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AE1108-II Aerospace Mechanics of Materials

17 April 2014 09.00h - 12.00h

Answer sheets

Last name and initials: Answermodel

Student no.:

NOTE:

a) **This exam consists of FOUR problems**. Each problem carries equal weight, all problems should be attempted.

b) **All problems must be solved in the provided answer sheets.** Additional sheets (ie: blank sheets for rough work) will not be accepted!

c) **Write your name and study number on EVERY page of the answer sheets**. Sheets without name or study number will not be accepted.

d) If you are in any doubt about the interpretation of the question, state the assumptions you have made in answering the question.

e) **Neatness** of presentation of your answer will be considered in the marking

f) All answers must be given with the appropriate **SI units**

Exam AE1108-II: Aerospace Mechanics of Materials

Answer sheets | Student no:

17 April 2014 | Name: Answermodel

Problem 1 (Weight 2.5 - approx. 45 minutes)
A simply supported beam ABCDE is subjected to the loads shown in Figure Q1(a). The beam has a uniform cross section comprising of a channel section adhesively bonded to a reinforcing plate as shown in Figure Q1(b).

Een scharnierend opgelegde balk ABCDE wordt belast zoals aangegeven in Figuur Q1(a). De balk heeft een uniforme doorsnede bestaande uit een hoedprofiel dat met lijm verbonden is aan een extra plaat ter versterking, zoals te zien is in Figuur Q1b.

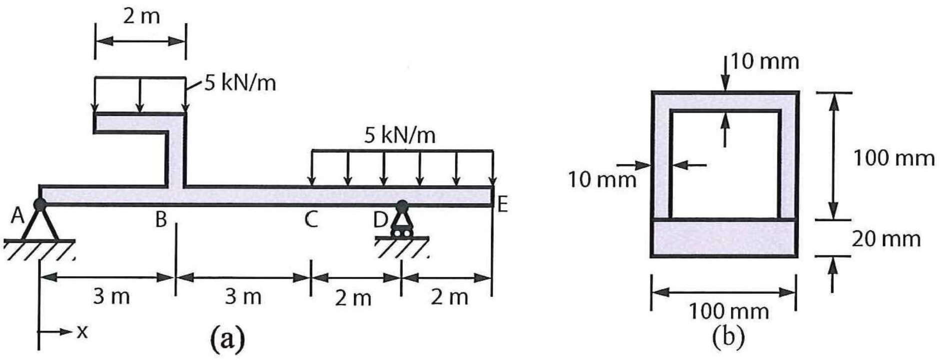

Figure Q1

Questions

a) Calculate the loading at B due to the uniformly loaded frame rigidly connected at B. Draw a Free Body Diagram of beam ABCDE on the next page indicating the reactions at B.
Bereken de belasting in punt B ten gevolge van het uniform belaste hoekframe dat in B onvervormbaar aan de balk vastzit en teken een vrijlichaamsdiagram van balk ABCDE waarin de reacties in punt B zijn getekend op de volgende pagina.

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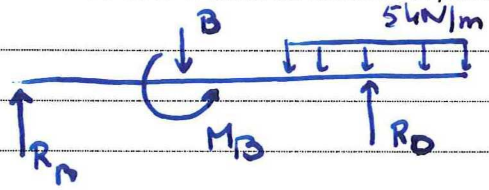
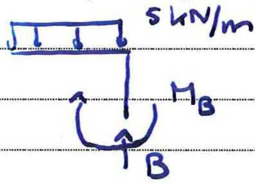




$\Sigma F_y \uparrow + : 0 = B - 10 \rightarrow \underline{B = 10\,kN}$

$\Sigma M_A \circlearrowleft + : 0 = -B \cdot 3 + M_B + R_D \cdot 8 - 5 \cdot 4 \cdot 8$

