# CHAPTER 15: Wave Motion

## Responses to Questions

1. Yes. A simple periodic wave travels through a medium, which must be in contact with or connected to the source for the wave to be generated. If the medium changes, the wave speed and wavelength can change but the frequency remains constant.

2. The speed of the transverse wave is the speed at which the wave disturbance propagates down the cord. The individual tiny pieces of cord will move perpendicular to the cord with an average speed of four times the amplitude divided by the period. The average velocity of the individual pieces of cord is zero, but the average speed is not the same as the wave speed.

3. The maximum climb distance (4.3 m) occurs when the tall boat is at a crest and the short boat is in a trough. If we define the height difference of the boats on level seas as $\Delta h$ and the wave amplitude as $A$, then $\Delta h + 2A = 4.3$ m. The minimum climb distance (2.5 m) occurs when the tall boat is in a trough and the short boat is at a crest. Then $\Delta h - 2A = 2.5$ m. Solving these two equations for $A$ gives a wave amplitude of 0.45 m.

4. (*a*) Striking the rod vertically from above will displace particles in a direction perpendicular to the rod and will set up primarily transverse waves.
   (*b*) Striking the rod horizontally parallel to its length will give the particles an initial displacement parallel to the rod and will set up primarily longitudinal waves.

5. The speed of sound in air obeys the equation $v = \sqrt{B/\rho}$. If the bulk modulus is approximately constant and the density of air decreases with temperature, then the speed of sound in air should increase with increasing temperature.

6. First, estimate the number of wave crests that pass a given point per second. This is the frequency of the wave. Then, estimate the distance between two successive crests, which is the wavelength. The product of the frequency and the wavelength is the speed of the wave.

7. The speed of sound is defined as $v = \sqrt{B/\rho}$, where $B$ is the bulk modulus and $\rho$ is the density of the material. The bulk modulus of most solids is at least $10^6$ times as great as the bulk modulus of air. This difference overcomes the larger density of most solids, and accounts for the greater speed of sound in most solids than in air.

8. One reason is that the wave energy is spread out over a larger area as the wave travels farther from the source, as can be seen by the increasing diameter of the circular wave. The wave does not gain energy as it travels, so if the energy is spread over a larger area, the amplitude of the wave must be smaller. Secondly, the energy of the wave dissipates due to damping, and the amplitude decreases.

9. If two waves have the same speed but one has half the wavelength of the other, the wave with the shorter wavelength must have twice the frequency of the other. The energy transmitted by a wave depends on the wave speed and the square of the frequency. The wave with the shorter wavelength will transmit four times the energy transmitted by the other wave.

10. Yes. Any function of $(x - vt)$ will represent wave motion because it will satisfy the wave equation, Eq. 15-16.

11. The frequency does not change at the boundary because the two sections of cord are tied to each other and they must oscillate together. The wavelength and wave speed can be different, but the frequency must remain constant across the boundary.

12. The transmitted wave has a shorter wavelength. If the wave is inverted upon reflection at the boundary between the two sections of rope, then the second section of rope must be heavier. Therefore, the transmitted wave (traveling in the heavier rope) will have a lower velocity than the incident wave or the reflected wave. The frequency does not change at the boundary, so the wavelength of the transmitted wave must also be smaller.

13. Yes, total energy is always conserved. The particles in the medium, which are set into motion by the wave, have both kinetic and potential energy. At the instant in which two waves interfere destructively, the displacement of the medium may be zero, but the particles of the medium will have velocity, and therefore kinetic energy.

14. Yes. If you touch the string at any node you will not disturb the motion. There will be nodes at each end as well as at the points one-third and two-thirds of the distance along the length of the string.

15. No. The energy of the incident and reflected wave is distributed around the antinodes, which exhibit large oscillations. The energy is a property of the wave as a whole, not of one particular point on the wave.

16. Yes. A standing wave is an example of a resonance phenomenon, caused by constructive interference between a traveling wave and its reflection. The wave energy is distributed around the antinodes, which exhibit large amplitude oscillations, even when the generating oscillations from the hand are small.

17. When a hand or mechanical oscillator vibrates a string, the motion of the hand or oscillator is not exactly the same for each vibration. This variation in the generation of the wave leads to nodes which are not quite "true" nodes. In addition, real cords have damping forces which tend to reduce the energy of the wave. The reflected wave will have a smaller amplitude than the incident wave, so the two waves will not completely cancel, and the node will not be a true node.

18. AM radio waves have a much longer wavelength than FM radio waves. How much waves bend, or diffract, around obstacles depends on the wavelength of the wave in comparison to the size of the obstacle. A hill is much larger than the wavelength of FM waves, and so there will be a "shadow" region behind the hill. However, the hill is not large compared to the wavelength of AM signals, so the AM radio waves will bend around the hill.

19. Waves exhibit diffraction. If a barrier is placed between the energy source and the energy receiver, and energy is still received, it is a good indication that the energy is being carried by waves. If placement of the barrier stops the energy transfer, it may be because the energy is being transferred by particles or that the energy is being transferred by waves with wavelengths smaller than the barrier.

## Solutions to Problems

1. The wave speed is given by $v = \lambda f$. The period is 3.0 seconds, and the wavelength is 8.0 m.

$$v = \lambda f = \lambda / T = (8.0\,\text{m})/(3.0\,\text{s}) = \boxed{2.7\,\text{m/s}}$$

2. The distance between wave crests is the wavelength of the wave.

$$\lambda = v/f = 343\,\text{m/s}/262\ \text{Hz} = \boxed{1.31\ \text{m}}$$

3. The elastic and bulk moduli are taken from Table 12-1. The densities are taken from Table 13-1.

(*a*) For water: $v = \sqrt{B/\rho} = \sqrt{\dfrac{2.0\times10^{9}\ \text{N/m}^2}{1.00\times10^{3}\ \text{kg/m}^3}} = \boxed{1400\,\text{m/s}}$

(*b*) For granite: $v = \sqrt{E/\rho} = \sqrt{\dfrac{45\times10^{9}\ \text{N/m}^2}{2.7\times10^{3}\ \text{kg/m}^3}} = \boxed{4100\,\text{m/s}}$

(*c*) For steel: $v = \sqrt{E/\rho} = \sqrt{\dfrac{200\times10^{9}\ \text{N/m}^2}{7.8\times10^{3}\ \text{kg/m}^3}} = \boxed{5100\,\text{m/s}}$

4. To find the wavelength, use $\lambda = v/f$.

AM: $\lambda_1 = \dfrac{v}{f_1} = \dfrac{3.00\times10^{8}\ \text{m/s}}{550\times10^{3}\,\text{Hz}} = 545\ \text{m}$ $\quad \lambda_2 = \dfrac{v}{f_2} = \dfrac{3.00\times10^{8}\ \text{m/s}}{1600\times10^{3}\,\text{Hz}} = 188\ \text{m}$ $\quad \boxed{\text{AM: 190 m to 550 m}}$

FM: $\lambda_1 = \dfrac{v}{f_1} = \dfrac{3.00\times10^{8}\ \text{m/s}}{88\times10^{6}\,\text{Hz}} = 3.41\,\text{m}$ $\quad \lambda_2 = \dfrac{v}{f_2} = \dfrac{3.00\times10^{8}\ \text{m/s}}{108\times10^{6}\,\text{Hz}} = 2.78\,\text{m}$ $\quad \boxed{\text{FM: 2.8 m to 3.4 m}}$

5. The speed of the longitudinal wave is given by Eq. 15-3, $v = \sqrt{E/\rho}$. The speed and the frequency are used to find the wavelength. The bulk modulus is found in Table 12-1, and the density is found in Table 13-1.

$$\lambda = \frac{v}{f} = \frac{\sqrt{\dfrac{E}{\rho}}}{f} = \frac{\sqrt{\dfrac{100\times10^{9}\ \text{N/m}^2}{7.8\times10^{3}\ \text{kg/m}^3}}}{5800\,\text{Hz}} = \boxed{0.62\,\text{m}}$$

6. To find the time for a pulse to travel from one end of the cord to the other, the velocity of the pulse on the cord must be known. For a cord under tension, we have Eq. 15-2, $v = \sqrt{F_T/\mu}$.

$$v = \frac{\Delta x}{\Delta t} = \sqrt{\frac{F_T}{\mu}} \quad\rightarrow\quad \Delta t = \frac{\Delta x}{\sqrt{\dfrac{F_T}{\mu}}} = \frac{8.0\,\text{m}}{\sqrt{\dfrac{140\,\text{N}}{(0.65\,\text{kg})/(8.0\,\text{m})}}} = \boxed{0.19\,\text{s}}$$

$\boxed{7.}$ For a cord under tension, we have from Eq. 15-2 that $v = \sqrt{F_T/\mu}$. The speed is also the displacement divided by the elapsed time, $v = \dfrac{\Delta x}{\Delta t}$. The displacement is the length of the cord.

$$v = \sqrt{\frac{F_T}{\mu}} = \frac{\Delta x}{\Delta t} \quad\rightarrow\quad F_T = \mu\frac{\ell^2}{(\Delta t)^2} = \frac{m}{\ell}\frac{\ell^2}{(\Delta t)^2} = \frac{m\ell}{(\Delta t)^2} = \frac{(0.40\,\text{kg})(7.8\,\text{m})}{(0.85\,\text{s})^2} = \boxed{4.3\,\text{N}}$$

8. The speed of the water wave is given by $v = \sqrt{B/\rho}$, where $B$ is the bulk modulus of water, from Table 12-1, and $\rho$ is the density of sea water, from Table 13-1. The wave travels twice the depth of the ocean during the elapsed time.

$$v = \frac{2\ell}{t} \rightarrow \ell = \frac{vt}{2} = \frac{t}{2}\sqrt{\frac{B}{\rho}} = \frac{2.8\,\text{s}}{2}\sqrt{\frac{2.0\times10^9\,\text{N/m}^2}{1.025\times10^3\,\text{kg/m}^3}} = \boxed{2.0\times10^3\,\text{m}}$$

9. (*a*) The speed of the pulse is given by

$$v = \frac{\Delta x}{\Delta t} = \frac{2(660\,\text{m})}{17\,\text{s}} = 77.65\,\text{m/s} \approx \boxed{78\,\text{m/s}}$$

(*b*) The tension is related to the speed of the pulse by $v = \sqrt{F_T/\mu}$. The mass per unit length of the cable can be found from its volume and density.

$$\rho = \frac{m}{V} = \frac{m}{\pi(d/2)^2\,\ell} \rightarrow$$

$$\mu = \frac{m}{\ell} = \pi\rho\left(\frac{d}{2}\right)^2 = \pi\left(7.8\times10^3\,\text{kg/m}^3\right)\left(\frac{1.5\times10^{-2}\,\text{m}}{2}\right)^2 = 1.378\,\text{kg/m}$$

$$v = \sqrt{F_T/\mu} \rightarrow F_T = v^2\mu = (77.65\,\text{m/s})^2(1.378\,\text{kg/m}) = \boxed{8300\,\text{N}}$$

10. (*a*) Both waves travel the same distance, so $\Delta x = v_1 t_1 = v_2 t_2$. We let the smaller speed be $v_1$, and the larger speed be $v_2$. The slower wave will take longer to arrive, and so $t_1$ is more than $t_2$.

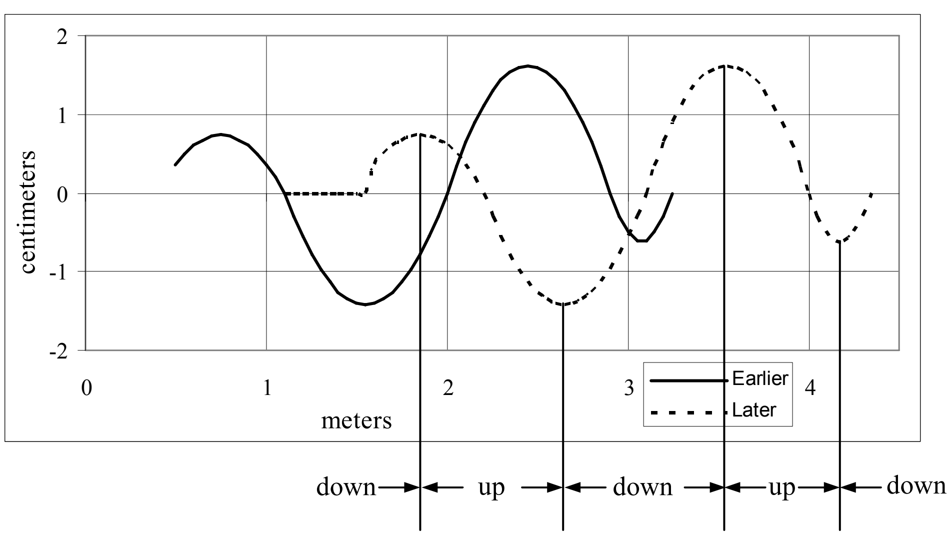
$$t_1 = t_2 + 1.7\,\text{min} = t_2 + 102\,\text{s} \rightarrow v_1(t_2 + 102\,\text{s}) = v_2 t_2 \rightarrow$$

$$t_2 = \frac{v_1}{v_2 - v_1}(102\,\text{s}) = \frac{5.5\,\text{km/s}}{8.5\,\text{km/s} - 5.5\,\text{km/s}}(102\,\text{s}) = 187\,\text{s}$$

$$\Delta x = v_2 t_2 = (8.5\,\text{km/s})(187\,\text{s}) = \boxed{1600\,\text{km}}$$

(*b*) This is not enough information to determine the epicenter. All that is known is the distance of the epicenter from the seismic station. The direction is not known, so the epicenter lies on a circle of radius $1.9\times10^3\,\text{km}$ from the seismic station. Readings from at least two other seismic stations are needed to determine the epicenter's position.

11. (*a*) The shape will not change. The wave will move 1.10 meters to the right in 1.00 seconds. See the graph. The parts of the string that are moving up or down are indicated.



(*b*) At the instant shown, the string at point A will be moving down. As the wave moves to the right, the string at point A will move down by 1 cm in the time it takes the "valley" between 1 m and 2 m to move to the right by about 0.25 m.

$$v = \frac{\Delta y}{\Delta t} = \frac{-1\,\text{cm}}{0.25\,\text{m}/1.10\,\text{m/s}} \approx \boxed{-4\,\text{cm/s}}$$

This answer will vary depending on the values read from the graph.

12. We assume that the wave will be transverse. The speed is given by Eq. 15-2. The tension in the wire is equal to the weight of the hanging mass. The linear mass density is the volume mass density times the cross-sectional area of the wire. The volume mass density is found in Table 13-1.

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{m_{\text{ball}}g}{\frac{\rho V}{\ell}}} = \sqrt{\frac{m_{\text{ball}}g}{\rho\frac{A\ell}{\ell}}} = \sqrt{\frac{(5.0\,\text{kg})(9.80\,\text{m/s}^2)}{(7800\,\text{kg/m}^3)\pi(0.50\times10^{-3}\,\text{m})^2}} = \boxed{89\,\text{m/s}}$$

$\boxed{13.}$ The speed of the waves on the cord can be found from Eq. 15-2, $v = \sqrt{F_T/\mu}$. The distance between the children is the wave speed times the elapsed time.

$$\Delta x = v\Delta t = \Delta t\sqrt{\frac{F_T}{m/\Delta x}} \rightarrow \Delta x = (\Delta t)^2\frac{F_T}{m} = (0.50\,\text{s})^2\frac{35\,\text{N}}{0.50\,\text{kg}} = \boxed{18\,\text{m}}$$

14. (*a*) We are told that the speed of the waves only depends on the acceleration due to gravity and the wavelength.

$$v = kg^\alpha\lambda^\gamma \rightarrow \left[\frac{L}{T}\right] = \left[\frac{L}{T^2}\right]^\alpha[L]^\gamma \quad T: -1 = -2\alpha \rightarrow \alpha = 1/2$$

$$L: 1 = \alpha + \gamma \rightarrow \gamma = 1 - \alpha = 1/2 \qquad \boxed{v = k\sqrt{g\lambda}}$$

(*b*) Here the speed of the waves depends only on the acceleration due to gravity and the depth of the water.

$$v = kg^\alpha h^\beta \rightarrow \left[\frac{L}{T}\right] = \left[\frac{L}{T^2}\right]^\alpha[L]^\beta \quad T: -1 = -2\alpha \rightarrow \alpha = 1/2$$

$$L: 1 = \alpha + \beta \rightarrow \beta = 1 - \alpha = 1/2 \qquad \boxed{v = k\sqrt{gh}}$$

15. From Eq. 15-7, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1 = E_2/E_1 = A_2^2/A_1^2 = 3 \rightarrow A_2/A_1 = \sqrt{3} = \boxed{1.73}$$

The more energetic wave has the larger amplitude.

16. (*a*) Assume that the earthquake waves spread out spherically from the source. Under those conditions, Eq. (15-8ab) applies, stating that intensity is inversely proportional to the square of the distance from the source of the wave.

$$I_{45\,\text{km}}/I_{15\,\text{km}} = (15\,\text{km})^2/(45\,\text{km})^2 = \boxed{0.11}$$

(*b*) The intensity is proportional to the square of the amplitude, and so the amplitude is inversely proportional to the distance from the source of the wave.

$$A_{45\,\text{km}}/A_{15\,\text{km}} = 15\,\text{km}/45\,\text{km} = \boxed{0.33}$$

17. We assume that all of the wave motion is outward along the surface of the water – no waves are propagated downwards. Consider two concentric circles on the surface of the water, centered on the place where the circular waves are generated. If there is no damping, then the power (energy per unit time) being transferred across the boundary of each of those circles must be the same. Or, the power associated with the wave must be the same at each circular boundary. The intensity depends on the amplitude squared, so for the power we have this.

$$P = I(2\pi r) = kA^2 2\pi r = \text{constant} \rightarrow A^2 = \frac{\text{constant}}{2\pi rk} \rightarrow \boxed{A \propto \frac{1}{\sqrt{r}}}$$

18. (*a*) Assuming spherically symmetric waves, the intensity will be inversely proportional to the square of the distance from the source. Thus $Ir^2$ will be constant.

$$I_{\text{near}} r_{\text{near}}^2 = I_{\text{far}} r_{\text{far}}^2 \rightarrow$$

$$I_{\text{near}} = I_{\text{far}} \frac{r_{\text{far}}^2}{r_{\text{near}}^2} = \left(3.0\times10^6\ \text{W/m}^2\right)\frac{(48\,\text{km})^2}{(1.0\,\text{km})^2} = 6.912\times10^9\ \text{W/m}^2 \approx \boxed{6.9\times10^9\ \text{W/m}^2}$$

(*b*) The power passing through an area is the intensity times the area.

$$P = IA = \left(6.912\times10^9\ \text{W/m}^2\right)\left(2.0\,\text{m}^2\right) = \boxed{1.4\times10^{10}\,\text{W}}$$

19. (*a*) The power transmitted by the wave is assumed to be the same as the output of the oscillator. That power is given by Eq. 15-6. The wave speed is given by Eq. 15-2. Note that the mass per unit length can be expressed as the volume mass density times the cross sectional area.

$$\overline{P} = 2\pi^2 \rho S v f^2 A^2 = 2\pi^2 \rho S \sqrt{\frac{F_T}{\mu}} f^2 A^2 = 2\pi^2 \rho S \sqrt{\frac{F_T}{\rho S}} f^2 A^2 = 2\pi^2 f^2 A^2 \sqrt{S\rho F_T}$$

$$= 2\pi^2 (60.0\,\text{Hz})^2 (0.0050\,\text{m})^2 \sqrt{\pi\left(5.0\times10^{-3}\,\text{m}\right)^2 \left(7800\,\text{kg/m}^3\right)(7.5\,\text{N})} = \boxed{0.38\,\text{W}}$$

(*b*) The frequency and amplitude are both squared in the equation. Thus is the power is constant, and the frequency doubles, the amplitude must be halved, and so be $\boxed{0.25\,\text{cm}}$.

20. Consider a wave traveling through an area *S* with speed *v*, much like Figure 15-11. Start with Eq. 15-7, and use Eq. 15-6.

$$I = \frac{\overline{P}}{S} = \frac{E}{St} = \frac{E\ell}{S\ell t} = \frac{E}{S\ell}\frac{\ell}{t} = \frac{\text{energy}}{\text{volume}} \times v$$

21. (*a*) We start with Eq. 15-6. The linear mass density is the mass of a given volume of the cord divided by the cross-sectional area of the cord.

$$\overline{P} = 2\pi^2 \rho S v f^2 A^2 \ ; \ \mu = \frac{m}{\ell} = \frac{\rho V}{\ell} = \frac{\rho S\ell}{\ell} = \rho S \rightarrow \overline{P} = 2\pi^2 \mu v f^2 A^2$$

(*b*) The speed of the wave is found from the given tension and mass density, according to Eq. 15-2.

$$\overline{P} = 2\pi^2 \mu v f^2 A^2 = 2\pi^2 f^2 A^2 \mu \sqrt{F_T/\mu} = 2\pi^2 f^2 A^2 \sqrt{\mu F_T}$$

$$= 2\pi^2 (120\,\text{Hz})^2 (0.020\,\text{m})^2 \sqrt{(0.10\,\text{kg/m})(135\,\text{N})} = \boxed{420\,\text{W}}$$

22. (a) The only difference is the direction of motion.

$$D(x,t) = 0.015\sin(25x + 1200t)$$

(*b*) The speed is found from the wave number and the angular frequency, Eq. 15-12.

$$v = \frac{\omega}{k} = \frac{1200\,\text{rad/s}}{25\,\text{rad/m}} = \boxed{48\,\text{m/s}}$$

23. To represent a wave traveling to the left, we replace $x$ by $x + vt$. The resulting expression can be given in various forms.

$$\boxed{\begin{aligned} D &= A\sin[2\pi(x+vt)/\lambda + \phi] = A\sin\left[2\pi\left(\frac{x}{\lambda}+\frac{vt}{\lambda}\right)+\phi\right] = A\sin\left[2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)+\phi\right] \\ &= A\sin(kx + \omega t + \phi) \end{aligned}}$$

24. The traveling wave is given by $D = 0.22\sin(5.6x + 34t)$.

(*a*) The wavelength is found from the coefficient of *x*.

$$5.6\,\text{m}^{-1} = \frac{2\pi}{\lambda} \quad\rightarrow\quad \lambda = \frac{2\pi}{5.6\,\text{m}^{-1}} = 1.122\,\text{m} \approx \boxed{1.1\,\text{m}}$$

(*b*) The frequency is found from the coefficient of *t*.

$$34\,\text{s}^{-1} = 2\pi f \quad\rightarrow\quad f = \frac{34\,\text{s}^{-1}}{2\pi} = 5.411\,\text{Hz} \approx \boxed{5.4\,\text{Hz}}$$

(*c*) The velocity is the ratio of the coefficients of *t* and *x*.

$$v = \lambda f = \frac{2\pi}{5.6\,\text{m}^{-1}}\frac{34\,\text{s}^{-1}}{2\pi} = 6.071\,\text{m/s} \approx \boxed{6.1\,\text{m/s}}$$

Because both coefficients are positive, the velocity is in the $\boxed{\text{negative } x \text{ direction}}$.

(*d*) The amplitude is the coefficient of the sine function, and so is $\boxed{0.22\,\text{m}}$.

(*e*) The particles on the cord move in simple harmonic motion with the same frequency as the wave. From Chapter 14, $v_{\text{max}} = D\omega = 2\pi fD$.

$$v_{\text{max}} = 2\pi fD = 2\pi\left(\frac{34\,\text{s}^{-1}}{2\pi}\right)(0.22\,\text{m}) = \boxed{7.5\,\text{m/s}}$$

The minimum speed is when a particle is at a turning point of its motion, at which time the speed is 0.

$$v_{\text{min}} = \boxed{0}$$

$\boxed{25.}$ The traveling wave is given by $D(x,t) = (0.026\,\text{m})\sin\left[(45\text{m}^{-1})x - (1570\text{s}^{-1})t + 0.66\right]$.

(*a*) $$v_x = \frac{\partial D(x,t)}{\partial t} = -(1570\text{s}^{-1})(0.026\,\text{m})\cos\left[(45\,\text{m}^{-1})x - (1570\,\text{s}^{-1})t + 0.66\right] \quad\rightarrow$$

$$(v_x)_{\text{max}} = (1570\text{s}^{-1})(0.026\text{m}) = \boxed{41\,\text{m/s}}$$

(*b*) $$a_x = \frac{\partial^2 D(x,t)}{\partial t^2} = -(1570\text{s}^{-1})^2(0.026\,\text{m})\sin\left[(45\,\text{m}^{-1})x - (1570\,\text{s}^{-1})t + 0.66\right] \quad\rightarrow$$

$$(a_x)_{\text{max}} = (1570\text{s}^{-1})^2(0.026\,\text{m}) = \boxed{6.4\times10^4\,\text{m/s}^2}$$

($c$) $v_x(1.00\,\text{m}, 2.50\,\text{s}) = -(1570\,\text{s}^{-1})(0.026\,\text{m})\cos\left[(45\,\text{m}^{-1})(1.00\,\text{m}) - (1570\,\text{s}^{-1})(2.50\,\text{s}) + 0.66\right]$

$$= \boxed{35\,\text{m/s}}$$

$$a_x(1.00\,\text{m}, 2.50\,\text{s}) = -(1570\,\text{s}^{-1})^2(0.026\,\text{m})\sin\left[(45\,\text{m}^{-1})(1.00\,\text{m}) - (1570\,\text{s}^{-1})(2.50\,\text{s}) + 0.66\right]$$

$$= \boxed{3.2\times10^4\,\text{m/s}^2}$$

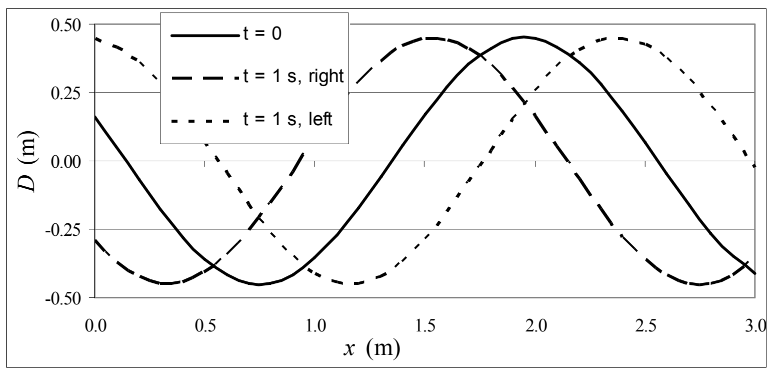
26. The displacement of a point on the cord is given by the wave, $D(x,t) = 0.12\sin(3.0x - 15.0t)$. The velocity of a point on the cord is given by $\dfrac{\partial D}{\partial t}$.

$$D(0.60\,\text{m}, 0.20\,\text{s}) = (0.12\,\text{m})\sin\left[(3.0\text{m}^{-1})(0.60\,\text{m}) - (15.0\,\text{s}^{-1})(0.20\,\text{s})\right] = \boxed{-0.11\,\text{m}}$$

$$\frac{\partial D}{\partial t} = (0.12\,\text{m})(-15.0\,\text{s}^{-1})\cos(3.0x - 15.0t)$$

$$\frac{\partial D}{\partial t}(0.60\,\text{m}, 0.20\,\text{s}) = (0.12\,\text{m})(-15.0\,\text{s}^{-1})\cos\left[(3.0\text{m}^{-1})(0.60\,\text{m}) - (15.0\,\text{s}^{-1})(0.20\,\text{s})\right] = \boxed{-0.65\,\text{m/s}}$$

27. (*a*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.27a."




(*b*) For motion to the right, replace $x$ by $x - vt$.

$$\boxed{D(x,t) = (0.45\,\text{m})\cos[2.6(x - 2.0t) + 1.2]}$$

(*c*) See the graph above.

(*d*) For motion to the left, replace $x$ by $x + vt$. Also see the graph above.

$$\boxed{D(x,t) = (0.45\,\text{m})\cos[2.6(x + 2.0t) + 1.2]}$$

28. (*a*) The wavelength is the speed divided by the frequency.

$$\lambda = \frac{v}{f} = \frac{345\,\text{m/s}}{524\,\text{Hz}} = \boxed{0.658\,\text{m}}$$

(*b*) In general, the phase change in degrees due to a time difference is given by $\dfrac{\Delta\phi}{360^\circ} = \dfrac{\Delta t}{T}$.

$$\frac{\Delta\phi}{360^\circ} = \frac{\Delta t}{T} = f\Delta t \rightarrow \Delta t = \frac{1}{f}\frac{\Delta\phi}{360^\circ} = \frac{1}{524\,\text{Hz}}\left(\frac{90^\circ}{360^\circ}\right) = \boxed{4.77\times10^{-4}\,\text{s}}$$

(*c*) In general, the phase change in degrees due to a position difference is given by $\frac{\Delta\phi}{360^\circ} = \frac{\Delta x}{\lambda}$.

$$\frac{\Delta\phi}{360^\circ} = \frac{\Delta x}{\lambda} \quad \rightarrow \quad \Delta\phi = \frac{\Delta x}{\lambda}(360^\circ) = \frac{0.044\,\text{m}}{0.658\,\text{m}}(360^\circ) = \boxed{24.1^\circ}$$

29. The amplitude is 0.020 cm, the wavelength is 0.658 m, and the frequency is 524 Hz. The displacement is at its most negative value at $x = 0$, $t = 0$, and so the wave can be represented by a cosine that is phase shifted by half of a cycle.

$$D(x,t) = A\cos(kx - \omega t + \phi)$$

$$A = 0.020\,\text{cm}\ ;\ k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{2\pi(524\,\text{Hz})}{345\,\text{m/s}} = 9.54\,\text{m}^{-1}\ ;\ \omega = 2\pi f = 2\pi(524\,\text{Hz}) = 3290\,\text{rad/s}$$

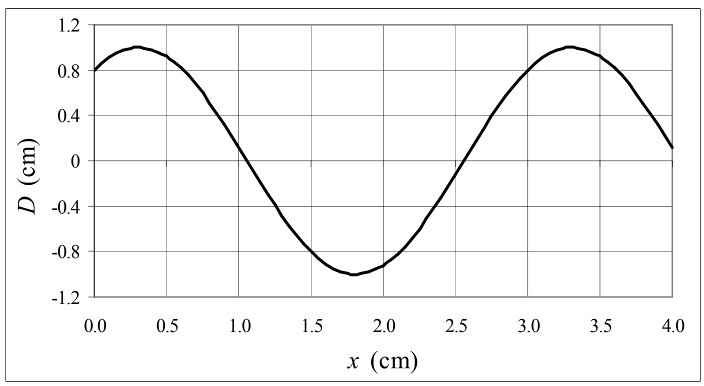
$$\boxed{D(x,t) = (0.020\,\text{cm})\cos\left[(9.54\,\text{m}^{-1})x - (3290\,\text{rad/s})t + \pi\right],\ x \text{ in m, } t \text{ in s}}$$

Other equivalent expressions include the following.

$$D(x,t) = -(0.020\,\text{cm})\cos\left[(9.54\,\text{m}^{-1})x - (3290\,\text{rad/s})t\right]$$

$$D(x,t) = (0.020\,\text{cm})\sin\left[(9.54\,\text{m}^{-1})x - (3290\,\text{rad/s})t + \tfrac{3}{2}\pi\right]$$

30. (*a*) For the particle of string at $x = 0$, the displacement is not at the full amplitude at $t = 0$. The particle is moving upwards, and so a maximum is approaching from the right. The general form of the wave is given by $D(x,t) = A\sin(kx + \omega t + \phi)$. At $x = 0$ and $t = 0$, $D(0,0) = A\sin\phi$ and so we can find the phase angle.




$$D(0,0) = A\sin\phi \quad \rightarrow \quad 0.80\,\text{cm} = (1.00\,\text{cm})\sin\phi \quad \rightarrow \quad \phi = \sin^{-1}(0.80) = 0.93$$

So we have $D(x,0) = A\sin\left(\frac{2\pi}{3.0}x + 0.93\right)$, $x$ in cm. See the graph. It matches the description given earlier. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.30a."

(*b*) We use the given data to write the wave function. Note that the wave is moving to the right, and that the phase angle has already been determined.

$$D(x,t) = A\sin(kx + \omega t + \phi)$$

$$A = 1.00\,\text{cm}\ ;\ k = \frac{2\pi}{3.00\,\text{cm}} = 2.09\,\text{cm}^{-1}\ ;\ \omega = 2\pi f = 2\pi(245\,\text{Hz}) = 1540\,\text{rad/s}$$

$$\boxed{D(x,t) = (1.00\,\text{cm})\sin\left[(2.09\,\text{cm}^{-1})x + (1540\,\text{rad/s})t + 0.93\right],\ x \text{ in cm, } t \text{ in s}}$$

31. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$D = A\sin kx \cos\omega t$$

$$\frac{\partial D}{\partial x} = kA\cos kx\cos\omega t \ ; \ \frac{\partial^2 D}{\partial x^2} = -k^2 A\sin kx\cos\omega t$$

$$\frac{\partial D}{\partial t} = -\omega A\sin kx\sin\omega t \ ; \ \frac{\partial^2 D}{\partial t^2} = -\omega^2 A\sin kx\cos\omega t$$

This gives $\frac{\partial^2 D}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 D}{\partial t^2}$, and since $v = \frac{\omega}{k}$ from Eq. 15-12, we have $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

Yes, the function is a solution.

32. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

(*a*) $D = A\ln(x + vt)$

$$\frac{\partial D}{\partial x} = \frac{A}{x+vt} \ ; \ \frac{\partial^2 D}{\partial x^2} = -\frac{A}{(x+vt)^2} \ ; \ \frac{\partial D}{\partial t} = \frac{Av}{x+vt} \ ; \ \frac{\partial^2 D}{\partial t^2} = -\frac{Av^2}{(x+vt)^2}$$

This gives $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so yes, the function is a solution.

(*b*) $D = (x - vt)^4$

$$\frac{\partial D}{\partial x} = 4(x-vt)^3 \ ; \ \frac{\partial^2 D}{\partial x^2} = 12(x-vt)^2 \ ; \ \frac{\partial D}{\partial t} = -4v(x-vt)^3 \ ; \ \frac{\partial^2 D}{\partial t^2} = 12v^2(x-vt)^2$$

This gives $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so yes, the function is a solution.

33. We find the various derivatives for the function from Eq. 15-13c.

$$D(x,t) = A\sin(kx + \omega t) \ ; \ \frac{\partial D}{\partial x} = Ak\cos(kx+\omega t) \ ; \ \frac{\partial^2 D}{\partial x^2} = -Ak^2\sin(kx+\omega t);$$

$$\frac{\partial D}{\partial t} = A\omega\cos(kx+\omega t) \ ; \ \frac{\partial^2 D}{\partial t^2} = -A\omega^2\sin(kx+\omega t)$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2} \ \rightarrow \ -Ak^2\sin(kx+\omega t) = \frac{1}{v^2}\left(-A\omega^2\sin(kx+\omega t)\right) \ \rightarrow \ k^2 = \frac{\omega^2}{v^2}$$

Since $v = \omega/k$, the wave equation is satisfied.

We find the various derivatives for the function from Eq. 15-15. Make the substitution that $u = x + vt$, and then use the chain rule.

$$D(x,t) = D(x+vt) = D(u) \quad ; \quad \frac{\partial D}{\partial x} = \frac{dD}{du}\frac{\partial u}{\partial x} = \frac{dD}{du} \quad ; \quad \frac{\partial^2 D}{\partial x^2} = \frac{\partial}{\partial x}\frac{dD}{du} = \left(\frac{d}{dx}\frac{dD}{du}\right)\frac{\partial u}{\partial x} = \frac{d^2 D}{du^2}$$

$$\frac{\partial D}{\partial t} = \frac{dD}{du}\frac{\partial u}{\partial t} = v\frac{dD}{du} \quad ; \quad \frac{\partial^2 D}{\partial t^2} = \frac{\partial}{\partial t}\left(v\frac{dD}{du}\right) = v\frac{\partial}{\partial t}\frac{dD}{du} = v\left(\frac{d}{du}\frac{dD}{du}\right)\frac{\partial u}{\partial t} = v\frac{d^2 D}{du^2}v = v^2\frac{d^2 D}{du^2}$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2} \quad \rightarrow \quad \frac{d^2 D}{du^2} = \frac{1}{v^2}v^2\frac{d^2 D}{du^2} = \frac{d^2 D}{du^2}$$

Since we have an identity, the wave equation is satisfied.

34. Find the various derivatives for the linear combination.

$$D(x,t) = C_1 D_1 + C_2 D_2 = C_1 f_1(x,t) + C_2 f_2(x,t)$$

$$\frac{\partial D}{\partial x} = C_1\frac{\partial f_1}{\partial x} + C_2\frac{\partial f_2}{\partial x} \quad ; \quad \frac{\partial^2 D}{\partial x^2} = C_1\frac{\partial^2 f_1}{\partial x^2} + C_2\frac{\partial^2 f_2}{\partial x^2}$$

$$\frac{\partial D}{\partial t} = C_1\frac{\partial f_1}{\partial t} + C_2\frac{\partial f_2}{\partial t} \quad ; \quad \frac{\partial^2 D}{\partial t^2} = C_1\frac{\partial^2 f_1}{\partial t^2} + C_2\frac{\partial^2 f_2}{\partial t^2}$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$. Use the fact that both $f_1$ and $f_2$ satisfy the wave equation.

$$\frac{\partial^2 D}{\partial x^2} = C_1\frac{\partial^2 f_1}{\partial x^2} + C_2\frac{\partial^2 f_2}{\partial x^2} = C_1\left[\frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2}\right] + C_2\left[\frac{1}{v^2}\frac{\partial^2 f_2}{\partial t^2}\right] = \frac{1}{v^2}\left[C_1\frac{\partial^2 f_1}{\partial t^2} + C_2\frac{\partial^2 f_2}{\partial t^2}\right] = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

Thus we see that $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so *D* satisfies the wave equation.

35. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$D = e^{-(kx-\omega t)^2} \quad ; \quad \frac{\partial D}{\partial x} = -2k(kx-\omega t)e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial x^2} = -2k(kx-\omega t)\left[-2k(kx-\omega t)e^{-(kx-\omega t)^2}\right] + \left(-2k^2\right)e^{-(kx-\omega t)^2} = 2k^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2}$$

$$\frac{\partial D}{\partial t} = 2\omega(kx-\omega t)e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial t^2} = 2\omega(kx-\omega t)\left[2\omega(kx-\omega t)e^{-(kx-\omega t)^2}\right] + \left(-2\omega^2\right)e^{-(kx-\omega t)^2} = 2\omega^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2} \quad \rightarrow \quad 2k^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2} = \frac{1}{v^2}2\omega^2\left[2(kx-\omega t)^2 - 1\right]e^{-(kx-\omega t)^2} \quad \rightarrow$$

$$k^2 = \frac{\omega^2}{v^2}$$

Since $v = \frac{\omega}{k}$, we have an identity. Yes, the function is a solution.

36. We assume that $A \ll \lambda$ for the wave given by $D = A\sin(kx - \omega t)$.

$$D = A\sin(kx - \omega t) \rightarrow v' = \frac{\partial D}{\partial t} = -\omega A\cos(kx - \omega t) \rightarrow v'_{max} = \omega A$$

$$A \ll \lambda \rightarrow \frac{v'_{max}}{\omega} \ll \lambda \rightarrow v'_{max} \ll \omega\lambda = v_{wave} \rightarrow \boxed{v'_{max} \ll v_{wave}}$$

$$\frac{v'_{max}}{v} = \frac{\omega A}{v} = \frac{2\pi f A}{v} = \frac{2\pi f \frac{\lambda}{100}}{f\lambda} = \boxed{\frac{\pi}{50} \approx 0.063}$$

37. (*a*) For the wave in the lighter cord, $D(x,t) = (0.050\,\text{m})\sin\left[\left(7.5\,\text{m}^{-1}\right)x - \left(12.0\,\text{s}^{-1}\right)t\right]$.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\left(7.5\,\text{m}^{-1}\right)} = \boxed{0.84\,\text{m}}$$

(*b*) The tension is found from the velocity, using Eq. 15-2.

$$v = \sqrt{\frac{F_T}{\mu}} \rightarrow F_T = \mu v^2 = \mu\frac{\omega^2}{k^2} = (0.10\,\text{kg/m})\frac{\left(12.0\,\text{s}^{-1}\right)^2}{\left(7.5\,\text{m}^{-1}\right)^2} = \boxed{0.26\,\text{N}}$$

(*c*) The tension and the frequency do not change from one section to the other.

$$F_{T1} = F_{T2} \rightarrow = \mu_1\frac{\omega_1^2}{k_1^2} = \mu_2\frac{\omega_2^2}{k_2^2} \rightarrow \lambda_2 = \lambda_1\sqrt{\frac{\mu_1}{\mu_2}} = \frac{2\pi}{k_1}\sqrt{\frac{\mu_1}{\mu_2}} = \frac{2\pi}{\left(7.5\,\text{m}^{-1}\right)}\sqrt{0.5} = \boxed{0.59\,\text{m}}$$

38. (*a*) The speed of the wave in a stretched cord is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The tensions must be the same in both parts of the cord. If they were not the same, then the net longitudinal force on the joint between the two parts would not be zero, and the joint would have to accelerate along the length of the cord.

$$v = \sqrt{F_T/\mu} \rightarrow \frac{v_H}{v_L} = \frac{\sqrt{F_T/\mu_H}}{\sqrt{F_T/\mu_L}} = \boxed{\sqrt{\frac{\mu_L}{\mu_H}}}$$

(*b*) The frequency must be the same in both sections. If it were not, then the joint between the two sections would not be able to keep the two sections together. The ends could not stay in phase with each other if the frequencies were different.

$$f = \frac{v}{\lambda} \rightarrow \frac{v_H}{\lambda_H} = \frac{v_L}{\lambda_L} \rightarrow \frac{\lambda_H}{\lambda_L} = \frac{v_H}{v_L} = \boxed{\sqrt{\frac{\mu_L}{\mu_H}}}$$

(*c*) The ratio under the square root sign is less than 1, and so the $\boxed{\text{lighter cord}}$ has the greater wavelength.

39. (*a*) The distance traveled by the reflected sound wave is found from the Pythagorean theorem.

$$d = 2\sqrt{D^2 + \left(\tfrac{1}{2}x\right)^2} = vt \rightarrow \boxed{t = \frac{2}{v}\sqrt{D^2 + \left(\tfrac{1}{2}x\right)^2}}$$

(*b*) Solve for $t^2$.

$$t^2 = \frac{4}{v^2}\left[D^2 + \left(\tfrac{1}{2}x\right)^2\right] = \frac{x^2}{v^2} + \frac{4}{v^2}D^2$$

A plot of $t^2$ vs $x^2$ would have a slope of $1/v^2$, which can be used to determine the value of *v*. The *y* intercept of that plot is $\frac{4}{v^2}D^2$. Knowing the *y* intercept and the value of *v*, the value of *D* can be determined.

40. The tension and the frequency do not change from one side of the knot to the other.

(*a*) We force the cord to be continuous at $x = 0$ for all times. This is done by setting the initial wave plus the reflected wave (the displacement of a point infinitesimally to the LEFT of $x = 0$) equal to the transmitted wave (the displacement of a point infinitesimally to the RIGHT of $x = 0$) for all times. We also use the facts that $\sin(-\theta) = -\sin\theta$ and $k_1 v_1 = k_2 v_2$.

$$D(0,t) + D_R(0,t) = D_T(0,t) \rightarrow A\sin(-k_1 v_1 t) + A_R \sin(k_1 v_1 t) = A_T \sin(-k_2 v_2 t) \rightarrow$$
$$-A\sin(k_1 v_1 t) + A_R \sin(k_1 v_1 t) = -A_T \sin(k_2 v_2 t) = -A_T \sin(k_1 v_1 t) \rightarrow$$
$$-A + A_R = -A_T \rightarrow \boxed{A = A_T + A_R}$$

(*b*) To make the slopes match for all times, we must have $\frac{\partial}{\partial x}\left[D(x,t) + D_R(x,t)\right] = \frac{\partial}{\partial x}\left[D_T(x,t)\right]$ when evaluated at the origin. We also use the result of the above derivation, and the facts that $\cos(-\theta) = \cos\theta$ and $k_1 v_1 = k_2 v_2$.

$$\frac{\partial}{\partial x}\left[D(x,t) + D_R(x,t)\right]\Big|_{x=0} = \frac{\partial}{\partial x}\left[D_T(x,t)\right]\Big|_{x=0} \rightarrow$$
$$k_1 A\cos(-k_1 v_1 t) + k_1 A_R \cos(k_1 v_1 t) = k_2 A_T \cos(-k_2 v_2 t) \rightarrow$$
$$k_1 A\cos(k_1 v_1 t) + k_1 A_R \cos(k_1 v_1 t) = k_2 A_T \cos(k_2 v_2 t) \rightarrow$$
$$k_1 A + k_1 A_R = k_2 A_T = k_2(A - A_R) \rightarrow \boxed{A_R = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)A}$$

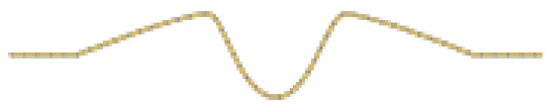
Use $k_2 = k_1 \frac{v_1}{v_2}$.

$$A_R = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)A = \left(\frac{k_1\frac{v_1}{v_2} - k_1}{k_1\frac{v_1}{v_2} + k_1}\right)A = \frac{k_1}{k_1}\left(\frac{\frac{v_1}{v_2} - 1}{\frac{v_1}{v_2} + 1}\right)A = \left(\frac{\frac{v_1}{v_2} - \frac{v_2}{v_2}}{\frac{v_1}{v_2} + \frac{v_2}{v_2}}\right)A = \boxed{\left(\frac{v_1 - v_2}{v_1 + v_2}\right)A}$$

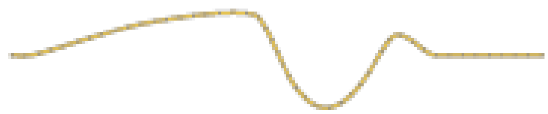
(*c*) Combine the results from the previous two parts.

$$A_T = A - A_R = A - \left(\frac{k_2 - k_1}{k_2 + k_1}\right)A = A\left[1 - \left(\frac{k_2 - k_1}{k_2 + k_1}\right)\right] = A\left[\left(\frac{k_2 + k_1}{k_2 + k_1}\right) - \left(\frac{k_2 - k_1}{k_2 + k_1}\right)\right] = \boxed{\left(\frac{2k_1}{k_2 + k_1}\right)A}$$
$$= \left(\frac{2k_1}{k_1\frac{v_1}{v_2} + k_1}\right)A = \boxed{\left(\frac{2v_2}{v_1 + v_2}\right)A}$$

41. (*a*)



(*b*)



(*c*) The energy is [all kinetic energy] at the moment when the string has no displacement. There is no elastic potential energy at that moment. Each piece of the string has speed but no displacement.

42. (*a*) The resultant wave is the algebraic sum of the two component waves.

$$D = D_1 + D_2 = A\sin(kx-\omega t) + A\sin(kx-\omega t+\phi) = A[\sin(kx-\omega t) + A\sin(kx-\omega t+\phi)]$$
$$= A\{2\sin\tfrac{1}{2}[(kx-\omega t)+(kx-\omega t+\phi)]\}\{\cos\tfrac{1}{2}[(kx-\omega t)-(kx-\omega t+\phi)]\}$$
$$= 2A\{\sin\tfrac{1}{2}(2kx-2\omega t+\phi)\}\{\cos\tfrac{1}{2}(\phi)\} = \boxed{\left(2A\cos\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right)}$$

(*b*) The amplitude is the absolute value of the coefficient of the sine function, $\boxed{\left|2A\cos\frac{\phi}{2}\right|}$. The wave is [purely sinusoidal] because the dependence on *x* and *t* is $\sin\left(kx-\omega t+\frac{\phi}{2}\right)$.

(*c*) If $\phi = 0, 2\pi, 4\pi, \cdots, 2n\pi$, then the amplitude is $\left|2A\cos\frac{\phi}{2}\right| = \left|2A\cos\frac{2n\pi}{2}\right| = |2A\cos n\pi| = |2A(\pm1)|$ $= 2A$, which is constructive interference. If $\phi = \pi, 3\pi, 5\pi, \cdots, (2n+1)\pi$, then the amplitude is $\left|2A\cos\frac{\phi}{2}\right| = \left|2A\cos\frac{(2n+1)\pi}{2}\right| = \left|2A\cos\left[(n+\tfrac{1}{2})\pi\right]\right| = 0$, which is destructive interference.

(*d*) If $\phi = \frac{\pi}{2}$, then the resultant wave is as follows.

$$D = \left(2A\cos\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right) = \left(2A\cos\frac{\pi}{4}\right)\sin\left(kx-\omega t+\frac{\pi}{4}\right) = \sqrt{2}A\sin\left(kx-\omega t+\frac{\pi}{4}\right)$$

This wave has an amplitude of $\sqrt{2}A$, is traveling in the positive *x* direction, and is shifted to the left by an eighth of a cycle. This is "halfway" between the two original waves. The displacement is $\tfrac{1}{2}A$ at the origin at $t = 0$.

[43.] The fundamental frequency of the full string is given by $f_{\text{unfingered}} = \frac{v}{2\ell} = 441\,\text{Hz}$. If the length is reduced to 2/3 of its current value, and the velocity of waves on the string is not changed, then the new frequency will be as follows.

$$f_{\text{fingered}} = \frac{v}{2(\tfrac{2}{3}\ell)} = \frac{3}{2}\frac{v}{2\ell} = \left(\frac{3}{2}\right)f_{\text{unfingered}} = \left(\frac{3}{2}\right)(441\,\text{Hz}) = \boxed{662\,\text{Hz}}$$

44. The frequencies of the harmonics of a string that is fixed at both ends are given by $f_n = nf_1$, and so the first four harmonics are $\boxed{f_1 = 294\,\text{Hz}, f_2 = 588\,\text{Hz}, f_3 = 882\,\text{Hz}, f_4 = 1176\,\text{Hz}}$.

45. The oscillation corresponds to the fundamental. The frequency of that oscillation is $f_1 = \frac{1}{T} = \frac{1}{1.5\,\text{s}} = \frac{2}{3}\,\text{Hz}$. The bridge, with both ends fixed, is similar to a vibrating string, and so $\boxed{f_n = nf_1 = \frac{2n}{3}\,\text{Hz},\ n = 1,2,3\ldots}$. The periods are the reciprocals of the frequency, and so $\boxed{T_n = \frac{1.5\,\text{s}}{n},\ n = 1,2,3\ldots}$.

46. Four loops is the standing wave pattern for the 4th harmonic, with a frequency given by $f_4 = 4f_1 = 280\ \text{Hz}$. Thus $\boxed{f_1 = 70\ \text{Hz}, f_2 = 140\ \text{Hz},\ f_3 = 210\ \text{Hz, and } f_5 = 350\ \text{Hz}}$ are all other resonant frequencies.

47. Each half of the cord has a single node, at the center of the cord. Thus each half of the cord is a half of a wavelength, assuming that the ends of the cord are also nodes. The tension is the same in both halves of the cord, and the wavelengths are the same based on the location of the node. Let subscript 1 represent the lighter density, and subscript 2 represent the heavier density.

$$v_1 = \sqrt{\frac{F_{T1}}{\mu_1}} = \lambda_1 f_1 \ ;\ v_2 = \sqrt{\frac{F_{T2}}{\mu_2}} = \lambda_2 f_2 \ ;\ \lambda_1 = \lambda_2 \ ;\ F_{T1} = F_{T2}$$

$$\frac{f_1}{f_2} = \frac{\frac{1}{\lambda_1}\sqrt{\frac{F_{T1}}{\mu_1}}}{\frac{1}{\lambda_2}\sqrt{\frac{F_{T2}}{\mu_2}}} = \sqrt{\frac{\mu_2}{\mu_1}} = \boxed{\sqrt{2}}$$

The frequency is higher on the lighter portion.

48. Adjacent nodes are separated by a half-wavelength, as examination of Figure 15-26 will show.

$$\lambda = \frac{v}{f} \ \rightarrow\ \Delta x_{\text{node}} = \tfrac{1}{2}\lambda = \frac{v}{2f} = \frac{96\,\text{m/s}}{2(445\,\text{Hz})} = \boxed{0.11\,\text{m}}$$

49. Since $f_n = nf_1$, two successive overtones differ by the fundamental frequency, as shown below.

$$\Delta f = f_{n+1} - f_n = (n+1)f_1 - nf_1 = f_1 = 320\,\text{Hz} - 240\,\text{Hz} = \boxed{80\,\text{Hz}}$$

50. The speed of waves on the string is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The resonant frequencies of a string with both ends fixed are given by Eq. 15-17b, $f_n = \frac{nv}{2\ell_{\text{vib}}}$, where $\ell_{\text{vib}}$ is the length of the portion that is actually vibrating. Combining these relationships allows the frequencies to be calculated.

$$f_n = \frac{n}{2\ell_{\text{vib}}}\sqrt{\frac{F_T}{\mu}} \qquad f_1 = \frac{1}{2(0.600\,\text{m})}\sqrt{\frac{520\,\text{N}}{(3.16\times10^{-3}\,\text{kg})/(0.900\,\text{m})}} = 320.7\,\text{Hz}$$

$$f_2 = 2f_1 = 641.4\,\text{Hz} \quad f_3 = 3f_1 = 962.1\,\text{Hz}$$

So the three frequencies are $\boxed{320\,\text{Hz}, 640\,\text{Hz}, 960\,\text{Hz}}$, to 2 significant figures.

51. The speed of the wave is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The wavelength of the fundamental is $\lambda_1 = 2\ell$. Thus the frequency of the fundamental is $f_1 = \frac{v}{\lambda_1} = \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu}}$. Each harmonic is present in a vibrating string, and so $f_n = nf_1 = \boxed{\frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}}}$, $n = 1, 2, 3, \ldots$.

52. The string must vibrate in a standing wave pattern to have a certain number of loops. The frequency of the standing waves will all be 120 Hz, the same as the vibrator. That frequency is also expressed by Eq. 15-17b, $f_n = \frac{nv}{2\ell}$. The speed of waves on the string is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The tension in the string will be the same as the weight of the masses hung from the end of the string, $F_T = mg$, ignoring the mass of the string itself. Combining these relationships gives an expression for the masses hung from the end of the string.

(*a*) $$f_n = \frac{nv}{2\ell} = \frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}} = \frac{n}{2\ell}\sqrt{\frac{mg}{\mu}} \rightarrow m = \frac{4\ell^2 f_n^2 \mu}{n^2 g}$$

$$m_1 = \frac{4(1.50\,\text{m})^2(120\,\text{Hz})^2(6.6\times10^{-4}\,\text{kg/m})}{1^2(9.80\,\text{m/s}^2)} = 8.728\,\text{kg} \approx \boxed{8.7\,\text{kg}}$$

(*b*) $$m_2 = \frac{m_1}{2^2} = \frac{8.728\,\text{kg}}{4} = \boxed{2.2\,\text{kg}}$$

(*c*) $$m_5 = \frac{m_1}{5^2} = \frac{8.728\,\text{kg}}{25} = \boxed{0.35\,\text{kg}}$$

53. The tension in the string is the weight of the hanging mass, $F_T = mg$. The speed of waves on the string can be found by $v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{mg}{\mu}}$, and the frequency is given as $f = 120\,\text{Hz}$. The wavelength of waves created on the string will thus be given by

$$\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{mg}{\mu}} = \frac{1}{120\,\text{Hz}}\sqrt{\frac{(0.070\,\text{kg})(9.80\,\text{m/s}^2)}{(6.6\times10^{-4}\,\text{kg/m})}} = 0.2687\,\text{m}.$$

The length of the string must be an integer multiple of half of the wavelength for there to be nodes at both ends and thus form a standing wave. Thus $\ell = \lambda/2, \lambda, 3\lambda/2, \cdots n\lambda/2$. The number of standing wave patterns is given by the number of integers that satisfy $0.10\,\text{m} < n\lambda/2 < 1.5\,\text{m}$.

$$0.10\,\text{m} < n\lambda/2 \rightarrow n > \frac{2(0.10\,\text{m})}{\lambda} = \frac{2(0.10\,\text{m})}{0.2687\,\text{m}} = 0.74$$

$$n\lambda/2 < 1.5\,\text{m} \;\rightarrow\; n < \frac{2(1.5\,\text{m})}{\lambda} = \frac{2(1.5\,\text{m})}{0.2687\,\text{m}} = 11.1$$

Thus we see that we must have *n* from 1 to 11, and so there are $\boxed{11}$ standing wave patterns that may be achieved.

54. The standing wave is given by $D = (2.4\,\text{cm})\sin(0.60x)\cos(42t)$.

(*a*) The distance between nodes is half of a wavelength.

$$d = \tfrac{1}{2}\lambda = \tfrac{1}{2}\frac{2\pi}{k} = \frac{\pi}{0.60\,\text{cm}^{-1}} = 5.236\,\text{cm} \approx \boxed{5.2\,\text{cm}}$$

(*b*) The component waves travel in opposite directions. Each has the same frequency and speed, and each has half the amplitude of the standing wave.

$$A = \tfrac{1}{2}(2.4\,\text{cm}) = \boxed{1.2\,\text{cm}}\;;\; f = \frac{\omega}{2\pi} = \frac{42\,\text{s}^{-1}}{2\pi} = 6.685\,\text{Hz} \approx \boxed{6.7\,\text{Hz}}\;;$$

$$v = \lambda f = 2d_{\text{node}} f = 2(5.236\,\text{cm})(6.685\,\text{Hz}) = 70.01\,\text{cm/s} \approx \boxed{70\,\text{cm/s}}\;(2\text{ sig. fig.})$$

(*c*) The speed of a particle is given by $\dfrac{\partial D}{\partial t}$.

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial t}\left[(2.4\,\text{cm})\sin(0.60x)\cos(42t)\right] = (-42\,\text{rad/s})(2.4\,\text{cm})\sin(0.60x)\sin(42t)$$

$$\frac{\partial D}{\partial t}(3.20\,\text{cm},\,2.5\,\text{s}) = (-42\,\text{rad/s})(2.4\,\text{cm})\sin\left[(0.60\,\text{cm}^{-1})(3.20\,\text{cm})\right]\sin\left[(42\,\text{rad/s})(2.5\,\text{s})\right]$$

$$= \boxed{92\,\text{cm/s}}$$

$\boxed{55.}$ (*a*) The given wave is $D_1 = 4.2\sin(0.84x - 47t + 2.1)$. To produce a standing wave, we simply need to add a wave of the same characteristics but traveling in the opposite direction. This is the appropriate wave.

$$\boxed{D_2 = 4.2\sin(0.84x + 47t + 2.1)}$$

(*b*) The standing wave is the sum of the two component waves. We use the trigonometric identity that $\sin\theta_1 + \sin\theta_2 = 2\sin\tfrac{1}{2}(\theta_1+\theta_2)\cos\tfrac{1}{2}(\theta_1-\theta_2)$.

$$D = D_1 + D_2 = 4.2\sin(0.84x - 47t + 2.1) + 4.2\sin(0.84x + 47t + 2.1)$$

$$= 4.2(2)\left\{\sin\tfrac{1}{2}\left[(0.84x - 47t + 2.1) + (0.84x + 47t + 2.1)\right]\right\}$$

$$\left\{\cos\tfrac{1}{2}\left[(0.84x - 47t + 2.1) - (0.84x + 47t + 2.1)\right]\right\}$$

$$= 8.4\sin(0.84x + 2.1)\cos(-47t) = \boxed{8.4\sin(0.84x + 2.1)\cos(47t)}$$

We note that the origin is NOT a node.

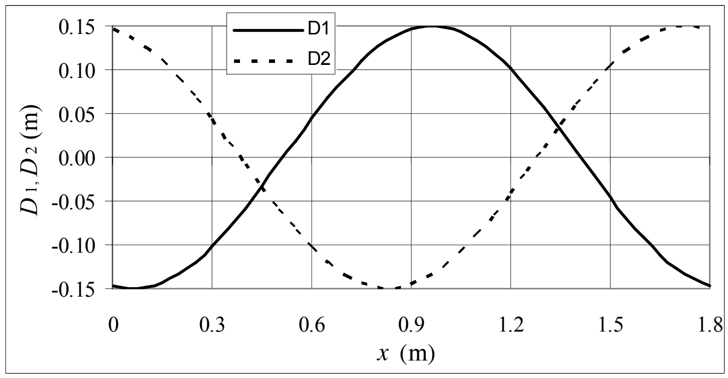
56. From the description of the water's behavior, there is an antinode at each end of the tub, and a node in the middle. Thus one wavelength is twice the tub length.

$$v = \lambda f = (2\ell_{\text{tub}})f = 2(0.45\,\text{m})(0.85\,\text{Hz}) = \boxed{0.77\,\text{m/s}}$$

57. The frequency is given by $f = \dfrac{v}{\lambda} = \dfrac{1}{\lambda}\sqrt{\dfrac{F}{\mu}}$. The wavelength and the mass density do not change when the string is tightened.

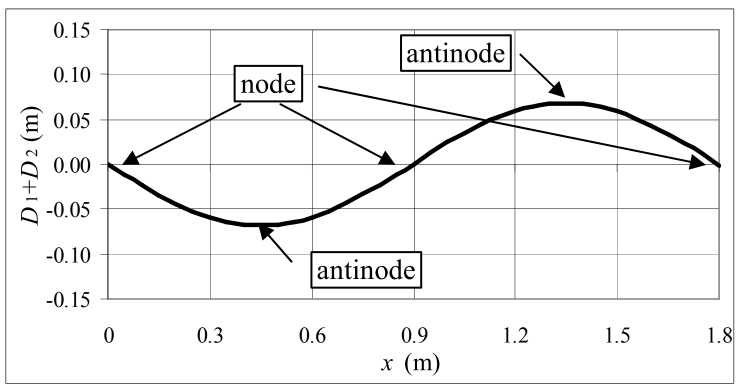
$$f = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{F}{\mu}} \quad \rightarrow \quad \frac{f_2}{f_1} = \frac{\frac{1}{\lambda}\sqrt{\frac{F_2}{\mu}}}{\frac{1}{\lambda}\sqrt{\frac{F_1}{\mu}}} = \sqrt{\frac{F_2}{F_1}} \quad \rightarrow \quad f_2 = f_1\sqrt{\frac{F_2}{F_1}} = (294\,\text{Hz})\sqrt{1.15} = \boxed{315\,\text{Hz}}$$

58. (*a*) Plotting one full wavelength means from $x = 0$ to $x = \lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{3.5\,\text{m}^{-1}} = 1.795\,\text{m}$ $\approx 1.8\,\text{m}$. The functions to be plotted are given below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.58."




$$D_1 = (0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x - 1.8\right] \text{ and } D_2 = (0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x + 1.8\right]$$

(*b*) The sum $D_1 + D_2$ is plotted, and the nodes and antinodes are indicated. The analytic result is given below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.58."




$$D_1 + D_2 = (0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x - 1.8\right] + (0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x - 1.8\right]$$
$$= (0.30\,\text{m})\sin\left(3.5\,\text{m}^{-1}x\right)\cos(1.8)$$

This expression should have nodes and antinodes at positions given by the following.

$$3.5\,\text{m}^{-1}x_{\text{node}} = n\pi, n = 0,1,2\ldots \quad \rightarrow \quad x = \frac{n\pi}{3.5} = 0,\ 0.90\,\text{m},\ 1.80\ \text{m}$$

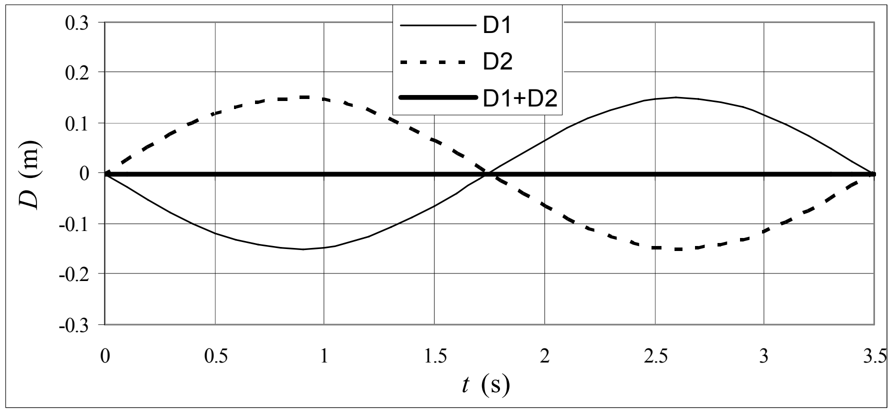
$$3.5\,\text{m}^{-1}x_{\text{antinode}} = \left(n + \tfrac{1}{2}\right)\pi, n = 0,1,2\ldots \quad \rightarrow \quad x = \frac{\left(n + \frac{1}{2}\right)\pi}{3.5} = 0.45\,\text{m},\ 1.35\ \text{m}$$

The graph agrees with the calculations.

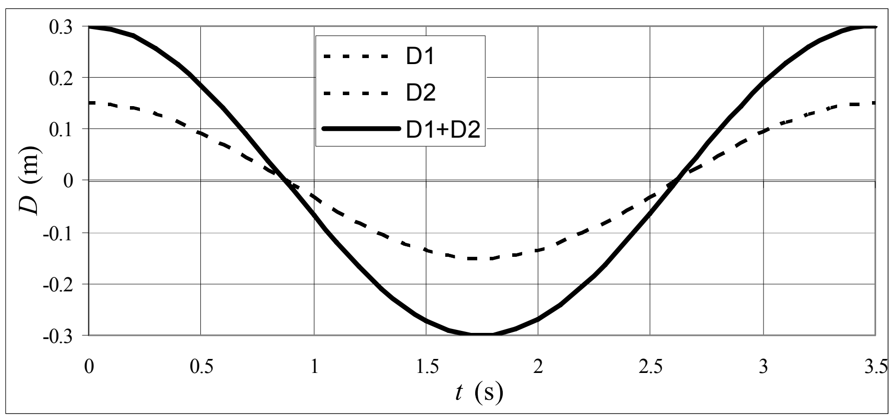
59. The standing wave formed from the two individual waves is given below. The period is given by $T = 2\pi/\omega = 2\pi/1.8\,\text{s}^{-1} = 3.5\,\text{s}$.

$$D_1 + D_2 = (0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x - \left(1.8\,\text{s}^{-1}\right)t\right] + (0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x - \left(1.8\,\text{s}^{-1}\right)t\right]$$
$$= (0.30\,\text{m})\sin\left(3.5\,\text{m}^{-1}x\right)\cos\left(1.8\,\text{s}^{-1}t\right)$$

(*a*) For the point $x = 0$, we see that the sum of the two waves is identically 0. This means that the point $x = 0$ is a node of the standing wave. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.59."




(*b*) For the point $x = \lambda/4$, we see that the amplitude of that point is twice the amplitude of either wave. Thus this point is an antinode of the standing wave. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.59."




60. (*a*) The maximum swing is twice the amplitude of the standing wave. Three loops is 1.5 wavelengths, and the frequency is given.

$$A = \tfrac{1}{2}(8.00\,\text{cm}) = 4.00\,\text{cm} \;;\; \omega = 2\pi f = 2\pi(120\,\text{Hz}) = 750\,\text{rad/s} \;;$$

$$k = \frac{2\pi}{\lambda} \;\rightarrow\; ;\; \tfrac{3}{2}\lambda = 1.64\,\text{m} \;\rightarrow\; \lambda = 1.09\,\text{m} \;;\; k = \frac{2\pi}{1.09\,\text{m}} = 5.75\,\text{m}^{-1}$$

$$D = A\sin(kx)\cos(\omega t) = \boxed{(4.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x\right]\cos\left[(750\,\text{rad/s})t\right]}$$

(*b*) Each component wave has the same wavelength, the same frequency, and half the amplitude of the standing wave.

$$\boxed{\begin{aligned} D_1 &= (2.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x - (750\,\text{rad/s})t\right] \\ D_2 &= (2.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x + (750\,\text{rad/s})t\right] \end{aligned}}$$

61. Any harmonic with a node directly above the pickup will NOT be "picked up" by the pickup. The pickup location is exactly 1/4 of the string length from the end of the string, so a standing wave with a frequency corresponding to 4 (or 8 or 12 etc.) loops will not excite the pickup. So $\boxed{n = 4, 8, \text{and } 12}$ will not excite the pickup.

62. The gap between resonant frequencies is the fundamental frequency (which is thus 300 Hz for this problem), and the wavelength of the fundamental is twice the string length.

$$v = \lambda f = (2\ell)(f_{n+1} - f_n) = 2(0.65\,\text{m})(300\,\text{Hz}) = \boxed{390\,\text{m/s}}$$

63. The standing wave is the sum of the two individual standing waves. We use the trigonometric identities for the cosine of a difference and a sum.

$$\cos(\theta_1 - \theta_2) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\ ;\ \cos(\theta_1 + \theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$$

$$\begin{aligned} D = D_1 + D_2 &= A\cos(kx - \omega t) + A\cos(kx + \omega t) = A\left[\cos(kx - \omega t) + \cos(kx + \omega t)\right] \\ &= A\left[\cos kx\cos\omega t + \sin kx\sin\omega t + \cos kx\cos\omega t - \sin kx\sin\omega t\right] \\ &= 2A\cos kx\cos\omega t \end{aligned}$$

Thus the standing wave is $D = 2A\cos kx\cos\omega t$. The nodes occur where the position term forces $D = 2A\cos kx\cos\omega t = 0$ for all time. Thus $\cos kx = 0 \rightarrow kx = \pm(2n+1)\frac{\pi}{2},\ n = 0,1,2,\cdots$. Thus, since $k = 2.0\,\text{m}^{-1}$, we have $\boxed{x = \pm\left(n + \tfrac{1}{2}\right)\frac{\pi}{2}\,\text{m},\ n = 0,1,2,\cdots}$.

64. The frequency for each string must be the same, to ensure continuity of the string at its junction. Each string will obey these relationships: $\lambda f = v$, $v = \sqrt{\frac{F_T}{\mu}}$, $\lambda = \frac{2\ell}{n}$. Combine these to find the nodes. Note that *n* is the number of "loops" in the string segment, and that *n* loops requires $n+1$ nodes.

$$\lambda f = v\ ,\ v = \sqrt{\frac{F_T}{\mu}}\ ,\ \lambda = \frac{2\ell}{n} \rightarrow \frac{2\ell}{n}f = \sqrt{\frac{F_T}{\mu}} \rightarrow f = \frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}}$$

$$\frac{n_{\text{Al}}}{2\ell_{\text{Al}}}\sqrt{\frac{F_T}{\mu_{\text{Al}}}} = \frac{n_{\text{Fe}}}{2\ell_{\text{Fe}}}\sqrt{\frac{F_T}{\mu_{\text{Fe}}}} \rightarrow \frac{n_{\text{Al}}}{n_{\text{Fe}}} = \frac{\ell_{\text{Al}}}{\ell_{\text{Fe}}}\sqrt{\frac{\mu_{\text{Al}}}{\mu_{\text{Fe}}}} = \frac{0.600\,\text{m}}{0.882\,\text{m}}\sqrt{\frac{2.70\,\text{g/m}}{7.80\,\text{g/m}}} = 0.400 = \frac{2}{5}$$

Thus there are 3 nodes on the aluminum, since $n_{\text{Al}} = 2$, and 6 nodes on the steel, since $n_{\text{Fe}} = 5$, but one node is shared so there are $\boxed{8\text{ total nodes}}$. Use the formula derived above to find the lower frequency.

$$f = \frac{n_{\text{Al}}}{2\ell_{\text{Al}}}\sqrt{\frac{F_{\text{Al}}}{\mu_{\text{Al}}}} = \frac{2}{2(0.600\,\text{m})}\sqrt{\frac{135\,\text{N}}{2.70\times10^{-3}\,\text{kg/m}}} = \boxed{373\,\text{Hz}}$$

65. The speed in the second medium can be found from the law of refraction, Eq. 15-19.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \rightarrow v_2 = v_1\frac{\sin\theta_2}{\sin\theta_1} = (8.0\,\text{km/s})\left(\frac{\sin 31^\circ}{\sin 52^\circ}\right) = \boxed{5.2\,\text{km/s}}$$

66. The angle of refraction can be found from the law of refraction, Eq. 15-19.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \rightarrow \sin\theta_2 = \sin\theta_1\frac{v_2}{v_1} = \sin 35^\circ\frac{2.5\,\text{m/s}}{2.8\,\text{m/s}} = 0.512 \rightarrow \theta_2 = \sin^{-1}0.419 = \boxed{31^\circ}$$

67. The angle of refraction can be found from the law of refraction, Eq. 15-19. The relative velocities can be found from the relationship given in the problem.

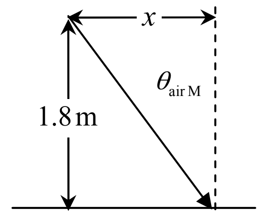
$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}=\frac{331+0.60T_2}{331+0.60T_1} \rightarrow \sin\theta_2=\sin 33^\circ\frac{331+0.60(-15)}{331+0.60(25)}=\sin 33^\circ\frac{322}{346}=0.5069$$

$$\theta_2=\sin^{-1}0.5069=\boxed{30^\circ}\ (2\text{ sig. fig.})$$

68. (*a*) Eq. 15-19 gives the relationship between the angles and the speed of sound in the two media. For total internal reflection (for no sound to enter the water), $\theta_{\text{water}}=90^\circ$ or $\sin\theta_{\text{water}}=1$. The air is the "incident" media. Thus the incident angle is given by the following.

$$\frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{water}}}=\frac{v_{\text{air}}}{v_{\text{water}}}\ ;\ \theta_{\text{air}}=\theta_{\text{i}}=\sin^{-1}\left[\sin\theta_{\text{water}}\frac{v_{\text{air}}}{v_{\text{water}}}\right] \rightarrow \boxed{\theta_{\text{iM}}=\sin^{-1}\left[\frac{v_{\text{air}}}{v_{\text{water}}}\right]=\sin^{-1}\left[\frac{v_{\text{i}}}{v_{\text{r}}}\right]}$$

(*b*) From the angle of incidence, the distance is found. See the diagram.




$$\theta_{\text{air M}}=\sin^{-1}\frac{v_{\text{air}}}{v_{\text{water}}}=\sin^{-1}\frac{343\text{ m/s}}{1440\text{ m/s}}=13.8^\circ$$

$$\tan\theta_{\text{air M}}=\frac{x}{1.8\text{ m}} \rightarrow x=(1.8\text{ m})\tan 13.8^\circ=\boxed{0.44\text{ m}}$$

69. The angle of refraction can be found from the law of refraction, Eq. 15-19. The relative velocities can be found from Eq. 15-3.

$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}=\frac{\sqrt{E/\rho_2}}{\sqrt{E/\rho_1}}=\sqrt{\frac{\rho_1}{\rho_2}}=\sqrt{\frac{SG_1\rho_{\text{water}}}{SG_2\rho_{\text{water}}}}=\sqrt{\frac{SG_1}{SG_2}}$$

$$\sin\theta_2=\sin\theta_1\sqrt{\frac{SG_1}{SG_2}}=\sin 38^\circ\sqrt{\frac{3.6}{2.8}}=0.70 \rightarrow \theta_2=\sin^{-1}0.70=\boxed{44^\circ}$$

70. The error of $2^\circ$ is allowed due to diffraction of the waves. If the waves are incident at the "edge" of the dish, they can still diffract into the dish if the relationship $\theta\approx\lambda/\ell$ is satisfied.

$$\theta\approx\frac{\lambda}{\ell} \rightarrow \lambda=\ell\theta=(0.5\text{ m})\left(2^\circ\times\frac{\pi\text{ rad}}{180^\circ}\right)=1.745\times10^{-2}\text{ m}\approx\boxed{2\times10^{-2}\text{ m}}$$

If the wavelength is longer than that, there will not be much diffraction, but "shadowing" instead.

71. The frequency is 880 Hz and the phase velocity is 440 m/s, so the wavelength is

$$\lambda=\frac{v}{f}=\frac{440\text{ m/s}}{880\text{ Hz}}=0.50\text{ m}.$$

(*a*) Use the ratio of distance to wavelength to define the phase difference.

$$\frac{x}{\lambda}=\frac{\pi/6}{2\pi} \rightarrow x=\frac{\lambda}{12}=\frac{0.50\text{ m}}{12}=\boxed{0.042\text{ m}}$$

(b) Use the ratio of time to period to define the phase difference.

$$\frac{t}{T}=\frac{\phi}{2\pi} \rightarrow \phi=\frac{2\pi t}{T}=2\pi tf=2\pi\left(1.0\times10^{-4}\text{ s}\right)(880\text{ Hz})=\boxed{0.55\text{ rad}}$$

72. The frequency at which the water is being shaken is about 1 Hz. The sloshing coffee is in a standing wave mode, with antinodes at each edge of the cup. The cup diameter is thus a half-wavelength, or $\lambda = 16$ cm. The wave speed can be calculated from the frequency and the wavelength.

$$v = \lambda f = (16\text{ cm})(1\text{ Hz}) = \boxed{16\text{ cm/s}}$$

73. The speed of a longitudinal wave in a solid is given by Eq. 15-3, $v = \sqrt{E/\rho}$. Let the density of the less dense material be $\rho_1$, and the density of the more dense material be $\rho_2$. The less dense material will have the higher speed, since the speed is inversely proportional to the square root of the density.

$$\frac{v_1}{v_2} = \frac{\sqrt{E/\rho_1}}{\sqrt{E/\rho_2}} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{2.5} \approx \boxed{1.6}$$

74. From Eq. 15-7, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1 = P_2/P_1 = A_2^2/A_1^2 = 2.5 \;\rightarrow\; A_2/A_1 = \sqrt{2.5} = \boxed{1.6}$$

The more energetic wave has the larger amplitude.

75. (*a*) The amplitude is half the peak-to-peak distance, so $\boxed{0.05\text{ m}}$.

(*b*) The maximum kinetic energy of a particle in simple harmonic motion is the total energy, which is given by $E_{total} = \frac{1}{2}kA^2$.

Compare the two kinetic energy maxima.

$$\frac{K_{2\,max}}{K_{1\,max}} = \frac{\frac{1}{2}kA_2^2}{\frac{1}{2}kA_1^2} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{0.075\text{ m}}{0.05\text{ m}}\right)^2 = \boxed{2.25}$$

76. From Eq. 15-17b, $f_n = \frac{nv}{2L}$, we see that the frequency is proportional to the wave speed on the stretched string. From Eq. 15-2, $v = \sqrt{F_T/\mu}$, we see that the wave speed is proportional to the square root of the tension. Thus the frequency is proportional to the square root of the tension.

$$\sqrt{\frac{F_{T2}}{F_{T1}}} = \frac{f_2}{f_1} \;\rightarrow\; F_{T2} = \left(\frac{f_2}{f_1}\right)^2 F_{T1} = \left(\frac{247\text{ Hz}}{255\text{ Hz}}\right)^2 F_{T1} = 0.938 F_{T1}$$

Thus the tension should be $\boxed{\text{decreased by 6.2\%}}$.

77. We assume that the earthquake wave is moving the ground vertically, since it is a transverse wave. An object sitting on the ground will then be moving with SHM, due to the two forces on it – the normal force upwards from the ground and the weight downwards due to gravity. If the object loses contact with the ground, then the normal force will be zero, and the only force on the object will be its weight. If the only force is the weight, then the object will have an acceleration of *g* downwards. Thus the limiting condition for beginning to lose contact with the ground is when the maximum acceleration caused by the wave is greater than *g*. Any larger downward acceleration and the ground would "fall" quicker than the object. The maximum acceleration is related to the amplitude and the frequency as follows.

$$a_{max} = \omega^2 A > g \;\rightarrow\; A > \frac{g}{\omega^2} = \frac{g}{4\pi^2 f^2} = \frac{9.80\text{ m/s}^2}{4\pi^2(0.60\text{ Hz})^2} = \boxed{0.69\text{ m}}$$

78. (*a*) The speed of the wave at a point *h* above the lower end depends on the tension at that point and the linear mass density of the cord. The tension must equal the mass of the lower segment if the lower segment is in equilibrium. Use Eq. 15-2 for the wave speed.

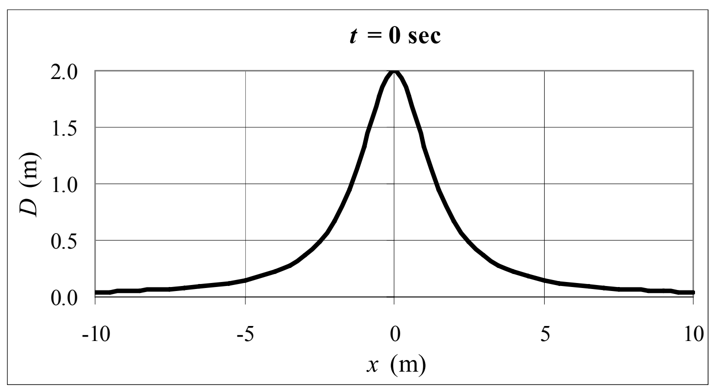
$$F_T = m_{\text{segment}} g = \frac{h}{\ell} mg \ ; \ v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{\frac{h}{\ell} mg}{\frac{m}{\ell}}} = \boxed{\sqrt{hg}}$$

(*b*) We treat *h* as a variable, measured from the bottom of the cord. The wave speed at that point is given above as $v = \sqrt{hg}$. The distance a wave would travel up the cord during a time *dt* is then $dh = vdt = \sqrt{hg}\, dt$. To find the total time for a wave to travel up the cord, integrate over the length of the cord.

$$dh = vdt = \sqrt{hg}dt \ \rightarrow \ dt = \frac{dh}{\sqrt{hg}} \ \rightarrow \ \int_0^{t_{\text{total}}} dt = \int_0^L \frac{dh}{\sqrt{hg}} \ \rightarrow$$

$$t_{\text{total}} = \int_0^L \frac{dh}{\sqrt{hg}} = 2\sqrt{\frac{h}{g}}\Bigg|_0^L = \boxed{2\sqrt{\frac{L}{g}}}$$

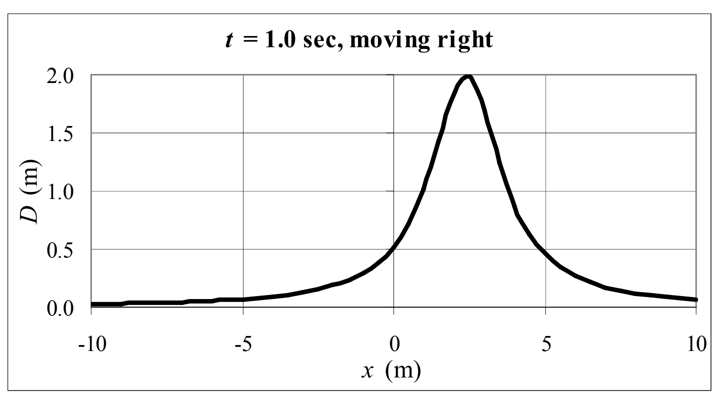
79. (*a*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.79."




(*b*) The wave function is found by replacing *x* in the pulse by $x - vt$.

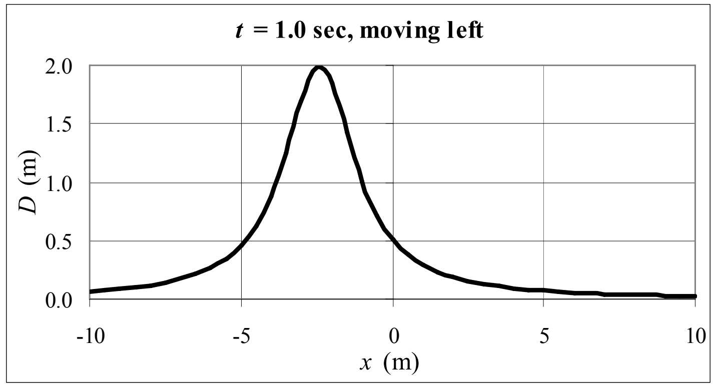
$$\boxed{D(x,t) = \frac{4.0\,\text{m}^3}{\left[x - (2.4\,\text{m/s})t\right]^2 + 2.0\,\text{m}^2}}$$

(*c*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.79."



(*d*) The wave function is found by replacing $x$ in the pulse by $x+vt$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.79."

$$\boxed{D=\frac{4.0\,\text{m}^3}{\left[x+(2.4\,\text{m/s})t\right]^2+2.0\,\text{m}^2}}$$




80. (*a*) The frequency is related to the tension by Eqs. 15-1 and 15-2.

$$f=\frac{v}{\lambda}=\frac{1}{\lambda}\sqrt{\frac{F_\text{T}}{\mu}}\;\rightarrow\;\frac{df}{dF_\text{T}}=\frac{1}{2\lambda}\sqrt{\frac{1}{\mu F_\text{T}}}=\frac{1}{2F_\text{T}}\frac{1}{\lambda}\sqrt{\frac{F_\text{T}}{\mu}}=\frac{f}{2F_\text{T}}$$

$$\frac{\Delta f}{\Delta F_\text{T}}\approx\frac{f}{2F_\text{T}}\;\rightarrow\;\boxed{\Delta f\approx\tfrac{1}{2}\left(\frac{\Delta F_\text{T}}{F_\text{T}}\right)f}$$

(*b*) $\dfrac{\Delta f}{\Delta F_\text{T}}\approx\dfrac{f}{2F_\text{T}}\;\rightarrow\;\dfrac{\Delta F_\text{T}}{F_\text{T}}\approx 2\dfrac{\Delta f}{f}=2\left(\dfrac{6}{436}\right)=0.0275=\boxed{3\%}$

(*c*) The only change in the expression $\dfrac{1}{\lambda}\sqrt{\dfrac{F_\text{T}}{\mu}}$ as the overtone changes is the wavelength, and the wavelength does not influence the final result. So $\boxed{\text{yes}}$, the formula still applies.

81. (*a*) The overtones are given by $f_n=nf_1, n=2,3,4\ldots$

$$\text{G}:\;f_2=2(392\,\text{Hz})=\boxed{784\,\text{Hz}}\qquad f_3=3(392\,\text{Hz})=1176\,\text{Hz}\approx\boxed{1180\,\text{Hz}}$$

$$\text{B}:\;f_2=2(494\,\text{Hz})=\boxed{988\,\text{Hz}}\qquad f_3=3(440\,\text{Hz})=1482\,\text{Hz}\approx\boxed{1480\,\text{Hz}}$$

(*b*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different masses for the strings.

$$\frac{f_\text{G}}{f_\text{A}}=\frac{v_\text{G}/\lambda}{v_\text{A}/\lambda}=\frac{v_\text{G}}{v_\text{A}}=\frac{\sqrt{\dfrac{F_\text{T}}{m_\text{G}/\ell}}}{\sqrt{\dfrac{F_\text{T}}{m_\text{A}/\ell}}}=\sqrt{\frac{m_\text{A}}{m_\text{G}}}\;\rightarrow\;\frac{m_\text{G}}{m_\text{A}}=\left(\frac{f_\text{A}}{f_\text{G}}\right)^2=\left(\frac{494}{392}\right)^2=\boxed{1.59}$$

(*c*) If the two strings have the same mass per unit length and the same tension, then the wave speed on both strings is the same. The frequency difference is then due to a difference in wavelength. For the fundamental, the wavelength is twice the length of the string.

$$\frac{f_\text{G}}{f_\text{B}}=\frac{v/\lambda_\text{G}}{v/\lambda_\text{B}}=\frac{\lambda_\text{B}}{\lambda_\text{G}}=\frac{2\ell_\text{B}}{2\ell_\text{G}}\;\rightarrow\;\frac{\ell_\text{G}}{\ell_\text{B}}=\frac{f_\text{B}}{f_\text{G}}=\frac{494}{392}=\boxed{1.26}$$

(*d*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different tensions for the strings.

$$\frac{f_B}{f_A}=\frac{v_B/\lambda}{v_A/\lambda}=\frac{v_B}{v_A}=\frac{\sqrt{\frac{F_{TB}}{m/L}}}{\sqrt{\frac{F_{TA}}{m/L}}}=\sqrt{\frac{F_{TB}}{F_{TA}}}\rightarrow\frac{F_{TB}}{F_{TA}}=\left(\frac{f_B}{f_A}\right)^2=\left(\frac{392}{494}\right)^2=\boxed{0.630}$$

82. Relative to the fixed needle position, the ripples are moving with a linear velocity given by

$$v=\left(33\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi(0.108\text{ m})}{1\text{ rev}}\right)=0.3732\text{ m/s}$$

This speed is the speed of the ripple waves moving past the needle. The frequency of the waves is

$$f=\frac{v}{\lambda}=\frac{0.3732\text{ m/s}}{1.55\times10^{-3}\text{m}}=240.77\text{ Hz}\approx\boxed{240\text{ Hz}}$$

83. The speed of the pulses is found from the tension and mass per unit length of the wire.

$$v=\sqrt{\frac{F_T}{\mu}}=\sqrt{\frac{255\text{ N}}{0.152\text{ kg}/10.0\text{ m}}}=129.52\text{ m/s}$$

The total distance traveled by the two pulses will be the length of the wire. The second pulse has a shorter time of travel than the first pulse, by 20.0 ms.

$$\ell=d_1+d_2=vt_1+vt_2=vt_1+v\left(t_1-2.00\times10^{-2}\right)$$

$$t_1=\frac{\ell+2.00\times10^{-2}v}{2v}=\frac{(10.0\text{ m})+2.00\times10^{-2}(129.52\text{ m/s})}{2(129.52\text{ m/s})}=4.8604\times10^{-2}\text{s}$$

$$d_1=vt_1=(129.52\text{ m/s})\left(4.8604\times10^{-2}\text{s}\right)=6.30\text{ m}$$

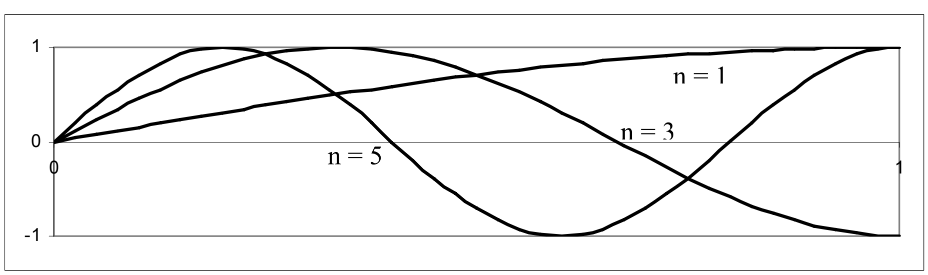
The two pulses meet $\boxed{6.30\text{ m}}$ from the end where the first pulse originated.

84. We take the wave function to be $D(x,t)=A\sin(kx-\omega t)$. The wave speed is given by $v=\frac{\omega}{k}=\frac{\lambda}{f}$, while the speed of particles on the cord is given by $\frac{\partial D}{\partial t}$.

$$\frac{\partial D}{\partial t}=-\omega A\cos(kx-\omega t)\quad\rightarrow\quad\left(\frac{\partial D}{\partial t}\right)_{max}=\omega A$$

$$\omega A=v=\frac{\omega}{k}\quad\rightarrow\quad A=\frac{1}{k}=\frac{\lambda}{2\pi}=\frac{10.0\text{ cm}}{2\pi}=\boxed{1.59\text{ cm}}$$

85. For a resonant condition, the free end of the string will be an antinode, and the fixed end of the string will be a node. The minimum distance from a node to an antinode is $\lambda/4$. Other wave patterns that fit the boundary conditions of a node at



one end and an antinode at the other end include $3\lambda/4$ , $5\lambda/4$ , … . See the diagrams. The general relationship is $\ell = (2n-1)\lambda/4$ , $n = 1,2,3,\cdots$. Solving for the wavelength gives

$$\boxed{\lambda = \frac{4\ell}{2n-1} \text{ , } n = 1,2,3,\cdots}.$$

86. The addition of the support will force the bridge to have its lowest mode of oscillation to have a node at the center of the span, which would be the first overtone of the fundamental frequency. If the wave speed in the bridge material remains constant, then the resonant frequency will double, to $\boxed{6.0 \text{ Hz}}$. Since earthquakes don't do significant shaking at that frequency, the modifications would be effective at keeping the bridge from having large oscillations during an earthquake.

87. From the figure, we can see that the amplitude is 3.5 cm, and the wavelength is 20 cm. The maximum of the wave at $x = 0$ has moved to $x = 12$ cm at $t = 0.80$ s, which is used to find the velocity. The wave is moving to the right. Finally, since the displacement is a maximum at $x = 0$ and $t = 0$, we can use a cosine function without a phase angle.

$$A = 3.5\,\text{cm};\ \lambda = 20\,\text{cm} \ \rightarrow\ k = \frac{2\pi}{\lambda} = 0.10\pi\,\text{cm}^{-1};\ v = \frac{12\,\text{cm}}{0.80\,\text{s}} = 15\,\text{cm/s};\ \omega = vk = 1.5\pi\,\text{rad/s}$$

$$D(x,t) = A\cos(kx - \omega t) = \boxed{(3.5\,\text{cm})\cos(0.10\pi x - 1.5\pi t),\ x \text{ in cm, } t \text{ in s}}$$

88. From the given data, $A = 0.50\,\text{m}$ and $v = 2.5\,\text{m}/4.0\,\text{s} = 0.625\,\text{m/s}$. We use Eq. 15-6 for the average power, with the density of sea water from Table 13-1. We estimate the area of the chest as $(0.30\,\text{m})^2$. Answers may vary according to the approximation used for the area of the chest.

