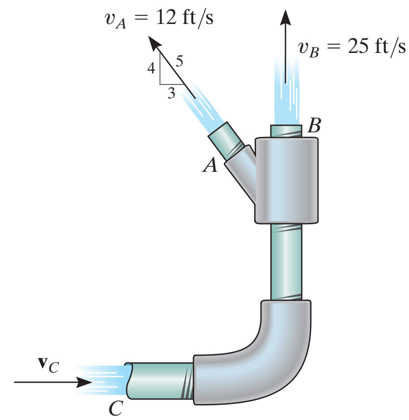
$$F_y = 1196.75\ \text{N} = 1.20\ \text{kN} \qquad \textbf{Ans.}$$



Ans:
$F_x = 795$ N
$F_y = 1.20$ kN

15–121.

The gauge pressure of water at C is 40 lb/in^2. If water flows out of the pipe at A and B with velocities $v_A = 12$ ft/s and $v_B = 25$ ft/s, determine the horizontal and vertical components of force exerted on the elbow necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 0.75 in. at C, and at A and B the diameter is 0.5 in. $\gamma_w = 62.4$ lb/ft^3.



SOLUTION

$$\frac{dm_A}{dt} = \frac{62.4}{32.2}(12)(\pi)\left(\frac{0.25}{12}\right)^2 = 0.03171 \text{ slug/s}$$

$$\frac{dm_B}{dt} = \frac{62.4}{32.2}(25)(\pi)\left(\frac{0.25}{12}\right)^2 = 0.06606 \text{ slug/s}$$

$$\frac{dm_C}{dt} = 0.03171 + 0.06606 = 0.09777 \text{ slug/s}$$

$$v_C A_C = v_A A_A + v_B A_B$$

$$v_C(\pi)\left(\frac{0.375}{12}\right)^2 = 12(\pi)\left(\frac{0.25}{12}\right)^2 + 25(\pi)\left(\frac{0.25}{12}\right)^2$$

$$v_C = 16.44 \text{ ft/s}$$

$$\xrightarrow{+} \Sigma F_x = \frac{dm_B}{dt} v_{B_s} + \frac{dm_A}{dt} v_{A_s} - \frac{dm_C}{dt} v_{C_s}$$

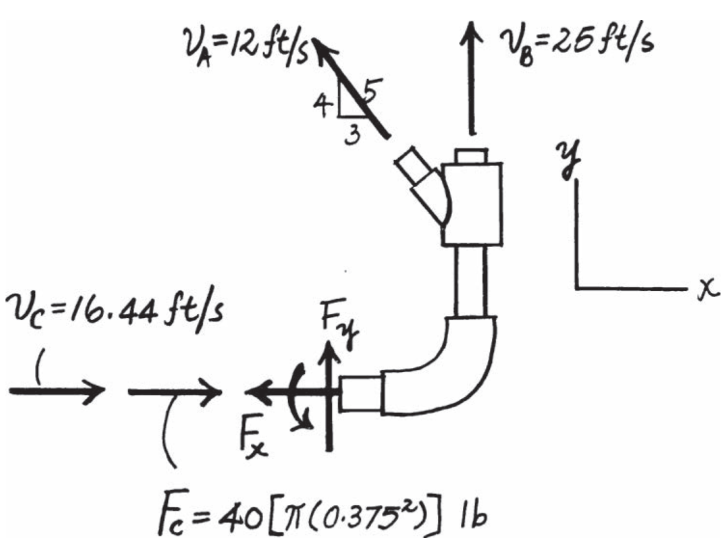
$$40(\pi)(0.375)^2 - F_x = 0 - 0.03171(12)\left(\frac{3}{5}\right) - 0.09777(16.44)$$

$F_x = 19.5$ lb **Ans.**

$$+\uparrow \Sigma F_y = \frac{dm_B}{dt} v_{B_y} + \frac{dm_A}{dt} v_{A_y} - \frac{dm_C}{dt} v_{C_y}$$

$$F_y = 0.06606(25) + 0.03171\left(\frac{4}{5}\right)(12) - 0$$

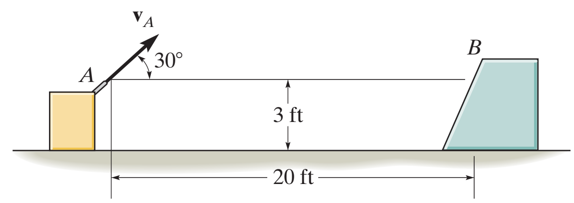
$F_y = 1.9559 = 1.96$ lb **Ans.**



Ans:
$F_x = 19.5$ lb
$F_y = 1.96$ lb

15–122.

The fountain shoots water in the direction shown. If the water is discharged at 30° from the horizontal, and the cross-sectional area of the water stream is approximately 2 in^2, determine the force it exerts on the concrete wall at B. $\gamma_w = 62.4\ \text{lb/ft}^3$.



SOLUTION

$(\xrightarrow{+})\qquad s = s_0 + v_0 t$

$$20 = 0 + v_A \cos 30° t$$

$(+\uparrow)\qquad v = v_0 + a_c t$

$$-(v_A \sin 30°) = (v_A \sin 30°) - 32.2t$$

Solving,

$t = 0.8469\ \text{s}$

$v_A = v_B = 27.27\ \text{ft/s}$

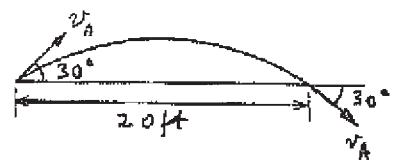
At B:

$$\frac{dm}{dt} = \rho v A = \left(\frac{62.4}{32.2}\right)(27.27)\left(\frac{2}{144}\right) = 0.7340\ \text{slug/s}$$

$$+\searrow \Sigma F = \frac{dm}{dt}(v_A - v_B)$$

$$-F = 0.7340(0 - 27.27)$$

$F = 20.0\ \text{lb}$ **Ans.**



Ans:
$F = 20.0\ \text{lb}$

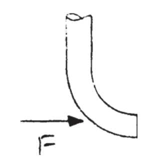
15–123.

A plow located on the front of a locomotive scoops up snow at the rate of $10\ \text{ft}^3/\text{s}$ and stores it in the train. If the locomotive is traveling at a constant speed of 12 ft/s, determine the resistance to motion caused by the shoveling. The specific weight of snow is $\gamma_s = 6\ \text{lb/ft}^3$.

SOLUTION

$$\Sigma F_x = m\frac{dv}{dt} + v_{D/t}\frac{dm_t}{dt}$$

$$F = 0 + (12 - 0)\left(\frac{10(6)}{32.2}\right)$$

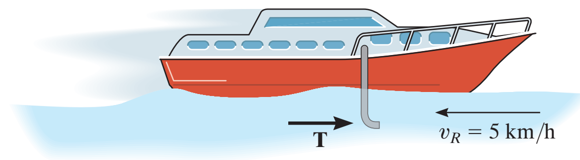


$F = 22.4$ lb **Ans.**

Ans:
$F = 22.4$ lb

***15–124.**

The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured *relative* to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust T on the tube that is required to overcome the resistance due to the water collection and yet maintain the constant speed of the boat. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

$$\frac{dm}{dt} = \frac{40}{80} = 0.5\ \text{kg/s}$$

$$v_{D/t} = (70)\left(\frac{1000}{3600}\right) = 19.444\ \text{m/s}$$

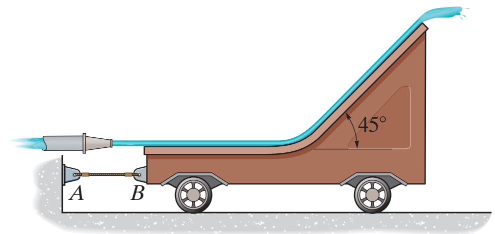
$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$T = 0 + 19.444(0.5) = 9.72\ \text{N}$$ **Ans.**

Ans:
$T = 9.72$ N

15–125.

Water is discharged from a nozzle with a velocity of 12 m/s and strikes the blade mounted on the 20-kg cart. Determine the tension developed in the cord, needed to hold the cart stationary, and the normal reaction of the wheels on the cart. The nozzle has a diameter of 50 mm and the density of water is $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

Steady Flow Equation: Here, the mass flow rate at sections A and B of the control volume is $\dfrac{dm}{dt} = \rho_W Q = \rho_W A v = 1000\left[\dfrac{\pi}{4}(0.05^2)\right](12) = 7.5\pi\ \text{kg/s}$

Referring to the free-body diagram of the control volume shown in Fig. a,

$$\overset{+}{\rightarrow}\ \Sigma F_x = \frac{dm}{dt}[(v_B)_x - (v_A)_x]; \qquad -F_x = 7.5\pi(12\cos 45° - 12)$$

$$F_x = 82.81\ \text{N}$$

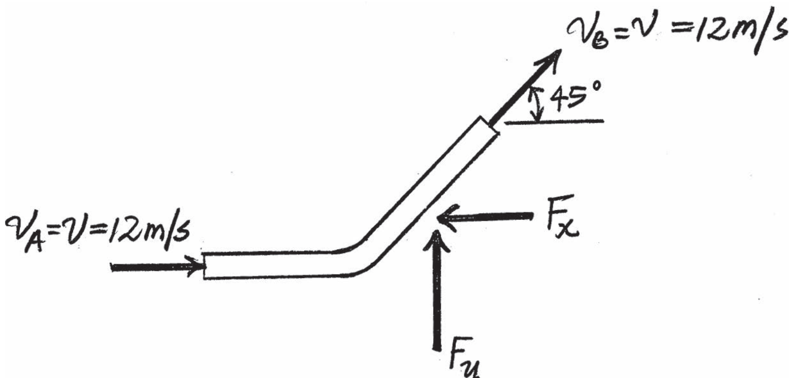
$$+\uparrow \Sigma F_y = \frac{dm}{dt}[(v_B)_y - (v_A)_y]; \qquad F_y = 7.5\pi(12\sin 45° - 0)$$

$$F_y = 199.93\ \text{N}$$

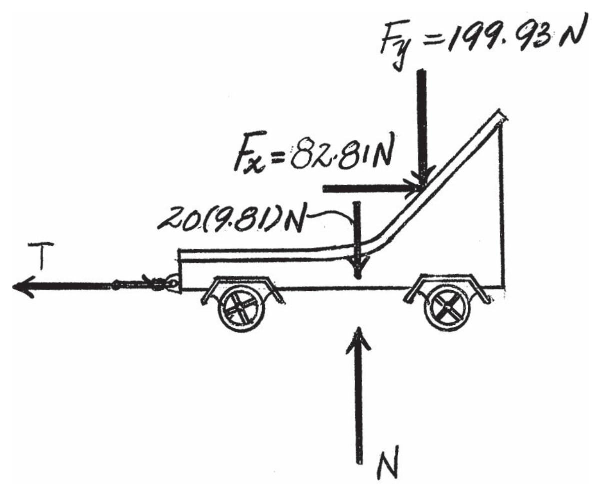
Equilibrium: Using the results of $\mathbf{F}_x$ and $\mathbf{F}_y$ and referring to the free-body diagram of the cart shown in Fig. b,

$$\overset{+}{\rightarrow}\ \Sigma F_x = 0; \qquad 82.81 - T = 0 \qquad T = 82.8\ \text{N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad N - 20(9.81) - 199.93 = 0 \qquad N = 396\ \text{N} \qquad \textbf{Ans.}$$



(a)

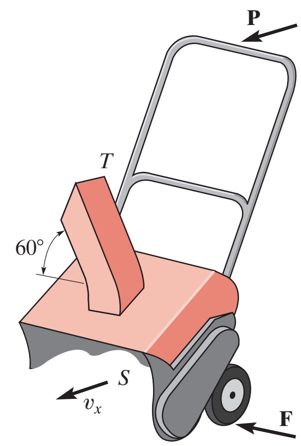


(b)

Ans:
$T = 82.8$ N
$N = 396$ N

15–126.

A snowblower having a scoop S with a cross-sectional area of $A_s = 0.12\ \text{m}^3$ is pushed into snow with a speed of $v_s = 0.5\ \text{m/s}$. The machine discharges the snow through a tube T that has a cross-sectional area of $A_T = 0.03\ \text{m}^2$ and is directed 60° from the horizontal. If the density of snow is $\rho_s = 104\ \text{kg/m}^3$, determine the horizontal force P required to push the blower forward, and the resultant frictional force F of the wheels on the ground, necessary to prevent the blower from moving sideways. The wheels roll freely.



SOLUTION

$$\frac{dm}{dt} = \rho v_s A_s = (104)(0.5)(0.12) = 6.24\ \text{kg/s}$$

$$v_s = \frac{dm}{dt}\left(\frac{1}{\rho A_r}\right) = \left(\frac{6.24}{104(0.03)}\right) = 2.0\ \text{m/s}$$

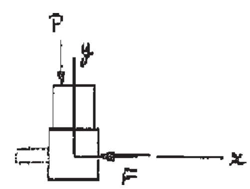
$$\Sigma F_x = \frac{dm}{dt}(v_{T_2} - v_{S_2})$$

$$-F = 6.24(-2\cos 60° - 0)$$

$$F = 6.24\ \text{N}$$ **Ans.**

$$\Sigma F_y = \frac{dm}{dt}(v_{T_2} - v_{S_2})$$

$$-P = 6.24(0 - 0.5)$$

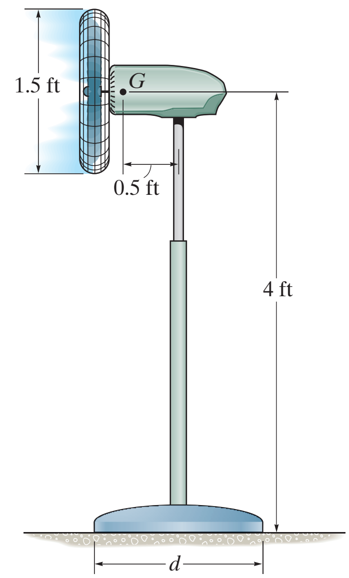


$$P = 3.12\ \text{N}$$ **Ans.**

Ans:
$F = 6.24$ N
$P = 3.12$ N

15–127.

The fan blows air at 6000 ft^3/min. If the fan has a weight of 30 lb and a center of gravity at G, determine the smallest diameter d of its base so that it will not tip over. The specific weight of air is $\gamma = 0.076\ \text{lb/ft}^3$.



SOLUTION

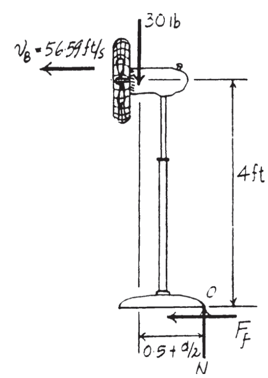
Equations of Steady Flow: Here $Q = \left(\frac{6000\ \text{ft}^3}{\text{min}}\right) \times \left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 100\ \text{ft}^3/\text{s}$. Then,

$v = \frac{Q}{A} = \frac{100}{\frac{\pi}{4}(1.5^2)} = 56.59\ \text{ft/s}$. Also, $\frac{dm}{dt} = \rho_a Q = \frac{0.076}{32.2}(100) = 0.2360\ \text{slug/s}$.

Applying Eq. 15–26 we have

$$\text{a} +\Sigma M_O = \frac{dm}{dt}(d_{OB}\, v_B - d_{OA}\, v_A); \quad 30\left(0.5 + \frac{d}{2}\right) = 0.2360\,[4(56.59) - 0]$$

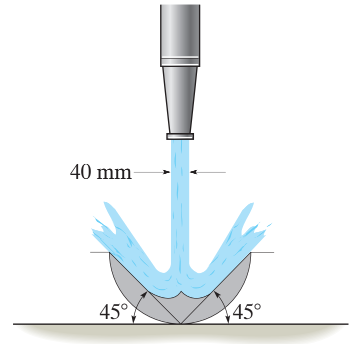
$$d = 2.56\ \text{ft} \qquad \textbf{Ans.}$$



Ans:
$d = 2.56$ ft

***15–128.**

The nozzle has a diameter of 40 mm. If it discharges water uniformly with a downward velocity of 20 m/s against the fixed blade, determine the vertical force exerted by the water on the blade. $\rho_w = 1\ \text{Mg/m}^3$.



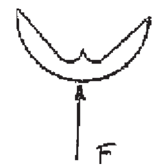
SOLUTION

$$\frac{dm}{dt} = \rho v A = (1000)(20)(\pi)(0.02)^2 = 25.13\ \text{kg/s}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}\left(v_{B[\text{placeholder}]} - v_{Ay}\right)$$

$$F = (25.13)(20 \sin 45° - (-20))$$

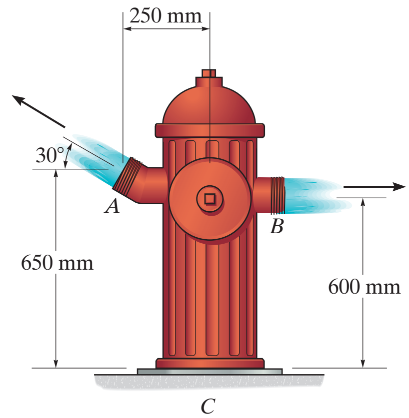
$$F = 858\ \text{N}$$ **Ans.**



Ans:
$F = 858\ \text{N}$

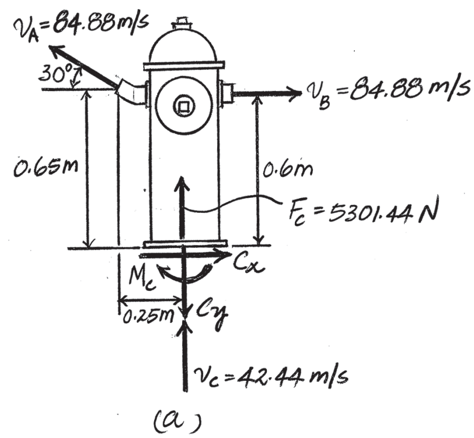
15–129.

The water flow enters below the hydrant at C at the rate of $0.75\ \text{m}^3/\text{s}$. It is then divided equally between the two outlets at A and B. If the gauge pressure at C is 300 kPa, determine the horizontal and vertical force reactions and the moment reaction on the fixed support at C. The diameter of the two outlets at A and B is 75 mm, and the diameter of the inlet pipe at C is 150 mm. The density of water is $\rho_w = 1000\ \text{kg/m}^3$. Neglect the mass of the contained water and the hydrant.



SOLUTION

Free-Body Diagram: The free-body diagram of the control volume is shown in Fig. *a*. The force exerted on section A due to the water pressure is $F_C = p_C A_C = 300(10^3)\left[\frac{\pi}{4}(0.15^2)\right] = 5301.44\ \text{N}$. The mass flow rate at sections A, B, and C, are $\frac{dm_A}{dt} = \frac{dm_B}{dt} = \rho_W\left(\frac{Q}{2}\right) = 1000\left(\frac{0.75}{2}\right) = 375\ \text{kg/s}$ and $\frac{dm_C}{dt} = \rho_W Q = 1000(0.75) = 750\ \text{kg/s}$.



The speed of the water at sections A, B, and C are

$$v_A = v_B = \frac{Q/2}{A_A} = \frac{0.75/2}{\frac{\pi}{4}(0.075^2)} = 84.88\ \text{m/s} \qquad v_C = \frac{Q}{A_C} = \frac{0.75}{\frac{\pi}{4}(0.15^2)} = 42.44\ \text{m/s}.$$

Steady Flow Equation: Writing the force steady flow equations along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm_A}{dt}(v_A)_x + \frac{dm_B}{dt}(v_B)_x - \frac{dm_C}{dt}(v_C)_x;$$

$$C_x = -375(84.88\cos 30^\circ) + 375(84.88) - 0$$

$$C_x = 4264.54\ \text{N} = 4.26\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \frac{dm_A}{dt}(v_A)_y + \frac{dm_B}{dt}(v_B)_y - \frac{dm_C}{dt}(v_C)_y;$$

$$-C_y + 5301.44 = 375(84.88\sin 30^\circ) + 0 - 750(42.44)$$

$$Cy = 21\,216.93\ \text{N} = 2.12\ \text{kN} \qquad \textbf{Ans.}$$

Writing the steady flow equation about point C,

$$+\Sigma M_C = \frac{dm_A}{dt} dv_A + \frac{dm_B}{dt} dv_B - \frac{dm_C}{dt} dv_C;$$

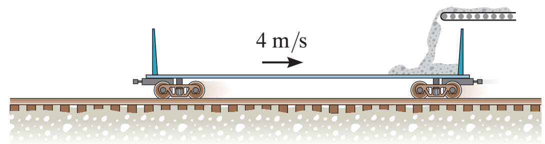
$$-M_C = 375(0.65)(84.88\cos 30^\circ) - 375(0.25)(84.88\sin 30^\circ) + [-375(0.6)(84.88)] - 0$$

$$M_C = 5159.28\ \text{N}\cdot\text{m} = 5.16\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$C_x = 4.26$ kN
$C_y = 2.12$ kN
$M_C = 5.16$ kN·m

15–130.

Sand drops onto the 2-Mg empty rail car at 50 kg/s from a conveyor belt. If the car is initially coasting at 4 m/s, determine the speed of the car as a function of time.



SOLUTION

Gains Mass System. Here the sand drops vertically onto the rail car. Thus $(v_i)_x = 0$. Then

$$\mathbf{V}_D = \mathbf{V}_i + \mathbf{V}_{D/i}$$

$$(\overset{+}{\rightarrow})\quad v = (v_i)_x + (v_{D/i})_x$$

$$v = 0 + (v_{D/i})_x$$

$$(v_{D/i})_x = v$$

Also, $\dfrac{dm_i}{dt} = 50 \text{ kg/s}$ and $m = 2000 + 50t$

$$\Sigma F_x = m\frac{dv}{dt} + (v_{D/i})_x \frac{dm_i}{dt};$$

$$0 = (2000 + 50t)\frac{dv}{dt} + v(50)$$

$$\frac{dv}{v} = -\frac{50\,dt}{2000 + 50t}$$

Integrate this equation with initial condition $v = 4$ m/s at $t = 0$.

$$\int_{4\text{ m/s}}^{v} \frac{dv}{v} = -50\int_0^t \frac{dt}{2000 + 50t}$$

$$\ln v\Big|_{4\text{ m/s}}^{v} = -\ln(2000 + 50t)\Big|_0^t$$

$$\ln\frac{v}{4} = \ln\left(\frac{2000}{2000 + 50t}\right)$$

$$\frac{v}{4} = \frac{2000}{2000 + 50t}$$

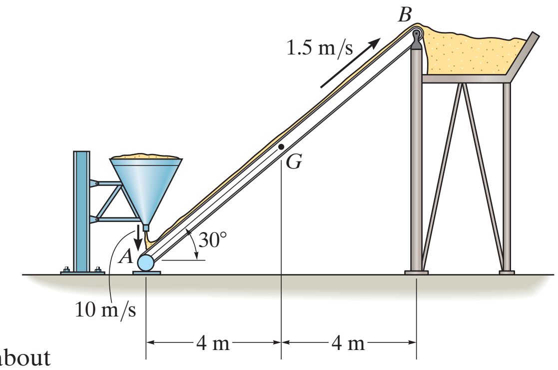
$$v = \left\{\frac{8000}{2000 + 50t}\right\} \text{ m/s} \qquad \textbf{Ans.}$$

Ans:

$$v = \left\{\frac{8000}{2000 + 50t}\right\} \text{ m/s}$$

15–131.

Sand is discharged from the silo at A at a rate of 50 kg/s with a vertical velocity of 10 m/s onto the conveyor belt, which is moving with a constant velocity of 1.5 m/s. If the conveyor system and the sand on it have a total mass of 750 kg and center of mass at point G, determine the horizontal and vertical components of reaction at the pin support B roller support A. Neglect the thickness of the conveyor.



SOLUTION

Steady Flow Equation: The moment steady flow equation will be written about point B to eliminate $\mathbf{B}_x$ and $\mathbf{B}_y$. Referring to the free-body diagram of the control volume shown in Fig. a,

$$+\Sigma M_B = \frac{dm}{dt}(dv_B - dv_A); \qquad 750(9.81)(4) - A_y(8) = 50[0 - 8(5)]$$

$$A_y = 4178.5 \text{ N} = 4.18 \text{ kN} \qquad \textbf{Ans.}$$

Writing the force steady flow equation along the x and y axes,

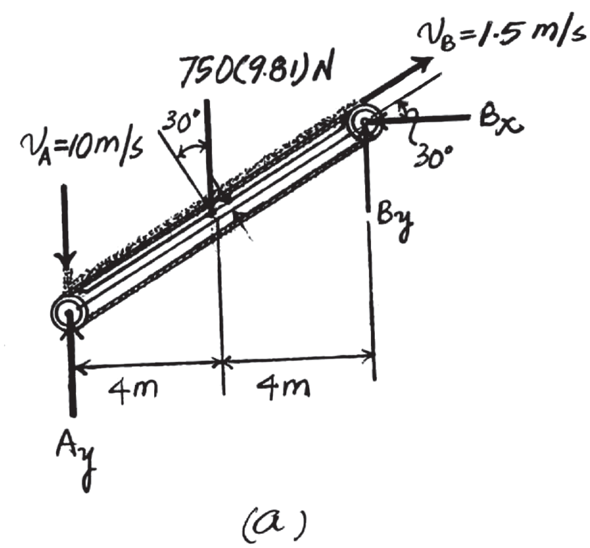
$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}[(v_B)_x - (v_A)_x]; \qquad -B_x = 50(1.5 \cos 30° - 0)$$

$$B_x = |-64.95 \text{ N}| = 65.0 \text{ N} \rightarrow \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}[(v_B)_y - (v_A)_y]; \qquad B_y + 4178.5 - 750(9.81)$$

$$= 50[1.5 \sin 30° - (-10)]$$

$$B_y = 3716.25 \text{ N} = 3.72 \text{ kN} \uparrow \qquad \textbf{Ans.}$$

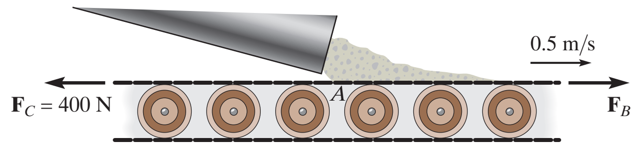


(a)

Ans:
$A_y = 4.18$ kN
$B_x = 65.0$ N $\rightarrow$
$B_y = 3.72$ kN $\uparrow$

***15–132.**

Sand is deposited from a chute onto a conveyor belt which is moving at 0.5 m/s. If the sand is assumed to fall vertically onto the belt at A at the rate of 4 kg/s, determine the belt tension F_B to the right of A. The belt is free to move over the conveyor rollers and its tension to the left of A is $F_C = 400$ N.



SOLUTION

$$(\overset{+}{\rightarrow})\ \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$

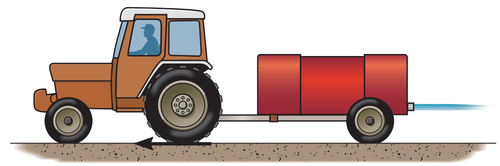
$$F_B - 400 = 4(0.5 - 0)$$

$$F_B = 2 + 400 = 402 \text{ N}$$ **Ans.**

Ans:
$F_B = 402$ N

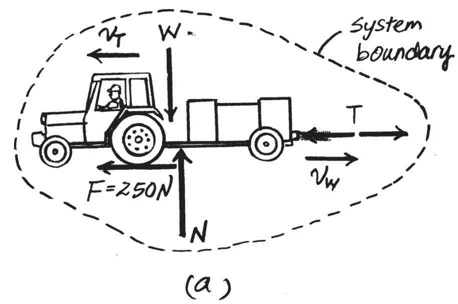
15–133.

The tractor together with the empty tank has a total mass of 4 Mg. The tank is filled with 2 Mg of water. The water is discharged at a constant rate of 50 kg/s with a constant velocity of 5 m/s, measured relative to the tractor. If the tractor starts from rest, and the rear wheels provide a resultant traction force of 250 N, determine the velocity and acceleration of the tractor at the instant the tank becomes empty.



SOLUTION

The free-body diagram of the tractor and water jet is shown in Fig. *a*. The pair of thrust **T** cancel each other since they are internal to the system. The mass of the tractor and the tank at any instant t is given by $m = (4000 + 2000) - 50t = (6000 - 50t)\text{kg}$.



$$\stackrel{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad 250 = (6000 - 50t)\frac{dv}{dt} - 5(50)$$

$$a = \frac{dv}{dt} = \frac{10}{120 - t} \qquad \textbf{(1)}$$

The time taken to empty the tank is $t = \dfrac{2000}{50} = 40$ s. Substituting the result of t into Eq. (1),

$$a = \frac{10}{120 - 40} = 0.125\text{m/s}^2 \qquad \textbf{Ans.}$$

Integrating Eq. (1),

$$\int_0^v dv = \int_0^{40\text{ s}} \frac{10}{120 - t}dt$$

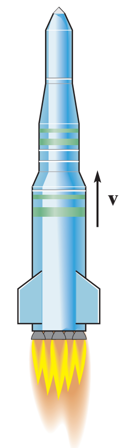
$$v = -10\ln(120 - t)\Big|_0^{40\text{ s}}$$

$$= 4.05\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$a = 0.125\text{ m/s}^2$
$v = 4.05\text{ m/s}$

15–134.

A rocket has an empty weight of 500 lb and carries 300 lb of fuel. If the fuel is burned at the rate of 15 lb/s and ejected with a relative velocity of 4400 ft/s, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.



SOLUTION

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{[placeholder]}\frac{dm_e}{dt}$$

At n time t, $m = m_0 - ct$, where $c = \dfrac{dm_e}{dt}$. In space the weight of the rocket is zero.

$$0 = (m_0 - ct)\frac{dv}{dt} - v_{[placeholder]}[placeholder]$$

$$\int_0^v dv = \int_0^1 \left(\frac{c v_{[placeholder]}}{m_0 - ct}\right)dt$$

$$v = v_{[placeholder]} \ln\left(\frac{m_0}{m_0 - ct}\right) \qquad \textbf{(1)}$$

The maximum speed occurs when all the fuel is consumed, that is, when $t = \dfrac{300}{15} = 20$ s. Here, $m_0 = \dfrac{500 + 300}{32.2} = 24.8447$ slug, $c = \dfrac{15}{32.2} = 0.4658$ slug/s, $v_{[placeholder]} = 4400$ ft/s. Substitute the numerical values into Eq. (1);

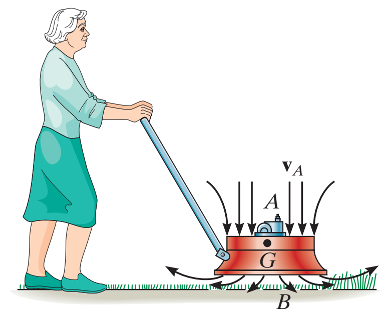
$$v_{max} = 4400 \ln\left(\frac{24.8447}{24.8447 - 0.4658(20)}\right)$$

$$v_{max} = 2068 \text{ ft/s} = 2.07(10^3) \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v_{max} = 2.07\,(10^3)$ ft/s

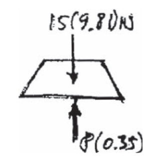
15–135.

A power lawn mower hovers very close over the ground. This is done by drawing air in at a speed of 6 m/s through an intake unit A, which has a cross-sectional area of $A_A = 0.25\ \text{m}^2$, and then discharging it at the ground, B, where the cross-sectional area is $A_B = 0.35\ \text{m}^2$. If air at A is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has a mass of 15 kg with center of mass at G. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$.



SOLUTION

$$\frac{dm}{dt} = \rho\, A_A v_A = 1.22(0.25)(6) = 1.83\ \text{kg/s}$$



$$+\uparrow \Sigma F_y = \frac{dm}{dt}((v_B)_y - (v_A)_y)$$

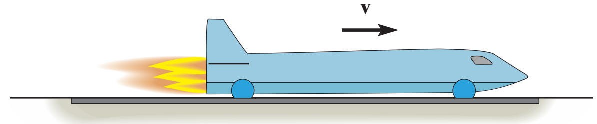
$$\text{pressure} = (0.35) - 15(9.81) = 1.83(0 - (-6))$$

$$\text{pressure} = 452\ \text{Pa}$$ **Ans.**

Ans:
452 Pa

***15–136.**

The rocket car has a mass of 2 Mg (empty) and carries 120 kg of fuel. If the fuel is consumed at a constant rate of 6 kg/s and ejected from the car with a relative velocity of 800 m/s, determine the maximum speed attained by the car starting from rest. The drag resistance due to the atmosphere is $F_D = (6.8v^2)$ N, where v is the speed in m/s.



SOLUTION

$$\Sigma F_s = m\frac{dv}{dt} - v_{[\text{placeholder}]}[\text{placeholder}]$$

At time t_1 the mass of the car is $m_0 - ct_1$ where $c = \dfrac{dm_c}{dt} = 6$ kg/s

Set $F = kv^2$, then

$$-kv^2 = (m_0 - ct)\frac{dv}{dt} - v_{D/e}c$$

$$\int_0^t \frac{dv}{(cv_{D/e} - kv^2)} = \int_0^t \frac{dt}{(m_0 - ct)}$$

$$\left(\frac{1}{2\sqrt{cv_{D/e}k}}\right)\ln\left[\frac{\sqrt{\dfrac{cv_{D/e}}{k}} + v}{\sqrt{\dfrac{cv_{D/e}}{k}} - v}\right]_0^v = -\frac{1}{c}\ln(m_0 - ct)\Big|_0^t$$

$$\left(\frac{1}{2\sqrt{cv_{D/e}k}}\right)\ln\left(\frac{\sqrt{\dfrac{cv_{D/e}}{k}} + v}{\sqrt{\dfrac{cv_{D/e}}{k}} - v}\right) = -\frac{1}{c}\ln\left(\frac{m_0 - ct}{m_0}\right)$$

Maximum speed occurs at the instant the fuel runs out

$$t = \frac{120}{6} = 20 \text{ s}$$

Thus,

$$\left(\frac{1}{2\sqrt{(6)(800)(6.8)}}\right)\ln\left(\frac{\sqrt{\dfrac{(6)(800)}{6.8}} + v}{\sqrt{\dfrac{(6)(800)}{6.8}} - v}\right) = -\frac{1}{6}\ln\left(\frac{2120 - 6(20)}{2120}\right)$$

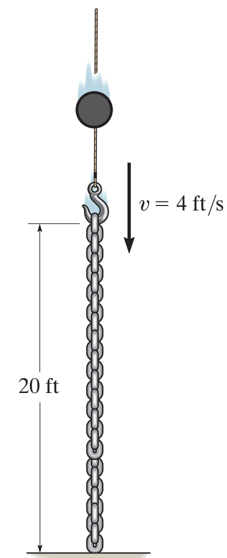
Solving,

$v = 25.0$ m/s **Ans.**

Ans:
$v = 25.0$ m/s

15–137.

If the chain is lowered at a constant speed $v = 4$ ft/s, determine the normal reaction exerted on the floor as a function of time. The chain has a weight of 5 lb/ft and a total length of 20 ft.



SOLUTION

At time t, the weight of the chain on the floor is $W = mg(vt)$

$$\frac{dv}{dt} = 0, \qquad m_i = m(vt)$$

$$\frac{dm_i}{dt} = mv$$

$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$R - mg(vt) = 0 + v(mv)$$

$$R = m(gvt + v^2)$$

$$R = \frac{5}{32.2}\left(32.2(4)(t) + (4)^2\right)$$

$$R = (20t + 2.48) \text{ lb}$$ **Ans.**

Ans:
$R = \{20t + 2.48\}$ lb

15–138.

The second stage of a two-stage rocket weighs 2000 lb (empty) and is launched from the first stage with a velocity of 3000 mi/h. The fuel in the second stage weighs 1000 lb. If it is consumed at the rate of 50 lb/s and ejected with a relative velocity of 8000 ft/s, determine the acceleration of the second stage just after the engine is fired. What is the rocket's acceleration just before all the fuel is consumed? Neglect the effect of gravitation.

SOLUTION

Initially,

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\left(\frac{dm_e}{dt}\right)$$

$$0 = \frac{3000}{32.2}a - 8000\left(\frac{50}{32.2}\right)$$

$$a = 133 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Finally,

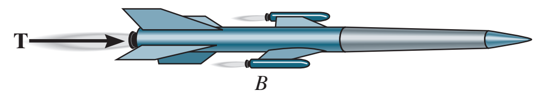
$$0 = \frac{2000}{32.2}a - 8000\left(\frac{50}{32.2}\right)$$

$$a = 200 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_i = 133 \text{ ft/s}^2$
$a_f = 200 \text{ ft/s}^2$

15–139.

The missile weighs 40 000 lb. The constant thrust provided by the turbojet engine is $T = 15\,000$ lb. Additional thrust is provided by *two* rocket boosters B. The propellant in each booster is burned at a constant rate of 150 lb/s, with a relative exhaust velocity of 3000 ft/s. If the mass of the propellant lost by the turbojet engine can be neglected, determine the velocity of the missile after the 4-s burn time of the boosters. The initial velocity of the missile is 300 mi/h.



SOLUTION

$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

At a time t, $m = m_0 - ct$, where $c = \dfrac{dm_e}{dt}$.

$$T = (m_0 - ct)\frac{dv}{dt} - v_{D/e}c$$

$$\int_{v_0}^{v} dv = \int_0^t \left(\frac{T + cv_{D/e}}{m_0 - ct}\right)dt$$

$$v = \left(\frac{T + cv_{D/e}}{c}\right)\ln\left(\frac{m_0}{m_0 - ct}\right) + v_0 \qquad \textbf{(1)}$$

Here, $m_0 = \dfrac{40\,000}{32.2} = 1242.24$ slug, $c = 2\left(\dfrac{150}{32.2}\right) = 9.3168$ slug/s, $v_{D/e} = 3000$ ft/s,

$t = 4$ s, $v_0 = \dfrac{300(5280)}{3600} = 440$ ft/s.

Substitute the numerical values into Eq. (1):

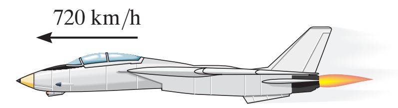
$$v_{max} = \left(\frac{15\,000 + 9.3168(3000)}{9.3168}\right)\ln\left(\frac{1242.24}{1242.24 - 9.3168(4)}\right) + 440$$

$$v_{max} = 580 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v_{\max} = 580$ ft/s

***15–140.**

The jet is traveling at a speed of 720 km/h. If the fuel is being spent at 0.8 kg/s, and the engine takes in air at 200 kg/s, whereas the exhaust gas (air and fuel) has a relative speed of 12 000 m/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (55\,v^2)$, where the speed is measured in m/s. The jet has a mass of 7 Mg.



SOLUTION

Since the mass enters and exits the plane at the same time, we can combine Eqs. 15-29 and 15-30 which resulted in

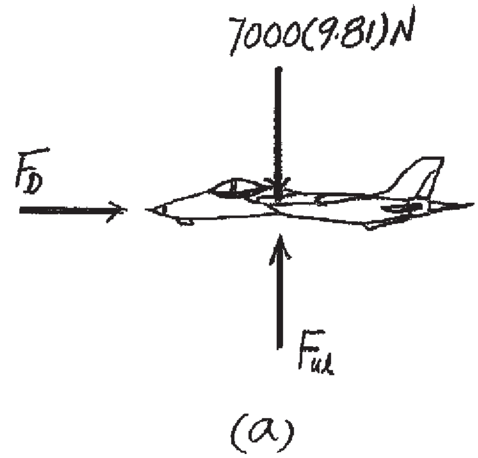
$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$

Here $m = 7000$ kg, $\dfrac{dv}{dt} = a$, $v_{D/e} = 12000$ m/s, $\dfrac{dm_e}{dt} = 0.8 + 200 = 200.8$ kg/s

$$v = \left(720\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 200\text{ m/s},\ v_{D/i} = v = 200\text{ m/s},$$

$$\frac{dm_i}{dt} = 200\text{ kg/s}$$

and $F_D = 55(200^2) = 2.2(10^6)$ N. Referring to the FBD of the jet, Fig. *a*



$$(\overset{+}{\leftarrow})\ \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt};$$

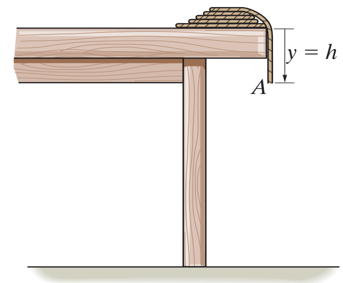
$$-2.2(10^6) = 7000a - 12000(200.8) + 200(200)$$

$$a = 24.23\text{ m/s}^2 = 24.2\text{ m/s}^2$$ **Ans.**

Ans:
$a = 24.2\text{ m/s}^2$

15–141.

The rope has a mass m' per unit length. If the end length $y = h$ is draped off the edge of the table, and released, determine the velocity of its end A for any position y, as the rope uncoils and begins to fall.



SOLUTION

$$+\downarrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time t, $m = m'y$ and $\dfrac{dm_i}{dt} = \dfrac{m'dy}{dt} = m'v$. Here, $v_{D/i} = v$, $\dfrac{dv}{dt} = g$.

$$m'gy = m'y\frac{dv}{dt} + v(m'v)$$

$$gy = y\frac{dv}{dt} + v^2 \qquad \text{since } v = \frac{dy}{dt}, \text{ then } dt = \frac{dy}{v}$$

$$gy = vy\frac{dv}{dy} + v^2$$

Multiply both sides by $2ydy$

$$2gy^2\,dy = 2vy^2\,dv + 2yv^2\,dy$$

$$\int 2gy^2\,dy = \int d\left(v^2y^2\right)$$

$$\tfrac{2}{3}gy^3 + C = v^2y^2$$

$$v = 0 \text{ at } y = h \qquad \tfrac{2}{3}gh^3 + C = 0 \qquad C = -\tfrac{2}{3}gh^3$$

$$\tfrac{2}{3}gy^3 - \tfrac{2}{3}gh^3 = v^2y^2$$

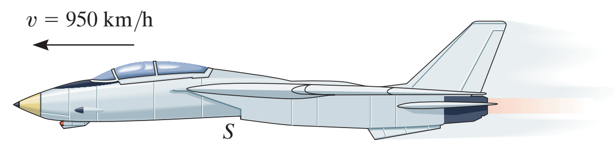
$$v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)} \qquad \textbf{Ans.}$$

Ans:

$$v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)}$$

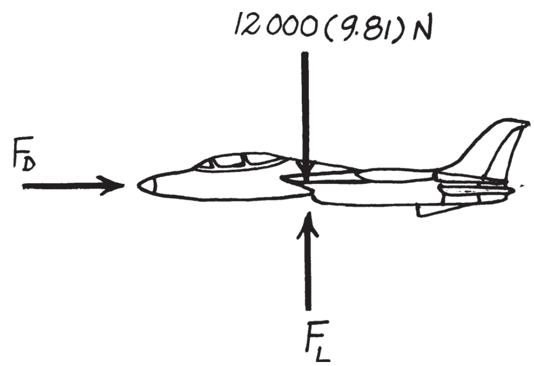
15–142.

The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops S at the rate of 50 m^3/s. If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m^3. *Hint:* Since mass both enters and exits the plane, Eqs. 15–28 and 15–29 must be combined to yield $\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$.



SOLUTION

$$\Sigma F_s = m\frac{dv}{dt} - \frac{dm_e}{dt}(v_{D/E}) + \frac{dm_i}{dt}(v_{D/i}) \qquad \textbf{(1)}$$



$$v = 950 \text{ km/h} = 0.2639 \text{ km/s}, \qquad \frac{dv}{dt} = 0$$

$$v_{D/E} = 0.45 \text{ km/s}$$

$$v_{D/t} = 0.2639 \text{ km/s}$$

$$\frac{dm_t}{dt} = 50(1.22) = 61.0 \text{ kg/s}$$

$$\frac{dm_e}{dt} = 0.4 + 61.0 = 61.4 \text{ kg/s}$$

Forces T and R are incorporated into Eq. (1) as the last two terms in the equation.

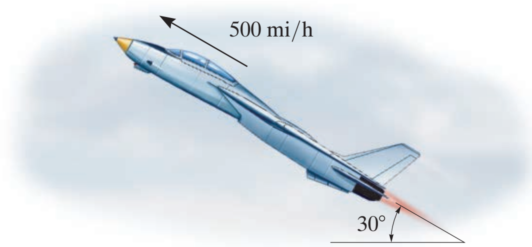
$$(\overset{+}{\leftarrow}) \quad -F_D = 0 - (0.45)(61.4) + (0.2639)(61)$$

$$F_D = 11.5 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_D = 11.5$ kN

15–143.

The jet is traveling at a speed of 500 mi/h, 30° with the horizontal. If the fuel is being spent at 3 lb/s, and the engine takes in air at 400 lb/s, whereas the exhaust gas (air and fuel) has a relative speed of 32 800 ft/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (0.7v^2)$ lb, where the speed is measured in ft/s. The jet has a weight of 15 000 lb. *Hint:* See Prob. 15–142.

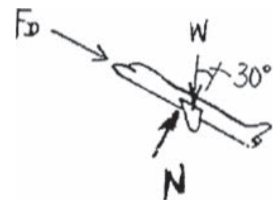


SOLUTION

$$\frac{dm_i}{dt} = \frac{400}{32.2} = 12.42 \text{ slug/s}$$

$$\frac{dm_e}{dt} = \frac{403}{32.2} = 12.52 \text{ slug/s}$$

$$v = v_{D/i} = 500 \text{ mi/h} = 733.3 \text{ ft/s}$$



$$\nwarrow + \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$

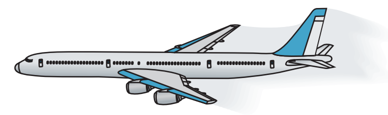
$$-(15\,000)\sin 30° - 0.7(733.3)^2 = \frac{15\,000}{32.2}\frac{dv}{dt} - 32\,800(12.52) + 733.3(12.42)$$

$$a = \frac{dv}{dt} = 37.5 \text{ ft/s}^2$$ **Ans.**

Ans:
$a = 37.5 \text{ ft/s}^2$

***15–144.**

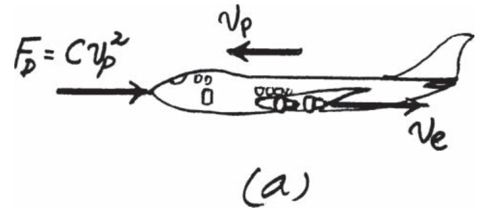
A four-engine commercial jumbo jet is cruising at a constant speed of 800 km/h in level flight when all four engines are in operation. Each of the engines is capable of discharging combustion gases with a velocity of 775 m/s relative to the plane. If during a test two of the engines, one on each side of the plane, are shut off, determine the new cruising speed of the jet. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. Neglect the loss of mass due to fuel consumption.



SOLUTION

Steady Flow Equation: Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is

$$(\overset{+}{\rightarrow}) \qquad v_e + v_p + v_{e/p}$$



When the four engines are in operation, the airplane has a constant speed of $v_p = \left[800(10^3)\,\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 222.22$ m/s. Thus,

$$(\overset{+}{\rightarrow}) \qquad v_e = -222.22 + 775 = 552.78\text{ m/s} \rightarrow$$

Referring to the free-body diagram of the airplane shown in Fig. *a*,

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \qquad C(222.22^2) = 4\frac{dm}{dt}(552.78 - 0)$$

$$C = 0.044775\frac{dm}{dt}$$

When only two engines are in operation, the exit speed of the air is

$$(\overset{+}{\rightarrow}) \qquad v_e = -v_p + 775$$

Using the result for C,

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \quad \left(0.044775\frac{dm}{dt}\right)\left(v_p{}^2\right) = 2\frac{dm}{dt}\left[-v_p + 775\right) - 0\right]$$

$$0.044775v_p{}^2 + 2v_p - 1550 = 0$$

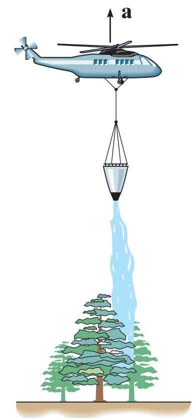
Solving for the positive root,

$v_p = 165.06\text{ m/s} = 594\text{ km/h}$ **Ans.**

Ans:
$v_P = 594$ km/h

15–145.

The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to thehelicopter, determine the initial upward accelerationthe helicopter experiences as the water is being released.



SOLUTION

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

Initially, the bucket is full of water, hence $m = 10(10^3) + 0.5(10^3) = 10.5(10^3)$ kg

$0 = 10.5(10^3)\,a - (10)(50)$

$a = 0.0476 \text{ m/s}^2$ **Ans.**

Ans:
$a = 0.0476 \text{ m/s}^2$

15–146.

A rocket has an empty weight of 500 lb and carries 300 lb of fuel. If the fuel is burned at the rate of 1.5 lb/s and ejected with a velocity of 4400 ft/s relative to the rocket, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.

SOLUTION

$$+\uparrow \Sigma F_s = \frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

At a time t, $m = m_0 - ct$, where $c = \dfrac{dm_e}{dt}$. In space the weight of the rocket is zero.

$$0 = (m_0 - ct)\frac{dv}{dt} - v_{D/e}\,c$$

$$\int_0^v dv = \int_0^t \left(\frac{cv_{D/e}}{m_0 - ct}\right)dt$$

$$v = v_{D/e}\ln\left(\frac{m_0}{m_0 - ct}\right) \qquad \textbf{(1)}$$

The maximum speed occurs when all the fuel is consumed, that is, when $t = \frac{300}{1.5} = 200$ s.

Here, $m_0 = \frac{500 + 300}{32.2} = 24.8447$ slug, $c = \frac{1.5}{32.2} = 0.04658$ slug/s, $v_{D/e} = 4400$ ft/s.

Substitute the numerical into Eq. (1):

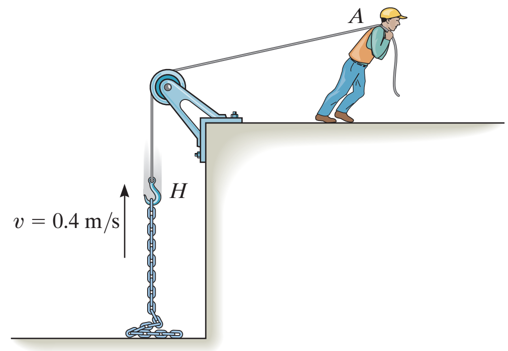
$$v_{max} = 4400\ln\left(\frac{24.8447}{24.8447 - (0.04658(200))}\right)$$

$$v_{max} = 2068 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v_{max} = 2.07(10^3)$ ft/s

15–147.

Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed $v = 0.4$ m/s. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.



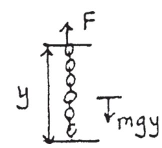
SOLUTION

$$\frac{dv}{dt} = 0, \qquad y = vt$$

$$m_i = my = mvt$$

$$\frac{dm_i}{dt} = mv$$

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\left(\frac{dm_i}{dt}\right)$$



$$F - mgvt = 0 + v(mv)$$

$$F = m(gvt + v^2)$$

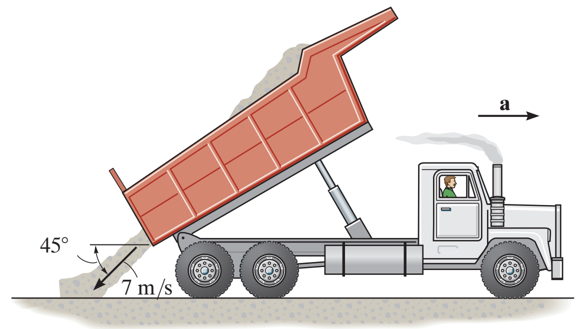
$$= 2[9.81(0.4)t + (0.4)^2]$$

$$F = (7.85t + 0.320)\ \text{N}$$ **Ans.**

Ans:
$F = \{7.85t + 0.320\}$ N

***15–148.**

The truck has a mass of 50 Mg when empty. When it is unloading 5 m^3 of sand at a constant rate of 0.8 m^3/s, the sand flows out the back at a speed of 7 m/s, measured relative to the truck, in the direction shown. If the truck is free to roll, determine its initial acceleration just as the load begins to empty. Neglect the mass of the wheels and any frictional resistance to motion. The density of sand is $\rho_s = 1520\ \text{kg/m}^3$.



SOLUTION

A System That Loses Mass: Initially, the total mass of the truck is $m = 50(10^3) + 5(1520) = 57.6(10^3)$ kg and $\dfrac{dm_e}{dt} = 0.8(1520) = 1216$ kg/s. Applying Eq. 15–29, we have

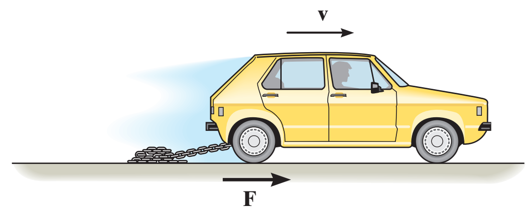
$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad 0 = 57.6(10^3)a - (0.8\cos 45°)(1216)$$

$$a = 0.104\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.104\ \text{m/s}^2$

15–149.

The car has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force F that must be supplied by the rear wheels of the car, necessary to maintain a constant speed v while the chain is being drawn out.



SOLUTION

$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time t, $m = m_0 + ct$, where $c = \dfrac{dm_i}{dt} = \dfrac{m'dx}{dt} = m'v$.

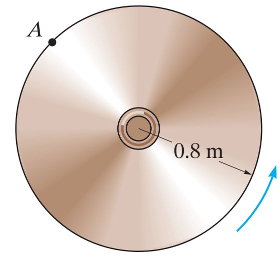
Here, $v_{D/i} = v$, $\quad \dfrac{dv}{dt} = 0$.

$$F = (m_0 - m'v\,)(0) + v(m'v) = m'v^2 \qquad \textbf{Ans.}$$

Ans:
$F = m'v^2$

16–1.

The angular velocity of the disk is defined by $\omega = (5t^2 + 2)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and acceleration of point A on the disk when $t = 0.5$ s.



SOLUTION

$$\omega = (5\,t^2 + 2)\ \text{rad/s}$$

$$\alpha = \frac{d\omega}{dt} = 10\,t$$

$$t = 0.5\ \text{s}$$

$$\omega = 3.25\ \text{rad/s}$$

$$\alpha = 5\ \text{rad/s}^2$$

$$v_A = \omega r = 3.25(0.8) = 2.60\ \text{m/s} \qquad \textbf{Ans.}$$

$$a_z = \alpha r = 5(0.8) = 4\ \text{m/s}^2$$

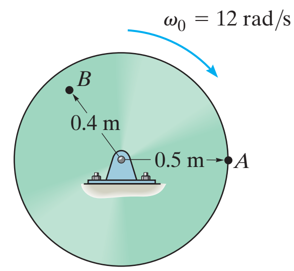
$$a_n = \omega^2 r = (3.25)^2(0.8) = 8.45\ \text{m/s}^2$$

$$a_A = \sqrt{(4)^2 + (8.45)^2} = 9.35\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_A = 2.60\ \text{m/s}$
$a_A = 9.35\ \text{m/s}^2$

16–2.

The angular acceleration of the disk is defined by $\alpha = 3t^2 + 12$ rad/s, where t is in seconds. If the disk is originally rotating at $\omega_0 = 12$ rad/s, determine the magnitude of the velocity and the n and t components of acceleration of point A on the disk when $t = 2$ s.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined by integrating $d\omega = \alpha\, dt$ with the initial condition $\omega = 12$ rad/s at $t = 0$.

$$\int_{12\text{ rad/s}}^{\omega} d\omega = \int_0^{2s} (3t^2 + 12)dt$$

$$\omega - 12 = (t^3 + 12t)\Big|_0^{2s}$$

$$\omega = 44.0 \text{ rad/s}$$

Motion of Point A. The magnitude of the velocity is

$$v_A = \omega r_A = 44.0(0.5) = 22.0 \text{ m/s}$$ **Ans.**

At $t = 2$ s, $\alpha = 3(2^2) + 12 = 24$ rad/s^2. Thus, the tangential and normal components of the acceleration are

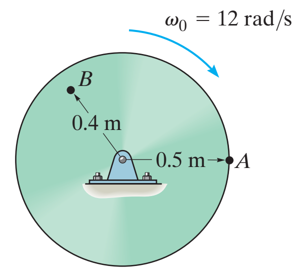
$$(a_A)_t = \alpha r_A = 24(0.5) = 12.0 \text{ m/s}^2$$ **Ans.**

$$(a_A)_n = \omega^2 r_A = (44.0^2)(0.5) = 968 \text{ m/s}^2$$ **Ans.**

Ans:
$v_A = 22.0$ m/s
$(a_A)_t = 12.0$ m/s^2
$(a_A)_n = 968$ m/s^2

16–3.

The disk is originally rotating at $\omega_0 = 12$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 20$ rad/s^2, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant $t = 2$ s.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined using

$$\omega = \omega_0 + \alpha_c t; \qquad \omega = 12 + 20(2) = 52 \text{ rad/s}$$

Motion of Point A. The magnitude of the velocity is

$$v_A = \omega r_A = 52(0.5) = 26.0 \text{ m/s}$$ **Ans.**

The tangential and normal component of acceleration are

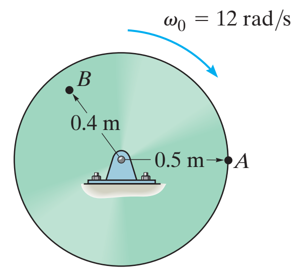
$$(a_A)_t = \alpha r = 20(0.5) = 10.0 \text{ m/s}^2$$ **Ans.**

$$(a_A)_n = \omega^2 r = \left(52^2\right)(0.5) = 1352 \text{ m/s}^2$$ **Ans.**

Ans:
$v_A = 26.0$ m/s
$\left(a_A\right)_t = 10.0$ m/s^2
$\left(a_A\right)_n = 1352$ m/s^2

***16–4.**

The disk is originally rotating at $\omega_0 = 12$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 20\ \text{rad/s}^2$, determine the magnitudes of the velocity and the n and t components of acceleration of point B when the disk undergoes 2 revolutions.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined using

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0); \qquad \omega^2 = 12^2 + 2(20)[2(2\pi) - 0]$$

$$\omega = 25.43\ \text{rad/s}$$

Motion of Point B. The magnitude of the velocity is

$$v_B = \omega r_B = 25.43(0.4) = 10.17\ \text{m/s} = 10.2\ \text{m/s} \qquad \textbf{Ans.}$$

The tangential and normal components of acceleration are

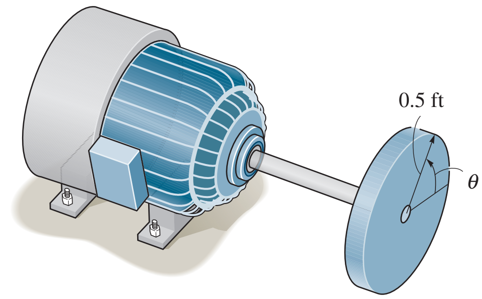
$$(a_B)_t = \alpha r_B = 20(0.4) = 8.00\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$(a_B)_n = \omega^2 r_B = \left(25.43^2\right)(0.4) = 258.66\ \text{m/s}^2 = 259\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_B = 10.2$ m/s
$(a_B)_t = 8.00\ \text{m/s}^2$
$(a_B)_n = 259\ \text{m/s}^2$

16–5.

The disk is driven by a motor such that the angular position of the disk is defined by $\theta = (20t + 4t^2)$ rad, where t is in seconds. Determine the number of revolutions, the angular velocity, and angular acceleration of the disk when $t = 90$ s.



SOLUTION

Angular Displacement: At $t = 90$ s.

$$\theta = 20(90) + 4\left(90^2\right) = (34200 \text{ rad}) \times \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 5443 \text{ rev} \qquad \textbf{Ans.}$$

Angular Velocity: Applying Eq. 16–1. we have

$$\omega = \frac{d\theta}{dt} = 20 + 8t\bigg|_{t=90\ s} = 740 \text{ rad/s} \qquad \textbf{Ans.}$$

Angular Acceleration: Applying Eq. 16–2. we have

$$\alpha = \frac{d\omega}{dt} = 8 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\theta = 5443$ rev
$\omega = 740$ rad/s
$\alpha = 8$ rad/s^2

16–6.

A wheel has an initial clockwise angular velocity of 10 rad/s and a constant angular acceleration of 3 rad/s^2. Determine the number of revolutions it must undergo to acquire a clockwise angular velocity of 15 rad/s. What time is required?

SOLUTION

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$(15)^2 = (10)^2 + 2(3)(\theta - 0)$$

$$\theta = 20.83 \text{ rad} = 20.83\left(\frac{1}{2\pi}\right) = 3.32 \text{ rev.} \qquad \textbf{Ans.}$$

$$\omega = \omega_0 + \alpha_c t$$

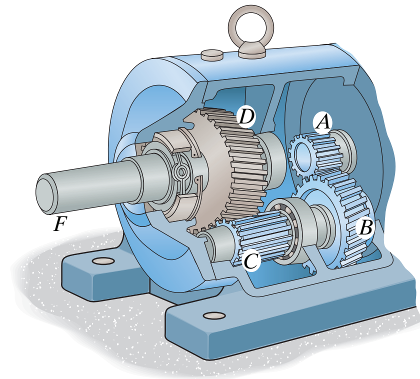
$$15 = 10 + 3t$$

$$t = 1.67 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$\theta = 3.32$ rev
$t = 1.67$ s

16–7.

If gear A rotates with a constant angular acceleration of $\alpha_A = 90\ \text{rad/s}^2$, starting from rest, determine the time required for gear D to attain an angular velocity of 600 rpm. Also, find the number of revolutions of gear D to attain this angular velocity. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



SOLUTION

Gear B is in mesh with gear A. Thus,

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{15}{50}\right)(90) = 27\ \text{rad/s}^2$$

Since gears C and B share the same shaft, $\alpha_C = \alpha_B = 27\ \text{rad/s}^2$. Also, gear D is in mesh with gear C. Thus,

$$\alpha_D r_D = \alpha_C r_C$$

$$\alpha_D = \left(\frac{r_C}{r_D}\right)\alpha_C = \left(\frac{25}{75}\right)(27) = 9\ \text{rad/s}^2$$

The final angular velocity of gear D is $\omega_D = \left(\frac{600\ \text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 20\pi\ \text{rad/s}$. Applying the constant acceleration equation,

$$\omega_D = (\omega_D)_0 + \alpha_D t$$

$$20\pi = 0 + 9t$$

$$t = 6.98\ \text{s} \qquad \textbf{Ans.}$$

and

$$\omega_D{}^2 = (\omega_D)_0{}^2 + 2\alpha_D[\theta_D - (\theta_D)_0]$$

$$(20\pi)^2 = 0^2 + 2(9)(\theta_D - 0)$$

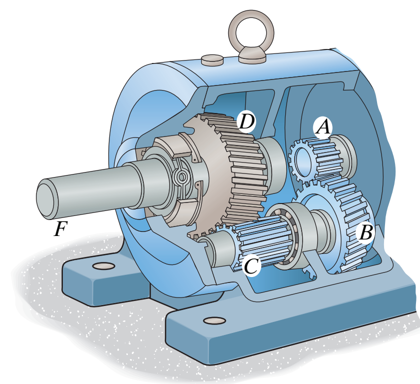
$$\theta_D = (219.32\ \text{rad})\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)$$

$$= 34.9\ \text{rev} \qquad \textbf{Ans.}$$

Ans:
$t = 6.98\ \text{s}$
$\theta_D = 34.9\ \text{rev}$

***16–8.**

If gear A rotates with an angular velocity of $\omega_A = (\theta_A + 1)$ rad/s, where θ_A is the angular displacement of gear A, measured in radians, determine the angular acceleration of gear D when $\theta_A = 3$ rad, starting from rest. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



SOLUTION

Motion of Gear A:

$$\alpha_A d\theta_A = \omega_A d\omega_A$$

$$\alpha_A d\theta_A = (\theta_A + 1)\, d(\theta_A + 1)$$

$$\alpha_A d\theta_A = (\theta_A + 1)\, d\theta_A$$

$$\alpha_A = (\theta_A + 1)$$

At $\theta_A = 3$ rad,

$$\alpha_A = 3 + 1 = 4 \text{ rad/s}^2$$

Motion of Gear D: Gear A is in mesh with gear B. Thus,

$$\alpha_B r_B = \alpha_A r_A$$
$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{15}{50}\right)(4) = 1.20 \text{ rad/s}^2$$

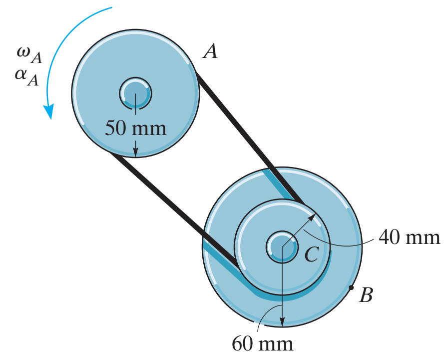
Since gears C and B share the same shaft $\alpha_C = \alpha_B = 1.20$ rad/s^2. Also, gear D is in mesh with gear C. Thus,

$$\alpha_D r_D = \alpha_C r_C$$
$$\alpha_D = \left(\frac{r_C}{r_D}\right)\alpha_C = \left(\frac{25}{75}\right)(1.20) = 0.4 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\alpha_D = 0.4$ rad/s^2

16–9.

At the instant $\omega_A = 5$ rad/s, pulley A is given an angular acceleration $\alpha = (0.8\theta)$ rad/s^2, where θ is in radians. Determine the magnitude of acceleration of point B on pulley C when A rotates 3 revolutions. Pulley C has an inner hub which is fixed to its outer one and turns with it.



SOLUTION

Angular Motion. The angular velocity of pulley A can be determined by integrating $\omega\, d\omega = \alpha\, d\theta$ with the initial condition $\omega_A = 5$ rad/s at $\theta_A = 0$.

$$\int_{5\text{ rad/s}}^{\omega_A} \omega\, d\omega = \int_0^{\theta_A} 0.8\theta d\theta$$

$$\left.\frac{\omega^2}{2}\right|_{5\text{ rad/s}}^{\omega_A} = \left(0.4\theta^2\right)\Big|_0^{\theta_A}$$

$$\frac{\omega_A^2}{2} - \frac{5^2}{2} = 0.4\theta_A^2$$

$$\omega_A = \left\{\sqrt{0.8\theta_A^2 + 25}\right\} \text{ rad/s}$$

At $\theta_A = 3(2\pi) = 6\pi$ rad,

$$\omega_A = \sqrt{0.8(6\pi)^2 + 25} = 17.585 \text{ rad/s}$$

$$\alpha_A = 0.8(6\pi) = 4.8\pi \text{ rad/s}^2$$

Since pulleys A and C are connected by a non-slip belt,

$$\omega_C r_C = \omega_A r_A; \qquad \omega_C(40) = 17.585(50)$$

$$\omega_C = 21.982 \text{ rad/s}$$

$$\alpha_C r_C = \alpha_A r_A; \qquad \alpha_C(40) = (4.8\pi)(50)$$

$$\alpha_C = 6\pi \text{ rad/s}^2$$

Motion of Point B. The tangential and normal components of acceleration of point B can be determined from

$$(a_B)_t = \alpha_C r_B = 6\pi(0.06) = 1.1310 \text{ m/s}^2$$

$$(a_B)_n = \omega_C^2 r_B = \left(21.982^2\right)(0.06) = 28.9917 \text{ m/s}^2$$

Thus, the magnitude of a_B is

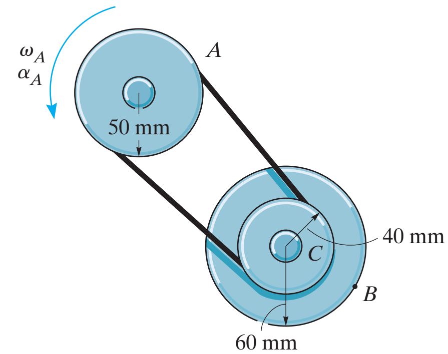
$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{1.1310^2 + 28.9917^2}$$

$$= 29.01 \text{ m/s}^2 = 29.0 \text{ m/s}^2$$ **Ans.**

Ans:
$a_B = 29.0 \text{ m/s}^2$

16–10.

At the instant $v_A = 5$ rad/s, pulley A is given a constant angular acceleration $\alpha_A = 6$ rad/s^2. Determine the magnitude of acceleration of point B on pulley C when A rotates 2 revolutions. Pulley C has an inner hub which is fixed to its outer one and turns with it.



SOLUTION

Angular Motion. Since the angular acceleration of pulley A is constant, we can apply

$$\omega_A^2 = (\omega_A)_0^2 + 2\alpha_A[\theta_A - (\theta_A)_0];$$

$$\omega_A^2 = 5^2 + 2(6)[2(2\pi) - 0]$$

$$\omega_A = 13.2588 \text{ rad/s}$$

Since pulleys A and C are connected by a non-slip belt,

$$\omega_C r_C = \omega_A r_A; \qquad \omega_C(40) = 13.2588(50)$$

$$\omega_C = 16.5735 \text{ rad/s}$$

$$\alpha_C r_C = \alpha_A r_A; \qquad \alpha_C(40) = 6(50)$$

$$\alpha_C = 7.50 \text{ rad/s}^2$$

Motion of Point B. The tangential and normal component of acceleration of point B can be determined from

$$(a_B)_t = \alpha_C r_B = 7.50(0.06) = 0.450 \text{ m/s}^2$$

$$(a_B)_n = \omega_C^2 r_B = (16.5735^2)(0.06) = 16.4809 \text{ m/s}^2$$

Thus, the magnitude of a_B is

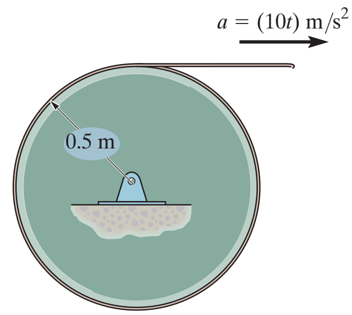
$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{0.450^2 + 16.4809^2}$$

$$= 16.4871 \text{ m/s}^2 = 16.5 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_B = 16.5 \text{ m/s}^2$

16–11.

The cord, which is wrapped around the disk, is given an acceleration of $a = (10t)\ \mathrm{m/s^2}$, where t is in seconds. Starting from rest, determine the angular displacement, angular velocity, and angular acceleration of the disk when $t = 3$ s.



SOLUTION

Motion of Point P. The tangential component of acceleration of a point on the rim is equal to the acceleration of the cord. Thus

$$(a_t) = \alpha r; \qquad 10t = \alpha(0.5)$$

$$\alpha = \{20t\}\ \mathrm{rad/s^2}$$

When $t = 3$ s,

$$\alpha = 20(3) = 60\ \mathrm{rad/s^2} \qquad \textbf{Ans.}$$

Angular Motion. The angular velocity of the disk can be determined by integrating $d\omega = \alpha\, dt$ with the initial condition $\omega = 0$ at $t = 0$.

$$\int_0^{\omega} d\omega = \int_0^t 20t\, dt$$

$$\omega = \{10t^2\}\ \mathrm{rad/s}$$

When $t = 3$ s,

$$\omega = 10(3^2) = 90.0\ \mathrm{rad/s} \qquad \textbf{Ans.}$$

The angular displacement of the disk can be determined by integrating $d\theta = \omega\, dt$ with the initial condition $\theta = 0$ at $t = 0$.

$$\int_0^{\theta} d\theta = \int_0^t 10t^2\, dt$$

$$\theta = \left\{\frac{10}{3}t^3\right\}\ \mathrm{rad}$$

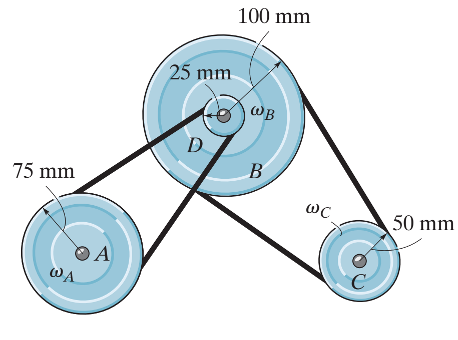
When $t = 3$ s,

$$\theta = \frac{10}{3}(3^3) = 90.0\ \mathrm{rad} \qquad \textbf{Ans.}$$

Ans:
$\alpha = 60\ \mathrm{rad/s^2}$
$\omega = 90.0\ \mathrm{rad/s}$
$\theta = 90.0\ \mathrm{rad}$

***16–12.**

The power of a bus engine is transmitted using the belt-and-pulley arrangement shown. If the engine turns pulley A at $\omega_A = (20t + 40)$ rad/s, where t is in seconds, determine the angular velocities of the generator pulley B and the air-conditioning pulley C when $t = 3$ s.



SOLUTION

When $t = 3$ s

$$\omega_A = 20(3) + 40 = 100 \text{ rad/s}$$

The speed of a point P on the belt wrapped around A is

$$v_P = \omega_A r_A = 100(0.075) = 7.5 \text{ m/s}$$

$$\omega_B = \frac{v_P}{r_D} = \frac{7.5}{0.025} = 300 \text{ rad/s} \qquad \textbf{Ans.}$$

The speed of a point P' on the belt wrapped around the outer periphery of B is

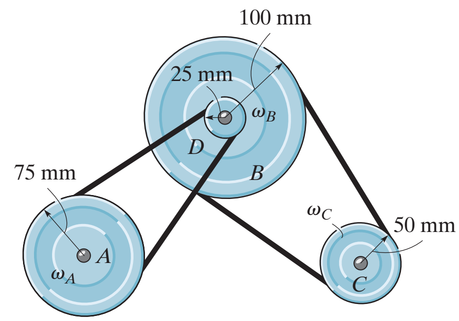
$$v'_p = \omega_B r_B = 300(0.1) = 30 \text{ m/s}$$

Hence, $\omega_C = \dfrac{v'_P}{r_C} = \dfrac{30}{0.05} = 600 \text{ rad/s}$ **Ans.**

Ans:
$\omega_B = 300$ rad/s
$\omega_C = 600$ rad/s

16–13.

The power of a bus engine is transmitted using the belt-and-pulley arrangement shown. If the engine turns pulley A at $\omega_A = 60$ rad/s, determine the angular velocities of the generator pulley B and the air-conditioning pulley C. The hub at D is rigidly *connected* to B and turns with it.



SOLUTION

The speed of a point P on the belt wrapped around A is

$$v_P = \omega_A r_A = 60(0.075) = 4.5 \text{ m/s}$$

$$\omega_B = \frac{v_P}{r_D} = \frac{4.5}{0.025} = 180 \text{ rad/s} \qquad \textbf{Ans.}$$

The speed of a point P' on the belt wrapped around the outer periphery of B is

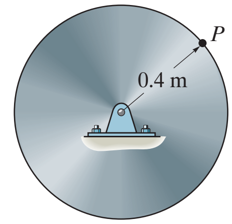
$$v'_P = \omega_B r_B = 180(0.1) = 18 \text{ m/s}$$

Hence, $\omega_C = \dfrac{v'_P}{r_C} = \dfrac{18}{0.05} = 360 \text{ rad/s}$ **Ans.**

Ans:
$\omega_B = 180$ rad/s
$\omega_C = 360$ rad/s

16–14.

The disk starts from rest and is given an angular acceleration $\alpha = (2t^2)\ \text{rad/s}^2$, where t is in seconds. Determine the angular velocity of the disk and its angular displacement when $t = 4$ s.



SOLUTION

$$\alpha = \frac{d\omega}{dt} = 2t^2$$

$$\int_0^\omega d\omega = \int_0^t 2t^2 dt$$

$$\omega = \frac{2}{3}t^3\Big|_0^t$$

$$\omega = \frac{2}{3}t^3$$

When $t = 4$ s,

$$\omega = \frac{2}{3}(4)^3 = 42.7\ \text{rad/s}$$ **Ans.**

$$\int_0^\theta d\theta = \int_0^t \frac{2}{3}t^3 dt$$

$$\theta = \frac{1}{6}t^4$$

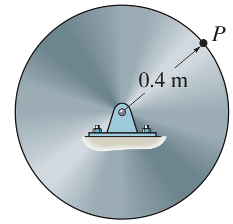
When $t = 4$ s,

$$\theta = \frac{1}{6}(4)^4 = 42.7\ \text{rad}$$ **Ans.**

Ans:
$\omega = 42.7$ rad/s
$\theta = 42.7$ rad

16–15.

The disk starts from rest and is given an angular acceleration $\alpha = (5t^{1/2})$ rad/s^2, where t is in seconds. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $t = 2$ s.



SOLUTION

Motion of the Disk: Here, when $t = 0, \omega = 0$.

$$d\omega = \alpha dt$$

$$\int_0^{\omega} d\omega = \int_0^t 5t^{\frac{1}{2}}dt$$

$$\omega\Big|_0^{\omega} = \frac{10}{3}t^{\frac{3}{2}}\Big|_0^t$$

$$\omega = \left\{\frac{10}{3}t^{\frac{3}{2}}\right\} \text{ rad/s}$$

When $t = 2$ s,

$$\omega = \frac{10}{3}\left(2^{\frac{3}{2}}\right) = 9.428 \text{ rad/s}$$

When $t = 2$ s,

$$\alpha = 5\left(2^{\frac{1}{2}}\right) = 7.071 \text{ rad/s}^2$$

Motion of point P: The tangential and normal components of the acceleration of point P when $t = 2$ s are

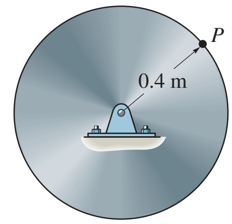
$$a_t = \alpha r = 7.071(0.4) = 2.83 \text{ m/s}^2$$ **Ans.**

$$a_n = \omega^2 r = 9.428^2(0.4) = 35.6 \text{ m/s}^2$$ **Ans.**

Ans:
$a_t = 2.83$ m/s^2
$a_n = 35.6$ m/s^2

***16–16.**

The disk starts at $\omega_0 = 1$ rad/s when $\theta = 0$, and is given an angular acceleration $\alpha = (0.3\theta)$ rad/s^2, where θ is in radians. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $\theta = 1$ rev.



SOLUTION

$$\alpha = 0.3\theta$$

$$\int_1^\omega \omega d\omega = \int_0^\theta 0.3\theta d\theta$$

$$\frac{1}{2}\omega^2\Big|_1^\omega = 0.15\theta^2\Big|_0^\theta$$

$$\frac{\omega^2}{2} - 0.5 = 0.15\theta^2$$

$$\omega = \sqrt{0.3\theta^2 + 1}$$

At $\theta = 1 \text{ rev} = 2\pi$ rad

$$\omega = \sqrt{0.3(2\pi)^2 + 1}$$

$$\omega = 3.584 \text{ rad/s}$$

$$a_t = \alpha r = 0.3(2\pi) \text{ rad/s}^2(0.4 \text{ m}) = 0.7540 \text{ m/s}^2$$ **Ans.**

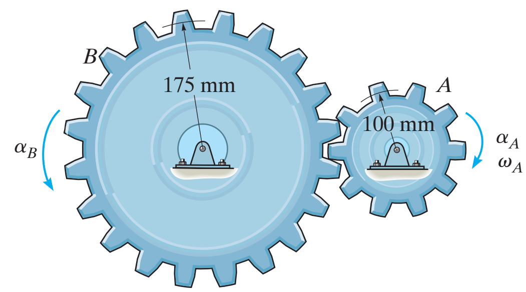
$$a_n = \omega^2 r = (3.584 \text{ rad/s})^2(0.4 \text{ m}) = 5.137 \text{ m/s}^2$$ **Ans.**

$$a_p = \sqrt{(0.7540)^2 + (5.137)^2} = 5.19 \text{ m/s}^2$$

Ans:
$a_t = 0.7540 \text{ m/s}^2$
$a_n = 5.137 \text{ m/s}^2$

16–17.

A motor gives gear A an angular acceleration of $\alpha_A = (2 + 0.006\,\theta^2)\ \text{rad/s}^2$, where θ is in radians. If this gear is initially turning at $\omega_A = 15\ \text{rad/s}$, determine the angular velocity of gear B after A undergoes an angular displacement of 10 rev.



SOLUTION

Angular Motion. The angular velocity of the gear A can be determined by integrating $\omega\,d\omega = \alpha\,d\theta$ with initial condition $\omega_A = 15\ \text{rad/s}$ at $\theta_A = 0$.

$$\int_{15\ \text{rad/s}}^{\omega_A} \omega\,d\omega = \int_0^{\theta_A} \left(2 + 0.006\,\theta^2\right)d\theta$$

$$\left.\frac{\omega^2}{2}\right|_{15\ \text{rad/s}}^{\omega_A} = \left.\left(2\theta + 0.002\,\theta^3\right)\right|_0^{\theta_A}$$

$$\frac{\omega_A^2}{2} - \frac{15^2}{2} = 2\theta_A + 0.002\,\theta_A^3$$

$$\omega_A = \sqrt{0.004\,\theta_A^3 + 4\,\theta + 225}\ \text{rad/s}$$

At $\theta_A = 10(2\pi) = 20\pi$ rad,

$$\omega_A = \sqrt{0.004(20\pi)^3 + 4(20\pi) + 225}$$

$$= 38.3214\ \text{rad/s}$$

Since gear B is meshed with gear A,

$$\omega_B r_B = \omega_A r_A; \qquad \omega_B(175) = 38.3214(100)$$

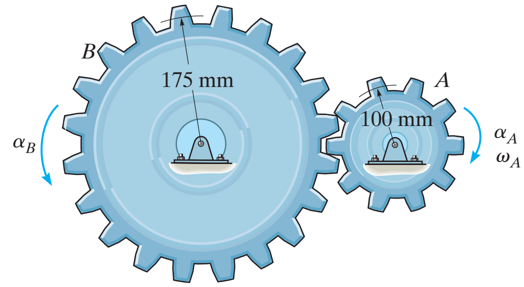
$$\omega_B = 21.8979\ \text{rad/s}$$

$$= 21.9\ \text{rad/s}\ \circlearrowright \qquad \textbf{Ans.}$$

Ans:
$\omega_B = 21.9\ \text{rad/s}\ \circlearrowright$

16–18.

A motor gives gear A an angular acceleration of $\alpha_A = (2t^3)$ rad/s^2, where t is in seconds. If this gear is initially turning at $\omega_A = 15$ rad/s, determine the angular velocity of gear B when $t = 3$ s.



SOLUTION

Angular Motion. The angular velocity of gear A can be determined by integrating $d\omega = \alpha\, dt$ with initial condition $\omega_A = 15$ rad/s at $t = 0$ s.

$$\int_{15\text{ rad/s}}^{\omega_A} d\omega = \int_0^t 2t^3\, dt$$

$$\omega_A - 15 = \frac{1}{2}t^4\Big|_0^t$$

$$\omega_A = \left\{\frac{1}{2}t^4 + 15\right\}\text{ rad/s}$$

At $t = 3$ s,

$$\omega_A = \frac{1}{2}(3^4) + 15 = 55.5\text{ rad/s}$$

Since gear B meshed with gear A,

$$\omega_B r_B = \omega_A r_A; \qquad \omega_B(175) = 55.5(100)$$

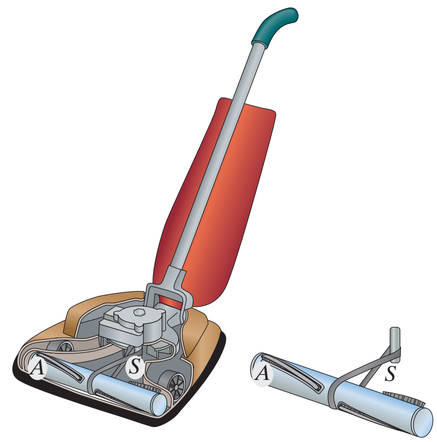
$$\omega_B = 31.7143\text{ rad/s}$$

$$= 31.7\text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

Ans:
$\omega_B = 31.7$ rad/s $\circlearrowright$

16–19.

The vacuum cleaner's armature shaft S rotates with an angular acceleration of $\alpha = 4\omega^{3/4}$ rad/s², where ω is in rad/s. Determine the brush's angular velocity when $t = 4$ s, starting from $\omega_0 = 1$ rad/s, at $\theta = 0$. The radii of the shaft and the brush are 0.25 in. and 1 in., respectively. Neglect the thickness of the drive belt.



SOLUTION

Motion of the Shaft: The angular velocity of the shaft can be determined from

$$\int dt = \int \frac{d\omega_S}{\alpha_S}$$

$$\int_0^t dt = \int_1^{\omega_s} \frac{d\omega_S}{4\omega_S{}^{3/4}}$$

$$t\Big|_0^t = \omega_S{}^{1/4}\Big|_1^{\omega_s}$$

$$t = \omega_S{}^{1/4} - 1$$

$$\omega_S = (t+1)^4$$

When $t = 4$ s

$$\omega_s = 5^4 = 625 \text{ rad/s}$$

Motion of the Beater Brush: Since the brush is connected to the shaft by a non-slip belt, then

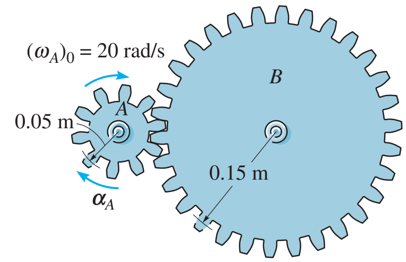
$$\omega_B r_B = \omega_s r_s$$

$$\omega_B = \left(\frac{r_s}{r_B}\right)\omega_s = \left(\frac{0.25}{1}\right)(625) = 156 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_B = 156$ rad/s

***16–20.**

A motor gives gear A an angular acceleration of $\alpha_A = (4t^3)\ \text{rad/s}^2$, where t is in seconds. If this gear is initially turning at $(\omega_A)_0 = 20\ \text{rad/s}$, determine the angular velocity of gear B when $t = 2$ s.



SOLUTION

$\alpha_A = 4\,t^3$

$dw = \alpha\, dt$

$$\int_{20}^{w_A} dw_A = \int_0^t \alpha_A\, dt = \int_0^t 4\,t^3\, dt$$

$w_A = t^4 + 20$

When $t = 2$ s,

$w_A = 36\ \text{rad/s}$

$w_A\, r_A = w_B\, r_B$

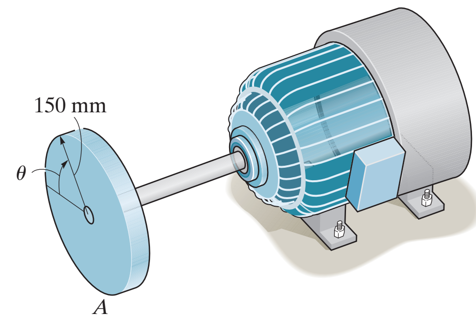
$36(0.05) = w_B(0.15)$

$w_B = 12\ \text{rad/s}$ **Ans.**

Ans:
$w_B = 12\ \text{rad/s}$

16–21.

The motor turns the disk with an angular velocity of $\omega = (5t^2 + 3t)$ rad/s, where t is in seconds. Determine the magnitudes of the velocity and the n and t components of acceleration of the point A on the disk when $t = 3$ s.



SOLUTION

Angular Motion. At $t = 3$ s,

$$\omega = 5(3^2) + 3(3) = 54 \text{ rad/s}$$

The angular acceleration of the disk can be determined using

$$\alpha = \frac{d\omega}{dt}; \qquad \alpha = \{10t + 3\} \text{ rad/s}^2$$

At $t = 3$ s,

$$\alpha = 10(3) + 3 = 33 \text{ rad/s}^2$$

Motion of Point A. The magnitude of the velocity is

$$v_A = \omega r_A = 54(0.15) = 8.10 \text{ m/s}$$ **Ans.**

The tangential and normal component of acceleration are

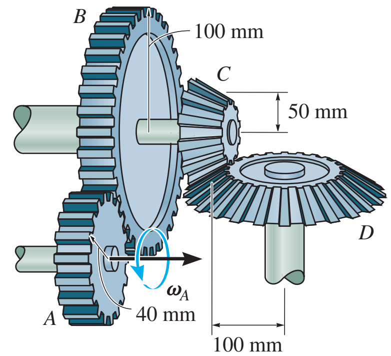
$$(a_A)_t = \alpha r_A = 33(0.15) = 4.95 \text{ m/s}^2$$ **Ans.**

$$(a_A)_n = \omega^2 r_A = (54^2)(0.15) = 437.4 \text{ m/s}^2 = 437 \text{ m/s}^2$$ **Ans.**

Ans:
$v_A = 8.10$ m/s
$(a_A)_t = 4.95$ m/s^2
$(a_A)_n = 437$ m/s^2

16–22.

If the motor turns gear A with an angular acceleration of $\alpha_A = 2\ \text{rad/s}^2$ when the angular velocity is $\omega_A = 20\ \text{rad/s}$, determine the angular acceleration and angular velocity of gear D.



SOLUTION

Angular Motion: The angular velocity and acceleration of gear B must be determined first. Here, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{40}{100}\right)(20) = 8.00\ \text{rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{40}{100}\right)(2) = 0.800\ \text{rad/s}^2$$

Since gear C is attached to gear B, then $\omega_C = \omega_B = 8\ \text{rad/s}$ and $\alpha_C = \alpha_B = 0.8\ \text{rad/s}^2$. Realizing that $\omega_C r_C = \omega_D r_D$ and $\alpha_C r_C = \alpha_D r_D$, then

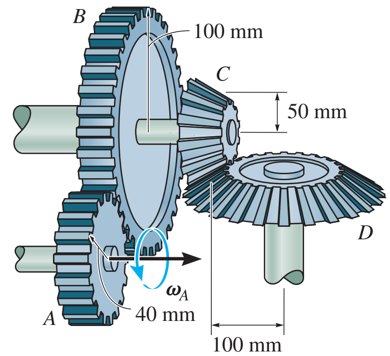
$$\omega_D = \frac{r_C}{r_D}\omega_C = \left(\frac{50}{100}\right)(8.00) = 4.00\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\alpha_D = \frac{r_C}{r_D}\alpha_C = \left(\frac{50}{100}\right)(0.800) = 0.400\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_D = 4.00\ \text{rad/s}$
$\alpha_D = 0.400\ \text{rad/s}^2$

16–23.

If the motor turns gear A with an angular acceleration of $\alpha_A = 3\ \text{rad/s}^2$ when the angular velocity is $\omega_A = 60\ \text{rad/s}$, determine the angular acceleration and angular velocity of gear D.



SOLUTION

Angular Motion: The angular velocity and acceleration of gear B must be determined first. Here, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{40}{100}\right)(60) = 24.0\ \text{rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{40}{100}\right)(3) = 1.20\ \text{rad/s}^2$$

Since gear C is attached to gear B, then $\omega_C = \omega_B = 24.0\ \text{rad/s}$ and $\alpha_C = \alpha_B = 1.20\ \text{rad/s}^2$. Realizing that $\omega_C r_C = \omega_D r_D$ and $\alpha_C r_C = \alpha_D r_D$, then

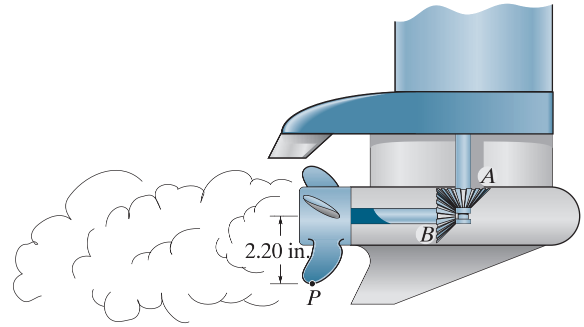
$$\omega_D = \frac{r_C}{r_D}\omega_C = \left(\frac{50}{100}\right)(24.0) = 12.0\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\alpha_D = \frac{r_C}{r_D}\alpha_C = \left(\frac{50}{100}\right)(1.20) = 0.600\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_D = 12.0\ \text{rad/s}$
$\alpha_D = 0.600\ \text{rad/s}^2$

***16–24.**

The gear A on the drive shaft of the outboard motor has a radius $r_A = 0.5$ in. and the meshed pinion gear B on the propeller shaft has a radius $r_B = 1.2$ in. Determine the angular velocity of the propeller in $t = 1.5$ s, if the drive shaft rotates with an angular acceleration $\alpha = (400t^3)$ rad/s^2, where t is in seconds. The propeller is originally at rest and the motor frame does not move.



SOLUTION

Angular Motion: The angular velocity of gear A at $t = 1.5$ s must be determined first. Applying Eq. 16–2, we have

$$d\omega = \alpha dt$$

$$\int_0^{\omega_A} d\omega = \int_0^{1.5\,s} 400t^3\,dt$$

$$\omega_A = 100t^4\big|_0^{1.5\,s} = 506.25 \text{ rad/s}$$

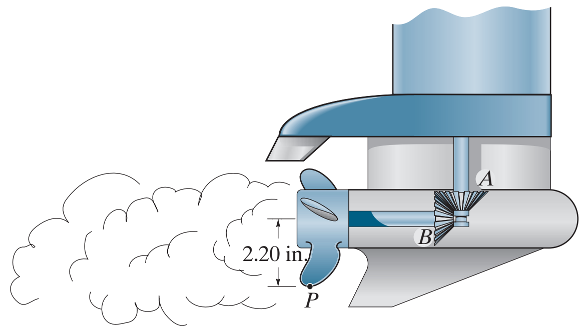
However, $\omega_A r_A = \omega_B r_B$ where ω_B is the angular velocity of propeller. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{0.5}{1.2}\right)(506.25) = 211 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_B = 211$ rad/s

16–25.

For the outboard motor in Prob. 16–24, determine the magnitude of the velocity and acceleration of point P located on the tip of the propeller at the instant $t = 0.75$ s.



SOLUTION

Angular Motion: The angular velocity of gear A at $t = 0.75$ s must be determined first. Applying Eq. 16–2, we have

$$d\omega = \alpha dt$$

$$\int_0^{\omega_A} d\omega = \int_0^{0.75\text{ s}} 400t^3\, dt$$

$$\omega_A = 100t^4 \Big|_0^{0.75\text{ s}} = 31.64\text{ rad/s}$$

The angular acceleration of gear A at $t = 0.75$ s is given by

$$\alpha_A = 400\left(0.75^3\right) = 168.75\text{ rad/s}^2$$

However, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$ where ω_B and α_B are the angular velocity and acceleration of propeller. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{0.5}{1.2}\right)(31.64) = 13.18\text{ rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{0.5}{1.2}\right)(168.75) = 70.31\text{ rad/s}^2$$

Motion of P: The magnitude of the velocity of point P can be determined using Eq. 16–8.

$$v_P = \omega_B r_P = 13.18\left(\frac{2.20}{12}\right) = 2.42\text{ ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of point P can be determined using Eqs. 16–11 and 16–12, respectively.

$$a_r = \alpha_B r_P = 70.31\left(\frac{2.20}{12}\right) = 12.89\text{ ft/s}^2$$

$$a_n = \omega_B^2 r_P = \left(13.18^2\right)\left(\frac{2.20}{12}\right) = 31.86\text{ ft/s}^2$$

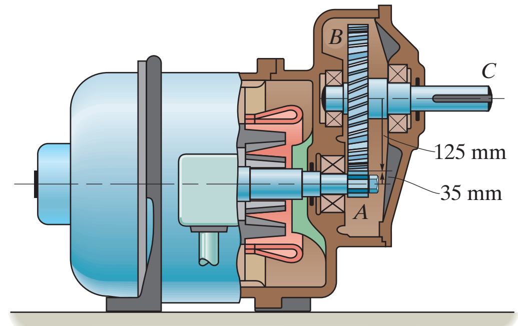
The magnitude of the acceleration of point P is

$$a_P = \sqrt{a_r^2 + a_n^2} = \sqrt{12.89^2 + 31.86^2} = 34.4\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_P = 2.42$ ft/s
$a_P = 34.4$ ft/s^2

16–26.

The pinion gear A on the motor shaft is given a constant angular acceleration $\alpha = 3\text{ rad/s}^2$. If the gears A and B have the dimensions shown, determine the angular velocity and angular displacement of the output shaft C, when $t = 2$ s starting from rest. The shaft is fixed to B and turns with it.



SOLUTION

$$\omega = \omega_0 + \alpha_c t$$

$$\omega_A = 0 + 3(2) = 6\text{ rad/s}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$\theta_A = 0 + 0 + \frac{1}{2}(3)(2)^2$$

$$\theta_A = 6\text{ rad}$$

$$\omega_A r_A = \omega_B r_B$$

$$6(35) = \omega_B(125)$$

$$\omega_C = \omega_B = 1.68\text{ rad/s} \qquad \textbf{Ans.}$$

$$\theta_A r_A = \theta_B r_B$$

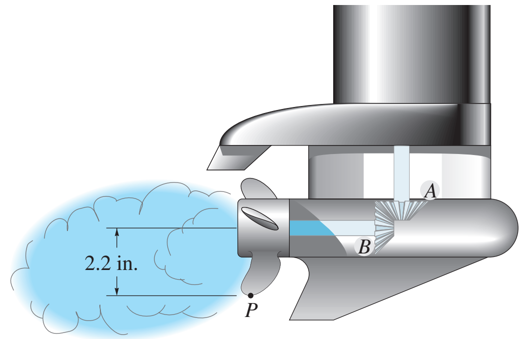
$$6(35) = \theta_B(125)$$

$$\theta_C = \theta_B = 1.68\text{ rad} \qquad \textbf{Ans.}$$

Ans:
$\omega_C = 1.68\text{ rad/s}$
$\theta_C = 1.68\text{ rad}$

16–27.

The gear A on the drive shaft of the outboard motor has a radius $r_A = 0.7$ in. and the meshed pinion gear B on the propeller shaft has a radius $r_B = 1.4$ in. Determine the angular velocity of the propeller in $t = 1.3$ s if the drive shaft rotates with an angular acceleration $\alpha = (300\sqrt{t})$ rad/s^2, where t is in seconds. The propeller is originally at rest and the motor frame does not move.



SOLUTION

$$\alpha_A r_A = \alpha_B r_B$$

$$(300\sqrt{t})(0.7) = \alpha_p(1.4)$$

$$\alpha_P = 150\sqrt{t}$$

$$d\omega = \alpha\, dt$$

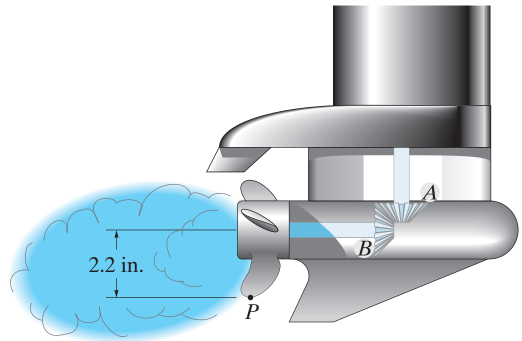
$$\int_0^\omega d\omega = \int_0^t 150\sqrt{t}\, dt$$

$$\omega = 100t^{3/2}\big|_{t=1.3} = 148 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 148$ rad/s

***16–28.**

The gear A on the drive shaft of the outboard motor has a radius $r_A = 0.7$ in. and the meshed pinion gear B on the propeller shaft has a radius $r_B = 1.4$ in. Determine the magnitudes of the velocity and acceleration of a point P located on the tip of the propeller at the instant $t = 0.75$ s. the drive shaft rotates with an angular acceleration $\alpha = (300\sqrt{t})$ rad/s^2, where t is in seconds. The propeller is originally at rest and the motor frame does not move.



SOLUTION

Angular Motion: The angular velocity of gear A at $t = 0.75$ s must be determined first. Applying Eq. 16–2, we have

$$d\omega = \alpha dt$$

$$\int_0^{\omega_A} d\omega = \int_0^{0.75\text{ s}} 300\sqrt{t}\, dt$$

$$\omega_A = 200\, t^{3/2}\Big|_0^{0.75\text{ s}} = 129.9 \text{ rad/s}$$

The angular acceleration of gear A at $t = 0.75$ s is given by

$$\alpha_A = 300\sqrt{0.75} = 259.81 \text{ rad/s}^2$$

However, $\omega_A r_A = \omega_B r_B$ and $\alpha_A r_A = \alpha_B r_B$ where ω_B and α_B are the angular velocity and acceleration of propeller. Then,

$$\omega_B = \frac{r_A}{r_B}\omega_A = \left(\frac{0.7}{1.4}\right)(129.9) = 64.95 \text{ rad/s}$$

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{0.7}{1.4}\right)(259.81) = 129.9 \text{ rad/s}^2$$

Motion of P: The magnitude of the velocity of point P can be determined using Eq. 16–8.

$$v_P = \omega_B r_P = 64.95\left(\frac{2.20}{12}\right) = 11.9 \text{ ft/s}$$

The tangential and normal components of the acceleration of point P can be determained using Eqs. 16–11 and 16–12, respectively.

$$a_t = \alpha_B r_P = 129.9\left(\frac{2.20}{12}\right) = 23.82 \text{ ft/s}^2$$

$$a_n = \omega_B^2 r_P = (64.95)^2\left(\frac{2.20}{12}\right) = 773.44 \text{ ft/s}^2$$

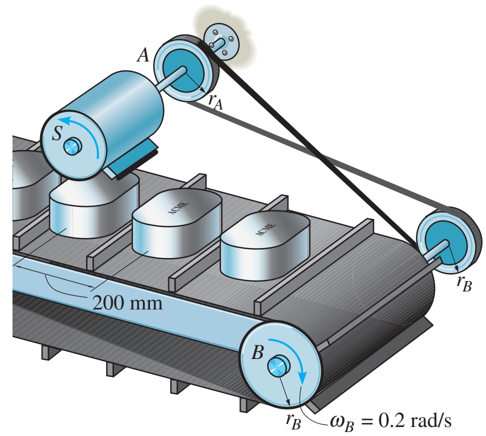
The magnitude of the acceleration of point P is

$$a_P = \sqrt{a_t^2 + a_n^2} = \sqrt{(23.82)^2 + (773.44)^2} = 774 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a_P = 774 \text{ ft/s}^2$

16–29.

A stamp S, located on the revolving drum, is used to label canisters. If the canisters are centered 200 mm apart on the conveyor, determine the radius r_A of the driving wheel A and the radius r_B of the conveyor belt drum so that for each revolution of the stamp it marks the top of a canister. How many canisters are marked per minute if the drum at B is rotating at $\omega_B = 0.2$ rad/s? Note that the driving belt is twisted as it passes between the wheels.



SOLUTION

$l = 2\pi(r_A)$

$r_A = \dfrac{200}{2\pi} = 31.8$ mm **Ans.**

For the drum at B:

$l = 2\pi(r_B)$

$r_B = \dfrac{200}{2\pi} = 31.8$ mm **Ans.**

In $t = 60$ s

$\theta = \theta_0 + \omega_0 t$

$\theta = 0 + 0.2(60) = 12$ rad

$l = \theta r_B = 12(31.8) = 382.0$ mm

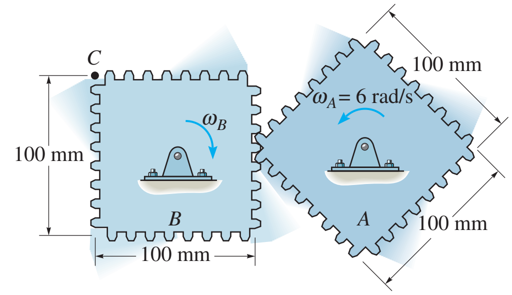
Hence,

$n = \dfrac{382.0}{200} = 1.91$ canisters marked per minute **Ans.**

Ans:
$r_A = 31.8$ mm
$r_B = 31.8$ mm
1.91 canisters per minute

16–30.

At the instant shown, gear A is rotating with a constant angular velocity of $\omega_A = 6\ \text{rad/s}$. Determine the largest angular velocity of gear B and the maximum speed of point C.



SOLUTION

$(r_B)_{max} = (r_A)_{max} = 50\sqrt{2}\ \text{mm}$

$(r_B)_{min} = (r_A)_{min} = 50\ \text{mm}$

When r_A is max., r_B is min.

$\omega_B(r_B) = \omega_A r_A$

$$(\omega_B)_{max} = 6\left(\frac{r_A}{r_B}\right) = 6\left(\frac{50\sqrt{2}}{50}\right)$$

$(\omega_B)_{max} = 8.49\ \text{rad/s}$ **Ans.**

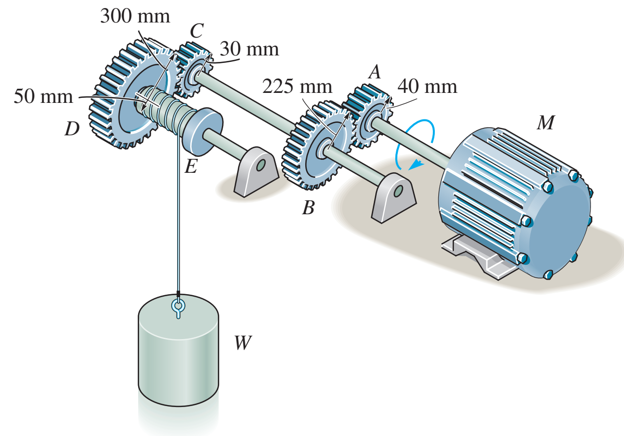
$v_C = (\omega_B)_{max}\, r_C = 8.49\big(0.05\sqrt{2}\big)$

$v_C = 0.6\ \text{m/s}$ **Ans.**

Ans:
$(\omega_B)_{\text{max}} = 8.49\ \text{rad/s}$
$(v_C)_{\text{max}} = 0.6\ \text{m/s}$

16–31.

Determine the distance the load W is lifted in $t = 5$ s using the hoist. The shaft of the motor M turns with an angular velocity $\omega = 100(4 + t)$ rad/s, where t is in seconds.



SOLUTION

Angular Motion: The angular displacement of gear A at $t = 5$ s must be determined first. Applying Eq. 16–1, we have

$$d\theta = \omega dt$$

$$\int_0^{\theta_A} d\theta = \int_0^{5\,s} 100(4 + t)\,dt$$

$$\theta_A = 3250 \text{ rad}$$

Here, $r_A\,\theta_A = r_B\,\theta_B$. Then, the angular displacement of gear B is given by

$$\theta_B = \frac{r_A}{r_B}\theta_A = \left(\frac{40}{225}\right)(3250) = 577.78 \text{ rad}$$

Since gear C is attached to the same shaft as gear B, then $\theta_C = \theta_B = 577.78$ rad. Also, $r_D\,\theta_D = r_C\,\theta_C$, then, the angular displacement of gear D is given by

$$\theta_D = \frac{r_C}{r_D}\theta_C = \left(\frac{30}{300}\right)(577.78) = 57.78 \text{ rad}$$

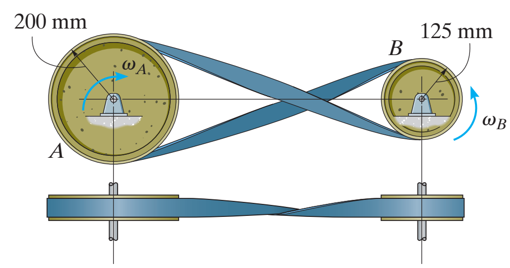
Since shaft E is attached to gear D, $\theta_E = \theta_D = 57.78$ rad. The distance at which the load W is lifted is

$$s_W = r_E\theta_E = (0.05)(57.78) = 2.89 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s_W = 2.89$ m

***16–32.**

The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If A has a constant angular acceleration of $\alpha_A = 30\ \text{rad/s}^2$, determine the tangential and normal components of acceleration of a point located at the rim of B when $t = 3$ s, starting from rest.



SOLUTION

Motion of Wheel A: Since the angular acceleration of wheel A is constant, its angular velocity can be determined from

$$\omega_A = (\omega_A)_0 + \alpha_C t$$

$$= 0 + 30(3) = 90\ \text{rad/s}$$

Motion of Wheel B: Since wheels A and B are connected by a nonslip belt, then

$$\omega_B r_B = \omega_A r_A$$

$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{200}{125}\right)(90) = 144\ \text{rad/s}$$

and

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{200}{125}\right)(30) = 48\ \text{rad/s}^2$$

Thus, the tangential and normal components of the acceleration of point P located at the rim of wheel B are

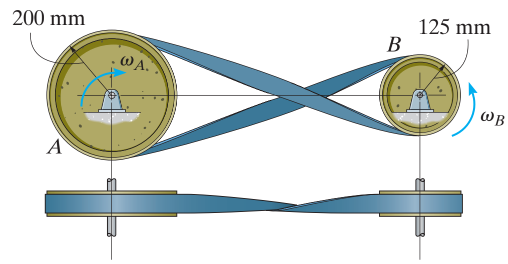
$$(a_p)_t = \alpha_B r_B = 48(0.125) = 6\ \text{m/s}^2$$ **Ans.**

$$(a_p)_n = \omega_B{}^2 r_B = (144^2)(0.125) = 2592\ \text{m/s}^2$$ **Ans.**

Ans:
$(a_p)_t = 6\ \text{m/s}^2$
$(a_p)_n = 2592\ \text{m/s}^2$

16–33.

The driving belt is twisted so that pulley B rotates in the opposite direction to that of drive wheel A. If the angular displacement of A is $\theta_A = (5t^3 + 10t^2)$ rad, where t is in seconds, determine the angular velocity and angular acceleration of B when $t = 3$ s.



SOLUTION

Motion of Wheel A: The angular velocity and angular acceleration of wheel A can be determined from

$$\omega_A = \frac{d\theta_A}{dt} = \left(15t^2 + 20t\right) \text{ rad/s}$$

and

$$\alpha_A = \frac{d\omega_A}{dt} = \left(30t + 20\right) \text{ rad/s}$$

When $t = 3$ s,

$$\omega_A = 15\left(3^2\right) + 20(3) = 195 \text{ rad/s}$$

$$\alpha_A = 30(3) + 20 = 110 \text{ rad/s}$$

Motion of Wheel B: Since wheels A and B are connected by a nonslip belt, then

$$\omega_B r_B = \omega_A r_A$$

$$\omega_B = \left(\frac{r_A}{r_B}\right)\omega_A = \left(\frac{200}{125}\right)(195) = 312 \text{ rad/s} \qquad \textbf{Ans.}$$

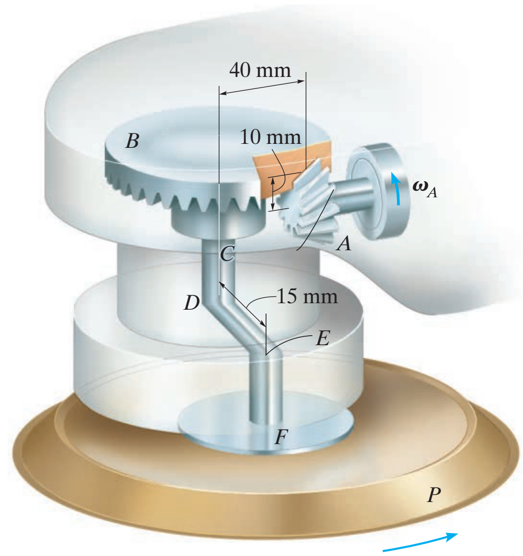
$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{200}{125}\right)(110) = 176 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_B = 312$ rad/s
$\alpha_B = 176$ rad/s^2

16–34.

For a short time a motor of the random-orbit sander drives the gear A with an angular velocity of $\omega_A = 40(t^3 + 6t)$ rad/s, where t is in seconds. This gear is connected to gear B, which is fixed connected to the shaft CD. The end of this shaft is connected to the eccentric spindle EF and pad P, which causes the pad to orbit around shaft CD at a radius of 15 mm. Determine the magnitudes of the velocity and the tangential and normal components of acceleration of the spindle EF when $t = 2$ s after starting from rest.



SOLUTION

$$\omega_A r_A = \omega_B r_B$$

$$\omega_A (10) = \omega_B (40)$$

$$\omega_B = \frac{1}{4}\omega_A$$

$$v_E = \omega_B r_E = \frac{1}{4}\omega_A (0.015) = \frac{1}{4}(40)\left(t^3 + 6t\right)(0.015)\Big|_{t=2}$$

$$v_E = 3 \text{ m/s}$$ **Ans.**

$$\alpha_A = \frac{d\omega_A}{dt} = \frac{d}{dt}\left[40\left(t^3 + 6t\right)\right] = 120t^2 + 240$$

$$\alpha_A r_A = \alpha_B r_B$$

$$\alpha_A (10) = \alpha_B (40)$$

$$\alpha_B = \frac{1}{4}\alpha_A$$

$$(a_E)_t = \alpha_B r_E = \frac{1}{4}\left(120t^2 + 240\right)(0.015)\Big|_{t=2}$$

$$(a_E)_t = 2.70 \text{ m/s}^2$$ **Ans.**

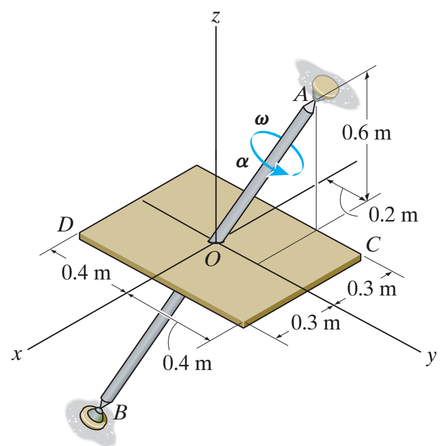
$$(a_E)_n = \omega_B^2 r_E = \left[\frac{1}{4}(40)\left(t^3 + 6t\right)\right]^2 (0.015)\Big|_{t=2}$$

$$(a_E)_n = 600 \text{ m/s}^2$$ **Ans.**

Ans:
$v_E = 3$ m/s
$(a_E)_t = 2.70$ m/s^2
$(a_E)_n = 600$ m/s^2

16–35.

If the shaft and plate rotates with a constant angular velocity of $\omega = 14$ rad/s, determine the velocity and acceleration of point C located on the corner of the plate at the instant shown. Express the result in Cartesian vector form.



SOLUTION

We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

Since ω is constant

$$\alpha = 0$$

For convenience, $\mathbf{r}_C = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point C can be determined from

$$\mathbf{v}_C = \omega \times \mathbf{r}_C$$

$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})$$

$$= [-4.8\mathbf{i} - 3.6\mathbf{j} - 1.2\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_C = \alpha \times \mathbf{r}_C + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_c)$$

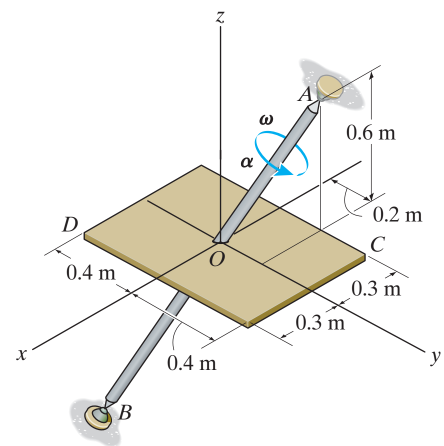
$$= 0 + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$

$$= [38.4\mathbf{i} - 64.8\mathbf{j} + 40.8\mathbf{k}] \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{-4.8\mathbf{i} - 3.6\mathbf{j} - 1.2\mathbf{k}\}$ m/s
$\mathbf{a}_C = \{38.4\mathbf{i} - 64.8\mathbf{j} + 40.8\mathbf{k}\}$ m/s^2

***16–36.**

At the instant shown, the shaft and plate rotates with an angular velocity of $\omega = 14$ rad/s and angular acceleration of $\alpha = 7$ rad/s^2. Determine the velocity and acceleration of point D located on the corner of the plate at this instant. Express the result in Cartesian vector form.



SOLUTION

We will first express the angular velocity ω of the plate in Cartesian vector form. The unit vector that defines the direction of ω and α is

$$\mathbf{u}_{OA} = \frac{-0.3\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + 0.6^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Thus,

$$\omega = \omega\mathbf{u}_{OA} = 14\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

$$\alpha = \alpha\mathbf{u}_{OA} = 7\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right) = [-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}] \text{ rad/s}$$

For convenience, $\mathbf{r}_D = [-0.3\mathbf{i} + 0.4\mathbf{j}]$ m is chosen. The velocity and acceleration of point D can be determined from

$$\mathbf{v}_D = \omega \times r_D$$

$$= (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (0.3\mathbf{i} - 0.4\mathbf{j})$$

$$= [4.8\mathbf{i} + 3.6\mathbf{j} + 1.2\mathbf{k}] \text{ m/s}$$ **Ans.**

and

$$\mathbf{a}_D = \alpha \times \mathbf{r}_D - \omega^2\,\mathbf{r}_D$$

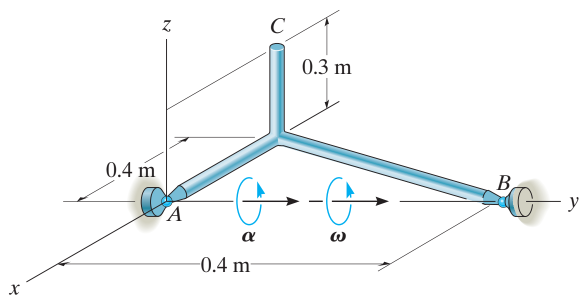
$$= (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j}) + (-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times [(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}) \times (-0.3\mathbf{i} + 0.4\mathbf{j})]$$

$$= [-36.0\mathbf{i} + 66.6\mathbf{j} - 40.2\mathbf{k}] \text{ m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_D = [4.8\mathbf{i} + 3.6\mathbf{j} + 1.2\mathbf{k}]$ m/s
$\mathbf{a}_D = [-36.0\mathbf{i} + 66.6\mathbf{j} - 40.2\mathbf{k}]$ m/s^2

16–37.

The rod assembly is supported by ball-and-socket joints at A and B. At the instant shown it is rotating about the y axis with an angular velocity $\omega = 5\text{ rad/s}$ and has an angular acceleration $\alpha = 8\text{ rad/s}^2$. Determine the magnitudes of the velocity and acceleration of point C at this instant. Solve the problem using Cartesian vectors and Eqs. 16–9 and 16–13.



SOLUTION

$$v_C = \omega \times \mathbf{r}$$

$$v_C = 5\mathbf{j} \times (-0.4\mathbf{i} + 0.3\mathbf{k}) = \{1.5\mathbf{i} + 2\mathbf{k}\}\text{ m/s}$$

$$v_C = \sqrt{1.5^2 + 2^2} = 2.50\text{ m/s} \qquad \textbf{Ans.}$$

$$\text{a}_C = \text{a} \times \mathbf{r} - \omega^2\mathbf{r}$$

$$= 8\mathbf{j} \times (-0.4\mathbf{i} + 0.3\mathbf{k}) - 5^2(-0.4\mathbf{i} + 0.3\mathbf{k})$$

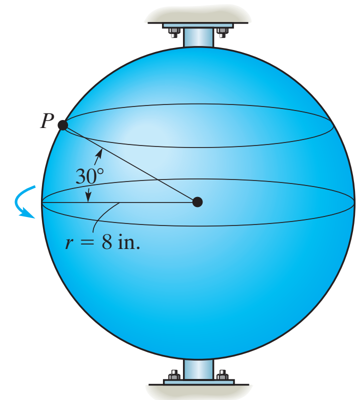
$$= \{12.4\mathbf{i} - 4.3\mathbf{k}\}\text{ m/s}^2$$

$$a_C = \sqrt{12.4^2 + (-4.3)^2} = 13.1\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_C = 2.50\text{ m/s}$
$a_C = 13.1\text{ m/s}^2$

16–38.

The sphere starts from rest at $\theta = 0°$ and rotates with an angular acceleration of $\alpha = (4\theta + 1)$ rad/s^2, where θ is in radians. Determine the magnitudes of the velocity and acceleration of point P on the sphere at the instant $\theta = 6$ rad.



SOLUTION

$$\omega\, d\omega = \alpha\, d\theta$$

$$\int_0^\omega \omega\, d\omega = \int_0^\theta (4\theta + 1)\, d\theta$$

$$\omega = 2\theta$$

At $\theta = 6$ rad,

$$\alpha = 4(6) + 1 = 25 \text{ rad/s}^2, \qquad \omega = \sqrt{4(6)^2 + 2(6)} = 12.49 \text{ rad/s}$$

$$v = ar' = 12.49(8 \cos 30°) = 86.53 \text{ in./s}$$

$v = 7.21$ ft/s **Ans.**

$$a_r = \frac{v^2}{r^2} = \frac{(86.53)^2}{(8 \cos 30°)} = 1080.8 \text{ in./s}^2$$

$$a_r = \alpha r^2 = 25(8 \cos 30°) = 173.21 \text{ in./s}^2$$

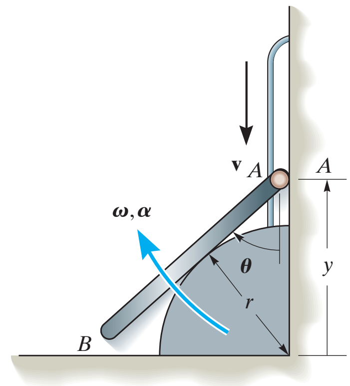
$$a = \sqrt{(1080.8)^2 + (173.21)^2} = 1094.59 \text{ in./s}^2$$

$a = 91.2$ ft/s^2 **Ans.**

Ans:
$v = 7.21$ ft/s
$a = 91.2$ ft/s^2

16–39.

The end A of the bar is moving downward along the slotted guide with a constant velocity $\mathbf{v}_A$. Determine the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the bar as a function of its position y.



SOLUTION

Position coordinate equation:

$$\sin\theta = \frac{r}{y}$$

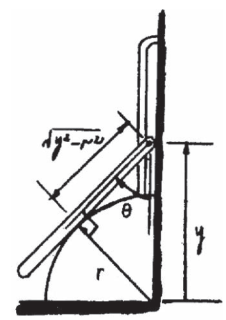
Time derivatives:

$$\cos\theta\dot{\theta} = -\frac{r}{y^2}\dot{y} \text{ however, } \cos\theta = \frac{\sqrt{y^2 - r^2}}{y} \text{ and } \dot{y} = -v_A, \dot{\theta} = \omega$$

$$\left(\frac{\sqrt{y^2 - r^2}}{y}\right)\omega = \frac{r}{y^2}v_A \qquad \omega = \frac{rv_A}{y\sqrt{y^2 - r^2}} \qquad \textbf{Ans.}$$

$$\alpha = \dot{\omega} = rv_A\left[-y^{-2}\dot{y}\left(y^2 - r^2\right)^{-\frac{1}{2}} + \left(y^{-1}\right)\left(-\tfrac{1}{2}\right)\left(y^2 - r^2\right)^{-\frac{3}{2}}(2y\dot{y})\right]$$

$$\alpha = \frac{rv_A^2\,(2y^2 - r^2)}{y^2(y^2 - r^2)^{\frac{3}{2}}} \qquad \textbf{Ans.}$$



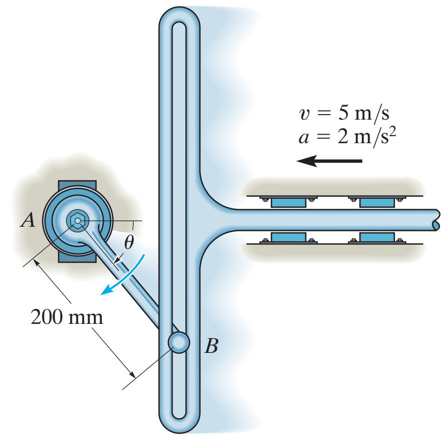
Ans:

$$\omega = \frac{rv_A}{y\sqrt{y^2 - r^2}}$$

$$\alpha = \frac{rv_A^2\left(2y^2 - r^2\right)}{y^2\left(y^2 - r^2\right)^{3/2}}$$

***16–40.**

At the instant $\theta = 60°$, the slotted guide rod is moving to the left with an acceleration of $2\ \text{m/s}^2$ and a velocity of $5\ \text{m/s}$. Determine the angular acceleration and angular velocity of link AB at this instant.



SOLUTION

Position Coordinate Equation. The rectilinear motion of the guide rod can be related to the angular motion of the crank by relating x and θ using the geometry shown in Fig. a, which is

$$x = 0.2 \cos \theta \text{ m}$$

Time Derivatives. Using the chain rule,

$$\dot{x} = -0.2(\sin \theta)\dot{\theta} \qquad \textbf{(1)}$$

$$\ddot{x} = -0.2[(\cos \theta)\dot{\theta}^2 + (\sin \theta)\ddot{\theta}] \qquad \textbf{(2)}$$

Here $\dot{x} = v$, $\ddot{x} = a$, $\dot{\theta} = \omega$ and $\ddot{\theta} = \alpha$ when $\theta = 60°$. Realizing that the velocity and acceleration of the guide rod are directed toward the negative sense of x, $v = -5\ \text{m/s}$ and $a = -2\ \text{m/s}^2$. Then Eq (1) gives

$$-s = (-0.2(\sin 60°)\omega$$

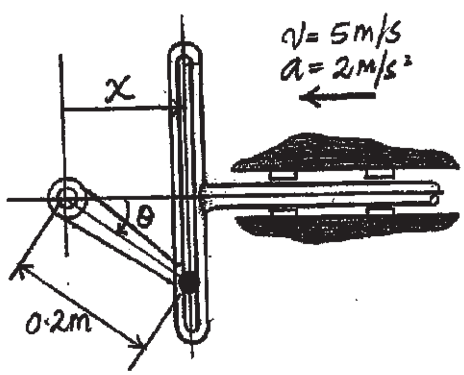
$$\omega = 28.87 \text{ rad/s} = 28.9 \text{ rad/s} \curvearrowright \qquad \textbf{Ans.}$$

Subsequently, Eq. (2) gives

$$-2 = -0.2[\cos 60°(28.87^2) + (\sin 60°)\alpha]$$

$$\alpha = -469.57 \text{ rad/s}^2 = 470 \text{ rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

The negative sign indicates that $\boldsymbol{\alpha}$ is directed in the negative sense of θ.

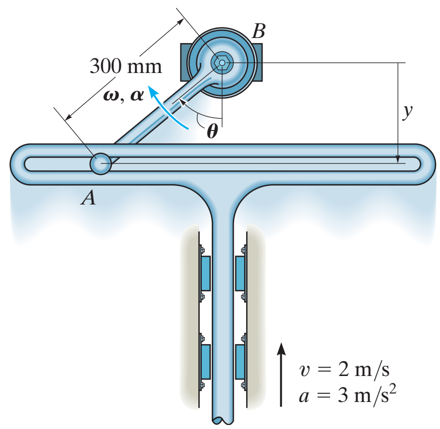


(a)

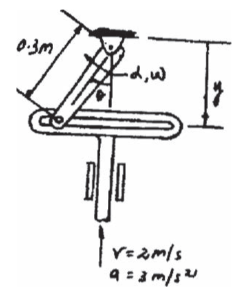
Ans:
$\omega = 28.9$ rad/s $\curvearrowright$
$\alpha = 470$ rad/s^2 $\circlearrowleft$

16–41.

At the instant $\theta = 50°$, the slotted guide is moving upward with an acceleration of $3\ \mathrm{m/s^2}$ and a velocity of $2\ \mathrm{m/s}$. Determine the angular acceleration and angular velocity of link AB at this instant. *Note:* The upward motion of the guide is in the negative y direction.



SOLUTION



$$y = 0.3\cos\theta$$

$$\dot{y} = v_y = -0.3\sin\theta\dot{\theta}$$

$$\ddot{y} = a_y = -0.3\left(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right)$$

Here $v_y = -2\ \mathrm{m/s}$, $a_y = -3\ \mathrm{m/s^2}$, and $\dot{\theta} = \omega, \dot{\theta} = \omega, \ddot{\theta} = \alpha, \theta = 50°$.

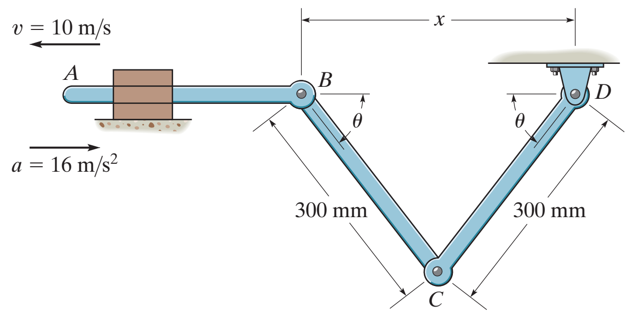
$-2 = -0.3\sin 50°(\omega)$ $\qquad \omega = 8.70\ \mathrm{rad/s}$ **Ans.**

$-3 = -0.3[\sin 50°(\alpha) + \cos 50°(8.70)^2]$ $\qquad \alpha = -50.5\ \mathrm{rad/s^2}$ **Ans.**

Ans:
$\omega = 8.70\ \mathrm{rad/s}$
$\alpha = -50.5\ \mathrm{rad/s^2}$

16–42.

At the instant shown, $\theta = 60°$, and rod AB is subjected to a deceleration of $16\ \text{m/s}^2$ when the velocity is $10\ \text{m/s}$. Determine the angular velocity and angular acceleration of link CD at this instant.

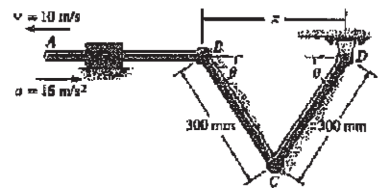


SOLUTION

$x = 2(0.3)\cos\theta$

$x = -0.6\sin\theta(\dot{\theta})$ **(1)**

$\ddot{x} = -0.6\cos\theta(\dot{\theta})^2 - 0.6\sin\theta(\ddot{\theta})$ **(2)**



Using Eqs. (1) and (2) at $\theta = 60°$, $\dot{x} = 10\ \text{m/s}$, $\ddot{x} = -16\ \text{m/s}^2$.

$10 = -0.6\sin 60°(\omega)$

$\omega = -19.245 = -19.2\ \text{rad/s}$ **Ans.**

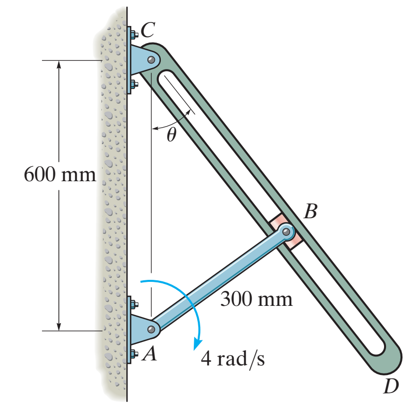
$-16 = -0.6\cos 60°(-19.245)^2 - 0.6\sin 60°(\alpha)$

$\alpha = -183\ \text{rad/s}^2$ **Ans.**

Ans:
$\omega = -19.2\ \text{rad/s}$
$\alpha = -183\ \text{rad/s}^2$

16–43.

The crank AB is rotating with a constant angular velocity of 4 rad/s. Determine the angular velocity of the connecting rod CD at the instant $\theta = 30°$.

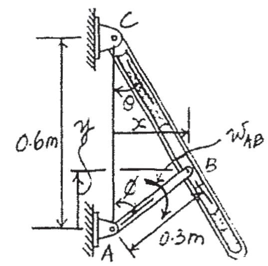


SOLUTION

Position Coordinate Equation: From the geometry,

$$0.3 \sin \phi = (0.6 - 0.3 \cos \phi) \tan \theta \qquad \textbf{[1]}$$

Time Derivatives: Taking the time derivative of Eq. [1], we have



$$0.3 \cos \phi \frac{d\phi}{dt} = 0.6\sec^2\theta\frac{d\theta}{dt} - 0.3\left(\cos \theta \sec^2 \theta\frac{d\theta}{dt} - \tan \theta \sin \theta\frac{d\phi}{dt}\right)$$

$$\frac{d\theta}{dt} = \left[\frac{0.3(\cos \phi - \tan \theta \sin \phi)}{0.3\sec^2\theta(2 - \cos \phi)}\right]\frac{d\phi}{dt} \qquad \textbf{[2]}$$

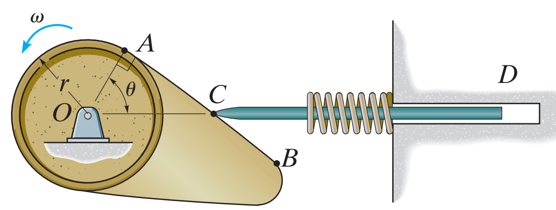
However, $\dfrac{d\theta}{dt} = \omega_{BC}$, $\dfrac{d\phi}{dt} = \omega_{AB} = 4$ rad/s. At the instant $\theta = 30°$, from Eq. [3], $\phi = 60.0°$. Substitute these values into Eq. [2] yields

$$\omega_{BC} = \left[\frac{0.3(\cos 60.0° - \tan 30° \sin 60.0°)}{0.3 \sec^2 30° (2 - \cos 60.0°)}\right](4) = 0 \qquad \textbf{Ans.}$$

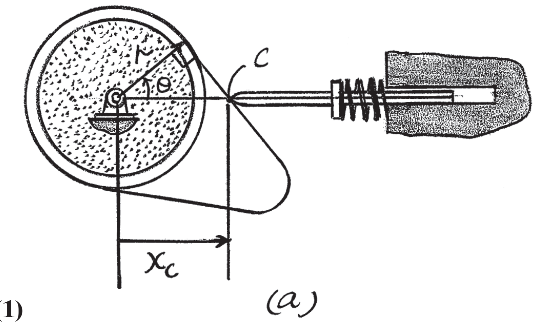
Ans:
$\omega_{AB} = 0$

***16–44.**

Determine the velocity and acceleration of the follower rod CD as a function of θ when the contact between the cam and follower is along the straight region AB on the face of the cam. The cam rotates with a constant counterclockwise angular velocity $\boldsymbol{\omega}$.



SOLUTION



Position Coordinate: From the geometry shown in Fig. a,

$$x_C = \frac{r}{\cos\theta} = r\sec\theta$$

Time Derivative: Taking the time derivative,

$$v_{CD} = \dot{x}_C = r\sec\theta\tan\theta\dot{\theta} \qquad \textbf{(1)}$$

Here, $\dot{\theta} = +\omega$ since ω acts in the positive rotational sense of θ. Thus, Eq. (1) gives

$$v_{CD} = r\omega\sec\theta\tan\theta \rightarrow \qquad \textbf{Ans.}$$

The time derivative of Eq. (1) gives

$$a_{CD} = \ddot{x}_C = r\{\sec\theta\tan\theta\ddot{\theta} + \dot{\theta}[\sec\theta(\sec^2\theta\dot{\theta}) + \tan\theta(\sec\theta\tan\theta\dot{\theta})]\}$$

$$a_{CD} = r[\sec\theta\tan\theta\ddot{\theta} + (\sec^3\theta + \sec\theta\tan^2\theta)\dot{\theta}^2]$$

Since $\dot{\theta} = \omega$ is constant, $\ddot{\theta} = \alpha = 0$. Then,

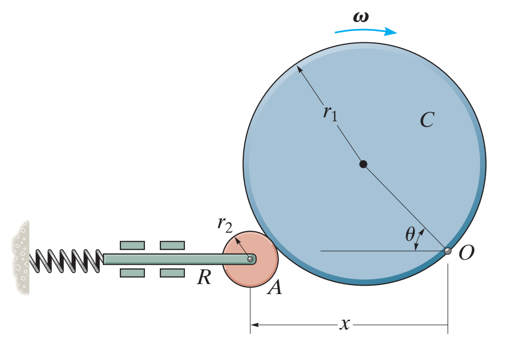
$$a_{CD} = r[\sec\theta\tan\theta(0) + (\sec^3\theta + \sec\theta\tan^2\theta)\omega^2]$$

$$= r\omega^2\left(\sec^3\theta + \sec\theta\tan^2\theta\right) \rightarrow \qquad \textbf{Ans.}$$

Ans:
$v_{CD} = r\omega\sec\theta\tan\theta \rightarrow$
$a_{CD} = r\omega^2\left(\sec^3\theta + \sec\theta\tan^2\theta\right) \rightarrow$

16–45.

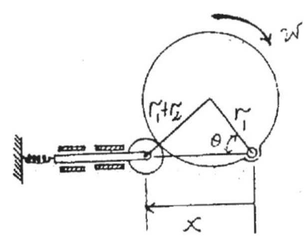
Determine the velocity of rod R for any angle θ of the cam C if the cam rotates with a constant angular velocity $\boldsymbol{\omega}$. The pin connection at O does not cause an interference with the motion of A on C.



SOLUTION

Position Coordinate Equation: Using law of cosine.

$$(r_1 + r_2)^2 = x^2 + r_1^2 - 2r_1 x\cos\theta \qquad \textbf{(1)}$$



Time Derivatives: Taking the time derivative of Eq. (1). we have

$$0 = 2x\frac{dx}{dt} - 2r_1\left(-x\sin\theta\frac{d\theta}{dt} + \cos\theta\frac{dx}{dt}\right) \qquad \textbf{(2)}$$

However $v = \dfrac{dx}{dt}$ and $\omega = \dfrac{d\theta}{dt}$. From Eq.(2),

$$0 = xv - r_1(v\cos\theta - x\omega\sin\theta)$$

$$v = \frac{r_1 x\omega\sin\theta}{r_1\cos\theta - x} \qquad \textbf{(3)}$$

However, the positive root of Eq.(1) is

$$x = r_1\cos\theta + \sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1r_2}$$

Substitute into Eq.(3), we have

$$v = -\left(\frac{r_1^2\omega\sin 2\theta}{2\sqrt{r_1^2\cos^2\theta + r_2^2 + 2\dot{r}_1 r_2}} + r_1\omega\sin\theta\right) \qquad \textbf{Ans.}$$

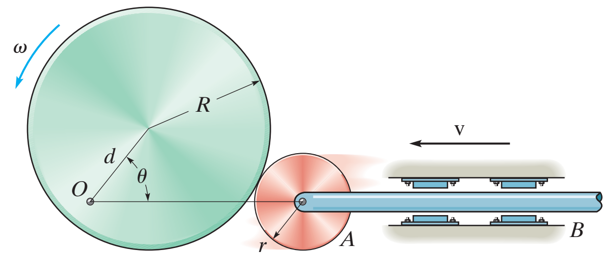
Note: Negative sign indicates that v is directed in the opposite direction to that of positive x.

Ans:

$$v = -\left(\frac{r_1^2\omega\sin 2\theta}{2\sqrt{r_1^2\cos^2\theta + r_2^2 + 2r_1r_2}} + r_1\omega\sin\theta\right)$$

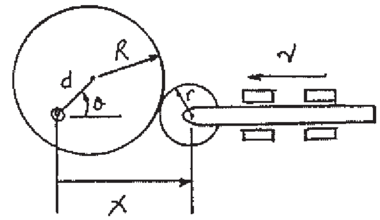
16–46.

The circular cam rotates about the fixed point O with a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity v of the follower rod AB as a function of θ.



SOLUTION

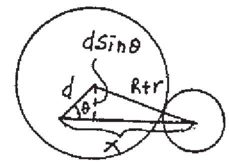
$$x = d\cos\theta + \sqrt{(R+r)^2 - (d\sin\theta)^2}$$



$$\dot{x} = v_{AB} = -d\sin\theta\dot{\theta} - \frac{d^2\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\dot{\theta}$$ Where $\dot{\theta} = \omega$ and $v_{AB} = -v$

$$-v = -d\sin\theta(\omega) - \frac{d^2\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\omega$$

$$v = \omega d\left(\sin\theta + \frac{d\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\right)$$ **Ans.**

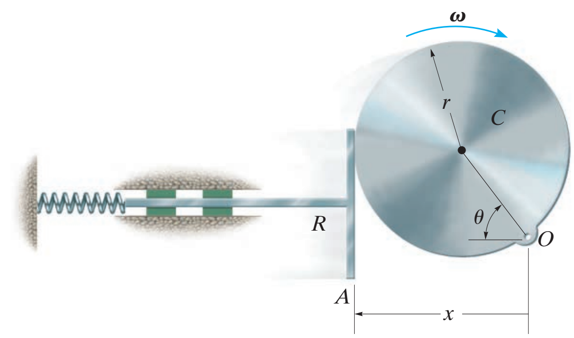


Ans:

$$v = \omega d\left(\sin\theta + \frac{d\sin 2\theta}{2\sqrt{(R+r)^2 - d^2\sin^2\theta}}\right)$$

16–47.

Determine the velocity of the rod R for any angle θ of cam C as the cam rotates with a constant angular velocity $\boldsymbol{\omega}$. The pin connection at O does not cause an interference with the motion of plate A on C.



SOLUTION

$$x = r + r\cos\theta$$

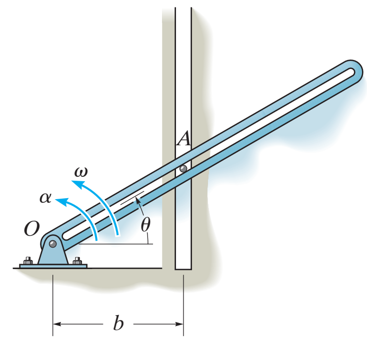
$$x = -r\sin\theta\theta$$

$$v = -r\omega\sin\theta \qquad \textbf{Ans.}$$

Ans:
$v = -r\omega \sin\theta$

***16–48.**

Determine the velocity and acceleration of the peg A which is confined between the vertical guide and the rotating slotted rod.



SOLUTION

Position Coordinate Equation. The rectilinear motion of peg A can be related to the angular motion of the slotted rod by relating y and θ using the geometry shown in Fig. a, which is

$$y = b \tan \theta$$

Time Derivatives. Using the chain rule,

$$\dot{y} = b(\sec^2 \theta)\dot{\theta} \qquad \textbf{(1)}$$

$$\ddot{y} = b[2 \sec \theta(\sec \theta \tan \theta\dot{\theta})\dot{\theta} + \sec^2 \theta\ddot{\theta}]$$

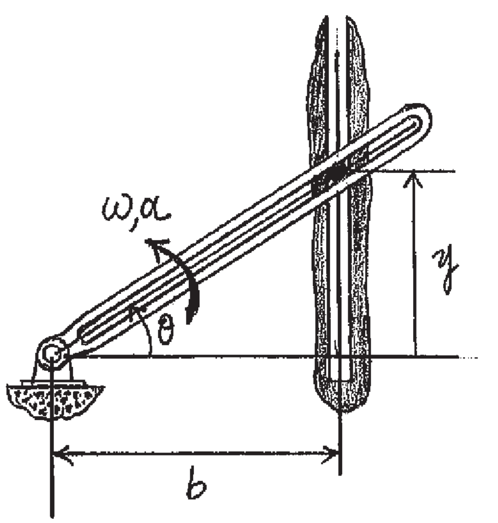
$$\ddot{y} = b(2 \sec^2 \theta \tan \theta\dot{\theta}^2 + \sec^2 \theta\ddot{\theta})$$

$$\ddot{y} = b \sec^2 \theta(2 \tan \theta\dot{\theta}^2 + \ddot{\theta}) \qquad \textbf{(2)}$$

Here, $\dot{y} = v, \ddot{y} = a, \dot{\theta} = \omega$ and $\ddot{\theta} = \alpha$. Then Eqs. (1) and (2) become

$$v = \omega b \sec^2 \theta \qquad \textbf{Ans.}$$

$$a = b \sec^2 \theta(2\omega^2 \tan \theta + \alpha) \qquad \textbf{Ans.}$$

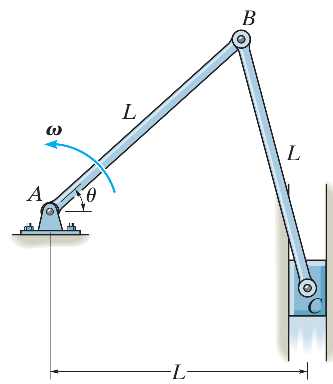


(a)

Ans:
$v = \omega b \sec^2 \theta$
$a = b \sec^2 \theta(2\omega^2 \tan \theta + \alpha)$

16–49.

Bar AB rotates uniformly about the fixed pin A with a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity and acceleration of block C, at the instant $\theta = 60°$.



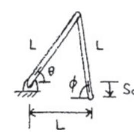
SOLUTION

$$L\cos\theta + L\cos\phi = L$$

$$\cos\theta + \cos\phi = 1$$

$$\sin\theta\,\dot{\theta}] + \sin\phi\,\dot{\phi} = 0 \qquad \textbf{(1)}$$

$$\cos\theta(\dot{\theta})^2 + \sin\theta\ddot{\theta} + \sin\phi\ddot{\phi} + \cos\phi\ (\dot{\phi})^2 = 0 \qquad \textbf{(2)}$$



When $\theta = 60°, \phi = 60°$,

thus, $\dot{\theta} = -\dot{\phi} = \omega$ (from Eq. (1))

$\ddot{\theta} = 0$

$\ddot{\phi} = -1.155\omega^2$ (from Eq.(2))

Also, $s_C = L\sin\phi - L\sin\theta$

$$v_C = L\cos\phi\,\dot{\phi} - L\cos\theta\,\dot{\theta}$$

$$a_C = -L\sin\phi\,(\dot{\phi})^2 + L\cos\phi\,(\ddot{\phi}) - L\cos\theta(\ddot{\theta}) + L\sin\theta(\dot{\theta})^2$$

At $\theta = 60°, \phi = 60°$

$$s_C = 0$$

$$v_C = L(\cos 60°)(-\omega) - L\cos 60°(\omega) = -L\omega = L\omega\uparrow \qquad \textbf{Ans.}$$

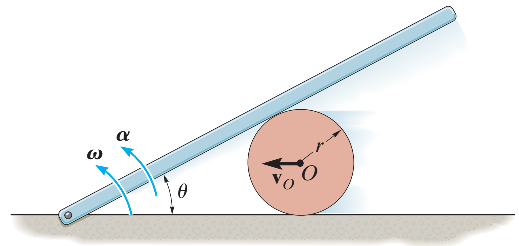
$$a_C = -L\sin 60°(-\omega)^2 + L\cos 60°(-1.155\omega^2) + 0 + L\sin 60°(\omega)^2$$

$$a_C = -0.577\,L\omega^2 = 0.577\,L\omega^2\uparrow \qquad \textbf{Ans.}$$

Ans:
$v_C = L\omega\uparrow$
$a_C = 0.577\,L\omega^2\uparrow$

16–50.

The center of the cylinder is moving to the left with a constant velocity $\mathbf{v}_0$. Determine the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the bar. Neglect the thickness of the bar.



SOLUTION

Position Coordinate Equation. The rectilinear motion of the cylinder can be related to the angular motion of the rod by relating x and θ using the geometry shown in Fig. a, which is

$$x = \frac{r}{\tan\theta/2} = r\cot\theta/2$$

Time Derivatives. Using the chain rule,

$$\dot{x} = r\left[\left(-\csc^2\theta/2\right)\left(\frac{1}{2}\dot{\theta}\right)\right]$$

$$\dot{x} = -\frac{r}{2}\left(\csc^2\theta/2\right)\dot{\theta} \qquad \textbf{(1)}$$

$$\ddot{x} = -\frac{r}{2}\left[2\csc\theta/2\left(-\csc\theta/2\cot\theta/2\right)\left(\frac{1}{2}\dot{\theta}\right)\dot{\theta} + \left(\csc^2\theta/2\right)\ddot{\theta}\right]$$

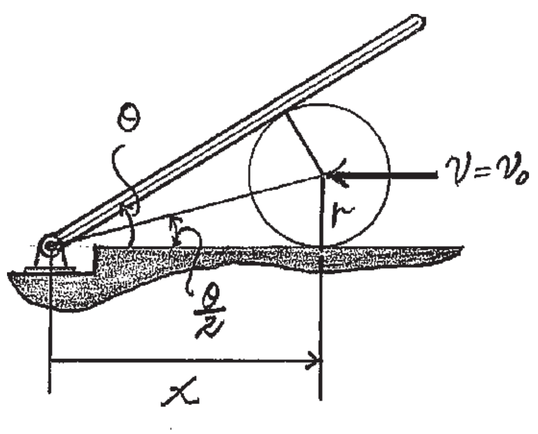
$$\ddot{x} = \frac{r}{2}\left[\left(\csc^2\theta/2\cot\theta/2\right)\dot{\theta}^2 - \left(\csc^2\theta/2\right)\ddot{\theta}\right]$$

$$\ddot{x} = \frac{r\csc^2\theta/2}{2}\left[\left(\cot\theta/2\right)\dot{\theta}^2 - \ddot{\theta}\right] \qquad \textbf{(2)}$$

Here $\dot{x} = -v_0$ since $\mathbf{v}_0$ is directed toward the negative sense of x and $\dot{\theta} = \omega$. Then Eq. (1) gives,

$$-v_0 = -\frac{r}{2}\left(\csc^2\theta/2\right)\omega$$

$$\omega = \frac{2v_0}{r}\sin^2\theta/2 \qquad \textbf{Ans.}$$



(a)

15–50. Continued

Also, $\ddot{x} = 0$ since v is constant and $\ddot{\theta} = \alpha$. Substitute the results of ω into Eq. (2):

$$0 = \frac{r \csc^2 \theta/2}{2}\left[(\cot \theta/2)\left(\frac{2v_0}{r} \sin^2 \theta/2\right)^2 - \alpha\right]$$

$$\alpha = (\cot \theta/2)\left(\frac{2r_0}{r} \sin^2 \theta/2\right)^2$$

$$\alpha = \left(\frac{\cos \theta/2}{\sin \theta/2}\right)\left(\frac{4v_0^2}{r^2} \sin^4 \theta/2\right)$$

$$\alpha = \frac{4v_0^2}{r^2}(\sin^3 \theta/2)(\cos \theta/2)$$

$$\alpha = \frac{2v_0^2}{r^2}(2 \sin \theta/2 \cos \theta/2)(\sin^2 \theta/2)$$

Since $\sin \theta = 2 \sin \theta/2 \cos \theta/2$, then

$$\alpha = \frac{2v_0^2}{r^2}(\sin \theta)(\sin^2 \theta/2) \qquad \textbf{Ans.}$$

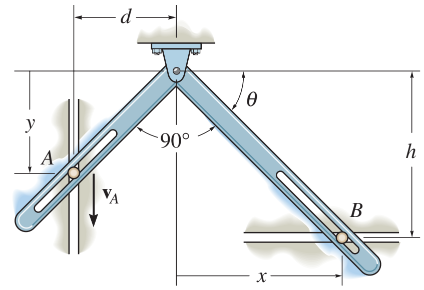
Ans:

$$\omega = \frac{2v_0}{r} \sin^2 \theta/2$$

$$\alpha = \frac{2v_0^2}{r^2}(\sin \theta)(\sin^2 \theta/2)$$

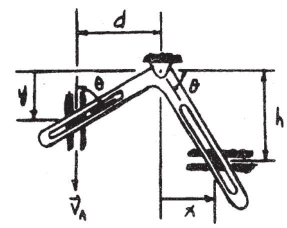
16–51.

The pins at A and B are confined to move in the vertical and horizontal tracks. If the slotted arm is causing A to move downward at $\mathbf{v}_A$, determine the velocity of B at the instant shown.



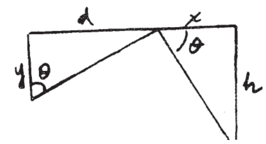
SOLUTION

Position coordinate equation:



$$\tan\theta = \frac{h}{x} = \frac{d}{y}$$

$$x = \left(\frac{h}{d}\right)y$$



Time derivatives:

$$\dot{x} = \left(\frac{h}{d}\right)\dot{y}$$

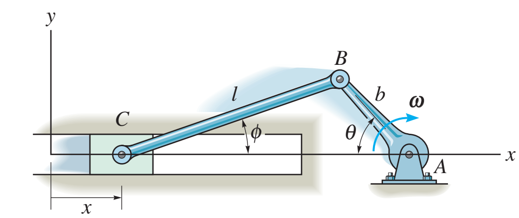
$$v_B = \left(\frac{h}{d}\right)v_A \qquad \textbf{Ans.}$$

Ans:

$v_B = \left(\frac{h}{d}\right)v_A$

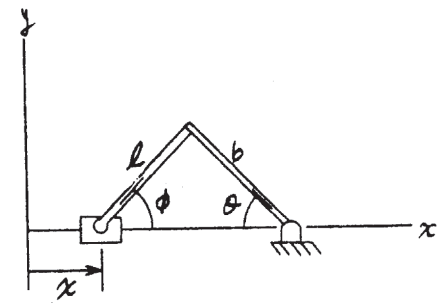
***16–52.**

The crank AB has a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity and acceleration of the slider at C as a function of θ. *Suggestion:* Use the x coordinate to express the motion of C and the ϕ coordinate for CB. $x = 0$ when $\phi = 0°$.



SOLUTION

$$x = l + b - (L\cos\phi + b\cos\theta)$$



$$l\sin\phi = b\sin\theta \text{ or } \sin\phi = \frac{b}{l}\sin\theta$$

$$v_C = \dot{x} = l\sin\phi\dot{\phi} + b\sin\theta\dot{\theta} \quad (1)$$

$$\cos\phi\dot{\phi} = \frac{b}{l}\cos\theta\dot{\theta} \quad (2)$$

Since $\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\left(\frac{b}{l}\right)^2\sin^2\theta}$

then,

$$\dot{\phi} = \frac{\left(\frac{b}{l}\right)\cos\theta\omega}{\sqrt{1-\left(\frac{b}{l}\right)^2\sin^2\theta}} \quad (3)$$

$$v_C = b\omega\left[\frac{\left(\frac{b}{l}\right)\sin\theta\cos\theta}{\sqrt{1-\left(\frac{b}{l}\right)^2\sin^2\theta}}\right] + b\omega\sin\theta \qquad \textbf{Ans.}$$

From Eq. (1) and (2):

$$a_C = \dot{v}_C = l\ddot{\phi}\sin\phi + l\dot{\phi}\cos\phi\dot{\phi} + b\cos\theta\left(\dot{\theta}\right)^2 \quad (4)$$

$$-\sin\phi\dot{\phi}^2 + \cos\phi\ddot{\phi} = -\left(\frac{b}{l}\right)\sin\theta\dot{\theta}^2$$

$$\ddot{\phi} = \frac{\dot{\phi}^2\sin\phi - \frac{b}{l}\omega^2\sin\theta}{\cos\phi} \quad (5)$$

Substituting Eqs. (1), (2), (3) and (5) into Eq. (4) and simplifying yields

$$a_C = b\omega^2\left[\frac{\left(\frac{b}{l}\right)\left(\cos 2\theta + \left(\frac{b}{l}\right)^2\sin^4\theta\right)}{\left(1-\left(\frac{b}{l}\right)^2\sin^2\theta\right)^{\frac{3}{2}}} + \cos\theta\right] \qquad \textbf{Ans.}$$

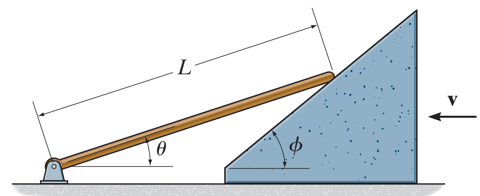
Ans:

$$v_C = b\omega\left[\frac{\left(\frac{b}{l}\right)\sin\theta\cos\theta}{\sqrt{1-\left(\frac{b}{l}\right)^2\sin^2\theta}}\right] + b\omega\sin\theta$$

$$a_C = b\omega^2\left[\frac{\left(\frac{b}{l}\right)\left(\cos 2\theta + \left(\frac{b}{l}\right)^2\sin^4\theta\right)}{\left(1-\left(\frac{b}{l}\right)^2\sin^2\theta\right)^{\frac{3}{2}}} + \cos\theta\right]$$

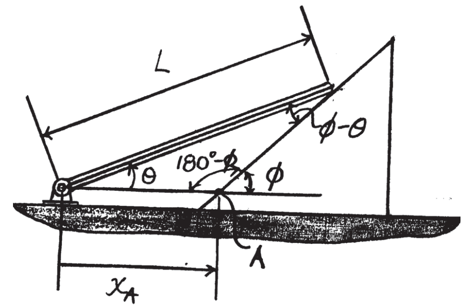
16–53.

If the wedge moves to the left with a constant velocity $\mathbf{v}$, determine the angular velocity of the rod as a function of θ.



SOLUTION

Position Coordinates: Applying the law of sines to the geometry shown in Fig. *a*,



(a)

$$\frac{x_A}{\sin(\phi - \theta)} = \frac{L}{\sin\left(180^\circ - \phi\right)}$$

$$x_A = \frac{L\sin(\phi - \theta)}{\sin\left(180^\circ - \phi\right)}$$

However, $\sin\left(180^\circ - \phi\right) = \sin\phi$. Therefore,

$$x_A = \frac{L\sin(\phi - \theta)}{\sin\phi}$$

Time Derivative: Taking the time derivative,

$$\dot{x}_A = \frac{L\cos(\phi - \theta)(-\dot{\theta})}{\sin\phi}$$

$$v_A = \dot{x}_A = -\frac{L\cos(\phi - \theta)\dot{\theta}}{\sin\phi} \qquad \textbf{(1)}$$

Since point A is on the wedge, its velocity is $v_A = -v$. The negative sign indicates that $\mathbf{v}_A$ is directed towards the negative sense of x_A. Thus, Eq. (1) gives

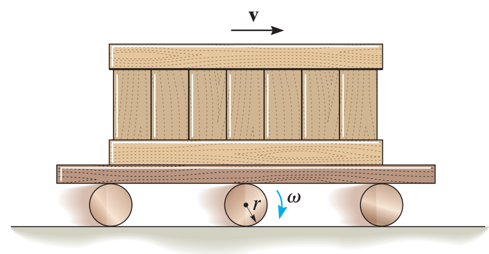
$$\dot{\theta} = \frac{v\sin\phi}{L\cos(\phi - \theta)} \qquad \textbf{Ans.}$$

Ans:

$$\dot{\theta} = \frac{v\sin\phi}{L\cos(\phi - \theta)}$$

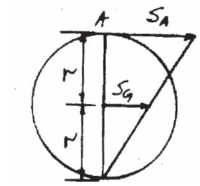
16–54.

The crate is transported on a platform which rests on rollers, each having a radius r. If the rollers do not slip, determine their angular velocity if the platform moves forward with a velocity $\mathbf{v}$.



SOLUTION

Position coordinate equation: From Example 163, $s_G = r\theta$. Using similar triangles



$$s_A = 2s_G = 2r\theta$$

Time derivatives:

$$s_A = v = 2r\,\dot{\theta} \quad \text{Where } \dot{\theta} = \omega$$

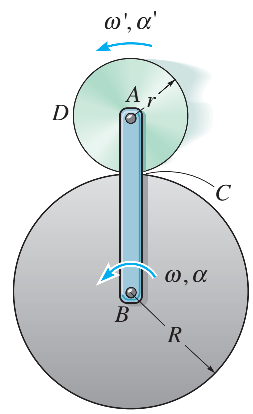
$$\omega = \frac{v}{2r}$$ **Ans.**

Ans:

$\omega = \dfrac{v}{2r}$

16–55.

Arm AB has an angular velocity of $\boldsymbol{\omega}$ and an angular acceleration of $\boldsymbol{\alpha}$. If no slipping occurs between the disk D and the fixed curved surface, determine the angular velocity and angular acceleration of the disk.



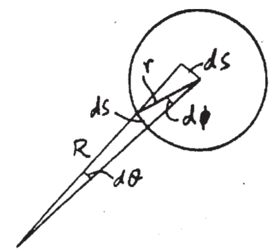
SOLUTION

$$ds = (R + r)\,d\theta = r\,d\phi$$

$$(R + r)\left(\frac{d\theta}{dt}\right) = r\left(\frac{d\phi}{dt}\right)$$

$$\omega' = \frac{(R + r)\omega}{r}$$ **Ans.**

$$\alpha' = \frac{(R + r)\alpha}{r}$$ **Ans.**



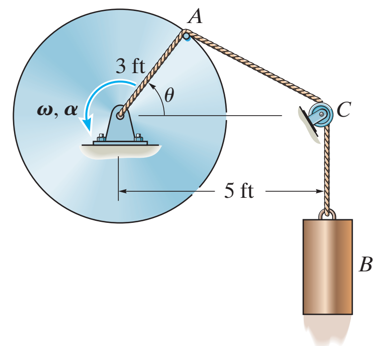
Ans:

$$\omega' = \frac{(R + r)\omega}{r}$$

$$\alpha' = \frac{(R + r)\alpha}{r}$$

***16–56.**

At the instant shown, the disk is rotating with an angular velocity of $\boldsymbol{\omega}$ and has an angular acceleration of $\boldsymbol{\alpha}$. Determine the velocity and acceleration of cylinder B at this instant. Neglect the size of the pulley at C.

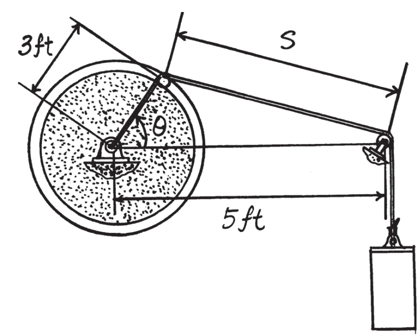


SOLUTION

$$s = \sqrt{3^2 + 5^2 - 2(3)(5)\cos\theta}$$

$$v_B = \dot{s} = \frac{1}{2}(34 - 30\cos\theta)^{-\frac{1}{2}}(30\sin\theta)\dot{\theta}$$

$$v_B = \frac{15\,\omega\sin\theta}{(34 - 30\cos\theta)^{\frac{1}{2}}}$$ **Ans.**



$$a_B = \ddot{s} = \frac{15\,\omega\cos\theta\dot{\theta} + 15\dot{\omega}\sin\theta}{\sqrt{34 - 30\cos\theta}} + \frac{\left(-\frac{1}{2}\right)(15\omega\sin\theta)\left(30\sin\theta\dot{\theta}\right)}{(34 - 30\cos\theta)^{\frac{3}{2}}}$$

$$= \frac{15\,(\omega^2\cos\theta + \alpha\sin\theta)}{(34 - 30\cos\theta)^{\frac{1}{2}}} - \frac{225\,\omega^2\sin^2\theta}{(34 - 30\cos\theta)^{\frac{3}{2}}}$$ **Ans.**

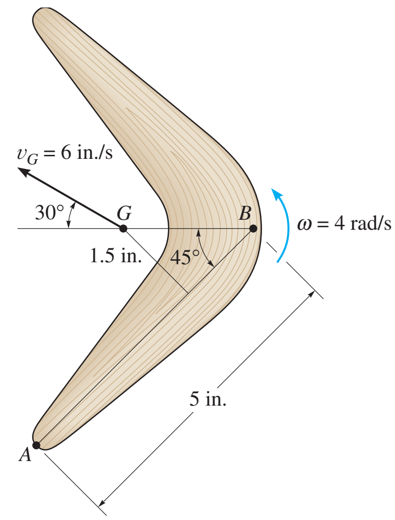
Ans:

$$v_B = \frac{15\,\omega\sin\theta}{(34 - 30\cos\theta)^{\frac{1}{2}}}$$

$$a_B = \frac{15\,(\omega^2\cos\theta + \alpha\sin\theta)}{(34 - 30\cos\theta)^{\frac{1}{2}}} - \frac{225\,\omega^2\sin^2\theta}{(34 - 30\cos\theta)^{\frac{3}{2}}}$$

16–57.

At the instant shown the boomerang has an angular velocity $\omega = 4$ rad/s, and its mass center G has a velocity $v_G = 6$ in./s. Determine the velocity of point B at this instant.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_G + \mathbf{v}_{B/G}$

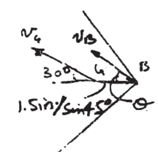
$\mathbf{v}_B = \underset{30°\measuredangle}{6} + \underset{\uparrow}{[4(1.5/\sin 45°) = 8.4852]}$

$(\overset{+}{\leftarrow})\quad (v_B)_x = 6\cos 30° + 0 = 5.196$ in./s

$(+\uparrow)(v_B)_y = 6\sin 30° + 8.4852 = 11.485$ in./s

$v_B = \sqrt{(5.196)^2 + (11.485)^2} = 12.3$ in./s **Ans.**

$\theta = \tan^{-1}\dfrac{11.485}{5.196} = 65.7°$



Also;

$\mathbf{v}_B = \mathbf{v}_G + \omega \times \mathbf{r}_{B/G}$

$(v_B)_x\mathbf{i} + (v_B)_y\mathbf{j} = (-6\cos 30°\mathbf{i} + 6\sin 30°\mathbf{j}) + (4\mathbf{k}) \times (1.5/\sin 45°)\mathbf{i}$

$(v_B)_x = -6\cos 30° = -5.196$ in./s

$(v_B)_y = 6\sin 30° + 8.4853 = 11.485$ in./s

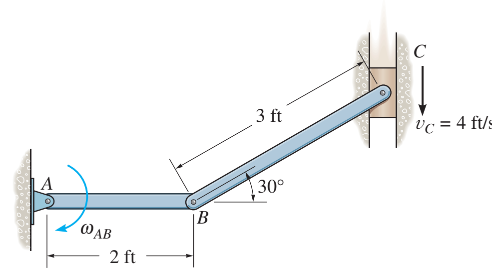
$v_B = \sqrt{(5.196)^2 + (11.485)^2} = 12.6$ in./s **Ans.**

$\theta = \tan^{-1}\dfrac{11.485}{5.196} = 65.7°$

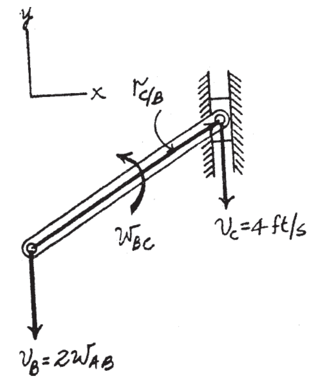
Ans:
$v_B = 12.6$ in./s
$65.7° \measuredangle$

16–58.

If the block at C is moving downward at 4 ft/s, determine the angular velocity of bar AB at the instant shown.



SOLUTION



***Kinematic Diagram*:** Since link AB is rotating about fixed point A, then v_B is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 2\omega_{AB}$. At the instant shown, v_B is directed towards the *negative y* axis. Also, block C is moving downward vertically due to the constraint of the guide. Then v_c is directed toward *negative* y axis.

***Velocity Equation*:** Here, $\mathbf{r}_{C/B} = \{3 \cos 30°\mathbf{i} + 3 \sin 30°\mathbf{j}\}$ ft $= \{2.598\mathbf{i} + 1.50\mathbf{j}\}$ ft. Applying Eq. 16–16, we have

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$-4\mathbf{j} = -2\omega_{AB}\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (2.598\mathbf{i} + 1.50\mathbf{j})$$

$$-4\mathbf{j} = -1.50\omega_{BC}\mathbf{i} + (2.598\omega_{BC} - 2\omega_{AB})\mathbf{j}$$

Equating **i** and **j** components gives

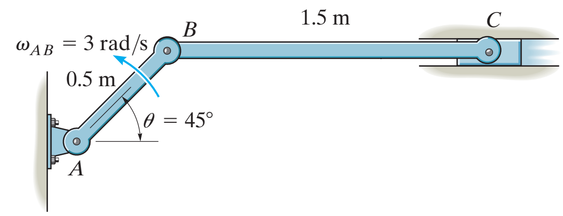
$$0 = -1.50\omega_{BC} \qquad \omega_{BC} = 0$$

$$-4 = 2.598(0) - 2\omega_{AB} \qquad \omega_{AB} = 2.00 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_{AB} = 2.00$ rad/s

16–59.

The link AB has an angular velocity of 3 rad/s. Determine the velocity of block C and the angular velocity of link BC at the instant $\theta = 45°$. Also, sketch the position of link BC when $\theta = 60°$, 45°, and 30° to show its general plane motion.



SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. *a*.

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{AB}$$
$$= (3\mathbf{k}) \times (0.5 \cos 45°\mathbf{i} + 0.5 \sin 45°\mathbf{j})$$
$$= \{-1.0607\mathbf{i} + 1.0607\mathbf{j}\} \text{ m/s}$$

General Plane Motion. For link BC, refer to Fig. *b*. Applying the relative velocity equation,

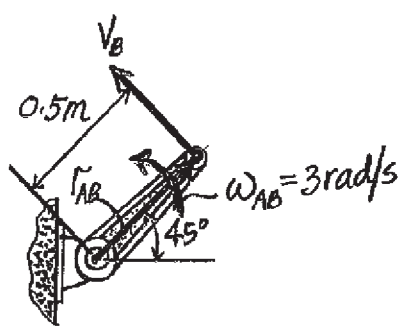
$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$
$$-v_C\mathbf{i} = (-1.0607\mathbf{i} + 1.0607\mathbf{j}) + (-\omega_{BC}\mathbf{k}) \times (1.5\mathbf{i})$$
$$-v_C\mathbf{i} = -1.0607\mathbf{i} + (1.0607 - 1.5\omega_{BC})\mathbf{j}$$

Equating **i** and **j** components;

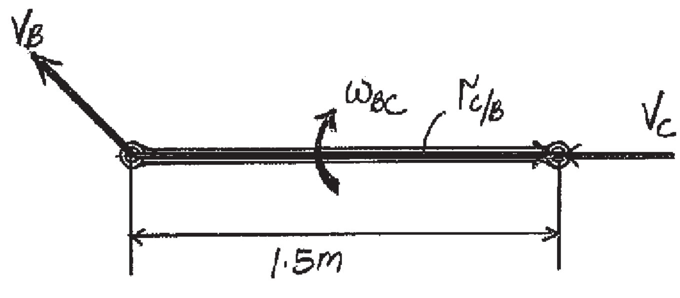
$-v_C = -1.0607 \qquad v_C = 1.0607 \text{ m/s} = 1.06 \text{ m/s}$ **Ans.**

$0 = 1.0607 - 1.5\omega_{BC} \qquad \omega_{BC} = 0.7071 \text{ rad/s} = 0.707 \text{ rad/s}$ **Ans.**

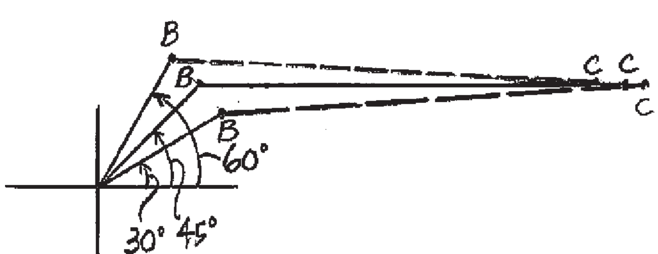
The general plane motion of link BC is described by its orientation when $\theta = 30°$, 45° and 60° shown in Fig. *c*.



(a)



(b)

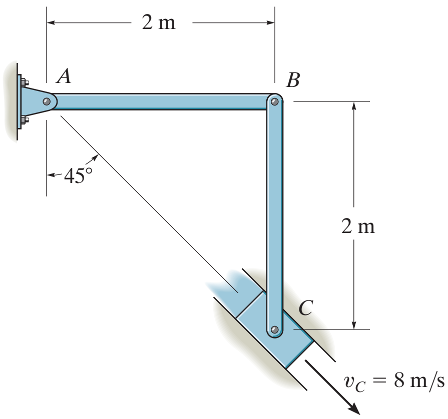


(c)

Ans:
$v_C = 1.06$ m/s ←
$\omega_{BC} = 0.707$ rad/s ↺

***16–60.**

The slider block C moves at 8 m/s down the inclined groove. Determine the angular velocities of links AB and BC, at the instant shown.



SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. a.

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$

$$\mathbf{v}_B = (-\omega_{AB}\mathbf{k}) \times (2\mathbf{i}) = -2\omega_{AB}\mathbf{j}$$

General Plane Motion. For link BC, refer to Fig. b. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

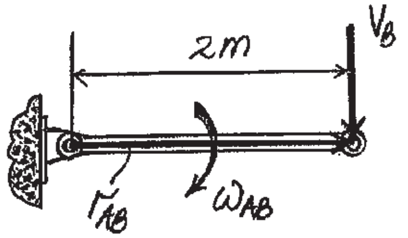
$$-2\omega_{AB}\mathbf{j} = (8 \sin 45°\mathbf{i} - 8 \cos 45°\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (2\mathbf{j})$$

$$-2\omega_{AB}\mathbf{j} = (8 \sin 45° - 2\omega_{BC})\mathbf{i} - 8 \cos 45°\mathbf{j}$$

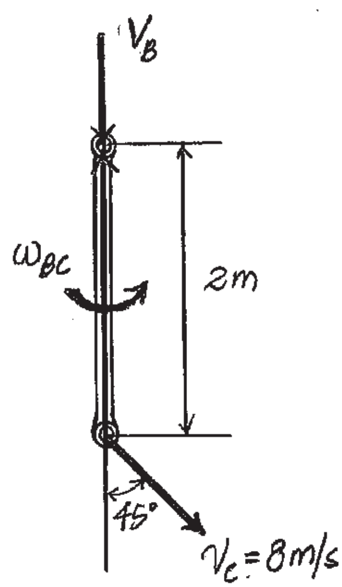
Equating **i** and **j** components,

$0 = 8 \sin 45° - 2\omega_{BC}$ $\quad \omega_{BC} = 2.828 \text{ rad/s} = 2.83 \text{ rad/s}$ ↺ **Ans.**

$-2\omega_{AB} = -8 \cos 45°$ $\quad \omega_{AB} = 2.828 \text{ rad/s} = 2.83 \text{ rad/s}$ ↻ **Ans.**



(a)

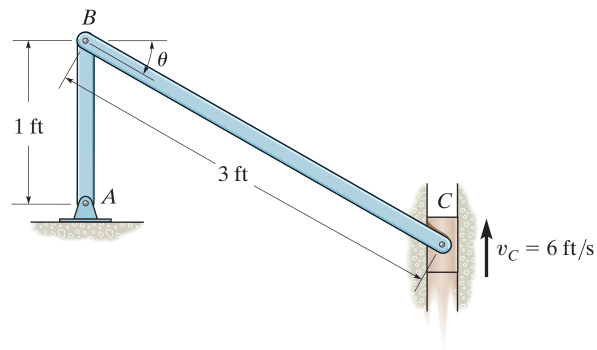


(b)

Ans:
$\omega_{BC} = 2.83 \text{ rad/s}$ ↺
$\omega_{AB} = 2.83 \text{ rad/s}$ ↻

16–61.

Determine the angular velocity of links AB and BC at the instant $\theta = 30°$. Also, sketch the position of link BC when $\theta = 55°, 45°$, and $30°$ to show its general plane motion.



SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. a.

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$

$$\mathbf{v}_B = (\omega_{AB}\mathbf{k}) \times \mathbf{j} = -\omega_{AB}\,\mathbf{i}$$

General Plane Motion. For link BC, refer to Fig. b. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$-\omega_{AB}\mathbf{i} = 6\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (-3\cos 30°\mathbf{i} + 3\sin 30°\mathbf{j})$$

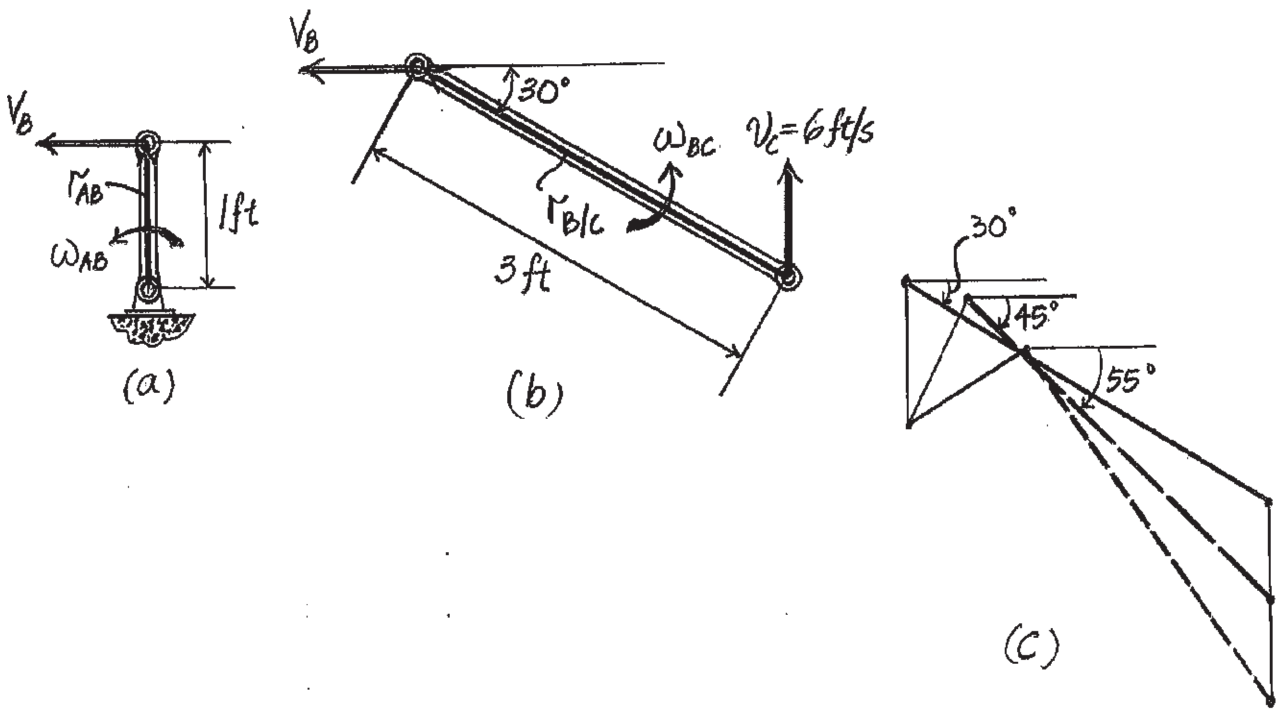
$$-\omega_{AB}\mathbf{i} = -1.5\omega_{BC}\mathbf{i} + (6 - 2.5981\,\omega_{BC})\mathbf{j}$$

Equating **i** and **j** components,

$$0 = 6 - 2.5981\,\omega_{BC}; \qquad \omega_{BC} = 2.3094 \text{ rad/s} = 2.31 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

$$-\omega_{AB} = -1.5(2.3094); \qquad \omega_{AB} = 3.4641 \text{ rad/s} = 3.46 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

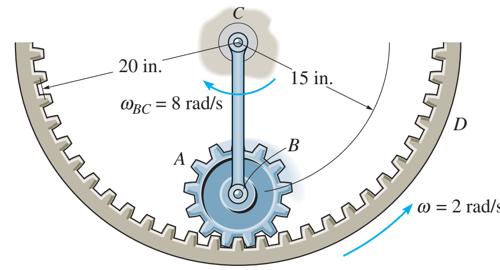
The general plane motion of link BC is described by its orientation when $\theta = 30°$, 45° and 55° shown in Fig. c.



Ans:
$\omega_{BC} = 2.31$ rad/s $\circlearrowleft$
$\omega_{AB} = 3.46$ rad/s $\circlearrowleft$

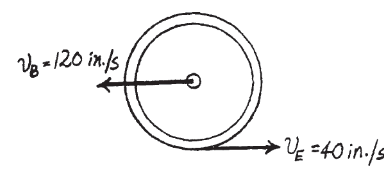
16–62.

The planetary gear A is pinned at B. Link BC rotates clockwise with an angular velocity of 8 rad/s, while the outer gear rack rotates counterclockwise with an angular velocity of 2 rad/s. Determine the angular velocity of gear A.



SOLUTION

***Kinematic Diagram*:** Since link BC is rotating about fixed point C. then v_B is always directed perpendicular to link BC and its magnitude is $v_B = \omega_{BC} r_{BC} = 8(15) = 120$ in./s. At the instant shown. v_B is directed to the left. Also, at the same instant, point E is moving to the right with a speed of $v_E = \omega_B r_{CE} = 2(20) = 40$ in./s.



***Velocity Equation*:** Here, $v_{B/E} = \omega_A r_{B/E} = 5\omega_A$ which is directed to the left. Applying Eq. 16–15, we have

$$\mathbf{v}_B = \mathbf{v}_E + \mathbf{v}_{B/E}$$

$$\begin{bmatrix} \underset{\leftarrow}{120} \end{bmatrix} = \begin{bmatrix} \underset{\rightarrow}{40} \end{bmatrix} + \begin{bmatrix} \underset{\leftarrow}{5\omega_A} \end{bmatrix}$$

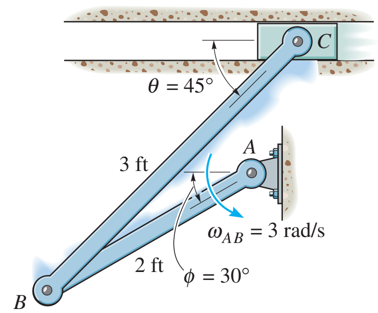
$$(\overset{+}{\rightarrow}) \qquad -120 = 40 - 5\omega_A$$

$$\omega_A = 32.0 \text{ rad/s}$$ **Ans.**

Ans:
$\omega_A = 32.0$ rad/s

16–63.

If the angular velocity of link AB is $\omega_{AB} = 3$ rad/s, determine the velocity of the block at C and the angular velocity of the connecting link CB at the instant $\theta = 45°$ and $\phi = 30°$.



SOLUTION

$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

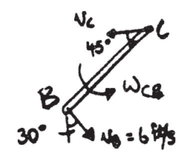
$$\begin{bmatrix} v_C \\ \leftarrow \end{bmatrix} = \begin{bmatrix} 6 \\ 30° \searrow \end{bmatrix} + \begin{bmatrix} \omega_{CB}(3) \\ 45° \nearrow \end{bmatrix}$$

$(\overset{+}{\rightarrow})$ $\quad -v_C = 6 \sin 30° - \omega_{CB}(3)\cos 45°$

$(+\uparrow)$ $\quad 0 = -6\cos 30° + \omega_{CB}(3)\sin 45°$

$\omega_{CB} = 2.45$ rad/s $\circlearrowleft$ **Ans.**

$v_C = 2.20$ ft/s $\leftarrow$ **Ans.**



Also,

$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$-v_C\,\mathbf{i} = (6\sin 30°\mathbf{i} - 6\cos 30°\mathbf{j}) + (\omega_{CB}\mathbf{k}) \times (3\cos 45°\mathbf{i} + 3\sin 45°\mathbf{j})$

$(\overset{+}{\rightarrow})$ $\quad -v_C = 3 - 2.12\omega_{CB}$

$(+\uparrow)$ $\quad 0 = -5.196 + 2.12\omega_{CB}$

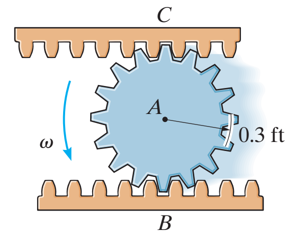
$\omega_{CB} = 2.45$ rad/s $\circlearrowleft$ **Ans.**

$v_C = 2.20$ ft/s $\leftarrow$ **Ans.**

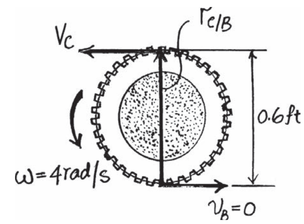
Ans:
$\omega_{CB} = 2.45$ rad/s $\circlearrowleft$
$v_C = 2.20$ ft/s $\leftarrow$

***16–64.**

The pinion gear A rolls on the fixed gear rack B with an angular velocity $\omega = 4$ rad/s. Determine the velocity of the gear rack C.



SOLUTION



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$(\overset{+}{\leftarrow})$ $\quad v_C = 0 + 4(0.6)$

$v_C = 2.40 \text{ ft/s}$ **Ans.**

Also:

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

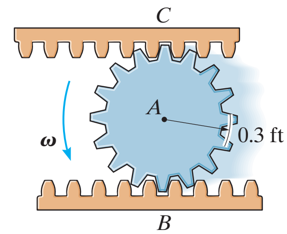
$$-v_C\,\mathbf{i} = 0 + (4\mathbf{k}) \times (0.6\mathbf{j})$$

$v_C = 2.40 \text{ ft/s}$ **Ans.**

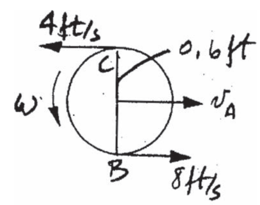
Ans:
$v_C = 2.40 \text{ ft/s}$
$v_C = 2.40 \text{ ft/s}$

16–65.

The pinion gear rolls on the gear racks. If B is moving to the right at 8 ft/s and C is moving to the left at 4 ft/s, determine the angular velocity of the pinion gear and the velocity of its center A.



SOLUTION



$$v_C = v_B + v_{C/B}$$

$(\xrightarrow{+})$ $$-4 = 8 - 0.6(\omega)$$

$$\omega = 20 \text{ rad/s}$$ **Ans.**

$$v_A = v_B + v_{A/B}$$

$(\xrightarrow{+})$ $$v_A = 8 - 20(0.3)$$

$$v_A = 2 \text{ ft/s} \rightarrow$$ **Ans.**

Also:

$$v_C = v_B + \omega \times \mathbf{r}_{C/B}$$

$$-4\mathbf{i} = 8\mathbf{i} + (\omega\mathbf{k}) \times (0.6\mathbf{j})$$

$$-4 = 8 - 0.6\omega$$

$$\omega = 20 \text{ rad/s}$$ **Ans.**

$$v_A = v_B + \omega \times \mathbf{r}_{A/B}$$

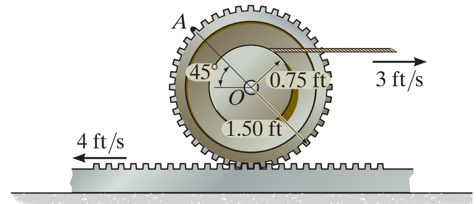
$$v_A\mathbf{i} = 8\mathbf{i} + 20\mathbf{k} \times (0.3\mathbf{j})$$

$$v_A = 2 \text{ ft/s} \rightarrow$$ **Ans.**

Ans:
$\omega = 20$ rad/s
$v_A = 2$ ft/s $\rightarrow$

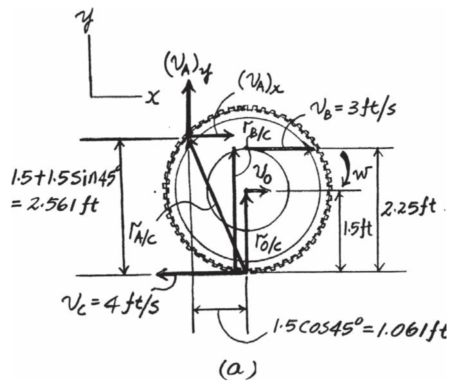
16–66.

Determine the angular velocity of the gear and the velocity of its center O at the instant shown.



SOLUTION

General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of the gear shown in Fig. a,



$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{B/C}$$

$$3\mathbf{i} = -4\mathbf{i} + (-\omega\mathbf{k}) \times (2.25\mathbf{j})$$

$$3\mathbf{i} = (2.25\omega - 4)\mathbf{i}$$

Equating the $\mathbf{i}$ components yields

$$3 = 2.25\omega - 4 \qquad \textbf{(1)}$$

$$\omega = 3.111 \text{ rad/s} \qquad \textbf{Ans. (2)}$$

For points O and C,

$$\mathbf{v}_O = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{O/C}$$

$$= -4\mathbf{i} + (-3.111\mathbf{k}) \times (1.5\mathbf{j})$$

$$= [0.6667\mathbf{i}] \text{ ft/s}$$

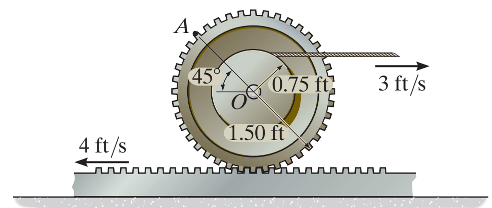
Thus,

$$v_O = 0.667 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

Ans:
$\omega = 3.11$ rad/s
$v_O = 0.667$ ft/s $\rightarrow$

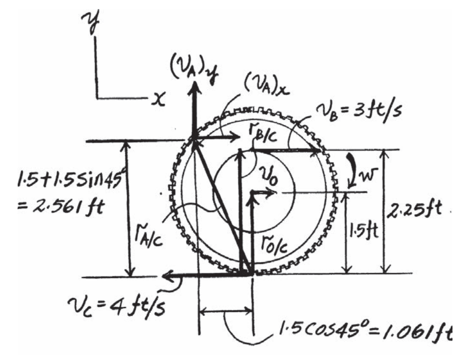
16–67.

Determine the velocity of point A on the rim of the gear at the instant shown.



SOLUTION

General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of the gear shown in Fig. a,



(a)

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{B/C}$$

$$3\mathbf{i} = -4\mathbf{i} + \left(-\omega\mathbf{k}\right) \times \left(2.25\mathbf{j}\right)$$

$$3\mathbf{i} = \left(2.25\omega - 4\right)\mathbf{i}$$

Equating the $\mathbf{i}$ components yields

$$3 = 2.25\omega - 4 \qquad \textbf{(1)}$$

$$\omega = 3.111 \text{ rad/s} \qquad \textbf{(2)}$$

For points A and C,

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{A/C}$$

$$\left(v_A\right)_x \mathbf{i} + \left(v_A\right)_y \mathbf{j} = -4\mathbf{i} + \left(-3.111\mathbf{k}\right) \times \left(-1.061\mathbf{i} + 2.561\mathbf{j}\right)$$

$$\left(v_A\right)_x \mathbf{i} + \left(v_A\right)_y \mathbf{j} = 3.9665\mathbf{i} + 3.2998\mathbf{j}$$

Equating the $\mathbf{i}$ and $\mathbf{j}$ components yields

$$\left(v_A\right)_x = 3.9665 \text{ ft/s} \qquad \left(v_A\right)_y = 3.2998 \text{ ft/s}$$

Thus, the magnitude of v_A is

$$v_A = \sqrt{\left(v_A\right)_x{}^2 + \left(v_A\right)_y{}^2} = \sqrt{3.9665^2 + 3.2998^2} = 5.16 \text{ ft/s} \qquad \textbf{Ans.}$$

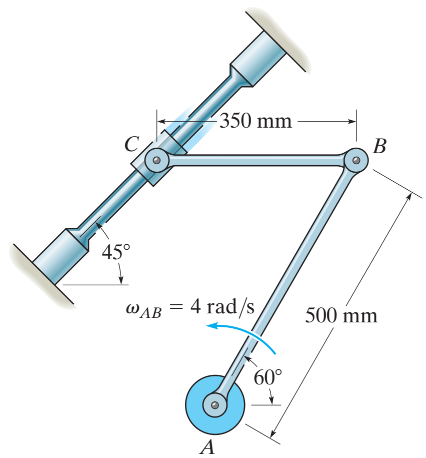
and its direction is

$$\theta = \tan^{-1}\left[\frac{\left(v_A\right)_y}{\left(v_A\right)_x}\right] = \tan^{-1}\left(\frac{3.2998}{3.9665}\right) = 39.8° \qquad \textbf{Ans.}$$

Ans:
$v_A = 5.16$ ft/s
$\theta = 39.8°$ ∠

***16–68.**

Knowing that angular velocity of link AB is $\omega_{AB} = 4$ rad/s, determine the velocity of the collar at C and the angular velocity of link CB at the instant shown. Link CB is horizontal at this instant.



SOLUTION

$$v_B = \omega_{AB} r_{AB}$$
$$= 4(0.5) = 2 \text{ m/s}$$

$\mathbf{v}_B = \{-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}\}$ m/s $\quad \mathbf{v}_C = -v_C \cos 45°\mathbf{i} - v_C \sin 45°\mathbf{j}$

$\omega = \omega_{BC}\mathbf{k} \qquad \mathbf{r}_{C/B} = \{-0.35\mathbf{i}\}$ m

$$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$$

$$-v_C \cos 45°\mathbf{i} - v_C \sin 45°\mathbf{j} = (-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (-0.35\mathbf{i})$$

$$-v_C \cos 45°\mathbf{i} - v_C \sin 45°\mathbf{j} = -2\cos 30°\mathbf{i} + (2\sin 30° - 0.35\omega_{BC})\mathbf{j}$$

Equating the **i** and **j** components yields:

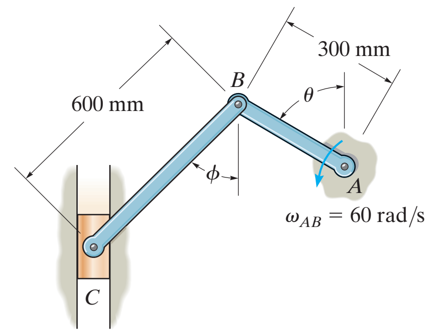
$-v_C \cos 45° = -2\cos 30° \qquad v_C = 2.45$ m/s **Ans.**

$-2.45 \sin 45° = 2\sin 30° - 0.35\omega_{BC} \qquad \omega_{BC} = 7.81$ rad/s **Ans.**

Ans:
$v_C = 2.45$ m/s
$\omega_{BC} = 7.81$ rad/s

16–69.

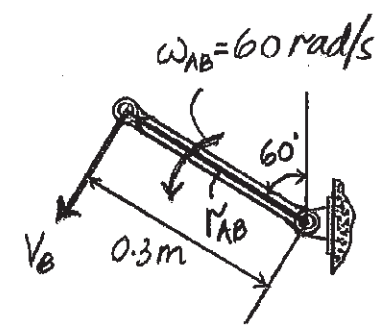
Rod AB is rotating with an angular velocity of $\omega_{AB} = 60$ rad/s. Determine the velocity of the slider C at the instant $\theta = 60^\circ$ and $\phi = 45^\circ$. Also, sketch the position of bar BC when $\theta = 30^\circ$, 60° and 90° to show its general plane motion.



SOLUTION

Rotation About Fixed Axis. For link AB, refer to Fig. a.

$$\mathbf{V}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$
$$= (60\mathbf{k}) \times (-0.3 \sin 60^\circ \mathbf{i} + 0.3 \cos 60^\circ \mathbf{j})$$
$$= \{-9\mathbf{i} - 9\sqrt{3}\mathbf{j}\} \text{ m/s}$$



(a)

General Plane Motion. For link BC, refer to Fig. b. Applying the relative velocity equation,

$$\mathbf{V}_C = \mathbf{V}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$
$$-v_C\mathbf{j} = (-9\mathbf{i} - 9\sqrt{3}\,\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (-0.6 \sin 45^\circ \mathbf{i} - 0.6 \cos 45^\circ \mathbf{j})$$
$$-v_C\mathbf{j} = \left(0.3\sqrt{2}\omega_{BC} - 9\right)\mathbf{i} + \left(-0.3\sqrt{2}\omega_{BC} - 9\sqrt{3}\right)\mathbf{j}$$

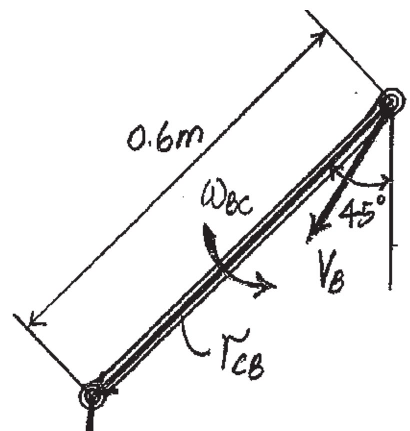
Equating **i** components,

$$0 = 0.3\sqrt{2}\omega_{BC} - 9; \qquad \omega_{BC} = 15\sqrt{2} \text{ rad/s} = 21.2 \text{ rad/s} \circlearrowright$$

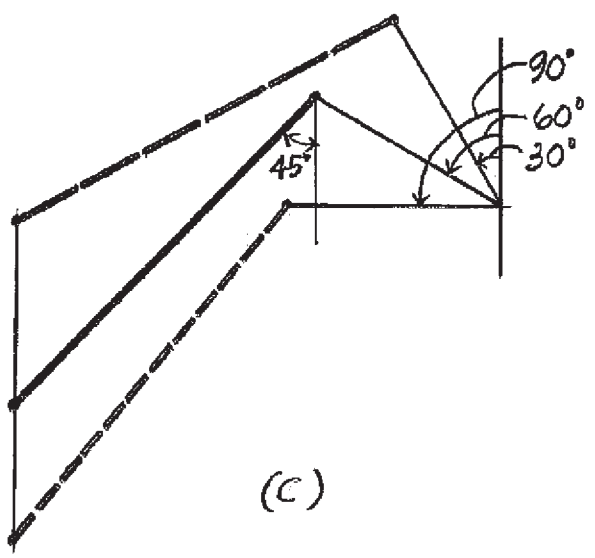
Then, equating **j** components,

$$-v_C = \left(-0.3\sqrt{2}\right)\left(15\sqrt{2}\right) - 9\sqrt{3}; \quad v_C = 24.59 \text{ m/s} = 24.6 \text{ m/s}\downarrow \quad \textbf{Ans.}$$

The general plane motion of link BC is described by its orientation when $\theta = 30^\circ$, 60° and 90° shown in Fig. c.



(b)

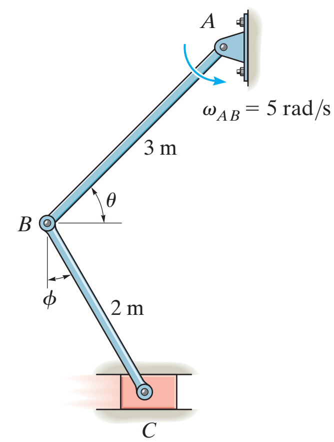


(c)

Ans:
$v_C = 24.6$ m/s$\downarrow$

16–70.

The angular velocity of link AB is $\omega_{AB} = 5$ rad/s. Determine the velocity of block C and the angular velocity of link BC at the instant $\theta = 45°$ and $\phi = 30°$. Also, sketch the position of link CB when $\theta = 45°, 60°$, and $75°$ to show its general plane motion.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB}$$
$$= (5\mathbf{k}) \times (-3\cos 45°\mathbf{i} - 3\sin 45°\mathbf{j})$$
$$= \left\{\frac{15\sqrt{2}}{2}\mathbf{i} - \frac{15\sqrt{2}}{2}\mathbf{j}\right\} \text{ m/s}$$

General Plane Motion. For link BC, refer to Fig. b. Applying the relative velocity equation,

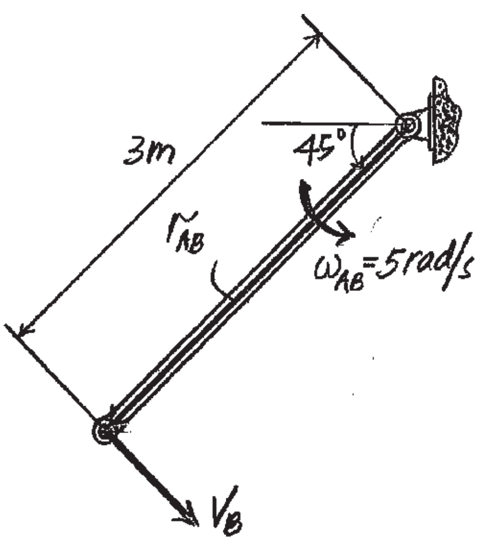
$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$
$$v_C\mathbf{i} = \left(\frac{15\sqrt{2}}{2}\mathbf{i} - \frac{15\sqrt{2}}{2}\mathbf{j}\right) + (\omega_{BC}\mathbf{k}) \times (2\sin 30°\,\mathbf{i} - 2\cos 30°\mathbf{j})$$
$$v_C\mathbf{i} = \left(\frac{15\sqrt{2}}{2} + \sqrt{3}\,\omega_{BC}\right)\mathbf{i} + \left(\omega_{BC} - \frac{15\sqrt{2}}{2}\right)\mathbf{j}$$

Equating $\mathbf{j}$ components,

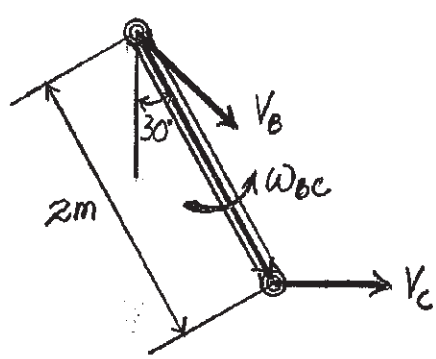
$$O = \omega_{BC} - \frac{15\sqrt{2}}{2}; \omega_{BC} = \frac{15\sqrt{2}}{2}\text{ rad/s} = 10.6 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Then, equating $\mathbf{i}$ components,

$$v_C = \frac{15\sqrt{2}}{2} + \sqrt{3}\left(\frac{15\sqrt{2}}{2}\right) = 28.98 \text{ m/s} = 29.0 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



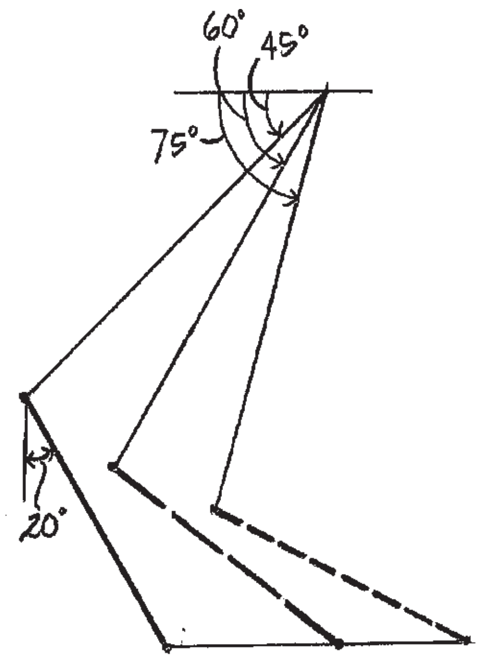
(a)



(b)

16–70. Continued

The general plane motion of link BC is described by its orientation when $\theta = 45°$, 60° and 75° shown in Fig. c

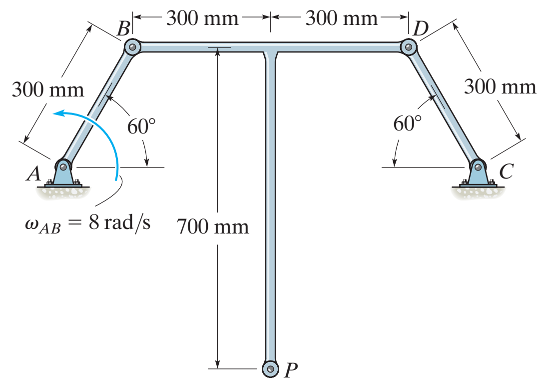


(c)

Ans:
$\omega_{BC} = 10.6$ rad/s ↺
$v_C = 29.0$ m/s →

16–71.

The similar links AB and CD rotate about the fixed pins at A and C. If AB has an angular velocity $\omega_{AB} = 8$ rad/s, determine the angular velocity of BDP and the velocity of point P.



