1 M.A.S.O

INTRODUCTION TO AEROSPACE ENGINEERING

Layer	Lapse Rate	km
Troposphere	-6.5	0 - 11
Tropopause	+0.0	11 - 20
Stratosphere	+1.0	20 - 32
Stratosphere	+2.8	32 - 47
Stratopause	+0.0	47 - 51
Mesosphere	-2.8	51 - 71
Mesosphere	-2.0	~~71~~ - 84
Mesopause	-	84 - x

72 RULE

Doubling time $T_2 = \dfrac{72}{r}$ $r \longrightarrow$ growth percentile.

HISTORY

300 AD Kong-Ming Lanterns Zhuge Liang

This lanterns used hot air to raise in the sky.

1782 Joseph and Étienne Montgolfier

Manned flight balloon

ECUATION OF STATE

$$p \cdot V = n \cdot \mathcal{R} \cdot T$$

GAS LAW

p = presure
V = volume
n = number of moles
$\mathcal{R}$ = constant of gas
T = temperature
ρ = density
M = molar mass
m = mass
R_{air} = constant of air

$\mathcal{R} = 8.3145\ J/mol\,k$
$M_{air} = 0.02897\ kg/mol$
$\frac{\mathcal{R}}{M}\ air = 287.00\ J/kg = R$

$V = \dfrac{m}{\rho}$ $n = \dfrac{m}{M}$

$$p \cdot \frac{m}{\rho} = \frac{m}{M} \cdot \mathcal{R} \cdot T$$

$$\boxed{R = \frac{\mathcal{R}}{M}}$$

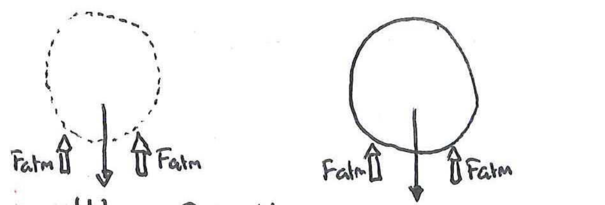
$$P = \rho \cdot \frac{\mathcal{R}}{M} \cdot T$$

$$\boxed{P = \rho \cdot R \cdot T}$$

LIFT EQUATION : BALLOONS DIFERENT GAS

same pressure but different weights.

object in water same effect ass helium in air



$W_{air} = \rho_{atm} \cdot V \cdot g$ $W_{gas} = \rho_{gas} \cdot V \cdot g$

$$L_G = W_{air} - W_{gas}$$

$$L_G = \rho_{atm} \cdot V \cdot g - \rho_{gas} \cdot V \cdot g$$

$$\boxed{L_G = \rho_{atm} \cdot V \cdot g \left(1 - \frac{\rho_{gas}}{\rho_{atm}}\right)} \longrightarrow \boxed{L_G = \rho_{atm} V \cdot g \left(1 - \frac{M_{gas}}{M_{atm}}\right)}$$

$\rho_{atm} \to 0.001285$

LIFT ECUATION BALLOON TEMPERATURE

$$L_G = \rho_{atm} \cdot V \cdot g \left(1 - \frac{\rho_{hot\ air}}{\rho_{atm}}\right) :$$

$$\rho = \frac{P}{R \cdot T} \qquad \frac{\frac{P}{R \cdot T_{hot}}}{\frac{P}{R \cdot T_{atm}}} = \frac{\frac{1}{T_{hot}}}{\frac{1}{T_{atm}}}$$

$$L_G = \rho_{atm} \cdot V \cdot g \left(1 - \frac{T_{atm}}{T_{hot}}\right)$$

$$\boxed{L_G = \rho_{atm} \cdot V \cdot g \left(\frac{\Delta T}{T_{atm} + \Delta T}\right)}$$

$$\frac{T_{atm} + \Delta T}{T_{atm} + \Delta T} - \frac{T_{atm}}{T_{atm} + \Delta T}$$

$$M \begin{cases} \text{Helium} = 4.00\ g/mol \\ \text{Hydrogen} = 2.02\ g/mol \\ \text{Air} = 28.97\ g/mol \end{cases}$$

2nd LECTURE

STANDARD ATMOSPHERE: it is used so everyone has the same reference conditions

- Depends on altitude
- 3 components
  - pressure
  - density
  - temperature

obeys
- gas law
- gravity law

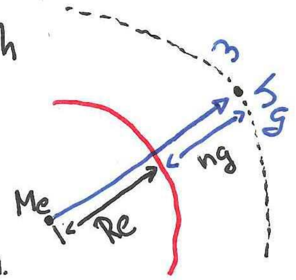
STANDARD SEA LEVEL

$$\left.\begin{array}{l} P = 101325.0\ Pa \\ T = 15^{\circ} = 288.15\ K \\ \rho = 1.225\ kg/m^3 \end{array}\right\} \text{ISA PROFILE}$$

ABSOLUTE ALTITUDE: altitude from the center of the earth

$$h_a = h_g + r$$

absolute geometric radius

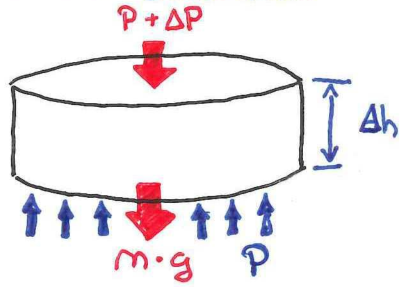


GEOMETRIC ALTITUDE: altitude from the surface of the earth.

GEOPOTENTIAL ALTITUDE: altitude from the surface but taking into account the gravity.

$$\boxed{g = g_0 \left(\frac{R_e}{h_a}\right)^2} \qquad \boxed{g = g_0 \left(\frac{R_e}{R_e + h_g}\right)^2}$$

HYDROSTATIC EQUATION



Equilibrium $\Sigma F_{down} = \Sigma F_{up}$

$$m \cdot g + (P + \Delta P) = (P) \cdot A$$

$$\rho \cdot V \cdot g + AP + A \cdot \Delta P = AP$$

$$\rho \cdot \Delta h \cdot \cancel{A} \cdot g = \cancel{AP} - \cancel{AP} - \cancel{A} \cdot \Delta P$$

$$\boxed{\Delta P = -\rho \cdot \Delta h \cdot g}$$

INTEGRATING THE HYDROSTATIC EQUATION $\boxed{\text{WHEN } a \neq 0}$

$$dp = -\rho \cdot \Delta h \cdot g$$

$$P = \rho \cdot R \cdot T$$

$$\rho = \frac{P}{R \cdot T}$$

Toussaint formula

$$T_1 = T_0 + a(h - h_0)$$

$$\frac{T_1 - T_0}{h_1 - h_0} = \boxed{a = \frac{dT}{dh}}$$

$$dp = -\frac{P}{RT} \cdot dh \cdot g$$

$$\frac{1}{P} dp = -\frac{g}{RT} \cdot dh$$

$$\frac{1}{P} dp = -\frac{g}{R \cdot a} \cdot \frac{1}{T} \cdot dT$$

$$\int_{P_0}^{P_1} \frac{1}{P} dP = \underbrace{-\frac{g}{R \cdot a}}_{\text{constant}} \int_{T_0}^{T_1} \frac{1}{T} \cdot dT$$

$$\ln P_1 - \ln P_2 = -\frac{g}{R \cdot a}(\ln T_1 - \ln T_0)$$

$$e^{\ln P_1 - \ln P_0} = e^{(\ln T_1 - \ln T_0) \cdot -\frac{g}{R \cdot a}}$$

WITH PRESSURE

$$\boxed{\frac{P_1}{P_0} = (T_1 / T_0)^{\frac{-g}{R \cdot a}}}$$

WITH DENSITY

$$\boxed{\frac{\rho_1}{\rho_0} = \left(\frac{T_1}{T_0}\right)^{\frac{-g}{R \cdot a} - 1}}$$

WHEN $\boxed{a = 0}$

$$\frac{1}{P} dP = \boxed{-\frac{g}{RT}} \cdot dh$$

get back to this step → now everything is constant

$$\int_{P_0}^{P_1} \frac{1}{P} dP = -\frac{g}{RT} \int_{H_0}^{H_1} dh$$

$$\ln P_1 - \ln P_0 = -\frac{g}{RT} \cdot (h_1 - h_0)$$

$$\boxed{\frac{P_1}{P_0} = e^{-\frac{g}{RT}(h_1 - h_0)}}$$

INTEGRATION OF THE GEOPOTENTIAL FORMULA

$$\boxed{dp = -\rho \cdot \Delta h \cdot g}$$

ALWAYS REMEMBER

$$dp = dp$$

$$-\rho \cdot g_0 \cdot dh = -\rho \cdot g_1 \cdot dh_g$$

$$\boxed{dh = \frac{g}{g_0} dh_g}$$

$$dh = \frac{Re^2}{(Re + h_g)^2} dh_g$$

$$\int_{h_0}^{h_1} dh = Re^2 \int_{h_{g0}}^{h_{g1}} \frac{1}{(Re + h_g)^2}$$

we consider this numbers as 0 (surface of earth)

$$= Re^2 \int_{h_{g0}}^{h_{g1}} (Re + h_g)^{-2} dh_g$$

$$= Re^2 \left[\frac{-1}{Re + h_g} - \frac{-1}{Re}\right]$$

$$= Re \left[\frac{-Re}{Re + h_g} - \frac{-Re}{Re}\right]$$

$$= Re \left[\frac{-Re}{Re + h_g} + 1\right] = \left[\frac{Re}{Re + h_g} \cdot h_g\right]$$

$$\boxed{\underset{\text{geopotential}}{h} = \frac{Re}{Re + h_g} \cdot h_g} \rightarrow \boxed{\underset{\text{geometric}}{h_g} = \frac{Re}{Re - h} \cdot h}$$

the geopotential altitude takes the variation of gravity into account

LIFT, AIRFOILS & WINGS

HISTORY:
- Otto Lilienthal (1848 – 1896), he designed gliders with a glide ratio of 12:1 he died after a crash.
- Samuel Langley: early pioneer.
- Wright Brothers: they could fly horizontal and even turn. They were the first in a lot of things.

FOUR MAIN FORCES ON AN AIRPLANE:

- WEIGHT (W): three components: ▸ aircraft empty weight ▸ fuel ▸ payload (passenger + luggage)
- THRUST (T): provided by the engines
- LIFT (L): generated by the wings mainly
- DRAG (D): caused by the fuselage, wings, tail and surfaces

LIFT FORCE: from Bernoulli's law

$$\boxed{L = C_L \cdot \frac{1}{2} \rho \cdot V^2 \cdot S}$$

CL coefficient depends on:
- shape and wind
- angle between airspeed vector and aircraft

COMPONENTS

- S – Surface area wings $\boxed{m^2}$
- V – Velocity (airspeed) $\boxed{m/s}$
- ρ – Air density $\boxed{kg/m^3}$

with this three variables we want to create $\boxed{\frac{kg \cdot m}{s^2}}$

- α – Angle of attack
- – airfoil (shape)
- – weather
- – Viscosity of fluid
- ΔP – Pressure difference

Benefits of reducing weight
- more payload
- more fuel (longer distances)
- improve performances

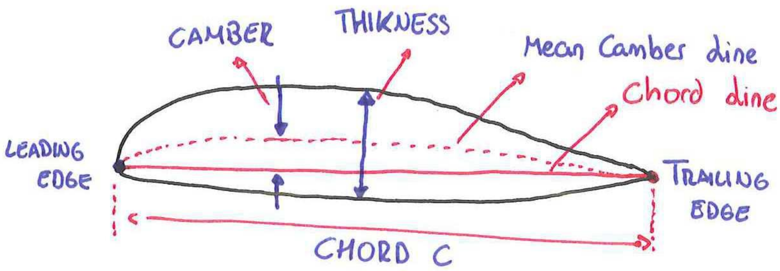
LIFT Parameters

C_L depends on α

ρ depends on altitude and temperature

V and S are design parameters.

NACA PROFILE



NACA 2412

- 2 % camber (of chord lenght)
- 0.4 of the chord location
- 12 % thikness / chord ratio

NACA 23012

- $\boxed{2}$ design lift coeficient $C_L = 0.3(2 \cdot 0.15)$
- $\boxed{30}$ location of max camber $30/2 = 15\%$ chord
- max thikness of 12 % chord $\boxed{12}$

VISCOSITY

$$Re = \frac{\rho \cdot V \cdot L}{\mu}$$

Re = Renolds Number

ρ = density

V = air speed

L = lenght chor c

μ = dynamic viscosity Ns/m^2

Moment equation

$$\boxed{M = C_m \cdot \frac{1}{2} \rho \cdot V^2 \cdot S \cdot c}$$

BERNOULLIS LAW

BERNOULLIS LAW: sum of static and dynamic pressure remains constant.

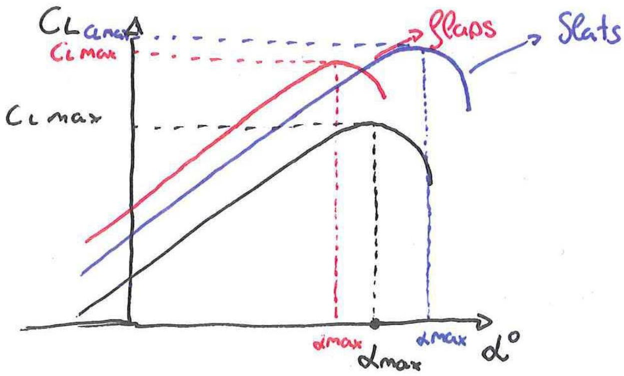
$$\boxed{P + \frac{1}{2} \rho \cdot V^2 = \text{constant}}$$

VELOCITY MEASUREMENTS

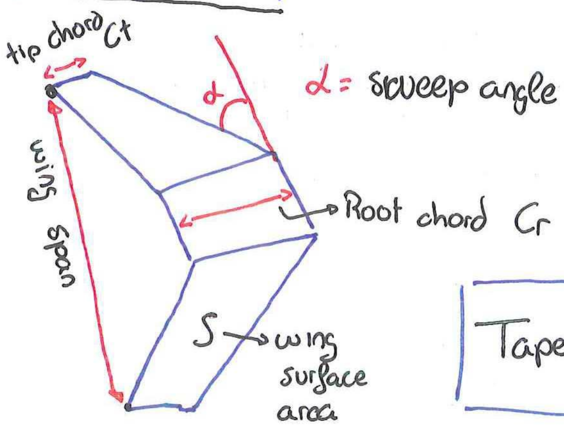
$$\boxed{V = \sqrt{\frac{2 \cdot (P_{tot} - P_s)}{\rho}}}$$

LIFT DEVICES

- Slats
- Flaps



WING GEOMETRY



$$\boxed{\text{Taper} = \frac{C_t}{C_r}}$$

6.

DRAG

$$\boxed{D = C_D \cdot \frac{1}{2}\rho \cdot v^2 \cdot S}$$

3 types of Drag

- FRICTION DRAG
- PRESSURE DRAG (INDUCED FROM LIFT)
- WAWE DRAG (TRANSONIC, SUPERSONIC SPEEDS)

AIR FLOWS

Laminar: stics to the airfoil

Turbulent:

$$\boxed{\frac{C_L}{C_D} = \frac{L}{D}}$$

DRAG COEFICIENT:

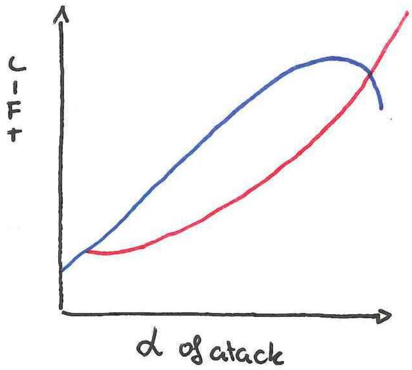
$C_D = C_{D0} + C_{D1}$ → induced drag

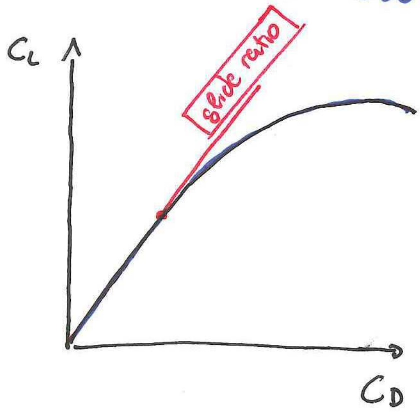
C_{D0} → zero lift drag

DRAG POLAR

$$C_D = C_{D0} + \frac{C_l^2}{\pi \cdot Ae}$$

$$\frac{C_L}{C_D}$$





$$C_L = \frac{L}{q_\infty \cdot S} \qquad C_D = \frac{D}{q_\infty \cdot S} \qquad C_M = \frac{M}{q_\infty \cdot S \cdot c}$$

DRAG POLAR EXPLAINED (INDUCED)

$$C_{DI} = \frac{(C_L)^2}{\pi \cdot e_0 \cdot AR}$$

C_{DI}: INDUCED DRAG COEFFICIENT

C_L → LIFT COEFFICIENT

AR → ASPECT RATIO

e_0 → spam efficiency factor.

$$AR = \frac{S^2}{A}$$

S → spam

A → Area

STABILITY AND CONTROL

1. CONTROLS

WRIGHT BROTHERS: deformed the wing to move made the roll control

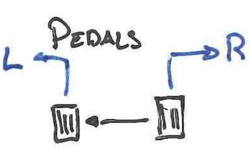
EUROPEAN: rigid wings:

First ailerons 1908 Antoinette

CLASIC CONTROL SYSTEM: made by cables

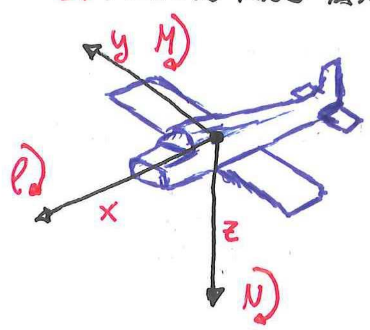
FLY BY WIRE:
- allows unestable planes.
- saves weight.

we can control 6 degrees of freedom with 4 controls



- Elevator
- Rudder
- Throttle
- Aileron L and R

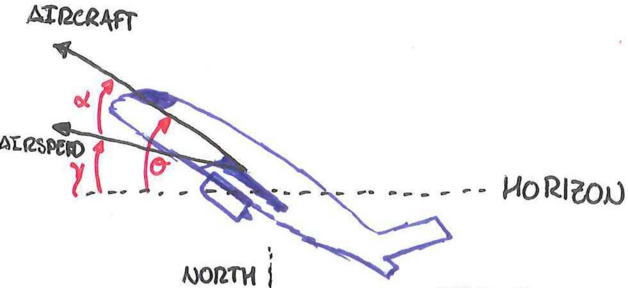
2. ANGLES AND AXES



Rotation about x axes = bank angle / Roll

Rotation about y axes = nose up / nose down / Pitch

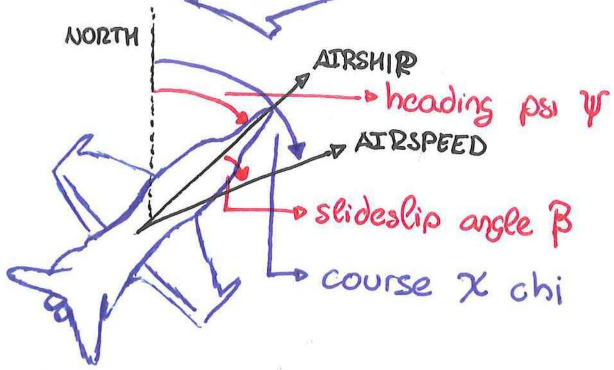
Rotation about z axes = Yaw



θ pitch angle

α angle of attack

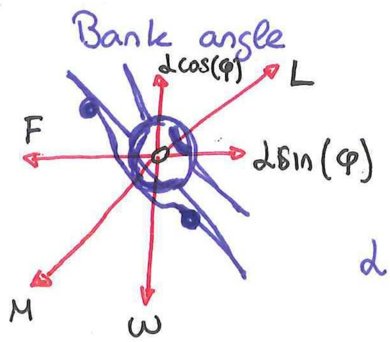
γ climbe angle



$$\boxed{\chi = \psi + \beta}$$

Load factor n:

$$\boxed{n = \frac{1}{\cos(\varphi)}}$$

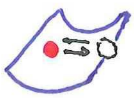


$$L \sin(\varphi) = F = \frac{W \cdot V^2}{g \cdot R_T}$$

R_T → radius of turn

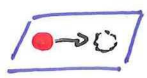
3. STABILITY

Positive



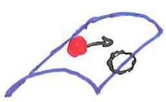
$$\boxed{\frac{\text{deviation}}{\text{reaction}} = \frac{+}{-} = \frac{-}{+} < 0}$$

Neutral



$$\boxed{\frac{+}{+} = \frac{-}{-} > 0}$$

Negative



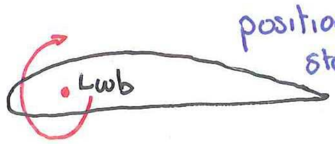
$$\boxed{0}$$

NEUTRAL POINT

$$\boxed{\frac{l_{np}}{c} = V_H \cdot \frac{C_{L\alpha h}}{C_{L\alpha}} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)}$$

DYNAMIC STABILITY: is harder to judge and calculate than static stability

4. STATIC STABILITY

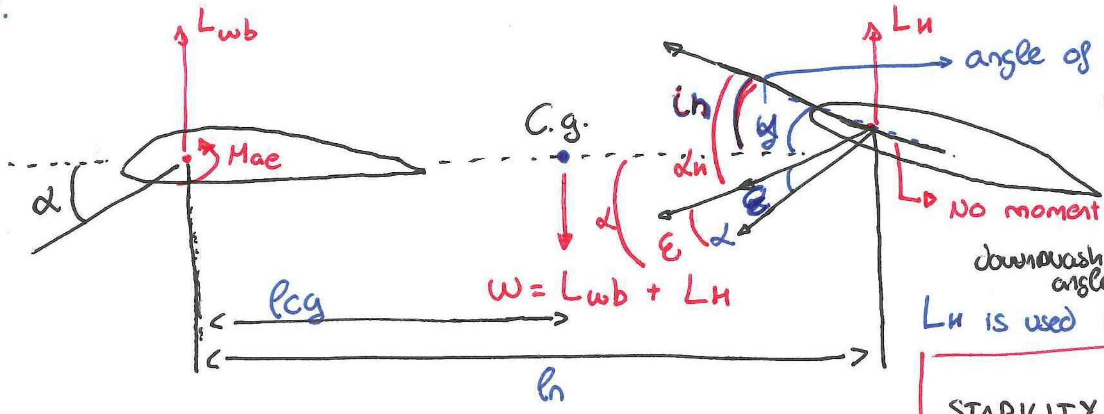


position in which the moment stays the same (no α)

indiferent

AERODINAMIC CENTER

8.



$$\boxed{\alpha_H = \alpha - \varepsilon + i_H}$$

downwash angle $\boxed{\varepsilon = \alpha \cdot \frac{d\varepsilon}{d\alpha}} + \varepsilon_0$

