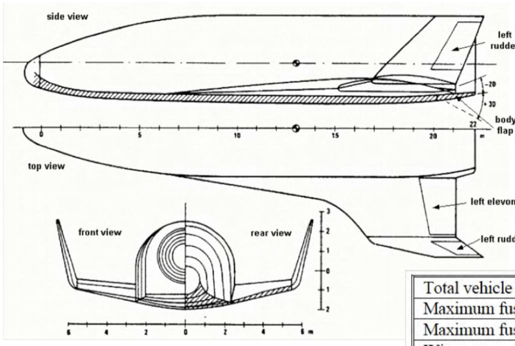
HORUS X - SPACE APPLICATIONS: ENTRY VEHICLES

Reference Vehicle (1)



Total vehicle length	25 m
Maximum fuselage width	5.4 m
Maximum fuselage height	4.5 m
Wing span	13.0 m
Wing area	110 m^2
Re-entry mass	26,029 kg

- Neglections for aerodynamic effects
  - No aeroelasticity
  - Influence of R.N included in drag coefficient
  - No especial landing aerodynamics
  - No inteference effects of the flaps due to hypersonic regime.
  - When $\beta > 2°$, β is simplified by linearization.

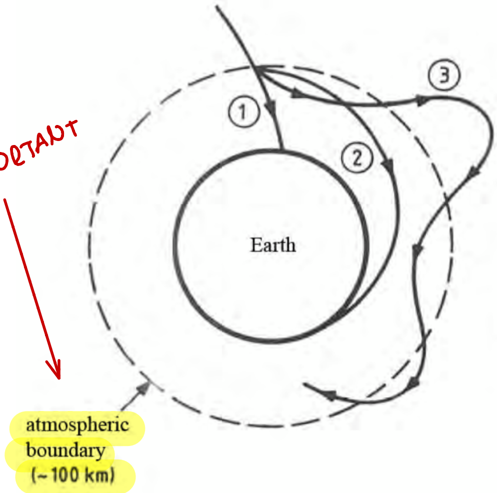
OPERATIONAL PARAMETER RANGES:

α [°]	M [-]	δ_r [°]	δ_e [°]	δ_b [°]
0.0	1.2	0.0	-40.0	-20.0
5.0	1.5	10.0	-30.0	-10.0
10.0	2.0	20.0	-20.0	0.0
15.0	3.0	30.0	-10.0	10.0
20.0	5.0	40.0	0.0	20.0
25.0	10.0		10.0	30.0
30.0	20.0		20.0	
35.0			30.0	
40.0			40.0	
45.0				

Graphs for pitching moments with and without flaps as a function of alpha and Mach are given aswell.

ATMOSPHERE RE-ENTRY

IMPORTANT



Trajectory types:
1 ballistic entry
2 gliding entry
3 skipping entry

VELOCITY REGIMES:

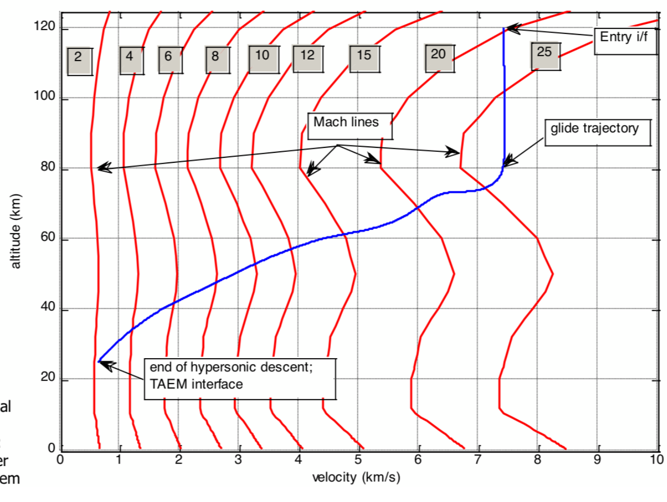
Hypersonic $M > 5$

Supersonic $1 < M < 5$

Transonic $0.8 < M < 1.2$

Subsonic $M < 1$ (effectively $M < 0.6$)

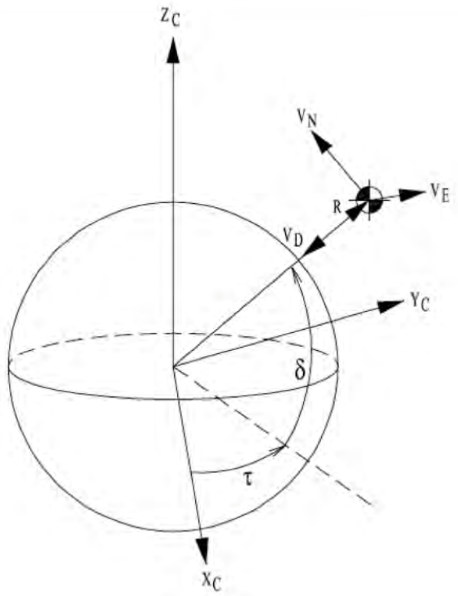
This is how the spacecraft looks on reentry.



TAEM=terminal area energy management: switch to other guidance system

Bank maneouvers are used to increase crossrange and for constraint control.

Non-linear EOM:



Translational motion

$$\dot{\delta} = \frac{V_N}{R} \qquad \dot{\tau} = \frac{V_E}{R \cdot \cos\delta} \qquad \dot{R} = -V_D$$

$$\dot{V}_N = \frac{F_{xE}}{m} - 2\Omega_t \cdot V_E \cdot \sin\delta - \Omega_t^2 \cdot R \cdot \sin\delta \cdot \cos\delta - \frac{V_E^2 \tan\delta - V_N \cdot V_D}{R}$$

$$\dot{V}_E = \frac{F_{yE}}{m} + 2\Omega_t \cdot (V_D \cos\delta + V_N \cdot \sin\delta) + \frac{V_E}{R} \cdot (V_N \cdot \tan\delta + V_D)$$

$$\dot{V}_D = \frac{F_{zE}}{m} - 2\Omega_t \cdot V_E \cdot \cos\delta - \Omega_t^2 \cdot R \cdot \cos^2\delta - \frac{V_E^2 + V_N^2}{R}$$

Rotational Motion

$$\dot{p} = \frac{I_{zz}}{I^*} M_x + \frac{I_{xz}}{I^*} M_z + \frac{(I_{xx} - I_{yy} + I_{zz}) I_{xz}}{I^*} pq + \frac{(I_{yy} - I_{zz}) I_{zz} - I_{xz}^2}{I^*} qr$$

$$\dot{q} = \frac{M_y}{I_{yy}} + \frac{I_{xz}}{I_{yy}} (r^2 - p^2) + \frac{I_{zz} - I_{xx}}{I_{yy}} pr$$

$$\dot{r} = \frac{I_{xz}}{I^*} M_x + \frac{I_{xx}}{I^*} M_z + \frac{(I_{xx} - I_{yy}) I_{xx} + I_{xz}^2}{I^*} pq + \frac{(-I_{xx} + I_{yy} - I_{zz}) I_{xz}}{I^*} qr$$

with $I^* = I_{xx} I_{zz} - I_{xz}^2$.

* two plans of mass symmetry.

$$\dot{p} = \frac{M_x}{I_{xx}} + \frac{I_{yy} - I_{zz}}{I_{xx}} qr$$

$$\dot{q} = \frac{M_y}{I_{yy}} + \frac{I_{zz} - I_{xx}}{I_{yy}} pr$$

$$\dot{r} = \frac{M_z}{I_{zz}} + \frac{I_{xx} - I_{yy}}{I_{zz}} pq$$

$$\boxed{\begin{aligned} \dot{\varphi} &= \tilde{p} + \sin\varphi \tan\theta \tilde{q} + \cos\varphi \tan\theta \tilde{r} \\ \dot{\theta} &= \cos\varphi \tilde{q} - \sin\varphi \tilde{r} \\ \dot{\psi} &= \frac{\sin\varphi}{\cos\theta} \tilde{q} + \frac{\cos\varphi}{\cos\theta} \tilde{r} \end{aligned}}$$

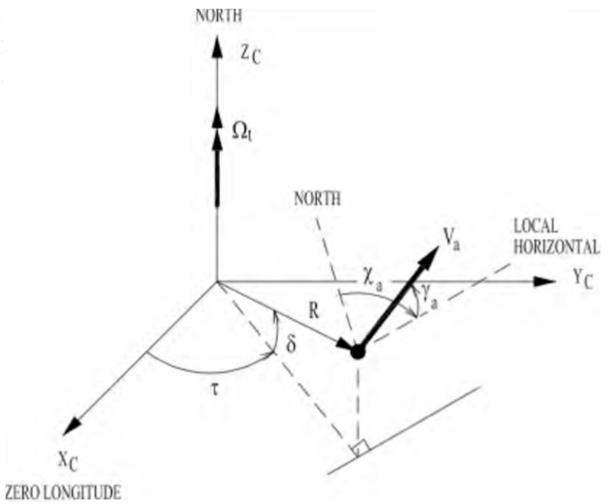
$$\tilde{p} = p + c\theta s\psi \dot{\delta} - [c\delta c\psi c\theta + s\delta s\theta] (\dot{\tau} + \Omega_t)$$

$$\tilde{q} = q + (s\psi s\theta s\varphi + c\psi c\varphi)\dot{\delta} - [c\delta(c\psi s\theta s\varphi - s\psi c\varphi) - s\delta c\theta s\varphi] (\dot{\tau} + \Omega_t)$$

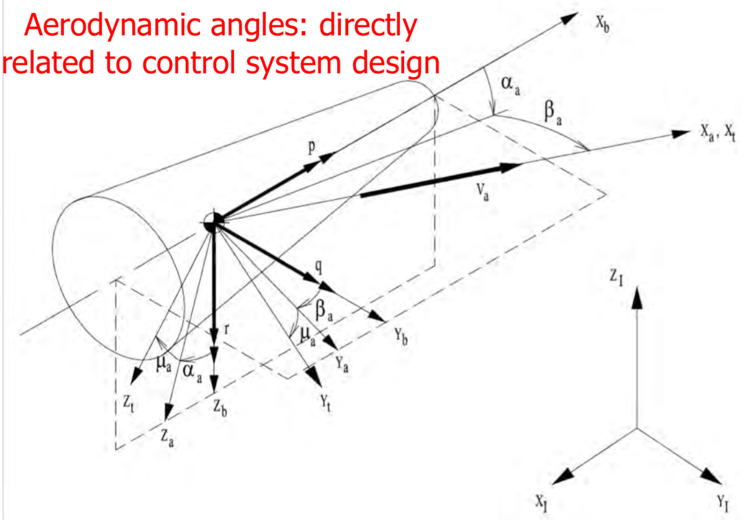
$$\tilde{r} = r + (s\psi c\varphi s\theta - c\psi s\varphi)\dot{\delta} - [c\delta(s\psi s\varphi + c\psi s\theta c\varphi) - s\delta c\theta c\varphi] (\dot{\tau} + \Omega_t)$$