SOLUTION

$\mathbf{v}_D = \mathbf{v}_B + \omega \times \mathbf{r}_{D/B}$

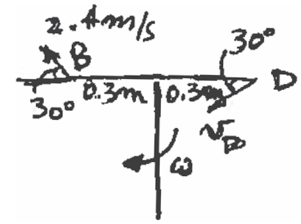
$-v_D \cos 30°\mathbf{i} - v_D \sin 30°\mathbf{j} = -2.4 \cos 30°\mathbf{i} + 2.4 \sin 30°\mathbf{j} + (\omega\mathbf{k}) \times (0.6\mathbf{i})$

$-v_D \cos 30° = -2.4 \cos 30°$

$-v_D \sin 30° = 2.4 \sin 30° + 0.6\omega$

$v_D = 2.4$ m/s

$\omega = -4$ rad/s **Ans.**



$\mathbf{v}_P = \mathbf{v}_B + \omega \times \mathbf{r}_{P/B}$

$\mathbf{v}_P = -2.4 \cos 30°\mathbf{i} + 2.4 \sin 30°\mathbf{j} + (-4\mathbf{k}) \times (0.3\mathbf{i} - 0.7\mathbf{j})$

$(v_P)_x = -4.88$ m/s

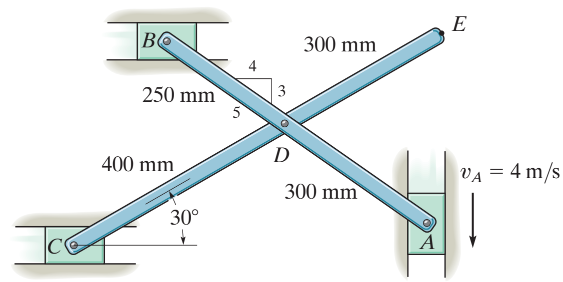
$(v_P)_y = 0$

$v_P = 4.88$ m/s $\leftarrow$ **Ans.**

Ans:
$v_P = 4.88$ m/s $\leftarrow$

***16–72.**

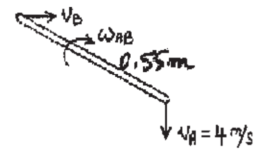
If the slider block A is moving downward at $v_A = 4\text{ m/s}$, determine the velocities of blocks B and C at the instant shown.



SOLUTION

$$v_B = v_A + v_{B/A}$$

$$\underset{\rightarrow}{v_B} = 4\downarrow + \omega_{AB}(0.55)$$

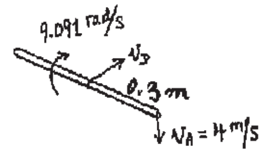


$$(\overset{+}{\rightarrow}) \qquad v_B = 0 + \omega_{AB}(0.55)\left(\frac{3}{5}\right)$$

$$(+\uparrow) \qquad 0 = -4 + \omega_{AB}(0.55)\left(\frac{4}{5}\right)$$

Solving,

$$\omega_{AB} = 9.091\text{ rad/s}$$



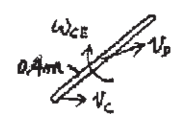
$$v_B = 3.00\text{ m/s} \qquad \textbf{Ans.}$$

$$v_D = v_A + v_{D/A}$$

$$\mathbf{v}_D = \underset{\downarrow}{4} + \underset{\ _4\nearrow^3_5}{[(0.3)(9.091) = 2.727]}$$

$$v_C = v_D + v_{C/D}$$

$$\underset{\rightarrow}{v_C} = \underset{\downarrow}{4} + \underset{\ ^5\nearrow^3_4}{2.727} + \underset{\nwarrow 30^\circ}{\omega_{CE}(0.4)}$$



$$(\overset{+}{\rightarrow}) \qquad v_C = 0 + 2.727\left(\frac{3}{5}\right) - \omega_{CE}(0.4)(\sin 30^\circ)$$

$$(+\uparrow) \qquad 0 = -4 + 2.727\left(\frac{4}{5}\right) + \omega_{CE}(0.4)(\cos 30^\circ)$$

$$\omega_{CE} = 5.249\text{ rad/s}$$

$$v_C = 0.587\text{ m/s} \qquad \textbf{Ans.}$$

Also:

$$v_B = v_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{i} = -4\mathbf{j} + (-\omega_{AB}\mathbf{k}) \times \left\{\frac{-4}{5}(0.55)\mathbf{i} + \frac{3}{5}(0.55)\mathbf{j}\right\}$$

***16–72. Continued**

$v_B = \omega_{AB}(0.33)$

$0 = -4 + 0.44\omega_{AB}$

$\omega_{AB} = 9.091 \text{ rad/s}$

$v_B = 3.00 \text{ m/s}$ **Ans.**

$\mathbf{v}_D = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$

$v_D = -4\mathbf{j} + (-9.091\mathbf{k}) \times \left\{\dfrac{-4}{5}(0.3)\mathbf{i} + \dfrac{3}{5}(0.3)\mathbf{j}\right\}$

$v_D = \{1.636\mathbf{i} - 1.818\mathbf{j}\} \text{ m/s}$

$v_C = v_D + \omega_{CE} \times \mathbf{r}_{C/D}$

$v_C\mathbf{i} = (1.636\mathbf{i} - 1.818\mathbf{j}) + (-\omega_{CE}\mathbf{k}) \times (-0.4 \cos 30°\mathbf{i} - 0.4 \sin 30°\mathbf{j})$

$v_C = 1.636 - 0.2\omega_{CE}$

$0 = -1.818 - 0.346\omega_{CE}$

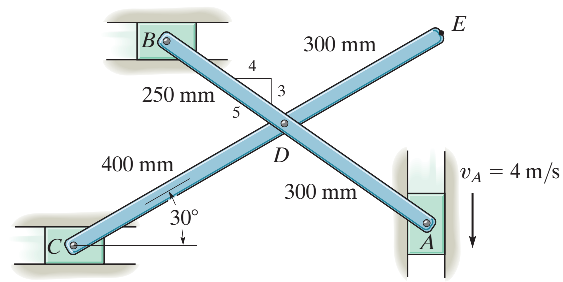
$\omega_{CE} = 5.25 \text{ rad/s}$

$v_C = 0.587 \text{ m/s}$ **Ans.**

Ans:
$v_B = 3.00 \text{ m/s}$
$v_C = 0.587 \text{ m/s}$
$v_B = 3.00 \text{ m/s}$
$v_C = 0.587 \text{ m/s}$

16–73.

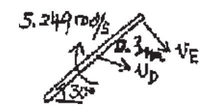
If the slider block A is moving downward at $v_A = 4$ m/s, determine the velocity of point E at the instant shown.



SOLUTION

See solution to Prob. 16–87.

$$v_E = v_D + v_{E/D}$$



$$\vec{v_E} = 4\downarrow + \underset{4\,\overset{3}{\nearrow}\,5}{2.727} + \underset{\measuredangle\, 30°}{(5.249)(0.3)}$$

$$(\xrightarrow{+}) \qquad (v_E)_x = 0 + 2.727\left(\frac{3}{5}\right) + 5.249(0.3)(\sin 30°)$$

$$(+\downarrow) \qquad (v_E)_y = 4 - 2.727\left(\frac{4}{5}\right) + 5.249(0.3)(\cos 30°)$$

$$(v_E)_x = 2.424 \text{ m/s} \rightarrow$$

$$(v_E)_y = 3.182 \text{ m/s} \downarrow$$

$$\mathbf{v}_E = \sqrt{(2.424)^2 + (3.182)^2} = 4.00 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{3.182}{2.424}\right) = 52.7° \qquad \textbf{Ans.}$$

Also:

See solution to Prob. 16–87.

$$\mathbf{v}_E = \mathbf{v}_D + \omega_{CE} \times \mathbf{r}_{E/D}$$

$$\mathbf{v}_E = (1.636\mathbf{i} - 1.818\mathbf{j}) + (-5.25\mathbf{k}) \times \{\cos 30°(0.3)\mathbf{i} - 0.4 \sin 30°(0.3)\mathbf{j}\}$$

$$\mathbf{v}_E = \{2.424\mathbf{i} - 3.182\mathbf{j}\} \text{ m/s}$$

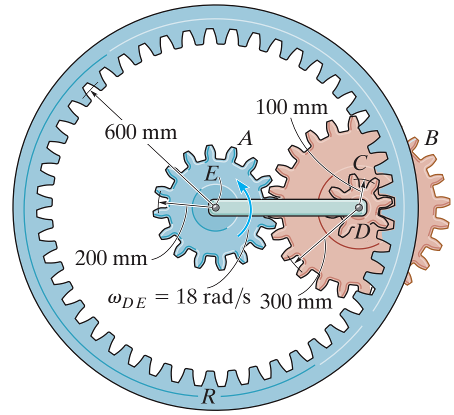
$$v_E = \sqrt{(2.424)^2 + (3.182)^2} = 4.00 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{3.182}{2.424}\right) = 52.7° \qquad \textbf{Ans.}$$

Ans:
$v_E = 4.00$ m/s
$\theta = 52.7°$

16–74.

The epicyclic gear train consists of the sun gear A which is in mesh with the planet gear B. This gear has an inner hub C which is fixed to B and in mesh with the fixed ring gear R. If the connecting link DE pinned to B and C is rotating at $\omega_{DE} = 18$ rad/s about the pin at E, determine the angular velocities of the planet and sun gears.



SOLUTION

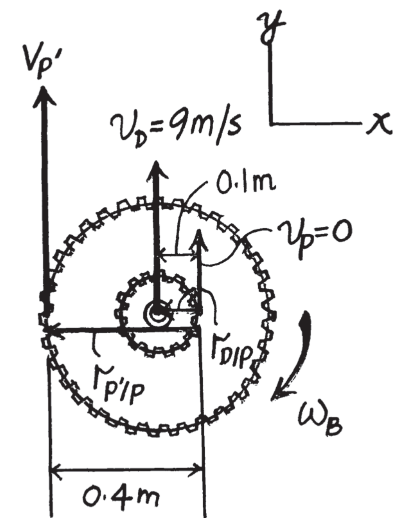
$v_D = r_{DE}\,\omega_{DE} = (0.5)(18) = 9$ m/s $\uparrow$

The velocity of the contact point P with the ring is zero.

$\mathbf{v}_D = \mathbf{v}_P + \omega \times \mathbf{r}_{D/P}$

$9\mathbf{j} = 0 + (-\omega_B\,\mathbf{k}) \times (-0.1\mathbf{i})$

$\omega_B = 90$ rad/s ↻ **Ans.**



Let P' be the contact point between A and B.

$\mathbf{v}_{P'} = \mathbf{v}_P + \omega \times \mathbf{r}_{P'/P}$

$v_{P'}\,\mathbf{j} = \mathbf{0} + (-90\mathbf{k}) \times (-0.4\mathbf{i})$

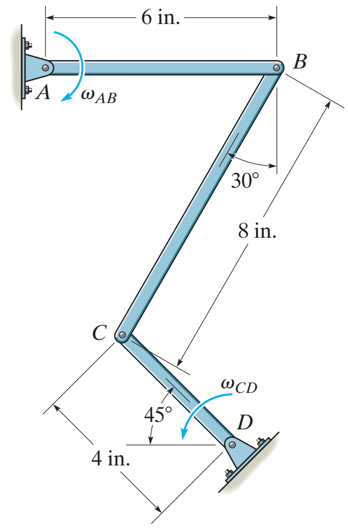
$v_{P'} = 36$ m/s $\uparrow$

$\omega_A = \dfrac{v_{P'}}{r_A} = \dfrac{36}{0.2} = 180$ rad/s ↺ **Ans.**

Ans:
$\omega_B = 90$ rad/s ↻
$\omega_A = 180$ rad/s ↺

16–75.

If link AB is rotating at $\omega_{AB} = 3$ rad/s, determine the angular velocity of link CD at the instant shown.



SOLUTION

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D}$$

$$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$$

$$(\omega_{CD}\mathbf{k}) \times (-4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j}) = (-3\mathbf{k}) \times (6\mathbf{i}) + (\omega_{BC}\mathbf{k}) \times (-8\sin 30°\mathbf{i} - 8\cos 30°\mathbf{j})$$

$$-2.828\omega_{CD} = 0 + 6.928\omega_{BC}$$

$$-2.828\omega_{CD} = -18 - 4\omega_{BC}$$

Solving,

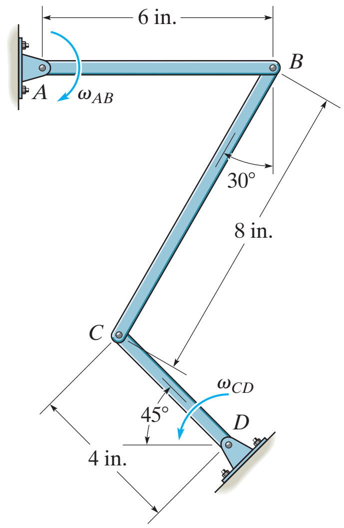
$$\omega_{BC} = -1.65 \text{ rad/s}$$

$$\omega_{CD} = 4.03 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_{CD} = 4.03$ rad/s

***16–76.**

If link CD is rotating at $\omega_{CD} = 5$ rad/s, determine the angular velocity of link AB at the instant shown.



SOLUTION

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D}$$

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

$$(-\omega_{AB}\mathbf{k}) \times (6\mathbf{i}) = (5\mathbf{k}) \times (-4\cos 45°\mathbf{i} + 4\sin 45°\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (8\sin 30°\mathbf{i} + 8\cos 30°\mathbf{j})$$

$$0 = -14.142 - 6.9282\omega_{BC}$$

$$-6\omega_{AB} = -14.142 + 4\omega_{BC}$$

Solving,

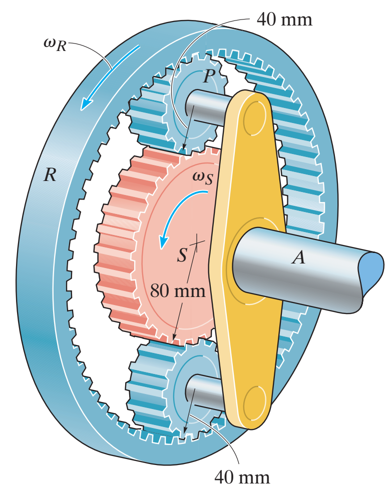
$$\omega_{AB} = 3.72 \text{ rad/s}$$ **Ans.**

$$\omega_{BC} = -2.04 \text{ rad/s}$$

Ans:
$\omega_{AB} = 3.72$ rad/s

16–77.

The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear R is held fixed, $\omega_R = 0$, and the sun gear S is rotating at $\omega_S = 5$ rad/s. Determine the angular velocity of each of the planet gears P and shaft A.



SOLUTION

$v_A = 5(80) = 400 \text{ mm/s} \leftarrow$

$v_B = 0$

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$0 = -400\mathbf{i} + (\omega_p\,\mathbf{k}) \times (80\mathbf{j})$

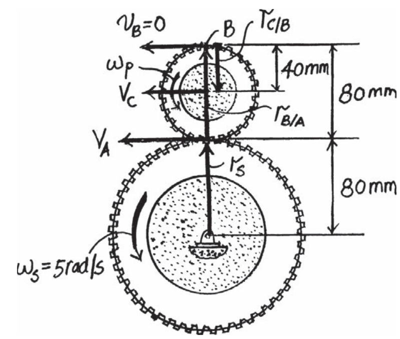
$0 = -400\mathbf{i} - 80\omega_p\,\mathbf{i}$

$\omega_P = -5 \text{ rad/s} = 5 \text{ rad/s}$ **Ans.**

$\mathbf{v}_C = \mathbf{v}_B + \omega \times \mathbf{r}_{C/B}$

$\mathbf{v}_C = 0 + (-5\mathbf{k}) \times (-40\mathbf{j}) = -200\mathbf{i}$

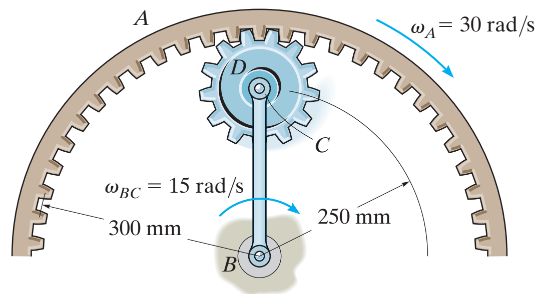
$\omega_A = \dfrac{200}{120} = 1.67 \text{ rad/s}$ **Ans.**



Ans:
$\omega_P = 5 \text{ rad/s}$
$\omega_A = 1.67 \text{ rad/s}$

16–78.

If the ring gear A rotates clockwise with an angular velocity of $\omega_A = 30$ rad/s, while link BC rotates clockwise with an angular velocity of $\omega_{BC} = 15$ rad/s, determine the angular velocity of gear D.



SOLUTION

Rotation About A Fixed Axis. The magnitudes of the velocity of Point E on the rim and center C of gear D are

$$v_E = \omega_A r_A = 30(0.3) = 9 \text{ m/s}$$

$$v_C = \omega_{BC} r_{BC} = 15(0.25) = 3.75 \text{ m/s}$$

General Plane Motion. Applying the relative velocity equation by referring to Fig. *a*,

$$\mathbf{v}_E = \mathbf{v}_C + \boldsymbol{\omega}_D \times \mathbf{r}_{E/C}$$

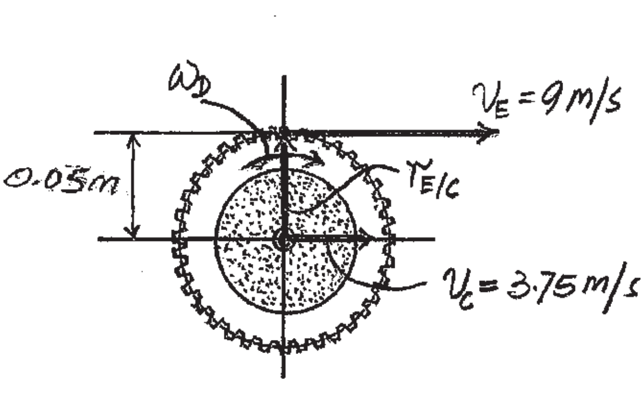
$$9\mathbf{i} = 3.75\mathbf{i} + (-\omega_D\mathbf{k}) \times (0.05\mathbf{j})$$

$$9\mathbf{i} = (3.75 + 0.05\omega_D)\mathbf{i}$$

Equating **i** component,

$$9 = 3.75 + 0.05\omega_D$$

$$\omega_D = 105 \text{ rad/s} \curvearrowright$$ **Ans.**

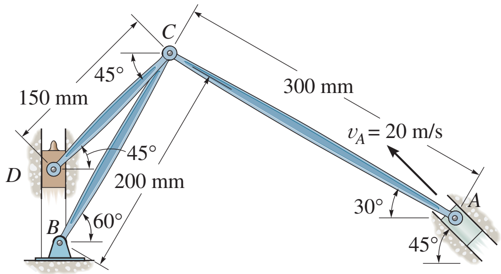


(a)

Ans:
$\omega_D = 105$ rad/s $\curvearrowright$

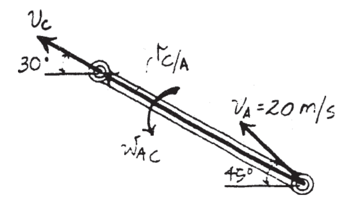
16–79.

The mechanism shown is used in a riveting machine. It consists of a driving piston A, three links, and a riveter which is attached to the slider block D. Determine the velocity of D at the instant shown, when the piston at A is traveling at $v_A = 20$ m/s.



SOLUTION

Kinematic Diagram: Since link BC is rotating about fixed point B, then v_C is always directed perpendicular to link BC. At the instant shown. $\mathbf{v}_C = -v_C \cos 30°\mathbf{i} + v_C \sin 30°\mathbf{j} = -0.8660\,v_C\mathbf{i} + 0.500\,v_C\,\mathbf{j}$. Also, block D is moving towards the *negative y* axis due to the constraint of the guide. Then. $\mathbf{v}_D = -v_D\mathbf{j}$.

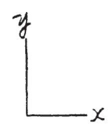


Velocity Equation: Here, $\mathbf{v}_A = \{-20 \cos 45°\mathbf{i} + 20 \sin 45°\mathbf{j}\}$m/s $= \{-14.14\mathbf{i} + 14.14\mathbf{j}\}$ m/s and $\mathbf{r}_{C/A} = \{-0.3\cos 30°\mathbf{i} + 0.3\sin 30°\mathbf{j}\}$m $= \{-0.2598\mathbf{i} + 0.150\mathbf{j}\}$ m. Applying Eq. 16–16 to link AC, we have

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega}_{AC} \times \mathbf{r}_{C/A}$$

$$-0.8660\,v_C\mathbf{i} + 0.500\,v_C\mathbf{j} = -14.14\mathbf{i} + 14.14\mathbf{j} + (\omega_{AC}\mathbf{k}) \times (-0.2598\mathbf{i} + 0.150\mathbf{j})$$

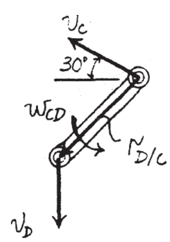
$$-0.8660\,v_C\mathbf{i} + 0.500\,v_C\mathbf{j} = -(14.14 + 0.150\,\omega_{AC})\mathbf{i} + (14.14 - 0.2598\omega_{AC})\mathbf{j}$$



Equating **i** and **j** components gives

$$-0.8660\,v_C = -(14.14 + 0.150\,\omega_{AC}) \qquad \textbf{[1]}$$

$$0.500\,v_C = 14.14 - 0.2598\,\omega_{AC} \qquad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields

$$\omega_{AC} = 17.25 \text{ rad/s} \qquad v_C = 19.32 \text{ m/s}$$

Thus, $\mathbf{v}_C = \{-19.32 \cos 30°\mathbf{i} + 19.32 \sin 30°\mathbf{j}\}$ m/s $= \{-16.73\mathbf{i} + 9.659\mathbf{j}\}$ m/s and $\mathbf{r}_{D/C} = \{-0.15\cos 45°\mathbf{i} - 0.15\sin 45°\mathbf{j}\}$m $= \{-0.1061\mathbf{i} - 0.1061\mathbf{j}\}$ m. Applying Eq. 16–16 to link CD, we have

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$$

$$-v_D\mathbf{j} = -16.73\mathbf{i} + 9.659\mathbf{j} + (\omega_{CD}\mathbf{k}) \times (-0.1061\mathbf{i} - 0.1061\mathbf{j})$$

$$-v_D\mathbf{j} = (0.1061\omega_{CD} - 16.73)\mathbf{i} + (9.659 - 0.1061\omega_{CD})\mathbf{j}$$

Equating **i** and **j** components gives

$$0 = 0.1061\omega_{CD} - 16.73 \qquad \textbf{[3]}$$

$$-v_D = 9.659 - 0.1061\,\omega_{CD} \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields

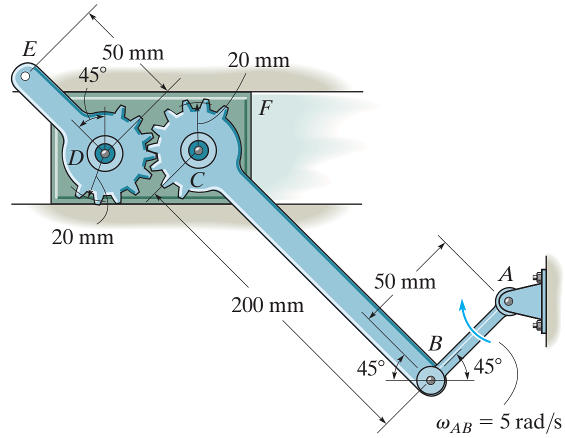
$$\omega_{CD} = 157.74 \text{ rad/s}$$

$$v_D = 7.07 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_D = 7.07$ m/s

***16–80.**

The mechanism is used on a machine for the manufacturing of a wire product. Because of the rotational motion of link AB and the sliding of block F, the segmental gear lever DE undergoes general plane motion. If AB is rotating at $\omega_{AB} = 5$ rad/s, determine the velocity of point E at the instant shown.



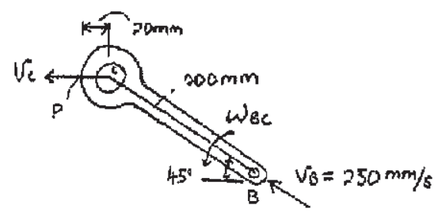
SOLUTION

$v_B = \omega_{AB} r_{AB} = 5(50) = 250$ mm/s $45° \searrow$

$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

$v_C = 250 + \omega_{BC}(200)$

$\leftarrow \quad 45° \searrow \quad 45° \swarrow$



$(+\uparrow) \quad 0 = 250 \sin 45° - \omega_{BC}(200) \sin 45°$

$(\overset{+}{\leftarrow}) \quad v_C = 250 \cos 45° + \omega_{BC}(200) \cos 45°$

Solving,

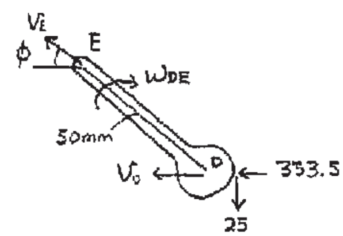
$v_C = 353.6$ mm/s; $\qquad \omega_{BC} = 1.25$ rad/s

$\mathbf{v}_p = \mathbf{v}_C + \mathbf{v}_{p/C}$

$\underset{\rightarrow}{v_p} = \underset{\leftarrow}{353.6} + [(1.25)(20) = \underset{\downarrow}{25}]$

$\mathbf{v}_D = \mathbf{v}_p + \mathbf{v}_{D/p}$

$\underset{\leftarrow}{v_D} = (\underset{\leftarrow}{353.6} + \underset{\downarrow}{25}) + \underset{\uparrow}{20\omega_{DE}}$



$(\overset{+}{\leftarrow}) \quad v_D = 353.6 + 0 + 0$

$(+\downarrow) \quad 0 = 0 + (1.25)(20) - \omega_{DE}(20)$

Solving,

$v_D = 353.6$ mm/s; $\qquad \omega_{DE} = 1.25$ rad/s

$\mathbf{v}_E = \mathbf{v}_D + \mathbf{v}_{E/D}$

$v_E = 353.6 + 1.25(50)$

$\phi \searrow \quad \leftarrow \quad \angle 45°$

$(\overset{+}{\leftarrow}) \quad v_E \cos \phi = 353.6 - 1.25(50) \cos 45°$

$(+\uparrow) \quad v_E \sin \phi = 0 + 1.25(50) \sin 45°$

Solving,

$v_E = 312$ mm/s **Ans.**

$\phi = 8.13°$ **Ans.**

***16–80. Continued**

Also;

$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A}$

$\mathbf{v}_C = \mathbf{v}_B + \omega_{BC} \times \mathbf{r}_{C/B}$

$-v_C\mathbf{i} = (-5\mathbf{k}) \times (-0.05 \cos 45°\mathbf{i} - 0.05 \sin 45°\mathbf{j}) + (\omega_{BC}\mathbf{k}) \times (-0.2 \cos 45°\mathbf{i} + 0.2 \sin 45°\mathbf{j})$

$-v_C = -0.1768 - 0.1414\omega_{BC}$

$0 = 0.1768 - 0.1414\omega_{BC}$

$\omega_{BC} = 1.25 \text{ rad/s}, \qquad v_C = 0.254 \text{ m/s}$

$\mathbf{v}_p = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{p/C}$

$\mathbf{v}_D = \mathbf{v}_p + \omega_{DE} \times \mathbf{r}_{D/p}$

$\mathbf{v}_D = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{p/C} + \omega_{DE} \times \mathbf{r}_{D/p}$

$v_D\mathbf{i} = -0.354\mathbf{i} + (1.25\mathbf{k}) \times (-0.02\mathbf{i}) + (\omega_{DE}\mathbf{k}) \times (-0.02\mathbf{i})$

$v_D = -0.354$

$0 = -0.025 - \omega_{DE}(0.02)$

$v_D = 0.354 \text{ m/s}, \qquad \omega_{DE} = 1.25 \text{ rad/s}$

$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{DE} \times \mathbf{r}_{E/D}$

$(v_E)_x\mathbf{i} + (v_E)_y\mathbf{j} = -0.354\mathbf{i} + (-1.25\mathbf{k}) \times (-0.05 \cos 45°\mathbf{i} + 0.05 \sin 45°\mathbf{j})$

$(v_E)_x = -0.354 + 0.0442 = -0.3098$

$(v_E)_y = 0.0442$

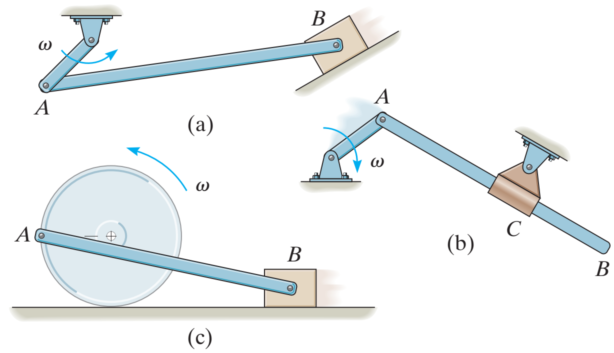
$v_E = \sqrt{(-0.3098)^2 + (0.0442)^2} = 312 \text{ mm/s}$ **Ans.**

$\phi = \tan^{-1}\left(\frac{0.0442}{0.3098}\right) = 8.13°$ **Ans.**

Ans:
$v_E = 312 \text{ mm/s}$
$\phi = 8.13°$
$v_E = 312 \text{ mm/s}$
$\phi = 8.13°$

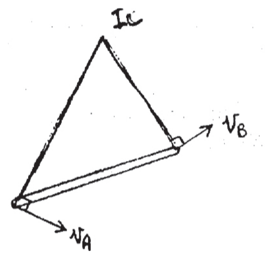
16–81.

In each case show graphically how to locate the instantaneous center of zero velocity of link AB. Assume the geometry is known.

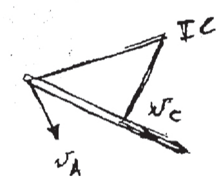


SOLUTION

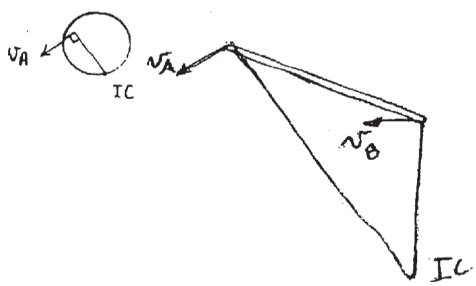
a)



b)

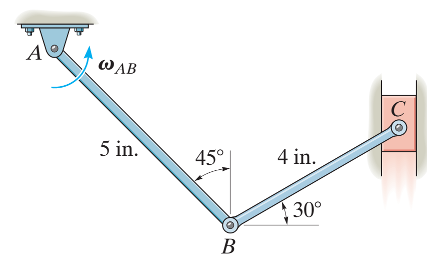


c)



16–82.

Determine the angular velocity of link AB at the instant shown if block C is moving upward at 12 in./s.



SOLUTION

$$\frac{4}{\sin 45^\circ} = \frac{r_{IC-B}}{\sin 30^\circ} = \frac{r_{IC-C}}{\sin 105^\circ}$$

$r_{IC-C} = 5.464$ in.

$r_{IC-B} = 2.828$ in.

$v_C = \omega_{BC}(r_{IC-C})$

$12 = \omega_{BC}(5.464)$

$\omega_{BC} = 2.1962$ rad/s

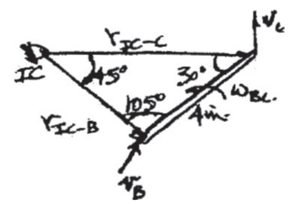
$v_B = \omega_{BC}(r_{IC-B})$

$\quad = 2.1962(2.828) = 6.211$ in./s

$v_B = \omega_{AB}\, r_{AB}$

$6.211 = \omega_{AB}(5)$

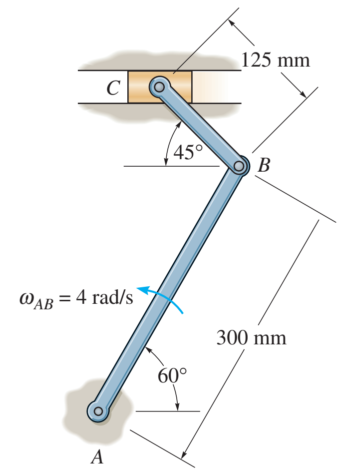
$\omega_{AB} = 1.24$ rad/s **Ans.**



Ans:
$\omega_{AB} = 1.24$ rad/s

16–83.

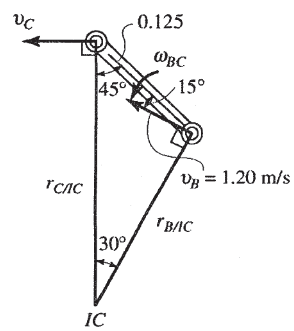
The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the angular velocity of the link CB at the instant shown, if the link AB is rotating at 4 rad/s.



SOLUTION

Kinematic Diagram: Since linke AB is rotating about fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 4(0.3) = 1.20$ m/s. At the instant shown, $\mathbf{v}_B$ is directed at an angle 30° with the horizontal. Also, block C is moving horizontally due to the constraint of the guide.

Instantaneous Center: The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have



$$\frac{r_{B/IC}}{\sin 45^\circ} = \frac{0.125}{\sin 30^\circ} \qquad r_{B/IC} = 0.1768 \text{ m}$$

$$\frac{r_{C/IC}}{\sin 105^\circ} = \frac{0.125}{\sin 30^\circ} \qquad r_{C/IC} = 0.2415 \text{ m}$$

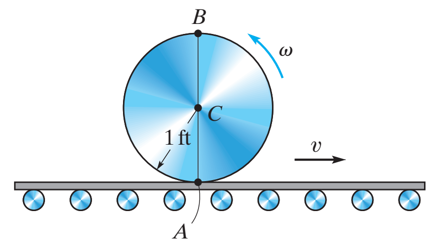
The angular velocity of bar BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.1768} = 6.79 \text{ rad/s} \qquad \textbf{Ans.}$$

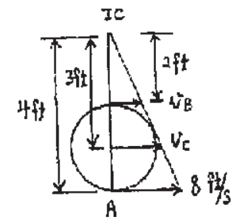
Ans:
$\omega_{BC} = 6.79$ rad/s

***16–84.**

The conveyor belt is moving to the right at $v = 8$ ft/s, and at the same instant the cylinder is rolling counterclockwise at $\omega = 2$ rad/s without slipping. Determine the velocities of the cylinder's center C and point B at this instant.



SOLUTION



$$r_{A-IC} = \frac{8}{2} = 4 \text{ ft}$$

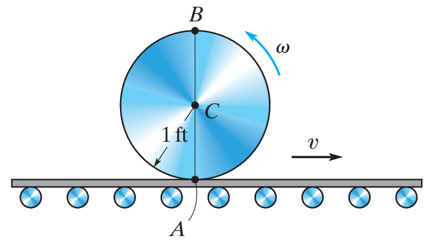
$v_C = 2(3) = 6.00$ ft/s $\rightarrow$ **Ans.**

$v_B = 2(2) = 4.00$ ft/s $\rightarrow$ **Ans.**

Ans:
$v_C = 6.00$ ft/s $\rightarrow$
$v_B = 4.00$ ft/s $\rightarrow$

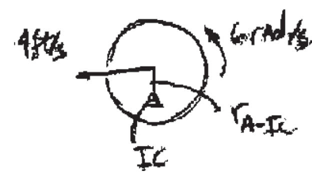
16–85.

The conveyor belt is moving to the right at $v = 12$ ft/s, and at the same instant the cylinder is rolling counterclockwise at $\omega = 6$ rad/s while its center has a velocity of 4 ft/s to the left. Determine the velocities of points A and B on the disk at this instant. Does the cylinder slip on the conveyor?



SOLUTION

$r_{A-IC} = \dfrac{4}{6} = 0.667$ ft



$v_A = 6(1 - 0.667) = 2$ ft/s $\rightarrow$ **Ans.**

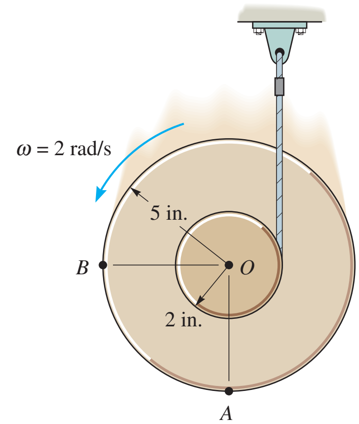
$v_B = 6(1 + 0.667) = 10$ ft/s $\leftarrow$ **Ans.**

Since $v_A \neq 12$ ft/s the cylinder slips on the conveyer. **Ans.**

Ans:
$v_A = 2$ ft/s $\rightarrow$
$v_B = 10$ ft/s $\leftarrow$
The cylinder slips.

16–86.

As the cord unravels from the wheel's inner hub, the wheel is rotating at $\omega = 2$ rad/s at the instant shown. Determine the velocities of points A and B.



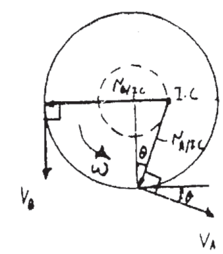
SOLUTION

$r_{B/IC} = 5 + 2 = 7$ in. $\qquad r_{A/IC} = \sqrt{2^2 + 5^2} = \sqrt{29}$ in.

$v_B = \omega\, r_{B/IC} = 2(7) = 14$ in./s $\downarrow$ **Ans.**

$v_A = \omega\, r_{A/IC} = 2\left(\sqrt{29}\right) = 10.8$ in./s **Ans.**

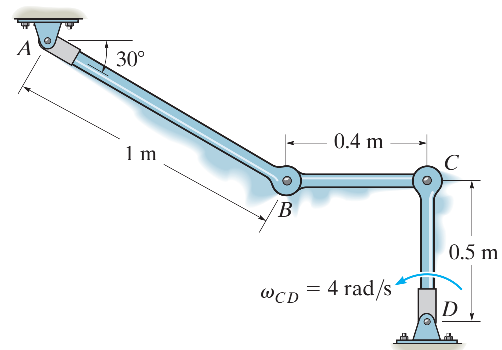
$\theta = \tan^{-1}\left(\dfrac{2}{5}\right) = 21.8°$ ⦩ **Ans.**



Ans:
$v_B = 14$ in./s$\downarrow$
$v_A = 10.8$ in./s
$\theta = 21.8°$ ⦩

16–87.

If rod CD is rotating with an angular velocity $\omega_{CD} = 4$ rad/s, determine the angular velocities of rods AB and CB at the instant shown.



SOLUTION

Rotation About A Fixed Axis. For links AB and CD, the magnitudes of the velocities of C and D are

$$v_C = \omega_{CD} r_{CD} = 4(0.5) = 2.00 \text{ m/s}$$

$$v_B = \omega_{AB} r_{AB} = \omega_{AB}(1)$$

And their direction are indicated in Fig. *a* and *b*.

General Plane Motion. With the results of $\mathbf{v}_C$ and $\mathbf{v}_B$, the IC for link BC can be located as shown in Fig. *c*. From the geometry of this figure,

$$r_{C/IC} = 0.4 \tan 30° = 0.2309 \text{ m} \qquad r_{B/IC} = \frac{0.4}{\cos 30°} = 0.4619 \text{ m}$$

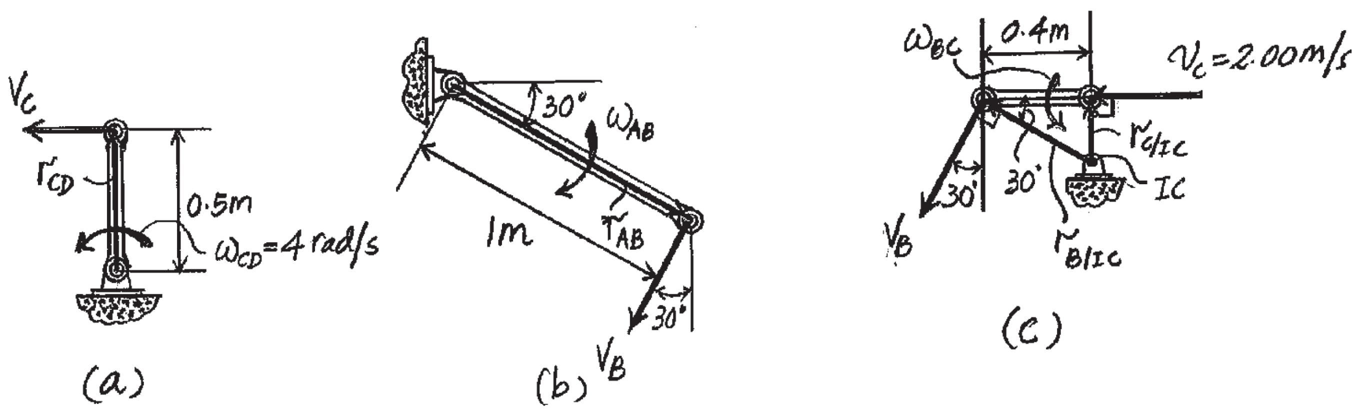
Then, the kinematics gives

$v_C = \omega_{BC} r_{C/IC};$ $\quad 2.00 = \omega_{BC}(0.2309)$

$$\omega_{BC} = 8.6603 \text{ rad/s} = 8.66 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

$v_B = \omega_{BC} r_{B/IC};$ $\quad \omega_{AB}(1) = 8.6603(0.4619)$

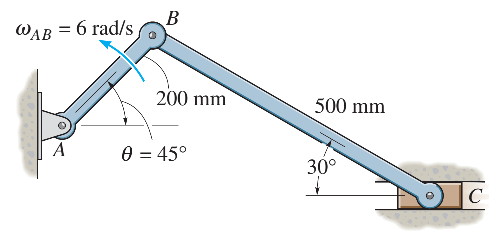
$$\omega_{AB} = 4.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



Ans:
$\omega_{BC} = 8.66$ rad/s $\circlearrowleft$
$\omega_{AB} = 4.00$ rad/s $\circlearrowright$

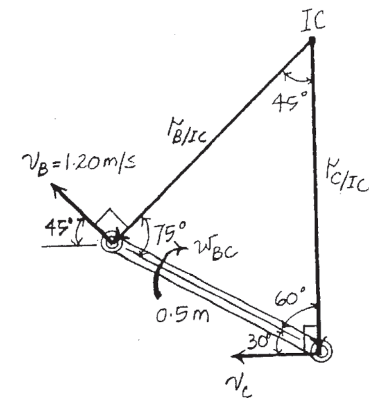
***16–88.**

If bar AB has an angular velocity $\omega_{AB} = 6$ rad/s, determine the velocity of the slider block C at the instant shown.



SOLUTION

Kinematic Diagram: Since link AB is rotating about fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB}\, r_{AB} = 6(0.2) = 1.20$ m/s. At the instant shown. $\mathbf{v}_B$ is directed with an angle 45° with the horizontal. Also, block C is moving horizontally due to the constraint of the guide.



Instantaneous Center: The instantaneous center of zero velocity of bar BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sine, we have

$$\frac{r_{B/IC}}{\sin 60°} = \frac{0.5}{\sin 45°} \qquad r_{B/IC} = 0.6124 \text{ m}$$

$$\frac{r_{C/IC}}{\sin 75°} = \frac{0.5}{\sin 45°} \qquad r_{C/IC} = 0.6830 \text{ m}$$

The angular velocity of bar BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.6124} = 1.960 \text{ rad/s}$$

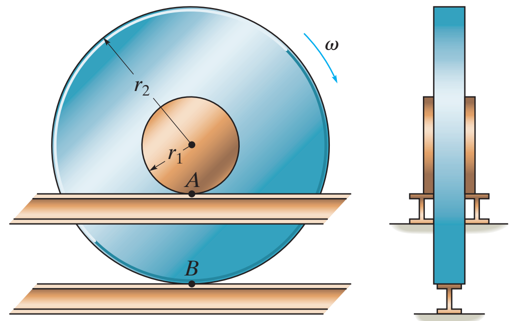
Thus, the velocity of block C is

$$v_C = \omega_{BC}\, r_{C/IC} = 1.960(0.6830) = 1.34 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

Ans:
$v_C = 1.34$ m/s $\leftarrow$

16–89.

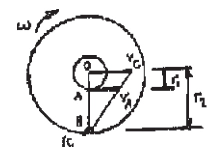
Show that if the rim of the wheel and its hub maintain contact with the three tracks as the wheel rolls, it is necessary that slipping occurs at the hub A if no slipping occurs at B. Under these conditions, what is the speed at A if the wheel has angular velocity $\boldsymbol{\omega}$?



SOLUTION

IC is at B.

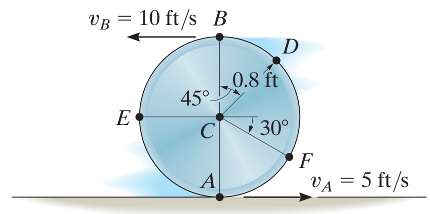
$v_A = \omega(r_2 - r_1) \rightarrow$ **Ans.**



Ans:
$v_A = \omega(r_2 - r_1)$

16–90.

Due to slipping, points A and B on the rim of the disk have the velocities shown. Determine the velocities of the center point C and point D at this instant.

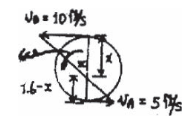


SOLUTION

$$\frac{1.6 - x}{5} = \frac{x}{10}$$

$$5x = 16 - 10x$$

$$x = 1.06667 \text{ ft}$$

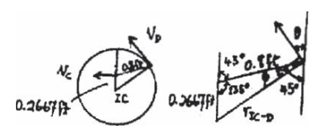


$$\omega = \frac{10}{1.06667} = 9.375 \text{ rad/s}$$

$$r_{IC-D} = \sqrt{(0.2667)^2 + (0.8)^2 - 2(0.2667)(0.8)\cos 135^\circ} = 1.006 \text{ ft}$$

$$\frac{\sin\phi}{0.2667} = \frac{\sin 135^\circ}{1.006}$$

$$\phi = 10.80^\circ$$



$$v_C = 0.2667(9.375) = 2.50 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

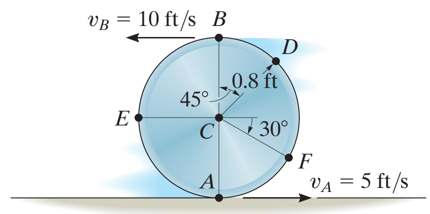
$$v_D = 1.006(9.375) = 9.43 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\theta = 45^\circ + 10.80^\circ = 55.8^\circ \measuredangle$$

Ans:
$v_C = 2.50$ ft/s $\leftarrow$
$v_D = 9.43$ ft/s
$\theta = 55.8^\circ \measuredangle$

16–91.

Due to slipping, points A and B on the rim of the disk have the velocities shown. Determine the velocities of the center point C and point E at this instant.



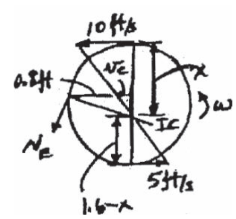
SOLUTION

$$\frac{1.6 - x}{5} = \frac{x}{10}$$

$$5x = 16 - 10x$$

$$x = 1.06667 \text{ ft}$$

$$\omega = \frac{10}{1.06667} = 9.375 \text{ rad/s}$$



$$v_C = \omega(r_{IC-C})$$

$$= 9.375(1.06667 - 0.8)$$

$$= 2.50 \text{ ft/s} \leftarrow$$ **Ans.**

$$v_E = \omega(r_{IC-E})$$

$$= 9.375\sqrt{(0.8)^2 + (0.26667)^2}$$

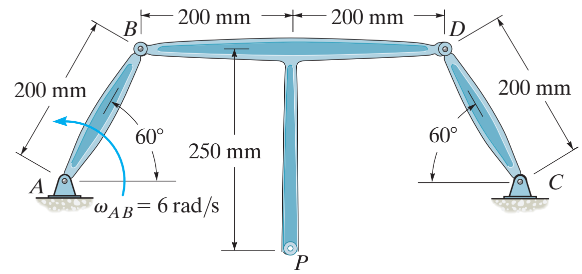
$$= 7.91 \text{ ft/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{0.26667}{0.8}\right) = 18.4° \measuredangle$$

Ans:
$v_C = 2.50$ ft/s $\leftarrow$
$v_E = 7.91$ ft/s
$\theta = 18.4° \measuredangle$

***16–92.**

Member AB is rotating at $\omega_{AB} = 6\text{ rad/s}$. Determine the velocity of point D and the angular velocity of members BPD and CD.



SOLUTION

Rotation About A Fixed Axis. For links AB and CD, the magnitudes of the velocities of B and D are

$v_B = \omega_{AB} r_{AB} = 6(0.2) = 1.20\text{ m/s} \qquad v_D = \omega_{CD}(0.2)$

And their directions are indicated in Figs. *a* and *b*.

General Plane Motion. With the results of $\mathbf{v}_B$ and $\mathbf{v}_D$, the IC for member BPD can be located as show in Fig. *c*. From the geometry of this figure,

$$r_{B/IC} = r_{D/IC} = 0.4\text{ m}$$

Then, the kinematics gives

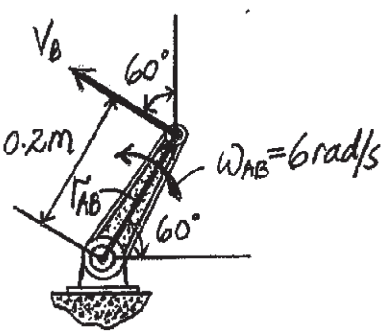
$$\omega_{BPD} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.4} = 3.00\text{ rad/s} \;\circlearrowright \qquad \textbf{Ans.}$$

$$v_D = \omega_{BPD} r_{D/IC} = (3.00)(0.4) = 1.20\text{ m/s} \swarrow \qquad \textbf{Ans.}$$

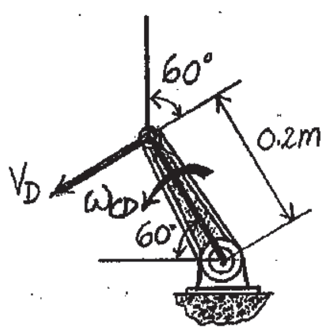
Thus,

$v_D = \omega_{CD}(0.2); \qquad 1.2 = \omega_{CD}(0.2)$

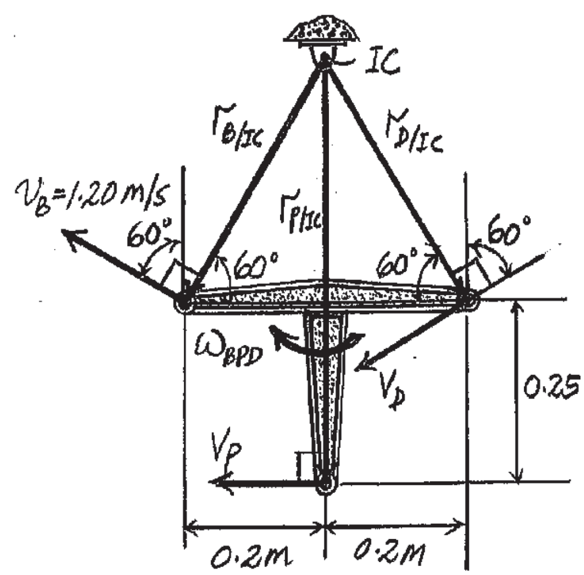
$$\omega_{CD} = 6.00\text{ rad/s} \;\circlearrowleft \qquad \textbf{Ans.}$$



(a)



(b)

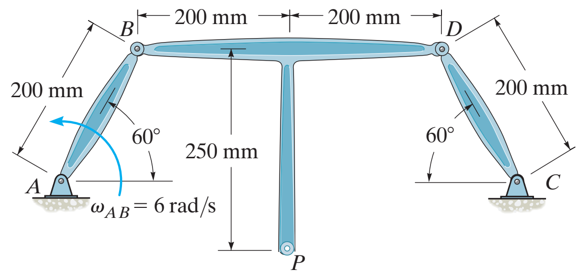


(c)

Ans:
$\omega_{BPD} = 3.00\text{ rad/s} \;\circlearrowright$
$v_D = 1.20\text{ m/s} \swarrow$
$\omega_{CD} = 6.00\text{ rad/s} \;\circlearrowleft$

16–93.

Member AB is rotating at $\omega_{AB} = 6$ rad/s. Determine the velocity of point P, and the angular velocity of member BPD.



SOLUTION

Rotation About A Fixed Axis. For links AB and CD, the magnitudes of the velocities of B and D are

$$v_B = \omega_{AB} r_{AB} = 6(0.2) = 1.20 \text{ m/s} \qquad v_D = \omega_{CD}(0.2)$$

And their direction are indicated in Fig. *a* and *b*

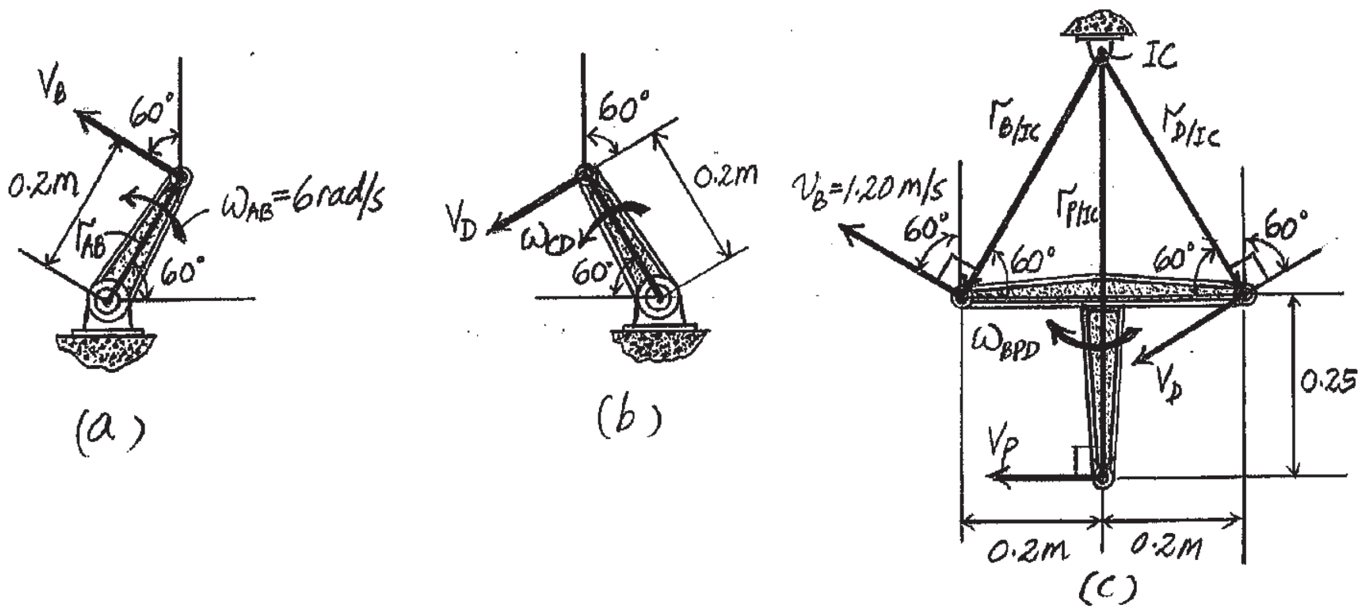
General Plane Motion. With the results of $\mathbf{v}_B$ and $\mathbf{v}_D$, the IC for member BPD can be located as shown in Fig. *c*. From the geometry of this figure

$$r_{B/IC} = 0.4 \text{ m} \qquad r_{P/IC} = 0.25 + 0.2 \tan 60° = 0.5964 \text{ m}$$

Then the kinematics give

$$\omega_{BPD} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.4} = 3.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

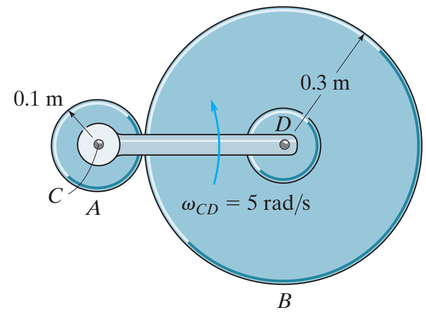
$$v_P = \omega_{BPD} r_{P/IC} = (3.00)(0.5964) = 1.7892 \text{ m/s} = 1.79 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$



Ans:
$\omega_{BPD} = 3.00$ rad/s $\circlearrowright$
$v_P = 1.79$ m/s $\leftarrow$

16–94.

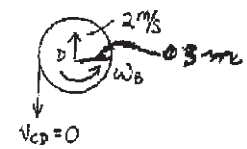
The cylinder B rolls on the fixed cylinder A without slipping. If connected bar CD is rotating with an angular velocity $\omega_{CD} = 5$ rad/s, determine the angular velocity of cylinder B. Point C is a fixed point.



SOLUTION

$v_D = 5(0.4) = 2$ m/s

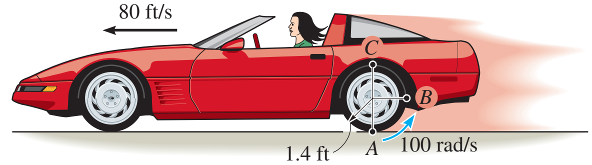
$\omega_B = \dfrac{2}{0.3} = 6.67$ rad/s **Ans.**



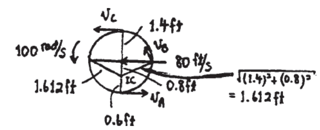
Ans:
$\omega_B = 6.67$ rad/s

16–95.

As the car travels forward at 80 ft/s on a wet road, due to slipping, the rear wheels have an angular velocity $\omega = 100$ rad/s. Determine the speeds of points A, B, and C caused by the motion.



SOLUTION



$$r = \frac{80}{100} = 0.8 \text{ ft}$$

$v_A = 0.6(100) = 60.0 \text{ ft/s} \rightarrow$ **Ans.**

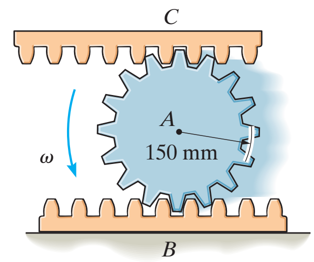
$v_C = 2.2(100) = 220 \text{ ft/s} \leftarrow$ **Ans.**

$v_B = 1.612(100) = 161 \text{ ft/s} \quad 60.3^\circ$ **Ans.**

Ans:
$v_A = 60.0 \text{ ft/s} \rightarrow$
$v_C = 220 \text{ ft/s} \leftarrow$
$v_B = 161 \text{ ft/s}$
$\theta = 60.3^\circ$

***16–96.**

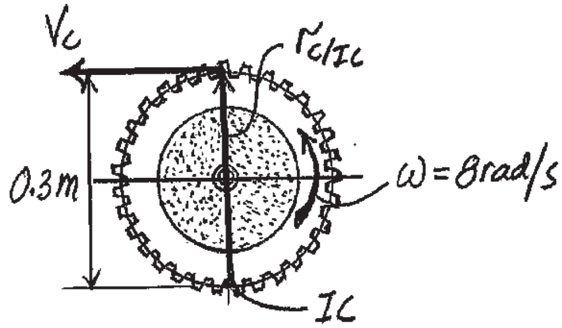
The pinion gear A rolls on the fixed gear rack B with an angular velocity $\omega = 8$ rad/s. Determine the velocity of the gear rack C.



SOLUTION

General Plane Motion. The location of IC for the gear is at the bottom of the gear where it meshes with gear rack B as shown in Fig. a. Thus,

$$v_C = \omega r_{C/IC} = 8(0.3) = 2.40 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

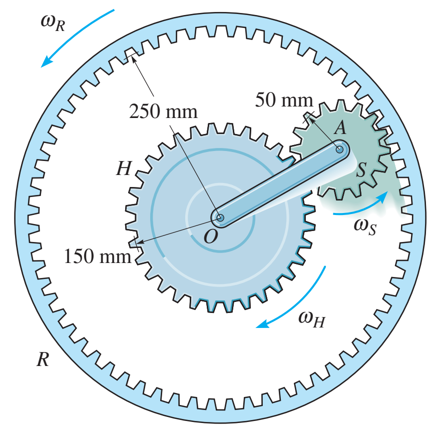


(a)

Ans:
$v_C = 2.40$ m/s $\leftarrow$

16–97.

If the hub gear H and ring gear R have angular velocities $\omega_H = 5$ rad/s and $\omega_R = 20$ rad/s, respectively, determine the angular velocity ω_S of the spur gear S and the angular velocity of its attached arm OA.

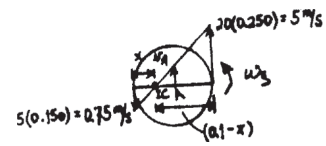


SOLUTION

$$\frac{5}{0.1 - x} = \frac{0.75}{x}$$

$$x = 0.01304 \text{ m}$$

$$\omega_S = \frac{0.75}{0.01304} = 57.5 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$



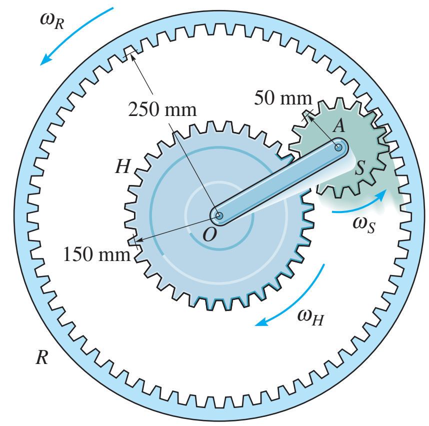
$$v_A = 57.5(0.05 - 0.01304) = 2.125 \text{ m/s}$$

$$\omega_{OA} = \frac{2.125}{0.2} = 10.6 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Ans:
$\omega_S = 57.5 \text{ rad/s} \circlearrowleft$
$\omega_{OA} = 10.6 \text{ rad/s} \circlearrowleft$

16–98.

If the hub gear H has an angular velocity $\omega_H = 5$ rad/s, determine the angular velocity of the ring gear R so that the arm OA attached to the spur gear S remains stationary ($\omega_{OA} = 0$). What is the angular velocity of the spur gear?

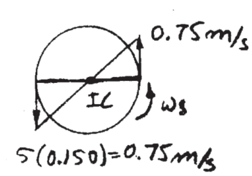


SOLUTION

The IC is at A.

$$\omega_S = \frac{0.75}{0.05} = 15.0 \text{ rad/s}$$ **Ans.**

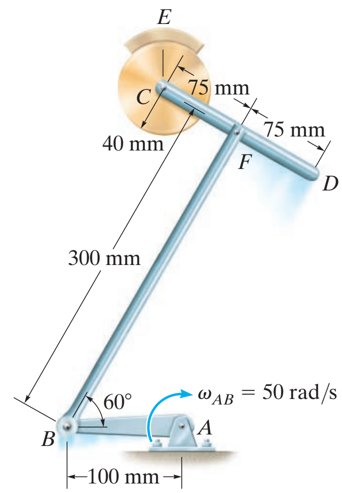
$$\omega_R = \frac{0.75}{0.250} = 3.00 \text{ rad/s}$$ **Ans.**



Ans:
$\omega_S = 15.0$ rad/s
$\omega_R = 3.00$ rad/s

16–99.

The crankshaft AB rotates at $\omega_{AB} = 50$ rad/s about the fixed axis through point A, and the disk at C is held fixed in its support at E. Determine the angular velocity of rod CD at the instant shown.



SOLUTION

$$r_{B/IC} = \frac{0.3}{\sin 30^\circ} = 0.6 \text{ m}$$

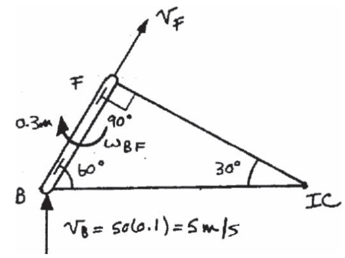
$$r_{F/IC} = \frac{0.3}{\tan 30^\circ} = 0.5196 \text{ m}$$

$$\omega_{BF} = \frac{5}{0.6} = 8.333 \text{ rad/s}$$

$$v_F = 8.333(0.5196) = 4.330 \text{ m/s}$$

Thus,

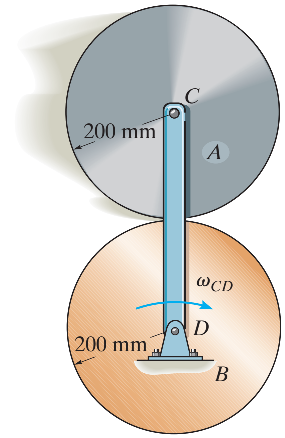
$$\omega_{CD} = \frac{4.330}{0.075} = 57.7 \text{ rad/s} \circlearrowleft$$ **Ans.**



Ans:
$\omega_{CD} = 57.7$ rad/s $\circlearrowleft$

***16–100.**

Cylinder A rolls on the *fixed cylinder B* without slipping. If bar CD is rotating with an angular velocity of $\omega_{CD} = 3$ rad/s, determine the angular velocity of A.



SOLUTION

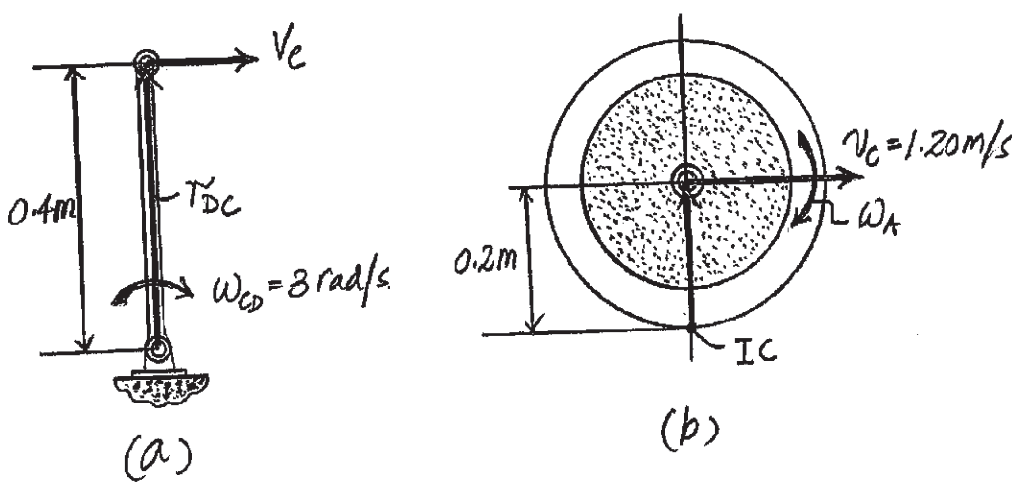
Rotation About A Fixed Axis. The magnitude of the velocity of C is

$$v_C = \omega_{CD} r_{DC} = 3(0.4) = 1.20 \text{ m/s} \rightarrow$$

General Plane Motion. The IC for cylinder A is located at the bottom of the cylinder where it contacts with cylinder B, since no slipping occurs here, Fig. *b*.

$$v_C = \omega_A \, r_{C/IC}; \qquad 1.20 = \omega_A(0.2)$$

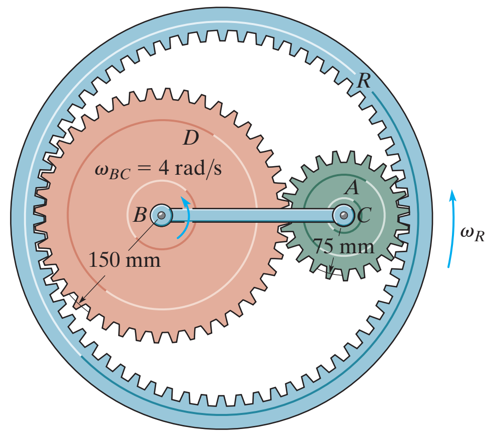
$$\omega_A = 6.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



Ans:
$\omega_A = 6.00$ rad/s $\circlearrowright$

16–101.

The planet gear A is pin connected to the end of the link BC. If the link rotates about the fixed point B at 4 rad/s, determine the angular velocity of the ring gear R. The sun gear D is fixed from rotating.



SOLUTION

Gear A:

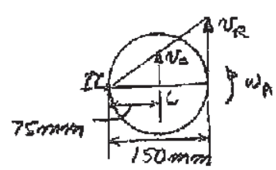
$v_C = 4(225) = 900$ mm/s

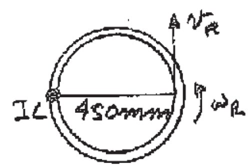
$$\omega_A = \frac{900}{75} = \frac{v_R}{150}$$

$v_R = 1800$ mm/s

Ring gear:

$$\omega_R = \frac{1800}{450} = 4 \text{ rad/s}$$ **Ans.**

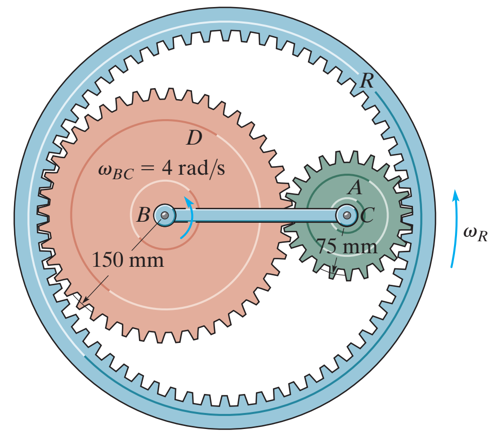




Ans:
$\omega_R = 4$ rad/s

16–102.

Solve Prob. 16–101 if the sun gear D is rotating clockwise at $\omega_D = 5$ rad/s while link BC rotates counterclockwise at $\omega_{BC} = 4$ rad/s.



SOLUTION

Gear A:

$$v_P = 5(150) = 750 \text{ mm/s}$$

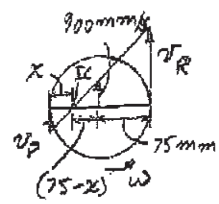
$$v_C = 4(225) = 900 \text{ mm/s}$$

$$\frac{x}{750} = \frac{75 - x}{900}$$

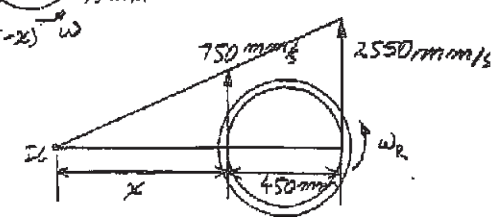
$$x = 34.09 \text{ mm}$$

$$\omega = \frac{750}{34.09} = 22.0 \text{ rad/s}$$

$$v_R = [75 + (75 - 34.09)](22) = 2550 \text{ mm/s}$$



Ring gear:



$$\frac{750}{x} = \frac{2550}{x + 450}$$

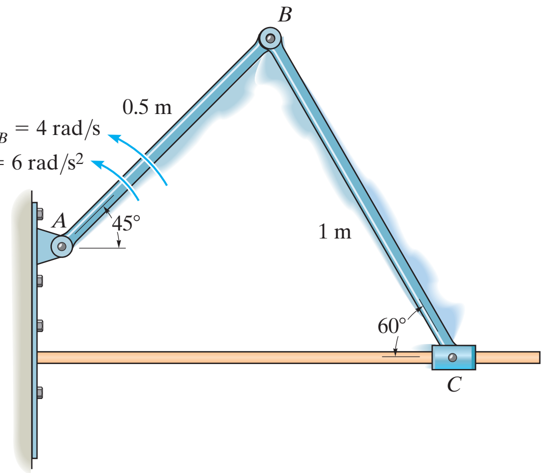
$$x = 187.5 \text{ mm}$$

$$\omega_R = \frac{750}{187.5} = 4 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Ans:
$\omega_R = 4$ rad/s

16–103.

Bar AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 4(0.5) = 2.00 \text{ m/s } 45^\circ \nwarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2\, \mathbf{r}_{AB}$$

$$= 6\mathbf{k} \times (0.5 \cos 45^\circ \mathbf{i} + 0.5 \sin 45^\circ \mathbf{j}) - 4^2(0.5 \cos 45^\circ \mathbf{i} + 0.5 \sin 45^\circ \mathbf{j})$$

$$= \{-5.5\sqrt{2}\mathbf{i} - 2.5\sqrt{2}\mathbf{j}\} \text{ m/s}^2$$

General Plane Motion. The IC of link BC can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. b. From the geometry of this figure,

$$\frac{r_{B/IC}}{\sin 30^\circ} = \frac{1}{\sin 45^\circ}; \qquad r_{B/IC} = \frac{\sqrt{2}}{2} \text{ m}$$

$$\frac{r_{C/IC}}{\sin 105^\circ} = \frac{1}{\sin 45^\circ}; \qquad r_{C/IC} = 1.3660 \text{ m}$$

Then the kinematics gives,

$$v_B = \omega_{BC}\, r_{B/IC}; \qquad 2 = \omega_{BC}\left(\frac{\sqrt{2}}{2}\right) \qquad \omega_{BC} = 2\sqrt{2} \text{ rad/s} \circlearrowright$$

$$v_C = \omega_{BC} r_{B/IC}; \qquad v_C = \left(2\sqrt{2}\right)(1.3660) = 3.864 \text{ m/s} = 3.86 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. c,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\, r_{C/B}$$

$$-a_C\mathbf{i} = \left(-5.5\sqrt{2}\mathbf{i} - 2.5\sqrt{2}\mathbf{j}\right) + (-\alpha_{BC}\mathbf{k}) \times (1 \cos 60^\circ \mathbf{i} - 1 \sin 60^\circ \mathbf{j})$$
$$- \left(2\sqrt{2}\right)^2(1 \cos 60^\circ \mathbf{i} - 1 \sin 60^\circ \mathbf{j})$$

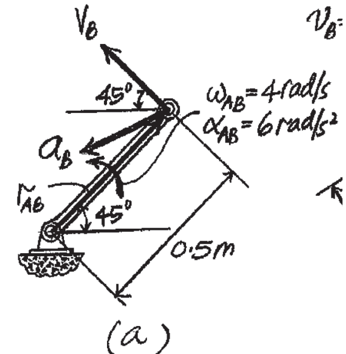
$$-a_C\mathbf{i} = \left(-\frac{\sqrt{3}}{2}\alpha_{BC} - 11.7782\right)\mathbf{i} + (3.3927 - 0.5\alpha_{BC})\mathbf{j}$$

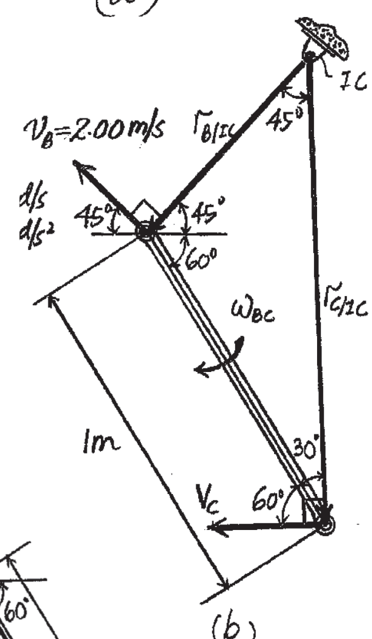
Equating $\mathbf{j}$ components,

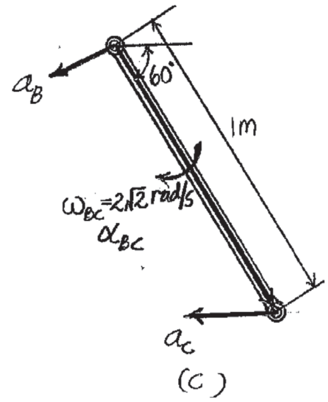
$$0 = 3.3927 - 0.5\alpha_{BC}; \qquad \alpha_{BC} = 6.7853 \text{ rad/s}^2 \circlearrowright$$

Then, $\mathbf{i}$ component gives

$$-a_C = -\frac{\sqrt{3}}{2}(6.7853) - 11.7782; \qquad a_C = 17.65 \text{ m/s}^2 = 17.7 \text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$



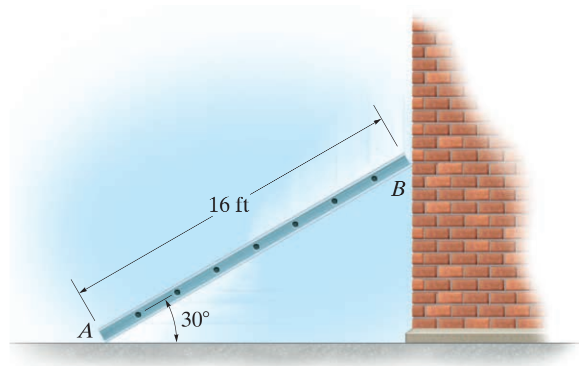




Ans:
$v_C = 3.86 \text{ m/s} \leftarrow$
$a_C = 17.7 \text{ m/s}^2 \leftarrow$

***16–104.**

At a given instant the bottom A of the ladder has an acceleration $a_A = 4\text{ ft/s}^2$ and velocity $v_A = 6\text{ ft/s}$, both acting to the left. Determine the acceleration of the top of the ladder, B, and the ladder's angular acceleration at this same instant.



SOLUTION

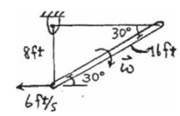
$$\omega = \frac{6}{8} = 0.75\text{ rad/s}$$

$$a_B = a_A + (a_{B/A})_n + (a_{B/A})_t$$

$$\underset{\downarrow}{a_B} = \underset{\leftarrow}{4} + \underset{30°\swarrow}{(0.75)^2(16)} + \underset{30°\nwarrow}{\alpha(16)}$$

$$(\overset{+}{\leftarrow})\qquad 0 = 4 + (0.75)^2(16)\cos 30° - \alpha(16)\sin 30°$$

$$(+\downarrow)\qquad a_B = 0 + (0.75)^2(16)\sin 30° + \alpha(16)\cos 30°$$

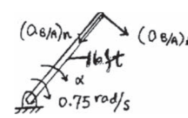


Solving,

$$\alpha = 1.47\text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$a_B = 24.9\text{ ft/s}^2\downarrow \qquad \textbf{Ans.}$$

Also:



$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\mathbf{r}_{B/A}$$

$$-a_B\mathbf{j} = -4\mathbf{i} + (\alpha\mathrm{k}) \times (16\cos 30°\mathbf{i} + 16\sin 30°\mathbf{j}) - (0.75)^2(16\cos 30°\mathbf{i} + 16\sin 30°\mathbf{j})$$

$$0 = -4 - 8\alpha - 7.794$$

$$-a_B = 13.856\alpha - 4.5$$

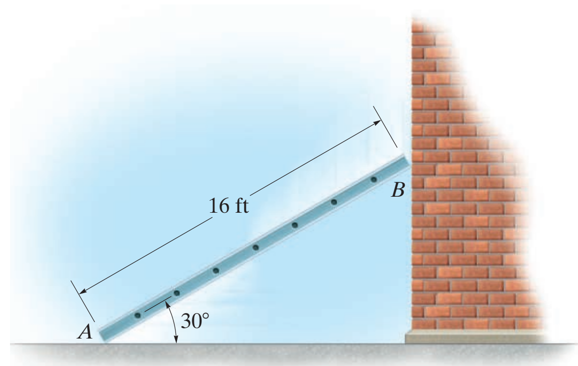
$$\alpha = 1.47\text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$a_B = 24.9\text{ ft/s}^2\downarrow \qquad \textbf{Ans.}$$

Ans:
$\alpha = 1.47\text{ rad/s}^2$
$a_B = 24.9\text{ ft/s}^2\downarrow$
$\alpha = 1.47\text{ rad/s}^2$
$a_B = 24.9\text{ ft/s}^2\downarrow$

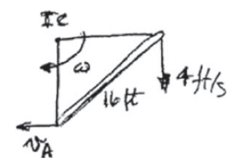
16–105.

At a given instant the top B of the ladder has an acceleration $a_B = 2\ \text{ft/s}^2$ and a velocity of $v_B = 4\ \text{ft/s}$, both acting downward. Determine the acceleration of the bottom A of the ladder, and the ladder's angular acceleration at this instant.



SOLUTION

$$\omega = \frac{4}{16 \cos 30°} = 0.288675\ \text{rad/s}$$



$$\mathbf{a}_A = \mathbf{a}_B + \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$$

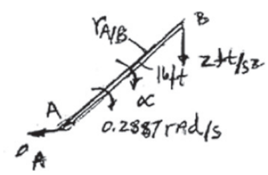
$$-a_A \mathbf{i} = -2\mathbf{j} + (\alpha \mathbf{k}) \times (-16 \cos 30° \mathbf{i} - 16 \sin 30° \mathbf{j}) - (0.288675)^2(-16 \cos 30° \mathbf{i} - 16 \sin 30° \mathbf{j})$$

$$-a_A = 8\alpha + 1.1547$$

$$0 = -2 - 13.856\,\alpha + 0.6667$$

$$\alpha = -0.0962\ \text{rad/s}^2 = 0.0962\ \text{rad/s}^2 \circlearrowright$$ **Ans.**

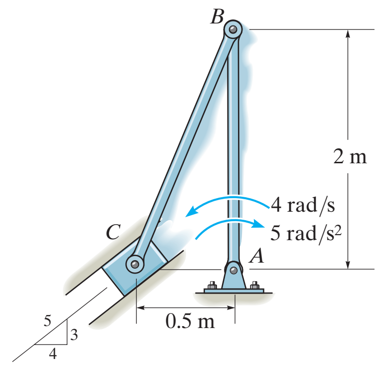
$$a_A = -0.385\ \text{ft/s}^2 = 0.385\ \text{ft/s}^2 \rightarrow$$ **Ans.**



Ans:
$\alpha = 0.0962\ \text{rad/s}^2 \circlearrowright$
$a_A = 0.385\ \text{ft/s}^2 \rightarrow$

16–106.

Member AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



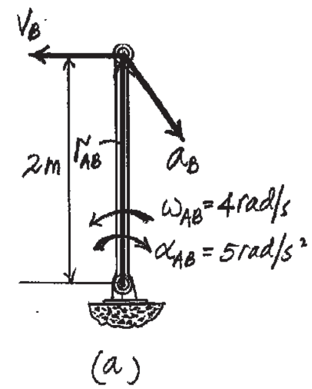
SOLUTION

Rotation About A Fixed Axis. For member AB, refer to Fig. *a*.

$$v_B = \omega_{AB} r_{AB} = 4(2) = 8 \text{ m/s} \leftarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (-5\mathbf{k}) \times (2\mathbf{j}) - 4^2(2\mathbf{j}) = \{10\mathbf{i} - 32\mathbf{j}\} \text{ m/s}^2$$



General Plane Motion. The IC for member BC can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. *b*. From the geometry of this figure

$$\phi = \tan^{-1}\left(\frac{3}{4}\right) = 36.87° \qquad \theta = 90° - \phi = 53.13°$$

Then

$$\frac{r_{B/IC} - 2}{0.5} = \tan 53.13; \qquad r_{B/IC} = 2.6667 \text{ m}$$

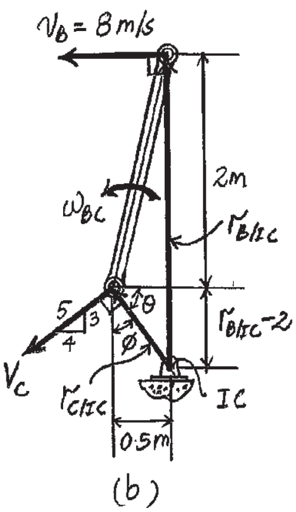
$$\frac{0.5}{r_{C/IC}} = \cos 53.13; \qquad r_{C/IC} = 0.8333 \text{ m}$$

The kinematics gives

$$v_B = \omega_{BC} r_{B/IC}; \qquad 8 = \omega_{BC}(2.6667)$$

$$\omega_{BC} = 3.00 \text{ rad/s} \circlearrowleft$$

$$v_C = \omega_{BC} r_{C/IC} = 3.00(0.8333) = 2.50 \text{ m/s} \swarrow \qquad \textbf{Ans.}$$



Applying the relative acceleration equation by referring to Fig. *c*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-a_C\left(\frac{4}{5}\right)\mathbf{i} - a_C\left(\frac{3}{5}\right)\mathbf{j} = (10\mathbf{i} - 32\mathbf{j}) + \alpha_{BC}\mathbf{k} \times (-0.5\mathbf{i} - 2\mathbf{j}) - \left(3.00^2\right)(-0.5\mathbf{i} - 2\mathbf{j})$$

$$-\frac{4}{5}a_C\mathbf{i} - \frac{3}{5}a_C\mathbf{j} = (2\alpha_{BC} + 14.5)\mathbf{i} + (-0.5\alpha_{BC} - 14)\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components

$$-\frac{4}{5}a_C = 2\alpha_{BC} + 14.5 \qquad \textbf{(1)}$$

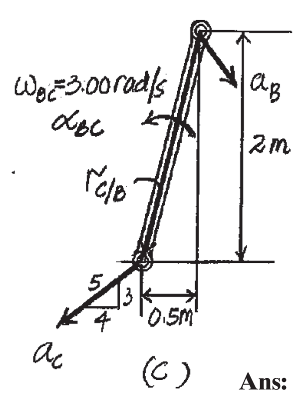
$$-\frac{3}{5}a_C = -0.5\alpha_{BC} - 14 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$a_C = 12.969 \text{ m/s}^2 = 13.0 \text{ m/s}^2 \swarrow \qquad \textbf{Ans.}$$

$$\alpha_{BC} = -12.4375 \text{ rad/s}^2 = 12.4 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

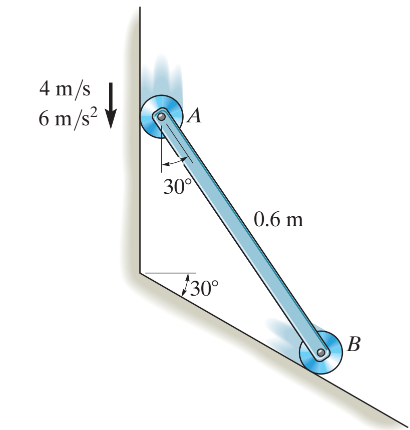
The negative sign indicates that α_{BC} is directed in the opposite sense from what is shown in Fig. (*c*).



Ans:
$a_C = 13.0 \text{ m/s}^2 \swarrow$
$\alpha_{BC} = 12.4 \text{ rad/s}^2 \circlearrowright$

16–107.

At a given instant the roller A on the bar has the velocity and acceleration shown. Determine the velocity and acceleration of the roller B, and the bar's angular velocity and angular acceleration at this instant.



SOLUTION

General Plane Motion. The IC of the bar can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$ as shown in Fig. *a*. From the geometry of this figure,

$$r_{A/IC} = r_{B/IC} = 0.6\text{ m}$$

Thus, the kinematics give

$$v_A = \omega r_{A/IC}; \qquad 4 = \omega(0.6)$$

$$\omega = 6.667\text{ rad/s} = 6.67\text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

$$v_B = \omega r_{B/IC} = 6.667(0.6) = 4.00\text{ m/s} \searrow \qquad \textbf{Ans.}$$

Applying the relative acceleration equation, by referring to Fig. *b*,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$a_B \cos 30°\mathbf{i} - a_B \sin 30°\mathbf{j} = -6\mathbf{j} + (\alpha\mathbf{k}) \times (0.6 \sin 30°\mathbf{i} - 0.6 \cos 30°\mathbf{j})$$
$$- \left(6.667^2\right)(0.6 \sin 30°\mathbf{i} - 0.6 \cos 30°\mathbf{j})$$

$$\frac{\sqrt{3}}{2}a_B\mathbf{i} - \frac{1}{2}a_B\mathbf{j} = \left(0.3\sqrt{3}\alpha - 13.33\right)\mathbf{i} + (0.3\alpha + 17.09)\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components,

$$\frac{\sqrt{3}}{2}a_B = 0.3\sqrt{3}\alpha - 13.33 \qquad \textbf{(1)}$$

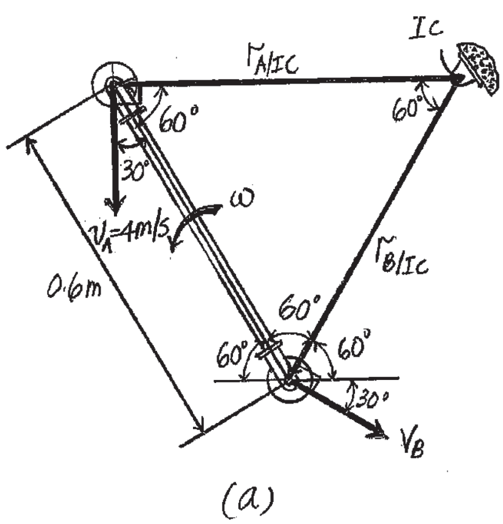
$$-\frac{1}{2}a_B = 0.3\alpha + 17.09 \qquad \textbf{(2)}$$

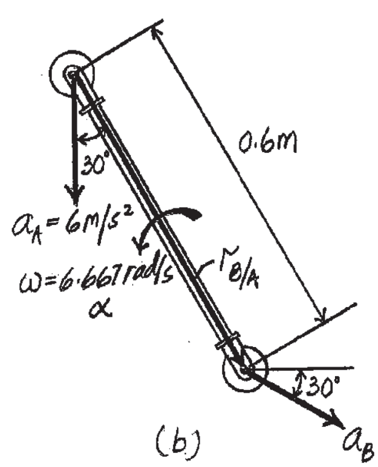
Solving Eqs. (1) and (2)

$$\alpha = -15.66\text{ rad/s}^2 = 15.7\text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$a_B = -24.79\text{ m/s}^2 = 24.8\text{ m/s}^2 \nwarrow \qquad \textbf{Ans.}$$

The negative signs indicate that $\boldsymbol{\alpha}$ and $\mathbf{a}_B$ are directed in the senses that opposite to those shown in Fig. *b*

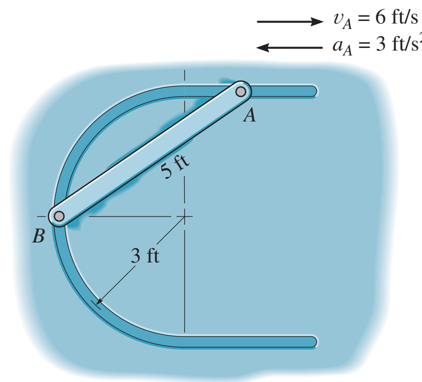




Ans:
$\omega = 6.67\text{ rad/s} \circlearrowright$
$v_B = 4.00\text{ m/s} \searrow$
$\alpha = 15.7\text{ rad/s}^2 \circlearrowright$
$a_B = 24.8\text{ m/s}^2 \nwarrow$

***16–108.**

The rod is confined to move along the path due to the pins at its ends. At the instant shown, point A has the motion shown. Determine the velocity and acceleration of point B at this instant.



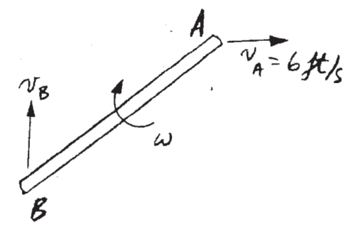
SOLUTION

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

$v_B\,\mathbf{j} = 6\mathbf{i} + (-\omega\mathbf{k}) \times (-4\mathbf{i} - 3\mathbf{j})$

$0 = 6 - 3\omega, \qquad \omega = 2 \text{ rad/s}$

$v_B = 4\omega = 4(2) = 8 \text{ ft/s} \uparrow$ **Ans.**



$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\,\mathbf{r}_{B/A}$

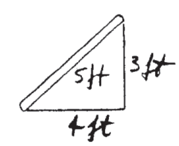
$21.33\mathbf{i} + (a_B)_t\,\mathbf{j} = -3\mathbf{i} + \alpha\mathbf{k} \times (-4\mathbf{i} - 3\mathbf{j}) - (-2)^2\,(-4\mathbf{i} - 3\mathbf{j})$

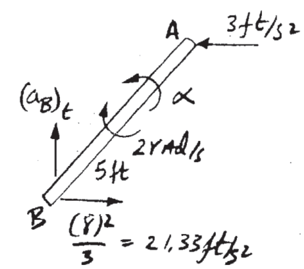
$(\overset{+}{\rightarrow}) \qquad 21.33 = -3 + 3\alpha + 16; \qquad \alpha = 2.778 \text{ rad/s}^2$

$(+\uparrow) \qquad (a_B)_t = -(2.778)(4) + 12 = 0.8889 \text{ ft/s}^2$

$a_B = \sqrt{(21.33)^2 + (0.8889)^2} = 21.4 \text{ ft/s}^2$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{0.8889}{21.33}\right) = 2.39^\circ$ ∠ **Ans.**

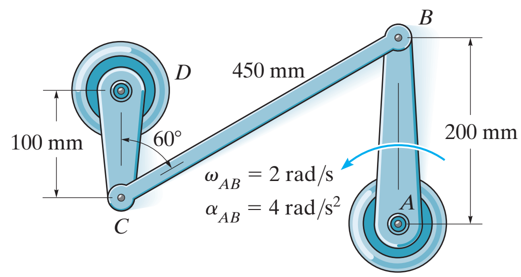




Ans:
$v_B = 8 \text{ ft/s} \uparrow$
$a_B = 21.4 \text{ ft/s}^2$
$\theta = 2.39^\circ$ ∠

16–109.

Member AB has the angular motions shown. Determine the angular velocity and angular acceleration of members CB and DC.



SOLUTION

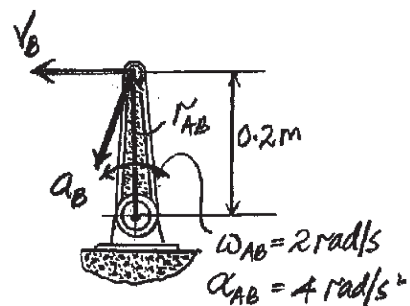
Rotation About A Fixed Axis. For crank AB, refer to Fig. *a*.

$$v_B = \omega_{AB} r_{AB} = 2(0.2) = 0.4 \text{ m/s} \leftarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (4\mathbf{k}) \times (0.2\mathbf{j}) - 2^2(0.2\mathbf{j})$$

$$= \{-0.8\mathbf{i} - 0.8\mathbf{j}\} \text{ m/s}^2$$



(a)

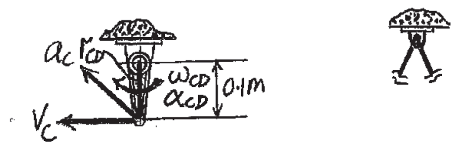
For link CD, refer to Fig. *b*.

$$v_C = \omega_{CD} r_{CD} = \omega_{CD}(0.1)$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$$

$$= (-\alpha_{CD}\mathbf{k}) \times (-0.1\mathbf{j}) - \omega_{CD}^2(-0.1\mathbf{j})$$

$$= -0.1\alpha_{CD}\mathbf{i} + 0.1\omega_{CD}^2\mathbf{j}$$



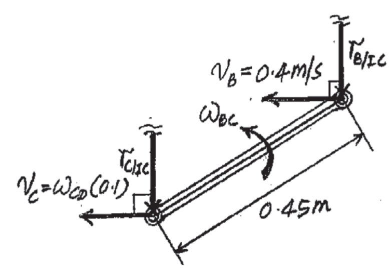
(b)

General Plane Motion. The IC of link CD can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ of which in this case is at infinity as indicated in Fig. *c*. Thus, $r_{B/IC} = r_{C/IC} = \infty$. Thus, kinematics gives

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{0.4}{\infty} = 0 \qquad \textbf{Ans.}$$

Then

$$v_C = v_B; \qquad \omega_{CD}(0.1) = 0.4 \qquad \omega_{CD} = 4.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



(c)

Applying the relative acceleration equation by referring to Fig. *d*,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-0.1\alpha_{CD}\mathbf{i} + 0.1(4.00^2)\mathbf{j} = (-0.8\mathbf{i} - 0.8\mathbf{j}) + (\alpha_{BC}\mathbf{k}) \times (-0.45 \sin 60°\mathbf{i} - 0.45 \cos 60°\mathbf{j}) - \mathbf{0}$$

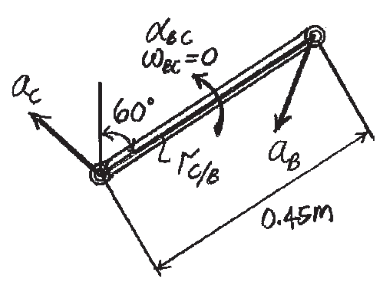
$$-0.1\alpha_{CD}\mathbf{i} + 1.6\mathbf{j} = (0.225\alpha_{BC} - 0.8)\mathbf{i} + (-0.8 - 0.3897\alpha_{BC})\mathbf{j}$$

Equating $\mathbf{j}$ components,

$$1.6 = -0.8 - 0.3897\alpha_{BC}; \qquad \alpha_{BC} = -6.1584 \text{ rad/s}^2 = 6.16 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Then $\mathbf{i}$ components give

$$-0.1\alpha_{CD} = 0.225(-6.1584) - 0.8; \qquad \alpha_{CD} = 21.86 \text{ rad/s}^2 = 21.9 \text{ rad/s}^2 \circlearrowright \quad \textbf{Ans.}$$

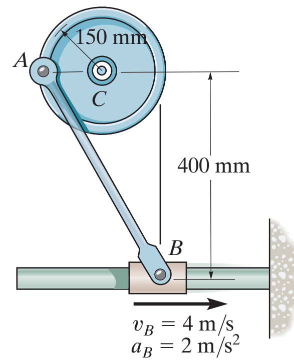


(d)

Ans:
$\omega_{BC} = 0$
$\omega_{CD} = 4.00 \text{ rad/s} \circlearrowright$
$\alpha_{BC} = 6.16 \text{ rad/s}^2 \circlearrowright$
$\alpha_{CD} = 21.9 \text{ rad/s}^2 \circlearrowright$

16–110.

The slider block has the motion shown. Determine the angular velocity and angular acceleration of the wheel at this instant.



SOLUTION

Rotation About A Fixed Axis. For wheel C, refer to Fig. a.

$$v_A = \omega_C r_C = \omega_C(0.15) \downarrow$$

$$\mathbf{a}_A = \boldsymbol{\alpha}_C \times \mathbf{r}_C - \omega_C^2 \mathbf{r}_C$$

$$\mathbf{a}_A = (\alpha_C \mathbf{k}) \times (-0.15\mathbf{i}) - \omega_C^2(-0.15\mathbf{i})$$

$$= 0.15\,\omega_C^2 \mathbf{i} - 0.15\alpha_C \mathbf{j}$$

General Plane Motion. The IC for crank AB can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$ as shown in Fig. b. Here

$$r_{A/IC} = 0.3 \text{ m} \quad r_{B/IC} = 0.4 \text{ m}$$

Then the kinematics gives

$$v_B = \omega_{AB}\, r_{B/IC}; \quad 4 = \omega_{AB}(0.4) \quad \omega_{AB} = 10.0 \text{ rad/s} \circlearrowright$$

$$v_A = \omega_{AB}\, r_{A/IC}; \quad \omega_C(0.15) = 10.0(0.3) \quad \omega_C = 20.0 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. c,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$2\mathbf{i} = 0.15(20.0^2)\mathbf{i} - 0.15\alpha_C \mathbf{j} + (\alpha_{AB}\mathbf{k}) \times (0.3\mathbf{i} - 0.4\mathbf{j}) - 10.0^2(0.3\mathbf{i} - 0.4\mathbf{j})$$

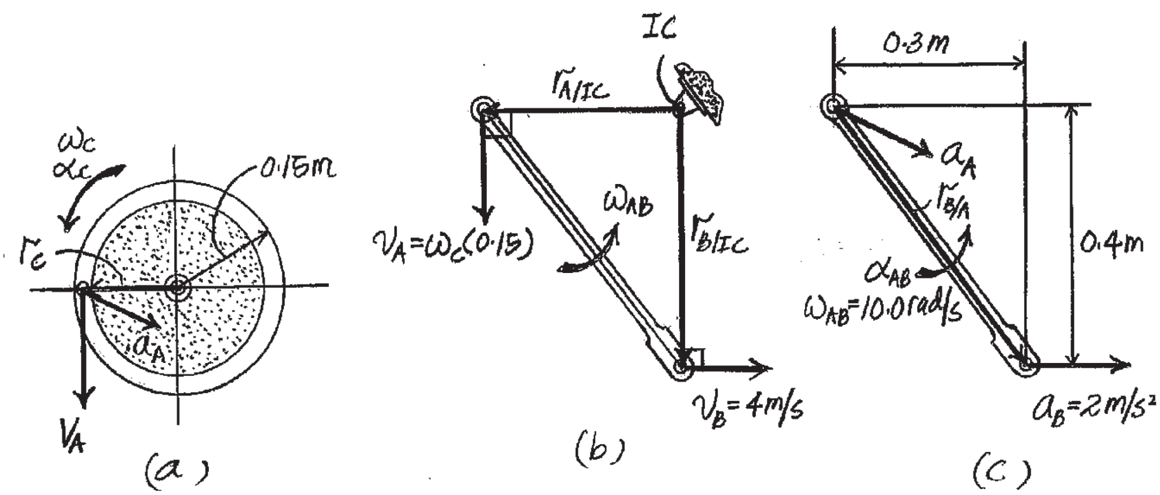
$$2\mathbf{i} = (0.4\alpha_{AB} + 30)\mathbf{i} + (0.3\alpha_{AB} - 0.15\alpha_C + 40)\mathbf{j}$$

Equating **i** and **j** components,

$$2 = 0.4\alpha_{AB} + 30; \quad \alpha_{AB} = -70.0 \text{ rad/s}^2 = 70.0 \text{ rad/s}^2 \circlearrowleft$$

$$0 = 0.3(-70.0) + 0.15\alpha_C + 40; \quad \alpha_C = -126.67 \text{ rad/s}^2 = 127 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

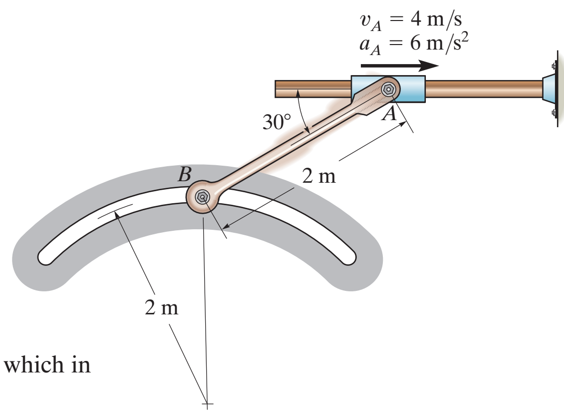
The negative signs indicate that $\boldsymbol{\alpha}_C$ and $\boldsymbol{\alpha}_{AB}$ are directed in the sense that those shown in Fig. a and c.



Ans:
$\omega_C = 20.0$ rad/s $\circlearrowright$
$\alpha_C = 127$ rad/s $\circlearrowleft$

16–111.

At a given instant the slider block A is moving to the right with the motion shown. Determine the angular acceleration of link AB and the acceleration of point B at this instant.



SOLUTION

General Plane Motion. The IC of the link can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$, which in this case is at infinity as shown in Fig. *a*. Thus

$$r_{A/IC} = r_{B/IC} = \infty$$

Then the kinematics gives

$$v_A = \omega\, r_{A/IC}; \quad 4 = \omega\,(\infty) \quad \omega = 0$$

$$v_B = v_A = 4 \text{ m/s}$$

Since B moves along a circular path, its acceleration will have tangential and normal components. Hence $(a_B)_n = \dfrac{v_B^2}{r_B} = \dfrac{4^2}{2} = 8 \text{ m/s}^2$

Applying the relative acceleration equation by referring to Fig. *b*,

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\,\mathbf{r}_{B/A}$$

$$(a_B)_t\mathbf{i} - 8\mathbf{j} = 6\mathbf{i} + (\alpha\mathbf{k}) \times (-2\cos 30°\mathbf{i} - 2\sin 30°\mathbf{j}) - 0$$

$$(a_B)_t\mathbf{i} - 8\mathbf{j} = (\alpha + 6)\mathbf{i} - \sqrt{3}\alpha\mathbf{j}$$

Equating **i** and **j** componenets,

$$-8 = -\sqrt{3}\alpha; \quad \alpha = \frac{8\sqrt{3}}{3}\text{ rad/s}^2 = 4.62 \text{ rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

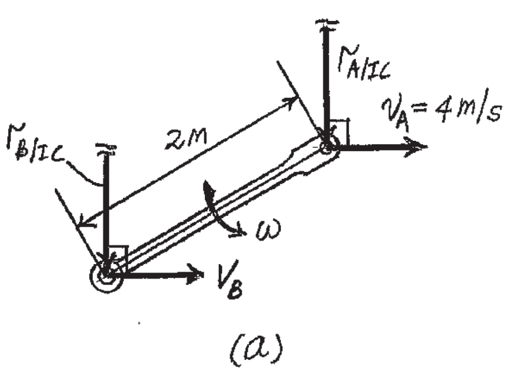
$$(a_B)_t = \alpha + 6; \quad (a_B)_t = \frac{8\sqrt{3}}{3} + 6 = 10.62 \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is

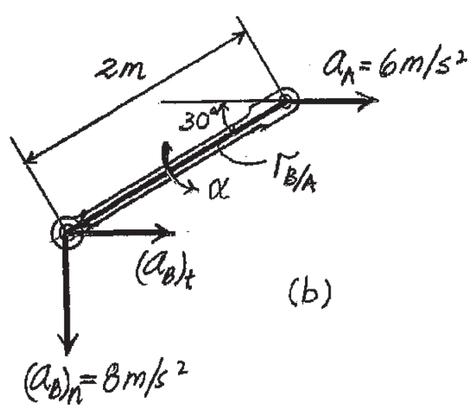
$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{10.62^2 + 8^2} = 13.30 \text{ m/s}^2 = 13.3 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left[\frac{(a_B)_n}{(a_B)_t}\right] = \tan^{-1}\left(\frac{8}{10.62}\right) = 36.99° = 37.0° \qquad \textbf{Ans.}$$



(a)

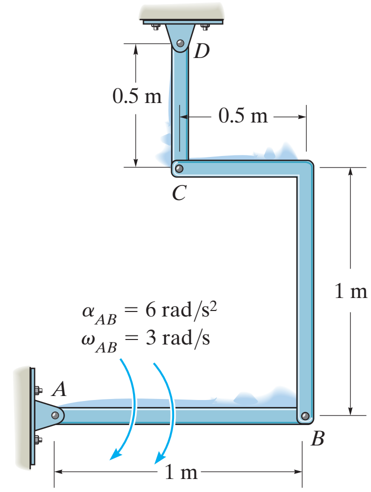


(b)

Ans:
$\alpha_{AB} = 4.62 \text{ rad/s}^2 \circlearrowleft$
$a_B = 13.3 \text{ m/s}^2$
$\theta = 37.0°$

***16–112.**

Determine the angular acceleration of link CD if link AB has the angular velocity and angular acceleration shown.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 3(1) = 3.00 \text{ m/s} \downarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^{\,2}\mathbf{r}_{AB}$$
$$= (-6\mathbf{k}) \times (1\mathbf{i}) - 3^2(1\mathbf{i})$$
$$= \{-9\mathbf{i} - 6\mathbf{j}\}\text{ m/s}$$

For link CD, refer to Fig. b

$$v_C = \omega_{CD} r_{DC} = \omega_{CD}(0.5) \rightarrow$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{DC} - \omega_{CD}^{\,2}\mathbf{r}_{DC}$$
$$= (\alpha_{CD}\mathbf{k}) \times (-0.5\mathbf{j}) - \omega_{CD}^{2}(-0.5\mathbf{j})$$
$$= 0.5\alpha_{CD}\mathbf{i} + 0.5\omega_{CD}^2\mathbf{j}$$

General Plane Motion. The IC of link BC can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$ as shown in Fig. c. Thus

$$r_{B/IC} = 0.5 \text{ m} \qquad r_{C/IC} = 1 \text{ m}$$

Then, the kinematics gives

$$v_B = \omega_{BC}\, r_{B/IC}; \quad 3 = \omega_{BC}(0.5) \quad \omega_{BC} = 6.00 \text{ rad/s} \circlearrowright$$

$$v_C = \omega_{BC}\, r_{C/IC}; \quad \omega_{CD}(0.5) = 6.00(1) \quad \omega_{CD} = 12.0 \text{ rad/s} \circlearrowleft$$

Applying the relative acceleration equation by referring to Fig. d,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times r_{C/B} - \omega_{BC}^{\,2}\,\mathbf{r}_{C/B}$$

$$0.5\alpha_{CD}\mathbf{i} + 0.5(12.0^2)\mathbf{j} = (-9\mathbf{i} - 6\mathbf{j}) + (-\alpha_{BC}\mathbf{k}) \times (-0.5\mathbf{i} + \mathbf{j}) - 6.00^2(-0.5\mathbf{i} + \mathbf{j})$$

$$0.5\alpha_{CD}\mathbf{i} + 72\mathbf{j} = (\alpha_{BC} + 9)\mathbf{i} + (0.5\alpha_{BC} - 42)\mathbf{j}$$

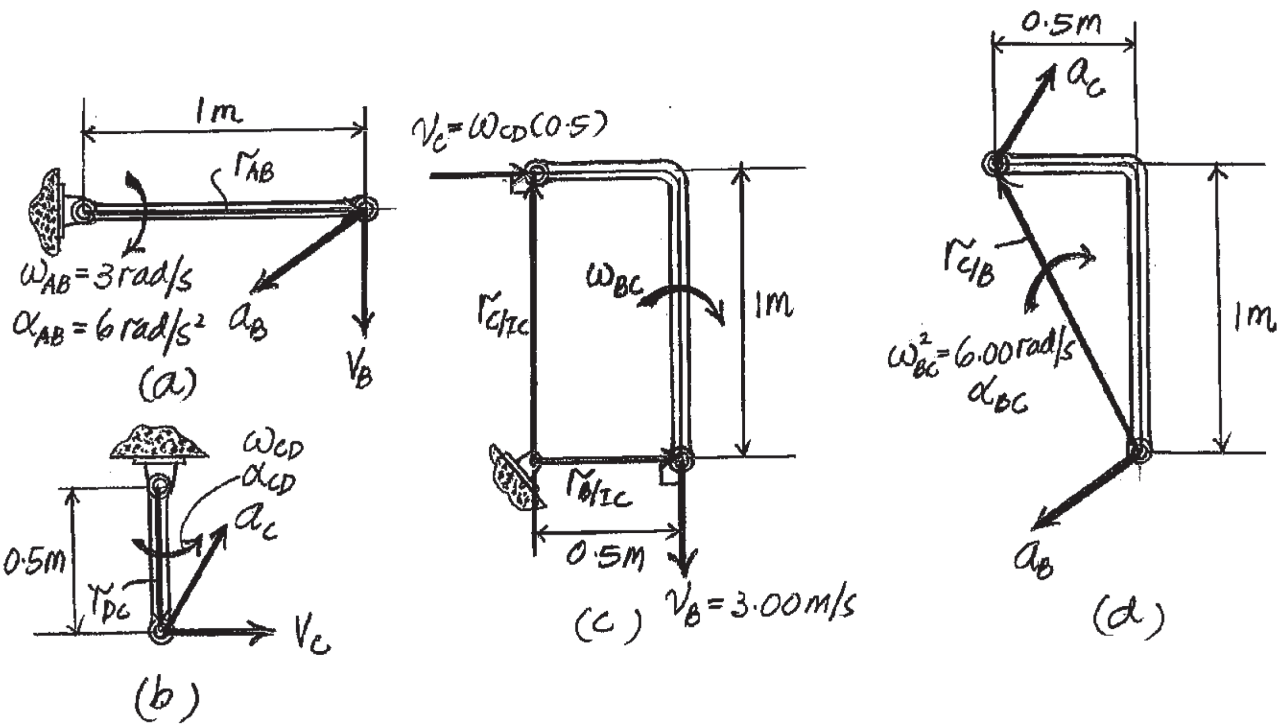
***16–112. Continued**

Equating **j** components,

$$72 = (0.5\alpha_{BC} - 42); \quad \alpha_{BC} = 228 \text{ rad/s}^2 \circlearrowright$$

Then **i** component gives

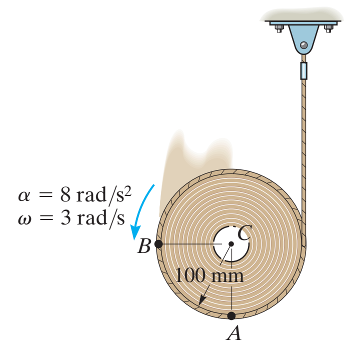
$$0.5\alpha_{CD} = 228 + 9; \quad \alpha_{CD} = 474 \text{ rad/s}^2 \circlearrowleft$$ **Ans.**



Ans:
$\alpha_{CD} = 474 \text{ rad/s}^2 \circlearrowleft$

16–113.

The reel of rope has the angular motion shown. Determine the velocity and acceleration of point A at the instant shown.



SOLUTION

General Plane Motion. The IC of the reel is located as shown in Fig. a. Here,

$$r_{A/IC} = \sqrt{0.1^2 + 0.1^2} = 0.1414 \text{ m}$$

Then, the Kinematics give

$$v_A = \omega r_{A/IC} = 3(0.1414) = 0.4243 \text{ m/s} = 0.424 \text{ m/s} \;\searrow 45^\circ \qquad \textbf{Ans.}$$

Here $a_C = \alpha r = 8(0.1) = 0.8 \text{ m/s}^2 \downarrow$. Applying the relative acceleration equation by referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{A/C} - \omega^2 \mathbf{r}_{A/C}$$

$$\mathbf{a}_A = -0.8\mathbf{j} + (8\mathbf{k}) \times (-0.1\mathbf{j}) - 3^2(-0.1\mathbf{j})$$

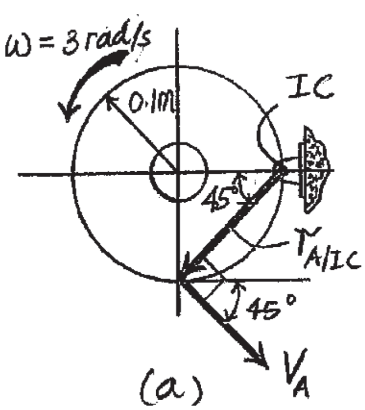
$$= \{0.8\mathbf{i} + 0.1\mathbf{j}\} \text{ m/s}^2$$

The magnitude of $\mathbf{a}_A$ is

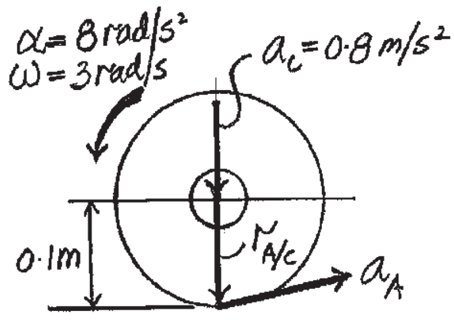
$$a_A = \sqrt{0.8^2 + 0.1^2} = 0.8062 \text{ m/s}^2 = 0.806 \text{ m/s}^2 \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{0.1}{0.8}\right) = 7.125^\circ = 7.13^\circ \measuredangle \qquad \textbf{Ans.}$$



(a)

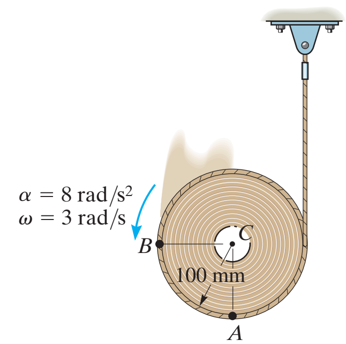


(b)

Ans:
$v_A = 0.424 \text{ m/s}$
$\theta_v = 45^\circ \searrow$
$a_A = 0.806 \text{ m/s}^2$
$\theta_a = 7.13^\circ \measuredangle$

16–114.

The reel of rope has the angular motion shown. Determine the velocity and acceleration of point B at the instant shown.



SOLUTION

General Plane Motion. The *IC* of the reel is located as shown in Fig. *a*. Here, $r_{B/FC} = 0.2$ m. Then the kinematics gives

$$v_B = \omega r_{B/IC} = (3)(0.2) = 0.6\ \mathrm{m/s}\downarrow \qquad \textbf{Ans.}$$

Here, $a_C = \alpha r = 8(0.1) = 0.8\ \mathrm{m/s^2}\downarrow$. Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

$$\mathbf{a}_B = -0.8\mathbf{j} + (8\mathbf{k}) \times (-0.1\mathbf{i}) - 3^2(-0.1\mathbf{i})$$

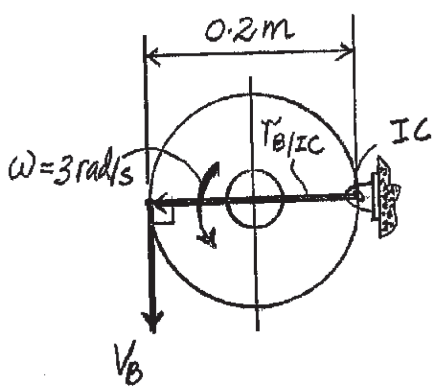
$$= \{0.9\mathbf{i} - 1.6\mathbf{j}\}\ \mathrm{m/s^2}$$

The magnitude of $\mathbf{a}_B$ is

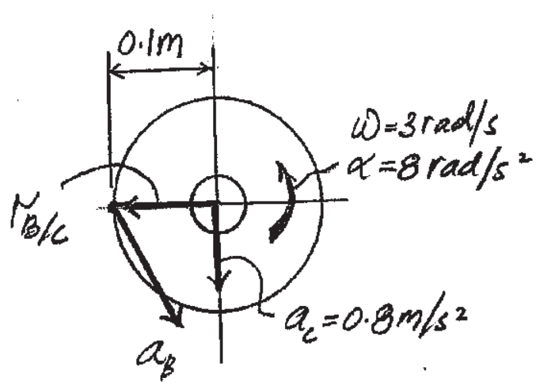
$$a_B = \sqrt{0.9^2 + (-1.6)^2} = 1.8358\ \mathrm{m/s^2} = 1.84\ \mathrm{m/s^2} \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{1.6}{0.9}\right) = 60.64° = 60.6° \measuredangle \qquad \textbf{Ans.}$$



(a)

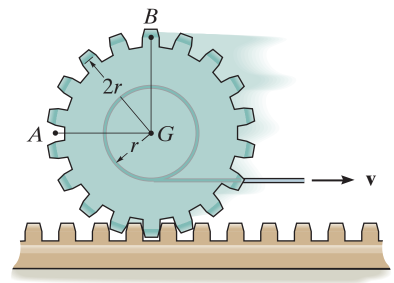


(b)

Ans:
$v_B = 0.6\ \mathrm{m/s}\downarrow$
$a_B = 1.84\ \mathrm{m/s^2}$
$\theta = 60.6° \measuredangle$

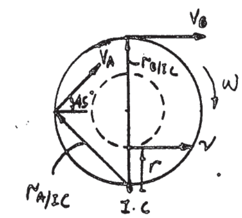
16–115.

A cord is wrapped around the inner spool of the gear. If it is pulled with a constant velocity **v**, determine the velocities and accelerations of points A and B. The gear rolls on the fixed gear rack.



SOLUTION

Velocity analysis:

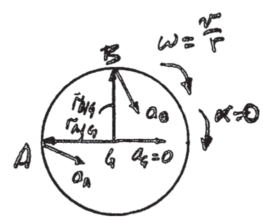


$$\omega = \frac{v}{r}$$

$$v_B = \omega r_{B/IC} = \frac{v}{r}(4r) = 4v \rightarrow$$ **Ans.**

$$v_A = \omega\, r_{A/IC} = \frac{v}{r}\left(\sqrt{(2r)^2 + (2r)^2}\right) = 2\sqrt{2}v \quad \measuredangle 45^\circ$$ **Ans.**

Acceleration equation: From Example 16–3, Since $a_G = 0, \alpha = 0$



$$\mathbf{r}_{B/G} = 2r\,\mathbf{j} \qquad \mathbf{r}_{A/G} = -2r\,\mathbf{i}$$

$$\mathbf{a}_B = \mathbf{a}_G + \alpha \times \mathbf{r}_{B/G} - \omega^2 \mathbf{r}_{B/G}$$

$$= 0 + 0 - \left(\frac{v}{r}\right)^2 (2r\mathbf{j}) = -\frac{2v^2}{r}\mathbf{j}$$

$$a_B = \frac{2v^2}{r} \downarrow$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_G + \alpha \times \mathbf{r}_{A/G} - \omega^2 \mathbf{r}_{A/G}$$

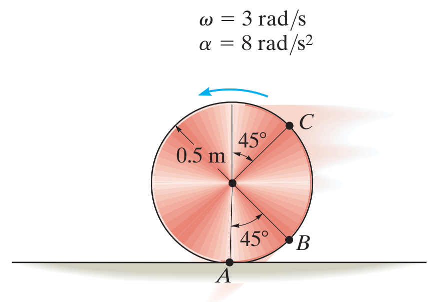
$$= 0 + 0 - \left(\frac{v}{r}\right)^2 (-2r\mathbf{i}) = \frac{2v^2}{r}\mathbf{i}$$

$$a_A = \frac{2v^2}{r} \rightarrow$$ **Ans.**

Ans:
$v_B = 4v \rightarrow$
$v_A = 2\sqrt{2}v$
$\theta = 45^\circ \measuredangle$
$a_B = \frac{2v^2}{r} \downarrow$
$a_A = \frac{2v^2}{r} \rightarrow$

***16–116.**

The disk has an angular acceleration $\alpha = 8\ \text{rad/s}^2$ and angular velocity $\omega = 3\ \text{rad/s}$ at the instant shown. If it does not slip at A, determine the acceleration of point B.



SOLUTION

General Plane Motion. Since the disk rolls without slipping, $a_O = \alpha r = 8(0.5) = 4\ \text{m/s}^2 \leftarrow$. Applying the relative acceleration equation by referring to Fig. a,

$$\mathbf{a}_B = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{B/O} - \omega^2 r_{B/O}$$

$$\mathbf{a}_B = (-4\mathbf{i}) + (8\mathbf{k}) \times (0.5 \sin 45°\mathbf{i} - 0.5 \cos 45°\mathbf{j}) - 3^2(0.5 \sin 45°\mathbf{i} - 0.5 \cos 45°\mathbf{j})$$

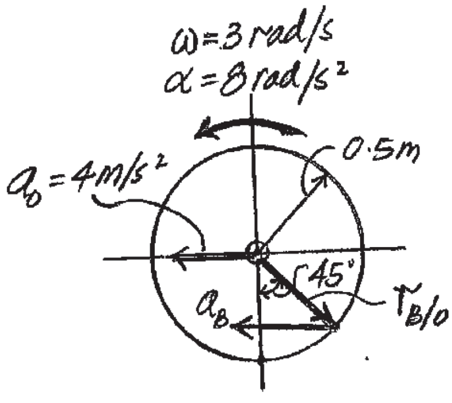
$$a_B = \{-4.354\mathbf{i} + 6.010\mathbf{j}\}\ \text{m/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$ is

$$a_B = \sqrt{(-4.354)^2 + 6.010^2} = 7.4215\ \text{m/s}^2 = 7.42\ \text{m/s}^2 \qquad \textbf{Ans.}$$

And its direction is given by

$$\theta = \tan^{-1}\left(\frac{6.010}{4.354}\right) = 54.08° = 54.1° \measuredangle \qquad \textbf{Ans.}$$

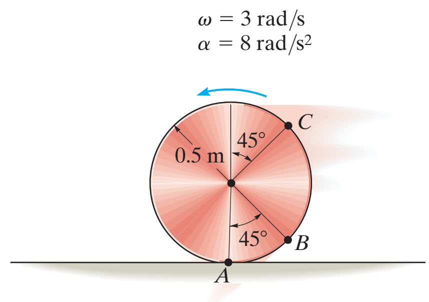


(a)

Ans:
$a_B = 7.42\ \text{m/s}^2$
$\theta = 54.1° \measuredangle$

16–117.

The disk has an angular acceleration $\alpha = 8 \text{ rad/s}^2$ and angular velocity $\omega = 3$ rad/s at the instant shown. If it does not slip at A, determine the acceleration of point C.



SOLUTION

General Plane Motion. Since the disk rolls without slipping, $a_O = \alpha r = 8(0.5) = 4 \text{ m/s}^2 \leftarrow$. Applying the relative acceleration equation by referring to Fig. *a*,

$$\mathbf{a}_C = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{C/O} - \omega^2 \mathbf{r}_{C/O}$$

$$\mathbf{a}_C = (-4\mathbf{i}) + (8\mathbf{k}) \times (0.5 \sin 45°\mathbf{i} + 0.5 \cos 45°\mathbf{j}) - 3^2(0.5 \sin 45°\mathbf{i} + 0.5 \cos 45°\mathbf{j})$$

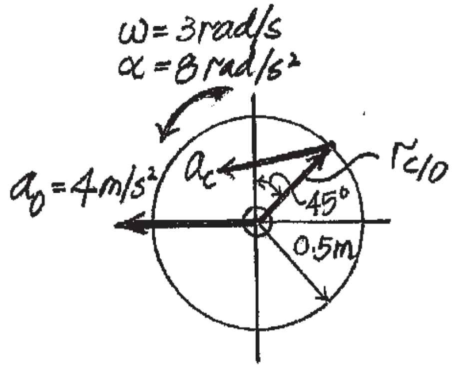
$$a_C = \{-10.0104\mathbf{i} - 0.3536\mathbf{j}\} \text{ m/s}^2$$

Thus, the magnitude of $\mathbf{a}_C$ is

$$a_C = \sqrt{(-10.0104)^2 + (-0.3536)^2} = 10.017 \text{ m/s}^2 = 10.0 \text{ m/s}^2 \quad \textbf{Ans.}$$

And its direction is defined by

$$\theta = \tan^{-1}\left\{\frac{0.3536}{10.0104}\right\} = 2.023° = 2.02° \quad \textbf{Ans.}$$

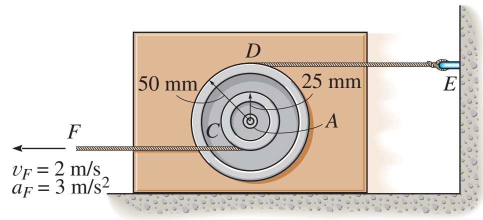


(a)

Ans:
$a_C = 10.0 \text{ m/s}^2$
$\theta = 2.02°$

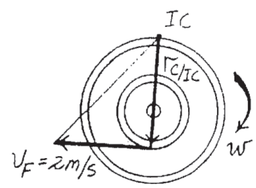
16–118.

A single pulley having both an inner and outer rim is pin-connected to the block at A. As cord CF unwinds from the inner rim of the pulley with the motion shown, cord DE unwinds from the outer rim. Determine the angular acceleration of the pulley and the acceleration of the block at the instant shown.



SOLUTION

Velocity Analysis: The angular velocity of the pulley can be obtained by using the method of instantaneous center of zero velocity. Since the pulley rotates without slipping about point D, i.e: $v_D = 0$, then point D is the location of the instantaneous center.

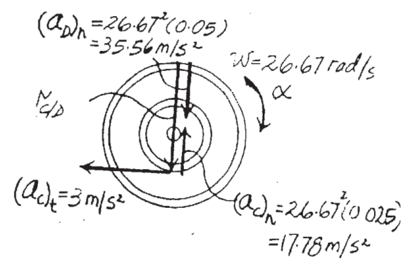


$$v_F = \omega r_{C/IC}$$

$$2 = \omega(0.075)$$

$$\omega = 26.67 \text{ rad/s}$$

Acceleration Equation: The angular acceleration of the gear can be obtained by analyzing the angular motion points C and D. Applying Eq. 16–18 with $\mathbf{r}_{C/D} = \{-0.075\mathbf{j}\}$ m, we have



$$\mathbf{a}_C = \mathbf{a}_D + \alpha \times \mathbf{r}_{C/D} - \omega^2 \mathbf{r}_{C/D}$$

$$-3\mathbf{i} + 17.78\mathbf{j} = -35.56\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.075\mathbf{j}) - 26.67^2\,(-0.075\mathbf{j})$$

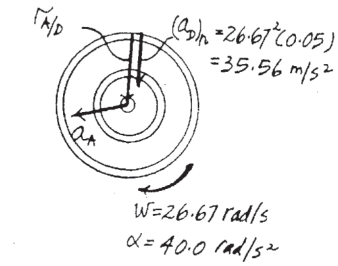
$$-3\mathbf{i} + 17.78\mathbf{j} = -0.075\,\alpha\mathbf{i} + 17.78\mathbf{j}$$

Equating **i** and **j** components, we have

$$-3 = -0.075\alpha \qquad \alpha = 40.0 \text{ rad/s}^2$$ **Ans.**

$$17.78 = 17.78 \ (\textit{Check!})$$

The acceleration of point A can be obtained by analyzing the angular motion points A and D. Applying Eq. 16–18 with $\mathbf{r}_{A/D} = \{-0.05\mathbf{j}\}$ m. we have



$$\mathbf{a}_A = \mathbf{a}_D + \alpha \times \mathbf{r}_{A/D} - \omega^2 \mathbf{r}_{A/D}$$

$$= -35.56\mathbf{j} + (-40.0\mathbf{k}) \times (-0.05\mathbf{j}) - 26.67^2(-0.05\mathbf{j})$$

$$= \{-2.00\mathbf{i}\} \text{ m/s}^2$$

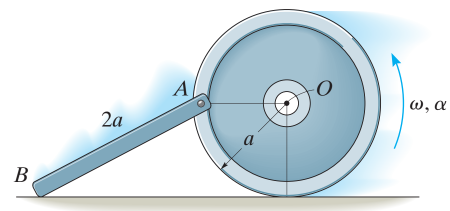
Thus,

$$a_A = 2.00 \text{ m/s}^2 \leftarrow$$ **Ans.**

Ans:
$\alpha = 40.0$ rad/s²
$a_A = 2.00$ m/s² $\leftarrow$

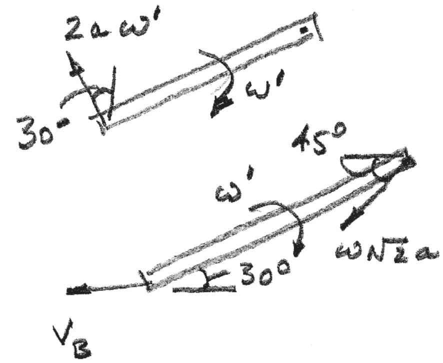
16–119.

The wheel rolls without slipping such that at the instant shown it has an angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$. Determine the velocity and acceleration of point B on the rod at this instant.



SOLUTION

$\bar{v}_B = \bar{v}_A + \bar{v}_{B/A\,(Pin)}$



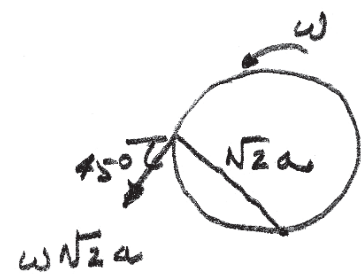
$\overset{+}{\leftarrow}\ v_B = \dfrac{1}{\sqrt{2}}\left(\omega\sqrt{2}a\right) + 2a\omega'\left(\dfrac{1}{2}\right)$

$+\uparrow\ O = -\dfrac{1}{\sqrt{2}}\left(\omega\sqrt{2}a\right) + 2a\omega'\left(\dfrac{\sqrt{3}}{2}\right)$

$\omega' = \dfrac{\omega}{\sqrt{3}}$

$v_B = 1.58\,\omega a$ **Ans.**

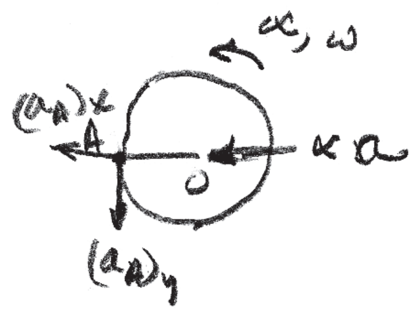
$\bar{a}_A = \bar{a}_O + \bar{a}_{A/O\,(Pin)}$



$$\underset{\leftarrow}{(a_A)_x} + \underset{\downarrow}{(a_A)_y} = \underset{\leftarrow}{\alpha a} + \underset{\downarrow}{\alpha(a)} + \underset{\rightarrow}{\omega^2 a}$$

$(a_A)_x = \alpha a - \omega^2 a$

$(a_A)_y = \alpha a$



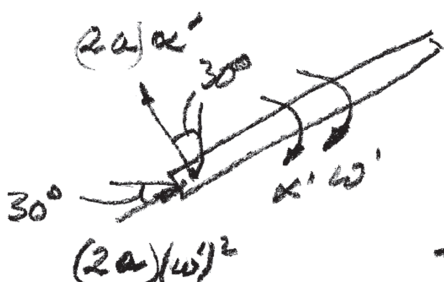
$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A\,(Pin)}$

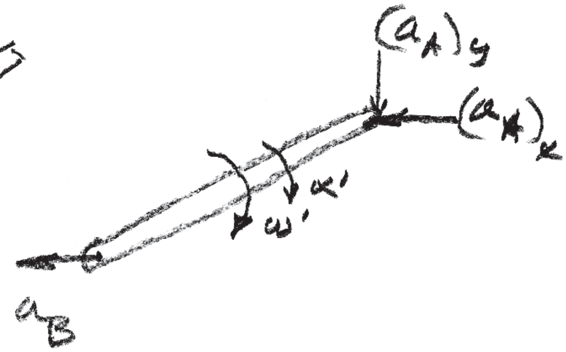
$a_B = \alpha a - \omega^2 a + 2a(\alpha')\left(\dfrac{1}{2}\right) - 2a\left(\dfrac{\omega}{\sqrt{3}}\right)^2 \dfrac{\sqrt{3}}{2}$

$O = -\alpha a + 2a\alpha'\left(\dfrac{2}{\sqrt{3}}\right) + 2a\left(\dfrac{\omega}{\sqrt{3}}\right)^2\left(\dfrac{1}{2}\right)$

$\alpha' = 0.577\alpha - 0.1925\omega^2$

$a_B = 1.58\alpha a - 1.77\omega^2 a$ **Ans.**

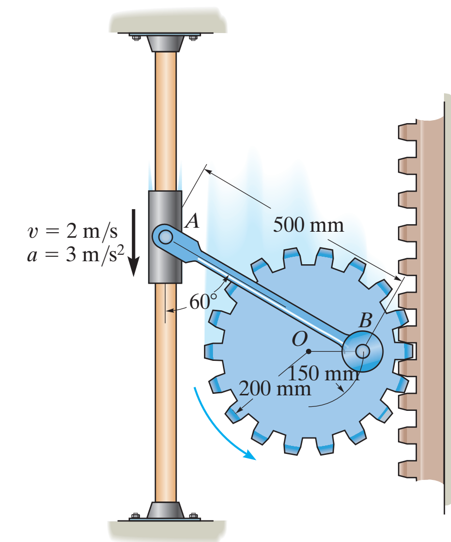




Ans:
$v_B = 1.58\omega a$
$a_B = 1.58\,\alpha a - 1.77\omega^2 a$

***16–120.**

The collar is moving downward with the motion shown. Determine the angular velocity and angular acceleration of the gear at the instant shown as it rolls along the fixed gear rack.



SOLUTION

General Plane Motion. For gear *C*, the location of its *IC* is indicate in Fig. *a*. Thus

$$v_B = \omega_C\, r_{B/(IC)_1} = \omega_C(0.05) \downarrow \qquad \textbf{(1)}$$

The *IC* of link *AB* can be located using $\mathbf{v}_A$ and $\mathbf{v}_B$, which in this case is at infinity. Thus

$$\omega_{AB} = \frac{v_A}{r_{A/(IC)_2}} = \frac{2}{\infty} = 0$$

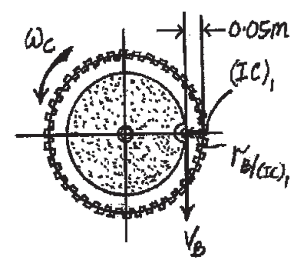
Then

$$v_B = v_A = 2 \text{ m/s} \downarrow$$

Substitute the result of v_B into Eq. (1)

$$2 = \omega_C(0.05)$$

$$\omega_C = 40.0 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$



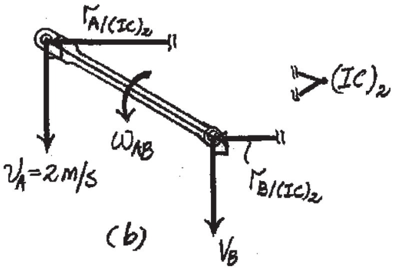
(a)

Applying the relative acceleration equation to gear *C*, Fig. *c*, with $a_O = \alpha_C r_C = \alpha_C(0.2) \downarrow$,

$$\mathbf{a}_B = \mathbf{a}_O + \boldsymbol{\alpha}_C \times \mathbf{r}_{B/O} - \omega_C^2\, \mathbf{r}_{B/O}$$

$$\mathbf{a}_B = -\alpha_C(0.2)\mathbf{j} + (\alpha_C\mathbf{k}) \times (0.15\mathbf{i}) - 40.0^2(0.15\mathbf{i})$$

$$= -240\mathbf{i} - 0.05\alpha_C\mathbf{j}$$



(b)

For link *AB*, Fig. *d*,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\, \mathbf{r}_{B/A}$$

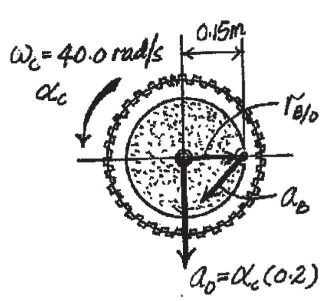
$$-240\mathbf{i} - 0.05\alpha_C\mathbf{j} = (-3\mathbf{j}) + (\alpha_{AB}\mathbf{k}) \times (0.5 \sin 60°\mathbf{i} - 0.5 \cos 60°\mathbf{j}) - \mathbf{0}$$

$$-240\mathbf{i} - 0.05\alpha_C = 0.25\alpha_{AB}\mathbf{i} + (0.25\sqrt{3}\alpha_{AB} - 3)\mathbf{j}$$

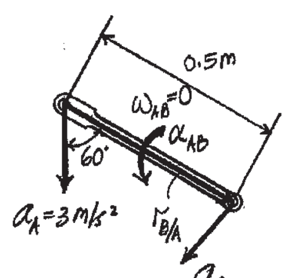
Equating **i** and **j** components

$$-240 = 0.25\alpha_{AB}; \quad \alpha_{AB} = -960 \text{ rad/s}^2 = 960 \text{ rad/s}^2 \circlearrowleft$$

$$-0.05\alpha_C = \left(0.25\sqrt{3}\right)(-960) - 3; \quad \alpha_C = 8373.84 \text{ rad/s}^2 = 8374 \text{ rad/s}^2 \circlearrowright \quad \textbf{Ans.}$$



(c)

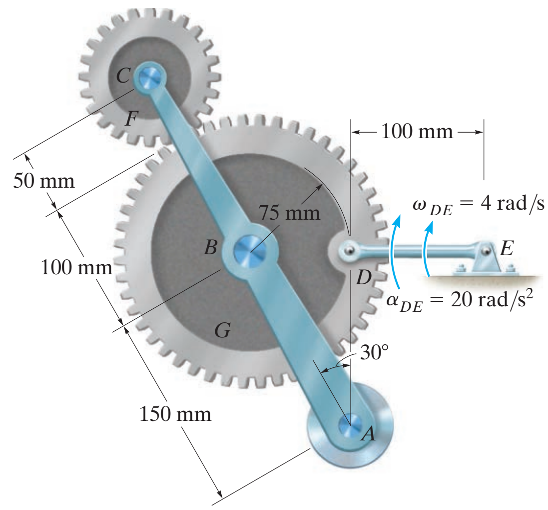


(d)

Ans:
$\omega_C = 40.0$ rad/s $\circlearrowright$
$\alpha_C = 8374$ rad/s² $\circlearrowright$

16–121.

The tied crank and gear mechanism gives rocking motion to crank AC, necessary for the operation of a printing press. If link DE has the angular motion shown, determine the respective angular velocities of gear F and crank AC at this instant, and the angular acceleration of crank AC.



SOLUTION

Velocity analysis:

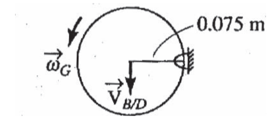
$$v_D = \omega_{DE} r_{D/E} = 4(0.1) = 0.4 \text{ m/s} \uparrow$$

$$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D}$$

$$\underset{\measuredangle 30^\circ}{v_B} = \underset{\uparrow}{0.4} + \underset{\downarrow}{(\omega_G)(0.075)}$$

$$(\overset{+}{\rightarrow}) \qquad v_B \cos 30^\circ = 0, \qquad v_B = 0$$

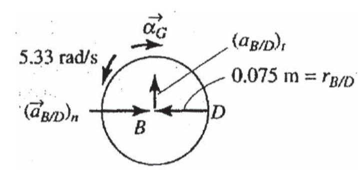
$$(+\uparrow) \quad \omega_G = 5.33 \text{ rad/s}$$



Since $v_B = 0, \qquad v_C = 0, \qquad \omega_{AC} = 0$ **Ans.**

$$\omega_F r_F = \omega_G r_G$$

$$\omega_F = 5.33\left(\frac{100}{50}\right) = 10.7 \text{ rad/s}$$ **Ans.**



Acceleration analysis:

$$(a_D)_n = (4)^2(0.1) = 1.6 \text{ m/s}^2 \rightarrow$$

$$(a_D)_t = (20)(0.1) = 2 \text{ m/s}^2 \uparrow$$

$$(\mathbf{a}_B)_n + (\mathbf{a}_B)_t = (\mathbf{a}_D)_n + (\mathbf{a}_D)_t + (\mathbf{a}_{B/D})_n + (\mathbf{a}_{B/D})_t$$

$$0 + \underset{\measuredangle 30^\circ}{(a_B)_t} = \underset{\rightarrow}{1.6} + \underset{\uparrow}{2} + \underset{\rightarrow}{(5.33)^2(0.075)} + \underset{\uparrow}{\alpha_G(0.075)}$$

$$(+\uparrow) \quad (a_B)_t \sin 30^\circ = 0 + 2 + 0 + \alpha_G(0.075)$$

$$(\overset{+}{\rightarrow}) \quad (a_B)_t \cos 30^\circ = 1.6 + 0 + (5.33)^2(0.075) + 0$$

Solving,

$$(a_B)_t = 4.31 \text{ m/s}^2, \qquad \alpha_G = 2.052 \text{ rad/s}^2 \circlearrowright$$

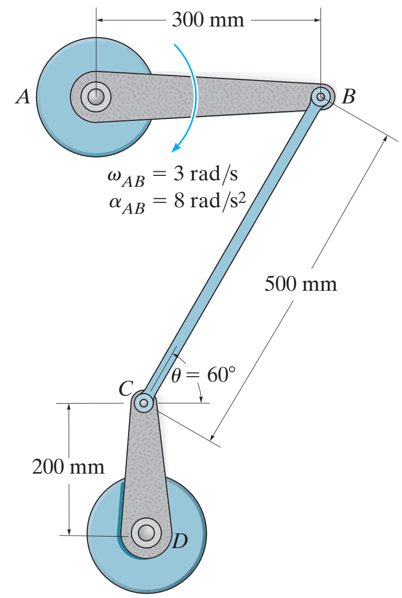
Hence,

$$\alpha_{AC} = \frac{(a_B)_t}{r_{B/A}} = \frac{4.31}{0.15} = 28.7 \text{ rad/s}^2 \circlearrowright$$ **Ans.**

Ans:
$\omega_{AC} = 0$
$\omega_F = 10.7 \text{ rad/s} \circlearrowright$
$\alpha_{AC} = 28.7 \text{ rad/s}^2 \circlearrowright$

16–122.

If member AB has the angular motion shown, determine the angular velocity and angular acceleration of member CD at the instant shown.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 3(0.3) = 0.9 \text{ m/s} \downarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (-8\mathbf{k}) \times (0.3\mathbf{i}) - 3^2(0.3\mathbf{i})$$

$$= \{-2.70\mathbf{i} - 2.40\mathbf{j}\} \text{ m/s}^2$$

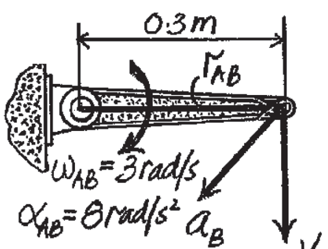
For link CD, refer to Fig. b.

$$v_C = \omega_{CD} r_{CD} = \omega_{CD}(0.2) \leftarrow$$

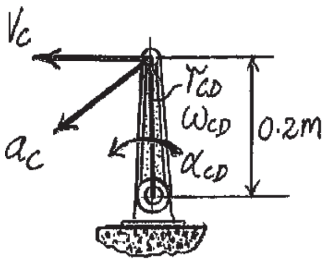
$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$$

$$\mathbf{a}_C = (\alpha_{CD}\mathbf{k}) \times (0.2\mathbf{j}) - \omega_{CD}^2(0.2\mathbf{j})$$

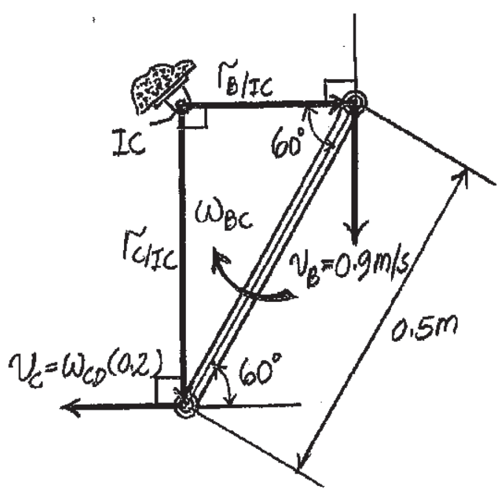
$$= -0.2\alpha_{CD}\mathbf{i} - 0.2\omega_{CD}^2\mathbf{j}$$



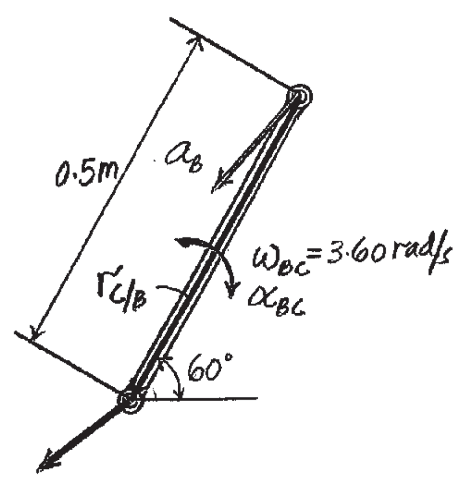
(a)



(b)



(c)



(d)

16–122. Continued

General Plane Motion. The IC of link BC can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. c. From the geometry of this figure,

$$r_{B/IC} = 0.5 \cos 60^\circ = 0.25 \text{ m} \qquad r_{C/IC} = 0.5 \sin 60^\circ = 0.25\sqrt{3} \text{ m}$$

Then kinematics gives

$$v_B = \omega_{BC} r_{B/IC}; \qquad 0.9 = \omega_{BC}(0.25) \qquad \omega_{BC} = 3.60 \text{ rad/s} \circlearrowright$$

$$v_C = \omega_{BC} r_{C/IC}; \qquad \omega_{CD}(0.2) = (3.60)\left(0.25\sqrt{3}\right)$$

$$\omega_{CD} = 7.7942 \text{ rad/s} = 7.79 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. d,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-0.2\alpha_{CD}\mathbf{i} - 0.2\left(7.7942^2\right)\mathbf{j} = (-2.70\mathbf{i} - 2.40\mathbf{j}) + (-\alpha_{BC}\mathbf{k}) \times (-0.5 \cos 60^\circ \mathbf{i} - 0.5 \sin 60^\circ \mathbf{j})$$
$$- 3.60^2(-0.5 \cos 60^\circ \mathbf{i} - 0.5 \sin 60^\circ \mathbf{j})$$

$$-0.2\alpha_{CD}\mathbf{i} - 12.15\mathbf{j} = \left(0.54 - 0.25\sqrt{3}\alpha_{BC}\right)\mathbf{i} + (3.2118 + 0.25\alpha_{BC})\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$-12.15 = 3.2118 + 0.25\alpha_{BC}; \qquad \alpha_{BC} = -61.45 \text{ rad/s}^2 = 61.45 \text{ rad/s}^2 \circlearrowleft$$

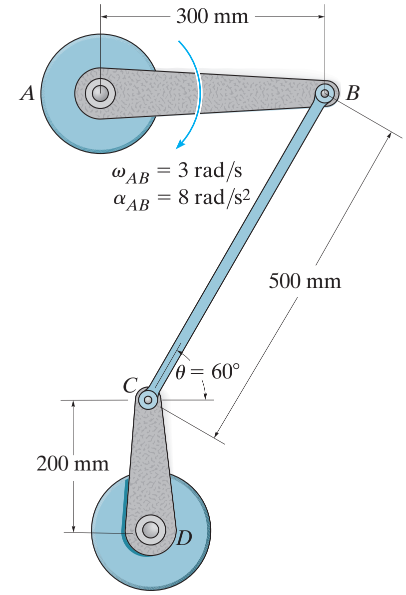
Then the $\mathbf{i}$ component gives

$$-0.2\alpha_{CD} = 0.54 - 0.25\sqrt{3}(-61.4474); \quad \alpha_{CD} = -135.74 \text{ rad/s}^2 = 136 \text{ rad/s}^2 \circlearrowright \quad \textbf{Ans.}$$

Ans:
$\omega_{CD} = 7.79 \text{ rad/s} \circlearrowleft$
$\alpha_{CD} = 136 \text{ rad/s}^2 \circlearrowright$

16–123.

If member AB has the angular motion shown, determine the velocity and acceleration of point C at the instant shown.



SOLUTION

Rotation About A Fixed Axis. For link AB, refer to Fig. a.

$$v_B = \omega_{AB} r_{AB} = 3(0.3) = 0.9 \text{ m/s} \downarrow$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$$

$$= (-8\mathbf{k}) \times (0.3\mathbf{i}) - 3^2(0.3\mathbf{i})$$

$$= \{-2.70\mathbf{i} - 2.40\mathbf{j}\} \text{ m/s}^2$$

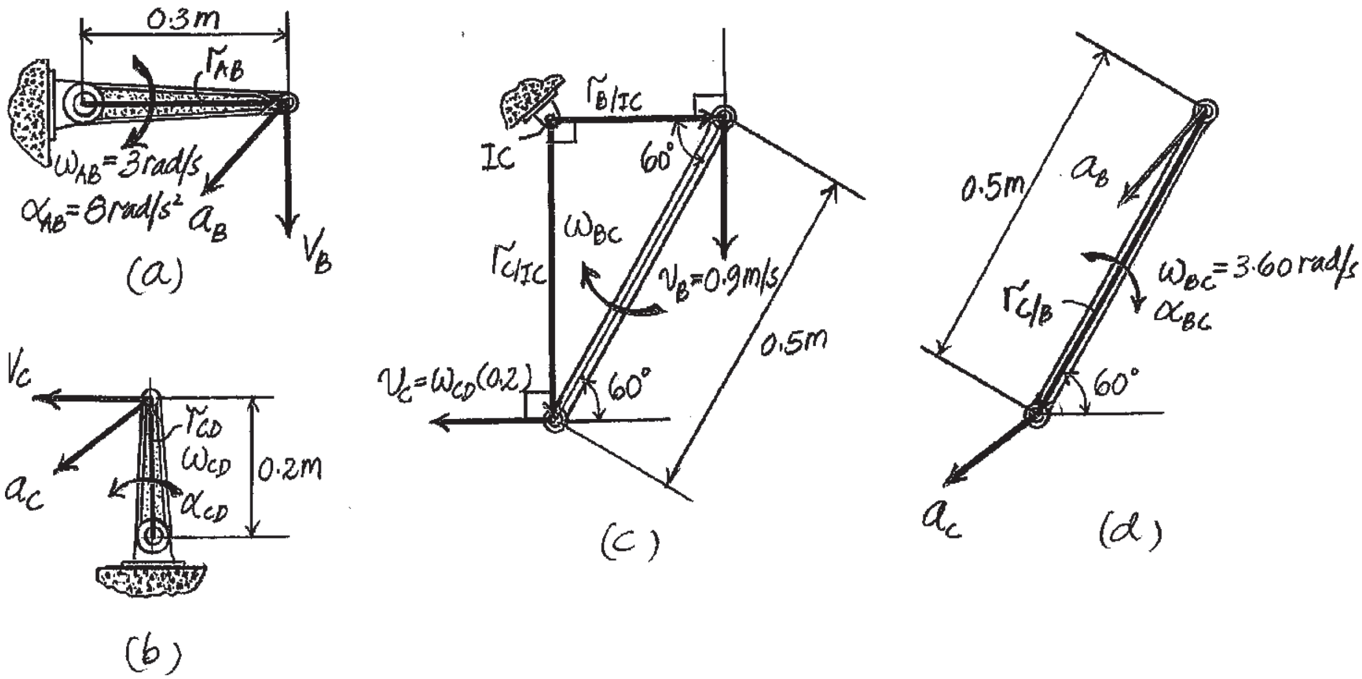
For link CD, refer to Fig. b.

$$v_C = \omega_{CD} r_{CD} = \omega_{CD}(0.2) \leftarrow$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{CD} - \omega_{CD}^2 \mathbf{r}_{CD}$$

$$\mathbf{a}_C = (\alpha_{CD}\mathbf{k}) \times (0.2\mathbf{j}) - \omega_{CD}^2(0.2\mathbf{j})$$

$$= -0.2\alpha_{CD}\mathbf{i} - 0.2\omega_{CD}^2\mathbf{j}$$



16–123. Continued

General Plane Motion. The IC of link BC can be located using $\mathbf{v}_B$ and $\mathbf{v}_C$ as shown in Fig. c. From the geometry of this figure,

$$r_{B/IC} = 0.5 \cos 60° = 0.25 \text{ m} \qquad r_{C/IC} = 0.5 \sin 60° = 0.25\sqrt{3} \text{ m}$$

Then kinematics gives

$$v_B = \omega_{BC} r_{B/IC}; \quad 0.9 = \omega_{BC}(0.25) \quad \omega_{BC} = 3.60 \text{ rad/s}$$

$$v_C = \omega_{BC} r_{C/IC}; \quad \omega_{CD}(0.2) = (3.60)\left(0.25\sqrt{3}\right)$$

$$\omega_{CD} = 7.7942 \text{ rad/s} = 7.79 \text{ rad/s} \qquad \textbf{Ans.}$$

Applying the relative acceleration equation by referring to Fig. d,

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$-0.2\alpha_{CD}\mathbf{i} - 0.2\left(7.7942^2\right)\mathbf{j} = (-2.70\mathbf{i} - 2.40\mathbf{j}) + (-\alpha_{BC}\mathbf{k}) \times (-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$
$$- 3.60^2(-0.5 \cos 60°\mathbf{i} - 0.5 \sin 60°\mathbf{j})$$

$$-0.2\alpha_{CD}\mathbf{i} - 12.15\mathbf{j} = \left(0.54 - 0.25\sqrt{3}\alpha_{BC}\right)\mathbf{i} + (3.2118 + 0.25\alpha_{BC})\mathbf{j}$$

Equating the $\mathbf{j}$ components,

$$-12.15 = 3.2118 + 0.25\alpha_{BC}; \qquad \alpha_{BC} = -61.45 \text{ rad/s}^2 = 61.45 \text{ rad/s}^2$$

Then the $\mathbf{i}$ component gives

$$-0.2\alpha_{CD} = 0.54 - 0.25\sqrt{3}(-61.4474); \quad \alpha_{CD} = -135.74 \text{ rad/s}^2 = 136 \text{ rad/s}^2 \quad \textbf{Ans.}$$

From the angular motion of CD,

$$v_C = w_{CD}(0.2) = (7.7942)(0.2) = 1.559 \text{ m/s} = 1.56 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = -0.2(-135.74)\mathbf{i} - 12.15j$$
$$= \{27.15\mathbf{i} - 12.15j\} \text{ m/s}$$

The magnitude of $\mathbf{a}_C$ is

$$a_C = \sqrt{27.15^2 + (-12.15)^2} = 29.74 \text{ m/s}^2 = 29.7 \text{ m/s}^2 \qquad \textbf{Ans.}$$

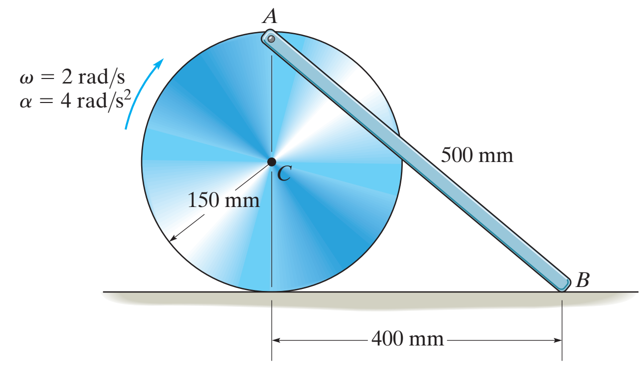
And its direction is defined by

$$\theta = \tan^{-1}\left(\frac{12.15}{27.15}\right) = 24.11° = 24.1° \qquad \textbf{Ans.}$$

Ans:
$v_C = 1.56 \text{ m/s} \leftarrow$
$a_C = 29.7 \text{ m/s}^2$
$\theta = 24.1°$

***16–124.**

The disk rolls without slipping such that it has an angular acceleration of $\alpha = 4\ \text{rad/s}^2$ and angular velocity of $\omega = 2\ \text{rad/s}$ at the instant shown. Determine the acceleration of points A and B on the link and the link's angular acceleration at this instant. Assume point A lies on the periphery of the disk, 150 mm from C.



SOLUTION

The IC is at ∞, so $\omega = 0$.

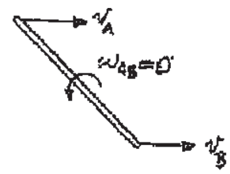
$$\mathbf{a}_A = \mathbf{a}_C + \alpha \times \mathbf{r}_{A/C} - \omega^2 \mathbf{r}_{A/C}$$

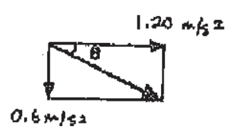
$$\mathbf{a}_A = 0.6\mathbf{i} + (-4\mathbf{k}) \times (0.15\mathbf{j}) - (2)^2(0.15\mathbf{j})$$

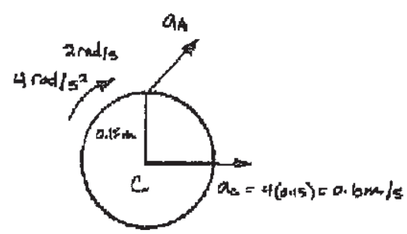
$$\mathbf{a}_A = (1.20\mathbf{i} - 0.6\mathbf{j})\ \text{m/s}^2$$

$$a_A = \sqrt{(1.20)^2 + (-0.6)^2} = 1.34\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{0.6}{1.20}\right) = 26.6^\circ \searrow \qquad \textbf{Ans.}$$



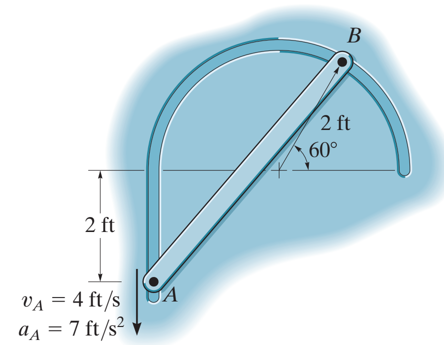




Ans:
$a_A = 1.34\ \text{m/s}^2$
$\theta = 26.6^\circ \searrow$

16–125.

The ends of the bar AB are confined to move along the paths shown. At a given instant, A has a velocity of $v_A = 4$ ft/s and an acceleration of $a_A = 7$ ft/s². Determine the angular velocity and angular acceleration of AB at this instant.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$v_B = 4 + \omega(4.788)$

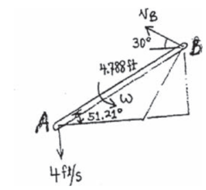
30° ↖ ↓ ↖ 51.21°

$(\xrightarrow{+})$ $-v_B \cos 30° = 0 - \omega(4.788) \sin 51.21°$

$(+\uparrow)$ $v_B \sin 30° = -4 + \omega(4.788) \cos 51.21°$

$v_B = 20.39$ ft/s 30° ↖

$\omega = 4.73$ rad/s ↺ **Ans.**



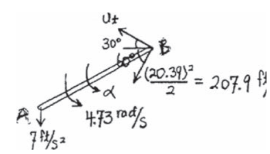
$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$a_t + 207.9 = 7 + 107.2 + 4.788(\alpha)$

30° ↖ 60° ↙ ↓ ↙ 51.21° ↖ 51.21°

$(\xleftarrow{+})$ $a_t \cos 30° + 207.9 \cos 60° = 0 + 107.2 \cos 51.21° + 4.788\alpha(\sin 51.21°)$

$(+\uparrow)$ $a_t \sin 30° - 207.9 \sin 60° = -7 - 107.2 \sin 51.21° + 4.788\alpha(\cos 51.21°)$



$a_t(0.866) - 3.732\alpha = -36.78$

$a_t(0.5) - 3\alpha = 89.49$

$a_t = -607$ ft/s²

$\alpha = -131$ rad/s² $= 131$ rad/s² ↻ **Ans.**

Also:

$\mathbf{v}_B = \mathbf{v}_A = \omega \times \mathbf{r}_{B/A}$

$-v_B \cos 30°\mathbf{i} + v_B \sin 30°\mathbf{j} = -4\mathbf{j} + (\omega\mathbf{k}) \times (3\mathbf{i} + 3.732\mathbf{j})$

$-v_B \cos 30° = -\omega(3.732)$

$v_B \sin 30° = -4 + \omega(3)$

$\omega = 4.73$ rad/s ↺ **Ans.**

$v_B = 20.39$ ft/s

$\mathbf{a}_B = \mathbf{a}_A - \omega^2\mathbf{r}_{B/A} + \alpha \times \mathbf{r}_{B/A}$

$(-a_t \cos 30°\mathbf{i} + a_t \sin 30°\mathbf{j}) + (-207.9 \cos 60°\mathbf{i} - 207.9 \sin 60°\mathbf{j}) = -7\mathbf{j} - (4.732)^2(3\mathbf{i} + 3.732\mathbf{j})$

$+(\alpha\mathbf{k}) \times (3\mathbf{i} + 3.732\mathbf{j})$

$-a_t \cos 30° - 207.9 \cos 60° = -(4.732)^2(3) - \alpha(3.732)$

$a_t \sin 30° - 207.9 \sin 60° = -7 - (4.732)^2(3.732) + \alpha(3)$

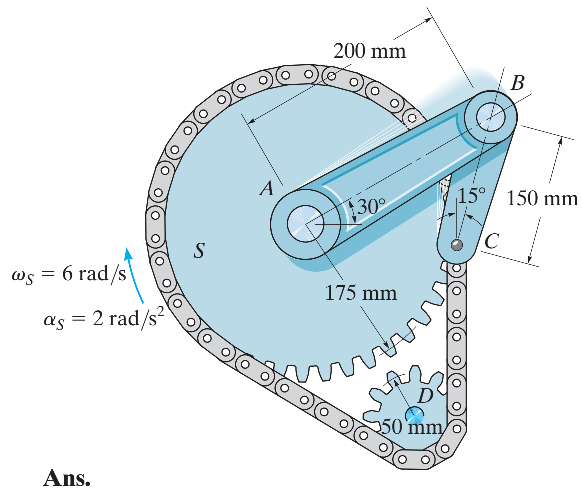
$a_t = -607$ ft/s²

$\alpha = -131$ rad/s² $= 131$ rad/s² ↻ **Ans.**

Ans:
$\omega = 4.73$ rad/s ↺
$\alpha = 131$ rad/s² ↻

16–126.

The mechanism produces intermittent motion of link AB. If the sprocket S is turning with an angular acceleration $\alpha_S = 2\ \text{rad/s}^2$ and has an angular velocity $\omega_S = 6\ \text{rad/s}$ at the instant shown, determine the angular velocity and angular acceleration of link AB at this instant. The sprocket S is mounted on a shaft which is *separate* from a collinear shaft attached to AB at A. The pin at C is attached to one of the chain links such that it moves vertically downward.

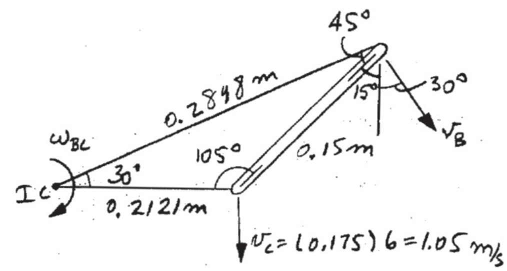


SOLUTION

$$\omega_{BC} = \frac{1.05}{0.2121} = 4.950\ \text{rad/s}$$

$$v_B = (4.95)(0.2898) = 1.434\ \text{m/s}$$

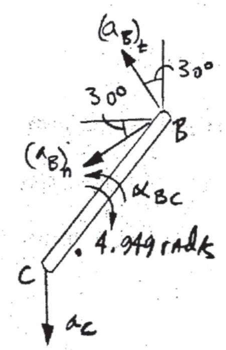
$$\omega_{AB} = \frac{1.435}{0.2} = 7.1722\ \text{rad/s} = 7.17\ \text{rad/s} \circlearrowright \qquad \textbf{Ans.}$$



$$a_C = \alpha_S r_S = 2(0.175) = 0.350\ \text{m/s}^2$$

$$(\mathbf{a}_B)_n + (\mathbf{a}_B)_t = \mathbf{a}_C + (\mathbf{a}_{B/C})_n + (\mathbf{a}_{B/C})_t$$

$$\begin{bmatrix} (7.172)^2(0.2) \\ 30° \swarrow \end{bmatrix} + \begin{bmatrix} (a_B)_t \\ \nwarrow 30° \end{bmatrix} = \begin{bmatrix} 0.350 \\ \downarrow \end{bmatrix} + \begin{bmatrix} (4.949)^2\ (0.15) \\ \swarrow 15° \end{bmatrix} + \begin{bmatrix} \alpha_{BC}\ (0.15) \\ 15° \nwarrow \end{bmatrix}$$



$(\overset{+}{\rightarrow})\quad -(10.29)\cos 30° - (a_B)_t \sin 30° = 0 - (4.949)^2(0.15)\sin 15° - \alpha_{BC}(0.15)\cos 15°$

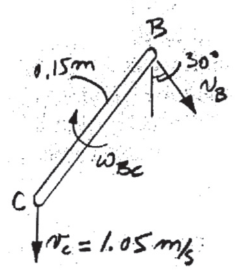
$(+\uparrow)\quad -(10.29)\sin 30° + (a_B)_t \cos 30° = -0.350 - (4.949)^2(0.15)\cos 15° + \alpha_{BC}(0.15)\sin 15°$

$\alpha_{BC} = 70.8\ \text{rad/s}^2, \qquad (a_B)_t = 4.61\ \text{m/s}^2$

Hence,

$$\alpha_{AB} = \frac{(a_B)_t}{r_{B/A}} = \frac{4.61}{0.2} = 23.1\ \text{rad/s}^2 \circlearrowleft \qquad \textbf{Ans.}$$

Also,



$v_C = \omega_S r_S = 6(0.175) = 1.05\ \text{m/s} \downarrow$

$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$

$v_B \sin 30°\mathbf{i} - v_B \cos 30°\mathbf{j} = -1.05\mathbf{j} + (-\omega_{BC}\mathbf{k}) \times (0.15 \sin 15°\mathbf{i} + 0.15 \cos 15°\mathbf{j})$

$(\overset{+}{\rightarrow})\quad v_B \sin 30° = 0 + \omega_{BC}(0.15)\cos 15°$

$(+\uparrow)\quad -v_B \cos 30° = -1.05 - \omega_{BC}(0.15)\sin 15°$

$v_B = 1.434\ \text{m/s}, \qquad \omega_{BC} = 4.950\ \text{rad/s}$

$$\omega_{AB} = \frac{v_B}{r_{B/A}} = \frac{1.434}{0.2} = 7.172 = 7.17\ \text{rad/s} \circlearrowright \qquad \textbf{Ans.}$$

16–126. Continued

$$\mathbf{a}_B = \mathbf{a}_C + \alpha_{BC} \times \mathbf{r}_{B/C} - \omega^2 \mathbf{r}_{B/C}$$

$$(\alpha_{AB}\mathbf{k}) \times (0.2 \cos 30° \mathbf{i} + 0.2 \sin 30° \mathbf{j}) - (7.172)^2 (0.2 \cos 30° \mathbf{i} + 0.2 \sin 30° \mathbf{j})$$

$$= -(2)(0.175)\mathbf{j} + (\alpha_{BC}\mathbf{k}) \times (0.15 \sin 15° \mathbf{i} + 0.15 \cos 15° \mathbf{j}) - (4.950)^2 (0.15 \sin 15° \mathbf{i} + 0.15 \cos 15° \mathbf{j})$$

$$\left(\xrightarrow{+}\right) \qquad -\alpha_{AB}(0.1) - 8.9108 = -0.1449\alpha_{BC} - 0.9512$$

$$(+\uparrow) \qquad \alpha_{AB}(0.1732) - 5.143 = -0.350 + 0.0388\alpha_{BC} - 3.550$$

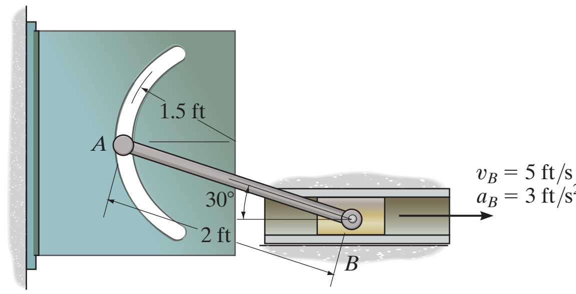
$\alpha_{AB} = 23.1 \text{ rad/s}^2$ ↺ **Ans.**

$\alpha_{BC} = 70.8 \text{ rad/s}^2$

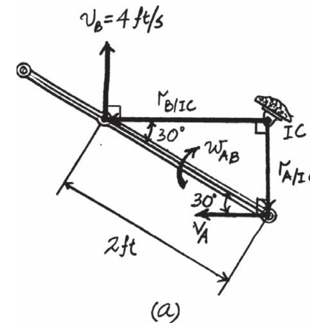
Ans:
$\omega_{AB} = 7.17 \text{ rad/s}$ ↻
$\alpha_{AB} = 23.1 \text{ rad/s}^2$ ↺

16–127.

The slider block moves with a velocity of $v_B = 5$ ft/s and an acceleration of $a_B = 3$ ft/s^2. Determine the angular acceleration of rod AB at the instant shown.



SOLUTION



Angular Velocity: The velocity of point A is directed along the tangent of the circular slot. Thus, the location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure,

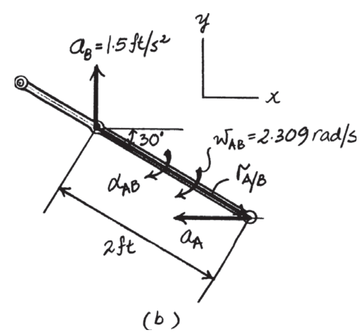
$$r_{B/IC} = 2 \sin 30° = 1 \text{ ft} \qquad r_{A/IC} = 2 \cos 30° = 1.732 \text{ ft}$$

Thus,

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{5}{1} = 5 \text{ rad/s}$$

Then

$$v_A = \omega_{AB}\, r_{A/IC} = 5(1.732) = 8.660 \text{ ft/s}$$



Acceleration and Angular Acceleration: Since point A travels along the circular slot, the normal component of its acceleration has a magnitude of $(a_A)_n = \dfrac{v_A{}^2}{\rho} = \dfrac{8.660^2}{1.5} = 50$ ft/s^2 and is directed towards the center of the circular slot. The tangential component is directed along the tangent of the slot. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - \omega_{AB}{}^2\, \mathbf{r}_{A/B}$$

$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = 3\mathbf{i} + (\alpha_{AB}\,\mathbf{k}) \times (-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}) - 5^2(-2\cos 30°\,\mathbf{i} + 2\sin 30°\mathbf{j})$$

$$50\mathbf{i} - (a_A)_t\,\mathbf{j} = (46.30 - \alpha_{AB})\mathbf{i} + (1.732\alpha_{AB} + 25)\mathbf{j}$$

Equating the **i** components,

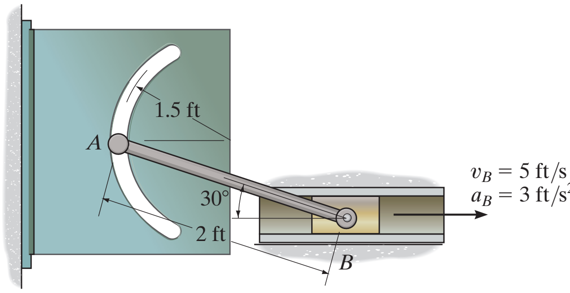
$$50 = 46.30 - \alpha_{AB}$$

$$\alpha_{AB} = -3.70 \text{ rad/s}^2 = 3.70 \text{ rad/s}^2 \circlearrowright$$ **Ans.**

Ans:
$\alpha_{AB} = 3.70$ rad/s^2 $\circlearrowright$

***16–128.**

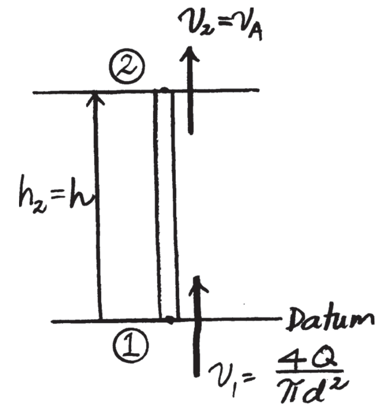
The slider block moves with a velocity of $v_B = 5$ ft/s and an acceleration of $a_B = 3$ ft/s^2. Determine the acceleration of A at the instant shown.



SOLUTION

Angualr Velocity: The velocity of point A is directed along the tangent of the circular slot. Thus, the location of the IC for rod AB is indicated in Fig. a. From the geometry of this figure,

$$r_{B/IC} = 2 \sin 30° = 1 \text{ ft} \qquad r_{A/IC} = 2 \cos 30° = 1.732 \text{ ft}$$



(a)

Thus,

$$\omega_{AB} = \frac{v_B}{r_{B/IC}} = \frac{5}{1} = 5 \text{ rad/s}$$

Then

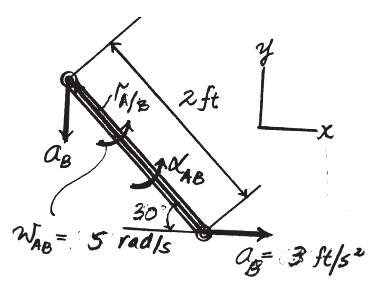
$$v_A = \omega_{AB}\, r_{A/IC} = 5(1.732) = 8.660 \text{ ft/s}$$

Acceleration and Angular Acceleration: Since point A travels along the circular slot, the normal component of its acceleration has a magnitude of $(a_A)_n = \frac{{v_A}^2}{\rho} = \frac{8.660^2}{1.5} = 50$ ft/s^2 and is directed towards the center of the circular slot. The tangential component is directed along the tangent of the slot. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha_{AB} \times \mathbf{r}_{A/B} - {\omega_{AB}}^2\, \mathbf{r}_{A/B}$$

$$50\mathbf{i} - (a_A)_t\mathbf{j} = 3\mathbf{i} + (\alpha_{AB}\mathbf{k}) \times (-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}) - 5^2(-2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j})$$

$$50\mathbf{i} - (a_A)_t\mathbf{j} = (46.30 - \alpha_{AB})\mathbf{i} - (1.732\alpha_{AB} + 25)\mathbf{j}$$



(b)

Equating the **i** and **j** components,

$$50 = 46.30 - \alpha_{AB}$$

$$-(a_A)_t = -(1.732\alpha_{AB} + 25)$$

Solving,

$$\alpha_{AB} = -3.70 \text{ rad/s}^2$$

$$(a_A)_t = 18.59 \text{ ft/s}^2 \downarrow$$

Thus, the magnitude of $\mathbf{a}_A$ is

$$a_A = \sqrt{(a_A)_t^{\,2} + (a_A)_n^{\,2}} = \sqrt{18.59^2 + 50^2} = 53.3 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

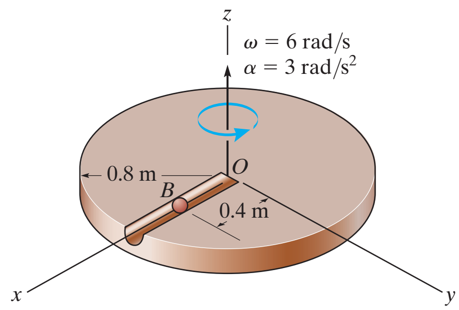
and its direction is

$$\theta = \tan^{-1}\left[\frac{(a_A)_t}{(a_A)_n}\right] = \tan^{-1}\left(\frac{18.59}{50}\right) = 20.4° \;⦩ \qquad \textbf{Ans.}$$

Ans:
$a_A = 53.3$ ft/s^2
$\theta = 20.4°$ ⦩

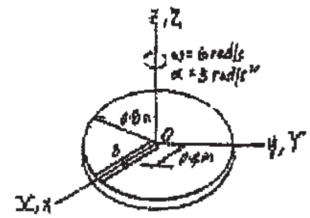
16–129.

At the instant shown, ball B is rolling along the slot in the disk with a velocity of 600 mm/s and an acceleration of 150 mm/s^2, both measured relative to the disk and directed away from O. If at the same instant the disk has the angular velocity and angular acceleration shown, determine the velocity and acceleration of the ball at this instant.



SOLUTION

Kinematic Equations:



$$v_B = v_O + \Omega \times \mathbf{r}_{B/O} + (v_{B/O})_{xyz} \qquad \textbf{(1)}$$

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (v_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \qquad \textbf{(2)}$$

$$v_O = 0$$

$$\mathbf{a}_O = 0$$

$$\Omega = \{6\mathbf{k}\}\text{ rad/s}$$

$$\dot{\Omega} = \{3\mathbf{k}\}\text{ rad/s}^2$$

$$\mathbf{r}_{B/O} = \{0.4\mathbf{i}\}\text{ m}$$

$$(v_{B/O})_{xyz} = \{0.6\mathbf{i}\}\text{ m/s}$$

$$(\mathbf{a}_{B/O})_{xyz} = \{0.15\mathbf{i}\}\text{ m/s}^2$$

Substitute the date into Eqs. (1) and (2) yields:

$$v_B = 0 + (6\mathbf{k}) \times (0.4\mathbf{i}) + (0.6\mathbf{i}) = \{0.6\mathbf{i} + 2.4\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

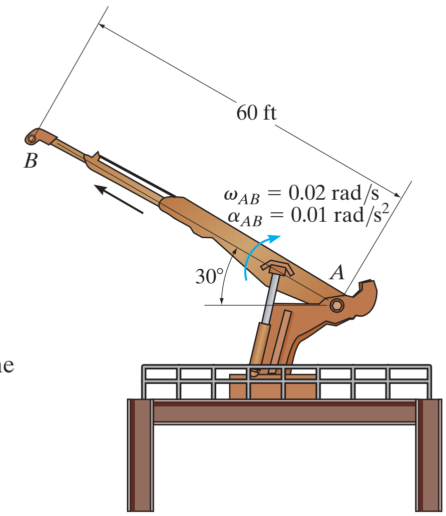
$$\mathbf{a}_B = 0 + (3\mathbf{k}) \times (0.4\mathbf{i}) + (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.4\mathbf{i})] + 2(6\mathbf{k}) \times (0.6\mathbf{i}) + (0.15\mathbf{i})$$

$$= \{-14.2\mathbf{i} + 8.40\mathbf{j}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_B = \{0.6\mathbf{i} + 2.4\mathbf{j}\}$ m/s
$\mathbf{a}_B = \{-14.2\mathbf{i} + 8.40\mathbf{j}\}$ m/s^2

16–130.

The crane's telescopic boom rotates with the angular velocity and angular acceleration shown. At the same instant, the boom is extending with a constant speed of 0.5 ft/s, measured relative to the boom. Determine the magnitudes of the velocity and acceleration of point B at this instant.



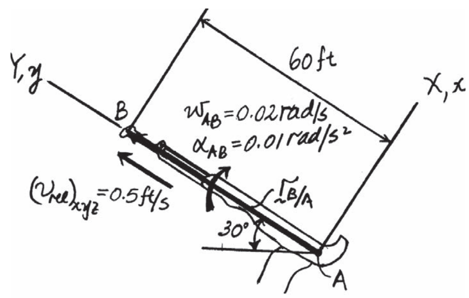
SOLUTION

Reference Frames: The xyz rotating reference frame is attached to boom AB and coincides with the XY fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xy frame with respect to the XY frame is

$\mathbf{v}_A = \mathbf{a}_A = \mathbf{0}$ $\qquad \omega_{AB} = [-0.02\mathbf{k}]$ rad/s $\qquad \dot{\omega}_{AB} = \alpha = [-0.01\mathbf{k}]$ rad/s^2

For the motion of point B with respect to the xyz frame, we have

$\mathbf{r}_{B/A} = [60\mathbf{j}]$ ft $\qquad (\mathbf{v}_{\text{rel}})_{xyz} = [0.5\mathbf{j}]$ ft/s $\qquad (\mathbf{a}_{\text{rel}})_{xyz} = \mathbf{0}$



(a)

Velocity: Applying the relative velocity equation,

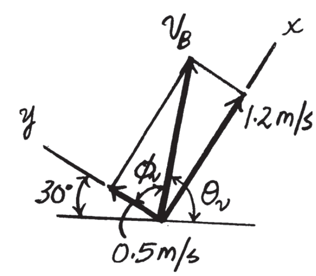
$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-0.02\mathbf{k}) \times (60\mathbf{j}) + 0.5\mathbf{j}$$

$$= [1.2\mathbf{i} + 0.5\mathbf{j}] \text{ ft / s}$$

Thus, the magnitude of $\mathbf{v}_B$, Fig. b, is

$$v_B = \sqrt{1.2^2 + 0.5^2} = 1.30 \text{ ft/s} \qquad \textbf{Ans.}$$



(b)

Acceleration: Applying the relative acceleration equation,

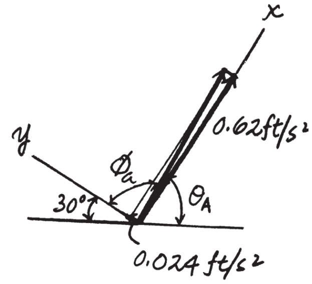
$$\mathbf{a}_B = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{B/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{B/A}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$= \mathbf{0} + (-0.01\mathbf{k}) \times (60\mathbf{j}) + (-0.02\mathbf{k}) \times [(-0.02\mathbf{k}) \times (60\mathbf{j})] + 2(-0.02\mathbf{k}) \times (0.5\mathbf{j}) + \mathbf{0}$$

$$= [0.62\mathbf{i} - 0.024\,\mathbf{j}] \text{ ft/s}^2$$

Thus, the magnitude of $\mathbf{a}_B$, Fig. c, is

$$a_B = \sqrt{0.62^2 + (-0.024)^2} = 0.6204 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

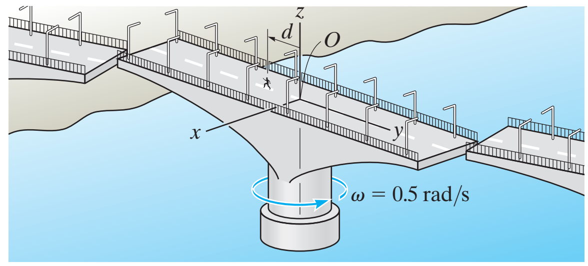


(c)

Ans:
$v_B = 1.30$ ft/s
$a_B = 0.6204$ ft/s^2

16–131.

While the swing bridge is closing with a constant rotation of 0.5 rad/s, a man runs along the roadway at a constant speed of 5 ft/s relative to the roadway. Determine his velocity and acceleration at the instant $d = 15$ ft.



SOLUTION

$\Omega = \{0.5\mathbf{k}\}$ rad/s

$\Omega = \mathbf{0}$

$\mathbf{r}_{m/o} = \{-15\,\mathbf{j}\}$ ft

$(v_{m/o})_{xyz} = \{-5\mathbf{j}\}$ ft/s

$(a_{m/o})_{xyz} = \mathbf{0}$

$v_m = \mathbf{v}_o + \Omega \times \mathbf{r}_{m/o} + (\mathbf{v}_{m/o})_{xyz}$

$v_m = 0 + (0.5\mathbf{k}) \times (-15\mathbf{j}) - 5\mathbf{j}$

$\mathbf{v}_m = \{7.5\mathbf{i} - 5\mathbf{j}\}$ ft/s **Ans.**

$\mathbf{a}_m = \mathbf{a}_O + \Omega \times \mathbf{r}_{m/O} + \Omega \times (\Omega \times \mathbf{r}_{m/O}) + 2\Omega \times (\mathbf{v}_{m/O})_{xyz} + (\mathbf{a}_{m/O})_{xyz}$

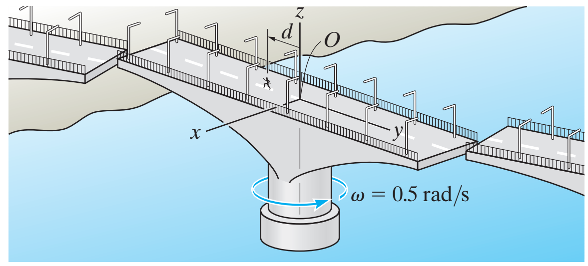
$\mathbf{a}_m = \mathbf{0} + \mathbf{0} + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (-15\mathbf{j})] + 2(0.5\mathbf{k}) \times (-5\mathbf{j}) + \mathbf{0}$

$\mathbf{a}_m = \{5\mathbf{i} + 3.75\mathbf{j}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_m = \{7.5\mathbf{i} - 5\mathbf{j}\}$ ft/s
$\mathbf{a}_m = \{5\mathbf{i} + 3.75\mathbf{j}\}$ ft/s^2

***16–132.**

While the swing bridge is closing with a constant rotation of 0.5 rad/s, a man runs along the roadway such that when $d = 10$ ft he is running outward from the center at 5 ft/s with an acceleration of 2 ft/s^2, both measured relative to the roadway. Determine his velocity and acceleration at this instant.



SOLUTION

$\Omega = \{0.5\mathbf{k}\}$ rad/s

$\dot{\Omega} = \mathbf{0}$

$\mathbf{r}_{m/o} = \{-10\,\mathbf{j}\}$ ft

$(v_{m/O})_{xyz} = \{-5\mathbf{j}\}$ ft/s

$(a_{m/O})_{xyz} = \{-2\mathbf{j}\}$ ft/s^2

$v_m = \mathbf{v}_o + \Omega \times \mathbf{r}_{m/o} + (\mathbf{v}_{m/o})_{xyz}$

$v_m = 0 + (0.5\mathbf{k}) \times (-10\mathbf{j}) - 5\mathbf{j}$

$\mathbf{v}_m = \{5\mathbf{i} - 5\mathbf{j}\}$ ft/s **Ans.**

$\mathbf{a}_m = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{m/O} + \Omega \times (\Omega \times \mathbf{r}_{m/O}) + 2\Omega \times (\mathbf{v}_{m/O})_{xyz} + (\mathbf{a}_{m/O})_{xyz}$

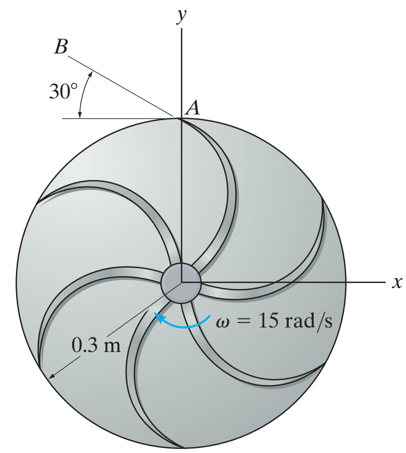
$\mathbf{a}_m = \mathbf{0} + \mathbf{0} + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (-10\mathbf{j})] + 2(0.5\mathbf{k}) \times (-5\mathbf{j}) - 2\mathbf{j}$

$\mathbf{a}_m = \{5\mathbf{i} + 0.5\mathbf{j}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_m = \{5\mathbf{i} - 5\mathbf{j}\}$ ft/s
$\mathbf{a}_m = \{5\mathbf{i} + 0.5\mathbf{j}\}$ ft/s^2

16–133.

Water leaves the impeller of the centrifugal pump with a velocity of 25 m/s and acceleration of 30 m/s^2, both measured relative to the impeller along the blade line AB. Determine the velocity and acceleration of a water particle at A as it leaves the impeller at the instant shown. The impeller rotates with a constant angular velocity of $\omega = 15$ rad/s.



SOLUTION

Reference Frame: The xyz rotating reference frame is attached to the impeller and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

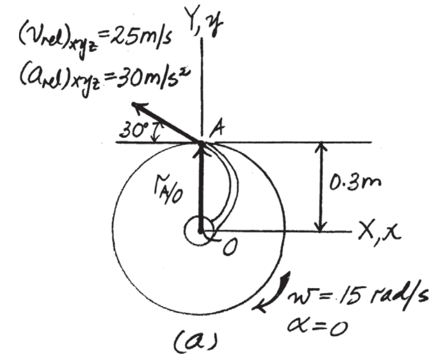
$$\mathbf{v}_O = \mathbf{a}_O = \mathbf{0} \qquad \omega = [-15\mathbf{k}]\ \text{rad/s} \qquad \dot{\omega} = \mathbf{0}$$

The motion of point A with respect to the xyz frame is

$$\mathbf{r}_{A/O} = [0.3\mathbf{j}]\ \text{m}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = (-25\cos 30°\,\mathbf{i} + 25\sin 30°\,\mathbf{j}) = [-21.65\mathbf{i} + 12.5\mathbf{j}]\ \text{m/s}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = (-30\cos 30°\,\mathbf{i} + 30\sin 30°\,\mathbf{j}) = [-25.98\mathbf{i} + 15\mathbf{j}]\ \text{m/s}^2$$



Velocity: Applying the relative velocity equation.

$$\begin{aligned}\mathbf{v}_A &= \mathbf{v}_O + \omega \times \mathbf{r}_{A/O} + (\mathbf{v}_{\text{rel}})_{xyz} \\ &= \mathbf{0} + (-15\mathbf{k}) \times (0.3\mathbf{j}) + (-21.65\mathbf{i} + 12.5\mathbf{j}) \\ &= [-17.2\mathbf{i} + 12.5\mathbf{j}]\ \text{m/s} \qquad \textbf{Ans.}\end{aligned}$$

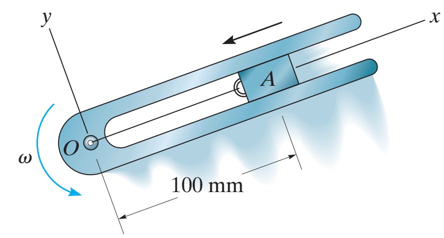
Acceleration: Applying the relative acceleration equation,

$$\begin{aligned}\mathbf{a}_A &= \mathbf{a}_O + \dot{\omega} \times \mathbf{r}_{A/O} + \omega \times (\omega \times \mathbf{r}_{A/O}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz} \\ &= \mathbf{0} + (-15\mathbf{k}) \times [(-15\mathbf{k}) \times (0.3\mathbf{j})] + 2(-15\mathbf{k}) \times (-21.65\mathbf{i} + 12.5\mathbf{j}) + (-25.98\mathbf{i} + 15\mathbf{j}) \\ &= [349\mathbf{i} + 597\mathbf{j}]\ \text{m/s}^2 \qquad \textbf{Ans.}\end{aligned}$$

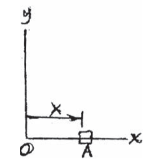
Ans:
$\mathbf{v}_A = \{-17.2\mathbf{i} + 12.5\mathbf{j}\}$ m/s
$\mathbf{a}_A = \{349\mathbf{i} + 597\mathbf{j}\}$ m/s^2

16–134.

Block A, which is attached to a cord, moves along the slot of a horizontal forked rod. At the instant shown, the cord is pulled down through the hole at O with an acceleration of $4\ \text{m/s}^2$ and its velocity is 2 m/s. Determine the acceleration of the block at this instant. The rod rotates about O with a constant angular velocity $\omega = 4$ rad/s.



SOLUTION



Motion of moving reference.

$\mathbf{v}_O = \mathbf{0}$

$\mathbf{a}_O = \mathbf{0}$

$\Omega = 4\mathbf{k}$

$\dot{\Omega} = \mathbf{0}$

Motion of A with respect to moving reference.

$\mathbf{r}_{A/O} = 0.1\mathbf{i}$

$\mathbf{v}_{A/O} = -2\mathbf{i}$

$\mathbf{a}_{A/O} = -4\mathbf{i}$

Thus,

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{A/O} + \Omega \times (\Omega \times \mathbf{r}_{A/O}) + 2\Omega \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$$

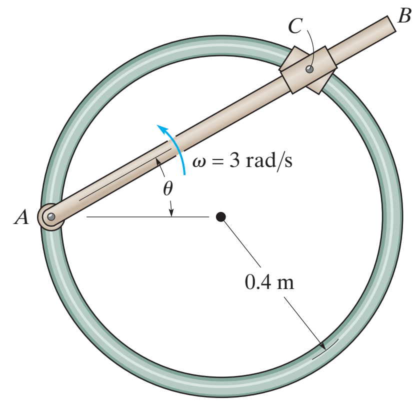
$$= \mathbf{0} + \mathbf{0} + (4\mathbf{k}) \times (4\mathbf{k} \times 0.1\mathbf{i}) + 2(4\mathbf{k} \times (-2\mathbf{i})) - 4\mathbf{i}$$

$\mathbf{a}_A = \{-5.60\mathbf{i} - 16\mathbf{j}\}\ \text{m/s}^2$ **Ans.**

Ans:
$\mathbf{a}_A = \{-5.60\mathbf{i} - 16\mathbf{j}\}\ \text{m/s}^2$

16–135.

Rod AB rotates counterclockwise with a constant angular velocity $\omega = 3$ rad/s. Determine the velocity of point C located on the double collar when $\theta = 30°$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod AB.

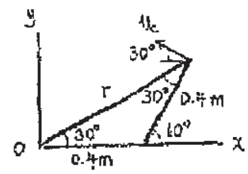


SOLUTION

$r = 2(0.4 \cos 30°) = 0.6928$ m

$\mathbf{r}_{C/A} = 0.6928 \cos 30°\mathbf{i} + 0.6928 \sin 30°\mathbf{j}$

$= \{0.600\mathbf{i} + 0.3464\mathbf{j}\}$ m



$\mathbf{v}_C = -0.866 v_C\mathbf{i} + 0.5 v_C\mathbf{j}$

$v_C = v_A + \Omega \times \mathbf{r}_{C/A} + (v_{C/A})_{xyz}$

$-0.866 v_C\mathbf{i} + 0.5 v_C\mathbf{j} = 0 + (3\mathbf{k}) \times (0.600\mathbf{i} + 0.3464\mathbf{j}) + (v_{C/A} \cos 30°\mathbf{i} + v_{C/A} \sin 30°\mathbf{j})$

$-0.866 v_C\mathbf{i} + 0.5 v_C\mathbf{j} = 0 - 1.039\mathbf{i} + 1.80\mathbf{j} + 0.866 v_{C/A}\mathbf{i} + 0.5 v_{C/A}\mathbf{j}$

$-0.866 v_C = -1.039 + 0.866 v_{C/A}$

$0.5 v_C = 1.80 + 0.5 v_{C/A}$

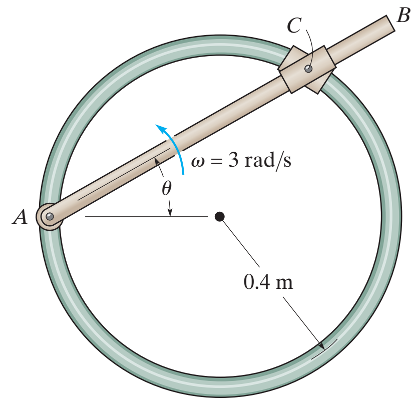
$v_C = 2.40$ m/s **Ans.**

$v_{C/A} = -1.20$ m/s

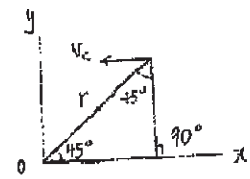
Ans:
$v_C = 2.40$ m/s
$\theta = 60°$ ◺

***16–136.**

Rod AB rotates counterclockwise with a constant angular velocity $\omega = 3$ rad/s. Determine the velocity and acceleration of point C located on the double collar when $\theta = 45°$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod AB.



SOLUTION



$\mathbf{r}_{C/A} = \{0.400\mathbf{i} + 0.400\mathbf{j}\}$

$\mathbf{v}_C = -v_C\mathbf{i}$

$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$-v_C\mathbf{i} = 0 + (3\mathbf{k}) \times (0.400\mathbf{i} + 0.400\mathbf{j}) + (v_{C/A}\cos 45°\mathbf{i} + v_{C/A}\sin 45°\mathbf{j})$

$-v_C\mathbf{i} = 0 - 1.20\mathbf{i} + 1.20\mathbf{j} + 0.707v_{C/A}\mathbf{i} + 0.707v_{C/A}\mathbf{j}$

$-v_C = -1.20 + 0.707v_{C/A}$

$0 = 1.20 + 0.707v_{C/A}$

$v_C = 2.40\text{ m/s}$ **Ans.**

$v_{C/A} = -1.697\text{ m/s}$

$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$$-(a_C)_t\mathbf{i} - \frac{(2.40)^2}{0.4}\mathbf{j} = 0 + 0 + 3\mathbf{k} \times [3\mathbf{k} \times (0.4\mathbf{i} + 0.4\mathbf{j})] + 2(3\mathbf{k}) \times [0.707(-1.697)\mathbf{i} + 0.707(-1.697)\mathbf{j}] + 0.707a_{C/A}\mathbf{i} + 0.707a_{C/A}\mathbf{j}$$

$-(a_C)_t\mathbf{i} - 14.40\mathbf{j} = 0 + 0 - 3.60\mathbf{i} - 3.60\mathbf{j} + 7.20\mathbf{i} - 7.20\mathbf{j} + 0.707a_{C/A}\mathbf{i} + 0.707a_{C/A}\mathbf{j}$

$-(a_C)_t = -3.60 + 7.20 + 0.707a_{C/A}$

$-14.40 = -3.60 - 7.20 + 0.707a_{C/A}$

$a_{C/A} = -5.09\text{ m/s}^2$

$(a_C)_t = 0$

Thus,

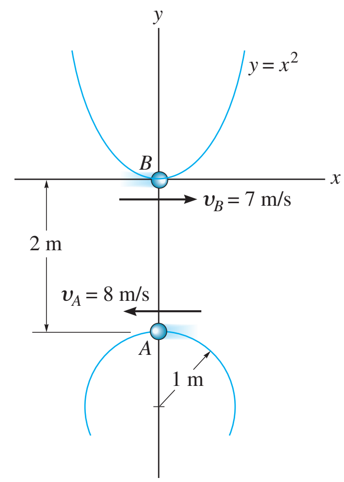
$$a_C = (a_C)_n = \frac{(2.40)^2}{0.4} = 14.4\text{ m/s}^2$$

$a_C = \{-14.4\mathbf{j}\}\text{ m/s}^2$ **Ans.**

Ans:
$v_C = 2.40\text{ m/s}$
$a_C = \{-14.4\mathbf{j}\}\text{ m/s}^2$

16–137.

Particles B and A move along the parabolic and circular paths, respectively. If B has a velocity of 7 m/s in the direction shown and its speed is increasing at 4 m/s^2, while A has a velocity of 8 m/s in the direction shown and its speed is decreasing at 6 m/s^2, determine the relative velocity and relative acceleration of B with respect to A.



SOLUTION

$$\Omega = \frac{8}{1} = 8 \text{ rad/s}^2, \qquad \Omega = \{-8\mathbf{k}\} \text{ rad/s}$$

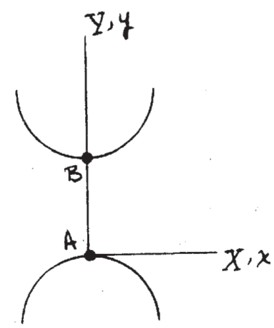
$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$7\mathbf{i} = -8\mathbf{i} + (8\mathbf{k}) \times (2\mathbf{j}) + (\mathbf{v}_{B/A})_{xyz}$$

$$7\mathbf{i} = -8\mathbf{i} - 16\mathbf{i} + (\mathbf{v}_{B/A})_{xyz}$$

$$(\mathbf{v}_{B/A})_{xyz} = \{31.0\mathbf{i}\} \text{ m/s}$$ **Ans.**

$$\dot{\Omega} = \frac{6}{1} = 6 \text{ rad/s}^2, \qquad \dot{\Omega} = \{-6\mathbf{k}\} \text{ rad/s}^2$$



$$(a_A)_n = \frac{(v_A)^2}{1} = \frac{(8)^2}{1} = 64 \text{ m/s}^2 \downarrow$$

$$y = x^2$$

$$\frac{dy}{dx} = 2x\Big|_{x=0} = 0$$

$$\frac{d^2y}{dx^2} = 2$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1+0]^{\frac{3}{2}}}{2} = \frac{1}{2}$$

$$(a_B)_n = \frac{(7)^2}{\frac{1}{2}} = 98 \text{ m/s}^2 \uparrow$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$4\mathbf{i} + 98\mathbf{j} = 6\mathbf{i} - 64\mathbf{j} + (-6\mathbf{k}) \times (2\mathbf{j}) + (8\mathbf{k}) \times (8\mathbf{k} \times 2\mathbf{j}) + 2(8\mathbf{k}) \times (31\mathbf{i}) + (\mathbf{a}_{B/A})_{xyz}$$

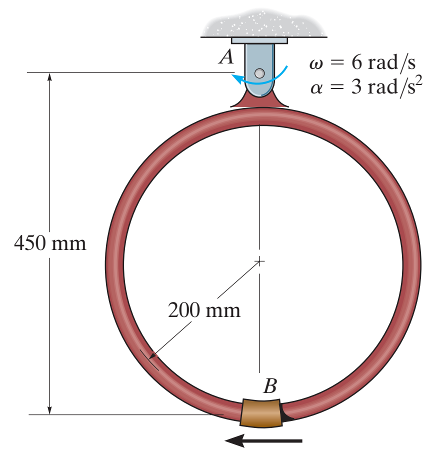
$$4\mathbf{i} + 98\mathbf{j} = 6\mathbf{i} - 64\mathbf{j} + 12\mathbf{i} - 128\mathbf{j} + 496\mathbf{j} + (\mathbf{a}_{B/A})_{xyz}$$

$$(\mathbf{a}_{B/A})_{xyz} = \{-14.0\mathbf{i} - 206\mathbf{j}\} \text{ m/s}^2$$ **Ans.**

Ans:
$(\mathbf{v}_{B/A})_{xyz} = \{31.0\mathbf{i}\}$ m/s
$(\mathbf{a}_{B/A})_{xyz} = \{-14.0\mathbf{i} - 206\mathbf{j}\}$ m/s^2

16–138.

Collar B moves to the left with a speed of 5 m/s, which is increasing at a constant rate of 1.5 m/s^2, relative to the hoop, while the hoop rotates with the angular velocity and angular acceleration shown. Determine the magnitudes of the velocity and acceleration of the collar at this instant.