L_H is used to trim aircraft

STABILITY $\begin{cases} C_{m_{CL}} > 0 \\ C_{m_\alpha} < 0 \end{cases}$

$$\Sigma M = M_{ae_{wb}} + L_{wb} \cdot l_{cg} - L_H (l_h - l_{cg})$$

$\bar{c}$ = average chord = $\frac{\text{wing area}}{\text{span}}$

$$\Sigma M = M_{ae_{wb}} + L_{wb} \cdot l_{cg} - L_H l_h + L_H l_{cg}$$

$$\Sigma M = M_{ae_{wb}} + L \cdot l_g - L_H \cdot l_h$$

$$C_m = \frac{M}{\frac{1}{2}\rho \cdot V^2 \cdot S \cdot \bar{c}}$$

$$C_m = C_{mac_{wb}} + \frac{L \cdot l_{cg}}{\frac{1}{2}\cdot\rho\cdot V^2 \cdot S \cdot \bar{c}} - \frac{L_H \cdot l_h}{\frac{1}{2}\cdot\rho\cdot V^2 \cdot S\bar{c}}$$

$$C_L = \frac{L}{\frac{1}{2}\cdot\rho\cdot V^2 \cdot S}$$

$$C_m = C_{mac_{wb}} + C_L \cdot \frac{l_{cg}}{\bar{c}} - \frac{S_h \cdot l_h}{S \cdot \bar{c}} \cdot C_{LH}$$

$$C_{LH} = \frac{L_H}{\frac{1}{2}\cdot\rho\cdot V^2 \cdot S_H} \qquad \boxed{C_{LH} = a_t \cdot \alpha_H}$$

$$C_m = C_{mac_{wb}} + C_L \cdot \frac{l_{cg}}{c} - C_{LH} \cdot V_H$$

$$V_H = \frac{S_h \cdot l_H}{S \cdot \bar{c}}$$

$$\frac{dC_m}{d\alpha} = \frac{dC_{mac_w}}{d\alpha} + \frac{dC_L}{d\alpha} \cdot \frac{l_{cg}}{c} - \frac{dC_{LH}}{d\alpha} \cdot V_H$$

$\frac{C_{mac_w}}{d\alpha} = 0$ Moment coefficient does not vary depending on the α

$$\frac{dC_m}{d\alpha} = + \frac{dC_L}{d\alpha} \cdot \frac{l_{cg}}{\bar{c}} - \frac{dC_{LH}}{d\alpha} \cdot V_H$$

We know $\frac{dC_{LH}}{d\alpha_H}$ but not $\frac{dC_{LH}}{d\alpha}$ so

$\alpha_H = \alpha - \varepsilon + i_H \rightarrow \frac{d\alpha_H}{d\alpha} = 1 - \frac{d\varepsilon}{d\alpha}$

$$\frac{dC_{LH}}{d\alpha} = \frac{dC_{LH}}{d\alpha_H} \cdot \frac{d\alpha_H}{d\alpha} = \frac{dC_{LH}}{d\alpha}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$\frac{dC_m}{d\alpha} = \frac{dC_L}{d\alpha} \cdot \frac{l_{cg}}{\bar{c}} - \frac{dC_{LH}}{d\alpha_H} \cdot V_H \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

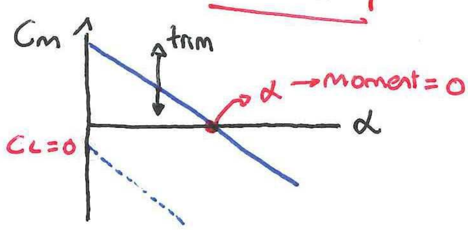
$\frac{dC_L}{d\alpha}$ and $\frac{dC_{LH}}{d\alpha_H}$ can be writen as a and a_t

$$C_{m_a} = a \cdot \frac{l_{cg}}{\bar{c}} - a_t \cdot V_H \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) < 0$$

in order to be stable smaller than 0

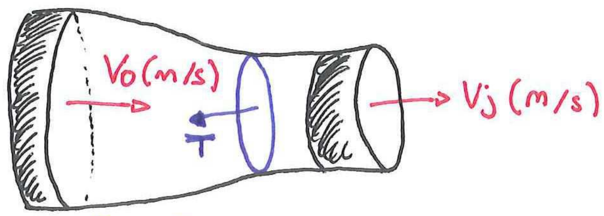
ANDERSON $\boxed{h = \frac{l_{cg}}{\bar{c}}}$

$$a \cdot \frac{l_{cg}}{c} < \left[a_t \cdot V_H \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)\right] \longrightarrow [\text{neutral point}]$$



Propulsión

Theory of Propulsion



$\dot{m}$ [kg/s]
masa de aire

momentum mass velocity

$$I = \dot{m} \cdot V$$

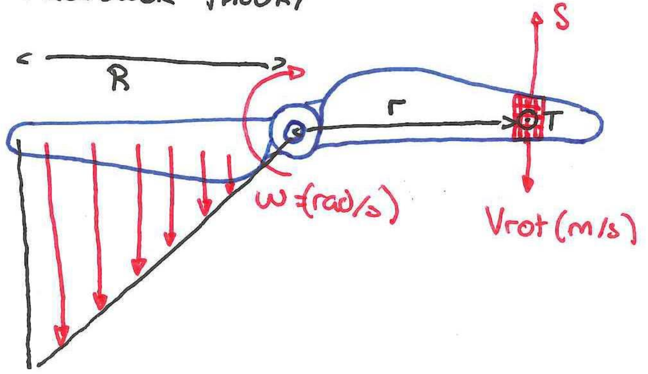
$$F = \frac{dI}{dt} \qquad F = m \cdot \frac{dV}{dt} = m \cdot a$$

$$\boxed{T = \dot{m}(V_j - V_o)} \quad \frac{kg}{s} \cdot \frac{m}{s} = N$$

Propulsion Systems

- Human Powered
- Electrical Engine
- Airbreathing Engine
  - Propeller propulsion
    - Counter Rotating
    - Rotating
      - Fixed Pitch
      - Variable Pitch
      - Ducted Fan
    - Piston Engines
      - Turbo-Compound Engine
    - Turbo Prop / Turbo Shaft
      - Turbo-Compound Engine
      - Turbo Engine with Propfan
  - Jet Propulsion
    - TurboFan
      - $\Delta \dot{m}$ FAN
      - Turbo Engine with Propfan
      - By-Pass Engine
    - Turbo Jet
      - By-Pass Engine
      - Turbo-RamJet
    - Ram Jet
      - Scram Jet
      - Turbo-RamJet
      - Ram Rocket
- Rocket Engine

Propeller Theory



$$\boxed{V_{rot} = \omega \cdot r}$$

T = Thrust (pointing at us)

S = Side force (drag for wings)

Available Power for Thrust

Work Performed: $W = T \cdot \underbrace{\Delta s}_{\text{distance}}$ J

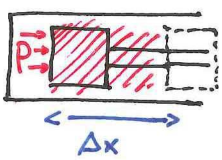
Available Power: $P_a = \dfrac{T \cdot \Delta s}{\Delta t} = \boxed{T \cdot V}$ J/sec

Brake (shaft) Power $P_{br} = \text{max power} \cdot \text{Trotle}$

Propulsive Efficiency $\eta = \dfrac{P_a}{P_{br}} = \dfrac{T \cdot V}{P_{br}}$

4 Stages of Airbreathing Engines

1. Intake suck
2. Compression squeeze
3. Work bang
4. Exhaust blow



More Volume

$W = P \cdot \underbrace{A \cdot \Delta x}$

$W = P \cdot \Delta V$

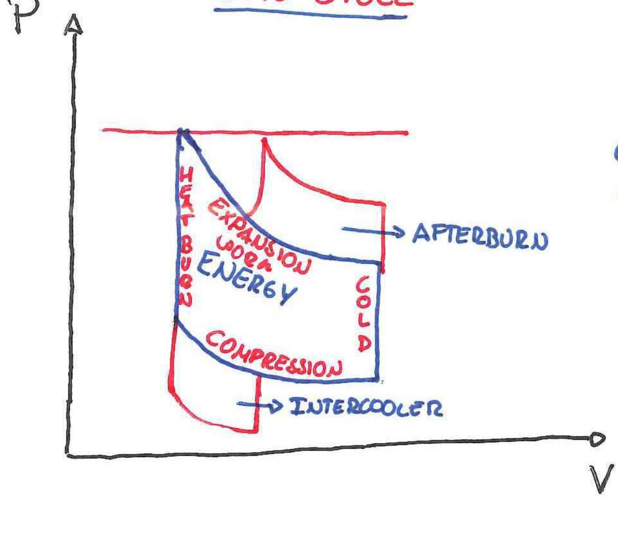
$W = \int_{V_1}^{V_2} P \cdot \Delta V$

P = Pressure

A = Area

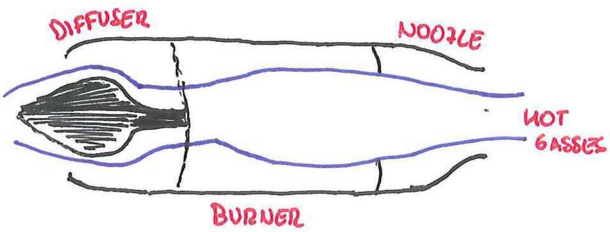
Δx = Displacement

OTTO CYCLE

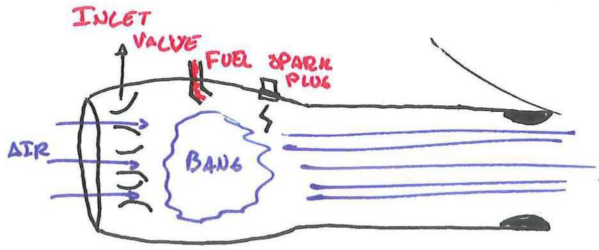


Compressors: used in order to have less volume which means more area.

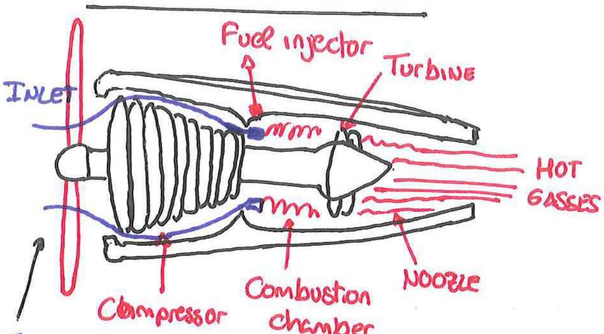
Ramjet



Pulsar Jet

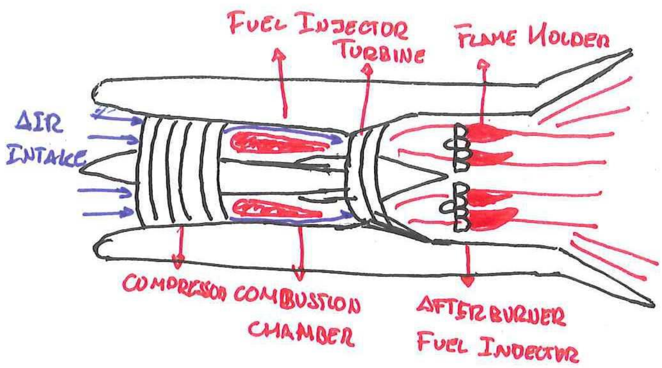


Pure Jet Engine



(Turbo Prop adds a prop in the inlet)

Jet Engine With Afterburner

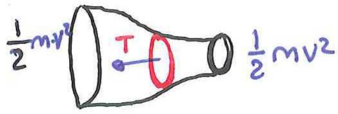


Turbo Fan: adds a second air stream to the engine around the compresor.

Jet Efficiency

$$P_j = \frac{1}{2}\dot{m}(V_j^2 - V_0^2) \qquad T = \dot{m}(V_j - V_0) \qquad P_a = T \cdot V_0$$

$$\eta_J = \frac{P_a}{P_j}$$



Less V_j = less noise

$$\eta_J = \frac{TV_0}{\frac{1}{2}\dot{m}(V_j^2 - V_0^2)} = \frac{\dot{m}\cancel{(V_j - V_0)}V_0}{\frac{1}{2}\dot{m}(V_j + V_0)\cancel{(V_j - V_0)}}$$

$$\boxed{\eta_J = \frac{2}{1 + \frac{V_j}{V_0}}}$$

Turbofan $\frac{2}{3}$ shaft more efficient

Structural Concepts

WEIGHT

Empty Weight:

- Structure
- Systems
- Crew and Flight attendants
- Operating items

Payload:

- Passengers
- Cargo

Fuel

Goal of aircraft design: minimize empty weight to maximize Payload and Fuel

Structure: Skeleton

- Many elements
- Several functions
- Coherence
- Joints
- Different materials

Main function

- Carrying loads
- Protection of the inside
- Framework to attach systems

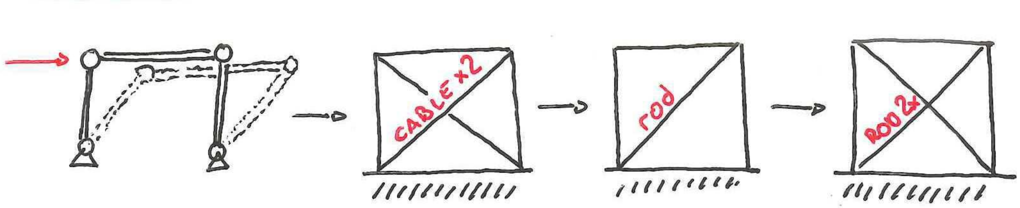
Requirements

- Stiffness: Resistence to get blended
- Strenght: endurance during its lifetime
- Light weight
- Durable
- Cheap and effective
- Availability and maintenance

History:

- 1903: Wright Flyer: cables, lath, wood, linen
- 1917: Sopwith Camel: truss, spars, ribs, steel rods, tubes
- 1924: Fokker F VII: wood: tripex, canvas
- 1934: Douglas DC3: stiffened shell structures Aluminum
- 1949: Comet: pressure cabin Al-alloys

From Trus to Beam

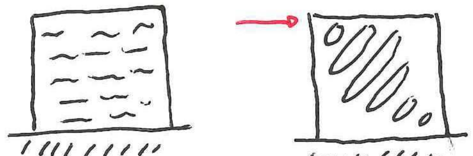


Convenient

- Fail safe

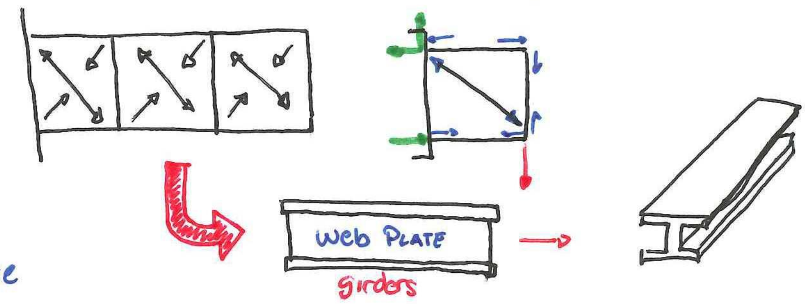
Inconvinients of 2x Rod

- More difficult to assemble
- More difficult to calculate
- Heavier



Rod replaced by a skin

Caused Buckling

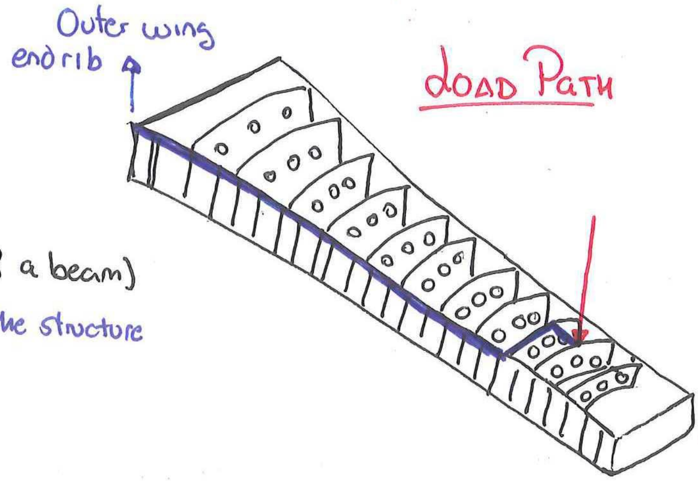


Elastic Buckling: not failure

Plastic Buckling: failure

Fail safe structure: reserve, if one element fails the airplane won't crush

Safe life structure: each element is strong enough to stay intact for the life cycle.



Anatomy of a Structure

- Structure: assembly of elements (derivates of a beam)
- Each element participates in the functions of the structure
- Structure has coherence
- Structural elements are joined together

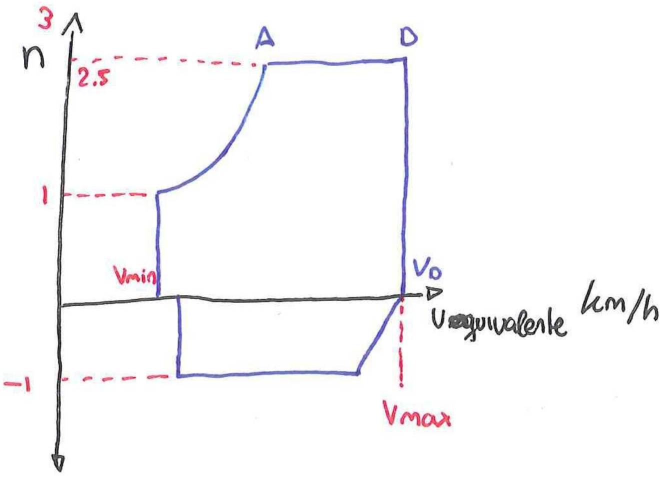
Principal Structural Elements

- P.S.E primary structure carry loads, failure is catastrophic
- Non P.S.E secondary structures, failure is not catastrophic
- Beamlike: wing ribs, stringers, fuselage frame.
- Shell structure: skin made of a metal sheet which is load carrying and has beamlike elements
- Sandwich structures: usually composites: acts like a beam in two directions. The face sheets carry tension and compression while the core material prevents buckling and carry shear loads.

Loads

$n = \frac{L}{W} = \boxed{6}$

(g)



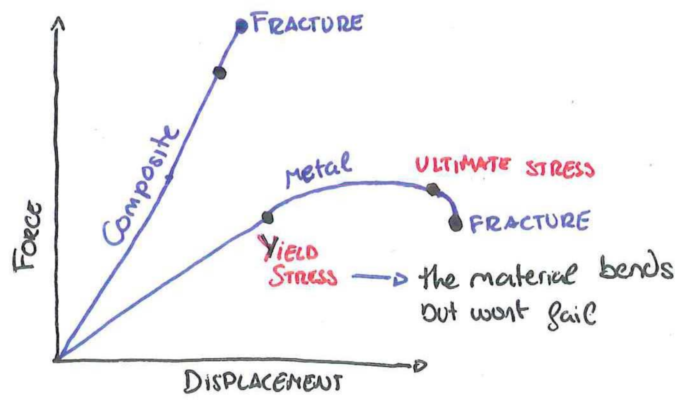
Limit Load:

- Load experienced once in a lifetime
- No remaining damage allowed

Ultimate Load

- Limit load × safety factor
- Failure allowed after 3 seconds

Failure Behavior Materials



Structural Conceps

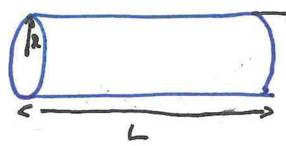
- **Forces to Stress**
  - Pressure or tension exerted

stress N/mm^2 ← $\sigma = \dfrac{F}{A_0}$; F → force N ; A_0 → area mm^2

- **Displacement to Strain**
  - Dimensionless elongation

strain ← $\varepsilon = \dfrac{\Delta L}{L_0}$; ΔL → change in length mm ; L_0 → original length mm

Stresses in a pressure vessel

- 

t = Skin thickness

ΔP = Pressure difference

- We want to determine
  - Circumference stress
  - Longitudonal stress

- Longitudinal stress

$$\sigma_{long} = \frac{\Delta P \cdot R}{2t}$$

- Circumferential stress

$$\sigma_{circ} = \frac{\Delta P \cdot R}{t}$$

- We assume
  - Fuselage with homogeneous stresses
  - No cut-outs (windows/door/holes)
  - Flat ends.

- Ratio

$$\frac{\sigma_{circ}}{\sigma_{long}} = 2$$

Cut-outs

- Ideal cylinder is disturbed by cut-outs
  - Windows
  - Passenger doors
  - Cargo doors
  - Landing Gear doors.

Materials: Substance or matter that has properties but no shape.

- Relation btw materials and structures

Materials ⟶ Semi Finished Parts ⟶ Structural Elements ⟶ Structure

- Four Main Material Groups
  - Metal alloys
  - Composites: fibers, resin, metal

  } Most used

  - Pure Polymers
  - Ceramics

  } Less used

Metal Properties

- Isotropic (properties always the same in all orientations)
- Can be strengthened (alloying, heat treatment)
- Plastic behaviour and Melting (recycling, welding)
- Good processibility
- Low costs
- Huge diversity in tension properties.

Polymeres

- Two main types
  - Thermoplastic: softening reversible, one component
  - Thermoset: curing irreversible, often more components
- Isotropic
- Low strength and stiffness
- Huge variety
- Plastic flow and Melting (recycling, welding)
- Good processibility
- Low costs

Calculations

$$\boxed{\text{Cross sectional area} = \text{Tensile load} \times \text{strenght of mat}}$$

$$\frac{F}{W} = \frac{\sigma_y \times A}{\rho \times A \cdot L} = \boxed{\frac{\sigma_y}{\rho}} \cdot \frac{1}{L}$$

F = required load
W = weight.

$\boxed{\frac{\sigma_y}{\rho}}$ → comparison of two materials

$$\text{Strain} = \frac{\text{stress}}{\text{E module}}$$

Composites

- Fiber reinforced polymeres
  - Polymers + Fibres
    - glass, carbon, aramid
    - length: short, long always continuous
- Hybrid Materials
  - Glare: composite and metal layers.
- Anisotropic: benefit reinforce softne faces.
- Layered structure laminate
- High strength and stiffness
- Low density
- Often expensive
- No plasticity
- Good productivity and processability

Continuous Fiber Composites

- Function of fibres:
  - Strength and stiffness
- Function of resin:
  - Support and protect
- Strong and stiff in Tension
- Fibers in 0° and 90° degrees = Cross ply
- UD = Unidirectional Composite

Sandwich Composites

- Two laminates - Facings
- Lightweight core.

Link Between Materials and Structures

- Load carrying capacity of a structure depends on:
  - Design, shape
  - Materials
  - Production techniques.