$\Sigma M_B \circlearrowleft + : 0 = -M_B + 10 \cdot 1$

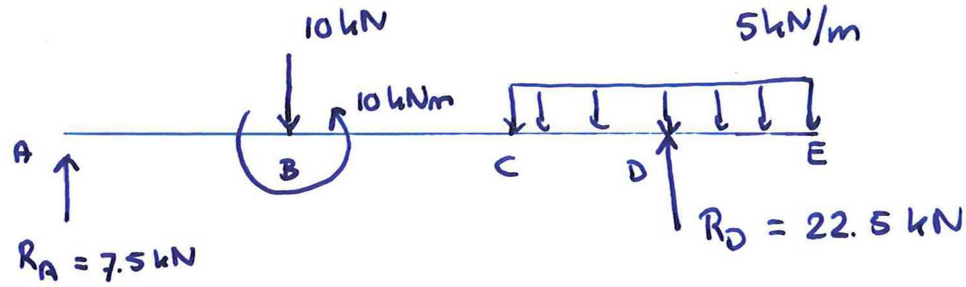
$\rightarrow \underline{M_B = 10\,kNm}$

$\rightarrow \underline{R_D = 22.5\,kN}$

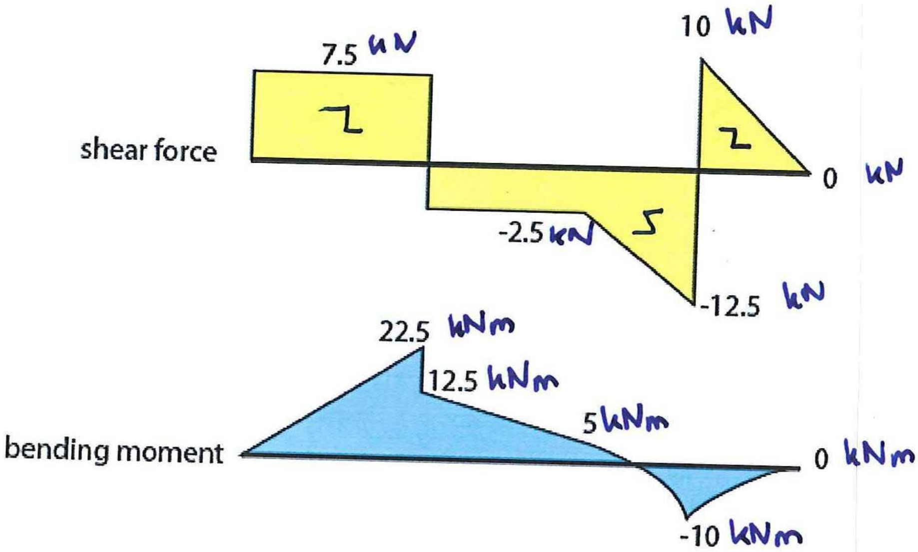
$\Sigma F_y \uparrow + : 0 = R_A + R_D - B - 5 \cdot 4$

$\rightarrow \underline{R_A = 7.5\,kN}$

(Problem 1 continued)



b) Draw the complete shear force and bending moment diagrams for the beam ABCDE below including the appropriate signs and values.
Teken de volledige dwarskrachten en momentenlijnen voor balk ABCDE hieronder inclusief de bijbehorende tekens en waardes.



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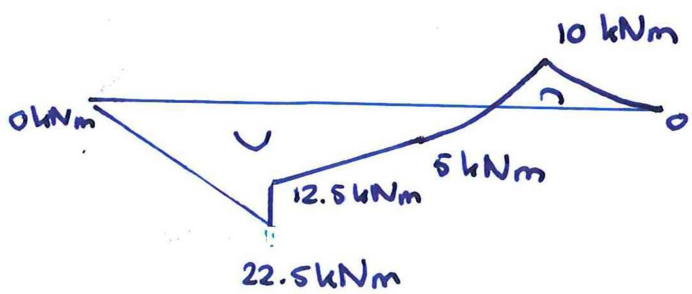
or



(Problem 1 continued)

c) Calculate the magnitude of the maximum normal stress σ in the beam and indicate where in the beam it occurs.
Bereken de grootte van de maximale normaalspanning σ die in de balk optreedt en geef aan waar deze in de balk optreedt.

$$\sigma = \frac{My}{I}$$

max. normal stress occurs when M & y are maximum

From diagram @ b) Mmax occurs just left of point B at x = 3 m, M_{max} = 22.5 kNm

Section properties:

$$\bar{y}_{wrt\ bottom} = \frac{(100 \cdot 120)\,60 - (90 \cdot 80)(20+45)}{(100 \cdot 120) - (90 \cdot 80)} = 52.5 \text{ mm from bottom}$$

$$I = \left[\frac{1}{12} \cdot (100)(120)^3 + (100)(120)(\bar{y} - 60)^2\right] - \left[\frac{1}{12} \cdot (80) \cdot (90)^3 + (80)(90)(\bar{y} - (20+45))^2\right]$$

$$= 9.09 \cdot 10^{-6} \text{ m}^4$$

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y_{max} is @ top of beam $y_{max} = 120 - \bar{y} = 67.5$ mm

$$|\sigma| = \left|\frac{M_{max}\, y_{max}}{I}\right| = \underline{167.1 \text{ MPa}}$$

(Problem 1 continued)

d) Calculate the minimum shear strength τ of the adhesive in order to prevent failure along the bonded interface. In the rounding of the answer be mindful of the design criterion.
Bereken de minimale schuifsterkte τ van de lijm om het begeven van de lijmlaag te voorkomen. Hou bij het afronden van het antwoord rekening met het ontwerpcriterium.