SOLUTION

Reference Frames: The xyz rotating reference frame is attached to the hoop and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the xyz frame with respect to the XYZ frame is

$$v_A = a_A = \mathbf{0} \qquad \omega = [-6\mathbf{k}]\ \text{rad/s} \qquad \dot{\omega} = \alpha = [-3\mathbf{k}]\ \text{rad/s}^2$$

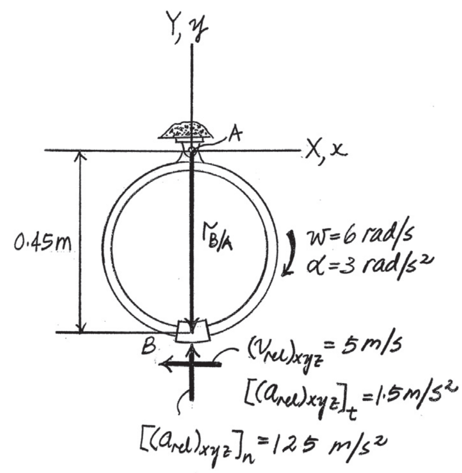
For the motion of collar B with respect to the xyz frame,

$$\mathbf{r}_{B/A} = [-0.45\mathbf{j}]\ \text{m}$$

$$(v_{\text{rel}})_{xyz} = [-5\mathbf{i}]\ \text{m/s}$$

The normal components of $(\mathbf{a}_{\text{rel}})_{xyz}$ is $[(a_{\text{rel}})_{xyz}]_n = \dfrac{(v_{\text{rel}})_{xyz}{}^2}{\rho} = \dfrac{5^2}{0.2} = 125\ \text{m/s}^2$. Thus,

$$(\mathbf{a}_{\text{rel}})_{xyz} = [-1.5\mathbf{i} + 125\mathbf{j}]\ \text{m/s}$$



(a)

Velocity: Applying the relative velocity equation,

$$\begin{aligned}
\mathbf{v}_B &= \mathbf{v}_A + \omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{xyz} \\
&= \mathbf{0} + (-6\mathbf{k}) \times (-0.45\mathbf{j}) + (-5\mathbf{i}) \\
&= [-7.7\mathbf{i}]\ \text{m/s}
\end{aligned}$$

Thus,

$$v_B = 7.7\ \text{m/s} \leftarrow \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\begin{aligned}
\mathbf{a}_B &= \mathbf{a}_A + \dot{\omega} \times \mathbf{r}_{B/A} + \omega \times (\omega \times \mathbf{r}_{B/A}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz} \\
&= \mathbf{0} + (-3\mathbf{k}) \times (-0.45\mathbf{j}) + (-6\mathbf{k}) \times [(-6\mathbf{k}) \times (-0.45\mathbf{j})] + 2(-6\mathbf{k}) \times (-5\mathbf{i}) + (-1.5\mathbf{i} + 125\mathbf{j}) \\
&= [-2.85\mathbf{i} + 201.2\mathbf{j}]\ \text{m/s}^2
\end{aligned}$$

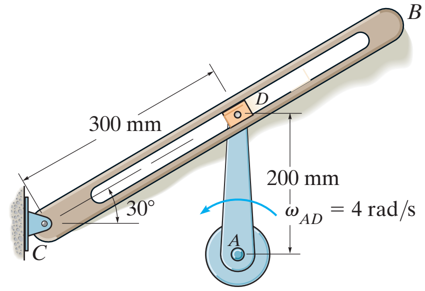
Thus, the magnitude of $\mathbf{a}_B$ is therefore

$$a_B = \sqrt{2.85^2 + 201.2^2} = 201\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_B = 7.7$ m/s
$a_B = 201$ m/s^2

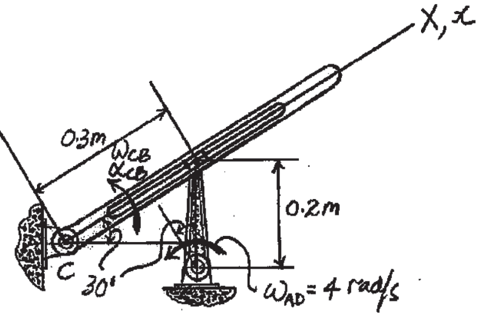
16–139.

Block D of the mechanism is confined to move within the slot of member CB. If link AD is rotating at a constant rate of $\omega_{AD} = 4$ rad/s, determine the angular velocity and angular acceleration of member CB at the instant shown.



SOLUTION

The fixed and rotating $X - Y$ and $x - y$ coordinate system are set to coincide with origin at C as shown in Fig. a. Here the $x - y$ coordinate system is attached to member CB. Thus



(a)

Motion of moving Reference	**Motion of Block D with respect to moving Reference**
$\mathbf{v}_C = \mathbf{0}$	$\mathbf{r}_{D/C} = \{0.3\mathbf{i}\}$ m
$\mathbf{a}_C = \mathbf{0}$	
$\boldsymbol{\Omega} = \boldsymbol{\omega}_{CB} = \omega_{CB}\mathbf{k}$	$(\mathbf{v}_{D/C})_{xyz} = (\mathbf{v}_{D/C})_{xyz}\mathbf{i}$
$\dot{\boldsymbol{\Omega}} = \boldsymbol{\alpha}_{CB} = \alpha_{CB}\mathbf{k}$	$(\mathbf{a}_{D/C})_{xyz} = (\mathbf{a}_{D/C})_{xyz}\mathbf{i}$

The Motions of Block D in the fixed frame are,

$$\mathbf{v}_D = \boldsymbol{\omega}_{A/D} \times \mathbf{r}_{D/A} = (4\mathbf{k}) \times (0.2 \sin 30°\mathbf{i} + 0.2 \cos 30°\mathbf{j}) = \{-0.4\sqrt{3}\mathbf{i} + 0.4\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_D = \boldsymbol{\alpha}_{AD} \times \mathbf{r}_{D/A} - \omega_{AD}^2(\mathbf{r}_{D/A}) = 0 - 4^2(0.2 \sin 30°\mathbf{i} + 0.2 \cos 30°\mathbf{j})$$
$$= \{-1.6\mathbf{i} - 1.6\sqrt{3}\mathbf{j}\} \text{ m/s}^2$$

Applying the relative velocity equation,

$$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\Omega} \times \mathbf{r}_{D/C} + (\mathbf{v}_{D/C})_{xyz}$$

$$-0.4\sqrt{3}\mathbf{i} + 0.4\mathbf{j} = \mathbf{0} + (\omega_{CB}\mathbf{k}) \times (0.3\mathbf{i}) + (v_{D/C})_{xyz}\mathbf{i}$$

$$-0.4\sqrt{3}\mathbf{i} + 0.4\mathbf{j} = (v_{D/C})_{xyz}\mathbf{i} + 0.3\,\omega_{CB}\mathbf{j}$$

Equating **i** and **j** components,

$$(v_{D/C})_{xyz} = -0.4\sqrt{3} \text{ m/s}$$

$$0.4 = 0.3\,\omega_{CB}; \quad \omega_{CB} = 1.3333 \text{ rad/s} = 1.33 \text{ rad/s} \circlearrowleft \qquad \textbf{Ans.}$$

Applying the relative acceleration equation,

$$\mathbf{a}_D = \mathbf{a}_C + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{D/C} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{D/C}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{D/C})_{xyz} + (\mathbf{a}_{D/C})_{xyz}$$

$$-1.6\mathbf{i} - 1.6\sqrt{3}\mathbf{j} = 0 + (\alpha_{CD}\mathbf{k}) \times (0.3\mathbf{i}) + (1.3333\mathbf{k}) \times (1.3333\mathbf{k} \times 0.3\mathbf{i})$$
$$+ 2(1.3333\mathbf{k}) \times (-0.4\sqrt{3}\mathbf{i}) + (a_{D/C})_{xyz}\mathbf{i}$$

$$1.6\mathbf{i} - 1.6\sqrt{3}\mathbf{j} = [(a_{D/C})_{xyz} - 0.5333]\mathbf{i} + (0.3\alpha_{CD} - 1.8475)\mathbf{j}$$

Equating **i** and **j** components

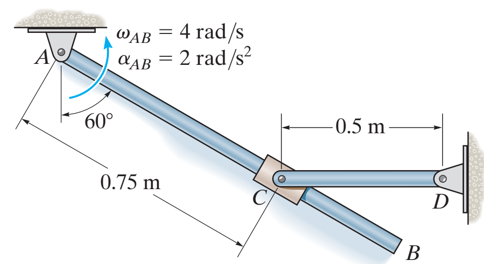
$$1.6 = [(a_{D/C})_{xyz} - 0.5333]; \quad (a_{D/C})_{xyz} = 2.1333 \text{ m/s}^2$$

$$-1.6\sqrt{3} = 0.3\,\alpha_{CD} - 1.8475; \quad \alpha_{CD} = -3.0792 \text{ rad/s}^2 = 3.08 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Ans:
$\omega_{CB} = 1.33$ rad/s $\circlearrowleft$
$\alpha_{CD} = 3.08$ rad/s^2 $\circlearrowright$

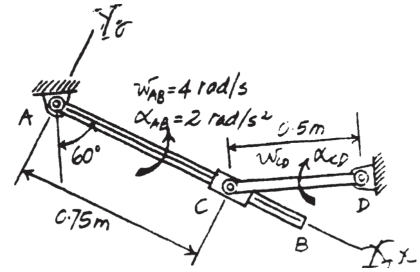
***16–140.**

At the instant shown rod AB has an angular velocity $\omega_{AB} = 4$ rad/s and an angular acceleration $\alpha_{AB} = 2$ rad/s^2. Determine the angular velocity and angular acceleration of rod CD at this instant. The collar at C is pin connected to CD and slides freely along AB.



SOLUTION

Coordinate Axes: The origin of both the fixed and moving frames of reference are located at point A. The x, y, z moving frame is attached to and rotate with rod AB since collar C slides along rod AB.



Kinematic Equation: Applying Eqs. 16–24 and 16–27, we have

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \qquad \textbf{(1)}$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz} \qquad \textbf{(2)}$$

Motion of moving reference	*Motion of C with respect to moving reference*
$\mathbf{v}_A = \mathbf{0}$	$r_{C/A} = \{0.75\mathbf{i}\}$m
$\mathbf{a}_A = \mathbf{0}$	$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{xyz}\,\mathbf{i}$
$\Omega = 4\mathbf{k}$ rad/s	$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{xyz}\,\mathbf{i}$
$\dot{\Omega} = 2\mathbf{k}$ rad/s^2	

The velocity and acceleration of collar C can be determined using Eqs. 16–9 and 16–14 with $\mathbf{r}_{C/D} = \{-0.5\cos 30°\mathbf{i} - 0.5\sin 30°\mathbf{j}\}$m $= \{-0.4330\mathbf{i} - 0.250\mathbf{j}\}$ m.

$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = -\omega_{CD}\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j})$$

$$= -0.250\omega_{CD}\mathbf{i} + 0.4330\omega_{CD}\mathbf{j}$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$$

$$= -\alpha_{CD}\,\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j}) - \omega_{CD}^2(-0.4330\mathbf{i} - 0.250\mathbf{j})$$

$$= \left(0.4330\omega_{CD}^2 - 0.250\,\alpha_{CD}\right)\mathbf{i} + \left(0.4330\alpha_{CD} + 0.250\omega_{CD}^2\right)\mathbf{j}$$

Substitute the above data into Eq.(1) yields

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$-0.250\,\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\mathbf{j} = \mathbf{0} + 4\mathbf{k} \times 0.75\mathbf{i} + (v_{C/A})_{xyz}\,\mathbf{i}$$

$$-0.250\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\,\mathbf{j} = (v_{C/A})_{xyz}\,\mathbf{i} + 3.00\mathbf{j}$$

Equating **i** and **j** components and solve, we have

$$(v_{C/A})_{xyz} = -1.732 \text{ m/s}$$

$$\omega_{CD} = 6.928 \text{ rad/s} = 6.93 \text{ rad/s} \qquad \textbf{Ans.}$$

***16–140. Continued**

Substitute the above data into Eq.(2) yields

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (v_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$\left[0.4330\left(6.928^2\right) - 0.250\,\alpha_{CD}\right]\mathbf{i} + \left[0.4330\alpha_{CD} + 0.250\left(6.928^2\right)\right]\mathbf{j}$$

$$= \mathbf{0} + 2\mathbf{k} \times 0.75\mathbf{i} + 4\mathbf{k} \times (4\mathbf{k} \times 0.75\mathbf{i}) + \mathbf{2}\,(4\mathbf{k}) \times (-1.732\mathbf{i}) + (a_{C/A})_{xyz}\,\mathbf{i}$$

$$(20.78 - 0.250\alpha_{CD})\mathbf{i} + (0.4330\,\alpha_{CD} + 12)\mathbf{j} = \left[(a_{C/A})_{xyz} - 12.0\right]\mathbf{i} - 12.36\mathbf{j}$$

Equating **i** and **j** components, we have

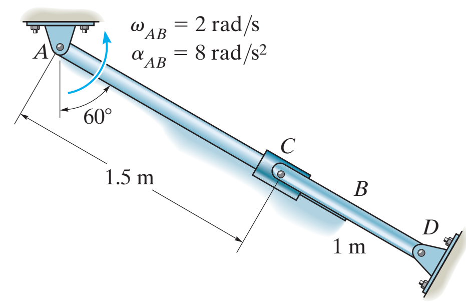
$$(a_{C/A})_{xyz} = 46.85 \text{ m/s}^2$$

$$\alpha_{CD} = -56.2 \text{ rad/s}^2 = 56.2 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Ans:
$\omega_{CD} = 6.93$ rad/s
$\alpha_{CD} = 56.2$ rad/s^2 $\circlearrowright$

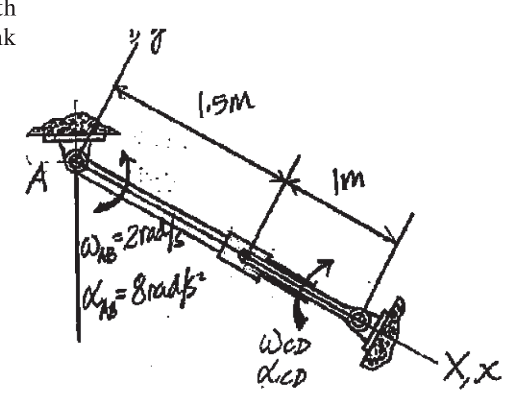
16–141.

The collar C is pinned to rod CD while it slides on rod AB. If rod AB has an angular velocity of 2 rad/s and an angular acceleration of 8 rad/s^2, both acting counterclockwise, determine the angular velocity and the angular acceleration of rod CD at the instant shown.



SOLUTION

The fixed and rotating $X - Y$ and $x - y$ coordinate systems are set to coincide with origin at A as shown in Fig. a. Here, the $x - y$ coordinate system is attached to link AC. Thus,



(a)

Motion of moving Reference	**Motion of collar C with respect to moving Reference**
$\mathbf{v}_A = \mathbf{0}$	$\mathbf{r}_{C/A} = \{1.5\mathbf{i}\}$ m
$\mathbf{a}_A = \mathbf{0}$	
$\mathbf{\Omega} = \boldsymbol{\omega}_{AB} = \{2\mathbf{k}\}$ rad/s	$(\mathbf{v}_{C/A})_{xyz} = (\mathbf{v}_{C/A})_{xyz}\mathbf{i}$
$\dot{\mathbf{\Omega}} = \boldsymbol{\alpha}_{AB} = \{8\mathbf{k}\}$ rad/s^2	$(\mathbf{a}_{C/A})_{xyz} = (\mathbf{a}_{C/A})_{xyz}\mathbf{i}$

The motions of collar C in the fixed system are

$$\mathbf{v}_C = \boldsymbol{\omega}_{CD} \times \mathbf{r}_{C/D} = (-\omega_{CD}\mathbf{k}) \times (-\mathbf{i}) = \omega_{CD}\mathbf{j}$$

$$a_C = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}{}^2\,\mathbf{r}_{C/D} = (-\alpha_{C/D}\mathbf{k}) \times (-\mathbf{i}) - \omega_{CD}^2(-\mathbf{i}) = \omega_{CD}^2\mathbf{i} + \alpha_{CD}\mathbf{j}$$

Applying the relative velocity equation,

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{\Omega} \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$\omega_{CD}\mathbf{j} = \mathbf{0} + (2\mathbf{k}) \times (1.5\mathbf{i}) = (v_{C/A})_{xyz}\mathbf{i}$$

$$\omega_{CD}\mathbf{j} = (v_{C/A})_{xyz}\mathbf{i} + 3\mathbf{j}$$

Equating **i** and **j** components

$$(v_{C/A})_{xyz} = 0$$

$$\omega_{CD} = 3.00 \text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

Applying the relative acceleration equation,

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{C/A} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$3.00^2\mathbf{i} + \alpha_{CD}\mathbf{j} = \mathbf{0} + (8\mathbf{k}) \times (1.5\mathbf{i}) + (2\mathbf{k}) \times (2\mathbf{k} \times 1.5\mathbf{i}) + 2(2\mathbf{k}) \times \mathbf{0} + (a_{C/A})_{xyz}\mathbf{i}$$

$$9\mathbf{i} + \alpha_{CD}\mathbf{j} = \left[(a_{C/A})_{xyz} - 6\right]\mathbf{i} + 12\mathbf{j}$$

Equating **i** and **j** components,

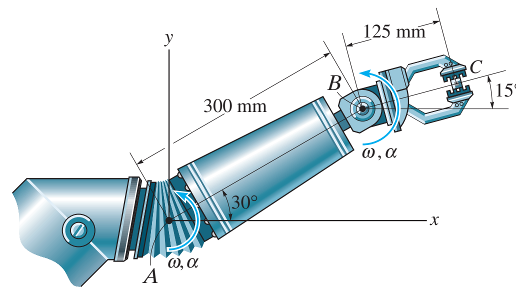
$$9 = (a_{C/A})_{xyz} - 6; \quad (a_{C/A})_{xyz} = 15 \text{ m/s}^2$$

$$\alpha_{CD} = 12.0 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Ans:
$\omega_{CD} = 3.00$ rad/s $\circlearrowright$
$\alpha_{CD} = 12.0$ rad/s^2 $\circlearrowright$

16–142.

At the instant shown, the robotic arm AB is rotating counterclockwise at $\omega = 5$ rad/s and has an angular acceleration $\alpha = 2$ rad/s^2. Simultaneously, the grip BC is rotating counterclockwise at $\omega' = 6$ rad/s and $\alpha' = 2$ rad/s^2, both measured relative to a *fixed* reference. Determine the velocity and acceleration of the object held at the grip C.



SOLUTION

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz} \quad \textbf{(1)}$$

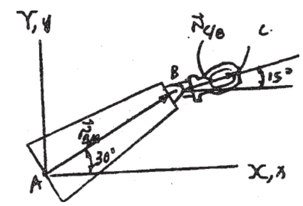
$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz} \quad \textbf{(2)}$$

Motion of moving reference | ***Motion of C with respect to moving reference***

$$\mathbf{r}_{C/B} = \{0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j}\} \text{ m}$$

$\Omega = \{6\mathbf{k}\}$ rad/s $\qquad (\mathbf{v}_{C/B})_{xyz} = 0$

$\dot{\Omega} = \{2\mathbf{k}\}$ rad/s^2 $\qquad (\mathbf{a}_{C/B})_{xyz} = 0$



Motion of *B*:

$$\mathbf{v}_B = \omega \times \mathbf{r}_{B/A}$$
$$= (5\mathbf{k}) \times (0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j})$$
$$= \{-0.75\mathbf{i} + 1.2990\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_B = \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$
$$= (2\mathbf{k}) \times (0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j}) - (5)^2(0.3 \cos 30°\mathbf{i} + 0.3 \sin 30°\mathbf{j})$$
$$= \{-6.7952\mathbf{i} - 3.2304\mathbf{j}\} \text{ m/s}^2$$

Substitute the data into Eqs. (1) and (2) yields:

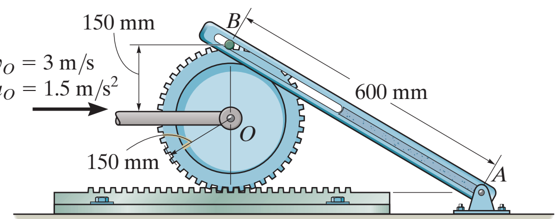
$$\mathbf{v}_C = (-0.75\mathbf{i} + 1.2990\mathbf{j}) + (6\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j}) + 0$$
$$= \{-0.944\mathbf{i} + 2.02\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = (-6.79527\mathbf{i} - 3.2304\mathbf{j}) + (2\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j})$$
$$+ (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.125 \cos 15°\mathbf{i} + 0.125 \sin 15°\mathbf{j})] + 0 + 0$$
$$= \{-11.2\mathbf{i} - 4.15\mathbf{j}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{-0.944\mathbf{i} + 2.02\mathbf{j}\}$ m/s
$\mathbf{a}_C = \{-11.2\mathbf{i} - 4.15\mathbf{j}\}$ m/s^2

16–143.

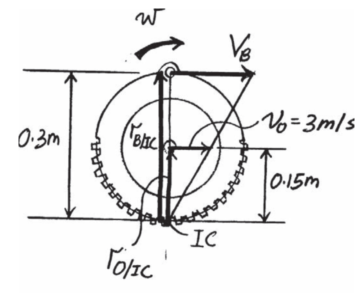
Peg B on the gear slides freely along the slot in link AB. If the gear's center O moves with the velocity and acceleration shown, determine the angular velocity and angular acceleration of the link at this instant.



SOLUTION

Gear Motion: The *IC* of the gear is located at the point where the gear and the gear rack mesh, Fig. *a*. Thus,

$$\omega = \frac{v_O}{r_{O/IC}} = \frac{3}{0.15} = 20 \text{ rad/s}$$



(a)

Then,

$$v_B = \omega r_{B/IC} = 20(0.3) = 6 \text{ m/s} \rightarrow$$

Since the gear rolls on the gear rack, $\alpha = \dfrac{a_O}{r} = \dfrac{1.5}{0.15} = 10$ rad/s. By referring to Fig. *b*,

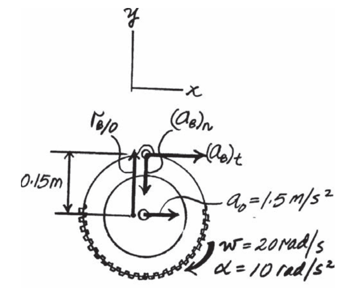
$$\mathbf{a}_B = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$(a_B)_t \mathbf{i} - (a_B)_n \mathbf{j} = 1.5\mathbf{i} + (-10\mathbf{k}) \times 0.15\mathbf{j} - 20^2(0.15\mathbf{j})$$

$$(a_B)_t \mathbf{i} - (a_B)_n \mathbf{j} = 3\mathbf{i} - 60\mathbf{j}$$

Thus,

$$(a_B)_t = 3 \text{ m/s}^2 \qquad (a_B)_n = 60 \text{ m/s}^2$$

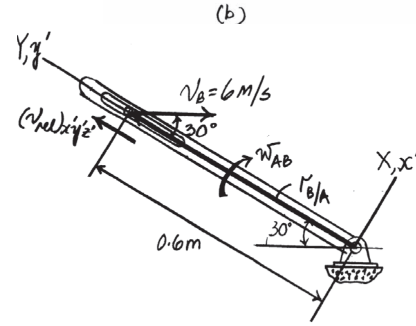


(b)

Reference Frame: The $x'y'z'$ rotating reference frame is attached to link AB and coincides with the XYZ fixed reference frame, Figs. *c* and *d*. Thus, $\mathbf{v}_B$ and $\mathbf{a}_B$ with respect to the XYZ frame is

$$\mathbf{v}_B = [6 \sin 30°\mathbf{i} - 6 \cos 30° \mathbf{j}] = [3\mathbf{i} - 5.196\mathbf{j}] \text{ m/s}$$

$$\mathbf{a}_B = (3 \sin 30° - 60 \cos 30°)\mathbf{i} + (-3 \cos 30° - 60 \sin 30°)\mathbf{j}$$
$$= [-50.46\mathbf{i} - 32.60\mathbf{j}] \text{ m/s}^2$$



(c)

For motion of the $x'y'z'$ frame with reference to the XYZ reference frame,

$$\mathbf{v}_A = \mathbf{a}_A = \mathbf{0} \qquad \boldsymbol{\omega}_{AB} = -\omega_{AB}\mathbf{k} \qquad \dot{\boldsymbol{\omega}}_{AB} = -\alpha_{AB}\,\mathbf{k}$$

For the motion of point B with respect to the $x'y'z'$ frame is

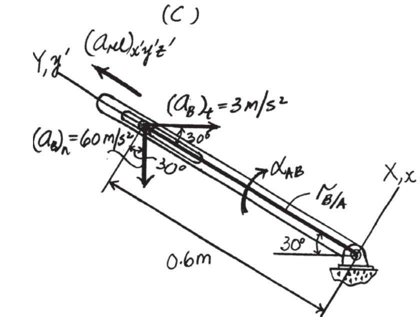
$$\mathbf{r}_{B/A} = [0.6\mathbf{j}]\text{m} \qquad (\mathbf{v}_{\text{rel}})_{x'y'z'} = (v_{\text{rel}})_{x'y'z'}\,\mathbf{j} \qquad (\mathbf{a}_{\text{rel}})_{x'y'z'} = (a_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} + (\mathbf{v}_{\text{rel}})_{x'y'z'}$$

$$3\mathbf{i} - 5.196\mathbf{j} = \mathbf{0} + (-\omega_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (v_{\text{rel}})_{x'y'z'}\,\mathbf{j}$$

$$3\mathbf{i} - 5.196\mathbf{j} = 0.6\omega_{AB}\,\mathbf{i} + (v_{\text{rel}})_{x'y'z'}\mathbf{j}$$



(d)

Equating the **i** and **j** components yields

$$3 = 0.6\omega_{AB} \qquad \omega_{AB} = 5 \text{ rad/s} \qquad \textbf{Ans.}$$

$$(v_{\text{rel}})_{x'y'z'} = -5.196 \text{ m/s}$$

Acceleration: Applying the relative acceleration equation.

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\omega}}_{AB} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\omega}_{AB} \times (\mathbf{v}_{\text{rel}})_{x'y'z'} + (\mathbf{a}_{\text{rel}})_{x'y'z'}$$

$$-50.46\mathbf{i} - 32.60\mathbf{j} = \mathbf{0} + (-\alpha_{AB}\mathbf{k}) \times (0.6\mathbf{j}) + (-5\mathbf{k}) \times [(-5\mathbf{k}) \times (0.6\mathbf{j})] + 2(-5\mathbf{k}) \times (-5.196\mathbf{j}) + (a_{\text{rel}})_{x'y'z'}\mathbf{j}$$

$$-50.46\mathbf{i} - 32.60\mathbf{j} = (0.6\alpha_{AB} - 51.96)\mathbf{i} + \left[(a_{\text{rel}})_{x'y'z'} - 15\right]\mathbf{j}$$

Equating the **i** components,

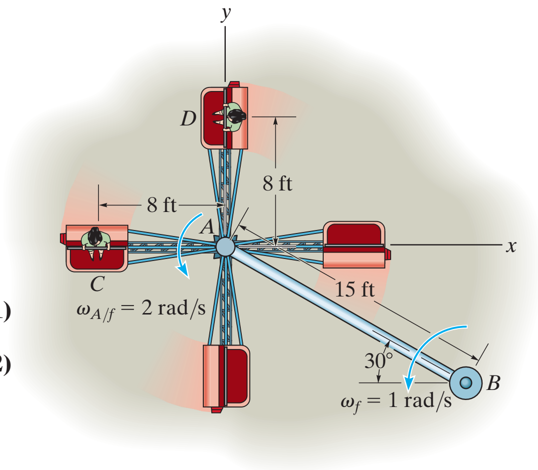
$$-50.46 = 0.6\alpha_{AB} - 51.96$$

$$\alpha_{AB} = 2.5 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega_{AB} = 5$ rad/s ↻
$\alpha_{AB} = 2.5$ rad/s^2 ↻

***16–144.**

The cars on the amusement-park ride rotate around the axle at A with a constant angular velocity $\omega_{A/f} = 2$ rad/s, measured relative to the frame AB. At the same time the frame rotates around the main axle support at B with a constant angular velocity $\omega_f = 1$ rad/s. Determine the velocity and acceleration of the passenger at C at the instant shown.

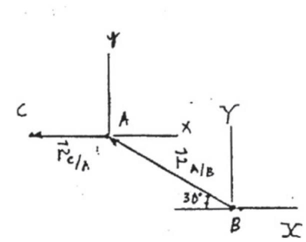


SOLUTION

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \quad \textbf{(1)}$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz} \quad \textbf{(2)}$$

Motion of moving refernce	*Motion of C with respect to moving reference*
$\Omega = \{3\mathbf{k}\}$ rad/s	$\mathbf{r}_{C/A} = \{-8\mathbf{i}\}$ ft
$\dot{\Omega} = \mathbf{0}$	$(\mathbf{v}_{C/A})_{xyz} = \mathbf{0}$
	$(\mathbf{a}_{C/A})_{xyz} = \mathbf{0}$



Motion of A:

$$\mathbf{v}_A = \omega \times \mathbf{r}_{A/B}$$
$$= (1\mathbf{k}) \times (-15\cos 30°\mathbf{i} + 15\sin 30°\mathbf{j})$$
$$= \{-7.5\mathbf{i} - 12.99\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$$
$$= \mathbf{0} - (1)^2(-15\cos 30°\mathbf{i} + 15\sin 30°\mathbf{j})$$
$$= \{12.99\mathbf{i} - 7.5\mathbf{j}\} \text{ ft/s}^2$$

Substitute the data into Eqs.(1) and (2) yields:

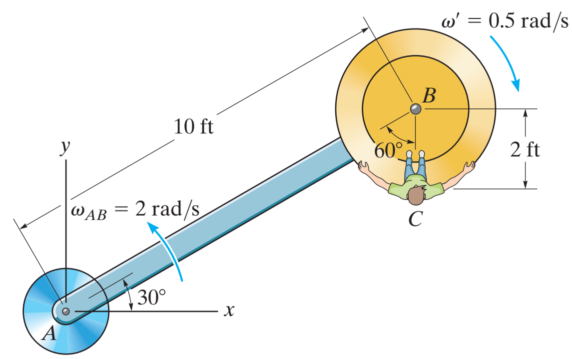
$$\mathbf{v}_C = (-7.5\mathbf{i} - 12.99\mathbf{j}) + (3\mathbf{k}) \times (-8\mathbf{i}) + \mathbf{0}$$
$$= \{-7.5\mathbf{i} - 37.0\mathbf{j}\} \text{ ft/s} \quad \textbf{Ans.}$$

$$\mathbf{a}_C = (12.99\mathbf{i} - 7.5\mathbf{j}) + \mathbf{0} + (3\mathbf{k}) \times [(3\mathbf{k}) \times (-8\mathbf{i}) + \mathbf{0} + \mathbf{0}]$$
$$= \{85.0\mathbf{i} - 7.5\mathbf{j}\} \text{ ft/s}^2 \quad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{-7.5\mathbf{i} - 37.0\mathbf{j}\}$ ft/s
$\mathbf{a}_C = \{85.0\mathbf{i} - 7.5\mathbf{j}\}$ ft/s^2

16–145.

A ride in an amusement park consists of a rotating arm AB having a constant angular velocity $\omega_{AB} = 2$ rad/s about point A and a car mounted at the end of the arm which has a constant angular velocity $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}$ rad/s, measured relative to the arm. At the instant shown, determine the velocity and acceleration of the passenger at C.



SOLUTION

$$\mathbf{r}_{B/A} = (10\cos 30^\circ\,\mathbf{i} + 10\sin 30^\circ\,\mathbf{j}) = \{8.66\mathbf{i} + 5\mathbf{j}\}\text{ ft}$$

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = 2\mathbf{k} \times (8.66\mathbf{i} + 5\mathbf{j}) = \{-10.0\mathbf{i} + 17.32\mathbf{j}\}\text{ ft/s}$$

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$$

$$= 0 - (2)^2(8.66\mathbf{i} + 5\mathbf{j}) = \{-34.64\mathbf{i} - 20\mathbf{j}\}\text{ ft/s}^2$$

$$\Omega = (2 - 0.5)\mathbf{k} = 1.5\mathbf{k}$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= -10.0\mathbf{i} + 17.32\mathbf{j} + 1.5\mathbf{k} \times (-2\mathbf{j}) + 0$$

$$= \{-7.00\mathbf{i} + 17.3\mathbf{j}\}\text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

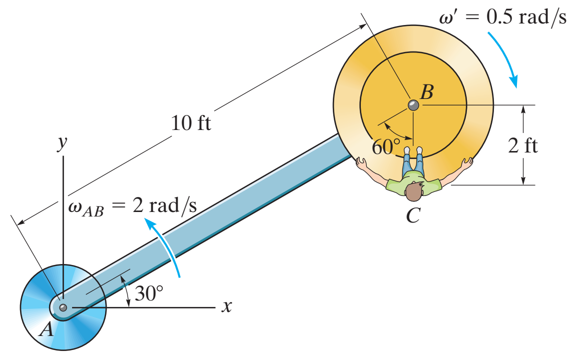
$$= -34.64\mathbf{i} - 20\mathbf{j} + 0 + (1.5\mathbf{k}) \times (1.5\mathbf{k}) \times (-2\mathbf{j}) + 0 + 0$$

$$= \{-34.6\mathbf{i} - 15.5\mathbf{j}\}\text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{-7.00\mathbf{i} + 17.3\mathbf{j}\}$ ft/s
$\mathbf{a}_C = \{-34.6\mathbf{i} - 15.5\mathbf{j}\}$ ft/s^2

16–146.

A ride in an amusement park consists of a rotating arm AB that has an angular acceleration of $\alpha_{AB} = 1\ \text{rad/s}^2$ when $\omega_{AB} = 2$ rad/s at the instant shown. Also at this instant the car mounted at the end of the arm has an angular acceleration of $\boldsymbol{\alpha} = \{-0.6\mathbf{k}\}\ \text{rad/s}^2$ and angular velocity of $\boldsymbol{\omega}' = \{-0.5\mathbf{k}\}$ rad/s, measured relative to the arm. Determine the velocity and acceleration of the passenger C at this instant.



SOLUTION

$$\mathbf{r}_{B/A} = (10\cos 30°\mathbf{i} + 10\sin 30°\mathbf{j}) = \{8.66\mathbf{i} + 5\mathbf{j}\}\ \text{ft}$$

$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = 2\mathbf{k} \times (8.66\mathbf{i} + 5\mathbf{j}) = \{-10.0\mathbf{i} + 17.32\mathbf{j}\}\ \text{ft/s}$$

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\,\mathbf{r}_{B/A}$$

$$= (1\mathbf{k}) \times (8.66\mathbf{i} + 5\mathbf{j}) - (2)^2(8.66\mathbf{i} + 5\mathbf{j}) = \{-39.64\mathbf{i} - 11.34\mathbf{j}\}\ \text{ft/s}^2$$

$$\Omega = (2-0.5)\mathbf{k} = 1.5\mathbf{k}$$

$$\dot{\Omega} = (1 - 0.6)\mathbf{k} = 0.4\mathbf{k}$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= -10.0\mathbf{i} + 17.32\mathbf{j} + 1.5\mathbf{k} \times (-2\mathbf{j}) + 0$$

$$= \{-7.00\mathbf{i} + 17.3\mathbf{j}\}\ \text{ft/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

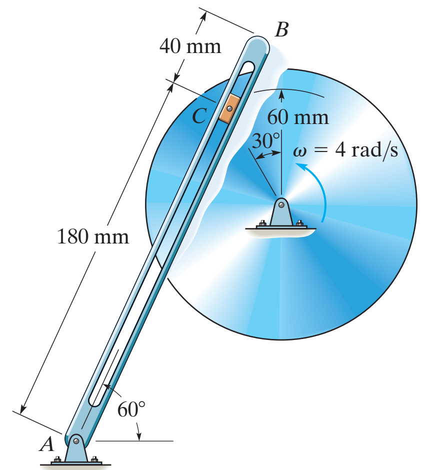
$$= -39.64\mathbf{i} - 11.34\mathbf{j} + (0.4\mathbf{k}) \times (-2\mathbf{j}) + (1.5\mathbf{k}) \times (1.5\mathbf{k}) \times (-2\mathbf{j}) + 0 + 0$$

$$= \{-38.8\mathbf{i} - 6.84\mathbf{j}\}\ \text{ft/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_C = \{-7.00\mathbf{i} + 17.3\mathbf{j}\}$ ft/s
$\mathbf{a}_C = \{-38.8\mathbf{i} - 6.84\mathbf{j}\}\ \text{ft/s}^2$

16–147.

If the slider block C is fixed to the disk that has a constant counterclockwise angular velocity of 4 rad/s, determine the angular velocity and angular acceleration of the slotted arm AB at the instant shown.

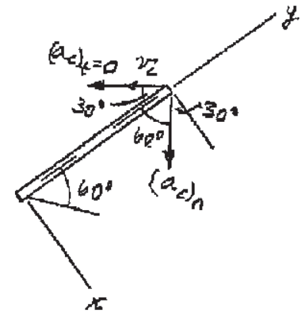


SOLUTION

$\mathbf{v}_C = -(4)(60)\sin 30°\mathbf{i} - 4(60)\cos 30°\mathbf{j} = -120\mathbf{i} - 207.85\mathbf{j}$

$\mathbf{a}_C = (4)^2(60)\sin 60°\mathbf{i} - (4)^2(60)\cos 60°\mathbf{j} = 831.38\mathbf{i} - 480\mathbf{j}$

Thus,



$\mathbf{v}_C = \mathbf{v}_A + \mathbf{\Omega} \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$

$-120\mathbf{i} - 207.85\mathbf{j} = \mathbf{0} + (\omega_{AB}\mathbf{k}) \times (180\mathbf{j}) - v_{C/A}\mathbf{j}$

$-120 = -180\omega_{AB}$

$\omega_{AB} = 0.667 \text{ rad/s} \circlearrowleft$ **Ans.**

$-207.85 = -v_{C/A}$

$v_{C/A} = 207.85 \text{ mm/s}$

$\mathbf{a}_C = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{C/A} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{C/A}) + 2\mathbf{\Omega} \times (v_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$

$831.38\mathbf{i} - 480\mathbf{j} = 0 + (\alpha_{AB}\mathbf{k}) \times (180\mathbf{j}) + (0.667\mathbf{k}) \times [(0.667\mathbf{k}) \times (180\mathbf{j})]$

$\quad + 2(0.667\mathbf{k}) \times (-207.85\mathbf{j}) - a_{C/A}\mathbf{j}$

$831.38\mathbf{i} - 480\mathbf{j} = -180\,\alpha_{AB}\mathbf{i} - 80\mathbf{j} + 277.13\mathbf{i} - a_{C/A}\mathbf{j}$

$831.38 = -180\alpha_{AB} + 277.13$

$\alpha_{AB} = -3.08$

Thus,

$\alpha_{AB} = 3.08 \text{ rad/s}^2 \circlearrowright$ **Ans.**

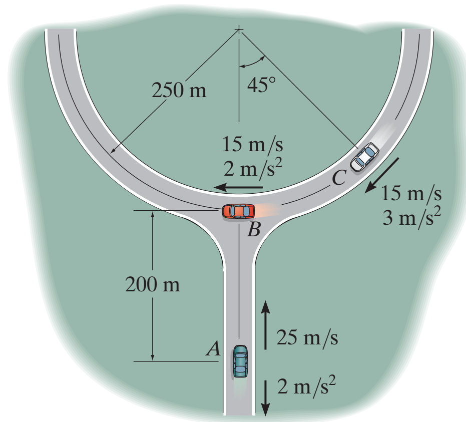
$-480 = -80 - a_{C/A}$

$a_{C/A} = 400 \text{ mm/s}^2$

Ans:
$\omega_{AB} = 0.667 \text{ rad/s} \circlearrowleft$
$\alpha_{AB} = 3.08 \text{ rad/s}^2 \circlearrowright$

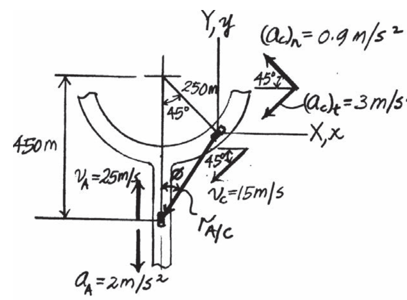
***16–148.**

At the instant shown, car A travels with a speed of $25\,\text{m/s}$, which is decreasing at a constant rate of $2\,\text{m/s}^2$, while car C travels with a speed of $15\,\text{m/s}$, which is increasing at a constant rate of $3\,\text{m/s}^2$. Determine the velocity and acceleration of car A with respect to car C.



SOLUTION

Reference Frame: The xyz rotating reference frame is attached to car C and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since car C moves along the circular road, its normal component of acceleration is $(a_C)_n = \dfrac{{v_C}^2}{\rho} = \dfrac{15^2}{250} = 0.9\ \text{m/s}^2$. Thus, the motion of car C with respect to the XYZ frame is



(a)

$$\mathbf{v}_C = -15\cos 45°\mathbf{i} - 15\sin 45°\mathbf{j} = [-10.607\mathbf{i} - 10.607\mathbf{j}]\ \text{m/s}$$

$$\mathbf{a}_C = (-0.9\cos 45° - 3\cos 45°)\mathbf{i} + (0.9\sin 45° - 3\sin 45°)\mathbf{j} = [-2.758\mathbf{i} - 1.485\mathbf{j}]\ \text{m/s}^2$$

Also, the angular velocity and angular acceleration of the xyz reference frame is

$$\omega = \frac{v_C}{\rho} = \frac{15}{250} = 0.06\ \text{rad/s} \qquad \omega = [-0.06\mathbf{k}]\ \text{rad/s}$$

$$\dot{\omega} = \frac{(a_C)_t}{\rho} = \frac{3}{250} = 0.012\ \text{rad/s}^2 \qquad \dot{\omega} = [-0.012\mathbf{k}]\ \text{rad/s}^2$$

The velocity and accdeleration of car A with respect to the XYZ frame is

$$\mathbf{v}_A = [25\mathbf{j}]\ \text{m/s} \qquad \mathbf{a}_A = [-2\mathbf{j}]\ \text{m/s}^2$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{A/C} = -250\sin 45°\mathbf{i} - (450 - 250\cos 45°)\mathbf{j} = [-176.78\mathbf{i} - 273.22\mathbf{j}]\ \text{m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_C + \omega \times \mathbf{r}_{A/C} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = (-10.607\mathbf{i} - 10.607\mathbf{j}) + (-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j}) + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$25\mathbf{j} = -27\mathbf{i} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$(\mathbf{v}_{\text{rel}})_{xyz} = [27\mathbf{i} + 25\mathbf{j}]\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_A = \mathbf{a}_C + \dot{\omega} \times r_{A/C} + \omega \times (\omega \times \mathbf{r}_{A/C}) + 2\omega \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-2\mathbf{j} = (-2.758\mathbf{i} - 1.485\mathbf{j}) + (-0.012\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j})$$
$$+ (-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 273.22\mathbf{j})] + 2(-0.06\mathbf{k}) \times (27\mathbf{i} + 25\mathbf{j}) + (\mathbf{a}_{\text{rel}})_{xyz}$$

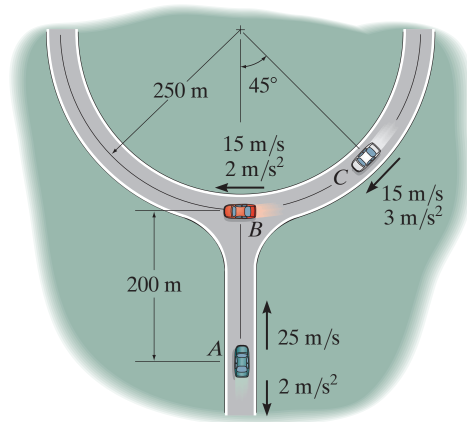
$$-2\mathbf{j} = -2.4\mathbf{i} - 1.62\mathbf{j} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$(\mathbf{a}_{\text{rel}})_{xyz} = [2.4\mathbf{i} - 0.38\mathbf{j}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$(\mathbf{v}_{\text{rel}})_{xyz} = [27\mathbf{i} + 25\mathbf{j}]\ \text{m/s}$
$(\mathbf{a}_{\text{rel}})_{xyz} = [2.4\mathbf{i} - 0.38\mathbf{j}]\ \text{m/s}^2$

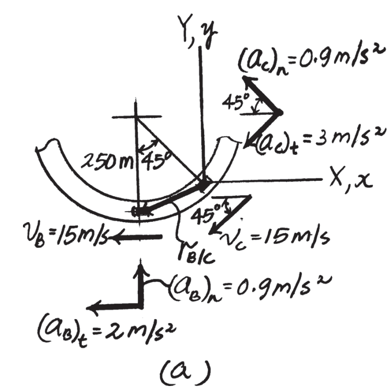
16–149.

At the instant shown, car B travels with a speed of 15 m/s, which is increasing at a constant rate of $2\,\mathrm{m/s^2}$, while car C travels with a speed of 15 m/s, which is increasing at a constant rate of $3\ \mathrm{m/s^2}$. Determine the velocity and acceleration of car B with respect to car C.



SOLUTION

Reference Frame: The xyz rotating reference frame is attached to C and coincides with the XYZ fixed reference frame at the instant considered, Fig. a. Since B and C move along the circular road, their normal components of acceleration are $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{15^2}{250} = 0.9\ \mathrm{m/s^2}$ and $(a_C)_n = \dfrac{v_C{}^2}{\rho} = \dfrac{15^2}{250} = 0.9\ \mathrm{m/s^2}$. Thus, the motion of cars B and C with respect to the XYZ frame are



$$\mathbf{v}_B = [-15\mathbf{i}]\ \mathrm{m/s}$$

$$\mathbf{v}_C = [-15\cos 45°\mathbf{i} - 15\sin 45°\mathbf{j}] = [-10.607\mathbf{i} - 10.607\mathbf{j}]\ \mathrm{m/s}$$

$$\mathbf{a}_B = [-2\mathbf{i} + 0.9\mathbf{j}]\ \mathrm{m/s^2}$$

$$\mathbf{a}_C = (-0.9\cos 45° - 3\cos 45°)\mathbf{i} + (0.9\sin 45° - 3\sin 45°)\mathbf{j} = [-2.758\mathbf{i} - 1.485\,\mathbf{j}]\ \mathrm{m/s^2}$$

Also, the angular velocity and angular acceleration of the xyz reference frame with respect to the XYZ reference frame are

$$\omega = \frac{v_C}{\rho} = \frac{15}{250} = 0.06\ \mathrm{rad/s} \qquad \omega = [-0.06\mathbf{k}]\ \mathrm{rad/s}$$

$$\dot{\omega} = \frac{(a_C)_t}{\rho} = \frac{3}{250} = 0.012\ \mathrm{rad/s^2} \qquad \dot{\omega} = [-0.012\mathbf{k}]\ \mathrm{rad/s^2}$$

From the geometry shown in Fig. a,

$$\mathbf{r}_{B/C} = -250\sin 45°\mathbf{i} - (250 - 250\cos 45°)\mathbf{j} = [-176.78\mathbf{i} - 73.22\,\mathbf{j}]\ \mathrm{m}$$

Velocity: Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times r_{B/C} + (\mathbf{v}_{\mathrm{rel}})_{xyz}$$

$$-15\mathbf{i} = (-10.607\mathbf{i} - 10.607\mathbf{j}) + (-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j}) + (\mathbf{v}_{\mathrm{rel}})_{xyz}$$

$$-15\mathbf{i} = -15\mathbf{i} + (\mathbf{v}_{\mathrm{rel}})_{xyz}$$

$$(\mathbf{v}_{\mathrm{rel}})_{xyz} = \mathbf{0} \qquad \textbf{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \dot{\omega} \times \mathbf{r}_{B/C} + \omega \times (\omega \times \mathbf{r}_{B/C}) + 2\omega \times (\mathbf{v}_{\mathrm{rel}})_{xyz} + (\mathbf{a}_{\mathrm{rel}})_{xyz}$$

$$-2\mathbf{i} + 0.9\mathbf{j} = (-2.758\mathbf{i} - 1.485\mathbf{j}) + (-0.012\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j})$$

$$+(-0.06\mathbf{k}) \times [(-0.06\mathbf{k}) \times (-176.78\mathbf{i} - 73.22\mathbf{j})] + 2(-0.06\mathbf{k}) \times \mathbf{0} + (\mathbf{a}_{\mathrm{rel}})_{xyz}$$

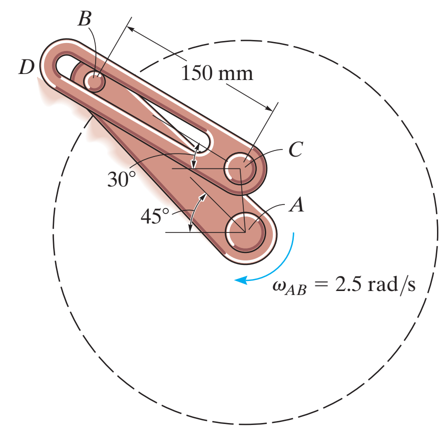
$$-2\mathbf{i} + 0.9\mathbf{j} = -3\mathbf{i} + 0.9\mathbf{j} + (\mathbf{a}_{\mathrm{rel}})_{xyz}$$

$$(a_{\mathrm{rel}})_{xyz} = [1\mathbf{i}]\ \mathrm{m/s^2} \qquad \textbf{Ans.}$$

Ans:
$(\mathbf{v}_{\mathrm{rel}})_{xyz} = \mathbf{0}$
$(\mathbf{a}_{\mathrm{rel}})_{xyz} = \{1\mathbf{i}\}\ \mathrm{m/s^2}$

16–150.

The two-link mechanism serves to amplify angular motion. Link AB has a pin at B which is confined to move within the slot of link CD. If at the instant shown, AB (input) has an angular velocity of $\omega_{AB} = 2.5$ rad/s, determine the angular velocity of CD (output) at this instant.



SOLUTION

$$\frac{r_{BA}}{\sin 120^\circ} = \frac{0.15\text{ m}}{\sin 45^\circ}$$

$$r_{BA} = 0.1837\text{ m}$$

$$\mathbf{v}_C = \mathbf{0}$$

$$\mathbf{a}_C = \mathbf{0}$$

$$\Omega = -\omega_{DC}\mathbf{k}$$

$$\dot{\Omega} = -\alpha_{DC}\mathbf{k}$$

$$\mathbf{r}_{B/C} = \{-0.15\,\mathbf{i}\}\text{ m}$$

$$(\mathbf{v}_{B/C})_{xyz} = (v_{B/C})_{xyz}\mathbf{i}$$

$$(\mathbf{a}_{B/C})_{xyz} = (a_{B/C})_{xyz}\mathbf{i}$$

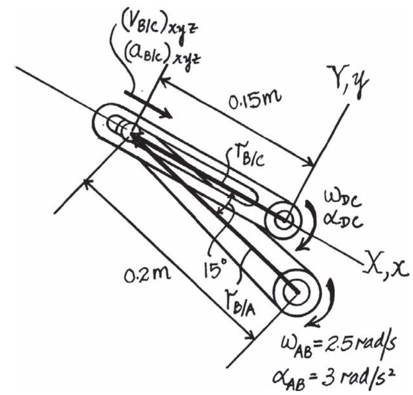
$$\mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{B/A} = (-2.5\mathbf{k}) \times (-0.1837\cos 15^\circ\mathbf{i} + 0.1837\sin 15^\circ\mathbf{j})$$

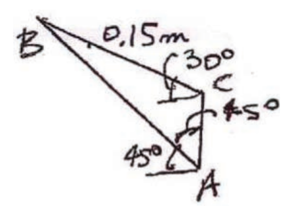
$$= \{0.1189\mathbf{i} + 0.4436\mathbf{j}\}\text{ m/s}$$

$$\mathbf{v}_B = \mathbf{v}_C + \Omega \times \mathbf{r}_{B/C} + (\mathbf{v}_{B/C})_{xyz}$$

$$0.1189\mathbf{i} + 0.4436\mathbf{j} = \mathbf{0} + (-\omega_{DC}\mathbf{k}) \times (-0.15\mathbf{i}) + (v_{B/C})_{xyz}\mathbf{i}$$

$$0.1189\mathbf{i} + 0.4436\mathbf{j} = (v_{B/C})_{xyz}\mathbf{i} + 0.15\omega_{DC}\mathbf{j}$$





Solving:

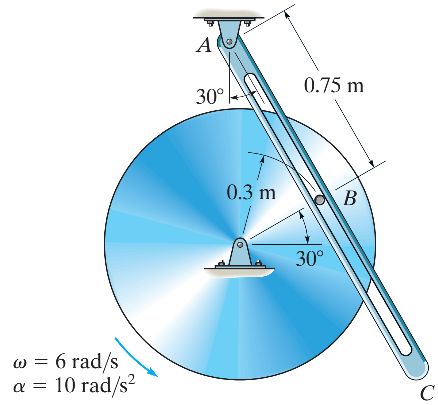
$$(v_{B/C})_{xyz} = 0.1189\text{ m/s}$$

$$\omega_{DC} = 2.96\text{ rad/s} \circlearrowright \qquad \textbf{Ans.}$$

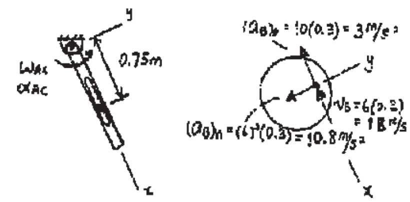
Ans:
$\omega_{DC} = 2.96$ rad/s $\circlearrowright$

16–151.

The disk rotates with the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link AC at this instant. The peg at B is fixed to the disk.



SOLUTION



$\mathbf{v}_B = -6(0.3)\mathbf{i} = -1.8\mathbf{i}$

$\mathbf{a}_B = -10(0.3)\mathbf{i} - (6)^2(0.3)\mathbf{j} = -3\mathbf{i} - 10.8\mathbf{j}$

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (v_{B/A})_{xyz}$

$-1.8\mathbf{i} = 0 + (\omega_{AC}\mathbf{k}) \times (0.75\mathbf{i}) - (v_{B/A})_{xyz}\mathbf{i}$

$-1.8\mathbf{i} = -(v_{B/A})_{xyz}$

$(v_{B/A})_{xyz} = 1.8\text{ m/s}$

$0 = \omega_{AC}(0.75)$

$\omega_{AC} = 0$ **Ans.**

$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (v_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

$-3\mathbf{i} - 10.8\mathbf{j} = \mathbf{0} + \alpha_{AC}\mathbf{k} \times (0.75\mathbf{i}) + \mathbf{0} + \mathbf{0} - a_{A/B}\mathbf{i}$

$-3 = -a_{A/B}$

$a_{A/B} = 3\text{ m/s}^2$

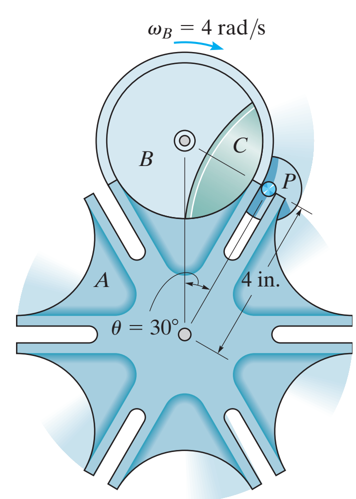
$-10.8 = \alpha_{A/C}(0.75)$

$\alpha_{A/C} = 14.4\text{ rad/s}^2 \circlearrowright$ **Ans.**

Ans:
$\omega_{AC} = 0$
$\alpha_{AC} = 14.4\text{ rad/s}^2 \circlearrowright$

***16–152.**

The Geneva mechanism is used in a packaging system to convert constant angular motion into intermittent angular motion. The star wheel A makes one sixth of a revolution for each full revolution of the driving wheel B and the attached guide C. To do this, pin P, which is attached to B, slides into one of the radial slots of A, thereby turning wheel A, and then exits the slot. If B has a constant angular velocity of $\omega_B = 4$ rad/s, determine $\boldsymbol{\omega}_A$ and $\boldsymbol{\alpha}_A$ of wheel A at the instant shown.



SOLUTION

The circular path of motion of P has a radius of

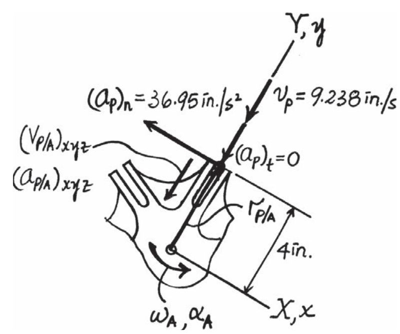
$$r_P = 4 \tan 30° = 2.309 \text{ in.}$$

Thus,

$$\mathbf{v}_P = -4(2.309)\mathbf{j} = -9.238\mathbf{j}$$

$$\mathbf{a}_P = -(4)^2(2.309)\mathbf{i} = -36.95\mathbf{i}$$

Thus,



$$\mathbf{v}_P = \mathbf{v}_A + \Omega \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$

$$-9.238\mathbf{j} = \mathbf{0} + (\omega_A\,\mathbf{k}) \times (4\mathbf{j}) - v_{P/A}\,\mathbf{j}$$

Solving,

$$\omega_A = 0 \qquad \textbf{Ans.}$$

$$v_{P/A} = 9.238 \text{ in./s}$$

$$\mathbf{a}_P = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{P/A} + \Omega \times (\Omega \times \mathbf{r}_{P/A}) + 2\Omega \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$

$$-36.95\mathbf{i} = \mathbf{0} + (\alpha_A\mathbf{k}) \times (4\mathbf{j}) + \mathbf{0} + \mathbf{0} - a_{P/A}\,\mathbf{j}$$

Solving,

$$-36.95 = -4\alpha_A$$

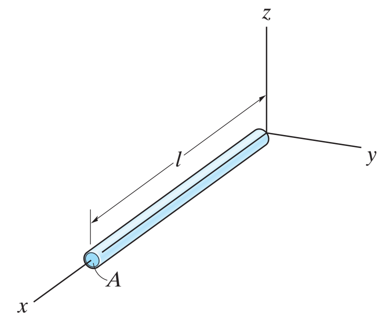
$$\alpha_A = 9.24 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$a_{P/A} = 0$$

Ans:
$\omega_A = 0$
$\alpha_A = 9.24 \text{ rad/s}^2 \circlearrowright$

17–1.

Determine the moment of inertia I_y for the slender rod. The rod's density ρ and cross-sectional area A are constant. Express the result in terms of the rod's total mass m.



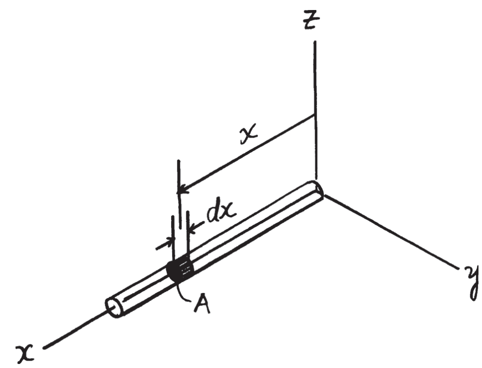
SOLUTION

$$I_y = \int_M x^2\, dm$$

$$= \int_0^l x^2\,(\rho\, A\, dx)$$

$$= \frac{1}{3}\rho\, A\, l^3$$

$$m = \rho\, A\, l$$



Thus,

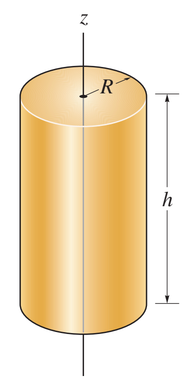
$$I_y = \frac{1}{3} m\, l^2 \qquad \textbf{Ans.}$$

Ans:

$I_y = \frac{1}{3} m\, l^2$

17–2.

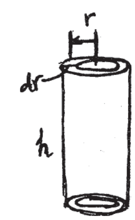
The solid cylinder has an outer radius R, height h, and is made from a material having a density that varies from its center as $\rho = k + ar^2$, where k and a are constants. Determine the mass of the cylinder and its moment of inertia about the z axis.



SOLUTION

Consider a shell element of radius r and mass

$$dm = \rho\, dV = \rho(2\pi\, r\, dr)h$$



$$m = \int_0^R (k + ar^2)(2\pi\, r\, dr)h$$

$$m = 2\pi\, h\left(\frac{kR^2}{2} + \frac{aR^4}{4}\right)$$

$$m = \pi\, h\, R^2\left(k + \frac{aR^2}{2}\right) \qquad \textbf{Ans.}$$

$$dI = r^2\, dm = r^2(\rho)(2\pi\, r\, dr)h$$

$$I_z = \int_0^R r^2(k + ar^2)(2\pi\, r\, dr)\, h$$

$$I_z = 2\pi h \int_0^R (k\, r^3 + a\, r^5)\, dr$$

$$I_z = 2\pi h\left[\frac{k\, R^4}{4} + \frac{aR^6}{6}\right]$$

$$I_z = \frac{\pi\, h\, R^4}{2}\left[k + \frac{2\, aR^2}{3}\right] \qquad \textbf{Ans.}$$

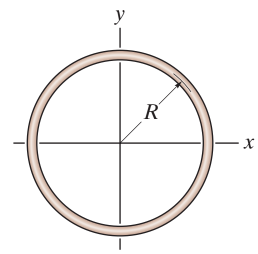
Ans:

$$m = \pi\, h\, R^2\left(k + \frac{aR^2}{2}\right)$$

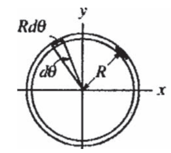
$$I_z = \frac{\pi\, h\, R^4}{2}\left[k + \frac{2\, aR^2}{3}\right]$$

17–3.

Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.



SOLUTION



$$I_z = \int_0^{2\pi} \rho A (R\,d\theta) R^2 = 2\pi\, \rho\, A\, R^3$$

$$m = \int_0^{2\pi} \rho\, A\, R\, d\theta = 2\pi\, \rho\, A\, R$$

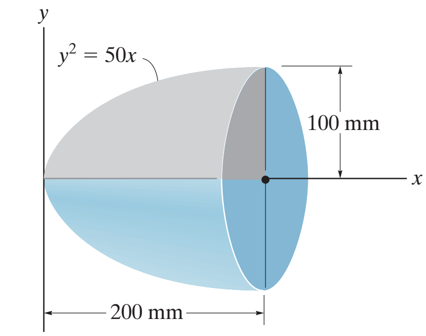
Thus,

$$I_z = m R^2 \qquad \textbf{Ans.}$$

Ans:
$I_z = mR^2$

***17–4.**

The paraboloid is formed by revolving the shaded area around the x axis. Determine the radius of gyration k_x. The density of the material is $\rho = 5\ \text{Mg/m}^3$.



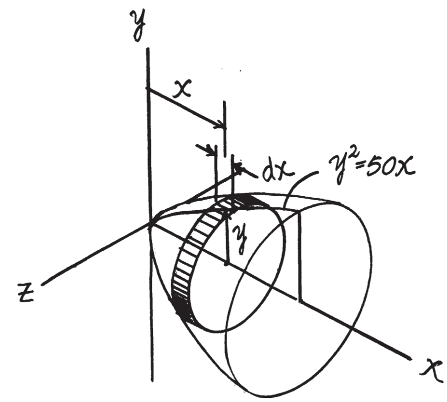
SOLUTION

$$dm = \rho\,\pi\,y^2\,dx = \rho\,\pi\,(50x)\,dx$$

$$I_x = \int \frac{1}{2} y^2\,dm = \frac{1}{2}\int_0^{200} 50\,x\,\{\pi\,\rho\,(50x)\}\,dx$$

$$= \rho\,\pi\left(\frac{50^2}{2}\right)\left[\frac{1}{3}x^3\right]_0^{200}$$

$$= \rho\,\pi\left(\frac{50^2}{6}\right)(200)^3$$



$$m = \int dm = \int_0^{200} \pi\,\rho\,(50x)\,dx$$

$$= \rho\,\pi\,(50)\left[\frac{1}{2}x^2\right]_0^{200}$$

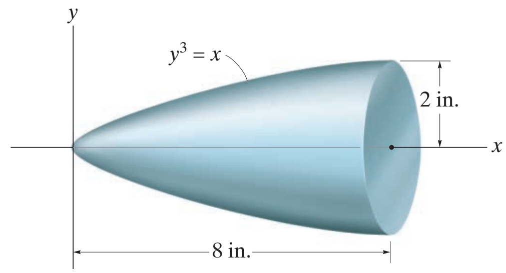
$$= \rho\,\pi\left(\frac{50}{2}\right)(200)^2$$

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{50}{3}(200)} = 57.7\ \text{mm}$$ **Ans.**

Ans:
$k_x = 57.7\ \text{mm}$

17–5.

Determine the radius of gyration k_x of the body. The specific weight of the material is $\gamma = 380\ \text{lb/ft}^3$.



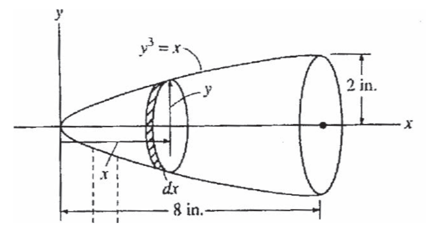
SOLUTION

$$dm = \rho\, dV = \rho\pi\, y^2\, dx$$

$$d\,I_x = \frac{1}{2}(dm)\, y^2 = \frac{1}{2}\pi\rho y^4\, dx$$

$$I_x = \int_0^8 \frac{1}{2}\pi\rho x^{4/3}\, dx = 86.17\rho$$

$$m = \int_0^8 \pi\rho x^{2/3}\, dx = 60.32\rho$$

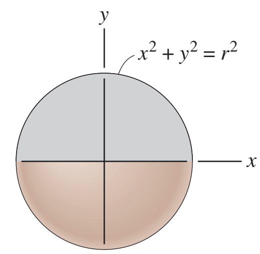


$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{86.17\rho}{60.32\rho}} = 1.20\ \text{in.}$$ **Ans.**

Ans:
$k_x = 1.20$ in.

17–6.

The sphere is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the sphere. The material has a constant density ρ.



SOLUTION

$$dI_x = \frac{y^2\,dm}{2}$$

$$dm = \rho\,dV = \rho(\pi y^2\,dx) = \rho\,\pi(r^2 - x^2)\,dx$$

$$dI_x = \frac{1}{2}\rho\,\pi(r^2 - x^2)^2\;dx$$

$$I_x = \int_{-r}^{r}\frac{1}{2}\rho\,\pi(r^2 - x^2)^2\,dx$$

$$= \frac{8}{15}\pi\rho\,r^5$$

$$m = \int_{-r}^{r}\rho\,\pi(r^2 - x^2)\,dx$$

$$= \frac{4}{3}\rho\,\pi\,r^3$$

Thus,

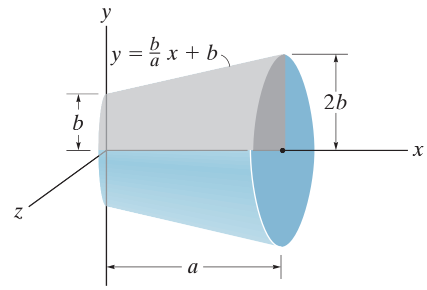
$$I_x = \frac{2}{5}m\,r^2$$ **Ans.**

Ans:

$I_x = \dfrac{2}{5}mr^2$

17–7.

The frustum is formed by rotating the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the frustum. The frustum has a constant density ρ.



SOLUTION

$$dm = \rho\, dV = \rho\pi y^2\, dx = \rho\pi\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx$$

$$dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}\rho\pi y^4\, dx$$

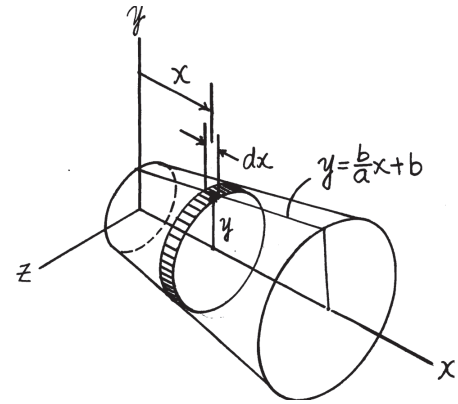
$$dI_x = \frac{1}{2}\rho\pi\left(\frac{b^4}{a^4}x^4 + \frac{4\,b^4}{a^3}x^3 + \frac{6\,b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$

$$I_x = \int dI_x = \frac{1}{2}\rho\pi\int_0^a\left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6\,b^4}{a^2}x^2 + \frac{4\,b^4}{a}x + b^4\right)dx$$

$$= \frac{31}{10}\rho\pi ab^4$$

$$m = \int_m dm = \rho\pi\int_0^a\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx = \frac{7}{3}\rho\pi ab^2$$

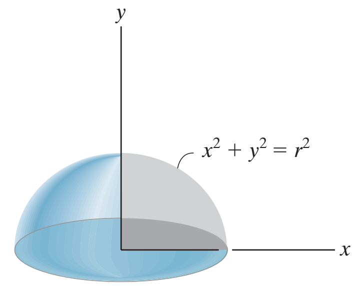
$$I_x = \frac{93}{70}mb^2$$ **Ans.**



Ans:
$I_x = \frac{93}{70}mb^2$

***17–8.**

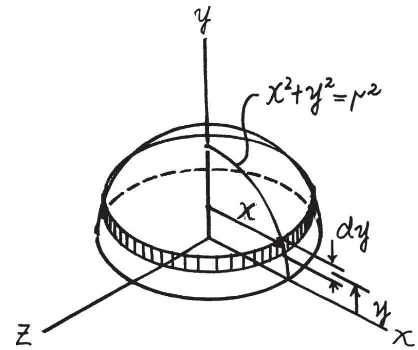
The hemisphere is formed by rotating the shaded area around the y axis. Determine the moment of inertia I_y and express the result in terms of the total mass m of the hemisphere. The material has a constant density ρ.



SOLUTION

$$m = \int_V \rho \, dV = \rho \int_0^r \pi x^2 dy = \rho\pi \int_0^r (r^2 - y^2) dy$$

$$= \rho\pi\left[r^2 y - \frac{1}{3}y^3\right]_0^r = \frac{2}{3}\rho\pi r^3$$



$$I_y = \int_m \frac{1}{2}(dm)\, x^2 = \frac{\rho}{2}\int_0^r \pi x^4 dy = \frac{\rho\pi}{2}\int_0^r (r^2 - y^2)^2 dy$$

$$= \frac{\rho\pi}{2}\left[r^4 y - \frac{2}{3}r^2 y^3 + \frac{y^5}{5}\right]_0^r = \frac{4\rho\pi}{15} r^5$$

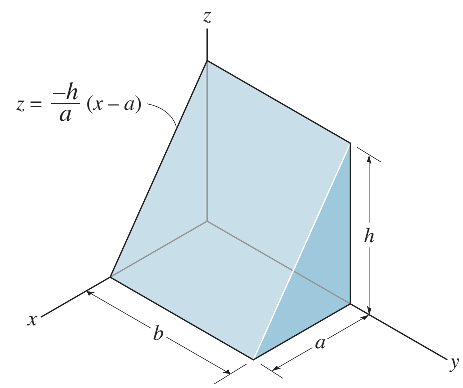
Thus,

$$I_y = \frac{2}{5} m r^2$$ **Ans.**

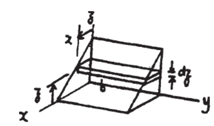
Ans:
$I_y = \frac{2}{5}mr^2$

17–9.

Determine the moment of inertia of the homogeneous triangular prism with respect to the y axis. Express the result in terms of the mass m of the prism. *Hint:* For integration, use thin plate elements parallel to the x–y plane and having a thickness dz.



SOLUTION



$$dV = bx\,dz = b(a)(1 - \frac{z}{h})\,dz$$

$$dI_y = dI_y + (dm)[(\frac{x}{2})^2 + z^2]$$

$$= \frac{1}{12}dm(x^2) + dm(\frac{x^2}{4}) + dmz^2$$

$$= dm(\frac{x^2}{3} + z^2)$$

$$= [b(a)(1 - \frac{z}{h})dz](\rho)[\frac{a^2}{3}(1 - \frac{z}{h})^2 + z^2]$$

$$I_y = ab\rho\int_0^k [\frac{a^3}{3}(\frac{h - z}{h})^3 + z^2(1 - \frac{z}{h})]dz$$

$$= ab\rho[\frac{a^2}{3h^3}(h^4 - \frac{3}{2}h^4 + h^4 - \frac{1}{4}h^4) + \frac{1}{h}(\frac{1}{3}h^4 - \frac{1}{4}h^4)]$$

$$= \frac{1}{12}abh\rho(a^2 + h^2)$$

$$m = \rho V = \frac{1}{2}abh\rho$$

Thus,

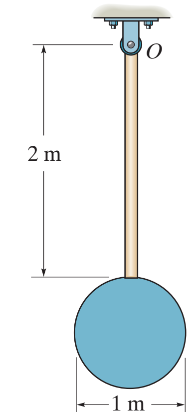
$$I_y = \frac{m}{6}(a^2 + h^2) \qquad \textbf{Ans.}$$

Ans:

$I_y = \frac{m}{6}(a^2 + h^2)$

17–10.

The pendulum consists of a 4-kg circular plate and a 2-kg slender rod. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.



SOLUTION

Using the parallel axis theorem by referring to Fig. a,

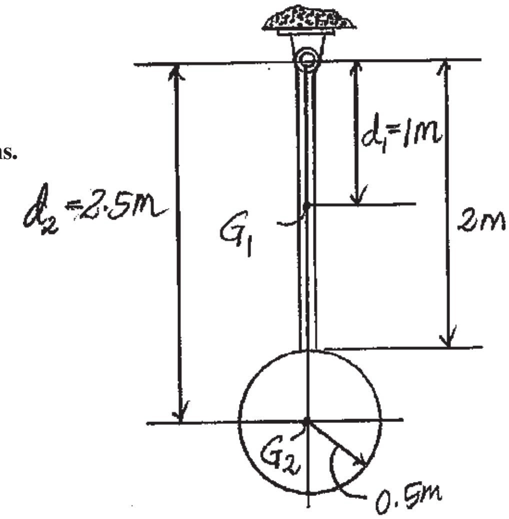
$$I_O = \Sigma\left(I_G + md^2\right)$$

$$= \left[\frac{1}{12}(2)\left(2^2\right) + 2\left(1^2\right)\right] + \left[\frac{1}{2}(4)\left(0.5^2\right) + 4\left(2.5^2\right)\right]$$

$$= 28.17 \text{ kg} \cdot \text{m}^2$$

Thus, the radius of gyration is

$$k_O = \sqrt{\frac{I_O}{m}} = \sqrt{\frac{28.17}{4 + 2}} = 2.167 \text{ m} = 2.17 \text{ m} \qquad \textbf{Ans.}$$

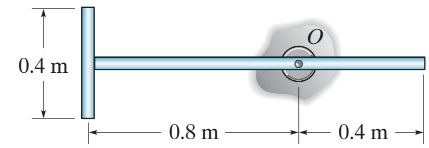


(a)

Ans:
$k_O = 2.17$ m

17–11.

The assembly is made of the slender rods that have a mass per unit length of 3 kg/m. Determine the mass moment of inertia of the assembly about an axis perpendicular to the page and passing through point O.



SOLUTION

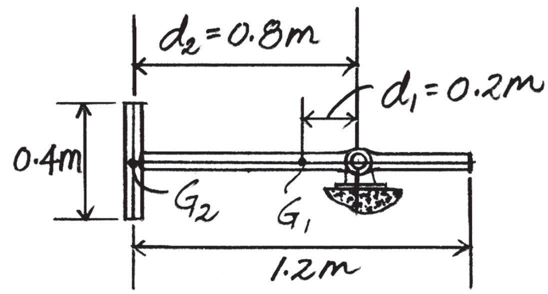
Using the parallel axis theorem by referring to Fig. *a*,

$$I_O = \Sigma\left(I_G + md^2\right)$$

$$= \left\{\frac{1}{12}[3(1.2)]\left(1.2^2\right) + [3(1.2)]\left(0.2^2\right)\right\}$$

$$+ \left\{\frac{1}{12}[3(0.4)]\left(0.4^2\right) + [3(0.4)]\left(0.8^2\right)\right\}$$

$$= 1.36 \text{ kg} \cdot \text{m}^2$$ **Ans.**

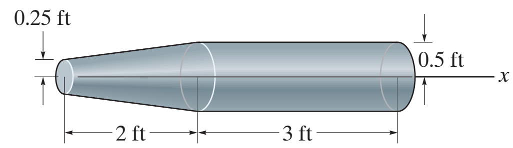


(a)

Ans:
$I_O = 1.36 \text{ kg} \cdot \text{m}^2$

***17–12.**

Determine the moment of inertia of the solid steel assembly about the x axis. Steel has a specific weight of $\gamma_{st} = 490\ \text{lb/ft}^3$.



SOLUTION

$$I_x = \frac{1}{2} m_1 (0.5)^2 + \frac{3}{10} m_2 (0.5)^2 - \frac{3}{10} m_3 (0.25)^2$$

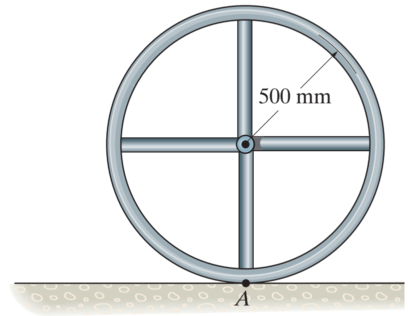
$$= \left[\frac{1}{2}\pi(0.5)^2(3)(0.5)^2 + \frac{3}{10}\left(\frac{1}{3}\right)\pi(0.5)^2(4)(0.5)^2 - \frac{3}{10}\left(\frac{1}{2}\right)\pi(0.25)^2(2)(0.25)^2\right]\left(\frac{490}{32.2}\right)$$

$$= 5.64\ \text{slug}\cdot\text{ft}^2$$ **Ans.**

Ans:
$I_x = 5.64\ \text{slug}\cdot\text{ft}^2$

17–13.

The wheel consists of a thin ring having a mass of 10 kg and four spokes made from slender rods, each having a mass of 2 kg. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point A.



SOLUTION

$$I_A = I_o + md^3$$

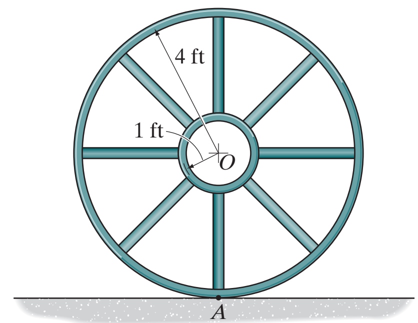
$$= \left[2\left[\frac{1}{12}(4)(1)^2\right] + 10(0.5)^2\right] + 18(0.5)^2$$

$$= 7.67 \text{ kg} \cdot \text{m}^2$$ **Ans.**

Ans:
$I_A = 7.67 \text{ kg} \cdot \text{m}^2$

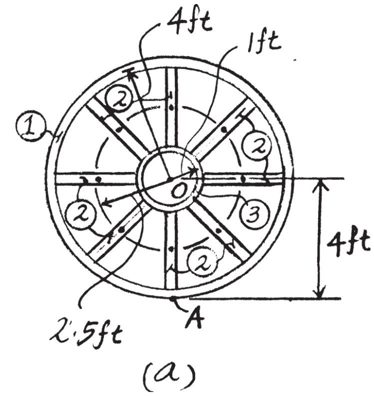
17–14.

If the large ring, small ring and each of the spokes weigh 100 lb, 15 lb, and 20 lb, respectively, determine the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A.



SOLUTION

Composite Parts: The wheel can be subdivided into the segments shown in Fig. *a*. The spokes which have a length of $(4 - 1) = 3$ ft and a center of mass located at a distance of $\left(1 + \frac{3}{2}\right)$ ft $= 2.5$ ft from point O can be grouped as segment (2).



Mass Moment of Inertia: First, we will compute the mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point O.

$$I_O = \left(\frac{100}{32.2}\right)(4^2) + 8\left[\frac{1}{12}\left(\frac{20}{32.2}\right)(3^2) + \left(\frac{20}{32.2}\right)(2.5^2)\right] + \left(\frac{15}{32.2}\right)(1^2)$$

$$= 84.94 \text{ slug} \cdot \text{ft}^2$$

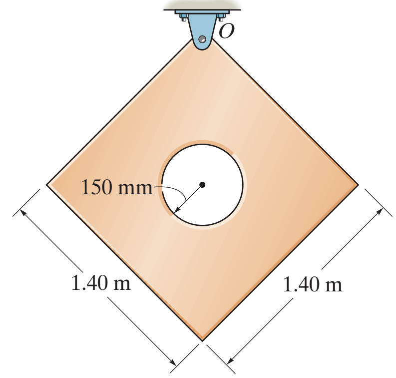
The mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point A can be found using the parallel-axis theorem $I_A = I_O + md^2$, where $m = \frac{100}{32.2} + 8\left(\frac{20}{32.2}\right) + \frac{15}{32.2} = 8.5404$ slug and $d = 4$ ft. Thus,

$$I_A = 84.94 + 8.5404(4^2) = 221.58 \text{ slug} \cdot \text{ft}^2 = 222 \text{ slug} \cdot \text{ft}^2 \qquad \textbf{Ans.}$$

Ans:
$I_A = 222 \text{ slug} \cdot \text{ft}^2$

17–15.

Determine the moment of inertia about an axis perpendicular to the page and passing through the pin at O. The thin plate has a hole in its center. Its thickness is 50 mm, and the material has a density $\rho = 50\ \text{kg/m}^3$.



SOLUTION

$$I_G = \frac{1}{12}\left[50(1.4)(1.4)(0.05)\right]\left[(1.4)^2 + (1.4)^2\right] - \frac{1}{2}\left[50(\pi)(0.15)^2(0.05)\right](0.15)^2$$

$$= 1.5987\ \text{kg}\cdot\text{m}^2$$

$$I_O = I_G + md^2$$

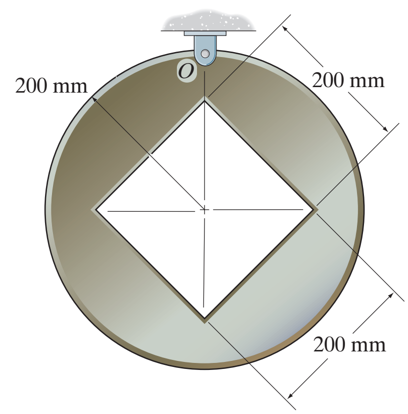
$$m = 50(1.4)(1.4)(0.05) - 50(\pi)(0.15)^2(0.05) = 4.7233\ \text{kg}$$

$$I_O = 1.5987 + 4.7233(1.4\sin 45°)^2 = 6.23\ \text{kg}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_O = 6.23\ \text{kg}\cdot\text{m}^2$

***17–16.**

Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \text{kg/m}^2$.



SOLUTION

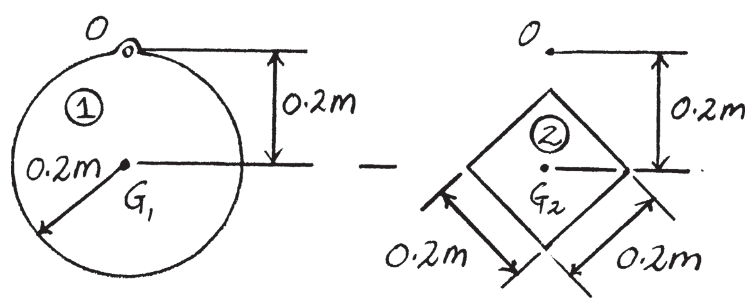
Composite Parts: The plate can be subdivided into two segments as shown in Fig. *a*. Since segment (2) is a hole, it should be considered as a negative part. The perpendicular distances measured from the center of mass of each segment to the point O are also indicated.

Mass Moment of Inertia: The moment of inertia of segments (1) and (2) are computed as $m_1 = \pi(0.2^2)(20) = 0.8\pi$ kg and $m_2 = (0.2)(0.2)(20) = 0.8$ kg. The moment of inertia of the plate about an axis perpendicular to the page and passing through point O for each segment can be determined using the parallel-axis theorem.

$$I_O = \Sigma I_G + md^2$$

$$= \left[\frac{1}{2}(0.8\pi)(0.2^2) + 0.8\pi(0.2^2)\right] - \left[\frac{1}{12}(0.8)(0.2^2 + 0.2^2) + 0.8(0.2^2)\right]$$

$$= 0.113\ \text{kg}\cdot\text{m}^2$$ **Ans.**

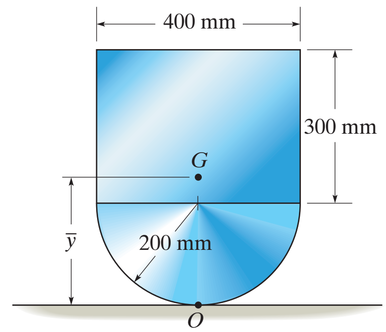


(a)

Ans:
$I_O = 0.113\ \text{kg}\cdot\text{m}^2$

17–17.

Determine the location $\bar{y}$ of the center of mass G of the assembly and then calculate the moment of inertia about an axis perpendicular to the page and passing through G. The block has a mass of 3 kg and the semicylinder has a mass of 5 kg.



SOLUTION

Moment inertia of the semicylinder about its center of mass:

$$(I_G)_{cyc} = \frac{1}{2}mR^2 - m\left(\frac{4R}{3\pi}\right)^2 = 0.3199mR^2$$

$$\bar{y} = \frac{\Sigma \tilde{y} m}{\Sigma m} = \frac{\left[0.2 - \frac{4(0.2)}{3\pi}\right](5) + 0.35(3)}{5 + 3} = 0.2032 \text{ m} = 0.203 \text{ m} \qquad \textbf{Ans.}$$

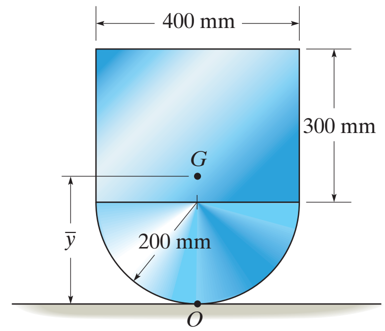
$$I_G = 0.3199(5)(0.2)^2 + 5\left[0.2032 - \left(0.2 - \frac{4(0.2)}{3\pi}\right)\right]^2 + \frac{1}{12}(3)\left(0.3^2 + 0.4^2\right) + 3(0.35 - 0.2032)^2$$

$$= 0.230 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

Ans:
$I_G = 0.230 \text{ kg} \cdot \text{m}^2$

17–18.

Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point O. The block has a mass of 3 kg, and the semicylinder has a mass of 5 kg.



SOLUTION

$$(I_G)_{cyl} = \frac{1}{2}mR^2 - m\left(\frac{4R}{3\pi}\right)^2 = 0.3199\,mR^2$$

$$I_O = 0.3199(5)(0.2)^2 + 5\left(0.2 - \frac{4(0.2)}{3\pi}\right)^2 + \frac{1}{12}(3)\left((0.3)^2 + (0.4)^2\right) + 3(0.350)^2$$

$$= 0.560 \text{ kg}\cdot\text{m}^2$$ **Ans.**

Also from the solution to Prob. 17–22,

$$I_O = I_G + md^2$$

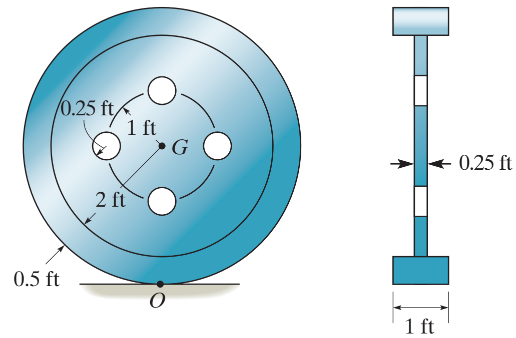
$$= 0.230 + 8(0.2032)^2$$

$$= 0.560 \text{ kg}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_O = 0.560 \text{ kg}\cdot\text{m}^2$

17–19.

Determine the moment of inertia of the wheel about an axis which is perpendicular to the page and passes through the center of mass G. The material has a specific weight $\gamma = 90\ \text{lb/ft}^3$.



SOLUTION

$$I_G = \frac{1}{2}\left[\frac{90}{32.2}(\pi)(2)^2(0.25)\right](2)^2 + \frac{1}{2}\left[\frac{90}{32.2}(\pi)(2.5)^2(1)\right](2.5)^2$$

$$-\frac{1}{2}\left[\frac{90}{32.2}(\pi)(2)^2(1)\right](2)^2 - 4\left[\frac{1}{2}\left(\frac{90}{32.2}\right)(\pi)(0.25)^2(0.25)\right](0.25)^2$$

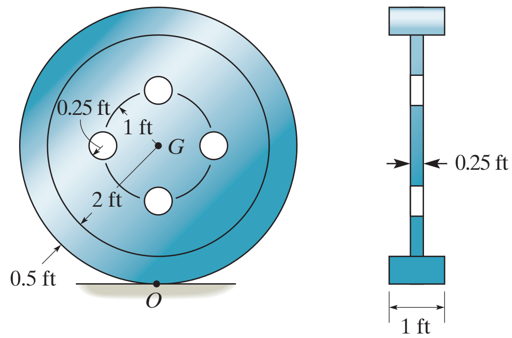
$$-4\left[\left(\frac{90}{32.2}\right)(\pi)(0.25)^2(0.25)\right](1)^2$$

$$= 118.25 = 118\ \text{slug}\cdot\text{ft}^2$$ **Ans.**

Ans:
$I_G = 118\ \text{slug}\cdot\text{ft}^2$

***17–20.**

Determine the moment of inertia of the wheel about an axis which is perpendicular to the page and passes through point O. The material has a specific weight $\gamma = 90\ \text{lb/ft}^3$.



SOLUTION

$$m = \frac{90}{32.2}\left[\pi(2)^2(0.25) + \pi\left\{(2.5)^2(1) - (2)^2(1)\right\} - 4\pi(0.25)^2(0.25)\right] = 27.99\ \text{slug}$$

From the solution to Prob. 17–18,

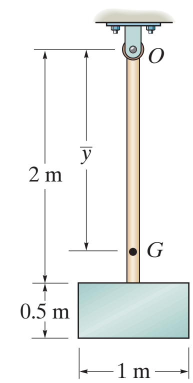
$I_G = 118.25\ \text{slug}\cdot\text{ft}^2$

$I_O = 118.25 + 27.99(2.5)^2 = 293\ \text{slug}\cdot\text{ft}^2$ **Ans.**

Ans:
$I_O = 293\ \text{slug}\cdot\text{ft}^2$

17–21.

The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



SOLUTION

$$\bar{y} = \frac{\Sigma\,\bar{y}m}{\Sigma\,m} = \frac{1(3) + 2.25(5)}{3 + 5} = 1.781\text{ m} = 1.78\text{ m} \qquad \textbf{Ans.}$$

$$I_G = \Sigma \bar{I}_G + md^2$$

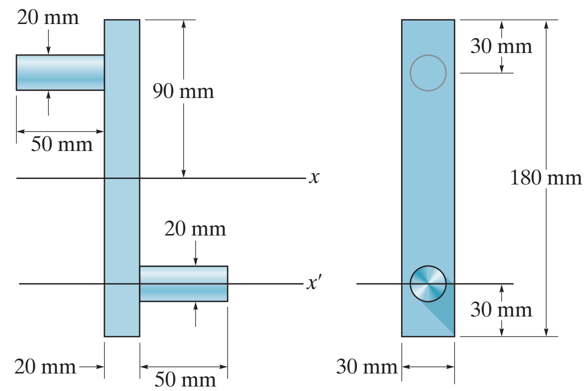
$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

$$= 4.45\text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

Ans:
$\bar{y} = 1.78$ m
$I_G = 4.45$ kg $\cdot$ m^2

17–22.

Determine the moment of inertia of the overhung crank about the x axis. The material is steel having a destiny of $\rho = 7.85 \text{ Mg/m}^3$.



SOLUTION

$$m_c = 7.85(10^3)\left((0.05)\pi(0.01)^2\right) = 0.1233 \text{ kg}$$

$$m_p = 7.85(10^3)((0.03)(0.180)(0.02)) = 0.8478 \text{ kg}$$

$$I_x = 2\left[\frac{1}{2}(0.1233)(0.01)^2 + (0.1233)(0.06)^2\right]$$

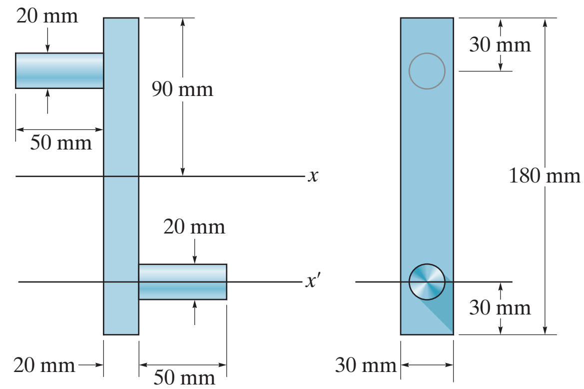
$$+ \left[\frac{1}{12}(0.8478)\left((0.03)^2 + (0.180)^2\right)\right]$$

$$= 0.00325 \text{ kg} \cdot \text{m}^2 = 3.25 \text{ g} \cdot \text{m}^2$$ **Ans.**

Ans:
$I_x = 3.25 \text{ g} \cdot \text{m}^2$

17–23.

Determine the moment of inertia of the overhung crank about the x' axis. The material is steel having a destiny of $\rho = 7.85\ \text{Mg/m}^3$.



SOLUTION

$$m_c = 7.85\left(10^3\right)\left((0.05)\pi(0.01)^2\right) = 0.1233\ \text{kg}$$

$$m_p = 7.85\left(10^3\right)\left((0.03)(0.180)(0.02)\right) = 0.8478\ \text{kg}$$

$$I_{x'} = \left[\frac{1}{2}(0.1233)(0.01)^2\right] + \left[\frac{1}{2}(0.1233)(0.02)^2 + (0.1233)(0.120)^2\right]$$

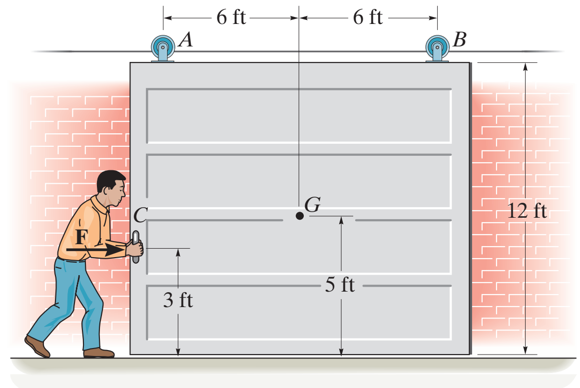
$$+ \left[\frac{1}{12}(0.8478)\left((0.03)^2 + (0.180)^2\right) + (0.8478)(0.06)^2\right]$$

$$= 0.00719\ \text{kg}\cdot\text{m}^2 = 7.19\ \text{g}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_{x'} = 7.19\ \text{g}\cdot\text{m}^2$

***17–24.**

The door has a weight of 200 lb and a center of gravity at G. Determine how far the door moves in 2 s, starting from rest, if a man pushes on it at C with a horizontal force $F = 30$ lb. Also, find the vertical reactions at the rollers A and B.

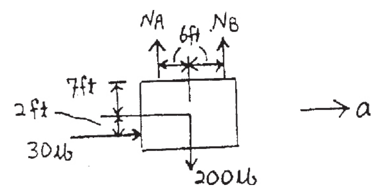


SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;$ $\qquad 30 = \left(\dfrac{200}{32.2}\right)a_G$

$$a_G = 4.83 \text{ ft/s}^2$$

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;$ $\qquad N_B(12) - 200(6) + 30(9) = \left(\dfrac{200}{32.2}\right)(4.83)(7)$



$N_B = 95.0$ lb **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\qquad N_A + 95.0 - 200 = 0$

$N_A = 105$ lb **Ans.**

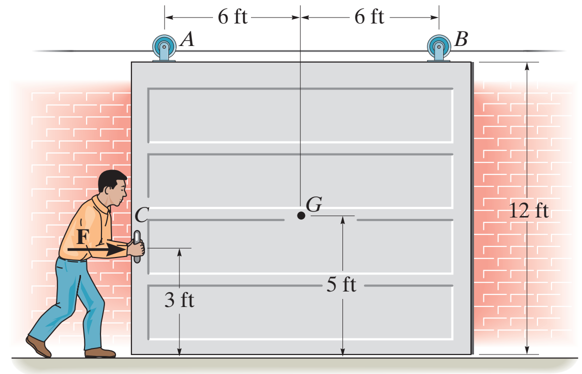
$(\xrightarrow{+})$ $\qquad s = s_0 + v_0 t + \dfrac{1}{2}a_G t^2$

$s = 0 + 0 + \dfrac{1}{2}(4.83)(2)^2 = 9.66$ ft **Ans.**

Ans:
$N_B = 95.0$ lb
$N_A = 105$ lb
$s = 9.66$ ft

17–25.

The door has a weight of 200 lb and a center of gravity at G. Determine the constant force F that must be applied to the door to push it open 12 ft to the right in 5 s, starting from rest. Also, find the vertical reactions at the rollers A and B.

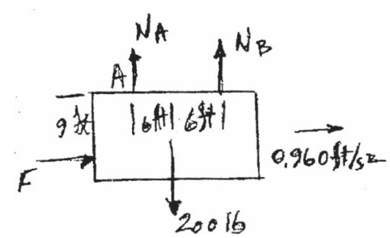


SOLUTION

$$(\overset{+}{\rightarrow})\, s = s_0 + v_0 t + \frac{1}{2}a_G t^2$$

$$12 = 0 + 0 + \frac{1}{2}a_G(5)^2$$

$$a_c = 0.960 \text{ ft/s}^2$$



$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x;\qquad F = \dfrac{200}{32.2}(0.960)$

$F = 5.9627 \text{ lb} = 5.96 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;\qquad N_B(12) - 200(6) + 5.9627(9) = \dfrac{200}{32.2}(0.960)(7)$

$N_B = 99.0 \text{ lb}$ **Ans.**

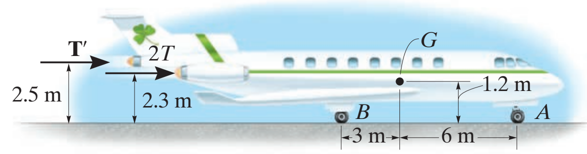
$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A + 99.0 - 200 = 0$

$N_A = 101 \text{ lb}$ **Ans.**

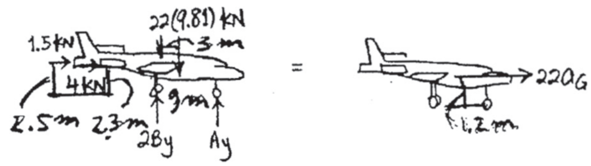
Ans:
$F = 5.96$ lb
$N_B = 99.0$ lb
$N_A = 101$ lb

17–26.

The jet aircraft has a total mass of 22 Mg and a center of mass at G. Initially at take-off the engines provide a thrust $2T = 4$ kN and $T' = 1.5$ kN. Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the *two* wing wheels located at B. Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m a_x; \qquad 1.5 + 4 = 22a_G$

$+\uparrow \Sigma F_y = 0; \qquad 2B_y + A_y - 22(9.81) = 0$

$\circlearrowleft + \Sigma M_B = \Sigma (M_K)_B; \qquad 4(2.3) - 1.5(2.5) - 22(9.81)(3) + A_y(9) = -22a_G(1.2)$

$A_y = 72.6 \text{ kN}$ **Ans.**

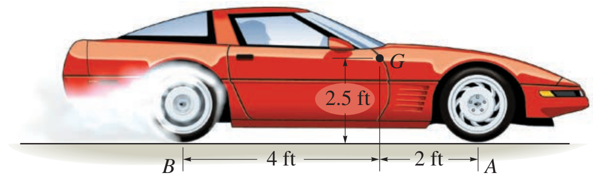
$B_y = 71.6 \text{ kN}$ **Ans.**

$a_G = 0.250 \text{ m/s}^2$ **Ans.**

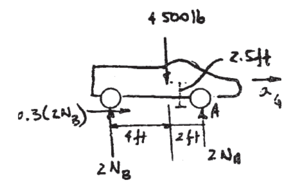
Ans:
$A_y = 72.6 \text{ kN}$
$B_y = 71.6 \text{ kN}$
$a_G = 0.250 \text{ m/s}^2$

17–27.

The sports car has a weight of 4500 lb and center of gravity at G. If it starts from rest it causes the rear wheels to slip as it accelerates. Determine how long it takes for it to reach a speed of 10 ft/s. Also, what are the normal reactions at each of the four wheels on the road? The coefficients of static and kinetic friction at the road are $\mu_s = 0.5$ and $\mu_k = 0.3$, respectively. Neglect the mass of the wheels.



SOLUTION



$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad -2N_B(6) + 4500(2) = \frac{-4500}{32.2}a_G(2.5)$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 0.3(2N_B) = \frac{4500}{32.2}a_G$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad 2N_B + 2N_A - 4500 = 0$$

Solving,

$N_A = 1393$ lb **Ans.**

$N_B = 857$ lb **Ans.**

$a_G = 3.68 \text{ ft/s}^2$

$(\xrightarrow{+}) \quad v = v_0 + a_c t$

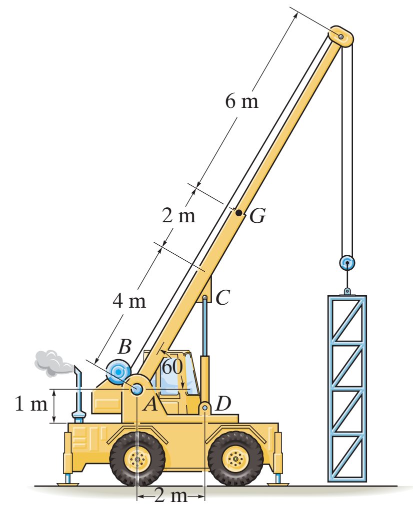
$10 = 0 + 3.68\,t$

$t = 2.72$ s **Ans.**

Ans:
$N_A = 1393$ lb
$N_B = 857$ lb
$t = 2.72$ s

***17–28.**

The assembly has a mass of 8 Mg and is hoisted using the boom and pulley system. If the winch at *B* draws in the cable with an acceleration of 2 m/s², determine the compressive force in the hydraulic cylinder needed to support the boom. The boom has a mass of 2 Mg and mass center at *G*.



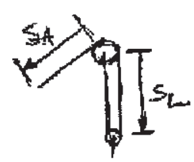
SOLUTION

$s_B + 2s_L = l$

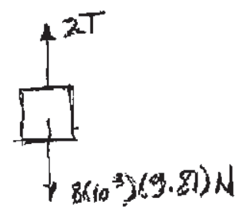
$a_B = -2a_L$

$2 = -2a_L$

$a_L = -1\ \text{m/s}^2$



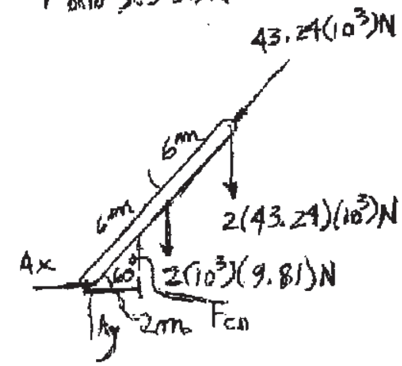
Assembly:



$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 8(10^3)(9.81) = 8(10^3)(1)$

$T = 43.24\ \text{kN}$

Boom:



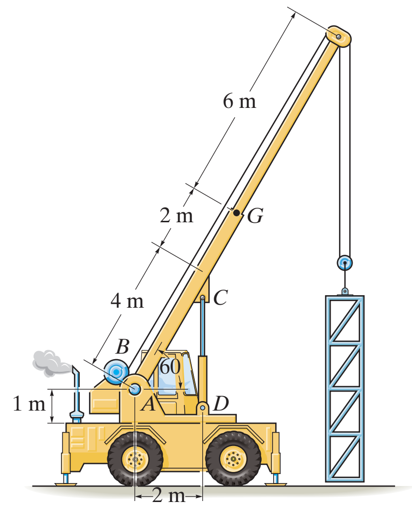
$\zeta + \Sigma M_A = 0; \quad F_{CD}(2) - 2(10^3)(9.81)(6\cos 60°) - 2(43.24)(10^3)(12\cos 60°) = 0$

$F_{CD} = 289\ \text{kN}$ **Ans.**

Ans:
$F_{CD} = 289\ \text{kN}$

17–29.

The assembly has a mass of 4 Mg and is hoisted using the winch at B. Determine the greatest acceleration of the assembly so that the compressive force in the hydraulic cylinder supporting the boom does not exceed 180 kN. What is the tension in the supporting cable? The boom has a mass of 2 Mg and mass center at G.

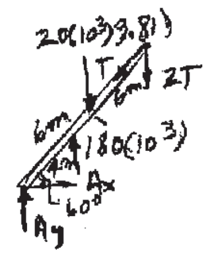


SOLUTION

Boom:

$\circlearrowleft + \Sigma M_A = 0; \qquad 180(10^3)(2) - 2(10^3)(9.81)(6\cos 60^\circ) - 2T(12\cos 60^\circ) = 0$

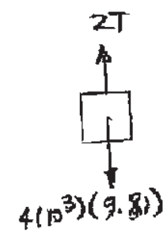
$T = 25\,095 \text{ N} = 25.1 \text{ kN}$ **Ans.**



Assembly:

$+\uparrow \Sigma F_y = ma_y; \qquad 2(25\,095) - 4(10^3)(9.81) = 4(10^3)\,a$

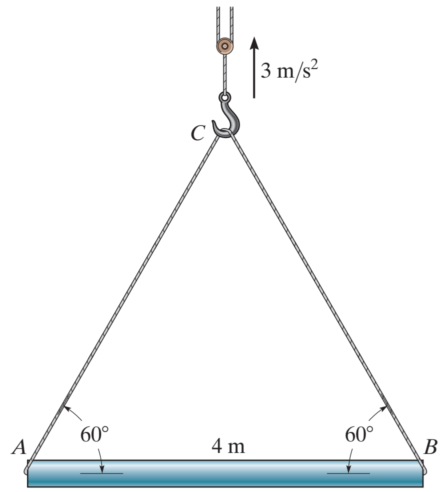
$a = 2.74 \text{ m/s}^2$ **Ans.**



Ans:
$a = 2.74 \text{ m/s}^2$
$T = 25.1 \text{ kN}$

17–30.

The uniform girder AB has a mass of 8 Mg. Determine the internal axial, shear, and bending-moment loadings at the center of the girder if a crane gives it an upward acceleration of 3 m/s^2.

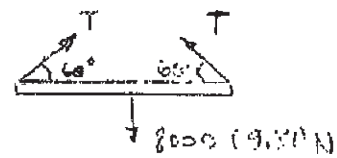


SOLUTION

Girder:

$+\uparrow \Sigma F_y = ma_y; \qquad 2T \sin 60° - 8000(9.81) = 8000(3)$

$T = 59\,166.86 \text{ N}$



Segment:

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 59\,166.86 \cos 60° - N = 0$

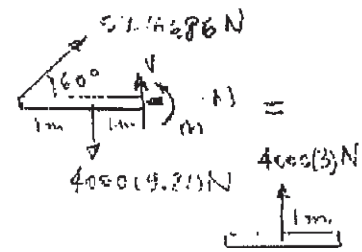
$N = 29.6 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = ma_y; \qquad 59\,166.86 \sin 60° - 4000(9.81) + V = 4000(3)$

$V = 0$ **Ans.**

$\zeta + \Sigma M_C = \Sigma (M_k)_C; \qquad M + 4000(9.81)(1) - 59\,166.86 \sin 60°(2) = -4000(3)(1)$

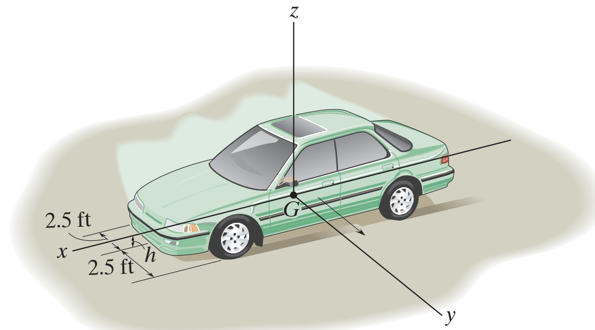
$M = 51.2 \text{ kN} \cdot \text{m}$ **Ans.**



Ans:
$N = 29.6 \text{ kN}$
$V = 0$
$M = 51.2 \text{ kN} \cdot \text{m}$

17–31.

A car having a weight of 4000 lb begins to skid and turn with the brakes applied to all four wheels. If the coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.8$, determine the maximum critical height h of the center of gravity G such that the car does not overturn. Tipping will begin to occur after the car rotates 90° from its original direction of motion and, as shown in the figure, undergoes *translation* while skidding. *Hint*: Draw a free-body diagram of the car viewed from the front. When tipping occurs, the normal reactions of the wheels on the right side (or passenger side) are zero.



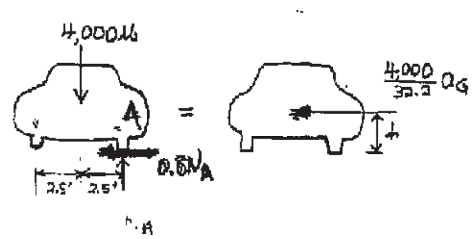
SOLUTION

N_A represents the reaction for both the front and rear wheels on the left side.

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0.8N_A = \frac{4000}{32.2} a_G$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 4000 = 0$$

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 4000(2.5) = \frac{4000}{32.2}(a_G)(h)$$



Solving,

$N_A = 4000$ lb

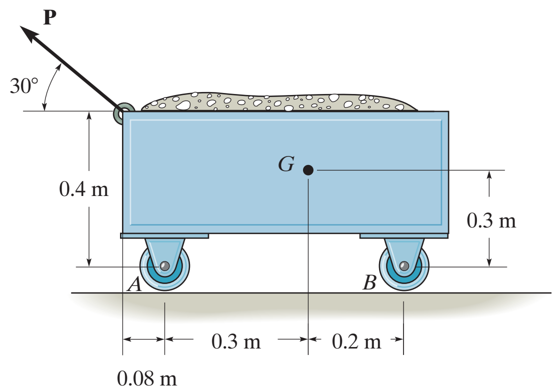
$a_G = 25.76$ ft/s^2

$h = 3.12$ ft **Ans.**

Ans:
$h = 3.12$ ft

***17–32.**

A force of $P = 300$ N is applied to the 60-kg cart. Determine the reactions at both the wheels at A and both the wheels at B. Also, what is the acceleration of the cart? The mass center of the cart is at G.



SOLUTION

Equations of Motions. Referring to the FBD of the cart, Fig. a,

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad 300 \cos 30° = 60a$

$$a = 4.3301 \text{ m/s}^2 = 4.33 \text{ m/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A + N_B + 300 \sin 30° - 60(9.81) = 60(0) \qquad \textbf{(1)}$

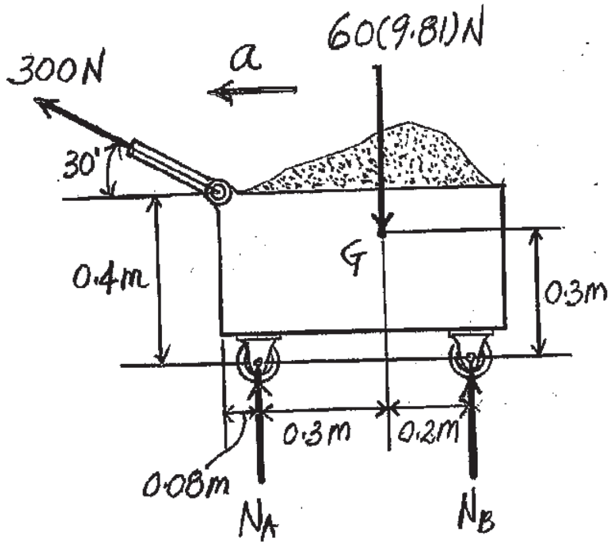
$\circlearrowleft + \Sigma M_G = 0;\qquad N_B(0.2) - N_A(0.3) + 300 \cos 30°(0.1)$

$$- 300 \sin 30°(0.38) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$N_A = 113.40 \text{ N} = 113 \text{ N} \qquad \textbf{Ans.}$$

$$N_B = 325.20 \text{ N} = 325 \text{ N} \qquad \textbf{Ans.}$$

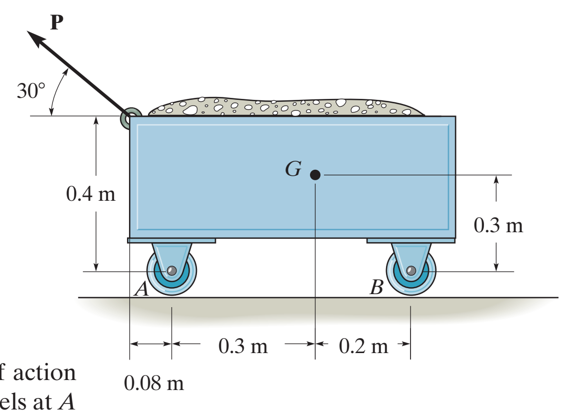


(a)

Ans:
$a = 4.33 \text{ m/s}^2 \leftarrow$
$N_A = 113$ N
$N_B = 325$ N

17–33.

Determine the largest force **P** that can be applied to the 60-kg cart, without causing one of the wheel reactions, either at A or at B, to be zero. Also, what is the acceleration of the cart? The mass center of the cart is at G.



SOLUTION

Equations of Motions. Since $(0.38\text{ m})\tan 30° = 0.22\text{ m} > 0.1\text{ m}$, the line of action of **P** passes *below* G. Therefore, **P** tends to rotate the cart clockwise. The wheels at A will leave the ground before those at B. Then, it is required that $N_A = 0$. Referring, to the FBD of the cart, Fig. *a*

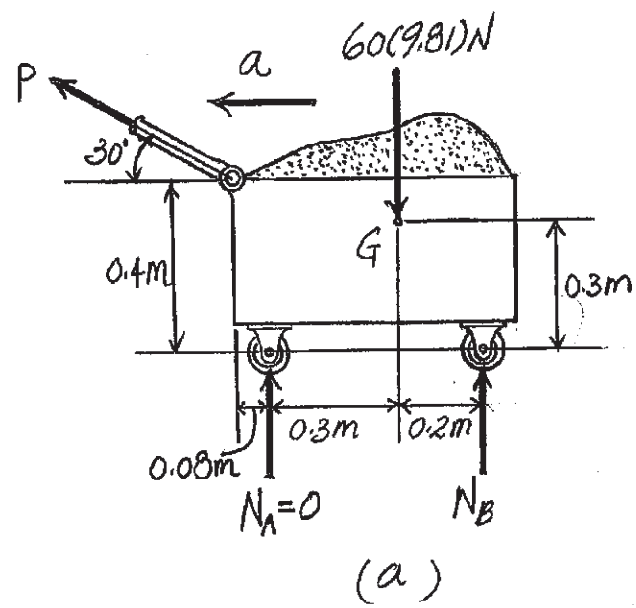
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_B + P\sin 30° - 60(9.81) = 60(0) \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_G = 0; \qquad P\cos 30°(0.1) - P\sin 30°(0.38) + N_B(0.2) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$P = 578.77\text{ N} = 579\text{ N} \qquad \textbf{Ans.}$$

$$N_B = 299.22\text{ N}$$

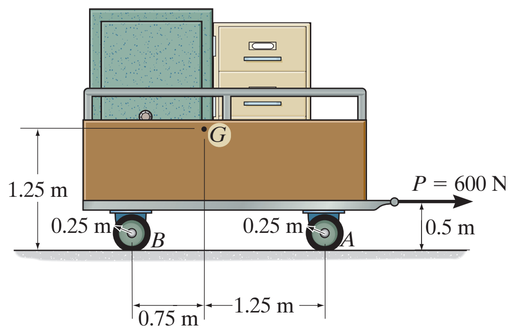


(a)

Ans:
$P = 579$ N

17–34.

The trailer with its load has a mass of 150 kg and a center of mass at G. If it is subjected to a horizontal force of $P = 600$ N, determine the trailer's acceleration and the normal force on the pair of wheels at A and at B. The wheels are free to roll and have negligible mass.



SOLUTION

Equations of Motion: Writing the force equation of motion along the x axis,

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 600 = 150a \quad\quad a = 4 \text{ m/s}^2 \rightarrow$ **Ans.**

Using this result to write the moment equation about point A,

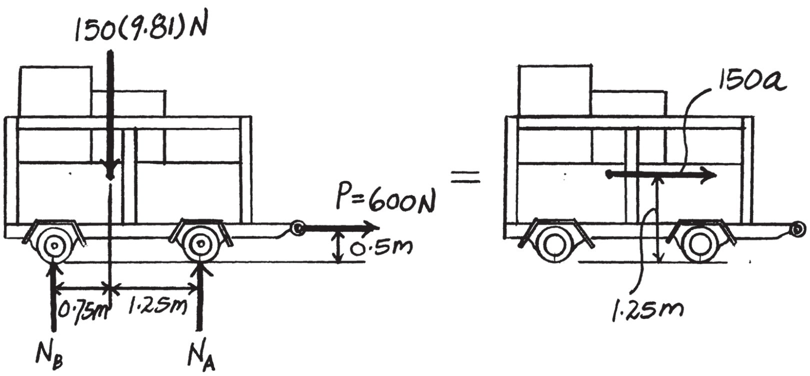
$\circlearrowleft +\Sigma M_A = (M_k)_A; \quad 150(9.81)(1.25) - 600(0.5) - N_B(2) = -150(4)(1.25)$

$N_B = 1144.69 \text{ N} = 1.14 \text{ kN}$ **Ans.**

Using this result to write the force equation of motion along the y axis,

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 1144.69 - 150(9.81) = 150(0)$

$N_A = 326.81 \text{ N} = 327 \text{ N}$ **Ans.**

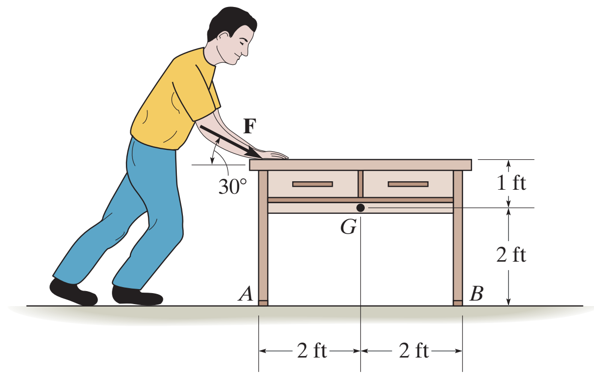


(a)

Ans:
$a = 4 \text{ m/s}^2 \rightarrow$
$N_B = 1.14 \text{ kN}$
$N_A = 327 \text{ N}$

17–35.

The desk has a weight of 75 lb and a center of gravity at G. Determine its initial acceleration if a man pushes on it with a force $F = 60$ lb. The coefficient of kinetic friction at A and B is $\mu_k = 0.2$.

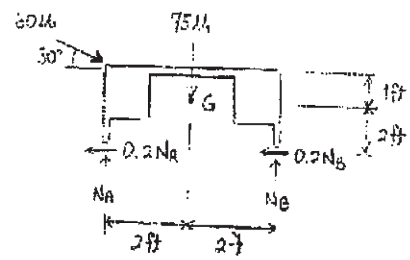


SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x;$ $\quad 60 \cos 30° - 0.2N_A - 0.2N_B = \dfrac{75}{32.2} a_G$

$+\uparrow \Sigma F_y = ma_y;$ $\quad N_A + N_B - 75 - 60 \sin 30° = 0$

$\circlearrowleft + \Sigma M_G = 0;$ $\quad 60 \sin 30°(2) - 60 \cos 30°(1) - N_A(2) + N_B(2) - 0.2N_A(2) - 0.2N_B(2) = 0$



Solving,

$a_G = 13.3 \text{ ft/s}^2$ **Ans.**

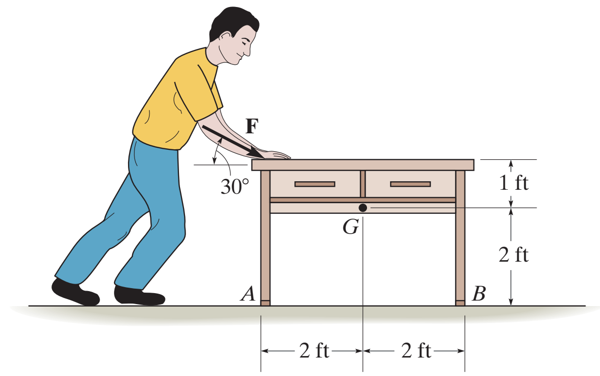
$N_A = 44.0 \text{ lb}$

$N_B = 61.0 \text{ lb}$

Ans:
$a_G = 13.3 \text{ ft/s}^2$

***17–36.**

The desk has a weight of 75 lb and a center of gravity at G. Determine the initial acceleration of a desk when the man applies enough force F to overcome the static friction at A and B. Also, find the vertical reactions on each of the two legs at A and at B. The coefficients of static and kinetic friction at A and B are $\mu_s = 0.5$ and $\mu_k = 0.2$, respectively.

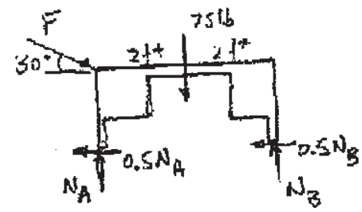


SOLUTION

Force required to start desk moving;

$\xrightarrow{+} \Sigma F_x = 0; \qquad F\cos 30^\circ - 0.5N_A - 0.5N_B = 0$

$+\uparrow \Sigma F_y = 0; \qquad N_A + N_B - F\sin 30^\circ - 75 = 0$



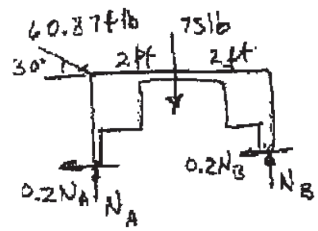
Solving for F by eliminating $N_A + N_B$,

$$F = 60.874 \text{ lb}$$

Desk starts to slide.

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 60.874\cos 30^\circ - 0.2N_A - 0.2N_B = \dfrac{75}{32.2}a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + N_B - 60.874\sin 30^\circ - 75 = 0$



Solving for a_G by eliminating $N_A + N_B$,

$$a_G = 13.58 = 13.6 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$\circlearrowleft + \Sigma M_A = \Sigma(M_K)_A; \qquad N_B(4) - 75(2) - 60.874\cos 30^\circ(3) = \dfrac{-75}{32.2}(13.58)(2)$

$$N_B = 61.2 \text{ lb}$$

So that

$$N_A = 44.2 \text{ lb}$$

For each leg,

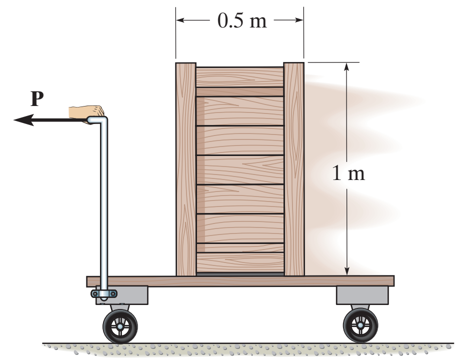
$$N_A' = \frac{44.2}{2} = 22.1 \text{ lb} \qquad \textbf{Ans.}$$

$$N_B' = \frac{61.2}{2} = 30.6 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$a_G = 13.6 \text{ ft/s}^2$
$N_A' = 22.1 \text{ lb}$
$N_B' = 30.6 \text{ lb}$

17–37.

The 150-kg uniform crate rests on the 10-kg cart. Determine the maximum force P that can be applied to the handle without causing the crate to tip on the cart. Slipping does not occur.



SOLUTION

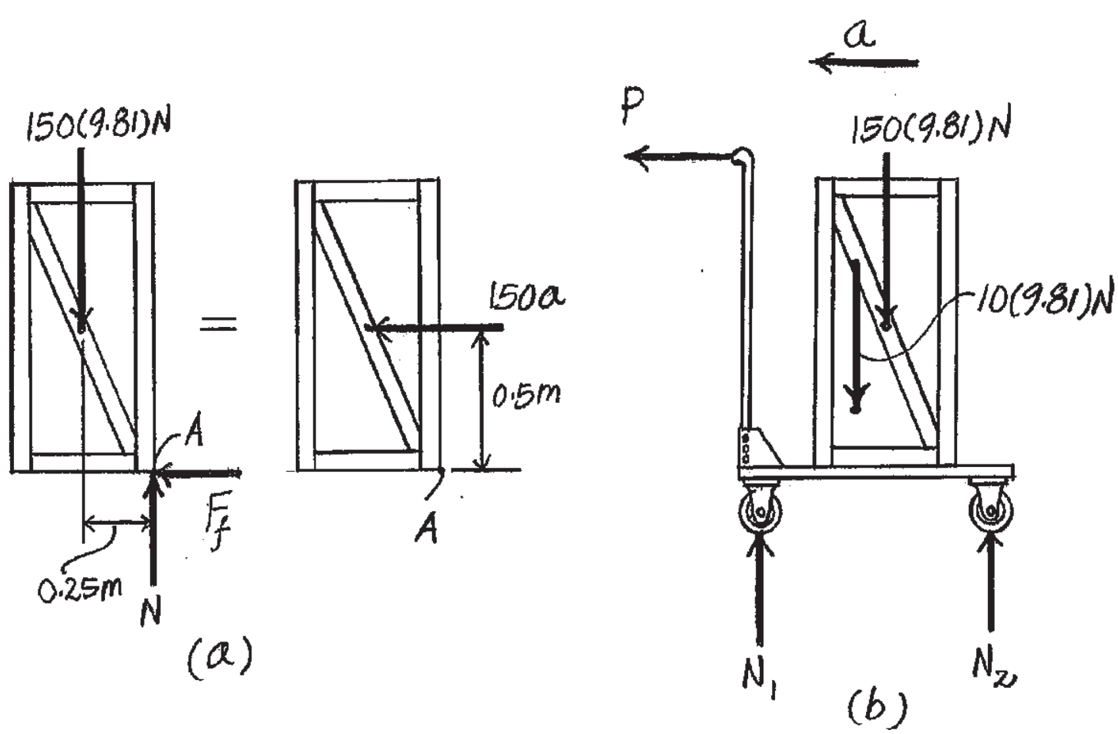
Equation of Motion. Tipping will occur about edge A. Referring to the FBD and kinetic diagram of the crate, Fig. a,

$$\zeta+\Sigma M_A = \Sigma(M_K)_A; \qquad 150(9.81)(0.25) = (150a)(0.5)$$

$$a = 4.905 \text{ m/s}^2$$

Using the result of a and refer to the FBD of the crate and cart, Fig. b,

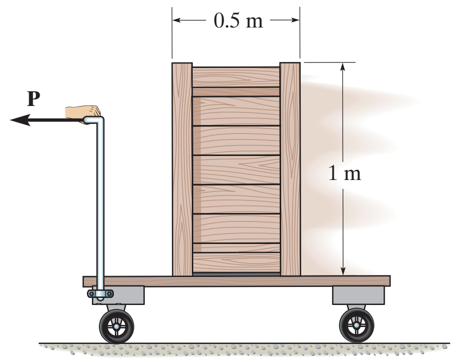
$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x \qquad P = (150 + 10)(4.905) = 784.8 \text{ N} = 785 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$P = 785$ N

17–38.

The 150-kg uniform crate rests on the 10-kg cart. Determine the maximum force P that can be applied to the handle without causing the crate to slip or tip on the cart. The coefficient of static friction between the crate and cart is $\mu_s = 0.2$.



SOLUTION

Equation of Motion. Assuming that the crate slips before it tips, then $F_f = \mu_s N = 0.2\,N$. Referring to the FBD and kinetic diagram of the crate, Fig. *a*

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 150\,(9.81) = 150\,(0) \qquad N = 1471.5\text{ N}$$

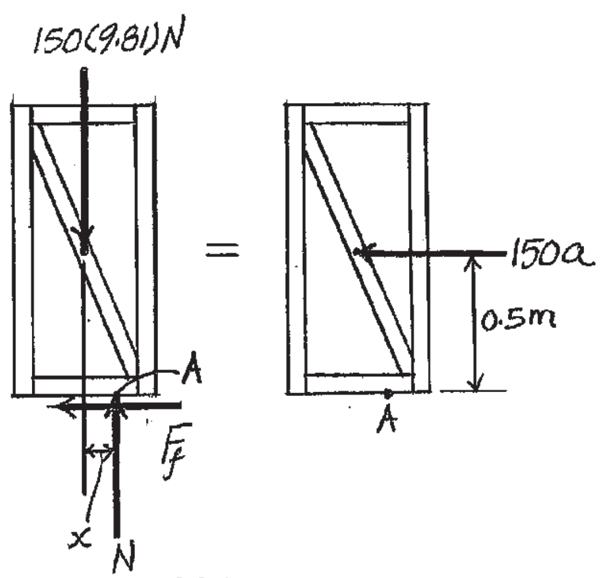
$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad 0.2(1471.5) = 150\,a \qquad a = 1.962\text{ m/s}^2$$

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 150(9.81)(x) = 150(1.962)(0.5)$$

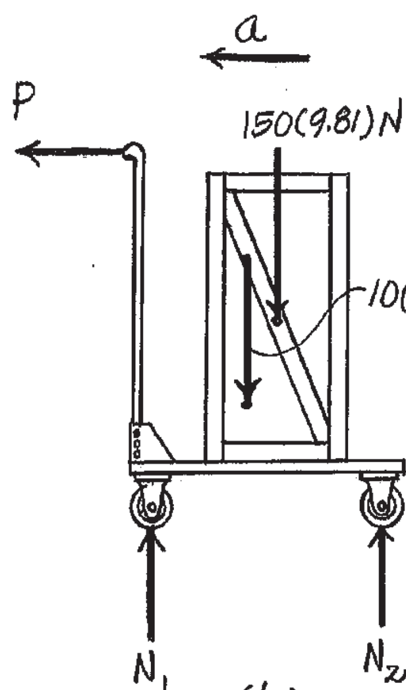
$$x = 0.1\text{ m}$$

Since $x = 0.1\text{ m} < 0.25\text{ m}$, the crate indeed slips before it tips. Using the result of a and refer to the FBD of the crate and cart, Fig. *b*,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P = (150 + 10)(1.962) = 313.92\text{ N} = 314\text{ N} \qquad \textbf{Ans.}$$



(a)

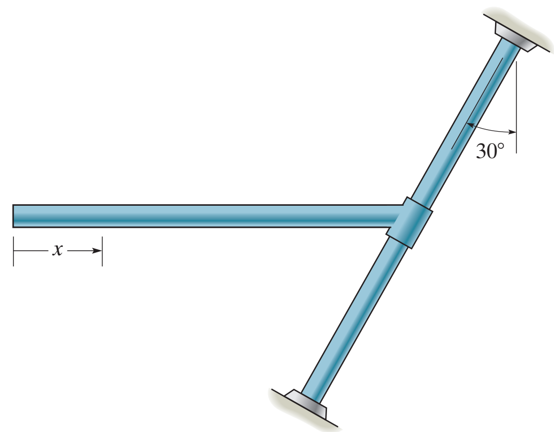


(b)

Ans:
$P = 314$ N

17–39.

The bar has a weight per length w and is supported by the smooth collar. If it is released from rest, determine the internal normal force, shear force, and bending moment in the bar as a function of x.



SOLUTION

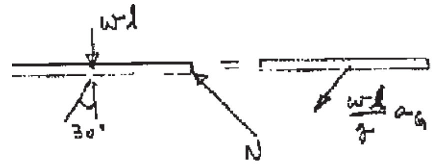
Entire bar:

$$\Sigma F_{x'} = m(a_G)_{x'}; \qquad wl\cos 30^\circ = \frac{wl}{g}(a_G)$$

$$a_G = g\cos 30^\circ$$

Segment:

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad N = (wx\cos 30^\circ)\sin 30^\circ = 0.433wx \qquad \textbf{Ans.}$$

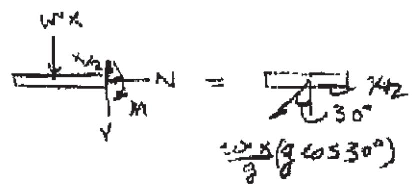


$$+\downarrow \Sigma F_y = m(a_G)_y; \qquad wx - V = wx\cos 30^\circ(\cos 30^\circ)$$

$$V = 0.25wx \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_S = \Sigma(M_k)_S; \qquad wx\left(\frac{x}{2}\right) - M = wx\cos 30^\circ(\cos 30^\circ)\left(\frac{x}{2}\right)$$

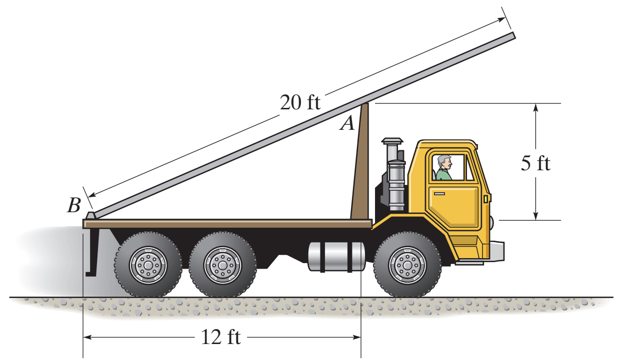
$$M = 0.125wx^2 \qquad \textbf{Ans.}$$



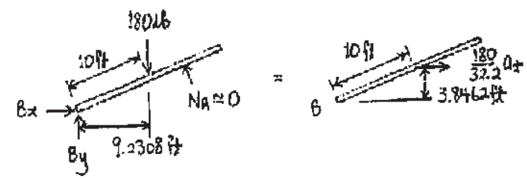
Ans:
$N = 0.433wx$
$V = 0.25wx$
$M = 0.125wx^2$

***17–40.**

The smooth 180-lb pipe has a length of 20 ft and a negligible diameter. It is carried on a truck as shown. Determine the maximum acceleration which the truck can have without causing the normal reaction at A to be zero. Also determine the horizontal and vertical components of force which the truck exerts on the pipe at B.



SOLUTION



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad B_x = \frac{180}{32.2} a_T$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - 180 = 0$$

$$\circlearrowleft + \Sigma M_B = \Sigma (M_k)_B; \qquad 180(10)\left(\frac{12}{13}\right) = \frac{180}{32.2} a_T (10)\left(\frac{5}{13}\right)$$

Solving,

$B_x = 432$ lb **Ans.**

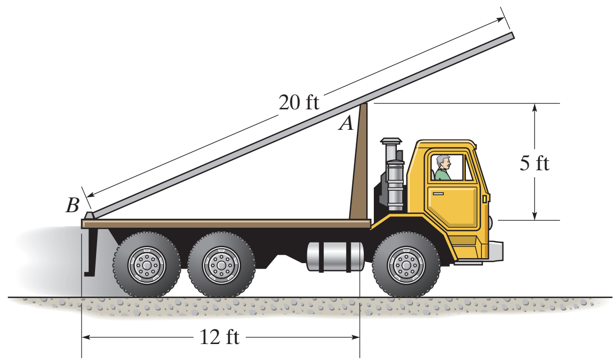
$B_y = 180$ lb **Ans.**

$a_T = 77.3 \text{ ft/s}^2$ **Ans.**

Ans:
$B_x = 432$ lb
$B_y = 180$ lb
$a_T = 77.3 \text{ ft/s}^2$

17–41.

The smooth 180-lb pipe has a length of 20 ft and a negligible diameter. It is carried on a truck as shown. If the truck accelerates at $a = 5\ \text{ft/s}^2$, determine the normal reaction at A and the horizontal and vertical components of force which the truck exerts on the pipe at B.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad B_x - N_A\left(\frac{5}{13}\right) = \frac{180}{32.2}(5)$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - 180 + N_A\left(\frac{12}{13}\right) = 0$$

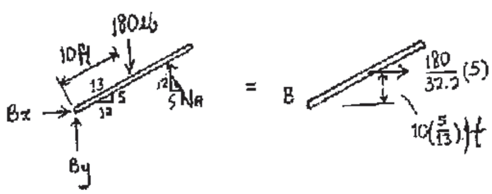
$$\zeta + \Sigma M_B = \Sigma (M_k)_B; \qquad -180(10)\left(\frac{12}{13}\right) + N_A(13) = -\frac{180}{32.2}(5)(10)\left(\frac{5}{13}\right)$$

Solving,

$B_x = 73.9\ \text{lb}$ **Ans.**

$B_y = 69.7\ \text{lb}$ **Ans.**

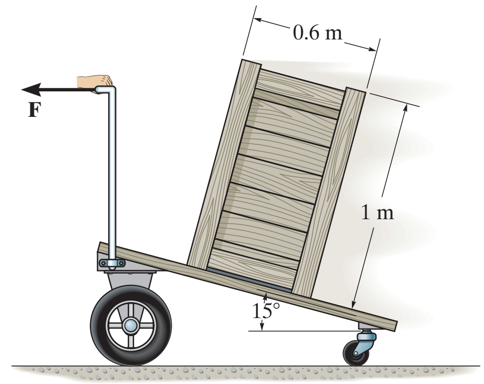
$N_A = 120\ \text{lb}$ **Ans.**



Ans:
$B_x = 73.9\ \text{lb}$
$B_y = 69.7\ \text{lb}$
$N_A = 120\ \text{lb}$

17–42.

The uniform crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine the smallest acceleration that will cause the crate either to tip or slip relative to the cart. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.



SOLUTION

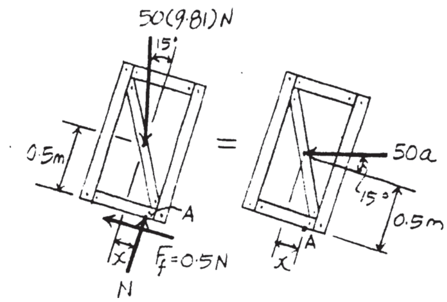
Equations of Motion: Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A$; $50(9.81)\cos 15°(x) - 50(9.81)\sin 15°(0.5)$

$= 50a\cos 15°(0.5) + 50a\sin 15°(x)$ **(1)**

$+\nearrow \Sigma F_{y'} = m(a_G)_{y'}$; $N - 50(9.81)\cos 15° = -50a\sin 15°$ **(2)**

$\searrow + \Sigma F_{x'} = m(a_G)_{x'}$; $50(9.81)\sin 15° - 0.5N = -50a\cos 15°$ **(3)**



Solving Eqs. (1), (2), and (3) yields

$$N = 447.81 \text{ N} \qquad x = 0.250 \text{ m}$$

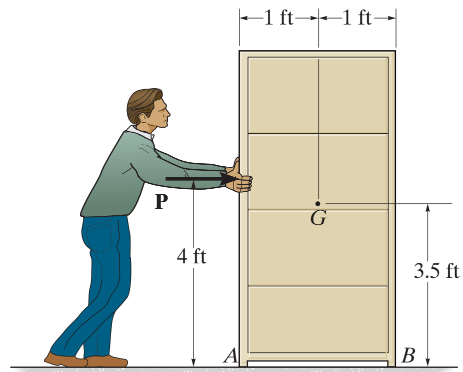
$$a = 2.01 \text{ m/s}^2$$ **Ans.**

Since $x < 0.3$ m, then crate will not tip. Thus, **the crate slips**. **Ans.**

Ans:
$a = 2.01 \text{ m/s}^2$
The crate slips.

17–43.

Determine the acceleration of the 150-lb cabinet and the normal reaction under the legs A and B if $P = 35$ lb. The coefficients of static and kinetic friction between the cabinet and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively. The cabinet's center of gravity is located at G.



SOLUTION

Equations of Equilibrium: The free-body diagram of the cabinet under the static condition is shown in Fig. a, where $\mathbf{P}$ is the unknown minimum force needed to move the cabinet. We will assume that the cabinet slides before it tips. Then, $F_A = \mu_s N_A = 0.2N_A$ and $F_B = \mu_s N_B = 0.2N_B$.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad P - 0.2N_A - 0.2N_B = 0 \qquad \textbf{(1)}$$

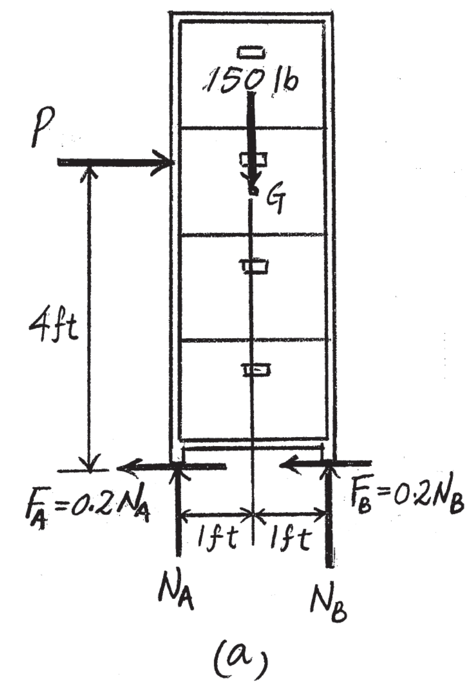
$$+\uparrow\Sigma F_y = 0; \qquad N_A + N_B - 150 = 0 \qquad \textbf{(2)}$$

$$+\Sigma M_A = 0; \qquad N_B(2) - 150(1) - P(4) = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

$$P = 30\text{ lb} \qquad N_A = 15\text{ lb} \qquad N_B = 135\text{ lb}$$

Since $P < 35$ lb and $\mathbf{N}_A$ is positive, the cabinet will slide.



Equations of Motion: Since the cabinet is in motion, $F_A = \mu_k N_A = 0.15N_A$ and $F_B = \mu_k N_B = 0.15N_B$. Referring to the free-body diagram of the cabinet shown in Fig. b,

$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad 35 - 0.15N_A - 0.15N_B = \left(\frac{150}{32.2}\right)a \qquad \textbf{(4)}$$

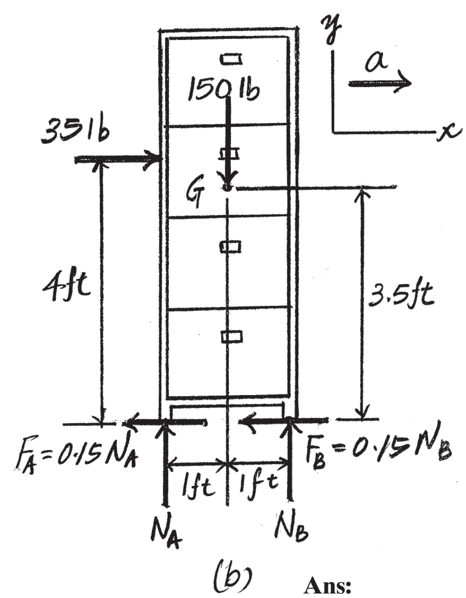
$$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad N_A + N_B - 150 = 0 \qquad \textbf{(5)}$$

$$+\Sigma M_G = 0; \qquad N_B(1) - 0.15N_B(3.5) - 0.15N_A(3.5) - N_A(1) - 35(0.5) = 0 \qquad \textbf{(6)}$$

Solving Eqs. (4), (5), and (6) yields

$$a = 2.68\text{ ft/s}^2 \qquad \textbf{Ans.}$$

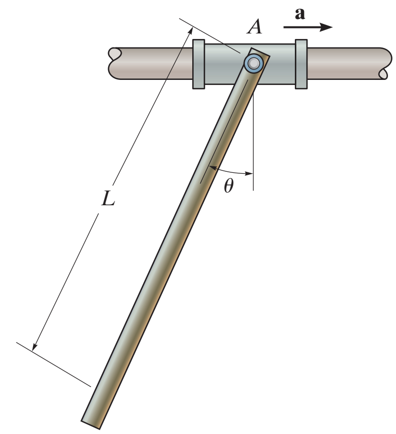
$$N_A = 26.9\text{ lb} \qquad N_B = 123\text{ lb} \qquad \textbf{Ans.}$$



Ans:
$a = 2.68\text{ ft/s}^2$
$N_A = 26.9$ lb
$N_B = 123$ lb

***17–44.**

The uniform bar of mass m is pin connected to the collar, which slides along the smooth horizontal rod. If the collar is given a constant acceleration of **a**, determine the bar's inclination angle θ. Neglect the collar's mass.

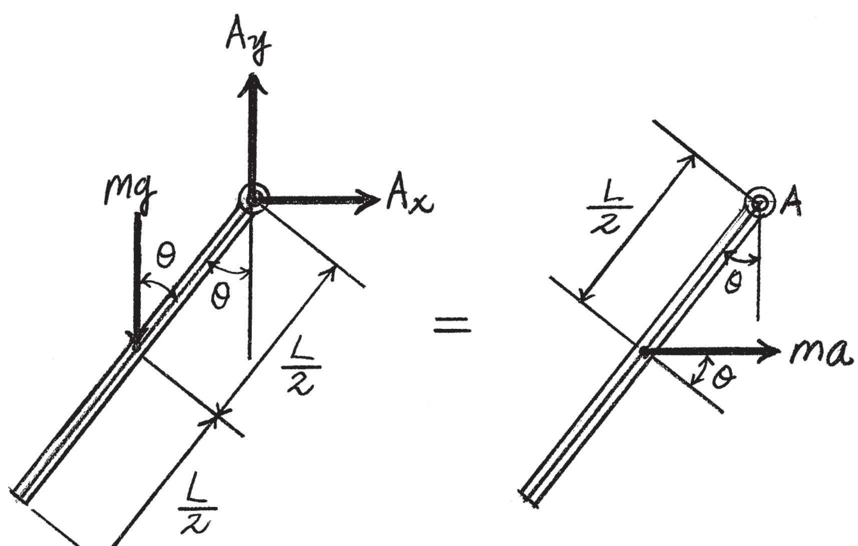


SOLUTION

Equations of Motion: Writing the moment equation of motion about point A,

$$+\Sigma M_A = (M_k)_A; \qquad mg \sin\theta\left(\frac{L}{2}\right) = ma\cos\theta\left(\frac{L}{2}\right)$$

$$\theta = \tan^{-1}\left(\frac{a}{g}\right) \qquad \textbf{Ans.}$$



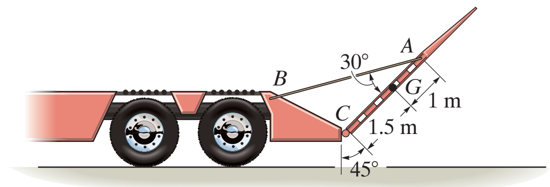
(a)

Ans:

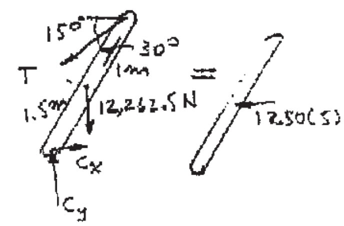
$\theta = \tan^{-1}\left(\frac{a}{g}\right)$

17–45.

The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at G. If it is supported by the cable AB and hinge at C, determine the tension in the cable when the truck begins to accelerate at 5 m/s^2. Also, what are the horizontal and vertical components of reaction at the hinge C?



SOLUTION



$\circlearrowleft + \Sigma M_C = \Sigma (M_k)_C;\quad T \sin 30°(2.5) - 12\,262.5(1.5 \cos 45°) = 1250(5)(1.5 \sin 45°)$

$T = 15\,708.4 \text{ N} = 15.7 \text{ kN}$ **Ans.**

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\quad -C_x + 15\,708.4 \cos 15° = 1250(5)$

$C_x = 8.92 \text{ kN}$ **Ans.**

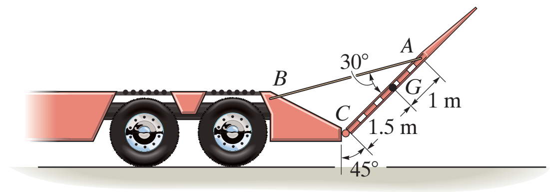
$+\uparrow \Sigma F_y = m(a_G)_y;\quad C_y - 12\,262.5 - 15\,708.4 \sin 15° = 0$

$C_y = 16.3 \text{ kN}$ **Ans.**

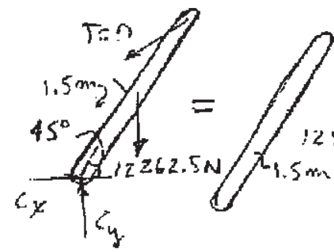
Ans:
$T = 15.7$ kN
$C_x = 8.92$ kN
$C_y = 16.3$ kN

17–46.

The drop gate at the end of the trailer has a mass of 1.25 Mg and mass center at G. If it is supported by the cable AB and hinge at C, determine the maximum deceleration of the truck so that the gate does not begin to rotate forward. What are the horizontal and vertical components of reaction at the hinge C?



SOLUTION



$\circlearrowleft +\Sigma M_C = \Sigma(M_k)_C; \qquad -12\,262.5(1.5 \cos 45°) = -1250(a)(1.5 \sin 45°)$

$a = 9.81 \text{ m/s}^2$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad C_x = 1250(9.81)$

$C_x = 12.3 \text{ kN}$ **Ans.**

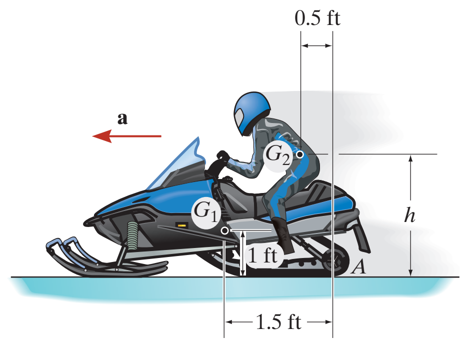
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad C_y - 12\,262.5 = 0$

$C_y = 12.3 \text{ kN}$ **Ans.**

Ans:
$a = 9.81 \text{ m/s}^2$
$C_x = 12.3 \text{ kN}$
$C_y = 12.3 \text{ kN}$

17–47.

The snowmobile has a weight of 250 lb, centered at G_1, while the rider has a weight of 150 lb, centered at G_2. If the acceleration is $a = 20\ \text{ft/s}^2$, determine the maximum height h of G_2 of the rider so that the snowmobile's front skid does not lift off the ground. Also, what are the traction (horizontal) force and normal reaction under the rear tracks at A?



SOLUTION

Equations of Motion: Since the front skid is required to be on the verge of lift off, $N_B = 0$. Writing the moment equation about point A and referring to Fig. a,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 250(1.5) + 150(0.5) = \frac{150}{32.2}(20)(h_{\text{max}}) + \frac{250}{32.2}(20)(1)$$

$$h_{\text{max}} = 3.163\ \text{ft} = 3.16\ \text{ft} \qquad \textbf{Ans.}$$

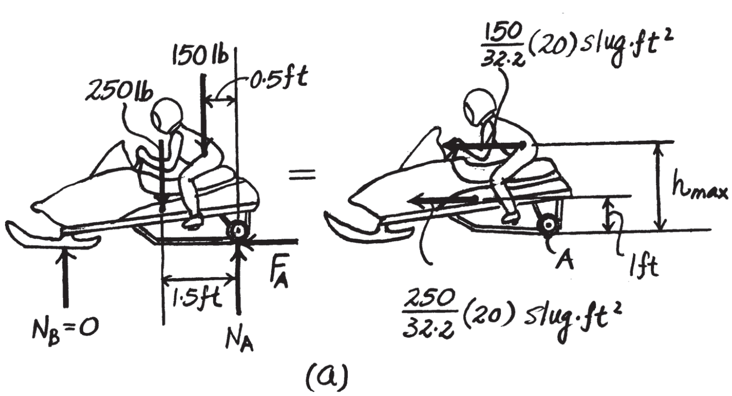
Writing the force equations of motion along the x and y axes,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_A = \frac{150}{32.2}(20) + \frac{250}{32.2}(20)$$

$$F_A = 248.45\ \text{lb} = 248\ \text{lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 250 - 150 = 0$$

$$N_A = 400\ \text{lb} \qquad \textbf{Ans.}$$

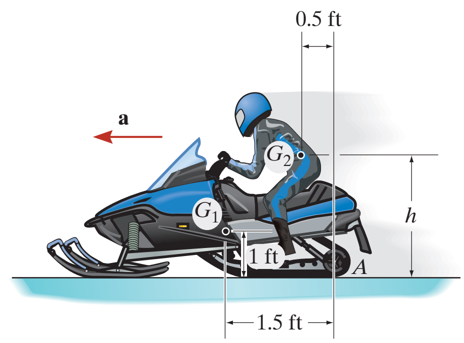


(a)

Ans:
$h_{\text{max}} = 3.16\ \text{ft}$
$F_A = 248\ \text{lb}$
$N_A = 400\ \text{lb}$

***17–48.**

The snowmobile has a weight of 250 lb, centered at G_1, while the rider has a weight of 150 lb, centered at G_2. If $h = 3$ ft, determine the snowmobile's maximum permissible acceleration **a** so that its front skid does not lift off the ground. Also, find the traction (horizontal) force and the normal reaction under the rear tracks at A.



SOLUTION

Equations of Motion: Since the front skid is required to be on the verge of lift off, $N_B = 0$. Writing the moment equation about point A and referring to Fig. a,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 250(1.5) + 150(0.5) = \left(\frac{150}{32.2}a_{\max}\right)(3) + \left(\frac{250}{32.2}a_{\max}\right)(1)$$

$$a_{\max} = 20.7 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

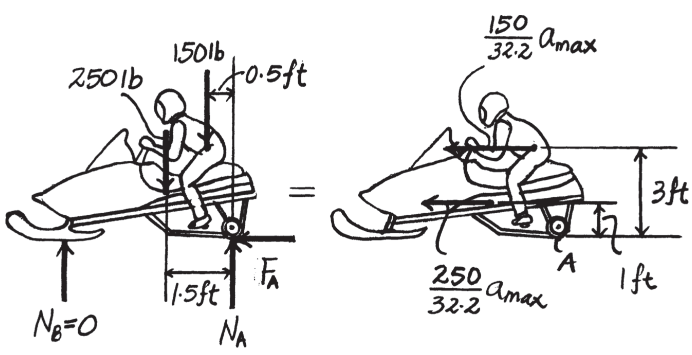
Writing the force equations of motion along the x and y axes and using this result, we have

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad F_A = \frac{150}{32.2}(20.7) + \frac{250}{32.2}(20.7)$$

$$F_A = 257.14 \text{ lb} = 257 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A - 150 - 250 = 0$$

$$N_A = 400 \text{ lb} \qquad \textbf{Ans.}$$

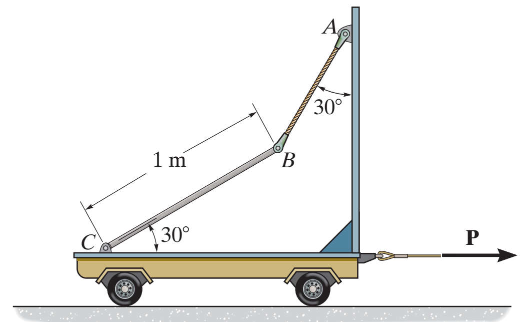


(a)

Ans:
$a_{\max} = 20.7 \text{ ft/s}^2$
$F_A = 257 \text{ lb}$
$N_A = 400 \text{ lb}$

17–49.

If the cart's mass is 30 kg and it is subjected to a horizontal force of $P = 90$ N, determine the tension in cord AB and the horizontal and vertical components of reaction on end C of the uniform 15-kg rod BC.



SOLUTION

Equations of Motion: The acceleration **a** of the cart and the rod can be determined by considering the free-body diagram of the cart and rod system shown in Fig. *a*.

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 90 = (15 + 30)a \qquad a = 2 \text{ m/s}^2$

The force in the cord can be obtained directly by writing the moment equation of motion about point C by referring to Fig. *b*.

$+\Sigma M_C = (M_k)_C; \quad F_{AB} \sin 30°(1) - 15(9.81)\cos 30°(0.5) = -15(2)\sin 30°(0.5)$

$F_{AB} = 112.44 \text{ N} = 112 \text{ N}$ **Ans.**

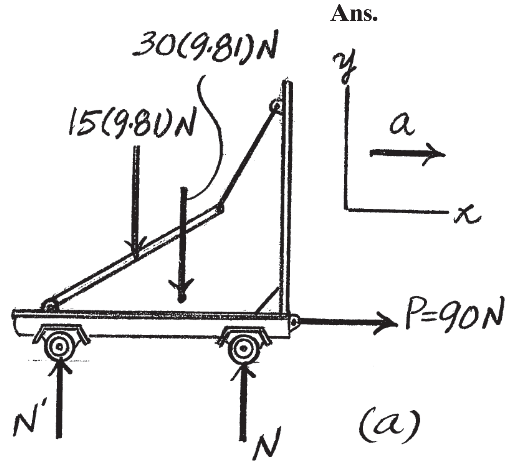
Using this result and applying the force equations of motion along the x and y axes,

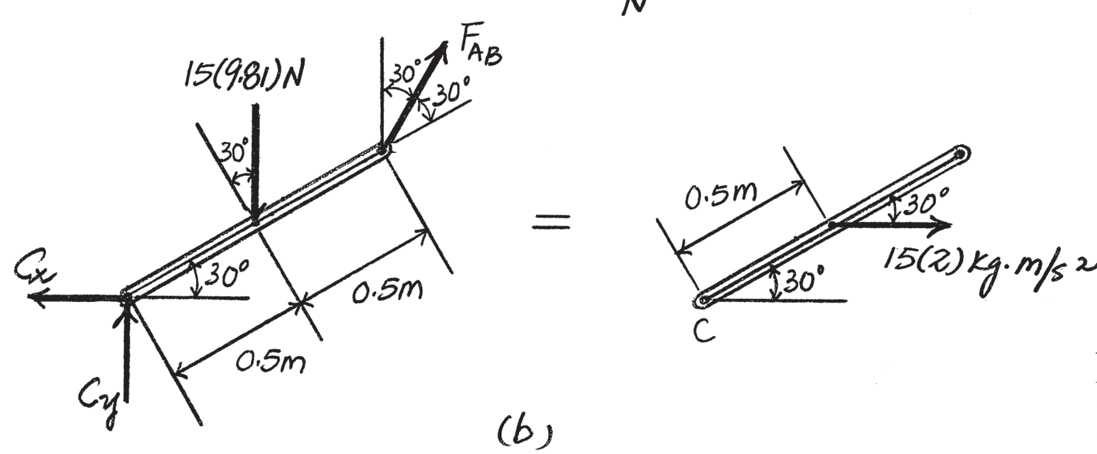
$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad -C_x + 112.44 \sin 30° = 15(2)$

$C_x = 26.22 \text{ N} = 26.2 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y; \quad C_y + 112.44 \cos 30° - 15(9.81) = 0$

$C_y = 49.78 \text{ N} = 49.8 \text{ N}$ **Ans.**

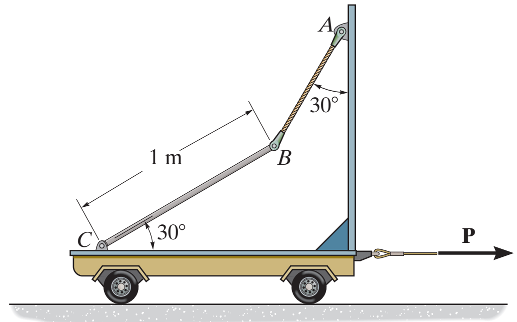




Ans:
$F_{AB} = 112$ N
$C_x = 26.2$ N
$C_y = 49.8$ N

17–50.

If the cart's mass is 30 kg, determine the horizontal force P that should be applied to the cart so that the cord AB just becomes slack. The uniform rod BC has a mass of 15 kg.



SOLUTION

Equations of Motion: Since cord AB is required to be on the verge of becoming slack, $F_{AB} = 0$. The corresponding acceleration $\mathbf{a}$ of the rod can be obtained directly by writing the moment equation of motion about point C. By referring to Fig. a.

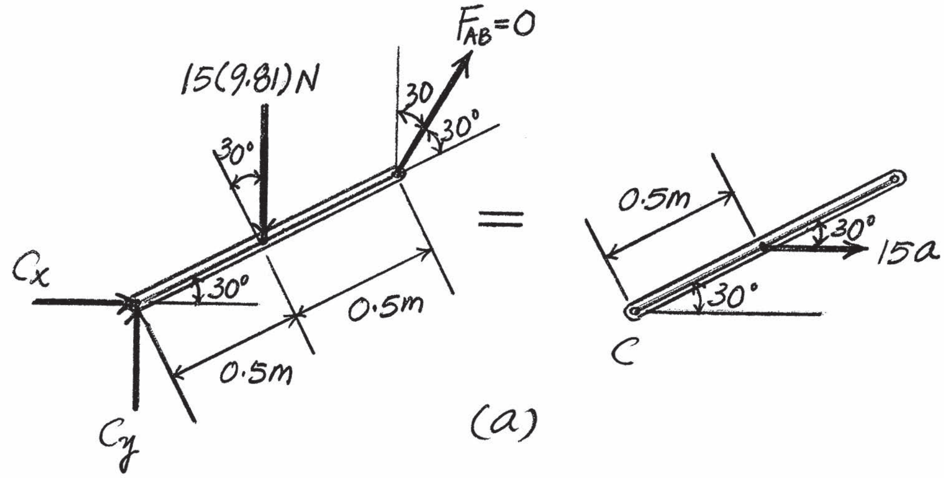
$$+\Sigma M_C = \Sigma(M_C)_A; \qquad -15(9.81)\cos 30°(0.5) = -15a\sin 30°(0.5)$$

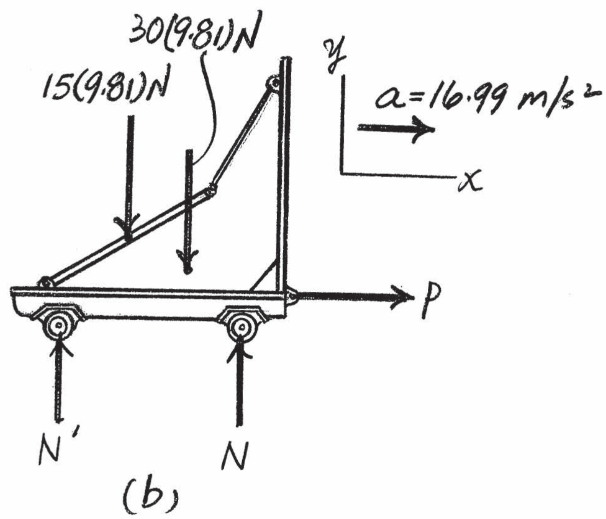
$$a = 16.99 \text{ m/s}^2$$

Using this result and writing the force equation of motion along the x axis and referring to the free-body diagram of the cart and rod system shown in Fig. b,

$$(\xrightarrow{+})\Sigma F_x = m(a_G)_x; \qquad P = (30 + 15)(16.99)$$

$$= 764.61 \text{ N} = 765 \text{ N} \qquad \textbf{Ans.}$$

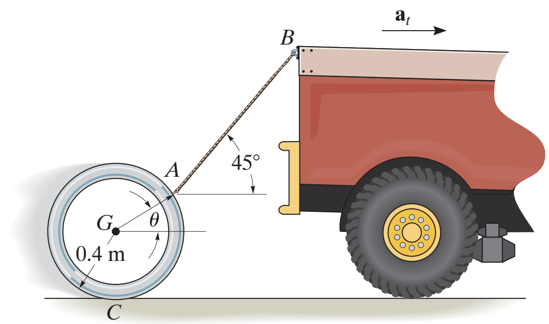




Ans:
$P = 765$ N

17–51.

The pipe has a mass of 800 kg and is being towed behind the truck. If the acceleration of the truck is $a_t = 0.5\ \text{m/s}^2$, determine the angle θ and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -0.1N_C + T\cos 45° = 800(0.5)$

$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 800(9.81) + T\sin 45° = 0$

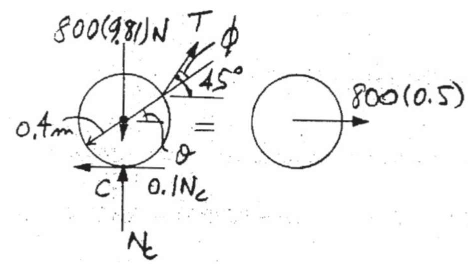
$\circlearrowleft + \Sigma M_G = 0; \qquad -0.1N_C(0.4) + T\sin\phi(0.4) = 0$

$N_C = 6770.9\ \text{N}$

$T = 1523.24\ \text{N} = 1.52\ \text{kN}$ **Ans.**

$\sin\phi = \dfrac{0.1(6770.9)}{1523.24} \qquad \phi = 26.39°$

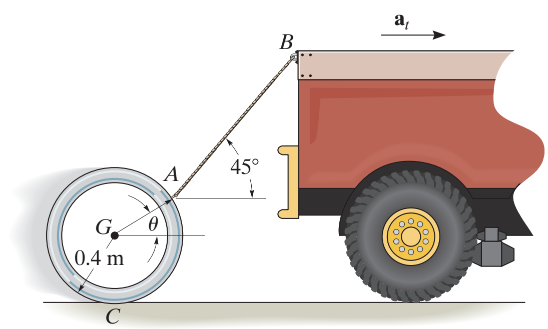
$\theta = 45° - \phi = 18.6°$ **Ans.**



Ans:
$T = 1.52\ \text{kN}$
$\theta = 18.6°$

***17–52.**

The pipe has a mass of 800 kg and is being towed behind a truck. If the angle $\theta = 30°$, determine the acceleration of the truck and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.



SOLUTION

$\xrightarrow{+} \Sigma F_x = m a_x; \qquad T \cos 45° - 0.1 N_C = 800a$

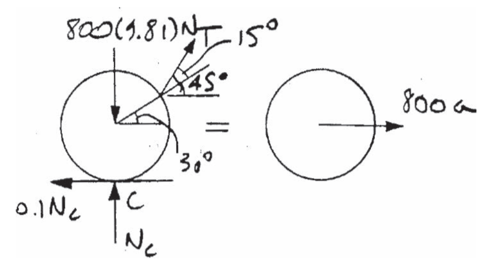
$+\uparrow \Sigma F_y = m a_y; \qquad N_C - 800(9.81) + T \sin 45° = 0$

$\circlearrowleft + \Sigma M_G = 0; \qquad T \sin 15°(0.4) - 0.1 N_C(0.4) = 0$

$N_C = 6161 \text{ N}$

$T = 2382 \text{ N} = 2.38 \text{ kN}$ **Ans.**

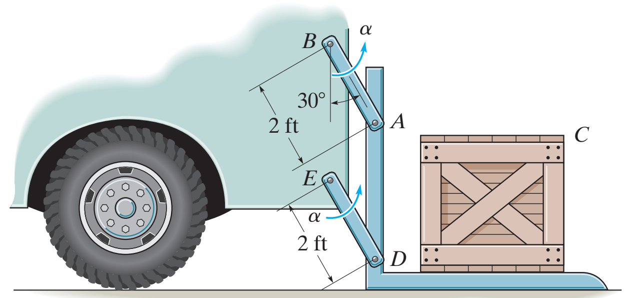
$a = 1.33 \text{ m/s}^2$ **Ans.**



Ans:
$T = 2.38 \text{ kN}$
$a = 1.33 \text{ m/s}^2$

17–53.

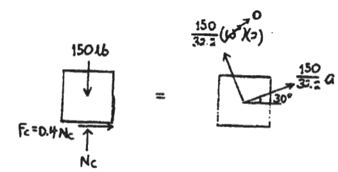
The crate C has a weight of 150 lb and rests on the truck elevator for which the coefficient of static friction is $\mu_s = 0.4$. Determine the largest initial angular acceleration α, starting from rest, which the parallel links AB and DE can have without causing the crate to slip. No tipping occurs.



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 0.4N_C = \dfrac{150}{32.2}(a)\cos 30°$

$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 150 = \dfrac{150}{32.2}(a)\sin 30°$



$N_C = 195.0 \text{ lb}$

$a = 19.34 \text{ ft/s}^2$

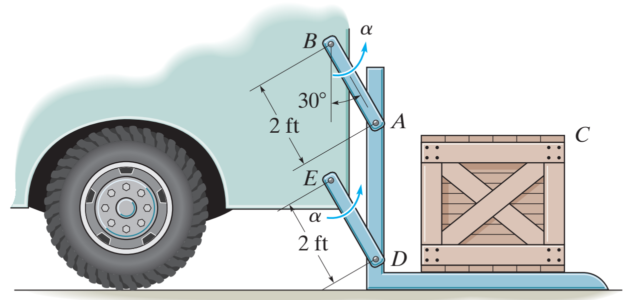
$19.34 = 2\alpha$

$\alpha = 9.67 \text{ rad/s}^2$ **Ans.**

Ans:
$\alpha = 9.67 \text{ rad/s}^2$

17–54.

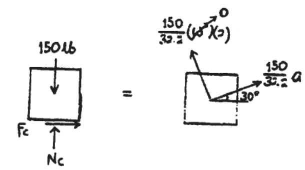
The crate C has a weight of 150 lb and rests on the truck elevator. Determine the initial friction and normal force of the elevator on the crate if the parallel links are given an angular acceleration $\alpha = 2\ \text{rad/s}^2$ starting from rest.



SOLUTION

$\alpha = 2\ \text{rad/s}^2$

$a = 2\alpha = 4\ \text{rad/s}^2$



$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad F_C = \dfrac{150}{32.2}(a)\cos 30°$

$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 150 = \dfrac{150}{32.2}(a)\sin 30°$

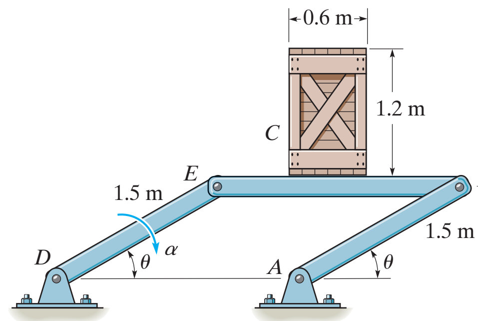
$F_C = 16.1\ \text{lb}$ **Ans.**

$N_C = 159\ \text{lb}$ **Ans.**

Ans:
$F_C = 16.1\ \text{lb}$
$N_C = 159\ \text{lb}$

17–55.

The 100-kg uniform crate C rests on the elevator floor where the coefficient of static friction is $\mu_s = 0.4$. Determine the largest initial angular acceleration α, starting from rest at $\theta = 90°$, without causing the crate to slip. No tipping occurs.



SOLUTION

Equations of Motion. The crate undergoes curvilinear translation. At $\theta = 90°$, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. However; $(a_G)_t = \alpha r = \alpha(1.5)$. Assuming that the crate slides before it tips, then, $F_f = \mu_s N = 0.4\ N$.

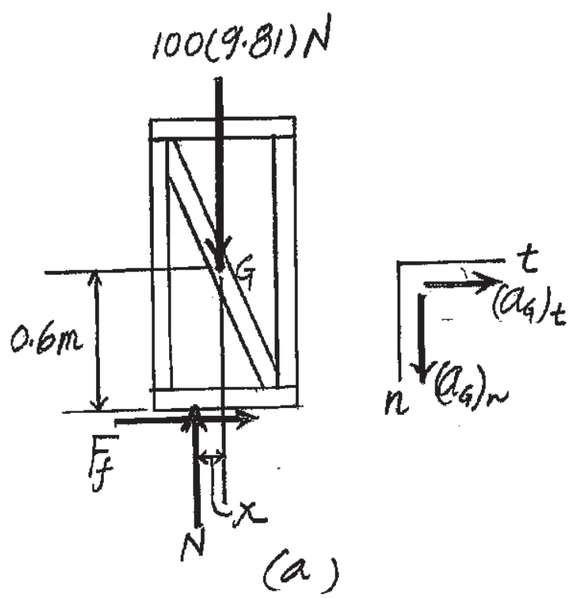
$\Sigma F_n = m(a_G)_n;\quad 100(9.81) - N = 100(0)\quad N = 981\text{ N}$

$\Sigma F_t = m(a_G)_t;\quad 0.4(981) = 100[\alpha(1.5)]\quad \alpha = 2.616\text{ rad/s}^2 = 2.62\text{ rad/s}^2$ **Ans.**

$\circlearrowleft + \Sigma M_G = 0;\quad 0.4(981)(0.6) - 981(x) = 0$

$x = 0.24\text{ m}$

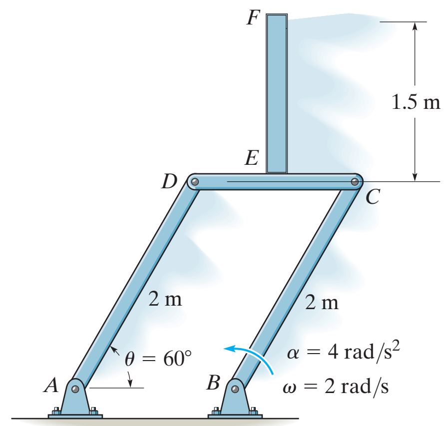
Since $x < 0.3$ m, the crate indeed slides before it tips, as assumed.



Ans:
$\alpha = 2.62\text{ rad/s}^2$

***17–56.**

The two uniform 4-kg bars DC and EF are fixed (welded) together at E. Determine the normal force N_E, shear force V_E, and moment M_E, which DC exerts on EF at E if at the instant $\theta = 60°$ BC has an angular velocity $\omega = 2$ rad/s and an angular acceleration $\alpha = 4$ rad/s^2 as shown.



SOLUTION

Equations of Motion. The rod assembly undergoes curvilinear motion. Thus, $(a_G)_t = \alpha r = 4(2) = 8\text{ m/s}^2$ and $(a_G)_n = \omega^2 r = (2^2)(2) = 8\text{ m/s}^2$. Referring to the FBD and kinetic diagram of rod EF, Fig. a

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad V_E = 4(8)\cos 30° + 4(8)\cos 60°$$

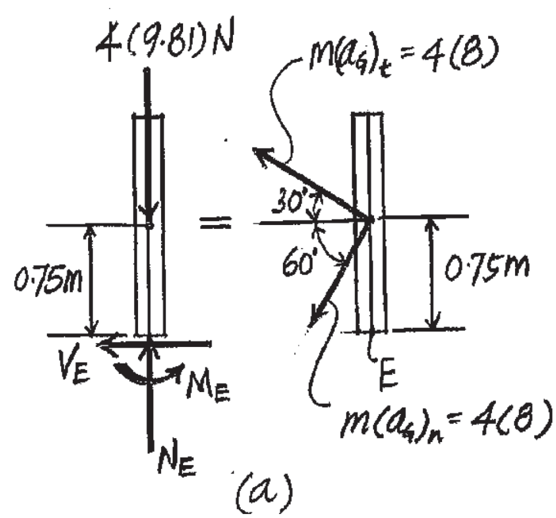
$$= 43.71\text{ N} = 43.7\text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_E - 4(9.81) = 4(8)\sin 30° - 4(8)\sin 60°$$

$$N_E = 27.53\text{ N} = 27.5\text{ N} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_E = \Sigma(M_k)_E; \qquad M_E = 4(8)\cos 30°(0.75) + 4(8)\cos 60°(0.75)$$

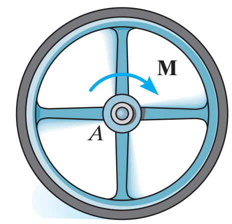
$$= 32.78\text{ N}\cdot\text{m} = 32.8\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



Ans:
$V_E = 43.7$ N
$N_E = 27.5$ N
$M_E = 32.8$ N·m

17–57.

The 10-kg wheel has a radius of gyration $k_A = 200$ mm. If the wheel is subjected to a moment $M = (5t)$ N·m, where t is in seconds, determine its angular velocity when $t = 3$ s starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.

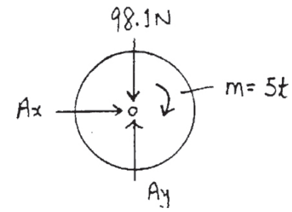


SOLUTION

$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x; \qquad A_x = 0$

$+\uparrow\Sigma F_y = m(a_G)_y; \qquad A_y - 10(9.81) = 0$

$\circlearrowright + \Sigma M_A = I_a\alpha; \qquad 5t = 10(0.2)^2\alpha$



$$\alpha = \frac{d\omega}{dt} = 12.5t$$

$$\omega = \int_0^3 12.5t\,dt = \frac{12.5}{2}(3)^2$$

$\omega = 56.2$ rad/s **Ans.**

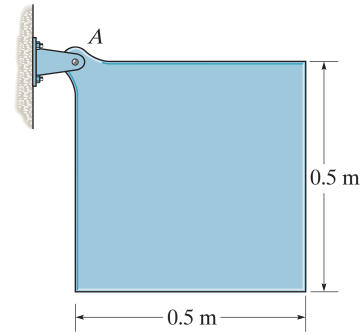
$A_x = 0$ **Ans.**

$A_y = 98.1$ N **Ans.**

Ans:
$\omega = 56.2$ rad/s
$A_x = 0$
$A_y = 98.1$ N

17–58.

The uniform 24-kg plate is released from rest at the position shown. Determine its initial angular acceleration and the horizontal and vertical reactions at the pin A.



SOLUTION

Equations of Motion. The mass moment of inertia of the plate about its center of gravity G is $I_G = \frac{1}{12}(24)(0.5^2 + 0.5^2) = 1.00\ \text{kg} \cdot \text{m}^2$. Since the plate is at rest initially $\omega = 0$. Thus, $(a_G)_n = \omega^2 r_G = 0$. Here $r_G = \sqrt{0.25^2 + 0.25^2} = 0.25\sqrt{2}$ m. Thus, $(a_G)_t = \alpha r_G = \alpha(0.25\sqrt{2})$. Referring to the FBD and kinetic diagram of the plate,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \quad -24(9.81)(0.25) = -24\left[\alpha(0.25\sqrt{2})\right](0.25\sqrt{2}) - 1.00\,\alpha$$

$$\alpha = 14.715\ \text{rad/s}^2 = 14.7\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Also, the same result can be obtained by applying $\Sigma M_A = I_A \alpha$ where $I_A = \frac{1}{12}(24)(0.5^2 + 0.5^2) + 24(0.25\sqrt{2})^2 = 4.00\ \text{kg} \cdot \text{m}^2$:

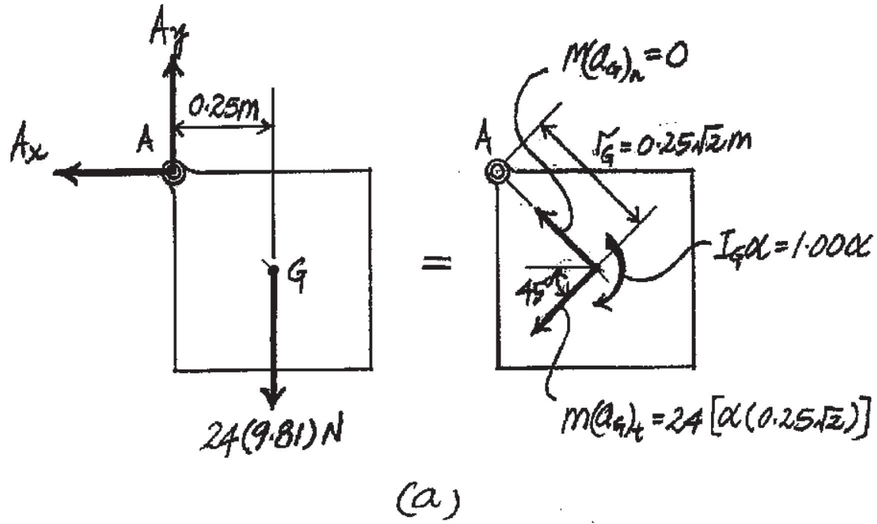
$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \quad -24(9.81)(0.25) = -4.00\,\alpha$$

$$\alpha = 14.715\ \text{rad/s}^2$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad A_x = 24\left[14.715(0.25\sqrt{2})\right]\cos 45^\circ = 88.29\ \text{N} = 88.3\ \text{N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad A_y - 24(9.81) = -24\left[14.715(0.25\sqrt{2})\right]\sin 45^\circ$$

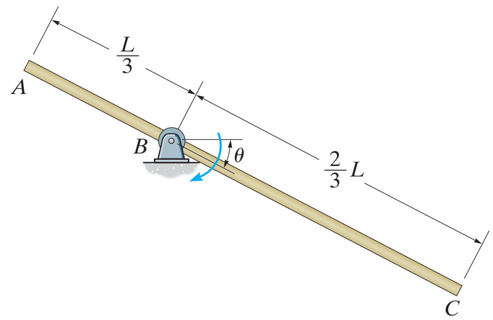
$$A_y = 147.15\ \text{N} = 147\ \text{N} \qquad \textbf{Ans.}$$



Ans:
$\alpha = 14.7\ \text{rad/s}^2$
$A_x = 88.3$ N
$A_y = 147$ N

17–59.

The uniform slender rod has a mass m. If it is released from rest when $\theta = 0°$, determine the magnitude of the reactive force exerted on it by pin B when $\theta = 90°$.



SOLUTION

Equations of Motion: Since the rod rotates about a fixed axis passing through point B, $(a_G)_t = \alpha r_G = \alpha\left(\frac{L}{6}\right)$ and $(a_G)_n = \omega^2 r_G = \omega^2\left(\frac{L}{6}\right)$. The mass moment of inertia of the rod about its G is $I_G = \frac{1}{12}mL^2$. Writing the moment equation of motion about point B,

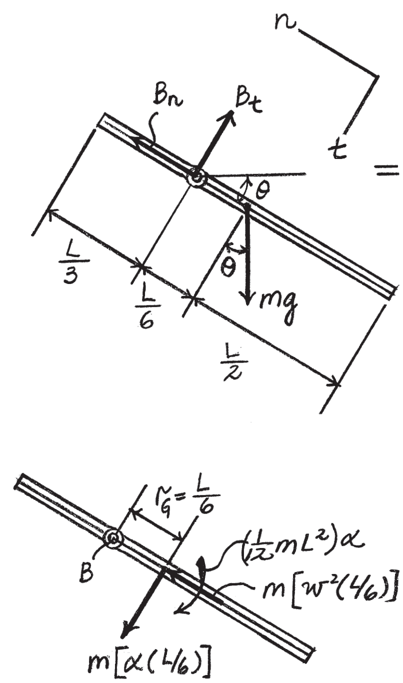
$$+\Sigma M_B = \Sigma(M_k)_B; \quad -mg\cos\theta\left(\frac{L}{6}\right) = -m\left[\alpha\left(\frac{L}{6}\right)\right]\left(\frac{L}{6}\right) - \left(\frac{1}{12}mL^2\right)\alpha$$

$$\alpha = \frac{3g}{2L}\cos\theta$$

This equation can also be obtained by applying $\Sigma M_B = I_B\alpha$, where $I_B = \frac{1}{12}mL^2 + m\left(\frac{L}{6}\right)^2 = \frac{1}{9}mL^2$. Thus,

$$+\Sigma M_B = I_B\alpha; \quad -mg\cos\theta\left(\frac{L}{6}\right) = -\left(\frac{1}{9}mL^2\right)\alpha$$

$$\alpha = \frac{3g}{2L}\cos\theta$$



(a)

Using this result and writing the force equation of motion along the n and t axes,

$$\Sigma F_t = m(a_G)_t; \quad mg\cos\theta - B_t = m\left[\left(\frac{3g}{2L}\cos\theta\right)\left(\frac{L}{6}\right)\right]$$

$$B_t = \frac{3}{4}mg\cos\theta \quad \textbf{(1)}$$

$$\Sigma F_n = m(a_G)_n; \quad B_n - mg\sin\theta = m\left[\omega^2\left(\frac{L}{6}\right)\right]$$

$$B_n = \frac{1}{6}m\omega^2 L + mg\sin\theta \quad \textbf{(2)}$$

Kinematics: The angular velocity of the rod can be determined by integrating

$$\int \omega\,d\omega = \int \alpha\,d\theta$$

$$\int_0^\omega \omega\,d\omega = \int_0^\theta \frac{3g}{2L}\cos\theta\,d\theta$$

$$\omega = \sqrt{\frac{3g}{L}\sin\theta}$$

When $\theta = 90°$, $\omega = \sqrt{\frac{3g}{L}}$. Substituting this result and $\theta = 90°$ into Eqs. (1) and (2),

$$B_t = \frac{3}{4}mg\cos 90° = 0$$

$$B_n = \frac{1}{6}m\left(\frac{3g}{L}\right)(L) + mg\sin 90° = \frac{3}{2}mg$$

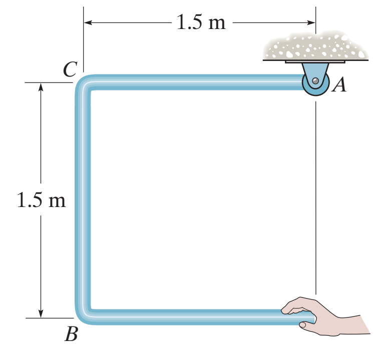
$$F_A = \sqrt{A_t^2 + A_n^2} = \sqrt{0^2 + \left(\frac{3}{2}mg\right)^2} = \frac{3}{2}mg \quad \textbf{Ans.}$$

Ans:

$F_A = \frac{3}{2}mg$

***17–60.**

The bent rod has a mass of 2 kg/m. If it is released from rest in the position shown, determine its initial angular acceleration and the horizontal and vertical components of reaction at A.



SOLUTION

Equations of Motion. Referring to Fig. a, the location of center of gravity G of the bent rod is at

$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m} = \frac{2[0.75(1.5)(2)] + 1.5(2)(1.5)}{3(1.5)(2)} = 1.00 \text{ m}$$

$$\bar{y} = \frac{1.5}{2} = 0.75 \text{ m}$$

The mass moment of inertia of the bent rod about its center of gravity is $I_G = 2\left[\frac{1}{12}(3)(1.5^2) + 3(0.25^2 + 0.75^2)\right] + \left[\frac{1}{12}(3)(1.5^2) + 3(0.5^2)\right] = 6.1875 \text{ kg}\cdot\text{m}^2$. Here, $r_G = \sqrt{1.00^2 + 0.75^2} = 1.25$ m. Since the bent rod is at rest initially, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r_G = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(1.25)$. Referring to the FBD and kinetic diagram of the plate,

$$\circlearrowleft + \Sigma M_A = (M_k)_A; \qquad 9(9.81)(1) = 9[\alpha(1.25)](1.25) + 6.1875\,\alpha$$

$$\alpha = 4.36 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Also, the same result can be obtained by applying $\Sigma M_A = I_A \alpha$ where

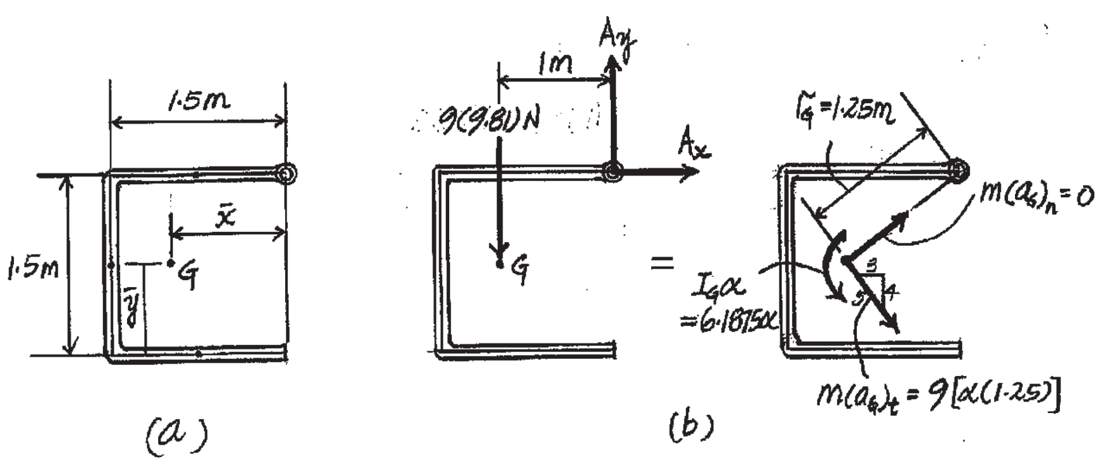
$$I_A = \frac{1}{12}(3)(1.5^2) + 3(0.75^2) + \frac{1}{12}(3)(1.5^2) + 3(1.5^2 + 0.75^2)$$
$$+ \frac{1}{12}(3)(1.5^2) + 3(1.5^2 + 0.75^2) = 20.25 \text{ kg}\cdot\text{m}^2:$$

$$\circlearrowleft + \Sigma M_A = I_A\alpha, \qquad 9(9.81)(1) = 20.25\,\alpha \qquad \alpha = 4.36 \text{ rad/s}^2$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad A_x = 9[4.36(1.25)]\left(\frac{3}{5}\right) = 29.43 \text{ N} = 29.4 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 9(9.81) = -9[4.36(1.25)]\left(\frac{4}{5}\right)$$

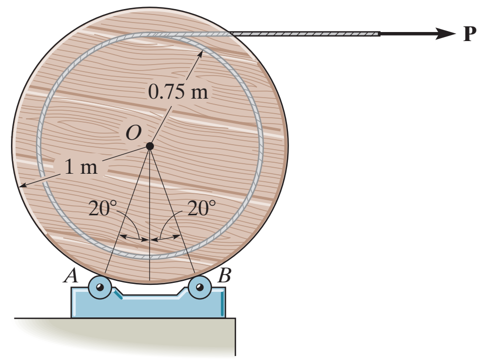
$$A_y = 49.05 \text{ N} = 49.1 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$\alpha = 4.36 \text{ rad/s}^2 \circlearrowright$
$A_x = 29.4$ N
$A_y = 49.1$ N

17–61.

If a horizontal force of $P = 100$ N is applied to the 300-kg reel of cable, determine its initial angular acceleration. The reel rests on rollers at A and B and has a radius of gyration of $k_O = 0.6$ m.



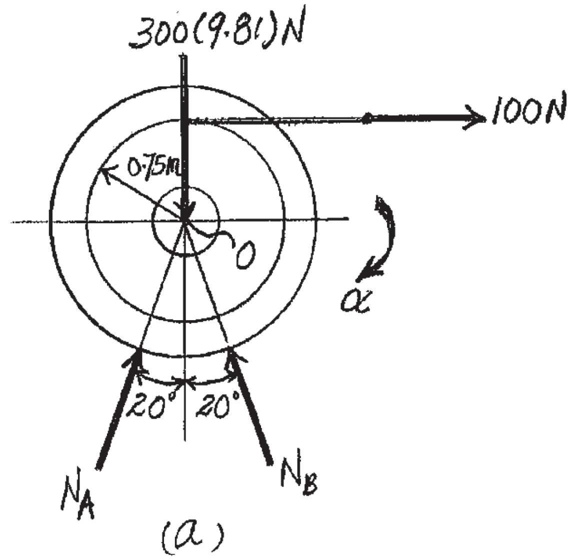
SOLUTION

Equations of Motions. The mass moment of inertia of the reel about O is $I_O = Mk_O^2 = 300\left(0.6^2\right) = 108\ \text{kg}\cdot\text{m}^2$. Referring to the FBD of the reel, Fig. a,

$$\zeta + \Sigma M_O = I_O\,\alpha; \qquad -100(0.75) = 108(-\alpha)$$

$$\alpha = 0.6944\ \text{rad/s}^2$$

$$= 0.694\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

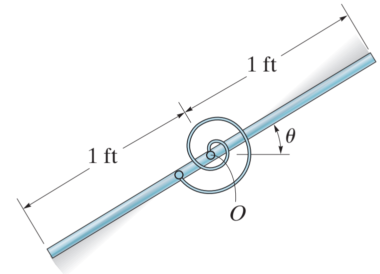


(a)

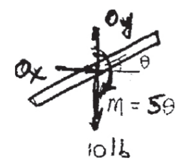
Ans:
$\alpha = 0.694\ \text{rad/s}^2$

17–62.

The 10-lb bar is pinned at its center O and connected to a torsional spring. The spring has a stiffness $k = 5 \text{ lb} \cdot \text{ft/rad}$, so that the torque developed is $M = (5\theta) \text{ lb} \cdot \text{ft}$, where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$.



SOLUTION



$$\circlearrowleft + \Sigma M_O = I_O\alpha; \quad -5\theta = \left[\frac{1}{12}\left(\frac{10}{32.2}\right)(2)^2\right]\alpha$$

$$-48.3\,\theta = \alpha$$

$$\alpha\, d\theta = \omega\, d\omega$$

$$-\int_{\frac{\pi}{2}}^{o} 48.3\,\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$$

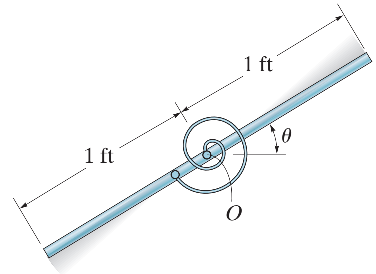
$$\frac{48.3}{2}\left(\frac{\pi}{2}\right)^2 = \frac{1}{2}\omega^2$$

$\omega = 10.9 \text{ rad/s}$ **Ans.**

Ans:
$\omega = 10.9 \text{ rad/s}$

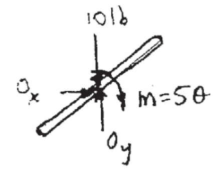
17–63.

The 10-lb bar is pinned at its center O and connected to a torsional spring. The spring has a stiffness $k = 5\ \text{lb}\cdot\text{ft/rad}$, so that the torque developed is $M = (5\theta)\ \text{lb}\cdot\text{ft}$, where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 45°$.



SOLUTION

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 5\theta = \left[\frac{1}{12}\left(\frac{10}{32.2}\right)(2)^2\right]\alpha$$



$$\alpha = -48.3\theta$$

$$\alpha\, d\theta = \omega\, d\omega$$

$$-\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 48.3\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$$

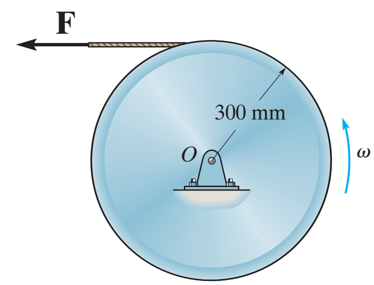
$$-24.15\left(\left(\frac{\pi}{4}\right)^2 - \left(\frac{\pi}{2}\right)^2\right) = \frac{1}{2}\omega^2$$

$$\omega = 9.45\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 9.45\ \text{rad/s}$

***17–64.**

A cord is wrapped around the outer surface of the 8-kg disk. If a force of $F = (\frac{1}{4}\theta^2)$ N, where θ is in radians, is applied to the cord, determine the disk's angular acceleration when it has turned 5 revolutions. The disk has an initial angular velocity of $\omega_0 = 1$ rad/s.



SOLUTION

Equations of Motion. The mass moment inertia of the disk about O is $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(8)(0.3^2) = 0.36\ \text{kg}\cdot\text{m}^2$. Referring to the FBD of the disk, Fig. a,

$$\zeta + \Sigma M_O = I_O\,\alpha; \qquad \left(\frac{1}{4}\theta^2\right)(0.3) = 0.36\,\alpha$$

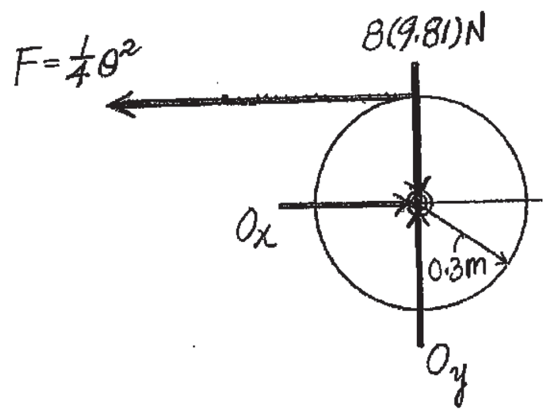
$$\alpha = (0.2083\,\theta^2)\ \text{rad/s}^2$$

Kinematics. Using the result of α, integrate $\omega d\omega = \alpha d\theta$ with the initial condition $\omega = 0$ when $\theta = 0$,

$$\int_1^{\omega} \omega d\omega = \int_0^{5(2\pi)} 0.2083\,\theta^2\,d\theta$$

$$\left(\frac{1}{2}\right)(\omega_2 - 1) = 0.06944\,\theta^3\Big|_0^{5(2\pi)}$$

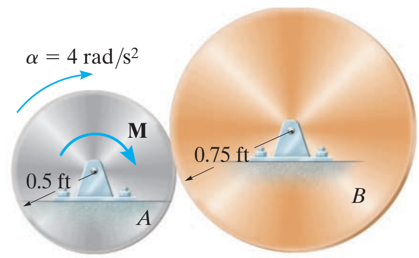
$$\omega = 65.63\ \text{rad/s} = 65.6\ \text{rad/s}$$ **Ans.**



Ans:
$\omega = 65.6$ rad/s

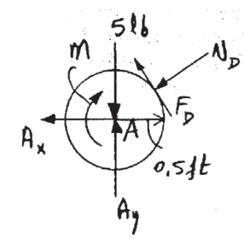
17–65.

Disk A has a weight of 5 lb and disk B has a weight of 10 lb. If no slipping occurs between them, determine the couple moment **M** which must be applied to disk A to give it an angular acceleration of 4 rad/s^2.



SOLUTION

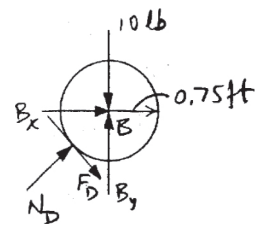
Disk A:



$$\circlearrowleft + \Sigma M_A = I_A \alpha_A; \qquad M - F_D(0.5) = \left[\frac{1}{2}\left(\frac{5}{32.2}\right)(0.5)^2\right](4)$$

Disk B:

$$+\Sigma M_B = I_B \alpha_B; \qquad F_D\,(0.75) = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(0.75)^2\right]\alpha_B$$



$$r_A \alpha_A = r_B \alpha_B$$

$$0.5(4) = 0.75\alpha_B$$

Solving:

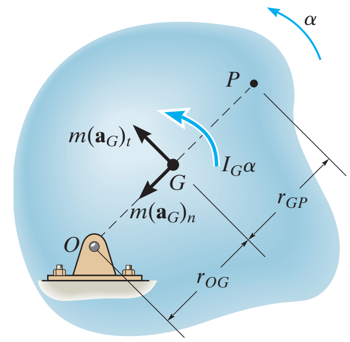
$$\alpha_B = 2.67 \text{ rad/s}^2; \qquad F_D = 0.311 \text{ lb}$$

$$M = 0.233 \text{ lb} \cdot \text{ft}$$ **Ans.**

Ans:
$M = 0.233 \text{ lb} \cdot \text{ft}$

17–66.

The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis passing through O is shown in the figure. Show that $I_G\boldsymbol{\alpha}$ may be eliminated by moving the vectors $m(\mathbf{a}_G)_t$ and $m(\mathbf{a}_G)_n$ to point P, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass G of the body. Here k_G represents the radius of gyration of the body about an axis passing through G. The point P is called the *center of percussion* of the body.



SOLUTION

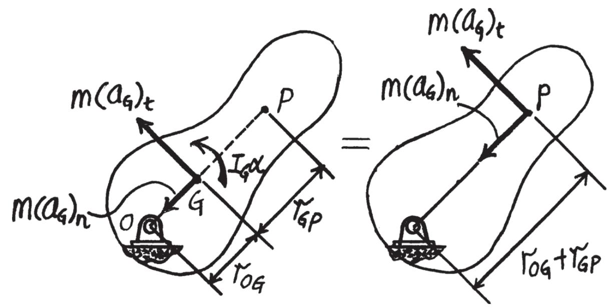
$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + \left(mk_G^2\right)\alpha$$

However,

$$k_G^2 = r_{OG}\, r_{GP} \text{ and } \alpha = \frac{(a_G)_t}{r_{OG}}$$

$$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t\, r_{OG} + (mr_{OG}\, r_{GP})\left[\frac{(a_G)_t}{r_{OG}}\right]$$

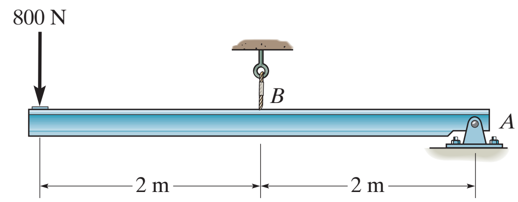
$$= m(a_G)_t(r_{OG} + r_{GP}) \qquad \textbf{Q.E.D.}$$



Ans:
$m(a_G)_t\, r_{OG} + I_G\,\alpha = m(a_G)_t(r_{OG} + r_{GP})$

17–67.

If the cord at B suddenly fails, determine the horizontal and vertical components of the initial reaction at the pin A, and the angular acceleration of the 120-kg beam. Treat the beam as a uniform slender rod.



SOLUTION

Equations of Motion. The mass moment of inertia of the beam about A is $I_A = \frac{1}{12}(120)(4^2) + 120(2^2) = 640\text{ kg}\cdot\text{m}^2$. Initially, the beam is at rest, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(2) = 2\alpha$. Referring to the FBD of the beam, Fig. a

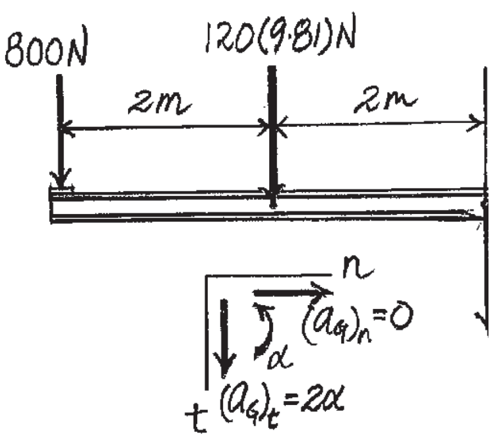
$\circlearrowleft + \Sigma M_A = I_A\alpha;\qquad 800(4) + 120(9.81)(2) = 640\,\alpha$

$\alpha = 8.67875\text{ rad/s}^2 = 8.68\text{ rad/s}^2$ **Ans.**

$\Sigma F_n = m(a_G)_n;\qquad A_n = 0$ **Ans.**

$\Sigma F_t = m(a_G)_t;\qquad 800 + 120(9.81) + A_t = 120[2(8.67875)]$

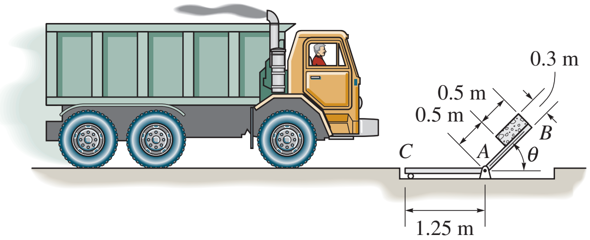
$A_t = 105.7\text{ N} = 106\text{ N}$ **Ans.**



Ans:
$\alpha = 8.68\text{ rad/s}^2$
$A_n = 0$
$A_t = 106\text{ N}$

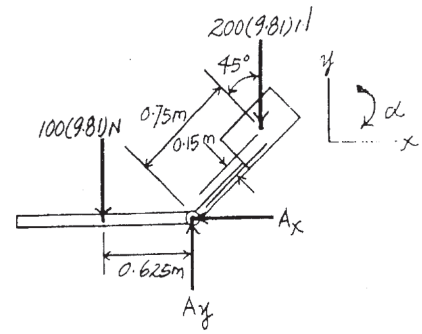
***17–68.**

The device acts as a pop-up barrier to prevent the passage of a vehicle. It consists of a 100-kg steel plate AC and a 200-kg counterweight solid concrete block located as shown. Determine the moment of inertia of the plate and block about the hinged axis through A. Neglect the mass of the supporting arms AB. Also, determine the initial angular acceleration of the assembly when it is released from rest at $\theta = 45°$.



