## AE3235 Thermodynamics and Gas Turbines

Governing equations

If 
$$M=0$$
 
$$T_a=T_0$$
 
$$T_0=T\left(1+\frac{\gamma-1}{2}M^2\right)$$
 
$$P_a=P_0$$
 
$$P_0=P\left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

Compression

$$T_2 = T_1 \left[ 1 + \frac{1}{\eta} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \right]$$

Mass flow

$$\dot{m}_{a_{duct}} + \dot{m}_{a_{core}} = \dot{m}_{a}$$
 
$$BPR = \frac{\dot{m}_{a_{duct}}}{\dot{m}_{a_{core}}}$$

**Fuel flow** 

$$\dot{m}_{fuel} = \frac{\dot{m}_{air} \cdot c_{pg} \cdot \Delta T_{cc}}{\eta_{cc} \cdot H_f}$$

Expansion

$$\frac{T_2}{T_1} = 1 - \eta \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \right] \rightarrow P_2 = P_1 \left[ \frac{1}{\eta} \left( \frac{T_2}{T_1} - 1 \right) + 1 \right]^{\frac{\gamma}{\gamma - 1}}$$

$$W \cdot \eta_{mech} = c_{na} \cdot \dot{m} \cdot (T_2 - T_1)$$

$$c_{pa} \cdot \dot{m} \cdot \Delta T_{comp} = \eta_{mech} \cdot c_{pg} \cdot \left( \dot{m}_{a_{core}} + \dot{m}_{fuel} \right) \cdot \Delta T_{turb}$$

Critical pressure ratio

$$\varepsilon_{kr} = \frac{P_1}{P_2} = \left[ \frac{1}{1 - \frac{\gamma - 1}{\eta_{is} \cdot (\gamma + 1)}} \right]^{\frac{\gamma}{\gamma - 1}}$$

With  $\varepsilon_{kr}$ , one can check if the nozzle is choked or not

If  $\frac{P}{P_a} < \varepsilon_{kr}$  the nozzle is **NOT choked** 

$$P_j = P_a$$
$$P_0 = P_{2.1}$$

 $T_i$  is found with isentropic relationship

$$c_{j} = \sqrt{2 \cdot C_{p} \cdot \Delta T}$$

$$F_{N} = \dot{m} \cdot (c_{j} - c_{0})$$

$$A_{j} = \frac{\dot{m} \cdot R \cdot T_{j}}{P_{j} \cdot c_{j}}$$

$$rac{P}{P_a} > arepsilon_{kr}$$
 the nozzle is **choked**

$$P = \rho \cdot R \cdot T$$

$$P_s = \frac{P}{P_a} = \frac{P}{\varepsilon_{kr}}$$

$$TR_{cr} = \frac{\gamma + 1}{2}$$
$$T_j = \frac{T}{TR_{cr}}$$

$$c_{j} = \sqrt{\gamma \cdot R \cdot T_{j}}$$

$$A_{j} = \frac{\dot{m} \cdot RT_{cr}}{P_{j} \cdot c_{j}}$$

$$F_{N} = \dot{m}_{j}(c_{j} - c_{0}) + A_{j}(P_{j} - P_{0})$$

Specific thrust

$$F_{s} = \frac{F_{N}}{\dot{m}}$$

Thrust specific fuel consumption

$$TSFC = \frac{\dot{m}_{fuel}}{F_N}$$

$$CD = \frac{A_{eff}}{A_{goniometric}}$$

First stage of an axial turbine.

$$T_0 = T_{st} + \frac{1}{2C_p}V^2$$

$$Sp\ Power = C_p \Delta T$$

$$\eta_{is} = \frac{\Delta T}{\Delta T_{is}}$$

$$\Delta T_{is} = T_0 \left( 1 - \left( \frac{1}{\varepsilon} \right)^{\frac{\gamma - 1}{\gamma}} \right)$$