Necessary shear stress of the adhesive is defined as the max. shear stress occurring at adhesive interface in the beam.

$\tau = \frac{VQ}{Ib}$ $V = V_{max}$ from V-diagram: $V = -12.5\,kN$

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$Q_{ad} = 20 \cdot 100\,(\bar{y} - 10) = 8.5 \cdot 10^{-5}\ m^3$

$b_{ad} = 10 + 10 = 20\ mm$

$\tau_{min_{allow}} = \frac{V_{max}\,Q_{ad}}{I\,b_{ad}} = 5{,}844\ MPa$

(rounded up)

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Problem 2 (Weight 2.5 - approx. 45 minutes)
A steel pipe with inner diameter d_i and outer diameter d_o, and a solid aluminium rod of diameter d form the three-segment system illustrated below.
Een stalen buis met binnendiameter d_i en buitendiameter d_o en een massieve aluminium staaf met diameter d vormen het onderstaande samengestelde systeem.

Relevant values:
$L_1 = L_2 = 0.5$ m; $L_3 = 1$ m; $\delta_{gap} = 0.75$ mm; $d_o = 50$mm; $d_i = 45$ mm; $d = 20$ mm; $E_{al} = 70$ GPa; $E_{steel} = 200$ GPa; $F = 50$kN;

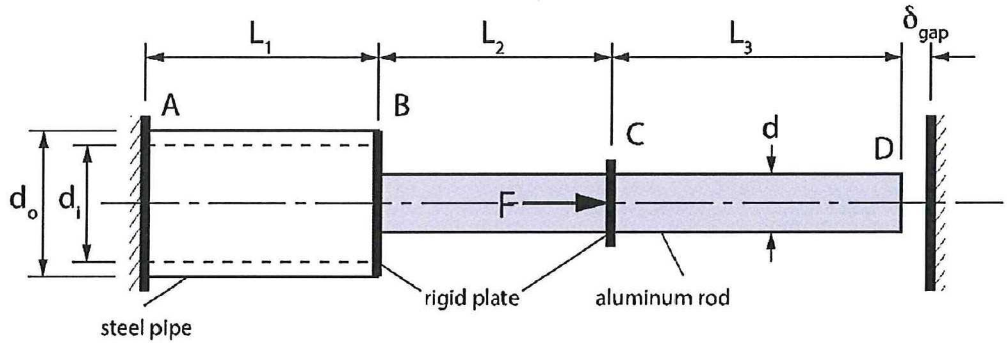

Figure Q2

Questions

a) Calculate the normal stress in the three segments due to the applied force F with the correct sign for tension (+) or compression (-) assuming D makes contact with the wall.
Bereken de normaalspanning ten gevolge van de aangebrachte kracht F in ieder van de drie segmenten met het juiste teken voor trek(+) en druk(-). Neem aan dat D contact maakt met de muur.

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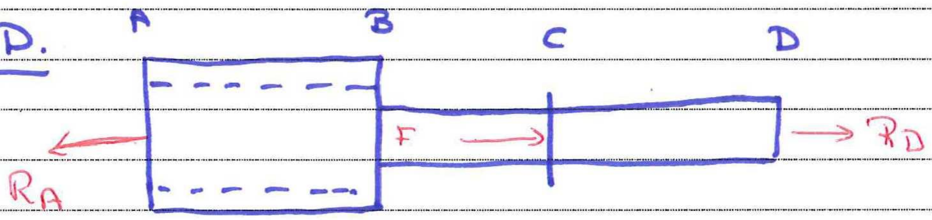
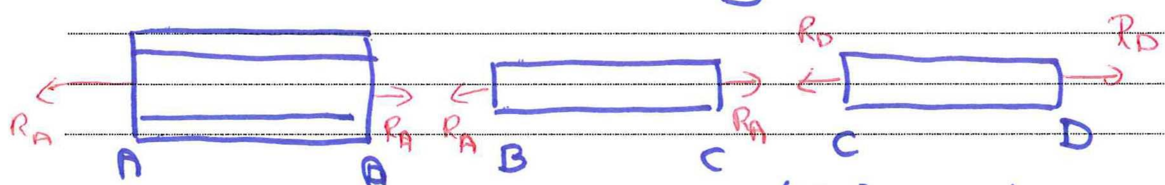
F.B.D.



