AEROSPACE FLIGHT DYNAMICS AND SIMULATION

AE3202

EXAMINATION - RESIT

August 28, 2012

Delft University of Technology
Faculty of Aerospace Engineering

This exam contains 5 questions.

You may use the formulas on the given formula sheets.

PLEASE NOTE

Always write down the correct units for each computed parameter value. Be mindful for any required conversion before making any computations. Always write down the derivations of your answers.

Question 1 (10 points)

Decide for every statement whether it is True or False. No explanation is required!

(a) For the equations of motion of a fast aircraft, the Earth can be considered flat and non-rotating.

(b) For a wing, the neutral point is the point where the line of action of the resultant aerodynamic force C_R crosses the mean aerodynamic chord.

(c) The main wing of a conventional aircraft reduces the local dynamic pressure at the horizontal tailplane.

(d) The contribution of the wing to C_{l_β} is caused by the difference in geometric angle of attack of the left and right hand halves of the wing in sideslipping flight.

(e) The contribution of the fuselage to C_{n_β} is destabilizing.

(f) The wing of a conventional aircraft has a negative contribution to C_{l_p}.

(g) In the phugoid motion, the pitch rate and airspeed vary in particular.

(h) A more negative C_{l_β} leads to a more damped Dutch roll motion.

(i) In general, an unstable spiral mode is preferable over an unstable Dutch roll mode.

Question 2 (23 points): static and dynamic transformations

In general Newton's Laws hold for a non-rotating inertial frame. If one wants to express the equations of motion in a rotating frame, one has to compensate for this rotation. This process involves both static and dynamic transformations. In the following, these transformations are addressed.

(a) **(3 points)** State the three (static) unit-axis transformation matrices for a rotation α around the X-, Y- and Z-axis.

(b) **(8 points)** In a dynamic rotation we assume that the angular displacement $d\alpha$ (the rotation) takes place in a certain time dt. Given the three frames A, B and C with corresponding rotations in Fig. 1 you are asked to:

i) **(2 points)** Set up the static transformation from frame A to C, $\mathbb{T}_{CA}$, in terms of the product of individual $\mathbb{T}$.

ii) **(6 points)** From the sequence of transformations, derive the angular rate of frame C with respect to frame A, $\mathbf{\Omega}_{CA}$. Make sure to express all components of $\mathbf{\Omega}_{CA}$ in components of frame C.

(c) **(12 points)** The position vector $\mathbf{R}$ is defined in the E-frame by $\mathbf{R} = -R\mathbf{z_E}$ (Fig. 2). The time derivative of $\mathbf{R}$ can be derived from this definition, and would include a component *relative* to the E-frame and one due to the *rotation* of the E-frame. You are asked to derive an expression for $\frac{d\mathbf{R}}{dt}$ expressed in components of the E-frame. Use can be made of the following:

$$\frac{d\mathbf{x_E}}{dt} = \mathbf{\Omega}^E_{EC} \times \mathbf{x_E} \qquad \frac{d\mathbf{y_E}}{dt} = \mathbf{\Omega}^E_{EC} \times \mathbf{y_E} \qquad \frac{d\mathbf{z_E}}{dt} = \mathbf{\Omega}^E_{EC} \times \mathbf{z_E}$$

