April 2014

Solution Manual Intro AE II exam 2013

AE1110-II Introduction to Aerospace Engineering, Exam from January 28, 2013

This solution manual has been created created in a joint effort by first-year students of 2013/2014. It may contain mistakes.
The questions and answers are given in blue and the solutions in black.

"Structures & Materials" Question A:

A spacecraft launched with a launch vehicle, may be represented by the concentrated mass at the end of a clamped beam, which could be simplified to a spring-mass system, both illustrated in Figure 1. The following parameters are specified: M = 1000 kg, E = 72 GPa, A = 5000 mm2 , L = 12 m and I = 1·109 mm4

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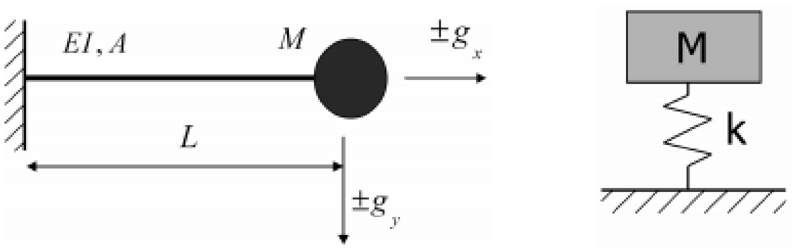


Figure 1 Schematic of a concentrated mass at the end of a clamped beam (left) and simplification to a
single degree of freedom mass-spring system (right)

For a beam loaded in axial and lateral direction, the deflections can be calculated respectively with

$$\delta_x = \frac{P_x L}{EA} \quad \text{and} \quad \delta_y = \frac{P_y L^3}{3EI}$$

1. (2 pts) What is the unit of the spring constant k?

a. N/m

b. Nm O

c. kg/m O

d. Hz O

2. (2 pts) Calculate the value of the spring constant in axial direction kx

a. 3·104 O

b. 3.3·10-5 O

c. 3·107

d. 3.3·10-8 O

$$k_x = \frac{EA}{L} = \frac{72\cdot10^9\cdot5000\cdot10^{-6}}{12} = 3\cdot10^7$$

3. (2 pts) Calculate the value of the spring constant in lateral direction ky

a. 8·10-3 O

b. 1.3·105

c. 8·10-6 O

d. 125 O

$$k_y = \frac{3EI}{L^3} = \frac{3\cdot72\cdot10^9\cdot(1\cdot10^9\cdot10^{-12})}{12^3} = 1.3\cdot10^5$$

4. (3 pts) Calculate the natural frequency fn in axial direction (alternative value for kx is 1·107)

a. 0.04 Hz O

b. 27.6 Hz

c. 12 Hz O

d. 15.9 Hz ← alternative

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{3\cdot10^7}{1000}} = 27.6\ s^{-1}$$

5. (3 pts) Calculate the maximum quasi-static stress in the beam in axial direction if the launcher is accelerating in this direction with an acceleration of gx = 6g

a. 11.8 MPa

b. 3.9 MPa O

c. 1.2 MPa O

d. 5.9 MPa O

$$\sigma = \frac{m\cdot6g}{A} = \frac{1000\cdot6\cdot9.81}{5000} = 11.8\ \frac{N}{mm^2}$$

6. (2 pts) Calculate the thickness t of the circumferential shell structure if the radius of this structure is R = 0.5 m

a. 0.006 mm O

b. 0.8 mm O

c. 1.59 mm

d. 0.63 mm O

$$A = 2\pi\cdot R\cdot t \Rightarrow t = \frac{A}{2\pi R}$$

7. (3 pts) Calculate the required minimum area A, if the natural frequency fn should be greater

than 31 Hz
a. 160 mm2 O
b. 6.3 m2 O
c. 0.16 m2 O
d. 6.3· 103 mm2

Substitute equation of exercise 2 in the equation of exercise 4 and rewrite it, use SI units:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{E \cdot A}{m \cdot L}} \Rightarrow A = \frac{L \cdot m \cdot (2\pi \cdot f_n)^2}{E} = \frac{12 \cdot 1000 \cdot (2\pi \cdot 31)^2}{72 \cdot 10^9} = 6.3 \cdot 10^{-3}\ m^2 = 6.3 \cdot 10^3\ mm^2$$

