

Introduction to AE

1. Eq of state

Gas law: $pV = nRT$ w, $p[\text{Pa}]$, $V[\text{m}^3]$, $n[\text{mol}]$, $R = 8,3145 [\text{J/molK}]$, $T[\text{K}]$.

$$n = \frac{m}{M} \text{ (molar mass)}, \quad V = \frac{m}{\rho}$$

$$p \frac{m}{\rho} = \frac{m}{M} RT \rightarrow p = \frac{\rho}{M} RT \quad \text{or} \quad \rho = \frac{p \cdot M}{RT}$$

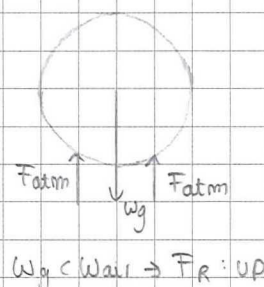
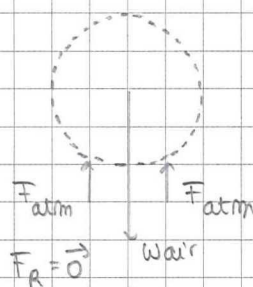
for air: $p = \rho_{\text{air}} R_{\text{air}} T$ w, $R_{\text{air}} = \frac{R}{M_{\text{air}}} = 287 [\text{J/molK}]$.

2. Aerostatics counter gravity w/ sth lighter than air

$$L = m_{\text{air}} g - m_{\text{gas}} g \quad \text{w, } m = \rho V$$

$$= \rho_{\text{air}} V g - \rho_{\text{gas}} V g$$

$$= \rho_{\text{air}} V g \left(1 - \frac{\rho_{\text{gas}}}{\rho_{\text{air}}} \right)$$



a) helium balloon

$$L = \rho_{\text{air}} V g \left(1 - \frac{\rho_{\text{He}}}{\rho_{\text{air}}} \right)$$

$$\frac{\rho_{\text{He}}}{\rho_{\text{air}}} = \frac{p_{\text{He}}}{p_{\text{air}}} \cdot \frac{M_{\text{He}}}{M_{\text{air}}} \cdot \frac{T_{\text{air}}}{T_{\text{He}}} \cdot \frac{R}{R} \quad \text{same } p \text{ \& } T \text{ in- \& outside the balloon}$$

$$L = \rho_{\text{air}} V g \left(1 - \frac{M_{\text{He}}}{M_{\text{air}}} \right)$$

b) hot air balloon

$$L = \rho_{\text{atm}} V g \left(1 - \frac{\rho_{\text{gas}}}{\rho_{\text{air}}} \right)$$

$$\frac{\rho_1}{\rho_2} = \frac{p_1}{p_2} \cdot \frac{M_1}{M_2} \cdot \frac{T_2}{T_1} \cdot \frac{R}{R} \quad \text{same } p \text{ \& } M \text{ in- \& outside the balloon.} \\ \hookrightarrow \text{same gas (air)}$$

$$L = \rho_{\text{atm}} V g \left(1 - \frac{T_{\text{atm}}}{T_{\text{hot}}} \right) \quad \text{w, } T_{\text{atm}} = T; \quad T_{\text{hot}} = T + \Delta T$$

$$= \rho_{\text{atm}} V g \left(\frac{\Delta T}{T + \Delta T} \right)$$

comparison for Helium: $L = \rho V g \frac{6}{7}$ to equal this w, hot air,

$$\frac{\Delta T}{T + \Delta T} = \frac{6}{7} \rightarrow \Delta T = 6T \text{ i.e. for external } T = 288\text{K}, \Delta T = 1800\text{K! (He > hot air)}$$



3. ISA (international standard atm)

* assumption: pure air (no humidity)

* depends on altitude $\rightarrow$ 3 components:

pressure (p [Pa])	101325
density (ρ [kg/m ³])	1.225
temperature (T [K])	288.15

* obeys: gas law
gravity

nb altitude

a) absolute altitude: from the center of the Earth.

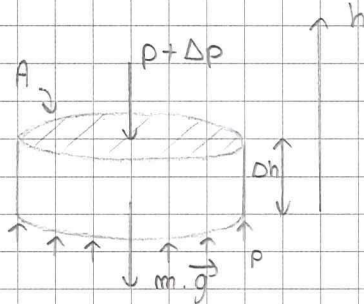
$$h_a = h_g + R_{\text{Earth}}$$

b) geometric altitude (h_g): from the surface of Earth.

c) geopotential altitude: from the surface of Earth taking gravity into account. ($\vec{g} \downarrow$ when $h \uparrow$)

$$g = g_0 \left(\frac{R_e}{h_a} \right)^2 \quad \text{or} \quad g = g_0 \left(\frac{R_e}{R_e + h_g} \right)^2; \quad h = \left(\frac{R_e}{R_e + h_g} \right) h_g$$

a) hydrostatic equation



Static equilibrium

$$(p + \Delta p)A + mg = p \cdot A$$

forces acting
downwards

force acting upwards

$$\text{w/ } m = \rho V = \rho A \Delta h$$

$$\cancel{pA} + \Delta p A + \rho A \Delta h g = \cancel{pA}$$

$$\Delta p + \rho \Delta h g = 0$$

$$\Delta p = -\rho \Delta h g$$

$$dp = -\rho g dh \rightarrow \text{integrate: } \int_{p_0}^{p_1} dp = \int_{h_0}^{h_1} -\rho g dh \quad \text{w/ } \rho = \frac{p}{RT}$$

$\rightarrow$ funct^o of $h \propto p$

$$dp = -\frac{p}{RT} g dh$$

$$\underbrace{\frac{1}{p} dp}_{\text{funct}^o \text{ of } p} = -g \underbrace{\frac{1}{RT} dh}_{\text{function of } h}$$

not possible to integrate because $-\rho g$
not only function of h (alt.)

b) integrating the hydrostatic eq.

gradient layer $\rightarrow T_1 = T_0 + a(h_1 - h_0)$ mb $a \neq 0$
a constant in a layer

$$\frac{1}{p} dp = -p \frac{1}{RT} dh \quad \text{w, } a = \frac{dT}{dh} \Rightarrow dh = \frac{dT}{a}$$

$$\int_{p_0}^{p_1} \frac{1}{p} dp = \int_{h_0}^{h_1} -p \frac{1}{RT} dh$$

$$\int_{p_0}^{p_1} \frac{1}{p} dp = \int_{T_0}^{T_1} -p \frac{1}{RT} \cdot \frac{1}{a} dT$$

$$\int_{p_0}^{p_1} \frac{1}{p} dp = -p \frac{1}{aR} \int_{T_0}^{T_1} \frac{1}{T} dT$$

$$\ln p_1 - \ln p_0 = -p \frac{1}{aR} (\ln T_1 - \ln T_0)$$

$$e^{(\ln p_1 - \ln p_0)} = e^{(\ln T_1 - \ln T_0) \cdot \frac{-g}{aR}}$$

$$\frac{e \ln p_1}{e \ln p_0} = \left(\frac{e \ln T_1}{e \ln T_0} \right)^{-g/aR}$$

$$\frac{p_1}{p_0} = \left(\frac{T_1}{T_0} \right)^{-g/aR}$$

$\rightarrow$ density: $p = eRT$

$$\frac{e_1}{e_0} \cdot \frac{R}{R} \cdot \frac{T_1}{T_0} = \left(\frac{T_1}{T_0} \right)^{-g/aR}$$

$$\frac{e_1}{e_0} = \left(\frac{T_1}{T_0} \right)^{\frac{-g}{aR} - 1}$$

isothermic layer $T = T_0 = T_1$

$$\int_{p_0}^{p_1} \frac{1}{p} dp = \int_{h_0}^{h_1} -g \cdot \frac{1}{RT} dh$$

$\downarrow$ $\downarrow$ $\downarrow$
 cst cst cst

$$\ln p_1 - \ln p_0 = -g \cdot \frac{1}{RT} (h_1 - h_0)$$

$$\frac{p_1}{p_0} = e^{-g/RT (h_1 - h_0)}$$

$\rightarrow$ density: $p = eRT$

$$\frac{e_1}{e_0} \cdot \frac{R}{R} \cdot \frac{T_1}{T_0} = \frac{p_1}{p_0}$$

$$\frac{p_1}{p_0} = \frac{e_1}{e_0} = e^{-g/RT (h_1 - h_0)}$$

4. Focus on an airplane

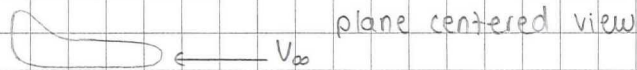
1. weight : aircraft empty weight + fuel + payload.
2. Thrust : provided by the engines
3. lift : mainly generated by the wings.
4. Drag : caused by the fuselage, wing, tail & surfaces.

Lift

$$L = \frac{1}{2} \rho V^2 S C_L$$

* ρ : air density, depends on altitude & temperature

* V : airspeed

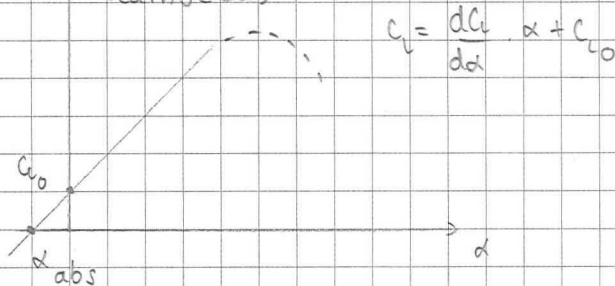


* S : surface area of the wing only, seen from top (design parameter)
(= wing span $\times$ chord)

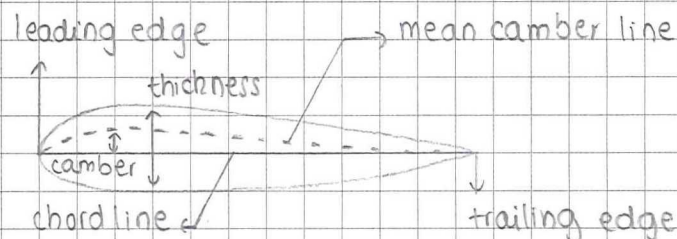
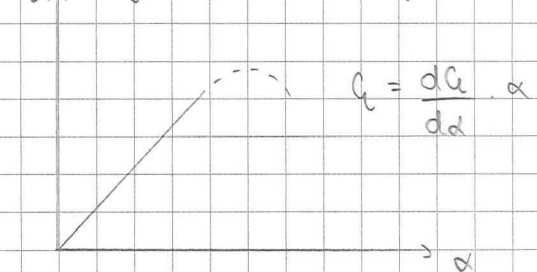
* C_L : lift coefficient, depends on α
the shape of the flow around the wing

it represents the lifting quality of the airfoil / ability to generate L .

C_L asymmetrical airfoil
(cambered)



C_L symmetrical airfoil



coding airfoils : NACA profile

NACA 2412 : 2% camber (of chord length) at
0.4 of the chord (from the leading edge)
12% of thickness / chord ratio.

mb NACA 00XX
symmetrical airfoil.



Reynold's number

scaling factor $\rightarrow$ viscosity effect (laminar / turbulent flow)

$$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

ν , L = typical length (chord length for airfoils)

μ = dynamic viscosity [Ns/m^2]

ν = kinematic viscosity

Bernoulli's principle

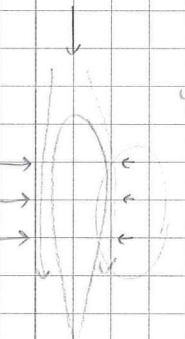
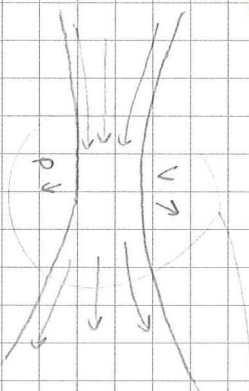
$\rightarrow$ only true for constant $\rho \rightarrow$ incompressible flow

$$\rho + \frac{1}{2} \rho V^2 = \text{constant along a streamline}$$

$$\rho_{\text{stat}} + \rho_{\text{dyn}} = \rho_{\text{tot}} = \rho_{\text{st}}$$

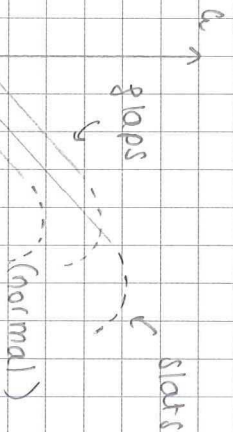
$$V = \sqrt{\frac{2(\rho_{\text{tot}} - \rho_{\text{st}})}{\rho}}$$

generation of lift



higher velocity on top
= lower local pressure
 $\rightarrow$ lift by pressure difference.

lift devices



slats keep the flow laminar along the wing.



used to fly at lower speed: ($w=L$ and $w_{\text{aircraft}} = c_{\text{st}}$)

$$w = L = \frac{1}{2} \rho V^2 S c_l \quad \text{to decrease } V \rightarrow \text{increase } c_l$$

$\rightarrow$ increase S



Drag

$$D = \frac{1}{2} \rho V^2 S C_D$$

scale the same way with density and speed as lift $\frac{L}{D} = \frac{C_L}{C_D}$

drag coefficient

$$C_D = C_{D_0} + C_{D_i}$$

w/ C_{D_0} = zero-lift drag: only depends on the airplane.

C_{D_i} = lift-induced drag

lift-drag polar

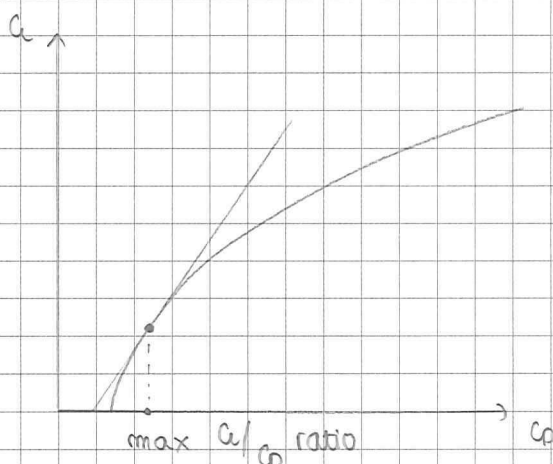
$$C_{D_i} = \frac{C_L^2}{\pi A e}$$

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e} \quad \text{w/ } A = \text{aspect ratio} = \frac{b^2}{S} = \frac{b}{c} = \frac{S}{c^2}$$

e = oswald efficiency factor



↗ scales



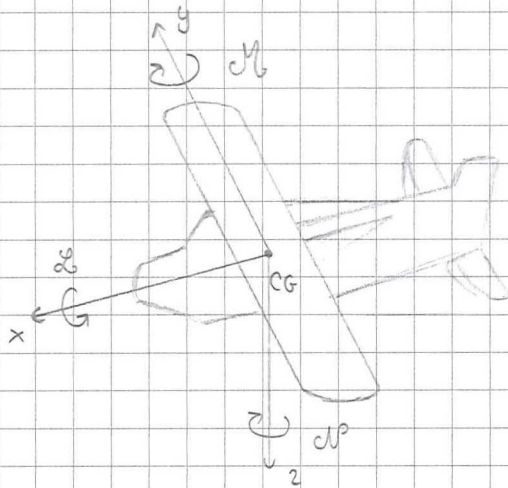
max $\frac{C_L}{C_D}$ ratio: minimum drag at a specific lift

↓
low fuel consumption

snowball effect: less weight
↓
less thrust → less lift needed
↓
less drag

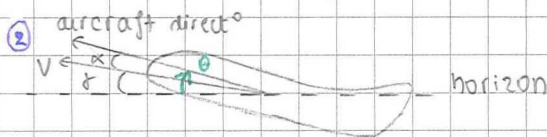


5. Stability and control : angles and axes

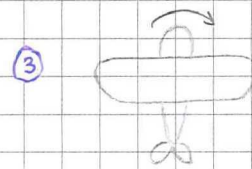


6 degrees of freedom

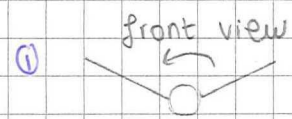
- ① rotation about x-axis:
bank angle / roll
- ② rotation about y-axis:
pitch (nose up = positive θ clockwise)
- ③ rotation about z-axis:
yaw angle



pitch angle $\theta = \alpha + \gamma$
↳ angle of attack

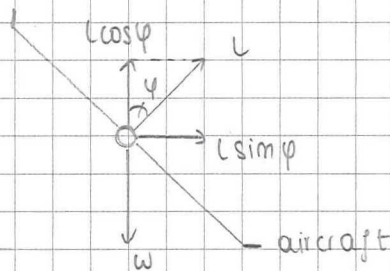


yaw angle ψ



roll angle ϕ .
bank

① horizontal, steady turn



$$L \cos \phi = W$$

↳ more lift needed to compensate W

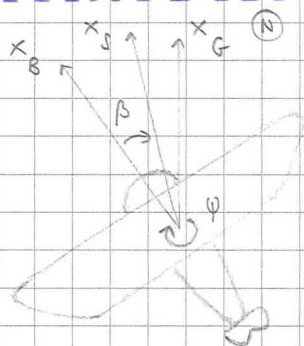
$$\text{load factor } n = \frac{1}{\cos \phi}$$

$L \sin \phi$ makes the plane turn:

$$L \sin \phi = \frac{W}{g} \cdot \frac{v^2}{R_T} \quad (W, R_T = \text{radius of turn})$$

mass $\vec{a}_c$

stability axes * body axes



x_B : direction of the aircraft

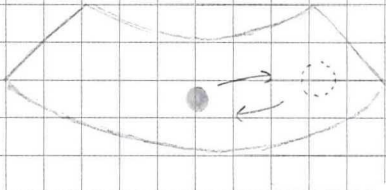
x_S : direction of the airspeed

x_G : North

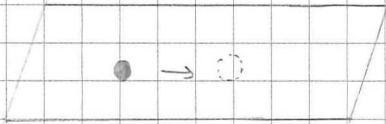
$\beta = \text{sideslip angle}$
 $\psi = \text{heading}$
course = $\psi + \beta$
(χ)

6. Stability

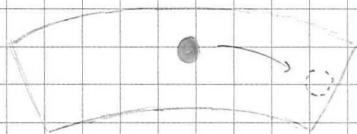
mb static stability is the initial tendency to return to the original state.



positive static stability:
reaction to disturbance is opposite to
the disturbance.



neutral static stability



negative static stability

longitudinal static stability

mb $C_L = \frac{L}{\frac{1}{2} \rho V^2 S}$ $w, S = \text{surface area of the wing}$

$$C_{L_H} = \frac{L_H}{\frac{1}{2} \rho V^2 S_H}$$

$w, S_H = \text{surface area of the tail} \Rightarrow L, D, \text{Moment}$
 Δ when generally speaking about forces on an aircraft
 (whole aircraft) only consider the wings bc tail not meant
 to generate lift. (1, 2 or 3 sets of wings)

$$C_m = \frac{M}{\frac{1}{2} \rho V^2 S \bar{c}}$$

$w, \bar{c} = \text{chord length} \Rightarrow M [Nm]$

$C_{m_{ac}}$ is constant for every angle of attack.

a.c. = aerodynamic center of the wing
 (generally $\frac{1}{4} c$)



condition for longitudinal static stability

1) positive change of pitching moment if negative change in α

react^o to dist.