Equilibrium: ΣF: $R_A = F + R_D$ (i)
1 # of stat. indeterminacy



$A_{al} = \pi\left(\frac{20}{2}\right)^2 = 314 \text{ mm}^2$ $A_{st} = \pi\left(\left(\frac{50}{2}\right)^2 - \left(\frac{45}{2}\right)^2\right) = 373 \text{ mm}^2$

(Problem 2 continued)

compatibility: $\delta_{AB} + \delta_{BC} + \delta_{CD} = \delta_{gap}$ (ii)

Force-displacement:

$\delta = \dfrac{PL}{EA} \rightarrow \quad \delta_{AB} = \dfrac{R_A L_1}{E_{st} A_{st}}$ (iii)

$\delta_{BC} = \dfrac{R_A L_2}{E_{al} E_{al}}$ (iv)

$\delta_{CD} = \dfrac{R_D L_3}{E_{al} A_{al}}$ (v)

substitute (iii), (iv) & (v) into (ii) gives

$$R_A \left(\frac{L_1}{E_{st} A_{st}} + \frac{L_2}{E_{al} A_{al}} \right) + \frac{R_D L_3}{E_{al} A_{al}} = \delta_{gap}$$

subst. (i) gives

$$(F + R_D)\left(\frac{L_1}{E_{st} A_{st}} + \frac{L_2}{E_{al} A_{al}} \right) + \frac{R_D L_3}{E_{al} A_{al}} = \delta_{gap}$$

gives: $R_D = \dfrac{\delta_{gap} - F\left(\dfrac{L_1}{E_{st} A_{st}} + \dfrac{L_2}{E_{al} A_{al}} \right)}{\left(\dfrac{L_1}{E_{st} A_{st}} + \dfrac{L_2}{E_{al} A_{al}} \right) + \dfrac{L_3}{E_{al} A_{al}}} = -9.6\,kN$

$R_A = F + R_D = 40.4\,kN$

$\sigma_{AB} = \dfrac{R_A}{A_{st}} = +108{,}3\,MPa$; $\sigma_{BC} = \dfrac{R_A}{A_{al}} = +128{,}7\,MPa$; $\sigma_{CD} = \dfrac{R_D}{A_{al}} = -30{,}5\,MPa$

(Problem 2 continued)

b) Calculate what minimum temperature change ΔT, in addition to the applied force F, would need to be applied to the entire system such that the resultant stress in segment *CD* is 0 MPa ($\alpha_{al} = 21 \times 10^{-6}/°C$; $\alpha_{steel} = 12 \times 10^{-6}/°C$). In the rounding of the answer be mindful of the design criterion.
Bereken welke minimale temperatuursverandering ΔT er, naast de al aangebrachte kracht F, te weeg moet worden gebracht in het totale systeem zodat de resulterende normaalspanning in deel CD gelijk is aan 0 MPa ($\alpha_{al} = 21 \times 10^{-6}/°C$; $\alpha_{steel} = 12 \times 10^{-6}/°C$).
Hou bij het afronden van het antwoord rekening met het ontwerpcriterium.

equilibrium & compatibility remain the same, however R_D is known

$\sigma_{CD} = 0 \rightarrow R_D = 0$ kN $\rightarrow R_A = F = 50$ kN

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$$\delta = \delta_{mech} + \delta_{thermal} = \frac{PL}{EA} + \alpha \Delta T \cdot L.$$

$$\delta_{AB} = \frac{R_A L_1}{E_{st} A_{st}} + \alpha_{st} \Delta T \cdot L_1 \quad \text{(vi)}$$

$$\delta_{BC} = \frac{R_A L_2}{E_{al} A_{al}} + \alpha_{st} \Delta T \cdot L_2 \quad \text{(vii)}$$

$$\delta_{CD} = \alpha_{al} \cdot \Delta T \cdot L_3 \quad \text{(viii)}$$

subst into comp eq. (ii) gives

$$\frac{R_A L_1}{E_{st} A_{st}} + \alpha_{steel} \Delta T L_1 + \frac{R_A L_2}{E_{al} A_{al}} + \alpha_{al} \Delta T L_2 + \alpha_{al} \Delta T L_3 = \delta_{gap}$$

$$\Delta T = \frac{\delta_{gap} - \left(\frac{R_A L_1}{E_{st} A_{st}} + \frac{R_A L_2}{E_{al} A_{al}} \right)}{\alpha_{st} L_1 + \alpha_{al} (L_2 + L_3)} = -19.25°$$

(rounded up)

(Problem 2 continued)

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Problem 3 (Weight 2.5 - approx. 45 minutes)

You are part of a small company designing a 20 m diameter two-bladed wind turbine shown schematically in Figure Q3(a). The wind turbine has a maximum power generation capacity of P = 100 kW produced at a turbine rotation speed of ω = 35 rpm.

