2.1 $\rho = p/RT = (1.2)(1.01 \times 10^5)/(287)(300)$

$\rho = 1.41 \text{ kg/m}^2$

$v = 1/\rho = 1/1.41 = \boxed{0.71 \text{ m}^3/\text{kg}}$

2.2 Mean kinetic energy of each atom =

$$\frac{3}{2}\ k\ T = \frac{3}{2}\ (1.38 \times 10^{-23})(500) = 1.035 \times 10^{-20} \text{ J}$$

One kg-mole, which has a mass of 4 kg, has 6.02 x 10^{26} atoms. Hence 1 kg has $\frac{1}{4}$ (6.02 x 10^{26}) = 1.505 x 10^{26} atoms

Total internal energy = (energy per atom)(number of atoms)

$= (1.035 \times 10^{-20})(1.505 \times 10^{26}) = \boxed{1.558 \times 10^6 \text{ J}}$

2.3 $$\rho = \frac{p}{RT} = \frac{2116}{(1716)(460+59)} = 0.00237 \frac{\text{slug}}{\text{ft}^3}$$

Volume of the room = (20)(15)(8) = 2400 ft^3

Total mass in the room = (2400)(0.00237) = 5.688 slug

Weight = $(5.688)(32.2) = \boxed{183 \text{ lb}}$

2.4 $$\rho = \frac{p}{RT} = \frac{2116}{(1716)(460-10)} = 0.00274 \frac{\text{slug}}{\text{ft}^3}$$

Since the volume of the room is the same, we can simply compare densities between the two problems.

$$\Delta\rho = 0.00274 - 0.00237 = 0.00037 \frac{\text{slug}}{\text{ft}^3}$$

$$\% \text{ change} = \frac{\Delta\rho}{\rho} = \frac{0.00037}{0.00237} \text{x } (100) = \boxed{15.6\% \text{ increase}}$$

2.5 First, calculate the density from the known mass and volume,

$\rho = 1500/900 = 1.67 \ lb_m/ft^3$

In consistent units, $\rho = 1.67/32.2 = 0.052 \ slug/ft^3$. Also, T = 70F = 70 + 460 = 530R.

Hence,

$p = \rho RT = (0.52)(1716)(530)$

$p = 47290 \ lb/ft^2$

or $p = 47290/2116 = \boxed{22.3 \text{ atm}}$

2.6 $p = \rho RT$

$\ell n p = \ell n \rho + \ell n R + \ell n T$

Differentiating with respect to time,

$$\frac{1}{p}\frac{dp}{dt} = \frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{T}\frac{dT}{dt}$$

or, $$\frac{dp}{dt} = \frac{p}{\rho}\frac{d\rho}{dt} + \frac{p}{T}\frac{dT}{dt}$$

or, $$\frac{dp}{dt} = RT\frac{d\rho}{dt} + \rho R\frac{dT}{dt} \qquad (1)$$

At the instant there is 1000 lb_m of air in the tank, the density is

$\rho = 1000/900 = 1.11 \ lb_m/ft^3$

$\rho = 1\ 11/32\ 2 = 0\ 0345$ slug/ft^3

Also, in consistent units, is is given that

$T = 50 + 460 = 510R$

and that

$$\frac{dT}{dt} = 1F/min = 1R/min = 0\ 0167R/sec$$

From the given pumping rate, and the fact that the volume of the tank is 900 ft^3, we also have

$$\frac{d\rho}{dt} = \frac{0 5\ lb_m / sec}{900\ ft^3} = 0\ 000556\ lb_m/(ft^3)(sec)$$

$$\frac{d\rho}{dt} = \frac{0.000556}{32\ 2} = 1\ 73 \times 10^{-5}\ slug/(ft^3)(sec)$$

Thus, from equation (1) above,

$$\frac{dp}{dt} = (1716)(510)(1\ 73 \times 10^{-5}) + (0\ 0345)(1716)(0\ 0167)$$

$$= 15\ 1 + 0\ 99 = 16\ 1\ lb/(ft^2)(sec) = \frac{16.1}{2116}$$

$$= \boxed{0.0076\ atm/sec}$$

2.7 In consistent units,

$T = -10 + 273 = 263K$

Thus,

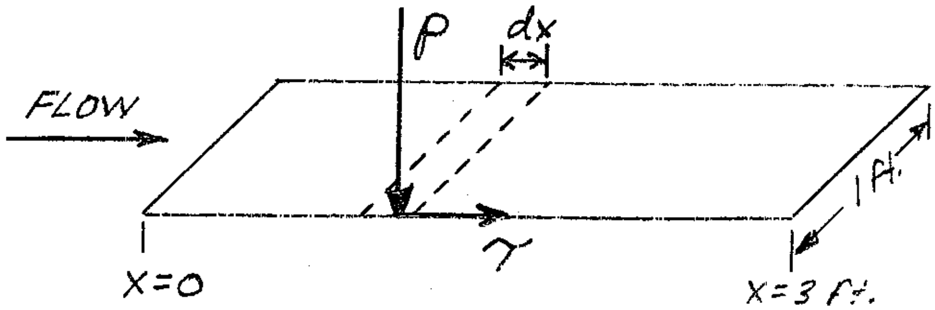
$\rho = p/RT = (1\ 7 \times 10^4)/(287)(263$

$\rho = \boxed{0.225\ kg/m^3}$

2.8 $\rho = p/RT = 0\ 5 \times 10^5/(287)(240) = 0\ 726\ kg/m^3$

$v = 1/\rho = 1/0\ 726 = \boxed{1.38\ m^3/kg}$

2.9

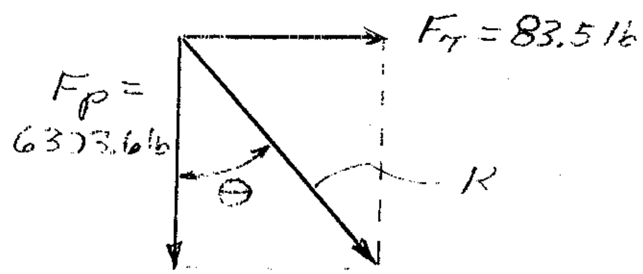


F_p = Force due to pressure = $\int_0^3 p\ dx = \int_0^3 (2116 - 10x)\ dx$

$= [2116x - 5x^2]_0^3 = 6303$ lb perpendicular to wall

F_τ = Force due to shear stress = $\int_0^3 \tau\ dx = \int_0^3 \frac{90}{(x+9)^{\frac{1}{2}}} dx$

$= [180\ (x+9)^{\frac{1}{2}}]_0^3 = 623\ 5 - 540 = 83\ 5$ lb tangential to wall



Magnitude of the resultant aerodynamic force =

$R = \sqrt{(6303)^2 + (83\ 5)^2} = \boxed{6303.6\ lb}$

$$\theta = \text{Arc Tan}\left(\frac{83.5}{6303}\right) = \boxed{0.76^\circ}$$

2.10 $V = \frac{3}{2} V_\infty \sin\theta$

Minimum velocity occurs when $\sin\theta = 0$, i.e. when $\theta = 0^\circ$ and 180°.

$\boxed{V_{min} = 0}$ at $\theta = 0^\circ$ and 180°, i.e., at its most forward and rearward points

Maximum velocity occurs when $\sin\theta = 1$, i.e. when $\theta = 90^\circ$ Hence

$$V_{max} = \frac{3}{2}(85)(1) = \boxed{127.5 \text{ mph}} \text{ at } \theta = 90^\circ,$$

i.e., the entire rim of the sphere in a plane perpendicular to the freestream direction.

2.11 The mass of air displaced is

$$M = (2.2)(0.002377) = 5.23 \times 10^{-3} \text{ slug}$$

The weight of this air is

$$W_{air} = (5.23 \times 10^{-3})(32.2) = 0.168 \text{ lb}$$

This is the lifting force on the balloon due to the outside air. However, the helium inside the balloon has weight, acting in the downward direction. The weight of the helium is less than that of air by the ratio of the molecular weights

$$W_{H_e} = (0.168)\frac{4}{28.8} = 0.0233 \text{ lb}.$$

Hence, the maximum weight that can be lifted by the balloon is

$$0.168 - 0.0233 = \boxed{0.145 \text{ lb}}$$

3.1 An examination of the standard temperature distribution through the atmosphere given in Figure 3.3 of the text shows that both 12 km and 18 km are in the same constant temperature region. Hence, the equations that apply are Eqs. (3.9) and (3.10) in the text. Since we are in the same isothermal region with therefore the same base values of p and ρ, these equations can be written as

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1} = e^{-(g_o/RT)(h_2-h_1)}$$

where points 1 and 2 are any two arbitrary points in the region. Hence, with g_o = 9.8 m/sec^2 and R = 287 joule/kgK, and letting points 1 and 2 correspond to 12 km and 18 km altitudes respectively,

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1} = e^{-\frac{9.8}{(287)(216.66)}(6000)} = 0.3884$$

Hence:

$$p_2 = (0.3884)(1.9399 \times 10^4) = \boxed{7.53 \times 10^3\ \text{N/m}^2}$$

$$\rho_2 = (0.3884)(3.1194 \times 10^{-1}) = \boxed{0.121\ \text{kg/m}^3}$$

and of course

$$T_2 = \boxed{216.66\text{K}}$$

These answers check the results listed in Appendix A of the text within round-off error.

3.2 From Appendix A of the text, we see immediately that p = 2.65 x 10^4 N/m^2 corresponds to 10,000 m, or 10 km, in the standard atmosphere. Hence,

$$\boxed{\text{pressure altitude} = 10\ \text{km}}$$

The outside air density is

$$\rho = \frac{p}{RT} = \frac{2.65x10^4}{(287)(220)} = 0\ 419\ kg/m^3$$

From Appendix A, this value of ρ corresponds to 9.88 km in the standard atmosphere.

