Introduction to Aerospace Engineering Exam (4/13/2015)

Part A:

$d = 3.95\ m$

$E = 72\ GPa$

$v = 0.3$

$t = 1.1mm = 1.1 \cdot 10^{-3}\ m$

$P_0 = 101.3\ kPa$ at 0km

$P_1 = 26.5\ kPa$ at 10km

The pressure altitude is kept at 3km

A1 $\sigma_{circ} = \dfrac{\Delta p \cdot R}{t}$

$$\boxed{\begin{aligned}\Delta p &= 79.5 - 26.5 = 53\ kPa = 53 \cdot 10^3\ Pa \\ R &= 3.95/2 = 1.975\ m \\ t &= 1.1 \cdot 10^{-3}\ m\end{aligned}}$$

$$\sigma_{circ} = \frac{53 \cdot 10^3 \cdot 1.975}{1.1 \cdot 10^{-3}}$$

$$= 95159090.91\ Pa \approx 95.2\ MPa$$

A2 $\sigma_{long} = \dfrac{\Delta p \cdot R}{2t}$, see values A1

$$\sigma_{long} = \frac{53 \cdot 10^3 \cdot 1.975}{2 \cdot 1.1 \cdot 10^{-3}} = 47579545.45\ Pa \approx 47.6\ MPa$$

A3 Given:

$\sigma_{circ} = \sigma_{long} = \sigma = 50\ MPa$

$E = 72\ GPa = 72 \cdot 10^3\ MPa$

$$\varepsilon = \frac{\sigma}{E} - v \cdot \frac{\sigma}{E} = \frac{50}{72 \cdot 10^3} - 0.3 \cdot \frac{50}{72 \cdot 10^3} = 4.8611 \cdot 10^{-4} \approx 0.00049$$

$$\approx 0.049\ \%$$

Since the fuselage is under bi-axial loading, we have to use the Poisson's ratio, v.

A4 $\sigma_{circ} = \sigma_{long}$ ("equally stressed")

So, the longitudinal strain is:

"the same as the circumferential strain."

A5 $q = \tau t = [Pa] \cdot [m] = \left[\frac{N}{m^2}\right] \cdot [m] = \left[\frac{N}{m}\right]$

A6

$$\boxed{\begin{aligned} M_T &= 1.2\cdot 10^6\ Nm \\ t &= 1.1\ mm = 1.1\cdot 10^{-3}\ m \\ d &= 3.95\ m \end{aligned}}$$

$$M_T = 2qA = 2\tau t A$$

$$\tau = \frac{M_T}{2At}$$

In this case A is the enclosed area, NOT THE CROSS-SECTIONAL AREA!

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{4}d^2$$
$$= \frac{\pi}{4}(3.95)^2 = 12.254\ m^2$$

$$\tau = \frac{1.2\cdot 10^6}{2\cdot 1.1\cdot 10^{-3}\cdot 12.254} = 44511731.91\ Pa \approx 44.5\ MPa$$

A7

$$\boxed{\begin{aligned} L &= 1m \\ W &= 800\ mm = 0.8m \\ t &= 1.1\ mm = 1.1\cdot 10^{-3}\ m \\ E &= 72\ GPa \\ \sigma_y &= 290\ MPa \\ \sigma_{ult} &= 435\ MPa \\ \varepsilon_{ult} &= 15\% \end{aligned}}$$

For this, we require the cross-sectional area:

$$A = wt = 0.8\cdot 1.1\cdot 10^{-3} = 8.8\cdot 10^{-4}\ m^2$$

$$\sigma_y = \frac{F_y}{A} \rightarrow F_y = \sigma_y A$$
$$= 290\cdot 10^3\cdot 8.8\cdot 10^{-4}$$
$$\approx 255.2\ kN$$

