Solution Manual Intro AE II resit 2013

AE1110-II Introduction to Aerospace Engineering, Resit from April 8, 2013

This solution manual has been created in a joint effort by many first-year students of 2013/2014. It may contain mistakes.
The questions and answers are given in blue and the solutions in black.

"Structures & Materials" Question A:

A fibre reinforced composite panel of 1 x 1 m is laminated in a symmetric lay-up with [0°/90°/0°/45°/-45°]s.
The nominal thickness of each ply is 0.132 mm. The strength and modulus of elasticity for a single ply is illustrated in Figure 1.

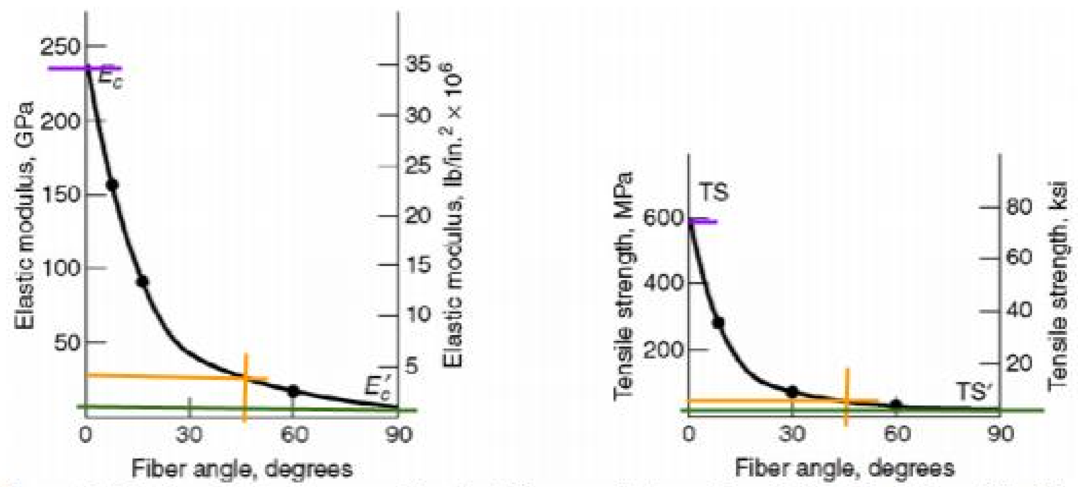


Figure 1 Relation between composite ply stiffness and strength and the orientation of loading

Note that it's a symmetric lay-up and there's a little s after the square bracket. This means: [0°/90°/0°/45°/-45°]s = [0°/90°/0°/45°/-45°/-45°/45°/0°/90°/0°], so we have **10 plies** with these fiber directions, stacked in the given sequence.
I denote the longitudinal direction with y and the transverse direction with x.

1. (2 pts) What is the fraction of the 0° fibre layers within the laminate?
a. 0.2 O
b. 0.3 O
c. 0.4
d. 0.5 O
All layers have the same thickness, 4 of 10 are 0°. 4/10 = 0.4

2. (2 pts) What is the tensile strength of the panel in longitudinal direction?

a. 266 MPa

b. 600 MPa O

c. 60 MPa O

d. 148 MPa O

Rule of mixtures, values from diagram:

0.4*600 + 0.4*60 + 0.2*10 = 266 MPa

3. (2 pts) What is the modulus of elasticity of the panel in transverse direction?

a. 240 GPa O

b. 107 GPa O

c. 25 GPa O

d. 60 GPa

Rule of mixtures, values from diagram. Since it's the transverse direction, the angles are inverted.

0.4*5 + 0.4*30 + 0.2*240 = 62 GPa = E_{xd}

4. (2 pts) Consider a biaxial load of 132 kN being applied to the panel in both directions, what is the strain in longitudinal direction if it can be assumed that vxy = vyx =0.33?

a. 9.3·10-4 O

b. 1.4·10-3

c. 3.7·10-4 O

d. 1.6·10-3 O

panel thickness t = 0.132 mm/ply * 10 plies = 1.32 mm

cross section S = t * b = 1.32 mm * 1000 mm = 1 320 mm ^2

$$\sigma_x = \sigma_y = \frac{F}{A} = \frac{132\,000\,[N]}{1\,320\,[mm^2]} = 100\left[\frac{N}{mm^2}\right] = 100\ [\text{MPa}]$$