Hence,

$$\boxed{\text{density altitude} = 9.88\ km}$$

3.3 At 35,000 ft, from Appendix B, we find that $p = 4.99 \times 10^2 = \boxed{499\ lb/ft^2}$.

3.4 From Appendix B in the text,

33,500 ft corresponds to $p = 535.89\ lb/ft^2$

32,000 ft corresponds to $\rho = 8.2704 \times 10^{-4}\ slug/ft^3$

Hence,

$$T = \frac{p}{\rho R} = \frac{535.89}{(8.2704 \times 10^{-4})(1716)} = \boxed{378\ R}$$

3.5 $\frac{|h - h_G|}{h} = 0.02 = \left|1 - \frac{h_G}{h}\right|$

From Eq. (3.6), the above equation becomes

$$\left|1 - \left(\frac{r + h_G}{r}\right)\right| = \left|1 - 1 - \frac{h_G}{r}\right| = 0.02$$

$$h_G = 0.02\ r = 0.02\ (6.357 \times 10^6)$$

$$h_G = 1.27 \times 10^5\ m = \boxed{127\ km}$$

3.6 $T = 15 - 0.0065h = 15 - 0.0065(5000) = -17.5°C = 255.5°K$

$$a = \frac{dT}{dh} = -0.0065$$

From Eq (3.12)

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-g_o/aR} = \left(\frac{255.5}{288}\right)^{-(9.8)/(-0.0065)(287)} = 0.533$$

$$p = 0.533\, p_1 = 0.533\,(1.01 \times 10^5) = \boxed{5.38 \times 10^4\ N/m^2}$$

3.7 $$\ell n\ \frac{p}{p_1} = -\frac{g}{RT}\ (h - h_1)$$

$$h - h_1 = -\frac{1}{g}\ RT\ \ell n\ \frac{p}{p_1} = -\frac{1}{24.9}\ (4157)(150)\ \ell n\ 0.5$$

Letting $h_1 = 0$ (the surface)

$$h = 17{,}358\ m = \boxed{17.358\ km}$$

3.8 A standard altitude of 25,000 ft falls within the first gradient region in the standard atmosphere. Hence, the variation of pressure and temperature are given by:

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-\frac{g}{aR}} \qquad (1)$$

and

$$T = T_1 + a\,(h - h_1) \qquad (2)$$

Differentiating Eq (1) with respect to time:

$$\frac{1}{p_1}\frac{dp}{dt} = \left(\frac{1}{T_1}\right)^{-\frac{g}{aR}} \left(-\frac{g}{AR}\right) T^{\left(-\frac{g}{aR}-1\right)} \frac{dT}{dt} \tag{3}$$

Differentiating Eq (2) with respect to time:

$$\frac{dT}{dt} = a\frac{dh}{dt} \tag{4}$$

Substitute Eq (4) into (3)

$$\frac{dp}{dt} = -\; p_1 (T_1)^{\frac{g}{aR}} \left(\frac{g}{R}\right) T^{-\left(\frac{g}{aR}+1\right)} \frac{dh}{dt} \tag{5}$$

In Eq. (5), dh/dt is the rate-of-climb, given by dh/dt = 500 ft/sec. Also, in the first gradient region, the lapse rate can be calculated from the tabulations in Appendix B. For example, take 0 ft and 10,000 ft, we find

$$a = \frac{T_2 - T_1}{h_2 - h_1} = \frac{483.04 - 518.69}{10{,}000 - 0} = -0.00357 \frac{°R}{ft}$$

Also from Appendix B, p_1 = 2116.2 lb/ft^2 at sea level, and T = 429.64 °R at 25,000 ft. Thus,

$$\frac{g}{aR} = \frac{32.2}{(-0.00357)(1716)} = -5.256$$

Hence, from Eq (5)

$$\frac{dp}{dt} = -(2116.2)(518.69)^{-5.256}\left(\frac{32.2}{1716}\right)(429.64)^{4.256}(500)$$

$$\frac{dp}{dt} = \boxed{-1717 \frac{lb}{ft^2\,sec}}$$

3.9 From the hydrostatic equation, Eq. (3.2) or (3.3),

$$dp = -\rho\, g_o\, dh$$

or $$\frac{dp}{dt} = -\rho\, g_o \frac{dh}{dt}$$

The upward speed of the elevator is dh/dt, which is

$$\frac{dh}{dt} = \frac{dp/dt}{-\rho\, g_o}$$

At sea level, $\rho = 1.225$ kg/m^3. Also, a one-percent change in presure per minute starting from sea level is

$$\frac{dp}{dt} = -(1.01 \times 10^5)(0.01) = -1.01 \times 10^3 \text{ N/m}^2 \text{ per minute}$$

Hence

$$\frac{dh}{dt} = \frac{-1.01 \times 10^3}{(1.225)(9.8)} = \boxed{84.1 \text{ meter per minute}}$$

3.10 From Appendix B:

At 35,500 ft: $p = 535.89$ lb/ft^2

At 34,000 ft: $p = 523.47$ lb/ft^2

For a pressure of 530 lb/ft^2, the pressure altitude is

$$33{,}500 + 500 \left(\frac{535.89 - 530}{535.89 - 523.47}\right) = \boxed{33737 \text{ ft}}$$

The density at the altitude at which the airplane is flying is

$$\rho = \frac{p}{RT} = \frac{530}{(1716)(390)} = 7.919 \times 10^{-4} \text{ slug/ft}^3$$

From Appendix B:

At 33,000 ft: $\rho = 7\ 9656 \times 10^{-4}$ slug/ft^3

At 33,500 ft: $\rho = 7\ 8165 \times 10^{-4}$ slug/ft^3

Hence, the density altitude is

$$33{,}000 + 500 \left(\frac{7.9656 - 7.919}{7\ 9656 - 7.8165}\right) = \boxed{33{,}156 \text{ ft}}$$

4.1 $A_1V_1 = A_2V_2$

Let points 1 and 2 denote the inlet and exit conditions respectively Then,

$$V_2 = V_1 \left(\frac{A_1}{A_2}\right) = (5)\left(\frac{1}{4}\right) = \boxed{1.25 \text{ ft/sec}}$$

4.2 From Bernoulli's equation,

$$p_1 + \rho \frac{V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2}$$

$$p_2 - p_1 = \frac{\rho}{2} (V_1^2 - V_2^2)$$

In consistent units,

$$\rho = \frac{62.4}{32.2} = 1\ 94 \text{ slug/ft}^3$$

Hence,

$$p_2 - p_1 = \frac{1.94}{2} [(5)^2 - (1\ 25)^2]$$

$$p_2 - p_1 = 0.97\ (23\ 4) = \boxed{22.7 \text{ lb/ft}^2}$$

At 33,000 ft: $\rho = 7\ 9656 \times 10^{-4}$ slug/ft^3

At 33,500 ft: $\rho = 7\ 8165 \times 10^{-4}$ slug/ft^3

Hence, the density altitude is

$$33{,}000 + 500 \left(\frac{7.9656 - 7.919}{7\ 9656 - 7.8165}\right) = \boxed{33{,}156 \text{ ft}}$$

4.1 $A_1V_1 = A_2V_2$

Let points 1 and 2 denote the inlet and exit conditions respectively Then,

$$V_2 = V_1 \left(\frac{A_1}{A_2}\right) = (5)\left(\frac{1}{4}\right) = \boxed{1.25 \text{ ft/sec}}$$

4.2 From Bernoulli's equation,

$$p_1 + \rho\frac{V_1^2}{2} = p_2 + \rho\frac{V_2^2}{2}$$

$$p_2 - p_1 = \frac{\rho}{2}(V_1^2 - V_2^2)$$

In consistent units,

$$\rho = \frac{62.4}{32.2} = 1\ 94 \text{ slug/ft}^3$$

Hence,

$$p_2 - p_1 = \frac{1.94}{2}[(5)^2 - (1\ 25)^2]$$

$$p_2 - p_1 = 0.97\ (23\ 4) = \boxed{22.7 \text{ lb/ft}^2}$$

4.3 From Appendix A; at 3000m altitude,

$p_1 = 7.01 \times 10^4 \text{ N/m}^2$

$\rho = 0.909 \text{ kg/m}^3$

From Bernoulli's equation,

$$p_2 = p_1 + \frac{\rho}{2}(V_1^2 - V_2^2)$$

$$p_2 = 7.01 \times 10^4 + \frac{0.909}{2}[60^2 - 70^2]$$

$$p_2 = 7.01 \times 10^4 - 0.059 \times 10^4 = \boxed{6.95 \times 10^4 \text{ N/m}^2}$$

4.4 From Bernoulli's equation,

$$p_1 + \frac{\rho}{2}V_1^2 = p_2 + \frac{\rho}{2}V_2^2$$

Also from the incompressible continuity equation

$$V_2 = V_1 (A_1/A_2)$$

Combining,

$$p_1 + \frac{\rho}{2}V_1^2 = p_2 + \frac{\rho}{2}(A_1/A_2)^2$$

$$V_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left[(A_1/A_2)^2 - 1\right]}}$$

At standard sea level, $\rho = 0.002377 \text{ slug/ft}^3$ Hence,

$$V_1 = \sqrt{\frac{2(80)}{(.002377)[(4)^2 - 1]}} = \boxed{67 \text{ ft/sec}}$$

Note that also $V_1 = 67 \left(\frac{60}{80}\right) = 46$ mi/h. (This is approximately the landing speed of World War I vintage aircraft)

4.5 $p_1 + \frac{1}{2}\rho\, V^2 = p_3 + \frac{1}{2}\rho\, V^2$

$$V_1^2 = \frac{2(p_3 - p_1)}{\rho} + V_3^2 \qquad (1)$$

$$A_1\, V_1 = A_3\, V_3, \text{ or } V_3 = \frac{A_1}{A_3} V_1 \qquad (2)$$

Substitute (2) into (1)

$$V_1^2 = \frac{2(p_3 - p_1)}{\rho} + \left(\frac{A_1}{A_3}\right)^2 V_1^2$$

or, $$V_1 = \sqrt{\frac{2(p_3 - p_1)}{\rho\left[1 - \left(\frac{A_1}{A_3}\right)^2\right]}} \qquad (3)$$

Also,

$$A_1\, V_1 = A_2\, V_2$$

or, $$V_2 = \left(\frac{A_1}{A_2}\right) V_1 \qquad (4)$$

Substitute (3) into (4)

$$V_1 = \frac{A_1}{A_2} \sqrt{\frac{2(p_3 - p_1)}{\rho\left[1 - \left(\frac{A_1}{A_3}\right)^2\right]}}$$

$$V_2 = \frac{3}{15}\sqrt{\frac{2(1.00 - 1.02)\ x\ 10^5}{(1.225)\left[1-\left(\frac{3}{2}\right)^2\right]}}$$

$$V_2 = \boxed{102.22 \text{ m/sec}}$$

Note: It takes a pressure difference of <u>only</u> 0.02 atm to produce such a high velocity.

4.6 $V_1 = 130 \text{ mph} = 130\left(\frac{88}{60}\right) = 190.7 \text{ ft/sec}$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

$$V_2^2 = \frac{2}{\rho}(p_1 - p_2) + V_1^2$$

$$V_2^2 = \frac{2(1760.9 - 1750.0)}{0.0020482} + (190.7)^2$$

$$V_2 = \boxed{216.8 \text{ ft/sec}}$$

4.7 From Bernoulli's equation,

$$p_1 - p_2 = \frac{\rho}{2}(V_2^2 - V_1^2)$$

And from the incompressible continuity equation,

$$V_2 = V_1 (A_1/A_2)$$

Combining:

$$p_1 - p_2 = \frac{\rho}{2} V_1^2 [(A_1/A_2)^2 - 1]$$

Hence, the maximum pressure difference will occur when simultaneously:

1. V_1 is maximum

2. ρ is maximum i.e sea level

The design maximum velocity is 90 m/sec, and $\rho = 1.225$ kg/m^3 at sea level. Hence,

$$p_1 - p_2 = \frac{1.225}{2}(90)^2\left[(1.3)^2 - 1\right] = \boxed{3423\ \text{N/m}^2}$$

Please note: In reality the airplane will most likely exceed 90 m/sec in a dive, so the airspeed indicator should be designed for a maximum velocity somewhat above 90 m/sec.

4.8 The isentropic relations are

$$\frac{p_e}{p_o} = \left(\frac{\rho_e}{\rho_o}\right)^{\gamma} = \left(\frac{T_e}{T_o}\right)^{\frac{\gamma}{\gamma-1}}$$

Hence,

$$T_e = T_o\left(\frac{p_e}{p_o}\right)^{\frac{\gamma-1}{\gamma}} = (300)\left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = \boxed{155\text{K}}$$

From the equation of state:

$$\rho_o = \frac{p_o}{RT_o} = \frac{(10)(1.01\text{x}10^5)}{(287)(300)} = 11.73\ \text{kg/m}^3$$

Thus,

$$\rho_e = \rho_o\left(\frac{p_e}{p_o}\right)^{\frac{1}{\gamma}} = 11.73\left(\frac{1}{10}\right)^{\frac{1}{1.4}} = \boxed{2.26\ \text{kg/m}^3}$$

As a check on the results, apply the equation of state at the exit.

