FLIGHT DYNAMICS AE3202

NEW STYLE

EXAMINATION

April 6, 2011

Delft University of Technology
Faculty of Aerospace Engineering
Control and Simulation Division

This exam contains 6 questions.

You may use the formulas on the given formula sheets.

Hand in your derivations

PLEASE NOTE

Always write down the correct units for each computed parameter value. Be mindful for any required conversion before making any computations. **Always** write down the derivations of your answers.

Question 1 (15 %)

In the accompanying figure three reference frames are indicated in relation with a schematic representation of a vehicle. These frames are the body frame (index b), the aerodynamic frame (index a) and the trajectory frame (index t). The relation between these frames can be described by the indicated angles (angle of attack α, angle of sideslip β and bank angle μ). In the figure, all angles are positive.

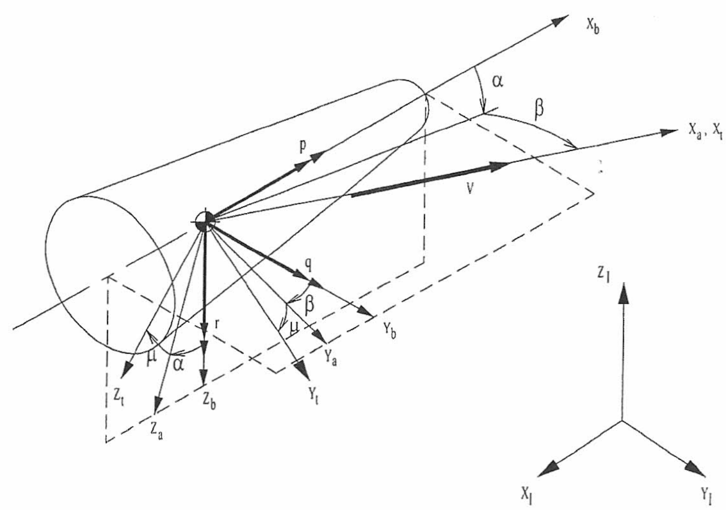


Figure 1: Reference frames.

(a) Derive the transformation matrix $\mathbb{T}_{bt}$ (from trajectory to body frame). Before composing the final transformation matrix $\mathbb{T}_{bt}$, specify the transformation matrices for each of the rotations.

(b) Suppose we also need the inverse transformation matrix. Explain (in words) how we can derive this matrix from $\mathbb{T}_{bt}$ found above. Carefully state the mathematical principle involved.

Question 2 (15 %)

(a) Sketch the elevator trim curve ($\delta_e - V$) if the following conditions hold:

- Statically stable, stick fixed
- $C_{m_0} > 0$

Clearly indicate the negative and positive side of the axes!

(b) Sketch the elevator control force curve ($F_e - V$) if the following conditions hold:

- Statically unstable, stick free
- $\delta_{t_e} < \delta_{t_0}$

Clearly indicate the negative and positive side of the axes!

Question 3 (10 %)

The characteristic equation for the asymmetric motions is:

$$A\lambda_b^4 + B\lambda_b^3 + C\lambda_b^2 + D\lambda_b + E = 0$$

According to the Routh-Hurwitz stability criteria it is necessary that,

$$A > 0 \qquad B > 0 \qquad C > 0 \qquad D > 0 \qquad E > 0$$

and,

$$BCD - AD^2 - B^2E > 0$$

Consider an aircraft which coefficients have the following numerical values:

$$A = 120 \qquad B = 40 \qquad C = 10 \qquad D = 2.5 \qquad E = 0.2$$

Which of the following statements is (are) correct:
(Hint: take into account the lateral stability diagram)

(a) Both the Dutch roll mode and the spiral mode are unstable, the aircraft does not possess spiral stability.

(b) The Dutch roll mode is stable, the spiral mode is unstable, and the aircraft does not possess spiral stability.

(c) All the eigenmotions are stable, but the aircraft does not possess spiral stability.

(d) The Dutch roll mode is not stable, the spiral mode is stable, and the aircraft possesses spiral stability.

(e) All the eigenmotions are stable, and the aircraft possesses spiral stability.

(f) The Dutch roll mode is unstable, the spiral mode is stable and the aircraft does not possess spiral stability.