8. (3 pts) Instead of increasing the area A, increasing the natural frequency fn to 31 Hz could also be achieved by
a. Substituting aluminium by magnesium O
b. Increasing the length L to 15 m O
c. Reducing the mass M to 700 kg
d. Reducing the axial acceleration to 5g O

Substitute equation of exercise 2 in the equation of exercise 4, that is take the intermediate formula from question 7, and analyse:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{E \cdot A}{m \cdot L}} \qquad E\downarrow \Rightarrow f_n\downarrow;\ L\uparrow \Rightarrow f_n\downarrow;\ m\downarrow \Rightarrow f_n\uparrow;$$ acceleration has no influence.

“Structures & Materials” Question B:

Consider a hanging aluminium beam with a cross section A = 100 mm2 clamped at the top, as illustrated in Figure 2. The aluminium properties are given by E = 72 GPa, ρ = 2780 kg/m3, σyield = 345 MPa, and σult = 480 MPa.

1. (2 pts) Calculate the maximum length of this beam before it breaks near the clamping
a. 12.7 km O
b. 176 m O
c. 17.6 km
d. 127 m O

$$\sigma_{ult} = \frac{F}{A} = \frac{g \cdot m}{A} = \frac{g \cdot A \cdot L \cdot \rho}{A} = g \cdot L \cdot \rho \Rightarrow \frac{\sigma_{ult}}{g \cdot \rho} = L = \frac{480 \cdot 10^6}{2780 \cdot 9.81} = 17\ 600\ m$$

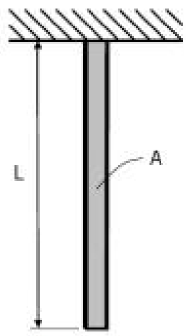


Figure 2

2. (2 pts) Calculate the elongation of the beam if it should remain elastic along its entire length and if all the mass could be considered concentrated at the lower end of the beam

a. 0.005 m O
b. 84 m O
c. 12700 m O
d. 61 m

$$L = \frac{\sigma_{yield}}{g \cdot \rho} = 12900 \quad \varepsilon = \frac{\sigma}{E} = 4.79 \cdot 10^{-3} \qquad \Delta L = L \cdot \varepsilon = 61[m]$$

3. (2 pts) If the cross sectional area would be increased to 250 mm2, the answer to question 1 would

a. decrease O
b. remain the same
c. increase O

If you look at the formula from exercise B1 the cross-sectional area is not in the formula so the stress is not dependent on that.

4. (2 pts) Multiplying the answer to question 1 with the acceleration of gravity g would result in the

a. breaking length O
b. ultimate strength O
c. specific strength
d. compression strength O

Rearrange the equation used in question 1:

$$\frac{\sigma_{ult}}{g \cdot \rho} = L \Leftrightarrow L \cdot g = \frac{\sigma_{ult}}{\rho} = \sigma_{ult,\,specific}$$

and you arrive at the definition of specific strength.

5. (total 12 pts) State whether the following statements are true or false

a) Safe life is the number of flights, landings or flight hours during which there is a low probability that the structure will degrade below its design strength **TRUE**

b) Composites do not corrode and therefore do not suffer from environmental conditions. **FALSE**

c) To provide the load path through a spacecraft structure two categories of structures can be distinguished: truss structures and stage structures **FALSE**

d) A sheet provides a diagonal function when loaded in shear **TRUE**

e) Structural safety is solely the responsibility of the aircraft manufacturer and the operator **FALSE**
The aviation authority is responsible as well.