> <

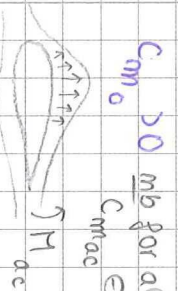
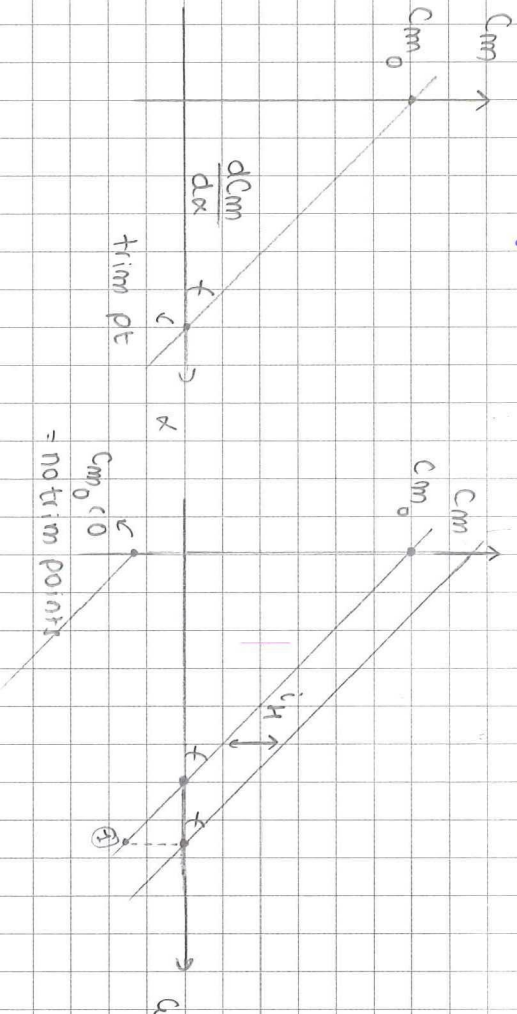
disturbance

and vice-versa.

$$\frac{dC_m}{d\alpha} = C_{m_\alpha} < 0.$$

2) pitching moment has to be positive (clockwise, nose up) when $\alpha = 0$. because C_{m_0} is descending, C_{m_0} has to be positive to have a trim pt (where the moment is 0)

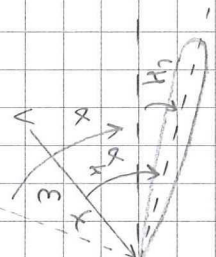
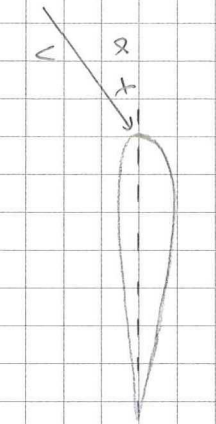
$C_m(\alpha)$ linear (before stall) $\rightarrow C_m(\alpha)$ curve = $C_m(\alpha)$ curve



$C_{m_0} > 0$ mb for a cambered airfoil
 $C_{mac} \ominus \rightarrow$ wing alone not stable

if we need a bigger α (see pt ①), C_m is negative $\rightarrow$ trim setting i_H (does not change α but influence the overall moment of the plane).

* calculate C_{m_α} (change in pitching moment due to change of α)



ϵ : downwash
 as you generate lift you push air downwash

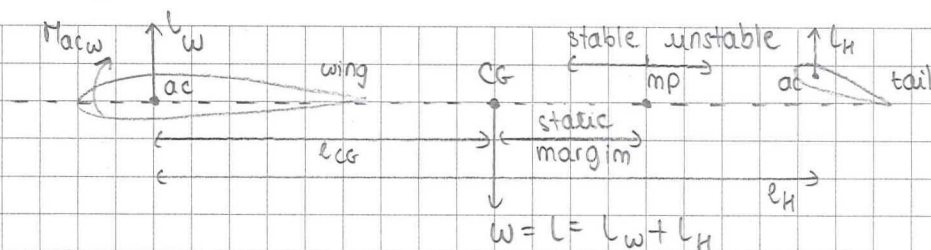
$$\alpha_H = \alpha - \epsilon + i_H$$

$$\frac{d\alpha_H}{d\alpha} = \frac{d}{d\alpha} (\alpha - \epsilon + i_H) = \frac{d\alpha}{d\alpha} - \frac{d\epsilon}{d\alpha} + \frac{di_H}{d\alpha}$$

$\rightarrow i_H$ does not vary w/ α

$$= 1 - \frac{d\epsilon}{d\alpha}$$





$$M_{CG} = \Sigma M = M_{acw} + L_w l_{CG} - l_H (l_H - l_{CG})$$

$$= Mac_w + L_w l_{co} - L_H l_H + L_H l_{co}$$

make it dimensionless = $\frac{Mac_w}{L} \frac{L}{L} \frac{L}{L} \frac{L}{L} = Mac_w + L \frac{L}{L} \frac{L}{L} \frac{L}{L}$

$$\frac{M_{CG}}{\frac{1}{2} eV^2 Sc} = \frac{M_{acw}}{\frac{1}{2} eV^2 Sc} + \frac{L l_{CG}}{\frac{1}{2} eV^2 Sc} - \frac{l_H l_H}{\frac{1}{2} eV^2 Sc}$$

$$W_H = C_H \frac{1}{2} \rho V^2 S_H$$

$$C_m = C_{macw} + C_L \frac{l_{CG}}{c} - C_H \frac{S_H l_H}{S_C} = v_H \quad \uparrow \text{dimensionless}$$

$$\text{differentiell } C_m = C_{mac_w} + C_g \frac{dc_g}{c} - C_H V_H$$

$$\hookrightarrow \frac{dC_m}{dx} = \frac{dC_{m,acw}}{dx} + \frac{dC_u}{dx} \frac{c_{cc}}{c} - \frac{dC_H}{dx} V_H \quad \frac{dC_H}{dx} \cdot \frac{dx_H}{dx} = \frac{dC_H}{dx} \cdot \frac{dx}{dx}$$

$$= \frac{dC_{CH}}{d\alpha_H} \left(1 - \frac{dE}{d\alpha} \right)$$

$$C_{m_2} = \frac{dC}{dx} \frac{L_0}{c} - V_H \left(1 - \frac{d\varepsilon}{dx} \right) \frac{dL_H}{dx}$$

↳ steepness of C_α curve
 $\underset{\text{not}}{=} C_\alpha = a$ not

↳ steepness of $G_H(\alpha_H)$ curve = $G_{\alpha_H} = a$ not t

$$C_{m\alpha} = a \frac{c_{co}}{c} - V_H \left(1 - \frac{d\varepsilon}{dx} \right) a_t$$

to satisfy criterion 1 : $C_{m_2} < 0$

$$\rightarrow a \frac{v_{cc}}{c} - v_H \left(1 - \frac{d\epsilon}{d\alpha} \right) \alpha < 0$$

$$a \frac{dc}{c} = v_H \left(1 - \frac{d\varepsilon}{dx}\right) a_T$$

neutral point: $a \frac{e_{mp}}{c} - v_H \left(1 - \frac{d\varepsilon}{dx} \right) a_T = 0$

$$\frac{e_{mp}}{c} = \frac{a_t}{a} V_H \left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad \begin{array}{l} \text{in \% of chord length} \\ \rightarrow \text{dimensionless} \end{array}$$

static margin: $\text{dist } l_{mp} \propto c_g = |l_{mp} - l_{cg}| \quad [m]$

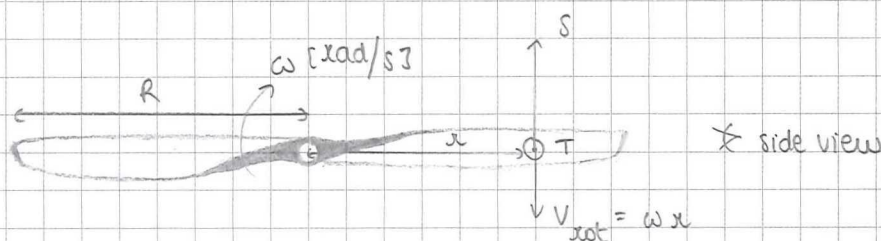
Amount of span you have to move your CG backwards before the plane becomes unstable.

7. propulsion

Air breathing engines

a) propeller theory

blades can be twisted such that the angle of attack remains constant.

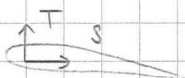


T (pointing at us) : useful force

S : side force

// side force is to thrust what drag is to lift.

* side view:



available power for thrust:

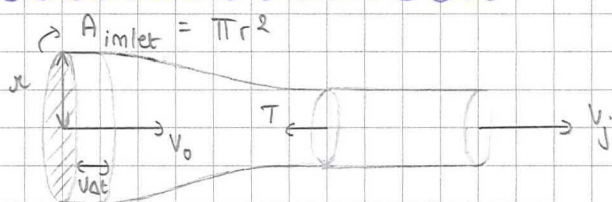
$$W = F \cdot \Delta x = T \cdot \Delta x$$

$$P_a = \frac{\Delta W}{\Delta t} = T \cdot \frac{\Delta x}{\Delta t} = T \cdot V$$

efficiency of propellers: propulsive efficiency

$$\eta = \frac{P_a}{P_{br}} \quad w, P_{br} = \text{brake (shaft) power.}$$

b) airbreathing jet engines



$$\text{bypass ratio} = \frac{\dot{m}_{\text{cold}}}{\dot{m}_{\text{hot}}}$$

$$T = \dot{m} (V_j - V_0) \quad w, \dot{m} = \rho A V$$

jet efficiency:

$$P_a = T \cdot V_0 = \dot{m} (V_j - V_0) \cdot V_0$$

Energy put in
kinetic E exit kinetic E entrance

$$P_{\text{jet}} = \frac{1}{2} \dot{m} V_j^2 - \frac{1}{2} \dot{m} V_0^2 = \frac{1}{2} \dot{m} (V_j^2 - V_0^2)$$

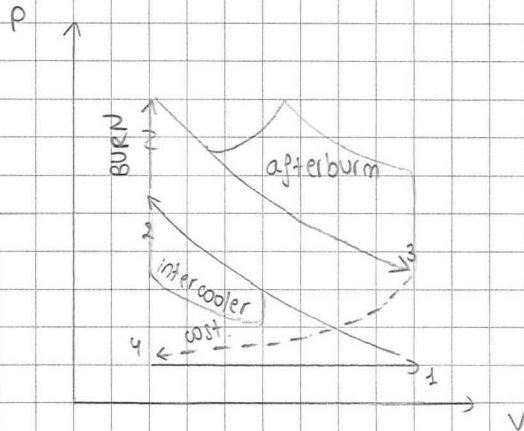
$$\eta_{\text{jet}} = \frac{P_a}{P_j} = \frac{\dot{m} (V_j - V_0) V_0}{\frac{1}{2} \dot{m} (V_j^2 - V_0^2)} = \frac{2(V_j - V_0)V_0}{(V_j - V_0)(V_j + V_0)} = \frac{2V_0}{V_0(1 + \frac{V_j}{V_0})} = \frac{2}{1 + \frac{V_j}{V_0}}$$

$$\eta_{\text{jet}} = 100\% \rightarrow T = 0 \quad (\text{a loss in } E_{\text{kin}} \text{ is needed}).$$

working cycle of piston engines:

most common: 4 strokes engines

- 1) intake such
- 2) compression squeeze
- 3) power bang
- 4) exhaust blow


 $p = \frac{mRT}{V}$ so the $p(V)$ curve looks like $\frac{1}{x}$.

$$W = p \cdot A \cdot \Delta x \rightarrow W = p \Delta V \rightarrow W = \int_{V_1}^{V_2} p dV$$

work = area under the curve

BURN $\rightarrow \uparrow T \rightarrow \uparrow p \rightarrow \uparrow W$ 3. Instrumentation

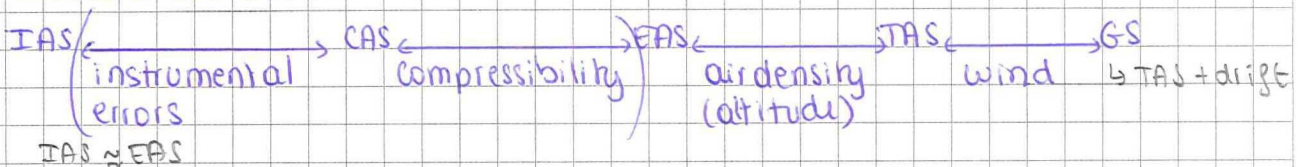
$$1 \text{ n.m.} = 1852 \text{ m} \quad \neq \quad 1 \text{ mile} = 1609 \text{ m}$$

 $\hookrightarrow$ nautic mile

$$1 \text{ kts} = 0,51444 \text{ m/s}$$

$$1 \text{ ft} = 0,3048 \text{ m} \rightarrow 1 \text{ ft/min} = \frac{0,3048}{60} \frac{\text{m}}{\text{s}}$$

$$M = \frac{V_{TAS}}{a} \quad w, \quad a = \sqrt{\gamma R T}; \quad \gamma_{\text{air}} = 1,4$$

equivalent airspeed
 $V_{EAS} \neq V_{TAS}$ because V_{EAS} does not vary w, ISA values

$$\frac{1}{2} \rho_0 V_{EAS}^2 = \frac{1}{2} \rho V_{TAS}^2$$

flight levels

used after a transition altitude 4500 ft = FL045

FL085 = 8500 ft above 101325 Pa altitude.

 $\times 100$

jet packs: exhaust above CG

ground effect: inverted downwash: $\downarrow C_L$; $\uparrow L$ (up to 40%)**STUDENT
FACTOR**

decrease arm when bend-over

Op zoek naar tips om een goede cv en motivatiebrief op te stellen? www.studentfactor.nl/adviezenhelicopters: need 2 rotors counter-rotating or 1 rotor & 1 tail-rotor $\rightarrow$ higher lift on sidedead-man's region $\uparrow$ (auto-rotating)

Structural concepts

1. Functions of a structure

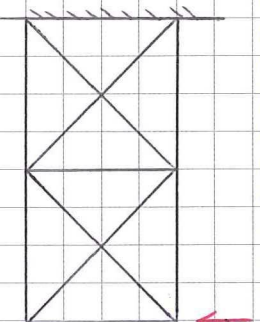
- carrying the loads: during an operation, an aircraft is loaded by many forces & moments (bending & torsion)
 - loads induced by
 - manoeuvres
 - environment (e.g. variation of temperature)
 - impacts
 - ⇒ make sure that the loads are in equilibrium.
- protective function: protect the payload or the passengers against impacts & environment (-60°C and $\downarrow$ 60-70%).
- framework: systems fixed to the structure, which keep them in place and support them.

2. Requirements

- strength: should be adapted to the mission of the aircraft.
 - ↳ how much load it can carry.
- stiffness: adequate stiffness depending on the structure
 - ↳ resistance against deformation.
- lightweight, $\downarrow$ empty weight $\rightarrow$ $\uparrow$ fuel (larger range) $\rightarrow$ $\uparrow$ payload (higher income)
- durable: should be able to resist
 - $\rightarrow$ deterioration or ageing effects
 - fatigue
 - corrosion
 - degradation
- cheap and cost effective: affordable & cheap in operation.
- ease of maintenance: a part that fails should be replaced quickly

3. Structural concepts

1. Truss structures



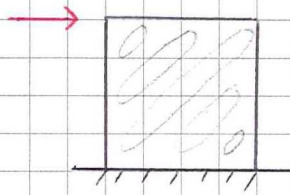
bending moment
(increasing from right to left)

+ torsion moment in 3D.

rods can withstand tension & compression
wires can withstand tension (only)



if rods are replaced by a thin sheet (no tension / compression)
→ shear forces on the skin.



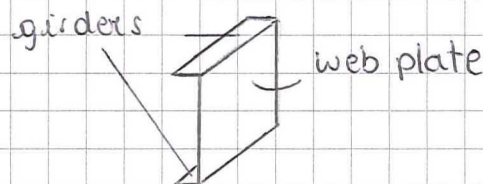
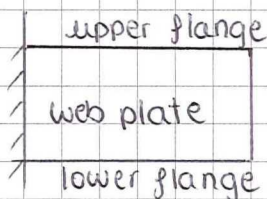
elastic buckling > plastic buckling
no failure failure

stiffeners can be used to reduce the buckling effect.

2 beams

web plate = metal sheet to replace the diagonal wires / rods of the truss structure → carries transverse forces by shear loading.

girders = upper & lower flange → carry bending moment.



4. Fail safe - Safe life

* safe life : strong enough to stay intact for the entire life cycle.
(1 load path with low probability of failure)
→ no inspection required but replaced after life cycle.

→ ie plus utilise

* Fail safe : one element may fail, other elements strong enough to stay intact for limited time.
(multiple load path for each force)
→ inspection required.

Safe life used if we cannot inspect a part.

5. Structural concepts

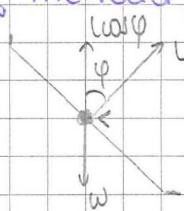
1. loads

load factor $m = \frac{L}{w} = 1$ (cruise)

$m > 1$: feeling heavy

$m < 1$: feeling light (can be 0)

when banking, increase of the load factor because $L \cos \varphi$ has to compensate weight.





* limit load : no remaining damage allowed.

* ultimate load: limit load $\times$ safety factor
 $\rightarrow$ failure allowed after 3s.

2. stresses and strain material properties : independant of the shape.