A8

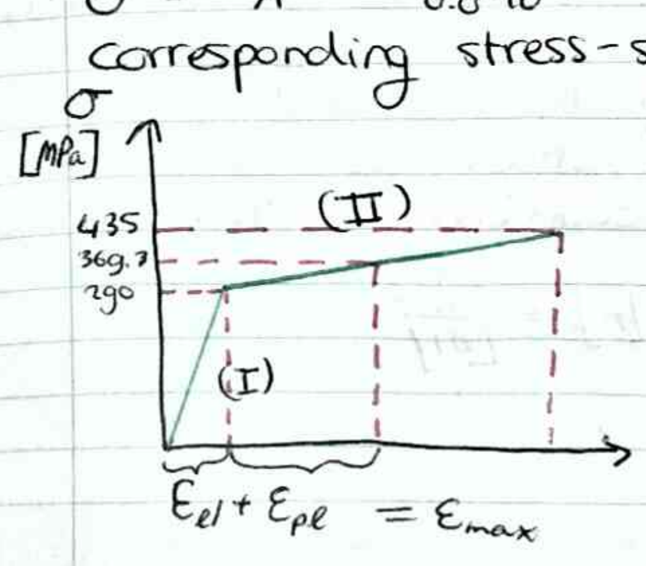
$$\varepsilon_y = \frac{\sigma_y}{E} = \frac{290}{72\cdot 10^3} = 4.0278\cdot 10^{-3} \approx 0.004 \approx 0.4\%$$

A9

The maximum strain consists of both plastic and elastic strain: $\varepsilon_{max} = \varepsilon_{pl} + \varepsilon_{el}$

$$\sigma = \frac{F}{A} = \frac{3.25\cdot 10^3}{8.8\cdot 10^{-4}} = 369318181.8\ Pa \approx 369.3\ MPa$$

corresponding stress-strain curve:



The slope of part II is:

$$a = \frac{435-290}{0.15 - 4.0278\cdot 10^{-3}} = 993.339$$

$$\varepsilon_{pl} = \frac{369.3-290}{993.339} = 0.07983$$

$$\varepsilon_{el} = \frac{\sigma_y}{E} = \frac{290}{72000} = 4.0278 \cdot 10^{-3}$$

$$\varepsilon_{max} = \varepsilon_{pl} + \varepsilon_{el} = 0.0798 + 4.0278 \cdot 10^{-3} = 0.084$$
$$\approx 8.4\%$$

A10 For permanent deformation, we need to look at the yield stress:

$$\sigma_y = \frac{F}{A} = \frac{F}{wt} \rightarrow t = \frac{F}{w\sigma_y} = \frac{325 \cdot 10^3}{0.8 \cdot 290 \cdot 10^6} = 1.4 \cdot 10^{-3} m$$
$$= 1.4 mm$$

Part B:

$m = 1200 kg$ $E = 72 GPa$ $A = 5000 mm^2$ $L = 12 m$	$I = 1 \cdot 10^9 mm^4$ $g = 9.81 m/s^2$	deflection: axial: $\delta_x = \frac{P_x L}{AE}$ lateral: $\delta_y = \frac{P_y L^3}{3EI}$

B1 Spring constant $k_x = \frac{F}{\delta_x} = \frac{P_x}{\left(\frac{P_x L}{AE}\right)} = \frac{AE}{L}$

$$k_x = \frac{AE}{L} = \frac{5000 \cdot 10^{-6} \cdot 72 \cdot 10^9}{12} = 3 \cdot 10^7 Pa$$

B2 Spring constant $k_y = \frac{F}{\delta_y} = \frac{P_y}{\left(\frac{P_y L^3}{3EI}\right)} = \frac{3EI}{L^3}$

$$k_y = \frac{3EI}{L^3} = \frac{3 \cdot 72 \cdot 10^9 \cdot 1 \cdot 10^9 \cdot 10^{-12}}{(12)^3} = 125000 Pa \approx 1.3 \cdot 10^5 Pa$$

B3 $K = \frac{F}{\delta} = \frac{[N]}{[m]} = \left[\frac{N}{m}\right]$

B4 $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi} \cdot \sqrt{\frac{3.3 \cdot 10^8}{1200}} = 83.5 Hz$