These equations are not for re-entry: NEXT PAGE

RE-ENTRY ANALYSIS



Aerodynamic angles: directly related to control system design



$$\dot V = -\frac{D}{m} + g\cdot\sin\gamma + \Omega_t^2\cdot R\cos\delta\,(\sin\gamma\cdot\cos\delta - \cos\gamma\cdot\sin\delta\cdot\cos\chi)$$

$$V\dot\gamma = \frac{L\cos\mu}{m} - g\cdot\cos\gamma + 2\Omega_t V\cdot\cos\delta\cdot\sin\chi + \frac{V^2}{R}\cos\gamma + \Omega_t^2\cdot R\cos\delta\,(\cos\gamma\cdot\cos\delta - \sin\gamma\cdot\cos\delta\cdot\cos\chi)$$

$$V\cos\gamma\,\dot\chi = \frac{L\sin\mu}{m} + 2\Omega_t V\cdot(\sin\delta\cdot\cos\gamma - \cos\delta\cdot\sin\gamma\cdot\cos\chi) + \frac{V^2}{R}\cos^2\gamma\tan\delta\cdot\sin\chi + \Omega_t^2 r\cos\delta\cdot\sin\delta\cdot\sin\chi$$

$$\dot R = \dot h = V\sin\gamma \qquad \dot\tau = \frac{V\cdot\sin\chi\cdot\cos\gamma}{R\cdot\cos\delta} \qquad \dot\delta = \frac{V\cdot\cos\chi\cos\gamma}{R}$$

$\gamma = \pm 90°$

$\delta = \pm 90°$

DYNAMIC EQUATIONS FOR ROTATIONAL MOTION ARE THE SAME

KINEMATIC EQUATIONS

Coupling due to large angle of attack

$$\dot\alpha\cos\beta = -p\cos\alpha\sin\beta + q\cos\beta - r\sin\alpha\sin\beta + \sin\mu\left[\dot\chi\cos\gamma - \dot\delta\sin\chi\sin\gamma + (\dot\tau + \Omega_t)(\cos\delta\cos\chi\sin\gamma - \sin\delta\cos\gamma)\right] - \cos\mu\left[\dot\gamma - \dot\delta\cos\chi - (\dot\tau + \Omega_t)\cos\delta\sin\chi\right]$$

$$\dot\beta = p\sin\alpha - r\cos\alpha + \sin\mu\left[\dot\gamma - \dot\delta\cos\chi - (\dot\tau + \Omega_t)\cos\delta\sin\chi\right] + \cos\mu\left[\dot\chi\cos\gamma - \dot\delta\sin\chi\sin\gamma + (\dot\tau + \Omega_t)(\cos\delta\cos\chi\sin\gamma - \sin\delta\cos\gamma)\right]$$

$$\dot\mu = -p\cos\alpha\cos\beta - q\sin\beta - r\sin\alpha\cos\beta + \dot\alpha\sin\beta - \dot\chi\sin\gamma - \dot\delta\sin\chi\cos\gamma + (\dot\tau + \Omega_t)(\cos\delta\cos\chi\cos\gamma + \sin\delta\sin\gamma)$$

Rotation of Earth $(\dot\delta, \dot\tau)$ and local horizontal plane $(\dot\gamma, \dot\chi)$

During bank reversals, bank angle coverage of 120°, creates large induced angle of sideslip. This sideslip produces coupling with pitch motion.

SUMMARY:

For simulation of re-entry:

(INERTIAL) CARTESIAN components for position and velocity.

For analysis of re-entry:

Spherical components for translational motion.

Aerodynamic angles for attitude.

BEWARE:

Flight path angle (±90°), latitude (±90°)

Sideslip angle (±90°)

Large angle of attack:

Coupling between roll-yaw

Coupling between rudder - ailerons.

LINEARIZATION:

ASSUMPTIONS: EoM

$\Omega_t = 0$: Rotation of earth equals 0. Because the vehicle is faster.

$J_{xz} = 0$: Mass symmetric vehicle.

Ignore smaller terms.

$$\dot{V} = -\frac{D}{m} - g\sin\gamma \qquad \dot{R} = V\sin\gamma$$

$$\dot{\gamma} = \left(\frac{V}{R} - \frac{g}{V}\right)\cos\gamma + \frac{L\cos\mu - S\sin\mu}{mV} \qquad \dot{\tau} = \frac{V\cos\gamma\sin\chi}{R\cos\delta}$$

$$\dot{\chi} = \frac{V}{R}\cos\gamma\tan\delta\sin\chi - \frac{L\sin\mu + S\cos\mu}{mV\cos\gamma} \qquad \dot{\delta} = \frac{V}{R}\cos\gamma\cos\chi$$

out-of-plane motion and coupling between longitudinal and lateral forces

No banking: heading change only through internal dynamics

MORE ASSUMPTIONS:

$\delta = 0°$ vehicle moves along equator

$\chi = 90°$ $\dot{\delta} = 0$

EoM

dynamic coupling longitudinal and lateral motion.

$$\dot{\alpha} = q - (p\cos\alpha + r\sin\alpha)\tan\beta - \frac{L - mg\cos\gamma\cos\mu}{mV\cos\beta}$$

$$\dot{\beta} = p\cdot\sin\alpha - r\cos\alpha - \frac{S + mg\cos\gamma\cdot\sin\mu}{mV}$$

$$\dot{\mu} = -\frac{p\cdot\cos\alpha + r\sin\alpha}{\cos\beta} - \frac{L - mg\cos\gamma\cdot\cos\mu}{mV}\tan\beta + \frac{L\sin\mu + S\cos\mu}{mV}\cdot\tan\gamma$$

Coupling between translational and rotational motion.

9 degrees of freedom: $V, \gamma, R, p, q, r, \alpha, \beta, \mu$

Taking first-order Taylor terms $\longrightarrow$ gives 9 coupled differential equations

We write them as $\Delta\dot{x} = A\Delta x + B\Delta u$ eigenvalues of A gives motion.

EXAMPLE:

METHOD 1

$$\dot{R} = V\cdot\sin\gamma \longrightarrow \dot{R}_0 + \Delta\dot{R} = (V_0 + \Delta V)\cdot\underbrace{\sin(\gamma_0 + \Delta\gamma)}_{\sin\gamma_0\cdot\cos\Delta\gamma + \cos\gamma_0\cdot\sin\Delta\gamma}$$

$$\cos\Delta\gamma = 1 \qquad \sin\Delta\gamma = \Delta\gamma$$

$$\dot{R}_0 + \Delta\dot{R} = V_0\cdot\sin\gamma_0 + V_0\cos\gamma_0\cdot\Delta\gamma + \sin\gamma_0\cdot\Delta V + \underbrace{\cos\gamma_0\,\Delta V\Delta\gamma}_{0}$$

0 because of second order term.

Substract from both sides the nominal.

$$\Delta\dot{R} = V_0\cos\gamma_0\,\Delta\gamma + \sin\gamma_0\cdot\Delta V$$

METHOD 2

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

$\dot{R} = V\sin\gamma$ $\quad f = V\sin\gamma$

$$\left.\frac{\partial f}{\partial V}\right|_0 = \sin\gamma_0 \qquad \frac{\partial f}{\partial \gamma} = V_0\cos\gamma_0$$

$$\Delta\dot{R} = V_0\cos\gamma_0\,\Delta\gamma + \sin\gamma_0\cdot\Delta V$$

LINEARIZATION:

We do the same with the rest of the variables: g

We also get nominal forces. D_0, L_0, S_0 and moments $\mathcal{L}_0, \mathcal{M}_0, \mathcal{N}_0$

$\Delta D, \Delta L, \Delta S$ $\qquad$ $\Delta\mathcal{L}, \Delta\mathcal{M}, \Delta\mathcal{N}$

Central gravity field: $g_0 + \Delta g = \dfrac{\mu}{(R_0+\Delta R)^2} = \dfrac{\mu}{R_0^2}\cdot\dfrac{1}{1+2\frac{\Delta R}{R_0}} \approx \dfrac{\mu}{R_0^2}\left(1-2\dfrac{\Delta R}{R_0}\right)$

$$\Delta g = -2g_0\frac{\Delta R}{R_0}$$

THE OTHER TRANSLATIONAL MOTION

$$\Delta\dot{V} = -\frac{\Delta D}{m} + 2\frac{g_0}{R_0}\sin\gamma_0\Delta R - g_0\cos\gamma_0\Delta\gamma$$

$D = C_D \cdot \bar{q} \cdot S_{ref}$

$q = \frac{1}{2}\rho V^2$

$$\boxed{-\dot{\gamma}_0 = \left(\frac{g_0}{V_0} - \frac{V_0}{R_0}\right) - \frac{L_0\cos\mu_0}{mV_0}}$$

$$\Delta\dot{\gamma} = \left(-\dot{\gamma}_0 + \frac{2V_0}{R_0}\cos\gamma_0\right)\frac{\Delta V}{V_0} + \left(\frac{2g_0}{R_0} - \frac{V_0^2}{R_0^2}\right)\frac{\cos\gamma_0}{V_0}\Delta R - \left(\frac{V_0^2}{R_0} - g_0\right)\frac{\sin\gamma_0}{V_0}\Delta\gamma +$$
$$-\frac{L_0}{mV_0}\sin\mu_0\Delta\mu + \frac{\cos\mu_0}{mV_0}\Delta L - \frac{\sin\mu_0}{mV_0}\Delta S$$

small negligible

0, bodyflap for trim only.