f) Tension joints can be created with mechanical fastening (bolting) and with welding **TRUE**

g) A hole in an infinite plate has a stress concentration Kt = 3 indifferent whether the hole is open or filled **FALSE**
When, the hole is filled, the stress concentration is smaller.

h) Expansion and contraction during respectively load decrease and increase is described by the coefficient of thermal expansion **FALSE**

i) During curing of thermoplastic polymers irreversible chain linking occurs. **FALSE**
Only during curing of thermoset polymers irreversible chain linking occurs.

j) The E-modulus of titanium can be significantly changed by alloying **FALSE**
It's not possible to significantly change the E-modulus of any material by alloying.

k) A disadvantage of ceramics is the high ductility **FALSE**
They are very brittle which is the opposite of ductile.

l) Milling is a process that removes "chips" from the material **TRUE**

m) Filament winding is a process that can be applied in dry and wet condition to short fibres only **FALSE**
Filament winding can only be done with long fibres.

n) The selection of a rib type depends predominantly on 4 aspects: loads, design philosophy, available equipment, and costs **TRUE**
The selection of any structural component depends predominantly on these 4 aspects.

o) The selection of materials for a wing skin application could depend on whether it is the upper or lower wing skin. **TRUE**
Yes, during flight the upper wing skin is loaded in compression and the lower wing skin in tension.

“SPACE” Questions:

1) General questions (10 points): True False

The thrust of a rocket engine is larger in vacuum than in air: ● O
The circular velocity of a satellite at 1000 km above the Earth or Mars is the same: O ●
$V = \sqrt{\frac{\mu}{r}}$ -> μ is smaller on Mars. and so is r
The orbital velocity of a satellite is dependent on its mass: O ●
The mass is not in the vis-viva equation.
In an elliptic orbit the velocity in apocentre is higher than in pericentre: O ●
Not true, highest V is always in the pericenter
The inclination of a Sun-synchronous orbit around the Earth is 90 deg: O ●
Drag losses of a vertically launched rocket are larger than gravity losses: O ●
Gravity losses of a vertically launched rocket are dependent of the burn time: ● O
The first man on the Moon was Youri Gagarin: O ●
He was the first man in space. The first man on the moon was Neil Armstrong.
The theoretical “Tsiolkowski” end velocity of a rocket is not dependent on its thrust: ● O
The “ground-track” of the ISS moves to the west during every orbit: ● O

2) (4 points): There are 6 “Kepler” parameters describing a satellite orbit. Which one is missing in the following series (semi-major axis, eccentricity, inclination, argument of perigee, time of last perigee passage)?
a. True anomaly O
b. Gravitational parameter O
c. Right Ascension of Ascending node ●
d. Eccentric anomaly O
See slides from lecture 5-6 pages 58 and 61.

3) (4 points): The orbital period of a geosynchronous satellite is 23 hours 56 minutes and 4 seconds. Why is it NOT 24 hours?
a. The Earth is not a perfect sphere O
b. The Earth is rotating around the Sun ●
c. The gravity of the Moon changes the orbital period O
d. The shape of the orbit of the satellite is elliptic O

4) (4 points): According to the law of Tsiolkowsky, the final velocity of a rocket Vfinal = c * ln (Mstart/Mfinal), where c is the exhaust velocity, Mstart is the starting mass and Mfinal is the final mass after all the propellants have been used. What is the velocity of the rocket when 50% of the propellants have been used?

a. < 0.5 * Vfinal ●
b. > 0.5 * Vfinal O
c. = 0.5 * Vfinal O
d. = 0.5 *c O

Example calculation with data from the Apollo lunar module ascent stage:
c = 3050 m/s; M_{total} = 4700 kg; M_{dry} = 2150 kg => M_{half} = 3425 kg

$$V_{final} = c \cdot ln\left(\frac{M_{total}}{M_{dry}}\right) = 3050 \cdot ln\left(\frac{4700}{2150}\right) = 2385\ m/s$$

$$V_{half} = c \cdot ln\left(\frac{M_{total}}{M_{half}}\right) = 3050 \cdot ln\left(\frac{4700}{3425}\right) = 965\ m/s < 0.5 \cdot 2385$$