Forces to stresses : pressure or tension exerted

$$\sigma = \frac{F \text{ [N]}}{A \text{ [mm}^2\text{]}} \quad \text{w, } A = \text{cross-sectional area} \\ \text{[N/mm}^2\text{]} = \text{[MPa]} \quad = \text{thickness} \times \text{width}$$

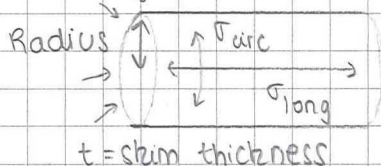
for composites: $\sigma = \% \text{ resin} \times \sigma_{\text{resin}} + \% \text{ fibers} \times \sigma_{\text{fibers}}$
Displacement to strain: dimensionless elongation

$$\epsilon = \frac{\Delta L}{L_0} = \frac{\text{change in length}}{\text{original length}}$$

Stresses in a pressure vessel (fuselage)

assumptions: * fuselage w homogeneous stress
 * no cut-outs (windows/doors/holes)
 * flat ends

circular cross sect^o = the best for a pressurized fuselage: pressure distributed evenly (no edges = no critical point).
 Δp = pressure difference (in $\leftrightarrow$ outside)



$$\sigma_{\text{circ}} = \frac{\Delta p R}{t} ; \sigma_{\text{long}} = \frac{\Delta p R}{2t}$$

mb for isotropic materials, if Δp , R and σ are known,

$$t = \frac{\Delta p R}{\sigma_{\text{circ}}} = \frac{\Delta p R}{2\sigma_{\text{long}}} : \text{take the bigger value.}$$

for anisotropic materials (composites)

$$t = \frac{\Delta p R}{\sigma_{\text{circ}}} + \frac{\Delta p R}{2\sigma_{\text{long}}}$$

Δ cut-outs $\rightarrow$ windows
 $\rightarrow$ doors (passenger, cargo, landing gear doors).

all the stresses that should be there if there was no hole goes around the hole.



the structural loads need to by-pass the cut out.



6. Materials

substance that has properties depending on its composition (and not on its shape)

material → semi-finished part → structural element → structure

four main material groups:

- * metal alloys
- * composites (metal, resin, fibers)
- * pure polymers } less used
- * ceramic

(PSE)

principal structural element:

primary structure

↓
beam-like elements

failure is catastrophic

(shell structure: skin made of a metal sheet which carries loads and has beam-like elements)

sandwich structure: usually composites; acts like a beam in two directions → face sheets: carry tension and compression

→ core material: prevents buckling & carries shear loads

def homogeneous: same composite everywhere in the material

isotropic: same properties in every direction

plastic behaviour: can apply a permanent deformation

metal alloys -

→ isotropic

- * can be strengthened (alloying, heat treatment)
- * plastic behaviour & melting (recycling, welding)
- * good processability (easy to transform)
- * low cost
- * huge diversity

polymers -

→ thermoset: curing irreversible (can't be shaped & reshaped)
more than 1 component

→ thermo plastics: softening reversible (can be shaped & reshaped)
one component

- * isotropic
- * low strength & stiffness → not used for PSE
- * huge variety
- * plastic flow & melting (recycling)
- * good processability
- * low costs

ceramics -

* high stiffness

* too brittle

* bad processing

} not used for PSE



composites: materials composed of $\neq$ constituents

mb UD = unidirectional composites

1. fibers reinforced polymers

- a) fibers: strong & stiff (in longitudinal direction) $\rightarrow$ glass, carbon, aramid
b) polymer:
 - * support of the fibers
 - * take care of the load transfer between fibers
 - * protects the fibers

2. hybrid materials GLARE

laminates of alternating metal & composite layers.

characteristics

- * anisotropic (can be adapted to the needs)
- * layered structure (laminati)
- * high strength & stiffness in the direct° of fibers
 $\rightarrow$ tension only
- * low density
- * expensive
- * no plasticity
- * good processibility but only once (thermoset)

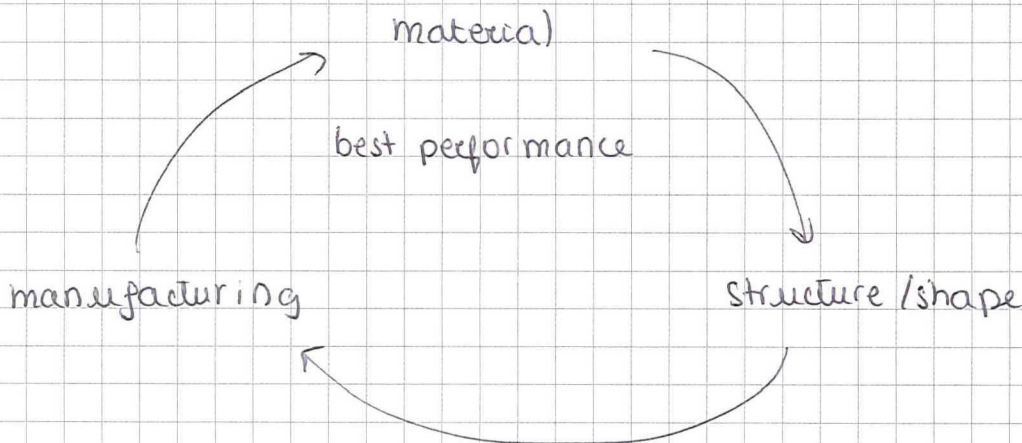
mb E-modulus = stiffness

yield stress: how much force can I apply before it starts plastic def.

failure strength: how much stress before it breaks (ultimate stress)

max strain: how much can I elongate before it breaks

$\downarrow$ divide by density
Specific values





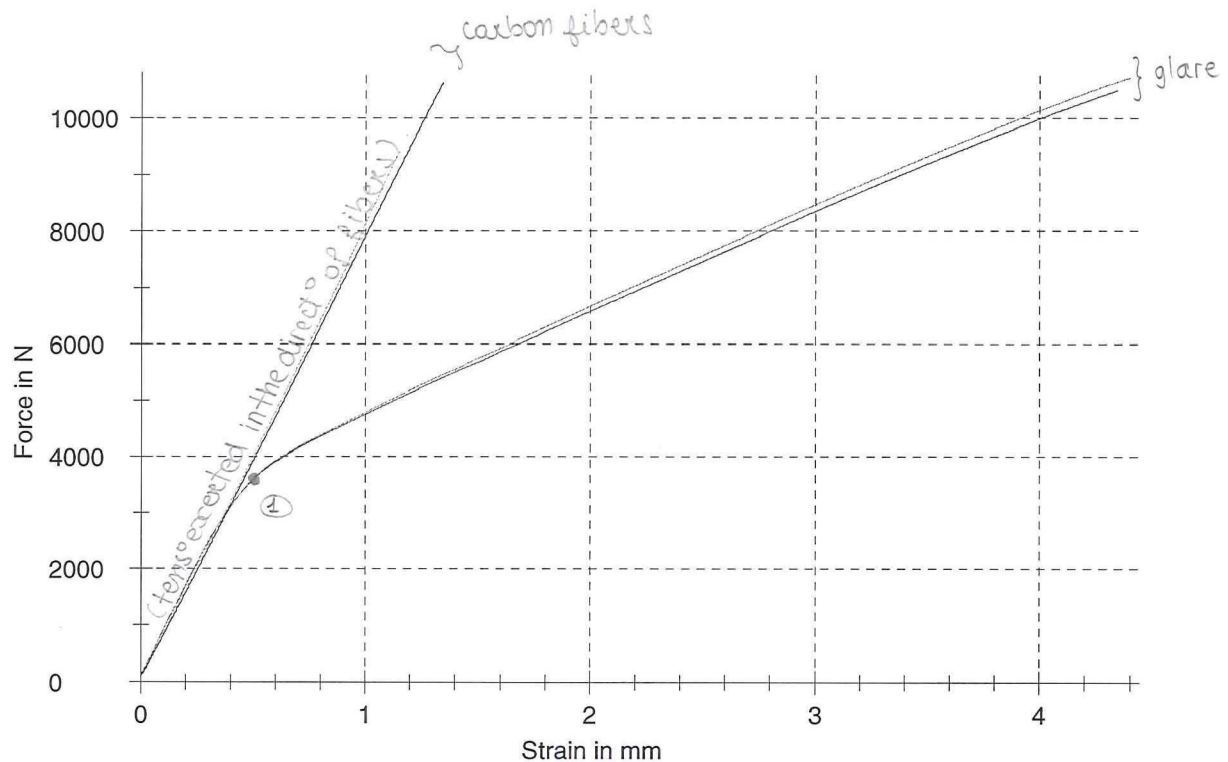
Datum: _____

This image shows a full page of blank graph paper. The grid consists of small squares formed by thin gray lines. A single vertical line runs down the left side of the page, creating a margin. There are no markings or text on the paper.

Results table:

Legend	Specimen ID	L_0 mm	F_{max} N	dL at F_{max} mm	F_{Break} N	dL at break mm	a_0 mm	b_0 mm	S_0 mm ²
	Glare1	100,02	10499	4,35	10500	4,35	1,05	12,48	13,10
	Glare2	100,00	10722	4,40	10720	4,40	1,06	12,48	13,23
	CBF1	100,02	9404,2	1,15	9404	1,15	0,92	19,96	18,36
	CBF2	100,00	10636	1,34	10640	1,34	0,93	19,99	18,59

Curve graph:



glare: at point ①, one of the components breaks but the other one still holds it, until it breaks

carbon fibers: where the graph stops is when the tension break occurs.
 → 2 breaks of carbon fibers

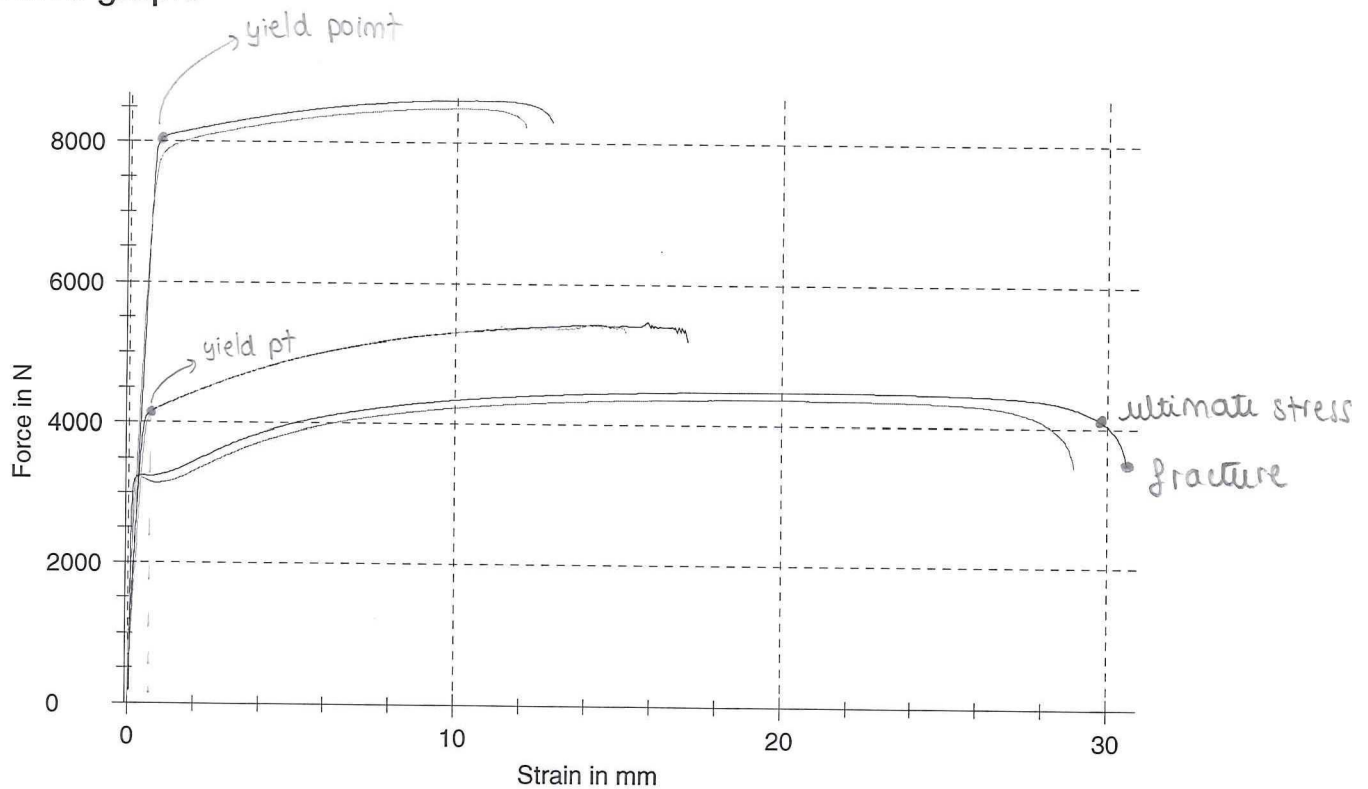
tension:
it can't hold it

compression:
all the tension comes back in the material (as if it was a spring)
and a compression break occurs right after the tension break

Results table:

Legend	Specimen ID	L_0 mm	F_{max} N	dL at F_{max} mm	F_{Break} N	dL at break mm	a_0 mm	b_0 mm	S_0 mm ²
	Steel1	100,00	4481,7	19,64	3491	30,59	0,99	12,53	12,40
	Steel2	100,00	4366,8	19,36	3438	28,98	0,99	12,5	12,38
	AL2-1	100,00	5463,1	15,89	5196	17,12	0,99	12,52	12,39
	AL2-2	100,00	5426,8	15,12	5322	15,22	0,99	12,48	12,36
	AL7-1	100,00	8609,2	10,57	8301	12,93	1,26	12,48	15,72
	AL7-2	100,00	8492,2	10,22	8227	12,12	1,24	12,49	15,49

Curve graph:



steepness of the curve before the yield point = stiffness.

after the yield point: the structure experiences a plastic deformation
→ becomes useless

Aerodynamics1. Fundamentals

a) pressure : normal force per unit area on a surface.

$$p_B = \lim_{dA \rightarrow 0} \frac{dF}{dA}$$

[Pa]

b) density

$$\rightarrow \text{specific volume } \nu = \frac{1}{\rho} = \frac{V}{m}$$

[m³/kg]

$$\rho_B = \lim_{dV \rightarrow 0} \frac{dm}{dV}$$

[kg/m³]

c) temperature

$$k_E = \frac{3}{2} kT \quad \text{w, } k = \text{boltzman's constant}$$

$\hookrightarrow$ kinetic energy

d) equation of state Δ for perfect gas

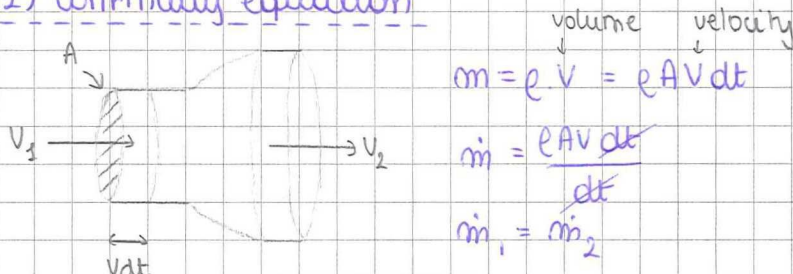
$$p = \rho R T$$

[Pa]

$$p_{\text{stat, sea level}} = 101325 \text{ Pa}$$

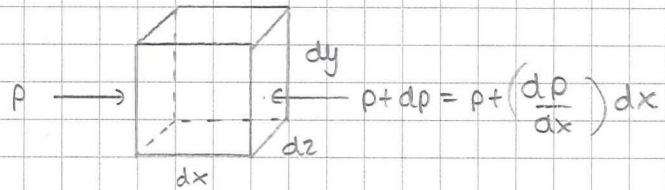
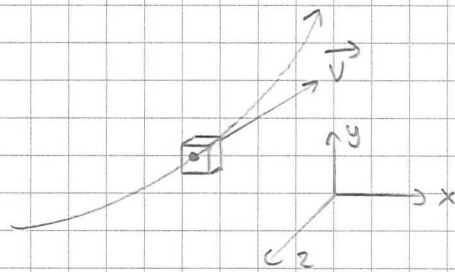
e) velocity of a small fluid element as it sweep through a point B

$$\vec{V}_B = \frac{d\vec{s}}{dt}$$

2. Fundamental equations1) continuity equation

$$\rho A V = \text{constant} \rightarrow \text{for incompressible flows } A V = \text{constant}$$

application : windtunnels.

2) Euler's equation or momentum equation

assumptions: * no friction force * steady flow
* no gravity force

$$\sum F_x^{\rightarrow}: \underbrace{p \, dy \, dz}_{\text{pressure}} - \underbrace{(p + dp) \, dy \, dz}_{\text{area}} = p \, dy \, dz - (p + \frac{dp}{dx} dx) \, dy \, dz = -\frac{dp}{dx} dx \, dy \, dz$$

$$\begin{cases} m = \rho V = \rho \, dx \, dy \, dz \\ a = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} v \end{cases}$$

$$F = m \cdot a \Rightarrow -\frac{dp}{dx} dx \, dy \, dz = \rho \, dx \, dy \, dz \cdot v \frac{dv}{dx}$$

$$dp = -\rho v \, dv \quad (\text{valid for both compressible \& incompressible flows})$$

3) Bernoulli's principle

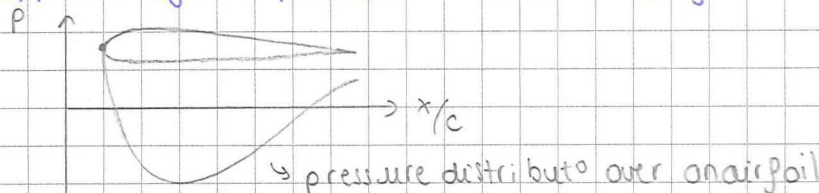
$$dp + \rho v \, dv = 0 \quad \text{along a streamline} \quad \Delta \text{assumpt}^{\circ}$$

$$\int_{p_1}^{p_2} dp + \int_{v_1}^{v_2} \rho v \, dv = 0$$

$$p_2 - p_1 + \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 = 0 \quad \text{static pressure}$$

$$p_2 + \frac{1}{2} \rho v_2^2 = p_1 + \frac{1}{2} \rho v_1^2 \rightarrow \underbrace{p + \frac{1}{2} \rho v^2}_{\text{dynamic pressure}} = p_{\text{tot}} \quad (\text{ist})$$

applicat^o of this principle: use the stagnation point to calculate p_{tot}



+ Pitot-static tube

Let's be remarkable.



