A MATLAB Script for Earth-to-Mars Mission Design

This document describes a MATLAB script named `e2m_matlab.m` that can be used to design and optimize ballistic interplanetary missions from Earth park orbit to encounter at Mars. The software assumes that interplanetary injection occurs *impulsively* from a circular Earth park orbit. The B-plane coordinates are expressed in a Mars-centered (areocentric) mean equator and IAU node of epoch coordinate system. B-plane targets are enforced using either a combination of periapsis radius and orbital inclination, individual B-plane coordinates ($\mathbf{B \bullet T}$ and $\mathbf{B \bullet R}$) of the arrival hyperbola, or entry interface (EI) conditions at Mars. The type of targeting and the target values are defined by the user.

The first part of this MATLAB script solves for the minimum delta-v using a *patched-conic*, two-body Lambert solution for the transfer trajectory from Earth to Mars. The second part implements a simple *shooting* method that attempts to optimize the characteristics of the geocentric injection hyperbola while numerically integrating the spacecraft's geocentric and heliocentric equations of motion and targeting to components of the B-plane relative to Mars.

The spacecraft motion within the Earth's SOI includes the Earth's J_2 oblate gravity effect and the point-mass perturbations of the sun and moon. The heliocentric equations of motion include the point-mass gravity of the sun and the first seven planets of the solar system.

The user can select one of the following delta-v optimization options for the two-body solution of the interplanetary transfer trajectory:

- minimize departure delta-v
- minimize arrival delta-v
- minimize total delta-v
- no optimization

The major computational steps implemented in this script are as follows:

- solve the two-body, patched-conic interplanetary Lambert problem for the energy C_3, declination (DLA) and asymptote (RLA) of the outgoing or departure hyperbola
- compute the orbital elements of the geocentric departure hyperbola and the components of the interplanetary injection delta-v vector using the two-body solution
- perform geocentric orbit propagation from perigee of the geocentric departure hyperbola to the Earth's sphere-of-influence (SOI; default value = 925,000 kilometers)
- perform an n-body heliocentric orbit propagation from the Earth's SOI to closest approach at Mars
- target to the user-defined B-plane coordinates while minimizing the magnitude of the hyperbolic v-infinity at Earth departure (equivalent to minimizing the departure energy since $C_3 = V_\infty^2$)

This MATLAB script uses the SNOPT nonlinear programming algorithm to solve both the patched-conic and numerically integrated trajectory optimization problems. The coordinates of the sun, moon and planets are computed using the JPL Development Ephemeris DE421.

Input file format and contents

This section describes a typical input data file for the `e2m_matlab` script. In the following discussion the actual input file contents are in `courier` font and all explanations are in *times italic* font. Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input. The time scale for all internal calculations is Barycentric Dynamical Time (TDB).

The software allows the user to specify an initial guess for the departure and arrival calendar dates and a search interval. For any guess for departure time t_L and user-defined search interval Δt, the departure time t is constrained as follows:

$$t_L - \Delta t \le t \le t_L + \Delta t$$

Likewise, for any guess for arrival time t_A and user-defined search interval, the arrival time t is constrained as follows:

$$t_A - \Delta t \le t \le t_A + \Delta t$$

For fixed departure and/or arrival times, the search interval should be set to 0.

The first six lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with six and only six initial text lines.

```
*****************************************
** interplanetary trajectory optimization
** script ==> e2m_matlab.m
** Mars '03 mars03.in
** December 26, 2012
*****************************************
```

The first numerical input is an integer that defines the type of patched-conic trajectory optimization performed by this script. Please note that option 4 simply solves Lambert's two-point boundary value problem (TPBVP) using the inputs for departure and arrival calendar dates provided by the user.

```
*******************
* simulation type *
*******************
 1 = minimize departure delta-v
 2 = minimize arrival delta-v
 3 = minimize total delta-v
 4 = no optimization
--------------------
1
```

The next input defines an initial guess for the departure calendar date. Please be sure to include all digits of the calendar year.

```
departure calendar date initial guess (month, day, year)
6,1,2003
```

This next number defines the lower and upper search interval for the departure calendar date.

```
departure date search boundary (days)
30
```

The next input defines an initial guess for the arrival calendar date. Please be sure to include all digits of the calendar year.

```
arrival calendar date initial guess (month, day, year)
12,1,2003
```

This number defines the lower and upper search interval for the arrival calendar date.

```
arrival date search boundary (days)
30
```

The next set of inputs defines several characteristics of the departure hyperbola and initial flight conditions. The perigee altitude and launch site latitude are with respect to a spherical Earth. The launch azimuth is measured positive clockwise from north.

```
***************************
* geocentric phase modeling
***************************

perigee altitude of launch hyperbola (kilometers)
185.32

launch azimuth (degrees)
93.0

launch site latitude (degrees)
28.5
```

The next input specifies the type of targeting at Mars performed by the `e2m_matlab` *script. Option 1 will target to components of the B-plane and option 2 will target to a Mars-centered hyperbola with a specified radius of closest approach (periapsis) and orbital inclination. Option 3 will target to user-defined entry interface (EI) conditions and option 4 will target a grazing flyby of Mars with a user-defined B-plane angle.*

```
****************************
* encounter planet targeting
****************************

type of targeting
(1 = B-plane, 2 = orbital elements, 3 = EI conditions, 4 = grazing flyby)
2
```

The next two inputs are the user-defined B-plane components used with targeting option 1.

```
B dot T
4607.4

B dot R
-7888.0
```

These next two inputs define the radius of closest approach and the orbital inclination of the encounter hyperbola at Mars. These flight conditions are used by targeting option 2. The radius of closest approach is with respect to a spherical Mars model and the orbital inclination is with respect to the mean equator of Mars.

```
radius of closest approach (kilometers)
5000.0

orbital inclination (degrees)
60.0
```

The next two inputs define the inertial flight path angle and altitude of the entry interface at Mars. These flight conditions are used by targeting option 3. The altitude is with respect to a spherical Mars model. Targeting option 3 will target to the orbital inclination defined above.

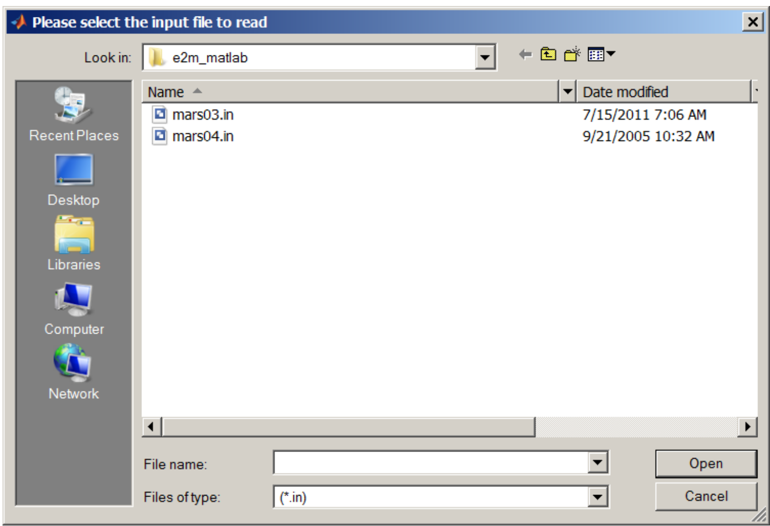
```
EI flight path angle (degrees)
-2.0

EI altitude (kilometers)
100.0
```

The final input is the user-defined B-plane angle for the grazing flyby option. Please note that the B-plane angle is measured positive clockwise from the **T** axis of the B-plane coordinate system.

```
user-defined b-plane angle (degrees)
-60.0
```

User interaction with the script

The `e2m_matlab` script will interactively prompt the user for the name of a simulation definition data file with a window similar to the following;



The default filename type is `*.in`. However, the script will read any compatible data file.

Script example

The following is the solution created with this MATLAB script for this example. The output is organized by the following major sections:

- Two-body/patched-conic pass
    1. two body Lambert solution
    2. departure hyperbola orbital elements and state vector
    3. heliocentric coordinates of Earth at departure and Mars at arrival
    4. heliocentric coordinates of the spacecraft on the transfer trajectory

- Targeting/optimization pass
    1. optimized characteristics of the departure hyperbola
    2. heliocentric coordinates of the spacecraft and Mars at closest approach
    3. geocentric and heliocentric coordinates of the spacecraft at the Earth SOI

The first output section illustrates the two-body Lambert solution including the SNOPT algorithm iterations and solution summary. The solution is provided in the heliocentric, Earth mean equator and equinox of J2000 (EME2000) coordinate system. The time scale is Universal Coordinated Time (UTC). Please see Appendix A for additional explanation about the provided information.

```
 Nonlinear constraints       0     Linear constraints       1
 Nonlinear variables         2     Linear variables         0
 Jacobian  variables         0     Objective variables      2
 Total constraints           1     Total variables          2


 The user has defined       0   out of       2   first  derivatives

 Major Minors     Step   nObj Feasible  Optimal      Objective    nS
     0      2               1           3.2E-03  3.0416787E+00     2   r
     1      1  7.1E-01      2           2.7E-03  3.0338679E+00     2 n rl
     2      1  7.1E-01      3           2.6E-03  3.0271837E+00     2 s  l
     3      1  1.0E+00      4           1.2E-03  2.9746321E+00     2
     4      1  1.0E+00      5           3.0E-04  2.9644391E+00     2
     5      1  1.0E+00      6           4.1E-05  2.9643144E+00     2
     6      1  1.0E+00      7           6.4E-06  2.9643112E+00     2
     7      1  1.0E+00      8          (1.7E-08) 2.9643112E+00     2       c
     7      2  1.0E+00      8          (3.3E-07) 2.9643112E+00     2       c

 SNOPTA EXIT   0 -- finished successfully
 SNOPTA INFO   1 -- optimality conditions satisfied
 Problem name
 No. of iterations                  10   Objective value      2.9643111870E+00
 No. of major iterations             7   Linear objective     0.0000000000E+00
 Penalty parameter           0.000E+00   Nonlinear objective  2.9643111870E+00
 No. of calls to funobj             37   No. of calls to funcon             37
 Calls with modes 1,2 (known g)      8   Calls with modes 1,2 (known g)      8
 Calls for forward differencing     16   Calls for forward differencing     16
 Calls for central differencing      4   Calls for central differencing      4
 No. of superbasics                  2   No. of basic nonlinears             0
 No. of degenerate steps             0   Percentage                       .00
 Max x                       2 1.5E+03   Max pi                      1 1.0E+00
 Max Primal infeas           0 0.0E+00   Max Dual infeas             1 6.5E-07


 Solution printed on file   9

Earth-to-Mars mission design

=========================
two-body Lambert solution
=========================

minimize departure delta-v

departure heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       2895.912618  meters/second
y-component of delta-v       -530.389043  meters/second
z-component of delta-v       -345.714312  meters/second

delta-v magnitude            2964.311187  meters/second
```

```
arrival heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v      -2063.021184  meters/second
y-component of delta-v       1164.270845  meters/second
z-component of delta-v       1311.949617  meters/second

delta-v magnitude            2707.913367  meters/second


heliocentric coordinates of the Earth at departure
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11507361

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.4965147327e+008  1.6237346597e-002  2.3439054671e+001  1.0245240440e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 7.2430848045e-004  1.5204742994e+002  2.5449983434e+002  3.6545322928e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -4.05626080325426e+007  -1.34199491174615e+008  -5.81817199031513e+007  +1.51789133777145e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +2.82279246201946e+001  -7.39786255837625e+000  -3.20748439559914e+000  +2.93569762568291e+001


spacecraft heliocentric coordinates after the first impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11507361

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8838714747e+008  1.9427720613e-001  2.3490037882e+001  2.5349091879e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.5596572332e-001  5.9131918561e-001  2.5408223798e+002  5.1616340901e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -4.05626080325426e+007  -1.34199491174615e+008  -5.81817199031513e+007  +1.51789133777145e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.11238372380865e+001  -7.92825160148060e+000  -3.55319870788600e+000  +3.23137066728192e+001


spacecraft heliocentric coordinates prior to the second impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003

UTC time             15:22:06.692

UTC Julian Date      2452998.14035523

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8838714747e+008  1.9427720613e-001  2.3490037882e+001  2.5349091879e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.5596572332e-001  1.5290995806e+002  4.6400876854e+001  5.1616340901e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +1.49990801462616e+008  +1.46776341464580e+008  +6.32690486133341e+007  +2.19188292227393e+008
```

```
        vx (kps)                vy (kps)                vz (kps)                vmag (kps)
 -1.46793406657452e+001  +1.56263833645881e+001  +6.84186935963794e+000  +2.25050677797786e+001


spacecraft heliocentric coordinates after the second impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003

UTC time             15:22:06.692

UTC Julian Date      2452998.14035523

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8838714747e+008  1.9427720613e-001  2.3490037882e+001  2.5349091879e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.5596572332e-001  1.5290995806e+002  4.6400876854e+001  5.1616340901e+002

        rx (km)                 ry (km)                 rz (km)                 rmag (km)
 +1.49990801462616e+008  +1.46776341464580e+008  +6.32690486133341e+007  +2.19188292227393e+008

        vx (kps)                vy (kps)                vz (kps)                vmag (kps)
 -1.67423618500392e+001  +1.67906542098226e+001  +8.15381897618937e+000  +2.50742400285223e+001


heliocentric coordinates of Mars at arrival
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003

UTC time             15:22:06.692

UTC Julian Date      2452998.14035523

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 2.2793930707e+008  9.3541889964e-002  2.4677224952e+001  3.3297923712e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 3.3716583265e+000  7.0759517389e+001  4.3738754512e+001  6.8697217107e+002

        rx (km)                 ry (km)                 rz (km)                 rmag (km)
 +1.49990801462616e+008  +1.46776341464580e+008  +6.32690486133341e+007  +2.19188292227394e+008

        vx (kps)                vy (kps)                vz (kps)                vmag (kps)
 -1.67423618500392e+001  +1.67906542098226e+001  +8.15381897618937e+000  +2.50742400285223e+001
```