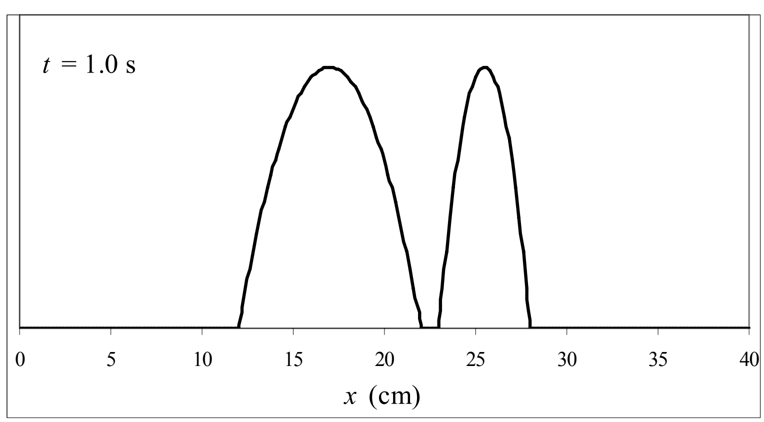
$$\overline{P} = 2\pi^2 \rho S v f^2 A^2 = 2\pi^2 (1025\,\text{kg/m}^3)(0.30\,\text{m})^2 (0.625\,\text{m/s})(0.25\,\text{Hz})^2 (0.50\,\text{m})^2$$

$$= \boxed{18\,\text{W}}$$

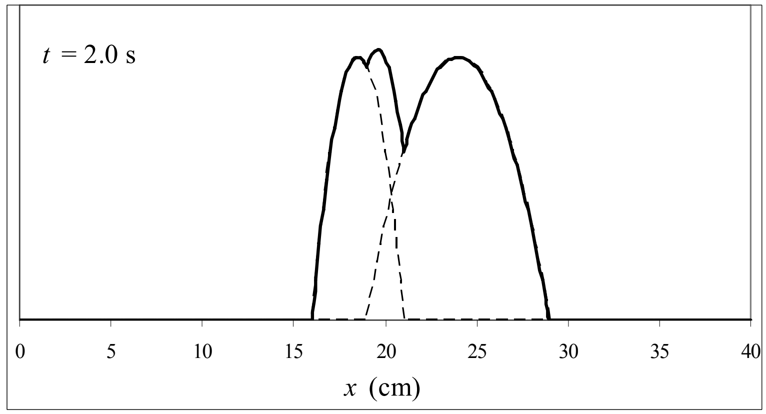
89. The unusual decrease of water corresponds to a trough in Figure 15-4. The crest or peak of the wave is then one-half wavelength from the shore. The peak is 107.5 km away, traveling at 550 km/hr.

$$\Delta x = vt \ \rightarrow\ t = \frac{\Delta x}{v} = \frac{\frac{1}{2}(215\,\text{km})}{550\,\text{km/hr}}\left(\frac{60\,\text{min}}{1\,\text{hr}}\right) = 11.7\,\text{min} \approx \boxed{12\,\text{min}}$$

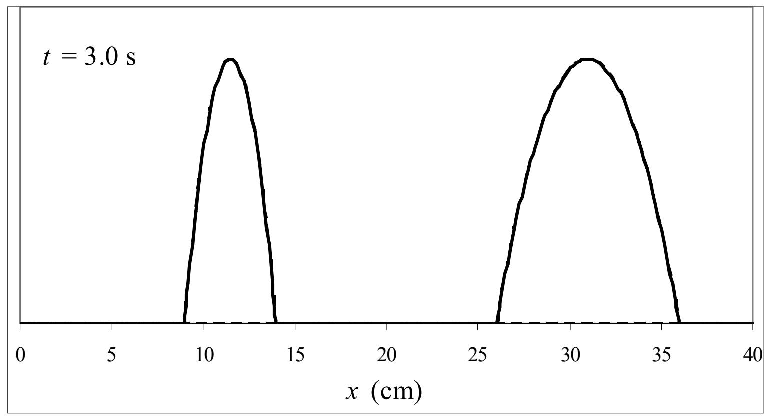
90. At $t = 1.0$ s, the leading edge of each wave is 1.0 cm from the other wave. They have not yet interfered. The leading edge of the wider wave is at 22 cm, and the leading edge of the narrower wave is at 23 cm.



At $t$ = 2.0 s, the waves are overlapping. The diagram uses dashed lines to show the parts of the original waves that are undergoing interference.




At $t$ = 3.0 s, the waves have "passed through" each other, and are no longer interfering.




91. Because the radiation is uniform, the same energy must pass through every spherical surface, which has the surface area $4\pi r^2$. Thus the intensity must decrease as $1/r^2$. Since the intensity is proportional to the square of the amplitude, the amplitude will decrease as $1/r$. The radial motion will be sinusoidal, and so we have $D = \left(\frac{A}{r}\right)\sin(kr - \omega t)$.

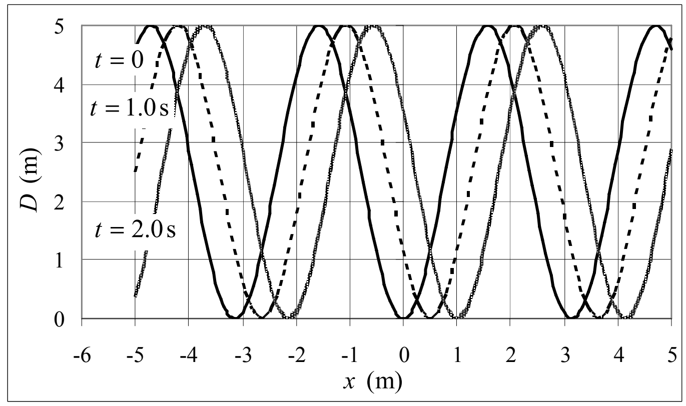
92. The wavelength is to be 1.0 m. Use Eq. 15-1.

$$v = f\lambda \rightarrow f = \frac{v}{\lambda} = \frac{344\,\text{m/s}}{1.0\,\text{m}} = \boxed{340\,\text{Hz}}$$

There will be significant diffraction only for wavelengths larger than the width of the window, and so waves with frequencies lower than 340 Hz would diffract when passing through this window.

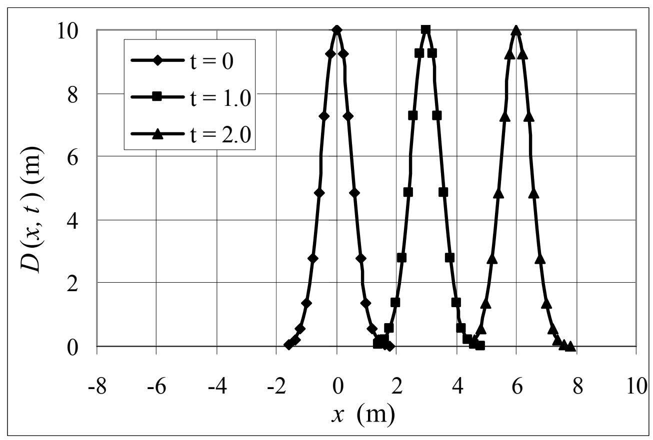
93. The value of $k$ was taken to be $1.0\,\text{m}^{-1}$ for this problem. The peak of the wave moves to the right by 0.50 m during each second that elapses. This can be seen qualitatively from the graph, and quantitatively from the spreadsheet data. Thus the wave speed is given by the constant $c$, $\boxed{0.50\,\text{m/s}}$. The direction of motion is in the positive $x$ direction. The wavelength is seen to be $\boxed{\lambda = \pi\,\text{m}}$. Note that this doesn't agree with the relationship $\lambda = \frac{2\pi}{k}$. The period of the function $\sin^2\theta$ is $\pi$, not $2\pi$ as is the case for $\sin\theta$. In a similar fashion the period of this function is $\boxed{T = 2\pi\,\text{s}}$. Note that this

doesn't agree with the relationship $kv = \omega = \frac{2\pi}{T}$, again because of the behavior of the $\sin^2\theta$ function. But the relationship $\frac{\lambda}{T} = v$ is still true for this wave function. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.93."

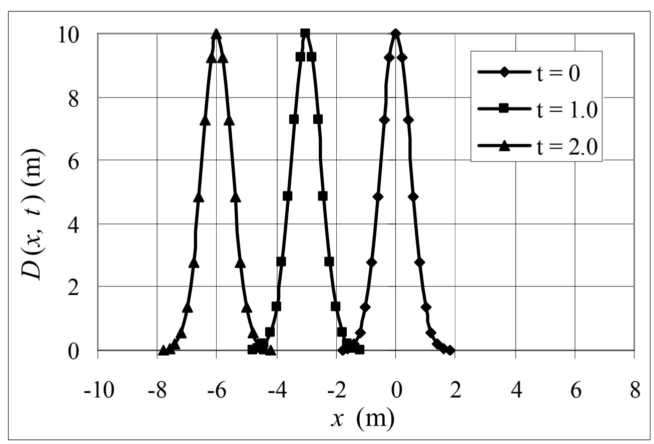



Further insight is gained by re-writing the function using the trigonometric identity $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$, because function $\cos 2\theta$ has a period of $\pi$.

94. (*a*) The graph shows the wave moving 3.0 m to the right each second, which is the expected amount since the speed of the wave is 3.0 m/s and the form of the wave function says the wave is moving to the right. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.94a."




(*b*) The graph shows the wave moving 3.0 m to the left each second, which is the expected amount since the speed of the wave is 3.0 m/s and the form of the wave function says the wave is moving to the left. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.94b."



# CHAPTER 16: Sound

## Responses to Questions

1. Sound exhibits diffraction, refraction, and interference effects that are characteristic of waves. Sound also requires a medium, a characteristic of mechanical waves.

2. Sound can cause objects to vibrate, which is evidence that sound is a form of energy. In extreme cases, sound waves can even break objects. (See Figure 14-24 showing a goblet shattering from the sound of a trumpet.)

3. Sound waves generated in the first cup cause the bottom of the cup to vibrate. These vibrations excite vibrations in the stretched string which are transmitted down the string to the second cup, where they cause the bottom of the second cup to vibrate, generating sound waves which are heard by the second child.

4. The wavelength will change. The frequency cannot change at the boundary since the media on both sides of the boundary are oscillating together. If the frequency were to somehow change, there would be a "pile-up" of wave crests on one side of the boundary.

5. If the speed of sound in air depended significantly on frequency, then the sounds that we hear would be separated in time according to frequency. For example, if a chord were played by an orchestra, we would hear the high notes at one time, the middle notes at another, and the lower notes at still another. This effect is not heard for a large range of distances, indicating that the speed of sound in air does not depend significantly on frequency.

6. Helium is much less dense than air, so the speed of sound in the helium is higher than in air. The wavelength of the sound produced does not change, because it is determined by the length of the vocal cords and other properties of the resonating cavity. The frequency therefore increases, increasing the pitch of the voice.

7. The speed of sound in a medium is equal to $v = \sqrt{B/\rho}$, where $B$ is the bulk modulus and $\rho$ is the density of the medium. The bulk moduli of air and hydrogen are very nearly the same. The density of hydrogen is less than the density of air. The reduced density is the main reason why sound travels faster in hydrogen than in air.

8. The intensity of a sound wave is proportional to the square of the frequency, so the higher-frequency tuning fork will produce more intense sound.

9. Variations in temperature will cause changes in the speed of sound and in the length of the pipes. As the temperature rises, the speed of sound in air increases, increasing the resonance frequency of the pipes, and raising the pitch of the sound. But the pipes get slightly longer, increasing the resonance wavelength and decreasing the resonance frequency of the pipes and lowering the pitch. As the temperature decreases, the speed of sound decreases, decreasing the resonance frequency of the pipes, and lowering the pitch of the sound. But the pipes contract, decreasing the resonance wavelength and increasing the resonance frequency of the pipes and raising the pitch. These effects compete, but the effect of temperature change on the speed of sound dominates.

10. A tube will have certain resonance frequencies associated with it, depending on the length of the tube and the temperature of the air in the tube. Sounds at frequencies far from the resonance

frequencies will not undergo resonance and will not persist. By choosing a length for the tube that isn't resonant for specific frequencies you can reduce the amplitude of those frequencies.

11. As you press on frets closer to the bridge, you are generating higher frequency (and shorter wavelength) sounds. The difference in the wavelength of the resonant standing waves decreases as the wavelengths decrease, so the frets must be closer together as you move toward the bridge.

12. Sound waves can diffract around obstacles such as buildings if the wavelength of the wave is large enough in comparison to the size of the obstacle. Higher frequency corresponds to shorter wavelength. When the truck is behind the building, the lower frequency (longer wavelength) waves bend around the building and reach you, but the higher frequency (shorter wavelength) waves do not. Once the truck has emerged from behind the building, all the different frequencies can reach you.

13. Standing waves are generated by a wave and its reflection. The two waves have a constant phase relationship with each other. The interference depends only on where you are along the string, on your position in space. Beats are generated by two waves whose frequencies are close but not equal. The two waves have a varying phase relationship, and the interference varies with time rather than position.

14. The points would move farther apart. A lower frequency corresponds to a longer wavelength, so the distance between points where destructive and constructive interference occur would increase.

15. According to the principle of superposition, adding a wave and its inverse produces zero displacement of the medium. Adding a sound wave and its inverse effectively cancels out the sound wave and substantially reduces the sound level heard by the worker.

16. (a) The closer the two component frequencies are to each other, the longer the wavelength of the beat. If the two frequencies are very close together, then the waves very nearly overlap, and the distance between a point where the waves interfere constructively and a point where they interfere destructively will be very large.

17. No. The Doppler shift is caused by relative motion between the source and observer.

18. No. The Doppler shift is caused by relative motion between the source and observer. If the wind is blowing, both the wavelength and the velocity of the sound will change, but the frequency of the sound will not.

19. The child will hear the highest frequency at position C, where her speed toward the whistle is the greatest.

20. The human ear can detect frequencies from about 20 Hz to about 20,000 Hz. One octave corresponds to a doubling of frequency. Beginning with 20 Hz, it takes about 10 doublings to reach 20,000 Hz. So, there are approximately 10 octaves in the human audible range.

21. If the frequency of the sound is halved, then the ratio of the frequency of the sound as the car recedes to the frequency of the sound as the car approaches is equal to ½. Substituting the appropriate Doppler shift equations in for the frequencies yields a speed for the car of $^1/_3$ the speed of sound.

## Solutions to Problems

In these solutions, we usually treat frequencies as if they are significant to the whole number of units. For example, 20 Hz is taken as to the nearest Hz, and 20 kHz is taken as to the nearest kHz. We also treat all decibel values as good to whole number of decibels. So 120 dB is good to the nearest decibel.

1. The round trip time for sound is 2.0 seconds, so the time for sound to travel the length of the lake is 1.0 seconds. Use the time and the speed of sound to determine the length of the lake.

$$d = vt = (343\,\text{m/s})(1.0\text{ s}) = 343\text{ m} \approx \boxed{340\,\text{m}}$$

2. The round trip time for sound is 2.5 seconds, so the time for sound to travel the length of the lake is 1.25 seconds. Use the time and the speed of sound in water to determine the depth of the lake.

$$d = vt = (1560\,\text{m/s})(1.25\text{ s}) = 1950\text{ m} = \boxed{2.0\times10^{3}\,\text{m}}$$

3. (*a*) $\lambda_{20\text{ Hz}} = \dfrac{v}{f} = \dfrac{343\,\text{m/s}}{20\text{ Hz}} = \boxed{17\text{ m}}$ $\quad\lambda_{20\text{ kHz}} = \dfrac{v}{f} = \dfrac{343\,\text{m/s}}{2.0\times10^{4}\,\text{Hz}} = \boxed{1.7\times10^{-2}\,\text{m}}$

So the range is from 1.7 cm to 17 m.

(*b*) $\lambda = \dfrac{v}{f} = \dfrac{343\,\text{m/s}}{15\times10^{6}\,\text{Hz}} = \boxed{2.3\times10^{-5}\,\text{m}}$

4. The distance that the sounds travels is the same on both days. That distance is equal to the speed of sound times the elapsed time. Use the temperature-dependent relationships for the speed of sound in air.

$$d = v_1t_1 = v_2t_2 \;\rightarrow\; \left[(331 + 0.6(27))\,\text{m/s}\right](4.70\,\text{s}) = \left[(331 + 0.6(T_2))\,\text{m/s}\right](5.20\,\text{s}) \;\rightarrow$$

$$T_2 = \boxed{-29^\circ\text{C}}$$

5. (*a*) The ultrasonic pulse travels at the speed of sound, and the round trip distance is twice the distance *d* to the object.

$$2d_{\text{min}} = vt_{\text{min}} \;\rightarrow\; d_{\text{min}} = \tfrac{1}{2}vt_{\text{min}} = \tfrac{1}{2}(343\,\text{m/s})(1.0\times10^{-3}\text{s}) = \boxed{0.17\,\text{m}}$$

(*b*) The measurement must take no longer than 1/15 s. Again, the round trip distance is twice the distance to the object.

$$2d_{\text{max}} = vt_{\text{max}} \;\rightarrow\; d_{\text{max}} = \tfrac{1}{2}vt_{\text{max}} = \tfrac{1}{2}(343\,\text{m/s})(\tfrac{1}{15}\text{s}) = \boxed{11\,\text{m}}$$

(*c*) The distance is proportional to the speed of sound. So the percentage error in distance is the same as the percentage error in the speed of sound. We assume the device is calibrated to work at $20^\circ\text{C}$.

$$\frac{\Delta d}{d} = \frac{\Delta v}{v} = \frac{v_{23^\circ\text{C}} - v_{20^\circ\text{C}}}{v_{20^\circ\text{C}}} = \frac{\left[331 + 0.60(23)\right]\text{m/s} - 343\,\text{m/s}}{343\,\text{m/s}} = 0.005248 \approx \boxed{0.5\%}$$

6. (*a*) For the fish, the speed of sound in seawater must be used.

$$d = vt \;\rightarrow\; t = \frac{d}{v} = \frac{1350\,\text{m}}{1560\,\text{m/s}} = \boxed{0.865\,\text{s}}$$

(*b*) For the fishermen, the speed of sound in air must be used.

$$d = vt \;\rightarrow\; t = \frac{d}{v} = \frac{1350\,\text{m}}{343\,\text{m/s}} = \boxed{3.94\,\text{s}}$$

7. The total time $T$ is the time for the stone to fall $\left(t_{\text{down}}\right)$ plus the time for the sound to come back to the top of the cliff $\left(t_{\text{up}}\right)$: $T = t_{\text{up}} + t_{\text{down}}$. Use constant acceleration relationships for an object dropped from rest that falls a distance $h$ in order to find $t_{\text{down}}$, with down as the positive direction. Use the constant speed of sound to find $t_{\text{up}}$ for the sound to travel a distance $h$.

$$\text{down: } y = y_0 + v_0 t_{\text{down}} + \tfrac{1}{2} a t_{\text{down}}^2 \rightarrow h = \tfrac{1}{2} g t_{\text{down}}^2 \qquad \text{up: } h = v_{\text{snd}} t_{\text{up}} \rightarrow t_{\text{up}} = \frac{h}{v_{\text{snd}}}$$

$$h = \tfrac{1}{2} g t_{\text{down}}^2 = \tfrac{1}{2} g \left(T - t_{\text{up}}\right)^2 = \tfrac{1}{2} g \left(T - \frac{h}{v_{\text{snd}}}\right)^2 \rightarrow h^2 - 2v_{\text{snd}}\left(\frac{v_{\text{snd}}}{g} + T\right)h + T^2 v_{\text{snd}}^2 = 0$$

This is a quadratic equation for the height. This can be solved with the quadratic formula, but be sure to keep several significant digits in the calculations.

$$h^2 - 2\left(343\,\text{m/s}\right)\left(\frac{343\,\text{m/s}}{9.80\,\text{m/s}^2} + 3.0\,\text{s}\right)h + \left(3.0\,\text{s}\right)^2\left(343\,\text{m/s}\right)^2 = 0 \rightarrow$$

$$h^2 - \left(26068\,\text{m}\right)h + 1.0588 \times 10^6\,\text{m}^2 = 0 \rightarrow h = 26028\,\text{m}, 41\,\text{m}$$

The larger root is impossible since it takes more than 3.0 sec for the rock to fall that distance, so the correct result is $h = \boxed{41\,\text{m}}$.

8. The two sound waves travel the same distance. The sound will travel faster in the concrete, and thus take a shorter time.

$$d = v_{\text{air}} t_{\text{air}} = v_{\text{concrete}} t_{\text{concrete}} = v_{\text{concrete}}\left(t_{\text{air}} - 0.75\,\text{s}\right) \rightarrow t_{\text{air}} = \frac{v_{\text{concrete}}}{v_{\text{concrete}} - v_{\text{air}}} 0.75\,\text{s}$$

$$d = v_{\text{air}} t_{\text{air}} = v_{\text{air}}\left(\frac{v_{\text{concrete}}}{v_{\text{concrete}} - v_{\text{air}}} 0.75\,\text{s}\right)$$

The speed of sound in concrete is obtained from Table 16-1 as 3000 m/s.

$$d = \left(343\,\text{m/s}\right)\left(\frac{3000\,\text{m/s}}{3000\,\text{m/s} - 343\,\text{m/s}}\left(0.75\,\text{s}\right)\right) = \boxed{290\,\text{m}}$$

9. The "5 second rule" says that for every 5 seconds between seeing a lightning strike and hearing the associated sound, the lightning is 1 mile distant. We assume that there are 5 seconds between seeing the lightning and hearing the sound.

(*a*) At 30°C, the speed of sound is $\left[331 + 0.60\left(30\right)\right]\text{m/s} = 349\,\text{m/s}$. The actual distance to the lightning is therefore $d = vt = \left(349\,\text{m/s}\right)\left(5\,\text{s}\right) = 1745\,\text{m}$. A mile is 1610 m.

$$\%\ \text{error} = \frac{1745 - 1610}{1745}\left(100\right) \approx \boxed{8\%}$$

(*b*) At 10°C, the speed of sound is $\left[331 + 0.60\left(10\right)\right]\text{m/s} = 337\,\text{m/s}$. The actual distance to the lightning is therefore $d = vt = \left(337\,\text{m/s}\right)\left(5\,\text{s}\right) = 1685\,\text{m}$. A mile is 1610 m.

$$\%\ \text{error} = \frac{1685 - 1610}{1685}\left(100\right) \approx \boxed{4\%}$$

10. The relationship between the pressure and displacement amplitudes is given by Eq. 16-5.

(*a*) $\Delta P_{\text{M}} = 2\pi\rho vAf \rightarrow A = \dfrac{\Delta P_{\text{M}}}{2\pi\rho vf} = \dfrac{3.0\times10^{-3}\,\text{Pa}}{2\pi\left(1.29\,\text{kg/m}^3\right)\left(331\,\text{m/s}\right)\left(150\,\text{Hz}\right)} = \boxed{7.5\times10^{-9}\,\text{m}}$

(*b*) $A = \dfrac{\Delta P_{\text{M}}}{2\pi\rho vf} = \dfrac{3.0\times10^{-3}\,\text{Pa}}{2\pi\left(1.29\,\text{kg/m}^3\right)\left(331\,\text{m/s}\right)\left(15\times10^3\,\text{Hz}\right)} = \boxed{7.5\times10^{-11}\,\text{m}}$

11. The pressure amplitude is found from Eq. 16-5. The density of air is $1.29\,\text{kg/m}^3$.

(*a*) $\Delta P_{\text{M}} = 2\pi\rho vAf = 2\pi\left(1.29\,\text{kg/m}^3\right)\left(331\,\text{m/s}\right)\left(3.0\times10^{-10}\,\text{m}\right)\left(55\,\text{Hz}\right) = \boxed{4.4\times10^{-5}\,\text{Pa}}$

(*b*) $\Delta P_{\text{M}} = 2\pi\rho vAf = 2\pi\left(1.29\,\text{kg/m}^3\right)\left(331\,\text{m/s}\right)\left(3.0\times10^{-10}\,\text{m}\right)\left(5500\,\text{Hz}\right) = \boxed{4.4\times10^{-3}\,\text{Pa}}$

12. The pressure wave can be written as Eq. 16-4.

(*a*) $\Delta P = -\Delta P_{\text{M}}\cos\left(kx - \omega t\right)$

$\Delta P_{\text{M}} = 4.4\times10^{-5}\,\text{Pa}$ ; $\omega = 2\pi f = 2\pi\left(55\,\text{Hz}\right) = 110\pi\,\text{rad/s}$ ; $k = \dfrac{\omega}{v} = \dfrac{110\pi\,\text{rad/s}}{331\,\text{m/s}} = 0.33\pi\,\text{m}^{-1}$

$\boxed{\Delta P = -\left(4.4\times10^{-5}\,\text{Pa}\right)\cos\left[\left(0.33\pi\,\text{m}^{-1}\right)x - \left(110\pi\,\text{rad/s}\right)t\right]}$

(*b*) All is the same except for the amplitude and $\omega = 2\pi f = 2\pi\left(5500\text{ Hz}\right) = 1.1\times10^4\pi\,\text{rad/s}$.

$\boxed{\Delta P = -\left(4.4\times10^{-3}\,\text{Pa}\right)\cos\left[\left(0.33\pi\,\text{m}^{-1}\right)x - \left(1.1\times10^4\pi\,\text{rad/s}\right)t\right]}$

13. The pressure wave is $\Delta P = \left(0.0035\,\text{Pa}\right)\sin\left[\left(0.38\pi\,\text{m}^{-1}\right)x - \left(1350\pi\,\text{s}^{-1}\right)t\right]$.

(*a*) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{0.38\pi\,\text{m}^{-1}} = \boxed{5.3\,\text{m}}$

(*b*) $f = \dfrac{\omega}{2\pi} = \dfrac{1350\pi\,\text{s}^{-1}}{2\pi} = \boxed{675\,\text{Hz}}$

(*c*) $v = \dfrac{\omega}{k} = \dfrac{1350\pi\,\text{s}^{-1}}{0.38\pi\,\text{m}^{-1}} = 3553\,\text{m/s} \approx \boxed{3600\,\text{m/s}}$

(*d*) Use Eq. 16-5 to find the displacement amplitude.

$\Delta P_{\text{M}} = 2\pi\rho vAf \rightarrow$

$$A = \frac{\Delta P_{\text{M}}}{2\pi\rho vf} = \frac{\left(0.0035\,\text{Pa}\right)}{2\pi\left(2300\,\text{kg/m}^3\right)\left(3553\,\text{m/s}\right)\left(675\,\text{Hz}\right)} = \boxed{1.0\times10^{-13}\,\text{m}}$$

14. $120\text{ dB} = 10\log\dfrac{I_{120}}{I_0} \rightarrow I_{120} = 10^{12}I_0 = 10^{12}\left(1.0\times10^{-12}\text{ W/m}^2\right) = \boxed{1.0\,\text{W/m}^2}$

$20\text{ dB} = 10\log\dfrac{I_{20}}{I_0} \rightarrow I_{20} = 10^{2}I_0 = 10^{2}\left(1.0\times10^{-12}\text{ W/m}^2\right) = \boxed{1.0\times10^{-10}\text{ W/m}^2}$

The pain level is $10^{10}$ times more intense than the whisper.

15. $\beta = 10\log\frac{I}{I_0} = 10\log\frac{2.0\times10^{-6}\ \mathrm{W/m^2}}{1.0\times10^{-12}\ \mathrm{W/m^2}} = \boxed{63\ \mathrm{dB}}$

16. From Figure 16-6, at 40 dB the low frequency threshold of hearing is about $\boxed{70-80\ \mathrm{Hz}}$. There is no intersection of the threshold of hearing with the 40 dB level on the high frequency side of the chart, and so a 40 dB signal can be heard all the way up to the highest frequency that a human can hear, $\boxed{20{,}000\ \mathrm{Hz}}$.

17. (*a*) From Figure 16-6, at 100 Hz, the threshold of hearing (the lowest detectable intensity by the ear) is approximately $5\times10^{-9}\ \mathrm{W/m^2}$. The threshold of pain is about $5\ \mathrm{W/m^2}$. The ratio of highest to lowest intensity is thus $\frac{5\ \mathrm{W/m^2}}{5\times10^{-9}\ \mathrm{W/m^2}} = \boxed{10^9}$.

    (*b*) At 5000 Hz, the threshold of hearing is about $10^{-13}\ \mathrm{W/m^2}$, and the threshold of pain is about $10^{-1}\ \mathrm{W/m^2}$. The ratio of highest to lowest intensity is $\frac{10^{-1}\ \mathrm{W/m^2}}{10^{-13}\ \mathrm{W/m^2}} = \boxed{10^{12}}$.

    Answers may vary due to estimation in the reading of the graph.

18. Compare the two power output ratings using the definition of decibels.

$$\beta = 10\log\frac{P_{150}}{P_{100}} = 10\log\frac{150\ \mathrm{W}}{100\ \mathrm{W}} = \boxed{1.8\ \mathrm{dB}}$$

This would barely be perceptible.

$\boxed{19.}$ The intensity can be found from the decibel value.

$$\beta = 10\log\frac{I}{I_0} \rightarrow I = 10^{\beta/10}I_0 = 10^{12}\left(10^{-12}\ \mathrm{W/m^2}\right) = 1.0\ \mathrm{W/m^2}$$

Consider a square perpendicular to the direction of travel of the sound wave. The intensity is the energy transported by the wave across a unit area perpendicular to the direction of travel, per unit time. So $I = \frac{\Delta E}{S\Delta t}$, where *S* is the area of the square. Since the energy is "moving" with the wave, the "speed" of the energy is *v*, the wave speed. In a time $\Delta t$, a volume equal to $\Delta V = Sv\Delta t$ would contain all of the energy that had been transported across the area *S*. Combine these relationships to find the energy in the volume.

$$I = \frac{\Delta E}{S\Delta t} \rightarrow \Delta E = IS\Delta t = \frac{I\Delta V}{v} = \frac{\left(1.0\ \mathrm{W/m^2}\right)\left(0.010\ \mathrm{m}\right)^3}{343\ \mathrm{m/s}} = \boxed{2.9\times10^{-9}\ \mathrm{J}}$$

20. From Example 12-4, we see that a sound level decrease of 3 dB corresponds to a halving of intensity. Thus the sound level for one firecracker will be $95\ \mathrm{dB} - 3\ \mathrm{dB} = \boxed{92\ \mathrm{dB}}$.

21. From Example 16-4, we see that a sound level decrease of 3 dB corresponds to a halving of intensity. Thus, if two engines are shut down, the intensity will be cut in half, and the sound level will be 127 dB. Then, if one more engine is shut down, the intensity will be cut in half again, and the sound level will drop by 3 more dB, to a final value of $\boxed{124\ \mathrm{dB}}$.

22. $62\,\text{dB} = 10\log\left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} \quad \rightarrow \quad \left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} = 10^{6.2} = \boxed{1.6\times10^{6}}$

$98\,\text{dB} = 10\log\left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} \quad \rightarrow \quad \left(I_{\text{Signal}}/I_{\text{Noise}}\right)_{\text{tape}} = 10^{9.8} = \boxed{6.3\times10^{9}}$

23. (*a*) According to Table 16-2, the intensity in normal conversation, when about 50 cm from the speaker, is about $3\times10^{-6}\ \text{W/m}^2$. The intensity is the power output per unit area, and so the power output can be found. The area is that of a sphere.

$$I = \frac{P}{A} \quad \rightarrow \quad P = IA = I\left(4\pi r^2\right) = \left(3\times10^{-6}\ \text{W/m}^2\right)4\pi\left(0.50\ \text{m}\right)^2 = 9.425\times10^{-6}\,\text{W} \approx \boxed{9.4\times10^{-6}\,\text{W}}$$

(*b*) $75\,\text{W}\left(\dfrac{1\ \text{person}}{9.425\times10^{-6}\,\text{W}}\right) = 7.96\times10^{6} \approx \boxed{8.0\times10^{6}\,\text{people}}$

24. (*a*) The energy absorbed per second is the power of the wave, which is the intensity times the area.

$$50\ \text{dB} = 10\log\frac{I}{I_0} \quad \rightarrow \quad I = 10^{5} I_0 = 10^{5}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = 1.0\times10^{-7}\ \text{W/m}^2$$

$$P = IA = \left(1.0\times10^{-7}\ \text{W/m}^2\right)\left(5.0\times10^{-5}\,\text{m}^2\right) = \boxed{5.0\times10^{-12}\,\text{W}}$$

(*b*) $1\ \text{J}\left(\dfrac{1\ \text{s}}{5.0\times10^{-12}\ \text{J}}\right)\left(\dfrac{1\ \text{yr}}{3.16\times10^{7}\,\text{s}}\right) = \boxed{6.3\times10^{3}\,\text{yr}}$

<u>25.</u> The intensity of the sound is defined to be the power per unit area. We assume that the sound spreads out spherically from the loudspeaker.

(*a*) $I_{250} = \dfrac{250\,\text{W}}{4\pi\left(3.5\,\text{m}\right)^2} = 1.624\,\text{W/m}^2 \qquad \beta_{250} = 10\log\dfrac{I_{250}}{I_0} = 10\log\dfrac{1.624\,\text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2} = \boxed{122\,\text{dB}}$

$I_{45} = \dfrac{45\,\text{W}}{4\pi\left(3.5\text{m}\right)^2} = 0.2923\,\text{W/m}^2 \qquad \beta_{45} = 10\log\dfrac{I_{45}}{I_0} = 10\log\dfrac{0.2923\,\text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2} = \boxed{115\,\text{dB}}$

(*b*) According to the textbook, for a sound to be perceived as twice as loud as another means that the intensities need to differ by a factor of 10. That is not the case here – they differ only by a factor of $\dfrac{1.624}{0.2598} \approx 6$. $\boxed{\text{The expensive amp will not sound twice as loud as the cheaper one.}}$

26. (*a*) Find the intensity from the 130 dB value, and then find the power output corresponding to that intensity at that distance from the speaker.

$$\beta = 130\ \text{dB} = 10\log\frac{I_{2.8\text{m}}}{I_0} \quad \rightarrow \quad I_{2.8\text{m}} = 10^{13} I_0 = 10^{13}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = 10\,\text{W/m}^2$$

$$P = IA = 4\pi r^2 I = 4\pi\left(2.2\,\text{m}\right)^2\left(10\,\text{W/m}^2\right) = 608\,\text{W} \approx \boxed{610\,\text{W}}$$

(*b*) Find the intensity from the 85 dB value, and then from the power output, find the distance corresponding to that intensity.

$$\beta = 85\,\text{dB} = 10\log\frac{I}{I_0} \quad \rightarrow \quad I = 10^{8.5} I_0 = 10^{8.5}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = 3.16\times10^{-4}\ \text{W/m}^2$$

$$P = 4\pi r^2 I \quad \rightarrow \quad r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{608\,\text{W}}{4\pi\left(3.16\times10^{-4}\ \text{W/m}^2\right)}} = \boxed{390\,\text{m}}$$

27. The first person is a distance of $r_1 = 100\,\text{m}$ from the explosion, while the second person is a distance $r_2 = \sqrt{5}(100\,\text{m})$ from the explosion. The intensity detected away from the explosion is inversely proportional to the square of the distance from the explosion.

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} = \left[\frac{\sqrt{5}(100\,\text{m})}{100\,\text{m}}\right]^2 = 5 \;;\; \beta = 10\log\frac{I_1}{I_2} = 10\log 5 = \boxed{7.0\,\text{dB}}$$

28. (*a*) The intensity is proportional to the square of the amplitude, so if the amplitude is 2.5 times greater, the intensity will $\boxed{\text{increase by a factor of } 6.25 \approx 6.3}$.

(*b*) $\beta = 10\log I/I_0 = 10\log 6.25 = \boxed{8\,\text{dB}}$

29. (*a*) The pressure amplitude is seen in Eq. 16-5 to be proportional to the displacement amplitude and to the frequency. Thus $\boxed{\text{the higher frequency wave has the larger pressure amplitude, by a factor of 2.6.}}$

(*b*) The intensity is proportional to the square of the frequency. Thus the ratio of the intensities is the square of the frequency ratio.

$$\frac{I_{2.6f}}{I_f} = \frac{(2.6f)^2}{f^2} = 6.76 \approx \boxed{6.8}$$

30. The intensity is given by Eq. 15-7, $I = 2\pi^2 v\rho f^2 A^2$, using the density of air and the speed of sound in air.

$$I = 2\rho v\pi^2 f^2 A^2 = 2\left(1.29\,\text{kg/m}^3\right)(343\,\text{m/s})\pi^2(380\,\text{Hz})^2\left(1.3\times10^{-4}\,\text{m}\right)^2 = 21.31\,\text{W/m}^2$$

$$\beta = 10\log\frac{I}{I_0} = 10\log\frac{21.31\,\text{W/m}^2}{1.0\times10^{-12}\,\text{W/m}^2} = 133\,\text{dB} \approx \boxed{130\,\text{dB}}$$

Note that this is above the threshold of pain.

31. (*a*) We find the intensity of the sound from the decibel value, and then calculate the displacement amplitude from Eq. 15-7.

$$\beta = 10\log\frac{I}{I_0} \;\rightarrow\; I = 10^{\beta/10} I_0 = 10^{12}\left(10^{-12}\,\text{W/m}^2\right) = 1.0\,\text{W/m}^2$$

$$I = 2\pi^2 v\rho f^2 A^2 \;\rightarrow$$

$$A = \frac{1}{\pi f}\sqrt{\frac{I}{2\rho v}} = \frac{1}{\pi(330\,\text{Hz})}\sqrt{\frac{1.0\,\text{W/m}^2}{2\left(1.29\,\text{kg/m}^3\right)(343\,\text{m/s})}} = \boxed{3.2\times10^{-5}\,\text{m}}$$

(*b*) The pressure amplitude can be found from Eq. 16-7.

$$I = \frac{(\Delta P_{\text{M}})^2}{2v\rho} \;\rightarrow$$

$$\Delta P_{\text{M}} = \sqrt{2v\rho I} = \sqrt{2(343\,\text{m/s})\left(1.29\,\text{kg/m}^3\right)\left(1.0\,\text{W/m}^2\right)} = \boxed{30\,\text{Pa}\,(2\text{ sig. fig.})}$$

32. (*a*) We assume that there has been no appreciable absorption in this 25 meter distance. The intensity is the power divide by the area of a sphere of radius 25 meters. We express the sound level in dB.

$$I=\frac{P}{4\pi r^2}\ ;\ \beta=10\log\frac{I}{I_0}=10\log\frac{P}{4\pi r^2 I_0}=10\log\frac{\left(5.0\times10^5\,\text{W}\right)}{4\pi\left(25\,\text{m}\right)^2\left(10^{-12}\,\text{W/m}^2\right)}=\boxed{138\,\text{dB}}$$

(*b*) We find the intensity level at the new distance, and subtract due to absorption.

$$\beta=10\log\frac{P}{4\pi r^2 I_0}=10\log\frac{\left(5.0\times10^5\,\text{W}\right)}{4\pi\left(1000\,\text{m}\right)^2\left(10^{-12}\,\text{W/m}^2\right)}=106\,\text{dB}$$

$$\beta_{\substack{\text{with}\\ \text{absorption}}}=106\,\text{dB}-\left(1.00\,\text{km}\right)\left(7.0\,\text{dB/km}\right)=\boxed{99\,\text{dB}}$$

(*c*) We find the intensity level at the new distance, and subtract due to absorption.

$$\beta=10\log\frac{P}{4\pi r^2 I_0}=10\log\frac{\left(5.0\times10^5\,\text{W}\right)}{4\pi\left(7500\,\text{m}\right)^2\left(10^{-12}\,\text{W/m}^2\right)}=88.5\,\text{dB}$$

$$\beta_{\substack{\text{with}\\ \text{absorption}}}=88.5\,\text{dB}-\left(7.50\,\text{km}\right)\left(7.0\,\text{dB/km}\right)=\boxed{36\,\text{dB}}$$

33. For a closed tube, Figure 16-12 indicates that $f_1=\frac{v}{4\ell}$. We assume the bass clarinet is at room temperature.

$$f_1=\frac{v}{4\ell}\ \rightarrow\ \ell=\frac{v}{4f_1}=\frac{343\,\text{m/s}}{4\left(69.3\,\text{Hz}\right)}=\boxed{1.24\,\text{m}}$$

34. For a vibrating string, the frequency of the fundamental mode is given by $f=\frac{v}{2L}=\frac{1}{2L}\sqrt{\frac{F_T}{m/L}}$.

$$f=\frac{1}{2L}\sqrt{\frac{F_T}{m/L}}\ \rightarrow\ F_T=4Lf^2m=4\left(0.32\,\text{m}\right)\left(440\,\text{Hz}\right)^2\left(3.5\times10^{-4}\,\text{kg}\right)=\boxed{87\,\text{N}}$$

35. (*a*) If the pipe is closed at one end, only the odd harmonic frequencies are present, and are given by $f_n=\frac{nv}{4L}=nf_1, n=1,3,5\cdots$.

$$f_1=\frac{v}{4L}=\frac{343\,\text{m/s}}{4\left(1.24\text{ m}\right)}=\boxed{69.2\text{ Hz}}$$

$$f_3=3f_1=\boxed{207\,\text{Hz}}\qquad f_5=5f_1=\boxed{346\,\text{Hz}}\qquad f_7=7f_1=\boxed{484\,\text{Hz}}$$

(*b*) If the pipe is open at both ends, all the harmonic frequencies are present, and are given by $f_n=\frac{nv}{2\ell}=nf_1$.

$$f_1=\frac{v}{2\ell}=\frac{343\,\text{m/s}}{2\left(1.24\,\text{m}\right)}=138.3\,\text{Hz}\approx\boxed{138\,\text{Hz}}$$

$$f_2=2f_1=\frac{v}{\ell}=\boxed{277\,\text{Hz}}\qquad f_3=3f_1=\frac{3v}{2\ell}=\boxed{415\,\text{Hz}}\qquad f_4=4f_1=\frac{2v}{\ell}=\boxed{553\,\text{Hz}}$$

36. (*a*) The length of the tube is one-fourth of a wavelength for this (one end closed) tube, and so the wavelength is four times the length of the tube.

$$f = \frac{v}{\lambda} = \frac{343\,\text{m/s}}{4(0.21\,\text{m})} = \boxed{410\,\text{Hz}}$$

(*b*) If the bottle is one-third full, then the effective length of the air column is reduced to 14 cm.

$$f = \frac{v}{\lambda} = \frac{343\,\text{m/s}}{4(0.14\,\text{m})} = \boxed{610\,\text{Hz}}$$

37. For a pipe open at both ends, the fundamental frequency is given by $f_1 = \frac{v}{2\ell}$, and so the length for a given fundamental frequency is $\ell = \frac{v}{2f_1}$.

$$\ell_{20\ \text{Hz}} = \frac{343\,\text{m/s}}{2(20\ \text{Hz})} = \boxed{8.6\ \text{m}} \qquad \ell_{20\ \text{kHz}} = \frac{343\,\text{m/s}}{2(20{,}000\ \text{Hz})} = \boxed{8.6\times10^{-3}\,\text{m}}$$

38. We approximate the shell as a closed tube of length 20 cm, and calculate the fundamental frequency.

$$f = \frac{v}{4\ell} = \frac{343\,\text{m/s}}{4(0.20\,\text{m})} = 429\,\text{Hz} \approx \boxed{430\,\text{Hz}}$$

39. (*a*) We assume that the speed of waves on the guitar string does not change when the string is fretted. The fundamental frequency is given by $f = \frac{v}{2\ell}$, and so the frequency is inversely proportional to the length.

$$f \propto \frac{1}{\ell} \quad \rightarrow \quad f\ell = \text{constant}$$

$$f_E\ell_E = f_A\ell_A \quad \rightarrow \quad \ell_A = \ell_E\frac{f_E}{f_A} = (0.73\ \text{m})\left(\frac{330\ \text{Hz}}{440\ \text{Hz}}\right) = 0.5475\ \text{m}$$

The string should be fretted a distance $0.73\ \text{m} - 0.5475\ \text{m} = 0.1825\,\text{m} \approx \boxed{0.18\ \text{m}}$ from the nut of the guitar.

(*b*) The string is fixed at both ends and is vibrating in its fundamental mode. Thus the wavelength is twice the length of the string (see Fig. 16-7).

$$\lambda = 2\ell = 2(0.5475\ \text{m}) = 1.095\,\text{m} \approx \boxed{1.1\ \text{m}}$$

(*c*) The frequency of the sound will be the same as that of the string, $\boxed{440\ \text{Hz}}$. The wavelength is given by the following.

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{440\ \text{Hz}} = \boxed{0.78\ \text{m}}$$

40. (*a*) At $T = 15^\circ\text{C}$, the speed of sound is given by $v = (331 + 0.60(15))\,\text{m/s} = 340\,\text{m/s}$ (with 3 significant figures). For an open pipe, the fundamental frequency is given by $f = \frac{v}{2\ell}$.

$$f = \frac{v}{2\ell} \quad \rightarrow \quad \ell = \frac{v}{2f} = \frac{340\,\text{m/s}}{2(262\ \text{Hz})} = \boxed{0.649\,\text{m}}$$

(*b*) The frequency of the standing wave in the tube is $\boxed{262\ \text{Hz}}$. The wavelength is twice the length of the pipe, $\boxed{1.30\,\text{m}}$.

(*c*) The wavelength and frequency are the same in the air, because it is air that is resonating in the organ pipe. The frequency is $\boxed{262\text{ Hz}}$ and the wavelength is $\boxed{1.30\,\text{m}}$.

41. The speed of sound will change as the temperature changes, and that will change the frequency of the organ. Assume that the length of the pipe (and thus the resonant wavelength) does not change.

$$f_{22} = \frac{v_{22}}{\lambda} \qquad f_{5.0} = \frac{v_{5.0}}{\lambda} \qquad \Delta f = f_{5.0} - f_{22} = \frac{v_{5.0} - v_{22}}{\lambda}$$

$$\frac{\Delta f}{f} = \frac{\dfrac{v_{5.0} - v_{22}}{\lambda}}{\dfrac{v_{22}}{\lambda}} = \frac{v_{5.0}}{v_{22}} - 1 = \frac{331 + 0.60(5.0)}{331 + 0.60(22)} - 1 = -2.96\times10^{-2} = \boxed{-3.0\%}$$

42. A flute is a tube that is open at both ends, and so the fundamental frequency is given by $f = \dfrac{v}{2\ell}$, where $\ell$ is the distance from the mouthpiece (antinode) to the first open side hole in the flute tube (antinode).

$$f = \frac{v}{2\ell} \quad\rightarrow\quad \ell = \frac{v}{2f} = \frac{343\,\text{m/s}}{2(349\,\text{Hz})} = \boxed{0.491\,\text{m}}$$

$\boxed{43.}$ For a tube open at both ends, all harmonics are allowed, with $f_n = nf_1$. Thus consecutive harmonics differ by the fundamental frequency. The four consecutive harmonics give the following values for the fundamental frequency.

$$f_1 = 523\,\text{Hz} - 392\,\text{Hz} = 131\,\text{Hz},\ \ 659\,\text{Hz} - 523\,\text{Hz} = 136\,\text{Hz},\ \ 784\,\text{Hz} - 659\,\text{Hz} = 125\,\text{Hz}$$

The average of these is $f_1 = \frac{1}{3}(131\,\text{Hz} + 136\,\text{Hz} + 125\,\text{Hz}) \approx 131\,\text{Hz}$. We use that for the fundamental frequency.

(*a*) $f_1 = \dfrac{v}{2\ell} \quad\rightarrow\quad \ell = \dfrac{v}{2f_1} = \dfrac{343\,\text{m/s}}{2(131\,\text{Hz})} = \boxed{1.31\,\text{m}}$

Note that the bugle is coiled like a trumpet so that the full length fits in a smaller distance.

(*b*) $f_n = nf_1 \quad\rightarrow\quad n_{\text{G4}} = \dfrac{f_{\text{G4}}}{f_1} = \dfrac{392\,\text{Hz}}{131\,\text{Hz}} = 2.99\,;\ n_{\text{C5}} = \dfrac{f_{\text{C5}}}{f_1} = \dfrac{523\,\text{Hz}}{131\,\text{Hz}} = 3.99\,;$

$n_{\text{E5}} = \dfrac{f_{\text{E5}}}{f_1} = \dfrac{659\,\text{Hz}}{131\,\text{Hz}} = 5.03\,;\ n_{\text{G5}} = \dfrac{f_{\text{G5}}}{f_1} = \dfrac{784\,\text{Hz}}{131\,\text{Hz}} = 5.98$

The harmonics are $\boxed{3,\ 4,\ 5,\ \text{and}\ 6}$.

44. (*a*) The difference between successive overtones for this pipe is 176 Hz. The difference between successive overtones for an open pipe is the fundamental frequency, and each overtone is an integer multiple of the fundamental. Since 264 Hz is not a multiple of 176 Hz, 176 Hz cannot be the fundamental, and so the pipe cannot be open. Thus it must be a $\boxed{\text{closed}}$ pipe.

(*b*) For a closed pipe, the successive overtones differ by twice the fundamental frequency. Thus 176 Hz must be twice the fundamental, so the fundamental is $\boxed{88\,\text{Hz}}$. This is verified since 264 Hz is 3 times the fundamental, 440 Hz is 5 times the fundamental, and 616 Hz is 7 times the fundamental.

45. The tension and mass density of the string do not change, so the wave speed is constant. The frequency ratio for two adjacent notes is to be $2^{1/12}$. The frequency is given by $f = \frac{v}{2\ell}$.

$$f = \frac{v}{2\ell} \rightarrow \frac{f_{\substack{1st\\fret}}}{f_{unfingered}} = \frac{\dfrac{v}{2\ell_{\substack{1st\\fret}}}}{\dfrac{v}{2\ell_{unfingered}}} = 2^{1/12} \rightarrow \ell_{\substack{1st\\fret}} = \frac{\ell_{unfingered}}{2^{1/12}} = \frac{65.0\,\text{cm}}{2^{1/12}} = 61.35\,\text{cm}$$

$$\ell_{\substack{2nd\\fret}} = \frac{\ell_{\substack{1st\\fret}}}{2^{1/12}} = \frac{\ell_{unfingered}}{2^{2/12}} \rightarrow \ell_{\substack{nth\\fret}} = \frac{\ell_{unfingered}}{2^{n/12}} \;;\; x_{\substack{nth\\fret}} = \ell_{unfingered} - \ell_{\substack{nth\\fret}} = \ell_{unfingered}\left(1 - 2^{-n/12}\right)$$

$$x_1 = (65.0\,\text{cm})\left(1 - 2^{-1/12}\right) = \boxed{3.6\,\text{cm}} \;;\; x_2 = (65.0\,\text{cm})\left(1 - 2^{-2/12}\right) = \boxed{7.1\,\text{cm}}$$

$$x_3 = (65.0\,\text{cm})\left(1 - 2^{-3/12}\right) = \boxed{10.3\,\text{cm}} \;;\; x_4 = (65.0\,\text{cm})\left(1 - 2^{-4/12}\right) = \boxed{13.4\,\text{cm}}$$

$$x_5 = (65.0\,\text{cm})\left(1 - 2^{-5/12}\right) = \boxed{16.3\,\text{cm}} \;;\; x_6 = (65.0\,\text{cm})\left(1 - 2^{-6/12}\right) = \boxed{19.0\,\text{cm}}$$

46. (*a*) The difference between successive overtones for an open pipe is the fundamental frequency.

$$f_1 = 330\,\text{Hz} - 275\,\text{Hz} = \boxed{55\text{ Hz}}$$

(*b*) The fundamental frequency is given by $f_1 = \frac{v}{2\ell}$. Solve this for the speed of sound.

$$v = 2\ell f_1 = 2(1.80\text{ m})(55\text{ Hz}) = 198\,\text{m/s} \approx \boxed{2.0\times10^2\text{ m/s}}$$

47. The difference in frequency for two successive harmonics is 40 Hz. For an open pipe, two successive harmonics differ by the fundamental, so the fundamental could be 40 Hz, with 240 Hz being the 6th harmonic and 280 Hz being the 7th harmonic. For a closed pipe, two successive harmonics differ by twice the fundamental, so the fundamental could be 20 Hz. But the overtones of a closed pipe are odd multiples of the fundamental, and both overtones are even multiples of 30 Hz. So the pipe must be an $\boxed{\text{open pipe}}$.

$$f = \frac{v}{2\ell} \rightarrow \ell = \frac{v}{2f} = \frac{[331 + 0.60(23.0)]\,\text{m/s}}{2(40\,\text{Hz})} = \boxed{4.3\,\text{m}}$$

48. (*a*) The harmonics for the open pipe are $f_n = \frac{nv}{2\ell}$. To be audible, they must be below 20 kHz.

$$\frac{nv}{2\ell} < 2\times10^4\,\text{Hz} \rightarrow n < \frac{2(2.48\,\text{m})\left(2\times10^4\,\text{Hz}\right)}{343\,\text{m/s}} = 289.2$$

Since there are 289 harmonics, there are $\boxed{\text{288 overtones}}$.

(*b*) The harmonics for the closed pipe are $f_n = \frac{nv}{4\ell}$, *n* odd. Again, they must be below 20 kHz.

$$\frac{nv}{4\ell} < 2\times10^4\,\text{Hz} \rightarrow n < \frac{4(2.48\,\text{m})\left(2\times10^4\,\text{Hz}\right)}{343\,\text{m/s}} = 578.4$$

The values of *n* must be odd, so *n* = 1, 3, 5, …, 577. There are 289 harmonics, and so there are $\boxed{\text{288 overtones}}$.

49. A tube closed at both ends will have standing waves with displacement nodes at each end, and so has the same harmonic structure as a string that is fastened at both ends. Thus the wavelength of the fundamental frequency is twice the length of the hallway, $\lambda_1 = 2\ell = 16.0\,\text{m}$.

$$f_1 = \frac{v}{\lambda_1} = \frac{343\,\text{m/s}}{16.0\,\text{m}} = \boxed{21.4\,\text{Hz}} \ ; \ f_2 = 2f_1 = \boxed{42.8\,\text{Hz}}$$

50. To operate with the first harmonic, we see from the figure that the thickness must be half of a wavelength, so the wavelength is twice the thickness. The speed of sound in the quartz is given by $v = \sqrt{G/\rho}$, analogous to Eqs. 15-3 and 15-4.

$$t = \tfrac{1}{2}\lambda = \tfrac{1}{2}\frac{v}{f} = \tfrac{1}{2}\frac{\sqrt{G/\rho}}{f} = \tfrac{1}{2}\frac{\sqrt{\left(2.95\times10^{10}\,\text{N/m}^2\right)/\left(2650\,\text{kg/m}^2\right)}}{12.0\times10^6\,\text{Hz}} = \boxed{1.39\times10^{-4}\,\text{m}}$$

51. The ear canal can be modeled as a closed pipe of length 2.5 cm. The resonant frequencies are given by $f_n = \dfrac{nv}{4\ell}$, $n$ odd. The first several frequencies are calculated here.

$$f_n = \frac{nv}{4\ell} = \frac{n\left(343\,\text{m/s}\right)}{4\left(2.5\times10^{-2}\,\text{m}\right)} = n\left(3430\text{ Hz}\right),\ n \text{ odd}$$

$$\boxed{f_1 = 3430\text{ Hz} \quad f_3 = 10{,}300\text{ Hz} \quad f_5 = 17{,}200\text{ Hz}}$$

In the graph, the most sensitive frequency is between 3000 and 4000 Hz. This corresponds to the fundamental resonant frequency of the ear canal. The sensitivity decrease above 4000 Hz, but is seen to "flatten out" around 10,000 Hz again, indicating higher sensitivity near 10,000 Hz than at surrounding frequencies. This 10,000 Hz relatively sensitive region corresponds to the first overtone resonant frequency of the ear canal.

52. From Eq. 15-7, the intensity is proportional to the square of the amplitude and the square of the frequency. From Fig. 16-14, the relative amplitudes are $\dfrac{A_2}{A_1} \approx 0.4$ and $\dfrac{A_3}{A_1} \approx 0.15$.

$$I = 2\pi^2 v\rho f^2 A^2 \ \rightarrow \ \frac{I_2}{I_1} = \frac{2\pi^2 v\rho f_2^2 A_2^2}{2\pi^2 v\rho f_1^2 A_1^2} = \frac{f_2^2 A_2^2}{f_1^2 A_1^2} = \left(\frac{f_2}{f_1}\right)^2\left(\frac{A_2}{A_1}\right)^2 = 2^2\left(0.4\right)^2 = \boxed{0.64}$$

$$\frac{I_3}{I_1} = \left(\frac{f_3}{f_1}\right)^2\left(\frac{A_3}{A_1}\right)^2 = 3^2\left(0.15\right)^2 = \boxed{0.20}$$

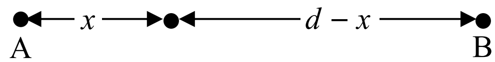
$$\beta_{2-1} = 10\log\frac{I_2}{I_1} = 10\log 0.64 = \boxed{-2\,\text{dB}} \ ; \ \beta_{3-1} = 10\log\frac{I_3}{I_1} = 10\log 0.24 = \boxed{-7\,\text{dB}}$$

53. The beat period is 2.0 seconds, so the beat frequency is the reciprocal of that, 0.50 Hz. Thus the other string is off in frequency by $\boxed{\pm 0.50\text{ Hz}}$. The beating does not tell the tuner whether the second string is too high or too low.

54. The beat frequency is the difference in the two frequencies, or $277\text{ Hz} - 262\text{ Hz} = \boxed{15\text{ Hz}}$. If the frequencies are both reduced by a factor of 4, then the difference between the two frequencies will also be reduced by a factor of 4, and so the beat frequency will be $\tfrac{1}{4}\left(15\text{ Hz}\right) = 3.75\,\text{Hz} \approx \boxed{3.8\text{ Hz}}$.

55. Since there are 4 beats/s when sounded with the 350 Hz tuning fork, the guitar string must have a frequency of either 346 Hz or 354 Hz. Since there are 9 beats/s when sounded with the 355 Hz tuning fork, the guitar string must have a frequency of either 346 Hz or 364 Hz. The common value is $\boxed{346\text{ Hz}}$.

56. (*a*) Since the sounds are initially $180^\circ$ out of phase, another $180^\circ$ of phase must be added by a path length difference. Thus the difference of the distances from the speakers to the point of constructive interference must be half of a wavelength. See the diagram.




$$(d-x)-x=\tfrac{1}{2}\lambda \;\rightarrow\; d=2x+\tfrac{1}{2}\lambda \;\rightarrow\; d_{\min}=\tfrac{1}{2}\lambda=\frac{v}{2f}=\frac{343\,\text{m/s}}{2(294\,\text{Hz})}=\boxed{0.583\,\text{m}}$$

This minimum distance occurs when the observer is right at one of the speakers. If the speakers are separated by more than 0.583 m, the location of constructive interference will be moved away from the speakers, along the line between the speakers.

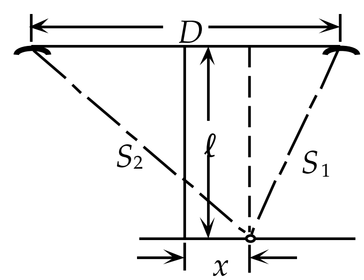
(*b*) Since the sounds are already $180^\circ$ out of phase, as long as the listener is equidistant from the speakers, there will be completely destructive interference. So even if the speakers have a tiny separation, the point midway between them will be a point of completely destructive interference. The minimum separation between the speakers is $\boxed{0}$.

57. Beats will be heard because the difference in the speed of sound for the two flutes will result in two different frequencies.

$$f_1=\frac{v_1}{2\ell}=\frac{[331+0.60(28)]\,\text{m/s}}{2(0.66\,\text{m})}=263.4\,\text{Hz}$$

$$f_2=\frac{v_2}{2\ell}=\frac{[331+0.60(5.0)]\,\text{m/s}}{2(0.66\,\text{m})}=253.0\,\text{Hz} \qquad \Delta f=263.4\,\text{Hz}-253.0\,\text{Hz}=\boxed{10\,\text{beats/sec}}$$

58. (*a*) The microphone must be moved to the right until the difference in distances from the two sources is half a wavelength. See the diagram. We square the expression, collect terms, isolate the remaining square root, and square again.




$$S_2-S_1=\tfrac{1}{2}\lambda \;\rightarrow$$

$$\sqrt{\left(\tfrac{1}{2}D+x\right)^2+\ell^2}-\sqrt{\left(\tfrac{1}{2}D-x\right)^2+\ell^2}=\tfrac{1}{2}\lambda \;\rightarrow$$

$$\sqrt{\left(\tfrac{1}{2}D+x\right)^2+\ell^2}=\tfrac{1}{2}\lambda+\sqrt{\left(\tfrac{1}{2}D-x\right)^2+\ell^2} \;\rightarrow$$

$$\left(\tfrac{1}{2}D+x\right)^2+\ell^2=\tfrac{1}{4}\lambda^2+2\left(\tfrac{1}{2}\lambda\right)\sqrt{\left(\tfrac{1}{2}D-x\right)^2+\ell^2}+\left(\tfrac{1}{2}D-x\right)^2+\ell^2 \;\rightarrow$$

$$2Dx-\tfrac{1}{4}\lambda^2=\lambda\sqrt{\left(\tfrac{1}{2}D-x\right)^2+\ell^2} \;\rightarrow\; 4D^2x^2-2(2Dx)\tfrac{1}{4}\lambda^2+\tfrac{1}{16}\lambda^4=\lambda^2\left[\left(\tfrac{1}{2}D-x\right)^2+\ell^2\right]$$

$$4D^2x^2-Dx\lambda^2+\tfrac{1}{16}\lambda^4=\tfrac{1}{4}D^2\lambda^2-Dx\lambda^2+x^2\lambda^2+\lambda^2\ell^2 \;\rightarrow\; x=\lambda\sqrt{\frac{\left(\tfrac{1}{4}D^2+\ell^2-\tfrac{1}{16}\lambda^2\right)}{\left(4D^2-\lambda^2\right)}}$$

The values are $D=3.00\,\text{m}$, $\ell=3.20\,\text{m}$, and $\lambda=v/f=(343\,\text{m/s})/(494\,\text{Hz})=0.694\,\text{m}$.

$$x=(0.694\,\text{m})\sqrt{\frac{\tfrac{1}{4}(3.00\,\text{m})^2+(3.20\,\text{m})^2-\tfrac{1}{16}(0.694\,\text{m})^2}{4(3.00\,\text{m})^2-(0.694\,\text{m})^2}}=\boxed{0.411\,\text{m}}$$

(*b*) When the speakers are exactly out of phase, the maxima and minima will be interchanged. The intensity $\boxed{\text{maxima are 0.411 m to the left or right of the midpoint}}$, and the intensity $\boxed{\text{minimum is at the midpoint}}$.

59. The beat frequency is 3 beats per 2 seconds, or 1.5 Hz. We assume the strings are the same length and the same mass density.

(*a*) The other string must be either $220.0\,\text{Hz} - 1.5\,\text{Hz} = \boxed{218.5\,\text{Hz}}$ or $220.0\,\text{Hz} + 1.5\,\text{Hz}$ $= \boxed{221.5\ \text{Hz}}$.

(*b*) Since $f = \dfrac{v}{2\ell} = \dfrac{1}{2\ell}\sqrt{\dfrac{F_T}{\mu}}$, we have $f \propto \sqrt{F_T} \rightarrow \dfrac{f}{\sqrt{F_T}} = \dfrac{f'}{\sqrt{F'_T}} \rightarrow F'_T = F_T\left(\dfrac{f'}{f}\right)^2$.

To change 218.5 Hz to 220.0 Hz: $F' = F_T\left(\dfrac{220.0}{218.5}\right)^2 = 1.014$, $\boxed{1.4\%\ \text{increase}}$.

To change 221.5 Hz to 220.0 Hz: $F' = F_T\left(\dfrac{220.0}{221.5}\right)^2 = 0.9865$, $\boxed{1.3\%\ \text{decrease}}$.

60. (*a*) To find the beat frequency, calculate the frequency of each sound, and then subtract the two frequencies.

$$f_{\text{beat}} = |f_1 - f_2| = \left|\frac{v}{\lambda_1} - \frac{v}{\lambda_2}\right| = (343\,\text{m/s})\left|\frac{1}{2.64\,\text{m}} - \frac{1}{2.72\,\text{m}}\right| = 3.821\,\text{Hz} \approx \boxed{4\,\text{Hz}}$$

(*b*) The speed of sound is 343 m/s, and the beat frequency is 3.821 Hz. The regions of maximum intensity are one “beat wavelength” apart.

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{3.821\,\text{Hz}} = 89.79\,\text{m} \approx \boxed{90\,\text{m}}\ (2\ \text{sig. fig.})$$

$\boxed{61.}$ (*a*) Observer moving towards stationary source.

$$f' = \left(1 + \frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f = \left(1 + \frac{30.0\,\text{m/s}}{343\,\text{m/s}}\right)(1350\ \text{Hz}) = \boxed{1470\ \text{Hz}}$$

(*b*) Observer moving away from stationary source.

$$f' = \left(1 - \frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f = \left(1 - \frac{30.0\,\text{m/s}}{343\,\text{m/s}}\right)(1350\ \text{Hz}) = \boxed{1230\ \text{Hz}}$$

62. The moving object can be treated as a moving “observer” for calculating the frequency it receives and reflects. The bat (the source) is stationary.

$$f'_{\text{object}} = f_{\text{bat}}\left(1 - \frac{v_{\text{object}}}{v_{\text{snd}}}\right)$$

Then the object can be treated as a moving source emitting the frequency $f'_{\text{object}}$, and the bat as a stationary observer.

$$f''_{\text{bat}} = \frac{f'_{\text{object}}}{\left(1 + \dfrac{v_{\text{object}}}{v_{\text{snd}}}\right)} = f_{\text{bat}}\frac{\left(1 - \dfrac{v_{\text{object}}}{v_{\text{snd}}}\right)}{\left(1 + \dfrac{v_{\text{object}}}{v_{\text{snd}}}\right)} = f_{\text{bat}}\frac{(v_{\text{snd}} - v_{\text{object}})}{(v_{\text{snd}} + v_{\text{object}})}$$

$$= \left(5.00\times10^{4}\,\text{Hz}\right)\frac{343\,\text{m/s}-30.0\,\text{m/s}}{343\,\text{m/s}+30.0\,\text{m/s}} = \boxed{4.20\times10^{4}\,\text{Hz}}$$

63. (*a*) For the 18 m/s relative velocity:

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \left(2300\ \text{Hz}\right)\frac{1}{\left(1-\dfrac{18\,\text{m/s}}{343\,\text{m/s}}\right)} = 2427\,\text{Hz} \approx \boxed{2430\,\text{Hz}}$$

$$f'_{\substack{\text{observer}\\\text{moving}}} = f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = \left(2300\ \text{Hz}\right)\left(1+\frac{18\,\text{m/s}}{343\,\text{m/s}}\right) = 2421\,\text{Hz} \approx \boxed{2420\,\text{Hz}}$$

The frequency shifts are slightly different, with $f'_{\substack{\text{source}\\\text{moving}}} > f'_{\substack{\text{observer}\\\text{moving}}}$. The two frequencies are close, but they are not identical. As a means of comparison, calculate the spread in frequencies divided by the original frequency.

$$\frac{f'_{\substack{\text{source}\\\text{moving}}} - f'_{\substack{\text{observer}\\\text{moving}}}}{f_{\text{source}}} = \frac{2427\,\text{Hz}-2421\,\text{Hz}}{2300\,\text{Hz}} = 0.0026 = 0.26\%$$

(*b*) For the 160 m/s relative velocity:

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \left(2300\ \text{Hz}\right)\frac{1}{\left(1-\dfrac{160\,\text{m/s}}{343\,\text{m/s}}\right)} = 4311\,\text{Hz} \approx \boxed{4310\,\text{Hz}}$$

$$f'_{\substack{\text{observer}\\\text{moving}}} = f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = \left(2300\ \text{Hz}\right)\left(1+\frac{160\,\text{m/s}}{343\,\text{m/s}}\right) = 3372\ \text{Hz} \approx \boxed{3370\,\text{Hz}}$$

The difference in the frequency shifts is much larger this time, still with $f'_{\substack{\text{source}\\\text{moving}}} > f'_{\substack{\text{observer}\\\text{moving}}}$.

$$\frac{f'_{\substack{\text{source}\\\text{moving}}} - f'_{\substack{\text{observer}\\\text{moving}}}}{f_{\text{source}}} = \frac{4311\,\text{Hz}-3372\,\text{Hz}}{2300\,\text{Hz}} = 0.4083 = 41\%$$

(*c*) For the 320 m/s relative velocity:

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)} = \left(2300\ \text{Hz}\right)\frac{1}{\left(1-\dfrac{320\,\text{m/s}}{343\,\text{m/s}}\right)} = \boxed{34,300\,\text{Hz}}$$

$$f'_{\substack{\text{observer}\\\text{moving}}} = f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = \left(2300\ \text{Hz}\right)\left(1+\frac{320\,\text{m/s}}{343\,\text{m/s}}\right) = 4446\,\text{Hz} \approx \boxed{4450\,\text{Hz}}$$

The difference in the frequency shifts is quite large, still with $f'_{\substack{\text{source}\\\text{moving}}} > f'_{\substack{\text{observer}\\\text{moving}}}$.

$$\frac{f'_{\substack{\text{source}\\\text{moving}}} - f'_{\substack{\text{observer}\\\text{moving}}}}{f_{\text{source}}} = \frac{34,300\,\text{Hz}-4446\,\text{Hz}}{2300\,\text{Hz}} = 12.98 = 1300\%$$

(*d*) The Doppler formulas are asymmetric, with a larger shift for the moving source than for the moving observer, when the two are getting closer to each other. In the following derivation, assume $v_{\text{src}} \ll v_{\text{snd}}$, and use the binomial expansion.

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1-\frac{v_{\text{src}}}{v_{\text{snd}}}\right)} = f\left(1-\frac{v_{\text{src}}}{v_{\text{snd}}}\right)^{-1} \approx f\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right) = f'_{\substack{\text{observer}\\\text{moving}}}$$

64. The frequency received by the stationary car is higher than the frequency emitted by the stationary car, by $\Delta f = 4.5\text{ Hz}$.

$$f_{\text{obs}} = f_{\text{source}} + \Delta f = \frac{f_{\text{source}}}{\left(1-\frac{v_{\text{source}}}{v_{\text{snd}}}\right)} \quad \rightarrow$$

$$f_{\text{source}} = \Delta f\left(\frac{v_{\text{snd}}}{v_{\text{source}}}-1\right) = (4.5\text{ Hz})\left(\frac{343\text{ m/s}}{15\text{ m/s}}-1\right) = \boxed{98\text{ Hz}}$$

65. (*a*) The observer is stationary, and the source is moving. First the source is approaching, then the source is receding.

$$120.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 33.33\text{ m/s}$$

$$f'_{\substack{\text{source}\\\text{moving}\\\text{towards}}} = f\frac{1}{\left(1-\frac{v_{\text{src}}}{v_{\text{snd}}}\right)} = (1280\text{ Hz})\frac{1}{\left(1-\frac{33.33\text{ m/s}}{343\text{ m/s}}\right)} = \boxed{1420\text{ Hz}}$$

$$f'_{\substack{\text{source}\\\text{moving}\\\text{away}}} = f\frac{1}{\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right)} = (1280\text{ Hz})\frac{1}{\left(1+\frac{33.33\text{ m/s}}{343\text{ m/s}}\right)} = \boxed{1170\text{ Hz}}$$

(*b*) Both the observer and the source are moving, and so use Eq. 16-11.

$$90.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 25\text{ m/s}$$

$$f'_{\text{approaching}} = f\frac{(v_{\text{snd}}+v_{\text{obs}})}{(v_{\text{snd}}-v_{\text{src}})} = (1280\text{ Hz})\frac{(343\text{ m/s}+25\text{ m/s})}{(343\text{ m/s}-33.33\text{ m/s})} = \boxed{1520\text{ Hz}}$$

$$f'_{\text{receding}} = f\frac{(v_{\text{snd}}-v_{\text{obs}})}{(v_{\text{snd}}+v_{\text{src}})} = (1280\text{ Hz})\frac{(343\text{ m/s}-25\text{ m/s})}{(343\text{ m/s}+33.33\text{ m/s})} = \boxed{1080\text{ Hz}}$$

(*c*) Both the observer and the source are moving, and so again use Eq. 16-11.

$$80.0\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 22.22\text{ m/s}$$

$$f'_{\substack{\text{police}\\\text{car}\\\text{approaching}}} = f\frac{(v_{\text{snd}}-v_{\text{obs}})}{(v_{\text{snd}}-v_{\text{src}})} = (1280\text{ Hz})\frac{(343\text{ m/s}-22.22\text{ m/s})}{(343\text{ m/s}-33.33\text{ m/s})} = \boxed{1330\text{ Hz}}$$

$$f'_{\substack{\text{police}\\\text{car}\\\text{receding}}} = f\frac{(v_{\text{snd}}+v_{\text{obs}})}{(v_{\text{snd}}+v_{\text{src}})} = (1280\text{ Hz})\frac{(343\text{ m/s}+22.22\text{ m/s})}{(343\text{ m/s}+33.33\text{ m/s})} = \boxed{1240\text{ Hz}}$$

66. The wall can be treated as a stationary "observer" for calculating the frequency it receives. The bat is flying toward the wall.

$$f'_{\text{wall}} = f_{\text{bat}} \frac{1}{\left(1 - \frac{v_{\text{bat}}}{v_{\text{snd}}}\right)}$$

Then the wall can be treated as a stationary source emitting the frequency $f'_{\text{wall}}$, and the bat as a moving observer, flying toward the wall.

$$f''_{\text{bat}} = f'_{\text{wall}}\left(1 + \frac{v_{\text{bat}}}{v_{\text{snd}}}\right) = f_{\text{bat}} \frac{1}{\left(1 - \frac{v_{\text{bat}}}{v_{\text{snd}}}\right)}\left(1 + \frac{v_{\text{bat}}}{v_{\text{snd}}}\right) = f_{\text{bat}} \frac{(v_{\text{snd}} + v_{\text{bat}})}{(v_{\text{snd}} - v_{\text{bat}})}$$

$$= \left(3.00\times10^4\,\text{Hz}\right)\frac{343\,\text{m/s} + 7.0\,\text{m/s}}{343\,\text{m/s} - 7.0\,\text{m/s}} = \boxed{3.13\times10^4\,\text{Hz}}$$

67. We assume that the comparison is to be made from the frame of reference of the stationary tuba. The stationary observers would observe a frequency from the moving tuba of

$$f_{\text{obs}} = \frac{f_{\text{source}}}{\left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}}\right)} = \frac{75\,\text{Hz}}{\left(1 - \frac{12.0\,\text{m/s}}{343\,\text{m/s}}\right)} = 78\,\text{Hz} \qquad f_{\text{beat}} = 78\,\text{Hz} - 75\,\text{Hz} = \boxed{3\,\text{Hz}}.$$

68. For the sound to be shifted up by one note, we must have $f'_{\substack{\text{source}\\\text{moving}}} = f\left(2^{1/12}\right)$.

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1 - \frac{v_{\text{src}}}{v_{\text{snd}}}\right)} = f\left(2^{1/12}\right) \rightarrow$$

$$v_{\text{src}} = \left(1 - \frac{1}{2^{1/12}}\right)v_{\text{snd}} = \left(1 - \frac{1}{2^{1/12}}\right)(343\,\text{m/s}) = 19.25\,\text{m/s}\left(\frac{3.6\,\text{km/h}}{\text{m/s}}\right) = \boxed{69.3\,\text{km/h}}$$

69. The ocean wave has $\lambda = 44\,\text{m}$ and $v = 18\,\text{m/s}$ relative to the ocean floor. The frequency of the ocean wave is then $f = \frac{v}{\lambda} = \frac{18\,\text{m/s}}{44\,\text{m}} = 0.409\,\text{Hz}$.

(*a*) For the boat traveling west, the boat will encounter a Doppler shifted frequency, for an observer moving towards a stationary source. The speed $v = 18\,\text{m/s}$ represents the speed of the waves in the stationary medium, and so corresponds to the speed of sound in the Doppler formula. The time between encountering waves is the period of the Doppler shifted frequency.

$$f'_{\substack{\text{observer}\\\text{moving}}} = \left(1 + \frac{v_{\text{obs}}}{v_{\text{snd}}}\right)f = \left(1 + \frac{15\,\text{m/s}}{18\,\text{m/s}}\right)(0.409\,\text{Hz}) = 0.750\,\text{Hz} \rightarrow$$

$$T = \frac{1}{f} = \frac{1}{0.750\,\text{Hz}} = \boxed{1.3\,\text{s}}$$

(*b*) For the boat traveling east, the boat will encounter a Doppler shifted frequency, for an observer moving away from a stationary source.

$$f'_{\substack{\text{observer}\\\text{moving}}} = \left(1 - \frac{v_{\text{obs}}}{v_{\text{snd}}}\right) f = \left(1 - \frac{15\,\text{m/s}}{18\,\text{m/s}}\right)(0.409\,\text{Hz}) = 0.0682\,\text{Hz} \quad \rightarrow$$

$$T = \frac{1}{f} = \frac{1}{0.0682\,\text{Hz}} = \boxed{15\,\text{s}}$$

70. The Doppler effect occurs only when there is relative motion of the source and the observer along the line connecting them. In the first four parts of this problem, the whistle and the observer are not moving relative to each other and so there is no Doppler shift. The wind speed increases (or decreases) the velocity of the waves in the direction of the wind, as if the speed of sound were different, but the frequency of the waves doesn't change. We do a detailed analysis of this claim in part (*a*).

(*a*) The wind velocity is a movement of the medium, and so adds or subtracts from the speed of sound in the medium. Because the wind is blowing away from the observer, the effective speed of sound is $v_{\text{snd}} - v_{\text{wind}}$. The wavelength of the waves traveling towards the observer is $\lambda_a = (v_{\text{snd}} - v_{\text{wind}})/f_0$, where $f_0$ is the frequency emitted by the factory whistle. This wavelength approaches the observer at a relative speed of $v_{\text{snd}} - v_{\text{wind}}$. Thus the observer hears the frequency calculated here.

$$f_a = \frac{v_{\text{snd}} - v_{\text{wind}}}{\lambda_a} = \frac{v_{\text{snd}} - v_{\text{wind}}}{\dfrac{v_{\text{snd}} - v_{\text{wind}}}{f_0}} = f_0 = \boxed{720\,\text{Hz}}$$

(*b*) Because the wind is blowing towards the observer, the effective speed of sound is $v_{\text{snd}} + v_{\text{wind}}$. The same kind of analysis as applied in part (a) gives that $f_b = \boxed{720\,\text{Hz}}$.

(*c*) Because the wind is blowing perpendicular to the line towards the observer, the effective speed of sound along that line is $v_{\text{snd}}$. Since there is no relative motion of the whistle and observer, there will be no change in frequency, and so $f_c = \boxed{720\,\text{Hz}}$.

(*d*) This is just like part (*c*), and so $f_d = \boxed{720\,\text{Hz}}$.

(*e*) Because the wind is blowing toward the cyclist, the effective speed of sound is $v_{\text{snd}} + v_{\text{wind}}$. The wavelength traveling toward the cyclist is $\lambda_e = (v_{\text{snd}} + v_{\text{wind}})/f_0$. This wavelength approaches the cyclist at a relative speed of $v_{\text{snd}} + v_{\text{wind}} + v_{\text{cycle}}$. The cyclist will hear the following frequency.

$$f_e = \frac{(v_{\text{snd}} + v_{\text{wind}} + v_{\text{cycle}})}{\lambda_e} = \frac{(v_{\text{snd}} + v_{\text{wind}} + v_{\text{cycle}})}{(v_{\text{snd}} + v_{\text{wind}})} f_0 = \frac{(343 + 15.0 + 12.0)\,\text{m/s}}{(343 + 15.0)}(720\,\text{Hz})$$

$$= \boxed{744\,\text{Hz}}$$

(*f*) Since the wind is not changing the speed of the sound waves moving towards the cyclist, the speed of sound is 343 m/s. The observer is moving towards a stationary source with a speed of 12.0 m/s.

$$f' = f\left(1 + \frac{v_{\text{obs}}}{v_{\text{sns}}}\right) = (720\ \text{Hz})\left(1 + \frac{12.0\,\text{m/s}}{343\,\text{m/s}}\right) = \boxed{745\,\text{Hz}}$$

71. The maximum Doppler shift occurs when the heart has its maximum velocity. Assume that the heart is moving away from the original source of sound. The beats arise from the combining of the original 2.25 MHz frequency with the reflected signal which has been Doppler shifted. There are two Doppler shifts – one for the heart receiving the original signal (observer moving away from stationary source) and one for the detector receiving the reflected signal (source moving away from stationary observer).

$$f'_{\text{heart}} = f_{\text{original}}\left(1 - \frac{v_{\text{heart}}}{v_{\text{snd}}}\right) \qquad f''_{\text{detector}} = \frac{f'_{\text{heart}}}{\left(1 + \frac{v_{\text{heart}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(1 - \frac{v_{\text{heart}}}{v_{\text{snd}}}\right)}{\left(1 + \frac{v_{\text{heart}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(v_{\text{snd}} - v_{\text{heart}}\right)}{\left(v_{\text{snd}} + v_{\text{heart}}\right)}$$

$$\Delta f = f_{\text{original}} - f''_{\text{detector}} = f_{\text{original}} - f_{\text{original}}\frac{\left(v_{\text{snd}} - v_{\text{blood}}\right)}{\left(v_{\text{snd}} + v_{\text{blood}}\right)} = f_{\text{original}}\frac{2v_{\text{blood}}}{\left(v_{\text{snd}} + v_{\text{blood}}\right)} \quad \rightarrow$$

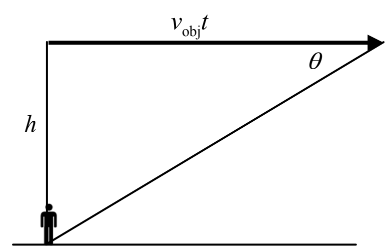
$$v_{\text{blood}} = v_{\text{snd}}\frac{\Delta f}{2f_{\text{original}} - \Delta f} = \left(1.54\times10^{3}\ \text{m/s}\right)\frac{260\ \text{Hz}}{2\left(2.25\times10^{6}\ \text{Hz}\right) - 260\ \text{Hz}} = \boxed{8.9\times10^{-2}\ \text{m/s}}$$

If instead we had assumed that the heart was moving towards the original source of sound, we would get $v_{\text{blood}} = v_{\text{snd}}\dfrac{\Delta f}{2f_{\text{original}} + \Delta f}$. Since the beat frequency is much smaller than the original frequency, the $\Delta f$ term in the denominator does not significantly affect the answer.

72. (*a*) The angle of the shock wave front relative to the direction of motion is given by Eq. 16-12.

$$\sin\theta = \frac{v_{\text{snd}}}{v_{\text{obj}}} = \frac{v_{\text{snd}}}{2.0v_{\text{snd}}} = \frac{1}{2.0} \quad \rightarrow \quad \theta = \sin^{-1}\frac{1}{2.0} = \boxed{30^{\circ}} \ \text{(2 sig. fig.)}$$

(*b*) The displacement of the plane $\left(v_{\text{obj}}t\right)$ from the time it passes overhead to the time the shock wave reaches the observer is shown, along with the shock wave front. From the displacement and height of the plane, the time is found.




$$\tan\theta = \frac{h}{v_{\text{obj}}t} \quad \rightarrow \quad t = \frac{h}{v_{\text{obj}}\tan\theta} = \frac{6500\ \text{m}}{(2.0)(310\ \text{m/s})\tan 30^{\circ}} = \boxed{18\,\text{s}}$$

73. (*a*) The Mach number is the ratio of the object's speed to the speed of sound.

$$M = \frac{v_{\text{obs}}}{v_{\text{sound}}} = \frac{\left(1.5\times10^{4}\ \text{km/hr}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/hr}}\right)}{45\,\text{m/s}} = 92.59 \approx \boxed{93}$$

(*b*) Use Eq. 16-125 to find the angle.

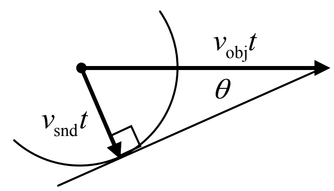
$$\theta = \sin^{-1}\frac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\frac{1}{M} = \sin^{-1}\frac{1}{92.59} = \boxed{0.62^{\circ}}$$

74. From Eq. 16-12, $\sin\theta = \dfrac{v_{\text{snd}}}{v_{\text{obj}}}$.

(*a*) $\theta = \sin^{-1}\dfrac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\dfrac{343\,\text{m/s}}{8800\,\text{m/s}} = \boxed{2.2^\circ}$

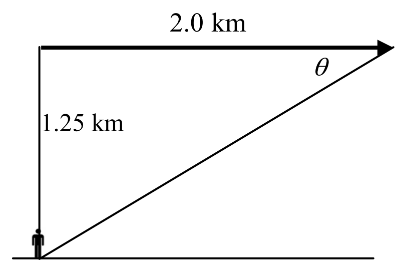
(*b*) $\theta = \sin^{-1}\dfrac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\dfrac{1560\,\text{m/s}}{8800\,\text{m/s}} = \boxed{10^\circ}$ (2 sig. fig.)

75. Consider one particular wave as shown in the diagram, created at the location of the black dot. After a time $t$ has elapsed from the creation of that wave, the supersonic source has moved a distance $v_{\text{obj}}t$, and the wave front has moved a distance $v_{\text{snd}}t$. The line from the position of the source at time $t$ is tangent to all of the wave fronts, showing the location of the shock wave. A tangent to a circle at a point is perpendicular to the radius connecting that point to the center, and so a right angle is formed. From the right triangle, the angle $\theta$ can be defined.




$$\boxed{\sin\theta = \frac{v_{\text{snd}}t}{v_{\text{obj}}t} = \frac{v_{\text{snd}}}{v_{\text{obj}}}}$$

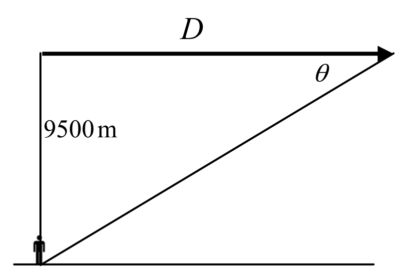
76. (*a*) The displacement of the plane from the time it passes overhead to the time the shock wave reaches the listener is shown, along with the shock wave front. From the displacement and height of the plane, the angle of the shock wave front relative to the direction of motion can be found. Then use Eq. 16-12.




$$\tan\theta = \frac{1.25\text{ km}}{2.0\text{ km}} \rightarrow \theta = \tan^{-1}\frac{1.25}{2.0} = \boxed{32^\circ}$$

(*b*) $M = \dfrac{v_{\text{obj}}}{v_{\text{snd}}} = \dfrac{1}{\sin\theta} = \dfrac{1}{\sin 32^\circ} = \boxed{1.9}$

77. Find the angle of the shock wave, and then find the distance the plane has traveled when the shock wave reaches the observer. Use Eq. 16-12.




$$\theta = \sin^{-1}\frac{v_{\text{snd}}}{v_{\text{obj}}} = \sin^{-1}\frac{v_{\text{snd}}}{2.2v_{\text{snd}}} = \sin^{-1}\frac{1}{2.2} = 27^\circ$$

$$\tan\theta = \frac{9500\,\text{m}}{D} \rightarrow D = \frac{9500\,\text{m}}{\tan 27^\circ} = 18616\,\text{m} = \boxed{19\,\text{km}}$$

78. The minimum time between pulses would be the time for a pulse to travel from the boat to the maximum distance and back again. The total distance traveled by the pulse will be 150 m, at the speed of sound in fresh water, 1440 m/s.

$$d = vt \rightarrow t = \frac{d}{v} = \frac{150\,\text{m}}{1440\,\text{m/s}} = \boxed{0.10\,\text{s}}$$

79. Assume that only the fundamental frequency is heard. The fundamental frequency of an open pipe is given by $f = \frac{v}{2L}$.

(*a*) $f_{3.0} = \frac{v}{2L} = \frac{343\,\text{m/s}}{2(3.0\text{ m})} = \boxed{57\text{ Hz}}$ $\quad f_{2.5} = \frac{v}{2L} = \frac{343\,\text{m/s}}{2(2.5\text{ m})} = \boxed{69\text{ Hz}}$

$f_{2.0} = \frac{v}{2L} = \frac{343\,\text{m/s}}{2(2.0\text{ m})} = \boxed{86\text{ Hz}}$ $\quad f_{1.5} = \frac{v}{2L} = \frac{343\,\text{m/s}}{2(1.5\text{ m})} = 114.3\,\text{Hz} \approx \boxed{110\text{ Hz}}$

$f_{1.0} = \frac{v}{2L} = \frac{343\,\text{m/s}}{2(1.0\text{ m})} = 171.5\text{ Hz} \approx \boxed{170\text{ Hz}}$

(*b*) On a noisy day, there are a large number of component frequencies to the sounds that are being made – more people walking, more people talking, etc. Thus it is more likely that the frequencies listed above will be a component of the overall sound, and then the resonance will be more prominent to the hearer. If the day is quiet, there might be very little sound at the desired frequencies, and then the tubes will not have any standing waves in them to detect.

80. The single mosquito creates a sound intensity of $I_0 = 1\times10^{-12}\text{ W/m}^2$. Thus 100 mosquitoes will create a sound intensity of 100 times that of a single mosquito.

$$I = 100I_0 \qquad \beta = 10\log\frac{100I_0}{I_0} = 10\log 100 = \boxed{20\,\text{dB}}.$$

81. The two sound level values must be converted to intensities, then the intensities added, and then converted back to sound level.

$$I_{82}:\quad 82\,\text{dB} = 10\log\frac{I_{82}}{I_0} \;\rightarrow\; I_{82} = 10^{8.2}I_0 = 1.585\times10^8 I_0$$

$$I_{89}:\quad 89\,\text{dB} = 10\log\frac{I_{87}}{I_0} \;\rightarrow\; I_{89} = 10^{8.9}I_0 = 7.943\times10^8 I_0$$

$$I_{\text{total}} = I_{82} + I_{89} = \left(9.528\times10^8\right)I_0 \;\rightarrow$$

$$\beta_{\text{total}} = 10\log\frac{9.528\times10^8 I_0}{I_0} = 10\log 6.597\times10^8 = 89.8\,\text{dB} \approx \boxed{90\,\text{dB}}\;(2\text{ sig. fig.})$$

82. The power output is found from the intensity, which is the power radiated per unit area.

$$115\text{ dB} = 10\log\frac{I}{I_0} \;\rightarrow\; I = 10^{11.5}I_0 = 10^{11.5}\left(1.0\times10^{-12}\text{ W/m}^2\right) = 3.162\times10^{-1}\text{ W/m}^2$$

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} \;\rightarrow\; P = 4\pi r^2 I = 4\pi(9.00\text{ m})^2\left(3.162\times10^{-1}\text{ W/m}^2\right) = \boxed{322\text{ W}}$$

83. Relative to the 1000 Hz output, the 15 kHz output is –12 dB.

$$-12\text{ dB} = 10\log\frac{P_{15\,\text{kHz}}}{175\,\text{W}} \;\rightarrow\; -1.2 = \log\frac{P_{15\,\text{kHz}}}{175\text{ W}} \;\rightarrow\; 10^{-1.2} = \frac{P_{15\,\text{kHz}}}{175\,\text{W}} \;\rightarrow\; P_{15\,\text{kHz}} = \boxed{11\,\text{W}}$$

84. The 130 dB level is used to find the intensity, and the intensity is used to find the power. It is assumed that the jet airplane engine radiates equally in all directions.

$$\beta = 130\,\text{dB} = 10\log\frac{I}{I_0} \rightarrow I = 10^{13} I_0 = 10^{13}\left(1.0\times10^{-12}\ \text{W/m}^2\right) = 1.0\times10^{1}\ \text{W/m}^2$$

$$P = IA = I\pi r^2 = \left(1.0\times10^{1}\ \text{W/m}^2\right)\pi\left(2.0\times10^{-2}\right)^2 = \boxed{0.013\,\text{W}}$$

85. The gain is given by $\beta = 10\log\frac{P_{\text{out}}}{P_{\text{in}}} = 10\log\frac{125\ \text{W}}{1.0\times10^{-3}\,\text{W}} = \boxed{51\,\text{dB}}$.

86. It is desired that the sound from the speaker arrives at a listener 30 ms after the sound from the singer arrives. The fact that the speakers are 3.0 m behind the singer adds in a delay of $\frac{3.0\,\text{m}}{343\,\text{m/s}} =$ $8.7\times10^{-3}\,\text{s}$, or about 9 ms. Thus there must be $\boxed{21\ \text{ms}}$ of delay added into the electronic circuitry.