$p_e = \rho_e RT_e$?

$1.01 \times 10^5 = (2.26)(287)(155)$

$1.01 \times 10^5 = 1.01 \times 10^5$ It checks!

4.9 Since the velocity is essentially zero in the reservoir, the energy equation written between the reservoir and the exit is

$$h_o = h_e + \frac{V_e^2}{2}$$

or, $$V_e^2 = 2\,(h_o - h_e) \qquad (1)$$

However, $h = c_p T$. Thus Eq. (1) becomes

$$V_e^2 = 2\,c_p\,(T_o - T_e)$$

$$V_e^2 = 2\,c_p\,T_o\left(1 - \frac{T_e}{T_o}\right) \qquad (2)$$

However, the flow is isentropic, hence

$$\frac{T_e}{T_o} = \left(\frac{p_e}{p_o}\right)^{\frac{\gamma-1}{\gamma}} \qquad (3)$$

Substitute (3) into (1)

$$V_e = \sqrt{2\,c_p T_o\left[1 - \left(\frac{p_e}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad (4)$$

This is the desired result. Note from Eq. (4) that V_e increases as T_o increases, and as p_e/p_o decreases. Equation (4) is a useful formula for rocket engine performance analysis.

4.10 The flow velocity is certainly large enough that the flow must be treated as compressible. From the energy equation,

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_2 \frac{V_2^2}{2} \quad (1)$$

At a standard altitude of 5 km, from Appendix A,

$$p_1 = 5\ 4 \times 10^4\ \text{N/m}^2$$

$$T_1 = 255.7\ \text{K}$$

Also, for air, $c_p = 1005$ joule/(kg)(K). Hence, from Eq. (1) above,

$$T_2 = T_1 + \frac{V_1^2 - V_2^2}{2\ c_p}$$

$$T_2 = 255\ 7 + \frac{(270)^2 - (330)^2}{2(1005)}$$

$$T_2 = 255\ 7 - 17.9 = 237\ 8\text{K}$$

Since the flow is also isentropic,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Thus,

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}} = 5\ 4 \times 10^4 \left(\frac{237.8}{255\ 7}\right)^{\frac{1\ 4}{1\ 4-1}}$$

$$p_2 = \boxed{4.19 \times 10^4\ \text{N/m}^2}$$

Please note: This problem and problem 4 3 ask the same question However, the flow velocities in the present problem require a compressible analysis. Make certain to examine the solutions of both problems 4.10 and 4.3 in order to contrast compressible versus incompressible analyses

---

4.11 From the energy equation

$$c_p T_o = c_p T_e + \frac{V_e^2}{2}$$

or, $$T_e = T_o - \frac{V_e^2}{2\ c_p}$$

$$T_e = 1000 - \frac{1500^2}{2(6000)} = 812.5R$$

In the reservoir, the density is

$$\rho_o = \frac{p_o}{RT_o} = \frac{(7)(2116)}{(1716)(1000)} = 0.0086 \text{ slug/ft}^3$$

From the isentropic relation,

$$\frac{\rho_e}{\rho_o} = \left(\frac{T_e}{T_o}\right)^{\frac{1}{\gamma-1}}$$

$$\rho_e = 0.0086 \left(\frac{812.5}{1000}\right)^{\frac{1}{1.4-1}} = 0.0051 \text{ slug/ft}^3$$

From the continuity equation,

$$\dot{m} = \rho_e A_e V_e$$

Thus, $$A_e = \frac{\dot{m}}{\rho_e V_e}$$

In consistent units,

$$\dot{m} = \frac{1.5}{32.2} = 0.047 \text{ slug/sec.}$$

Hence,

$$A_e = \frac{\dot{m}}{\rho_e V_e} = \frac{0.047}{(0\ 0051)(1500)} = \boxed{0.0061\ ft^2}$$

4.12 $V_1 = 1500\ mph = 1500\left(\frac{88}{60}\right) = 2200\ ft/sec$

$$C_p\, T_1 + \frac{V_1^2}{2} C_p\, T_2\, \frac{V_2^2}{2}$$

$$V_2^2 = 2\, C_p\, (T_1 - T_2) + V_1^2$$

$$V_2^2 = 2\, (6000)(389\ 99 - 793\ 32) + (2200)^2$$

$$V_2 = \boxed{6.3\ ft/sec}$$

Note: This is a very small velocity compared to the initial freestream velocity of 2200 ft/sec At the point in question, the velocity is very near zero, and hence the point is nearly a stagnation point

---

4.13 At the inlet, the mass flow of air is

$$\dot{m}_{air} = \rho AV = (3\ 6391 \times 10^{-4})(20)(2200) = 16\ 0/slug/sec$$

$$\dot{m}_{fuel} = (0\ 05)(16\ 01) = 0\ 8\ slug/sec$$

Total mass flow at exit = 16 01 + 0 8 = $\boxed{16.81\ slug/sec}$

4.14 From problem 4 11,

$V_e = 1500\ ft/sec$

$T_e = 812\ 5R$

Hence,

$$a_e = \sqrt{\gamma R T_e} = \sqrt{(1.4)(1716)(812.5)}$$

$$= 1397 \text{ ft/sec}$$

Thus, $M_e = \dfrac{V_e}{a_e} = \dfrac{1500}{1397} = \boxed{1.07}$

Note that the nozzle of problem 4.11 is just barely supersonic.

4.15 From Appendix A,

$$T_\infty = 216.66\text{K}$$

Hence,

$$a_\infty = \sqrt{\gamma R T}$$

$$= \sqrt{(1.4)(287)(216.66)} = 295 \text{ m/sec}$$

Thus, $M_\infty = \dfrac{V_\infty}{a_\infty} = \dfrac{250}{295} = \boxed{0.847}$

4.16 At standard sea level, $T_\infty = 518.69\text{R}$

$$a_\infty = \sqrt{\gamma R T} = \sqrt{(1.4)(1716)(518.69)} = 1116 \text{ ft/sec}$$

$$V_\infty = M_\infty a_\infty = (3)(1116) = 3348 \text{ ft/sec}$$

Since 60 mi/hr - 88 ft/sec., then

$$V_\infty = 3348\ (60/88) = \boxed{2283 \text{ mi/h}}$$

4.17 $V = 2200$ ft/sec

$$a = \sqrt{\gamma RT} = \sqrt{(1.4)(1716)(389.99)} = 967.94 \text{ ft/sec}$$

$$M = \frac{V}{a} = \frac{2200}{967.94} = \boxed{2.27}$$

4.18 The test section density is

$$\rho = \frac{p}{RT} = \frac{1.01 \times 10^5}{(287)(300)} = 1.173 \text{ kg/m}^3$$

Since the flow is low speed, consider it to be incompressible, i.e., with the above density throughout.

$$p_1 - p_2 = \frac{\rho}{2} V_2^2 [1 - (A_2/A_1)^2] \qquad (1)$$

In terms of the manometer reading,

$$p_1 - p_2 = \omega \Delta h \qquad (2)$$

where $\omega = 1.33 \times 10^5$ N/m^3 for mercury.

Thus, combining Eqs. (1) and (2),

$$\Delta h = \frac{\rho}{2\omega} V_2^2 [1 - (A_2/A_1)^2]$$

$$= \frac{1.173}{(2)(1.33 \times 10^5)} (80)^2 [1 - (1/20)^2]$$

$$\Delta h = 0.028 \text{m} = \boxed{2.8 \text{ cm}}$$

4.19 $V_2 = 200 \text{ mph} = 300 \left(\frac{88}{60}\right) = 293.3 \text{ ft/sec}$

(a) $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$

$A_1 V_1 = A_2 V_2$

$V_1 = \frac{A_2}{A_1} V_2$

$p_1 + \frac{1}{2}\rho \left(\frac{A_2}{A_1}\right)^2 V_2^2 = p_2 + \frac{1}{2}\rho V_2^2$

$p_1 - p_2 = \frac{1}{2}\rho \left[1 - \left(\frac{A_2}{A_1}\right)^2\right] V_2^2$

$p_1 - p_2 = \frac{0.002377}{2}\left[1 - \left(\frac{4}{20}\right)^2\right](293.3)^2$

$p_1 - p_2 = \boxed{98.15 \text{ lb/ft}^2}$

(b) $p_1 + \frac{1}{2}\rho V_1^2 = p_3 + \frac{1}{2}\rho V_3^3$

$A_1 V_1 = A_2 V_2 \quad : \quad V_1 = \frac{A_2}{A_1} V_2$

$A_2 V_2 = A_3 V_3 \quad : \quad V_3 = \frac{A_2}{A_3} V_2$

$p_1 - p_3 = \frac{1}{2}\rho \left[\left(\frac{A_2}{A_3}\right)^2 - \left(\frac{A_2}{A_1}\right)^2\right] V_2^2$

$p_1 - p_3 = \frac{0.002377}{2}\left[\left(\frac{4}{18}\right)^2 - \left(\frac{4}{20}\right)^2\right](293.3)^2$

$p_1 - p_3 = \boxed{0.959 \text{ lb/ft}^2}$

Note: By the addition of a diffuser, the required pressure difference was reduced by an order of magnitude Since it costs money to produce a pressure difference (say by running compresors or vacuum pumps), then a diffuser, the purpose of which is to improve the aerodynamic efficiency, allows the wind tunnel to be operated more economically.

4.20 In the test section

$$\rho = \frac{p}{RT} = \frac{2116}{(1716)(70+460)} = 0.00233 \text{ slug/ft}^3$$

The flow velocity is low enough so that incompressible flow can be assumed Hence, from Bernoulli's equation,

$$p_o = p + \frac{1}{2}\,\rho\,V^2$$

$$p_o = 2116 + \frac{1}{2}\,(0.00233)\,[150\,(88/60)]^2$$

(Remember that 88 ft/sec = 60 mi/h)

$$p_o = 2116 + \frac{1}{2}\,(0.00233)(220)^2$$

$$p_o = \boxed{2172 \text{ lb/ft}^2}$$

4.21 The altimeter meaasures pressure altitude. Thus, from Appendix B, p = 1572 lb/ft^2. The air density is then

$$\rho = \frac{p}{RT} = \frac{1572}{(1716)(500)} = 0.00183 \text{ slug/ft}^3$$

Hence, from Bernoulli's equation,

$$V_{true} = \sqrt{\frac{2(p_o - p)}{\rho}} = \sqrt{\frac{2(1650 - 1572)}{0.00183}}$$

$$V_{true} = 292 \text{ ft/sec}$$

The equivalent airspeed is

$$V_e = \sqrt{\frac{2(p_o - p)}{\rho_s}} = \sqrt{\frac{2(1650 - 1572)}{0.002377}}$$

$$V_e = \boxed{256 \text{ ft/sec}}$$

4.22 The altimeter measures pressure altitude. Thus, from Appendix A, $p = 7.95 \times 10^4$ N/m^2. Hence,

$$\rho = \frac{p}{RT} = \frac{7.95 \times 10^4}{(287)(280)} = 0.989 \text{ kg/m}^3$$

The relation between V_{true} and V_e is

$$V_{true}/V_e = \sqrt{\rho_s / \rho}$$

Hence,

$$V_{true} = 50 \sqrt{(1.225)/0.989} = \boxed{56 \text{ m/sec}}$$

4.23 In the test section,

$$a = \sqrt{\gamma RT} = \sqrt{(1.4)(287)(270)} = 329 \text{ m/sec}$$

$$M = V/a = 250/329 = 0.760$$

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = [1 + 0.2\,(0.760)^2]^{3.5} = 1.47$$