The following output summarizes the orbital characteristics of the initial circular park orbit and the departure hyperbola for the two-body solution.

```
park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

park orbit
----------

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 6.5634578000e+003  0.0000000000e+000  2.8644284856e+001  0.0000000000e+000

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 2.0356396962e+000  1.9503978483e+002  1.9503978483e+002  6.1248616627e-002

        rx (km)                 ry (km)                 rz (km)                 rmag (km)
 -6.28153849210695e+003  -1.71891801652489e+003  -8.16439414179488e+002  +6.56345780000000e+003

        vx (kps)                vy (kps)                vz (kps)                vmag (kps)
 +2.25553181463621e+000  -6.52893260021440e+000  -3.60775090896881e+000  +7.79296165049872e+000
```

```
departure hyperbola
-------------------

c3                              8.787141  km^2/sec^2

v-infinity                   2964.311187  meters/second

asymptote right ascension     349.621254  degrees

asymptote declination          -6.697391  degrees

perigee altitude              185.320000  kilometers

launch azimuth                 93.000000  degrees

launch site latitude           28.500000  degrees


UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11507361

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -4.5361790595e+004  1.1446913297e+000  2.8644284856e+001  1.9503978483e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 2.0356396962e+000  3.6000000000e+002  1.9503978483e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -6.28153849210695e+003  -1.71891801652488e+003  -8.16439414179486e+002  +6.56345780000000e+003

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.30317326874917e+000  -9.56146816398225e+000  -5.28346631401135e+000  +1.14126089648913e+001


hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1047.641454  meters/sec
y-component of delta-v      -3032.535564  meters/sec
z-component of delta-v      -1675.715405  meters/sec

delta-v magnitude            3619.647314  meters/sec
```

The following program output summarizes the flight conditions determined by the n-body, numerically integrated optimized solution. The initial illustrates the SNOPT algorithm characteristics.

```
 Nonlinear constraints       2     Linear constraints       1
 Nonlinear variables         3     Linear variables         0
 Jacobian  variables         3     Objective variables      0
 Total constraints           3     Total variables          3


 The user has defined       0   out of       6   first  derivatives

 Major Minors     Step   nCon Feasible  Optimal  MeritFunction    nS Penalty
     0      1               1  6.7E+00  6.7E-04  2.9643112E+00                r
     1      0  1.3E-01      2  5.8E+00  7.8E-05  2.9642649E+00              n r
     2      1  2.7E-02      3  5.7E+00  1.9E-03  2.9644118E+00     1 2.2E-07 s
     3      1  1.3E-01      4  4.9E+00  1.2E-03  2.9645811E+00     1 5.0E-07
     4      1  1.7E-01      5  4.1E+00  1.3E-03  2.9646285E+00     1 6.1E-07
     5      1  2.3E-01      6  3.2E+00  1.4E-03  2.9646570E+00     1 7.1E-07
     6      1  3.5E-01      7  2.1E+00  1.1E-03  2.9646794E+00     1 8.5E-07
     7      1  1.0E+00      8  1.8E-02  1.2E-04  2.9646958E+00     1 1.0E-06
     8      1  3.3E-01      9  1.2E-02  1.2E-05  2.9646955E+00     1 1.0E-06
     9      1  1.0E+00     10 (5.9E-07) 3.9E-06  2.9646955E+00     1 1.0E-06
     9      2  1.0E+00     10 (5.9E-07) 2.7E-06  2.9646955E+00     1 1.0E-06       c
 Major Minors     Step   nCon Feasible  Optimal  MeritFunction    nS Penalty
    10      1  1.0E+00     11 (5.9E-08)(2.0E-06) 2.9646955E+00     1 1.0E-06       c
```

```
 SNOPTA EXIT   0 -- finished successfully
 SNOPTA INFO   1 -- optimality conditions satisfied
 Problem name
 No. of iterations                  11   Objective value      2.9646954867E+00
 No. of major iterations            10   Linear objective     2.9646954867E+00
 Penalty parameter           1.048E-06   Nonlinear objective  0.0000000000E+00
 No. of calls to funobj             75   No. of calls to funcon             75
 Calls with modes 1,2 (known g)     11   Calls with modes 1,2 (known g)     11
 Calls for forward differencing     30   Calls for forward differencing     30
 Calls for central differencing     12   Calls for central differencing     12
 No. of superbasics                  1   No. of basic nonlinears             2
 No. of degenerate steps             0   Percentage                        .00
 Max x                       2 6.1E+00   Max pi                      3 1.0E+00
 Max Primal infeas           0 0.0E+00   Max Dual infeas             2 2.4E-05
 Nonlinear constraint violn    3.7E-07


 Solution printed on file   9

=======================
optimal n-body solution
=======================

orbital element targeting

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

park orbit
----------

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 6.5634578000e+003  0.0000000000e+000  2.8644284856e+001  0.0000000000e+000

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 2.6756890559e+000  1.9474156131e+002  1.9474156131e+002  6.1248616627e-002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -6.27206698422076e+003  -1.76044686286978e+003  -8.00612815810907e+002  +6.56345780000000e+003

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +2.28959382658945e+000  -6.51429704867106e+000  -3.61274761878861e+000  +7.79296165049872e+000


departure hyperbola
-------------------

c3                             8.789419  km^2/sec^2

v-infinity                  2964.695487  meters/second

asymptote right ascension     349.992893  degrees

asymptote declination          -6.838513  degrees


UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11581649

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -4.5350031280e+004  1.1447288484e+000  2.8644284856e+001  1.9474156131e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 2.6756890559e+000  0.0000000000e+000  1.9474156131e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -6.27206698422076e+003  -1.76044686286978e+003  -8.00612815810909e+002  +6.56345780000000e+003

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.35308561234210e+000  -9.54011818810613e+000  -5.29083015550749e+000  +1.14127087889404e+001
```

```
hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1063.491786  meters/sec
y-component of delta-v      -3025.821139  meters/sec
z-component of delta-v      -1678.082537  meters/sec

delta-v magnitude            3619.747138  meters/sec

transfer time                 201.331876  days

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

UTC calendar date    23-Dec-2003

UTC time             22:43:36.414

UTC Julian Date      2452997.44694924

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -5.8449879968e+003  1.8554337083e+000  5.9999989954e+001  1.1389589769e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 1.0573209493e+002  1.3620882543e-006  1.1389589905e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -1.65092434229199e+003  -2.56925600851159e+003  +3.95896071550230e+003  +4.99999975681355e+003

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +2.19014537923133e+000  -4.08068259552846e+000  -1.73493855313629e+000  +4.94557570064152e+000


B-plane coordinates at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

b-magnitude                 9135.090295  kilometers
b dot r                    -7888.074747  kilometers
b dot t                     4607.401814  kilometers
b-plane angle                300.289078  degrees
v-infinity                  2706.911098  meters/second
r-periapsis                 4999.999757  kilometers
decl-asymptote                 7.541645  degrees
rasc-asymptote               281.348325  degrees

flight path angle              0.000001  degrees

spacecraft heliocentric coordinates at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003

UTC time             22:43:36.414

UTC Julian Date      2452997.44694924

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8440870022e+008  2.3701914695e-001  1.9041618478e+001  2.7135359758e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 3.4391054196e+002  1.5010004094e+002  6.1453638518e+001  4.9989917381e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +1.50992608621866e+008  +1.45762886035126e+008  +6.27807294766759e+007  +2.19059368202660e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 -1.35221557125005e+001  +1.70344597592078e+001  +4.35557459015284e+000  +2.21809049502397e+001
```

```
heliocentric coordinates of Mars at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003

UTC time             22:43:36.414

UTC Julian Date      2452997.44694924

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 2.2793935977e+008  9.3542132991e-002  2.4677224880e+001  3.3297930887e+002
     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 3.3716581632e+000  7.0367974211e+001  4.3347283078e+001  6.8697240931e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +1.50990439112200e+008  +1.45767094614877e+008  +6.27791229021031e+007  +2.19060212836825e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 -1.66286361833180e+001  +1.69011945189360e+001  +8.20144750419211e+000  +2.50883570336538e+001
```

The final program output summarizes both the geocentric and heliocentric spacecraft trajectory at the Earth's sphere-of-influence.

```
spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003

UTC time             18:20:28.041

UTC Julian Date      2452799.26421343

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -4.5602518184e+004  1.1432734956e+000  2.8506946409e+001  1.9469994993e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 2.6886247296e+000  1.4947862406e+002  3.4417857399e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +8.99373337242110e+005  -1.79625903668514e+005  -1.20362514884332e+005  +9.25000000000012e+005

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.03072408676696e+000  -5.32307861774739e-001  -3.65996859413918e-001  +3.09880522958728e+000


spacecraft heliocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003

UTC time             18:20:28.041

UTC Julian Date      2452799.26421343

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.9070346904e+008  2.0396240805e-001  2.3500011747e+001  2.5358976947e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 5.2634029254e-001  3.7899708609e+000  2.5737974033e+002  5.2571237318e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -3.19305377694382e+007  -1.36202140051031e+008  -5.90923703870906e+007  +1.51863393991742e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.16290560341318e+001  -6.53285287954192e+000  -2.96678275764469e+000  +3.24326556465401e+001
```

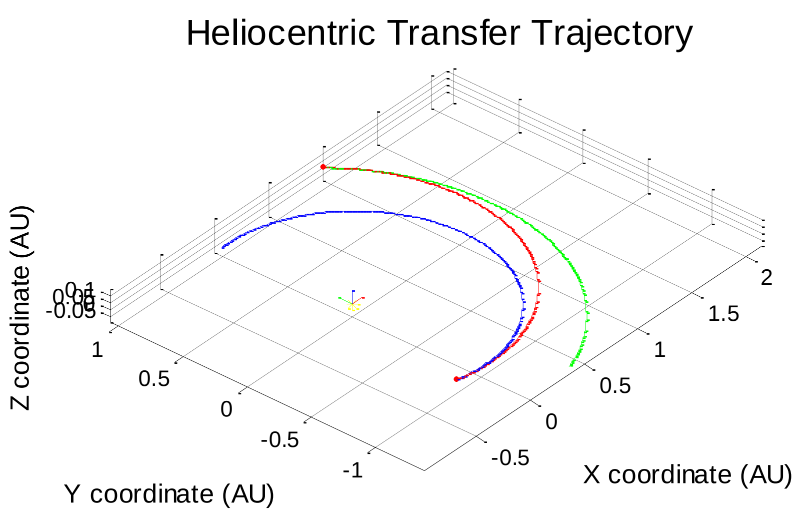
This MATLAB script will also create a graphics display of the interplanetary and encounter trajectories using a function named `marsplot.m`. The interactive graphic features of MATLAB will allow the user to rotate and "zoom" the displays. These capabilities allow the user to interactively find the "best" viewpoint as well as verify orbital geometry of the heliocentric and areocentric trajectories.