E_y = 0.4*240 + 0.4*30 + 0.2*5 = 109 GPa

$$\varepsilon_y = \frac{\sigma_y}{E_y} - \frac{\sigma_x}{E_x}\nu_{xy} = \frac{100\,[MPa]}{109\,000\,[MPa]} - \frac{100\,[MPa]}{60\,000\,[MPa]}0.33 = 3.7 \cdot 10^{-4}$$

5. (2 pts) What is the maximum strain to failure that can occur under this biaxial load case if linear elastic material behaviour may be assumed?

a. 2.5· 10-3

b. 1.2·10-2 O

c. 2.9·10-3 O

d. 3.4·10-3 O

$$\varepsilon_{max} = \frac{\sigma_{max}}{E_y} = \frac{266}{105\,000} = 2.5\overline{3} \cdot 10^{-3} \text{ or } \varepsilon_{max} = \frac{\sigma_{yieldtransvers}}{E_x} = \frac{148}{60\,000} = 2.46 \cdot 10^{-3}$$

6. (2 pts) If a circular cut-out would be created in the centre of this panel with a diameter of 125 mm, what would be the nominal stress in longitudinal direction for the given biaxial loading case?

a. 100 MPa O

b. 87.5 MPa O

c. 114,3 MPa

d. 125 MPa O

The NOMINAL stress does not depend on K_t, so the only thing we have to take into account is the reduced cross section:

panel thickness t = 1.32 mm from 4.

cross section S = t * (b - D) = 1.32 mm * (1000 - 125) mm = 1 155 mm²

load in axial and lateral direction from question 4: F = 132 kN

$$\sigma_x = \sigma_y = \frac{F}{A} = \frac{132\,000\,[N]}{1\,155\,[mm^2]} = 114.3\left[\frac{N}{mm^2}\right] = 114.3\ [\text{MPa}]$$

7. (2 pts) For the same configuration as question 6, what would be the nominal stress in transverse direction for the given biaxial loading case?

a. 100 MPa O

b. 87.5 MPa O

c. 114,3 MPa

d. 125 MPa O

Same as 6 because the composition of the material has no influence on the applied stress. The applied stress (=F/S) is the same in both directions since both loads and both cross-sections are the same in this case. (from part 4: load is 132 kN in <u>both</u> directions; from introduction to question A “panel of 1 x 1 m”)

8. (2 pts) Consider an isotropic plate with infinite length and width. What would be the magnitude of the stress concentration factor Kt if the plate contains an elliptical cut-out with a/b=2 while being uniaxially loaded?

a. 1 O

b. 2 O

c. 3 O

d. 5

$$K_t = 1 + 2\frac{a}{b} = 1 + 2 \cdot 2 = 5$$

“Structures & Materials” Question B:

A single-aisle aircraft has a fuselage diameter of 3.95 m with a fuselage skin thickness of 1 mm.

1. (2 pts) Calculate the hoop stress if the pressure differential is 45 kPa

a. 89 MPa

b. 0.2 MPa O

c. 178 MPa O

d. 45 MPa O

$$\sigma_{hoop} = \frac{\Delta P \cdot r}{t} = \frac{45000\,Pa \cdot 1.975\,m}{0.001\,m} = 89\text{ MPa}$$

2. (2 pts) If the hoop stress level is the maximum design stress level, will it be possible to increase the maximum cruise altitude of the aircraft?

a. Yes, but the cabin pressure should also be increased O

b. No, that would increase the pressure differential and thus increase the hoop stress O

c. Yes, but the cabin pressure altitude should be increased

d. No, the cruising altitude pressure is limited by the pressure differential O

A higher cabin pressure altitude (= altitude where the valves are closed) means a lower cabin pressure. By increasing this altitude the maximum pressure difference can be kept constant.

3. (2 pts) If it is assumed that the aluminium used as fuselage skin material has a Poisson’s ratio of 0.3 what would be the strain ratio εlong/εhoop ?

a. 0.24

b. 1 O

c. 4.25 O

d. 0.5 O

$$\sigma_{long} = \tfrac{1}{2}\,\sigma_{hoop}$$

$$\varepsilon_{hoop} = \frac{\sigma_{hoop}}{E} - \frac{\nu\,\sigma_{long}}{E} = \frac{\sigma_{hoop}}{E} \cdot \left(1 - \frac{\nu}{2}\right) = 0.00108$$

$$\varepsilon_{long} = \frac{\sigma_{long}}{E} - \frac{\nu\,\sigma_{hoop}}{E} = \frac{\sigma_{hoop}}{E} \cdot \left(\frac{1}{2} - \nu\right) = 0.000254$$

$$\frac{\varepsilon_{long}}{\varepsilon_{hoop}} = \frac{0.000254}{0.00108} = 0.24$$

Where σ_{hoop}= 89MPa from 1 and v=0.3,

also E = 70 GPa but it doesn’t matter which value you use. The result is always the same.

