Ballooning and Aerodynamics Formulas

Lift for a Balloon.

Q=volume

$$L_B = \omega_{at}Q = \rho_{at}gQ$$

$$L_N = L_B - W_{gas} = gQ(\rho_{at} - \rho_{gas})$$

Hot air balloon

$$L_N = \rho_{at}gQ\left(1 - \frac{\rho_{gas}}{\rho_{at}}\right) = \rho_{at}gQ\left(1 - \frac{T_{at}}{T_{gas}}\right) = \rho_{at}gQ\frac{\Delta T}{T_{at} + \Delta T}$$

$$L_N = \rho_{at}gQ\left(1 - \frac{\rho_{gas}}{\rho_{at}}\right) = L_N = \rho_{at}gQ\left(1 - \frac{\hat{M}_{gas}}{\hat{M}_{at}}\right)$$

$\hat{M}$ =molar mass

Ideal gas law

$$p \cdot V = n \cdot R \cdot T \qquad R = 8.3145\, j/molK$$

$$p = \rho \cdot R_{air} \cdot T \qquad M_{air} = 28.97 g/mol$$

$$p = \rho RT \qquad R_{air} = 8.3145/0.02897 = 287.0 J/kgK$$

$$g = g0\left(\frac{r}{h_a}\right)^2 = g0\left(\frac{r}{r + h_G}\right)^2$$

$$dp = -\rho g dh_G = -\rho g0 dh \longrightarrow 1 = \frac{g0}{g}\frac{dh}{dh_G}$$

$$dh = \frac{g}{g0}dh_G$$

$$dh = \frac{r^2}{(r + h_G)^2}dh_G$$

$$\int_0^h dh = \int_0^{h_G}\frac{r^2}{(r + h_G)^2}dh_G$$

$$h = r^2\left(\frac{-1}{(r + h_G)^2}\right)_0^{h_G} = r^2\left(\frac{-1}{r + h_G} + \frac{1}{r}\right) = r^2\left(\frac{-r + r + h_G}{(r + h_G)r}\right)$$

$$h = \frac{r}{r + h_G}h_G$$

ISA

$$\frac{p}{p1} = \left(\frac{T}{T1}\right)^{-g0/aR}$$

$$\frac{\rho}{\rho1} = \left(\frac{T}{T1}\right)^{-((g0/aR)+1)}$$

Lift

$$L = C_L \cdot \frac{1}{2}\rho V^2 \cdot S$$

Drag

$$D = C_D \cdot \frac{1}{2}\rho V^2 \cdot S$$

$$C_D = C_{D0} + C_{Di}$$

$$C_D = C_{D0} + \frac{C_L^2}{\pi e AR}$$

$$C_{Di} = \frac{L^2}{\pi e AR \frac{1}{4}\rho^2 V^4 S^2}$$

$$D_i = C_{Di} \cdot \frac{1}{2}\rho V^2 \cdot S = \frac{L^2}{\pi e AR \frac{1}{2}\rho V^2 S}$$

Wing aspect ratio

$$AR = \frac{b^2}{S}$$

Pressure cabin

$$\sigma_{circ} = \frac{(p2 - p1)\cdot r}{t}$$

$$\sigma_{long} = \frac{(p2 - p1)\cdot r}{2t}$$

$$ratio\, \frac{\sigma_{circ}}{\sigma_{long}} = 2$$

Momentum equation

$$T = \dot{m}\cdot\left(V_j - V_0\right)$$

Jet efficiency

$$P_j = \frac{1}{2}\dot{m}\cdot\left(V_j^{\,2} - V_0^{\,2}\right)$$

$$P_a = TV_0$$

$$\eta_j = \frac{P_a}{P_j} = \frac{TV_0}{\frac{1}{2}\dot{m}\cdot\left(V_j^{\,2} - V_0^{\,2}\right)} = \frac{2}{1 + \frac{V_j}{V_0}}$$