The script will ask you for the step size at which to plot the graphics data with the following prompt:

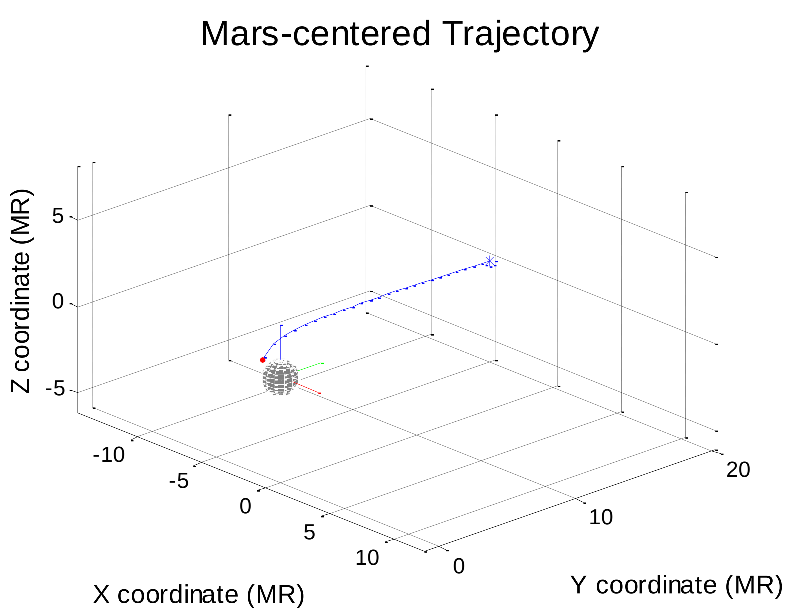
```
please input the plot step size (days)
?
```

A value between 1 and 5 days is recommended.

The following is a heliocentric, ecliptic view of the transfer and planetary orbits. The x-axis of this system is red, the y-axis green and the z-axis is blue.



This next plot is a view of the encounter trajectory in the Mars-centered mean equator and IAU node of epoch coordinate system. The small red dot is the periapsis of the encounter hyperbola. The asterisk indicates the location of the spacecraft when it is 50,000 kilometers from Mars.



Please note that the `marsplot` function can be run stand-alone from within the MATLAB command window. This allows the user the option to experiment with different graphic step sizes without having to re-run the main script.

Technical discussion

This section describes the main numerical methods implemented in the `e2m_matlab` script.

Solving the two body Lambert problem

Lambert's problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamic problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis *a* of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length *c* of the chord joining these two positions. This relationship can be stated as follows:

$$tof = tof\left(r_i + r_f, c, a\right)$$

From the following form of Kepler's equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}\left(E - e\sin E\right)}$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}}\left[E - E_0 - e\left(\sin E - \sin E_0\right)\right]$$

where *E* is the eccentric anomaly associated with radius *r*, E_0 is the eccentric anomaly at r_0, and $t = 0$ when $r = r_0$.

At this point we need to introduce the following trigonometric sun and difference identities:

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{a+\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha-\beta}{2}\sin\frac{a+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha-\beta}{2}\cos\frac{a+\beta}{2}$$

If we let $E = \alpha$ and $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following equation:

$$t=\sqrt{\frac{a^3}{\mu}}\left\{E-E_0-2\sin\frac{E-E_0}{2}\left(e\cos\frac{E+E_0}{2}\right)\right\}$$

With the two substitutions given by

$$e\cos\frac{E+E_0}{2}=\cos\frac{\alpha+\beta}{2}$$

$$\sin\frac{E-E_0}{2}=\sin\frac{\alpha-\beta}{2}$$

the time equation becomes

$$t=\sqrt{\frac{a^3}{\mu}\left\{(\alpha-\beta)-2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}\right\}}$$

From the elliptic relationships given by

$$r=a(1-e\cos E)$$

$$x=a(\cos E-e)$$

$$y=a\sin E\sqrt{1-e^2}$$

and some more manipulation, we have the following equations:

$$\cos\alpha=\left(1-\frac{r+r_0}{2a}\right)-\frac{c}{2a}=1-\frac{r+r_0+c}{2a}=1-\frac{s}{a}$$

$$\sin\beta=\left(1-\frac{r+r_0}{2a}\right)+\frac{c}{2a}=1-\frac{r+r_0-c}{2a}=1-\frac{s-c}{a}$$

This part of the derivation makes use of the following three relationships:

$$\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}=1-\frac{r+r_0}{2}$$

$$\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}=\sin\frac{E-E_0}{2}\sqrt{1-\left(e\cos\frac{E+E_0}{2}\right)^2}$$

$$\left(\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}\right)^2=\left(\frac{x-x_0}{2a}\right)^2+\left(\frac{y-y_0}{2a}\right)^2=\left(\frac{c}{2a}\right)^2$$

With the use of the half angle formulas given by

$$\sin\frac{\alpha}{2} = \sqrt{\frac{s}{2a}} \qquad \sin\frac{\beta}{2} = \sqrt{\frac{s-c}{2a}}$$

and several additional substitutions, we have the time-of-flight form of Lambert's theorem

$$t = \sqrt{\frac{a^3}{\mu}}\left[(\alpha-\beta)-(\sin\alpha-\sin\beta)\right]$$

A discussion about the angles α and β can be found in "Geometrical Interpretation of the Angles α and β in Lambert's Problem" by J. E. Prussing, AIAA *Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this MATLAB script is based on the method described in "A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem" by R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48:** 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde, and involve one or more revolutions about the central body.

Designing the departure hyperbola

This section describes the algorithm used to determine the Earth-centered-inertial (ECI) state vector of a departure hyperbola for interplanetary missions. In the discussion that follows, interplanetary injection is assumed to occur *impulsively* at perigee of the departure hyperbola.

The departure trajectory for interplanetary missions can be defined using the specific (per unit mass) orbital energy C_3, and the right ascension α_∞ (RLA) and declination δ_∞ (DLA) of the outgoing asymptote. The perigee radius of the departure hyperbola is calculated using the user's value for perigee altitude.

The orbital inclination is computed from the user-defined launch azimuth Σ_L and launch site geocentric latitude ϕ_L using this equation

$$i = \cos^{-1}\left(\cos\phi_L \sin\Sigma_L\right)$$

The algorithm used to design the departure hyperbola only works for geocentric orbit inclinations that satisfy the following constraint

$$|i| > |\delta_\infty|$$

If this inequality is not satisfied, the software will print the following error message

```
park orbit error!!

|inclination| must be > |asymptote declination|
```

The code will also print the inclination of the park orbit, the declination of the departure hyperbola and stop. The user can then change either the azimuth or launch site latitude to satisfy this constraint and restart the script.

A unit vector in the direction of the departure asymptote is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where

$\alpha_\infty =$ right ascension of departure asymptote (RLA)

$\delta_\infty =$ declination of departure asymptote (DLA)

The T-axis direction of the B-plane coordinate system is determined from the following vector cross product:

$$\hat{\mathbf{T}} = \hat{\mathbf{S}} \times \hat{\mathbf{u}}_z$$

where $\hat{\mathbf{u}}_z = \left[0\,0\,1\right]^T$ is a unit vector perpendicular to the Earth's equator.

The following cross product operation completes the B-plane coordinate system.

$$\hat{\mathbf{R}} = \hat{\mathbf{S}} \times \hat{\mathbf{T}}$$

The B-plane angle is determined from the orbital inclination of the departure hyperbola i and the declination of the outgoing asymptote according to

$$\cos\theta = \cos i / \cos\delta_\infty$$

The unit angular momentum vector of the departure hyperbola is given by

$$\hat{\mathbf{h}} = \hat{\mathbf{T}} \sin\theta - \hat{\mathbf{R}} \cos\theta$$

The sine and cosine of the true anomaly at infinity are given by the next two equations

$$\cos\theta_\infty = -\frac{\mu}{r_p V_\infty^2 + \mu} \qquad \sin\theta_\infty = \sqrt{1 - \cos^2\theta_\infty}$$

where $V_\infty = \sqrt{C_3} = V_L - V_p$ is the spacecraft's velocity at infinity (v-infinity), V_L is the heliocentric departure velocity determined from the Lambert solution, V_p is the heliocentric velocity of the departure planet, and r_p is the user-specified perigee radius of the departure hyperbola.

A unit vector in the direction of perigee of the departure hyperbola is determined from

$$\hat{\mathbf{r}}_p = \hat{\mathbf{S}} \cos\theta_\infty - \left(\hat{\mathbf{h}} \times \hat{\mathbf{S}}\right) \sin\theta_\infty$$

The ECI position vector at perigee is equal to $\mathbf{r}_p = r_p \hat{\mathbf{r}}_p$.

The scalar magnitude of the perigee velocity can be determined from

$$V_p = \sqrt{\frac{2\mu}{r_p} + V_\infty^2}$$

A unit vector aligned with the velocity vector at perigee is

$$\hat{\mathbf{v}}_p = \hat{\mathbf{h}} \times \hat{\mathbf{r}}_p$$

The ECI velocity vector at perigee of the departure hyperbola is given by

$$\mathbf{v}_p = V_p \hat{\mathbf{v}}_p$$

Finally, the classical orbital elements of the departure hyperbola can be determined from the position and velocity vectors at perigee. The injection delta-v vector and magnitude can be determined from the velocity difference between the local circular velocity of the park orbit and departure hyperbola at the orbital location of the impulsive maneuver.

Propagating the spacecraft's trajectory

The spacecraft's orbital motion is modeled with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this Earth-centered-inertial (ECI) inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time (TT).

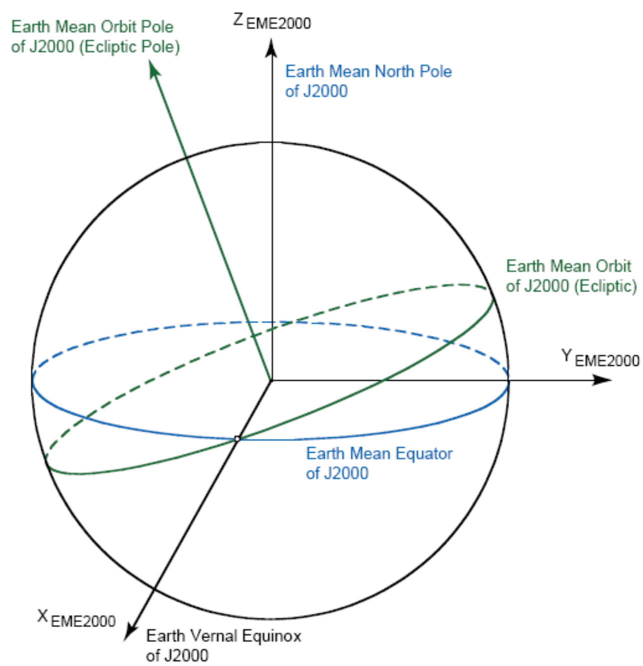


Figure 1. Earth mean equator and equinox of J2000 coordinate system

Geocentric trajectory propagation

This part of the trajectory analysis implements a *special perturbation* technique which numerically integrates the vector system of second-order, nonlinear differential equations of motion of a spacecraft given by

$$\vec{a}\left(\vec{r},\vec{v},t\right)=\ddot{\vec{r}}\left(\vec{r},\dot{\vec{r}},t\right)=\vec{a}_g\left(\vec{r}\right)+\vec{a}_m\left(\vec{r},t\right)+\vec{a}_s\left(\vec{r},t\right)$$

where

$t =$ time

$\vec{r} =$ inertial position vector of the satellite

$\vec{v} =$ inertial velocity vector of the satellite

$\vec{a}_g =$ acceleration due to Earth gravity

$\vec{a}_m =$ acceleration due to the Moon

$\vec{a}_s =$ acceleration due to the Sun

The system of six first-order differential equations subject to Earth gravity is defined by

$$\dot{y}_1 = v_x = y_4 \qquad \dot{y}_2 = v_y = y_5 \qquad \dot{y}_2 = v_z = y_6$$

$$\dot{y}_4 = -\mu\frac{r_x}{r^3}\left\{1+\frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(1-\frac{5r_z^2}{r^2}\right)\right\}$$

$$\dot{y}_5 = -\mu\frac{r_y}{r^3}\left\{1+\frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(1-\frac{5r_z^2}{r^2}\right)\right\}$$

$$\dot{y}_6 = -\mu\frac{r_z}{r^3}\left\{1+\frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(3-\frac{5r_z^2}{r^2}\right)\right\}$$

where $r=\sqrt{r_x^2+r_y^2+r_z^2}=\sqrt{y_1^2+y_2^2+y_3^2}$. In these equations μ and r_{eq} are the gravitational constant and equatorial radius of the Earth, respectively and J_2 is the oblateness gravity coefficient.

The acceleration contribution of the Moon represented by a *point mass* is given by

$$\vec{a}_m\left(\vec{r},t\right)=-\mu_m\left(\frac{\vec{r}_{m-b}}{\left|\vec{r}_{m-b}\right|^3}+\frac{\vec{r}_{e-m}}{\left|\vec{r}_{e-m}\right|^3}\right)$$

where

$\mu_m =$ gravitational constant of the Moon

$\vec{r}_{m-b} =$ position vector from the Moon to the satellite

$\vec{r}_{e-m} =$ position vector from the Earth to the Moon

The acceleration contribution of the sun represented by a *point mass* is given by

$$\mathbf{a}_s(\mathbf{r},t) = -\mu_s\left(\frac{\mathbf{r}_{s-sc}}{\left|\mathbf{r}_{s-sc}\right|^3} + \frac{\mathbf{r}_{e-s}}{\left|\mathbf{r}_{e-s}\right|^3}\right)$$

where

$\mu_s =$ gravitational constant of the sun

$\mathbf{r}_{s-sc} =$ position vector from the sun to the spacecraft

$\mathbf{r}_{e-s} =$ position vector from the Earth to the sun

The `e2m_matlab` script uses Battin's $F(q)$ function described in the next section to compute the point-mass gravitational effect of the Sun and Moon.