4. (2 pts) If a lateral force of 1 kN would act on the vertical tail plane at a distance of 5.5 m from the fuselage centreline, what would be the torsional moment acting on the rear fuselage?
a. 3950 Nm O
b. 5500 Nm
c. 1550 N/m O
d. 1550 Nm O

$M = F \cdot d = 1000\,N \cdot 5.5\,m = 5500\,Nm$

5. (2 pts) Calculate the shear stress in the rear fuselage due to a torsional moment of MT = 3000 Nm
a. 3.67 N/mm O
b. 122 MPa O
c. 0.12 N/mm O
d. 0.12 MPa

$\tau = \frac{F}{S} = \frac{M_T/r}{2\pi r \cdot t} = \frac{3000\,Nm}{2\pi \cdot 1.975^2 m^2 \cdot 0.001\,m}$ = 122 407 Pa = 0.12 MPa

6. (2 pts) With the aircraft standing on the platform, the shear stress in the front fuselage skin due to this torsional moment is:
a. Zero
b. Less than the answer to question 5, but greater than zero O
c. Equal to the answer to question 5 O
d. Greater than the answer to question 5 O

The load path goes from the tail through the rear landing gear. It does NOT pass through the front fuselage. The front tyre of the landing gear is not capable of creating a moment since the lever to the fuselage center is zero (if we disregard friction).

7. (2 pts) Assume the wheel base of the main landing gear is 12.5 m. Calculate the reaction forces if the torsional moment would be 6000 Nm.
a. 125 N O
b. 240 N O
c. 480 N
d. 351 N O

Since the aircraft is symmetrical, the reaction forces on the left and right wheel will each cancel out half of the applied moment. Note that this question is rather superficial because there will also be reaction forces to the aircraft's weight.

$M = F \cdot s \Rightarrow F_1 = -F_2 = \frac{M}{s} = \frac{6000/2}{12.5/2} = 480\,N$

8. (2 pts) If the aircraft is designed to sustain loads during operation in presence of any damage that potentially may occur until such damage is detected and repaired, this design philosophy is called

a. Fail safe O
b. Safe life O
c. Damage tolerance
d. Durability O

9. (2 pts) Which parties or entities are involved in assuring continued airworthiness of aircraft?

a. Airworthiness authority and aircraft manufacturer O
b. Aircraft manufacturer and operator O
c. Operator and airworthiness authority O
d. Airworthiness authority, operator and manufacturer

10. (2 pts) During final assembly, the aluminium fuselage barrel can be joined by

a. Either mechanical fastening or adhesive bonding O
b. Either welding or mechanical fastening O
c. Either adhesive bonding or welding O
d. Mechanical fastening
For the other mentioned methods extremely accurate tolerances would be required.

11. (2 pts) In a riveted joint the actual stress concentration is lower than what would be expected for the open hole geometry. What reduces the stress concentration? (multiple answers possible)

a. Hole filling
b. Secondary bending O
c. Interference
d. Bearing O
e. Friction O

12. (2 pts) If mechanical fastening would be applied to unidirectional fibre reinforced composite with fibres oriented in loading direction; which failure mode would occur?

a. Net section failure O
b. Bearing failure O
c. Shear out failure
d. Rivet failure O

"SPACE" Questions:

1) General questions (12 points): True False

Large rockets always achieve a higher end-velocity than small rockets: **FALSE**

No, $\Lambda = \frac{M_{Liftoff}}{M_{Burnout}}$ is decisive.

The solar radiation energy decreases linearly with the distance to the Sun: **FALSE**

No, it decreases quadratically.

The circular orbital velocity of a satellite increases with altitude: **FALSE**

No, It decreases.

An orbit with an eccentricity e=1 is a hyperbola: **FALSE**

No, it's a parabola. A hyperbola has e>1

The inclination of an orbit can never be larger than 180 deg: **TRUE**

True, else the node you are looking at is no longer the ascending node

Drag losses of a rocket are dependent of its absolute size: **TRUE**

Yes, predominantly on its cross sectional area. => Big rocket, small drag losses.