Perfomance depends on
- Materials
- Design / Shape
- Manufacturing techniques
- Other politics, cost, experience

Composites Vs Metal

- Metals
  - \+ Plastic Behavior - Damage Tolerant - Joining
  - \+ Chaeap Materials - Easy processing
  - \- Labor intensive

Manufactoring
- Plastic deformation
- Forging
- Casting

Designs
- Stiffened shell
- structure

- Composites
  - \+ High spec. strength and stiffness - low weight
  - \+ High integration possible
  - \- Expensive materials → compensated by production methods

Manufacturing
- Laminating
- Filament winding

Designs
- Sandwich

Exploring the Limits

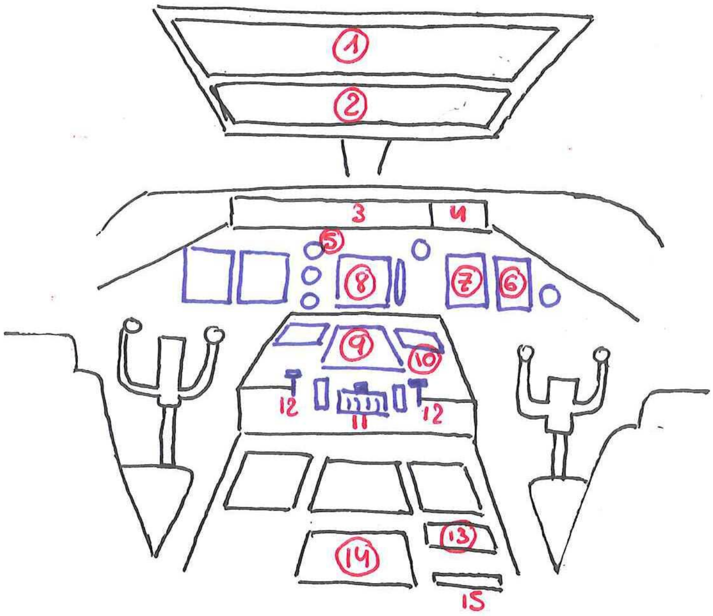
X-Planes: experimental planes.
- Bell X1: fly supersonic, sound "barrier" 1947 Charles Yeager Mach 1,015
- X-57: latest
- X-15: hypersonic flight / high altitude Mach 6,72, Altitude 107,9 km,
  - Aerodynamic heating > 650°C needed: stainless steel
    titanium alloys
    special steel alloys.

Blackbird: 1964
- Fastest aircraft
- Titanium → 1668°C melting
- Leading edge → 400°C

Space Shuttle
- Thermal protection system
- Up to 1650°
- T > 1260° Reinforced Carbon-Carbon.

Cockpit and Systems

- BOEING 727
- AIRBUS A320 → glass cockpit fly by wire
- BOEING 747-400



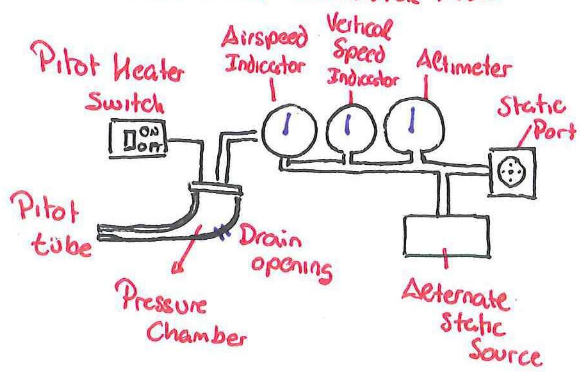
1. Circuit breakers
2. Overhead control panel
3. Mode control panel
4. Display control panel
5. Stand-by systems
6. Primary Flight Display
7. Navigation display
8. Upper EICAS
9. Lower EICAS
10. Command and display unit. (CDU)
11. Throttle levers
12. Flaps & Speed brake
13. Radio Control Panel
14. Trim Panel
15. Printer

INSTRUMENTATION

Units for speed, altitude and vertical speed.

- Circumference earth is 40000 km
- 1 nm = 1 minute = 40000 / 360 / 60 = $\boxed{1852\,m}$
- 1 knts = 1 nm/hr = 1852 m / 3600 s = $\boxed{0.51444\ m/s}$
- $M = V/a \quad a = \sqrt{\gamma \cdot R \cdot T} \quad \gamma = 1.4$
- 1 ft = 0.3048 m
- 1 ft/min = 0.00508 m/s

AIR DATA COMPUTER ADC



▷ STATIC PRESSURE, DYNAMIC PRESSURE, TOTAL PRESSURE

- Speed from differende static and total pressure
- Altitude from difference btw static pressure and reference as set by pilot (QNH setting) based on definition in standard atmosphere.
- V/S as change in P_{st}

$$P_{TOT} = P_{ST} + P_{dyn}$$

$$P_{dyn} = \frac{1}{2}\rho V^2$$

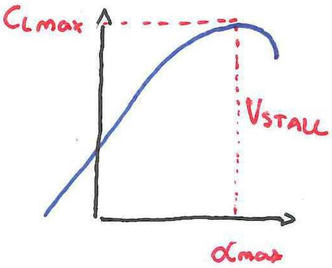
Equivalent Airspeed

$$q_{\text{dynamic pressure}} = \frac{1}{2}\rho \cdot V^2$$

$$\boxed{W = L = C_L \cdot \frac{1}{2} \cdot \rho_0 V_{EAS}^2\, S = C_L \cdot \frac{1}{2}\rho \cdot V_{TAS}^2}$$

$$\frac{1}{2}\rho_0 \underline{V_{EAS}}^2 = \frac{1}{2}\rho \cdot \underline{V_{TAS}}^2$$

$$\boxed{EAS = TAS\sqrt{\frac{\text{actual air density}}{\text{standard air density}}}}$$



$$\boxed{V_{STALL} = \sqrt{\frac{W}{C_{L\,max} \cdot \frac{1}{2}\rho \cdot V_{TAS}^2}}}$$

NO CHANGE IN DENSITY

$$\boxed{V_{STALL\ EQUI} = \sqrt{\frac{W}{C_{L\,max} \cdot \frac{1}{2} \cdot \rho_0 \cdot V^2}}}$$

IAS: Indicated air speed
EAS: Equivalent air speed
TAS: True airspeed
GS: Ground speed
M: Mach number = V_{TAS}/a

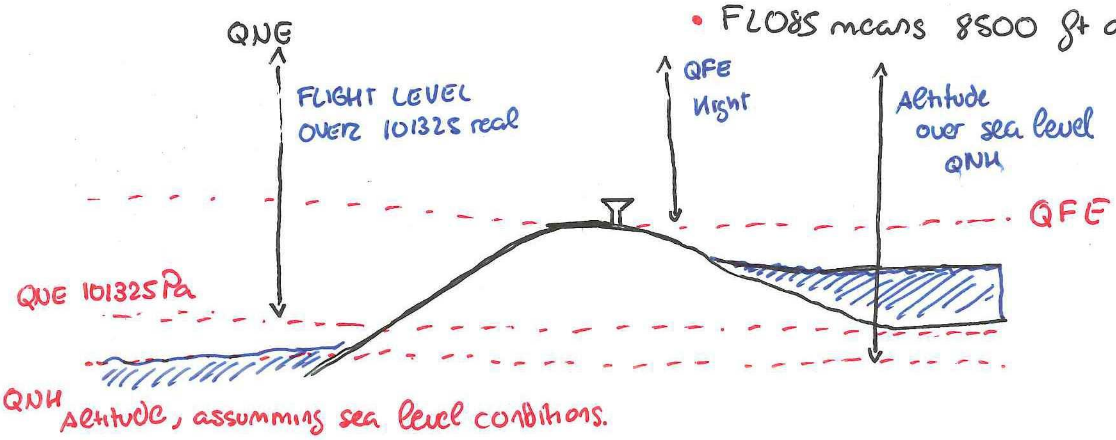
Altitude

- Pressure altitude is not real altitude
- Adjust for pressure at sea level (QNH)
- Transition altitude: 4500 ft = FL045 ×100

Sea level: 101325 Pa

Flight Levels

- Used above a so called 'transition altitude'.
- FL085 means 8500 ft above the 101325 Pa Pressure.



Control Loops

- Manual Control

Pilot → FCS → aircraft

speeds and angles
positions and trajectories

- Autopilot, mode control panel, flight mode panel

Pilot → AP & AT → FCS → aircraft →

speed, altitude, vert speed, course

position, trajectory / route.

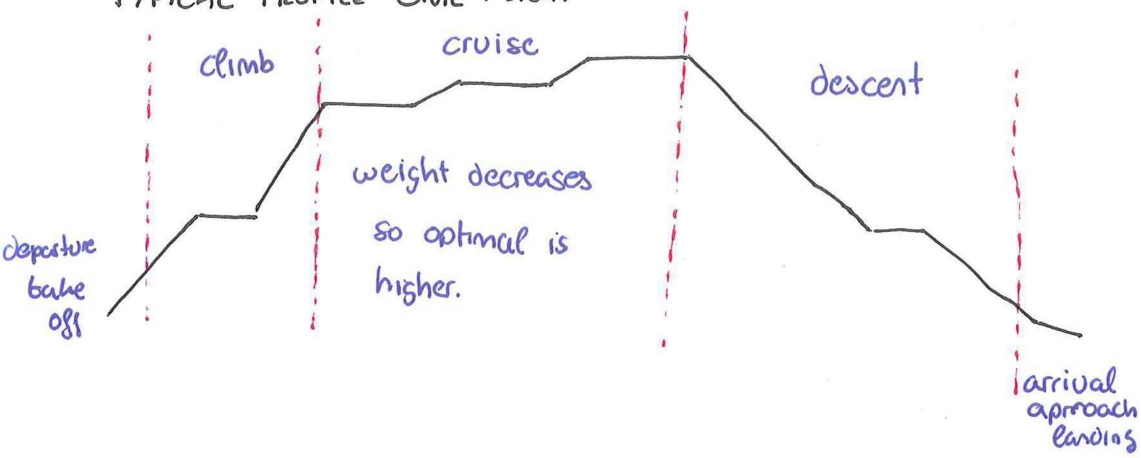
- FMS LNAV & VNAV FLIGHT

Pilot → FMS → AP & AT → FCS → AIRCRAFT

speed, altitude, vert speed, course

Position trajectory / route

Typical Profile Civil Flight

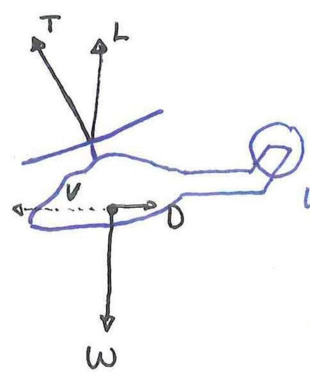


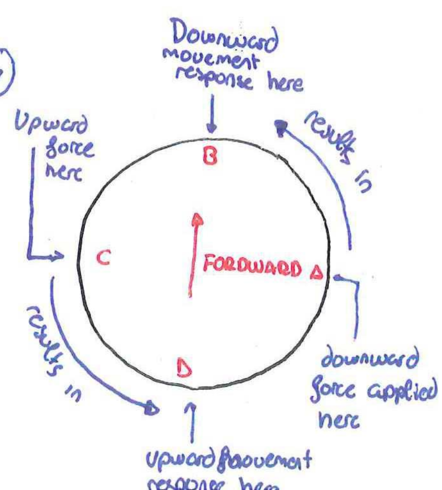
Aircraft Types

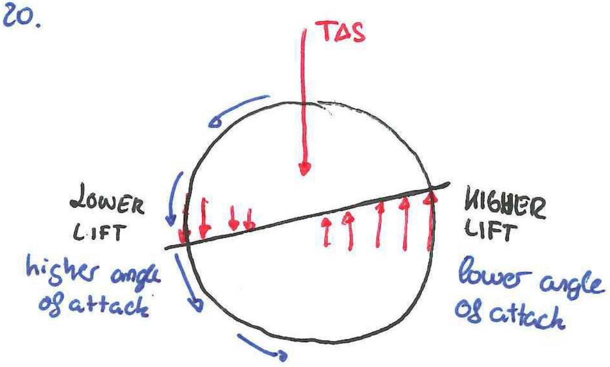
- Ground Effect Aircrafts   Russian KM-Ekranoplan "The caspia sea monster"
  - Ground effect?
    - No vertical speed at ground level
    - As if mirrored aircraft generates lift with its inverted downwash
    - Increases in lift can be up to 40% (less than span of the wing)
    - Reduces induced drag 10% at half the wingspan above the ground.

- Helicopters
  - More noise
  - Less fuel efficient
  - Less environmentally friendly
  - More expensive to buy and operate
  - Less comfortable for passengers
  - Harder to fly.

  - Land and take off anywhere
  - Hover.







Rotor: controls, Thrust, Lift.

Tail rotor: compensates moment created by the rotor.

3. STOVL VTOL Bae Sea Harrier, McDonnel Douglas AV-8B, F-35B

- Vertical landing and take off

F-35A normal
F-35C carrier variant

4. ROCKETS FOR LIFT ON AIRCRAFT JATO rockets B-47B

- Jet Pack
  - Problem with moments

5. UAV - UNMANNED AERIAL VEHICLE (DRONES) GLOBAL HAWK

- One man controlling multiple aircrafts. (goal)
- 4 man controlling one UAV (current situation)
- UAV crash more often.
- R/C toy airplane is a UAV but a UAV is not a toy.

· Surveillance UAV
· 65 380 ft
· 30hr 24min

Issues
- Reliability, responsability
- Less controllers per vehicle
- More acceptance

6. MICRO AERIAL VEHICLE DELFLY AND DELFLY NANO

7. PERSONAL AIR VEHICLE

- PAL-V: moto-helicopter
- AEROMOVIL 3.0

- Real problem is to create a airspace
-

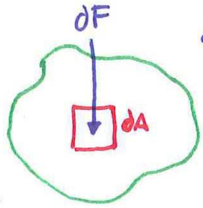
8. HYPERSONIC PLANES ASTOX (Really fast but not eco friendly)

9. GREEN AIRCRAFT

- New engines
- New propulsion methods

AERODYNAMICS

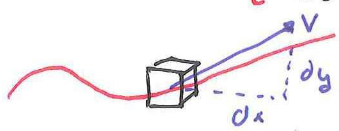
• FUNDAMENTAL QUANTITIES



Pressure in point B

$$p = \lim_{dA \to 0} \left(\frac{dF}{dA} \right) \frac{N}{m^2}$$

- Pressure: normal force per unit area on a surface
- Density: of a substance is its mass per volume $\rho = \lim_{dv \to 0} \left(\frac{dm}{dv} \right) \frac{kg}{m^3}$
- Temperature: measure of the average kinetic energy of the particles in the gas. $\left[KE = \frac{3}{2} kT \right]$
- Velocity: velocity of an infinitesimally small fluid elements as it sweeps through B $\left[V = \frac{d\vec{s}}{dt} \right]$



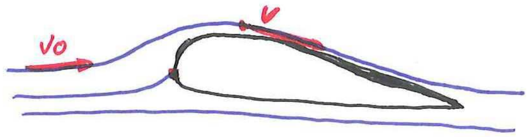
• PERFECT GAS: in which intermolecular forces are negligible

EQUATION $P = \rho \cdot R \cdot T$

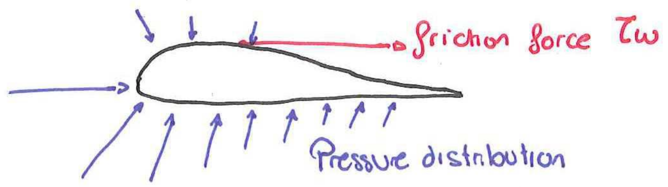
• Non-perfect GAS: $\frac{P}{\rho \cdot R \cdot T} = 1 + \frac{ap}{T} - \frac{bp}{T^3}$

• SPECIFIC VOLUME : volume of 1 kilo of mass $\left[v = \frac{1}{\rho} \right]$ $\left[p\, v = RT \right]$

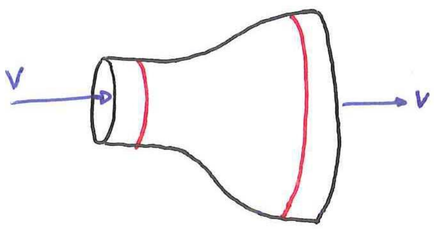
VELOCITY AND STREAMLINES



FORCES



EQUATIONS



CONTINUITY EQUATION

$\rho_1 \cdot A_1 \cdot V_1 = \rho_2 \cdot A_2 \cdot V_2$

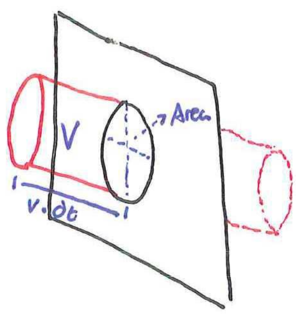
$\rho_1 = \rho_2 \rightarrow$ no change in density

$\boxed{AV = \text{constant}}$ incompressible flow

steady fluid flow

$\dot{m}_{in} = \dot{m}_{out}$

$\dot{m}$ = mass flow per second



$$\dot{m} = \frac{\rho \cdot A V \, dt}{dt} \quad \left[\dot{m} = \rho \cdot AV \right]$$

ρ = densidad

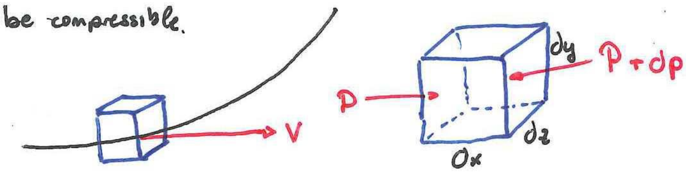
A = Area

V = Velocity

Euler Equation

- Gravity forces are neglected
- Viscosity is neglected
- Steady flow
- Flow may be compressible.

Newton 2nd law : $F = m \cdot a$



3 Forces

- Pressure
- Friction
- Gravity

} we neglect this two for our derivation.

$$F = m \cdot a$$

$$* \; F = P \cdot dy \cdot dz - (P + dP)\, dy \cdot dz$$

$$* \; dp = \frac{dP}{dx} \cdot dx$$

$$* \; F = P \cdot dy \cdot dz - \left(P + \frac{dP}{dx} \cdot dx\right) dy \cdot dz$$

$$* \; F = -\frac{dP}{dx}\, dx \cdot dy \cdot dz$$

$$* \; F = -\frac{dP}{dx} \cdot \text{Volume}$$

$$* \; m = \rho \cdot \text{volume} = \rho \cdot dx \cdot dy \cdot dz$$

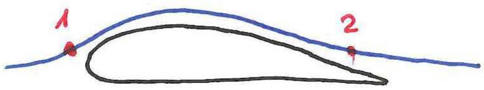
$$* \; a = \frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt} = \frac{dV}{dx} \cdot V$$

$$F \quad = \quad m \quad \cdot \quad a$$

$$-\frac{dP}{dx}(dx \cdot dy \cdot dz) = \rho \cdot (dx \cdot dy \cdot dz) \cdot V \cdot \frac{dV}{dx}$$

$$\boxed{dP = -\rho \cdot V \cdot dV}$$

1. Relation between Force and Momentum
2. Momentum equation

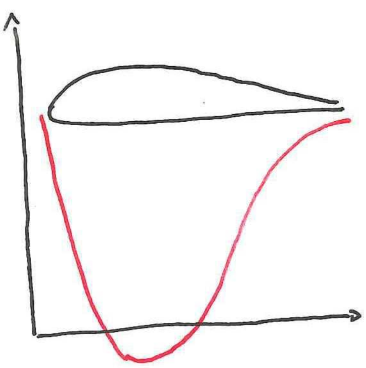


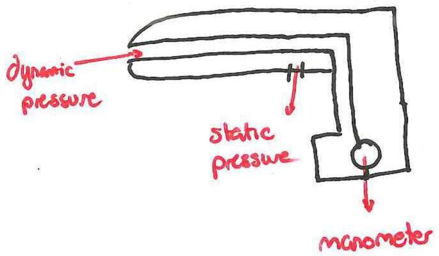
$$P_{TOT} = \underbrace{P}_{\text{static pressure}} + \underbrace{\frac{1}{2}\rho v^2}_{\text{dinamic pressure}}$$

$$\int_{P_1}^{P_2} dp = \int_{V_1}^{V_2} \rho V \cdot dV = 0$$

$$\left[P_2 - P_1 + \rho \cdot \left(\frac{1}{2} V_2^2 - \frac{1}{2} V_1^2\right) = 0\right]$$

$$\left[P_1 + \frac{1}{2}\rho V^2 = \text{constant}\right]$$





Ptot tube

Terms

- inviscid = frictionless
- incompressible flow → P = constant
- compressible → Euler

COMPRESSIBILITY

δq
heat added

e : internal energy of the system.

$$\boxed{de = \delta q + \delta w}$$ FIRST LAWS OF THERMODYNAMICS

e can change

- Heat is added or taken away from the system
- Work is done on, or by, the system.

- OTHER FORM OF FIRST LAW

$w = F \cdot \Delta x$

$w = P \cdot A \cdot \Delta x$

$\delta w = \int_A p \cdot dA \cdot \Delta x$ $\quad$ $\delta w = P \cdot \left[\int_A s \cdot dA \right]$ change of specific volume

$$\boxed{de = \delta q - P \cdot dv}$$

- ENTHALPY

~~heat~~ $h = e + pV$

$dh = de + d(p \cdot v)$

$dh = de + pdv + vdp$

$de = \delta q - pdv$

$$\boxed{dh = \delta q + vdp}$$

- ESPECIFIC HEAT

$$c \equiv \frac{\delta q}{\partial T}$$

2 TYPES OF PROCESS

Heat capacity at

- CONSTANT VOLUME

$$C_V = \frac{\delta q}{\partial T}$$

$\cdot\, de = \delta q - pdv \quad dv = 0$

$de = \delta q$

$de = C_v \cdot dT$

$$\boxed{e = C_v \cdot T}$$

- CONSTANT PRESSURE

$$C_P = \frac{\delta q}{\partial T}$$

$\cdot\, dh = \delta q + vdp \quad dp = 0$

$\delta q = dh$

$dh = c_p \cdot dT$

$$\boxed{h = C_p \cdot T}$$

C_v : how much the energy of a substance takes in or gives of when its temperature changes. Amount of energy (heat) required to rise the Temperature 1° degree when the volume is constant.

SAME

C_p : amount of heat required to increase temperature by 1 degree celsius. when heat is given at a constant pressure.

Equations for a Perfect Gas

- $de = C_v \cdot dT$
- $dh = C_p \cdot dT$
- $e = C_v \cdot T$
- $h = C_p \cdot T$

AIR

$C_v = 720\ J/kg\,K$

$C_p = 1008\ J/(kg/K)$

Isentropic Flow

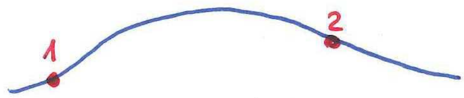
- Adiabatic Process : $\delta q = 0$
- Reversible Process : no frictional or dissipative effect
- Isentropic Process : adiabatic and reversible

Ratio specific heat $\boxed{\frac{C_p}{C_v} = \boxed{\gamma = 1.4}}$

$\delta h = dq + dp \cdot v \qquad \delta q = 0$

$dh = v \cdot dp \qquad dh = C_p \cdot dT$

$C_p \cdot dT = v \cdot dp \qquad v \cdot dp = C_p \cdot dT$



$\frac{-P \cdot dv}{v\,dp} = \frac{C_v}{C_p} \rightarrow \underbrace{\frac{dP}{P} = -\gamma \cdot \frac{dv}{v}}_{\text{start point}} \rightarrow \boxed{\frac{dP}{P} = -}$

start point $\longrightarrow$ INTEGRATE $\longrightarrow$ $\boxed{\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-\gamma}}$

$\boxed{\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}} \qquad \boxed{\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{+\gamma}}$

Energy can neither be created nor destroyed

- Energy equation for frictionless adiabatic flow.