<u>Heliocentric trajectory propagation</u>

The general vector equation for *point-mass* perturbations such as the Moon or planets is given by

$$\ddot{\mathbf{r}} = -\sum_{j=1}^{n}\mu_j\left[\frac{\mathbf{d}_j}{d_j^3} + \frac{\mathbf{s}_j}{s_j^3}\right]$$

In this equation, $\mathbf{s}_j$ is the vector from the primary body to the secondary body *j*, μ_j is the gravitational constant of the secondary body and $\mathbf{d}_j = \mathbf{r} - \mathbf{s}_j$, where $\mathbf{r}$ is the position vector of the spacecraft relative to the primary body.

To avoid numerical problems, use is made of Richard Battin's $f(q)$ function given by

$$f(q_k) = q_k\left[\frac{3 + 3q_k + q_k^2}{1 + \left(\sqrt{1 + q_k}\right)^3}\right]$$

where

$$q_k = \frac{\mathbf{r}^T(\mathbf{r} - 2\mathbf{s}_k)}{\mathbf{s}_k^T\mathbf{s}_k}$$

The point-mass acceleration due to *n* gravitational bodies can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^{n}\frac{\mu_k}{d_k^3}\left[\mathbf{r} + f(q_k)\mathbf{s}_k\right]$$

In these equations, $\mathbf{s}_k$ is the vector from the primary body to the secondary body, μ_k is the gravitational constant of the secondary body and $\mathbf{d}_k = \mathbf{r} - \mathbf{s}_k$, where $\mathbf{r}$ is the position vector of the spacecraft relative to the primary body. The derivation of the $f(q)$ functions is described in Section 8.4 of "An

Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition", by Richard H. Battin, AIAA Education Series, 1999.

In this MATLAB script the heliocentric coordinates of the Moon, planets and sun are based on the JPL Development Ephemeris DE421. These coordinates are evaluated in the Earth mean equator and equinox of J2000 coordinate system (EME2000).

Predicting the conditions at the Earth's sphere of influence

The trajectory conditions at the boundary of the Earth's sphere of influence are determined during the numerical integration of the spacecraft's geocentric equations of motion by finding the time at which the difference between the geocentric distance and the user-defined value is essentially zero. This scalar mission constraint is computed as follows

$$\Delta r = \left|\mathbf{r}_{sc}\right|_p - r_{soi_u} \approx 0$$

where $\left|\mathbf{r}_{sc}\right|_p$ is the scalar magnitude of the *predicted* geocentric position vector of the spacecraft and r_{soi_u} is the user-defined value of the geocentric distance of the SOI boundary.

In this MATLAB script, the sphere-of-influence calculations use the event-finding feature of the `ode45` MATLAB function. The following is the source code that performs this calculation.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% solve for geocentric sphere-of-influence conditions
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% set up for ode45

options = odeset('RelTol', 1.0e-10, 'AbsTol', 1.0e-10, 'Events', @soi_event);

% define maximum search time (seconds)

tof = 25.0 * 86400.0;

[t, ysol, tevent, yevent, ie] = ode45(@e2m_eqm1, [0 tof], [rhyper vhyper], options);
```

The following is the MATLAB source code that evaluates the SOI event function.

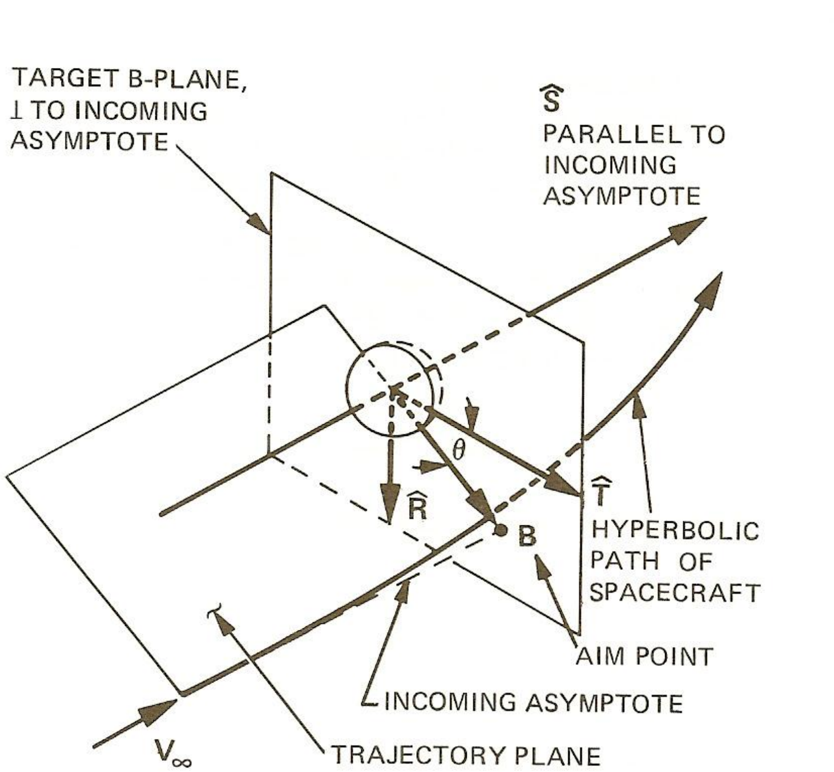
```
function [value, isterminal, direction] = soi_event(t, y)

% sphere-of-influence event function

% input

%  t = current simulation time
%  y = current spacecraft geocentric state vector (km & km/sec)

% output

%  value = difference between current position and soi (kilometers)

% global

%  rsoi = Earth sphere-of-influence (kilometers)

% Orbital Mechanics with Matlab
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

global rsoi

% difference between current geocentric distance and soi (kilometers)

value = norm(y(1:3)) - rsoi;

isterminal = 1;

direction =  [];
```

This script uses a "hard-wired" value of 925,000 kilometers for the Earth's SOI.

B-plane targeting

The derivation of B-plane coordinates is described in the classic JPL reports, "A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories" and "Some Orbital Elements Useful in Space Trajectory Calculations", both by William Kizner. The following diagram illustrates the fundamental geometry of the B-plane coordinate system.



Given the user-defined closest approach radius r_{ca} and orbital inclination *i*, the incoming v-infinity magnitude v_∞, and the right ascension α_∞ (RLA) and declination δ_∞ (DLA) of the incoming asymptote vector at moment of closest approach, the following series of equations can be used to determine the required B-plane target vector:

$$\mathbf{B{\bullet}T} = b_t \cos\theta \qquad \mathbf{B{\bullet}R} = b_t \sin\theta$$

where

$$b_t = \sqrt{\frac{2\mu r_{ca}}{v_\infty^2} + r_{ca}^2} = r_{ca}\sqrt{1 + \frac{2\mu}{r_{ca} v_\infty^2}}$$

and

$$\cos\theta = \frac{\cos i}{\cos\delta_\infty} \qquad \sin\theta = -\sqrt{1-\cos^2\theta} \qquad \to \theta = \tan^{-1}\left(\sin\theta, \cos\theta\right)$$

$$\sin\delta_\infty = \left|\hat{\mathbf{s}} \times \hat{\mathbf{z}}\right| = \sqrt{s_x^2 + s_y^2}$$

$$\hat{\mathbf{z}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where δ_∞ and α_∞ are the declination and right ascension of the asymptote of the incoming hyperbola.

Important note!!

This technique only works for aerocentric orbit inclinations that satisfy

$$|i| > |\delta_\infty|$$

If this inequality is not satisfied, the software will print the following error message

```
b-plane targeting error!!

|inclination| must be > |asymptote declination|
```

It will also display the actual declination of the asymptote and stop. The user should then edit the input file, include a valid orbital inclination and restart the simulation.

The following computational steps summarize the calculation of the *predicted* B-plane vector from a planet-centered position vector **r** and velocity vector **v** at closest approach.

angular momentum vectors

$$\mathbf{h} = \mathbf{r} \times \mathbf{v} \qquad \hat{\mathbf{h}} = \frac{\mathbf{h}}{|\mathbf{h}|}$$

radius rate

$$\dot{r} = \mathbf{r} \bullet \mathbf{v} / |\mathbf{r}|$$

semiparameter

$$p = h^2/\mu$$

semimajor axis

$$a = \frac{r}{\left(2 - \frac{r v^2}{\mu}\right)}$$

orbital eccentricity

$$e = \sqrt{1 - p/a}$$

true anomaly

$$\cos\theta = \frac{p - r}{e r} \qquad \sin\theta = \frac{\dot{r} h}{e\mu} \quad \rightarrow \theta = \tan^{-1}\left(\sin\theta, \cos\theta\right)$$

B-plane magnitude

$$B = \sqrt{p|a|}$$

fundamental vectors

$$\hat{\mathbf{z}} = \frac{r\,\mathbf{v} - \dot{r}\,\mathbf{r}}{h}$$

$$\hat{\mathbf{p}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\mathbf{z}} \qquad \hat{\mathbf{q}} = \sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\mathbf{z}}$$

S vector

$$\mathbf{S} = -\frac{a}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{p}} + \frac{b}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{q}}$$

B vector

$$\mathbf{B} = \frac{b^2}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{p}} + \frac{ab}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{q}}$$

T vector

$$\mathbf{T} = \frac{\left(S_y^2, -S_x^2, 0\right)^T}{\sqrt{S_x^2 + S_y^2}}$$

R vector

$$\mathbf{R} = \mathbf{S} \times \mathbf{T} = \left(-S_z T_y,\ S_z T_x,\ S_x T_y - S_y T_x\right)^T$$

Targeting to the Mars-centered periapsis radius and orbital inclination

For this targeting option, the two equality constraints enforced by the SNOPT nonlinear programming algorithm are

$$r_p - r_{ca} = 0$$

$$\cos i - \hat{\mathbf{h}}_z = 0$$

where r_p and i are the user-defined periapsis radius and orbital inclination, respectively, and $\hat{\mathbf{h}}_z$ is the z-component of the unit angular momentum vector at closest approach to Mars.

The mission elapsed time at which the spacecraft reaches closest approach to Mars is predicted using the event prediction capability of the MATLAB `ode45` algorithm. During the numerical integration of the spacecraft's heliocentric equations of motion, the `ode45` numerical method searches for the time at which the flight path angle *with respect to Mars* is nearly zero within a small tolerance. This constraint corresponds to closest approach to Mars.

Closest approach is predicted with the following *mission constraint*

$$\sin \gamma = \frac{\mathbf{r} \bullet \mathbf{v}}{\left| \mathbf{r} \bullet \mathbf{v} \right|}$$

where **r** and **v** are the Mars-centered position and velocity vectors, respectively.

Targeting to user-defined B-plane coordinates

For this targeting option, the two nonlinear equality constraints enforced by the SNOPT nonlinear programming algorithm are

$$(\mathbf{B} \bullet \mathbf{T})_p - (\mathbf{B} \bullet \mathbf{T})_u = 0$$

$$(\mathbf{B} \bullet \mathbf{R})_p - (\mathbf{B} \bullet \mathbf{R})_u = 0$$

where the *p* subscript refers to coordinates predicted by the software and the *u* subscript denotes coordinates provided by the user. The *predicted* B-plane coordinates are based on the Mars-centered flight conditions at closest approach.

Targeting to user-defined entry interface (EI) conditions

For this targeting option, the following equations can be used to determine the required B-plane components based on the user-defined EI targets which consist of the inertial flight path angle and altitude relative to a spherical Mars model.

$$\mathbf{B} \bullet \mathbf{T} = b_t \cos \theta$$

$$\mathbf{B} \bullet \mathbf{R} = b_t \sin \theta$$

where

$$b_t = \cos \gamma_{ei} \sqrt{\frac{2 \mu r_{ei}}{v_\infty^2} + r_{ei}^2}$$

In these equations, γ_{ei} is the user-defined flight path angle at the entry interface, and r_{ei} is the Mars-centered radius at the entry interface (sum of Mars equatorial radius plus user-defined EI altitude).

Entry interface at Mars is determined during the numerical integration of the spacecraft's heliocentric equations of motion by finding the time at which the difference between the *predicted* Mars-centered flight path angle and the user-defined *inertial* EI flight path angle is zero. This mission constraint is enforced as follows

$$\sin^{-1}\left(\frac{\mathbf{r}\bullet\mathbf{v}}{\left|\mathbf{r}\bullet\mathbf{v}\right|}\right)-\gamma_t=0$$

where **r** and **v** are the Mars-centered position and velocity vectors, respectively, and γ_t is the user-defined EI flight path angle. The event prediction capability of the MATLAB `ode45` algorithm is used to determine the time at which the spacecraft reaches the entry interface condition at Mars.

The following is the MATLAB source code that determines the time and trajectory conditions at closest approach at Mars.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% solve for closest approach conditions
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% set up for ode45

options = odeset('RelTol', 1.0e-10, 'AbsTol', 1.0e-10, 'Events', @e2m_fpa_event);

% define maximum search time (days)

tof = 500.0;

[t, ysol, tevent, yevent, ie] = ode45(@e2m_eqm2, [0 tof], [rsc_soi vsc_soi], options);

rsc_ca = yevent(1:3);

vsc_ca = yevent(4:6);

% julian date at closest approach

jdtdb_ca = jdtdb_soi + tevent;
```