$$\Delta D = \frac{\partial D}{\partial M}\Delta M + \frac{\partial D}{\partial \alpha}\Delta\alpha + \cancel{\frac{\partial D}{\partial h}\Delta h} + \cancel{\frac{\partial D}{\partial \delta_e}\Delta\delta_e} + \cancel{\frac{\partial D}{\partial \delta_a}\Delta\delta_a} + \frac{\partial D}{\partial \delta_r}\Delta\delta_r + \cancel{\frac{\partial D}{\partial \delta_b}\Delta\delta_b}$$

Variation due to Mach and angle of attack greatest.

$$\Delta D = \frac{\partial D}{\partial M}\Delta M + \frac{\partial D}{\partial \alpha}\Delta\alpha \qquad \frac{\partial D}{\partial M} = \frac{\partial C_D}{\partial M}\cdot\bar{q}\,S_{ref} + \frac{\partial \bar{q}}{\partial M}C_D\cdot S_{ref} \quad \text{with } M = \frac{V}{a}$$

$$\frac{\partial q}{\partial M} = \frac{1}{2}\frac{\partial(\rho M^2 a^2)}{\partial M} = \rho_0 a_0^2 M_0 = \frac{2\bar{q}_0}{M_0}$$

$$\boxed{\Delta D = \left(\frac{M_0}{V_0}\cdot\frac{\partial C_D}{\partial M} + \frac{2C_D}{V_0}\right)\bar{q}_0\cdot S_{ref}\Delta V + \frac{\partial C_D}{\partial \alpha}\bar{q}_0 S_{ref}\Delta\alpha}$$

$$\Delta\mathcal{L} = \frac{\partial C_l}{\partial\beta}\bar{q}_0 S_{ref} b_{ref}\Delta\beta + \frac{\partial C_l}{\partial\delta_a}\bar{q}_0 S_{ref} b_{ref}\Delta\delta_a$$

$$\Delta\mathcal{M} = \frac{M_0}{V_0}\frac{\partial C_m}{\partial M}\bar{q}_0 S_{ref} c_{ref}\Delta V + \frac{\partial C_m}{\partial\alpha}\bar{q}_0 S_{ref} c_{ref}\Delta\alpha + \frac{\partial C_m}{\partial\delta_e}\bar{q}_0 S_{ref} c_{ref}\Delta\delta_e$$

$$\Delta\mathcal{N} = \frac{\partial C_n}{\partial\beta}\bar{q}_0 S_{ref} b_{ref}\Delta\beta + \frac{\partial C_n}{\partial\delta_r}\bar{q}_0 S_{ref} b_{ref}\Delta\delta_r + \frac{\partial C_n}{\partial\delta_a}\bar{q}_0 S_{ref} b_{ref}\Delta\delta_a$$

FINAL STATE SPACE MODEL

$$\begin{pmatrix}\Delta\dot{V}\\ \Delta\dot{\gamma}\\ \Delta\dot{R}\\ \Delta\dot{p}\\ \Delta\dot{q}\\ \Delta\dot{r}\\ \Delta\dot{\alpha}\\ \Delta\dot{\beta}\\ \Delta\dot{\mu}\end{pmatrix} = \begin{bmatrix}
a_{VV} & a_{V\gamma} & a_{VR} & \cancel{a_{Vp}} & \cancel{a_{Vq}} & \cancel{a_{Vr}} & a_{V\alpha} & \cancel{a_{V\beta}} & \cancel{a_{V\mu}}\\
a_{\gamma V} & a_{\gamma\gamma} & a_{\gamma R} & \cancel{a_{\gamma p}} & \cancel{a_{\gamma q}} & \cancel{a_{\gamma r}} & a_{\gamma\alpha} & a_{\gamma\beta} & a_{\gamma\mu}\\
a_{RV} & a_{R\gamma} & \cancel{a_{RR}} & \cancel{a_{Rp}} & \cancel{a_{Rq}} & \cancel{a_{Rr}} & \cancel{a_{R\alpha}} & \cancel{a_{R\beta}} & \cancel{a_{R\mu}}\\
\cancel{a_{pV}} & \cancel{a_{p\gamma}} & \cancel{a_{pR}} & \cancel{a_{pp}} & \cancel{a_{pq}} & \cancel{a_{pr}} & \cancel{a_{p\alpha}} & a_{p\beta} & \cancel{a_{p\mu}}\\
a_{qV} & \cancel{a_{q\gamma}} & \cancel{a_{qR}} & \cancel{a_{qp}} & \cancel{a_{qq}} & \cancel{a_{qr}} & a_{q\alpha} & \cancel{a_{q\beta}} & \cancel{a_{q\mu}}\\
\cancel{a_{rV}} & \cancel{a_{r\gamma}} & \cancel{a_{rR}} & \cancel{a_{rp}} & \cancel{a_{rq}} & \cancel{a_{rr}} & \cancel{a_{r\alpha}} & a_{r\beta} & \cancel{a_{r\mu}}\\
a_{\alpha V} & a_{\alpha\gamma} & a_{\alpha R} & \cancel{a_{\alpha p}} & a_{\alpha q} & \cancel{a_{\alpha r}} & a_{\alpha\alpha} & \cancel{a_{\alpha\beta}} & a_{\alpha\mu}\\
a_{\beta V} & a_{\beta\gamma} & a_{\beta R} & a_{\beta p} & \cancel{a_{\beta q}} & a_{\beta r} & \cancel{a_{\beta\alpha}} & a_{\beta\beta} & a_{\beta\mu}\\
a_{\mu V} & a_{\mu\gamma} & \cancel{a_{\mu R}} & a_{\mu p} & \cancel{a_{\mu q}} & a_{\mu r} & a_{\mu\alpha} & a_{\mu\beta} & a_{\mu\mu}
\end{bmatrix}\begin{pmatrix}\Delta V\\ \Delta\gamma\\ \Delta R\\ \Delta p\\ \Delta q\\ \Delta r\\ \Delta\alpha\\ \Delta\beta\\ \Delta\mu\end{pmatrix} +$$

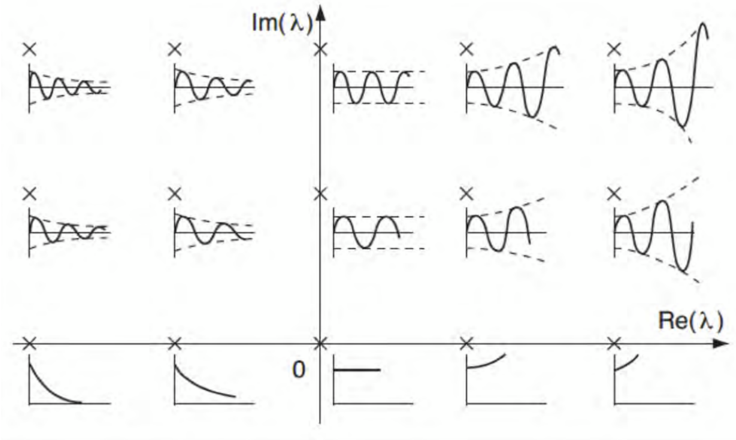
$$+\begin{bmatrix}
\cancel{b_{Ve}} & \cancel{b_{Va}} & \cancel{b_{Vr}} & \cancel{b_{Vx}} & \cancel{b_{Vy}} & \cancel{b_{Vz}}\\
\cancel{b_{\gamma e}} & \cancel{b_{\gamma a}} & \cancel{b_{\gamma r}} & \cancel{b_{\gamma x}} & \cancel{b_{\gamma y}} & \cancel{b_{\gamma z}}\\
\cancel{b_{Re}} & \cancel{b_{Ra}} & \cancel{b_{Rr}} & \cancel{b_{Rx}} & \cancel{b_{Ry}} & \cancel{b_{Rz}}\\
\cancel{b_{pe}} & b_{pa} & \cancel{b_{pr}} & b_{px} & \cancel{b_{py}} & \cancel{b_{pz}}\\
b_{qe} & \cancel{b_{qa}} & \cancel{b_{qr}} & \cancel{b_{qx}} & b_{qy} & \cancel{b_{qz}}\\
\cancel{b_{re}} & b_{ra} & b_{rr} & \cancel{b_{rx}} & \cancel{b_{ry}} & b_{rz}\\
\cancel{b_{\alpha e}} & \cancel{b_{\alpha a}} & \cancel{b_{\alpha r}} & \cancel{b_{\alpha x}} & \cancel{b_{\alpha y}} & \cancel{b_{\alpha z}}\\
\cancel{b_{\beta e}} & \cancel{b_{\beta a}} & \cancel{b_{\beta r}} & \cancel{b_{\beta x}} & \cancel{b_{\beta y}} & \cancel{b_{\beta z}}\\
\cancel{b_{\mu e}} & \cancel{b_{\mu a}} & \cancel{b_{\mu r}} & \cancel{b_{\mu x}} & \cancel{b_{\mu y}} & \cancel{b_{\mu z}}
\end{bmatrix}\begin{pmatrix}\Delta\delta_e\\ \Delta\delta_a\\ \Delta\delta_r\\ \Delta M_{T,x}\\ \Delta M_{T,y}\\ \Delta M_{T,z}\end{pmatrix}$$

EIGENMOTION:

$$\dot{x} = Ax + Bu$$

$$A\mu = \lambda\mu \qquad (A-\lambda I)\mu = 0 \qquad |A-\lambda I| = 0$$

$$x_\lambda(t) = e^{\lambda t}\mu \qquad x_\lambda(t) = \sum_{i=1}^{q} c_i e^{\lambda_i t}\mu_i$$



Impulse responses for various eigenvalue locations

- Period P $\qquad P = \dfrac{2\pi}{Im(\lambda)}$
- Halving time $T_{1/2}$ $\qquad T_{\frac{1}{2}} = \dfrac{\ln\frac{1}{2}}{Re(\lambda)}$
- Doubling time T_2 $\qquad T_2 = \dfrac{\ln 2}{Re(\lambda)}$
- Damping ratio ζ $\qquad \zeta = -\dfrac{Re(\lambda)}{\sqrt{Re(\lambda)^2 + Im(\lambda)^2}}$
- Natural frequency ω_n $\qquad \omega_n = \sqrt{Re(\lambda)^2 + Im(\lambda)^2}$

$$P = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$$

These are **dimensional** parameters!