Hence,

$p_o = 1.47p = 1.47\,(1.01 \times 10^5) = \boxed{1.48 \times 10^5\ N/m^2}$

4.24 $p = 1.94 \times 10^4\ N/m^2$ from Appendix A

$$M_1{}^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_o}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = \frac{2}{1.4 - 1}\left[\left(\frac{2.96 \times 10^4}{1.94 \times 10^4}\right)^{0.286} - 1\right]$$

$M_1{}^2 = 0.642$

$M_1 = \boxed{0.801}$

4.25 $$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_o}{p} = [1 + 0.2\,(0.65)^2]^{3.5} = 1.328$$

$$p = \frac{p_o}{1.328} = \frac{2339}{1.328} = 1761\ lb/ft^2$$

From Appendix B, this pressure corresponds to a pressure altitude, hence altimeter reading of $\boxed{5000\ ft.}$

4.26 At standard sea level,

$T = 518.69R$

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2}M^2 = 1 + 0.2\,(0.96)^2 = 1.184$$

$T_o = 1.184T = 1.184\,(518.69)$

$$\boxed{T_o = 614.3R = 154.3F}$$

4.27 $a_1 = \sqrt{\gamma R T_1} = \sqrt{(1.4)(287)(220)} = 297 \text{ m/sec}$

$M_1 = V_1/a_1 = 596/197 = 2.0$

The flow is supersonic. Hence, the Rayleigh Pitot tube formula must be used.

$$\frac{p_{o_2}}{p_1} = \left[\frac{(\gamma+1)^2 M_1^2}{4\gamma\, M_1^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right]$$

$$\frac{p_{o_2}}{p_1} = \left[\frac{(2.4)^2(2)^2}{4(1.4)(2)^2 - 2(0.4)}\right]^{3.5} \left[\frac{1-1.4+2(1.4)(2)^2}{2.4}\right]$$

$$\frac{p_{o_2}}{p_1} = 5.64$$

$p_1 = 2.65 \times 10^4 \text{ N/m}^2$ from Appendix A.

Hence,

$$p_{o_2} = 5.64\,(2.65 \times 10^4) = \boxed{1.49 \times 10^5 \text{ N/m}^2}$$

4.28 $q = \frac{1}{2}\rho V^2 = \frac{1}{2}\left(\frac{\gamma p}{\gamma p}\right)\rho V^2 = \frac{\gamma}{2} p\left(\frac{\rho}{\gamma p}\right) V^2 = \frac{\gamma}{2}\, p\, \frac{V^2}{a^2}$

Hence:

$$\boxed{q = \frac{\gamma}{2}\, p\, M^2}$$

4.29 $q_\infty = \frac{\gamma}{2}\, p_\infty M_\infty^2 = 0.7\, p_\infty M_\infty^2$ (1)

Use Appendix A to obtain the values of p_∞ corresponding to the given values of h.

Then use Eq. (1) above to calculate q_∞.

h(km)	60	50	40	30	20
p_∞(N/m^2)	25.6	87.9	299.8	1.19×10^3	5.53×10^3
M	17	9.5	5.5	3	1
q_∞(N/M^2)	5.2×10^3	5.6×10^3	6.3×10^3	7.5×10^3	3.9×10^3

Note that q_∞ progressively increases as the shuttle penetrates deeper into the atmosphere, that it peaks at a slightly supersonic Mach number, and then decreases as the shuttle completes its entry.

4.30 Recall that total pressure is defined as that pressure that would exist if the flow were slowed isentropically to zero velocity. This is a definition; it applies to all flows -- subsonic or supersonic. Hence, Eq. (4.74) applies, no matter whether the flow is subsonic or supersonic.

$$\frac{p_o}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\gamma/(\gamma-1)} = [1 + 0.2\,(2)^2]^{1.4/0.4} = 7.824$$

Hence:

$$p_o = 7.824\, p_\infty = 7.824\,(2116) = \boxed{1.656 \times 10^4 \ \frac{\text{lb}}{\text{ft}^2}}$$

Note that the above value is not the pressure at a stagnation point at the nose of a blunt body, because in slowing to zero velocity, the flow has to go through a shock wave, which is non-isentropic. The stagnation pressure at the nose of a body in a Mach 2 stream is the total pressure behind a normal shock wave, which is lower than the total pressure of the freestream, as calculated above. This stagnation pressure at the nose of a blunt body is given by Eq (4 79).

$$\frac{p_{o_2}}{p_1} = \left[\frac{(\gamma+1)^2 M_\infty^2)}{4\gamma\ M_2^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right] =$$

$$= \left[\frac{(2.4)^2(2)^2}{4(1.4)(2)^2 - 2(0.4)}\right]^{1.4/0.4} \left[\frac{1-1.4+2(1.4)(2)^2}{2.4}\right] = 5.639$$

Hence,

$$p_{o_2} = 5.639\ p_\infty = 5.639\ (2116) = \boxed{1.193 \times 10^4\ \frac{lb}{ft^2}}$$

If Bernoulli's equation is used, the following wrong result for total pressure is obtained.

$$p_o = p_\infty + q_\infty = p_\infty + \frac{1}{2}\ \rho\ V_\infty^2 = p_\infty + \frac{\gamma}{2}\ p_\infty\ M_\infty^2$$

$$p_o = 2116 + 0.7\ (2116)\ (2)^2 = 0.804 \times 10^4\ \frac{lb}{ft^2}$$

Compared to the correct result of $1.656 \times 10^4\ \frac{lb}{ft^2}$, this leads to an error 51%.

4.31 $$\frac{p_e}{p_o} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{-\gamma}{\gamma-1}}$$

$$p_e = 5(1.01 \times 10^5)\ [1 + 0.2\ (3)^2]^{-3.5}$$

$$p_e = \boxed{1.37 \times 10^4\ N/m^2}$$

$$\frac{T_e}{T_o} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{-1} = [1 + 0.2\ (3)^2]^{-1}$$

$T_e = (500)(0.357) = \boxed{178.6K}$

$$\rho_e = \frac{p_e}{RT_e} = \frac{1.37 \times 10^4}{(287)(178.6)} = \boxed{0.267\ kg/m^3}$$

4.32 $$\frac{p_e}{p_o} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{-\gamma}{\gamma-1}}$$

Hence,

$$M_e^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_e}{p_o}\right)^{\frac{1-\gamma}{\gamma}} - 1\right]$$

$M_e^2 = 5[(0.2)^{-0.286} - 1] = 2.92$

$M_e = 1.71$

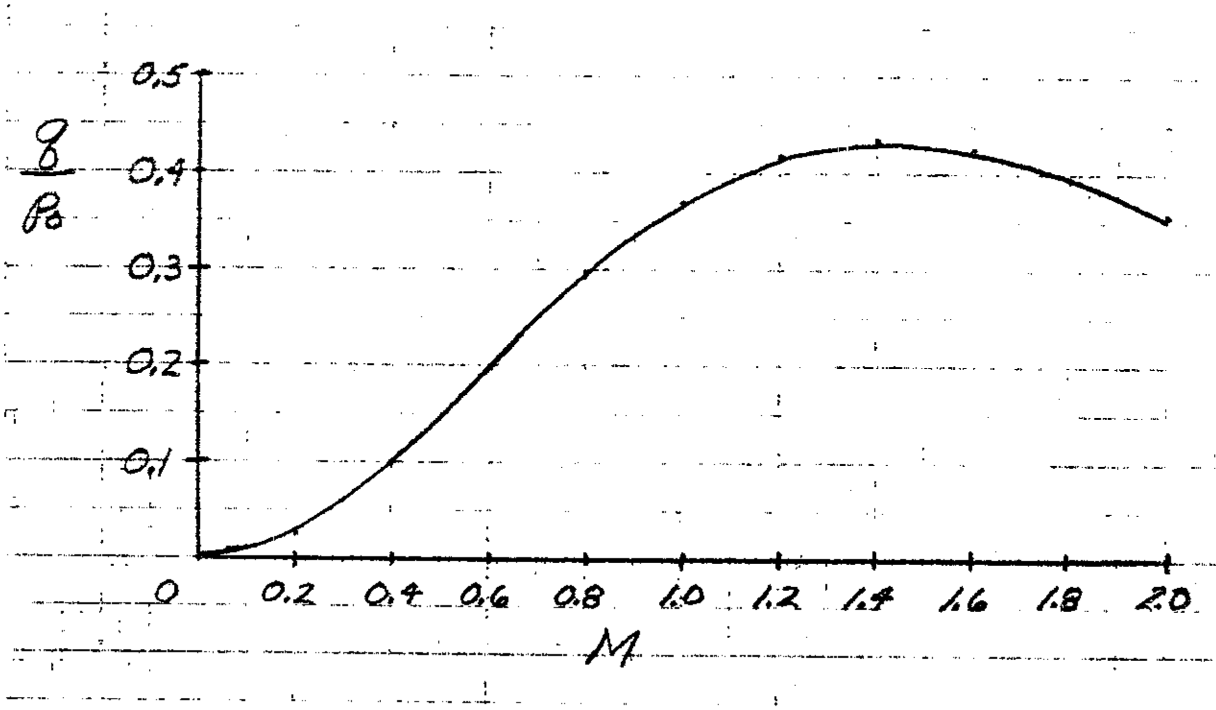
$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{\gamma}\right) M^2\right]^{(\gamma+1)/(\gamma-1)}$$

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{(1.71)^2}\ [(0.833)(1 + 0.2\ (1.71)^2)]^6$$

$$\frac{A_e}{A_t} = \boxed{1.35}$$

4.33 $$\frac{q}{p_o} = \frac{\gamma}{2}\frac{p}{p_o} M^2 = \frac{\gamma}{2} M^2\left[1 + \frac{\gamma - 1}{2} M^2\right]^{-\gamma/(\gamma-1)} = 0.7\, M^2\, (1 + 0.2\, M^2)^{-3.5}$$

M	M^2	q/p_o
0	0	0
0.2	0.04	0.027
0.4	0.16	0.100
0.6	0.36	0.198
0.8	0.64	0.294
1.0	1.0	0.370
1.2	1.44	0.416
1.4	1.96	0.431
1.6	2.56	0.422
1.8	3.24	0.395
2.0	4.00	0.358



Note that the dynamic pressure increases with Mach number for $M < 1.4$ but decreases with Mach number for $M > 1.4$. I.e., in an isentropic nozzle expansion, there is a peak local dynamic pressure which occurs at $M = 1.4$

4.34 First, calculate the value of the Reynolds number.

$$\mathrm{Re}_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{(1.225)(200)(3)}{(1.7894 \times 10^{-5})} = 4.10 \times 10^7$$

The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(200)^2 = 2.45 \times 10^4 \text{ N/m}^2$$

Hence,

$$\delta_L = \frac{5.2L}{\sqrt{\mathrm{Re}_L}} = \frac{5.2(3)}{\sqrt{4.1 \times 10^7}} = 0.0024\text{m} = \boxed{0.24 \text{ cm}}$$

and

$$C_f = \frac{1.328}{\sqrt{\mathrm{Re}_L}} = \frac{1.328}{\sqrt{4.1 \times 10^7}} = 0.00021$$