Je bent onderdeel van een klein bedrijf dat een 20 m diameter, twee-bladige windturbine aan het ontwerpen is, zoals getoond in figuur Q3(a). De windturbine heeft een maximaal vermogenopwekkingscapaciteit van P = 100 kW dat geproduceerd wordt bij een turbine hoeksnelheid van ω = 35 rpm.

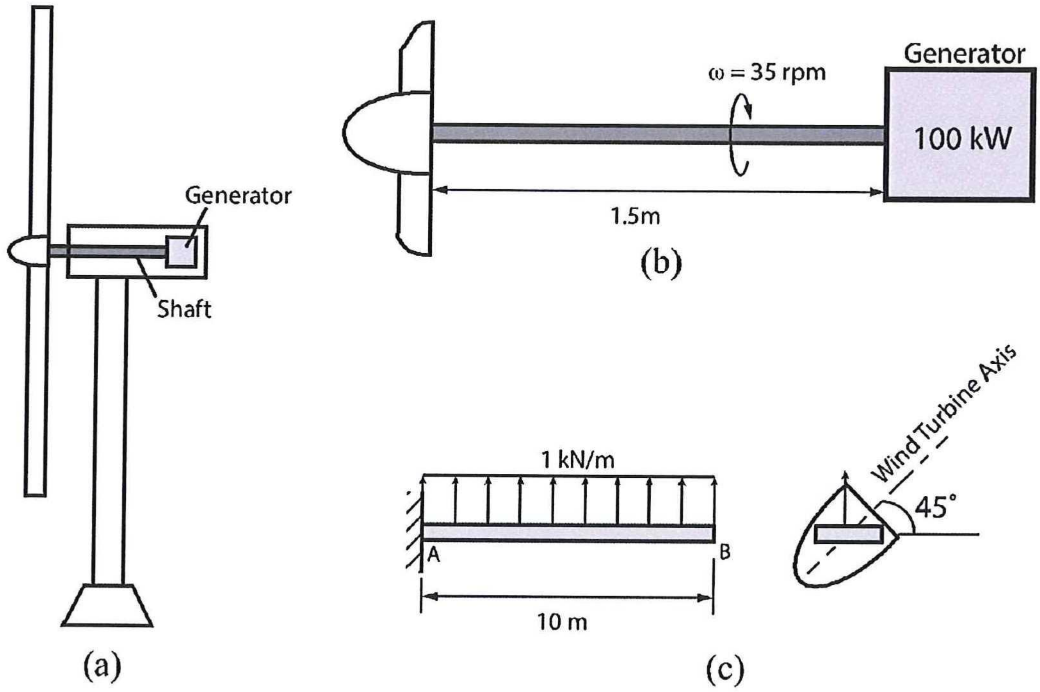

Figure Q3

(Problem 3 continued)

Questions

a) The generator is driven by a 1.5 m long shaft connected directly to the wind turbine rotor, as shown Figure Q3(b). Calculate the minimum outer diameter (to the nearest mm) of this shaft if it is constrained to an outside-diameter-to-inside-diameter ratio of $d_o/d_i = 1.25$, and a maximum shear stress on the outer surface of the shaft of 100 MPa. Shaft material properties: G = 80GPa, σ_y = 300MPa. In the rounding of the answer be mindful of the design criterion. (Note: for power transmission shafts, P = ωT; P = power, ω = shaft rotation speed, T = shaft torque)
De generator wordt aangedreven door een 1.5 m lange as, die direct aan de windturbine verbonden is, zoals getoond in figuur Q3(b). Bereken de minimale buitendiameter (op de dichtstbijzijnde mm) van deze as, als deze beperkt is tot een buiten/binnendiameter ratio van d_o/d_i = 1.25 en een maximale schuifspanning aan de buitenkant van de as van 100 MPa. Materiaal eigenschappen van de as: G = 80GPa, σ_y = 300MPa. Hou bij het afronden van het antwoord rekening met het ontwerpcriterium. (Let op: voor vermogen overbrengende assen P = ωT; P = vermogen, ω = hoeksnelheid van de as, T = torsie van de as)

$\omega = 35\text{ rpm} = 3.665 \frac{\text{rad}}{\text{s}}$ $\quad P = 100\text{ kW}$

$T = \frac{P}{\omega} = 27.284\text{ kNm}$

$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L}$ From torsion formula

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$J = \frac{\pi}{2}(r_o^4 - r_i^4)$ & $\frac{d_o}{d_i} = \frac{r_o}{r_i} = 1.25 \rightarrow r_i = 0.8 r_o$