5) (4 points): A GPS satellite circles around the Earth with an orbital period of 12 hours. What is the altitude of the satellite above the surface of the Earth?

a. 17892 km O
b. 20232 km ●
c. 6378 km O
d. 26610 km O

$$T = 2\pi \cdot \sqrt{\frac{a^3}{\mu_{earth}}} \Rightarrow$$

$$a = \sqrt[3]{\left(\frac{T}{2\pi}\right)^2 \cdot \mu_{earth}} = \sqrt[3]{\left(\frac{12 \cdot 3600}{2\pi}\right)^2 \cdot 398601.3} = 26610\ km$$

$$h = a - r = 26610 - 6378 = 20232\ km$$

6) (4 points): A rocket departs in vertical direction from the surface of the Moon. The starting mass of the rocket, Mstart = 1000 kg. What is the minimum thrust of the rocket motor to achieve lift-off?

a. 10 kN O
b. 1622 N ●
c. 1000 N O
d. 9810 N O

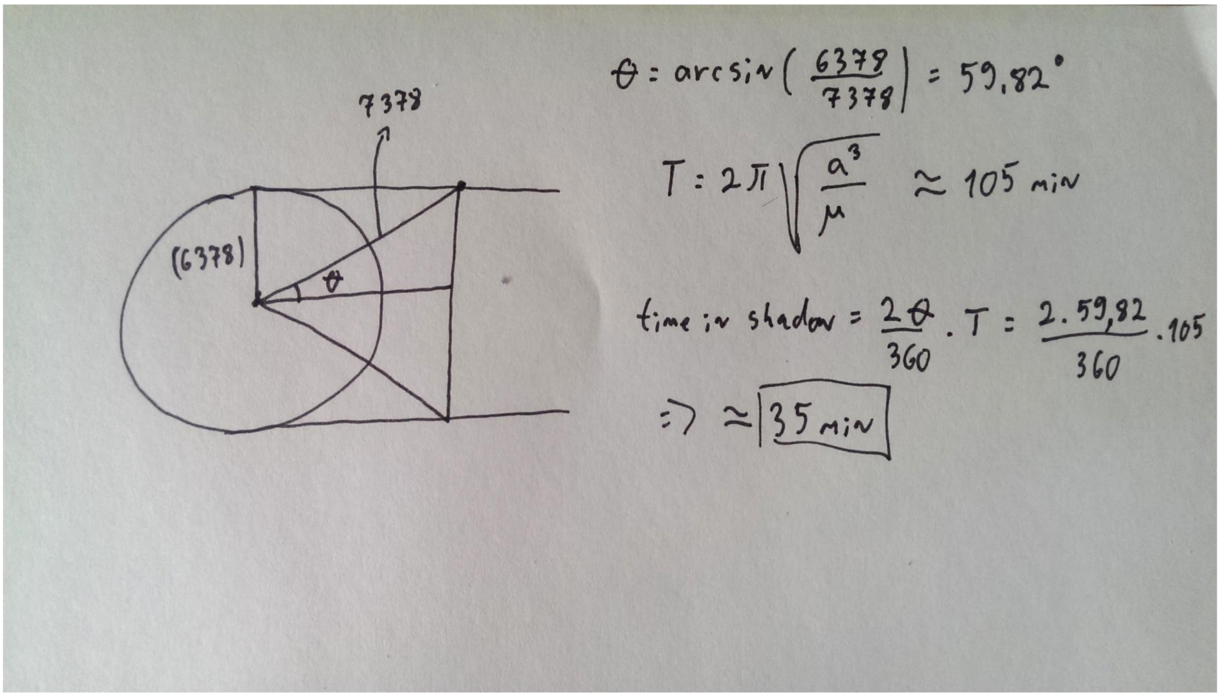
For equilibrium, that is, when the rocket is hovering:

$$T_{min} = m \cdot a = m \cdot g_{Moon} = 1000 \cdot 1.622 = 1622\ N$$

$$g_{Moon} = \frac{\mu}{r^2} = \frac{4.9 \cdot 10^3}{1738^2} = 1.622 \cdot 10^{-3} \left[\frac{km}{s^2}\right] \Rightarrow 1.622 \left[\frac{m}{s^2}\right]$$