87. The strings are both tuned to the same frequency, and both have the same length. The mass per unit length is the density times the cross sectional area. The frequency is related to the tension by Eqs. 15-1 and 15-2.

$$f = \frac{v}{2\ell}\ ;\ v = \sqrt{\frac{F_{\text{T}}}{\mu}} \rightarrow f = \frac{1}{2\ell}\sqrt{\frac{F_{\text{T}}}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{F_{\text{T}}}{\rho\pi r^2}} \rightarrow F_{\text{T}} = 4\ell^2\rho f^2\pi r^2 \rightarrow$$

$$\frac{F_{\text{T high}}}{F_{\text{T low}}} = \frac{4\ell^2\rho f^2\pi r_{\text{high}}^2}{4\ell^2\rho f^2\pi r_{\text{low}}^2} = \left(\frac{r_{\text{high}}}{r_{\text{low}}}\right)^2 = \left(\frac{\frac{1}{2}d_{\text{high}}}{\frac{1}{2}d_{\text{low}}}\right)^2 = \left(\frac{0.724\,\text{mm}}{0.699\,\text{mm}}\right)^2 = \boxed{1.07}$$

88. The strings are both tuned to the same frequency, and both have the same length. The mass per unit length is the density times the cross sectional area. The frequency is related to the tension by Eqs. 15-1 and 15-2.

$$f = \frac{v}{2\ell}\ ;\ v = \sqrt{\frac{F_{\text{T}}}{\mu}} \rightarrow f = \frac{1}{2\ell}\sqrt{\frac{F_{\text{T}}}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{F_{\text{T}}}{\rho\pi r^2}} \rightarrow F_{\text{T}} = 4\ell^2\rho f^2\pi r^2 \rightarrow$$

$$\frac{F_{\text{T acoustic}}}{F_{\text{T electric}}} = \frac{4\ell^2\rho_{\text{acoustic}} f^2\pi r_{\text{acoustic}}^2}{4\ell^2\rho_{\text{electric}} f^2\pi r_{\text{electric}}^2} = \frac{\rho_{\text{acoustic}} r_{\text{acoustic}}^2}{\rho_{\text{electric}} r_{\text{electric}}^2} = \left(\frac{\rho_{\text{acoustic}}}{\rho_{\text{electric}}}\right)\left(\frac{d_{\text{acoustic}}}{d_{\text{electric}}}\right)^2$$

$$= \left(\frac{7760\,\text{kg/m}^3}{7990\,\text{kg/m}^3}\right)\left(\frac{0.33\,\text{m}}{0.25\,\text{m}}\right)^2 = \boxed{1.7}$$

89. (*a*) The wave speed on the string can be found from the length and the fundamental frequency.

$$f = \frac{v}{2\ell} \rightarrow v = 2\ell f = 2\left(0.32\ \text{m}\right)\left(440\ \text{Hz}\right) = 281.6\,\text{m/s} \approx \boxed{280\,\text{m/s}}$$

The tension is found from the wave speed and the mass per unit length.

$$v = \sqrt{\frac{F_{\text{T}}}{\mu}} \rightarrow F_{\text{T}} = \mu v^2 = \left(7.21\times10^{-4}\ \text{kg/m}\right)\left(281.6\,\text{m/s}\right)^2 = \boxed{57\,\text{N}}$$

(*b*) The length of the pipe can be found from the fundamental frequency and the speed of sound.

$$f = \frac{v}{4\ell} \rightarrow \ell = \frac{v}{4f} = \frac{343\,\text{m/s}}{4\left(440\ \text{Hz}\right)} = 0.1949\,\text{m} \approx \boxed{0.19\,\text{m}}$$

(*c*) The first overtone for the string is twice the fundamental. $\boxed{880\ \text{Hz}}$

The first overtone for the open pipe is 3 times the fundamental. $\boxed{1320\ \text{Hz}}$

90. The apparatus is a closed tube. The water level is the closed end, and so is a node of air displacement. As the water level lowers, the distance from one resonance level to the next corresponds to the distance between adjacent nodes, which is one-half wavelength.

$$\Delta\ell = \tfrac{1}{2}\lambda \ \rightarrow\ \lambda = 2\Delta\ell = 2(0.395\,\text{m} - 0.125\,\text{m}) = 0.540\ \text{m}$$

$$f = \frac{v}{\lambda} = \frac{343\,\text{m/s}}{0.540\ \text{m}} = \boxed{635\ \text{Hz}}$$

$\boxed{91.}$ The fundamental frequency of a tube closed at one end is given by $f_1 = \dfrac{v}{4\ell}$. The change in air temperature will change the speed of sound, resulting in two different frequencies.

$$\frac{f_{30.0^\circ\text{C}}}{f_{25.0^\circ\text{C}}} = \frac{\dfrac{v_{30.0^\circ\text{C}}}{4\ell}}{\dfrac{v_{25.0^\circ\text{C}}}{4\ell}} = \frac{v_{30.0^\circ\text{C}}}{v_{25.0^\circ\text{C}}} \ \rightarrow\ f_{30.0^\circ\text{C}} = f_{25.0^\circ\text{C}}\left(\frac{v_{30.0^\circ\text{C}}}{v_{25.0^\circ\text{C}}}\right)$$

$$\Delta f = f_{30.0^\circ\text{C}} - f_{25.0^\circ\text{C}} = f_{25.0^\circ\text{C}}\left(\frac{v_{30.0^\circ\text{C}}}{v_{25.0^\circ\text{C}}} - 1\right) = (349\,\text{Hz})\left(\frac{331 + 0.60(30.0)}{331 + 0.60(25.0)} - 1\right) = \boxed{3\,\text{Hz}}$$

92. Call the frequencies of four strings of the violin $f_\text{A}, f_\text{B}, f_\text{C}, f_\text{D}$ with $f_\text{A}$ the lowest pitch. The mass per unit length will be named $\mu$. All strings are the same length and have the same tension. For a string with both ends fixed, the fundamental frequency is given by $f_1 = \dfrac{v}{2\ell} = \dfrac{1}{2\ell}\sqrt{\dfrac{F_\text{T}}{\mu}}$.

$$f_\text{B} = 1.5 f_\text{A} \ \rightarrow\ \frac{1}{2\ell}\sqrt{\frac{F_\text{T}}{\mu_\text{B}}} = 1.5\frac{1}{2\ell}\sqrt{\frac{F_\text{T}}{\mu_\text{A}}} \ \rightarrow\ \mu_\text{B} = \frac{\mu_\text{A}}{(1.5)^2} = \boxed{0.44\mu_\text{A}}$$

$$f_\text{C} = 1.5 f_\text{B} = (1.5)^2 f_\text{A} \ \rightarrow\ \frac{1}{2\ell}\sqrt{\frac{F_\text{T}}{\mu_\text{C}}} = (1.5)^2\frac{1}{2\ell}\sqrt{\frac{F_\text{T}}{\mu_\text{A}}} \ \rightarrow\ \mu_\text{C} = \frac{\mu_\text{A}}{(1.5)^4} = \boxed{0.20\mu_\text{A}}$$

$$f_\text{D} = 1.5 f_\text{C} = (1.5)^3 f_\text{A} \ \rightarrow\ \frac{1}{2\ell}\sqrt{\frac{F_\text{T}}{\mu_\text{D}}} = (1.5)^3\frac{1}{2\ell}\sqrt{\frac{F_\text{T}}{\mu_\text{A}}} \ \rightarrow\ \mu_\text{D} = \frac{\mu_\text{A}}{(1.5)^6} = \boxed{0.088\mu_\text{A}}$$

93. The effective length of the tube is $\ell_\text{eff.} = \ell + \tfrac{1}{3}D = 0.60\,\text{m} + \tfrac{1}{3}(0.030\,\text{m}) = 0.61\,\text{m}$.

Uncorrected frequencies: $f_n = \dfrac{(2n-1)v}{4\ell}, n = 1,2,3\ldots \ \rightarrow$

$$f_{1-4} = (2n-1)\frac{343\,\text{m/s}}{4(0.60\,\text{m})} = 143\,\text{Hz},\ 429\,\text{Hz},\ 715\,\text{Hz},\ 1000\,\text{Hz}$$

Corrected frequencies: $f_n = \dfrac{(2n-1)v}{4\ell_\text{eff}}, n = 1,2,3\ldots \ \rightarrow$

$$f_{1-4} = (2n-1)\frac{343\,\text{m/s}}{4(0.61\,\text{m})} = \boxed{141\,\text{Hz},\ 422\,\text{Hz},\ 703\,\text{Hz},\ 984\,\text{Hz}}$$

94. Since the sound is loudest at points equidistant from the two sources, the two sources must be in phase. The difference in distance from the two sources must be an odd number of half-wavelengths for destructive interference.

$$0.28\,\text{m} = \lambda/2 \;\rightarrow\; \lambda = 0.56\,\text{m} \qquad f = v/\lambda = 343\,\text{m/s}/0.56\,\text{m} = \boxed{610\,\text{Hz}}$$

$$0.28\,\text{m} = 3\lambda/2 \;\rightarrow\; \lambda = 0.187\,\text{m} \qquad f = v/\lambda = 343\,\text{m/s}/0.187\,\text{m} = 1840\,\text{Hz}\;(\text{out of range})$$

95. As the train approaches, the observed frequency is given by $f'_{\text{approach}} = f\Big/\left(1-\frac{v_{\text{train}}}{v_{\text{snd}}}\right)$. As the train recedes, the observed frequency is given by $f'_{\text{recede}} = f\Big/\left(1+\frac{v_{\text{train}}}{v_{\text{snd}}}\right)$. Solve each expression for $f$, equate them, and then solve for $v_{\text{train}}$.

$$f'_{\text{approach}}\left(1-\frac{v_{\text{train}}}{v_{\text{snd}}}\right) = f'_{\text{recede}}\left(1+\frac{v_{\text{train}}}{v_{\text{snd}}}\right) \;\rightarrow$$

$$v_{\text{train}} = v_{\text{snd}}\frac{\left(f'_{\text{approach}} - f'_{\text{recede}}\right)}{\left(f'_{\text{approach}} + f'_{\text{recede}}\right)} = (343\,\text{m/s})\frac{(552\,\text{Hz} - 486\,\text{Hz})}{(552\,\text{Hz} + 486\,\text{Hz})} = \boxed{22\,\text{m/s}}$$

96. The Doppler shift is 3.5 Hz, and the emitted frequency from both trains is 516 Hz. Thus the frequency received by the conductor on the stationary train is 519.5 Hz. Use this to find the moving train's speed.

$$f' = f\frac{v_{\text{snd}}}{\left(v_{\text{snd}} - v_{\text{source}}\right)} \;\rightarrow\; v_{\text{source}} = \left(1-\frac{f}{f'}\right)v_{\text{snd}} = \left(1-\frac{516\text{ Hz}}{519.5\text{ Hz}}\right)(343\,\text{m/s}) = \boxed{2.31\,\text{m/s}}$$

97. (*a*) Since both speakers are moving towards the observer at the same speed, both frequencies have the same Doppler shift, and the observer hears no beats.

(*b*) The observer will detect an increased frequency from the speaker moving towards him and a decreased frequency from the speaker moving away. The difference in those two frequencies will be the beat frequency that is heard.

$$f'_{\text{towards}} = f\frac{1}{\left(1-\frac{v_{\text{train}}}{v_{\text{snd}}}\right)} \qquad f'_{\text{away}} = f\frac{1}{\left(1+\frac{v_{\text{train}}}{v_{\text{snd}}}\right)}$$

$$f'_{\text{towards}} - f'_{\text{away}} = f\frac{1}{\left(1-\frac{v_{\text{train}}}{v_{\text{snd}}}\right)} - f\frac{1}{\left(1+\frac{v_{\text{train}}}{v_{\text{snd}}}\right)} = f\left[\frac{v_{\text{snd}}}{\left(v_{\text{snd}} - v_{\text{train}}\right)} - \frac{v_{\text{snd}}}{\left(v_{\text{snd}} + v_{\text{train}}\right)}\right]$$

$$(348\text{ Hz})\left[\frac{343\,\text{m/s}}{(343\,\text{m/s} - 10.0\,\text{m/s})} - \frac{343\,\text{m/s}}{(343\,\text{m/s} + 10.0\,\text{m/s})}\right] = \boxed{20\,\text{Hz}}\;(2\text{ sig. fig.})$$

(*c*) Since both speakers are moving away from the observer at the same speed, both frequencies have the same Doppler shift, and the observer hears no beats.

98. For each pipe, the fundamental frequency is given by $f = \frac{v}{2\ell}$. Find the frequency of the shortest pipe.

$$f = \frac{v}{2\ell} = \frac{343\,\text{m/s}}{2(2.40\,\text{m})} = 71.46\,\text{Hz}$$

The longer pipe has a lower frequency. Since the beat frequency is 8.0 Hz, the frequency of the longer pipe must be 63.46 Hz. Use that frequency to find the length of the longer pipe.

$$f = \frac{v}{2\ell} \quad\rightarrow\quad \ell = \frac{v}{2f} = \frac{343\,\text{m/s}}{2(63.46\,\text{Hz})} = \boxed{2.70\,\text{m}}$$

99. Use Eq. 16-11, which applies when both source and observer are in motion. There will be two Doppler shifts in this problem – first for the emitted sound with the bat as the source and the moth as the observer, and then the reflected sound with the moth as the source and the bat as the observer.

$$f'_{\text{moth}} = f_{\text{bat}}\frac{(v_{\text{snd}} + v_{\text{moth}})}{(v_{\text{snd}} - v_{\text{bat}})} \qquad f''_{\text{bat}} = f'_{\text{moth}}\frac{(v_{\text{snd}} + v_{\text{bat}})}{(v_{\text{snd}} - v_{\text{moth}})} = f_{\text{bat}}\frac{(v_{\text{snd}} + v_{\text{moth}})}{(v_{\text{snd}} - v_{\text{bat}})}\frac{(v_{\text{snd}} + v_{\text{bat}})}{(v_{\text{snd}} - v_{\text{moth}})}$$

$$= (51.35\,\text{kHz})\frac{(343+5.0)}{(343-7.5)}\frac{(343+7.5)}{(343-5.0)} = \boxed{55.23\,\text{kHz}}$$

100. The beats arise from the combining of the original 3.80 MHz frequency with the reflected signal which has been Doppler shifted. There are two Doppler shifts – one for the blood cells receiving the original frequency (observer moving away from stationary source) and one for the detector receiving the reflected frequency (source moving away from stationary observer).

$$f'_{\text{blood}} = f_{\text{original}}\left(1 - \frac{v_{\text{blood}}}{v_{\text{snd}}}\right) \qquad f''_{\text{detector}} = \frac{f'_{\text{blood}}}{\left(1 + \frac{v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(1 - \frac{v_{\text{blood}}}{v_{\text{snd}}}\right)}{\left(1 + \frac{v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{(v_{\text{snd}} - v_{\text{blood}})}{(v_{\text{snd}} + v_{\text{blood}})}$$

$$\Delta f = f_{\text{original}} - f''_{\text{detector}} = f_{\text{original}} - f_{\text{original}}\frac{(v_{\text{snd}} - v_{\text{blood}})}{(v_{\text{snd}} + v_{\text{blood}})} = f_{\text{original}}\frac{2v_{\text{blood}}}{(v_{\text{snd}} + v_{\text{blood}})}$$

$$= (3.80\times10^{6}\,\text{Hz})\frac{2(0.32\,\text{m/s})}{(1.54\times10^{3}\,\text{m/s} + 0.32\,\text{m/s})} = \boxed{1600\,\text{Hz}}$$

101. It is 70.0 ms from the start of one chirp to the start of the next. Since the chirp itself is 3.0 ms long, it is 67.0 ms from the end of a chirp to the start of the next. Thus the time for the pulse to travel to the moth and back again is 67.0 ms. The distance to the moth is half the distance that the sound can travel in 67.0 ms, since the sound must reach the moth and return during the 67.0 ms.

$$d = v_{\text{snd}}t = (343\,\text{m/s})\tfrac{1}{2}(67.0\times10^{-3}\,\text{s}) = \boxed{11.5\text{ m}}$$

102. (*a*) We assume that $v_{\text{src}} \ll v_{\text{snd}}$, and use the binomial expansion.

$$f'_{\substack{\text{source}\\\text{moving}}} = f\frac{1}{\left(1 - \frac{v_{\text{src}}}{v_{\text{snd}}}\right)} = f\left(1 - \frac{v_{\text{src}}}{v_{\text{snd}}}\right)^{-1} \approx f\left(1 + \frac{v_{\text{src}}}{v_{\text{snd}}}\right) = f'_{\substack{\text{observer}\\\text{moving}}}$$

(*b*) We calculate the percent error in general, and then substitute in the given relative velocity.

$$\%\,\text{error} = \left(\frac{\text{approx.} - \text{exact}}{\text{exact}}\right)100 = 100\left(\frac{f\left(1+\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right) - f\dfrac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)}}{f\dfrac{1}{\left(1-\dfrac{v_{\text{src}}}{v_{\text{snd}}}\right)}}\right)$$

$$= 100\left[\left(1+\frac{v_{\text{src}}}{v_{\text{snd}}}\right)\left(1-\frac{v_{\text{src}}}{v_{\text{snd}}}\right) - 1\right] = -100\left(\frac{v_{\text{src}}}{v_{\text{snd}}}\right)^2 = -100\left(\frac{18.0\,\text{m/s}}{343\,\text{m/s}}\right)^2 = \boxed{-0.28\%}$$

The negative sign indicates that the approximate value is less than the exact value.

103. The person will hear a frequency $f'_{\text{towards}} = f\left(1+\dfrac{v_{\text{walk}}}{v_{\text{snd}}}\right)$ from the speaker that they walk towards.

The person will hear a frequency $f'_{\text{away}} = f\left(1-\dfrac{v_{\text{walk}}}{v_{\text{snd}}}\right)$ from the speaker that they walk away from.

The beat frequency is the difference in those two frequencies.

$$f'_{\text{towards}} - f'_{\text{away}} = f\left(1+\frac{v_{\text{walk}}}{v_{\text{snd}}}\right) - f\left(1-\frac{v_{\text{walk}}}{v_{\text{snd}}}\right) = 2f\frac{v_{\text{walk}}}{v_{\text{snd}}} = 2(282\,\text{Hz})\frac{1.4\,\text{m/s}}{343\,\text{m/s}} = \boxed{2.3\,\text{Hz}}$$

104. There will be two Doppler shifts in this problem – first for a stationary source with a moving "observer" (the blood cells), and then for a moving source (the blood cells) and a stationary "observer" (the receiver). Note that the velocity component of the blood parallel to the sound transmission is $v_{\text{blood}}\cos 45° = \frac{1}{\sqrt{2}}v_{\text{blood}}$. It is that component that causes the Doppler shift.

$$f'_{\text{blood}} = f_{\text{original}}\left(1-\frac{\frac{1}{\sqrt{2}}v_{\text{blood}}}{v_{\text{snd}}}\right)$$

$$f''_{\text{detector}} = \frac{f'_{\text{blood}}}{\left(1+\dfrac{\frac{1}{\sqrt{2}}v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(1-\dfrac{\frac{1}{\sqrt{2}}v_{\text{blood}}}{v_{\text{snd}}}\right)}{\left(1+\dfrac{\frac{1}{\sqrt{2}}v_{\text{blood}}}{v_{\text{snd}}}\right)} = f_{\text{original}}\frac{\left(v_{\text{snd}} - \frac{1}{\sqrt{2}}v_{\text{blood}}\right)}{\left(v_{\text{snd}} + \frac{1}{\sqrt{2}}v_{\text{blood}}\right)} \rightarrow$$

$$v_{\text{blood}} = \sqrt{2}\frac{\left(f_{\text{original}} - f''_{\text{detector}}\right)}{\left(f''_{\text{detector}} + f_{\text{original}}\right)}v_{\text{snd}}$$

Since the cells are moving away from the transmitter / receiver combination, the final frequency received is less than the original frequency, by 780 Hz. Thus $f''_{\text{detector}} = f_{\text{original}} - 780\,\text{Hz}$.

$$v_{\text{blood}} = \sqrt{2}\frac{\left(f_{\text{original}} - f''_{\text{detector}}\right)}{\left(f''_{\text{detector}} + f_{\text{original}}\right)}v_{\text{snd}} = \sqrt{2}\frac{(780\,\text{Hz})}{\left(2f_{\text{original}} - 780\,\text{Hz}\right)}v_{\text{snd}}$$

$$= \sqrt{2}\frac{(780\,\text{Hz})}{\left[2\left(5.0\times10^6\,\text{Hz}\right) - 780\,\text{Hz}\right]}(1540\,\text{m/s}) = \boxed{0.17\,\text{m/s}}$$

105. The apex angle is 15°, so the shock wave angle is 7.5°. The angle of the shock wave is also given by $\sin\theta = v_{\text{wave}}/v_{\text{object}}$.

$$\sin\theta = v_{\text{wave}}/v_{\text{object}} \quad\rightarrow\quad v_{\text{object}} = v_{\text{wave}}/\sin\theta = 2.2\,\text{km/h}/\sin 7.5^{\circ} = \boxed{17\,\text{km/h}}$$

106. First, find the path difference in the original configuration. Then move the obstacle to the right by $\Delta d$ so that the path difference increases by $\frac{1}{2}\lambda$. Note that the path difference change must be on the same order as the wavelength, and so $\Delta d \ll d, \ell$ since $\lambda \ll \ell, d$.

$$(\Delta D)_{\text{initial}} = 2\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2} - \ell \;;\; (\Delta D)_{\text{final}} = 2\sqrt{(d+\Delta d)^2 + \left(\tfrac{1}{2}\ell\right)^2} - \ell$$

$$(\Delta D)_{\text{final}} - (\Delta D)_{\text{initial}} = \tfrac{1}{2}\lambda = \left(2\sqrt{(d+\Delta d)^2 + \left(\tfrac{1}{2}\ell\right)^2} - \ell\right) - \left(2\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2} - \ell\right) \rightarrow$$

$$2\sqrt{(d+\Delta d)^2 + \left(\tfrac{1}{2}\ell\right)^2} = \tfrac{1}{2}\lambda + 2\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2}$$

Square the last equation above.

$$4\left[d^2 + 2d\Delta d + (\Delta d)^2 + \left(\tfrac{1}{2}\ell\right)^2\right] = \tfrac{1}{4}\lambda^2 + 2\left(\tfrac{1}{2}\lambda\right)2\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2} + 4\left[d^2 + \left(\tfrac{1}{2}\ell\right)^2\right]$$

We delete terms that are second order in the small quantities $\Delta d$ and $\lambda$.

$$8d\Delta d = 2\lambda\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2} \quad\rightarrow\quad \boxed{\Delta d = \frac{\lambda}{4d}\sqrt{d^2 + \left(\tfrac{1}{2}\ell\right)^2}}$$

107. (*a*) The "singing" rod is manifesting standing waves. By holding the rod at its midpoint, it has a node at its midpoint, and antinodes at its ends. Thus the length of the rod is a half wavelength. The speed of sound in aluminum is found in Table 16-1.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{5100\,\text{m/s}}{1.50\text{ m}} = \boxed{3400\text{ Hz}}$$

(*b*) The wavelength of sound in the rod is twice the length of the rod, $\boxed{1.50\,\text{m}}$.

(*c*) The wavelength of the sound in air is determined by the frequency and the speed of sound in air.

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{3400\,\text{Hz}} = \boxed{0.10\,\text{m}}$$

108. The displacement amplitude is related to the intensity by Eq. 15-7. The intensity can be calculated from the decibel value. The medium is air.

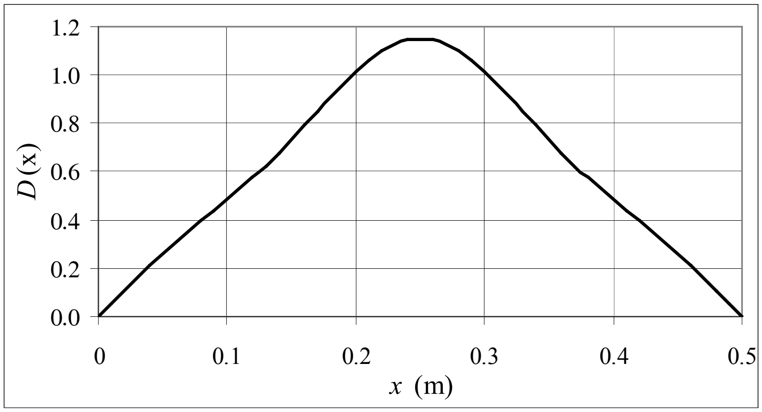
$$\beta = 10\log\frac{I}{I_0} \quad\rightarrow\quad I = \left(10^{\beta/10}\right)I_0 = 10^{10.5}\left(10^{-12}\,\text{W/m}^2\right) = 0.0316\,\text{W/m}^2$$

(*a*) $I = 2\pi^2 v\rho f^2 A^2 \rightarrow$

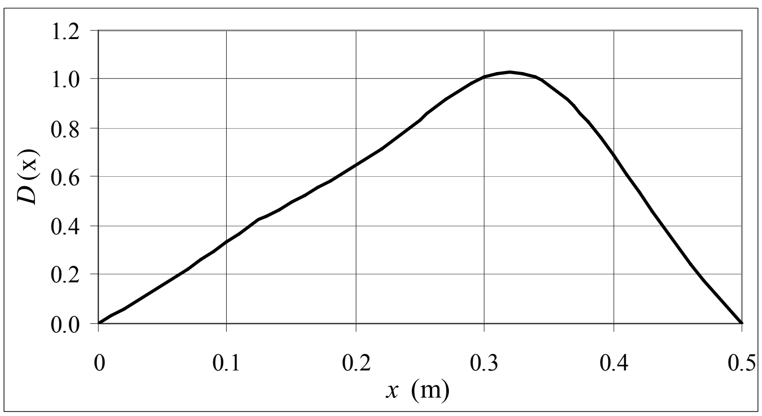
$$A = \frac{1}{\pi f}\sqrt{\frac{I}{2v\rho}} = \frac{1}{\pi\left(8.0\times10^3\,\text{Hz}\right)}\sqrt{\frac{0.0316\,\text{W/m}^2}{2(343\,\text{m/s})\left(1.29\,\text{kg/m}^3\right)}} = \boxed{2.4\times10^{-7}\text{m}}$$

(*b*) $$A = \frac{1}{\pi f}\sqrt{\frac{I}{2v\rho}} = \frac{1}{\pi(35\,\text{Hz})}\sqrt{\frac{0.0316\,\text{W/m}^2}{2(343\,\text{m/s})\left(1.29\,\text{kg/m}^3\right)}} = \boxed{5.4\times10^{-5}\text{m}}$$

109. (*a*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH16.XLS," on tab "Problem 16.109a."




(*b*) The spreadsheet used for this problem can be found on the Media Manager, with filename \ "PSE4_ISM_CH16.XLS," on tab "Problem 16.109b."



# CHAPTER 21: Electric Charges and Electric Field

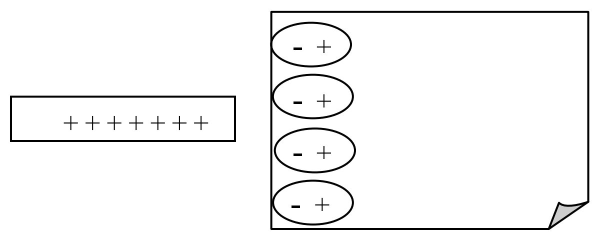
## Responses to Questions

1. Rub a glass rod with silk and use it to charge an electroscope. The electroscope will end up with a net positive charge. Bring the pocket comb close to the electroscope. If the electroscope leaves move farther apart, then the charge on the comb is positive, the same as the charge on the electroscope. If the leaves move together, then the charge on the comb is negative, opposite the charge on the electroscope.

2. The shirt or blouse becomes charged as a result of being tossed about in the dryer and rubbing against the dryer sides and other clothes. When you put on the charged object (shirt), it causes charge separation within the molecules of your skin (see Figure 21-9), which results in attraction between the shirt and your skin.

3. Fog or rain droplets tend to form around ions because water is a polar molecule, with a positive region and a negative region. The charge centers on the water molecule will be attracted to the ions (positive to negative).

4. See also Figure 21-9 in the text. The negatively charged electrons in the paper are attracted to the positively charged rod and move towards it within their molecules. The attraction occurs because the negative charges in the paper are closer to the positive rod than are the positive charges in the paper, and therefore the attraction between the unlike charges is greater than the repulsion between the like charges.




5. A plastic ruler that has been rubbed with a cloth is charged. When brought near small pieces of paper, it will cause separation of charge in the bits of paper, which will cause the paper to be attracted to the ruler. On a humid day, polar water molecules will be attracted to the ruler and to the separated charge on the bits of paper, neutralizing the charges and thus eliminating the attraction.

6. The *net charge* on a conductor is the difference between the total positive charge and the total negative charge in the conductor. The “free charges” in a conductor are the electrons that can move about freely within the material because they are only loosely bound to their atoms. The “free electrons” are also referred to as “conduction electrons.” A conductor may have a zero net charge but still have substantial free charges.

7. Most of the electrons are strongly bound to nuclei in the metal ions. Only a few electrons per atom (usually one or two) are free to move about throughout the metal. These are called the “conduction electrons.” The rest are bound more tightly to the nucleus and are not free to move. Furthermore, in the cases shown in Figures 21-7 and 21-8, not all of the conduction electrons will move. In Figure 21-7, electrons will move until the attractive force on the remaining conduction electrons due to the incoming charged rod is balanced by the repulsive force from electrons that have already gathered at the left end of the neutral rod. In Figure 21-8, conduction electrons will be repelled by the incoming rod and will leave the stationary rod through the ground connection until the repulsive force on the remaining conduction electrons due to the incoming charged rod is balanced by the attractive force from the net positive charge on the stationary rod.

8. The electroscope leaves are connected together at the top. The horizontal component of this tension force balances the electric force of repulsion. (Note: The vertical component of the tension force balances the weight of the leaves.)

9. Coulomb's law and Newton's law are very similar in form. The electrostatic force can be either attractive or repulsive; the gravitational force can only be attractive. The electrostatic force constant is also much larger than the gravitational force constant. Both the electric charge and the gravitational mass are properties of the material. Charge can be positive or negative, but the gravitational mass only has one form.

10. The gravitational force between everyday objects on the surface of the Earth is extremely small. (Recall the value of G: $6.67 \times 10^{-11}$ $Nm^2/kg^2$.) Consider two objects sitting on the floor near each other. They are attracted to each other, but the force of static fiction for each is much greater than the gravitational force each experiences from the other. Even in an absolutely frictionless environment, the acceleration resulting from the gravitational force would be so small that it would not be noticeable in a short time frame. We are aware of the gravitational force between objects if at least one of them is very massive, as in the case of the Earth and satellites or the Earth and you.

    The electric force between two objects is typically zero or close to zero because ordinary objects are typically neutral or close to neutral. We are aware of electric forces between objects when the objects are charged. An example is the electrostatic force (static cling) between pieces of clothing when you pull the clothes out of the dryer.

11. Yes, the electric force is a conservative force. Energy is conserved when a particle moves under the influence of the electric force, and the work done by the electric force in moving an object between two points in space is independent of the path taken.

12. Coulomb observed experimentally that the force between two charged objects is directly proportional to the charge on each one. For example, if the charge on either object is tripled, then the force is tripled. This is not in agreement with a force that is proportional to the *sum* of the charges instead of to the *product* of the charges. Also, a charged object is not attracted to or repelled from a neutral object, which would be the case if the numerator in Coulomb's law were proportional to the sum of the charges.

13. When a charged ruler attracts small pieces of paper, the charge on the ruler causes a separation of charge in the paper. For example, if the ruler is negatively charged, it will force the electrons in the paper to the edge of the paper farthest from the ruler, leaving the near edge positively charged. If the paper touches the ruler, electrons will be transferred from the ruler to the paper, neutralizing the positive charge. This action leaves the paper with a net negative charge, which will cause it to be repelled by the negatively charged ruler.

14. The test charges used to measure electric fields are small in order to minimize their contribution to the field. Large test charges would substantially change the field being investigated.

15. When determining an electric field, it is best, but not required, to use a positive test charge. A negative test charge would be fine for determining the magnitude of the field. But the direction of the electrostatic force on a negative test charge will be opposite to the direction of the electric field. The electrostatic force on a positive test charge will be in the same direction as the electric field. In order to avoid confusion, it is better to use a positive test charge.

16. See Figure 21-34b. A diagram of the electric field lines around two negative charges would be just like this diagram except that the arrows on the field lines would point towards the charges instead of away from them. The distance between the charges is *l*.

17. The electric field will be strongest to the right of the positive charge (between the two charges) and weakest to the left of the positive charge. To the right of the positive charge, the contributions to the field from the two charges point in the same direction, and therefore add. To the left of the positive charge, the contributions to the field from the two charges point in opposite directions, and therefore subtract. Note that this is confirmed by the density of field lines in Figure 21-34a.

18. At point C, the positive test charge would experience zero net force. At points A and B, the direction of the force on the positive test charge would be the same as the direction of the field. This direction is indicated by the arrows on the field lines. The strongest field is at point A, followed (in order of decreasing field strength) by B and then C.

19. Electric field lines can never cross because they give the direction of the electrostatic force on a positive test charge. If they were to cross, then the force on a test charge at a given location would be in more than one direction. This is not possible.

20. The field lines must be directed radially toward or away from the point charge (see rule 1). The spacing of the lines indicates the strength of the field (see rule 2). Since the magnitude of the field due to the point charge depends only on the distance from the point charge, the lines must be distributed symmetrically.

21. The two charges are located along a line as shown in the diagram.


(*a*) If the signs of the charges are opposite then the point on the line where $E = 0$ will lie to the left of Q. In that region the electric fields from the two charges will point in opposite directions, and the point will be closer to the smaller charge.

(*b*) If the two charges have the same sign, then the point on the line where $E = 0$ will lie between the two charges, closer to the smaller charge. In this region, the electric fields from the two charges will point in opposite directions.

22. The electric field at point P would point in the negative *x*-direction. The magnitude of the field would be the same as that calculated for a positive distribution of charge on the ring:

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Qx}{\left(x^2 + a^2\right)^{3/2}}$$

23. The velocity of the test charge will depend on its initial velocity. The field line gives the direction of the change in velocity, not the direction of the velocity. The acceleration of the test charge will be along the electric field line.

24. The value measured will be slightly less than the electric field value at that point before the test charge was introduced. The test charge will repel charges on the surface of the conductor and these charges will move along the surface to increase their distances from the test charge. Since they will then be at greater distances from the point being tested, they will contribute a smaller amount to the field.

25. The motion of the electron in Example 21-16 is projectile motion. In the case of the gravitational force, the acceleration of the projectile is in the same direction as the field and has a value of $g$; in the case of an electron in an electric field, the direction of the acceleration of the electron and the field direction are opposite, and the value of the acceleration varies.

26. Initially, the dipole will spin clockwise. It will "overshoot" the equilibrium position (parallel to the field lines), come momentarily to rest and then spin counterclockwise. The dipole will continue to oscillate back and forth if no damping forces are present. If there are damping forces, the amplitude will decrease with each oscillation until the dipole comes to rest aligned with the field.

27. If an electric dipole is placed in a nonuniform electric field, the charges of the dipole will experience forces of different magnitudes whose directions also may not be exactly opposite. The addition of these forces will leave a net force on the dipole.

## Solutions to Problems

1. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.602\times10^{-19}\,\text{C}\right)\left(26\times1.602\times10^{-19}\,\text{C}\right)}{\left(1.5\times10^{-12}\,\text{m}\right)^2} = \boxed{2.7\times10^{-3}\,\text{N}}$$

2. Use the charge per electron to find the number of electrons.

$$\left(-38.0\times10^{-6}\text{C}\right)\left(\frac{1\ \text{electron}}{-1.602\times10^{-19}\,\text{C}}\right) = \boxed{2.37\times10^{14}\,\text{electrons}}$$

3. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(25\times10^{-6}\text{C}\right)\left(2.5\times10^{-3}\text{C}\right)}{\left(0.28\,\text{m}\right)^2} = \boxed{7200\,\text{N}}$$

4. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.602\times10^{-19}\,\text{C}\right)^2}{\left(4.0\times10^{-15}\,\text{m}\right)^2} = \boxed{14\,\text{N}}$$

5. The charge on the plastic comb is negative, so the comb has gained electrons.

$$\frac{\Delta m}{m} = \frac{\left(3.0\times10^{-6}\text{C}\right)\left(\frac{1\text{e}^-}{1.602\times10^{-19}\,\text{C}}\right)\left(\frac{9.109\times10^{-31}\,\text{kg}}{1\text{e}^-}\right)}{0.035\,\text{kg}} = 4.9\times10^{-16} = \boxed{4.9\times10^{-14}\%}$$

6. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the distance is multiplied by a factor of 1/8, the force will be multiplied by a factor of 64.

$$F = 64F_0 = 64\left(3.2\times10^{-2}\,\text{N}\right) = \boxed{2.0\ \text{N}}$$

7. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the force is tripled, the distance has been reduced by a factor of $\sqrt{3}$.

$$r = \frac{r_0}{\sqrt{3}} = \frac{8.45\text{ cm}}{\sqrt{3}} = \boxed{4.88\text{ cm}}$$

8. Use the charge per electron and the mass per electron.

$$\left(-46\times10^{-6}\text{C}\right)\left(\frac{1\text{ electron}}{-1.602\times10^{-19}\text{C}}\right) = 2.871\times10^{14} \approx \boxed{2.9\times10^{14}\text{electrons}}$$

$$\left(2.871\times10^{14}\text{e}^-\right)\left(\frac{9.109\times10^{-31}\text{kg}}{1\text{ e}^-}\right) = \boxed{2.6\times10^{-16}\text{kg}}$$

9. To find the number of electrons, convert the mass to moles, the moles to atoms, and then multiply by the number of electrons in an atom to find the total electrons. Then convert to charge.

$$15\text{kg Au} = \left(15\text{kg Au}\right)\left(\frac{1\text{mole Al}}{0.197\text{kg}}\right)\left(\frac{6.022\times10^{23}\text{atoms}}{1\text{ mole}}\right)\left(\frac{79\text{electrons}}{1\text{molecule}}\right)\left(\frac{-1.602\times10^{-19}\text{C}}{\text{electron}}\right)$$

$$= \boxed{-5.8\times10^{8}\text{C}}$$

The net charge of the bar is $\boxed{0\text{C}}$, since there are equal numbers of protons and electrons.

10. Take the ratio of the electric force divided by the gravitational force.

$$\frac{F_\text{E}}{F_\text{G}} = \frac{k\frac{Q_1Q_2}{r^2}}{G\frac{m_1m_2}{r^2}} = \frac{kQ_1Q_2}{Gm_1m_2} = \frac{\left(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.602\times10^{-19}\text{C}\right)^2}{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(9.11\times10^{-31}\text{kg}\right)\left(1.67\times10^{-27}\text{kg}\right)} = \boxed{2.3\times10^{39}}$$

The electric force is about $2.3\times10^{39}$ times stronger than the gravitational force for the given scenario.

11. (*a*) Let one of the charges be $q$, and then the other charge is $Q_\text{T} - q$. The force between the charges is $F_\text{E} = k\frac{q\left(Q_\text{T}-q\right)}{r^2} = \frac{k}{r^2}\left(qQ_\text{T} - q^2\right)$. To find the maximum and minimum force, set the first derivative equal to 0. Use the second derivative test as well.

$$F_\text{E} = \frac{k}{r^2}\left(qQ_\text{T} - q^2\right) \ ; \ \frac{dF_\text{E}}{dq} = \frac{k}{r^2}\left(Q_\text{T} - 2q\right) = 0 \ \rightarrow \ q = \tfrac{1}{2}Q_\text{T}$$

$$\frac{d^2F_\text{E}}{dq^2} = -\frac{2k}{r^2} < 0 \ \rightarrow \ q = \tfrac{1}{2}Q_\text{T} \text{ gives } \left(F_\text{E}\right)_\text{max}$$

So $\boxed{q_1 = q_2 = \tfrac{1}{2}Q_\text{T}}$ gives the maximum force.

(*b*) If one of the charges has all of the charge, and the other has no charge, then the force between them will be 0, which is the minimum possible force. So $\boxed{q_1 = 0,\ q_2 = Q_\text{T}}$ gives the minimum force.

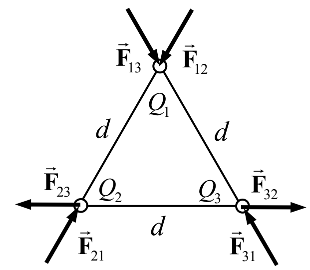
12. Let the right be the positive direction on the line of charges. Use the fact that like charges repel and unlike charges attract to determine the direction of the forces. In the following expressions, $k = 8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

$$\vec{\mathbf{F}}_{+75} = -k\frac{(75\mu\text{C})(48\mu\text{C})}{(0.35\,\text{m})^2}\hat{\mathbf{i}} + k\frac{(75\mu\text{C})(85\mu\text{C})}{(0.70\,\text{m})^2}\hat{\mathbf{i}} = -147.2\,\text{N}\,\hat{\mathbf{i}} \approx \boxed{-150\,\text{N}\,\hat{\mathbf{i}}}$$

$$\vec{\mathbf{F}}_{+48} = k\frac{(75\mu\text{C})(48\mu\text{C})}{(0.35\,\text{m})^2}\hat{\mathbf{i}} + k\frac{(48\mu\text{C})(85\mu\text{C})}{(0.35\,\text{m})^2}\hat{\mathbf{i}} = 563.5\,\text{N}\,\hat{\mathbf{i}} \approx \boxed{560\,\text{N}\,\hat{\mathbf{i}}}$$

$$\vec{\mathbf{F}}_{-85} = -k\frac{(85\mu\text{C})(75\mu\text{C})}{(0.70\,\text{m})^2}\hat{\mathbf{i}} - k\frac{(85\mu\text{C})(48\mu\text{C})}{(0.35\,\text{m})^2}\hat{\mathbf{i}} = -416.3\,\text{N}\,\hat{\mathbf{i}} \approx \boxed{-420\,\text{N}\,\hat{\mathbf{i}}}$$

13. The forces on each charge lie along a line connecting the charges. Let the variable *d* represent the length of a side of the triangle. Since the triangle is equilateral, each angle is 60°. First calculate the magnitude of each individual force.




$$F_{12} = k\frac{|Q_1Q_2|}{d^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(7.0\times10^{-6}\text{C})(8.0\times10^{-6}\text{C})}{(1.20\,\text{m})^2}$$
$$= 0.3495\,\text{N}$$

$$F_{13} = k\frac{|Q_1Q_3|}{d^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(7.0\times10^{-6}\text{C})(6.0\times10^{-6}\text{C})}{(1.20\,\text{m})^2}$$
$$= 0.2622\,\text{N}$$

$$F_{23} = k\frac{|Q_2Q_3|}{d^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(8.0\times10^{-6}\text{C})(6.0\times10^{-6}\text{C})}{(1.20\,\text{m})^2} = 0.2996\,\text{N} = F_{32}$$

Now calculate the net force on each charge and the direction of that net force, using components.

$$F_{1x} = F_{12x} + F_{13x} = -(0.3495\,\text{N})\cos60^\circ + (0.2622\,\text{N})\cos60^\circ = -4.365\times10^{-2}\,\text{N}$$

$$F_{1y} = F_{12y} + F_{13y} = -(0.3495\,\text{N})\sin60^\circ - (0.2622\,\text{N})\sin60^\circ = -5.297\times10^{-1}\,\text{N}$$

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \boxed{0.53\,\text{N}} \qquad \theta_1 = \tan^{-1}\frac{F_{1y}}{F_{1x}} = \tan^{-1}\frac{-5.297\times10^{-1}\,\text{N}}{-4.365\times10^{-2}\,\text{N}} = \boxed{265^\circ}$$

$$F_{2x} = F_{21x} + F_{23x} = (0.3495\,\text{N})\cos60^\circ - (0.2996\,\text{N}) = -1.249\times10^{-1}\,\text{N}$$

$$F_{2y} = F_{21y} + F_{23y} = (0.3495\,\text{N})\sin60^\circ + 0 = 3.027\times10^{-1}\,\text{N}$$

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = \boxed{0.33\,\text{N}} \qquad \theta_2 = \tan^{-1}\frac{F_{2y}}{F_{2x}} = \tan^{-1}\frac{3.027\times10^{-1}\,\text{N}}{-1.249\times10^{-1}\,\text{N}} = \boxed{112^\circ}$$

$$F_{3x} = F_{31x} + F_{32x} = -(0.2622\,\text{N})\cos60^\circ + (0.2996\,\text{N}) = 1.685\times10^{-1}\,\text{N}$$

$$F_{3y} = F_{31y} + F_{32y} = (0.2622\,\text{N})\sin60^\circ + 0 = 2.271\times10^{-1}\,\text{N}$$

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \boxed{0.26\,\text{N}} \qquad \theta_3 = \tan^{-1}\frac{F_{3y}}{F_{3x}} = \tan^{-1}\frac{2.271\times10^{-1}\,\text{N}}{1.685\times10^{-1}\,\text{N}} = \boxed{53^\circ}$$

14. (*a*) If the force is repulsive, both charges must be positive since the total charge is positive. Call the total charge $Q$.

$$Q_1 + Q_2 = Q \qquad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ_1(Q-Q_1)}{d^2} \rightarrow Q_1^2 - QQ_1 + \frac{Fd^2}{k} = 0$$

$$Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2} = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}$$

$$= \tfrac{1}{2}\left[\left(90.0\times10^{-6}\,\text{C}\right) \pm \sqrt{\left(90.0\times10^{-6}\,\text{C}\right)^2 - 4\frac{(12.0\,\text{N})(1.16\,\text{m})^2}{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}}\right]$$

$$= \boxed{60.1\times10^{-6}\,\text{C}\,,\ 29.9\times10^{-6}\,\text{C}}$$

(*b*) If the force is attractive, then the charges are of opposite sign. The value used for $F$ must then be negative. Other than that, the solution method is the same as for part (*a*).

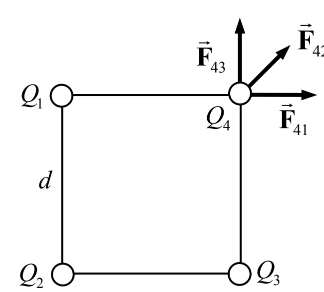
$$Q_1 + Q_2 = Q \qquad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ_1(Q-Q_1)}{d^2} \rightarrow Q_1^2 - QQ_1 + \frac{Fd^2}{k} = 0$$

$$Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2} = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}$$

$$= \tfrac{1}{2}\left[\left(90.0\times10^{-6}\,\text{C}\right) \pm \sqrt{\left(90.0\times10^{-6}\,\text{C}\right)^2 - 4\frac{(-12.0\,\text{N})(1.16\,\text{m})^2}{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}}\right]$$

$$= \boxed{106.8\times10^{-6}\,\text{C}\,,\ -16.8\times10^{-6}\,\text{C}}$$

15. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other three charges. The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable $d$ represent the 0.100 m length of a side of the square, and let the variable $Q$ represent the 4.15 mC charge at each corner.




$$F_{41} = k\frac{Q^2}{d^2} \rightarrow F_{41x} = k\frac{Q^2}{d^2}\,,\ F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \rightarrow F_{42x} = k\frac{Q^2}{2d^2}\cos45^\circ = k\frac{\sqrt{2}Q^2}{4d^2}\,,\ F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \rightarrow F_{43x} = 0\,,\ F_{43y} = k\frac{Q^2}{d^2}$$

Add the $x$ and $y$ components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 = k\frac{Q^2}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right) = F_{4y}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k\frac{Q^2}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q^2}{d^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

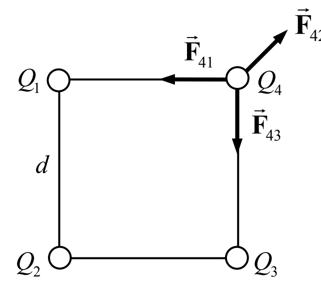
$$=\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(4.15\times10^{-3}\,\text{C}\right)^2}{\left(0.100\,\text{m}\right)^2}\left(\sqrt{2}+\frac{1}{2}\right)=\boxed{2.96\times10^{7}\,\text{N}}$$

$\theta=\tan^{-1}\dfrac{F_{4y}}{F_{4x}}=\boxed{45^\circ}$ above the $x$-direction.

For each charge, the net force will be the magnitude determined above, and will lie along the line from the center of the square out towards the charge.

16. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other charges.

The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable $d$ represent the 0.100 m length of a side of the square, and let the variable $Q$ represent the 4.15 mC charge at each corner.




$$F_{41}=k\frac{Q^2}{d^2}\;\rightarrow\;F_{41x}=-k\frac{Q^2}{d^2},\;F_{41y}=0$$

$$F_{42}=k\frac{Q^2}{2d^2}\;\rightarrow\;F_{42x}=k\frac{Q^2}{2d^2}\cos45^\circ=k\frac{\sqrt{2}Q^2}{4d^2},\;F_{42y}=k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43}=k\frac{Q^2}{d^2}\;\rightarrow\;F_{43x}=0,\;F_{43y}=-k\frac{Q^2}{d^2}$$

Add the $x$ and $y$ components together to find the total force, noting that $F_{4x}=F_{4y}$.

$$F_{4x}=F_{41x}+F_{42x}+F_{43x}=-k\frac{Q^2}{d^2}+k\frac{\sqrt{2}Q^2}{4d^2}+0=k\frac{Q^2}{d^2}\left(-1+\frac{\sqrt{2}}{4}\right)=-0.64645k\frac{Q^2}{d^2}=F_{4y}$$

$$F_4=\sqrt{F_{4x}^2+F_{4y}^2}=k\frac{Q^2}{d^2}\left(0.64645\right)\sqrt{2}=k\frac{Q^2}{d^2}\left(0.9142\right)$$

$$=\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(4.15\times10^{-3}\,\text{C}\right)^2}{\left(0.100\,\text{m}\right)^2}\left(0.9142\right)=\boxed{1.42\times10^{7}\,\text{N}}$$

$\theta=\tan^{-1}\dfrac{F_{4y}}{F_{4x}}=\boxed{225^\circ}$ from the $x$-direction, or exactly towards the center of the square.

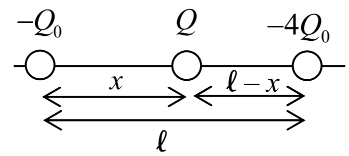
For each charge, there are two forces that point towards the adjacent corners, and one force that points away from the center of the square. Thus for each charge, the net force will be the magnitude of $\boxed{1.42\times10^{7}\,\text{N}}$ and will lie along the line from the charge inwards towards the center of the square.

17. The spheres can be treated as point charges since they are spherical, and so Coulomb's law may be used to relate the amount of charge to the force of attraction. Each sphere will have a magnitude $Q$ of charge, since that amount was removed from one sphere and added to the other, being initially uncharged.

$$F=k\frac{Q_1Q_2}{r^2}=k\frac{Q^2}{r^2}\;\rightarrow\;Q=r\sqrt{\frac{F}{k}}=\left(0.12\text{ m}\right)\sqrt{\frac{1.7\times10^{-2}\,\text{N}}{8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2}}$$

$$=1.650\times10^{-7}\,\text{C}\left(\frac{1\text{ electron}}{1.602\times10^{-19}\,\text{C}}\right)=\boxed{1.0\times10^{12}\,\text{electrons}}$$

18. The negative charges will repel each other, and so the third charge must put an opposite force on each of the original charges. Consideration of the various possible configurations leads to the conclusion that the third charge must be positive and must be between the other two charges. See the diagram for the definition of variables.




For each negative charge, equate the magnitudes of the two forces on the charge. Also note that $0 < x < \ell$.

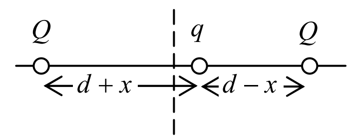
$$\text{left: } k\frac{Q_0Q}{x^2} = k\frac{4Q_0^2}{\ell^2} \qquad \text{right: } k\frac{4Q_0Q}{(\ell - x)^2} = k\frac{4Q_0^2}{\ell^2} \quad \rightarrow$$

$$k\frac{Q_0Q}{x^2} = k\frac{4Q_0Q}{(\ell - x)^2} \quad \rightarrow \quad x = \tfrac{1}{3}\ell$$

$$k\frac{Q_0Q}{x^2} = k\frac{4Q_0^2}{\ell^2} \quad \rightarrow \quad Q = 4Q_0\frac{x^2}{\ell^2} = Q_0\frac{4}{(3)^2} = \tfrac{4}{9}Q_0$$

Thus the charge should be of magnitude $\boxed{\tfrac{4}{9}Q_0}$, and a distance $\boxed{\tfrac{1}{3}\ell \text{ from } -Q_0 \text{ towards } -4Q_0}$.

19. (*a*) The charge will experience a force that is always pointing towards the origin. In the diagram, there is a greater force of $\dfrac{Qq}{4\pi\varepsilon_0(d-x)^2}$ to the left, and a lesser force of $\dfrac{Qq}{4\pi\varepsilon_0(d+x)^2}$ to the right. So the net force is towards the origin. The same would be true if the mass were to the left of the origin. Calculate the net force.




$$F_{\text{net}} = \frac{Qq}{4\pi\varepsilon_0(d+x)^2} - \frac{Qq}{4\pi\varepsilon_0(d-x)^2} = \frac{Qq}{4\pi\varepsilon_0(d+x)^2(d-x)^2}\left[(d-x)^2 - (d+x)^2\right]$$

$$= \frac{-4Qqd}{4\pi\varepsilon_0(d+x)^2(d-x)^2}x = \frac{-Qqd}{\pi\varepsilon_0(d+x)^2(d-x)^2}x$$

We assume that $x \ll d$.

$$F_{\text{net}} = \frac{-Qqd}{\pi\varepsilon_0(d+x)^2(d-x)^2}x \approx \frac{-Qq}{\pi\varepsilon_0 d^3}x$$

This has the form of a simple harmonic oscillator, where the "spring constant" is $k_{\text{elastic}} = \dfrac{Qq}{\pi\varepsilon_0 d^3}$.

The spring constant can be used to find the period. See Eq. 14-7b.

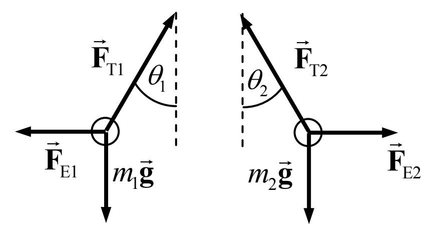
$$T = 2\pi\sqrt{\frac{m}{k_{\text{elastic}}}} = 2\pi\sqrt{\frac{m}{\dfrac{Qq}{\pi\varepsilon_0 d^3}}} = \boxed{2\pi\sqrt{\frac{m\pi\varepsilon_0 d^3}{Qq}}}$$

(*b*) Sodium has an atomic mass of 23.

$$T = 2\pi\sqrt{\frac{m\pi\varepsilon_0 d^3}{Qq}} = 2\pi\sqrt{\frac{(29)(1.66\times10^{-27}\,\text{kg})\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)(3\times10^{-10}\,\text{m})^3}{(1.60\times10^{-19}\,\text{C})^2}}$$

$$= 2.4\times10^{-13}\,\text{s}\left(\frac{10^{12}\,\text{ps}}{1\,\text{s}}\right) = 0.24\,\text{ps} \approx \boxed{0.2\,\text{ps}}$$

20. If all of the angles to the vertical (in both cases) are assumed to be small, then the spheres only have horizontal displacement, and so the electric force of repulsion is always horizontal. Likewise, the small angle condition leads to $\tan\theta \approx \sin\theta \approx \theta$ for all small angles. See the free-body diagram for each sphere, showing the three forces of gravity, tension, and the electrostatic force. Take to the right to be the positive horizontal direction, and up to be the positive vertical direction. Since the spheres are in equilibrium, the net force in each direction is zero.



(*a*) $\sum F_{1x} = F_{T1}\sin\theta_1 - F_{E1} = 0 \;\rightarrow\; F_{E1} = F_{T1}\sin\theta_1$

$$\sum F_{1y} = F_{T1}\cos\theta_1 - m_1 g \;\rightarrow\; F_{T1} = \frac{m_1 g}{\cos\theta_1} \;\rightarrow\; F_{E1} = \frac{m_1 g}{\cos\theta_1}\sin\theta_1 = m_1 g\tan\theta_1 = m_1 g\theta_1$$

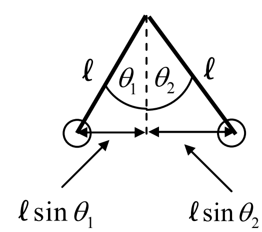
A completely parallel analysis would give $F_{E2} = m_2 g\theta_2$. Since the electric forces are a Newton's third law pair, they can be set equal to each other in magnitude.

$$F_{E1} = F_{E2} \;\rightarrow\; m_1 g\theta_1 = m_2 g\theta_2 \;\rightarrow\; \theta_1/\theta_2 = m_2/m_1 = \boxed{1}$$

(*b*) The same analysis can be done for this case.

$$F_{E1} = F_{E2} \;\rightarrow\; m_1 g\theta_1 = m_2 g\theta_2 \;\rightarrow\; \theta_1/\theta_2 = m_1/m_1 = \boxed{2}$$

(*c*) The horizontal distance from one sphere to the other is s by the small angle approximation. See the diagram. Use the relationship derived above that $F_E = mg\theta$ to solve for the distance.



Case 1: $d = \ell(\theta_1 + \theta_2) = 2\ell\theta_1 \;\rightarrow\; \theta_1 = \dfrac{d}{2\ell}$

$$m_1 g\theta_1 = F_{E1} = \frac{kQ(2Q)}{d^2} = mg\frac{d}{2\ell} \;\rightarrow\; \boxed{d = \left(\frac{4\ell kQ^2}{mg}\right)^{1/3}}$$

Case 2: $d = \ell(\theta_1 + \theta_2) = \dfrac{3}{2}\ell\theta_1 \;\rightarrow\; \theta_1 = \dfrac{2d}{3\ell}$

$$m_1 g\theta_1 = F_{E1} = \frac{kQ(2Q)}{d^2} = mg\frac{2d}{3\ell} \;\rightarrow\; \boxed{d = \left(\frac{3\ell kQ^2}{mg}\right)^{1/3}}$$

21. Use Eq. 21–3 to calculate the force. Take east to be the positive *x* direction.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} \;\rightarrow\; \vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(-1.602\times10^{-19}\,\mathrm{C}\right)\left(1920\,\mathrm{N/C}\,\hat{\mathbf{i}}\right) = -3.08\times10^{-16}\,\mathrm{N}\,\hat{\mathbf{i}} = \boxed{3.08\times10^{-16}\,\mathrm{N\ west}}$$

22. Use Eq. 21–3 to calculate the electric field. Take north to be the positive *y* direction.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{-2.18\times10^{-14}\,\mathrm{N}\,\hat{\mathbf{j}}}{1.602\times10^{-19}\,\mathrm{C}} = -1.36\times10^{5}\,\mathrm{N/C}\,\hat{\mathbf{j}} = \boxed{1.36\times10^{5}\,\mathrm{N/C\ south}}$$

23. Use Eq. 21–4a to calculate the electric field due to a point charge.

$$E = k\frac{Q}{r^2} = \left(8.988\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\frac{33.0\times10^{-6}\,\mathrm{C}}{\left(0.164\,\mathrm{m}\right)^2} = \boxed{1.10\times10^{7}\,\mathrm{N/C\ up}}$$

Note that the electric field points away from the positive charge.

24. Use Eq. 21–3 to calculate the electric field.

$$\vec{\mathbf{E}}=\frac{\vec{\mathbf{F}}}{q}=\frac{8.4\,\text{N down}}{-8.8\times10^{-6}\,\text{C}}=\boxed{9.5\times10^{5}\ \text{N/C up}}$$

25. Use the definition of the electric field, Eq. 21-3.

$$\vec{\mathbf{E}}=\frac{\vec{\mathbf{F}}}{q}=\frac{\left(7.22\times10^{-4}\,\text{N}\,\hat{\mathbf{j}}\right)}{4.20\times10^{-6}\,\text{C}}=\boxed{172\,\text{N/C}\,\hat{\mathbf{j}}}$$

26. Use the definition of the electric field, Eq. 21-3.

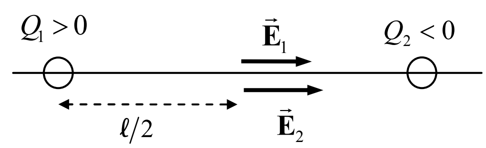
$$\vec{\mathbf{E}}=\frac{\vec{\mathbf{F}}}{q}=\frac{\left(3.0\hat{\mathbf{i}}-3.9\hat{\mathbf{j}}\right)\times10^{-3}\,\text{N}}{1.25\times10^{-6}\,\text{C}}=\boxed{\left(2400\,\hat{\mathbf{i}}-3100\,\hat{\mathbf{j}}\right)\text{N/C}}$$

27. Assuming the electric force is the only force on the electron, then Newton's second law may be used to find the acceleration.

$$\vec{\mathbf{F}}_{\text{net}}=m\vec{\mathbf{a}}=q\vec{\mathbf{E}}\ \rightarrow\ a=\frac{|q|}{m}E=\frac{\left(1.602\times10^{-19}\,\text{C}\right)}{\left(9.109\times10^{-31}\,\text{kg}\right)}\left(576\,\text{N/C}\right)=\boxed{1.01\times10^{14}\ \text{m/s}^2}$$

Since the charge is negative, the direction of the acceleration is $\boxed{\text{opposite to the field}}$.

28. The electric field due to the negative charge will point toward the negative charge, and the electric field due to the positive charge will point away from the positive charge. Thus both fields point in the same direction, towards the negative charge, and so can be added.

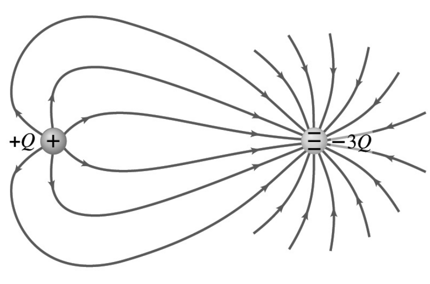



$$E=|E_1|+|E_2|=k\frac{|Q_1|}{r_1^2}+k\frac{|Q_2|}{r_2^2}=k\frac{|Q_1|}{(\ell/2)^2}+k\frac{|Q_2|}{(\ell/2)^2}=\frac{4k}{\ell^2}\left(|Q_1|+|Q_2|\right)$$

$$=\frac{4\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(0.080\,\text{m}\right)^2}\left(8.0\times10^{-6}\,\text{C}+5.8\times10^{-6}\,\text{C}\right)=\boxed{7.8\times10^{7}\ \text{N/C}}$$

The direction is $\boxed{\text{towards the negative charge}}$.

29.




30. Assuming the electric force is the only force on the electron, then Newton's second law may be used to find the electric field strength.

$$F_{net}=ma=qE\ \rightarrow\ E=\frac{ma}{q}=\frac{\left(1.673\times10^{-27}\,\text{kg}\right)\left(1.8\times10^{6}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.602\times10^{-19}\,\text{C}\right)}=\boxed{0.18\,\text{N/C}}$$

31. The field at the point in question is the vector sum of the two fields shown in Figure 21-56. Use the results of Example 21-11 to find the field of the long line of charge.

$$\vec{\mathbf{E}}_{\text{thread}} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\hat{\mathbf{j}} \ ; \ \vec{\mathbf{E}}_Q = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\left(-\cos\theta\hat{\mathbf{i}} - \sin\theta\hat{\mathbf{j}}\right) \ \rightarrow$$

$$\vec{\mathbf{E}} = \left(-\frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\cos\theta\right)\hat{\mathbf{i}} + \left(\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y} - \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\sin\theta\right)\hat{\mathbf{j}}$$

$$d^2 = (0.070\,\text{m})^2 + (0.120\,\text{m})^2 = 0.0193\,\text{m}^2 \ ; \ y = 0.070\,\text{m} \ ; \ \theta = \tan^{-1}\frac{12.0\,\text{cm}}{7.0\,\text{cm}} = 59.7°$$

$$E_x = -\frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\cos\theta = -\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(2.0\,\text{C})}{0.0193\,\text{m}^2}\cos 59.7° = -4.699\times10^{11}\,\text{N/C}$$

$$E_y = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y} - \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\sin\theta = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\lambda}{y} - \frac{|Q|}{d^2}\sin\theta\right)$$

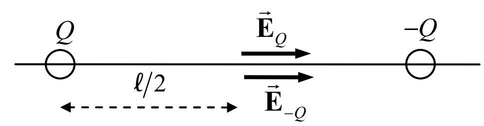
$$= \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{2(2.5\,\text{C/m})}{0.070\,\text{cm}} - \frac{(2.0\,\text{C})}{0.0193\,\text{m}^2}\sin 59.7°\right] = -1.622\times10^{11}\,\text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{\left(-4.7\times10^{11}\,\text{N/C}\right)\hat{\mathbf{i}} + \left(-1.6\times10^{11}\,\text{N/C}\right)\hat{\mathbf{j}}}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(-4.699\times10^{11}\,\text{N/C}\right)^2 + \left(-1.622\times10^{11}\,\text{N/C}\right)^2} = \boxed{5.0\times10^{11}\,\text{N/C}}$$

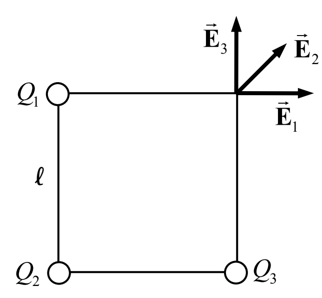
$$\theta_E = \tan^{-1}\frac{\left(-1.622\times10^{11}\,\text{N/C}\right)}{\left(-4.699\times10^{11}\,\text{N/C}\right)} = \boxed{199°}$$

32. The field due to the negative charge will point towards the negative charge, and the field due to the positive charge will point towards the negative charge. Thus the magnitudes of the two fields can be added together to find the charges.




$$E_{\text{net}} = 2E_Q = 2k\frac{Q}{(\ell/2)^2} = \frac{8kQ}{\ell^2} \ \rightarrow \ Q = \frac{E\ell^2}{8k} = \frac{(586\,\text{N/C})(0.160\,\text{m})^2}{8\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} = \boxed{2.09\times10^{-10}\,\text{C}}$$

33. The field at the upper right corner of the square is the vector sum of the fields due to the other three charges. Let the variable $\ell$ represent the 1.0 m length of a side of the square, and let the variable $Q$ represent the charge at each of the three occupied corners.




$$E_1 = k\frac{Q}{\ell^2} \ \rightarrow \ E_{1x} = k\frac{Q}{\ell^2} \ , E_{1y} = 0$$

$$E_2 = k\frac{Q}{2\ell^2} \ \rightarrow \ E_{2x} = k\frac{Q}{2\ell^2}\cos 45° = k\frac{\sqrt{2}Q}{4\ell^2} \ , E_{2y} = k\frac{\sqrt{2}Q}{4\ell^2}$$

$$E_3 = k\frac{Q}{\ell^2} \ \rightarrow \ E_{3x} = 0 \ , E_{1y} = k\frac{Q}{\ell^2}$$

Add the *x* and *y* components together to find the total electric field, noting that $E_x = E_y$.

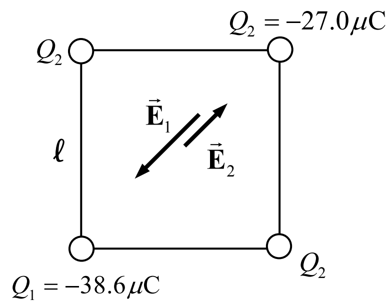
$$E_x = E_{1x} + E_{2x} + E_{3x} = k\frac{Q}{\ell^2} + k\frac{\sqrt{2}Q}{4\ell^2} + 0 = k\frac{Q}{\ell^2}\left(1+\frac{\sqrt{2}}{4}\right) = E_y$$

$$E = \sqrt{E_x^2 + E_y^2} = k\frac{Q}{\ell^2}\left(1+\frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q}{\ell^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(2.25\times10^{-6}\text{C}\right)}{\left(1.22\,\text{m}\right)^2}\left(\sqrt{2}+\frac{1}{2}\right) = \boxed{2.60\times10^4\ \text{N/C}}$$

$\theta = \tan^{-1}\dfrac{E_y}{E_x} = \boxed{45.0^\circ}$ from the $x$-direction.

34. The field at the center due to the two $-27.0\mu\text{C}$ negative charges on opposite corners (lower right and upper left in the diagram) will cancel each other, and so only the other two charges need to be considered. The field due to each of the other charges will point directly toward the charge. Accordingly, the two fields are in opposite directions and can be combined algebraically.




$$E = E_1 - E_2 = k\frac{|Q_1|}{\ell^2/2} - k\frac{|Q_2|}{\ell^2/2} = k\frac{|Q_1|-|Q_2|}{\ell^2/2}$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(38.6-27.0\right)\times10^{-6}\text{C}}{\left(0.525\,\text{m}\right)^2/2}$$

$$= \boxed{7.57\times10^6\ \text{N/C, towards the } -38.6\mu\text{C charge}}$$

35. Choose the rightward direction to be positive. Then the field due to $+Q$ will be positive, and the field due to $-Q$ will be negative.

$$E = k\frac{Q}{(x+a)^2} - k\frac{Q}{(x-a)^2} = kQ\left(\frac{1}{(x+a)^2} - \frac{1}{(x-a)^2}\right) = \boxed{\frac{-4kQxa}{\left(x^2-a^2\right)^2}}$$

The negative sign means the field points to the $\boxed{\text{left}}$.

36. For the net field to be zero at point P, the magnitudes of the fields created by $Q_1$ and $Q_2$ must be equal. Also, the distance $x$ will be taken as positive to the left of $Q_1$. That is the only region where the total field due to the two charges can be zero. Let the variable $\ell$ represent the 12 cm distance, and note that $|Q_1| = \frac{1}{2}Q_2$.

$$\left|\vec{\mathbf{E}}_1\right| = \left|\vec{\mathbf{E}}_2\right| \rightarrow k\frac{|Q_1|}{x^2} = k\frac{Q_2}{(x+\ell)^2} \rightarrow$$

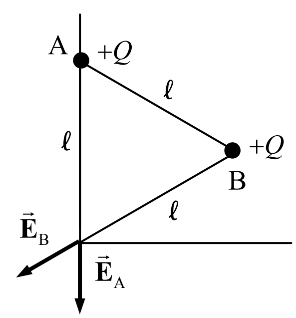
$$x = \ell\frac{\sqrt{|Q_1|}}{\left(\sqrt{Q_2}-\sqrt{|Q_1|}\right)} = (12\,\text{cm})\frac{\sqrt{25\mu\text{C}}}{\left(\sqrt{45\mu\text{C}}-\sqrt{25\mu\text{C}}\right)} = \boxed{35\,\text{cm}}$$

37. Make use of Example 21-11. From that, we see that the electric field due to the line charge along the $y$ axis is $\vec{\mathbf{E}}_1 = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}\hat{\mathbf{i}}$. In particular, the field due to that line of charge has no $y$ dependence. In a similar fashion, the electric field due to the line charge along the $x$ axis is $\vec{\mathbf{E}}_2 = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\hat{\mathbf{j}}$. Then the total field at $(x, y)$ is the vector sum of the two fields.

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}\hat{\mathbf{i}} + \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\hat{\mathbf{j}} = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{x}\hat{\mathbf{i}} + \frac{1}{y}\hat{\mathbf{j}}\right)$$

$$E = \frac{\lambda}{2\pi\varepsilon_0}\sqrt{\frac{1}{x^2} + \frac{1}{y^2}} = \boxed{\frac{\lambda}{2\pi\varepsilon_0 xy}\sqrt{x^2 + y^2}} \; ; \; \theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}}{\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}} = \boxed{\tan^{-1}\frac{x}{y}}$$

38. (*a*) The field due to the charge at A will point straight downward, and the field due to the charge at B will point along the line from A to the origin, 30° below the negative $x$ axis.




$$E_A = k\frac{Q}{\ell^2} \rightarrow E_{Ax} = 0 \, , E_{Ax} = -k\frac{Q}{\ell^2}$$

$$E_B = k\frac{Q}{\ell^2} \rightarrow E_{Bx} = -k\frac{Q}{\ell^2}\cos 30^\circ = -k\frac{\sqrt{3}Q}{2\ell^2},$$

$$E_{By} = -k\frac{Q}{\ell^2}\sin 30^\circ = -k\frac{Q}{2\ell^2}$$

$$E_x = E_{Ax} + E_{Bx} = -k\frac{\sqrt{3}Q}{2\ell^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{3Q}{2\ell^2}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4\ell^4} + \frac{9k^2Q^2}{4\ell^4}} = \sqrt{\frac{12k^2Q^2}{4\ell^4}} = \boxed{\frac{\sqrt{3}kQ}{\ell^2}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{-k\frac{3Q}{2\ell^2}}{-k\frac{\sqrt{3}Q}{2\ell^2}} = \tan^{-1}\frac{-3}{-\sqrt{3}} = \tan^{-1}\sqrt{3} = \boxed{240^\circ}$$

(*b*) Now reverse the direction of $\vec{\mathbf{E}}_A$

$$E_A = k\frac{Q}{\ell^2} \rightarrow E_{Ax} = 0 \, , E_{Ax} = -k\frac{Q}{\ell^2}$$

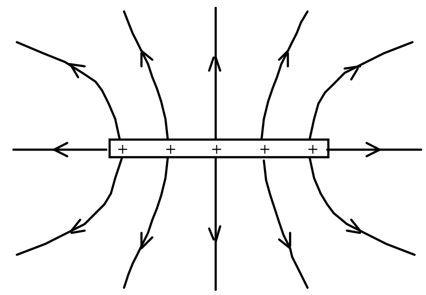
$$E_B = k\frac{Q}{\ell^2} \rightarrow E_{Bx} = k\frac{Q}{\ell^2}\cos 30^\circ = k\frac{\sqrt{3}Q}{2\ell^2} \, , E_{By} = k\frac{Q}{\ell^2}\sin 30^\circ = k\frac{Q}{2\ell^2}$$

$$E_x = E_{Ax} + E_{Bx} = k\frac{\sqrt{3}Q}{2\ell^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{Q}{2\ell^2}$$

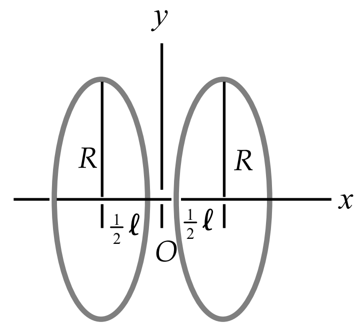
$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4\ell^4} + \frac{k^2Q^2}{4\ell^4}} = \sqrt{\frac{4k^2Q^2}{4\ell^4}} = \boxed{\frac{kQ}{\ell^2}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{k\dfrac{Q}{2\ell^2}}{-k\dfrac{\sqrt{3}Q}{2\ell^2}} = \tan^{-1}\frac{1}{-\sqrt{3}} = \boxed{330^\circ}$$

39. Near the plate, the lines should come from it almost vertically, because it is almost like an infinite line of charge when the observation point is close. When the observation point is far away, it will look like a point charge.



40. Consider Example 21-9. We use the result from this example, but shift the center of the ring to be at $x = \frac{1}{2}\ell$ for the ring on the right, and at $x = -\frac{1}{2}\ell$ for the ring on the left. The fact that the original expression has a factor of *x* results in the interpretation that the sign of the field expression will give the direction of the field. No special consideration needs to be given to the location of the point at which the field is to be calculated.




$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_{\text{right}} + \vec{\mathbf{E}}_{\text{left}}$$

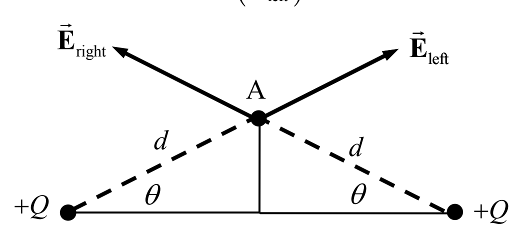
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q\left(x-\frac{1}{2}\ell\right)}{\left[\left(x-\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}}\hat{\mathbf{i}} + \frac{1}{4\pi\varepsilon_0}\frac{Q\left(x+\frac{1}{2}\ell\right)}{\left[\left(x+\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}}\hat{\mathbf{i}}$$

$$= \hat{\mathbf{i}}\frac{Q}{4\pi\varepsilon_0}\left\{\frac{\left(x-\frac{1}{2}\ell\right)}{\left[\left(x-\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}} + \frac{\left(x+\frac{1}{2}\ell\right)}{\left[\left(x+\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}}\right\}$$

41. Both charges must be of the same sign so that the electric fields created by the two charges oppose each other, and so can add to zero. The magnitudes of the two electric fields must be equal.

$$E_1 = E_2 \rightarrow k\frac{Q_1}{(\ell/3)^2} = k\frac{Q_2}{(2\ell/3)^2} \rightarrow 9Q_1 = \frac{9Q_2}{4} \rightarrow \frac{Q_1}{Q_2} = \boxed{\frac{1}{4}}$$

42. In each case, find the vector sum of the field caused by the charge on the left $\left(\vec{\mathbf{E}}_{\text{left}}\right)$ and the field caused by the charge on the right $\left(\vec{\mathbf{E}}_{\text{right}}\right)$



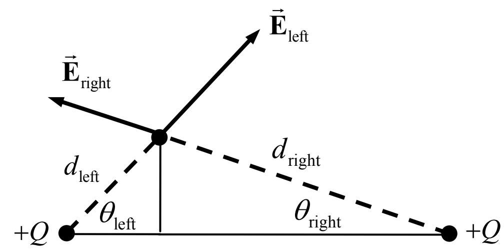

Point A: From the symmetry of the geometry, in calculating the electric field at point A only the vertical components of the fields need to be considered. The horizontal components will cancel each other.

$$\theta = \tan^{-1}\frac{5.0}{10.0} = 26.6^\circ$$

$$d = \sqrt{(5.0\,\text{cm})^2 + (10.0\,\text{cm})^2} = 0.1118\,\text{m}$$

$$E_A = 2\frac{kQ}{d^2}\sin\theta = 2\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.7\times10^{-6}\,\text{C}}{(0.1118\,\text{m})^2}\sin 26.6° = \boxed{3.7\times10^6\ \text{N/C}} \qquad \theta_A = \boxed{90°}$$

Point B: Now the point is not symmetrically placed, and so horizontal and vertical components of each individual field need to be calculated to find the resultant electric field.




$$\theta_{left} = \tan^{-1}\frac{5.0}{5.0} = 45° \qquad \theta_{right} = \tan^{-1}\frac{5.0}{15.0} = 18.4°$$

$$d_{left} = \sqrt{(5.0\,\text{cm})^2 + (5.0\,\text{cm})^2} = 0.0707\,\text{m}$$

$$d_{right} = \sqrt{(5.0\,\text{cm})^2 + (15.0\,\text{cm})^2} = 0.1581\,\text{m}$$

$$E_x = \left(\vec{E}_{left}\right)_x + \left(\vec{E}_{right}\right)_x = k\frac{Q}{d_{left}^2}\cos\theta_{left} - k\frac{Q}{d_{right}^2}\cos\theta_{right}$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.7\times10^{-6}\text{C}\right)\left[\frac{\cos 45°}{(0.0707\,\text{m})^2} - \frac{\cos 18.4°}{(0.1581\,\text{m})^2}\right] = 5.30\times10^6\ \text{N/C}$$

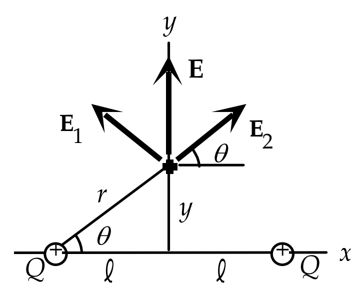
$$E_y = \left(\vec{E}_{left}\right)_y + \left(\vec{E}_{right}\right)_y = k\frac{Q}{d_{left}^2}\sin\theta_{left} + k\frac{Q}{d_{right}^2}\sin\theta_{right}$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.7\times10^{-6}\text{C}\right)\left[\frac{\sin 45°}{(0.0707\,\text{m})^2} + \frac{\sin 18.4°}{(0.1581\,\text{m})^2}\right] = 7.89\times10^6\ \text{N/C}$$

$$E_B = \sqrt{E_x^2 + E_y^2} = \boxed{9.5\times10^6\ \text{N/C}} \qquad \theta_B = \tan^{-1}\frac{E_y}{E_x} = \boxed{56°}$$

The results are consistent with Figure 21-34b. In the figure, the field at Point A points straight up, matching the calculations. The field at Point B should be to the right and vertical, matching the calculations. Finally, the field lines are closer together at Point B than at Point A, indicating that the field is stronger there, matching the calculations.

43. (*a*) See the diagram. From the symmetry of the charges, we see that the net electric field points along the *y* axis.




$$\vec{E} = 2\frac{Q}{4\pi\varepsilon_0\left(\ell^2 + y^2\right)}\sin\theta\,\hat{\mathbf{j}} = \boxed{\frac{Qy}{2\pi\varepsilon_0\left(\ell^2 + y^2\right)^{3/2}}\hat{\mathbf{j}}}$$

(*b*) To find the position where the magnitude is a maximum, set the first derivative with respect to *y* equal to 0, and solve for the *y* value.

$$E = \frac{Qy}{2\pi\varepsilon_0\left(\ell^2 + y^2\right)^{3/2}} \rightarrow$$

$$\frac{dE}{dy} = \frac{Q}{2\pi\varepsilon_0\left(\ell^2 + y^2\right)^{3/2}} + \left(-\tfrac{3}{2}\right)\frac{Qy}{2\pi\varepsilon_0\left(\ell^2 + y^2\right)^{5/2}}(2y) = 0 \rightarrow$$

$$\frac{1}{\left(\ell^2 + y^2\right)^{3/2}} = \frac{3y^2}{\left(\ell^2 + y^2\right)^{5/2}} \rightarrow y^2 = \tfrac{1}{2}\ell^2 \rightarrow y = \boxed{\pm\ell/\sqrt{2}}$$

This has to be a maximum, because the magnitude is positive, the field is 0 midway between the charges, and $E \to 0$ as $y \to \infty$.

44. From Example 21-9, the electric field along the $x$-axis is $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{\left(x^2+a^2\right)^{\frac{3}{2}}}$. To find the position where the magnitude is a maximum, we differentiate and set the first derivative equal to zero.

$$\frac{dE}{dx} = \frac{Q}{4\pi\varepsilon_0}\frac{\left(x^2+a^2\right)^{\frac{3}{2}} - x\frac{3}{2}\left(x^2+a^2\right)^{\frac{1}{2}}2x}{\left(x^2+a^2\right)^3} = \frac{Q}{4\pi\varepsilon_0\left(x^2+a^2\right)^{\frac{5}{2}}}\left[\left(x^2+a^2\right)-3x^2\right]$$

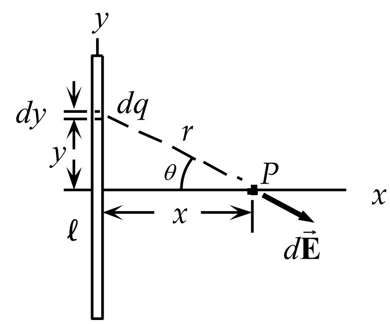
$$= \frac{Q}{4\pi\varepsilon_0\left(x^2+a^2\right)^{\frac{5}{2}}}\left[a^2-2x^2\right] = 0 \quad \to \quad \boxed{x_M = \pm\frac{a}{\sqrt{2}}}$$

Note that $E=0$ at $x=0$ and $x=\infty$, and that $|E|>0$ for $0<|x|<\infty$. Thus the value of the magnitude of $E$ at $x = x_M$ must be a maximum. We could also show that the value is a maximum by using the second derivative test.

45. Because the distance from the wire is much smaller than the length of the wire, we can approximate the electric field by the field of an infinite wire, which is derived in Example 21-11.

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{x} = \left(8.988\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{2\left(\dfrac{4.75\times10^{-6}\,\text{C}}{2.0\,\text{m}}\right)}{\left(2.4\times10^{-2}\,\text{m}\right)} = \boxed{\begin{array}{l}1.8\times10^6\ \text{N/C},\\ \text{away from the wire}\end{array}}$$

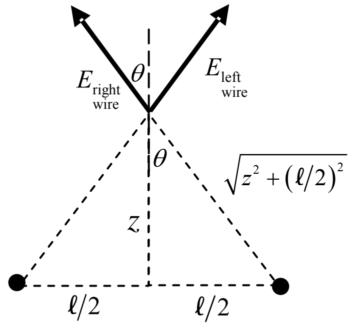
46. This is essentially Example 21-11 again, but with different limits of integration. From the diagram here, we see that the maximum angle is given by $\sin\theta = \dfrac{\ell/2}{\sqrt{x^2+(\ell/2)^2}}$. We evaluate the results at that angle.




$$E = \frac{\lambda}{4\pi\varepsilon_0 x}\left(\sin\theta\right)\Bigg|_{\sin\theta = \frac{-\ell/2}{\sqrt{x^2+(\ell/2)^2}}}^{\sin\theta = \frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 x}\left[\frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}} - \left(-\frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}}\right)\right] = \frac{\lambda\ell}{4\pi\varepsilon_0 x\sqrt{x^2+(\ell/2)^2}} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\frac{\ell}{x\left(4x^2+\ell^2\right)^{1/2}}}$$

47. If we consider just one wire, then from the answer to problem 46, we would have the following. Note that the distance from the wire to the point in question is $x = \sqrt{z^2+(\ell/2)^2}$.




$$E_{\text{wire}} = \frac{\lambda}{2\pi\varepsilon_0}\frac{\ell}{\sqrt{z^2+(\ell/2)^2}\left(4\left[z^2+(\ell/2)^2\right]+\ell^2\right)^{1/2}}$$

But the total field is not simply four times the above expression, because the fields due to the four wires are not parallel to each other.

Consider a side view of the problem. The two dots represent two parallel wires, on opposite sides of the square. Note that only the vertical component of the field due to each wire will actually contribute to the total field. The horizontal components will cancel.

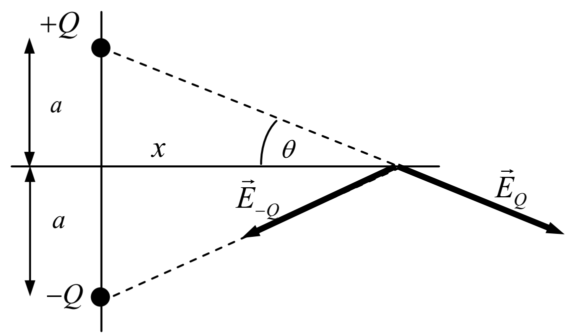
$$E_{\text{wire}} = 4\left(E_{\text{wire}}\right)\cos\theta = 4\left(E_{\text{wire}}\right)\frac{z}{\sqrt{z^2+\left(\ell/2\right)^2}}$$

$$E_{\text{wire}} = 4\left[\frac{\lambda}{2\pi\varepsilon_0}\frac{\ell}{\sqrt{z^2+\left(\ell/2\right)^2}\left(4\left[z^2+\left(\ell/2\right)^2\right]+\ell^2\right)^{1/2}}\right]\frac{z}{\sqrt{z^2+\left(\ell/2\right)^2}}$$

$$= \boxed{\frac{8\lambda\ell z}{\pi\varepsilon_0\left(4z^2+\ell^2\right)\left(4z^2+2\ell^2\right)^{1/2}}}$$

The direction is vertical, perpendicular to the loop.

48. From the diagram, we see that the $x$ components of the two fields will cancel each other at the point P. Thus the net electric field will be in the negative $y$-direction, and will be twice the $y$-component of either electric field vector.

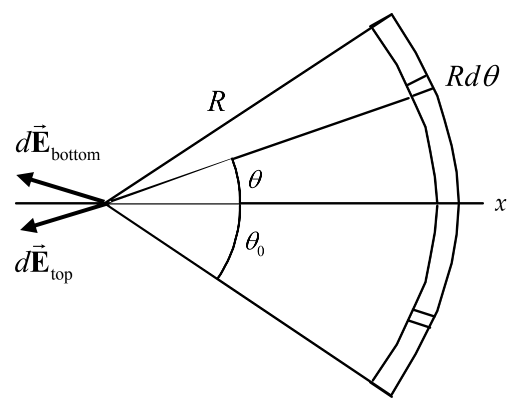



$$E_{\text{net}} = 2E\sin\theta = 2\frac{kQ}{x^2+a^2}\sin\theta$$

$$= \frac{2kQ}{x^2+a^2}\frac{a}{\left(x^2+a^2\right)^{1/2}}$$

$$= \boxed{\frac{2kQa}{\left(x^2+a^2\right)^{3/2}} \text{ in the negative } y \text{ direction}}$$

49. Select a differential element of the arc which makes an angle of $\theta$ with the $x$ axis. The length of this element is $Rd\theta$, and the charge on that element is $dq = \lambda R d\theta$. The magnitude of the field produced by that element is $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\theta}{R^2}$. From the diagram, considering pieces of the arc that are symmetric with respect to the $x$ axis, we see that the total field will only have an $x$ component. The vertical components of the field due to symmetric portions of the arc will cancel each other. So we have the following.

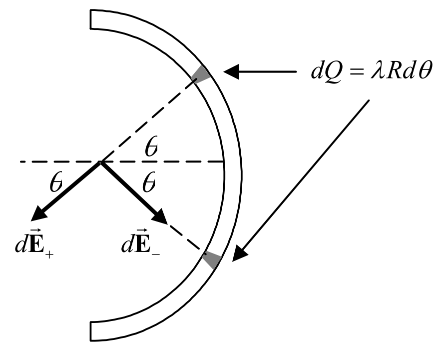



$$dE_{\text{horizontal}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\theta}{R^2}\cos\theta$$

$$E_{\text{horizontal}} = \int_{-\theta_0}^{\theta_0}\frac{1}{4\pi\varepsilon_0}\cos\theta\frac{\lambda R d\theta}{R^2} = \frac{\lambda}{4\pi\varepsilon_0 R}\int_{-\theta_0}^{\theta_0}\cos\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\left[\sin\theta_0 - \sin\left(-\theta_0\right)\right] = \frac{2\lambda\sin\theta_0}{4\pi\varepsilon_0 R}$$

The field points in the negative $x$ direction, so $E = \boxed{-\frac{2\lambda\sin\theta_0}{4\pi\varepsilon_0 R}\hat{\mathbf{i}}}$

50. (*a*) Select a differential element of the arc which makes an angle of $\theta$ with the *x* axis. The length of this element is $Rd\theta$, and the charge on that element is $dq = \lambda R d\theta$. The magnitude of the field produced by that element is $dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda R d\theta}{R^2}$. From the diagram, considering pieces of the arc that are symmetric with respect to the *x* axis, we see that the total field will only have a *y* component, because the magnitudes of the fields due to those two pieces are the same. From the diagram we see that the field will point down. The horizontal components of the field cancel.




$$dE_{\text{vertical}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\theta}{R^2}\sin\theta = \frac{\lambda_0}{4\pi\varepsilon_0 R}\sin^2\theta\, d\theta$$

$$E_{\text{vertical}} = \int_{-\pi/2}^{\pi/2}\frac{\lambda_0}{4\pi\varepsilon_0 R}\sin^2\theta\, d\theta = \frac{\lambda_0}{4\pi\varepsilon_0 R}\int_{-\pi/2}^{\pi/2}\sin^2\theta\, d\theta = \frac{\lambda_0}{4\pi\varepsilon_0 R}\left(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)_{-\pi/2}^{\pi/2}$$

$$= \frac{\lambda_0}{4\pi\varepsilon_0 R}\left(\tfrac{1}{2}\pi\right) = \frac{\lambda_0}{8\varepsilon_0 R} \quad\rightarrow\quad \vec{\mathbf{E}} = \boxed{-\frac{\lambda_0}{8\varepsilon_0 R}\hat{\mathbf{j}}}$$

(*b*) The force on the electron is given by Eq. 21-3. The acceleration is found from the force.

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} = -\frac{q\lambda_0}{8\varepsilon_0 R}\hat{\mathbf{j}} \quad\rightarrow$$

$$\vec{\mathbf{a}} = -\frac{q\lambda_0}{8m\varepsilon_0 R}\hat{\mathbf{j}} = \frac{e\lambda_0}{8m\varepsilon_0 R}\hat{\mathbf{j}} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.0\times10^{-6}\,\text{C/m}\right)}{8\left(9.11\times10^{-31}\,\text{kg}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(0.010\,\text{m}\right)}\hat{\mathbf{j}}$$

$$= \boxed{2.5\times10^{17}\,\text{m/s}^2\,\hat{\mathbf{j}}}$$

51. (*a*) If we follow the first steps of Example 21-11, and refer to Figure 21-29, then the differential electric field due to the segment of wire is still $dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\, dy}{\left(x^2+y^2\right)}$. But now there is no symmetry, and so we calculate both components of the field.

$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dy}{\left(x^2+y^2\right)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x\, dy}{\left(x^2+y^2\right)^{3/2}}$$

$$dE_y = -dE\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dy}{\left(x^2+y^2\right)}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda y\, dy}{\left(x^2+y^2\right)^{3/2}}$$

The anti-derivatives needed are in Appendix B4.

$$E_x = \int_0^{\ell}\frac{1}{4\pi\varepsilon_0}\frac{\lambda x\, dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\int_0^{\ell}\frac{dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\left(\frac{y}{x^2\sqrt{x^2+y^2}}\right)_0^{\ell}$$

$$= \boxed{\frac{\lambda\ell}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}}$$

$$E_y = -\int_0^{\ell} \frac{1}{4\pi\varepsilon_0} \frac{\lambda y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0} \int_0^{\ell} \frac{y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{\sqrt{x^2+y^2}}\right)_0^{\ell}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+\ell^2}} - \frac{1}{x}\right) = \boxed{\frac{\lambda}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}\left(x-\sqrt{x^2+\ell^2}\right)}$$

Note that $E_y < 0,$ and so the electric field points to the right and down.

(*b*) The angle that the electric field makes with the *x* axis is given as follows.

$$\tan\theta = \frac{E_y}{E_x} = \frac{\dfrac{\lambda}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}\left(x-\sqrt{x^2+\ell^2}\right)}{\dfrac{\lambda\ell}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}} = \frac{x-\sqrt{x^2+\ell^2}}{\ell} = \frac{x}{\ell} - \sqrt{1+\frac{x^2}{\ell^2}}$$

As $\ell \to \infty$, the expression becomes $\tan\theta = -1$, and so the field makes an angle of $\boxed{45° \text{ below the } x \text{ axis}}$.

52. Please note: the first printing of the textbook gave the length of the charged wire as 6.00 m, but it should have been 6.50 m. That error has been corrected in later printings, and the following solution uses a length of 6.50 m.

(*a*) If we follow the first steps of Example 21-11, and refer to Figure 21-29, then the differential electric field due to the segment of wire is still $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}$. But now there is no symmetry, and so we calculate both components of the field.