3. Thermodynamics assumption: perfect gas: no molecular forces

1. first law of thermodynamics

$$\underset{\substack{\downarrow \\ \text{energy}}}{\delta e} = \overset{\substack{\uparrow \text{heat}}}{\delta q} + \underset{\substack{\downarrow \text{work}}}{\delta w}$$

$$w = F \cdot \Delta x = p \cdot \Delta A \cdot \Delta x \rightarrow \delta w = \int_A p \cdot dA \cdot x = p \int_A x \cdot dA = -p \, dV$$

constant on the area.
change of specific volume

$$\delta e = \delta q - p \, dV$$

enthalpy: $h = e + pv$

$$dh = de + d(pv) = de + v \, dp + p \, dv$$

$$= \underbrace{dq}_{de} - p \, dv + p \, dv + v \, dp$$

$$dh = dq + v \, dp$$

specific energy $c = \frac{\delta q}{dT}$

$[J/kgK] \, c_v = \left(\frac{\delta q}{dT} \right)_{\text{at constant volume}} \rightarrow dV=0 \rightarrow \delta q = de \rightarrow de = c_v \, dT \Rightarrow e = c_v T$

$[J/kgK] \, c_p = \left(\frac{\delta q}{dT} \right)_{\text{at constant pressure}} \rightarrow dp=0 \rightarrow \delta q = dh \rightarrow dh = c_p \, dT \Rightarrow h = c_p T$

$c_{v, \text{air}} = 720 \, J/kgK$ } for $T < 600K$. ratio $\frac{c_p}{c_v} = 1.4 = \gamma$ (for air).
 $c_{p, \text{air}} = 1008 \, J/kgK$

2. Isentropic flows compressible flows

def: isentropic = adiabatic (ie $\delta q = 0$)
 = reversible (no frictional or dissipative effects)

since $\delta q = 0$, $\delta e = -p \, dV$ and $de = c_v \, dT \Rightarrow -p \, dV = c_v \, dT$

since $\delta q = 0$, $dh = v \, dp$ and $dh = c_p \, dT \Rightarrow v \, dp = c_p \, dT$

$$\frac{-p \, dV}{v \, dp} = \frac{c_v \, dT}{c_p \, dT} \Rightarrow \frac{dp}{p} = -\left(\frac{c_p}{c_v} \right) \frac{dV}{V} \Rightarrow \frac{dp}{p} = -\gamma \frac{dV}{V}$$

→ integrating: $\frac{p_2}{p_1} = \left(\frac{V_2}{V_1} \right)^{-\gamma}$ or $V = \frac{1}{\rho} \rightarrow \frac{p_2}{p_1} = \left(\frac{\rho_1}{\rho_2} \right)^{\gamma}$

with eq of state: $\left(\frac{p_2}{p_1} \right) = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$

Let's be remarkable.





6. if $V=0$ (anywhere in the flow)

stagnat° pt / reservoir

Energy equation:

$$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_0 + \frac{1}{2} V_0^2$$

$= 0$

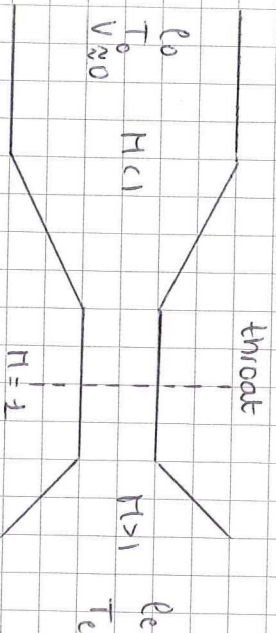
$$\frac{T_0}{T_1} = 1 + \frac{\gamma-1}{2} \frac{V_1^2}{a_1^2} = 1 + \frac{\gamma-1}{2} M_1^2$$

$$\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{\frac{\gamma}{\gamma-1}} = \frac{\rho_0}{\rho_1}$$

→ second form of isentropic relations:
use it between 2 stations where 1 of the 2 has $V=0$

mb for $M < 0.3$, the change in density is less than 5% (± incompressible)

7. Supersonic wind tunnels



area-velocity relation

$$\frac{dA}{A} = (M^2 - 1) \frac{du}{u}$$

1) subsonic flow $M < 1$, $du >$, $dA <$
 $V \uparrow \Leftrightarrow A \downarrow$

2) sonic flow

$$\frac{du}{u} = \left(\frac{1}{\gamma} \right) \frac{dA}{A}$$

$$\frac{du}{u} = \infty \rightarrow \text{impossible} \rightarrow \frac{dA}{A} = 0 \text{ since } \frac{du}{u} = 0 \text{ (finite)}$$

→ in the throat, $M=1$

3) supersonic flow $M > 1$, $du >$, $dA >$
 $V \uparrow \Leftrightarrow A \uparrow$ (ex nozzle)

mb $M \uparrow$: drop in ρ, ϵ, T

air reacting to that change with the aircraft when it goes through
→ condensation.

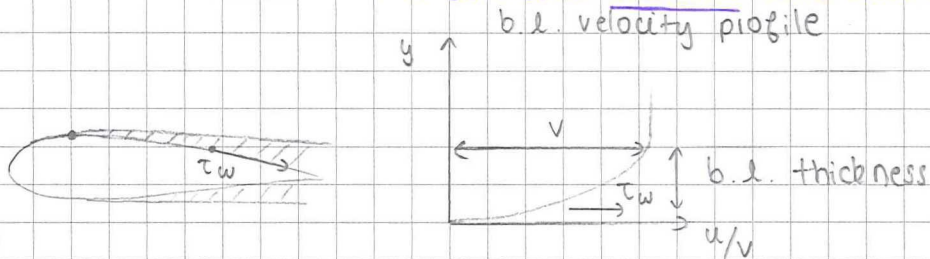




8 Laminar and turbulent flow

viscous flow (friction): the flow adheres to the surface
 → right on the surface, the velocity is 0.

boundary layer thin region of retarded flow
 mb the pressure $\perp$ to the surface is constant
 — Bernoulli's law is not valid
 v is constant outside the boundary layer.



$$\tau_w = \mu \left(\frac{du}{dy} \right)_{y=0}$$

μ = absolute viscosity c.
 $= 1,789 \cdot 10^{-5} \text{ kg/ms}$

laminar flow streamlines are smooth & regular, a fluid element moves smoothly along a streamline.

turbulent flow streamlines break up, a fluid element moves in a random, irregular way.

1. laminar flow

$$\delta = \frac{5,2x}{\sqrt{Re_x}} \quad \text{boundary layer thickness}$$

$$C_{f_x} = \frac{0,664}{\sqrt{Re_x}} \quad \text{local skin friction coeff} \rightarrow C_f = \frac{1,328}{\sqrt{Re_L}} \quad \text{tot skin fr. drag coeff}$$

2. turbulent flow

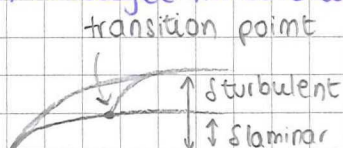
$$\delta = \frac{0,37x}{(Re_x)^{0,2}} \quad \text{thickness}$$

$$C_f = \frac{0,074}{Re_L^{0,2}} \quad \text{total skin friction drag coefficient}$$

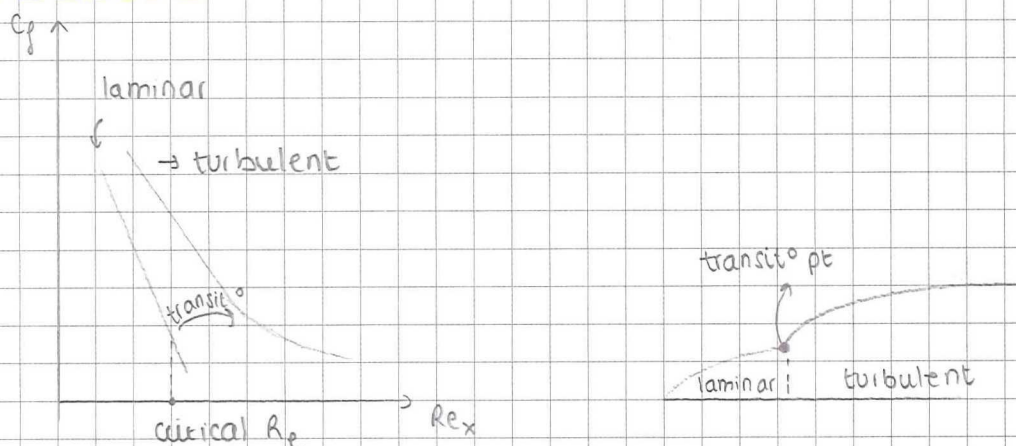
note: $Re = \frac{\rho V L}{\mu}$ (L = chord for airfoils, diameters for spheres, length for a flat plate).

same Re = same flow conditions.

⇒ friction is larger in a turbulent than in a laminar flow



3 Transition



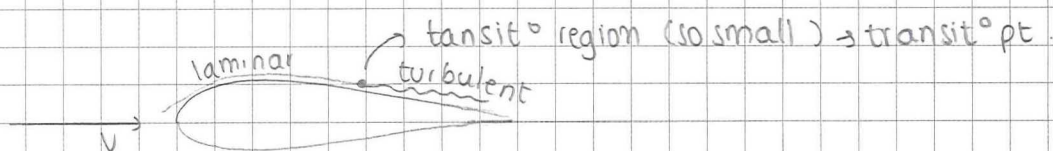
The Energy of the boundary layer decreases as the flow goes along the flat plate: E is subtracted from the flow.
unstable flow (waves) $\rightarrow$ turbulent spots in laminar flow $\rightarrow$ turbulent flow

on airfoils

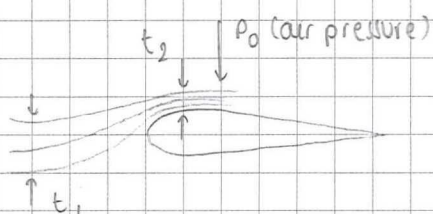
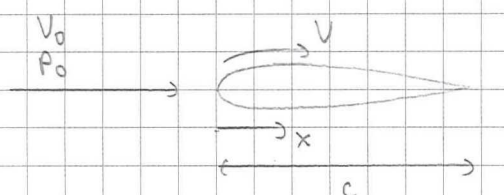
laminar B.L. $\rightarrow$ thin
 $\rightarrow$ low skin friction drag

turbulent B.L. $\rightarrow$ thick
 $\rightarrow$ high skin friction drag

creat° of airfoils where the flow stays laminar longer.

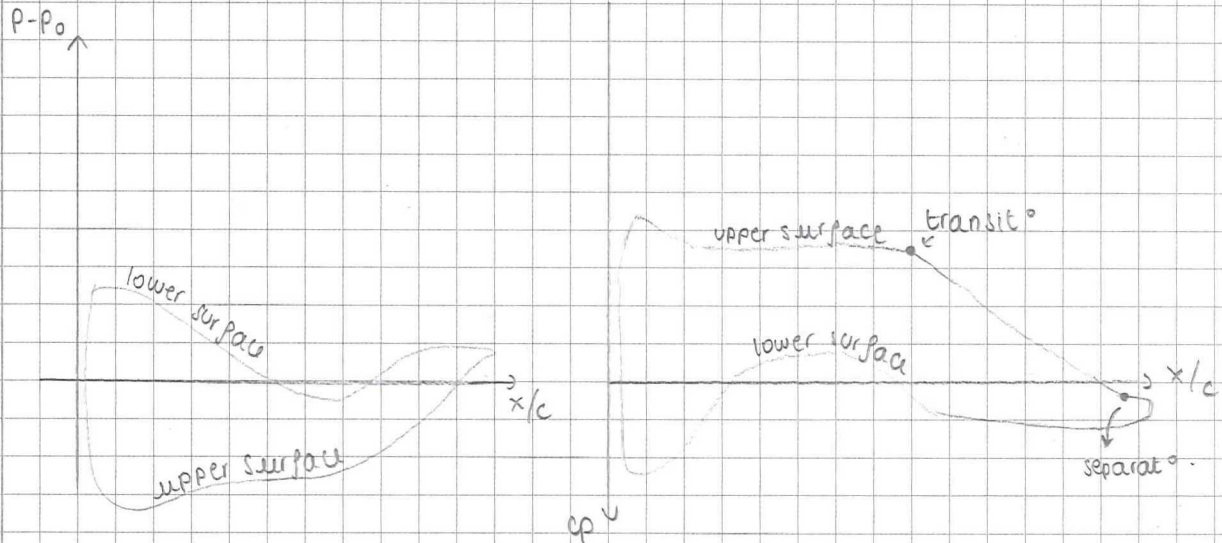


9 Pressure distribution



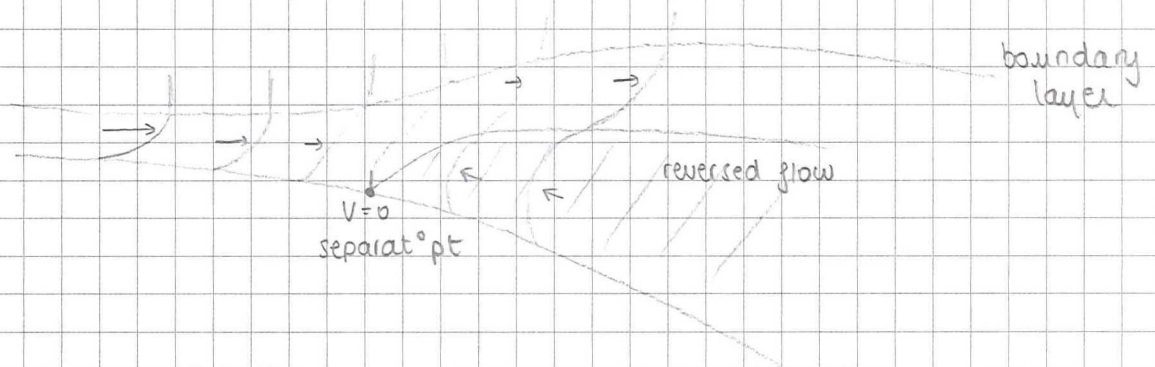
$t_1 < t_2$ (streamlines squeezed)

$V \uparrow \rightarrow p \downarrow$

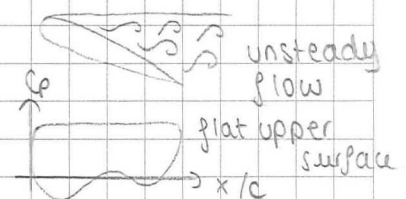
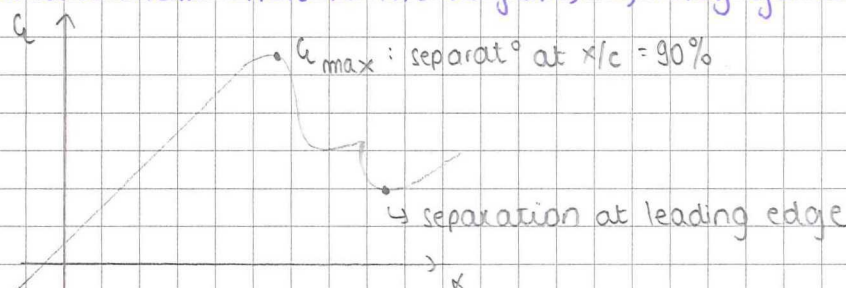


≠ graphs for each $V_0 \rightarrow C_p = \frac{P - P_0}{\frac{1}{2} \rho V_0^2}$ (graph not dependant on V)
 ≠ graphs for each α
 ≠ graphs for each Re

adverse pressure gradient : increasing pressure
 → particles get "pushed back" (loss of Energy)
 The only E a particle has in a streamline is its velocity → decrease in V
 velocity profile gets thinner → $V=0$ → reversed.
 separation point

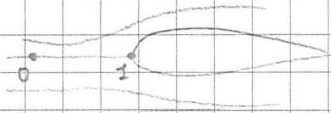


when $V=0$, there is (no force) not enough energy to bend the streamlines (to make them stick to the airfoil) → they go with the flow



lot of vibration → reduce the lifetime of the structure

at the stagnation point



($V_1 = 0$)

$$P_0 + \frac{1}{2} \rho V_0^2 = P_1 + \frac{1}{2} \rho V_1^2$$

$$P_1 - P_0 = \frac{1}{2} \rho V_0^2$$

$$C_p = \frac{P_1 - P_0}{\frac{1}{2} \rho V_0^2} = 1$$

or $P_0 + \frac{1}{2} \rho V^2 = P_t$

$P_t - P_0 = q_0 \rightarrow C_p = \frac{P - P_0}{q_0}$ in stagnat° pt: static pressure = tot pressure (because $V=0$).

in terms of velocity: $C_p = \frac{P - P_0}{q_0} = \frac{(P_t - P_0) - (P_t - P)}{q_0} = \frac{q_0 - q}{q_0} = 1 - \frac{q}{q_0}$

$$C_p = 1 - \frac{\frac{1}{2} \rho V^2}{\frac{1}{2} \rho V_0^2} = 1 - \frac{V^2}{V_0^2}$$

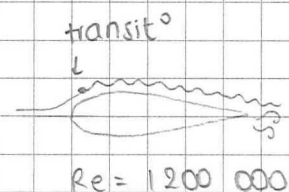
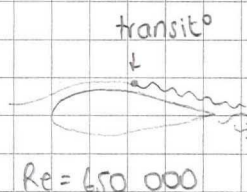
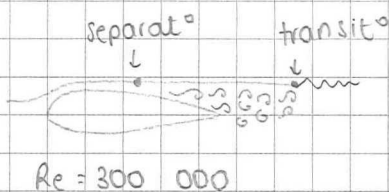
10. separation

- * loss in lift
- * increase in pressure drag
- * generation of unsteady loads (promotes fatigue)

separation $\rightarrow$ laminar separation (low Re < 500 000)

high viscosity

$\rightarrow$ turbulent separation (high Re : promotes the transition process)



really high pressure change
 $\rightarrow$ separat°

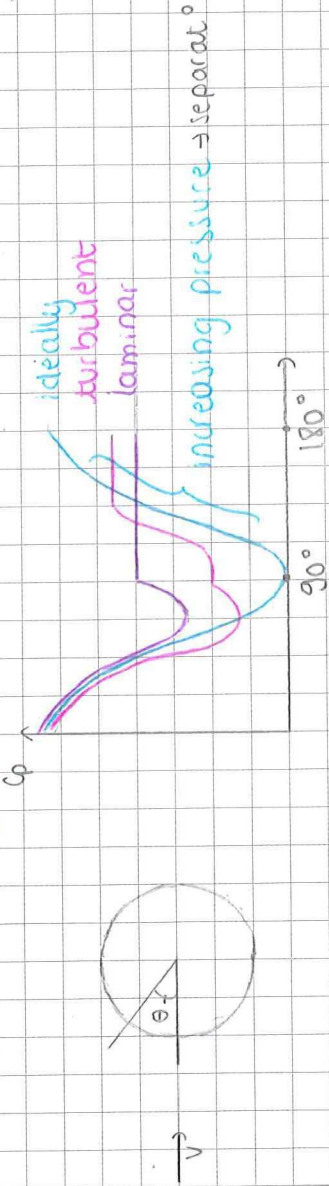
HIGH C_d

LOWER C_d

Let's be remarkable.