B5 $g_x = 6.5 g = 63.765 m/s^2$

$A = 5000 mm^2$

$$\sigma = \frac{F}{A} = \frac{m \cdot g_x}{A} = \frac{1200 \cdot 63.765}{5000 \cdot 10^{-6}} = 15303600 Pa \approx 15.3 MPa$$

B6 $\boxed{\begin{array}{l} R = 0.5\ m = 500\ mm \\ A = 5000\ mm^2 \end{array}}$

$$A = \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = 5000\ mm^2$$

$$r = \sqrt{\frac{\pi \cdot R^2 - A}{\pi}} = \sqrt{\frac{\pi \cdot (0.5)^2 - 5000 \cdot 10^{-6}}{\pi}} = 0.4984059\ m$$

$$= 498.4059\ mm$$

$$t = 500 - 498.41 = 1.5941\ m \approx 1.6\ mm$$

B7 $\left.\begin{array}{l} f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \\ k_x = \frac{AE}{L}\ (B1) \end{array}\right\} f_n = \frac{1}{2\pi}\sqrt{\frac{AE}{mL}}$

$$f_n \leq \frac{1}{2\pi}\sqrt{\frac{AE}{mL}} \rightarrow 31 \leq \frac{1}{2\pi}\sqrt{\frac{AE}{mL}}$$

$$194.78 \leq \sqrt{\frac{AE}{mL}}$$

$$37938.76 \leq \frac{AE}{mL}$$

$$A \geq \frac{mL}{E} \cdot 37938.76 = \frac{1200 \cdot 12}{72 \cdot 10^9} \cdot 37938.76 = 7.6 \cdot 10^{-3}\ m^2 = 7.6 \cdot 10^3\ mm^2$$

B8 $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ → reducing the mass will lead to a higher natural frequency.

B9 See answers on BrightSpace.

Part C:

C See answers on Bright Space.

Part D

D1 $h = 1\ m$

$m_{begin} = 1000\ kg$

$m_{propellant} = 300\ kg$ (consumed propellant)

$I_{sp} = 250\ s$

Also, given: $m_{propellant} = m_{begin} \cdot \left[1 - e^{-\frac{\mu \cdot \Delta t}{I_{sp} \cdot g_0 \cdot r^2}}\right]$

Rearranging the equation:

$$\Delta t = -\frac{I_{sp} \cdot g_0 \cdot r^2 \ln\left(1 - \frac{m_p}{m_b}\right)}{\mu}$$

The formula sheet gives:

$\mu_{mars} = 4.28 \cdot 10^4\ km^3/s^2$

$g_0 = 9.80665\ m/s^2 = 0.00980665\ km/s^2$

$r_{mars} = 3402\ km$

$$\Delta t = -\frac{250 \cdot 0.00980665 \cdot (3402)^2 \cdot \ln\left(1 - \frac{300}{1000}\right)}{4.28 \cdot 10^4} \simeq 236.465 \simeq 3.94\ min$$

D2 $\boxed{\begin{array}{l} h = 200\ km \\ V_{//} = 11.20\ km/s \end{array}}$

Formula sheet gives:
$\mu_{EARTH} = 398600.441\ km^3/s^2$
$R_e = 6378.361\ km$

VIS-VIVA equation:

$$\frac{1}{2}V^2 - \frac{\mu}{r_p} = -\frac{\mu}{2a}$$

$$2a = -\frac{\mu}{\frac{1}{2}V^2 - \frac{\mu}{r_p}} = -\frac{\mu}{\left(\frac{\frac{1}{2}V^2 r_p - \mu}{r_p}\right)} = -\frac{\mu r_p}{\frac{1}{2}V^2 r_p - \mu}$$

$$a = -\frac{\mu r_p}{v^2 r_p - 2\mu} = -\frac{398600.441 \cdot (6378.136 + 200)}{(11.2)^2 \cdot (6378.136 + 200) - 2 \cdot 398600.441}$$