$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x\,dy}{\left(x^2+y^2\right)^{3/2}}$$

$$dE_y = -dE\cos\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda y\,dy}{\left(x^2+y^2\right)^{3/2}}$$

The anti-derivatives needed are in Appendix B4.

$$E_x = \int_{y_{\min}}^{y_{\max}} \frac{1}{4\pi\varepsilon_0}\frac{\lambda x\,dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\int_{y_{\min}}^{y_{\max}} \frac{dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\left(\frac{y}{x^2\sqrt{x^2+y^2}}\right)_{y_{\min}}^{y_{\max}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 x}\left(\frac{y_{\max}}{\sqrt{x^2+y_{\max}^2}} - \frac{y_{\min}}{\sqrt{x^2+y_{\min}^2}}\right)$$

$$= \left(8.99\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{\left(3.15\times10^{-6}\,\text{C}\right)/\left(6.50\,\text{m}\right)}{\left(0.250\,\text{m}\right)}$$

$$\left(\frac{2.50\,\text{m}}{\sqrt{\left(0.250\,\text{m}\right)^2+\left(2.50\,\text{m}\right)^2}} - \frac{\left(-4.00\,\text{m}\right)}{\sqrt{\left(0.250\,\text{m}\right)^2+\left(-4.00\,\text{m}\right)^2}}\right)$$

$$= 3.473\times10^4\ \text{N/C} \approx \boxed{3.5\times10^4\ \text{N/C}}$$

$$E_y = -\int_{y_{\min}}^{y_{\max}} \frac{1}{4\pi\varepsilon_0} \frac{\lambda y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0} \int_{y_{\min}}^{y_{\max}} \frac{y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{\sqrt{x^2+y^2}}\right)_{y_{\min}}^{y_{\max}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y_{\max}^2}} - \frac{1}{\sqrt{x^2+y_{\min}^2}}\right)$$

$$= \left(8.99\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{\left(3.15\times10^{-6}\,\text{C}\right)}{\left(6.50\,\text{m}\right)}$$

$$\left(\frac{1}{\sqrt{\left(0.250\,\text{m}\right)^2+\left(2.50\,\text{m}\right)^2}} - \frac{1}{\sqrt{\left(0.250\,\text{m}\right)^2+\left(-4.00\,\text{m}\right)^2}}\right)$$

$$= 647\,\text{N/C} \approx \boxed{650\,\text{N/C}}$$

(*b*) We calculate the infinite line of charge result, and calculate the errors.

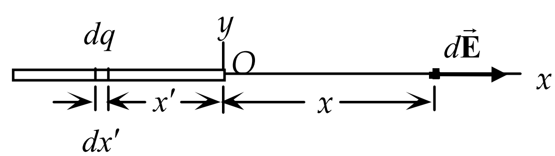
$$E = \frac{\lambda}{2\pi\varepsilon_0 x} = \frac{2\lambda}{4\pi\varepsilon_0 x} = 2\left(8.99\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{\left(3.15\times10^{-6}\,\text{C}\right)}{\left(6.50\,\text{m}\right)\left(0.250\,\text{m}\right)} = 3.485\times10^4\,\text{N/m}$$

$$\frac{E_x - E}{E} = \frac{\left(3.473\times10^4\,\text{N/C}\right)-\left(3.485\times10^4\,\text{N/m}\right)}{\left(3.485\times10^4\,\text{N/m}\right)} = \boxed{-0.0034}$$

$$\frac{E_y}{E} = \frac{\left(647\,\text{N/C}\right)}{\left(3.485\times10^4\,\text{N/m}\right)} = \boxed{0.019}$$

And so we see that $E_x$ is only about 0.3% away from the value obtained from the infinite line of charge, and $E_y$ is only about 2% of the value obtained from the infinite line of charge. The field of an infinite line of charge result would be a good approximation for the field due to this wire segment.

53. Choose a differential element of the rod $dx'$ a distance $x'$ from the origin, as shown in the diagram. The charge on that differential element is $dq = \frac{Q}{\ell}dx'$. The variable $x'$ is treated as positive,




so that the field due to this differential element is $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\left(x+x'\right)^2} = \frac{Q}{4\pi\varepsilon_0\ell}\frac{dx'}{\left(x+x'\right)^2}$. Integrate along the rod to find the total field.

$$E = \int dE = \int_0^{\ell} \frac{Q}{4\pi\varepsilon_0\ell}\frac{dx'}{\left(x+x'\right)^2} = \frac{Q}{4\pi\varepsilon_0\ell}\int_0^{\ell}\frac{dx'}{\left(x+x'\right)^2} = \frac{Q}{4\pi\varepsilon_0\ell}\left(-\frac{1}{x+x'}\right)_0^{\ell} = \frac{Q}{4\pi\varepsilon_0\ell}\left(\frac{1}{x}-\frac{1}{x+\ell}\right)$$

$$= \boxed{\frac{Q}{4\pi\varepsilon_0 x\left(x+\ell\right)}}$$

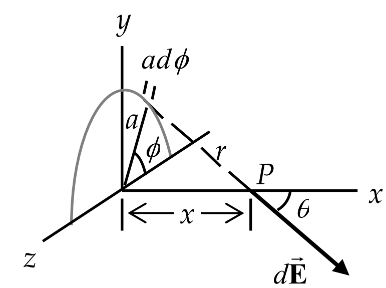
54. As suggested, we divide the plane into long narrow strips of width $dy$ and length $\ell$. The charge on the strip is the area of the strip times the charge per unit area: $dq = \sigma\ell dy$. The charge per unit length on the strip is $\lambda = \frac{dq}{\ell} = \sigma dy$. From Example 21-11, the field due to that narrow strip is $dE = \frac{\lambda}{2\pi\varepsilon_0\sqrt{y^2+z^2}} = \frac{\sigma dy}{2\pi\varepsilon_0\sqrt{y^2+z^2}}$. From Figure 21-68 in the textbook, we see that this field does not point vertically. From the symmetry of the plate, there is another long narrow strip a distance $y$ on the other side of the origin, which would create the same magnitude electric field. The horizontal components of those two fields would cancel each other, and so we only need calculate the vertical component of the field. Then we integrate along the $y$ direction to find the total field.

$$dE = \frac{\sigma dy}{2\pi\varepsilon_0\sqrt{y^2+z^2}} \quad ; \quad dE_z = dE\cos\theta = \frac{\sigma z dy}{2\pi\varepsilon_0\left(y^2+z^2\right)}$$

$$E = E_z = \int_{-\infty}^{\infty}\frac{\sigma z dy}{2\pi\varepsilon_0\left(y^2+z^2\right)} = \frac{\sigma z}{2\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{dy}{\left(y^2+z^2\right)} = \frac{\sigma z}{2\pi\varepsilon_0}\frac{1}{z}\left(\tan^{-1}\frac{y}{z}\right)_{-\infty}^{\infty}$$

$$= \frac{\sigma}{2\pi\varepsilon_0}\left[\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right] = \frac{\sigma}{2\pi\varepsilon_0}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \boxed{\frac{\sigma}{2\varepsilon_0}}$$

55. Take Figure 21-28 and add the angle $\phi$, measured from the $-z$ axis, as indicated in the diagram. Consider an infinitesimal length of the ring $ad\phi$. The charge on that infinitesimal length is $dq = \lambda(ad\phi) = \frac{Q}{\pi a}(ad\phi) = \frac{Q}{\pi}d\phi$. The charge creates an infinitesimal electric field, $d\vec{\mathbf{E}}$, with magnitude $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{Q}{\pi}d\phi}{x^2+a^2}$. From the symmetry of the figure, we see that the $z$ component of $d\vec{\mathbf{E}}$ will be cancelled by the $z$ component due to the piece of the ring that is on the opposite side of the y axis. The trigonometric relationships give $dE_x = dE\cos\theta$ and $dE_y = -dE\sin\theta\sin\phi$. The factor of $\sin\phi$ can be justified by noting that $dE_y = 0$ when $\phi = 0$, and $dE_y = -dE\sin\theta$ when $\phi = \pi/2$.




$$dE_x = dE\cos\theta = \frac{Q}{4\pi^2\varepsilon_0}\frac{d\phi}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}} = \frac{Qx}{4\pi^2\varepsilon_0}\frac{d\phi}{\left(x^2+a^2\right)^{3/2}}$$

$$E_x = \frac{Qx}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\int_0^{\pi}d\phi = \boxed{\frac{Qx}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}}$$

$$dE_y = -dE\sin\theta\sin\phi = -\frac{Q}{4\pi^2\varepsilon_0}\frac{d\phi}{x^2+a^2}\frac{a}{\sqrt{x^2+a^2}}\sin\phi = -\frac{Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\sin\phi d\phi$$

$$E_y = -\frac{Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\int_0^{\pi}\sin\phi d\phi = -\frac{Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\left[(-\cos\pi) - (-\cos 0)\right]$$

$$= \boxed{-\frac{2Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}}$$

We can write the electric field in vector notation.

$$\vec{\mathbf{E}} = \frac{Qx}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}\hat{\mathbf{i}} - \frac{2Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\hat{\mathbf{j}} = \boxed{\frac{Q}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}\left(x\hat{\mathbf{i}} - \frac{2a}{\pi}\hat{\mathbf{j}}\right)}$$

56. (*a*) Since the field is uniform, the electron will experience a constant force in the direction opposite to its velocity, so the acceleration is constant and negative. Use constant acceleration relationships with a final velocity of 0.

$$F = ma = qE = -eE \;\rightarrow\; a = -\frac{eE}{m} \;;\; v^2 = v_0^2 + 2a\Delta x = 0 \;\rightarrow$$

$$\Delta x = -\frac{v_0^2}{2a} = -\frac{v_0^2}{2\left(-\frac{eE}{m}\right)} = \frac{mv_0^2}{2eE} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(27.5\times10^6\,\text{m/s}\right)^2}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(11.4\times10^3\,\text{N/C}\right)} = \boxed{0.189\,\text{m}}$$

(*b*) Find the elapsed time from constant acceleration relationships. Upon returning to the original position, the final velocity will be the opposite of the initial velocity.

$$v = v_0 + at \;\rightarrow$$

$$t = \frac{v - v_0}{a} = \frac{-2v_0}{\left(-\frac{eE}{m}\right)} = \frac{2mv_0}{eE} = \frac{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(27.5\times10^6\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(11.4\times10^3\,\text{N/C}\right)} = \boxed{2.75\times10^{-8}\,\text{s}}$$

57. (*a*) The acceleration is produced by the electric force.

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} = -e\vec{\mathbf{E}} \;\rightarrow$$

$$\vec{\mathbf{a}} = -\frac{e}{m}\vec{\mathbf{E}} = -\frac{\left(1.60\times10^{-19}\,\text{C}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)}\left[\left(2.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}\right)\times10^4\,\text{N/C}\right] = \left(-3.513\times10^{15}\hat{\mathbf{i}} - 1.405\times10^{16}\hat{\mathbf{j}}\right)\text{m/s}^2$$

$$\approx \boxed{-3.5\times10^{15}\,\text{m/s}^2\,\hat{\mathbf{i}} - 1.4\times10^{16}\,\text{m/s}^2\,\hat{\mathbf{j}}}$$

(*b*) The direction is found from the components of the velocity.

$$\vec{\mathbf{v}} = \vec{\mathbf{v}} + \vec{\mathbf{a}}t = \left(8.0\times10^4\,\text{m/s}\right)\hat{\mathbf{j}} + \left[\left(-3.513\times10^{15}\hat{\mathbf{i}} - 1.405\times10^{16}\hat{\mathbf{j}}\right)\text{m/s}^2\right]\left(1.0\times10^{-9}\,\text{s}\right)$$

$$= \left(-3.513\times10^6\hat{\mathbf{i}} - 1.397\times10^7\hat{\mathbf{j}}\right)\text{m/s}$$

$$\tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-1.397\times10^7\,\text{m/s}}{-3.513\times10^6\,\text{m/s}}\right) = 256^\circ \text{ or } -104^\circ$$

This is the direction relative to the $x$ axis. The direction of motion relative to the initial direction is measured from the $y$ axis, and so is $\boxed{\theta = 166^\circ \text{ counter-clockwise}}$ from the initial direction.

58. (*a*) The electron will experience a force in the opposite direction to the electric field. Since the electron is to be brought to rest, the electric field must be in the same direction as the initial velocity of the electron, and so is to the $\boxed{\text{right}}$.

(*b*) Since the field is uniform, the electron will experience a constant force, and therefore have a constant acceleration. Use constant acceleration relationships to find the field strength.

$$F = qE = ma \;\rightarrow\; a = \frac{qE}{m} \qquad v^2 = v_0^2 + 2a\Delta x = v_0^2 + 2\frac{qE}{m}\Delta x \;\rightarrow$$

$$E = \frac{m\left(v^2 - v_0^2\right)}{2q\Delta x} = \frac{-mv_0^2}{2q\Delta x} = -\frac{\left(9.109\times10^{-31}\,\text{kg}\right)\left(7.5\times10^5\,\text{m/s}\right)^2}{2\left(-1.602\times10^{-19}\,\text{C}\right)\left(0.040\,\text{m}\right)} = \boxed{40\,\text{N/C}}\ \ (2\text{ sig. fig.})$$

59. The angle is determined by the velocity. The *x* component of the velocity is constant. The time to pass through the plates can be found from the *x* motion. Then the *y* velocity can be found using constant acceleration relationships.

$$x = v_0 t \;\rightarrow\; t = \frac{x}{v_0}\ ;\ v_y = v_{y0} + a_y t = -\frac{eE}{m}\frac{x}{v_0}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{-\frac{eE}{m}\frac{x}{v_0}}{v_0} = -\frac{eEx}{mv_0^2} = -\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(5.0\times10^3\,\text{N/C}\right)\left(0.049\,\text{m}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.00\times10^7\,\text{m/s}\right)^2} = -.4303 \;\rightarrow$$

$$\theta = \tan^{-1}\left(-0.4303\right) = \boxed{-23^\circ}$$

60. Since the field is constant, the force on the electron is constant, and so the acceleration is constant. Thus constant acceleration relationships can be used. The initial conditions are $x_0 = 0$, $y_0 = 0$, $v_{x0} = 1.90\,\text{m/s}$, and $v_{y0} = 0$.

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} \;\rightarrow\; \vec{\mathbf{a}} = \frac{q}{m}\vec{\mathbf{E}} = -\frac{e}{m}\vec{\mathbf{E}}\ ;\ a_x = -\frac{e}{m}E_x\ ,\ a_y = -\frac{e}{m}E_y$$

$$x = x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2 = v_{x0}t - \frac{eE_x}{2m}t^2$$

$$= \left(1.90\,\text{m/s}\right)\left(2.0\,\text{s}\right) - \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(2.00\times10^{-11}\,\text{N/C}\right)}{2\left(9.11\times10^{-31}\,\text{kg}\right)}\left(2.0\,\text{s}\right)^2 = \boxed{-3.2\,\text{m}}$$

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = -\frac{eE_y}{2m}t^2 = -\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(-1.20\times10^{-11}\,\text{N/C}\right)}{2\left(9.11\times10^{-31}\,\text{kg}\right)}\left(2.0\,\text{s}\right)^2 = \boxed{4.2\,\text{m}}$$

61. (*a*) The field along the axis of the ring is given in Example 21-9, with the opposite sign because this ring is negatively charged. The force on the charge is the field times the charge *q*. Note that if *x* is positive, the force is to the left, and if *x* is negative, the force is to the right. Assume that $x \ll R$.

$$F = qE = \frac{q}{4\pi\varepsilon_0}\frac{(-Q)x}{\left(x^2 + R^2\right)^{3/2}} = \frac{-qQx}{4\pi\varepsilon_0}\frac{1}{\left(x^2 + R^2\right)^{3/2}} \approx \frac{-qQx}{4\pi\varepsilon_0 R^3}$$

This has the form of a simple harmonic oscillator, where the "spring constant" is $k_{\text{elastic}} = \dfrac{Qq}{4\pi\varepsilon_0 R^3}$.

(*b*) The spring constant can be used to find the period. See Eq. 14-7b.

$$T = 2\pi\sqrt{\frac{m}{k_{\text{elastic}}}} = 2\pi\sqrt{\frac{m}{\frac{Qq}{4\pi\varepsilon_0 R^3}}} = 2\pi\sqrt{\frac{m4\pi\varepsilon_0 R^3}{Qq}} = 4\pi\sqrt{\frac{m\pi\varepsilon_0 R^3}{Qq}}$$

62. (*a*) The dipole moment is given by the product of the positive charge and the separation distance.

$$p = Q\ell = \left(1.60\times10^{-19}\,\text{C}\right)\left(0.68\times10^{-9}\,\text{m}\right) = 1.088\times10^{-28}\,\text{C}\bullet\text{m} \approx \boxed{1.1\times10^{-28}\,\text{C}\bullet\text{m}}$$

(*b*) The torque on the dipole is given by Eq. 21-9a.

$$\tau = pE\sin\theta = \left(1.088\times10^{-28}\,\text{C}\bullet\text{m}\right)\left(2.2\times10^{4}\ \text{N/C}\right)\left(\sin 90^\circ\right) = \boxed{2.4\times10^{-24}\,\text{C}\bullet\text{m}}$$

(*c*) $\tau = pE\sin\theta = \left(1.088\times10^{-28}\,\text{C}\bullet\text{m}\right)\left(2.2\times10^{4}\ \text{N/C}\right)\left(\sin 45^\circ\right) = \boxed{1.7\times10^{-24}\,\text{N}\bullet\text{m}}$

(*d*) The work done by an external force is the change in potential energy. Use Eq. 21-10.

$$W = \Delta U = \left(-pE\cos\theta_{\text{final}}\right) - \left(-pE\cos\theta_{\text{initial}}\right) = pE\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right)$$

$$= \left(1.088\times10^{-28}\,\text{C}\bullet\text{m}\right)\left(2.2\times10^{4}\ \text{N/C}\right)\left[1-(-1)\right] = \boxed{4.8\times10^{-24}\,\text{J}}$$

63. (*a*) The dipole moment is the effective charge of each atom times the separation distance.

$$p = Q\ell \quad\rightarrow\quad Q = \frac{p}{\ell} = \frac{3.4\times10^{-30}\,\text{C}\bullet\text{m}}{1.0\times10^{-10}\,\text{m}} = \boxed{3.4\times10^{-20}\,\text{C}}$$

(*b*) $\dfrac{Q}{e} = \dfrac{3.4\times10^{-20}\,\text{C}}{1.60\times10^{-19}\,\text{C}} = 0.21$ $\boxed{\text{No}}$, the net charge on each atom is not an integer multiple of $e$. This is an indication that the H and Cl atoms are not ionized – they haven't fully gained or lost an electron. But rather, the electrons spend more time near the Cl atom than the H atom, giving the molecule a net dipole moment. The electrons are not distributed symmetrically about the two nuclei.

(*c*) The torque is given by Eq. 21-9a.

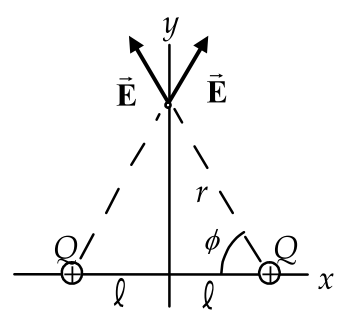
$$\tau = pE\sin\theta \quad\rightarrow\quad \tau_{\text{max}} = pE = \left(3.4\times10^{-30}\,\text{C}\bullet\text{m}\right)\left(2.5\times10^{4}\ \text{N/C}\right) = \boxed{8.5\times10^{-26}\,\text{N}\bullet\text{m}}$$

(*d*) The energy needed from an external force is the change in potential energy. Use Eq. 21-10.

$$W = \Delta U = \left(-pE\cos\theta_{\text{final}}\right) - \left(-pE\cos\theta_{\text{initial}}\right) = pE\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right)$$

$$= \left(3.4\times10^{-30}\,\text{C}\bullet\text{m}\right)\left(2.5\times10^{4}\ \text{N/C}\right)\left[1-\cos 45^\circ\right] = \boxed{2.5\times10^{-26}\,\text{J}}$$

64. (*a*) From the symmetry in the diagram, we see that the resultant field will be in the $y$ direction. The vertical components of the two fields add together, while the horizontal components cancel.




$$E_{\text{net}} = 2E\sin\phi = 2\frac{Q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)}\frac{r}{\left(r^2+\ell^2\right)^{1/2}}$$

$$= \frac{2Qr}{4\pi\varepsilon_0\left(r^2+\ell^2\right)^{3/2}} \approx \frac{2Qr}{4\pi\varepsilon_0\left(r^3\right)} = \boxed{\frac{2Q}{4\pi\varepsilon_0 r^2}}$$

(*b*) Both charges are the same sign. A long distance away from the charges, they will look like a single charge of magnitude $2Q$, and so $E = k\dfrac{q}{r^2} = \dfrac{2Q}{4\pi\varepsilon_0 r^2}$.

65. (*a*) There will be a torque on the dipole, in a direction to decrease $\theta$. That torque will give the dipole an angular acceleration, in the opposite direction of $\theta$.

$$\tau = -pE\sin\theta = I\alpha \quad\rightarrow\quad \alpha = \frac{d^2\theta}{dt^2} = -\frac{pE}{I}\sin\theta$$

If $\boxed{\theta \text{ is small}}$, so that $\sin\theta \approx \theta$, then the equation is in the same form as Eq. 14-3, the equation of motion for the simple harmonic oscillator.

$$\frac{d^2\theta}{dt^2} = -\frac{pE}{I}\sin\theta \approx -\frac{pE}{I}\theta \quad \to \quad \frac{d^2\theta}{dt^2} + \frac{pE}{I}\theta = 0$$

(*b*) The frequency can be found from the coefficient of $\theta$ in the equation of motion.

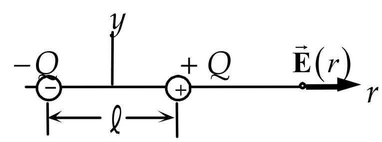
$$\omega^2 = \frac{pE}{I} \quad \to \quad f = \boxed{\frac{1}{2\pi}\sqrt{\frac{pE}{I}}}$$

66. If the dipole is of very small extent, then the potential energy is a function of position, and so Eq. 21-10 gives $U(x) = -\vec{\mathbf{p}}\bullet\vec{\mathbf{E}}(x)$. Since the potential energy is known, we can use Eq. 8-7.

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left[-\vec{\mathbf{p}}\bullet\vec{\mathbf{E}}(x)\right] = \vec{\mathbf{p}}\bullet\frac{d\vec{\mathbf{E}}}{dx}$$

Since the field does not depend on the *y* or *z* coordinates, all other components of the force will be 0. Thus $\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} = \boxed{\left(\vec{\mathbf{p}}\bullet\frac{d\vec{\mathbf{E}}}{dx}\right)\hat{\mathbf{i}}}$.

67. (*a*) Along the *x* axis the fields from the two charges are parallel so the magnitude is found as follows.




$$E_{\text{net}} = E_{+Q} + E_{-Q} = \frac{Q}{4\pi\varepsilon_0\left(r-\frac{1}{2}\ell\right)^2} + \frac{(-Q)}{4\pi\varepsilon_0\left(r+\frac{1}{2}\ell\right)^2}$$

$$= \frac{Q\left[\left(r+\frac{1}{2}\ell\right)^2 - \left(r-\frac{1}{2}\ell\right)^2\right]}{4\pi\varepsilon_0\left(r+\frac{1}{2}\ell\right)^2\left(r-\frac{1}{2}\ell\right)^2}$$

$$= \frac{Q(2r\ell)}{4\pi\varepsilon_0\left(r+\frac{1}{2}\ell\right)^2\left(r-\frac{1}{2}\ell\right)^2} \approx \frac{Q(2r\ell)}{4\pi\varepsilon_0 r^4} = \frac{2Q\ell}{4\pi\varepsilon_0 r^3} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}}$$

The same result is obtained if the point is to the left of $-Q$.

(*b*) The electric field points in the $\boxed{\text{same direction as the dipole}}$ moment vector.

68. Set the magnitude of the electric force equal to the magnitude of the force of gravity and solve for the distance.

$$F_{\text{E}} = F_{\text{G}} \quad \to \quad k\frac{e^2}{r^2} = mg \quad \to$$

$$r = e\sqrt{\frac{k}{mg}} = \left(1.602\times10^{-19}\,\text{C}\right)\sqrt{\frac{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}} = \boxed{5.08\,\text{m}}$$

69. Water has an atomic mass of 18, so 1 mole of water molecules has a mass of 18 grams. Each water molecule contains 10 protons.

$$65\,\text{kg}\left(\frac{6.02\times10^{23}\,\text{H}_2\text{O molecules}}{0.018\,\text{kg}}\right)\left(\frac{10\,\text{protons}}{1\text{ molecule}}\right)\left(\frac{1.60\times10^{-19}\,\text{C}}{\text{proton}}\right) = \boxed{3.5\times10^9\,\text{C}}$$

70. Calculate the total charge on all electrons in 3.0 g of copper, and compare $38\mu\text{C}$ to that value.

$$\text{Total electron charge} = 3.0\,\text{g}\left(\frac{1\,\text{mole}}{63.5\,\text{g}}\right)\left(\frac{6.02\times10^{23}\,\text{atoms}}{\text{mole}}\right)\left(\frac{29\,\text{e}}{\text{atoms}}\right)\left(\frac{1.602\times10^{-19}\,\text{C}}{1\,\text{e}}\right) = 1.32\times10^{5}\,\text{C}$$

$$\text{Fraction lost} = \frac{38\times10^{-6}\,\text{C}}{1.32\times10^{5}\,\text{C}} = \boxed{2.9\times10^{-10}}$$

71. Use Eq. 21-4a to calculate the magnitude of the electric charge on the Earth.

$$E = k\frac{Q}{r^2} \;\rightarrow\; Q = \frac{Er^2}{k} = \frac{(150\ \text{N/C})(6.38\times10^{6}\,\text{m})^2}{8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{6.8\times10^{5}\,\text{C}}$$

Since the electric field is pointing towards the Earth's center, the charge must be $\boxed{\text{negative}}$.

72. (*a*) From problem 71, we know that the electric field is pointed towards the Earth's center. Thus an electron in such a field would experience an upwards force of magnitude $F_E = eE$. The force of gravity on the electron will be negligible compared to the electric force.

$$F_E = eE = ma \;\rightarrow$$

$$a = \frac{eE}{m} = \frac{(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})}{(9.11\times10^{-31}\,\text{kg})} = 2.638\times10^{13}\ \text{m/s}^2 \approx \boxed{2.6\times10^{13}\ \text{m/s}^2\text{, up}}$$

(*b*) A proton in the field would experience a downwards force of magnitude $F_E = eE$. The force of gravity on the proton will be negligible compared to the electric force.

$$F_E = eE = ma \;\rightarrow$$

$$a = \frac{eE}{m} = \frac{(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})}{(1.67\times10^{-27}\,\text{kg})} = 1.439\times10^{10}\ \text{m/s}^2 \approx \boxed{1.4\times10^{10}\ \text{m/s}^2\text{, down}}$$

(*c*) Electron: $\frac{a}{g} = \frac{2.638\times10^{13}\ \text{m/s}^2}{9.80\,\text{m/s}^2} \approx \boxed{2.7\times10^{12}}$; Proton: $\frac{a}{g} = \frac{1.439\times10^{10}\ \text{m/s}^2}{9.80\,\text{m/s}^2} \approx \boxed{1.5\times10^{9}}$

73. For the droplet to remain stationary, the magnitude of the electric force on the droplet must be the same as the weight of the droplet. The mass of the droplet is found from its volume times the density of water. Let *n* be the number of excess electrons on the water droplet.

$$F_E = |q|E = mg \;\rightarrow\; neE = \tfrac{4}{3}\pi r^3 \rho g \;\rightarrow$$

$$n = \frac{4\pi r^3 \rho g}{3eE} = \frac{4\pi(1.8\times10^{-5}\,\text{m})^3(1.00\times10^{3}\ \text{kg/m}^3)(9.80\,\text{m/s}^2)}{3(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})} = 9.96\times10^{6} \approx \boxed{1.0\times10^{7}\,\text{electrons}}$$

74. There are four forces to calculate. Call the rightward direction the positive direction. The value of *k* is $8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$ and the value of *e* is $1.602\times10^{-19}\,\text{C}$.

$$F_{net} = F_{CH} + F_{CN} + F_{OH} + F_{ON} = \frac{k(0.40e)(0.20e)}{(1\times10^{-9}\,\text{m})^2}\left[-\frac{1}{(0.30)^2} + \frac{1}{(0.40)^2} + \frac{1}{(0.18)^2} - \frac{1}{(0.28)^2}\right]$$

$$= 2.445\times10^{-10}\,\text{N} \approx \boxed{2.4\times10^{-10}\,\text{N}}$$

75. Set the Coulomb electrical force equal to the Newtonian gravitational force on one of the bodies (the Moon).

$$F_{\text{E}} = F_{\text{G}} \quad \rightarrow \quad k\frac{Q^2}{r_{\text{orbit}}^2} = G\frac{M_{\text{Moon}}M_{\text{Earth}}}{r_{\text{orbit}}^2} \quad \rightarrow$$

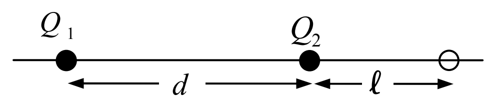
$$Q = \sqrt{\frac{GM_{\text{Moon}}M_{\text{Earth}}}{k}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}} = \boxed{5.71\times10^{13}\,\text{C}}$$

76. The electric force must be a radial force in order for the electron to move in a circular orbit.

$$F_{\text{E}} = F_{\text{radial}} \quad \rightarrow \quad k\frac{Q^2}{r_{\text{orbit}}^2} = \frac{mv^2}{r_{\text{orbit}}} \quad \rightarrow$$

$$r_{\text{orbit}} = k\frac{Q^2}{mv^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.602\times10^{-19}\,\text{C}\right)^2}{\left(9.109\times10^{-31}\,\text{kg}\right)\left(2.2\times10^6\ \text{m/s}\right)^2} = \boxed{5.2\times10^{-11}\,\text{m}}$$

77. Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge $(Q_2)$. Also, in between the two charges, the fields due to the two charges are parallel to each other and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, this means that $\ell$ must be positive.




$$E = -k\frac{|Q_2|}{\ell^2} + k\frac{Q_1}{(\ell + d)^2} = 0 \quad \rightarrow \quad |Q_2|(\ell + d)^2 = Q_1\ell^2 \quad \rightarrow$$

$$\ell = \frac{\sqrt{|Q_2|}}{\sqrt{Q_1} - \sqrt{|Q_2|}}d = \frac{\sqrt{5.0\times10^{-6}\,\text{C}}}{\sqrt{2.5\times10^{-5}\,\text{C}} - \sqrt{5.0\times10^{-6}\,\text{C}}}(2.0\,\text{m}) = \boxed{\begin{array}{l}1.6\,\text{m from } Q_2,\\ 3.6\,\text{m from } Q_1\end{array}}$$

78. We consider that the sock is only acted on by two forces – the force of gravity, acting downward, and the electrostatic force, acting upwards. If charge $Q$ is on the sweater, then it will create an electric field of $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q/A}{2\varepsilon_0}$, where $A$ is the surface area of one side of the sweater. The same magnitude of charge will be on the sock, and so the attractive force between the sweater and sock is $F_E = QE = \frac{Q^2}{2\varepsilon_0 A}$. This must be equal to the weight of the sweater. We estimate the sweater area as 0.10 m$^2$, which is roughly a square foot.

$$F_E = QE = \frac{Q^2}{2\varepsilon_0 A} = mg \quad \rightarrow$$

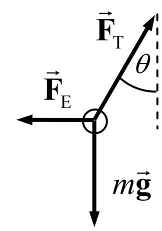
$$Q = \sqrt{2\varepsilon_0 Amg} = \sqrt{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)\left(0.10\,\text{m}^2\right)\left(0.040\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)} = \boxed{8\times10^{-7}\,\text{C}}$$

$\boxed{79.}$ The sphere will oscillate sinusoidally about the equilibrium point, with an amplitude of 5.0 cm. The angular frequency of the sphere is given by $\omega = \sqrt{k/m} = \sqrt{126\,\text{N/m}/0.650\,\text{kg}} = 13.92\,\text{rad/s}$. The distance of the sphere from the table is given by $r = \left[0.150 - 0.0500\cos(13.92t)\right]\text{m}$. Use this distance and the charge to give the electric field value at the tabletop. That electric field will point upwards at all times, towards the negative sphere.

$$E = k\frac{|Q|}{r^2} = \frac{\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(3.00\times10^{-6}\,\mathrm{C}\right)}{\left[0.150-0.0500\cos(13.92t)\right]^2\,\mathrm{m^2}} = \frac{2.70\times10^4}{\left[0.150-0.0500\cos(13.92t)\right]^2}\,\mathrm{N/C}$$

$$= \boxed{\frac{1.08\times10^7}{\left[3.00-\cos(13.9t)\right]^2}\,\mathrm{N/C,\ upwards}}$$

80. The wires form two sides of an isosceles triangle, and so the two charges are separated by a distance $\ell = 2(78\,\mathrm{cm})\sin 26° = 68.4\,\mathrm{cm}$ and are directly horizontal from each other. Thus the electric force on each charge is horizontal. From the free-body diagram for one of the spheres, write the net force in both the horizontal and vertical directions and solve for the electric force. Then write the electric force by Coulomb's law, and equate the two expressions for the electric force to find the charge.




$$\sum F_y = F_T\cos\theta - mg = 0 \rightarrow F_T = \frac{mg}{\cos\theta}$$

$$\sum F_x = F_T\sin\theta - F_E = 0 \rightarrow F_E = F_T\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$$

$$F_E = k\frac{(Q/2)^2}{\ell^2} = mg\tan\theta \rightarrow Q = 2\ell\sqrt{\frac{mg\tan\theta}{k}}$$

$$= 2(0.684\,\mathrm{m})\sqrt{\frac{\left(24\times10^{-3}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\tan 26°}{\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)}} = 4.887\times10^{-6}\,\mathrm{C} \approx \boxed{4.9\times10^{-6}\,\mathrm{C}}$$

81. The electric field at the surface of the pea is given by Eq. 21-4a. Solve that equation for the charge.

$$E = k\frac{Q}{r^2} \rightarrow Q = \frac{Er^2}{k} = \frac{\left(3\times10^6\,\mathrm{N/C}\right)\left(3.75\times10^{-3}\,\mathrm{m}\right)^2}{8.988\times10^9\,\mathrm{N\cdot m^2/C^2}} = \boxed{5\times10^{-9}\,\mathrm{C}}$$

This corresponds to about 3 billion electrons.

82. There will be a rightward force on $Q_1$ due to $Q_2$, given by Coulomb's law. There will be a leftward force on $Q_1$ due to the electric field created by the parallel plates. Let right be the positive direction.

$$\sum F = k\frac{|Q_1Q_2|}{x^2} - |Q_1|E$$

$$= \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(6.7\times10^{-6}\,\mathrm{C}\right)\left(1.8\times10^{-6}\,\mathrm{C}\right)}{(0.34\,\mathrm{m})^2} - \left(6.7\times10^{-6}\,\mathrm{C}\right)\left(7.3\times10^4\,\mathrm{N/C}\right)$$

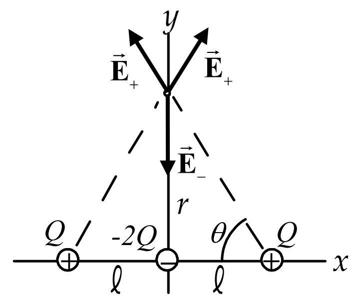
$$= \boxed{0.45\,\mathrm{N,\ right}}$$

83. The weight of the sphere is the density times the volume. The electric force is given by Eq. 21-1, with both spheres having the same charge, and the separation distance equal to their diameter.

$$mg = k\frac{Q^2}{(d)^2} \rightarrow \rho\tfrac{4}{3}\pi r^3 g = \frac{kQ^2}{(2r)^2} \rightarrow$$

$$Q = \sqrt{\frac{16\rho\pi g r^5}{3k}} = \sqrt{\frac{16\left(35\,\text{kg/m}^3\right)\pi\left(9.80\,\text{m/s}^2\right)\left(1.0\times10^{-2}\,\text{m}\right)^5}{3\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}} = \boxed{8.0\times10^{-9}\,\text{C}}$$

84. From the symmetry, we see that the resultant field will be in the $y$ direction. So we take the vertical component of each field.




$$E_{\text{net}} = 2E_+\sin\theta - E_- = 2\frac{Q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)}\frac{r}{\left(r^2+\ell^2\right)^{1/2}} - \frac{2Q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{2Q}{4\pi\varepsilon_0}\left[\frac{r}{\left(r^2+\ell^2\right)^{3/2}} - \frac{1}{r^2}\right]$$

$$= \frac{2Q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)^{3/2}r^2}\left[r^3 - \left(r^2+\ell^2\right)^{3/2}\right]$$

$$= \frac{2Qr^3\left[1-\left(1+\frac{\ell^2}{r^2}\right)^{3/2}\right]}{4\pi\varepsilon_0 r^5\left(1+\frac{\ell^2}{r^2}\right)^{3/2}}$$

Use the binomial expansion, assuming $r \gg \ell$.

$$E_{\text{net}} = \frac{2Qr^3\left[1-\left(1+\frac{\ell^2}{r^2}\right)^{3/2}\right]}{4\pi\varepsilon_0 r^5\left(1+\frac{\ell^2}{r^2}\right)^{3/2}} \approx \frac{2Qr^3\left[1-\left(1+\frac{3}{2}\frac{\ell^2}{r^2}\right)\right]}{4\pi\varepsilon_0 r^5\left(1+\frac{3}{2}\frac{\ell^2}{r^2}\right)} = \frac{2Qr^3\left(-\frac{3}{2}\frac{\ell^2}{r^2}\right)}{4\pi\varepsilon_0 r^5(1)} = \boxed{-\frac{3Q\ell^2}{4\pi\varepsilon_0 r^4}}$$

Notice that the field points toward the negative charges.

$\boxed{85.}$ This is a constant acceleration situation, similar to projectile motion in a uniform gravitational field. Let the width of the plates be $\ell$, the vertical gap between the plates be $h$, and the initial velocity be $v_0$. Notice that the vertical motion has a maximum displacement of $h/2$. Let upwards be the positive vertical direction. We calculate the vertical acceleration produced by the electric field and the time $t$ for the electron to cross the region of the field. We then use constant acceleration equations to solve for the angle.

$$F_y = ma_y = qE = -eE \rightarrow a_y = -\frac{eE}{m} \;;\; \ell = v_0\cos\theta_0(t) \rightarrow t = \frac{\ell}{v_0\cos\theta_0}$$

$$v_{y_{\text{top}}} = v_{0y} + a_y t_{\text{top}} \rightarrow 0 = v_0\sin\theta_0 - \frac{eE}{m}\left(\tfrac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right) \rightarrow v_0^2 = \frac{eE}{2m}\left(\frac{\ell}{\sin\theta_0\cos\theta_0}\right)$$

$$y_{\text{top}} = y_0 + v_{0y}t_{\text{top}} + \tfrac{1}{2}a_y t^2 \;\rightarrow\; \tfrac{1}{2}h = v_0 \sin\theta_0\left(\tfrac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right) - \tfrac{1}{2}\frac{eE}{m}\left(\tfrac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right)^2 \;\rightarrow$$

$$h = \ell\tan\theta_0 - \frac{eE\ell^2}{4m\cos^2\theta_0}\frac{1}{v_0^2}^{2} = \ell\tan\theta_0 - \frac{eE\ell^2}{4m\cos^2\theta_0}\frac{1}{\frac{eE}{2m}\left(\frac{\ell}{\sin\theta_0\cos\theta_0}\right)} = \ell\tan\theta_0 - \tfrac{1}{2}\ell\tan\theta_0$$

$$h = \tfrac{1}{2}\ell\tan\theta_0 \;\rightarrow\; \theta_0 = \tan^{-1}\frac{2h}{\ell} = \tan^{-1}\frac{2(1.0\,\text{cm})}{6.0\,\text{cm}} = \boxed{18^\circ}$$

86. (*a*) The electric field from the long wire is derived in Example 21-11.

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\text{, radially away from the wire}$$

(*b*) The force on the electron will point radially in, producing a centripetal acceleration.

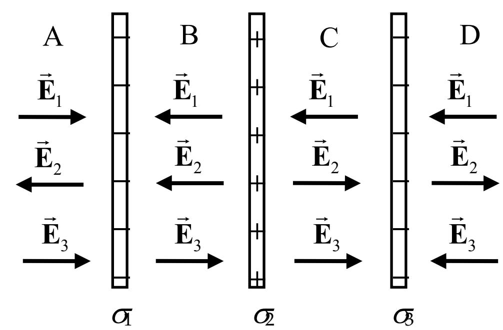
$$|F| = |qE| = \frac{e}{2\pi\varepsilon_0}\frac{\lambda}{r} = \frac{mv^2}{r} \;\rightarrow$$

$$v = \sqrt{2\frac{1}{4\pi\varepsilon_0}\frac{e\lambda}{m}} = \sqrt{2\left(8.99\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.14\times10^{-6}\,\text{C/m}\right)}{9.11\times10^{-31}\,\text{kg}}}$$

$$= \boxed{2.1\times10^7\,\text{m/s}}$$

Note that this speed is independent of *r*.

87. We treat each of the plates as if it were infinite, and then use Eq. 21-7. The fields due to the first and third plates point towards their respective plates, and the fields due to the second plate point away from it. See the diagram. The directions of the fields are given by the arrows, so we calculate the magnitude of the fields from Eq. 21-7. Let the positive direction be to the right.




$$E_A = E_1 - E_2 + E_3 = \frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

$$= \frac{(0.50 - 0.25 + 0.35)\times10^{-6}\,\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)} = \boxed{3.4\times10^4\,\text{N/C, to the right}}$$

$$E_B = -E_1 - E_2 + E_3 = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

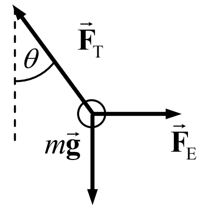
$$= \frac{(-0.50 - 0.25 + 0.35)\times10^{-6}\,\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)} = -2.3\times10^4\,\text{N/C} = \boxed{2.3\times10^4\,\text{N/C to the left}}$$

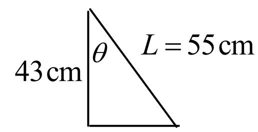
$$E_C = -E_1 + E_2 + E_3 = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

$$= \frac{(-0.50 + 0.25 + 0.35)\times10^{-6}\,\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)} = \boxed{5.6\times10^3\,\text{N/C to the right}}$$

$$E_D = -E_1 + E_2 - E_3 = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$
$$= \frac{(-0.50+0.25-0.35)\times10^{-6}\ \text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)} = -3.4\times10^4\ \text{N/C} = \boxed{3.4\times10^3\ \text{N/C to the left}}$$

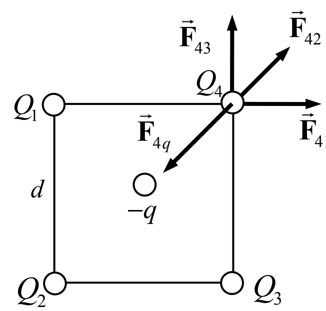
88. Since the electric field exerts a force on the charge in the same direction as the electric field, the charge is positive. Use the free-body diagram to write the equilibrium equations for both the horizontal and vertical directions, and use those equations to find the magnitude of the charge.




$$\theta = \cos^{-1}\frac{43\ \text{cm}}{55\ \text{cm}} = 38.6^\circ$$
$$\sum F_x = F_E - F_T\sin\theta = 0 \rightarrow F_E = F_T\sin\theta = QE$$
$$\sum F_y = F_T\cos\theta - mg = 0 \rightarrow F_T = \frac{mg}{\cos\theta} \rightarrow QE = mg\tan\theta$$
$$Q = \frac{mg\tan\theta}{E} = \frac{(1.0\times10^{-3}\text{kg})(9.80\ \text{m/s}^2)\tan38.6^\circ}{(1.5\times10^4\ \text{N/C})} = \boxed{5.2\times10^{-7}\text{C}}$$

89. A negative charge must be placed at the center of the square. Let $Q = 8.0\ \mu\text{C}$ be the charge at each corner, let $-q$ be the magnitude of negative charge in the center, and let $d = 9.2\ \text{cm}$ be the side length of the square. By the symmetry of the problem, if we make the net force on one of the corner charges be zero, the net force on each other corner charge will also be zero.




$$F_{41} = k\frac{Q^2}{d^2} \rightarrow F_{41x} = k\frac{Q^2}{d^2},\ F_{41y} = 0$$
$$F_{42} = k\frac{Q^2}{2d^2} \rightarrow F_{42x} = k\frac{Q^2}{2d^2}\cos45^\circ = k\frac{\sqrt{2}Q^2}{4d^2},\ F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$
$$F_{43} = k\frac{Q^2}{d^2} \rightarrow F_{43x} = 0,\ F_{43y} = k\frac{Q^2}{d^2}$$
$$F_{4q} = k\frac{qQ}{d^2/2} \rightarrow F_{4qx} = -k\frac{2qQ}{d^2}\cos45^\circ = -k\frac{\sqrt{2}qQ}{d^2} = F_{4qy}$$

The net force in each direction should be zero.

$$\sum F_x = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 - k\frac{\sqrt{2}qQ}{d^2} = 0 \rightarrow$$
$$q = Q\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = (8.0\times10^{-6}\text{C})\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = 7.66\times10^{-6}\text{C}$$

So the charge to be placed is $-q = \boxed{-7.7\times10^{-6}\text{C}}$.

This is an $\boxed{\text{unstable equilibrium}}$. If the center charge were slightly displaced, say towards the right, then it would be closer to the right charges than the left, and would be attracted more to the right. Likewise the positive charges on the right side of the square would be closer to it and would be attracted more to it, moving from their corner positions. The system would not have a tendency to return to the symmetric shape, but rather would have a tendency to move away from it if disturbed.

90. (*a*) The force of sphere B on sphere A is given by Coulomb's law.

$$F_{AB} = \boxed{\frac{kQ^2}{R^2}, \text{ away from B}}$$

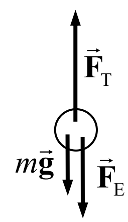
(*b*) The result of touching sphere B to uncharged sphere C is that the charge on B is shared between the two spheres, and so the charge on B is reduced to $Q/2$. Again use Coulomb's law.

$$F_{AB} = k\frac{QQ/2}{R^2} = \boxed{\frac{kQ^2}{2R^2}, \text{ away from B}}$$

(*c*) The result of touching sphere A to sphere C is that the charge on the two spheres is shared, and so the charge on A is reduced to $3Q/4$. Again use Coulomb's law.

$$F_{AB} = k\frac{(3Q/4)(Q/2)}{R^2} = \boxed{\frac{3kQ^2}{8R^2}, \text{ away from B}}$$

$\boxed{91.}$ (*a*) The weight of the mass is only about 2 N. Since the tension in the string is more than that, there must be a downward electric force on the positive charge, which means that the electric field must be pointed $\boxed{\text{down}}$. Use the free-body diagram to write an expression for the magnitude of the electric field.




$$\sum F = F_T - mg - F_E = 0 \;\rightarrow\; F_E = QE = F_T - mg \;\rightarrow$$

$$E = \frac{F_T - mg}{Q} = \frac{5.18\,\text{N} - (0.210\,\text{kg})(9.80\,\text{m/s}^2)}{3.40\times10^{-7}\text{C}} = \boxed{9.18\times10^6\ \text{N/C}}$$

(*b*) Use Eq. 21-7.

$$E = \frac{\sigma}{2\varepsilon_0} \;\rightarrow\; \sigma = 2E\varepsilon_0 = 2(9.18\times10^6\ \text{N/C})(8.854\times10^{-12}) = \boxed{1.63\times10^{-4}\ \text{C/m}^2}$$

92. (*a*) The force will be attractive. Each successive charge is another distance *d* farther than the previous charge. The magnitude of the charge on the electron is *e*.

$$F = k\frac{eQ}{(d)^2} + k\frac{eQ}{(2d)^2} + k\frac{eQ}{(3d)^2} + k\frac{eQ}{(4d)^2} + \cdots = k\frac{eQ}{d^2}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\right)$$

$$= k\frac{eQ}{d^2}\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{4\pi\varepsilon_0}\frac{eQ}{d^2}\frac{\pi^2}{6} = \boxed{\frac{\pi eQ}{24\varepsilon_0 d^2}}$$

(*b*) Now the closest *Q* is a distance of 3*d* from the electron.

$$F = k\frac{eQ}{(3d)^2} + k\frac{eQ}{(4d)^2} + k\frac{eQ}{(5d)^2} + k\frac{eQ}{(6d)^2} + \cdots = k\frac{eQ}{d^2}\left(\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\cdots\right)$$

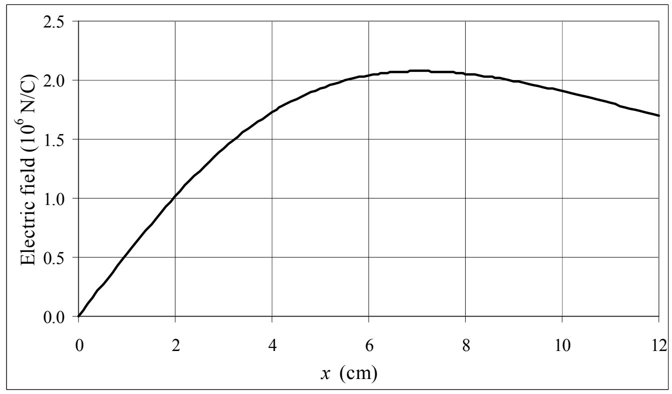
$$= k\frac{eQ}{d^2}\sum_{n=3}^{\infty}\frac{1}{n^2} = k\frac{eQ}{d^2}\left[\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right) - \frac{1}{1^2} - \frac{1}{2^2}\right] = k\frac{eQ}{d^2}\left[\frac{\pi^2}{6}-\frac{5}{4}\right] = \boxed{\frac{eQ}{4\pi\varepsilon_0 d^2}\left[\frac{\pi^2}{6}-\frac{5}{4}\right]}$$

93. (*a*) Take $\frac{dE}{dx}$, set it equal to 0, and solve for the location of the maximum.

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}}$$

$$\frac{dE}{dx}=\frac{Q}{4\pi\varepsilon_0}\left[\frac{\left(x^2+a^2\right)^{3/2}-x\tfrac{3}{2}\left(x^2+a^2\right)^{1/2}2x}{\left(x^2+a^2\right)^3}\right]=\frac{Q}{4\pi\varepsilon_0}\frac{\left(a^2-2x^2\right)}{\left(x^2+a^2\right)^{5/2}}=0\ \rightarrow\ a^2-2x^2=0\ \rightarrow$$

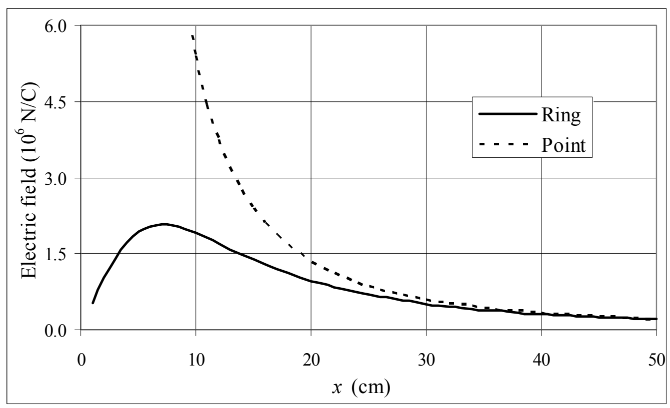
$$x=\frac{a}{\sqrt{2}}=\frac{10.0\,\text{cm}}{\sqrt{2}}=\boxed{7.07\,\text{cm}}$$




(*b*) $\boxed{\text{Yes}}$, the maximum of the graph does coincide with the analytic maximum. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.93b."

(*c*) The field due to the ring is $E_{\text{ring}}=\frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}}$.

(*d*) The field due to the point charge is $E_{\text{ring}}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}$. Both are plotted on the graph. The graph shows that the two fields converge at large distances from the origin. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.93cd."

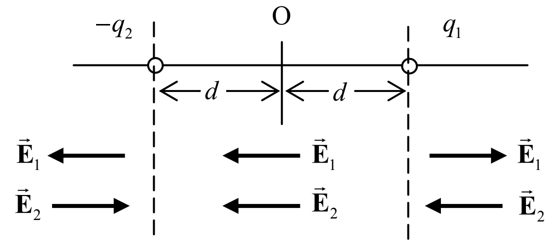



(*e*) According to the spreadsheet, $E_{\text{ring}}=0.9E_{\text{point}}$ at about $\boxed{37\text{ cm}}$. An analytic calculation gives the same result.

$$E_{\text{ring}}=0.9E_{\text{point}}\ \rightarrow\ \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}}=0.9\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}\ \rightarrow$$

$$x^3=0.9\left(x^2+a^2\right)^{3/2}=0.9x^3\left(1+\frac{a^2}{x^2}\right)^{3/2}\ \rightarrow\ x=\frac{a}{\sqrt{\left(\frac{1}{0.9}\right)^{2/3}-1}}=\frac{10.0\,\text{cm}}{\sqrt{\left(\frac{1}{0.9}\right)^{2/3}-1}}=37.07\,\text{cm}$$

94. (*a*) Let $q_1 = 8.00\mu\text{C}$, $q_2 = 2.00\mu\text{C}$, and $d = 0.0500\,\text{m}$. The field directions due to the charges are shown in the diagram. We take care with the signs of the *x* coordinate used to calculate the magnitude of the field.

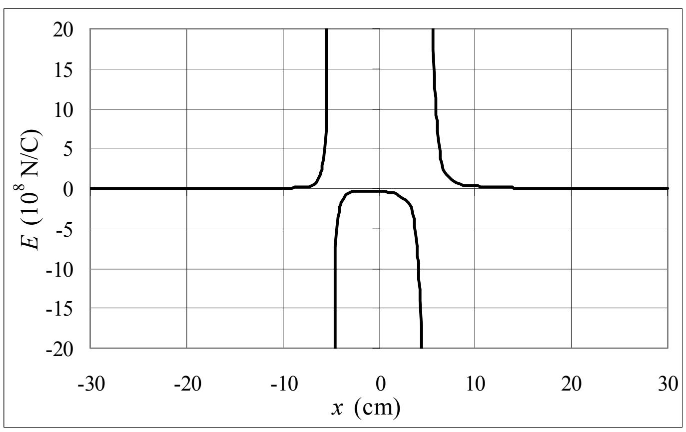



$$E_{x<-d} = E_2 - E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(|x|-d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(|x|+d\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(-x-d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(-x+d\right)^2}$$

$$E_{-d<x<0} = -E_2 - E_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(d-|x|\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(|x|+d\right)^2} = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(d+x\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(-x+d\right)^2}$$

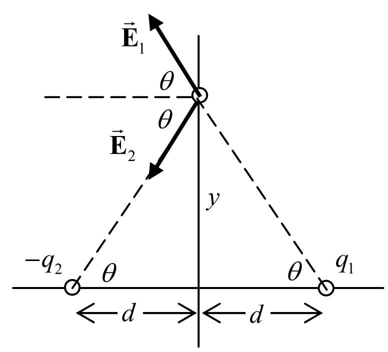
$$E_{0<x<d} = -E_2 - E_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(d+x\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(d-x\right)^2}$$

$$E_{d<x} = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(x-d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(x+d\right)^2}$$




The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.94a."

(*b*) Now for points on the y axis. See the diagram for this case.




$$E_x = -E_1\cos\theta - E_2\cos\theta$$

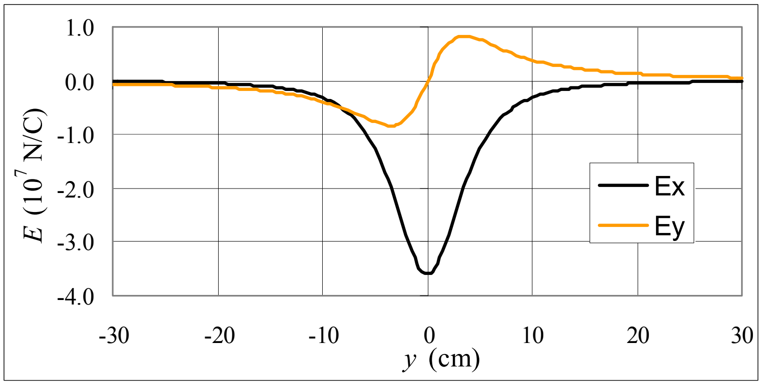
$$= -\frac{1}{4\pi\varepsilon_0}\frac{q_1\cos\theta}{\left(d^2+y^2\right)} - \frac{1}{4\pi\varepsilon_0}\frac{q_2\cos\theta}{\left(d^2+y^2\right)}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{\left(q_1+q_2\right)}{\left(d^2+y^2\right)}\cos\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\left(q_1+q_2\right)}{\left(d^2+y^2\right)}\frac{d}{\sqrt{d^2+y^2}}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{\left(q_1+q_2\right)d}{\left(d^2+y^2\right)^{3/2}}$$

$$E_y = E_1\sin\theta - E_2\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{q_1\sin\theta}{\left(d^2+y^2\right)} - \frac{1}{4\pi\varepsilon_0}\frac{q_2\sin\theta}{\left(d^2+y^2\right)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\left(q_1-q_2\right)}{\left(d^2+y^2\right)}\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{\left(q_1-q_2\right)}{\left(d^2+y^2\right)}\frac{y}{\sqrt{d^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\left(q_1-q_2\right)y}{\left(d^2+y^2\right)^{3/2}}$$




The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.94b."

# CHAPTER 22: Gauss's Law

## Responses to Questions

1. No. If the net electric flux through a surface is zero, then the net charge contained in the surface is zero. However, there may be charges both inside and outside the surface that affect the electric field at the surface. The electric field could point outward from the surface at some points and inward at others. Yes. If the electric field is zero for all points on the surface, then the net flux through the surface must be zero and no net charge is contained within the surface.

2. No. The electric field in the expression for Gauss's law refers to the *total* electric field, not just the electric field due to any enclosed charge. Notice, though, that if the electric field is due to a charge outside the Gaussian surface, then the net flux through the surface due to this charge will be zero.

3. The electric flux will be the same. The flux is equal to the net charge enclosed by the surface divided by $\varepsilon_0$. If the same charge is enclosed, then the flux is the same, regardless of the shape of the surface.

4. The net flux will be zero. An electric dipole consists of two charges that are equal in magnitude but opposite in sign, so the net charge of an electric dipole is zero. If the closed surface encloses a zero net charge, than the net flux through it will be zero.

5. Yes. If the electric field is zero for all points on the surface, then the integral of $\vec{\mathbf{E}}\bullet d\vec{\mathbf{A}}$ over the surface will be zero, the flux through the surface will be zero, and no net charge will be contained in the surface. No. If a surface encloses no net charge, then the net electric flux through the surface will be zero, but the electric field is not necessarily zero for all points on the surface. The integral of $\vec{\mathbf{E}}\bullet d\vec{\mathbf{A}}$ over the surface must be zero, but the electric field itself is not required to be zero. There may be charges outside the surface that will affect the values of the electric field at the surface.

6. The electric flux through a surface is the scalar (dot) product of the electric field vector and the area vector of the surface. Thus, in magnitude, $\Phi_{\mathrm{E}} = EA\cos\theta$ . By analogy, the gravitational flux through a surface would be the product of the gravitational field (or force per unit mass) and the area, or $\Phi_{\mathrm{g}} = gA\cos\theta$ . Any mass, such as a planet, would be a "sink" for gravitational field. Since there is not "anti-gravity" there would be no sources.

7. No. Gauss's law is most useful in cases of high symmetry, where a surface can be defined over which the electric field has a constant value and a constant relationship to the direction of the outward normal to the surface. Such a surface cannot be defined for an electric dipole.

8. When the ball is inflated and charge is distributed uniformly over its surface, the field inside is zero. When the ball is collapsed, there is no symmetry to the charge distribution, and the calculation of the electric field strength and direction inside the ball is difficult (and will most likely give a non-zero result).

9. For an infinitely long wire, the electric field is radially outward from the wire, resulting from contributions from all parts of the wire. This allows us to set up a Gaussian surface that is cylindrical, with the cylinder axis parallel to the wire. This surface will have zero flux through the top and bottom of the cylinder, since the net electric field and the outward surface normal are perpendicular at all points over the top and bottom. In the case of a short wire, the electric field is not radially outward from the wire near the ends; it curves and points directly outward along the axis of

the wire at both ends. We cannot define a useful Gaussian surface for this case, and the electric field must be computed directly.

10. In Example 22-6, there is no flux through the flat ends of the cylindrical Gaussian surface because the field is directed radially outward from the wire. If instead the wire extended only a short distance past the ends of the cylinder, there would be a component of the field through the ends of the cylinder. The result of the example would be altered because the value of the field at a given point would now depend not only on the radial distance from the wire but also on the distance from the ends.

11. The electric flux through the sphere remains the same, since the same charge is enclosed. The electric field at the surface of the sphere is changed, because different parts of the sphere are now at different distances from the charge. The electric field will not have the same magnitude for all parts of the sphere, and the direction of the electric field will not be parallel to the outward normal for all points on the surface of the sphere. The electric field will be stronger on the side closer to the charge and weaker on the side further from the charge.

12. (*a*) A charge of $(Q-q)$ will be on the outer surface of the conductor. The total charge $Q$ is placed on the conductor but since $+q$ will reside on the inner surface, the leftover, $(Q-q)$, will reside on the outer surface.
    (*b*) A charge of $+q$ will reside on the inner surface of the conductor since that amount is attracted by the charge $-q$ in the cavity. (Note that E must be zero inside the conductor.)

13. Yes. The charge $q$ will induce a charge $-q$ on the inside surface of the thin metal shell, leaving the outside surface with a charge $+q$. The charge $Q$ outside the sphere will feel the same electric force as it would if the metal shell were not present.

14. The total flux through the balloon's surface will not change because the enclosed charge does not change. The flux per unit surface area will decrease, since the surface area increases while the total flux does not change.

## Solutions to Problems

1. The electric flux of a uniform field is given by Eq. 22-1b.

(*a*) $\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}} = EA\cos\theta = (580\,\mathrm{N/C})\pi(0.13\,\mathrm{m})^2\cos 0 = \boxed{31\,\mathrm{N\cdot m^2/C}}$

(*b*) $\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}} = EA\cos\theta = (580\,\mathrm{N/C})\pi(0.13\,\mathrm{m})^2\cos 45^\circ = \boxed{22\,\mathrm{N\cdot m^2/C}}$

(*c*) $\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}} = EA\cos\theta = (580\,\mathrm{N/C})\pi(0.13\,\mathrm{m})^2\cos 90^\circ = \boxed{0}$

2. Use Eq. 22-1b for the electric flux of a uniform field. Note that the surface area vector points radially outward, and the electric field vector points radially inward. Thus the angle between the two is 180°.

$$\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}} = EA\cos\theta = (150\,\mathrm{N/C})4\pi R_E^2\cos 180^\circ = -4\pi(150\,\mathrm{N/C})\left(6.38\times10^6\,\mathrm{m}\right)^2$$
$$= \boxed{-7.7\times10^{16}\,\mathrm{N\cdot m^2/C}}$$

3. (*a*) Since the field is uniform, no lines originate or terminate inside the cube, and so the net flux is $\Phi_{net} = \boxed{0}$.

(*b*) There are two opposite faces with field lines perpendicular to the faces. The other four faces have field lines parallel to those faces. For the faces parallel to the field lines, no field lines enter or exit the faces. Thus $\Phi_{parallel} = \boxed{0}$.

Of the two faces that are perpendicular to the field lines, one will have field lines entering the cube, and so the angle between the field lines and the face area vector is 180°. The other will have field lines exiting the cube, and so the angle between the field lines and the face area vector is 0°. Thus we have $\Phi_{entering} = \vec{E}\bullet\vec{A} = E_0 A\cos 180° = \boxed{-E_0\ell^2}$ and $\Phi_{leaving} = \vec{E}\bullet\vec{A} = E_0 A\cos 0° = \boxed{E_0\ell^2}$.

4. (*a*) From the diagram in the textbook, we see that the flux outward through the hemispherical surface is the same as the flux inward through the circular surface base of the hemisphere. On that surface all of the flux is perpendicular to the surface. Or, we say that on the circular base, $\vec{\mathbf{E}} \parallel \vec{\mathbf{A}}$. Thus $\Phi_E = \vec{\mathbf{E}}\bullet\vec{\mathbf{A}} = \boxed{\pi r^2 E}$.

(b) $\vec{\mathbf{E}}$ is perpendicular to the axis, then every field line would both enter through the hemispherical surface and leave through the hemispherical surface, and so $\Phi_E = \boxed{0}$.

5. Use Gauss's law to determine the enclosed charge.

$$\Phi_E = \frac{Q_{encl}}{\varepsilon_o} \rightarrow Q_{encl} = \Phi_E\varepsilon_o = \left(1840\,\text{N}\cdot\text{m}^2/\text{C}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{1.63\times10^{-8}\,\text{C}}$$

6. The net flux through each closed surface is determined by the net charge inside. Refer to the picture in the textbook.

$$\Phi_1 = \left(+Q-3Q\right)/\varepsilon_0 = \boxed{-2Q/\varepsilon_0}\ ;\ \Phi_2 = \left(+Q+2Q-3Q\right)/\varepsilon_0 = \boxed{0}\ ;$$

$$\Phi_3 = \left(+2Q-3Q\right)/\varepsilon_0 = \boxed{-Q/\varepsilon_0}\ ;\ \Phi_4 = \boxed{0}\ ;\ \Phi_5 = \boxed{+2Q/\varepsilon_0}$$

$\boxed{7.}$ (*a*) Use Gauss's law to determine the electric flux.

$$\Phi_E = \frac{Q_{encl}}{\varepsilon_o} = \frac{-1.0\times10^{-6}\,\text{C}}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{-1.1\times10^5\,\text{N}\cdot\text{m}^2/\text{C}}$$

(*b*) Since there is no charge enclosed by surface $A_2$, $\Phi_E = \boxed{0}$.

8. The net flux is only dependent on the charge enclosed by the surface. Since both surfaces enclose the same amount of charge, the flux through both surfaces is the same. Thus the ratio is $\boxed{1:1}$.

9. The only contributions to the flux are from the faces perpendicular to the electric field. Over each of these two surfaces, the magnitude of the field is constant, so the flux is just $\vec{\mathbf{E}}\bullet\vec{\mathbf{A}}$ on each of these two surfaces.

$$\Phi_E = \left(\vec{\mathbf{E}}\bullet\vec{\mathbf{A}}\right)_{right} + \left(\vec{\mathbf{E}}\bullet\vec{\mathbf{A}}\right)_{left} = E_{right}\ell^2 - E_{left}\ell^2 = \frac{Q_{encl}}{\varepsilon_0} \rightarrow$$

$$Q_{\text{encl}} = \left(E_{\text{right}} - E_{\text{left}}\right)\ell^2\varepsilon_0 = \left(410\,\text{N/C} - 560\,\text{N/C}\right)\left(25\,\text{m}\right)^2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{-8.3\times10^{-7}\,\text{C}}$$

10. Because of the symmetry of the problem one sixth of the total flux will pass through each face.

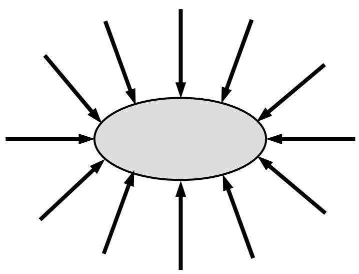
$$\Phi_{\text{face}} = \tfrac{1}{6}\Phi_{\text{total}} = \tfrac{1}{6}\frac{Q_{\text{encl}}}{\varepsilon_0} = \boxed{\frac{Q_{\text{encl}}}{6\varepsilon_0}}$$

Notice that the side length of the cube did not enter into the calculation.

11. The charge density can be found from Eq. 22-4, Gauss's law. The charge is the charge density times the length of the rod.

$$\Phi = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\lambda\ell}{\varepsilon_0} = \quad\rightarrow\quad \lambda = \frac{\Phi\varepsilon_0}{\ell} = \frac{\left(7.3\times10^5\,\text{N}\cdot\text{m}^2/\text{C}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}{0.15\,\text{m}} = \boxed{4.3\times10^{-5}\,\text{C/m}}$$

12.



13. The electric field can be calculated by Eq. 21-4a, and that can be solved for the magnitude of the charge.

$$E = k\frac{Q}{r^2} \quad\rightarrow\quad Q = \frac{Er^2}{k} = \frac{\left(6.25\times10^2\,\text{N/C}\right)\left(3.50\times10^{-2}\,\text{m}\right)^2}{8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 8.52\times10^{-11}\,\text{C}$$

This corresponds to about $5\times10^8$ electrons. Since the field points toward the ball, the charge must be negative. Thus $Q = \boxed{-8.52\times10^{-11}\,\text{C}}$.

14. The charge on the spherical conductor is uniformly distributed over the surface area of the sphere, so $\sigma = \dfrac{Q}{4\pi R^2}$. The field at the surface of the sphere is evaluated at $r = R$.

$$E\left(r = R\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2\sigma}{R^2} = \boxed{\frac{\sigma}{\varepsilon_0}}$$

15. The electric field due to a long thin wire is given in Example 22-6 as $E = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{R}$.

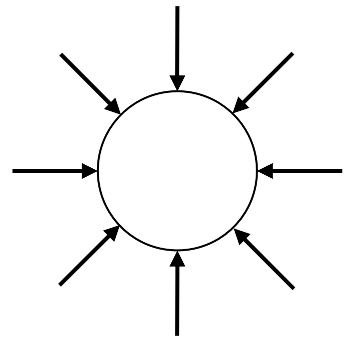
(*a*) $$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{R} = \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{2\left(-7.2\times10^{-6}\,\text{C/m}\right)}{\left(5.0\,\text{m}\right)} = \boxed{-2.6\times10^4\,\text{N/C}}$$

The negative sign indicates the electric field is pointed $\boxed{\text{towards the wire}}$.

(*b*) $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{R} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{2\left(-7.2\times10^{-6}\ \mathrm{C/m}\right)}{(1.5\,\mathrm{m})} = \boxed{-8.6\times10^4\ \mathrm{N/C}}$

The negative sign indicates the electric field is pointed towards the wire.

16. Because the globe is a conductor, the net charge of -1.50 mC will be arranged symmetrically around the sphere.



17. Due to the spherical symmetry of the problem, the electric field can be evaluated using Gauss's law and the charge enclosed by a spherical Gaussian surface of radius *r*.

$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad\rightarrow\quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2}$$

Since the charge densities are constant, the charge enclosed is found by multiplying the appropriate charge density times the volume of charge enclosed by the Gaussian sphere. Let $r_1 = 6.0\,\mathrm{cm}$ and $r_2 = 12.0\,\mathrm{cm}$.

(*a*) Negative charge is enclosed for $r < r_1$.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_{(-)}\left(\frac{4}{3}\pi r^3\right)}{r^2} = \frac{\rho_{(-)}r}{3\varepsilon_0} = \frac{\left(-5.0\,\mathrm{C/m^3}\right)r}{3\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)}$$

$$= \boxed{\left(-1.9\times10^{11}\ \mathrm{N/C\bullet m}\right)r}$$

(*b*) In the region $r_1 < r < r_2$, all of the negative charge and part of the positive charge is enclosed.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_{(-)}\left(\frac{4}{3}\pi r_1^3\right)+\rho_{(+)}\left[\frac{4}{3}\pi\left(r^3-r_1^3\right)\right]}{r^2} = \frac{\left(\rho_{(-)}-\rho_{(+)}\right)\left(r_1^3\right)}{3\varepsilon_0 r^2} + \frac{\rho_{(+)}r}{3\varepsilon_0}$$

$$= \frac{\left[\left(-5.0\,\mathrm{C/m^3}\right)-\left(8.0\,\mathrm{C/m^3}\right)\right](0.060\,\mathrm{m})^3}{3\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)r^2} + \frac{\left(8.0\,\mathrm{C/m^3}\right)r}{3\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)}$$

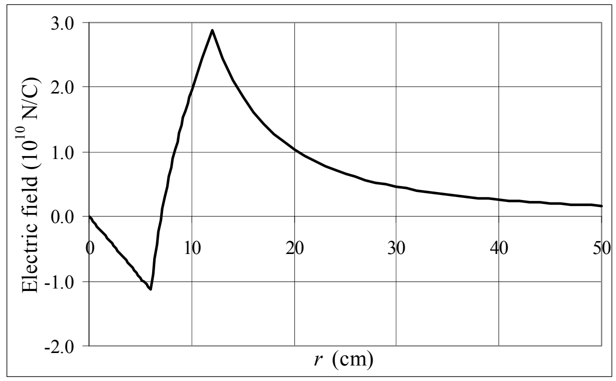
$$= \boxed{\frac{\left(-1.1\times10^8\ \mathrm{N\bullet m^2/C}\right)}{r^2} + \left(3.0\times10^{11}\ \mathrm{N/C\bullet m}\right)r}$$

(*c*) In the region $r_2 < r$, all of the charge is enclosed.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_{(-)}\left(\frac{4}{3}\pi r_1^3\right)+\rho_{(+)}\left[\frac{4}{3}\pi\left(r_2^3-r_1^3\right)\right]}{r^2} = \frac{\left(\rho_{(-)}-\rho_{(+)}\right)\left(r_1^3\right)+\rho_{(+)}\left(r_2^3\right)}{3\varepsilon_0 r^2} =$$

$$= \frac{\left[\left(-5.0\,\mathrm{C/m^3}\right)-\left(8.0\,\mathrm{C/m^3}\right)\right](0.060\,\mathrm{m})^3+\left(8.0\,\mathrm{C/m^3}\right)(0.120\,\mathrm{m})^3}{3\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)r^2} = \boxed{\frac{\left(4.1\times10^8\ \mathrm{N\bullet m^2/C}\right)}{r^2}}$$

(*d*) See the adjacent plot. The field is continuous at the edges of the layers. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.17d."




18. See Example 22-3 for a detailed discussion related to this problem.
(*a*) Inside a solid metal sphere the electric field is $\boxed{0}$.
(*b*) Inside a solid metal sphere the electric field is $\boxed{0}$.
(*c*) Outside a solid metal sphere the electric field is the same as if all the charge were concentrated at the center as a point charge.

$$|E| = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2} = \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(5.50\times10^{-6}\,\mathrm{C}\right)}{\left(3.10\,\mathrm{m}\right)^2} = \boxed{5140\,\mathrm{N/C}}$$

The field would point towards the center of the sphere.
(*d*) Same reasoning as in part (*c*).

$$|E| = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2} = \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(5.50\times10^{-6}\,\mathrm{C}\right)}{\left(8.00\,\mathrm{m}\right)^2} = \boxed{772\,\mathrm{N/C}}$$

The field would point towards the center of the sphere.
(*e*) The answers would be $\boxed{\text{no different}}$ for a thin metal shell.
(*f*) The solid sphere of charge is dealt with in Example 22-4. We see from that Example that the field inside the sphere is given by $|E| = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r_0^3}r.$ Outside the sphere the field is no different.

So we have these results for the solid sphere.

$$|E(r=0.250\,\mathrm{m})| = \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{5.50\times10^{-6}\,\mathrm{C}}{\left(3.00\,\mathrm{m}\right)^3}\left(0.250\,\mathrm{m}\right) = \boxed{458\,\mathrm{N/C}}$$

$$|E(r=2.90\,\mathrm{m})| = \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{5.50\times10^{-6}\,\mathrm{C}}{\left(3.00\,\mathrm{m}\right)^3}\left(2.90\,\mathrm{m}\right) = \boxed{5310\,\mathrm{N/C}}$$

$$|E(r=3.10\,\mathrm{m})| = \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{5.50\times10^{-6}\,\mathrm{C}}{\left(3.10\,\mathrm{m}\right)^2} = \boxed{5140\,\mathrm{N/C}}$$

$$|E(r=8.00\,\mathrm{m})| = \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{5.50\times10^{-6}\,\mathrm{C}}{\left(3.10\,\mathrm{m}\right)^2} = \boxed{772\,\mathrm{N/C}}$$

All point towards the center of the sphere.

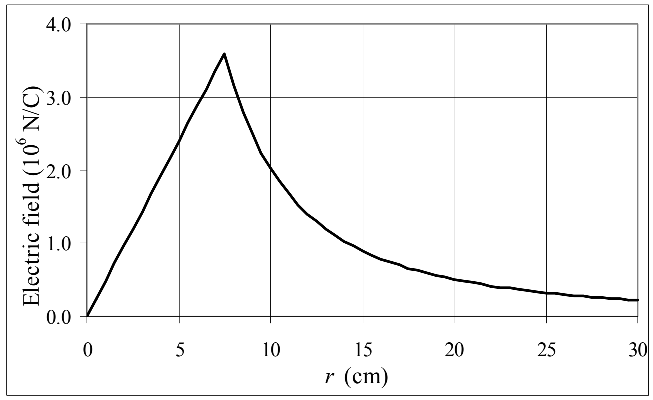
19. For points inside the nonconducting spheres, the electric field will be determined by the charge inside the spherical surface of radius *r*.

$$Q_{\text{encl}} = Q\left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r_0^3}\right) = Q\left(\frac{r}{r_0}\right)^3$$

The electric field for $r \le r_0$ can be calculated from Gauss's law.

$$E(r \le r_0) = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$$

$$= Q\left(\frac{r}{r_0}\right)^3 \frac{1}{4\pi\varepsilon_0 r^2} = \left(\frac{Q}{4\pi\varepsilon_0 r_0^3}\right) r$$




The electric field outside the sphere is calculated from Gauss's law with $Q_{\text{encl}} = Q$.

$$E(r \ge r_0) = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.19."

20. (*a*) When close to the sheet, we approximate it as an infinite sheet, and use the result of Example 22-7. We assume the charge is over both surfaces of the aluminum.

$$E = \frac{\sigma}{2\varepsilon_o} = \frac{\dfrac{275\times10^{-9}\,\text{C}}{(0.25\,\text{m})^2}}{2(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)} = \boxed{2.5\times10^5\,\text{N/C, away from the sheet}}$$

(*b*) When far from the sheet, we approximate it as a point charge.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = (8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{275\times10^{-9}\,\text{C}}{(15\,\text{m})^2} = \boxed{11\,\text{N/C, away from the sheet}}$$

21. (*a*) Consider a spherical gaussian surface at a radius of 3.00 cm. It encloses all of the charge.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E(4\pi r^2) = \frac{Q}{\varepsilon_0} \rightarrow$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = (8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.50\times10^{-6}\,\text{C}}{(3.00\times10^{-2}\,\text{m})^2} = \boxed{5.49\times10^7\,\text{N/C, radially outward}}$$

(*b*) A radius of 6.00 cm is inside the conducting material, and so the field must be $\boxed{0}$. Note that there must be an induced charge of $-5.50\times10^{-6}\,\text{C}$ on the surface at $r = 4.50$ cm, and then an induced charge of $5.50\times10^{-6}\,\text{C}$ on the outer surface of the sphere.

(*c*) Consider a spherical gaussian surface at a radius of 3.00 cm. It encloses all of the charge.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E(4\pi r^2) = \frac{Q}{\varepsilon_0} \rightarrow$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = (8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.50\times10^{-6}\,\text{C}}{(30.0\times10^{-2}\,\text{m})^2} = \boxed{5.49\times10^5\,\text{N/C, radially outward}}$$

22. (*a*) Inside the shell, the field is that of the point charge, $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$.

(*b*) There is no field inside the conducting material: $\boxed{E = 0}$.

(*c*) Outside the shell, the field is that of the point charge, $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$.

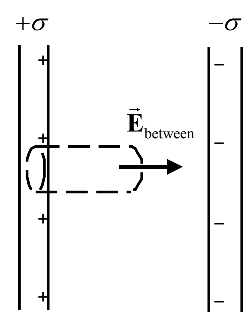
(*d*) The shell does not affect the field due to *Q* alone, except in the shell material, where the field is 0. The charge *Q* does affect the shell – it polarizes it. There will be an induced charge of –*Q* uniformly distributed over the inside surface of the shell, and an induced charge of +*Q* uniformly distributed over the outside surface of the shell.

23. (*a*) There can be no field inside the conductor, and so there must be an induced charge of $\boxed{-8.00\mu\text{C}}$ on the surface of the spherical cavity.

(*b*) Any charge on the conducting material must reside on its boundaries. If the net charge of the cube is $-6.10\mu\text{C}$, and there is a charge of $-8.00\mu\text{C}$ on its inner surface, there must be a charge of $\boxed{+1.90\mu\text{C}}$ on the outer surface.

24. Since the charges are of opposite sign, and since the charges are free to move since they are on conductors, the charges will attract each other and move to the inside or facing edges of the plates. There will be no charge on the outside edges of the plates. And there cannot be charge in the plates themselves, since they are conductors. All of the charge must reside on surfaces. Due to the symmetry of the problem, all field lines must be perpendicular to the plates, as discussed in Example 22-7.

(*a*) To find the field between the plates, we choose a gaussian cylinder, perpendicular to the plates, with area A for the ends of the cylinder. We place one end inside the left plate (where the field must be zero), and the other end between the plates. No flux passes through the curved surface of the cylinder.

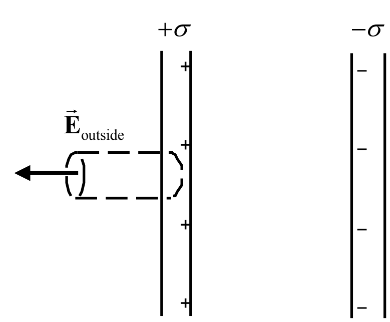



$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\substack{\text{right}\\\text{end}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$E_{\text{between}}A = \frac{\sigma A}{\varepsilon_0} \rightarrow \boxed{E_{\text{between}} = \frac{\sigma}{\varepsilon_0}}$$

The field lines between the plates leave the inside surface of the left plate, and terminate on the inside surface of the right plate. A similar derivation could have been done with the right end of the cylinder inside of the right plate, and the left end of the cylinder in the space between the plates.

(*b*) If we now put the cylinder from above so that the right end is inside the conducting material, and the left end is to the left of the left plate, the only possible location for flux is through the left end of the cylinder. Note that there is NO charge enclosed by the Gaussian cylinder.

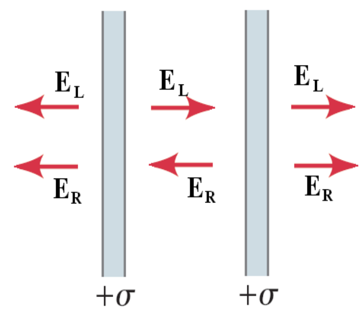



$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\\text{end}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$E_{\text{outside}}A = \frac{0}{\varepsilon_0} \quad \rightarrow \quad \boxed{E_{\text{outside}} = \frac{0}{\varepsilon_0}}$$

(*c*) If the two plates were nonconductors, the results would not change. The charge would be distributed over the two plates in a different fashion, and the field inside of the plates would not be zero, but the charge in the empty regions of space would be the same as when the plates are conductors.

25. Example 22-7 gives the electric field from a positively charged plate as $E = \sigma / 2\varepsilon_0$ with the field pointing away from the plate. The fields from the two plates will add, as shown in the figure.




(*a*) Between the plates the fields are equal in magnitude, but point in opposite directions.

$$E_{\text{between}} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \boxed{0}$$

(*b*) Outside the two plates the fields are equal in magnitude and point in the same direction.

$$E_{\text{outside}} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}}$$

(*c*) When the plates are conducting the charge lies on the surface of the plates. For nonconducting plates the same charge will be spread across the plate. This will not affect the electric field between or outside the two plates. It will, however, allow for a non-zero field inside each plate.

26. Because 3.0 cm << 1.0 m, we can consider the plates to be infinite in size, and ignore any edge effects. We use the result from Problem 24(*a*).

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0} \quad \rightarrow \quad Q = EA\varepsilon_0 = (160\,\text{N/C})(1.0\,\text{m})^2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{1.4\times10^{-9}\,\text{C}}$$

27. (*a*) In the region $0 < r < r_1$, a gaussian surface would enclose no charge. Thus, due to the spherical symmetry, we have the following.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} = 0 \quad \rightarrow \quad E = \boxed{0}$$

(*b*) In the region $r_1 < r < r_2$, only the charge on the inner shell will be enclosed.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma_1 4\pi r_1^2}{\varepsilon_0} \quad \rightarrow \quad E = \boxed{\frac{\sigma_1 r_1^2}{\varepsilon_0 r^2}}$$

(*c*) In the region $r_2 < r$, the charge on both shells will be enclosed.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma_1 4\pi r_1^2 + \sigma_2 4\pi r_2^2}{\varepsilon_0} \quad \rightarrow \quad E = \boxed{\frac{\sigma_1 r_1^2 + \sigma_2 r_2^2}{\varepsilon_0 r^2}}$$

(*d*) To make $E = 0$ for $r_2 < r$, we must have $\boxed{\sigma_1 r_1^2 + \sigma_2 r_2^2 = 0}$. This implies that the shells are of opposite charge.

(*e*) To make $E = 0$ for $r_1 < r < r_2$, we must have $\boxed{\sigma_1 = 0}$. Or, if a charge $Q = -4\pi\sigma_1 r_1^2$ were placed at the center of the shells, that would also make $E = 0$.

28. If the radius is to increase from $r_0$ to $2r_0$ linearly during an elapsed time of $T$, then the rate of increase must be $r_0/T$. The radius as a function of time is then $r = r_0 + \frac{r_0}{T}t = r_0\left(1+\frac{t}{T}\right)$. Since the balloon is spherical, the field outside the balloon will have the same form as the field due to a point charge.

(*a*) Here is the field just outside the balloon surface.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2\left(1+\frac{t}{T}\right)^2}}$$

(*b*) Since the balloon radius is always smaller than $3.2r_0$, the total charge enclosed in a gaussian surface at $r = 3.2r_0$ does not change in time.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(3.2r_0\right)^2}}$$

29. Due to the spherical symmetry of the problem, Gauss's law using a sphere of radius $r$ leads to the following.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{encl}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{encl}}{4\pi\varepsilon_0 r^2}$$

(*a*) For the region $0 < r < r_1$, the enclosed charge is 0.

$$E = \frac{Q_{encl}}{4\pi\varepsilon_0 r^2} = \boxed{0}$$

(*b*) For the region $r_1 < r < r_0$, the enclosed charge is the product of the volume charge density times the volume of charged material enclosed. The charge density is given by $\rho = \frac{Q}{\frac{4}{3}\pi r_0^3 - \frac{4}{3}\pi r_1^3}$ $= \frac{3Q}{4\pi\left(r_0^3 - r_1^3\right)}$.

$$E = \frac{Q_{encl}}{4\pi\varepsilon_0 r^2} = \frac{\rho V_{encl}}{4\pi\varepsilon_0 r^2} = \frac{\rho\left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3\right]}{4\pi\varepsilon_0 r^2} = \frac{\frac{3Q}{4\pi\left(r_0^3 - r_1^3\right)}\left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3\right]}{4\pi\varepsilon_0 r^2} = \boxed{\frac{Q}{4\pi\varepsilon_0 r^2}\frac{\left(r^3 - r_1^3\right)}{\left(r_0^3 - r_1^3\right)}}$$

(*c*) For the region $r > r_0$, the enclosed charge is the total charge, $Q$.

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

30. By the superposition principle for electric fields (Section 21-6), we find the field for this problem by adding the field due to the point charge at the center to the field found in Problem 29. At any location $r > 0$, the field due to the point charge is $E = \frac{q}{4\pi\varepsilon_0 r^2}$.

(*a*) $E = E_q + E_Q = \dfrac{q}{4\pi\varepsilon_0 r^2} + 0 = \boxed{\dfrac{q}{4\pi\varepsilon_0 r^2}}$

(*b*) $E = E_q + E_Q = \dfrac{q}{4\pi\varepsilon_0 r^2} + \dfrac{Q}{4\pi\varepsilon_0 r^2}\dfrac{\left(r^3 - r_1^3\right)}{\left(r_0^3 - r_1^3\right)} = \boxed{\dfrac{1}{4\pi\varepsilon_0 r^2}\left[q + Q\dfrac{\left(r^3 - r_1^3\right)}{\left(r_0^3 - r_1^3\right)}\right]}$

(*c*) $E = E_q + E_Q = \dfrac{q}{4\pi\varepsilon_0 r^2} + \dfrac{Q}{4\pi\varepsilon_0 r^2} = \boxed{\dfrac{q+Q}{4\pi\varepsilon_0 r^2}}$

31. (*a*) Create a gaussian surface that just encloses the inner surface of the spherical shell. Since the electric field inside a conductor must be zero, Gauss's law requires that the enclosed charge be zero. The enclosed charge is the sum of the charge at the center and charge on the inner surface of the conductor.

$$Q_{\text{enc}} = q + Q_{\text{inner}} = 0$$

Therefore $Q_{\text{inner}} = \boxed{-q}$.

(*b*) The total charge on the conductor is the sum of the charges on the inner and outer surfaces.

$$Q = Q_{\text{outer}} + Q_{\text{inner}} \quad \rightarrow \quad Q_{\text{outer}} = Q - Q_{\text{inner}} = \boxed{Q + q}$$

(*c*) A gaussian surface of radius $r < r_1$ only encloses the center charge, *q*. The electric field will therefore be the field of the single charge.

$$\boxed{E(r < r_1) = \frac{q}{4\pi\varepsilon_0 r^2}}$$

(*d*) A gaussian surface of radius $r_1 < r < r_0$ is inside the conductor so $\boxed{E = 0}$.

(*e*) A gaussian surface of radius $r > r_0$ encloses the total charge $q + Q$. The electric field will then be the field from the sum of the two charges.

$$\boxed{E(r > r_0) = \frac{q+Q}{4\pi\varepsilon_0 r^2}}$$

32. (*a*) For points inside the shell, the field will be due to the point charge only.

$$E\left(r < r_0\right) = \boxed{\frac{q}{4\pi\varepsilon_0 r^2}}$$

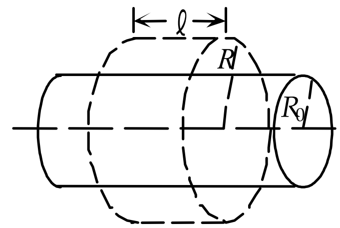
(*b*) For points outside the shell, the field will be that of a point charge, equal to the total charge.

$$E\left(r > r_0\right) = \boxed{\frac{q+Q}{4\pi\varepsilon_0 r^2}}$$

(*c*) If $q = Q$, we have $E\left(r < r_0\right) = \boxed{\dfrac{Q}{4\pi\varepsilon_0 r^2}}$ and $E\left(r > r_0\right) = \boxed{\dfrac{2Q}{4\pi\varepsilon_0 r^2}}$.

(*d*) If $q = -Q$, we have $E\left(r < r_0\right) = \boxed{\dfrac{-Q}{4\pi\varepsilon_0 r^2}}$ and $E\left(r > r_0\right) = \boxed{0}$.

33. We follow the development of Example 22-6. Because of the symmetry, we expect the field to be directed radially outward (no fringing effects near the ends of the cylinder) and to depend only on the perpendicular distance, *R*, from the symmetry axis of the shell. Because of the cylindrical symmetry, the field will be the same at all points on a gaussian surface that is a cylinder whose axis coincides with the axis of the shell. The gaussian surface is of radius *r* and length $\ell$. $\vec{\mathbf{E}}$ is perpendicular to this surface at all points. In order to apply Gauss's law, we need a closed surface, so we include the flat ends of the cylinder. Since $\vec{\mathbf{E}}$ is parallel to the flat ends, there is no flux through the ends. There is only flux through the curved wall of the gaussian cylinder.




$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma A_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{\sigma A_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

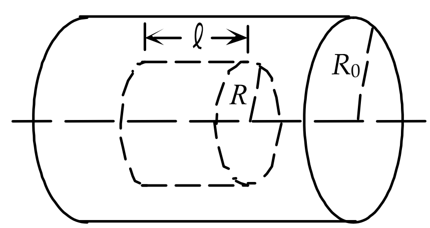
(*a*) For $R > R_0$, the enclosed surface area of the shell is $A_{\text{encl}} = 2\pi R_0\ell$.

$$E = \frac{\sigma A_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\sigma 2\pi R_0\ell}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\sigma R_0}{\varepsilon_0 R}\text{, radially outward}}$$

(*b*) For $R < R_0$, the enclosed surface area of the shell is $A_{\text{encl}} = 0$, and so $E = \boxed{0}$.

(*c*) The field for $R > R_0$ due to the shell is the same as the field due to the long line of charge, if we substitute $\boxed{\lambda = 2\pi R_0\sigma}$.

34. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.




$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho_{\text{E}} V_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{\rho_{\text{E}} V_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(*a*) For $R > R_0$, the enclosed volume of the shell is $V_{\text{encl}} = \pi R_0^2\ell$.

$$E = \frac{\rho_{\text{E}} V_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_{\text{E}} R_0^2}{2\varepsilon_0 R}\text{, radially outward}}$$

(*b*) For $R < R_0$, the enclosed volume of the shell is $V_{\text{encl}} = \pi R^2\ell$.

$$E = \frac{\rho_{\text{E}} V_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_{\text{E}} R}{2\varepsilon_0}\text{, radially outward}}$$

35. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details. We choose the gaussian cylinder to be the same length as the cylindrical shells.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(*a*) For $0 < R < R_1$, no charge is enclosed, and so $E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{0}$.

(*b*) For $R_1 < R < R_2$, charge $+Q$ is enclosed, and so $E = \boxed{\dfrac{Q}{2\pi\varepsilon_0 R\ell}\text{, radially outward}}$.

(*c*) For $R > R_2$, both charges of $+Q$ and $-Q$ are enclosed, and so $E = \dfrac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{0}$.

(*d*) The force on an electron between the cylinders points in the direction opposite to the electric field, and so the force is inward. The electric force produces the centripetal acceleration for the electron to move in the circular orbit.

$$F_{\text{centrip}} = eE = \frac{eQ}{2\pi\varepsilon_0 R\ell} = m\frac{v^2}{R} \quad \rightarrow \quad K = \tfrac{1}{2}mv^2 = \boxed{\frac{eQ}{4\pi\varepsilon_0 \ell}}$$

Note that this is independent of the actual value of the radius, as long as $R_1 < R < R_2$.

36. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details. We choose the gaussian cylinder to be the same length as the cylindrical shells.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(2\pi R\ell\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(*a*) At a distance of $R = 3.0\,\text{cm}$, no charge is enclosed, and so $E = \dfrac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \boxed{0}$.

(*b*) At a distance of $R = 7.0\,\text{cm}$, the charge on the inner cylinder is enclosed.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{2}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{R\ell} = 2\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(-0.88\times10^{-6}\text{C}\right)}{\left(0.070\,\text{m}\right)\left(5.0\,\text{m}\right)} = \boxed{-4.5\times10^4\,\text{N/C}}$$

The negative sign indicates that the field points radially inward.

(*c*) At a distance of $R = 12.0\,\text{cm}$, the charge on both cylinders is enclosed.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{2}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{R\ell} = 2\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.56-0.88\right)\times10^{-6}\text{C}}{\left(0.120\,\text{m}\right)\left(5.0\,\text{m}\right)} = \boxed{2.0\times10^4\,\text{N/C}}$$

The field points radially outward.

37. (*a*) The final speed can be calculated from the work-energy theorem, where the work is the integral of the force on the electron between the two shells.

$$W = \int \vec{F} \cdot d\vec{r} = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$

Setting the force equal to the electric field times the charge on the electron, and inserting the electric field from Problem 36 gives the work done on the electron.

$$W = \int_{R_1}^{R_2} \frac{qQ}{2\pi\varepsilon_0 \ell R}\,dR = \frac{qQ}{2\pi\varepsilon_0 \ell}\ln\left(\frac{R_2}{R_1}\right)$$

$$= \frac{\left(-1.60\times10^{-19}\text{C}\right)\left(-0.88\,\mu\text{C}\right)}{2\pi\left(8.85\times10^{-12}\text{C}^2/\text{Nm}^2\right)\left(5.0\,\text{m}\right)}\ln\left(\frac{9.0\,\text{cm}}{6.5\,\text{cm}}\right) = 1.65\times10^{-16}\ \text{J}$$

Solve for the velocity from the work-energy theorem.

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\left(1.65\times10^{-16}\ \text{J}\right)}{9.1\times10^{-31}\ \text{kg}}} = \boxed{1.9\times10^7\,\text{m/s}}$$

(*b*) The electric force on the proton provides its centripetal acceleration.

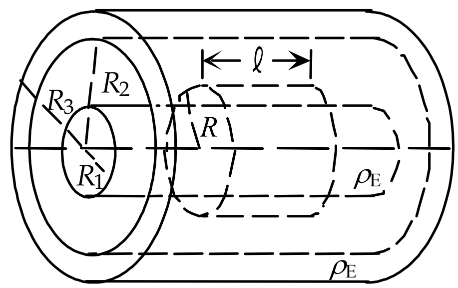
$$F_c = \frac{mv^2}{R} = qE = \frac{|qQ|}{2\pi\varepsilon_0 \ell R}$$

The velocity can be solved for from the centripetal acceleration.

$$v = \sqrt{\frac{(1.60\times10^{-19}\,\text{C})(0.88\,\mu\text{C})}{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{Nm}^2)(1.67\times10^{-27}\,\text{kg})(5.0\text{ m})}} = \boxed{5.5\times10^5\,\text{m/s}}$$

Note that as long as the proton is between the two cylinders, the velocity is independent of the radius.

38. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.




$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(*a*) For $0 < R < R_1$, the enclosed charge is the volume of charge enclosed, times the charge density.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R^2 \ell}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R}{2\varepsilon_0}}$$

(*b*) For $R_1 < R < R_2$, the enclosed charge is all of the charge on the inner cylinder.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R_1^2}{2\varepsilon_0 R}}$$

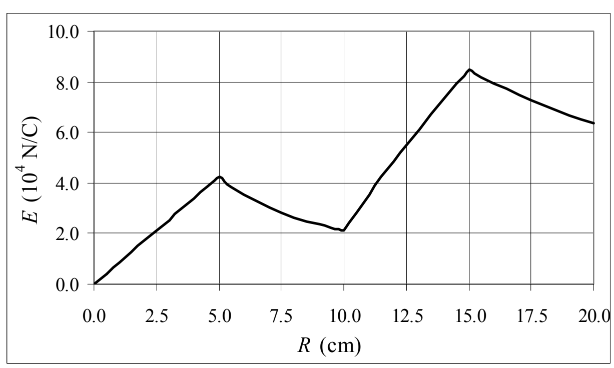
(*c*) For $R_2 < R < R_3$, the enclosed charge is all of the charge on the inner cylinder, and the part of the charge on the shell that is enclosed by the gaussian cylinder.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell + \rho_E(\pi R^2\ell - \pi R_2^2 \ell)}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E(R_1^2 + R^2 - R_2^2)}{2\varepsilon_0 R}}$$

(*d*) For $R > R_3$, the enclosed charge is all of the charge on both the inner cylinder and the shell.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell + \rho_E(\pi R_3^2\ell - \pi R_2^2 \ell)}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E(R_1^2 + R_3^2 - R_2^2)}{2\varepsilon_0 R}}$$

(*e*) See the graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.38e."



39. Due to the spherical symmetry of the geometry, we have the following to find the electric field at any radius *r*. The field will point either radially out or radially in.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$$

(*a*) For $0 < r < r_0$, the enclosed charge is due to the part of the charged sphere that has a radius smaller than *r*.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{\rho_{\text{E}}\left(\frac{4}{3}\pi r^3\right)}{4\pi\varepsilon_0 r^2} = \boxed{\frac{\rho_{\text{E}} r}{3\varepsilon_0}}$$

(*b*) For $r_0 < r < r_1$, the enclosed charge is due to the entire charged sphere of radius $r_0$.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{\rho_{\text{E}}\left(\frac{4}{3}\pi r_0^3\right)}{4\pi\varepsilon_0 r^2} = \boxed{\frac{\rho_{\text{E}} r_0^3}{3\varepsilon_0 r^2}}$$

(*c*) For $r_1 < r < r_2$, *r* is in the interior of the conducting spherical shell, and so $E = \boxed{0}$. This implies that $Q_{\text{encl}} = 0$, and so there must be an induced charge of magnitude $-\frac{4}{3}\rho_{\text{E}}\pi r_0^3$ on the inner surface of the conducting shell, at $r_1$.

(*d*) For $r > r_2$, the enclosed charge is the total charge of both the sphere and the shell.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{Q + \rho_{\text{E}}\left(\frac{4}{3}\pi r_0^3\right)}{4\pi\varepsilon_0 r^2} = \boxed{\left(\frac{Q}{4\pi\varepsilon_0} + \frac{\rho_{\text{E}} r_0^3}{3\varepsilon_0}\right)\frac{1}{r^2}}$$

40. The conducting outer tube is uncharged, and the electric field is 0 everywhere within the conducting material. Because there will be no electric field inside the conducting material of the outer cylinder tube, the charge on the inner nonconducting cylinder will induce an oppositely signed, equal magnitude charge on the inner surface of the conducting tube. This charge will NOT be uniformly distributed, because the inner cylinder is not in the center of the tube. Since the conducting tube has no net charge, there will be an induced charge on the OUTER surface of the conducting tube, equal in magnitude to the charge on the inner cylinder, and of the same sign. This charge will be uniformly distributed. Since there is no electric field in the conducting material of the tube, there is no way for the charges in the region interior to the tube to influence the charge distribution on the outer surface. Thus the field outside the tube is due to a cylindrically symmetric distribution of charge. Application of Gauss's law as in Example 22-6, for a Gaussian cylinder with a radius larger than the conducting tube, and a length $\ell$ leads to $E\left(2\pi R\ell\right) = \frac{Q_{\text{encl}}}{\varepsilon_0}$. The enclosed charge is the amount of charge on the inner cylinder.

$$Q_{\text{encl}} = \rho_{\text{E}}\pi R_1^2\ell \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{\varepsilon_0\left(2\pi R\ell\right)} = \boxed{\frac{\rho_{\text{E}} R_1^2}{2\varepsilon_0 R}}$$

41. We treat the source charge as a disk of positive charge of radius concentric with a disk of negative charge of radius $R_0$. In order for the net charge of the inner space to be 0, the charge per unit area of the source disks must both have the same magnitude but opposite sign. The field due to the annulus is then the sum of the fields due to both the positive and negative rings.

(*a*) At a distance of $0.25R_0$ from the center of the ring, we can approximate both of the disks as infinite planes, each producing a uniform field. Since those two uniform fields will be of the same magnitude and opposite sign, the net field is $\boxed{0}$.

(*b*) At a distance of $75R_0$ from the center of the ring, it appears to be approximately a point charge, and so the field will approximate that of a point charge, $E = \boxed{\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{\left(75R_0\right)^2}}$

42. The conducting sphere is uncharged, and the electric field is 0 everywhere within its interior, except for in the cavities. When charge $Q_1$ is placed in the first cavity, a charge $-Q_1$ will be drawn from the conducting material to the inner surface of the cavity, and the electric field will remain 0 in the conductor. That charge $-Q_1$ will NOT be distributed symmetrically on the cavity surface. Since the conductor is neutral, a compensating charge $Q_1$ will appear on the outer surface of the conductor (charge can only be on the surfaces of conductors in electrostatics). Likewise, when charge $Q_2$ is placed in the second cavity, a charge $-Q_2$ will be drawn from the conducting material, and a compensating charge $Q_2$ will appear on the outer surface. Since there is no electric field in the conducting material, there is no way for the charges in the cavities to influence the charge distribution on the outer surface. So the distribution of charge on the outer surface is uniform, just as it would be if there were no inner charges, and instead a charge $Q_1 + Q_2$ were simply placed on the conductor. Thus the field outside the conductor is due to a spherically symmetric distribution of $Q_1 + Q_2$. Application of Gauss's law leads to $E = \boxed{\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1+Q_2}{r^2}}$. If $Q_1 + Q_2 > 0$, the field will point radially outward. If $Q_1 + Q_2 < 0$, the field will point radially inward.

$\boxed{43.}$ (*a*) Choose a cylindrical gaussian surface with the flat ends parallel to and equidistant from the slab. By symmetry the electric field must point perpendicularly away from the slab, resulting in no flux passing through the curved part of the gaussian cylinder. By symmetry the flux through each end of the cylinder must be equal with the electric field constant across the surface. $\oint \vec{E}\cdot d\vec{A} = 2EA$

The charge enclosed by the surface is the charge density of the slab multiplied by the volume of the slab enclosed by the surface.

$$q_{enc} = \rho_E\left(Ad\right)$$

Gauss's law can then be solved for the electric field.

$$\oint \vec{E}\cdot d\vec{A} = 2EA = \frac{\rho_E Ad}{\varepsilon_0} \quad\rightarrow\quad E = \boxed{\frac{\rho_E d}{2\varepsilon_0}}$$

Note that this electric field is independent of the distance from the slab.

(*b*) When the coordinate system of this problem is changed to axes parallel $\left(\hat{\mathbf{z}}\right)$ and perpendicular $\left(\hat{\mathbf{r}}\right)$ to the slab, it can easily be seen that the particle will hit the slab if the initial perpendicular velocity is sufficient for the particle to reach the slab before the acceleration decreases its velocity to zero. In the new coordinate system the axes are rotated by 45°.

$$\vec{r}_0 = y_0\cos 45°\hat{\mathbf{r}} + y_0\sin 45°\hat{\mathbf{z}} = \frac{y_0}{\sqrt{2}}\hat{\mathbf{r}} + \frac{y_0}{\sqrt{2}}\hat{\mathbf{z}}$$

$$\vec{v}_0 = -v_0 \sin 45°\hat{\mathbf{r}} + v_0 \cos 45°\hat{\mathbf{z}} = -\frac{v_0}{\sqrt{2}}\hat{\mathbf{r}} + \frac{v_0}{\sqrt{2}}\hat{\mathbf{z}}$$

$$\vec{a} = qE/m\hat{\mathbf{r}}$$

The perpendicular components are then inserted into Eq. 2-12c, with the final velocity equal to zero.

$$0 = v_{r0}^2 - 2a(r - r_0) = \frac{v_0^2}{2} - 2\frac{q}{m}\left(\frac{\rho_E d}{2\varepsilon_0}\right)\left(\frac{y_0}{\sqrt{2}} - 0\right)$$

Solving for the velocity gives the minimum speed that the particle can have to reach the slab.

$$v_0 \geq \boxed{\sqrt{\frac{\sqrt{2}q\rho_E d y_0}{m\varepsilon_0}}}$$

44. Due to the spherical symmetry of the problem, Gauss's law using a sphere of radius *r* leads to the following.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}$$

(*a*) For the region $0 < r < r_1$, the enclosed charge is 0.

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \boxed{0}$$

(*b*) For the region $r_1 < r < r_0$, the enclosed charge is the product of the volume charge density times the volume of charged material enclosed. The charge density is given by $\rho = \rho_0 \frac{r_1}{r}$. We must integrate to find the total charge. We follow the procedure given in Example 22-5. We divide the sphere up into concentric thin shells of thickness *dr*, as shown in Fig. 22-14. We then integrate to find the charge.

$$Q_{\text{encl}} = \int \rho_{\text{E}} dV = \int_{r_1}^{r} \rho_0 \frac{r_1}{r'} 4\pi\left(r'\right)^2 dr' = 4\pi r_1 \rho_0 \int_{r_1}^{r} r' dr' = 2\pi r_1 \rho_0 \left(r^2 - r_1^2\right)$$

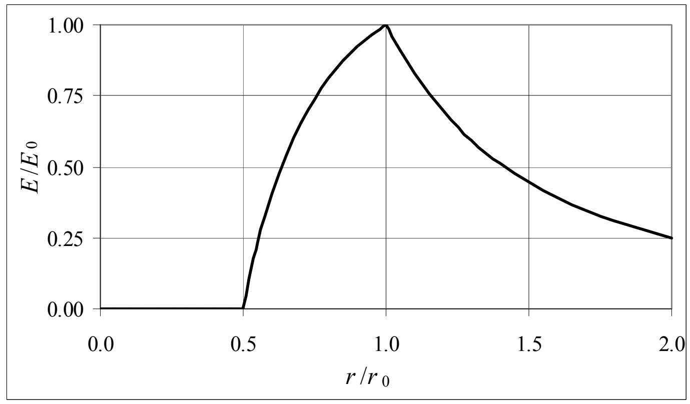
$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{2\pi r_1 \rho_0 \left(r^2 - r_1^2\right)}{4\pi\varepsilon_0 r^2} = \boxed{\frac{\rho_0 r_1 \left(r^2 - r_1^2\right)}{2\varepsilon_0 r^2}}$$

(*c*) For the region $r > r_0$, the enclosed charge is the total charge, found by integration in a similar fashion to part (*b*).

$$Q_{\text{encl}} = \int \rho_{\text{E}} dV = \int_{r_1}^{r_0} \rho_0 \frac{r_1}{r'} 4\pi\left(r'\right)^2 dr' = 4\pi r_1 \rho_0 \int_{r_1}^{r_0} r' dr' = 2\pi r_1 \rho_0 \left(r_0^2 - r_1^2\right)$$

$$E = \frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{2\pi r_1 \rho_0 \left(r_0^2 - r_1^2\right)}{4\pi\varepsilon_0 r^2} = \boxed{\frac{\rho_0 r_1 \left(r_0^2 - r_1^2\right)}{2\varepsilon_0 r^2}}$$

(*d*) See the attached graph. We have chosen $r_1 = \frac{1}{2} r_0$. Let $E_0 = E(r = r_0) = \dfrac{\rho_0 r_1 \left(r_0^2 - r_1^2\right)}{2\varepsilon_0 r_0^2}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.44d."




45. (*a*) The force felt by one plate will be the charge on that plate multiplied by the electric field caused by the other plate. The field due to one plate is found in Example 22-7. Let the positive plate be on the left, and the negative plate on the right. We find the force on the negative plate due to the positive plate.

$$F_{\substack{\text{on}\\\text{plate}\\\text{"b"}}} = q_{\substack{\text{on}\\\text{plate}\\\text{"b"}}} E_{\substack{\text{due to}\\\text{plate}\\\text{"a"}}} = (\sigma_b A_b) E_a = (\sigma_b A_b)\frac{\sigma_a}{2\varepsilon_0}$$

$$= \frac{\left(-15\times10^{-6}\,\text{C/m}^2\right)\left(1.0\,\text{m}^2\right)\left(-15\times10^{-6}\,\text{C/m}^2\right)}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{Nm}^2\right)} = -12.71\,\text{N}$$

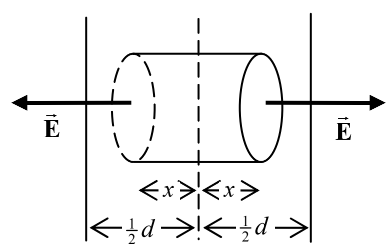
$$\approx \boxed{13\,\text{N, towards the other plate}}$$

(*b*) Since the field due to either plate is constant, the force on the other plate is constant, and then the work is just the force times the distance. Since the plates are oppositely charged, they will attract, and so a force equal to and opposite the force above will be required to separate them. The force will be in the same direction as the displacement of the plates.

$$W = \vec{\mathbf{F}}\bullet\Delta\vec{\mathbf{x}} = (12.71\,\text{N})(\cos 0^\circ)\left(5.0\times10^{-3}\,\text{m}\right) = \boxed{0.064\,\text{J}}$$

46. Because the slab is very large, and we are considering only distances from the slab much less than its height or breadth, the symmetry of the slab results in the field being perpendicular to the slab, with a constant magnitude for a constant distance from the center. We assume that $\rho_E > 0$ and so the electric field points away from the center of the slab.

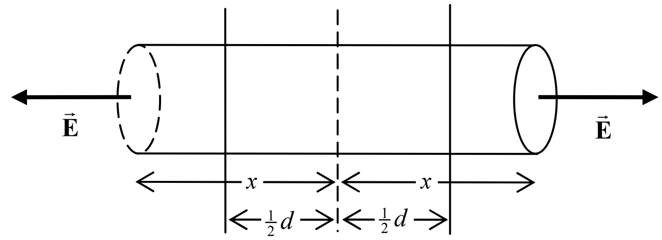
(*a*) To determine the field inside the slab, choose a cylindrical gaussian surface, of length $2x < d$ and cross-sectional area $A$. Place it so that it is centered in the slab. There will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on both ends, and is the same magnitude on both ends. Apply Gauss's law to find the electric field at a distance $x < \frac{1}{2}d$ from the center of the slab. See the first diagram.




$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow 2EA = \frac{\rho(2xA)}{\varepsilon_0} \rightarrow$$

$$\boxed{E_{\text{inside}} = \frac{\rho x}{\varepsilon_0};\ |x| < \tfrac{1}{2}d}$$

(*b*) Use a similar arrangement to determine the field outside the slab. Now let $2x > d$. See the second diagram.



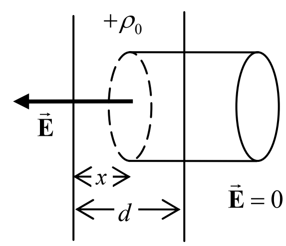

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$2EA = \frac{\rho(dA)}{\varepsilon_0} \rightarrow \boxed{E_{\text{outside}} = \frac{\rho d}{2\varepsilon_0};\ |x| > \tfrac{1}{2}d}$$

Notice that electric field is continuous at the boundary of the slab.

47. (*a*) In Problem 46, it is shown that the field outside a flat slab of nonconducting material with a uniform charge density is given by $E = \dfrac{\rho d}{2\varepsilon_0}$. If the charge density is positive, the field points away from the slab, and if the charge density is negative, the field points towards the slab. So for this problem's configuration, the field outside of both half-slabs is the vector sum of the fields from each half-slab. Since those fields are equal in magnitude and opposite in direction, the field outside the slab is $\boxed{0}$.

(*b*) To find the field in the positively charged half-slab, we use a cylindrical gaussian surface of cross sectional area *A*. Place it so that its left end is in the positively charged half-slab, a distance $x > 0$ from the center of the slab. Its right end is external to the slab. Due to the symmetry of the configuration, there will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on the left end, and is 0 on the right end. We assume that the electric field is pointing to the left. Apply Gauss's law to find the electric field a distance $0 < x < d$ from the center of the slab. See the diagram.

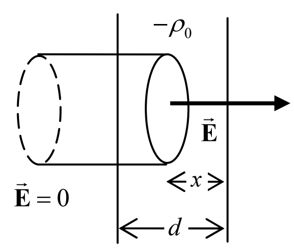



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\ \text{end}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$EA = \frac{\rho_0 (d - x) A}{\varepsilon_0} \rightarrow E_{x>0} = \frac{\rho_0 (d - x)}{\varepsilon_0}$$

Since the field is pointing to the left, we can express this as $\boxed{\vec{E}_{x>0} = -\dfrac{\rho_0 (d - x)}{\varepsilon_0} \hat{\mathbf{i}}}$.

(*c*) To find the field in the negatively charged half-slab, we use a cylindrical gaussian surface of cross sectional area *A*. Place it so that its right end is in the negatively charged half-slab, a distance $x < 0$ from the center of the slab. Its left end is external to the slab. Due to the symmetry of the configuration, there will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on the left end, and is 0 on the right end. We assume that the electric field is pointing to the right. Apply Gauss's law to find the electric field at a distance $-d < x < 0$ from the center of the slab. See the diagram.



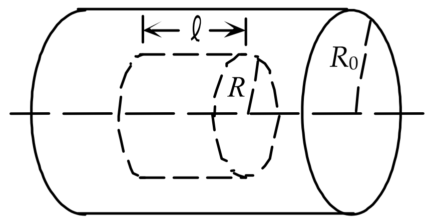

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\substack{\text{right}\\ \text{end}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$EA = \frac{-\rho_0 (d+x) A}{\varepsilon_0} \quad \rightarrow \quad E_{x<0} = \frac{-\rho_0 (d+x)}{\varepsilon_0}$$

Since the field is pointing to the left, we can express this as $\boxed{E_{x<0} = -\frac{\rho_0 (d+x)}{\varepsilon_0}\hat{\mathbf{i}}}$.

Notice that the field is continuous at all boundaries. At the left edge $(x=-d)$, $E_{x<0} = E_{\text{outside}}$. At the center $(x=0)$, $E_{x<0} = E_{>0}$. And at the right edge $(x=d)$, $E_{x>0} = E_{\text{outside}}$.

48. We follow the development of Example 22-6. Because of the symmetry, we expect the field to be directed radially outward (no fringing effects near the ends of the cylinder) and to depend only on the perpendicular distance, *R*, from the symmetry axis of the cylinder. Because of the cylindrical symmetry, the field will be the same at all points on a gaussian surface that is a cylinder whose axis coincides with the axis of the cylinder.




The gaussian surface is of radius *r* and length $\ell$. $\vec{\mathbf{E}}$ is perpendicular to this surface at all points. In order to apply Gauss's law, we need a closed surface, so we include the flat ends of the cylinder. Since $\vec{\mathbf{E}}$ is parallel to the flat ends, there is no flux through the ends. There is only flux through the curved wall of the gaussian cylinder.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

To find the field inside the cylinder, we must find the charge enclosed in the gaussian cylinder. We divide the gaussian cylinder up into coaxial thin cylindrical shells of length $\ell$ and thickness *dR*. That shell has volume $dV = 2\pi R\ell dR$. The total charge in the gaussian cylinder is found by integration.

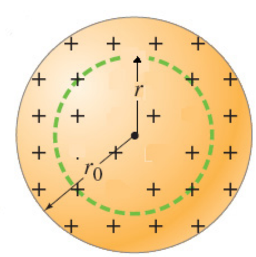
$$R < R_0: \quad Q_{\text{encl}} = \int_0^R \rho_E dV = \int_0^R \rho_0 \left(\frac{R}{R_0}\right)^2 2\pi R\ell\, dR = \frac{2\pi\rho_0\ell}{R_0^2}\int_0^R R^3\, dR = \frac{\pi\rho_0\ell R^4}{2R_0^2}$$

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\frac{\pi\rho_0\ell R^4}{2R_0^2}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_0 R^3}{4\varepsilon_0 R_0^2}\text{, radially out}}$$

$$R < R_0: \quad Q_{\text{encl}} = \int_0^{R_0} \rho_E dV = \frac{2\pi\rho_0\ell}{R_0^2}\int_0^{R_0} R^3\, dR = \frac{\pi\rho_0\ell R_0^2}{2}$$

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\frac{\pi\rho_0\ell R_0^2}{2}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_0 R_0^2}{4\varepsilon_0 R}\text{, radially out}}$$

49. The symmetry of the charge distribution allows the electric field inside the sphere to be calculated using Gauss's law with a concentric gaussian sphere of radius $r \le r_0$. The enclosed charge will be found by integrating the charge density over the enclosed volume.




$$Q_{\text{encl}} = \int \rho_E dV = \int_0^r \rho_0 \left(\frac{r'}{r_0}\right) 4\pi r'^2 dr' = \frac{\rho_0 \pi r^4}{r_0}$$

The enclosed charge can be written in terms of the total charge by setting

$r = r_0$ and solving for the charge density in terms of the total charge.

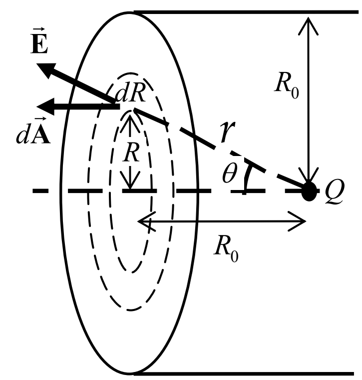
$$Q = \frac{\rho_0 \pi r_0^4}{r_0} = \rho_0 \pi r_0^3 \rightarrow \rho_0 = \frac{Q}{\pi r_0^3} \rightarrow Q_{\text{encl}}(r) = \frac{\rho_0 \pi r^4}{r_0} = Q\left(\frac{r}{r_0}\right)^4$$

The electric field is then found from Gauss's law

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}\left(\frac{r}{r_0}\right)^4 \rightarrow \boxed{E = \frac{Q}{4\pi\varepsilon_0}\frac{r^2}{r_0^4}}$$

The electric field points radially outward since the charge distribution is positive.

50. By Gauss's law, the total flux through the cylinder is $Q/\varepsilon_0$. We find the flux through the ends of the cylinder, and then subtract that from the total flux to find the flux through the curved sides. The electric field is that of a point charge. On the ends of the cylinder, that field will vary in both magnitude and direction. Thus we must do a detailed integration to find the flux through the ends of the cylinder. Divide the ends into a series of concentric circular rings, of radius *R* and thickness *dR*. Each ring will have an area of $2\pi R dR$. The angle between $\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$ is $\theta$, where $\tan\theta = R/R_0$. See the diagram of the left half of the cylinder.




$$\Phi_{\substack{\text{left}\\\text{end}}} = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_0^{R_0} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\cos\theta\left(2\pi R\right)dR$$

The flux integral has three variables: *r*, *R*, and $\theta$. We express *r* and $\theta$ in terms of *R* in order to integrate. The anti-derivative is found in Appendix B-4.

$$r = \sqrt{R^2 + R_0^2} \ ; \ \cos\theta = \frac{R_0}{r} = \frac{R_0}{\sqrt{R^2 + R_0^2}}$$

$$\Phi_{\substack{\text{left}\\\text{end}}} = \int_0^{R_0} \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(R^2 + R_0^2\right)}\frac{R_0}{\sqrt{R^2 + R_0^2}}\left(2\pi R\right)dR = \frac{2\pi Q R_0}{4\pi\varepsilon_0}\int_0^{R_0}\frac{R\,dR}{\left(R^2 + R_0^2\right)^{3/2}} = \frac{QR_0}{2\varepsilon_0}\left[-\frac{1}{\sqrt{R^2 + R_0^2}}\right]_0^{R_0}$$

$$= \frac{Q}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{2}}\right] \ ; \ \Phi_{\substack{\text{both}\\\text{ends}}} = 2\Phi_{\substack{\text{left}\\\text{end}}} = \frac{Q}{\varepsilon_0}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$\Phi_{\text{total}} = \frac{Q}{\varepsilon_0} = \Phi_{\text{sides}} + \Phi_{\substack{\text{both}\\\text{ends}}} \rightarrow \Phi_{\text{sides}} = \frac{Q}{\varepsilon_0} - \Phi_{\substack{\text{both}\\\text{ends}}} = \frac{Q}{\varepsilon_0} - \frac{Q}{\varepsilon_0}\left[1 - \frac{1}{\sqrt{2}}\right] = \boxed{\frac{Q}{\sqrt{2}\varepsilon_0}}$$

51. The gravitational field a distance *r* from a point mass *M* is given by Eq. 6-8, $\mathbf{g} = -\frac{GM}{r^2}\hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ is a unit vector pointing radially outward from mass *M*. Compare this to the electric field of a point charge, $\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$. To change the electric field to the gravitational field, we would make these changes: $\vec{\mathbf{E}} \rightarrow \vec{\mathbf{g}}$ ; $Q/\varepsilon_0 \rightarrow -4\pi GM$. Make these substitutions in Gauss's law.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow \boxed{\oint \vec{\mathbf{g}} \cdot d\vec{\mathbf{A}} = -4\pi G M_{\text{encl}}}$$

52. (*a*) We use Gauss's law for a spherically symmetric charge distribution, and assume that all the charge is on the surface of the Earth. Note that the field is pointing radially inward, and so the dot product introduces a negative sign.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = -E\left(4\pi r^2\right) = Q_{\text{encl}}/\varepsilon_0 \quad \rightarrow$$

$$Q_{\text{encl}} = -4\pi\varepsilon_0 E R_{\text{Earth}}^2 = \frac{-\left(150\,\text{N/C}\right)\left(6.38\times10^6\,\text{m}\right)^2}{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} = -6.793\times10^5\,\text{C} \approx \boxed{-6.8\times10^5\,\text{C}}$$

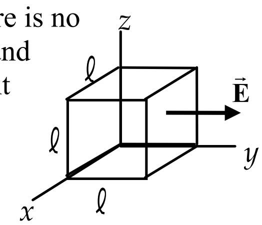
(*b*) Find the surface density of electrons. Let *n* be the total number of electrons.

$$\sigma = \frac{Q}{A} = -\frac{ne}{A} \quad \rightarrow$$

$$\frac{n}{A} = -\frac{Q}{eA} = -\frac{-4\pi\varepsilon_0 E R_{\text{Earth}}^2}{e\left(4\pi R_{\text{Earth}}^2\right)} = \frac{\varepsilon_0 E}{e} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(150\,\text{N/C}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)}$$

$$= \boxed{8.3\times10^9\,\text{electrons/m}^2}$$

53. The electric field is strictly in the *y* direction. So, referencing the diagram, there is no flux through the top, bottom, front, or back faces of the cube. Only the "left" and "right" faces will have flux through them. And since the flux is only dependent on the *y* coordinate, the flux through each of those two faces is particularly simple. Calculate the flux and use Gauss's law to find the enclosed charge.




$$\Phi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\\text{face}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\substack{\text{right}\\\text{face}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$= \int_{\substack{\text{left}\\\text{face}}} b\hat{\mathbf{j}} \cdot \left(-\hat{\mathbf{j}}dA\right) + \int_{\substack{\text{right}\\\text{face}}} \left(a\ell + b\right)\hat{\mathbf{j}} \cdot \left(\hat{\mathbf{j}}dA\right) = -b\ell^2 + a\ell^3 + b\ell^2$$

$$= a\ell^3 = Q_{\text{encl}}/\varepsilon_0 \quad \rightarrow \quad Q_{\text{encl}} = \boxed{\varepsilon_0 a\ell^3}$$

54. (*a*) Find the value of *b* by integrating the charge density over the entire sphere. Follow the development given in Example 22-5.

$$Q = \int \rho_{\text{E}} dV = \int_0^{r_0} br\left(4\pi r^2 dr\right) = 4\pi b\left(\tfrac{1}{4}r_0^4\right) \quad \rightarrow \quad b = \boxed{\frac{Q}{\pi r_0^4}}$$

(*b*) To find the electric field inside the sphere, we apply Gauss's law to an imaginary sphere of radius *r*, calculating the charge enclosed by that sphere. The spherical symmetry allows us to evaluate the flux integral simply.

$$\Phi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \;;\; Q = \int \rho_{\text{E}} dV = \int_0^r \frac{Q}{\pi r_0^4} r\left(4\pi r^2 dr\right) = \frac{Qr^4}{r_0^4} \quad \rightarrow$$

$$E = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Qr^2}{r_0^4},\, r < r_0}$$

(*c*) As discussed in Example 22-4, the field outside a spherically symmetric distribution of charge is the same as that for a point charge of the same magnitude located at the center of the sphere.

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2},\, r > r_0}$$

55. The flux through a gaussian surface depends only on the charge enclosed by the surface. For both of these spheres the two point charges are enclosed within the sphere. Therefore the flux is the same for both spheres.

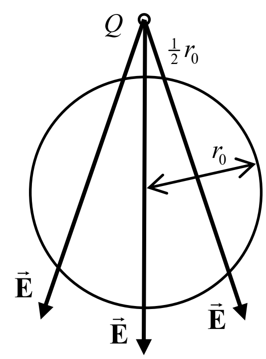
$$\Phi = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\left(9.20\times10^{-9}\,\text{C}\right)+\left(-5.00\times10^{-9}\,\text{C}\right)}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{475\,\text{N}\cdot\text{m}^2/\text{C}}$$

56. (*a*) The flux through any closed surface containing the total charge must be the same, so the flux through the larger sphere is the same as the flux through the smaller sphere, $\boxed{+235\,\text{N}\cdot\text{m}^2/\text{C}}$.

(*b*) Use Gauss's law to determine the enclosed charge.

$$\Phi = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow Q_{\text{encl}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(+235\,\text{N}\cdot\text{m}^2/\text{C}\right) = \boxed{2.08\times10^{-9}\,\text{C}}$$

57. (*a*) There is no charge enclosed within the sphere, and so no flux lines can originate or terminate inside the sphere. All field lines enter and leave the sphere. Thus the net flux is $\boxed{0}$.




(*b*) The maximum electric field will be at the point on the sphere closest to $Q$, which is the top of the sphere. The minimum electric field will be at the point on the sphere farthest from Q, which is the bottom of the sphere.

$$E_{\max} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_{\text{closest}}^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\frac{1}{2}r_0\right)^2} = \boxed{\frac{1}{\pi\varepsilon_0}\frac{Q}{r_0^2}}$$

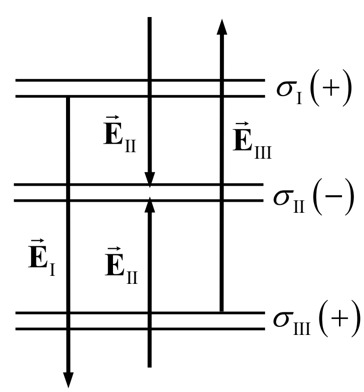
$$E_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_{\text{farthest}}^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\frac{5}{2}r_0\right)^2} = \boxed{\frac{1}{25\pi\varepsilon_0}\frac{Q}{r_0^2}}$$

Thus the range of values is $\boxed{\frac{1}{\pi\varepsilon_0}\frac{Q}{r_0^2} \le E_{\substack{\text{sphere}\\ \text{surface}}} \le \frac{1}{25\pi\varepsilon_0}\frac{Q}{r_0^2}}$.

(*c*) $\vec{\mathbf{E}}$ is $\boxed{\text{not perpendicular}}$ at all points. It is only perpendicular at the two points already discussed: the point on the sphere closest to the point charge, and the point on the sphere farthest from the point charge.

(*d*) The electric field is not perpendicular or constant over the surface of the sphere. Therefore Gauss's law is $\boxed{\text{not useful}}$ for obtaining E at the surface of the sphere because a gaussian surface cannot be chosen that simplifies the flux integral.

58. The force on a sheet is the charge on the sheet times the average electric field due to the other sheets: But the fields due to the "other" sheets is uniform, so the field is the same over the entire sheet. The force per unit area is then the charge per unit area, times the field due to the other sheets.




$$F_{\substack{\text{on}\\ \text{sheet}}} = q_{\substack{\text{on}\\ \text{sheet}}}\bar{E}_{\substack{\text{other}\\ \text{sheets}}} = q_{\substack{\text{on}\\ \text{sheet}}}E_{\substack{\text{other}\\ \text{sheets}}} \rightarrow$$

$$\left(\frac{F}{A}\right)_{\substack{\text{on}\\ \text{sheet}}} = \left(\frac{q}{A}\right)_{\substack{\text{on}\\ \text{sheet}}}E_{\substack{\text{other}\\ \text{sheets}}} = \sigma_{\substack{\text{on}\\ \text{sheet}}}E_{\substack{\text{other}\\ \text{sheets}}}$$

The uniform fields from each of the three sheets are indicated on the diagram. Take the positive direction as upwards. We take the direction from the diagram, and so use the absolute value of each charge density. The electric field magnitude due to each sheet is given by $E = \sigma/2\varepsilon_0$.

$$\left(\frac{F}{A}\right)_{\mathrm{I}} = \sigma_{\mathrm{I}}\left(E_{\mathrm{III}} - E_{\mathrm{II}}\right) = \frac{\sigma_{\mathrm{I}}}{2\varepsilon_0}\left(|\sigma_{\mathrm{III}}| - |\sigma_{\mathrm{II}}|\right) = \frac{6.5\times10^{-9}\ \mathrm{C/m^2}}{2\left(8.85\times10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)}\left[(5.0-2.0)\times10^{-9}\ \mathrm{C/m^2}\right]$$

$$= \boxed{1.1\times10^{-6}\ \mathrm{N/m^2}\ \ (\mathrm{up})}$$

$$\left(\frac{F}{A}\right)_{\mathrm{II}} = \sigma_{\mathrm{II}}\left(E_{\mathrm{III}} - E_{\mathrm{I}}\right) = \frac{\sigma_{\mathrm{II}}}{2\varepsilon_0}\left(|\sigma_{\mathrm{III}}| - |\sigma_{\mathrm{I}}|\right) = \frac{-2.0\times10^{-9}\ \mathrm{C/m^2}}{2\left(8.85\times10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)}\left[(5.0-6.5)\times10^{-9}\ \mathrm{C/m^2}\right]$$

$$= \boxed{1.7\times10^{-7}\ \mathrm{N/m^2}\ \ (\mathrm{up})}$$

$$\left(\frac{F}{A}\right)_{\mathrm{III}} = \sigma_{\mathrm{III}}\left(E_{\mathrm{III}} - E_{\mathrm{I}}\right) = \frac{\sigma_{\mathrm{III}}}{2\varepsilon_0}\left(|\sigma_{\mathrm{II}}| - |\sigma_{\mathrm{I}}|\right) = \frac{5.0\times10^{-9}\ \mathrm{C/m^2}}{2\left(8.85\times10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)}\left[(2.0-6.5)\times10^{-9}\ \mathrm{C/m^2}\right]$$

$$= \boxed{-1.3\times10^{-6}\ \mathrm{N/m^2}\ \ (\mathrm{down})}$$

59. (*a*) The net charge inside a sphere of radius $a_0$ will be made of two parts – the positive point charge at the center of the sphere, and some fraction of the total negative charge, since the negative charge is distributed over all space, as described by the charge density. To evaluate the portion of the negative charge inside the sphere, we must determine the coefficient *A*. We do that by integrating the charge density over all space, in the manner of Example 22-5. Use an integral from Appendix B-5.

$$-e = \int \rho_E dV = \int_0^\infty \left(-Ae^{-2r/a_0}\right)\left(4\pi r^2 dr\right) = -4\pi A\int_0^\infty e^{-2r/a_0} r^2 dr = -4\pi A\frac{2!}{\left(2/a_0\right)^3} = -\pi A a_0^3 \quad\rightarrow$$

$$A = \frac{e}{\pi a_0^3}$$

Now we find the negative charge inside the sphere of radius $a_0$, using an integral from Appendix B-4. We are indicating the elementary charge by $(e)$, so as to not confuse it with the base of the natural logarithms.

$$Q_{\mathrm{neg}} = \int_0^{a_0}\left(-Ae^{-2r/a_0}\right)\left(4\pi r^2 dr\right) = -\frac{4\pi(e)}{\pi a_0^3}\int_0^{a_0} e^{-2r/a_0} r^2 dr$$

$$= -\frac{4\pi(e)}{\pi a_0^3}\left\{\left[-\frac{e^{-2r/a_0}}{\left(2/a_0\right)^3}\right]\left[\left(2/a_0\right)^2 r^2 + 2\left(2/a_0\right)r + 2\right]\right\}_0^{a_0} = (e)\left[5e^{-2} - 1\right]$$

$$Q_{\mathrm{net}} = Q_{\mathrm{neg}} + Q_{\mathrm{pos}} = (e)\left[5e^{-2}-1\right] + (e) = (e)5e^{-2} = \left(1.6\times10^{-19}\mathrm{C}\right)5e^{-2} = 1.083\times10^{-19}\mathrm{C}$$

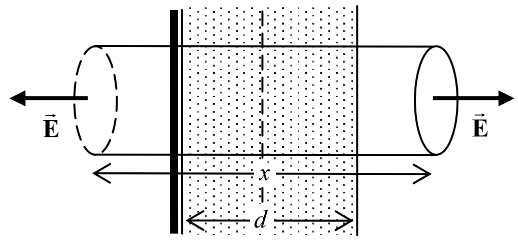
$$\approx \boxed{1.1\times10^{-19}\mathrm{C}}$$

(*b*) The field at a distance $r = a_0$ is that of a point charge of magnitude $Q_{\mathrm{net}}$ at the origin, because of the spherical symmetry and Gauss's law.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\mathrm{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\mathrm{net}}}{a_0^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(1.083\times10^{-19}\mathrm{C}\right)}{\left(0.53\times10^{-10}\mathrm{m}\right)^2} = \boxed{3.5\times10^{11}\ \mathrm{N/C}}$$

60. The field due to the plane is $E_{\text{plane}} = \dfrac{\sigma}{2\varepsilon_0}$, as discussed in Example 22-7. Because the slab is very large, and we assume that we are considering only distances from the slab much less than its height or breadth, the symmetry of the slab results in its field being perpendicular to the slab, with a constant magnitude for a constant distance from its center. We also assume that $\rho_E > 0$ and so the electric field of the slab points away from the center of the slab.

(*a*) To determine the field to the left of the plane, we choose a cylindrical gaussian surface, of length $x > d$ and cross-sectional area $A$. Place it so that the plane is centered inside the cylinder. See the diagram. There will be no flux through the curved wall of the cylinder. From the symmetry, the electric field is parallel to the surface area vector on both ends. We already know that the field due to the plane is the same on both ends, and by the symmetry of the problem, the field due to the slab must also be the same on both ends. Thus the total field is the same magnitude on both ends.



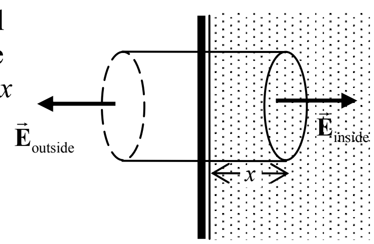

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow 2E_{\text{outside}}A = \frac{\sigma A + \rho_E dA}{\varepsilon_0} \rightarrow$$

$$E_{\text{outside}} = E_{\substack{\text{left} \\ \text{of plane}}} = \boxed{\frac{\sigma + \rho_E d}{2\varepsilon_0}}$$

(*b*) As argued above, the field is symmetric on the outside of the charged matter.

$$E_{\substack{\text{right} \\ \text{of plane}}} = \boxed{\frac{\sigma + \rho_E d}{2\varepsilon_0}}$$

(*c*) To determine the field inside the slab, we choose a cylindrical gaussian surface of cross-sectional area $A$ with one face to the left of the plane, and the other face inside the slab, a distance $x$ from the plane. Due to symmetry, the field again is parallel to the surface area vector on both ends, has a constant value on each end, and no flux pierces the curved walls. Apply Gauss's law.




$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left} \\ \text{end}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\substack{\text{right} \\ \text{end}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E_{\text{outside}}A + E_{\text{inside}}A + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0}$$

$$Q_{\text{encl}} = \sigma A + \rho_E xA \rightarrow \left(\frac{\sigma + \rho_E d}{2\varepsilon_0}\right)A + E_{\text{inside}}A = \frac{\sigma A + \rho_E xA}{\varepsilon_0} \rightarrow$$

$$E_{\text{inside}} = \boxed{\frac{\sigma + \rho_E(2x - d)}{2\varepsilon_0}}, \ 0 < x < d$$

Notice that the field is continuous from "inside" to "outside" at the right edge of the slab, but not at the left edge of the slab. That discontinuity is due to the surface charge density.

61. Consider this sphere as a combination of two spheres. Sphere 1 is a solid sphere of radius $r_0$ and charge density $\rho_E$ centered at A and sphere 2 is a second sphere of radius $r_0/2$ and density $-\rho_E$ centered at C.

(*a*) The electric field at A will have zero contribution from sphere 1 due to its symmetry about point A. The electric field is then calculated by creating a gaussian surface centered at point C with radius $r_0/2$.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0} \rightarrow E \cdot 4\pi\left(\tfrac{1}{2}r_0\right)^2 = \frac{\left(-\rho_E\right)\frac{4}{3}\pi\left(\frac{1}{2}r_0\right)^3}{\varepsilon_0} \rightarrow \boxed{E = -\frac{\rho_E r_0}{6\varepsilon_0}}$$

Since the electric field points into the gaussian surface (negative) the electric field at point A points to the right.

(*b*) At point B the electric field will be the sum of the electric fields from each sphere. The electric field from sphere 1 is calculated using a gaussian surface of radius $r_0$ centered at A.

$$\oint \vec{\mathbf{E}}_1 \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0} \rightarrow E_1 \cdot 4\pi r_0^2 = \frac{\frac{4}{3}\pi r_0^3\left(\rho_E\right)}{\varepsilon_0} \rightarrow E_1 = \frac{\rho_E r_0}{3\varepsilon_0}$$

At point B the field from sphere 1 points toward the left. The electric field from sphere 2 is calculated using a gaussian surface centered at C of radius $3r_0/2$.

$$\oint \vec{\mathbf{E}}_2 \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0} \rightarrow E_2 \cdot 4\pi\left(\tfrac{3}{2}r_0\right)^2 = \frac{\left(-\rho_E\right)\frac{4}{3}\pi\left(\frac{1}{2}r_0\right)^3}{\varepsilon_0} \rightarrow E_2 = -\frac{\rho_E r_0}{54\varepsilon_0}$$

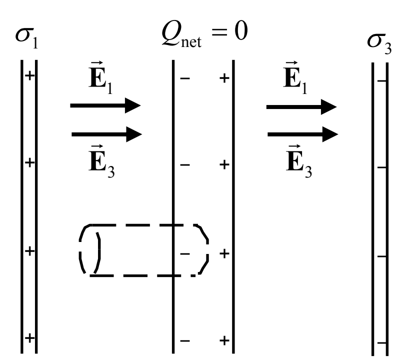
At point B, the electric field from sphere 2 points toward the right. The net electric field is the sum of these two fields. The net field points to the left.

$$E = E_1 + E_2 = \frac{\rho_E r_0}{3\varepsilon_0} + \frac{-\rho_E r_0}{54\varepsilon_0} = \boxed{\frac{17\rho_E r_0}{54\varepsilon_0}}.$$

62. We assume the charge is uniformly distributed, and so the field of the pea is that of a point charge.

$$E\left(r = R\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \rightarrow$$

$$Q = E4\pi\varepsilon_0 R^2 = \left(3\times10^6\ \text{N/C}\right)4\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)\left(0.00375\,\text{m}\right)^2 = \boxed{5\times10^{-9}\,\text{C}}$$




63. (*a*) In an electrostatic situation, there is no electric field inside a conductor. Thus $E = \boxed{0}$ inside the conductor.

(*b*) The positive sheet produces an electric field, external to itself, directed away from the plate with a magnitude as given in Example 22-7, of $E_1 = \frac{|\sigma_1|}{2\varepsilon_0}$. The negative sheet produces an electric field, external to itself, directed towards the plate with a magnitude of $E_2 = \frac{|\sigma_2|}{2\varepsilon_0}$. Between the left and middle sheets, those two fields are parallel and so add to each other.

$$E_{\substack{\text{left}\\\text{middle}}} = E_1 + E_2 = \frac{|\sigma_1| + |\sigma_2|}{2\varepsilon_0} = \frac{2\left(5.00\times10^{-6}\ \text{C/m}^2\right)}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)} = \boxed{5.65\times10^5\ \text{N/C}}, \text{ to the right}$$

(*c*) The same field is between the middle and right sheets. See the diagram.

$$E_{\substack{\text{middle}\\\text{right}}} = \boxed{5.65\times10^{5}\ \text{N/C}}\text{, to the right}$$

(*d*) To find the charge density on the surface of the left side of the middle sheet, choose a gaussian cylinder with ends of area *A*. Let one end be inside the conducting sheet, where there is no electric field, and the other end be in the area between the left and middle sheets. Apply Gauss's law in the manner of Example 22-16. Note that there is no flux through the curved sides of the cylinder, and there is no flux through the right end since it is in conducting material. Also note that the field through the left end is in the opposite direction as the area vector of the left end.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\\text{end}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\substack{\text{right}\\\text{end}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = -E_{\substack{\text{left}\\\text{middle}}}A + 0 + 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma_{\text{left}}A}{\varepsilon_0} \rightarrow$$

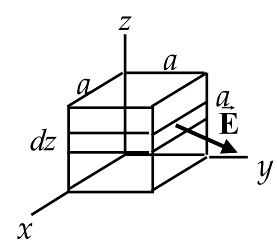
$$\sigma_{\text{left}} = -\varepsilon_0 E_{\substack{\text{left}\\\text{middle}}} = -\varepsilon_0\left(\frac{|\sigma_1|+|\sigma_2|}{2\varepsilon_0}\right) = \boxed{-5.00\times10^{-6}\ \text{C/m}^2}$$

(*e*) Because the middle conducting sheet has no net charge, the charge density on the right side must be the opposite of the charge density on the left side.

$$\sigma_{\text{right}} = -\sigma_{\text{left}} = \boxed{5.00\times10^{-6}\ \text{C/m}^2}$$

Alternatively, we could have applied Gauss's law on the right side in the same manner that we did on the left side. The same answer would result.

64. Because the electric field has only *x* and *y* components, there will be no flux through the top or bottom surfaces. For the other faces, we choose a horizontal strip of height *dz* and width *a* for a differential element and integrate to find the flux. The total flux is used to determine the enclosed charge.




$$\Phi_{\substack{\text{front}\\(x=a)}} = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_0^a \left[E_0\left(1+\frac{z}{a}\right)\hat{\mathbf{i}} + E_0\left(\frac{z}{a}\right)\hat{\mathbf{j}}\right]\cdot\left(adz\hat{\mathbf{i}}\right)$$

$$= E_0 a\int_0^a \left(1+\frac{z}{a}\right)dz = E_0 a\left(z+\frac{z^2}{2a}\right)_0^a = \tfrac{3}{2}E_0a^2$$

$$\Phi_{\substack{\text{back}\\(x=0)}} = \int_0^a \left[E_0\left(1+\frac{z}{a}\right)\hat{\mathbf{i}} + E_0\left(\frac{z}{a}\right)\hat{\mathbf{j}}\right]\cdot\left(-adz\hat{\mathbf{i}}\right) = -\tfrac{3}{2}E_0a^2$$

$$\Phi_{\substack{\text{right}\\(y=a)}} = \int_0^a \left[E_0\left(1+\frac{z}{a}\right)\hat{\mathbf{i}} + E_0\left(\frac{z}{a}\right)\hat{\mathbf{j}}\right]\cdot\left(adz\hat{\mathbf{j}}\right) = E_0 a\int_0^a\left(\frac{z}{a}\right)dz = E_0 a\left(\frac{z^2}{2a}\right)_0^a = \tfrac{1}{2}E_0a^2$$

$$\Phi_{\substack{\text{left}\\(y=a)}} = \int_0^a \left[E_0\left(1+\frac{z}{a}\right)\hat{\mathbf{i}} + E_0\left(\frac{z}{a}\right)\hat{\mathbf{j}}\right]\cdot\left(-adz\hat{\mathbf{j}}\right) = -\tfrac{1}{2}E_0a^2$$

$$\Phi_{\text{total}} = \Phi_{\text{front}} + \Phi_{\text{back}} + \Phi_{\text{right}} + \Phi_{\text{left}} + \Phi_{\text{top}} + \Phi_{\text{bottom}} = \tfrac{3}{2}E_0a^2 - \tfrac{3}{2}E_0a^2 + \tfrac{1}{2}E_0a^2 - \tfrac{1}{2}E_0a^2 = 0 + 0$$

$$= 0 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow \boxed{Q_{\text{encl}} = 0}$$

65. (*a*) Because the shell is a conductor, there is no electric field in the conducting material, and all charge must reside on its surfaces. All of the field lines that originate from the point charge at the center must terminate on the inner surface of the shell. Therefore the inner surface must have an equal but opposite charge to the point charge at the center. Since the conductor has the same magnitude of charge as the point charge at the center, all of the charge on the conductor is $\boxed{\text{on the inner surface of the shell}}$, in a spherically symmetric distribution.

(*b*) By Gauss's law and the spherical symmetry of the problem, the electric field can be calculated by $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_{\text{encl}}}{r^2}$.

$$r < 0.10\,\text{m}:\ E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.0\times10^{-6}\,\text{C}\right)}{r^2} = \boxed{\frac{2.7\times10^4\,\text{N}\cdot\text{m}^2/\text{C}}{r^2}}$$

$$r > 0.15\,\text{m}:\ E = \boxed{0}$$

And since there is no electric field in the shell, we could express the second answer as $r > 0.10\,\text{m}:\ E = \boxed{0}$.

66. (*a*) At a strip such as is marked in the textbook diagram, $d\vec{\mathbf{A}}$ is perpendicular to the surface, and $\vec{\mathbf{E}}$ is inclined at an angle $\theta$ relative to $d\vec{\mathbf{A}}$.

$$\Phi_{\text{hemisphere}} = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int_0^{\pi/2} E\cos\theta\left(2\pi R^2\sin\theta\,d\theta\right)$$

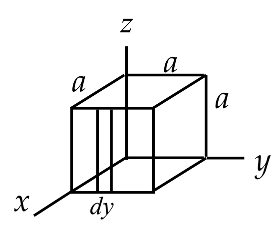
$$= 2\pi R^2 E\int_0^{\pi/2}\cos\theta\sin\theta\,d\theta = 2\pi R^2 E\left(\tfrac{1}{2}\sin^2\theta\right)_0^{\pi/2} = \pi R^2 E$$

(*b*) Choose a closed gaussian surface consisting of the hemisphere and the circle of radius *R* at the base of the hemisphere. There is no charge inside that closed gaussian surface, and so the total flux through the two surfaces (hemisphere and base) must be zero. The field lines are all perpendicular to the circle, and all of the same magnitude, and so that flux is very easy to calculate.

$$\Phi_{\text{circle}} = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int E\left(\cos 180^\circ\right)d\vec{\mathbf{A}} = -EA = -E\pi R^2$$

$$\Phi_{\text{total}} = 0 = \Phi_{\text{circle}} + \Phi_{\text{hemisphere}} = -E\pi R^2 + \Phi_{\text{hemisphere}} \quad\rightarrow\quad \Phi_{\text{hemisphere}} = \boxed{\pi R^2 E}$$

67. The flux is the sum of six integrals, each of the form $\iint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}$. Because the electric field has only *x* and *y* components, there will be no flux through the top or bottom surfaces. For the other faces, we choose a vertical strip of height *a* and width *dy* (for the front and back faces) or *dx* (for the left and right faces). See the diagram for an illustration of a strip on the front face. The total flux is then calculated, and used to determine the enclosed charge.




$$\Phi_{\substack{\text{front}\\(x=a)}} = \int_0^a \left(E_{x0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{i}} + E_{y0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{j}}\right)\cdot a\,dy\,\hat{\mathbf{i}} = aE_{x0}\int_0^a e^{-\left(\frac{a+y}{a}\right)^2}dy$$

This integral does not have an analytic anti-derivative, and so must be integrated numerically. We approximate the integral by a sum: $\displaystyle\int_0^a e^{-\left(\frac{a+y}{a}\right)^2}dy \approx \sum_{i=1}^{n} e^{-\left(\frac{a+y_i}{a}\right)^2}\Delta y$. The region of integration is divided into $n$ elements, and so $\Delta y = \dfrac{a-0}{n}$ and $y_i = i\Delta y$. We initially evaluate the sum for $n$ = 10. Then we evaluate it for $n$ = 20. If the two sums differ by no more than 2%, we take that as the value of the integral. If they differ by more than 2%, we choose a larger $n$, compute the sum, and compare that to the result for $n$ = 20. We continue until a difference of 2% or less is reached. This integral, for $n$ = 100 and $a$ = 1.0 m, is 0.1335 m. So we have this intermediate result.

$$\underset{(x=a)}{\Phi_{\text{front}}} = aE_{x0}\sum_{i=1}^{n} e^{-\left(\frac{a+y_i}{a}\right)^2}\Delta y = (1.0\,\text{m})(50\,\text{N/C})(0.1335\,\text{m}) = 6.675\,\text{N}\bullet\text{m}^2/\text{C}$$

Now do the integral over the back face.

$$\underset{(x=0)}{\Phi_{\text{back}}} = \int_0^a \left(E_{x0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{i}} + E_{y0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{j}}\right)\bullet\left(-a\,dy\,\hat{\mathbf{i}}\right) = -aE_{x0}\int_0^a e^{-\left(\frac{y}{a}\right)^2}dy$$

We again get an integral that cannot be evaluated analytically. A similar process to that used for the front face is applied again, and so we make this approximation: $-aE_{x0}\int_0^a e^{-\left(\frac{y}{a}\right)^2}dy \approx -aE_{x0}\sum_{i=1}^{n} e^{-\left(\frac{y_i}{a}\right)^2}\Delta y$.

The numeric integration gives a value of 0.7405 m.

$$\underset{(x=0)}{\Phi_{\text{back}}} = -aE_{x0}\sum_{i=1}^{n} e^{-\left(\frac{y_i}{a}\right)^2}\Delta y = -(1.0\,\text{m})(50\,\text{N/C})(0.7405\,\text{m}) = -37.025\,\text{N}\bullet\text{m}^2/\text{C}.$$

Now consider the right side.

$$\underset{(y=a)}{\Phi_{\text{right}}} = \int_0^a \left(E_{x0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{i}} + E_{y0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{j}}\right)\bullet a\,dx\,\hat{\mathbf{j}} = aE_{y0}\int_0^a e^{-\left(\frac{x+a}{a}\right)^2}dx$$

Notice that the same integral needs to be evaluated as for the front side. All that has changed is the variable name. Thus we have the following.

$$\underset{(y=a)}{\Phi_{\text{right}}} = aE_{y0}\int_0^a e^{-\left(\frac{x+a}{a}\right)^2}dx \approx (1.0\,\text{m})(25\,\text{N/C})(0.1335\,\text{m}) = 3.3375\,\text{N}\bullet\text{m}^2/\text{C}$$

Finally, do the left side, following the same process. The same integral arises as for the back face.

$$\underset{(y=0)}{\Phi_{\text{left}}} = \int_0^a \left(E_{x0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{i}} + E_{y0}e^{-\left(\frac{x+y}{a}\right)^2}\hat{\mathbf{j}}\right)\bullet\left(-a\,dx\,\hat{\mathbf{j}}\right) = -aE_{y0}\int_0^a e^{-\left(\frac{x}{a}\right)^2}dx$$

$$\approx -(1.0\,\text{m})(25\,\text{N/C})(0.7405\,\text{m}) = -18.5125\,\text{N}\bullet\text{m}^2/\text{C}$$

Sum to find the total flux, and multiply by $\varepsilon_0$ to find the enclosed charge.

$$\Phi_{\text{total}} = \Phi_{\text{front}} + \Phi_{\text{back}} + \Phi_{\text{right}} + \Phi_{\text{left}} + \Phi_{\text{top}} + \Phi_{\text{bottom}}$$

$$= (6.675 - 37.025 + 3.3375 - 18.5125)\,\text{N}\bullet\text{m}^2/\text{C} = -45.525\,\text{N}\bullet\text{m}^2/\text{C} \approx \boxed{-46\,\text{N}\bullet\text{m}^2/\text{C}}$$

$$Q_{\text{encl}} = \varepsilon_0\Phi_{\text{total}} = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(-45.525\,\text{N}\bullet\text{m}^2/\text{C}\right) = \boxed{-4.0\times10^{-10}\,\text{C}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.67."

# CHAPTER 23: Electric Potential

## Responses to Questions

1. Not necessarily. If two points are at the same potential, then no *net* work is done in moving a charge from one point to the other, but work (both positive and negative) could be done at different parts of the path. No. It is possible that positive work was done over one part of the path, and negative work done over another part of the path, so that these two contributions to the net work sum to zero. In this case, a non-zero force would have to be exerted over both parts of the path.

2. The negative charge will move toward a region of higher potential and the positive charge will move toward a region of lower potential. In both cases, the potential energy of the charge will decrease.

3. (*a*) The electric potential is the electric potential energy per unit charge. The electric potential is a scalar. The electric field is the electric force per unit charge, and is a vector.
   (*b*) Electric potential is the electric potential energy per unit charge.

4. Assuming the electron starts from rest in both cases, the final speed will be twice as great. If the electron is accelerated through a potential difference that is four times as great, then its increase in kinetic energy will also be greater by a factor of four. Kinetic energy is proportional to the square of the speed, so the final speed will be greater by a factor of two.

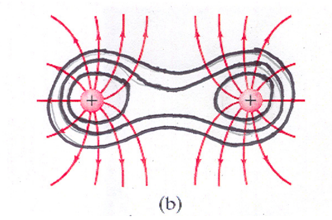
5. Yes. If the charge on the particle is negative and it moves from a region of low electric potential to a region of high electric potential, its electric potential energy will decrease.

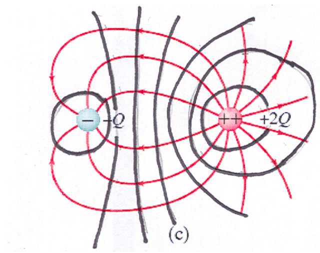
6. No. Electric potential is the *potential energy* per unit charge at a point in space and electric field is the *electric force* per unit charge at a point in space. If one of these quantities is zero, the other is not necessarily zero. For example, the point exactly between two charges with equal magnitudes and opposite signs will have a zero electric potential because the contributions from the two charges will be equal in magnitude and opposite in sign. (Net electric potential is a *scalar* sum.) This point will not have a zero electric field, however, because the electric field contributions will be in the same direction (towards the negative and away from the positive) and so will add. (Net electric field is a *vector* sum.) As another example, consider the point exactly between two equal positive point charges. The electric potential will be positive since it is the sum of two positive numbers, but the electric field will be zero since the field contributions from the two charges will be equal in magnitude but opposite in direction.

7. (*a*) $V$ at other points would be lower by 10 V. $E$ would be unaffected, since $E$ is the negative gradient of $V$, and a change in $V$ by a constant value will not change the value of the gradient.
   (*b*) If $V$ represents an absolute potential, then yes, the fact that the Earth carries a net charge would affect the value of $V$ at the surface. If $V$ represents a potential difference, then no, the net charge on the Earth would not affect the choice of $V$.

8. No. An equipotential line is a line connecting points of equal electric potential. If two equipotential lines crossed, it would indicate that their intersection point has two different values of electric potential simultaneously, which is impossible. As an analogy, imagine contour lines on a topographic map. They also never cross because one point on the surface of the Earth cannot have two different values for elevation above sea level.

9. The equipotential lines (in black) are perpendicular to the electric field lines (in red).




10. The electric field is zero in a region of space where the electric potential is constant. The electric field is the gradient of the potential; if the potential is constant, the gradient is zero.

11. The Earth's gravitational equipotential lines are roughly circular, so the orbit of the satellite would have to be roughly circular.

12. The potential at point P would be unchanged. Each bit of positive charge will contribute an amount to the potential based on its charge and its distance from point P. Moving charges to different locations on the ring does not change their distance from P, and hence does not change their contributions to the potential at P.

    The value of the electric field will change. The electric field is the vector sum of all the contributions to the field from the individual charges. When the charge $Q$ is distributed uniformly about the ring, the $y$-components of the field contributions cancel, leaving a net field in the $x$-direction. When the charge is not distributed uniformly, the $y$-components will not cancel, and the net field will have both $x$- and $y$-components, and will be larger than for the case of the uniform charge distribution. There is no discrepancy here, because electric potential is a scalar and electric field is a vector.

13. The charge density and the electric field strength will be greatest at the pointed ends of the football because the surface there has a smaller radius of curvature than the middle.

14. No. You cannot calculate electric potential knowing only electric field at a point and you cannot calculate electric field knowing only electric potential at a point. As an example, consider the uniform field between two charged, conducting plates. If the potential difference between the plates is known, then the distance between the plates must also be known in order to calculate the field. If the field between the plates is known, then the distance to a point of interest between the plates must also be known in order to calculate the potential there. In general, to find $V$, you must know E and be able to integrate it. To find E, you must know $V$ and be able to take its derivative. Thus you need E or $V$ in the region around the point, not just at the point, in order to be able to find the other variable.

15. (*a*) Once the two spheres are placed in contact with each other, they effectively become one larger conductor. They will have the same potential because the potential everywhere on a conducting surface is constant.
    (*b*) Because the spheres are identical in size, an amount of charge $Q/2$ will flow from the initially charged sphere to the initially neutral sphere so that they will have equal charges.
    (*c*) Even if the spheres do not have the same radius, they will still be at the same potential once they are brought into contact because they still create one larger conductor. However, the amount of charge that flows will not be exactly equal to half the total charge. The larger sphere will end up with the larger charge.

16. If the electric field points due north, the change in the potential will be (*a*) greatest in the direction opposite the field, south; (*b*) least in the direction of the field, north; and (*c*) zero in a direction perpendicular to the field, east and west.

17. Yes. In regions of space where the equipotential lines are closely spaced, the electric field is stronger than in regions of space where the equipotential lines are farther apart.

18. If the electric field in a region of space is uniform, then you can infer that the electric potential is increasing or decreasing uniformly in that region. For example, if the electric field is 10 V/m in a region of space then you can infer that the potential difference between two points 1 meter apart (measured parallel to the direction of the field) is 10 V. If the electric potential in a region of space is uniform, then you can infer that the electric field there is zero.

19. The electric potential energy of two unlike charges is negative. The electric potential energy of two like charges is positive. In the case of unlike charges, work must be done to separate the charges. In the case of like charges, work must be done to move the charges together.

## Solutions to Problems

1. Energy is conserved, so the change in potential energy is the opposite of the change in kinetic energy. The change in potential energy is related to the change in potential.

$$\Delta U = q\Delta V = -\Delta K \quad \rightarrow$$

$$\Delta V = \frac{-\Delta K}{q} = \frac{K_{\text{initial}} - K_{\text{final}}}{q} = \frac{mv^2}{2q} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(5.0\times10^{5}\,\text{m/s}\right)^2}{2\left(-1.60\times10^{-19}\,\text{C}\right)} = \boxed{-0.71\,\text{V}}$$

The final potential should be lower than the initial potential in order to stop the electron.

2. The work done by the electric field can be found from Eq. 23-2b.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} \quad \rightarrow \quad W_{\text{ba}} = -qV_{\text{ba}} = -\left(1.60\times10^{-19}\,\text{C}\right)\left[-55\,\text{V} - 185\,\text{V}\right] = \boxed{3.84\times10^{-17}\,\text{J}}$$

3. The kinetic energy gained by the electron is the work done by the electric force. Use Eq. 23-2b to calculate the potential difference.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} = -\frac{5.25\times10^{-16}\,\text{J}}{\left(-1.60\times10^{-19}\,\text{C}\right)} = \boxed{3280\,\text{V}}$$

The electron moves from low potential to high potential, so $\boxed{\text{plate B}}$ is at the higher potential.

4. By the work energy theorem, the total work done, by the external force and the electric field together, is the change in kinetic energy. The work done by the electric field is given by Eq. 23-2b.

$$W_{\text{external}} + W_{\text{electric}} = \text{KE}_{\text{final}} - \text{KE}_{\text{initial}} \quad \rightarrow \quad W_{\text{external}} - q\left(V_{\text{b}} - V_{\text{a}}\right) = \text{KE}_{\text{final}} \quad \rightarrow$$

$$\left(V_{\text{b}} - V_{\text{a}}\right) = \frac{W_{\text{external}} - \text{KE}_{\text{final}}}{q} = \frac{7.00\times10^{-4}\,\text{J} - 2.10\times10^{-4}\,\text{J}}{-9.10\times10^{-6}\,\text{C}} = \boxed{-53.8\,\text{V}}$$

Since the potential difference is negative, we see that $V_{\text{a}} > V_{\text{b}}$.

5. As an estimate, the length of the bolt would be the voltage difference of the bolt divided by the breakdown electric field of air.

$$\frac{1\times10^{8}\,\text{V}}{3\times10^{6}\,\text{V/m}}=33\,\text{m}\approx\boxed{30\,\text{m}}$$

6. The distance between the plates is found from Eq. 23-4b, using the magnitude of the electric field.

$$|E|=\frac{V_{\text{ba}}}{d}\;\rightarrow\; d=\frac{V_{\text{ba}}}{|E|}=\frac{45\,\text{V}}{1300\,\text{V/m}}=\boxed{3.5\times10^{-2}\,\text{m}}$$

7. The maximum charge will produce an electric field that causes breakdown in the air. We use the same approach as in Examples 23-4 and 23-5.

$$V_{\text{surface}}=r_0E_{\text{breakdown}}\text{ and }V_{\text{surface}}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}\;\rightarrow$$

$$Q=4\pi\varepsilon_0r_0^2E_{\text{breakdown}}=\left(\frac{1}{8.99\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2}\right)(0.065\,\text{m})^2\left(3\times10^{6}\,\text{V/m}\right)=\boxed{1.4\times10^{-6}\text{C}}$$

8. We assume that the electric field is uniform, and so use Eq. 23-4b, using the magnitude of the electric field.

$$E=\frac{V_{\text{ba}}}{d}=\frac{110\,\text{V}}{4.0\times10^{-3}\,\text{m}}=\boxed{2.8\times10^{4}\,\text{V/m}}$$

9. To find the limiting value, we assume that the E-field at the radius of the sphere is the minimum value that will produce breakdown in air. We use the same approach as in Examples 23-4 and 23-5.

$$V_{\text{surface}}=r_0E_{\text{breakdown}}\;\rightarrow\; r_0=\frac{V_{\text{surface}}}{E_{\text{breakdown}}}=\frac{35{,}000\,\text{V}}{3\times10^{6}\,\text{V/m}}=0.0117\,\text{m}\approx\boxed{0.012\,\text{m}}$$

$$V_{\text{surface}}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}\;\rightarrow\; Q=4\pi\varepsilon_0V_{\text{surface}}r_0=\left(\frac{1}{8.99\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2}\right)(35{,}000\,\text{V})(0.0117\,\text{m})$$

$$=\boxed{4.6\times10^{-8}\text{C}}$$

10. If we assume the electric field is uniform, then we can use Eq. 23-4b to estimate the magnitude of the electric field. From Problem 22-24 we have an expression for the electric field due to a pair of oppositely charged planes. We approximate the area of a shoe as 30 cm x 8 cm.

$$E=\frac{V}{d}=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0A}\;\rightarrow$$

$$Q=\frac{\varepsilon_0AV}{d}=\frac{\left(8.85\times10^{-12}\text{C}^2/\text{Nm}^2\right)\left(0.024\text{m}^2\right)\left(5.0\times10^{3}\text{V}\right)}{1.0\times10^{-3}\,\text{m}}=\boxed{1.1\times10^{-6}\text{C}}$$

11. Since the field is uniform, we may apply Eq. 23-4b. Note that the electric field always points from high potential to low potential.

(*a*) $\boxed{V_{\text{BA}}=0}$. The distance between the two points is exactly perpendicular to the field lines.

(*b*) $V_{\text{CB}}=V_{\text{C}}-V_{\text{B}}=(-4.20\,\text{N/C})(7.00\,\text{m})=\boxed{-29.4\,\text{V}}$

(*c*) $V_{\text{CA}}=V_{\text{C}}-V_{\text{A}}=V_{\text{C}}-V_{\text{B}}+V_{\text{B}}-V_{\text{A}}=V_{\text{CB}}+V_{\text{BA}}=-29.4\,\text{V}+0=\boxed{-29.4\,\text{V}}$

12. From Example 22-7, the electric field produced by a large plate is uniform with magnitude $E = \frac{\sigma}{2\varepsilon_0}$.

The field points away from the plate, assuming that the charge is positive. Apply Eq. 23-41.

$$V(x) - V(0) = V(x) - V_0 = -\int_0^x \vec{\mathbf{E}} \cdot (d\vec{\boldsymbol{\ell}}) = -\int_0^x \left(\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{i}}\right) \cdot (dx\hat{\mathbf{i}}) = -\frac{\sigma x}{2\varepsilon_0} \rightarrow \boxed{V(x) = V_0 - \frac{\sigma x}{2\varepsilon_0}}$$

13. (*a*) The electric field at the surface of the Earth is the same as that of a point charge, $E = \frac{Q}{4\pi\varepsilon_0 r_0^2}$.

The electric potential at the surface, relative to $V(\infty) = 0$ is given by Eq. 23-5. Writing this in terms of the electric field and radius of the earth gives the electric potential.

$$V = \frac{Q}{4\pi\varepsilon_0 r_0} = Er_0 = (-150\,\text{V/m})(6.38 \times 10^6\,\text{m}) = \boxed{-0.96\text{ GV}}$$

(*b*) Part (*a*) demonstrated that the potential at the surface of the earth is 0.96 GV lower than the potential at infinity. Therefore if the potential at the surface of the Earth is taken to be zero, the potential at infinity must be $V(\infty) = \boxed{0.96\text{ GV}}$. If the charge of the ionosphere is included in the calculation, the electric field outside the ionosphere is basically zero. The electric field between the earth and the ionosphere would remain the same. The electric potential, which would be the integral of the electric field from infinity to the surface of the earth, would reduce to the integral of the electric field from the ionosphere to the earth. This would result in a negative potential, but of a smaller magnitude.

14. (*a*) The potential at the surface of a charged sphere is derived in Example 23-4.

$$V_0 = \frac{Q}{4\pi\varepsilon_0 r_0} \rightarrow Q = 4\pi\varepsilon_0 r_0 V_0 \rightarrow$$

$$\sigma = \frac{Q}{\text{Area}} = \frac{Q}{4\pi r_0^2} = \frac{4\pi\varepsilon_0 r_0 V_0}{4\pi r_0^2} = \frac{V_0\varepsilon_0}{r_0} = \frac{(680\,\text{V})(8.85 \times 10^{-12}\ \text{C}^2/\text{Nm}^2)}{(0.16\,\text{m})} = 3.761 \times 10^{-8}\,\text{C/m}^2$$

$$\approx \boxed{3.8 \times 10^{-8}\,\text{C/m}^2}$$

(*b*) The potential away from the surface of a charged sphere is also derived in Example 23-4.

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{4\pi\varepsilon_0 r_0 V_0}{4\pi\varepsilon_0 r} = \frac{r_0 V_0}{r} \rightarrow r = \frac{r_0 V_0}{V} = \frac{(0.16\,\text{m})(680\,\text{V})}{(25\,\text{V})} = 4.352\,\text{m} \approx \boxed{4.4\,\text{m}}$$

15. (*a*) After the connection, the two spheres are at the $\boxed{\text{same potential}}$. If they were at different potentials, then there would be a flow of charge in the wire until the potentials were equalized.

(*b*) We assume the spheres are so far apart that the charge on one sphere does not influence the charge on the other sphere. Another way to express this would be to say that the potential due to either of the spheres is zero at the location of the other sphere. The charge splits between the spheres so that their potentials (due to the charge on them only) are equal. The initial charge on sphere 1 is $Q$, and the final charge on sphere 1 is $Q_1$.

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1} \ ; \ V_2 = \frac{Q - Q_1}{4\pi\varepsilon_0 r_2} \ ; \ V_1 = V_2 \rightarrow \frac{Q_1}{4\pi\varepsilon_0 r_1} = \frac{Q - Q_1}{4\pi\varepsilon_0 r_2} \rightarrow Q_1 = Q\frac{r_1}{(r_1 + r_2)}$$

Charge transferred $Q - Q_1 = Q - Q\frac{r_1}{(r_1 + r_2)} = \boxed{Q\frac{r_2}{(r_1 + r_2)}}$

16. From Example 22-6, the electric field due to a long wire is radial relative to the wire, and is of magnitude $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$. If the charge density is positive, the field lines point radially away from the wire. Use Eq. 23-41 to find the potential difference, integrating along a line that is radially outward from the wire.

$$V_b - V_a = -\int_{R_a}^{R_b} \vec{\mathbf{E}}\bullet\left(d\vec{\boldsymbol{\ell}}\right) = -\int_{R_a}^{R_b} \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}dR = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(R_b - R_a\right) = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_a}{R_b}}$$

17. (*a*) The width of the end of a finger is about 1 cm, and so consider the fingertip to be a part of a sphere of diameter 1 cm. We assume that the electric field at the radius of the sphere is the minimum value that will produce breakdown in air. We use the same approach as in Examples 23-4 and 23-5.

$$V_{\text{surface}} = r_0 E_{\text{breakdown}} = \left(0.005\,\text{m}\right)\left(3\times10^6\ \text{V/m}\right) = \boxed{15{,}000\,\text{V}}$$

Since this is just an estimate, we might expect anywhere from 10,000 V to 20,000 V.

(*b*) $V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi r_0^2\sigma}{r_0} \quad\rightarrow$

$$\sigma = V_{\text{surface}}\frac{\varepsilon_0}{r_0} = \left(15{,}000\,\text{V}\right)\frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)}{0.005\,\text{m}} = \boxed{2.7\times10^{-5}\ \text{C/m}^2}$$

Since this is an estimate, we might say the charge density is on the order of $30\,\mu\text{C/m}^2$.

18. We assume the field is uniform, and so Eq. 23-4b applies.

$$E = \frac{V}{d} = \frac{0.10\,\text{V}}{10\times10^{-9}\,\text{m}} = \boxed{1\times10^7\ \text{V/m}}$$

19. (*a*) The electric field outside a charged, spherically symmetric volume is the same as that for a point charge of the same magnitude of charge. Integrating the electric field from infinity to the radius of interest will give the potential at that radius.

$$E\left(r \ge r_0\right) = \frac{Q}{4\pi\varepsilon_0 r^2} \quad ; \quad V\left(r \ge r_0\right) = -\int_{\infty}^{r}\frac{Q}{4\pi\varepsilon_0 r^2}dr = \left.\frac{Q}{4\pi\varepsilon_0 r}\right|_{\infty}^{r} = \boxed{\frac{Q}{4\pi\varepsilon_0 r}}$$

(*b*) Inside the sphere the electric field is obtained from Gauss's Law using the charge enclosed by a sphere of radius *r*.

$$4\pi r^2 E = \frac{Q}{\varepsilon_0}\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r_0^3} \quad\rightarrow\quad E\left(r < r_0\right) = \frac{Qr}{4\pi\varepsilon_0 r_0^3}$$

Integrating the electric field from the surface to $r < r_0$ gives the electric potential inside the sphere.

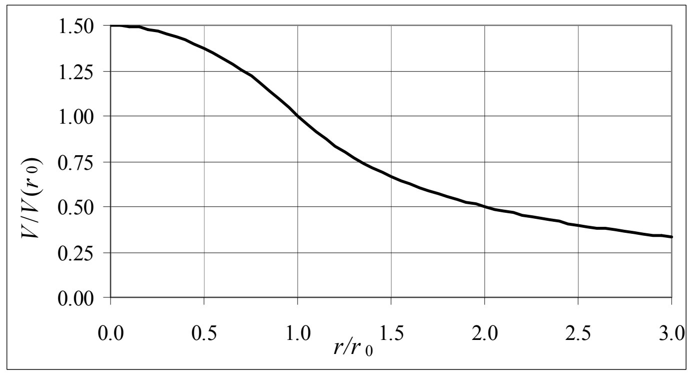
$$V\left(r < r_0\right) = V\left(r_0\right) - \int_{r_0}^{r}\frac{Qr}{4\pi\varepsilon_0 r_0^3}dr = \frac{Q}{4\pi\varepsilon_0 r_0} - \left.\frac{Qr^2}{8\pi\varepsilon_0 r_0^3}\right|_{r_0}^{r} = \boxed{\frac{Q}{8\pi\varepsilon_0 r_0}\left(3 - \frac{r^2}{r_0^2}\right)}$$

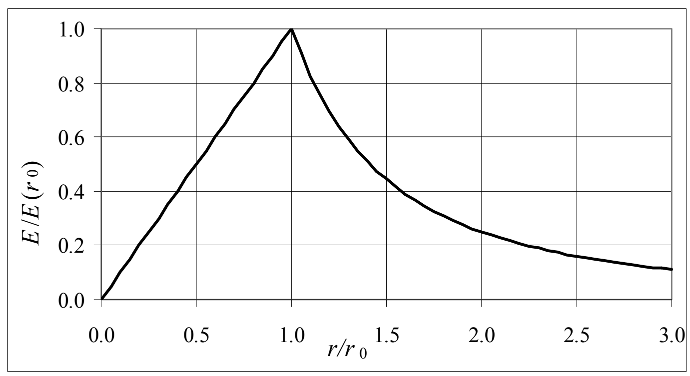
(*c*) To plot, we first calculate $V_0 = V\left(r = r_0\right) = \frac{Q}{4\pi\varepsilon_0 r_0}$ and $E_0 = E\left(r = r_0\right) = \frac{Q}{4\pi\varepsilon_0 r_0^2}$. Then we plot $V/V_0$ and $E/E_0$ as functions of $r/r_0$.

For $r < r_0$: $V/V_0 = \dfrac{\dfrac{Q}{8\pi\varepsilon_0 r_0}\left(3 - \dfrac{r^2}{r_0^2}\right)}{\dfrac{Q}{4\pi\varepsilon_0 r_0}} = \frac{1}{2}\left(3 - \dfrac{r^2}{r_0^2}\right)$ ; $E/E_0 = \dfrac{\dfrac{Qr}{4\pi\varepsilon_0 r_0^3}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}} = \dfrac{r}{r_0}$

For $r > r_0$: $V/V_0 = \dfrac{\dfrac{Q}{4\pi\varepsilon_0 r}}{\dfrac{Q}{4\pi\varepsilon_0 r_0}} = \dfrac{r_0}{r} = \left(r/r_0\right)^{-1}$ ; $E/E_0 = \dfrac{\dfrac{Q}{4\pi\varepsilon_0 r^2}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}} = \dfrac{r_0^2}{r^2} = \left(r/r_0\right)^{-2}$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.19c."




20. We assume the total charge is still $Q$, and let $\rho_E = kr^2$. We evaluate the constant $k$ by calculating the total charge, in the manner of Example 22-5.

$$Q = \int \rho_E dV = \int_0^{r_0} kr^2\left(4\pi r^2 dr\right) = \tfrac{4}{5}k\pi r_0^5 \quad \rightarrow \quad k = \frac{5Q}{4\pi r_0^5}$$

(*a*) The electric field outside a charged, spherically symmetric volume is the same as that for a point charge of the same magnitude of charge. Integrating the electric field from infinity to the radius of interest gives the potential at that radius.

$$E\left(r \geq r_0\right) = \frac{Q}{4\pi\varepsilon_0 r^2} \; ; \; V\left(r \geq r_0\right) = -\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r^2}dr = \left.\frac{Q}{4\pi\varepsilon_0 r}\right|_\infty^r = \boxed{\frac{Q}{4\pi\varepsilon_0 r}}$$

(*b*) Inside the sphere the electric field is obtained from Gauss's Law using the charge enclosed by a sphere of radius $r$.

$$4\pi r^2 E = \frac{Q_{\text{encl}}}{\varepsilon_0} \; ; \; Q_{\text{encl}} = \int \rho_E dV = \frac{5Q}{4\pi r_0^5}\int_0^r r^2\left(4\pi r^2 dr\right) = \frac{5Q}{4\pi r_0^5}\tfrac{4}{5}\pi r^5 = \frac{Qr^5}{r_0^5} \quad \rightarrow$$

$$E\left(r<r_0\right)=\frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}=\frac{Qr^3}{4\pi\varepsilon_0 r_0^5}$$

Integrating the electric field from the surface to $r<r_0$ gives the electric potential inside the sphere.

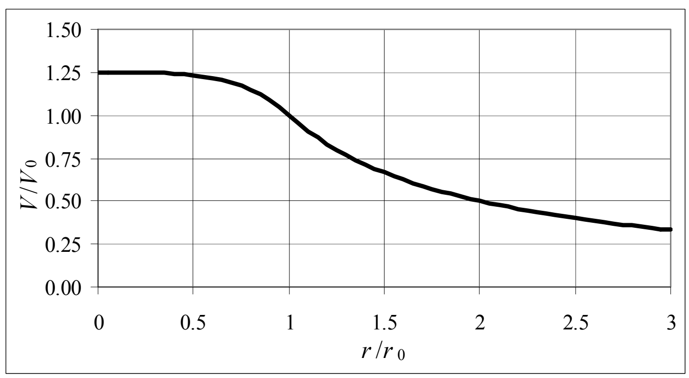
$$V\left(r<r_0\right)=V\left(r_0\right)-\int_{r_0}^{r}\frac{Qr^3}{4\pi\varepsilon_0 r_0^5}dr=\frac{Q}{4\pi\varepsilon_0 r_0}-\left.\frac{Qr^4}{16\pi\varepsilon_0 r_0^5}\right|_{r_0}^{r}=\frac{Q}{16\pi\varepsilon_0 r_0}\left(5-\frac{r^4}{r_0^4}\right)$$

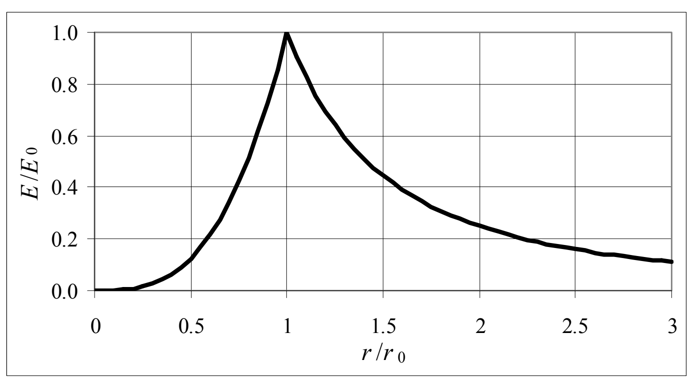
(*c*) To plot, we first calculate $V_0=V\left(r=r_0\right)=\frac{Q}{4\pi\varepsilon_0 r_0}$ and $E_0=E\left(r=r_0\right)=\frac{Q}{4\pi\varepsilon_0 r_0^2}$. Then we plot $V/V_0$ and $E/E_0$ as functions of $r/r_0$.

For $r<r_0$: $\quad V/V_0=\dfrac{\dfrac{Q}{16\pi\varepsilon_0 r_0}\left(5-\dfrac{r^4}{r_0^4}\right)}{\dfrac{Q}{4\pi\varepsilon_0 r_0}}=\frac{1}{4}\left(5-\dfrac{r^4}{r_0^4}\right)$ ; $E/E_0=\dfrac{\dfrac{Qr^3}{4\pi\varepsilon_0 r_0^5}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}}=\dfrac{r^3}{r_0^3}$

For $r>r_0$: $\quad V/V_0=\dfrac{\dfrac{Q}{4\pi\varepsilon_0 r}}{\dfrac{Q}{4\pi\varepsilon_0 r_0}}=\dfrac{r_0}{r}=\left(r/r_0\right)^{-1}$ ; $E/E_0=\dfrac{\dfrac{Q}{4\pi\varepsilon_0 r^2}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}}=\dfrac{r_0^2}{r^2}=\left(r/r_0\right)^{-2}$

The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH23.XLS,” on tab “Problem 23.20c.”



21. We first need to find the electric field. Since the charge distribution is spherically symmetric, Gauss's law tells us the electric field everywhere.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\rm encl}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\rm encl}}{r^2}$$

If $r < r_0$, calculate the charge enclosed in the manner of Example 22-5.

$$Q_{\rm encl} = \int \rho_{\rm E}\,dV = \int_0^r \rho_0\left[1-\frac{r^2}{r_0^2}\right]4\pi r^2 dr = 4\pi\rho_0\int_0^r\left[r^2-\frac{r^4}{r_0^2}\right]dr = 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5r_0^2}\right]$$

The total charge in the sphere is the above expression evaluated at $r = r_0$.

$$Q_{\rm total} = 4\pi\rho_0\left[\frac{r_0^3}{3}-\frac{r_0^5}{5r_0^2}\right] = \frac{8\pi\rho_0 r_0^3}{15}$$

Outside the sphere, we may treat it as a point charge, and so the potential at the surface of the sphere is given by Eq. 23-5, evaluated at the surface of the sphere.

$$V\left(r=r_0\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\rm total}}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{8\pi\rho_0 r_0^3}{15}}{r_0} = \frac{2\rho_0 r_0^2}{15\varepsilon_0}$$

The potential inside is found from Eq. 23-4a. We need the field inside the sphere to use Eq. 23-4a. The field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = E dr$.

$$E\left(r<r_0\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\rm encl}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5r_0^2}\right]}{r^2} = \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3}-\frac{r^3}{5r_0^2}\right]$$

$$V_r - V_{r_0} = -\int_{r_0}^{r}\vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = -\int_{r_0}^{r}E\,dr = -\int_{r_0}^{r}\frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3}-\frac{r^3}{5r_0^2}\right]dr = -\frac{\rho_0}{\varepsilon_0}\left[\frac{r^2}{6}-\frac{r^4}{20r_0^2}\right]_{r_0}^{r}$$

$$V_r = V_{r_0} + \left(-\frac{\rho_0}{\varepsilon_0}\left[\frac{r^2}{6}-\frac{r^4}{20r_0^2}\right]_{r_0}^{r}\right) = \frac{2\rho_0 r_0^2}{15\varepsilon_0} - \frac{\rho_0}{\varepsilon_0}\left[\left(\frac{r^2}{6}-\frac{r^4}{20r_0^2}\right)-\left(\frac{r_0^2}{6}-\frac{r_0^4}{20r_0^2}\right)\right]$$

$$= \boxed{\frac{\rho_0}{\varepsilon_0}\left(\frac{r_0^2}{4}-\frac{r^2}{6}+\frac{r^4}{20r_0^2}\right)}$$

22. Because of the spherical symmetry of the problem, the electric field in each region is the same as that of a point charge equal to the net enclosed charge.

(*a*) For $r > r_2$: $E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\rm encl}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{3}{2}Q}{r^2} = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r^2}}$

For $r_1 < r < r_2$: $E = \boxed{0}$, because the electric field is 0 inside of conducting material.

For $0 < r < r_1$: $E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\rm encl}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{1}{2}Q}{r^2} = \boxed{\frac{1}{8\pi\varepsilon_0}\frac{Q}{r^2}}$

(*b*) For $r > r_2$, the potential is that of a point charge at the center of the sphere.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{3}{2}Q}{r} = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r}},\ r > r_2$$

(*c*) For $r_1 < r < r_2$, the potential is constant and equal to its value on the outer shell, because there is no electric field inside the conducting material.

$$V = V\left(r = r_2\right) = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r_2}},\ r_1 < r < r_2$$

(*d*) For $0 < r < r_1$, we use Eq. 23-4a. The field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}}\bullet d\vec{\boldsymbol{\ell}} = E dr$.

$$V_r - V_{r_1} = -\int_{r_1}^{r} \vec{\mathbf{E}}\bullet d\vec{\boldsymbol{\ell}} = -\int_{r_1}^{r} E\,dr = -\int_{r_1}^{r} \frac{1}{8\pi\varepsilon_0}\frac{Q}{r^2}dr = \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

$$V_r = V_{r_1} + \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right) = \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{2r_1} + \frac{1}{r}\right) = \boxed{\frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r_2} + \frac{1}{r}\right)},\ 0 < r < r_1$$

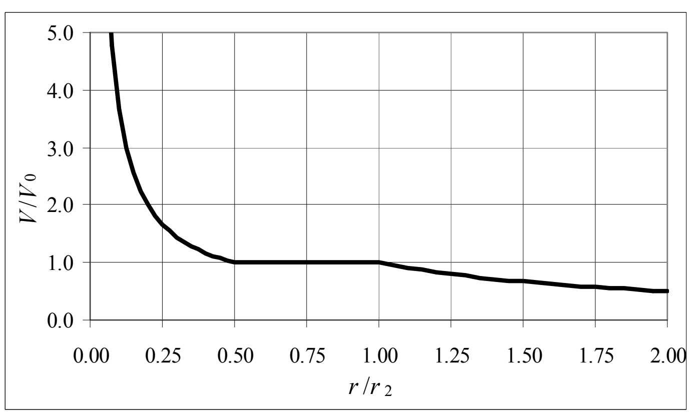
(*e*) To plot, we first calculate $V_0 = V\left(r = r_2\right) = \dfrac{3Q}{8\pi\varepsilon_0 r_2}$ and $E_0 = E\left(r = r_2\right) = \dfrac{3Q}{8\pi\varepsilon_0 r_2^2}$. Then we plot $V/V_0$ and $E/E_0$ as functions of $r/r_2$.