Influence of Re on the airfoil performance
 $Re \uparrow : C_d \downarrow$ and $C_{l,max} \uparrow$ (general tendency)

mb a sphere w, a diameter "d" has as much drag as an airfoil w, a thickness of "10d"

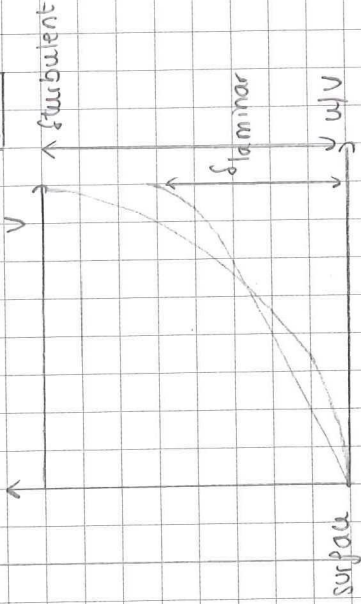


* laminar separation = high C_d

* the separat° when the flow is turbulent occurs after the separation when the flow is laminar

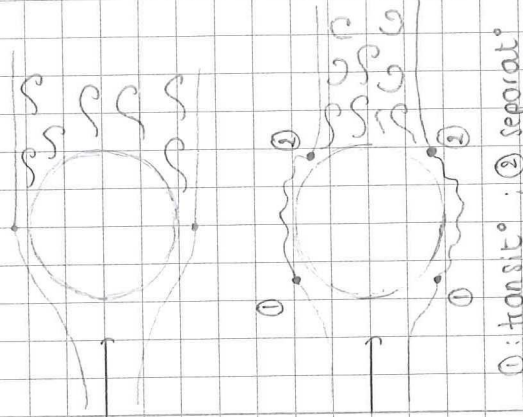
mb drag distribut°,

pressure
p_{stato}



a turbulent boundary layer has a higher velocity (= more energy) close to the surface than a laminar BL

stays attached longer: more energy to bend the streamlines



ex: dimples on a golf ball: disturb the flow $\downarrow$ turbulent

* no laminar separat°
 * transit° occurs later

$\Rightarrow$ $\downarrow$ pressure drag (main cause of drag on bluff bodies)

ASML

Be part of progress



* low Re: high influence of viscosity (friction)

↓
for bluff bodies: laminar separation = high pressure drag.

↗ Re:
(more energy)

↓ of the B.L. thickness

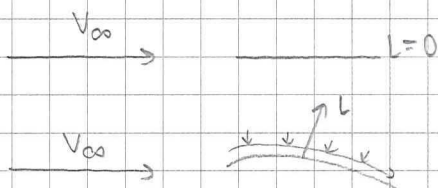
↓ c_f

transition from laminar to turbulent moves forward (leading e) + separat° moves backwards

↓ drag (3 wings over 2 for an airfoil)

↗ $c_{l,max}$ (because flows stays attached longer)

lift slope goes up (less de-cambering of the airfoil)



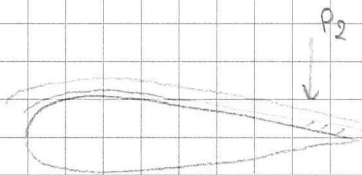
streamlines curved: a force must have moved them down

↳ act° is minus reaction: if a force (downwards) is applied on the flow, the flow exerts a force (upwards on the plate).

the thicker the boundary layer, the less lift:



no big curvature: P_1 (pressure needed to curve the streamline) small → react° (lift) small.



big curvature: P_2 big → react° (lift) big.

Laminar / turbulent airfoil

↓
transit° locat° near to the leading edge } due to ≠
transit° locat° near to the trailing edge } pressure distribut°

better $\frac{L}{D}$

↓
turbulent flow: more drag (for the same c_L)

$\tau_{w,turbulent} > \tau_{w,laminar}$

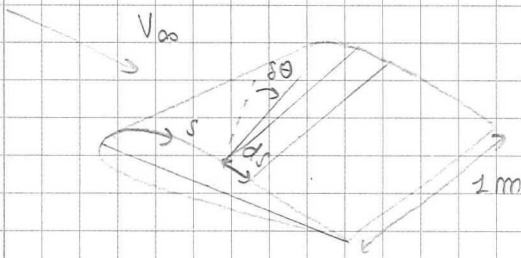
↗ shifted upwards.

mb cambered airfoils: better $c_L(\alpha) \propto c_d(c_L)$ than symmetrical airfoils.

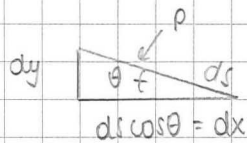
Let's be remarkable.



BOSCH



$$N = \int_{\text{leading edge}}^{\text{trailing edge}} p_e \cos \theta \, ds - \int_{LE}^{TE} p_u \cos \theta$$

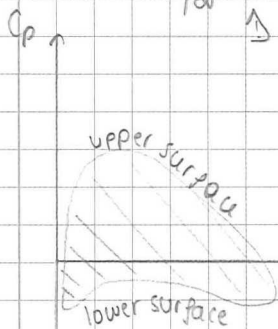


$$N = \int_{x=0}^{x=c} p_e \, dx - \int_{x=0}^{x=c} p_u \, dx$$

$$C_m = \frac{N}{q_\infty \cdot c} \rightarrow C_m = \int_0^1 \frac{p_e}{q_\infty} d\left(\frac{x}{c}\right) - \int_0^1 \frac{p_u}{q_\infty} d\left(\frac{x}{c}\right)$$

$$= \int_0^1 \left(\frac{p_e - p_\infty}{q_\infty} \right) d\left(\frac{x}{c}\right) - \int_0^1 \left(\frac{p_u - p_\infty}{q_\infty} \right) d\left(\frac{x}{c}\right)$$

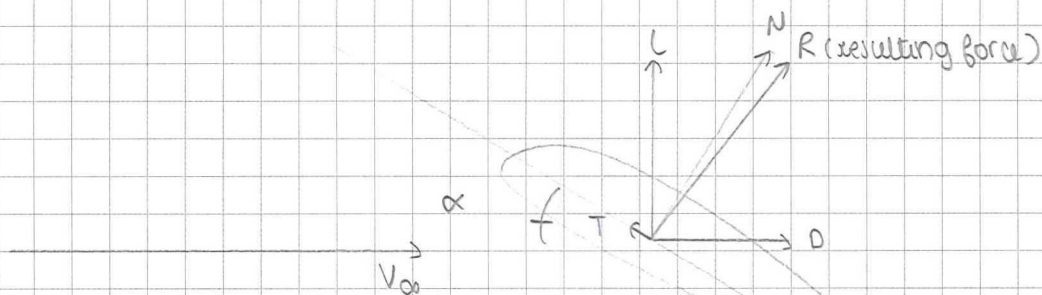
pressure coeff.



$C_m = \text{area}$

$$= \int_0^1 (C_{p_e} - C_{p_u}) d\left(\frac{x}{c}\right)$$

w, only the pressure distributo: not the shape of the airfoil.



L : $\perp$ to V_∞

N : $\perp$ to chord

ΔT = tangential force

mb: $\alpha < 5^\circ \rightarrow L = N$

$$L = N \cos \alpha - T \sin \alpha$$

$$D = N \sin \alpha + T \cos \alpha$$

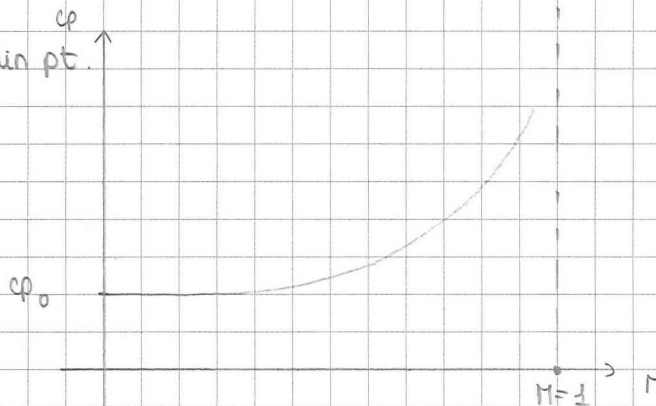
($\rightarrow$ pressure drag!)

$$C_L = C_m \cos \alpha - C_E \sin \alpha$$

$$C_D = C_m \sin \alpha + C_E \cos \alpha$$

compressibility correction of the pressure coefficient

at a certain pt.



$$C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}}$$

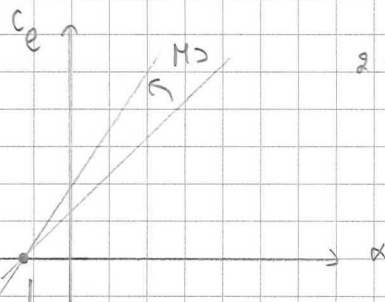
$C_{p,0}$: incompressible pressure coef for $M \approx 0$

valid for C_e airfoils, not for wings and only when $M < 0.7$.

$$C_e = \int_0^1 \frac{(C_{pe} - C_{pu})_0 d(\frac{x}{c})}{\sqrt{1-M_\infty^2}} = \frac{1}{\sqrt{1-M_\infty^2}} \int_0^1 (C_{pe} - C_{pu})_0 d(\frac{x}{c}) = \frac{C_{e,0}}{\sqrt{1-M_\infty^2}}$$

$$C_{e,0} = \int_0^1 (C_{pe} - C_{pu})_0 d(\frac{x}{c})$$

nb $C_{e,0}$: incompressible lift coeff. for $M \approx 0$



2 lift curves for the same α , $\neq M$

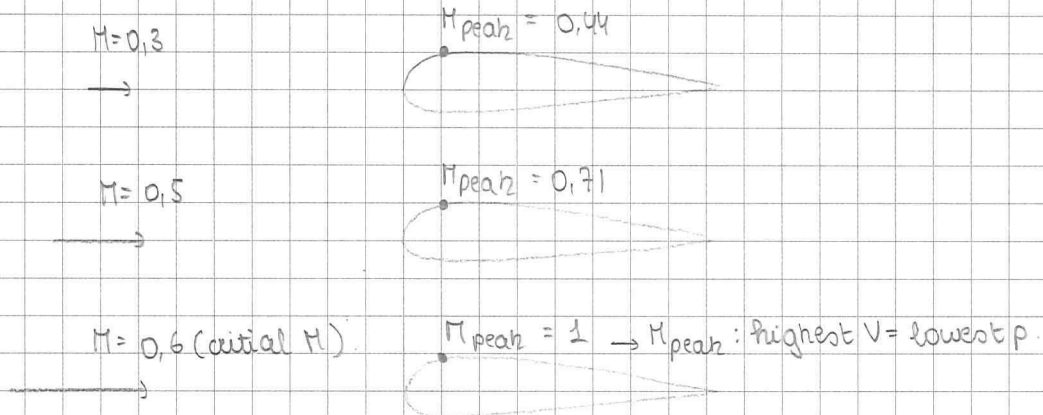
same point for both curves

ex $\alpha = 4^\circ$, $C_{e,0} = 0.9$, $M = 0.7$

$$C_e = \frac{0.9}{\sqrt{1-0.7^2}} = 1.26$$

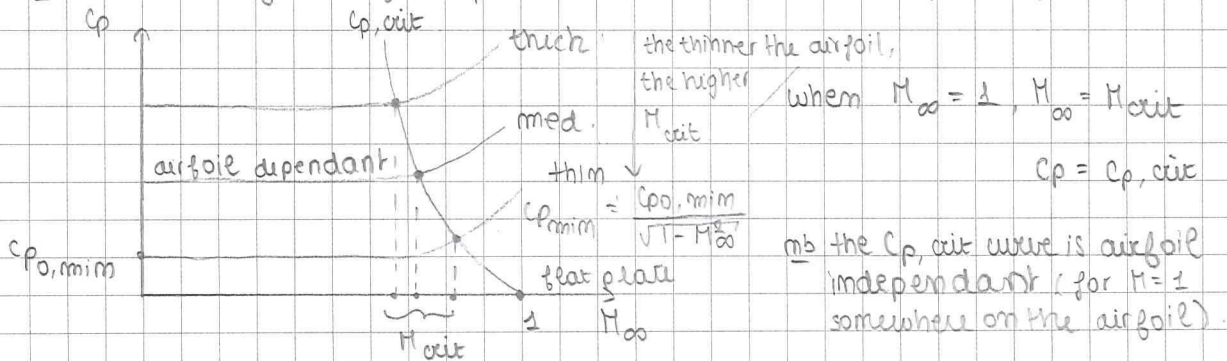


critical M & critical pressure coefficient
for a particular airfoil

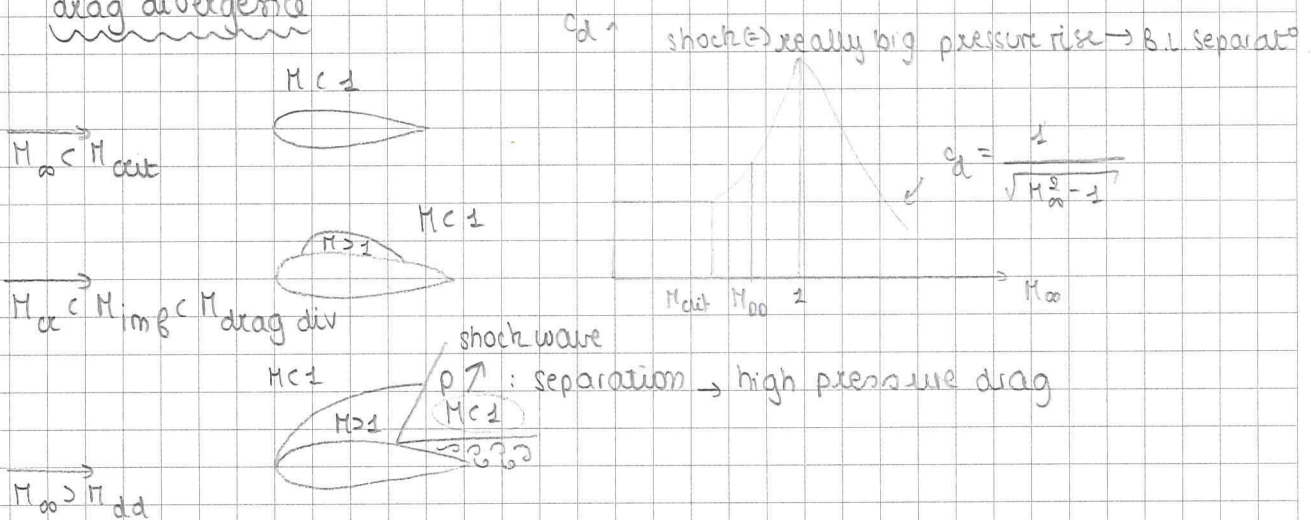


critical M : M of the incoming flow for which $M = 1$ on the airfoil

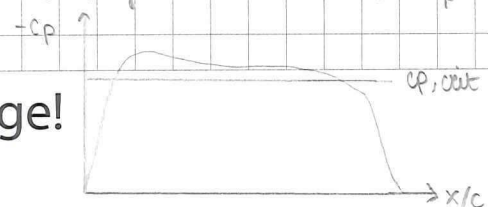
nb critical M for a flat plate = 1 (doesn't speed up the flow)



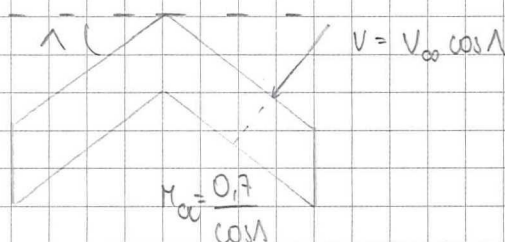
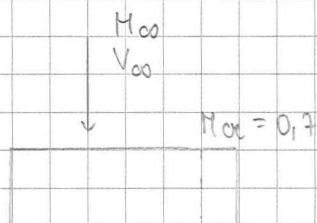
drag divergence



Supercritical airfoil: the shock occurs later on the airfoil: minimize the C_p rise
 $\rightarrow$ restrain upper surface thickness.

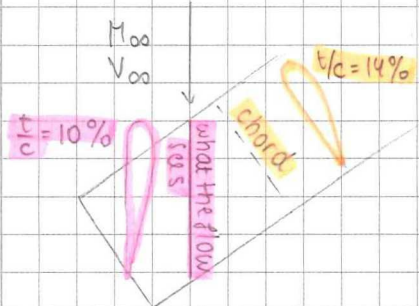


Swept wings : dealing w/ $M_{\infty} \propto M_{crit}$



by sweeping the wings for subsonic aircraft, M_{∞} is "delayed" to higher Mach numbers

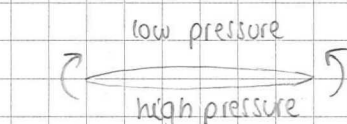
$$M_{cr \text{ swept}} = \frac{M_{cr \text{ not swept}}}{\cos \Lambda}$$



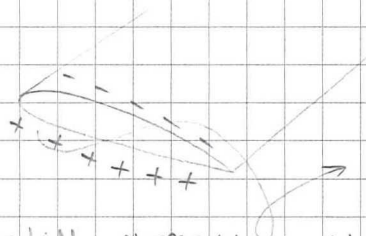
a thinner airfoil has higher $M_{cr} \propto M_{\infty}$

mb $\frac{L}{D} \downarrow$ because of the sweep

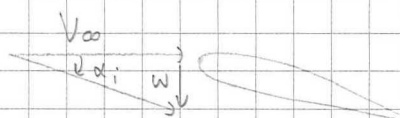
Finite wings



(the higher the pressure $\neq$, the $\uparrow$ the lift = strong tip vortices)



the trailing tip vortex causes downwash : changes the local flow vector (w)