$dh - v\,dp = 0$

$dp = -\rho \cdot V\,dV$

$0 = dh + v \cdot \rho \cdot V \cdot dV \qquad v = \frac{1}{\rho}$ specific volume $\qquad dh + V\,dV = 0 \longrightarrow$ INTEGRATE $\boxed{h_2 + \frac{1}{2}V_2^2 = h_1 + \frac{1}{2}V_1^2}$ constant

$h = C_pT \rightarrow \boxed{C_pT + \frac{1}{2}V^2 = \text{constant}}$

Steady frictionless incompressive

- Continuity equation $A_1 V_1 = A_2 V_2$
- Bernoulli's equation $P_1 + \frac{1}{2}\rho \cdot V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$
- Equation of state $P_1 = \rho_1 \cdot R \cdot T_1 \quad P_2 = \rho_2 \cdot R \cdot T_2$

Steady, isentropic compressible flow

- Continuity equation: $\rho_1 \cdot A_1 \cdot V_1 = \rho_2 \cdot A_2 \cdot V_2$
- Isentropic relations: $\frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$
- Energy equation: $C_p \cdot T_1 + \frac{1}{2}V_1^2 = C_p \cdot T_2 + \frac{1}{2}V_2^2$
- Equation of state: $P_1 = \rho_1 \cdot R \cdot T_1 \quad P_2 = \rho_2 \cdot R \cdot T_2$

SPEED OF SOUND

STATIC OBSERVER

P	$P+dP$
ρ	$\rho+d\rho$
T	$T+dT$
$\leftarrow a$	

moving sound wave with speed a into a stagnant gas.

OBSERVER MOVING WITH SOUND WAVE

P	$P+dP$
ρ	$\rho+d\rho$
T	$T+dT$
$a \rightarrow$	$a+da \rightarrow$

motionless sound wave

$$\rho_1 \cdot A_1 \cdot V_1 = \rho_2 \cdot A_2 \cdot V_2 \longrightarrow \rho \cdot A_1 \cdot a = (\rho + d\rho) A_2 (a + da) \longrightarrow a = -\rho \cdot \frac{da}{d\rho}$$

$$dp = -\rho V dV \longrightarrow da = -\frac{1}{\rho a} dp \qquad a^2 = \frac{dP}{d\rho}$$

$$\boxed{a = \sqrt{\frac{dP}{d\rho}}} \text{ ISENTROPIC}$$

$$\boxed{a = \sqrt{\gamma \cdot R \cdot T}}$$

$$\boxed{M = \frac{V}{a}}$$

- $M < 1$ ⟶ subsonic
- $M = 1$ ⟶ sonic
- M around 1 ⟶ transonic
- $M > 1$ ⟶ supersonic
- $M > 5$ ⟶ hypersonic

▷ EQUATIONS FOR A PERFECT GAS

$$h = e + pv$$

$$h = C_p T \quad \bullet \quad e = C_v T \quad \bullet \quad pv = RT$$

$$C_p T = C_v \cdot T + RT$$

$$\boxed{C_p = \frac{\gamma R}{\gamma - 1}}$$

FLOW FROM A RESERVOIR

$$\boxed{C_p T + \frac{1}{2} V^2 = \text{constant}}$$

$$\frac{T_0}{T_1} = 1 + \frac{V_1^2}{2 C_p T_1} \qquad V_0 = 0$$

$$\boxed{\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} \cdot \frac{V_1^2}{\gamma \cdot R \cdot T_1}}$$

$$\boxed{\frac{P_0}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma - 1}}}$$

$$\boxed{\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{1/\gamma - 1}}$$

$M < 0.3$ incompressible less than 5%

$M > 0.3$ compressible

In a nozzle, the Mach number always equals to 1

$M > 1$ shock waves are formed

SHOCK WAVE

$M_1 > M_2$

$P_1 < P_2$

$V_1 > V_2$

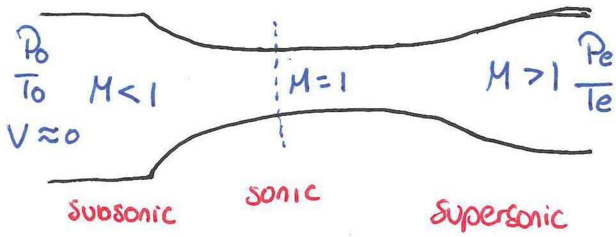
$T_1 < T_2$

$P_{T1} > P_{T2}$

$T_{T1} = T_{T2}$

SUPERSONIC WIND TUNNEL

CONTINUITY AND EULER EQUATION



Supersonic tunnel and rocket engine

$\rho V A = \text{constant}$

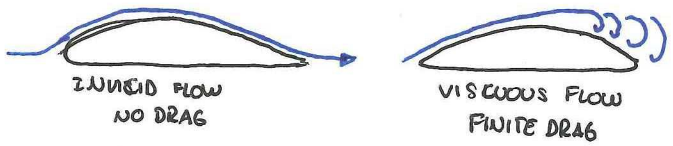
$$\ln\rho + \ln V + \ln A = \ln(\text{constant}) \longrightarrow \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \Rightarrow \boxed{\rho = -\frac{dp}{V dV}}$$

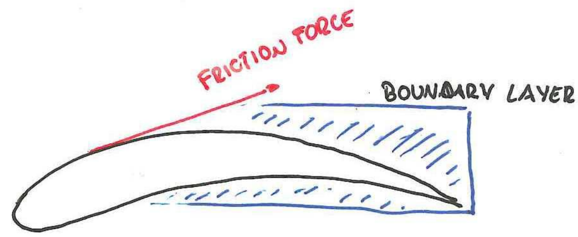
$$\frac{-d\rho V dV}{dp} + \frac{dA}{A} + \frac{dV}{V} = 0 \longrightarrow \frac{d\rho}{dp} = \frac{1}{dp/d\rho} = \frac{1}{a^2}$$

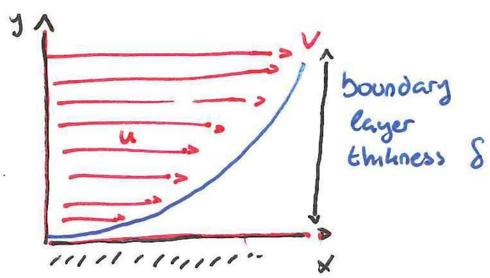
$$\frac{dA}{A} = \frac{V^2 dV}{V \cdot a^2} - \frac{dV}{V} \longrightarrow \frac{dA}{A} = \frac{M^2 dV}{V} - \frac{dV}{V} \qquad \boxed{\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}}$$

- CASE A: subsonic flow $M<1$ $dV >$ $dA <$ (increase V we need to decrease A and viceversa)
- CASE B supersonic flow $M>1$ $dV >$ $dA >$ (increase V we need to increase A
- CASE C Sonic flow $M=1$ $\frac{x}{0}$ no possible so $\boxed{\frac{dA}{A} = 0} \longrightarrow$ THROAT

LAMINAR AND TURBULENT FLOWS







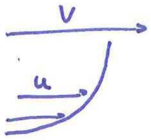
u: local velocity

- Boundary Layer: vicinity of the surface, thin region of retarded flow
- The pressure through the bundary layer in a direction perpendicular to the surface is constant.

27.

SHEAR STRESS

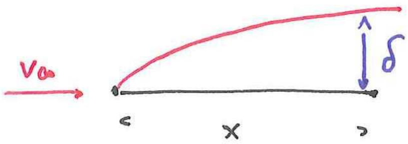
$$\tau_w = \mu \left(\frac{du}{dy}\right)_{y=0}$$



μ = absolute viscosity coefficient or viscosity

$\mu = 1.789 \cdot 10^{-5}$

viscosity: how compact the flow is



$$Re = \frac{\rho_\infty \cdot V_\infty \cdot x}{\mu_\infty}$$

$$Re = \frac{V_\infty \cdot x}{\nu_\infty}$$

Laminar: streamlines are smoth and regular and a fluid element moves smoothly along a streamline.

Turbulent: streamlines brake up and a fluid element moves in a random irregular way.

LAMINAR BOUNDARY

$$\delta = \underbrace{\frac{5.2\, x}{\sqrt{Re_x}}}_{\text{local}} \quad \boxed{\text{FLAT PLATE}}$$

[NO EXACT SOLUTION FOR TURBULENT BOUNDARY LAYERS]

LOCAL FRICTION COEFFICIENT

$$C_f = \underbrace{\frac{1.328}{\sqrt{Re_L}}}_{\text{all the length}}$$

$$\left[C_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho_\infty \cdot V_\infty^2} = \frac{\tau_w}{q_\infty}\right]$$

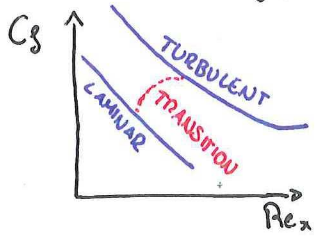
$$C_{f_x} = \frac{0.664}{\sqrt{Re_x}}$$

C_{f_x} and τ_w decrease as $\sqrt{x}$ increases.

$$\left[D_f = \int_0^L \tau_w \cdot dx = \int_0^L C_{f_x} \cdot q_\infty \cdot dx\right] \left[D_f = 0.664 \cdot q_\infty \int_0^L \frac{dx}{\sqrt{Re_x}}\right]$$

TOTAL SKIN FRICTION COEFFICIENT

$$\left[C_f = \frac{D_f}{q_\infty \cdot S}\right]$$



C_f varies as

- $L^{-1/5}$ for turbulent
- $L^{-1/2}$ for laminar flow

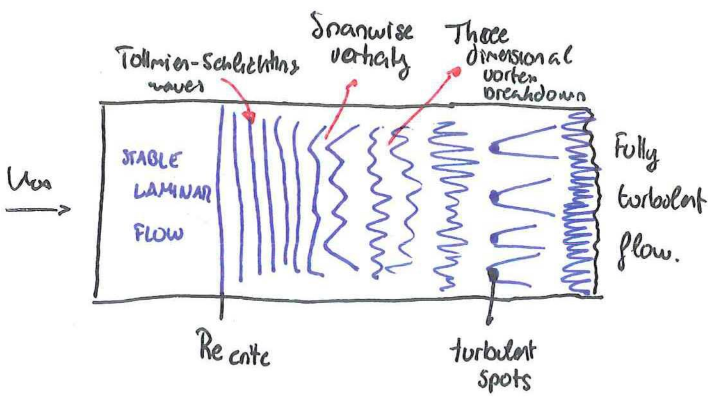
AIRFOILS

$$Re = \frac{V_\infty \cdot c}{\nu_\infty}$$

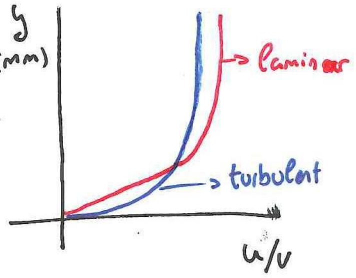
CYLINDERS

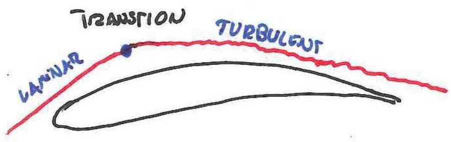
$$Re = \frac{V_\infty \cdot d}{\nu_\infty}$$

$$Re = \frac{\rho_\infty \cdot V_\infty \cdot x}{\mu}$$



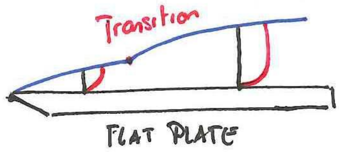
TRANSITION





SKIN FRICTION

$$\tau = \mu \cdot \frac{du}{dy}$$

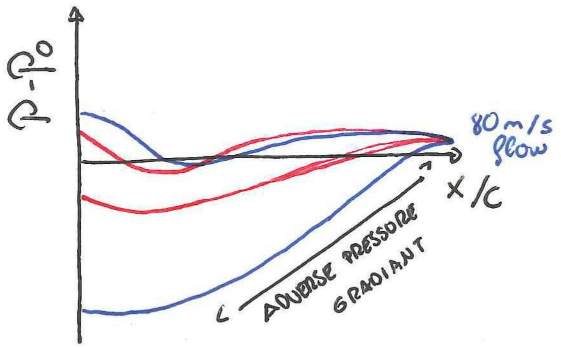


- Critical Reynolds number at which transitions occurs is difficult to find.
- Has to be from experimental data.

[MAJORITY OF FLOWS IS TURBULENT
we have to create laminar flow airfoils]

LAMINAR: boundary layer: thin, low skin friction drag

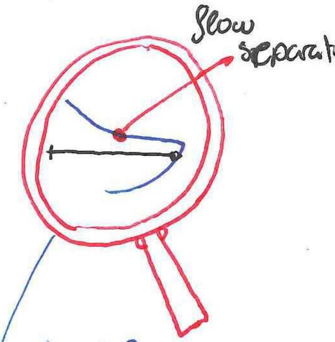
TURBULENT: boundary layer: thick, high skin friction drag.



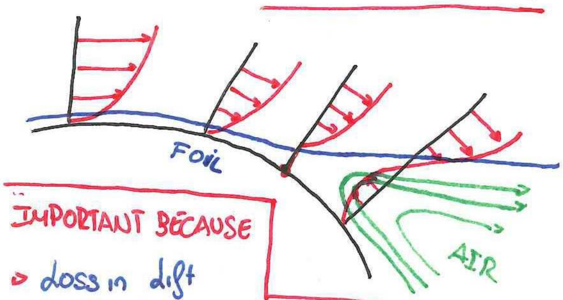
Higher velocity means more pressure in both sides.

- Flip the graph to have upper surface up and lower surface down.

$$\boxed{C_p = \frac{P - P_\infty}{q_\infty}} \qquad \boxed{C_p = 1 - \frac{V_1^2}{V_0^2}}$$



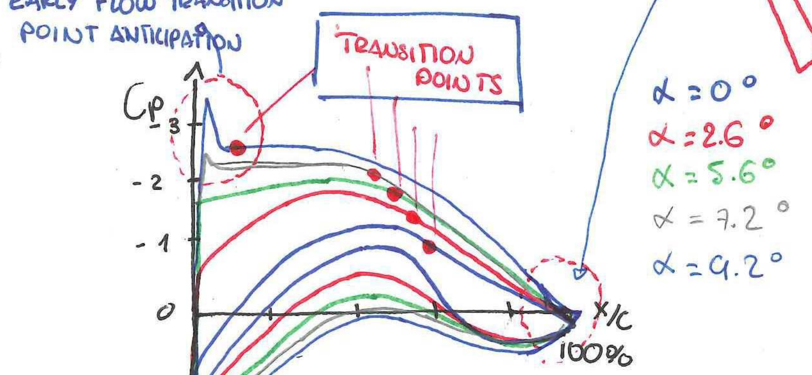
SEPARATION OF FLOW



IMPORTANT BECAUSE

- Loss in lift
- Increase in pressure drag
- Generation of unsteady loads

EARLY FLOW TRANSITION POINT ANTICIPATION



[IN THE STAGNATION POINT $C_p = 1$]

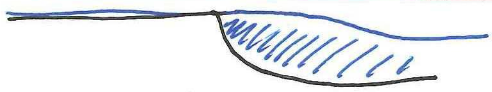
SEPARATION CAN BE

- LAMINAR low Re
- TURBULENT high Re

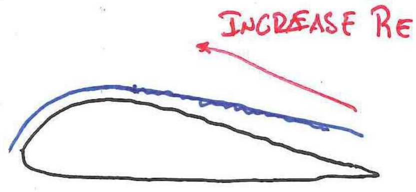
$$Re = \frac{V_\infty \cdot c}{\mu_\infty} \qquad Re = \frac{\text{mechanical forces in the flow}}{\text{viscous forces}}$$

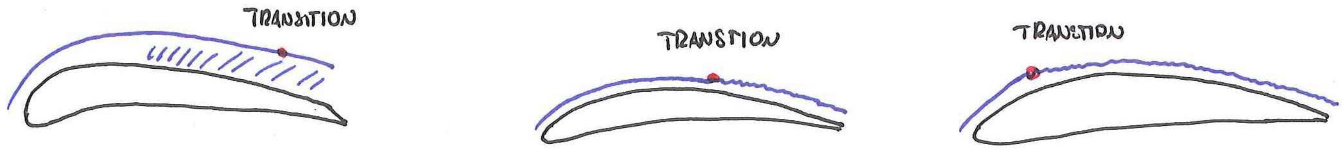
[laminar flow < 500,000]

LAMINAR SEPARATION OVER AN EXPANSION



Low Re number to high Re number →





Re = 300 000
Cd = high

Re = 650 000
Cd : lower

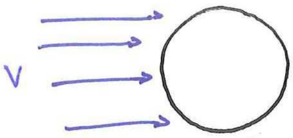
Re = 1000 000
Cd ≈ Cd at 650000

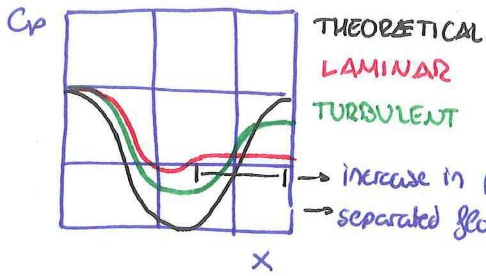
[On the same airfoil, a higher Re means more Cl at a higher angle of attack, and more Cl at a lower Cd.]

Drag and Friction

[Cd = Cd pressure + Cd friction]

Drag to viscous effects = friction drag + pressure drag = profile drag



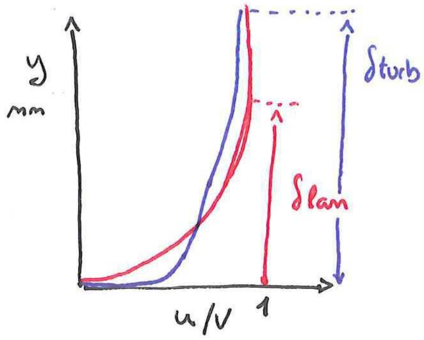


→ increase in pressure = +slope
→ separated flow = more drag

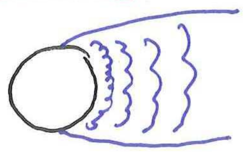
Pressure drag > friction drag

less pressure in the ~~back~~ than in the back, the cylinder is sucked back

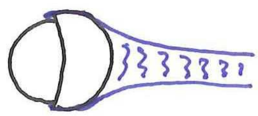
FROM THE GRAPH: Turbulent boundary layer has more flow kinetic energy near the surface also, the flow separation is postponed



Transitions



> DRAG >



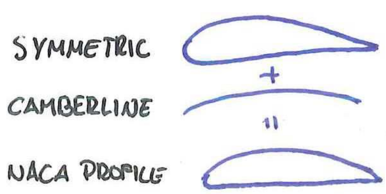
FREE TRANSITIONS

ARTIFICIAL TRANSITION

Airfoils, Lifts from Pressure Distributions and Critical Mach Numbers

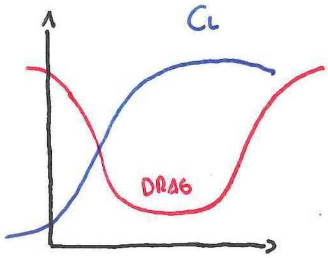
Re: INFLUENCE IN VISCOSITY EFFECT

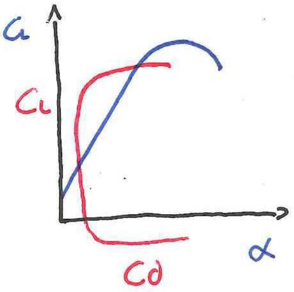
CREATION OF NACA



Increasing Re

- Influence of viscosity decreases
- Transition location moves forward on an airfoil
- Boundary layer thickness and Cf decrease
- Drag goes down (3 wins over 2 for an airfoil)
- Lift slope goes up (less de-cambering of the airfoil)

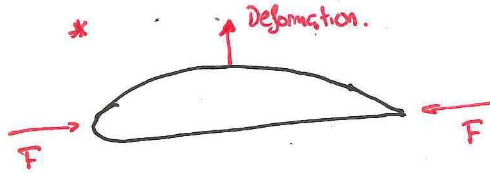




INFLUENCE OF CAMBER

If camber increases, more force is needed to curve the air down which results in more lift

* Deformation.



[The more thickness the lower Cl for the same α and the lower Cl for the same Cd]

↑ THICKNESS INFLUENCE ↑

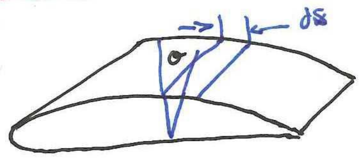
TURBULENT VS LAMINAR AIRFOIL

[For a turbulent airfoil there is a bigger max Cl for the same α

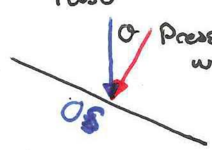
And a higher max Cl for the same Cd.]

CAMBER { The more camber the higher Cl for the same α ; The more camber the higher Cl for the same Cd } The curve is bringed upwards.