V	=	51.82 m/sec	m	=	5897 kg	$\bar{c}$	=	2.134 m
S	=	21.37 m^2				μ_c	=	105.56
K_Y^2	=	0.8979	x_{cg}	=	0.32 $\bar{c}$			
C_{X_0}	=	0	C_{Z_0}	=	−1.640			
C_{X_u}	=	0	C_{Z_u}	=	−3.72	C_{m_u}	=	−0.004
C_{X_α}	=	0.580	C_{Z_α}	=	−5.5296	C_{m_α}	=	−0.660
$C_{X_{\dot{\alpha}}}$	=	0	$C_{Z_{\dot{\alpha}}}$	=	−0.80	$C_{m_{\dot{\alpha}}}$	=	−2.50
C_{X_q}	=	0	C_{Z_q}	=	−2.050	C_{m_q}	=	−6.75
$C_{X_{\delta_e}}$	=	0	$C_{Z_{\delta_e}}$	=	−0.400	$C_{m_{\delta_e}}$	=	−0.980

Table 1: Symmetric stability and control derivatives (question 4)

Question 4 (20 %)

(a) How many eigenmotions does a conventional aircraft have and what are they called?

(b) What is the name of the lateral eigenmotion which is characterized by an oscillation? Draw the time response of the eigenmotion during three periods in a figure with p and r on the axes taking into account the following assumptions:

- The eigenmotion is stable.
- Only this eigenmotion is excited and that the initialization phase (i.e. the time interval when the eigenmotion is introduced) has passed.
- The time to half amplitude is equal to the period.
- The maximal values of p and r is 0.08 rad/s during the considering time interval.

Clearly indicate the direction of time by using arrows on the lines, i.e. —▷—▷—.

Consider the aircraft data presented in table 1.

(c) Derive the simplified EOM for the short period mode assuming that the initial steady flight condition is level, i.e. $\gamma_0 = 0$, and the airspeed is constant (**Note** the flight path angle is allowed to vary!).

(d) Compute the dimensionless eigenvalues and the dimensional time to half amplitude, period, damped natural frequency, and the damping ratio for this eigenmotion.

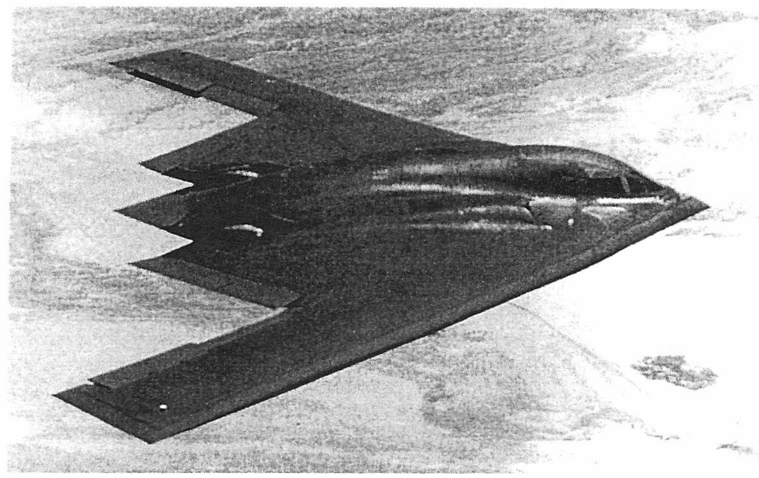


Figure 2: Northrop B-2 Spirit

Question 5 (20 %)

A flying wing is an aircraft without a tailplane, see Figure 2. As a result, the characteristic modes of the flying wing differ significantly from those of a conventional aircraft. In this question the dynamics of the short period mode of a flying wing will be examined.

(a) From the full linearized longitudinal equations of motion, derive a simplified form describing the short period motion. Assume that $\hat{u} = 0$ and that $\theta = 0$. Additionally, the stability derivatives $C_{m_{\dot{\alpha}}}$, $C_{Z_{\dot{\alpha}}}$, C_{m_q} and C_{Z_q} are assumed to be zero.

(b) Derive the characteristic equation of the simplified short period motion and determine the non-dimensional eigenvalues. Does the flying wing have a stable short period motion? Use the numerical data from Table 2.