$$a = -93776.87\ km$$

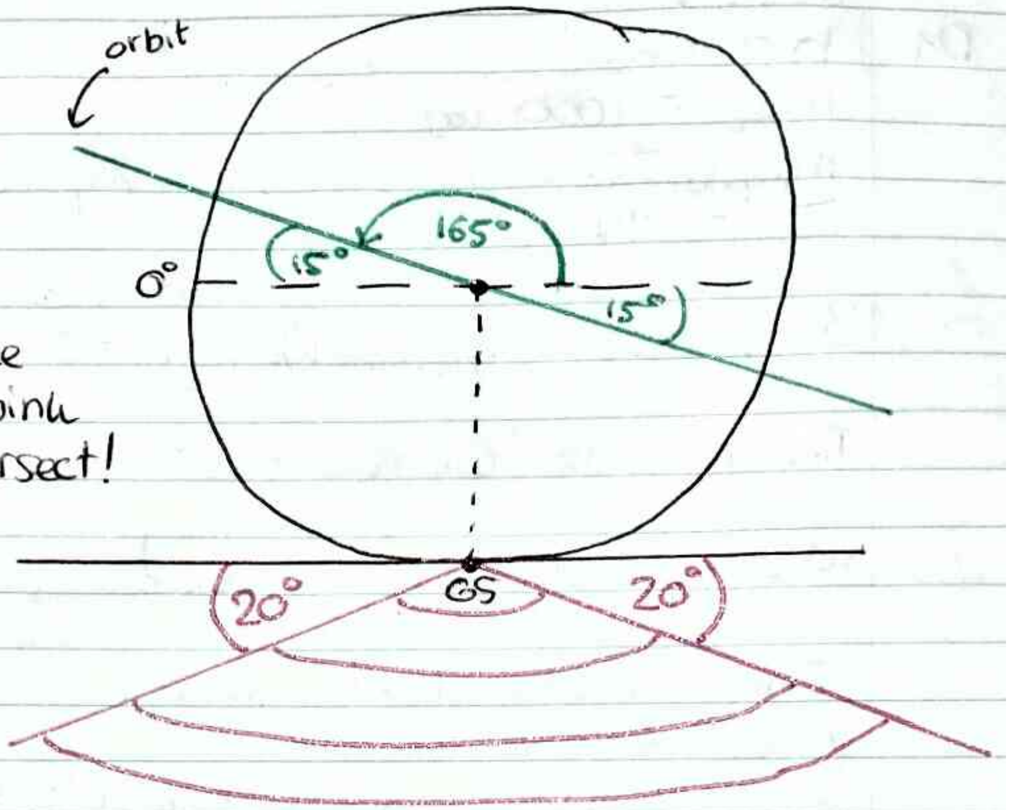
$$r_p = R_e + h = a(1 - e)$$

$$e = 1 - \frac{R_e + h}{a} = 1 - \frac{6378.136 + 200}{-93776.87} = 1.07$$

D3

$$\boxed{\begin{array}{l} i = 165^\circ \\ \varepsilon_{min} = 20^\circ \end{array}}$$

This is simply not possible since the green and pink line never intersect!



↑ GS-view

D4

$$\boxed{\begin{array}{l} m_{start} = 1000\,kg \\ m_{end} = 600\,kg \\ I_{sp} = 250\,s \\ V_{end} = 1.0906\ km/s \end{array}}$$

Use formula sheet equation:

$$V_{end} = g_0 \cdot I_{sp} \cdot \left[\ln(\Lambda) - \left(\frac{1}{\Psi_0}\right)\left(1 - \frac{1}{\Lambda}\right)\right]$$

$$\Lambda = \frac{m_{begin}}{m_{end}}$$

$$1.0906 = 0.00980665 \cdot 250 \cdot \left[\ln\left(\frac{1000}{600}\right) - \left(\frac{1}{\Psi_0}\right)\left(1 - \frac{600}{1000}\right)\right]$$

$$\Psi_0 = 6.0$$

D5 There are two kinds of days.
The one we are most familiar with is technically a solar day. A solar day is exactly 24 hours. It is defined as the amount of time it takes for the Earth to make one full rotation with respect to the Sun.
However, a sidereal day is 23h, 56 min and 4s long. It is defined as the amount of time it takes for the Earth to make one full rotation with respect to the stars.