The following is the MATLAB source code which calculates the flight path angle relative to Mars.

```
function [value, isterminal, direction] = e2m_fpa_event(t, y)

% flight path angle event function

% input

%  t = time since soi (days)
%  y = spacecraft heliocentric state vector (au, au/day)

% output

%  value = mars-centered flight path angle (radians)

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
global jdtdb_soi fpa_tar

% mars heliocentric state vector at current time t

jdate = jdtdb_soi + t;

svmars = jplephem(jdate, 4, 11);

rmars = svmars(1:3);

vmars = svmars(4:6);

% form the mars-centered spacecraft position and velocity vectors

rm2sc = y(1:3) - rmars(1:3);

vm2sc = y(4:6) - vmars(1:3);

tmatrix = mme2000(jdate);

rsc = tmatrix * rm2sc;

vsc = tmatrix * vm2sc;

% flight path angle (radians)

fpa = dot(rsc', vsc) / (norm(rsc) * norm(vsc));

value = fpa - fpa_tar;

isterminal = 1;

direction =  [];
```

Targeting to a Mars-centered grazing flyby

The general expression for the periapsis radius of an encounter hyperbola at Mars is given by

$$\tilde{r}_p = \frac{1}{\tilde{v}_\infty^2}\left(\sqrt{1+\tilde{b}_\infty^2\tilde{v}_\infty^4} - 1\right)$$

where the *normalized* quantities are

$$\begin{aligned}
\tilde{r}_p &= \text{ normalized periapsis radius} = r_p / r_m \\
\tilde{b}_\infty &= \text{ normalized b-plane magnitude} = b_\infty / r_m \\
\tilde{v}_\infty &= \text{ normalized v-infinity speed} = v_\infty / v_{lc} \\
v_{lc} &= \text{ local circular speed at Mars} = \sqrt{\mu_m / r_m} \\
r_m &= \text{radius of Mars} \\
\mu_m &= \text{gravitational constant of Mars}
\end{aligned}$$

For a grazing flyby, $\tilde{r}_p = 1$ and the normalized B-plane distance is equal to

$$\tilde{b}_\infty = \sqrt{1 + \frac{2}{\tilde{v}_\infty^2}}$$

The required B-plane equality constraints are computed from

$$\mathbf{B} \bullet \mathbf{T} = b_\infty \cos\theta$$

$$\mathbf{B} \bullet \mathbf{R} = b_\infty \sin\theta$$

where θ is the user-defined B-plane angle of the grazing trajectory. Please note that the B-plane angle is measured positive clockwise from the **T** axis of the B-plane coordinate system. The two equality constraints for this program option are simply the difference between the predicted and required $\mathbf{B} \bullet \mathbf{T}$ and $\mathbf{B} \bullet \mathbf{R}$ components.

Geocentric-to-areocentric coordinate transformation

This section describes the transformation of coordinates between the Earth mean equator and equinox of J2000 and areocentric mean equator and IAU node of epoch coordinate systems. This transformation is used to compute the B-plane coordinates at encounter.

A unit vector in the direction of the pole of Mars can be determined from

$$\hat{\mathbf{p}}_{Mars} = \begin{bmatrix} \cos\alpha_p \cos\delta_p \\ \sin\alpha_p \cos\delta_p \\ \sin\delta_p \end{bmatrix}$$

The IAU 2000 right ascension and declination of the pole of Mars in the Earth mean equator and equinox of J2000 (EME2000) coordinate system are given by the following expressions

$$\alpha_p = 317.68143 - 0.1061T$$

$$\delta_p = 52.88650 - 0.0609T$$

where T is the time in Julian centuries given by $T = \left(JD - 2451545.0\right)/36525$ and JD is the TDB Julian Date.

The unit vector in the direction of the *IAU-defined* x-axis is computed from

$$\hat{\mathbf{x}} = \hat{\mathbf{p}}_{J2000} \times \hat{\mathbf{p}}_{Mars}$$

where $\hat{\mathbf{p}}_{J2000} = \left[0\;0\;1\right]^T$ is unit vector in the direction of the pole of the J2000 coordinate system.

The unit vector in the y-axis direction of this coordinate system is

$$\hat{\mathbf{y}} = \hat{\mathbf{p}}_{Mars} \times \hat{\mathbf{x}}$$

Finally, the components of the matrix that transforms coordinates from the EME2000 system to the Mars-centered mean equator and IAU node of epoch system are as follows:

$$\mathbf{M} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{p}}_{Mars} \end{bmatrix}$$

Leap seconds calculation

The difference between International Atomic Time (TAI) and Universal Coordinated Time (UTC) is the number of current leap seconds. International Atomic Time (TAI, Temps Atomique International) is a physical time scale with the unit of the SI (System International) second and derived from a statistical timescale based on a large number of atomic clocks. Coordinated Universal Time (UTC) is the time scale available from broadcast time signals. It is a compromise between the highly stable atomic time and the irregular earth rotation. UTC is the international basis of civil and scientific time.

The calculation of leap seconds in this MATLAB script is performed by a function that reads a simple ASCII data file and evaluates the current value of leap seconds. The leap second function must be initialized by including the following statements in the main script.

```
% read leap seconds data file

readleap;
```

The `readleap` MATLAB function reads the contents of the following simple comma-separated-variable (csv) two column data file. The name of this file is `tai-utc.dat`.

```
2441317.5,  10.0
2441499.5,  11.0
2441683.5,  12.0
2442048.5,  13.0
2442413.5,  14.0
2442778.5,  15.0
2443144.5,  16.0
2443509.5,  17.0
2443874.5,  18.0
2444239.5,  19.0
2444786.5,  20.0
2445151.5,  21.0
2445516.5,  22.0
2446247.5,  23.0
2447161.5,  24.0
2447892.5,  25.0
2448257.5,  26.0
2448804.5,  27.0
2449169.5,  28.0
2449534.5,  29.0
2450083.5,  30.0
2450630.5,  31.0
2451179.5,  32.0
2453736.5,  33.0
2454832.5,  34.0
```

The first column of this data file is the Julian date, on the UTC time scale, at which the leap second became valid. The second column is the leap second value, in seconds.

Note that this data is passed between the leap second MATLAB functions by way of a global statement.

```
global jdateleap leapsec
```

The MATLAB function that actually reads and evaluates the current value of leap seconds has the following syntax and single argument.

```
function leapsecond = findleap(jdate)

% find number of leap seconds for utc julian date

% input

%  jdate = utc julian date

% input via global

%  jdateleap = array of utc julian dates
%  leapsec   = array of leap seconds

% output

%  leapsecond = number of leap seconds
```

Please note that this function does not extrapolate outside the range of dates contained in the data file.

The leap seconds data file should be updated whenever the International Earth Rotation and Reference Systems Service (IERS) announces a new leap second.

SNOPT algorithm implementation

This section provides details about the parts of the MATLAB script that solve these nonlinear programming (NLP) problems using the SNOPT algorithm. In the classic patched-conic trajectory optimization problem, the departure and arrival calendar dates are the *control variables* and the user-specified ΔV is the *objective function* or *performance index*.

MATLAB versions of SNOPT for several computer platforms can be found at Professor Philip Gill's web site which is located at http://scicomp.ucsd.edu/~peg/. Professor Gill's web site also includes a PDF version of the SNOPT software user's guide.

The SNOPT algorithm requires an initial guess for the control variables. For the two-body Lambert problem they are given by

```
xg(1) = jdtdb_tip - jdtdb0;

xg(2) = jdtdb_arrival - jdtdb0;

xg = xg';
```

where `jdtdb_tip` and `jdtdb_arrival` are the initial user-provided departure and arrival date guesses, and `jdtdb0` is a reference Julian Date (on the Barycentric Dynamical Time scale) equal to `2451544.5` (January 1, 2000). This offset value is used to *scale* the control variables.

The algorithm also requires lower and upper bounds for the control variables. These are determined from the initial guesses and user-defined search boundaries as follows:

```
% bounds on control variables

xlwr(1) = xg(1) - ddays1;

xupr(1) = xg(1) + ddays1;

xlwr(2) = xg(2) - ddays2;

xupr(2) = xg(2) + ddays2;

xlwr = xlwr';

xupr = xupr';

xlwr = xlwr';

xupr = xupr';
```

where `ddays1` and `ddays2` are the user-defined departure and arrival search boundaries, respectively.

The algorithm also requires lower and upper bounds on the objective function. For this problem these bounds are given by

```
% bounds on objective function

flow(1) = 0.0d0;

fupp(1) = +Inf;
```

The actual call to the SNOPT MATLAB interface function (`snopt.m`) is as follows

```
[x, f, inform, xmul, fmul] = snopt(xg, xlwr, xupr, flow, fupp, 'e2m_deltav');
```

where `e2m_deltav` is the name of the MATLAB function that solves Lambert's problem and computes the current value of the objective function.

The following is the MATLAB source code for this function.

```
function [f, g] = e2m_deltav (x)

% two-body, patched-conic delta-v objective function

% input

%  x = current values for launch and arrival julian dates wrt reference

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

global aunit smu ip1 ip2 jdtdb0

global otype rito vito dv1 dv2

% current julian dates

jdtdb_tip = x(1) + jdtdb0;

jdtdb_arrival = x(2) + jdtdb0;
```

```
% time-of-flight

taud = jdtdb_arrival - jdtdb_tip;

tof = taud * 86400.0;

% compute initial state vector

svi = jplephem(jdtdb_tip, ip1, 11);

ri = aunit * svi(1:3);

vi = aunit * svi(4:6) / 86400.0;

% compute final state vector

svf = jplephem(jdtdb_arrival, ip2, 11);

rf = aunit * svf(1:3);

vf = aunit * svf(4:6) / 86400.0;

% solve Lambert's problem

revmax = 0;

sv1(1:3) = ri;

sv1(4:6) = vi;

sv2(1:3) = rf;

sv2(4:6) = vf;

[vito, vfto] = glambert(smu, sv1, sv2, tof, revmax);

rito = ri;

% calculate departure delta-v

dv1(1) = vito(1) - vi(1);
dv1(2) = vito(2) - vi(2);
dv1(3) = vito(3) - vi(3);

dvm1 = norm(dv1);

% propagate transfer orbit

[r2, v2] = twobody2 (smu, tof, ri, vito);

% calculate arrival delta-v

dv2(1) = vf(1) - v2(1);
dv2(2) = vf(2) - v2(2);
dv2(3) = vf(3) - v2(3);

dvm2 = norm(dv2);


% load scalar objective function

switch otype

case 1

   % launch
```

```
    f(1) = dvm1;

case 2

    % arrival

    f(1) = dvm2;

case 3

    % launch + arrival

    f(1) = dvm1 + dvm2;

case 4

    f(1) = dvm1 + dvm2;

end

% no derivatives

g = [];
```

The following is the MATLAB source code snippet for the initialization of the n-body optimization algorithm. The control variables for this part of the computations are the v-infinity magnitude, and the right ascension (RLA) and declination (DLA) of the outgoing asymptote. The objective function for these calculations is the scalar magnitude of the departure v-infinity. The initial guess vector `xg` uses the values of v-infinity, RLA and DLA computed by the patched-conic Lambert solution.

The bounds for v-infinity are in units of kilometers per second and the bounds for RLA and DLA are in units of radians. The user can edit these bounds at this point in the source code.

```
% define lower and upper bounds for vinf, rla and dla

xlwr(1) = xg(1) - 0.05;

xupr(1) = xg(1) + 0.05;

xlwr(2) = xg(2) - 10.0 * dtr;

xupr(2) = xg(2) + 10.0 * dtr;

xlwr(3) = xg(3) - 1.0 * dtr;

xupr(3) = xg(3) + 1.0 * dtr;

if (otype == 4)

    % transpose bounds

    xlwr = xlwr';

    xupr = xupr';

end

% bounds on objective function

flow(1) = 0.0d0;
```

```
fupp(1) = +Inf;

% bounds on final b-plane/orbital element equality constraints

flow(2) = 0.0d0;

fupp(2) = 0.0d0;

flow(3) = 0.0d0;

fupp(3) = 0.0d0;

flow = flow';

fupp = fupp';
```

The actual call to the SNOPT MATLAB interface function for this part of the script is as follows

```
[x, f, inform, xmul, fmul] = snopt(xg, xlwr, xupr, flow, fupp, 'e2m_shoot');
```

where `e2m_shoot` is the name of the MATLAB function that implements the simple shooting method used to compute the time and flight characteristics at closest approach to Mars.