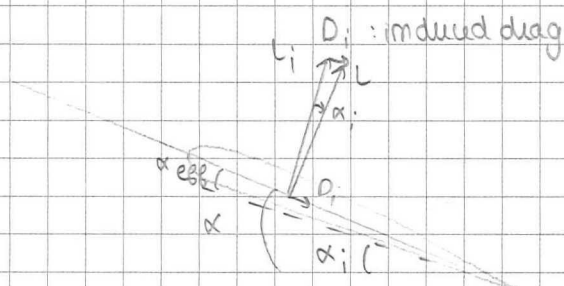
w = downwash

α_i : induced angle of attack

$\alpha - \alpha_i = \alpha_e$ (α effective) -

$D_i = L \sin \alpha_i$

if α_i is small : $\sin \alpha_i \approx \alpha_i \rightarrow D_i = L \alpha_i$





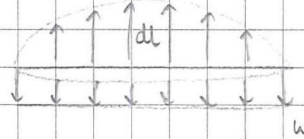
elliptical lift distribution $\Rightarrow$ constant downwash = constant α_i

$$\alpha_i = \frac{C_l}{\pi A}$$

$$C_{D_i} = C_l \alpha_i$$

$$C_{D_i} = \frac{C_l^2}{\pi A}$$

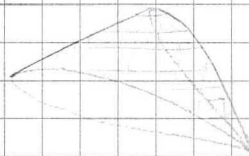
$$w, A = \frac{b^2}{S}$$



how to get

elliptical loading $\rightarrow$ elliptically shaped wings

$\rightarrow$ twist



span efficiency factor

$$C_{D_i} = \frac{C_l^2}{\pi A e_e}$$

e_e : span efficiency factor $\neq e$ (oswald eff factor)

elliptical loading: $e = 1$ (min. C_{D_i})

non-ell. loading: $e > 1$ ($\nearrow C_{D_i}$)

for a wing

$$C_D = C_{D_p} + \frac{C_l^2}{\pi A e_e}$$

$\downarrow$
profile C_{D_p} (lower case d)



for a complete aircraft

$$\hat{=} C_D = C_{D_0} + \frac{C_l^2}{\pi A e_e}$$

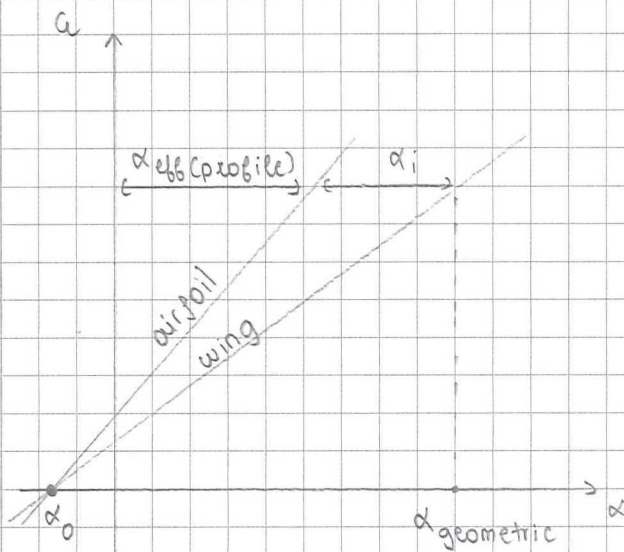
$\downarrow$
drag coeff at $C_l = 0$

reduce C_{D_i} : $\nearrow A$ (high $A =$ low C_{D_i})
approach elliptical loads

effect on the lift curve slope

having a finite wing ($\neq$ airfoil) reduce the lift curve slope

$$\alpha_i = \frac{57,3 C_l}{\pi A e_e} \quad (\alpha_i \text{ in } ^\circ) \quad \text{or} \quad \alpha_i = \frac{C_l}{\pi A e_e} \quad (\alpha_i \text{ in rad})$$



$$a_0 = \frac{dC_l}{d\alpha} \quad (\text{airfoil})$$

$$a = \frac{dC_l}{d\alpha} \quad (\text{wing})$$

$$\alpha_{\text{eff}} \text{ or } \alpha_{\text{profile}} = \alpha - \alpha_i$$

↓
 $\alpha_{\text{geometric}}$

$$\frac{dC_l}{d(\alpha - \alpha_i)} = a_0 \rightarrow \text{integrate: } C_l = a_0 (\alpha - \alpha_i) + C^{\text{st}}$$

$$\text{w, } \alpha_i = \frac{C_l}{\pi A e_1} : C_l = a_0 \left(\alpha - \frac{C_l}{\pi A e_1} \right) + C$$

$$\left(1 + \frac{a_0}{\pi A e_1} \right) C_l = a_0 \alpha + C$$

$$C_l = \frac{a_0 \alpha}{1 + \frac{a_0}{\pi A e_1}} + \frac{C}{1 + \frac{a_0}{\pi A e_1}} \rightarrow \text{deriv: } \frac{dC_l}{d\alpha} = \frac{a_0}{1 + \frac{a_0}{\pi A e_1}} \quad \nearrow \text{rads}$$

$$= \frac{a_0}{1 + \frac{57.3 a_0}{\pi A e_1}} \quad (\text{deg}).$$

$$C_l = \frac{dC_l}{d\alpha} (\alpha - \alpha_0) = a (\alpha - \alpha_0).$$

$$a_{0, \text{comp}} = \frac{a_0}{\sqrt{1 - M_{\infty}^2}} \quad (\text{incomp.})$$

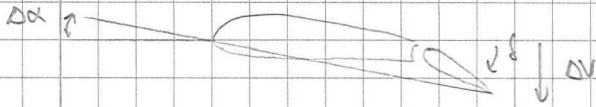
$$\underline{\text{mb}} \quad C_D = \underbrace{C_{D, \text{fr.}}}_{\text{friction}} + \underbrace{C_{D, \text{pressure}}}_{\text{separat.}} + \underbrace{C_{D, \text{wave}}}_{\text{shock waves}} \rightarrow C_D = C_{D_0} + \frac{C_l^2}{\pi A e_1}.$$

Flaps

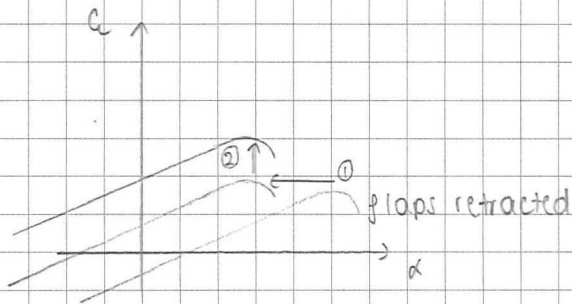
$$W = L = \frac{1}{2} \rho v^2 S C_L$$

$$V_{\infty} = \sqrt{\frac{2W'}{\rho S C_L}}$$

$$V_{\text{stall}} = \sqrt{\frac{2W'}{\rho S C_{L_{\text{max}}}}}$$



- 1) increase in effective $\alpha \rightarrow$ curve goes up
- 2) increase the curvature of the flow (camber) $\rightarrow$ curve goes up



if the flap is too much deflected: separation
 $\rightarrow$ slots: the flow stays attached longer

$\downarrow$
 can deflect the flow to a higher level.

This sheet gives the equations you have to know by heart. All other equations necessary for solving the aerodynamic problem will be given at the exam

Equation of state	$p = \rho RT \quad [\text{N/m}^2]$
Aerofoil drag force	$D = C_d \frac{1}{2} \rho V^2 c \quad [\text{N/m}]$
Aerofoil lift force	$L = C_l \frac{1}{2} \rho V^2 c \quad [\text{N/m}]$
Aspect ratio	$A = \frac{b^2}{S}$
Bernoulli equation	$p + \frac{1}{2} \rho V^2 = \text{Constant} \quad (\text{along a streamline})$
Continuity equation	$\rho AV = \text{Constant}$
Dynamic pressure	$q = \frac{1}{2} \rho V^2 \quad [\text{N/m}^2]$
Energy equation	$C_p T + \frac{1}{2} V^2 = \text{Constant}$
Mach number	$M = \frac{V}{a}$
Pressure coefficient	$C_p = \frac{p - p_o}{q_o} \quad \text{also} \quad C_p = \frac{p - p_\infty}{q_\infty}$
Reynolds number	$\text{Re} = \frac{\rho V L}{\mu} = \frac{V L}{\nu} \quad \text{for spheres and cylinders } L=d$ For airfoils and wings $L=c$ (the chord)
Speed of sound	$a = \sqrt{\gamma R T} \quad [\text{m/s}]$
Wing lift force	$L = C_l \frac{1}{2} \rho V^2 S \quad [\text{N}]$

Wing drag force

$$D = C_D \frac{1}{2} \rho V^2 S \quad [\text{N}]$$

Wing drag coefficient

$$C_D = C_d + \frac{C_L^2}{\pi A e_1}$$

2nd form of isentropic relations

$$\frac{T_o}{T_i} = 1 + \frac{\gamma - 1}{2} M_i^2$$

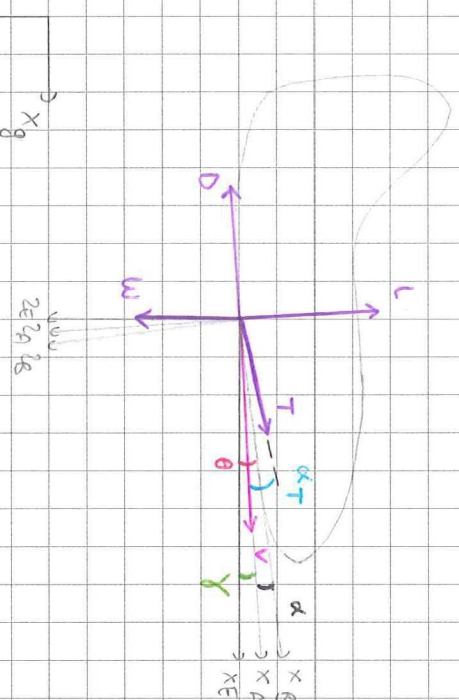
Prandtl-Glauert correction

$$C_l = \frac{C_{l,0}}{\sqrt{(1 - M^2)}}$$

Furthermore you should be able to derive the Bernoulli equation from the Euler equation ($dp = -\rho V dV$)

Flight mechanics

1. axis-system, free body diagram, kinetic diagram



$\Rightarrow$ angle of attack

$$\theta = \alpha + \gamma$$

$\Rightarrow$ flight path angle

Pitch attitude

X_E, Z_E : moving earth axis sys.

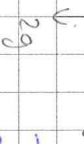
X_G, Z_G : ground axis sys.

X_A, Z_A : air path axis sys.

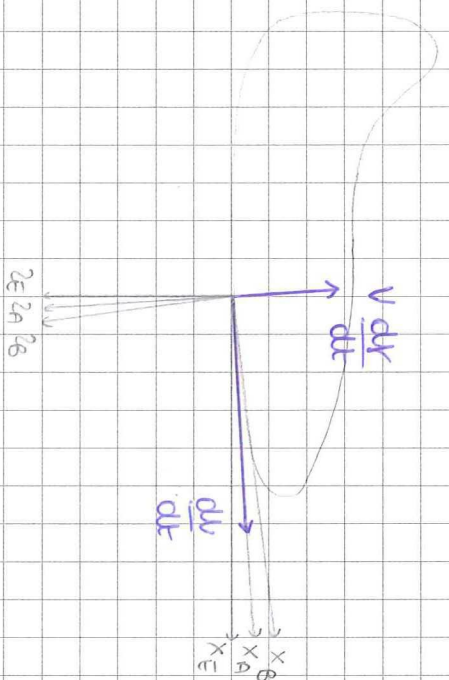
X_E, Z_E : body axis sys.

$\Rightarrow$ goes through the nose

α_T : thrust angle of attack:
engines not always // to the nose.



The ground axis system is an inertial frame of reference if
* earth is considered flat (ignore $\vec{a}_e$)
* earth is not rotating (assume that when going from South to North, you're moving in a straight path)



$$a = \frac{V^2}{R} \quad V = \omega R \quad \omega = \frac{d\theta}{dt}$$

$$U = \frac{d\theta}{dt} \cdot R$$

$$a = V \frac{d\theta}{dt}$$



2. Equations of motion $F = m a$ (inertial frame of reference)

$$\sum F_{\parallel} : \frac{w}{g} \frac{dV}{dt} = T \cos(\alpha_T) - D - w \sin(\gamma)$$

$$\sum F_{\perp} : \underbrace{\frac{w}{g}}_m \cdot \underbrace{V \frac{d\alpha}{dt}}_a = L + T \sin(\alpha_T) - w \cos(\gamma)$$

variables: * t (time) independent variable

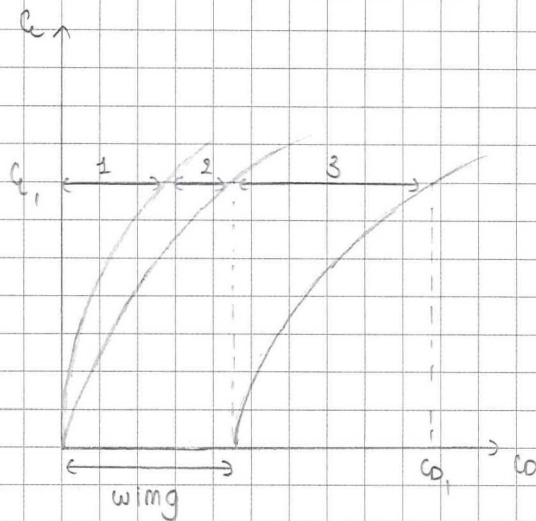
* γ, V (direct $\propto$ magnitude of airspeed) state variables
* $\theta, T(L, w, \alpha_T) \rightarrow$ to express in terms of γ and V .

Drag

lift-drag polar: $c_D = c_{D_0} + \frac{C_L^2}{\pi A e}$ (for the entire aircraft)

mb for wing only: $c_D = c_{D_0} + \frac{C_L^2}{\pi A e_1}$

where $e_1 = 1$ for elliptical load ($e_1 = e$)



1. profile drag coefficient (c_{D_0})
2. induced drag coefficient
3. parasite drag coefficient

The lift-drag polar is a parabolic relation $\rightarrow c_D = c_{D_0} + k_1 C_L + k_2 C_L^2$

$\rightarrow$ we want to express the drag as a function of airspeed.

assumption steady horizontal flight

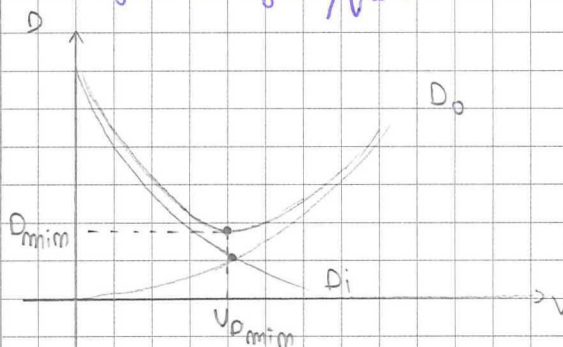
$$L = W = \frac{1}{2} \rho V^2 S C_L \rightarrow C_L = \frac{W}{\frac{1}{2} \rho V^2 S} \quad (\text{at a given altitude, } \rho = \text{const} : C_L \text{ only function of airspeed})$$

$$D = \frac{1}{2} \rho V^2 S c_D = \frac{1}{2} \rho V^2 S (c_{D_0} + k_1 C_L + k_2 C_L^2)$$

$$= \frac{1}{2} \rho V^2 S c_{D_0} + \frac{1}{2} \rho V^2 S k_1 \left(\frac{W}{\frac{1}{2} \rho V^2 S} \right) + \frac{1}{2} \rho V^2 S k_2 \left(\frac{W}{\frac{1}{2} \rho V^2 S} \right)^2$$

$$= \underbrace{\frac{1}{2} \rho V^2 S c_{D_0} + k_1 W}_{D_0} + \underbrace{\frac{2 k_2 W^2}{\rho V^2 S}}_{D_{\text{induced}}}$$

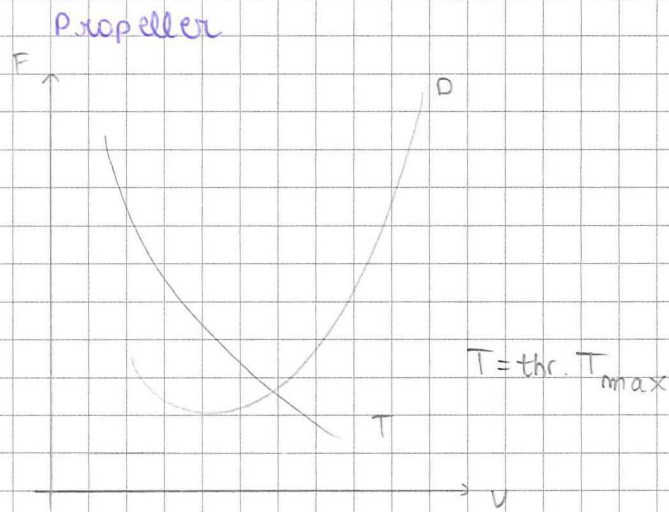
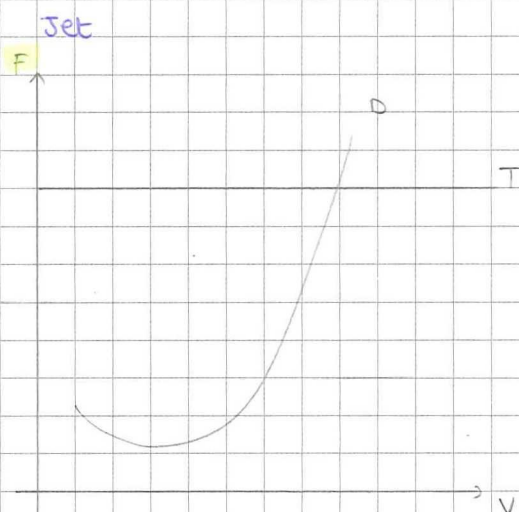
$$D = f(V^2) + f(1/V^2)$$



at low speed: induced drag important
 $\rightarrow$ high aspect ratio (slender wings)

at high speed: D_0 important

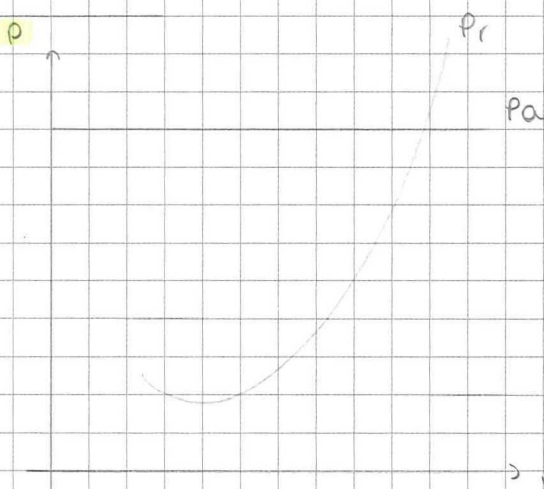
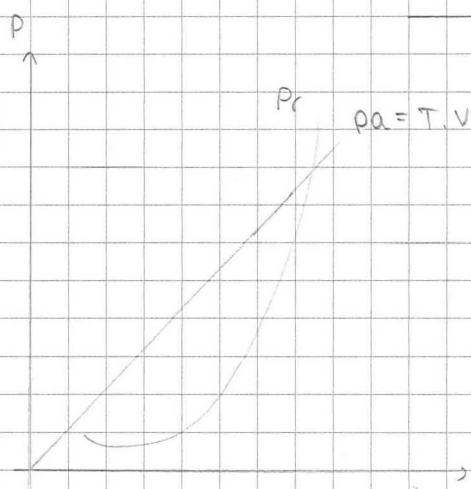
3. Performance diagrams



$$T = \dot{m} (V_j - V_0)$$

$\uparrow$ $\downarrow$
 $\quad \quad \quad =$

propeller blade $\approx$ wing
The lift generated by a propeller blade can be decomposed in thrust & torque (rotate needed).



$$P_r = D \cdot V$$

$$P_a = T \cdot V$$

$$\eta_{th} = \frac{P_j}{Q} \quad w, Q = \dot{m}_f H$$

$$\eta_j = \frac{P_a}{P_j} \quad w, P_j = \frac{1}{2} \dot{m} (V_j^2 - V_0^2)$$

$$= \frac{2}{1 + V_j/V_0}$$

$$\eta_{tot} = \eta_{thermal} \cdot \eta_j = \frac{P_a}{Q}$$

which % of fuel E is used for transport

mb loss of kinetic E needed:
if $\eta_j = 1$, $V_j = V_0 \rightarrow T = 0$.