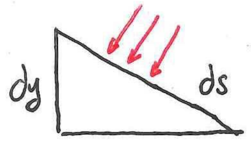
OBTAINING LIFT FROM PRESSURE DISTRIBUTION



[N force is perpendicular to the chord]



$$N = \int_{LE}^{TE} P_l \cos\theta \, ds - \int_{LE}^{TE} P_u \cos\theta \, ds$$



$dx = d\cos\theta$

$\cos\theta \, ds = dx$

$$N = \int_{x=0}^{x=c} P_l \, dx - \int_{x=0}^{x=c} P_u \, dx$$

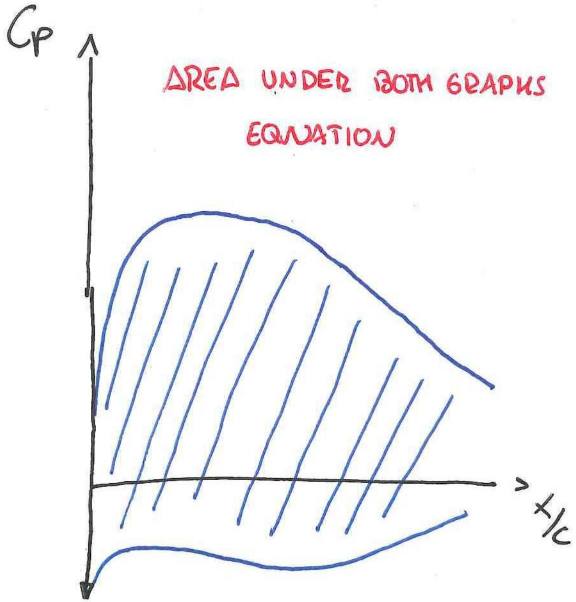
Normal force coefficient

$$C_n = \frac{N}{\frac{1}{2}\rho V_\infty^2 \cdot c} = \frac{N}{q_\infty \cdot c}$$

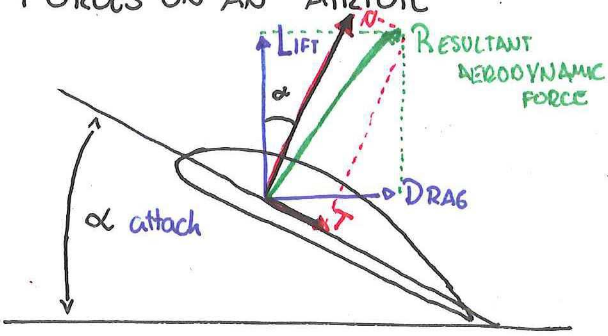
$$C_n = \int_0^1 \frac{P_l}{q_\infty} d\left(\frac{x}{c}\right) - \int_0^1 \frac{P_u}{q_\infty} d\left(\frac{x}{c}\right)$$

$$C_n = \int_0^1 \frac{P_l - P_\infty}{q_\infty} d\left(\frac{x}{c}\right) - \int_0^1 \frac{P_u - P_\infty}{q_\infty} d\left(\frac{x}{c}\right)$$

$$\boxed{C_n = \int_0^1 (C_{p_l} - C_{p_u}) \, d\left(\frac{x}{c}\right)}$$



FORCES ON AN AIRFOIL



N = Normal Force
T = Tangential Force

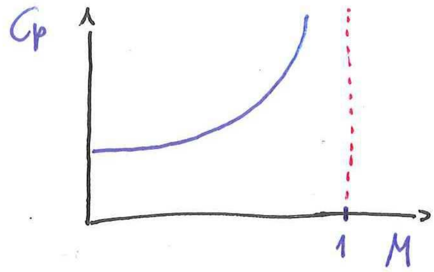
$$\boxed{\begin{aligned} L &= N\cdot\cos\alpha - T\cdot\sin\alpha \\ D &= N\cdot\sin\alpha + T\cos\alpha \end{aligned}}$$

$$\boxed{\begin{aligned} C_l &= C_n\cdot\cos\alpha - C_T\cdot\sin\alpha \\ C_D &= C_n\cdot\sin\alpha + C_T\cdot\cos\alpha \end{aligned}}$$

[$C_l \approx C_n$ for small angless of attack $\alpha < 5$]

COMPRESSIBILITY CORRECTIONS

$$\boxed{C_p = \frac{C_{p,0}}{\sqrt{1-M_\infty^2}}}$$ new C_p for airfoils valid for $M < 0.7$



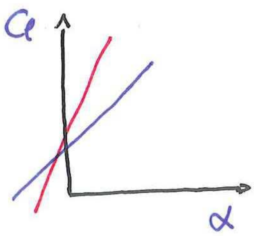
C_p: pressure coefficient

FOR C_l

$$C_l \approx \int_0^1 \frac{(C_{pl} - C_{pu})}{\sqrt{1-M_\infty^2}}\, d\left(\frac{x}{c}\right) = \frac{1}{\sqrt{1-M_\infty^2}}\int_0^1 (C_{pl} - C_{pu})\, d\left(\frac{x}{c}\right)$$

$$\boxed{C_l = \frac{C_{l,0}}{\sqrt{1-M_\infty^2}}}$$

$C_{l,0}$ = incompressible c_l



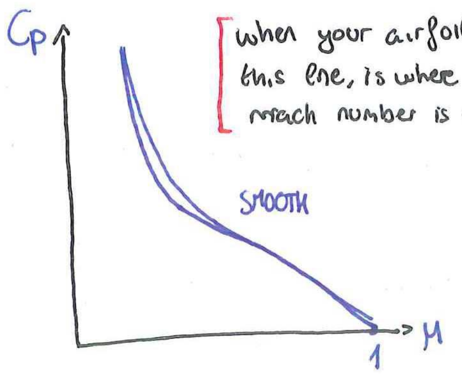
lower mach
Higher mach

CRITICAL MACH NUMBER

lowest Mach number at which the airflow over some point in the aircraft reaches Mach 1

[Thin airfoils have a higher critical mach]

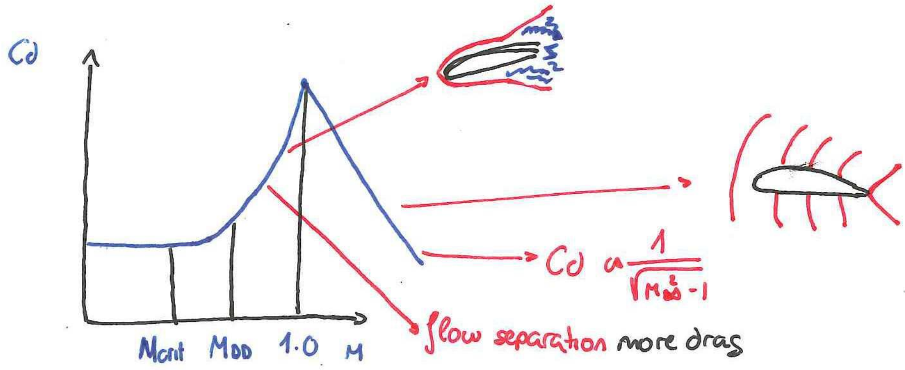
$$C_p = \frac{2}{\gamma\cdot M_\infty^2}\left[\left(\frac{1+\frac{1}{2}(\gamma-1)M_\infty^2}{1+\frac{1}{2}(\gamma-1)M^2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

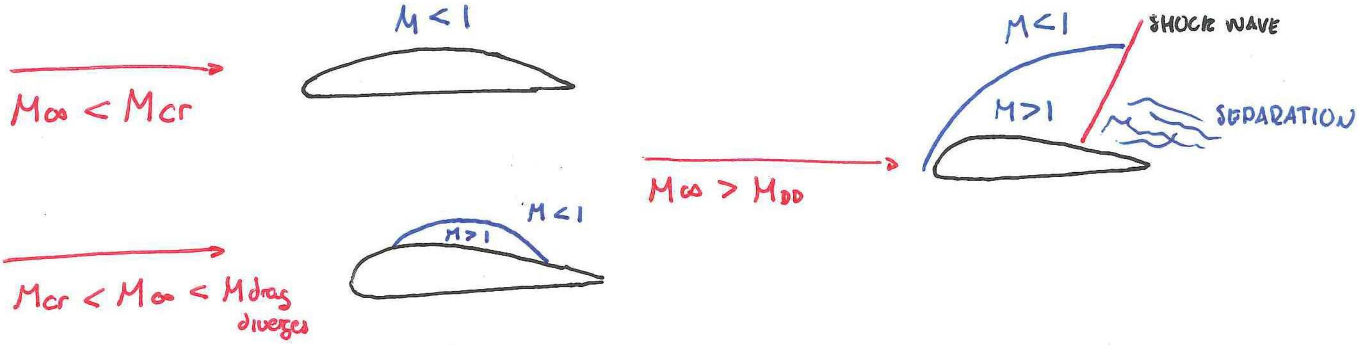


[when your airfoil crosses this line, is where the mach number is critical.]

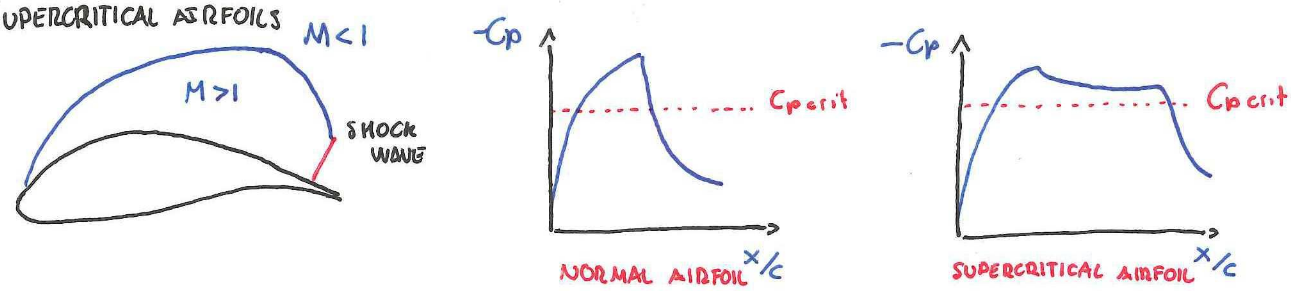
Drag divergence after crossing critical Mach number

Called sound barrier because is hard to cross without enough thrust

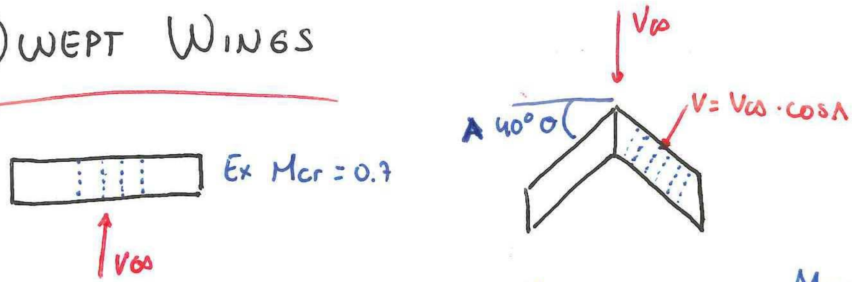




Supercritical airfoils



Swept Wings



the same airfoil is receiving a different speed because of the swept angle.

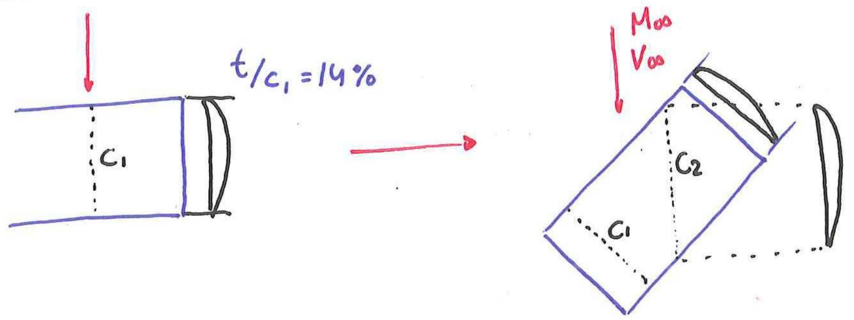
$$M_{cr} \text{ for swept wing} = \frac{0.7}{\cos\Lambda} = 0.91$$

$$\boxed{M_{cr} \text{ for swept wing} = \frac{M_{crit}}{\cos\Lambda}}$$

By sweeping the wings of a subsonic aircraft, the drag divergence is delayed to higher Mach numbers.

[Lift-Drag ratio goes down]

ANOTHER WAY OF LOOKING AT SWEPT WINGS

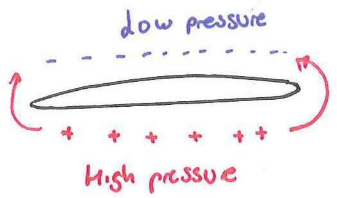


$t/c_2 = 10\%$

the second airfoil is thinner which means that has a higher M_{crit} and M_{DD}

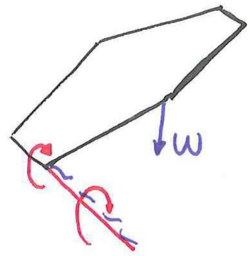
FINITE WINGS

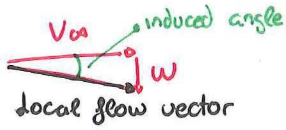
Flow around the wing tip generates tip vortices

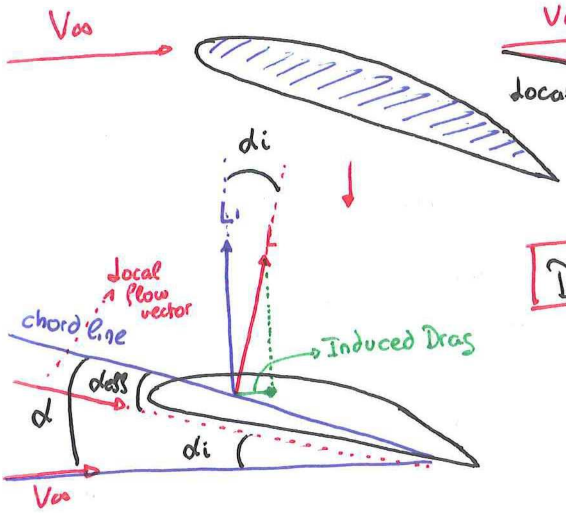


A vortex causes induced velocities

W: the trailing tip vortex causes downwash







WING TIP VORTICES CAUSE

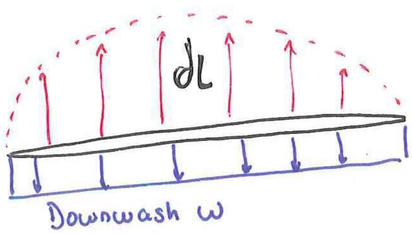
- EFFECT ON LIFT
- EFFECT ON DRAG

$$\boxed{D_i = L \cdot \sin \alpha_i}$$

↳ α is small $\quad \alpha_i \approx \sin \alpha_i \longrightarrow D_i = L \cdot \alpha_i$

$$C_D = C_L \cdot \alpha_i$$

EXAMPLE: ELLIPTICAL LIFT DISTRIBUTION: it creates a constant downwash and a constant induced angle of attack.



$$\alpha_i = \frac{C_L}{\pi \cdot A} \quad \left[C_{Di} = C_L \alpha_i\right] \quad \left[C_{Di} = \frac{C_L^2}{\pi A}\right] \quad A = \frac{b^2}{S}$$

ELLIPTIC LOADING

- ELLIPTICALLY SHAPED WINGS
- Introduce twist in wings with straight Trailing Edge and LE

SPAN EFFICIENCY FACTOR : e_1

$$C_{Di} = \frac{C_L^2}{\pi A e_1}$$

Elliptical loading ⟶ $e = 1$ minimum induced drag

Non elliptical loading ⟶ $e < 1$ higher induced drag

TOTAL DRAG OF THE WING

$$C_D = \underbrace{C_d}_{\text{Profile drag}} + \frac{C_L^2}{\underbrace{\pi \cdot A \cdot e_1}_{\text{Induced drag}}}$$

TOTAL DRAG FOR AN AIRCRAFT

$$C_D = \underbrace{C_{D_0}}_{C_D \text{ at } C_L = 0} + \frac{C_L^2}{\pi \cdot A \cdot \underbrace{e}_{\text{oswald factor}}}$$

[High aspect ratio ▷ low induced drag

low aspect ratio ▷ High induced drag] How aspect ratio affects induced drag.

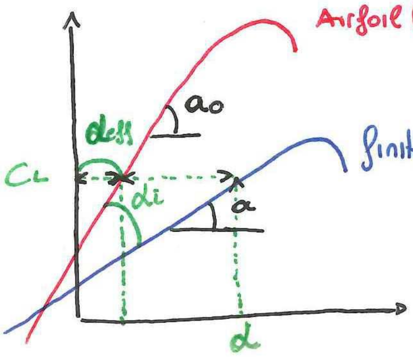
LIFT CURVE SLOPE

α eff of a finite wing is to reduce the wing lift curve slope

$$\boxed{\alpha_{eff} = \alpha - \alpha_i}$$

The induced angle of attack reduces the local effective angle of attack.

RADIANS	DEGREES
$\alpha_i = \dfrac{C_L}{\pi \cdot A \cdot e_1}$	$\alpha_i = \dfrac{C_L \cdot 57.3}{\pi \cdot A \cdot e_1}$



Airfoil (infinite wing) effective angle of attack $a_0 = \dfrac{dC_l}{d\alpha}$

finite wing (geometric angle of attack) $a = \dfrac{dC_L}{d\alpha}$

[Thin airfoil theory $a_0 = 2\pi$]

IMPORTANT EQUATIONS

$$\boxed{a = \frac{dC_L}{d\alpha} = \frac{a_0}{1 + \frac{a_0}{\pi \cdot A \cdot e_1}}}$$

$$\boxed{a_0 = \frac{dC_l}{d\alpha}}$$

$$\boxed{\frac{dC_L}{d\alpha} = \frac{a_0}{1 + \frac{a_0 \cdot 57.3}{\pi \cdot A \cdot e_1}}}$$

[a_0 per degree]

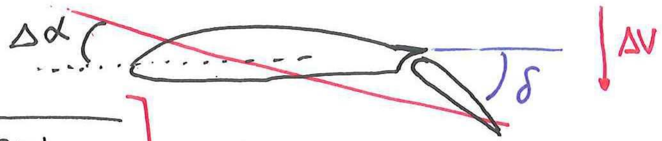
TOTAL DRAG

$$\boxed{C_D = C_d + \frac{C_L^2}{\pi A e_1}}$$

$$\boxed{C_d = \underbrace{C_{df}}_{\text{friction}} + \underbrace{C_{d\,press}}_{\text{pressure}} + \underbrace{C_{dw}}_{\text{shock waves}}}$$

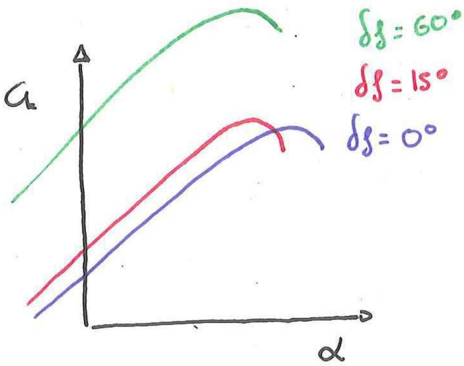
FLAPS : Flaps produce an increase in effective angle of attack and camber

$$W = L = C_L \cdot \frac{1}{2} \rho V_\infty^2 \cdot S$$



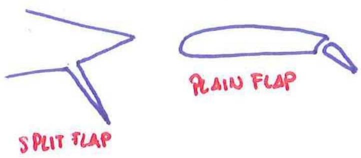
$$\left[V_\infty = \sqrt{\frac{2W}{\rho_\infty \cdot S \cdot C_L}} \right] \quad \left[V_{STALL} = \sqrt{\frac{2W}{\rho_\infty \cdot S \cdot C_{L\,max}}} \right]$$

The landing speed is decreased when the maximum lift coefficient is increased



SLATS : in the front of the wing.

FLAPS



FLIGHT MECHANICS

INTRO

EQUATION OF MOTION

$$\vec{F} = m \cdot \vec{a}$$

- aerodynamic forces
- weight forces
- propulsion forces
- atmosphere
- pilot

MOTION OF AIRCRAFT RELATIVE TO EARTH

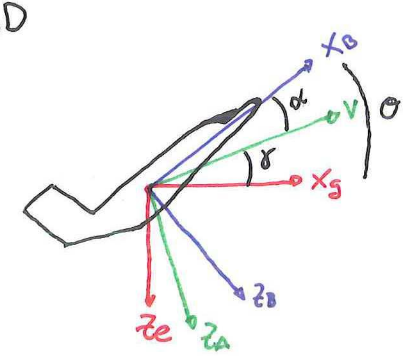
$$a = \frac{V^2}{R} \qquad R = R_e + h$$

$$R = 6.371 + 10 = 6381$$

$$a = \frac{250^2}{6381 \cdot 10^3} = 0.097 \, m/s^2$$ neglectable

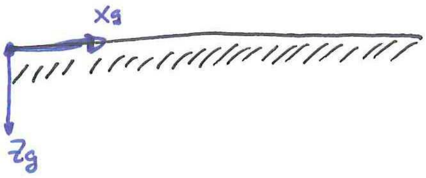
ASSUMPTIONS: earth is not rotation
earth is flat

FBD



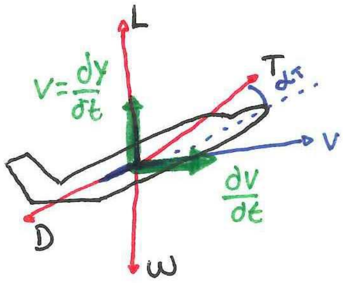
γ : Flight path angle
α : angle of attack
θ = pitch attitude
E = moving earth
G = ground (inertial)
B = body
A = Air Path

Fly path angle and speed gives me the position of the airplane



Forcing acting on airplane

$F = m \cdot a \qquad y = \gamma$



W: weight
L = lift - perpendicular about airspeed
D = Parallel to airspeed.
T = aligned
α_T = angle of attack thrust $(\alpha_T = \alpha + \underbrace{i}_{\text{constant}})$

$$\left[a = \frac{V^2}{R} \quad \omega R = V \quad \omega = \frac{d\gamma}{dt} \right] \quad V = \frac{d\gamma}{dt} R \quad a = \frac{V \cdot V}{R}$$

$$\boxed{a = V \cdot \frac{d\gamma}{dt}}$$

Derivations of equations of motion

$$\left[\Sigma F_{/\!/ V}: \frac{W}{g} \cdot \frac{dV}{dt} = T\cos\alpha_T - D - W\sin\gamma \right] \quad \Sigma F = m \cdot a$$

$$\left[\Sigma F_{\perp V}: \frac{W}{g} \cdot V \cdot \frac{d\gamma}{dt} = L - W \cdot \cos\gamma + T\sin\alpha_T \right] \quad \Sigma F = m \cdot a$$

VARIABLES:

INDEPENDENT: t STATE γ V OTHER $T\ L\ W\ D\ \alpha_T$ we want to express the behaviour as V and γ and t.