SOLUTION

Mass Moment of Inertia:



$$I_A = \frac{1}{12}(100)\left(1.25^2\right) + 100\left(0.625^2\right) + \frac{1}{12}(200)\left(0.5^2 + 0.3^2\right) + 200\left(\sqrt{0.75^2 + 0.15^2}\right)^2$$

$$= 174.75 \text{ kg}\cdot\text{m}^2 = 175 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

Equation of Motion: Applying Eq. 17–16, we have

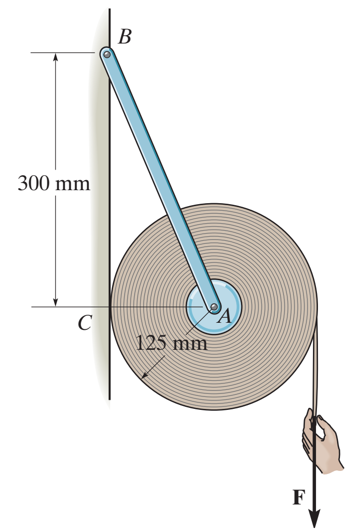
$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad 100(9.81)(0.625) + 200(9.81)\sin 45°(0.15) - 200(9.81)\cos 45°(0.75) = -174.75\alpha$$

$$\alpha = 1.25 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$I_A = 175 \text{ kg}\cdot\text{m}^2$
$\alpha = 1.25 \text{ rad/s}^2$

17–69.

The 20-kg roll of paper has a radius of gyration $k_A = 90$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$ and a vertical force $F = 30$ N is applied to the end of the paper, determine the angular acceleration of the roll as the paper unrolls.



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad N_C - T_{AB}\cos 67.38^\circ = 0$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_{AB}\sin 67.38^\circ - 0.2N_C - 20(9.81) - 30 = 0$

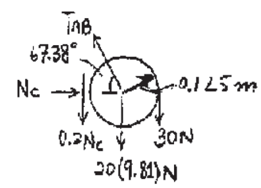
$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad -0.2N_C(0.125) + 30(0.125) = 20(0.09)^2\alpha$

Solving:

$N_C = 103$ N

$T_{AB} = 267$ N

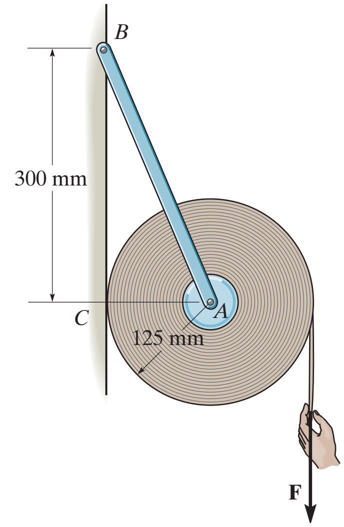
$\alpha = 7.28$ rad/s^2 **Ans.**



Ans:
$\alpha = 7.28$ rad/s^2

17–70.

The 20-kg roll of paper has a radius of gyration $k_A = 90$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$, determine the constant vertical force F that must be applied to the roll to pull off 1 m of paper in $t = 3$ s starting from rest. Neglect the mass of paper that is removed.



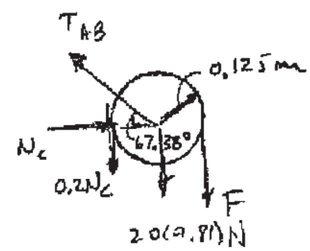
SOLUTION

$$(+\downarrow)\ s = s_0 + v_0 t + \frac{1}{2}a_C t^2$$

$$1 = 0 + 0 + \frac{1}{2}a_C(3)^2$$

$$a_C = 0.222 \text{ m/s}^2$$

$$\alpha = \frac{a_C}{0.125} = 1.778 \text{ rad/s}^2$$



$$\xrightarrow{+}\ \Sigma F_x = m(a_{G\,x}); \qquad N_C - T_{AB}\cos 67.38^\circ = 0$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_{AB}\sin 67.38^\circ - 0.2N_C - 20(9.81) - F = 0$$

$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad -0.2N_C(0.125) + F(0.125) = 20(0.09)^2(1.778)$$

Solving:

$$N_C = 99.3 \text{ N}$$

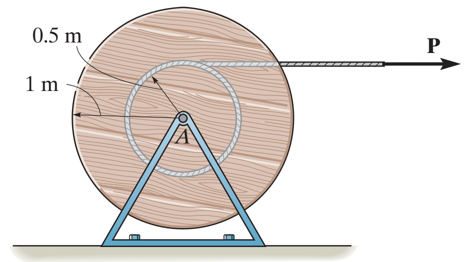
$$T_{AB} = 258 \text{ N}$$

$$F = 22.1 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$F = 22.1$ N

17–71.

The reel of cable has a mass of 400 kg and a radius of gyration of $k_A = 0.75$ m. Determine its angular velocity when $t = 2$ s, starting from rest, if the force $\mathbf{P} = (20t^2 + 80)$ N, when t is in seconds. Neglect the mass of the unwound cable, and assume it is always at a radius of 0.5 m.



SOLUTION

Equations of Motion. The mass moment of inertia of the reel about A is $I_A = Mk_A^2 = 400(0.75^2) = 225 \text{ kg}\cdot\text{m}^2$. Referring to the FBD of the reel, Fig. *a*

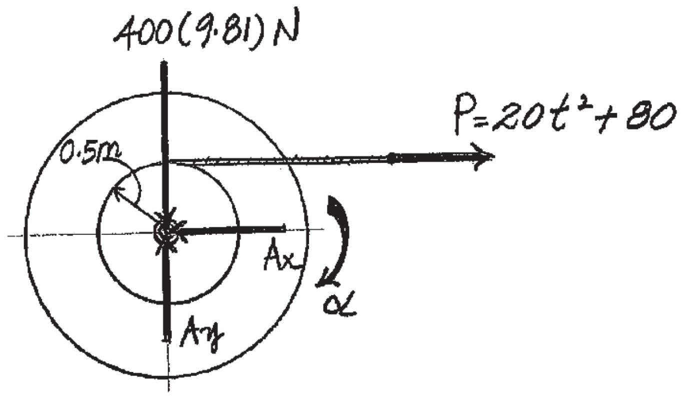
$\circlearrowleft + \Sigma M_A = I_A\alpha;\quad -(20t^2 + 80)(0.5) = 225(-\alpha)$

$$\alpha = \frac{2}{45}(t^2 + 4) \text{ rad/s}^2$$

Kinematics. Using the result of α, integrate $d\omega = \alpha dt$, with the initial condition $\omega = 0$ at $t = 0$,

$$\int_0^{\omega} d\omega = \int_0^{2s} \frac{2}{45}(t^2 + 4)\, dt$$

$$\omega = 0.4741 \text{ rad/s} = 0.474 \text{ rad/s} \qquad \textbf{Ans.}$$

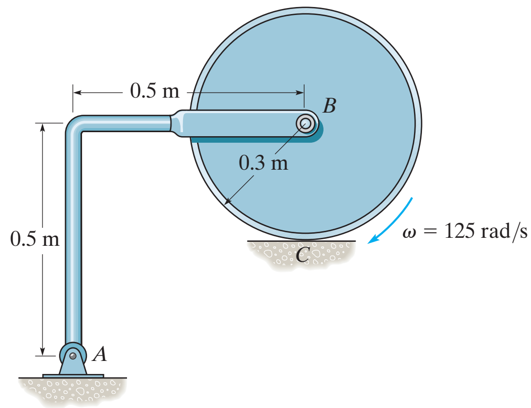


(a)

Ans:
$\omega = 0.474$ rad/s

***17–72.**

The 30-kg disk is originally spinning at $\omega = 125$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of force which the member AB exerts on the pin at A during this time? Neglect the mass of AB.



SOLUTION

Equations of Motion. The mass moment of inertia of the disk about B is $I_B = \frac{1}{2}mr^2 = \frac{1}{2}(30)(0.3^2) = 1.35\ \text{kg}\cdot\text{m}^2$. Since it is required to slip at C, $F_f = \mu_C N_C = 0.5\,N_C$. Referring to the FBD of the disk, Fig. a,

$\xrightarrow{+}\Sigma F_x = m(a_G)_x;\qquad 0.5N_C - F_{AB}\cos 45° = 30(0)\qquad$ **(1)**

$+\uparrow\Sigma F_y = m(a_G)_y;\qquad N_C - F_{AB}\sin 45° - 30(9.81) = 30(0)\qquad$ **(2)**

Solving Eqs. (1) and (2),

$N_C = 588.6\ \text{N}\qquad F_{AB} = 416.20\ \text{N}$

Subsequently,

$\circlearrowleft + \Sigma M_B = I_B\alpha;\qquad 0.5(588.6)(0.3) = 1.35\alpha$

$$\alpha = 65.4\ \text{rad/s}^2 \circlearrowleft$$

Referring to the FBD of pin A, Fig. b,

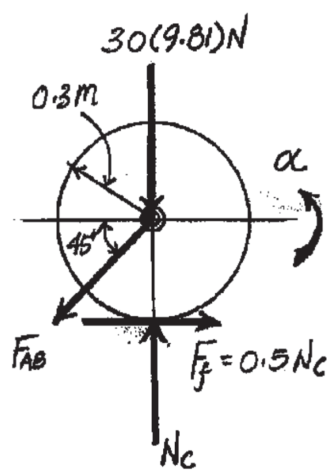
$\xrightarrow{+}\Sigma F_x = 0;\qquad 416.20\cos 45° - A_x = 0\qquad A_x = 294.3\ \text{N} = 294\ \text{N}\qquad$ **Ans.**

$+\uparrow\Sigma F_y = 0;\qquad 416.20\sin 45° - A_y = 0\qquad A_y = 294.3\ \text{N} = 294\ \text{N}\qquad$ **Ans.**

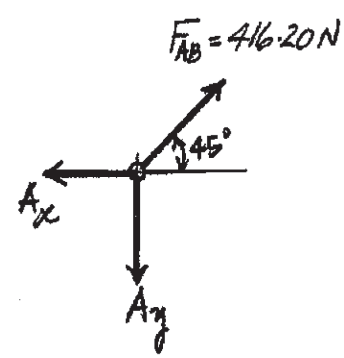
Kinematic. Using the result of α,

$+\circlearrowright\ \omega = \omega_0 + \alpha t;\qquad 0 = 125 + (-65.4)t$

$t = 1.911\ \text{s} = 1.91\ \text{s}\qquad$ **Ans.**



(a)

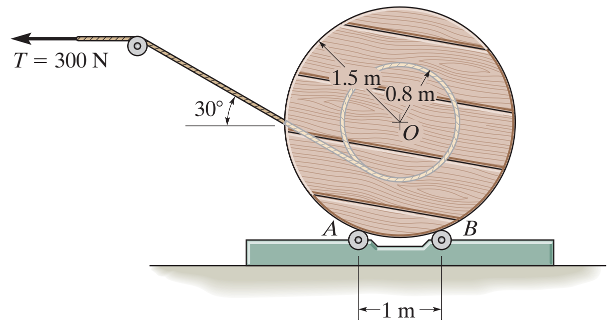


(b)

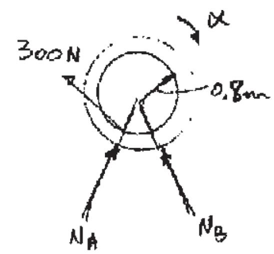
Ans:
$A_x = 294$ N
$A_y = 294$ N
$t = 1.91$ s

17–73.

Cable is unwound from a spool supported on small rollers at A and B by exerting a force $T = 300$ N on the cable. Compute the time needed to unravel 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and a radius of gyration of $k_O = 1.2$ m. For the calculation, neglect the mass of the cable being unwound and the mass of the rollers at A and B. The rollers turn with no friction.



SOLUTION



$I_O = mk_O^2 = 600(1.2)^2 = 864\ \text{kg}\cdot\text{m}^2$

$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 300(0.8) = 864(\alpha) \qquad \alpha = 0.2778\ \text{rad/s}^2$

The angular displacement $\theta = \dfrac{s}{r} = \dfrac{5}{0.8} = 6.25$ rad.

$$\theta = \theta_0 + \omega_0\, r + \frac{1}{2}\alpha_c t^2$$

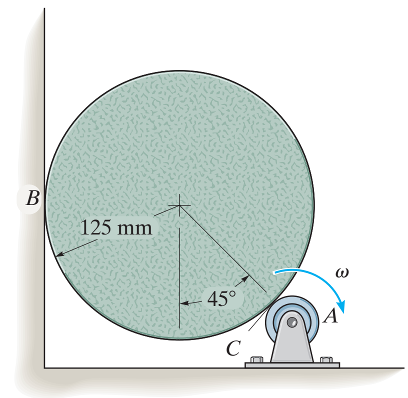
$$6.25 = 0 + 0 + \frac{1}{2}(0.27778)t^2$$

$$t = 6.71\ \text{s}$$ **Ans.**

Ans:
$t = 6.71$ s

17–74.

The 5-kg cylinder is initially at rest when it is placed in contact with the wall B and the rotor at A. If the rotor always maintains a constant clockwise angular velocity $\omega = 6$ rad/s, determine the initial angular acceleration of the cylinder. The coefficient of kinetic friction at the contacting surfaces B and C is $\mu_k = 0.2$.



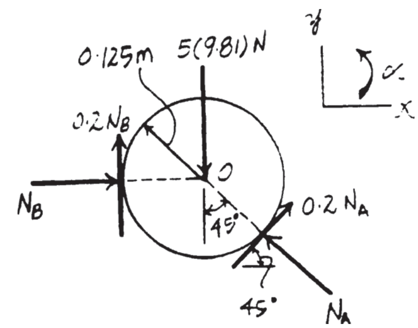
SOLUTION

Equations of Motion: The mass moment of inertia of the cylinder about point O is given by $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(5)(0.125^2) = 0.0390625 \text{ kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have

$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x;\qquad N_B + 0.2N_A\cos 45^\circ - N_A\sin 45^\circ = 0$ **(1)**

$+\uparrow\Sigma F_y = m(a_G)_y;\qquad 0.2N_B + 0.2N_A\sin 45^\circ + N_A\cos 45^\circ - 5(9.81) = 0$ **(2)**

$\circlearrowleft +\Sigma M_O = I_O\alpha;\qquad 0.2N_A(0.125) - 0.2N_B(0.125) = 0.0390625\alpha$ **(3)**



Solving Eqs. (1), (2), and (3) yields;

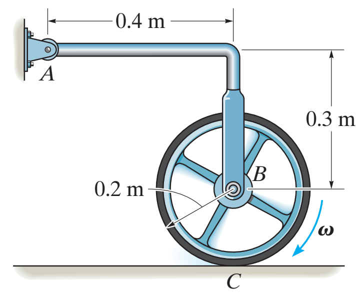
$$N_A = 51.01 \text{ N}\qquad N_B = 28.85 \text{ N}$$

$$\alpha = 14.2 \text{ rad/s}^2$$ **Ans.**

Ans:
$\alpha = 14.2 \text{ rad/s}^2$

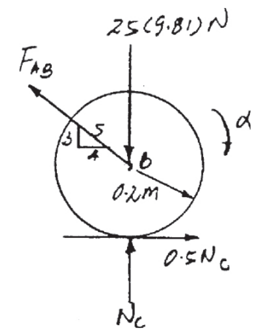
17–75.

The wheel has a mass of 25 kg and a radius of gyration $k_B = 0.15$ m. It is originally spinning at $\omega = 40$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at A exerts on AB during this time? Neglect the mass of AB.



SOLUTION

$I_B = mk_B^2 = 25(0.15)^2 = 0.5625\ \text{kg}\cdot\text{m}^2$



$+\uparrow \Sigma F_y = m(a_G)_y; \qquad \left(\frac{3}{5}\right) F_{AB} + N_C - 25(9.81) = 0 \qquad$ **(1)**

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad 0.5N_C - \left(\frac{4}{5}\right) F_{AB} = 0 \qquad$ **(2)**

$\circlearrowleft + \Sigma M_B = I_B\alpha; \qquad 0.5N_C(0.2) = 0.5625(-\alpha) \qquad$ **(3)**

Solvings Eqs. (1), (2) and (3) yields:

$F_{AB} = 111.48\ \text{N} \qquad N_C = 178.4\ \text{N}$

$\alpha = -31.71\ \text{rad/s}^2$

$A_x = \frac{4}{5} F_{AB} = 0.8(111.48) = 89.2\ \text{N}$ **Ans.**

$A_y = \frac{3}{5} F_{AB} = 0.6(111.48) = 66.9\ \text{N}$ **Ans.**

$\omega = \omega_0 + \alpha_c t$

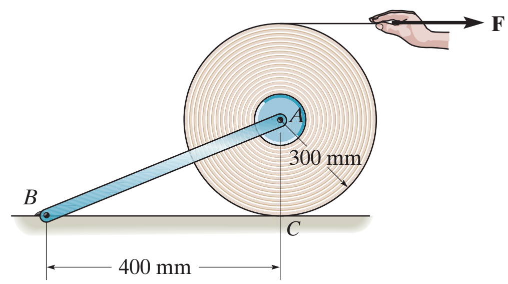
$0 = 40 + (-31.71)\, t$

$t = 1.26\ \text{s}$ **Ans.**

Ans:
$A_x = 89.2$ N
$A_y = 66.9$ N
$t = 1.25$ s

***17–76.**

The 20-kg roll of paper has a radius of gyration $k_A = 120$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. The roll rests on the floor, for which the coefficient of kinetic friction is $\mu_k = 0.2$. If a horizontal force $F = 60$ N is applied to the end of the paper, determine the initial angular acceleration of the roll as the paper unrolls.



SOLUTION

Equations of Motion. The mass moment of inertia of the paper roll about A is $I_A = mk_A^2 = 20(0.12^2) = 0.288\ \text{kg}\cdot\text{m}^2$. Since it is required to slip at C, the friction is $F_f = \mu_k N = 0.2$ N. Referring to the FBD of the paper roll, Fig. *a*

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.2\text{ N} - F_{AB}\left(\frac{4}{5}\right) + 60 = 20(0) \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad \text{N} - F_{AB}\left(\frac{3}{5}\right) - 20(9.81) = 20(0) \qquad \textbf{(2)}$$

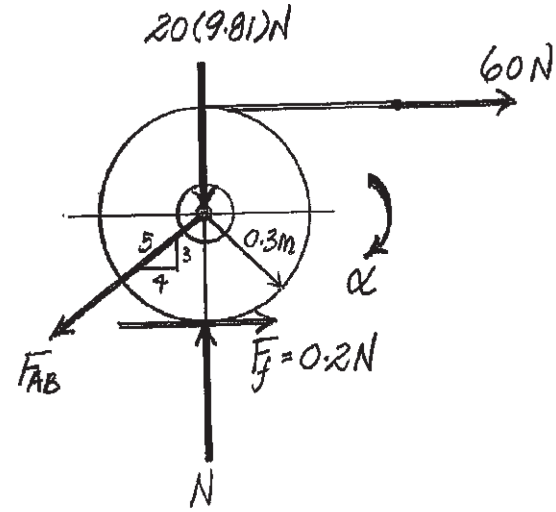
Solving Eqs. (1) and (2)

$$F_{AB} = 145.94\text{ N} \qquad \text{N} = 283.76\text{ N}$$

Subsequently

$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad 0.2(283.76)(0.3) - 60(0.3) = 0.288(-\alpha)$$

$$\alpha = 3.3824\text{ rad/s}^2 = 3.38\text{ rad/s}^2 \qquad \textbf{Ans.}$$

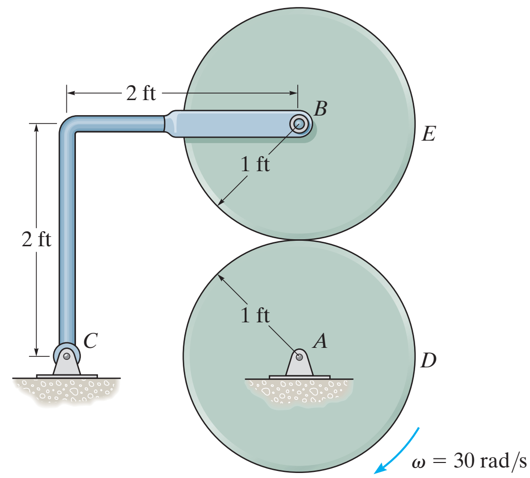


(a)

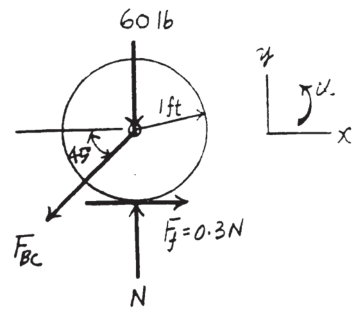
Ans:
$\alpha = 3.38\text{ rad/s}^2$

17–77.

Disk D turns with a constant clockwise angular velocity of 30 rad/s. Disk E has a weight of 60 lb and is initially at rest when it is brought into contact with D. Determine the time required for disk E to attain the same angular velocity as disk D. The coefficient of kinetic friction between the two disks is $\mu_k = 0.3$. Neglect the weight of bar BC.



SOLUTION



Equations of Motion: The mass moment of inertia of disk E about point B is given by $I_B = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{60}{32.2}\right)(1^2) = 0.9317 \text{ slug}\cdot\text{ft}^2$. Applying Eq. 17–16, we have

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;$ $\qquad 0.3N - F_{BC}\cos 45° = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\qquad N - F_{BC}\sin 45° - 60 = 0 \qquad$ **(2)**

$\circlearrowleft + \Sigma M_O = I_O\,\alpha;$ $\qquad 0.3N(1) = 0.9317\alpha \qquad$ **(3)**

Solving Eqs. (1), (2) and (3) yields:

$$F_{BC} = 36.37 \text{ lb} \qquad N = 85.71 \text{ lb} \qquad \alpha = 27.60 \text{ rad/s}^2$$

Kinematics: Applying equation $\omega = \omega_0 + \alpha_t$, we have

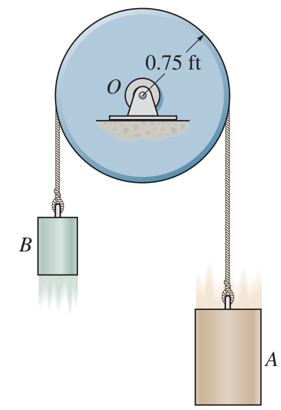
$(\circlearrowleft +)$ $\qquad 30 = 0 + 27.60t$

$t = 1.09 \text{ s}$ **Ans.**

Ans:
$t = 1.09$ s

17–78.

Two cylinders A and B, having a weight of 10 lb and 5 lb, respectively, are attached to the ends of a cord which passes over a 3-lb pulley (disk). If the cylinders are released from rest, determine their speed in $t = 0.5$ s. The cord does not slip on the pulley. Neglect the mass of the cord. *Suggestion*: Analyze the "system" consisting of both the cylinders and the pulley.

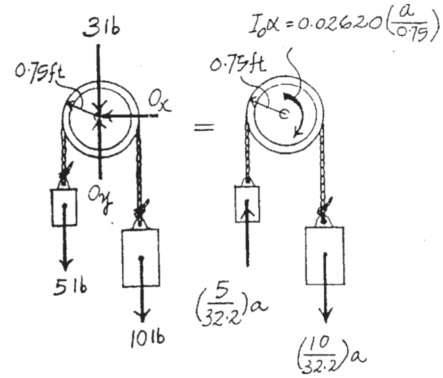


SOLUTION

Equation of Motion: The mass moment of inertia of the pulley (disk) about point O is given by $I_O = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{3}{32.2}\right)(0.75^2) = 0.02620 \text{ slug} \cdot \text{ft}^2$. Here, $a = \alpha r$ or $\alpha = \frac{a}{r} = \frac{a}{0.75}$. Applying Eq. 17–16, we have

$$\zeta + \Sigma M_O = I_O\alpha; \qquad 5(0.75) - 10(0.75) = -0.02620\left(\frac{a}{0.75}\right) - \left[\left(\frac{5}{32.2}\right)a\right](0.75) - \left[\left(\frac{10}{32.2}\right)a\right](0.75)$$

$$a = 9.758 \text{ ft/s}^2$$



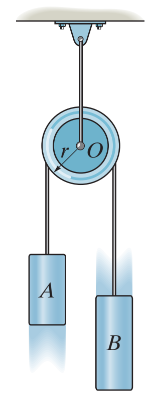
Kinematic: Applying equation $v = v_0 + at$, we have

$$v = 0 + 9.758(0.5) = 4.88 \text{ ft/s}$$ **Ans.**

Ans:
$v = 4.88 \text{ ft/s}$

17–79.

The two blocks A and B have a mass of 5 kg and 10 kg, respectively. If the pulley can be treated as a disk of mass 3 kg and radius 0.15 m, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.



SOLUTION

Kinematics: Since the pulley rotates about a fixed axis passes through point O, its angular acceleration is

$$\alpha = \frac{a}{r} = \frac{a}{0.15} = 6.6667a$$

The mass moment of inertia of the pulley about point O is

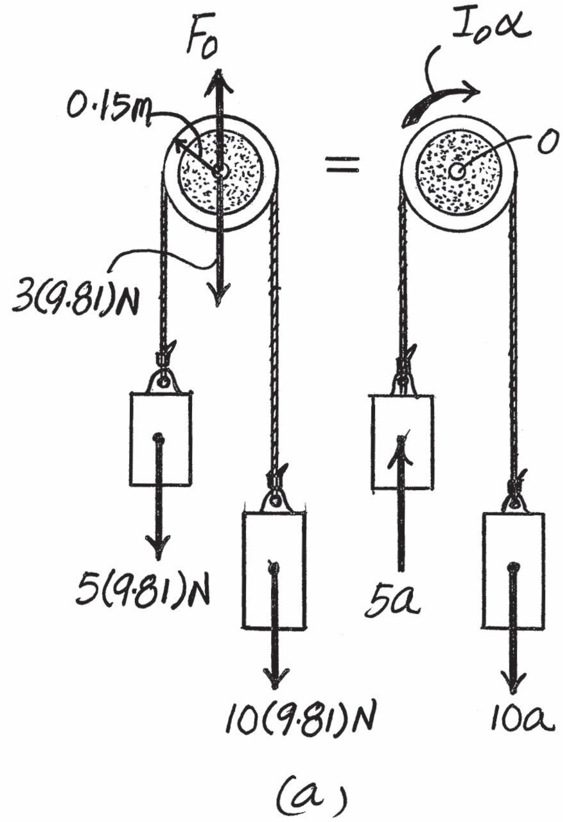
$$I_o = \frac{1}{2}Mr^2 = \frac{1}{2}(3)(0.15^2) = 0.03375\ \text{kg}\cdot\text{m}^2$$

Equation of Motion: Write the moment equation of motion about point O by referring to the free-body and kinetic diagram of the system shown in Fig. a,

$$\circlearrowleft + \Sigma M_o = \Sigma (M_k)_o; \qquad 5(9.81)(0.15) - 10(9.81)(0.15)$$

$$= -0.03375(6.6667a) - 5a(0.15) - 10a(0.15)$$

$$a = 2.973\ \text{m/s}^2 = 2.97\ \text{m/s}^2 \qquad \textbf{Ans.}$$

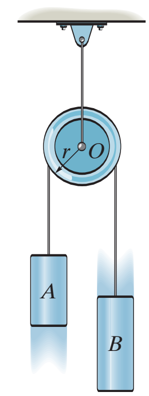


(a)

Ans:
$a = 2.97\ \text{m/s}^2$

***17–80.**

The two blocks A and B have a mass m_A and m_B, respectively, where $m_B > m_A$. If the pulley can be treated as a disk of mass M, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.



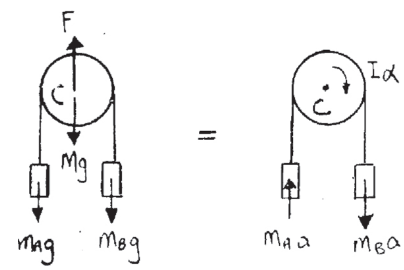
SOLUTION

$$a = \alpha r$$

$$\circlearrowleft +\Sigma M_C = \Sigma (M_k)_C; \qquad m_B\, g(r) - m_A g(r) = \left(\frac{1}{2}Mr^2\right)\alpha + m_B r^2\alpha + m_A r^2\alpha$$

$$\alpha = \frac{g(m_B - m_A)}{r\left(\frac{1}{2}M + m_B + m_A\right)}$$

$$a = \frac{g(m_B - m_A)}{\left(\frac{1}{2}M + m_B + m_A\right)} \qquad \textbf{Ans.}$$

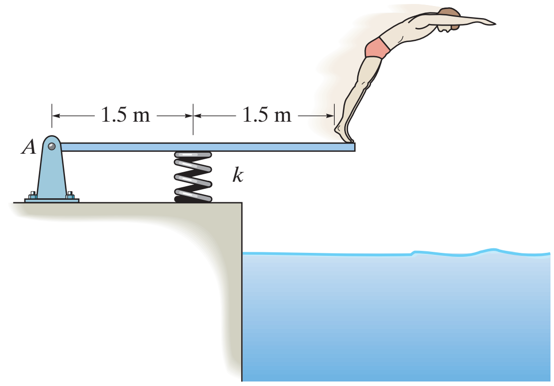


Ans:

$$a = \frac{g(m_B - m_A)}{\left(\frac{1}{2}M + m_B + m_A\right)}$$

17–81.

Determine the angular acceleration of the 25-kg diving board and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount of 200 mm, $\omega = 0$, and the board is horizontal. Take $k = 7$ kN/m.



SOLUTION

$$\circlearrowleft + \sum M_A = I_A\alpha; \qquad 1.5(1400 - 245.25) = \left[\frac{1}{3}(25)(3)^2\right]\alpha$$

$$+\uparrow \sum F_t = m(a_G)_t\,; \qquad 1400 - 245.25 - A_y = 25(1.5\alpha)$$

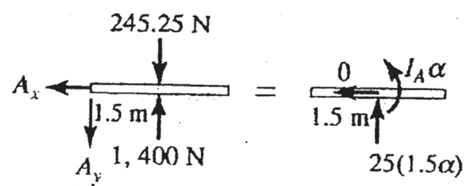
$$\stackrel{+}{\leftarrow} \sum F_n = m(a_G)_n\,; \qquad A_x = 0$$

Solving,

$$A_x = 0 \qquad \textbf{Ans.}$$

$$A_y = 289 \text{ N} \qquad \textbf{Ans.}$$

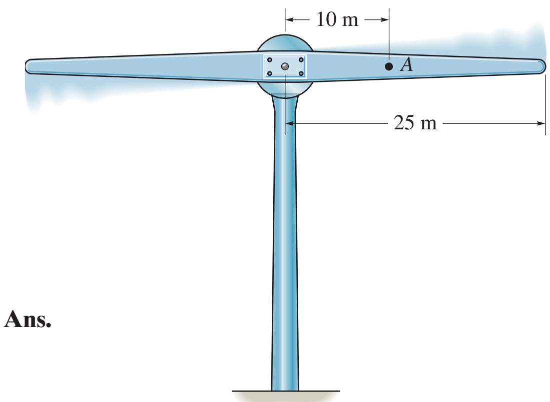
$$\alpha = 23.1 \text{ rad/s}^2 \qquad \textbf{Ans.}$$



Ans:
$A_x = 0$
$A_y = 289$ N
$\alpha = 23.1$ rad/s^2

17–82.

The lightweight turbine consists of a rotor which is powered from a torque applied at its center. At the instant the rotor is horizontal it has an angular velocity of 15 rad/s and a clockwise angular acceleration of 8 rad/s^2. Determine the internal normal force, shear force, and moment at a section through A. Assume the rotor is a 50-m-long slender rod, having a mass of 3 kg/m.



SOLUTION

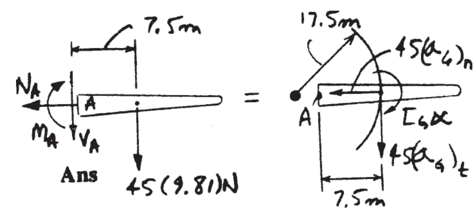
$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n;$ $\quad N_A = 45(15)^2(17.5) = 177\text{kN}$ **Ans.**

$+\downarrow \Sigma F_t = m(a_G)_t;$ $\quad V_A + 45(9.81) = 45(8)(17.5)$

$V_A = 5.86 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;$ $\quad M_A + 45(9.81)(7.5) = \left[\frac{1}{12}(45)(15)^2\right](8) + [45(8)(17.5)](7.5)$

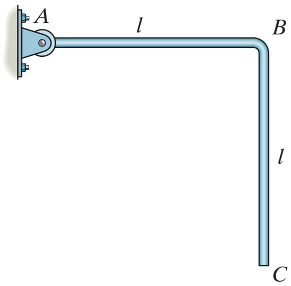
$M_A = 50.7 \text{ kN} \cdot \text{m}$ **Ans.**



Ans:
$N_A = 177 \text{ kN}$
$V_A = 5.86 \text{ kN}$
$M_A = 50.7 \text{ kN} \cdot \text{m}$

17–83.

The two-bar assembly is released from rest in the position shown. Determine the initial bending moment at the fixed joint B. Each bar has a mass m and length l.

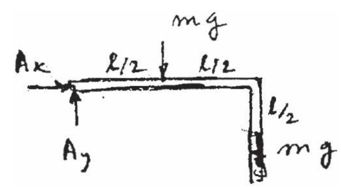


SOLUTION

Assembly:

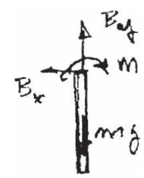
$$I_A = \frac{1}{3}ml^2 + \frac{1}{12}(m)(l)^2 + m(l^2 + (\frac{l}{2})^2)$$

$$= 1.667\ ml^2$$



$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad mg(\frac{l}{2}) + mg(l) = (1.667ml^2)\alpha$$

$$\alpha = \frac{0.9\ g}{l}$$

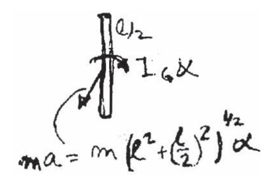


Segment BC:

$$\circlearrowleft + \Sigma M_B = \Sigma(M_k)_B; \qquad M = \left[\frac{1}{12}ml^2\right]\alpha + m(l^2 + (\frac{l}{2})^2)^{1/2}\alpha(\frac{l/2}{l^2 + (\frac{l}{2})^2})(\frac{l}{2})$$

$$M = \frac{1}{3}ml^2\,\alpha = \frac{1}{3}ml^2(\frac{0.9g}{l})$$

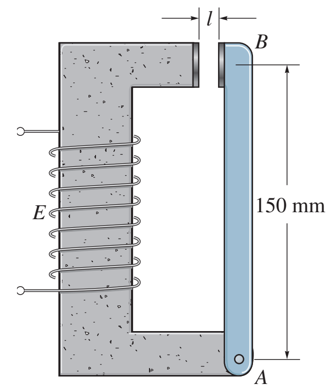
$$M = 0.3gml \qquad \textbf{Ans.}$$



Ans:
$M = 0.3gml$

***17–84.**

The armature (slender rod) AB has a mass of 0.2 kg and can pivot about the pin at A. Movement is controlled by the electromagnet E, which exerts a horizontal attractive force on the armature at B of $F_B = (0.2(10^{-3})l^{-2})$ N, where l in meters is the gap between the armature and the magnet at any instant. If the armature lies in the horizontal plane, and is originally at rest, determine the speed of the contact at B the instant $l = 0.01$ m. Originally $l = 0.02$ m.



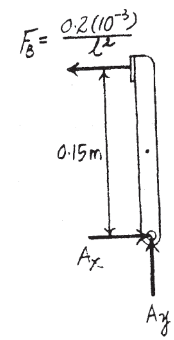
SOLUTION

Equation of Motion: The mass moment of inertia of the armature about point A is given by $I_A = I_G + mr_G^2 = \frac{1}{12}(0.2)\left(0.15^2\right) + 0.2\left(0.075^2\right) = 1.50\left(10^{-3}\right)\text{kg}\cdot\text{m}^2$

Applying Eq. 17–16, we have

$$\zeta+\Sigma M_A = I_A\alpha; \qquad \frac{0.2(10^{-3})}{l^2}(0.15) = 1.50\left(10^{-3}\right)\alpha$$

$$\alpha = \frac{0.02}{l^2}$$



Kinematic: From the geometry, $l = 0.02 - 0.15\theta$. Then $dl = -0.15\,d\theta$ or $d\theta = -\frac{dl}{0.15}$. Also, $\omega = \frac{v}{0.15}$ hence $d\omega = \frac{dv}{0.15}$. Substitute into equation $\omega d\omega = \alpha d\theta$, we have

$$\frac{v}{0.15}\left(\frac{dv}{0.15}\right) = \alpha\left(-\frac{dl}{0.15}\right)$$

$$vdv = -0.15\,\alpha dl$$

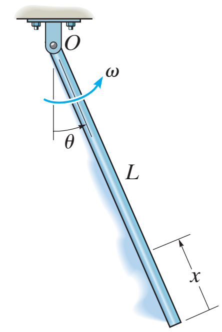
$$\int_0^v vdv = \int_{0.02\text{ m}}^{0.01\text{ m}} -0.15\left(\frac{0.02}{l^2}\right)dl$$

$$v = 0.548\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 0.548$ m/s

17–85.

The bar has a weight per length of w. If it is rotating in the vertical plane at a constant rate ω about point O, determine the internal normal force, shear force, and moment as a function of x and θ.

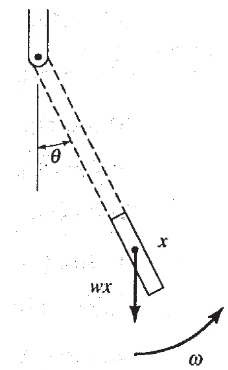


SOLUTION

$$a = \omega^2\left(L - \frac{x}{z}\right)\overset{\theta}{\nwarrow}$$

Forces:

$$\frac{wx}{g}\omega^2\left(L - \frac{x}{z}\right)\overset{\theta}{\nwarrow} = N\overset{\theta}{\nwarrow} + S\measuredangle\theta + wx\downarrow \qquad \textbf{(1)}$$



Moments:

$$I\alpha = M - S\left(\frac{x}{2}\right)$$

$$O = M - \frac{1}{2}Sx \qquad \textbf{(2)}$$

Solving (1) and (2),

$$N = wx\left[\frac{\omega^2}{g}\left(L - \frac{x}{2}\right) + \cos\theta\right] \qquad \textbf{Ans.}$$

$$V = wx\sin\theta \qquad \textbf{Ans.}$$

$$M = \frac{1}{2}wx^2\sin\theta \qquad \textbf{Ans.}$$

Ans:

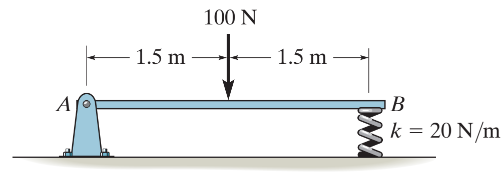
$$N = wx\left[\frac{\omega^2}{g}\left(L - \frac{x}{2}\right) + \cos\theta\right]$$

$$V = wx\sin\theta$$

$$M = \frac{1}{2}wx^2\sin\theta$$

17–86.

The 4-kg slender rod is initially supported horizontally by a spring at B and pin at A. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the 100-N force is applied.



SOLUTION

Equation of Motion. The mass moment of inertia of the rod about A is $I_A = \frac{1}{12}(4)(3^2) + 4(1.5^2) = 12.0\ \text{kg}\cdot\text{m}^2$. Initially, the beam is at rest, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(1.5)$. The force developed in the spring before the application of the 100 N force is $F_{sp} = \frac{4(9.81)\ \text{N}}{2} = 19.62$ N. Referring to the FBD of the rod, Fig. a,

$$\circlearrowleft + M_A = I_A\alpha; \qquad 19.62(3) - 100(1.5) - 4(9.81)(1.5) = 12.0(-\alpha)$$

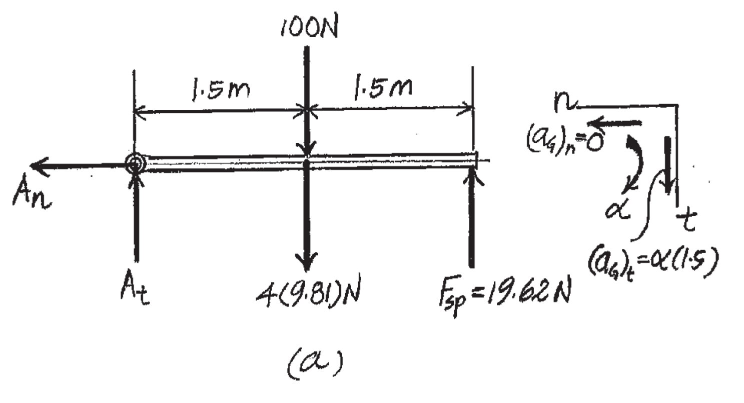
$$\alpha = 12.5\ \text{rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

Then

$$(a_G)_t = 12.5(1.5) = 18.75\ \text{m/s}^2 \downarrow$$

Since $(a_G)_n = 0$. Then

$$a_G = (a_G)_t = 18.75\ \text{m/s}^2 \downarrow \qquad \textbf{Ans.}$$

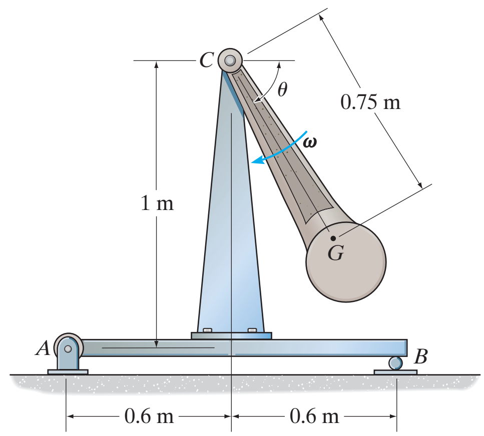


(a)

Ans:
$\alpha = 12.5\ \text{rad/s}^2 \circlearrowright$
$a_G = 18.75\ \text{m/s}^2 \downarrow$

17–87.

The 100-kg pendulum has a center of mass at G and a radius of gyration about G of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin A and the normal reaction of the roller B at the instant $\theta = 90°$ when the pendulum is rotating at $\omega = 8$ rad/s. Neglect the weight of the beam and the support.



SOLUTION

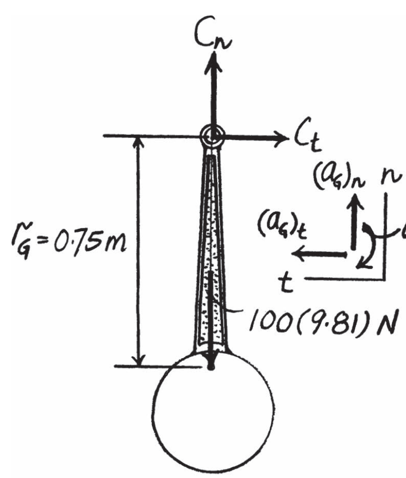
Equations of Motion: Since the pendulum rotates about the fixed axis passing through point C, $(a_G)_t = \alpha r_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 8^2(0.75) = 48\ \text{m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25)^2 + 100(0.75^2) = 62.5\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point C and referring to the free-body diagram of the pendulum, Fig. a, we have

$$\circlearrowleft + \Sigma M_C = I_C\alpha; \qquad 0 = 62.5\alpha \qquad \alpha = 0$$

Using this result to write the force equations of motion along the n and t axes,

$$\overset{+}{\leftarrow} \Sigma F_t = m(a_G)_t; \qquad -C_t = 100[0(0.75)] \qquad C_t = 0$$

$$+\uparrow \Sigma F_n = m(a_G)_n; \qquad C_n - 100(9.81) = 100(48) \qquad C_n = 5781\ \text{N}$$



(a)

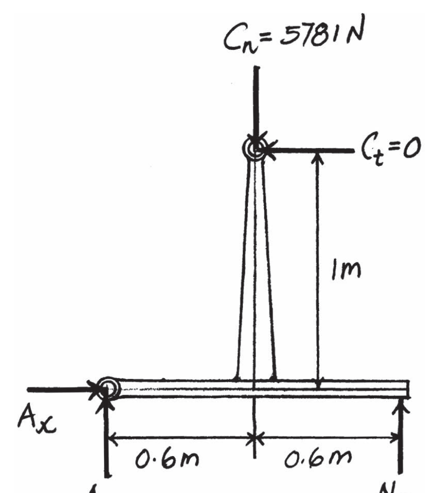
Equilibrium: Writing the moment equation of equilibrium about point A and using the free-body diagram of the beam in Fig. b, we have

$$+\Sigma M_A = 0; \qquad N_B(1.2) - 5781(0.6) = 0 \qquad N_B = 2890.5\ \text{N} = 2.89\ \text{kN} \qquad \textbf{Ans.}$$

Using this result to write the force equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 2890.5 - 5781 = 0 \qquad A_y = 2890.5\ \text{N} = 2.89\ \text{kN} \qquad \textbf{Ans.}$$

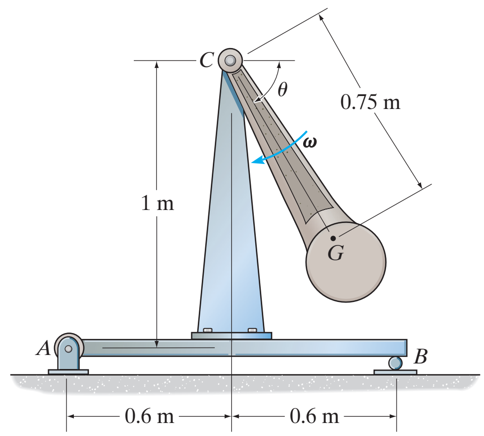


(b)

Ans:
$N_B = 2.89$ kN
$A_x = 0$
$A_y = 2.89$ kN

***17–88.**

The 100-kg pendulum has a center of mass at G and a radius of gyration about G of $k_G = 250$ mm. Determine the horizontal and vertical components of reaction on the beam by the pin A and the normal reaction of the roller B at the instant $\theta = 0°$ when the pendulum is rotating at $\omega = 4$ rad/s. Neglect the weight of the beam and the support.



SOLUTION

Equations of Motion: Since the pendulum rotates about the fixed axis passing through point C, $(a_G)_t = \alpha r_G = \alpha(0.75)$ and $(a_G)_n = \omega^2 r_G = 4^2(0.75) = 12\ \text{m/s}^2$. Here, the mass moment of inertia of the pendulum about this axis is $I_C = 100(0.25^2) + 100(0.75)^2 = 62.5\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point C and referring to the free-body diagram shown in Fig. a,

$$\circlearrowleft + \Sigma M_C = I_C\alpha; \qquad -100(9.81)(0.75) = -62.5\alpha \qquad \alpha = 11.772\ \text{rad/s}^2$$

Using this result to write the force equations of motion along the n and t axes, we have

$$+\uparrow \Sigma F_t = m(a_G)_t; \qquad C_t - 100(9.81) = -100[11.772(0.75)] \qquad C_t = 98.1\ \text{N}$$

$$\overset{+}{\leftarrow} \Sigma F_n = m(a_G)_n; \qquad C_n = 100(12) \qquad C_n = 1200\ \text{N}$$

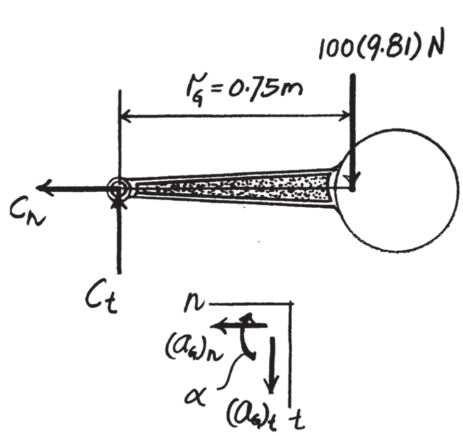
Equilibrium: Writing the moment equation of equilibrium about point A and using the free-body diagram of the beam in Fig. b,

$$+\Sigma M_A = 0; \qquad N_B(1.2) - 98.1(0.6) - 1200(1) = 0 \qquad N_B = 1049.05\ \text{N} = 1.05\ \text{kN} \qquad \textbf{Ans.}$$

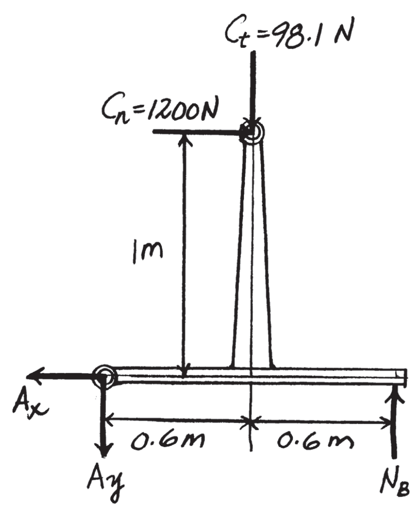
Using this result to write the force equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1200 - A_x = 0 \qquad A_x = 1200\ \text{N} = 1.20\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 1049.05 - 98.1 - A_y = 0 \qquad A_y = 950.95\ \text{N} = 951\ \text{N} \qquad \textbf{Ans.}$$



(a)

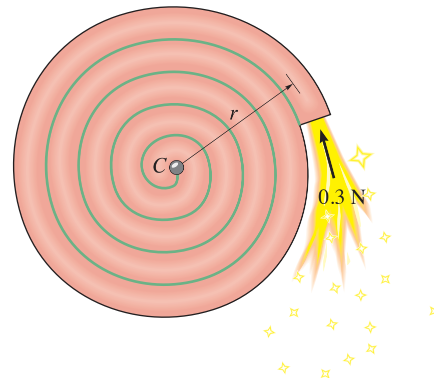


(b)

Ans:
$N_B = 1.05$ kN
$A_x = 1.20$ kN
$A_y = 951$ N

17–89.

The "Catherine wheel" is a firework that consists of a coiled tube of powder which is pinned at its center. If the powder burns at a constant rate of 20 g/s such as that the exhaust gases always exert a force having a constant magnitude of 0.3 N, directed tangent to the wheel, determine the angular velocity of the wheel when 75% of the mass is burned off. Initially, the wheel is at rest and has a mass of 100 g and a radius of r = 75 mm. For the calculation, consider the wheel to always be a thin disk.

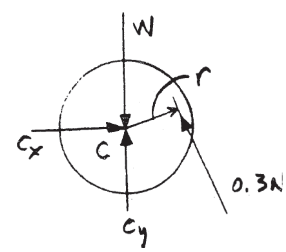


SOLUTION

Mass of wheel when 75% of the powder is burned = 0.025 kg

$$\text{Time to burn off 75 \%} = \frac{0.075\ \text{kg}}{0.02\ \text{kg/s}} = 3.75\ \text{s}$$

$$m(t) = 0.1 - 0.02t$$



Mass of disk per unit area is

$$\rho_0 = \frac{m}{A} = \frac{0.1\ \text{kg}}{\pi(0.075\ \text{m})^2} = 5.6588\ \text{kg/m}^2$$

At any time t,

$$5.6588 = \frac{0.1 - 0.02t}{\pi r^2}$$

$$r(t) = \sqrt{\frac{0.1 - 0.02t}{\pi(5.6588)}}$$

$$+\Sigma M_C = I_C\alpha; \qquad 0.3r = \frac{1}{2}mr^2\alpha$$

$$\alpha = \frac{0.6}{mr} = \frac{0.6}{(0.1 - 0.02t)\sqrt{\dfrac{0.1 - 0.02t}{\pi(5.6588)}}}$$

$$\alpha = 0.6\left(\sqrt{\pi(5.6588)}\right)[0.1 - 0.02t]^{-\frac{3}{2}}$$

$$\alpha = 2.530[0.1 - 0.02t]^{-\frac{3}{2}}$$

$$d\omega = \alpha\, dt$$

$$\int_0^{\omega} d\omega = 2.530\int_0^{t}[0.1 - 0.02t]^{-\frac{3}{2}}\, dt$$

$$\omega = 253\left[(0.1 - 0.02t)^{-\frac{1}{2}} - 3.162\right]$$

For $t = 3.75$ s,

$$\omega = 800\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 800$ rad/s

17–90.

If the disk in Fig. 17–21*a rolls without slipping*, show that when moments are summed about the instantaneous center of zero velocity, *IC*, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where I_{IC} represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

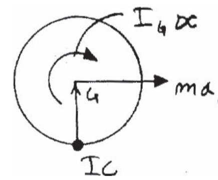
SOLUTION

$\circlearrowleft + \Sigma M_{IC} = \Sigma(M_K)_{IC};\qquad \Sigma M_{IC} = I_G\alpha + (ma_G)r$

Since there is no slipping, $a_G = \alpha r$

Thus, $\Sigma M_{IC} = \left(I_G + mr^2\right)\alpha$

By the parallel–axis thoerem, the term in parenthesis represents I_{IC}. Thus,

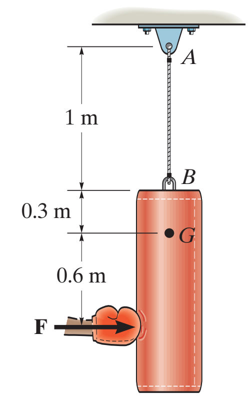


$\Sigma M_{IC} = I_{IC}\alpha$ **Q.E.D.**

Ans:
$\Sigma M_{IC} = I_{IC}\alpha$

17–91.

The 20-kg punching bag has a radius of gyration about its center of mass G of $k_G = 0.4$ m. If it is initially at rest and is subjected to a horizontal force $F = 30$ N, determine the initial angular acceleration of the bag and the tension in the supporting cable AB.



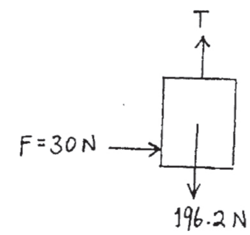
SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 30 = 20(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T - 196.2 = 20(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad 30(0.6) = 20(0.4)^2\alpha$

$\alpha = 5.62 \text{ rad/s}^2$ **Ans.**



$(a_G)_x = 1.5 \text{ m/s}^2$

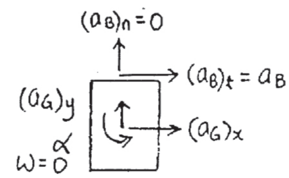
$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$a_B\mathbf{i} = (a_G)_y\mathbf{j} + (a_G)_x\mathbf{i} - \alpha(0.3)\mathbf{i}$

$(+\uparrow) \qquad (a_G)_y = 0$

Thus,

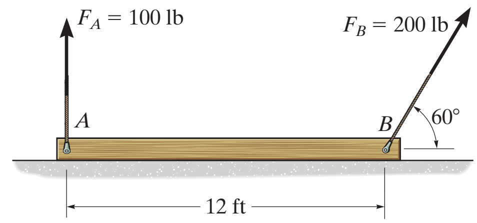
$T = 196$ N **Ans.**



Ans:
$\alpha = 5.62 \text{ rad/s}^2$
$T = 196$ N

***17–92.**

The uniform 150-lb beam is initially at rest when the forces are applied to the cables. Determine the magnitude of the acceleration of the mass center and the angular acceleration of the beam at this instant.



SOLUTION

Equations of Motion: The mass moment of inertia of the beam about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{150}{32.2}\right)(12^2) = 55.90 \text{ slug} \cdot \text{ft}^2$.

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 200 \cos 60° = \frac{150}{32.2}(a_G)_x$$

$$(a_G)_x = 21.47 \text{ ft/s}^2$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad 100 + 200 \sin 60° - 150 = \frac{150}{32.2}(a_G)_y$$

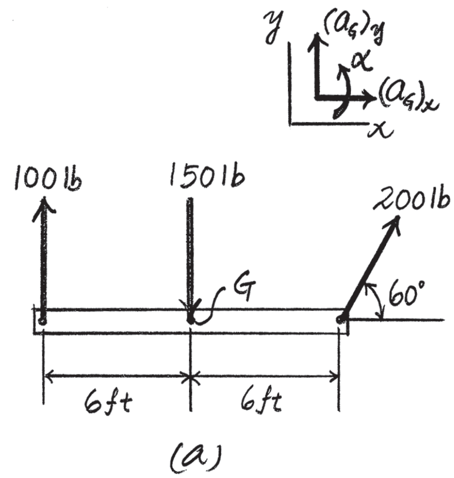
$$(a_G)_y = 26.45 \text{ ft/s}^2$$

$$+\Sigma M_G = I_G\alpha; \quad 200 \sin 60°(6) - 100(6) = 55.90\alpha$$

$$\alpha = 7.857 \text{ rad/s}^2 = 7.86 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Thus, the magnitude of $\mathbf{a}_G$ is

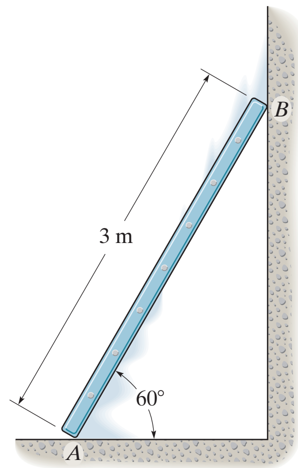
$$a_G = \sqrt{(a_G)_x^2 + (a_G)_y^2} = \sqrt{21.47^2 + 26.45^2} = 34.1 \text{ ft/s}^2 \qquad \textbf{Ans.}$$



Ans:
$\alpha = 7.86 \text{ rad/s}^2$
$a_G = 34.1 \text{ ft/s}^2$

17–93.

The slender 12-kg bar has a clockwise angular velocity of $\omega = 2$ rad/s when it is in the position shown. Determine its angular acceleration and the normal reactions of the smooth surface A and B at this instant.



SOLUTION

Equations of Motion. The mass moment of inertia of the rod about its center of gravity G is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(12)(3^2) = 9.00\ \text{kg}\cdot\text{m}^2$. Referring to the FBD and kinetic diagram of the rod, Fig. a

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad N_B = 12(a_G)_x \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 12(9.81) = -12(a_G)_y \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_O = (M_k)_O; \quad -12(9.81)(1.5\cos 60^\circ) = -12(a_G)_x(1.5\sin 60^\circ) - 12(a_G)_y(1.5\cos 60^\circ) - 9.00\alpha$$

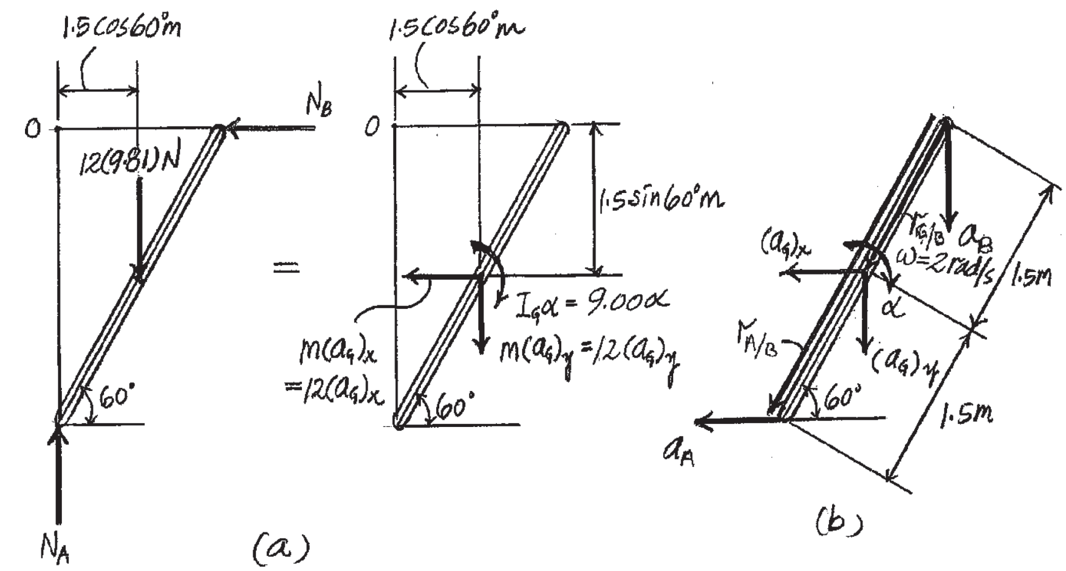
$$\sqrt{3}(a_G)_x + (a_G)_y + \alpha = 9.81 \qquad \textbf{(3)}$$

Kinematics. Applying the relative acceleration equation relating $\mathbf{a}_G$ and $\mathbf{a}_B$ by referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{G/B} - \omega^2\mathbf{r}_{G/B}$$

$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = -a_B\mathbf{j} + (-\alpha\mathbf{k}) \times (-1.5\cos 60^\circ\mathbf{i} - 1.5\sin 60^\circ\mathbf{j}) - 2^2(-1.5\cos 60^\circ\mathbf{i} - 1.5\sin 60^\circ\mathbf{j})$$

$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = (3 - 0.75\sqrt{3}\alpha)\mathbf{i} + (0.75\alpha - a_B + 3\sqrt{3})\mathbf{j}$$



17–93. Continued

Equating **i** and **j** components,

$$-(a_G)_x = 3 - 0.75\sqrt{3}\alpha \qquad \textbf{(4)}$$

$$-(a_G)_y = 0.75\alpha - a_B + 3\sqrt{3} \qquad \textbf{(5)}$$

Also, relate $\mathbf{a}_B$ and $\mathbf{a}_A$,

$$\mathbf{a}_A = \mathbf{a}_B + \alpha \times \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B}$$

$$-a_A\mathbf{i} = -a_B\mathbf{j} + (-\alpha\mathbf{k}) \times (-3\cos 60°\mathbf{i} - 3\sin 60°\mathbf{j})$$

$$-2^2(-3\cos 60°\mathbf{i} - 3\sin 60°\mathbf{j})$$

$$-a_A\mathbf{i} = (6 - 1.5\sqrt{3}\alpha)\mathbf{i} + (1.5\alpha - a_B + 6\sqrt{3})\mathbf{j}$$

Equating **j** components,

$$0 = 1.5\alpha - a_B + 6\sqrt{3}; \qquad a_B = 1.5\alpha + 6\sqrt{3} \qquad \textbf{(6)}$$

Substituting Eq. (6) into (5)

$$(a_G)_y = 0.75\alpha + 3\sqrt{3} \qquad \textbf{(7)}$$

Substituting Eq. (4) and (7) into (3)

$$\sqrt{3}\left(0.75\sqrt{3}\alpha - 3\right) + 0.75\alpha + 3\sqrt{3} + \alpha = 9.81$$

$$\alpha = 2.4525 \text{ rad/s}^2 = 2.45 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Substituting this result into Eqs. (4) and (7)

$$-(a_G)_x = 3 - (0.75\sqrt{3})(2.4525); \quad (a_G)_x = 0.1859 \text{ m/s}^2$$

$$(a_G)_y = 0.75(2.4525) + 3\sqrt{3}; \quad (a_G)_y = 7.0355 \text{ m/s}^2$$

Substituting these results into Eqs. (1) and (2)

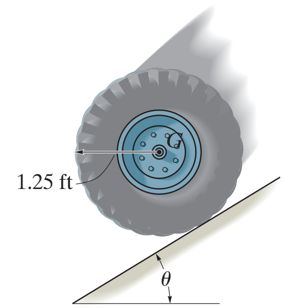
$$N_B = 12(0.1859); \quad N_B = 2.2307 \text{ N} = 2.23 \text{ N} \qquad \textbf{Ans.}$$

$$N_A - 12(9.81) = -12(7.0355); \quad N_A = 33.2937 \text{ N} = 33.3 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$\alpha = 2.45 \text{ rad/s}^2$
$N_B = 2.23$ N
$N_A = 33.3$ N

17–94.

The tire has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, determine the tire's angular acceleration as it rolls down the incline. Set $\theta = 12°$.

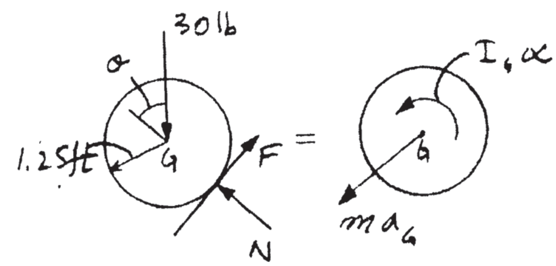


SOLUTION

$+\swarrow \Sigma F_x = m(a_G)_x;\qquad 30 \sin 12° - F = \left(\frac{30}{32.2}\right)a_G$

$+\nwarrow \Sigma F_y = m(a_G)_y;\qquad N - 30 \cos 12° = 0$

$\circlearrowleft + \Sigma M_G = I_G\,\alpha;\qquad F(1.25) = \left[\left(\frac{30}{32.2}\right)(0.6)^2\right]\alpha$



Assume the wheel does not slip.

$$a_G = (1.25)\alpha$$

Solving:

$F = 1.17$ lb

$N = 29.34$ lb

$a_G = 5.44\ \text{ft/s}^2$

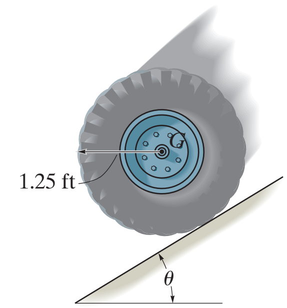
$\alpha = 4.35\ \text{rad/s}^2$ **Ans.**

$F_{\max} = 0.2(29.34) = 5.87\ \text{lb} > 1.17\ \text{lb}$ **OK**

Ans:
$\alpha = 4.32\ \text{rad/s}^2$

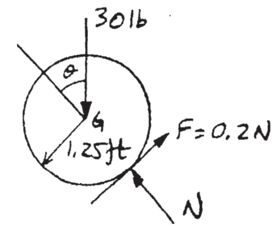
17–95.

The tire has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, determine the maximum angle θ of the inclined plane so that the tire rolls without slipping.



SOLUTION

Since wheel is on the verge of slipping:



$$+\swarrow\Sigma F_x = m(a_G)_x; \qquad 30 \sin\theta - 0.2N = \left(\frac{30}{32.2}\right)(1.25\alpha) \qquad \textbf{(1)}$$

$$+\nwarrow\Sigma F_y = m(a_G)_y; \qquad N - 30\cos\theta = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft +\Sigma M_C = I_G\,\alpha; \qquad 0.2N(1.25) = \left[\left(\frac{30}{32.2}\right)(0.6)^2\right]\alpha \qquad \textbf{(3)}$$

Substituting Eqs.(2) and (3) into Eq. (1),

$$30\sin\theta - 6\cos\theta = 26.042\cos\theta$$

$$30\sin\theta = 32.042\cos\theta$$

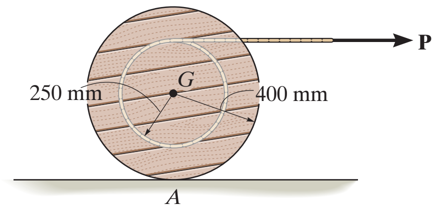
$$\tan\theta = 1.068$$

$$\theta = 46.9^\circ \qquad \textbf{Ans.}$$

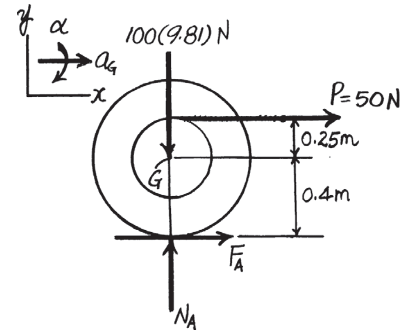
Ans:
$\theta = 46.9^\circ$

***17–96.**

The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 50 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A - 100(9.81) = 0$

$\circlearrowright + \Sigma M_G = I_G\alpha;\qquad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\alpha$

$\alpha = 1.30 \text{ rad/s}^2$ **Ans.**

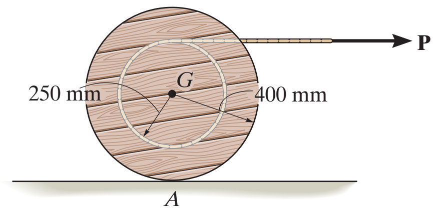
$a_G = 0.520 \text{ m/s}^2 \qquad N_A = 981 \text{ N} \qquad F_A = 2.00 \text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 2.00 \text{ N}$ **OK**

Ans:
$\alpha = 1.30 \text{ rad/s}^2$

17–97.

Solve Prob. 17–96 if the cord and force $P = 50\text{ N}$ are directed vertically upwards.



SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)x; \quad F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 50 - 100(9.81) = 0$

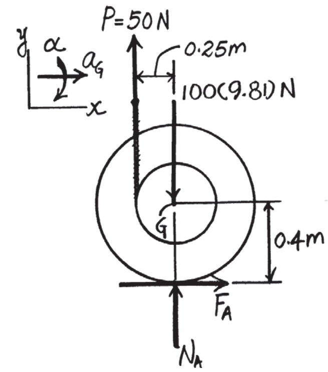
$\circlearrowleft + \Sigma M_G = I_G\alpha; \quad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\,\alpha$

$$\alpha = 0.500 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$a_G = 0.2 \text{ m/s}^2 \qquad N_A = 931 \text{ N} \qquad F_A = 20 \text{ N}$$

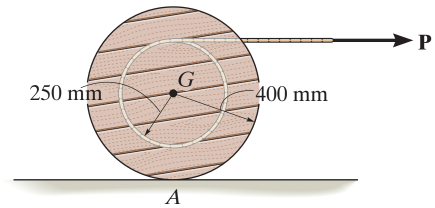
Since $(F_A)_{max} = 0.2(931) = 186.2 \text{ N} > 20 \text{ N}$ **OK**



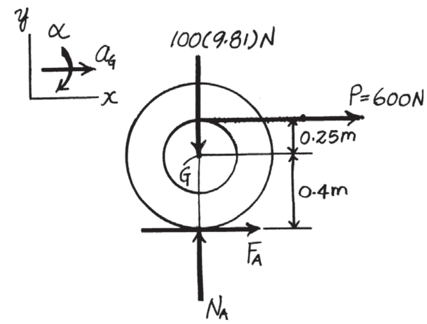
Ans:
$\alpha = 0.500 \text{ rad/s}^2$

17–98.

The spool has a mass of 100 kg and a radius of gyration $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 600$ N.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 600 + F_A = 100 a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G \, \alpha; \qquad 600(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\alpha$

$\alpha = 15.6 \text{ rad/s}^2$ **Ans.**

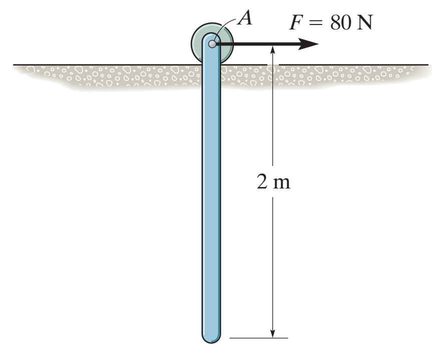
$a_G = 6.24 \text{ m/s}^2 \qquad N_A = 981 \text{ N} \qquad F_A = 24.0 \text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 24.0 \text{ N}$ **OK**

Ans:
$\alpha = 15.6 \text{ rad/s}^2$

17–99.

The 12-kg uniform bar is supported by a roller at A. If a horizontal force of $F = 80$ N is applied to the roller, determine the acceleration of the center of the roller at the instant the force is applied. Neglect the weight and the size of the roller.



SOLUTION

Equations of Motion. The mass moment of inertia of the bar about its center of gravity G is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(12)(2^2) = 4.00\ \text{kg}\cdot\text{m}^2$. Referring to the FBD and kinetic diagram of the bar, Fig. *a*

$\overset{+}{\rightarrow}\Sigma F_x = m(a_G)_x;\qquad 80 = 12(a_G)_x \qquad (a_G)_x = 6.6667\ \text{m/s}^2 \rightarrow$

$\circlearrowleft +\Sigma M_A = (\mu_k)_A;\qquad 0 = 12(6.6667)(1) - 4.00\,\alpha \qquad \alpha = 20.0\ \text{rad/s}^2 \circlearrowright$

Kinematic. Since the bar is initially at rest, $\omega = 0$. Applying the relative acceleration equation by referring to Fig. *b*,

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$$

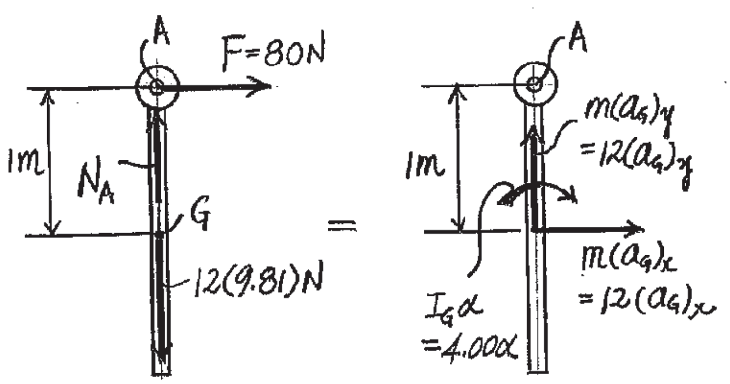
$$6.6667\mathbf{i} + (a_G)_y\mathbf{j} = a_A\mathbf{i} + (-20.0\mathbf{k}) \times (-\mathbf{j}) - \mathbf{0}$$

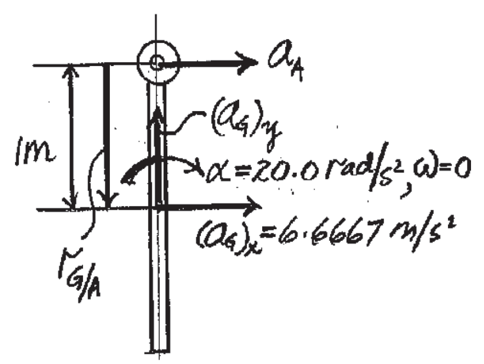
$$6.6667\mathbf{i} + (a_G)_y\mathbf{j} = (a_A - 20)\mathbf{i}$$

Equating **i** and **j** components,

$6.6667 = a_A - 20; \qquad a_A = 26.67\ \text{m/s}^2 = 26.7\ \text{m/s}^2 \rightarrow$ **Ans.**

$(a_G)_y = 0$



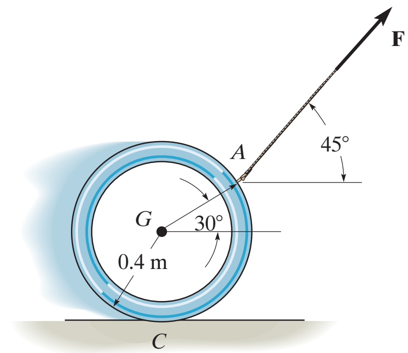


(b)

Ans:
$a_A = 26.7\ \text{m/s}^2 \rightarrow$

***17–100.**

A force of $F = 10$ N is applied to the 10-kg ring as shown. If slipping does not occur, determine the ring's initial angular acceleration, and the acceleration of its mass center, G. Neglect the thickness of the ring.



SOLUTION

Equations of Motion. The mass moment of inertia of the ring about its center of gravity G is $I_G = mr^2 = 10(0.4^2) = 1.60\ \text{kg}\cdot\text{m}^2$. Referring to the FBD and kinetic diagram of the ring, Fig. a,

$$\circlearrowleft + \Sigma M_C = (\mu_k)_C; \quad (10 \sin 45°)(0.4 \cos 30°) - (10 \cos 45°)[0.4(1 + \sin 30°)]$$

$$= -(10a_G)(0.4) - 1.60\alpha$$

$$4a_G + 1.60\alpha = 1.7932 \qquad \textbf{(1)}$$

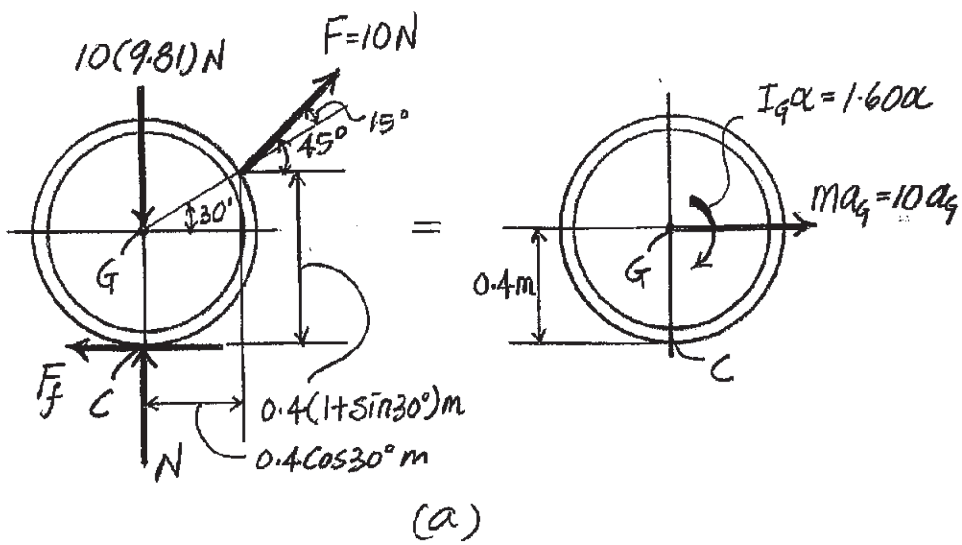
Kinematics. Since the ring rolls without slipping,

$$a_G = \alpha r = \alpha(0.4) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$\alpha = 0.5604\ \text{rad/s}^2 = 0.560\ \text{rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

$$a_G = 0.2241\ \text{m/s}^2 = 0.224\ \text{m/s}^2 \rightarrow \qquad \textbf{Ans.}$$

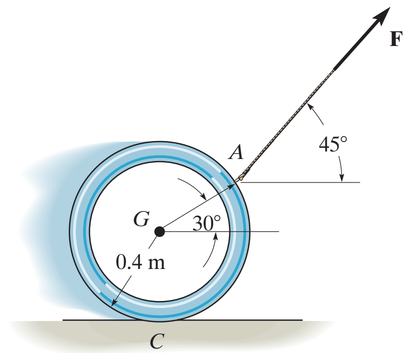


(a)

Ans:
$\alpha = 0.560\ \text{rad/s}^2 \circlearrowright$
$a_G = 0.224\ \text{m/s}^2 \rightarrow$

17–101.

If the coefficient of static friction at C is $\mu_s = 0.3$, determine the largest force **F** that can be applied to the 5-kg ring, without causing it to slip. Neglect the thickness of the ring.



SOLUTION

Equations of Motion: The mass moment of inertia of the ring about its center of gravity G is $I_G = mr^2 = 10(0.4^2) = 1.60\ \text{kg}\cdot\text{m}^2$. Here, it is required that the ring is on the verge of slipping at C, $F_f = \mu_s N = 0.3\ \text{N}$. Referring to the FBD and kinetic diagram of the ring, Fig. a

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad F \sin 45° + N - 10(9.81) = 10(0) \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad F \cos 45° - 0.3\ \text{N} = 10a_G \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad F \sin 15°(0.4) - 0.3\ \text{N}(0.4) = -1.60\alpha \qquad \textbf{(3)}$$

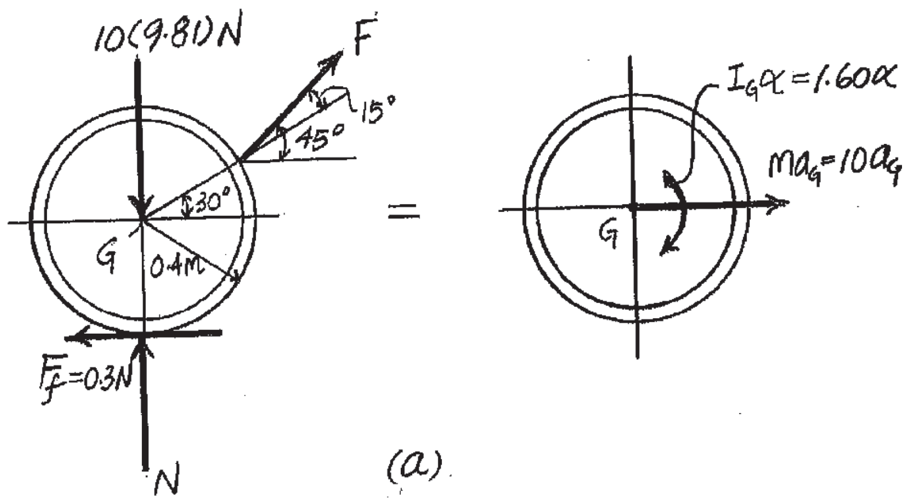
Kinematics. Since the ring rolls without slipping,

$$a_G = \alpha r = \alpha(0.4) \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4),

$$F = 42.34\ \text{N} = 42.3\ \text{N} \qquad \textbf{Ans.}$$

$N = 68.16\ \text{N} \qquad \alpha = 2.373\ \text{rad/s}^2 \circlearrowright \qquad a_G = 0.9490\ \text{m/s}^2 \rightarrow$

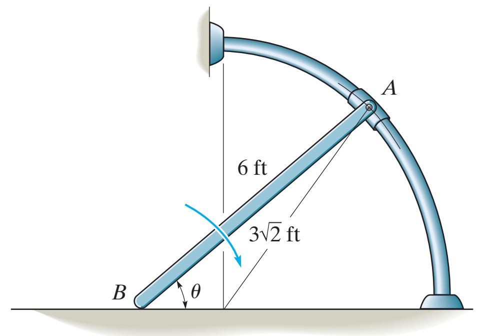


(a)

Ans:
$F = 42.3\ \text{N}$

17–102.

The 25-lb slender rod has a length of 6 ft. Using a collar of negligible mass, its end A is confined to move along the smooth circular bar of radius $3\sqrt{2}$ ft. End B rests on the floor, for which the coefficient of kinetic friction is $\mu_B = 0.4$. If the bar is released from rest when $\theta = 30°$, determine the angular acceleration of the bar at this instant.



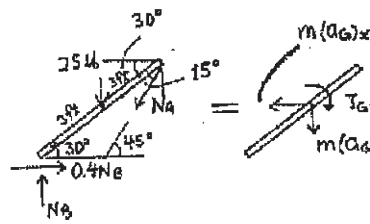
SOLUTION

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\quad -0.4\,N_B + N_A \cos 45° = \dfrac{25}{32.2}(a_G)_x$ **(1)**

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_B - 25 - N_A \sin 45° = \dfrac{-25}{32.2}(a_G)_y$ **(2)**

$\circlearrowleft + \Sigma M_G = I_G\,\alpha;\quad N_B(3\cos 30°) - 0.4\,N_B\,(3\sin 30°)$

$+ N_A \sin 15°(3) = \dfrac{1}{12}\left(\dfrac{25}{32.2}\right)(6)^2\alpha$ **(3)**



$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

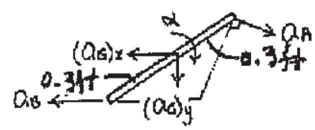
$\underset{\leftarrow}{a_B} = \underset{\measuredangle 45°}{a_A} + \underset{\measuredangle 30°}{6\alpha}$

$(+\uparrow)\quad 0 = -a_A \sin 45° + 6\alpha(\cos 30°)$

$a_A = 7.34847\alpha$

$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}$

$\underset{\leftarrow}{(a_G)_x} + \underset{\downarrow}{(a_G)_y} = \underset{\measuredangle 45°}{7.34847\alpha} + \underset{\measuredangle 30°}{3\alpha}$



$(\overset{+}{\leftarrow})\qquad (a_G)_x = -5.196\alpha + 1.5\alpha = -3.696\alpha$ **(4)**

$(+\downarrow)\qquad (a_G)_y = 5.196\alpha - 2.598\alpha = 2.598\alpha$ **(5)**

Solving Eqs. (1)–(5) yields:

$N_B = 9.01$ lb

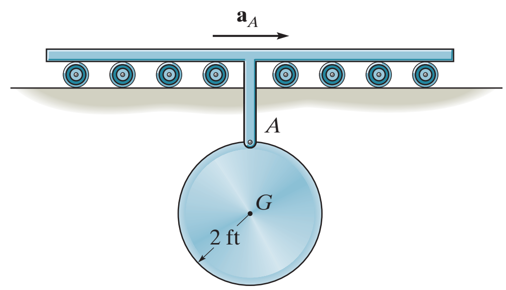
$N_A = -11.2$ lb

$\alpha = 4.01\ \text{rad/s}^2$ **Ans.**

Ans:
$\alpha = 4.01\ \text{rad/s}^2$

17–103.

The 15-lb circular plate is suspended from a pin at A. If the pin is connected to a track which is given an acceleration $a_A = 5\ \text{ft/s}^2$, determine the horizontal and vertical components of reaction at A and the angular acceleration of the plate. The plate is originally at rest.

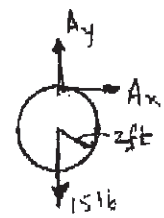


SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad A_x = \dfrac{15}{32.2}(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad A_y - 15 = \dfrac{15}{32.2}(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad A_x(2) = \left[\dfrac{1}{2}\left(\dfrac{15}{32.2}\right)(2)^2\right]\alpha$

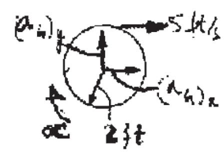


$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}$

$\mathbf{a}_G = 5\mathbf{i} - 2\alpha\mathbf{i}$

$(+\uparrow) \qquad (a_G)_y = 0$

$(\overset{+}{\rightarrow}) \qquad (a_G)_x = 5 - 2\alpha$



Thus,

$A_y = 15.0\ \text{lb}$ **Ans.**

$A_x = 0.776\ \text{lb}$ **Ans.**

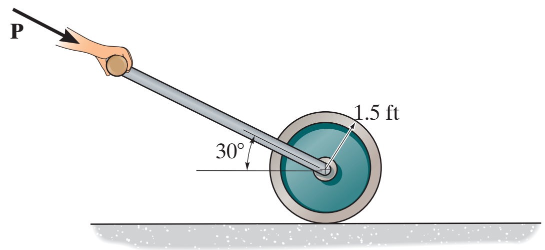
$\alpha = 1.67\ \text{rad/s}^2$ **Ans.**

$a_G = (a_G)_x = 1.67\ \text{ft/s}^2$

Ans:
$A_y = 15.0\ \text{lb}$
$A_x = 0.776\ \text{lb}$
$\alpha = 1.67\ \text{rad/s}^2$

***17–104.**

If $P = 30$ lb, determine the angular acceleration of the 50-lb roller. Assume the roller to be a uniform cylinder and that no slipping occurs.



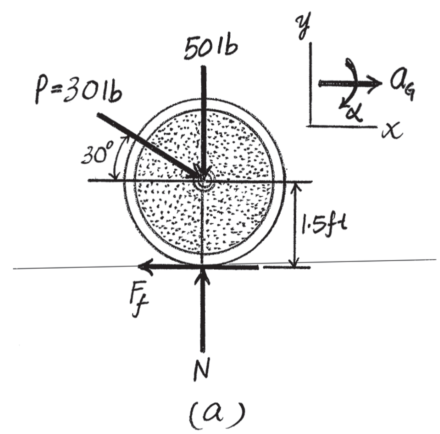
SOLUTION

Equations of Motion: The mass moment of inertia of the roller about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{50}{32.2}\right)\left(1.5^2\right) = 1.7469 \text{ slug}\cdot\text{ft}^2$. We have

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \quad 30\cos 30° - F_f = \frac{50}{32.2}a_G \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - 50 - 30\sin 30° = 0 \qquad N = 65 \text{ lb}$$

$$+\Sigma M_G = I_G\alpha; \qquad F_f(1.5) = 1.7469\alpha \qquad \textbf{(2)}$$



Since the roller rolls without slipping,

$$a_G = \alpha r = \alpha(1.5) \qquad \textbf{(3)}$$

Solving Eqs. (1) through (3) yields

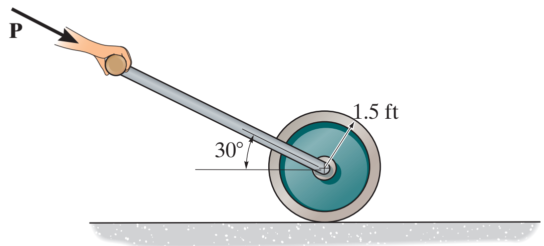
$$\alpha = 7.436 \text{ rad/s}^2 = 7.44 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$F_f = 8.660 \text{ lb} \qquad a_G = 11.15 \text{ ft/s}^2$$

Ans:
$\alpha = 7.44 \text{ rad/s}^2$

17–105.

If the coefficient of static friction between the 50-lb roller and the ground is $\mu_s = 0.25$, determine the maximum force P that can be applied to the handle, so that roller rolls on the ground without slipping. Also, find the angular acceleration of the roller. Assume the roller to be a uniform cylinder.



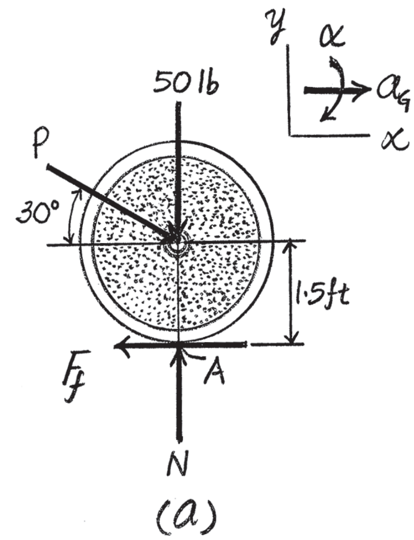
SOLUTION

Equations of Motion: The mass moment of inertia of the roller about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{50}{32.2}\right)(1.5^2) = 1.7469\ \text{slug}\cdot\text{ft}^2$. We have

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad P\cos 30° - F_f = \frac{50}{32.2}a_G \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - P\sin 30° - 50 = 0 \qquad \textbf{(2)}$$

$$+\Sigma M_G = I_G\alpha; \quad F_f(1.5) = 1.7469\alpha \qquad \textbf{(3)}$$



Since the roller is required to be on the verge of slipping,

$$a_G = \alpha r = \alpha(1.5) \qquad \textbf{(4)}$$

$$F_f = \mu_s N = 0.25N \qquad \textbf{(5)}$$

Solving Eqs. (1) through (5) yields

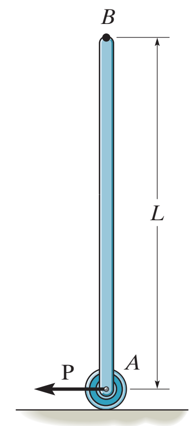
$\alpha = 18.93\ \text{rad/s}^2 = 18.9\ \text{rad/s}^2 \qquad P = 76.37\ \text{lb} = 76.4\ \text{lb}$ **Ans.**

$N = 88.18\ \text{lb} \qquad a_G = 28.39\ \text{ft/s}^2 \qquad F_f = 22.05\ \text{lb}$

Ans:
$\alpha = 18.9\ \text{rad/s}^2$
$P = 76.4\ \text{lb}$

17–106.

The uniform bar of mass m and length L is balanced in the vertical position when the horizontal force **P** is applied to the roller at A. Determine the bar's initial angular acceleration and the acceleration of its top point B.



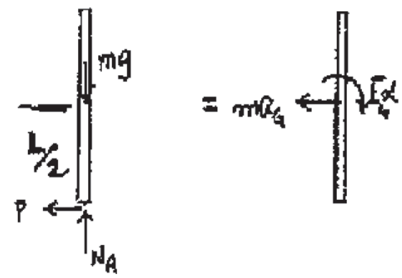
SOLUTION

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P = ma_G$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad P\left(\frac{L}{2}\right) = \left(\frac{1}{12}mL^2\right)\alpha$

$P = \frac{1}{6}mL\alpha$

$\alpha = \frac{6P}{mL}$ **Ans.**



$a_G = \frac{P}{m}$

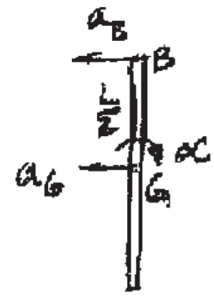
$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$-a_B\mathbf{i} = \frac{-P}{m}\mathbf{i} + \frac{L}{2}\alpha\,\mathbf{i}$

$(\overset{+}{\leftarrow}) \qquad a_B = \frac{P}{m} - \frac{L\alpha}{2}$

$= \frac{P}{m} - \frac{L}{2}\left(\frac{6P}{mL}\right)$

$a_B = -\frac{2P}{m} = \frac{2P}{m}$ **Ans.**



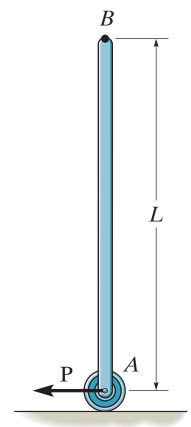
Ans:

$\alpha = \frac{6P}{mL}$

$a_B = \frac{2P}{m}$

17–107.

Solve Prob. 17–106 if the roller is removed and the coefficient of kinetic friction at the ground is μ_k.



SOLUTION

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P - \mu_k N_A = m a_G$

$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad (P - \mu_k N_A)\dfrac{L}{2} = \left(\dfrac{1}{12} m L^2\right)\alpha$

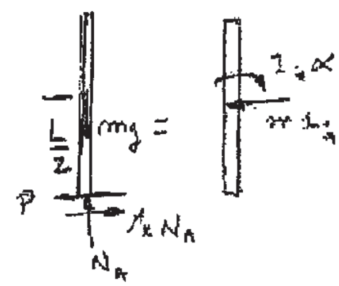
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - mg = 0$

Solving,

$$N_A = mg$$

$$a_G = \frac{L}{6}\alpha$$

$$\alpha = \frac{6(P - \mu_k mg)}{mL} \qquad \textbf{Ans.}$$



$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$(\overset{+}{\rightarrow}) \quad a_B = -\dfrac{L}{6}\alpha + \dfrac{L}{2}\alpha$

$a_B = \dfrac{L\alpha}{3}$

$$a_B = \frac{2(P - \mu_k mg)}{m} \qquad \textbf{Ans.}$$

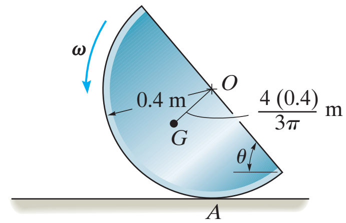
Ans:

$\alpha = \dfrac{6(P - \mu_k mg)}{mL}$

$a_B = \dfrac{2(P - \mu_k mg)}{m}$

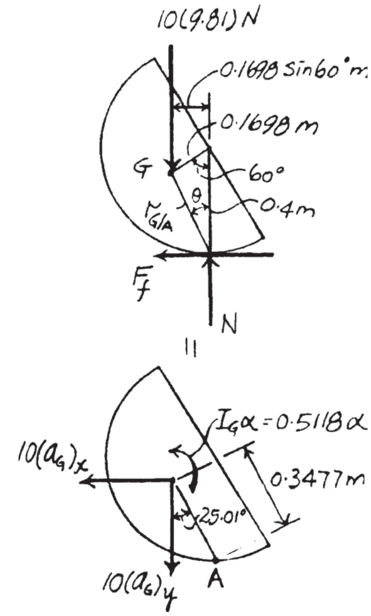
***17–108.**

The semicircular disk having a mass of 10 kg is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. If the coefficient of static friction at A is $\mu_s = 0.5$, determine if the disk slips at this instant.



SOLUTION

Equations of Motion: The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)\left(0.4^2\right) - 10\,(0.1698^2) = 0.5118\ \text{kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)\,(0.4)\cos 60°} = 0.3477$ m Also, using law of sines, $\dfrac{\sin\theta}{0.1698} = \dfrac{\sin 60°}{0.3477}$, $\theta = 25.01°$. Applying Eq. 17–16, we have



$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad 10(9.81)(0.1698\sin 60°) = 0.5118\alpha$$
$$+\,10(a_G)_x\cos 25.01°(0.3477)$$
$$+\,10(a_G)_y\sin 25.01°(0.3477) \qquad \textbf{(1)}$$

$$\xrightarrow{+}\ \Sigma F_x = m(a_G)_x; \qquad F_f = 10(a_G)_x \qquad \textbf{(2)}$$

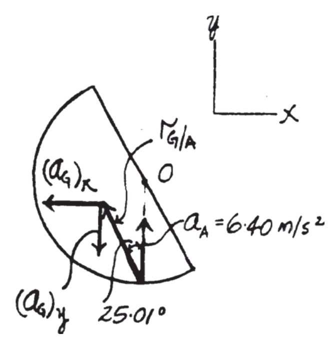
$$+\uparrow F_y = m(a_G)_y; \qquad N - 10(9.81) = -10(a_G)_y \qquad \textbf{(3)}$$

Kinematics: Assume that the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477\sin 25.01°\mathbf{i} + 0.3477\cos 25.01°\mathbf{j}\}$ m $= \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16–18, we have

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2\mathbf{r}_{G/A}$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k} \times (-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$

$$-(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = (2.3523 - 0.3151\,\alpha)\,\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$



Equating **i** and **j** components, we have

$$(a_G)_x = 0.3151\alpha - 2.3523 \qquad \textbf{(4)}$$

$$(a_G)_y = 0.1470\alpha - 1.3581 \qquad \textbf{(5)}$$

Solving Eqs. (1), (2), (3), (4), and (5) yields:

$$\alpha = 13.85\ \text{rad/s}^2 \qquad (a_G)_x = 2.012\ \text{m/s}^2 \qquad (a_G)_y = 0.6779\ \text{m/s}^2$$

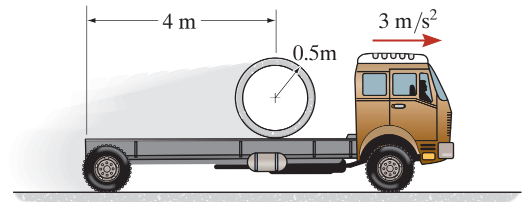
$$F_f = 20.12\ \text{N} \qquad N = 91.32\ \text{N}$$

Since $F_f < (F_f)_{\max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular **disk does not slip**. **Ans.**

Ans:
Since $F_f < (F_f)_{\max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular **disk does not slip**.

17–109.

The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of $3\,\text{m/s}^2$, determine the culvert's angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.



SOLUTION

Equations of Motion: The mass moment of inertia of the culvert about its mass center is $I_G = mr^2 = 500\left(0.5^2\right) = 125\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A using Fig. a,

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 0 = 125\alpha - 500a_G(0.5) \qquad \textbf{(1)}$$

Kinematics: Since the culvert does not slip at A, $(a_A)_t = 3\ \text{m/s}^2$. Applying the relative acceleration equation and referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 r_{G/A}$$

$$a_G\mathbf{i} = 3\mathbf{i} + (a_A)_n\,\mathbf{j} + (\alpha\mathbf{k} \times 0.5\mathbf{j}) - \omega^2(0.5\mathbf{j})$$

$$a_G\mathbf{i} = (3 - 0.5\alpha)\mathbf{i} + \left[(a_A)_n - 0.5\omega^2\right]\mathbf{j}$$

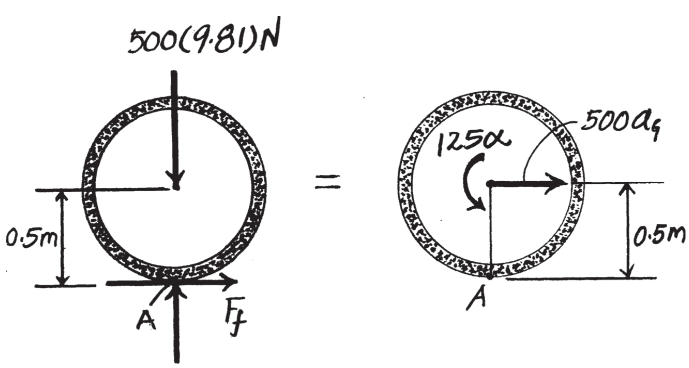
Equating the **i** components,

$$a_G = 3 - 0.5\alpha \qquad \textbf{(2)}$$

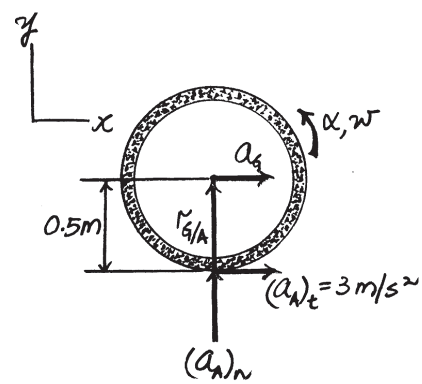
Solving Eqs. (1) and (2) yields

$$a_G = 1.5\ \text{m/s}^2 \rightarrow$$

$$\alpha = 3\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



(a)

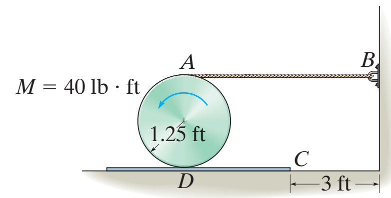


(b)

Ans:
$\alpha = 3\ \text{rad/s}^2$

17–110.

The 15-lb disk rests on the 5-lb plate. A cord is wrapped around the periphery of the disk and attached to the wall at B. If a torque $M = 40$ lb · ft is applied to the disk, determine the angular acceleration of the disk and the time needed for the end C of the plate to travel 3 ft and strike the wall. Assume the disk does not slip on the plate and the plate rests on the surface at D having a coefficient of kinetic friction of $\mu_k = 0.2$. Neglect the mass of the cord.



SOLUTION

Disk:

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad T - F_P = \frac{15}{32.2} a_G$$

$$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad -F_P(1.25) + 40 - T(1.25) = \left[\frac{1}{2}\left(\frac{15}{32.2}\right)(1.25)^2\right]\alpha$$

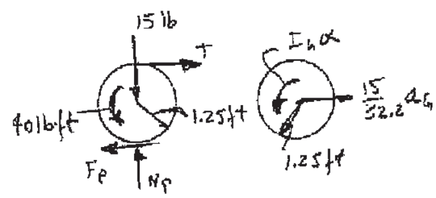


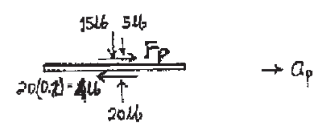
Plate:

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad F_P - 4 = \frac{5}{32.2} a_P$$

$$\mathbf{a}_P = \mathbf{a}_G + \mathbf{a}_{P/G}$$

$$(\overset{+}{\rightarrow}) \qquad a_P = a_G + \alpha(1.25)$$

$$a_G = \alpha(1.25)$$



Thus

$$a_P = 2.5\alpha$$

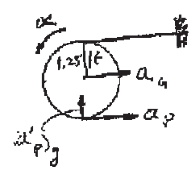
Solving,

$$F_P = 9.65 \text{ lb}$$

$$a_P = 36.367 \text{ ft/s}^2$$

$$\alpha = 14.5 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$T = 18.1 \text{ lb}$$



$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

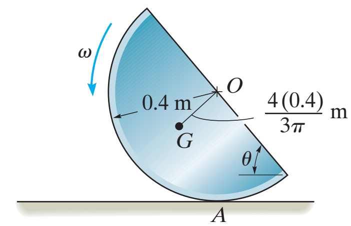
$$3 = 0 + 0 + \frac{1}{2}(36.367)t^2$$

$$t = 0.406 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$\alpha = 14.5 \text{ rad/s}^2$
$t = 0.406$ s

17–111.

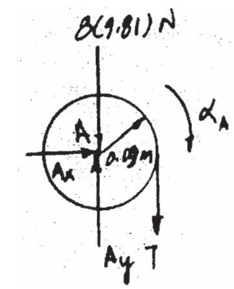
The semicircular disk having a mass of 10 kg is rotating at $\omega = 4$ rad/s at the instant $\theta = 60°$. If the coefficient of static friction at A is $\mu_s = 0.5$, determine if the disk slips at this instant.



SOLUTION

For roll A.

$$\circlearrowright + \Sigma M_A = I_A\,\alpha; \qquad T(0.09) = \tfrac{1}{2}(8)(0.09)^2\alpha_A \qquad \textbf{(1)}$$



For roll B

$$\circlearrowleft + \Sigma M_O = \Sigma(M_k)_O; \qquad 8(9.81)(0.09) = \tfrac{1}{2}(8)(0.09)^2\alpha_B + 8a_B(0.09) \qquad \textbf{(2)}$$

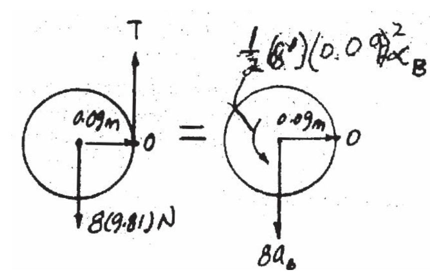
$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T - 8(9.81) = -8a_B \qquad \textbf{(3)}$$

Kinematics:

$$\mathbf{a}_B = \mathbf{a}_O + (\mathbf{a}_{B/O})_t + (\mathbf{a}_{B/O})_n$$

$$\begin{bmatrix} a_B \\ \downarrow \end{bmatrix} = \begin{bmatrix} a_O \\ \downarrow \end{bmatrix} + \begin{bmatrix} \alpha_B\,(0.09) \\ \downarrow \end{bmatrix} + [0]$$

$$(+\downarrow) \qquad a_B = a_O + 0.09\alpha_B \qquad \textbf{(4)}$$



also,

$$(+\downarrow) \qquad a_O = \alpha_A\,(0.09) \qquad \textbf{(5)}$$

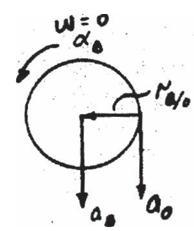
Solving Eqs. (1)–(5) yields:

$\alpha_A = 43.6\ \text{rad/s}^2$ **Ans.**

$\alpha_B = 43.6\ \text{rad/s}^2$ **Ans.**

$T = 15.7$ N **Ans.**

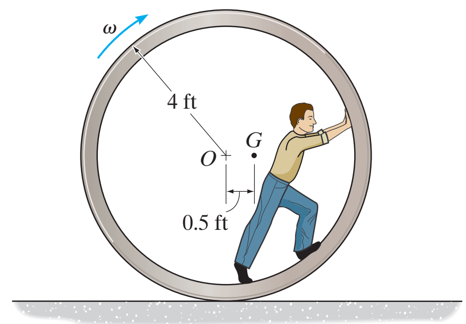
$a_B = 7.85\ \text{m/s}^2 \qquad a_O = 3.92\ \text{m/s}^2$



Ans:
The disk does not slip.

***17–112.**

The circular concrete culvert rolls with an angular velocity of $\omega = 0.5$ rad/s when the man is at the position shown. At this instant the center of gravity of the culvert and the man is located at point G, and the radius of gyration about G is $k_G = 3.5$ ft. Determine the angular acceleration of the culvert. The combined weight of the culvert and the man is 500 lb. Assume that the culvert rolls without slipping, and the man does not move within the culvert.



SOLUTIONS

Equations of Motion: The mass moment of inertia of the system about its mass center is $I_G = mk_G^2 = \frac{500}{32.2}(3.5^2) = 190.22 \text{ slug}\cdot\text{ft}^2$. Writing the moment equation of motion about point A, Fig. a,

$$+\ \Sigma M_A = \Sigma(M_k)_A;\quad -500(0.5) = -\frac{500}{32.2}(a_G)_x(4) - \frac{500}{32.2}(a_G)_y(0.5) - 190.22\alpha \quad \textbf{(1)}$$

Kinematics: Since the culvert rolls without slipping,

$$a_0 = \alpha r = \alpha(4) \rightarrow$$

Applying the relative acceleration equation and referrring to Fig. b,

$$a_G = a_O + \alpha \times r_{G/O} - \omega^2 \mathbf{r}_{G/A}$$

$$(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = 4\alpha\mathbf{i} + (-\alpha\mathbf{k}) \times (0.5\mathbf{i}) - (0.5^2)(0.5\mathbf{i})$$

$$(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = (4\alpha - 0.125)\mathbf{i} - 0.5\alpha\mathbf{j}$$

Equation the **i** and **j** components,

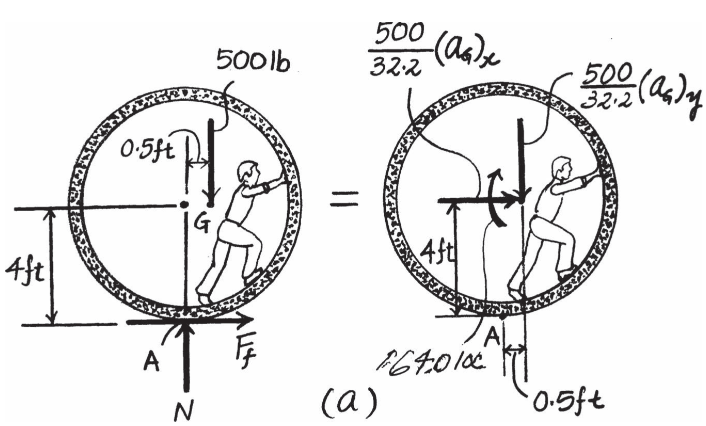
$$(a_G)_x = 4\alpha - 0.125 \quad \textbf{(2)}$$

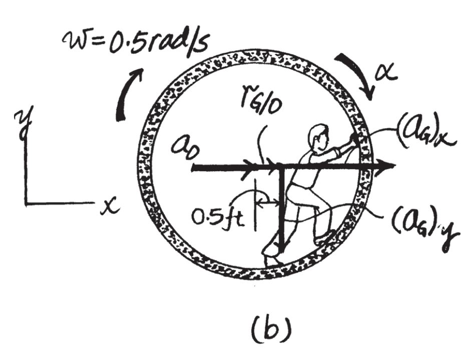
$$(a_G)_y = 0.5\alpha \quad \textbf{(3)}$$

Subtituting Eqs. (2) and (3) into Eq. (1),

$$-500(0.5) = -\frac{500}{32.2}(4\alpha - 0.125)(4) - \frac{500}{32.2}(0.5\alpha)(0.5) - 190.22\alpha$$

$$\alpha = 0.582 \text{ rad/s}^2 \quad \textbf{Ans.}$$

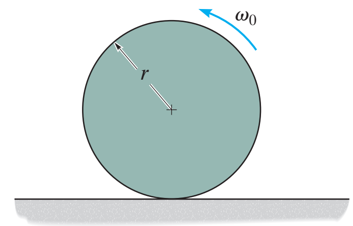




Ans:
$\alpha = 0.582 \text{ rad/s}^2$

17–113.

The uniform disk of mass m is rotating with an angular velocity of ω_0 when it is placed on the floor. Determine the initial angular acceleration of the disk and the acceleration of its mass center. The coefficient of kinetic friction between the disk and the floor is μ_k.



SOLUTION

Equations of Motion. Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$. We have

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N - mg = 0 \qquad N = mg$

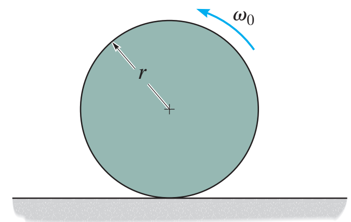
$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad \mu_k(mg) = ma_G \qquad a_G = \mu_k g \leftarrow$ **Ans.**

$\circlearrowleft + \Sigma M_G = I_G\alpha;\qquad -\mu_k(mg)r = \left(\frac{1}{2}mr^2\right)\alpha \qquad \alpha = \frac{2\mu_k g}{r} \circlearrowright$ **Ans.**

Ans:
$a_G = \mu_k g \leftarrow$
$\alpha = \frac{2\mu_k g}{r} \circlearrowright$

17–114.

The uniform disk of mass m is rotating with an angular velocity of ω_0 when it is placed on the floor. Determine the time before it starts to roll without slipping. What is the angular velocity of the disk at this instant? The coefficient of kinetic friction between the disk and the floor is μ_k.



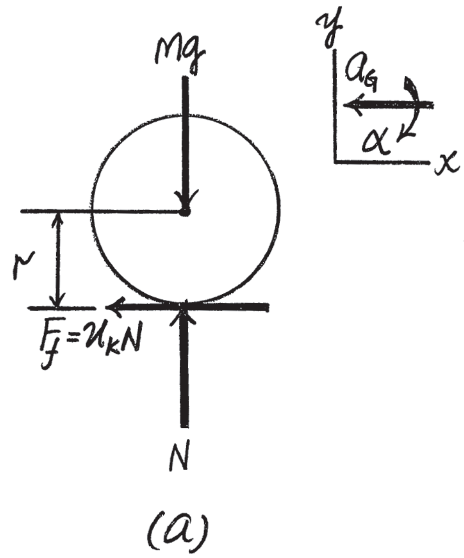
SOLUTION

Equations of Motion: Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$.

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - mg = 0 \quad N = mg$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad \mu_k(mg) = ma_G \quad a_G = \mu_k g$

$+\Sigma M_G = I_G\alpha; \quad -\mu_k(mg)r = -\left(\frac{1}{2}mr^2\right)\alpha \quad \alpha = \frac{2\mu_k g}{r}$



Kinematics: At the instant when the disk rolls without slipping, $v_G = \omega r$. Thus,

$(\overset{+}{\leftarrow})$

$$v_G = (v_G)_0 + a_G t$$
$$\omega r = 0 + \mu_k g t$$
$$t = \frac{\omega r}{\mu_k g} \quad \textbf{(1)}$$

and

$$\omega = \omega_0 + \alpha t$$

$(\circlearrowleft +)$ $$\omega = \omega_0 + \left(-\frac{2\mu_k g}{r}\right)t \quad \textbf{(2)}$$

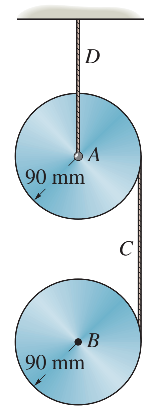
Solving Eqs. (1) and (2) yields

$$\omega = \frac{1}{3}\omega_0 \qquad t = \frac{\omega_0 r}{3\mu_k g} \qquad \textbf{Ans.}$$

Ans:
$\omega = \frac{1}{3}\omega_0$
$t = \frac{\omega_0 r}{3\mu_k g}$

17–115.

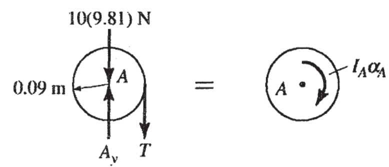
A cord is wrapped around each of the two 10-kg disks. If they are released from rest, determine the angular acceleration of each disk and the tension in the cord C. Neglect the mass of the cord.



SOLUTION

For A:

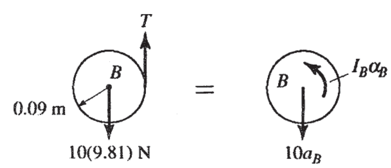
$$\circlearrowright +\Sigma M_A = I_A\alpha_A; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_A \qquad \textbf{(1)}$$



For B:

$$\circlearrowleft +\Sigma M_B = I_B\alpha_B; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_B \qquad \textbf{(2)}$$

$$+\downarrow \Sigma F_y = m(a_B)_y; \qquad 10(9.81) - T = 10a_B \qquad \textbf{(3)}$$



$$\mathbf{a}_B = \mathbf{a}_P + (\mathbf{a}_{B/P})_t + (\mathbf{a}_{B/P})_n$$

$$(+\downarrow)\ a_B = 0.09\alpha_A + 0.09\alpha_B + 0 \qquad \textbf{(4)}$$

Solving,

$a_B = 7.85\ \text{m/s}^2$

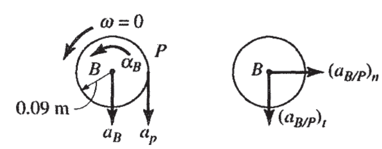
$\alpha_A = 43.6\ \text{rad/s}^2$ **Ans.**

$\alpha_B = 43.6\ \text{rad/s}^2$ **Ans.**

$T = 19.6\ \text{N}$ **Ans.**

$A_y = 10(9.81) + 19.62$

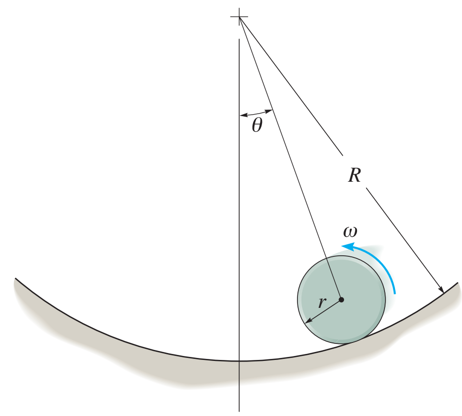
$= 118\ \text{N}$



Ans:
$\alpha_A = 43.6\ \text{rad/s}^2$ ↻
$\alpha_B = 43.6\ \text{rad/s}^2$ ↺
$T = 19.6\ \text{N}$

***17–116.**

The disk of mass m and radius r rolls without slipping on the circular path. Determine the normal force which the path exerts on the disk and the disk's angular acceleration if at the instant shown the disk has an angular velocity of $\boldsymbol{\omega}$.

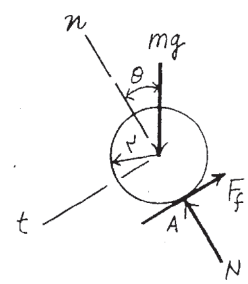


SOLUTION

Equation of Motion: The mass moment of inertia of the disk about its center of mass is given by $I_G = \frac{1}{2}mr^2$. Applying Eq. 17–16, we have

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad mg\sin\theta(r) = \left(\frac{1}{2}mr^2\right)\alpha + m(a_G)_t\,(r) \qquad \textbf{[1]}$$

$$\Sigma F_n = m(a_G)_n; \qquad N - mg\cos\theta = m(a_G)_n \qquad \textbf{[2]}$$



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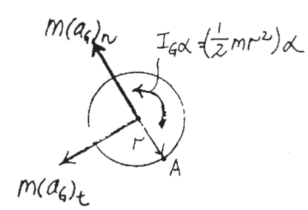


Kinematics: Since the semicircular disk does not slip at A, then $v_G = \omega r$ and $(a_G)_t = \alpha r$. Substitute $(a_G)_t = \alpha r$ into Eq. [1] yields

$$mg\sin\theta(r) = \left(\frac{1}{2}mr^2\right)\alpha + m(\alpha r)(r)$$

$$\alpha = \frac{2g}{3r}\sin\theta \qquad \textbf{Ans.}$$

Also, the center of the mass for the disk moves around a circular path having a radius of $\rho = R - r$. Thus, $(a_G)_n = \frac{v_G^2}{\rho} = \frac{\omega^2 r^2}{R - r}$. Substitute into Eq. [2] yields

$$N - mg\cos\theta = m\left(\frac{\omega^2 r^2}{R - r}\right)$$

$$N = m\left(\frac{\omega^2 r^2}{R - r} + g\cos\theta\right) \qquad \textbf{Ans.}$$

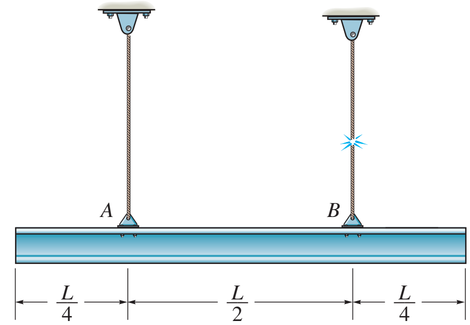
Ans:

$\alpha = \frac{2g}{3r}\sin\theta$

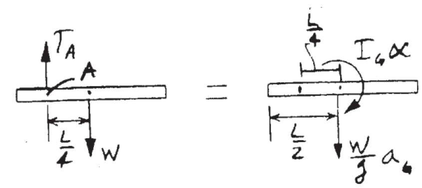
$N = m\left(\frac{\omega^2 r^2}{R - r} + g\cos\theta\right)$

17–117.

The uniform beam has a weight W. If it is originally at rest while being supported at A and B by cables, determine the tension in cable A if cable B suddenly fails. Assume the beam is a slender rod.



SOLUTION



$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_A - W = -\frac{W}{g}a_G$$

$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad W\left(\frac{L}{4}\right) = \left[\frac{1}{12}\left(\frac{W}{g}\right)L^2\right]\alpha + \frac{W}{g}\left(\frac{L}{4}\right)\alpha\left(\frac{L}{4}\right)$$

$$1 = \frac{1}{g}\left(\frac{L}{4} + \frac{L}{3}\right)\alpha$$

Since $a_G = \alpha\left(\frac{L}{4}\right)$.

$$\alpha = \frac{12}{7}\left(\frac{g}{L}\right)$$

$$T_A = W - \frac{W}{g}(\alpha)\left(\frac{L}{4}\right) = W - \frac{W}{g}\left(\frac{12}{7}\right)\left(\frac{g}{L}\right)\left(\frac{L}{4}\right)$$

$$T_A = \frac{4}{7}W \qquad \textbf{Ans.}$$

Also,

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad T_A - W = -\frac{W}{g}a_G$$

$$\circlearrowleft + \Sigma M_G = I_G\alpha; \qquad T_A\left(\frac{L}{4}\right) = \left[\frac{1}{12}\left(\frac{W}{g}\right)L^2\right]\alpha$$

Since $a_G = \frac{L}{4}\alpha$

$$T_A = \frac{1}{3}\left(\frac{W}{g}\right)L\alpha$$

$$\frac{1}{3}\left(\frac{W}{g}\right)L\alpha - W = -\frac{W}{g}\left(\frac{L}{4}\right)\alpha$$

$$\alpha = \frac{12}{7}\left(\frac{g}{L}\right)$$

$$T_A = \frac{1}{3}\left(\frac{W}{g}\right)L\left(\frac{12}{7}\right)\left(\frac{g}{L}\right)$$

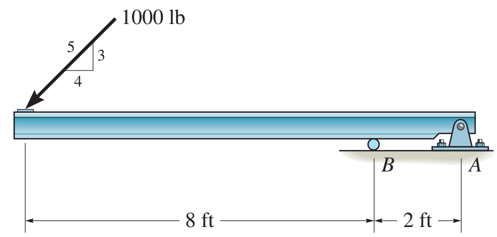
$$T_A = \frac{4}{7}W \qquad \textbf{Ans.}$$

Ans:

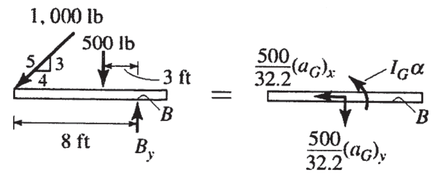
$T_A = \frac{4}{7}W$

17–118.

The 500-lb beam is supported at A and B when it is subjected to a force of 1000 lb as shown. If the pin support at A suddenly fails, determine the beam's initial angular acceleration and the force of the roller support on the beam. For the calculation, assume that the beam is a slender rod so that its thickness can be neglected.



SOLUTION



$$\overset{+}{\leftarrow}\sum F_x = m(a_G)_x; \qquad 1000\left(\frac{4}{5}\right) = \frac{500}{32.2}(a_G)_x$$

$$+\downarrow\sum F_y = m(a_G)_y; \qquad 1000\left(\frac{3}{5}\right) + 500 - B_y = \frac{500}{32.2}(a_G)_y$$

$$\circlearrowleft + \sum M_B = \sum(M_k)_B; \quad 500(3) + 1000\left(\frac{3}{5}\right)(8) = \frac{500}{32.2}(a_G)_y(3) + \left[\frac{1}{12}\left(\frac{500}{32.2}\right)(10)^2\right]\alpha$$

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$$

$$-a_B\mathbf{i} = -(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} + \alpha(3)\mathbf{j}$$

$$(+\downarrow) \quad (a_G)_y = \alpha(3)$$

$\alpha = 23.4 \text{ rad/s}^2$ **Ans.**

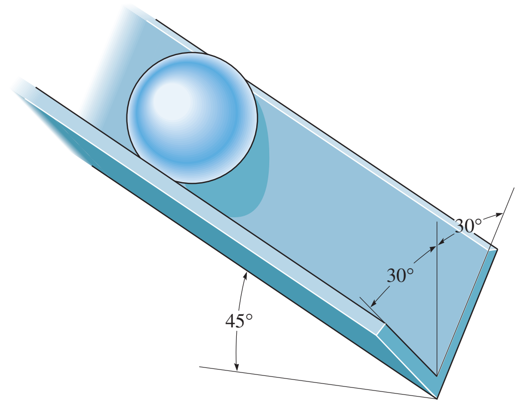
$B_y = 9.62 \text{ lb}$ **Ans.**

$B_y > 0$ means that the beam stays in contact with the roller support.

Ans:
$\alpha = 23.4 \text{ rad/s}^2$
$B_y = 9.62 \text{ lb}$

17–119.

The solid ball of radius r and mass m rolls without slipping down the 60° trough. Determine its angular acceleration.

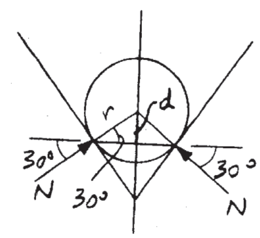


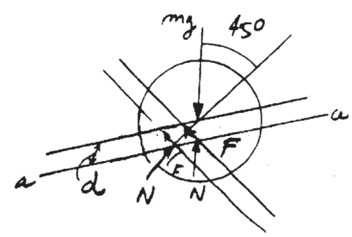
SOLUTION

$$d = r \sin 30° = \frac{r}{2}$$

$$\Sigma M_{a-a} = \Sigma(M_k)_{a-a}; \qquad mg \sin 45°\left(\frac{r}{2}\right) = \left[\frac{2}{5}mr^2 + m\left(\frac{r}{2}\right)^2\right]\alpha$$

$$\alpha = \frac{10g}{13\sqrt{2}\,r} \qquad \textbf{Ans.}$$



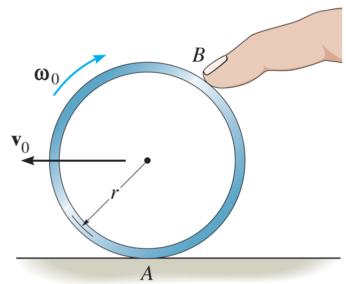


Ans:

$$\alpha = \frac{10g}{13\sqrt{2}\,r}$$

***17–120.**

By pressing down with the finger at B, a thin ring having a mass m is given an initial velocity $\mathbf{v_0}$ and a backspin $\boldsymbol{\omega}_0$ when the finger is released. If the coefficient of kinetic friction between the table and the ring is μ_κ, determine the distance the ring travels forward before backspinning stops.



SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad N_A - mg = 0$$

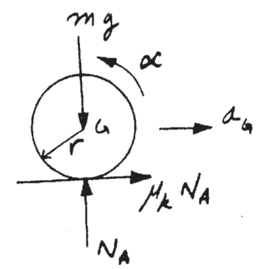
$$N_A = mg$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad \mu_k\,(mg) = m(a_G)$$

$$a_G = \mu_k\, g$$

$$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad \mu_k\,(mg)r = mr^2\alpha$$

$$\alpha = \frac{\mu_k\, g}{r}$$



$$(\circlearrowright +) \qquad \omega = \omega_0 + \alpha_c\, t$$

$$0 = \omega_0 - \left(\frac{\mu_k\, g}{r}\right) t$$

$$t = \frac{\omega_0\, r}{\mu_k\, g}$$

$$(\overset{+}{\leftarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c\, t^2$$

$$s = 0 + v_0\left(\frac{\omega_0\, r}{\mu_k\, g}\right) - \left(\frac{1}{2}\right)(\mu_k\, g)\left(\frac{\omega_0^2\, r^2}{\mu_k^2\, g^2}\right)$$

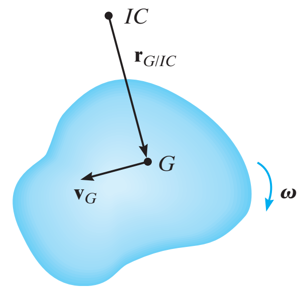
$$s = \left(\frac{\omega_0\, r}{\mu_k\, g}\right)\left(v_0 - \frac{1}{2}\,\omega_0\, r\right) \qquad \textbf{Ans.}$$

Ans:

$$s = \left(\frac{\omega_0\, r}{\mu_k\, g}\right)\left(v_0 - \frac{1}{2}\,\omega_0\, r\right)$$

18–1.

At a given instant the body of mass m has an angular velocity $\boldsymbol{\omega}$ and its mass center has a velocity $\mathbf{v}_G$. Show that its kinetic energy can be represented as $T = \frac{1}{2}I_{IC}\omega^2$, where I_{IC} is the moment of inertia of the body determined about the instantaneous axis of zero velocity, located a distance $r_{G/IC}$ from the mass center as shown.



SOLUTION

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\,\omega^2 \qquad \text{where } v_G = \omega r_{G/IC}$$

$$= \frac{1}{2}m(\omega r_{G/IC})^2 + \frac{1}{2}I_G\,\omega^2$$

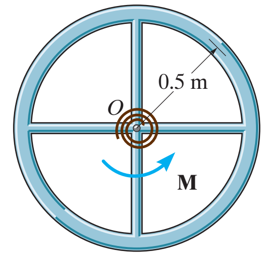
$$= \frac{1}{2}\left(mr_{G/IC}^2 + I_G\right)\omega^2 \qquad \text{However } mr_{G/IC}^2 + I_G = I_{IC}$$

$$= \frac{1}{2}I_{IC}\,\omega^2$$

Q.E.D.

18–2.

The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2\ \text{N}\cdot\text{m/rad}$, and the wheel is rotated until the torque $M = 25\ \text{N}\cdot\text{m}$ is developed, determine the maximum angular velocity of the wheel if it is released from rest.



SOLUTION

Kinetic Energy and Work: The mass moment of inertia of the wheel about point O is

$$I_O = m_R r^2 + 2\left(\frac{1}{12}m_r l^2\right)$$
$$= 5(0.5^2) + 2\left[\frac{1}{12}(2)(1^2)\right]$$
$$= 1.5833\ \text{kg}\cdot\text{m}^2$$

Thus, the kinetic energy of the wheel is

$$T = \frac{1}{2}I_O\,\omega^2 = \frac{1}{2}(1.5833)\,\omega^2 = 0.79167\,\omega^2$$

Since the wheel is released from rest, $T_1 = 0$. The torque developed is $M = k\theta = 2\theta$. Here, the angle of rotation needed to develop a torque of $M = 25\ \text{N}\cdot\text{m}$ is

$$2\theta = 25 \qquad \theta = 12.5\ \text{rad}$$

The wheel achieves its maximum angular velocity when the spacing is unwound that is when the wheel has rotated $\theta = 12.5$ rad. Thus, the work done by $\underset{\infty}{\mathbf{M}}$ is

$$U_M = \int M\,d\theta = \int_0^{12.5\ \text{rad}} 2\theta\,d\theta$$
$$= \theta^2\Big|_0^{12.5\ \text{rad}} = 156.25\ \text{J}$$

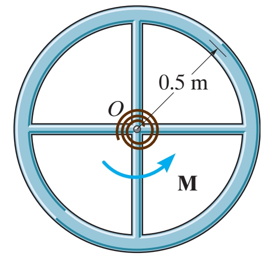
Principle of Work and Energy:

$$T_1 + \Sigma u_{1-2} = T_2$$
$$0 + 156.25 = 0.79167\,\omega^2$$
$$\omega = 14.0\ \text{rad/s}$$ **Ans.**

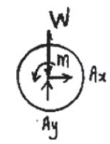
Ans:
$\omega = 14.0\ \text{rad/s}$

18–3.

The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2\ \text{N}\cdot\text{m/rad}$, so that the torque on the center of the wheel is $M = (2\theta)\ \text{N}\cdot\text{m}$, where θ is in radians, determine the maximum angular velocity of the wheel if it is rotated two revolutions and then released from rest.



SOLUTION



$$I_o = 2\left[\frac{1}{12}(2)(1)^2\right] + 5(0.5)^2 = 1.583$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{4\pi} 2\theta\, d\theta = \frac{1}{2}(1.583)\,\omega^2$$

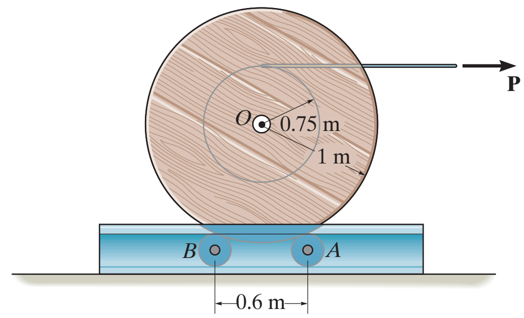
$$(4\pi)^2 = 0.7917\omega^2$$

$\omega = 14.1$ rad/s **Ans.**

Ans:
$\omega = 14.1$ rad/s

***18–4.**

A force of $P = 60$ N is applied to the cable, which causes the 200-kg reel to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. Assume the radius of gyration of the reel about its center axis remains constant at $k_O = 0.6$ m.



SOLUTION

Kinetic Energy. Since the reel is at rest initially, $T_1 = 0$. The mass moment of inertia of the reel about its center O is $I_0 = mk_0^2 = 200\left(0.6^2\right) = 72.0\ \text{kg} \cdot \text{m}^2$. Thus,

$$T_2 = \frac{1}{2}I_0\omega^2 = \frac{1}{2}(72.0)\omega^2 = 36.0\,\omega^2$$

Work. Referring to the FBD of the reel, Fig. *a*, only force **P** does positive work. When the reel rotates 2 revolution, force **P** displaces $S = \theta r = 2(2\pi)(0.75) = 3\pi$ m. Thus

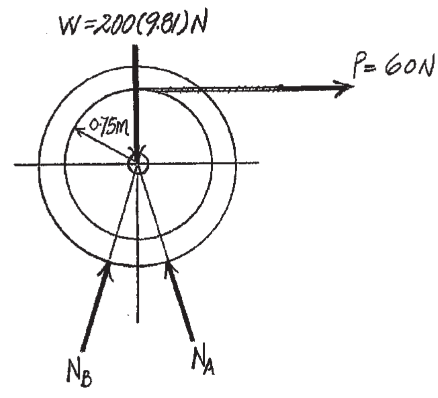
$$U_p = P_s = 60(3\pi) = 180\pi\ \text{J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 180\pi = 36.0\,\omega^2$$

$$\omega = 3.9633\ \text{rad/s} = 3.96\ \text{rad/s}$$ **Ans.**

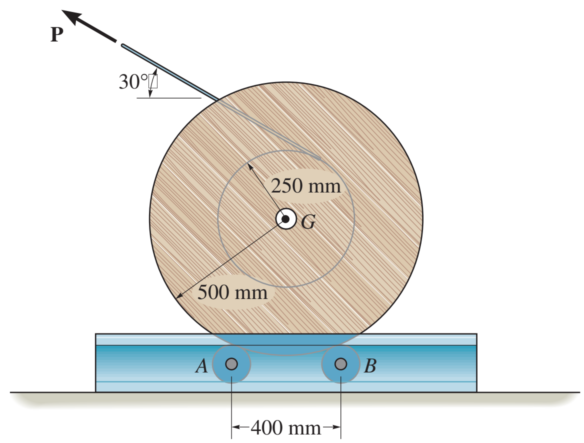


(a)

Ans:
$\omega = 3.96\ \text{rad/s}$

18–5.

A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.

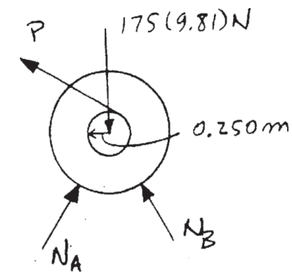


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 20(2)(2\pi)(0.250) = \frac{1}{2}\left[175(0.42)^2\right]\omega^2$$

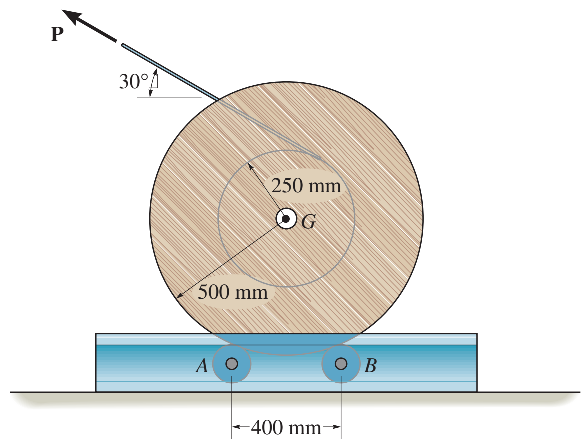
$$\omega = 2.02 \text{ rad/s}$$ **Ans.**



Ans:
$\omega = 2.02$ rad/s

18–6.

A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn without slipping on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the cable. Each roller can be considered as an 18-kg cylinder, having a radius of 0.1 m. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.



SOLUTION

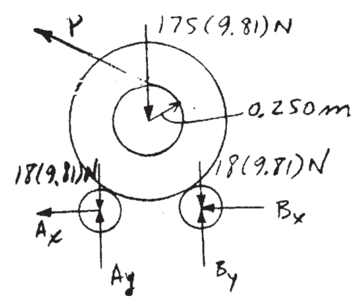
System:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$[0 + 0 + 0] + 20(2)(2\pi)(0.250) = \frac{1}{2}\left[175(0.42)^2\right]\omega^2 + 2\left[\frac{1}{2}(18)(0.1)^2\right]\omega_r^2$$

$$v = \omega_r(0.1) = \omega(0.5)$$

$$\omega_r = 5\omega$$



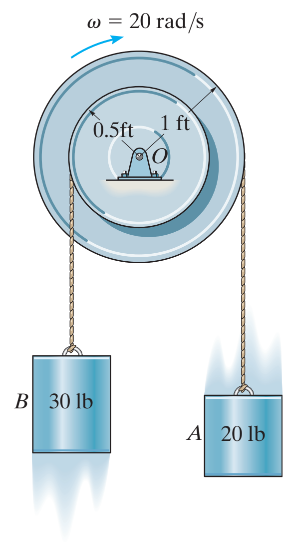
Solving:

$\omega = 1.78$ rad/s **Ans.**

Ans:
$\omega = 1.78$ rad/s

18–7.

The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a radius of gyration about its center of $k_O = 0.6$ ft and is turning with an angular velocity of 20 rad/s clockwise. Determine the kinetic energy of the system. Assume that neither cable slips on the pulley.



SOLUTION

$$T = \frac{1}{2} I_O\,\omega_O^2 + \frac{1}{2} m_A\,v_A^2 + \frac{1}{2} m_B\,v_B^2$$

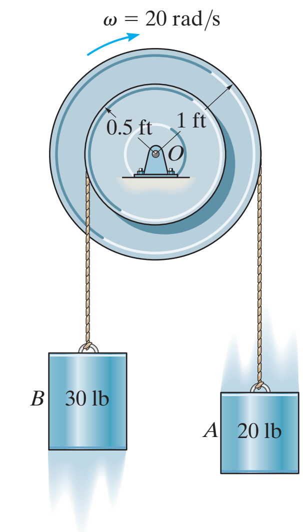
$$T = \frac{1}{2}\left(\frac{50}{32.2}(0.6)^2\right)(20)^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)\left[(20)(1)\right]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)\left[(20)(0.5)\right]^2$$

$$= 283 \text{ ft}\cdot\text{lb}$$ **Ans.**

Ans:
$T = 283$ ft · lb

***18–8.**

The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a centroidal radius of gyration of $k_O = 0.6$ ft and is turning with an angular velocity of 20 rad/s clockwise. Determine the angular velocity of the pulley at the instant the 20-lb weight moves 2 ft downward.



SOLUTION

Kinetic Energy and Work: Since the pulley rotates about a fixed axis, $v_A = \omega r_A = \omega(1)$ and $v_B = \omega r_B = \omega(0.5)$. The mass moment of inertia of the pulley about point O is $I_O = mk_O{}^2 = \left(\frac{50}{32.2}\right)(0.6^2) = 0.5590 \text{ slug} \cdot \text{ft}^2$. Thus, the kinetic energy of the system is

$$T = \frac{1}{2} I_O \omega^2 + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B{}^2$$

$$= \frac{1}{2}(0.5590)\omega^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)[\omega(1)]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)[\omega(0.5)]^2$$

$$= 0.7065\omega^2$$

Thus, $T_1 = 0.7065(20^2) = 282.61 \text{ ft} \cdot \text{lb}$. Referring to the FBD of the system shown in Fig. *a*, we notice that $\mathbf{O}_x$, $\mathbf{O}_y$, and $\mathbf{W}_p$ do no work while $\mathbf{W}_A$ does positive work and $\mathbf{W}_B$ does negative work. When A moves 2 ft downward, the pulley rotates

$$\theta = \frac{S_A}{r_A} = \frac{S_B}{r_B}$$

$$\frac{2}{1} = \frac{S_B}{0.5}$$

$$S_B = 2(0.5) = 1 \text{ ft} \uparrow$$

Thus, the work of $\mathbf{W}_A$ and $\mathbf{W}_B$ are

$$U_{W_A} = W_A S_A = 20(2) = 40 \text{ ft} \cdot \text{lb}$$

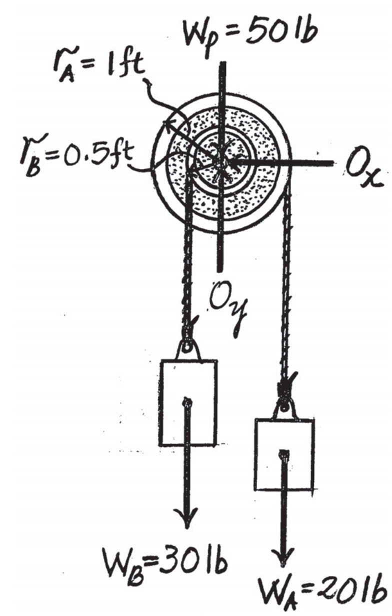
$$U_{W_B} = -W_B S_B = -30(1) = -30 \text{ ft} \cdot \text{lb}$$

Principle of Work and Energy:

$$T_1 + U_{1-2} = T_2$$

$$282.61 + [40 + (-30)] = 0.7065\,\omega^2$$

$$\omega = 20.4 \text{ rad/s} \qquad \textbf{Ans.}$$

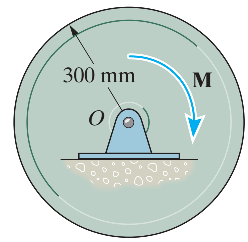


(a)

Ans:
$\omega = 20.4$ rad/s

18–9.

The disk, which has a mass of 20 kg, is subjected to the couple moment of $M = (2\theta + 4)\ \text{N}\cdot\text{m}$, where θ is in radians. If it starts from rest, determine its angular velocity when it has made two revolutions.



SOLUTION

Kinetic Energy. Since the disk starts from rest, $T_1 = 0$. The mass moment of inertia of the disk about its center O is $I_0 = \frac{1}{2}mr^2 = \frac{1}{2}(20)(0.3^2) = 0.9\ \text{kg}\cdot\text{m}^2$. Thus

$$T_2 = \frac{1}{2}I_0\omega^2 = \frac{1}{2}(0.9)\,\omega^2 = 0.45\,\omega^2$$

Work. Referring to the FBD of the disk, Fig. *a*, only couple moment **M** does work, which it is positive

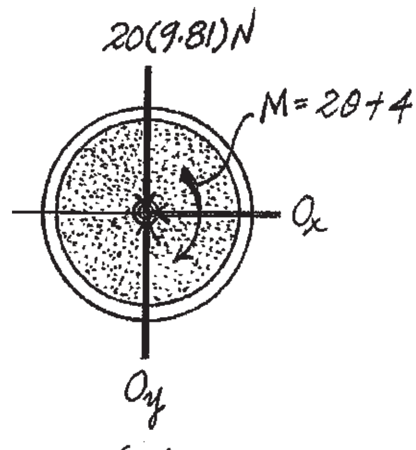
$$U_M = \int M\,d\theta = \int_0^{2(2\pi)} (2\theta + 4)d\theta = \theta^2 + 4\theta\Big|_0^{4\pi} = 208.18\ \text{J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 208.18 = 0.45\,\omega^2$$

$$\omega = 21.51\ \text{rad/s} = 21.5\ \text{rad/s}$$ **Ans.**

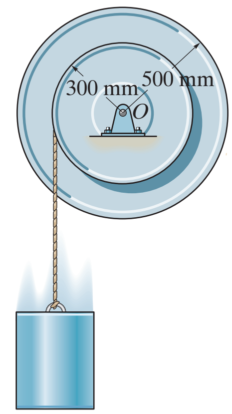


(a)

Ans:
$\omega = 21.5\ \text{rad/s}$

18–10.

The spool has a mass of 40 kg and a radius of gyration of $k_O = 0.3$ m. If the 10-kg block is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 15$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



SOLUTION

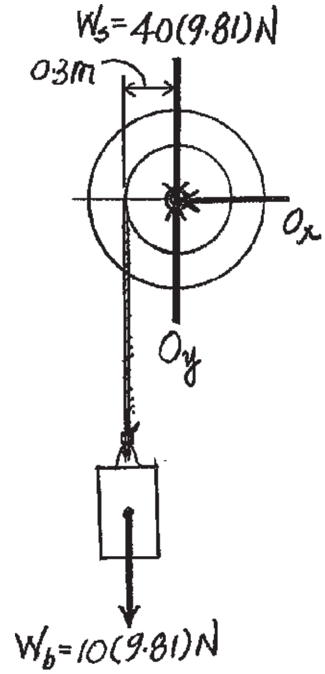
Kinetic Energy. Since the system is released from rest, $T_1 = 0$. The final velocity of the block is $v_b = \omega r = 15(0.3) = 4.50$ m/s. The mass moment of inertia of the spool about O is $I_0 = mk_0^2 = 40\left(0.3^2\right) = 3.60\ \text{Kg}\cdot\text{m}^2$. Thus

$$T_2 = \frac{1}{2}I_0\omega^2 + \frac{1}{2}m_b v_b^2$$

$$= \frac{1}{2}(3.60)\left(15^2\right) + \frac{1}{2}(10)\left(4.50^2\right)$$

$$= 506.25\ \text{J}$$

For the block, $T_1 = 0$ and $T_2 = \dfrac{1}{2}m_b v_b^2 = \dfrac{1}{2}\left(10\right)\left(4.50^2\right) = 101.25$ J



(a)

Work. Referring to the FBD of the system Fig. *a*, only $\mathbf{W}_b$ does work when the block displaces s vertically downward, which it is positive.

$$U_{W_b} = W_b s = 10(9.81)s = 98.1\ s$$

Referring to the FBD of the block, Fig. *b*. $\mathbf{W}_b$ does positive work while $\mathbf{T}$ does negative work.

$$U_T = -Ts$$

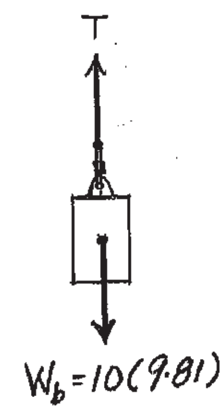
$$U_{W_b} = W_b s = 10(9.81)(s) = 98.1\ s$$

Principle of Work and Energy. For the system,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 98.1s = 506.25$$

$$s = 5.1606\ \text{m} = 5.16\ \text{m}$$ **Ans.**



(b)

For the block using the result of s,

$$T_1 + \Sigma U_{1-2} = T_2$$

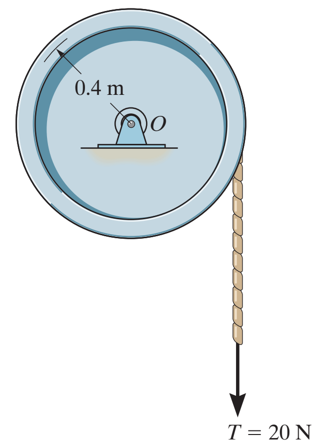
$$0 + 98.1(5.1606) - T(5.1606) = 101.25$$

$$T = 78.48\ \text{N} = 78.5\ \text{N}$$ **Ans.**

Ans:
$s = 5.16$ m
$T = 78.5$ N

18–11.

The force of $T = 20$ N is applied to the cord of negligible mass. Determine the angular velocity of the 20-kg wheel when it has rotated 4 revolutions starting from rest. The wheel has a radius of gyration of $k_O = 0.3$ m.



SOLUTION

Kinetic Energy. Since the wheel starts from rest, $T_1 = 0$. The mass moment of inertia of the wheel about point O is $I_0 = mk_0^2 = 20\left(0.3^2\right) = 1.80\ \text{kg}\cdot\text{m}^2$. Thus,

$$T_2 = \frac{1}{2} I_0 \omega^2 = \frac{1}{2}(1.80)\,\omega^2 = 0.9\,\omega^2$$

Work. Referring to the FBD of the wheel, Fig. *a*, only force **T** does work. This work is positive since **T** is required to displace vertically downward, $s_T = \theta r = 4(2\pi)(0.4) = 3.2\pi$ m.

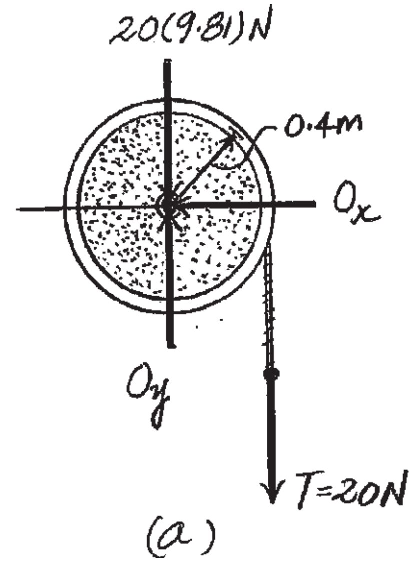
$$U_T = Ts_T = 20(3.2\pi) = 64\pi\ \text{J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 64\pi = 0.9\,\omega^2$$

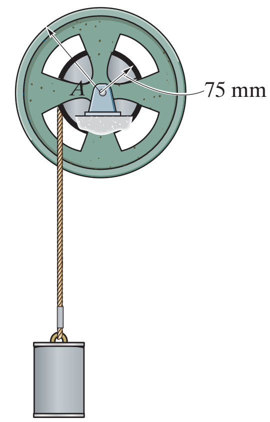
$$\omega = 14.94\ \text{rad/s} = 14.9\ \text{rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega = 14.9$ rad/s

***18–12.**

Determine the velocity of the 50-kg cylinder after it has descended a distance of 2 m. Initially, the system is at rest. The reel has a mass of 25 kg and a radius of gyration about its center of mass A of $k_A = 125$ mm.

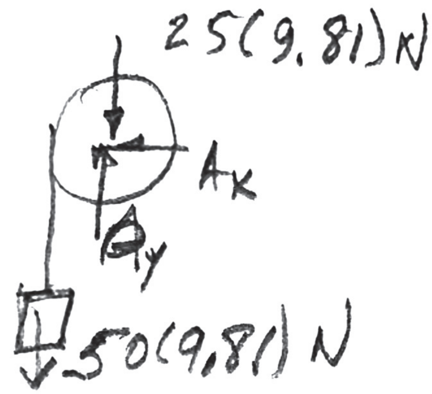


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 50(9.81)(2) = \frac{1}{2}[(25)(0.125)^2]\left(\frac{v}{0.075}\right)^2 + \frac{1}{2}(50)\,v^2$$

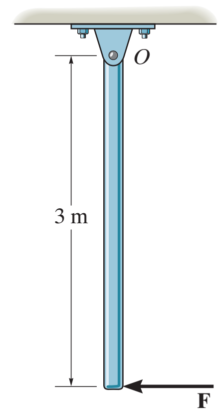
$$v = 4.05 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 4.05$ m/s

18–13.

The 10-kg uniform slender rod is suspended at rest when the force of $F = 150$ N is applied to its end. Determine the angular velocity of the rod when it has rotated 90° clockwise from the position shown. The force is always perpendicular to the rod.



SOLUTION

Kinetic Energy. Since the rod starts from rest, $T_1 = 0$. The mass moment of inertia of the rod about O *is* $I_0 = \frac{1}{12}(10)(3^2) + 10(1.5^2) = 30.0 \text{ kg} \cdot \text{m}^2$. Thus,

$$T_2 = \frac{1}{2} I_0 \omega^2 = \frac{1}{2}(30.0)\,\omega^2 = 15.0\,\omega^2$$

Work. Referring to the FBD of the rod, Fig. *a*, when the rod undergoes an angular displacement θ, force **F** does positive work whereas **W** does negative work. When $\theta = 90°$, $S_W = 1.5$ m and $S_F = \theta r = \left(\frac{\pi}{2}\right)(3) = \frac{3\pi}{2}$ m. Thus

$$U_F = 150\left(\frac{3\pi}{2}\right) = 225\pi \text{ J}$$

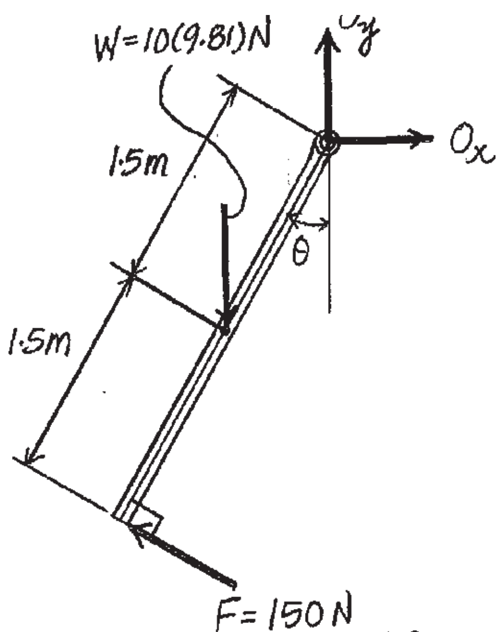
$$U_W = -10(9.81)(1.5) = -147.15 \text{ J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 225\pi + (-147.15) = 15.0\,\omega^2$$

$$\omega = 6.1085 \text{ rad/s} = 6.11 \text{ rad/s}$$ **Ans.**

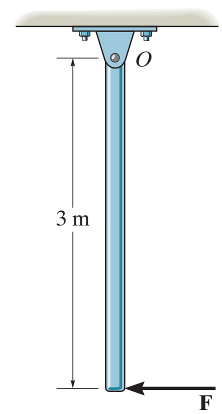


(a)

Ans:
$\omega = 6.11$ rad/s

18–14.

The 10-kg uniform slender rod is suspended at rest when the force of $F = 150$ N is applied to its end. Determine the angular velocity of the rod when it has rotated 180° clockwise from the position shown. The force is always perpendicular to the rod.



SOLUTION

Kinetic Energy. Since the rod starts from rest, $T_1 = 0$. The mass moment of inertia of the rod about O is $I_0 = \frac{1}{12}(10)(3^2) + 10(1.5^2) = 30.0\ \text{kg}\cdot\text{m}^2$. Thus,

$$T_2 = \frac{1}{2}I_0\omega^2 = \frac{1}{2}(30.0)\,\omega^2 = 15.0\,\omega^2$$

Work. Referring to the FBD of the rod, Fig. a, when the rod undergoes an angular displacement θ, force **F** does positive work whereas **W** does negative work. When $\theta = 180°$, $S_W = 3$ m and $S_F = \theta r = \pi(3) = 3\pi$ m. Thus

$$U_F = 150(3\pi) = 450\pi\ \text{J}$$

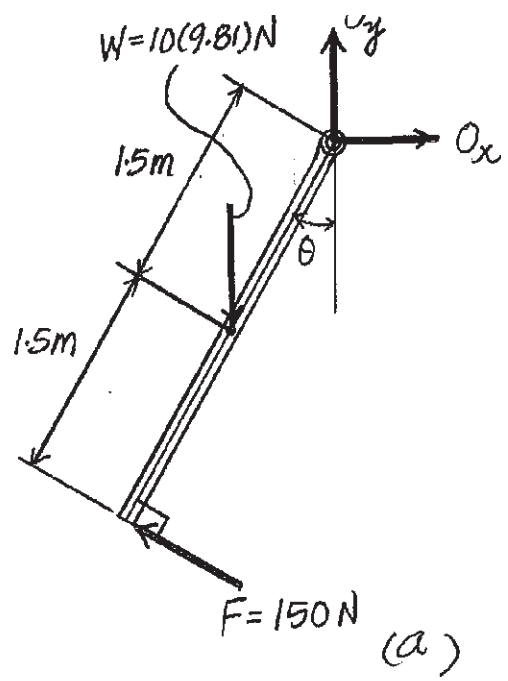
$$U_W = -10(9.81)(3) = -294.3\ \text{J}$$

Principle of Work and Energy. Applying Eq. 18,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 450\pi + (-294.3) = 15.0\,\omega^2$$

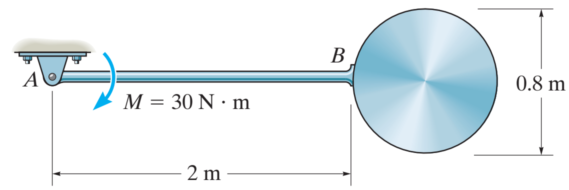
$$\omega = 8.6387\ \text{rad/s} = 8.64\ \text{rad/s}$$ **Ans.**



Ans:
$\omega = 8.64$ rad/s

18–15.

The pendulum consists of a 10-kg uniform disk and a 3-kg uniform slender rod. If it is released from rest in the position shown, determine its angular velocity when it rotates clockwise 90°.



SOLUTION

Kinetic Energy. Since the assembly is released from rest, initially, $T_1 = 0$. The mass moment of inertia of the assembly about A is

$$I_A = \left[\frac{1}{12}(3)(2^2) + 3(1^2)\right] + \left[\frac{1}{2}(10)(0.4^2) + 10(2.4^2)\right] = 62.4 \text{ kg} \cdot \text{m}^2. \text{Thus,}$$

$$T_2 = \frac{1}{2}I_A\omega^2 = \frac{1}{2}(62.4)\,\omega^2 = 31.2\,\omega^2$$

Work. Referring to the FBD of the assembly, Fig. *a*. Both $\mathbf{W}_r$ and $\mathbf{W}_d$ do positive work, since they displace vertically downward $S_r = 1$ m and $S_d = 2.4$ m, respectively. Also, couple moment $\mathbf{M}$ does positive work

$$U_{W_r} = W_r S_r = 3(9.81)(1) = 29.43 \text{ J}$$

$$U_{W_d} = W_d S_d = 10(9.81)(2.4) = 235.44 \text{ J}$$

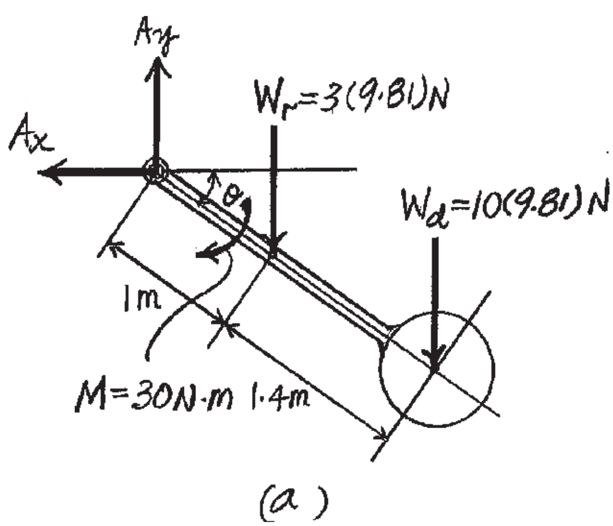
$$U_M = M\theta = 30\left(\frac{\pi}{2}\right) = 15\pi \text{ J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 29.43 + 235.44 + 15\pi = 31.2\,\omega^2$$

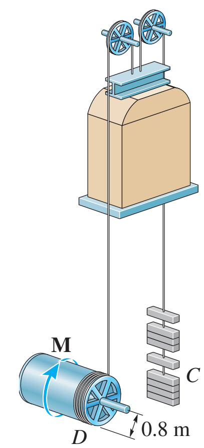
$$\omega = 3.1622 \text{ rad/s} = 3.16 \text{ rad/s}$$ **Ans.**



Ans:
$\omega = 3.16$ rad/s

***18–16.**

A motor supplies a constant torque $M = 6\ \text{kN}\cdot\text{m}$ to the winding drum that operates the elevator. If the elevator has a mass of 900 kg, the counterweight C has a mass of 200 kg, and the winding drum has a mass of 600 kg and radius of gyration about its axis of $k = 0.6$ m, determine the speed of the elevator after it rises 5 m starting from rest. Neglect the mass of the pulleys.



SOLUTION

$$v_E = v_C$$

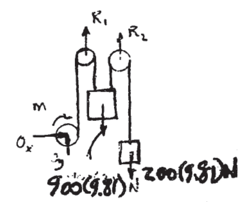
$$\theta = \frac{s}{r} = \frac{5}{0.8}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 6000\left(\frac{5}{0.8}\right) - 900(9.81)(5) + 200(9.81)(5) = \frac{1}{2}(900)(v)^2 + \frac{1}{2}(200)(v)^2$$

$$+ \frac{1}{2}\left[600(0.6)^2\right]\left(\frac{v}{0.8}\right)^2$$

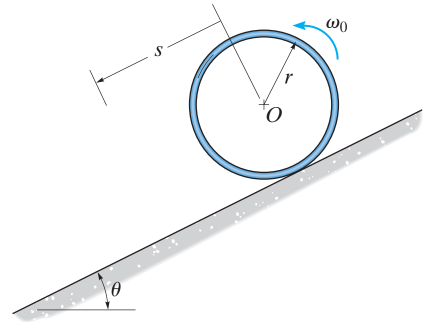
$$v = 2.10\ \text{m/s} \qquad \textbf{Ans.}$$



Ans:
$v = 2.10\ \text{m/s}$

18–17.

The center O of the thin ring of mass m is given an angular velocity of ω_0. If the ring rolls without slipping, determine its angular velocity after it has traveled a distance of s down the plane. Neglect its thickness.

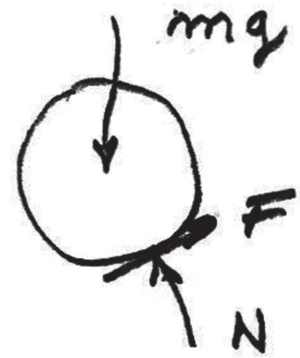


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(mr^2 + mr^2)\omega_0{}^2 + mg(s \sin\theta) = \frac{1}{2}(mr^2 + mr^2)\omega^2$$

$$\omega = \sqrt{\omega_0{}^2 + \frac{g}{r^2}s \sin\theta}$$ **Ans.**

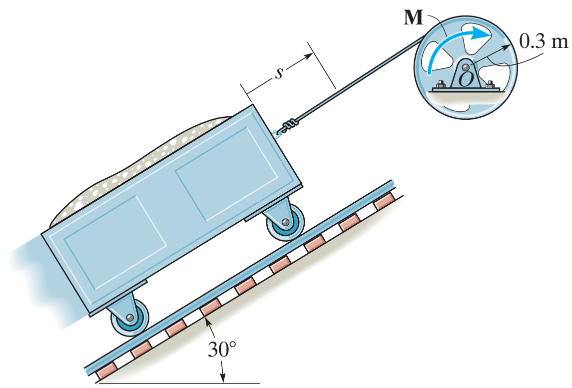


Ans:

$$\omega = \sqrt{\omega_0^2 + \frac{g}{r^2}s \sin\theta}$$

18–18.

The wheel has a mass of 100 kg and a radius of gyration of $k_O = 0.2$ m. A motor supplies a torque $M = (40\theta + 900)$ N·m, where θ is in radians, about the drive shaft at O. Determine the speed of the loading car, which has a mass of 300 kg, after it travels $s = 4$ m. Initially the car is at rest when $s = 0$ and $\theta = 0°$. Neglect the mass of the attached cable and the mass of the car's wheels.



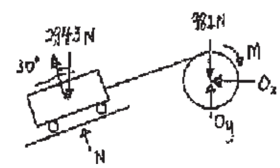
SOLUTION

$s = 0.3\theta = 4$

$\theta = 13.33$ rad

$T_1 + \Sigma U_{1-2} = T_2$

$$[0 + 0] + \int_0^{13.33} (40\theta + 900)d\theta - 300(9.81)\sin 30°\,(4) = \frac{1}{2}(300)v_C^2 + \frac{1}{2}\left[100(0.20)^2\right]\left(\frac{v_C}{0.3}\right)^2$$

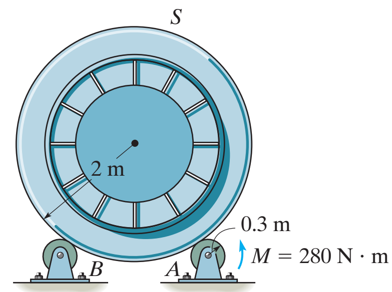


$v_C = 7.49$ m/s **Ans.**

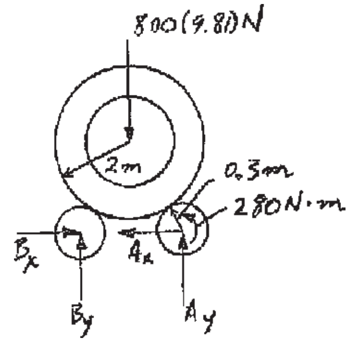
Ans:
$v_C = 7.49$ m/s

18–19.

The rotary screen S is used to wash limestone. When empty it has a mass of 800 kg and a radius of gyration of $k_G = 1.75$ m. Rotation is achieved by applying a torque of $M = 280\ \text{N}\cdot\text{m}$ about the drive wheel at A. If no slipping occurs at A and the supporting wheel at B is free to roll, determine the angular velocity of the screen after it has rotated 5 revolutions. Neglect the mass of A and B.



SOLUTION



$$T_S + \Sigma U_{1-2} = T_2$$

$$0 + 280(\theta_A) = \frac{1}{2}[800(1.75)^2]\,\omega^2$$

$$\theta_S(2) = \theta_A(0.3)$$

$$5(2\pi)(2) = \theta_A(0.3)$$

$$\theta_A = 209.4 \text{ rad}$$

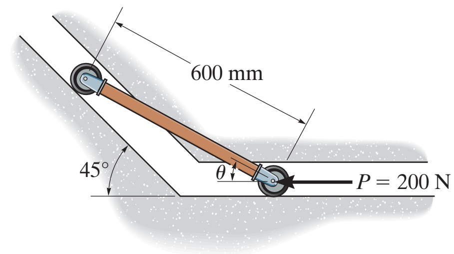
Thus

$$\omega = 6.92 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 6.92$ rad/s

***18–20.**

If $P = 200$ N and the 15-kg uniform slender rod starts from rest at $\theta = 0°$, determine the rod's angular velocity at the instant just before $\theta = 45°$.



SOLUTION

Kinetic Energy and Work: Referring to Fig. *a*,

$$r_{A/IC} = 0.6 \tan 45° = 0.6 \text{ m}$$

Then

$$r_{G/IC} = \sqrt{0.3^2 + 0.6^2} = 0.6708 \text{ m}$$

Thus,

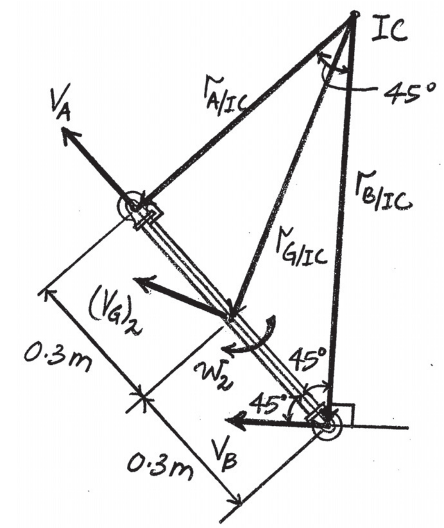
$$(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(0.6708)$$

The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(15)(0.6^2) = 0.45 \text{ kg} \cdot \text{m}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2^2 + \frac{1}{2}I_G\,\omega_2^2$$

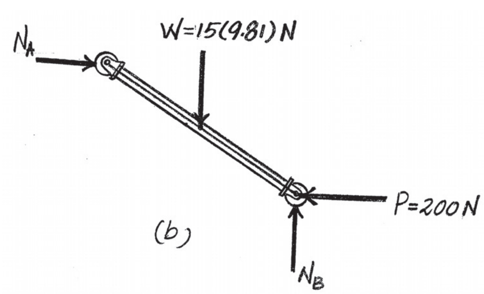
$$= \frac{1}{2}(15)[w_2(0.6708)]^2 + \frac{1}{2}(0.45)\,\omega_2^2$$

$$= 3.6\omega_2^2$$



(a)

Since the rod is initially at rest, $T_1 = 0$. Referring to Fig. *b*, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while **P** does positive work and **W** does negative work. When $\theta = 45°$, **P** displaces through a horizontal distance $s_P = 0.6$ m and W displaces vertically upwards through a distance of $h = 0.3 \sin 45°$, Fig. *c*. Thus, the work done by **P** and **W** is



$$U_P = Ps_P = 200(0.6) = 120 \text{ J}$$

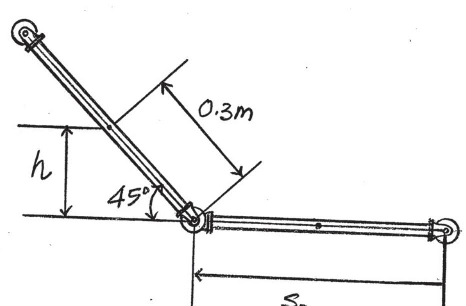
$$U_W = -Wh = -15(9.81)(0.3 \sin 45°) = -31.22 \text{ J}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [120 - 31.22] = 3.6\omega_2^2$$

$$\omega_2 = 4.97 \text{ rad/s} \qquad \textbf{Ans.}$$

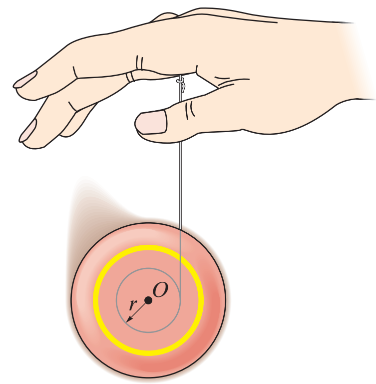


(c)

Ans:
$\omega_2 = 4.97$ rad/s

18–21.

A yo-yo has a weight of 0.3 lb and a radius of gyration $k_O = 0.06$ ft. If it is released from rest, determine how far it must descend in order to attain an angular velocity $\omega = 70$ rad/s. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is $r = 0.02$ ft.



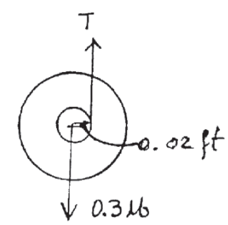
SOLUTION

$v_G = (0.02)70 = 1.40$ ft/s

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + (0.3)(s) = \frac{1}{2}\left(\frac{0.3}{32.2}\right)(1.40)^2 + \frac{1}{2}\left[(0.06)^2\left(\frac{0.3}{32.2}\right)\right](70)^2$$

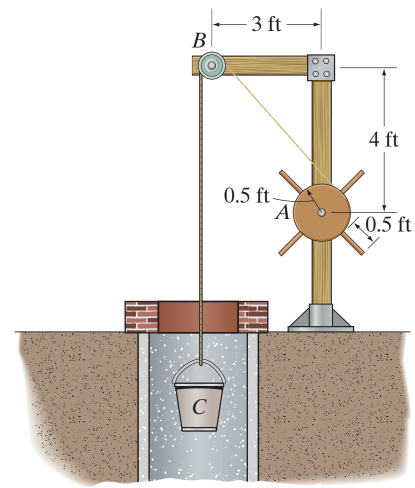
$s = 0.304$ ft **Ans.**



Ans:
$s = 0.304$ ft

18–22.

If the 50-lb bucket is released from rest, determine its velocity after it has fallen a distance of 10 ft. The windlass A can be considered as a 30-lb cylinder, while the spokes are slender rods, each having a weight of 2 lb. Neglect the pulley's weight.



SOLUTION

Kinetic Energy and Work: Since the windlass rotates about a fixed axis, $v_C = \omega_A r_A$ or $\omega_A = \dfrac{v_C}{r_A} = \dfrac{v_C}{0.5} = 2v_C$. The mass moment of inertia of the windlass about its mass center is

$$I_A = \frac{1}{2}\left(\frac{30}{32.2}\right)(0.5^2) + 4\left[\frac{1}{12}\left(\frac{2}{32.2}\right)(0.5^2) + \frac{2}{32.2}(0.75^2)\right] = 0.2614 \text{ slug}\cdot\text{ft}^2$$

Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

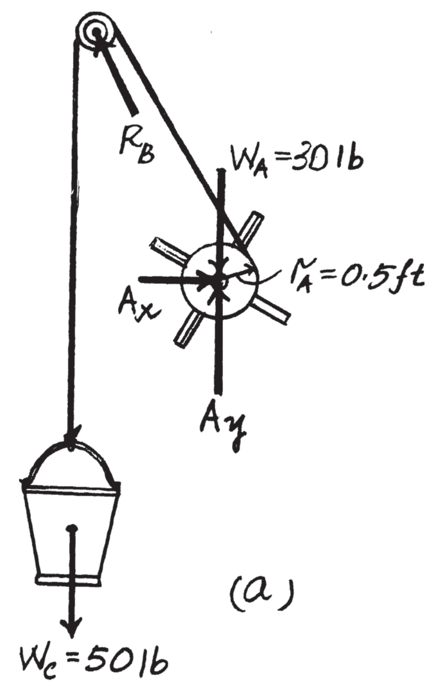
$$= \frac{1}{2}I_A\omega^2 + \frac{1}{2}m_C v_C{}^2$$

$$= \frac{1}{2}(0.2614)(2v_C)^2 + \frac{1}{2}\left(\frac{50}{32.2}\right)v_C{}^2$$

$$= 1.2992v_C{}^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. a, $\mathbf{W}_A$, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{R}_B$ do no work, while $\mathbf{W}_C$ does positive work. Thus, the work done by $\mathbf{W}_C$, when it displaces vertically downward through a distance of $s_C = 10$ ft, is

$$U_{W_C} = W_C s_C = 50(10) = 500 \text{ ft}\cdot\text{lb}$$



Principle of Work and Energy:

$$T_1 + \Sigma U_{1\text{–}2} = T_2$$

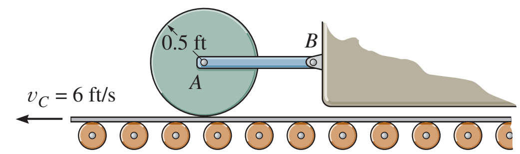
$$0 + 500 = 1.2992v_C{}^2$$

$$v_C = 19.6 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v_C = 19.6$ ft/s

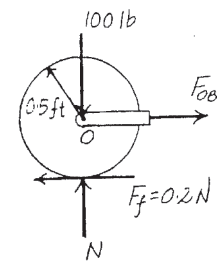
18–23.

The coefficient of kinetic friction between the 100-lb disk and the surface of the conveyor belt is $\mu_A = 0.2$. If the conveyor belt is moving with a speed of $v_C = 6$ ft/s when the disk is placed in contact with it, determine the number of revolutions the disk makes before it reaches a constant angular velocity.



SOLUTION

Equation of Motion: In order to obtain the friction developed at point A of the disk, the normal reaction N_A must be determine first.



$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - 100 = 0 \qquad N = 100 \text{ lb}$$

Work: The friction $F_f = \mu_k N = 0.2(100) = 20.0$ lb develops a constant couple moment of $M = 20.0(0.5) = 10.0$ lb · ft about point O when the disk is brought in contact with the conveyor belt. This couple moment does *positive* work of $U = 10.0(\theta)$ when the disk undergoes an angular displacement θ. The normal reaction N, force F_{OB} and the weight of the disk do no work since point O does not displace.

Principle of Work and Energy: The disk achieves a constant angular velocity when the points on the rim of the disk reach the speed of that of the conveyor belt, i.e; $v_C = 6$ ft/s. This constant angular velocity is given by $\omega = \dfrac{v_C}{r} = \dfrac{6}{0.5} = 12.0$ rad/s. The mass moment inertia of the disk about point O is $I_O = \dfrac{1}{2}mr^2 = \dfrac{1}{2}\left(\dfrac{100}{32.2}\right)(0.5^2) = 0.3882 \text{ slug}\cdot\text{ft}^2$. Applying Eq.18–13, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + U = \frac{1}{2} I_O \omega^2$$

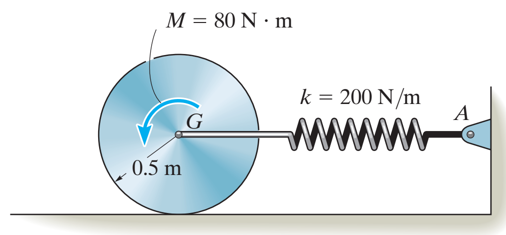
$$0 + 10.0\theta = \frac{1}{2}(0.3882)\left(12.0^2\right)$$

$$\theta = 2.80 \text{ rad} \times \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 0.445 \text{ rev} \qquad \textbf{Ans.}$$

Ans:
$\theta = 0.445$ rev

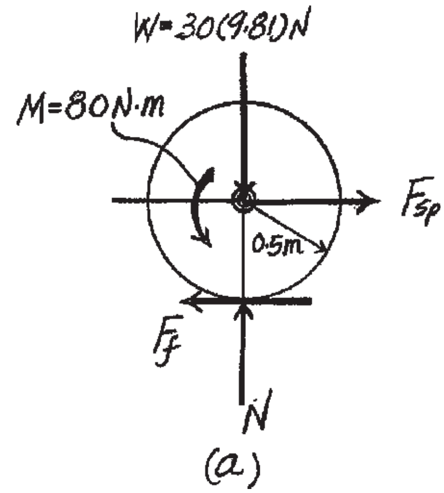
***18–24.**

The 30-kg disk is originally at rest, and the spring is unstretched. A couple moment of $M = 80\text{ N}\cdot\text{m}$ is then applied to the disk as shown. Determine its angular velocity when its mass center G has moved 0.5 m along the plane. The disk rolls without slipping.



SOLUTION

Kinetic Energy. Since the disk is at rest initially, $T_1 = 0$. The disk rolls without slipping. Thus, $v_G = \omega_r = \omega(0.5)$. The mass moment of inertia of the disk about its center of gravity G is $I_G = \frac{1}{2}mr = \frac{1}{2}(30)(0.5^2) = 3.75\text{ kg}\cdot\text{m}^2$. Thus,



$$T_2 = \frac{1}{2}I_G\,\omega^2 + \frac{1}{2}Mv_G^2$$

$$= \frac{1}{2}(3.75)\omega^2 + \frac{1}{2}(30)[\omega(0.5)]^2$$

$$= 5.625\,\omega^2$$

Work. Since the disk rolls without slipping, the friction $\mathbf{F}_f$ does no work. Also when the center of the disk moves $S_G = 0.5$ m, the disk rotates $\theta = \frac{s_G}{r} = \frac{0.5}{0.5} = 1.00$ rad. Here, couple moment $\mathbf{M}$ does positive work whereas the spring force does negative work.

$$U_M = M\theta = 80(1.00) = 80.0\text{ J}$$

$$U_{F_{sp}} = -\frac{1}{2}kx^2 = -\frac{1}{2}(200)(0.5^2) = -25.0\text{ J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

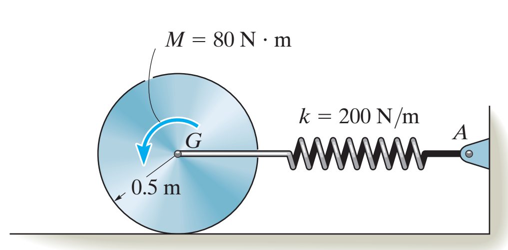
$$0 + 80 + (-25.0) = 5.625\,\omega^2$$

$$\omega = 3.127\text{ rad/s} = 3.13\text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 3.13\text{ rad/s}$

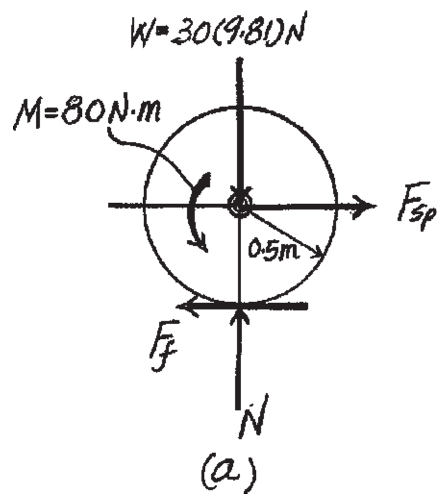
18–25.

The 30-kg disk is originally at rest, and the spring is unstretched. A couple moment $M = 80\text{ N}\cdot\text{m}$ is then applied to the disk as shown. Determine how far the center of mass of the disk travels along the plane before it momentarily stops. The disk rolls without slipping.



SOLUTION

Kinetic Energy. Since the disk is at rest initially and required to stop finally, $T_1 = T_2 = 0$.



Work. Since the disk rolls without slipping, the friction $\mathbf{F}_f$ does no work. Also, when the center of the disk moves s_G, the disk rotates $\theta = \frac{s_G}{r} = \frac{s_G}{0.5} = 2\,s_G$. Here, couple moment $\mathbf{M}$ does positive work whereas the spring force does negative work.

$$U_M = M\theta = 80(2s_G) = 160\,s_G$$

$$U_{F_{sp}} = -\frac{1}{2}kx^2 = -\frac{1}{2}(200)\,s_G^2 = -100\,s_G^2$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 160\,s_G + \left(-100\,s_G^2\right) = 0$$

$$160\,s_G - 100\,s_G^2 = 0$$

$$s_G(160 - 100\,s_G) = 0$$

Since $s_G \neq 0$, then

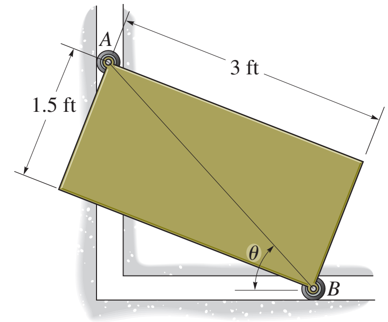
$$160 - 100\,s_G = 0$$

$$s_G = 1.60\text{ m} \qquad \textbf{Ans.}$$

Ans:
$s_G = 1.60\text{ m}$

18–26.

Two wheels of negligible weight are mounted at corners A and B of the rectangular 75-lb plate. If the plate is released from rest at $\theta = 90°$, determine its angular velocity at the instant just before $\theta = 0°$.



SOLUTION

Kinetic Energy and Work: Referring Fig. a,

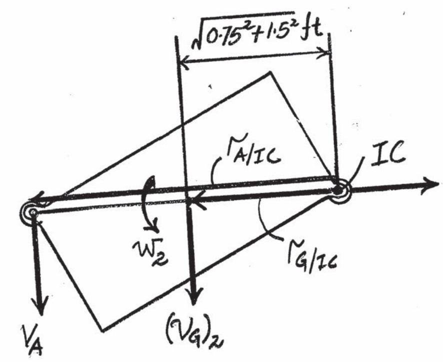
$$(v_G)_2 = \omega r_{A/IC} = \omega\left(\sqrt{0.75^2 + 1.5^2}\right) = 1.677\omega^2$$

The mass moment of inertia of the plate about its mass center is $I_G = \frac{1}{12}m(a^2 + b^2) = \frac{1}{12}\left(\frac{75}{32.2}\right)(1.5^2 + 3^2) = 2.1836\text{ slug}\cdot\text{ft}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}\omega_2{}^2$$

$$= \frac{1}{2}\left(\frac{75}{32.2}\right)(1.677\omega_2)^2 + \frac{1}{2}I_G(2.1836)\omega_2{}^2$$

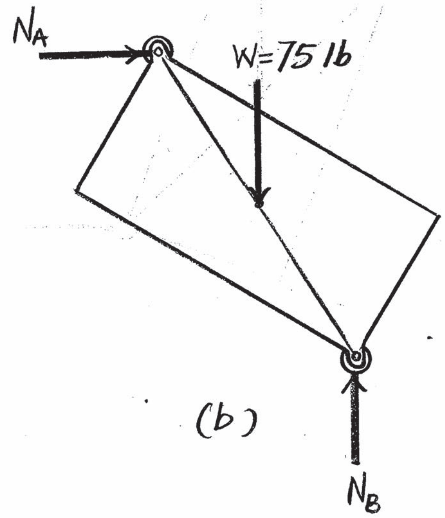
$$= 4.3672\omega_2{}^2$$



(a)

Since the plate is initially at rest, $T_1 = 0$. Referring to Fig. b, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while $\mathbf{W}$ does positive work. When $\theta = 0°$, $\mathbf{W}$ displaces vertically through a distance of $h = \sqrt{0.75^2 + 1.5^2} = 1.677$ ft, Fig. c. Thus, the work done by $\mathbf{W}$ is

$$U_W = Wh = 75(1.677) = 125.78\text{ ft}\cdot\text{lb}$$



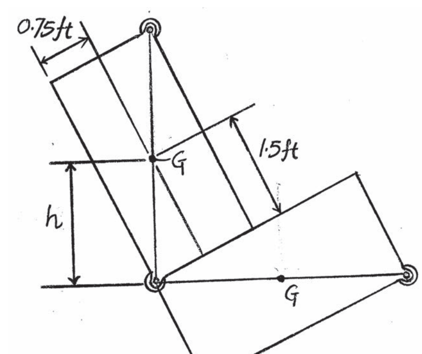
(b)

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 125.78 = 4.3672\omega_2{}^2$$

$$\omega_2 = 5.37\text{ rad/s} \qquad \textbf{Ans.}$$

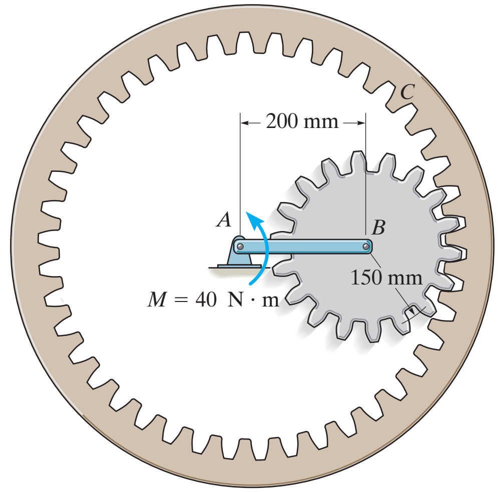


(c)

Ans:
$\omega_2 = 5.37$ rad/s

18–27.

The link AB is subjected to a couple moment of $M = 40\ \text{N} \cdot \text{m}$. If the ring gear C is fixed, determine the angular velocity of the 15-kg inner gear when the link has made two revolutions starting from rest. Neglect the mass of the link and assume the inner gear is a disk. Motion occurs in the vertical plane.



SOLUTION

Kinetic Energy. The mass moment of inertia of the inner gear about its center B is $I_B = \frac{1}{2}mr^2 = \frac{1}{2}(15)(0.15^2) = 0.16875\ \text{kg} \cdot \text{m}^2$. Referring to the kinematics diagram of the gear, the velocity of center B of the gear can be related to the gear's angular velocity, which is

$$v_B = \omega r_{B/IC}; \qquad v_B = \omega(0.15)$$

Thus,

$$T = \frac{1}{2}I_B\omega^2 + \frac{1}{2}Mv_G^2$$

$$= \frac{1}{2}(0.16875)\,\omega^2 + \frac{1}{2}(15)[\omega(0.15)]^2$$

$$= 0.253125\,\omega^2$$

Since the gear starts from rest, $T_1 = 0$.

Work. Referring to the FBD of the gear system, we notice that **M** does positive work whereas **W** does no work, since the gear returns to its initial position after the link completes two revolutions.

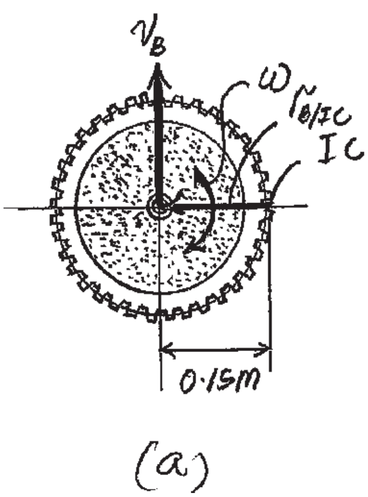
$$U_M = M_\theta = 40[2(2\pi)] = 160\pi\ \text{J}$$

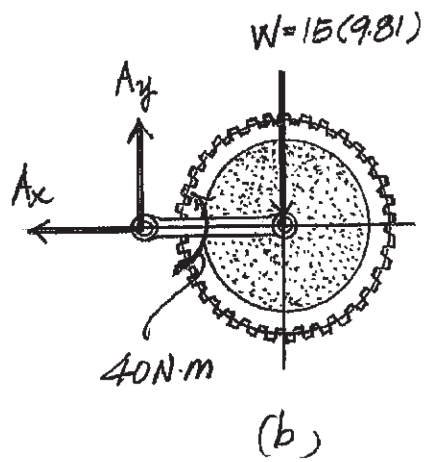
Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 160\pi = 0.253125\,\omega^2$$

$$\omega = 44.56\ \text{rad/s} = 44.6\ \text{rad/s} \qquad \textbf{Ans.}$$

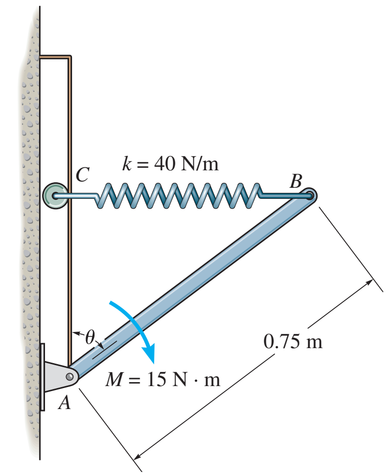




Ans:
$\omega = 44.6\ \text{rad/s}$

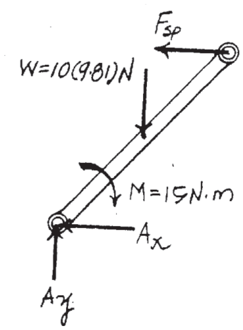
***18–28.**

The 10-kg rod AB is pin-connected at A and subjected to a couple moment of $M = 15$ N·m. If the rod is released from rest when the spring is unstretched at $\theta = 30°$, determine the rod's angular velocity at the instant $\theta = 60°$. As the rod rotates, the spring always remains horizontal, because of the roller support at C.



SOLUTION

***Free Body Diagram*:** The spring force F_{sP} does *negative* work since it acts in the opposite direction to that of its displacement s_{sp}, whereas the weight of the cylinder acts in the same direction of its displacement s_w and hence does *positive* work. Also, the couple moment **M** does positive work as it acts in the same direction of its angular displacement θ. The reactions A_x and A_y do no work since point A does not displace. Here, $s_{sp} = 0.75 \sin 60° - 0.75 \sin 30° = 0.2745$ m and $s_W = 0.375 \cos 30° - 0.375 \cos 60° = 0.1373$ m.



Principle of Work and Energy: The mass moment of inertia of the cylinder about point A is $I_A = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}(10)\left(0.75^2\right) + 10\left(0.375^2\right) = 1.875\ \text{kg}\cdot\text{m}^2$. Applying Eq.18–13, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + Ws_W + M\theta - \frac{1}{2}ks_P^2 = \frac{1}{2}I_A\,\omega^2$$

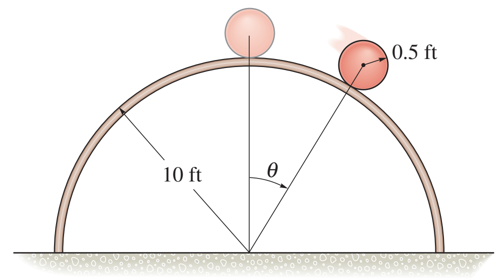
$$0 + 10(9.81)(0.1373) + 15\left(\frac{\pi}{3} - \frac{\pi}{6}\right) - \frac{1}{2}(40)\left(0.2745^2\right) = \frac{1}{2}(1.875)\,\omega^2$$

$$\omega = 4.60\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 4.60$ rad/s

18–29.

The 10-lb sphere starts from rest at $\theta = 0°$ and rolls without slipping down the cylindrical surface which has a radius of 10 ft. Determine the speed of the sphere's center of mass at the instant $\theta = 45°$.



SOLUTION

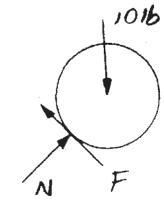
Kinematics:

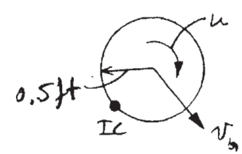
$v_G = 0.5\omega_G$

$T_1 + \Sigma U_{1-2} = T_2$

$$0 + 10(10.5)(1 - \cos 45°) = \frac{1}{2}\left(\frac{10}{32.2}\right)v_G^2 + \frac{1}{2}\left[\frac{2}{5}\left(\frac{10}{32.2}\right)(0.5)^2\right]\left(\frac{v_G}{0.5}\right)^2$$

$v_G = 11.9$ ft/s **Ans.**

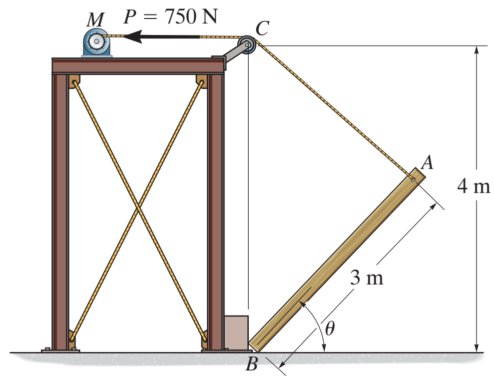




Ans:
$v_G = 11.9$ ft/s

18–30.

Motor M exerts a constant force of $P = 750$ N on the rope. If the 100-kg post is at rest when $\theta = 0°$, determine the angular velocity of the post at the instant $\theta = 60°$. Neglect the mass of the pulley and its size, and consider the post as a slender rod.



SOLUTION

Kinetic Energy and Work: Since the post rotates about a fixed axis, $v_G = \omega r_G = \omega(1.5)$. The mass moment of inertia of the post about its mass center is $I_G = \frac{1}{12}(100)(3^2) = 75\text{ kg}\cdot\text{m}^2$. Thus, the kinetic energy of the post is

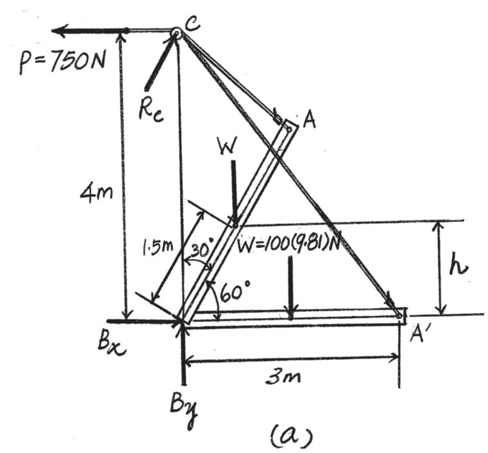
$$T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\omega^2$$

$$= \frac{1}{2}(100)[\omega(1.5)]^2 + \frac{1}{2}(75)\omega^2$$

$$= 150\omega^2$$

This result can also be obtained by applying $T = \frac{1}{2}I_B\omega^2$, where $I_B = \frac{1}{12}(100)(3^2) + 100(1.5^2) = 300\text{ kg}\cdot\text{m}^2$. Thus,

$$T = \frac{1}{2}I_B\omega^2 = \frac{1}{2}(300)\omega^2 = 150\omega^2$$



Since the post is initially at rest, $T_1 = 0$. Referring to Fig. *a*, $\mathbf{B}_x$, $\mathbf{B}_y$, and $\mathbf{R}_C$ do no work, while $\mathbf{P}$ does positive work and $\mathbf{W}$ does negative work. When $\theta = 60°$, $\mathbf{P}$ displaces $s_P = A'C - AC$, where $AC = \sqrt{4^2 + 3^2 - 2(4)(3)\cos 30°} = 2.053$ m and $A'C = \sqrt{4^2 + 3^2} = 5$ m. Thus, $s_P = 5 - 2.053 = 2.947$ m. Also, $\mathbf{W}$ displaces vertically upwards through a distance of $h = 1.5\sin 60° = 1.299$ m. Thus, the work done by $\mathbf{P}$ and $\mathbf{W}$ is

$$U_P = Ps_P = 750(2.947) = 2210.14\text{ J}$$

$$U_W = -Wh = -100(9.81)(1.299) = -1274.36\text{ J}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

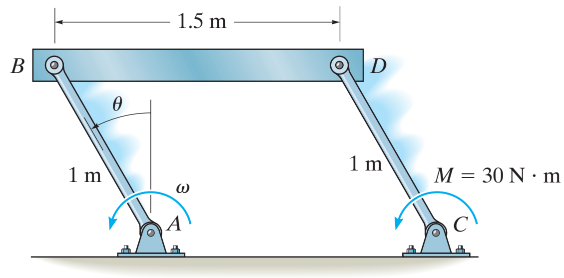
$$0 + [2210.14 - 1274.36] = 150\omega^2$$

$$\omega = 2.50\text{ rad/s}$$ **Ans.**

Ans:
$\omega = 2.50$ rad/s

18–31.

The linkage consists of two 6-kg rods AB and CD and a 20-kg bar BD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega = 2$ rad/s. If rod CD is subjected to a couple moment of $M = 30$ N·m, determine ω_{AB} at the instant $\theta = 90°$.



SOLUTION

Kinetic Energy. The mass moment of inertia of each link about the axis of rotation is $I_A = \frac{1}{12}(6)(1^2) + 6(0.5^2) = 2.00\ \text{kg}\cdot\text{m}$. The velocity of the center of mass of the bar is $v_G = \omega r = \omega(1)$. Thus,

$$T = 2\left(\frac{1}{2}I_A\omega^2\right) + \frac{1}{2}M_b v_G^2$$

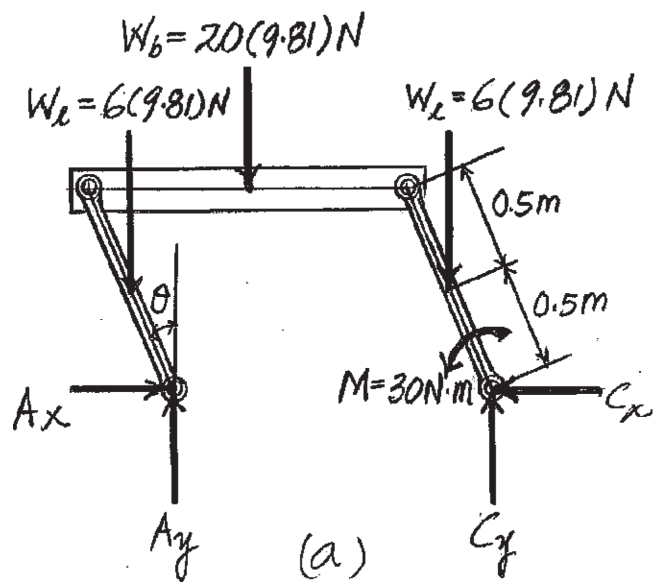
$$= 2\left[\frac{1}{2}(2.00)\omega^2\right] + \frac{1}{2}(20)[\omega(1)]^2$$

$$= 12.0\,\omega^2$$

Initially, $\omega = 2$ rad/s. Then

$$T_1 = 12.0(2^2) = 48.0\ \text{J}$$

Work. Referring to the FBD of the assembly, Fig. *a*, the weights W_b, W_c and couple moment $\mathbf{M}$ do positive work when the links undergo an angular displacement θ. When $\theta = 90^\circ = \frac{\pi}{2}$ rad,



$$U_{W_b} = W_b s_b = 20(9.81)(1) = 196.2\ \text{J}$$

$$U_{W_c} = W_c s_c = 6(9.81)(0.5) = 29.43\ \text{J}$$

$$U_M = M\theta = 30\left(\frac{\pi}{2}\right) = 15\pi\ \text{J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

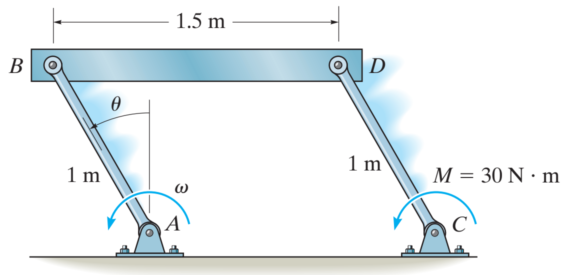
$$48.0 + [196.2 + 2(29.43) + 15\pi] = 12.0\,\omega^2$$

$$\omega = 5.4020\ \text{rad/s} = 5.40\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 5.40$ rad/s

***18–32.**

The linkage consists of two 6-kg rods AB and CD and a 20-kg bar BD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega = 2$ rad/s. If rod CD is subjected to a couple moment $M = 30$ N·m, determine ω at the instant $\theta = 45°$.



SOLUTION

Kinetic Energy. The mass moment of inertia of each link about the axis of rotation is $I_A = \frac{1}{12}(6)(1^2) + 6(0.5^2) = 2.00 \text{ kg}\cdot\text{m}^2$. The velocity of the center of mass of the bar is $v_G = \omega r = \omega(1)$. Thus,

$$T = 2\left(\frac{1}{2}I_A\omega_A^2\right)^2 + \frac{1}{2}m_b v_G^2$$

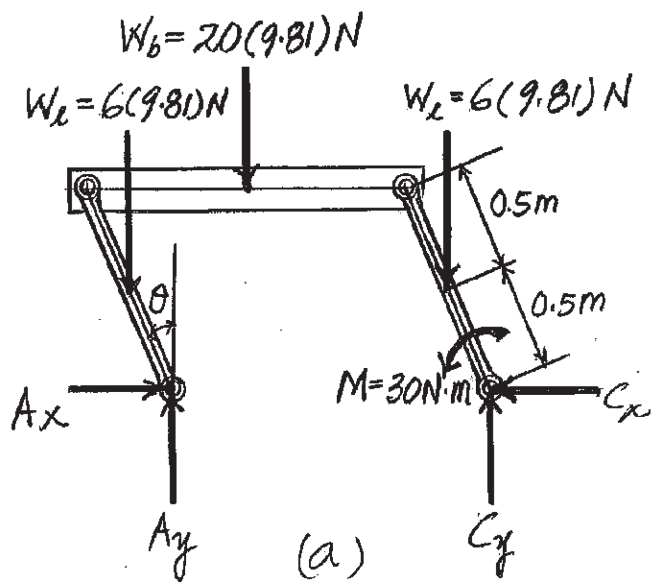
$$= 2\left[\frac{1}{2}(2.00)\omega^2\right] + \frac{1}{2}(20)[\omega(1)]^2$$

$$= 12.0\,\omega^2$$

Initially, $\omega = 2$ rad/s. Then

$$T_1 = 12.0(2^2) = 48.0 \text{ J}$$

Work. Referring to the FBD of the assembly, Fig. *a*, the weights W_b, W_c and couple moment $\mathbf{M}$ do positive work when the links undergo an angular displacement θ. when $\theta = 45^\circ = \frac{\pi}{4}$ rad,



$$U_{W_b} = W_b s_b = 20(9.81)(1 - \cos 45^\circ) = 57.47 \text{ J}$$

$$U_{W_c} = W_c s_c = 6(9.81)[0.5(1 - \cos 45°)] = 8.620 \text{ J}$$

$$U_M = M\theta = 30\left(\frac{\pi}{4}\right) = 7.5\pi \text{ J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

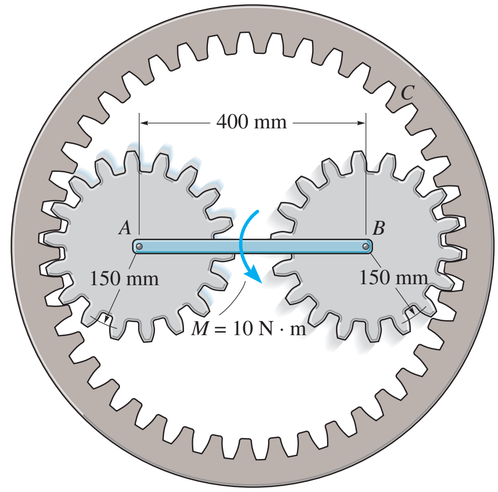
$$48.0 + [57.47 + 2(8.620) + 7.5\pi] = 12.0\,\omega^2$$

$$\omega = 3.4913 \text{ rad/s} = 3.49 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 3.49$ rad/s

18–33.