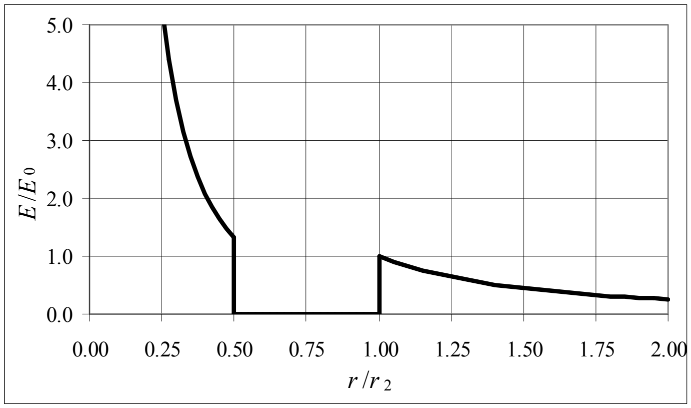
For $0 < r < r_1$: $$\frac{V}{V_0} = \frac{\dfrac{Q}{8\pi\varepsilon_0}\left(\dfrac{1}{r_2} + \dfrac{1}{r}\right)}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = \tfrac{1}{3}\left[1 + \left(r/r_2\right)^{-1}\right]\ ;\ \frac{E}{E_0} = \frac{\dfrac{1}{8\pi\varepsilon_0}\dfrac{Q}{r^2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = \tfrac{1}{3}\frac{r_2^2}{r^2} = \tfrac{1}{3}\left(r/r_2\right)^{-2}$$

For $r_1 < r < r_2$: $$\frac{V}{V_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r_2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = 1\ ;\ \frac{E}{E_0} = \frac{0}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = 0$$

For $r > r_2$: $$\frac{V}{V_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = \frac{r_2}{r} = \left(r/r_2\right)^{-1}\ ;\ \frac{E}{E_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r^2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = \frac{r_2^2}{r^2} = \left(r/r_2\right)^{-2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.22e."






23. The field is found in Problem 22-33. The field inside the cylinder is 0, and the field outside the cylinder is $\dfrac{\sigma R_0}{\varepsilon_0 R}$.

(*a*) Use Eq. 23-4a to find the potential. Integrate along a radial line, so that $\vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = EdR$.

$$V_R - V_{R_0} = -\int_{R_0}^{R} \vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = -\int_{R_0}^{R} E\,dR = -\int_{R_0}^{R} \frac{\sigma R_0}{\varepsilon_0 R}\,dR = -\frac{\sigma R_0}{\varepsilon_0}\ln\frac{R}{R_0} \rightarrow$$

$$V_R = \boxed{V_0 - \frac{\sigma R_0}{\varepsilon_0}\ln\frac{R}{R_0}},\ R > R_0$$

(*b*) The electric field inside the cylinder is 0, so the potential inside is constant and equal to the potential on the surface, $\boxed{V_0}$.

(*c*) $\boxed{\text{No}}$, we are not able to assume that $V = 0$ at $R = \infty$. $V \neq 0$ because there would be charge at infinity for an infinite cylinder. And from the formula derived in (*a*), if $R = \infty$, $V_R = -\infty$.

24. Use Eq. 23-5 to find the charge.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \rightarrow Q = (4\pi\varepsilon_0) rV = \left(\frac{1}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\right)(0.15\ \text{m})(185\,\text{V}) = \boxed{3.1\times10^{-9}\,\text{C}}$$

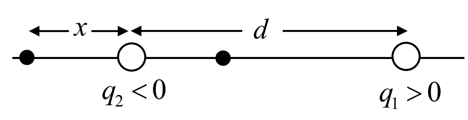
$\boxed{25.}$ (*a*) The electric potential is given by Eq. 23-5.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{1.60\times10^{-19}\,\text{C}}{0.50\times10^{-10}\,\text{m}} = 28.77\,\text{V} \approx \boxed{29\,\text{V}}$$

(*b*) The potential energy of the electron is the charge of the electron times the electric potential due to the proton.

$$U = QV = \left(-1.60\times10^{-19}\text{C}\right)(28.77\,\text{V}) = \boxed{-4.6\times10^{-18}\,\text{J}}$$

26. (*a*) Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge $(q_2)$. Also, in between the two charges, the fields due to the two charges are parallel to each other (both to the left) and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge,

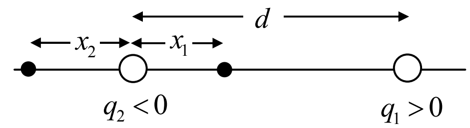


but not between them. In the diagram, this is the point to the left of $q_2$. Take rightward as the positive direction.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{x^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{(d+x)^2} = 0 \rightarrow |q_2|(d+x)^2 = q_1 x^2 \rightarrow$$

$$x = \frac{\sqrt{|q_2|}}{\sqrt{q_1}-\sqrt{|q_2|}}d = \frac{\sqrt{2.0\times10^{-6}\,\text{C}}}{\sqrt{3.4\times10^{-6}\,\text{C}}-\sqrt{2.0\times10^{-6}\,\text{C}}}(5.0\,\text{cm}) = \boxed{16\,\text{cm left of } q_2}$$

(*b*) The potential due to the positive charge is positive everywhere, and the potential due to the negative charge is negative everywhere. Since the negative charge is smaller in magnitude than the positive charge, any point where the potential is zero must be closer to the negative charge. So consider locations between the charges (position $x_1$) and to the left of the negative charge (position $x_2$) as shown in the diagram.




$$V_{\text{location 1}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(d-x_1)} + \frac{q_2}{x_1}\right] = 0 \rightarrow x_1 = \frac{q_2 d}{(q_2 - q_1)} = \frac{(-2.0\times10^{-6}\,\text{C})(5.0\,\text{cm})}{(-5.4\times10^{-6}\,\text{C})} = 1.852\,\text{cm}$$

$$V_{\text{location 2}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(d+x_2)} + \frac{q_2}{x_2}\right] = 0 \rightarrow$$

$$x_2 = -\frac{q_2 d}{(q_1+q_2)} = -\frac{(-2.0\times10^{-6}\,\text{C})(5.0\,\text{cm})}{(1.4\times10^{-6}\,\text{C})} = 7.143\,\text{cm}$$

So the two locations where the potential is zero are **1.9 cm from the negative charge towards the positive charge, and 7.1 cm from the negative charge away from the positive charge.**

27. The work required is the difference in the potential energy of the charges, calculated with the test charge at the two different locations. The potential energy of a pair of charges is given in Eq. 23-10 as $U = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r}$. So to find the work, calculate the difference in potential energy with the test charge at the two locations. Let *Q* represent the $25\mu\text{C}$ charge, let *q* represent the $0.18\mu\text{C}$ test charge, *D* represent the 6.0 cm distance, and let *d* represent the 1.0 cm distance. Since the potential energy of the two $25\mu\text{C}$ charges doesn't change, we don't include it in the calculation.

$$U_{\text{initial}} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{D/2} + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{D/2} \qquad U_{\text{final}} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2-d]} + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2+d]}$$

$$\text{Work}_{\substack{\text{external}\\\text{force}}} = U_{\text{final}} - U_{\text{initial}} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2-d]} + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2+d]} - 2\left(\frac{1}{4\pi\varepsilon_0}\frac{Qq}{D/2}\right)$$

$$= \frac{2Qq}{4\pi\varepsilon_0}\left[\frac{1}{[D-2d]} + \frac{1}{[D+2d]} - \frac{1}{D/2}\right]$$

$$= 2(8.99\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2)(25\times10^{-6}\,\text{C})(0.18\times10^{-6}\,\text{C})\left[\frac{1}{0.040\,\text{m}} + \frac{1}{0.080\,\text{m}} - \frac{1}{0.030\,\text{m}}\right]$$

$$= \boxed{0.34\,\text{J}}$$

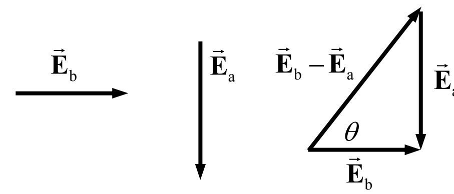
An external force needs to do positive work to move the charge.

28. (*a*) The potential due to a point charge is given by Eq. 23-5.

$$V_{ba} = V_b - V_a = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_b} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_a} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$= \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(-3.8\times10^{-6}\text{C}\right)\left(\frac{1}{0.36\,\text{m}} - \frac{1}{0.26\,\text{m}}\right) = \boxed{3.6\times10^4\,\text{V}}$$

(*b*) The magnitude of the electric field due to a point charge is given by Eq. 21-4a. The direction of the electric field due to a negative charge is towards the charge, so the field at point a will point downward, and the field at point b will point to the right. See the vector diagram.




$$\vec{\mathbf{E}}_b = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r_b^2}\hat{\mathbf{i}} = \frac{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(3.8\times10^{-6}\text{C}\right)}{\left(0.36\,\text{m}\right)^2}\hat{\mathbf{i}} = 2.636\times10^5\ \text{V/m}\,\hat{\mathbf{i}}$$

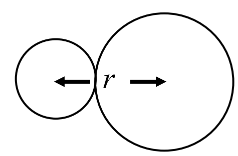
$$\vec{\mathbf{E}}_a = -\frac{1}{4\pi\varepsilon_0}\frac{|q|}{r_a^2}\hat{\mathbf{j}} = -\frac{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(3.8\times10^{-6}\text{C}\right)}{\left(0.26\,\text{m}\right)^2}\hat{\mathbf{j}} = -5.054\times10^5\ \text{V/m}\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}_b - \vec{\mathbf{E}}_a = 2.636\times10^5\ \text{V/m}\,\hat{\mathbf{i}} + 5.054\times10^5\ \text{V/m}\,\hat{\mathbf{j}}$$

$$\left|\vec{\mathbf{E}}_b - \vec{\mathbf{E}}_a\right| = \sqrt{\left(2.636\times10^5\ \text{V/m}\right)^2 + \left(5.054\times10^5\ \text{V/m}\right)^2} = \boxed{5.7\times10^5\ \text{V/m}}$$

$$\theta = \tan^{-1}\frac{-E_a}{E_b} = \tan^{-1}\frac{5.054\times10^5}{2.636\times10^5} = \boxed{62^\circ}$$

29. We assume that all of the energy the proton gains in being accelerated by the voltage is changed to potential energy just as the proton's outer edge reaches the outer radius of the silicon nucleus.




$$U_{\text{initial}} = U_{\text{final}} \quad\rightarrow\quad eV_{\text{initial}} = \frac{1}{4\pi\varepsilon_0}\frac{e(14e)}{r} \quad\rightarrow$$

$$V_{\text{initial}} = \frac{1}{4\pi\varepsilon_0}\frac{14e}{r} = \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(14)\left(1.60\times10^{-19}\text{C}\right)}{(1.2+3.6)\times10^{-15}\text{m}} = \boxed{4.2\times10^6\,\text{V}}$$

30. By energy conservation, all of the initial potential energy of the charges will change to kinetic energy when the charges are very far away from each other. By momentum conservation, since the initial momentum is zero and the charges have identical masses, the charges will have equal speeds in opposite directions from each other as they move. Thus each charge will have the same kinetic energy.

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad U_{\text{initial}} = K_{\text{final}} \quad\rightarrow\quad \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r} = 2\left(\tfrac{1}{2}mv^2\right) \quad\rightarrow$$

$$v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{mr}} = \sqrt{\frac{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(5.5\times10^{-6}\text{C}\right)^2}{\left(1.0\times10^{-6}\text{kg}\right)\left(0.065\,\text{m}\right)}} = \boxed{2.0\times10^3\ \text{m/s}}$$

31. By energy conservation, all of the initial potential energy will change to kinetic energy of the electron when the electron is far away. The other charge is fixed, and so has no kinetic energy. When the electron is far away, there is no potential energy.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow U_{\text{initial}} = K_{\text{final}} \rightarrow \frac{(-e)(Q)}{4\pi\varepsilon_0 r} = \tfrac{1}{2}mv^2 \rightarrow$$

$$v = \sqrt{\frac{2(-e)(Q)}{(4\pi\varepsilon_0)mr}} = \sqrt{\frac{2\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(-1.60\times10^{-19}\text{C}\right)\left(-1.25\times10^{-10}\text{C}\right)}{\left(9.11\times10^{-31}\text{kg}\right)(0.425\,\text{m})}}$$

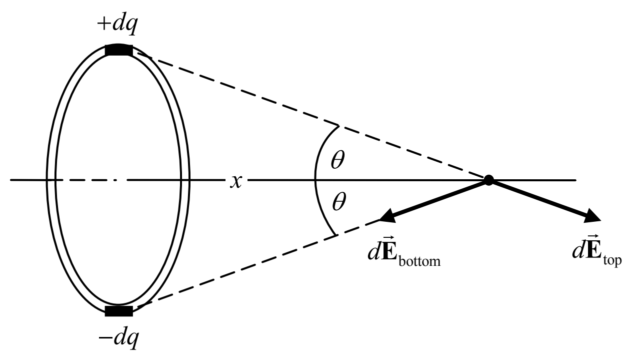
$$= \boxed{9.64\times10^5\ \text{m/s}}$$

32. Use Eq. 23-2b and Eq. 23-5.

$$V_{\text{BA}} = V_{\text{B}} - V_{\text{A}} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d-b} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{b}\right) - \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{b} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{d-b}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{d-b} - \frac{1}{b} - \frac{1}{b} + \frac{1}{d-b}\right) = 2\frac{1}{4\pi\varepsilon_0}q\left(\frac{1}{d-b} - \frac{1}{b}\right) = \boxed{\frac{2q(2b-d)}{4\pi\varepsilon_0 b(d-b)}}$$

33. (*a*) For every element $dq$ as labeled in Figure 23-14 on the top half of the ring, there will be a diametrically opposite element of charge $-dq$. The potential due to those two infinitesimal elements will cancel each other, and so the potential due to the entire ring is $\boxed{0}$.




(*b*) We follow Example 21-9 from the textbook. But because the upper and lower halves of the ring are oppositely charged, the parallel components of the fields from diametrically opposite infinitesimal segments of the ring will cancel each other, and the perpendicular components add, in the negative $y$ direction. We know then that $\boxed{E_x = 0}$.

$$dE_y = -dE\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\frac{Q}{2\pi R}d\ell}{\left(x^2+R^2\right)}\frac{R}{\left(x^2+R^2\right)^{1/2}} = -\frac{Q}{8\pi^2\varepsilon_0}\frac{d\ell}{\left(x^2+R^2\right)^{3/2}}$$

$$E_y = \int_0^{2\pi R} dE_y = -\frac{Q}{8\pi^2\varepsilon_0}\frac{1}{\left(x^2+R^2\right)^{3/2}}\int_0^{2\pi R} d\ell = -\frac{Q}{4\pi\varepsilon_0}\frac{R}{\left(x^2+R^2\right)^{3/2}} \rightarrow$$

$$\vec{\mathbf{E}} = \boxed{-\frac{Q}{4\pi\varepsilon_0}\frac{R}{\left(x^2+R^2\right)^{3/2}}\hat{\mathbf{j}}}$$

Note that for $x \gg R$, this reduces to $\vec{\mathbf{E}} = -\frac{Q}{4\pi\varepsilon_0}\frac{R}{x^3}\hat{\mathbf{j}}$, which has the typical distance dependence for the field of a dipole, along the axis of the dipole.

34. The potential at the corner is the sum of the potentials due to each of the charges, using Eq. 23-5.

$$V=\frac{1}{4\pi\varepsilon_0}\frac{(3Q)}{\ell}+\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}\ell}+\frac{1}{4\pi\varepsilon_0}\frac{(-2Q)}{\ell}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}\left(1+\frac{1}{\sqrt{2}}\right)=\boxed{\frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}Q}{2\ell}\left(\sqrt{2}+1\right)}$$

35. We follow the development of Example 23-9, with Figure 23-15. The charge on a thin ring of radius $R$ and thickness $dR$ is $dq=\sigma dA=\sigma(2\pi R dR)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{\sigma(2\pi R dR)}{\sqrt{x^2+R^2}}=\frac{\sigma}{2\varepsilon_0}\int_{R_1}^{R_2}\frac{R}{\sqrt{x^2+R^2}}dR=\frac{\sigma}{2\varepsilon_0}\left(x^2+R^2\right)^{1/2}\Big|_{R_1}^{R_2}$$

$$=\boxed{\frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2+R_2^2}-\sqrt{x^2+R_1^2}\right)}$$

36. All of the charge is the same distance from the center of the semicircle – the radius of the semicircle. Use Eq 23-6b to calculate the potential.

$$\ell=\pi r_0\ \rightarrow\ r_0=\frac{\ell}{\pi}\ ;\ V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0 r_0}\int dq=\frac{Q}{4\pi\varepsilon_0\frac{\ell}{\pi}}=\boxed{\frac{Q}{4\varepsilon_0\ell}}$$

37. The electric potential energy is the product of the point charge and the electric potential at the location of the charge. Since all points on the ring are equidistant from any point on the axis, the electric potential integral is simple.

$$U=qV=q\int\frac{dq}{4\pi\varepsilon_0\sqrt{r^2+x^2}}=\frac{q}{4\pi\varepsilon_0\sqrt{r^2+x^2}}\int dq=\frac{qQ}{4\pi\varepsilon_0\sqrt{r^2+x^2}}$$

Energy conservation is used to obtain a relationship between the potential and kinetic energies at the center of the loop and at a point 2.0 m along the axis from the center.

$$K_0+U_0=K+U$$

$$0+\frac{qQ}{4\pi\varepsilon_0\sqrt{r^2}}=\tfrac{1}{2}mv^2+\frac{qQ}{4\pi\varepsilon_0\sqrt{r^2+x^2}}$$

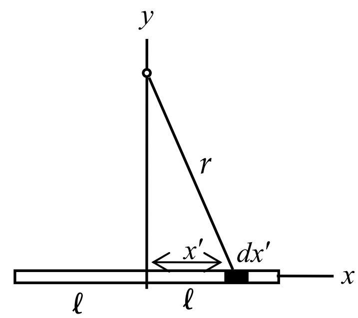
This is equation is solved to obtain the velocity at $x$ = 2.0 m.

$$v=\sqrt{\frac{qQ}{2\pi\varepsilon_0 m}\left(\frac{1}{r}-\frac{1}{\sqrt{r^2+x^2}}\right)}$$

$$=\sqrt{\frac{(3.0\,\mu\text{C})(15.0\,\mu\text{C})}{2\pi\left(8.85\times10^{-12}\text{C}^2/\text{Nm}^2\right)\left(7.5\times10^{-3}\text{kg}\right)}\left(\frac{1}{0.12\,\text{m}}-\frac{1}{\sqrt{(0.12\,\text{m})^2+(2.0\,\text{m})^2}}\right)}$$

$$=\boxed{29\text{ m/s}}$$

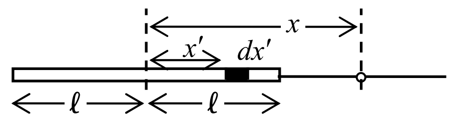
38. Use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length $dx'$ at position $x'$ along the rod. The charge on the element is $dq = \frac{Q}{2\ell}dx'$, and the element is a distance $r = \sqrt{x'^2 + y^2}$ from a point on the $y$ axis. Use an indefinite integral from Appendix B-4, page A-7.




$$V_{y\,\text{axis}} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{\frac{Q}{2\ell}dx'}{\sqrt{x'^2+y^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\ell}\left[\ln\left(\sqrt{x'^2+y^2}+x'\right)\right]_{-\ell}^{\ell} = \boxed{\frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{\sqrt{\ell^2+y^2}+\ell}{\sqrt{\ell^2+y^2}-\ell}\right)\right]}$$

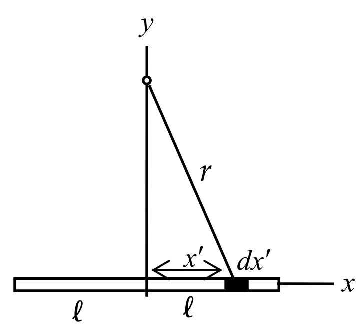
39. Use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length $dx'$ at position $x'$ along the rod. The charge on the element is $dq = \frac{Q}{2\ell}dx'$, and the element is a distance $x - x'$ from a point outside the rod on the $x$ axis.




$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{\frac{Q}{2\ell}dx'}{x-x'} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\ell}\left[-\ln\left(x-x'\right)\right]_{-\ell}^{\ell} = \boxed{\frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{x+\ell}{x-\ell}\right)\right]},\ x>\ell$$

40. For both parts of the problem, use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length $dx'$ at position $x'$ along the rod. The charge on the element is $dq = \lambda dx' = ax'dx'$.

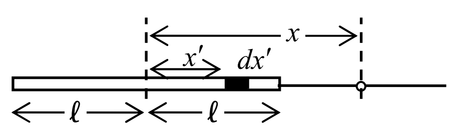
(*a*) The element is a distance $r = \sqrt{x'^2 + y^2}$ from a point on the $y$ axis.




$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{ax'dx'}{\sqrt{x'^2+y^2}} = 0$$

The integral is equal to 0 because the region of integration is "even" with respect to the origin, while the integrand is "odd." Alternatively, the antiderivative can be found, and the integral can be shown to be 0. This is to be expected since the potential from points symmetric about the origin would cancel on the $y$ axis.

(*b*) The element is a distance $x - x'$ from a point outside the rod on the $x$ axis.




$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{ax'dx'}{x-x'} = \frac{a}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{x'dx'}{x-x'}$$

A substitution of $z = x - x'$ can be used to do the integration.

$$V = \frac{a}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{x'dx'}{x-x'} = \frac{a}{4\pi\varepsilon_0}\int_{x+\ell}^{x-\ell}\frac{(x-z)(-dz)}{z} = \frac{a}{4\pi\varepsilon_0}\int_{x-\ell}^{x+\ell}\left(\frac{x}{z}-1\right)dz$$

$$= \frac{a}{4\pi\varepsilon_0}\left(x\ln z - z\right)_{x-\ell}^{x+\ell} = \boxed{\frac{a}{4\pi\varepsilon_0}\left[x\ln\left(\frac{x+\ell}{x-\ell}\right)-2\ell\right]},\ x>\ell$$

41. We follow the development of Example 23-9, with Figure 23-15. The charge on a thin ring of radius $R$ and thickness $dR$ will now be $dq = \sigma dA = \left(aR^2\right)\left(2\pi R dR\right)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_0^{R_0}\frac{\left(aR^2\right)\left(2\pi R dR\right)}{\sqrt{x^2+R^2}} = \frac{a}{2\varepsilon_0}\int_0^{R_0}\frac{R^3dR}{\sqrt{x^2+R^2}}$$

A substitution of $x^2+R^2=u^2$ can be used to do the integration.

$$x^2+R^2=u^2 \quad\rightarrow\quad R^2=u^2-x^2\ ;\ 2RdR=2udu$$

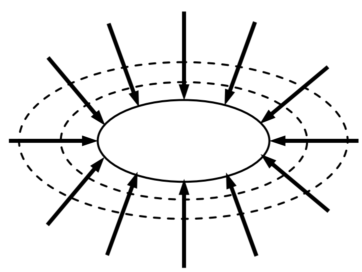
$$V = \frac{a}{2\varepsilon_0}\int_0^{R_0}\frac{R^3dR}{\sqrt{x^2+R^2}} = \frac{a}{2\varepsilon_0}\int_{R=0}^{R=R_0}\frac{\left(u^2-x^2\right)udu}{u} = \frac{a}{2\varepsilon_0}\left[\tfrac{1}{3}u^3-ux^2\right]_{R=0}^{R=R_0}$$

$$= \frac{a}{2\varepsilon_0}\left[\tfrac{1}{3}\left(x^2+R^2\right)^{3/2}-x^2\left(x^2+R^2\right)^{1/2}\right]_{R=0}^{R=R_0}$$

$$= \frac{a}{2\varepsilon_0}\left[\left\{\tfrac{1}{3}\left(x^2+R_0^2\right)^{3/2}-x^2\left(x^2+R_0^2\right)^{1/2}\right\}+\tfrac{2}{3}x^3\right]$$

$$= \boxed{\frac{a}{6\varepsilon_0}\left[\left(R_0^2-2x^2\right)\left(x^2+R_0^2\right)^{1/2}+2x^3\right]},\ x>0$$

42.



43. The electric field from a large plate is uniform with magnitude $E=\sigma/2\varepsilon_0$, with the field pointing away from the plate on both sides. Equation 23-4(a) can be integrated between two arbitrary points to calculate the potential difference between those points.

$$\Delta V = -\int_{x_0}^{x_1}\frac{\sigma}{2\varepsilon_0}dx = \frac{\sigma(x_0-x_1)}{2\varepsilon_0}$$

Setting the change in voltage equal to 100 V and solving for $x_0-x_1$ gives the distance between field lines.

$$x_0-x_1 = \frac{2\varepsilon_0\Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right)\left(100\ \text{V}\right)}{0.75\times10^{-6}\,\text{C/m}^2} = 2.36\times10^{-3}\,\text{m} \approx \boxed{2\ \text{mm}}$$

44. The potential at the surface of the sphere is $V_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}$. The potential outside the sphere is $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = V_0\frac{r_0}{r}$, and decreases as you move away from the surface. The difference in potential between a given location and the surface is to be a multiple of 100 V.

$$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} = \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(\frac{0.50\times10^{-6}\,\text{C}}{0.44\,\text{m}}\right) = 10{,}216\,\text{V}$$

$$V_0 - V = V_0 - V_0\frac{r_0}{r} = (100\,\text{V})n \quad\rightarrow\quad r = \frac{V_0}{\left[V_0 - (100\,\text{V})n\right]}r_0$$

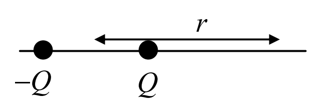
(*a*) $r_1 = \frac{V_0}{\left[V_0 - (100\,\text{V})1\right]}r_0 = \frac{10{,}216\,\text{V}}{10{,}116\,\text{V}}(0.44\,\text{m}) = \boxed{0.444\,\text{m}}$

Note that to within the appropriate number of significant figures, this location is at the surface of the sphere. That can be interpreted that we don't know the voltage well enough to be working with a 100-V difference.

(*b*) $r_{10} = \frac{V_0}{\left[V_0 - (100\,\text{V})10\right]}r_0 = \frac{10{,}216\,\text{V}}{9{,}216\,\text{V}}(0.44\,\text{m}) = \boxed{0.49\,\text{m}}$

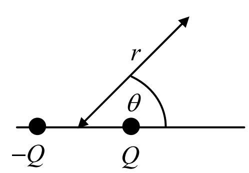
(*c*) $r_{100} = \frac{V_0}{\left[V_0 - (100\,\text{V})100\right]}r_0 = \frac{10{,}216\,\text{V}}{216\,\text{V}}(0.44\,\text{m}) = \boxed{21\,\text{m}}$

45. The potential due to the dipole is given by Eq. 23-7.



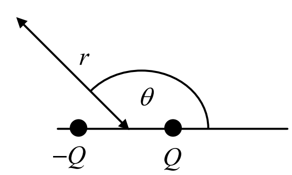

(*a*) $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(4.8\times10^{-30}\text{C}\bullet\text{m}\right)\cos 0}{\left(4.1\times10^{-9}\text{m}\right)^2}$

$= \boxed{2.6\times10^{-3}\,\text{V}}$




(*b*) $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(4.8\times10^{-30}\text{C}\bullet\text{m}\right)\cos 45^\circ}{\left(4.1\times10^{-9}\text{m}\right)^2}$

$= \boxed{1.8\times10^{-3}\,\text{V}}$




(*c*) $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(4.8\times10^{-30}\text{C}\bullet\text{m}\right)\cos 135^\circ}{\left(1.1\times10^{-9}\text{m}\right)^2}$

$= \boxed{-1.8\times10^{-3}\,\text{V}}$

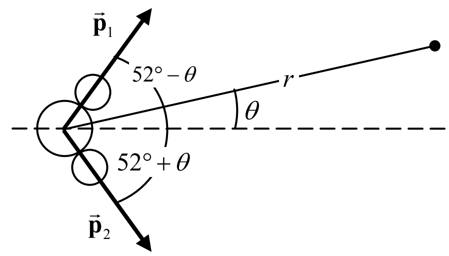
46. (*a*) We assume that $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$ are equal in magnitude, and that each makes a $52^\circ$ angle with $\vec{\mathbf{p}}$. The magnitude of $\vec{\mathbf{p}}_1$ is also given by $p_1 = qd$, where $q$ is the net charge on the hydrogen atom, and $d$ is the distance between the H and the O.

$$p = 2p_1\cos 52^\circ \quad\rightarrow\quad p_1 = \frac{p}{2\cos 52^\circ} = qd \quad\rightarrow$$

$$q = \frac{p}{2d\cos 52^\circ} = \frac{6.1\times10^{-30}\text{C}\bullet\text{m}}{2\left(0.96\times10^{-10}\text{m}\right)\cos 52^\circ} = \boxed{5.2\times10^{-20}\text{C}}$$

This is about 0.32 times the charge on an electron.

(*b*) Since we are considering the potential far from the dipoles, we will take the potential of each dipole to be given by Eq. 23-7. See the diagram for the angles involved. From part (a), $p_1 = p_2 = \dfrac{p}{2\cos 52^\circ}$.




$$V = V_{p_1} + V_{p_2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{p_1\cos(52^\circ - \theta)}{r} + \frac{1}{4\pi\varepsilon_0}\frac{p_2\cos(52^\circ + \theta)}{r}$$
$$= \frac{1}{4\pi\varepsilon_0 r}\frac{p}{2\cos 52^\circ}\left[\cos(52^\circ - \theta) + \cos(52^\circ + \theta)\right]$$
$$= \frac{1}{4\pi\varepsilon_0 r}\frac{p}{2\cos 52^\circ}\left(\cos 52^\circ \cos\theta + \sin 52^\circ \cos\theta + \cos 52^\circ \cos\theta - \sin 52^\circ \cos\theta\right)$$
$$= \frac{1}{4\pi\varepsilon_0 r}\frac{p}{2\cos 52^\circ}\left(2\cos 52^\circ \cos\theta\right) = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r}}$$

47. $E = -\dfrac{dV}{dr} = -\dfrac{d}{dr}\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}\right) = -\dfrac{q}{4\pi\varepsilon_0}\dfrac{d}{dr}\left(\dfrac{1}{r}\right) = -\dfrac{q}{4\pi\varepsilon_0}\left(-\dfrac{1}{r^2}\right) = \boxed{\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}}$

48. The potential gradient is the negative of the electric field. Outside of a spherically symmetric charge distribution, the field is that of a point charge at the center of the distribution.

$$\frac{dV}{dr} = -E = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = -\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(92)\left(1.60\times10^{-19}\text{C}\right)}{\left(7.5\times10^{-15}\text{m}\right)^2} = \boxed{-2.4\times10^{21}\ \text{V/m}}$$

49. The electric field between the plates is obtained from the negative derivative of the potential.

$$E = -\frac{dV}{dx} = -\frac{d}{dx}\left[(8.0\ \text{V/m})\,x\ +\ 5.0\ \text{V}\right] = -8.0\ \text{V/m}$$

The charge density on the plates (assumed to be conductors) is then calculated from the electric field between two large plates, $E = \sigma/\varepsilon_0$.

$$\sigma = E\varepsilon_0 = (8.0\,\text{V/m})\left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right) = \boxed{7.1\times10^{-11}\text{C/m}^2}$$

The plate at the origin has the charge $-7.1\times10^{-11}\text{C/m}^2$ and the other plate, at a positive *x*, has charge $+7.1\times10^{-11}\text{C/m}^2$ so that the electric field points in the negative direction.

50. We use Eq. 23-9 to find the components of the electric field.

$$E_x = -\frac{\partial V}{\partial x} = 0\ ;\ E_z = -\frac{\partial V}{\partial z} = 0$$
$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{by}{\left(a^2+y^2\right)}\right] = -\frac{\left(a^2+y^2\right)b - by(2y)}{\left(a^2+y^2\right)^2} = \frac{\left(y^2-a^2\right)b}{\left(a^2+y^2\right)^2}$$
$$\vec{\mathbf{E}} = \boxed{\frac{\left(y^2-a^2\right)b}{\left(a^2+y^2\right)^2}\hat{\mathbf{j}}}$$

51. We use Eq. 23-9 to find the components of the electric field.

$$E_x = -\frac{\partial V}{\partial x} = -2.5y + 3.5yz \;;\; E_y = -\frac{\partial V}{\partial y} = -2y - 2.5x + 3.5xz \;;\; E_z = -\frac{\partial V}{\partial z} = 3.5xy$$

$$\vec{\mathbf{E}} = \boxed{\left(-2.5y + 3.5yz\right)\hat{\mathbf{i}} + \left(-2y - 2.5x + 3.5xz\right)\hat{\mathbf{j}} + \left(3.5xy\right)\hat{\mathbf{k}}}$$

52. We use the potential to find the electric field, the electric field to find the force, and the force to find the acceleration.

$$E_x = -\frac{\partial V}{\partial x} \;;\; F_x = qE_x \;;\; a_x = \frac{F_x}{m} = \frac{qE_x}{m} = -\frac{q}{m}\frac{\partial V}{\partial x} = -\frac{q}{m}\frac{\partial V}{\partial x}$$

$$a_x\left(x = 2.0\,\text{m}\right) = -\frac{2.0\times10^{-6}\,\text{C}}{5.0\times10^{-5}\,\text{kg}}\left[2\left(2.0\,\text{V/m}^2\right)\left(2.0\,\text{m}\right) - 3\left(3.0\,\text{V/m}^3\right)\left(2.0\,\text{m}\right)^2\right] = \boxed{1.1\,\text{m/s}^2}$$

53. (*a*) The potential along the *y* axis was derived in Problem 38.

$$V_{y\,\text{axis}} = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{\sqrt{\ell^2+y^2}+\ell}{\sqrt{\ell^2+y^2}-\ell}\right)\right] = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\sqrt{\ell^2+y^2}+\ell\right) - \ln\left(\sqrt{\ell^2+y^2}-\ell\right)\right]$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{Q}{8\pi\varepsilon_0\ell}\left[\frac{\frac{1}{2}\left(\ell^2+y^2\right)^{-1/2}2y}{\sqrt{\ell^2+y^2}+\ell} - \frac{\frac{1}{2}\left(\ell^2+y^2\right)^{-1/2}2y}{\sqrt{\ell^2+y^2}-\ell}\right] = \frac{Q}{4\pi\varepsilon_0 y\sqrt{\ell^2+y^2}}$$

From the symmetry of the problem, this field will point along the *y* axis.

$$\vec{\mathbf{E}} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{y\sqrt{\ell^2+y^2}}\hat{\mathbf{j}}}$$

Note that for $y \gg \ell$, this reduces to the field of a point charge at the origin.

(*b*) The potential along the *x* axis was derived in Problem 39.

$$V_{x\,\text{axis}} = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{x+\ell}{x-\ell}\right)\right] = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(x+\ell\right) - \ln\left(x-\ell\right)\right]$$

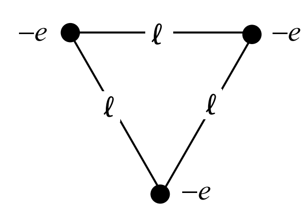
$$E_x = -\frac{\partial V}{\partial x} = -\frac{Q}{8\pi\varepsilon_0\ell}\left[\frac{1}{x+\ell} - \frac{1}{x-\ell}\right] = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{x^2-\ell^2}\right)$$

From the symmetry of the problem, this field will point along the *x* axis.

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{x^2-\ell^2}\right)\hat{\mathbf{i}}$$

Note that for $x \gg \ell$, this reduces to the field of a point charge at the origin.

54. Let the side length of the equilateral triangle be $\ell$. Imagine bringing the electrons in from infinity one at a time. It takes no work to bring the first electron to its final location, because there are no other charges present. Thus $W_1 = 0$. The work done in bringing in the second electron to its final location is equal to the charge on the electron times the potential (due to the first electron) at the final location of the second electron.




Thus $W_2 = \left(-e\right)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{\ell}\right) = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{L}$. The work done in bringing the third electron to its final

location is equal to the charge on the electron times the potential (due to the first two electrons).
Thus $W_3 = (-e)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{\ell} - \frac{1}{4\pi\varepsilon_0}\frac{e}{\ell}\right) = \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\ell}$. The total work done is the sum $W_1 + W_2 + W_3$.

$$W = W_1 + W_2 + W_3 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\ell} + \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\ell} = \frac{1}{4\pi\varepsilon_0}\frac{3e^2}{\ell} = \frac{3\left(8.99\times10^9\ \mathrm{N{\bullet}m^2/C^2}\right)\left(1.60\times10^{-19}\mathrm{C}\right)^2}{\left(1.0\times10^{-10}\mathrm{m}\right)}$$

$$= \boxed{6.9\times10^{-18}\mathrm{J}} = 6.9\times10^{-18}\mathrm{J}\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\mathrm{J}}\right) = \boxed{43\,\mathrm{eV}}$$

55. The gain of kinetic energy comes from a loss of potential energy due to conservation of energy, and the magnitude of the potential difference is the energy per unit charge. The helium nucleus has a charge of $2e$.

$$\Delta V = \frac{\Delta U}{q} = -\frac{\Delta K}{q} = -\frac{125\times10^3\mathrm{eV}}{2e} = \boxed{-62.5\,\mathrm{kV}}$$

The negative sign indicates that the helium nucleus had to go from a higher potential to a lower potential.

56. The kinetic energy of the particle is given in each case. Use the kinetic energy to find the speed.

(*a*) $\frac{1}{2}mv^2 = K \rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(1500\,\mathrm{eV}\right)\left(1.60\times10^{-19}\ \mathrm{J/eV}\right)}{9.11\times10^{-31}\mathrm{kg}}} = \boxed{2.3\times10^7\ \mathrm{m/s}}$

(*b*) $\frac{1}{2}mv^2 = K \rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(1500\,\mathrm{eV}\right)\left(1.60\times10^{-19}\ \mathrm{J/eV}\right)}{1.67\times10^{-27}\mathrm{kg}}} = \boxed{5.4\times10^5\ \mathrm{m/s}}$

57. The potential energy of the two-charge configuration (assuming they are both point charges) is given by Eq. 23-10.

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\Delta U = U_{\mathrm{final}} - U_{\mathrm{initial}} = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r_{\mathrm{initial}}} - \frac{1}{r_{\mathrm{final}}}\right)$$

$$= \left(8.99\times10^9\ \mathrm{N{\bullet}m^2/C^2}\right)\left(1.60\times10^{-19}\mathrm{C}\right)^2\left(\frac{1}{0.110\times10^{-9}\mathrm{m}} - \frac{1}{0.100\times10^{-9}\mathrm{m}}\right)\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\mathrm{J}}\right)$$

$$= -1.31\,\mathrm{eV}$$

Thus $\boxed{1.3\ \mathrm{eV}}$ of potential energy was lost.

58. The kinetic energy of the alpha particle is given. Use the kinetic energy to find the speed.

$$\tfrac{1}{2}mv^2 = K \rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(5.53\times10^6\mathrm{eV}\right)\left(1.60\times10^{-19}\ \mathrm{J/eV}\right)}{6.64\times10^{-27}\mathrm{kg}}} = \boxed{1.63\times10^7\ \mathrm{m/s}}$$

59. Following the same method as presented in Section 23-8, we get the following results.

(*a*) 1 charge: No work is required to move a single charge into a position, so $U_1 = 0$.

2 charges: This represents the interaction between $Q_1$ and $Q_2$.

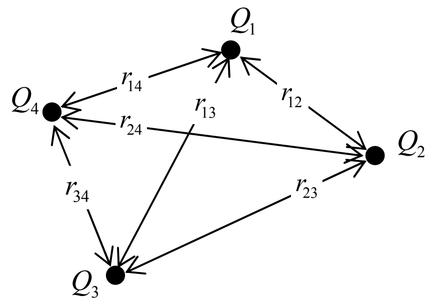
$$U_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}}$$

3 charges: This now adds the interactions between $Q_1$ & $Q_3$ and $Q_2$ & $Q_3$.

$$U_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}}+\frac{Q_1Q_3}{r_{13}}+\frac{Q_2Q_3}{r_{23}}\right)$$

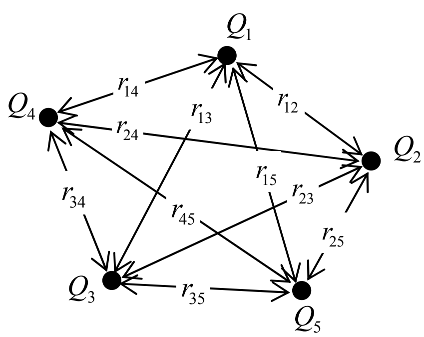
4 charges: This now adds the interaction between $Q_1$ & $Q_4$, $Q_2$ & $Q_4$, and $Q_3$ & $Q_4$.

$$\boxed{U_4 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}}+\frac{Q_1Q_3}{r_{13}}+\frac{Q_1Q_4}{r_{14}}+\frac{Q_2Q_3}{r_{23}}+\frac{Q_2Q_4}{r_{24}}+\frac{Q_3Q_4}{r_{34}}\right)}$$




(*b*) 5 charges: This now adds the interaction between $Q_1$ & $Q_5$, $Q_2$ & $Q_5$, $Q_3$ & $Q_5$, and $Q_4$ & $Q_5$.

$$\boxed{U_5 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}}+\frac{Q_1Q_3}{r_{13}}+\frac{Q_1Q_4}{r_{14}}+\frac{Q_1Q_5}{r_{15}}+\frac{Q_2Q_3}{r_{23}}+\frac{Q_2Q_4}{r_{24}}+\frac{Q_2Q_5}{r_{25}}+\frac{Q_3Q_4}{r_{34}}+\frac{Q_3Q_5}{r_{35}}+\frac{Q_4Q_5}{r_{45}}\right)}$$




60. (*a*) The potential energy of the four-charge configuration was derived in Problem 59. Number the charges clockwise, starting in the upper right hand corner of the square.

$$U_4 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}}+\frac{Q_1Q_3}{r_{13}}+\frac{Q_1Q_4}{r_{14}}+\frac{Q_2Q_3}{r_{23}}+\frac{Q_2Q_4}{r_{24}}+\frac{Q_3Q_4}{r_{34}}\right)$$

$$= \frac{Q^2}{4\pi\varepsilon_0}\left(\frac{1}{b}+\frac{1}{\sqrt{2}b}+\frac{1}{b}+\frac{1}{b}+\frac{1}{\sqrt{2}b}+\frac{1}{b}\right) = \boxed{\frac{Q^2}{4\pi\varepsilon_0 b}\left(4+\sqrt{2}\right)}$$

(*b*) The potential energy of the fifth charge is due to the interaction between the fifth charge and each of the other four charges. Each of those interaction terms is of the same magnitude since the fifth charge is the same distance from each of the other four charges.

$$U_{\substack{\text{5th}\\\text{charge}}} = \boxed{\frac{Q^2}{4\pi\varepsilon_0 b}\left(4\sqrt{2}\right)}$$

(*c*) If the center charge were moved away from the center, it would be moving closer to 1 or 2 of the other charges. Since the charges are all of the same sign, by moving closer, the center charge would be repelled back towards its original position. Thus it is in a place of $\boxed{\text{stable}}$ equilibrium.

(*d*) If the center charge were moved away from the center, it would be moving closer to 1 or 2 of the other charges. Since the corner charges are of the opposite sign as the center charge, the center charge would be attracted towards those closer charges, making the center charge move even farther from the center. So it is in a place of $\boxed{\text{unstable}}$ equilibrium.

61. (*a*) The electron was accelerated through a potential difference of 1.33 kV (moving from low potential to high potential) in gaining 1.33 keV of kinetic energy. The proton is accelerated through the opposite potential difference as the electron, and has the exact opposite charge. Thus the proton gains the same kinetic energy, $\boxed{1.33\text{ keV}}$.

(*b*) Both the proton and the electron have the same KE. Use that to find the ratio of the speeds.

$$\tfrac{1}{2}m_{\text{p}}v_{\text{p}}^2 = \tfrac{1}{2}m_{\text{e}}v_{\text{e}}^2 \quad\rightarrow\quad \frac{v_{\text{e}}}{v_{\text{p}}} = \sqrt{\frac{m_{\text{p}}}{m_e}} = \sqrt{\frac{1.67\times10^{-27}\text{ kg}}{9.11\times10^{-31}\text{ kg}}} = \boxed{42.8}$$

The lighter electron is moving about 43 times faster than the heavier proton.

62. We find the energy by bringing in a small amount of charge at a time, similar to the method given in Section 23-8. Consider the sphere partially charged, with charge $q < Q$. The potential at the surface of the sphere is $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_0}$, and the work to add a charge $dq$ to that sphere will increase the potential energy by $dU = Vdq$. Integrate over the entire charge to find the total potential energy.

$$U = \int dU = \int_0^Q \frac{1}{4\pi\varepsilon_0}\frac{q}{r_0}dq = \boxed{\frac{1}{8\pi\varepsilon_0}\frac{Q^2}{r_0}}$$

63. The two fragments can be treated as point charges for purposes of calculating their potential energy. Use Eq. 23-10 to calculate the potential energy. Using energy conservation, the potential energy is all converted to kinetic energy as the two fragments separate to a large distance.

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad U_{\text{initial}} = K_{\text{final}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}V$$

$$= \left(8.99\times10^9\text{ N}\bullet\text{m}^2/\text{C}^2\right)\frac{(38)(54)\left(1.60\times10^{-19}\text{C}\right)^2}{\left(5.5\times10^{-15}\text{m}\right)+\left(6.2\times10^{-15}\text{m}\right)}\left(\frac{1\text{eV}}{1.60\times10^{-19}\text{J}}\right) = 250\times10^6\text{eV}$$

$$= \boxed{250\text{ MeV}}$$

This is about 25% greater than the observed kinetic energy of 200 MeV.

64. We find the energy by bringing in a small amount of spherically symmetric charge at a time, similar to the method given in Section 23-8. Consider that the sphere has been partially constructed, and so has a charge $q < Q$, contained in a radius $r < r_0$. Since the sphere is made of uniformly charged material, the charge density of the sphere must be $\rho_E = \frac{Q}{\frac{4}{3}\pi r_0^3}$. Thus the partially constructed sphere also satisfies $\rho_E = \frac{q}{\frac{4}{3}\pi r^3}$, and so $\frac{q}{\frac{4}{3}\pi r^3} = \frac{Q}{\frac{4}{3}\pi r_0^3} \rightarrow q = \frac{Qr^3}{r_0^3}$. The potential at the surface of that sphere can now found.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{Qr^3}{r_0^3}}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Qr^2}{r_0^3}$$

We now add another infinitesimally thin shell to the partially constructed sphere. The charge of that shell is $dq = \rho_E 4\pi r^2 dr$. The work to add charge $dq$ to the sphere will increase the potential energy by $dU = Vdq$. Integrate over the entire sphere to find the total potential energy.

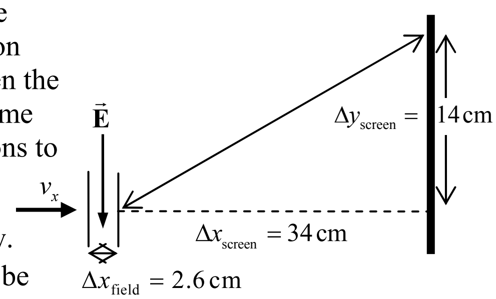
$$U = \int dU = \int Vdq = \int_0^{r_0} \frac{1}{4\pi\varepsilon_0}\frac{Qr^2}{r_0^3}\rho_E 4\pi r^2 dr = \frac{\rho_E}{\varepsilon_0}\frac{Q}{r_0^3}\int_0^{r_0} r^4 dr = \boxed{\frac{3Q^2}{20\pi\varepsilon_0 r_0}}$$

65. The ideal gas model, from Eq. 18-4, says that $K = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$.

$$K = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT \rightarrow v_{\substack{rms \\ 273\,K}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\,J/K)(273\,K)}{9.11\times10^{-31}\,kg}} = \boxed{1.11\times10^5\,m/s}$$

$$v_{\substack{rms \\ 2700\,K}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\,J/K)(2700\,K)}{9.11\times10^{-31}\,kg}} = \boxed{3.5\times10^5\,m/s}$$

66. If there were no deflecting field, the electrons would hit the center of the screen. If an electric field of a certain direction moves the electrons towards one extreme of the screen, then the opposite field will move the electrons to the opposite extreme of the screen. So we solve for the field to move the electrons to one extreme of the screen. Consider three parts to the electron's motion, and see the diagram, which is a top view. First, during the horizontal acceleration phase, energy will be conserved and so the horizontal speed of the electron $v_x$ can be found from the accelerating potential $V$. Secondly, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron a leftward velocity, $v_y$. We assume that there is very little leftward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the left edge of the screen.




Acceleration:

$$U_{initial} = K_{final} \rightarrow eV = \frac{1}{2}mv_x^2 \rightarrow v_x = \sqrt{\frac{2eV}{m}}$$

Deflection:

time in field: $\Delta x_{\text{field}} = v_x t_{\text{field}} \rightarrow t_{\text{field}} = \dfrac{\Delta x_{\text{field}}}{v_x}$

$$F_y = eE = ma_y \rightarrow a_y = \frac{eE}{m} \qquad v_y = v_0 + a_y t_{\text{field}} = 0 + \frac{eE\Delta x_{\text{field}}}{mv_x}$$

Screen:

$$\Delta x_{\text{screen}} = v_x t_{\text{screen}} \rightarrow t_{\text{screen}} = \frac{\Delta x_{\text{screen}}}{v_x} \qquad \Delta y_{\text{screen}} = v_y t_{\text{screen}} = v_y \frac{\Delta x_{\text{screen}}}{v_x}$$

$$\frac{\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}} = \frac{v_y}{v_x} = \frac{\frac{eE\Delta x_{\text{field}}}{mv_x}}{v_x} = \frac{eE\Delta x_{\text{field}}}{mv_x^2} \rightarrow$$

$$E = \frac{\Delta y_{\text{screen}} m v_x^2}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{\Delta y_{\text{screen}} m \frac{2eV}{m}}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{2V\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}\Delta x_{\text{field}}} = \frac{2\left(6.0\times10^3\,\text{V}\right)\left(0.14\,\text{m}\right)}{\left(0.34\text{ m}\right)\left(0.026\,\text{m}\right)}$$

$$= 1.90\times10^5\text{ V/m} \approx 1.9\times10^5\text{ V/m}$$

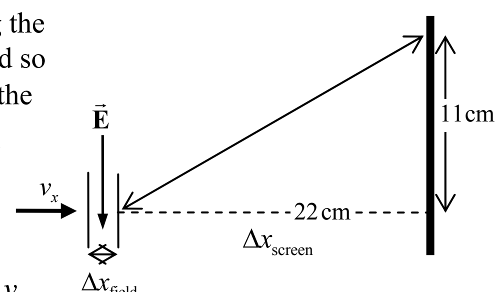
As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$\Delta y = v_0 t_{\text{field}} + \tfrac{1}{2}a_y t_{\text{field}}^2 = 0 + \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \frac{eE\left(\Delta x_{\text{field}}\right)^2}{2m\left(\frac{2eV}{m}\right)} = \frac{E\left(\Delta x_{\text{field}}\right)^2}{4V}$$

$$= \frac{\left(1.90\times10^5\text{ V/m}\right)\left(0.026\,\text{m}\right)^2}{4\left(6000\,\text{V}\right)} = 5.4\times10^{-3}\,\text{m}$$

This is about 4% of the total 15 cm vertical deflection, and so for an estimation, our approximation is acceptable. And so the field must vary from $\boxed{+1.9\times10^5\text{ V/m to }-1.9\times10^5\text{ V/m}}$

67. Consider three parts to the electron's motion. First, during the horizontal acceleration phase, energy will be conserved and so the horizontal speed of the electron $v_x$ can be found from the accelerating potential, $V$. Secondly, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron an upward velocity, $v_y$.




We assume that there is very little upward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the top of the screen.

Acceleration:

$$U_{\text{initial}} = K_{\text{final}} \rightarrow eV = \tfrac{1}{2}mv_x^2 \rightarrow v_x = \sqrt{\frac{2eV}{m}}$$

Deflection:

time in field: $\Delta x_{\text{field}} = v_x t_{\text{field}} \quad \rightarrow \quad t_{\text{field}} = \dfrac{\Delta x_{\text{field}}}{v_x}$

$$F_y = eE = ma_y \quad \rightarrow \quad a_y = \frac{eE}{m} \qquad v_y = v_0 + a_y t_{\text{field}} = 0 + \frac{eE\Delta x_{\text{field}}}{mv_x}$$

Screen:

$$\Delta x_{\text{screen}} = v_x t_{\text{screen}} \quad \rightarrow \quad t_{\text{screen}} = \frac{\Delta x_{\text{screen}}}{v_x} \qquad \Delta y_{\text{screen}} = v_y t_{\text{screen}} = v_y \frac{\Delta x_{\text{screen}}}{v_x}$$

$$\frac{\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}} = \frac{v_y}{v_x} = \frac{\frac{eE\Delta x_{\text{field}}}{mv_x}}{v_x} = \frac{eE\Delta x_{\text{field}}}{mv_x^2} \quad \rightarrow$$

$$E = \frac{\Delta y_{\text{screen}} m v_x^2}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{\Delta y_{\text{screen}} m \frac{2eV}{m}}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{2V\Delta y_{\text{screen}}}{\Delta x_{\text{screen}} \Delta x_{\text{field}}} = \frac{2(7200\,\text{V})(0.11\,\text{m})}{(0.22\,\text{m})(0.028\,\text{m})}$$

$$= 2.57 \times 10^5\ \text{V/m} \approx \boxed{2.6 \times 10^5\ \text{V/m}}$$

As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$\Delta y = v_0 t_{\text{field}} + \tfrac{1}{2} a_y t_{\text{field}}^2 = 0 + \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \frac{eE(\Delta x_{\text{field}})^2}{2m\left(\frac{2eV}{m}\right)} = \frac{E(\Delta x_{\text{field}})^2}{4V}$$

$$= \frac{(2.97 \times 10^5\ \text{V/m})(0.028\,\text{m})^2}{4(7200\,\text{V})} = 8.1 \times 10^{-3}\,\text{m}$$

This is about 7% of the total 11 cm vertical deflection, and so for an estimation, our approximation is acceptable.

68. The potential of the earth will increase because the "neutral" Earth will now be charged by the removing of the electrons. The excess charge will be the elementary charge times the number of electrons removed. We approximate this change in potential by using a spherical Earth with all the excess charge at the surface.

$$Q = \left(\frac{1.602 \times 10^{-19}\,\text{C}}{e^-}\right)\left(\frac{10\ e^-}{\text{H}_2\text{O molecule}}\right)\left(\frac{6.02 \times 10^{23}\,\text{molecules}}{0.018\,\text{kg}}\right)\left(\frac{1000\,\text{kg}}{\text{m}^3}\right)\tfrac{4}{3}\pi(0.00175\,\text{m})^3$$

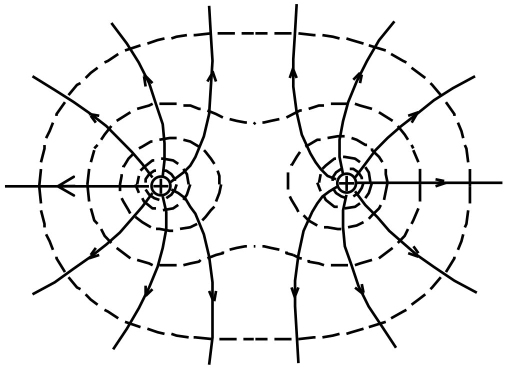
$$= 1203\,\text{C}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_{\text{Earth}}} = (8.99 \times 10^9\ \text{N}\bullet\text{m}^2/\text{C}^2)\frac{1203\,\text{C}}{6.38 \times 10^6\,\text{m}} = \boxed{1.7 \times 10^6\,\text{V}}$$

69. The potential at the surface of a charged sphere is that of a point charge of the same magnitude, located at the center of the sphere.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = (8.99 \times 10^9\ \text{N}\bullet\text{m}^2/\text{C}^2)\frac{(1 \times 10^{-8}\,\text{C})}{(0.15\,\text{m})} = 599.3\,\text{V} \approx \boxed{600\,\text{V}}$$

70.



71. Let $d_1$ represent the distance from the left charge to point b, and let $d_2$ represent the distance from the right charge to point b. Let *Q* represent the positive charges, and let *q* represent the negative charge that moves. The change in potential energy is given by Eq. 23-2b.

$$d_1 = \sqrt{12^2 + 14^2}\,\text{cm} = 18.44\text{ cm} \qquad d_2 = \sqrt{14^2 + 24^2}\,\text{cm} = 27.78\text{ cm}$$

$$U_\text{b} - U_\text{a} = q\left(V_\text{b} - V_\text{a}\right) = q\frac{1}{4\pi\varepsilon_0}\left[\left(\frac{Q}{0.1844\,\text{m}} + \frac{Q}{0.2778\,\text{m}}\right) - \left(\frac{Q}{0.12\,\text{m}} + \frac{Q}{0.24\,\text{m}}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}Qq\left[\left(\frac{1}{0.1844\,\text{m}} + \frac{1}{0.2778\,\text{m}}\right) - \left(\frac{1}{0.12\,\text{m}} + \frac{1}{0.24\,\text{m}}\right)\right]$$

$$= \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(-1.5\times10^{-6}\text{C}\right)\left(33\times10^{-6}\text{C}\right)\left(-3.477\,\text{m}^{-1}\right) = 1.547\,\text{J} \approx \boxed{1.5\,\text{J}}$$

72. (*a*) All eight charges are the same distance from the center of the cube. Use Eq. 23-5 for the potential of a point charge.

$$V_\text{center} = 8\frac{1}{4\pi\varepsilon_0}\frac{Q}{\frac{\sqrt{3}}{2}\ell} = \boxed{\frac{16}{\sqrt{3}}\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}} \approx 9.24\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}$$

(*b*) For the seven charges that produce the potential at a corner, three are a distance $\ell$ away from that corner, three are a distance $\sqrt{2}\ell$ away from that corner, and one is a distance $\sqrt{3}\ell$ away from that corner.

$$V_\text{corner} = 3\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell} + 3\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}\ell} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{3}\ell} = \boxed{\left(3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}} \approx 5.70\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}$$

(*c*) The total potential energy of the system is half the energy found by multiplying each charge times the potential at a corner. The factor of half comes from the fact that if you took each charge times the potential at a corner, you would be counting each pair of charges twice.

$$U = \tfrac{1}{2}8\left(QV_\text{corner}\right) = \boxed{4\left(3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\ell}} \approx 22.8\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\ell}$$

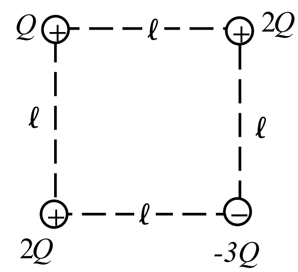
73. The electric force on the electron must be the same magnitude as the weight of the electron. The magnitude of the electric force is the charge on the electron times the magnitude of the electric field. The electric field is the potential difference per meter: $E = V/d$.

$$F_E = mg \ ; \ F_E = |q|E = eV/d \ \rightarrow \ eV/d = mg \ \rightarrow$$

$$V = \frac{mgd}{e} = \frac{(9.11\times10^{-31}\,\text{kg})(9.80\,\text{m/s}^2)(0.035\,\text{m})}{1.60\times10^{-19}\text{C}} = \boxed{2.0\times10^{-12}\,\text{V}}$$

Since it takes such a tiny voltage to balance gravity, the thousands of volts in a television set are more than enough (by many orders of magnitude) to move electrons upward against the force of gravity.

74. From Problem 59, the potential energy of a configuration of four charges is $U = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_3Q_4}{r_{34}}\right)$.




Let a side of the square be ℓ, and number the charges clockwise starting with the upper left corner.

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_3Q_4}{r_{34}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q(2Q)}{\ell} + \frac{Q(-3Q)}{\sqrt{2}\ell} + \frac{Q(2Q)}{\ell} + \frac{(2Q)(-3Q)}{\ell} + \frac{(2Q)(2Q)}{\sqrt{2}\ell} + \frac{(-3Q)(2Q)}{\ell}\right)$$

$$= \frac{Q^2}{4\pi\varepsilon_0\ell}\left(\frac{1}{\sqrt{2}} - 8\right) = \left(8.99\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(3.1\times10^{-6}\text{C})^2}{0.080\,\text{m}}\left(\frac{1}{\sqrt{2}} - 8\right) = \boxed{-7.9\,\text{J}}$$

75. The kinetic energy of the electrons (provided by the UV light) is converted completely to potential energy at the plate since they are stopped. Use energy conservation to find the emitted speed, taking the 0 of PE to be at the surface of the barium.

$$\text{KE}_{\text{initial}} = \text{PE}_{\text{final}} \ \rightarrow \ \tfrac{1}{2}mv^2 = qV \ \rightarrow$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(-1.60\times10^{-19}\,\text{C})(-3.02\,\text{V})}{9.11\times10^{-31}\,\text{kg}}} = \boxed{1.03\times10^6\,\text{m/s}}$$

76. To find the angle, the horizontal and vertical components of the velocity are needed. The horizontal component can be found using conservation of energy for the initial acceleration of the electron. That component is not changed as the electron passes through the plates. The vertical component can be found using the vertical acceleration due to the potential difference of the plates, and the time the electron spends between the plates.

Horizontal:

$$\text{PE}_{\text{inital}} = \text{KE}_{\text{final}} \ \rightarrow \ qV = \tfrac{1}{2}mv_x^2 \qquad t = \frac{\Delta x}{v_x}$$

Vertical:

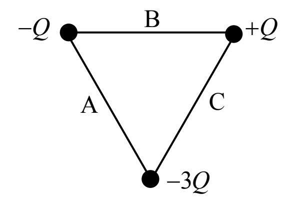
$$F_E = qE_y = ma = m\frac{(v_y - v_{y0})}{t} \ \rightarrow \ v_y = \frac{qE_yt}{m} = \frac{qE_y\Delta x}{mv_x}$$

Combined:

$$\tan\theta = \frac{v_y}{v_x} = \frac{\dfrac{qE_y\Delta x}{mv_x}}{v_x} = \frac{qE_y\Delta x}{mv_x^2} = \frac{qE_y\Delta x}{2qV} = \frac{E_y\Delta x}{2V} = \frac{\left(\dfrac{250\,\text{V}}{0.013\,\text{m}}\right)(0.065\,\text{m})}{2(5500\,\text{V})} = 0.1136$$

$$\theta = \tan^{-1} 0.1136 = \boxed{6.5^\circ}$$

77. Use Eq. 23-5 to find the potential due to each charge. Since the triangle is equilateral, the 30-60-90 triangle relationship says that the distance from a corner to the midpoint of the opposite side is $\sqrt{3}\ell/2$.




$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-3Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(Q)}{\sqrt{3}\ell/2} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\ell}\left(-4 + \frac{1}{\sqrt{3}}\right)$$

$$= \boxed{\frac{Q}{\pi\varepsilon_0\ell}\left(\frac{\sqrt{3}}{6} - 2\right)}$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-3Q)}{\sqrt{3}\ell/2} = -\frac{1}{4\pi\varepsilon_0}\frac{6Q}{\sqrt{3}\ell} = \boxed{-\frac{\sqrt{3}Q}{2\pi\varepsilon_0\ell}}$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{(Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-3Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{\sqrt{3}\ell/2} = -\frac{1}{4\pi\varepsilon_0}\frac{2Q}{\ell}\left(2 + \frac{1}{\sqrt{3}}\right) = \boxed{-\frac{Q}{\pi\varepsilon_0\ell}\left(1 + \frac{\sqrt{3}}{6}\right)}$$

78. Since the E-field points downward, the surface of the Earth is a lower potential than points above the surface. Call the surface of the Earth 0 volts. Then a height of 2.00 m has a potential of 300 V. We also call the surface of the Earth the 0 location for gravitational PE. Write conservation of energy relating the charged spheres at 2.00 m (where their speed is 0) and at ground level (where their electrical and gravitational potential energies are 0).

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad mgh + qV = \tfrac{1}{2}mv^2 \quad\rightarrow\quad v = \sqrt{2\left(gh + \frac{qV}{m}\right)}$$

$$v_+ = \sqrt{2\left[(9.80\,\text{m/s}^2)(2.00\,\text{m}) + \frac{(4.5\times10^{-4}\,\text{C})(300\,\text{V})}{(0.340\,\text{kg})}\right]} = 6.3241\,\text{m/s}$$

$$v_- = \sqrt{2\left[(9.80\,\text{m/s}^2)(2.00\,\text{m}) + \frac{(-4.5\times10^{-4}\,\text{C})(300\,\text{V})}{(0.340\,\text{kg})}\right]} = 6.1972\,\text{m/s}$$

$$v_+ - v_- = 6.3241\,\text{m/s} - 6.1972\,\text{m/s} = \boxed{0.13\,\text{m/s}}$$

79. (*a*) The energy is related to the charge and the potential difference by Eq. 23-3.

$$\Delta U = q\Delta V \quad\rightarrow\quad \Delta V = \frac{\Delta U}{q} = \frac{4.8\times10^6\,\text{J}}{4.0\,\text{C}} = \boxed{1.2\times10^6\,\text{V}}$$

(*b*) The energy (as heat energy) is used to raise the temperature of the water and boil it. Assume that room temperature is 20°C.

$$Q = mc\Delta T + mL_f \quad\rightarrow$$

$$m = \frac{Q}{c\Delta T + L_f} = \frac{4.8\times10^6\,\text{J}}{\left(4186\,\frac{\text{J}}{\text{kg}\bullet\text{C}^\circ}\right)\left(80\,\text{C}^\circ\right) + \left(22.6\times10^5\,\frac{\text{J}}{\text{kg}}\right)} = \boxed{1.8\,\text{kg}}$$

80. Use Eq. 23-7 for the dipole potential, and use Eq. 23-9 to determine the electric field.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{p}{4\pi\varepsilon_0}\frac{\frac{x}{\left(x^2+y^2\right)^{1/2}}}{x^2+y^2} = \frac{p}{4\pi\varepsilon_0}\frac{x}{\left(x^2+y^2\right)^{3/2}}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{p}{4\pi\varepsilon_0}\left[\frac{\left(x^2+y^2\right)^{3/2} - x\frac{3}{2}\left(x^2+y^2\right)^{1/2}2x}{\left(x^2+y^2\right)^3}\right] = \boxed{\frac{p}{4\pi\varepsilon_0}\left[\frac{2x^2-y^2}{\left(x^2+y^2\right)^{5/2}}\right]}$$

$$= \frac{p}{4\pi\varepsilon_0}\left[\frac{2\cos^2\theta - \sin^2\theta}{r^3}\right]$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{px}{4\pi\varepsilon_0}\left[-\frac{3}{2}\left(x^2+y^2\right)^{-5/2}2y\right] = \boxed{\frac{p}{4\pi\varepsilon_0}\left[\frac{3xy}{\left(x^2+y^2\right)^{5/2}}\right]}\frac{p}{4\pi\varepsilon_0}\left[\frac{3\cos\theta\sin\theta}{r^3}\right]$$

Notice the $\frac{1}{r^3}$ dependence in both components, which is indicative of a dipole field.

81. (*a*) Since the reference level is given as $V = 0$ at $r = \infty$, the potential outside the shell is that of a point charge with the same total charge.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_E\left(\frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3\right)}{r} = \boxed{\frac{\rho_E}{3\varepsilon_0}\frac{\left(r_2^3 - r_1^3\right)}{r}},\ r > r_2$$

Note that the potential at the surface of the shell is $V_{r_2} = \frac{\rho_E}{3\varepsilon_0}\left(r_2^2 - \frac{r_1^3}{r_2}\right)$.

(*b*) To find the potential in the region $r_1 < r < r_2$, we need the electric field in that region. Since the charge distribution is spherically symmetric, Gauss's law may be used to find the electric field.

$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad\rightarrow\quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_E\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3\right)}{r^2} = \frac{\rho_E}{3\varepsilon_0}\frac{\left(r^3 - r_1^3\right)}{r^2}$$

The potential in that region is found from Eq. 23-4a. The electric field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}}\bullet d\vec{\boldsymbol{\ell}} = E dr$.

$$V_r - V_{r_2} = -\int_{r_2}^{r}\vec{\mathbf{E}}\bullet d\vec{\boldsymbol{\ell}} = -\int_{r_2}^{r}E\,dr = -\int_{r_2}^{r}\frac{\rho_E}{3\varepsilon_0}\frac{\left(r^3 - r_1^3\right)}{r^2}dr = -\frac{\rho_E}{3\varepsilon_0}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)dr = -\frac{\rho_E}{3\varepsilon_0}\left(\frac{1}{2}r^2 + \frac{r_1^3}{r}\right)\Bigg|_{r_2}^{r}$$

$$V_r = V_{r_2} + \left[-\frac{\rho_E}{3\varepsilon_0}\left(\frac{1}{2}r^2 + \frac{r_1^3}{r}\right)\Bigg|_{r_2}^{r}\right] = \frac{\rho_E}{3\varepsilon_0}\left(\frac{3}{2}r_2^2 - \frac{1}{2}r^2 - \frac{r_1^3}{r}\right) = \boxed{\frac{\rho_E}{\varepsilon_0}\left(\frac{1}{2}r_2^2 - \frac{1}{6}r^2 - \frac{1}{3}\frac{r_1^3}{r}\right)},\ r_1 < r < r_2$$

(*c*) Inside the cavity there is no electric field, so the potential is constant and has the value that it has on the cavity boundary.

$$V_{r_1} = \frac{\rho_E}{\varepsilon_0}\left(\tfrac{1}{2}r_2^2 - \tfrac{1}{6}r_1^2 - \tfrac{1}{3}\frac{r_1^3}{r_1}\right) = \boxed{\frac{\rho_E}{2\varepsilon_0}\left(r_2^2 - r_1^2\right)}, \; r < r_1$$

The potential is continuous at both boundaries.

82. We follow the development of Example 23-9, with Figure 23-15. The charge density of the ring is $\sigma = \left(\frac{Q}{\pi R_0^2 - \pi\left(\frac{1}{2}R_0\right)^2}\right) = \frac{4Q}{3\pi R_0^2}$. The charge on a thin ring of radius *R* and thickness *dR* is $dq = \sigma dA = \frac{4Q}{3\pi R_0^2}\left(2\pi R dR\right)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{\frac{1}{2}R_0}^{R_0}\frac{\frac{4Q}{3\pi R_0^2}\left(2\pi R dR\right)}{\sqrt{x^2+R^2}} = \frac{2Q}{3\varepsilon_0\pi R_0^2}\int_{\frac{1}{2}R_0}^{R_0}\frac{R}{\sqrt{x^2+R^2}}dR = \frac{2Q}{3\varepsilon_0\pi R_0^2}\left(x^2+R^2\right)^{1/2}\Big|_{\frac{1}{2}R_0}^{R_0}$$

$$= \frac{2Q}{3\varepsilon_0\pi R_0^2}\left(\sqrt{x^2+R_0^2} - \sqrt{x^2+\tfrac{1}{4}R_0^2}\right)$$

83. From Example 22-6, the electric field due to a long wire is radial relative to the wire, and is of magnitude $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$. If the charge density is positive, the field lines point radially away from the wire. Use Eq. 23-41 to find the potential difference, integrating along a line that is radially outward from the wire.

$$V_a - V_b = -\int_{R_b}^{R_a}\vec{\mathbf{E}}\bullet\left(d\vec{\boldsymbol{\ell}}\right) = -\int_{R_b}^{R_a}\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}dR = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(R_a - R_b\right) = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_b}{\mathrm{R_a}}}$$

84. (*a*) We may treat the sphere as a point charge located at the center of the field. Then the electric field at the surface is $E_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2}$, and the potential at the surface is $V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}$.

$$V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} = E_{\text{surface}}r_0 = E_{\text{breakdown}}r_0 = \left(3\times10^6\,\mathrm{V/m}\right)\left(0.20\,\mathrm{m}\right) = \boxed{6\times10^5\,\mathrm{V}}$$

(*b*) $$V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} \rightarrow Q = \left(4\pi\varepsilon_0\right)r_0V_{\text{surface}} = \frac{\left(0.20\,\mathrm{m}\right)\left(6\times10^5\,\mathrm{V}\right)}{\left(8.99\times10^9\,\mathrm{N\bullet m^2/C^2}\right)} = 1.33\times10^{-5}\,\mathrm{C} \approx \boxed{1\times10^{-5}\,\mathrm{C}}$$

85. (*a*) The voltage at $x = 0.20$ m is obtained by inserting the given data directly into the voltage equation.

$$V\left(0.20\,\mathrm{m}\right) = \frac{B}{\left(x^2+R^2\right)^2} = \frac{150\,\mathrm{V\bullet m^4}}{\left[\left(0.20\,\mathrm{m}\right)^2 + \left(0.20\,\mathrm{m}\right)^2\right]^2} = \boxed{23\text{ kV}}$$

(*b*) The electric field is the negative derivative of the potential.

$$\vec{\mathbf{E}}(x) = -\frac{d}{dx}\left[\frac{B}{\left(x^2+R^2\right)^2}\right]\hat{\mathbf{i}} = \boxed{\frac{4Bx\,\hat{\mathbf{i}}}{\left(x^2+R^2\right)^3}}$$

Since the voltage only depends on *x* the electric field points in the positive *x* direction.

(*c*) Inserting the given values in the equation of part (*b*) gives the electric field at $x = 0.20\text{ m}$

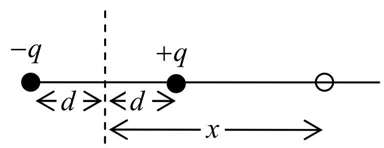
$$\vec{\mathbf{E}}(0.20\text{ m}) = \frac{4\left(150\,\text{V}\bullet\text{m}^4\right)\left(0.20\,\text{m}\right)\hat{\mathbf{i}}}{\left[\left(0.20\,\text{m}\right)^2+\left(0.20\,\text{m}\right)^2\right]^3} = \boxed{2.3\times10^5\ \text{V/m}\,\hat{\mathbf{i}}}$$

86. Use energy conservation, equating the energy of charge $-q_1$ at its initial position to its final position at infinity. Take the speed at infinity to be 0, and take the potential of the point charges to be 0 at infinity.

$$E_{\text{initial}} = E_{\text{final}} \;\rightarrow\; K_{\text{initial}} + U_{\text{initial}} = K_{\text{final}} + U_{\text{final}} \;\rightarrow\; \tfrac{1}{2}mv_0^2 + \left(-q_1\right)V_{\substack{\text{initial}\\\text{point}}} = \tfrac{1}{2}mv_{\text{final}}^2 + \left(-q_1\right)V_{\substack{\text{final}\\\text{point}}}$$

$$\tfrac{1}{2}mv_0^2 + \left(-q_1\right)\frac{1}{4\pi\varepsilon_0}\frac{2q_2}{\sqrt{a^2+b^2}} = 0 + 0 \;\rightarrow\; v_0 = \boxed{\sqrt{\frac{1}{m\pi\varepsilon_0}\frac{q_1q_2}{\sqrt{a^2+b^2}}}}$$

87. (*a*) From the diagram, the potential at *x* is the potential of two point charges.



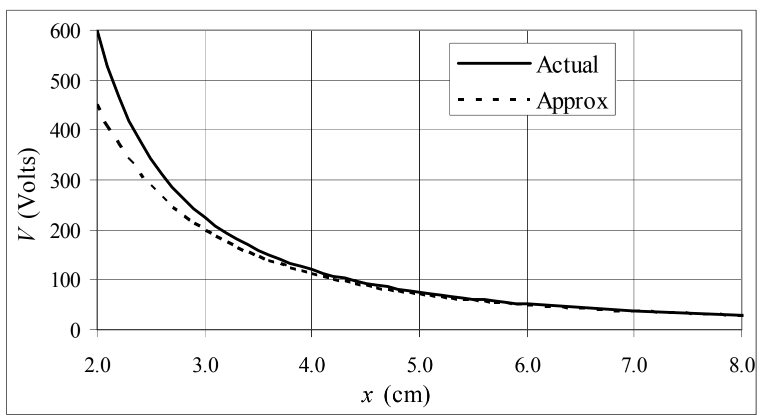

$$V_{\text{exact}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{x-d}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{x+d}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{2qd}{\left(x^2-d^2\right)}\right]\ ,\quad q = 1.0\times10^{-9}\,\text{C},\ d = 0.010\,\text{m}$$

(*b*) The approximate potential is given by Eq. 23-7, with $\theta = 0$, $p = 2qd$, and $r = x$.

$$V_{\text{approx}} = \frac{1}{4\pi\varepsilon_0}\frac{2qd}{x^2}$$

To make the difference at small distances more apparent, we have plotted from 2.0 cm to 8.0 cm. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.87."



88. The electric field can be determined from the potential by using Eq. 23-8, differentiating with respect to $x$.

$$E(x) = -\frac{dV(x)}{dx} = -\frac{d}{dx}\left[\frac{Q}{2\pi\varepsilon_0 R_0^2}\left[\left(x^2+R_0^2\right)^{1/2} - x\right]\right] = -\frac{Q}{2\pi\varepsilon_0 R_0^2}\left[\tfrac{1}{2}\left(x^2+R_0^2\right)^{-1/2}(2x) - 1\right]$$

$$= \boxed{\frac{Q}{2\pi\varepsilon_0 R_0^2}\left[1 - \frac{x}{\left(x^2+R_0^2\right)^{1/2}}\right]}$$

Express $V$ and $E$ in terms of $x/R_0$. Let $X = x/R_0$.

$$V(x) = \frac{Q}{2\pi\varepsilon_0 R_0^2}\left[\left(x^2+R_0^2\right)^{1/2} - x\right] = \frac{2Q}{4\pi\varepsilon_0 R_0}\left(\sqrt{X^2+1} - X\right)$$

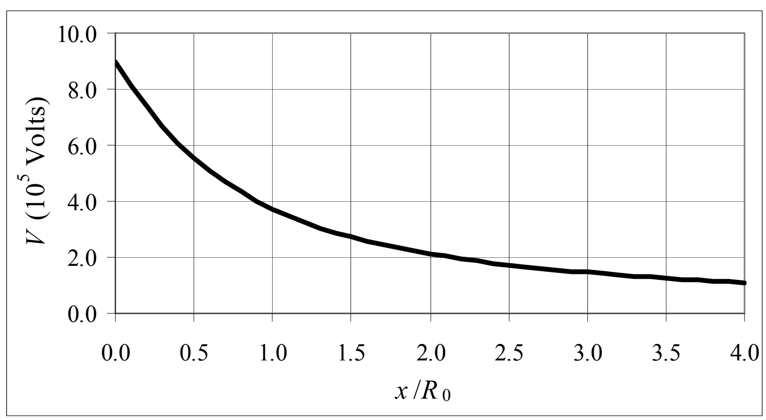
$$= \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{2\left(5.0\times10^{-6}\text{C}\right)}{0.10\,\text{m}}\left(\sqrt{X^2+1} - X\right) = \left(8.99\times10^5\text{V}\right)\left(\sqrt{X^2+1} - X\right)$$

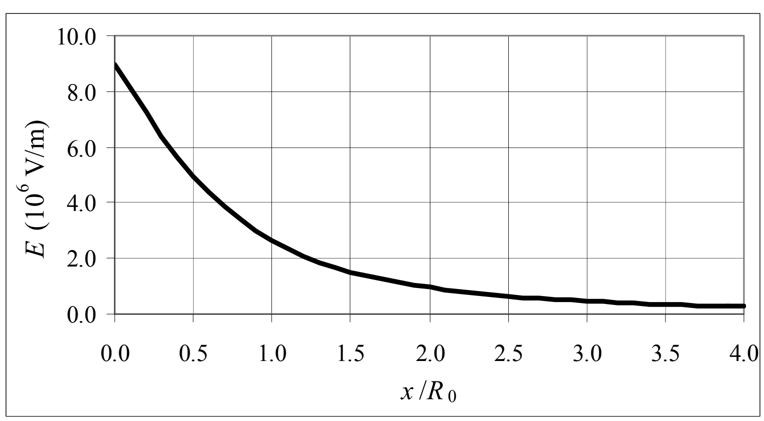
$$E(x) = \frac{Q}{2\pi\varepsilon_0 R_0^2}\left[1 - \frac{x}{\left(x^2+R_0^2\right)^{1/2}}\right] = \frac{2Q}{4\pi\varepsilon_0 R_0^2}\left[1 - \frac{X}{\sqrt{X^2+1}}\right]$$

$$= \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{2\left(5.0\times10^{-6}\text{C}\right)}{\left(0.10\,\text{m}\right)^2}\left[1 - \frac{X}{\sqrt{X^2+1}}\right]$$

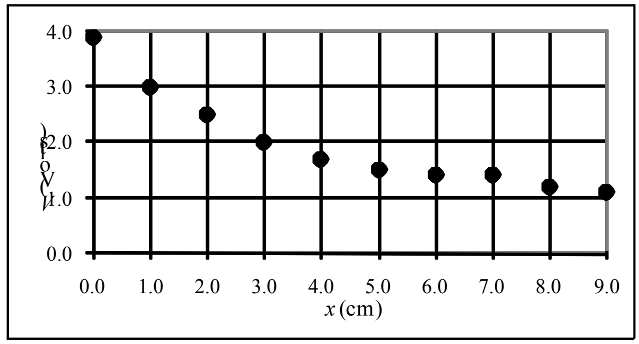
$$= \left(8.99\times10^6\ \text{V/m}\right)\left[1 - \frac{X}{\sqrt{X^2+1}}\right]$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.88."



89. (*a*) If the field is caused by a point charge, the potential will have a graph that has the appearance of $1/r$ behavior, which means that the potential change per unit of distance will decrease as potential is measured farther from the charge. If the field is caused by a sheet of charge, the potential will have a linear decrease with distance. The graph indicates that the field is caused by a point charge. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.89a."

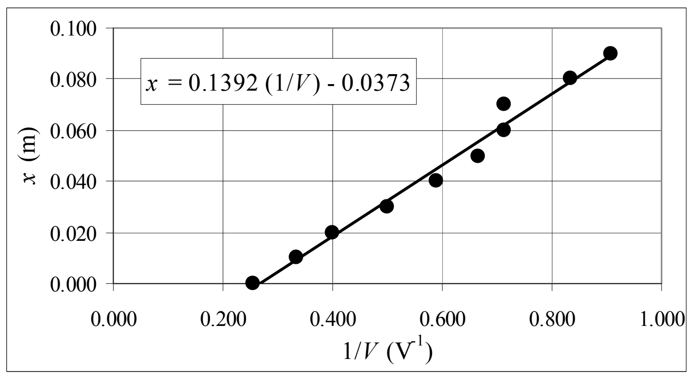



(*b*) Assuming the field is caused by a point charge, we assume the charge is at $x = d$, and then the potential is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x-d}$. This can be rearranged to the following.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x-d} \quad \rightarrow$$

$$x = \frac{1}{V}\frac{Q}{4\pi\varepsilon_0} + d$$

If we plot $x$ vs. $\frac{1}{V}$, the slope is $\frac{Q}{4\pi\varepsilon_0}$, which can be used to determine the charge.




$$\text{slope} = 0.1392\,\text{m}\bullet\text{V} = \frac{Q}{4\pi\varepsilon_0} \quad \rightarrow$$

$$Q = 4\pi\varepsilon_0 \left(0.1392\,\text{m}\bullet\text{V}\right) = \frac{\left(0.1392\,\text{m}\bullet\text{V}\right)}{\left(8.99\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2\right)} = \boxed{1.5\times10^{-11}\,\text{C}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.89b."

(*c*) From the above equation, the *y* intercept of the graph is the location of the charge, *d*. So the charge is located at $x = d = -0.0373\,\text{m} \approx \boxed{3.7\,\text{cm from the first measured position}}$.

# CHAPTER 24: Capacitance, Dielectrics, Electric Energy Storage

## Responses to Questions

1. Yes. If the conductors have different shapes, then even if they have the same charge, they will have different charge densities and therefore different electric fields near the surface. There can be a potential difference between them. The definition of capacitance $C = Q/V$ cannot be used here because it is defined for the case where the charges on the two conductors of the capacitor are equal and opposite.

2. Underestimate. If the separation between the plates is not very small compared to the plate size, then fringing cannot be ignored and the electric field (for a given charge) will actually be smaller. The capacitance is inversely proportional to potential and, for parallel plates, also inversely proportional to the field, so the capacitance will actually be larger than that given by the formula.

3. Ignoring fringing field effects, the capacitance would decrease by a factor of 2, since the area of overlap decreases by a factor of 2. (Fringing effects might actually be noticeable in this configuration.)

4. When a capacitor is first connected to a battery, charge flows to one plate. Because the plates are separated by an insulating material, charge cannot cross the gap. An equal amount of charge is therefore repelled from the opposite plate, leaving it with a charge that is equal and opposite to the charge on the first plate. The two conductors of a capacitor will have equal and opposite charges even if they have different sizes or shapes.

5. Charge a parallel-plate capacitor using a battery with a known voltage $V$. Let the capacitor discharge through a resistor with a known resistance $R$ and measure the time constant. This will allow calculation of the capacitance $C$. Then use $C = \varepsilon_0 A/d$ and solve for $\varepsilon_0$.

6. Parallel. The equivalent capacitance of the three capacitors in parallel will be greater than that of the same three capacitors in series, and therefore they will store more energy when connected to a given potential difference if they are in parallel.

7. If a large copper sheet of thickness $\ell$ is inserted between the plates of a parallel-plate capacitor, the charge on the capacitor will appear on the large flat surfaces of the copper sheet, with the negative side of the copper facing the positive side of the capacitor. This arrangement can be considered to be two capacitors in series, each with a thickness of $\frac{1}{2}(d-\ell)$. The new net capacitance will be $C' = \varepsilon_0 A/(d-\ell)$, so the capacitance of the capacitor will be reduced.

8. A force is required to increase the separation of the plates of an isolated capacitor because you are pulling a positive plate away from a negative plate. The work done in increasing the separation goes into increasing the electric potential energy stored between the plates. The capacitance decreases, and the potential between the plates increases since the charge has to remain the same.

9. (*a*) The energy stored quadruples since the potential difference across the plates doubles and the capacitance doesn't change: $U = \frac{1}{2}CV^2$.
   (*b*) The energy stored quadruples since the charge doubles and the capacitance doesn't change: $U = \frac{1}{2}\frac{Q^2}{C}$.

(*c*) If the separation between the plates doubles, the capacitance is halved. The potential difference across the plates doesn't change if the capacitor remains connected to the battery, so the energy stored is also halved: $U = \frac{1}{2}CV^2$.

10. (*c*) If the voltage across a capacitor is doubled, the amount of energy it can store is quadrupled: $U = \frac{1}{2}CV^2$.

11. The dielectric will be pulled into the capacitor by the electrostatic attractive forces between the charges on the capacitor plates and the polarized charges on the dielectric's surface. (Note that the addition of the dielectric decreases the energy of the system.)

12. If the battery remains connected to the capacitor, the energy stored in the electric field of the capacitor will increase as the dielectric is inserted. Since the energy of the system increases, work must be done and the dielectric will have to be pushed into the area between the plates. If it is released, it will be ejected.

13. (*a*) If the capacitor is isolated, $Q$ remains constant, and $U = \frac{1}{2}\frac{Q^2}{C}$ becomes $U' = \frac{1}{2}\frac{Q^2}{KC}$ and the stored energy decreases.

(*b*) If the capacitor remains connected to a battery so $V$ does not change, $U = \frac{1}{2}CV^2$ becomes $U' = \frac{1}{2}KCV^2$, and the stored energy increases.

14. For dielectrics consisting of polar molecules, one would expect the dielectric constant to decrease with temperature. As the thermal energy increases, the molecular vibrations will increase in amplitude, and the polar molecules will be less likely to line up with the electric field.

15. When the dielectric is removed, the capacitance decreases. The potential difference across the plates remains the same because the capacitor is still connected to the battery. If the potential difference remains the same and the capacitance decreases, the charge on the plates and the energy stored in the capacitor must also decrease. (Charges return to the battery.) The electric field between the plates will stay the same because the potential difference across the plates and the distance between the plates remain constant.

16. For a given configuration of conductors and dielectrics, $C$ is the proportionality constant between the voltage between the plates and the charge on the plates.

17. The dielectric constant is the ratio of the capacitance of a capacitor with the dielectric between the plates to the capacitance without the dielectric. If a conductor were inserted between the plates of a capacitor such that it filled the gap and touched both plates, the capacitance would drop to zero since charge would flow from one plate to the other. So, the dielectric constant of a good conductor would be zero.

## Solutions to Problems

1. The capacitance is found from Eq. 24-1.

$$Q = CV \quad \rightarrow \quad C = \frac{Q}{V} = \frac{2.8\times10^{-3}\,\text{C}}{930\,\text{V}} = 3.0\times10^{-6}\,\text{F} = \boxed{3.0\,\mu\text{F}}$$

2. We assume the capacitor is fully charged, according to Eq. 24-1.

$$Q = CV = \left(12.6\times10^{-6}\,\text{F}\right)\left(12.0\,\text{V}\right) = \boxed{1.51\times10^{-4}\,\text{C}}$$

3. The capacitance is found from Eq. 24-1.

$$Q = CV \quad\rightarrow\quad C = \frac{Q}{V} = \frac{75\times10^{-12}\,\text{C}}{24.0\,\text{V}} = 3.1\times10^{-12}\,\text{F} = \boxed{3.1\,\text{pF}}$$

4. Let $Q_1$ and $V_1$ be the initial charge and voltage on the capacitor, and let $Q_2$ and $V_2$ be the final charge and voltage on the capacitor. Use Eq. 24-1 to relate the charges and voltages to the capacitance.

$$Q_1 = CV_1 \qquad Q_2 = CV_2 \qquad Q_2 - Q_1 = CV_2 - CV_1 = C\left(V_2 - V_1\right) \quad\rightarrow$$

$$C = \frac{Q_2 - Q_1}{V_2 - V_1} = \frac{26\times10^{-6}\,\text{C}}{50\,\text{V}} = 5.2\times10^{-7}\,\text{F} = \boxed{0.52\,\mu\text{F}}$$

5. After the first capacitor is disconnected from the battery, the total charge must remain constant. The voltage across each capacitor must be the same when they are connected together, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the final potential difference to find the value of the second capacitor.

$$Q_{\text{Total}} = C_1 V_{\substack{1\\ \text{initial}}} \qquad Q_{\substack{1\\ \text{final}}} = C_1 V_{\text{final}} \qquad Q_{\substack{2\\ \text{final}}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = Q_{\substack{1\\ \text{final}}} + Q_{\substack{2\\ \text{final}}} = \left(C_1 + C_2\right)V_{\text{final}} \quad\rightarrow\quad C_1 V_{\substack{1\\ \text{initial}}} = \left(C_1 + C_2\right)V_{\text{final}} \quad\rightarrow$$

$$C_2 = C_1\left(\frac{V_{\substack{1\\ \text{initial}}}}{V_{\text{final}}} - 1\right) = \left(7.7\times10^{-6}\,\text{F}\right)\left(\frac{125\,\text{V}}{15\,\text{V}} - 1\right) = 5.6\times10^{-5}\,\text{F} = \boxed{56\,\mu\text{F}}$$

6. The total charge will be conserved, and the final potential difference across the capacitors will be the same.

$$Q_0 = Q_1 + Q_2 \ ; \ V_1 = V_2 \quad\rightarrow\quad \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_0 - Q_1}{C_2} \quad\rightarrow\quad \boxed{Q_1 = Q_0\frac{C_1}{C_1 + C_2}}$$

$$Q_2 = Q_0 - Q_1 = Q_0 - Q_0\frac{C_1}{C_1 + C_2} = \boxed{Q_2 = Q_0\left(\frac{C_2}{C_1 + C_2}\right)}$$

$$V_1 = V_2 = \frac{Q_1}{C_1} = \frac{Q_0\dfrac{C_1}{C_1 + C_2}}{C_1} = \boxed{V = \frac{Q_0}{C_1 + C_2}}$$

7. The work to move the charge between the capacitor plates is $W = qV$, where $V$ is the voltage difference between the plates, assuming that $q \ll Q$ so that the charge on the capacitor does not change appreciably. The charge is then found from Eq. 24-1. The assumption that $q \ll Q$ is justified.

$$W = qV = q\left(\frac{Q}{C}\right) \quad\rightarrow\quad Q = \frac{CW}{q} = \frac{\left(15\,\mu\text{F}\right)\left(15\,\text{J}\right)}{0.20\,\text{mC}} = \boxed{1.1\,\text{C}}$$

8. (*a*) The total charge on the combination of capacitors is the sum of the charges on the two individual capacitors, since there is no battery connected to them to supply additional charge, and there is no neutralization of charge by combining positive and negative charges. The voltage across each capacitor must be the same after they are connected, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the fact of equal potentials to find the charge on each capacitor and the common potential difference.

$$Q_{\substack{1\\\text{initial}}} = C_1 V_{\substack{1\\\text{initial}}} \quad Q_{\substack{2\\\text{initial}}} = C_2 V_{\substack{2\\\text{initial}}} \quad Q_{\substack{1\\\text{final}}} = C_1 V_{\text{final}} \quad Q_{\substack{2\\\text{final}}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = Q_{\substack{1\\\text{initial}}} + Q_{\substack{2\\\text{initial}}} = Q_{\substack{1\\\text{final}}} + Q_{\substack{2\\\text{final}}} = C_1 V_{\substack{1\\\text{initial}}} + C_2 V_{\substack{2\\\text{initial}}} = C_1 V_{\text{final}} + C_2 V_{\text{final}} \rightarrow$$

$$V_{\text{final}} = \frac{C_1 V_{\substack{1\\\text{initial}}} + C_2 V_{\substack{2\\\text{initial}}}}{C_1 + C_2} = \frac{\left(2.70\times10^{-6}\,\text{F}\right)\left(475\,\text{V}\right) + \left(4.00\times10^{-6}\,\text{F}\right)\left(525\,\text{V}\right)}{\left(6.70\times10^{-6}\,\text{F}\right)}$$

$$= 504.85\,\text{V} \approx \boxed{505\,\text{V}} = V_1 = V_2$$

$$Q_{\substack{1\\\text{final}}} = C_1 V_{\text{final}} = \left(2.70\times10^{-6}\,\text{F}\right)\left(504.85\,\text{V}\right) = \boxed{1.36\times10^{-3}\,\text{C}}$$

$$Q_{\substack{2\\\text{final}}} = C_2 V_{\text{final}} = \left(4.00\times10^{-6}\,\text{F}\right)\left(504.85\,\text{V}\right) = \boxed{2.02\times10^{-3}\,\text{C}}$$

(*b*) By connecting plates of opposite charge, the total charge will be the difference of the charges on the two individual capacitors. Once the charges have equalized, the two capacitors will again be at the same potential.

$$Q_{\substack{1\\\text{initial}}} = C_1 V_{\substack{1\\\text{initial}}} \quad Q_{\substack{2\\\text{initial}}} = C_2 V_{\substack{2\\\text{initial}}} \quad Q_{\substack{1\\\text{final}}} = C_1 V_{\text{final}} \quad Q_{\substack{2\\\text{final}}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = \left|Q_{\substack{1\\\text{initial}}} - Q_{\substack{2\\\text{initial}}}\right| = Q_{\substack{1\\\text{final}}} + Q_{\substack{2\\\text{final}}} \rightarrow \left|C_1 V_{\substack{1\\\text{initial}}} - C_2 V_{\substack{2\\\text{initial}}}\right| = C_1 V_{\text{final}} + C_2 V_{\text{final}} \rightarrow$$

$$V_{\text{final}} = \frac{\left|C_1 V_{\substack{1\\\text{initial}}} - C_2 V_{\substack{2\\\text{initial}}}\right|}{C_1 + C_2} = \frac{\left|\left(2.70\times10^{-6}\,\text{F}\right)\left(475\,\text{V}\right) - \left(4.00\times10^{-6}\,\text{F}\right)\left(525\,\text{V}\right)\right|}{\left(6.70\times10^{-6}\,\text{F}\right)}$$

$$= 122.01\,\text{V} \approx \boxed{120\,\text{V}} = V_1 = V_2$$

$$Q_{\substack{1\\\text{final}}} = C_1 V_{\text{final}} = \left(2.70\times10^{-6}\,\text{F}\right)\left(122.01\,\text{V}\right) = \boxed{3.3\times10^{-4}\,\text{C}}$$

$$Q_{\substack{2\\\text{final}}} = C_2 V_{\text{final}} = \left(4.00\times10^{-6}\,\text{F}\right)\left(122.01\,\text{V}\right) = \boxed{4.9\times10^{-4}\,\text{C}}$$

9. Use Eq. 24-1.

$$\Delta Q = C\Delta V \;;\; t = \frac{\Delta Q}{\Delta Q/\Delta t} = \frac{C\Delta V}{\Delta Q/\Delta t} = \frac{\left(1200\,\text{F}\right)\left(6.0\,\text{V}\right)}{1.0\times10^{-3}\,\text{C/s}} = 7.2\times10^{6}\,\text{s}\left(\frac{1\,\text{d}}{86{,}400\,\text{s}}\right) = \boxed{83\,\text{d}}$$

10. (*a*) The absolute value of the charge on each plate is given by Eq. 24-1. The plate with electrons has a net negative charge.

$$Q = CV \quad\rightarrow\quad N\left(-e\right) = -CV \quad\rightarrow$$

$$N=\frac{CV}{e}=\frac{\left(35\times10^{-15}\,\text{F}\right)\left(1.5\,\text{V}\right)}{1.60\times10^{-19}\,\text{C}}=3.281\times10^{5}\approx\boxed{3.3\times10^{5}\,\text{electrons}}$$

(*b*) Since the charge is directly proportional to the potential difference, a 1.0% decrease in potential difference corresponds to a 1.0% decrease in charge.