The reason for this discrepancy is that the Earth orbits the Sun and after one full rotation, the Sun no longer occupies the same spot in the sky. Thus, answer B.

D6 $I = 1367\ W/m^2$ at 1 AU

$\eta = 0.15$

$P_{generated} = 420W$

$$P = \frac{I \cdot S \cdot \eta}{(0.5)^2} \rightarrow S = \frac{420 \cdot (0.5)^2}{1367 \cdot 0.15} \approx 0.5\ m^2$$

D7 $T = 5\ kN$

$a_{final} = 3g = 29.42\ m/s^2$

For maximum lift-off mass $T = W = 5 \cdot 10^3 N = m \cdot g.80665$, so

$$m = \frac{5 \cdot 10^3}{9.80665} \approx 510\ kg$$

Also, $T = ma$

$$m_{burn-out} = \frac{T}{a_{final}} = \frac{5 \cdot 10^3}{3 \cdot 9.80665} \approx 170\ kg$$

D8 $T_{mars} = 1.881$ years

$d_{mars-sun} = 1.524$ AU

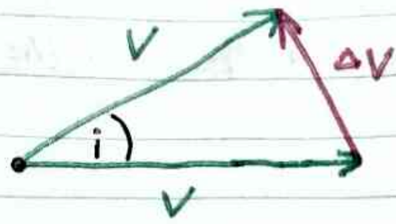
$d_{neptune-sun} = 30.110$ AU

$$T = 2\pi\sqrt{\frac{a^3}{\mu_{sun}}} \rightarrow \mu_{sun} = \frac{a^3}{\left(\frac{T}{2\pi}\right)^2} = \frac{(1.524)^3}{\left(\frac{1.881}{2\pi}\right)^2} = 39.5\ AU^3/y^2$$

$$T = 2\pi\sqrt{\frac{a^3}{\mu_{sun}}} = 2\pi\sqrt{\frac{(30.110)^3}{39.5}} \approx 165.188 \text{ years}$$

This can also be done by converting AU to km, but consistancy with the units leads to the same.

D9 Remember this: cosine-rule:



$$\Delta V^2 = V^2 + V^2 - 2V^2\cos(i)$$
$$= 2V^2 - 2V^2\cos(i) \quad (\mathrm{I})$$

However, we require the orbital velocity:

$$V_{185} = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398600.441}{6378.136+185}} = 7.793 \text{ km/s}$$

(I) gives:

$$\Delta V_{185} = \sqrt{2V^2 - 2V^2\cos(i)} = \sqrt{2\cdot(7.793)^2 - 2\cdot(7.793)^2\cos(28.5^\circ)} = 3.837 \text{ km/s}$$

$$V_{35800} = \sqrt{\frac{398600.441}{6378.136+35800}} = 3.074 \text{ km/s}$$

$$\Delta V_{35800} = \sqrt{2\cdot(3.074)^2 - 2\cdot(3.074)^2\cos(28.5^\circ)} = 1.513 \text{ km/s}$$

D10

$$\boxed{\begin{aligned} d &= 120 \text{ AU} \\ f &= 2.6 \text{ GHz} \\ P_{transmission} &= 20\text{W} \\ D &= 72 \text{ m} \end{aligned}}$$

Formula sheet:

$$D = \frac{6000d}{f}\cdot\sqrt{\frac{b}{P}}$$

$$b = \left(\frac{Df}{6000d}\right)^2 \cdot P = \left(\frac{72\cdot 2.6\cdot 10^9}{6000\cdot 120\cdot 149597870.66}\right)^2 \cdot 20 = 6\cdot 10^{-5} \text{ bits/s}$$

D11

$$\boxed{\begin{aligned} &h = 800 \text{ km} \\ &i = 62.5^\circ \\ &\text{GS at } 60^\circ \text{ north latitude} \end{aligned}}$$

Eventually, the satellite will go to a lower inclination over time. Thus, the maximum elevation is 90°.