In vertical flight, the apogee altitude of a rocket depends on its initial thrust-to-weight ratio: **TRUE**

The first man in orbit around the Earth was John Glenn: **FALSE**

He was the first American in orbit. The first man was Yuri Gagarin.

The end velocity of a rocket in vertical flight is dependent on the burn time: **TRUE**

True, the burn time times the gravitational force decreases the ideal velocity

The Space Shuttle could carry more payload than the Saturn-V rocket: **FALSE**

Saturn-V was the largest rocket ever built and also the one with the highest payload.

The largest risk of space debris is caused by small objects (5-10 cm): **TRUE**

True, they have the highest energy/possibility of impact ratio (not sure if that is the correct explanation) also, they are small, so cannot be detected, and are very hard to "tidy" up.

The specific impulse (Isp) of a rocket engine is higher than that of a jet engine **FALSE**

Isp is an efficiency measure. Since a rocket craft has to carry an oxidizer in addition to the fuel it is in general less efficient. Jet engines take the oxygen from the air.

2) (4 points): There are 6 "Kepler" parameters describing a satellite orbit. Which one is missing in the following series (semi-major axis, eccentricity, inclination, argument of perigee, right ascension of ascending node)?

a. Mean anomaly O

b. Gravitational parameter O

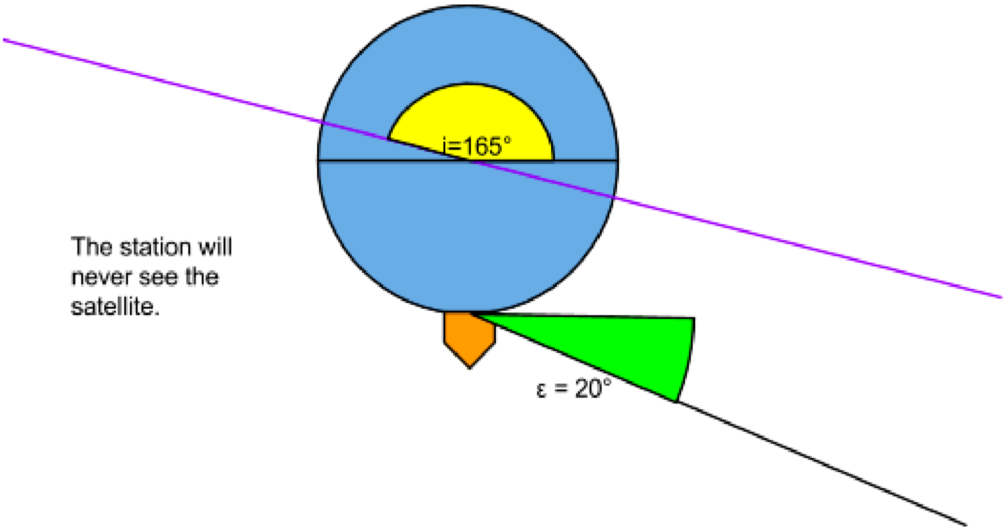
c. Orbital period O

d. Time of last perigee passage

See slides from lecture 5-6 pages 58 and 61.

3) (4 points): A satellite is launched in a circular orbit around the Earth with an inclination of 165 deg. It needs to make radio contact with a station at the South Pole. The satellite needs to be at an elevation angle of at least 20 deg above the horizon of the station. What is the minimum height of this circular orbit to be able to make this radio contact?

a. 68766 km O
b. 75144 km O
c. Infinity O
d. No answer possible



4) (4 points): Two almost identical rockets (A and B) are launched with a different payload mass. ONE difference is that the thrust of rocket B is twice as large as the thrust of rocket A, but the specific impulse, Isp, is the same. The other difference is that the payload mass of rocket B is twice as large as that of rocket A. What is the ideal "tsiolkowsky" end velocity of rocket B with respect to rocket A?

a. B < A
b. B = A O
c. B > A O
d. B = 2*A O

rocket B	**rocket A**
2Ta = m w -> 2Ta ~ m (since w is same for rocket a and b; one can take out the constant)	Ta = m w -> Ta ~ m
Ta ~ m/2	Ta ~ m (the mass flow in rocket A is double!)
$\Delta V = I_{sp} \cdot g_0 \cdot ln\left(\frac{M_{Begin}}{M_{End}}\right)$ $\frac{M_{Begin}}{M_{End}} = \frac{2\,M_A}{2\,M_A - ½\,m\,t}$	$\Delta V = I_{sp} \cdot g_0 \cdot ln\left(\frac{M_{Begin}}{M_{End}}\right)$ $\frac{M_{Begin}}{M_{End}} = \frac{M_A}{M_A - m\,t}$

the change in velocity will be biggest if the ratio of Mbegin/Mend is biggest.
Comparing both equations, one can clearly see that the ratio is bigger in rocket A.