Total Drag Polar

$$C_D = \underbrace{C_{D0}}_{\text{0 LIFT D}} + \underbrace{\frac{C_L^2}{\pi \cdot A\,\phi}}_{\text{INDUCED DRAG}}$$

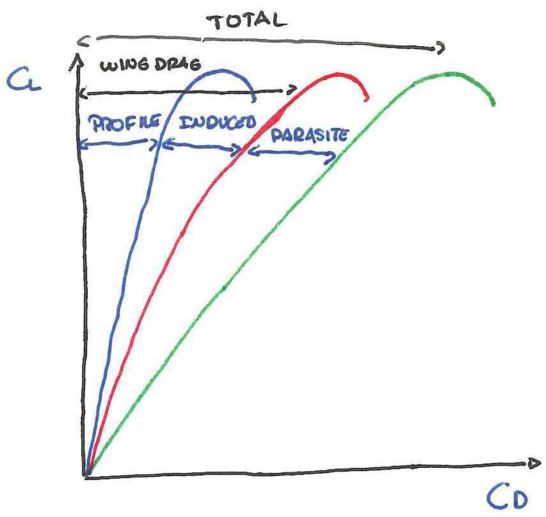
$A = \frac{b^2}{S}$: aspect ratio

ϕ : Span efficiency → e oswald efficiency factor

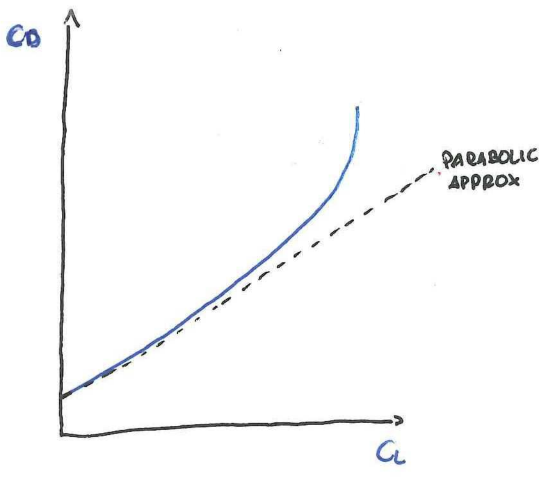
Parabolic equation

$y = ax^2 + b$

$y = ax^2 + bx + c$



PROFILE
WING
AIRCRAFT



INTEGRATION OF DRAG VS SPEED

$$L = W = C_L \tfrac{1}{2} \rho V^2 \cdot S \qquad C_L = \frac{W^2}{S \rho V^2}$$

$$D = C_D \cdot \tfrac{1}{2} \rho V^2 S$$

$$D = \left(C_{D0} + \frac{C_L^2}{\pi A e}\right) \tfrac{1}{2} \rho V^2 S$$

$$D = C_{D0} \cdot \tfrac{1}{2} \cancel{\rho V^2} \cdot S + \frac{W^2}{\cancel{S^2}} \cdot \frac{4}{e\cancel{\rho^2}} \cdot \frac{1}{V^2} \cdot \frac{1}{\pi \cdot \frac{b^2}{S}} \cdot \cancel{\tfrac{1}{2}} \cdot \cancel{\rho} \cdot \cancel{V^2} \cdot \cancel{S^2}$$

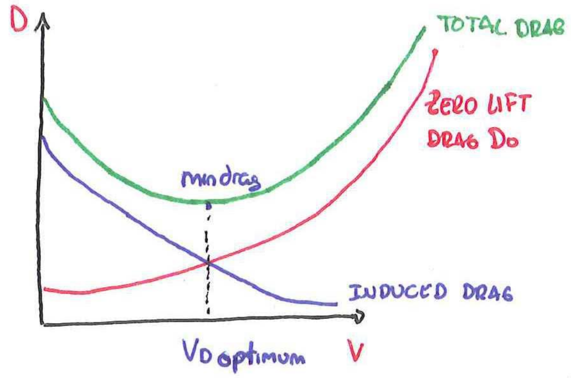
$$D = C_{D0} \cdot \tfrac{1}{2} \rho S \cdot V^2 + \frac{W^2}{b^2} \cdot \frac{2}{\rho} \cdot \frac{1}{\pi e} \cdot \frac{1}{V^2}$$

$$D = k_1 V^2 + k_2 \cdot \frac{1}{V^2} \qquad D = k_1 \cdot V^2 + \frac{k_2}{V^2}$$

$$\boxed{C_D = C_{D0} + k_1 \cdot C_L + k_2 \cdot C_L^2}$$

If we are looking for a fast vehicle we have to look at $C_{D0} \cdot \frac{1}{2} \cdot \rho \cdot S$

If we ae looking for low speeds we have to look at $\frac{W^2}{b^2} \cdot \frac{2}{\rho} \cdot \frac{1}{\pi e}$

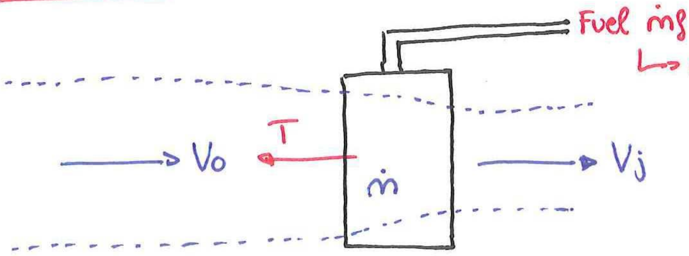


$$\left.\begin{aligned} D &= C_D \tfrac{1}{2} \cdot \rho V^2 \cdot S \\ D &= k_1 \cdot V^2 + k_2 \cdot \frac{1}{V^2} \end{aligned}\right\} \; C_D \tfrac{1}{2} \rho V^2 \cdot S = k_1 \cdot V^2 + \frac{k_2}{V^2}$$

$$C_D = \frac{C_{D0} \cdot \frac{1}{2} \rho V^2 \cdot S}{\frac{1}{2} \rho V^2 \cdot S} + \frac{\frac{2W^2}{b^2 \cdot \rho \cdot \pi \cdot e} \cdot \frac{1}{V_2}}{\frac{1}{2} \rho V^2 \cdot S}$$

$$C_D = C_{D0} + \frac{4W^2}{\rho^2 \pi \cdot b^2 \cdot e \cdot S \cdot V^4} \cdot \frac{S}{S} = C_{D0} + \frac{C_L^2}{\pi A e} = \boxed{C_{D0} + k_2 \cdot C_L^2}$$

PROPULSION

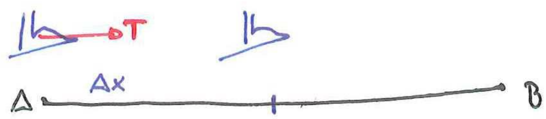


we put mass in the engine and accelerate it

$$T = (\dot m + \dot m_f) V_j - \dot m V_0$$

$$\boxed{T = \dot m (V_j - V_0)}$$

TOTAL EFFICIENCY



$$\left[P_a = \frac{W}{\Delta t} = T \cdot \frac{\Delta x}{\Delta t} = T \cdot V\right]$$

$$\left.\begin{aligned} W &= T \cdot \Delta x \\ P_a &= T \cdot V \\ Q &= \dot m_f \cdot H \end{aligned}\right\} \; n_t = \frac{P_a}{Q}$$

H: amount of energy per unit fuel

TOTAL EFFICIENCY $\frac{P_{available}}{Q}$ 100% idealy

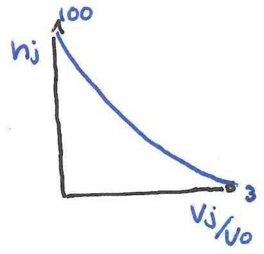
Jet Power: $\frac{1}{2} \dot m V_j^2 - \frac{1}{2} \dot m V_0^2$

$$n_t = \frac{P_a}{Q} = \frac{P_a}{Q} \cdot \frac{P_j}{P_j}$$

$\rightarrow \frac{P_j}{Q} = n_{th}$ thermo power

$\rightarrow \frac{P_a}{P_j}$ how well the increase in energy results in motion.

$$\left[n_j = \frac{P_a}{P_j} = \frac{T \cdot V}{\frac{1}{2}\dot{m}V_j^2 - \frac{1}{2}\dot{m}V_0^2} = \frac{\dot{m}(V_j - V_0)V}{\frac{1}{2}\dot{m}(V_j^2 - V_0^2)} = \frac{2V}{(V_j + V_0)} = \frac{2}{1 + \frac{V_j}{V_0}} \right]$$



Velocity vs efficiency

$V = 100\,m/s$ $T = 100\,N$ $\dot{m} = 1\,kg/sec^2$	$V = 200\,m/s$ $T = 100\,N$ $\dot{m} = 1\,kg/sec^2$
$T = \dot{m}(V_j - V_0)$	$T = \dot{m}(V_j - V_0)$
$V_j = 200\,m/s$	$V_j = 300\,m/s$
$n_j = \frac{2}{1 + \frac{200}{100}} = 66\%$	$n_j = \frac{2}{1 + \frac{300}{200}} = 80\%$

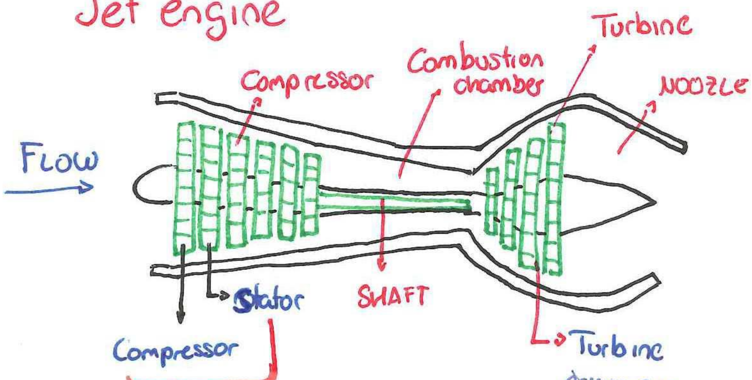
The higher the speed the higher efficiency.

Propeller: low speed
Jet: fast speed
} same propulsive efficiency.

$$F = C_p \cdot P_{br} = \frac{C_p}{\eta_{JET}} \cdot P_a =$$

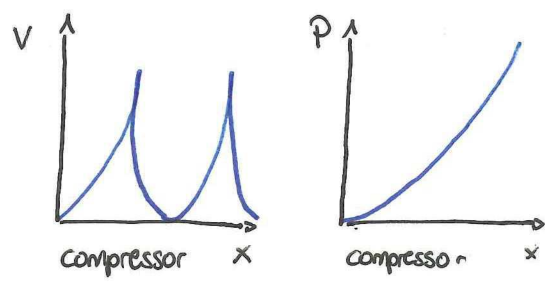
Propulsion Systems

Jet engine

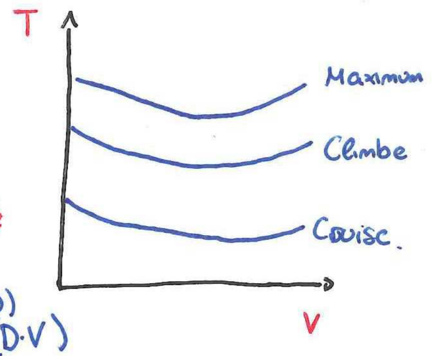


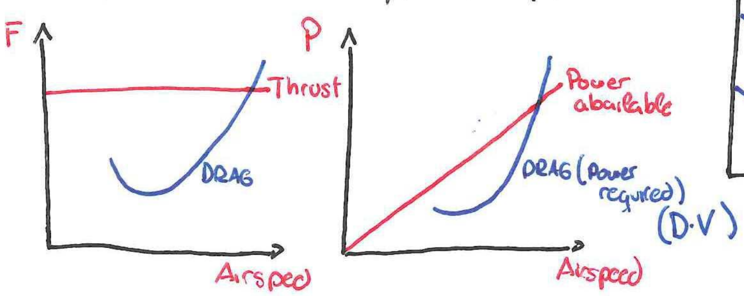
the velocities increases are back to 0 because of the stator. This energy is transformed in Pressure.

CC: 30 atm
1500 k

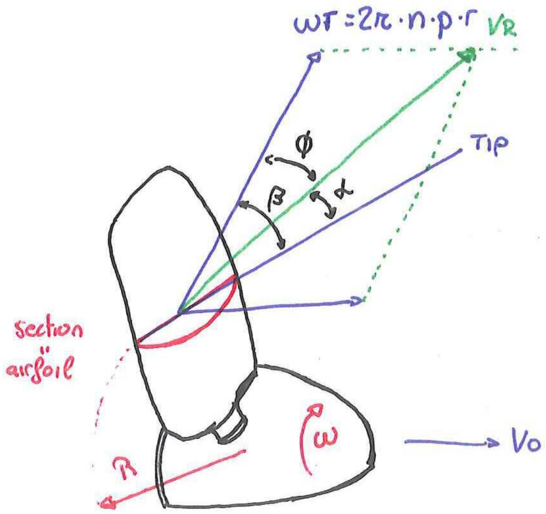


Jet engine thrust: $T = \dot{m} \cdot (V_j - V_0)$
$= \uparrow_+ = \uparrow_+$





PROPELLER

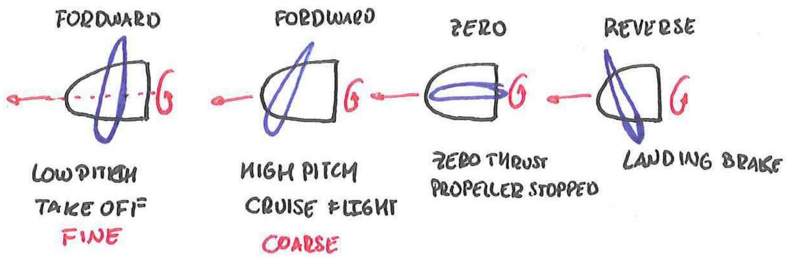


IN THE BLADES, THE AIRFOIL

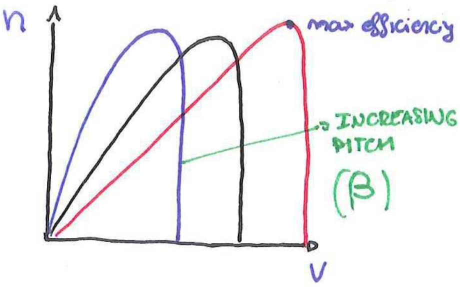
Lift → Thrust

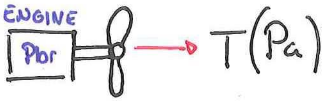
Drag → Torque

PITCH GEOMETRY



VARIABLE PITCH EFFICIENCY





$T(P_a)$ $\quad P_a = \eta \cdot P_{br}$

P_{br} = Power by engine

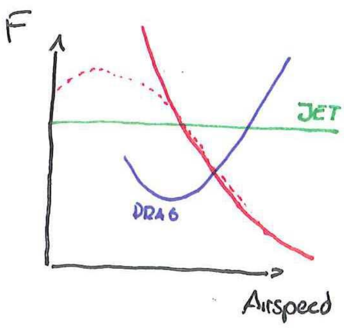
P_a = Power abailable

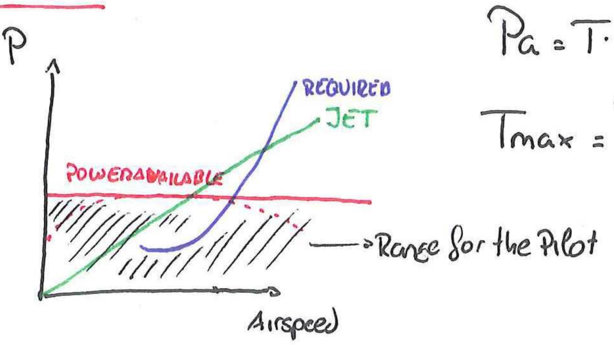
η = constant. $\quad P_{br}$ = constant

P_a = constant.

$$\left[\eta_j = \frac{P_a}{P_{br}}\right]$$

IDEAL PROPELLER PERFORMANCE





$$P_a = T \cdot V$$

$$T_{max} = \frac{P_{a\,max}}{V} = \frac{\text{constant}}{\propto \text{function.}}$$

HORIZONTAL FLIGHT PERFORMANCE

Steady: Flight in which the forces and moments acting on an aircraft do not vary on time, magnitude nor direction. $dV/dt = 0$

Horizontal: Aircraft remains at a constant altitude $\gamma = 0$

Symmetric: Flight in which both the angles of slideslip is zero and the plane of symmetry of the aircraft is perpendicular to the earth. $\beta = 0$ not turning

EQUATIONS HORIZONTAL SYMMETRIC FLIGHT

$$\Sigma F_{/\!/ V}: \cancel{\frac{W}{g}\frac{dV}{dt}}^{\to 0 \text{ STEADY}} = T \cdot \cancel{\cos\alpha_T}^{\to 1} - D - W\cancel{\sin\gamma}^{\to 0}$$

$$\Sigma F_{\perp V}: \cancel{\frac{W}{g}\frac{dV}{dt}}^{\to 0} V = L - W\cancel{\cos\gamma}^{\to 1} + T\cancel{\sin\alpha_T}^{\to 0}$$

$$\boxed{T = D \quad L = W}$$

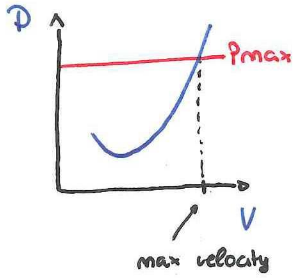
40.

SAINT LOUIS

DATA: $C_D = 0.0686 - 0.0880 \cdot C_L + 0.169 \cdot C_L^2$

$C_{L max} = 1.24$ $P_a = 175.24\ kW$

Surface: $29.7\ m^2$ $W_{max}: 22.8\ kN$ $W_{min}: 10.7\ kN$



Data in P – V

$$V = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}}$$

USE RANDOM VALUES

$C_L = 1$ $V = 24.3\ m/s$

Calculate C_D with formula

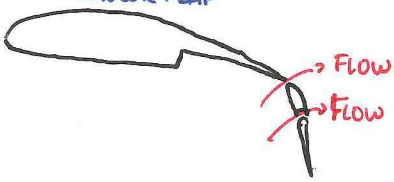
Calculate D

Calculate P with $[F \cdot V]$

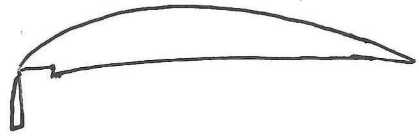
High Lift Devices

PLAIN FLAP

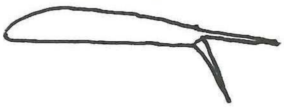
DOUBLE SLOTTED FOWLER FLAP



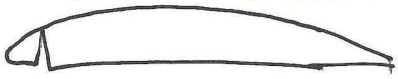
KRUEGER FLAP



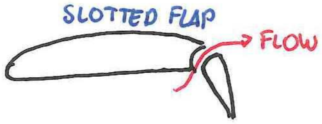
SPLIT FLAP



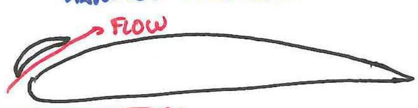
LEADING EDGE DROOP



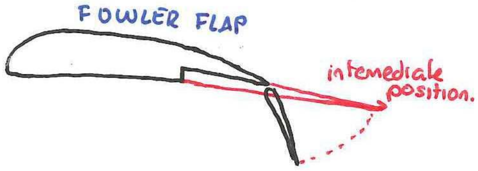
SLOTTED FLAP



HANDLEY-PAGE SLOT



FOWLER FLAP



$$\boxed{V_{opt} = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_{L opt}}} \qquad C_{L opt} = \sqrt{\frac{C_{D0}}{k_2}}}$$

$$P_{a max} = P_{required}$$
$$= D \cdot V$$
$$= C_D \cdot \frac{1}{2} \rho V^3 \cdot S$$
$$= C_D \cdot \frac{1}{2} \rho \frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L} \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}} \cdot S$$
$$= \frac{C_D}{C_L} \cdot W \cdot \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}}$$
$$= \sqrt{\frac{W^3}{S} \cdot \frac{2}{\rho} \cdot \frac{C_D^2}{C_L^3}}$$

$$P_{a max} = \sqrt{\frac{W^3}{S} \cdot \frac{2}{\rho} \cdot \frac{C_{D0} + k_1 \cdot C_L + k_2 \cdot C_L^2}{C_L^3}}$$

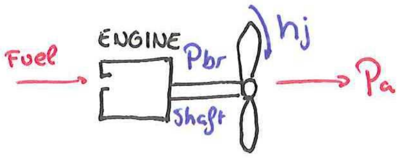
[C_L is the only constant]

PROPELLER MINIMUM SPEED

$$\boxed{V_{min} = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_{L max}}}}$$

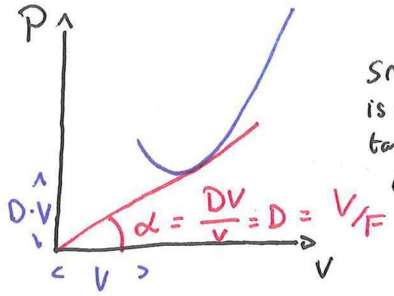
Range

Maximum or optimum $= \frac{V}{F} = \frac{m/s}{kg/sec} = \boxed{\frac{m}{kg}}$ maximize specific range

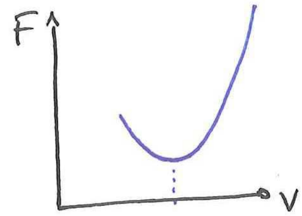


$$F \hat{=} C_p \cdot P_{br} = C_p/_h \cdot P_a = \frac{C_p}{h} \cdot D \cdot V$$

$$\frac{V}{F} = \frac{h}{C_p} \cdot \frac{V}{D \cdot V} = \frac{1}{D}$$ Drag is minimum we will have max $\frac{V}{F}$



smallest angle
is when drag is minimum
tangent to the graph from
(0,0)



min drag corresponds to the min point of graph F-V

Jet

Jet minimum airspeed

$$V_{min} = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_{L\,max}}}$$

Jet Maximum airspeed

Find max C_L] max airspeed occurs when T_{max}

$T_{max} = D \qquad T_{max} = D \cdot \frac{L}{L} = \frac{C_D}{C_L} W$

$$T_{max} = \frac{C_{D0} + k_1 \cdot C_L + k_2 C_L^2}{C_L} W$$

$$\frac{C_L \cdot T_{max}}{W} = C_{D0} + k_1 \cdot C_L + k_2 C_L^2$$

$$\left[k_2 C_L^2 + \left(k_1 - \frac{T_{max}}{W}\right) C_L + C_{D0} = 0 \right]$$

Jet max endurance

$$\boxed{C_{L\,opt} = \sqrt{\frac{C_{D0}}{k_2}}}$$

Range Max

$$[F = C_T \cdot T]$$

$$\left(\frac{V}{F}\right) = \frac{V}{C_T \cdot D}$$

to ~~maximize~~
maximize V/F
we need to maximize
$\frac{V}{C_T \cdot D}$ and $\frac{V}{D}$ max $= \frac{D}{V}$ min

$$\left(\frac{D}{V}\right)_{min} = \frac{D}{L} \cdot L \cdot \frac{1}{V} \qquad \frac{L}{L}$$

$$\left(\frac{D}{V}\right)_{min} = \frac{C_D}{C_L} \cdot W \cdot \frac{1}{\sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}}}$$

$$\frac{D}{V_{min}} = \left(\frac{C_L}{C_D^2}\right)_{max}$$ ⟶ max when derivate $= 0$

$$\frac{d}{dC_L}\left(\frac{C_L}{C_D^2}\right) = 0$$

$$\frac{C_D^2 \cdot 1 - C_L \cdot 2 C_D \cdot \frac{dC_D}{dC_L}}{C_D^4}$$

$$\frac{1\,C_D}{2\,C_L} \quad \left[\frac{d}{dC_L}\left(C_{D0} + k_1 C_L + k_2 C_L^2\right)\right] \rightarrow \left[k_1 + 2k_2 \cdot C_L\right]$$

$$\boxed{3 k_2 C_L^2 + k_1 C_L - C_{D0} = 0}$$

$$\left[C_L = \frac{-k_1 \pm \sqrt{k_1^2 + 12 k_2 C_{D0}}}{6 k_2} \right]$$

Range for propeller continued

Drag minimum $D = C_D \cdot \frac{1}{2}\rho V^2 \cdot S$ $\quad V = \sqrt{\frac{2W}{\rho \cdot C_L \cdot S}}$

$$D = C_D \cdot \frac{1}{\cancel{2}}\cancel{\rho} \frac{W}{\cancel{S}} \cdot \frac{\cancel{2}}{\cancel{\rho}} \cdot \frac{1}{C_L} \cdot \cancel{S}$$

$$D = \frac{C_D \cdot W}{C_L}$$

W is a constant

For a minimum drag we need a $\frac{C_D}{C_L}$ min, or a $\frac{C_L}{C_D}$ max $\quad \frac{C_D}{C_L} = \frac{C_{D0}}{C_L} + \frac{k_1 \cancel{C_L}}{\cancel{C_L}} + \frac{k_2 \cdot C_L^{\cancel{2}}}{\cancel{C_L}}$

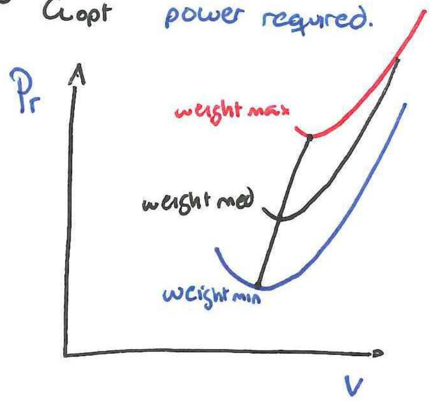
derivate: $\frac{d}{dC_L}\cdot\left(\frac{C_D}{C_L}\right) = -\frac{C_{D0}}{C_L^2} + k_2 = 0 \rightarrow \boxed{C_{L\,opt\,R} = \sqrt{\frac{C_{D0}}{k_2}}}$ $\boxed{C_{L\,opt\,R} = \sqrt{C_{D0}\cdot\pi A e}}$

C_L optimum

Effect of weight

Decrease $\downarrow D = \frac{C_D}{C_L}\cdot W \downarrow$ decrease $\quad V_{opt}\downarrow = \sqrt{\frac{W\downarrow}{S}\cdot\frac{2}{\rho}\cdot\frac{1}{C_{Lopt}}}$

As weight reduces, we need less fuel because we need less power required.



$$P_r \approx W\sqrt{W} \quad \begin{cases} V \approx \sqrt{W} \\ D \sim W \end{cases}$$

DECREASE DEPENDENCE

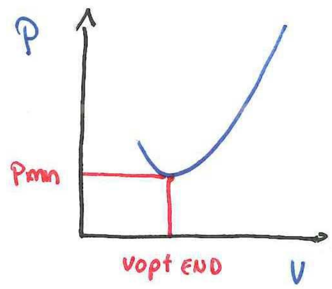
Endurance

HOW LONG IN THE AIR

Fuel flow as low as possible [minimum power means max endurance]

$$P_r = D\cdot V = \frac{C_D}{C_L}\cdot W \cdot \sqrt{\frac{W}{S}\cdot\frac{2}{\rho}\cdot\frac{1}{C_L}}$$

$\sqrt{\frac{W^3}{S}\cdot\frac{2}{\rho}\cdot\frac{C_D^2}{C_L^3}}$ For a P_{min} we have to maximize $\frac{C_L^3}{C_D^2}$ max $\left[\frac{d}{dC_L}\left(\frac{C_L^3}{C_D^2}\right) = 0\right]$



$$\frac{3C_L^2\cdot C_D^2 - 2C_D\cdot C_L^3\cdot\frac{dC_D}{dC_L}}{C_D^4} = 0 \longrightarrow 3\cancel{C_L^2}\cdot C_D^{\cancel{2}} - 2\cancel{C_D}\cdot C_L^{\cancel{3}}\cdot\frac{dC_D}{dC_L} = 0$$

C_D^4 greater than 0

$$3C_D - 2C_L\cdot\frac{dC_D}{dC_L} \rightarrow \frac{3}{2}\frac{C_D}{C_L} = \frac{dC_D}{dC_L} : \boxed{-k_2C_L^2 + k_1C_L + 3C_{D0} = 0}$$

$$\boxed{V_{opt} = \sqrt{\frac{W\cdot 2\cdot}{S\cdot\rho\cdot C_{Lopt}}}} \quad \boxed{C_{L\,opt} = \frac{k_1 \pm \sqrt{k_1^2 + 12k_2C_{D0}}}{2k_2}}$$

Speed Stability and Inestability

Exam check!