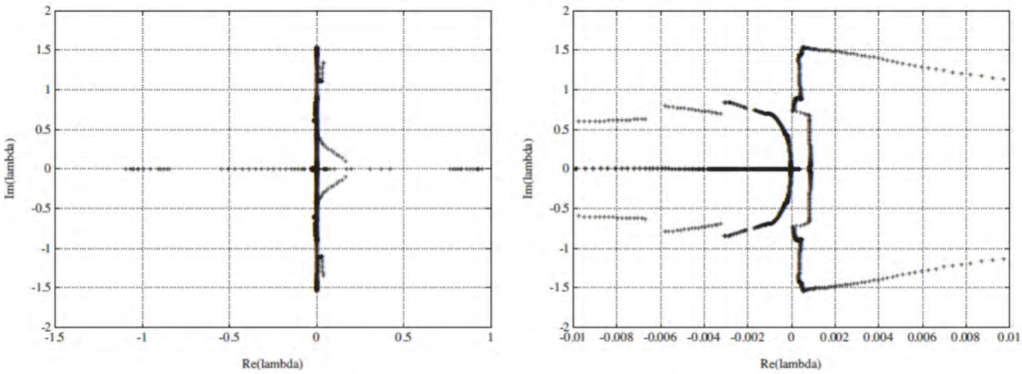
SUMMARY:

- Conventional aircraft has five different modes, i.e., two longitudinal and three lateral ones

- Longitudinal modes (pitching):
  - Short-period – fast, periodic motion, usually well damped
  - Phugoid – very slow oscillation, usually poorly damped
- Lateral modes (rolling):
  - Lateral oscillation (or Dutch roll) – moderately to well-damped oscillation, yaw rotation induces large forces
  - Rolling convergence, strongly damped, aperiodic motion
  - Spiral mode, stable or unstable aperiodic motion with large time constant

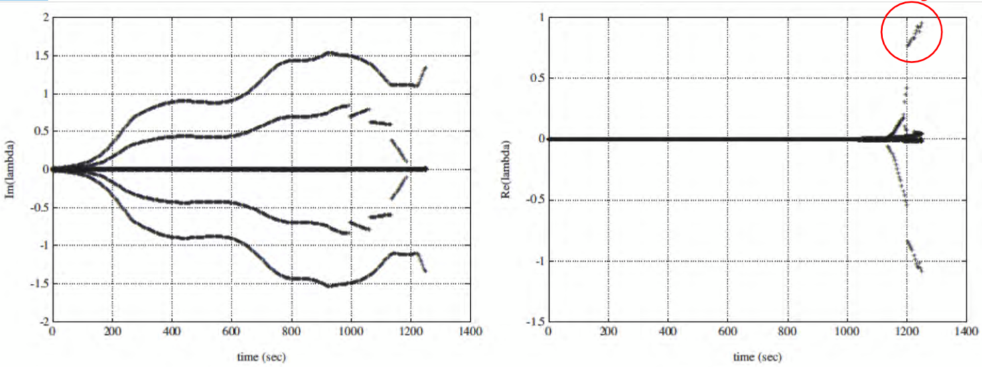
- Longitudinal and lateral modes are decoupled

CHECK EXAMPLE OF PENDULUM ON LECTURE 15:



(a) Imaginary versus real parts $\qquad$ (b) Detail centred around the origin

Unstable towards the end of the flight



(c) Imaginary parts $\qquad$ (d) Real parts

APOLLO X – SPACE APPLICATIONS: ENTRY VEHICLES EIGENMOTIONS

To analyze a eigenmotion: select time points and evaluate the eigenvalues and eigenmotion of state A matrix.

	short-period *) oscillation		phugoid		lateral oscillation		aperiodic roll mode		height mode
λ_i Re	$-0.7302\cdot10^{-7}$		$0.4025\cdot10^{-4}$		$0.8677\cdot10^{-3}$		$-0.1735\cdot10^{-2}$	$-0.7925\cdot10^{-18}$	$-0.8402\cdot10^{-4}$
Im	$\pm0.7415\cdot10^{-2}$		$\pm0.1212\cdot10^{-2}$		±0.0168		-	-	-
P (s)	847.3		5,183.0		373.8		∞	∞	∞
$T_{1/2}$ (s)	$0.949\cdot10^{7}$		-17,220.6		-798.9		399.4	$0.875\cdot10^{18}$	8,250.0
ζ (-)	$0.985\cdot10^{-5}$		-0.033		-0.052		-	-	-
ω_n (rad/s)	$0.742\cdot10^{-2}$		$0.121\cdot10^{-2}$		0.017		-	-	-
μ_i	z	θ (°)	z	θ (°)	z	θ (°)	z	z	z
ΔV	**0.0129**	**10.7**	**$0.1271\cdot10^{-2}$**	**288.9**	$0.1134\cdot10^{-15}$	6.2	$0.1678\cdot10^{-16}$	$0.2112\cdot10^{-15}$	$0.6573\cdot10^{-3}$
$\Delta\gamma$	$0.9637\cdot10^{-6}$	331.7	$0.1632\cdot10^{-6}$	18.1	$0.4382\cdot10^{-20}$	325.8	$0.3441\cdot10^{-20}$	$0.6882\cdot10^{-21}$	$0.1351\cdot10^{-7}$
ΔR	**1.0000**	**63.3**	**1.0000**	**287.7**	$0.2211\cdot10^{-14}$	51.1	$0.1475\cdot10^{-13}$	$0.3129\cdot10^{-12}$	**1.0000**
Δp	$0.8551\cdot10^{-19}$	277.7	$0.2260\cdot10^{-23}$	291.9	**0.0206**	**287.8**	$0.2144\cdot10^{-2}$	$0.8153\cdot10^{-3}$	$0.1007\cdot10^{-22}$
Δq	**$0.1678\cdot10^{-3}$**	**6.1**	$0.1804\cdot10^{-9}$	300.2	$0.3156\cdot10^{-17}$	318.3	$0.4950\cdot10^{-19}$	$0.4129\cdot10^{-19}$	$0.2798\cdot10^{-9}$
Δr	$0.9334\cdot10^{-20}$	291.6	$0.7187\cdot10^{-24}$	0.4	**$0.1542\cdot10^{-2}$**	**287.8**	$0.1606\cdot10^{-3}$	$0.9716\cdot10^{-3}$	$0.8683\cdot10^{-23}$
$\Delta\alpha$	**0.0226**	**276.1**	$0.3150\cdot10^{-7}$	281.9	$0.3891\cdot10^{-16}$	345.0	$0.3712\cdot10^{-16}$	$0.1735\cdot10^{-16}$	$0.1527\cdot10^{-7}$
$\Delta\beta$	$0.1940\cdot10^{-17}$	21.6	$0.2444\cdot10^{-22}$	272.3	**0.7277**	**20.8**	$0.7814\cdot10^{-2}$	$0.1235\cdot10^{-15}$	$0.2045\cdot10^{-22}$
$\Delta\mu$	$0.2609\cdot10^{-17}$	11.3	$0.5346\cdot10^{-21}$	52.2	**1.0000**	**11.7**	**1.0000**	**1.0000**	$0.9654\cdot10^{-20}$

t = 0 s

	former short-period oscillation		periodic pitch/roll mode		lateral oscillation		pitch/roll divergence	spiral mode	
λ_i Re	-0.5078	0.3695	-0.0157		0.0229		0.0814	$-0.2892\cdot10^{-3}$	$0.9450\cdot10^{-14}$
Im	-	-	$\pm0.7035\cdot10^{-2}$		±1.113		-	-	-
P (s)	∞	∞	893.1		5.6		∞	∞	∞
$T_{1/2}$ (s)	1.4	-1.9	44.3		-30.3		-8.5	2,396.8	$-0.734\cdot10^{14}$
ζ (-)	-	-	0.912		-0.021		-	-	-
ω_n (rad/s)	-	-	0.017		1.113		-	-	-
μ_i	z	z	z	θ (°)	z	θ (°)	z	z	z
ΔV	**0.6721**	**0.5181**	$0.1758\cdot10^{-2}$	9.3	0.0256	284.8	**0.1215**	$0.3228\cdot10^{-4}$	$0.3896\cdot10^{-5}$
$\Delta\gamma$	$0.4118\cdot10^{-3}$	$0.2972\cdot10^{-3}$	$0.1598\cdot10^{-4}$	14.0	$0.1027\cdot10^{-2}$	17.9	$0.6490\cdot10^{-4}$	$0.2695\cdot10^{-6}$	$0.3286\cdot10^{-9}$
ΔR	**1.0000**	**1.0000**	**1.0000**	**349.9**	**1.0000**	**286.9**	**1.0000**	**1.0000**	**1.0000**
Δp	$0.2036\cdot10^{-4}$	$0.1595\cdot10^{-4}$	$0.4226\cdot10^{-6}$	48.5	**0.0808**	**19.2**	$0.3478\cdot10^{-5}$	$0.3717\cdot10^{-8}$	$0.5878\cdot10^{-9}$
Δq	**$0.3447\cdot10^{-2}$**	**$0.1822\cdot10^{-2}$**	$0.1025\cdot10^{-6}$	278.8	$0.7476\cdot10^{-4}$	278.2	$0.2963\cdot10^{-4}$	$0.6599\cdot10^{-8}$	$0.2753\cdot10^{-8}$
Δr	$0.3419\cdot10^{-5}$	$0.2680\cdot10^{-5}$	$0.7098\cdot10^{-7}$	48.5	**0.0136**	**19.2**	$0.5842\cdot10^{-6}$	$0.6244\cdot10^{-9}$	$0.1219\cdot10^{-8}$
$\Delta\alpha$	**$0.7534\cdot10^{-2}$**	**$0.4372\cdot10^{-2}$**	**$0.3333\cdot10^{-5}$**	**9.5**	$0.4170\cdot10^{-3}$	16.0	**$0.2428\cdot10^{-3}$**	$0.6126\cdot10^{-7}$	$0.7396\cdot10^{-8}$
$\Delta\beta$	$0.1604\cdot10^{-5}$	$0.9150\cdot10^{-6}$	$0.1125\cdot10^{-8}$	72.7	**0.0140**	**290.4**	$0.4397\cdot10^{-7}$	$0.1669\cdot10^{-12}$	$0.7179\cdot10^{-21}$
$\Delta\mu$	$0.2726\cdot10^{-4}$	$0.7653\cdot10^{-4}$	**$0.1671\cdot10^{-4}$**	**48.9**	**0.0722**	**288.1**	**$0.4241\cdot10^{-4}$**	**$0.3279\cdot10^{-6}$**	**$0.2092\cdot10^{-6}$**

t = 1196 s

Differences on eigenmodes compared to aircraft are due to the large velocity and trajectory.