$P_r = D \cdot V$ (power needed to overcome the aerodynamic drag)

$P_a = \text{constant}$: you can change the propeller pitch such that $\eta_j = \text{cst}$
 $P_a = \text{constant}$

$$\eta_j = \frac{P_a}{P_{br}}$$

(shaft power : cst)

change of propeller pitch $\rightarrow \eta_j \approx \text{cst}$

4. Horizontal flight performanceflight conditions1) steady: V does not vary in time, magnitude or direction

$$\frac{dV}{dt} = 0$$

2) horizontal: constant altitude; $y = 0$ and $\dot{\alpha}_T = 0$

$$\frac{dy}{dt} = 0$$

3) symmetric: the aircraft is not turning
(sideslip angle) $= 0$ eq. of motions

$$\Sigma F_{\parallel V}: T \cos \alpha_T - D - W \sin \gamma = \frac{W}{g} \frac{dV}{dt} \Rightarrow \Sigma F_{\parallel V}: T = D$$

$$\Sigma F_{\perp V}: L - W \cos \gamma + T \sin \alpha_T = \frac{W}{g} V \frac{d\gamma}{dt} \Rightarrow \Sigma F_{\perp V}: L = W$$

1. minimum airspeed occurs at $C_{L_{max}}$

$$L = W = \frac{1}{2} \rho V^2 S C_L \quad (\Rightarrow) \quad V = \sqrt{\frac{2W}{\rho S C_L}}$$

$$V_{min} = \sqrt{\frac{2W}{\rho S C_{L_{max}}}}$$

2. maximum airspeeda) for a propeller aircraft: occurs at $P_a = P_r$

$$P_a = P_r$$

$$= D \cdot V \quad \text{w, } D = \frac{1}{2} \rho V^2 S C_D \quad \& \quad L = W = \frac{1}{2} \rho V^2 S C_L \rightarrow D = \frac{C_D}{C_L} \cdot W$$

$$= \frac{C_D}{C_L} W \cdot \sqrt{\frac{2W}{\rho S C_L}}$$

$$P_a^2 = \frac{2W^3}{\rho S} \cdot \frac{C_D^2}{C_L^3} \quad \text{w, } C_D = C_{D0} + k_1 C_L + k_2 C_L^2$$

$$P_a^2 = \frac{2W^3}{\rho S} \cdot \frac{(C_{D0} + k_1 C_L + k_2 C_L^2)^2}{C_L^3}$$

solve for C_L & take the smallest positive value.

b) for a jet aircraft : occurs at $T_{\max}$

$$T_{\max} = 0 \quad (\text{steady, horizontal flight : } T=0)$$

$$= \frac{C_0}{C_L} W$$

$$= \frac{C_{D0} + k_1 C_L + k_2 C_L^2}{C_L} W$$

$$T_{\max} C_L = (C_{D0} + k_1 C_L + k_2 C_L^2) W$$

$$0 = C_{D0} + k_1 C_L + k_2 C_L^2 - \frac{T_{\max}}{W} C_L$$

$$0 = C_{D0} + \left(k_1 - \frac{T_{\max}}{W}\right) C_L + k_2 C_L^2$$

solve for C_L & take the smallest positive value.

3 maximum range (distance)

a) for a propeller aircraft : occurs at $\left(\frac{V}{F}\right)_{\max} \rightarrow \text{at } D_{\min}$

$$F = \frac{C_P}{\eta} P_r \quad (\text{fuel flow}) \quad F \equiv \frac{C_P}{\eta} P_{br} W, \quad \eta = \frac{P_a}{P_{br}} \text{ and } P_a = P_r \text{ (because } T=0)$$

$$\left(\frac{V}{F}\right)_{\max} = \frac{V}{\frac{C_P}{\eta} P_r} = \frac{V}{\frac{C_P}{\eta} D V} = \frac{\eta}{C_P} \frac{1}{D_{\min}}$$

const.

at which V does $D_{\min}$ occur?

$$D = C_D \frac{1}{2} \rho V^2 S = \frac{C_D}{C_L} W \rightarrow D_{\min} = \left(\frac{C_D}{C_L}\right)_{\min} W = \left(\frac{C_D}{C_L}\right)_{\max} W$$

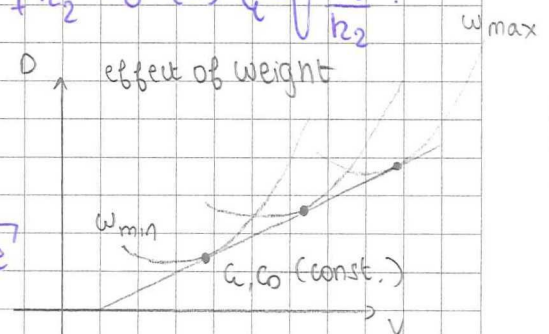
$$\frac{C_D}{C_L} = \frac{C_{D0} + k_1 C_L + k_2 C_L^2}{C_L} = \frac{C_{D0}}{C_L} + k_1 + k_2 C_L$$

$$\frac{d}{dC_L} \left(\frac{C_D}{C_L} \right) = 0 \quad (\text{find extr})$$

$$\frac{d}{dC_L} \left(\frac{C_{D0}}{C_L} + k_1 + k_2 C_L \right) = -\frac{C_{D0}}{C_L^2} + k_2 \rightarrow -\frac{C_{D0}}{C_L^2} + k_2 = 0 \quad (\Rightarrow) C_L = \sqrt{\frac{C_{D0}}{k_2}}$$

$$\text{mb } C_D = C_{D0} + \frac{C_L^2}{\pi A e} \rightarrow \frac{C_D}{C_L} = \frac{C_{D0}}{C_L} + \frac{C_L}{\pi A e}$$

$$\frac{d}{dC_L} \left(\frac{C_D}{C_L} \right) = -\frac{C_{D0}}{C_L^2} + \frac{1}{\pi A e} = 0 \quad (\Rightarrow) C_L = \sqrt{C_{D0} \pi A e}$$



Let's be remarkable.

$$V = \frac{2W}{\rho S C_L}$$

$$D = \frac{C_D}{C_L} W$$

→ the graph shifts upward and right



BOSCH

Technologie voor het leven



b) for a jet aircraft: occurs at $\left(\frac{U}{F}\right)_{\max} \rightarrow \text{at } \left(\frac{D}{V}\right)_{\min}$

$F \equiv C_T \cdot T = C_T \cdot D$ (horizontal steady flight: $T=D$)

$\frac{U}{F} = \frac{U}{C_T \cdot D}$; as C_T is constant, to have $\left(\frac{U}{F}\right)_{\max}$, you need $\left(\frac{D}{V}\right)_{\min}$

at which velocity does $\left(\frac{D}{V}\right)_{\min}$ occur?

$$\left(\frac{D}{V}\right)_{\min} = \frac{D}{L} \cdot \frac{1}{V} = \frac{C_D}{C_L} \cdot \frac{1}{V} = \frac{C_D}{C_L} \cdot \frac{1}{\sqrt{\frac{2W}{\rho S C_L}}} = \left(\frac{C_D}{C_L^2}\right)_{\max}$$

$$\left(\frac{C_D}{C_L^2}\right)_{\max} \rightarrow \frac{d}{dC_L} \left(\frac{C_D}{C_L^2}\right) = 0 \quad \text{quotient rule}$$

$$\frac{C_D^2 - C_L \cdot 2C_D \cdot \frac{dC_D}{dC_L}}{C_L^4} = 0 \quad \text{chain rule}$$

$$w, \frac{dC_D}{dC_L} = \frac{d}{dC_L} (C_{D0} + k_1 C_L + k_2 C_L^2)$$

$$= k_1 + 2k_2 C_L$$

$$C_D^2 - 2C_L C_D (k_1 + 2k_2 C_L) = 0$$

$$2C_L (k_1 + 2k_2 C_L) = C_{D0} + k_1 C_L + k_2 C_L^2$$

$$3k_2 C_L^2 + k_1 C_L - C_{D0} = 0$$

$$\Rightarrow C_L = \frac{-k_1 \pm \sqrt{k_1^2 + 12k_2 C_{D0}}}{6k_2}$$

4 maximum endurance (time)

a) for a propeller aircraft: occurs at $F_{\min} \rightarrow P_{r \min}$

$$F = C_p \cdot P_{br} = \frac{C_p}{\eta} \cdot P_a = \frac{C_p}{\eta} \cdot P_r$$

$$P_{r \min} = D \cdot V = \frac{C_D}{C_L} \cdot \frac{W}{\sqrt{\frac{2W}{\rho S C_L}}} = \sqrt{\frac{2W^3}{\rho S}} \cdot \left(\frac{C_D}{C_L^3}\right)_{\min} \rightarrow \left(\frac{C_D^3}{C_L^2}\right)_{\max}$$

$$\frac{d}{dC_L} \left(\frac{C_D^3}{C_L^2}\right) = \frac{C_D^3 \cdot 3C_L^2 - C_L^3 \cdot 2C_D \frac{dC_D}{dC_L}}{C_L^4} = 0$$

$$\frac{3}{2} \frac{C_D}{C_L} = \frac{dC_D}{dC_L} \quad \Rightarrow \quad \frac{3}{2} \frac{(C_{D0} + k_1 C_L + k_2 C_L^2)}{C_L} = k_1 + 2k_2 C_L$$

$$3C_{D0} + 3k_1 C_L + 3k_2 C_L^2 = 2C_L k_1 + 4k_2 C_L^2$$

$$k_2 C_L^2 - k_1 C_L - 3C_{D0} = 0 \quad \Rightarrow \quad C_L = \frac{k_1 \pm \sqrt{k_1^2 + 12k_2 C_{D0}}}{2k_2}$$



b) for a jet aircraft: occurs at $F_{\min} \rightarrow$ at $D_{\min}$

$$F = c_T \cdot T = c_T \cdot D$$

$$D_{\min} = \left(\frac{C_D}{C_L} \right)_{\min} \cdot W$$

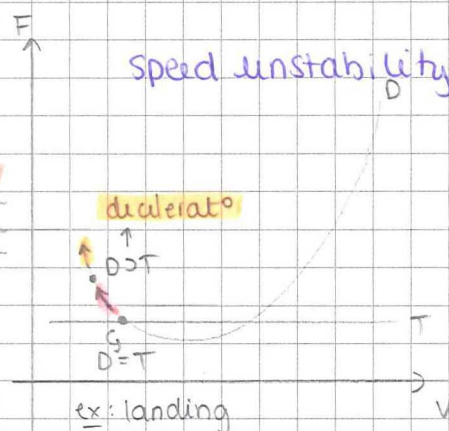
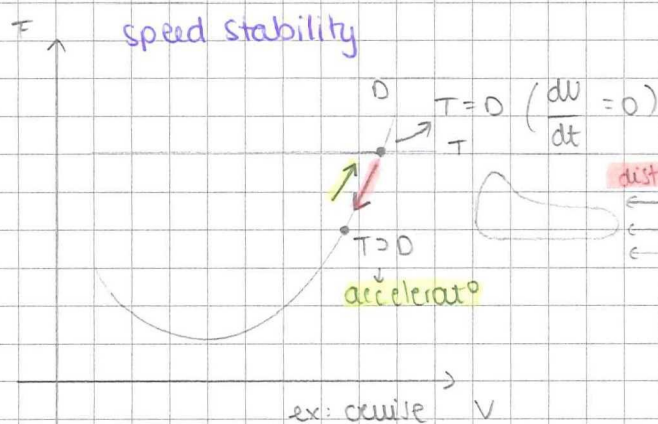
$$\left(\frac{C_D}{C_L} \right)_{\min} \rightarrow \frac{d}{dC_L} \left(\frac{C_D}{C_L} \right) = 0$$

$$\frac{d}{dC_L} \left(\frac{C_{D0} + k_1 C_L + k_2 C_L^2}{C_L} \right) = 0$$

$$\frac{d}{dC_L} \left(\frac{C_{D0}}{C_L} + k_1 + k_2 C_L \right) = 0$$

$$-\frac{C_{D0}}{C_L^2} + k_2 = 0 \Rightarrow C_L = \sqrt{\frac{C_{D0}}{k_2}}$$

5. Speed stability

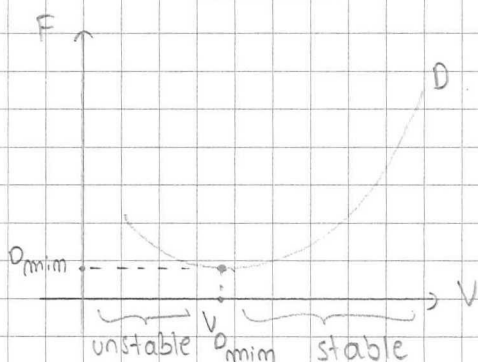


disturbance $\rightarrow$ plane slows down

$D \downarrow$
acceleratio

disturbance $\rightarrow$ plane slows down

$D \uparrow$
plane slows down even further



a disturb in speed is amplified by the system $\rightarrow$ adjust the throttle setting

The drag curve is a parabola
 $\rightarrow$ every plane has an unstable speed region

The transition from stable to unstable is at $V_{\min}$



6. Climbing flight performanceflight conditions

- 1) straight path: $\gamma \neq 0$ but $\frac{d\gamma}{dt} = 0$
- 2) small γ ($< 15^\circ$) for commercial flights
 $\cos \gamma \approx 1$ but $\sin \gamma \neq 0$.
- 3) steady flight: $V = \text{cst}$ so $\frac{dV}{dt} = 0$
- 4) $T \parallel V$ ($\alpha_T = 0$)

eq of motion

$$\sum F_{\perp V}: L - \cancel{W \cos \gamma} + \cancel{T \sin \alpha_T} = \cancel{\frac{W}{g} V \frac{d\gamma}{dt}} \Rightarrow \sum F_{\perp V}: L = W$$

$$\sum F_{\parallel V}: \cancel{T \cos \alpha_T} - D - \cancel{W \sin \gamma} = \cancel{\frac{W}{g} \frac{dV}{dt}} \Rightarrow \sum F_{\parallel V}: \frac{T - D}{W} = \sin \gamma$$

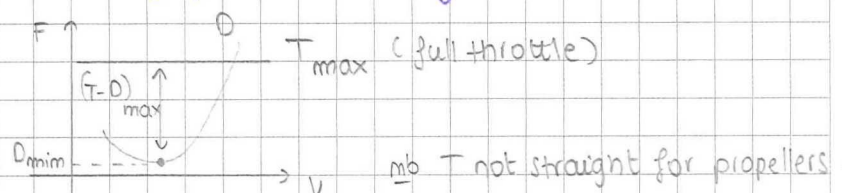
$$\text{or } V \sin \gamma = \frac{P_a - P_r}{W} = R.O.C$$

$\Rightarrow \theta \neq \gamma$: at low speed, a big angle of attack is needed to have sufficient lift $\rightarrow$ nose up but not necessarily climbing.

1. max climb angle no energy conversion during a steep climb
 $\rightarrow$ add E using propulsion system

for jet aircrafts:

$$\sin(\gamma_{\max}) = \frac{(T - D)_{\max}}{W}$$



$\gamma_{\max}$ occurs at $(T - D)_{\max} \rightarrow D_{\min}$, as T is constant.

$$C_{L_{\text{opt}}} = \sqrt{\frac{C_{D0}}{k_2}}$$

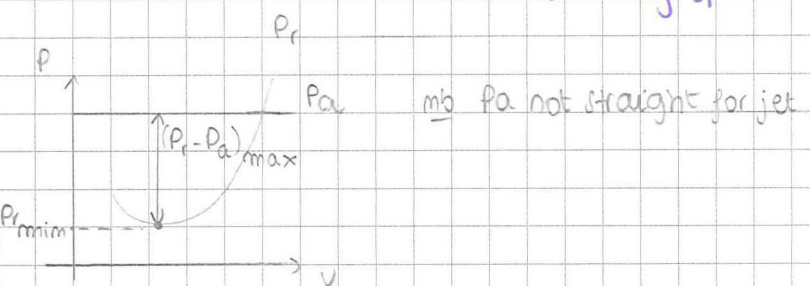
2. max rate of climb fast climb = trading kin E for pot E but stay above stalling speed.

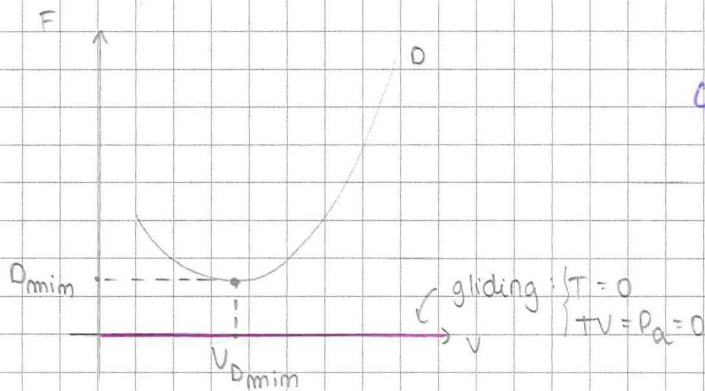
$$V \sin(\gamma) = \frac{P_a - P_r}{W} = R.O.C$$

for propeller aircrafts:

$R.O.C_{\max}$ occurs at $P_{r_{\min}}$

$$C_{L_{\text{opt}}} = \frac{k_1 \pm \sqrt{k_1^2 + 12 k_2 C_{D0}}}{2 k_2}$$



3. minimum descend angle in gliding flight, for all aircrafts.