The two 2-kg gears A and B are attached to the ends of a 3-kg slender bar. The gears roll within the fixed ring gear C, which lies in the horizontal plane. If a 10-N·m torque is applied to the center of the bar as shown, determine the number of revolutions the bar must rotate starting from rest in order for it to have an angular velocity of $\omega_{AB} = 20$ rad/s. For the calculation, assume the gears can be approximated by thin disks. What is the result if the gears lie in the vertical plane?



SOLUTION

Energy equation (where G refers to the center of one of the two gears):

$$M\theta = T_2$$

$$10\theta = 2\left(\frac{1}{2}I_G\omega_{\text{gear}}^2\right) + 2\left(\frac{1}{2}m_{\text{gear}}\right)(0.200\omega_{AB})^2 + \frac{1}{2}I_{AB}\omega_{AB}^2$$

Using $m_{\text{gear}} = 2$ kg, $I_G = \frac{1}{2}(2)(0.150)^2 = 0.0225 \text{ kg}\cdot\text{m}^2$,

$I_{AB} = \frac{1}{12}(3)(0.400)^2 = 0.0400 \text{ kg}\cdot\text{m}^2$ and $\omega_{\text{gear}} = \frac{200}{150}\omega_{AB}$,

$$10\theta = 0.0225\left(\frac{200}{150}\right)^2\omega_{AB}^2 + 2(0.200)^2\omega_{AB}^2 + 0.0200\omega_{AB}^2$$

When $\omega_{AB} = 20$ rad/s,

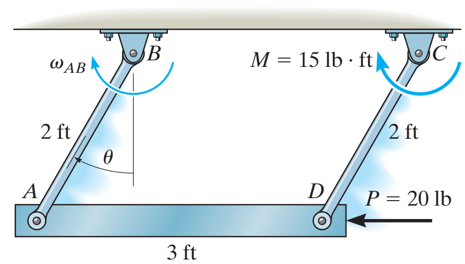
$$\theta = 5.60 \text{ rad}$$

$$= 0.891 \text{ rev, regardless of orientation} \qquad \textbf{Ans.}$$

Ans:
$\theta = 0.891$ rev,
regardless of orientation

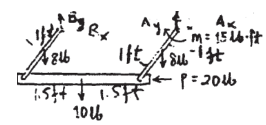
18–34.

The linkage consists of two 8-lb rods AB and CD and a 10-lb bar AD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega_{AB} = 2$ rad/s. If rod CD is subjected to a couple moment $M = 15$ lb · ft and bar AD is subjected to a horizontal force $P = 20$ lb as shown, determine ω_{AB} at the instant $\theta = 90°$.



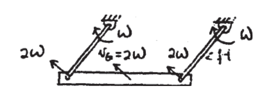
SOLUTION

$T_1 + \Sigma U_{1-2} = T_2$



$$2\left[\frac{1}{2}\left\{\frac{1}{3}\left(\frac{8}{32.2}\right)(2)^2\right\}(2)^2\right] + \frac{1}{2}\left(\frac{10}{32.2}\right)(4)^2 + \left[20(2) + 15\left(\frac{\pi}{2}\right) - 2(8)(1) - 10(2)\right]$$

$$= 2\left[\frac{1}{2}\left\{\frac{1}{3}\left(\frac{8}{32.2}\right)(2)^2\right\}\omega^2\right] + \frac{1}{2}\left(\frac{10}{32.2}\right)(2\omega)^2$$

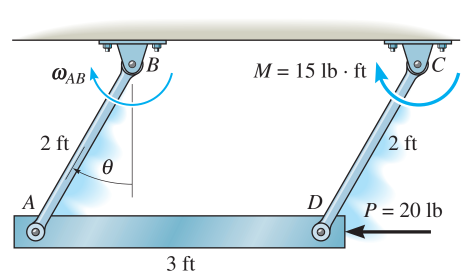


$\omega = 5.74$ rad/s **Ans.**

Ans:
$\omega = 5.74$ rad/s

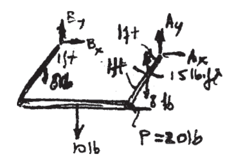
18–35.

The linkage consists of two 8-lb rods AB and CD and a 10-lb bar AD. When $\theta = 0°$, rod AB is rotating with an angular velocity $\omega_{AB} = 2$ rad/s. If rod CD is subjected to a couple moment $M = 15$ lb·ft and bar AD is subjected to a horizontal force $P = 20$ lb as shown, determine ω_{AB} at the instant $\theta = 45°$.



SOLUTION

$T_1 + \Sigma U_{1-2} = T_2$



$$2\left[\frac{1}{2}\left\{\frac{1}{3}\left(\frac{8}{32.2}\right)(2)^2\right\}(2)^2\right] + \frac{1}{2}\left(\frac{10}{32.2}\right)(4)^2$$

$$+\left[20(2\sin 45°) + 15\left(\frac{\pi}{4}\right) - 2(8)(1 - \cos 45°) - 10(2 - 2\cos 45°)\right]$$

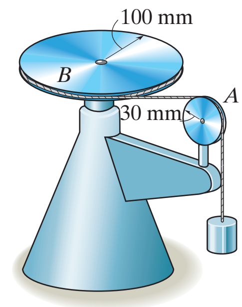
$$= 2\left[\frac{1}{2}\left\{\frac{1}{3}\left(\frac{8}{32.2}\right)(2)^2\right\}\omega^2\right] + \frac{1}{2}\left(\frac{10}{32.2}\right)(2\omega)^2$$

$\omega = 5.92$ rad/s **Ans.**

Ans:
$\omega_{AB} = 5.92$ rad/s

***18–36.**

The assembly consists of a 3-kg pulley A and 10-kg pulley B. If a 2-kg block is suspended from the cord, determine the block's speed after it descends 0.5 m starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.



SOLUTION

$T_1 + V_1 = T_2 + V_2$

$$[0 + 0 + 0] + [0] = \frac{1}{2}\left[\frac{1}{2}(3)(0.03)^2\right]\omega_A^2 + \frac{1}{2}\left[\frac{1}{2}(10)(0.1)^2\right]\omega_B^2 + \frac{1}{2}(2)(v_C)^2 - 2(9.81)(0.5)$$

$v_C = \omega_B(0.1) = 0.03\omega_A$

Thus,

$\omega_B = 10v_C$

$\omega_A = 33.33v_C$

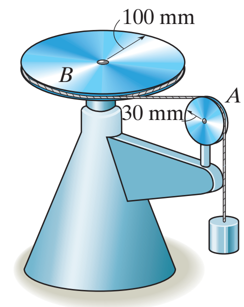
Substituting and solving yields,

$v_C = 1.52 \text{ m/s}$ **Ans.**

Ans:
$v_C = 1.52 \text{ m/s}$

18–37.

The assembly consists of a 3-kg pulley A and 10-kg pulley B. If a 2-kg block is suspended from the cord, determine the distance the block must descend, starting from rest, in order to cause B to have an angular velocity of 6 rad/s. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.



SOLUTION

$v_C = \omega_B(0.1) = 0.03\,\omega_A$

If $\omega_B = 6$ rad/s then

$\omega_A = 20$ rad/s

$v_C = 0.6$ m/s

$T_1 + V_1 = T_2 + V_2$

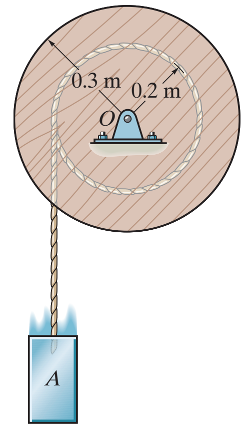
$$[0 + 0 + 0] + [0] = \frac{1}{2}\left[\frac{1}{2}(3)(0.03)^2\right](20)^2 + \frac{1}{2}\left[\frac{1}{2}(10)(0.1)^2\right](6)^2 + \frac{1}{2}(2)(0.6)^2 - 2(9.81)s_C$$

$s_C = 78.0$ mm **Ans.**

Ans:
$s_C = 78.0$ mm

18–38.

The spool has a mass of 50 kg and a radius of gyration $k_O = 0.280$ m. If the 20-kg block A is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 5$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



SOLUTION

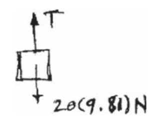
$v_A = 0.2\omega = 0.2(5) = 1$ m/s

System:

$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0] + 0 = \frac{1}{2}(20)(1)^2 + \frac{1}{2}[50(0.280)^2](5)^2 - 20(9.81)\,s$$

$s = 0.30071$ m $= 0.301$ m **Ans.**



Block:

$$T_1 + \Sigma U_{1-2} = T_2$$

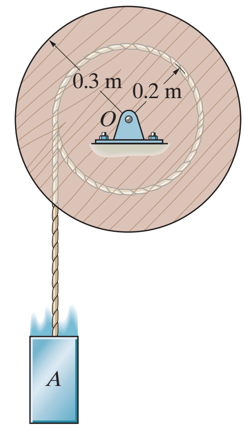
$$0 + 20(9.81)(0.30071) - T(0.30071) = \frac{1}{2}(20)(1)^2$$

$T = 163$ N **Ans.**

Ans:
$s = 0.301$ m
$T = 163$ N

18–39.

The spool has a mass of 50 kg and a radius of gyration $k_O = 0.280$ m. If the 20-kg block A is released from rest, determine the velocity of the block when it descends 0.5 m.



SOLUTION

Potential Energy: With reference to the datum established in Fig. *a*, the gravitational potential energy of block A at position 1 and 2 are

$$V_1 = (V_g)_1 = W_A y_1 = 20(9.81)(0) = 0$$

$$V_2 = (V_g)_2 = -W_A y_2 = -20(9.81)(0.5) = -98.1 \text{ J}$$

Kinetic Energy: Since the spool rotates about a fixed axis, $\omega = \dfrac{v_A}{r_A} = \dfrac{v_A}{0.2} = 5v_A$. Here, the mass moment of inertia about the fixed axis passes through point O is $I_O = mk_O^2 = 50(0.280)^2 = 3.92 \text{ kg}\cdot\text{m}^2$. Thus, the kinetic energy of the system is

$$T = \frac{1}{2}I_O\omega^2 + \frac{1}{2}m_A v_A^2$$

$$= \frac{1}{2}(3.92)(5v_A)^2 + \frac{1}{2}(20)v_A^2 = 59v_A^2$$

Since the system is at rest initially, $T_1 = 0$

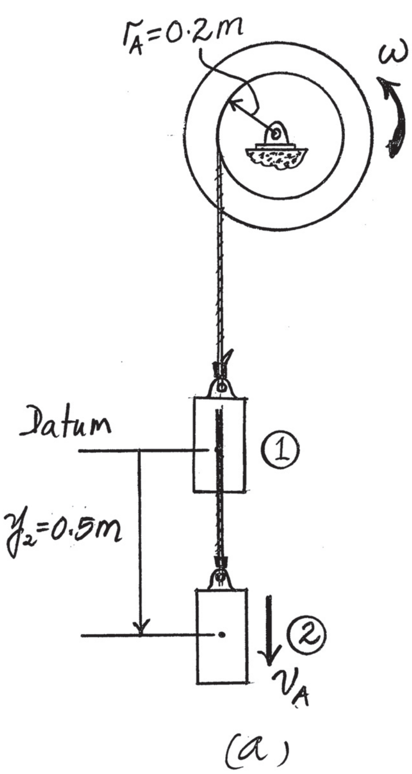
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 59v_A^2 + (-98.1)$$

$$v_A = 1.289 \text{ m/s}$$

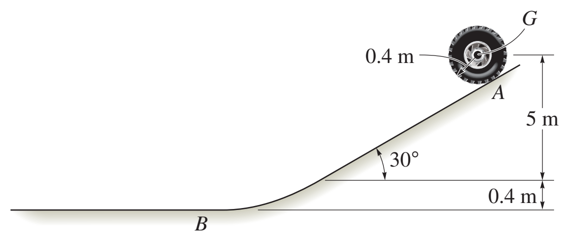
$$= 1.29 \text{ m/s}$$ **Ans.**



Ans:
$v_A = 1.29$ m/s

***18–40.**

An automobile tire has a mass of 7 kg and radius of gyration $k_G = 0.3$ m. If it is released from rest at A on the incline, determine its angular velocity when it reaches the horizontal plane. The tire rolls without slipping.



SOLUTION

$$v_G = 0.4\omega$$

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

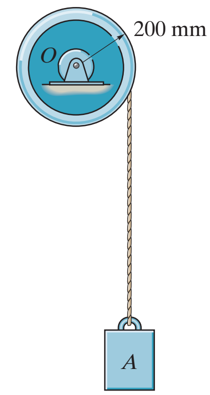
$$0 + 7(9.81)(5) = \frac{1}{2}(7)(0.4\omega)^2 + \frac{1}{2}[7\,(0.3)^2]\omega^2 + 0$$

$$\omega = 19.8 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 19.8$ rad/s

18–41.

The spool has a mass of 20 kg and a radius of gyration of $k_O = 160$ mm. If the 15-kg block A is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 8$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.



SOLUTION

Kinetic Energy. The mass moment of inertia of the spool about its center O is $I_0 = mk_0^2 = 20(0.16^2) = 0.512\ \text{kg}\cdot\text{m}^2$. The velocity of the block is $v_b = \omega_s r_s = \omega_s(0.2)$. Thus,

$$\begin{aligned}T &= \frac{1}{2}I_0\omega^2 + \frac{1}{2}m_b v_b^2\\ &= \frac{1}{2}(0.512)\omega_s^2 + \frac{1}{2}(15)[\omega_s(0.2)]^2\\ &= 0.556\,\omega_s^2\end{aligned}$$

Since the system starts from rest, $T_1 = 0$. When $\omega_s = 8$ rad/s,

$$T_2 = 0.556(8^2) = 35.584\ \text{J}$$

Potential Energy. With reference to the datum set in Fig. *a*, the initial and final gravitational potential energy of the block are

$$(V_g)_1 = m_b g y_1 = 0$$

$$(V_g)_2 = m_b g(-y_2) = 15(9.81)(-s_b) = -147.15 s_b$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 35.584 + (-147.15 s_b)$$

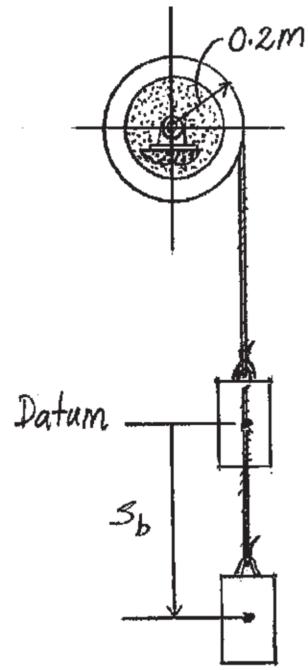
$$s_b = 0.2418\ \text{m} = 242\ \text{mm} \qquad \textbf{Ans.}$$

Principle of Work and Energy. The final velocity of the block is $(v_b)_2 = (\omega_s)_2 r_s = 8(0.2) = 1.60$ m/s. Referring to the FBD of the block, Fig. *b* and using the result of S_b,

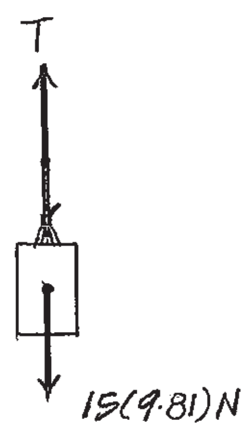
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 15(9.81)(0.2418) - T(0.2418) = \frac{1}{2}(15)(1.60^2)$$

$$T = 67.75\quad N = 67.8\ \text{N} \qquad \textbf{Ans.}$$



(a)

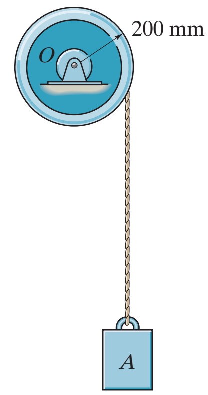


(b)

Ans:
$s_b = 242$ mm
$T = 67.8$ N

18–42.

The spool has a mass of 20 kg and a radius of gyration of $k_O = 160$ mm. If the 15-kg block A is released from rest, determine the velocity of the block when it descends 600 mm.



SOLUTION

Kinetic Energy. The mass moment of inertia of the spool about its center O is $I_0 = mk_0^2 = 20\left(0.16^2\right) = 0.512\ \text{kg}\cdot\text{m}^2$. The angular velocity of the spool is $\omega_s = \dfrac{v_b}{r_s} = \dfrac{v_b}{0.2} = 5v_b$. Thus,

$$T = \frac{1}{2}I_0\omega^2 + \frac{1}{2}m_b v_b^2$$

$$= \frac{1}{2}(0.512)(5v_b)^2 + \frac{1}{2}(15)v_b^2$$

$$= 13.9v_b^2$$

Since the system starts from rest, $T_1 = 0$.

Potential Energy. With reference to the datum set in Fig. *a*, the initial and final gravitational potential energies of the block are

$$(V_g)_1 = m_b g y_1 = 0$$

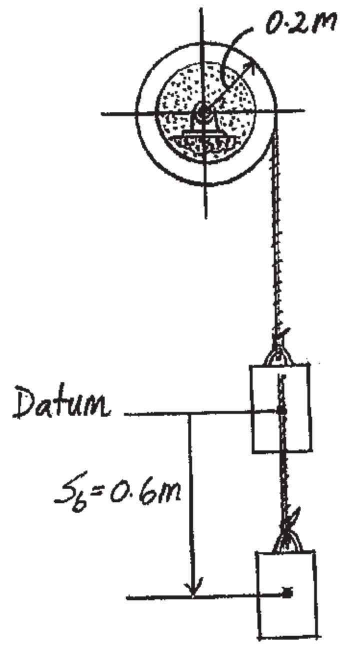
$$(V_g)_2 = m_b g y_2 = 15(9.81)(-0.6) = -88.29\ \text{J}$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 13.9v_b^2 + (-88.29)$$

$$v_b = 2.5203\ \text{m/s} = 2.52\ \text{m/s} \qquad \textbf{Ans.}$$

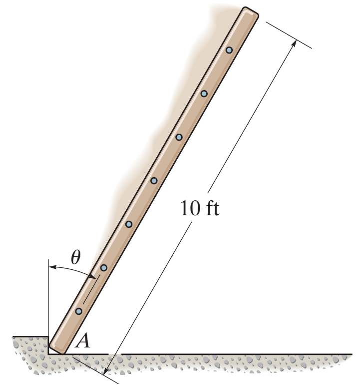


(a)

Ans:
$v_b = 2.52$ m/s

18–43.

A uniform ladder having a weight of 30 lb is released from rest when it is in the vertical position. If it is allowed to fall freely, determine the angle θ at which the bottom end A starts to slide to the right of A. For the calculation, assume the ladder to be a slender rod and neglect friction at A.



SOLUTION

Potential Energy: Datum is set at point A. When the ladder is at its initial and final position, its center of gravity is located 5 ft and $(5\cos\theta)$ ft *above* the datum. Its initial and final gravitational potential energy are $30(5) = 150\text{ ft}\cdot\text{lb}$ and $30(5\cos\theta) = 150\cos\theta\text{ ft}\cdot\text{lb}$, respectively. Thus, the initial and final potential energy are

$$V_1 = 150\text{ ft}\cdot\text{lb} \qquad V_2 = 150\cos\theta\text{ ft}\cdot\text{lb}$$

Kinetic Energy: The mass moment inertia of the ladder about point A is $I_A = \frac{1}{12}\left(\frac{30}{32.2}\right)(10^2) + \left(\frac{30}{32.2}\right)(5^2) = 31.06\text{ slug}\cdot\text{ft}^2$. Since the ladder is initially at rest, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}I_A\omega^2 = \frac{1}{2}(31.06)\omega^2 = 15.53\omega^2$$

Conservation of Energy: Applying Eq. 18–18, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 150 = 15.53\omega^2 + 150\cos\theta$$

$$\omega^2 = 9.66(1 - \cos\theta)$$

Equation of Motion: The mass moment inertia of the ladder about its mass center is $I_G = \frac{1}{12}\left(\frac{30}{32.2}\right)(10^2) = 7.764\text{ slug}\cdot\text{ft}^2$. Applying Eq. 17–16, we have

$$+\Sigma M_A = \Sigma(M_k)_A; \qquad -30\sin\theta(5) = -7.764\alpha - \left(\frac{30}{32.2}\right)[\alpha(5)](5)$$

$$\alpha = 4.83\sin\theta$$

$$\xrightarrow{+}\Sigma F_x = m(a_G)_x; \qquad A_x = -\frac{30}{32.2}[9.66(1 - \cos\theta)(5)]\sin\theta + \frac{30}{32.2}[4.83\sin\theta(5)]\cos\theta$$

$$A_x = -\frac{30}{32.2}(48.3\sin\theta - 48.3\sin\theta\cos\theta - 24.15\sin\theta\cos\theta)$$

$$= 45.0\sin\theta\,(1 - 1.5\cos\theta) = 0$$

If the ladder begins to slide, then $A_x = 0$. Thus, for $\theta > 0$,

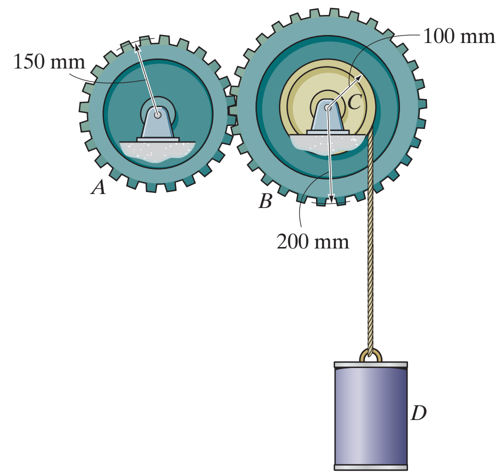
$$45.0\sin\theta\,(1 - 1.5\cos\theta) = 0$$

$$\theta = 48.2^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 48.2^\circ$

***18–44.**

Determine the speed of the 50-kg cylinder after it has descended a distance of 2 m, starting from rest. Gear A has a mass of 10 kg and a radius of gyration of 125 mm about its center of mass. Gear B and drum C have a combined mass of 30 kg and a radius of gyration about their center of mass of 150 mm.



SOLUTION

Potential Energy: With reference to the datum shown in Fig. a, the gravitational potential energy of block D at position (1) and (2) is

$$V_1 = (V_g)_1 = W_D(y_D)_1 = 50(9.81)(0) = 0$$

$$V_2 = (V_g)_2 = -W_D(y_D)_2 = -50(9.81)(2) = -981 \text{ J}$$

Kinetic Energy: Since gear B rotates about a fixed axis, $\omega_B = \dfrac{v_D}{r_D} = \dfrac{v_D}{0.1} = 10v_D$. Also, since gear A is in mesh with gear B, $\omega_A = \left(\dfrac{r_B}{r_A}\right)\omega_B = \left(\dfrac{0.2}{0.15}\right)(10v_D) = 13.33v_D$. The mass moment of inertia of gears A and B about their mass centers are $I_A = m_A k_A^2 = 10(0.125^2) = 0.15625 \text{ kg}\cdot\text{m}^2$ and $I_B = m_B k_B^2 = 30(0.15^2) = 0.675 \text{ kg}\cdot\text{m}^2$. Thus, the kinetic energy of the system is

$$\begin{aligned} T &= \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2 + \frac{1}{2}m_D v_D^2 \\ &= \frac{1}{2}(0.15625)(13.33v_D)^2 + \frac{1}{2}(0.675)(10v_D)^2 + \frac{1}{2}(50)v_D^2 \\ &= 72.639v_D^2 \end{aligned}$$

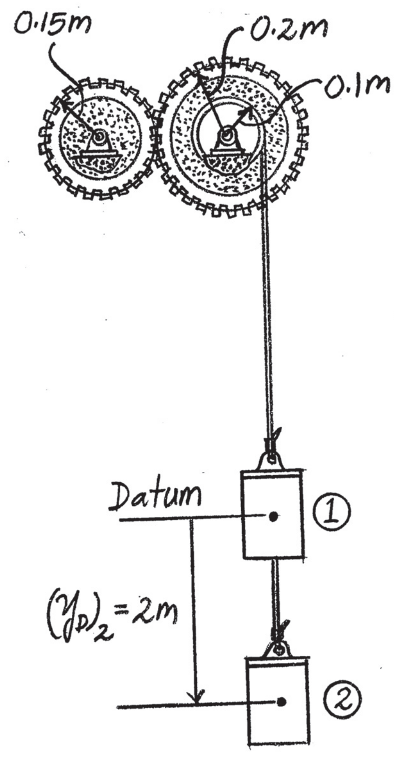
Since the system is initially at rest, $T_1 = 0$.

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 72.639v_D^2 - 981$$

$$v_D = 3.67 \text{ m/s} \qquad \textbf{Ans.}$$

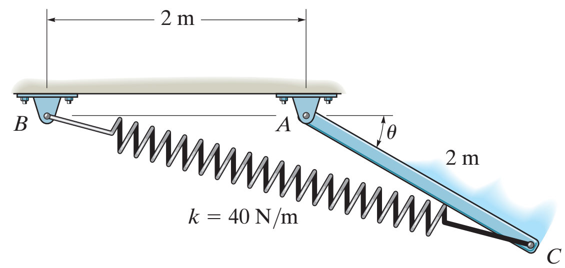


(a)

Ans:
$v_D = 3.67$ m/s

18–45.

The 12-kg slender rod is attached to a spring, which has an unstretched length of 2 m. If the rod is released from rest when $\theta = 30°$, determine its angular velocity at the instant $\theta = 90°$.



SOLUTION

Kinetic Energy. The mass moment of inertia of the rod about A is $I_A = \frac{1}{12}(12)(2^2) + 12(1^2) = 16.0 \text{ kg}\cdot\text{m}^2$. Then

$$T = \frac{1}{2} I_A \omega^2 = \frac{1}{2}(16.0)\,\omega^2 = 8.00\,\omega^2$$

Since the rod is released from rest, $T_1 = 0$.

Potential Energy. With reference to the datum set in Fig. *a*, the gravitational potential energies of the rod at positions ① and ② are

$$(V_g)_1 = mg(-y_1) = 12(9.81)(-1 \sin 30°) = -58.86 \text{ J}$$

$$(V_g)_2 = mg(-y_2) = 12(9.81)(-1) = -117.72 \text{ J}$$

The stretches of the spring when the rod is at positions ① and ② are

$$x_1 = 2(2 \sin 75°) - 2 = 1.8637 \text{ m}$$

$$x_2 = \sqrt{2^2 + 2^2} - 2 = 0.8284 \text{ m}$$

Thus, the initial and final elastic potential energies of the spring are

$$(V_e)_1 = \frac{1}{2} kx_1^2 = \frac{1}{2}(40)\left(1.8637^2\right) = 69.47 \text{ J}$$

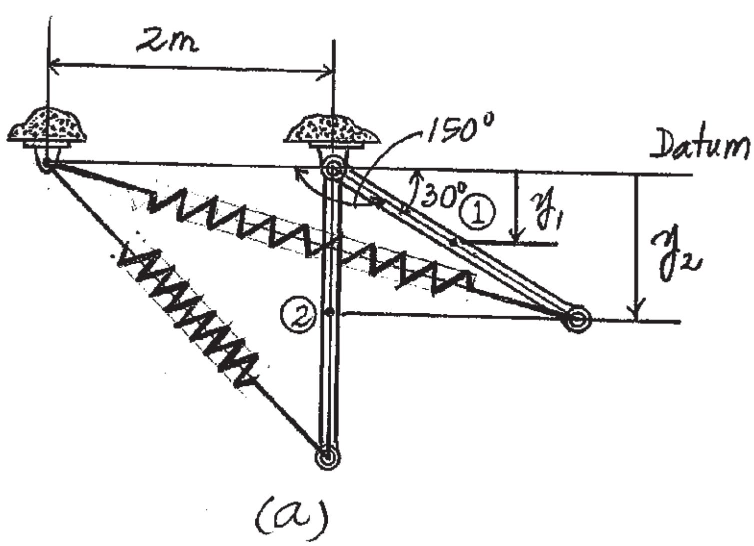
$$(V_e)_2 = \frac{1}{2} kx_2^2 = \frac{1}{2}(40)\left(0.8284^2\right) = 13.37 \text{ J}$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-58.86) + 69.47 = 8.00\omega^2 + (-117.72) + 13.73$$

$$\omega = 3.7849 \text{ rad/s} = 3.78 \text{ rad/s}$$ **Ans.**

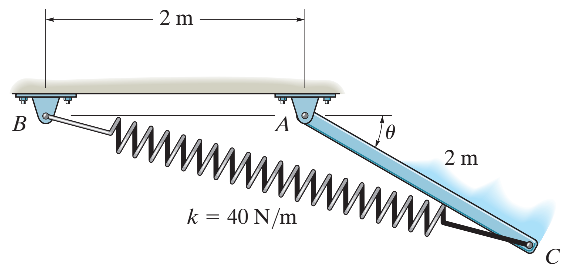


(a)

Ans:
$\omega = 3.78$ rad/s

18–46.

The 12-kg slender rod is attached to a spring, which has an unstretched length of 2 m. If the rod is released from rest when $\theta = 30°$, determine the angular velocity of the rod the instant the spring becomes unstretched.



SOLUTION

Kinetic Energy. The mass moment of inertia of the rod about A is $I_A = \frac{1}{12}(12)(2^2) + 12(1^2) = 16.0 \text{ kg}\cdot\text{m}^3$. Then

$$T = \frac{1}{2} I_A \omega^2 = \frac{1}{2}(16.0)\omega^2 = 8.00\,\omega^2$$

Since the rod is release from rest, $T_1 = 0$.

Potential Energy. When the spring is unstretched, the rod is at position ② shown in Fig. *a*. with reference to the datum set, the gravitational potential energies of the rod at positions ① and ② are

$$(V_g)_1 = mg(-y_1) = 12(9.81)(-1 \sin 30°) = -58.86 \text{ J}$$

$$(V_g)_2 = mg(-y_2) = 12(9.81)(-1 \sin 60°) = -101.95 \text{ J}$$

The stretch of the spring when the rod is at position ① is

$$x_1 = 2(2 \sin 75°) - 2 = 1.8637 \text{ m}$$

It is required that $x_2 = 0$. Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2} k x_1^2 = \frac{1}{2}(40)(1.8637^2) = 69.47 \text{ J}$$

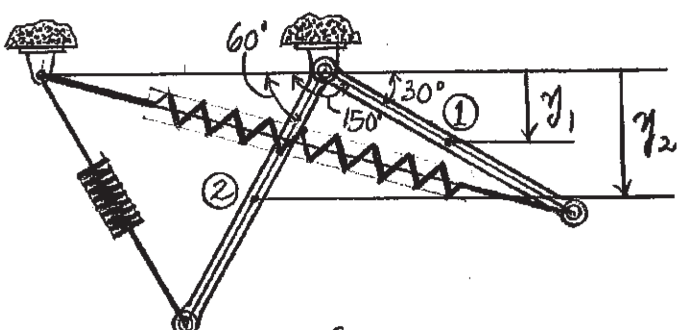
$$(V_e)_2 = \frac{1}{2} k x_2^2 = 0$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-58.86) + 69.47 = 8.00\omega^2 + (-101.95) + 0$$

$$\omega = 3.7509 \text{ rad/s} = 3.75 \text{ rad/s}$$ **Ans.**

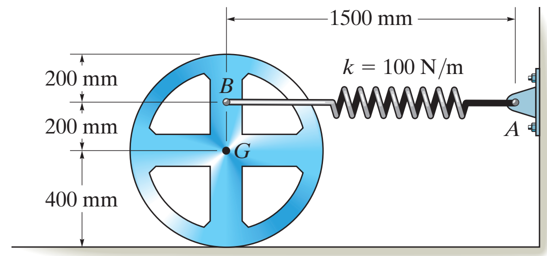


(a)

Ans:
$\omega = 3.75$ rad/s

18–47.

The 40-kg wheel has a radius of gyration about its center of gravity G of $k_G = 250$ mm. If it rolls without slipping, determine its angular velocity when it has rotated clockwise 90° from the position shown. The spring AB has a stiffness $k = 100$ N/m and an unstretched length of 500 mm. The wheel is released from rest.



SOLUTION

Kinetic Energy. The mass moment of inertia of the wheel about its center of mass G is $I_G = mk_G^2 = 40\left(0.25^2\right) = 2.50\ \text{kg}\cdot\text{m}^2$, since the wheel rolls without slipping, $v_G = \omega r_G = \omega(0.4)$. Thus

$$T = \frac{1}{2} I_G\,\omega^2 + \frac{1}{2} m v_G^2$$

$$= \frac{1}{2}(2.50)\omega^2 + \frac{1}{2}(40)[\omega(0.4)]^2 = 4.45\,\omega^2$$

Since the wheel is released from rest, $T_1 = 0$.

Potential Energy. When the wheel rotates 90° clockwise from position ① to ②, Fig. *a*, its mass center displaces $S_G = \theta r_G = \frac{\pi}{2}(0.4) = 0.2\pi$ m. Then $x^y = 1.5 - 0.2 - 0.2\pi = 0.6717$ m. The stretches of the spring when the wheel is at positions ① and ② are

$$x_1 = 1.50 - 0.5 = 1.00\ \text{m}$$

$$x_2 = \sqrt{0.6717^2 + 0.2^2} - 0.5 = 0.2008\ \text{m}$$

Thus, the initial and final elastic potential energies are

$$(V_e)_1 = \frac{1}{2} k x_1^2 = \frac{1}{2}(100)\left(1^2\right) = 50\ \text{J}$$

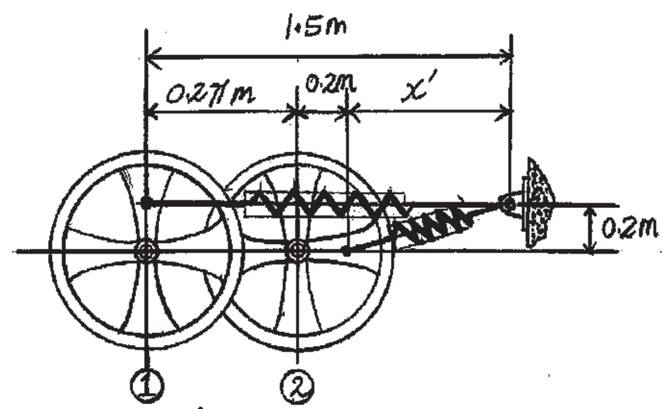
$$(V_e)_2 = \frac{1}{2} k x_2^2 = \frac{1}{2}(100)\left(0.2008^2\right) = 2.0165\ \text{J}$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 50 = 4.45\omega^2 + 2.0165$$

$$\omega = 3.2837\ \text{rad/s} = 3.28\ \text{rad/s} \qquad \textbf{Ans.}$$

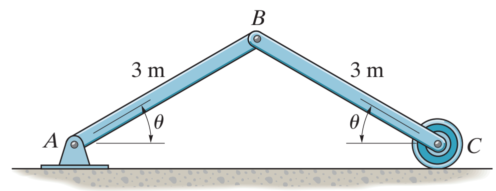


(a)

Ans:
$\omega = 3.28$ rad/s

18–48.

The assembly consists of two 10-kg bars which are pin connected. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 0°$. The 5-kg disk at C has a radius of 0.5 m and rolls without slipping.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of bar BC at the final position, Fig. a, we found that IC is located at C. Thus, $(v_c)_2 = 0$. Also,

$$(v_B)_2 = (\omega_{BC})_2\, r_{B/IC}; \qquad (v_b)_2 = (\omega_{BC})_2(3)$$

$$(v_G)_2 = (\omega_{BC})_2\, r_{G/IC}; \qquad (v_G)_2 = (\omega_{BC})_2(1.5)$$

Then for bar AB,

$$(v_B)_2 = (\omega_{AB})_2\, r_{AB}; \quad (\omega_{BC})_2(3) = (\omega_{AB})_2(3)$$

$$(\omega_{AB})_2 = (\omega_{BC})_2$$

For the disk, since the velocity of its center $(v_c)_2 = 0$, then $(\omega_d)_2 = 0$. Thus

$$T_2 = \frac{1}{2}I_A(\omega_{AB})_2^2 + \frac{1}{2}I_G(\omega_{BC})_2^2 + \frac{1}{2}m_r(v_G)_2^2$$

$$= \frac{1}{2}\left[\frac{1}{3}(10)(3^2)\right](\omega_{BC})_2^2 + \frac{1}{2}\left[\frac{1}{12}(10)(3^2)\right](\omega_{BC})_2^2 + \frac{1}{2}(10)[(\omega_{BC})_2(1.5)]^2$$

$$= 30.0(\omega_{BC})_2^2$$

Potential Energy. With reference to the datum set in Fig. b, the initial and final gravitational potential energies of the system are

$$(V_g)_1 = 2mgy_1 = 2[10(9.81)(1.5 \sin 60°)] = 254.87 \text{ J}$$

$$(V_g)_2 = 2mgy_2 = 0$$

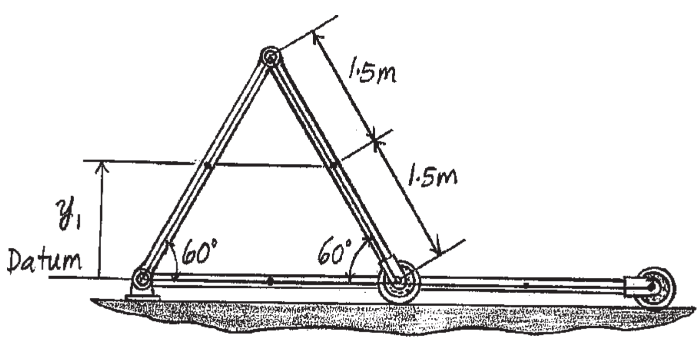
Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

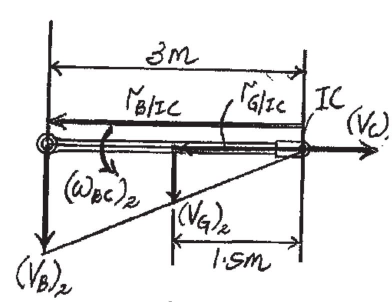
$$0 + 254.87 = 30.0(\omega_{BC})_2^2 + 0$$

$$(\omega_{BC})_2 = 2.9147 \text{ rad/s} = 2.91 \text{ rad/s} \qquad \textbf{Ans.}$$

$$(\omega_{AB})_2 = (\omega_{BC})_2 = 2.91 \text{ rad/s} \qquad \textbf{Ans.}$$



(b)

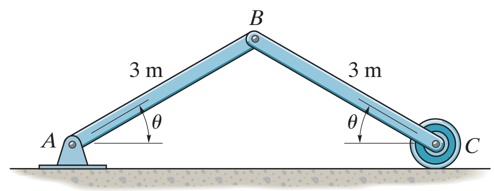


(a)

Ans:
$(\omega_{BC})_2 = 2.91$ rad/s
$(\omega_{AB})_2 = 2.91$ rad/s

18–49.

The assembly consists of two 10-kg bars which are pin connected. If the bars are released from rest when $\theta = 60°$, determine their angular velocities at the instant $\theta = 30°$. The 5-kg disk at C has a radius of 0.5 m and rolls without slipping.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of bar BC at final position with IC so located, Fig. a,

$$r_{B/IC} = r_{C/IC} = 3 \text{ m} \qquad r_{G/IC} = 3 \sin 60° = 1.5\sqrt{3} \text{ m}$$

Thus,

$$(v_B)_2 = (\omega_{BC})_2 r_{B/IC}; \qquad (v_B)_2 = (\omega_{BC})_2(3)$$

$$(v_C)_2 = (\omega_{BC})_2 r_{C/IC}; \qquad (v_C)_2 = (\omega_{BC})_2(3)$$

$$(v_G)_2 = (\omega_{BC})_2 r_{G/IC}; \qquad (v_G)_2 = (\omega_{BC})_2\left(1.5\sqrt{3}\right)$$

Then for rod AB,

$$(v_B)_2 = (\omega_{AB})_2 r_{AB}; \qquad (\omega_{BC})_2(3) = (\omega_{AB})_2(3)$$

$$(\omega_{AB})_2 = (\omega_{BC})_2$$

For the disk, since it rolls without slipping,

$$(v_C)_2 = (\omega_d)_2 r_d; \qquad (\omega_{BC})_2(3) = (\omega_d)_2(0.5)$$

$$(\omega_d)_2 = 6(\omega_{BC})_2$$

Thus, the kinetic energy of the system at final position is

$$T_2 = \frac{1}{2}I_A(\omega_{AB})_2^2 + \frac{1}{2}I_G(\omega_{BC})_2^2 + \frac{1}{2}m_r(v_G)_2^2 + \frac{1}{2}I_C(\omega_d)_2^2 + \frac{1}{2}m_d(v_C)_2^2$$

$$= \frac{1}{2}\left[\frac{1}{3}(10)\left(3^2\right)\right](\omega_{BC})_2^2 + \frac{1}{2}\left[\frac{1}{12}(10)\left(3^2\right)\right](\omega_{BC})_2^2 + \frac{1}{2}(10)\left[(\omega_{BC})_2\left(1.5\sqrt{3}\right)\right]^2$$

$$+ \frac{1}{2}\left[\frac{1}{2}(5)\left(0.5^2\right)\right][6(\omega_{BC})_2]^2 + \frac{1}{2}(5)[(\omega_{BC})_2(3)]^2$$

$$= 86.25(\omega_{BC})_2^2$$

Potential Energy. With reference to the datum set in Fig. b, the initial and final gravitational potential energies of the system are

$$(V_g)_1 = 2mgy_1 = 2[10(9.81)(1.5 \sin 60°)] = 254.87 \text{ J}$$

$$(V_g)_2 = 2mgy_2 = 2[10(9.81)(1.5 \sin 30°)] = 147.15 \text{ J}$$

18–49. Continued

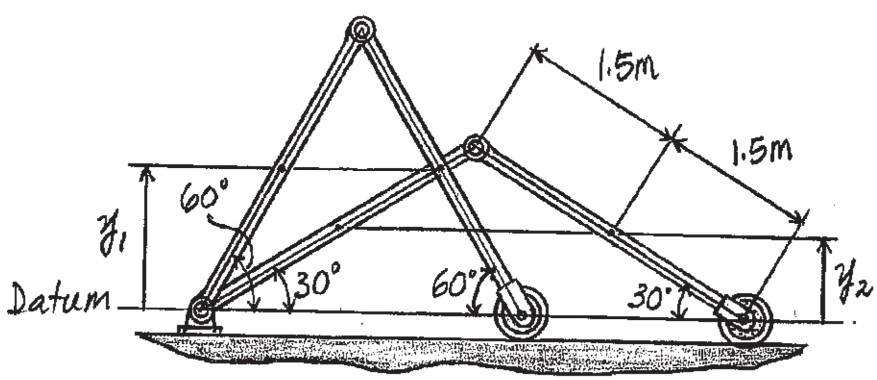
Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

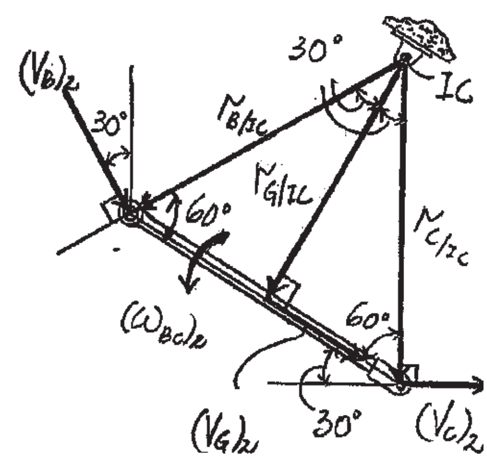
$$0 + 254.87 = 86.25(\omega_{BC})_2^2 + 147.15$$

$$(\omega_{BC})_2 = 1.1176 \text{ rad/s} = 1.12 \text{ rad/s} \qquad \textbf{Ans.}$$

$$(\omega_{AB})_2 = (\omega_{BC})_2 = 1.12 \text{ rad/s} \qquad \textbf{Ans.}$$



(b)

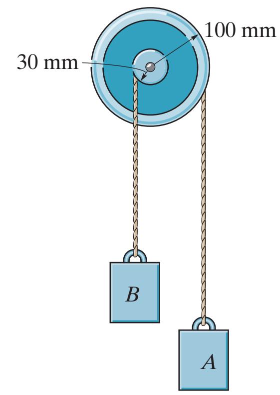


(a)

Ans:
$(\omega_{AB})_2 = (\omega_{BC})_2 = 1.12 \text{ rad/s}$

18–50.

The compound disk pulley consists of a hub and attached outer rim. If it has a mass of 3 kg and a radius of gyration $k_G = 45$ mm, determine the speed of block A after A descends 0.2 m from rest. Blocks A and B each have a mass of 2 kg. Neglect the mass of the cords.

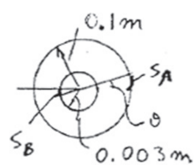


SOLUTION

$T_1 + V_1 = T_2 + V_2$

$$[0 + 0 + 0] + [0 + 0] = \frac{1}{2}[3(0.045)^2]\omega^2 + \frac{1}{2}(2)(0.03\omega)^2 + \frac{1}{2}(2)(0.1\omega)^2 - 2(9.81)s_A + 2(9.81)s_B$$

$$\theta = \frac{s_B}{0.03} = \frac{s_A}{0.1}$$



$s_B = 0.3\, s_A$

Set $s_A = 0.2$ m, $s_B = 0.06$ m

Substituting and solving yields,

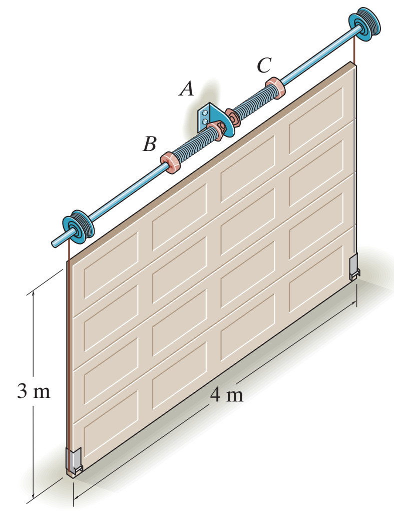
$\omega = 14.04$ rad/s

$v_A = 0.1(14.04) = 1.40$ m/s **Ans.**

Ans:
$v_A = 1.40$ m/s

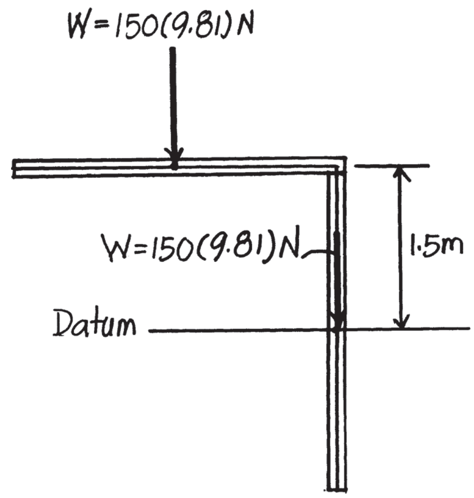
18–51.

The uniform garage door has a mass of 150 kg and is guided along smooth tracks at its ends. Lifting is done using the two springs, each of which is attached to the anchor bracket at A and to the counterbalance shaft at B and C. As the door is raised, the springs begin to unwind from the shaft, thereby assisting the lift. If each spring provides a torsional moment of $M = (0.7\theta)\ \text{N}\cdot\text{m}$, where θ is in radians, determine the angle θ_0 at which both the left-wound and right-wound spring should be attached so that the door is completely balanced by the springs, i.e., when the door is in the vertical position and is given a slight force upwards, the springs will lift the door along the side tracks to the horizontal plane with no final angular velocity. *Note:* The elastic potential energy of a torsional spring is $V_e = \frac{1}{2}k\theta^2$, where $M = k\theta$ and in this case $k = 0.7\ \text{N}\cdot\text{m/rad}$.



SOLUTION

Datum at initial position.



$$T_1 + V_1 = T_2 + V_2$$

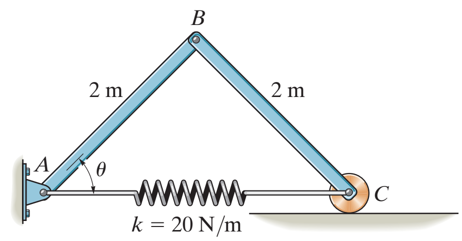
$$0 + 2\left[\frac{1}{2}(0.7)\theta_0^2\right] + 0 = 0 + 150(9.81)(1.5)$$

$$\theta_0 = 56.15 \text{ rad} = 8.94 \text{ rev.}$$ **Ans.**

Ans:
$\theta_0 = 8.94$ rev

***18–52.**

The two 12-kg slender rods are pin connected and released from rest at the position $\theta = 60^\circ$. If the spring has an unstretched length of 1.5 m, determine the angular velocity of rod BC, when the system is at the position $\theta = 0^\circ$. Neglect the mass of the roller at C.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of rod BC at the final position, Fig. *a* we found that IC is located at C. Thus, $(v_C)_2 = 0$. Also,

$$(v_B)_2 = (\omega_{BC})_2 r_{B/IC}; \qquad (v_B)_2 = (\omega_{BC})_2(2)$$

$$(v_G)_2 = (\omega_{BC})_2 r_{C/IC}; \qquad (v_G)_2 = (\omega_{BC})_2(1)$$

Then for rod AB,

$$(v_B)_2 = (\omega_{AB})_2 r_{AB}; \qquad (\omega_{BC})_2(2) = (\omega_{AB})_2(2)$$

$$(\omega_{AB})_2 = (\omega_{BC})_2$$

Thus,

$$T_2 = \frac{1}{2}I_A(\omega_{AB})_2^2 + \frac{1}{2}I_G(\omega_{BC})_2^2 + \frac{1}{2}m_r(v_G)_2^2$$

$$= \frac{1}{2}\left[\frac{1}{3}(12)(2^2)\right](\omega_{BC})_2^2 + \frac{1}{2}\left[\frac{1}{12}(12)(2^2)\right](\omega_{BC})_2^2 + \frac{1}{2}(12)[(\omega_{BC})_2(1)]^2$$

$$= 16.0(\omega_{BC})_2^2$$

Potential Energy. With reference to the datum set in Fig. *b*, the initial and final gravitational potential energies of the system are

$$(V_g)_1 = 2mgy_1 = 2[12(9.81)(1 \sin 60^\circ)] = 203.90 \text{ J}$$

$$(V_g)_2 = 2mgy_2 = 0$$

The stretch of the spring when the system is at initial and final position are

$$x_1 = 2(2 \cos 60^\circ) - 1.5 = 0.5 \text{ m}$$

$$x_2 = 4 - 1.5 = 2.50 \text{ m}$$

Thus, the initial and final elastic potential energies of the spring is

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(20)(0.5^2) = 2.50 \text{ J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(20)(2.50^2) = 62.5 \text{ J}$$

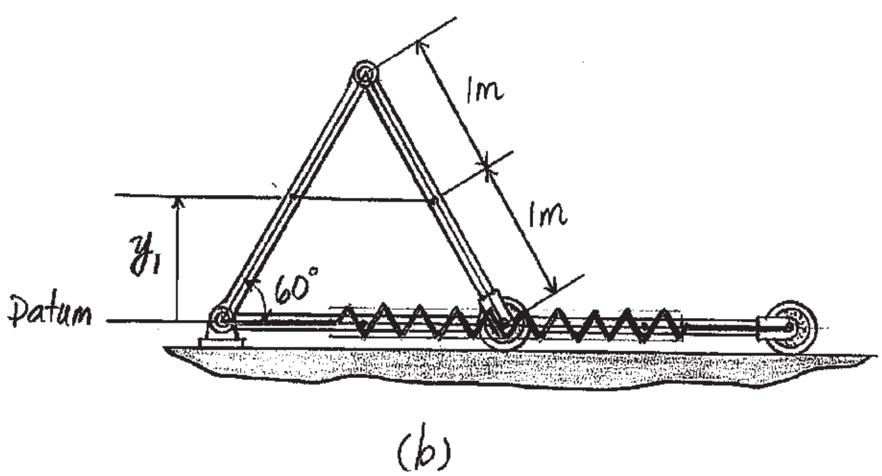
18–52. Continued

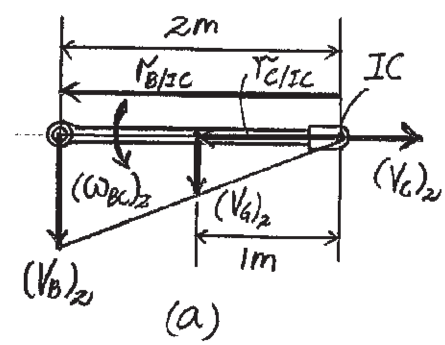
Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (203.90 + 2.50) = 16.0(\omega_{BC})_2^2 + (0 + 62.5)$$

$$(\omega_{BC})_2 = 2.9989 \text{ rad/s} = 3.00 \text{ rad/s}$$ **Ans.**

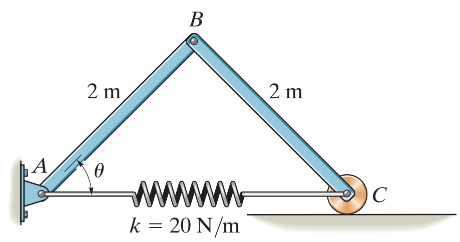




Ans:
$(\omega_{BC})_2 = 3.00$ rad/s

18–53.

The two 12-kg slender rods are pin connected and released from rest at the position $\theta = 60°$. If the spring has an unstretched length of 1.5 m, determine the angular velocity of rod BC, when the system is at the position $\theta = 30°$.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of rod BC at final position with IC so located, Fig. *a*

$$r_{B/IC} = r_{C/IC} = 2 \text{ m} \qquad r_{G/IC} = 2 \sin 60° = \sqrt{3} \text{ m}$$

Thus,

$$(V_B)_2 = (\omega_{BC})_2 r_{B/IC}; \qquad (V_B)_2 = (\omega_{BC})_2(2)$$

$$(V_C)_2 = (\omega_{BC})_2 r_{C/IC}; \qquad (V_C)_2 = (\omega_{BC})_2(2)$$

$$(V_G)_2 = (\omega_{BC})_2 r_{G/IC}; \qquad (V_G)_2 = (\omega_{BC})_2\left(\sqrt{3}\right)$$

Then for rod AB,

$$(V_B)_2 = (\omega_{AB})_2 r_{AB}; \qquad (\omega_{BC})_2(2) = (\omega_{AB})_2(2)$$

$$(\omega_{AB})_2 = (\omega_{BC})_2$$

Thus,

$$T_2 = \frac{1}{2}I_A(\omega_{AB})_2^2 + \frac{1}{2}I_G(\omega_{BC})_2^2 + \frac{1}{2}m_r(V_G)_2^2$$

$$= \frac{1}{2}\left[\frac{1}{3}(12)\left(2^2\right)\right](\omega_{BC})_2^2 + \frac{1}{2}\left[\frac{1}{12}(12)\left(2^2\right)\right](\omega_{BC})_2^2 + \frac{1}{2}(12)[(\omega_{BC})_2\sqrt{3}]^2$$

$$= 28.0(\omega_{BC})_2^2$$

Potential Energy. With reference to the datum set in Fig. *b*, the initial and final gravitational potential energy of the system are

$$(V_g)_1 = 2mgy_1 = 2[12(9.81)(1 \sin 60°)] = 203.90 \text{ J}$$

$$(V_g)_2 = 2mgy_2 = 2[12(9.81)(1 \sin 30°)] = 117.72 \text{ J}$$

The stretch of the spring when the system is at initial and final position are

$$x_1 = 2(2 \cos 60°) - 1.5 = 0.5 \text{ m}$$

$$x_2 = 2(2 \cos 30°) - 1.5 = 1.9641 \text{ m}$$

Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(20)\left(0.5^2\right) = 2.50 \text{ J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(20)\left(1.9641^2\right) = 38.58 \text{ J}$$

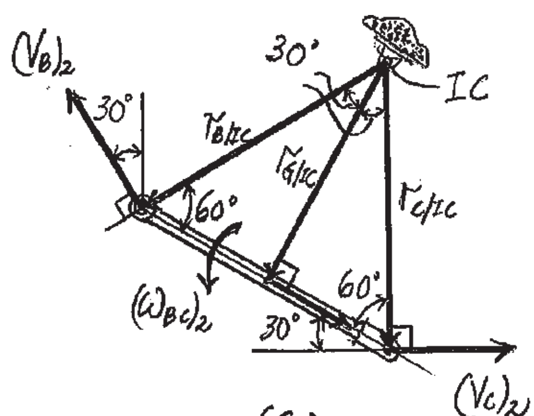
18–53. Continued

Conservation of Energy.

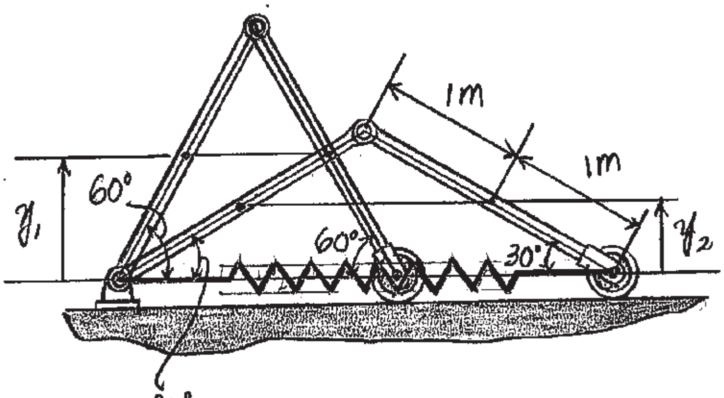
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (203.90 + 2.50) = 28.0(\omega_{BC})_2^2 + (117.72 + 38.58)$$

$$\omega_{BC} = 1.3376 \text{ rad/s} = 1.34 \text{ rad/s} \qquad \textbf{Ans.}$$



(a)

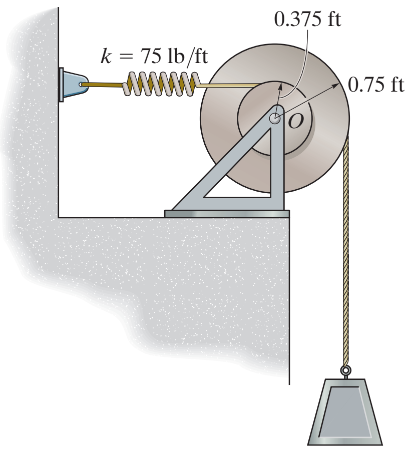


(b)

Ans:
$\omega_{BC} = 1.34$ rad/s

18–54.

If the 250-lb block is released from rest when the spring is unstretched, determine the velocity of the block after it has descended 5 ft. The drum has a weight of 50 lb and a radius of gyration of $k_O = 0.5$ ft about its center of mass O.



SOLUTION

Potential Energy: With reference to the datum shown in Fig. *a*, the gravitational potential energy of the system when the block is at position 1 and 2 is

$$(V_g)_1 = W(y_G)_1 = 250(0) = 0$$

$$(V_g)_2 = -W(y_G)_2 = -250(5) = -1250 \text{ ft} \cdot \text{lb}$$

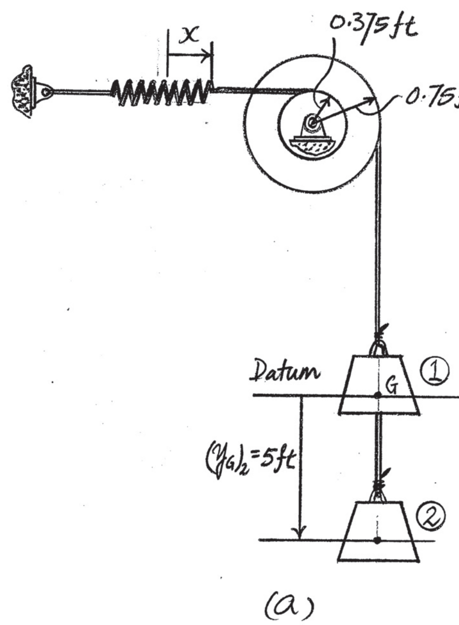
When the block descends $s_b = 5$ ft, the drum rotates through an angle of $\theta = \dfrac{s_b}{r_b} = \dfrac{5}{0.75} = 6.667$ rad. Thus, the stretch of the spring is $x = s + s_0 = r_{sp}\theta + 0 = 0.375(6.667) = 2.5$ ft. The elastic potential energy of the spring is

$$(V_e)_2 = \frac{1}{2}kx^2 = \frac{1}{2}(75)(2.5^2) = 234.375 \text{ ft} \cdot \text{lb}$$

Since the spring is initially unstretched, $(V_e)_1 = 0$. Thus,

$$V_1 = (V_g)_1 + (V_e)_1 = 0$$

$$V_2 = (V_g)_2 + (V_e)_2 = -1250 + 234.375 = -1015.625 \text{ ft} \cdot \text{lb}$$



Kinetic Energy: Since the drum rotates about a fixed axis passing through point O, $\omega = \dfrac{v_b}{r_b} = \dfrac{v_b}{0.75} = 1.333v_b$. The mass moment of inertia of the drum about its mass center is $I_O = mk_O{}^2 = \dfrac{50}{32.2}\left(0.5^2\right) = 0.3882 \text{ slug} \cdot \text{ft}^2$.

$$\begin{aligned} T &= \frac{1}{2}I_O\omega^2 + \frac{1}{2}m_b v_b{}^2 \\ &= \frac{1}{2}(0.3882)(1.333v_b)^2 + \frac{1}{2}\left(\frac{250}{32.2}\right)v_b{}^2 \\ &= 4.2271v_b{}^2 \end{aligned}$$

Since the system is initially at rest, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

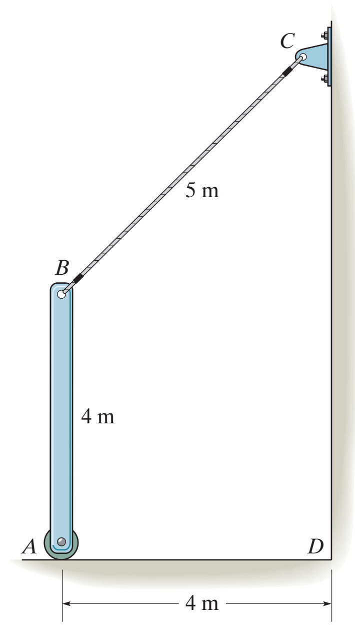
$$0 + 0 = 4.2271v_b{}^2 - 1015.625$$

$$v_b = 15.5 \text{ ft/s} \quad \downarrow \qquad \textbf{Ans.}$$

Ans:
$v_b = 15.5$ ft/s

18–55.

The slender 15-kg bar is initially at rest and standing in the vertical position when the bottom end A is displaced slightly to the right. If the track in which it moves is smooth, determine the speed at which end A strikes the corner D. The bar is constrained to move in the vertical plane. Neglect the mass of the cord BC.



SOLUTION

$$x^2 + y^2 = 5^2$$

$$x^2 + (7 - y)^2 = 4^2$$

Thus, $\quad y = 4.1429 \text{ m}$

$$x = 2.7994 \text{ m}$$

$$(5)^2 = (4)^2 + (7)^2 - 2(4)(7)\cos\phi$$

$$\phi = 44.42^\circ$$

$$h^2 = (2)^2 + (7)^2 - 2(2)(7)\cos 44.42^\circ$$

$$h = 5.745 \text{ m}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 147.15(2) = \frac{1}{2}\left[\frac{1}{12}(15)(4)^2\right]\omega^2 + \frac{1}{2}(15)(5.745\omega)^2 + 147.15\left(\frac{7 - 4.1429}{2}\right)$$

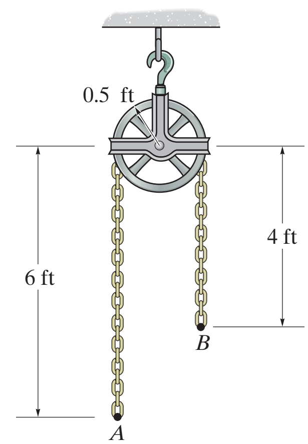
$$\omega = 0.5714 \text{ rad/s}$$

$v_A = 0.5714(7) = 4.00 \text{ m/s}$ **Ans.**

Ans:
$v_A = 4.00 \text{ m/s}$

***18–56.**

If the chain is released from rest from the position shown, determine the angular velocity of the pulley after the end B has risen 2 ft. The pulley has a weight of 50 lb and a radius of gyration of 0.375 ft about its axis. The chain weighs 6 lb/ft.



SOLUTION

Potential Energy: $(y_{G1})_1 = 2$ ft, $(y_{G2})_1 = 3$ ft, $(y_{G1})_2 = 1$ ft, and $(y_{G2})_2 = 4$ ft. With reference to the datum in Fig. *a*, the gravitational potential energy of the chain at position 1 and 2 is

$$V_1 = (V_g)_1 = W_1(y_{G1})_1 - W_2(y_{G2})_1$$

$$= -6(4)(2) - 6(6)(3) = -156 \text{ ft} \cdot \text{lb}$$

$$V_2 = (V_g)_2 = -W_1(y_{G1})_2 + W_2(y_{G2})_2$$

$$= -6(2)(1) - 6(8)(4) = -204 \text{ ft} \cdot \text{lb}$$

Kinetic Energy: Since the system is initially at rest, $T_1 = 0$. The pulley rotates about a fixed axis, thus, $(V_{G1})_2 = (V_{G2})_2 = \omega_2 r = \omega_2(0.5)$. The mass moment of inertia of the pulley about its axis is $I_O = mk_O{}^2 = \dfrac{50}{32.2}(0.375^2) = 0.2184 \text{ slug} \cdot \text{ft}^2$. Thus, the final kinetic energy of the system is

$$T = \frac{1}{2}I_O\omega_2{}^2 + \frac{1}{2}m_1(V_{G1})_2{}^2 + \frac{1}{2}m_2\,(V_{G2})_2{}^2$$

$$= \frac{1}{2}(0.2184)\omega_2{}^2 + \frac{1}{2}\left[\frac{6(2)}{32.2}\right][\omega_2(0.5)]^2 + \frac{1}{2}\left[\frac{6(8)}{32.2}\right][\omega_2(0.5)]^2 + \frac{1}{2}\left[\frac{6(0.5)(\pi)}{32.2}\right][\omega_2(0.5)]^2$$

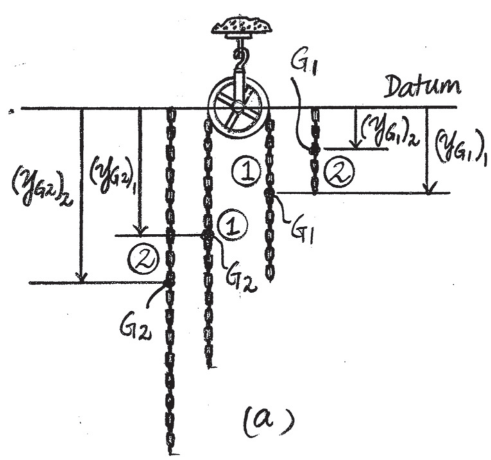
$$= 0.3787\omega_2{}^2$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-156) = 0.3787\omega_2^2 = (-204)$$

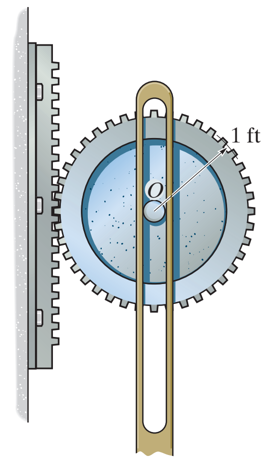
$$\omega_2 = 11.3 \text{ rad/s}$$ **Ans.**



Ans:
$\omega_2 = 11.3$ rad/s

18–57.

If the gear is released from rest, determine its angular velocity after its center of gravity O has descended a distance of 4 ft. The gear has a weight of 100 lb and a radius of gyration about its center of gravity of $k = 0.75$ ft.



SOLUTION

Potential Energy: With reference to the datum in Fig. *a*, the gravitational potential energy of the gear at position 1 and 2 is

$$V_1 = (V_g)_1 = W(y_0)_1 = 100(0) = 0$$

$$V_2 = (V_g)_2 = -W_1(y_0)_2 = -100(4) = -400 \text{ ft} \cdot \text{lb}$$

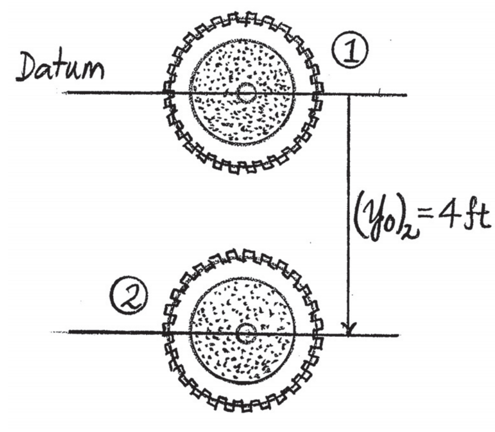
Kinetic Energy: Referring to Fig. *b*, we obtain $v_O = \omega r_{O/IC} = \omega(1)$. The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2 = \dfrac{100}{32.2}(0.75^2) = 1.7469 \text{ kg} \cdot \text{m}^2$.

Thus,

$$T = \frac{1}{2}mv_O{}^2 + \frac{1}{2}I_O\omega^2$$

$$= \frac{1}{2}\left(\frac{100}{32.2}\right)[\omega(1)]^2 + \frac{1}{2}(1.7469)\omega^2$$

$$= 2.4262\omega^2$$



(a)

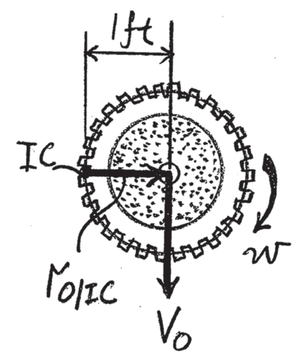
Since the gear is initially at rest, $T_1 = 0$.

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 2.4262\omega^2 - 400$$

$$\omega = 12.8 \text{ rad/s} \qquad \textbf{Ans.}$$

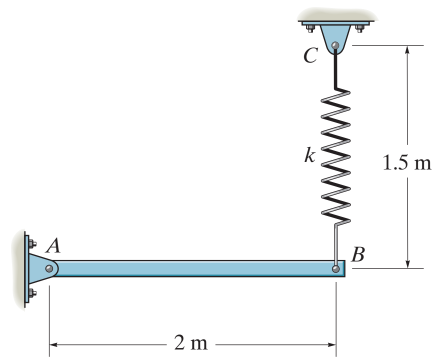


(b)

Ans:
$\omega = 12.8$ rad/s

18–58.

The slender 6-kg bar AB is horizontal and at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated clockwise 90° after being released.



SOLUTION

Kinetic Energy. The mass moment of inertia of the bar about A is

$I_A = \frac{1}{12}(6)(2^2) + 6(1^2) = 8.00 \text{ kg} \cdot \text{m}^2$. Then

$$T = \frac{1}{2} I_A \,\omega^2 = \frac{1}{2}(8.00)\,\omega^2 = 4.00\,\omega^2$$

Since the bar is at rest initially and required to stop finally, $T_1 = T_2 = 0$.

Potential Energy. With reference to the datum set in Fig. a, the gravitational potential energies of the bar when it is at positions ① and ② are

$$(V_g)_1 = mgy_1 = 0$$

$$(V_g)_2 = mgy_2 = 6(9.81)(-1) = -58.86 \text{ J}$$

The stretch of the spring when the bar is at position ② is

$$x_2 = \sqrt{2^2 + 3.5^2} - 1.5 = 2.5311 \text{ m}$$

Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = 0$$

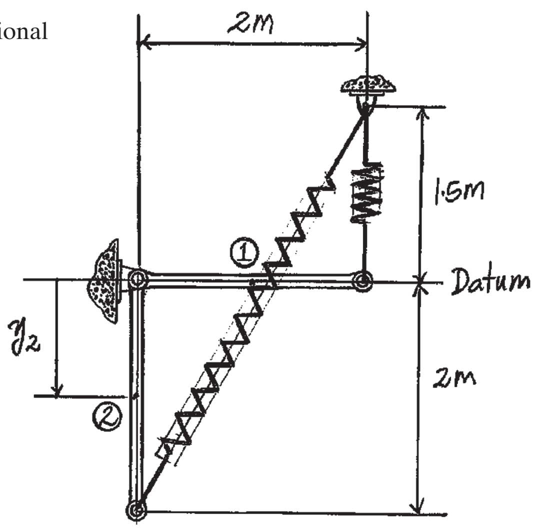
$$(V_e)_2 = \frac{1}{2}k(2.5311^2) = 3.2033k$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0 + 0) = 0 + (-58.86) + 3.2033k$$

$$k = 18.3748 \text{ N/m} = 18.4 \text{ N/m}$$ **Ans.**

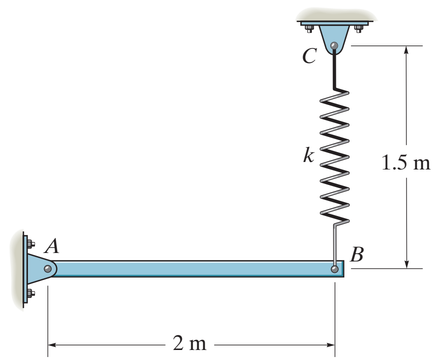


(a)

Ans:
$k = 18.4$ N/m

18–59.

The slender 6-kg bar AB is horizontal and at rest and the spring is unstretched. Determine the angular velocity of the bar when it has rotated clockwise 45° after being released. The spring has a stiffness of $k = 12$ N/m.



SOLUTION

Kinetic Energy. The mass moment of inertia of the bar about A is $I_A = \frac{1}{12}(6)(2^2) + 6(1^2) = 8.00\ \text{kg}\cdot\text{m}^2$. Then

$$T = \frac{1}{2}I_A\,\omega^2 = \frac{1}{2}(8.00)\,\omega^2 = 4.00\,\omega^2$$

Since the bar is at rest initially, $T_1 = 0$.

Potential Energy. with reference to the datum set in Fig. *a*, the gravitational potential energies of the bar when it is at positions ① and ② are

$$(V_g)_1 = mgy_1 = 0$$

$$(V_g)_2 = mgy_2 = 6(9.81)(-1\sin 45°) = -41.62\ \text{J}$$

From the geometry shown in Fig. *a*,

$$a = \sqrt{2^2 + 1.5^2} = 2.5\ \text{m} \qquad \phi = \tan^{-1}\left(\frac{1.5}{2}\right) = 36.87°$$

Then, using cosine law,

$$l = \sqrt{2.5^2 + 2^2 - 2(2.5)(2)\cos(45° + 36.87°)} = 2.9725\ \text{m}$$

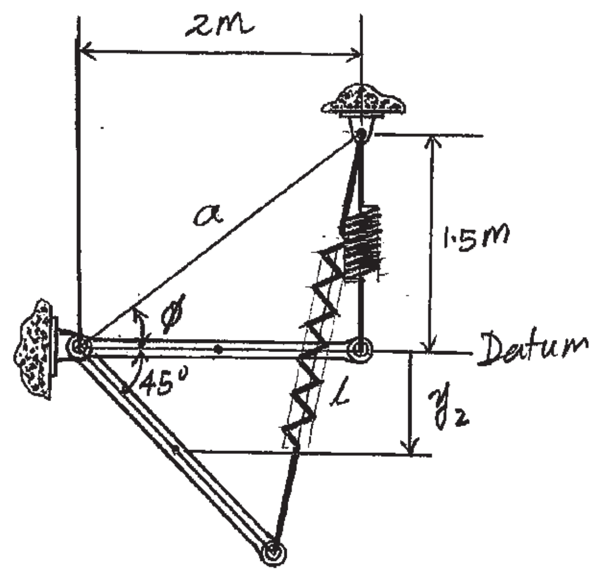
Thus, the stretch of the spring when the bar is at position ② is

$$x_2 = 2.9725 - 1.5 = 1.4725\ \text{m}$$

Thus, the initial and final elastic potential energies of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = 0$$

$$(V_e)_2 = \frac{1}{2}(12)(1.4725^2) = 13.01\ \text{J}$$



(a)

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0 + 0) = 4.00\,\omega^2 + (-41.62) + 13.01$$

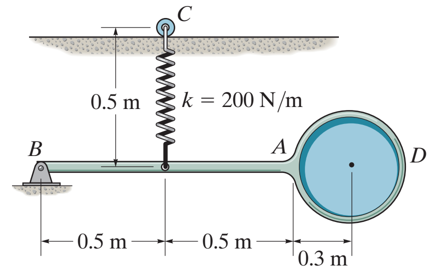
$$\omega = 2.6744\ \text{rad/s} = 2.67\ \text{rad/s}$$

Ans.

Ans:
$\omega = 2.67$ rad/s

***18–60.**

The pendulum consists of a 6-kg slender rod fixed to a 15-kg disk. If the spring has an unstretched length of 0.2 m, determine the angular velocity of the pendulum when it is released from rest and rotates clockwise 90° from the position shown. The roller at C allows the spring to always remain vertical.



SOLUTION

Kinetic Energy. The mass moment of inertia of the pendulum about B is $I_B = \left[\frac{1}{12}(6)(1^2) + 6(0.5^2)\right] + \left[\frac{1}{2}(15)(0.3^2) + 15(1.3^2)\right] = 28.025\ \text{kg}\cdot\text{m}^2$. Thus

$$T = \frac{1}{2}I_B\,\omega^2 = \frac{1}{2}(28.025)\,\omega^2 = 14.0125\,\omega^2$$

Since the pendulum is released from rest, $T_1 = 0$.

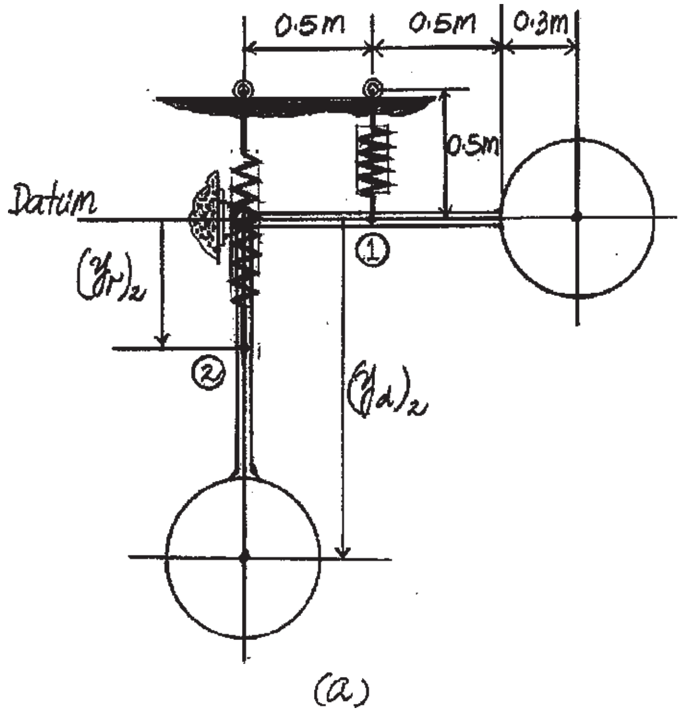
Potential Energy. with reference to the datum set in Fig. a, the gravitational potential energies of the pendulum when it is at positions ① and ② are

$$(V_g)_1 = m_r g(y_r)_1 + m_d g(y_d)_1 = 0$$

$$(V_g)_2 = m_r g(y_r)_2 + m_d g(y_d)_2$$

$$= 6(9.81)(-0.5) + 15(9.81)(-1.3)$$

$$= -220.725\ \text{J}$$



The stretch of the spring when the pendulum is at positions ① and ② are

$$x_1 = 0.5 - 0.2 = 0.3\ \text{m}$$

$$x_2 = 1 - 0.2 = 0.8\ \text{m}$$

Thus, the initial and final elastic potential energies of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(200)(0.3^2) = 9.00\ \text{J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(200)(0.8^2) = 64.0\ \text{J}$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0 + 9.00) = 14.0125\omega^2 + (-220.725) + 64.0$$

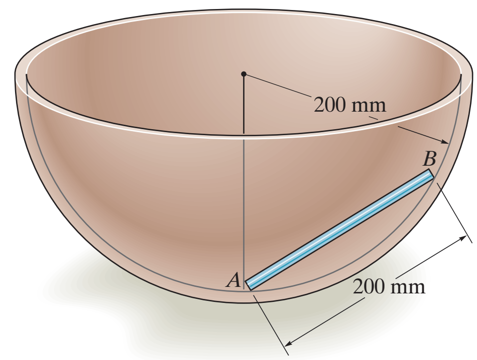
$$\omega = 3.4390\ \text{rad/s} = 3.44\ \text{rad/s}$$

Ans.

Ans:
$\omega = 3.44\ \text{rad/s}$

18–61.

The 500-g rod AB rests along the smooth inner surface of a hemispherical bowl. If the rod is released from rest from the position shown, determine its angular velocity at the instant it swings downward and becomes horizontal.



SOLUTION

Select datum at the bottom of the bowl.

$$\theta = \sin^{-1}\left(\frac{0.1}{0.2}\right) = 30^\circ$$

$$h = 0.1 \sin 30^\circ = 0.05$$

$$CE = \sqrt{(0.2)^2 - (0.1)^2} = 0.1732 \text{ m}$$

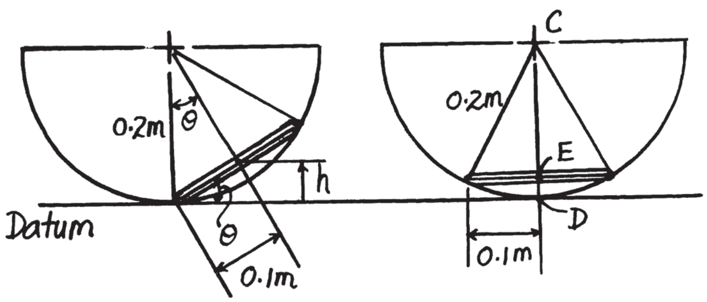
$$ED = 0.2 - 0.1732 = 0.02679$$

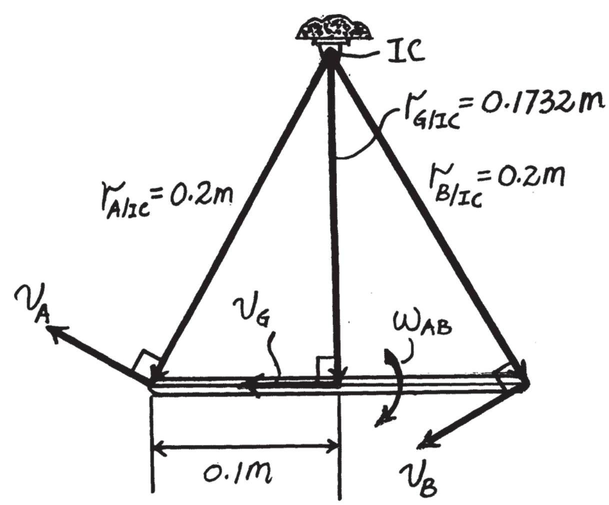
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0.5)(9.81)(0.05) = \frac{1}{2}\left[\frac{1}{12}(0.5)(0.2)^2\right]\omega_{AB}^2 + \frac{1}{2}(0.5)(v_G)^2 + (0.5)(9.81)(0.02679)$$

Since $v_G = 0.1732\omega_{AB}$

$$\omega_{AB} = 3.70 \text{ rad/s} \qquad \textbf{Ans.}$$

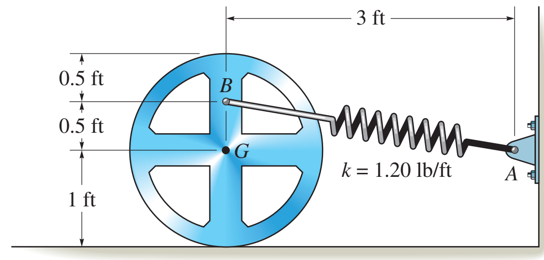




Ans:
$\omega_{AB} = 3.70$ rad/s

18–62.

The 50-lb wheel has a radius of gyration about its center of gravity G of $k_G = 0.7$ ft. If it rolls without slipping, determine its angular velocity when it has rotated clockwise 90° from the position shown. The spring AB has a stiffness $k = 1.20$ lb/ft and an unstretched length of 0.5 ft. The wheel is released from rest.



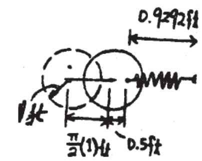
SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(1.20)\left[\sqrt{(3)^2 + (0.5)^2} - 0.5\right]^2 = \frac{1}{2}\left[\frac{50}{32.2}(0.7)^2\right]\omega^2 + \frac{1}{2}\left(\frac{50}{32.2}\right)(1\omega)^2$$

$$+ \frac{1}{2}(1.20)(0.9292 - 0.5)^2$$

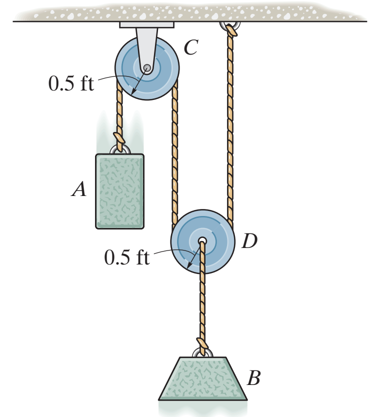
$$\omega = 1.80 \text{ rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega = 1.80$ rad/s

18–63.

The system consists of 60-lb and 20-lb blocks A and B, respectively, and 5-lb pulleys C and D that can be treated as thin disks. Determine the speed of block A after block B has risen 5 ft, starting from rest. Assume that the cord does not slip on the pulleys, and neglect the mass of the cord.



SOLUTION

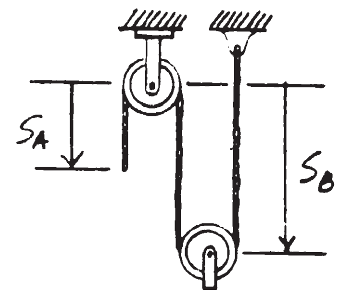
Kinematics: The speed of block A and B can be related using the position coordinate equation.

$$s_A + 2s_B = l \qquad \textbf{(1)}$$

$$\Delta s_A + 2\Delta s_B = 0 \qquad \Delta s_A + 2(5) = 0 \qquad \Delta s_A = -10 \text{ ft} = 10 \text{ ft}\downarrow$$

Taking time derivative of Eq. (1), we have

$$v_A + 2v_B = 0 \qquad v_B = -0.5v_A$$



Potential Energy: Datum is set at fixed pulley C. When blocks A and B (pulley D) are at their initial position, their centers of gravity are located at s_A and s_B. Their initial gravitational potential energies are $-60s_A$, $-20s_B$, and $-5s_B$. When block B (pulley D) rises 5 ft, block A decends 10 ft. Thus, the final position of blocks A and B (pulley D) are $(s_A + 10)$ ft and $(s_B - 5)$ ft *below* datum. Hence, their respective final gravitational potential energy are $-60(s_A + 10)$, $-20(s_B - 5)$, and $-5(s_B - 5)$. Thus, the initial and final potential energy are

$$V_1 = -60s_A - 20s_B - 5s_B = -60s_A - 25s_B$$

$$V_2 = -60(s_A + 10) - 20(s_B - 5) - 5(s_B - 5) = -60s_A - 25s_B - 475$$

Kinetic Energy: The mass moment inertia of the pulley about its mass center is $I_G = \frac{1}{2}\left(\frac{5}{32.2}\right)(0.5^2) = 0.01941 \text{ slug}\cdot\text{ft}^2$. Since pulley D rolls without slipping, $\omega_D = \frac{v_B}{r_D} = \frac{v_B}{0.5} = 2v_B = 2(-0.5v_A) = -v_A$. Pulley C rotates about the fixed point hence $\omega_C = \frac{v_A}{r_C} = \frac{v_A}{0.5} = 2v_A$. Since the system is at initially rest, the initial kinectic energy is $T_1 = 0$. The final kinetic energy is given by

$$T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}m_D v_B^2 + \frac{1}{2}I_G\omega_D^2 + \frac{1}{2}I_G\,\omega_C^2$$

$$= \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)(-0.5v_A)^2 + \frac{1}{2}\left(\frac{5}{32.2}\right)(-0.5v_A)^2$$

$$+ \frac{1}{2}(0.01941)(-v_A)^2 + \frac{1}{2}(0.01941)(2v_A)^2$$

$$= 1.0773v_A^2$$

Conservation of Energy: Applying Eq. 18–19, we have

$$T_1 + V_1 = T_2 + V_2$$

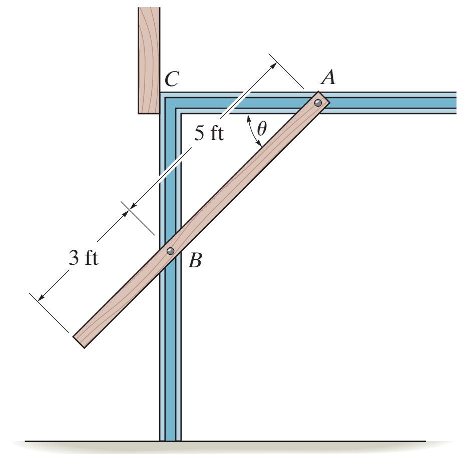
$$0 + (-60s_A - 25s_B) = 1.0773v_A^2 + (-60s_A - 25s_B - 475)$$

$$v_4 = 21.0 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 21.0$ ft/s

***18–64.**

The door is made from one piece, whose ends move along the horizontal and vertical tracks. If the door is in the open position, $\theta = 0°$, and then released, determine the speed at which its end A strikes the stop at C. Assume the door is a 180-lb thin plate having a width of 10 ft.



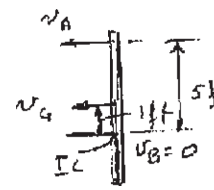
SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left[\frac{1}{12}\left(\frac{180}{32.2}\right)(8)^2\right]\omega^2 + \frac{1}{2}\left(\frac{180}{32.2}\right)(1\omega)^2 - 180(4)$$

$$\omega = 6.3776 \text{ rad/s}$$

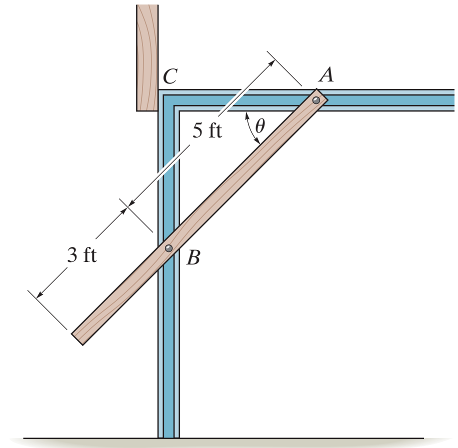
$$v_c = \omega(5) = 6.3776(5) = 31.9 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v_c = 31.9$ m/s

18–65.

The door is made from one piece, whose ends move along the horizontal and vertical tracks. If the door is in the open position, $\theta = 0°$, and then released, determine its angular velocity at the instant $\theta = 30°$. Assume the door is a 180-lb thin plate having a width of 10 ft.



SOLUTION

$T_1 + V_1 = T_2 + V_2$

$$0 + 0 = \frac{1}{2}\left[\frac{1}{12}\left(\frac{180}{32.2}\right)(8)^2\right]\omega^2 + \frac{1}{2}\left(\frac{180}{32.2}\right)v_G^2 - 180(4 \sin 30°) \qquad \textbf{(1)}$$

$r_{IC-G} = \sqrt{(1)^2 + (4.3301)^2 - 2(1)(4.3301)\cos 30°}$

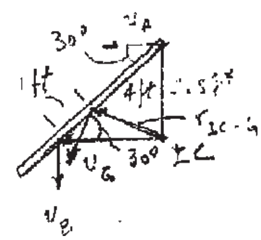
$r_{IC-G} = 3.50 \text{ m}$

Thus,

$v_G = 3.50\,\omega$

Substitute into Eq. (1) and solving,

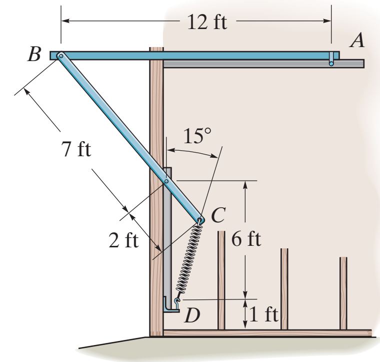
$\omega = 2.71 \text{ rad/s}$ **Ans.**



Ans:
$\omega = 2.71 \text{ rad/s}$

18–66.

The end A of the garage door AB travels along the horizontal track, and the end of member BC is attached to a spring at C. If the spring is originally unstretched, determine the stiffness k so that when the door falls downward from rest in the position shown, it will have zero angular velocity the moment it closes, i.e., when it and BC become vertical. Neglect the mass of member BC and assume the door is a thin plate having a weight of 200 lb and a width and height of 12 ft. There is a similar connection and spring on the other side of the door.



SOLUTION

$(2)^2 = (6)^2 + (CD)^2 - 2(6)(CD)\cos 15°$

$CD^2 - 11.591CD + 32 = 0$

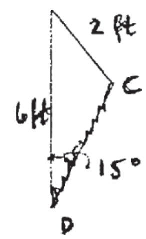
Selecting the smaller root:

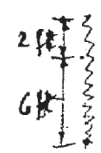
$CD = 4.5352$ ft

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = 0 + 2\left[\frac{1}{2}(k)(8 - 4.5352)^2\right] - 200(6)$

$k = 100$ lb/ft **Ans.**

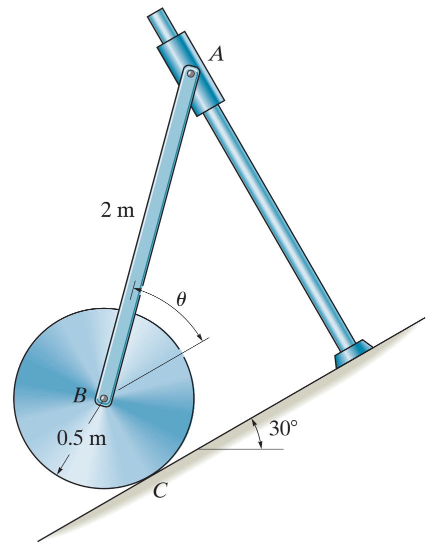




Ans:
$k = 100$ lb/ft

18–67.

The system consists of a 30-kg disk, 12-kg slender rod BA, and a 5-kg smooth collar A. If the disk rolls without slipping, determine the velocity of the collar at the instant $\theta = 0°$. The system is released from rest when $\theta = 45°$.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of the rod at its final position, Fig. *a*, we found that IC is located at B. Thus, $(v_B)_2 = 0$. Also

$$(v_A)_2 = (\omega_r)_2 r_{A/IC}; \qquad (v_A)_2 = (\omega_r)_2(2) \qquad (\omega_r)_2 = \frac{(v_A)_2}{2}$$

Then

$$(v_{Gr})_2 = (\omega_r)_2(r_{Gr/IC}); \qquad (v_{Gr})_2 = \frac{(v_A)_2}{2}(1) = \frac{(v_A)_2}{2}$$

For the disk, since the velocity of its center $(v_B)_2 = 0$, $(\omega_d)_2 = 0$. Thus,

$$T_2 = \frac{1}{2}m_r(v_{Gr})_2^2 + \frac{1}{2}I_{Gr}(\omega_r)_2^2 + \frac{1}{2}m_c(v_A)_2^2$$

$$= \frac{1}{2}(12)\left[\frac{(v_A)_2}{2}\right]^2 + \frac{1}{2}\left[\frac{1}{12}(12)(2^2)\right]\left[\frac{(v_A)_2}{2}\right]^2 + \frac{1}{2}(5)(v_A)_2^2$$

$$= 4.50(v_A)_2^2$$

Potential Energy. Datum is set as shown in Fig. *a*. Here,

$$S_B = 2 - 2\cos 45° = 0.5858 \text{ m}$$

Then

$$(y_d)_1 = 0.5858 \sin 30° = 0.2929 \text{ m}$$

$$(y_r)_1 = 0.5858 \sin 30° + 1 \sin 75° = 1.2588 \text{ m}$$

$$(y_r)_2 = 1 \sin 30° = 0.5 \text{ m}$$

$$(y_c)_1 = 0.5858 \sin 30° + 2 \sin 75° = 2.2247 \text{ m}$$

$$(y_c)_2 = 2 \sin 30° = 1.00 \text{ m}$$

Thus, the gravitational potential energies of the disk, rod and collar at the initial and final positions are

$$(V_d)_1 = m_d g(y_d)_1 = 30(9.81)(0.2929) = 86.20 \text{ J}$$

$$(V_d)_2 = m_d g(y_d)_2 = 0$$

$$(V_r)_1 = m_r g(y_r)_1 = 12(9.81)(1.2588) = 148.19 \text{ J}$$

$$(V_r)_2 = m_r g(y_r)_2 = 12(9.81)(0.5) = 58.86 \text{ J}$$

$$(V_c)_1 = m_c g(y_c)_1 = 5(9.81)(2.2247) = 109.12 \text{ J}$$

$$(V_c)_2 = m_c g(y_c)_2 = 5(9.81)(1.00) = 49.05 \text{ J}$$

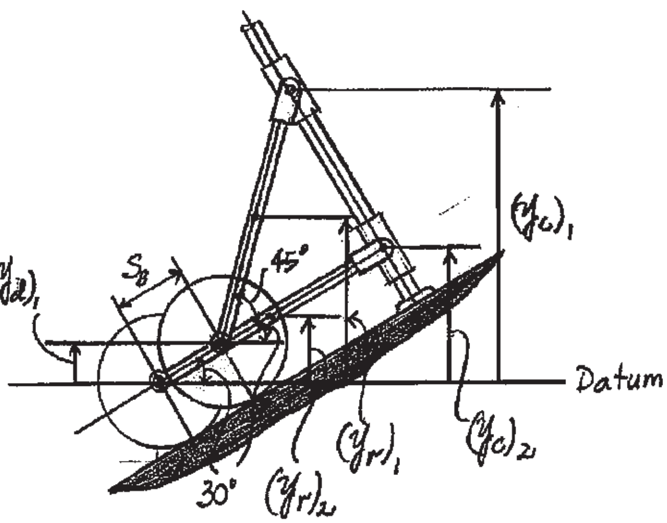
18–67. Continued

Conservation of Energy.

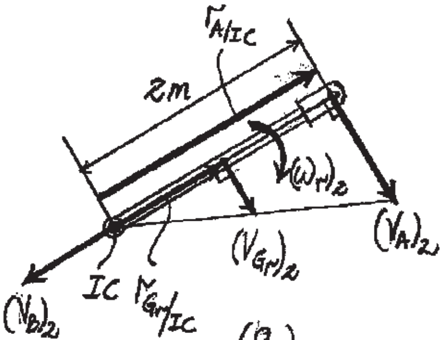
$T_1 + V_1 = T_2 + V_2$

$0 + (86.20 + 148.19 + 109.12) = 4.50(v_A)_2^2 + (0 + 58.86 + 49.05)$

$(v_A)_2 = 7.2357\ \text{m/s} = 7.24\ \text{m/s}$ **Ans.**



(b)

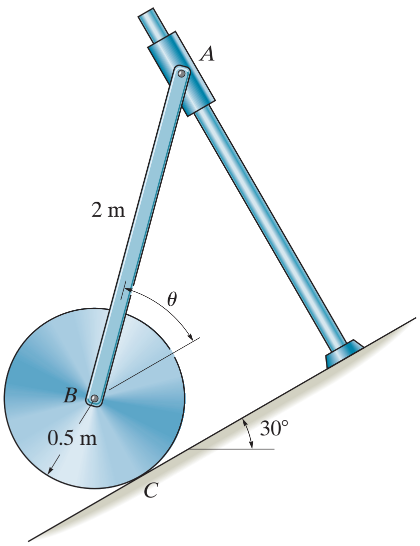


(a)

Ans:
$(v_A)_2 = 7.24\ \text{m/s}$

***18–68.**

The system consists of a 30-kg disk A, 12-kg slender rod BA, and a 5-kg smooth collar A. If the disk rolls without slipping, determine the velocity of the collar at the instant $\theta = 30°$. The system is released from rest when $\theta = 45°$.



SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. Referring to the kinematics diagram of the rod at final position with IC so located, Fig. a,

$$r_{A/IC} = 2\cos 30° = 1.7321 \text{ m} \qquad r_{B/IC} = 2\cos 60° = 1.00 \text{ m}$$

$$r_{Gr/IC} = \sqrt{1^2 + 1.00^2 - 2(1)(1.00)\cos 60°} = 1.00 \text{ m}$$

Then

$$(v_A)_2 = (\omega_r)_2(r_{A/IC}); \qquad (v_A)_2 = (\omega_r)_2(1.7321) \qquad (\omega_r)_2 = 0.5774(v_A)_2$$

$$(v_B)_2 = (\omega_r)_2(r_{B/IC}); \qquad (v_B)_2 = [0.5774(v_A)_2](1.00) = 0.5774(v_A)_2$$

$$(v_{Gr})_2 = (\omega_r)_2(r_{Gr/IC}); \qquad (v_{Gr})_2 = [0.5774(v_A)_2](1.00) = 0.5774(v_A)_2$$

Since the disk rolls without slipping,

$$(v_B)_2 = \omega_d r_d; \qquad 0.5774(v_A)_2 = (\omega_d)_2(0.5)$$

$$(\omega_d)_2 = 1.1547(v_A)_2$$

Thus, the kinetic energy of the system at final position is

$$T_2 = \frac{1}{2}m_r(v_{Gr})_2^2 + \frac{1}{2}I_{Gr}(\omega_r)_2^2 + \frac{1}{2}m_d(v_B)_2^2 + \frac{1}{2}I_B(\omega_d)_2^2 + \frac{1}{2}m_c(v_A)_2^2$$

$$= \frac{1}{2}(12)[0.5774(v_A)_2]^2 + \frac{1}{2}\left[\frac{1}{12}(12)(2^2)\right][0.5774(v_A)_2]^2$$

$$+ \frac{1}{2}(3.0)[0.5774(v_A)_2]^2 + \frac{1}{2}\left[\frac{1}{2}(30)(0.5^2)\right][1.1547(v_A)_2]^2$$

$$+ \frac{1}{2}(5)(v_A)_2^2$$

$$= 12.6667(v_A)_2^2$$

Potential Energy. Datum is set as shown in Fig. a. Here,

$$S_B = 2\cos 30° - 2\cos 45° = 0.3178 \text{ m}$$

Then

$$(y_d)_1 = 0.3178 \sin 30° = 0.1589 \text{ m}$$

$$(y_r)_1 = 0.3178 \sin 30° + 1 \sin 75° = 1.1248 \text{ m}$$

$$(y_r)_2 = 1 \sin 60° = 0.8660 \text{ m}$$

$$(y_c)_1 = 0.3178 \sin 30° + 2 \sin 75° = 2.0908 \text{ m}$$

$$(y_c)_2 = 2 \sin 60° = 1.7321 \text{ m}$$

***18–68. Continued**

Thus, the gravitational potential energies of the disk, rod and collar at initial and final position are

$$(V_d)_1 = m_d g(y_d)_1 = 30(9.81)(0.1589) = 46.77 \text{ J}$$

$$(V_d)_2 = m_d g(y_d)_2 = 0$$

$$(V_r)_1 = m_r g(y_r)_1 = 12(9.81)(1.1248) = 132.42 \text{ J}$$

$$(V_r)_2 = m_r g(y_r)_2 = 12(9.81)(0.8660) = 101.95 \text{ J}$$

$$(V_c)_1 = m_c g(y_c)_1 = 5(9.81)(2.0908) = 102.55 \text{ J}$$

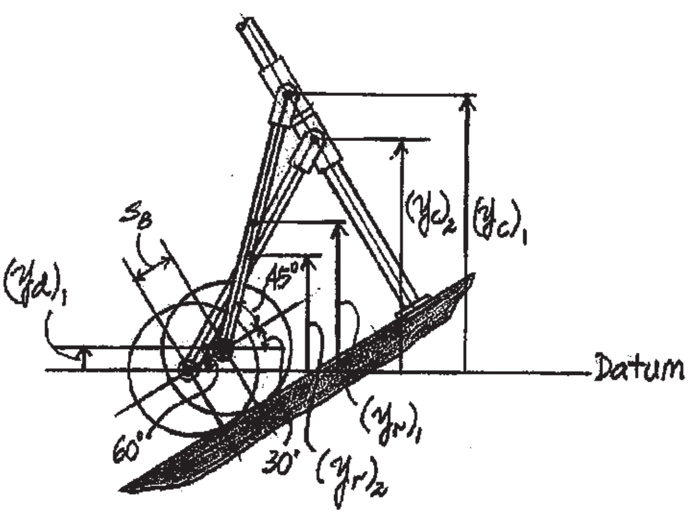
$$(V_c)_2 = m_c g(y_c)_2 = 5(9.81)(1.7321) = 84.96 \text{ J}$$

Conservation of Energy.

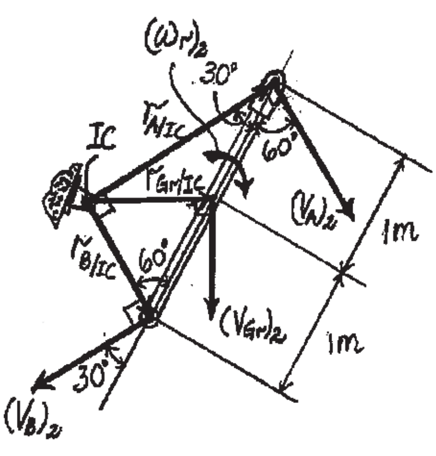
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (46.77 + 132.42 + 102.55) = 12.6667(v_A)_2^2 + (0 + 101.95 + 84.96)$$

$$(v_A)_2 = 2.7362 \text{ m/s} = 2.74 \text{ m/s} \qquad \textbf{Ans.}$$



(b)

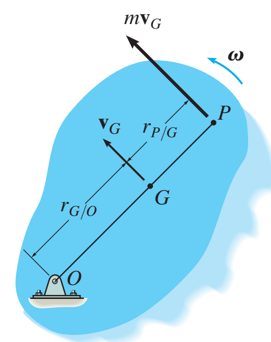


(a)

Ans:
$(v_A)_2 = 2.74 \text{ m/s}$

19–1.

The rigid body (slab) has a mass m and rotates with an angular velocity $\boldsymbol{\omega}$ about an axis passing through the fixed point O. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude mv_G and acting through point P, called the *center of percussion,* which lies at a distance $r_{P/G} = k_G^2/r_{G/O}$ from the mass center G. Here k_G is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through G.



SOLUTION

$$H_O = (r_{G/O} + r_{P/G})\, mv_G = r_{G/O}\,(mv_G) + I_G\omega, \qquad \text{where} \quad I_G = mk_G^2$$

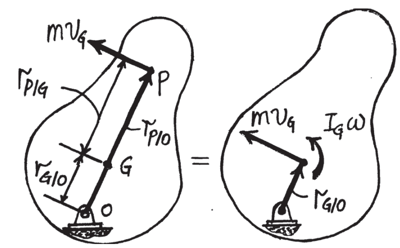
$$r_{G/O}\,(mv_G) + r_{P/G}\,(mv_G) = r_{G/O}\,(mv_G) + (mk_G^2)\,\omega$$

$$r_{P/G} = \frac{k_G^2}{v_G/\omega}$$

However, $v_G = \omega r_{G/O}$ or $r_{G/O} = \dfrac{v_G}{\omega}$

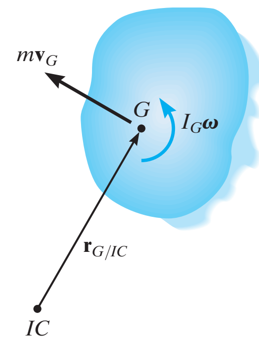
$$r_{P/G} = \frac{k_G^2}{r_{G/O}}$$

Q.E.D.



19–2.

At a given instant, the body has a linear momentum $\mathbf{L} = m\mathbf{v}_G$ and an angular momentum $\mathbf{H}_G = I_G\boldsymbol{\omega}$ computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity IC can be expressed as $\mathbf{H}_{IC} = I_{IC}\boldsymbol{\omega}$, where I_{IC} represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the IC is located at a distance $r_{G/IC}$ away from the mass center G.



SOLUTION

$$H_{IC} = r_{G/IC}\,(mv_G) + I_G\,\omega, \qquad \text{where} \quad v_G = \omega r_{G/IC}$$

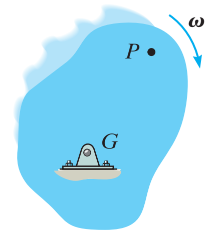
$$= r_{G/IC}\,(m\omega r_{G/IC}) + I_G\,\omega$$

$$= (I_G + mr^2_{G/IC})\,\omega$$

$$= I_{IC}\,\omega \qquad \textbf{Q.E.D.}$$

19–3.

Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center G, the angular momentum is the same when computed about any other point P.



SOLUTION

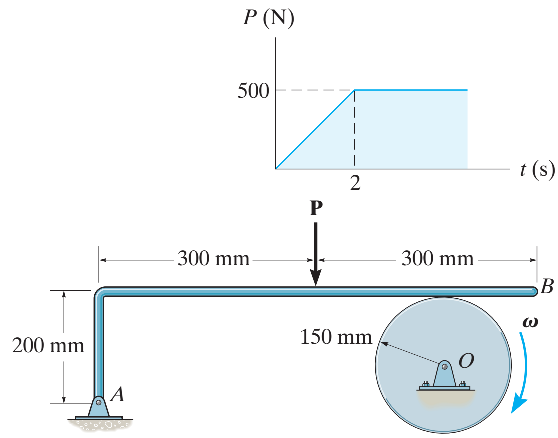
Since $v_G = 0$, the linear momentum $L = mv_G = 0$. Hence the angular momentum about any point P is

$$H_P = I_G\,\omega$$

Since ω is a free vector, so is $\mathbf{H}_P$. **Q.E.D.**

***19–4.**

The 40-kg disk is rotating at $\boldsymbol{\omega} = 100$ rad/s. When the force $\mathbf{P}$ is applied to the brake as indicated by the graph. If the coefficient of kinetic friction at B is $\mu_k = 0.3$, determine the time t needed to stay the disk from rotating. Neglect the thickness of the brake.



SOLUTION

Equilibrium. Since slipping occurs at brake pad, $F_f = \mu_k N = 0.3\,N$. Referring to the FBD the brake's lever, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad N(0.6) - 0.3\,N(0.2) - P(0.3) = 0$$

$$N = 0.5556\,P$$

Thus,

$$F_f = 0.3(0.5556\,P) = 0.1667\,P$$

Principle of Impulse and Momentum. The mass moment of inertia of the disk about its center O is $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(40)\left(0.15^2\right) = 0.45\ \text{kg}\cdot\text{m}^2$.

$$I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O dt = I_O\omega_2$$

It is required that $\omega_2 = 0$. Assuming that $t > 2$ s,

$$0.45(100) + \int_0^t [-0.1667\,P(0.15)]dt = 0.45(0)$$

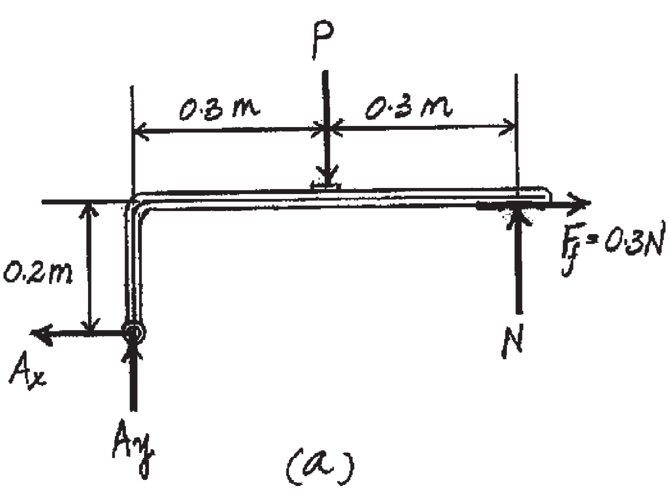
$$0.025\int_0^t P\,dt = 45$$

$$\int_0^t P\,dt = 1800$$

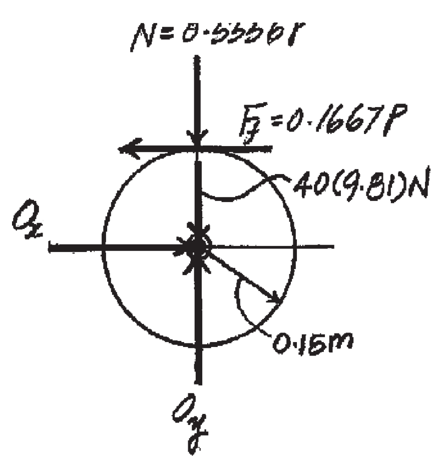
$$\frac{1}{2}(500)(2) + 500(t - 2) = 1800$$

$$t = 4.60 \text{ s} \qquad \textbf{Ans.}$$

Since $t > 2$ s, the assumption was correct.



(a)

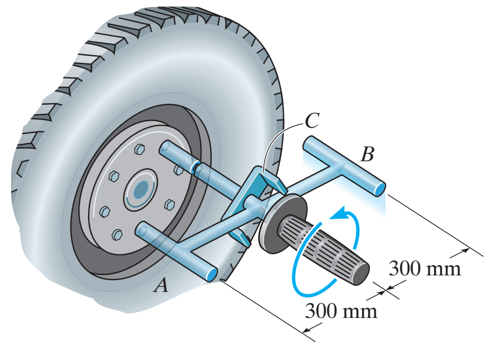


(b)

Ans:
$t = 4.60$ s

19–5.

The impact wrench consists of a slender 1-kg rod AB which is 580 mm long, and cylindrical end weights at A and B that each have a diameter of 20 mm and a mass of 1 kg. This assembly is free to turn about the handle and socket, which are attached to the lug nut on the wheel of a car. If the rod AB is given an angular velocity of 4 rad/s and it strikes the bracket C on the handle without rebounding, determine the angular impulse imparted to the lug nut.



SOLUTION

$$I_{\text{axle}} = \frac{1}{12}(1)(0.6 - 0.02)^2 + 2\left[\frac{1}{2}(1)(0.01)^2 + 1(0.3)^2\right] = 0.2081 \text{ kg} \cdot \text{m}^2$$

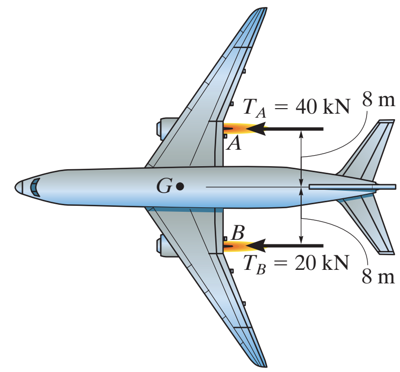
$$\int M\,dt = I_{\text{axle}}\,\omega = 0.2081(4) = 0.833 \text{ kg} \cdot \text{m}^2/\text{s}$$ **Ans.**

Ans:

$$\int M\,dt = 0.833 \text{ kg} \cdot \text{m}^2/\text{s}$$

19–6.

The airplane is traveling in a straight line with a speed of 300 km/h, when the engines A and B produce a thrust of $T_A = 40$ kN and $T_B = 20$ kN, respectively. Determine the angular velocity of the airplane in $t = 5$ s. The plane has a mass of 200 Mg, its center of mass is located at G, and its radius of gyration about G is $k_G = 15$ m.



SOLUTION

Principle of Angular Impulse and Momentum: The mass moment of inertia of the airplane about its mass center is $I_G = mk_G^2 = 200(10^3)(15^2) = 45(10^6)\text{ kg}\cdot\text{m}^2$. Applying the angular impulse and momentum equation about point G,

$$I_z\omega_1 + \Sigma\int_{t_1}^{t_2} M_G dt = I_G\omega_2$$

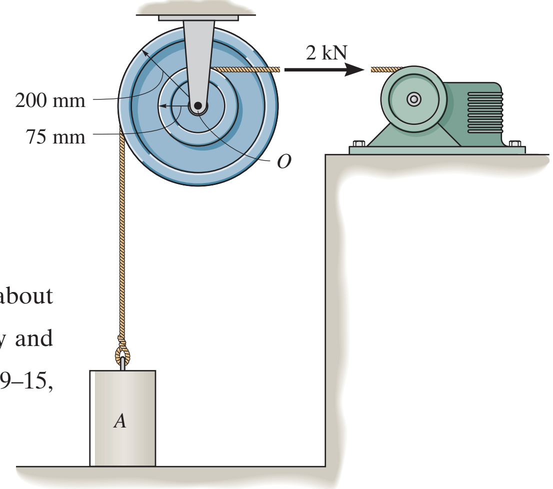
$$0 + 40(10^3)(5)(8) - 20(10^3)(5)(8) = 45(10^6)\omega$$

$$\omega = 0.0178 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 0.0178$ rad/s

19–7.

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration of $k_O = 110$ mm. If the block at A has a mass of 40 kg, determine the speed of the block in 3 s after a constant force of 2 kN is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest.



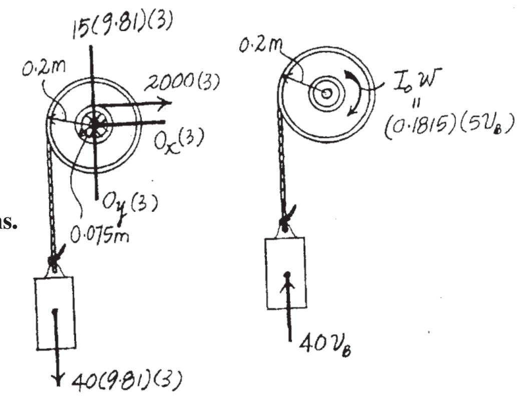
SOLUTION

Principle of Impulse and Momentum: The mass moment inertia of the pulley about point O is $I_O = 15\left(0.11^2\right) = 0.1815\ \text{kg}\cdot\text{m}^2$. The angular velocity of the pulley and the velocity of the block can be related by $\omega = \dfrac{v_B}{0.2} = 5v_B$. Applying Eq. 19–15, we have

$$\left(\sum \text{syst. angular momentum}\right)_{O_1} + \left(\sum \text{syst. angular impulse}\right)_{O_{1-2}} = \left(\sum \text{syst. angular momentum}\right)_{O_2}$$

$$(\circlearrowleft +) \qquad 0 + [40(9.81)(3)](0.2) - [2000(3)](0.075) = -40v_B(0.2) - 0.1815(5v_B)$$

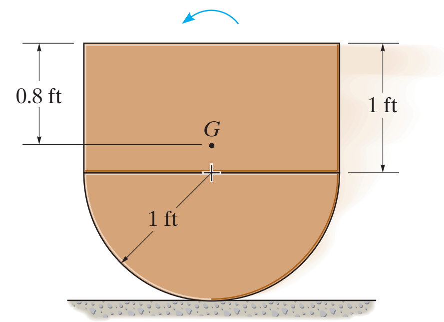
$$v_B = 24.1\ \text{m/s} \qquad \textbf{Ans.}$$



Ans:
$v_B = 24.1$ m/s

***19–8.**

The assembly weighs 10 lb and has a radius of gyration $k_G = 0.6$ ft about its center of mass G. The kinetic energy of the assembly is 31 ft·lb when it is in the position shown. If it is rolling counterclockwise on the surface without slipping, determine its linear momentum at this instant.



SOLUTION

$$I_G = (0.6)^2\left(\frac{10}{32.2}\right) = 0.1118 \text{ slug}\cdot\text{ft}^2$$

$$T = \frac{1}{2}\left(\frac{10}{32.2}\right)v_G^2 + \frac{1}{2}(0.1118)\,\omega^2 = 31 \qquad \textbf{(1)}$$

$$v_G = 1.2\,\omega$$

Substitute into Eq. (1),

$$\omega = 10.53 \text{ rad/s}$$

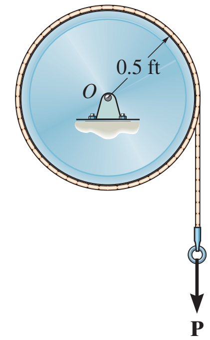
$$v_G = 10.53(1.2) = 12.64 \text{ ft/s}$$

$$L = mv_G = \frac{10}{32.2}(12.64) = 3.92 \text{ slug}\cdot\text{ft/s} \qquad \textbf{Ans.}$$

Ans:
$L = 3.92 \text{ slug}\cdot\text{ft/s}$

19–9.

The disk has a weight of 10 lb and is pinned at its center O. If a vertical force of $P = 2$ lb is applied to the cord wrapped around its outer rim, determine the angular velocity of the disk in four seconds starting from rest. Neglect the mass of the cord.

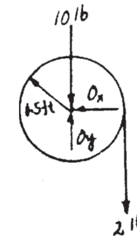


SOLUTION

$(\circlearrowleft +)$ $$I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = I_O\omega_2$$

$$0 + 2(0.5)(4) = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2\right]\omega_2$$

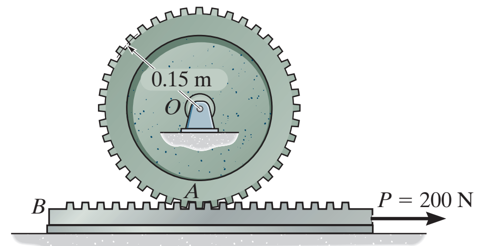
$$\omega_2 = 103 \text{ rad/s}$$ **Ans.**



Ans:
$\omega_2 = 103$ rad/s

19–10.

The 30-kg gear A has a radius of gyration about its center of mass O of $k_O = 125$ mm. If the 20-kg gear rack B is subjected to a force of $P = 200$ N, determine the time required for the gear to obtain an angular velocity of 20 rad/s, starting from rest. The contact surface between the gear rack and the horizontal plane is smooth.



SOLUTION

Kinematics: Since the gear rotates about the fixed axis, the final velocity of the gear rack is required to be

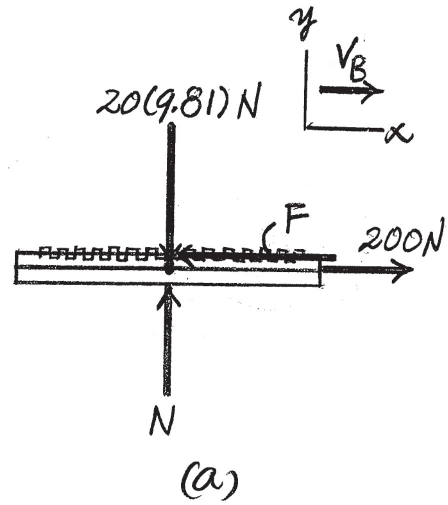
$$(v_B)_2 = \omega_2 r_B = 20(0.15) = 3 \text{ m/s} \rightarrow$$

Principle of Impulse and Momentum: Applying the linear impulse and momentum equation along the x axis using the free-body diagram of the gear rack shown in Fig. a,

$$(\overset{+}{\rightarrow}) \qquad m(v_B)_1 + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_B)_2$$

$$0 + 200(t) - F(t) = 20(3)$$

$$F(t) = 200t - 60 \qquad \textbf{(1)}$$



The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2 = 30(0.125^2) = 0.46875 \text{ kg} \cdot \text{m}^2$. Writing the angular impulse and momentum equation about point O using the free-body diagram of the gear shown in Fig. b,

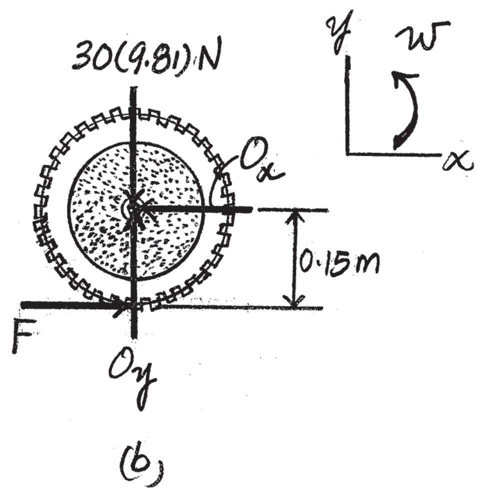
$$I_O\omega_1 + \Sigma \int_{t_1}^{t_2} M_O dt = I_O\omega_2$$

$$0 + F(t)(0.15) = 0.46875(20)$$

$$F(t) = 62.5 \qquad \textbf{(2)}$$

Substituting Eq. (2) into Eq. (1) yields

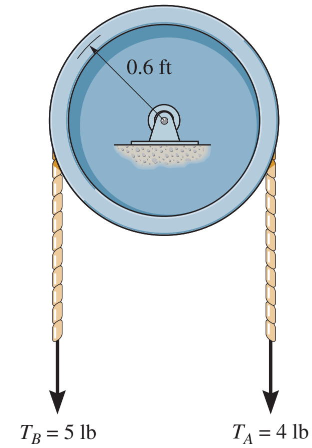
$$t = 0.6125 \text{ s} \qquad \textbf{Ans.}$$



Ans:
$t = 0.6125$ s

19–11.

The pulley has a weight of 8 lb and may be treated as a thin disk. A cord wrapped over its surface is subjected to forces $T_A = 4$ lb and $T_B = 5$ lb. Determine the angular velocity of the pulley when $t = 4$ s if it starts from rest when $t = 0$. Neglect the mass of the cord.



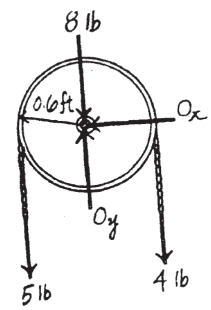
SOLUTION

Principle of Impulse and Momentum: The mass moment inertia of the pulley about its mass center is $I_o = \frac{1}{2}\left(\frac{8}{32.2}\right)(0.6^2) = 0.04472 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 19–14, we have

$$I_O\omega_1 + \Sigma \int_{t_1}^{t_2} M_o dt = I_o\omega_2$$

$$(\circlearrowleft +) \qquad 0 + [5(4)](0.6) - [4(4)](0.6) = 0.04472\omega_2$$

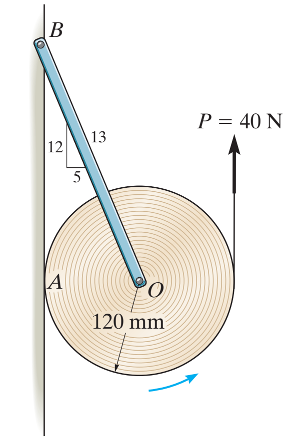
$$\omega_2 = 53.7 \text{ rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega_2 = 53.7$ rad/s

***19–12.**

The 40-kg roll of paper rests along the wall where the coefficient of kinetic friction is $\mu_k = 0.2$. If a vertical force of $P = 40$ N is applied to the paper, determine the angular velocity of the roll when $t = 6$ s starting from rest. Neglect the mass of the unraveled paper and take the radius of gyration of the spool about the axle O to be $k_O = 80$ mm.



SOLUTION

Principle of Impulse and Momentum. The mass moment of inertia of the paper roll about its center is $I_O = mk_O^2 = 40\left(0.08^2\right) = 0.256\ \text{kg}\cdot\text{m}^2$. Since the paper roll is required to slip at point of contact, $F_f = \mu_k N = 0.2$ N. Referring to the FBD of the paper roll, Fig. a,

$$(\overset{+}{\rightarrow})\qquad m[(v_O)_x]_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m[(v_O)_x]_2$$

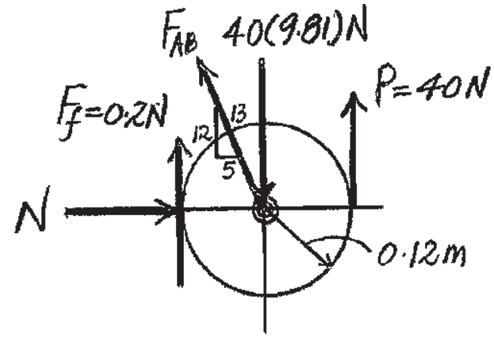
$$0 + N(6) - F_{AB}\left(\frac{5}{13}\right)(6) = 0$$

$$F_{AB} = \frac{13}{5}\text{N} \qquad \textbf{(1)}$$

$$(+\uparrow)\qquad m[(v_O)_y]_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m[(v_O)_y]_2$$

$$0 + 0.2\ \text{N}(6) + F_{AB}\left(\frac{12}{13}\right)(6) + 40(6) - 40(9.81)(6) = 0$$

$$0.2\ \text{N} + \frac{12}{13}F_{AB} = 352.4 \qquad \textbf{(2)}$$



(a)

Solving Eqs. (1) and (2)

$$N = 135.54\ \text{N} \qquad F_{AB} = 352.4\ \text{N}$$

Subsequently

$$\circlearrowleft+\qquad I_O\omega_1 + \Sigma\int M_O\,dt = I_O\omega_2$$

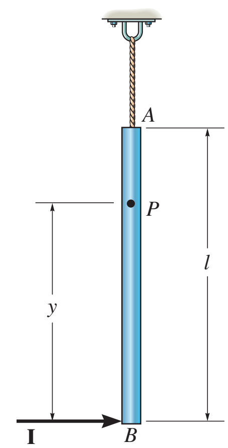
$$0.256(0) + 40(0.12)(6) - 0.2(135.54)(0.12)(6) = 0.256\omega$$

$$\omega = 36.26\ \text{rad/s} = 36.3\ \text{rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 36.3$ rad/s

19–13.

The slender rod has a mass m and is suspended at its end A by a cord. If the rod receives a horizontal blow giving it an impulse $\mathbf{I}$ at its bottom B, determine the location y of the point P about which the rod appears to rotate during the impact.



SOLUTION

Principle of Impulse and Momentum:

$$(\circlearrowleft +) \qquad I_G\omega_1 + \Sigma \int_{t_1}^{t_2} M_G\,dt = I_G\,\omega_2$$

$$0 + I\left(\frac{l}{2}\right) = \left[\frac{1}{12}ml^2\right]\omega \qquad I = \frac{1}{6}ml\omega$$

$$(\xrightarrow{+}) \qquad m(v_{Ax})_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_{Ax})_2$$

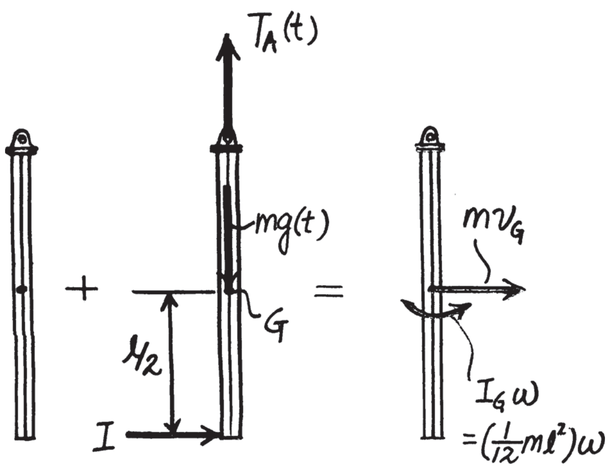
$$0 + \frac{1}{6}ml\omega = mv_G \qquad v_G = \frac{l}{6}\,\omega$$

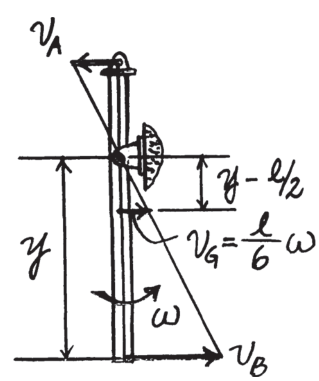
Kinematics: Point P is the IC.

$$v_B = \omega\,y$$

Using similar triangles,

$$\frac{\omega\,y}{y} = \frac{\frac{l}{6}\omega}{y - \frac{l}{2}} \qquad y = \frac{2}{3}l \qquad \textbf{Ans.}$$

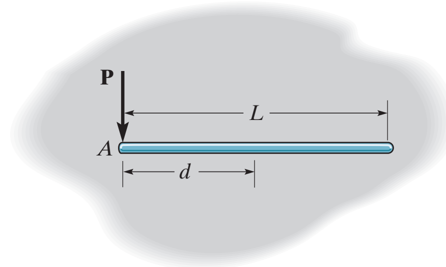




Ans:
$y = \frac{2}{3}l$

19–14.

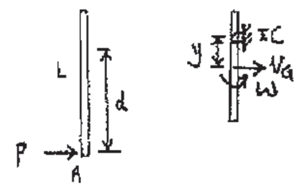
The rod of length L and mass m lies on a smooth horizontal surface and is subjected to a force $\mathbf{P}$ at its end A as shown. Determine the location d of the point about which the rod begins to turn, i.e, the point that has zero velocity.



SOLUTION

$(\overset{+}{\rightarrow})\qquad m(v_{Gx})_1 + \Sigma\int F_x\,dt = m(v_{Gx})_2$

$$0 + P(t) = m(v_G)_x$$



$(+\uparrow)\qquad m(v_{G_y})_1 + \Sigma\int F_y\,dt = m(v_{G_y})_2$

$$0 + 0 = m(v_G)_y$$

$(\circlearrowleft +)\qquad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

$$0 + P(t)\left(\frac{L}{2}\right) = \frac{1}{12}mL^2\omega$$

$$v_G = y\omega$$

$$m(v_G)_x\left(\frac{L}{2}\right) = \frac{1}{12}mL^2\omega$$

$$(v_G)_x = \frac{L}{6}\omega$$

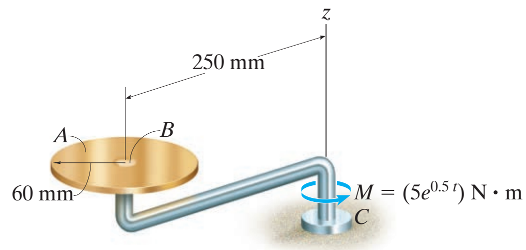
$$y = \frac{L}{6}$$

$$d = \frac{L}{2} + \frac{L}{6} = \frac{2}{3}L \qquad \textbf{Ans.}$$

Ans:
$d = \dfrac{2}{3}l$

19–15.

A 4-kg disk A is mounted on arm BC, which has a negligible mass. If a torque of $M = (5e^{0.5t})$ N·m, where t is in seconds, is applied to the arm at C, determine the angular velocity of BC in 2 s starting from rest. Solve the problem assuming that (a) the disk is set in a smooth bearing at B so that it rotates with curvilinear translation, (b) the disk is fixed to the shaft BC, and (c) the disk is given an initial freely spinning angular velocity of $\boldsymbol{\omega}_D = \{-80\mathbf{k}\}$ rad/s prior to application of the torque.



SOLUTION

a)

$$(H_z)_1 + \Sigma \int M_z\, dt = (H_z)_2$$

$$0 + \int_0^2 5e^{0.5t} dt = 4(v_B)(0.25)$$

$$\frac{5}{0.5} e^{0.5t}\Big|_0^2 = v_B$$

$$v_B = 17.18 \text{ m/s}$$

Thus,

$$\omega_{BC} = \frac{17.18}{0.25} = 68.7 \text{ rad/s} \qquad \textbf{Ans.}$$

b)

$$(H_z)_1 + \Sigma \int M_z\, dt = (H_z)_2$$

$$0 + \int_0^2 5e^{0.5t} dt = 4(v_B)(0.25) + \left[\frac{1}{2}(4)(0.06)^2\right]\omega_{BC}$$

Since $v_B = 0.25\,\omega_{BC}$, then

$$\omega_{BC} = 66.8 \text{ rad/s} \qquad \textbf{Ans.}$$

c)

$$(H_z)_1 + \Sigma \int M_z\, dt = (H_z)_2$$

$$-\left[\frac{1}{2}(4)(0.06)^2\right](80) + \int_0^2 5e^{0.5t} dt = 4(v_B)(0.25) - \left[\frac{1}{2}(4)(0.06)^2\right](80)$$

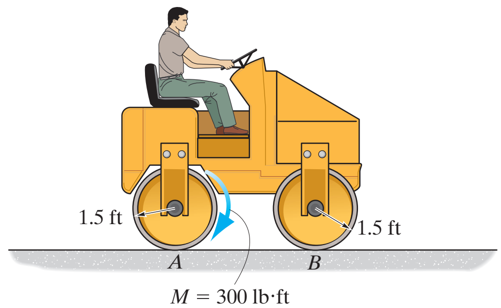
Since $v_B = 0.25\,\omega_{BC}$,

$$\omega_{BC} = 68.7 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
(a) $\omega_{BC} = 68.7$ rad/s
(b) $\omega_{BC} = 66.8$ rad/s
(c) $\omega_{BC} = 68.7$ rad/s

***19–16.**

The frame of a tandem drum roller has a weight of 4000 lb excluding the two rollers. Each roller has a weight of 1500 lb and a radius of gyration about its axle of 1.25 ft. If a torque of $M = 300$ lb·ft is supplied to the rear roller A, determine the speed of the drum roller 10 s later, starting from rest.



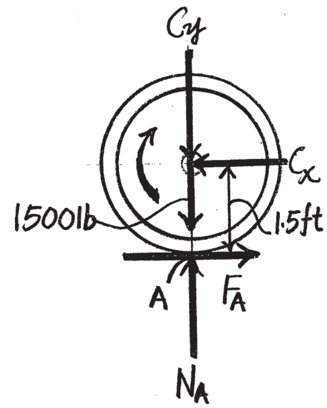
SOLUTION

Principle of Impulse and Momentum: The mass moments of inertia of the rollers about their mass centers are $I_C = I_D = \dfrac{1500}{32.2}\left(1.25^2\right) = 72.787$ slug·ft². Since the rollers roll without slipping, $\omega = \dfrac{v}{r} = \dfrac{v}{1.5} = 0.6667v$. Using the free-body diagrams of the rear roller and front roller, Figs. *a* and *b*, and the momentum diagram of the rollers, Fig. *c*,

$$(H_A)_1 + \Sigma\int_{t_1}^{t_2} M_A dt = (H_A)_2$$

$$0 + 300(10) - C_x(10)(1.5) = \frac{1500}{32.2}v(1.5) + 72.787(0.6667v)$$

$$C_x = 200 - 7.893v \qquad \textbf{(1)}$$



(a)

and

$$(H_B)_1 + \Sigma\int_{t_1}^{t_2} M_B dt = (H_B)_2$$

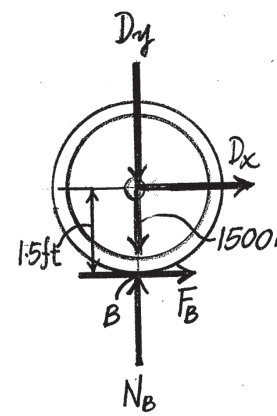
$$0 + D_x(10)(1.5) = \frac{1500}{32.2}v(1.5) + 72.787(0.6667v)$$

$$D_x = 7.893v \qquad \textbf{(2)}$$

Referring to the free-body diagram of the frame shown in Fig. *d*,

$$\xrightarrow{+}\qquad m\left[(v_G)_x\right]_1 + \Sigma\int_{t_1}^{t_2} F_x dt = m\left[(v_G)_x\right]_2$$

$$0 + C_x(10) - D_x(10) = \frac{4000}{32.2}v \qquad \textbf{(3)}$$

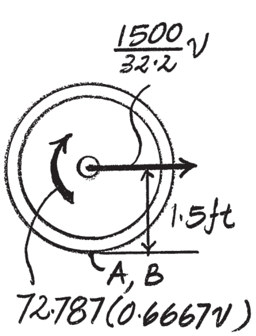


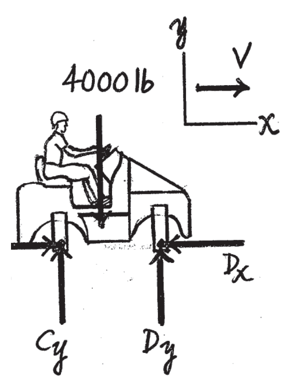
(b)

Substituting Eqs. (1) and (2) into Eq. (3),

$$(200 - 7.893v)(10) - 7.893v(10) = \frac{4000}{32.2}v$$

$$v = 7.09 \text{ ft/s} \qquad \textbf{Ans.}$$



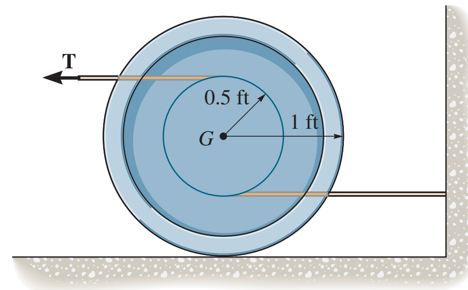


(c) (d)

Ans:
$v = 7.09$ ft/s

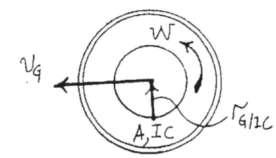
19–17.

The 100-lb wheel has a radius of gyration of $k_G = 0.75$ ft. If the upper wire is subjected to a tension of $T = 50$ lb, determine the velocity of the center of the wheel in 3 s, starting from rest. The coefficient of kinetic friction between the wheel and the surface is $\mu_k = 0.1$.



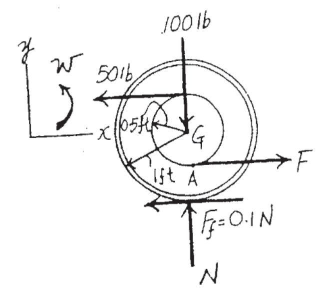
SOLUTION

Principle of Impulse and Momentum: We can eliminate the force F from the analysis if we apply the principle of impulse and momentum about point A. The mass moment inertia of the wheel about point A is $I_A = \frac{100}{32.2}\left(0.75^2\right) + \frac{100}{32.2}\left(0.5^2\right) = 2.523 \text{ slug}\cdot\text{ft}^2$. Applying Eq. 19–14, we have



$$m\left(v_{G_y}\right)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m\left(v_{G_y}\right)_2$$

$(+\uparrow)$ $\qquad 0 + N(t) - 100(t) = 0 \qquad N = 100 \text{ lb}$



$$I_A\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_A\,dt = I_A\,\omega_2$$

$(\circlearrowleft +)$ $\qquad 0 + [50(3)](1) - [0.1(100)(3)](0.5) = 2.523\,\omega_2 \qquad$ **[1]**

Kinematics: Since the wheel rolls without slipping at point A, the instantaneous center of zero velocity is located at point A. Thus,

$$v_G = \omega_2\, r_{G/IC}$$

$$\omega_2 = \frac{v_G}{r_{G/IC}} = \frac{v_G}{0.5} = 2v_G \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

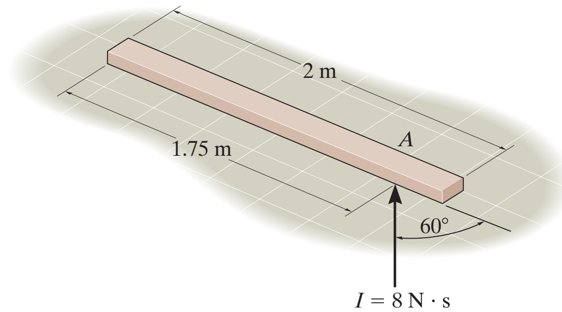
$$v_G = 26.8 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\omega_2 = 53.50 \text{ rad/s}$$

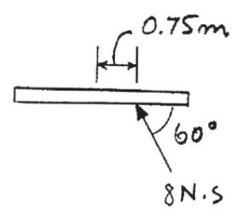
Ans:
$v_G = 26.8$ ft/s

19–18.

The 4-kg slender rod rests on a smooth floor. If it is kicked so as to receive a horizontal impulse $I = 8\ \text{N}\cdot\text{s}$ at point A as shown, determine its angular velocity and the speed of its mass center.



SOLUTION



$$(\overset{+}{\rightarrow}) \qquad m(v_{Gx})_1 + \Sigma \int F_x\, dt = m(v_{Gx})_2$$

$$0 + 8 \cos 60° = 4(v_G)_x$$

$$(v_G)_x = 1 \text{ m/s}$$

$$(+\uparrow) \qquad m(v_{Gy})_1 + \Sigma \int F_y\, dt = m(v_{Gy})_2$$

$$0 + 8 \sin 60° = 4(v_G)_y$$

$$(v_G)_y = 1.732 \text{ m/s}$$

$$v_G = \sqrt{(1.732)^2 + (1)^2} = 2 \text{ m/s} \qquad \textbf{Ans.}$$

$$(\circlearrowleft +) \qquad (H_G)_1 + \Sigma \int M_G\, dt = (H_G)_2$$

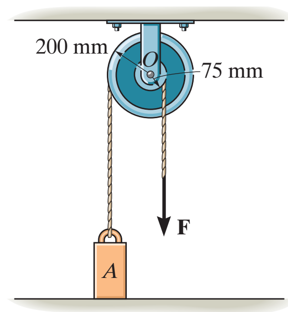
$$0 + 8 \sin 60°(0.75) = \left[\frac{1}{12}(4)(2)^2\right]\omega$$

$$\omega = 3.90 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$v_G = 2 \text{ m/s}$
$\omega = 3.90 \text{ rad/s}$

19–19.

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration $k_O = 110$ mm. If the block at A has a mass of 40 kg, determine the speed of the block in 3 s after a constant force $F = 2$ kN is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest. Neglect the mass of the rope.



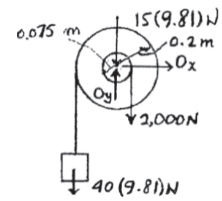
SOLUTION

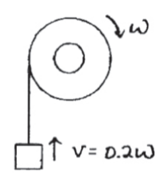
$(\circlearrowright +) \qquad (H_O)_1 + \Sigma \int M_O\, dt = (H_O)_2$

$$0 + 2000(0.075)(3) - 40(9.81)(0.2)(3) = 15(0.110)^2\omega + 40(0.2\omega)\,(0.2)$$

$$\omega = 120.4 \text{ rad/s}$$

$$v_A = 0.2(120.4) = 24.1 \text{ m/s} \qquad \textbf{Ans.}$$

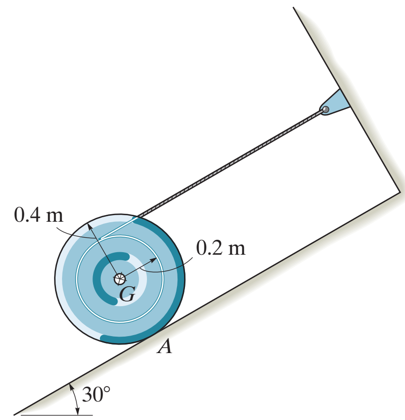




Ans:
$v_A = 24.1$ m/s

***19–20.**

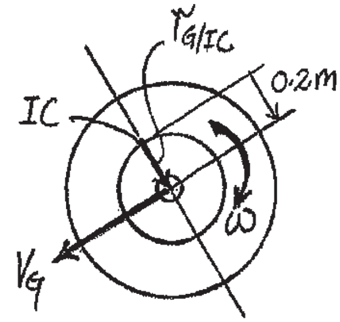
The 100-kg spool is resting on the inclined surface for which the coefficient of kinetic friction is $\mu_k = 0.1$. Determine the angular velocity of the spool when $t = 4$ s after it is released from rest. The radius of gyration about the mass center is $k_G = 0.25$ m.



SOLUTION

Kinematics. The IC of the spool is located as shown in Fig. a. Thus

$$v_G = \omega r_{G/IC} = \omega(0.2)$$



(a)

Principle of Impulse and Momentum. The mass moment of inertia of the spool about its mass center is $I_G = mk_G^2 = 100\left(0.25^2\right) = 6.25\ \text{kg}\cdot\text{m}^2$. Since the spool is required to slip, $F_f = \mu_k N = 0.1\ N$. Referring to the FBD of the spool, Fig. b,

$$\nwarrow^{+} \qquad m[(v_G)_y]_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m[(v_G)_y]_2$$

$$0 + N(4) - 100(9.81)\cos 30°(4) = 0$$

$$N = 849.57\ \text{N}$$

$$\overset{+}{\nearrow} \qquad m[(v_G)_x]_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m[(v_G)_x]_2$$

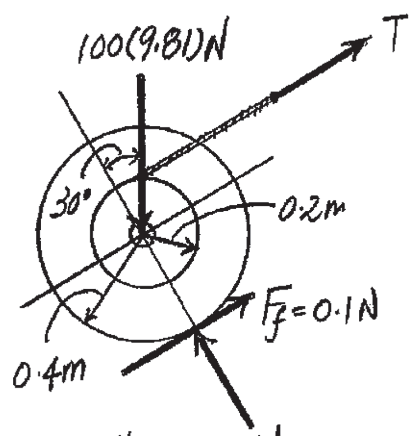
$$0 + T(4) + 0.1(849.57)(4) - 100(9.81)\sin 30°(4) = 100[-\omega(0.2)]$$

$$T + 5\omega = 405.54 \qquad \textbf{(1)}$$

$$\circlearrowleft + \qquad I_G\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\,\omega_2$$

$$0 + 0.1(849.57)(0.4)(4) - T(0.2)(4) = -6.25\,\omega_2$$

$$0.8T - 6.25\omega_2 = 135.93 \qquad \textbf{(2)}$$



(b)

Solving Eqs. (1) and (2),

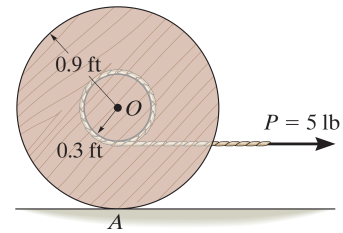
$$\omega = 18.39\ \text{rad/s} = 18.4\ \text{rad/s} \circlearrowright \qquad \textbf{Ans.}$$

$$T = 313.59\ \text{N}$$

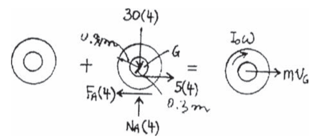
Ans:
$\omega = 18.4\ \text{rad/s} \circlearrowright$

19–21.

The spool has a weight of 30 lb and a radius of gyration $k_O = 0.45$ ft. A cord is wrapped around its inner hub and the end subjected to a horizontal force $P = 5$ lb. Determine the spool's angular velocity in 4 s starting from rest. Assume the spool rolls without slipping.



SOLUTION



$(+\uparrow)\qquad m(v_y) + \Sigma\int F_y\,dt = m(v_y)_2$

$$0 + N_A(4) - 30(4) = 0$$

$$N_A = 30 \text{ lb}$$

$(\overset{+}{\rightarrow})\qquad m(v_x)_1 + \Sigma\int F_x dt = m(v_x)_2$

$$0 + 5(4) - F_A(4) = \frac{30}{32.2}v_G$$

$(\circlearrowleft +)\qquad (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$

$$0 + F_A(4)(0.9) - 5(4)(0.3) = \frac{30}{32.2}(0.45)^2\,\omega$$

Since no slipping occurs

Set $v_G = 0.9\,\omega$

$F_A = 2.33$ lb

$\omega = 12.7$ rad/s **Ans.**

Also,

$(\circlearrowleft +)\qquad (H_A)_1 + \Sigma M_A\,dt = (H_A)_2$

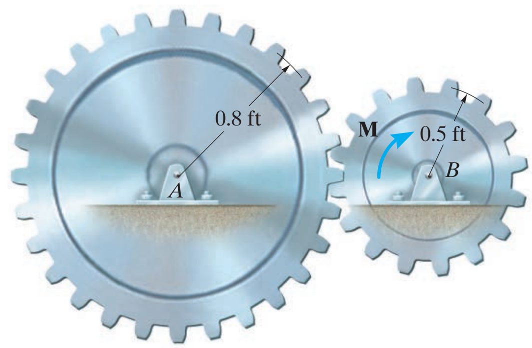
$$0 + 5(4)(0.6) = \left[\frac{30}{32.2}(0.45)^2 + \frac{30}{32.2}(0.9)^2\right]\omega$$

$\omega = 12.7$ rad/s **Ans.**

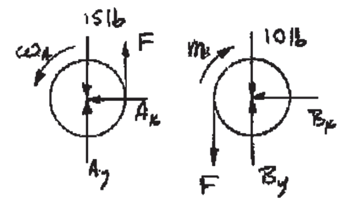
Ans:
$\omega = 12.7$ rad/s

19–22.

The two gears A and B have weights and radii of gyration of $W_A = 15$ lb, $k_A = 0.5$ ft and $W_B = 10$ lb, $k_B = 0.35$ ft, respectively. If a motor transmits a couple moment to gear B of $M = 2(1 - e^{-0.5t})$ lb · ft, where t is in seconds, determine the angular velocity of gear A in $t = 5$ s, starting from rest.



SOLUTION



$\omega_A(0.8) = \omega_B(0.5)$

$\omega_B = 1.6\omega_A$

Gear B:

$$(\circlearrowright +) \qquad (H_B)_1 + \Sigma \int M_B\, dt = (H_B)_2$$

$$0 + \int_0^5 2\left(1 - e^{-0.5t}\right)dt - \int 0.5F\, dt = \left[\left(\frac{10}{32.2}\right)(0.35)^2\right](1.6\omega_A)$$

$$6.328 = 0.5\int F\, dt + 0.06087\omega_A \qquad \textbf{(1)}$$

Gear A:

$$0 = 0.8\int F\, dt - 0.1165\omega_A \qquad \textbf{(2)}$$

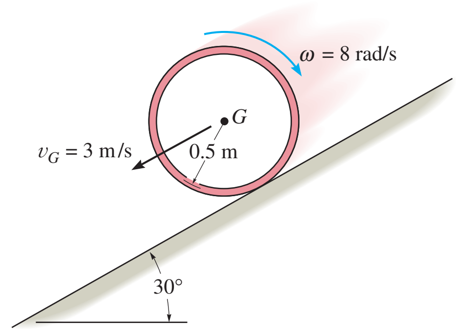
$(\circlearrowleft +)$ $\quad (H_A)_1 + \Sigma \int M_A\, dt = (H_A)_2$ $\qquad$ Eliminate $\int F\, dt$ between Eqs. (1) and (2), and solving for ω_A.

$$0 + \int 0.8F\, dt = \left[\left(\frac{15}{32.2}\right)(0.5)^2\right]\omega_A \qquad \omega_A = 47.3 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_A = 47.3$ rad/s

19–23.

The hoop (thin ring) has a mass of 5 kg and is released down the inclined plane such that it has a backspin $\omega = 8$ rad/s and its center has a velocity $v_G = 3$ m/s as shown. If the coefficient of kinetic friction between the hoop and the plane is $\mu_k = 0.6$, determine how long the hoop rolls before it stops slipping.



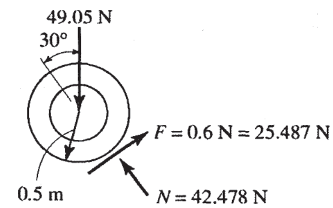
SOLUTION

$\swarrow +$ $\quad mv_{x1} + \sum \int F_x dt = mv_{x2}$

$$5(3) + 49.05 \sin 30^\circ (t) - 25.487t = 5v_G$$

$\circlearrowleft +$ $\quad (H_G)_1 + \sum \int M_G dt = (H_G)_2$

$$-5(0.5)^2(8) + 25.487(0.5)(t) = 5(0.5)^2\left(\frac{v_G}{0.5}\right)$$



Solving,

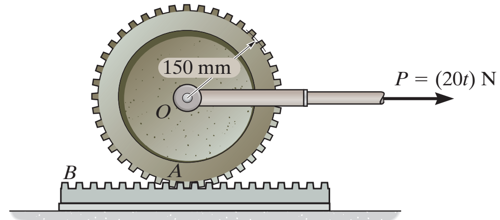
$$v_G = 2.75 \text{ m/s}$$

$$t = 1.32 \text{ s}$$ **Ans.**

Ans:
$t = 1.32$ s

***19–24.**

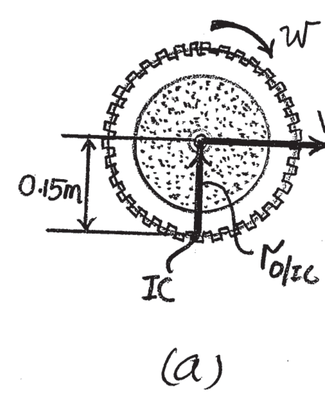
The 30-kg gear is subjected to a force of $P = (20t)$ N, where t is in seconds. Determine the angular velocity of the gear at $t = 4$ s, starting from rest. Gear rack B is fixed to the horizontal plane, and the gear's radius of gyration about its mass center O is $k_O = 125$ mm.



SOLUTION

Kinematics: Referring to Fig. *a*,

$$v_O = \omega r_{O/IC} = \omega(0.15)$$



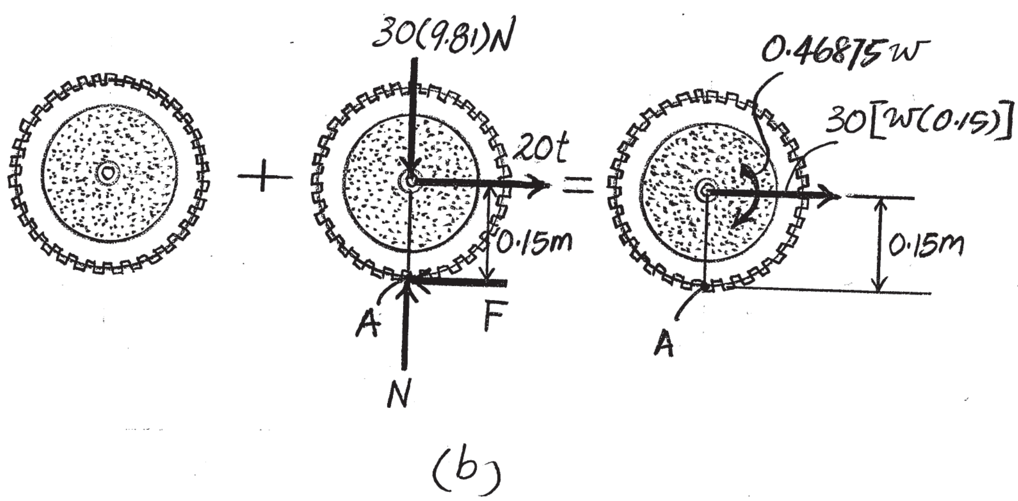
Principle of Angular Impulse and Momentum: The mass moment of inertia of the gear about its mass center is $I_O = mk_O{}^2 = 30(0.125^2) = 0.46875 \text{ kg} \cdot \text{m}^2$. Writing the angular impulse and momentum equation about point A shown in Fig. *b*,

$$(H_A)_1 + \Sigma \int_{t_1}^{t_2} M_A \, dt = (H_A)_2$$

$$0 + \int_0^{4\text{ s}} 20t(0.15)dt = 0.46875\omega + 30\,[\omega(0.15)]\,(0.15)$$

$$1.5t^2 \Big|_0^{4\text{ s}} = 1.14375\omega$$

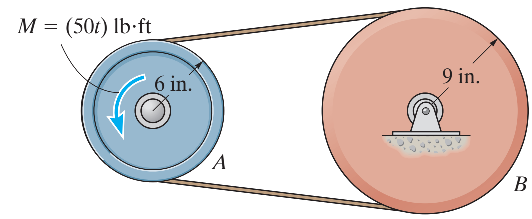
$$\omega = 21.0 \text{ rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega = 21.0$ rad/s

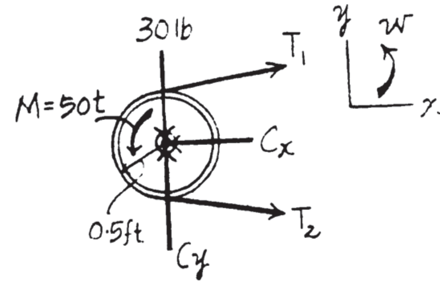
19–25.

The 30-lb flywheel A has a radius of gyration about its center of 4 in. Disk B weighs 50 lb and is coupled to the flywheel by means of a belt which does not slip at its contacting surfaces. If a motor supplies a counterclockwise torque to the flywheel of $M = (50t)$ lb · ft, where t is in seconds, determine the time required for the disk to attain an angular velocity of 60 rad/s starting from rest.



SOLUTION

Principle of Impulse and Momentum: The mass moment inertia of the flywheel about point C is $I_C = \frac{30}{32.2}\left(\frac{4}{12}\right)^2 = 0.1035 \text{ slug} \cdot \text{ft}^2$. The angular velocity of the flywheel is $\omega_A = \frac{r_B}{r_A}\omega_B = \frac{0.75}{0.5}(60) = 90.0 \text{ rad/s}$. Applying Eq. 19–14 to the flywheel [FBD(a)], we have



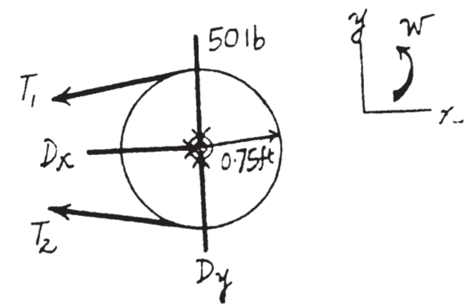
(a)

$$I_C\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_C\,dt = I_C\,\omega_2$$

$$(\circlearrowleft +)\quad 0 + \int_0^t 50t\,dt + \left[\int T_2\,(dt)\right](0.5) - \left[\int T_1(dt)\right](0.5) = 0.1035(90)$$

$$25t^2 + 0.5\int (T_2 - T_1)dt = 9.317 \qquad \textbf{(1)}$$

The mass moment inertia of the disk about point D is $I_D = \frac{1}{2}\left(\frac{50}{32.2}\right)(0.75^2) = 0.4367 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 19–14 to the disk [FBD(b)], we have



(b)

$$I_D\,\omega_1 + \Sigma\int_{t_1}^{t_2} M_D\,dt = I_D\,\omega_2$$

$$(\circlearrowleft +)\qquad 0 + \left[\int T_1\,(dt)\right](0.75) - \left[\int T_2\,(dt)\right](0.75) = 0.4367(60)$$

$$\int (T_2 - T_1)dt = -34.94 \qquad \textbf{(2)}$$

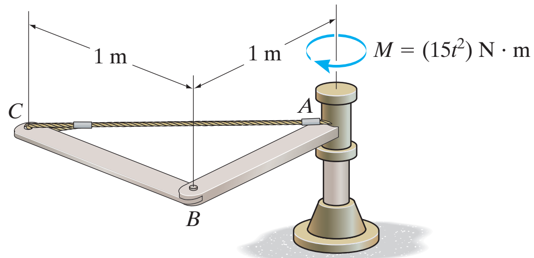
Substitute Eq. (2) into Eq. (1) and solving yields

$$t = 1.04 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 1.04$ s

19–26.

If the shaft is subjected to a torque of $M = (15t^2)\ \text{N}\cdot\text{m}$, where t is in seconds, determine the angular velocity of the assembly when $t = 3$ s, starting from rest. Rods AB and BC each have a mass of 9 kg.



SOLUTION

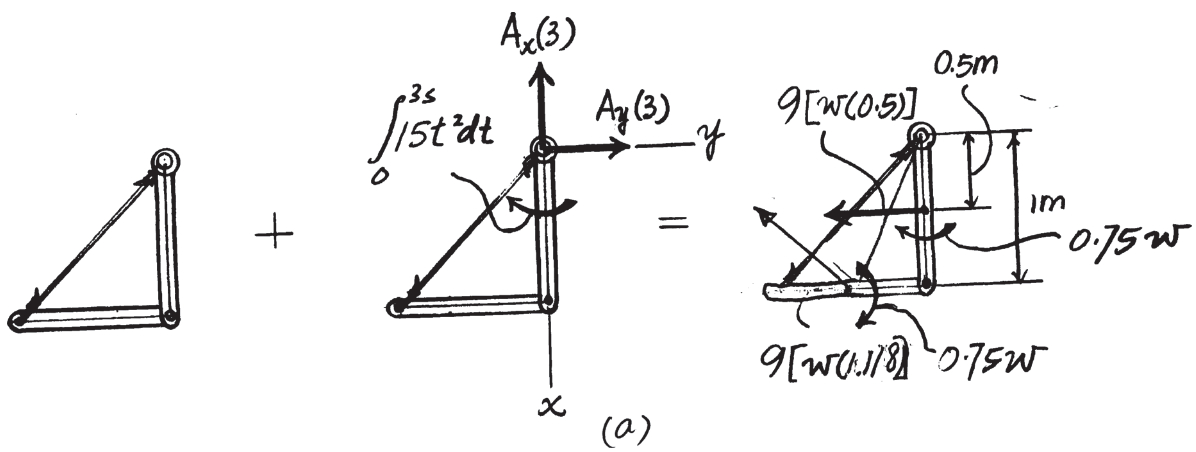
Principle of Impulse and Momentum: The mass moment of inertia of the rods about their mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(9)(1^2) = 0.75\ \text{kg}\cdot\text{m}^2$. Since the assembly rotates about the fixed axis, $(v_G)_{AB} = \omega(r_G)_{AB} = \omega(0.5)$ and $(v_G)_{BC} = \omega(r_G)_{BC} = \omega\left(\sqrt{1^2 + (0.5)^2}\right) = \omega(1.118)$. Referring to Fig. a,

$$\circlearrowleft + \qquad (H_z)_1 + \Sigma\int_{t_1}^{t_2} M_z\,dt = (H_z)_2$$

$$0 + \int_0^{3\text{s}} 15t^2 dt = 9\big[\omega(0.5)\big](0.5) + 0.75\omega + 9\big[\omega(1.118)\big](1.118) + 0.75\omega$$

$$5t^3\Big|_0^{3\text{ s}} = 15\omega$$

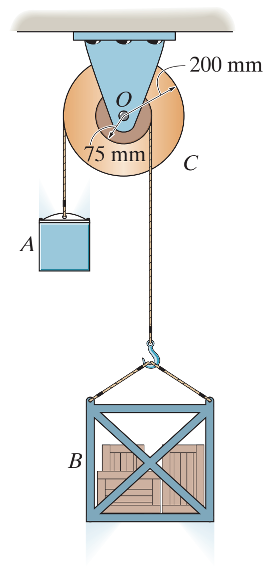
$$\omega = 9\ \text{rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega = 9\ \text{rad/s}$

19–27.

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 15 kg and a radius of gyration of $k_O = 110$ mm. If the block at A has a mass of 40 kg and the container at B has a mass of 85 kg, including its contents, determine the speed of the container when $t = 3$ s after it is released from rest.



SOLUTION

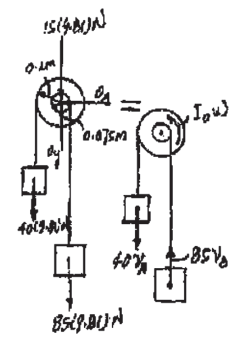
The angular velocity of the pulley can be related to the speed of container B by $\omega = \dfrac{v_A}{0.075} = 13.333\, v_B$. Also the speed of block A $v_A = \omega(0.2) = 13.33\, v_B(0.2) - 2.667\, v_B$,

$$(\circlearrowleft +)\left(\Sigma \text{Syst. Ang. Mom.}\right)_{O1} + \left(\Sigma \text{Syst. Ang. Imp.}\right)_{O(1-2)} = \left(\Sigma \text{Syst. Ang. Mom.}\right)_{O2}$$

$$0 + 40(9.81)(0.2)(3) - 85(9.81)(0.075)(3)$$

$$= \left[15(0.110)^2\right](13.333\, v_B) + 85\, v_B(0.075) + 40(2.667\, v_B)(0.2)$$

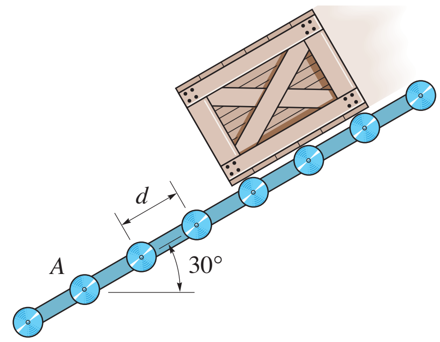
$$v_B = 1.59 \text{ m/s} \qquad \textbf{Ans.}$$



Ans:
$v_B = 1.59$ m/s

***19–28.**

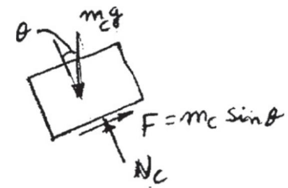
The crate has a mass m_c. Determine the constant speed v_0 it acquires as it moves down the conveyor. The rollers each have a radius of r, mass m, and are spaced d apart. Note that friction causes each roller to rotate when the crate comes in contact with it.



SOLUTION

The number of rollers per unit length is $1/d$.

Thus in one second, $\dfrac{v_0}{d}$ rollers are contacted.



If a roller is brought to full angular speed of $\omega = \dfrac{v_0}{r}$ in t_0 seconds, then the moment of inertia that is effected is

$$I' = I\left(\frac{v_0}{d}\right)(t_0) = \left(\frac{1}{2}m\,r^2\right)\left(\frac{v_0}{d}\right)t_0$$

Since the frictional impluse is

$F = m_c \sin\theta$ then

$$\circlearrowleft + (H_G)_1 + \Sigma\int M_G\,dt = (H_G)_2$$

$$0 + (m_c \sin\theta)\,r\,t_0 = \left[\left(\frac{1}{2}m\,r^2\right)\left(\frac{v_0}{d}\right)t_0\right]\left(\frac{v_0}{r}\right)$$

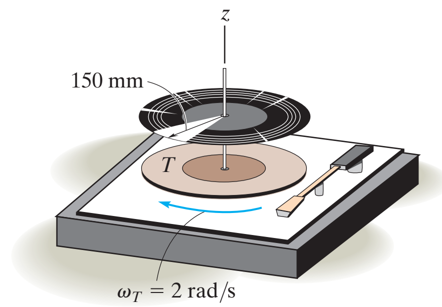
$$v_0 = \sqrt{(2\,g\sin\theta\,d)\left(\frac{m_c}{m}\right)}$$ **Ans.**

Ans:

$$v_0 = \sqrt{(2\,g\sin\theta\,d)\left(\frac{m_c}{m}\right)}$$

19–29.

The turntable T of a record player has a mass of 0.75 kg and a radius of gyration $k_z = 125$ mm. It is *turning freely* at $\omega_T = 2$ rad/s when a 50-g record (thin disk) falls on it. Determine the final angular velocity of the turntable just after the record stops slipping on the turntable.



SOLUTION

$(H_z)_1 = (H_z)_2$

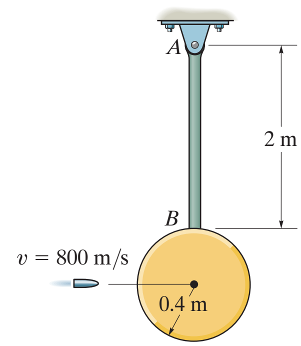
$$0.75(0.125)^2(2) = \left[0.75(0.125)^2 + \frac{1}{2}(0.050)(0.150)^2\right]\omega$$

$\omega = 1.91$ rad/s **Ans.**

Ans:
$\omega = 1.91$ rad/s

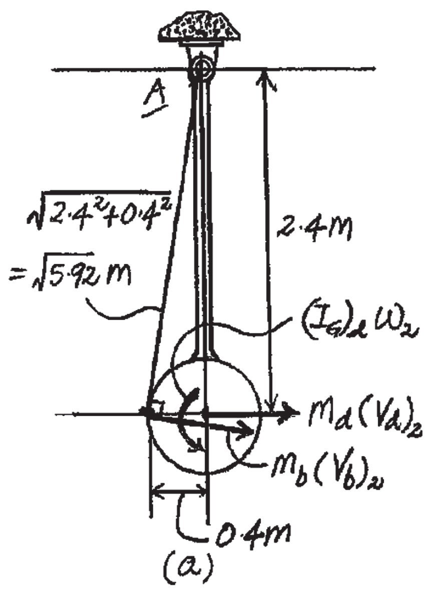
19–30.

The 10-g bullet having a velocity of 800 m/s is fired into the edge of the 5-kg disk as shown. Determine the angular velocity of the disk just after the bullet becomes embedded into its edge. Also, calculate the angle θ the disk will swing when it stops. The disk is originally at rest. Neglect the mass of the rod AB.



SOLUTION

Conservation of Angular Momentum. The mass moment of inertia of the disk about its mass center is $(I_G)_d = \frac{1}{2}(5)(0.4^2) = 0.4\ \text{kg}\cdot\text{m}^2$. Also, $(v_b)_2 = \omega_2\left(\sqrt{5.92}\right)$ and $(v_d)_2 = \omega_2(2.4)$. Referring to the momentum diagram with the embedded bullet, Fig. a,



$$\Sigma(H_A)_1 = \Sigma(H_A)_2$$

$$M_b(v_b)_1(r_b)_1 = (I_G)_d\,\omega_2 + M_d(v_d)_2(r_2) + M_b(v_b)_2(r_b)_2$$

$$0.01(800)(2.4) = 0.4\omega_2 + 5\left[\omega_2(2.4)\right](2.4) + 0.01\left[\omega_2\left(\sqrt{5.92}\right)\right]\left(\sqrt{5.92}\right)$$

$$\omega_2 = 0.6562\ \text{rad/s} = 0.656\ \text{rad/s} \qquad \textbf{Ans.}$$

Kinetic Energy. Since the system is required to stop finally, $T_3 = 0$. Here

$$T_2 = \frac{1}{2}(I_G)_d\,\omega_2^2 + \frac{1}{2}Md(v_d)_2^2 + \frac{1}{2}M_b(v_b)_2^2$$

$$= \frac{1}{2}(0.4)\left(0.6562^2\right) + \frac{1}{2}(5)[0.6562(2.4)]^2 + \frac{1}{2}(0.01)\left[0.6562\left(\sqrt{5.92}\right)\right]^2$$

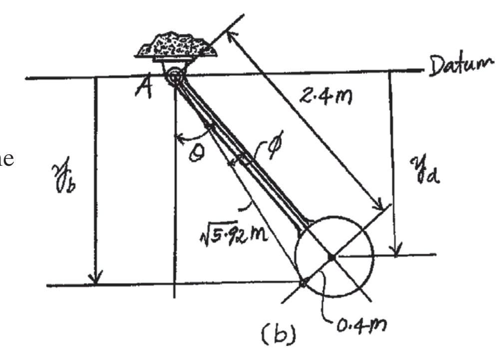
$$= 6.2996\ \text{J}$$

Potential Energy. Datum is set as indicated on Fig. b.

Here $\phi = \tan^{-1}\left(\frac{0.4}{2.4}\right) = 9.4623$. Hence

$$y_1 = 2.4\cos\theta \qquad y_b = \sqrt{5.92}\cos(\theta - 9.4623°)$$

Thus, the gravitational potential energy of the disk and bullet with reference to the datum is



$$(V_g)_d = M_d\,g\,(y_d) = 5(9.81)(-2.4\cos\theta) = -117.72\cos\theta$$

$$(V_g)_b = M_b g(y_b) = 0.01(9.81)\left[\left(-\sqrt{5.92}\cos(\theta - 9.4623°)\right]\right.$$

$$= -0.0981\sqrt{5.92}\cos(\theta - 9.4623°)$$

At $\theta = 0°$,

$$[(V_g)_d]_2 = -117.72\cos 0° = -117.72\ \text{J}$$

$$[(V_g)_b]_2 = -0.0981\sqrt{5.92}\cos(0 - 9.4623°) = -0.23544\ \text{J}$$

Conservation of Energy.

$$T_2 + V_2 = T_3 + V_3$$

$$6.2996 + (-117.72) + (-0.23544) = 0 + (-117.72\cos\theta) + \left[-0.0981\sqrt{5.92}\cos(\theta - 9.4623°)\right]$$

$$117.72\cos\theta + 0.0981\sqrt{5.92}\cos(\theta - 9.4623°) = 111.66$$

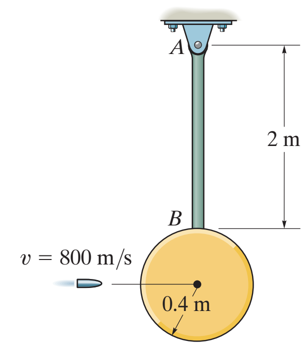
Solved by numerically,

$$\theta = 18.83° = 18.8° \qquad \textbf{Ans.}$$

Ans:
$\omega_2 = 0.656$ rad/s
$\theta = 18.8°$

19–31.

The 10-g bullet having a velocity of 800 m/s is fired into the edge of the 5-kg disk as shown. Determine the angular velocity of the disk just after the bullet becomes embedded into its edge. Also, calculate the angle θ the disk will swing when it stops. The disk is originally at rest. The rod AB has a mass of 3 kg.



SOLUTION

Conservation of Angular Momentum. The mass moments of inertia of the disk and rod about their respective mass centers are $(I_G)_d = \frac{1}{2}(5)(0.4^2) = 0.4\ \text{kg}\cdot\text{m}^2$ and $(I_G)_r = \frac{1}{12}(3)(2^2) = 1.00\ \text{kg}\cdot\text{m}^2$. Also, $(v_b)_2 = \omega_2(\sqrt{5.92})$, $(v_d)_2 = \omega_2(2.4)$ and $(v_r)_2 = \omega_2(1)$. Referring to the momentum diagram with the embedded bullet, Fig. a,

$$\Sigma(H_A)_1 = \Sigma(H_A)_2$$

$$M_b(v_b)_1(r_b)_1 = (I_G)_d\,\omega_2 + M_d(v_d)_2(r_d) + (I_G)_r\,\omega_2 + m_r(v_r)_2(\mathbf{r}) + M_b(v_b)_2(r_b)_2$$

$$0.01(800)(2.4) = 0.4\omega_2 + 5[\omega_2(2.4)](2.4) + 1.00\omega_2 + 3[\omega_2(1)](1)$$

$$+\ 0.01\left[\omega_2(\sqrt{5.92})\right](\sqrt{5.92})\Big]$$

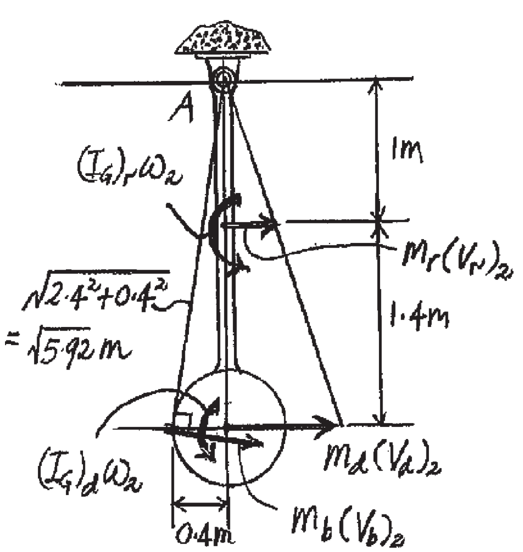
$$\omega_2 = 0.5773\ \text{rad/s} = 0.577\ \text{rad/s} \qquad \textbf{Ans.}$$

Kinetic Energy. Since the system is required to stop finally, $T_3 = 0$. Here

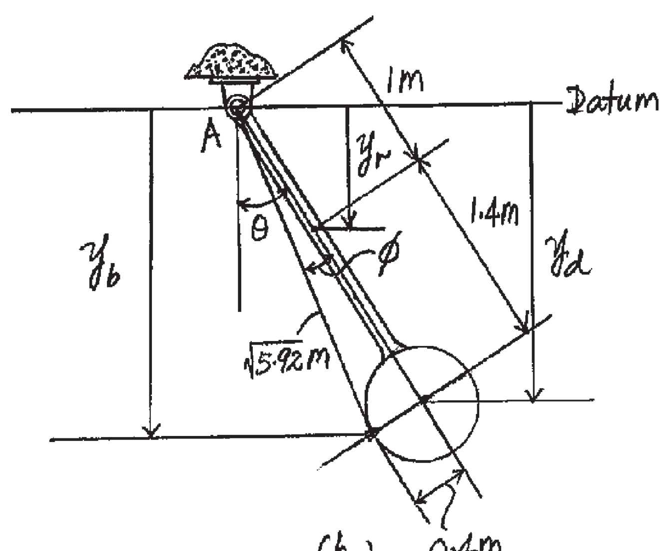
$$T_2 = \frac{1}{2}(I_G)_d\,\omega_2^2 + \frac{1}{2}M_d(v_d)_2^2 + \frac{1}{2}(I_G)_r\,\omega_2^2 + \frac{1}{2}M_r(v_r)_2^2 + \frac{1}{2}M_b(v_b)_2^2$$

$$= \frac{1}{2}(0.4)(0.5773^2) + \frac{1}{2}(5)[0.5773(2.4)]^2 + \frac{1}{2}(1.00)(0.5773^2) + \frac{1}{2}(3)[0.5773(1)]^2 + \frac{1}{2}(0.01)\left[0.5773(\sqrt{5.92})\right]^2$$

$$= 5.5419\ \text{J}$$



(a)



(b)

19–31. Continued

Potential Energy. Datum is set as indicated on Fig. *b*.

Here $\phi = \tan^{-1}\left(\frac{0.4}{2.4}\right) = 9.4623°$. Hence

$y_d = 2.4 \cos \theta,\ y_r = \cos \theta,\ y_b = \sqrt{5.92} \cos (\theta - 9.4623°)$

Thus, the gravitational potential energy of the disk, rod and bullet with reference to the datum is

$$(V_g)_d = M_d\, g\, y_d = 5(9.81)(-2.4 \cos \theta) = -117.72 \cos \theta$$

$$(V_g)_r = M_r\, g\, y_r = 3(9.81)(-\cos \theta) = -29.43 \cos \theta$$

$$(V_g)_b = m_b\, g\, y_b = 0.01(9.81)\left[-\sqrt{5.92} \cos (\theta - 94623°)\right]$$

$$= -0.0981\sqrt{5.92} \cos(\theta - 9.4623°)$$

At $\theta = 0°$,

$$[(V_g)_d]_2 = -117.72 \cos 0° = -117.72 \text{ J}$$

$$[(V_g)_r]_2 = -29.43 \cos 0° = -29.43 \text{ J}$$

$$[(V_g)_b]_2 = -0.0981\sqrt{5.92} \cos (0° - 9.4623°) = -0.23544 \text{ J}$$

Conservation of Energy.

$$T_2 + V_2 = T_3 + V_3$$

$$5.5419 + (-117.72) + (-29.43) + (-0.23544) = 0 + (-117.72 \cos \theta)$$

$$+ (-29.43 \cos \theta) + \left[-0.0981\sqrt{5.92} \cos (\theta - 9.4623°)\right]$$

$$147.15 \cos \theta + 0.0981\sqrt{5.92} \cos (\theta - 9.4623°) = 141.84$$

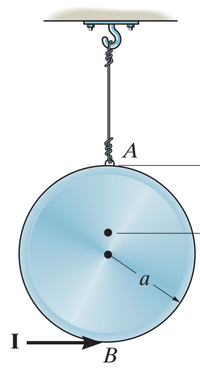
Solved numerically,

$$\theta = 15.78° = 15.8° \qquad \textbf{Ans.}$$

Ans:
$\omega_2 = 0.577$ rad/s
$\theta = 15.8°$

***19–32.**

The circular disk has a mass m and is suspended at A by the wire. If it receives a horizontal impulse $\mathbf{I}$ at its edge B, determine the location y of the point P about which the disk appears to rotate during the impact.



SOLUTION

Principle of Impulse and Momentum. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}ma^2$

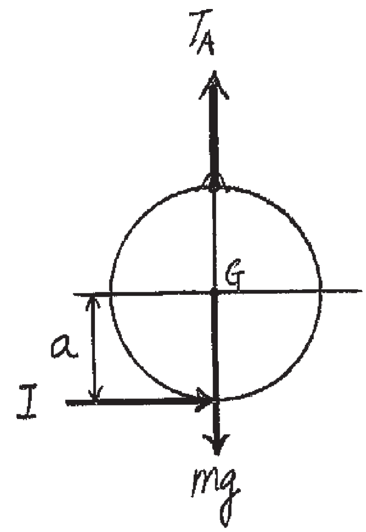
$$\circlearrowleft + \quad I_G\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,dt = I_G\,\omega_2$$

$$0 + I_a = \left(\frac{1}{2}ma^2\right)\omega$$

$$I = \frac{1}{2}ma\,\omega \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow}) \quad m\left[(v_G)_x\right]_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m\left[(v_G)_x\right]_2$$

$$0 + I = mv_G \qquad \textbf{(2)}$$



(a)

Equating Eqs. (1) and (2),

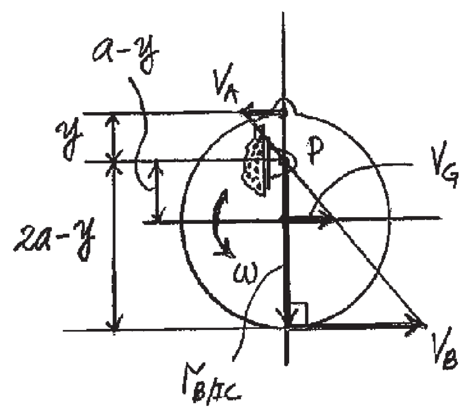
$$\frac{1}{2}ma\,\omega = mv_G$$

$$v_G = \frac{a}{2}\omega$$

Kinematics. Here, IC is located at P, Fig. b. Thus, $v_B = \omega\, r_{B/IC} = \omega(2a - y)$. Using similar triangles,

$$\frac{a-y}{v_G} = \frac{2a-y}{v_B}; \qquad \frac{a-y}{\frac{a}{2}\omega} = \frac{2a-y}{\omega(2a-y)}$$

$$y = \frac{1}{2}a \qquad \textbf{Ans.}$$



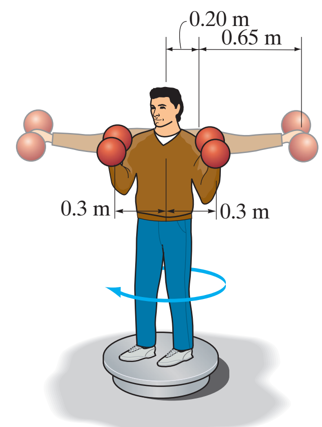
(b)

Ans:

$y = \frac{1}{2}a$

19–33.

The 80-kg man is holding two dumbbells while standing on a turntable of negligible mass, which turns freely about a vertical axis. When his arms are fully extended, the turntable is rotating with an angular velocity of 0.5 rev/s. Determine the angular velocity of the man when he retracts his arms to the position shown. When his arms are fully extended, approximate each arm as a uniform 6-kg rod having a length of 650 mm, and his body as a 68-kg solid cylinder of 400-mm diameter. With his arms in the retracted position, assume the man as an 80-kg solid cylinder of 450-mm diameter. Each dumbbell consists of two 5-kg spheres of negligible size.



SOLUTION

Conservation of Angular Momentum: Since no external angular impulse acts on the system during the motion, angular momentum about the axis of rotation (z axis) is conserved. The mass moment of inertia of the system when the arms are in the fully extended position is

$$(I_z)_1 = 2\left[10(0.85^2)\right] + 2\left[\frac{1}{12}(6)(0.65^2) + 6(0.525^2)\right] + \frac{1}{2}(68)(0.2^2)$$

$$= 19.54 \text{ kg} \cdot \text{m}^2$$

And the mass moment of inertia of the system when the arms are in the retracted position is

$$(I_z)_2 = 2\left[10(0.3^2)\right] + \frac{1}{2}(80)(0.225^2)$$

$$= 3.825 \text{ kg} \cdot \text{m}^2$$

Thus,

$$(H_z)_1 = (H_z)_2$$

$$(I_z)_1\omega_1 = (I_z)_2\omega_2$$

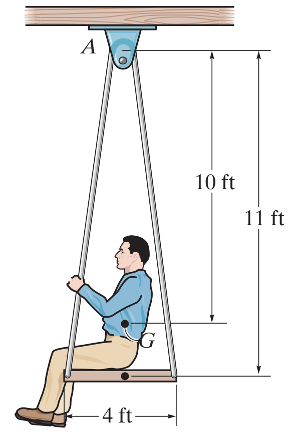
$$19.54(0.5) = 3.825\omega_2$$

$$\omega_2 = 2.55 \text{ rev/s}$$ **Ans.**

Ans:
$\omega_2 = 2.55 \text{ rev/s}$

19–34.

The platform swing consists of a 200-lb flat plate suspended by four rods of negligible weight. When the swing is at rest, the 150-lb man jumps off the platform when his center of gravity G is 10 ft from the pin at A. This is done with a horizontal velocity of 5 ft/s, measured relative to the swing at the level of G. Determine the angular velocity he imparts to the swing just after jumping off.

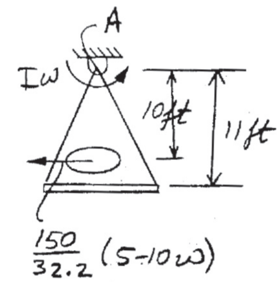


SOLUTION

$(\circlearrowleft +) \qquad (H_A)_1 = (H_A)_2$

$$0 + 0 = \left[\frac{1}{12}\left(\frac{200}{32.2}\right)(4)^2 + \frac{200}{32.2}(11)^2\right]\omega - \left[\left(\frac{150}{32.2}\right)(5 - 10\omega)\right](10)$$

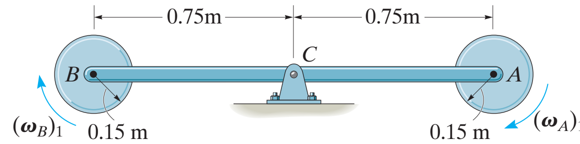
$\omega = 0.190 \text{ rad/s}$ **Ans.**



Ans:
$\omega = 0.190 \text{ rad/s}$

19–35.

The 2-kg rod ACB supports the two 4-kg disks at its ends. If both disks are given a clockwise angular velocity $(\omega_A)_1 = (\omega_B)_1 = 5$ rad/s while the rod is held stationary and then released, determine the angular velocity of the rod after both disks have stopped spinning relative to the rod due to frictional resistance at the pins A and B. Motion is in the *horizontal plane*. Neglect friction at pin C.



SOLUTION

$\circlearrowleft+ \qquad H_1 = H_2$

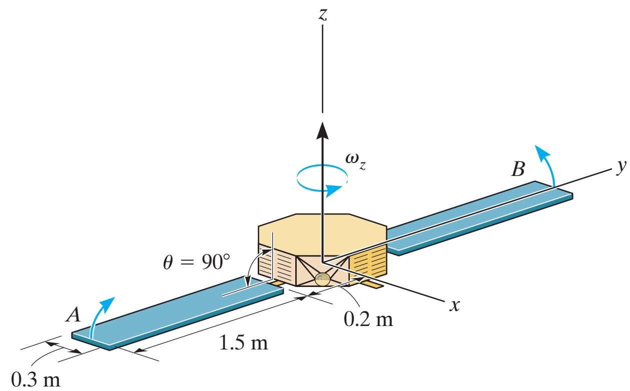
$$2\left[\frac{1}{2}(4)(0.15)^2\right](5) = 2\left[\frac{1}{2}(4)(0.15)^2\right]\omega + 2[4(0.75\,\omega)(0.75)] + \left[\frac{1}{12}(2)(1.50)^2\right]\omega$$

$\omega = 0.0906$ rad/s **Ans.**

Ans:
$\omega = 0.0906$ rad/s

***19–36.**

The satellite has a mass of 200 kg and a radius of gyration about z axis of $k_z = 0.1$ m, excluding the two solar panels A and B. Each solar panel has a mass of 15 kg and can be approximated as a thin plate. If the satellite is originally spinning about the z axis at a constant rate $\omega_z = 0.5$ rad/s when $\theta = 90°$, determine the rate of spin if both panels are raised and reach the upward position, $\theta = 0°$, at the same instant.



SOLUTION

Conservation of Angular Momentum. When $\theta = 90°$, the mass moment of inertia of the entire satellite is

$$I_z = 200(0.1^2) + 2\left[\frac{1}{12}(15)(0.3^2 + 1.5^2) + 15(0.95^2)\right] = 34.925 \text{ kg}\cdot\text{m}^2$$

when $\theta = 0°$, $I'_z = 200(0.1^2) + 2\left[\frac{1}{12}(15)(0.3^2) + 15(0.2^2)\right] = 3.425 \text{ kg}\cdot\text{m}^2$

Thus

$$(H_z)_1 = (H_z)_2$$

$$I_z(\omega_z)_1 = I'_z(\omega_z)_2$$

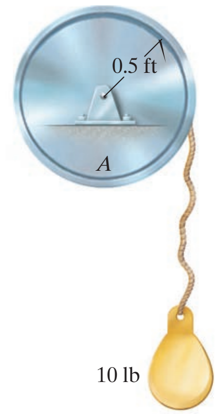
$$34.925(0.5) = 3.425(\omega_z)_2$$

$$(\omega_z)_2 = 5.0985 \text{ rad/s} = 5.10 \text{ rad/s}$$ **Ans.**

Ans:
$(\omega_z)_2 = 5.10$ rad/s

19–37.

Disk A has a weight of 20 lb. An inextensible cable is attached to the 10-lb weight and wrapped around the disk. The weight is dropped 2 ft before the slack is taken up. If the impact is perfectly elastic, i.e., $e = 1$, determine the angular velocity of the disk just after impact.



SOLUTION

For the weight

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 10(2) = \frac{1}{2}\left(\frac{10}{32.2}\right) v_2^2$$

$$v_2 = 11.35 \text{ ft/s}$$

$$(H_A)_2 = (H_A)_3$$

$$mv_2(0.5) + 0 = m v_3 (0.5) + I_A\, \omega$$

$$\left(\frac{10}{32.2}\right)(11.35)(0.5) + 0 = \left(\frac{10}{32.2}\right) v_3 (0.5) + \left[\frac{1}{2}\left(\frac{20}{32.2}\right)(0.5)^2\right]\omega \qquad \textbf{[1]}$$

$$(+\downarrow) \qquad e = \frac{0.5\omega - v_3}{v_2 - 0} \qquad 1 = \frac{0.5\omega - v_3}{11.35 - 0} \qquad 11.35 = 0.5\omega - v_3 \qquad \textbf{[2]}$$

Solving Eqs.[1] and [2] yields:

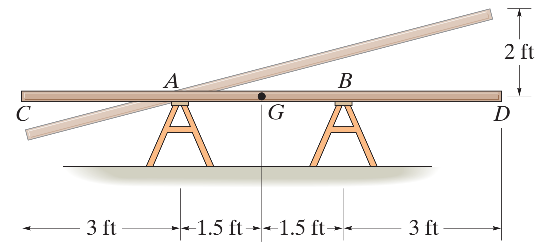
$$\omega = 22.7 \text{ rad/s} \qquad \textbf{Ans.}$$

$$v_3 = 0$$

Ans:
$\omega = 22.7 \text{ rad/s}$

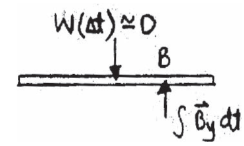
19–38.

The plank has a weight of 30 lb, center of gravity at G, and it rests on the two sawhorses at A and B. If the end D is raised 2 ft above the top of the sawhorses and is released from rest, determine how high end C will rise from the top of the sawhorses after the plank falls so that it rotates clockwise about A, strikes and pivots on the sawhorses at B, and rotates clockwise off the sawhorse at A.



SOLUTION

Establishing a datum through AB, the angular velocity of the plank just before striking B is

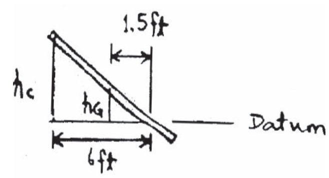


$$T_1 + V_1 = T_2 + V_2$$

$$0 + 30\left[\frac{2}{6}(1.5)\right] = \frac{1}{2}\left[\frac{1}{12}\left(\frac{30}{32.2}\right)(9)^2 + \frac{30}{32.2}(1.5)^2\right](\omega_{CD})_2^2 + 0$$

$$(\omega_{CD})_2 = 1.8915 \text{ rad/s}$$

$$(v_G)_2 = 1.8915(1.5) = 2.837 \text{ m/s}$$



$$(\circlearrowleft +) \qquad (H_B)_2 = (H_B)_3$$

$$\left[\frac{1}{12}\left(\frac{30}{32.2}\right)(9)^2\right](1.8915) - \frac{30}{32.2}(2.837)(1.5) = \left[\frac{1}{2}\left(\frac{30}{32.2}\right)(9)^2\right](\omega_{AB})_3 + \frac{30}{32.2}(v_G)_3(1.5)$$

Since $(v_G)_3 = 1.5(\omega_{AB})_3$

$$(\omega_{AB})_3 = 0.9458 \text{ rad/s}$$

$$(v_G)_3 = 1.4186 \text{ m/s}$$

$$T_3 + V_3 = T_4 + V_4$$

$$\frac{1}{2}\left[\frac{1}{12}\left(\frac{30}{32.2}\right)(9)^2\right](0.9458)^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)(1.4186)^2 + 0 = 0 + 30h_G$$

$$h_G = 0.125$$

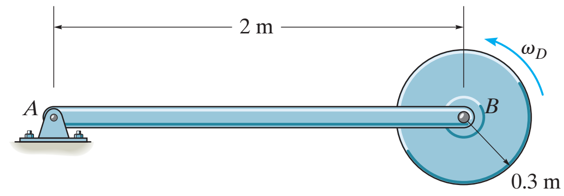
Thus,

$$h_C = \frac{6}{1.5}(0.125) = 0.500 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$h_C = 0.500$ ft

19–39.

The 12-kg rod AB is pinned to the 40-kg disk. If the disk is given an angular velocity $\omega_D = 100$ rad/s while the rod is held stationary, and the assembly is then released, determine the angular velocity of the rod after the disk has stopped spinning relative to the rod due to frictional resistance at the bearing B. Motion is in the *horizontal plane*. Neglect friction at the pin A.



SOLUTION

Conservation of Angular Momentum

Initial: Since the rod is stationary and the disk is not translating, the total angular momentum about A equals the angular momentum of the disk about B, where for the disk $I_B = \frac{1}{2} m_D r^2 = \frac{1}{2}(40)(0.3)^2 = 1.80 \text{ kg} \cdot \text{m}^2$.

$$(H_A)_1 = I_B \omega_1 = 1.80(100) = 180 \text{ kg} \cdot \text{m}^2/\text{s}$$

Final: The rod and disk move as a single unit for which $I_A = \frac{1}{3} m_R l^2 + \frac{1}{2} m_D r^2 + m_D l^2$

$= \frac{1}{3}(12)(2)^2 + \frac{1}{2}(40)(0.3)^2 + (40)(2)^2 = 177.8 \text{ kg} \cdot \text{m}^2$.

$$(H_A)_2 = I_A\,\omega_2 = 177.8\,\omega_2$$

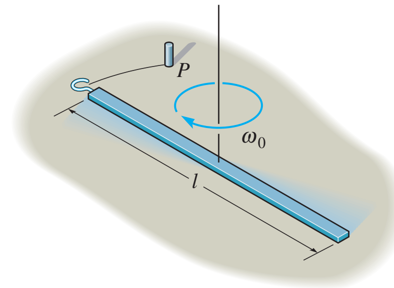
Setting $(H_A)_1 = (H_A)_2$ and solving,

$\omega_2 = 1.012 \text{ rad/s} = 1.01 \text{ rad/s}$ **Ans.**

Ans:
$\omega_2 = 1.01$ rad/s

***19–40.**

A thin rod of mass m has an angular velocity $\boldsymbol{\omega}_0$ while rotating on a smooth surface. Determine its new angular velocity just after its end strikes and hooks onto the peg and the rod starts to rotate about P without rebounding. Solve the problem (a) using the parameters given, (b) setting $m = 2$ kg, $\omega_0 = 4$ rad/s, $l = 1.5$ m.



SOLUTION

(a)

$$\Sigma\,(H_P)_0 = \Sigma(H_P)_1$$

$$\left[\frac{1}{12}ml^2\right]\omega_0 = \left[\frac{1}{3}ml^2\right]\omega$$

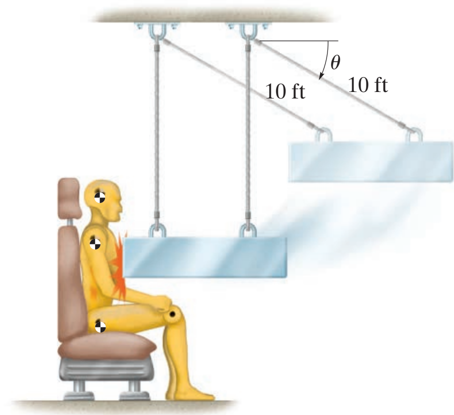
$$\omega = \frac{1}{4}\omega_0 \qquad \textbf{Ans.}$$

(b) From part (a) $\qquad \omega = \dfrac{1}{4}\omega_0 = \dfrac{1}{4}(4) = 1 \text{ rad/s}$ **Ans.**

Ans:
$\omega = \frac{1}{4}\omega_0$
$\omega = 1$ rad/s

19–41.

Tests of impact on the fixed crash dummy are conducted using the 300-lb ram that is released from rest at $\theta = 30°$, and allowed to fall and strike the dummy at $\theta = 90°$. If the coefficient of restitution between the dummy and the ram is $e = 0.4$, determine the angle θ to which the ram will rebound before momentarily coming to rest.