SHORT PERIOD: longer period, poorly damped due to low atmosphere

+ time + close to aircraft
+ aerodynamic force + damping

At 1196 s short period splits { strongly damped / strongly diverging

+ time + influence of bank.

PHUGOID: even slower (1 hr period) unstable (poorly damped)

+ time + damping

when bank changes from 0 to non 0 // coupling between the symmetric and asymmetric.

LATERAL OSCILLATION: unstable, but similar

Unstable but stable at some time (596 s)

+ time – period

@ 396 s // coupling between lateral and longitudinal motion.

APERIODIC ROLL MOTION:

Changes @ 396 s to periodic roll motion with small coupling to α.

When starts banking, height becomes dominant.

Next time point: 1. break up into two aperiodic roll modes 2. stable periodic roll mode

3. two aperiodic modes (unstable coupled with α)

CONCLUSIONS:

Similar to aircraft

Slower with no atmosphere

Coupling between asymmetrical and symmetrical.

Control necessary due to oscillatory nature.

Not distinguishable eigenmodes in open-loop simulation.

OTHER CASES: APOLLO

Hyperbolic entry velocity

- Axisymmetric (= rotational symmetric) vehicle
- Asymmetry introduced by shift of centre of mass
  - z_{cg} = -0.137 m
  - As a result: hypersonic trim angle of attack α = -24.5°
  - Lift-to-drag ratio L/D ≈ 0.3 (without shift: L/D = 0): large aerodynamic loads (up to 6g)
- Only one plane of mass symmetry, $I_{xz} \neq 0$ (full set of Euler equations needed!)
  - I_{xx} = 5618 kg m², $I_{yy} = I_{zz}$ = 4455 kg m²
  - $I_{xy} = I_{yz}$ = 0.0 kg m² and I_{xz} = 1752 kg m²

- All coeficients are a function of M, α, β.
- $\Omega t = 0$
- $\delta = 0,\ \dot{\delta} = 0,\ \chi = 90°$

In this case there are some differences with the previous A, B matrices:

Extra terms due to I_{xz}

Some aerodynamic derivatives are 0.

No aerodynamic control surfaces

$$a_{pp} = I_{p_1} q_0$$

$$a_{pq} = I_{p_1} p_0 + I_{p_2} r_0$$

$$a_{pr} = I_{p_2} q_0$$

$$a_{qp} = -2\frac{I_{xz}}{I_{yy}} p_0 + \frac{I_{zz} - I_{xx}}{I_{yy}} q_0$$

$$a_{qr} = \frac{I_{zz} - I_{xx}}{I_{yy}} p_0 + 2\frac{I_{xz}}{I_{yy}} r_0$$

$$a_{rp} = I_{r_1} q_0$$

$$a_{rq} = I_{r_1} p_0 + I_{r_2} r_0$$

$$a_{rr} = I_{r_2} q_0$$

$$I_{p_1} = \frac{I_{xx} - I_{yy} + I_{zz}}{I^*}$$

$$I_{p_2} = \frac{(I_{yy} - I_{zz}) I_{zz} - I_{xz}^2}{I^*}$$

$$I_{r_1} = \frac{(I_{xx} - I_{yy}) I_{xx} - I_{xz}^2}{I^*}$$

$$I_{r_2} = \frac{(-I_{xx} + I_{yy} - I_{zz}) I_{xz}}{I^*}$$

$$I^* = I_{xx} I_{zz} - I_{xz} I_{xz}$$

$$b_{px} = \frac{I_{zz}}{I^*}$$

$$b_{pz} = \frac{I_{xz}}{I^*}$$

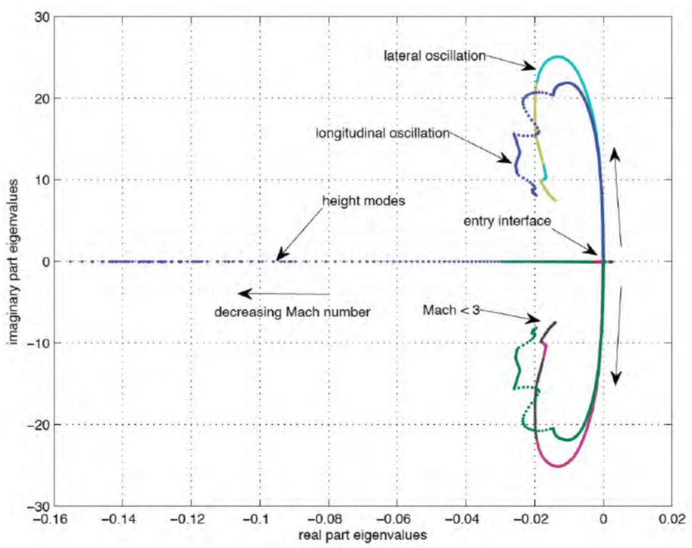
$$b_{qy} = \frac{1}{I_{yy}}$$

$$b_{rx} = \frac{I_{xz}}{I^*}$$

$$b_{rz} = \frac{I_{xx}}{I^*}$$

(Strong) coupling between X and Z axis: more difficult for controller

(Stronger) coupling between symmetric and asymmetric motion



- The 5 eigenmodes can not be found.
- Longitudinal and lateral oscillation show coupling.
- + time + high frequency oscillations
- Aperiodic height modes
- All eigenmodes are stable
- Very short periods

- Longitudinal oscillation:
  - Marginally (at entry interface) to reasonably damped motion
  - At entry interface, motion is driven by inertia only with P = 885 s
  - Due to strongly increasing dynamic pressure, the frequency of oscillations increases, until P = 0.4 s (!) between t = 200 and 250 s
  - Strong (inertial) coupling with lateral motion for t < 100 s
  - Coupling with altitude (t<100 s) and velocity (t>50 s) due to non-zero nominal bank angle
- Lateral oscillation:
  - Coupling between the symmetric and asymmetric motion, as well as a coupling with translational motion
  - Slightly worse damped than the longitudinal oscillation
  - Overall, slightly shorter period (P_{min} ≈ 0.3 s)

- Lateral bank/height modes:
  - For t<100 s there are two very slow aperiodic modes (one stable and one unstable), driven by bank angle and height
  - The modes change in "pure" height modes
  - Even though the amplitude half time reduces, the unstable motion remains slow. $T½_{min}$ = 268 s for the stable mode.
- Height/velocity modes:
  - For t<150 s: unstable height oscillation and stable aperiodic height mode, both slow. Height oscillation becomes stable at t ≈ 150 s
  - The height oscillation breaks up in a stable height/velocity mode (T½ = 55 s), and a stable height mode (T½ = 550 s)
  - The height/velocity mode becomes "well" damped towards end of the flight (T½ = 5 s)
  - The stable height mode becomes unstable (t=150 s) but remains slow

Formula Sheet

T or F:

Fly by Wire, reversible? FALSE

Fast aircraft, Earth non-rotating? FALSE

Phugoid, pitch rate and airspeed vary? FALSE

A more negative C_{l_β} leads to more damped Dutch roll! False

When aircraft dynamically ustable must be statically unstable: FALSE

Linearized E.O.M describe exactly motion of Aircraft: FALSE

C_{Y_β} of an aircraft, mostly produced by wing: FALSE

C_{Z_α} and C_{X_α} have a significant effect on S.P.M and P.M: FALSE

Shifting the x_{cg} fwd will decrease the required control forces: FALSE increases longi. stability

C_{l_β} is cause by $\Delta\alpha$ between the wings? TRUE

Wings of conventional airplane, negative C_{l_β}? TRUE

Unstable spiral preffered over unstable D.R.? True

Most pilots, unpleasant to fly if $\frac{\partial F_e}{\partial V} < 0$ True

Newtons laws motion only valid for inertial R.F True

Positive deflections of control surfaces cause negative roll, pitch and yaw: True

Contribution of fuselage to C_{n_β} is destabilizing True

$C_{m_{\dot\alpha}}$ is used to account for time delay of ε hit the horizontal tail. True

ω_n is at most equal or less than the undamped frequency True

$$\omega_n = \omega_0 \cdot \underbrace{\sqrt{1-\zeta^2}}_{\text{always less than } 1}$$

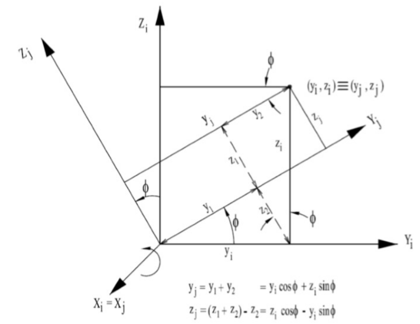
$$\boxed{\omega_n < \omega_0}$$

Table with coefficients:

Coefficient	Sign	Change
C_{Y_β}	-	+
C_{l_β}	-	+
C_{n_β}	+	+
C_{Y_p}	-	+
C_{l_p}	-	+
C_{n_p}	-	-
C_{Y_r}	+	+
C_{l_r}	+	+
C_{n_r}	-	+
$C_{Y_{\delta_r}}$	+	+
$C_{l_{\delta_r}}$	+	+
$C_{n_{\delta_r}}$	-	+

Axis Transformations:

$$T_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \qquad T_y = \begin{bmatrix} c(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \qquad T_z = \begin{bmatrix} c(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & c(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



From B to C $\rightarrow T_{CB}$ "order inversed but same direction"

- Use right hand rule to see if positive or negative.

Angular rate of D w.r.t. A $\longrightarrow \Omega^D_{DA} = \Omega^D_{DC} + \Omega^C_{CB} + \Omega^B_{BA}$

from A to B stille for the angle!

usually it can be broken down into

$$\Omega^D_{DA} = \dot{\gamma} \cdot z_D + \dot{\beta} \cdot Y_C + \dot{\alpha} \cdot X_B$$

Have to be expressed in D coordinates.