Time scales

This section is a brief explanation of the time scales used in this MATLAB script.

Coordinated Universal Time, UTC

Coordinated Universal Time (UTC) is the time scale available from broadcast time signals. It is a compromise between the highly stable atomic time and the irregular earth rotation. UTC is the international basis of civil and scientific time.

Terrestrial Time, TT

Terrestrial Time is the time scale that would be kept by an ideal clock on the geoid - approximately, sea level on the surface of the Earth. Since its unit of time is the SI (atomic) second, TT is independent of the variable rotation of the Earth. TT is meant to be a smooth and continuous "coordinate" time scale independent of Earth rotation. In practice TT is derived from International Atomic Time (TAI), a time scale kept by real clocks on the Earth's surface, by the relation **TT = TAI + $32^s.184$.** It is the time scale now used for the precise calculation of future astronomical events observable from Earth.

TT = TAI + 32.184 seconds

TT = UTC + (number of leap seconds) + 32.184 seconds

Barycentric Dynamical Time, TDB

Barycentric Dynamical Time is the time scale that would be kept by an ideal clock, free of gravitational fields, co-moving with the solar system barycenter. It is always within 2 milliseconds of TT, the difference caused by relativistic effects. TDB is the time scale now used for investigations of the dynamics of solar system bodies.

TDB = TT + periodic corrections

where typical periodic corrections (USNO Circular 179) are

$$\begin{aligned}TDB = TT &+ 0.001657\sin\left(628.3076T + 6.2401\right)\\ &+ 0.000022\sin\left(575.3385T + 4.2970\right)\\ &+ 0.000014\sin\left(1256.6152T + 6.1969\right)\\ &+ 0.000005\sin\left(606.9777T + 4.0212\right)\\ &+ 0.000005\sin\left(52.9691T + 0.4444\right)\\ &+ 0.000002\sin\left(21.3299T + 5.5431\right)\\ &+ 0.000010T\sin\left(628.3076T + 4.2490\right) + \cdots\end{aligned}$$

In this equation, the coefficients are in seconds, the angular arguments are in radians, and T is the number of Julian centuries of TT from J2000; T = (Julian Date(TT) – 2451545.0) / 36525.

The following is the MATLAB source code for a function that computes a Julian Date on the TDB scale from a Julian Date of the UTC time scale.

```
function jdtdb = utc2tdb (jdutc, tai_utc)

% convert UTC julian date to TDB julian date

% input

%  jdutc   = UTC julian date
%  tai_utc = TAI-UTC (seconds)

% output

%  jdtdb = TDB julian date

% Reference Frames in Astronomy and Geophysics
% J. Kovalevsky et al., 1989, pp. 439-442

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

dtr = pi / 180.0;

% TDT julian date

corr = (tai_utc + 32.184) / 86400.0;

jdtdt = jdutc + corr;

% time argument for correction

t = (jdtdt - 2451545.0) / 36525.0;

% compute correction in microseconds

corr = 1656.675     * sin(dtr * (35999.3729 * t + 357.5287))...
       + 22.418     * sin(dtr * (32964.467  * t + 246.199))...
       + 13.84      * sin(dtr * (71998.746  * t + 355.057))...
       +  4.77      * sin(dtr * ( 3034.906  * t +  25.463))...
       +  4.677     * sin(dtr * (34777.259  * t + 230.394))...
       + 10.216 * t * sin(dtr * (35999.373  * t + 243.451))...
       +  0.171 * t * sin(dtr * (71998.746  * t + 240.98 ))...
```

```
       +  0.027 * t * sin(dtr * ( 1222.114  * t + 194.661))...
       +  0.027 * t * sin(dtr * ( 3034.906  * t + 336.061))...
       +  0.026 * t * sin(dtr * (  -20.186  * t +   9.382))...
       +  0.007 * t * sin(dtr * (29929.562  * t + 264.911))...
       +  0.006 * t * sin(dtr * (  150.678  * t +  59.775))...
       +  0.005 * t * sin(dtr * ( 9037.513  * t + 256.025))...
       +  0.043 * t * sin(dtr * (35999.373  * t + 151.121));

% convert corrections to days

corr = 0.000001 * corr / 86400.0;

% TDB julian date

jdtdb = jdtdt + corr;
```

Algorithm resources

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"IERS Conventions (2003)", IERS Technical Note 32, November 2003.

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"Interplanetary Mission Design Handbook, Volume 1, Part 2", JPL Publication 82-43, September 15, 1983.

"User's Guide for SNOPT Version 6, A Fortran Package for Large-Scale Nonlinear Programming", Philip E. Gill, Walter Murray and Michael A. Saunders, December 2002.

"User's Guide for SNOPT Version 7, A Fortran Package for Large-Scale Nonlinear Programming", Philip E. Gill, Walter Murray and Michael A. Saunders, March 20, 2006.

APPENDIX A

Contents of the Simulation Summary

This appendix is a brief summary of the information contained in the simulation summary screen displays produced by the `e2m_matlab` software.

The simulation summary screen display contains the following information:

UTC calendar date = UTC calendar date of trajectory event

UTC time = UTC time of trajectory event

UTC Julian Date = Julian Date of trajectory event on UTC time scale

sma (km) = semimajor axis in kilometers

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of periapsis in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (days) = orbital period in days

rx (km) = x-component of the spacecraft's position vector in kilometers

ry (km) = y-component of the spacecraft's position vector in kilometers

rz (km) = z-component of the spacecraft's position vector in kilometers

rmag (km) = scalar magnitude of the spacecraft's position vector in kilometers

vx (kps) = x-component of the spacecraft's velocity vector in kilometers per second

vy (kps) = y-component of the spacecraft's velocity vector in kilometers per second

vz (ksp) = z-component of the spacecraft's velocity vector in kilometers per second

vmag (kps) = scalar magnitude of the spacecraft's velocity vector in kilometers per second

b-magnitude = magnitude of the b-plane vector in kilometers

b dot r = dot product of the b-vector and r-vector in kilometers

b dot t = dot product of the b-vector and t-vector in kilometers

b-plane angle = orientation of the b-plane vector in degrees

v-infinity = magnitude of outgoing or incoming v-infinity vector in kilometers/second

r-periapsis = periapsis radius of incoming hyperbola in kilometers

decl-asymptote = declination of incoming v-infinity vector in degrees

rasc-asymptote = right ascension of incoming v-infinity vector in degrees

flight path angle = flight path angle in degrees

APPENDIX B

Entry Interface Example

This appendix summarizes the trajectory characteristics of a typical entry interface simulation. The following is the input data file (`mars_ei.in`) for this example.

```
*****************************************
** interplanetary trajectory optimization
** script ==> e2m_matlab.m
** Mars EI mars_ei.in
** December 26, 2012
*****************************************

*******************
* simulation type *
*******************
 1 = minimize departure delta-v
 2 = minimize arrival delta-v
 3 = minimize total delta-v
 4 = no optimization
--------------------
1

departure calendar date initial guess (month, day, year)
6,1,2003

departure date search boundary (days)
30

arrival calendar date initial guess (month, day, year)
12,1,2003

arrival date search boundary (days)
30

***************************
* geocentric phase modeling
***************************

perigee altitude of launch hyperbola (kilometers)
185.32

launch azimuth (degrees)
93.0

launch site latitude (degrees)
28.5

****************************
* encounter planet targeting
****************************

type of targeting
(1 = B-plane, 2 = orbital elements, 3 = EI conditions, 4 = grazing flyby)
3

B dot T
10965.197268

B dot R
-6109.036804

radius of closest approach (kilometers)
5000.0

orbital inclination (degrees)
60.0

EI flight path angle (degrees)
-2.0
```

```
EI altitude (kilometers)
100.0

user-defined b-plane angle (degrees)
-60.0
```

The following is the two-body and n-body optimal solution for this example.

```
 Nonlinear constraints       0     Linear constraints       1
 Nonlinear variables         2     Linear variables         0
 Jacobian  variables         0     Objective variables      2
 Total constraints           1     Total variables          2


 The user has defined       0   out of       2   first  derivatives

 Major Minors     Step   nObj Feasible  Optimal      Objective    nS
     0      2              1           3.2E-03  3.0416787E+00     2   r
     1      1  7.1E-01     2           2.7E-03  3.0338679E+00     2 n rl
     2      1  7.1E-01     3           2.6E-03  3.0271837E+00     2 s  l
     3      1  1.0E+00     4           1.2E-03  2.9746321E+00     2
     4      1  1.0E+00     5           3.0E-04  2.9644391E+00     2
     5      1  1.0E+00     6           4.1E-05  2.9643144E+00     2
     6      1  1.0E+00     7           6.4E-06  2.9643112E+00     2
     7      1  1.0E+00     8          (1.7E-08) 2.9643112E+00     2       c
     7      2  1.0E+00     8          (3.3E-07) 2.9643112E+00     2       c

 SNOPTA EXIT   0 -- finished successfully
 SNOPTA INFO   1 -- optimality conditions satisfied
 Problem name
 No. of iterations                  10   Objective value      2.9643111870E+00
 No. of major iterations             7   Linear objective     0.0000000000E+00
 Penalty parameter           0.000E+00   Nonlinear objective  2.9643111870E+00
 No. of calls to funobj             37   No. of calls to funcon             37
 Calls with modes 1,2 (known g)      8   Calls with modes 1,2 (known g)      8
 Calls for forward differencing     16   Calls for forward differencing     16
 Calls for central differencing      4   Calls for central differencing      4
 No. of superbasics                  2   No. of basic nonlinears             0
 No. of degenerate steps             0   Percentage                        .00
 Max x                       2 1.5E+03   Max pi                      1 1.0E+00
 Max Primal infeas           0 0.0E+00   Max Dual infeas             1 6.5E-07


 Solution printed on file   9

Earth-to-Mars mission design

=========================
two-body Lambert solution
=========================

minimize departure delta-v

departure heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       2895.912618  meters/second
y-component of delta-v       -530.389043  meters/second
z-component of delta-v       -345.714312  meters/second

delta-v magnitude            2964.311187  meters/second


arrival heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v      -2063.021184  meters/second
y-component of delta-v       1164.270845  meters/second
z-component of delta-v       1311.949617  meters/second

delta-v magnitude            2707.913367  meters/second
```

```
heliocentric coordinates of the Earth at departure
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11507361

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.4965147327e+008  1.6237346597e-002  2.3439054671e+001  1.0245240440e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 7.2430848045e-004  1.5204742994e+002  2.5449983434e+002  3.6545322928e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -4.05626080325426e+007  -1.34199491174615e+008  -5.81817199031513e+007  +1.51789133777145e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +2.82279246201946e+001  -7.39786255837625e+000  -3.20748439559914e+000  +2.93569762568291e+001


spacecraft heliocentric coordinates after the first impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11507361

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8838714747e+008  1.9427720613e-001  2.3490037882e+001  2.5349091879e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.5596572332e-001  5.9131918561e-001  2.5408223798e+002  5.1616340901e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -4.05626080325426e+007  -1.34199491174615e+008  -5.81817199031513e+007  +1.51789133777145e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.11238372380865e+001  -7.92825160148060e+000  -3.55319870788600e+000  +3.23137066728192e+001


spacecraft heliocentric coordinates prior to the second impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003

UTC time             15:22:06.692

UTC Julian Date      2452998.14035523

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8838714747e+008  1.9427720613e-001  2.3490037882e+001  2.5349091879e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.5596572332e-001  1.5290995806e+002  4.6400876854e+001  5.1616340901e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +1.49990801462616e+008  +1.46776341464580e+008  +6.32690486133341e+007  +2.19188292227393e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 -1.46793406657452e+001  +1.56263833645881e+001  +6.84186935963794e+000  +2.25050677797786e+001


spacecraft heliocentric coordinates after the second impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003
```

```
UTC time             15:22:06.692

UTC Julian Date      2452998.14035523

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8838714747e+008  1.9427720613e-001  2.3490037882e+001  2.5349091879e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.5596572332e-001  1.5290995806e+002  4.6400876854e+001  5.1616340901e+002

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +1.49990801462616e+008  +1.46776341464580e+008  +6.32690486133341e+007  +2.19188292227393e+008

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 -1.67423618500392e+001  +1.67906542098226e+001  +8.15381897618937e+000  +2.50742400285223e+001


heliocentric coordinates of Mars at arrival
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003

UTC time             15:22:06.692

UTC Julian Date      2452998.14035523

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 2.2793930707e+008  9.3541889964e-002  2.4677224952e+001  3.3297923712e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 3.3716583265e+000  7.0759517389e+001  4.3738754512e+001  6.8697217107e+002

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +1.49990801462616e+008  +1.46776341464580e+008  +6.32690486133341e+007  +2.19188292227394e+008

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 -1.67423618500392e+001  +1.67906542098226e+001  +8.15381897618937e+000  +2.50742400285223e+001


park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

park orbit
----------

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 6.5634578000e+003  0.0000000000e+000  2.8644284856e+001  0.0000000000e+000

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 2.0356396962e+000  1.9503978483e+002  1.9503978483e+002  6.1248616627e-002

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.28153849210695e+003  -1.71891801652489e+003  -8.16439414179488e+002  +6.56345780000000e+003

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +2.25553181463621e+000  -6.52893260021440e+000  -3.60775090896881e+000  +7.79296165049872e+000


departure hyperbola
-------------------

c3                             8.787141  km^2/sec^2

v-infinity                  2964.311187  meters/second

asymptote right ascension     349.621254  degrees

asymptote declination          -6.697391  degrees

perigee altitude              185.320000  kilometers

launch azimuth                 93.000000  degrees
```

```
launch site latitude           28.500000  degrees


UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11507361

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -4.5361790595e+004  1.1446913297e+000  2.8644284856e+001  1.9503978483e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 2.0356396962e+000  3.6000000000e+002  1.9503978483e+002

        rx (km)                ry (km)               rz (km)               rmag (km)
 -6.28153849210695e+003  -1.71891801652488e+003  -8.16439414179486e+002  +6.56345780000000e+003

        vx (kps)               vy (kps)              vz (kps)              vmag (kps)
 +3.30317326874917e+000  -9.56146816398225e+000  -5.28346631401135e+000  +1.14126089648913e+001


hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1047.641454  meters/sec
y-component of delta-v      -3032.535564  meters/sec
z-component of delta-v      -1675.715405  meters/sec

delta-v magnitude            3619.647314  meters/sec
```