5) (5 points): A spacecraft departs from a circular parking orbit at 200 km altitude around the Earth to reach Venus via a Hohman transfer orbit around the Sun. This requires a Delta-V to change the parking orbit into a hyperbolic orbit with respect to the Earth. The required hyperbolic excess velocity, V∞, to enter the Hohman orbit around the Sun to Venus is 2.50 km/s. What is the required velocity change, Delta-V, to change the circular parking orbit into the hyperbolic orbit?

a. 2.50 km/s O

b. 3.50 km/s

c. 10.28 km/s O

d. 11.29 km/s O

$$V^2 = V_{esc}^{\ 2} + V_{\infty}^{\ 2}$$

Vis-Viva for parabolic orbit: $V_{esc} = \sqrt{\frac{2\mu}{r}} = \sqrt{\frac{2\cdot 398601.3}{6378+200}} = 11\ km/s$

$$V_{end} = \sqrt{V_{esc}^2 + V_{\infty}^2} = \sqrt{11^2 + 2.5^2} = 11.29\ km/s$$

Vis-Viva for circular orbit: $V_{parking} = \sqrt{\frac{\mu}{a}} = \sqrt{\frac{398601.3}{200+6378}} = 7.78\ km/s = V_{initial}$

$$\Delta V = V_{end} - V_{parking} = 3.5\ km/s$$

6) (6 points): A rocket departs in vertical direction from the surface of the Moon. The starting mass of the rocket, Mstart = 1000 kg and the burn-out mass, Mend = 600 kg. The specific impulse of the engine, Isp = 250 s. The rocket burns out with a velocity of 1.0906 km/s. What is the initial thrust-to weight ratio ψ0 of the rocket? (HINT: First compute the burn time of the rocket)

a. 1.0 O

b. 4.0 O

c. 6.0

d. 8.0 O

(question: how is g0 = 9.81? its launched from the moon… answer: g0 is always 9.81, it has nothing to do with the gravity of the planet, it's just a relational parameter)

$$\Delta V = Isp \cdot g_0 \cdot ln\left(\frac{M_{begin}}{M_{end}}\right) = 250 \cdot 9.81 \cdot ln\left(\frac{1000}{600}\right) = 1252.8\ m/s$$

$$V_{burn} = \Delta V - g_0 \cdot t_b \ \Rightarrow$$

$$t_b = \frac{\Delta V - V_{burn}}{g_0} = \frac{1252.8 - 1090.6}{9.81} = 16.5\ s$$

$$t_b = \frac{I_{sp}}{\psi_0} \cdot \left(1 - \frac{1}{\Lambda}\right) \Rightarrow \psi_0 = \frac{I_{sp}}{t_b} \cdot \left(1 - \frac{1}{\Lambda}\right) = \frac{250}{16.5} \cdot \left(1 - \frac{600}{1000}\right) = 6$$