SOLUTION

Datum through pin support at ceiling.

$$T_1 + V_1 = T_2 + V_2$$

$$0 - 300(10 \sin 30°) = \frac{1}{2}\left(\frac{300}{32.2}\right)(v)^2 - 300(10)$$

$$v = 17.944 \text{ ft/s}$$

$$(\overset{+}{\rightarrow}) \qquad e = 0.4 = \frac{v' - 0}{0 - (-17.944)}$$

$$v' = 7.178 \text{ ft/s}$$

$$T_2 + V_2 = T_3 + V_3$$

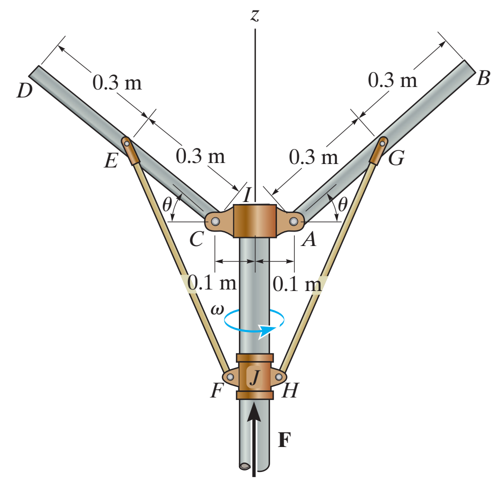
$$\frac{1}{2}\left(\frac{300}{32.2}\right)(7.178)^2 - 300(10) = 0 - 300(10 \sin \theta)$$

$$\theta = 66.9° \qquad \textbf{Ans.}$$

Ans:
$\theta = 66.9°$

19–42.

The vertical shaft is rotating with an angular velocity of 3 rad/s when $\theta = 0°$. If a force **F** is applied to the collar so that $\theta = 90°$, determine the angular velocity of the shaft. Also, find the work done by force **F**. Neglect the mass of rods GH and EF and the collars I and J. The rods AB and CD each have a mass of 10 kg.



SOLUTION

Conservation of Angular Momentum: Referring to the free-body diagram of the assembly shown in Fig. *a*, the sum of the angular impulses about the z axis is zero. Thus, the angular momentum of the system is conserved about the axis. The mass moments of inertia of the rods about the z axis when $\theta = 0°$ and $90°$ are

$$(I_z)_1 = 2\left[\frac{1}{12}(10)(0.6^2) + 10(0.3 + 0.1)^2\right] = 3.8\ \text{kg}\cdot\text{m}^2$$

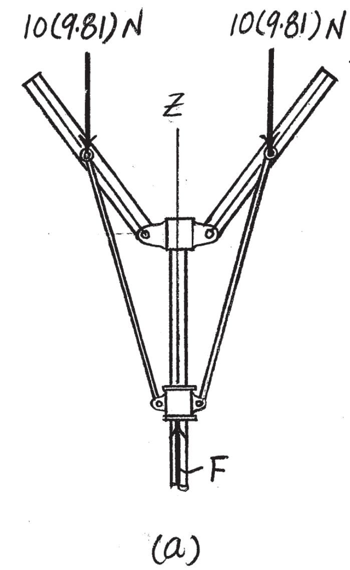
$$(I_z)_2 = 2\left[10(0.1^2)\right] = 0.2\ \text{kg}\cdot\text{m}^2$$

Thus,

$$(H_z)_1 = (H_z)_2$$

$$3.8(3) = 0.2\omega_2$$

$$\omega_2 = 57\ \text{rad/s} \qquad \textbf{Ans.}$$

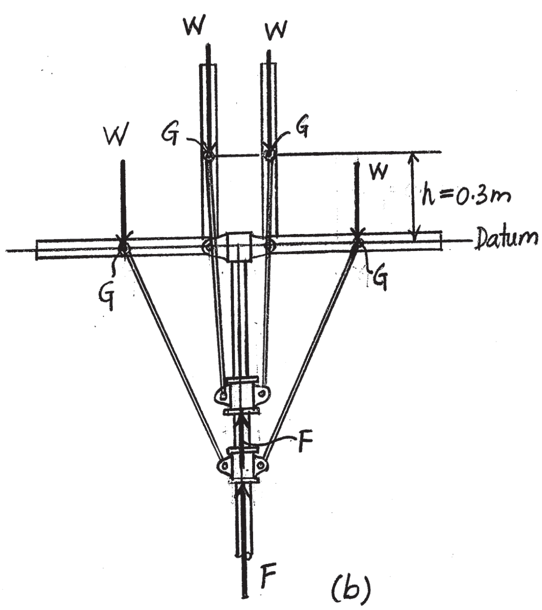


Principle of Work and Energy: As shown on the free-body diagram of the assembly, Fig. *b*, **W** does negative work, while **F** does positive work. The work of **W** is $U_W = -Wh = -10(9.81)(0.3) = -29.43$ J. The initial and final kinetic energy of the assembly is $T_1 = \frac{1}{2}(I_z)_1\omega_1^2 = \frac{1}{2}(3.8)(3^2) = 17.1$ J and $T_2 = \frac{1}{2}(I_z)_2\omega_2^2 = \frac{1}{2}(0.2)(57^2) = 324.9$ J. Thus,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$17.1 + 2(-29.43) + U_F = 324.9$$

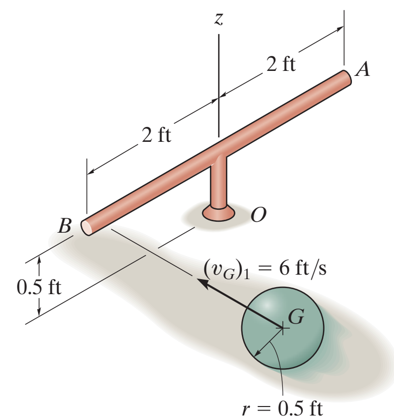
$$U_F = 367\ \text{J} \qquad \textbf{Ans.}$$



Ans:
$\omega_2 = 57$ rad/s
$U_F = 367$ J

19–43.

The mass center of the 3-lb ball has a velocity of $(v_G)_1 = 6$ ft/s when it strikes the end of the smooth 5-lb slender bar which is at rest. Determine the angular velocity of the bar about the z axis just after impact if $e = 0.8$.



SOLUTION

Conservation of Angular Momentum: Since force F due to the impact is *internal* to the system consisting of the slender bar and the ball, it will cancel out. Thus, angular momentum is conserved about the z axis. The mass moment of inertia of the slender bar about the z axis is $I_z = \frac{1}{12}\left(\frac{5}{32.2}\right)(4^2) = 0.2070 \text{ slug} \cdot \text{ft}^2$. Here, $\omega_2 = \frac{(v_B)_2}{2}$.
Applying Eq. 19–17, we have

$$(H_z)_1 = (H_z)_2$$

$$\left[m_b\,(v_G)_1\right](r_b) = I_z\,\omega_2 + \left[m_b\,(v_G)_2\right](r_b)$$

$$\left(\frac{3}{32.2}\right)(6)(2) = 0.2070\left[\frac{(v_B)_2}{2}\right] + \left(\frac{3}{32.2}\right)(v_G)_2(2) \quad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{(v_B)_2 - (v_G)_2}{(v_G)_1 - (v_B)_1}$$

$$0.8 = \frac{(v_B)_2 - (v_G)_2}{6 - 0} \quad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_G)_2 = 2.143 \text{ ft/s} \qquad (v_B)_2 = 6.943 \text{ ft/s}$$

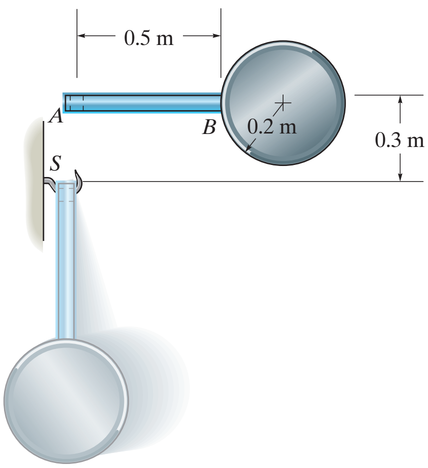
Thus, the angular velocity of the slender rod is given by

$$\omega_2 = \frac{(v_B)_2}{2} = \frac{6.943}{2} = 3.47 \text{ rad/s} \quad \textbf{Ans.}$$

Ans:
$\omega_2 = 3.47$ rad/s

***19–44.**

The pendulum consists of a slender 2-kg rod AB and 5-kg disk. It is released from rest without rotating. When it falls 0.3 m, the end A strikes the hook S, which provides a permanent connection. Determine the angular velocity of the pendulum after it has rotated 90°. Treat the pendulum's weight during impact as a nonimpulsive force.



SOLUTION

$$T_0 + V_0 = T_1 + V_1$$

$$0 + 2(9.81)(0.3) + 5(9.81)(0.3) = \frac{1}{2}(2)(v_G)_1^2 + \frac{1}{2}(5)(v_G)_1^2$$

$(v_G)_1 = 2.4261$ m/s

$$\Sigma(H_s)_1 = \Sigma(H_s)_2$$

$$2(2.4261)(0.25) + 5(2.4261)(0.7) = \left[\frac{1}{12}(2)(0.5)^2 + 2(0.25)^2 + \frac{1}{2}(5)(0.2)^2 + 5(0.7)^2\right]\omega$$

$\omega = 3.572$ rad/s

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left[\frac{1}{12}(2)(0.5)^2 + 2(0.25) + \frac{1}{2}(5)(0.2)^2 + 5(0.7)^2\right](3.572)^2 + 0$$

$$= \frac{1}{2}\left[\frac{1}{12}(2)(0.5)^2 + 2(0.25)^2 + \frac{1}{2}(5)(0.2)^2 + 5(0.7)^2\right]\omega^2$$

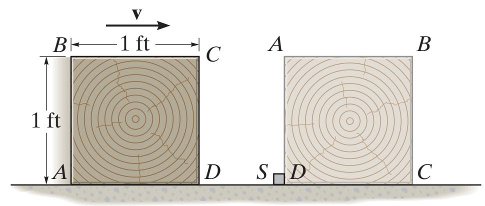
$$+ 2(9.81)(-0.25) + 5(9.81)(-0.7)$$

$$\omega = 6.45 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 6.45$ rad/s

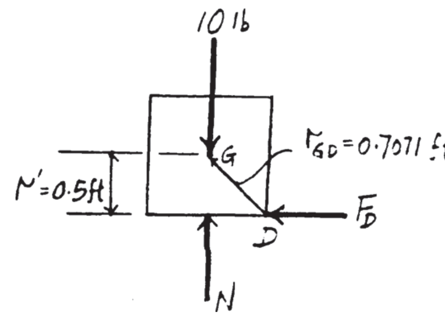
19–45.

The 10-lb block slides on the smooth surface when the corner D hits a stop block S. Determine the minimum velocity $\mathbf{v}$ the block should have which would allow it to tip over on its side and land in the position shown. Neglect the size of S. *Hint:* During impact consider the weight of the block to be nonimpulsive.



SOLUTION

Conservation of Energy: If the block tips over about point D, it must at least achieve the dash position shown. Datum is set at point D. When the block is at its initial and final position, its center of gravity is located 0.5 ft and 0.7071 ft *above* the datum. Its initial and final potential energy are $10(0.5) = 5.00\ \text{ft} \cdot \text{lb}$ and $10(0.7071) = 7.071\ \text{ft} \cdot \text{lb}$. The mass moment of inertia of the block about point D is



$$I_D = \frac{1}{12}\left(\frac{10}{32.2}\right)\left(1^2 + 1^2\right) + \left(\frac{10}{32.2}\right)\left(\sqrt{0.5^2 + 0.5^2}\right)^2 = 0.2070\ \text{slug} \cdot \text{ft}^2$$

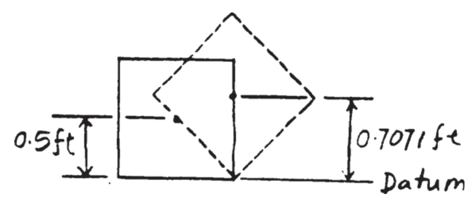
The initial kinetic energy of the block (after the impact) is $\frac{1}{2} I_D \omega_2^2 = \frac{1}{2}(0.2070)\ \omega_2^2$. Applying Eq. 18–18, we have

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(0.2070)\ \omega_2^2 + 5.00 = 0 + 7.071$$

$$\omega_2 = 4.472\ \text{rad/s}$$

Conservation of Angular Momentum: Since the weight of the block and the normal reaction N are *nonimpulsive* forces, the angular momentum is conserves about point D. Applying Eq. 19–17, we have



$$(H_D)_1 = (H_D)_2$$

$$(mv_G)(r') = I_D\ \omega_2$$

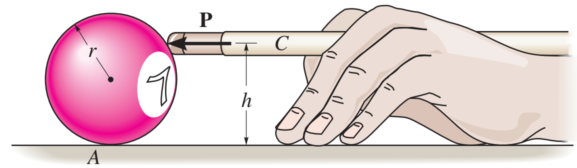
$$\left[\left(\frac{10}{32.2}\right)v\right](0.5) = 0.2070(4.472)$$

$$v = 5.96\ \text{ft/s} \qquad \textbf{Ans.}$$

Ans:
$v = 5.96\ \text{ft/s}$

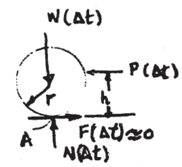
19–46.

Determine the height h at which a billiard ball of mass m must be struck so that no frictional force develops between it and the table at A. Assume that the cue C only exerts a horizontal force **P** on the ball.



SOLUTION

For the ball



$$(\overset{+}{\leftarrow})\ mv_1 + \Sigma\int F\,dt = mv_2$$

$$0 + P(\Delta t) = mv_2 \qquad \textbf{(1)}$$

$$\circlearrowleft+ \quad (H_A)_1 + \Sigma\int M_{A\,dt=(HA)2}$$

$$0 + (P)\Delta t(h) = \left[\frac{2}{5}m r^2 + m r^2\right]\omega_2 \qquad \textbf{(2)}$$

Require $v_2 = \omega_2 r$ **(3)**

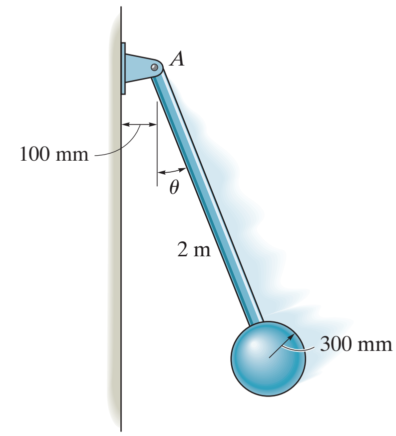
Solving Eqs. (1)–(3) for h yields

$$h = \frac{7}{5}r \qquad \textbf{Ans.}$$

Ans:
$h = \dfrac{7}{5}r$

19–47.

The pendulum consists of a 15-kg solid ball and 6-kg rod. If it is released from rest when $\theta_1 = 90°$, determine the angle θ_2 after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest. Take $e = 0.6$.



SOLUTION

Kinetic Energy. The mass moment of inertia of the pendulum about A is $I_A = \frac{1}{3}(6)(2^2) + \left[\frac{2}{5}(15)(0.3^2) + 15(2.3^2)\right] = 87.89\ \text{kg}\cdot\text{m}^2$.

Thus,

$$T = \frac{1}{2}I_A\omega^2 = \frac{1}{2}(87.89)\,\omega^2 = 43.945\omega^2$$

Potential Energy. With reference to datum set in Fig. *a*, the gravitational potential energy of the pendulum is

$$V_g = m_r g y_r + m_s g y_s$$
$$= 6(9.81)(-\cos\theta) + 15(9.81)(-2.3\cos\theta)$$
$$= -397.305\cos\theta$$

Coefficient of Restitution. The velocity of the mass center of the ball is $v_b = \omega r_G = \omega(2.3)$. Thus

$$(\overset{+}{\leftarrow})\ e = \frac{(v_w)_2 - (v_b)_2'}{(v_b)_2 - (v_w)_1}; \qquad 0.6 = \frac{0 - [-\omega_2'(2.3)]}{\omega_2(2.3) - 0}$$

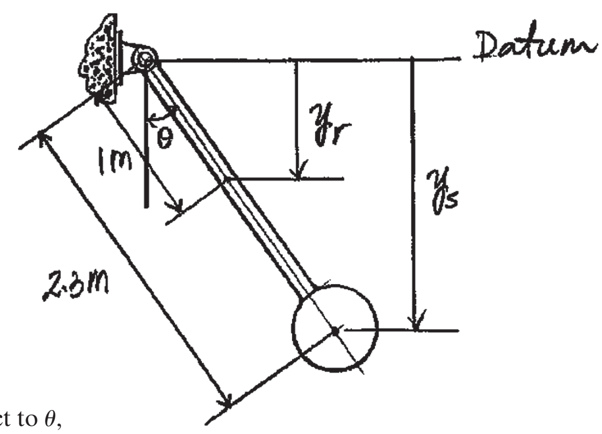
$$\omega_2' = 0.6\omega_2 \qquad \textbf{(1)}$$

Conservation of Energy. Consider the pendulum swing from the position $\theta = 90°$ to $\theta = 0°$ just before the impact,

$$(V_g)_1 = -397.305\cos 90° = 0$$
$$(V_g)_2 = -397.305\cos 0° = -397.305\ \text{J}$$
$$T_1 = 0 \qquad T_2 = 43.945\omega_2^2$$

Then

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 0 = 43.945\omega_2^2 + (-397.305)$$
$$\omega_2 = 3.0068\ \text{rad/s}$$



(a)

Thus, just after the impact, from Eq. (1)

$$\omega_2' = 0.6(3.0068) = 1.8041\ \text{rad/s}$$

Consider the pendulum swing from position $\theta = 90°$ just after the impact to θ,

$$(V_g)_2' = (V_g)_2 = -397.305\ \text{J}$$
$$(V_g)_3 = -397.305\cos\theta$$
$$T_2' = 43.945(1.8041^2) = 143.03\ \text{J}$$
$$T_3 = 0\ \ (\text{required})$$

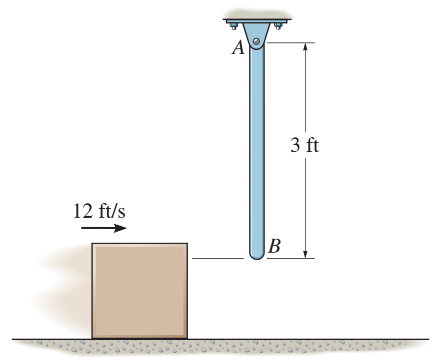
Then

$$T_2' + V_2' = T_3 + V_3$$
$$143.03 + (-397.305) = 0 + (-397.305\cos\theta)$$
$$\theta = 50.21° = 50.2° \qquad \textbf{Ans.}$$

Ans:
$\theta = 50.2°$

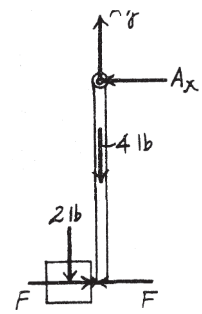
***19–48.**

The 4-lb rod AB is hanging in the vertical position. A 2-lb block, sliding on a smooth horizontal surface with a velocity of 12 ft/s, strikes the rod at its end B. Determine the velocity of the block immediately after the collision. The coefficient of restitution between the block and the rod at B is $e = 0.8$.



SOLUTION

Conservation of Angular Momentum: Since force F due to the impact is *internal* to the system consisting of the slender rod and the block, it will cancel out. Thus, angular momentum is conserved about point A. The mass moment of inertia of the slender rod about point A is $I_A = \frac{1}{12}\left(\frac{4}{32.2}\right)(3^2) + \frac{4}{32.2}(1.5^2) = 0.3727 \text{ slug} \cdot \text{ft}^2$. Here, $\omega_2 = \frac{(v_B)_2}{3}$. Applying Eq. 19–17, we have



$$(H_A)_1 = (H_A)_2$$

$$[m_b(v_b)_1](r_b) = I_A\omega_2 + [m_b(v_b)_2](r_b)$$

$$\left(\frac{2}{32.2}\right)(12)(3) = 0.3727\left[\frac{(v_B)_2}{3}\right] + \left(\frac{2}{32.2}\right)(v_b)_2(3) \qquad \textbf{[1]}$$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{(v_B)_2 - (v_b)_2}{(v_b)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.8 = \frac{(v_B)_2 - (v_b)_2}{12 - 0} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

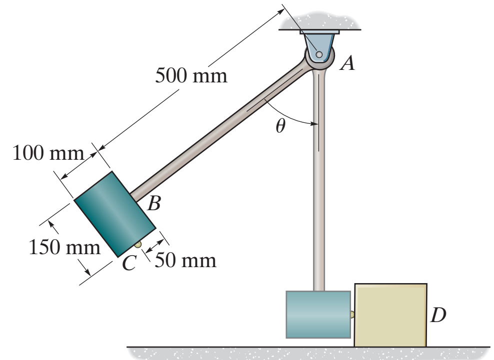
$$(v_b)_2 = 3.36 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

$$(v_B)_2 = 12.96 \text{ ft/s} \rightarrow$$

Ans:
$(v_b)_2 = 3.36 \text{ ft/s} \rightarrow$

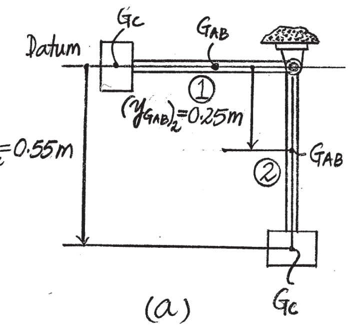
19–49.

The hammer consists of a 10-kg solid cylinder C and 6-kg uniform slender rod AB. If the hammer is released from rest when $\theta = 90°$ and strikes the 30-kg block D when $\theta = 0°$, determine the velocity of block D and the angular velocity of the hammer immediately after the impact. The coefficient of restitution between the hammer and the block is $e = 0.6$.



SOLUTION

Conservation of Energy: With reference to the datum in Fig. *a*, $V_1 = (V_g)_1 = W_{AB}(y_{GAB})_1 + W_C(y_{GC})_1 = 0$ and $V_2 = (V_g)_2 = -W_{AB}(y_{GAB})_2 - W_C(y_{GC})_2 = -6(9.81)(0.25) - 10(9.81)(0.55) = -68.67$ J. Initially, $T_1 = 0$. Since the hammer rotates about the fixed axis, $(v_{GAB})_2 = \omega_2 r_{GAB} = \omega_2(0.25)$ and $(v_{GC})_2 = \omega_2 r_{GC} = \omega_2(0.55)$. The mass moment of inertia of rod AB and cylinder C about their mass centers is $I_{GAB} = \frac{1}{12}ml^2 = \frac{1}{12}(6)(0.5^2) = 0.125\ \text{kg}\cdot\text{m}^2$ and $I_C = \frac{1}{12}m(3r^2 + h^2) = \frac{1}{12}(10)\left[3(0.05^2) + 0.15^2\right] = 0.025\ \text{kg}\cdot\text{m}^2$. Thus,



$$T_2 = \frac{1}{2}I_{GAB}\omega_2^2 + \frac{1}{2}m_{AB}(v_{GAB})_2^2 + \frac{1}{2}I_{GC}\,\omega_2^2 + \frac{1}{2}m_C(v_{GC})_2^2$$

$$= \frac{1}{2}(0.125)\omega_2^2 + \frac{1}{2}(6)\left[\omega_2(0.25)\right]^2 + \frac{1}{2}(0.025)\omega_2^2 + \frac{1}{2}(10)\left[\omega_2(0.55)\right]^2$$

$$= 1.775\,\omega_2^2$$

Then,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 1.775\omega_2^2 + (-68.67)$$

$$\omega_2 = 6.220\ \text{rad/s}$$

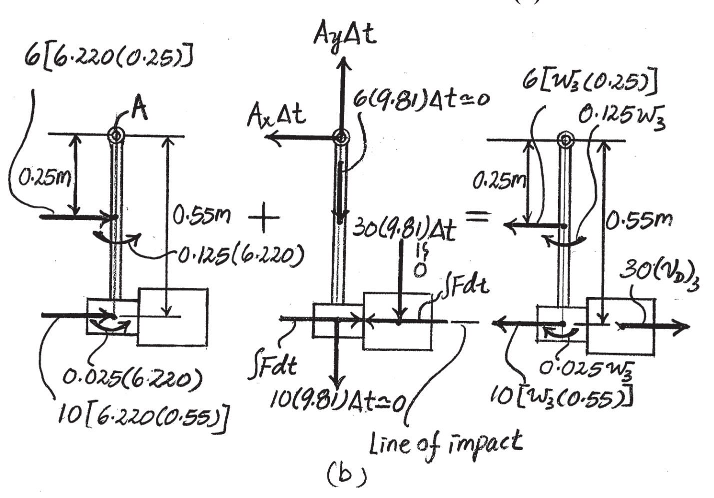
Conservation of Angular Momentum: The angular momentum of the system is conserved point A. Then,

$$(H_A)_1 = (H_A)_2$$

$$0.125(6.220) + 6[6.220(0.25)](0.25) + 0.025(6.220) + 10[6.220(0.55)](0.55)$$

$$= 30v_D(0.55) - 0.125\omega_3 - 6[\omega_3(0.25)](0.25) - 0.025\omega_3 - 10[\omega_3(0.55)](0.55)$$

$$16.5v_D - 3.55\omega_3 = 22.08 \qquad \textbf{(1)}$$



19–49. Continued

Coefficient of Restitution: Referring to Fig. *c*, the components of the velocity of the impact point P just before and just after impact along the line of impact are $\left[(v_P)_x\right]_2 = (v_{GC})_2 = \omega_2 r_{GC} = 6.220(0.55) = 3.421\text{ m/s} \rightarrow$ and $\left[(v_P)_x\right]_3 = (v_{GC})_3 = \omega_3 r_{GC} = \omega_3(0.55) \leftarrow$. Thus,

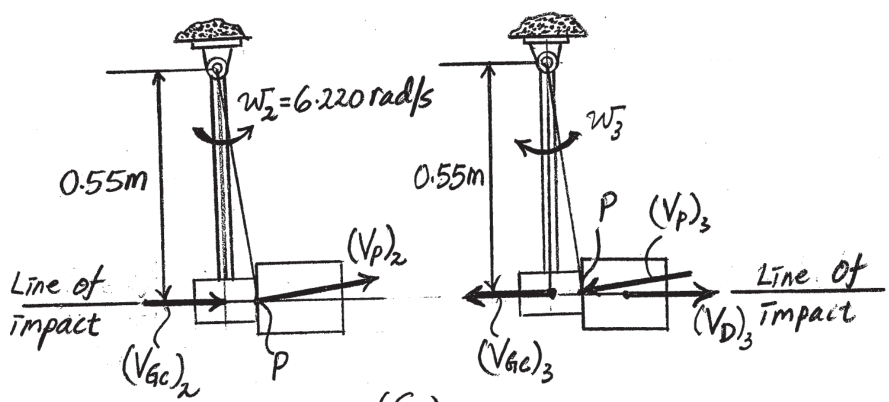
$$\overset{+}{\rightarrow} \qquad e = \frac{(v_D)_3 - \left[(v_P)_x\right]_3}{\left[(v_P)_x\right]_2 - (v_D)_2}$$

$$0.6 = \frac{(v_D)_3 - \left[-\omega_3(0.55)\right]}{3.421 - 0}$$

$$(v_D)_3 + 0.55\omega_3 = 2.053 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$(v_D)_3 = 1.54\text{ m/s} \qquad \omega_3 = 0.934\text{ rad/s} \qquad \textbf{Ans.}$$

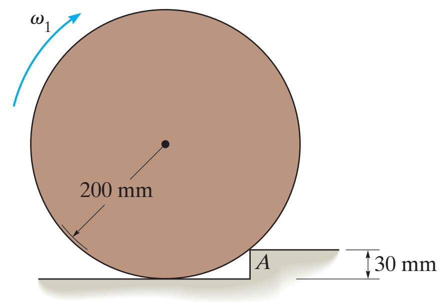


(C)

Ans:
$(v_D)_3 = 1.54\text{ m/s}$
$\omega_3 = 0.934\text{ rad/s}$

19–50.

The 20-kg disk strikes the step without rebounding. Determine the largest angular velocity ω_1 the disk can have and not lose contact with the step, A.



SOLUTION

Conservation of Angular Momentum. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}(20)(0.2^2) = 0.4\ \text{kg}\cdot\text{m}^2$. Since no slipping occurs, $v_G = \omega r = \omega(0.2)$. Referring to the impulse and momentum diagram, Fig. a, we notice that angular moment is conserved about point A since **W** is nonimpulsive. Thus,

$$(H_A)_1 = (H_A)_2$$

$$20[\omega_1(0.2)](0.17) + 0.4\,\omega_1 = 0.4\,\omega_2 + 20[\omega_2(0.2)](0.2)$$

$$\omega_1 = 1.1111\,\omega_2 \qquad \textbf{(1)}$$

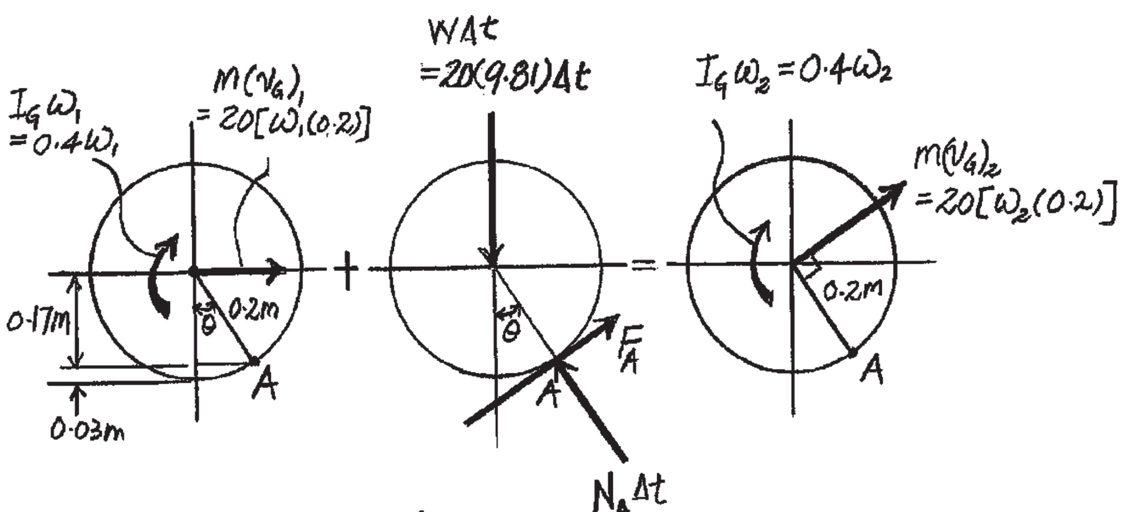
Equations of Motion. Since the requirement is the disk is about to lose contact with the step when it rotates about A, $N_A \simeq 0$. Here $\theta = \cos^{-1}\left(\frac{0.17}{0.2}\right) = 31.79°$. Consider the motion along n direction,

$$+\searrow \Sigma F_n = M(a_G)_n; \qquad 20(9.81)\cos 31.79° = 20\left[\omega_2^2(0.2)\right]$$

$$\omega_2 = 6.4570\ \text{rad/s}$$

Substitute this result into Eq. (1)

$$\omega_1 = 1.1111(6.4570) = 7.1744\ \text{rad/s} = 7.17\ \text{rad/s} \qquad \textbf{Ans.}$$

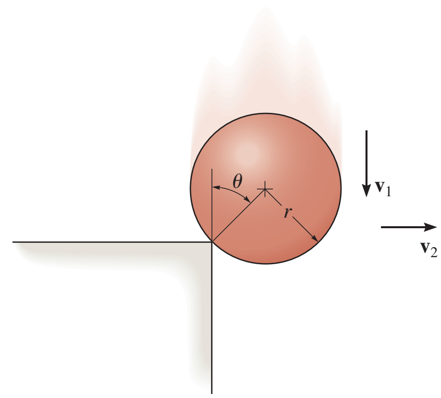


(a)

Ans:
$\omega_1 = 7.17\ \text{rad/s}$

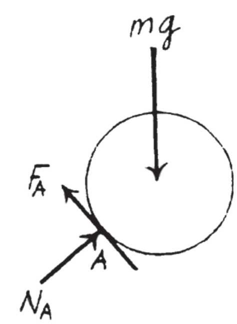
19–51.

The solid ball of mass m is dropped with a velocity $\mathbf{v}_1$ onto the edge of the rough step. If it rebounds horizontally off the step with a velocity $\mathbf{v}_2$, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is e.



SOLUTION

Conservation of Angular Momentum: Since the weight of the solid ball is a *nonimpulsive force*, then angular momentum is conserved about point A. The mass moment of inertia of the solid ball about its mass center is $I_G = \frac{2}{5}mr^2$. Here, $\omega_2 = \frac{v_2 \cos\theta}{r}$. Applying Eq. 19–17, we have



$$(H_A)_1 = (H_A)_2$$

$$\left[m_b\,(v_b)_1\right](r') = I_G\,\omega_2 + \left[m_b\,(v_b)_2\right](r'')$$

$$(mv_1)(r\sin\theta) = \left(\frac{2}{5}mr^2\right)\left(\frac{v_2\cos\theta}{r}\right) + (mv_2)(r\cos\theta)$$

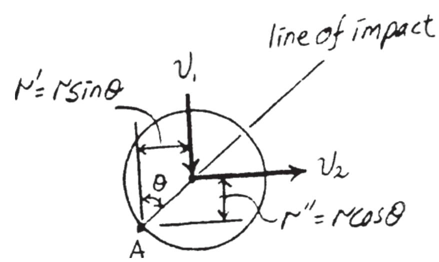
$$\frac{v_2}{v_1} = \frac{5}{7}\tan\theta \qquad \textbf{(1)}$$

Coefficient of Restitution: Applying Eq. 19–20, we have

$$e = \frac{0 - (v_b)_2}{(v_b)_1 - 0}$$

$$e = \frac{-(v_2\sin\theta)}{-v_1\cos\theta}$$

$$\frac{v_2}{v_1} = \frac{e\cos\theta}{\sin\theta} \qquad \textbf{(2)}$$



Equating Eqs. (1) and (2) yields

$$\frac{5}{7}\tan\theta = \frac{e\cos\theta}{\sin\theta}$$

$$\tan^2\theta = \frac{7}{5}e$$

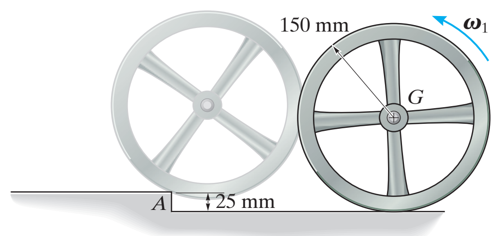
$$\theta = \tan^{-1}\left(\sqrt{\frac{7}{5}e}\right) \qquad \textbf{Ans.}$$

Ans:

$\theta = \tan^{-1}\left(\sqrt{\frac{7}{5}e}\right)$

***19–52.**

The wheel has a mass of 50 kg and a radius of gyration of 125 mm about its center of mass G. Determine the minimum value of the angular velocity $\boldsymbol{\omega}_1$ of the wheel, so that it strikes the step at A without rebounding and then rolls over it without slipping.



SOLUTION

Conservation of Angular Momentum: Referring to Fig. a, the sum of the angular impulses about point A is zero. Thus, angular momentum of the wheel is conserved about this point. Since the wheel rolls without slipping, $(v_G)_1 = \omega_1 r = \omega_1(0.15)$ and $(v_G)_2 = \omega_2 r = \omega_2(0.15)$. The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2 = 50(0.125^2) = 0.78125 \text{ kg}\cdot\text{m}^2$. Thus,

$$(H_A)_1 = (H_A)_2$$

$$50[\omega_1(0.15)](0.125) + 0.78125\omega_1 = 50[\omega_2(0.15)](0.15) + 0.78125\omega_2$$

$$\omega_1 = 1.109\omega_2 \qquad \textbf{(1)}$$

Conservation of Energy: With reference to the datum in Fig. a, $V_2 = (V_g)_2 = W(y_G)_2 = 0$ and $V_3 = (V_g)_3 = W(y_G)_3 = 50(9.81)(0.025) = 12.2625$ J. Since the wheel is required to be at rest in the final position, $T_3 = 0$. The initial kinetic energy of the wheel is $T_2 = \frac{1}{2}m(v_G)_2{}^2 + \frac{1}{2}I_G\omega_2{}^2 = \frac{1}{2}(50)[\omega_2(0.15)]^2 + \frac{1}{2}(0.78125)(\omega_2{}^2) = 0.953125\omega_2{}^2$. Then

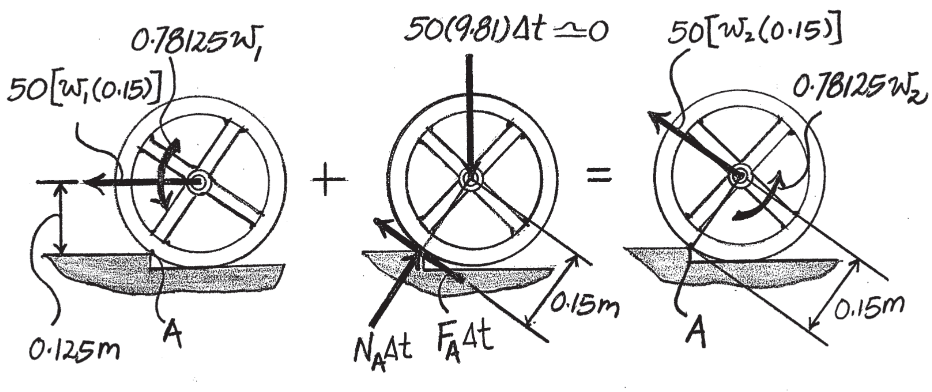
$$T_2 + V_2 = T_3 + V_3$$

$$0.953125\omega_2{}^2 + 0 = 0 + 12.2625$$

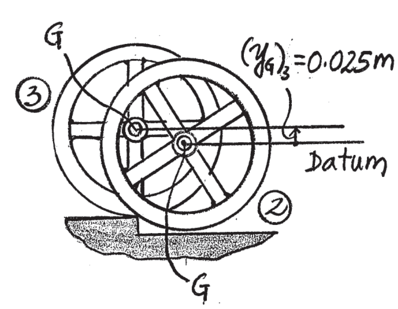
$$\omega_2 = 3.587 \text{ rad/s}$$

Substituting this result into Eq. (1), we obtain

$$\omega_1 = 3.98 \text{ rad/s} \qquad \textbf{Ans.}$$



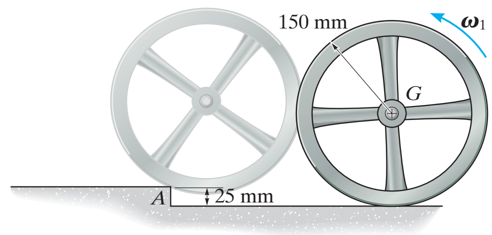
(a)



(b)

19–53.

The wheel has a mass of 50 kg and a radius of gyration of 125 mm about its center of mass G. If it rolls without slipping with an angular velocity of $\boldsymbol{\omega}_1 = 5$ rad/s before it strikes the step at A, determine its angular velocity after it rolls over the step. The wheel does not loose contact with the step when it strikes it.



SOLUTION

Conservation of Angular Momentum: Referring to Fig. a, the sum of the angular impulses about point A is zero. Thus, angular momentum of the wheel is conserved about this point. Since the wheel rolls without slipping, $(v_G)_1 = \omega_1 r = (5)(0.15) = 0.75$ m/s and $\omega_2 = \omega_2 r = \omega_2(0.15)$. The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2 = 50(0.125^2) = 0.78125\ \text{kg}\cdot\text{m}^2$. Thus,

$$(H_A)_1 = (H_A)_2$$

$$50(0.75)(0.125) + 0.78125(5) = 50[\omega_2(0.15)](0.15) + 0.78125\omega_2$$

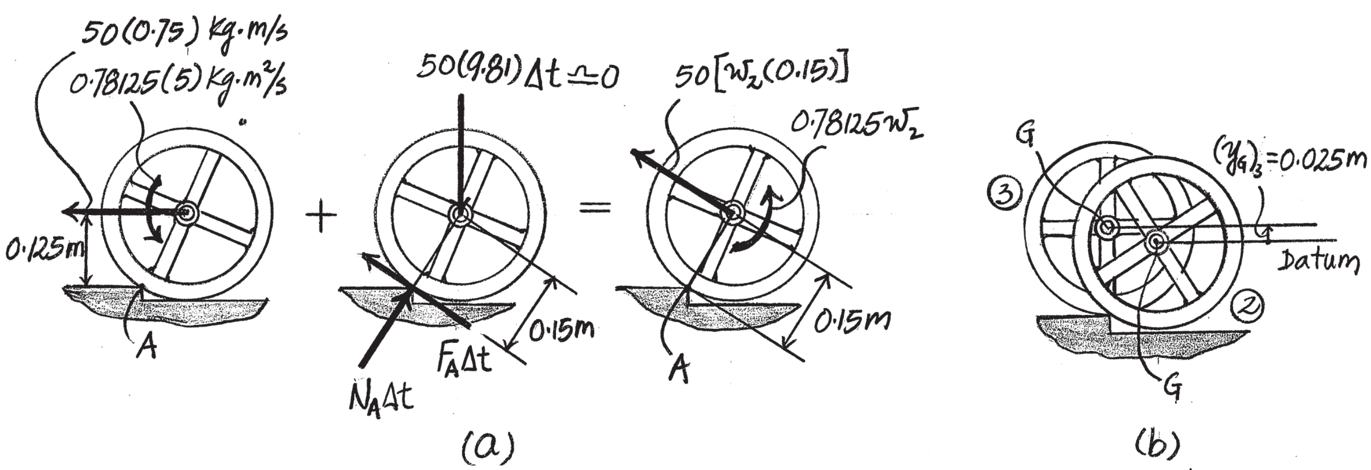
$$\omega_2 = 4.508\ \text{rad/s} \qquad \textbf{(1)}$$

Conservation of Energy: With reference to the datum in Fig. a, $V_2 = (V_g)_2 = W(y_G)_2 = 0$ and $V_3 = (V_g)_3 = W(y_G)_3 = 50(9.81)(0.025) = 12.2625$ J. The initial kinetic energy of the wheel is $T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}(50)[\omega(0.15)]^2 + \frac{1}{2}(0.78125)\omega^2 = 0.953125\omega^2$. Thus, $T_2 = 0.953125\omega_2{}^2 = 0.953125(4.508^2) = 19.37$ J and $T_3 = 0.953125\omega_3{}^2$.

$$T_2 + V_2 = T_3 + V_3$$

$$19.37 + 0 = 0.953125\omega_3{}^2 + 12.2625$$

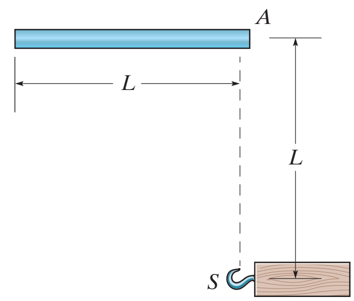
$$\omega_3 = 2.73\ \text{rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega_3 = 2.73$ rad/s

19–54.

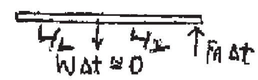
The rod of mass m and length L is released from rest without rotating. When it falls a distance L, the end A strikes the hook S, which provides a permanent connection. Determine the angular velocity ω of the rod after it has rotated 90°. Treat the rod's weight during impact as a nonimpulsive force.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + mgL = \frac{1}{2}mv_G^2 + 0$$



$$v_G = \sqrt{2gL}$$

$$H_1 = H_2$$

$$m\sqrt{2gL}\left(\frac{L}{2}\right) = \frac{1}{3}mL^2(\omega_2)$$

$$\omega_2 = \frac{3}{2}\frac{\sqrt{2gL}}{L}$$

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left(\frac{1}{3}mL^2\right)\frac{9(2gL)}{4L^2} + 0 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 - mg\left(\frac{L}{2}\right)$$

$$\frac{3}{4}gL = \frac{1}{6}L^2\omega^2 - g\left(\frac{L}{2}\right)$$

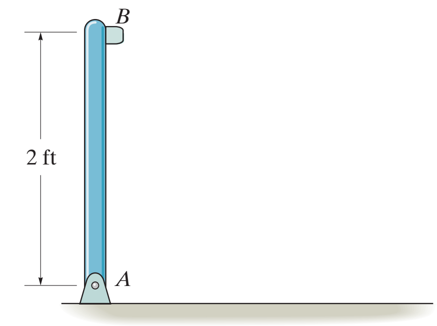
$$\omega = \sqrt{7.5\frac{g}{L}} \qquad \textbf{Ans.}$$

Ans:

$$\omega = \sqrt{7.5\,\frac{g}{L}}$$

19–55.

The 15-lb rod AB is released from rest in the vertical position. If the coefficient of restitution between the floor and the cushion at B is $e = 0.7$, determine how high the end of the rod rebounds after impact with the floor.



SOLUTION

$T_1 = V_1 = T_2 + V_2$

$$0 + 15(1) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{15}{32.2}\right)(2)^2\right]\omega_2^2$$

$\omega_2 = 6.950 \text{ rad/s} \qquad \text{Hence } (v_B)_2 = 6.950(2) = 13.90 \text{ rad/s}$

$$(+\downarrow) \qquad e = \frac{0 - (v_B)_3}{(v_B)_2 - 0}; \qquad 0.7 = \frac{0 - (v_B)_3}{13.90}$$

$$(v_B)_3 = -9.730 \text{ ft/s} = 9.730 \text{ ft/s} \uparrow$$

$$\omega_3 = \frac{(v_B)_3}{2} = \frac{9.730}{2} = 4.865 \text{ rad/s}$$

$T_3 + V_3 = T_4 + V_4$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{15}{32.2}\right)(2)^2\right](4.865)^2 = 0 + 15(h_G)$$

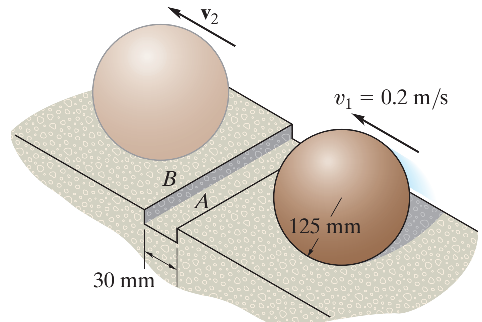
$h_G = 0.490 \text{ ft}$

$h_B = 2h_G = 0.980 \text{ ft}$ **Ans.**

Ans:
$h_B = 0.980 \text{ ft}$

***19–56.**

A ball having a mass of 8 kg and initial speed of $v_1 = 0.2$ m/s rolls over a 30-mm-long depression. Assuming that the ball rolls off the edges of contact first A, then B, without slipping, determine its final velocity $\mathbf{v}_2$ when it reaches the other side.



SOLUTION

$$\omega_1 = \frac{0.2}{0.125} = 1.6 \text{ rad/s} \qquad \omega_2 = \frac{v_2}{0.125} = 8v_2$$

$$\theta = \sin^{-1}\left(\frac{15}{125}\right) = 6.8921^\circ$$

$$h = 125 - 125 \cos 6.8921^\circ = 0.90326 \text{ mm}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(8)(0.2)^2 + \frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](1.6)^2 + 0$$

$$= -(0.90326)(10^{-3})8(9.81) + \frac{1}{2}(8)\omega^2(0.125)^2 + \frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](\omega)^2$$

$$\omega = 1.836 \text{ rad/s}$$

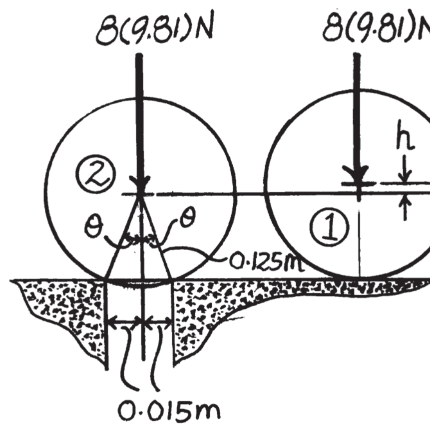
$$(H_B)_2 = (H_B)_3$$

$$\left[\frac{2}{5}(8)(0.125)^2\right](1.836) + 8(1.836)(0.125)\cos 6.892^\circ(0.125\cos 6.892^\circ)$$

$$- 8(0.22948 \sin 6.892^\circ)(0.125 \sin 6.892^\circ)$$

$$= \left[\frac{2}{5}(8)(0.125)^2\right]\omega_3 + 8(0.125)\omega_3(0.125)$$

$$\omega_3 = 1.7980 \text{ rad/s}$$



$$T_3 + V_3 = T_4 + V_4$$

$$\frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](1.7980)^2 + \frac{1}{2}(8)(1.7980)^2(0.125)^2 + 0$$

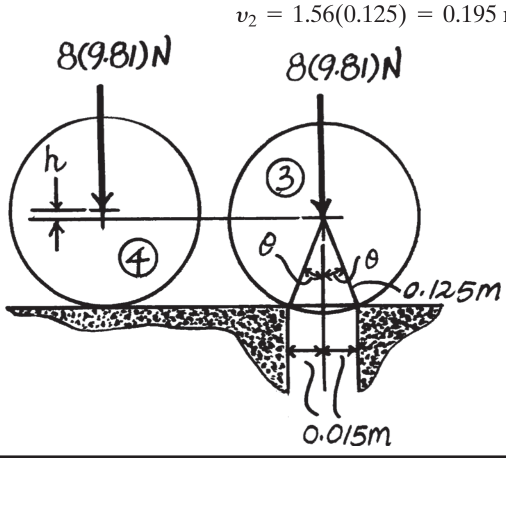
$$= 8(9.81)(0.90326(10^{-3})) + \frac{1}{2}\left[\frac{2}{5}(8)(0.125)^2\right](\omega_4)^2$$

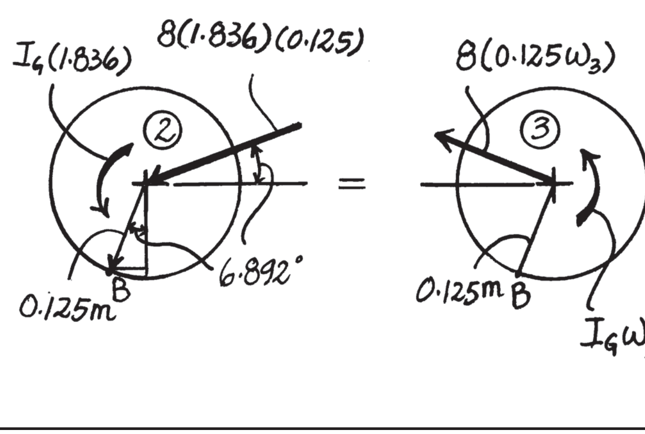
$$+ \frac{1}{2}(8)(\omega_4)^2(0.125)^2$$

$$\omega_4 = 1.56 \text{ rad/s}$$

So that

$$v_2 = 1.56(0.125) = 0.195 \text{ m/s} \qquad \textbf{Ans.}$$

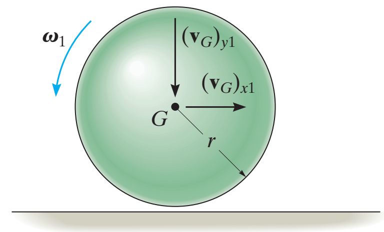




Ans:
$v_2 = 0.195$ m/s

19–57.

A solid ball with a mass m is thrown on the ground such that at the instant of contact it has an angular velocity $\boldsymbol{\omega}_1$ and velocity components $(\mathbf{v}_G)_{x1}$ and $(\mathbf{v}_G)_{y1}$ as shown. If the ground is rough so no slipping occurs, determine the components of the velocity of its mass center just after impact. The coefficient of restitution is e.



SOLUTION

Coefficient of Restitution (y direction):

$$(+\downarrow) \qquad e = \frac{0 - (v_G)_{y2}}{(v_G)_{y1} - 0} \qquad (v_G)_{y2} = -e(v_G)_{y1} = e(v_G)_{y1} \uparrow \qquad \textbf{Ans.}$$

Conservation of angular momentum about point on the ground:

$$(\circlearrowleft +) \qquad (H_A)_1 = (H_A)_2$$

$$-\frac{2}{5}mr^2\omega_1 + m(v_G)_{x1}r = \frac{2}{5}mr^2\omega_2 + m(v_G)_{x2}r$$

Since no slipping, $(v_G)_{x2} = \omega_2 r$ then,

$$\omega_2 = \frac{5\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right)}{7r}$$

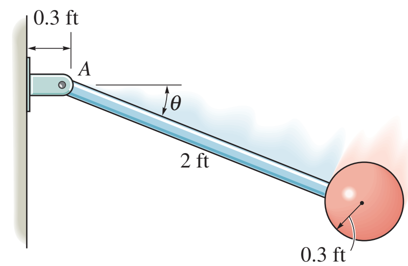
Therefore

$$(v_G)_{x2} = \frac{5}{7}\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right) \qquad \textbf{Ans.}$$

Ans:
$(v_G)_{y2} = e(v_G)_{y1} \uparrow$
$(v_G)_{x2} = \frac{5}{7}\left((v_G)_{x1} - \frac{2}{5}\omega_1 r\right) \leftarrow$

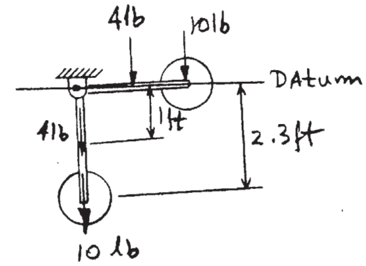
19–58.

The pendulum consists of a 10-lb solid ball and 4-lb rod. If it is released from rest when $\theta_0 = 0°$, determine the angle θ_1 of rebound after the ball strikes the wall and the pendulum swings up to the point of momentary rest. Take $e = 0.6$.



SOLUTION

$$I_A = \frac{1}{3}\left(\frac{4}{32.2}\right)(2)^2 + \frac{2}{5}\left(\frac{10}{32.2}\right)(0.3)^2 + \left(\frac{10}{32.2}\right)(2.3)^2 = 1.8197 \text{ slug}\cdot\text{ft}^2$$



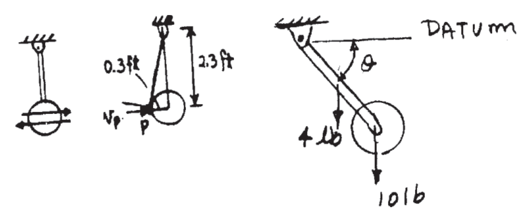
Just before impact:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 + \frac{1}{2}(1.8197)\omega^2 - 4(1) - 10(2.3)$$

$$\omega = 5.4475 \text{ rad/s}$$

$$v_P = 2.3(5.4475) = 12.529 \text{ ft/s}$$



Since the wall does not move,

$$\left(\overset{+}{\rightarrow}\right) \qquad e = 0.6 = \frac{(v_P) - 0}{0 - (-12.529)}$$

$$(v_P) = 7.518 \text{ ft/s}$$

$$\omega' = \frac{7.518}{2.3} = 3.2685 \text{ rad/s}$$

$$T_3 + V_3 = T_4 + V_4$$

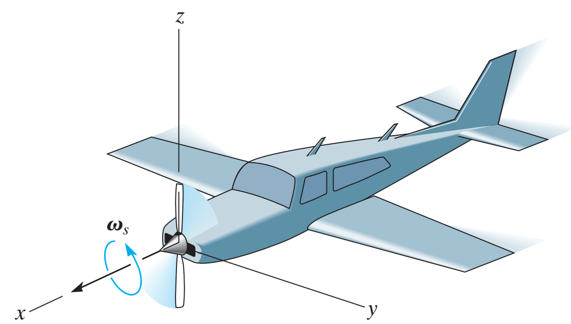
$$\frac{1}{2}(1.8197)(3.2685)^2 - 4(1) - 10(2.3) = 0 - 4(1)\sin\theta_1 - 10(2.3\sin\theta_1)$$

$$\theta_1 = 39.8° \qquad \textbf{Ans.}$$

Ans:
$\theta_1 = 39.8°$

20–1.

The propeller of an airplane is rotating at a constant speed $\omega_s \mathbf{i}$, while the plane is undergoing a turn at a constant rate ω_t. Determine the angular acceleration of the propeller if (a) the turn is horizontal, i.e., $\omega_t \mathbf{k}$, and (b) the turn is vertical, downward, i.e., $\omega_t \mathbf{j}$.



SOLUTION

(a) For ω_s, $\Omega = \omega_t \mathbf{k}$.

$$(\dot{\omega}_s)_{XYZ} = (\dot{\omega}_s)_{xyz} + \Omega \times \omega_s$$

$$= \mathbf{0} + (\omega_t \mathbf{k}) \times (\omega_s \mathbf{i}) = \omega_s \omega_t \mathbf{j}$$

For ω_t, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_t)_{XYZ} = (\dot{\omega}_t)_{xyz} + \Omega \times \omega_t \mathbf{k} = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

$$\alpha = \dot{\omega} = (\dot{\omega}_s)_{XYZ} + (\dot{\omega}_t)_{XYZ}$$

$$\alpha = \omega_s \omega_t \mathbf{j} + \mathbf{0} = \omega_s \omega_t \mathbf{j}$$ **Ans.**

(b) For ω_s, $\Omega = \omega_t \mathbf{j}$.

$$(\dot{\omega}_s)_{XYZ} = (\dot{\omega}_s)_{xyz} + \Omega \times \omega_s$$

$$= 0 + (\omega_t \mathbf{j}) \times (\omega_s \mathbf{i}) = -\omega_s \omega_t \mathbf{k}$$

For ω_t, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_t)_{XYZ} = (\dot{\omega}_t)_{xyz} + \Omega \times \omega_t = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

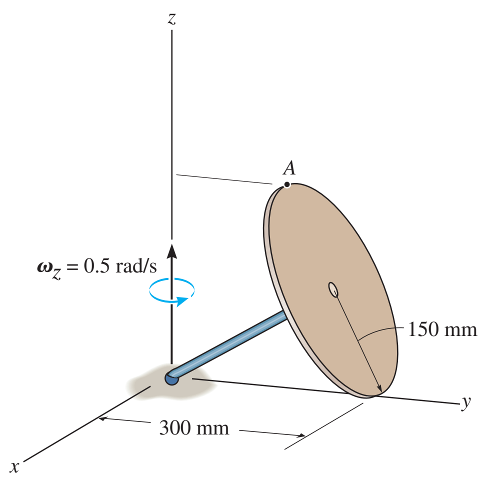
$$\alpha = \dot{\omega} = (\dot{\omega}_s)_{XYZ} + (\dot{\omega}_t)_{XYZ}$$

$$\alpha = -\omega_s \omega_t \mathbf{k} + \mathbf{0} = -\omega_s \omega_t \mathbf{k}$$ **Ans.**

Ans:
(a) $\boldsymbol{\alpha} = \omega_s \omega_t \mathbf{j}$
(b) $\boldsymbol{\alpha} = -\omega_s \omega_t \mathbf{k}$

20–2.

The disk rotates about the z axis at a constant rate $\omega_z = 0.5$ rad/s without slipping on the horizontal plane. Determine the velocity and the acceleration of point A on the disk.



SOLUTION

Angular Velocity: The coordinate axes for the fixed frame (X, Y, Z) and rotating frame (x, y, z) at the instant shown are set to be coincident. Thus, the angular velocity of the disk at this instant (with reference to X, Y, Z) can be expressed in terms of **i**, **j**, **k** components. Since the disk rolls without slipping, then its angular velocity $\omega = \omega_s + \omega_z$ is always directed along the instantaneuos axis of zero velocity (y axis). Thus,

$$\omega = \omega_s + \omega_z$$

$$-\omega\mathbf{j} = -\omega_s \cos 30°\mathbf{j} - \omega_s \sin 30°\mathbf{k} + 0.5\mathbf{k}$$

Equating **k** and **j** components, we have

$$0 = -\omega_s \sin 30° + 0.5 \qquad \omega_s = 1.00 \text{ rad/s}$$

$$-\omega = -1.00 \cos 30° \qquad \omega = 0.8660 \text{ rad/s}$$

Angular Acceleration: The angular acceleration α will be determined by investigating the time rate of change of *angular velocity* with respect to the fixed XYZ frame. Since ω always lies in the fixed X–Y plane, then $\omega = \{-0.8660\mathbf{j}\}$ rad/s is observed to have a *constant direction* from the rotating xyz frame if this frame is rotating at $\Omega = \omega_z = \{0.5\mathbf{k}\}$ rad/s. Applying Eq. 20–6 with $(\dot{\omega})_{xyz} = \mathbf{0}$, we have

$$\alpha = \dot{\omega} = (\dot{\omega})_{xyz} + \omega_z \times \omega = \mathbf{0} + 0.5\mathbf{k} \times (-0.8660\mathbf{j}) = \{0.4330\mathbf{i}\} \text{ rad/s}^2$$

Velocity and Acceleration: Applying Eqs. 20–3 and 20–4 with the ω and α obtained above and $\mathbf{r}_A = \{(0.3 - 0.3 \cos 60°)\mathbf{j} + 0.3 \sin 60°\mathbf{k}\}$ m $= \{0.15\mathbf{j} + 0.2598\mathbf{k}\}$ m, we have

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (-0.8660\mathbf{j}) \times (0.15\mathbf{j} + 0.2598\mathbf{k}) = \{-0.225\mathbf{i}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times (\omega \times \mathbf{r}_A)$$

$$= (0.4330\mathbf{i}) \times (0.15\mathbf{j} + 0.2598\mathbf{k})$$

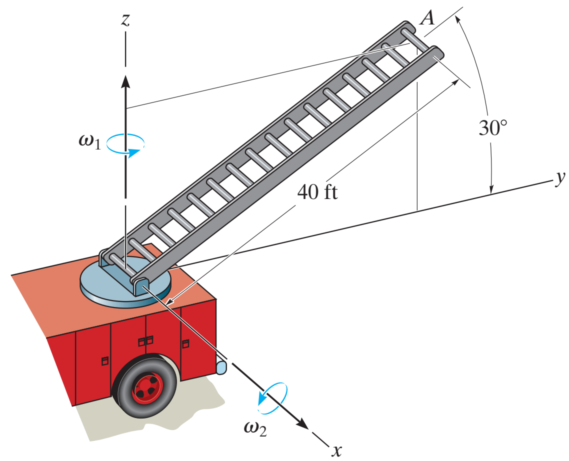
$$+ (-0.8660\mathbf{j}) \times [(-0.8660\mathbf{j}) \times (0.15\mathbf{j} + 0.2598\mathbf{k})]$$

$$= \{-0.1125\mathbf{j} - 0.130\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-0.225\mathbf{i}\}$ m/s
$\mathbf{a}_A = \{-0.1125\mathbf{j} - 0.130\mathbf{k}\}$ m/s^2

20–3.

The ladder of the fire truck rotates around the z axis with an angular velocity $\omega_1 = 0.15$ rad/s, which is increasing at 0.8 rad/s^2. At the same instant it is rotating upward at a constant rate $\omega_2 = 0.6$ rad/s. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.



SOLUTION

$$\omega = \omega_1 + \omega_2 = 0.15\mathbf{k} + 0.6\mathbf{i} = \{0.6\mathbf{i} + 0.15\mathbf{k}\} \text{ rad/s}$$

Angular acceleration: For ω_1, $\omega = \omega_1 = \{0.15\mathbf{k}\}$ rad/s.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \omega \times \omega_2$$

$$= 0 + (0.15\mathbf{k}) \times (0.6\mathbf{i}) = \{0.09\mathbf{j}\} \text{ rad/s}^2$$

For ω_1, $\Omega = 0$.

$$(\dot{\omega}_1)_{XYZ} = (\dot{\omega}_1)_{xyz} + \omega \times \omega_1 = (0.8\mathbf{k}) + 0 = \{0.8\mathbf{k}\} \text{ rad/s}^2$$

$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = 0.8\mathbf{k} + 0.09\mathbf{j} = \{0.09\mathbf{j} + 0.8\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_A = 40 \cos 30°\mathbf{j} + 40 \sin 30°\mathbf{k} = \{34.641\mathbf{j} + 20\mathbf{k}\} \text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$

$$= (0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$$

$$= \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r} + \omega \times \mathbf{v}_A$$

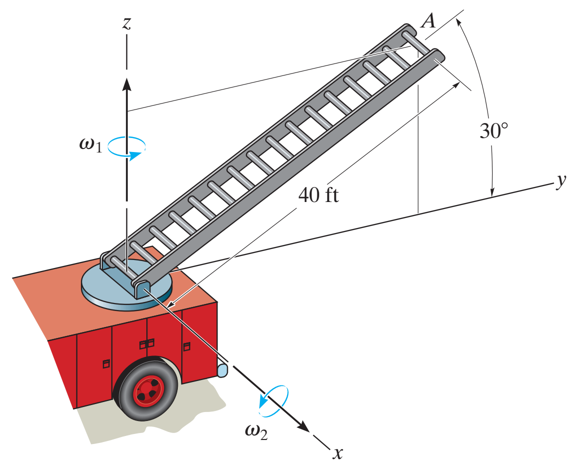
$$= (0.09\mathbf{j} + 0.8\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k}) + (0.6\mathbf{i} + 0.15\mathbf{k}) \times (-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k})$$

$$= \{-24.1\mathbf{i} - 13.3\mathbf{j} - 7.20\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-24.1\mathbf{i} - 13.3\mathbf{j} - 7.20\mathbf{k}\}$ ft/s^2

***20–4.**

The ladder of the fire truck rotates around the z axis with an angular velocity of $\omega_1 = 0.15$ rad s, which is increasing at 0.2 rad/s^2. At the same instant it is rotating upward at $\omega_2 = 0.6$ rad/s while increasing at 0.4 rad/s^2. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.



SOLUTION

$\mathbf{r}_{A/O} = 40 \cos 30°\mathbf{j} + 40 \sin 30°\mathbf{k}$

$\mathbf{r}_{A/O} = \{34.641\mathbf{j} + 20\mathbf{k}\}$ ft

$\Omega = \omega_1 \mathbf{k} + \omega_2 \mathbf{i} = \{0.6\mathbf{i} + 0.15\mathbf{k}\}$ rad/s

$\dot{\omega} = \dot{\omega}_1 \mathbf{k} + \dot{\omega}_2 \mathbf{i} + \omega_1 \mathbf{k} \times \omega_2 \mathbf{i}$

$\dot{\Omega} = 0.2\mathbf{k} + 0.4\mathbf{i} + 0.15\mathbf{k} \times 0.6\mathbf{i} = \{0.4\mathbf{i} + 0.09\mathbf{j} + 0.2\mathbf{k}\}$ rad/s^2

$\mathbf{v}_A = \Omega \times \mathbf{r}_{A/O} = (0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$

$\mathbf{v}_A = \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}$ ft/s **Ans.**

$\mathbf{a}_A = \Omega \times (\Omega \times \mathbf{r}_{A/O}) + \dot{\omega} \times \mathbf{r}_{A/O}$

$\mathbf{a}_A = (0.6\mathbf{i} + 0.15\mathbf{k}) \times [(0.6\mathbf{i} + 0.15\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})]$

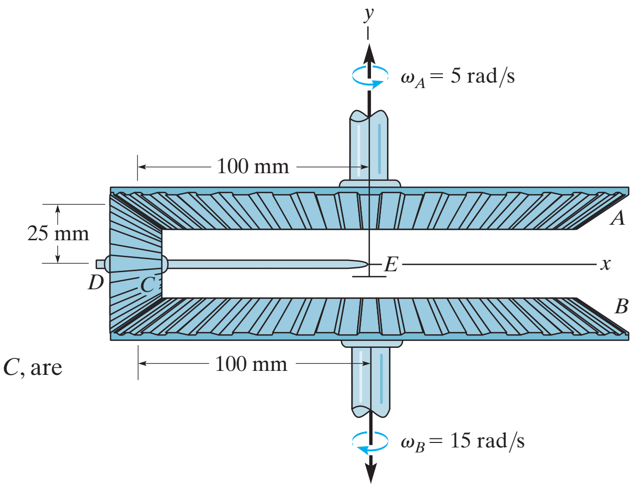
$\quad + (0.4\mathbf{i} + 0.09\mathbf{j} + 0.2\mathbf{k}) \times (34.641\mathbf{j} + 20\mathbf{k})$

$\mathbf{a}_A = \{-3.33\mathbf{i} - 21.3\mathbf{j} + 6.66\mathbf{k}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_A = \{-5.20\mathbf{i} - 12\mathbf{j} + 20.8\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-3.33\mathbf{i} - 21.3\mathbf{j} + 6.66\mathbf{k}\}$ ft/s^2

20–5.

If the plate gears A and B are rotating with the angular velocities shown, determine the angular velocity of gear C about the shaft DE. What is the angular velocity of DE about the y axis?



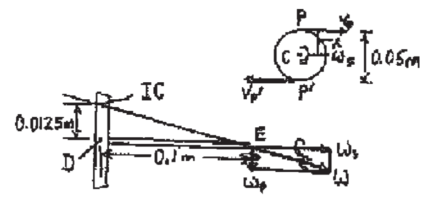
SOLUTION

The speeds of points P and P', located at the top and bottom of gear C, are

$$v_p = (5)(0.1) = 0.5 \text{ m/s}$$

$$v_{p'} = (15)(0.1) = 1.5 \text{ m/s}$$

The IC is located as shown.



$$\frac{0.5}{x} = \frac{1.5}{(0.05 - x)}; \quad x = 0.0125 \text{ m}$$

$$\frac{\omega_s}{0.1} = \frac{\omega_p}{0.0125}; \quad \omega_s = 8\,\omega_p$$

$$\omega = \omega_s\mathbf{i} - \omega_p\mathbf{j} = \omega_s\mathbf{i} - \frac{1}{8}\omega_p\mathbf{j}$$

$$\mathbf{v} = \omega \times \mathbf{r}$$

$$0.5\mathbf{k} = \left(\omega_p\mathbf{i} - \frac{1}{8}\omega_s\mathbf{j}\right) \times (-0.1\mathbf{i} + 0.025\mathbf{j})$$

$$0.5\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_s & -\frac{1}{8}\omega_s & 0 \\ -0.1 & 0.025 & 0 \end{vmatrix} = 0.0125\omega_s\,\mathbf{k}$$

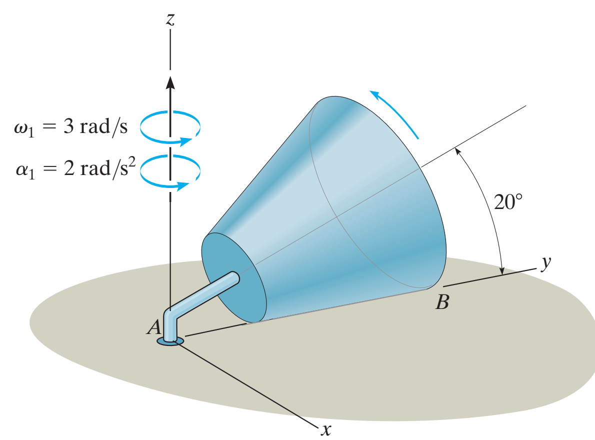
$$\omega_s = \frac{0.5}{0.0125} = 40 \text{ rad/s}$$ **Ans.** (Angular velocity of C about DE)

$$\omega_p = \frac{1}{8}(40) = 5 \text{ rad/s}$$ **Ans.** (Angular velocity of DE about y axis)

Ans:
$(\omega_C)_{DE} = 40$ rad/s
$(\omega_{DE})_y = 5$ rad/s

20–6.

The conical spool rolls on the plane without slipping. If the axle has an angular velocity of $\omega_1 = 3$ rad/s and an angular acceleration of $\alpha_1 = 2$ rad/s^2 at the instant shown, determine the angular velocity and angular acceleration of the spool at this instant.



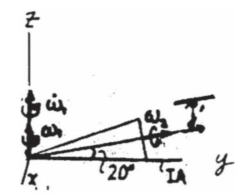
SOLUTION

$$\omega_1 = 3 \text{ rad/s}$$

$$\omega_2 = -\frac{3}{\sin 20^\circ} = -8.7714 \text{ rad/s}$$

$$\omega = \omega_1 + \omega_2 = 3\mathbf{k} - 8.7714 \cos 20^\circ \mathbf{j} - 8.7714 \sin 20^\circ \mathbf{k}$$

$$= \{-8.24\mathbf{j}\} \text{ rad/s}$$ **Ans.**



$$\left(\dot{\omega}_1\right)_{xyz} = 2 \text{ rad/s}^2$$

$$\left(\dot{\omega}_2\right)_{xyz} = -\frac{2}{\sin 20^\circ} = -5.8476 \text{ rad/s}^2$$

$$\alpha = \dot{\omega} = \left(\dot{\omega}_1\right)_{xyz} + \omega_1 \times \omega_1 + \left(\dot{\omega}_2\right)_{xyz} + \omega_1 \times \omega_2$$

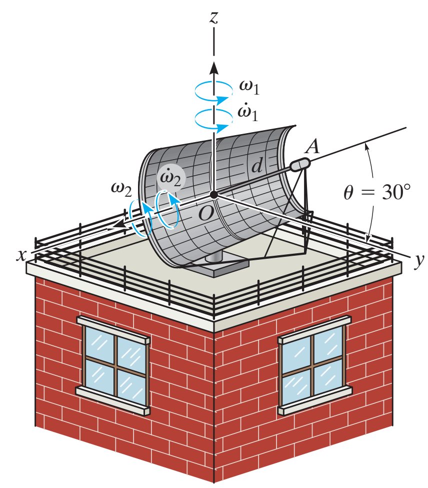
$$= 2\mathbf{k} + \mathbf{0} + (-5.8476 \cos 20^\circ \mathbf{j} - 5.8476 \sin 20^\circ \mathbf{k}) + (3\mathbf{k}) \times (-8.7714 \cos 20^\circ \mathbf{j} - 8.7714 \sin 20^\circ \mathbf{k})$$

$$\alpha = \{24.7\mathbf{i} - 5.49\mathbf{j}\} \text{ rad/s}^2$$ **Ans.**

Ans:
$\boldsymbol{\omega} = \{-8.24\mathbf{j}\}$ rad/s
$\boldsymbol{\alpha} = \{24.7\mathbf{i} - 5.49\mathbf{j}\}$ rad/s^2

20–7.

At a given instant, the antenna has an angular motion $\omega_1 = 3$ rad/s and $\dot{\omega}_1 = 2$ rad/s^2 about the z axis. At this same instant $\theta = 30°$, the angular motion about the x axis is $\omega_2 = 1.5$ rad/s, and $\dot{\omega}_2 = 4$ rad/s^2. Determine the velocity and acceleration of the signal horn A at this instant. The distance from O to A is $d = 3$ ft.



SOLUTION

$\mathbf{r}_A = 3\cos 30°\mathbf{j} + 3\sin 30°\mathbf{k} = \{2.598\mathbf{j} + 1.5\mathbf{k}\}$ ft

$\Omega = \omega_1 + \omega_2 = 3\mathbf{k} + 1.5\mathbf{i}$

$\mathbf{v}_A = \Omega \times \mathbf{r}_A$

$\mathbf{v}_A = (3\mathbf{k} + 1.5\mathbf{i}) \times (2.598\mathbf{j} + 1.5\mathbf{k})$

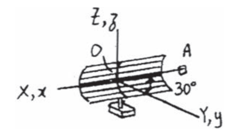
$= -7.794\mathbf{i} + 3.897\mathbf{k} - 2.25\mathbf{j}$

$= \{-7.79\mathbf{i} - 2.25\mathbf{j} + 3.90\mathbf{k}\}$ ft/s **Ans.**

$\dot{\Omega} = \dot{\omega}_1 + \dot{\omega}_2$

$= (2\mathbf{k} + 0) + (4\mathbf{i} + 3\mathbf{k} \times 1.5\mathbf{i})$

$= 4\mathbf{i} + 4.5\mathbf{j} + 2\mathbf{k}$



$\mathbf{a}_A = \dot{\omega} \times r_A + \Omega \times \mathbf{v}_A$

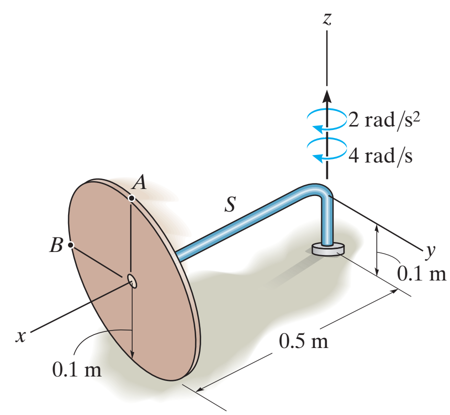
$\mathbf{a}_A = (4\mathbf{i} + 4.5\mathbf{j} + 2\mathbf{k}) \times (2.598\mathbf{j} + 1.5\mathbf{k}) + (3\mathbf{k} + 1.5\mathbf{i}) \times (-7.794\mathbf{i} - 2.25\mathbf{j} + 3.879\mathbf{k})$

$\mathbf{a}_A = \{8.30\mathbf{i} - 35.2\mathbf{j} + 7.02\mathbf{k}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_A = \{-7.79\mathbf{i} - 2.25\mathbf{j} + 3.90\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{8.30\mathbf{i} - 35.2\mathbf{j} + 7.02\mathbf{k}\}$ ft/s^2

***20–8.**

The disk rotates about the shaft S, while the shaft is turning about the z axis at a rate of $\omega_z = 4$ rad/s, which is increasing at 2 rad/s². Determine the velocity and acceleration of point A on the disk at the instant shown. No slipping occurs.



SOLUTION

Angular Velocity. The instantaneous axis of zero velocity (IA) is indicated in Fig. a, Here, the resultant angular velocity is always directed along IA. The fixed XYZ reference frame is set to coincide with the rotating xyz frame.

$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \omega_2$$

$$\frac{5}{\sqrt{26}}\omega\mathbf{i} - \frac{1}{\sqrt{26}}\omega\mathbf{k} = -4\mathbf{k} + \omega_2\mathbf{i}$$

Equating **k** and **i** components,

$$-\frac{1}{\sqrt{26}}\omega = -4 \qquad \omega = 4\sqrt{26}\text{ rad/s}$$

$$\frac{5}{\sqrt{26}}\left(4\sqrt{26}\right) = \omega_2 \quad \omega_2 = 20\text{ rad/s}$$

Thus, $\boldsymbol{\omega} = \dfrac{5}{\sqrt{26}}(4\sqrt{26})\mathbf{i} - \dfrac{1}{\sqrt{26}}(4\sqrt{26})\mathbf{k} = \{20\mathbf{i} - 4\mathbf{k}\}$ rad/s

Angular Acceleration. The direction of $\boldsymbol{\omega}_2$ does not change with reference to xyz rotating frame if this frame rotates with $\boldsymbol{\Omega} = \boldsymbol{\omega}_1 = \{-4\mathbf{k}\}$ rad/s. Here

$$\frac{(\dot{\omega}_2)_{xyz}}{(\dot{\omega}_1)_{xyz}} = \frac{5}{1}; \quad (\dot{\omega}_2)_{xyz} = 5(\dot{\omega}_1)_{xyz} = 5(2) = 10\text{ rad/s}^2$$

Therefore

$$\dot{\boldsymbol{\omega}}_2 = (\dot{\omega}_2)_{xyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega}_2$$
$$= 10\mathbf{i} + (-4\mathbf{k}) \times (20\mathbf{i})$$
$$= \{10\mathbf{i} - 80\mathbf{j}\}\text{ rad/s}^2$$

Since the direction of $\boldsymbol{\omega}_1$ will not change that is always along z axis when $\boldsymbol{\Omega} = \boldsymbol{\omega}_1$, then

$$\dot{\boldsymbol{\omega}}_1 = (\dot{\boldsymbol{\omega}}_1)_{xyz} + \boldsymbol{\omega}_1 \times \boldsymbol{\omega}_1$$
$$\dot{\boldsymbol{\omega}}_1 = (\dot{\omega}_1)_{xyz} = \{-2\mathbf{k}\}\text{ rad/s}^2$$

***20–8. Continued**

Finally,

$$\boldsymbol{\alpha} = \dot{\omega}_1 + \dot{\omega}_2$$

$$= -2\mathbf{k} + 10\mathbf{i} - 80\mathbf{j}$$

$$= \{10\mathbf{i} - 80\mathbf{j} - 2\mathbf{k}\} \text{ rad/s}^2$$

Velocity and Acceleration. Here $r_A = \{0.5\mathbf{i} + 0.1\mathbf{k}\}$ m

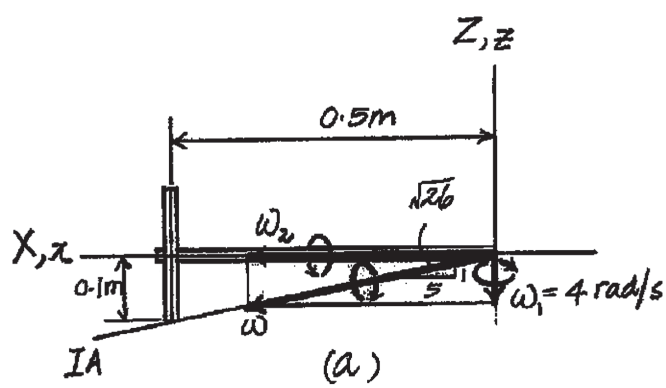
$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A = (20\mathbf{i} - 4\mathbf{k}) \times (0.5\mathbf{i} + 0.1\mathbf{k})$$

$$= \{-4.00\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times r_A)$$

$$= (10\mathbf{i} - 80\mathbf{j} - 2\mathbf{k}) \times (0.5\mathbf{i} + 0.1\mathbf{k}) + (20\mathbf{i} - 4\mathbf{k}) \times (-4.00\mathbf{j})$$

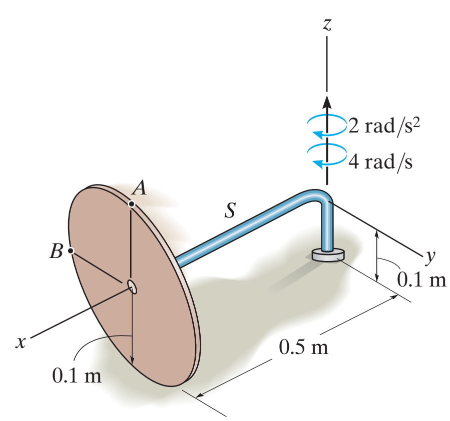
$$= \{-24\mathbf{i} - 2\mathbf{j} - 40\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$



Ans:
$\mathbf{v}_A = \{-4.00\mathbf{j}\}$ m/s
$\mathbf{a}_A = \{-24\mathbf{i} - 2\mathbf{j} - 40\mathbf{k}\}$ m/s^2

20–9.

The disk rotates about the shaft S, while the shaft is turning about the z axis at a rate of $\omega_z = 4$ rad/s, which is increasing at 2 rad/s². Determine the velocity and acceleration of point B on the disk at the instant shown. No slipping occurs.



SOLUTION

Angular velocity. The instantaneous axis of zero velocity (IA) is indicated in Fig. a. Here the resultant angular velocity is always directed along IA. The fixed XYZ reference frame is set coincide with the rotating xyz frame.

$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$$

$$\frac{5}{\sqrt{26}}\omega\mathbf{i} - \frac{1}{\sqrt{26}}\omega\mathbf{k} = -4\mathbf{k} + \omega_2\mathbf{i}$$

Equating **k** and **i** components,

$$-\frac{1}{\sqrt{26}}\omega = -4 \qquad \omega = 4\sqrt{26}\text{ rad/s}$$

$$\frac{5}{\sqrt{26}}\left(4\sqrt{26}\right) = \omega_2 \quad \omega_2 = 20\text{ rad/s}$$

Thus, $\omega = \dfrac{5}{\sqrt{26}}\left(4\sqrt{26}\right)\mathbf{i} - \dfrac{1}{\sqrt{26}}\left(4\sqrt{26}\right)\mathbf{k} = \{20\mathbf{i} - 4\mathbf{k}\}$ rad/s

Angular Acceleration. The direction of $\boldsymbol{\omega}_2$ does not change with reference to xyz rotating frame if this frame rotates with $\boldsymbol{\Omega} = \boldsymbol{\omega}_1 = \{-4\mathbf{k}\}$ rad/s. Here

$$\frac{(\dot{\omega}_2)_{xyz}}{(\dot{\omega}_1)_{xyz}} = \frac{5}{1}; \quad (\dot{\omega}_2)_{xyz} = 5(\dot{\omega}_1)_{xyz} = 5(2) = 10\text{ rad/s}^2$$

Therefore,

$$\dot{\omega}_2 = (\dot{\omega}_2)_{xyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega}_2$$

$$= 10\mathbf{i} + (-4\mathbf{k}) \times (20\mathbf{i})$$

$$= \{10\mathbf{i} - 80\mathbf{j}\}\text{ rad/s}^2$$

Since the direction of $\boldsymbol{\omega}_1$ will not change that is always along z axis when $\boldsymbol{\Omega} = \boldsymbol{\omega}_1$, then

$$\dot{\boldsymbol{\omega}}_1 = (\dot{\omega}_1)_{xyz} + \boldsymbol{\omega}_1 \times \omega_1$$

$$\dot{\boldsymbol{\omega}}_1 = (\dot{\omega}_1)_{xyz} = \{-2\mathbf{k}\}\text{ rad/s}^2$$

20–9. Continued

Finally,

$$\alpha = \dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2$$

$$= -2\mathbf{k} + (10\mathbf{i} - 80\mathbf{j})$$

$$= \{10\mathbf{i} - 80\mathbf{j} - 2\mathbf{k}\} \text{ rad/s}^2$$

Velocity and Acceleration. Here $r_B = \{0.5\mathbf{i} - 0.1\mathbf{j}\}$ m

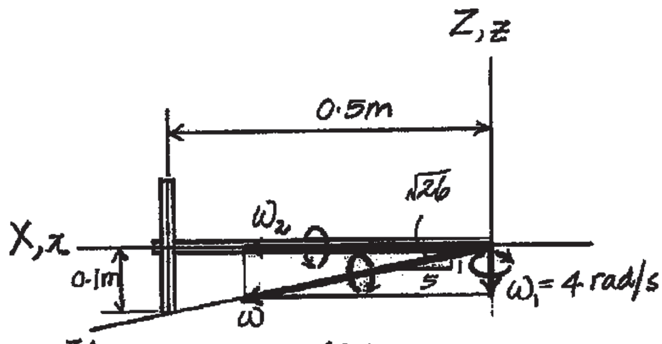
$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_B = (20\mathbf{i} - 4\mathbf{k}) \times (0.5\mathbf{i} - 0.1\mathbf{j})$$

$$= \{-0.4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_B = \boldsymbol{\alpha} \times \mathbf{r}_B + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_B)$$

$$= (10\mathbf{i} - 80\mathbf{j} - 2\mathbf{k}) \times (0.5\mathbf{i} - 0.1\mathbf{j}) + (20\mathbf{i} - 4\mathbf{k}) \times (-0.4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

$$= \{-8.20\mathbf{i} + 40.6\mathbf{j} - \mathbf{k}\} \text{ rad/s}^2 \qquad \textbf{Ans.}$$

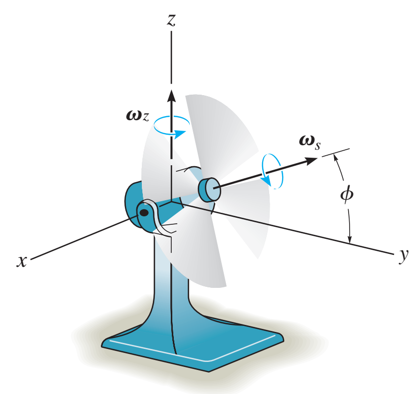


(a)

Ans:
$\mathbf{v}_B = \{-0.4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}\}$ m/s
$\mathbf{a}_B = \{-8.20\mathbf{i} + 40.6\mathbf{j} - \mathbf{k}\}$ rad/s^2

20–10.

The electric fan is mounted on a swivel support such that the fan rotates about the z axis at a constant rate of $\omega_z = 1$ rad/s and the fan blade is spinning at a constant rate $\omega_s = 60$ rad/s. If $\phi = 45°$ for the motion, determine the angular velocity and the angular acceleration of the blade.



SOLUTION

$\omega = \omega_z + \omega_s$

$= 1\mathbf{k} + 60\cos 45°\mathbf{j} + 60\sin 45°\mathbf{k}$

$= 42.426\mathbf{j} + 43.426\mathbf{k}$

$= \{42.4\mathbf{j} + 43.4\mathbf{k}\}$ rad/s **Ans.**

$\dot{\omega} = \dot{\omega}_z + \dot{\omega}_s$

$= 0 + 0 + \omega_z \times \omega_s$

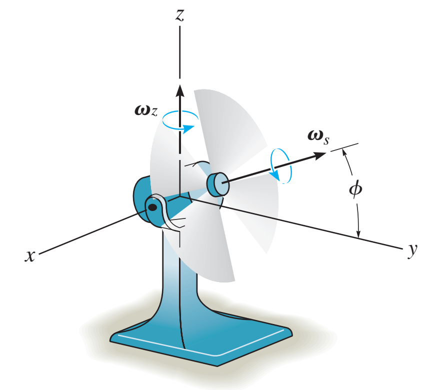
$= 1\mathbf{k} \times 42.426\mathbf{j} + 43.426\mathbf{k}$ **Ans.**

$= \{-42.4\mathbf{i}\}$ rad/s^2 **Ans.**

Ans:
$\boldsymbol{\omega} = \{42.4\mathbf{j} + 43.4\mathbf{k}\}$ rad/s
$\boldsymbol{\alpha} = \{-42.4\mathbf{i}\}$ rad/s^2

20–11.

The electric fan is mounted on a swivel support such that the fan rotates about the z axis at a constant rate of $\omega_z = 1$ rad/s and the fan blade is spinning at a constant rate $\omega_s = 60$ rad/s. If at the instant $\phi = 45°$, $\dot{\phi} = 2$ rad/s for the motion, determine the angular velocity and the angular acceleration of the blade.



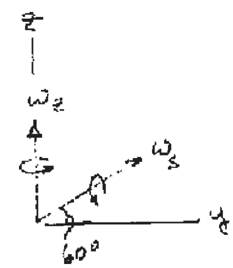
SOLUTION

$\omega = \omega_z + \omega_s + \omega_x$

$= 1\mathbf{k} + 60\cos 45°\mathbf{j} + 60\sin 45°\mathbf{k} + 2\mathbf{i}$

$= 2\mathbf{i} + 42.426\mathbf{j} + 43.426\mathbf{k}$

$= \{2\mathbf{i} + 42.4\mathbf{j} + 43.4\mathbf{k}\}$ rad/s **Ans.**



$\dot{\omega} = \dot{\omega}_z + \dot{\omega}_s + \dot{\omega}_x$

$= 0 + (\omega_z + \omega_x) \times \omega_s + \omega_z \times \omega_x$

$= 0 + (1\mathbf{k} + 2\mathbf{i}) \times (42.426\mathbf{j} + 43.426\mathbf{k}) + 1\mathbf{k} \times (2\mathbf{i})$

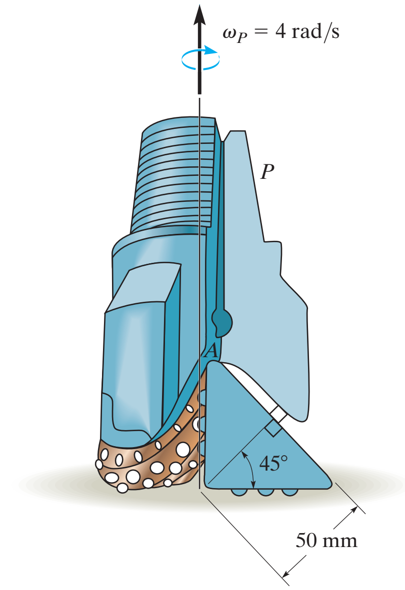
$= -42.426\mathbf{i} + 84.853\mathbf{k} - 84.853\mathbf{j} + 2\mathbf{j}$

$= \{-42.4\mathbf{i} - 82.9\mathbf{j} + 84.9\mathbf{k}\}$ rad/s^2 **Ans.**

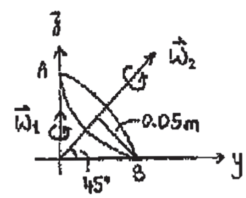
Ans:
$\boldsymbol{\omega} = \{2\mathbf{i} + 42.4\mathbf{j} + 43.4\mathbf{k}\}$ rad/s
$\boldsymbol{\alpha} = \{-42.4\mathbf{i} - 82.9\mathbf{j} + 84.9\mathbf{k}\}$ rad/s^2

***20–12.**

The drill pipe P turns at a constant angular rate $\omega_P = 4$ rad/s. Determine the angular velocity and angular acceleration of the conical rock bit, which rolls without slipping. Also, what are the velocity and acceleration of point A?



SOLUTION



$\omega = \omega_1 + \omega_2$

Since ω acts along the instantaneous axis of zero velocity,

$\omega\mathbf{j} = \omega_1\mathbf{k} + \omega_2 \cos 45°\mathbf{j} + \omega_2 \sin 45°\mathbf{k}$

Setting $\omega_1 = 4$ rad/s

$\omega\mathbf{j} = 4\mathbf{k} + 0.707\omega_2\mathbf{j} + 0.707\omega_2\mathbf{k}$

Equating components

$\omega = 0.707\omega_2$

$0 = 4 + 0.707\omega_2$

$\omega = -4$ rad/s

$\omega_2 = -5.66$ rad/s

Thus,

$\omega = \{-4.00\mathbf{j}\}$ rad/s **Ans.**

$\Omega = \omega_1$

$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$

$= 0 + \omega_1 \times (\omega)$

$= 0 + (4\mathbf{k}) \times (-4\mathbf{j})$

$\alpha = \dot{\omega} = \{16.01\}$ rad/s^2 **Ans.**

$\mathbf{v}_A = \omega \times \mathbf{r}_A$

$= (-4\mathbf{j}) \times [100(0.707)\mathbf{k}]$

$= \{-282.81\}$ mm/s

$\mathbf{v}_A = \{-0.283\mathbf{i}\}$ m/s **Ans.**

$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A$

$= (16\mathbf{i}) \times (100)(0.707)\mathbf{k} + (-4\mathbf{j}) \times (-282.8\mathbf{i})$

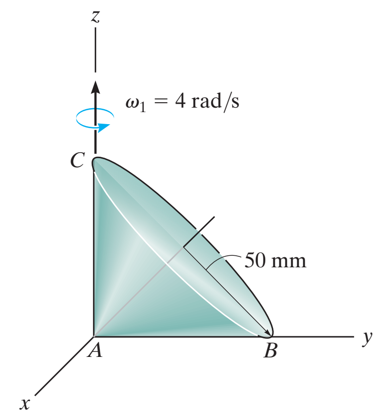
$= \{-1131.2\mathbf{j} - 1131.2\mathbf{k}\}$ mm/s^2

$\mathbf{a}_A = \{-1.13\mathbf{j} - 1.13\mathbf{k}\}$ m/s^2 **Ans.**

Ans:
$\omega = \{-4.00\mathbf{j}\}$ rad/s
$\alpha = \{16.01\}$ rad/s^2
$\mathbf{v}_A = \{-0.283\mathbf{i}\}$ m/s
$\mathbf{a}_A = \{-1.13\mathbf{j} - 1.13\mathbf{k}\}$ m/s^2

20–13.

The right circular cone rotates about the z axis at a constant rate of $\omega_1 = 4$ rad/s without slipping on the horizontal plane. Determine the magnitudes of the velocity and acceleration of points B and C.



SOLUTION

$\omega = \omega_1 + \omega_2$

Since ω acts along the instantaneous axis of zero velocity

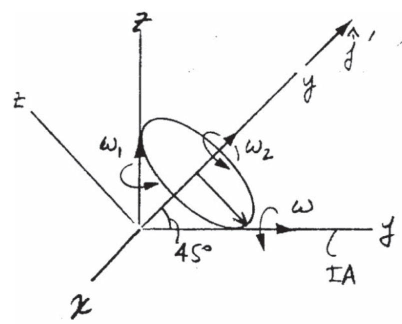
$\omega\mathbf{j} = 4\mathbf{k} + \omega_2 \cos 45°\,\mathbf{j} + \omega_2 \sin 45°\mathbf{k}.$

Equating components,

$\omega = 0.707\,\omega_2$

$0 = 4 + 0.707\,\omega_2$

$\omega = -4$ rad/s, $\qquad \omega_2 = -5.66$ rad/s



Thus,

$\omega = \{-4\mathbf{j}\}$ rad/s

$\Omega = \omega_1$

$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \mathbf{0} + \omega_1 \times \omega_2$

$\quad = \mathbf{0} + (4\mathbf{k}) \times (-5.66 \cos 45°\mathbf{j} - 5.66 \sin 45°\mathbf{k})$

$\alpha = \dot{\omega} = \{16\mathbf{i}\}$ rad/s^2

$\mathbf{v}_B = \omega \times \mathbf{r}_B = (-4\mathbf{j}) \times (0.1(0.707)\mathbf{j}) = \mathbf{0}$

$v_B = 0$ **Ans.**

$\mathbf{v}_C = \omega \times \mathbf{r}_C = (-4\mathbf{j}) \times (0.1(0.707)\mathbf{k}) = \{-0.2828\mathbf{i}\}$ m/s

$v_C = 0.283$ m/s **Ans.**

$\mathbf{a}_B = \alpha \times \mathbf{r}_B + \omega \times \mathbf{v}_B = 16\mathbf{i} \times (0.1)(0.707)\mathbf{j} + \mathbf{0}$

$\mathbf{a}_B = \{1.131\mathbf{k}\}$ m/s

$a_B = 1.13$ m/s^2 **Ans.**

$\mathbf{a}_C = \alpha \times \mathbf{r}_C + \omega \times \mathbf{v}_C = 16\mathbf{i} \times (0.1)(0.707)\mathbf{k} + (-4\mathbf{j}) \times (-0.2828\mathbf{i})$

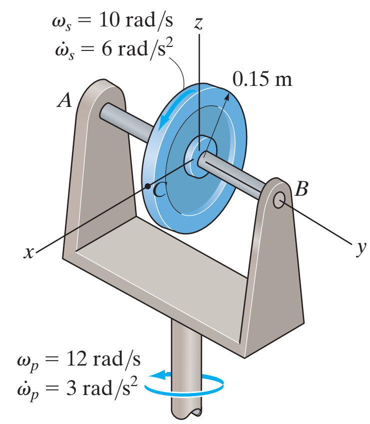
$\mathbf{a}_C = \{-1.131\mathbf{j} - 1.131\mathbf{k}\}$ m/s^2

$a_C = 1.60$ m/s^2 **Ans.**

Ans:
$v_B = 0$
$v_C = 0.283$ m/s
$a_B = 1.13$ m/s^2
$a_C = 1.60$ m/s^2

20–14.

The wheel is spinning about shaft AB with an angular velocity of $\omega_s = 10$ rad/s, which is increasing at a constant rate of $\dot{\omega}_s = 6$ rad/s^2, while the frame precesses about the z axis with an angular velocity of $\omega_p = 12$ rad/s, which is increasing at a constant rate of $\omega_p = 3$ rad/s^2. Determine the velocity and acceleration of point C located on the rim of the wheel at this instant.



SOLUTION

The XYZ fixed reference frame is set to coincide with the rotating xyz reference frame at the instant considered. Thus, the angular velocity of the wheel at this instant can be obtained by vector addition of ω_s and ω_p.

$$\omega = \omega_s + \omega_p = [10\mathbf{j} + 12\mathbf{k}] \text{ rad/s}$$

The angular acceleration of the disk is determined from

$$\alpha = \dot{\omega} = \dot{\omega}_s + \dot{\omega}_p$$

If we set the xyz rotating frame to have an angular velocity of $\Omega = \omega_p = [12\mathbf{k}]$ rad/s, the direction of ω_s will remain unchanged with respect to the xyz rotating frame which is along the y axis. Thus,

$$\dot{\omega}_s = (\dot{\omega}_s)_{xyz} + \omega_p \times \omega_s = 6\mathbf{j} + (12\mathbf{k}) \times (10\mathbf{j}) = [-120\mathbf{i} + \mathbf{6j}] \text{ rad/s}^2$$

Since ω_p is always directed along the Z axis where $\Omega = \omega_p$, then

$$\dot{\omega}_p = \left(\dot{\omega}_p\right)_{xyz} + \omega_p \times \omega_p = 3\mathbf{k} + \mathbf{0} = [3\mathbf{k}] \text{ rad/s}^2$$

Thus, $\alpha = (-120\mathbf{i} + 6\mathbf{j}) + 3\mathbf{k} = [-120\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}]$ rad/s^2

Here, $\mathbf{r}_C = [0.15\mathbf{i}]$ m, so that

$$v_C = \omega \times \mathbf{r}_C = (10\mathbf{j} + 12\mathbf{k}) \times (0.15\mathbf{i}) = [1.8\mathbf{j} - 1.5\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

and

$$a_C = \alpha \times \mathbf{r}_C + \omega \times (\omega \times \mathbf{r}_C)$$

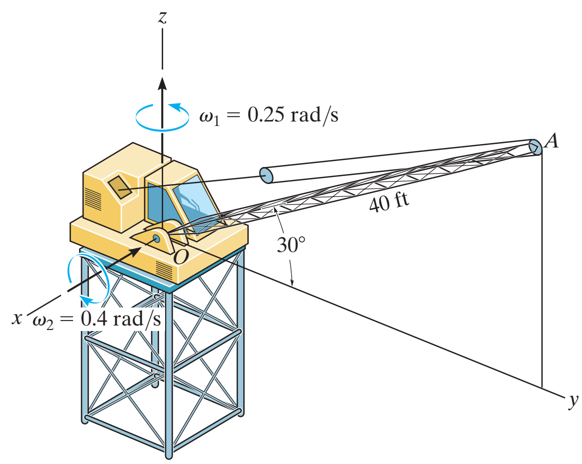
$$= (-120\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) \times (0.15\mathbf{i}) + (10\mathbf{j} + 12\mathbf{k}) \times [(10\mathbf{j} + 12\mathbf{k}) \times (0.15\mathbf{i})]$$

$$= [-36.6\mathbf{i} + 0.45\mathbf{j} - 0.9\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{1.8\mathbf{j} - 1.5\mathbf{k}\}$ m/s
$\mathbf{a}_C = \{-36.6\mathbf{i} + 0.45\mathbf{j} - 0.9\mathbf{k}\}$ m/s^2

20–15.

At the instant shown, the tower crane rotates about the z axis with an angular velocity $\omega_1 = 0.25$ rad/s, which is increasing at 0.6 rad/s^2. The boom OA rotates downward with an angular velocity $\omega_2 = 0.4$ rad/s, which is increasing at 0.8 rad/s^2. Determine the velocity and acceleration of point A located at the end of the boom at this instant.



SOLUTION

$$\omega = \omega_1 + \omega_2 = \{-0.4\,\mathbf{i} + 0.25\mathbf{k}\}\text{ rad/s}$$

$$\Omega = \{0.25\,\mathbf{k}\}\text{ rad/s}$$

$$\dot{\omega} = (\dot{\omega})_{xyz} + \Omega \times \omega = (-0.8\,\mathbf{i} + 0.6\mathbf{k}) + (0.25\mathbf{k}) \times (-0.4\,\mathbf{i} + 0.25\mathbf{k})$$

$$= \{-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}\}\text{ rad/s}^2$$

$$\mathbf{r}_A = 40\cos 30^\circ\mathbf{j} + 40\sin 30^\circ\mathbf{k} = \{34.64\mathbf{j} + 20\mathbf{k}\}\text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = (1 - 0.4\,\mathbf{i} + 0.25\,\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$$

$$\mathbf{v}_A = \{-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k}\}\text{ ft/s}$$ **Ans.**

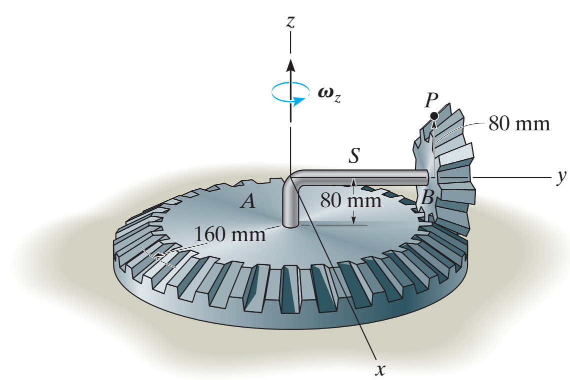
$$\mathbf{a}_A = \alpha \cdot \mathbf{r}_A + \omega \times \mathbf{v}_A = (-0.8\mathbf{i} - 0.1\mathbf{j} + 0.6\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k}) + (-0.4\mathbf{i} + 0.25\mathbf{k}) \times (-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k})$$

$$\mathbf{a}_A = \{-24.8\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}\text{ ft/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_A = \{-8.66\mathbf{i} + 8.00\mathbf{j} - 13.9\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-24.8\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}$ ft/s^2

***20–16.**

Gear A is fixed while gear B is free to rotate on the shaft S. If the shaft is turning about the z axis at $\omega_z = 5$ rad/s, while increasing at 2 rad/s^2, determine the velocity and acceleration of point P at the instant shown. The face of gear B lies in a vertical plane.



SOLUTION

$\Omega = \{5\mathbf{k} - 10\mathbf{j}\}$ rad/s

$\dot{\Omega} = \{50\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\}$ rad/s^2

$\mathbf{v}_P = \Omega \times \mathbf{r}_P$

$\mathbf{v}_P \quad (5\mathbf{k} - 10\mathbf{j} \times (160\mathbf{j} + 80\mathbf{k})$

$\mathbf{v}_P = \{-1600\mathbf{i}\}$ mm/s

$= \{-1.60\mathbf{i}\}$ m/s **Ans.**

$\mathbf{a}_P = \Omega \times \mathbf{v}_P + \dot{\Omega} \times \mathbf{r}_P$

$\mathbf{a}_P = \{50\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}\} \times (160\mathbf{j} + 80\mathbf{k}) + (-10\mathbf{j} + 5\mathbf{k}) \times (-1600\mathbf{i})$

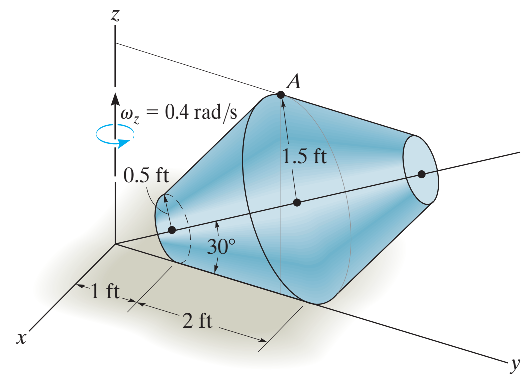
$\mathbf{a}_P = \{-640\mathbf{i} - 12000\mathbf{j} - 8000\mathbf{k}\}$ mm/s^2

$\mathbf{a}_P = \{-0.640\mathbf{i} - 12.0\mathbf{j} - 8.00\mathbf{k}\}$ m/s^2 **Ans.**

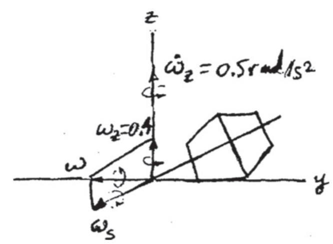
Ans:
$\mathbf{v}_P = \{-1.60\mathbf{i}\}$ m/s
$\mathbf{a}_P = \{-0.640\mathbf{i} - 12.0\mathbf{j} - 8.00\mathbf{k}\}$ m/s^2

20–17.

The truncated double cone rotates about the z axis at $\omega_z = 0.4$ rad/s without slipping on the horizontal plane. If at this same instant ω_z is increasing at $\dot{\omega}_z = 0.5$ rad/s^2, determine the velocity and acceleration of point A on the cone.



SOLUTION



$$\theta = \sin^{-1}\left(\frac{0.5}{1}\right) = 30^\circ$$

$$\omega_s = \frac{0.4}{\sin 30^\circ} = 0.8 \text{ rad/s}$$

$$\omega = 0.8 \cos 30^\circ = 0.6928 \text{ rad/s}$$

$$\omega = \{-0.6928\mathbf{j}\} \text{ rad/s}$$

$$\Omega = 0.4\mathbf{k}$$

$$\dot{\omega} = (\dot{\omega})_{xyz} + \Omega \times \omega \qquad \textbf{(1)}$$

$$= 0.5\mathbf{k} + (0.4\mathbf{k}) \times (-0.6928\mathbf{j})$$

$$\dot{\omega} = 0.2771\mathbf{i} + 0.5\mathbf{k}$$

$$\mathbf{r}_A = (3 - 3 \sin 30^\circ)\mathbf{j} + 3 \cos 30^\circ\mathbf{k}$$

$$= (1.5\mathbf{j} + 2.598\mathbf{k}) \text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A$$

$$= (-0.6928\mathbf{j}) \times (1.5\mathbf{j} + 2.598\mathbf{k})$$

$$\mathbf{v}_A = \{-1.80\mathbf{i}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A$$

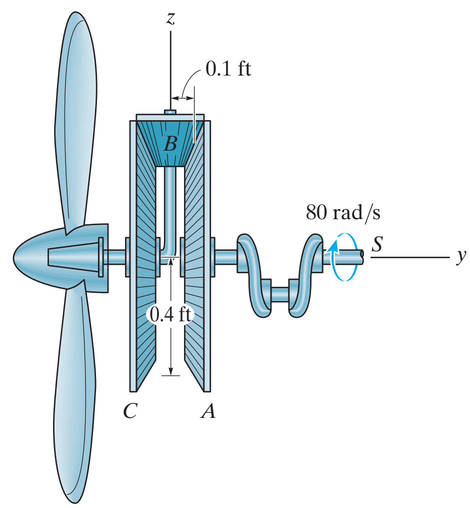
$$= (0.2771\mathbf{i} + 0.5\mathbf{k}) \times (1.5\mathbf{j} + 2.598\mathbf{k}) + (-0.6928\mathbf{j}) \times (-1.80\mathbf{i})$$

$$\mathbf{a}_A = \{-0.750\mathbf{i} - 0.720\mathbf{j} - 0.831\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-1.80\mathbf{i}\}$ ft/s
$\mathbf{a}_A = \{-0.750\mathbf{i} - 0.720\mathbf{j} - 0.831\mathbf{k}\}$ ft/s^2

20–18.

Gear A is fixed to the crankshaft S, while gear C is fixed. Gear B and the propeller are free to rotate. The crankshaft is turning at 80 rad/s about its axis. Determine the magnitudes of the angular velocity of the propeller and the angular acceleration of gear B.

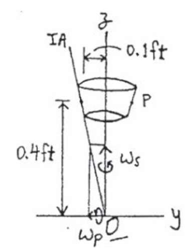


SOLUTION

Point P on gear B has a speed of

$v_P = 80(0.4) = 32 \text{ ft/s}$

The IA is located along the points of contant of B and C



$$\frac{\omega_P}{0.1} = \frac{\omega_s}{0.4}$$

$\omega_s = 4\omega_P$

$\omega = -\omega_P\,\mathbf{j} + \omega_s\,\mathbf{k}$

$\quad = -\omega_P\mathbf{j} + 4\omega_P\mathbf{k}$

$\mathbf{r}_{P/O} = 0.1\mathbf{j} = 0.4\mathbf{k}$

$\mathbf{v}_P = -32\mathbf{i}$

$\mathbf{v}_P = \omega \times \mathbf{r}_{P/O}$

$$-32\mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -\omega_P & 4\omega_P \\ 0 & 0.1 & 0.4 \end{vmatrix}$$

$-32\mathbf{i} = -0.8\omega_P\mathbf{i}$

$\omega_P = 40 \text{ rad/s}$

$\omega_P = \{-40\mathbf{j}\} \text{ rad/s}$ **Ans.**

$\omega_s = 4(40)\,\mathbf{k} = \{160\mathbf{k}\} \text{ rad/s}$

Thus,

$\omega = \omega_P + \omega_s$

Let the x,y,z axes have an angular velocity of $\Omega \times \omega_P$, then

$\alpha = \dot{\omega} = \dot{\omega}_P + \dot{\omega}_s = \mathbf{0} + \omega_P \times (\omega_s + \omega_P)$

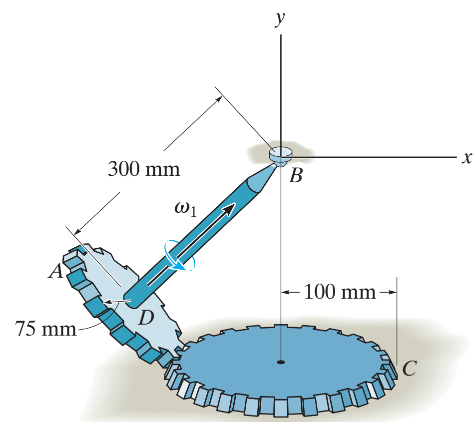
$\alpha = (-40\mathbf{j}) \times (160\mathbf{k} - 40\mathbf{j})$

$\alpha = \{-6400\mathbf{i}\} \text{ rad/s}^2$ **Ans.**

Ans:
$\boldsymbol{\omega}_P = \{-40\mathbf{j}\} \text{ rad/s}$
$\boldsymbol{\alpha}_B = \{-6400\mathbf{i}\} \text{ rad/s}^2$

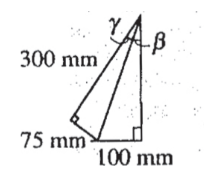
20–19.

Shaft BD is connected to a ball-and-socket joint at B, and a beveled gear A is attached to its other end. The gear is in mesh with a fixed gear C. If the shaft and gear A are *spinning* with a constant angular velocity $\omega_1 = 8$ rad/s, determine the angular velocity and angular acceleration of gear A.



SOLUTION

$$\gamma = \tan^{-1}\frac{75}{300} = 14.04^\circ \qquad \beta = \sin^{-1}\frac{100}{\sqrt{300^2 + 75^2}} = 18.87^\circ$$



The resultant angular velocity $\omega = \omega_1 + \omega_2$ is always directed along the instantaneous axis of zero velocity IA.

$$\frac{\omega}{\sin 147.09^\circ} = \frac{8}{\sin 18.87^\circ} \qquad \omega = 13.44 \text{ rad/s}$$

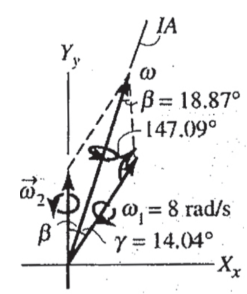
$$\boldsymbol{\omega} = 13.44 \sin 18.87^\circ\,\mathbf{i} + 13.44 \cos 18.87^\circ\,\mathbf{j}$$

$$= \{4.35\mathbf{i} + 12.7\mathbf{j}\} \text{ rad/s} \qquad \textbf{Ans.}$$

$$\frac{\omega_2}{\sin 14.04^\circ} = \frac{8}{\sin 18.87^\circ} \qquad \omega_2 = 6.00 \text{ rad/s}$$

$$\boldsymbol{\omega}_2 = \{6\mathbf{j}\} \text{ rad/s}$$

$$\boldsymbol{\omega}_1 = 8 \sin 32.91^\circ\,\mathbf{i} + 8 \cos 32.91^\circ\,\mathbf{j} = \{4.3466\mathbf{i} + 6.7162\mathbf{j}\} \text{ rad/s}$$



For ω_1, $\Omega = \omega_2 = \{6\mathbf{j}\}$ rad/s

$$(\dot{\omega}_1)_{xyz} = (\dot{\omega}_1)_{xyz} + \Omega \times \omega_1$$

$$= \mathbf{0} + (6\mathbf{j}) \times (4.3466\mathbf{i} + 6.7162\mathbf{j})$$

$$= \{-26.08\mathbf{k}\} \text{ rad/s}^2$$

For ω_2, $\Omega = \mathbf{0}$.

$$(\dot{\omega}_2)_{XYZ} = (\dot{\omega}_2)_{xyz} + \Omega \times \omega_2 = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

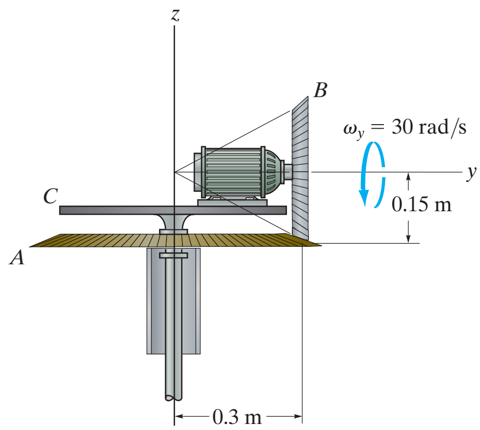
$$\alpha = \dot{\omega} = (\dot{\omega}_1)_{XYZ} + (\dot{\omega}_2)_{XYZ}$$

$$\alpha = \mathbf{0} + (-26.08\mathbf{k}) = \{-26.1\mathbf{k}\} \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\boldsymbol{\omega} = \{4.35\mathbf{i} + 12.7\mathbf{j}\}$ rad/s
$\boldsymbol{\alpha} = \{-26.1\mathbf{k}\}$ rad/s^2

***20–20.**

Gear B is driven by a motor mounted on turntable C. If gear A is held fixed, and the motor shaft rotates with a constant angular velocity of $\omega_y = 30$ rad/s, determine the angular velocity and angular acceleration of gear B.

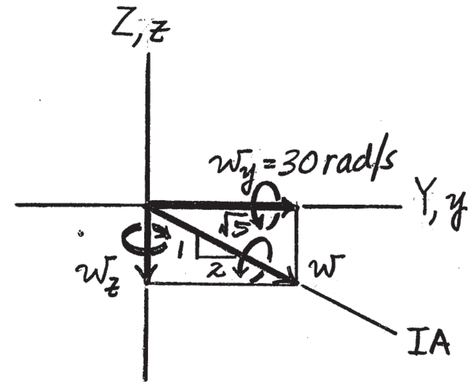


SOLUTION

The angular velocity ω of gear B is directed along the instantaneous axis of zero velocity, which is along the line where gears A and B mesh since gear A is held fixed. From Fig. a, the vector addition gives

$$\omega = \omega_y + \omega_z$$

$$\frac{2}{\sqrt{5}}\omega\mathbf{j} - \frac{1}{\sqrt{5}}\omega\mathbf{k} = 30\mathbf{j} - \omega_z\mathbf{k}$$



(a)

Equating the $\mathbf{j}$ and $\mathbf{k}$ components gives

$$\frac{2}{\sqrt{5}}\omega = 30 \qquad \omega = 15\sqrt{5}\text{ rad/s}$$

$$-\frac{1}{\sqrt{5}}\left(15\sqrt{5}\right) = -\omega_z \qquad \omega_z = 15\text{ rad/s}$$

Thus,

$$\omega = [30\mathbf{j} - 15\mathbf{k}]\text{ rad/s} \qquad \textbf{Ans.}$$

Here, we will set the XYZ fixed reference frame to coincide with the xyz rotating frame at the instant considered. If the xyz frame rotates with an angular velocity of $\Omega = \omega_z = [-15\mathbf{k}]$ rad/s, then ω_y will always be directed along the y axis with respect to the xyz frame. Thus,

$$\dot{\omega}_y = \left(\dot{\omega}_y\right)_{xyz} + \omega_z \times \omega_y = 0 + (-15\mathbf{k}) \times (30\mathbf{j}) = [450\mathbf{i}]\text{ rad/s}^2$$

When $\Omega = \omega_z$, ω_z is always directed along the z axis. Therefore,

$$\dot{\omega}_z = \left(\dot{\omega}_z\right)_{xyz} + \omega_z \times \omega_z = 0 + 0 = 0$$

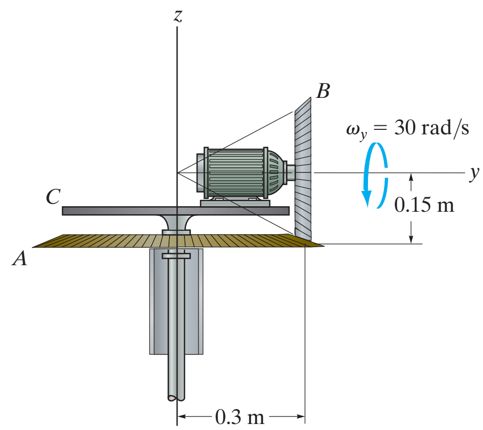
Thus,

$$\alpha = \dot{\omega}_y + \dot{\omega}_z = (450\mathbf{i}) + 0 = [450\mathbf{i}]\text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\omega = [30\mathbf{j} - 15\mathbf{k}]$ rad/s
$\alpha = [450\mathbf{i}]$ rad/s^2

20–21.

Gear B is driven by a motor mounted on turntable C. If gear A and the motor shaft rotate with constant angular speeds of $\omega_A = \{10\mathbf{k}\}$ rad/s and $\omega_y = \{30\mathbf{j}\}$ rad/s, respectively, determine the angular velocity and angular acceleration of gear B.



SOLUTION

If the angular velocity of the turn-table is ω_z, then the angular velocity of gear B is

$$\omega = \omega_y + \omega_z = [30\mathbf{j} + \omega_z\mathbf{k}] \text{ rad/s}$$

Since gear A rotates about the fixed axis (z axis), the velocity of the contact point **P** between gears A and B is

$$\mathbf{v}_p = \omega_A \times r_A = (10\mathbf{k}) \times (0.3\mathbf{j}) = [-3\mathbf{i}] \text{ m/s}$$

Since gear B rotates about a fixed point O, the origin of the xyz frame, then $\mathbf{r}_{OP} = [0.3\mathbf{j} - 0.15\mathbf{k}]$ m.

$$\mathbf{v}_p = \omega \times \mathbf{r}_{OP}$$

$$-3\mathbf{i} = (30\mathbf{j} + \omega_z\mathbf{k}) \times (0.3\mathbf{j} - 0.15\mathbf{k})$$

$$-3\mathbf{i} = -(4.5 + 0.3\omega_z)\mathbf{i}$$

Thus,

$$-3 = -(4.5 + 0.3\omega_z)$$

$$\omega_z = -5 \text{ rad/s}$$

Then,

$$\omega = [30\mathbf{j} - 5\mathbf{k}] \text{ rad/s} \qquad \textbf{Ans.}$$

Here, we will set the XYZ fixed reference frame to conincide with the xyz rotating frame at the instant considered. If the xyz frame rotates with an angular velocity of $\Omega = \omega_z = [-5\mathbf{k}]$ rad/s, then ω_y will always be directed along the y axis with respect to the xyz frame. Thus,

$$\dot{\omega}_y = \left(\dot{\omega}_y\right)_{xyz} + \omega_z \times \omega_y = \mathbf{0} + (-5\mathbf{k}) \times (30\mathbf{j}) = [150\mathbf{i}] \text{ rad/s}^2$$

When $\Omega = \omega_z$, ω_z is always directed along the z axis. Therefore,

$$\dot{\omega}_z = \left(\dot{\omega}_z\right)_{xyz} + \omega_z \times \omega_z = \mathbf{0} + \mathbf{0} = \mathbf{0}$$

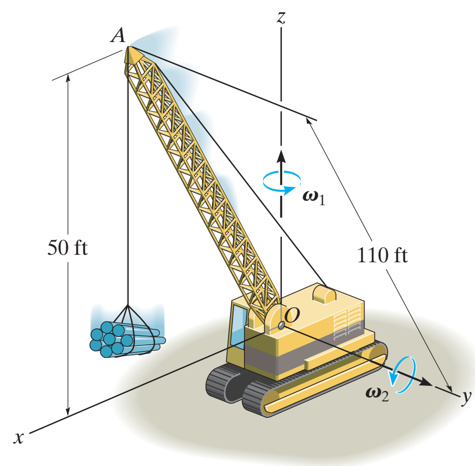
Thus,

$$\alpha = \dot{\omega}_y + \dot{\omega}_z = (150\mathbf{i} + 0) = [150\mathbf{i}] \text{ rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\boldsymbol{\omega} = \{30\mathbf{j} - 5\mathbf{k}\}$ rad/s
$\boldsymbol{\alpha} = \{150\mathbf{i}\}$ rad/s^2

20–22.

The crane boom OA rotates about the z axis with a constant angular velocity of $\omega_1 = 0.15$ rad/s, while it is rotating downward with a constant angular velocity of $\omega_2 = 0.2$ rad/s. Determine the velocity and acceleration of point A located at the end of the boom at the instant shown.



SOLUTION

$$\omega = \omega_1 + \omega_2 = \{0.2\mathbf{j} + 0.15\mathbf{k}\} \text{ rad/s}$$

Let the x, y, z axes rotate at $\Omega = \omega_1$, then

$$\dot{\omega} = (\dot{\omega})_{xyz} + \omega_1 \times \omega_2$$

$$\dot{\omega} = \mathbf{0} + 0.15\mathbf{k} \times 0.2\mathbf{j} = \{-0.03\mathbf{i}\} \text{ rad/s}^2$$

$$\mathbf{r}_A = \left[\sqrt{(110)^2 - (50)^2}\right]\mathbf{i} + 50\mathbf{k} = \{97.98\mathbf{i} + 50\mathbf{k}\} \text{ ft}$$

$$\mathbf{v}_A = \omega \times \mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 97.98 & 0 & 50 \end{vmatrix}$$

$$\mathbf{v}_A = \{10\mathbf{i} + 14.7\mathbf{j} - 19.6\mathbf{k}\} \text{ ft/s}$$ **Ans.**

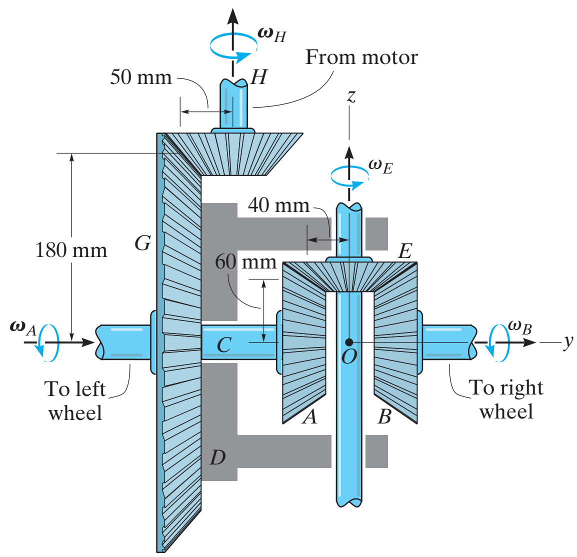
$$\mathbf{a}_A = \alpha \times \mathbf{r}_A + \omega \times \mathbf{v}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.03 & 0 & 0 \\ 97.98 & 0 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.2 & 0.15 \\ 10 & 14.7 & -19.6 \end{vmatrix}$$

$$\mathbf{a}_A = \{-6.12\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\} \text{ ft/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_A = \{10\mathbf{i} + 14.7\mathbf{j} - 19.6\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-6.12\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}$ ft/s^2

20–23.

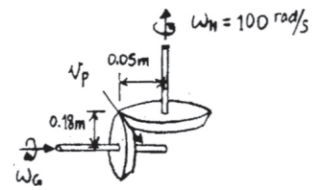
The differential of an automobile allows the two rear wheels to rotate at different speeds when the automobile travels along a curve. For operation, the rear axles are attached to the wheels at one end and have beveled gears A and B on their other ends. The differential case D is placed over the left axle but can rotate about C independent of the axle. The case supports a pinion gear E on a shaft, which meshes with gears A and B. Finally, a ring gear G is *fixed* to the differential case so that the case rotates with the ring gear when the latter is driven by the drive pinion H. This gear, like the differential case, is free to rotate about the left wheel axle. If the drive pinion is turning at $\omega_H = 100$ rad/s and the pinion gear E is spinning about its shaft at $\omega_E = 30$ rad/s, determine the angular velocity, ω_A and ω_B, of each axle.



SOLUTION

$$v_P = \omega_H r_H = 100(50) = 5000 \text{ mm/s}$$

$$\omega_G = \frac{5000}{180} = 27.78 \text{ rad/s}$$

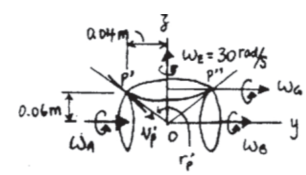


Point O is a fixed point of rotation for gears A, E, and B.

$$\Omega = \omega_G + \omega_E = \{27.78\mathbf{j} + 30\mathbf{k}\} \text{ rad/s}$$

$$\mathbf{v}_{P'} = \Omega \times \mathbf{r}_{P'} = (27.78\mathbf{j} + 30\mathbf{k}) \times (-40\mathbf{j} + 60\mathbf{k}) = \{2866.7\mathbf{i}\} \text{ mm/s}$$

$$\omega_A = \frac{2866.7}{60} = 47.8 \text{ rad/s}$$ **Ans.**



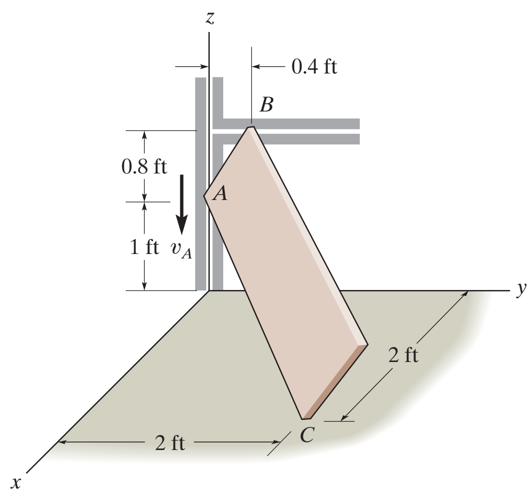
$$\mathbf{v}_{P''} = \Omega \times \mathbf{r}_{P''} = (27.78\mathbf{j} + 30\mathbf{k}) \times (40\mathbf{j} + 60\mathbf{k}) = \{466.7\mathbf{i}\} \text{ mm/s}$$

$$\omega_B = \frac{466.7}{60} = 7.78 \text{ rad/s}$$ **Ans.**

Ans:
$\omega_A = 47.8$ rad/s
$\omega_B = 7.78$ rad/s

***20–24.**

The end C of the plate rests on the horizontal plane, while end points A and B are restricted to move along the grooved slots. If at the instant shown A is moving downward with a constant velocity of $v_A = 4$ ft/s, determine the angular velocity of the plate and the velocities of points B and C.



SOLUTION

Velocity equation:

$\mathbf{v}_A = \{-4\mathbf{k}\}$ ft/s $\quad \mathbf{v}_B = -v_B\mathbf{j} \quad \omega = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$\mathbf{r}_{B/A} = \{0.4\mathbf{j} + 0.8\mathbf{k}\}$ ft $\quad \mathbf{r}_{C/A} = \{2\mathbf{i} + 2\mathbf{j} - 1\mathbf{k}\}$ ft

$\mathbf{v}_C = (v_C)_x\mathbf{i} + (v_C)_y\mathbf{j}$

$\mathbf{v}_B = v_A + \omega \times \mathbf{r}_{B/A}$

$$-v_B\mathbf{j} = (-4\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & 0.4 & 0.8 \end{vmatrix}$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components

$0.8\omega_y - 0.4\omega_z = 0$ **(1)**

$0.8\omega_x = v_B$ **(2)**

$0.4\omega_x - 4 = 0$ **(3)**

$$\mathbf{v}_C = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{C/A}$$

$$(v_C)_x\mathbf{i} + (v_C)_y\mathbf{j} = (-4\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 1 & 2 & -1 \end{vmatrix}$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components

$-\omega_y - 2\omega_z = (v_c)_x$ **(4)**

$2\omega_z + \omega_x = (v_C)_y$ **(5)**

$2\omega_x - 2\omega_y - 4 = 0$ **(6)**

Solving Eqs. [1] to [6] yields:

$\omega_x = 10$ rad/s $\quad \omega_y = 8$ rad/s $\quad \omega_z = 16$ rad/s $\quad v_B = 8$ ft/s

$(v_C)_x = -40$ ft/s $\quad (v_C)_y = 42$ ft/s

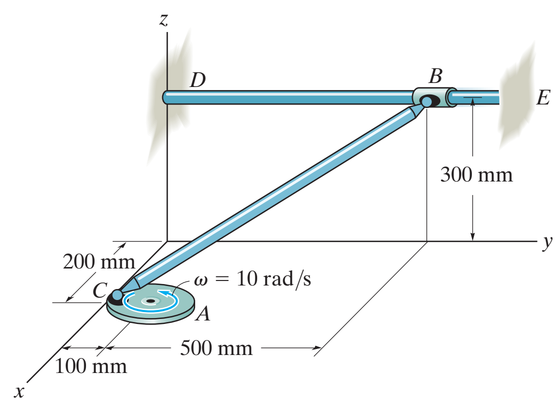
Then $\mathbf{v}_B = \{-8\mathbf{j}\}$ ft/s $\quad \mathbf{v}_C = \{-40\mathbf{i} + 42\mathbf{j}\}$ ft/s **Ans.**

$\omega = \{10\mathbf{i} + 8\mathbf{j} + 16\mathbf{k}\}$ rad/s **Ans.**

Ans:
$\mathbf{v}_B = \{-8\mathbf{j}\}$ ft/s $\quad \mathbf{v}_C = \{-40\mathbf{i} + 42\mathbf{j}\}$ ft/s
$\omega = \{10\mathbf{i} + 8\mathbf{j} + 16\mathbf{k}\}$ rad/s

20–25.

Disk A rotates at a constant angular velocity of 10 rad/s. If rod BC is joined to the disk and a collar by ball-and-socket joints, determine the velocity of collar B at the instant shown. Also, what is the rod's angular velocity $\boldsymbol{\omega}_{BC}$ if it is directed perpendicular to the axis of the rod?



SOLUTION

$\mathbf{v}_C = \{1\mathbf{i}\}$ m/s $\qquad \mathbf{v}_B = -v_B\mathbf{j} \qquad \omega_{BC} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$

$$r_{B/C} = \{-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \mathbf{v}_C + \omega_{BC} \times \mathbf{r}_{B/C}$$

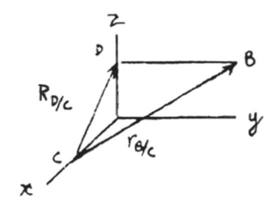
$$-\mathbf{v}_B = 1\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.2 & 0.6 & 0.3 \end{vmatrix}$$

Equating **i**, **j**, and **k** components

$$1 - 0.3\omega_y - 0.6\omega_z = 0 \qquad \textbf{(1)}$$

$$0.3\omega_x + 0.2\omega_z = v_B \qquad \textbf{(2)}$$

$$0.6\omega_x + 0.2\omega_y = 0 \qquad \textbf{(3)}$$



Since ω_{BC} is perpendicular to the axis of the rod,

$$\omega_{BC} \cdot \mathbf{r}_{B/C} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\mathbf{k}) \cdot (-0.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}) = 0$$

$$-0.2\omega_x + 0.6\omega_y + 0.3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$\omega_x = 0.204$ rad/s $\qquad \omega_y = -0.612$ rad/s $\qquad \omega_z = 1.36$ rad/s $\qquad v_B = 0.333$ m/s

Then

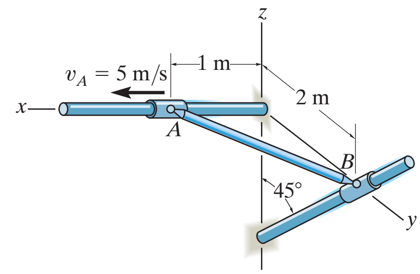
$$\omega_{BC} = \{0.204\mathbf{i} - 0.612\mathbf{j} + 1.36\mathbf{k}\}\text{ rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-0.333\mathbf{j}\}\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$\boldsymbol{\omega}_{BC} = \{0.204\mathbf{i} - 0.612\mathbf{j} + 1.36\mathbf{k}\}$ rad/s
$\mathbf{v}_B = \{-0.333\mathbf{j}\}$ m/s

20–26.

Rod AB is attached to collars at its ends by using ball-and-socket joints. If collar A moves along the fixed rod at $v_A = 5$ m/s, determine the angular velocity of the rod and the velocity of collar B at the instant shown. Assume that the rod's angular velocity is directed perpendicular to the axis of the rod.



SOLUTION

The velocities of collars A and B are

$$\mathbf{v}_A = \{5\mathbf{i}\} \text{ m/s} \qquad \mathbf{v}_B = v_B \sin 45°\mathbf{j} + v_B \cos 45°\mathbf{k} = \frac{1}{\sqrt{2}} v_B\mathbf{j} + \frac{1}{\sqrt{2}} v_B\mathbf{k}$$

Also, $\mathbf{r}_{B/A} = (0 - 1)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = \{-1\mathbf{i} + 2\mathbf{j}\}$ m and $\boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\frac{1}{\sqrt{2}} v_B\mathbf{j} + \frac{1}{\sqrt{2}} v_B\mathbf{k} = 5\mathbf{i} + (\omega_x\mathbf{i} + \omega_y\mathbf{j})\omega_z\mathbf{k} \times (-1\mathbf{i} + 2\mathbf{j})$$

$$\frac{1}{\sqrt{2}} v_B\mathbf{j} + \frac{1}{\sqrt{2}} v_B\mathbf{k} = (5 - 2\omega_z)\mathbf{i} - \omega_z\mathbf{j} + (2\omega_x + \omega_y)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = 5 - 2\,\omega_z \qquad \textbf{(1)}$$

$$\frac{1}{\sqrt{2}} v_B = -\omega_z \qquad \textbf{(2)}$$

$$\frac{1}{\sqrt{2}} v_B = 2\omega_x + \omega_y \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is directed perpendicular to the axis of rod AB, then

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (-1\mathbf{i} + 2\mathbf{j}) = 0$$

$$-\omega_x + 2\omega_y = 0 \qquad \textbf{(4)}$$

Solving Eqs. 1 to 4,

$$\omega_x = -1.00 \text{ rad/s} \quad \omega_y = -0.500 \text{ rad/s} \quad \omega_z = 2.50 \text{ rad/s} \quad v_B = -2.50\sqrt{2} \text{ m/s}$$

Then

$$\boldsymbol{\omega}_{AB} = \{-1.00\mathbf{i} - 0.500\mathbf{j} + 2.50\mathbf{k}\} \text{ rad/s} \qquad \textbf{Ans.}$$

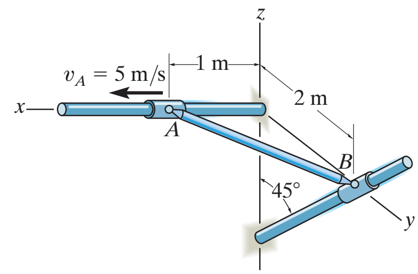
$$\mathbf{v}_B = \frac{1}{\sqrt{2}}\left(-2.50\sqrt{2}\right)\mathbf{j} + \frac{1}{\sqrt{2}}\left(-2.50\sqrt{2}\right)\mathbf{k} = \{-2.50\mathbf{j} - 2.50\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

Note: v_B can be obtained by solving Eqs. 1 and 2 without knowing the direction of $\boldsymbol{\omega}_{AB}$.

Ans:
$\boldsymbol{\omega}_{AB} = \{-1.00\mathbf{i} - 0.500\mathbf{j} + 2.50\mathbf{k}\}$ rad/s
$\mathbf{v}_B = \{-2.50\mathbf{j} - 2.50\mathbf{k}\}$ m/s

20–27.

Rod AB is attached to collars at its ends by using ball-and-socket joints. If collar A moves along the fixed rod with a velocity of $v_A = 5$ m/s and has an acceleration $a_A = 2$ m/s^2 at the instant shown, determine the angular acceleration of the rod and the acceleration of collar B at this instant. Assume that the rod's angular velocity and angular acceleration are directed perpendicular to the axis of the rod.



SOLUTION

The velocities of collars A and B are

$$\mathbf{v}_A = \{5\mathbf{i}\}\ \text{m/s} \qquad \mathbf{v}_B = v_B \sin 45°\mathbf{j} + v_B \cos 45°\mathbf{k} = \frac{1}{\sqrt{2}}v_B\mathbf{j} + \frac{1}{\sqrt{2}}v_B\mathbf{k}$$

Also, $\mathbf{r}_{B/A} = (0 - 1)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = \{-1\mathbf{i} + 2\mathbf{j}\}$ m and $\boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\frac{1}{\sqrt{2}}v_B\mathbf{j} + \frac{1}{\sqrt{2}}v_B\mathbf{k} = 5\mathbf{i} + (\omega_x\mathbf{i} + \omega_y\mathbf{j})\omega_z\mathbf{k} \times (-1\mathbf{i} + 2\mathbf{j})$$

$$\frac{1}{\sqrt{2}}v_B\mathbf{j} + \frac{1}{\sqrt{2}}v_B\mathbf{k} = (5 - 2\omega_z)\mathbf{i} - \omega_z\mathbf{j} + (2\omega_x + \omega_y)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = 5 - 2\,\omega_z \qquad \textbf{(1)}$$

$$\frac{1}{\sqrt{2}}v_B = -\omega_z \qquad \textbf{(2)}$$

$$\frac{1}{\sqrt{2}}v_B = 2\omega_x + \omega_y \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is directed perpendicular to the axis of rod AB, then

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (-1\mathbf{i} + 2\mathbf{j}) = 0$$

$$-\omega_x + 2\omega_y = 0 \qquad \textbf{(4)}$$

Solving Eqs. 1 to 4,

$$\omega_x = -1.00\ \text{rad/s} \quad \omega_y = -0.500\ \text{rad/s} \quad \omega_z = 2.50\ \text{rad/s} \quad v_B = -2.50\sqrt{2}\ \text{m/s}$$

Then

$$\boldsymbol{\omega}_{AB} = \{-1.00\mathbf{i} - 0.500\mathbf{j} + 2.50\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \frac{1}{\sqrt{2}}\left(-2.50\sqrt{2}\right)\mathbf{j} + \frac{1}{\sqrt{2}}\left(-2.50\sqrt{2}\right)\mathbf{k} = \{-2.50\mathbf{j} - 2.50\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans.}$$

Note: v_B can be obtained by solving Eqs. 1 and 2 without knowing the direction of $\boldsymbol{\omega}_{AB}$.

20–27. Continued

The accelerations of collars A and B are

$$a_A = \{2\mathbf{i}\}\ \text{m/s}^2 \quad a_B = a_B \sin 45°\mathbf{j} + a_B \cos 45°\mathbf{k} = \frac{1}{\sqrt{2}}a_B\mathbf{j} + \frac{1}{\sqrt{2}}a_B\mathbf{k}$$

Also, $\alpha_{AB} = \alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A})$$

$$\frac{1}{\sqrt{2}}a_B\mathbf{j} + \frac{1}{\sqrt{2}}a_B\mathbf{k} = 2\mathbf{i} + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (-1\mathbf{i} + 2\mathbf{j})$$

$$+ (-1.00\mathbf{i} - 0.500\mathbf{j} + 2.50\mathbf{k}) \times [(-1.00\mathbf{i} - 0.500\mathbf{j} + 2.50\mathbf{k}) \times (-1\mathbf{i} + 2\mathbf{j})]$$

$$\frac{1}{\sqrt{2}}a_B\mathbf{j} + \frac{1}{\sqrt{2}}a_B\mathbf{k} = (9.5 - 2\alpha_z)\mathbf{i} + (-\alpha_z - 15)\mathbf{j} + (2\alpha_x + \alpha_y)\mathbf{k}$$

Equating **i**, **j** and **k** components

$$0 = 9.5 - 2\alpha_z \qquad \textbf{(5)}$$

$$\frac{1}{\sqrt{2}}a_B = -\alpha_z - 15 \qquad \textbf{(6)}$$

$$\frac{1}{\sqrt{2}}a_B = z\alpha_x + \alpha_y \qquad \textbf{(7)}$$

Assuming that $\boldsymbol{\alpha}_{AB}$ is directed perpendicular to the axis of rod AB, then

$$\boldsymbol{\alpha}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \cdot (-1\mathbf{i} + 2\mathbf{j}) = 0$$

$$-\alpha_x + 2x_y = 0 \qquad \textbf{(8)}$$

Solving Eqs. 5 to 8,

$$\alpha_x = -7.9\ \text{rad/s}^2 \quad \alpha_y = -3.95\ \text{rad/s}^2 \quad \alpha_z = 4.75\ \text{rad/s}^2 \quad a_B = -19.75\sqrt{2}\ \text{m/s}^2$$

Thus,

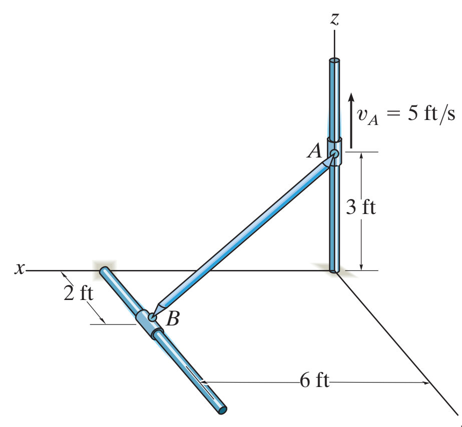
$$\boldsymbol{\alpha}_{AB} = \{-7.9\mathbf{i} - 3.95\mathbf{j} + 4.75\mathbf{k}\}\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

$$a_B = \frac{1}{\sqrt{2}}(-19.75\sqrt{2})\mathbf{j} + \frac{1}{\sqrt{2}}(-19.75\sqrt{2})\mathbf{j} = \{-19.75\mathbf{j} - 19.75\mathbf{k}\}\ \text{m/s}^2 \ \textbf{Ans.}$$

Ans:
$\boldsymbol{\alpha}_{AB} = \{-7.9\mathbf{i} - 3.95\mathbf{j} + 4.75\mathbf{k}\}\ \text{rad/s}^2$
$\mathbf{a}_B = \{-19.75\mathbf{j} - 19.75\mathbf{k}\}\ \text{m/s}^2$

***20–28.**

If the rod is attached with ball-and-socket joints to smooth collars A and B at its end points, determine the velocity of B at the instant shown if A is moving upward at a constant speed of $v_A = 5$ ft/s. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.



SOLUTION

The velocities of collars A and B are

$$\mathbf{v}_A = \{5\mathbf{k}\}\ \text{ft/s} \quad \mathbf{v}_B = -v_B\mathbf{j}$$

Also, $\mathbf{r}_{B/A} = (6-0)\mathbf{i} + (2-0)\mathbf{j} + (0-3)\mathbf{k} = \{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\}$ ft and $\boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$-v_B\mathbf{j} = 5\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

$$-v_B\mathbf{j} = (-3\omega_y - 2\omega_z)\mathbf{i} + (3\omega_x + 6\omega_z)\mathbf{j} + (2\omega_x - 6\omega_y + 5)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = -3\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$-v_B = 3\omega_x + 6\omega_z \qquad \textbf{(2)}$$

$$0 = 2\omega_x - 6\omega_y + 5 \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is directed perpendicular to the axis of rod AB, then,

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) = 0$$

$$6\omega_x + 2\omega_y - 3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. 1 to 4,

$$\omega_x = -\frac{65}{98}\ \text{rad/s} = -0.6633\ \text{rad/s} \quad \omega_y = \frac{30}{49}\ \text{rad/s} = 0.6122\ \text{rad/s}$$

$$\omega_z = -\frac{45}{49}\ \text{rad/s} = -0.9183\ \text{rad/s} \quad v_B = 7.50\ \text{ft/s}$$

Thus,

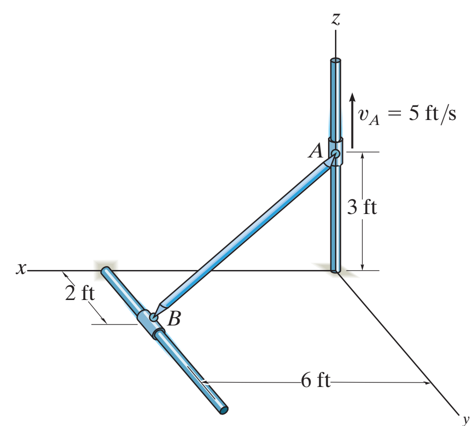
$$\boldsymbol{\omega}_{AB} = \{-0.663\mathbf{i} + 0.612\mathbf{j} - 0.918\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-7.50\mathbf{j}\}\ \text{ft/s} \qquad \textbf{Ans.}$$

Ans:
$\boldsymbol{\omega}_{AB} = \{-0.663\mathbf{i} + 0.612\mathbf{j} - 0.918\mathbf{k}\}$ rad/s
$\mathbf{v}_B = \{-7.50\mathbf{j}\}$ ft/s

20–29.

If the collar at A in Prob. 20–28 is moving upward with an acceleration of $\mathbf{a}_A = \{-2\mathbf{k}\}\ \text{ft/s}^2$, at the instant its speed is $v_A = 5$ ft/s, determine the acceleration of the collar at B at this instant.



SOLUTION

The velocities of collars A and B are

$$\mathbf{v}_A = \{5\mathbf{k}\}\ \text{ft/s} \qquad \mathbf{v}_B = -v_B\mathbf{j}$$

Also, $\mathbf{r}_{B/A} = (6-0)\mathbf{i} + (2-0)\mathbf{j} + (0-3)\mathbf{k} = \{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\}$ ft and $\boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$-v_B\mathbf{j} = 5\mathbf{k} + \left(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}\right) \times (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

$$-v_B\mathbf{j} = \left(-3\omega_y - 2\omega_z\right)\mathbf{i} + \left(3\omega_x + 6\omega_z\right)\mathbf{j} + \left(2\omega_x - 6\omega_y + 5\right)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = -3\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$-v_B = 3\omega_x + 6\omega_z \qquad \textbf{(2)}$$

$$0 = 2\omega_x - 6\omega_y + 5 \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is directed perpendicular to the axis of rod AB, then,

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$\left(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}\right) \cdot (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) = 0$$

$$6\omega_x + 2\omega_y - 3\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. 1 to 4,

$$\omega_x = -\frac{65}{98}\ \text{rad/s} = -0.6633\ \text{rad/s} \quad \omega_y = \frac{30}{49}\text{rad/s} = 0.6122\ \text{rad/s}$$

$$\omega_z = -\frac{45}{49}\ \text{rad/s} = -0.9183\ \text{rad/s} \quad v_B = 7.50\ \text{ft/s}$$

Thus,

$$\boldsymbol{\omega}_{AB} = \{-0.663\mathbf{i} + 0.612\mathbf{j} - 0.918\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

$$\mathbf{v}_B = \{-7.50\mathbf{j}\}\ \text{ft/s} \qquad \textbf{Ans.}$$

20–29. Continued

The accelerations of collars A and B are

$$\mathbf{a}_A = \{-2\mathbf{k}\}\ \text{ft/s}^2 \quad \mathbf{a}_B = a_B\mathbf{j}$$

Also, $\alpha_{AB} = \alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}_{AB} \times \left(\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}\right)$$

$$a_B\mathbf{j} = -2\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) + (-0.6633\mathbf{i} + 0.6122\mathbf{j} - 0.9183\mathbf{k}) \times [(-0.6633\mathbf{i} + 0.6122\mathbf{j} - 0.9183\mathbf{k}) \times (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})]$$

$$a_B\mathbf{j} = (-3\alpha_y - 2\alpha_z - 9.9490)\mathbf{i} + (3\alpha_x + 6\alpha_z - 3.3163)\mathbf{j} + (2\alpha_x - 6\alpha_y + 2.9745)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = -3\alpha_y - 2\alpha_z - 9.9490 \qquad \textbf{(5)}$$

$$a_B = 3\alpha_x + 6\alpha_z - 3.3163 \qquad \textbf{(6)}$$

$$0 = 2\alpha_x - 6\alpha_y + 2.9745 \qquad \textbf{(7)}$$

Eliminate α_y from Eqs. 5 and 7

$$2\alpha_x + 4\alpha_z = -22.8724 \qquad \textbf{(8)}$$

Multiply Eq. 6 by $\frac{2}{3}$ and rearrange,

$$2\alpha_x + 4\alpha_z = \frac{2}{3}a_B + 2.2109 \qquad \textbf{(9)}$$

Equating Eqs. (8) and (9)

$$-22.8724 = \frac{2}{3}a_B + 2.2109$$

$$a_B = -37.625\ \text{ft/s}^2$$

Thus,

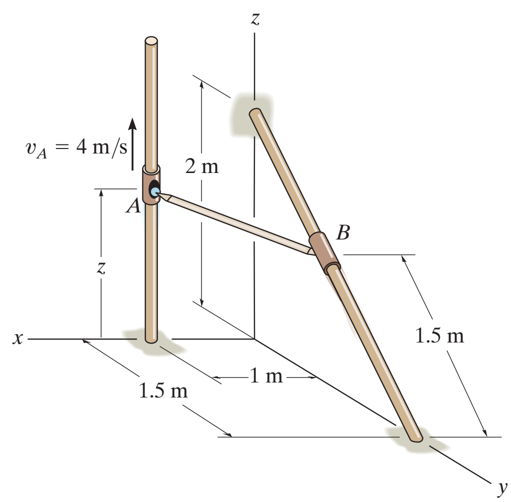
$$a_B = \{-37.6\mathbf{j}\}\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Note: There is no need to know the direction of $\boldsymbol{\alpha}_{AB}$ to determine $\mathbf{a}_B$.

Ans:
$\mathbf{a}_B = \{-37.6\mathbf{j}\}\ \text{ft/s}^2$

20–30.

Rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a speed $v_A = 4$ m/s, determine the speed of collar B at the instant $z = 2$ m. Assume the angular velocity of the rod is directed perpendicular to the rod.



SOLUTION

$$v_B = v_A + \omega \times r_{B/A}$$

The velocities of collars A and B are

$$\mathbf{v}_A = \{4\mathbf{k}\}\ \text{m/s} \qquad \mathbf{v}_B = -v_B\left(\frac{3}{5}\right)\mathbf{j} + v_B\left(\frac{4}{5}\right)\mathbf{k} = -\frac{3}{5}v_B\mathbf{j} + \frac{4}{5}v_B\mathbf{k}$$

Also, the coordinates of points A and B are $A(1, 0, 2)$ m and $B\left\{0, \left[1.5 - 1.5\left(\frac{3}{5}\right)\right], 1.5\left(\frac{4}{5}\right)\right] = B(0, 0.6, 1.2)$ m. Thus, $\mathbf{r}_{B/A} = (0 - 1)\mathbf{i} + (0.6 - 0)\mathbf{j} + (1.2 - 2)\mathbf{k}$ $= \{-1\mathbf{i} + 0.6\mathbf{j} - 0.8\mathbf{k}\}$ m. Also $\omega_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$-\frac{3}{5}v_B\mathbf{j} + \frac{4}{5}v_B\mathbf{k} = 4\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (-1\mathbf{i} + 0.6\mathbf{j} - 0.8\mathbf{k})$$

$$-\frac{3}{5}v_B\mathbf{j} + \frac{4}{5}v_B\mathbf{k} = (-0.8\omega_y - 0.6\omega_z)\mathbf{i} + (0.8\omega_x - \omega_z)\mathbf{j} + (0.6\omega_x + \omega_y + 4)\mathbf{k}$$

Equating **i**, **j** and **k** components

$$0 = -0.8\omega_y - 0.6\omega_z \qquad \textbf{(1)}$$

$$-\frac{3}{5}v_B = 0.8\omega_x - \omega_z \qquad \textbf{(2)}$$

$$\frac{4}{5}v_B = 0.6\omega_x + \omega_y + 4 \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is perpendicular to the axis of the rod AB, then

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (-1\mathbf{i} + 0.6\mathbf{j} - 0.8\mathbf{k}) = 0$$

$$-\omega_x + 0.6\omega_y - 0.8\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4),

$$\omega_x = -1.20\ \text{rad/s} \quad \omega_y = -0.720\ \text{rad/s} \quad \omega_z = 0.960\ \text{rad/s}$$

$$v_B = 3.20\ \text{m/s}$$

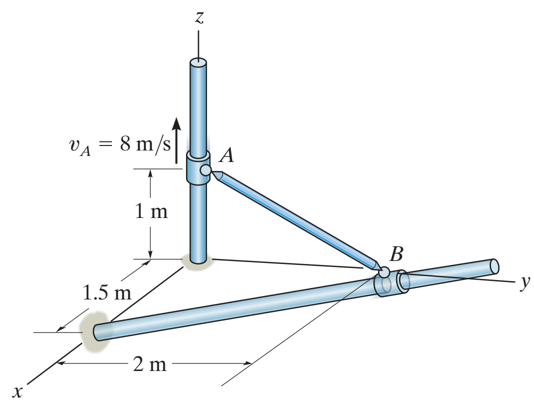
Then $v_B = -\frac{3}{5}(3.20)\mathbf{j} - \frac{4}{5}(3.20)\mathbf{k} = \{-1.92\mathbf{j} + 2.56\mathbf{k}\}$ m/s **Ans.**

Note: v_B can also be obtained by Solving Eqs. (1) to (3) without knowing the direction of $\boldsymbol{\omega}_{AB}$.

Ans:
$\mathbf{v}_B = \{-1.92\mathbf{j} + 2.56\mathbf{k}\}$ m/s

20–31.

The rod is attached to smooth collars A and B at its ends using ball-and-socket joints. Determine the speed of B at the instant shown if A is moving at $v_A = 8$ m/s. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.



SOLUTION

$\mathbf{v}_B = v_A + \mathbf{r}_{B/A}$

The velocities of collars A and B are

$$\mathbf{v}_A = \{8\mathbf{k}\} \text{ m/s} \qquad \mathbf{v}_B = v_B\left(\frac{3}{5}\right)\mathbf{i} - v_B\left(\frac{4}{5}\right)\mathbf{j} = \frac{3}{5}v_B\mathbf{i} - \frac{4}{5}v_B\mathbf{j}$$

Also, $\mathbf{r}_{B/A} = (0 - 0)\mathbf{i} + (2 - 0)\mathbf{j} + (0 - 1)\mathbf{k} = \{2\mathbf{j} - 1\mathbf{k}\}$ m and $\boldsymbol{\omega}_{AB} + \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\frac{3}{5}v_B\,\mathbf{i} - \frac{4}{5}v_B\,\mathbf{j} = 8\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$\frac{3}{5}v_B\mathbf{i} - \frac{4}{5}v_B\mathbf{j} = (-\omega_y - 2\omega_z)\mathbf{i} + \omega_x\mathbf{j} + (2\omega_x + 8)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$\frac{3}{5}v_B = -\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$-\frac{4}{5}v_B = \omega_x \qquad \textbf{(2)}$$

$$0 = 2\omega_x + 8 \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is perpendicular to the axis of rod AB, then

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{j} - 1\mathbf{k}) = 0$$

$$2\omega_y - \omega_z = 0 \qquad \textbf{(4)}$$

Solving Eq (1) to (4)

$$\omega_x = -4.00 \text{ rad/s} \qquad \omega_y = -0.600 \text{ rad/s} \qquad \omega_z = -1.20 \text{ rad/s}$$

$$v_B = 5.00 \text{ m/s} \qquad \textbf{Ans.}$$

Then,

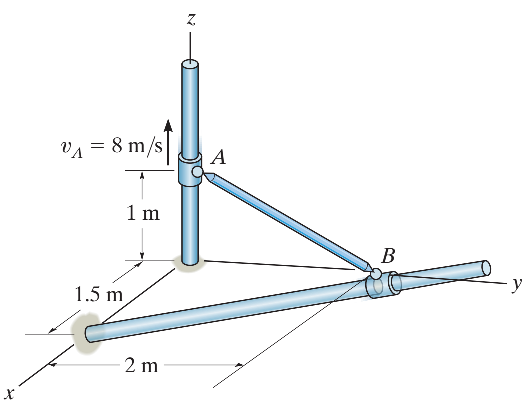
$$\omega_{AB} = \{-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}\} \text{ rad/s} \qquad \textbf{Ans.}$$

Note. v_B can be obtained by solving Eqs (2) and (3) without knowing the direction of $\boldsymbol{\omega}_{AB}$.

Ans:
$v_B = 5.00$ m/s
$\boldsymbol{\omega}_{AB} = \{-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}\}$ rad/s

***20–32.**

If the collar A in Prob. 20–31 has a deceleration of $\mathbf{a}_A = \{-5\mathbf{k}\}\ \text{m/s}^2$, at the instant shown, determine the acceleration of collar B at this instant.



SOLUTION

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{r}_{B/A}$

The velocities of collars A and B are

$$\mathbf{v}_A = \{8\mathbf{k}\}\ \text{m/s} \qquad \mathbf{v}_B = v_B\left(\frac{3}{5}\right)\mathbf{i} - v_B\left(\frac{4}{5}\right)\mathbf{j} = \frac{3}{5}v_B\mathbf{i} - \frac{4}{5}v_B\mathbf{j}$$

Also, $\mathbf{r}_{B/A} = (0-0)\mathbf{i} + (2-0)\mathbf{j} + (0-1)\mathbf{k} = \{2\mathbf{j} - 1\mathbf{k}\}$ m and $\boldsymbol{\omega}_{AB} + \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\frac{3}{5}v_B\mathbf{i} - \frac{4}{5}v_B\mathbf{j} = 8\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$

$$\frac{3}{5}v_B\mathbf{i} - \frac{4}{5}v_B\mathbf{j} = (-\omega_y - 2\omega_z)\mathbf{i} + \omega_x\mathbf{j} + (2\omega_x + 8)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$\frac{3}{5}v_B = -\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$-\frac{4}{5}v_B = \omega_x \qquad \textbf{(2)}$$

$$0 = 2\omega_x + 8 \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{AB}$ is perpendicular to the axis of rod AB, then

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{r}_{B/A} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (2\mathbf{j} - 1\mathbf{k}) = 0$$

$$2\omega_y - \omega_z = 0 \qquad \textbf{(4)}$$

Solving Eq (1) to (4)

$$\omega_x = -4.00\ \text{rad/s} \qquad \omega_y = -0.600\ \text{rad/s} \qquad \omega_z = -1.20\ \text{rad/s}$$

$$v_B = 5.00\ \text{m/s} \qquad \textbf{Ans.}$$

Then,

$$\omega_{AB} = \{-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}\}\ \text{rad/s} \qquad \textbf{Ans.}$$

Note. v_B can be obtained by solving Eqs (2) and (3) without knowing the direction of $\boldsymbol{\omega}_{AB}$.

***20–32. Continued**

The accelerations of collars A and B are

$$\mathbf{a}_A = \{-5\mathbf{k}\}\ \text{m/s}^2 \qquad \mathbf{a}_B = -a_B\left(\frac{3}{5}\right)\mathbf{i} + a_B\left(\frac{4}{5}\right)\mathbf{j} = -\frac{3}{5}a_B\,\mathbf{i} + \frac{4}{5}a_B\,\mathbf{j}$$

Also, $\alpha_{AB} = \alpha_x\,\mathbf{i} + \alpha_y\,\mathbf{j} + \alpha_z\mathbf{k}$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B})$$

$$-\frac{3}{5}a_B\mathbf{i} + \frac{4}{5}a_B\mathbf{j} = -5\mathbf{k} + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})$$
$$+ (-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}) \times [(-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}) \times (2\mathbf{j} - 1\mathbf{k})]$$

$$-\frac{3}{5}a_B\mathbf{i} + \frac{4}{5}a_B\mathbf{j} = (-\alpha_y - 2\alpha_z)\mathbf{i} + (\alpha_x - 35.6)\mathbf{j} + (2\alpha_x + 12.8)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$-\frac{3}{5}a_B = -\alpha_y + 2\alpha_z \qquad \textbf{(5)}$$

$$\frac{4}{5}a_B = \alpha_x - 35.6 \qquad \textbf{(6)}$$

$$0 = 2\alpha x + 12.8 \qquad \textbf{(7)}$$

Solving Eqs (6) and (7),

$$\alpha_x = -6.40\ \text{rad/s}^2 \qquad a_B = -52.5\ \text{m/s}^2$$

Then

$$a_B = -\frac{3}{5}(-52.5)\mathbf{i} + \frac{4}{5}(-52.5)\mathbf{j}$$

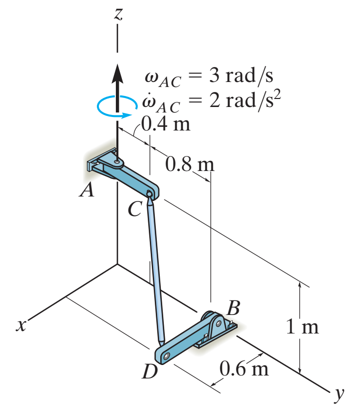
$$= \{31.5\mathbf{i} - 42.0\mathbf{j}\}\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Note. It is not necessary to know the direction of $\boldsymbol{\alpha}_{AB}$, if only $\mathbf{a}_B$ needs to be determined.

Ans:
$v_B = 5.00$ m/s
$\omega_{AB} = \{-4.00\mathbf{i} - 0.600\mathbf{j} - 1.20\mathbf{k}\}$ rad/s
$a_B = \{31.5\mathbf{i} - 42.0\mathbf{j}\}$ m/s^2

20–33.

Rod CD is attached to the rotating arms using ball-and-socket joints. If AC has the motion shown, determine the angular velocity of link BD at the instant shown.



SOLUTION

$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{D/C}$

The velocities of points C and D are

$\mathbf{v}_C = \boldsymbol{\omega}_{AC} \times \mathbf{r}_{AC} = 3\mathbf{k} \times 0.4\mathbf{j} = \{-1.2\mathbf{i}\}$ m/s

$\mathbf{v}_D = \boldsymbol{\omega}_{BD} \times \mathbf{r}_{BD} = \omega_{BD}\mathbf{j} \times 0.6\mathbf{i} = -0.6\omega_{BD}\mathbf{k}$

Also, $\mathbf{r}_{D/C} = (0.6 - 0)\mathbf{i} + (1.2 - 0.4)\mathbf{j} + (0.1)\mathbf{k} = \{0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k}\}$ m and $\omega_{CD} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$

$-0.6\omega_{BD}\mathbf{k} = -1.2\mathbf{i} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k})$

$-0.6\omega_{BD}\mathbf{k} = (-\omega_y - 0.8\omega_z - 1.2)\mathbf{i} + (\omega_x + 0.6\omega_z)\mathbf{j} + (0.8\omega_x - 0.6\omega_y)\mathbf{k}.$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components,

$$-\omega_y - 0.8\omega_z - 1.2 = 0 \qquad \textbf{(1)}$$

$$\omega_x + 0.6\omega_z = 0 \qquad \textbf{(2)}$$

$$0.8\omega_x - 0.6\omega_y = -0.6\omega_{BD} \qquad \textbf{(3)}$$

Assuming that $\boldsymbol{\omega}_{CD}$ is perpendicular to the axis of rod CD, then

$\boldsymbol{\omega}_{CD} \cdot \mathbf{r}_{D/C} = 0$

$(\omega_x\mathbf{i} + \omega_y\mathbf{j}) + \omega_z\mathbf{k}) \cdot (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k}) = 0$

$$0.6\omega_x + 0.8\omega_y - \omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs (1) to (4)

$\omega_x = 0.288$ rad/s $\qquad \omega_y = -0.816$ rad/s $\qquad \omega_z = -0.480$ rad/s

$\omega_{BD} = -1.20$ rad/s

Thus

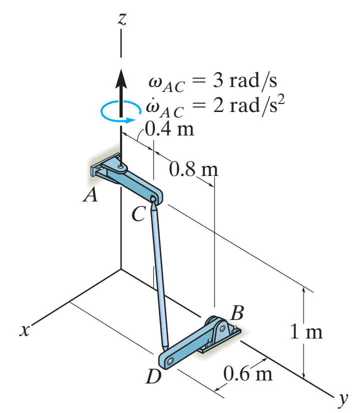
$$\omega_{BD} = \{-1.20\mathbf{j}\} \text{ rad/s} \qquad \textbf{Ans.}$$

Note: $\boldsymbol{\omega}_{BD}$ can be obtained by solving Eqs 1 to 3 without knowing the direction of $\boldsymbol{\omega}_{AB}$.

Ans:
$\boldsymbol{\omega}_{BD} = \{-1.20\mathbf{j}\}$ rad/s

20–34.

Rod CD is attached to the rotating arms using ball-and-socket joints. If AC has the motion shown, determine the angular acceleration of link BD at this instant.



SOLUTION

$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{D/C}$

The velocities of points C and D are

$\mathbf{v}_C = \boldsymbol{\omega}_{AC} \times \mathbf{r}_{AC} = 3\mathbf{k} \times 0.4\mathbf{j} = \{-1.2\mathbf{i}\}$ m/s

$\mathbf{v}_D = \boldsymbol{\omega}_{BD} \times \mathbf{r}_{BD} = \omega_{BD}\mathbf{j} \times 0.6\mathbf{i} = -0.6\omega_{BD}\mathbf{k}$

Also, $\mathbf{r}_{D/C} = (0.6 - 0)\mathbf{i} + (1.2 - 0.4)\mathbf{j} + (0.1)\mathbf{k} = \{0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k}\}$ m and $\boldsymbol{\omega}_{CD} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$\mathbf{v}_D = \mathbf{v}_C + \boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}$

$-0.6\omega_{BD}\mathbf{k} = -1.2\mathbf{i} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k})$

$-0.6\omega_{BD}\mathbf{k} = (-\omega_y - 0.8\omega_z - 1.2)\mathbf{i} + (\omega_x + 0.6\omega_z)\mathbf{j} + (0.8\omega_x - 0.6\omega_y)\mathbf{k}.$

Equating **i**, **j** and **k** components,

$-\omega_y - 0.8\omega_z - 1.2 = 0$ **(1)**

$\omega_x + 0.6\omega_z = 0$ **(2)**

$0.8\omega_x - 0.6\omega_y = -0.6\omega_{BD}$ **(3)**

Assuming that $\boldsymbol{\omega}_{CD}$ is perpendicular to the axis of rod CD, then

$\boldsymbol{\omega}_{CD} \cdot \mathbf{r}_{D/C} = 0$

$(\omega_x\mathbf{i} + \omega_y\mathbf{j}) + \ \omega_z\mathbf{k}) \cdot (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k}) = 0$

$0.6\omega_x + 0.8\omega_y - \omega_z = 0$ **(4)**

Solving Eqs (1) to (4)

$\omega_x = 0.288$ rad/s $\qquad \omega_y = -0.816$ rad/s $\qquad \omega_z = -0.480$ rad/s

$\omega_{BD} = -1.20$ rad/s

Thus

$\omega_{BD} = \{-1.20\mathbf{j}\}$ rad/s **Ans.**

Note: $\boldsymbol{\omega}_{BD}$ can be obtained by solving Eqs 1 to 3 without knowing the direction of $\boldsymbol{\omega}_{CD}$.

20–34. Continued

The accelerations of points C and D are

$\mathbf{a}_C = \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{AC} - \omega_{AC}^2\mathbf{r}_{AC} = (2\mathbf{k} \times 0.4\mathbf{j}) - 3^2(0.4\mathbf{j}) = \{-0.8\mathbf{i}, -3.6\mathbf{j}\}\ \text{m/s}$

$\mathbf{a}_D = \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{BD} - \omega_{BD}^2\mathbf{r}_{BD} = (\alpha_{BD}\mathbf{j} \times 0.6\mathbf{i}) - 1.20^2(0.6\mathbf{i}) = -0.864\mathbf{i} - 0.6\alpha_{BD}\mathbf{k}$

Also, $\alpha_{CD} = \alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}$ and $\boldsymbol{\omega}_{CD} = \{0.288\mathbf{i} - 0.816\mathbf{j} - 0.480\mathbf{k}\}\ \text{rad/s}$

Applying the relative acceleration equation,

$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{D/C} + \boldsymbol{\omega}_{CD} \times (\boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C})$$

$$-0.864\mathbf{i} - 0.6\alpha_{BD}\mathbf{k} = (-0.8\mathbf{i} - 3.6\mathbf{j}) + (\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \times (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k})$$
$$+ (0.288\mathbf{i} - 0.816\mathbf{j} - 0.480\mathbf{k}) \times [(0.288\mathbf{i} - 0.816\mathbf{j} - 0.480\mathbf{k}) \times (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k})]$$

$$-0.864\mathbf{i} - 0.6\alpha_{BD}\mathbf{k} = (-\alpha_y - 0.8\alpha_z - 1.38752)\mathbf{i} + (\alpha_x + 0.6\alpha_z - 4.38336)\mathbf{j} + (0.8\alpha_x - 0.6\alpha_y + 0.9792)\mathbf{k}$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components,

$$-0.864 = -\alpha_y - 0.8\alpha_z - 1.38752 \qquad \textbf{(5)}$$

$$0 = \alpha_x + 0.6\alpha_z - 4.38336 \qquad \textbf{(6)}$$

$$-0.6\alpha_{BD} = 0.8\alpha_x - 0.6\alpha_y + 0.9792 \qquad \textbf{(7)}$$

Assuming that $\boldsymbol{\alpha}_{CD}$ is perpendicular to the axis of rod CD, then

$$\boldsymbol{\alpha}_{CD} \cdot \mathbf{r}_{D/C} = 0$$

$$(\alpha_x\mathbf{i} + \alpha_y\mathbf{j} + \alpha_z\mathbf{k}) \cdot (0.6\mathbf{i} + 0.8\mathbf{j} - \mathbf{k}) = 0$$

$$0.6\alpha_x + 0.8\alpha_y - \alpha_z = 0 \qquad \textbf{(8)}$$

Solving Eqs. 5 to 8,

$$\alpha_x = 3.72\ \text{rad/s}^2 \qquad \alpha_y = -1.408\ \text{rad/s}^2 \qquad \alpha_z = 1.1056\ \text{rad/s}^2$$
$$\alpha_{BD} = -8.00\ \text{rad/s}^2$$

Thus,

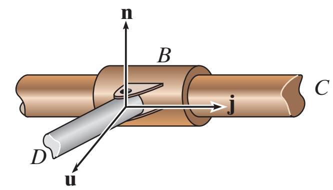
$$\alpha_{BD} = \{-8.00\mathbf{j}\}\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Note: α_{BD} can be obtained by solving Eqs 5 to 7 without knowing the direction of $\boldsymbol{\alpha}_{CD}$.

Ans:
$\boldsymbol{\alpha}_{BD} = \{-8.00\mathbf{j}\}\ \text{rad/s}^2$

20–35.

Solve Prob. 20–28 if the connection at B consists of a pin as shown in the figure below, rather than a ball-and-socket joint. *Hint:* The constraint allows rotation of the rod both along the bar (**j** direction) and along the axis of the pin (**n** direction). Since there is no rotational component in the **u** direction, i.e., perpendicular to **n** and **j** where $\mathbf{u} = \mathbf{j} \times \mathbf{n}$, an additional equation for solution can be obtained from $\boldsymbol{\omega} \cdot \mathbf{u} = 0$. The vector **n** is in the same direction as $\mathbf{r}_{D/B} \times \mathbf{r}_{C/B}$.



SOLUTION

The velocities of collars A and B are

$$\mathbf{v}_A = \{5\,\mathrm{k}\}\ \mathrm{ft/s} \qquad \mathbf{v}_B = -v_B\mathbf{j}$$

Also, $\mathbf{r}_{B/A} = (6-0)\mathbf{i} + (2-0)\mathbf{j} + (0-3)\mathbf{k} = \{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\}$ ft and $\omega_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$-v_B\mathbf{j} = 5\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

$$-v_B\mathbf{j} = (-3\omega_y - 2\omega_z)\mathbf{i} + (3\omega_x + 6\omega_z)\mathbf{j} + (2\omega_x - 6\omega_y + 5)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = -3\omega_y - 2\omega_z \qquad \textbf{(1)}$$

$$-v_B = 3\omega_x + 6\omega_z \qquad \textbf{(2)}$$

$$0 = 2\omega_x - 6\omega_y + 5 \qquad \textbf{(3)}$$

Here,

$$\mathbf{r}_{B/A} \times \mathbf{j} = (6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \times \mathbf{j} = 3\mathbf{i} + 6\mathbf{k}$$

Then

$$\mathbf{n} = \frac{\mathbf{r}_{B/A} \times \mathbf{j}}{|\mathbf{r}_{B/A} \times \mathbf{j}|} = \frac{3\mathbf{i} + 6\mathbf{k}}{\sqrt{3^2 + 6^2}} = \frac{3}{\sqrt{45}}\mathbf{i} + \frac{6}{\sqrt{45}}\mathbf{k}$$

Thus

$$\mathbf{u} = \mathbf{j} \times \mathbf{n} = \mathbf{j} \times \left(\frac{3}{\sqrt{45}}\mathbf{i} + \frac{6}{\sqrt{45}}\mathbf{k}\right) = \frac{6}{\sqrt{45}}\mathbf{i} - \frac{3}{\sqrt{45}}\mathbf{k}$$

It is required that

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{u} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot \left(\frac{6}{\sqrt{45}}\mathbf{i} - \frac{3}{\sqrt{45}}\mathbf{k}\right) = 0$$

$$\frac{6}{\sqrt{45}}\omega_x - \frac{3}{\sqrt{45}}\omega_z = 0$$

$$2\omega_x - \omega_z = 0 \qquad \textbf{(4)}$$

20–35. Continued

Solving Eqs (1) to (4)

$$\omega_x = -0.500 \text{ rad/s} \qquad \omega_y = 0.6667 \text{ rad/s} \qquad \omega_z = -1.00 \text{ rad/s}$$

$$v_B = 7.50 \text{ ft/s}$$

Thus,

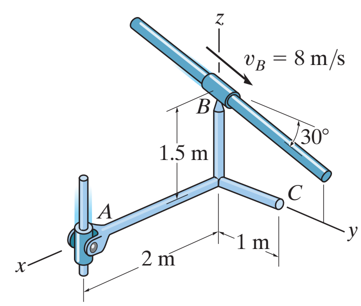
$\boldsymbol{\omega}_{AB} = \{-0.500\mathbf{i} + 0.667\mathbf{j} - 1.00\mathbf{k}\} \text{ rad/s}$ **Ans.**

$v_B = \{-7.50\mathbf{j}\} \text{ ft/s}$ **Ans.**

Ans:
$\boldsymbol{\omega}_{AB} = \{-0.500\mathbf{i} + 0.667\mathbf{j} - 1.00\mathbf{k}\} \text{ rad/s}$
$\mathbf{v}_B = \{-7.50\mathbf{j}\} \text{ ft/s}$

***20–36.**

Member ABC is pin connected at A and has a ball-and-socket joint at B. If the collar at B is moving along the inclined rod at $v_B = 8$ m/s, determine the velocity of point C at the instant shown. *Hint*: See Prob. 20–35.



SOLUTION

Velocities of collars A and B are

$\mathbf{v}_A = v_A\,\mathbf{k} \qquad \mathbf{v}_B = 8\cos 30°\,\mathbf{j} - 8\sin 30°\,\mathbf{k} = \{4\sqrt{3}\,\mathbf{j} - 4\,\mathbf{k}\}$ m/s

Also, $\mathbf{r}_{A/B} = \{2\,\mathbf{i} - 1.5\,\mathbf{k}\}$ m and $\omega_{AB} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$. Applying the relative velocity equation,

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{k} = \left(4\sqrt{3}\mathbf{j} - 4\mathbf{k}\right) + (\omega_x\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \times (2\mathbf{i} - 1.5\mathbf{k})$$

$$v_A\mathbf{k} = -1.5\omega_y\mathbf{i} + \left(1.5\omega_x + 2\omega_z + 4\sqrt{3}\right)\mathbf{j} + (-2\omega_y - 4)\mathbf{k}$$

Equating **i**, **j** and **k** components,

$$0 = -1.5\,\omega_y \qquad \omega_y = 0$$

$$0 = 1.5\,\omega_x + 2\omega_z + 4\sqrt{3} \qquad \textbf{(1)}$$

$$v_A = -2(0) - 4 \qquad v_A = -4 \text{ m/s}$$

Here $\mathbf{n} = \mathbf{j}$. Then $\mathbf{u} = \mathbf{k} \times \mathbf{n} = \mathbf{k} \times \mathbf{j} = -\mathbf{i}$. It is required that

$$\boldsymbol{\omega}_{AB} \cdot \mathbf{u} = 0$$

$$(\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \cdot (-\mathbf{i}) = 0$$

$$-\omega_x = 0 \qquad \omega_x = 0$$

Substitute this result into Eq (1),

$$0 = 1.5(0) + 2\,\omega_z + 4\sqrt{3}$$

$$\omega_z = -2\sqrt{3} \text{ rad/s}$$

Thus,

$$\omega_{AB} = \{-2\sqrt{3}\mathbf{k}\} \text{ rad/s}$$

Here, $\mathbf{r}_{C/B} = \{1\mathbf{j} - 1.5\mathbf{k}\}$. Using the result of $\boldsymbol{\omega}_{AB}$,

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{C/B}$$

$$\mathbf{v}_C = (4\sqrt{3}\mathbf{j} - 4\mathbf{k}) + (-2\sqrt{3}\mathbf{k}) \times (1\mathbf{j} - 1.5\mathbf{k})$$

$$= \{2\sqrt{3}\mathbf{i} + 4\sqrt{3}\mathbf{j} - 4\mathbf{k}\} \text{ m/s}$$

$$= \{3.46\mathbf{i} + 6.93\mathbf{j} - 4\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{3.46\mathbf{i} + 6.93\mathbf{j} - 4\mathbf{k}\}$ m/s

20–37.

Solve Example 20.5 such that the x, y, z axes move with curvilinear translation, $\Omega = \mathbf{0}$ in which case the collar appears to have both an angular velocity $\mathbf{\Omega}_{xyz} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$ and radial motion.

SOLUTION

Relative to XYZ, let xyz have

$$\Omega = 0 \qquad \dot{\Omega} = 0$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \{2\mathbf{j}\}\text{ m/s}$$

$$\mathbf{a}_B = \{0.75\mathbf{j} + 8\mathbf{k}\}\text{ m/s}^2$$

Relative to xyz, let $x'\ y'\ z'$ be coincident with xyz and be fixed to BD. Then

$$\Omega_{xyz} = \omega_1 + \omega_2 = \{4\mathbf{i} + 5\mathbf{k}\}\text{ rad/s} \qquad \dot{\omega}_{xyz} = \dot{\omega}_1 + \dot{\omega}_2 = \{1.5\mathbf{i} - 6\mathbf{k}\}\text{ rad/s}^2$$

$$(\mathbf{r}_{C/B})_{xyz} = \{0.2\mathbf{j}\}\text{ m}$$

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\mathbf{r}_{C/B})_{xyz}$$

$$= 3\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})$$

$$= \{-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k}\}\text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = (\ddot{\mathbf{r}}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{x'y'z'} + (\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{x'y'z'}\right]$$

$$+\left[(\dot{\omega}_1 + \dot{\omega}_2) \times (\mathbf{r}_{C/B})_{xyz}\right] + \left[(\omega_1 + \omega_2) \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right]$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[2\mathbf{j} + (4\mathbf{i} + 5\mathbf{k}) \times 3\mathbf{j}\right] + \left[(1.5\mathbf{i} - 6\mathbf{k}) \times 0.2\mathbf{j}\right] + \left[(4\mathbf{i} + 5\mathbf{k}) \times (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})\right]$$

$$= \{-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k}\}\text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + 0 + (-1\mathbf{i} + 3\mathbf{j} + 0.8\mathbf{k})$$

$$= \{-1.00\mathbf{i} + 5.00\mathbf{j} + 0.800\mathbf{k}\}\text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (0.75\mathbf{j} + 8\mathbf{k}) + \mathbf{0} + \mathbf{0} + \mathbf{0} + (-28.8\mathbf{i} - 6.2\mathbf{j} + 24.3\mathbf{k})$$

$$= \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_C = \{-1.00\mathbf{i} + 5.00\mathbf{j} + 0.800\mathbf{k}\}$ m/s
$\mathbf{a}_C = \{-28.8\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}$ m/s^2

20–38.

Solve Example 20.5 by fixing x, y, z axes to rod BD so that $\boldsymbol{\Omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$. In this case the collar appears only to move radially outward along BD; hence $\boldsymbol{\Omega}_{xyz} = \mathbf{0}$.

SOLUTION

Relative to XYZ, let $x'\ y'\ z'$ be concident with XYZ and have $\Omega' = \omega_1$ and $\dot{\Omega}' = \dot{\omega}_1$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \{4\mathbf{i} + 5\mathbf{k}\}\text{ rad/s}$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = \left[\left(\dot{\omega}_1\right)_{x'y'z'} + \omega_1 \times \omega_1\right] + \left[\left(\omega_2\right)_{x'y'z'} + \omega_1 \times \omega_2\right]$$

$$= (1.5\mathbf{i} + \mathbf{0}) + \left[-6\mathbf{k} + (4\mathbf{i}) \times (5\mathbf{k})\right] = \{1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}\}\text{ rad/s}^2$$

$$\mathbf{r}_B = \{-0.5\mathbf{k}\}\text{ m}$$

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = \left(\dot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \mathbf{r}_B = \mathbf{0} + (4\mathbf{i}) \times (-0.5\mathbf{k}) = \{2\mathbf{j}\}\text{ m/s}$$

$$\mathbf{a}_B = \dot{\mathbf{r}}_B = \left[\left(\ddot{\mathbf{r}}_B\right)_{x'y'z'} + \omega_1 \times \left(\dot{\mathbf{r}}_{\mathbf{B}}\right)_{x'y'z'}\right] + \dot{\omega}_1 \times r_B + \omega_1 \times \dot{\mathbf{r}}_B$$

$$= \mathbf{0} + \mathbf{0} + \left[(1.5\mathbf{i}) \times (-0.5\mathbf{k})\right] + (4\mathbf{i} \times 2\mathbf{j}) = \{0.75\mathbf{j} + 8\mathbf{k}\}\text{ m/s}^2$$

Relative to $x'y'z'$, let xyz have

$$\Omega_{x'y'z'} = \mathbf{0}; \qquad \dot{\Omega}_{x'y'z'} = \mathbf{0};$$

$$\left(r_{C/B}\right)_{xyz} = \{0.2\mathbf{j}\}\text{ m}$$

$$(\mathbf{v}_{C/B})_{xyz} = \{3\mathbf{j}\}\text{ m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \{2\mathbf{j}\}\text{ m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= 2\mathbf{j} + \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 3\mathbf{j}$$

$$= \{-1\mathbf{i} + 5\mathbf{j} + 0.8\mathbf{k}\}\text{m/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \Omega \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

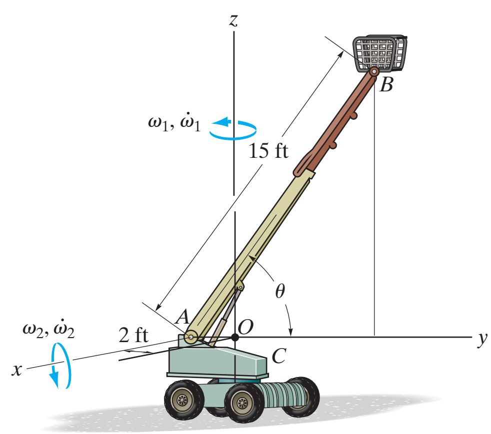
$$= (0.75\mathbf{j} + 8\mathbf{k}) + \left[(1.5\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}) \times (0.2\mathbf{j})\right] + (4\mathbf{i} + 5\mathbf{k}) \times \left[(4\mathbf{i} + 5\mathbf{k}) \times (0.2\mathbf{j})\right] + 2\left[(4\mathbf{i} + 5\mathbf{k}) \times (3\mathbf{j})\right] + 2\mathbf{j}$$

$$\mathbf{a}_C = \{-28.2\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}\text{ m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_C = \{-1\mathbf{i} + 5\mathbf{j} + 0.8\mathbf{k}\}$ m/s
$\mathbf{a}_C = \{-28.2\mathbf{i} - 5.45\mathbf{j} + 32.3\mathbf{k}\}$ m/s^2

20–39.

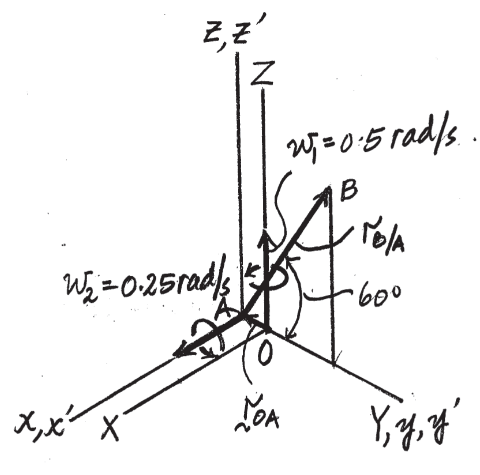
At the instant $\theta = 60°$, the telescopic boom AB of the construction lift is rotating with a constant angular velocity about the z axis of $\omega_1 = 0.5$ rad/s and about the pin at A with a constant angular speed of $\omega_2 = 0.25$ rad/s. Simultaneously, the boom is extending with a velocity of 1.5 ft/s, and it has an acceleration of 0.5 ft/s^2, both measured relative to the construction lift. Determine the velocity and acceleration of point B located at the end of the boom at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = \{0.5\mathbf{k}\} \text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = 0$$



(a)

Since point A rotates about a fixed axis (Z axis), its motion can determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (0.5\mathbf{k}) \times (-2\mathbf{j}) = \{1\mathbf{i}\} \text{ ft/s}$$

and

$$\begin{aligned} a_A &= \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA}) \\ &= 0 + (0.5\mathbf{k}) \times (0.5\mathbf{k}) \times (-2\mathbf{j}) \\ &= \{0.5\mathbf{j}\} \text{ ft/s}^2 \end{aligned}$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity of $\Omega' = \omega_2 = \{0.25\mathbf{i}\}$ rad/s, the direction of $\mathbf{r}_{B/A}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $\mathbf{r}_{B/A}$,

$$\begin{aligned} (\mathbf{v}_{B/A})_{xyz} &= (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times \mathbf{r}_{B/A}\right] \\ &= (1.5\cos 60°\mathbf{j} + 1.5\sin 60°\mathbf{k}) + 0.25\mathbf{i} \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k}) \\ &= \{-2.4976\mathbf{j} + 3.1740\mathbf{k}\} \text{ ft/s} \end{aligned}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega} = \dot{\omega}_2 = 0$. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$\begin{aligned} (\mathbf{a}_{B/A})_{xyz} &= (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times \mathbf{r}_{B/A} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz} \\ &= \left[(0.5\cos 60°\mathbf{j} + 0.5\sin 60°\mathbf{k}) + 0.25\mathbf{i} \times (1.5\cos 60°\mathbf{j} + 1.5\sin 60°\mathbf{k})\right] + 0.25\mathbf{i} \times (-2.4976\mathbf{j} + 3.1740\mathbf{k}) \\ &= \{-0.8683\mathbf{j} - 0.003886\mathbf{k}\} \text{ ft/s}^2 \end{aligned}$$

20–39. Continued

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (1\mathbf{i}) + (0.5\mathbf{k}) \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k}) + (-2.4976\mathbf{j} + 3.1740\mathbf{k})$$

$$= \{-2.75\mathbf{i} - 2.50\mathbf{j} + 3.17\mathbf{k}\}\ \text{m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= 0.5\mathbf{j} + 0 + 0.5\mathbf{k} \times \left[(0.5\mathbf{k}) \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k})\right]$$

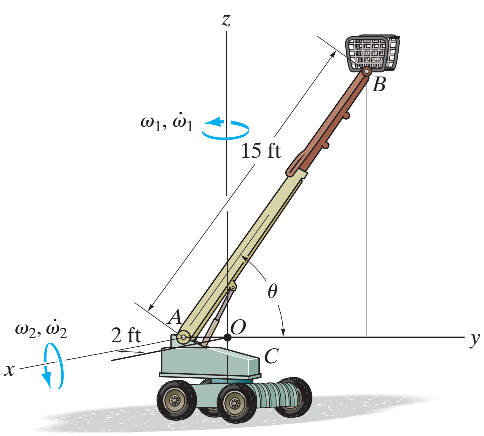
$$+ 2(0.5\mathbf{k}) \times (-2.4976\mathbf{j} + 3.1740\mathbf{k}) + (-0.8683\mathbf{j} - 0.003886\mathbf{k})$$

$$= \{2.50\mathbf{i} - 2.24\mathbf{j} - 0.00389\mathbf{k}\}\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_B = \{-2.75\mathbf{i} - 2.50\mathbf{j} + 3.17\mathbf{k}\}$ m/s
$\mathbf{a}_B = \{2.50\mathbf{i} - 2.24\mathbf{j} - 0.00389\mathbf{k}\}$ ft/s^2

***20–40.**

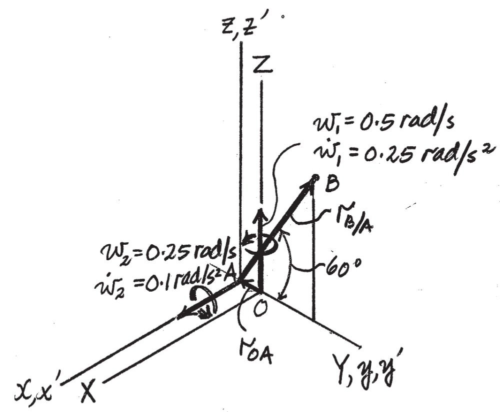
At the instant $\theta = 60°$, the construction lift is rotating about the z axis with an angular velocity of $\omega_1 = 0.5$ rad/s and an angular acceleration of $\dot{\omega}_1 = 0.25$ rad/s^2 while the telescopic boom AB rotates about the pin at A with an angular velocity of $\omega_2 = 0.25$ rad/s and angular acceleration of $\dot{\omega}_2 = 0.1$ rad/s^2. Simultaneously, the boom is extending with a velocity of 1.5 ft/s, and it has an acceleration of 0.5 ft/s^2, both measured relative to the frame. Determine the velocity and acceleration of point B located at the end of the boom at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = \{0.5\mathbf{k}\}\text{ rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \{0.25\mathbf{k}\}\text{ rad/s}^2$$



(a)

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (0.5\mathbf{k}) \times (-2\mathbf{j}) = \{1\mathbf{i}\}\text{ ft/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= (0.25\mathbf{k}) \times (-2\mathbf{j}) + (0.5\mathbf{k}) \times [0.5\mathbf{k} \times (-2\mathbf{j})]$$

$$= \{0.5\mathbf{i} + 0.5\mathbf{j}\}\text{ ft/s}^2$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity of $\Omega' = \omega_2 = [0.25\mathbf{i}]$ rad/s, the direction of $\mathbf{r}_{B/A}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $\mathbf{r}_{B/A}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times \mathbf{r}_{B/A}\right]$$

$$= (1.5\cos 60°\mathbf{j} + 1.5\sin 60°\mathbf{k}) + [0.25\mathbf{i} \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k})]$$

$$= \{-2.4976\mathbf{j} + 3.1740\mathbf{k}\}\text{ ft/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega} = \dot{\omega}_2 = [0.1\mathbf{i}]$ rad/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$(\mathbf{a}_{B/A})_{xyz} = (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\omega}_2 \times \mathbf{r}_{B/A}$$

$$+ \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= (0.5\cos 60°\mathbf{j} + 0.5\sin 60°\mathbf{k}) + (0.25\mathbf{i}) \times (1.5\cos 60°\mathbf{j} + 1.5\sin 60°\mathbf{k})$$

$$+ (0.1\mathbf{i}) \times (15\cos 60°\mathbf{j} + 15\sin 60°\mathbf{k}) + (0.25\mathbf{i}) \times (-2.4976\mathbf{j} + 3.1740\mathbf{k})$$

$$= \{-2.1673\mathbf{j} + 0.7461\mathbf{k}\}\text{ ft/s}^2$$

20–40. Continued

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= [1\mathbf{i}] + (0.5\mathbf{k}) \times (15\cos 60^\circ\mathbf{j} + 15\sin 60^\circ\mathbf{k}) + (-2.4976\mathbf{j} + 3.1740\mathbf{k})$$

$$= \{-2.75\mathbf{i} - 2.50\mathbf{j} + 3.17\mathbf{k}\}\ \text{ft/s}$$ **Ans.**

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

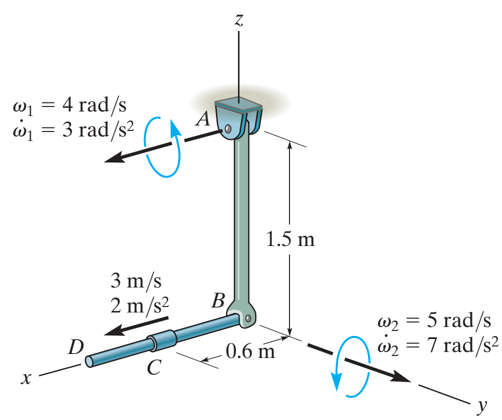
$$= (0.5\mathbf{i} + 0.5\mathbf{j}) + (0.25\mathbf{k}) \times (15\cos 60^\circ\mathbf{j} + 15\sin 60^\circ\mathbf{k}) + (0.5\mathbf{k}) \times [(0.5\mathbf{k}) \times (15\cos 60^\circ\mathbf{j} + 15\sin 60^\circ\mathbf{k})] + 2\,(0.5\mathbf{k}) \times (-2.4976\mathbf{j} + 3.1740\mathbf{k}) + (-2.1673\mathbf{j} + 0.7461\mathbf{k})$$

$$= \{1.12\mathbf{i} - 3.54\mathbf{j} + 0.746\mathbf{k}\}\ \text{ft/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_B = \{-2.75\mathbf{i} - 2.50\mathbf{j} + 3.17\mathbf{k}\}$ ft/s
$\mathbf{a}_B = \{1.12\mathbf{i} - 3.54\mathbf{j} + 0.746\mathbf{k}\}$ ft/s^2

20–41.

At the instant shown, the arm AB is rotating about the fixed pin A with an angular velocity $\omega_1 = 4$ rad/s and angular acceleration $\dot{\omega}_1 = 3$ rad/s². At this same instant, rod BD is rotating relative to rod AB with an angular velocity $\omega_2 = 5$ rad/s, which is increasing at $\dot{\omega}_2 = 7$ rad/s². Also, the collar C is moving along rod BD with a velocity of 3 m/s and an acceleration of 2 m/s², both measured relative to the rod. Determine the velocity and acceleration of the collar at this instant.



SOLUTION

$$\Omega = \{4\mathbf{i}\}\ \text{rad/s}$$

$$\dot{\Omega} = \{3\mathbf{i}\}\ \text{rad/s}^2$$

$$\mathbf{r}_B = \{-1.5\mathbf{k}\}\ \text{m}$$

$$\mathbf{v}_B = (\dot{\mathbf{r}}_B)_{xyz} + \Omega \times \mathbf{r}_B$$

$$= 0 + (4\mathbf{i}) \times (-1.5\mathbf{k})$$

$$= \{6\mathbf{j}\}\ \text{m/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times (\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$

$$= \mathbf{0} + \mathbf{0} + (3\mathbf{i}) \times (-1.5\mathbf{k}) + (4\mathbf{i}) \times (6\mathbf{j})$$

$$= \{4.5\mathbf{j} + 24\mathbf{k}\}\ \text{m/s}^2$$

$$\Omega_{C/B} = \{5\mathbf{j}\}\ \text{rad/s}$$

$$\dot{\Omega}_{C/B} = \{7\mathbf{j}\}\ \text{rad/s}^2$$

$$\mathbf{r}_{C/B} = \{0.6\mathbf{i}\}\ \text{m}$$

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} + \Omega_{C/B} \times \mathbf{r}_{C/B}$$

$$= (3\mathbf{i}) + (5\mathbf{j}) \times (0.6\mathbf{i})$$

$$= \{3\mathbf{i} - 3\mathbf{k}\}\ \text{m/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{xyz} + \Omega_{C/B} \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right] + \dot{\Omega}_{C/B} \times \mathbf{r}_{C/B} + \Omega_{C/B} \times \dot{\mathbf{r}}_{C/B}$$

$$= (2\mathbf{i}) + (5\mathbf{j}) \times (3\mathbf{i}) + (7\mathbf{j}) \times (0.6\mathbf{i}) + (5\mathbf{j}) \times (3\mathbf{i} - 3\mathbf{k})$$

$$= \{-13\mathbf{i} - 34.2\mathbf{k}\}\ \text{m/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= (6\mathbf{j}) + (4\mathbf{i}) \times (0.6\mathbf{i}) + (3\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{v}_C = \{3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}\ \text{m/s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

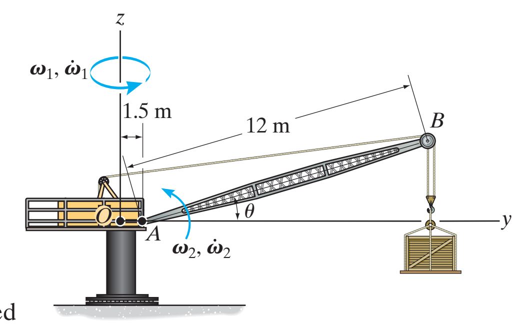
$$= (4.5\mathbf{j} + 24\mathbf{k}) + (3\mathbf{i}) \times (0.6\mathbf{i}) + (4\mathbf{i}) \times [(4\mathbf{i}) \times (0.6\mathbf{i})] + 2(4\mathbf{i}) \times (3\mathbf{i} - 3\mathbf{k}) + (-13\mathbf{i} - 34.2\mathbf{k})$$

$$\mathbf{a}_C = \{-13.0\mathbf{i} + 28.5\mathbf{j} - 10.2\mathbf{k}\}\ \text{m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_C = \{3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}$ m/s
$\mathbf{a}_C = \{-13.0\mathbf{i} + 28.5\mathbf{j} - 10.2\mathbf{k}\}$ m/s²

20–42.