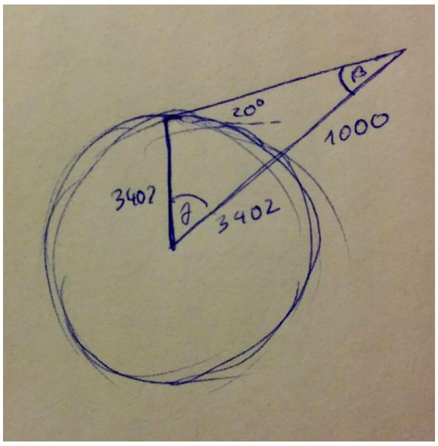
7) (5 points): A satellite is in a circular orbit around the Earth at 1000 km altitude. By chance, the Sun-Earth vector lies exactly in the orbital plane of the satellite. Assume that the Sun is a point light source and that its distance to the Earth is infinite. How many minutes per orbit does the satellite stay in the shadow of
the Earth?

a. 35 min ●
b. 17.5 min O
c. 105 min O
d. 52.5 min O



8) (5 points): A satellite is in a circular orbit around Mars at an altitude of 1000 km. A rover at the surface of Mars wants to make radio contact with the satellite when it is 20 deg above the horizon. What is the distance between the satellite and the rover at this moment?

a. 2794 km O
b. 3402 km O
c. 4402 km O
d. 1863 km ●



$$\frac{\sin\beta}{3402} = \frac{\sin(110°)}{4402} \Rightarrow \beta = 46.57° \Rightarrow \gamma = 180° - 110° - \beta = 23.43°$$

$$c = \sqrt{3402^2 + 4402^2 - 2 \cdot 3402 \cdot 4402 \cdot cos(23.43°)} = 1863\ km$$

9) (4 points): Near the Earth, the solar radiation energy is 1400 W/m2. The Earth is at 1 AU distance from the Sun. Solar cells have an efficiency of 15%. What is the required surface area of a solar panel of a spacecraft at a distance of 0.5 AU from the Sun to generate 420 W of electrical power?

a. 2.0 m2 O

b. 1.0 m2 O

c. 0.5 m2 ●

d. 5.0 m2 O

$$A = \frac{P_{req}}{S_{0.5AU} \cdot \eta} = \frac{P_{req}}{(S_{1\ AU} \cdot 4) \cdot \eta} = \frac{420}{1400 \cdot 4 \cdot 15} = 0.5\ m^2$$

to explain where did the 4 come from -> ½

$$1400 \left[\frac{W}{m^2} \right] \Rightarrow 1400 \left[\frac{W}{(½m)^2} \right] \Rightarrow 1400 \cdot 4 \left[\frac{W}{m^2} \right]$$

You can imagine that the light from the sun is distributed over the sphere surface area 4 D^2 π, hence the power density diminishes with the square of the distance.

10) (7 points): A spacecraft is traveling from Earth to Venus in an elliptical, but NON-Hohman transfer orbit, starting at the apocentre at 1 AU. The semi-major axis of this elliptical orbit, a = 0.75 AU. After a few months the orbit of the spacecraft crosses (!) the orbit of Venus. What is the "exact" travel time to this crossing point in days?
HINT: First compute the eccentricity, e, then the "true anomaly, theta, when the spacecraft crosses the orbit of Venus, then the "eccentric anomaly", E, and finally the travel time.
a. 112 days O
b. 43 days O
c. 119 days O
d. 76 days ●

The orbits of the planets are approximated as circular orbits. The needed constants are given on the sheet with the formulas.
Set your calculator to Radians!