The skin friction drag on one side of the plate is:

$$D_f = q_\infty S c_f = (2.45 \times 10^4)(3)(17.5)(0.00021)$$

$$D_f = 270\text{N}$$

The total skin friction drag, accounting for both the top and the bottom of the plate is twice this value, namely

$$\text{Total } D_f = \boxed{540\text{N}}$$

4.35 $$\delta = \frac{0.37L}{(\mathrm{Re}_L)^{0.2}} = \frac{0.37(3)}{(4.1 \times 10^7)^{0.2}} = 0.033\text{m} = \boxed{3.3 \text{ cm}}$$

From problem 4.24, we find

$$\delta_{turbulent}/\delta_{laminar} = \frac{3.3}{0.24} = \boxed{13.75}$$

The turbulent boundary layer is more than an order of magnitude thicker than the laminar boundary layer.

$$C_f = \frac{0.074}{(Re_L)^{0.2}} = \frac{0.074}{(4.1 \times 10^7)^{0.2}} = 0.0022$$

The skin friction drag on one side is then

$$D_f = q_\infty S c_f = (2/45 \times 10^4)(3)(17.5)(0.0022)$$

$$D_f = 2830N$$

The total, accounting for both top and bottom is

$$\text{Total } D_f = \boxed{5660N}$$

From problem 4.24, we find

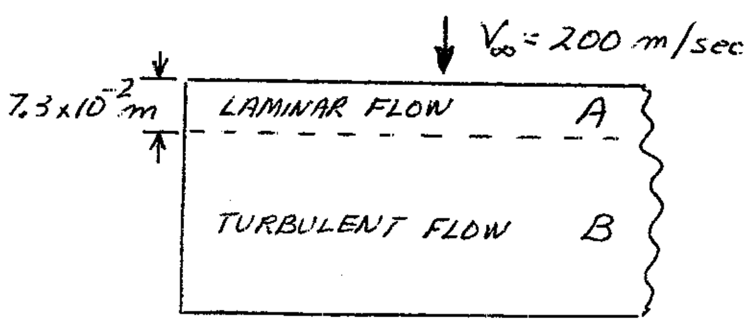
$$\left(D_{f_{turbulent}}\right)/\left(D_{f_{laminar}}\right) = \frac{5660}{540} = \boxed{10.5}$$

The turbulent skin friction drag is an order of magnitude larger than the laminar value.

4.36 $$R_{e_{x_{cr}}} = \frac{\rho_\infty V_\infty x_{cr}}{\mu_\infty}$$

$$x_{cr} = Re_{x_{cr}}\left(\frac{\mu_\infty}{\rho_\infty V_\infty}\right) = \frac{(10^6)(1.789 \times 10^{-5})}{(1.225)(200)}$$

$$x_{cr} = 7.3 \times 10^{-2} m$$



The turbulent drag that would exist over the first 7.3 x 10^{-2}m of chord length from the leading edge (area A) is

$$D_{f_A} = \frac{0.074}{(Re_{cr})^{0.2}} q_\infty S_A \quad \text{(on one side)}$$

$$D_{f_A} = \frac{0.074}{(10^6)^{0.2}} (2.45 \times 10^4)(7.3 \times 10^{-2})(17.5)$$

$$D_{f_A} = 146\text{N} \qquad \text{(on one side)}$$

From problem 4.25, the turbulent drag on one side, assuming both areas A and B to be turbulent, is 2830N. Hence, the turbulent drag on area B alone is:

$$D_{f_B} = 2830 - 146 - 2684\text{N} \qquad \text{(turbulent)}$$

The laminar drag on area A is

$$D_{f_A} = \frac{1.328}{(Re_{cr})^{0.5}} q_\infty S$$

$$D_{f_A} = \frac{1.328}{(10^6)^{0.5}} (2.45 \times 10^4)(7.3 \times 10^{-2})(17.5)$$

$$D_{f_A} = 42\text{N} \qquad \text{(laminar)}$$

Hence, the skin friction drag on one side, assuming area A to be laminar and area B to be turbulent is

$$D_f = D_{f_A}\text{ (laminar)} + D_{f_B}\text{ (turbulent)}$$

$$D_f = 42 + 2684 = 2726\text{N}$$

The total drag, accounting for both sides, is

$$\text{Total } D_f = \boxed{5452\text{N}}$$

Note: By comparing the results of this problem with those of problem 4.25, we see that the flow over the wing is mostly turbulent, which is usually the case for real airplanes in flight.

4.37 The relation between changes in pressure and velocity at a point in an inviscid flow is given by the Euler equation, Eq. (4 8)

$$dp = -\rho \, V \, d \, V$$

Letting s denote distance along the streamline through the point, Eq. (4 8) can be written as

$$\frac{dp}{ds} = -\rho \, V \, \frac{dV}{ds}$$

or,
$$\frac{dp}{ds} = -\rho \, V^2 \, \frac{(dV / V)}{ds}$$

(a)
$$\frac{(dV / V)}{ds} = 0\ 02 \text{ per millimeter}$$

Hence,

$$\frac{dp}{ds} = -(1\ 1)(100)^2(0\ 02) = 220 \, \frac{N}{m^2} \text{ per millimeter}$$

(b)
$$\frac{dp}{ds} = -(1.1)(1000)^2(0.02) = 22{,}000 \, \frac{N}{m^2} \text{ per millimeter}$$

Conclusion: At a point in a high-speed flow, it requires a much larger pressure gradient to achieve a given percentage change in velocity than for a low speed flow, everything else being equal

---

4.38 We use the fact that total pressure is constant in an isentropic flow. From Eq. (4 74) applied in the freestream.

$$\frac{p_o}{p_\infty} = \left(1 + \frac{\gamma - 1}{\gamma} M_\infty^{\ 2}\right)^{\frac{\gamma}{\gamma-1}} = [1 + 0\ 2\ (0\ 7)^2]^{3.5} = 1\ 387$$

From Eq (4 74) applied at the point on the wing,

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = [1 + 0.2\ (1.1)^2]^{3\ 5} = 2.135$$

Hence,

$$p = \left[\left(\frac{p_o}{p_\infty}\right) / \left(\frac{p_o}{p}\right)\right] p_\infty = \left(\frac{1.387}{2.135}\right) p_\infty = 0\,65\ p_\infty$$

At a standard altitude of 3 km, from Appendix A, $p_\infty = 7.0121 \times 10^4\ N/m^2$. Hence,

$$p = (0\ 65)(7\ 0121 \times 10^4) = \boxed{4.555 \times 10^4\ N/m^2}$$

4.39 This problem is simply asking what is the equivalent airspeed, as discussed in Section 4 12. Hence,

$$V_e = V \left(\frac{\rho}{\rho_s}\right)^{1/2} = (800) \left(\frac{1.0663 \times 10^{-3}}{2.3769 \times 10^{-3}}\right)^{1/2} = \boxed{535.8\ \text{ft.sec}}$$

4.40 (a) From Eq. (4 88)

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^{\ 2}}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2} M_e^{\ 2}\right)\right]^{\frac{\gamma+1}{\gamma-1}} = \frac{1}{(10)^2}\left\{\frac{2}{2.4}\left[1 + 0 2\ (10)^2\right]\right\}^6 = 2\ 87 \times 10^5$$

Hence:

$$\frac{A_e}{A_t} = \sqrt{2.87 \times 10^5} = \boxed{535.9}$$

(b) From Eq. (4.87)

$$\frac{p_o}{p_e} = \left(1 + \frac{\gamma - 1}{2} M_e^{\,2}\right)^{\frac{\gamma}{\gamma-1}} = [1 + 0.2\,(10)^2]^{3.5} = 4.244 \times 10^4$$

At a standard altitude of 55 km, p = 48.373 N/m² Hence

$$p_o = (4.244 \times 10^4)(48.373) = 2.053 \times 10^6 \text{ N/m}^2 = \boxed{20.3 \text{ atm}}$$

(c) From Eq. (4.85)

$$\frac{T_o}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^{\,2} = 1 + 0.2\,(10)^2 = 21$$

At a standard altitude of 55 km, T = 275.78 K Hence,

$$T_o = 275.78\,(21) = \boxed{5791 \text{ K}}$$

Examining the above results, we note that:

1 The required expansion ratio of 535.9 is <u>huge</u>, but is readily manufactured.

2 The required reservoir pressure of 20.3 atm is large, but can be handled by proper design of the reservoir chamber.

3 The required reservoir temperature of 5791 K is tremendously large, especially when you remember that the surface temperature of the sun is about 6000 K For a continuous flow hypersonic tunnel, such high reservoir tempertures can not be handled. In practice, a reservoir temperature of about half this value or less is employed, with the sacrifice made that "true temperature" simulation in the test stream is not obtained.

4.41 The speed of sound in the test stream is

$$a_e = \sqrt{\gamma\ R\ T_e} = \sqrt{(1.4)(287)(275.78} = 332.9\ \text{m/sec}$$

Hence,

$$V_e = M_e\ a_e = 10\ (332.9) = \boxed{3329\ \text{m/sec}}$$

4.42 (a) From Eq. 4.88, for $M_e = 20$

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^{\,2}}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^{\,2}\right)\right]^{\frac{\gamma+1}{\gamma-1}} = \frac{1}{(20)^2}\left\{\frac{2}{2.4}\left[1+0.2\ (20)^2\right]\right\}^6 = 2.365 \times 10^8$$

Hence:

$$\frac{A_e}{A_t} = \boxed{15{,}377}$$

(b) From Eq. (4.85)

$$\frac{T_o}{T_e} = \left(1+\frac{\gamma-1}{2}M_e^{\,2}\right)^{-1} = [1+(0.2)(20)^2]^{-1} = 0.01235$$

Hence,

$$T_e = (5791)(0.01235) = 71.5\ \text{K}$$

$$a_e = \sqrt{\gamma\ R\ T_e} = \sqrt{(1.4)(287)(71.5)} = 169.5\ \text{m/sec}$$

$$V_e = M_e\ a_e = 20\ (169.5) = \boxed{3390\ \text{m/sec}}$$

<u>Comments</u>:

1. To obtain Mach 20, i.e., to double the Mach number in this case, the expansion ratio must be increased by a factor of 15,377/535.9 = 28.7. High hypersonic Mach numbers demand wind tunnels with very large exit-to-throat ratios. In practice, this is usually obtained by designing the nozzle with a <u>small throat area</u>.