At the instant $\theta = 30°$, the frame of the crane and the boom AB rotate with a constant angular velocity of $\omega_1 = 1.5$ rad/s and $\omega_2 = 0.5$ rad/s, respectively. Determine the velocity and acceleration of point B at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. The angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [1.5\mathbf{k}]\ \text{rad/s} \qquad \dot{\Omega} = \dot{\omega}_1 = \mathbf{0}$$

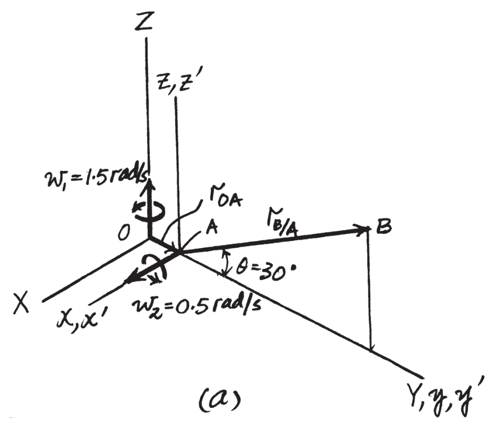
Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (1.5\mathbf{k}) \times (1.5\mathbf{j}) = [-2.25\mathbf{i}]\ \text{m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= \mathbf{0} + (1.5\mathbf{k}) \times [(1.5\mathbf{k}) \times (1.5\mathbf{j})]$$

$$= [-3.375\mathbf{j}]\ \text{m/s}^2$$



In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [0.5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (0.5\mathbf{i}) \times (12 \cos 30°\ \mathbf{j} + 12 \sin 30°\mathbf{k})$$

$$= [-3\mathbf{j} + 5.196\mathbf{k}]\ \text{m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = \mathbf{0}$. Taking the time derivative of $(\dot{\mathbf{r}}_{A/B})_{xyz}$,

$$(\mathbf{a}_{A/B})_{xyz} = (\ddot{\mathbf{r}}_{A/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{A/B})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{A/B})_{x'y'z'}\right] + \dot{\omega}_2 \times (\mathbf{r}_{A/B})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{A/B})_{xyz}$$

$$= [0 + 0] + 0 + (0.5\mathbf{i}) \times (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-2.598\mathbf{j} - 1.5\mathbf{k}]\ \text{m/s}^2$$

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (-2.25\mathbf{i}) + 1.5\mathbf{k} \times (12 \cos 30°\ \mathbf{j} + 12 \sin 30°\ \mathbf{k}) + (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}]\ \text{m/s} \qquad \textbf{Ans.}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{AB})_{xyz} + (\mathbf{a}_{AB})_{xyz}$$

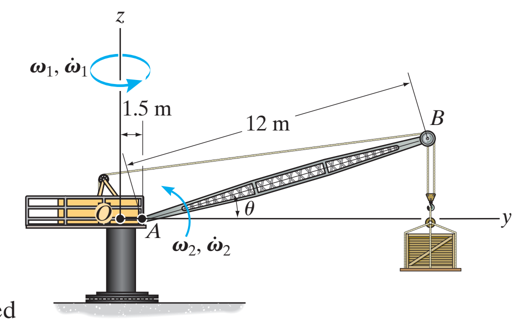
$$= (-3.375\mathbf{j}) + 0 + 1.5\mathbf{k} \times \left[(1.5\mathbf{k}) \times (12 \cos 30°\ \mathbf{j} + 12 \sin 30°\ \mathbf{k})\right] + 2(1.5\mathbf{k}) \times (-3\mathbf{j} + 5.196\mathbf{k}) + (-2.598\mathbf{j} - 1.5\mathbf{k})$$

$$= [9\mathbf{i} - 29.4\mathbf{j} - 1.5\mathbf{k}]\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_B = \{-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}\}$ m/s
$\mathbf{a}_B = \{9\mathbf{i} - 29.4\mathbf{j} - 1.5\mathbf{k}\}$ m/s^2

20–43.

At the instant $\theta = 30°$, the frame of the crane is rotating with an angular velocity of $\omega_1 = 1.5$ rad/s and angular acceleration of $\dot{\omega}_1 = 0.5$ rad/s^2, while the boom AB rotates with an angular velocity of $\omega_2 = 0.5$ rad/s and angular acceleration of $\dot{\omega}_2 = 0.25$ rad/s^2. Determine the velocity and acceleration of point B at this instant.



SOLUTION

The xyz rotating frame is set parallel to the fixed XYZ frame with its origin attached to point A, Fig. a. Thus, the angular velocity and angular acceleration of this frame with respect to the XYZ frame are

$$\Omega = \omega_1 = [1.5\mathbf{k}] \text{ rad/s} \qquad \dot{\Omega} = [0.5\mathbf{k}] \text{ rad/s}^2$$

Since point A rotates about a fixed axis (Z axis), its motion can be determined from

$$\mathbf{v}_A = \omega_1 \times \mathbf{r}_{OA} = (1.5\mathbf{k}) \times (1.5\mathbf{j}) = [-2.25\mathbf{i}] \text{ m/s}$$

$$\mathbf{a}_A = \dot{\omega}_1 \times \mathbf{r}_{OA} + \omega_1 \times (\omega_1 \times \mathbf{r}_{OA})$$

$$= (0.5\mathbf{k}) \times (1.5\mathbf{j}) + (1.5\mathbf{k}) \times \left[(1.5\mathbf{k}) \times (1.5\mathbf{j})\right]$$

$$= [-0.75\mathbf{i} - 3.375\mathbf{j}] \text{ m/s}^2$$

In order to determine the motion of point B relative to point A, it is necessary to establish a second $x'y'z'$ rotating frame that coincides with the xyz frame at the instant considered, Fig. a. If we set the $x'y'z'$ frame to have an angular velocity relative to the xyz frame of $\Omega' = \omega_2 = [0.5\mathbf{i}]$ rad/s, the direction of $(\mathbf{r}_{B/A})_{xyz}$ will remain unchanged with respect to the $x'y'z'$ frame. Taking the time derivative of $(\mathbf{r}_{B/A})_{xyz}$,

$$(\mathbf{v}_{B/A})_{xyz} = (\dot{\mathbf{r}}_{B/A})_{xyz} = \left[(\dot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\mathbf{r}_{B/A})_{xyz}\right]$$

$$= \mathbf{0} + (0.5\mathbf{i}) \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30° \,\mathbf{k})$$

$$= [-3\mathbf{j} + 5.196\mathbf{k}] \text{ m/s}$$

Since $\Omega' = \omega_2$ has a constant direction with respect to the xyz frame, then $\dot{\Omega}' = \dot{\omega}_2 = [0.25\mathbf{i}]$ m/s^2. Taking the time derivative of $(\dot{\mathbf{r}}_{B/A})_{xyz}$,

$$(\mathbf{a}_{B/A}) = (\ddot{\mathbf{r}}_{B/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{B/A})_{x'y'z'} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{x'y'z'}\right] + \dot{\Omega}_2 \times (\mathbf{r}_{B/A})_{xyz} + \omega_2 \times (\dot{\mathbf{r}}_{B/A})_{xyz}$$

$$= [0 + 0] + (0.25\mathbf{i}) \times (12 \cos 30° \,\mathbf{j} + 12 \sin 30°\mathbf{k}) + 0.5\mathbf{i} \times (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-4.098\mathbf{j} + 1.098\mathbf{k}] \text{ m/s}^2$$

20–43. Continued

Thus,

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$= (-2.25\mathbf{i}) + 1.5\mathbf{k} \times (12 \cos 30^\circ\, \mathbf{j} + 12 \sin 30^\circ \mathbf{k}) + (-3\mathbf{j} + 5.196\mathbf{k})$$

$$= [-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}] \text{ m/s} \qquad \textbf{Ans.}$$

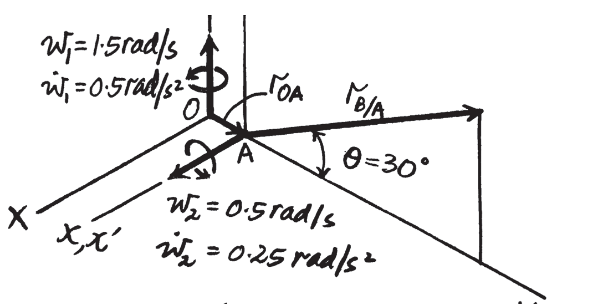
and

$$\mathbf{a}_B = \mathbf{a}_A = \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

$$= (-0.75\mathbf{i} - 3.375\mathbf{j}) + 0.5\mathbf{k} \times (12 \cos 30^\circ \mathbf{j} + 12 \sin 30^\circ\, \mathbf{k}) + (1.5\mathbf{k}) \times \left[(1.5\mathbf{k}) \times (12 \cos 30^\circ\, \mathbf{j} + 12 \sin 30^\circ\, \mathbf{k})\right]$$

$$+2(1.5\mathbf{k}) \times (-3\mathbf{j} + 5.196\mathbf{k}) + (-4.098\mathbf{j} + 1.098\mathbf{k})$$

$$= [3.05\mathbf{i} - 30.9\mathbf{j} + 1.10\mathbf{k}] \text{ m/s}^2 \qquad \textbf{Ans.}$$

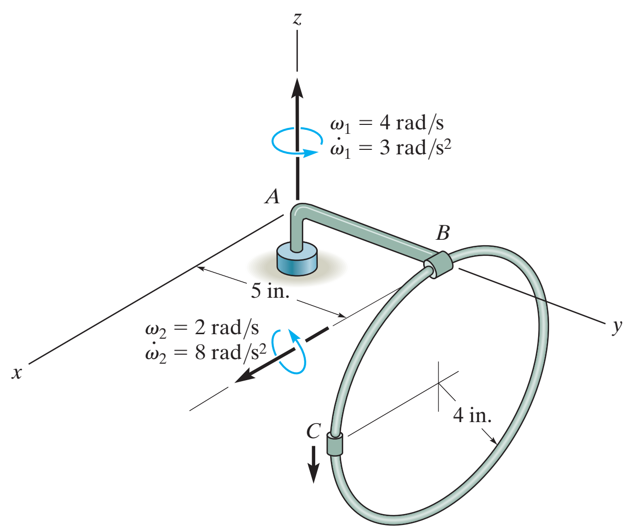


(a)

Ans:
$\mathbf{v}_B = \{-17.8\mathbf{i} - 3\mathbf{j} + 5.20\mathbf{k}\}$ m/s
$\mathbf{a}_B = \{3.05\mathbf{i} - 30.9\mathbf{j} + 1.10\mathbf{k}\}$ m/s^2

***20–44.**

At the instant shown, the rod AB is rotating about the z axis with an angular velocity $\omega_1 = 4$ rad/s and an angular acceleration $\dot{\omega}_1 = 3$ rad/s^2. At this same instant, the circular rod has an angular motion relative to the rod as shown. If the collar C is moving down around the circular rod with a speed of 3 in./s, which is increasing at 8 in./s^2, both measured relative to the rod, determine the collar's velocity and acceleration at this instant.



SOLUTION

$$\Omega = \{4\mathbf{k}\} \text{ rad/s}$$

$$\dot{\Omega} = \{3\mathbf{k}\} \text{ rad/s}^2$$

$$\mathbf{r}_B = \{5\mathbf{j}\} \text{ in.}$$

$$\mathbf{v}_B = (4\mathbf{k}) \times (5\mathbf{j}) = \{-20\mathbf{i}\} \text{ in./s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times (\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$

$$= (4\mathbf{k}) \times (-20\mathbf{i}) + (3\mathbf{k}) \times (5\mathbf{j})$$

$$= \{-15\mathbf{i} - 80\mathbf{j}\} \text{ in./s}^2$$

$$\Omega_{C/B} = \{2\mathbf{i}\} \text{ rad/s}$$

$$\dot{\Omega}_{C/B} = \{8\mathbf{i}\} \text{ rad/s}^2$$

$$\mathbf{r}_{C/B} = \{4\mathbf{i} - 4\mathbf{k}\} \text{ in.}$$

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} + \Omega_{C/B} \times \mathbf{r}_{C/B}$$

$$= (-3\mathbf{k}) + (2\mathbf{i}) \times (4\mathbf{i} - 4\mathbf{k})$$

$$= \{8\mathbf{j} - 3\mathbf{k}\} \text{ in./s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{xyz} + \Omega_{C/B} \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right] + \dot{\Omega}_{C/B} \times \mathbf{r}_{C/B} + \Omega_{C/B} \times \dot{\mathbf{r}}_{C/B}$$

$$= \left(-\frac{3^2}{4}\mathbf{j} - 8\mathbf{k}\right) + (2\mathbf{i}) \times (-3\mathbf{k}) + (8\mathbf{i}) \times (4\mathbf{i} - 4\mathbf{k})$$
$$+ (2\mathbf{i}) \times (8\mathbf{j} - 3\mathbf{k})$$

$$= \{-2.25\mathbf{i} + 44\mathbf{j} + 8\mathbf{k}\} \text{ in./s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \Omega \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= (-20\mathbf{i}) + (4\mathbf{k}) \times (4\mathbf{i} - 4\mathbf{k}) + (8\mathbf{j} - 3\mathbf{k})$$

$$\mathbf{v}_C = \{-20\mathbf{i} + 24\mathbf{j} - 3\mathbf{k}\} \text{ in./s}$$ **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{C/B} + \Omega \times (\Omega \times \mathbf{r}_{C/B}) + 2\Omega \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

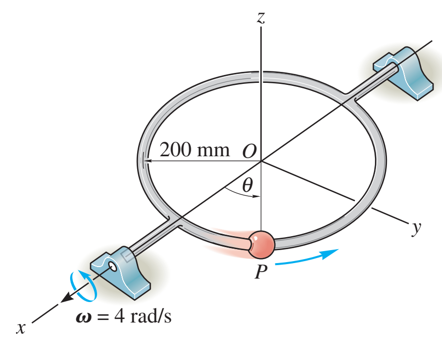
$$= (-15\mathbf{i} - 80\mathbf{j}) + (3\mathbf{k}) \times (4\mathbf{i} - 4\mathbf{k}) + (4\mathbf{k}) \times \left[(4\mathbf{k}) \times (4\mathbf{i} - 4\mathbf{k})\right]$$
$$+ 2(4\mathbf{k}) \times (8\mathbf{j} - 3\mathbf{k}) + (-2.25\mathbf{i} + 44\mathbf{j} + 8\mathbf{k})$$

$$\mathbf{a}_C = \{-145\mathbf{i} - 24\mathbf{j} + 8\mathbf{k}\} \text{ in./s}^2$$ **Ans.**

Ans:
$\mathbf{v}_C = \{-20\mathbf{i} + 24\mathbf{j} - 3\mathbf{k}\}$ in./s
$\mathbf{a}_C = \{-145\mathbf{i} - 24\mathbf{j} + 8\mathbf{k}\}$ in./s^2

20–45.

The particle P slides around the circular hoop with a constant angular velocity of $\dot{\theta} = 6$ rad/s, while the hoop rotates about the x axis at a constant rate of $\omega = 4$ rad/s. If at the instant shown the hoop is in the x–y plane and the angle $\theta = 45°$, determine the velocity and acceleration of the particle at this instant.



SOLUTION

Relative to XYZ, let xyz have

$\Omega = \omega = \{4\mathbf{i}\}$ rad/s, $\quad \dot{\Omega} = \dot{\omega} = \mathbf{0}$ (Ω does not change direction relative to XYZ.)

$\mathbf{r}_O = \mathbf{0}; \quad \mathbf{v}_O = \mathbf{0}; \quad \mathbf{a}_O = \mathbf{0}$

Relative to xyz, let coincident x', y', z', have

$\Omega_{xyz} = \{6\mathbf{k}\}$ rad/s, $\quad \dot{\Omega}_{xyz} = \mathbf{0}$ (Ω_{xyz} does not change direction relative to XYZ.)

$(\mathbf{r}_{P/O})_{xyz} = 0.2 \cos 45°\,\mathbf{i} + 0.2 \sin 45°\mathbf{j} = \{0.1414\mathbf{i} + 0.1414\mathbf{j}\}$ m

$(\mathbf{r}_{P/O})_{xyz}$ ($(\mathbf{r}_{P/O})_{xyz}$ changes direction relative to XYZ.)

$$(\mathbf{v}_{P/O})_{xyz} = \left(\dot{\mathbf{r}}_{P/O}\right)_{xyz} = \left(\dot{\mathbf{r}}_{P/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\mathbf{r}_{P/O}\right)_{xyz} = \mathbf{0} + (6\mathbf{k}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j})$$

$$= \{-0.8485\mathbf{i} + 0.8485\mathbf{j}\} \text{ m/s}$$

$$(\mathbf{a}_{P/O})_{xyz} = \left(\ddot{\mathbf{r}}_{P/O}\right)_{xyz} = \left[\left(\ddot{\mathbf{r}}_{P/O}\right)_{x'y'z'} + \Omega_{xyz} \times \left(\dot{\mathbf{r}}_{P/O}\right)_{x'y'z'}\right] + \Omega \times \left(\mathbf{r}_{P/O}\right)_{xyz} + \Omega \times \left(\dot{\mathbf{r}}_{P/O}\right)_{xyz}$$

$$= \left[\mathbf{0} + \mathbf{0}\right] + \mathbf{0} + (6\mathbf{k}) \times (-0.8485\mathbf{i} + 0.8485\mathbf{j}) = \{-5.0912\mathbf{i} - 5.0912\mathbf{j}\} \text{ m/s}^2$$

Thus,

$$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz} = \mathbf{0} + (4\mathbf{i}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j}) - 0.8485\mathbf{i} + 0.8485\mathbf{j}$$

$$= \{-0.849\mathbf{i} + 0.849\mathbf{j} + 0.566\mathbf{k}\} \text{ m/s}$$ **Ans.**

$$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times (\Omega \times \mathbf{r}_{P/O}) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$$

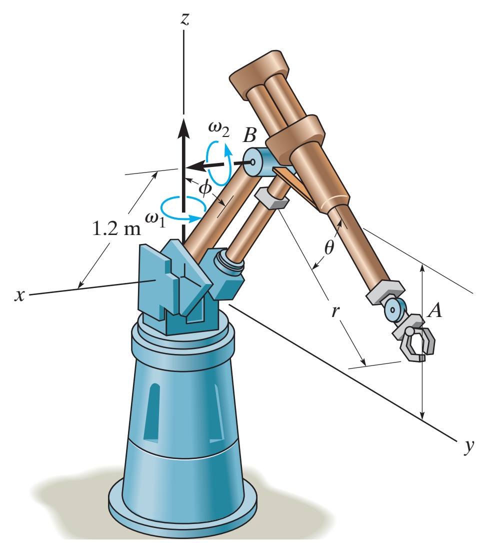
$$= \mathbf{0} + \mathbf{0} + (4\mathbf{i}) \times \left[(4\mathbf{i}) \times (0.1414\mathbf{i} + 0.1414\mathbf{j})\right] + 2(4\mathbf{i}) \times (-0.8485\mathbf{i} + 0.8485\mathbf{j}) - 5.0912\mathbf{i} - 5.0912\mathbf{j}$$

$$= \{-5.09\mathbf{i} - 7.35\mathbf{j} + 6.79\mathbf{k}\} \text{ m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_P = \{-0.849\mathbf{i} + 0.849\mathbf{j} + 0.566\mathbf{k}\}$ m/s
$\mathbf{a}_P = \{-5.09\mathbf{i} - 7.35\mathbf{j} + 6.79\mathbf{k}\}$ m/s^2

20–46.

At the instant shown, the industrial manipulator is rotating about the z axis at $\omega_1 = 5$ rad/s, and about joint B at $\omega_2 = 2$ rad/s. Determine the velocity and acceleration of the grip A at this instant, when $\phi = 30°$, $\theta = 45°$, and $r = 1.6$ m.



SOLUTION

$$\Omega = \{5\mathbf{k}\} \text{ rad/s} \qquad \dot{\Omega} = \mathbf{0}$$

$$\mathbf{r}_B = 1.2 \sin 30°\mathbf{j} + 1.2 \cos 30°\mathbf{k} = \{0.6\mathbf{j} + 1.0392\mathbf{k}\} \text{ m}$$

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \Omega \times \mathbf{r}_B = \mathbf{0} + (5\mathbf{k}) \times (0.6\mathbf{j} + 1.0392\mathbf{k}) = \{-3\mathbf{i}\} \text{ m/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times (\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$

$$= [\mathbf{0} + \mathbf{0}] + \mathbf{0} + [(5\mathbf{k}) \times (-3\mathbf{i})]$$

$$= \{-15\mathbf{j}\} \text{ m/s}^2$$

$$\Omega_{xyz} = \{2\mathbf{i}\} \text{ rad/s} \qquad \dot{\Omega}_{xyz} = \mathbf{0}$$

$$\mathbf{r}_{A/B} = 1.6 \cos 45°\mathbf{j} - 1.6 \sin 45°\mathbf{k} = \{1.1314\mathbf{j} - 1.1314\mathbf{k}\} \text{ m}$$

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{A/B}$$

$$= \mathbf{0} + (2\mathbf{i}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})$$

$$= \{2.2627\mathbf{j} + 2.2627\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \left[\dot{\Omega}_{xyz} \times \mathbf{r}_{A/B}\right] + \left[\Omega_{xyz} \times \dot{\mathbf{r}}_{A/B}\right]$$

$$(\mathbf{a}_{A/B})_{xyz} = [\mathbf{0} + \mathbf{0}] + \mathbf{0} + \left[(2\mathbf{i}) \times (2.2627\mathbf{j} + 2.2627\mathbf{k})\right]$$

$$= \{-4.5255\mathbf{j} + 4.5255\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$$

$$= (-3\mathbf{i}) + \left[(5\mathbf{k}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})\right] + (2.2627\mathbf{j} + 2.2627\mathbf{k})$$

$$= \{-8.66\mathbf{i} + 2.26\mathbf{j} + 2.26\mathbf{k}\} \text{ m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

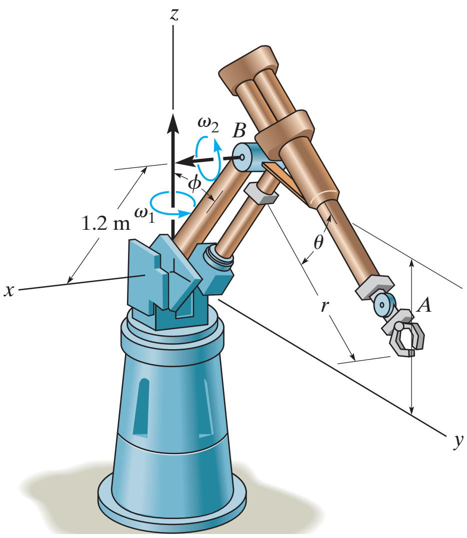
$$= (-15\mathbf{j}) + \mathbf{0} + (5\mathbf{k}) \times [(5\mathbf{k}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})] + [2(5\mathbf{k}) \times (2.2627\mathbf{j} + 2.2627\mathbf{k})] + (-4.5255\mathbf{j} + 4.5255\mathbf{k})$$

$$= \{-22.6\mathbf{i} - 47.8\mathbf{j} + 4.53\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-8.66\mathbf{i} + 2.26\mathbf{j} + 2.26\mathbf{k}\}$ m/s
$\mathbf{a}_A = \{-22.6\mathbf{i} - 47.8\mathbf{j} + 45.3\mathbf{k}\}$ m/s^2

20–47.

At the instant shown, the industrial manipulator is rotating about the z axis at $\omega_1 = 5$ rad/s, and $\dot{\omega}_1 = 2$ rad/s^2; and about joint B at $\omega_2 = 2$ rad/s and $\dot{\omega}_2 = 3$ rad/s^2. Determine the velocity and acceleration of the grip A at this instant, when $\phi = 30°$, $\theta = 45°$, and $r = 1.6$ m.



SOLUTION

$\Omega = \{5\mathbf{k}\}$ rad/s $\qquad \dot{\Omega} = \{2\mathbf{k}\}$ rad/s^2

$\mathbf{r}_B = 1.2 \sin 30°\mathbf{j} + 1.2 \cos 30°\mathbf{k} = \{0.6\mathbf{j} + 1.0392\mathbf{k}\}$ m

$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \Omega \times \mathbf{r}_B = \mathbf{0} + (5\mathbf{k}) \times (0.6\mathbf{j} + 1.0392\mathbf{k}) = \{-3\mathbf{i}\}$ m/s

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \Omega \times (\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\Omega} \times \mathbf{r}_B + \Omega \times \dot{\mathbf{r}}_B$$

$$= [\mathbf{0} + \mathbf{0}] + (2\mathbf{k}) \times (0.6\mathbf{j} + 1.0392\mathbf{k}) + [(5\mathbf{k}) \times (-3\mathbf{i})]$$

$$= \{-1.2\mathbf{i} - 15\mathbf{j}\} \text{ m/s}^2$$

$\Omega_{xyz} = \{2\mathbf{i}\}$ rad/s $\qquad \dot{\Omega}_{xyz} = \{3\mathbf{i}\}$ rad/s^2

$\mathbf{r}_{A/B} = 1.6 \cos 45°\mathbf{j} - 1.6 \sin 45°\mathbf{k} = \{1.1314\mathbf{j} - 1.1314\mathbf{k}\}$ m

$$(\mathbf{v}_{A/B})_{xyz} = \dot{\mathbf{r}}_{A/B} = (\dot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times \mathbf{r}_{A/B}$$

$$= \mathbf{0} + (2\mathbf{i}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})$$

$$= \{2.2627\mathbf{j} + 2.2627\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{A/B})_{xyz} = \ddot{\mathbf{r}}_{A/B} = \left[(\ddot{\mathbf{r}}_{A/B})_{xyz} + \Omega_{xyz} \times (\dot{\mathbf{r}}_{A/B})_{xyz}\right] + \left[\dot{\Omega}_{xyz} \times \mathbf{r}_{A/B}\right] + \left[\Omega_{xyz} \times \dot{\mathbf{r}}_{A/B}\right]$$

$$(\mathbf{a}_{A/B})_{xyz} = [\mathbf{0} + \mathbf{0}] + (3\mathbf{i}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k}) + [(2\mathbf{i}) \times (2.2627\mathbf{j} + 2.2627\mathbf{k})]$$

$$= \{-1.1313\mathbf{j} + 7.9197\mathbf{k}\} \text{ m/s}^2$$

$$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$$

$$= (-3\mathbf{i}) + [(5\mathbf{k}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})] + (2.2627\mathbf{j} + 2.2627\mathbf{k})$$

$$= \{-8.66\mathbf{i} + 2.26\mathbf{j} + 2.26\mathbf{k}\} \text{ m/s}$$ **Ans.**

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\Omega} \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

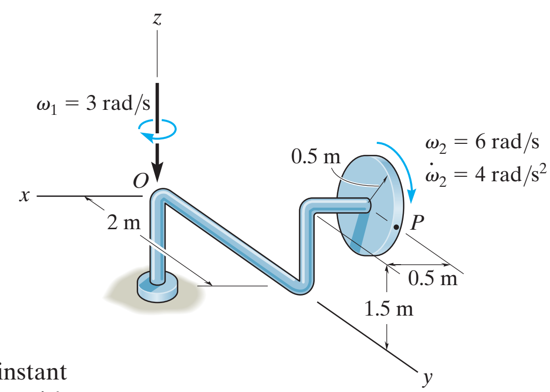
$$= (-1.2\mathbf{i} - 15\mathbf{j}) + (2\mathbf{k}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k}) + (5\mathbf{k}) \times [(5\mathbf{k}) \times (1.1314\mathbf{j} - 1.1314\mathbf{k})]$$
$$+ [2(5\mathbf{k}) \times (2.2627\mathbf{j} + 2.2627\mathbf{k})] + (-1.1313\mathbf{j} + 7.9197\mathbf{k})$$

$$= \{-26.1\mathbf{i} - 44.4\mathbf{j} + 7.92\mathbf{k}\} \text{ m/s}^2$$ **Ans.**

Ans:
$\mathbf{v}_A = \{-8.66\mathbf{i} + 2.26\mathbf{j} + 2.26\mathbf{k}\}$ m/s
$\mathbf{a}_A = \{-26.1\mathbf{i} - 44.4\mathbf{j} + 7.92\mathbf{k}\}$ m/s^2

***20–48.**

At the given instant, the rod is turning about the z axis with a constant angular velocity $\omega_1 = 3$ rad/s. At this same instant, the disk is spinning at $\omega_2 = 6$ rad/s when $\dot{\omega}_2 = 4$ rad/s^2, both measured *relative* to the rod. Determine the velocity and acceleration of point P on the disk at this instant.



SOLUTION

Motion of point A. Point A is located at the center of the disk. At the instant consider, the fixed XYZ frame and rotating $x'\,y'\,z'$ frame are set coincident with origin at Point O. The $x'\,y'\,z'$ frame is set rotate with constant angular velocity of $\mathbf{\Omega}' = \boldsymbol{\omega}_1 = \{-3\mathbf{k}\}$ rad/s of which the direction does not change relative to XYZ frame. Since $\mathbf{\Omega}'$ is constant $\dot{\mathbf{\Omega}}' = \mathbf{0}$. Here $\mathbf{r}_A = \{-0.5\mathbf{i} + 2\mathbf{j} + 1.5\mathbf{k}\}$ m and its direction changes relative to XYZ frame but does not change relative to $x'y'z'$ frame. Thus,

$$\mathbf{v}_A = (\dot{r}_A)_{x'y'z'} + \mathbf{\Omega}' \times \mathbf{r}_A = \mathbf{0} + (-3\mathbf{k}) \times (-0.5\mathbf{i} + 2\mathbf{j} + 1.5\mathbf{k}) = \{6\mathbf{i} + 1.5\mathbf{j}\} \text{ m/s}$$

$$\mathbf{a}_A = \left[(\ddot{r}_A)_{x'y'z'} + \mathbf{\Omega}' \times (\dot{r}_A)_{x'y'z'}\right] + \dot{\mathbf{\Omega}}' \times \mathbf{r}_A + \mathbf{\Omega}' \times \dot{\mathbf{r}}_A$$

$$= [\mathbf{0} + \mathbf{0}] + 0 + (-3\mathbf{k}) \times (6\mathbf{i} + 1.5\mathbf{j}) = \{4.5\mathbf{i} - 18\mathbf{j}\} \text{ m/s}^2$$

Motion of P with Respect to A. The xyz and $x''\,y''\,z''$ frame are set coincident with origin at A. Here $x''\,y''\,z''$ frame is set to rotate at $\mathbf{\Omega}'' = \boldsymbol{\omega}_z = \{-6\mathbf{i}\}$ rad/s of which its direction does not change relative to xyz frame. Also, $\Omega'' = \{-4\mathbf{i}\}$ rad/s^2. Here $(r_{P/A})_{xyz} = \{0.5\mathbf{j}\}$ m and its direction changes with respect to xyz frame but does not change relative to $x''\,y''\,z''$ frame.

$$(\mathbf{v}_{P/A})_{xyz} = (\dot{r}_{P/A})_{xyz} = (\dot{r}_{P/A})_{x''y''z''} + \mathbf{\Omega} \times (r_{P/A})_{xyz}$$

$$= \mathbf{0} + (-6\mathbf{i}) \times (0.5\mathbf{j}) = \{-3\mathbf{k}\} \text{ m/s}$$

$$(\mathbf{a}_{P/A})_{xyz} = (\ddot{r}_{P/A})_{xyz} = \left[(\ddot{r}_{P/A})_{x''y''z''} + \Omega'' \times (\dot{r}_{P/A})_{x''y''z''}\right] + \dot{\mathbf{\Omega}}'' \times (\mathbf{r}_{P/A})_{xyz} + \Omega'' \times (\dot{r}_{P/A})_{xyz}$$

$$= [0 + \mathbf{0}] + (-4\mathbf{i}) \times (0.5\mathbf{j}) + (-6\mathbf{i}) \times (-3\mathbf{k})$$

$$= \{-18\mathbf{j} - 2\mathbf{k}\} \text{ m/s}^2$$

Motion of P. Here, $\mathbf{\Omega} = \boldsymbol{\omega}_1 = \{-3\mathbf{k}\}$ rad/s and $\dot{\mathbf{\Omega}} = \mathbf{0}$.

$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{\Omega} \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$

$$= (6\mathbf{i} + 1.5\mathbf{j}) + (-3\mathbf{k}) \times (0.5\mathbf{j}) + (-3\mathbf{k})$$

$$= \{7.50\mathbf{i} + 1.50\mathbf{j} - 3.00\mathbf{k}\} \text{m/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_P = \mathbf{a}_A + \dot{\mathbf{\Omega}} \times \mathbf{r}_{P/A} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{P/A}) + 2\mathbf{\Omega} \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$

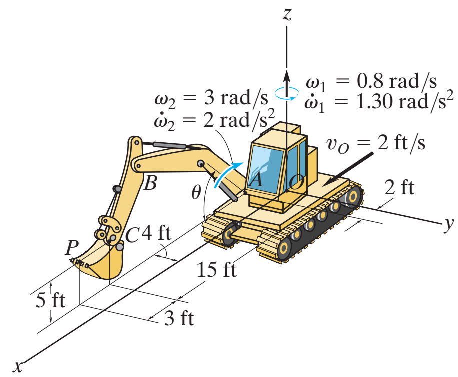
$$= (4.5\mathbf{i} - 18\mathbf{j}) + 0 + (-3\mathbf{k}) \times (-3\mathbf{k} \times 0.5\mathbf{j}) + 2(-3\mathbf{k}) \times (-3\mathbf{k}) + (-18\mathbf{j} - 2\mathbf{k})$$

$$= \{4.50\mathbf{i} - 40.5\mathbf{j} - 2.00\mathbf{k}\} \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_P = \{7.50\mathbf{i} + 1.50\mathbf{j} - 3.00\mathbf{k}\}$ m/s
$\mathbf{a}_P = \{4.50\mathbf{i} - 40.5\mathbf{j} - 2.00\mathbf{k}\}$ m/s^2

20–49.

At the instant shown, the backhoe is traveling forward at a constant speed $v_O = 2$ ft/s, and the boom ABC is rotating about the z axis with an angular velocity $\omega_1 = 0.8$ rad/s and an angular acceleration $\dot{\omega}_1 = 1.30$ rad/s^2. At this same instant the boom is rotating with $\omega_2 = 3$ rad/s when $\dot{\omega}_2 = 2$ rad/s^2, both measured relative to the frame. Determine the velocity and acceleration of point P on the bucket at this instant.



SOLUTION

$\boldsymbol{\Omega} = \{0.8\mathbf{k}\}$ rad/s

$\dot{\boldsymbol{\Omega}} = \{1.3\mathbf{k}\}$ rad/s^2

$\mathbf{r}_A = \{2\mathbf{i} - 4\mathbf{j}\}$ ft

$$\mathbf{v}_A = (\dot{\mathbf{r}}_A)_{xyz} + \boldsymbol{\Omega} \times \mathbf{r}_A$$
$$= (2\mathbf{i}) + (0.8\mathbf{k}) \times (2\mathbf{i} - 4\mathbf{j})$$
$$= \{5.20\mathbf{i} + 1.60\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{a}_A = \left[(\ddot{\mathbf{r}}_A)_{xyz} + \boldsymbol{\Omega} \times (\dot{\mathbf{r}}_A)_{xyz}\right] + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_A + \boldsymbol{\Omega} \times \dot{\mathbf{r}}_A$$
$$= \mathbf{0} + (0.8\mathbf{k}) \times (2\mathbf{i}) + (1.3\mathbf{k}) \times (2\mathbf{i} - 4\mathbf{j}) + (0.8\mathbf{k}) \times (5.20\mathbf{i} + 1.60\mathbf{j})$$
$$= \{3.92\mathbf{i} + 8.36\mathbf{j}\} \text{ ft/s}^2$$

$\boldsymbol{\Omega}_{P/A} = \{-3\mathbf{j}\}$ rad/s

$\dot{\boldsymbol{\Omega}}_{P/A} = \{-2\mathbf{j}\}$ rad/s^2

$\mathbf{r}_{P/A} = \{16\mathbf{i} + 5\mathbf{k}\}$ ft

$$(\mathbf{v}_{P/A})_{xyz} = (\dot{\mathbf{r}}_{P/A})_{xyz} + \boldsymbol{\Omega}_{P/A} \times \mathbf{r}_{P/A}$$
$$= \mathbf{0} + (-3\mathbf{j}) \times (16\mathbf{i} + 5\mathbf{k})$$
$$= \{-15\mathbf{i} + 48\mathbf{k}\} \text{ ft/s}$$

$$(\mathbf{a}_{P/A})_{xyz} = \left[(\ddot{\mathbf{r}}_{P/A})_{xyz} + \boldsymbol{\Omega}_{P/A} \times (\dot{\mathbf{r}}_{P/A})_{xyz}\right] + \dot{\boldsymbol{\Omega}}_{P/A} \times \mathbf{r}_{P/A} + \boldsymbol{\Omega}_{P/A} \times \dot{\mathbf{r}}_{P/A}$$
$$= \mathbf{0} + \mathbf{0} + (-2\mathbf{j}) \times (16\mathbf{i} + 5\mathbf{k}) + (-3\mathbf{j}) \times (-15\mathbf{i} + 48\mathbf{k})$$
$$= \{-154\mathbf{i} - 13\mathbf{k}\} \text{ ft/s}^2$$

$$\mathbf{v}_P = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{P/A} + (\mathbf{v}_{P/A})_{xyz}$$
$$= (5.2\mathbf{i} + 1.6\mathbf{j}) + (0.8\mathbf{k}) \times (16\mathbf{i} + 5\mathbf{k}) + (-15\mathbf{i} + 48\mathbf{k})$$

$\mathbf{v}_P = \{-9.80\mathbf{i} + 14.4\mathbf{j} + 48.0\mathbf{k}\}$ ft/s **Ans.**

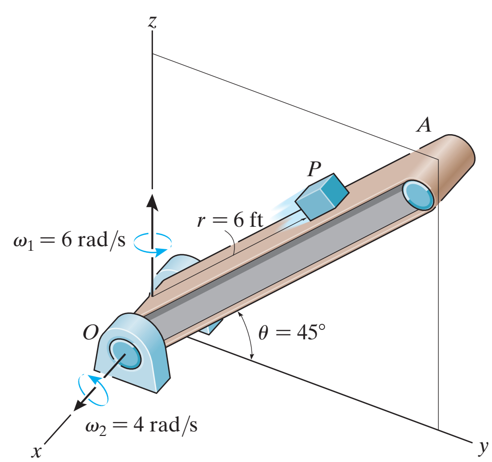
$$\mathbf{a}_P = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{P/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{P/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{P/A})_{xyz} + (\mathbf{a}_{P/A})_{xyz}$$
$$= (3.92\mathbf{i} + 8.36\mathbf{j}) + (1.3\mathbf{k}) \times (16\mathbf{i} + 5\mathbf{k}) + (0.8\mathbf{k}) \times [(0.8\mathbf{k}) \times (16\mathbf{i} + 5\mathbf{k})]$$
$$+ 2(0.8\mathbf{k}) \times (-15\mathbf{i} + 48\mathbf{k}) + (-154\mathbf{i} - 13\mathbf{j})$$

$\mathbf{a}_P = \{-160\mathbf{i} + 5.16\mathbf{j} - 13\mathbf{k}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_P = \{-9.80\mathbf{i} + 14.4\mathbf{j} + 48.0\mathbf{k}\}$ ft/s
$\mathbf{a}_P = \{-160\mathbf{i} + 5.16\mathbf{j} - 13\mathbf{k}\}$ ft/s^2

20–50.

At the instant shown, the arm OA of the conveyor belt is rotating about the z axis with a constant angular velocity $\omega_1 = 6$ rad/s, while at the same instant the arm is rotating upward at a constant rate $\omega_2 = 4$ rad/s. If the conveyor is running at a constant rate $\dot{r} = 5$ ft/s, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



SOLUTION

$\Omega = \omega_1 = \{6\mathbf{k}\}$ rad/s

$\dot{\Omega} = \mathbf{0}$

$\mathbf{r}_O = \mathbf{v}_O = \mathbf{a}_O = \mathbf{0}$

$\Omega_{P/O} = \{4\mathbf{i}\}$ rad/s

$\dot{\Omega}_{P/O} = \mathbf{0}$

$\mathbf{r}_{P/O} = \{4.243\mathbf{j} + 4.243\mathbf{k}\}$ ft

$$(\mathbf{v}_{P/O})_{xyz} = (\dot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times \mathbf{r}_{P/O}$$
$$= (5\cos 45°\mathbf{j} + 5\sin 45°\mathbf{k}) + (4\mathbf{i}) \times (4.243\mathbf{j} + 4.243\mathbf{k})$$
$$= \{-13.44\mathbf{j} + 20.51\mathbf{k}\} \text{ ft/s}$$

$$(\mathbf{a}_{P/O}) = (\ddot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times (\dot{\mathbf{r}}_{P/O})_{xyz} + \dot{\Omega}_{P/O} \times \mathbf{r}_{P/O} + \Omega_{P/O} \times \dot{\mathbf{r}}_{P/O}$$
$$= \mathbf{0} + (4\mathbf{i}) \times (3.536\mathbf{j} + 3.536\mathbf{k}) + \mathbf{0} + (4\mathbf{i}) \times (-13.44\mathbf{j} + 20.51\mathbf{k})$$
$$= \{-96.18\mathbf{j} - 39.60\mathbf{k}\} \text{ ft/s}^2$$

$$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{r}_{P/O})_{xyz}$$
$$= \mathbf{0} + (6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k}) + (-13.44\mathbf{j} + 20.51\mathbf{k})$$

$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s **Ans.**

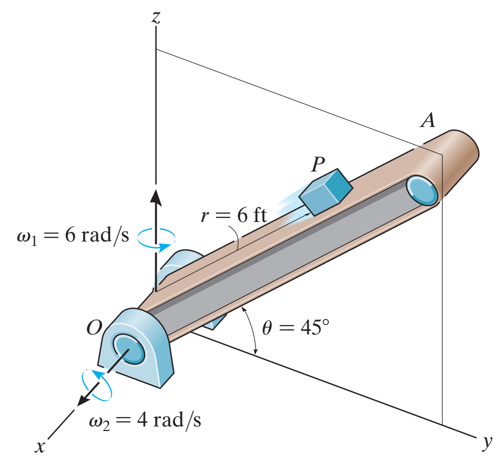
$$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times (\Omega \times \mathbf{r}_{P/O}) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$$
$$= \mathbf{0} + \mathbf{0} + (6\mathbf{k}) \times [(6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k})] + 2(6\mathbf{k}) \times (-13.44\mathbf{j} + 20.51\mathbf{k}) + (-96.18\mathbf{j} - 39.60\mathbf{k})$$

$\mathbf{a}_P = \{161\mathbf{i} - 249\mathbf{j} - 39.6\mathbf{k}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s
$\mathbf{a}_P = \{161\mathbf{i} - 249\mathbf{j} - 39.6\mathbf{k}\}$ ft/s^2

20–51.

At the instant shown, the arm OA of the conveyor belt is rotating about the z axis with a constant angular velocity $\omega_1 = 6$ rad/s, while at the same instant the arm is rotating upward at a constant rate $\omega_2 = 4$ rad/s. If the conveyor is running at a rate $\dot{r} = 5$ ft/s, which is increasing at $\ddot{r} = 8$ ft/s^2, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



SOLUTION

$\Omega = \omega_1 = \{6\mathbf{k}\}$ rad/s

$\dot{\Omega} = \mathbf{0}$

$\mathbf{r}_O = \mathbf{v}_O = \mathbf{a}_O = \mathbf{0}$

$\Omega_{P/O} = \{4\mathbf{i}\}$ rad/s

$\dot{\Omega}_{P/O} = \mathbf{0}$

$\mathbf{r}_{P/O} = \{4.243\mathbf{j} + 4.243\mathbf{k}\}$ ft

$$(\mathbf{v}_{P/O})_{xyz} = (\dot{\mathbf{r}}_{P/O})_{xyz} + \Omega_{P/O} \times \mathbf{r}_{P/O}$$

$$= (5\cos 45°\mathbf{j} + 5\sin 45°\mathbf{k}) + (4\mathbf{i}) \times (4.243\mathbf{j} + 4.243\mathbf{k})$$

$$= \{-13.44\mathbf{j} + 20.51\mathbf{k}\} \text{ ft/s}$$

$$(\mathbf{a}_{P/O})_{xyz} = 8\cos 45\mathbf{j} + 8\sin 45°\mathbf{k} - 96.18\mathbf{j} - 39.60\mathbf{k}$$

$$= \{-90.52\mathbf{j} - 33.945\mathbf{k}\} \text{ ft/s}^2$$

$$\mathbf{v}_P = \mathbf{v}_O + \Omega \times \mathbf{r}_{P/O} + (\mathbf{v}_{P/O})_{xyz}$$

$$= \mathbf{0} + (6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k}) + (-13.44\mathbf{j} + 20.51\mathbf{k})$$

$$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_P = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{P/O} + \Omega \times (\Omega \times \mathbf{r}_{P/O}) + 2\Omega \times (\mathbf{v}_{P/O})_{xyz} + (\mathbf{a}_{P/O})_{xyz}$$

$$= \mathbf{0} + \mathbf{0} + (6\mathbf{k}) \times [(6\mathbf{k}) \times (4.243\mathbf{j} + 4.243\mathbf{k})] + 2(6\mathbf{k}) \times (-13.44\mathbf{j} + 20.51\mathbf{k}) + (-90.52\mathbf{j} - 33.945\mathbf{k})$$

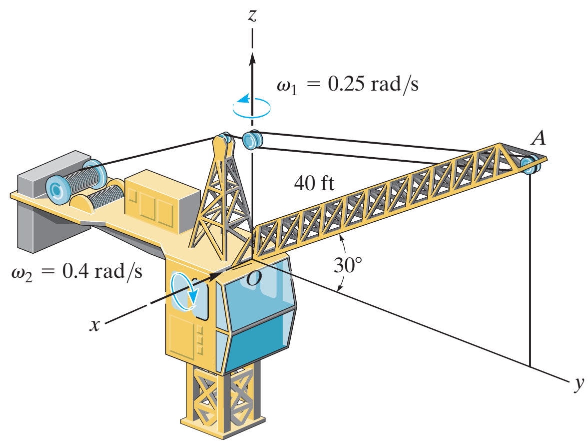
$$= -152.75\mathbf{j} + 161.23\mathbf{i} - 90.52\mathbf{j} - 33.945\mathbf{k}$$

$$\mathbf{a}_P = \{161\mathbf{i} - 243\mathbf{j} - 33.9\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_P = \{-25.5\mathbf{i} - 13.4\mathbf{j} + 20.5\mathbf{k}\}$ ft/s
$\mathbf{a}_P = \{161\mathbf{i} - 243\mathbf{j} - 33.9\mathbf{k}\}$ ft/s^2

***20–52.**

The crane is rotating about the z axis with a constant rate $\omega_1 = 0.25$ rad/s, while the boom OA is rotating downward with a constant rate $\omega_2 = 0.4$ rad/s. Compute the velocity and acceleration of point A located at the top of the boom at the instant shown.



SOLUTION

$\mathbf{\Omega} = \{0.25\mathbf{k}\}$ rad/s

$\dot{\mathbf{\Omega}} = 0$

$\mathbf{r}_O = 0$

$\mathbf{v}_O = 0$

$\mathbf{a}_O = 0$

$\mathbf{\Omega}_{A/O} = \{-0.4\mathbf{i}\}$ rad/s

$\dot{\mathbf{\Omega}}_{A/O} = 0$

$$\mathbf{r}_{A/O} = 4\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k}$$
$$= 34.64\mathbf{j} + 20\mathbf{k}$$

$$(\mathbf{v}_{A/O})_{xyz} = (\dot{\mathbf{r}}_{A/O})_{xyz} + \mathbf{\Omega}_{A/O} \times \mathbf{r}_{A/O}$$
$$= \mathbf{0} + (-0.4\mathbf{i}) \times (34.64\mathbf{j} + 20\mathbf{k})$$
$$= 8\mathbf{j} - 13.856\mathbf{k}$$

$$(\mathbf{a}_{A/O})_{xyz} = \left[(\ddot{\mathbf{r}}_{A/O})_{xyz} + \mathbf{\Omega}_{A/O} \times (\dot{\mathbf{r}}_{A/O})_{xyz}\right] + \dot{\mathbf{\Omega}}_{A/O} \times \mathbf{r}_{A/O} + \mathbf{\Omega}_{A/O} \times \dot{\mathbf{r}}_{A/O}$$
$$= 0 + 0 + 0 + (-4\mathbf{i}) \times (8\mathbf{j} - 13.86\mathbf{k})$$
$$= -5.542\mathbf{j} - 3.2\mathbf{k}$$

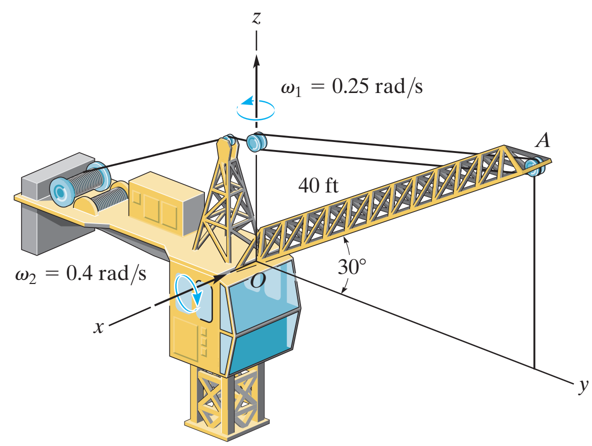
$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{\Omega} \times \mathbf{r}_{A/O} + (\mathbf{v}_{A/O})_{xyz}$$
$$= 0 + 0.25\mathbf{k} \times (34.64\mathbf{j} + 20\mathbf{k}) + (8\mathbf{j} - 13.856\mathbf{k})$$
$$= \{-8.66\mathbf{i} + 8\mathbf{j} - 13.9\mathbf{k}\} \text{ ft/s} \qquad \textbf{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_O + \dot{\mathbf{\Omega}} \times \mathbf{r}_{A/O} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{A/O}) + 2\mathbf{\Omega} \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$$
$$= 0 + 0 + (0.25\mathbf{k}) \times (0.25\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$$
$$+ 2(0.25\mathbf{k}) \times (8\mathbf{j} - 13.856\mathbf{k}) - 5.542\mathbf{j} - 3.2\mathbf{k}$$
$$\mathbf{a}_A = \{-4\mathbf{i} - 7.71\mathbf{j} - 3.20\mathbf{k}\} \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-8.66\mathbf{i} + 8\mathbf{j} - 13.9\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-4\mathbf{i} - 7.71\mathbf{j} - 3.20\mathbf{k}\}$ ft/s^2

20–53.

Solve Prob. 20–52 if the angular motions are increasing at $\dot{\omega}_1 = 0.4\ \text{rad/s}^2$ and $\dot{\omega}_2 = 0.8\ \text{rad/s}^2$ at the instant shown.



SOLUTION

$\mathbf{\Omega} = \{0.25\mathbf{k}\}$ rad/s

$\dot{\mathbf{\Omega}} = \{0.4\mathbf{k}\}$ rad/s^2

$\mathbf{r}_O = 0$

$\mathbf{v}_O = 0$

$\mathbf{a}_O = 0$

$\mathbf{\Omega}_{A/O} = \{-0.4\mathbf{i}\}$ rad/s

$\dot{\mathbf{\Omega}}_{A/O} = \{-0.8\mathbf{i}\}$ rad/s^2

$$\mathbf{r}_{A/O} = 4\cos 30°\mathbf{j} + 40\sin 30°\mathbf{k}$$
$$= 34.64\mathbf{j} + 20\mathbf{k}$$

$$(\mathbf{v}_{A/O})_{xyz} = (\dot{\mathbf{r}}_{A/O})_{xyz} + \Omega_{A/O} \times \mathbf{r}_{A/O}$$
$$= 0 + (-0.4\mathbf{i}) \times (34.64\mathbf{j} + 20\mathbf{k})$$
$$= 8\mathbf{j} - 13.856\mathbf{k}$$

$$(\mathbf{a}_{A/O})_{xyz} = \left[(\ddot{\mathbf{r}}_{A/O})_{xyz} + \mathbf{\Omega}_{A/O} \times (\dot{\mathbf{r}}_{A/O})_{xyz}\right] + \dot{\mathbf{\Omega}}_{A/O} \times \mathbf{r}_{A/O} + \mathbf{\Omega}_{A/O} \times \dot{\mathbf{r}}_{A/O}$$
$$= 0 + 0 + (-0.8\mathbf{i}) \times (34.64\mathbf{j} + 20\mathbf{k}) + (-4\mathbf{i}) \times (8\mathbf{j} - 13.86\mathbf{k})$$
$$= 10.457\mathbf{j} - 30.913\mathbf{k}$$

$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{\Omega} \times \mathbf{r}_{A/O} + (\mathbf{v}_{A/O})_{xyz}$$
$$= 0 + 0.25\mathbf{k} \times (34.64\mathbf{j} + 20\mathbf{k}) + (8\mathbf{j} - 13.856\mathbf{k})$$
$$= \{-8.66\mathbf{i} + 8\mathbf{j} - 13.9\mathbf{k}\}\ \text{ft/s} \qquad \textbf{Ans.}$$

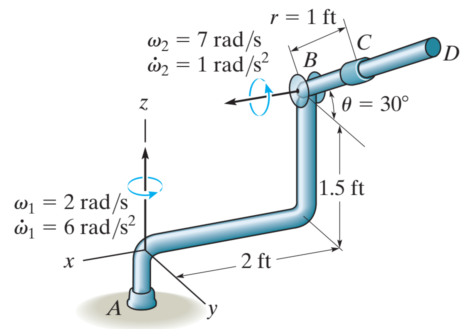
$$\mathbf{a}_A = \mathbf{a}_O + \dot{\mathbf{\Omega}} \times \mathbf{r}_{A/O} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{A/O}) + 2\mathbf{\Omega} \times (\mathbf{v}_{A/O})_{xyz} + (\mathbf{a}_{A/O})_{xyz}$$
$$= 0 + (0.4\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})$$
$$+ (0.25\mathbf{k}) \times [(0.25\mathbf{k}) \times (34.64\mathbf{j} + 20\mathbf{k})] + 2(0.25\mathbf{k}) \times (8\mathbf{j} - 13.856\mathbf{k}) + 10.457\mathbf{j} - 30.913\mathbf{k}$$

$$\mathbf{a}_A = \{-17.9\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}\ \text{ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\mathbf{v}_A = \{-8.66\mathbf{i} + 8\mathbf{j} - 13.9\mathbf{k}\}$ ft/s
$\mathbf{a}_A = \{-17.9\mathbf{i} + 8.29\mathbf{j} - 30.9\mathbf{k}\}$ ft/s^2

20–54.

At the instant shown, the arm AB is rotating about the fixed bearing with an angular velocity $\omega_1 = 2$ rad/s and angular acceleration $\dot{\omega}_1 = 6$ rad/s^2. At the same instant, rod BD is rotating relative to rod AB at $\omega_2 = 7$ rad/s, which is increasing at $\dot{\omega}_2 = 1$ rad/s^2. Also, the collar C is moving along rod BD with a velocity $\dot{r} = 2$ ft/s and a *deceleration* $\ddot{r} = -0.5$ ft/s^2, both measured relative to the rod. Determine the velocity and acceleration of the collar at this instant.



SOLUTION

$\mathbf{\Omega} = \{2\mathbf{k}\}$ rad/s

$\dot{\mathbf{\Omega}} = \{6\mathbf{k}\}$ rad/s^2

$\mathbf{r}_B = \{-2\mathbf{i} + 1.5\mathbf{k}\}$ ft

$$\mathbf{v}_B = \dot{\mathbf{r}}_B = (\dot{\mathbf{r}}_B)_{xyz} + \mathbf{\Omega} \times \mathbf{r}_B$$

$$= (2\mathbf{k}) \times (-2\mathbf{i} + 1.5\mathbf{k})$$

$$= \{-4\mathbf{j}\} \text{ ft/s}$$

$$\mathbf{a}_B = \ddot{\mathbf{r}}_B = \left[(\ddot{\mathbf{r}}_B)_{xyz} + \mathbf{\Omega} \times (\dot{\mathbf{r}}_B)_{xyz}\right] + \dot{\mathbf{\Omega}} \times \mathbf{r}_B + \mathbf{\Omega} \times \dot{\mathbf{r}}_B$$

$$= (6\mathbf{k}) \times (-2\mathbf{i} + 1.5\mathbf{k}) + (2\mathbf{k}) \times (-4\mathbf{j})$$

$$= \{8\mathbf{i} - 12\mathbf{j}\} \text{ ft/s}^2$$

$\mathbf{\Omega}_{C/B} = \{7\mathbf{i}\}$ rad/s

$\dot{\mathbf{\Omega}}_{C/B} = \{1\mathbf{i}\}$ rad/s^2

$\mathbf{r}_{C/B} = 1 \cos 30° \mathbf{j} + 1 \sin 30° \mathbf{k} = \{0.866\mathbf{j} + 0.5\mathbf{k}\}$ ft

$$(\mathbf{v}_{C/B})_{xyz} = (\dot{\mathbf{r}}_{C/B})_{xyz} + \mathbf{\Omega}_{C/B} \times \mathbf{r}_{C/B}$$

$$= (2 \cos 30° \mathbf{j} + 2 \sin 30° \mathbf{k}) + (7\mathbf{i}) \times (0.866\mathbf{j} + 0.5\mathbf{k})$$

$$= \{-1.768\mathbf{j} + 7.062\mathbf{k}\} \text{ ft/s}$$

$$(\mathbf{a}_{C/B})_{xyz} = \left[(\ddot{\mathbf{r}}_{C/B})_{xyz} + \mathbf{\Omega}_{C/B} \times (\dot{\mathbf{r}}_{C/B})_{xyz}\right] + \dot{\mathbf{\Omega}}_{C/B} \times \mathbf{r}_{C/B} + \mathbf{\Omega}_{C/B} \times \dot{\mathbf{r}}_{C/B}$$

$$= (-0.5 \cos 30° \mathbf{j} - 0.5 \sin 30° \mathbf{k}) + (7\mathbf{i}) \times (1.732\mathbf{j} + 1\mathbf{k}) + (1\mathbf{i}) \times (0.866\mathbf{j} + 0.5\mathbf{k}) + (7\mathbf{i}) \times (-1.768\mathbf{j} + 7.06\mathbf{k})$$

$$= \{-57.37\mathbf{j} + 0.3640\mathbf{k}\} \text{ ft/s}^2$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{\Omega} \times \mathbf{r}_{C/B} + (\mathbf{v}_{C/B})_{xyz}$$

$$= (-4\mathbf{j}) + (2\mathbf{k}) \times (0.866\mathbf{j} + 0.5\mathbf{k}) + (-1.768\mathbf{j} + 7.06\mathbf{k})$$

$\mathbf{v}_C = \{-1.73\mathbf{i} - 5.77\mathbf{j} + 7.06\mathbf{k}\}$ ft/s **Ans.**

$$\mathbf{a}_C = \mathbf{a}_B + \dot{\mathbf{\Omega}} \times \mathbf{r}_{C/B} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{C/B}) + 2\mathbf{\Omega} \times (\mathbf{v}_{C/B})_{xyz} + (\mathbf{a}_{C/B})_{xyz}$$

$$= (8\mathbf{i} - 12\mathbf{j}) + (6\mathbf{k}) \times (0.866\mathbf{j} + 0.5\mathbf{k}) + (2\mathbf{k}) \times \left[(2\mathbf{k}) \times (0.866\mathbf{j} + 0.5\mathbf{k})\right] + 2(2\mathbf{k}) \times (-1.768\mathbf{j} + 7.062\mathbf{k}) + (-57.37\mathbf{j} + 0.364\mathbf{k})$$

$\mathbf{a}_C = \{9.88\mathbf{i} - 72.8\mathbf{j} + 0.365\mathbf{k}\}$ ft/s^2 **Ans.**

Ans:
$\mathbf{v}_C = \{-1.73\mathbf{i} - 5.77\mathbf{j} + 7.06\mathbf{k}\}$ ft/s
$\mathbf{a}_C = \{9.88\mathbf{i} - 72.8\mathbf{j} + 0.365\mathbf{k}\}$ ft/s^2

21–1.

Show that the sum of the moments of inertia of a body, $I_{xx} + I_{yy} + I_{zz}$, is independent of the orientation of the x, y, z axes and thus depends only on the location of its origin.

SOLUTION

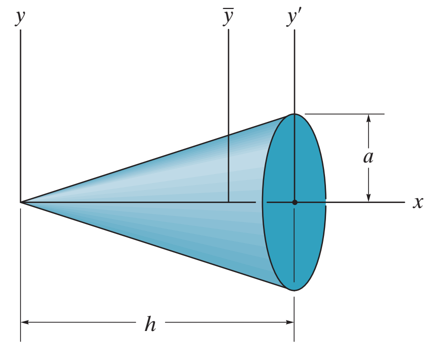
$$I_{xx} + I_{yy} + I_{zz} = \int_m (y^2 + z^2)dm + \int_m (x^2 + z^2)dm + \int_m (x^2 + y^2)dm$$

$$= 2\int_m (x^2 + y^2 + z^2)dm$$

However, $x^2 + y^2 + z^2 = r^2$, where r is the distance from the origin O to dm. Since $|r|$ is constant, it does not depend on the orientation of the x, y, z axis. Consequently, $I_{xx} + I_{yy} + I_{zz}$ is also indepenent of the orientation of the x, y, z axis. **Q.E.D.**

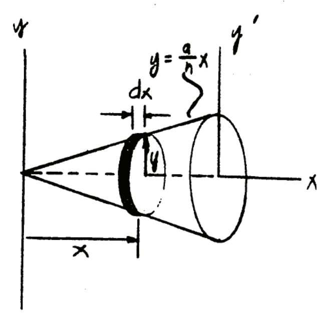
21–2.

Determine the moment of inertia of the cone with respect to a vertical $\overline{y}$ axis passing through the cone's center of mass. What is the moment of inertia about a parallel axis y' that passes through the diameter of the base of the cone? The cone has a mass m.



SOLUTION

The mass of the differential element is $dm = \rho dV = \rho(\pi y^2)\,dx = \dfrac{\rho\pi a^2}{h^2}x^2 dx.$



$$dI_y = \frac{1}{4}dmy^2 + dmx^2$$

$$= \frac{1}{4}\left[\frac{\rho\pi a^2}{h^2}x^2\,dx\right]\left(\frac{a}{h}x\right)^2 + \left(\frac{\rho\pi a^2}{h^2}x^2\right)x^2\,dx$$

$$= \frac{\rho\,\pi a^2}{4h^4}(4h^2 + a^2)\,x^4\,dx$$

$$I_y = \int dI_y = \frac{\rho\pi a^2}{4h^4}(4h^2 + a^2)\int_0^h x^4 dx = \frac{\rho\,\pi a^2 h}{20}(4h^2 + a^2)$$

However,

$$m = \int_m dm = \frac{\rho\,\pi a^2}{h^2}\int_0^h x^2\,dx = \frac{\rho\,\pi a^2 h}{3}$$

Hence,

$$I_y = \frac{3m}{20}(4h^2 + a^2)$$

Using the parallel axis theorem:

$$I_y = I_{\overline{y}} + md^2$$

$$\frac{3m}{20}(4h^2 + a^2) = I_{\overline{y}} + m\left(\frac{3h}{4}\right)^2$$

$$I_{\overline{y}} = \frac{3m}{80}(h^2 + 4a^2) \qquad \textbf{Ans.}$$

$$I_{y'} = I_{\overline{y}} + md^2$$

$$= \frac{3m}{80}(h^2 + 4a^2) + m\left(\frac{h}{4}\right)^2$$

$$= \frac{m}{20}(2h^2 + 3a^2) \qquad \textbf{Ans.}$$

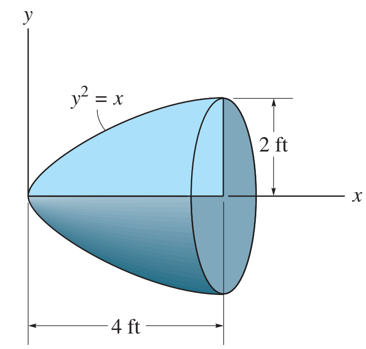
Ans:

$I_{\overline{y}} = \dfrac{3m}{80}(h^2 + 4a^2)$

$I_{y'} = \dfrac{m}{20}(2h^2 + 3a^2)$

21–3.

Determine moment of inertia I_y of the solid formed by revolving the shaded area around the x axis. The density of the material is $\rho = 12\ \text{slug/ft}^3$.



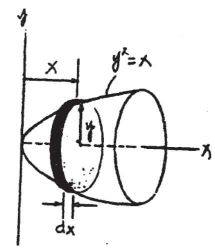
SOLUTION

The mass of the differential element is $dm = \rho dV = \rho\left(\pi y^2\right) dx = \rho\pi x dx$.

$$dI_y = \tfrac{1}{2}\, dm y^2 + dm x^2$$

$$= \tfrac{1}{4}[\rho\pi x dx](x) + (\rho\pi x dx)x^2$$

$$= \rho\pi\left(\frac{1}{4}x^2 + x^3\right) dx$$



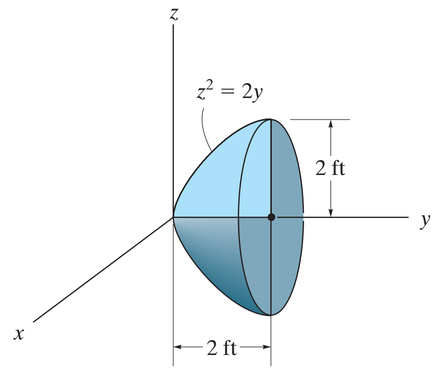
$$I_y = \int dI_y = \rho\pi \int_0^4 \left(\frac{1}{4}x^2 + x^3\right) dx = 69.33\ \pi\rho$$

$$= 69.33(\pi)(12) = 2614\ \text{slug}\cdot\text{ft}^2$$ **Ans.**

Ans:
$I_y = 2614\ \text{slug}\cdot\text{ft}^2$

***21–4.**

Determine the moments of inertia I_x and I_y of the paraboloid of revolution. The mass of the paraboloid is 20 slug.



SOLUTION

The mass of the differential element is $dm = \rho dV = \rho\left(\pi z^2\right) dy = 2\rho\pi y dy$.

$$m = 20 = \int_m dm = \int_0^2 2\rho\pi y dy$$

$$20 = 4\rho\pi \qquad \rho = \tfrac{5}{\pi}\ \text{slug/ft}^3$$

$$dI_x = \tfrac{1}{4} dm z^2 + dm\left(y^2\right)$$

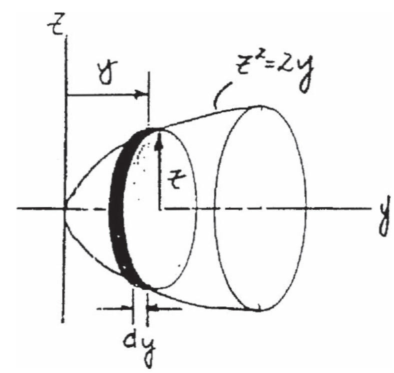
$$= \tfrac{1}{4}[2\rho\pi y dy](2y) + [2\rho\pi y dy]y^2$$

$$= \left(5y^2 + 10y^3\right) dy$$

$$I_x = \int dI_x = \int_0^2 \left(5y^2 + 10y^3\right) dy = 53.3\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans.}$$

$$dI_y = \tfrac{1}{2} dm z^2 = 2\rho\pi y^2\, dy = 10y^2\, dy$$

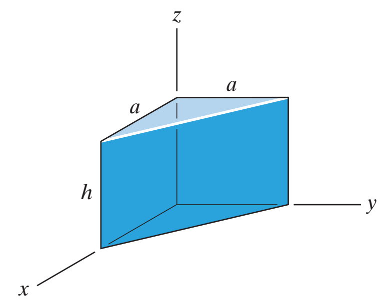
$$I_y = \int dI_y = \int_0^2 10y^2\, dy = 26.7\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans.}$$



Ans:
$I_x = 53.3\ \text{slug}\cdot\text{ft}^2$
$I_y = 26.7\ \text{slug}\cdot\text{ft}^2$

21–5.

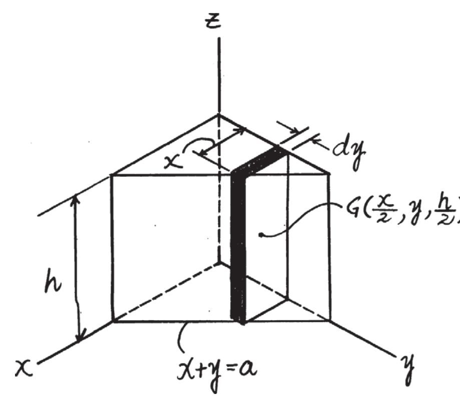
Determine by direct integration the product of inertia I_{yz} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the total mass m of the prism.



SOLUTION

The mass of the differential element is $dm = \rho dV = \rho h x dy = \rho h(a - y)dy$.

$$m = \int_m dm = \rho h \int_0^a (a - y)dy = \frac{\rho a^2 h}{2}$$



Using the parallel axis theorem:

$$dI_{yz} = (dI_{y'z'})_G + dm\, y_G z_G$$

$$= 0 + (\rho h x dy)\,(y)\left(\frac{h}{2}\right)$$

$$= \frac{\rho h^2}{2} x y dy$$

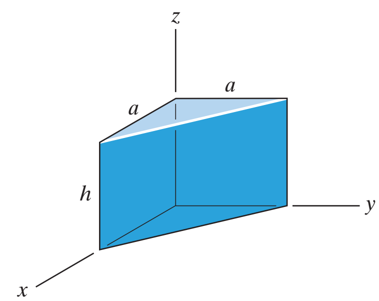
$$= \frac{\rho h^2}{2}(ay - y^2)\, dy$$

$$I_{yz} = \frac{\rho h^2}{2}\int_0^a (ay - y^2)\, dy = \frac{\rho a^3 h^2}{12} = \frac{1}{6}\left(\frac{\rho a^2 h}{2}\right)(ah) = \frac{m}{6} ah \qquad \textbf{Ans.}$$

Ans:
$I_{yz} = \frac{m}{6} ah$

21–6.

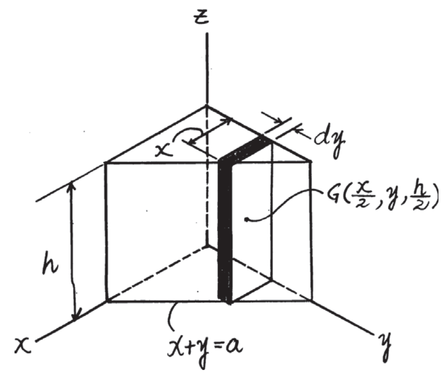
Determine by direct integration the product of inertia I_{xy} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the total mass m of the prism.



SOLUTION

The mass of the differential element is $dm = \rho dV = \rho h x dy = \rho h(a - y)dy$.

$$m = \int_m dm = \rho h \int_0^a (a - y)dy = \frac{\rho a^2 h}{2}$$



Using the parallel axis theorem:

$$dI_{xy} = (dI_{x'y'})_G + dm x_G y_G$$

$$= 0 + (\rho h x dy)\left(\frac{x}{2}\right)(y)$$

$$= \frac{\rho h^2}{2} x^2 y dy$$

$$= \frac{\rho h^2}{2}(y^3 - 2ay^2 + a^2 y)\, dy$$

$$I_{xy} = \frac{\rho h}{2}\int_0^a (y^3 - 2ay^2 + a^2 y)\, dy$$

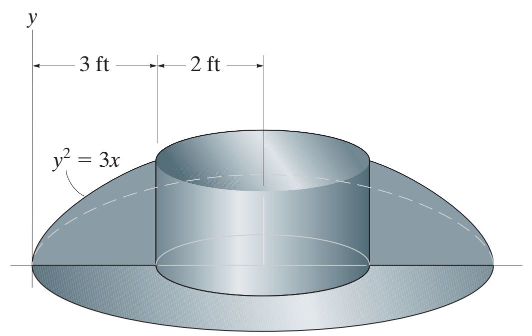
$$= \frac{\rho a^4 h}{24} = \frac{1}{12}\left(\frac{\rho a^2 h}{2}\right)a^2 = \frac{m}{12}a^2$$ **Ans.**

Ans:

$$I_{xy} = \frac{m}{12}a^2$$

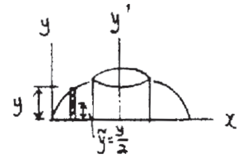
21–7.

Determine the product of inertia I_{xy} of the object formed by revolving the shaded area about the line $x = 5$ ft. Express the result in terms of the density of the material, ρ.



SOLUTION

$$\int_0^3 dm = \rho 2\pi \int_0^3 (5 - x)y\,dx = \rho 2\pi \int_0^3 (5 - x)\sqrt{3x}\,dx = 38.4\rho\pi$$



$$\int_0^3 \tilde{y}\,dm = \rho 2\pi \int_0^3 \frac{y}{2}(5 - x)y\,dx$$
$$= \rho\pi \int_0^3 (5 - x)(3x)\,dx$$
$$= 40.5\rho\pi$$

Thus, $\overline{y} = \dfrac{40.5\rho\pi}{38.4\rho\pi} = 1.055$ ft

The solid is symmetric about y, thus

$I_{xy'} = 0$

$I_{xy} = I_{xy'} + \overline{x}\,\overline{y}m$

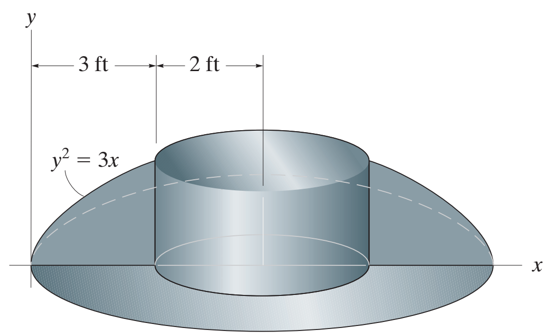
$= 0 + 5(1.055)(38.4\rho\pi)$

$I_{xy} = 636\rho$ **Ans.**

Ans:
$I_{xy} = 636\rho$

***21–8.**

Determine the moment of inertia I_y of the object formed by revolving the shaded area about the line $x = 5$ ft. Express the result in terms of the density of the material, ρ.



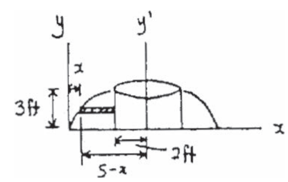
SOLUTION

$$I_{y'} = \int_0^3 \frac{1}{2}\, dm\, r^2 - \frac{1}{2}(m')(2)^2$$

$$\int_0^3 \frac{1}{2} dm\, r^2 = \frac{1}{2}\int_0^3 \rho\,\pi(5 - x)^4\, dy$$

$$= \frac{1}{2}\rho\,\pi\int_0^3\left(5 - \frac{y^2}{3}\right)^4 dy$$

$$= 490.29\,\rho\,\pi$$



$$m' = \rho\,\pi\,(2)^2(3) = 12\,\rho\,\pi$$

$$I_{y'} = 490.29\,\rho\,\pi - \frac{1}{2}(12\,\rho\,\pi)(2)^2 = 466.29\,\rho\,\pi$$

Mass of body;

$$m = \int_0^3 \rho\,\pi\,(5 - x)^2\, dy - m'$$

$$= \int_0^3 \rho\,\pi\,(5 - \frac{y^2}{3})^2\, dy - 12\,\rho\,\pi$$

$$= 38.4\,\rho\,\pi$$

$$I_y = 466.29\,\rho\,\pi + (38.4\,\rho\,\pi)(5)^2$$

$$= 1426.29\,\rho\,\pi$$

$$I_y = 4.48(10^3)\,\rho \qquad \textbf{Ans.}$$

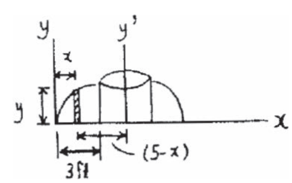
Also,

$$I_{y'} = \int_0^3 r^2\, dm$$

$$= \int_0^3 (5 - x)^2\,\rho\,(2\pi)(5 - x)y\, dx$$

$$= 2\,\rho\,\pi\int_0^3 (5 - x)^3(3x)^{1/2}\, dx$$

$$= 466.29\,\rho\,\pi$$



$$m = \int_0^3 dm$$

$$= 2\,\rho\,\pi\int_0^3 (5 - x)y\, dx$$

$$= 2\,\rho\,\pi\int_0^3 (5 - x)(3x)^{1/2}\, dx$$

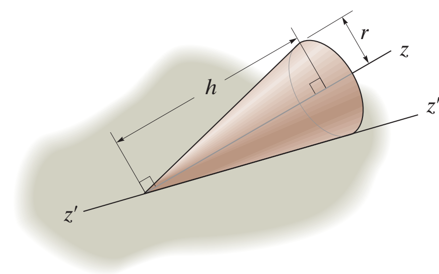
$$= 38.4\,\rho\,\pi$$

$$I_y = 466.29\,\rho\,\pi + 38.4\,\rho\,\pi(5)^2 = 4.48(10^3)\rho \qquad \textbf{Ans.}$$

Ans:
$I_y = 4.48(10^3)\,\rho$
$I_y = 4.48(10^3)\,\rho$

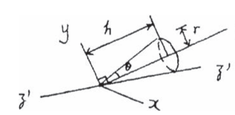
21–9.

Determine the moment of inertia of the cone about the z' axis. The weight of the cone is 15 lb, the *height* is $h = 1.5$ ft, and the radius is $r = 0.5$ ft.



SOLUTION

$$\theta = \tan^{-1}\left(\frac{0.5}{1.5}\right) = 18.43°$$



$$I_{xx} = I_{yy} = \left[\frac{3}{80}m\{4(0.5)^2 + (1.5)^2\}\right] + m\left[1.5 - \left(\frac{1.5}{4}\right)\right]^2$$

$$I_{xx} = I_{yy} = 1.3875\ m$$

$$I_z = \frac{3}{10}m(0.5)^2 = 0.075\ m$$

$$I_{xy} = I_{yz} = I_{zx} = 0$$

Using Eq. 21–5.

$$I_{z'z'} = u_{z'x}^2 I_{xx} + u_{z'y}^2 I_{yy} + u_{z'z}^2 I_{zz}$$

$$= 0 + [\cos(108.43°)]^2(1.3875m) + [\cos(18.43°)]^2(0.075m)$$

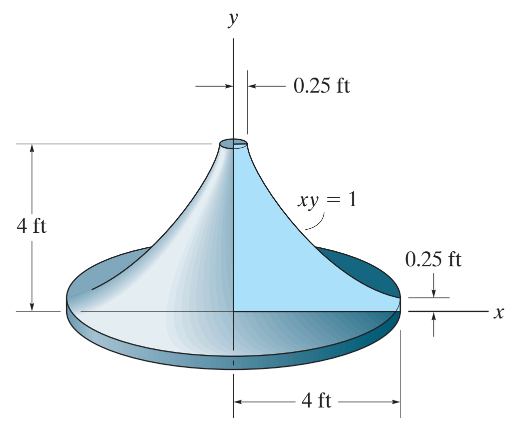
$$= 0.2062m$$

$$I_{z'z'} = 0.2062\left(\frac{15}{32.2}\right) = 0.0961\ \text{slug}\cdot\text{ft}^2$$ **Ans.**

Ans:
$I_{z'z'} = 0.0961\ \text{slug}\cdot\text{ft}^2$

21–10.

Determine the radii of gyration k_x and k_y for the solid formed by revolving the shaded area about the y axis. The density of the material is ρ.



SOLUTION

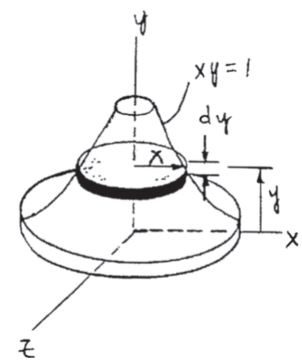
For k_y: The mass of the differential element is $dm = \rho dV = \rho(\pi x^2)\,dy = \rho\pi\dfrac{dy}{y^2}$.

$$dI_y = \tfrac{1}{2}dmx^2 = \tfrac{1}{2}\left[\rho\pi\tfrac{dy}{y^2}\right]\left(\tfrac{1}{y^2}\right) = \tfrac{1}{2}\rho\pi\tfrac{dy}{y^4}$$

$$I_y = \int dI_y = \tfrac{1}{2}\rho\pi\int_{0.25}^{4}\tfrac{dy}{y^4} + \frac{1}{2}\left[\rho(\pi)(4)^2(0.25)\right](4)^2$$
$$= 134.03\rho$$

However, $$m = \int_m dm = \rho\pi\int_{0.25}^{4}\tfrac{dy}{y^2} + \rho\left[\pi(4)^2(0.25)\right] = 24.35\rho$$

Hence, $$k_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{134.03\rho}{24.35\rho}} = 2.35\text{ ft}$$ **Ans.**



For k_x: $0.25\text{ ft} < y \leq 4\text{ ft}$

$$dI'_x = \frac{1}{4}dmx^2 + dmy^2$$
$$= \frac{1}{4}\left[\rho\pi\tfrac{dy}{y}\right]\left(\frac{I}{y^2}\right) + \left(\rho\pi\tfrac{dy}{y}\right)y^2$$
$$= \rho\pi\left(\tfrac{1}{4y^4} + 1\right)dy$$

$$I'_x = \int dI'_x = \rho\pi\int_{0.25}^{4}\left(\tfrac{1}{4y^4} + 1\right)dy = 28.53\rho$$

$$I''_x = \tfrac{1}{4}\left[\rho\pi(4)^2(0.25)\right](4)^2 + \left[\rho\pi(4)^2(0.25)\right](0.125)^2$$
$$= 50.46\rho$$

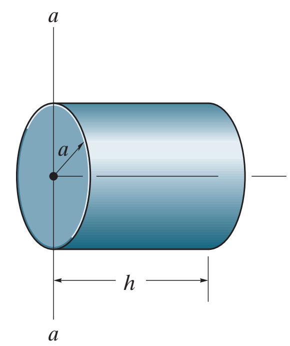
$$I_x = I'_x + I''_x = 28.53\rho + 50.46\rho = 78.99\rho$$

Hence, $$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{78.99\rho}{24.35\rho}} = 1.80\text{ ft}$$ **Ans.**

Ans:
$k_y = 2.35$ ft
$k_x = 1.80$ ft

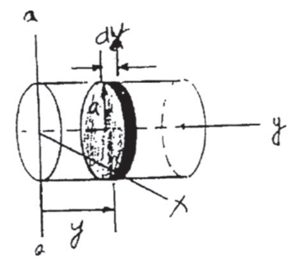
21–11.

Determine the moment of inertia of the cylinder with respect to the a–a axis of the cylinder. The cylinder has a mass m.



SOLUTION

The mass of the differential element is $dm = \rho dV = \rho\left(\pi a^2\right) dy$.



$$dI_{aa} = \tfrac{1}{4}dma^2 + dm\left(y^2\right)$$

$$= \tfrac{1}{4}\left[\rho\left(\pi a^2\right) dy\right]a^2 + \left[\rho\left(\pi a^2\right) dy\right]y^2$$

$$= \left(\tfrac{1}{4}\rho\pi a^4 + \rho\pi a^2 y^2\right) dy$$

$$I_{aa} = \int dI_{aa} = \int_0^h \left(\tfrac{1}{4}\rho\pi a^4 + \rho\pi a^2 y^2\right) dy$$

$$= \frac{\rho\pi a^2 h}{12}(3a^2 + 4h^2)$$

However, $$m = \int_m dm = \int_0^h \rho\left(\pi a^2\right) dy = \rho\pi a^2 h$$

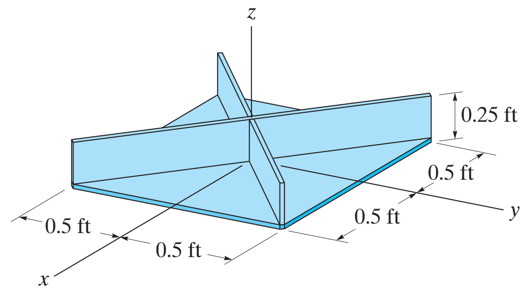
Hence, $$I_{aa} = \frac{m}{12}\left(3a^2 + 4h^2\right)$$ **Ans.**

Ans:

$I_{aa} = \dfrac{m}{12}\left(3a^2 + 4h^2\right)$

***21–12.**

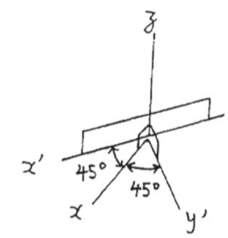
Determine the moment of inertia I_x of the composite plate assembly. The plates have a specific weight of 6 lb/ft^2.



SOLUTION

Horizontial plate:

$$I_{xx} = \frac{1}{12}\left(\frac{6(1)(1)}{32.2}\right)(1)^2 = 0.0155$$



Vertical plates:

$$l_{xx'} = 0.707, \qquad l_{xy'} = 0.707, \qquad l_{xz'} = 0$$

$$I_{x'x'} = \frac{1}{3}\left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)\left(\frac{1}{4}\right)^2 = 0.001372$$

$$I_{y'y'} = \left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)\left(\frac{1}{12}\right)\left[\left(\frac{1}{4}\right)^2 + (1\sqrt{2})^2\right] + \left(\frac{6(\frac{1}{4})(1\sqrt{2})}{32.2}\right)\left(\frac{1}{8}\right)^2$$

$$= 0.01235$$

Using Eq. 21–5,

$$I_{xx} = (0.707)^2(0.001372) + (0.707)^2(0.01235)$$

$$= 0.00686$$

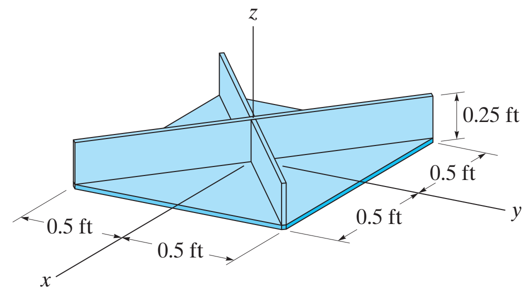
Thus,

$$I_{xx} = 0.0155 + 2(0.00686) = 0.0292 \text{ slug} \cdot \text{ft}^2$$ **Ans.**

Ans:
$I_{xx} = 0.0292 \text{ slug} \cdot \text{ft}^2$

21–13.

Determine the product of inertia I_{yz} of the composite plate assembly. The plates have a weight of 6 lb/ft^2.



SOLUTION

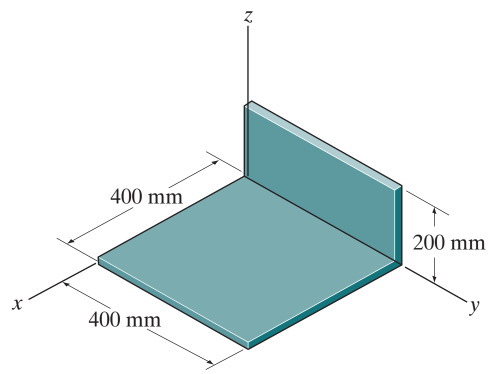
Due to symmetry,

$I_{yz} = 0$ **Ans.**

Ans:
$I_{yz} = 0$

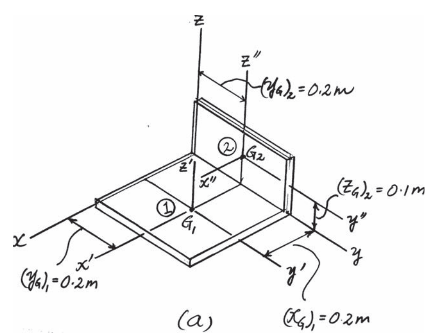
21–14.

Determine the products of inertia I_{xy}, I_{yz}, and I_{xz}, of the thin plate. The material has a density per unit area of $50\ \text{kg/m}^2$.



SOLUTION

The masses of segments (1) and (2) shown in Fig. *a* are $m_1 = 50(0.4)(0.4) = 8$ kg and $m_2 = 50(0.4)(0.2) = 4$ kg. Due to symmetry $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{x'z'} = 0$ for segment (1) and $\bar{I}_{x''y''} = \bar{I}_{y''z''} = \bar{I}_{x''z''} = 0$ for segment (2).



$$I_{xy} = \Sigma \bar{I}_{x'y'} + m x_G y_G$$

$$= \left[0 + 8(0.2)(0.2)\right] + \left[0 + 4(0)(0.2)\right]$$

$$= 0.32\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_{yz} = \Sigma \bar{I}_{y'z'} + m y_G z_G$$

$$= \left[0 + 8(0.2)(0)\right] + \left[0 + 4(0.2)(0.1)\right]$$

$$= 0.08\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

$$I_{xz} = \Sigma \bar{I}_{x'z'} + m x_G z_G$$

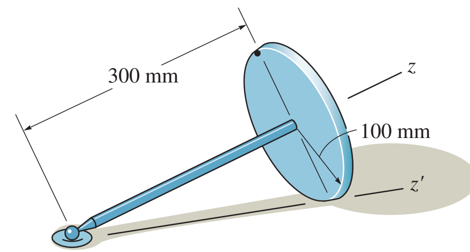
$$= \left[0 + 8(0.2)(0)\right] + \left[0 + 4(0)(0.1)\right]$$

$$= 0 \qquad \textbf{Ans.}$$

Ans:
$I_{xy} = 0.32\ \text{kg}\cdot\text{m}^2$
$I_{yz} = 0.08\ \text{kg}\cdot\text{m}^2$
$I_{xz} = 0$

21–15.

Determine the moment of inertia of both the 1.5-kg rod and 4-kg disk about the z' axis.

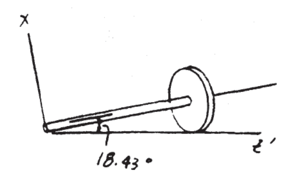


SOLUTION

Due to symmetry $I_{xy} = I_{yz} + I_{zx} = 0$

$$I_y = I_x = \left[\frac{1}{4}(4)(0.1)^2 + 4(0.3)^2\right] + \frac{1}{3}(1.5)(0.3)^2$$

$$= 0.415 \text{ kg} \cdot \text{m}^2$$



$$I_z = \frac{1}{2}(4)(0.1)^2 = 0.02 \text{ kg} \cdot \text{m}^2$$

$u_z = \cos(18.43°) = 0.9487, \qquad u_y = \cos 90° = 0,$

$u_x = \cos(90° + 18.43°) = -0.3162$

$$I_{z'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

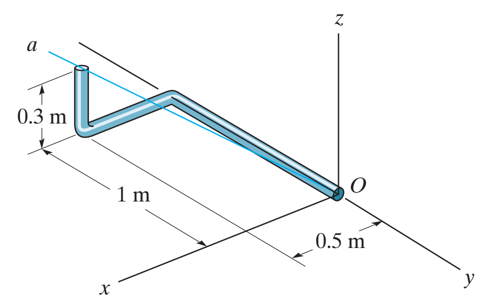
$$= 0.415(-0.3162)^2 + 0 + 0.02(0.9487)^2 - 0 - 0 - 0$$

$$= 0.0595 \text{ kg} \cdot \text{m}^2$$ **Ans.**

Ans:
$I_{z'} = 0.0595 \text{ kg} \cdot \text{m}^2$

***21–16.**

The bent rod has a mass of 3 kg/m. Determine the moment of inertia of the rod about the O–a axis.



SOLUTION

The bent rod is subdivided into three segments and the location of center of mass for each segment is indicated in Fig. a. The mass of each segments is $m_1 = 3(1) = 3$ kg, $m_2 = 3(0.5) = 1.5$ kg and $m_3 = 3(0.3) = 0.9$ kg.

$$I_{xx} = \left[\frac{1}{12}(3)(1^2) + 3(0.5^2)\right] + \left[0 + 1.5(1^2)\right] + \left[\frac{1}{12}(0.9)(0.3^2) + 0.9(0.15^2 + 1^2)\right]$$

$$= 3.427 \text{ kg}\cdot\text{m}^2$$

$$I_{yy} = 0 + \left[\frac{1}{12}(1.5)(0.5^2) + 1.5(0.25^2)\right] + \left[\frac{1}{12}(0.9)(0.3^2) + 0.9(0.15^2 + 0.5^2)\right]$$

$$= 0.377 \text{ kg}\cdot\text{m}^2$$

$$I_{zz} = \left[\frac{1}{12}(3)(1^2) + 3(0.5^2)\right] + \left[\frac{1}{12}(1.5)(0.5^2) + 1.5(1^2 + 0.25^2)\right] + \left[0 + 0.9(1^2 + 0.5^2)\right]$$

$$= 3.75 \text{ kg}\cdot\text{m}^2$$

$$I_{xy} = [0 + 0] + [0 + 1.5(0.25)(-1)] + [0 + 0.9(0.5)(-1)] = -0.825 \text{ kg}\cdot\text{m}^2$$

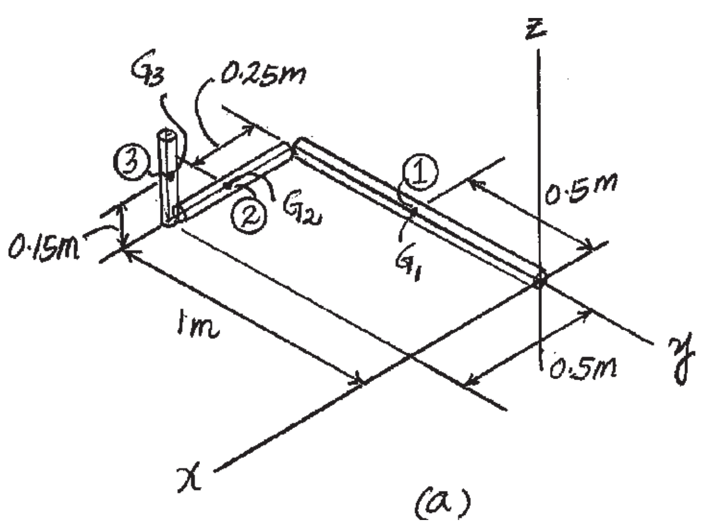
$$I_{yz} = [0 + 0] + [0 + 0] + [0 + 0.9(-1)(0.15)] = -0.135 \text{ kg}\cdot\text{m}^2$$

$$I_{zx} = [0 + 0] + [0 + 0] + [0 + 0.9(0.15)(0.5)] = 0.0675 \text{ kg}\cdot\text{m}^2$$

The unit vector that defines the direction of the O_a axis is

$$\mathbf{U}_{O_a} = \frac{0.5\mathbf{i} - 1\mathbf{j} + 0.3\mathbf{k}}{\sqrt{0.5^2 + (-1)^2 + 0.3^2}} = \frac{0.5}{\sqrt{1.34}}\mathbf{i} - \frac{1}{\sqrt{1.34}}\mathbf{j} + \frac{0.3}{\sqrt{1.34}}\mathbf{k}$$

Thus, $\quad u_x = \dfrac{0.5}{\sqrt{1.34}} \qquad u_y = -\dfrac{1}{\sqrt{1.34}} \qquad u_z = \dfrac{0.3}{\sqrt{1.34}}$



(a)

***21–16. Continued**

Then

$$I_{O_a} = I_{xx}\,u_x^2 + I_{yy}\,u_y^2 + I_{zz}\,u_z^2 - 2I_{xy}\,u_x u_y - 2I_{yz}\,u_y u_z - 2I_{zx}\,u_z u_x$$

$$= 3.427\left(\frac{0.5}{\sqrt{1.34}}\right)^2 + 0.377\left(-\frac{1}{\sqrt{1.34}}\right)^2 + 3.75\left(\frac{0.3}{\sqrt{1.34}}\right)^2 - 2(-0.825)\left(\frac{0.5}{\sqrt{1.34}}\right)\left(-\frac{1}{\sqrt{1.34}}\right)$$

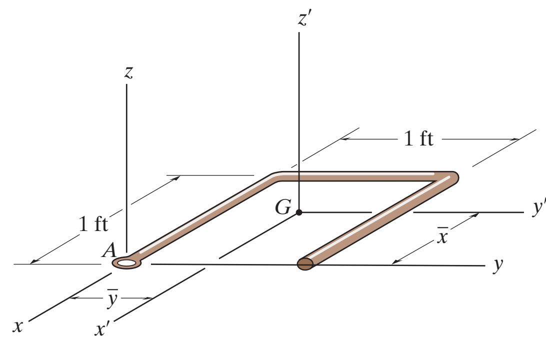
$$-2(-0.135)\left(-\frac{1}{\sqrt{1.34}}\right)\left(\frac{0.3}{\sqrt{1.34}}\right) - 2(0.0675)\left(\frac{0.3}{\sqrt{1.34}}\right)\left(\frac{0.5}{\sqrt{1.34}}\right)$$

$$= 0.4813 \text{ kg}\cdot\text{m}^2 = 0.481 \text{ kg}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_{O_a} = 0.481 \text{ kg}\cdot\text{m}^2$

21–17.

The bent rod has a weight of 1.5 lb/ft. Locate the center of gravity $G(\bar{x}, \bar{y})$ and determine the principal moments of inertia $I_{x'}$, $I_{y'}$, and $I_{z'}$ of the rod with respect to the x', y', z' axes.



SOLUTION

Due to symmetry

$\bar{y} = 0.5$ ft **Ans.**

$$\bar{x} = \frac{\Sigma \bar{x} W}{\Sigma w} = \frac{(-1)(1.5)(1) + 2\left[(-0.5)(1.5)(1)\right]}{3\left[1.5(1)\right]} = -0.667 \text{ ft}$$ **Ans.**

$$I_{x'} = 2\left[\left(\frac{1.5}{32.2}\right)(0.5)^2\right] + \frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2$$

$= 0.0272 \text{ slug} \cdot \text{ft}^2$ **Ans.**

$$I_{y'} = 2\left[\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.667 - 0.5)^2\right] + \left(\frac{1.5}{32.2}\right)(1 - 0.667)_2$$

$= 0.0155 \text{ slug} \cdot \text{ft}^2$ **Ans.**

$$I_{z'} = 2\left[\frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.5^2 + 0.1667^2)\right]$$

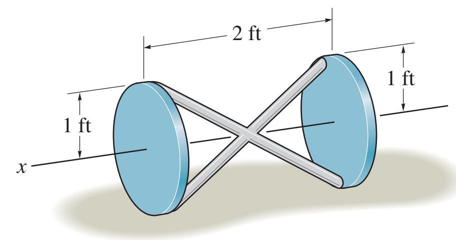
$$+ \frac{1}{12}\left(\frac{1.5}{32.2}\right)(1)^2 + \left(\frac{1.5}{32.2}\right)(0.3333)^2$$

$= 0.0427 \text{ slug} \cdot \text{ft}^2$ **Ans.**

Ans:
$\bar{y} = 0.5$ ft
$\bar{x} = -0.667$ ft
$I_{x'} = 0.0272 \text{ slug} \cdot \text{ft}^2$
$I_{y'} = 0.0155 \text{ slug} \cdot \text{ft}^2$
$I_{z'} = 0.0427 \text{ slug} \cdot \text{ft}^2$

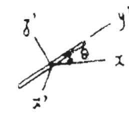
21–18.

Determine the moment of inertia of the rod-and-disk assembly about the x axis. The disks each have a weight of 12 lb. The two rods each have a weight of 4 lb, and their ends extend to the rims of the disks.



SOLUTION

For a rod:



$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$$

$$u_{x'} = \cos 90^\circ = 0, \qquad u_{y'} = \cos 45^\circ = 0.7071, \qquad u_{z'} = \cos(90^\circ + 45^\circ) = -0.7071$$

$$I_{x'} = I_{z'} = \left(\frac{1}{12}\right)\left(\frac{4}{32.2}\right)\left[(2)^2 + (2)^2\right] = 0.08282 \text{ slug}\cdot\text{ft}^2$$

$$I_{y'} = 0$$

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

$$I_x = 0 + 0 + (0.08282)(-0.7071)^2 = 0.04141 \text{ slug}\cdot\text{ft}^2$$

For a disk:

$$I_x = \left(\frac{1}{2}\right)\left(\frac{12}{32.2}\right)(1)^2 = 0.1863 \text{ slug}\cdot\text{ft}^2$$

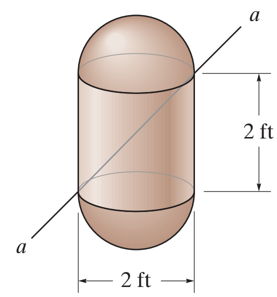
Thus.

$$I_x = 2(0.04141) + 2(0.1863) = 0.455 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans.}$$

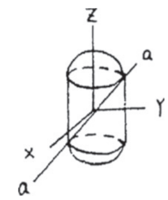
Ans:
$I_x = 0.455 \text{ slug}\cdot\text{ft}^2$

21–19.

Determine the moment of inertia of the composite body about the *aa* axis. The cylinder weighs 20 lb, and each hemisphere weighs 10 lb.



SOLUTION



$u_{az} = 0.707$

$u_{ax} = 0$

$u_{ay} = 0.707$

$$I_{zz} = \frac{1}{2}(\frac{20}{32.2})(1)^2 + 2[\frac{2}{5}(\frac{10}{32.2})(1)^2]$$

$$= 0.5590 \text{ slug} \cdot \text{ft}^2$$

$$I_{xx} = I_{yy} = \frac{1}{12}(\frac{20}{32.2})[3(1)^2 + (2)^2] + 2[0.259(\frac{10}{32.2})(1)^2 + \frac{10}{32.2}(\frac{11}{8})^2]$$

$$I_{xx} = I_{yy} = 1.6975 \text{ slug} \cdot \text{ft}^2$$

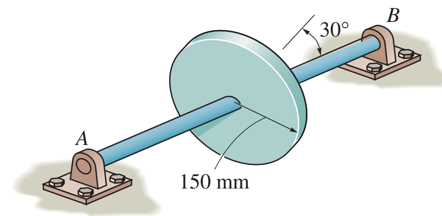
$$I_{aa} = 0 + (0.707)^2(1.6975) + (0.707)^2(0.559)$$

$$I_{aa} = 1.13 \text{ slug} \cdot \text{ft}^2$$ **Ans.**

Ans:
$I_{aa} = 1.13 \text{ slug} \cdot \text{ft}^2$

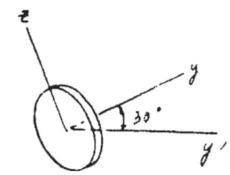
***21–20.**

Determine the moment of inertia of the disk about the axis of shaft AB. The disk has a mass of 15 kg.



SOLUTION

Due to symmetry



$$I_{xy} = I_{yz} = I_{zx} = 0$$

$$I_x = I_z = \frac{1}{4}(15)(0.15)^2 = 0.084375\ \text{kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{2}(15)(0.15)^2 = 0.16875\ \text{kg}\cdot\text{m}^2$$

$$u_x = \cos 90^\circ = 0, \qquad u_y = \cos 30^\circ = 0.8660$$

$$u_z = \cos(30^\circ + 90^\circ) = -0.5$$

$$I_{y'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy}u_x u_y - 2I_{yz}u_y u_z - 2I_{zx}u_z u_x$$

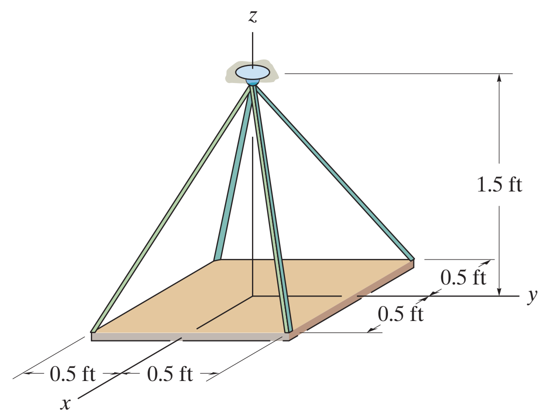
$$= 0 + 0.16875(0.8660)^2 + 0.084375(-0.5)^2 - 0 - 0 - 0$$

$$= 0.148\ \text{kg}\cdot\text{m}^2$$ **Ans.**

Ans:
$I_{y'} = 0.148\ \text{kg}\cdot\text{m}^2$

21–21.

The thin plate has a weight of 5 lb and each of the four rods weighs 3 lb. Determine the moment of inertia of the assembly about the z axis.



SOLUTION

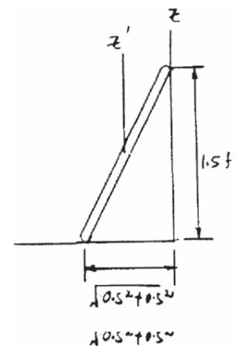
For the rod:

$$I_{z'} = \frac{1}{12}\left(\frac{3}{32.2}\right)\left(\sqrt{(0.5^2 + (0.5)^2}\right)^2 = 0.003882 \text{ slug}\cdot\text{ft}^2$$

For the composite assembly of rods and disks:

$$I_z = 4\left[0.003882 + \left(\frac{3}{32.2}\right)\left(\frac{\sqrt{0.5^2 + 0.5^2}}{2}\right)^2\right] + \frac{1}{12}\left(\frac{5}{32.2}\right)(1^2 + 1^2)$$

$$= 0.0880 \text{ slug}\cdot\text{ft}^2$$ **Ans.**



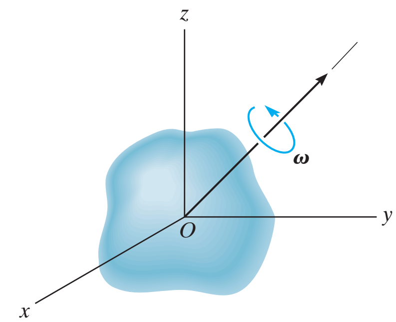
Ans:
$I_z = 0.0880 \text{ slug}\cdot\text{ft}^2$

21–22.

If a body contains *no planes of symmetry,* the principal moments of inertia can be determined mathematically. To show how this is done, consider the rigid body which is spinning with an angular velocity $\boldsymbol{\omega}$, directed along one of its principal axes of inertia. If the principal moment of inertia about this axis is I, the angular momentum can be expressed as $\mathbf{H} = I\boldsymbol{\omega} = I\omega_x\mathbf{i} + I\omega_y\mathbf{j} + I\omega_z\mathbf{k}$. The components of $\mathbf{H}$ may also be expressed by Eqs. 21–10, where the inertia tensor is assumed to be known. Equate the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components of both expressions for $\mathbf{H}$ and consider ω_x, ω_y, and ω_z to be unknown. The solution of these three equations is obtained provided the determinant of the coefficients is zero. Show that this determinant, when expanded, yields the cubic equation

$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2)I - (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}I_{xy}^2) = 0$$

The three positive roots of I, obtained from the solution of this equation, represent the principal moments of inertia I_x, I_y, and I_z.



SOLUTION

$$\mathbf{H} = I\omega = I\omega_x\,\mathbf{i} + I\omega_y\,\mathbf{j} + I\omega_z\,\mathbf{k}$$

Equating the $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components to the scalar equations (Eq. 21–10) yields

$$(I_{xx} - I)\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z = 0$$

$$-I_{xx}\,\omega_x + (I_{xy} - I)\,\omega_y - I_{yz}\,\omega_z = 0$$

$$-I_{zx}\,\omega_z - I_{zy}\,\omega_y + (I_{zz} - I)\,\omega_z = 0$$

Solution for ω_x, ω_y, and ω_z requires

$$\begin{vmatrix} (I_{xx} - I) & -I_{xy} & -I_{xz} \\ -I_{yx} & (I_{yy} - I) & -I_{yz} \\ -I_{zx} & -I_{zy} & (I_{zz} - I) \end{vmatrix} = 0$$

Expanding

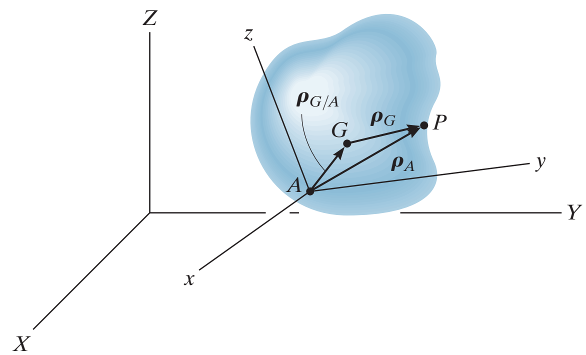
$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + \left(I_{xx}\,I_{yy} + I_{yy}\,I_{zz} + I_{zz}\,I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2\right)I - \left(I_{xx}\,I_{yy}\,I_{zz} - 2I_{xy}\,I_{yz}\,I_{zx} - I_{xx}\,I_{yz}^2 - I_{yy}I_{zx}^2 - I_{zz}\,I_{xy}^2\right) = 0 \textbf{ Q.E.D.}$$

Ans:

$$I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + \left(I_{xx}\,I_{yy} + I_{yy}\,I_{zz} + I_{zz}\,I_{xx} - I_{xy}^2 - I_{yz}^2 - I_{zx}^2\right)I - \left(I_{xx}\,I_{yy}\,I_{zz} - 2I_{xy}\,I_{yz}\,I_{zx} - I_{xx}\,I_{yz}^2 - I_{yy}\,I_{zx}^2 - I_{zz}\,I_{xy}^2\right) = 0 \textbf{ Q.E.D.}$$

21–23.

Show that if the angular momentum of a body is determined with respect to an arbitrary point A, then $\mathbf{H}_A$ can be expressed by Eq. 21–9. This requires substituting $\boldsymbol{\rho}_A = \boldsymbol{\rho}_G + \boldsymbol{\rho}_{G/A}$ into Eq. 21–6 and expanding, noting that $\int \boldsymbol{\rho}_G \, dm = \mathbf{0}$ by definition of the mass center and $\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega} \times \boldsymbol{\rho}_{G/A}$.



SOLUTION

$$\mathbf{H}_A = \left(\int_m \rho_A \, dm\right) \times \mathbf{v}_A + \int_m \rho_A \times (\omega \times \rho_A) dm$$

$$= \left(\int_m (\rho_G + \rho_{G/A}) \, dm\right) \times \mathbf{v}_A + \int_m (\rho_G + \rho_{G/A}) \times \left[\omega \times \rho_G + \rho_{G/A})\right] dm$$

$$= \left(\int_m \rho_G \, dm\right) \times \mathbf{v}_A + (\rho_{G/A} \times \mathbf{v}_A) \int_m dm + \int_m \rho_G \times (\omega \times \rho_G) \, dm$$

$$+ \left(\int_m \rho_G dm\right) \times (\omega \times \rho_{G/A}) + \rho_{G/A} \times \left(\omega \times \int_m \rho_G \, dm\right) + \rho_{G/A} \times (\omega \times \rho_{G/A}) \int_m dm$$

Since $\int_m \rho_G \, dm = 0$ and from Eq. 21–8 $\mathbf{H}_G = \int_m \rho_G \times (\omega \times \rho_G) dm$

$$\mathbf{H}_A = (\rho_{G/A} \times \mathbf{v}_A)m + \mathbf{H}_G + \rho_{G/A} \times (\omega \times \rho_{G/A})m$$

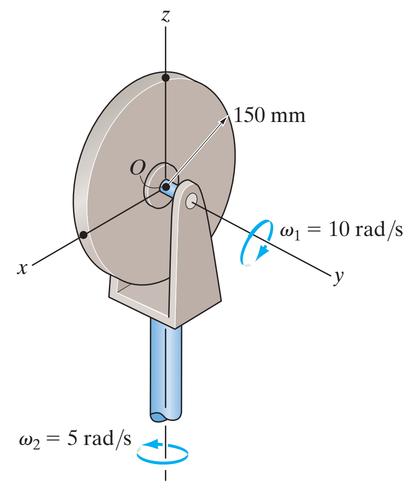
$$= \rho_{G/A} \times (\mathbf{v}_A + (\omega \times \rho_{G/A}))m + \mathbf{H}_G$$

$$= (\rho_{G/A} \times m\mathbf{v}_G) + \mathbf{H}_G \qquad \textbf{Q.E.D.}$$

Ans:
$\mathbf{H}_A = (\rho_{G/A} \times m\mathrm{v}_G) + \mathbf{H}_G$

***21–24.**

The 15-kg circular disk spins about its axle with a constant angular velocity of $\omega_1 = 10$ rad/s. Simultaneously, the yoke is rotating with a constant angular velocity of $\omega_2 = 5$ rad/s. Determine the angular momentum of the disk about its center of mass O, and its kinetic energy.



SOLUTION

The mass moments of inertia of the disk about the x, y, and z axes are

$$I_x = I_z = \frac{1}{4}mr^2 = \frac{1}{4}(15)(0.15^2) = 0.084375 \text{ kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{2}mr^2 = \frac{1}{2}(15)(0.15^2) = 0.16875 \text{ kg}\cdot\text{m}^2$$

Due to symmetry,

$$I_{xy} = I_{yz} = I_{xz} = 0$$

Here, the angular velocity of the disk can be determined from the vector addition of ω_1 and ω_2. Thus,

$$\omega = \omega_1 + \omega_2 = [-10\mathbf{j} + 5\mathbf{k}] \text{ rad/s}$$

so that

$$\omega_x = 0 \qquad \omega_y = -10 \text{ rad/s} \qquad \omega_z = 5 \text{ rad/s}$$

Since the disk rotates about a fixed point O, we can apply

$$H_x = I_x\omega_x = 0.084375(0) = 0$$

$$H_y = I_y\omega_y = 0.16875(-10) = -1.6875 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$H_z = I_z\omega_z = 0.084375(5) = 0.421875 \text{ kg}\cdot\text{m}^2/\text{s}$$

Thus,

$$H_O = [-1.69\mathbf{j} + 0.422\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

The kinetic energy of the disk can be determined from

$$T = \frac{1}{2}I_x\omega_x^{\,2} + \frac{1}{2}I_y\omega_y^{\,2} + \frac{1}{2}I_z\omega_z^{\,2}$$

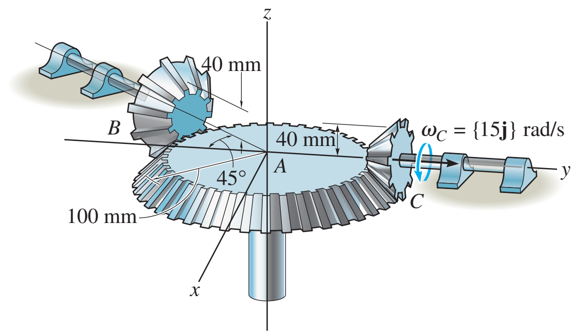
$$= \frac{1}{2}(0.084375)(0^2) + \frac{1}{2}(0.16875)(-10)^2 + \frac{1}{2}(0.084375)(5^2)$$

$$= 9.49 \text{ J}$$ **Ans.**

Ans:
$H_O = [-1.69\mathbf{j} + 0.422\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s}$
$T = 9.49$ **J**

21–25.

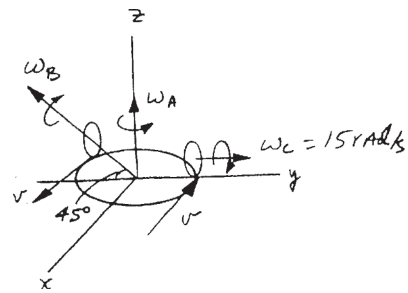
The large gear has a mass of 5 kg and a radius of gyration of $k_z = 75$ mm. Gears B and C each have a mass of 200 g and a radius of gyration about the axis of their connecting shaft of 15 mm. If the gears are in mesh and C has an angular velocity of $\boldsymbol{\omega}_C = \{15\mathbf{j}\}$ rad/s, determine the total angular momentum for the system of three gears about point A.



SOLUTION

$$I_A = 5(0.075)^2 = 28.125\left(10^{-3}\right)\text{ kg}\cdot\text{m}^2$$

$$I_B = I_C = 0.2(0.015)^2 = 45\left(10^{-6}\right)\text{ kg}\cdot\text{m}^2$$



Kinematics:

$$\omega_C = \omega_B = 15\text{ rad/s}$$

$$v = (0.04)(15) = 0.6\text{ m/s}$$

$$\omega_A = \left(\frac{0.6}{0.1}\right) = 6\text{ rad/s}$$

$$H_B = I_B\,\omega_B = \left(45\left(10^{-6}\right)\right)(15) = 675\left(10^{-6}\right)$$

$$\mathbf{H}_B = -675\left(10^{-6}\right)\sin 45°\mathbf{i} - 675\left(10^{-6}\right)\cos 45°\mathbf{j}$$

$$\mathbf{H}_B = -477.3\left(10^{-6}\right)\mathbf{i} - 477.3\left(10^{-6}\right)\mathbf{j}$$

$$H_C = I_C\,\omega_C = \left(45\left(10^{-6}\right)\right)(15) = 675\left(10^{-6}\right)$$

$$\mathbf{H}_C = 675\left(10^{-6}\right)\mathbf{j}$$

$$H_A = I_A\,\omega_A = 28.125\left(10^{-3}\right)(6) = 0.16875$$

$$\mathbf{H}_A = 0.16875\mathbf{k}$$

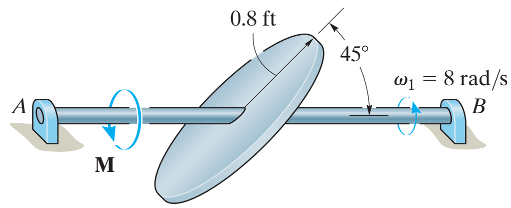
The total angular momentum is therefore,

$$\mathbf{H} = \mathbf{H}_B + \mathbf{H}_C + \mathbf{H}_A = \left\{-477\left(10^{-6}\right)\mathbf{i} + 198\left(10^{-6}\right)\mathbf{j} + 0.169\mathbf{k}\right\}\text{ kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{H} = \{-477(10^{-6})\mathbf{i} + 198(10^{-6})\mathbf{j} + 0.169\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$

21–26.

The circular disk has a weight of 15 lb and is mounted on the shaft AB at an angle of 45° with the horizontal. Determine the angular velocity of the shaft when $t = 3$ s if a constant torque $M = 2\ \text{lb}\cdot\text{ft}$ is applied to the shaft. The shaft is originally spinning at $\omega_1 = 8$ rad/s when the torque is applied.



SOLUTION

Due to symmetry

$$I_{xy} = I_{yz} = I_{zx} = 0$$

$$I_y = I_z = \frac{1}{4}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.07453\ \text{slug}\cdot\text{ft}^2$$

$$I_x = \frac{1}{2}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.1491\ \text{slug}\cdot\text{ft}^2$$

For x' axis

$u_x = \cos 45° = 0.7071 \qquad u_y = \cos 45° = 0.7071$

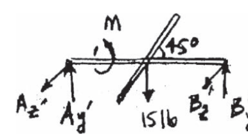
$u_z = \cos 90° = 0$

$$I_{z'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

$$= 0.1491(0.7071)^2 + 0.07453(0.7071)^2 + 0 - 0 - 0 - 0$$

$$= 0.1118\ \text{slug}\cdot\text{ft}^2$$

Principle of impulse and momentum:



$$(H_{x'})_1 + \Sigma\int M_{x'}\,dt = (H_{x'})_2$$

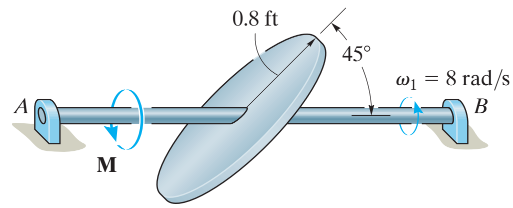
$$0.1118(8) + 2(3) = 0.1118\,\omega_2$$

$\omega_2 = 61.7$ rad/s **Ans.**

Ans:
$\omega_2 = 61.7$ rad/s

21–27.

The circular disk has a weight of 15 lb and is mounted on the shaft AB at an angle of 45° with the horizontal. Determine the angular velocity of the shaft when $t = 2$ s if a torque $M = (4e^{0.1t})$ lb · ft, where t is in seconds, is applied to the shaft. The shaft is originally spinning at $\omega_1 = 8$ rad/s when the torque is applied.



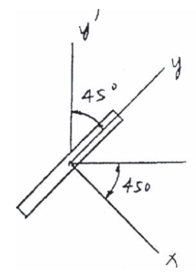
SOLUTION

Due to symmetry

$I_{xy} = I_{yz} = I_{zx} = 0$

$I_y = I_z = \frac{1}{4}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.07453 \text{ slug} \cdot \text{ft}^2$

$I_x = \frac{1}{2}\left(\frac{15}{32.2}\right)(0.8)^2 = 0.1491 \text{ slug} \cdot \text{ft}^2$



For x' axis

$u_x = \cos 45° = 0.7071 \qquad u_y = \cos 45° = 0.7071$

$u_z = \cos 90° = 0$

$$I_{z'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

$$= 0.1491(0.7071)^2 + 0.07453(0.7071)^2 + 0 - 0 - 0 - 0$$

$$= 0.1118 \text{ slug} \cdot \text{ft}^2$$

Principle of impulse and momentum:

$$(H_{x'})_1 + \Sigma \int M_{x'}\,dt = (H_{x'})_2$$

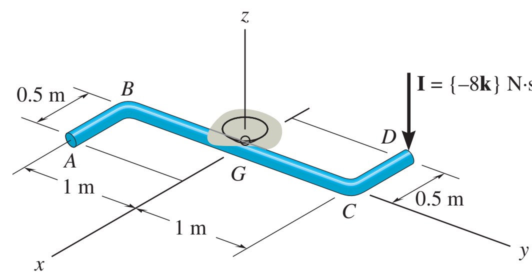
$$0.1118(8) + \int_0^2 4e^{0.1\,t}\,dt = 0.1118\omega_2$$

$\omega_2 = 87.2$ rad/s **Ans.**

Ans:
$\omega_2 = 87.2$ rad/s

***21–28.**

The rod assembly is supported at G by a ball-and-socket joint. Each segment has a mass of 0.5 kg/m. If the assembly is originally at rest and an impulse of $\mathbf{I} = \{-8\mathbf{k}\}$ N·s is applied at D, determine the angular velocity of the assembly just after the impact.



SOLUTION

Moments and products of inertia:

$$I_{xx} = \frac{1}{12}\left[2(0.5)\right](2)^2 + 2\left[0.5(0.5)\right](1)^2 = 0.8333 \text{ kg}\cdot\text{m}^2$$

$$I_{yy} = \frac{1}{12}\left[1(0.5)\right](1)^2 = 0.04166 \text{ kg}\cdot\text{m}^2$$

$$I_{zz} = \frac{1}{12}\left[2(0.5)\right](2)^2 + 2\left[\frac{1}{12}\left[0.5(0.5)\right](0.5)^2 + \left[0.5(0.5)\right](1^2 + 0.25^2)\right]$$

$$= 0.875 \text{ kg}\cdot\text{m}^2$$

$$I_{xy} = \left[0.5(0.5)\right](-0.25)(1) + \left[0.5(0.5)\right](0.25)(-1) = -0.125 \text{ kg}\cdot\text{m}^2$$

$$I_{yz} = I_{xz} = 0$$

From Eq. 21–10

$$H_x = 0.8333\omega_x + 0.125\omega_y$$

$$H_y = 0.125\omega_x + 0.04166\omega_y$$

$$H_z = 0.875\omega_z$$

$$(\mathrm{H}_G)_1 + \Sigma\int_{t_1}^{t_2} \mathrm{M}_G dt = (\mathrm{H}_G)_2$$

$$\mathbf{0} + (-0.5\mathbf{i} + 1\mathbf{j}) \times (-8\mathbf{k}) = (0.8333\omega_x + 0.125\omega_y)\mathbf{i} + (0.125\omega_x + 0.04166\omega_y)\mathbf{j} + 0.875\omega_z\mathbf{k}$$

Equating **i**, **j** and **k** components

$$-8 = 0.8333\omega_x + 0.125\omega_y \qquad \textbf{(1)}$$

$$-4 = 0.125\omega_x + 0.04166\omega_y \qquad \textbf{(2)}$$

$$0 = 0.875\omega_z \qquad \textbf{(3)}$$

Solving Eqs. (1) to (3) yields:

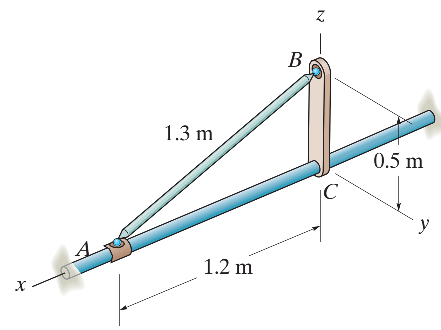
$\omega_x = 8.73$ rad/s $\qquad \omega_y{-}12\bar{2}$ rad/s $\qquad \omega_z = 0$

Then $\qquad \omega = \{8.73\mathbf{i} - 122\mathbf{j}\}$ rad/s $\qquad$ **Ans.**

Ans:
$\omega = \{8.73\mathbf{i} - 122\mathbf{j}\}$ rad/s

21–29.

The 4-lb rod AB is attached to the 1-lb collar at A and a 2-lb link BC using ball-and-socket joints. If the rod is released from rest in the position shown, determine the angular velocity of the link after it has rotated 180°.

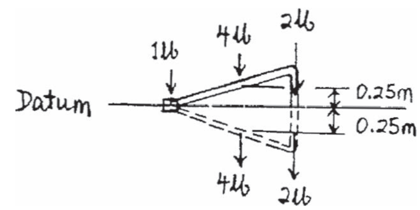


SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 4(0.25) + 2(0.25) = T_2 - 4(0.25) - 2(0.25)$$

$$T_2 = 3$$

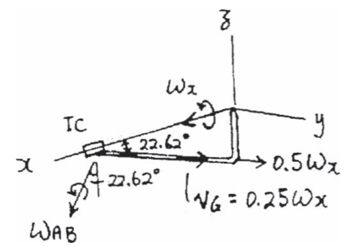


$$\omega_{AB} = \frac{0.5\omega_x}{1.3} = 0.3846\omega_x$$

$$T_2 = \frac{1}{2}\left[\frac{1}{3}\left(\frac{2}{32.2}\right)(0.5)^2\right]\omega_x^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{4}{32.2}\right)(1.3)^2\right](0.3846\omega_x)^2 + \frac{1}{2}\left(\frac{4}{32.2}\right)(0.25\omega_x)^2$$

$$T_2 = 0.007764\omega_x^2 = 3$$

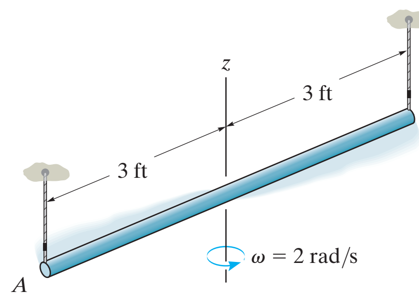
$$\omega_x = 19.7 \text{ rad/s} \qquad \textbf{Ans.}$$



Ans:
$\omega_x = 19.7 \text{ rad/s}$

21–30.

The rod weighs 3 lb/ft and is suspended from parallel cords at A and B. If the rod has an angular velocity of 2 rad/s about the z axis at the instant shown, determine how high the center of the rod rises at the instant the rod momentarily stops swinging.



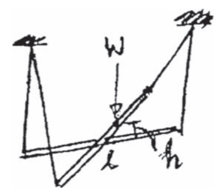
SOLUTION

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left[\frac{1}{12}\frac{W}{g}l^2\right]\omega^2 + 0 = 0 + Wh$$

$$h = \frac{1}{24}\frac{l^2\omega^2}{g} = \frac{1}{24}\frac{(6)^2(2)^2}{(32.2)}$$

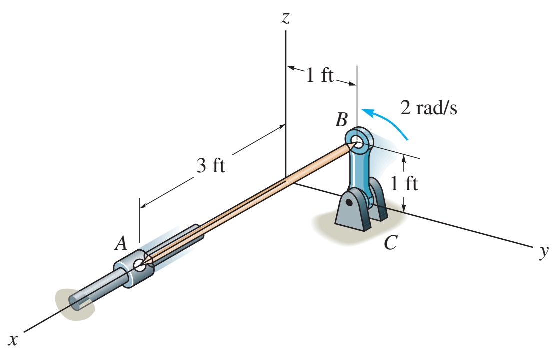
$h = 0.1863 \text{ ft} = 2.24 \text{ in.}$ **Ans.**



Ans:
$h = 2.24$ in.

21–31.

The 4-lb rod AB is attached to the rod BC and collar A using ball-and-socket joints. If BC has a constant angular velocity of 2 rad/s, determine the kinetic energy of AB when it is in the position shown. Assume the angular velocity of AB is directed perpendicular to the axis of AB.



SOLUTION

$$\mathbf{v}_A = v_A\mathbf{i} \qquad \mathbf{v}_B + \{-2\mathbf{j}\}\ \text{ft/s} \qquad \boldsymbol{\omega}_{AB} = \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$$

$$\mathbf{r}_{B/A} = \{-3\mathbf{i} + 1\mathbf{j} + 1\mathbf{k}\}\ \text{ft} \qquad \mathbf{r}_{G/B} = \{1.5\mathbf{i} - 0.5\mathbf{j} - 0.5\mathbf{k}\}$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$-2\mathbf{j} = v_A\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -3 & 1 & 1 \end{vmatrix}$$

Equating **i, j** and **k** components

$$\omega_y - \omega_z + v_A = 0 \qquad \textbf{(1)}$$

$$\omega_x + 3\omega_z = 2 \qquad \textbf{(2)}$$

$$\omega_x + 3\omega_y = 0 \qquad \textbf{(3)}$$

Since ω_{AB} is perpendicular to the axis of the rod,

$$\boldsymbol{\omega}_{AB} \quad \mathbf{r}_{B/A} = (\omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}) \quad (-3\mathbf{i} + 1\mathbf{j} + 1\mathbf{k}) = 0$$

$$-3\omega_x + 1\omega_y + 1\omega_z = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4) yields:

$$\omega_x = 0.1818\ \text{rad/s} \qquad \omega_y = -0.06061\ \text{rad/s} \qquad \omega_z = 0.6061\ \text{rad/s}$$

$$v_A = 0.6667\ \text{ft/s}$$

Hence $\boldsymbol{\omega}_{AB} = \{0.1818\mathbf{i} - 0.06061\mathbf{j} + 0.6061\mathbf{k}\}$ rad/s $\qquad \mathbf{v}_A = \{0.6667\mathbf{i}\}$ ft/s

$$\mathbf{v}_G = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{G/B}$$

$$= -2\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1818 & -0.06061 & 0.6061 \\ 1.5 & -0.5 & -0.5 \end{vmatrix}$$

$$= \{0.3333\mathbf{i} - 1.0\mathbf{j}\}\ \text{ft/s}$$

$$\omega_{AB}^2 = 0.1818^2 + (-0.06061)^2 + 0.6061^2 = 0.4040$$

$$v_G^2 = (0.3333)^2 + (-1.0)^2 = 1.111$$

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\,\omega_{AB}^2$$

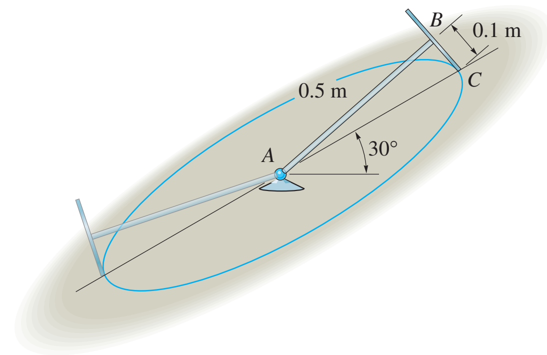
$$= \frac{1}{2}\left(\frac{4}{32.2}\right)(1.111) + \frac{1}{2}\left[\frac{1}{12}\left(\frac{4}{32.2}\right)\left(\sqrt{3^2 + 1^2 + 1^2}\right)^2\right](0.4040)$$

$$= 0.0920\ \text{ft} \cdot \text{lb} \qquad \textbf{Ans.}$$

Ans:
$T = 0.0920$ ft · lb

***21–32.**

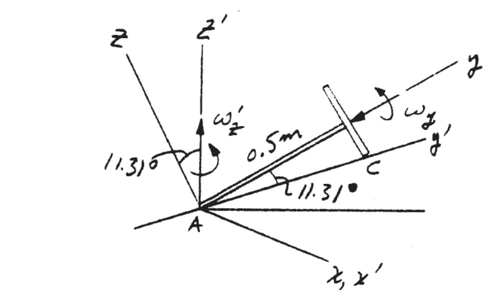
The 2-kg thin disk is connected to the slender rod which is fixed to the ball-and-socket joint at A. If it is released from rest in the position shown, determine the spin of the disk about the rod when the disk reaches its lowest position. Neglect the mass of the rod. The disk rolls without slipping.



SOLUTION

$$I_x = I_z = \frac{1}{4}(2)(0.1)^2 + 2(0.5)^{2-} = 0.505 \text{ kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{2}(2)(0.1)^2 = 0.01 \text{ kg}\cdot\text{m}^2$$



$$\omega = \omega_y + \omega_z = -\omega_v \mathbf{j} + \omega_z \cdot \sin 11.31° \mathbf{j} + \omega_z \cdot \cos 11.31° \mathbf{k}$$

$$= (0.19612\omega_z - \omega_y)\mathbf{j} + (0.98058\omega_z \cdot)\mathbf{k}$$

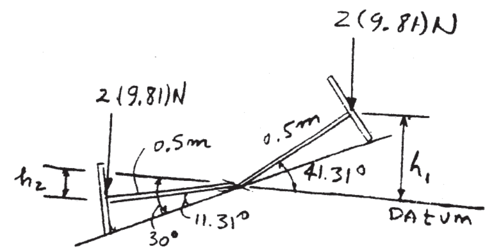
Since $\mathbf{v}_A = \mathbf{v}_C = \mathbf{0}$, then

$$\mathbf{v}_C = \mathbf{v}_A + \omega \times \mathbf{r}_{C/A}$$

$$\mathbf{0} = \mathbf{0} + \left[(0.19612\omega_z \cdot - \omega_y)\mathbf{j} + (0.98058\omega_z \cdot)\mathbf{k}\right] \times (0.5\mathbf{j} - 0.1\mathbf{k})$$

$$0 = -0.019612\omega_z + 0.1\omega_y - 0.49029\omega_z \cdot$$

$$\omega_z = 0.19612\omega_y$$



Thus,

$$\omega = -0.96154\omega_y \mathbf{j} \div 0.19231\omega_y \mathbf{k}$$

$$h_1 = 0.5 \sin 41.31° = 0.3301 \text{ m}, \qquad h_2 = 0.5 \sin 18.69° = 0.1602 \text{ m}$$

$$T_1 + V_1 = T_2 + V_2$$

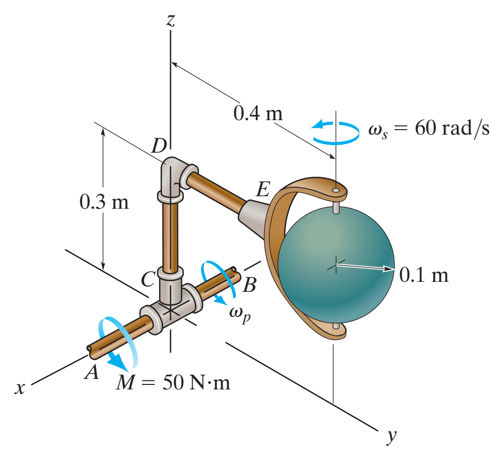
$$0 + 2(9.81)(0.3301) = \left[0 + \frac{1}{2}(0.01)(-0.96154\omega_y)^2 + \frac{1}{2}(0.505)(0.19231\omega_y)^2\right] - 2(9.81)(0.1602$$

$\omega_y = 26.2$ rad/s **Ans.**

Ans:
$\omega_y = 26.2$ rad/s

21–33.

The 20-kg sphere rotates about the axle with a constant angular velocity of $\omega_s = 60$ rad/s. If shaft AB is subjected to a torque of $M = 50$ N·m, causing it to rotate, determine the value of ω_p after the shaft has turned 90° from the position shown. Initially, $\omega_p = 0$. Neglect the mass of arm CDE.

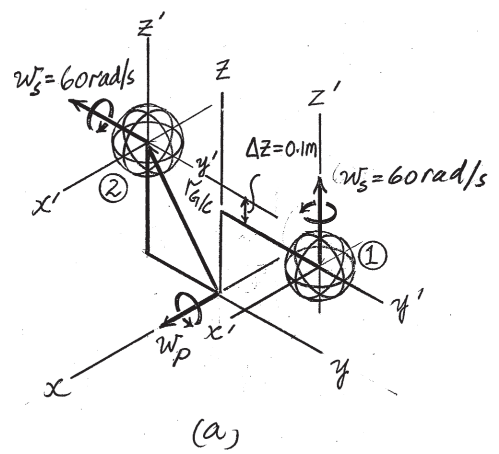


SOLUTION

The mass moments of inertia of the sphere about the x', y', and z' axes are

$$I_{x'} = I_{y'} = I_{z'} = \frac{2}{5}mr^2 = \frac{2}{5}(20)(0.1^2) = 0.08 \text{ kg}\cdot\text{m}^2$$

When the sphere is at position (1), Fig. a, $\omega_p = 0$. Thus, the velocity of its mass center is zero and its angular velocity is $\omega_1 = [60\mathbf{k}]$ rad/s. Thus, its kinetic energy at this position is



$$T = \frac{1}{2}m(v_G)_1^2 + \frac{1}{2}I_{x'}(\omega_1)_{x'}^2 + \frac{1}{2}I_{y'}(\omega_1)_{y'}^2 + \frac{1}{2}I_{z'}(\omega_1)_{z'}^2$$

$$= 0 + 0 + 0 + \frac{1}{2}(0.08)(60^2)$$

$$= 144 \text{ J}$$

When the sphere is at position (2), Fig. a, $\omega_p = \omega_p\mathbf{i}$. Then the velocity of its mass center is $(\mathbf{v}_G)_2 = \omega_p \times \mathbf{v}_{G/C} = (\omega_p\mathbf{i}) \times (-0.3\mathbf{j} + 0.4\mathbf{k}) = -0.4\omega_p\mathbf{j} - 0.3\omega_p\mathbf{k}$. Then $(v_G)_2{}^2 = (-0.4\omega_p)^2 + (-0.3\omega_p)^2 = 0.25\omega_p^2$. Also, its angular velocity at this position is $\omega_2 = \omega_p\mathbf{i} - 60\mathbf{j}$. Thus, its kinetic energy at this position is

$$T = \frac{1}{2}m(v_G)_2^2 + \frac{1}{2}I_{x'}(\omega_2)_{x'}^2 + \frac{1}{2}I_{y'}(\omega_2)_{y'}^2 + \frac{1}{2}I_{z'}(\omega_2)_{z'}^2$$

$$= \frac{1}{2}(20)\left(0.25\omega_p^2\right) + \frac{1}{2}(0.08)\left(\omega_p^2\right) + \frac{1}{2}(0.08)(-60)^2$$

$$= 2.54\omega_p^2 + 144$$

When the sphere moves from position (1) to position (2), its center of gravity raises vertically $\Delta z = 0.1$ m. Thus, its weight $\mathbf{W}$ does negative work.

$$U_W = -W\Delta z = -20(9.81)(0.1) = -19.62 \text{ J}$$

Here, the couple moment M does positive work.

$$U_W = M\theta = 50\left(\frac{\pi}{2}\right) = 25\pi\text{J}$$

Applying the principle of work and energy,

$$T_1 + \Sigma U_{1-2} = T_2$$

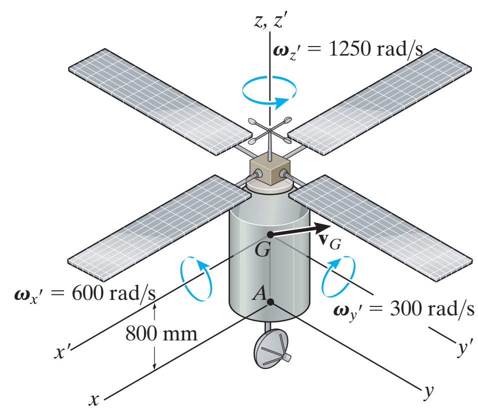
$$144 + 25\pi + (-19.62) = 2.54\omega_p^2 + 144$$

$$\omega_p = 4.82 \text{ rad/s}$$ **Ans.**

Ans:
$\omega_p = 4.82$ rad/s

21–34.

The 200-kg satellite has its center of mass at point G. Its radii of gyration about the z', x', y' axes are $k_{z'} = 300$ mm, $k_{x'} = k_{y'} = 500$ mm, respectively. At the instant shown, the satellite rotates about the x', y', and z' axes with the angular velocity shown, and its center of mass G has a velocity of $\mathbf{v}_G = \{-250\mathbf{i} + 200\mathbf{j} + 120\mathbf{k}\}$ m/s. Determine the angular momentum of the satellite about point A at this instant.



SOLUTION

The mass moments of inertia of the satellite about the x', y', and z' axes are

$$I_{x'} = I_{y'} = 200\left(0.5^2\right) = 50 \text{ kg}\cdot\text{m}^2$$

$$I_{z'} = 200\left(0.3^2\right) = 18 \text{ kg}\cdot\text{m}^2$$

Due to symmetry, the products of inertia of the satellite with respect to the x', y', and z' coordinate system are equal to zero.

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

The angular velocity of the satellite is

$$\omega = [600\mathbf{i} + 300\mathbf{j} + 1250\mathbf{k}] \text{ rad/s}$$

Thus,

$$\omega_{x'} = 600 \text{ rad/s} \qquad \omega_{y'} = -300 \text{ rad/s} \qquad \omega_{z'} = 1250 \text{ rad/s}$$

Then, the components of the angular momentum of the satellite about its mass center G are

$$(H_G)_{x'} = I_{x'}\omega_{x'} = 50(600) = 30\,000 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(H_G)_{y'} = I_{y'}\omega_{y'} = 50(-300) = -15\,000 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(H_G)_{z'} = I_{z'}\omega_{z'} = 18(1250) = 22\,500 \text{ kg}\cdot\text{m}^2/\text{s}$$

Thus,

$$\mathbf{H}_G = [30\,000\mathbf{i} - 15\,000\mathbf{j} + 22\,500\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s}$$

The angular momentum of the satellite about point A can be determined from

$$\mathbf{H}_A = \mathbf{r}_{G/A} \times m\mathbf{v}_G + \mathbf{H}_G$$

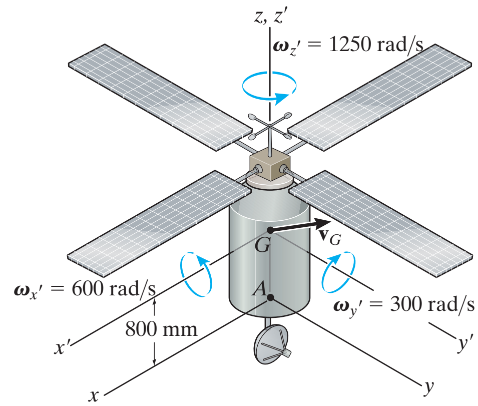
$$= (0.8\mathbf{k}) \times 200(-250\mathbf{i} + 200\mathbf{j} + 120\mathbf{k}) + (30\,000\mathbf{i} - 15\,000\mathbf{j} + 22\,500\mathbf{k})$$

$$= [-2000\mathbf{i} - 55\,000\mathbf{j} + 22\,500\mathbf{k}] \text{ kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{H}_A = \{-2000\mathbf{i} - 55\,000\mathbf{j} + 22\,500\mathbf{k}\}$ kg·m²/s

21–35.

The 200-kg satellite has its center of mass at point G. Its radii of gyration about the z', x', y' axes are $k_{z'} = 300$ mm, $k_{x'} = k_{y'} = 500$ mm, respectively. At the instant shown, the satellite rotates about the x', y', and z' axes with the angular velocity shown, and its center of mass G has a velocity of $\mathbf{v}_G = \{-250\mathbf{i} + 200\mathbf{j} + 120\mathbf{k}\}$ m/s. Determine the kinetic energy of the satellite at this instant.



SOLUTION

The mass moments of inertia of the satellite about the x', y', and z' axes are

$$I_{x'} = I_{y'} = 200\left(0.5^2\right) = 50 \text{ kg} \cdot \text{m}^2$$

$$I_{z'} = 200\left(0.3^2\right) = 18 \text{ kg} \cdot \text{m}^2$$

Due to symmetry, the products of inertia of the satellite with respect to the x', y', and z, coordinate system are equal to zero.

$$I_{x'y'} = I_{y'z'} = I_{x'z'} = 0$$

The angular velocity of the satellite is

$$\omega = [600\mathbf{i} - 300\mathbf{j} + 1250\mathbf{k}] \text{ rad/s}$$

Thus,

$$\omega_{x'} = 600 \text{ rad/s} \qquad \omega_{y'} = -300 \text{ rad/s} \qquad \omega_{z'} = 1250 \text{ rad/s}$$

Since $v_G{}^2 = (-250)^2 + 200^2 + 120^2 = 116\,900 \text{ m}^2/\text{s}^2$, the kinetic energy of the satellite can be determined from

$$T = \frac{1}{2}mv_G{}^2 + \frac{1}{2}I_{x'}\omega_{x'}{}^2 + \frac{1}{2}I_{y'}\omega_{y'}{}^2 + \frac{1}{2}I_{z'}\omega_{z'}{}^2$$

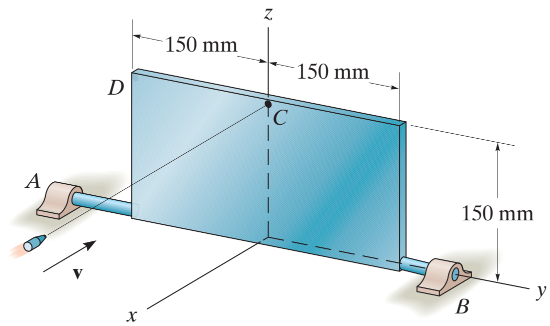
$$= \frac{1}{2}(200)(116\,900) + \frac{1}{2}(50)\left(600^2\right) + \frac{1}{2}(50)(-300)^2 + \frac{1}{2}(18)\left(1250^2\right)$$

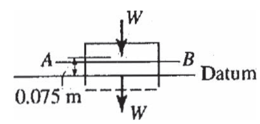
$$= 37.0025\left(10^6\right)\text{J} = 37.0 \text{ MJ}$$ **Ans.**

Ans:
$T = 37.0$ MJ

***21–36.**

The 15-kg rectangular plate is free to rotate about the y axis because of the bearing supports at A and B. When the plate is balanced in the vertical plane, a 3-g bullet is fired into it, perpendicular to its surface, with a velocity $\mathbf{v} = \{-2000\mathbf{i}\}$ m/s. Compute the angular velocity of the plate at the instant it has rotated 180°. If the bullet strikes corner D with the same velocity $\mathbf{v}$, instead of at C, does the angular velocity remain the same? Why or why not?





SOLUTION

Consider the projectile and plate as an entire system.

Angular momentum is conserved about the AB axis.

$$(\mathbf{H}_{AB})_1 = -(0.003)(2000)(0.15)\mathbf{j} = \{-0.9\mathbf{j}\}$$

$$(\mathbf{H}_{AB})_1 = (\mathbf{H}_{AB})_2$$

$$-0.9\mathbf{j} = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

Equating components,

$$\omega_x = 0$$

$$\omega_z = 0$$

$$\omega_y = \frac{-0.9}{\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right]} = -8 \text{ rad/s}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right](8)^2 + 15(9.81)(0.15)$$

$$= \frac{1}{2}\left[\frac{1}{12}(15)(0.15)^2 + 15(0.075)^2\right]\omega_{AB}^2$$

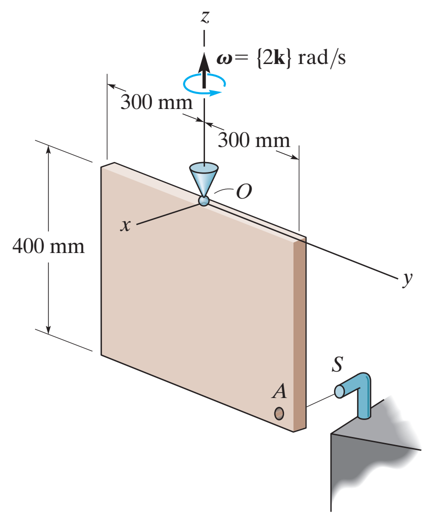
$\omega_{AB} = 21.4$ rad/s **Ans.**

If the projectile strikes the plate at D, the angular velocity is the same, only the impulsive reactions at the bearing supports A and B will be different.

Ans:
$\omega_{AB} = 21.4$ rad/s

21–37.

The 5-kg thin plate is suspended at O using a ball-and-socket joint. It is rotating with a constant angular velocity $\boldsymbol{\omega} = \{2\mathbf{k}\}$ rad/s when the corner A strikes the hook at S, which provides a permanent connection. Determine the angular velocity of the plate immediately after impact.



SOLUTION

Angular momentum is conserved about the OA axis.

$$(\mathbf{H}_O)_1 = I_z\omega_z\mathbf{k}$$

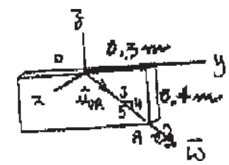
$$= \left[\frac{1}{12}(5)(0.6)^2\right](2)\mathbf{k} = 0.30\mathbf{k}$$

$$\mathbf{u}_{OA} = \{0.6\mathbf{j} - 0.8\mathbf{k}\}$$

$$(\mathbf{H}_{OA})_1 = (\mathbf{H}_O)_1 \cdot \mathbf{u}_{OA}$$

$$= (0.30)(-0.8) = -0.24$$

$$\omega = 0.6\omega\mathbf{j} - 0.8\omega\mathbf{k} \qquad \textbf{(1)}$$



$$(\mathbf{H}_O)_2 = I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$= \frac{1}{3}(5)(0.4)^2\omega_y\mathbf{j} + \frac{1}{12}(5)(0.6)^2\omega_z\mathbf{k}$$

$$= 0.2667\omega_y\mathbf{j} + 0.150\omega_z\mathbf{k}$$

From Eq. (1),

$$\omega_y = 0.6\omega$$

$$\omega_z = -0.8\omega$$

$$(\mathbf{H}_O)_2 = 0.16\omega\mathbf{j} - 0.120\omega\mathbf{k}$$

$$(\mathbf{H}_{OA})_2 = (\mathbf{H}_O)_2 - \mathbf{u}_{OA}$$

$$= 0.16\omega(0.6) + (0.12\omega)(0.8) = 0.192\omega$$

Thus,

$$(\mathbf{H}_{OA})_1 = (\mathbf{H}_{OA})_2$$

$$-0.24 = 0.192\omega$$

$$\omega = -1.25 \text{ rad/s}$$

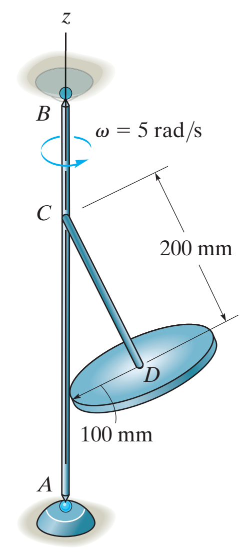
$$\omega = -1.25\mathbf{u}_{OA}$$

$$\omega = \{-0.750\mathbf{j} + 1.00\mathbf{k}\} \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\boldsymbol{\omega} = \{-0.750\mathbf{j} + 1.00\mathbf{k}\}$ rad/s

21–38.

Determine the kinetic energy of the 7-kg disk and 1.5-kg rod when the assembly is rotating about the z axis at $\omega = 5$ rad/s.



SOLUTION

Due to symmetry

$$I_{xy} = I_{yz} = I_{zx} = 0$$

$$I_y = I_z = \left[\frac{1}{4}(7)(0.1)^2 + 7(0.2)^2\right] + \frac{1}{3}(1.5)(0.2)^2$$

$$= 0.3175 \text{ kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{2}(7)(0.1)^2 = 0.035 \text{ kg}\cdot\text{m}^2$$

For z' axis

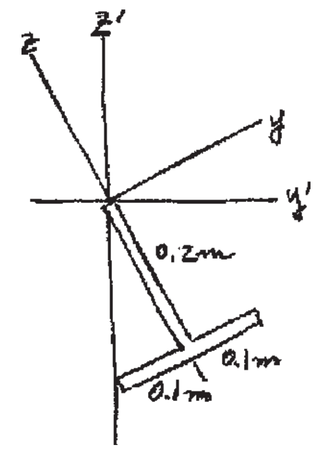
$u_z = \cos 26.56° = 0.8944 \qquad u_y = \cos 116.57° = -0.4472$

$u_x = \cos 90° = 0$

$$I_{z'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy}u_x u_y - 2I_{yz}u_y u_z - 2I_{zx}u_z u_x$$

$$= 0 + 0.3175(-0.4472)^2 + 0.035(0.8944)^2 - 0 - 0 - 0$$

$$= 0.0915 \text{ kg}\cdot\text{m}^2$$



$$T = \frac{1}{2}I_{x'}\omega_{x'}^2 + \frac{1}{2}I_{y'}\omega_{y'}^2 + \frac{1}{2}I_{z'}\omega_{z'}^2$$

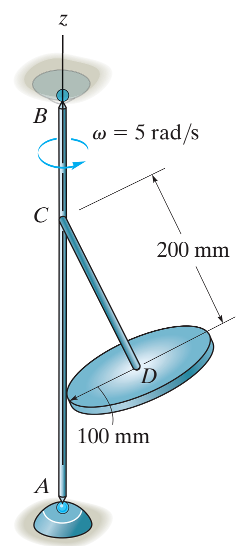
$$= 0 + 0 + \frac{1}{2}(0.0915)(5)^2$$

$$= 1.14 \text{ J}$$ **Ans.**

Ans:
$T = 1.14$ J

21–39.

Determine the angular momentum $\mathbf{H}_z$ of the 7-kg disk and 1.5-kg rod when the assembly is rotating about the z axis at $\omega = 5$ rad/s.



SOLUTION

Due to symmetry $\qquad I_{xy} = I_{yz} = I_{zx} = 0$

$$I_y = I_x = \left[\frac{1}{4}(7)(0.1)^2 + 7(0.2)^2\right] + \frac{1}{3}(1.5)(0.2)^2$$

$$= 0.3175 \text{ kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{2}(7)(0.1)^2 = 0.035 \text{ kg}\cdot\text{m}^2$$

For z' axis

$u_z = \cos 26.56° = 0.8944 \qquad u_y = \cos 116.57° = -0.4472$

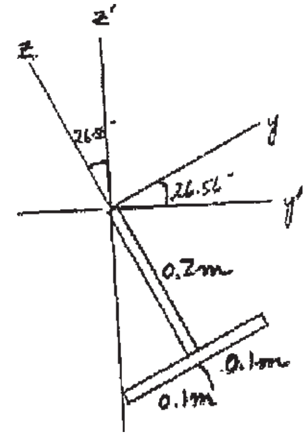
$u_x = \cos 90° = 0$

$$I_{z'} = I_x u_x^2 + I_y u_y^2 + I_z u_z^2 - 2I_{xy} u_x u_y - 2I_{yz} u_y u_z - 2I_{zx} u_z u_x$$

$$= 0 + 0.3175(-0.4472)^2 + 0.035(0.8944)^2 - 0 - 0 - 0$$

$$= 0.0915 \text{ kg}\cdot\text{m}^2$$

$\omega_x = \omega_y = 0$



$$H_z = -I_{zx}\omega_x - I_{xy}\omega_y + I_{zz}\omega_z$$

$$= -0 - 0 + 0.0915(5)$$

$$= 0.4575 \text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Ans:
$H_z = 0.4575 \text{ kg}\cdot\text{m}^2/\text{s}$

***21–40.**

Derive the scalar form of the rotational equation of motion about the x axis if $\mathbf{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *not constant* with respect to time.

SOLUTION

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\,\mathbf{i} + H_y\,\mathbf{j} + H_z\,\mathbf{k})$$

$$= \left(\dot{H}_x\,\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\,\mathbf{k}\right)_{xyz} + \Omega \times (H_x\,\mathbf{i} + H_y\mathbf{j} + H_z\,\mathbf{k})$$

Substitute $\Omega = \Omega_x\,\mathbf{i} + \Omega_y\,\mathbf{j} + \Omega_z\,\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z\,H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x\,H_z + \Omega_z H_x\right)\mathbf{j}$$

$$+ \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y\,H_x + \Omega_x\,H_y\right)\mathbf{k}$$

Subsitute H_x, H_y and H_z using Eq. 21–10. For the $\mathbf{i}$ component,

$$\Sigma M_x = \frac{d}{dt}(I_x\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\omega_x)$$

$$+ \Omega_y\,(I_z\,\omega_z - I_{zx}\,\omega_x - I_{zy}\,\omega_y) \qquad \textbf{Ans.}$$

One can obtain y and z components in a similar manner.

Ans:

$$\Sigma M_x = \frac{d}{dt}(I_x\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z)$$
$$- \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\omega_x)$$
$$+ \Omega_y\,(I_z\,\omega_z - I_{zx}\,\omega_x - I_{zy}\,\omega_y)$$

21–41.

Derive the scalar form of the rotational equation of motion about the x axis if $\mathbf{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *constant* with respect to time.

SOLUTION

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\,\mathbf{i} + H_y\,\mathbf{j} + H_z\,\mathbf{k})$$

$$= \left(\dot{H}_x\,\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\,\mathbf{k}\right)_{xyz} + \Omega \times (H_x\,\mathbf{i} + H_y\mathbf{j} + H_z\,\mathbf{k})$$

Substitute $\Omega = \Omega_x\,\mathbf{i} + \Omega_y\,\mathbf{j} + \Omega_z\,\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z\,H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x\,H_z + \Omega_z H_x\right)\mathbf{j} + \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y\,H_x + \Omega_x\,H_y\right)\mathbf{k}$$

Substitute H_x, H_y and H_z using Eq. 21–10. For the $\mathbf{i}$ component

$$\Sigma M_x = \frac{d}{dt}(I_x\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\omega_x) + \Omega_y\,(I_z\,\omega_z - I_{zx}\,\omega_x - I_{zy}\,\omega_y)$$

For constant inertia, expanding the time derivative of the above equation yields

$$\Sigma M_x = (I_x\,\dot{\omega}_x - I_{xy}\,\dot{\omega}_y - I_{xz}\dot{\omega}_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\,\omega_x) + \Omega_y\,(I_z\,\omega_z - I_{zx}\omega_x - I_{zy}\,\omega_y) \qquad \textbf{Ans.}$$

One can obtain y and z components in a similar manner.

Ans:
$\Sigma M_x = (I_x\dot{\omega}_x - I_{xy}\dot{\omega}_y - I_{xz}\dot{\omega}_z)$
$-\Omega_z(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x)$
$+\Omega_y(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y)$
Similarly for ΣM_y and ΣM_z.

21–42.

Derive the Euler equations of motion for $\mathbf{\Omega} \neq \boldsymbol{\omega}$, i.e., Eqs. 21–26.

SOLUTION

In general

$$\mathbf{M} = \frac{d}{dt}(H_x\,\mathbf{i} + H_y\,\mathbf{j} + H_z\,\mathbf{k})$$

$$= \left(\dot{H}_x\,\mathbf{i} + \dot{H}_y\mathbf{j} + \dot{H}_z\,\mathbf{k}\right)_{xyz} + \Omega \times (H_x\,\mathbf{i} + H_y\mathbf{j} + H_z\,\mathbf{k})$$

Substitute $\Omega = \Omega_x\,\mathbf{i} + \Omega_y\,\mathbf{j} + \Omega_z\,\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(\left(\dot{H}_x\right)_{xyz} - \Omega_z\,H_y + \Omega_y H_z\right)\mathbf{i} + \left(\left(\dot{H}_y\right)_{xyz} - \Omega_x\,H_z + \Omega_z H_x\right)\mathbf{j} + \left(\left(\dot{H}_z\right)_{xyz} - \Omega_y\,H_x + \Omega_x\,H_y\right)\mathbf{k}$$

Substitute H_x, H_y and H_z using Eq. 21–10. For the **i** component

$$\Sigma M_x = \frac{d}{dt}(I_x\,\omega_x - I_{xy}\,\omega_y - I_{xz}\,\omega_z) - \Omega_z\,(I_y\,\omega_y - I_{yz}\,\omega_z - I_{yx}\omega_x) + \Omega_y\,(I_z\,\omega_z - I_{zx}\,\omega_x - I_{zy}\,\omega_y)$$

Set $I_{xy} = I_{yz} = I_{zx} = 0$ and require I_x, I_y, I_z to be constant. This yields

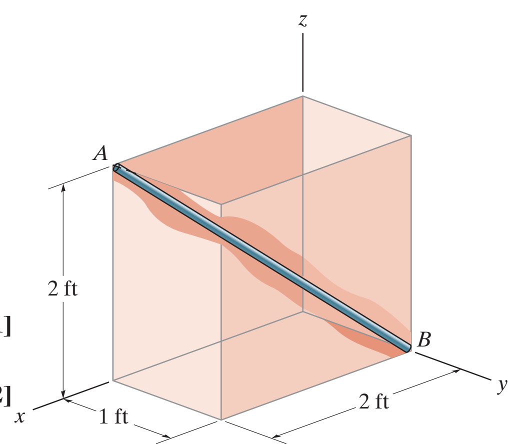
$$\Sigma M_x = I_x\,\dot{\omega}_x - I_y\,\Omega_z\,\omega_y + I_z\,\Omega_y\omega_z \qquad \textbf{Ans.}$$

One can obtain y and z components in a similar manner.

Ans:
$\Sigma M_x = I_x\,\dot{\omega}_x - I_y\,\Omega_z\,\omega_y + I_z\,\Omega_y\omega_z$

21–43.

The 4-lb bar rests along the smooth corners of an open box. At the instant shown, the box has a velocity $\mathbf{v} = \{3\mathbf{j}\}$ ft/s and an acceleration $\mathbf{a} = \{-6\mathbf{j}\}$ ft/s^2. Determine the x, y, z components of force which the corners exert on the bar.

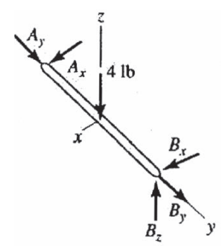


SOLUTION

$$\Sigma F_x = m(a_G)_x; \qquad A_x + B_x = 0 \qquad [1]$$

$$\Sigma F_y = m(a_G)_y; \qquad A_y + B_y = \left(\frac{4}{32.2}\right)(-6) \qquad [2]$$

$$\Sigma F_z = m(a_G)_z; \qquad B_z - 4 = 0 \qquad B_z = 4 \text{ lb} \qquad \textbf{Ans.}$$



Applying Eq. 21–25 with $\omega_x = \omega_y = \omega_z = 0 \;\; \dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$

$$\Sigma(M_G)_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z; \qquad B_y(1) - A_y(1) + 4(0.5) = 0 \qquad [3]$$

$$\Sigma(M_G)_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x; \qquad A_x(1) - B_x(1) + 4(1) = 0 \qquad [4]$$

Solving Eqs. [1] to [4] yields:

$A_x = -2.00$ lb $\qquad A_y = 0.627$ lb $\qquad B_x = 2.00$ lb $\qquad B_y = -1.37$ lb $\qquad$ **Ans.**

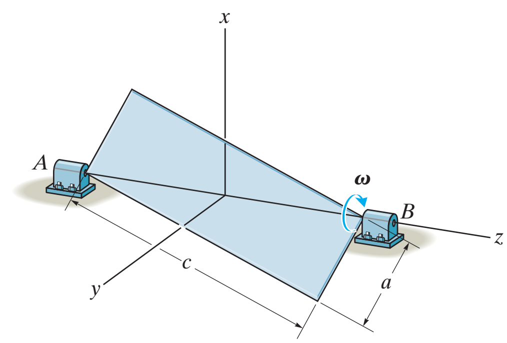
$$\Sigma(M_G)_z = I_z\dot{\omega}_z - (I_x - I_y)\,\omega_x\omega_y;$$

$$(-2.00)(0.5) - (2.00)(0.5) - (-1.37)(1) + (0.627)(1) = 0 \qquad \textbf{(O.K!)}$$

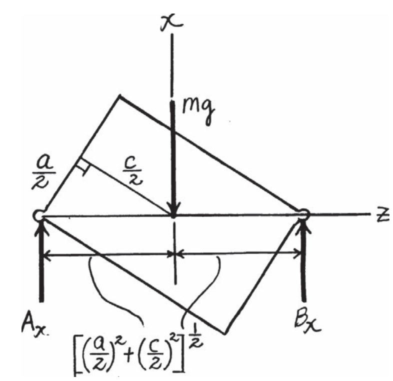
Ans:
$B_z = 4$ lb
$A_x = -2.00$ lb
$A_y = 0.627$ lb
$B_x = 2.00$ lb
$B_y = -1.37$ lb

***21–44.**

The uniform rectangular plate has a mass of $m = 2$ kg and is given a rotation of $\omega = 4$ rad/s about its bearings at A and B. If $a = 0.2$ m and $c = 0.3$ m, determine the vertical reactions at the instant shown. Use the x, y, z axes shown and note that $I_{zx} = -\left(\frac{mac}{12}\right)\left(\frac{c^2 - a^2}{c^2 + a^2}\right)$.



SOLUTION



$\omega_x = 0, \qquad \omega_y = 0, \qquad \omega_z = -4$

$\dot{\omega}_x = 0, \qquad \dot{\omega}_y = 0, \qquad \dot{\omega}_z = 0$

$$\Sigma M_y = I_{yy}\,\dot{\omega}_y - (I_{zz} - I_{xx})\omega_z\,\omega_x - I_{yz}\left(\dot{\omega}_z - \omega_x\,\omega_y\right)$$

$$- I_{zx}\left(\omega_z^2 - \omega_x^2\right) - I_{xy}\left(\dot{\omega}_x + \omega_y\,\omega_z\right)$$

$$B_x\left[\left(\frac{a}{2}\right)^2 + \left(\frac{c}{2}\right)^2\right]^{\frac{1}{2}} - A_x\left[\left(\frac{a}{2}\right)^2 + \left(\frac{c}{2}\right)^2\right]^{\frac{1}{2}} = -I_{zx}\,(\omega)^2$$

$$B_x - A_x = \left(\frac{mac}{6}\right)\left(\frac{c^2 - a^2}{\left[a^2 + c^2\right]^{\frac{3}{2}}}\right)\omega^2$$

$$\Sigma F_x = m(a_G)_x\,; \qquad A_x + B_x - mg = 0$$

Substitute the data,

$$B_x - A_x = \frac{2(0.2)(0.3)}{6}\left[\frac{(0.3)^2 - (0.2)^2}{\left[(0.3)^2 + (0.2)^2\right]^{\frac{3}{2}}}\right](-4)^2 = 0.34135$$

$$A_x + B_x = 2(9.81)$$

Solving:

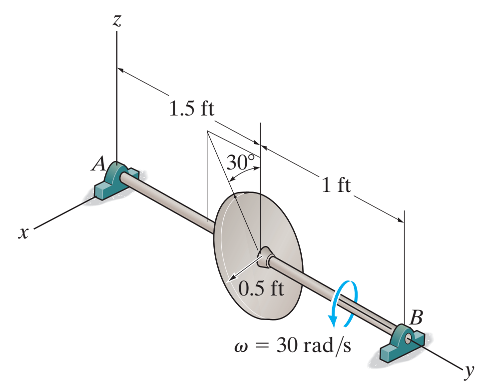
$A_x = 9.64$ N **Ans.**

$B_x = 9.98$ N **Ans.**

Ans:
$A_x = 9.64$ N
$B_x = 9.98$ N

21–45.

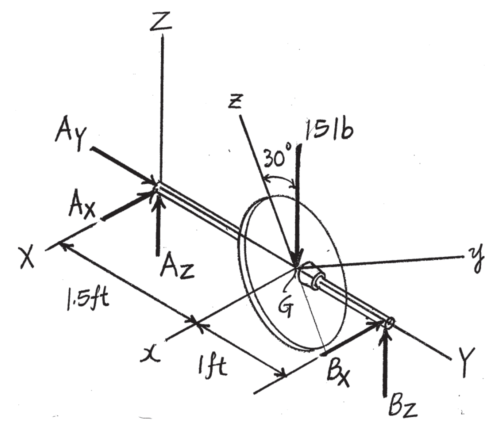
If the shaft AB is rotating with a constant angular velocity of $\omega = 30$ rad/s, determine the X, Y, Z components of reaction at the thrust bearing A and journal bearing B at the instant shown. The disk has a weight of 15 lb. Neglect the weight of the shaft AB.



SOLUTION

The rotating xyz frame is set with its origin at the plate's mass center, Fig. *a*. This frame will be fixed to the disk so that its angular velocity is $\Omega = \omega$ and the x, y, and z axes will always be the principle axes of inertia of the disk. Referring to Fig. *b*,

$$\omega = [30\cos 30°\mathbf{j} - 30\sin 30°\mathbf{k}]\text{ rad/s} = [25.98\mathbf{j} - 15\mathbf{k}]\text{ rad/s}$$



(a)

Thus,

$$\omega_x = 0 \qquad \omega_y = 25.98\text{ rad/s} \qquad \omega_z = -15\text{ rad/s}$$

Since ω is always directed towards the $+Y$ axis and has a constant magnitude, $\dot{\omega} = 0$. Also, since $\Omega = \omega$, $\left(\dot{\omega}_{xyz}\right) = \dot{\omega} = 0$. Thus,

$$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$$

The mass moments of intertia of the disk about the x, y, z axes are

$$I_x = I_z = \frac{1}{4}\left(\frac{15}{32.2}\right)\left(0.5^2\right) = 0.02911\text{ slug}\cdot\text{ft}^2$$

$$I_y = \frac{1}{2}\left(\frac{15}{32.2}\right)\left(0.5^2\right) = 0.05823\text{ slug}\cdot\text{ft}^2$$

Applying the equations of motion,

$$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z;\ B_Z(1) - A_Z(1.5) = 0 - (0.05823 - 0.02911)(25.98)(-15)$$

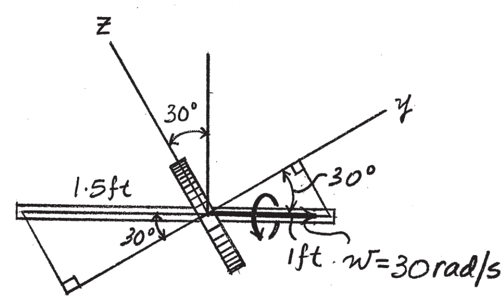
$$B_Z - 1.5A_Z = 11.35 \qquad \textbf{(1)}$$

$$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x;\ B_X(1\sin 30°) - A_X(1.5\sin 30°) = 0 - 0$$

$$B_X - 1.5A_X = 0 \qquad \textbf{(2)}$$

$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y;\ B_X(1\cos 30°) - A_X(1.5\cos 30°) = 0 - 0$$

$$B_X - 1.5A_X = 0$$



(b)

$$\Sigma F_X = m(a_G)_X; \qquad A_X + B_X = 0 \qquad \textbf{(3)}$$

$$\Sigma F_Y = m(a_G)_Y; \qquad A_Y = 0 \qquad \textbf{Ans.}$$

$$\Sigma F_Z = m(a_G)_Z; \qquad A_Z + B_Z - 15 = 0 \qquad \textbf{(4)}$$

Solving Eqs. (1) through (4),

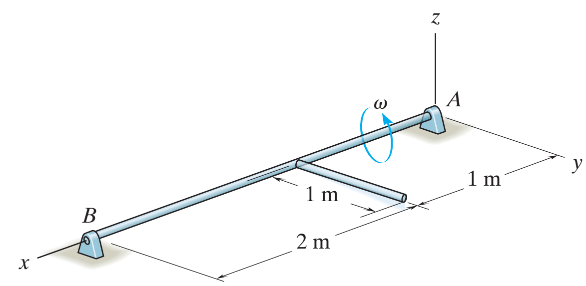
$$A_Z = 1.461\text{ lb} \qquad B_Z = 13.54\text{ lb} = 13.5\text{ lb} \qquad \textbf{Ans.}$$

$$A_X = B_X = 0 \qquad \textbf{Ans.}$$

Ans:
$A_Z = 1.46$ lb
$B_Z = 13.5$ lb
$A_X = A_Y = B_X = 0$

21–46.

The assembly is supported by journal bearings at A and B, which develop only y and z force reactions on the shaft. If the shaft is rotating in the direction shown at $\boldsymbol{\omega} = \{2\mathbf{i}\}$ rad/s, determine the reactions at the bearings when the assembly is in the position shown. Also, what is the shaft's angular acceleration? The mass per unit length of each rod is 5 kg/m.



SOLUTION

Equations of Motion. The inertia properties of the assembly are

$$I_x = \frac{1}{3}[5(1)](1^2) = 1.6667\ \text{kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{3}[5(3)](3^2) + [0 + 5(1)(1^2)] = 50.0\ \text{kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{3}[5(3)](3^2) + \left\{\frac{1}{12}[5(1)](1^2) + 5(1)(1^2 + 0.5^2)\right\} = 51.67\ \text{kg}\cdot\text{m}^2$$

$$I_{xy} = 0 + [5(1)](1)(0.5) = 2.50\ \text{kg}\cdot\text{m}^2 \qquad I_{yz} = I_{zx} = 0$$

with $\dot{\omega}_x = 2$ rad/s, $\omega_y = \omega_z = 0$ and $\dot{\omega}_y = \dot{\omega}_z = 0$ by referring to the FBD of the assembly, Fig. a,

$\Sigma M_x = I_x\,\dot{\omega}_x; \qquad -[5(1)](9.81)(0.5) = 1.6667\,\dot{\omega}_x$

$\dot{\omega}_x = -14.715\ \text{rad/s}^2 = -14.7\ \text{rad/s}^2$ **Ans.**

$\Sigma M_y = -I_{xy}\,\dot{\omega}_x; \quad [5(1)](9.81)(1) + [5(2)](9.81)(1.5) - B_z(3) = -2.50(-14.715)$

$B_z = 77.6625\ \text{N} = 77.7\ \text{N}$ **Ans.**

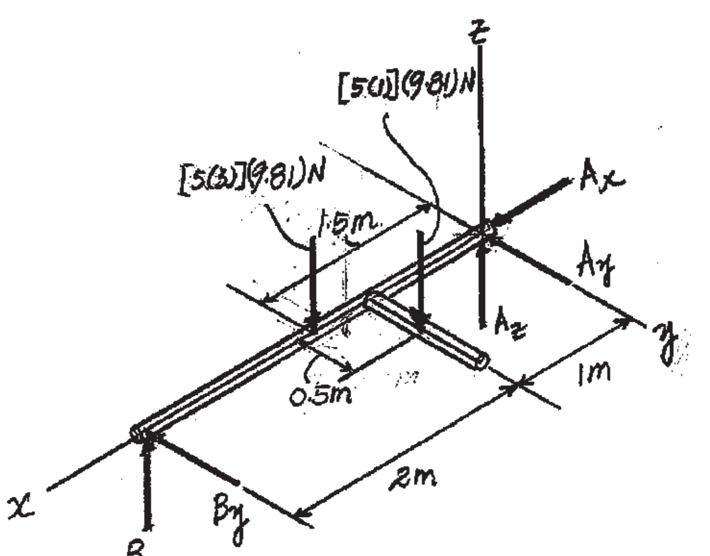
$\Sigma M_z = -I_{xy}\,\omega_x^2; \quad -B_y(3) = -2.50(2^2) \qquad B_y = 3.3333\ \text{N} = 3.33\ \text{N}$ **Ans.**

$\Sigma F_x = M(a_G)_x; \qquad A_x = 0$ **Ans.**

$\Sigma F_y = M(a_G)_y; \quad -A_y - 3.333 = [5(1)][-2^2(0.5)] \quad A_y = 6.667\ \text{N} = 6.67\ \text{N}$ **Ans.**

$\Sigma F_z = M(a_G)_z; \quad A_z + 77.6625 - [5(3)](9.81) - [5(1)](9.81) = [5(1)][-14.715(0.5)]$

$A_z = 81.75\ \text{N}$ **Ans.**



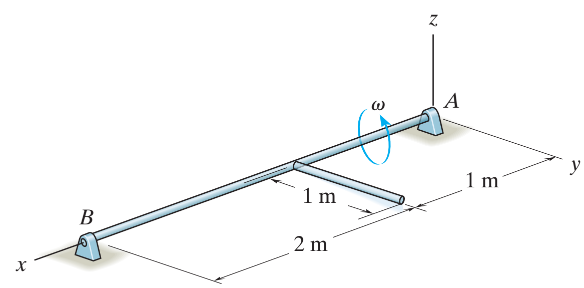
(a)

(1.5, 2, 0)
(1, 0.5, 0)

Ans:
$\dot{\omega}_x = -14.7\ \text{rad/s}^2$
$B_z = 77.7\ \text{N}$
$B_y = 3.33\ \text{N}$
$A_x = 0$
$A_y = 6.67\ \text{N}$
$A_z = 81.75\ \text{N}$

21–47.

The assembly is supported by journal bearings at A and B, which develop only y and z force reactions on the shaft. If the shaft A is subjected to a couple moment $\mathbf{M} = \{40\mathbf{i}\}\ \text{N}\cdot\text{m}$, and at the instant shown the shaft has an angular velocity of $\boldsymbol{\omega} = \{2\mathbf{i}\}\ \text{rad/s}$, determine the reactions at the bearings of the assembly at this instant. Also, what is the shaft's angular acceleration? The mass per unit length of each rod is 5 kg/m.



SOLUTION

Equations of Motions. The inertia properties of the assembly are

$$I_x = \frac{1}{3}[5(1)](1^2) = 1.6667\ \text{kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{3}[5(3)](3^2) + \left[0 + [5(1)](1^2)\right] = 50.0\ \text{kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{3}[5(3)](3^2) + \left\{\frac{1}{12}[5(1)](1^2) + [5(1)](1^2 + 0.5^2)\right\} = 51.67\ \text{kg}\cdot\text{m}^2$$

$$I_{xy} = 0 + [5(1)](1)(0.5) = 2.50\ \text{kg}\cdot\text{m}^2 \qquad I_{yz} = I_{zx} = 0$$

with $\omega_x = 2\ \text{rad/s}$, $\omega_y = \omega_z = 0$ and $\dot{\omega}_y = \dot{\omega}_z = 0$ by referring to the FBD of the assembly, Fig. *a*,

$\Sigma M_x = I_x\dot{\omega}_x;\qquad 40 - [5(1)](9.81)(0.5) = 1.6667\,\dot{\omega}_x$

$\dot{\omega}_x = 9.285\ \text{rad/s}^2$ **Ans.**

$\Sigma M_y = -I_{xy}\,\dot{\omega}_x;\qquad [5(1)](9.81)(1) + [5(3)](9.81)(1.5) - B_z(3) = -2.50(9.285)$

$B_z = 97.6625\ \text{N} = 97.7\ \text{N}$ **Ans.**

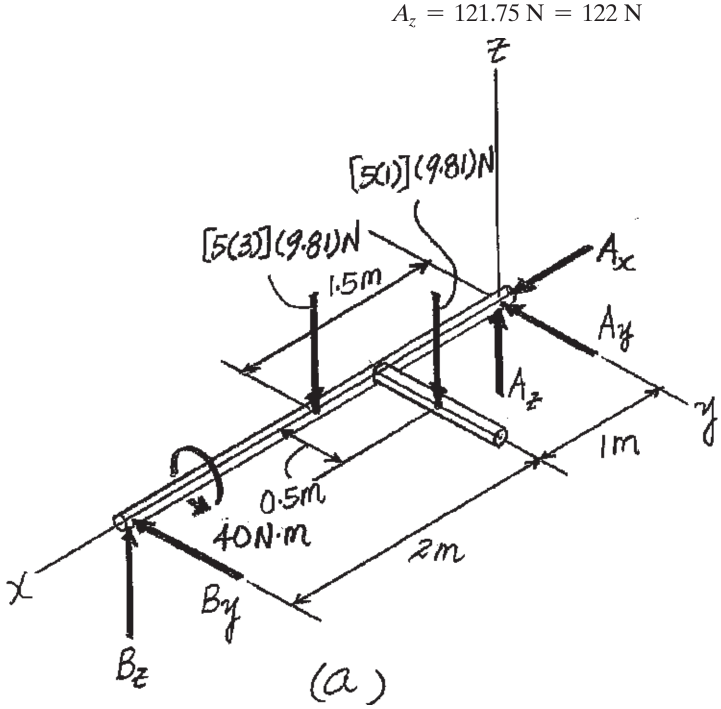
$\Sigma M_z = -I_{xy}\,\omega_x^2;\quad -B_y(3) = -2.50(2^2) \qquad B_y = 3.3333\ \text{N} = 3.33\ \text{N}$ **Ans.**

$\Sigma F_x = M(a_G)_x;\qquad A_x = 0$ **Ans.**

$\Sigma F_y = M(a_G)_y;\ -A_y - 3.3333 = [5(1)]\left[-2^2(0.5)\right]\ \ A_y = 6.6667\ \text{N} = 6.67\ \text{N}$ **Ans.**

$\Sigma F_z = M(a_G)_z;\ \ A_z + 97.6625 - [5(3)](9.81) - [5(1)](9.81) = [5(1)][9.285(0.5)]$

$A_z = 121.75\ \text{N} = 122\ \text{N}$ **Ans.**

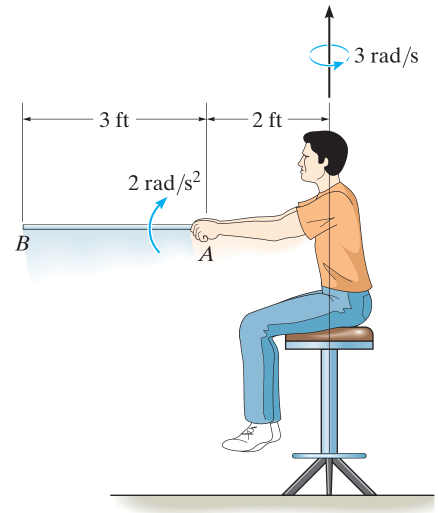


(a)

Ans:
$\dot{\omega}_x = 9.285\ \text{rad/s}^2$
$B_z = 97.7\ \text{N}$
$B_y = 3.33\ \text{N}$
$A_x = 0$
$A_y = 6.67\ \text{N}$
$A_z = 122\ \text{N}$

***21–48.**

The man sits on a swivel chair which is rotating with a constant angular velocity of 3 rad/s. He holds the uniform 5-lb rod AB horizontal. He suddenly gives it an angular acceleration of $2\ \text{rad/s}^2$, measured relative to him, as shown. Determine the required force and moment components at the grip, A, necessary to do this. Establish axes at the rod's center of mass G, with $+z$ upward, and $+y$ directed along the axis of the rod towards A.



SOLUTION

$$I_x = I_z = \frac{1}{12}\left(\frac{5}{32.2}\right)(3)^2 = 0.1165\ \text{ft}^4$$

$$I_y = 0$$

$$\Omega = \omega = 3\mathbf{k}$$

$$\omega_x = \omega_y = 0$$

$$\omega_z = 3\ \text{rad/s}$$

$$\dot{\Omega} = (\dot{\omega}_{xyz}) + \Omega \times \omega = -2\mathbf{i} + 0$$

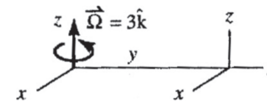
$$\dot{\omega}_x = -2\ \text{rad/s}^2$$

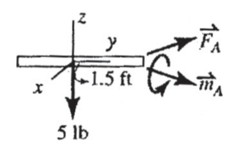
$$\dot{\omega}_y = \dot{\omega}_z = 0$$

$$(a_G)_x = 0$$

$$(a_G)_y = (3.5)(3)^2 = 31.5\ \text{ft/s}^2$$

$$(a_G)_z = 2(1.5) = 3\ \text{ft/s}^2$$





$\Sigma F_x = m(a_G)_x;\qquad A_x = 0$ **Ans.**

$\Sigma F_y = m(a_G)_y;\qquad A_y = \dfrac{5}{32.2}(31.5) = 4.89\ \text{lb}$ **Ans.**

$\Sigma F_z = m(a_G)_z;\qquad -5 + A_z = \dfrac{5}{32.2}(3)$

$A_z = 5.47\ \text{lb}$ **Ans.**

$$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\,\omega_z;$$

$$M_x + 5.47(1.5) = 0.1165(-2) - 0$$

$M_x = -8.43\ \text{lb}\cdot\text{ft}$ **Ans.**

$$\Sigma M_y = I_y\dot{\omega}_x - (I_z - I_x)\omega_z\,\omega_x;$$

$$0 + M_y = 0 - 0$$

$M_y = 0$ **Ans.**

$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\,\omega_y;$$

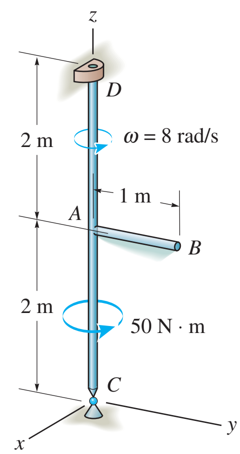
$$M_z = 0 - 0$$

$M_z = 0$ **Ans.**

Ans:
$A_x = 0$
$A_y = 4.89\ \text{Ib}$
$A_z = 5.47\ \text{Ib}$
$M_x = -8.43\ \text{Ib}\cdot\text{ft}$
$M_y = 0$
$M_z = 0$

21–49.

The rod assembly is supported by a ball-and-socket joint at C and a journal bearing at D, which develops only x and y force reactions. The rods have a mass of 0.75 kg/m. Determine the angular acceleration of the rods and the components of reaction at the supports at the instant $\omega = 8$ rad/s as shown.



SOLUTION

$\Omega = \omega = 8\mathbf{k}$

$\omega_x = \omega_y = 0, \qquad \omega_z = 8 \text{ rad/s}$

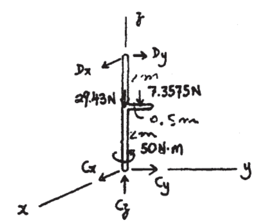
$\dot{\omega}_x = \dot{\omega}_y = 0, \qquad \dot{\omega}_z = \dot{\omega}_z$

$I_{xz} = I_{xy} = 0$

$I_{yz} = 0.75(1)(2)(0.5) = 0.75 \text{ kg} \cdot \text{m}^2$

$I_{zz} = \frac{1}{3}(0.75)(1)(1)^2 = 0.25 \text{ kg} \cdot \text{m}^2$

Eqs. 21–24 become



$\Sigma M_x = I_{yz}\omega_z^2$

$\Sigma M_y = -I_{yz}\dot{\omega}_z$

$\Sigma M_z = I_{zz}\dot{\omega}_z$

Thus,

$-D_y(4) - 7.3575(0.5) = 0.75(8)^2$

$D_y = -12.9 \text{ N}$ **Ans.**

$D_x(4) = -0.75\,\omega_z$

$50 = 0.25\,\dot{\omega}_z$

$\dot{\omega}_z = 200 \text{ rad/s}^2$ **Ans.**

$D_x = -37.5 \text{ N}$ **Ans.**

$\Sigma F_x = m(a_G)_x; \qquad C_x - 37.5 = -1(0.75)(200)(0.5)$

$C_x = -37.5 \text{ N}$ **Ans.**

$\Sigma F_y = m(a_G)_y; \qquad C_y - 12.9 = -(1)(0.75)(8)^2(0.5)$

$C_y = -11.1 \text{ N}$ **Ans.**

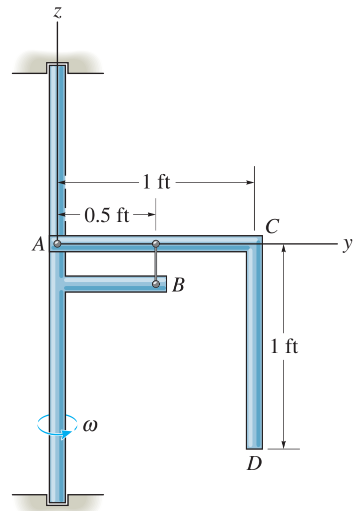
$\Sigma F_z = m(a_G)_z; \qquad C_z - 7.3575 - 29.43 = 0$

$C_z = 36.8 \text{ N}$ **Ans.**

Ans:
$\dot{\omega}_z = 200 \text{ rad/s}^2$
$D_y = -12.9 \text{ N}$
$D_x = -37.5 \text{ N}$
$C_x = -37.5 \text{ N}$
$C_y = -11.1 \text{ N}$
$C_z = 36.8 \text{ N}$

21–50.

The bent uniform rod *ACD* has a weight of 5 lb/ft and is supported at *A* by a pin and at *B* by a cord. If the vertical shaft rotates with a constant angular velocity $\omega = 20$ rad/s, determine the *x, y, z* components of force and moment developed at *A* and the tension in the cord.



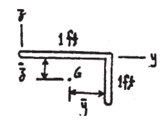
SOLUTION

$w_x = w_y = 0$

$w_z = 20$ rad/s

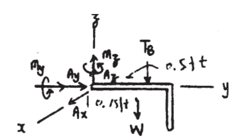
$\dot{w}_x = \dot{w}_y = \dot{w}_z = 0$

The center of mass is located at



$$\bar{z} = \frac{5(1)\left(\frac{1}{2}\right)}{5(2)} = 0.25 \text{ ft}$$

$\bar{y} = 0.25$ ft (symmetry)



$$I_{yz} = \frac{5}{32.2}(-0.5)(1) = -0.0776 \text{ slug} \cdot \text{ft}^2$$

$I_{zx} = 0$

Eqs. 21–24 reduce to

$\Sigma M_x = I_{yz}(w_z)^2;$

$-T_B(0.5) - 10(0.75) = -0.0776(20)^2$

$T_B = 47.1$ lb **Ans.**

$\Sigma M_y = 0; \qquad M_y = 0$ **Ans.**

$\Sigma M_z = 0; \qquad M_z = 0$ **Ans.**

$\Sigma F_x = ma_x; \qquad A_x = 0$ **Ans.**

$$\Sigma F_y = ma_y; \qquad A_y = -\left(\frac{10}{32.2}\right)(20)^2(1 - 0.25)$$

$A_y = -93.2$ lb **Ans.**

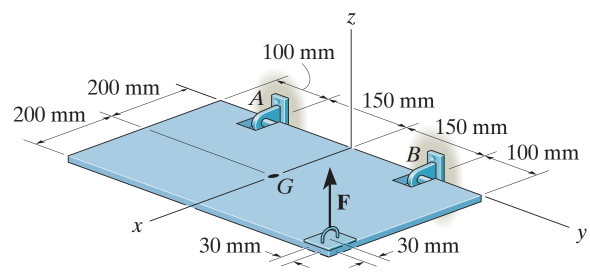
$\Sigma F_z = ma_z; \qquad A_z - 47.1 - 10 = 0$

$A_z = 57.1$ lb **Ans.**

Ans:
$T_B = 47.1$ lb
$M_y = 0$
$M_z = 0$
$A_x = 0$
$A_y = -93.2$ lb
$A_z = 57.1$ lb

21–51.

The uniform hatch door, having a mass of 15 kg and a mass center at G, is supported in the horizontal plane by bearings at A and B. If a vertical force $F = 300$ N is applied to the door as shown, determine the components of reaction at the bearings and the angular acceleration of the door. The bearing at A will resist a component of force in the y direction, whereas the bearing at B will not. For the calculation, assume the door to be a thin plate and neglect the size of each bearing. The door is originally at rest.

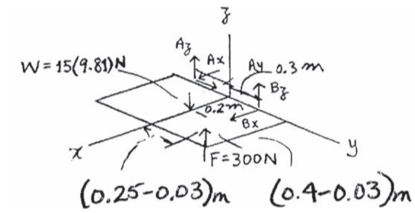


SOLUTION

$$\omega_x = \omega_y = \omega_z = 0$$

$$\dot{\omega}_x = \dot{\omega}_z = 0$$

Eqs. 21–25 reduce to



$\Sigma M_x = 0;\qquad 300(0.25 - 0.03) + B_z(0.15) - A_z(0.15) = 0$

$B_z - A_z = -440$ **(1)**

$\Sigma M_y = I_y\dot{\omega}_y;\qquad 15(9.81)(0.2) - (300)(0.4 - 0.03) = [\frac{1}{12}(15)(0.4)^2 + 15(0.2)^2]\dot{\omega}_y$

$\dot{\omega}_y = -102\text{ rad/s}^2$ **Ans.**

$\Sigma M_z = 0;\qquad -B_x(0.15) + A_x(0.15) = 0$

$\Sigma F_x = m(a_G)_x;\qquad -A_x + B_x = 0$

$A_x = B_x = 0$ **Ans.**

$\Sigma F_y = m(a_G)_y;\qquad A_y = 0$ **Ans.**

$\Sigma F_z = m(a_G)_z;\qquad 300 - 15(9.81) + B_z + A_z = 15(101.96)(0.2)$

$B_z + A_z = 153.03$ **(2)**

Solving Eqs. (1) and (2) yields

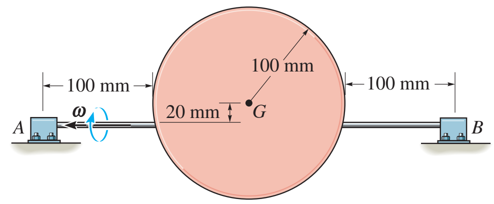
$$A_z = 297\text{ N}\qquad \textbf{Ans.}$$

$$B_z = -143\text{ N}\qquad \textbf{Ans.}$$

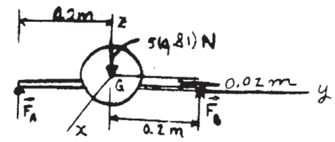
Ans:
$\dot{\omega}_y = -102\text{ rad/s}^2$
$A_x = B_x = 0$
$A_y = 0$
$A_z = 297\text{ N}$
$B_z = -143\text{ N}$

***21–52.**

The 5-kg circular disk is mounted off center on a shaft which is supported by bearings at A and B. If the shaft is rotating at a constant rate of $\omega = 10$ rad/s, determine the vertical reactions at the bearings when the disk is in the position shown.



SOLUTION



$$\omega_x = 0, \qquad \omega_y = -10 \text{ rad/s}, \qquad \omega_z = 0$$

$$\dot{\omega}_x = 0, \qquad \dot{\omega}_y = 0, \qquad \dot{\omega}_z = 0$$

$$\circlearrowleft + \Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\omega_y \omega_z$$

$$-(0.2)(F_A) + (0.2)(F_B) = 0$$

$$F_A = F_B$$

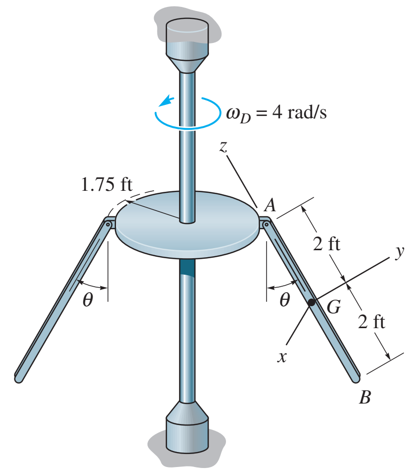
$$+\downarrow \Sigma F_z = ma_z; \qquad F_A + F_B - 5(9.81) = -5(10)^2 (0.02)$$

$$F_A = F_B = 19.5 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$F_A = F_B = 19.5$ N

21–53.

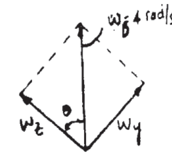
Two uniform rods, each having a weight of 10 lb, are pin connected to the edge of a rotating disk. If the disk has a constant angular velocity $\omega_D = 4$ rad/s, determine the angle θ made by each rod during the motion, and the components of the force and moment developed at the pin A. *Suggestion:* Use the x, y, z axes oriented as shown.



SOLUTION

$$I_y = \frac{1}{12}\left(\frac{10}{32.2}\right)(4)^2 = 0.4141 \text{ slug}\cdot\text{ft}^2 \qquad I_z = 0$$

Applying Eq. 21–25 with



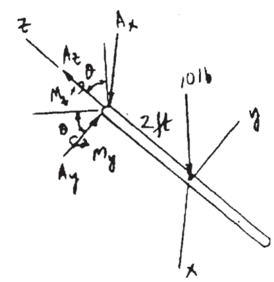
$$\omega_y = 4 \sin\theta \qquad \omega_z = 4\cos\theta \qquad \omega_x = 0$$

$$\dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_z = 0$$

$$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z; \qquad -A_y(2) = 0 - (0.4141 - 0)(4\sin\theta)(4\cos\theta) \qquad \textbf{(1)}$$

$$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x; \qquad M_y + A_x(2) = 0 \qquad \textbf{(2)}$$

$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y; \qquad M_z = 0 \qquad \textbf{Ans.}$$



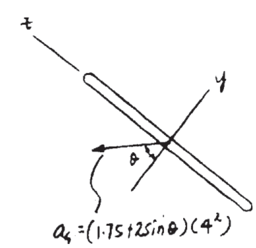
Also,

$$\Sigma F_x = m(a_G)_x; \qquad A_x = 0 \qquad \textbf{Ans.}$$

From Eq. (2) $\qquad M_y = 0 \qquad$ **Ans.**

$$\Sigma F_y = m(a_G)_y; \qquad A_y - 10\sin\theta = -\left(\frac{10}{32.2}\right)(1.75 + 2\sin\theta)(4)^2\cos\theta \qquad \textbf{(3)}$$

$$\Sigma F_z = m(a_G)_z; \qquad A_z - 10\cos\theta = \left(\frac{10}{32.2}\right)(1.75 + 2\sin\theta)(4)^2\sin\theta \qquad \textbf{(4)}$$



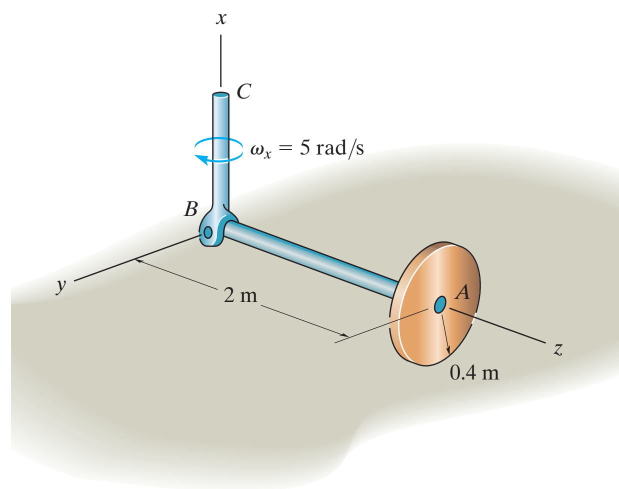
Solving Eqs. (1), (3) and (4) yields:

$$\theta = 64.1^\circ \qquad A_y = 1.30 \text{ lb} \qquad A_z = 20.2 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$M_z = 0$
$A_x = 0$
$M_y = 0$
$\theta = 64.1^\circ$
$A_y = 1.30$ lb
$A_z = 20.2$ lb

21–54.

The 10-kg disk turns around the shaft AB, while the shaft rotates about BC at a constant rate of $\omega_x = 5$ rad/s. If the disk does not slip, determine the normal and frictional force it exerts on the ground. Neglect the mass of shaft AB.



SOLUTION

Kinematics. The instantaneous axis of zero velocity (IA) is indicated in Fig. a. Here the resultant angular velocity is always directed along IA. The fixed reference frame is set to coincide with the rotating xyz frame using the similar triangle,

$$\frac{\omega_z}{2} = \frac{\omega_x}{0.4}; \qquad \omega_z = \frac{2}{0.4}(5) = 25.0 \text{ rad/s}$$

Thus,

$$\boldsymbol{\omega} = \boldsymbol{\omega}_x + \boldsymbol{\omega}_z = \{-5\mathbf{i} + 25.0\mathbf{k}\} \text{ rad/s}$$

Here, $(\dot{\omega}_x)_{xyz} = (\dot{\omega}_z)_{xyz} = 0$ since ω_x is constant. The direction of $\boldsymbol{\omega}_x$ will not change that always along x axis when $\Omega = \omega_x$. Then

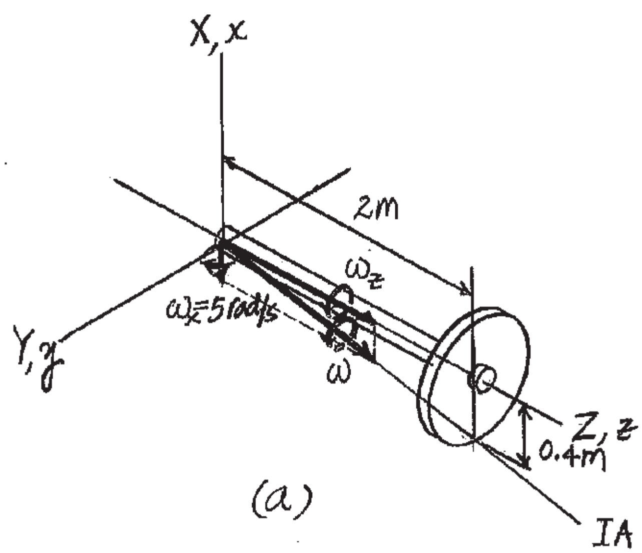
$$\dot{\boldsymbol{\omega}}_x = (\dot{\boldsymbol{\omega}}_x)_{xyz} + \boldsymbol{\omega}_x \times \boldsymbol{\omega}_x = \mathbf{0}$$

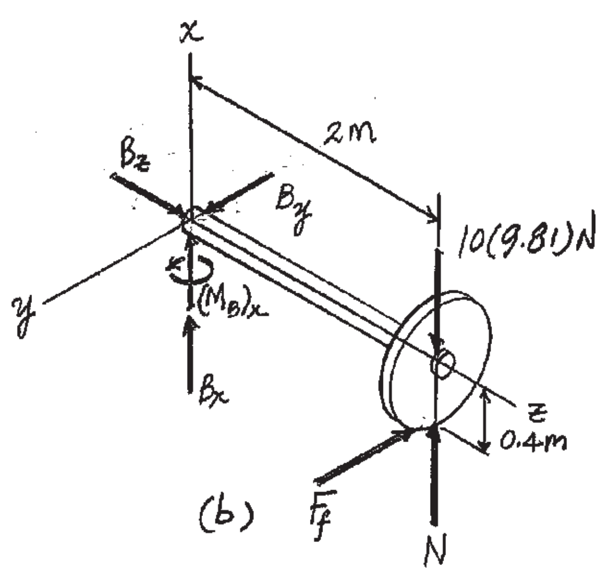
The direction of $\boldsymbol{\omega}_b$ does not change with reference to the xyz rotating frame if this frame rotates with $\Omega = \omega_x = \{-5\mathbf{i}\}$ rad/s. Then

$$\dot{\boldsymbol{\omega}}_z = (\dot{\omega}_z)_{xyz} + \boldsymbol{\omega}_x \times \omega_z$$
$$= 0 + (-5\mathbf{i}) \times (25.0\mathbf{k})$$
$$= \{125\mathbf{j}\} \text{ rad/s}^2$$

Finally

$$\dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}_x + \dot{\boldsymbol{\omega}}_z = \mathbf{0} + 125\mathbf{j} = \{125\mathbf{j}\} \text{ rad/s}^2$$





21–54. Continued

Equations of Motion. The mass moments of inertia of the disk about the x, y and z axes are

$$I_x = I_y = \frac{1}{4}(10)\left(0.4^2\right) + 10\left(2^2\right) = 40.4\ \text{kg}\cdot\text{m}^2$$

$$I_z = \frac{1}{2}(10)\left(0.4^2\right) = 0.800\ \text{kg}\cdot\text{m}^2$$

By referring to the FBD of the disk, Fig. b, $\Sigma M_y = I_y\,\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x$;

$$\text{N}(2) - 10(9.81)(2) = 40.4(125) - (0.8 - 40.4)(25.0)(-5)$$

$$N = 148.1\ \text{N} = 148\ \text{N} \qquad \textbf{Ans.}$$

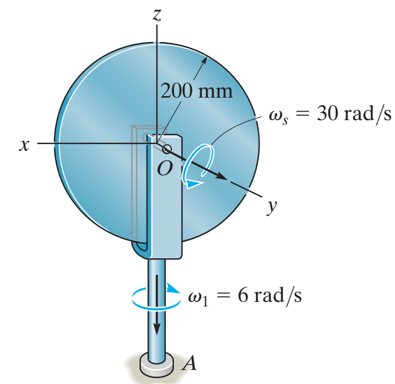
$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y;\qquad F_f(0.4) = 0 - 0$

$$F_f = 0 \qquad \textbf{Ans.}$$

Ans:
$N = 148\ \text{N}$
$F_f = 0$

21–55.

The 20-kg disk is spinning on its axle at $\omega_s = 30$ rad/s, while the forked rod is turning at $\omega_1 = 6$ rad/s. Determine the x and z moment components the axle exerts on the disk during the motion.