Method 1

1. Calculate eccentricity: $e = \frac{r_a - r_p}{r_a + r_p} = \frac{1\,AU - 0.5\,AU}{1\,AU + 0.5\,AU} = \frac{0.5}{1.5} = \frac{1}{3}$

2. now you can calculate the eccentricity Anomaly: $r = a(1 - e \cdot cos(E))$

$$E = arccos\left(\frac{\frac{r}{a} - 1}{-e}\right) = arccos\left(\frac{\frac{0.723\,AU}{0.75\,AU} - 1}{-\frac{1}{3}}\right) = 1.46\;rad$$

3. Calculate (t-tp): $(t - t_p) = \frac{E - e \cdot sin(E)}{n}$

$$n = \sqrt{\frac{\mu_{Sun}}{a^3}} = \sqrt{\frac{1.33 \cdot 10^{11}}{(0.75 \cdot 150 \cdot 10^6)^3}} = 3.0563 \cdot 10^{-7}\;s$$

$$(t - t_p) = \frac{1.46 - 1/3 \cdot sin(1.46)}{3.06 \cdot 10^{-7}} = 3\,688\,596\;s$$

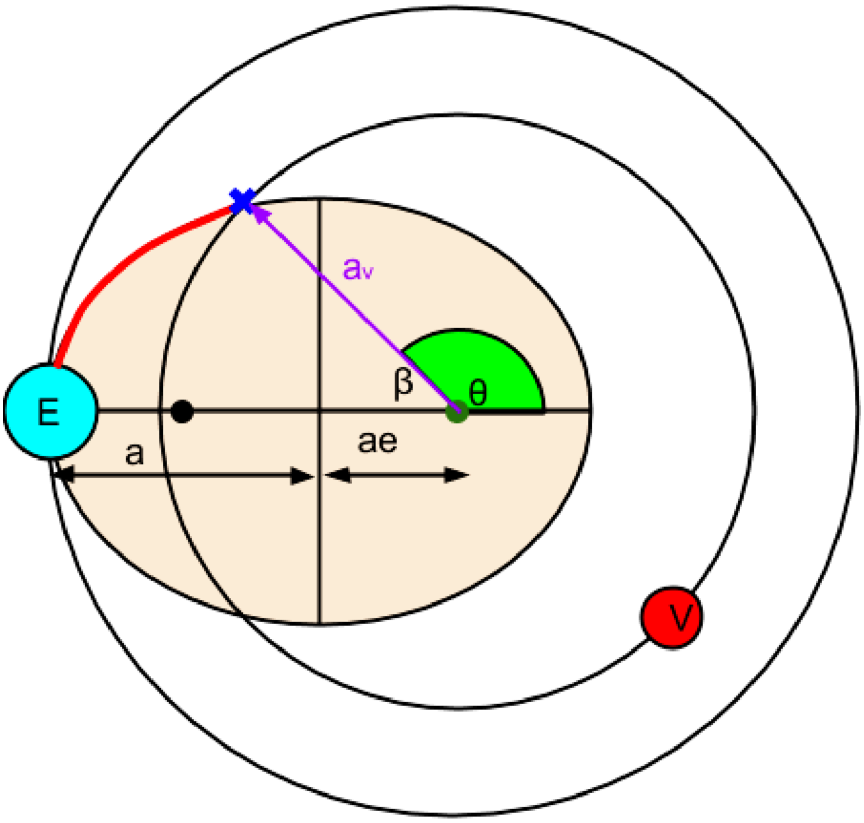
this is the time from the pericenter to the point where it crosses the venus orbit.