$= \frac{\pi}{2}(r_o^4 - (0.8r_o)^4) = 0.927 r_o^4$

$\tau_{max} = 100\text{ MPa}$ $\quad \frac{\tau}{r_o} = \frac{T}{0.927 r_o^4}$

$r_o = \sqrt[3]{\frac{T}{\tau_{max}\, 0.927}} = 66.518\text{ mm} \rightarrow \underline{d_o = 134\text{ mm}}$

(Problem 3 continued)

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(Problem 3 continued)

b) Based on your shaft sizing from part a), your colleague has determined that, in addition to the shear stress due to torsion, the shaft will be experiencing a compressive axial stress of 150 MPa due to aerodynamic loading. Draw Mohr's circle for the outer surface of the shaft and determine if the shaft material yield stress of 300 MPa is sufficient for the design according to Tresca's failure criterion.
Gebaseerd op jouw berekeningen van de as uit a) heeft je collega bepaald dat, naast de schuifspanning door torsie, de as ook een axiale drukspanning ondergaat van 150 MPa ten gevolge van de aerodynamische belasting. Teken de cirkel van Mohr voor de buitenkant van de as en bepaal of de vloeispanning van het materiaal van de as van 300 MPa voldoende is voor het ontwerp volgens het Criterium van Tresca.

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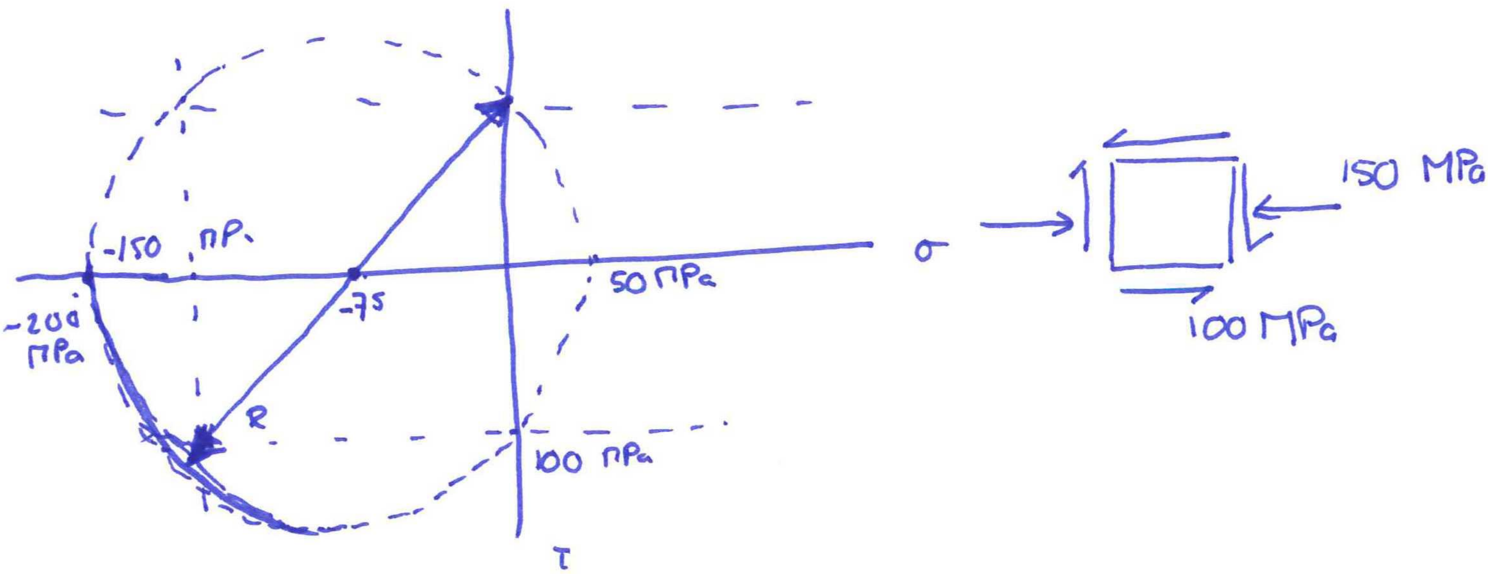

$$\sigma_{av} = \frac{0-150}{2} = -75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 125 \text{ MPa} \rightarrow \sigma_1 = 50 \text{ MPa}$$