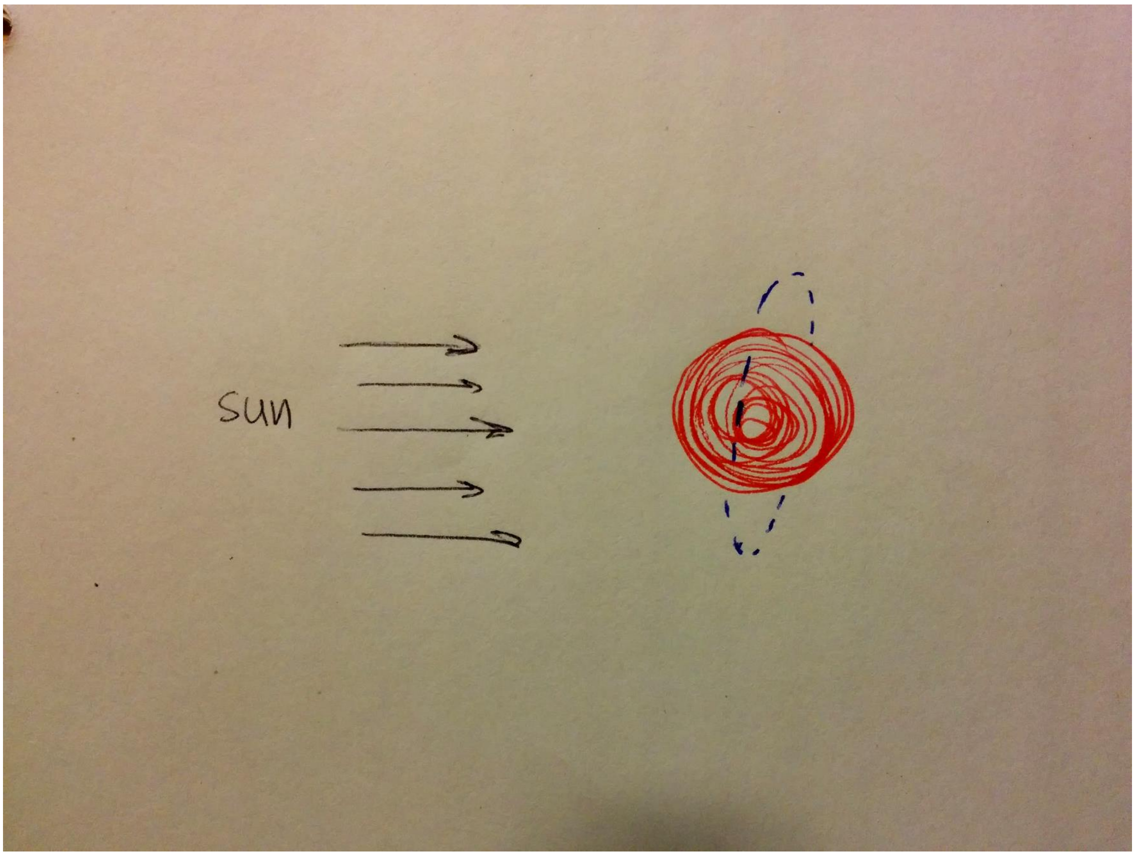
7) (4 points): A satellite is in a circular orbit around Mars at 1000 km altitude. The inclination is 90 deg (polar orbit). By chance, the Sun-Mars vector is exactly perpendicular to the orbital plane of the satellite.
Assume that the Sun is a point light source and that its distance to Mars is infinite. How many minutes per orbit does the satellite stay in the shadow of Mars?

a. 42 min O

b. 0 min

c. 148 min O

d. 74 min O



8) (7 points): A spacecraft is on its way from Earth to Jupiter in a Hohman trajectory. What is the true anomaly angle (theta) it travelled when it is half time during its mission? (HINT: First calculate the half-waytime, then use 3 iterations to convert time to eccentric anomaly via the scheme E = M + e*sin(E), and then convert to theta.)

a. 90.0 deg O
b. 153.5 deg
c. 156.3 deg O
d. 123.5 deg O

Set your calculator to RADIANS!

Since the time of the mission is half of the period of the Hohman orbit, half of that is ¼ of the period. Since M for the whole period is 2π, M that we are looking for is ½π. The eccentricity is

$$e = \frac{2ae}{2a} = \frac{a_J - a_E}{a_J + a_E} = \frac{5.2-1}{5.2+1} = 0.677$$

$E = \frac{1}{2}\pi + 0.677\sin(E)$ start with E e.g. 0.75π and just substitute in the sin with the Answer. E is aproximately 2.14

Question: I don't understand why you chose 0.75pi for the first iteration? Thanks! It does not really matter, the closer you choose your first E, the faster you'll have your answer (less iterations). Try it yourself, eventually you'll get 2.14. Or if you have a graphical or scientific calculator it can do the work for you.

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right) \quad \theta = 153.5$$

<u>Explanation of the iterative process:</u>

take the equation a = b + sin(a)

This is a so called transcendental equation, it doesn't have an analytical solution, however it may be solved numerically. Say, b = 2. (Keep in mind that a is in [rad])

a = 2 + sin(a);

0: choose a close enough value for a0 = ¾ π

1: a1 = 2 + sin(a0) = 2 + sin(¾ π) = 2.707…

2: a2 = 2 + sin(a1) = 2 + sin(2.707) = 2.421..