4. Calculate T: $T = \frac{2\pi}{n} = \frac{2\pi}{3.06 \cdot 10^{-7}} = 20\,558\,143\;s$

5. To get the time from the crossing point to the Apocenter you have to subtract (t-tp) from half of the Orbital period:

$$(t_a - t) = \frac{T}{2} - (t - t_p) = \frac{20\,558\,143\,s}{2} - 3\,688\,596\;s = 6\,590\,476\;s = 76.3\;d$$

Method 2, using mostly formulas given on the exam:



1. Eccentricity — From the sketch we have:

$$r_a = a + ae = a \cdot (1+e) \Rightarrow e = \frac{r_a}{a} - 1 = \frac{1\,AU}{0.75\,AU} - 1 = ⅓$$

2. True Anomaly — The spacecraft crosses Venus' orbit when both have the same distance from the sun.

$$a_V = a\,\frac{1-e^2}{1+e\cdot cos\theta} \Rightarrow 1 + \ e\,cos\theta = \frac{a}{a_V}\,(1-e^2)$$

$$cos\theta = \frac{1}{e}\left(\frac{a}{a_V}\,(1-e^2) - 1\right)$$

$$cos\theta = 3\left(\frac{0.75\,AU}{0.732\,AU}\left(1 - \frac{1}{3^2}\right) - 1\right) \Rightarrow \theta = 1.8067\,rad$$

3. Eccentric Anomaly

$$tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}} \cdot tan\frac{E}{2} \Rightarrow tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}} \cdot tan\frac{\theta}{2}$$

$$tan\frac{E}{2} = \sqrt{\frac{2/3}{4/3}} \cdot tan\frac{1.8067}{2} = 0.8972 \Rightarrow E = 1.4626\,rad$$

4. Mean Anomaly — M = E - e sin E = 1.4626 - ⅓ sin 1.4626 = 1.1312 rad

5. Mean Motion $n = \sqrt{\frac{\mu_{Sun}}{a^3}} = \sqrt{\frac{1.33 \cdot 10^{11}}{(0.75 \cdot 150 \cdot 10^6)^3}} = 3.0563 \cdot 10^{-7}\ s$

6. Time $\Delta t = \frac{M_\beta}{n} = \frac{M_{(\theta=\pi)} - M}{n} = \frac{\pi - M}{n} = \frac{\pi - 1.1312}{3.0563 \cdot 10^{-7}}$

$\Delta t = 6\,577\,864\ s = 1827\ h = 76.1\ d$

11) (4 points): A satellite is in an elliptical orbit around the Earth. The orbit lies in the equatorial plane (inclination = 0 deg). The satellite has to make a manoeuvre (short rocket burn) to change its inclination to 5 deg. What is the best point in the orbit to do this (minimum delta V)?

a. At perigee O

b. At "true anomaly" (theta) = 90 deg O

c. At "true anomaly" (theta) = 180 deg ●

d. Does not matter (Anywhere in the orbit) O

For an inclination change only the direction of the velocity vector has to be changed. The smaller the velocity, the smaller the required delta V. The velocity minimum is at apogee. Apogee is at theta = 180 deg.

12) (5 points): A spacecraft in orbit around Mars with a total mass (including propellants!) of 1000 kg needs to make a velocity change (Delta V) of 1 km/s. It uses a rocket engine with a specific impulse, Isp = 250 s. How much propellant is consumed during this manoeuvre?

a. 250 kg O

b. 661 kg O

c. 335 kg ●

d. 100 kg O

$$\Delta V = I_{sp} \cdot g_0 \cdot ln(\Lambda) \Rightarrow$$

$$\frac{\Delta V}{I_{sp} \cdot g_0} = ln\left(\frac{M_{begin}}{M_{end}}\right) \Rightarrow$$

$$e^{\frac{\Delta V}{I_{sp} \cdot g_0}} = \frac{M_{begin}}{M_{end}} \Rightarrow M_{end} = \frac{M_{begin}}{e^{\frac{\Delta V}{I_{sp} \cdot g_0}}} = \frac{1000}{e^{\frac{1000}{250 \cdot 9.81}}} = 665\ kg$$

$$M_{propellant} = 1000 - 665 = 335\ kg$$

For calculating the exhaust velocity you <u>always</u> have to use g_0=9.81 m/s². That's because of the definition of I_{sp}.