$\Delta Q=0.01Q$ ;

$$\Delta t=\frac{\Delta Q}{\Delta Q/\Delta t}=\frac{0.01Q}{\Delta Q/\Delta t}=\frac{0.01CV}{\Delta Q/\Delta t}=\frac{0.01\left(35\times10^{-15}\,\text{F}\right)\left(1.5\,\text{V}\right)}{0.30\times10^{-15}\,\text{C/s}}=1.75\,\text{s}\approx\boxed{1.8\,\text{s}}$$

11. Use Eq. 24-2.

$$C=\varepsilon_0\frac{A}{d}\quad\rightarrow\quad A=\frac{Cd}{\varepsilon_0}=\frac{\left(0.40\times10^{-6}\,\text{F}\right)\left(2.8\times10^{-3}\,\text{m}\right)}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)}=126.6\,\text{m}^2\approx\boxed{130\,\text{m}^2}$$

If the capacitor plates were square, they would be about 11.2 m on a side.

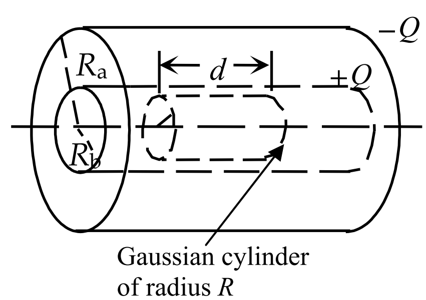
12. The capacitance per unit length of a coaxial cable is derived in Example 24-2

$$\frac{C}{\ell}=\frac{2\pi\varepsilon_0}{\ln\left(R_{\text{outside}}/R_{\text{inside}}\right)}=\frac{2\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)}{\ln\left(5.0\,\text{mm}/1.0\,\text{mm}\right)}=\boxed{3.5\times10^{-11}\,\text{F/m}}$$

13. Inserting the potential at the surface of a spherical conductor into Eq. 24.1 gives the capacitance of a conducting sphere. Then inserting the radius of the Earth yields the Earth's capacitance.

$$C=\frac{Q}{V}=\frac{Q}{\left(Q/4\pi\varepsilon_0 r\right)}=4\pi\varepsilon_0 r=4\pi\left(8.85\times10^{-12}\,\text{F/m}\right)\left(6.38\times10^{6}\,\text{m}\right)=\boxed{7.10\times10^{-4}\,\text{F}}$$

14. From the symmetry of the charge distribution, any electric field must be radial, away from the cylinder axis, and its magnitude must be independent of the location around the axis (for a given radial location). We assume the cylinders have charge of magnitude $Q$ in a length $\ell$. Choose a Gaussian cylinder of length $d$ and radius $R$, centered on the capacitor's axis, with $d\ll\ell$ and the Gaussian cylinder far away from both ends of the capacitor. On the ends of this cylinder, $\vec{\mathbf{E}}\perp d\vec{\mathbf{A}}$ and so there is no flux through the ends. On the curved side of the cylinder, the field has a constant magnitude and $\vec{\mathbf{E}}\parallel d\vec{\mathbf{A}}$. Thus $\vec{\mathbf{E}}\bullet d\vec{\mathbf{A}}=EdA$. Write Gauss's law.




$$\oint\!\!\oint\vec{\mathbf{E}}\bullet d\vec{\mathbf{A}}=E_{\substack{\text{curved}\\\text{walls}}}A_{\substack{\text{curved}\\\text{walls}}}=E\left(2\pi Rd\right)=\frac{Q_{\text{encl}}}{\varepsilon_0}$$

For $R<R_b$, $Q_{\text{encl}}=0\ \rightarrow\ E\left(2\pi Rd\right)\varepsilon_0=0\ \rightarrow\ E=0.$

For $R>R_a$, $Q_{\text{encl}}=\frac{Q}{\ell}d+\frac{-Q}{\ell}d=0$, and so $Q_{\text{encl}}=0\ \rightarrow\ E\left(2\pi Rd\right)\varepsilon_0=0\ \rightarrow\ E=0.$

15. We assume there is a uniform electric field between the capacitor plates, so that $V=Ed$, and then use Eqs. 24-1 and 24-2.

$$Q_{\text{max}}=CV_{\text{max}}=\varepsilon_0\frac{A}{d}\left(E_{\text{max}}d\right)=\varepsilon_0 AE_{\text{max}}=\left(8.85\times10^{-12}\,\text{F/m}\right)\left(6.8\times10^{-4}\,\text{m}^2\right)\left(3.0\times10^{6}\,\text{V/m}\right)$$

$$=\boxed{1.8\times10^{-8}\,\text{C}}$$

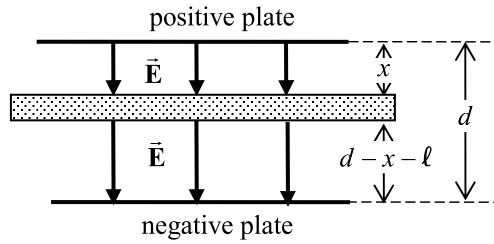
16. We assume there is a uniform electric field between the capacitor plates, so that $V = Ed$, and then use Eqs. 24-1 and 24-2.

$$Q = CV = \varepsilon_0 \frac{A}{d}(Ed) = \varepsilon_0 AE = \left(8.85\times10^{-12}\,\text{F/m}\right)\left(21.0\times10^{-4}\,\text{m}^2\right)\left(4.80\times10^{5}\,\text{V/m}\right)$$
$$= \boxed{8.92\times10^{-9}\,\text{C}}$$

17. We assume there is a uniform electric field between the capacitor plates, so that $V = Ed$, and then use Eqs. 24-1 and 24-2.

$$Q = CV = CEd \;\rightarrow\; E = \frac{Q}{Cd} = \frac{92\times10^{-6}\,\text{C}}{\left(0.80\times10^{-6}\,\text{F}\right)\left(2.0\times10^{-3}\,\text{m}\right)} = \boxed{5.8\times10^{4}\,\text{V/m}}$$

18. (*a*) The uncharged plate will polarize so that negative charge will be drawn towards the positive capacitor plate, and positive charge will be drawn towards the negative capacitor plate. The same charge will be on each face of the plate as on the original capacitor plates. The same electric field will be in the gaps as before the plate was inserted. Use that electric field to determine the potential difference between the two original plates, and the new capacitance. Let *x* be the distance from one original plate to the nearest face of the sheet, and so $d - \ell - x$ is the distance from the other original plate to the other face of the sheet.




$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \;;\; V_1 = Ex = \frac{Qx}{A\varepsilon_0} \;;\; V_2 = E(d-\ell-x) = \frac{Q(d-\ell-x)}{A\varepsilon_0}$$

$$\Delta V = V_1 + V_2 = \frac{Qx}{A\varepsilon_0} + \frac{Q(d-\ell-x)}{A\varepsilon_0} = \frac{Q(d-\ell)}{A\varepsilon_0} = \frac{Q}{C} \;\rightarrow\; C = \boxed{\varepsilon_0\frac{A}{(d-\ell)}}$$

(*b*) $$C_{\text{initial}} = \varepsilon_0\frac{A}{d} \;;\; C_{\text{final}} = \varepsilon_0\frac{A}{(d-\ell)} \;;\; \frac{C_{\text{final}}}{C_{\text{initial}}} = \frac{\varepsilon_0\frac{A}{(d-\ell)}}{\varepsilon_0\frac{A}{d}} = \frac{d}{d-\ell} = \frac{d}{d-0.40d} = \frac{1}{0.60} = \boxed{1.7}$$

19. (*a*) The distance between plates is obtained from Eq. 24-2.

$$C = \frac{\varepsilon_0 A}{x} \;\rightarrow\; x = \frac{\varepsilon_0 A}{C}$$

Inserting the maximum capacitance gives the minimum plate separation and the minimum capacitance gives the maximum plate separation.

$$x_{\text{min}} = \frac{\varepsilon_0 A}{C_{\text{max}}} = \frac{\left(8.85\,\text{pF/m}\right)\left(25\times10^{-6}\,\text{m}^2\right)}{1000.0\times10^{-12}\,\text{F}} = 0.22\,\mu\text{m}$$

$$x_{\text{max}} = \frac{\varepsilon_o A}{C_{\text{min}}} = \frac{\left(8.85\,\text{pF/m}\right)\left(25\times10^{-6}\,\text{m}^2\right)}{1.0\,\text{pF}} = 0.22\,\text{mm} = 220\,\mu\text{m}$$

So $\boxed{0.22\,\mu\text{m} \le x \le 220\,\mu\text{m}}$.

(*b*) Differentiating the distance equation gives the approximate uncertainty in distance.

$$\Delta x \approx \frac{dx}{dC}\Delta C = \frac{d}{dC}\left[\frac{\varepsilon_0 A}{C}\right]\Delta C = -\frac{\varepsilon_0 A}{C^2}\Delta C .$$

The minus sign indicates that the capacitance increases as the plate separation decreases. Since only the magnitude is desired, the minus sign can be dropped. The uncertainty is finally written in terms of the plate separation using Eq. 24-2.

$$\Delta x \approx \frac{\varepsilon_0 A}{\left(\frac{\varepsilon_0 A}{x}\right)^2}\Delta C = \boxed{\frac{x^2 \Delta C}{\varepsilon_0 A}}$$

(*c*) The percent uncertainty in distance is obtained by dividing the uncertainty by the separation distance.

$$\frac{\Delta x_{\text{min}}}{x_{\text{min}}}\times 100\% = \frac{x_{\text{min}}\Delta C}{\varepsilon_o A}\times 100\% = \frac{(0.22\,\mu\text{m})(0.1\,\text{pF})(100\%)}{(8.85\,\text{pF/m})(25\,\text{mm}^2)} = \boxed{0.01\%}$$

$$\frac{\Delta x_{\text{max}}}{x_{\text{max}}}\times 100\% = \frac{x_{\text{max}}\Delta C}{\varepsilon_o A}\times 100\% = \frac{(0.22\,\text{mm})(0.1\,\text{pF})(100\%)}{(8.85\,\text{pF/m})(25\,\text{mm}^2)} = \boxed{10\%}$$

20. The goal is to have an electric field of strength $E_S$ at a radial distance of 5.0 $R_b$ from the center of the cylinder. Knowing the electric field at a specific distance allows us to calculate the linear charge density on the inner cylinder. From the linear charge density and the capacitance we can find the potential difference needed to create the field. From the cylindrically symmetric geometry and Gauss's law, the field in between the cylinders is given by $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$. The capacitance of a cylindrical capacitor is given in Example 24-2.

$$E(R = 5.0R_b) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{(5.0R_b)} = E_S \;\rightarrow\; \lambda = 2\pi\varepsilon_0 (5.0R_b) E_S = \frac{Q}{\ell}$$

$$Q = CV \;\rightarrow\; V = \frac{Q}{C} = \frac{Q}{\dfrac{2\pi\varepsilon_0 \ell}{\ln(R_a/R_b)}} = \frac{Q}{\ell}\frac{\ln(R_a/R_b)}{2\pi\varepsilon_0} = \left[2\pi\varepsilon_0 (5.0R_b) E_S\right]\frac{\ln(R_a/R_b)}{2\pi\varepsilon_0}$$

$$= (5.0R_b) E_S \ln(R_a/R_b) = \left[5.0(1.0\times10^{-4}\,\text{m})\right](2.7\times10^{6}\,\text{N/C})\ln\left(\frac{0.100\,\text{m}}{1.0\times10^{-4}\,\text{m}}\right) = \boxed{9300\,\text{V}}$$

21. To reduce the net capacitance, another capacitor must be added in series.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \;\rightarrow\; \frac{1}{C_2} = \frac{1}{C_{\text{eq}}} - \frac{1}{C_1} = \frac{C_1 - C_{\text{eq}}}{C_1 C_{\text{eq}}} \;\rightarrow$$

$$C_2 = \frac{C_1 C_{\text{eq}}}{C_1 - C_{\text{eq}}} = \frac{(2.9\times10^{-9}\,\text{F})(1.6\times10^{-9}\,\text{F})}{(2.9\times10^{-9}\,\text{F}) - (1.6\times10^{-9}\,\text{F})} = 3.57\times10^{-9}\,\text{F} \approx \boxed{3600\,\text{pF}}$$

$\boxed{\text{Yes}}$, an existing connection needs to be broken in the process. One of the connections of the original capacitor to the circuit must be disconnected in order to connect the additional capacitor in series.

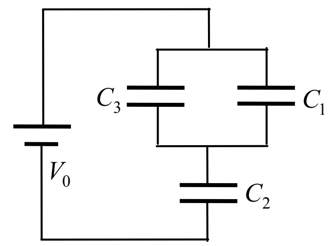
22. (*a*) Capacitors in parallel add according to Eq. 24-3.

$$C_{\text{eq}} = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 = 6(3.8\times10^{-6}\,\text{F}) = \boxed{2.28\times10^{-5}\,\text{F}} = 22.8\mu\text{F}$$

(*b*) Capacitors in series add according to Eq. 24-4.

$$C_{eq} = \left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}+\frac{1}{C_5}+\frac{1}{C_6}\right)^{-1} = \left(\frac{6}{3.8\times10^{-6}\,\text{F}}\right)^{-1} = \frac{3.8\times10^{-6}\,\text{F}}{6} = \boxed{6.3\times10^{-7}\,\text{F}}$$
$$= 0.63\,\mu\text{F}$$

23. We want a small voltage drop across $C_1$. Since $V = Q/C$, if we put the smallest capacitor in series with the battery, there will be a large voltage drop across it. Then put the two larger capacitors in parallel, so that their equivalent capacitance is large and therefore will have a small voltage drop across them. So put $C_1$ and $C_3$ in parallel with each other, and then put that combination in series with $C_2$. See the diagram. To calculate the voltage across $C_1$, find the equivalent capacitance and the net charge. That charge is used to find the voltage drop across $C_2$, and then that voltage is subtracted from the battery voltage to find the voltage across the parallel combination.




$$\frac{1}{C_{eq}} = \frac{1}{C_2}+\frac{1}{C_1+C_3} = \frac{C_1+C_2+C_3}{C_2(C_1+C_3)} \rightarrow C_{eq} = \frac{C_2(C_1+C_3)}{C_1+C_2+C_3} \;;\; Q_{eq} = C_{eq}V_0 \;;\; V_2 = \frac{Q_2}{C_2} = \frac{Q_{eq}}{C_2} \;;$$

$$V_1 = V_0 - V_2 = V_0 - \frac{Q_{eq}}{C_2} = V_0 - \frac{C_{eq}V_0}{C_2} = V_0 - \frac{\frac{C_2(C_1+C_3)}{C_1+C_2+C_3}V_0}{C_2} = \frac{C_2}{C_1+C_2+C_3}V_0 = \frac{1.5\,\mu\text{F}}{6.5\,\mu\text{F}}(12\,\text{V})$$
$$= \boxed{2.8\,\text{V}}$$

24. The capacitors are in parallel, and so the potential is the same for each capacitor, and the total charge on the capacitors is the sum of the individual charges. We use Eqs. 24-1 and 24-2.

$$Q_1 = C_1V = \varepsilon_0\frac{A_1}{d_1}V \;;\; Q_2 = C_2V = \varepsilon_0\frac{A_2}{d_2}V \;;\; Q_3 = C_3V = \varepsilon_0\frac{A_3}{d_3}V$$

$$Q_{total} = Q_1+Q_2+Q_3 = \varepsilon_0\frac{A_1}{d_1}V + \varepsilon_0\frac{A_2}{d_2}V + \varepsilon_0\frac{A_3}{d_3}V = \left(\varepsilon_0\frac{A_1}{d_1}+\varepsilon_0\frac{A_2}{d_2}+\varepsilon_0\frac{A_3}{d_3}\right)V$$

$$C_{net} = \frac{Q_{total}}{V} = \frac{\left(\varepsilon_0\frac{A_1}{d_1}+\varepsilon_0\frac{A_2}{d_2}+\varepsilon_0\frac{A_3}{d_3}\right)V}{V} = \left(\varepsilon_0\frac{A_1}{d_1}+\varepsilon_0\frac{A_2}{d_2}+\varepsilon_0\frac{A_3}{d_3}\right) = C_1+C_2+C_3$$

25. Capacitors in parallel add linearly, and so adding a capacitor in parallel will increase the net capacitance without removing the $5.0\,\mu\text{F}$ capacitor.

$$5.0\,\mu\text{F} + C = 16\,\mu\text{F} \rightarrow C = \boxed{11\,\mu\text{F connected in parallel}}$$

26. (*a*) The two capacitors are in $\boxed{\text{parallel}}$. Both capacitors have their high voltage plates at the same potential (the middle plate), and both capacitors have their low voltage plates at the same potential (the outer plates, which are connected).

(*b*) The capacitance of two capacitors in parallel is the sum of the individual capacitances.

$$C = C_1 + C_2 = \frac{\varepsilon_0 A}{d_1} + \frac{\varepsilon_0 A}{d_2} = \boxed{\varepsilon_0 A\left(\frac{1}{d_1}+\frac{1}{d_2}\right)} = \varepsilon_0 A\left(\frac{d_1+d_2}{d_1 d_2}\right)$$

(*c*) Let $\ell = d_1 + d_2 =$ constant. Then $C = \dfrac{\varepsilon_0 A\ell}{d_1 d_2} = \dfrac{\varepsilon_0 A\ell}{d_1(\ell - d_1)}$. We see that $C \to \infty$ as $d_1 \to 0$ or $d_1 \to \ell$ (which is $d_2 \to 0$). Of course, a real capacitor would break down as the plates got too close to each other. To find the minimum capacitance, set $\dfrac{dC}{d(d_1)} = 0$ and solve for $d_1$.

$$\frac{dC}{d(d_1)} = \frac{d}{d(d_1)}\left[\frac{\varepsilon_0 A\ell}{d_1\ell - d_1^2}\right] = \varepsilon_0 A\ell \frac{(\ell - 2d_1)}{(d_1\ell - d_1^2)^2} = 0 \;\rightarrow\; d_1 = \tfrac{1}{2}\ell = d_2$$

$$C_{min} = \varepsilon_0 A\left(\frac{d_1 + d_2}{d_1 d_2}\right)_{d_1 = \frac{1}{2}\ell} = \varepsilon_0 A\left(\frac{\ell}{(\frac{1}{2}\ell)(\frac{1}{2}\ell)}\right) = \varepsilon_0 A\left(\frac{4}{\ell}\right) = \varepsilon_0 A\left(\frac{4}{d_1 + d_2}\right)$$

$$\boxed{C_{min} = \frac{4\varepsilon_0 A}{d_1 + d_2}\;;\; C_{max} = \infty}$$

27. The maximum capacitance is found by connecting the capacitors in parallel.

$$C_{max} = C_1 + C_2 + C_3 = 3.6\times10^{-9}\,\text{F} + 5.8\times10^{-9}\,\text{F} + 1.00\times10^{-8}\,\text{F} = \boxed{1.94\times10^{-8}\,\text{F in parallel}}$$

The minimum capacitance is found by connecting the capacitors in series.

$$C_{min} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \left(\frac{1}{3.6\times10^{-9}\,\text{F}} + \frac{1}{5.8\times10^{-9}\,\text{F}} + \frac{1}{1.00\times10^{-8}\,\text{F}}\right)^{-1} = \boxed{1.82\times10^{-9}\,\text{F in series}}$$

28. When the capacitors are connected in series, they each have the same charge as the net capacitance.

(*a*) $$Q_1 = Q_2 = Q_{eq} = C_{eq}V = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} V = \left(\frac{1}{0.50\times10^{-6}\,\text{F}} + \frac{1}{0.80\times10^{-6}\,\text{F}}\right)^{-1}(9.0\,\text{V})$$

$$= 2.769\times10^{-6}\,\text{C}$$

$$V_1 = \frac{Q_1}{C_1} = \frac{2.769\times10^{-6}\,\text{C}}{0.50\times10^{-6}\,\text{F}} = 5.538\,\text{V} \approx \boxed{5.5\,\text{V}} \qquad V_2 = \frac{Q_2}{C_2} = \frac{2.769\times10^{-6}\,\text{C}}{0.80\times10^{-6}\,\text{F}} = 3.461\,\text{V} \approx \boxed{3.5\,\text{V}}$$

(*b*) $Q_1 = Q_2 = Q_{eq} = 2.769\times10^{-6}\,\text{C} \approx \boxed{2.8\times10^{-6}\,\text{C}}$

When the capacitors are connected in parallel, they each have the full potential difference.

(*c*) $V_1 = \boxed{9.0\,\text{V}}$ $\quad V_2 = \boxed{9.0\,\text{V}}$ $\quad Q_1 = C_1V_1 = (0.50\times10^{-6}\,\text{F})(9.0\,\text{V}) = \boxed{4.5\times10^{-6}\,\text{C}}$

$Q_2 = C_2V_2 = (0.80\times10^{-6}\,\text{F})(9.0\,\text{V}) = \boxed{7.2\times10^{-6}\,\text{C}}$

29. (*a*) From the diagram, we see that $C_1$ and $C_2$ are in series. That combination is in parallel with $C_3$, and then that combination is in series with $C_4$. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.

$$\frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \;\rightarrow\; C_{12} = \tfrac{1}{2}C\;;\; C_{123} = C_{12} + C_3 = \tfrac{1}{2}C + C = \tfrac{3}{2}C\;;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{2}{3C} + \frac{1}{C} = \frac{5}{3C} \;\rightarrow\; C_{1234} = \boxed{\tfrac{3}{5}C}$$

(*b*) The charge on the equivalent capacitor $C_{1234}$ is given by $Q_{1234} = C_{1234}V = \frac{3}{5}CV$. This is the charge on both of the series components of $C_{1234}$.

$$Q_{123} = \tfrac{3}{5}CV = C_{123}V_{123} = \tfrac{3}{2}CV_{123} \rightarrow V_{123} = \tfrac{2}{5}V$$
$$Q_4 = \tfrac{3}{5}CV = C_4V_4 \rightarrow V_4 = \tfrac{3}{5}V$$

The voltage across the equivalent capacitor $C_{123}$ is the voltage across both of its parallel components. Note that the sum of the charges across the two parallel components of $C_{123}$ is the same as the total charge on the two components, $\frac{3}{5}CV$.

$$V_{123} = \tfrac{2}{5}V = V_{12} \;;\; Q_{12} = C_{12}V_{12} = \left(\tfrac{1}{2}C\right)\left(\tfrac{2}{5}V\right) = \tfrac{1}{5}CV$$
$$V_{123} = \tfrac{2}{5}V = V_3 \;;\; Q_3 = C_3V_3 = C\left(\tfrac{2}{5}V\right) = \tfrac{2}{5}CV$$

Finally, the charge on the equivalent capacitor $C_{12}$ is the charge on both of the series components of $C_{12}$.

$$Q_{12} = \tfrac{1}{5}CV = Q_1 = C_1V_1 \rightarrow V_1 = \tfrac{1}{5}V \;;\; Q_{12} = \tfrac{1}{5}CV = Q_2 = C_1V_2 \rightarrow V_2 = \tfrac{1}{5}V$$

Here are all the results, gathered together.

$$\boxed{\begin{array}{l} Q_1 = Q_2 = \tfrac{1}{5}CV \;;\; Q_3 = \tfrac{2}{5}CV \;;\; Q_4 = \tfrac{3}{5}CV \\ V_1 = V_2 = \tfrac{1}{5}V \;;\; V_3 = \tfrac{2}{5}V \;;\; V_4 = \tfrac{3}{5}V \end{array}}$$

30. $C_1$ and $C_2$ are in series, so they both have the same charge. We then use that charge to find the voltage across each of $C_1$ and $C_2$. Then their combined voltage is the voltage across $C_3$. The voltage across $C_3$ is used to find the charge on $C_3$.

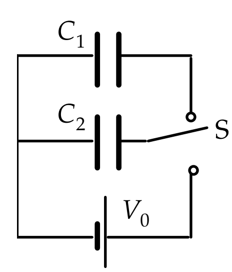
$$Q_1 = Q_2 = 12.4\mu\text{C} \;;\; V_1 = \frac{Q_1}{C_1} = \frac{12.4\mu\text{C}}{16.0\mu\text{F}} = 0.775\,\text{V} \;;\; V_2 = \frac{Q_2}{C_2} = \frac{12.4\mu\text{C}}{16.0\mu\text{F}} = 0.775\,\text{V}$$
$$V_3 = V_1 + V_2 = 1.55\,\text{V} \;;\; Q_3 = C_3V_3 = (16.0\mu\text{F})(1.55\,\text{V}) = 24.8\mu\text{C}$$

From the diagram, $C_4$ must have the same charge as the sum of the charges on $C_1$ and $C_3$. Then the voltage across the entire combination is the sum of the voltages across $C_4$ and $C_3$.

$$Q_4 = Q_1 + Q_3 = 12.4\mu\text{C} + 24.8\mu\text{C} = 37.2\mu\text{C} \;;\; V_4 = \frac{Q_4}{C_4} = \frac{37.2\mu\text{C}}{28.5\mu\text{F}} = 1.31\,\text{V}$$
$$V_{ab} = V_4 + V_3 = 1.31\,\text{V} + 1.55\,\text{V} = 2.86\,\text{V}$$

Here is a summary of all results.

$$\boxed{\begin{array}{l} Q_1 = Q_2 = 12.4\mu\text{C} \;;\; Q_3 = 24.8\mu\text{C} \;;\; Q_4 = 37.2\mu\text{C} \\ V_1 = V_2 = 0.775\,\text{V} \;;\; V_3 = 1.55\,\text{V} \;;\; V_4 = 1.31\,\text{V} \;;\; V_{ab} = 2.86\,\text{V} \end{array}}$$




31. When the switch is down the initial charge on $C_2$ is calculated from Eq. 24-1.

$$Q_2 = C_2V_0$$

When the switch is moved up, charge will flow from $C_2$ to $C_1$ until the voltage across the two capacitors is equal.

$$V = \frac{Q_2'}{C_2} = \frac{Q_1'}{C_1} \rightarrow Q_2' = Q_1'\frac{C_2}{C_1}$$

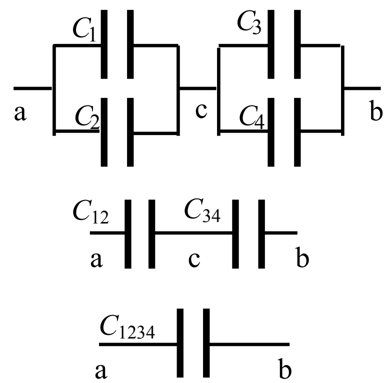
The sum of the charges on the two capacitors is equal to the initial charge on $C_2$.

$$Q_2 = Q_2' + Q_1' = Q_1' \frac{C_2}{C_1} + Q_1' = Q_1' \left( \frac{C_2 + C_1}{C_1} \right)$$

Inserting the initial charge in terms of the initial voltage gives the final charges.

$$Q_1' \left( \frac{C_2 + C_1}{C_1} \right) = C_2 V_0 \quad \rightarrow \quad Q_1' = \boxed{\frac{C_1 C_2}{C_2 + C_1} V_0} \ ; \ Q_2' = Q_1' \frac{C_2}{C_1} = \boxed{\frac{C_2^2}{C_2 + C} V_0}$$

32. (*a*) From the diagram, we see that $C_1$ and $C_2$ are in parallel, and $C_3$ and $C_4$ are in parallel. Those two combinations are then in series with each other. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.




$$C_{12} = C_1 + C_2 \ ; \ C_{34} = C_3 + C_4 \ ;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{12}} + \frac{1}{C_{34}} = \frac{1}{C_1 + C_2} + \frac{1}{C_3 + C_4} \quad \rightarrow$$

$$C_{1234} = \boxed{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}$$

(*b*) The charge on the equivalent capacitor $C_{1234}$ is given by $Q_{1234} = C_{1234} V$. This is the charge on both of the series components of $C_{1234}$. Note that $V_{12} + V_{34} = V$.

$$Q_{12} = C_{1234} V = C_{12} V_{12} \quad \rightarrow \quad V_{12} = \frac{C_{1234}}{C_{12}} V = \frac{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}{C_1 + C_2} V = \frac{(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)} V$$

$$Q_{34} = C_{1234} V = C_{34} V_{34} \quad \rightarrow \quad V_{34} = \frac{C_{1234}}{C_{34}} V = \frac{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}{C_3 + C_4} V = \frac{(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)} V$$

The voltage across the equivalent capacitor $C_{12}$ is the voltage across both of its parallel components, and the voltage across the equivalent $C_{34}$ is the voltage across both its parallel components.

$$V_{12} = \boxed{V_1 = V_2 = \frac{(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)} V} \ ;$$

$$C_1 V_1 = \boxed{Q_1 = \frac{C_1 (C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)} V} \ ; \ C_2 V_2 = \boxed{Q_2 = \frac{C_2 (C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)} V}$$

$$V_{34} = \boxed{V_3 = V_2 = \frac{(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)} V} \ ;$$

$$C_3 V_3 = \boxed{Q_3 = \frac{C_3 (C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)} V} \ ; \ C_4 V_4 = \boxed{Q_4 = \frac{C_4 (C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)} V}$$

33. (*a*) The voltage across $C_3$ and $C_4$ must be the same, since they are in parallel.

$$V_3 = V_4 \rightarrow \frac{Q_3}{C_3} = \frac{Q_4}{C_4} \rightarrow Q_4 = Q_3\frac{C_4}{C_3} = (23\mu\text{C})\frac{16\mu\text{F}}{8\mu\text{F}} = \boxed{46\mu\text{C}}$$

The parallel combination of $C_3$ and $C_4$ is in series with the parallel combination of $C_1$ and $C_2$, and so $Q_3 + Q_4 = Q_1 + Q_2$. That total charge then divides between $C_1$ and $C_2$ in such a way that $V_1 = V_2$.

$$Q_1 + Q_2 = Q_3 + Q_4 = 69\mu\text{C} \;;\; V_1 = V_2 \rightarrow \frac{Q_1}{C_1} = \frac{Q_4}{C_4} = \frac{69\mu\text{C} - Q_1}{C_4} \rightarrow$$

$$Q_1 = \frac{C_1}{C_4 + C_1}(69\mu\text{C}) = \frac{8.0\mu\text{F}}{24.0\mu\text{F}}(69\mu\text{C}) = \boxed{23\mu\text{C}} \;;\; Q_2 = 69\mu\text{C} - 23\mu\text{C} = \boxed{46\mu\text{C}}$$

Notice the symmetry in the capacitances and the charges.

(*b*) Use Eq. 24-1.

$$V_1 = \frac{Q_1}{C_1} = \frac{23\mu\text{C}}{8.0\mu\text{F}} = 2.875\,\text{V} \approx \boxed{2.9\,\text{V}} \;;\; V_2 = V_1 = \boxed{2.9\,\text{V}}$$

$$V_3 = \frac{Q_3}{C_3} = \frac{23\mu\text{C}}{8.0\mu\text{F}} = 2.875\,\text{V} \approx \boxed{2.9\,\text{V}} \;;\; V_4 = V_3 = \boxed{2.9\,\text{V}}$$

(*c*) $V_{\text{ba}} = V_1 + V_3 = 2.875\,\text{V} + 2.875\,\text{V} = 5.75\text{V} \approx \boxed{5.8\,\text{V}}$

34. We have $C_\text{P} = C_1 + C_2$ and $\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2}$. Solve for $C_1$ and $C_2$ in terms of $C_\text{P}$ and $C_\text{S}$.

$$\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_1} + \frac{1}{C_\text{P} - C_1} = \frac{(C_\text{P} - C_1) + C_1}{C_1(C_\text{P} - C_1)} = \frac{C_\text{P}}{C_1(C_\text{P} - C_1)} \rightarrow$$

$$\frac{1}{C_\text{S}} = \frac{C_\text{P}}{C_1(C_\text{P} - C_1)} \rightarrow C_1^2 - C_\text{P}C_1 + C_\text{P}C_\text{S} = 0 \rightarrow$$

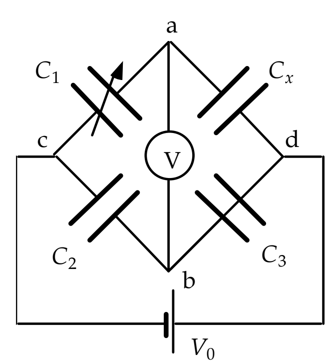
$$C_1 = \frac{C_\text{P} \pm \sqrt{C_\text{P}^2 - 4C_\text{P}C_\text{S}}}{2} = \frac{35.0\mu\text{F} \pm \sqrt{(35.0\mu\text{F})^2 - 4(35.0\mu\text{F})(5.5\mu\text{F})}}{2}$$

$$= 28.2\mu\text{F},\ 6.8\mu\text{F}$$

$$C_2 = C_\text{P} - C_1 = 35.0\mu\text{F} - 28.2\mu\text{F} = 6.8\mu\text{F} \text{ or } 35.0\mu\text{F} - 6.8\mu\text{F} = 28.2\mu\text{F}$$

So the two values are $\boxed{28.2\mu\text{F and } 6.8\mu\text{F}}$.

35. Since there is no voltage between points a and b, we can imagine there being a connecting wire between points a and b. Then capacitors $C_1$ and $C_2$ are in parallel, and so have the same voltage. Also capacitors $C_3$ and $C_x$ are in parallel, and so have the same voltage.




$$V_1 = V_2 \rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \;;\; V_3 = V_x \rightarrow \frac{Q_3}{C_3} = \frac{Q_x}{C_x}$$

Since no charge flows through the voltmeter, we could also remove it from the circuit and have no change in the circuit. In that case, capacitors $C_1$ and $C_x$ are in series and so have the same charge. Likewise capacitors $C_2$ and $C_3$ are in series, and so have the same charge.

$$Q_1 = Q_x \;;\; Q_2 = Q_3$$

Solve this system of equations for $C_x$.

$$\frac{Q_3}{C_3} = \frac{Q_x}{C_x} \;\rightarrow\; C_x = C_3\frac{Q_x}{Q_3} = C_3\frac{Q_1}{Q_2} = C_3\frac{C_1}{C_2} = (4.8\mu\text{F})\left(\frac{8.9\mu\text{F}}{18.0\mu\text{F}}\right) = \boxed{2.4\mu\text{F}}$$

36. The initial equivalent capacitance is the series combination of the two individual capacitances. Each individual capacitor will have the same charge as the equivalent capacitance. The sum of the two initial charges will be the sum of the two final charges, because charge is conserved. The final potential of both capacitors will be equal.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \;\rightarrow$$

$$C_{\text{eq}} = \frac{C_1C_2}{C_1 + C_2} \;;\; Q_{\text{eq}} = C_{\text{eq}}V_0 = \frac{C_1C_2}{C_1 + C_2}V_0 = \frac{(3200\,\text{pF})(1800\,\text{pF})}{5000\,\text{pF}}(12.0\,\text{V}) = 13{,}824\,\text{pC}$$

$$Q_{1\,\text{final}} + Q_{2\,\text{final}} = 2Q_{\text{eq}} \;;\; V_{1\,\text{final}} = V_{2\,\text{final}} \;\rightarrow\; \frac{Q_{1\,\text{final}}}{C_1} = \frac{Q_{2\,\text{final}}}{C_2} = \frac{2Q_{\text{eq}} - Q_{1\,\text{final}}}{C_2} \;\rightarrow$$

$$Q_{1\,\text{final}} = 2\frac{C_1}{C_1 + C_2}Q_{\text{eq}} = 2\frac{3200\,\text{pF}}{5000\,\text{pF}}(13{,}824\,\text{pC}) = 17{,}695\,\text{pC} \approx \boxed{1.8\times10^{-8}\,\text{C}}$$

$$Q_{2\,\text{final}} = 2Q_{\text{eq}} - Q_{1\,\text{final}} = 2(13{,}824\,\text{pC}) - 17{,}695\,\text{pC} = 9953\,\text{pC} \approx \boxed{1.0\times10^{-8}\,\text{C}}$$

37. (*a*) The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{\text{eq}} = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = C_1 + \left(\frac{C_3}{C_2C_3} + \frac{C_2}{C_2C_3}\right)^{-1} = C_1 + \left(\frac{C_2 + C_3}{C_2C_3}\right)^{-1} = \boxed{C_1 + \frac{C_2C_3}{C_2 + C_3}}$$

(*b*) For each capacitor, the charge is found by multiplying the capacitance times the voltage. For $C_1$, the full 35.0 V is across the capacitance, so $Q_1 = C_1V = (24.0\times10^{-6}\,\text{F})(35.0\,\text{V}) =$ $\boxed{8.40\times10^{-4}\,\text{C}}$. The equivalent capacitance of the series combination of $C_2$ and $C_3$ has the full 35.0 V across it, and the charge on the series combination is the same as the charge on each of the individual capacitors.

$$C_{\text{eq}} = \left(\frac{1}{C} + \frac{1}{C/2}\right)^{-1} = \frac{C}{3} \qquad Q_{\text{eq}} = C_{\text{eq}}V = \tfrac{1}{3}(24.0\times10^{-6}\,\text{F})(35.0\,\text{V}) = \boxed{2.80\times10^{-4}\,\text{C}} = Q_2 = Q_3$$

38. From the circuit diagram, we see that $C_1$ is in parallel with the voltage, and so $\boxed{V_1 = 24\,\text{V}}$.

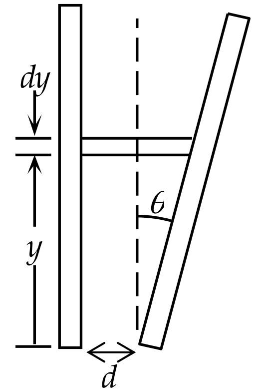
Capacitors $C_2$ and $C_3$ both have the same charge, so their voltages are inversely proportional to their capacitance, and their voltages must total to 24.0 V.

$$Q_2 = Q_3 \;\rightarrow\; C_2V_2 = C_3V_3 \;;\; V_2 + V_3 = V$$

$$V_2 + \frac{C_2}{C_3}V_2 = V \;\rightarrow\; V_2 = \frac{C_3}{C_2 + C_3}V = \frac{4.00\mu\text{F}}{7.00\mu\text{F}}(24.0\,\text{V}) = \boxed{13.7\,\text{V}}$$

$$V_3 = V - V_2 = 24.0\,\text{V} - 13.7\,\text{V} = \boxed{10.3\,\text{V}}$$

39. For an infinitesimal area element of the capacitance a distance $y$ up from the small end, the distance between the plates is $d + x = d + y\tan\theta \approx d + y\theta$. Since the capacitor plates are square, they are of dimension $\sqrt{A}\times\sqrt{A}$, and the area of the infinitesimal strip is $dA = \sqrt{A}\,dy$. The infinitesimal capacitance $dC$ of the strip is calculated, and then the total capacitance is found by adding together all of the infinitesimal capacitances, in parallel with each other.




$$C = \varepsilon_0 \frac{A}{d} \rightarrow dC = \varepsilon_0 \frac{dA}{d+y\theta} = \varepsilon_0 \frac{\sqrt{A}\,dy}{d+y\theta}$$

$$C = \int dC = \int_0^{\sqrt{A}} \varepsilon_0 \frac{\sqrt{A}\,dy}{d+y\theta} = \frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(d+y\theta\right)\Big|_0^{\sqrt{A}}$$

$$= \frac{\varepsilon_0\sqrt{A}}{\theta}\left[\ln\left(d+\theta\sqrt{A}\right) - \ln d\right] = \frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(\frac{d+\theta\sqrt{A}}{d}\right) = \frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(1+\frac{\theta\sqrt{A}}{d}\right)$$

We use the approximation from page A-1 that $\ln(1+x) \approx x - \frac{1}{2}x^2$.

$$C = \frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(1+\frac{\theta\sqrt{A}}{d}\right) = \frac{\varepsilon_0\sqrt{A}}{\theta}\left[\frac{\theta\sqrt{A}}{d} - \frac{1}{2}\left(\frac{\theta\sqrt{A}}{d}\right)^2\right] = \boxed{\frac{\varepsilon_0 A}{d}\left(1-\frac{\theta\sqrt{A}}{2d}\right)}$$

40. No two capacitors are in series or in parallel in the diagram, and so we may not simplify by that method. Instead use the hint as given in the problem. We consider point a as the higher voltage. The equivalent capacitance must satisfy $Q_{\text{tot}} = C_{\text{eq}}V$.

(*a*) The potential between a and b can be written in three ways. Alternate but equivalent expressions are shown in parentheses.

$$V = V_2 + V_1 \;;\; V = V_2 + V_3 + V_4 \;;\; V = V_5 + V_4 \quad \left(V_2 + V_3 = V_5 \;;\; V_3 + V_4 = V_1\right)$$

There are also three independent charge relationships. Alternate but equivalent expressions are shown in parentheses. Convert the charge expressions to voltage – capacitance expression.

$$Q_{\text{tot}} = Q_2 + Q_5 \;;\; Q_{\text{tot}} = Q_4 + Q_1 \;;\; Q_2 = Q_1 + Q_3 \quad \left(Q_4 = Q_3 + Q_5\right)$$

$$C_{\text{eq}}V = C_2V_2 + C_5V_5 \;;\; C_{\text{eq}}V = C_4V_4 + C_1V_1 \;;\; C_2V_2 = C_1V_1 + C_3V_3$$

We have a set of six equations: $V = V_2 + V_1$ (1) ; $V = V_2 + V_3 + V_4$ (2) ; $V = V_5 + V_4$ (3)

$C_{\text{eq}}V = C_2V_2 + C_5V_5$ (4) ; $C_{\text{eq}}V = C_4V_4 + C_1V_1$ (5) ; $C_2V_2 = C_1V_1 + C_3V_3$ (6)

Solve for $C_{\text{eq}}$ as follows.

(i) From Eq. (1), $V_1 = V - V_2$. Rewrite equations (5) and (6). $V_1$ has been eliminated.

$$C_{\text{eq}}V = C_4V_4 + C_1V - C_1V_2 \;(5) \;;\; C_2V_2 = C_1V - C_1V_2 + C_3V_3 \;(6)$$

(ii) From Eq. (3), $V_5 = V - V_4$. Rewrite equation (4). $V_5$ has been eliminated.

$$C_{\text{eq}}V = C_2V_2 + C_5V - C_5V_4 \;(4)$$

(iii) From Eq. (2), $V_3 = V - V_2 - V_4$. Rewrite equation (6). $V_3$ has been eliminated.

$$C_2V_2 = C_1V - C_1V_2 + C_3V - C_3V_2 - C_3V_4 \;(6) \rightarrow$$

$$\left(C_1 + C_2 + C_3\right)V_2 + C_3V_4 = \left(C_1 + C_3\right)V \;(6)$$

Here is the current set of equations.

$$C_{eq}V = C_2V_2 + C_5V - C_5V_4 \quad (4)$$

$$C_{eq}V = C_4V_4 + C_1V - C_1V_2 \quad (5)$$

$$(C_1 + C_2 + C_3)V_2 + C_3V_4 = (C_1 + C_3)V \quad (6)$$

(iv) From Eq. (4), $V_4 = \frac{1}{C_5}\left(C_2V_2 + C_5V - C_{eq}V\right)$. Rewrite equations (5) and (6).

$$C_5C_{eq}V = C_4\left[\left(C_2V_2 + C_5V - C_{eq}V\right)\right] + C_5C_1V - C_5C_1V_2 \quad (5)$$

$$C_5(C_1 + C_2 + C_3)V_2 + C_3\left[\left(C_2V_2 + C_5V - C_{eq}V\right)\right] = C_5(C_1 + C_3)V \quad (6)$$

(v) Group all terms by common voltage.

$$\left(C_5C_{eq} + C_4C_{eq} - C_4C_5 - C_5C_1\right)V = \left(C_4C_2 - C_5C_1\right)V_2 \quad (5)$$

$$\left[C_5(C_1 + C_3) + C_3C_{eq} - C_3C_5\right]V = \left[C_5(C_1 + C_2 + C_3) + C_3C_2\right]V_2 \quad (6)$$

(vi) Divide the two equations to eliminate the voltages, and solve for the equivalent capacitance.

$$\frac{\left(C_5C_{eq} + C_4C_{eq} - C_4C_5 - C_5C_1\right)}{\left[C_5(C_1 + C_3) + C_3C_{eq} - C_3C_5\right]} = \frac{\left(C_4C_2 - C_5C_1\right)}{\left[C_5(C_1 + C_2 + C_3) + C_3C_2\right]} \rightarrow$$

$$C_{eq} = \boxed{\frac{C_1C_2C_3 + C_1C_2C_4 + C_1C_2C_5 + C_1C_3C_5 + C_1C_4C_5 + C_2C_3C_4 + C_2C_4C_5 + C_3C_4C_5}{C_1C_3 + C_1C_4 + C_1C_5 + C_2C_3 + C_2C_4 + C_2C_5 + C_3C_4 + C_3C_5}}$$

(*b*) Evaluate with the given data. Since all capacitances are in $\mu$F, and the expression involves capacitance cubed terms divided by capacitance squared terms, the result will be in $\mu$F.

$$C_{eq} = \frac{C_1C_2C_3 + C_1C_2C_4 + C_1C_2C_5 + C_1C_3C_5 + C_1C_4C_5 + C_2C_3C_4 + C_2C_4C_5 + C_3C_4C_5}{C_1C_3 + C_1C_4 + C_1C_5 + C_2C_3 + C_2C_4 + C_2C_5 + C_3C_4 + C_3C_5}$$

$$= \frac{C_1\left[C_2(C_3 + C_4 + C_5) + C_5(C_3 + C_4)\right] + C_4\left(C_2C_3 + C_2C_5 + C_3C_5\right)}{C_1(C_3 + C_4 + C_5) + C_2(C_3 + C_4 + C_5) + C_3(C_4 + C_5)}$$

$$= \frac{(4.5)\{(8.0)(17.0) + (4.5)(12.5)\} + (8.0)\left[(8.0)(4.5) + (8.0)(4.5) + (4.5)(4.5)\right]}{(4.5)(17.0) + (8.0)(17.0) + (4.5)(12.5)}\mu\text{F}$$

$$= \boxed{6.0\mu\text{F}}$$

41. The stored energy is given by Eq. 24-5.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(2.8\times10^{-9}\,\text{F}\right)\left(2200\,\text{V}\right)^2 = \boxed{6.8\times10^{-3}\,\text{J}}$$

42. The energy density is given by Eq. 24-6.

$$u = \tfrac{1}{2}\varepsilon_0E^2 = \tfrac{1}{2}\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(150\,\text{V/m}\right)^2 = \boxed{1.0\times10^{-7}\,\text{J/m}^3}$$

[43.] The energy stored is obtained from Eq. 24-5, with the capacitance of Eq. 24-2.

$$U = \frac{Q^2}{2C} = \frac{Q^2d}{2\varepsilon_0A} = \frac{\left(4.2\times10^{-4}\,\text{C}\right)^2(0.0013\,\text{m})}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)(0.080\,\text{m})^2} = \boxed{2.0\times10^3\,\text{J}}$$

44. (*a*) The charge is constant, and the tripling of separation reduces the capacitance by a factor of 3.

$$\frac{U_2}{U_1}=\frac{\dfrac{Q^2}{2C_2}}{\dfrac{Q^2}{2C_1}}=\frac{C_1}{C_2}=\frac{\varepsilon_0\dfrac{A}{d}}{\varepsilon_0\dfrac{A}{3d}}=\boxed{3}$$

(*b*) The work done is the change in energy stored in the capacitor.

$$U_2-U_1=3U_1-U_1=2U_1=2\frac{Q^2}{2C_1}=\frac{Q^2}{\varepsilon_0\dfrac{A}{d}}=\boxed{\frac{Q^2d}{\varepsilon_0 A}}$$

45. The equivalent capacitance is formed by $C_1$ in parallel with the series combination of $C_2$ and $C_3$. Then use Eq. 24-5 to find the energy stored.

$$C_{\text{net}}=C_1+\frac{C_2C_3}{C_2+C_3}=C+\frac{C^2}{2C}=\tfrac{3}{2}C$$

$$U=\tfrac{1}{2}C_{\text{net}}V^2=\tfrac{3}{4}CV^2=\tfrac{3}{4}\left(22.6\times10^{-6}\,\text{F}\right)\left(10.0\,\text{V}\right)^2=\boxed{1.70\times10^{-3}\,\text{J}}$$

46. (*a*) Use Eqs. 24-3 and 24-5.

$$U_{\text{parallel}}=\tfrac{1}{2}C_{\text{eq}}V^2=\tfrac{1}{2}\left(C_1+C_2\right)V^2=\tfrac{1}{2}\left(0.65\times10^{-6}\,\text{F}\right)\left(28\,\text{V}\right)^2=2.548\times10^{-4}\,\text{J}\approx\boxed{2.5\times10^{-4}\,\text{J}}$$

(*b*) Use Eqs. 24-4 and 24-5.

$$U_{\text{series}}=\tfrac{1}{2}C_{\text{eq}}V^2=\tfrac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)V^2=\tfrac{1}{2}\left(\frac{\left(0.45\times10^{-6}\,\text{F}\right)\left(0.20\times10^{-6}\,\text{F}\right)}{0.65\times10^{-6}\,\text{F}}\right)\left(28\,\text{V}\right)^2$$

$$=5.428\times10^{-5}\,\text{J}\approx\boxed{5.4\times10^{-5}\,\text{J}}$$

(*c*) The charge can be found from Eq. 24-5.

$$U=\tfrac{1}{2}QV\;\rightarrow\;Q=\frac{2U}{V}\;\rightarrow\;Q_{\text{parallel}}=\frac{2\left(2.548\times10^{-4}\,\text{J}\right)}{28\,\text{V}}=\boxed{1.8\times10^{-5}\,\text{C}}$$

$$Q_{\text{series}}=\frac{2\left(5.428\times10^{-5}\,\text{J}\right)}{28\,\text{V}}=\boxed{3.9\times10^{-6}\,\text{C}}$$

47. The capacitance of a cylindrical capacitor is given in Example 24-2 as $C=\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_a/R_b\right)}$.

(*a*) If the charge is constant, the energy can be calculated by $U=\tfrac{1}{2}\dfrac{Q^2}{C}$.

$$\frac{U_2}{U_1}=\frac{\tfrac{1}{2}\dfrac{Q^2}{C_2}}{\tfrac{1}{2}\dfrac{Q^2}{C_1}}=\frac{C_1}{C_2}=\frac{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_a/R_b\right)}}{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(3R_a/R_b\right)}}=\boxed{\frac{\ln\left(3R_a/R_b\right)}{\ln\left(R_a/R_b\right)}}>1$$

The energy comes from the work required to separate the capacitor components.

(*b*) If the voltage is constant, the energy can be calculated by $U = \frac{1}{2}CV^2$.

$$\frac{U_2}{U_1} = \frac{\frac{1}{2}C_2V^2}{\frac{1}{2}C_1V^2} = \frac{C_2}{C_1} = \frac{\dfrac{2\pi\varepsilon_0\ell}{\ln(3R_a/R_b)}}{\dfrac{2\pi\varepsilon_0\ell}{\ln(R_a/R_b)}} = \boxed{\frac{\ln(R_a/R_b)}{\ln(3R_a/R_b)}} < 1$$

Since the voltage remained constant, and the capacitance decreased, the amount of charge on the capacitor components decreased. Charge flowed back into the battery that was maintaining the constant voltage.

48. (*a*) Before the capacitors are connected, the only stored energy is in the initially-charged capacitor. Use Eq. 24-5.

$$U_1 = \tfrac{1}{2}C_1V_0^2 = \tfrac{1}{2}\left(2.20\times10^{-6}\,\text{F}\right)\left(12.0\,\text{V}\right)^2 = 1.584\times10^{-4}\,\text{J} \approx \boxed{1.58\times10^{-4}\,\text{J}}$$

(*b*) The total charge available is the charge on the initial capacitor. The capacitance changes to the equivalent capacitance of the two capacitors in parallel.

$$Q = Q_1 = C_1V_0 \;;\; C_{\text{eq}} = C_1 + C_2 \;;\; U_2 = \tfrac{1}{2}\frac{Q^2}{C_{\text{eq}}} = \tfrac{1}{2}\frac{C_1^2V_0^2}{C_1+C_2} = \tfrac{1}{2}\frac{\left(2.20\times10^{-6}\,\text{F}\right)^2\left(12.0\,\text{V}\right)^2}{\left(5.70\times10^{-6}\,\text{F}\right)}$$

$$= 6.114\times10^{-5}\,\text{J} \approx \boxed{6.11\times10^{-5}\,\text{J}}$$

(*c*) $\Delta U = U_2 - U_1 = 6.114\times10^{-5}\,\text{J} - 1.584\times10^{-4}\,\text{J} = \boxed{-9.73\times10^{-5}\,\text{J}}$

49. (*a*) With the plate inserted, the capacitance is that of two series capacitors of plate separations $d_1 = x$ and $d_2 = d - \ell - x$.

$$C_i = \left[\frac{x}{\varepsilon_0 A} + \frac{d - x - \ell}{\varepsilon_0 A}\right]^{-1} = \frac{\varepsilon_0 A}{d - \ell}$$

With the plate removed the capacitance is obtained directly from Eq. 24-2.

$$C_f = \frac{\varepsilon_0 A}{d}$$

Since the voltage remains constant the energy of the capacitor will be given by Eq. 24-5 written in terms of voltage and capacitance. The work will be the change in energy as the plate is removed.

$$W = U_f - U_i = \tfrac{1}{2}\left(C_f - C_i\right)V^2$$

$$= \tfrac{1}{2}\left(\frac{\varepsilon_0 A}{d} - \frac{\varepsilon_0 A}{d-\ell}\right)V^2 = \boxed{-\frac{\varepsilon_0 A\ell V^2}{2d\left(d-\ell\right)}}$$

The net work done is negative. Although the person pulling the plate out must do work, charge is returned to the battery, resulting in a net negative work done.

(*b*) Since the charge now remains constant, the energy of the capacitor will be given by Eq. 24-5 written in terms of capacitance and charge.

$$W = \frac{Q^2}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right) = \frac{Q^2}{2}\left(\frac{d}{\varepsilon_0 A} - \frac{d-\ell}{\varepsilon_0 A}\right) = \frac{Q^2\ell}{2\varepsilon_0 A}$$

The original charge is $Q = CV_0 = \dfrac{\varepsilon_0 A}{d-\ell}V_0$ and so $W = \dfrac{\left(\dfrac{\varepsilon_0 A}{d-\ell}V_0\right)^2 \ell}{2\varepsilon_0 A} = \boxed{\dfrac{\varepsilon_0 A V_0^2 \ell}{2\left(d-\ell\right)^2}}$.

50. (*a*) The charge remains constant, so we express the stored energy as $U = \frac{1}{2}\dfrac{Q^2}{C} = \frac{1}{2}\dfrac{Q^2 x}{\varepsilon_0 A}$, where $x$ is the separation of the plates. The work required to increase the separation by $dx$ is $dW = Fdx$, where $F$ is the force on one plate exerted by the other plate. That work results in an increase in potential energy, $dU$.

$$dW = Fdx = dU = \frac{1}{2}\frac{Q^2 dx}{\varepsilon_0 A} \quad \rightarrow \quad \boxed{F = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A}}$$

(*b*) We cannot use $F = QE = Q\dfrac{\sigma}{\varepsilon_0} = Q\dfrac{Q}{\varepsilon_0 A} = \dfrac{Q^2}{\varepsilon_0 A}$ because the electric field is due to both plates, and charge cannot put a force on itself by the field it creates. By the symmetry of the geometry, the electric field at one plate, due to just the other plate, is $\frac{1}{2}E$. See Example 24-10.

51. (*a*) The electric field outside the spherical conductor is that of an equivalent point charge at the center of the sphere, so $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}, r > R$. Consider a differential volume of radius $dr$, and volume $dV = 4\pi r^2 dr$, as used in Example 22-5. The energy in that volume is $dU = udV$. Integrate over the region outside the conductor.

$$U = \int dU = \int udV = \frac{1}{2}\varepsilon_0 \int E^2 dV = \frac{1}{2}\varepsilon_0 \int_R^\infty \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right)^2 4\pi r^2 dr = \frac{Q^2}{8\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}dr = -\frac{Q^2}{8\pi\varepsilon_0}\frac{1}{r}\bigg|_R^\infty$$

$$= \boxed{\frac{Q^2}{8\pi\varepsilon_0 R}}$$

(*b*) Use Eq. 24-5 with the capacitance of an isolated sphere, from the text immediately after Example 24-3.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R} = \boxed{\frac{Q^2}{8\pi\varepsilon_0 R}}$$

(*c*) When there is a charge $q < Q$ on the sphere, the potential of the sphere is $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}$. The work required to add a charge $dq$ to the sphere is then $dW = Vdq = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}dq$. That work increase the potential energy by the same amount, so $dU = dW = Vdq = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}dq$. Build up the entire charge from 0 to $Q$, calculating the energy as the charge increases.

$$U = \int dU = \int dW = \int Vdq = \int_0^Q \frac{1}{4\pi\varepsilon_0}\frac{q}{R}dq = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q qdq = \boxed{\frac{Q^2}{8\pi\varepsilon_0 R}}$$

52. In both configurations, the voltage across the combination of capacitors is the same. So use $U = \frac{1}{2}CV^2$.

$$U_P = \tfrac{1}{2}C_P V^2 = \tfrac{1}{2}\left(C_1 + C_2\right)V^2 \;;\; U_S = \tfrac{1}{2}C_S V^2 = \tfrac{1}{2}\frac{C_1C_2}{\left(C_1 + C_2\right)}V^2$$

$$U_P = 5\,U_S \;\rightarrow\; \tfrac{1}{2}\left(C_1 + C_2\right)V^2 = 5\left(\tfrac{1}{2}\right)\frac{C_1C_2}{\left(C_1 + C_2\right)}V^2 \;\rightarrow\; \left(C_1 + C_2\right)^2 = 5C_1C_2 \;\rightarrow$$

$$C_1^2 - 3C_1C_2 + C_2^2 = 0 \;\rightarrow\; C_1 = \frac{3C_2 \pm \sqrt{9C_2^2 - 4C_2^2}}{2} = C_2\frac{3 \pm \sqrt{5}}{2} \;\rightarrow$$

$$\frac{C_1}{C_2} = \boxed{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2} = 2.62, 0.382}$$

53. First find the ratio of energy requirements for a logical operation in the past to the current energy requirements for a logical operation.

$$\frac{E_{\text{past}}}{E_{\text{present}}} = \frac{N\left(\frac{1}{2}CV^2\right)_{\text{past}}}{N\left(\frac{1}{2}CV^2\right)_{\text{present}}} = \left(\frac{C_{\text{past}}}{C_{\text{present}}}\right)\left(\frac{V_{\text{past}}}{V_{\text{present}}}\right)^2 = \left(\frac{20}{1}\right)\left(\frac{5.0}{1.5}\right)^2 = 220$$

So past operations would have required 220 times more energy. Since 5 batteries in the past were required to hold the same energy as a present battery, it would have taken 1100 times as many batteries in the past. And if it takes 2 batteries for a modern PDA, it would take $\boxed{\text{2200 batteries}}$ to power the PDA in the past. It $\boxed{\text{would not fit}}$ in a pocket or purse. The volume of a present-day battery is $V = \pi r^2 \ell = \pi\left(0.5\,\text{cm}\right)^2\left(4\,\text{cm}\right) = 3\,\text{cm}^3$. The volume of 2200 of them would be $6600\,\text{cm}^3$, which would require a cube about 20 cm in side length.

54. Use Eq. 24-8 to calculate the capacitance with a dielectric.

$$C = K\varepsilon_0\frac{A}{d} = \left(2.2\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\frac{\left(4.2\times10^{-2}\,\text{m}\right)^2}{\left(1.8\times10^{-3}\,\text{m}\right)} = \boxed{1.9\times10^{-11}\,\text{F}}$$

$\boxed{55.}$ The change in energy of the capacitor is obtained from Eq. 24-5 in terms of the constant voltage and the capacitance.

$$\Delta U = U_f - U_i = \tfrac{1}{2}C_0V^2 - \tfrac{1}{2}KC_0V^2 = -\tfrac{1}{2}\left(K-1\right)C_0V^2$$

The work done by the battery in maintaining a constant voltage is equal to the voltage multiplied by the change in charge, with the charge given by Eq. 24-1.

$$W_{\text{battery}} = V\left(Q_f - Q_i\right) = V\left(C_0V - KC_0V\right) = -\left(K-1\right)C_0V^2$$

The work done in pulling the dielectric out of the capacitor is equal to the difference between the change in energy of the capacitor and the energy done by the battery.

$$W = \Delta U - W_{\text{battery}} = -\tfrac{1}{2}\left(K-1\right)C_0V^2 + \left(K-1\right)C_0V^2$$

$$= \tfrac{1}{2}\left(K-1\right)C_0V^2 = \left(3.4-1\right)\left(8.8\times10^{-9}\,\text{F}\right)\left(100\,\text{V}\right)^2 = \boxed{1.1\times10^{-4}\,\text{J}}$$

56. We assume the charge and dimensions are the same as in Problem 43. Use Eq. 24-5 with charge and capacitance.

$$U = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}\frac{Q^2}{KC_0} = \tfrac{1}{2}\frac{Q^2 d}{K\varepsilon_0 A} = \tfrac{1}{2}\frac{\left(420\times10^{-6}\,\text{C}\right)^2\left(0.0013\,\text{m}\right)}{(7)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(64\times10^{-4}\,\text{m}^2\right)} = 289.2\,\text{J} \approx \boxed{290\,\text{J}}$$

57. From Problem 10, we have $C = 35\times10^{-15}\,\text{F}$. Use Eq. 24-8 to calculate the area.

$$C = K\varepsilon_0\frac{A}{d} \;\rightarrow\; A = \frac{Cd}{K\varepsilon_0} = \frac{\left(35\times10^{-15}\,\text{F}\right)\left(2.0\times10^{-9}\,\text{m}\right)}{(25)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = 3.164\times10^{-13}\,\text{m}^2\left(\frac{10^6\,\mu\text{m}}{1\,\text{m}}\right)^2$$

$$= 0.3164\,\mu\text{m}^2 \approx \boxed{0.32\,\mu\text{m}^2}$$

Half of the area of the cell is used for capacitance, so $1.5\,\text{cm}^2$ is available for capacitance. Each capacitor is one "bit."

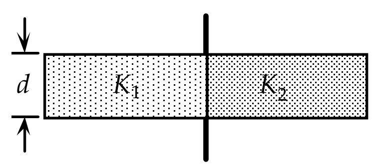
$$1.5\,\text{cm}^2\left(\frac{10^6\,\mu\text{m}}{10^2\,\text{cm}}\right)^2\left(\frac{1\,\text{bit}}{0.32\,\mu\text{m}^2}\right)\left(\frac{1\,\text{byte}}{8\,\text{bits}}\right) = 5.86\times10^7\,\text{bytes} \approx \boxed{59\,\text{Mbytes}}$$

58. The initial charge on the capacitor is $Q_{\text{initial}} = C_{\text{initial}}V$. When the mica is inserted, the capacitance changes to $C_{\text{final}} = KC_{\text{initial}}$, and the voltage is unchanged since the capacitor is connected to the same battery. The final charge on the capacitor is $Q_{\text{final}} = C_{\text{final}}V$.

$$\Delta Q = Q_{\text{final}} - Q_{\text{initial}} = C_{\text{final}}V - C_{\text{initial}}V = (K-1)C_{\text{initial}}V = (7-1)\left(3.5\times10^{-9}\,\text{F}\right)(32\,\text{V})$$

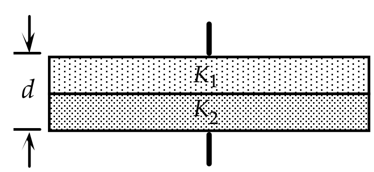
$$= \boxed{6.7\times10^{-7}\,\text{C}}$$

59. The potential difference is the same on each half of the capacitor, so it can be treated as two capacitors in parallel. Each parallel capacitor has half of the total area of the original capacitor.



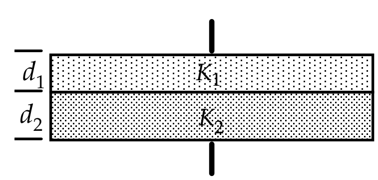

$$C = C_1 + C_2 = K_1\varepsilon_0\frac{\tfrac{1}{2}A}{d} + K_2\varepsilon_0\frac{\tfrac{1}{2}A}{d} = \boxed{\tfrac{1}{2}\left(K_1+K_2\right)\varepsilon_0\frac{A}{d}}$$

60. The intermediate potential at the boundary of the two dielectrics can be treated as the "low" potential plate of one half and the "high" potential plate of the other half, so we treat it as two capacitors in series. Each series capacitor has half of the inter-plate distance of the original capacitor.




$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\tfrac{1}{2}d}{K_1\varepsilon_0 A} + \frac{\tfrac{1}{2}d}{K_2\varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\frac{K_1+K_2}{K_1K_2} \;\rightarrow\; C = \boxed{\frac{2\varepsilon_0 A}{d}\frac{K_1K_2}{K_1+K_2}}$$

61. The capacitor can be treated as two series capacitors with the same areas, but different plate separations and dielectrics. Substituting Eq. 24-8 into Eq. 24-4 gives the effective capacitance.




$$C = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{d_1}{K_1A\varepsilon_0} + \frac{d_2}{K_2A\varepsilon_0}\right)^{-1} = \boxed{\frac{A\varepsilon_0K_1K_2}{d_1K_2 + d_2K_1}}$$

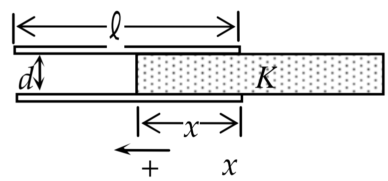
62. (*a*) Since the capacitors each have the same charge and the same voltage in the initial situation, each has the same capacitance of $C = \frac{Q_0}{V_0}$. When the dielectric is inserted, the total charge of $2Q_0$ will not change, but the charge will no longer be divided equally between the two capacitors. Some charge will move from the capacitor without the dielectric $(C_1)$ to the capacitor with the dielectric $(C_2)$. Since the capacitors are in parallel, their voltages will be the same.

$$V_1 = V_2 \rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \rightarrow \frac{Q_1}{C} = \frac{2Q_0 - Q_1}{KC} \rightarrow$$

$$Q_1 = \frac{2}{(K+1)}Q_0 = \frac{2}{4.2}Q_0 = \boxed{0.48Q_0} \ ; \ Q_2 = \boxed{1.52Q_0}$$

(*b*) $$V_1 = V_2 = \frac{Q_1}{C_1} = \frac{0.48Q_0}{Q_0/V_0} = \boxed{0.48V_0} = \frac{Q_2}{C_2} = \frac{1.52Q_0}{3.2Q_0/V_0}$$

63. (*a*) We treat this system as two capacitors, one with a dielectric, and one without a dielectric. Both capacitors have their high voltage plates in contact and their low voltage plates in contact, so they are in parallel. Use Eq. 24-2 and 24-8 for the capacitance. Note that *x* is measured from the right edge of the capacitor, and is positive to the left in the diagram.




$$C = C_1 + C_2 = \varepsilon_0 \frac{\ell(\ell - x)}{d} + K\varepsilon_0 \frac{\ell x}{d} = \boxed{\varepsilon_0 \frac{\ell^2}{d}\left[1 + (K-1)\frac{x}{\ell}\right]}$$

(*b*) Both "capacitors" have the same potential difference, so use $U = \frac{1}{2}CV^2$.

$$U = \tfrac{1}{2}(C_1 + C_2)V_0^2 = \boxed{\varepsilon_0 \frac{\ell^2}{2d}\left[1 + (K-1)\frac{x}{\ell}\right]V_0^2}$$

(*c*) We must be careful here. When the voltage across a capacitor is constant and a dielectric is inserted, charge flows from the battery to the capacitor. So the battery will lose energy and the capacitor gain energy as the dielectric is inserted. As in Example 24-10, we assume that work is done by an external agent $(W_{nc})$ in such a way that the dielectric has no kinetic energy. Then the work-energy principle (Chapter 8) can be expressed as $W_{nc} = \Delta U$ or $dW_{nc} = dU$. This is analogous to moving an object vertically at constant speed. To increase (decrease) the gravitational potential energy, positive (negative) work must be done by an outside, non-gravitational source.

In this problem, the potential energy of the voltage source and the potential energy of the capacitor both change as *x* changes. Also note that the change in charge stored on the capacitor is the opposite of the change in charge stored in the voltage supply.

$$dW_{nc} = dU = dU_{cap} + dU_{battery} \rightarrow F_{nc}dx = d\left(\tfrac{1}{2}CV_0^2\right) + d\left(Q_{battery}V_0\right) \rightarrow$$

$$F_{nc} = \tfrac{1}{2}V_0^2\frac{dC}{dx} + V_0\frac{dQ_{battery}}{dx} = \tfrac{1}{2}V_0^2\frac{dC}{dx} - V_0\frac{dQ_{cap}}{dx} = \tfrac{1}{2}V_0^2\frac{dC}{dx} - V_0^2\frac{dC}{dx} = -\tfrac{1}{2}V_0^2\frac{dC}{dx}$$

$$= -\tfrac{1}{2}V_0^2\varepsilon_0\frac{\ell^2}{d}\left[\frac{(K-1)}{\ell}\right] = -\frac{V_0^2\varepsilon_0\ell}{2d}(K-1)$$

Note that this force is in the opposite direction of $dx$, and so is to the right. Since this force is being applied to keep the dielectric from accelerating, there must be a force of equal magnitude to the left pulling on the dielectric. This force is due to the attraction of the charged plates and the induced charge on the dielectric. The magnitude and direction of this attractive force are $\boxed{\frac{V_0^2\varepsilon_0\ell}{2d}(K-1),\ \text{left}}$.

64. (*a*) We consider the cylinder as two cylindrical capacitors in parallel. The two "negative plates" are the (connected) halves of the inner cylinder (half of which is in contact with liquid, and half of which is in contact with vapor). The two "positive plates" are the (connected) halves of the outer cylinder (half of which is in contact with liquid, and half of which is in contact with vapor). Schematically, it is like Figure 24-30 in Problem 59. The capacitance of a cylindrical capacitor is given in Example 24-2.

$$C = C_{\text{liq}} + C_{\text{V}} = \frac{2\pi\varepsilon_0 K_{\text{liq}} h}{\ln(R_{\text{a}}/R_{\text{b}})} + \frac{2\pi\varepsilon_0 K_{\text{V}}(\ell - h)}{\ln(R_{\text{a}}/R_{\text{b}})} = \frac{2\pi\varepsilon_0\ell}{\ln(R_{\text{a}}/R_{\text{b}})}\left[(K_{\text{liq}} - K_{\text{V}})\frac{h}{\ell} + K_{\text{V}}\right] = C \quad \rightarrow$$

$$\frac{h}{\ell} = \boxed{\frac{1}{(K_{\text{liq}} - K_{\text{V}})}\left[\frac{C\ln(R_{\text{a}}/R_{\text{b}})}{2\pi\varepsilon_0\ell} - K_{\text{V}}\right]}$$

(*b*) For the full tank, $\frac{h}{\ell} = 1$, and for the empty tank, $\frac{h}{\ell} = 0$.

Full: $$C = \frac{2\pi\varepsilon_0\ell}{\ln(R_{\text{a}}/R_{\text{b}})}\left[(K_{\text{liq}} - K_{\text{V}})\frac{h}{\ell} + K_{\text{V}}\right] = \frac{2\pi\varepsilon_0\ell K_{\text{liq}}}{\ln(R_{\text{a}}/R_{\text{b}})}$$

$$= \frac{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.0\,\text{m})(1.4)}{\ln(5.0\,\text{mm}/4.5\,\text{mm})} = \boxed{1.5\times10^{-9}\,\text{F}}$$

Empty: $$C = \frac{2\pi\varepsilon_0\ell}{\ln(R_{\text{a}}/R_{\text{b}})}\left[(K_{\text{liq}} - K_{\text{V}})\frac{h}{\ell} + K_{\text{V}}\right] = \frac{2\pi\varepsilon_0\ell K_{\text{V}}}{\ln(R_{\text{a}}/R_{\text{b}})}$$

$$= \frac{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.0\,\text{m})(1.0)}{\ln(5.0\,\text{mm}/4.5\,\text{mm})} = \boxed{1.1\times10^{-9}\,\text{F}}$$

65. Consider the dielectric as having a layer of equal and opposite charges at each side of the dielectric. Then the geometry is like three capacitors in series. One air gap is taken to be $d_1$, and then the other air gap is $d - d_1 - \ell$.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d_1}{\varepsilon_0 A} + \frac{\ell}{K\varepsilon_0 A} + \frac{d - d_1 - \ell}{\varepsilon_0 A} = \frac{1}{\varepsilon_0 A}\left(\left[\frac{\ell}{K} + (d - \ell)\right]\right) \quad \rightarrow$$

$$C = \frac{\varepsilon_0 A}{\left[\frac{\ell}{K} + (d - \ell)\right]} = \frac{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.50\times10^{-2}\,\text{m}^2)}{\left[\frac{1.00\times10^{-3}\,\text{m}}{3.50} + (1.00\times10^{-3}\,\text{m})\right]} = \boxed{1.72\times10^{-10}\,\text{F}}$$

66. By leaving the battery connected, the voltage will not change when the dielectric is inserted, but the amount of charge will change. That will also change the electric field.

(*a*) Use Eq. 24-2 to find the capacitance.

$$C_0 = \varepsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(\frac{2.50\times10^{-2}\,\mathrm{m^2}}{2.00\times10^{-3}\,\mathrm{m}}\right) = 1.106\times10^{-10}\,\mathrm{F} \approx \boxed{1.11\times10^{-10}\,\mathrm{F}}$$

(*b*) Use Eq. 24-1 to find the initial charge on each plate.

$$Q_0 = C_0 V = \left(1.106\times10^{-10}\,\mathrm{F}\right)\left(150\,\mathrm{V}\right) = 1.659\times10^{-8}\,\mathrm{C} \approx \boxed{1.66\times10^{-8}\,\mathrm{C}}$$

In Example 24-12, the charge was constant, so it was simple to calculate the induced charge and then the electric fields from those charges. But now the voltage is constant, and so we calculate the fields first, and then calculate the charges. So we are solving the problem parts in a different order.

(*d*) We follow the same process as in part (*f*) of Example 24-12.

$$V = E_0\left(d-\ell\right) + E_{\mathrm{D}}\ell = E_0\left(d-\ell\right) + \frac{E_0}{K}\ell \;\rightarrow$$

$$E_0 = \frac{V}{d-\ell+\dfrac{\ell}{K}} = \frac{\left(150\,\mathrm{V}\right)}{\left(2.00\times10^{-3}\,\mathrm{m}\right)-\left(1.00\times10^{-3}\,\mathrm{m}\right)+\dfrac{\left(1.00\times10^{-3}\,\mathrm{m}\right)}{\left(3.50\right)}} = 1.167\times10^{5}\,\mathrm{V/m}$$

$$\approx \boxed{1.17\times10^{5}\,\mathrm{V/m}}$$

(*e*) $$E_{\mathrm{D}} = \frac{E_0}{K} = \frac{1.167\times10^{5}\,\mathrm{V/m}}{3.50} = 3.333\times10^{4}\,\mathrm{V/m} \approx \boxed{3.33\times10^{4}\,\mathrm{V/m}}$$

(*h*) $$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \;\rightarrow$$

$$Q = EA\varepsilon_0 = \left(1.167\times10^{5}\,\mathrm{V/m}\right)\left(0.0250\,\mathrm{m^2}\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right) = 2.582\times10^{-8}\,\mathrm{C}$$

$$\approx \boxed{2.58\times10^{-8}\,\mathrm{C}}$$

(*c*) $$Q_{\mathrm{ind}} = Q\left(1-\frac{1}{K}\right) = \left(2.582\times10^{-8}\,\mathrm{C}\right)\left(1-\frac{1}{3.50}\right) = \boxed{1.84\times10^{-8}\,\mathrm{C}}$$

(*f*) Because the battery voltage does not change, the potential difference between the plates is unchanged when the dielectric is inserted, and so is $V = \boxed{150\,\mathrm{V}}$.

(*g*) $$C = \frac{Q}{V} = \frac{2.582\times10^{-8}\,\mathrm{C}}{150\,\mathrm{V}} = \boxed{1.72\times10^{-10}\,\mathrm{pF}}$$

Notice that the capacitance is the same as in Example 24-12. Since the capacitance is a constant (function of geometry and material, not charge and voltage), it should be the same value.

$\boxed{67.}$ The capacitance will be given by $C = Q/V$. When a charge $Q$ is placed on one plate and a charge $-Q$ is placed on the other plate, an electric field will be set up between the two plates. The electric field in the air-filled region is just the electric field between two charged plates, $E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$. The electric field in the dielectric is equal to the electric field in the air, divided by the dielectric constant: $E_D = \dfrac{E_0}{K} = \dfrac{Q}{KA\varepsilon_0}$.

The voltage drop between the two plates is obtained by integrating the electric field between the two plates. One plate is set at the origin with the dielectric touching this plate. The dielectric ends at $x = \ell$. The rest of the distance to $x = d$ is then air filled.

$$V = -\int_0^d \vec{\mathbf{E}} \cdot d\vec{\mathbf{x}} = \int_0^\ell \frac{Q\,dx}{KA\varepsilon_0} + \int_\ell^d \frac{Q\,dx}{A\varepsilon_0} = \frac{Q}{A\varepsilon_0}\left(\frac{\ell}{K} + (d-\ell)\right)$$

The capacitance is the ratio of the voltage to the charge.

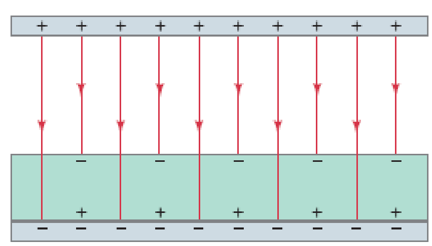
$$C = \frac{Q}{V} = \frac{Q}{\dfrac{Q}{A\varepsilon_0}\left(\dfrac{\ell}{K} + (d-\ell)\right)} = \boxed{\frac{\varepsilon_0 A}{d - \ell + \dfrac{\ell}{K}}}$$

68. Find the energy in each region from the energy density and the volume. The energy density in the "gap" is given by $u_{\text{gap}} = \frac{1}{2}\varepsilon_0 E_{\text{gap}}^2$, and the energy density in the dielectric is given by $u_{\text{D}} = \frac{1}{2}\varepsilon_D E_{\text{D}}^2$ $= \frac{1}{2}K\varepsilon_0\left(\dfrac{E_{\text{gap}}}{K}\right)^2 = \frac{1}{2}\varepsilon_0\dfrac{E_{\text{gap}}^2}{K}$, where Eq. 24-10 is used.

$$\frac{U_{\text{D}}}{U_{\text{total}}} = \frac{U_{\text{D}}}{U_{\text{gap}} + U_{\text{D}}} = \frac{u_{\text{D}}\text{Vol}_{\text{D}}}{u_{\text{gap}}\text{Vol}_{\text{gap}} + u_{\text{D}}\text{Vol}_{\text{D}}} = \frac{\frac{1}{2}\varepsilon_0\dfrac{E_{\text{gap}}^2}{K}A\ell}{\frac{1}{2}\varepsilon_0 E_{\text{gap}}^2 A(d-\ell) + \frac{1}{2}\varepsilon_0\dfrac{E_{\text{gap}}^2}{K}A\ell}$$

$$= \frac{\dfrac{\ell}{K}}{(d-\ell) + \dfrac{\ell}{K}} = \frac{\ell}{(d-\ell)K + \ell} = \frac{(1.00\,\text{mm})}{(1.00\,\text{mm})(3.50) + (1.00\,\text{mm})} = \boxed{0.222}$$

69. There are two uniform electric fields – one in the air, and one in the gap. They are related by Eq. 24-10. In each region, the potential difference is the field times the distance in the direction of the field over which the field exists.



$$V = E_{\text{air}}d_{\text{air}} + E_{\text{glass}}d_{\text{glass}} = E_{\text{air}}d_{\text{air}} + \frac{E_{\text{air}}}{K_{\text{glass}}}d_{\text{glass}} \rightarrow$$

$$E_{\text{air}} = V\frac{K_{\text{glass}}}{d_{\text{air}}K_{\text{glass}} + d_{\text{glass}}}$$

$$= (90.0\,\text{V})\frac{5.80}{(3.00\times10^{-3}\,\text{m})(5.80) + (2.00\times10^{-3}\,\text{m})}$$

$$= \boxed{2.69\times10^4\ \text{V/m}}$$

$$E_{\text{glass}} = \frac{E_{\text{air}}}{K_{\text{glass}}} = \frac{2.69\times10^4\ \text{V/m}}{5.80} = \boxed{4.64\times10^3\ \text{V/m}}$$

The charge on the plates can be calculated from the field at the plate, using Eq. 22-5. Use Eq. 24-11b to calculate the charge on the dielectric.

$$E_{\text{air}} = \frac{\sigma_{\text{plate}}}{\varepsilon_0} = \frac{Q_{\text{plate}}}{\varepsilon_0 A} \rightarrow$$

$$Q_{\text{plate}} = E_{\text{air}}\varepsilon_0 A = \left(2.69\times10^{4}\ \text{V/m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.45\,\text{m}^2\right) = \boxed{3.45\times10^{-7}\,\text{C}}$$

$$Q_{\text{ind}} = Q\left(1-\frac{1}{K}\right) = \left(3.45\times10^{-7}\,\text{C}\right)\left(1-\frac{1}{5.80}\right) = \boxed{2.86\times10^{-7}\,\text{C}}$$

70. (*a*) The capacitance of a single isolated conducting sphere is given after example 24-3.

$$C = 4\pi\varepsilon_0 r \rightarrow$$

$$\frac{C}{r} = 4\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right) = \left(1.11\times10^{-10}\ \frac{\text{F}}{\text{m}}\right)\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)\left(\frac{10^{12}\,\text{pF}}{1\,\text{F}}\right) = 1.11\,\text{pF/cm}$$

And so $C = \left(1.11\,\text{pF/cm}\right)r \rightarrow \boxed{C(\text{pF}) \approx r(\text{cm})}$.

(*b*) We assume that the human body is a sphere of radius 100 cm. Thus the rule $C(\text{pF}) \approx r(\text{cm})$ says that the capacitance of the human body is about $\boxed{100\,\text{pF}}$.

(*c*) A 0.5-cm spark would require a potential difference of about 15,000 V. Use Eq. 24-1.

$$Q = CV = \left(100\,\text{pF}\right)\left(15{,}000\,\text{V}\right) = \boxed{1.5\mu\text{C}}$$

71. Use Eq. 24-5 to find the capacitance.

$$U = \tfrac{1}{2}CV^2 \rightarrow C = \frac{2U}{V^2} = \frac{2\left(1200\,\text{J}\right)}{\left(7500\,\text{V}\right)^2} = \boxed{4.3\times10^{-5}\,\text{F}}$$

72. (*a*) We approximate the configuration as a parallel-plate capacitor, and so use Eq. 24-2 to calculate the capacitance.

$$C = \varepsilon_0\frac{A}{d} = \varepsilon_0\frac{\pi r^2}{d} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\frac{\pi\left[\left(4.5\,\text{in}\right)\left(0.0254\,\text{m/in}\right)\right]^2}{0.050\,\text{m}} = 7.265\times10^{-12}\,\text{F}$$

$$\approx \boxed{7\times10^{-12}\,\text{F}}$$

(*b*) Use Eq. 24-1.

$$Q = CV = \left(7.265\times10^{-12}\,\text{F}\right)\left(9\,\text{V}\right) = 6.539\times10^{-11}\,\text{C} \approx \boxed{7\times10^{-11}\,\text{C}}$$

(*c*) The electric field is uniform, and is the voltage divided by the plate separation.

$$E = \frac{V}{d} = \frac{9\,\text{V}}{0.050\,\text{m}} = 180\,\text{V/m} \approx \boxed{200\,\text{V/m}}$$

(*d*) The work done by the battery to charge the plates is equal to the energy stored by the capacitor. Use Eq. 24-5.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(7.265\times10^{-12}\,\text{F}\right)\left(9\,\text{V}\right)^2 = 2.942\times10^{-10}\,\text{J} \approx \boxed{3\times10^{-10}\,\text{J}}$$

(*e*) The electric field will stay the same, because the voltage will stay the same (since the capacitor is still connected to the battery) and the plate separation will stay the same. The capacitance changes, and so the charge changes (by Eq. 24-1), and so the work done by the battery changes (by Eq. 24-5).

73. Since the capacitor is disconnected from the battery, the charge on it cannot change. The capacitance of the capacitor is increased by a factor of *K*, the dielectric constant.

$$Q = C_{\text{initial}}V_{\text{initial}} = C_{\text{final}}V_{\text{final}} \rightarrow V_{\text{final}} = V_{\text{initial}}\frac{C_{\text{initial}}}{C_{\text{final}}} = V_{\text{initial}}\frac{C_{\text{initial}}}{KC_{\text{initial}}} = (34.0\,\text{V})\frac{1}{2.2} = \boxed{15\,\text{V}}$$

74. The energy is given by Eq. 24-5. Calculate the energy difference for the two different amounts of charge, and then solve for the difference.

$$U = \tfrac{1}{2}\frac{Q^2}{C} \rightarrow \Delta U = \tfrac{1}{2}\frac{(Q+\Delta Q)^2}{C} - \tfrac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left[(Q+\Delta Q)^2 - Q^2\right] = \frac{\Delta Q}{2C}[2Q+\Delta Q] \rightarrow$$

$$Q = \frac{C\Delta U}{\Delta Q} - \tfrac{1}{2}\Delta Q = \frac{(17.0\times10^{-6}\,\text{F})(18.5\,\text{J})}{(13.0\times10^{-3}\,\text{C})} - \tfrac{1}{2}(13.0\times10^{-3}\,\text{C}) = \boxed{17.7\times10^{-3}\,\text{C}} = 17.7\,\text{mC}$$

75. The energy in the capacitor, given by Eq. 24-5, is the heat energy absorbed by the water, given by Eq. 19-2.

$$U = Q_{\text{heat}} \rightarrow \tfrac{1}{2}CV^2 = mc\Delta T \rightarrow$$

$$V = \sqrt{\frac{2mc\Delta T}{C}} = \sqrt{\frac{2(3.5\,\text{kg})\left(4186\frac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(95^\circ\text{C} - 22^\circ\text{C})}{3.0\,\text{F}}} = 844\,\text{V} \approx \boxed{840\,\text{V}}$$

76. (*a*) The capacitance per unit length of a cylindrical capacitor with no dielectric is derived in Example 24-2, as $\frac{C}{\ell} = \frac{2\pi\varepsilon_0}{\ln(R_{\text{outside}}/R_{\text{inside}})}$. The addition of a dielectric increases the capacitance by a factor of *K*.

$$\frac{C}{\ell} = \boxed{\frac{2\pi\varepsilon_0 K}{\ln(R_{\text{outside}}/R_{\text{inside}})}}$$

(*b*) $$\frac{C}{\ell} = \frac{2\pi\varepsilon_0 K}{\ln(R_{\text{outside}}/R_{\text{inside}})} = \frac{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)2.6}{\ln(9.0\,\text{mm}/2.5\,\text{mm})} = \boxed{1.1\times10^{-10}\,\text{F/m}}$$

77. The potential can be found from the field and the plate separation. Then the capacitance is found from Eq. 24-1, and the area from Eq. 24-8.

$$E = \frac{V}{d} \;;\; Q = CV = CEd \rightarrow$$

$$C = \frac{Q}{Ed} = \frac{(0.675\times10^{-6}\,\text{C})}{(9.21\times10^4\,\text{V/m})(1.95\times10^{-3}\,\text{m})} = 3.758\times10^{-9}\,\text{F} \approx \boxed{3.76\times10^{-9}\,\text{F}}$$

$$C = K\varepsilon_0\frac{A}{d} \rightarrow A = \frac{Cd}{K\varepsilon_0} = \frac{(3.758\times10^{-9}\,\text{F})(1.95\times10^{-3}\,\text{m})}{(3.75)(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)} = \boxed{0.221\,\text{m}^2}$$

78. (*a*) If *N* electrons flow onto the plate, the charge on the top plate is $-Ne$, and the positive charge associated with the capacitor is $Q = Ne$. Since $Q = CV$, we have $Ne = CV \rightarrow \boxed{V = Ne/C}$, showing that *V* is proportional to *N*.

(*b*) Given $\Delta V = 1\,\text{mV}$ and we want $\Delta N = 1$, solve for the capacitance.

$$V = \frac{Ne}{C} \rightarrow \Delta V = \frac{e\Delta N}{C} \rightarrow$$

$$C = e\frac{\Delta N}{\Delta V} = \left(1.60\times10^{-19}\,\text{C}\right)\frac{1}{1\times10^{-3}\,\text{V}} = 1.60\times10^{-16}\,\text{F} \approx \boxed{2\times10^{-16}\,\text{F}}$$

(*c*) Use Eq. 24-8.

$$C = \varepsilon_0 K\frac{A}{d} = \varepsilon_0 K\frac{\ell^2}{d} \rightarrow$$

$$\ell = \sqrt{\frac{Cd}{\varepsilon_0 K}} = \sqrt{\frac{\left(1.60\times10^{-16}\,\text{F}\right)\left(100\times10^{-9}\,\text{m}\right)}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(3)}} = 7.76\times10^{-7}\,\text{m}\left(\frac{10^6\,\mu\text{m}}{1\,\text{m}}\right) = \boxed{0.8\,\mu\text{m}}$$

79. The relative change in energy can be obtained by inserting Eq. 24-8 into Eq. 24-5.

$$\frac{U}{U_0} = \frac{\frac{Q^2}{2C}}{\frac{Q^2}{2C_0}} = \frac{C_0}{C} = \frac{\frac{A\varepsilon_0}{d}}{\frac{KA\varepsilon_0}{\left(\frac{1}{2}d\right)}} = \boxed{\frac{1}{2K}}$$

The dielectric is attracted to the capacitor. As such, the dielectric will gain kinetic energy as it enters the capacitor. An external force is necessary to stop the dielectric. The negative work done by this force results in the decrease in energy within the capacitor.

Since the charge remains constant, and the magnitude of the electric field depends on the charge, and not the separation distance, the electric field will not be affected by the change in distance between the plates. The electric field between the plates will be reduced by the dielectric constant, as given in Eq. 24-10.

$$\frac{E}{E_0} = \frac{E_0/K}{E_0} = \boxed{\frac{1}{K}}$$

80. (*a*) Use Eq. 24-2.