We write z_D, Y_C and X_B because the angle rotates about this axes.

$Y_C = T_{CD} \cdot Y_D$ Then group into X_D, Y_D, z_D.

Show that

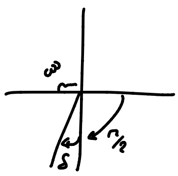
$$\dot{\phi} = p + \sin\phi \cdot \tan\theta \cdot q + \cos\phi \cdot \tan\theta \cdot r$$
$$\dot{\theta} = \cos\phi \cdot q - \sin\phi \cdot r$$
$$\dot{\psi} = \frac{\sin\phi}{\cos\theta} q + \frac{\cos\phi}{\cos\theta} r$$

From before: $\Omega_{BI} = \Omega^3_{B1} + \Omega^1_{12} + \Omega^2_{2I}$

$$\Omega_{BI} = \dot{\phi} X_B + \dot{\theta} Y_1 + \dot{\psi} z_2$$

2 ways

- either change Y_1, z_2
- or change Ω^1_{12}, and Ω^2_{2I}



$$\boxed{\begin{aligned} \sin(-\alpha) &= -\sin(\alpha) \\ \cos(-\alpha) &= \cos(\alpha) \\ \sin\left(\tfrac{\pi}{2} + \delta\right) &= \cos(\delta) \\ \cos\left(\tfrac{\pi}{2} + \delta\right) &= -\sin(\delta) \\ \sin\left(-\tfrac{\pi}{2} - \delta\right) &= -\cos(\delta) \\ \cos\left(-\tfrac{\pi}{2} - \delta\right) &= \sin(\delta) \end{aligned}}$$

Once you have the matrix:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi \cos\theta \\ 0 & -\sin\phi & \cos\phi \cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

How to inverse matrix

1st

$$\begin{bmatrix} \begin{vmatrix} \cos\phi & \sin\phi \cdot \cos\theta \\ -\sin\phi & \cos\phi \cdot \cos\theta \end{vmatrix} & \begin{vmatrix} 0 & \sin\phi \cdot \cos\theta \\ 0 & \cos\phi \cdot \cos\theta \end{vmatrix} & \begin{vmatrix} 0 & \cos\phi \\ 0 & -\sin\phi \end{vmatrix} \\ \begin{vmatrix} 0 & -\sin\theta \\ -\sin\phi & \cos\phi \cos\theta \end{vmatrix} & \begin{vmatrix} 1 & -\sin\theta \\ 0 & \cos\phi \cdot \cos\theta \end{vmatrix} & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$$

2nd

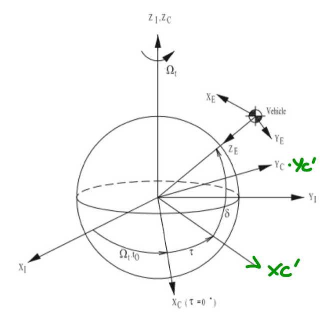
$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

3rd

$$\begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix}$$

4th: Calculate determinant: $\frac{1}{\det} \cdot \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$

From E to C for example $\quad T_{CE} = T_{CC'} \cdot T_{C'E} = T_z(-\tau) \cdot T_y\left(\frac{\pi}{2} + \delta\right)$



First keep in mind that right hand rule must work with E to C so then, from E to C' we see that, if we roll the fingers in the same direction as the angle moves (E to C'), our thumb points in the Yc' direction. Thus we write...

$$T_{C'E} = T_y\left(\delta + \frac{\pi}{2}\right)$$

$\frac{\pi}{2}$ is only added to δ

For C' to C if we roll the fingers from C' to C we see that the thumb points opposite to the z axis thus...

$$T_{C'C} = T_z(-\tau)$$

AXIS TRANSFORMATIONS:

Explain how α, β and μ are obtained. Given T_{bt}, T_{et} and T_{eb}

$$T_{bt} = f(\alpha, \beta, \mu) \qquad T_{bt} = T_y(\alpha) \cdot T_z(-\beta) \cdot T_x(-\mu) \text{ from transformations}$$

$$\boxed{T_{bt} = T_{eb}^{-1} \cdot T_{et}}$$

T_x → lets call it like this

$$T_{bt} = \begin{bmatrix} c\alpha \cdot c\beta & -c\alpha \cdot s\beta \cdot c\mu - s\alpha \cdot s\mu & +c\alpha \cdot s\beta \cdot s\mu - s\alpha \cdot c\mu \\ s(\beta) & c\beta \cdot c\mu & -c\beta \cdot s\mu \\ s\alpha \cdot c\beta & -s\alpha \cdot s\beta \cdot c\mu + c\alpha \cdot s\mu & +s\alpha\, s\beta\, s\mu + c\alpha \cdot c\mu \end{bmatrix}$$

to find α, β, μ

$$\frac{\sin(\alpha) \cdot \cancel{\cos(\beta)}}{\cos(\alpha) \cdot \cancel{\cos(\beta)}} = \tan(\alpha) = \frac{T_x(1,3)}{T_x(1,1)} \qquad \boxed{\alpha = \arctan\left(\frac{T_x(1,3)}{T_x(1,1)}\right)}$$

Same for the others.

STARTING FROM:

$$m \cdot \frac{dV_c^c}{dt} = F_{ext}^c$$

$$F_{ext}^T = \begin{bmatrix} -D - mg \cdot \sin\gamma \\ L - mg \cdot \cos\gamma \end{bmatrix}$$

arrive to:

$$m \cdot \frac{dV}{\partial t} = -D - mg \cdot \sin\gamma$$

$$m \cdot V \cdot \frac{d\gamma}{\partial t} = L - m \cdot g \cdot \cos\gamma \cdot \left(1 - \frac{V^2}{V_c^2}\right)$$

$$\frac{\partial V_c^c}{\partial t} = \frac{d}{dt}(V x_T^T) = \frac{dV}{dt} x_T^T + V \cdot \frac{dx_T^T}{dt}$$

Explain the following terms: Is not coriolis, dragging acceleration

$-2m\Omega_{CI}^c \times V_c^c$: Coriolis acceleration, due to the motion of the object in a rotating frame

$-m \cdot \Omega_{CI}^c \times r_{CM}^c$: Apparent centripetal acceleration due to the angular motion of the moving frame.

How to transform $\frac{\partial V_c^c}{\partial t}$ into E frame:

Differenciating their regular components, applying the chain rule and then finding expressions for the time derivatives of the unit vectors

---

c) $r = -R z_E$ $\quad \frac{\partial r}{\partial t}$ in E frame = ?

$$\frac{\partial r}{\partial t} = -\dot{R} z_E - R \cdot \frac{\partial z_E}{\partial t} \quad \text{where} \quad \frac{\partial z_E}{\partial t} = \Omega_{EC}^E \times z_E$$

$$\Omega_{EC} = -\dot{\delta} y_E + \dot{\tau} z_c \qquad \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = T_{CE} \begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix}$$

$z_c = \cos\delta\, x_E - \sin\delta\, z_E$ $\quad$ substitude

Final expression only x_E, y_E, z_E!

LINEARIZATION:

Find A and B:

IMPORTANT, ALL VARIABLES AFTER DIFFERENTIATION $\boxed{x_0}$

$$A = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}\right|_{x_0} \qquad B = \left.\begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_m} \end{bmatrix}\right|_{u_0}$$

Indicate state variables: for the control variables read the assumptions

$$\Delta x = [\Delta V, \Delta\alpha, \Delta q, \Delta p, \Delta r, \Delta\beta, \ldots]^T \qquad \Delta u = [\Delta\alpha, \Delta M_{T,z}, \Delta M_{T,y}]$$

$L = \bar{q} \cdot S \cdot C_L \rightarrow \Delta V, \Delta\alpha$

$D = \bar{q} \cdot S \cdot C_D$

$$\boxed{\begin{gathered} \frac{\partial q}{\partial V} = \rho V \quad \frac{\partial q}{\partial V} = \frac{2q}{V} \qquad M = \frac{V}{a} \quad \frac{\partial M}{\partial V} = \frac{1}{a} \cdot \frac{V}{V} = \frac{M}{V} \\ \frac{\partial C_L}{\partial V} \cdot \frac{\partial M}{\partial M} = \frac{\partial C_L}{\partial M} \cdot \frac{\partial M}{\partial V} = \frac{\partial C_L}{\partial M} \cdot \frac{M}{V} \end{gathered}}$$

$M_x = \mathcal{L} + M_{T,x} \qquad \mathcal{L} = C_p \cdot \bar{q} \cdot S_{ref} \cdot d_{ref} \longrightarrow \Delta V, \underbrace{\Delta\alpha}, \Delta\beta$

$M_z = \mathcal{N} + M_{T,z} \qquad \mathcal{N} = C_n \cdot \bar{q} \cdot S_{ref} \cdot d_{ref} \longrightarrow \Delta V, \Delta\alpha, \Delta\beta$

$\longrightarrow C_{p_\alpha}, C_{n_\alpha}$ only.

Some derivatives:

$\sin(\alpha) \longrightarrow \cos(\alpha)$	$\frac{1}{\cos(x)} = \frac{\sin(x)}{\cos^2(x)}$	$\frac{1}{x^2} = \frac{-2x}{x^4} = -\frac{2}{x^3}$
$\cos(\alpha) \longrightarrow -\sin(\alpha)$		
$\tan(\alpha) \longrightarrow 1/\cos^2\alpha$	$\frac{1}{\sin(x)} = -\frac{\cos(x)}{\cos^2(x)}$	

$\longrightarrow \dfrac{\frac{\partial 1}{\partial x} \cdot x^2 - \frac{\partial x^2}{\partial x} \cdot 1}{(x^2)^2}$

remember that is always possible to have only $\frac{1}{V^2} \cdot \ldots$

Example

$\dot{V} = -\frac{D}{m} - g \cdot \sin\gamma$

First identify variables: $\Delta V, \Delta\alpha, \Delta\gamma$

$\underbrace{D = q \cdot S \cdot C_D \text{ where } q = \frac{1}{2}\rho V^2}$

In these two, $\Delta V, \Delta\alpha$ are variables.

C_D changes with ΔV and $\Delta\alpha$ while q changes with V.