$$\sigma_2 = -200 \text{ MPa}$$

$$\tau_{max} = 125 \text{ MPa}$$

Tresca: $\tau_{max} \leq \frac{\sigma_y}{2} = 150 \text{ MPa}$

$125 \text{ MPa} \leq 150 \text{ MPa}$ → Tresca is met

shaft will <u>not</u> fail

(Problem 3 continued)

c) Excessive deformation of the wind turbine blades can adversely affect their efficiency. A design constraint has been imposed limiting the blade tip deflection in the direction of the wind turbine axis to 0.5 m. Approximating each blade as a prismatic beam oriented 45° to the wind turbine axis, with a uniform distributed load as shown in Figure Q3(c), calculate the required flexural rigidity of the blades. In the rounding of the answer be mindful of the design criterion.
Excessieve deformatie van de windturbine bladen kan hun effectiviteit negatief beïnvloeden. Een ontwerprestrictie is opgelegd die de doorbuiging van de tip van het turbineblad in de richting van de turbine as beperkt tot 0.5m. Als we ieder blad modelleren als een prismatische balk met een verdeelde belasting onder een hoek van 45° met de windturbine as, zoals getoond in figuur Q3(c), bereken de waarde van de benodigde stijfheid van de bladen. Hou bij het afronden van het antwoord rekening met het ontwerpcriterium.

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δ_{Tip} in direction of wind turbine axis: $\delta_{T_{WTA}} = 0.5\,m$

δ_{Tip} in direction of loading: $\delta_{T_{LD}} = \frac{\delta_{T_{WTA}}}{\cos 45°} = \frac{1}{2}\sqrt{2}\,m$

$\delta = \frac{wL^4}{8EI} \rightarrow EI = \frac{wL^4}{8\delta_{Tip_{LD}}}$

$= \frac{5}{4}\sqrt{2} \cdot 10^6\ Nm^2 = 1.77 \cdot 10^6\ Nm^2$

(round up)

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Problem 4 (Weight 2.5 - approx. 45 minutes)

A uniform prismatic beam has a total length of *4L* and flexural rigidity of EI. The beam is built in at A and simply supported at B, and is subjected to a uniform distributed load, *q*, as illustrated in figure Q4.

Een uniforme prismatische balk heeft een totale lengte van 4L en een buigstijfheid EI. De balk is ingeklemd in A en wordt ondersteund door een roloplegging in B. De balk wordt onderworpen aan een uniform verdeelde belasting q zoals aangegeven in figuur Q4.

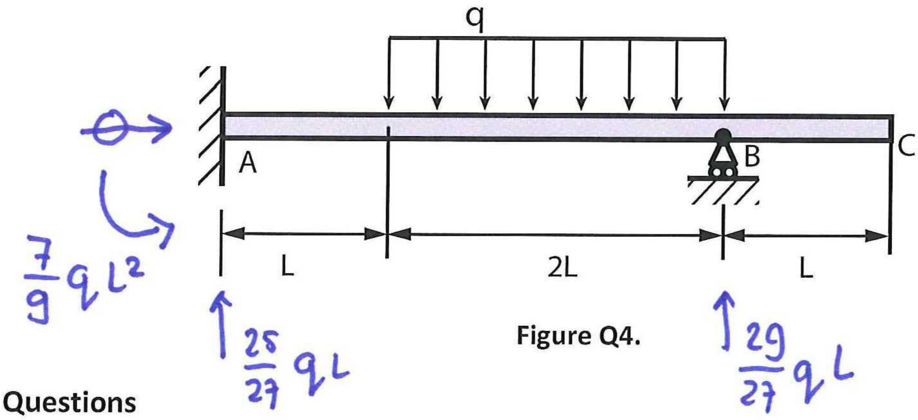


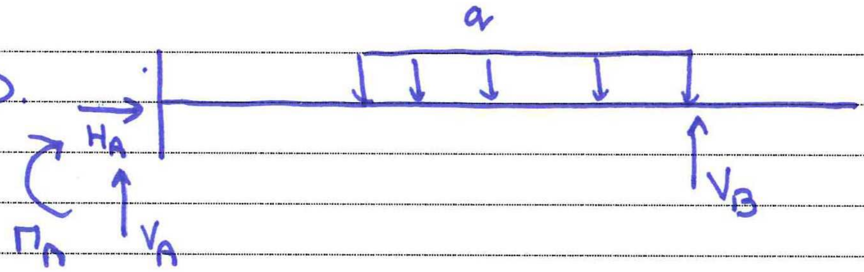
Figure Q4.

Questions

a) Calculate the reaction forces and moments at A and B and indicate them in the figure ~~below~~ above in the direction in which they act.

Bereken de reactiekrachten en momenten in A en B en geef ze aan in de bovenstaande figuur in de richting waarin ze werken.