(c) In order to improve the flying qualities of the flying wing, a simple pitch-rate feedback controller is integrated with the flight control system. This feedback controller has the following definition: $\delta_e = k_q \frac{q\bar{c}}{V}$. Derive the new characteristic equation for the simplified short period motion from part (b).

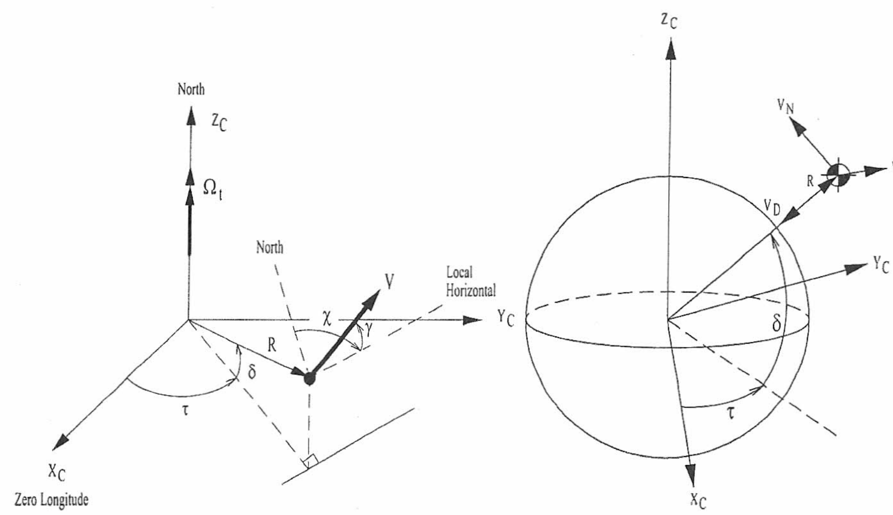
(d) For what range of the feedback gain k_q is the short period both stable and periodic? Use the numerical data from Table 2.

$V = 180[ms^{-1}]$	$\mu_c = 205$	$K_Y^2 = 0.98$	$C_{X_0} = 0$	$C_{X_u} = -0.22$	$C_{X_\alpha} = 0.465$
$C_{Z_0} = -1.14$	$C_{Z_u} = -2.72$	$C_{Z_\alpha} = 0.5$	$C_{Z_q} = 0$	$C_{Z_{\dot{\alpha}}} = 0$	$C_{Z_{\delta_e}} = -0.44$
$C_{m_\alpha} = -0.45$	$C_{m_u} = 0$	$C_{m_{\dot{\alpha}}} = 0$	$C_{m_{\delta_e}} = -1.28$	$C_{m_q} = 0$	

Table 2: Aircraft data for question 5

Question 6 (20 %)

For the analysis of re-entry problems it is common to express the position and velocity in spherical components, whereas for aircraft studies usually Cartesian velocity components are used. In the accompanying figure the two definitions of the state variables are given.



(a) Definition of spherical position and velocity state variables.

(b) Definition of spherical position and Cartesian velocity.

(a) Derive expressions for the spherical velocity components (velocity modulus V, flight-path angle γ and heading χ) as a function of the north, east and down velocity (V_N, V_E and V_D).

(b) A simplified expressions for the velocity derivative is given by

$$\dot{V} = -\frac{D}{m} - g\sin\gamma \tag{1}$$

Linearize this equation, where you can assume that the gravitational acceleration g is constant, i.e., $g = g_0$ and the drag is a function of angle of attack and Mach number. Express the resulting equation in state perturbations (note: the Mach number is *not* a state).

(c) For a vehicle entering the atmosphere the following eigenvalue λ and eigenvector $\boldsymbol{\mu}$ are calculated: $\lambda = 0.8723\,10^{-3} \pm 0.3257j$ and $\boldsymbol{\mu} = (0, 0, 0, 0.4007, 0, 0.0291, 0.7225, 1.0)^T$. The non-zero elements in $\boldsymbol{\mu}$ are related to $\Delta p, \Delta r, \Delta\beta$ and $\Delta\mu$.
(i) Identify the eigenmode.
(ii) Calculate the period P and damping ratio ζ of this eigenmode.
(iii) Is this mode stable? Explain why.