Reproduce definitions: steady, horizontal, straight, symmetric

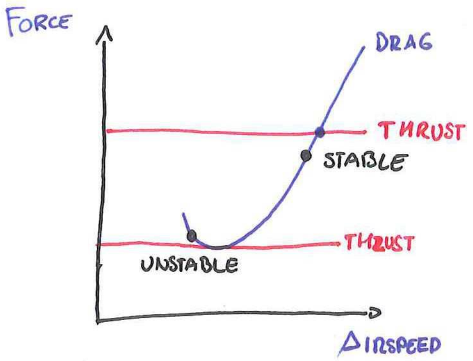
- Calculate minimum airspeed
- Calculate maximum airspeed
- Calculate especific range
- Calculate specific endurance
- Explain speed stability effect.

- Equations of motion
  - $\Sigma F_{\parallel V}: \frac{W}{g}\cdot\frac{dV}{dt} = T\cancel{\cos\alpha_T}^{1} - D - W\cancel{\sin\gamma}^{0} = \boxed{T-D}$
  - $\Sigma F_{\perp}: \frac{W}{g}\cancel{V\cdot\frac{d\gamma}{dt}}^{0} = L - W\cancel{\cos\gamma}^{1} + T\cancel{\sin\alpha_T}^{0} = \boxed{L = W}$

$[L = W]$ $\quad \left[\sin\gamma = \frac{T-D}{W}\right]$ $\quad \left[V\cdot\sin\gamma = \frac{P_a - P_r}{W} = [ROC]\right]$

[Maximum climb angle — Pitch attitude]

Perfomance Diagram



If thrust is larger than drag you will accererate
Speed stable: it will autocorrect

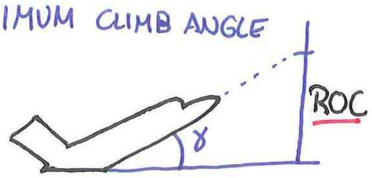
If drag is larger than thrust you will decelerate until you reach stall speed
Speed unstable: it won't autocorrect.

[Relevant in landing and take off]

Climbing and Descending Flight

$P_a > P_r$ so the airplane will climb

Maximum climb angle



$\Sigma F_{\parallel V}: \frac{W}{g}\cdot\frac{dV}{dt} = T\cos\alpha_T - D - W\cdot\cos\gamma = T - D - W\sin\gamma$

$\Sigma F_{\perp V}: \frac{W}{g}\cdot V\cdot\frac{d\gamma}{dt} = L - W\cos\gamma + T\sin\alpha_T = L - W\cancel{\cos\gamma}^{1}$

usually γ don't exceed $15°$

$\Sigma F_{\perp V}: L = W$

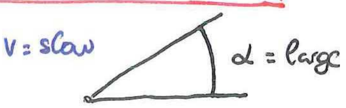
multiply by (V)

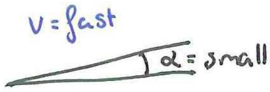
$\Sigma F_{\parallel V}: \sin\gamma = \frac{T-D}{W} = \left[V\cdot\sin\gamma = \frac{P_a - P_r}{W} = ROC\right]$

$V\sin\gamma = \frac{dh}{dt} = \frac{P_a - P_r}{W}$

$m\cdot g\,\frac{dh}{dt} = P_a - P_r$

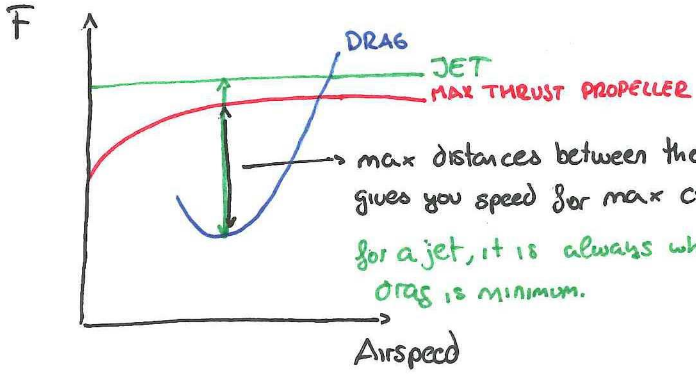
$\boxed{\text{Pitch does not tell climb angle}}$





$$L = W$$

$$\sin \gamma_{max} = \frac{(T-D)_{max}}{W}$$



max distances between the graphs gives you speed for max climb.

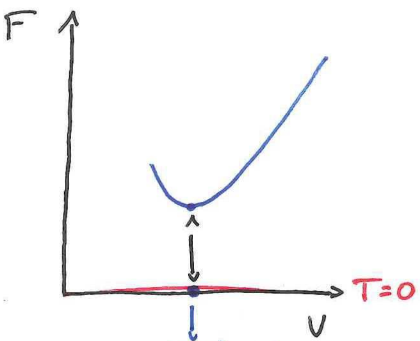
for a jet, it is always when the drag is minimum.

JET

$$C_{L opt} = \sqrt{\frac{C_{D0}}{k_2}}$$

$$C_D = C_{D0} + k_1 \cdot C_L + k_2 \cdot C_L^2 \qquad V_{opt} = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_{L opt}}}$$

PERFOMANCE WITH NOTHRUST GLIDING



we need a drag min.

if you maximize γ you minimize $-\gamma$

speed optimun ⟶ gives you less drag but also less $-\gamma$ angle.

$$\frac{\cancel{T}^{\,0} - D}{W} = \sin\gamma \qquad \frac{D}{W} = \sin\gamma \qquad \bar{\gamma} = \left(\arcsin\left(\frac{C_D}{C_L}\right)\right)$$

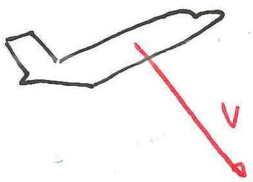
the smallest γ will be given by the min $\frac{C_D}{C_L}$

or also the max $\frac{C_L}{C_D}$

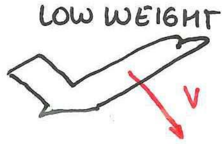
$$\boxed{C_{L opt} = \sqrt{\frac{C_{D0}}{k_2}}}$$

MASS DIFFERENCES

HIGH WEIGHT



LOW WEIGHT



both airplanes will cover the same distance but with a higher weight it will be cover in less time.

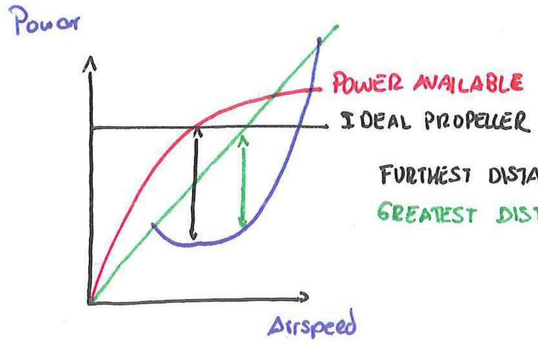
RATE OF CLIMB

$$L = W \qquad \sin\gamma = \frac{T-D}{W} \qquad V \cdot \sin\gamma = \frac{P_a - P_r}{W} = ROC$$

3 FACTORS

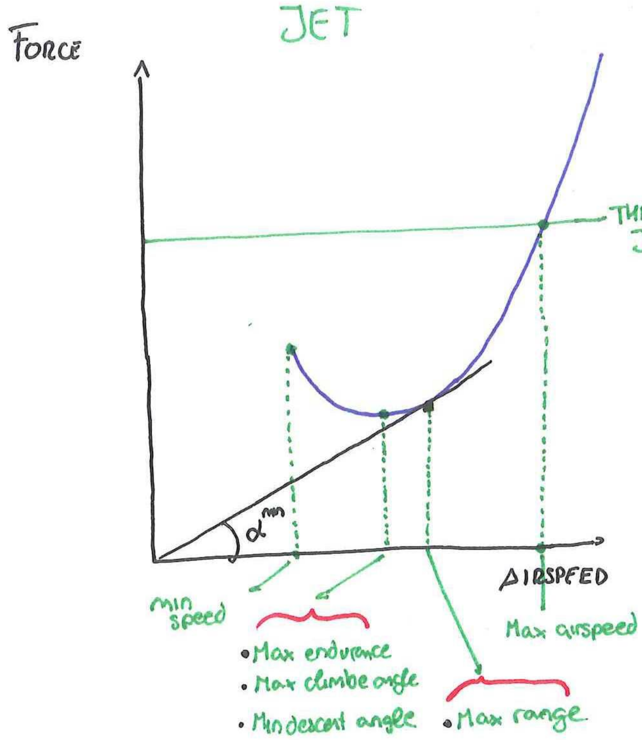
- Power available
- Power required
- Aircraft weight.

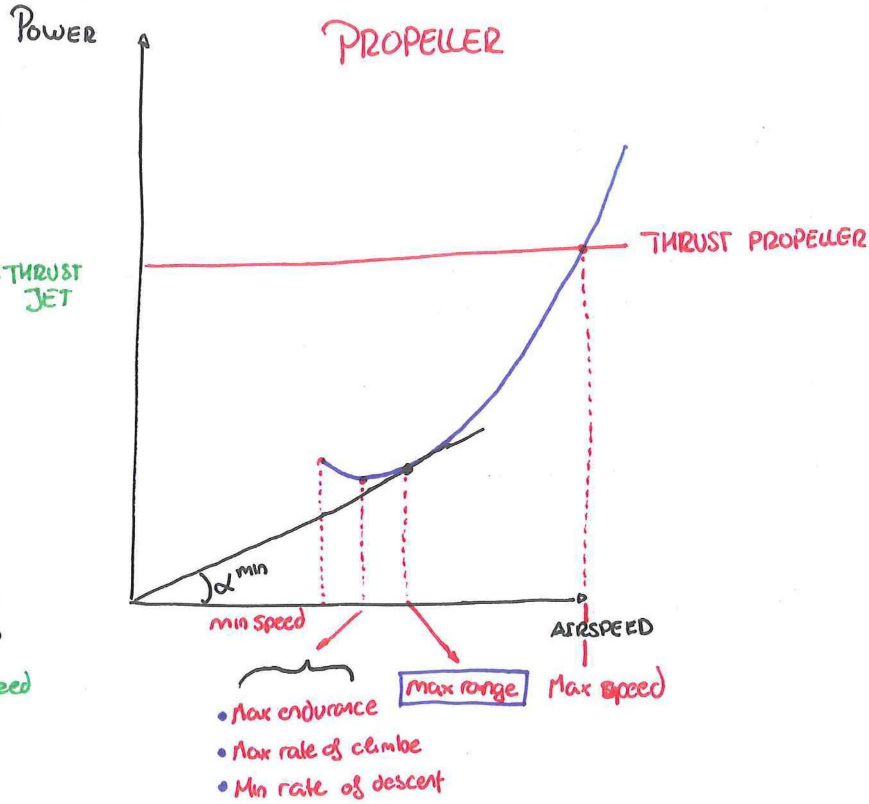
PERFOMANCE DIAGRAMS



FURTHEST DISTANCE GREATER [ROC] (min Drag)

GREATEST DISTANCE → [ROC]



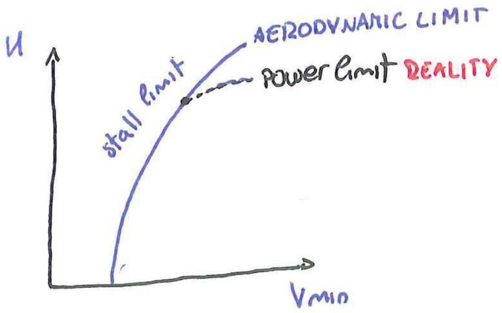


EFFECT OF ALTITUDE ON PERFORMANCE

MINIMUM AIRSPEED

H increases → ρ decreases

$$\underbrace{V_{min}}_{+} = \sqrt{\frac{W}{S} \cdot \underbrace{\frac{2}{\rho}}_{-} \cdot \frac{1}{C_{L\,max}}}$$



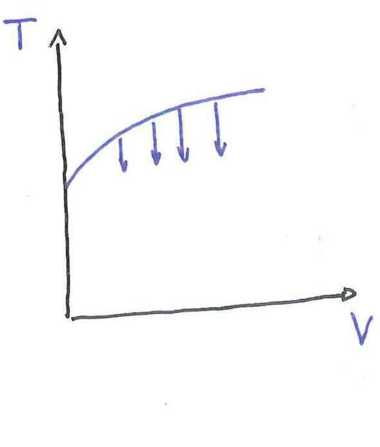
[~~MAXIMUM AIRSPEED~~]

EFFECT ON PROPULSION

$$\underbrace{T}_{\downarrow -} = \underbrace{\dot{m}}_{\downarrow -} \underbrace{(V_j - V_0)}_{\uparrow +}$$

$$\dot{m} = A \cdot V \cdot \rho$$

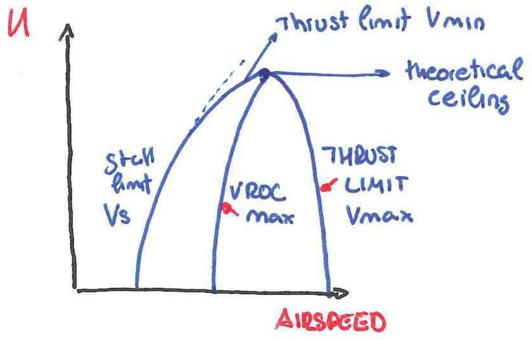
$\dot{m}$ ↓ − ; ρ ↓ −



$$\boxed{\frac{T}{T_0} = \frac{\rho n}{\rho_0}}$$

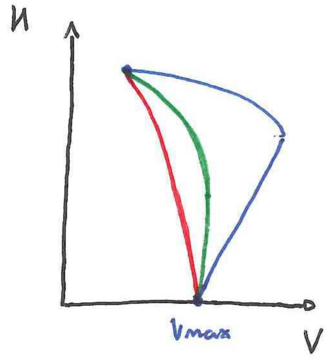
$$\boxed{\frac{P}{P_0} = \frac{\rho n}{\rho_0}}$$

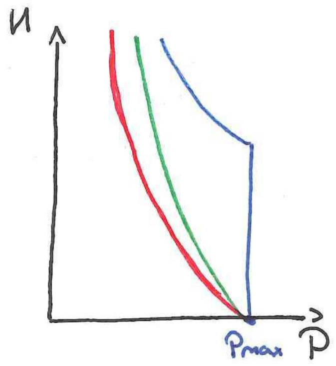
[~~MIN AIRSPEED CONTINUED~~]



MAX AIRSPEED: EFFECT ON ALTITUDE

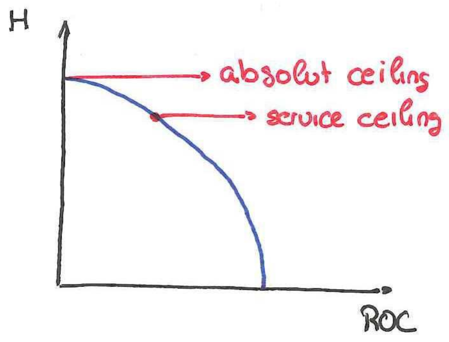
more altitude → shifted to the right

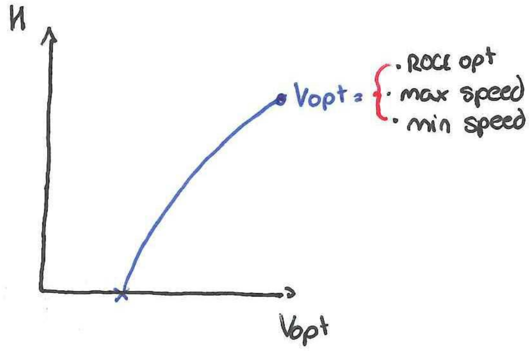




- SUPERCHARGER
- PISTON ENGINE
- TURBO PROP

MAX RATE OF CLIMB





MIN DENSITY

$$C_D = C_{D0} + \frac{C_L^2}{\pi \cdot A \cdot e} \qquad RC = 0 \quad \frac{P_a - P_r}{W} \qquad P_a - P_r \rightarrow T = D$$

$$\frac{T_0}{\rho_0} \cdot \rho = \frac{C_D}{C_L} \cdot W \qquad \rho = \frac{\rho_0}{T_0} \cdot \frac{C_D}{C_L} W$$

ρ minimum we need $\left(\frac{C_L}{C_D}\right)_{max}$

$$\rho = \frac{\rho_0}{T_0} \cdot \frac{\sqrt{4 \cdot C_{D0}^2}}{\sqrt{C_{D0} \pi \cdot Ae}} \cdot W \qquad \boxed{\rho = \frac{\rho_0}{T_0} \cdot \sqrt{\frac{4 C_{D0}}{\pi Ae}} \cdot W}$$

- FOR HIGH ALTITUDES WE NEED
  - High aspect ratio
    $A \uparrow +$
  - low zero-lift drag:
    $C_{D0} \downarrow -$
  - Propulsion sistem
    $\rho \downarrow -$
    $T \downarrow -$

Operational limits

- Manoeuvre loads

$$\frac{W}{g}\cdot V\cdot\frac{d\gamma}{dt}=L-W\cdot\cos\gamma$$

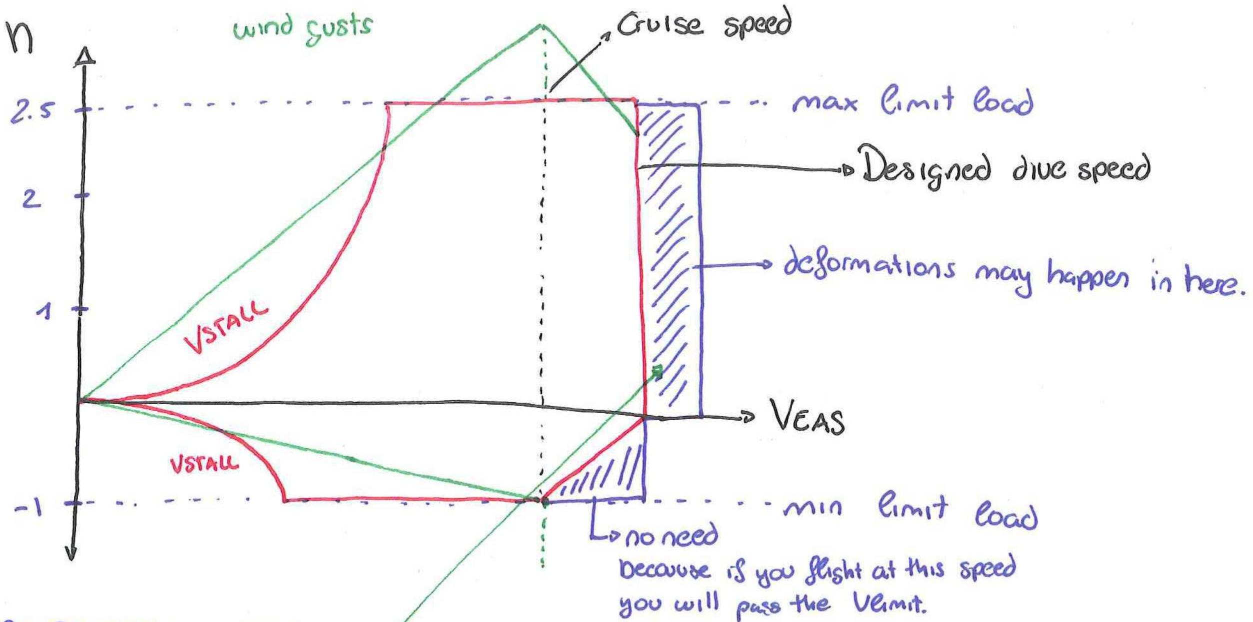
$L > W$: creates a change in flight path angle with time.

Load Factor

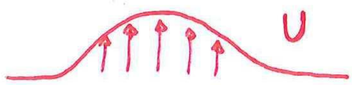
$$\boxed{n=\frac{L}{W}}$$

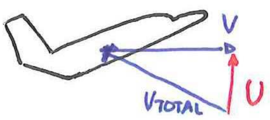
$$n=\frac{C_L}{\frac{W}{S}\cdot\frac{2}{\rho}\cdot\frac{1}{V^2}}\propto V^2$$

[EQUIVALENT AIRSPEED IS USED TO NOT CHANGE THE GRAPH CONTINIOUSLY.]