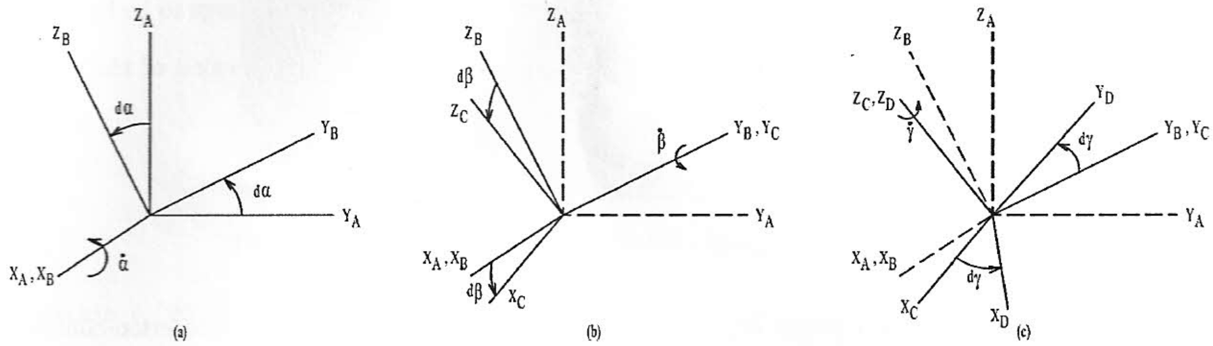

Figure 1: Transformation from A to C frame

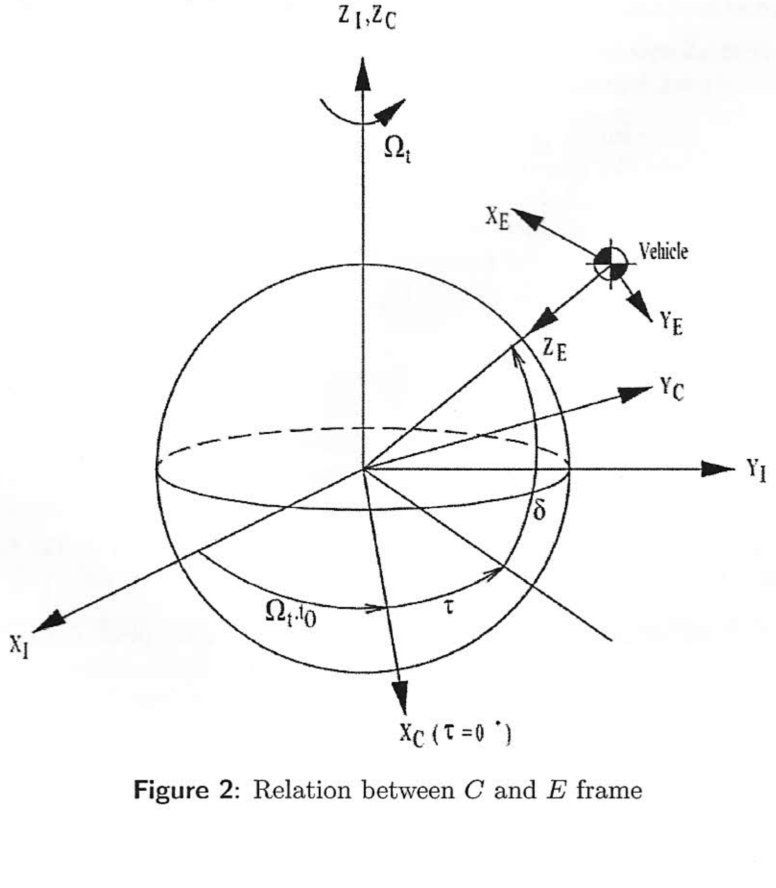

Figure 2: Relation between C and E frame

Question 3 (22 points): linearization

The open-loop flight behavior of an entry capsule is characterized by very strong oscillations around all three axes. Individual components of motion are hard to distinguish, because of a strong dynamic coupling, partially the result of an offset in the location of the center of mass in Z direction. This offset gives a product of inertia I_{xz} that is too large to be ignored.

(a) **(16 points)** The following set of equations describes the pitch motion of the entry capsule:

$$\dot{q} = \frac{M_y}{I_{yy}} + \frac{I_{xz}}{I_{yy}}\left(r^2 - p^2\right) + \frac{I_{zz} - I_{xx}}{I_{yy}} pr \tag{1}$$

$$\dot{\alpha} \approx q - \frac{L}{mV} \tag{2}$$

The external moment M_y contains an aerodynamic moment $\mathcal{M}$ and a reaction-control moment $M_{T,y}$.

Assume the following (READ THIS CAREFULLY):

- Both pitch moment $\mathcal{M}$ and lift L are a function of the angle of attack, α, and Mach number, M.
- The nominal state is a *trimmed* condition. Realize what this means for the nominal pitch moment.
- For the rotational motion considered, the atmospheric properties are constant.
- Due to the oscillatory nature of the rotational motion, the nominal angular rates p_0, q_0 and r_0 cannot be considered small
- The Mach number is *not* a state and has to be properly linearized.
- Translational and rotational motion are *not* decoupled.
- Consider all state and control variables (even though remaining state equations are not shown here)

You are asked to linearize the above equation and put it in state-space form. To facilitate you, follow the questions below. Clearly explain what you are doing.

(i) **(2 points)** Given the formulation for state-space form, $\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta\mathbf{u}$, explain how you obtain the matrices $\mathbf{A}$ and $\mathbf{B}$ from a set of n state equations $\mathbf{f}$.

(ii) **(2 points)** Indicate the state variables $\Delta\mathbf{x}$ and control variables $\Delta\mathbf{u}$ in Eqs. (1) and (2).

(iii) **(2 points)** As mentioned, the pitch-moment is, amongst others, a function of Mach number, but of course also of dynamic pressure. Derive the partial derivatives for Mach number and dynamic pressure w.r.t. the related state variables (again: the atmospheric properties are constant). Formulate your answer such that the atmospheric properties do *not* appear directly in your answer.

(iv) **(10 points)** Linearize both equations and write your answer in the form $\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta\mathbf{u}$ as two scalar equations.