Then write:

$$\Delta\dot{V} = \left.\frac{\partial\dot{V}}{\partial V}\right|_0 \Delta V + \left.\frac{\partial\dot{V}}{\partial\alpha}\right|_0 \Delta\alpha + \left.\frac{\partial\dot{V}}{\partial\gamma}\right|_0 \Delta\gamma$$

$$\left.\frac{\partial\dot{V}}{\partial\gamma}\right|_0 = -g \cdot \cos\gamma_0$$

write the 0 after all variables.

$$\left.\frac{\partial\dot{V}}{\partial\alpha}\right|_0 = -\underbrace{\frac{q_0 \cdot S}{m}}_{\text{this is the same}} \cdot \frac{C_D}{\partial\alpha} \qquad \left.\frac{\partial\dot{V}}{\partial V}\right|_0 = -\frac{1}{m} \cdot \frac{\partial q}{\partial V} \cdot S \cdot C_D - \frac{1}{m} \cdot q_0 \cdot S \cdot \frac{C_D}{\partial V}$$

$\boxed{\text{these two are derived as above in green.}}$

Dynamic stability:

1. Start with equations of motion. And remove the coefficients neglected by assumptions.
2. Derive the characteristic equation.

If there is D_c or D_b, substitute them by λ

Else, add on the diagonals $-\lambda$

$$\boxed{\lambda = \xi_c + \eta_c \lambda}$$

If ξ_c is negative the motion is stable

If ξ_c is positive the motion is unstable

$$\boxed{A = \dots \lambda^2 \quad B = \dots \lambda \quad C = \dots}$$

→ If $\frac{-B}{2A} < 0$ the motion is stable

If $B^2 - 4AC < 0$ the motion is periodic

↳ you need a complex eigenvalue.

3. Formulas on eigenvalues: add $\frac{\bar{c}}{V}$ when they give the values on the problem.

$$P = \frac{2\pi}{\eta_c} \cdot \frac{\bar{c}}{V}$$

→ complex part because if it's not complex, no period

$$T(1/2) = \frac{\ln(1/2)}{\xi_c} \cdot \frac{\bar{c}}{V}$$

damping, so real part.

$$C_{1/2} = \frac{T_{1/2}}{P}$$

$$\zeta = -\frac{\xi_c}{\sqrt{\xi_c^2 + \eta_c^2}} \qquad \omega_n = \frac{2\pi}{P} \qquad \omega_0 = \frac{\omega_n}{\sqrt{1-\zeta^2}}$$

some variants:

Add flight control system: $\delta_e = k_q \cdot \frac{q\bar{c}}{V}$ add it into the main matrix because of $\frac{q\bar{c}}{V}$

Also make sure that coefficients of δ_e are also moved.

FROM ASSYMETRIC EQUATION OF MOTION TO STATIONARY STRAIGHT FLIGHT

$$\begin{bmatrix} C_{Y_\beta} + \cancel{(C_{Y_{\dot\beta}} - 2\mu_b) D_b} & C_L & C_{Y_p} & C_{Y_r} - 4\mu_b \\ 0 & \cancel{-\frac{1}{2} D_b} & 1 & 0 \\ C_{\ell_\beta} & 0 & C_{\ell_p} - 4\mu_b K_X^2 D_b & C_{\ell_r} + 4\mu_b K_{XZ} D_b \\ C_{n_\beta} + \cancel{C_{n_{\dot\beta}} D_b} & 0 & C_{n_p} + 4\mu_b K_{XZ} D_b & C_{n_r} - 4\mu_b K_Z^2 D_b \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ \cancel{\frac{pb}{2V}} \\ \cancel{\frac{rb}{2V}} \end{bmatrix} = \begin{bmatrix} -C_{Y_{\delta_a}} & -C_{Y_{\delta_r}} \\ 0 & 0 \\ -C_{\ell_{\delta_a}} & -C_{\ell_{\delta_r}} \\ -C_{n_{\delta_a}} & -C_{n_{\delta_r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

$p = 0$ $\quad D_b = 0$

$r = 0$

δ_a = variable

δ_r = variable

$$\begin{bmatrix} C_{Y_\beta} & C_L & C_{Y_{\delta_a}} & C_{Y_{\delta_r}} \\ C_{\ell_\beta} & 0 & C_{\ell_{\delta_a}} & C_{\ell_{\delta_r}} \\ C_{n_\beta} & 0 & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ C_{n_e} \end{bmatrix}$$

in case of engine failure. and ΔT known.

→ If ΔT inside not know, ΔT can be added as a variable.

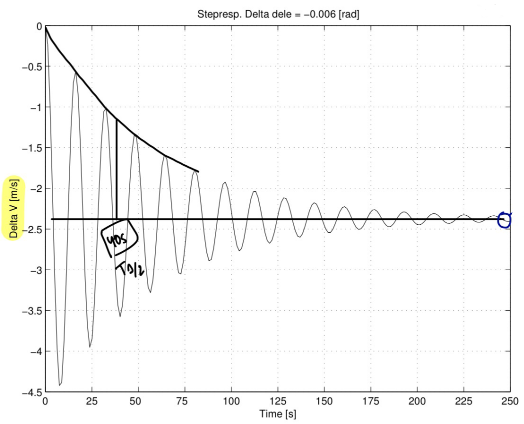
$$C_{n_e} = k \cdot \frac{\Delta T \cdot y_e}{\frac{1}{2}\rho V^2 \cdot S \cdot b}$$

b) Yaw rate of 180 deg per minute...

Make equations of motion add $\frac{rb}{2V}$ on the variables given! Only modifyable state variables as state variables.

Dynamic stability:

$\Delta \delta_e = -0.006$



1. Which eigenmotion?

Either phugoid or short period, because of elevator input.

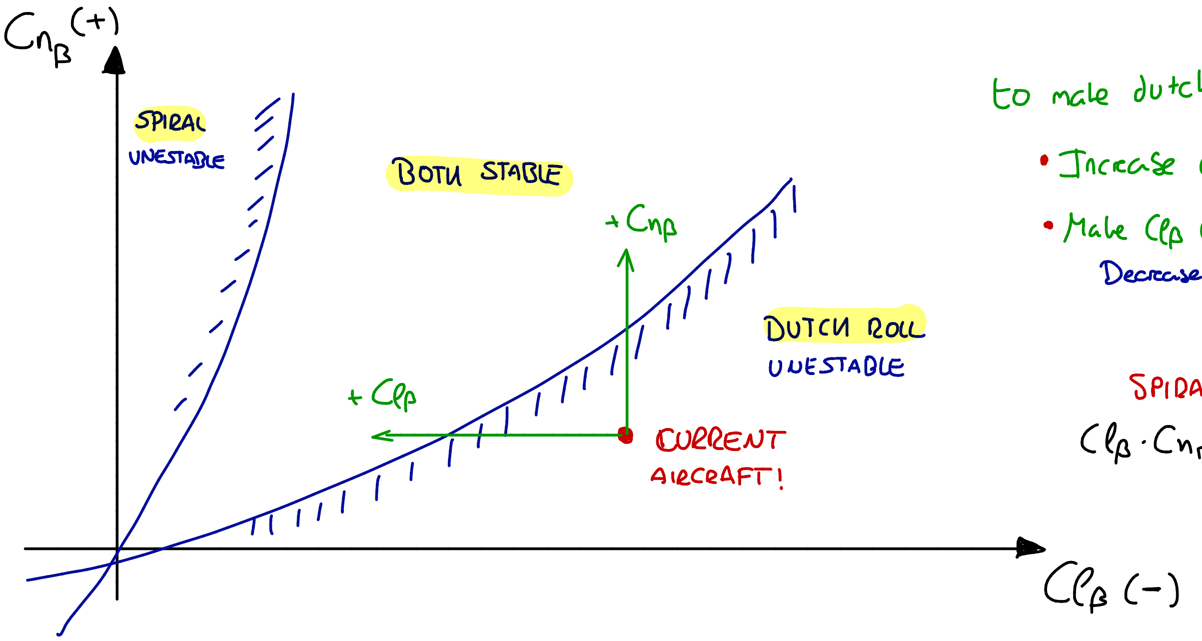
2. Period? T(1/2)?

Do average. $\frac{\text{time}}{\#\text{periods}} = 16s$ Time at $\frac{(0-(-2.4))}{2} \approx 40s$

3. Value of static elevator trim stability $\frac{d\delta_e}{\partial V}$? Stable?

$$\frac{\Delta \delta_e}{\Delta V} = \frac{-0.006}{-2.4} = 0.0025 \ \frac{rad}{m/s}$$

bigger than 0. Stable remember graph!



to make dutch roll stable

- Increase C_{n_β} $\boxed{+S_v}$
- Make C_{l_p} less negative.
  Decrease dihedral

SPIRAL STABILITY:

$$C_{l_\beta} \cdot C_{n_r} - C_{n_\beta} \cdot C_{l_r} > 0$$

C_{n_p} = yaw due to roll

For a positive roll motion yaw is negative so $C_{n_p} < 0$

When the aircraft is rolling experiences more angle of attack in the down going wing to the left thus meaning that part of the left acts forward, creating a yawing moment negative

Eigenmodes:

- Short period motion
- Phugoid oscillation
- Spiral motion
- Aperiodic roll
- Dutch roll

Routh - Hurwitz criteria:

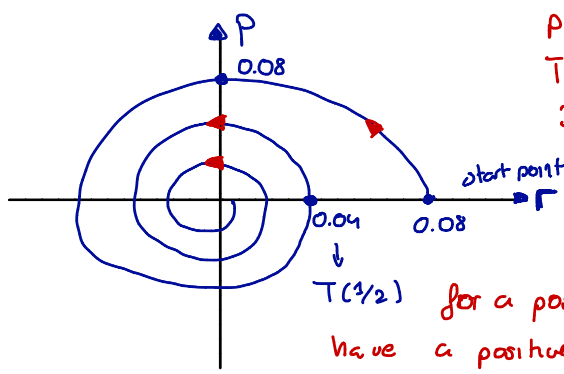
A > 0 B > 0 C > 0 D > 0 E > 0

$BCD - AD^2 - B^2E > 0$ Then all eigenvalues are negative. Stability!

Both spiral and dutch stable if this happens:

BCD ··· < 0 → Only spiral stability.