3: a3 = 2 + sin(a2) = 2 + sin(2.421) = 2.659

n: an = 2 + sin(a[n-1])

This example is converging slowly, but after many iterations you will get 2.55, which is the answer to the 3rd significant digit.

Personally, I write in my calculator b+sin(a0), press =, then b+sin(ANS) and just repeat until the first 3 digits stop changing. Doing this with a not-scientific calculator is tedious, however.(You can always just plot the function)

9) (4 points): Near the Earth, the solar radiation energy is 1400 W/m2. The Earth is at 1 AU distance from the Sun. Solar cells have an efficiency of 20%. An interplanetary spacecraft with 10 m2 solar panels needs a minimum electrical power of 100 W. What is its maximum distance of the spacecraft to the Sun to generate enough electrical power?

a. 11.8 AU O
b. 140.0 AU O
c. 5.3 AU
d. 28.0 AU O

$$A = \frac{P_{req}}{S \cdot \eta} \Rightarrow 10 = \frac{100}{\frac{1400}{x^2} \cdot 0.2} \Rightarrow x = \sqrt{\frac{10 \cdot 0.2 \cdot 1400}{100}} = 5.3\ AU$$

alternative:
use $L = b \cdot 4 \cdot \pi \cdot d^2$ (IB-astrophysics)
1st: you know $b = 1400$ and $d = 1\ [AU]$ (convert this to meters) >> $L = 3.96 \cdot 10^{26}\ [W]$
2nd: $Input\ \cdot 0.2 = 100\ [W]$ >> Input must be $500\ [Wm^{-2}]$
3rd: you know L, the new b (=Input), you find new d >> $d = 7.88 \cdot 10^{11}\ [m] = 5.29\ [AU]$

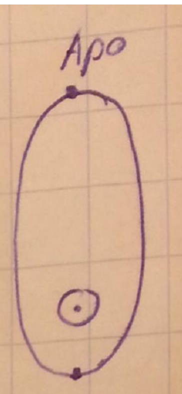
10) (4 points): A spacecraft is making a fly-by of Mars in a hyperbolic orbit with an eccentricity, e =1.4. The lowest point above the surface of Mars is 200 km. What is the velocity of the spacecraft at the lowest point above Mars?

a. 3.5 km/s O
b. 4.9 km/s O
c. 5.3 km/s
d. 5.9 km/s O

$$r_p = a \cdot (1-e) \Rightarrow a = \frac{r_p}{1-e} = \frac{200+3402}{1-1.4} = -9005$$
$$V^2 = \mu \cdot (\frac{2}{r} - \frac{1}{a})$$
$$V = \sqrt{4.28 \cdot 10^4 \cdot (\frac{2}{200+3402} - \frac{1}{-9005}} = 5.3\ km/s$$

11) (6 points): A satellite is in an orbit around the Earth with perigee height = 200 km and apogee height = 36000 km. The orbit lies in the equatorial plane. The satellite has to make a SINGLE manoeuvre (short rocket burn) at apogee to change its inclination to 28 deg and to circularize the orbit. What is the total Delta- V of this manoeuvre?

a. 1.48 km/s O
b. 1.82 km/s
c. 2.25 km/s O
d. 0.77 km/s O



$$2a = 36000 + 200 + 6370 + 6370$$

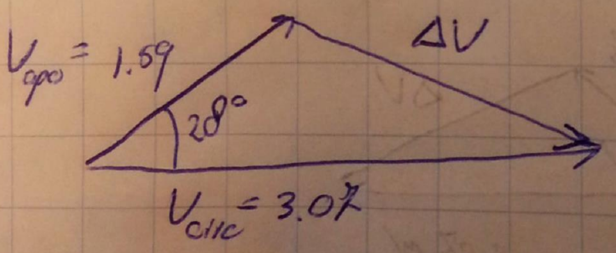
$$a = 24470$$

$$V_A^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$V_{apo} = \sqrt{398600\left(\frac{2}{36000+6370} - \frac{1}{24470}\right)} = 1.59 \text{ km/s}$$

$$V_{circ.} = \sqrt{\frac{\mu}{r}}$$

$$= \sqrt{\frac{398600}{36000+6370}} = 3.07$$



$$\Delta V^2 = 1.59^2 + 3.07^2 - 2 \cdot 1.59 \cdot 3.07 \cdot \cos(28^\circ)$$

$$\Delta V = 1.82$$