- AEROELASTIC EFFECTS
- WING GUSTS



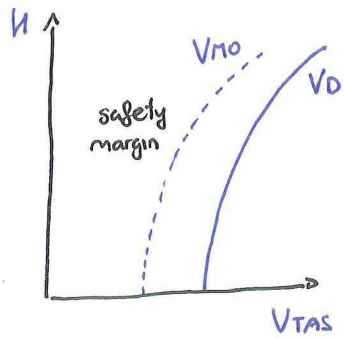


$$n=\frac{L}{W}\rightarrow\Delta n=\frac{\Delta L}{W}=\frac{\Delta C_L\cdot\frac{1}{2}\cdot\rho\left(V^2+U^2\right)\cdot S}{W}$$

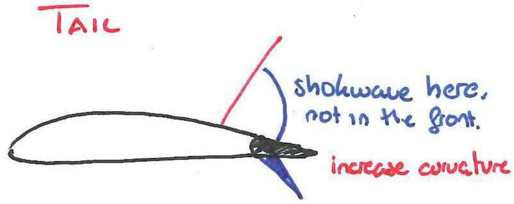
$$\left[\Delta n=\frac{dC_L}{d\alpha}\cdot\frac{\rho}{2}\cdot\frac{S}{W}\cdot U\cdot V\right]$$

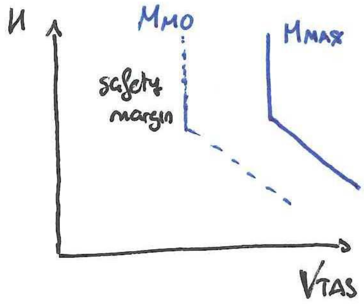
- Design dive speed

$$V_{d,tas}=\sqrt{\frac{\rho_0}{\rho}}\cdot V_{d,eas}$$



- **MAXIMUM MACH NUMBER**
    - Shock waves appear
        - Can cause buffet (vibrations)
    - The aerodynamic center shifts
        - Can cause a severe pitch motion.
    - Aerodynamic control surfaces become less effective

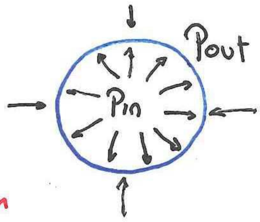




- **PRESURIZED CABIN**

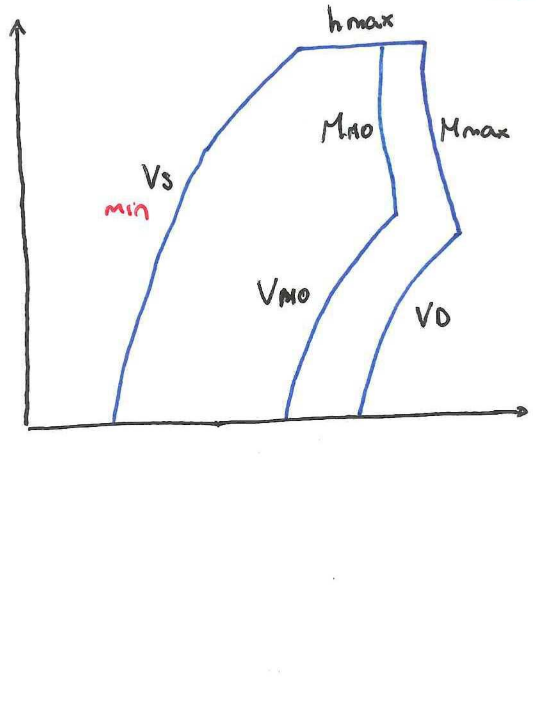
$P_{in} > P_{out}$

Pressure defines maximum height.



FLIGHT ENVELOPE

DEPENDS ON WEIGHT



- AIRSPEED INDICATOR   PITOT TUBE   VEAS $\boxed{\rho_0}$

STALL SPEED INDICATION

$$\underbrace{P_T - P}_{\Delta P_{meas}} = \frac{1}{2}\rho V^2$$

$$V_{E\,min} = \sqrt{\frac{W}{S}\cdot\frac{2}{\rho_0}\cdot\frac{1}{C_{L\,max}}}$$

[IT IS ALWAYS WRONG BUT IT TELLS YOU THE LIMIT CORRECTLY VSTALL]

EQUATIONS

EQUATION OF STATE

$$\boxed{P \cdot V = n \cdot R \cdot T}$$

n = moles
R = constant gas
R = constant air

$$\boxed{R = \frac{R}{M}} \quad P = \rho \cdot R \cdot T$$

M = molar mass

LIFT BALLON GAS

$$L_G = \rho_{atm} \cdot V \cdot g \cdot \left(1 - \frac{\rho_{gas}}{\rho_{atm}}\right)$$

$$L_G = \rho_{atm} \cdot V \cdot g \cdot \left(1 - \frac{M_{gas}}{M_{atm}}\right)$$

LIFT BALLON TEMPERATURE

$$L_G = \rho_{atm} \cdot V \cdot g \left(\frac{\Delta T}{T_{atm} + \Delta T}\right) \text{ NEWTONS}$$

HYDROSTATIC EQUATION

$$\Delta P = -\rho \cdot \Delta h \cdot g \quad Pa$$

$$a = \frac{dT}{dh}$$

WHEN $a \neq 0$

$$\frac{P_1}{P_0} = \left(\frac{T_1}{T_0}\right)^{-\frac{g}{a \cdot R}} \quad Pa$$

$$\frac{\rho_1}{\rho_0} = \left(\frac{T_1}{T_0}\right)^{-\frac{g}{a \cdot R} - 1} \quad kg/m^3$$

WHEN $a = 0$

$$\frac{P_1}{P_0} = (e)^{-\frac{g}{R \cdot T} \cdot (h_1 - h_0)} \quad Pa$$

$$\frac{\rho_1}{\rho_0} = e^{-\frac{g}{R \cdot T}(h_1 - h_0)} \quad kg/m^3$$

GEOPOTENTIAL FORMULAS

Re = radius earth m

$$h_{geop} = \frac{Re}{Re + h_{geom}} \cdot h_{geom} \quad m$$

$$h_{geom} = \frac{Re}{Re - h_{geop}} \cdot h_{geop} \quad m$$

WING GEOMETRY

$$\text{TAPER} = \frac{C_t}{C_r}$$

C_t = chord tip m
C_r = chord root m

LIFT EQUATION

$$L = C_L \cdot \frac{1}{2} \cdot \rho \cdot V^2 \cdot S \quad N$$

$$\boxed{\frac{L}{D} = \frac{C_L}{C_D}}$$

DRAG EQUATION

$$D = C_D \cdot \frac{1}{2} \cdot \rho \cdot V^2 \cdot S \quad N$$

MOMENT EQUATION

$$M = C_M \cdot \frac{1}{2} \cdot \rho \cdot V^2 \cdot S \cdot \bar{c} \quad N \cdot m$$

VISCOSITY EQUATION

$$Re = \frac{\rho \cdot V \cdot L}{\mu}$$

Re: Reynolds number

V = air speed m/s
μ = dinamic viscosity
L = chord lenght m

BERNOULLIS LAW

$$P + \frac{1}{2}\rho \cdot v^2 = \text{constant}$$

VELOCITY BASED ON BERNOULLIS

$$V = \sqrt{\frac{2 \cdot (P_{tot} - P_s)}{\rho}} \quad m/s$$

P_{tot} = total pressure Pa
P_s = static pessure Pa

DRAG COEFFICIENT

$$C_D = C_{D0} + C_{DI}$$

$$C_{DI} = \frac{C_L^2}{\pi \cdot e_0 \cdot AR}$$

$$AR = \frac{S^2}{A}$$

C_{D0} = zero lift drag
C_{DI} = induced drag
e_0 = span efficiency factor
AR = aspect ratio
S = span m
A = area m^2

ANGLES AND AXES

$$\theta = \alpha + \gamma$$

$$\chi = \psi + \beta$$

θ = pitch angle
α = angle of attack
γ = climbe angle
χ = course angle
ψ = heading angle
β = slideship angle

$$L \cdot \sin(\varphi) = F = \frac{W \cdot V^2}{g \cdot R_T}$$

$$n = \frac{1}{\cos(\varphi)} = \text{load}$$

φ angle of turn with the vertical
L = lift N
W = weight N
R_T = radius of turn m
n = load factor
V = velocity m/s

STABILITY (NEUTRAL POINT)

$$\frac{l_{np}}{c} = V_H \cdot \frac{C_{L\alpha H}}{C_{L\alpha}} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

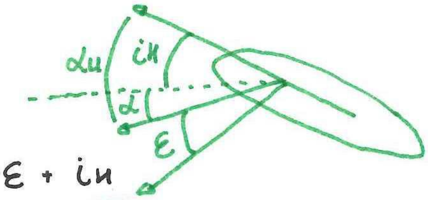
l_{np} = neutral point m
c = chord m

$$V_H = \frac{S_H \cdot l_H}{S \cdot c}$$

$$\frac{C_{LH}}{C_\alpha} = \boxed{a_t}$$

$C_{L\alpha H}$ = lift coefficient in tail/about the α
$C_{L\alpha}$ = lift coefficient in wings/change angle
$\frac{d\varepsilon}{d\alpha}$ = usually given.

STATIC STABILITY



$$\Sigma M_{cg} = M_{ac_{wb}} + L_{wb} \cdot l_{cg} - L_H (l_H - l_{cg})$$

$$\alpha_H = \alpha - \varepsilon + i_H$$

$$\Sigma M_{cg} = M_{ac_{wb}} + L \cdot l_{cg} - L_H (l_H)$$

$$a = \frac{dC_L}{d\alpha}$$

$$\varepsilon = \varepsilon_0 + \alpha \cdot \frac{d\varepsilon}{d\alpha}$$

$$a_t = \frac{dC_{LH}}{d\alpha_H}$$

$$C_{m_{cg}} = C_{m_{ac_{wb}}} + C_L \cdot \frac{l_{cg}}{c} - C_{LH} \cdot V_H$$

$$C_{LH} = a_t \cdot \alpha_H$$

$$V_H = \frac{S_H \cdot l_H}{S \cdot c}$$

$$C_{m\alpha} = a \cdot \frac{l_{cg}}{c} - a_t \cdot V_H \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) < 0$$

PROPULSION

$$I = m \cdot V$$

$$T = \dot{m}(V_j - V_0)$$

I = momentum $kg \cdot m/s$
m = mass kg
V = velocity m/s
$\dot{m}$ = mass of air kg
V_j = outlet velocity m/s
V_0 = inlet velocity m/s
T = trust N

PROPELLER

$$V_{rot} = \omega \cdot r$$

V = velocity m/s
ω = angular velocity rad/s

WORK PERFORMED

$$W = T \cdot \Delta s$$

$$W = P \cdot \Delta V$$

Δs = displacement m
W = work $kg \cdot \frac{m^2}{s^2}$
T = Thrust N
ΔV = volume m^3
P = pressure Pa

BRAKE (SHFT) POWER : P_{br}

AVAILABLE POWER

$$P_a = \frac{T \cdot \Delta s}{\Delta t} = T \cdot V$$

P_a = pascal = ~~power~~ P_a pressure Δt = time s
T = thrust N
Δs = displacement m

PROPULSIVE EFFICIENCY

$$\eta = \frac{P_a}{P_{br}} = \frac{T \cdot V}{P_{br}}$$

JET EFFICIENCY

$$\eta_J = \frac{P_a}{P_j}$$

$$\eta_J = \frac{2}{1 + \frac{V_j}{V_0}}$$

- $P_a = T \cdot V_0$
- $T = \dot{m}(V_j - V_0)$
- $P_j = \frac{1}{2} \dot{m} (V_j^2 - V_0^2)$
- $\dot{m} = \rho_{atm} \cdot A_{inlet} \cdot V_{TAS}$

STRESS

$$\sigma = \frac{F}{A}$$

F = force N
A = area mm^2
σ = stress N/mm^2

DISPLACEMENT TO STRAIN

$$\varepsilon = \frac{\Delta L}{L_0}$$

ε = strain
ΔL = change in length mm
L_0 = initial length mm

LOAD

$$n = \frac{L}{W} = \boxed{\frac{C_L}{C_D}}$$

remember that L and W is the same in static situations.

L = Lift N
W = weight N

Longitudonal stress

$$\sigma_{long} = \frac{\Delta P \cdot R}{2t}$$

R = Radius m ΔP = pressure Pa
t = thikness mm or m

Circumferential stress

$$\sigma_{circ} = \frac{\Delta P \cdot R}{t}$$

INSTRUMENTATION and VELOCITIES

$$M = \frac{V^{TAS}}{a} \qquad a = \sqrt{\gamma \cdot R \cdot T}$$

γ = 1.4 constant
R = gas constant air 287
T = temperature $K°$

- **EQUIVALENT AIR SPEED**

$$EAS = TAS \sqrt{\frac{\rho_1}{\rho_0}}$$

- **STALL VELOCITY**

$$V_{STALL\ EQUIVALENT} = \sqrt{\frac{W}{C_{Lmax} \cdot \frac{1}{2} \cdot \rho_0 \cdot \cancel{V^2} \cdot S}}$$

$$V_{STALL} = \sqrt{\frac{W}{C_{Lmax} \cdot \frac{1}{2} \cdot \rho \cdot S}}$$

Formula Sheet: Flight Mechanics

Equation of motion

Horizontal symetric flight

Steady

ROC

$$\Sigma F \parallel V: \quad \frac{W}{g} \cdot \cancelto{0}{\frac{dV}{dt}} = T \cdot \cancelto{1}{\cos\alpha_T} - D - W \cancelto{0}{\sin\gamma}$$

$$\Sigma F \perp V: \quad \cancelto{0}{\frac{W}{g} \cdot V \cdot \frac{d\gamma}{dt}} = L - W \cdot \cancelto{1}{\cos\gamma} + T \cancelto{0}{\sin\alpha_T}$$

Drag polar

$$C_D = C_{D0} + \frac{C_L^2}{\pi A \cdot e_{oswald}} \quad \text{aircraft}$$

$$C_D = C_{d0} + \frac{C_L^2}{\pi \cdot A \cdot e_{span}} \quad \text{wing}$$

Drag vs speed

$$L = W = C_L \cdot \frac{1}{2} \rho V^2 \cdot S$$

$$D = C_D \cdot \frac{1}{2} \rho V^2 \cdot S \qquad C_D = C_{D0} + \frac{C_L^2}{\pi \cdot A \cdot e_0}$$

$$D = \left(C_{D0} + \frac{C_L^2}{\pi A \cdot e_0}\right) \frac{1}{2} \rho V^2 \cdot S \qquad \text{unfold } C_L \text{ and } A$$

$$D = C_{D0} \cdot \frac{1}{2} \rho V^2 \cdot S + \frac{W^2}{S^2} \cdot \frac{4}{e} \cdot \frac{1}{V^2} \cdot \frac{1}{\pi \cdot \frac{b^2}{S}} \cdot \frac{1}{2} \rho \cdot V^2 \cdot S^2$$

$$D = C_{D0} \cdot \frac{1}{2} \rho V^2 S + \frac{W^2}{b^2} \cdot \frac{2}{\rho} \cdot \frac{1}{\pi e} \cdot \frac{1}{V^2}$$

$$\boxed{D = k_1 \cdot V^2 + \frac{k_2}{V^2}}$$

Propulsion efficiency

$$\boxed{P_a = \frac{W}{\Delta t} = T \cdot \frac{\Delta x}{\Delta t} = T \cdot V}$$

$$\boxed{\eta_t = \frac{P_a}{Q}} \cdot \frac{P_j}{P_j} = \frac{P_j}{Q} \cdot \frac{P_a}{P_j}$$

$$\eta_j = \frac{2}{1 + \frac{V_j}{V_0}} \qquad \eta_{TH} \frac{P_j}{Q}$$

Jet minimum airspeed

$$V_{min} = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_{L_{max}}}}$$

Jet maximum airspeed

max airspeed when T_{max}

$$T_{max} = D \cdot \frac{L}{L} = \frac{C_D}{C_L} \cdot W$$

$$T_{max} = \frac{C_{D0} + k_1 C_L + k_2 C_L^2}{C_L}$$

$$\frac{C_L \cdot T_{max}}{W} = C_{D0} + k_1 C_L + k_2 C_L^2$$

$$\boxed{k_2 \cdot C_L^2 + \left(k_1 - \frac{T_{max}}{W}\right) C_L + C_{D0} = 0}$$

P_a max

$$P_{a\,max} = P_{required}$$

$$= D \cdot V$$

$$= C_D \cdot \frac{1}{2} \rho V^3 \cdot S$$

$$= C_D \cdot \frac{1}{2} \rho \frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L} \cdot \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{S}{C_L}}$$

$$= \frac{C_D}{C_L} \cdot W \cdot \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}}$$

$$\boxed{= \sqrt{\frac{W^3}{S} \cdot \frac{2}{\rho} \cdot \frac{C_D^2}{C_L^3}}}$$

Propeller minumum airspeed

$$V_{min} = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_{L_{max}}}}$$

MAXIMUM RANGE JET

minimum amout of fuel per unit distance

$Fuel = C_T \cdot T$ $\quad T = D$ steady straight horizontal symetric

$$\frac{V}{F} = \frac{V}{C_T \cdot D}$$

include Velocity in both sides. to maximize $\frac{V}{F}$ we need to maximize $\frac{V}{D}$ or minimize $\frac{D}{V}$

$$\left(\frac{D}{V}\right)_{min} = \frac{D}{V} \cdot \frac{L}{L} = \frac{C_D}{C_L} \cdot W \cdot \frac{1}{V} = \frac{C_D}{C_L} \cdot W \cdot \frac{1}{\sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}}} = W \cdot \frac{1}{\sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{C_L}{C_D^2}}}$$

$\left(\frac{D}{V}\right)_{min}$ will happen when $\left(\frac{C_L}{C_D^2}\right)_{max}$ $\boxed{\text{differentiate}}$ $\quad \frac{d}{dC_L}\left(\frac{C_L}{C_D^2}\right) = 0$

$$\frac{C_D^2 \cdot 1 - C_L \cdot 2 \cdot C_D \frac{dC_D}{dC_L}}{C_D^4} = \frac{dC_D}{dC_L} = \frac{1}{2}\frac{C_D}{C_L}$$

use drag polar to find dC_D $\quad \boxed{k_1 + 2k_2 C_L}$

$$k_1 + 2k_2 C_L = \frac{1}{2}\frac{C_{D0} + k_1 C_L + k_2 C_L^2}{C_L} = \boxed{3k_2 \cdot C_L^2 + k_1 C_L - C_{D0} = 0}$$

MAXIMUM ENDURANCE JET

min amount of fuel per unit of time

$Fuel = C_T \cdot D$

$F_{min} = D_{min}$

$D_{min} = D \cdot \frac{L}{L}$

$D_{min} = \frac{C_D}{C_L} \cdot W$ $\quad \frac{C_D}{C_L}_{min} = \frac{C_L}{C_D}_{max}$

$$\frac{d}{dC_L}\left(\frac{C_D}{C_L}\right) = \frac{d}{dC_L}\left(\frac{C_{D0} + k_1 C_L + k_2 C_L^2}{C_L}\right)$$

$$\frac{d}{dC_L} \cdot \left(\frac{C_D}{C_L}\right) = -\frac{C_{D0}}{C_L^2} + k_2$$

$$\boxed{C_{L\,opt} = \sqrt{\frac{C_{D0}}{k_2}}}$$

RANGE FOR PROPELLER

$D = C_D \cdot \frac{1}{2}\rho V^2 \cdot S$ $\quad$ DRAG MINIMUM

$$D = C_D \cdot \frac{1}{2}\rho \cdot \frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L} \cdot S$$

$D = \frac{C_D \cdot W}{C_L}$ $\quad$ W is a constant $\quad \frac{C_D}{C_L} = \frac{C_{D0}}{C_L} + k_1 + k_2 \cdot C_L$

$$\frac{d}{dC_L} \cdot \left(\frac{C_D}{C_L}\right) = -\frac{C_{D0}}{C_L^2} + k_2 = 0$$

$$\boxed{C_{L\,opt} = \sqrt{\frac{C_{D0}}{k_2}}} \qquad \boxed{C_{L\,opt} = \sqrt{C_{D0} \cdot \pi \cdot A \cdot e}}$$

Load Factor

$$n = \frac{L}{W} \Rightarrow \Delta n = \frac{\Delta L}{W} = \frac{\Delta C_L \cdot \frac{1}{2} \cdot \rho (V^2 + U^2) \cdot S}{W}$$

$$\boxed{\Delta n = \frac{dC_L}{d\alpha} \cdot \frac{\rho}{2} \cdot \frac{S}{W} \cdot U \cdot V}$$

$$V_{D\,TAS} = V_{D\,EAS} \cdot \sqrt{\frac{\rho_0}{\rho}}$$

ENDURANCE FOR PROPELLER

fuel flow as low as possible

minimum power

$$P_r = D \cdot V = \frac{C_D}{C_L} \cdot W \cdot \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}} \rightarrow \sqrt{\frac{W^3}{S} \cdot \frac{2}{\rho} \cdot \frac{C_D^2}{C_L^3}}$$

For P_{min}, we have to maximize C_L^3/C_D^2 max

$$\frac{d}{dC_L} \cdot \left(\frac{C_L^3}{C_D^2}\right) = 0 \rightarrow \frac{3C_L^2 \cdot C_D^2 - 2C_D \cdot C_L^3 \cdot \frac{dC_D}{dC_L}}{C_D^4} \rightarrow 3C_D - 2C_L \cdot \frac{dC_D}{dC_L}$$

$$\frac{3}{2} \cdot \frac{C_D}{C_L} = \frac{dC_D}{dC_L} = \boxed{-k_2 C_L^2 + k_1 \cdot C_L + 3C_{D0} = 0}$$

RATE OF CLIMB FOR JET

corresponds to the max endurance of a JET

$$ROC = \frac{P_a - P_r}{W} = V \cdot \sin\gamma$$

$$\boxed{C_{L\,opt} = \sqrt{\frac{C_{D0}}{k_2}}}$$

$$ROC = \frac{C_D}{C_L} \quad \text{NO THRUST}$$

$$P_a - P_r = m \cdot g \cdot \frac{dh}{dt}$$

RATE OF CLIMB FOR PROPELLER

corresponds to the max endurance of a propeller

$$ROC = \frac{P_a - P_r}{W}$$

$$\boxed{-k_2 C_L^2 + k_1 C_L + 3C_{D0} = 0}$$

$$\boxed{C_{L\,opt} = \frac{k_1 \pm \sqrt{k_1^2 + 12k_2 C_{D0}}}{2k_2}}$$

$$ROC = \frac{C_D}{C_L} \quad \text{NO THRUST}$$

$$P_a - P_r = m \cdot g \cdot \frac{dh}{dt}$$

EFFECT ON ALTITUDE

V_{min} increases V_{max} decreases

$$\underbrace{V_{min}}_{+} = \sqrt{\frac{W}{S} \cdot \underbrace{\frac{2}{\rho}}_{-} \cdot \frac{1}{C_{L\,max}}}$$

$$\underbrace{T}_{-} = \underbrace{\dot{m}}_{-}\underbrace{(V_j - V_0)}_{+}$$

$$\frac{T}{T_0} = \frac{\rho^n}{\rho_0}$$

$$\frac{P}{P_0} = \frac{\rho^n}{\rho_0}$$

MINIMUM DENSITY

$$C_D = C_{D0} + \frac{C_L^2}{\pi \cdot A \cdot e} \qquad P_a - P_r = T - D$$

$$\frac{T_0}{P_0} \cdot \rho = \frac{C_D}{C_L} \cdot W \qquad \rho = \frac{P_0}{T_0} \cdot \frac{C_D}{C_L} \cdot W$$

$$\rho = \frac{P_0}{T_0} \cdot \frac{\sqrt{4 \cdot C_{D0}^2}}{\sqrt{C_{D0} \pi \cdot Ae}} \cdot W \qquad \rho = \frac{P_0}{T_0} \cdot \sqrt{\frac{4\,C_{D0}}{\pi A e}} \cdot W$$