Mach number

$$\sin\mu = 1/M$$

Equivalent airspeed

$$\frac{1}{2}\rho 0 V_{EAS}^{\,2} = \frac{1}{2}\rho V_{TAS}^{\,2}$$

$$C_L \frac{1}{2}\rho 0 V_{EAS}^{\,2} S = C_L \frac{1}{2}\rho V_{TAS}^{\,2} S$$

Aerodynamics

$$KE = \frac{3}{2}kT$$

$$k = 1.38 \times 10^{-23}\, J/K$$

Equation of state for actual gas

$$\frac{p}{\rho RT} 1 + \frac{ap}{T} - \frac{bp}{T^3}$$

Specific volume

$$v = \frac{1}{\rho}$$

$$pv = RT$$

Continuity

$$\dot{M}_{in} = \dot{M}_{out}$$

$$\dot{M} = \rho AV$$

$$A_1 V_1 = A_2 V_2$$

Euler equation

$$F = m \cdot a$$

$$F = pdydz - \left(p + \frac{dp}{dx} dx \right) dydz$$

$$F = -\frac{dp}{dx} dxdydz$$

$$m = \rho V = \rho dxdydz$$

$$a = \frac{dV}{dt} = \frac{dVdx}{dxdt} = \frac{dV}{dx} V$$

$$F = m \cdot a$$

$$-\frac{dp}{dx} dxdydz = \rho(dxdydz)V \frac{dV}{dx} \Rightarrow$$

$$dp = -\rho V dV$$

Bernoulli's law

$$\int_{p1}^{p2} dp + \int_{V1}^{V2} \rho V dV = 0 \rightarrow$$

$$(p2 - p1) + \rho \left(\frac{1}{2} V_2^2 - \frac{1}{2} V_1^2 \right) = 0$$

$$p1 + \frac{1}{2} \rho V_1^2 = p2 + \frac{1}{2} \rho V_2^2$$

First Law thermodynamics

$$de = \delta q + \delta w$$

$$\delta w = \int_A pdAs = p \int_A sdA = -pdv$$

$$\delta q = de - \delta w = de + pdv$$

$$enthalpy = h = e + pv$$

$$dh = de + pdv + vdp$$

$$\delta q = de + pdv = dh - pdv - vdp + pdv$$

$$\delta q = dh - vdp$$

Specific heat

For constant volume dv=0:
$$de = c_v dT$$
$$e = c_v T$$

For constant pressure dp=0:
$$dh = c_p dT$$
$$h = c_p T$$

Isotropic flow relations

Since $\delta q = 0 \Rightarrow \delta q = de + pdv = 0 \Rightarrow -pdv = de = c_v dT$

Since $\delta q = 0 \Rightarrow \delta q = dh - vdp = 0 \Rightarrow vdp = dh = c_p dT$

$$\frac{-pdv}{vdp} = \frac{c_v}{c_p} \Rightarrow \frac{dp}{p} = -\left(\frac{c_v}{c_p}\right)\frac{dv}{v} \Rightarrow \frac{dp}{p} = -\gamma\frac{dv}{v}$$

$$\frac{c_v}{c_p} = \gamma = 1.4 \, for - air$$

$$\int_1^2 \frac{dp}{p} = -\gamma\int_1^2 \frac{dv}{v} \Rightarrow \left[\ln p\right]_1^2 = -\gamma\left[\ln v\right]_1^2 \Rightarrow$$

$$\ln p2 - \ln p1 = -\gamma\left[\ln v2 - \ln v1\right]$$

$$\ln\left(\frac{p2}{p1}\right) = -\gamma \ln\left(\frac{v2}{v1}\right) \Rightarrow \left(\frac{p2}{p1}\right) = \left(\frac{v2}{v1}\right)^{-\gamma}$$

$$v1 = \frac{1}{\rho 1}$$

so

$$v2 = \frac{1}{\rho 2}$$

$$\left(\frac{p2}{p1}\right) = \left(\frac{\rho 1}{\rho 2}\right)^{-\gamma} \Rightarrow \frac{p2}{p1} = \left(\frac{\rho 2}{\rho 1}\right)^{\gamma}$$

with p/ρ=RT we write

$$\frac{p2}{p1} = \left(\frac{p2}{RT2}\frac{RT1}{p1}\right)^{\gamma} = \left(\frac{p2}{p1}\frac{T1}{T2}\right)^{\gamma} \Rightarrow \left(\frac{p2}{p1}\right)^{1-\gamma} = \left(\frac{T1}{T2}\right)^{\gamma}$$

$$\left(\frac{p2}{p1}\right)^{1-\gamma} = \left(\frac{T2}{T1}\right)^{-\gamma} \Rightarrow \frac{p2}{p1} = \left(\frac{T2}{T1}\right)^{\frac{\gamma}{\gamma-1}}$$

Energy equation

$$\delta q = dh - vdp$$

for adiabatic $\delta q = 0$

$$dh - vdp = 0$$

Combining gives:

$$dp = -\rho VdV$$

$$0 = dh + v\rho VdV$$

Now integrating

$$dh + VdV = 0$$

$$\int_1^2 dh + \int_1^2 VdV = 0 \Rightarrow h2 - h1 + \frac{1}{2}\left(V_2^2 - V_1^2\right) = 0 \Rightarrow$$

h=CpT

$$h2 + \frac{1}{2}V_2^2 = h1 + \frac{1}{2}V_1^2$$

$$C_p T + \frac{1}{2}V^2 = constant$$

Isentropic flow relations second form

$$C_p T + \frac{1}{2} V^2 = \text{constant}$$

$$C_p T1 + \frac{1}{2} V1^2 = C_p T0 + \frac{1}{2} V0^2 \Rightarrow V0 = 0$$

$$C_p T1 + \frac{1}{2} V1^2 = C_p T0 \Rightarrow \frac{To}{T1} = 1 + \frac{V1^2}{2 C_p T1} \Rightarrow$$

$$\frac{To}{T1} = 1 + \frac{\gamma - 1}{2} \frac{V1^2}{\gamma R T1}$$

$$a_1^2 = \gamma R T1$$

$$\frac{To}{T1} = 1 + \frac{\gamma - 1}{2} \frac{V1^2}{a_1^2} = 1 + \frac{\gamma - 1}{2} M_1^2$$

now with isentropic relations

$$\frac{p0}{p1} = \left(\frac{\rho 0}{\rho 1} \right)^{\gamma} = \left(\frac{To}{T1} \right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{p0}{p1} = \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho 0}{\rho 1} = \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\frac{1}{\gamma - 1}}$$

Reynolds number

$$\text{Re}_x = \frac{\rho_\infty V_\infty x}{\mu_\infty}$$

Boundary layer thickness

$$\delta = \frac{5.2x}{\sqrt{\text{Re}_x}}$$

Total skin drag coefficient

$$C_f = \frac{1.328}{\sqrt{\text{Re}_x}}$$