1) F.B.D.



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2) equilibrium : 4 unknowns (H_A, V_A, M_A & V_B), 3 eq. eq
→ 1 # of stat. indeterminacy

$\Sigma F_x \overset{+}{\rightarrow} : 0 = H_A \rightarrow H_A = 0$

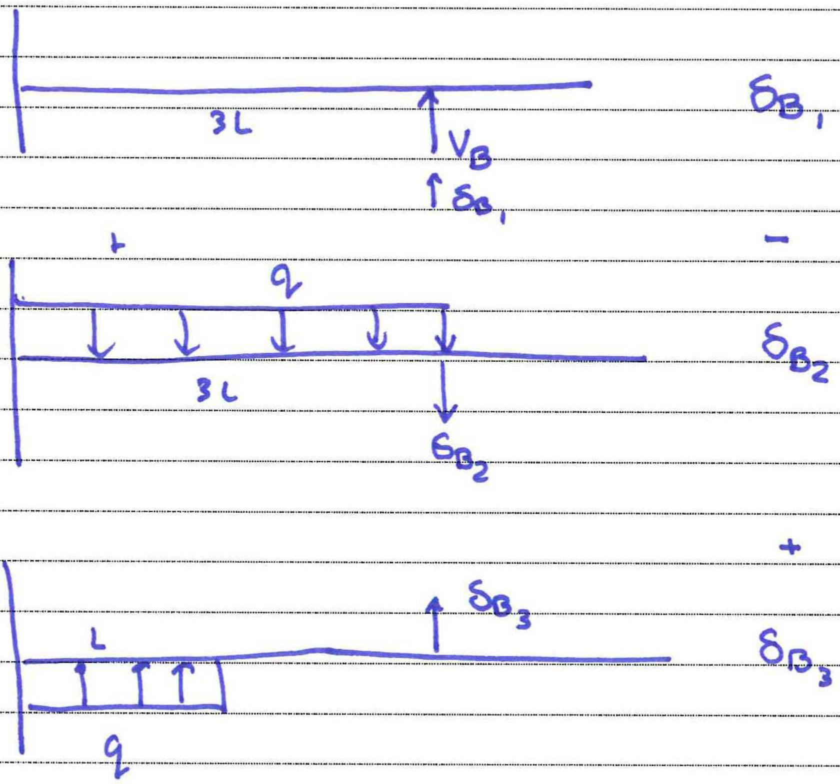
$\Sigma F_y \uparrow + : 0 = V_A + V_B - 2qL \rightarrow V_A + V_B = 2qL \quad (i)$

$\Sigma M_A \circlearrowright : 0 = M_A + q \cdot 2L \cdot 2L - V_B \cdot 3L \rightarrow M_A + 4qL^2 = 3L\, V_B \quad (ii)$

(Problem 4 continued)

3) compatibility: $\delta_B = 0$

or:



$$\delta_{B_1} = \frac{V_B \cdot (3L)^3}{3EI} \uparrow = \frac{27}{3}\frac{V_B L^3}{EI} \uparrow$$

$$\delta_{B_2} = \frac{q(3L)^4}{8EI} \downarrow = \frac{81}{8}\frac{qL^4}{EI} \downarrow$$

$$\delta_{B_3} = \frac{qL^4}{8EI} + \frac{qL^3}{6EI}\cdot 2L \uparrow = \frac{11}{24}\frac{qL^4}{EI} \uparrow$$

$$\delta_B = 0 = \frac{27}{3}\frac{V_B L^3}{EI} - \frac{81}{8}\frac{qL^4}{EI} + \frac{11}{24}\frac{qL^4}{EI} = 0$$

$$\rightarrow V_B = \frac{29}{27} qL$$

$$\rightarrow V_A = \frac{25}{27} qL$$

$$\rightarrow M_A = -\frac{7}{9} qL^2$$

(Problem 4 continued)

b) Calculate the slope and deflection of the beam at point C and indicate their direction.

Bereken de rotatie en de doorbuiging van de balk in punt C en geef de richtingen waarin ze werken aan.

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$$\theta_C = \theta_B$$

$$= \theta_{B_1} - \theta_{B_2} + \theta_{B_3}$$

$$= \frac{V_B(2L)^2}{2EI} - \frac{q(3L)^3}{6EI} + \frac{qL^3}{6EI}$$

$$= \frac{qL^3}{2EI} \; (+)$$

$$\delta_C = \delta_B + L\theta_B$$

$$= 0 + L\theta_B$$

$$= \frac{qL^4}{2EI} \; \uparrow +$$