2 Of particular interest is that the exit velocity is increased by a very small amount, namely by only 61 m/sec, although the exit Mach number has been doubled. The higher Mach number of 20 is achieved not by a large increase in exit velocity by rather by a large decrease in the speed of sound at the exit. This is characteristic of most conventional hypersonic wind tunnels -- the higher Mach numbers are not associated with corresponding increases in the test section flow velocities

---

5.1 Assume the moment is governed by

$$M = f(V_\infty, \rho_\infty, S, \mu_\infty, a_\infty)$$

More specifically:

$$M = Z\, V_\infty^{\,a}\, \rho_\infty^{\,b}\, S^d\, a_\infty^{\,e}\, \mu_\infty^{\,f}$$

Equating the dimensions of mass, m, length, ℓ, and time t, and considering Z dimensionless,

$$\frac{m\ell^2}{t^2} = \left(\frac{\ell}{t}\right)^a \left(\frac{m}{\ell^3}\right)^b \left(\ell^2\right)^d \left(\frac{\ell}{t}\right)^e \left(\frac{m}{\ell t}\right)^f$$

$1 = b + f$ (For mass)

$2 = a - 3b + 2d + e - f$ (For length)

$-2 = -a-e-f$ (for time)

Solving a, b, and d in terms of e and f,

$b = 1 - f$

and, $a = 2 - e - f$

2 Of particular interest is that the exit velocity is increased by a very small amount, namely by only 61 m/sec, although the exit Mach number has been doubled. The higher Mach number of 20 is achieved not by a large increase in exit velocity by rather by a large decrease in the speed of sound at the exit. This is characteristic of most conventional hypersonic wind tunnels -- the higher Mach numbers are not associated with corresponding increases in the test section flow velocities

---

5.1 Assume the moment is governed by

$$M = f(V_\infty, \rho_\infty, S, \mu_\infty, a_\infty)$$

More specifically:

$$M = Z\, V_\infty^{\,a}\, \rho_\infty^{\,b}\, S^d\, a_\infty^{\,e}\, \mu_\infty^{\,f}$$

Equating the dimensions of mass, m, length, ℓ, and time t, and considering Z dimensionless,

$$\frac{m\ell^2}{t^2} = \left(\frac{\ell}{t}\right)^a \left(\frac{m}{\ell^3}\right)^b \left(\ell^2\right)^d \left(\frac{\ell}{t}\right)^e \left(\frac{m}{\ell t}\right)^f$$

$1 = b + f$ (For mass)

$2 = a - 3b + 2d + e - f$ (For length)

$-2 = -a - e - f$ (for time)

Solving a, b, and d in terms of e and f,

$b = 1 - f$

and, $a = 2 - e - f$

and, $2 = 2 - e - f - 3 + 3f + 2d + e - f$

or $0 = -3 + f + 2d$

$$d = \frac{3 - f}{2}$$

Hence,

$$M = Z\, V_\infty^{2-e-f}\, \rho_\infty^{1-f}\, S^{(3-f)}/2\;\; a_\infty^{\,e}\; \mu_\infty^{\,f}$$

$$= Z\, V_\infty^{2}\, \rho_\infty\, S S^{1/2} \left(\frac{a_\infty}{V_\infty}\right)^{e} \left(\frac{\mu_\infty}{V_\infty \rho_\infty S^{1/2}}\right)^{f}$$

Note that $S^{1/2}$ is a characteristic length; denote it by the chord, c

$$M = \rho_\infty\, V_\infty^{2}\, S\, c\, Z \left(\frac{a_\infty}{V_\infty}\right)^{e} \left(\frac{\mu_\infty}{V_\infty \rho_\infty c}\right)^{f}$$

However, $a_\infty/V_\infty = 1/M_\infty$

and $\dfrac{\mu_\infty}{V_\infty \rho_\infty c} = \dfrac{1}{Re}$

Let

$$Z \left(\frac{1}{M_\infty}\right)^{e} \left(\frac{1}{Re}\right)^{f} = \frac{c_m}{2}$$

where c_m is the moment coefficient Then, as was to be derived, we have

$$M = \frac{1}{2}\rho_\infty\, V_\infty^{2}\, c\, c_m$$

or, $M = q_\infty\, S\, c\, c_m$

5.2 From Appendix D, at 5° angle of attack,

$c_\ell = 0.67$

$c_{m_{c/4}} = -0.025$

(Note: Two sets of lift and moment coefficient data are given for the NACA 1412 airfoil -- with and without flap deflection. Make certain to read the code properly, and use only the unflapped data, as given above. Also, note that the scale for $c_{m_{c/4}}$ is different than that for c_ℓ -- be careful in reading the data.)

With regard to c_d, first check the Reynolds number,

$$\mathrm{Re} = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(0.002377)(100)(3)}{(3.7373 \times 10^{-7})}$$

$$\mathrm{Re} = 1.9 \times 10^6$$

In the airfoil data, the closest Re is 3×10^6. Use c_d for this value.

$c_d = 0.007$ (for $c_\ell = 0.67$)

The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(100)^2 = 11.9 \text{ lb/ft}^2$$

The area per unit span is $S = 1(c) = (1)(3) = 3 \text{ ft}^2$

Hence, per unit span,

$$L = q_\infty S c_\ell = (11.9)(3)(0.67) = \boxed{23.9 \text{ lb}}$$

$$D = q_\infty S c_d = (11.9)(3)(0.007) = \boxed{0.25 \text{ lb}}$$

$$M_{c/4} = q_\infty S c\, c_{m_{c/4}} = (11.9)(3)(3)(-0.025) = \boxed{-2.68 \text{ ft.lb}}$$

5.3 $\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{(1.01 \times 10^5)}{(287)(303)} = 1.61 \text{ kg/m}^3$

From Appendix D,

$$c_\ell = 0.98$$

$$c_{m_{c/4}} = -0.012$$

Checking the Reynolds number, using the viscosity coefficient from the curve given in Chapter 4,

$$\mu_\infty = 1.82 \times 10^{-5} \text{ kg/m sec} \quad \text{at } T = 303K,$$

$$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(1.157)(42)(0.3)}{182 \times 10^{-5}} = 8 \times 10^5$$

This Reynolds number is considerably less than the lowest value of 3×10^6 for which data is given for the NACA 23012 airfoil in Appendix D. Hence, we can use this data only to give an educated guess; use

$c_d \approx 0.01$, which is about 10 percent higher than the value of 0.009 given for $Re = 3 \times 10^6$

The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.161)(42)^2 = 1024 \text{ N/m}^2$$

The area per unit span is $S = (1)(0.3) = 0.3 \text{ m}^2$. Hence,

$$L = q_\infty S c_\ell = (1024)(0.3)(0.98) = \boxed{301N}$$

$$D = q_\infty S c_d = (1024)(0.3)(0.01) = \boxed{3.07N}$$

$$M_{c/4} = q_\infty S c c_m = (1024)(0.3)(0.3)(-0.012) = \boxed{-1.1Nm}$$

5.4 From the previous problem, $q_\infty = 1020 \text{ N/m}^2$

$L = q_\infty S\ c_\ell$

Hence,

$$c_\ell = \frac{L}{q_\infty S}$$

The wing area S = (2)(0.3) = 0.6 m^2

Hence,

$$c_\ell = \frac{200}{(1024)(0.6)} = 0.33$$

From Appendix D, the angle of attack which corresponds to this lift coefficient is

$$\boxed{\alpha = 2^\circ}$$

5.5 From Appendix D, at $\alpha = 4^\circ$,

$c_\ell = 0.4$

Also, $V_\infty = 120 \left(\frac{88}{60}\right) = 176$ ft/sec

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(176)^2 = 36.8 \text{ lb/ft}^2$$

$L = q_\infty S\ c_\ell$

$$S = \frac{L}{q_\infty c_\ell} = \frac{29.5}{(36.8)(0.4)} = \boxed{2 \text{ ft}^2}$$

5.6 $L = q_\infty S\ c_\ell$

$D = q_\infty S\ c_d$

Hence,

$$\frac{L}{D} = \frac{q_\infty S\, c_\ell}{q_\infty S\, c_d} = \frac{c_\ell}{c_d}$$

We must tabulate the values of c_ℓ/c_d for various angles of attack, and find where the maximum occurs. For example, from Appendix D, at $\alpha = 0°$,

$$c_\ell = 0.25$$

$$c_d = 0.006$$

Hence

$$\frac{L}{D} = \frac{c_\ell}{c_d} = \frac{0.25}{0.006} = 41.7$$

A tabulation follows

α	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
c_ℓ	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.05	1.15
c_d	0.006	0.006	0.006	0.0065	0.0072	0.0075	0.008	0.0085	0.0095	0.0105
$\frac{c_\ell}{c_d}$	41.7	58.3	75	84.6	90.3	100	106	112	111	110

From the above tabulation,

$$\left(\frac{L}{D}\right)_{max} \approx \boxed{112}$$

5.7 At sea level

$$\rho_\infty = 1.225 \text{ kg/m}^3$$

$p_\infty = 1.01 \times 10^5 \text{ N/m}^2$

Hence,

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(50)^2 = 1531 \text{ N/m}^2$$

From the definition of pressure coefficient,

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{(0.95 - 1.01) \times 10^5}{1531} = \boxed{-3.91}$$

5.8 The speed is low enough that incompressible flow can be assumed From Bernoulli's equation,

$$p + \frac{1}{2}\rho V_\infty^2 = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2 = p_\infty + q_\infty$$

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{q_\infty - \frac{1}{2}\rho V^2}{q_\infty} = 1 - \frac{\frac{1}{2}\rho V^2}{\frac{1}{2}\rho_\infty V_\infty^2}$$

Since $\rho = \rho_\infty$ (constant density)

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(\frac{62}{55}\right)^2 = 1 - 1.27 = \boxed{-0.27}$$

5.9 The flow is low speed, hence assumed to be incompressible From problem 5.8,

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(\frac{195}{160}\right)^2 = \boxed{-0.485}$$

5.10 The speed of sound is

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(510)} = 1107 \text{ ft/sec}$$

Hence,

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{700}{1107} = 0.63$$

In problem 5.9, the pressure coefficient at the given point was calculated as -0.485. However, the conditions of problem 5.9 were low speed, hence we identify

$$C_{p_o} = -0.485$$

At the new, higher free stream velocity, the pressure coefficient must be corrected for compressibility. Using the Prandtl-Glauert Rule, the high speed pressure coefficient is

$$C_p = \frac{C_{p_o}}{\sqrt{1 - M_\infty^2}} = \frac{-0.485}{\sqrt{1 - (0.63)^2}} = \boxed{-0.625}$$

5.11 The formula derived in problem 5.8, namely

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2,$$

utilized Bernoulli's equation in the derivation. Hence, it is <u>not valid</u> for compressible flow. In the present problem, check the Mach number.

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(505)} = 1101 \text{ ft/sec}$$

$$M_\infty = \frac{780}{1101} = 0.708$$

The flow is clearly compressible! To obtain the pressure coefficient, first calculate ρ_∞ from the equation of state.

$$\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{2116}{(1716)(505)} = 0.00244 \text{ slug/ft}^3$$

To find the pressure at the point on the wing where V = 850 ft/sec, first find the temperature from the energy equation

$$c_p T + \frac{V^2}{V} = c_p T_\infty + \frac{V_\infty{}^2}{2}$$

$$T = T_\infty + \frac{V_\infty{}^2 - V^2}{2c_p}$$

The specific heat at constant pressure for air is

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(1716)}{(14-1)} = 6006 \frac{\text{ft lb}}{\text{slug R}}$$

Hence,

$$T = 505 + \frac{780^2 - 850^2}{2(6006)} = 505 - 9\ 5 = 495\ 5\text{R}$$

Assuming isentropic flow

$$\frac{p}{p} = \left(\frac{T}{T_\infty}\right)^{\frac{\gamma}{\gamma-1}}$$

$$p = (2116)\left(\frac{495.5}{505}\right)^{3\ 5} = 1980 \text{ lb/ft}^2$$

From the definition of C_p

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty{}^2} = \frac{1980 - 2116}{\frac{1}{2}(0\ 00244)(780)^2}$$

$$C_p = \boxed{-0.183}$$

5.12 A velocity of 100 ft/sec is low speed. Hence, the desired pressure coefficient is a low speed value, C_{p_o}.

$$C_p = \frac{C_{p_o}}{\sqrt{1 - M_\infty^2}}$$

From problem 5 11,

$$C_p = -0\ 183 \text{ and } M_\infty = 0\ 708 \text{. Thus, } 0\ 183 = \frac{C_{p_o}}{\sqrt{1 - (0.708)^2}}$$

$$C_{p_o} = (-0.183)(0\ 706) = \boxed{-0.129}$$

5.13 Recall that the airfoil data in Appendix D is for low speeds. Hence, at $\alpha = 4^\circ$, c_{ℓ_o} = 0.58

Thus, from the Prandtl-Glauert rule,

$$c_\ell = \frac{c_{\ell_o}}{\sqrt{1 - M_\infty^2}} = \frac{0.58}{\sqrt{1 - (0\ 8)^2}} = \boxed{0.97}$$

5.14 The lift coefficient measured is the high speed value, c_ℓ. Its low speed counterpart is c_{ℓ_o}, where

$$c_\ell = \frac{c_{\ell_o}}{\sqrt{1 - M_\infty^2}}$$

Hence,

$$c_{\ell_o} = (0\ 85)\ \sqrt{1 - (0\ 7)^2} = 0\ 607$$

For this value, the low speed data in Appendix D yield

$$\boxed{\alpha = 2^\circ}$$

5.15 First, obtain a curve of $C_{p,cr}$ versus M_∞ from

$$C_{p,cr} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{2+(\gamma-1)M_\infty^2}{\gamma+1}\right)^{\gamma/(\gamma-1)} - 1\right]$$

Some values are tabulated below for $\gamma = 1.4$.