SOLUTION

Solution I

Kinematics. The fixed reference XYZ frame is set coincident with the rotating xyz frame. Here, this rotating frame is set to rotate with $\mathbf{\Omega} = \boldsymbol{\omega}_p = \{-6\mathbf{k}\}$ rad/s. The angular velocity of the disk with respect to the XYZ frame is

$$\boldsymbol{\omega} = \boldsymbol{\omega}_p + \boldsymbol{\omega}_s = \{30\mathbf{j} - 6\mathbf{k}\} \text{ rad/s}$$

Then

$$\omega_x = 0 \qquad \omega_y = 30 \text{ rad/s} \qquad \omega_z = -6 \text{ rad/s}$$

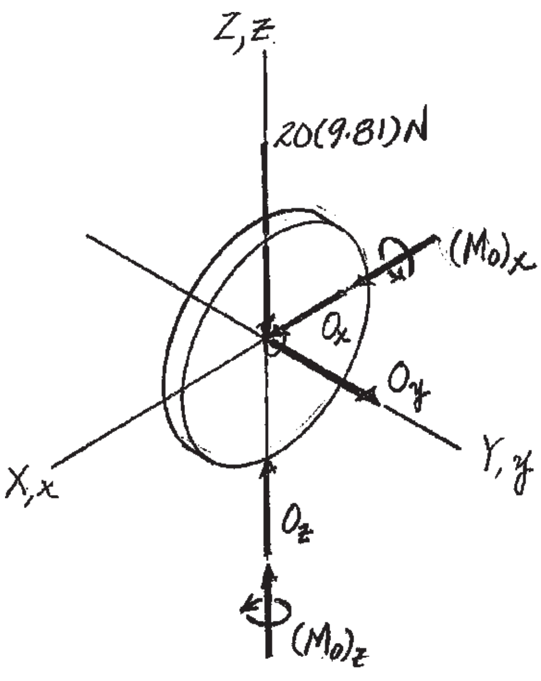
Since $\boldsymbol{\omega}_p$ and $\boldsymbol{\omega}_s$ does not change with respect to xyz frame, $\dot{\boldsymbol{\omega}}$ with respect to this frame is $\dot{\boldsymbol{\omega}} = \mathbf{0}$. Then

$$\dot{\omega}_x = 0 \qquad \dot{\omega}_y = 0 \qquad \dot{\omega}_z = 0$$

Equation of Motion. Although the disk spins about the y axis, but the mass moment of inertia of the disk remain constant with respect to the xyz frame.

$$I_x = I_z = \frac{1}{4}(20)\left(0.2^2\right) = 0.2 \text{ kg}\cdot\text{m}^2$$

$$I_y = \frac{1}{2}(20)\left(0.2^2\right) = 0.4 \text{ kg}\cdot\text{m}^2$$



(a)

21–55. Continued

With $\mathbf{\Omega} \neq \boldsymbol{\omega}$ with $\Omega_x = 0$, $\Omega_y = 0$ and $\Omega_z = -6$ rad/s

$\Sigma M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z;$

$$(M_0)_x = 0 - 0.4(-6)(30) + 0 \qquad (M_0)_x = 72.0\ \mathrm{N \cdot m} \qquad \textbf{Ans.}$$

$\Sigma M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x$

$$0 = 0 \quad \text{(Satisfied!)}$$

$\Sigma M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y;$

$$(M_0)_z = 0 - 0 + 0 = 0 \qquad \textbf{Ans.}$$

Solution II

Here, the xyz frame is set to rotate with $\mathbf{\Omega} = \boldsymbol{\omega} = \{30\mathbf{j} - 6\mathbf{k}\}$ rad/s. Setting another $x'y'z'$ frame coincide with xyz and XYZ frame to have an angular velocity of $\mathbf{\Omega}' = \boldsymbol{\omega}_p = \{-6\mathbf{k}\}$ rad/s,

$$\dot{\omega} = (\dot{\omega})_{xyz} = (\dot{\omega})_{x'y'z'} + \mathbf{\Omega}' \times \omega$$
$$= 0 + (-6\mathbf{k}) \times (30\mathbf{j} - 6\mathbf{k}) = \{180\mathbf{i}\}\ \mathrm{rad/s^2}$$

Thus

$$\dot{\omega}_x = 180\ \mathrm{rad/s} \qquad \dot{\omega}_y = 0 \qquad \dot{\omega}_z = 0$$

with $\Omega = \omega$,

$$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z; \qquad (M_0)_x = 0.2(180) - (0.4 - 0.2)(30)(-6)$$
$$= 72.0\ \mathrm{N \cdot m}$$

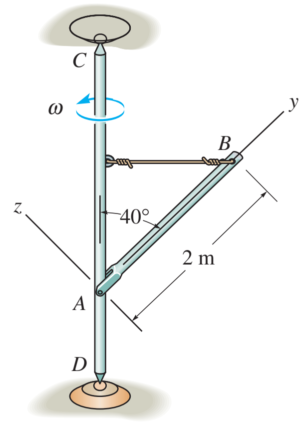
$$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x; \qquad 0 = 0 - 0 \quad \text{(Satisfied)}$$

$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y \qquad (M_0)_z = 0 - 0 = 0 \qquad \textbf{Ans.}$$

Ans:
$(M_0)_x = 72.0\ \mathrm{N \cdot m}$
$(M_0)_z = 0$

***21–56.**

The 4-kg slender rod AB is pinned at A and held at B by a cord. The axle CD is supported at its ends by ball-and-socket joints and is rotating with a constant angular velocity of 2 rad/s. Determine the tension developed in the cord and the magnitude of force developed at the pin A.



SOLUTION

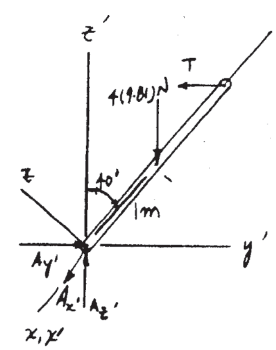
$$I_z = \frac{1}{3}(4)(2)^2 = 5.3333 \text{ kg}\cdot\text{m}^2 \qquad I_y = 0$$

Applying the third of Eq. 21–25 with

$$\omega_y = 2\cos 40^\circ = 1.5321 \text{ rad/s}$$

$$\omega_z = 2\sin 40^\circ = 1.2856 \text{ rad/s}$$

$$\Sigma M_x = I_x\,\dot{\omega}_x - (I_y - I_z)\,\omega_y\,\omega_z;$$



$$T(2\cos 40^\circ) - 4(9.81)(1\sin 40^\circ) = 0 - (0 - 5.3333)(1.5321)(1.2856)$$

$$T = 23.3 \text{ N} \qquad \textbf{Ans.}$$

Also,

$$\Sigma F_{x'} = m(a_G)_{x'}; \qquad A_{x'} = 0$$

$$\Sigma F_{y'} = m(a_G)_{y'}; \qquad A_{y'} - 23.32 = -4(2)^2(1\sin 40^\circ) \qquad A_{y'} = 13.03 \text{ N}$$

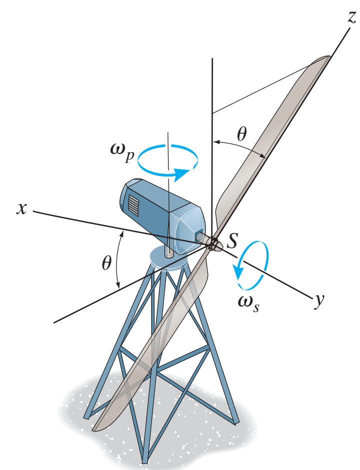
$$\Sigma F_{z'} = m(a_G)_{z'}; \qquad A_{z'} - 4(9.81) = 0 \qquad A_{z'} = 39.24 \text{ N}$$

$$F_A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{0^2 + 13.03^2 + 39.24^2} = 41.3 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$T = 23.3$ N
$F_A = 41.3$ N

21–57.

The blades of a wind turbine spin about the shaft S with a constant angular speed of ω_s, while the frame precesses about the vertical axis with a constant angular speed of ω_p. Determine the x, y, and z components of moment that the shaft exerts on the blades as a function of θ. Consider each blade as a slender rod of mass m and length l.

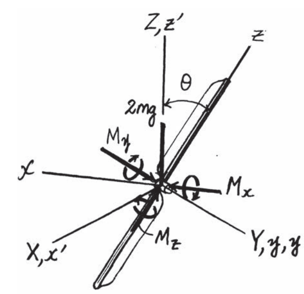


SOLUTION

The rotating xyz frame shown in Fig. a will be attached to the blade so that it rotates with an angular velocity of $\Omega = \omega$, where $\omega = \omega_s + \omega_p$. Referring to Fig. b $\omega_p = \omega_p \sin\theta \mathbf{i} + \omega_p \cos\theta \mathbf{k}$. Thus, $\omega = \omega_p \sin\theta\mathbf{i} + \omega_s \mathbf{j} + \omega_p \cos\theta \mathbf{k}$. Then

$$\omega_x = \omega_p \sin\theta \qquad \omega_y = \omega_s \;\; \omega_z = \omega_p \cos\theta$$

The angular acceleration of the blade $\dot{\omega}$ with respect to the XYZ frame can be obtained by setting another $x'y'z'$ frame having an angular velocity of $\Omega' = \omega_p = \omega_p \sin\theta\mathbf{i} + \omega_p\cos\theta\mathbf{k}$. Thus,



(a)

$$\dot{\omega} = \left(\dot{\omega}_{x'y'z'}\right) + \Omega' \times \omega$$

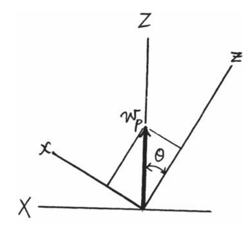
$$= (\dot{\omega}_1)_{x'y'z'} + (\dot{\omega}_2)_{x'y'z'} + \Omega' \times \omega_S + \Omega' \times \omega_P$$

$$= \mathbf{0} + \mathbf{0} + \left(\omega_p \sin\theta\mathbf{i} + \omega_p\cos\theta\mathbf{k}\right) \times (\omega_s\mathbf{j}) + \mathbf{0}$$

$$= -\omega_s\omega_p \cos\theta\mathbf{i} + \omega_s\omega_p\sin\theta\mathbf{k}$$

Since $\Omega = \omega$, $\dot{\omega}_{x'y'z'} = \dot{\omega}$. Thus,

$$\dot{\omega}_x = -\omega_s\omega_p\cos\theta \qquad \dot{\omega}_y = 0 \qquad \dot{\omega}_z = \omega_s\omega_p\sin\theta$$



(b)

Also, the x, y, and z axes will remain as principle axes of inertia for the blade. Thus,

$$I_x = I_y = \frac{1}{12}(2m)(2l)^2 = \frac{2}{3}ml^2 \qquad I_z = 0$$

Applying the moment equations of motion and referring to the free-body diagram shown in Fig. a,

$$\Sigma M_x = I_x\dot{\omega}_x - \left(I_y - I_z\right)\omega_y\omega_z; \qquad M_x = \frac{2}{3}ml^2(-\omega_s\omega_p\cos\theta) - \left(\frac{2}{3}ml^2 - 0\right)(\omega_s)(\omega_p\cos\theta)$$

$$= -\frac{4}{3}ml^2\,\omega_s\omega_p\cos\theta \qquad \textbf{Ans.}$$

$$\Sigma M_y = I_y\dot{\omega}_y - \left(I_z - I_x\right)\omega_z\omega_x; \qquad M_y = 0 - \left(0 - \frac{2}{3}ml^2\right)(\omega_p\cos\theta)(\omega_p\sin\theta)$$

$$= \frac{1}{3}ml^2\,{\omega_p}^2\sin 2\theta \qquad \textbf{Ans.}$$

$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y; \qquad M_z = 0 - 0 = 0 \qquad \textbf{Ans.}$$

Ans:

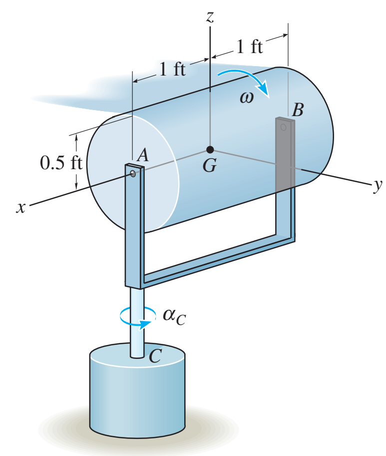
$M_x = -\frac{4}{3}ml^2\omega_s\omega_p\cos\theta$

$M_y = \frac{1}{3}ml^2\omega_p^2\sin 2\theta$

$M_z = 0$

21–58.

The 15-lb cylinder is rotating about shaft AB with a constant angular speed $\omega = 4$ rad/s. If the supporting shaft at C, initially at rest, is given an angular acceleration $a_C = 12$ rad/s^2, determine the components of reaction at the bearings A and B. The bearing at A cannot support a force component along the x axis, whereas the bearing at B does.



SOLUTION

$$\omega = \{-4\mathbf{i}\} \text{ rad/s}$$

$$\dot{\omega} = \dot{\omega}_{xyz} + \Omega \times \omega = 12\mathbf{k} + \mathbf{0} \times (-4\mathbf{i}) = \{12\mathbf{k}\} \text{ rad/s}^2$$

Hence

$\omega_x = -4, \qquad \omega_y = \omega_z = 0,$

$\dot{\omega}_x = \dot{\omega}_y = 0, \qquad \dot{\omega}_z = 12 \text{ rad/s}^2$

$\Sigma M_x = I_x \dot{\omega}_x - (I_y - I_z)\,\omega_y\,\omega_z, \qquad 0 = 0$

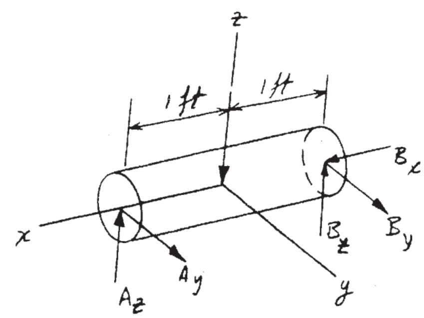
$\Sigma M_y = I_y \dot{\omega}_y - (I_z - I_x)\,\omega_z\,\omega_x, \qquad B_z(1) - A_z(1) = 0$

$\Sigma M_z = I_z \dot{\omega}_z - (I_x - I_y)\,\omega_x\,\omega_y, \qquad A_y(1) - B_y(1) = \left[\frac{1}{12}\left(\frac{15}{32.2}\right)\left(3(0.5)^2 + (2)^2\right)\right](12) - 0$

$\Sigma F_x = m(a_G)_x; \qquad B_x = 0$ **Ans.**

$\Sigma F_y = m(a_G)_y; \qquad A_y + B_y = -\left(\frac{15}{32.2}\right)(1)(12)$

$\Sigma F_z = m(a_G)_z; \qquad A_z + B_z - 15 = 0$



Solving,

$A_y = -1.69$ lb **Ans.**

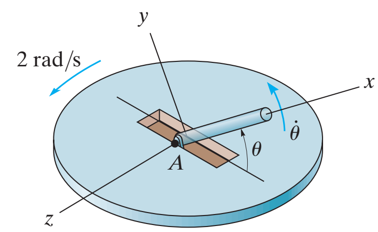
$B_y = -3.90$ lb **Ans.**

$A_z = B_z = 7.5$ lb **Ans.**

Ans:
$B_x = 0$
$B_y = -3.90$ lb
$A_y = -1.69$ lb
$A_z = B_z = 7.5$ lb

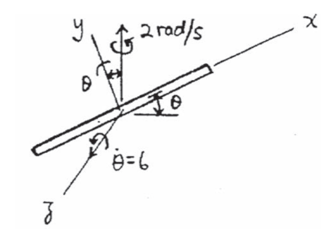
21–59.

The *thin rod* has a mass of 0.8 kg and a total length of 150 mm. It is rotating about its midpoint at a constant rate $\dot{\theta} = 6$ rad/s, while the table to which its axle A is fastened is rotating at fastened is rotating at 2 rad/s. Determine the x, y, z moment components which the axle exerts on the rod when the rod is in any position θ.



SOLUTION

The *x,y,z axes* are fixed as shown.



$$\omega_x = 2\sin\theta$$

$$\omega_y = 2\cos\theta$$

$$\omega_z = \dot{\theta} = 6$$

$$\dot{\omega}_x = 2\dot{\theta}\cos\theta = 12\cos\theta$$

$$\dot{\omega}_y = -2\dot{\theta}\sin\theta = -12\sin\theta$$

$$\dot{\omega}_z = 0$$

$$I_x = 0$$

$$I_y = I_z = \frac{1}{12}(0.8)(0.15)^2 = 1.5(10^{-3})$$

Using Eqs. 21–25:

$$\Sigma M_x = 0 - 0 = 0 \qquad \textbf{Ans.}$$

$$\Sigma M_y = 1.5(10^{-3})(-12\sin\theta) - [1.5(10^{-3}) - 0](6)(2\sin\theta)$$

$$\Sigma M_y = (-0.036\sin\theta)\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

$$\Sigma M_z = 0 - [0 - 1.5(10^{-3})](2\sin\theta)(2\cos\theta)$$

$$\Sigma M_z = 0.006\sin\theta\cos\theta = (0.003\sin 2\theta)\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\Sigma M_x = 0$
$\Sigma M_y = (-0.036\sin\theta)\ \text{N}\cdot\text{m}$
$\Sigma M_z = (0.003\sin 2\theta)\ \text{N}\cdot\text{m}$

***21–60.**

Show that the angular velocity of a body, in terms of Euler angles ϕ, θ, and ψ, can be expressed as $\omega = (\dot{\phi} \sin\theta \sin\psi + \dot{\theta} \cos\psi)\mathbf{i} + (\dot{\phi} \sin\theta \cos\psi - \dot{\theta} \sin\psi)\mathbf{j} + (\dot{\phi} \cos\theta + \dot{\psi})\mathbf{k}$, where **i**, **j**, and **k** are directed along the x, y, z axes as shown in Fig. 21–15d.

SOLUTION

From Fig. 21–15b. due to rotation ϕ, the x, y, z components of $\dot{\phi}$ are simply $\dot{\phi}$ along z axis.

From Fig 21–15c, due to rotation θ, the x, y, z components of $\dot{\phi}$ and $\dot{\theta}$ are $\dot{\phi} \sin\theta$ in the y direction, $\dot{\phi} \cos\theta$ in the z direction, and $\dot{\theta}$ in the x direction.

Lastly, rotation ψ. Fig. 21–15d, produces the final components which yields

$$\omega = \left(\dot{\phi} \sin\theta \sin\psi + \dot{\theta} \cos\psi\right)\mathbf{i} + \left(\dot{\phi} \sin\theta \cos\psi - \dot{\theta} \sin\psi\right)\mathbf{j} + \left(\dot{\phi} \cos\theta + \dot{\psi}\right)\mathbf{k} \quad \textbf{Q.E.D.}$$

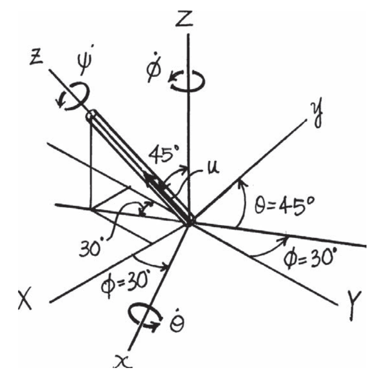
Ans:

$$\omega = \left(\dot{\phi} \sin\theta \sin\psi + \dot{\theta} \cos\psi\right)\mathbf{i} + \left(\dot{\phi} \sin\theta \cos\psi - \dot{\theta} \sin\psi\right)\mathbf{j} + \left(\dot{\phi} \cos\theta + \dot{\psi}\right)\mathbf{k}$$

21–61.

A thin rod is initially coincident with the Z axis when it is given three rotations defined by the Euler angles $\phi = 30°$, $\theta = 45°$, and $\psi = 60°$. If these rotations are given in the order stated, determine the coordinate direction angles α, β, γ of the axis of the rod with respect to the X, Y, and Z axes. Are these directions the same for any order of the rotations? Why?

SOLUTION



$\mathbf{u} = (1 \sin 45°) \sin 30° \, \mathbf{i} - (1 \sin 45°) \cos 30° \mathbf{j} + 1 \cos 45° \, \mathbf{k}$

$\mathbf{u} = 0.3536\mathbf{i} - 0.6124\mathbf{j} + 0.7071\mathbf{k}$

$\alpha = \cos^{-1} 0.3536 = 69.3°$ **Ans.**

$\beta = \cos^{-1}(-0.6124) = 128°$ **Ans.**

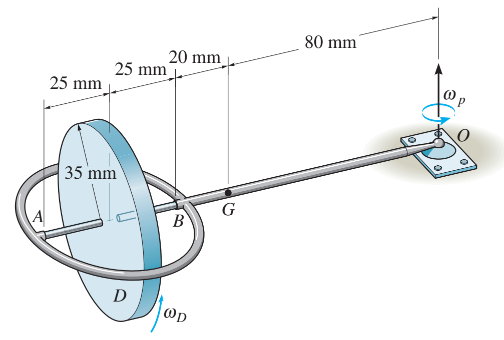
$\gamma = \cos^{-1}(0.7071) = 45°$ **Ans.**

No, the orientation of the rod will not be the same for any order of rotation, because finite rotations are not vectors.

Ans:
$\alpha = 69.3°$
$\beta = 128°$
$\gamma = 45°$
No, the orientation will not be the same for any order. Finite rotations are not vectors.

21–62.

The gyroscope consists of a uniform 450-g disk D which is attached to the axle AB of negligible mass. The supporting frame has a mass of 180 g and a center of mass at G. If the disk is rotating about the axle at $\omega_D = 90$ rad/s, determine the constant angular velocity ω_p at which the frame precesses about the pivot point O. The frame moves in the horizontal plane.

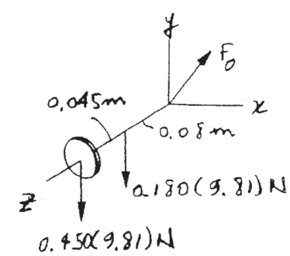


SOLUTION

$$\Sigma M_x = I_z \, \Omega_y \, \omega_z$$

$$(0.450)(9.81)(0.125) + (0.180)(9.81)(0.080) = \frac{1}{2}(0.450)(0.035)^2 \, \omega_P \, (90)$$

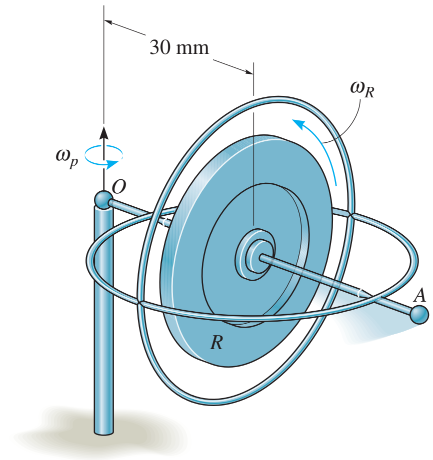
$\omega_P = 27.9$ rad/s **Ans.**



Ans:
$\omega_P = 27.9$ rad/s

21–63.

The toy gyroscope consists of a rotor R which is attached to the frame of negligible mass. If it is observed that the frame is precessing about the pivot point O at $\omega_p = 2$ rad/s, determine the angular velocity ω_R of the rotor. The stem OA moves in the horizontal plane. The rotor has a mass of 200 g and a radius of gyration $k_{OA} = 20$ mm about OA.

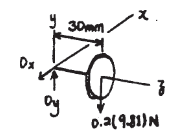


SOLUTION

$\Sigma M_x = I_z \Omega_y w_z$

$(0.2)(9.81)(0.03) = \left[0.2(0.02)^2\right](2)(w_R)$

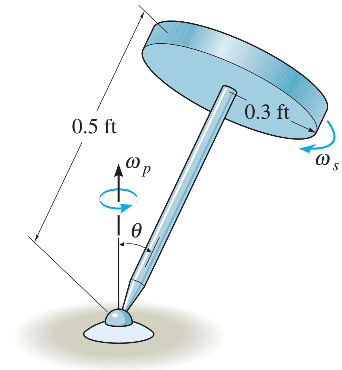
$\omega_R = 368$ rad/s **Ans.**



Ans:
$\omega_R = 368$ rad/s

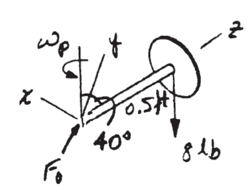
***21–64.**

The top consists of a thin disk that has a weight of 8 lb and a radius of 0.3 ft. The rod has a negligible mass and a length of 0.5 ft. If the top is spinning with an angular velocity $\omega_s = 300$ rad/s, determine the steady-state precessional angular velocity ω_p of the rod when $\theta = 40°$.



SOLUTION

$$\Sigma M_x = -I\,\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta(\dot{\phi}\cos\theta + \dot{\psi})$$



$$8(0.5\sin 40°) = -\left[\frac{1}{4}\left(\frac{8}{32.2}\right)(0.3)^2 + \left(\frac{8}{32.2}\right)(0.5)^2\right]\omega_p^2 \sin 40° \cos 40°$$

$$+\left[\frac{1}{2}\left(\frac{8}{32.2}\right)(0.3)^2\right]\omega_p \sin 40°\,(\omega_p \cos 40° + 300)$$

$$0.02783\omega_p^2 - 2.1559\omega_p + 2.571 = 0$$

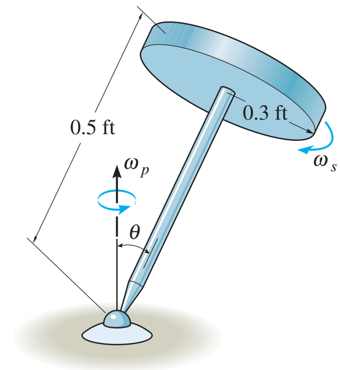
$\omega_p = 1.21$ rad/s **Ans.** (Low precession)

$\omega_p = 76.3$ rad/s **Ans.** (High precession)

Ans:
$w_p = 1.21$ rad/s
$w_p = 76.3$ rad/s

21–65.

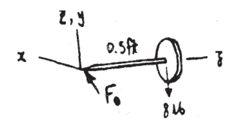
Solve Prob. 21–64 when $\theta = 90°$.



SOLUTION

$$\Sigma M_x = I_z \Omega_y w_z$$

$$8(0.5) = \left[\frac{1}{2}\left(\frac{8}{32.2}\right)(0.3)^2\right] w_p(300)$$

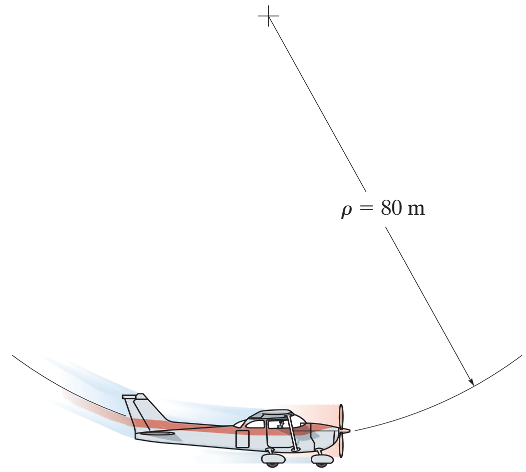


$\omega_p = 1.19$ rad/s **Ans.**

Ans:
$\omega_P = 1.19$ rad/s

21–66.

The propeller on a single-engine airplane has a mass of 15 kg and a centroidal radius of gyration of 0.3 m computed about the axis of spin. When viewed from the front of the airplane, the propeller is turning clockwise at 350 rad/s about the spin axis. If the airplane enters a vertical curve having a radius of 80 m and is traveling at 200 km/h, determine the gyroscopic bending moment which the propeller exerts on the bearings of the engine when the airplane is in its lowest position.



SOLUTION

$$\omega_s = 350 \text{ rad/s} = \omega_z$$

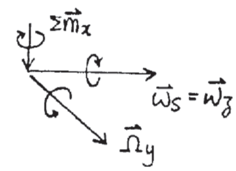
$$v = 200 \text{ km/h} = \frac{200(10^3)}{3600} = 55.56 \text{ m/s}$$

$$\Omega_y = \frac{55.56}{80} = 0.694 \text{ rad/s}$$

$$\Sigma M_x = I_z \Omega_y \omega_z$$

$$M_x = [15(0.3)^2](0.694)(350)$$

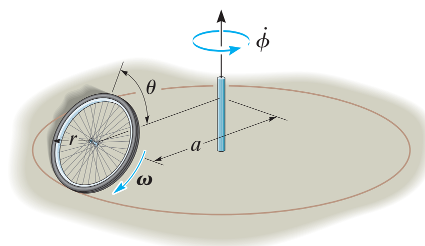
$$M_x = 328 \text{ N} \cdot \text{m}$$ **Ans.**



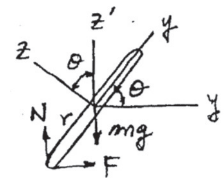
Ans:
$M_x = 328 \text{ N} \cdot \text{m}$

21–67.

A wheel of mass m and radius r rolls with constant spin $\boldsymbol{\omega}$ about a circular path having a radius a. If the angle of inclination is θ, determine the rate of precession. Treat the wheel as a thin ring. No slipping occurs.



SOLUTION



Since no slipping occurs,

$$(r)\dot{\psi} = (a + r\cos\theta)\dot{\phi}$$

or

$$\dot{\psi} = \left(\frac{a + r\cos\theta}{r}\right)\dot{\phi} \qquad \textbf{(1)}$$

Also,

$$\omega = \dot{\phi} + \dot{\psi}$$

$$\Sigma F_{y'} = m(a_G)_{y'}; \qquad F = m(a\,\dot{\phi}^2) \qquad \textbf{(2)}$$

$$\Sigma F_{z'} = m(a_G)_{z'}; \qquad N - mg = 0 \qquad \textbf{(3)}$$

$$I_x = I_y = \frac{mr^2}{2}, \qquad I_z = mr^2$$

$$\omega = \dot{\phi}\sin\theta\,\mathbf{j} + (-\dot{\psi} + \dot{\phi}\cos\theta)\mathbf{k}$$

Thus,

$$\omega_x = 0, \qquad \omega_y = \dot{\phi}\sin\theta, \qquad \omega_{z'} = -\dot{\psi} + \dot{\phi}\cos\theta$$

$$\dot{\omega} = \dot{\phi}\times\dot{\psi} = -\dot{\phi}\dot{\psi}\sin\theta$$

$$\dot{\omega}_x = -\dot{\phi}\dot{\psi}\sin\theta, \qquad \dot{\omega}_y = \dot{\omega}_z = 0$$

Applying

$$\Sigma M_x = I_x\,\dot{\omega}_x + (I_z - I_y)\omega_z\omega_y$$

$$F\,r\sin\theta - N\,r\cos\theta = \frac{m\,r^2}{2}(-\dot{\phi}\dot{\psi}\sin\theta) + \left(m\,r^2 - \frac{mr^2}{2}\right)(-\dot{\psi} + \dot{\phi}\cos\theta)(\dot{\phi}\sin\theta)$$

Using Eqs. (1), (2) and (3), and eliminating $\dot{\psi}$, we have

$$m\,a\,\dot{\phi}^2\,r\sin\theta - m\,g\,r\cos\theta = \frac{m\,r^2}{2}(-\dot{\phi})\sin\theta\left(\frac{a + r\cos\theta}{r}\right)\dot{\phi} + \frac{mr^2}{2}\left(\frac{-\dot{\phi}\,a}{r}\right)\dot{\phi}\sin\theta$$

$$m\,a\,\dot{\phi}^2\sin\theta\,r - m\,g\,r\cos\theta = \frac{m\,r^2}{2}\left(\frac{-\dot{\phi}^2 a}{r}\right)\sin\theta - \frac{m\,r^2}{2}(\dot{\phi}^2\sin\theta\cos\theta)$$

$$2\,g\cos\theta = a\,\dot{\phi}^2\sin\theta + r\,\dot{\phi}^2\sin\theta\cos\theta$$

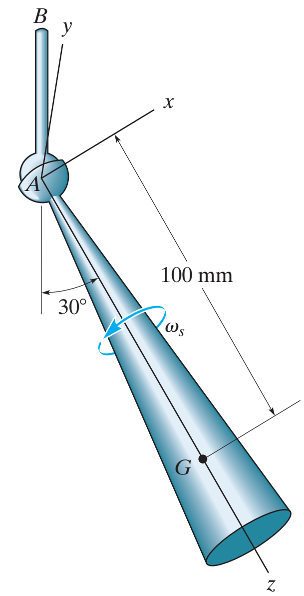
$$\dot{\phi} = \left(\frac{2\,g\cot\theta}{a + r\cos\theta}\right)^{1/2} \qquad \textbf{Ans.}$$

Ans:

$$\dot{\phi} = \left(\frac{2g\cos\theta}{a + r\cos\theta}\right)^{1/2}$$

***21–68.**

The conical top has a mass of 0.8 kg, and the moments of inertia are $I_x = I_y = 3.5(10^{-3})\ \text{kg}\cdot\text{m}^2$ and $I_z = 0.8(10^{-3})\ \text{kg}\cdot\text{m}^2$. If it spins freely in the ball-and socket joint at A with an angular velocity $\omega_s = 750$ rad/s, compute the precession of the top about the axis of the shaft AB.



SOLUTION

$\omega_s = 750$ rad/s

Using Eq. 21 – 30.

$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\,\dot{\phi}\sin\theta\,(\dot{\phi}\cos\theta + \dot{\phi})$$

$$0.1(0.8)(9.81)\sin 30° = -3.5(10^{-3})\dot{\phi}^2\sin 30°\cos 30° + 0.8(10^{-3})\dot{\phi}\sin 30°(\dot{\phi}\cos 30° + 750)$$

Thus,

$$1.160(10^{-3})\dot{\phi}^2 - 300(10^{-3})\dot{\phi} + 0.3924 = 0$$

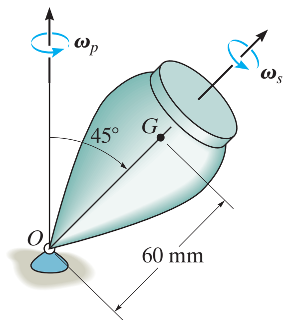
$\dot{\phi} = 1.31$ rad/s (low precession) **Ans.**

$\dot{\phi} = 255$ rad/s (high precession) **Ans.**

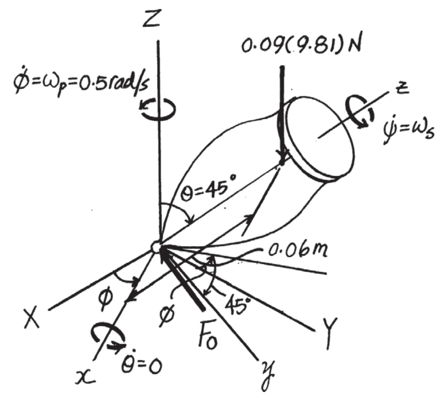
Ans:
$\dot{\phi} = 1.31$ rad/s
$\dot{\phi} = 255$ rad/s

21–69.

The top has a mass of 90 g, a center of mass at G, and a radius of gyration $k = 18$ mm about its axis of symmetry. About any transverse axis acting through point O the radius of gyration is $k_t = 35$ mm. If the top is connected to a ball-and-socket joint at O and the precession is $\omega_p = 0.5$ rad/s, determine the spin $\boldsymbol{\omega}_s$.



SOLUTION



$\omega_p = 0.5$ rad/s

$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta\left(\dot{\phi}\cos\theta + \dot{\psi}\right)$$

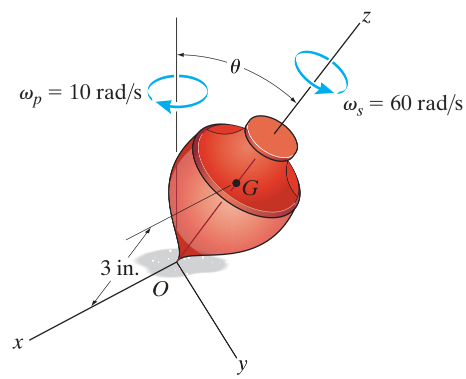
$$0.090(9.81)(0.06)\sin 45° = -0.090(0.035)^2\,(0.5)^2\,(0.7071)^2$$
$$+\ 0.090(0.018)^2(0.5)(0.7071)\left[0.5(0.7071) + \dot{\psi}\right]$$

$$\omega_s = \psi = 3.63\left(10^3\right)\text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_s = 3.63\left(10^3\right)$ rad/s

21–70.

The 1-lb top has a center of gravity at point G. If it spins about its axis of symmetry and precesses about the vertical axis at constant rates of $\omega_s = 60$ rad/s and $\omega_p = 10$ rad/s, respectively, determine the steady state angle θ. The radius of gyration of the top about the z axis is $k_z = 1$ in., and about the x and y axes it is $k_x = k_y = 4$ in.

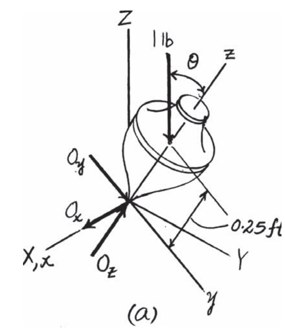


SOLUTION

Since $\dot{\psi} = \omega_s = 60$ rad/s and $\dot{\phi} = \omega_p = -10$ rad/s and θ are constant, the top undergoes steady precession.

$$I_z = \left(\frac{1}{32.2}\right)\left(\frac{1}{12}\right)^2 = 215.67\left(10^{-6}\right) \text{ slug} \cdot \text{ft}^2 \quad \text{and} \quad I = I_x = I_y = \left(\frac{1}{32.2}\right)\left(\frac{4}{12}\right)^2$$

$= 3.4507\left(10^{-3}\right)$ slug $\cdot$ ft^2.



Thus,

$$\Sigma M_x = -I\dot{\phi}^2 \sin\theta\cos\theta + I_z\dot{\phi}\sin\theta\left(\dot{\phi}\cos + \dot{\psi}\right)$$

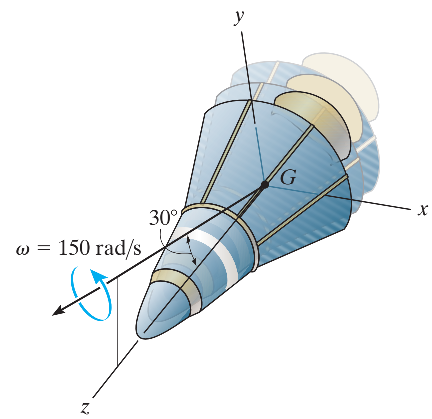
$$-1\sin\theta(0.25) = -3.4507\left(10^{-3}\right)(-10)^2\sin\theta\cos\theta + 215.67\left(10^{-6}\right)(-10)\sin\theta[(-10)\cos\theta + 60]$$

$$\theta = 68.1^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 68.1^\circ$

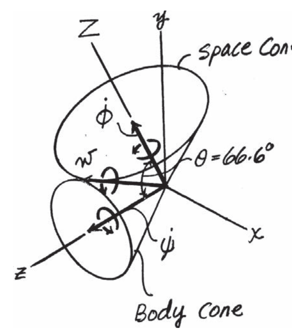
21–71.

The space capsule has a mass of 2 Mg, center of mass at G, and radii of gyration about its axis of symmetry (z axis) and its transverse axes (x or y axis) of $k_z = 2.75$ m and $k_x = k_y = 5.5$ m, respectively. If the capsule has the angular velocity shown, determine its precession $\dot{\phi}$ and spin $\dot{\psi}$. Indicate whether the precession is regular or retrograde. Also, draw the space cone and body cone for the motion.



SOLUTION

The only force acting on the space capsule is its own weight. Thus, it undergoes torque-free motion. $I_z = 2000\left(2.75^2\right) = 15\,125 \text{ kg}\cdot\text{m}^2$, $I = I_x = I_y = 2000\left(5.5^2\right) = 60\,500 \text{ kg}\cdot\text{m}^2$. Thus,



$$\omega_y = \frac{H_G \sin\theta}{I}$$

$$150 \sin 30° = \frac{H_G \sin\theta}{60\,500}$$

$$H_G \sin\theta = 4\,537\,500 \qquad \textbf{(1)}$$

$$\omega_z = \frac{H_G \cos\theta}{I_z}$$

$$150 \cos 30° = \frac{H_G \cos\theta}{15\,125}$$

$$H_G \cos\theta = 1\,964\,795.13 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$H_G = 4.9446\left(10^6\right) \text{ kg}\cdot\text{m}^2/\text{s} \qquad \theta = 66.59°$$

Using these results,

$$\dot{\phi} = \frac{H_G}{I} = \frac{H_G}{60\,500} = \frac{4.9446\left(10^6\right)}{60\,500} = 81.7 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta = \left[\frac{60\,500 - 15\,125}{60\,500(15125)}\right] 4.9446\left(10^6\right)\cos 30°$$

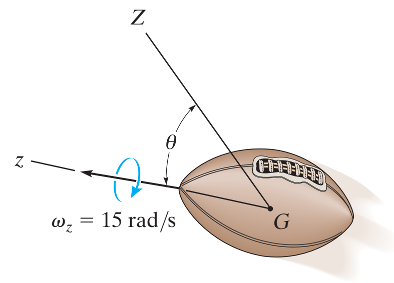
$$= 212 \text{ rad/s} \qquad \textbf{Ans.}$$

Since $I > I_z$, the motion is *regular precession*. **Ans.**

Ans:
$\dot{\phi} = 81.7$ rad/s
$\dot{\psi} = 212$ rad/s
regular precession

***21–72.**

The 0.25 kg football is spinning at $\omega_z = 15$ rad/s as shown. If $\theta = 40°$, determine the precession about the z axis. The radius of gyration about the spin axis is $k_z = 0.042$ m, and about a transverse axis is $k_y = 0.13$ m.



SOLUTION

Here, $\dot{\psi} = \omega_t = 15$ rad/s, $I = mk_y^2 = 0.25(0.13^2) = 0.004225\ \text{kg}\cdot\text{m}^2$ and $I_z = mk_z^2 = 0.25(0.042^2) = 0.000441\ \text{kg}\cdot\text{m}^2$.

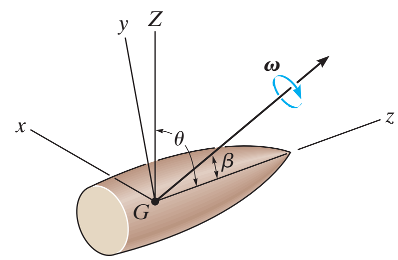
$$\dot{\psi} = \frac{I - I_z}{I_z}\dot{\phi}\cos\theta; \qquad 15 = \left(\frac{0.004225 - 0.000441}{0.000441}\right)\dot{\phi}\cos 40°$$

$$\dot{\phi} = 2.282\ \text{rad/s}^2 = 2.28\ \text{rad/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\dot{\phi} = 2.28\ \text{rad/s}^2$

21–73.

The projectile shown is subjected to torque-free motion. The transverse and axial moments of inertia are I and I_z, respectively. If θ represents the angle between the precessional axis Z and the axis of symmetry z, and β is the angle between the angular velocity $\boldsymbol{\omega}$ and the z axis, show that β and θ are related by the equation $\tan\theta = (I/I_z)\tan\beta$.



SOLUTION

From Eq. 21–34 $\omega_y = \dfrac{H_G \sin\theta}{I}$ and $\omega_z = \dfrac{H_G\cos\theta}{I_z}$ Hence $\dfrac{\omega_y}{\omega_z} = \dfrac{I_z}{I}\tan\theta$

However, $\omega_y = \omega\sin\beta$ and $\omega_z = \omega\cos\beta$

$$\frac{\omega_y}{\omega_z} = \tan\beta = \frac{I_z}{I}\tan\theta$$

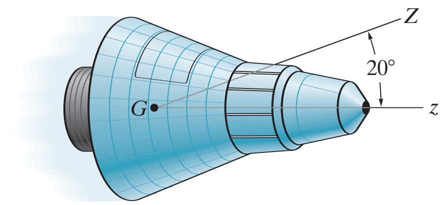
$$\tan\theta = \frac{I}{I_z}\tan\beta \qquad \textbf{Q.E.D.}$$

Ans:

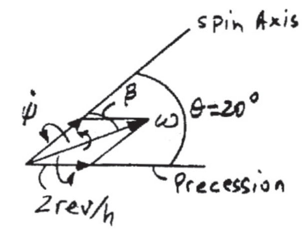
$\tan\theta = \dfrac{I}{I_z}\tan\beta$

21–74.

The radius of gyration about an axis passing through the axis of symmetry of the 1.6-Mg space capsule is $k_z = 1.2$ m, and about any transverse axis passing through the center of mass G, $k_t = 1.8$ m. If the capsule has a known steady-state precession of two revolutions per hour about the Z axis, determine the rate of spin about the z axis.



SOLUTION



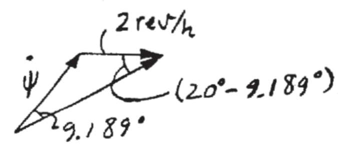
$$I = 1600(1.8)^2, \qquad I_z = 1600(1.2)^2$$

Use the result of Prob. 21–75.

$$\tan\theta = \left(\frac{I}{I_z}\right)\tan\beta$$

$$\tan 20^\circ = \left(\frac{1600(1.8)^2}{1600(1.2)^2}\right)\tan\beta$$

$$\beta = 9.189^\circ$$



Using the law of sines:

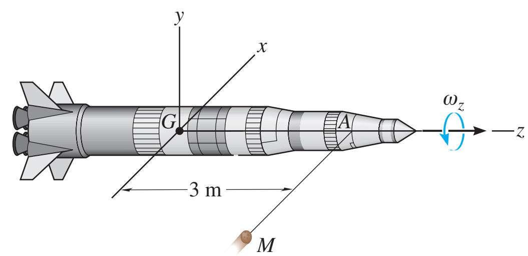
$$\frac{\sin 9.189^\circ}{2} = \frac{\sin(20^\circ - 9.189^\circ)}{\dot{\psi}}$$

$$\dot{\psi} = 2.35 \text{ rev/h} \qquad \textbf{Ans.}$$

Ans:
$\dot{\psi} = 2.35$ rev/h

21–75.

The rocket has a mass of 4 Mg and radii of gyration $k_z = 0.85$ m and $k_y = 2.3$ m. It is initially spinning about the z axis at $\omega_z = 0.05$ rad/s when a meteoroid M strikes it at A and creates an impulse $\mathbf{I} = \{300\mathbf{i}\}$ N·s. Determine the axis of precession after the impact.



SOLUTION

The impulse creates an angular momentum about the y axis of

$$H_y = 300(3) = 900\ \text{kg}\cdot\text{m}^2/\text{s}$$

Since

$$\omega_z = 0.05\ \text{rad/s}$$

then

$$\text{H}_G = 900\mathbf{j} + [4000(0.85)^2](0.05)\mathbf{k} = 900\mathbf{j} + 144.5\mathbf{k}$$

The axis of precession is defined by $\mathbf{H}_G$.

$$\text{u}_{H_G} = \frac{900\mathbf{j} + 144.5\mathbf{k}}{911.53} = 0.9874\mathbf{j} + 0.159\mathbf{k}$$

Thus,

$$\alpha = \cos^{-1}(0) = 90^\circ \qquad \textbf{Ans.}$$

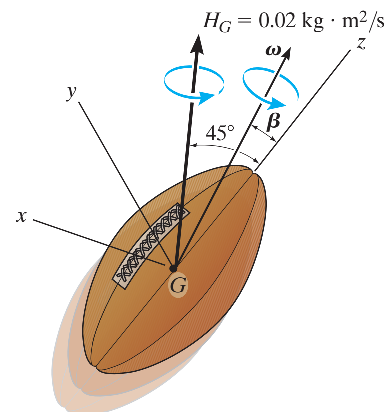
$$\beta = \cos^{-1}(0.9874) = 9.12^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}(0.159) = 80.9^\circ \qquad \textbf{Ans.}$$

Ans:
$\alpha = 90^\circ$
$\beta = 9.12^\circ$
$\gamma = 80.9^\circ$

***21–76.**

The football has a mass of 450 g and radii of gyration about its axis of symmetry (z axis) and its transverse axes (x or y axis) of $k_z = 30$mm and $k_x = k_y = 50$mm, respectively. If the football has an angular momentum of $H_G = 0.02\text{kg} \cdot \text{m}^2/\text{s}$, determine its precession ϕ and spin ψ. Also, find the angle β that the angular velocity vector makes with the z axis.



SOLUTION

Since the weight is the only force acting on the football, it undergoes torque-free motion. $I_z = 0.45(0.03^2) = 0.405(10^{-3})\ \text{kg}\cdot\text{m}^2$, $I = I_x = I_y = 0.45(0.05^2) = 1.125(10^{-3})\ \text{kg}\cdot\text{m}^2$, and $\theta = 45°$.

Thus,

$$\dot{\phi} = \frac{H_G}{I} = \frac{0.02}{1.125(10^{-3})} = 17.78 \text{ rad/s} = 17.8 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\dot{\psi} = \frac{I - I_z}{I I_z} H_G \cos\theta = \frac{1.125(10^{-3}) - 0.405(10^{-3})}{1.125(10^{-3})(0.405)(10^{-3})}(0.02)\cos 45°$$

$$= 22.35 \text{ rad/s} = 22.3 \text{ rad/s} \qquad \textbf{Ans.}$$

Also,

$$\omega_y = \frac{H_G \sin\theta}{I} = \frac{0.02 \sin 45°}{1.125(10^{-3})} = 12.57 \text{ rad/s}$$

$$\omega_z = \frac{H_G \cos\theta}{I_z} = \frac{0.02 \cos 45°}{0.405(10^{-3})} = 34.92 \text{ rad/s}$$

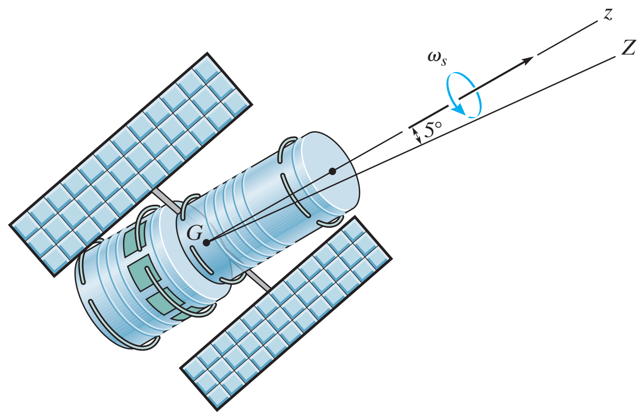
Thus,

$$\beta = \tan^{-1}\left(\frac{\omega_y}{\omega_z}\right) = \tan^{-1}\left(\frac{12.57}{34.92}\right) = 19.8° \qquad \textbf{Ans.}$$

Ans:
$\dot{\phi} = 17.8$ rad/s
$\dot{\psi} = 22.3$ rad/s
$\beta = 19.8°$

21–77.

The satellite has a mass of 1.8 Mg, and about axes passing through the mass center G the axial and transverse radii of gyration are $k_z = 0.8$ m and $k_t = 1.2$ m, respectively. If it is spinning at $\omega_s = 6$ rad/s when it is launched, determine its angular momentum. Precession occurs about the Z axis.



SOLUTION

$I = 1800(1.2)^2 = 2592\ \text{kg}\cdot\text{m}^2 \qquad I_z = 1800(0.8)^2 = 1152\ \text{kg}\cdot\text{m}^2$

Applying the third of Eqs. 21–36 with $\theta = 5°$ $\quad \dot{\psi} = 6$ rad/s

$$\psi = \frac{I - I_z}{H_z} H_G \cos\theta$$

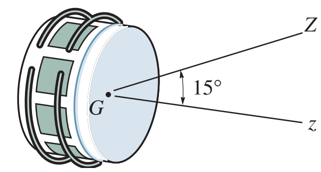
$$0 = \frac{2592 - 1152}{2592(1152)} H_G \cos 5°$$

$H_G = 12.5\ \text{Mg}\cdot\text{m}^2/\text{s}$ **Ans.**

Ans:
$H_G = 12.5\ \text{Mg}\cdot\text{m}^2/\text{s}$

21–78.

The radius of gyration about an axis passing through the axis of symmetry of the 1.2 Mg satellite is $k_z = 1.4$ m, and about any transverse axis passing through the center of mass G, $k_t = 2.20$ m. If the satelite has a known spin of 2700 rev/h about the z axis, determine the steady-state precession about the z axis.



SOLUTION

Gyroscopic Motion: Here, the spinning angular velocity $\dot{\psi} = \omega_s = \dfrac{2700(2\pi)}{3600} = 1.5\pi$ rad/s. The moment inertia of the satelite about the z axis is $I_z = 1200\left(1.4^2\right) = 2352\ \text{kg}\cdot\text{m}^2$ and the moment inertia of the satelite about its transverse axis is $I = 1200\left(2.20^2\right) = 5808\ \text{kg}\cdot\text{m}^2$. Applying the third of Eq. 21–36 with $\theta = 15°$, we have

$$\dot{\psi} = \frac{I - I_z}{I\,I_z} H_G \cos\theta$$

$$1.5\pi = \left[\frac{5808 - 2352}{5808(2352)}\right] H_G \cos 15°$$

$$H_G = 19.28\left(10^3\right)\ \text{kg}\cdot\text{m}^2/\text{s}$$

Applying the second of Eq. 21–36, we have

$$\dot{\phi} = \frac{H_G}{I} = \frac{19.28(10^3)}{5808} = 3.32\ \text{rad/s}$$ **Ans.**

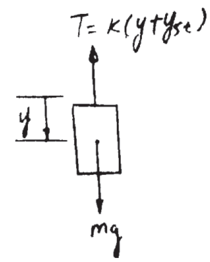
Ans:
$\dot{\phi} = 3.32$ rad/s

22–1.

A spring is stretched 175 mm by an 8-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 1.50 m/s, determine the differential equation which describes the motion. Assume that positive displacement is downward. Also, determine the position of the block when $t = 0.22$ s.

SOLUTION

$+\downarrow \Sigma F_y = ma_y; \qquad mg - k(y + y_{st}) = m\ddot{y} \qquad \text{where } ky_{st} = mg$



$$\ddot{y} + \frac{k}{m}y = 0$$

Hence $\qquad p = \sqrt{\frac{k}{m}} \qquad \text{Where } k = \frac{8(9.81)}{0.175} = 448.46 \text{ N/m}$

$$= \sqrt{\frac{448.46}{8}} = 7.487$$

$\therefore \qquad \ddot{y} + (7.487)^2 y = 0 \qquad \ddot{y} + 56.1y = 0$ **Ans.**

The solution of the above differential equation is of the form:

$y = A \sin pt + B \cos pt$ **(1)**

$v = \dot{y} = Ap \cos pt - Bp \sin pt$ **(2)**

At $t = 0$, $y = 0.1$ m and $v = v_0 = 1.50$ m/s

From Eq. (1) $\qquad 0.1 = A \sin 0 + B \cos 0 \qquad B = 0.1$ m

From Eq. (2) $\qquad v_0 = Ap \cos 0 - 0 \qquad A = \frac{v_0}{p} = \frac{1.50}{7.487} = 0.2003$ m

Hence $\qquad y = 0.2003 \sin 7.487t + 0.1 \cos 7.487t$

At $t = 0.22$ s, $\qquad y = 0.2003 \sin[7.487(0.22)] + 0.1 \cos[7.487(0.22)]$

$\qquad = 0.192$ m **Ans.**

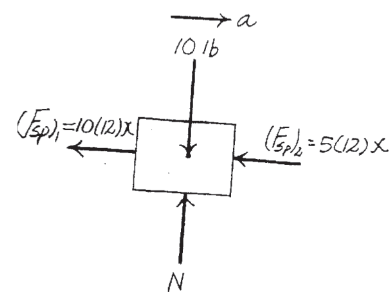
Ans:
$\ddot{y} + 56.1\,y = 0$
$y|_{t=0.22\text{ s}} = 0.192$ m

22–2.

A spring has a stiffness of 800 N/m. If a 2-kg block is attached to the spring, pushed 50 mm above its equilibrium position, and released from rest, determine the equation that describes the block's motion. Assume that positive displacement is downward.

SOLUTION

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{2}} = 20$$



$x = A \sin pt + B \cos pt$

$x = -0.05$ m when $t = 0$,

$-0.05 = 0 + B; \qquad B = -0.05$

$v = Ap \cos pt - Bp \sin pt$

$v = 0$ when $t = 0$,

$0 = A(20) - 0; \qquad A = 0$

Thus,

$x = -0.05 \cos (20t)$ **Ans.**

Ans:
$x = -0.05 \cos (20t)$

22–3.

A spring is stretched 200 mm by a 15-kg block. If the block is displaced 100 mm downward from its equilibrium position and given a downward velocity of 0.75 m/s, determine the equation which describes the motion. What is the phase angle? Assume that positive displacement is downward.

SOLUTION

$$k = \frac{F}{y} = \frac{15(9.81)}{0.2} = 735.75 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{735.75}{15}} = 7.00$$

$$y = A \sin \omega_n t + B \cos \omega_n t$$

$$y = 0.1 \text{ m when } t = 0,$$

$$0.1 = 0 + B; \qquad B = 0.1$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$v = 0.75 \text{ m/s when } t = 0,$$

$$0.75 = A(7.00)$$

$$A = 0.107$$

$$y = 0.107 \sin (7.00t) + 0.100 \cos (7.00t)$$ **Ans.**

$$\phi = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{0.100}{0.107}\right) = 43.0^\circ$$ **Ans.**

Ans:
$y = 0.107 \sin (7.00t) + 0.100 \cos (7.00t)$
$\phi = 43.0^\circ$

***22–4.**

When a 20-lb weight is suspended from a spring, the spring is stretched a distance of 4 in. Determine the natural frequency and the period of vibration for a 10-lb weight attached to the same spring.

SOLUTION

$$k = \frac{20}{\frac{4}{12}} = 60 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{60}{\frac{10}{32.2}}} = 13.90 \text{ rad/s} \qquad \textbf{Ans.}$$

$$\tau = \frac{2\pi}{\omega_n} = 0.452 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$\omega_n = 13.90$ rad/s
$\tau = 0.452$ s

22–5.

When a 3-kg block is suspended from a spring, the spring is stretched a distance of 60 mm. Determine the natural frequency and the period of vibration for a 0.2-kg block attached to the same spring.

SOLUTION

$$k = \frac{F}{\Delta x} = \frac{3(9.81)}{0.060} = 490.5 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{490.5}{0.2}} = 49.52 = 49.5 \text{ rad/s}$$ **Ans.**

$$f = \frac{\omega_n}{2\pi} = \frac{49.52}{2\pi} = 7.88 \text{ Hz}$$

$$\tau = \frac{1}{f} = \frac{1}{7.88} = 0.127 \text{ s}$$ **Ans.**

Ans:
$\omega_n = 49.5$ rad/s
$\tau = 0.127$ s

22–6.

An 8-kg block is suspended from a spring having a stiffness $k = 80\ \text{N/m}$. If the block is given an upward velocity of 0.4 m/s when it is 90 mm above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that positive displacement is measured downward.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{80}{8}} = 3.162\ \text{rad/s}$$

$$v = -0.4\ \text{m/s}, \qquad x = -0.09\ \text{m at } t = 0$$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$-0.09 = 0 + B$$

$$B = -0.09$$

$$v = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$-0.4 = A(3.162) - 0$$

$$A = -0.126$$

Thus, $x = -0.126 \sin (3.16t) - 0.09 \cos (3.16t)\ \text{m}$ **Ans.**

$C = \sqrt{A^2 + B^2} = \sqrt{(-0.126)^2 + (-0.09)} = 0.155\ \text{m}$ **Ans.**

Ans:
$x = \{-0.126 \sin (3.16t) - 0.09 \cos (3.16t)\}\ \text{m}$
$C = 0.155\ \text{m}$

22–7.

A 2-lb weight is suspended from a spring having a stiffness $k = 2$ lb/in. If the weight is pushed 1 in. upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What is the amplitude and the natural frequency of the vibration?

SOLUTION

$$k = 2(12) = 24 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{24}{\frac{2}{32.2}}} = 19.66 = 19.7 \text{ rad/s} \qquad \textbf{Ans.}$$

$$y = -\frac{1}{12}, \qquad v = 0 \text{ at } t = 0$$

From Eqs. 22–3 and 22–4,

$$-\frac{1}{12} = 0 + B$$

$$B = -0.0833$$

$$0 = A\omega_n + 0$$

$$A = 0$$

$$C = \sqrt{A^2 + B^2} = 0.0833 \text{ ft} = 1 \text{ in.} \qquad \textbf{Ans.}$$

Position equation,

$$y = (0.0833 \cos 19.7t) \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$\omega_n = 19.7$ rad/s
$C = 1$ in.
$y = (0.0833 \cos 19.7t)$ ft

***22–8.**

A 6-lb weight is suspended from a spring having a stiffness $k = 3\ \text{lb/in.}$ If the weight is given an upward velocity of 20 ft/s when it is 2 in. above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the weight, measured from the equilibrium position. Assume positive displacement is downward.

SOLUTION

$$k = 3(12) = 36\ \text{lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{36}{\frac{6}{32.2}}} = 13.90\ \text{rad/s}$$

$$t = 0, \qquad v = -20\ \text{ft/s}, \qquad y = -\frac{1}{6}\ \text{ft}$$

From Eq. 22–3,

$$-\frac{1}{6} = 0 + B$$

$$B = -0.167$$

From Eq. 22–4,

$$-20 = A(13.90) + 0$$

$$A = -1.44$$

Thus,

$$y = [-1.44 \sin (13.9t) - 0.167 \cos (13.9t)]\ \text{ft} \qquad \textbf{Ans.}$$

From Eq. 22–10,

$$C = \sqrt{A^2 + B^2} = \sqrt{(1.44)^2 + (-0.167)^2} = 1.45\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$y = [-1.44 \sin (13.9t) - 0.167 \cos (13.9t)]\ \text{ft}$
$C = 1.45\ \text{ft}$

22–9.

A 3-kg block is suspended from a spring having a stiffness of $k = 200$ N/m. If the block is pushed 50 mm upward from its equilibrium position and then released from rest, determine the equation that describes the motion. What are the amplitude and the frequency of the vibration? Assume that positive displacement is downward.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{3}} = 8.16 \text{ rad/s} \qquad \textbf{Ans.}$$

$$x = A \sin \omega_n t + B \cos \omega_n t$$

$$x = -0.05 \text{ m when } t = 0,$$

$$-0.05 = 0 + B; \qquad B = -0.05$$

$$v = Ap \cos \omega_n t - B\omega_n \sin \omega_n t$$

$$v = 0 \text{ when } t = 0,$$

$$0 = A(8.165) - 0; \qquad A = 0$$

Hence,

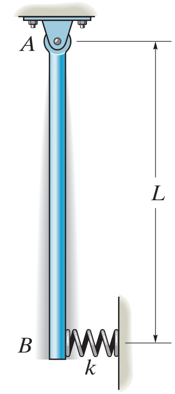
$$x = -0.05 \cos (8.16t) \qquad \textbf{Ans.}$$

$$C = \sqrt{A^2 + B^2} = \sqrt{(0)^2 + (-0.05)} = 0.05 \text{ m} = 50 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\omega_n = 8.16$ rad/s
$x = -0.05 \cos (8.16t)$
$C = 50$ mm

22–10.

The uniform rod of mass m is supported by a pin at A and a spring at B. If B is given a small sideward displacement and released, determine the natural period of vibration.



SOLUTION

Equation of Motion. The mass moment of inertia of the rod about A is $I_A = \frac{1}{3}mL^2$. Referring to the FBD. of the rod, Fig. a,

$$\zeta + \Sigma M_A = I_A\alpha; \qquad -mg\left(\frac{L}{2}\sin\theta\right) - (kx\cos\theta)(L) = \left(\frac{1}{3}mL^2\right)\alpha$$

However; $\quad x = L\sin\theta$. Then

$$\frac{-mgL}{2}\sin\theta - kL^2\sin\theta\cos\theta = \frac{1}{3}mL^2\alpha$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$,

$$\frac{-mgL}{2}\sin\theta - \frac{KL^2}{2}\sin 2\theta = \frac{1}{3}mL^2\alpha$$

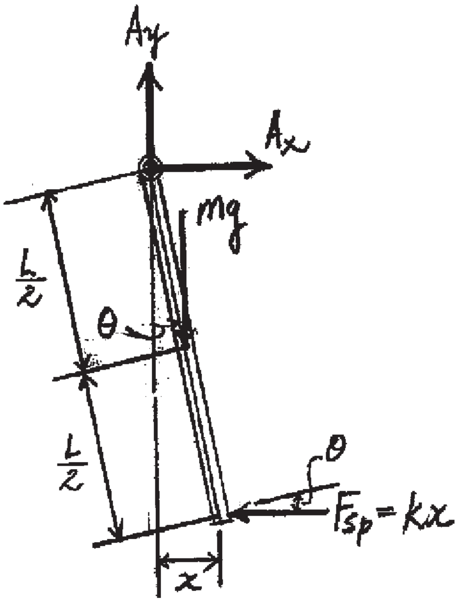
Here since θ is small $\sin\theta \simeq \theta$ and $\sin 2\theta \simeq 2\theta$. Also $\alpha = \ddot{\theta}$. Then the above equation becomes

$$\frac{1}{3}mL^2\ddot{\theta} + \left(\frac{mgL}{2} + kL^2\right)\theta = 0$$

$$\ddot{\theta} + \frac{3mg + 6kL}{2mL}\theta = 0$$

Comparing to that of the Standard form, $\omega_n = \sqrt{\dfrac{3mg + 6kL}{2mL}}$. Then

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2mL}{3mg + 6kL}} \qquad \textbf{Ans.}$$



(a)

$K L^2 \sin\theta \cos\theta$

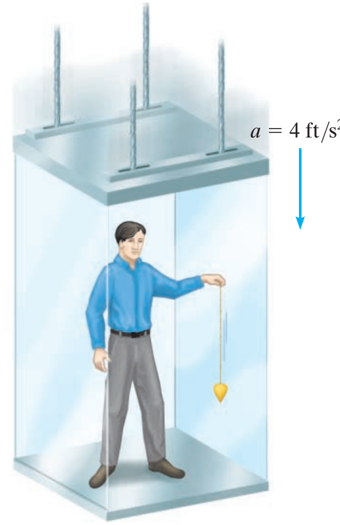
$\frac{kL^2}{2}$

Ans:

$\tau = 2\pi\sqrt{\dfrac{2mL}{3mg + 6kL}}$

22–11.

While standing in an elevator, the man holds a pendulum which consists of an 18-in. cord and a 0.5-lb bob. If the elevator is descending with an acceleration $a = 4\ \text{ft/s}^2$, determine the natural period of vibration for small amplitudes of swing.



SOLUTION

Since the acceleration of the pendulum is $(32.2 - 4) = 28.2\ \text{ft/s}^2$

Using the result of Example 22–1,

We have

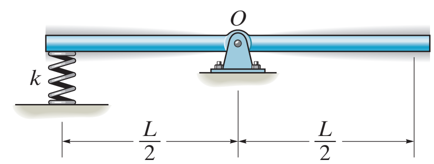
$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{28.2}{18/12}} = 4.336\ \text{rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{4.336} = 1.45\ \text{s} \qquad \textbf{Ans.}$$

Ans:
$\tau = 1.45\ \text{s}$

***22–12.**

Determine the natural period of vibration of the uniform bar of mass m when it is displaced downward slightly and released.



SOLUTION

Equation of Motion. The mass moment of inertia of the bar about O is $I_0 = \frac{1}{12}mL^2$. Referring to the FBD of the rod, Fig. a,

$$\circlearrowleft + \Sigma M_0 = I_0\alpha; \quad -ky\cos\theta\left(\frac{L}{2}\right) = \left(\frac{1}{12}mL^2\right)\alpha$$

However, $y = \frac{L}{2}\sin\theta$. Then

$$-k\left(\frac{L}{2}\sin\theta\right)\cos\theta\left(\frac{L}{2}\right) = \frac{1}{12}mL^2\alpha$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$, we obtain

$$\frac{1}{12}mL^2\alpha + \frac{kL^2}{8}\sin 2\theta = 0$$

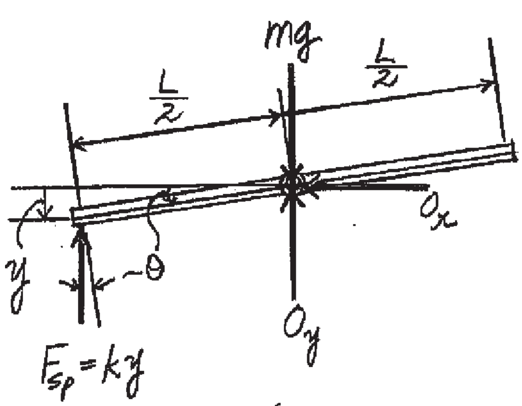
Here since θ is small, $\sin 2\theta \simeq 2\theta$. Also, $\alpha = \ddot{\theta}$. Then the above equation becomes

$$\frac{1}{12}mL^2\ddot{\theta} + \frac{kL^2}{4}\theta = 0$$

$$\ddot{\theta} + \frac{3k}{m}\theta = 0$$

Comparing to that of the Standard form, $\omega_n = \sqrt{\frac{3k}{m}}$. Then

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{3k}}$$ **Ans.**



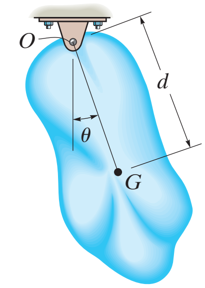
(a)

Ans:

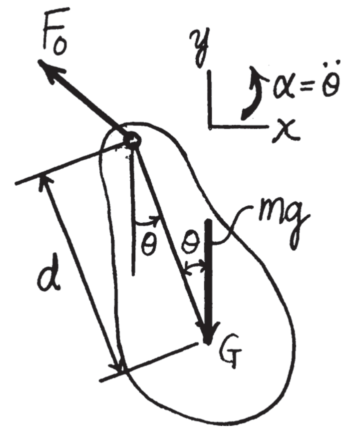
$$\tau = 2\pi\sqrt{\frac{m}{3k}}$$

22–13.

The body of arbitrary shape has a mass m, mass center at G, and a radius of gyration about G of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration.



SOLUTION



$\zeta\ +\Sigma M_O = I_O\,\alpha; \qquad -mgd\sin\theta = \left[mk_G^2 + md^2\right]\ddot{\theta}$

$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\sin\theta = 0$$

However, for small rotation $\sin\theta \approx \theta$. Hence

$$\ddot{\theta} + \frac{gd}{k_G^2 + d^2}\theta = 0$$

From the above differential equation, $\omega_n = \sqrt{\dfrac{gd}{k_G^2 + d^2}}$.

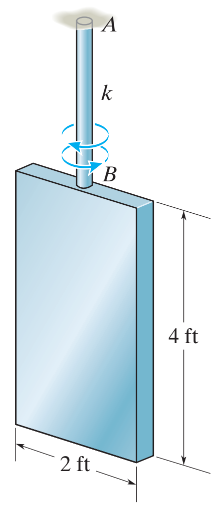
$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\dfrac{gd}{k_G^2 + d^2}}} = 2\pi\sqrt{\frac{k_G^2 + d^2}{gd}}$$ **Ans.**

Ans:

$\tau = 2\pi\sqrt{\dfrac{k_G^2 + d^2}{gd}}$

22–14.

The 20-lb rectangular plate has a natural period of vibration $\tau = 0.3$ s, as it oscillates around the axis of rod AB. Determine the torsional stiffness k, measured in lb · ft/rad, of the rod. Neglect the mass of the rod.



SOLUTION

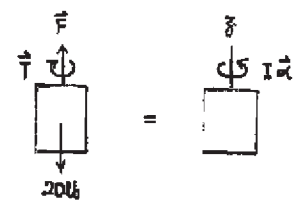
$T = k\theta$

$\Sigma M_z = I_z\alpha; \quad -k\theta = \frac{1}{12}\left(\frac{20}{32.2}\right)(2)^2\ddot{\theta}$

$\ddot{\theta} + k(4.83)\theta = 0$

$\tau = \dfrac{2\pi}{\sqrt{k(4.83)}} = 0.3$

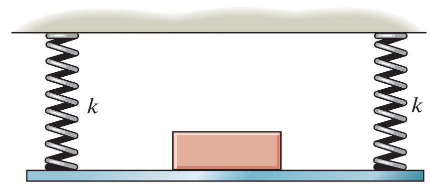
$k = 90.8 \text{ lb} \cdot \text{ft/rad}$ **Ans.**



Ans:
$k = 90.8 \text{ lb} \cdot \text{ft/rad}$

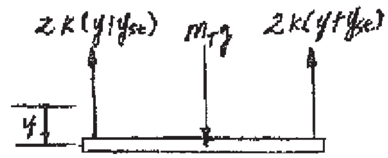
22–15.

A platform, having an unknown mass, is supported by *four* springs, each having the same stiffness k. When nothing is on the platform, the period of vertical vibration is measured as 2.35 s; whereas if a 3-kg block is supported on the platform, the period of vertical vibration is 5.23 s. Determine the mass of a block placed on the (empty) platform which causes the platform to vibrate vertically with a period of 5.62 s. What is the stiffness k of each of the springs?



SOLUTION

$+\downarrow \Sigma F_y = ma_y;\qquad m\tau g - 4k(y + y_{\tau s}) = m\tau \ddot{y}\qquad$ Where $4k\, y_{\tau s} = m_\tau g$



$$\ddot{y} + \frac{4k}{m\tau}y = 0$$

Hence
$$P = \sqrt{\frac{4k}{m\tau}}$$

$$\tau = \frac{2\pi}{P} = 2\pi\sqrt{\frac{m\tau}{4k}}$$

For empty platform $m\tau = m_P$, where m_P is the mass of the platform.

$$2.35 = 2\pi\sqrt{\frac{m_P}{4k}} \qquad \textbf{(1)}$$

When 3-kg block is on the platform $m_\tau = m_P + 3$.

$$5.23 = 2\pi\sqrt{\frac{m_P + 3}{4k}} \qquad \textbf{(2)}$$

When an unknown mass is on the platform $m_\tau = m_P + m_B$.

$$5.62 = 2\pi\sqrt{\frac{m_P + m_B}{4k}} \qquad \textbf{(3)}$$

Solving Eqs. (1) to (3) yields :

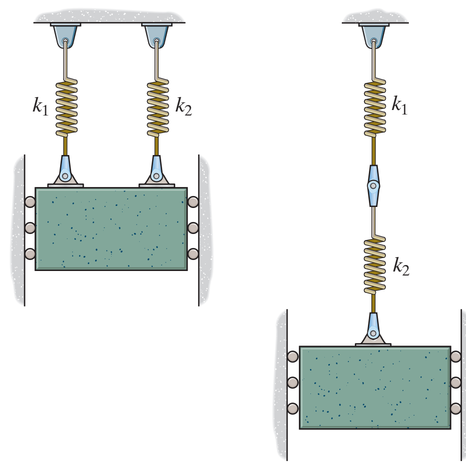
$$k = 1.36\ \text{N/m} \qquad m_B = 3.58\ \text{kg} \qquad \textbf{Ans.}$$

$$m_P = 0.7589\ \text{kg}$$

Ans:
$k = 1.36\ \text{N/m}$
$m_B = 3.58\ \text{kg}$

***22–16.**

A block of mass m is suspended from two springs having a stiffness of k_1 and k_2, arranged a) parallel to each other, and b) as a series. Determine the equivalent stiffness of a single spring with the same oscillation characteristics and the period of oscillation for each case.



(a) (b)

SOLUTION

(a) When the springs are arranged in parallel, the equivalent spring stiffness is

$$k_{eq} = k_1 + k_2$$ **Ans.**

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$$

Thus, the period of oscillation of the system is

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\dfrac{k_1 + k_2}{m}}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$ **Ans.**

(b) When the springs are arranged in a series, the equivalent stiffness of the system can be determined by equating the stretch of both spring systems subjected to the same load F.

$$\frac{F}{k_1} + \frac{F}{k_2} = \frac{F}{k_{eq}}$$

$$\frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k_{eq}}$$

$$\frac{k_2 + k_1}{k_1 k_2} = \frac{1}{k_{eq}}$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$ **Ans.**

The natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{\left(\dfrac{k_1 k_2}{k_2 + k_1}\right)}{m}}$$

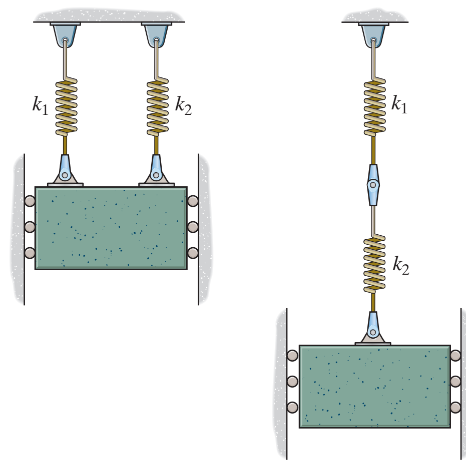
Thus, the period of oscillation of the system is

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\dfrac{\left(\dfrac{k_1 k_2}{k_2 + k_1}\right)}{m}}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$ **Ans.**

Ans:
$k_{eq} = k_1 + k_2$
$\tau = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}}$
$k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$
$\tau = 2\pi\sqrt{\dfrac{m(k_1 + k_2)}{k_1 k_2}}$

22–17.

The 15-kg block is suspended from two springs having a different stiffness and arranged a) parallel to each other, and b) as a series. If the natural periods of oscillation of the parallel system and series system are observed to be 0.5 s and 1.5 s, respectively, determine the spring stiffnesses k_1 and k_2.



(a) (b)

SOLUTION

The equivalent spring stiffness of the spring system arranged in parallel is $(k_{eq})_P = k_1 + k_2$ and the equivalent stiffness of the spring system arranged in a series can be determined by equating the stretch of the system to a single equivalent spring when they are subjected to the same load.

$$\frac{F}{k_1} + \frac{F}{k_2} = \frac{F}{(k_{eq})_S}$$

$$\frac{k_2 + k_1}{k_1 k_2} = \frac{1}{(k_{eq})_S}$$

$$(k_{eq})_S = \frac{k_1 k_2}{k_1 + k_2}$$

Thus the natural frequencies of the parallel and series spring system are

$$(\omega_n)_P = \sqrt{\frac{(k_{eq})_P}{m}} = \sqrt{\frac{k_1 + k_2}{15}}$$

$$(\omega_n)_S = \sqrt{\frac{(k_{eq})_S}{m}} = \sqrt{\frac{\left(\dfrac{k_1 k_2}{k_1 + k_2}\right)}{15}} = \sqrt{\frac{k_1 k_2}{15(k_1 + k_2)}}$$

Thus, the natural periods of oscillation are

$$\tau_P = \frac{2\pi}{(\omega_n)_P} = 2\pi\sqrt{\frac{15}{k_1 + k_2}} = 0.5 \qquad \textbf{(1)}$$

$$\tau_S = \frac{2\pi}{(\omega_n)_S} = 2\pi\sqrt{\frac{15(k_1 + k_2)}{k_1 k_2}} = 1.5 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

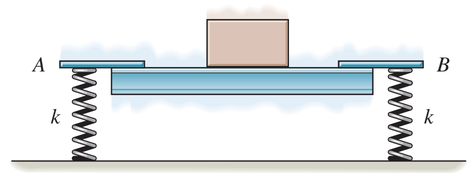
$k_1 = 2067$ N/m or 302 N/m **Ans.**

$k_2 = 302$ N/m or 2067 N/m **Ans.**

Ans:
$k_1 = 2067$ N/m
$k_2 = 302$ N/m
or vice versa

22–18.

The uniform beam is supported at its ends by two springs A and B, each having the same stiffness k. When nothing is supported on the beam, it has a period of vertical vibration of 0.83 s. If a 50-kg mass is placed at its center, the period of vertical vibration is 1.52 s. Compute the stiffness of each spring and the mass of the beam.



SOLUTION

$$\tau = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{\tau^2}{(2\pi)^2} = \frac{m}{k}$$

$$\frac{(0.83)^2}{(2\pi)^2} = \frac{m_B}{2k} \qquad \textbf{(1)}$$

$$\frac{(1.52)^2}{(2\pi)^2} = \frac{m_B + 50}{2k} \qquad \textbf{(2)}$$

Eqs. (1) and (2) become

$$m_B = 0.03490k$$

$$m_B + 50 = 0.1170k$$

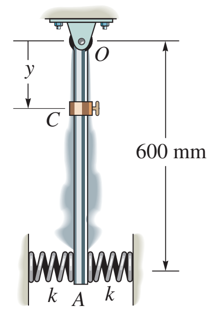
$$m_B = 21.2 \text{ kg} \qquad \textbf{Ans.}$$

$$k = 609 \text{ N/m} \qquad \textbf{Ans.}$$

Ans:
$m_B = 21.2$ kg
$k = 609$ N/m

22–19.

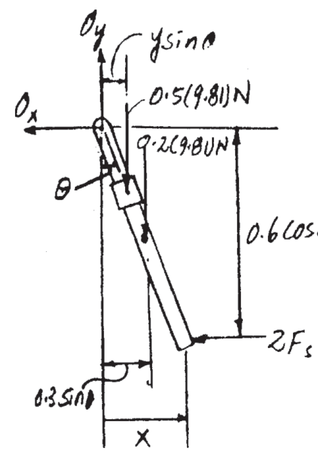
The slender rod has a mass of 0.2 kg and is supported at O by a pin and at its end A by two springs, each having a stiffness $k = 4$ N/m. The period of vibration of the rod can be set by fixing the 0.5-kg collar C to the rod at an appropriate location along its length. If the springs are originally unstretched when the rod is vertical, determine the position y of the collar so that the natural period of vibration becomes $\tau = 1$ s. Neglect the size of the collar.



SOLUTION

Moment of inertia about O:

$$I_O = \frac{1}{3}(0.2)(0.6)^2 + 0.5y^2 = 0.024 + 0.5y^2$$



Each spring force $F_s = kx = 4x$.

$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad -2(4x)(0.6\cos\theta) - 0.2(9.81)(0.3\sin\theta)$$

$$-0.5(9.81)(y\sin\theta) = (0.024 + 0.5y^2)\,\ddot{\theta}$$

$$-4.8x\cos\theta - (0.5886 + 4.905y)\sin\theta = (0.024 + 0.5y^2)\ddot{\theta}$$

However, for small displacement $x = 0.6\theta$, $\sin\theta \approx \theta$ and $\cos\theta = 1$. Hence

$$\ddot{\theta} + \frac{3.4686 + 4.905y}{0.024 + 0.5y^2}\theta = 0$$

From the above differential equation, $p = \sqrt{\dfrac{3.4686 + 4.905y}{0.024 + 0.5y^2}}$.

$$\tau = \frac{2\pi}{p}$$

$$1 = \frac{2\pi}{\sqrt{\dfrac{3.4686 + 4.905y}{0.024 + 0.5y^2}}}$$

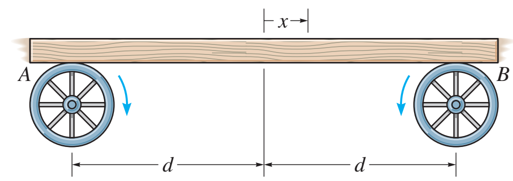
$$19.74y^2 - 4.905y - 2.5211 = 0$$

$$y = 0.503 \text{ m} = 503 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$y = 503$ mm

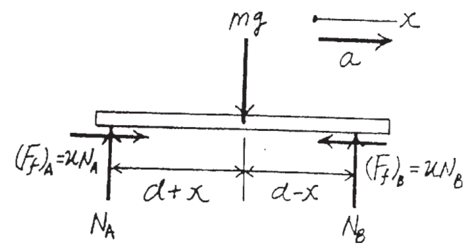
***22–20.**

A uniform board is supported on two wheels which rotate in opposite directions at a constant angular speed. If the coefficient of kinetic friction between the wheels and board is μ, determine the frequency of vibration of the board if it is displaced slightly, a distance x from the midpoint between the wheels, and released.



SOLUTION

Freebody Diagram: When the board is being displaced x to the right, the *restoring force* is due to the unbalance friction force at A and B $\left[(F_f)_B > (F_f)_A\right]$.



Equation of Motion:

$$\circlearrowleft + \Sigma M_A = \Sigma(M_A)_k; \qquad N_B(2d) - mg(d+x) = 0$$

$$N_B = \frac{mg(d+x)}{2d}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A + \frac{mg(d+x)}{2d} - mg = 0$$

$$N_A = \frac{mg(d-x)}{2d}$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad \mu\left[\frac{mg(d-x)}{2d}\right] - \mu\left[\frac{mg(d+x)}{2d}\right] = ma$$

$$a + \frac{\mu g}{d}x = 0 \qquad \textbf{(1)}$$

Kinematics: Since $a = \dfrac{d^2x}{dt^2} = \ddot{x}$, then substitute this value into Eq.(1), we have

$$\ddot{x} + \frac{\mu g}{d}x = 0 \qquad \textbf{(2)}$$

From Eq.(2), $\omega_n^{\,2} = \dfrac{\mu g}{d}$, thus, $\omega_n = \sqrt{\dfrac{\mu g}{d}}$. Applying Eq. 22–4, we have

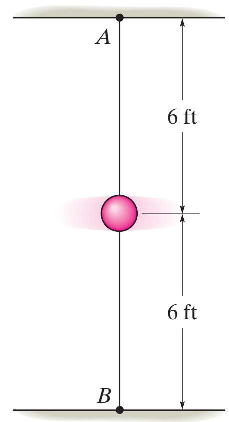
$$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\mu g}{d}} \qquad \textbf{Ans.}$$

Ans:

$$f = \frac{1}{2\pi}\sqrt{\frac{\mu g}{d}}$$

22–21.

If the wire AB is subjected to a tension of 20 lb, determine the equation which describes the motion when the 5-lb weight is displaced 2 in. horizontally and released from rest.



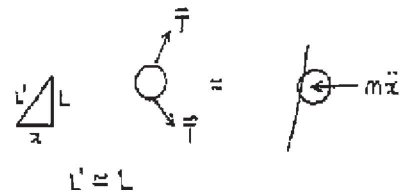
SOLUTION

$L' \equiv L$

$\overset{+}{\leftarrow} \Sigma F_x = m\,a_x; \qquad -2T\dfrac{x}{L} = m\ddot{x}$

$$\ddot{x} + \frac{2T}{Lm}x = 0$$

$$P = \sqrt{\frac{2T}{Lm}} = \sqrt{\frac{2(20)}{6\left(\frac{5}{32.2}\right)}} = 6.55 \text{ rad/s}$$



$x = A \sin pt + B \cos pt$

$x = \dfrac{1}{6}$ ft at $t = 0, \qquad$ Thus $B = \dfrac{1}{6} = 0.167$

$v = A\,p \cos pt - B\,p \sin pt$

$v = 0$ at $t = 0, \qquad$ Thus $A = 0$

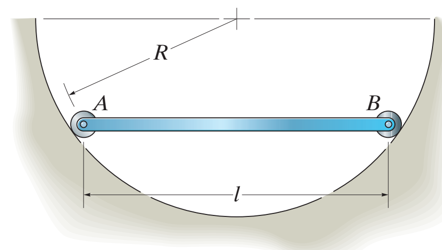
So that

$x = 0.167 \cos 6.55t$ **Ans.**

Ans:
$x = 0.167 \cos 6.55t$

22–22.

The bar has a length l and mass m. It is supported at its ends by rollers of negligible mass. If it is given a small displacement and released, determine the natural frequency of vibration.



SOLUTION

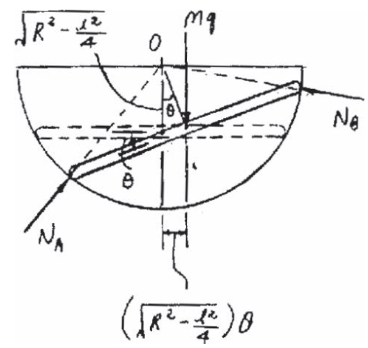
Moment of inertia about point O:

$$I_O = \frac{1}{12}ml^2 + m\left(\sqrt{R^2 - \frac{l^2}{4}}\right)^2 = m\left(R^2 - \frac{1}{6}l^2\right)$$

$$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad mg\left(\sqrt{R^2 - \frac{l^2}{4}}\right)\theta = -m\left(R^2 - \frac{1}{6}l^2\right)\ddot{\theta}$$

$$\ddot{\theta} + \frac{3g(4R^2 - l^2)^{\frac{1}{2}}}{6R^2 - l^2}\theta = 0$$

From the above differential equation, $\omega_n = \sqrt{\dfrac{3g(4R^2 - l^2)^{\frac{1}{2}}}{6R^2 - l^2}}$. **Ans.**

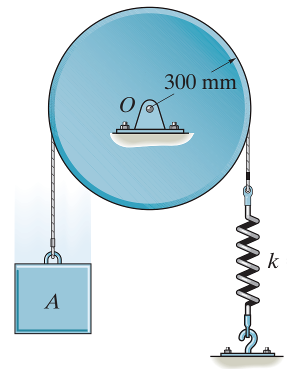


Ans:

$$\omega_n = \sqrt{\frac{3g(4R^2 - l^2)^{1/2}}{6R^2 - l^2}}$$

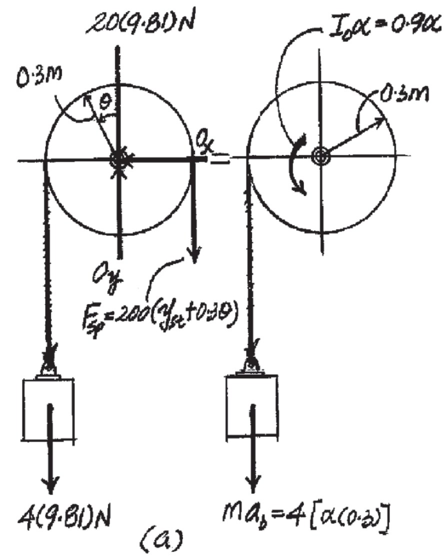
22–23.

The 20-kg disk, is pinned at its mass center O and supports the 4-kg block A. If the belt which passes over the disk is not allowed to slip at its contacting surface, determine the natural period of vibration of the system.



SOLUTION

Equation of Motion. The mass moment of inertia of the disk about its mass center O is $I_0 = \frac{1}{2}mr^2 = \frac{1}{2}(20)\left(0.3^2\right) = 0.9\ \text{kg}\cdot\text{m}^2$. When the disk undergoes a small angular displacement θ, the spring stretches further by $s = r\theta = 0.3\theta$. Thus, the total stretch is $y = y_{st} + 0.3\theta$. Then $F_{sp} = ky = 200(y_{st} + 0.3\theta)$. Referring to the FBD and kinetic diagram of the system, Fig. a,



$$\circlearrowleft + \Sigma M_0 = \Sigma(\mu_k)_0; \quad 4(9.81)(0.3) - 200(y_{st} + 0.3\theta)(0.3) = 0.90\alpha + 4[\alpha(0.3)](0.3)$$

$$11.772 - 60y_{st} - 18\theta = 1.26\alpha \qquad \textbf{(1)}$$

When the system is in equilibrium, $\theta = 0°$. Then

$$\circlearrowleft + \Sigma M_0 = 0; \qquad 4(9.81)(0.3) - 200(y_{st})(0.3) = 0$$

$$60y_{st} = 11.772$$

Substitute this result into Eq. (1), we obtain

$$-18\theta = 1.26\alpha$$

$$\alpha + 14.2857\theta = 0$$

Since $\alpha = \ddot{\theta}$, the above equation becomes

$$\ddot{\theta} + 14.2857\theta = 0$$

Comparing to that of standard form, $\omega_n = \sqrt{14.2857} = 3.7796$ rad/s.

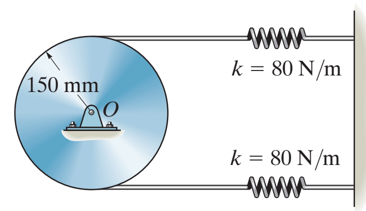
Thus,

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.7796} = 1.6623\ \text{s} = 1.66\ \text{s} \qquad \textbf{Ans.}$$

Ans:
$\tau = 1.66$ s

***22–24.**

The 10-kg disk is pin connected at its mass center. Determine the natural period of vibration of the disk if the springs have sufficient tension in them to prevent the cord from slipping on the disk as it oscillates. *Hint:* Assume that the initial stretch in each spring is δ_O.



SOLUTION

Equation of Motion. The mass moment of inertia of the disk about its mass center O is $I_0 = \frac{1}{2}Mr^2 = \frac{1}{2}(10)\left(0.15^2\right) = 0.1125\ \text{kg}\cdot\text{m}^2$. When the disk undergoes a small angular displacement θ, the top spring stretches further but the stretch of the spring is being reduced both by $s = r\theta = 0.15\theta$. Thus, $(F_{sp})_t = Kx_t = 80(\delta_0 - 0.15\theta)$ and $(F_{sp})_b = 80(\delta_0 - 0.15\theta)$. Referring to the FBD of the disk, Fig. *a*,

$$\circlearrowleft + \Sigma M_0 = I_0\alpha; \qquad -80(\delta_0 + 0.15\theta)(0.15) + 80(\delta_0 - 0.15\theta)(0.15) = 0.1125\alpha$$

$$-3.60\theta = 0.1125\alpha$$

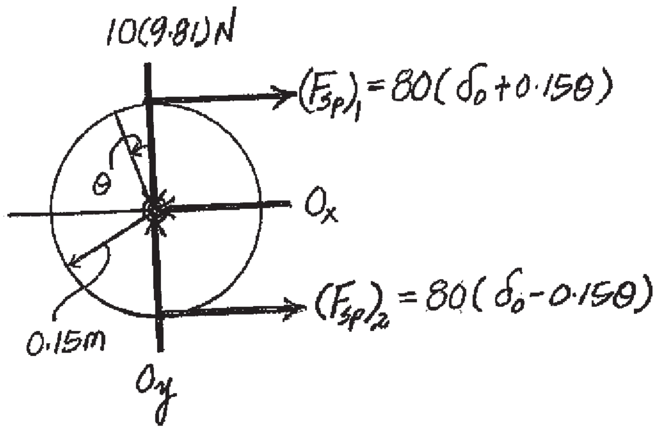
$$\alpha + 32\theta = 0$$

Since $\alpha = \ddot{\theta}$, this equation becomes

$$\ddot{\theta} + 32\theta = 0$$

Comparing to that of standard form, $\omega_n = \sqrt{32}$ rad/s. Then

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{32}} = 1.1107\ \text{s} = 1.11\ \text{s} \qquad \textbf{Ans.}$$

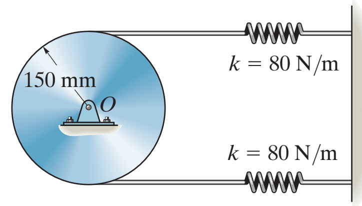


(a)

Ans:
$\tau = 1.11$ s

22–25.

If the disk in Prob. 22–24 has a mass of 10 kg, determine the natural frequency of vibration. *Hint:* Assume that the initial stretch in each spring is δ_O.



SOLUTION

Equation of Motion. The mass moment of inertia of the disk about its mass center O is $I_0 = \frac{1}{2}mr^2 = \frac{1}{2}(10)(0.15^2) = 0.1125\ \text{kg}\cdot\text{m}^2$ when the disk undergoes a small angular displacement θ, the top spring stretches but the bottom spring compresses, both by $s = r\theta = 0.15\theta$. Thus, $(F_{sp})_t = (F_{sp})_b = ks = 80(0.15\theta) = 12\theta$. Referring to the FBD of the disk, Fig. a,

$$\circlearrowleft + \Sigma M_0 = I_0\alpha; \qquad -12\theta(0.3) = 0.1125\alpha$$

$$-3.60\theta = 0.1125\alpha$$

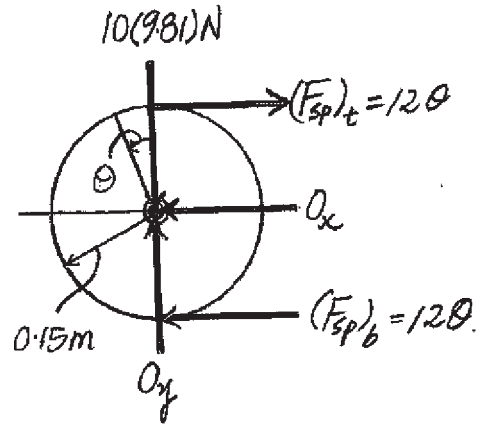
$$\alpha + 32\theta = 0$$

Since $\alpha = \ddot{\theta}$, this equation becomes

$$\ddot{\theta} + 32\theta = 0$$

Comparing to that of Standard form, $\omega_n = \sqrt{32}$ rad/s. Then

$$f = \frac{\omega_n}{2\pi} = \frac{\sqrt{32}}{2\pi} = 0.9003\text{ Hz} = 0.900\text{ Hz}$$

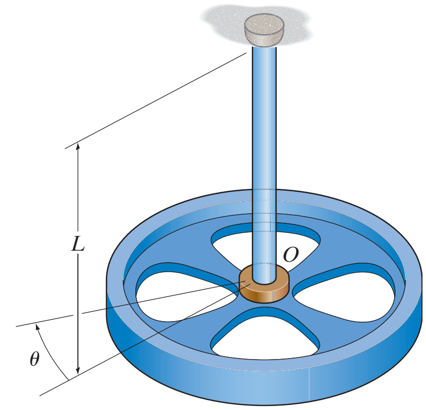


(a)

Ans:
$f = 0.900$ Hz

22–26.

A flywheel of mass m, which has a radius of gyration about its center of mass of k_O, is suspended from a circular shaft that has a torsional resistance of $M = C\theta$. If the flywheel is given a small angular displacement of θ and released, determine the natural period of oscillation.

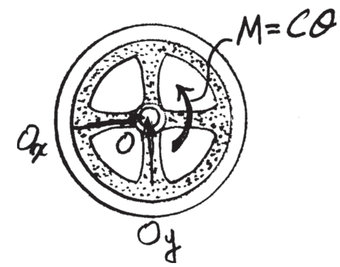


SOLUTION

Equation of Motion: The mass moment of inertia of the wheel about point O is $I_O = mk_O^2$. Referring to Fig. a,

$$\circlearrowleft + \quad \Sigma M_O = I_O\,\alpha; \qquad -C\theta = mk_O^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{C}{mk_O^2}\theta = 0$$



Comparing this equation to the standard equation, the natural circular frequency of the wheel is

$$\omega_n = \sqrt{\frac{C}{mk_O^2}} = \frac{1}{k_O}\sqrt{\frac{C}{m}}$$

Thus, the natural period of the oscillation is

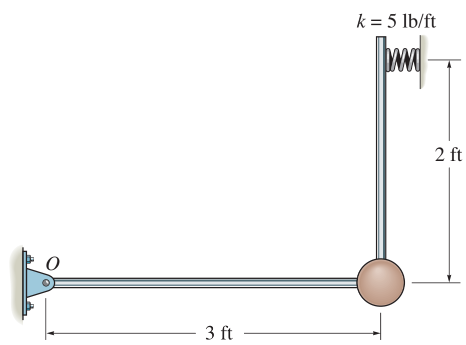
$$\tau = \frac{2\pi}{\omega_n} = 2\pi k_O\sqrt{\frac{m}{C}}$$ **Ans.**

Ans:

$$\tau = 2\pi k_O\sqrt{\frac{m}{C}}$$

22–27.

The 6-lb weight is attached to the rods of negligible mass. Determine the natural frequency of vibration of the weight when it is displaced slightly from the equilibrium position and released.

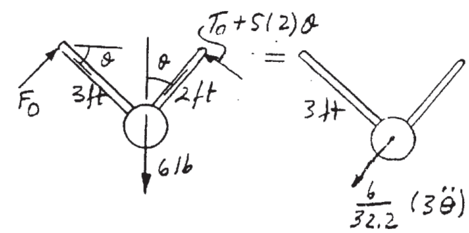


SOLUTION

T_O is the equilibrium force.

$$T_O = \frac{6(3)}{2} = 9 \text{ lb}$$

Thus, for small θ,



$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad 6(3) - \left[9 + 5(2)\theta\right](2) = \left(\frac{6}{32.2}\right)\left(3\ddot{\theta}\right)(3)$$

Thus,

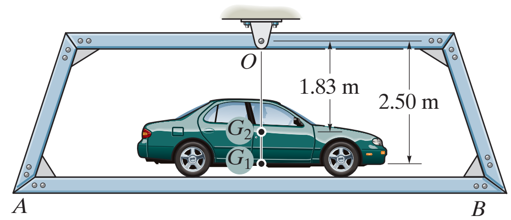
$$\ddot{\theta} + 11.926\theta = 0$$

$$\omega_n = \sqrt{11.926} = 3.45 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega_n = 3.45 \text{ rad/s}$

***22–28.**

The platform AB when empty has a mass of 400 kg, center of mass at G_1, and natural period of oscillation $\tau_1 = 2.38$ s. If a car, having a mass of 1.2 Mg and center of mass at G_2, is placed on the platform, the natural period of oscillation becomes $\tau_2 = 3.16$ s. Determine the moment of inertia of the car about an axis passing through G_2.



SOLUTION

Free-body Diagram: When an object arbitrary shape having a mass m is pinned at O and being displaced by an angular displacement of θ, the tangential component of its weight will create the *restoring moment* about point O.

Equation of Motion: Sum moment about point O to eliminate O_x and O_y.

$$\circlearrowleft + \Sigma M_O = I_O\alpha: \qquad -mg\sin\theta(l) = I_O\alpha \qquad \textbf{(1)}$$

Kinematics: Since $\alpha = \dfrac{d^2\theta}{dt^2} = \ddot{\theta}$ and $\sin\theta = \theta$ if θ is small, then substituting these values into Eq. (1), we have

$$-mgl\theta = I_O\ddot{\theta} \qquad \text{or} \qquad \ddot{\theta} + \frac{mgl}{I_O}\theta = 0 \qquad \textbf{(2)}$$

From Eq. (2), $\omega_n^2 = \dfrac{mgl}{I_O}$, thus, $\omega_n = \sqrt{\dfrac{mgl}{I_O}}$, Applying Eq. 22–12, we have

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_O}{mgl}} \qquad \textbf{(3)}$$

When the platform is empty, $\tau = \tau_1 = 2.38$ s, $m = 400$ kg and $l = 2.50$ m. Substituting these values into Eq. (3), we have

$$2.38 = 2\pi\sqrt{\frac{(I_O)_p}{400(9.81)(2.50)}} \qquad (I_O)_p = 1407.55 \text{ kg}\cdot\text{m}^2$$

When the car is on the platform, $\tau = \tau_2 = 3.16$ s, $m = 400 \text{ kg} + 1200 \text{ kg} = 1600$ kg. $l = \dfrac{2.50(400) + 1.83(1200)}{1600} = 1.9975$ m and $I_O = (I_O)_C + (I_O)_p = (I_O)_C + 1407.55$. Substituting these values into Eq. (3), we have

$$3.16 = 2\pi\sqrt{\frac{(I_O)_C + 1407.55}{1600(9.81)(1.9975)}} \qquad (I_O)_C = 6522.76 \text{ kg}\cdot\text{m}^2$$

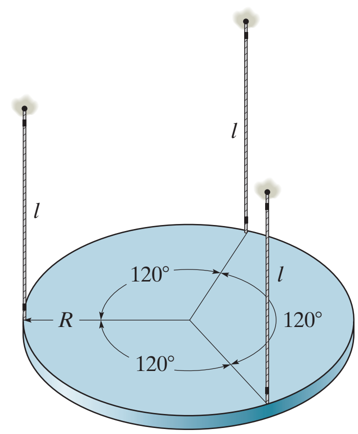
Thus, the mass moment inertia of the car about its mass center is

$$(I_G)_C = (I_O)_C - m_C d^2$$
$$= 6522.76 - 1200(1.83^2) = 2.50(10^3) \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

Ans:
$(I_G)_C = 2.50(10^3)$ kg $\cdot$ m^2

22–29.

The plate of mass m is supported by three symmetrically placed cords of length l as shown. If the plate is given a slight rotation about a vertical axis through its center and released, determine the natural period of oscillation.



SOLUTION

$$\Sigma M_z = I_z\alpha \qquad -3(T\sin\phi)R = \frac{1}{2}mR^2\ddot{\theta}$$

$$\sin\phi \equiv \phi$$

$$\ddot{\theta} + \frac{6T}{Rm}\phi = 0$$

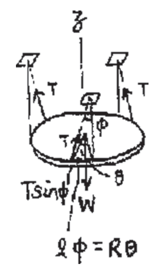
$$\Sigma F_z = 0 \qquad 3T\cos\phi - mg = 0$$

$$\phi = 0, \qquad T = \frac{mg}{3}, \qquad \phi = \frac{R}{l}\theta$$

$$\ddot{\theta} + \frac{6}{Rm}\left(\frac{mg}{3}\right)\left(\frac{R}{l}\theta\right) = 0$$

$$\ddot{\theta} + \frac{2g}{l}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{l}{2g}}$$ **Ans.**

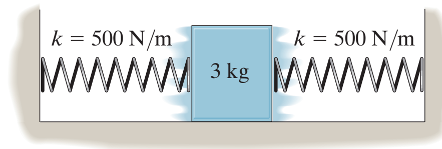


Ans:

$$\tau = 2\pi\sqrt{\frac{l}{2g}}$$

22–30.

Determine the differential equation of motion of the 3-kg block when it is displaced slightly and released. The surface is smooth and the springs are originally unstretched.



SOLUTION

$$T + V = \text{const.}$$

$$T = \frac{1}{2}(3)\dot{x}^2$$

$$V = \frac{1}{2}(500)x^2 + \frac{1}{2}(500)x^2$$

$$T + V = 1.5\dot{x}^2 + 500x^2$$

$$1.5(2\dot{x})\,\ddot{x} + 1000x\dot{x} = 0$$

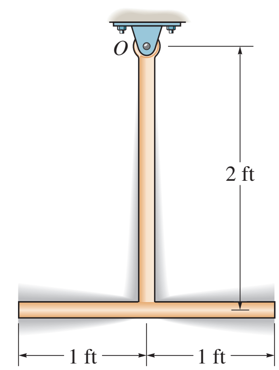
$$3\ddot{x} + 1000x = 0$$

$$\ddot{x} + 333x = 0 \qquad \textbf{Ans.}$$

Ans:
$\ddot{x} + 333x = 0$

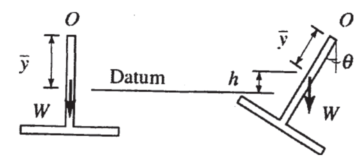
22–31.

Determine the natural period of vibration of the pendulum. Consider the two rods to be slender, each having a weight of 8 lb/ft.



SOLUTION

$$\bar{y} = \frac{1(8)(2) + 2(8)(2)}{8(2) + 8(2)} = 1.5 \text{ ft}$$



$$I_O = \frac{1}{32.2}\left[\frac{1}{12}(2)(8)(2)^2 + 2(8)(1)^2\right]$$

$$+ \frac{1}{32.2}\left[\frac{1}{12}(2)(8)(2)^2 + 2(8)(2)^2\right] = 2.8157 \text{ slug} \cdot \text{ft}^2$$

$$h = \bar{y}\,(1 - \cos\theta)$$

$$T + V = \text{const}$$

$$T = \frac{1}{2}(2.8157)(\dot{\theta})^2 = 1.4079\,\dot{\theta}^2$$

$$V = 8(4)(1.5)(1 - \cos\theta) = 48(1 - \cos\theta)$$

$$T + V = 1.4079\dot{\theta}^2 + 48(1 - \cos\theta)$$

$$1.4079\,(2\dot{\theta})\ddot{\theta} + 48(\sin\theta)\dot{\theta} = 0$$

For small θ, $\sin\theta = \theta$, then

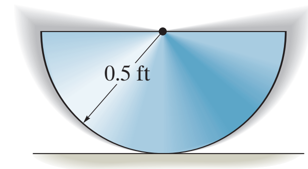
$$\ddot{\theta} + 17.047\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{17.047}} = 1.52 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$\tau = 1.52$ s

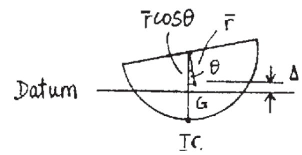
***22–32.**

Determine the natural period of vibration of the 10-lb semicircular disk.



SOLUTION

Datum at initial level of center of gravity of disk.



$$\Delta = \bar{r}(1 - \cos\theta)$$

$$E = T + V$$

$$= \frac{1}{2}I_{IC}(\dot{\theta})^2 + W\bar{r}(1 - \cos\theta)$$

$$\dot{E} = \dot{\theta}(I_{IC}\ddot{\theta} + W\bar{r}\sin\theta) = 0$$

For small θ, $\quad \sin\theta = \theta$

$$\ddot{\theta} + \frac{W\bar{r}}{I_{IC}}\theta = 0$$

$$\bar{r} = \frac{4(0.5)}{3\pi} = 0.212 \text{ ft}$$

$$I_A = I_G + m\bar{r}^2$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(0.5)^2 = I_G + \frac{10}{32.2}(0.212)^2$$

$$I_G = 0.02483 \text{ slug}\cdot\text{ft}^2$$

$$I_{IC} = I_G + m(r - \bar{r})^2$$

$$= 0.02483 + \frac{10}{32.2}(0.5 - 0.212)^2$$

$$= 0.05056 \text{ slug}\cdot\text{ft}^2$$

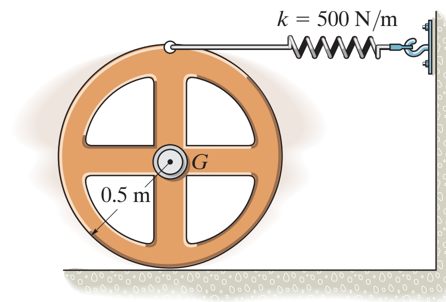
$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_{IC}}{W\bar{r}}} = 2\pi\sqrt{\frac{0.05056}{10(0.212)}}$$

$\tau = 0.970$ s **Ans.**

Ans:
$\tau = 0.970$ s

22–33.

If the 20-kg wheel is displaced a small amount and released, determine the natural period of vibration. The radius of gyration of the wheel is $k_G = 0.36$ m. The wheel rolls without slipping.



SOLUTION

Energy Equation. The mass moment of inertia of the wheel about its mass center is $I_G = mk_G = 20(0.361)^2 = 2.592\ \text{kg}\cdot\text{m}^2$. Since the wheel rolls without slipping, $v_G = \omega r = \omega(0.5)$. Thus,

$$T = \frac{1}{2}I_G\omega^2 + \frac{1}{2}mv_G^2$$

$$= \frac{1}{2}(2.592)\omega^2 + \frac{1}{2}(20)[\omega(0.5)]^2$$

$$= 3.796\,\omega^2 = 3.796\dot{\theta}^2$$

When the disk undergoes a small angular displacement θ, the spring stretches $s = \theta(1) = \theta$, Fig. *a*. Thus, the elastic potential energy is

$$V_e = \frac{1}{2}ks^2 = \frac{1}{2}(500)\theta^2 = 250\theta^2$$

Thus, the total energy is

$$E = T + V = 3.796\dot{\theta}^2 + 250\theta^2$$

Time Derivative. Taking the time derivative of the above equation,

$$7.592\dot{\theta}\ddot{\theta} + 500\theta\dot{\theta} = 0$$

$$\dot{\theta}(7.592\ddot{\theta} + 500\theta) = 0$$

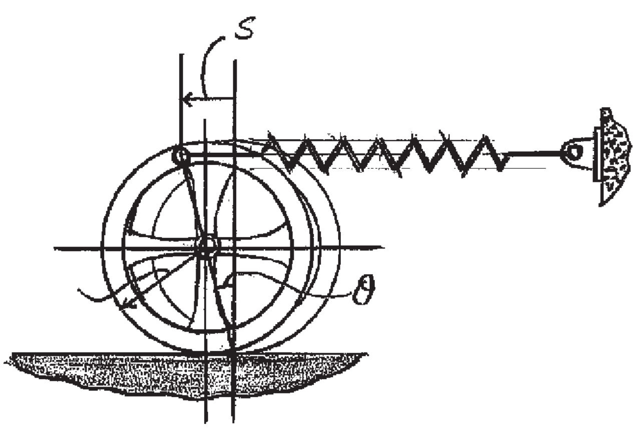
Since $\dot{\theta} \neq 0$, then

$$7.592\ddot{\theta} + 500\theta = 0$$

$$\ddot{\theta} + 65.8588\theta = 0$$

Comparing to that of standard form, $\omega_n = \sqrt{65.8588} = 8.1153$ rad/s. Thus,

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{8.1153} = 0.7742 \text{ s} = 0.774 \text{ s} \qquad \textbf{Ans.}$$

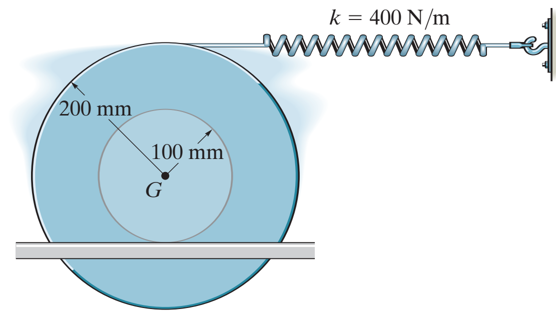


(a)

Ans:
$\tau = 0.774$ s

22–34.

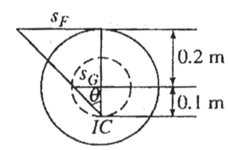
Determine the differential equation of motion of the 3-kg spool. Assume that it does not slip at the surface of contact as it oscillates. The radius of gyration of the spool about its center of mass is $k_G = 125$ mm.



SOLUTION

Kinematics: Since no slipping occurs, $s_G = 0.1\theta$ hence $s_F = \frac{0.3}{0.1}S_G = 0.3\theta$. Also,

$$v_G = 0.1\dot{\theta}.$$



$$E = T + V$$

$$E = \frac{1}{2}[(3)(0.125)^2]\dot{\theta}^2 + \frac{1}{2}(3)(0.1\dot{\theta})^2 + \frac{1}{2}(400)(0.3\theta)^2 = \text{const.}$$

$$= 0.038448\dot{\theta}^2 + 18\theta^2$$

$$0.076875\dot{\theta}\ddot{\theta} + 36\theta\dot{\theta} = 0$$

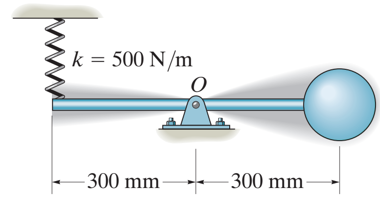
$$0.076875\dot{\theta}(\ddot{\theta} + 468.29\theta) = 0 \text{ Since } 0.076875\dot{\theta} \neq 0$$

$$\ddot{\theta} + 468\theta = 0$$ **Ans.**

Ans:
$\ddot{\theta} + 468\theta = 0$

22–35.

Determine the natural period of vibration of the 3-kg sphere. Neglect the mass of the rod and the size of the sphere.

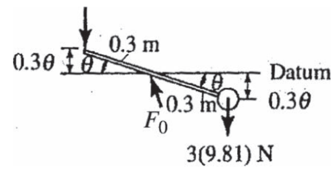


SOLUTION

$$E = T + V$$

$$= \frac{1}{2}(3)(0.3\dot{\theta})^2 + \frac{1}{2}(500)(\delta_{st} + 0.3\theta)^2 - 3(9.81)(0.3\theta)$$

$$E = \dot{\theta}[(3(0.3)^2\ddot{\theta} + 500(\delta_{st} + 0.3\theta)(0.3) - 3(9.81)(0.3)] = 0$$



By statics,

$$T(0.3) = 3(9.81)(0.3)$$

$$T = 3(9.81)\text{ N}$$

$$\delta_{st} = \frac{3(9.81)}{500}$$

Thus,

$$3(0.3)^2\ddot{\theta} + 500(0.3)^2\theta = 0$$

$$\ddot{\theta} + 166.67\theta = 0$$

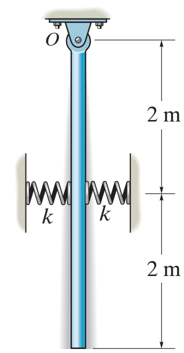
$$\omega_n = \sqrt{166.67} = 12.91\text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{12.91} = 0.487\text{ s}$$ **Ans.**

Ans:
$\tau = 0.487$ s

***22–36.**

If the lower end of the 6-kg slender rod is displaced a small amount and released from rest, determine the natural frequency of vibration. Each spring has a stiffness of $k = 200\ \text{N/m}$ and is unstretched when the rod is hanging vertically.



SOLUTION

Energy Equation. The mass moment of inertia of the rod about O is $I_0 = \frac{1}{3}ml^2 = \frac{1}{3}(6)\left(4^2\right) = 32\ \text{kg}\cdot\text{m}^2$. Thus, the Kinetic energy is

$$T = \frac{1}{2}I_0\omega^2 = \frac{1}{2}(32)\dot{\theta}^2 = 16\dot{\theta}^2$$

with reference to the datum set in Fig. *a*, the gravitational potential energy is

$$V_g = mgy = 6(9.81)(-2\cos\theta) = -117.72\cos\theta$$

When the rod undergoes a small angular displacement θ the spring deform $x = 2\sin\Omega$. Thus the elastic potential energy is

$$V_e = 2\left(\frac{1}{2}kx^2\right) = 2\left[\frac{1}{2}(200)(2\sin\theta)^2\right] = 800\sin^2\theta$$

Thus, the total energy is

$$E = T + V = 16\dot{\theta}^2 + 800\sin^2\theta - 117.72\cos\theta$$

Time Derivative. Taking the first time derivative of the above equation

$$32\dot{\theta}\ddot{\theta} + 1600(\sin\theta\cos\theta)\dot{\theta} + 117.72(\sin\theta)\dot{\theta} = 0$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$, we obtain

$$32\dot{\theta}\ddot{\theta} + 800(\sin 2\theta)\dot{\theta} + 117.72(\sin\theta)\dot{\theta} = 0$$

$$\dot{\theta}(32\ddot{\theta} + 800\sin 2\theta + 117.72\sin\theta) = 0$$

Since $\dot{\theta} \neq 0$,

$$32\ddot{\theta} + 800\sin 2\theta + 117.72\sin\theta) = 0$$

Since θ is small, $\sin 2\theta \simeq 2\theta$ and $\sin\theta = \theta$. The above equation becomes

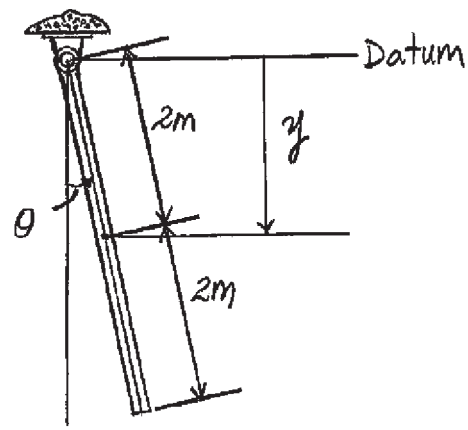
$$32\ddot{\theta} + 1717.72\theta = 0$$

$$\ddot{\theta} + 53.67875\theta = 0$$

Comparing to that of standard form, $\omega_n = \sqrt{53.67875} = 7.3266\ \text{rad/s}$.

Thus,

$$f = \frac{\omega_n}{2\pi} = \frac{7.3266}{2\pi} = 1.1661\ \text{Hz} = 1.17\ \text{Hz} \qquad \textbf{Ans.}$$

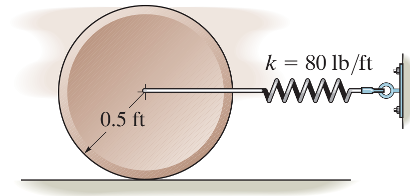


(a)

Ans:
$f = 1.17\ \text{Hz}$

22–37.

The disk has a weight of 30 lb and rolls without slipping on the horizontal surface as it oscillates about its equilibrium position. If the disk is displaced, by rolling it counterclockwise 0.2 rad, determine the equation which describes its oscillatory motion and the natural period when it is released.



SOLUTION

Energy Equation. The mass moment of inertia of the disk about its center of gravity is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{30}{32.2}\right)(0.5^2) = 0.11646 \text{ slug}\cdot\text{ft}^2$. Since the disk rolls without slipping, $v_G = \omega r = \omega(0.5)$. Thus

$$T = \frac{1}{2}I_G\omega^2 + \frac{1}{2}mv_G^2$$

$$= \frac{1}{2}(0.1146)\,\omega^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)[\omega(0.5)]^2$$

$$= 0.17469\,\omega^2 = 0.17469\dot{\theta}^2$$

When the disk undergoes a small angular displacement θ the spring stretches $s = \theta r = \theta(0.5)$, Fig. *a*. Thus, the elastic potential energy is

$$V_e = \frac{1}{2}ks^2 = \frac{1}{2}(80)[\theta(0.5)]^2 = 10\theta^2$$

Thus, the total energy is

$$E = T + V = 0.17469\dot{\theta}^2 + 10\theta^2$$

$$E = 0.175\dot{\theta}^2 + 10\theta^2 \qquad \textbf{Ans.}$$

Time Derivative. Taking the time derivative of the above equation,

$$0.34938\dot{\theta}\ddot{\theta} + 20\theta\dot{\theta} = 0$$

$$\dot{\theta}(0.34938\ddot{\theta} + 20\theta) = 0$$

Since $\dot{\theta} \neq 0$, then

$$0.34938\dot{\theta} + 20\theta = 0$$

$$\ddot{\theta} + 57.244\theta = 0$$

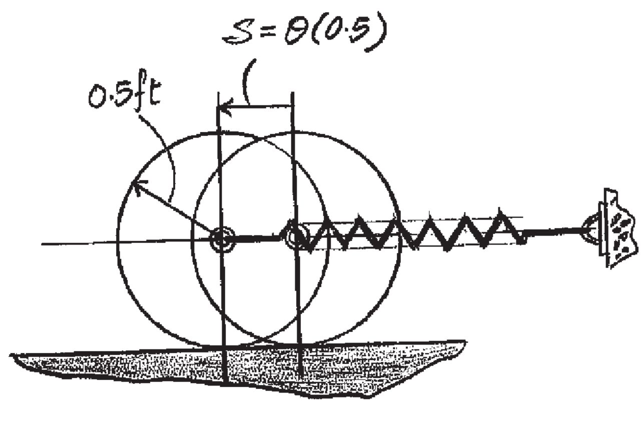
$$\ddot{\theta} = 57.2\theta = 0 \qquad \textbf{Ans.}$$

Comparing to that of standard form,

$\omega_n = \sqrt{57.2444} = 7.5660$ rad/s.

Thus

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{7.5660} = 0.8304 \text{ s} = 0.830 \text{ s} \qquad \textbf{Ans.}$$

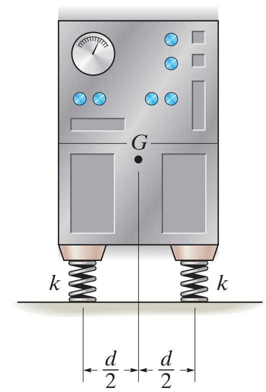


(a)

Ans:
$E = 0.175\dot{\theta}^2 + 10\,\theta^2$
$\tau = 0.830$ s

22–38.

The machine has a mass m and is uniformly supported by *four* springs, each having a stiffness k. Determine the natural period of vertical vibration.

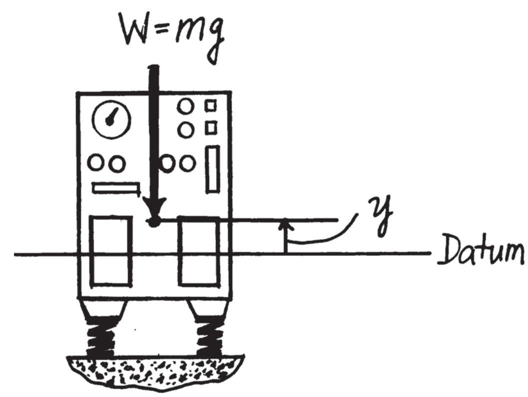


SOLUTION

$$T + V = \text{const.}$$

$$T = \frac{1}{2} m(\dot{y})^2$$

$$V = m\,g\,y + \frac{1}{2}(4k)(\Delta s - y)^2$$



$$T + V = \frac{1}{2} m(\dot{y})^2 + m\,g\,y + \frac{1}{2}(4k)(\Delta s - y)^2$$

$$m\,\dot{y}\,\ddot{y} + m\,g\,\dot{y} - 4k(\Delta s - y)\dot{y} = 0$$

$$m\,\ddot{y} + m\,g + 4ky - 4k\Delta s = 0$$

Since $\Delta s = \dfrac{mg}{4k}$

Then

$$m\ddot{y} + 4ky = 0$$

$$y + \frac{4k}{m} y = 0$$

$$\omega_n = \sqrt{\frac{4k}{m}}$$

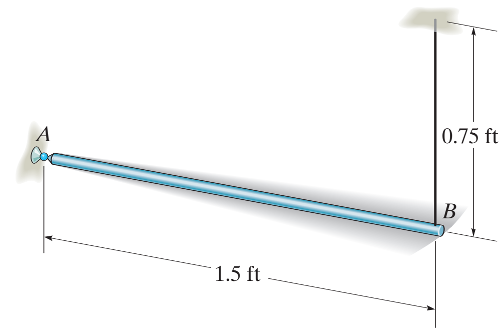
$$\tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{m}{k}}$$ **Ans.**

Ans:

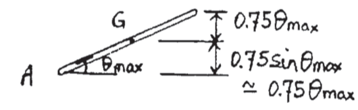
$\tau = \pi\sqrt{\dfrac{m}{k}}$

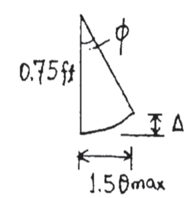
22–39.

The slender rod has a weight of 4 lb/ft. If it is supported in the horizontal plane by a ball-and-socket joint at A and a cable at B, determine the natural frequency of vibration when the end B is given a small horizontal displacement and then released.



SOLUTION





$$\phi = \frac{1.5\theta_{max}}{0.75}$$

$$\Delta = 0.75(1 - \cos\phi)$$

$$\cong 0.75\left(1 - 1 + \frac{\phi^2}{2}\right)$$

$$= 0.75\left(\frac{4\theta_{max}^2}{2}\right)$$

$$\Delta G = \frac{1}{2}\Delta = 0.75\theta_{max}^2$$

$$T_{max} = \frac{1}{2}I_A\,\omega_{max}^2$$

$$= \frac{1}{2}\left[\frac{1}{3}\left(\frac{4(1.5)}{32.2}\right)(1.5)^2\right]\omega_n^{\,2}\,\theta_{max}^2$$

$$= 0.0699\,\omega_n^{\,2}\,\theta_{max}^2$$

$$V_{max} = W\Delta_G = 4(1.5)(0.75\theta_{max}^2)$$

$$T_{max} = V_{max}$$

$$0.0699\omega_n^{\,2}\,\theta_{max}^2 = 4.5\,\theta_{max}^2$$

$$\omega_n^{\,2} = 64.40$$

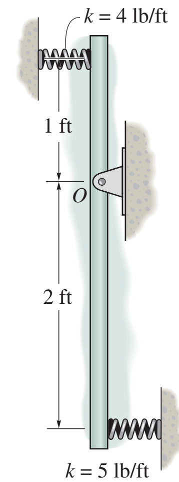
$$\omega_n = 8.025 \text{ rad/s}$$

$$f = \frac{\omega_n}{2\pi} = \frac{8.025}{2\pi} = 1.28 \text{ Hz}$$ **Ans.**

Ans:
$f = 1.28$ Hz

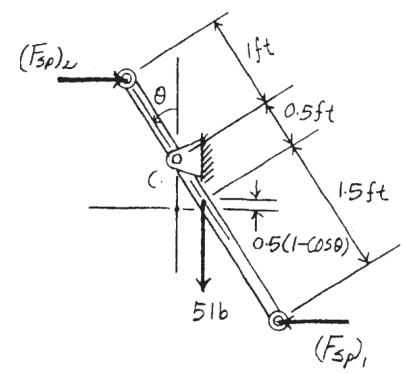
***22–40.**

If the slender rod has a weight of 5 lb, determine the natural frequency of vibration. The springs are originally unstretched.



SOLUTION

***Energy Equation*:** When the rod is being displaced a small angular displacement of θ, the compression of the spring at its ends can be approximated as $x_1 = 2\theta$ and $x_2 = 1\theta$. Thus, the elastic potential energy when the rod is at this position is $V_e = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2 = \frac{1}{2}(5)(2\theta)^2 + \frac{1}{2}(4)(1\theta)^2 = 12\theta^2$. The datum is set at the rod's mass center when the rod is at its original position. When the rod undergoes a small angular displacement θ, its mass center is $0.5(1 - \cos\theta)$ ft *above* the datum hence its gravitational potential energy is $V_g = 5[0.5(1 - \cos\theta)]$. Since θ is small, $\cos\theta$ can be approximated by the first two terms of the power series, that is, $\cos\theta = 1 - \frac{\theta^2}{2}$. Thus, $V_g = 2.5\left[1 - \left(1 - \frac{\theta^2}{2}\right)\right] = 1.25\theta^2$



$$V = V_e + V_g = 12\theta^2 + 1.25\theta^2 = 13.25\theta^2$$

The mass moment inertia of the rod about point O is $I_O = \frac{1}{12}\left(\frac{5}{32.2}\right)(3^2) + \frac{5}{32.2}\left(0.5^2\right) = 0.1553\ \text{slug}\cdot\text{ft}^2$. The kinetic energy is

$$T = \frac{1}{2}I_O\,\omega^2 = \frac{1}{2}(0.1553)\,\dot{\theta}^2 = 0.07764\dot{\theta}^2$$

The total energy of the system is

$$U = T + V = 0.07764\dot{\theta}^2 + 13.25\theta^2 \qquad \textbf{[1]}$$

***Time Derivative*:** Taking the time derivative of Eq.[1], we have

$$0.1553\dot{\theta}\ddot{\theta} + 26.5\theta\dot{\theta} = 0$$

$$\dot{\theta}\left(0.1553\ddot{\theta} + 26.5\theta\right) = 0$$

Since $\dot{\theta} \neq 0$, then

$$0.1553\ddot{\theta} + 26.5\theta = 0$$

$$\ddot{\theta} + 170.66\theta = 0 \qquad \textbf{[2]}$$

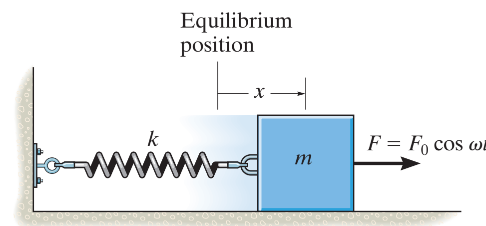
From Eq.[2], $p^2 = 170.66$, thus, $p = 13.06$ rad/s. Applying Eq.22–14, we have

$$f = \frac{p}{2\pi} = \frac{13.06}{2\pi} = 2.08\ \text{Hz} \qquad \textbf{Ans.}$$

Ans:
$f = 2.08$ Hz

22–41.

If the block-and-spring model is subjected to the periodic force $F = F_0 \cos \omega t$, show that the differential equation of motion is $\ddot{x} + (k/m)x = (F_0/m) \cos \omega t$, where x is measured from the equilibrium position of the block. What is the general solution of this equation?



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 \cos \omega t - kx = m\ddot{x}$

$$\ddot{x} + \frac{k}{m}x = \frac{F_0}{m} \cos \omega t \qquad \textbf{(Q.E.D.)}$$

$$\ddot{x} + p^2 x = \frac{F_0}{m} \cos \omega t \qquad \text{Where } p = \sqrt{\frac{k}{m}} \qquad \textbf{(1)}$$

The general solution of the above differential equation is of the form of $x = x_c + x_p$.

The complementary solution:

$x_c = A \sin pt + B \cos pt$

The particular solution:

$$s_p = .C \cos \omega t \qquad \textbf{(2)}$$

$$\ddot{x}_P = -C\omega^2 \cos \omega\, t \qquad \textbf{(3)}$$

Substitute Eqs. (2) and (3) into (1) yields:

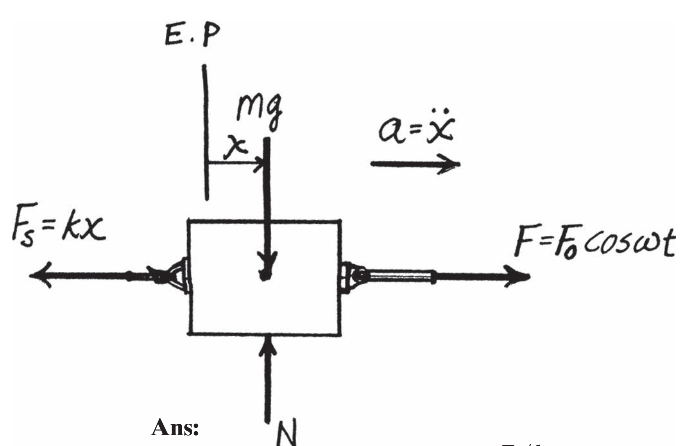
$$-C\omega^2 \cos \omega t + p^2 (C \cos \omega t) = \frac{F_0}{m} \cos \omega t$$

$$C = \frac{\frac{F_0}{m}}{p^2 - \omega^2} = \frac{F_0/k}{1 - \left(\frac{\omega}{p}\right)^2}$$

The general solution is therefore

$$s = A \sin pt + B \cos pt + \frac{F_0/k}{1 - \left(\frac{\omega}{p}\right)^2} \cos \omega t \qquad \textbf{Ans.}$$

The constants A and B can be found from the initial conditions.



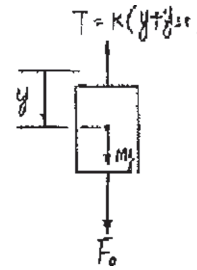
Ans:

$$x = A \sin \omega_n t + B \cos \omega_n t + \frac{F_o/k}{1 - (\omega/\omega_n)^2} \cos \omega t$$

22–42.

A block which has a mass m is suspended from a spring having a stiffness k. If an impressed downward vertical force $F = F_O$ acts on the weight, determine the equation which describes the position of the block as a function of time.

SOLUTION



$+\uparrow\Sigma F_y = ma_y; \qquad k(y_{st} + y) - mg - F_0 = -m\ddot{y}$

$$m\ddot{y} + ky + ky_{st} - mg = F_0$$

However, from equilibrium $ky_{st} - mg = 0$, therefore

$m\ddot{y} + ky = F_0$

$\ddot{y} + \dfrac{k}{m}y = \dfrac{F_0}{m} \qquad \text{where } \omega_n = \sqrt{\dfrac{k}{m}}$

$\ddot{y} + \omega_n^2 y = \dfrac{F_0}{m}$ **[1]**

The general solution of the above differential equation is of the form of $y = y_c + y_p$.

$y_c = A \sin \omega_n t + B \cos \omega_n t$

$y_P = C$ **[2]**

$\ddot{y}_P = 0$ **[3]**

Substitute Eqs. [2] and [3] into [1] yields :

$0 + \omega_n^2 C = \dfrac{F_0}{m} \qquad C = \dfrac{F_0}{mp^2} = \dfrac{F_0}{k}$

The general solution is therefore

$y = A \sin \omega_n t + B \cos \omega_n t + \dfrac{F_0}{k}$ **Ans.**

The constants A and B can be found from the initial conditions.

Ans:

$y = A \sin \omega_n t + B \cos \omega_n t + \dfrac{F_O}{k}$

22–43.

A 4-lb weight is attached to a spring having a stiffness $k = 10\ \text{lb/ft}$. The weight is drawn downward a distance of 4 in. and released from rest. If the support moves with a vertical displacement $\delta = (0.5 \sin 4t)$ in., where t is in seconds, determine the equation which describes the position of the weight as a function of time.

SOLUTION

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega_0 t$$

$$v = \dot{y} = A\omega_n \cos \omega_n t - B\omega_n \sin \omega_n t + \frac{\delta_0 \omega_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \cos \omega_0 t$$

The initial condition when $t = 0$, $y = y_0$, and $v = v_0$ is

$$y_0 = 0 + B + 0 \qquad B = y_0$$

$$v_0 = A\omega_n - 0 + \frac{\delta_0 \omega_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \qquad A = \frac{v_0}{\omega_n} - \frac{\delta_0 \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}}$$

Thus,

$$y = \left(\frac{v_0}{\omega_n} - \frac{\delta_0 \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}}\right) \sin \omega_n t + y_0 \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega_0 t$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10}{4/32.2}} = 8.972$$

$$\frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} = \frac{0.5/12}{1 - \left(\frac{4}{8.972}\right)^2} = 0.0520$$

$$\frac{v_0}{\omega_n} - \frac{\delta_0 \omega_0}{\omega_n - \frac{\omega_0^2}{\omega_n}} = 0 - \frac{(0.5/12)4}{8.972 - \frac{4^2}{8.972}} = -0.0232$$

$$y = (-0.0232 \sin 8.97t + 0.333 \cos 8.97t + 0.0520 \sin 4t)\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$y = \{-0.0232 \sin 8.97t + 0.333 \cos 8.97t + 0.0520 \sin 4t\}$ ft

***22–44.**

A 4-kg block is suspended from a spring that has a stiffness of $k = 600$ N/m. The block is drawn downward 50 mm from the equilibrium position and released from rest when $t = 0$. If the support moves with an impressed displacement of $\delta = (10 \sin 4t)$ mm, where t is in seconds, determine the equation that describes the vertical motion of the block. Assume positive displacement is downward.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{4}} = 12.25$$

The general solution is defined by Eq. 22–23 with $k\delta_0$ substituted for F_0.

$$y = A \sin \omega_n t + B \cos \omega_n t + \left(\frac{\delta_0}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]} \right) \sin \omega t$$

$\delta = (0.01 \sin 4t)$m, hence $\delta_0 = 0.01$, $\omega = 4$, so that

$y = A \sin 12.25t + B \cos 12.25t + 0.0112 \sin 4t$

$y = 0.05$ when $t = 0$

$0.05 = 0 + B + 0; \qquad B = 0.05$ m

$\dot{y} = A(12.25) \cos 12.25t - B(12.25) \sin 12.25t + 0.0112(4) \cos 4t$

$v = y = 0$ when $t = 0$

$0 = A(12.25) - 0 + 0.0112(4); \qquad A = -0.00366$ m

Expressing the result in mm, we have

$y = (-3.66 \sin 12.25t + 50 \cos 12.25t + 11.2 \sin 4t)$ mm **Ans.**

Ans:
$y = (-3.66 \sin 12.25t + 50 \cos 12.25t + 11.2 \sin 4t)$ mm

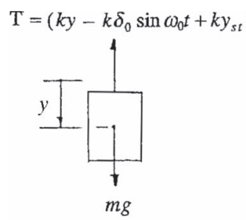
22–45.

Use a block-and-spring model like that shown in Fig. 22–14*a*, but suspended from a vertical position and subjected to a periodic support displacement $\delta = \delta_0 \sin \omega_0 t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.

SOLUTION

$+\uparrow \Sigma F_x = ma_x;\qquad k(y - \delta_0 \sin \omega_0 t + y_{st}) - mg = -m\ddot{y}$

$$m\ddot{y} + ky + ky_{st} - mg = k\delta_0 \sin \omega_0 t$$



However, from equilibrium

$$ky_{st} - mg = 0, \text{ therefore}$$

$$\ddot{y} + \frac{k}{m}y = \frac{k\delta_0}{m}\sin \omega t \qquad \text{where } \omega_n = \sqrt{\frac{k}{m}}$$

$$\ddot{y} + \omega_n^2 y = \frac{k\delta_0}{m} \sin \omega t \qquad \textbf{Ans. (1)}$$

The general solution of the above differential equation is of the form of $y = y_c + y_p$, where

$$y_c = A \sin \omega_n t + B \cos \omega_n t$$

$$y_p = C \sin \omega_0 t \qquad \textbf{(2)}$$

$$\ddot{y}_p = -C\omega_0^2 \sin \omega_0 t \qquad \textbf{(3)}$$

Substitute Eqs. (2) and (3) into (1) yields:

$$-C\omega^2 \sin \omega_0 t + \omega_n^2(C \sin \omega_0 t) = \frac{k\delta_0}{m} \sin \omega_0 t$$

$$C = \frac{\frac{k\delta_0}{m}}{\omega_n^2 - \omega_0^2} = \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}$$

The general solution is therefore

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \sin \omega t \qquad \textbf{Ans.}$$

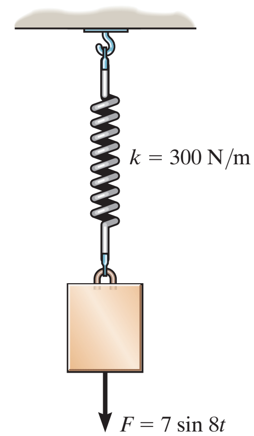
The constants A and B can be found from the initial conditions.

Ans:

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\delta_0}{1 - (\omega/\omega_n)^2} \sin \omega t$$

22–46.

A 5-kg block is suspended from a spring having a stiffness of 300 N/m. If the block is acted upon by a vertical force $F = (7 \sin 8t)$ N, where t is in seconds, determine the equation which describes the motion of the block when it is pulled down 100 mm from the equilibrium position and released from rest at $t = 0$. Assume that positive displacement is downward.



SOLUTION

The general solution is defined by:

$$y = A \sin \omega_n t + B \cos \omega_n t + \left(\frac{\frac{F_0}{k}}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \right) \sin \omega_0 t$$

Since

$F = 7 \sin 8t, \qquad F_0 = 7\text{ N}, \qquad \omega_0 = 8\text{ rad/s}, \qquad k = 300\text{ N/m}$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{5}} = 7.746\text{ rad/s}$$

Thus,

$$y = A \sin 7.746t + B \cos 7.746t + \left(\frac{\frac{7}{300}}{1 - \left(\frac{8}{7.746}\right)^2} \right) \sin 8t$$

$y = 0.1$ m when $t = 0$,

$0.1 = 0 + B - 0; \qquad B = 0.1\text{ m}$

$\dot{y} = A(7.746) \cos 7.746t - B(7.746) \sin 7.746t - (0.35)(8) \cos 8t$

$y = \dot{y} = 0$ when $t = 0$,

$\dot{y} = A(7.746) - 2.8 = 0; \qquad A = 0.361$

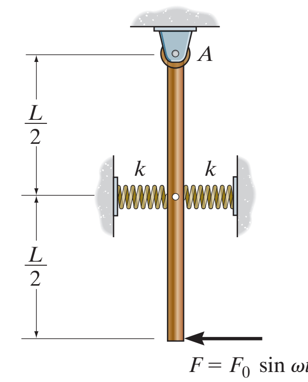
Expressing the results in mm, we have

$y = (361 \sin 7.75t + 100 \cos 7.75t - 350 \sin 8t)$ mm **Ans.**

Ans:
$y = (361 \sin 7.75t + 100 \cos 7.75t)$
$-350 \sin 8t)$ mm

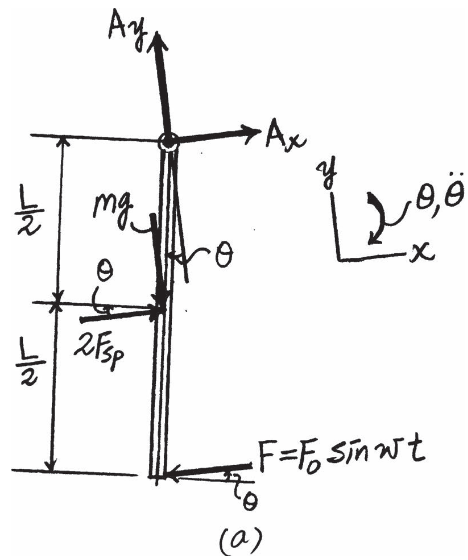
22–47.

The uniform rod has a mass of m. If it is acted upon by a periodic force of $F = F_0 \sin \omega t$, determine the amplitude of the steady-state vibration.



SOLUTION

Equation of Motion: When the rod rotates through a small angle θ, the springs compress and stretch $s = r_{AG}\theta = \frac{L}{2}\theta$. Thus, the force in each spring is $F_{sp} = ks = \frac{kL}{2}\theta$. The mass moment of inertia of the rod about point A is $I_A = \frac{1}{3}mL^2$. Referring to the free-body diagram of the rod shown in Fig. *a*,



$$+\Sigma M_A = I_A\alpha; \qquad F_O \sin \omega t \cos \theta(L) - mg \sin \theta\left(\frac{L}{2}\right) - 2\left(\frac{kL}{2}\theta\right)\cos \theta\left(\frac{L}{2}\right)$$

$$= \frac{1}{3}mL^2\dot{\theta}$$

Since θ is small, $\sin \theta \cong 0$ and $\cos \theta \cong 1$. Thus, this equation becomes

$$\frac{1}{3}mL\ddot{\theta} + \frac{1}{2}(mg + kL)\theta = F_O \sin \omega t$$

$$\ddot{\theta} + \frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right)\theta = \frac{3F_O}{mL}\sin \omega t \qquad \textbf{(1)}$$

The particular solution of this differential equation is assumed to be in the form of

$$\theta_p = C \sin \omega t \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2) twice,

$$\ddot{\theta}_p = -C\omega^2 \sin \omega t \qquad \textbf{(3)}$$

Substituting Eqs. (2) and (3) into Eq. (1),

$$-C\omega^2 \sin \omega t + \frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right)(C \sin \omega t) = \frac{3F_O}{mL}\sin \omega t$$

$$C\left[\frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right) - \omega^2\right]\sin \omega t = \frac{3F_O}{mL}\sin \omega t$$

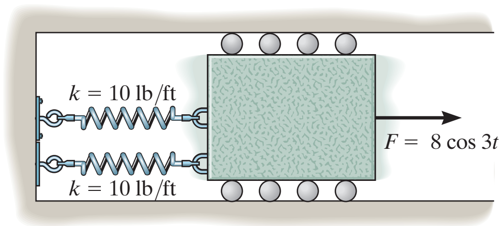
$$C = \frac{3F_O/mL}{\frac{3}{2}\left(\frac{g}{L} + \frac{k}{m}\right) - \omega^2}$$

$$C = \frac{3F_O}{\frac{3}{2}(mg + Lk) - mL\omega^2} \qquad \textbf{Ans.}$$

Ans:
$C = \frac{3F_O}{\frac{3}{2}(mg + Lk) - mL\omega^2}$

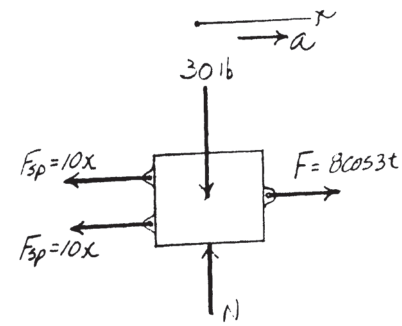
***22–48.**

The 30-lb block is attached to two springs having a stiffness of 10 lb/ft. A periodic force $F = (8 \cos 3t)$ lb, where t is in seconds, is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.



SOLUTION

Free-body Diagram: When the block is being displaced by amount x to the right, the *restoring force* that develops in both springs is $F_{sp} = kx = 10x$.



Equation of Motion:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -2(10x) + 8 \cos 3t = \frac{30}{32.2} a$$

$$a + 21.47x = 8.587 \cos 3t \qquad \textbf{[1]}$$

Kinematics: Since $a = \dfrac{d^2x}{dt^2} = \ddot{x}$, then substituting this value into Eq. [1], we have

$$\ddot{x} + 21.47x = 8.587 \cos 3t \qquad \textbf{[2]}$$

Since the friction will eventually dampen out the free vibration, we are only interested in the *particular solution* of the above differential equation which is in the form of

$$x_p = C \cos 3t$$

Taking second time derivative and substituting into Eq. [2], we have

$$-9C \cos 3t + 21.47C \cos 3t = 8.587 \cos 3t$$

$$C = 0.6888 \text{ ft}$$

Thus,

$$x_p = 0.6888 \cos 3t \qquad \textbf{[3]}$$

Taking the time derivative of Eq. [3], we have

$$v_p = \dot{x}_p = -2.0663 \sin 3t$$

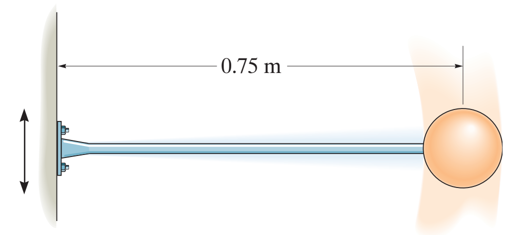
Thus,

$$\left(v_p\right)_{max} = 2.07 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$(v_p)_{max} = 2.07$ ft/s

22–49.

The light elastic rod supports a 4-kg sphere. When an 18-N vertical force is applied to the sphere, the rod deflects 14 mm. If the wall oscillates with harmonic frequency of 2 Hz and has an amplitude of 15 mm, determine the amplitude of vibration for the sphere.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{18}{0.014} = 1285.71 \text{ N/m}$$

$$\omega_0 = 2 \text{ Hz} = 2(2\pi) = 12.57 \text{ rad/s}$$

$$\delta_0 = 0.015 \text{ m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1285.71}{4}} = 17.93$$

Using Eq. 22–22, the amplitude is

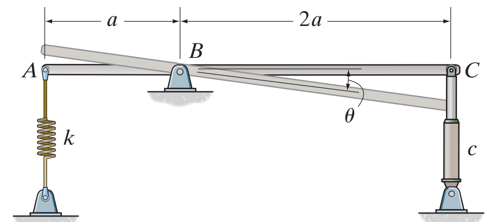
$$(x_p)_{max} = \left| \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} \right| = \left| \frac{0.015}{1 - \left(\frac{12.57}{17.93}\right)^2} \right|$$

$$(x_p)_{max} = 0.0295 \text{ m} = 29.5 \text{ mm}$$ **Ans.**

Ans:
$(x_p)_{\max} = 29.5 \text{ mm}$

22–50.

Find the differential equation for small oscillations in terms of θ for the uniform rod of mass m. Also show that if $c < \sqrt{mk}/2$, then the system remains underdamped. The rod is in a horizontal position when it is in equilibrium.



SOLUTION

Equation of Motion: When the rod is in equilibrium, $\theta = 0°$, $F_c = c\dot{y}_c = 0$ and $\ddot{\theta} = 0$. writing the moment equation of motion about point B by referring to the free-body diagram of the rod, Fig. a,

$$+\Sigma M_B = 0; \qquad -F_A(a) - mg\left(\frac{a}{2}\right) = 0 \qquad F_A = \frac{mg}{2}$$

Thus, the initial stretch of the spring is $s_O = \frac{F_A}{k} = \frac{mg}{2k}$. When the rod rotates about point B through a small angle θ, the spring stretches further by $s_1 = a\theta$. Thus, the force in the spring is $F_A = k(s_0 + s_1) = k\left(\frac{mg}{2k} + a\theta\right)$. Also, the velocity of end C of the rod is $v_c = \dot{y}_c = 2a\dot{\theta}$. Thus, $F_c = c\dot{y}_c = c(2a\dot{\theta})$. The mass moment of inertia of the rod about B is $I_B = \frac{1}{12}m(3a)^2 + m\left(\frac{a}{2}\right)^2 = ma^2$. Again, referring to Fig. a and writing the moment equation of motion about B,

$$\Sigma M_B = I_B\,\alpha; \qquad k\left(\frac{mg}{2k} + a\theta\right)\cos\theta(a) + \left(2a\dot{\theta}\right)\cos\theta(2a) - mg\cos\theta\left(\frac{a}{2}\right)$$

$$= -ma^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{4c}{m}\cos\theta\dot{\theta} + \frac{k}{m}(\cos\theta)\theta = 0$$

Since θ is small, $\cos\theta \cong 1$. Thus, this equation becomes

$$\ddot{\theta} + \frac{4c}{m}\dot{\theta} + \frac{k}{m}\theta = 0 \qquad \textbf{Ans.}$$

Comparing this equation to that of the standard form,

$$\omega_n = \sqrt{\frac{k}{m}} \qquad c_{eq} = 4c$$

Thus,

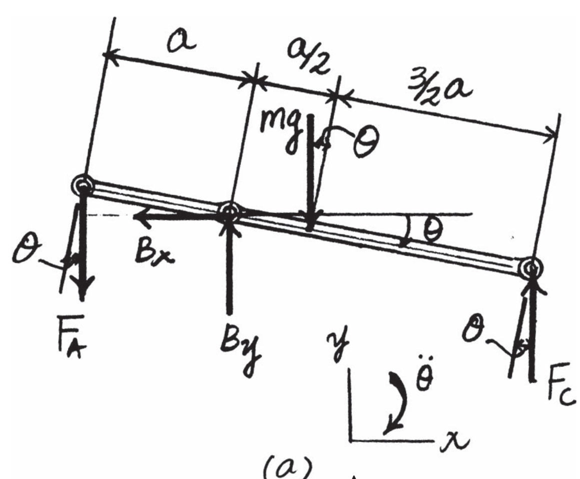
$$c_c = 2m\omega_n = 2m\sqrt{\frac{k}{m}} = 2\sqrt{mk}$$

For the system to be underdamped,

$$c_{eq} < c_c$$

$$4c < 2\sqrt{mk}$$

$$c < \frac{1}{2}\sqrt{mk} \qquad \textbf{Ans.}$$



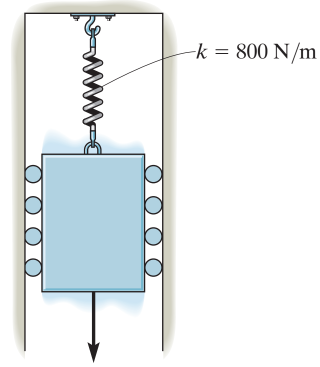
(a)

Ans:

$\ddot{\theta} + \frac{4c}{m}\dot{\theta} + \frac{k}{m}\dot{\theta} = 0$

22–51.

The 40-kg block is attached to a spring having a stiffness of 800 N/m. A force $F = (100 \cos 2t)$ N, where t is in seconds is applied to the block. Determine the maximum speed of the block for the steady-state vibration.



SOLUTION

For the steady-state vibration, the displacement is

$$y_p = \frac{F_0/k}{1 - (\omega_0/\omega_n)^2} \cos \omega t$$

Here $F_0 = 100$ N, $k = 800$ N/m, $\omega_0 = 2$ rad/s and

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{40}} = \sqrt{20} \text{ rad/s.}$$

Thus

$$y_p = \frac{100/800}{1 - (2/\sqrt{20})^2} \cos 2t$$

$$y_P = 0.15625 \cos 2t$$

Taking the time derivative of this equation

$$v_p = \dot{y}_p = -0.3125 \sin 2t \qquad \textbf{(2)}$$

v_p is maximum when $\sin 2t = 1$. Thus

$$(v_p)_{\max} = 0.3125 \text{ m/s} \qquad \textbf{Ans.}$$

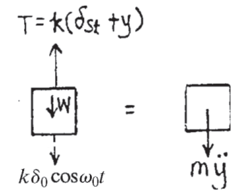
Ans:
$(v_p)_{\max} = 0.3125$ m/s

***22–52.**

Use a block-and-spring model like that shown in Fig. 22–14*a* but suspended from a vertical position and subjected to a periodic support displacement of $\delta = \delta_0 \cos \omega_0 t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.

SOLUTION

$+\downarrow \Sigma F_y = ma_y; \quad k\delta_0 \cos \omega_0 t + W - k\delta_{st} - ky = m\ddot{y}$



Since $W = k\delta_{st}$,

$$\ddot{y} + \frac{k}{m}y = \frac{k\delta_0}{m}\cos \omega_0 t \qquad \textbf{(1)}$$

$y_C = A \sin \omega_n y + B \cos \omega_n y$ (General sol.)

$y_P = C \cos \omega_0 t$ (Particular sol.)

Substitute y_p into Eq. (1)

$$C(-\omega_0^2 + \frac{k}{m}) \cos \omega_0 t = \frac{k\delta_0}{m}\cos \omega_0 t$$

$$C = \frac{\frac{k\delta_0}{m}}{\left(\frac{k}{m} - \omega_0^2\right)}$$

Thus, $y = y_C + y_P$

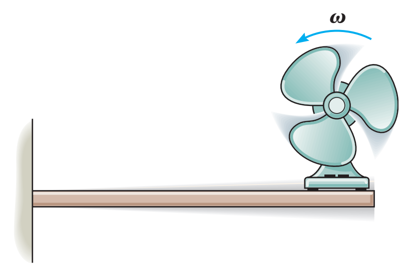
$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\frac{k\delta_0}{m}}{\left(\frac{k}{m} - \omega_0^2\right)}\cos \omega_0 t \qquad \textbf{Ans.}$$

Ans:

$$y = A \sin \omega_n t + B \cos \omega_n t + \frac{\frac{k\delta_0}{m}}{\left(\frac{k}{m} - \omega_0^2\right)}\cos \omega_0 t$$

22–53.

The fan has a mass of 25 kg and is fixed to the end of a horizontal beam that has a negligible mass. The fan blade is mounted eccentrically on the shaft such that it is equivalent to an unbalanced 3.5-kg mass located 100 mm from the axis of rotation. If the static deflection of the beam is 50 mm as a result of the weight of the fan, determine the angular velocity of the fan blade at which resonance will occur. *Hint:* See the first part of Example 22.8.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

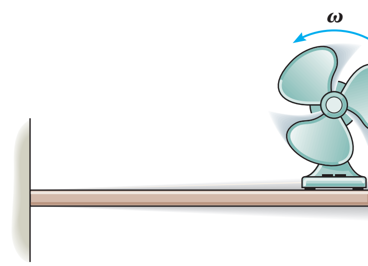
Resonance occurs when

$$\omega = \omega_n = 14.0 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 14.0 \text{ rad/s}$

22–54.

In Prob. 22–53, determine the amplitude of steady-state vibration of the fan if its angular velocity is 10 rad/s.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\,\omega^2 = 3.5(0.1)(10)^2 = 35 \text{ N}$$

Using Eq. 22–22, the amplitude is

$$(x_p)_{\max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2} \right|$$

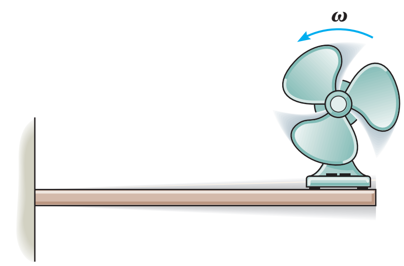
$$(x_p)_{\max} = \left| \frac{\frac{35}{4905}}{1 - \left(\frac{10}{14.01}\right)^2} \right| = 0.0146 \text{ m}$$

$$(x_p)_{\max} = 14.6 \text{ mm}$$ **Ans.**

Ans:
$(x_p)_{\max} = 14.6$ mm

22–55.

What will be the amplitude of steady-state vibration of the fan in Prob. 22–53 if the angular velocity of the fan blade is 18 rad/s? *Hint:* See the first part of Example 22.8.



SOLUTION

$$k = \frac{F}{\Delta y} = \frac{25(9.81)}{0.05} = 4905 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4905}{25}} = 14.01 \text{ rad/s}$$

The force caused by the unbalanced rotor is

$$F_0 = mr\omega^2 = 3.5(0.1)(18)^2 = 113.4 \text{ N}$$

Using Eq. 22–22, the amplitude is

$$(x_p)_{\max} = \left| \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{p}\right)^2} \right|$$

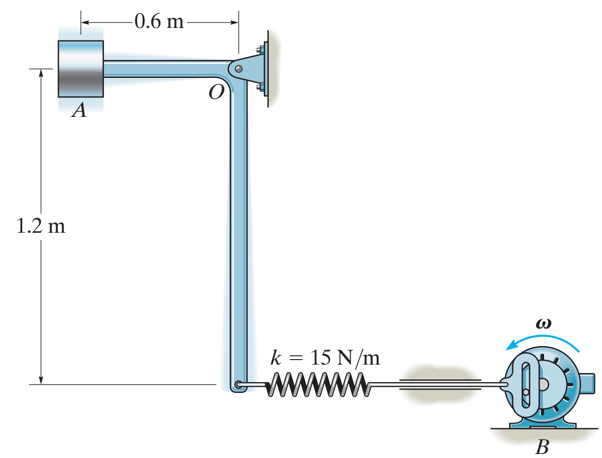
$$(x_p)_{\max} = \left| \frac{\frac{113.4}{4905}}{1 - \left(\frac{18}{14.01}\right)^2} \right| = 0.0355 \text{ m}$$

$$(x_p)_{\max} = 35.5 \text{ mm}$$ **Ans.**

Ans:
$(x_p)_{\max} = 35.5$ mm

***22–56.**

The small block at A has a mass of 4 kg and is mounted on the bent rod having negligible mass. If the rotor at B causes a harmonic movement $\delta_B = (0.1 \cos 15t)$ m, where t is in seconds, determine the steady-state amplitude of vibration of the block.



SOLUTION

$+\Sigma M_O = I_O\,\alpha; \qquad 4(9.81)(0.6) - F_s(1.2) = 4(0.6)^2\ddot{\theta}$

$$F_s = kx = 15(x + x_{st} - 0.1 \cos 15t)$$

$$x_{st} = \frac{4(9.81)(0.6)}{1.2(15)}$$

Thus,

$$-15(x - 0.1 \cos 15t)(1.2) = 4(0.6)^2\ddot{\theta}$$

$$x = 1.2\theta$$

$$\theta + 15\theta = 1.25 \cos 15t$$

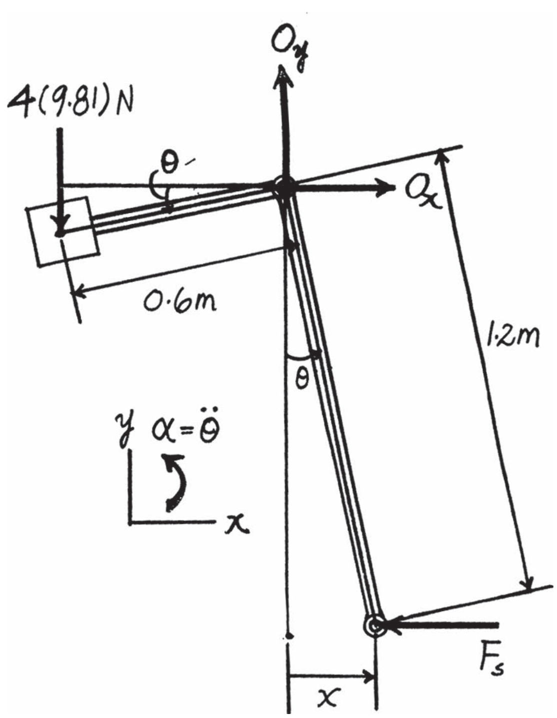
Set $x_p = C \cos 15t$

$$-C(15)^2 \cos 15t + 15(C \cos 15t) = 1.25 \cos 15t$$

$$C = \frac{1.25}{15 - (15)^2} = -0.00595 \text{ m}$$

$$\theta_{\max} = C = 0.00595 \text{ rad}$$

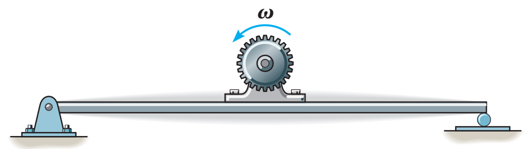
$$y_{\max} = (0.6 \text{ m})(0.00595 \text{ rad}) = 0.00357 \text{ rad} \qquad \textbf{Ans.}$$



Ans:
$y_{\max} = 0.00357$ rad

22–57.

The electric motor turns an eccentric flywheel which is equivalent to an unbalanced 0.25-lb weight located 10 in. from the axis of rotation. If the static deflection of the beam is 1 in. due to the weight of the motor, determine the angular velocity of the flywheel at which resonance will occur. The motor weights 150 lb. Neglect the mass of the beam.



SOLUTION

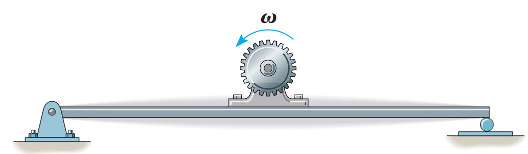
$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.66$$

Resonance occurs when $\omega = \omega_n = 19.7 \text{ rad/s}$ **Ans.**

Ans:
$\omega = 19.7 \text{ rad/s}$

22–58.

What will be the amplitude of steady-state vibration of the motor in Prob. 22–57 if the angular velocity of the flywheel is 20 rad/s?



SOLUTION

The constant value F_O of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_O = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)(20)^2 = 2.588 \text{ lb}$$

Hence $F = 2.588 \sin 20t$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

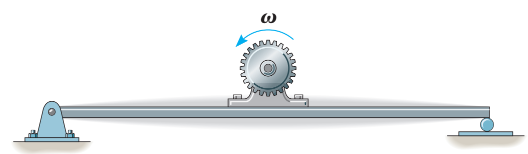
From Eq. 22–21, the amplitude of the steady state motion is

$$C = \left|\frac{F_0/k}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}\right| = \left|\frac{2.588/1800}{1 - \left(\frac{20}{19.657}\right)^2}\right| = 0.04085 \text{ ft} = 0.490 \text{ in.}$$ **Ans.**

Ans:
$C = 0.490$ in.

22–59.

Determine the angular velocity of the flywheel in Prob. 22–57 which will produce an amplitude of vibration of 0.25 in.



SOLUTION

The constant value F_O of the periodic force is due to the centrifugal force of the unbalanced mass.

$$F_O = ma_n = mr\omega^2 = \left(\frac{0.25}{32.2}\right)\left(\frac{10}{12}\right)\omega^2 = 0.006470\omega^2$$

$$F = 0.006470\omega^2 \sin \omega t$$

$$k = \frac{F}{\delta} = \frac{150}{1/12} = 1800 \text{ lb/ft} \qquad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{150/32.2}} = 19.657$$

From Eq. 22.21, the amplitude of the steady-state motion is

$$C = \left|\frac{F_0/k}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}\right|$$

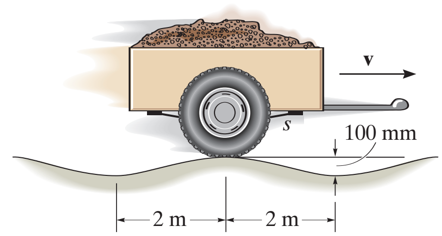
$$\frac{0.25}{12} = \left|\frac{0.006470\left(\dfrac{\omega^2}{1800}\right)}{1 - \left(\dfrac{\omega}{19.657}\right)^2}\right|$$

$$\omega = 19.0 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 19.0 \text{ rad/s}$

***22–60.**

The 450-kg trailer is pulled with a constant speed over the surface of a bumpy road, which may be approximated by a cosine curve having an amplitude of 50 mm and wave length of 4 m. If the two springs s which support the trailer each have a stiffness of 800 N/m, determine the speed v which will cause the greatest vibration (resonance) of the trailer. Neglect the weight of the wheels.



SOLUTION

The amplitude is $\delta_0 = 50 \text{ mm} = 0.05 \text{ m}$

The wave length is $\lambda = 4 \text{ m}$

$$k = 2(800) = 1600 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1600}{450}} = 1.89 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{1.89} = 3.33 \text{ s}$$

For maximum vibration of the trailer, resonance must occur, i.e.,

$$\omega_0 = \omega_n$$

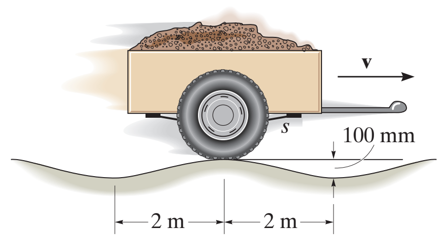
Thus, the trailer must travel $\lambda = 4$ m, in $\tau = 3.33$ s, so that

$$v_R = \frac{\lambda}{\tau} = \frac{4}{3.33} = 1.20 \text{ m.s} \qquad \textbf{Ans.}$$

Ans:
$v_R = 1.20$ m.s

22–61.

Determine the amplitude of vibration of the trailer in Prob. 22–60 if the speed $v = 15$ km/h.



SOLUTION

$$v = 15 \text{ km/h} = \frac{15(1000)}{3600} \text{ m/s} = 4.17 \text{ m/s}$$

$$\delta_0 = 0.05 \text{ m}$$

As shown in Prob. 22–50, the velocity is inversely proportional to the period. Since $\frac{1}{\tau} = f$ the the velocity is proportional of f, ω_n and ω_0

Hence, the amplitude of motion is

$$(x_p)_{max} = \left|\frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2}\right| = \left|\frac{\delta_0}{1 - \left(\frac{v}{v_R}\right)^2}\right|$$

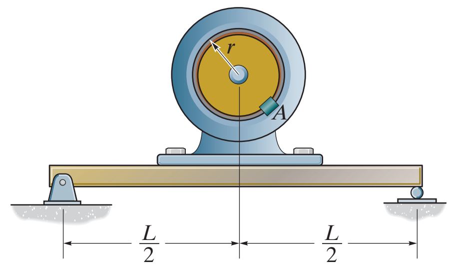
$$(x_p)_{max} = \left|\frac{0.05}{1 - \left(\frac{4.17}{1.20}\right)^2}\right| = 0.00453 \text{ m}$$

$$(x_p)_{max} = 4.53 \text{ mm}$$ **Ans.**

Ans:
$(x_p)_{\max} = 4.53$ mm

22–62.

The motor of mass M is supported by a simply supported beam of negligible mass. If block A of mass m is clipped onto the rotor, which is turning at constant angular velocity of ω, determine the amplitude of the steady-state vibration. *Hint:* When the beam is subjected to a concentrated force of P at its mid-span, it deflects $\delta = PL^3/48EI$ at this point. Here E is Young's modulus of elasticity, a property of the material, and I is the moment of inertia of the beam's cross-sectional area.



SOLUTION

In this case, $P = k_{eq}\delta$. Then, $k_{eq} = \dfrac{P}{\delta} = \dfrac{P}{PL^3/48EI} = \dfrac{48EI}{L^3}$. Thus, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{\frac{48EI}{L^3}}{M}} = \sqrt{\frac{48EI}{ML^3}}$$

Here, $F_O = ma_n = m(\omega^2 r)$. Thus,

$$Y = \frac{F_O/k_{eq}}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}$$

$$Y = \frac{\dfrac{m(\omega^2 r)}{48EI/L^3}}{1 - \dfrac{\omega^2}{48EI/ML^3}}$$

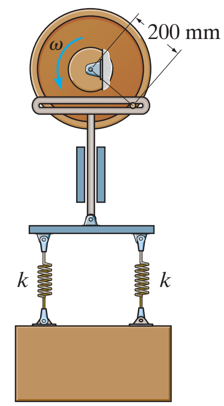
$$Y = \frac{mr\omega^2 L^3}{48EI - M\omega^2 L^3} \qquad \textbf{Ans.}$$

Ans:

$$Y = \frac{mr\omega^2 L^3}{48EI - M\omega^2 L^3}$$

22–63.

The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\boldsymbol{\omega}$. If the amplitude of the steady-state vibration is observed to be 400 mm, and the springs each have a stiffness of $k = 2500$ N/m, determine the two possible values of $\boldsymbol{\omega}$ at which the wheel must rotate. The block has a mass of 50 kg.



SOLUTION

In this case, $k_{eq} = 2k = 2(2500) = 5000$ N/m Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{5000}{50}} = 10 \text{ rad/s}$$

Here, $\delta_O = 0.2$ m and $(Y_P)_{\max} = \pm 0.4$ m, so that

$$(Y_P)_{\max} = \frac{\delta_O}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}$$

$$\pm 0.4 = \frac{0.2}{1 - \left(\dfrac{\omega}{10}\right)^2}$$

$$\frac{\omega^2}{100} = 1 \pm 0.5$$

Thus,

$$\frac{\omega^2}{100} = 1.5 \qquad \omega = 12.2 \text{ rad/s} \qquad \textbf{Ans.}$$

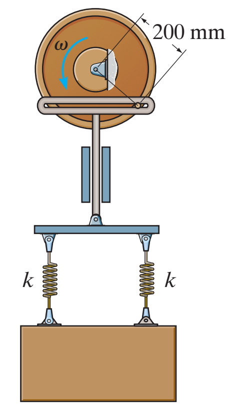
or

$$\frac{\omega^2}{100} = 0.5 \qquad \omega = 7.07 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 12.2$ rad/s
$\omega = 7.07$ rad/s

***22–64.**

The spring system is connected to a crosshead that oscillates vertically when the wheel rotates with a constant angular velocity of $\omega = 5$ rad/s. If the amplitude of the steady-state vibration is observed to be 400 mm, determine the two possible values of the stiffness k of the springs. The block has a mass of 50 kg.



SOLUTION

In this case, $k_{eq} = 2k$ Thus, the natural circular frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{2k}{50}} = \sqrt{0.04k}$$

Here, $\delta_O = 0.2$ m and $(Y_P)_{\max} = \pm 0.4$ m, so that

$$(Y_P)_{\max} = \frac{\delta_O}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$\pm 0.4 = \frac{0.2}{1 - \left(\frac{5}{\sqrt{0.04k}}\right)^2}$$

$$\frac{625}{k} = 1 \pm 0.5$$

Thus,

$$\frac{625}{k} = 1.5 \qquad k = 417 \text{ N/m} \qquad \textbf{Ans.}$$

or

$$\frac{625}{k} = 0.5 \qquad k = 1250 \text{ N/m} \qquad \textbf{Ans.}$$

Ans:
$k = 417$ N/m
$k = 1250$ N/m

22–65.

A 7-lb block is suspended from a spring having a stiffness of $k = 75$ lb/ft. The support to which the spring is attached is given simple harmonic motion which may be expressed as $\delta = (0.15 \sin 2t)$ ft, where t is in seconds. If the damping factor is $c/c_c = 0.8$, determine the phase angle ϕ of forced vibration.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$$\delta = 0.15 \sin 2t$$

$$\delta_0 = 0.15, \omega = 2$$

$$\phi' = \tan^{-1}\left(\frac{2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right) = \tan^{-1}\left(\frac{2(0.8)\left(\frac{2}{18.57}\right)}{1 - \left(\frac{2}{18.57}\right)^2}\right)$$

$$\phi' = 9.89^\circ \qquad \textbf{Ans.}$$

Ans:
$\phi' = 9.89^\circ$

22–66.

Determine the magnification factor of the block, spring, and dashpot combination in Prob. 22–65.

SOLUTION

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{\left(\frac{7}{32.2}\right)}} = 18.57$$

$$\delta = 0.15 \sin 2t$$

$$\delta_0 = 0.15, \quad \omega = 2$$

$$\text{MF} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_n}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}} = \frac{1}{\sqrt{\left[1 - \left(\frac{2}{18.57}\right)^2\right]^2 + \left[2(0.8)\left(\frac{2}{18.57}\right)\right]^2}}$$

$\text{MF} = 0.997$ **Ans.**

Ans:
$\text{MF} = 0.997$

22–67.

A block having a mass of 7 kg is suspended from a spring that has a stiffness $k = 600$ N/m. If the block is given an upward velocity of 0.6 m/s from its equilibrium position at $t = 0$, determine its position as a function of time. Assume that positive displacement of the block is downward and that motion takes place in a medium which furnishes a damping force $F = (50|v|)$ N, where v is in m/s.

SOLUTION

$$c = 50\text{ N s/m} \quad k = 600\text{ N/m} \quad m = 7\text{ kg}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{7}} = 9.258\text{ rad/s}$$

$$c_c = 2m\omega_n = 2(7)(9.258) = 129.6\text{ N}\cdot\text{s/m}$$

Since $c < c_z$, the system is underdamped,

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 9.258\sqrt{1 - \left(\frac{50}{129.6}\right)^2} = 8.542\text{ rad/s}$$

$$\frac{c}{2m} = \frac{50}{2(7)} = 3.751$$

From Eq. 22-32

$$y = D\left[e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

$$v = \dot{y} = D\left[e^{-\left(\frac{c}{2m}\right)t}\,\omega_d\,\cos(\omega_d t + \phi) + \left(-\frac{c}{2m}\right)e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_d t + \phi)\right]$$

$$v = De^{-\left(\frac{c}{2m}\right)t}\left[\omega_d\cos(\omega_d t + \phi) - \frac{c}{2m}\sin(\omega_d t + \phi)\right]$$

Applying the initial condition at $t = 0$, $y = 0$ and $v = -0.6$ m/s.

$$0 = D[e^{-0}\sin(0 + \phi)] \qquad \text{since} \quad D \neq 0$$

$$\sin\phi = 0 \qquad \phi = 0°$$

$$-0.6 = De^{-0}[8.542\cos 0° - 0]$$

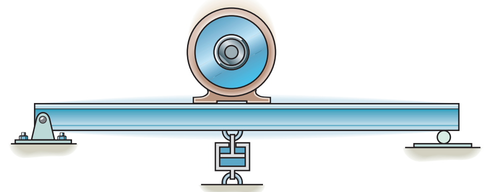
$$D = -0.0702\text{ m}$$

$$y = [-0.0702\,e^{-3.57t}\sin(8.540)]\text{ m} \qquad \textbf{Ans.}$$

Ans:
$y = \{-0.0702\,e^{-3.57t}\sin(8.540)\}$ m

***22–68.**

The 200-lb electric motor is fastened to the midpoint of the simply supported beam. It is found that the beam deflects 2 in. when the motor is not running. The motor turns an eccentric flywheel which is equivalent to an unbalanced weight of 1 lb located 5 in. from the axis of rotation. If the motor is turning at 100 rpm, determine the amplitude of steady-state vibration. The damping factor is $c/c_c = 0.20$. Neglect the mass of the beam.



SOLUTION

$$\delta = \frac{2}{12} = 0.167 \text{ ft}$$

$$\omega = 100\left(\frac{2\pi}{60}\right) = 10.47 \text{ rad/s}$$

$$k = \frac{200}{\frac{2}{12}} = 1200 \text{ lb/ft}$$

$$F_O = mr\omega^2 = \left(\frac{1}{32.2}\right)\left(\frac{5}{12}\right)(10.47)^2 = 1.419 \text{ lb}$$

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{\frac{200}{32.2}}} = 13.90 \text{ rad/s}$$

$$C' = \frac{\frac{F_0}{k}}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{p}\right)\right]^2}}$$

$$= \frac{\frac{1.419}{1200}}{\sqrt{\left[1 - \left(\frac{10.47}{13.90}\right)^2\right]^2 + \left[2(0.20)\left(\frac{10.47}{13.90}\right)\right]^2}}$$

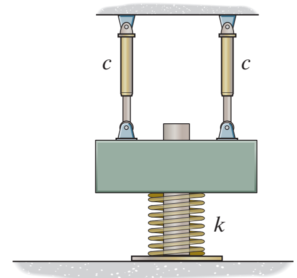
$$= 0.00224 \text{ ft}$$

$C' = 0.0269$ in. **Ans.**

Ans:
$C' = 0.0269$ in.

22–69.

Two identical dashpots are arranged parallel to each other, as shown. Show that if the damping coefficient $c < \sqrt{mk}$, then the block of mass m will vibrate as an underdamped system.



SOLUTION

When the two dash pots are arranged in parallel, the piston of the dashpots have the same velocity. Thus, the force produced is

$$F = c\dot{y} + c\dot{y} = 2c\dot{y}$$

The equivalent damping coefficient c_{eq} of a single dashpot is

$$c_{eq} = \frac{F}{\dot{y}} = \frac{2c\dot{y}}{\dot{y}} = 2c$$

For the vibration to occur (underdamped system), $c_{eq} < c_c$. However, $c_c = 2m\omega_n = 2m\sqrt{\frac{k}{m}}$. Thus,

$$c_{eq} < c_c$$

$$2c < 2m\sqrt{\frac{k}{m}}$$

$$c < \sqrt{mk}$$ **Ans.**

Ans:
$F = 2c\dot{y}$
$c_c = 2m\sqrt{\frac{k}{m}}$
$c < \sqrt{mk}$

22–70.

The damping factor, c/c_c, may be determined experimentally by measuring the successive amplitudes of vibrating motion of a system. If two of these maximum displacements can be approximated by x_1 and x_2, as shown in Fig. 22–16, show that $\ln\ x_1/x_2 = 2\pi(c/c_c)/\sqrt{1 - (c/c_c)^2}$. The quantity $\ln\ x_1/x_2$ is called the *logarithmic decrement*.

SOLUTION

Using Eq. 22–32,

$$x = D\left[e^{-\left(\frac{c}{2m}\right)t}\sin\left(\omega_d t + \phi\right)\right]$$

The maximum displacement is

$$x_{max} = De^{-\left(\frac{c}{2m}\right)t}$$

At $t = t_1$, and $t = t_2$

$$x_1 = De^{-\left(\frac{c}{2m}\right)t_1}$$

$$x_2 = De^{-\left(\frac{c}{2m}\right)t2}$$

Hence,

$$\frac{x_1}{x_2} = \frac{De^{-\left(\frac{c}{2m}\right)t1}}{De^{-\left(\frac{c}{2m}\right)t_2} = e^{-\left(\frac{c}{2m}\right)(t_1 - t_2)}}$$

Since $\omega_d t_2 - \omega_d t_1 = 2\pi$

then $t_2 - t_1 = \dfrac{2\pi}{\omega_d}$

so that $\ln\left(\dfrac{x_1}{x_2}\right) = \dfrac{c\pi}{m\omega_d}$

Using Eq. 22–33, $c_c = 2m\omega_n$

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = \frac{c_c}{2m}\sqrt{1 - \left(\frac{c}{c_r}\right)^2}$$

So that,

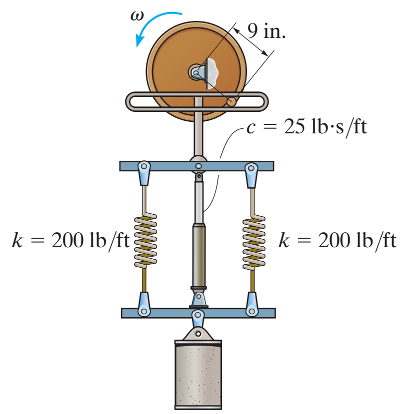
$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{c}{c_c}\right)}{\sqrt{1 - \left(\frac{c}{c_c}\right)^2}} \qquad \textbf{Q.E.D.}$$

Ans:

$$\text{In}\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{c}{c_c}\right)}{\sqrt{1 - \left(\frac{c}{c_c}\right)^2}}$$

22–71.

If the amplitude of the 50-lb cylinder's steady-state vibration is 6 in., determine the wheel's angular velocity ω.



SOLUTION

In this case, $Y = \dfrac{6}{12} = 0.5$ ft, $\delta_O = \dfrac{9}{12} = 0.75$ ft, and $k_{eq} = 2k = 2(200) = 400$ lb/ft. Then

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{400}{(50/32.2)}} = 16.05 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2\left(\frac{50}{32.2}\right)(16.05) = 49.84 \text{ lb}\cdot\text{s/ft}$$

$$\frac{c}{c_c} = \frac{25}{49.84} = 0.5016$$

$$Y = \frac{\delta_O}{\sqrt{\left[1 - \left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left(\dfrac{2(c/c_c)\omega}{\omega_n}\right)^2}}$$

$$0.5 = \frac{0.75}{\sqrt{\left[1 - \left(\dfrac{\omega}{16.05}\right)^2\right]^2 + \left(\dfrac{2(0.5016)\omega}{16.05}\right)^2}}$$

$$15.07(10^{-6})\omega^4 - 3.858(10^{-3})\omega^2 - 1.25 = 0$$

Solving for the positive root of this equation,

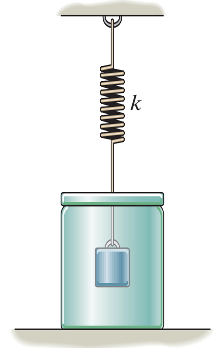
$$\omega^2 = 443.16$$

$$\omega = 21.1 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 21.1$ rad/s

***22–72.**

The block, having a weight of 12 lb, is immersed in a liquid such that the damping force acting on the block has a magnitude of $F = (0.7|v|)$ lb, where v is in ft/s. If the block is pulled down 0.62 ft and released from rest, determine the position of the block as a function of time. The spring has a stiffness of $k = 53$ lb/ft. Assume that positive displacement is downward.



SOLUTION

$$c = 0.7 \text{ lb} \cdot \text{s/ft} \qquad k = 53 \text{ lb/ft} \qquad m = \frac{12}{32.2} = 0.3727 \text{ slug}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{53}{0.3727}} = 11.925 \text{ rad/s}$$

$$c_c = 2m\omega_n = 2(0.3727)(11.925) = 8.889 \text{ lb} \cdot \text{s/ft}$$

Since $c < c_c$ the system is underdamped.

$$\omega_d = \omega_n\sqrt{1 - \left(\frac{c}{c_c}\right)^2} = 11.925\sqrt{1 - \left(\frac{0.7}{8.889}\right)^2} = 11.888 \text{ rad/s}$$

$$\frac{c}{2m} = \frac{0.7}{2(0.3727)} = 0.9392$$

From Eq. 22–32 $y = D\left[e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$

$$v = \dot{y} = D\left[e^{-\left(\frac{c}{2m}\right)t}\omega_d \cos(\omega_d t + \phi) + \left(-\frac{c}{2m}\right)e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)\right]$$

$$v = De^{-\left(\frac{c}{2m}\right)t}\left[\omega_d \cos(\omega_d t + \phi) - \frac{c}{2m} \sin(\omega_d t + \phi)\right]$$

Appling the initial condition at $t = 0$, $y = 0.62$ ft and $v = 0$.

$$0.62 = D\left[e^{-0} \sin(0 + \phi)\right]$$

$$D \sin\phi = 0.62 \qquad \textbf{(1)}$$

$$0 = De^{-0}\left[11.888 \cos(0 + \phi) - 0.9392 \sin(0 + \phi)\right] \qquad \text{since } D \neq 0$$

$$11.888 \cos\phi - 0.9392 \sin\theta = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

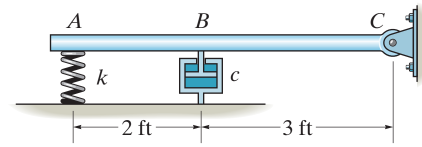
$$\phi = 85.5° = 1.49 \text{ rad} \qquad D = 0.622 \text{ ft}$$

$$y = 0.622[e^{-0.939t} \sin(11.9t + 1.49)] \qquad \textbf{Ans.}$$

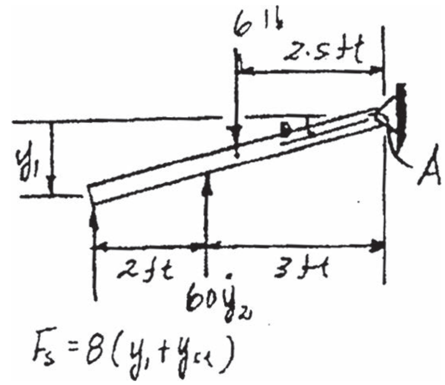
Ans:
$y = 0.622\,[e^{-0.939t} \sin(11.9t + 1.49)]$

22–73.

The bar has a weight of 6 lb. If the stiffness of the spring is $k = 8\ \text{lb/ft}$ and the dashpot has a damping coefficient $c = 60\ \text{lb}\cdot\text{s/ft}$, determine the differential equation which describes the motion in terms of the angle θ of the bar's rotation. Also, what should be the damping coefficient of the dashpot if the bar is to be critically damped?



SOLUTION



$$\circlearrowleft + \Sigma M_A = I_A\,\alpha; \qquad 6(2.5) - (60\dot{y}_2)(3) - 8(y_1 + y_{st})(5) = \left[\frac{1}{3}\left(\frac{6}{32.2}\right)(5)^2\right]\ddot{\theta}$$

$$1.5528\ddot{\theta} + 180\dot{y}_2 + 40y_1 + 40y_{st} - 15 = 0 \qquad \textbf{[1]}$$

From equilibrium $40y_{st} - 15 = 0$. Also, for small θ, $y_1 = 5\theta$ and $y_2 = 3\theta$ hence $\dot{y}_2 = 3\dot{\theta}$.

From Eq. [1] $\qquad 1.5528\ddot{\theta} + 180(3\dot{\theta}) + 40(5\theta) = 0$

$$1.55\ddot{\theta} + 540\dot{\theta} + 200\theta = 0 \qquad \textbf{Ans.}$$

By comparing the above differential equation to Eq. 22-27

$$m = 1.55 \qquad k = 200 \qquad \omega_n = \sqrt{\frac{200}{1.55}} = 11.35\ \text{rad/s} \qquad c = 9c_{d\cdot p}$$

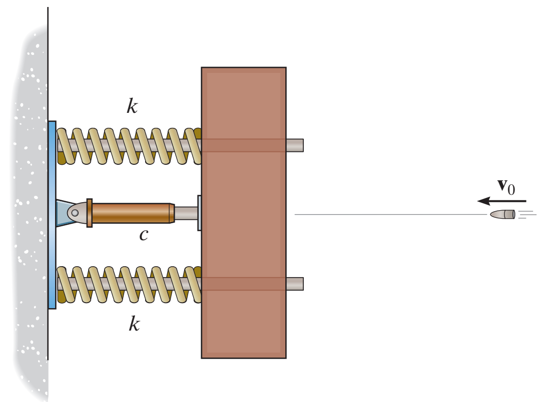
$$\left(\frac{9(c_{d\cdot p})_c}{2m}\right)^2 - \frac{k}{m} = 0$$

$$(c_{d\cdot p})_c = \frac{2}{9}\sqrt{km} = \frac{2}{9}\sqrt{200(1.55)} = 3.92\ \text{lb}\cdot\text{s/ft} \qquad \textbf{Ans.}$$

Ans:
$1.55\ddot{\theta} + 540\dot{\theta} + 200\theta = 0$
$(c_{dp})_c = 3.92\ \text{lb}\cdot\text{s/ft}$

22–74.

A bullet of mass m has a velocity of $\mathbf{v}_0$ just before it strikes the target of mass M. If the bullet embeds in the target, and the vibration is to be critically damped, determine the dashpot's critical damping coefficient, and the springs' maximum compression. The target is free to move along the two horizontal guides that are "nested" in the springs.



SOLUTION

Since the springs are arranged in parallel, the equivalent stiffness of the single spring system is $k_{eq} = 2k$. Also, when the bullet becomes embedded in the target, $m_T = m + M$. Thus, the natural frequency of the system is

$$\omega_n = \sqrt{\frac{k_{eq}}{m_T}} = \sqrt{\frac{2k}{m + M}}$$

When the system is critically damped

$$c = c_c = 2m_T\omega_n = 2(m + M)\sqrt{\frac{2k}{m + M}} = \sqrt{8(m + M)k} \qquad \textbf{Ans.}$$

The equation that describes the critically dampened system is

$$x = (A + Bt)e^{-\omega_n t}$$

When $t = 0$, $x = 0$. Thus,

$$A = 0$$

Then,

$$x = Bte^{-\omega_n t} \qquad \textbf{(1)}$$

Taking the time derivative,

$$v = \dot{x} = Be^{-\omega_n t} - B\omega_n te^{-\omega_n t}$$

$$v = Be^{-\omega_n t}(1 - \omega_n t) \qquad \textbf{(2)}$$

Since linear momentum is conserved along the horizontal during the impact, then

$$(\stackrel{+}{\leftarrow}) \qquad mv_0 = (m + M)v$$

$$v = \left(\frac{m}{m + M}\right)v_0$$

Here, when $t = 0$, $v = \left(\frac{m}{m + M}\right)v_0$. Thus, Eq. (2) gives

$$B = \left(\frac{m}{m + M}\right)v_0$$

And Eqs. (1) and (2) become

$$x = \left[\left(\frac{m}{m + M}\right)v_0\right]te^{-\omega_n t} \qquad \textbf{(3)}$$

$$v = \left[\left(\frac{m}{m + M}\right)v_0\right]e^{-\omega_n t}(1 - \omega_n t) \qquad \textbf{(4)}$$

22–74. Continued

The maximum compression of the spring occurs when the block stops. Thus, Eq. (4) gives

$$0 = \left[\left(\frac{m}{m + M}\right)v_0\right](1 - \omega_n t)$$

Since $\left(\dfrac{m}{m + M}\right)v_0 \neq 0$, then

$$1 - \omega_n t = 0$$

$$t = \frac{1}{\omega_n} = \sqrt{\frac{m + M}{2k}}$$

Substituting this result into Eq. (3)

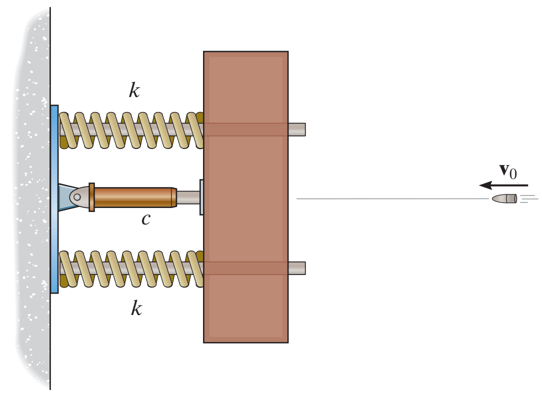
$$x_{\max} = \left[\left(\frac{m}{m + M}\right)v_0\right]\left(\sqrt{\frac{m + M}{2k}}\right)e^{-1}$$

$$= \left[\frac{m}{e}\sqrt{\frac{1}{2k(m + M)}}\right]v_0 \qquad \textbf{Ans.}$$

Ans:
$c_c = \sqrt{8(m + M)k}$
$x_{\max} = \left[\dfrac{m}{e}\sqrt{\dfrac{1}{2k(m + M)}}\right]v_0$

22–75.

A bullet of mass m has a velocity $\mathbf{v}_0$ just before it strikes the target of mass M. If the bullet embeds in the target, and the dashpot's damping coefficient is $0 < c \ll c_c$, determine the springs' maximum compression. The target is free to move along the two horizontal guides that are "nested" in the springs.



SOLUTION

Since the springs are arranged in parallel, the equivalent stiffness of the single spring system is $k_{eq} = 2k$. Also, when the bullet becomes embedded in the target, $m_T = m + M$. Thus, the natural circular frequency of the system

$$\omega_n = \sqrt{\frac{k_{eq}}{m_T}} = \sqrt{\frac{2k}{m + M}}$$

The equation that describes the underdamped system is

$$x = Ce^{-(c/2m_T)t}\sin(\omega_d t + \phi) \qquad \textbf{(1)}$$

When $t = 0$, $x = 0$. Thus, Eq. (1) gives

$$0 = C\sin\phi$$

Since $C \neq 0$, $\sin\phi = 0$. Then $\phi = 0$. Thus, Eq. (1) becomes

$$x = Ce^{-(c/2m_T)t}\sin\omega_d t \qquad \textbf{(2)}$$

Taking the time derivative of Eq. (2),

$$v = \dot{x} = C\left[\omega_d e^{-(c/2m_T)t}\cos\omega_d t - \frac{c}{2m_T}e^{-(c/2m_T)t}\sin\omega_d t\right]$$

$$v = Ce^{-(c/2m_T)t}\left[\omega_d\cos\omega_d t - \frac{c}{2m_T}\sin\omega_d t\right] \qquad \textbf{(3)}$$

Since linear momentum is conserved along the horizontal during the impact, then

$(\overset{+}{\leftarrow})$ $$mv_0 = (m + M)v$$

$$v = \left(\frac{m}{m + M}\right)v_0$$

When $t = 0$, $v = \left(\frac{m}{m + M}\right)v_0$. Thus, Eq. (3) gives

$$\left(\frac{m}{m + M}\right)v_0 = C\omega_d \qquad C = \left(\frac{m}{m + M}\right)\frac{v_0}{\omega_d}$$

And Eqs. (2) becomes

$$x = \left[\left(\frac{m}{m + M}\right)\frac{v_0}{\omega_d}\right]e^{-(c/2m_T)t}\sin\omega_d t \qquad \textbf{(4)}$$

22–75. Continued

The maximum compression of the spring occurs when

$$\sin \omega_d t = 1$$

$$\omega_d t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2\omega_d}$$

Substituting this result into Eq. (4),

$$x_{\max} = \left[\left(\frac{m}{m + M}\right)\frac{v_0}{\omega_d}\right] e^{-[c/2(m+M)]\left(\frac{\pi}{2\omega_d}\right)}$$

However, $\omega_d = \sqrt{\frac{k_{eq}}{m_T} - \left(\frac{c}{2m_T}\right)^2} = \sqrt{\frac{2k}{m + M} - \frac{c^2}{4(m + M)^2}} = \frac{1}{2(m + M)}$ $\sqrt{8k(m + M) - c^2}$. Substituting this result into Eq. (5),

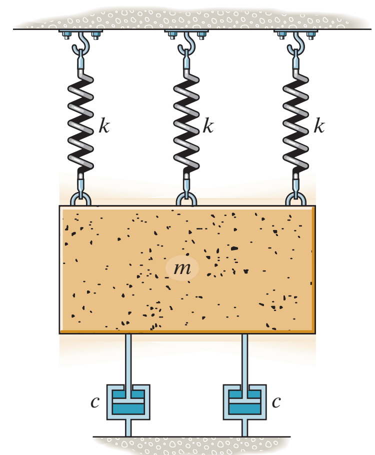
$$x_{\max} = \frac{2mv_0}{\sqrt{8k(m + M) - c^2}} e^{-\left[\frac{\pi c}{2\sqrt{8k(m + M) - c^2}}\right]} \qquad \textbf{Ans.}$$

Ans:

$$x_{\max} = \frac{2mv_0}{\sqrt{8k(m + M) - c^2}} e^{-\pi c/\left(2\sqrt{8k(m+M)-c^2}\right)}$$

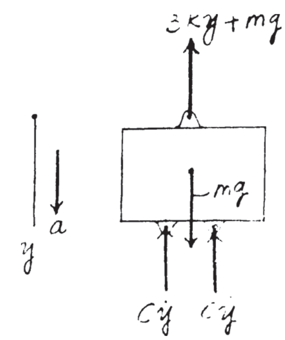
***22–76.**

Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs? Take $k = 100$ N/m, $c = 200$ N·s/m, $m = 25$ kg.



SOLUTION

Free-body Diagram: When the block is being displaced by an amount y vertically downward, the *restoring force* is developed by the three springs attached the block.



Equation of Motion:

$+\uparrow \Sigma F_x = 0;$ $$3ky + mg + 2c\dot{y} - mg = -m\ddot{y}$$

$$m\ddot{y} + 2c\dot{y} + 3ky = 0 \qquad \textbf{(1)}$$

Here, $m = 25$ kg, $c = 200$ N·s/m and $k = 100$ N/m. Substituting these values into Eq. (1) yields

$$25\ddot{y} + 400\dot{y} + 300y = 0$$

$$\ddot{y} + 16\dot{y} + 12y = 0 \qquad \textbf{Ans.}$$

Comparing the above differential equation with Eq. 22–27, we have $m = 1$kg, $c = 16$ N·s/m and $k = 12$ N/m. Thus, $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{12}{1}} = 3.464$ rad/s

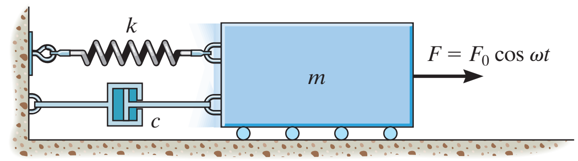
$$c_c = 2m\omega_n = 2(1)(3.464) = 6.928 \text{ N}\cdot\text{s/m}$$

Since $c > c_c$, the system will not vibrate. Therefore it is **overdamped.** **Ans.**

Ans:
$\ddot{y} + 16\dot{y} + 12y = 0$
Since $c > c_c$, the system will not vibrate. Therefore it is **overdamped.**

22–77.

Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.



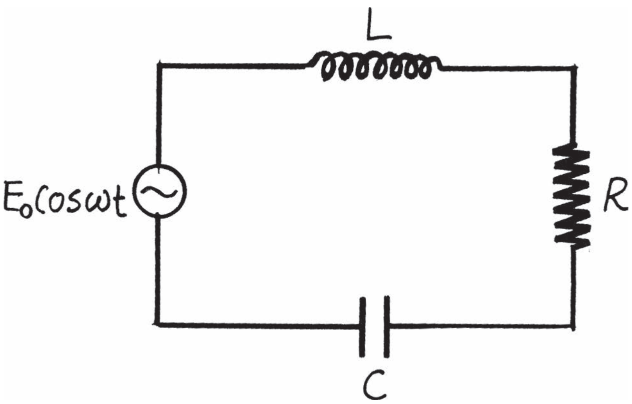
SOLUTION

For the block,

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

Using Table 22–1,

$$L\ddot{q} + R\dot{q} + \left(\frac{1}{C}\right)q = E_0 \cos \omega t \qquad \textbf{Ans.}$$

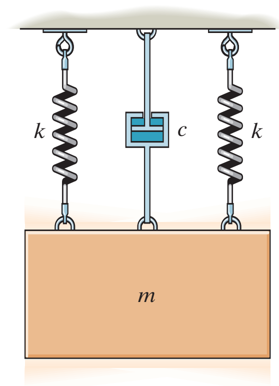


Ans:

$L\ddot{q} + R\dot{q} + \left(\frac{1}{C}\right)q = E_0 \cos \omega t$

22–78.

Draw the electrical circuit that is equivalent to the mechanical system shown. What is the differential equation which describes the charge q in the circuit?



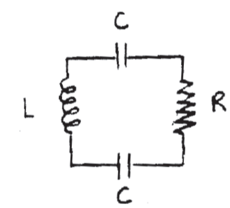
SOLUTION

For the block,

$$m\ddot{x} + c\dot{x} + 2k = 0$$

Using Table 22–1,

$$L\ddot{q} + R\dot{q} + \left(\frac{2}{C}\right)q = 0 \qquad \textbf{Ans.}$$

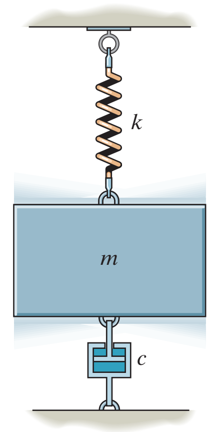


Ans:

$L\ddot{q} + R\dot{q} + \left(\dfrac{2}{C}\right)q = 0$

22–79.

Draw the electrical circuit that is equivalent to the mechanical system shown. Determine the differential equation which describes the charge q in the circuit.



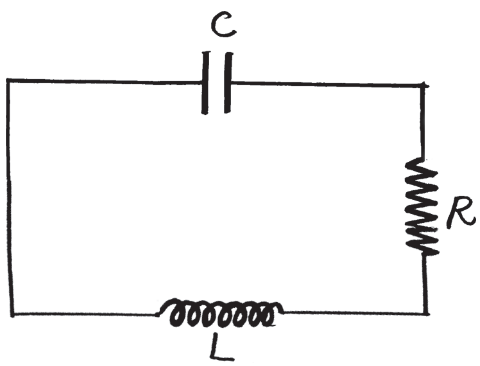
SOLUTION

For the block

$$m\ddot{y} + c\dot{y} + ky = 0$$

Using Table 22–1

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0 \qquad \textbf{Ans.}$$



Ans:

$L\ddot{q} + R\dot{q} + \frac{1}{C}q = 0$