please wait, solving b-plane targeting problem ...

```
 Nonlinear constraints       2     Linear constraints       1
 Nonlinear variables         3     Linear variables         0
 Jacobian  variables         3     Objective variables      0
 Total constraints           3     Total variables          3


 The user has defined       0   out of       6   first  derivatives

 Major Minors     Step   nCon Feasible  Optimal  MeritFunction    nS Penalty
     0      1               1  6.8E+00  6.8E-04  2.9643112E+00                r
     1      0  1.9E-01      2  5.5E+00  6.2E-05  2.9642660E+00              n r
     2      1  7.8E-03      3  5.4E+00  1.8E-03  2.9645829E+00     1 5.3E-07 s
     3      1  2.2E-01      4  4.2E+00  9.1E-04  2.9646732E+00     1 7.0E-07
     4      1  2.0E-01      5  3.4E+00  1.3E-03  2.9647009E+00     1 8.0E-07
     5      1  3.0E-01      6  2.4E+00  1.1E-03  2.9647252E+00     1 9.3E-07
     6      1  4.5E-01      7  1.3E+00  8.5E-04  2.9647400E+00     1 1.1E-06
     7      1  4.9E-01      8  6.6E-01  2.5E-04  2.9647451E+00     1 1.3E-06
     8      1  4.0E-01      9  4.0E-01  3.0E-04  2.9647457E+00     1 1.4E-06
     9      1  1.0E+00     10  6.9E-04  3.5E-05  2.9647461E+00     1 1.6E-06
 Major Minors     Step   nCon Feasible  Optimal  MeritFunction    nS Penalty
    10      1  1.0E+00     11  9.3E-06  4.6E-06  2.9647461E+00     1 1.6E-06
    11      1  4.4E-01     12  5.3E-06 (4.4E-07) 2.9647461E+00     1 1.6E-06       c
    11      2  4.4E-01     12  5.3E-06 (1.2E-07) 2.9647461E+00     1 1.6E-06       c
    12      1  1.0E+00     13 (1.2E-08)(3.6E-07) 2.9647461E+00     1 1.6E-06       c

 SNOPTA EXIT   0 -- finished successfully
 SNOPTA INFO   1 -- optimality conditions satisfied
 Problem name
 No. of iterations                  13   Objective value      2.9647460654E+00
 No. of major iterations            12   Linear objective     2.9647460654E+00
 Penalty parameter           1.600E-06   Nonlinear objective  0.0000000000E+00
 No. of calls to funobj             87   No. of calls to funcon             87
 Calls with modes 1,2 (known g)     13   Calls with modes 1,2 (known g)     13
 Calls for forward differencing     36   Calls for forward differencing     36
 Calls for central differencing     12   Calls for central differencing     12
 No. of superbasics                  1   No. of basic nonlinears             2
 No. of degenerate steps             0   Percentage                        .00
 Max x                       2 6.1E+00   Max pi                      3 1.0E+00
 Max Primal infeas           0 0.0E+00   Max Dual infeas             2 4.0E-06
 Nonlinear constraint violn    7.7E-08
```

```
=======================
optimal n-body solution
=======================

EI conditions targeting

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

park orbit
----------

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 6.5634578000e+003  0.0000000000e+000  2.8644284856e+001  0.0000000000e+000

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 2.6903592783e+000  1.9473189137e+002  1.9473189137e+002  6.1248616627e-002

        rx (km)                ry (km)               rz (km)               rmag (km)
 -6.27194162914130e+003  -1.76112681254354e+003  -8.00099270819202e+002  +6.56345780000000e+003

        vx (kps)               vy (kps)              vz (kps)              vmag (kps)
 +2.29000490920140e+000  -6.51406359991660e+000  -3.61290800024285e+000  +7.79296165049872e+000


departure hyperbola
-------------------

c3                              8.789719  km^2/sec^2

v-infinity                   2964.746065  meters/second

asymptote right ascension     349.998560  degrees

asymptote declination          -6.843242  degrees


UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11581649

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -4.5348483948e+004  1.1447337866e+000  2.8644284856e+001  1.9473189137e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 2.6903592783e+000  2.5444437452e-014  1.9473189137e+002

        rx (km)                ry (km)               rz (km)               rmag (km)
 -6.27194162914130e+003  -1.76112681254355e+003  -8.00099270819205e+002  +6.56345780000000e+003

        vx (kps)               vy (kps)              vz (kps)              vmag (kps)
 +3.35369149938573e+000  -9.53978728766862e+000  -5.29107112381805e+000  +1.14127219279328e+001


hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1063.686590  meters/sec
y-component of delta-v      -3025.723688  meters/sec
z-component of delta-v      -1678.163124  meters/sec

delta-v magnitude            3619.760277  meters/sec

transfer time                 201.338062  days

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------
UTC calendar date    23-Dec-2003

UTC time             22:52:30.924
```

```
UTC Julian Date      2452997.45313570

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -5.8453821649e+003  1.5975209005e+000  6.0000002602e+001  1.2003329012e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 1.0572928214e+002  3.5674756364e+002  1.1678085376e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -1.07509790852788e+003  -1.93939042764115e+003  +2.70302681879439e+003  +3.49620147114963e+003

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +2.64769020371662e+000  -4.39126585922395e+000  -2.35230563702043e+000  +5.64152648413538e+000


B-plane coordinates at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

b-magnitude                 7282.307041  kilometers
b dot r                    -6288.198385  kilometers
b dot t                     3672.949349  kilometers
b-plane angle                300.289287  degrees
v-infinity                  2706.819830  meters/second
r-periapsis                 3492.738015  kilometers
decl-asymptote                 7.544509  degrees
rasc-asymptote               281.343827  degrees

flight path angle             -2.000401  degrees

spacecraft heliocentric coordinates at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003

UTC time             22:52:30.924

UTC Julian Date      2452997.45313570

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8526300159e+008  2.3907269141e-001  1.8140235173e+001  2.7906205531e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 3.3805648364e+002  1.4794457995e+002  6.7006635260e+001  5.0337698241e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +1.50983176184419e+008  +1.45773193835115e+008  +6.27844916988370e+007  +2.19060804183466e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 -1.33074334383538e+001  +1.74558460362384e+001  +3.67543723023642e+000  +2.22554079807692e+001


heliocentric coordinates of Mars at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003

UTC time             22:52:30.924

UTC Julian Date      2452997.45313570

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 2.2793935930e+008  9.3542130829e-002  2.4677224881e+001  3.3297930824e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 3.3716581647e+000  7.0371469509e+001  4.3350777749e+001  6.8697240720e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +1.50981550662964e+008  +1.45776128214147e+008  +6.27835065468567e+007  +2.19061354172798e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 -1.66296550460305e+001  +1.69002108160439e+001  +8.20102384003258e+000  +2.50882311971466e+001
```

```
spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003

UTC time             18:20:24.394

UTC Julian Date      2452799.26417123

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -4.5600973241e+004  1.1432795490e+000  2.8507275422e+001  1.9469053155e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 2.7031447139e+000  1.4947809276e+002  3.4416862431e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +8.99381442533899e+005  -1.79535006188754e+005  -1.20437545551405e+005  +9.25000000000000e+005

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.03079389733870e+000  -5.32010514078975e-001  -3.66255579015599e-001  +3.09885301109881e+000


spacecraft heliocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003

UTC time             18:20:24.394

UTC Julian Date      2452799.26417123

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.9070386464e+008  2.0396404968e-001  2.3500187727e+001  2.5359028106e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 5.2785475026e-001  3.7880289553e+000  2.5737831002e+002  5.2571400900e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -3.19306339534222e+007  -1.36202027271382e+008  -5.90924359334799e+007  +1.51863338571408e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.16291214353697e+001  -6.53257433106946e+000  -2.96704963878467e+000  +3.24326877351126e+001
```

APPENDIX C

Grazing Flyby Example

This appendix summarizes the trajectory characteristics of a typical grazing flyby simulation. The following is the input data file (`mars_graze.in`) for this example.

```
*****************************************
** interplanetary trajectory optimization
** script ==> e2m_matlab.m
** Mars grazing flyby mars_graze.in
** December 27, 2012
*****************************************

*******************
* simulation type *
*******************
 1 = minimize departure delta-v
 2 = minimize arrival delta-v
 3 = minimize total delta-v
 4 = no optimization
--------------------
1

departure calendar date initial guess (month, day, year)
6,1,2003

departure date search boundary (days)
30

arrival calendar date initial guess (month, day, year)
12,1,2003

arrival date search boundary (days)
30

***************************
* geocentric phase modeling
***************************

perigee altitude of launch hyperbola (kilometers)
185.32

launch azimuth (degrees)
93.0

launch site latitude (degrees)
28.5

****************************
* encounter planet targeting
****************************

type of targeting
(1 = B-plane, 2 = orbital elements, 3 = EI conditions, 4 = grazing flyby)
4

B dot T
10965.197268

B dot R
-6109.036804

radius of closest approach (kilometers)
5000.0

orbital inclination (degrees)
60.0

EI flight path angle (degrees)
-2.0
```

```
EI altitude (kilometers)
100.0

user-defined b-plane angle (degrees)
-60.0
```