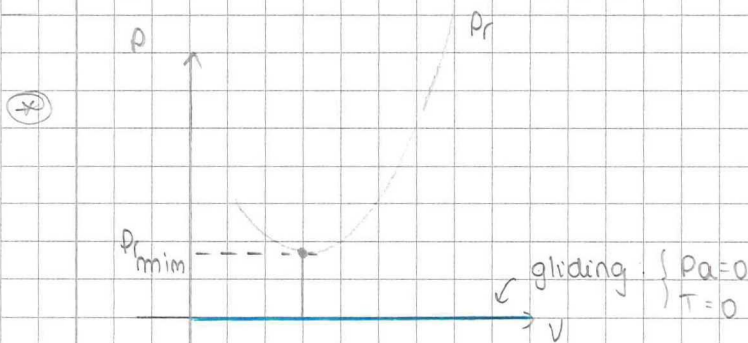
$$C_L = \sqrt{\frac{C_{D0}}{k_2}} \text{ for } D_{\min}$$

$$\sin \gamma = \frac{T-D}{W} \text{ but } T=0 \rightarrow \sin \gamma = \frac{-D}{W} = \frac{-D}{L} \text{ (because } W=L) = \frac{-C_D}{C_L}$$

$$C_D = C_{D0} + k_1 C_L + k_2 C_L^2$$

$$= C_{D0} + k_1 \sqrt{\frac{C_{D0}}{k_2}} + \cancel{k_2} \cdot \frac{C_{D0}}{\cancel{k_2}} = 2C_{D0} + k_1 \sqrt{\frac{C_{D0}}{k_2}}$$

$$\gamma = \arcsin\left(\frac{-C_D}{C_L}\right) = \arcsin\left(\frac{2C_{D0}}{\sqrt{\frac{C_{D0}}{k_2}}} + k_1\right) = \arcsin(2\sqrt{C_{D0}k_2} + k_1)$$

4. minimum rate of descend in gliding flightoccurs at $P_{\min}$ for prop

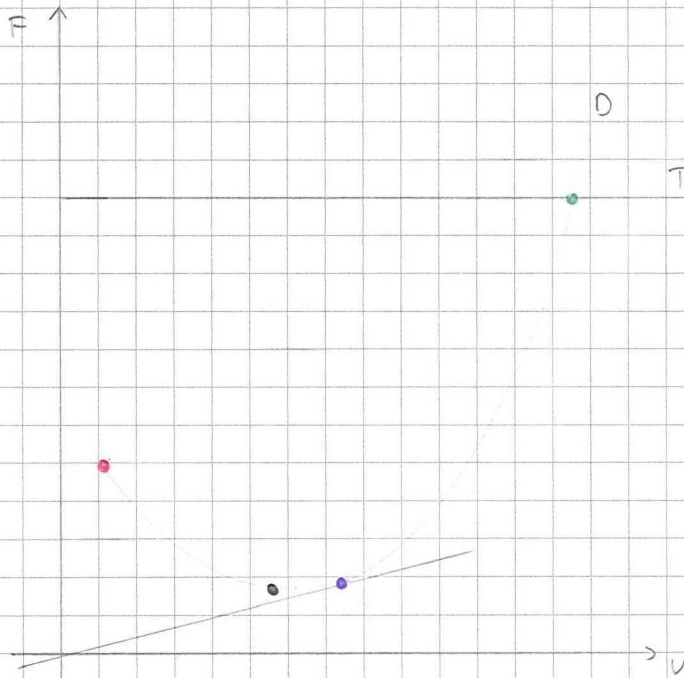
$$ROD = \frac{P_a - P_r}{W} \text{ but } P_a=0 \rightarrow ROD = \frac{D}{W} = \frac{C_D}{C_L} V_{\text{opt}}$$

$$V = \sqrt{\frac{2W}{\rho S C_{L_{\text{opt}}}}} \rightarrow \downarrow \text{ when } \rho \uparrow \text{ (i.e. when alt } \downarrow \text{ closer to the ground)}$$



7. Summary

1. Ideal jet



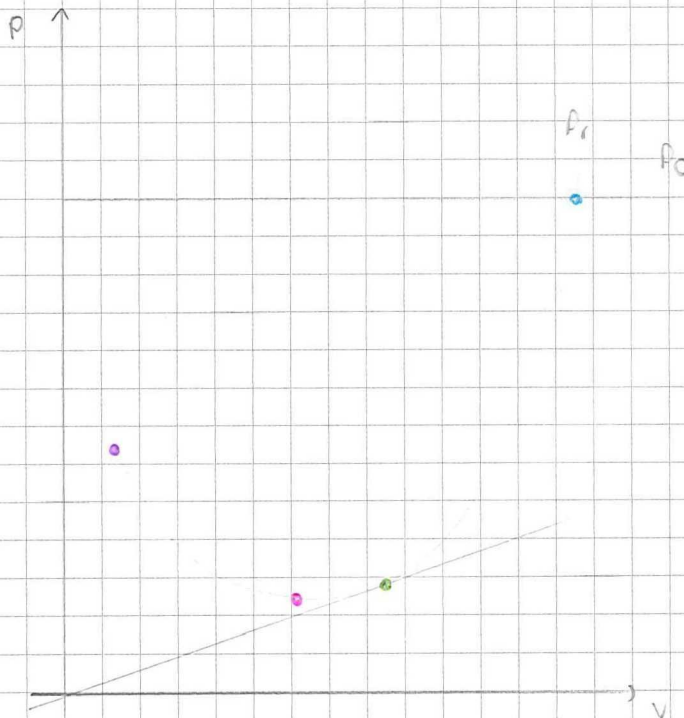
1. min speed u_{max}

2. max endurance $C_L = \sqrt{\frac{C_{D0}}{k_2}}$
max climb angle
min descend angle

3. max range $C_L = -k_1 = \frac{\sqrt{k_1^2 + 12k_2C_{D0}}}{6k_2}$

4. max airspeed

2. Ideal propeller



1. min speed u_{max}

2. max endurance $C_L = \frac{k_1 \pm \sqrt{k_1^2 + 12k_2C_{D0}}}{2k_2}$
max rate of climb
min rate of descend

3. max range min descend angle $C_L = \sqrt{\frac{C_{D0}}{k_2}}$

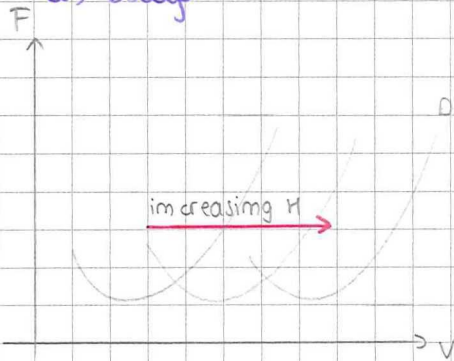
4. max airspeed



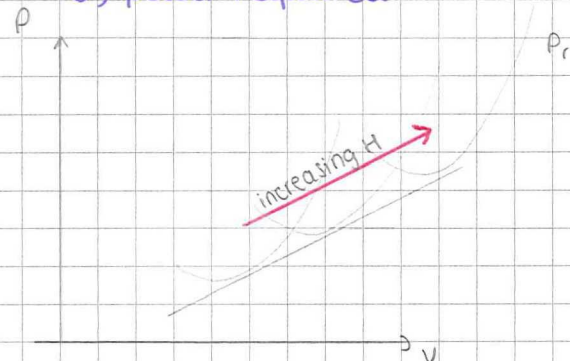
8. Performance limits

1. altitude effects on aircraft performance

a) Drag



b) power required



$$\left. \begin{aligned} D_{H1} &= \frac{C_{D1}}{C_{L1}} W \\ D_{H2} &= \frac{C_{D2}}{C_{L2}} W \end{aligned} \right\} D_{H1} = D_{H2}$$

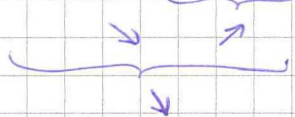
$$\left. \begin{aligned} V_{H1} &= \sqrt{\frac{2W}{\rho_1 S C_{L1}}} \\ V_{H2} &= \sqrt{\frac{2W}{\rho_2 S C_{L2}}} \end{aligned} \right\} \frac{V_{H1}}{V_{H2}} = \sqrt{\frac{\rho_2}{\rho_1}}$$

if $H_2 > H_1 \rightarrow \rho_2 < \rho_1$ then $V_2 > V_1$
V should increase to keep the same lift.

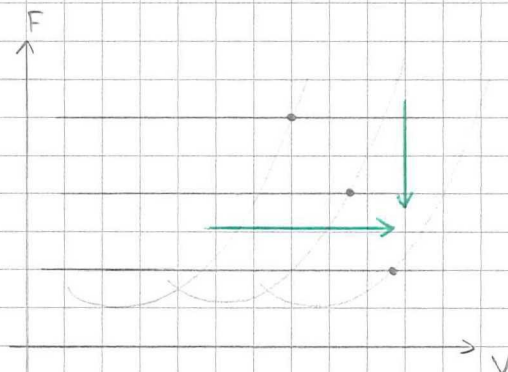
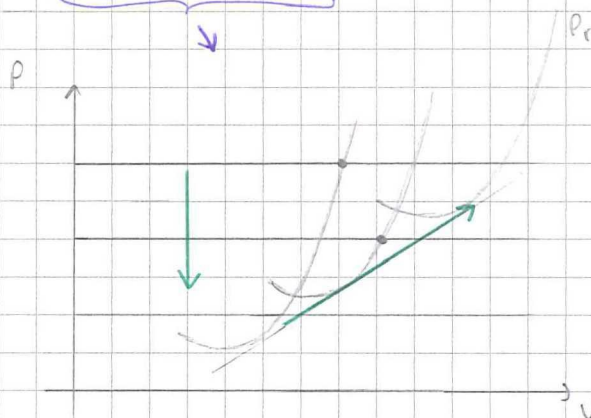
$$\left. \begin{aligned} P_{X1} &= D_1 V_1 \\ P_{X2} &= D_2 V_2 \end{aligned} \right\} D_1 = D_2 \rightarrow \frac{P_{X1}}{P_{X2}} = \frac{V_1}{V_2}$$

2. altitude effects on propulsion

$$T = \dot{m} (V_j - V_o)$$



mb $H \uparrow \rightarrow \rho \downarrow \rightarrow \dot{m} \downarrow$
 $H \uparrow \rightarrow T \downarrow \rightarrow V_j \uparrow$
 $\left. \begin{aligned} &\dot{m} \downarrow \\ &V_j \uparrow \end{aligned} \right\} \dot{m} \text{ goes down more than } V_j \text{ goes up}$



model the change in engine Power with: $\frac{P}{P_0} = \frac{\rho}{\rho_0}^m$

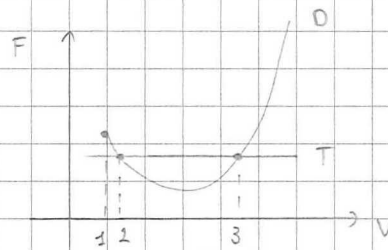
$\frac{T}{T_0} = \frac{\rho}{\rho_0}^m$



3. Performance limits

1. minimum airspeed

$$V_{min} = \sqrt{\frac{2W}{\rho S C_{L_{max}}}}$$



1) V_{min} but $D > T$

2) V_{min} : 2nd sol. of V_{max} eq.

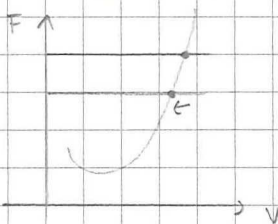
3) V_{max}

at high altitude: less air $\rightarrow$ to generate the same lift, C_L can not be changed as you already fly at $C_{L_{max}}$ thus you have to fly faster and $T \downarrow$ w, h so V_{min} also limited by thrust

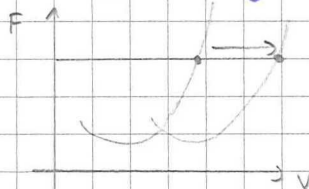
2. max airspeed

it depends:

a) if propulsion effect is dominant: $V_{max} \downarrow$ w, H

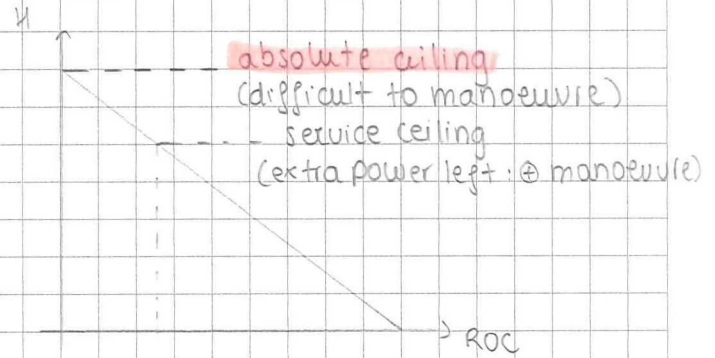
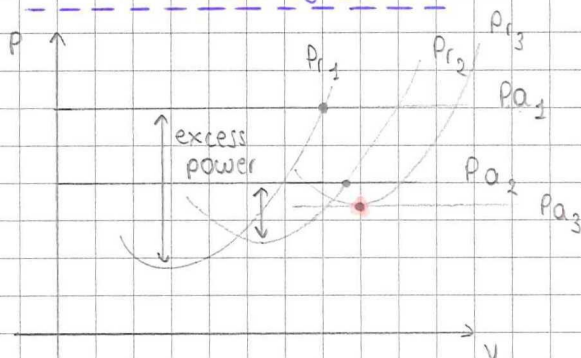


b) if aerodynamic effect is dominant: $V_{max} \uparrow$ w, H



but in general, V_{max} reduces w, H

3. max rate of climb



$$\frac{1}{T_s} = \frac{e}{l_s} \quad m=1 \quad (\text{stratosphere } s)$$

$$\underbrace{e \frac{T_s}{l_s}}_T = \underbrace{\frac{C_0}{C}}_D \cdot W$$

$$e_{\min} = \frac{l_s}{T_s} \cdot W \left(\frac{C_0}{C} \right)_{\min} \rightarrow C = \sqrt{\frac{C_{D0}}{k_2}} \text{ or } C = \sqrt{C_{D0} \pi A e}$$

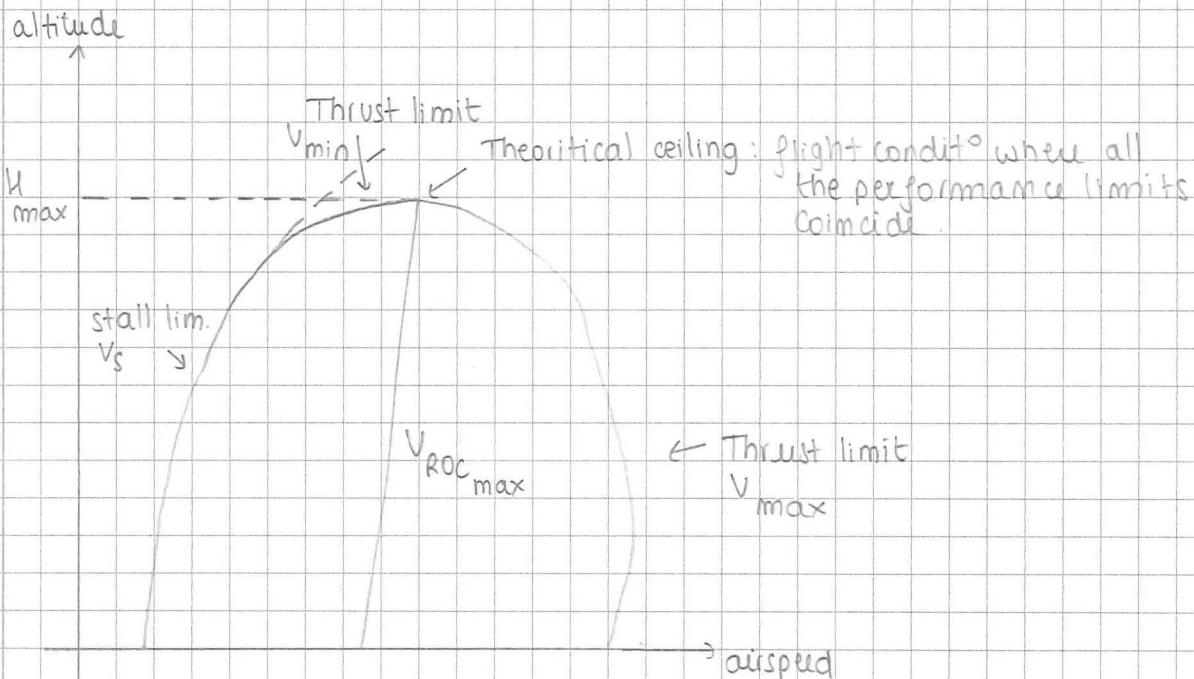
$$C_0 = C_{D0} + \frac{W^2}{\pi A e} = C_{D0} + \frac{C_{D0} \pi A e}{\pi A e} = 2C_{D0}$$

$\approx$ glider w, powerful engines

$$e_{\min} = \frac{l_s}{T_s} \cdot W \cdot \frac{2C_{D0}}{\sqrt{C_{D0} \pi A e}}$$

low
powerful high good

S. flight envelope



gratis
collegeblok

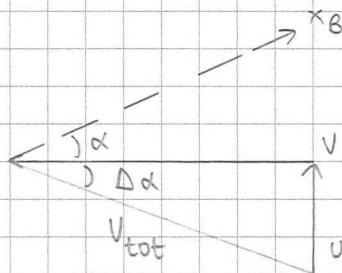
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1. Manoeuvre loads

$$\text{load factor } m = \frac{L}{W} \rightarrow m = \frac{C_L}{\frac{2W}{\rho S V^2}} : m = f(V^2)$$

2. Aeroelastic effects V_{DE}

3. Gust loads



$$m = \frac{L}{W}$$

$$\Delta m = \frac{\Delta L}{W} = \frac{\Delta C_L}{W} \cdot \frac{1}{2} \rho (V^2 + U^2) S$$

$$\Delta C_L = \frac{dC_L}{d\alpha} \Delta \alpha$$

$$\Delta \alpha = \frac{U}{V}$$

$$= \frac{dC_L}{d\alpha} \frac{U}{V} \frac{1}{W} \cdot \frac{\rho}{2} S V^2$$

$$\Delta m = \frac{dC_L}{d\alpha} \frac{\rho S}{2W} UV$$

4. Dive speed V_D

$$V_{D, TAS} = V_{D, EAS} \sqrt{\frac{\rho_0}{\rho}}$$

5. Max mach number

$$M = \frac{V}{a} \quad w, \quad a = \sqrt{\gamma R T} \quad w, \quad T = f(h)$$

buffeting: pressure induced vibrat°

($\Rightarrow$) $V = M \cdot a$ $\rightarrow$ limit on M because the local velocity over the wing (at high V_{∞}) can surpass the speed of sound $\rightarrow$ shockwaves = buffeting $\rightarrow$ damage and the aerodynamic control surfaces become less effective

6. Pressurized cabin

$P_{in} > P_{out} \rightarrow P_{out} \downarrow$ when $H \uparrow \rightarrow$ altitude limit.

7. Flight envelope