M_∞	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$C_{p,cr}$	-3.66	-2.13	-1.29	-0.779	-0.435	-0.188	0

Now, obtain the variation of the minimum pressure coefficient, C_p, with M_∞, where C_{p_o} = -0.90. From the Prandtl-Glauert rule,

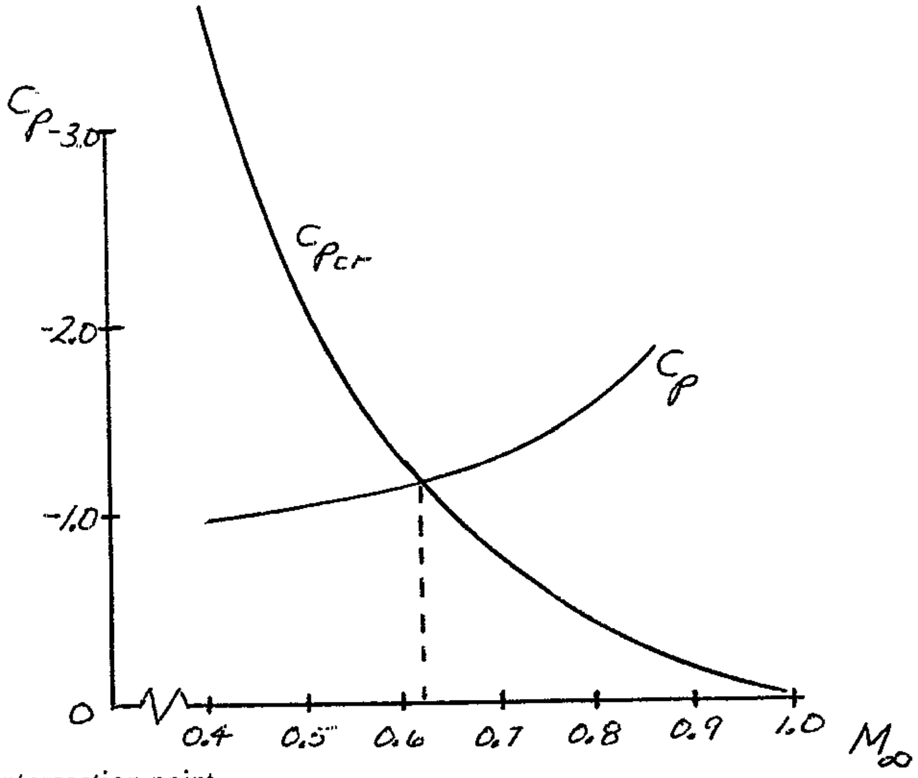
$$C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^2}}$$

$$C_p = \frac{-0.90}{\sqrt{1-M_\infty^2}}$$

Some tabulated values are:

M_∞	0.4	0.5	0.6	0.7	0.8	0.9
C_p	-0.98	-1.04	-1.125	-1.26	-1.5	-2.06

A plot of the two curves is given on the next page.



From the intersection point,

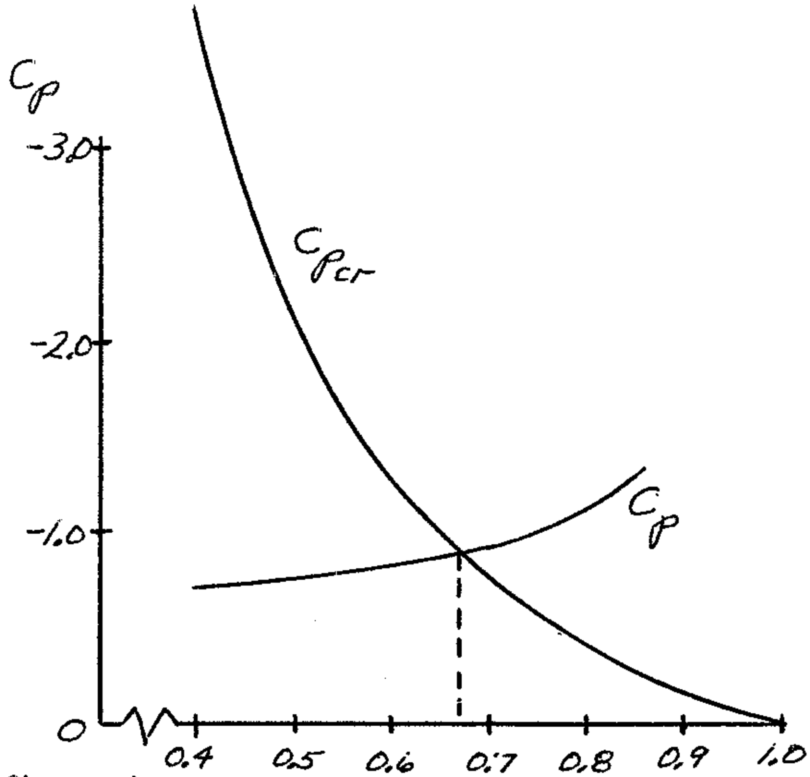
$$M_{cr} = \boxed{0.62}$$

5.16 The curve of $C_{p,cr}$ versus M_∞ has already been obtained in the previous problem; it is a universal curve, and hence can be used for this and all other problems. We simply have to obtain the variation of C_p with M_∞ from the Prandtl-Glauert rule, as follows:

$$C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^{\ 2}}} = \frac{-0.65}{\sqrt{1-M_\infty^{\ 2}}}$$

M_∞	0.4	0.5	0.6	0.7	0.8	0.9
C_p	-0.71	0.75	-0.81	-0.91	-1.08	-1.49

The results are plotted below.



From the point of intersection,

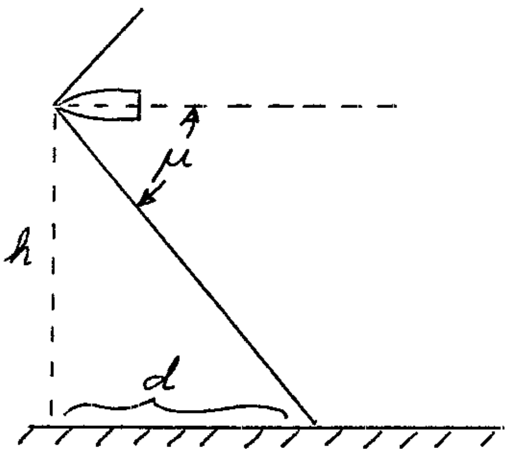
$M_{cr} = \boxed{0.68}$

Please note that, comparing problems 5.15 and 5.16, the critical Mach number for a given airfoil is somewhat dependent on angle of attack for the simple reason that the value of the minimum pressure coefficient is a function of angle of attack. When a critical Mach number is stated for a given airfoil in the literature, it is usually for a small (cruising) angle of attack.

5.17 Mach angle = μ = arc sin (1/M)

μ = arc sin (1/2)= $\boxed{30°}$

5.18



$$\mu = \mathrm{Sin}^{-1}\left(\frac{1}{M}\right) = \mathrm{Sin}^{-1}\left(\frac{1}{2.5}\right) = 23.6^\circ$$

$$d = h/\mathrm{Tan}\mu = \frac{10\text{km}}{0.436} = \boxed{22.9 \text{ km}}$$

5.19 At 36,000 ft, from Appendix B,

$T_\infty = 390.5^\circ R$

$\rho_\infty = 7.1 \times 10^{-4}$ slug/ft^3

Hence,

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(390.5)} = 969 \text{ ft / sec}$$

$$V_\infty = a_\infty M_\infty = (969)(2.2) = 2132 \text{ ft/sec}$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(7.1 \times 10^{-4})(2132)^2 = 1614 \text{ lb/ft}^2$$

In level flight, the airplane's lift must balance its weight, hence

$L = W = 16,000$ lb

From the definition of lift coefficient,

$C_L = L/q_\infty S = 16,000/(1614)(210) = 0.047$

Assume that all the lift is derived from the wings (this is not really true because the fuselage and horizontal tail also contribute to the airplane lift.) Moreover, assume the wings can be approximated by a thin flat plate Hence, the lift coefficient is given approximately by

$$C_L = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

Solve for α,

$$\alpha = \frac{1}{4} C_L \sqrt{M_\infty^2 - 1} = \frac{1}{4}(0\ 047)\sqrt{(2\ 2)^2 - 1}$$

$\alpha = 0\ 023$ radians (or 1 2 degrees)

The wave drag coefficient is approximated by

$$C_{D_w} = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.023)^2}{\sqrt{(2\ 2)^2 - 1}} = 0\ 00108$$

Hence,

$$D_w = q_\infty\ S\ C_{D_w} = (1614)(210)(0\ 00108)$$

$$D_w = \boxed{366\ \text{lb}}$$

5.20 (a) At 50,000 ft, $\rho_\infty = 3\ 6391 \times 10^{-4}$ slug/ft^3 and $T_\infty = 390°R$ Hence,

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1\ 4)(1716)(390)} = 968 \text{ ft / sec}$$

and $V_\infty = a_\infty M_\infty = (968)(2\ 2) = 2130$ ft/sec

The viscosity coefficient at $T_\infty = 390°R = 216\ 7K$ can be estimated from an extrapolation of the straight line given in Fig 4 30. The slope of this line is

$$\frac{d\mu}{dT} = \frac{(2.12 - 1.54)\ x\ 10^{-5}}{(350 - 250)} = 5.8\ x\ 10^{-8} \frac{kg}{(m)(sec)(K)}$$

Extrapolating from the sea level value of $\mu = 1.7894 \text{ x } 10^{-5}$ kg/(m)(sec), we have at T_∞ = 216.7 K.

$$\mu_\infty = 1.7894 \text{ x } 10^{-5} - (5.8 \text{ x } 10^{-8})\ (288 - 216.7)$$

$$\mu_\infty = 1.37 \text{ x } 10^{-5} \text{ kg/(m)(sec)}$$

Converting to english engineering units, using the information in Chapter 4, we have

$$\mu_\infty = \frac{1.37\ x\ 10^{-5}}{1.7894\ x\ 10^{-5}} (3.7373\ x\ 10^{-7} \frac{slug}{ft\ sec}) = 2.86\ x\ 10^{-7} \frac{slug}{ft\ sec}$$

Finally, we can calculate the Reynolds number for the flat plate:

$$Re_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{3.6391\ x\ 10^{-4} (2130)(202)}{2.86\ x\ 10^{-7}} = 5.47 \text{ x } 10^8$$

Thus, from Eq. (4.100) reduced by 20 percent

$$C_f = (0.8)\ \frac{0.074}{(Re_L)^{0.2}} = (0.8) \frac{0.074}{(5.74\ x\ 10^8)^{0.2}} = 0.00106$$

The wave drag coefficient is estimated from

$$c_{d,w} = \frac{4\alpha^2}{\sqrt{M_\infty^{\ 2} - 1}}$$

where $\alpha = \frac{2}{57.3} = 0.035$ rad.