The following is the two-body and n-body optimal solution for this example.

```
 Nonlinear constraints       0     Linear constraints       1
 Nonlinear variables         2     Linear variables         0
 Jacobian  variables         0     Objective variables      2
 Total constraints           1     Total variables          2


 The user has defined       0   out of       2   first  derivatives

 Major Minors     Step   nObj Feasible  Optimal      Objective    nS
     0      2              1           3.2E-03  3.0416787E+00     2   r
     1      1  7.1E-01      2           2.7E-03  3.0338679E+00     2 n rl
     2      1  7.1E-01      3           2.6E-03  3.0271837E+00     2 s  l
     3      1  1.0E+00      4           1.2E-03  2.9746321E+00     2
     4      1  1.0E+00      5           3.0E-04  2.9644391E+00     2
     5      1  1.0E+00      6           4.1E-05  2.9643144E+00     2
     6      1  1.0E+00      7           6.4E-06  2.9643112E+00     2
     7      1  1.0E+00      8          (1.7E-08) 2.9643112E+00     2       c
     7      2  1.0E+00      8          (3.3E-07) 2.9643112E+00     2       c

 SNOPTA EXIT   0 -- finished successfully
 SNOPTA INFO   1 -- optimality conditions satisfied
 Problem name
 No. of iterations                  10   Objective value      2.9643111870E+00
 No. of major iterations             7   Linear objective     0.0000000000E+00
 Penalty parameter           0.000E+00   Nonlinear objective  2.9643111870E+00
 No. of calls to funobj             37   No. of calls to funcon             37
 Calls with modes 1,2 (known g)      8   Calls with modes 1,2 (known g)      8
 Calls for forward differencing     16   Calls for forward differencing     16
 Calls for central differencing      4   Calls for central differencing      4
 No. of superbasics                  2   No. of basic nonlinears             0
 No. of degenerate steps             0   Percentage                        .00
 Max x                       2 1.5E+03   Max pi                      1 1.0E+00
 Max Primal infeas           0 0.0E+00   Max Dual infeas             1 6.5E-07


 Solution printed on file   9

Earth-to-Mars mission design

=========================
two-body Lambert solution
=========================

minimize departure delta-v

departure heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       2895.912618  meters/second
y-component of delta-v       -530.389043  meters/second
z-component of delta-v       -345.714312  meters/second

delta-v magnitude            2964.311187  meters/second


arrival heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v      -2063.021184  meters/second
y-component of delta-v       1164.270845  meters/second
z-component of delta-v       1311.949617  meters/second

delta-v magnitude            2707.913367  meters/second
```

```
heliocentric coordinates of the Earth at departure
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11507361

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.4965147327e+008  1.6237346597e-002  2.3439054671e+001  1.0245240440e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 7.2430848045e-004  1.5204742994e+002  2.5449983434e+002  3.6545322928e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -4.05626080325426e+007  -1.34199491174615e+008  -5.81817199031513e+007  +1.51789133777145e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +2.82279246201946e+001  -7.39786255837625e+000  -3.20748439559914e+000  +2.93569762568291e+001


spacecraft heliocentric coordinates after the first impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11507361

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8838714747e+008  1.9427720613e-001  2.3490037882e+001  2.5349091879e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.5596572332e-001  5.9131918561e-001  2.5408223798e+002  5.1616340901e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -4.05626080325426e+007  -1.34199491174615e+008  -5.81817199031513e+007  +1.51789133777145e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.11238372380865e+001  -7.92825160148060e+000  -3.55319870788600e+000  +3.23137066728192e+001


spacecraft heliocentric coordinates prior to the second impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003

UTC time             15:22:06.692

UTC Julian Date      2452998.14035523

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8838714747e+008  1.9427720613e-001  2.3490037882e+001  2.5349091879e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.5596572332e-001  1.5290995806e+002  4.6400876854e+001  5.1616340901e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +1.49990801462616e+008  +1.46776341464580e+008  +6.32690486133341e+007  +2.19188292227393e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 -1.46793406657452e+001  +1.56263833645881e+001  +6.84186935963794e+000  +2.25050677797786e+001


spacecraft heliocentric coordinates after the second impulse
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003
```

```
UTC time             15:22:06.692

UTC Julian Date      2452998.14035523

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8838714747e+008  1.9427720613e-001  2.3490037882e+001  2.5349091879e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.5596572332e-001  1.5290995806e+002  4.6400876854e+001  5.1616340901e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +1.49990801462616e+008  +1.46776341464580e+008  +6.32690486133341e+007  +2.19188292227393e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 -1.67423618500392e+001  +1.67906542098226e+001  +8.15381897618937e+000  +2.50742400285223e+001


heliocentric coordinates of Mars at arrival
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    24-Dec-2003

UTC time             15:22:06.692

UTC Julian Date      2452998.14035523

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 2.2793930707e+008  9.3541889964e-002  2.4677224952e+001  3.3297923712e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 3.3716583265e+000  7.0759517389e+001  4.3738754512e+001  6.8697217107e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 +1.49990801462616e+008  +1.46776341464580e+008  +6.32690486133341e+007  +2.19188292227394e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 -1.67423618500392e+001  +1.67906542098226e+001  +8.15381897618937e+000  +2.50742400285223e+001


park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

park orbit
----------

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 6.5634578000e+003  0.0000000000e+000  2.8644284856e+001  0.0000000000e+000

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 2.0356396962e+000  1.9503978483e+002  1.9503978483e+002  6.1248616627e-002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -6.28153849210695e+003  -1.71891801652489e+003  -8.16439414179488e+002  +6.56345780000000e+003

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +2.25553181463621e+000  -6.52893260021440e+000  -3.60775090896881e+000  +7.79296165049872e+000


departure hyperbola
-------------------

c3                              8.787141  km^2/sec^2

v-infinity                   2964.311187  meters/second

asymptote right ascension     349.621254  degrees

asymptote declination          -6.697391  degrees

perigee altitude              185.320000  kilometers

launch azimuth                 93.000000  degrees
```

```
launch site latitude           28.500000  degrees


UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11507361

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -4.5361790595e+004  1.1446913297e+000  2.8644284856e+001  1.9503978483e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 2.0356396962e+000  3.6000000000e+002  1.9503978483e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -6.28153849210695e+003  -1.71891801652488e+003  -8.16439414179486e+002  +6.56345780000000e+003

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.30317326874917e+000  -9.56146816398225e+000  -5.28346631401135e+000  +1.14126089648913e+001


hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1047.641454  meters/sec
y-component of delta-v      -3032.535564  meters/sec
z-component of delta-v      -1675.715405  meters/sec

delta-v magnitude            3619.647314  meters/sec
```

please wait, solving b-plane targeting problem ...

```
 Nonlinear constraints       2     Linear constraints       1
 Nonlinear variables         3     Linear variables         0
 Jacobian  variables         3     Objective variables      0
 Total constraints           3     Total variables          3


 The user has defined       0   out of       6   first  derivatives

 Major Minors     Step   nCon Feasible  Optimal  MeritFunction    nS Penalty
     0      1               1  6.8E+00  6.8E-04  2.9643112E+00                 r
     1      0  1.9E-01      2  5.5E+00  6.1E-05  2.9642679E+00               n r
     2      1  7.7E-03      3  5.5E+00  1.8E-03  2.9645800E+00     1 5.2E-07 s
     3      1  2.1E-01      4  4.3E+00  8.3E-04  2.9646753E+00     1 7.0E-07
     4      1  1.7E-01      5  3.6E+00  1.3E-03  2.9647007E+00     1 7.8E-07
     5      1  2.8E-01      6  2.6E+00  1.1E-03  2.9647264E+00     1 9.0E-07
     6      1  4.1E-01      7  1.5E+00  9.5E-04  2.9647430E+00     1 1.1E-06
     7      1  4.8E-01      8  7.8E-01  3.2E-04  2.9647496E+00     1 1.3E-06
     8      1  4.0E-01      9  4.7E-01  3.5E-04  2.9647506E+00     1 1.4E-06
     9      1  1.0E+00     10  9.8E-04  5.2E-05  2.9647511E+00     1 1.6E-06
 Major Minors     Step   nCon Feasible  Optimal  MeritFunction    nS Penalty
    10      1  1.0E+00     11  2.4E-05  7.3E-06  2.9647511E+00     1 1.6E-06
    11      1  1.0E+00     12 (3.6E-07) 2.4E-06  2.9647511E+00     1 1.6E-06
    11      2              12 (3.6E-07)(2.6E-07) 2.9647511E+00     1 1.6E-06       c

 SNOPTA EXIT   0 -- finished successfully
 SNOPTA INFO   1 -- optimality conditions satisfied
 Problem name
 No. of iterations                  12   Objective value      2.9647510828E+00
 No. of major iterations            11   Linear objective     2.9647510828E+00
 Penalty parameter           1.560E-06   Nonlinear objective  0.0000000000E+00
 No. of calls to funobj             78   No. of calls to funcon             78
 Calls with modes 1,2 (known g)     12   Calls with modes 1,2 (known g)     12
 Calls for forward differencing     36   Calls for forward differencing     36
 Calls for central differencing      6   Calls for central differencing      6
 No. of superbasics                  1   No. of basic nonlinears             2
 No. of degenerate steps             0   Percentage                        .00
 Max x                       2 6.1E+00   Max pi                      3 1.0E+00
 Max Primal infeas           0 0.0E+00   Max Dual infeas             2 3.1E-06
 Nonlinear constraint violn    2.3E-06
```

```
=======================
optimal n-body solution
=======================

grazing flyby targeting

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

park orbit
----------

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 6.5634578000e+003  0.0000000000e+000  2.8644284856e+001  0.0000000000e+000

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 2.6909576655e+000  1.9473172714e+002  1.9473172714e+002  6.1248616627e-002

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.27192876444002e+003  -1.76117658967180e+003  -8.00090548727318e+002  +6.56345780000000e+003

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +2.29005159543795e+000  -6.51404567702608e+000  -3.61291072322492e+000  +7.79296165049872e+000


departure hyperbola
-------------------

c3                              8.789749  km^2/sec^2

v-infinity                   2964.751083  meters/second

asymptote right ascension     349.998973  degrees

asymptote declination          -6.843340  degrees


UTC calendar date    05-Jun-2003

UTC time             14:45:42.360

UTC Julian Date      2452796.11581649

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -4.5348330455e+004  1.1447342765e+000  2.8644284856e+001  1.9473172714e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 2.6909576655e+000  0.0000000000e+000  1.9473172714e+002

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -6.27192876444002e+003  -1.76117658967181e+003  -8.00090548727318e+002  +6.56345780000000e+003

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.35376025398030e+000  -9.53976212926521e+000  -5.29107571587870e+000  +1.14127232313459e+001


hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v       1063.708659  meters/sec
y-component of delta-v      -3025.716452  meters/sec
z-component of delta-v      -1678.164993  meters/sec

delta-v magnitude            3619.761581  meters/sec

transfer time                 201.339223  days

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

UTC calendar date    23-Dec-2003
```

```
UTC time             22:54:11.212

UTC Julian Date      2452997.45429644

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -5.8453983255e+003  1.5810036924e+000  6.0285878821e+001  1.2054044790e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 1.0567851428e+002  8.4376619634e-006  1.2054045634e+002

        rx (km)                 ry (km)                rz (km)                rmag (km)
 -9.29550191967783e+002  -2.05336698911750e+003  +2.54042936812996e+003  +3.39619801039957e+003

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +2.71136361950206e+000  -4.34249213541916e+000  -2.51783409559746e+000  +5.70510465778583e+000


B-plane coordinates at Mars closest approach
(Mars mean equator and IAU node of epoch)
-----------------------------------------

b-magnitude                 7158.101791  kilometers
b dot r                    -6199.092835  kilometers
b dot t                     3579.059831  kilometers
b-plane angle                300.000083  degrees
v-infinity                  2706.816088  meters/second
r-periapsis                 3396.198010  kilometers
decl-asymptote                 7.544582  degrees
rasc-asymptote               281.343501  degrees

flight path angle              0.000005  degrees

spacecraft heliocentric coordinates at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003

UTC time             22:54:11.212

UTC Julian Date      2452997.45429644

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.8676872198e+008  2.3213766688e-001  1.8010194369e+001  2.8156719241e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 3.3704467436e+002  1.4640196205e+002  6.7969154466e+001  5.0952621038e+002

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +1.50981601076301e+008  +1.45775001191182e+008  +6.27851157228745e+007  +2.19061100134614e+008

        vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 -1.33673600301203e+001  +1.75961792231026e+001  +3.57279415940309e+000  +2.23847424718685e+001


heliocentric coordinates of Mars at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    23-Dec-2003

UTC time             22:54:11.212

UTC Julian Date      2452997.45429644

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 2.2793935921e+008  9.3542130423e-002  2.4677224881e+001  3.3297930812e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 3.3716581650e+000  7.0372125312e+001  4.3351433435e+001  6.8697240681e+002

        rx (km)                 ry (km)                rz (km)                rmag (km)
 +1.50979882901719e+008  +1.45777823089998e+008  +6.27843290055788e+007  +2.19061568319402e+008
```

```
       vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 -1.66298462022952e+001  +1.69000262431251e+001  +8.20094434723467e+000  +2.50882075867678e+001


spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003

UTC time             18:20:24.029

UTC Julian Date      2452799.26416700

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -4.5600818575e+004  1.1432801476e+000  2.8507286889e+001  1.9469038362e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 2.7037370098e+000  1.4947804028e+002  3.4416842391e+002

       rx (km)                 ry (km)                rz (km)                rmag (km)
 +8.99382544681222e+005  -1.79528458527220e+005  -1.20439075476231e+005  +9.25000000000000e+005

       vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.03080180918775e+000  -5.31989345771324e-001  -3.66261328960530e-001  +3.09885779468531e+000


spacecraft heliocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

UTC calendar date    08-Jun-2003

UTC time             18:20:24.029

UTC Julian Date      2452799.26416700

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.9070391796e+008  2.0396427127e-001  2.3500194498e+001  2.5359042174e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 5.2791303861e-001  3.7878307686e+000  2.5737825251e+002  5.2571422947e+002

       rx (km)                 ry (km)                rz (km)                rmag (km)
 -3.19306432952744e+007  -1.36202018532337e+008  -5.90924365136045e+007  +1.51863332923545e+008

       vx (kps)                vy (kps)               vz (kps)               vmag (kps)
 +3.16291289056473e+001  -6.53255504540109e+000  -2.96705620606298e+000  +3.24326917365979e+001
```