(b) **(4 points)** A numerical representation of the eigenvalues for pitch motion is $\lambda_{1,2} = -1.9744 \cdot 10^{-2} \pm 1.5534 \cdot 10^{1} j$. You are asked to calculate the *dimensional* values of the damping factor, ζ, period, P, and amplitude half (or double) time, $T_{\frac{1}{2}}$ or T_2, depending on the nature of the eigenmotion. First state the used equation, then your numerical value.

(c) **(2 points)** Is the mode under (c) stable or unstable? Explain why.

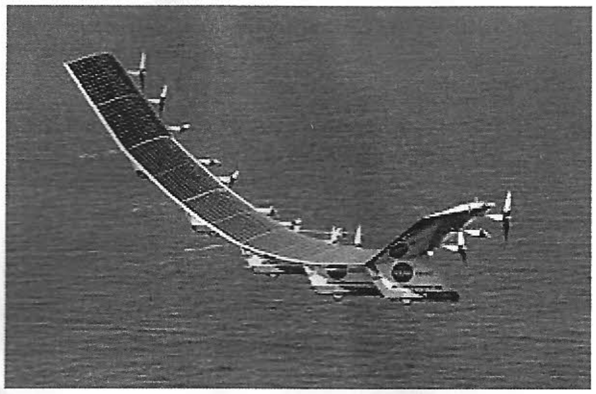

Figure 3: Helios - NASA solar powered flying wing

Question 4 (25 %)

A flying wing is an aircraft without a tailplane, see Figure 3. The characteristic modes of a flying wing differ significantly from those of a conventional aircraft. In this question the dynamics of the short period mode of a flying wing will be examined.

(a) From the full linearized longitudinal equations of motion, derive a simplified form describing the short period motion, such that the state vector becomes $[\alpha \;\; \frac{q\bar{c}}{V}]^T$. Additionally, the stability derivatives $C_{m_{\dot{\alpha}}}$, $C_{Z_{\dot{\alpha}}}$, C_{m_q} and C_{Z_q} are assumed to be zero.

(b) The characteristic equation of the simplified short period motion is given as $p(\lambda_c) = A\lambda_c^2 + B\lambda_c + C$. Derive symbolic values of A, B and C and then numerically calculate the corresponding eigenvalues. Does the flying wing have a stable short period motion? Use the numerical data from Table 1.

(c) In order to improve the flying qualities of the flying wing, an alpha feedback controller is integrated with the flight control system. This feedback controller has the following definition: $\delta_e = k_\alpha \alpha$. Derive the new characteristic equation of the form $p(\lambda_c) = A\lambda_c^2 + B\lambda_c + C$ for the simplified short period motion from part (b).

(d) For what range of the feedback gain k_α is the simplified short period stable? Use the numerical data from Table 1.

(e) For what range of the feedback gain k_α is the simplified short period both stable and periodic? Use the numerical data from Table 1.

$V = 80[ms^{-1}]$	$\mu_c = 50$	$K_Y^2 = 0.78$	$C_{X_0} = 0$	$C_{X_u} = -0.24$	$C_{X_\alpha} = 0.463$
$C_{Z_0} = -1.15$	$C_{Z_u} = -2.64$	$C_{Z_\alpha} = 0.51$	$C_{Z_q} = 0$	$C_{Z_{\dot{\alpha}}} = 0$	$C_{Z_{\delta_e}} = -0.41$
$C_{m_\alpha} = -0.78$	$C_{m_u} = 0$	$C_{m_{\dot{\alpha}}} = 0$	$C_{m_{\delta_e}} = -0.83$	$C_{m_q} = 0$	

Table 1: Aircraft data for question 4

Question 5 (20 %)

(a) Draw the lateral stability diagram for a conventional aircraft. Clearly indicate which parameters are on the axes. Also indicate the regions where the Dutch roll and spiral motion are stable and where they are unstable.

(b) Imagine an aircraft that has a convergent spiral motion while the Dutch roll is unstable. If you could change the geometric properties of just one part of the aircraft (wings, tail, fuselage, vertical stabilizer or horizontal stabilizer) what alteration is required to make the Dutch roll stable while keeping a convergent spiral motion? Present two possible alterations and explain their effect.

(c) Clearly indicate in the figure of question (5a) in which region the aircraft of question (5b) falls before the alteration (mark the location with a circle). Indicate by means of an arrow which path is followed through the lateral stability diagram when performing each alteration presented in question (5b). Indicate which arrow belongs to which alteration.

(d) What is the sign of the aerodynamic coefficient C_{n_p} for a conventional aircraft which has a positive wing sweep? Explain what the contribution of the wing is for this aerodynamic coefficient. Clearly mention how the aerodynamic forces which define this contribution are generated.