Thus,

$$c_{d,w} = \frac{4(0.035)^2}{\sqrt{(2.2)^2 - 1}} = 0.0025$$

Total drag coefficient = 0.0025 + (2)(0.00106) = **0.00462**

Note: In the above, C_f is multiplied by two, because Eq. (4.100) applied to only one side of the flat plate. In flight, both the top and bottom of the plate will experience skin friction, hence that total skin friction coefficient is 2(0.00106) = 0.00212.

(b) If α is increased to 5 degrees, where α = 5/57.3 - 0.00873 rad, then

$$c_{d,w} = \frac{4(0.0873)^2}{\sqrt{(2.2)^2 - 1}} = 0.01556$$

Total drag coefficient = 0.01556 + 2(0.00106) = $\boxed{0.0177}$

(c) In case (a) where the angle of attack is 2 degrees, the wave drag coefficient (0.0025) and the skin friction drag coefficient acting on both sides of the plate (2 x 0.00106 = 0.00212) are about the same. However, in case (b) where the angle of attack is higher, the wave drag coefficient (0.0177) is about eight times the total skin friction coefficient. This is because, as α increases, the strength of the leading edge shock increases rapidly. In this case, wave drag dominates the overall drag for the plate.

5.21 $V_\infty = 251 \text{ km/h} = \left(251\frac{\text{km}}{\text{h}}\right)\left(\frac{1\text{h}}{3600\,\text{sec}}\right)\left(\frac{1000\text{m}}{1\text{km}}\right) = 69.7 \text{ m/sec}$

$$\rho_\infty = 1.225 \text{ kg/m}^3$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(69.7)^2 = 2976 \text{ N/m}^2$$

$$C_L = \frac{L}{q_\infty S} = \frac{9800}{(2976)(16.2)} = 0.203$$

$$C_{D_i} = \frac{C_L^{\,2}}{\pi e AR} = \frac{(0.203)^2}{\pi(0.62)(7.31)} = 0.002894$$

$D_i = q_\infty \, S \, C_{D_i} = (2976)(16.2)(0.002894) = \boxed{139.5 \text{ N}}$

5.22 $V_\infty = 85.5$ km/h = 23.75 m/sec

$$q_\infty = \frac{1}{2} \rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(23.75)^2 = 345 \text{ N/m}^2$$

$$C_L = \frac{L}{q_\infty S} = \frac{9800}{(345)(16.2)} = 1.75$$

$$C_{D_i} = \frac{C_L^2}{\pi e AR} = \frac{(1.75)^2}{\pi(0.62)(7.31)} = 0.215$$

$$D_i = q_\infty \, S \, C_{D_i} = (345)(16.2)(0.215) = \boxed{1202 \text{ N}}$$

Note: The induced drag at low speeds, such as near stalling velocity, is considerably larger than at high speeds, near maximum velocity. Compare the results of problems 5.20 and 5.21

5.23 First, obtain the infinite wing lift slope. From Appendix D for a NACA 65-210 airfoil,

$$C_\ell = 1.05 \text{ at } \alpha = 8^\circ$$

$$C_\ell = 0 \text{ at } \alpha_{L=0} = -1.5^\circ$$

Hence,

$$a_o = \frac{1.05 - 0}{8 - (-1.5)} = 0.11 \text{ per degree}$$

The lift slope for the finite wing is

$$a = \frac{a_o}{1 + \frac{57\ 3\ a_o}{\pi\ e_1 AR}} = \frac{0.11}{1 + \frac{57.3(0.11)}{\pi\ (.9)(5)}} = 0\ 076 \text{ per degree}$$

At $\alpha = 6^\circ$,

$$C_L = a(\alpha - \alpha_{L=0}) = (0\ 076)\ [6 - (-1\ 5)] = \boxed{0.57}$$

The total drag coefficient is

$$C_D = c_d + \frac{C_L^{\ 2}}{\pi e AR} = (0\ 004) + \frac{(0.57)^2}{\pi\ (0\ 9)(5)}$$

$$C_D = 0\ 004 + 0\ 023 = \boxed{0.027}$$

5.24 $q_\infty = \frac{1}{2} \rho_\infty\ V_\infty^{\ 2} = \frac{1}{2}(0\ 002377)(100)^2 = 11\ 9\ \text{lb/ft}^2$

at $\alpha = 10^\circ$, L = 17 9 lb Hence

$$C_L = \frac{L}{q_\infty S} = \frac{17.9}{(11\ 9)(1\ 5)} = 1\ 0$$

at $\alpha = -2^\circ$, L = 0 Hence $\alpha_{L=0} = -2^\circ$

$$a = \frac{dC_L}{d\alpha} = \frac{1.0 - 0}{[10 - (-2)]} = 0\ 083 \text{ per degree}$$

This is the finite wing lift slope

$$a = \frac{a_o}{1 + \frac{57\ 3\ a_o}{\pi\ eAR}}$$

Solve for a_o

$$a_o = \frac{a}{1 - \frac{57.3\ a}{\pi\ eAR}} = \frac{0.083}{1 + \frac{57.3(0.083)}{\pi\ (0.95)(6)}}$$

$$a_o = \boxed{0.11 \text{ per degree}}$$

5.25 At $\alpha = -1^\circ$, the lift is zero. Hence, the total drag is simply the profile drag.

$$C_D = c_d + \frac{C_L^2}{\pi eAR} = c_d + 0 = c_d$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(130)^2 = 20.1 \text{ lb/ft}^2$$

Thus, at $\alpha = \alpha_{L=0} = -1^\circ$

$$c_d = \frac{D}{q_\infty S} = \frac{0.181}{(20.1)(1.5)} = 0.006$$

At $\alpha = 2^\circ$, assume that c_d has not materially changed, i.e., the "drag bucket" of the profile drag curve (see Appendix D) extends at least from -1° to 2°, where c_d is essentially constant. Thus, at $\alpha = 2^\circ$,

$$C_L = \frac{L}{q_\infty S} = \frac{5}{(20.1)(1.5)} = 0.166$$

$$C_D = \frac{D}{q_\infty S} = \frac{0.23}{(20.1)(1.5)} = 0.00763$$

However:

$$C_D = c_d + \frac{C_L^2}{\pi eAR}$$

$$0.00763 = 0.006 + \frac{(0.166)^2}{\pi\ e(6)} = 0.006 + \frac{0.00146}{e}$$

$e = \boxed{0.90}$

To obtain the lift slope of the airfoil (infinite wing), first calculate the finite wing lift slope.

$$a = \frac{(0.166 - 0)}{[2 - (-2)]} = 0.055 \text{ per degree}$$

$$a_o = \frac{a}{1 - \frac{57.3\ a}{\pi\ eAR}} = \frac{0.055}{1 - \frac{57.3(0055)}{\pi\ (0.9)(6)}}$$

$$a_o = \boxed{0.068 \text{ per degree}}$$

5.26 $V_{stall} = \sqrt{\frac{2W}{\rho_\infty S\ C_{L_{max}}}} = \sqrt{\frac{2(7780)}{(1.225)(16.6)(2.1)}}$

$$V_{stall} = \boxed{19.1 \text{ m/sec} = 68.7 \text{ km/h}}$$

5.27 (a) $\alpha = \frac{5}{57.3} = 0.087$ radians

$$c_\ell = 2\ \pi\ \alpha = 2\ \pi\ (0.087) = \boxed{0.548}$$

(b) Using the Prandtl-Glauert rule,

$$c_\ell = \frac{c_{\ell_o}}{\sqrt{1 - M_\infty^2}} = \frac{0.548}{\sqrt{1 - (0.7)^2}} = \boxed{0.767}$$

(c) From Eq. (5.50)

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.087)}{\sqrt{(2)^2 - 1}} = \boxed{0.2}$$

5.28 For $V_\infty = 21.8$ ft/sec at sea level

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(21.8)^2 = 0.565 \text{ lb/ft}^2$$

1 ounce = 1/16 lb = 0.0625 lb.

$$C_L = \frac{L}{q_\infty S} = \frac{0.0625}{(0.565)(1)} = \boxed{0.11}$$

For a flat plate airfoil

$$c_\ell = 2\,\pi\,\alpha = 2\,\pi\,(3/57.3) = \boxed{0.329}$$

The difference between the higher value predicted by thin airfoil theory and the lower value measured by Cayley is due to the low aspect ratio of Cayley's test wing, and viscous effects at low Reynolds number

---

5.29 From Eqs. (5.1) and (5.2), writtten in coefficient form

$$C_L = C_N \cos\alpha - C_A \sin\alpha$$

$$C_D = C_N \sin\alpha + C_A \cos\alpha$$

Hence:

$$C_L = 0.8 \cos 6^\circ - 0.06 \sin 6^\circ = 0.7956 - 0.00627 = \boxed{0.789}$$

$$C_D = 0.8 \sin\alpha + 0.06 \cos\alpha = 0.0836 + 0.0597 = \boxed{0.1433}$$

Note: At the relatively small angles of attack associated with normal airplane flight, C_L and C_N are essentially the same value, as shown in this example.

5.30 First solve for the angle of attack and the profile drag coefficient, which stay the same in this problem.

$$c_L = a\,\alpha = \frac{a_o\,\alpha}{1 + 57.3\ a_o/(\pi\ e_1 AR)}$$

or, $$\alpha = \frac{C_L}{a_o}[1 + 57\ 3\ a_o/\pi\ e_1\ AR)]$$

$$= \frac{0.35}{0\,11}\ \{1 + 57\ 3\ (0.11)/[\pi\ (0\ 9)(7)]\} = 4.2°$$

The profile drag can be obtained as follows

$$C_D = \frac{C_L}{(C_L / C_D)} = \frac{0.35}{29} = 0\,012$$

$$C_D = c_d + \frac{C_L{}^2}{\pi e AR}$$

or, $$c_d = C_D - \frac{C_L{}^2}{\pi e AR} = 0\ 012 - \frac{(0.35)^2}{\pi\ (9)(7)} = 0.0062$$

Increasing the aspect ratio at the same angle of attack increases C_L and reduces C_D For AR = 10, we have

$$C_L = a\,\alpha = \frac{a_o\,\alpha}{1 + 57.3\ a_o/(\pi\ e_1 AR)}$$

$$= \frac{(0.11)(4.2)}{1 + 57.3\ (0\,11)/[\pi\ (0\,9)(10)]} = 0.3778$$

$$C_D = c_d + \frac{C_L{}^2}{\pi e AR} = 0.062 - \frac{(0.3778)^2}{\pi\ (9)(10)} = 0.0062\ + 0.005048 = 0\ 112$$

Hence, the new value of L/D is

$$\frac{C_L}{C_D} = \frac{0.3778}{0\,0112} = \boxed{33.7}$$