Dutch roll motion: p vs r



Stable

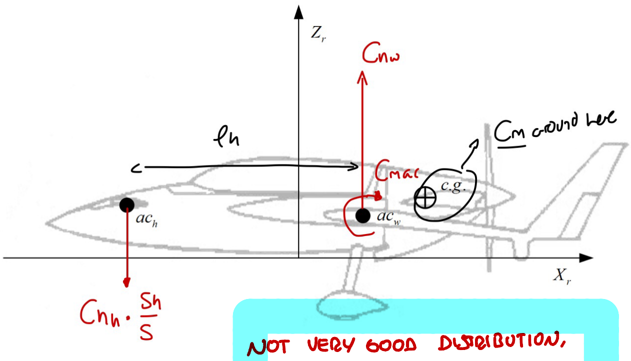
p, r = 0.08 max

T(1/2) = P

3 Periods

for a positive yaw we have a positive roll!

Static Stability:



NOT VERY GOOD DISTRIBUTION,
C_{Nh} positive plus cg in between.

1. Draw the coefficients:

Find C_m

NOT SURE IF NECESARY TO SHOW STEPS FOR $\left(\frac{V_h}{V}\right)^2$

2. $$C_m = C_{mac} + C_{Nw}\left(\frac{x_{cg} - x_{ac}}{\bar{c}}\right) - C_{Nh} \cdot \frac{S_h}{S} \cdot \frac{\ell_h}{\bar{c}} \cdot \left(\frac{V_h}{V}\right)^2$$

Remember $\bar{c}$

3. Find $C_{m\alpha}$

$$C_{m\alpha} = 0 + C_{Nw\alpha} \cdot \left(\frac{x_{cg} - x_{ac}}{\bar{c}}\right) - C_{Nh\alpha} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \cdot \frac{S_h}{S} \cdot \frac{\ell_h}{\bar{c}} \cdot \left(\frac{V_h}{V}\right)^2$$

4. Derive X_{nfix}

To here, start with this equation:

$$C_N = C_{Nw} - C_{Nh} \cdot \frac{S_h}{S} \cdot \left(\frac{V_h}{V}\right)^2 \longrightarrow \text{Multiply by } \left(\frac{X_{nfix} - X_w}{c}\right)$$

$$C_{N\alpha} = C_{Nw\alpha} - C_{Nh\alpha} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \cdot \frac{S_h}{S} \cdot \left(\frac{V_h}{V}\right)^2$$

(2) $$C_{N\alpha} \cdot \frac{X_{nfix} - X_w}{\bar{c}} = C_{Nw\alpha} \cdot \left(\frac{X_{nfix} - X_w}{\bar{c}}\right) - C_{Nh\alpha} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(\frac{X_{nfix} - X_w}{\bar{c}}\right) \cdot \frac{S_h}{S} \cdot \left(\frac{V_h}{V}\right)^2$$

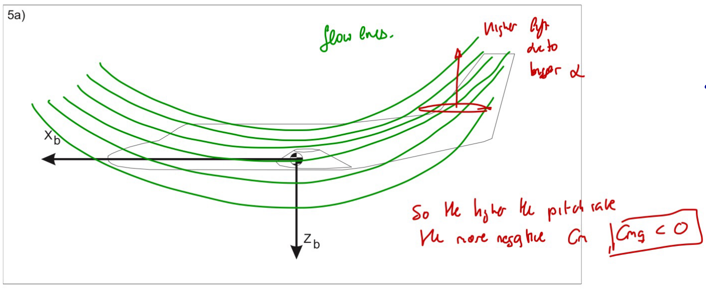
(1) From point 3, adapt for X_{nfix}

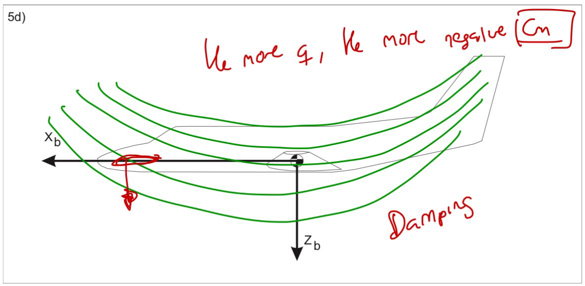
$$0 = C_{Nw\alpha}\left(\frac{X_{nfix} - X_w}{\bar{c}}\right) - C_{Nh\alpha} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(\frac{X_{nfix} - X_h}{\bar{c}}\right) \cdot \frac{S_h}{S} \cdot \left(\frac{V_h}{V}\right)^2$$

(1) − (2) : $\ell_h = X_w - X_h$

$$\frac{X_{nfix} - X_w}{\bar{c}} = \frac{C_{Nh\alpha}}{C_{N\alpha}} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \cdot \frac{S_h}{S} \cdot \left(\frac{V_h}{V}\right)^2 \cdot \frac{\ell_h}{\bar{c}}$$

Static Stability:





2. Derive relation for C_{m_q} fixed in terms of $\Delta\alpha$ and $C_{N_{h\alpha}}$.

$$\sin(\Delta\alpha) = \frac{x - x_{cg}}{R}$$

Since $R >> \ell_h$ we can say that

$$\Delta\alpha = \frac{x - x_{cg}}{R} \qquad q \cdot R = V \qquad R = \frac{V}{q}$$

$$\Delta\alpha = (x - x_{cg})\frac{q}{V} = \frac{(x - x_{cg})}{\bar{c}} \cdot \frac{q \cdot \bar{c}}{V}$$

At the tail $\rightarrow$ $\boxed{\Delta\alpha_h = \frac{\ell_h}{\bar{c}} \cdot \frac{q\bar{c}}{V}}$

Change of C_{N_h} due to $\Delta\alpha_h$

Full term for q

$$\Delta C_{N_h} = C_{N_{h\alpha}}\left(\frac{V_h}{V}\right)^2 \cdot \frac{S_h}{S} \cdot \Delta\alpha_h = C_{N_{h\alpha}} \cdot \left(\frac{V_h}{V}\right)^2 \cdot \frac{S_h}{S} \cdot \frac{\ell_h}{\bar{c}} \cdot \frac{q\bar{c}}{V}$$

$$\Delta C_{m_q} = \Delta C_{N_h} \cdot \frac{\ell_h}{\bar{c}} \qquad \frac{\partial C_m}{\partial q} = C_{N_{h\alpha}} \cdot \left(\frac{V_h}{V}\right)^2 \cdot \frac{S_h}{S} \cdot \frac{\ell_h^2}{\bar{c}^2}$$

IMPORTANT

Stick fixed lies after stick free

Conventional Aircraft during trimmed horizontal flight:

If augmentation added: Pulls column forward with 50N. $F_e = F_e + F_{e_{aug}}$

Remember to change x_{cg} and $[W]$ if the aircraft removes weight!

USE FORMULAS!

c) Effect of weight release on the phugoid and short period motion in terms of damping?

Period is reduced, the motions are better damped since the center of gravity shifts forward.

REMEMBER $\left.\frac{\partial F}{\partial V}\right|_0$ has to be found for when $\boxed{F_e = 0}$ to find stick force stability

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Where would the center of gravity be located?

(a) The center of gravity of the rutan voyager should lie in between the main wing and the canard. In this way, the both the canard and the main wing can generate positive lift, which increases the efficiency of the aircraft.

Why is it longitudinally stable?

(b) The neutral point of the Rutan voyager lies aft of the center of mass/gravity.

(c) The advantage of having a canard instead of a horizontal stabilizer is:

- A canard uses positive lift to counteract the negative moment generated by the main wing while a horizontal stabilizer in a large part of the flight envelope uses negative lift. Therefore the performance will increase when using a canard.
- Due to a higher efficiency of producing lift, the overall drag component becomes less in magnitude.
- A canard operates in cleaner air since it is in front of the main wing. This means that the control surfaces become more efficient.

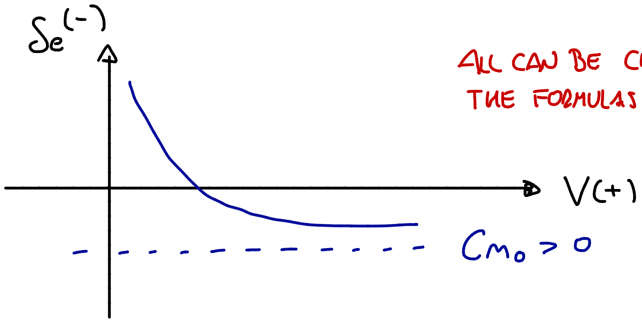
(d) The pilots of the Voyager had to constantly trim the aircraft because the mass fraction of its fuel was very high. As fuel is burned, the position of the center of gravity changes. Static stability can be lost if the center of gravity moves behind the center of pressure. The pilots must prevent this by pumping fuel from tank to tank, thereby ensuring that the c.g. stays in front of the center of pressure.

Remember, change of trim due to c.g. moved in flight.

Elevator Trim Curve

Trim curve ($\delta e - V$) if:

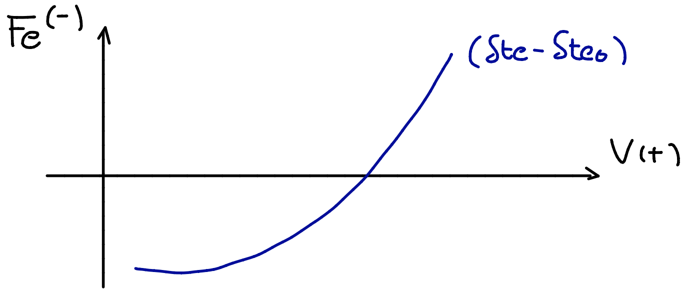
- Statically stable, stick fixed
- $C_{m_0} > 0$



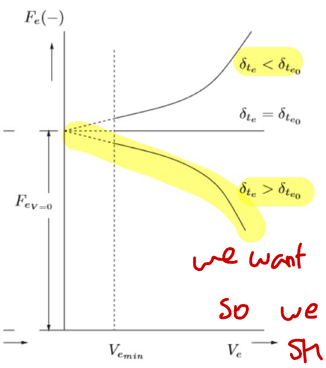
All can be checked with the formulas apparently!

Control force curve ($Fe - V$) if:

- Statically unstable, stick free
- $\delta te < \delta te_0$



VARIANTS:



we want $F_e = 0$ so we can have stick free