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(120\times10^6\,\text{m}^2\right)}{\left(1500\,\text{m}\right)} = 7.08\times10^{-7}\,\text{F} \approx \boxed{7.1\times10^{-7}\,\text{F}}$$

(*b*) Use Eq. 24-1.

$$Q = CV = \left(7.08\times10^{-7}\,\text{F}\right)\left(3.5\times10^7\,\text{V}\right) = 24.78\,\text{C} \approx \boxed{25\,\text{C}}$$

(*c*) Use Eq. 24-5.

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}\left(24.78\,\text{C}\right)\left(3.5\times10^7\,\text{V}\right) = 4.337\times10^8\,\text{J} \approx \boxed{4.3\times10^8\,\text{J}}$$

81. We treat this as *N* capacitors in parallel, so that the total capacitance is *N* times the capacitance of a single capacitor. The maximum voltage and dielectric strength are used to find the plate separation of a single capacitor.

$$d = \frac{V}{E_S} = \frac{100\,\text{V}}{30\times10^6\,\text{V/m}} = 3.33\times10^{-6}\,\text{m} \;;\; N = \frac{\ell}{d} = \frac{6.0\times10^{-3}\,\text{m}}{3.33\times10^{-6}\,\text{m}} = 1800$$

$$C_{\text{eq}} = NC = N\varepsilon_0 K\frac{A}{d} \rightarrow$$

$$K = \frac{C_{eq} d}{N\varepsilon_0 A} = \frac{\left(1.0\times10^{-6}\,\mathrm{F}\right)\left(3.33\times10^{-6}\,\mathrm{m}\right)}{1800\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(12.0\times10^{-3}\,\mathrm{m}\right)\left(14.0\times10^{-3}\,\mathrm{m}\right)} = 1.244 \approx \boxed{1.2}$$

82. The total charge doesn't change when the second capacitor is connected, since the two-capacitor combination is not connected to a source of charge. The final voltage across the two capacitors must be the same. Use Eq. 24-1.

$$Q_0 = C_1 V_0 = Q_1 + Q_2 = C_1 V_1 + C_2 V_2 = C_1 V_1 + C_2 V_1$$

$$C_2 = C_1 \frac{\left(V_0 - V_1\right)}{V_1} = \left(3.5\mu\mathrm{F}\right)\left(\frac{12.4\,\mathrm{V} - 5.9\,\mathrm{V}}{5.9\,\mathrm{V}}\right) = 3.856\mu\mathrm{F} \approx \boxed{3.9\mu\mathrm{F}}$$

83. (*a*) Use Eq. 24-5 to calculate the stored energy.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(8.0\times10^{-8}\,\mathrm{F}\right)\left(2.5\times10^4\,\mathrm{V}\right)^2 = \boxed{25\,\mathrm{J}}$$

(*b*) The power is the energy converted per unit time.

$$P = \frac{\text{Energy}}{\text{time}} = \frac{0.15\left(25\,\mathrm{J}\right)}{4.0\times10^{-6}\,\mathrm{s}} = 9.38\times10^5\,\mathrm{W} \approx \boxed{940\,\mathrm{kW}}$$

84. The pressure is the force per unit area on a face of the dielectric. The force is related to the potential energy stored in the capacitor by Eq. 8-7, $F = -\frac{dU}{dx}$, where *x* is the separation of the capacitor plates.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(K\varepsilon_0 \frac{A}{x}\right)V^2 \;\rightarrow\; F = -\frac{dU}{dx} = \frac{K\varepsilon_0 A V^2}{2x^2}\;;\; P = \frac{F}{A} = \frac{K\varepsilon_0 V^2}{2x^2} \;\rightarrow$$

$$V = \sqrt{\frac{2x^2 P}{K\varepsilon_0}} = \sqrt{\frac{2\left(1.0\times10^{-4}\,\mathrm{m}\right)^2\left(40.0\,\mathrm{Pa}\right)}{\left(3.1\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)}} = \boxed{170\,\mathrm{V}}$$

85. (*a*) From the diagram, we see that one group of 4 plates is connected together, and the other group of 4 plates is connected together. This common grouping shows that the capacitors are connected $\boxed{\text{in parallel}}$.

(*b*) Since they are connected in parallel, the equivalent capacitance is the sum of the individual capacitances. The variable area will change the equivalent capacitance.

$$C_{eq} = 7C = 7\varepsilon_0 \frac{A}{d}$$

$$C_{min} = 7\varepsilon_0 \frac{A_{min}}{d} = 7\left(8.85\times10^{-12}\,\mathrm{C^2/N{\bullet}m^2}\right)\frac{\left(2.0\times10^{-4}\,\mathrm{m^2}\right)}{\left(1.6\times10^{-3}\,\mathrm{m}\right)} = 7.7\times10^{-12}\,\mathrm{F}$$

$$C_{max} = 7\varepsilon_0 \frac{A_{max}}{d} = 7\left(8.85\times10^{-12}\,\mathrm{C^2/N{\bullet}m^2}\right)\frac{\left(9.0\times10^{-4}\,\mathrm{m^2}\right)}{\left(1.6\times10^{-3}\,\mathrm{m}\right)} = 3.5\times10^{-11}\,\mathrm{F}$$

And so the range is $\boxed{\text{from } 7.7\,\mathrm{pF} \text{ to } 35\,\mathrm{pF}}$.

86. (*a*) Since the capacitor is charged and then disconnected from the power supply, the charge is constant. Use Eq. 24-1 to find the new voltage.

$$Q = CV = \text{constant} \;\rightarrow\; C_1V_1 = C_2V_2 \;\rightarrow\; V_2 = V_1\frac{C_1}{C_2} = (7500\,\text{V})\frac{8.0\,\text{pF}}{1.0\,\text{pF}} = \boxed{6.0\times10^4\,\text{V}}$$

(*b*) In using this as a high voltage power supply, once it discharges, the voltage drops, and it needs to be recharged. So it is not a constant source of high voltage. You would also have to be sure it was designed to not have breakdown of the capacitor material when the voltage gets so high. Another disadvantage is that it has only a small amount of energy stored: $U = \frac{1}{2}CV^2$ $= \frac{1}{2}(1.0\times10^{-12}\,\text{C})(6.0\times10^4\,\text{V})^2 = 1.8\times10^{-3}\,\text{J}$ , and so could actually only supply a small amount of power unless the discharge time was extremely short.

87. Since the two capacitors are in series, they will both have the same charge on them.

$$Q_1 = Q_2 = Q_{\text{series}} \;;\; \frac{1}{C_{\text{series}}} = \frac{V}{Q_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} \;\rightarrow$$

$$C_2 = \frac{Q_{\text{series}}C_1}{C_1V - Q_{\text{series}}} = \frac{(125\times10^{-12}\,\text{C})(175\times10^{-12}\,\text{F})}{(175\times10^{-12}\,\text{F})(25.0\,\text{V}) - (125\times10^{-12}\,\text{C})} = \boxed{5.15\times10^{-12}\,\text{F}}$$

88. (*a*) The charge can be determined from Eqs. 24-1 and 24-2.

$$Q = CV = \varepsilon_0\frac{A}{d}V = (8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)\frac{(2.0\times10^{-4}\,\text{m}^2)}{(5.0\times10^{-4}\,\text{m})}(12\,\text{V}) = 4.248\times10^{-11}\,\text{C}$$

$$\approx \boxed{4.2\times10^{-11}\,\text{C}}$$

(*b*) Since the battery is disconnected, no charge can flow to or from the plates. Thus the charge is constant.

$$Q = \boxed{4.2\times10^{-11}\,\text{C}}$$

(*c*) The capacitance has changed and the charge has stayed constant, and so the voltage has changed.

$$Q = CV = \text{constant} \;\rightarrow\; C_1V_1 = C_0V_0 \;\rightarrow\; \varepsilon_0\frac{A}{d_1}V_1 = \varepsilon_0\frac{A}{d_0}V_0 \;\rightarrow$$

$$V_1 = \frac{d_1}{d_0}V_0 = \frac{0.75\,\text{mm}}{0.50\,\text{mm}}(12\,\text{V}) = \boxed{18\,\text{V}}$$

(*d*) The work is the change in stored energy.

$$W = \Delta U = \tfrac{1}{2}QV_1 - \tfrac{1}{2}QV_0 = \tfrac{1}{2}Q(V_1 - V_0) = \tfrac{1}{2}(4.248\times10^{-11}\,\text{C})(6.0\,\text{V}) = \boxed{1.3\times10^{-10}\,\text{J}}$$

89. The first capacitor is charged, and so has a certain amount of charge on its plates. Then, when the switch is moved, the capacitors are not connected to a source of charge, and so the final charge is equal to the initial charge. Initially treat capacitors $C_2$ and $C_3$ as their equivalent capacitance, $C_{23} = \frac{C_2C_3}{C_2 + C_3} = \frac{(2.0\mu\text{F})(2.4\mu\text{F})}{4.4\mu\text{F}} = 1.091\mu\text{F}$. The final voltage across $C_1$ and $C_{23}$ must be the same. The charge on $C_2$ and $C_3$ must be the same. Use Eq. 24-1.

$$Q_0 = C_1V_0 = Q_1 + Q_{23} = C_1V_1 + C_{23}V_{23} = C_1V_1 + C_{23}V_1 \quad \rightarrow$$

$$V_1 = \frac{C_1}{C_1 + C_{23}}V_0 = \frac{1.0\mu\text{F}}{1.0\mu\text{F} + 1.091\mu\text{F}}(24\,\text{V}) = 11.48\,\text{V} = V_1 = V_{23}$$

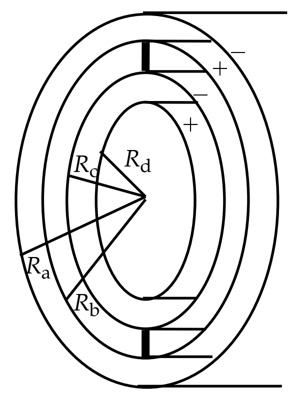
$$Q_1 = C_1V_1 = (1.0\mu\text{F})(11.48\,\text{V}) = 11.48\mu\text{C}$$

$$Q_{23} = C_{23}V_{23} = (1.091\mu\text{F})(11.48\,\text{V}) = 12.52\mu\text{C} = Q_2 = Q_3$$

$$V_2 = \frac{Q_2}{C_2} = \frac{12.52\mu\text{C}}{2.0\mu\text{F}} = 6.26\,\text{V} \;;\; V_3 = \frac{Q_3}{C_3} = \frac{12.52\mu\text{C}}{2.4\mu\text{F}} = 5.22\,\text{V}$$

To summarize: $\boxed{Q_1 = 11\mu\text{C}\,,\, V_1 = 11\,\text{V} \;;\; Q_2 = 13\mu\text{C}\,,\, V_2 = 6.3\,\text{V} \;;\; Q_3 = 13\mu\text{C}\,,\, V_3 = 5.2\,\text{V}}$

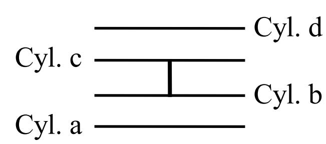
90. The metal conducting strips connecting cylinders b and c mean that b and c are at the same potential. Due to the positive charge on the inner cylinder and the negative charge on the outer cylinder, cylinders b and c will polarize according to the first diagram, with negative charge on cylinder c, and positive charge on cylinder b. This is then two capacitors in series, as illustrated in the second diagram. The capacitance per unit length of a cylindrical capacitor is derived in Example 24-2.




$$C_1 = \frac{2\pi\varepsilon_0\ell}{\ln(R_a/R_b)} \;;\; C_2 = \frac{2\pi\varepsilon_0\ell}{\ln(R_c/R_d)} \;;\; \frac{1}{C_{net}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \rightarrow$$

$$C_{net} = \frac{C_1C_2}{C_1 + C_2} = \frac{\left[\frac{2\pi\varepsilon_0\ell}{\ln(R_a/R_b)}\right]\left[\frac{2\pi\varepsilon_0\ell}{\ln(R_c/R_d)}\right]}{\frac{2\pi\varepsilon_0\ell}{\ln(R_a/R_b)} + \frac{2\pi\varepsilon_0\ell}{\ln(R_c/R_d)}}$$

$$= \frac{2\pi\varepsilon_0\ell}{\ln(R_c/R_d) + \ln(R_a/R_b)} = \frac{2\pi\varepsilon_0\ell}{\ln(R_aR_c/R_bR_d)} \quad \rightarrow$$




$$\frac{C}{\ell} = \boxed{\frac{2\pi\varepsilon_0}{\ln(R_aR_c/R_bR_d)}}$$

91. The force acting on one plate by the other plate is equal to the electric field produced by one charged plate multiplied by the charge on the second plate.

$$F = EQ = \left(\frac{Q}{2A\varepsilon_0}\right)Q = \frac{Q^2}{2A\varepsilon_0}$$

The force is attractive since the plates are oppositely charged. Since the force is constant, the work done in pulling the two plates apart by a distance $x$ is just the force times distance.

$$W = Fx = \boxed{\frac{Q^2x}{2A\varepsilon_0}}$$

The change in energy stored between the plates is obtained using Eq. 24-5.

$$W = \Delta U = \frac{Q^2}{2}\left(\frac{1}{C_2} - \frac{1}{C_1}\right) = \frac{Q^2}{2}\left(\frac{2x}{\varepsilon_0A} - \frac{x}{\varepsilon_0A}\right) = \boxed{\frac{Q^2x}{2\varepsilon_0A}}$$

The work done in pulling the plates apart is equal to the increase in energy between the plates.

92. Since the other values in this problem manifestly have 2 significant figures, we assume that the capacitance also has 2 significant figures.

(*a*) The number of electrons is found from the charge on the capacitor.

$$Q = CV = Ne \quad \rightarrow \quad N = \frac{CV}{e} = \frac{\left(30\times10^{-15}\,\text{F}\right)\left(1.5\,\text{V}\right)}{1.60\times10^{-19}\,\text{C}} = \boxed{2.8\times10^{5}\,e\text{'s}}$$

(*b*) The thickness is determined from the dielectric strength.

$$E_{\text{max}} = \frac{V}{d_{\text{min}}} \quad \rightarrow \quad d_{\text{min}} = \frac{V}{E_{\text{max}}} = \frac{1.5\,\text{V}}{1.0\times10^{9}\,\text{V/m}} = \boxed{1.5\times10^{-9}\,\text{m}}$$

(*c*) The area is found from Eq. 24-8.

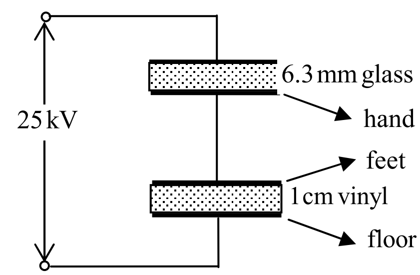
$$C = K\varepsilon_0\frac{A}{d} \quad \rightarrow \quad A = \frac{Cd}{K\varepsilon_0} = \frac{\left(30\times10^{-15}\,\text{F}\right)\left(1.5\times10^{-9}\,\text{m}\right)}{25\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{2.0\times10^{-13}\,\text{m}}$$

93. Use Eq. 24-2 for the capacitance.

$$C = \frac{\varepsilon_0 A}{d} \quad \rightarrow \quad d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.0\times10^{-4}\,\text{m}^2\right)}{\left(1\,\text{F}\right)} = \boxed{9\times10^{-16}\,\text{m}}$$

$\boxed{\text{No}}$, this is not practically achievable. The gap would have to be smaller than the radius of a proton.

94. See the schematic diagram for the arrangement. The two "capacitors" are in series, and so have the same charge. Thus their voltages, which must total 25kV, will be inversely proportional to their capacitances. Let $C_1$ be the glass-filled capacitor, and $C_2$ be the vinyl capacitor. The area of the foot is approximately twice the area of the hand, and since there are two feet on the floor and only one hand on the screen, the area ratio is $\frac{A_{\text{foot}}}{A_{\text{hand}}} = \frac{4}{1}$.




$$Q = C_1V_1 = C_2V_2 \quad \rightarrow \quad V_1 = V_2\frac{C_2}{C_1}$$

$$C_1 = \frac{\varepsilon_0 K_{\text{glass}} A_{\text{hand}}}{d_{\text{glass}}} \;;\; C_2 = \frac{\varepsilon_0 K_{\text{vinyl}} A_{\text{foot}}}{d_{\text{vinyl}}}$$

$$\frac{C_2}{C_1} = \frac{\dfrac{\varepsilon_0 K_{\text{vinyl}} A_{\text{foot}}}{d_{\text{vinyl}}}}{\dfrac{\varepsilon_0 K_{\text{glass}} A_{\text{hand}}}{d_{\text{glass}}}} = \frac{K_{\text{vinyl}} A_{\text{foot}} d_{\text{glass}}}{K_{\text{glass}} A_{\text{hand}} d_{\text{vinyl}}} = \frac{(3)(4)(0.63)}{(5)(1)(1.0)} = 1.5$$

$$V = V_1 + V_2 = V_2\frac{C_2}{C_1} + V_2 = 2.5V_2 = 25{,}000\,\text{V} \quad \rightarrow \quad V_2 = \boxed{10{,}000\,\text{V}}$$

95. (*a*) Use Eq. 24-2 to calculate the capacitance.

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.0\,\text{m}^2\right)}{\left(3.0\times10^{-3}\,\text{m}\right)} = \boxed{5.9\times10^{-9}\,\text{F}}$$

Use Eq. 24-1 to calculate the charge.

$$Q_0 = C_0V_0 = \left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right) = 2.655\times10^{-7}\,\text{C} \approx \boxed{2.7\times10^{-7}\,\text{C}}$$

The electric field is the potential difference divided by the plate separation.

$$E_0 = \frac{V_0}{d} = \frac{45\,\text{V}}{3.0\times10^{-3}\,\text{m}} = \boxed{15000\,\text{V/m}}$$

Use Eq. 24-5 to calculate the energy stored.

$$U_0 = \tfrac{1}{2}C_0V_0^2 = \tfrac{1}{2}\left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right)^2 = \boxed{6.0\times10^{-6}\,\text{J}}$$

(*b*) Now include the dielectric. The capacitance is multiplied by the dielectric constant.

$$C = KC_0 = 3.2\left(5.9\times10^{-9}\,\text{F}\right) = 1.888\times10^{-8}\,\text{F} \approx \boxed{1.9\times10^{-8}\,\text{F}}$$

The voltage doesn't change. Use Eq. 24-1 to calculate the charge.

$$Q = CV = KC_0V = 3.2\left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right) = 8.496\times10^{-7}\,\text{C} \approx \boxed{8.5\times10^{-7}\,\text{C}}$$

Since the battery is still connected, the voltage is the same as before, and so the electric field doesn't change.

$$E = E_0 = \boxed{15000\,\text{V/m}}$$

Use Eq. 24-5 to calculate the energy stored.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}KC_0V^2 = \tfrac{1}{2}(3.2)\left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right)^2 = \boxed{1.9\times10^{-5}\,\text{J}}$$

96. (*a*) For a plane conducting surface, the electric field is given by Eq. 22-5.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \rightarrow Q_{\text{max}} = E_S\varepsilon_0 A = \left(3\times10^6\,\text{N/C}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(150\times10^{-4}\,\text{m}^2\right)$$

$$= 3.98\times10^{-7}\,\text{C} \approx \boxed{4\times10^{-7}\,\text{C}}$$

(*b*) The capacitance of an isolated sphere is derived in the text, right after Example 24-3.

$$C = 4\pi\varepsilon_0 r = 4\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1\,\text{m}\right) = 1.11\times10^{-10}\,\text{F} \approx \boxed{1\times10^{-10}\,\text{F}}$$

(*c*) Use Eq. 24-1, with the maximum charge from part (a) and the capacitance from part (b).

$$Q = CV \rightarrow V = \frac{Q}{C} = \frac{3.98\times10^{-7}\,\text{C}}{1.11\times10^{-10}\,\text{F}} = 3586\,\text{V} \approx \boxed{4000\,\text{V}}$$

97. (*a*) The initial capacitance is obtained directly from Eq. 24-8.

$$C_0 = \frac{K\varepsilon_0 A}{d} = \frac{3.7\left(8.85\,\text{pF/m}\right)\left(0.21\,\text{m}\right)\left(0.14\,\text{m}\right)}{0.030\times10^{-3}\,\text{m}} = \boxed{32\,\text{nF}}$$

(*b*) Maximum charge will occur when the electric field between the plates is equal to the dielectric strength. The charge will be equal to the capacitance multiplied by the maximum voltage, where the maximum voltage is the electric field times the separation distance of the plates.

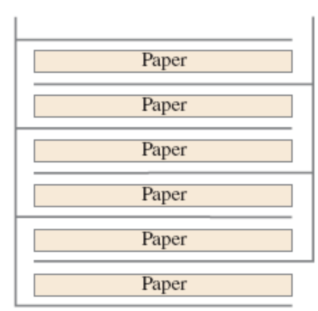
$$Q_{\text{max}} = C_0V = C_0Ed = \left(32\,\text{nF}\right)\left(15\times10^6\,\text{V/m}\right)\left(0.030\times10^{-3}\,\text{m}\right)$$

$$= \boxed{14\,\mu\text{C}}$$

(*c*) The sheets of foil would be separated by sheets of paper with alternating sheets connected together on each side. This capacitor would consist of 100 sheets of paper with 101 sheets of foil.

$$t = 101d_{\text{Al}} + 100d_{\text{paper}} = 101\left(0.040\,\text{mm}\right) + 100\left(0.030\,\text{mm}\right)$$

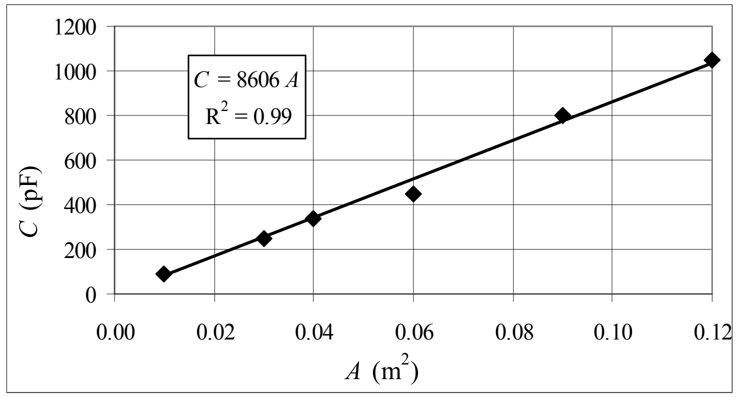
$$= \boxed{7.0\,\text{mm}}$$



(*d*) Since the capacitors are in parallel, each capacitor has the same voltage which is equal to the total voltage. Therefore breakdown will occur when the voltage across a single capacitor provides an electric field across that capacitor equal to the dielectric strength.

$$V_{max} = E_{max}d = \left(15\times10^{6}\,\text{V/m}\right)\left(0.030\times10^{-3}\,\text{m}\right) = \boxed{450\,\text{V}}$$

98. From Eq. 24-2, $C = \frac{\varepsilon_0}{d}A$. So if we plot $C$ vs. $A$, we should get a straight line with a slope of $\frac{\varepsilon_0}{d}$.




$$\frac{\varepsilon_0}{d} = \text{slope} \quad \rightarrow$$

$$d = \frac{\varepsilon_0}{\text{slope}} = \frac{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2}{8606\times10^{-12}\,\text{F/m}^2} = 1.03\times10^{-3}\,\text{m} \approx \boxed{1.0\,\text{mm}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH24.XLS," on tab "Problem 24.98."

# CHAPTER 25: Electric Currents and Resistance

## Responses to Questions

1. A battery rating in ampere-hours gives the total amount of charge available in the battery.

2. The chemical reactions within the cell cause electrons to pile up on the negative electrode. If the terminals of the battery are connected in a circuit, then electrons flow from the negative terminal because it has an excess of electrons. Once the electrons return to the cell, the electrolyte again causes them to move to the negative terminal.

3. When a flashlight is operated, the battery energy is being used up.

4. The terminal of the car battery connected to "ground" is actually connected to the metal frame of the car. This provides a large "sink" or "source" for charge. The metal frame serves as the common ground for all electrical devices in the car, and all voltages are measured with respect to the car's frame.

5. Generally, water is already in the faucet spout, but it will not come out until the faucet valve is opened. Opening the valve provides the pressure difference needed to force water out of the spout. The same thing is essentially true when you connect a wire to the terminals of a battery. Electrons already exist in the wires. The battery provides the potential that causes them to move, producing a current.

6. Yes. They might have the same resistance if the aluminum wire is thicker. If the lengths of the wires are the same, then the ratios of resistivity to cross-sectional area must also be the same for the resistances to be the same. Aluminum has a higher resistivity than copper, so if the cross-sectional area of the aluminum is also larger by the same proportion, the two wires will have the same resistance.

7. If the emf in a circuit remains constant and the resistance in the circuit is increased, less current will flow, and the power dissipated in the circuit will decrease. Both power equations support this result. If the current in a circuit remains constant and the resistance is increased, then the emf must increase and the power dissipated in the circuit will increase. Both equations also support this result. There is no contradiction, because the voltage, current, and resistance are related to each other by $V = IR$.

8. When a lightbulb burns out, the filament breaks, creating a gap in the circuit so that no current flows.

9. If the resistance of a small immersion heater were increased, it would slow down the heating process. The emf in the circuit made up of the heater and the wires that connect it to the wall socket is maintained at a constant rms value. If the resistance in the circuit is increased, less current will flow, and the power dissipated in the circuit will decrease, slowing the heating process.

10. Resistance is proportional to length and inversely proportional to cross-sectional area.
    (*a*) For the least resistance, you want to connect the wires to maximize area and minimize length. Therefore, connect them opposite to each other on the faces that are $2a$ by $3a$.
    (*b*) For the greatest resistance, you want to minimize area and maximize length. Therefore, connect the wires to the faces that are $1a$ by $2a$.

11. When a light is turned on, the filament is cool, and has a lower resistance than when it is hot. The current through the filament will be larger, due to the lower resistance. This momentary high current will heat the wire rapidly, possibly causing the filament to break due to thermal stress or vaporize. After the light has been on for some time, the filament is at a constant high temperature, with a higher resistance and a lower current. Since the temperature is constant, there is less thermal stress on the filament than when the light is first turned on.

12. When connected to the same potential difference, the 100-W bulb will draw more current ($P = IV$). The 75-W bulb has the higher resistance ($V = IR$ or $P = V^2/R$).

13. The electric power transferred by the lines is $P = IV$. If the voltage across the transmission lines is large, then the current in the lines will be small. The power lost in the transmission lines is $P = I^2R$. The power dissipated in the lines will be small, because $I$ is small.

14. If the circuit has a 15-A fuse, then it is rated to carry current of no more than 15 A. Replacing the 15-A fuse with a 25-A fuse will allow the current to increase to a level that is dangerously high for the wiring, which might result in overheating and possibly a fire.

15. The human eye and brain cannot distinguish the on-off cycle of lights when they are operated at the normal 60 Hz frequency. At much lower frequencies, such as 5 Hz, the eye and brain are able to process the on-off cycle of the lights, and they will appear to flicker.

16. The electrons are not "used up" as they pass through the lamp. Their energy is dissipated as light and heat, but with each cycle of the alternating voltage, their potential energy is raised again. As long as the electrons keep moving (converting potential energy into kinetic energy, light, and heat) the lamp will stay lit.

17. Immediately after the toaster is turned on, the Nichrome wire heats up and its resistance increases. Since the (rms) potential across the element remains constant, the current in the heating element must decrease.

18. No. Energy is dissipated in a resistor but current, the rate of flow of charge, is not "used up."

19. In the two wires described, the drift velocities of the electrons will be about the same, but the current density, and therefore the current, in the wire with twice as many free electrons per atom will be twice as large as in the other wire.

20. (*a*) If the length of the wire doubles, its resistance also doubles, and so the current in the wire will be reduced by a factor of two. Drift velocity is proportional to current, so the drift velocity will be halved.
    (*b*) If the wire's radius is doubled, the drift velocity remains the same. (Although, since there are more charge carriers, the current will quadruple.)
    (*c*) If the potential difference doubles while the resistance remains constant, the drift velocity and current will also double.

21. If you turn on an electric appliance when you are outside with bare feet, and the appliance shorts out through you, the current has a direct path to ground through your feet, and you will receive a severe shock. If you are inside wearing socks and shoes with thick soles, and the appliance shorts out, the current will not have an easy path to ground through you, and will most likely find an alternate path. You might receive a mild shock, but not a severe one.

## Solutions to Problems

1. Use the definition of current, Eq. 25-1a.

$$I=\frac{\Delta Q}{\Delta t} \rightarrow 1.30\,\text{A}=\frac{1.30\,\text{C}}{\text{s}}\times\frac{1\text{ electron}}{1.60\times10^{-19}\,\text{C}}=\boxed{8.13\times10^{18}\text{ electrons/s}}$$

2. Use the definition of current, Eq. 25-1a.

$$I=\frac{\Delta Q}{\Delta t} \rightarrow \Delta Q=I\Delta t=(6.7\,\text{A})(5.0\,\text{h})(3600\,\text{s/h})=\boxed{1.2\times10^{5}\,\text{C}}$$

3. Use the definition of current, Eq. 25-1a.

$$I=\frac{\Delta Q}{\Delta t}=\frac{(1200\text{ ions})(1.60\times10^{-19}\text{ C/ion})}{3.5\times10^{-6}\text{s}}=\boxed{5.5\times10^{-11}\,\text{A}}$$

4. Solve Eq. 25-2a for resistance.

$$R=\frac{V}{I}=\frac{120\,\text{V}}{4.2\,\text{A}}=\boxed{29\,\Omega}$$

5. (*a*) Use Eq. 25-2b to find the current.

$$V=IR \rightarrow I=\frac{V}{R}=\frac{240\,\text{V}}{8.6\,\Omega}=27.91\,\text{A}\approx\boxed{28\,\text{A}}$$

(*b*) Use the definition of current, Eq. 25-1a.

$$I=\frac{\Delta Q}{\Delta t} \rightarrow \Delta Q=I\Delta t=(27.91\,\text{A})(50\,\text{min})(60\,\text{s/min})=\boxed{8.4\times10^{4}\,\text{C}}$$

6. (*a*) Solve Eq. 25-2a for resistance.

$$R=\frac{V}{I}=\frac{120\,\text{V}}{9.5\,\text{A}}=12.63\,\Omega\approx\boxed{13\,\Omega}$$

(*b*) Use the definition of average current, Eq. 25-1a.

$$I=\frac{\Delta Q}{\Delta t} \rightarrow \Delta Q=I\Delta t=(9.5\,\text{A})(15\,\text{min})(60\,\text{s/min})=\boxed{8600\,\text{C}}$$

7. Use Ohm's Law, Eq. 25-2a, to find the current. Then use the definition of current, Eq. 25-1a, to calculate the number of electrons per minute.

$$I=\frac{V}{R}=\frac{\Delta Q}{\Delta t}=\frac{4.5\,\text{V}}{1.6\,\Omega}=\frac{2.8\,\text{C}}{\text{s}}\times\frac{1\text{ electron}}{1.60\times10^{-19}\,\text{C}}\times\frac{60\,\text{s}}{1\,\text{min}}=\boxed{1.1\times10^{21}\,\frac{\text{electrons}}{\text{minute}}}$$

8. Find the potential difference from the resistance and the current.

$$R=(2.5\times10^{-5}\,\Omega/\text{m})(4.0\times10^{-2}\,\text{m})=1.0\times10^{-6}\,\Omega$$

$$V=IR=(3100\,\text{A})(1.0\times10^{-6}\,\Omega)=\boxed{3.1\times10^{-3}\,\text{V}}$$

9\. (*a*) Use Eq. 25-2b to find the resistance.

$$R = \frac{V}{I} = \frac{12\,\text{V}}{0.60\,\text{A}} = \boxed{20\,\Omega} \text{ (2 sig. fig.)}$$

(*b*) An amount of charge $\Delta Q$ loses a potential energy of $(\Delta Q)V$ as it passes through the resistor. The amount of charge is found from Eq. 25-1a.

$$\Delta U = (\Delta Q)V = (I\Delta t)V = (0.60\,\text{A})(60\,\text{s})(12\,\text{V}) = \boxed{430\,\text{J}}$$

10\. (*a*) If the voltage drops by 15%, and the resistance stays the same, then by Eq. 25-2b, $V = IR$, the current will also drop by 15%.

$$I_{\text{final}} = 0.85 I_{\text{initial}} = 0.85(6.50\,\text{A}) = 5.525\,\text{A} \approx \boxed{5.5\,\text{A}}$$

(*b*) If the resistance drops by 15% (the same as being multiplied by 0.85), and the voltage stays the same, then by Eq. 25-2b, the current must be divided by 0.85.

$$I_{\text{final}} = \frac{I_{\text{initial}}}{0.85} = \frac{6.50\,\text{A}}{0.85} = 7.647\,\text{A} \approx \boxed{7.6\,\text{A}}$$

11\. Use Eq. 25-3 to find the diameter, with the area as $A = \pi r^2 = \pi d^2/4$.

$$R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2} \rightarrow d = \sqrt{\frac{4\ell\rho}{\pi R}} = \sqrt{\frac{4(1.00\,\text{m})(5.6\times10^{-8}\,\Omega\bullet\text{m})}{\pi(0.32\,\Omega)}} = \boxed{4.7\times10^{-4}\,\text{m}}$$

12\. Use Eq. 25-3 to calculate the resistance, with the area as $A = \pi r^2 = \pi d^2/4$.

$$R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2} = (1.68\times10^{-8}\,\Omega\bullet\text{m})\frac{4(4.5\,\text{m})}{\pi(1.5\times10^{-3}\,\text{m})^2} = \boxed{4.3\times10^{-2}\,\Omega}$$

$\boxed{13.}$ Use Eq. 25-3 to calculate the resistances, with the area as $A = \pi r^2 = \pi d^2/4$.

$$R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2}.$$

$$\frac{R_{\text{Al}}}{R_{\text{Cu}}} = \frac{\rho_{\text{Al}}\dfrac{4\ell_{\text{Al}}}{\pi d_{\text{Al}}^2}}{\rho_{\text{Cu}}\dfrac{4\ell_{\text{Cu}}}{\pi d_{\text{Cu}}^2}} = \frac{\rho_{\text{Al}}\ell_{\text{Al}}d_{\text{Cu}}^2}{\rho_{\text{Cu}}\ell_{\text{Cu}}d_{\text{Al}}^2} = \frac{(2.65\times10^{-8}\,\Omega\bullet\text{m})(10.0\,\text{m})(1.8\,\text{mm})^2}{(1.68\times10^{-8}\,\Omega\bullet\text{m})(20.0\,\text{m})(2.0\,\text{mm})^2} = \boxed{0.64}$$

14\. Use Eq. 25-3 to express the resistances, with the area as $A = \pi r^2 = \pi d^2/4$, and so $R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2}$.

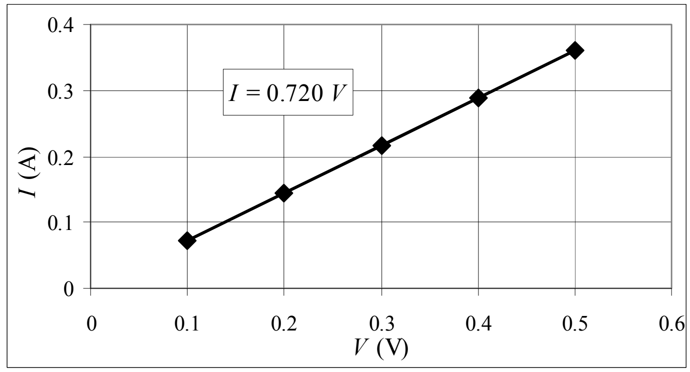
$$R_{\text{W}} = R_{\text{Cu}} \rightarrow \rho_{\text{W}}\frac{4\ell}{\pi d_{\text{W}}^2} = \rho_{\text{Cu}}\frac{4\ell}{\pi d_{\text{Cu}}^2} \rightarrow$$

$$d_{\text{W}} = d_{\text{Cu}}\sqrt{\frac{\rho_{\text{W}}}{\rho_{\text{Cu}}}} = (2.2\,\text{mm})\sqrt{\frac{5.6\times10^{-8}\,\Omega\bullet\text{m}}{1.68\times10^{-8}\,\Omega\bullet\text{m}}} = \boxed{4.0\,\text{mm}}$$

The diameter of the tungsten should be 4.0 mm.

15. (*a*) If the wire obeys Ohm's law, then $V = IR$ or $I = \frac{1}{R}V$, showing a linear relationship between $I$ and $V$. A graph of $I$ vs. $V$ should give a straight line with a slope of $\frac{1}{R}$ and a $y$-intercept of 0.

(*b*) From the graph and the calculated linear fit, we see that the wire obeys Ohm's law.




$$\text{slope} = \frac{1}{R} \quad \rightarrow$$

$$R = \frac{1}{0.720}\,\text{A/V} = \boxed{1.39\,\Omega}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH25.XLS," on tab "Problem 25.15b."

(*c*) Use Eq. 25-3 to find the resistivity.

$$R = \rho\frac{\ell}{A} \quad \rightarrow \quad \rho = \frac{AR}{\ell} = \frac{\pi d^2 R}{4\ell} = \frac{\pi\left(3.2\times10^{-4}\,\text{m}\right)^2\left(1.39\,\Omega\right)}{4\left(0.11\,\text{m}\right)} = \boxed{1.0\times10^{-6}\,\Omega\bullet\text{m}}$$

From Table 25-1, the material is $\boxed{\text{nichrome}}$.

16. Use Eq. 25-5 multiplied by $\ell/A$ so that it expresses resistance instead of resistivity.

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right] = 1.15R_0 \quad \rightarrow \quad 1+\alpha\left(T-T_0\right) = 1.15 \quad \rightarrow$$

$$T - T_0 = \frac{0.15}{\alpha} = \frac{0.15}{.0068\left(\text{C}°\right)^{-1}} = \boxed{22\,\text{C}°}$$

So raise the temperature by $22\,\text{C}°$ to a final temperature of $42°\text{C}$.

17. Since the resistance is directly proportional to the length, the length of the long piece must be 4.0 times the length of the short piece.

$$\ell = \ell_{\text{short}} + \ell_{\text{long}} = \ell_{\text{short}} + 4.0\ell_{\text{short}} = 5.0\ell_{\text{short}} \quad \rightarrow \quad \ell_{\text{short}} = 0.20\ell\,,\ \ell_{\text{long}} = 0.80\ell$$

Make the cut at $\boxed{\text{20\% of the length of the wire}}$.

$$\ell_{\text{short}} = 0.20\ell\,,\ \ell_{\text{long}} = 0.80\ell \quad \rightarrow \quad R_{\text{short}} = 0.2R = \boxed{2.0\,\Omega}\,,\ R_{\text{long}} = 0.8R = \boxed{8.0\,\Omega}$$

18. Use Eq. 25-5 for the resistivity.

$$\rho_{\text{T Al}} = \rho_{\text{0 Al}}\left[1+\alpha_{\text{Al}}\left(T-T_0\right)\right] = \rho_{\text{0 W}} \quad \rightarrow$$

$$T = T_0 + \frac{1}{\alpha_{\text{Al}}}\left(\frac{\rho_{\text{0 W}}}{\rho_{\text{0 Al}}} - 1\right) = 20°\text{C} + \frac{1}{0.00429\left(\text{C}°\right)^{-1}}\left(\frac{5.6\times10^{-8}\,\Omega\bullet\text{m}}{2.65\times10^{-8}\,\Omega\bullet\text{m}} - 1\right) = 279.49°\text{C} \approx \boxed{280°\text{C}}$$

$\boxed{19.}$ Use Eq. 25-5 multiplied by $\ell/A$ so that it expresses resistances instead of resistivity.

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right] \quad \rightarrow$$

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20°\text{C} + \frac{1}{0.0045\left(\text{C}°\right)^{-1}}\left(\frac{140\,\Omega}{12\,\Omega} - 1\right) = 2390°\text{C} \approx \boxed{2400°\text{C}}$$

20. Calculate the voltage drop by combining Ohm's Law (Eq. 25-2b) with the expression for resistance, Eq. 25-3.

$$V = IR = I\frac{\rho \ell}{A} = I\frac{4\rho\ell}{\pi d^2} = (12\,\text{A})\frac{4\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)(26\,\text{m})}{\pi\left(1.628\times10^{-3}\,\text{m}\right)^2} = \boxed{2.5\,\text{V}}$$

21. The wires have the same resistance and the same resistivity.

$$R_{\text{long}} = R_{\text{short}} \rightarrow \frac{\rho\ell_{\text{long}}}{A_1} = \frac{\rho\ell_{\text{short}}}{A_2} \rightarrow \frac{(4)2\ell_{\text{short}}}{\pi d_{\text{long}}^2} = \frac{4\ell_{\text{short}}}{\pi d_{\text{short}}^2} \rightarrow \boxed{\frac{d_{\text{long}}}{d_{\text{short}}} = \sqrt{2}}$$

22. In each case calculate the resistance by using Eq. 25-3 for resistance.

(*a*) $$R_x = \frac{\rho\ell_x}{A_{yz}} = \frac{\left(3.0\times10^{-5}\,\Omega\bullet\text{m}\right)\left(1.0\times10^{-2}\,\text{m}\right)}{\left(2.0\times10^{-2}\,\text{m}\right)\left(4.0\times10^{-2}\,\text{m}\right)} = 3.75\times10^{-4}\,\Omega \approx \boxed{3.8\times10^{-4}\,\Omega}$$

(*b*) $$R_y = \frac{\rho\ell_y}{A_{xz}} = \frac{\left(3.0\times10^{-5}\,\Omega\bullet\text{m}\right)\left(2.0\times10^{-2}\,\text{m}\right)}{\left(1.0\times10^{-2}\,\text{m}\right)\left(4.0\times10^{-2}\,\text{m}\right)} = \boxed{1.5\times10^{-3}\,\Omega}$$

(*c*) $$R_z = \frac{\rho\ell_z}{A_{xy}} = \frac{\left(3.0\times10^{-5}\,\Omega\bullet\text{m}\right)\left(4.0\times10^{-2}\,\text{m}\right)}{\left(1.0\times10^{-2}\,\text{m}\right)\left(2.0\times10^{-2}\,\text{m}\right)} = \boxed{6.0\times10^{-3}\,\Omega}$$

23. The original resistance is $R_0 = V/I_0$, and the high temperature resistance is $R = V/I$, where the two voltages are the same. The two resistances are related by Eq. 25-5, multiplied by $\ell/A$ so that it expresses resistance instead of resistivity.

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right] \rightarrow T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0}-1\right) = T_0 + \frac{1}{\alpha}\left(\frac{V/I}{V/I_0}-1\right) = T_0 + \frac{1}{\alpha}\left(\frac{I_0}{I}-1\right)$$

$$= 20.0^\circ\text{C} + \frac{1}{0.00429\left(\text{C}^\circ\right)^{-1}}\left(\frac{0.4212\,\text{A}}{0.3818\,\text{A}}-1\right) = \boxed{44.1^\circ\text{C}}$$

24. For the cylindrical wire, its (constant) volume is given by $V = \ell_0 A_0 = \ell A,$ and so $A = \frac{V}{\ell}.$ Combine this relationship with Eq. 25-3. We assume that $\Delta\ell \ll \ell_0$.

$$R_0 = \rho\frac{\ell_0}{A_0} = \rho\frac{\ell_0^2}{V_0} \;;\; R = \rho\frac{\ell}{A} = \rho\frac{\ell^2}{V_0} \;;\; \frac{dR}{d\ell} = 2\rho\frac{\ell}{V_0}$$

$$\Delta R \approx \frac{dR}{d\ell}\Delta\ell = 2\rho\frac{\ell}{V_0}\Delta\ell \rightarrow \Delta\ell = \frac{V_0\Delta R}{2\rho\ell} \rightarrow \frac{\Delta\ell}{\ell} = \frac{V_0\Delta R}{2\rho\ell^2} = \frac{\Delta R}{2\frac{\rho\ell^2}{V_0}} = \frac{1}{2}\frac{\Delta R}{R}$$

This is true for any initial conditions, and so $\boxed{\frac{\Delta\ell}{\ell_0} = \frac{1}{2}\frac{\Delta R}{R_0}}$

25. The resistance depends on the length and area as $R = \rho\ell/A$. Cutting the wire and running the wires side by side will halve the length and double the area.

$$R_2 = \frac{\rho\left(\frac{1}{2}\ell\right)}{2A} = \frac{1}{4}\frac{\rho\ell}{A} = \boxed{\tfrac{1}{4}R_1}$$

26. The total resistance is to be 3700 ohms $\left(R_{\text{total}}\right)$ at all temperatures. Write each resistance in terms of Eq.25-5 (with $T_0 = 0^\circ\text{C}$), multiplied by $\ell/A$ to express resistance instead of resistivity.

$$R_{\text{total}} = R_{0\text{C}}\left[1+\alpha_{\text{C}}T\right] + R_{0\text{N}}\left[1+\alpha_{\text{N}}T\right] = R_{0\text{C}} + R_{0\text{C}}\alpha_{\text{C}}T + R_{0\text{N}} + R_{0\text{N}}\alpha_{\text{N}}T$$
$$= R_{0\text{C}} + R_{0\text{N}} + \left(R_{0\text{C}}\alpha_{\text{C}} + R_{0\text{N}}\alpha_{\text{N}}\right)T$$

For the above to be true, the terms with a temperature dependence must cancel, and the terms without a temperature dependence must add to $R_{\text{total}}$. Thus we have two equations in two unknowns.

$$0 = \left(R_{0\text{C}}\alpha_{\text{C}} + R_{0\text{N}}\alpha_{\text{N}}\right)T \;\rightarrow\; R_{0\text{N}} = -\frac{R_{0\text{C}}\alpha_{\text{C}}}{\alpha_{\text{N}}}$$

$$R_{\text{total}} = R_{0\text{C}} + R_{0\text{N}} = R_{0\text{C}} - \frac{R_{0\text{C}}\alpha_{\text{C}}}{\alpha_{\text{N}}} = \frac{R_{0\text{C}}\left(\alpha_{\text{N}} - \alpha_{\text{C}}\right)}{\alpha_{\text{N}}} \;\rightarrow$$

$$R_{0\text{C}} = R_{\text{total}}\frac{\alpha_{\text{N}}}{\left(\alpha_{\text{N}} - \alpha_{\text{C}}\right)} = \left(3700\,\Omega\right)\frac{0.0004\left(\text{C}^\circ\right)^{-1}}{0.0004\left(\text{C}^\circ\right)^{-1} + 0.0005\left(\text{C}^\circ\right)^{-1}} = 1644\,\Omega \approx \boxed{1600\,\Omega}$$

$$R_{0\text{N}} = R_{\text{total}} - R_{0\text{C}} = 3700\,\Omega - 1644\,\Omega = 2056\,\Omega \approx \boxed{2100\,\Omega}$$

27. We choose a spherical shell of radius *r* and thickness *dr* as a differential element. The area of this element is $4\pi r^2$. Use Eq. 25-3, but for an infinitesimal resistance. Then integrate over the radius of the sphere.

$$R = \rho\frac{\ell}{A} \;\rightarrow\; dR = \rho\frac{d\ell}{A} = \frac{dr}{4\pi\sigma r^2} \;\rightarrow\; R = \int dR = \int_{r_1}^{r_2}\frac{dr}{4\pi\sigma r^2} = \frac{1}{4\pi\sigma}\left(-\frac{1}{r}\right)_{r_1}^{r_2} = \boxed{\frac{1}{4\pi\sigma}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

28. (*a*) Let the values at the lower temperature be indicated by a subscript "0". Thus $R_0 = \rho_0\frac{\ell_0}{A_0}$ $= \rho_0\frac{4\ell_0}{\pi d_0^2}$. The change in temperature results in new values for the resistivity, the length, and the diameter. Let $\alpha$ represent the temperature coefficient for the resistivity, and $\alpha_{\text{T}}$ represent the thermal coefficient of expansion, which will affect the length and diameter.

$$R = \rho\frac{\ell}{A} = \rho\frac{4\ell}{\pi d^2} = \rho_0\left[1+\alpha\left(T-T_0\right)\right]\frac{4\ell_0\left[1+\alpha_{\text{T}}\left(T-T_0\right)\right]}{\pi\left\{d_0\left[1+\alpha_{\text{T}}\left(T-T_0\right)\right]\right\}^2} = \rho_0\frac{4\ell_0}{\pi d_0^2}\frac{\left[1+\alpha\left(T-T_0\right)\right]}{\left[1+\alpha_{\text{T}}\left(T-T_0\right)\right]}$$
$$= R_0\frac{\left[1+\alpha\left(T-T_0\right)\right]}{\left[1+\alpha_{\text{T}}\left(T-T_0\right)\right]} \;\rightarrow\; R\left[1+\alpha_{\text{T}}\left(T-T_0\right)\right] = R_0\left[1+\alpha\left(T-T_0\right)\right] \;\rightarrow$$

$$T = T_0 + \frac{(R - R_0)}{(R_0\alpha - R\alpha_T)} = 20°\text{C} + \frac{(140\,\Omega - 12\,\Omega)}{\left[(12\,\Omega)(0.0045\,\text{C}^{\circ-1}) - (140\,\Omega)(5.5\times10^{-6}\,\text{C}^{\circ-1})\right]}$$

$$= 20°\text{C} + 2405°\text{C} = 2425°\text{C} \approx \boxed{2400°\text{C}}$$

(*b*) The net effect of thermal expansion is that both the length and diameter increase, which lowers the resistance.

$$\frac{R}{R_0} = \frac{\rho_0 \frac{4\ell}{\pi d^2}}{\rho_0 \frac{4\ell_0}{\pi d_0^2}} = \frac{\ell d_0^2}{\ell_0 d^2} = \frac{\ell_0\left[1 + \alpha_T (T - T_0)\right]}{\ell_0} \frac{d_0^2}{\left\{d_0\left[1 + \alpha_T (T - T_0)\right]\right\}^2} = \frac{1}{\left[1 + \alpha_T (T - T_0)\right]}$$

$$= \frac{1}{\left[1 + (5.5\times10^{-6}\,\text{C}^{\circ-1})(2405°\text{C})\right]} = 0.9869$$

$$\%\,\text{change} = \left(\frac{R - R_0}{R_0}\right)100 = \left(\frac{R}{R_0} - 1\right)100 = -1.31 \approx \boxed{-1.3\%}$$

The net effect of resistivity change is that the resistance increases.

$$\frac{R}{R_0} = \frac{\rho \frac{4\ell_0}{\pi d_0^2}}{\rho_0 \frac{4\ell_0}{\pi d_0^2}} = \frac{\rho}{\rho_0} = \frac{\rho_0\left[1 + \alpha (T - T_0)\right]}{\rho_0} = \left[1 + \alpha (T - T_0)\right] = \left[1 + (0.0045\,\text{C}^{\circ-1})(2405°\text{C})\right]$$

$$= 11.82$$

$$\%\,\text{change} = \left(\frac{R - R_0}{R_0}\right)100 = \left(\frac{R}{R_0} - 1\right)100 = 1082 \approx \boxed{1100\%}$$

29. (*a*) Calculate each resistance separately using Eq. 25-3, and then add the resistances together to find the total resistance.

$$R_{\text{Cu}} = \frac{\rho_{\text{Cu}}\ell}{A} = \frac{4\rho_{\text{Cu}}\ell}{\pi d^2} = \frac{4(1.68\times10^{-8}\,\Omega\bullet\text{m})(5.0\,\text{m})}{\pi(1.4\times10^{-3}\,\text{m})^2} = 0.054567\,\Omega$$

$$R_{\text{Al}} = \frac{\rho_{\text{Al}}\ell}{A} = \frac{4\rho_{\text{Al}}\ell}{\pi d^2} = \frac{4(2.65\times10^{-8}\,\Omega\bullet\text{m})(5.0\,\text{m})}{\pi(1.4\times10^{-3}\,\text{m})^2} = 0.086074\,\Omega$$

$$R_{\text{total}} = R_{\text{Cu}} + R_{\text{Al}} = 0.054567\,\Omega + 0.086074\,\Omega = 0.140641\,\Omega \approx \boxed{0.14\,\Omega}$$

(*b*) The current through the wire is the voltage divided by the total resistance.

$$I = \frac{V}{R_{\text{total}}} = \frac{85\times10^{-3}\,\text{V}}{0.140641\,\Omega} = 0.60438\,\text{A} \approx \boxed{0.60\,\text{A}}$$

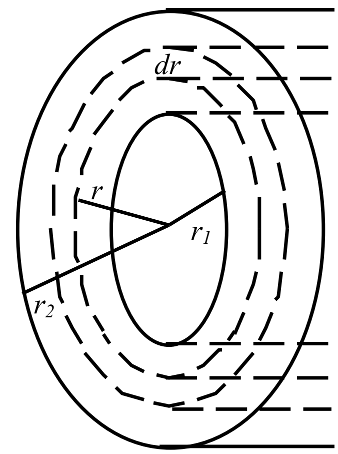
(*c*) For each segment of wire, Ohm's law is true. Both wires have the current found in (*b*) above.

$$V_{\text{Cu}} = IR_{\text{Cu}} = (0.60438\,\text{A})(0.054567\,\Omega) \approx \boxed{0.033\,\text{V}}$$

$$V_{\text{Al}} = IR_{\text{Al}} = (0.60438\,\text{A})(0.086074\,\Omega) \approx \boxed{0.052\,\text{V}}$$

Notice that the total voltage is 85 mV.

30. (*a*) Divide the cylinder up into concentric cylindrical shells of radius *r*, thickness *dr*, and length $\ell$. See the diagram. The resistance of one of those shells, from Eq. 25-3, is found. Note that the "length" in Eq. 25-3 is in the direction of the current flow, so we must substitute in *dr* for the "length" in Eq. 25-3. The area is the surface area of the thin cylindrical shell. Then integrate over the range of radii to find the total resistance.




$$R = \rho \frac{"\ell"}{A} \rightarrow dR = \rho \frac{dr}{2\pi r \ell} ;$$

$$R = \int dR = \int_{r_1}^{r_2} \rho \frac{dr}{2\pi r \ell} = \boxed{\frac{\rho}{2\pi \ell} \ln \frac{r_2}{r_1}}$$

(*b*) Use the data given to calculate the resistance from the above formula.

$$R = \frac{\rho}{2\pi \ell} \ln \frac{r_2}{r_1} = \frac{15 \times 10^{-5}\,\Omega \bullet \text{m}}{2\pi (0.024\,\text{m})} \ln \left( \frac{1.8\,\text{mm}}{1.0\,\text{mm}} \right) = \boxed{5.8 \times 10^{-4}\,\Omega}$$

(*c*) For resistance along the axis, we again use Eq. 25-3, but the current is flowing in the direction of length $\ell$. The area is the cross-sectional area of the face of the hollow cylinder.

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\pi \left( r_2^2 - r_1^2 \right)} = \frac{\left( 15 \times 10^{-5}\,\Omega \bullet \text{m} \right) \left( 0.024\,\text{m} \right)}{\pi \left[ \left( 1.8 \times 10^{-3}\,\text{m} \right)^2 - \left( 1.0 \times 10^{-3}\,\text{m} \right)^2 \right]} = \boxed{0.51\,\Omega}$$

31. Use Eq. 25-6 to find the power from the voltage and the current.

$$P = IV = (0.27\,\text{A})(3.0\,\text{V}) = \boxed{0.81\,\text{W}}$$

32. Use Eq. 25-7b to find the resistance from the voltage and the power.

$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(240\,\text{V})^2}{3300\,\text{W}} = \boxed{17\,\Omega}$$

33. Use Eq. 25-7b to find the voltage from the power and the resistance.

$$P = \frac{V^2}{R} \rightarrow V = \sqrt{RP} = \sqrt{(3300\,\Omega)(0.25\,\text{W})} = \boxed{29\,\text{V}}$$

34. Use Eq. 25-7b to find the resistance, and Eq. 25-6 to find the current.

(*a*) $$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(110\,\text{V})^2}{75\,\text{W}} = 161.3\,\Omega \approx \boxed{160\,\Omega}$$

$$P = IV \rightarrow I = \frac{P}{V} = \frac{75\,\text{W}}{110\,\text{V}} = 0.6818\,\text{A} \approx \boxed{0.68\,\text{A}}$$

(*b*) $$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(110\,\text{V})^2}{440\,\text{W}} = 27.5\,\Omega \approx \boxed{28\,\Omega}$$

$$P = IV \rightarrow I = \frac{P}{V} = \frac{440\,\text{W}}{110\,\text{V}} = \boxed{4.0\,\text{A}}$$

35. (*a*) From Eq. 25-6, if power $P$ is delivered to the transmission line at voltage $V$, there must be a current $I = P/V$. As this current is carried by the transmission line, there will be power losses of $I^2R$ due to the resistance of the wire. This power loss can be expressed as $\Delta P = I^2R$ $= \boxed{P^2R/V^2}$. Equivalently, there is a voltage drop across the transmission lines of $V' = IR$. Thus the voltage available to the users is $V - V'$, and so the power available to the users is $P' = (V - V')I = VI - V'I = VI - I^2R = P - I^2R$. The power loss is $\Delta P = P - P' = P - (P - I^2R)$ $= I^2R = \boxed{P^2R/V^2}$.

(*b*) Since $\Delta P \propto \dfrac{1}{V^2}$, $V$ should be $\boxed{\text{as large as possible}}$ to minimize $\Delta P$.

36. (*a*) Since $P = \dfrac{V^2}{R} \rightarrow R = \dfrac{V^2}{P}$ says that the resistance is inversely proportional to the power for a constant voltage, we predict that the 850 W setting has the higher resistance.

(*b*) $R = \dfrac{V^2}{P} = \dfrac{(120\,\text{V})^2}{850\,\text{W}} = \boxed{17\,\Omega}$

(*c*) $R = \dfrac{V^2}{P} = \dfrac{(120\,\text{V})^2}{1250\,\text{W}} = \boxed{12\,\Omega}$

37. (*a*) Use Eq. 25-6 to find the current.

$$P = IV \rightarrow I = \frac{P}{V} = \frac{95\,\text{W}}{115\,\text{V}} = \boxed{0.83\,\text{A}}$$

(*b*) Use Eq. 25-7b to find the resistance.

$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(115\,\text{V})^2}{95\,\text{W}} \approx \boxed{140\,\Omega}$$

38. The power (and thus the brightness) of the bulb is proportional to the square of the voltage, according to Eq. 25-7b, $P = \dfrac{V^2}{R}$. Since the resistance is assumed to be constant, if the voltage is cut in half from 240 V to 120V, the power will be reduced by a factor of 4. Thus the bulb will appear only about $\boxed{\text{1/4 as bright}}$ in the United States as in Europe.

39. To find the kWh of energy, multiply the kilowatts of power consumption by the number of hours in operation.

$$\text{Energy} = P(\text{in kW})\,t(\text{in h}) = (550\,\text{W})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(6.0\,\text{min})\left(\frac{1\,\text{h}}{60\,\text{min}}\right) = \boxed{0.055\,\text{kWh}}$$

To find the cost of the energy used in a month, multiply times 4 days per week of usage, times 4 weeks per month, times the cost per kWh.

$$\text{Cost} = \left(0.055\frac{\text{kWh}}{\text{d}}\right)\left(\frac{4\,\text{d}}{1\,\text{week}}\right)\left(\frac{4\,\text{week}}{1\,\text{month}}\right)\left(\frac{9.0\,\text{cents}}{\text{kWh}}\right) = \boxed{7.9\,\text{cents/month}}$$

40. To find the cost of the energy, multiply the kilowatts of power consumption by the number of hours in operation times the cost per kWh.

$$\text{Cost} = (25\,\text{W})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(365\,\text{day})\left(\frac{24\,\text{h}}{1\,\text{day}}\right)\left(\frac{\$0.095}{\text{kWh}}\right) \approx \boxed{\$21}$$

41. The $\text{A}\bullet\text{h}$ rating is the amount of charge that the battery can deliver. The potential energy of the charge is the charge times the voltage.

$$U = QV = (75\,\text{A}\bullet\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)(12\,\text{V}) = \boxed{3.2\times10^{6}\,\text{J}} = 0.90\,\text{kWh}$$

42. (*a*) Calculate the resistance from Eq. 25-2b and the power from Eq. 25-6.

$$R = \frac{V}{I} = \frac{3.0\,\text{V}}{0.38\,\text{A}} = 7.895\,\Omega \approx \boxed{7.9\,\Omega} \qquad P = IV = (0.38\,\text{A})(3.0\,\text{V}) = 1.14\,\text{W} \approx \boxed{1.1\,\text{W}}$$

(*b*) If four D-cells are used, the voltage will be doubled to 6.0 V. Assuming that the resistance of the bulb stays the same (by ignoring heating effects in the filament), the power that the bulb would need to dissipate is given by Eq. 25-7b, $P = \frac{V^2}{R}$. A doubling of the voltage means the power is increased by a factor of $\boxed{4}$. This should not be tried because the bulb is probably not rated for such a high wattage. The filament in the bulb would probably burn out, and the glass bulb might even explode if the filament burns violently.

$\boxed{43.}$ Each bulb will draw an amount of current found from Eq. 25-6.

$$P = IV \quad\rightarrow\quad I_{\text{bulb}} = \frac{P}{V}$$

The number of bulbs to draw 15 A is the total current divided by the current per bulb.

$$I_{\text{total}} = nI_{\text{bulb}} = n\frac{P}{V} \quad\rightarrow\quad n = \frac{VI_{\text{total}}}{P} = \frac{(120\,\text{V})(15\,\text{A})}{75\,\text{W}} = \boxed{24\text{ bulbs}}$$

44. Find the power dissipated in the cord by Eq. 25-7a, using Eq. 25-3 for the resistance.

$$P = I^2R = I^2\rho\frac{\ell}{A} = I^2\rho\frac{\ell}{\pi d^2/4} = I^2\rho\frac{4\ell}{\pi d^2} = (15.0\,\text{A})^2\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)\frac{4(5.4\,\text{m})}{\pi\left(0.129\times10^{-2}\,\text{m}\right)^2}$$

$$= 15.62\,\text{W} \approx \boxed{16\,\text{W}}$$

45. Find the current used to deliver the power in each case, and then find the power dissipated in the resistance at the given current.

$$P = IV \quad\rightarrow\quad I = \frac{P}{V} \qquad P_{\text{dissipated}} = I^2R = \frac{P^2}{V^2}R$$

$$P_{\substack{\text{dissipated}\\ 12{,}000\,\text{V}}} = \frac{\left(7.5\times10^{5}\,\text{W}\right)^2}{\left(1.2\times10^{4}\,\text{V}\right)^2}(3.0\,\Omega) = 11719\,\text{W}$$

$$P_{\substack{\text{dissipated}\\ 50{,}000\,\text{V}}} = \frac{\left(7.5\times10^{5}\,\text{W}\right)^2}{\left(5\times10^{4}\,\text{V}\right)^2}(3.0\,\Omega) = 675\,\text{W} \qquad \text{difference} = 11719\,\text{W} - 675\,\text{W} = \boxed{1.1\times10^{4}\,\text{W}}$$

46. (*a*) By conservation of energy and the efficiency claim, 75% of the electrical power dissipated by the heater must be the rate at which energy is absorbed by the water.

$$0.75_{\substack{\text{emitted by}\\ \text{electromagnet}}} = P_{\substack{\text{absorbed}\\ \text{by water}}} \rightarrow 0.75\left(IV\right) = \frac{Q_{\text{heat water}}}{t} = \frac{mc\Delta T}{t} \rightarrow$$

$$I = \frac{mc\Delta T}{0.75Vt} = \frac{(0.120\,\text{kg})(4186\,\text{J/kg})(95^\circ\text{C} - 25^\circ\text{C})}{(0.75)(12\,\text{V})(480\,\text{s})} = 8.139\,\text{A} \approx \boxed{8.1\,\text{A}}$$

(*b*) Use Ohm's law to find the resistance of the heater.

$$V = IR \rightarrow R = \frac{V}{I} = \frac{12\,\text{V}}{8.139\,\text{A}} = \boxed{1.5\,\Omega}$$

47. The water temperature rises by absorbing the heat energy that the electromagnet dissipates. Express both energies in terms of power, which is energy per unit time.

$$P_{\text{electric}} = P_{\substack{\text{to heat}\\ \text{water}}} \rightarrow IV = \frac{Q_{\text{heat water}}}{t} = \frac{mc\Delta T}{t} \rightarrow$$

$$\frac{m}{t} = \frac{IV}{c\Delta T} = \frac{(17.5\,\text{A})(240\,\text{V})}{(4186\,\text{J/kg}\bullet^\circ\text{C})(6.50\,\text{C}^\circ)} = 0.154\ \text{kg/s} \approx \boxed{0.15\,\text{kg/s}}$$

This is $154\,\text{mL/s}$.

48. For the wire to stay a constant temperature, the power generated in the resistor is to be dissipated by radiation. Use Eq. 25-7a and 19-18, both expressions of power (energy per unit time). We assume that the dimensions requested and dimensions given are those at the higher temperature, and do not take any thermal expansion effects into account. We also use Eq. 25-3 for resistance.

$$I^2R = \varepsilon\sigma A\left(T_{\text{high}}^4 - T_{\text{low}}^4\right) \rightarrow I^2\frac{4\rho\ell}{\pi d^2} = \varepsilon\sigma\pi d\ell\left(T_{\text{high}}^4 - T_{\text{low}}^4\right) \rightarrow$$

$$d = \left(\frac{4I^2\rho}{\pi^2\varepsilon\sigma\left(T_{\text{high}}^4 - T_{\text{low}}^4\right)}\right)^{1/3} = \left(\frac{4(15.0\,\text{A})^2(5.6\times10^{-8}\,\Omega\bullet\text{m})}{\pi^2(1.0)(5.67\times10^{-8}\,\text{W/m}^2\bullet\text{K}^4)\left[(3100\,\text{K})^4 - (293\,\text{K})^4\right]}\right)^{1/3}$$

$$= 9.92\times10^{-5}\,\text{m} \approx \boxed{0.099\,\text{mm}}$$

49. Use Ohm's law and the relationship between peak and rms values.

$$I_{\text{peak}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}\frac{V_{\text{rms}}}{R} = \sqrt{2}\frac{220\,\text{V}}{2700\,\Omega} = \boxed{0.12\,\text{A}}$$

50. Find the peak current from Ohm's law, and then find the rms current from Eq. 25-9a.

$$I_{\text{peak}} = \frac{V_{\text{peak}}}{R} = \frac{180\,\text{V}}{380\,\Omega} = 0.47368\,\text{A} \approx \boxed{0.47\,\text{A}} \qquad I_{\text{rms}} = I_{\text{peak}}\big/\sqrt{2} = (0.47368\,\text{A})\big/\sqrt{2} = \boxed{0.33\,\text{A}}$$

51. (*a*) When everything electrical is turned off, no current will be flowing into the house, even though a voltage is being supplied. Since for a given voltage, the more resistance, the lower the current, a zero current corresponds to an $\boxed{\text{infinite resistance}}$.

(*b*) Use Eq. 25-7a to calculate the resistance.

$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(120\,\text{V})^2}{2(75\,\text{W})} = \boxed{96\,\Omega}$$

52. The power and current can be used to find the peak voltage, and then the rms voltage can be found from the peak voltage.

$$\overline{P} = I_{\text{rms}} V_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}} V_{\text{rms}} \rightarrow V_{\text{rms}} = \frac{\sqrt{2}\overline{P}}{I_{\text{peak}}} = \frac{\sqrt{2}(1500\,\text{W})}{5.4\,\text{A}} = \boxed{390\,\text{V}}$$

53. Use the average power and rms voltage to calculate the peak voltage and peak current.

(*a*) $V_{\text{peak}} = \sqrt{2} V_{\text{rms}} = \sqrt{2}(660\,\text{V}) = 933.4\,\text{V} \approx \boxed{930\,\text{V}}$

(*b*) $\overline{P} = I_{\text{rms}} V_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}} V_{\text{rms}} \rightarrow I_{\text{peak}} = \frac{\sqrt{2}\overline{P}}{V_{\text{rms}}} = \frac{\sqrt{2}(1800\,\text{W})}{660\,\text{V}} = \boxed{3.9\,\text{A}}$

54. (*a*) We assume that the 2.5 hp is the average power, so the maximum power is twice that, or 5.0 hp, as seen in Figure 25-22.

$$5.0\,\text{hp}\left(\frac{746\,\text{W}}{1\,\text{hp}}\right) = 3730\,\text{W} \approx \boxed{3700\,\text{W}}$$

(*b*) Use the average power and the rms voltage to find the peak current.

$$\overline{P} = I_{\text{rms}} V_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}} V_{\text{rms}} \rightarrow I_{\text{peak}} = \frac{\sqrt{2}\overline{P}}{V_{\text{rms}}} = \frac{\sqrt{2}\left[\frac{1}{2}(3730\,\text{W})\right]}{240\,\text{V}} = \boxed{11\,\text{A}}$$

$\boxed{55.}$ (*a*) The average power used can be found from the resistance and the rms voltage by Eq. 25-10c.

$$\overline{P} = \frac{V_{\text{rms}}^2}{R} = \frac{(240\,\text{V})^2}{44\,\Omega} = 1309\,\text{W} \approx \boxed{1300\,\text{W}}$$

(*b*) The maximum power is twice the average power, and the minimum power is 0.

$$P_{\text{max}} = 2\overline{P} = 2(1309\,\text{W}) \approx \boxed{2600\,\text{W}} \qquad P_{\text{min}} = \boxed{0\,\text{W}}$$

56. (*a*) Find $V_{\text{rms}}$. Use an integral from Appendix B-4, page A-7.

$$V_{\text{rms}} = \left[\frac{1}{T}\int_0^T \left(V_0 \sin\frac{2\pi t}{T}\right)^2 dt\right]^{1/2} = \left[\frac{V_0^2}{T}\left(\frac{t}{2} - \frac{\sin\left(\frac{4\pi t}{T}\right)}{\frac{8\pi}{T}}\right)\Bigg|_0^T\right]^{1/2} = \left(\frac{V_0^2}{2}\right)^{1/2} = \boxed{\frac{V_0}{\sqrt{2}}}$$

(*b*) Find $V_{\text{rms}}$.

$$V_{\text{rms}} = \left[\frac{1}{T}\int_0^T V^2 dt\right]^{1/2} = \left[\frac{1}{T}\int_0^{T/2} V_0^2 dt + \frac{1}{T}\int_{T/2}^{T} (0)^2\, dt\right]^{1/2} = \left[\frac{V_0^2}{T}\frac{T}{2} + 0\right]^{1/2} = \boxed{\frac{V_0}{\sqrt{2}}}$$

57. (*a*) We follow the derivation in Example 25-14. Start with Eq. 25-14, in absolute value.

$$j = nev_{\text{d}} \rightarrow v_{\text{d}} = \frac{j}{ne} = \frac{I}{neA} = \frac{I}{\left(\frac{N(1\text{ mole})}{m(1\text{ mole})}\rho_{\text{D}}\right) e\left[\pi\left(\frac{1}{2}d\right)^2\right]} = \frac{4Im}{N\rho_{\text{D}} e\pi d^2}$$

$$v_{\text{d}} = \frac{4(2.3\times10^{-6}\,\text{A})(63.5\times10^{-3}\,\text{kg})}{(6.02\times10^{23})(8.9\times10^{3}\,\text{kg/m}^3)(1.60\times10^{-19}\,\text{C})\pi(0.65\times10^{-3}\,\text{m})^2} = \boxed{5.1\times10^{-10}\,\text{m/s}}$$

(*b*) Calculate the current density from Eq. 25-11.

$$j = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{4I}{\pi d^2} = \frac{4\left(2.3\times10^{-6}\,\text{A}\right)}{\pi\left(6.5\times10^{-4}\,\text{m}\right)^2} = 6.931\,\text{A/m}^2 \approx \boxed{6.9\,\text{A/m}^2}$$

(*c*) The electric field is calculated from Eq. 25-17.

$$j = \frac{1}{\rho}E \;\rightarrow\; E = \rho j = \left(1.68\times10^{-8}\,\Omega\text{•m}\right)\left(6.931\,\text{A/m}^2\right) = \boxed{1.2\times10^{-7}\,\text{V/m}}$$

58. (*a*) Use Ohm's law to find the resistance.

$$V = IR \;\rightarrow\; R = \frac{V}{I} = \frac{0.0220\,\text{V}}{0.75\,\text{A}} = 0.02933\,\Omega \approx \boxed{0.029\,\Omega}$$

(*b*) Find the resistivity from Eq. 25-3.

$$R = \frac{\rho \ell}{A} \;\rightarrow$$

$$\rho = \frac{RA}{\ell} = \frac{R\pi r^2}{\ell} = \frac{\left(0.02933\,\Omega\right)\pi\left(1.0\times10^{-3}\,\text{m}\right)^2}{\left(5.80\,\text{m}\right)} = 1.589\times10^{-8}\,\Omega\text{•m} \approx \boxed{1.6\times10^{-8}\,\Omega\text{•m}}$$

(*c*) Use Eq. 25-11 to find the current density.

$$j = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{0.75}{\pi\left(0.0010\,\text{m}\right)^2} = 2.387\times10^{5}\,\text{A/m}^2 \approx \boxed{2.4\times10^{5}\,\text{A/m}^2}$$

(*d*) Use Eq. 25-17 to find the electric field.

$$j = \frac{1}{\rho}E \;\rightarrow$$

$$E = \rho j = \left(1.589\times10^{-8}\,\Omega\text{•m}\right)\left(2.387\times10^{5}\,\text{A/m}^2\right) = 3.793\times10^{-3}\,\text{V/m} \approx \boxed{3.8\times10^{-3}\,\text{V/m}}$$

(*e*) Find the number of electrons per unit volume from the absolute value of Eq. 25-14.

$$j = nev_{\text{d}} \;\rightarrow\; n = \frac{j}{v_{\text{d}}e} = \frac{2.387\times10^{5}\,\text{A/m}^2}{\left(1.7\times10^{-5}\,\text{m/s}\right)\left(1.60\times10^{-19}\,\text{C}\right)} = \boxed{8.8\times10^{28}\,\text{e}^-/\text{m}^3}$$

59. We are given a charge density and a speed (like the drift speed) for both types of ions. From that we can use Eq. 25-13 (without the negative sign) to determine the current per unit area. Both currents are in the same direction in terms of conventional current – positive charge moving north has the same effect as negative charge moving south – and so they can be added.

$$I = neAv_{\text{d}} \;\rightarrow$$

$$\frac{I}{A} = \left(nev_{\text{d}}\right)_{\text{He}} + \left(nev_{\text{d}}\right)_{\text{O}} = \left[\left(2.8\times10^{12}\,\text{ions/m}^3\right)2\left(1.60\times10^{-19}\,\text{C/ion}\right)\left(2.0\times10^{6}\,\text{m/s}\right)\right] + \left[\left(7.0\times10^{11}\,\text{ions/m}^3\right)\left(1.60\times10^{-19}\,\text{C/ion}\right)\left(6.2\times10^{6}\,\text{m/s}\right)\right]$$

$$= 2.486\,\text{A/m}^2 \approx \boxed{2.5\,\text{A/m}^2\text{, North}}$$

60. The magnitude of the electric field is the voltage change per unit meter.

$$|E| = \frac{\Delta V}{\Delta x} = \frac{70\times10^{-3}\,\text{V}}{1.0\times10^{-8}\,\text{m}} = \boxed{7.0\times10^{6}\,\text{V/m}}$$

61. The speed is the change in position per unit time.

$$v = \frac{\Delta x}{\Delta t} = \frac{7.20\times10^{-2}\,\text{m} - 3.40\times10^{-2}\,\text{m}}{0.0063\,\text{s} - 0.0052\,\text{s}} = \boxed{35\,\text{m/s}}$$

Two measurements are needed because there may be a time delay from the stimulation of the nerve to the generation of the action potential.

62. The power is the work done per unit time. The work done to move a charge through a potential difference is the charge times the potential difference. The charge density must be multiplied by the surface area of the cell (the surface area of an open tube, length times circumference) to find the actual charge moved.

$$P = \frac{W}{t} = \frac{QV}{t} = \frac{Q}{t}V$$

$$= \left(3\times10^{-7}\,\frac{\text{mol}}{\text{m}^2\bullet\text{s}}\right)\left(6.02\times10^{23}\,\frac{\text{ions}}{\text{mol}}\right)\left(1.6\times10^{-19}\,\frac{\text{C}}{\text{ion}}\right)(0.10\,\text{m})\pi\left(20\times10^{-6}\,\text{m}\right)(0.030\,\text{V})$$

$$= \boxed{5.4\times10^{-9}\,\text{W}}$$

63. The energy supplied by the battery is the energy consumed by the lights.

$$E_{\text{supplied}} = E_{\text{consumed}} \quad\rightarrow\quad Q\Delta V = Pt \quad\rightarrow$$

$$t = \frac{Q\Delta V}{P} = \frac{(85\text{A}\bullet\text{h})(3600\,\text{s/h})(12\text{V})}{92\,\text{W}} = 39913\,\text{s}\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 11.09\,\text{h} \approx \boxed{11\,\text{h}}$$

64. The ampere-hour is a unit of charge.

$$(1.00\text{A}\bullet\text{h})\left(\frac{1\,\text{C/s}}{1\,\text{A}}\right)\left(\frac{3600\,\text{s}}{1\,\text{h}}\right) = \boxed{3600\,\text{C}}$$

65. Use Eqs. 25-3 and 25-7b.

$$R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2} = \frac{4\rho\ell}{\pi d^2} \;;\; P = \frac{V^2}{R} = \frac{V^2}{\frac{4\rho\ell}{\pi d^2}} \quad\rightarrow$$

$$\ell = \frac{V^2\pi d^2}{4\rho P} = \frac{(1.5\,\text{V})^2\,\pi\left(5.0\times10^{-4}\,\text{m}\right)^2}{4\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)(15\,\text{W})} = 1.753\,\text{m} \approx \boxed{1.8\,\text{m}}$$

If the voltage increases by a factor of 6 without the resistance changing, the power will increase by a factor of 36. The blanket would theoretically be able to deliver 540 W of power, which might make the material catch on fire or burn the occupant.

66. Use Eq. 25-6 to calculate the current.

$$P = IV \quad\rightarrow\quad I = \frac{P}{V} = \frac{746\,\text{W}}{120\,\text{V}} = \boxed{6.22\,\text{A}}$$

67. From Eq. 25-2b, if $R = V/I$, then $G = I/V$

$$G = \frac{I}{V} = \frac{0.48\,\text{A}}{3.0\,\text{V}} = \boxed{0.16\,\text{S}}$$

68. Use Eq. 25-7b to express the resistance in terms of the power, and Eq. 25-3 to express the resistance in terms of the wire geometry.

$$P=\frac{V^2}{R} \rightarrow R=\frac{V^2}{P} \qquad R=\rho\frac{\ell}{A}=\rho\frac{\ell}{\pi r^2}=4\rho\frac{\ell}{\pi d^2}$$

$$4\rho\frac{\ell}{\pi d^2}=\frac{V^2}{P} \rightarrow d=\sqrt{\frac{4\rho\ell P}{\pi V^2}}=\sqrt{\frac{4\left(9.71\times10^{-8}\,\Omega\bullet\text{m}\right)(3.5\,\text{m})(1500\,\text{W})}{\pi(110\,\text{V})^2}}=\boxed{2.3\times10^{-4}\,\text{m}}$$

69. (*a*) Calculate the total kWh used per day, and then multiply by the number of days and the cost per kWh.

$$(1.8\,\text{kW})(2.0\,\text{h/d})+4(0.1\,\text{kW})(6.0\,\text{h/d})+(3.0\,\text{kW})(1.0\,\text{h/d})+(2.0\,\text{kWh/d})$$
$$=11.0\,\text{kWh/d}$$

$$\text{Cost}=(11.0\,\text{kWh/d})(30\,\text{d})\left(\frac{\$0.105}{\text{kWh}}\right)=\$34.65\approx\boxed{\$35\text{ per month}}$$

(*b*) The energy required by the household is 35% of the energy that needs to be supplied by the power plant.

$$\text{Household Energy}=0.35(\text{coal mass})(\text{coal energy per mass}) \rightarrow$$

$$\text{coal mass}=\frac{\text{Household Energy}}{(0.35)(\text{coal energy per mass})}=\frac{(11.0\,\text{kWh/d})(365\,\text{d})\left(\frac{1000\,\text{W}}{\text{kW}}\right)\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)}{(0.35)\left(7500\frac{\text{kcal}}{\text{kg}}\right)\left(\frac{4186\,\text{J}}{1\,\text{kcal}}\right)}$$

$$=1315\,\text{kg}\approx\boxed{1300\,\text{kg of coal}}$$

70. To deliver 15 MW of power at 120 V requires a current of $I=\frac{P}{V}=\frac{15\times10^6\,\text{W}}{120\,\text{V}}=1.25\times10^5\,\text{A}$.

Calculate the power dissipated in the resistors using the current and the resistance.

$$P=I^2R=I^2\rho\frac{L}{A}=I^2\rho\frac{L}{\pi r^2}=4I^2\rho\frac{L}{\pi d^2}=4\left(1.25\times10^5\,\text{A}\right)^2\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)\frac{2(1.0\,\text{m})}{\pi\left(5.0\times10^{-3}\,\text{m}\right)^2}$$
$$=2.674\times10^7\,\text{W}$$

$$\text{Cost}=(\text{Power})(\text{time})(\text{rate per kWh})=\left(2.674\times10^7\,\text{W}\right)\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(1\,\text{h})\left(\frac{\$0.090}{\text{kWh}}\right)$$
$$=\$2407\approx\boxed{\$2,400\text{ per hour per meter}}$$

71. (*a*) Use Eq. 25-7b to relate the power to the voltage for a constant resistance.

$$P=\frac{V^2}{R} \rightarrow \frac{P_{105}}{P_{117}}=\frac{(105\,\text{V})^2/R}{(117\,\text{V})^2/R}=\frac{(105\,\text{V})^2}{(117\,\text{V})^2}=0.805\text{ or a }\boxed{19.5\%\text{ decrease}}$$

(*b*) The lower power output means that the resistor is generating less heat, and so the resistor's temperature would be lower. The lower temperature results in a lower value of the resistance, which would increase the power output at the lower voltages. Thus the decrease would be smaller than the value given in the first part of the problem.

72. Assume that we have a meter of wire, carrying 35 A of current, and dissipating 1.5 W of heat. The power dissipated is $P_R = I^2 R$, and the resistance is $R = \dfrac{\rho \ell}{A}$.

$$P_R = I^2 R = I^2 \frac{\rho \ell}{A} = I^2 \frac{\rho \ell}{\pi r^2} = I^2 \frac{4 \rho \ell}{\pi d^2} \quad \rightarrow$$

$$d = \sqrt{I^2 \frac{4\rho \ell}{P_R \pi}} = 2I\sqrt{\frac{\rho \ell}{P_R \pi}} = 2(35\,\text{A})\sqrt{\frac{(1.68\times 10^{-8}\,\Omega\bullet\text{m})(1.0\,\text{m})}{(1.5\,\text{W})\pi}} = \boxed{4.2\times 10^{-3}\,\text{m}}$$

73. (*a*) The resistance at the operating temperature can be calculated directly from Eq. 25-7.

$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} = \frac{(120\,\text{V})^2}{75\,\text{W}} = \boxed{190\,\Omega}$$

(*b*) The resistance at room temperature is found by converting Eq. 25-5 into an equation for resistances and solving for $R_0$.

$$R = R_0\left[1 + \alpha(T - T_0)\right]$$

$$R_0 = \frac{R}{\left[1 + \alpha(T - T_0)\right]} = \frac{192\,\Omega}{\left[1 + (0.0045\,\text{K}^{-1})(3000\,\text{K} - 293\,\text{K})\right]} = \boxed{15\,\Omega}$$

74. (*a*) The angular frequency is $\omega = 210\,\text{rad/s}$.

$$f = \frac{\omega}{2\pi} = \frac{210\,\text{rad/s}}{2\pi} = 33.42\,\text{Hz} \approx \boxed{33\,\text{Hz}}$$

(*b*) The maximum current is 1.80 A.

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{1.80\,\text{A}}{\sqrt{2}} = \boxed{1.27\,\text{A}}$$

(*c*) For a resistor, $V = IR$.

$$V = IR = (1.80\,\text{A})(\sin 210\,t)(24.0\,\Omega) = \boxed{(43.2\sin 210\,t)\,\text{V}}$$

75. (*a*) The power delivered to the interior is 65% of the power drawn from the source.

$$P_{\text{interior}} = 0.65 P_{\text{source}} \quad \rightarrow \quad P_{\text{source}} = \frac{P_{\text{interior}}}{0.65} = \frac{950\,\text{W}}{0.65} = 1462\,\text{W} \approx \boxed{1500\,\text{W}}$$

(*b*) The current drawn is current from the source, and so the source power is used to calculate the current.

$$P_{\text{source}} = IV_{\text{source}} \quad \rightarrow \quad I = \frac{P_{\text{source}}}{V_{\text{source}}} = \frac{1462\,\text{W}}{120\,\text{V}} = 12.18\,\text{A} \approx \boxed{12\,\text{A}}$$

76. The volume of wire is unchanged by the stretching. The volume is equal to the length of the wire times its cross-sectional area, and since the length was increased by a factor of 1.20, the area was decreased by a factor of 1.20. Use Eq. 25-3.

$$R_0 = \rho\frac{\ell_0}{A_0} \qquad \ell = 1.20\ell_0 \qquad A = \frac{A_0}{1.20} \qquad R = \rho\frac{\ell}{A} = \rho\frac{1.20\ell_0}{\frac{A_0}{1.20}} = (1.20)^2\rho\frac{\ell_0}{A_0} = 1.44R_0 = \boxed{1.44\,\Omega}$$

77. The long, thick conductor is labeled as conductor number 1, and the short, thin conductor is labeled as number 2. The power transformed by a resistor is given by Eq. 25-7b, $P = V^2/R$, and both have the same voltage applied.

$$R_1 = \rho\frac{\ell_1}{A_1} \quad R_2 = \rho\frac{\ell_2}{A_2} \quad \ell_1 = 2\ell_2 \quad A_1 = 4A_2 \ \left(\text{diameter}_1 = 2\text{diameter}_2\right)$$

$$\frac{P_1}{P_2} = \frac{V_1^2/R_1}{V_2^2/R_2} = \frac{R_2}{R_1} = \frac{\rho\ell_2/A_2}{\rho\ell_1/A_1} = \frac{\ell_2}{\ell_1}\frac{A_1}{A_2} = \frac{1}{2}\times 4 = 2 \quad \boxed{P_1 : P_2 = 2:1}$$

78. The heater must heat $108\,\text{m}^3$ of air per hour from $5^\circ\text{C}$ to $20^\circ\text{C}$, and also replace the heat being lost at a rate of 850 kcal/h. Use Eq. 19-2 to calculate the energy needed to heat the air. The density of air is found in Table 13-1.

$$Q = mc\Delta T \quad\rightarrow\quad \frac{Q}{t} = \frac{m}{t}c\Delta T = \left(108\frac{\text{m}^3}{\text{h}}\right)\left(1.29\frac{\text{kg}}{\text{m}^3}\right)\left(0.17\frac{\text{kcal}}{\text{kg}\bullet\text{C}^\circ}\right)\left(15\text{C}^\circ\right) = 355\frac{\text{kcal}}{\text{h}}$$

$$\text{Power required} = 355\frac{\text{kcal}}{\text{h}} + 850\frac{\text{kcal}}{\text{h}} = 1205\frac{\text{kcal}}{\text{h}}\left(\frac{4186\,\text{J}}{\text{kcal}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 1401\,\text{W} \approx \boxed{1400\,\text{W}}$$

79. (*a*) Use Eq. 25-7b.

$$P = \frac{V^2}{R} \quad\rightarrow\quad R = \frac{V^2}{P} = \frac{(240\,\text{V})^2}{2800\,\text{W}} = 20.57\,\Omega \approx \boxed{21\,\Omega}$$

(*b*) Only 75% of the heat from the oven is used to heat the water. Use Eq. 19-2.

$$0.75\left(P_{\text{oven}}\right)t = \text{Heat absorbed by water} = mc\Delta T \quad\rightarrow$$

$$t = \frac{mc\Delta T}{0.75\left(P_{\text{oven}}\right)} = \frac{(0.120\,\text{L})\left(\frac{1\,\text{kg}}{1\,\text{L}}\right)\left(4186\,\text{J/kg}\bullet\text{C}^\circ\right)\left(85\,\text{C}^\circ\right)}{0.75\left(2800\,\text{W}\right)} = 20.33\,\text{s} \approx \boxed{20\,\text{s}}\ (2\text{ sig. fig.})$$

(*c*) $$\frac{11\,\text{cents}}{\text{kWh}}\left(2.8\,\text{kW}\right)\left(20.33\,\text{s}\right)\frac{1\,\text{h}}{3600\,\text{s}} = \boxed{0.17\,\text{cents}}$$

80. (*a*) The horsepower required is the power dissipated by the frictional force, since we are neglecting the energy used for acceleration.

$$P = Fv = (240\,\text{N})(45\,\text{km/hr})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/hr}}\right) = 3000\,\text{W}\left(\frac{1\,\text{hp}}{746\,\text{W}}\right) = \boxed{4.0\,\text{hp}}$$

(*b*) The charge available by each battery is $Q = 95\,\text{A}\bullet\text{h} = 95\,\text{C/s}\bullet 3600\,\text{s} = 3.42\times10^5\,\text{C}$, and so the total charge available is 24 times that. The potential energy of that charge is the charge times the voltage. That energy must be delivered (batteries discharged) in a certain amount of time to produce the 3000 W necessary. The speed of the car times the discharge time is the range of the car between recharges.

$$P = \frac{U}{t} = \frac{QV}{t} \quad\rightarrow\quad t = \frac{QV}{P} = \frac{d}{v} \quad\rightarrow$$

$$d = vt = v\frac{QV}{P} = v\frac{QV}{Fv} = \frac{QV}{F} = \frac{24\left(3.42\times10^5\,\text{C}\right)(12\,\text{V})}{240\,\text{N}} = \boxed{410\,\text{km}}$$

81. The mass of the wire is the density of copper times the volume of the wire, and the resistance of the wire is given by Eq. 25-3. We represent the mass density by $\rho_m$ and the resistivity by $\rho$.

$$R = \rho\frac{\ell}{A} \rightarrow A = \frac{\rho\ell}{R} \qquad m = \rho_m \ell A = \rho_m \ell \frac{\rho\ell}{R} \rightarrow$$

$$\ell = \sqrt{\frac{mR}{\rho_m \rho}} = \sqrt{\frac{(0.0155\,\text{kg})(12.5\,\Omega)}{(8.9\times10^3\,\text{kg/m}^3)(1.68\times10^{-8}\,\Omega\bullet\text{m})}} = 35.997\,\text{m} \approx \boxed{36.0\,\text{m}}$$

$$A = \frac{\rho\ell}{R} = \pi\left(\tfrac{1}{2}d\right)^2 \rightarrow d = \sqrt{\frac{4\rho\ell}{\pi R}} = \sqrt{\frac{4(1.68\times10^{-8}\,\Omega\bullet\text{m})(35.997\,\text{m})}{\pi(12.5\,\Omega)}} = \boxed{2.48\times10^{-4}\,\text{m}}$$

82. The resistance can be calculated from the power and voltage, and then the diameter of the wire can be calculated from the resistance.

$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} \qquad R = \frac{\rho L}{A} = \frac{\rho L}{\pi\left(\frac{1}{2}d\right)^2} \rightarrow \frac{V^2}{P} = \frac{\rho L}{\pi\left(\frac{1}{2}d\right)^2} \rightarrow$$

$$d = \sqrt{\frac{4\rho LP}{\pi V^2}} = \sqrt{\frac{4(100\times10^{-8}\,\Omega\bullet\text{m})(3.8\,\text{m})(95\,\text{W})}{\pi(120\,\text{V})^2}} = 1.787\times10^{-4}\,\text{m} \approx \boxed{1.8\times10^{-4}\,\text{m}}$$

83. Use Eq. 25-7b.

(*a*) $P = \frac{V^2}{R} = \frac{(120\,\text{V})^2}{12\,\Omega} = \boxed{1200\,\text{W}}$

(*b*) $P = \frac{V^2}{R} = \frac{(120\,\text{V})^2}{140\,\Omega} = 103\,\text{W} \approx \boxed{100\,\text{W}}$ (2 sig. fig.)

84. Use Eq. 25-7b for the power in each case, assuming the resistance is constant.

$$\frac{P_{13.8\,\text{V}}}{P_{12.0\,\text{V}}} = \frac{(V^2/R)_{13.8\,\text{V}}}{(V^2/R)_{12.0\,\text{V}}} = \frac{13.8^2}{12.0^2} = 1.3225 = \boxed{32\%\text{ increase}}$$

85. Model the protons as moving in a continuous beam of cross-sectional area *A*. Then by Eq. 25-13, $I = neAv_d$, where we only consider the absolute value of the current. The variable *n* is the number of protons per unit volume, so $n = \frac{N}{A\ell}$, where *N* is the number of protons in the beam and $\ell$ is the circumference of the ring. The "drift" velocity in this case is the speed of light.

$$I = neAv_d = \frac{N}{A\ell}eAv_d = \frac{N}{\ell}ev_d \rightarrow$$

$$N = \frac{I\ell}{ev_d} = \frac{(11\times10^{-3})(6300\,\text{m})}{(1.60\times10^{-19}\,\text{C})(3.00\times10^8\,\text{m/s})} = \boxed{1.4\times10^{12}\,\text{protons}}$$

86. (*a*) The current can be found from Eq. 25-6.

$$I = P/V \qquad I_A = P_A/V_A = 40\text{W}/120\,\text{V} = \boxed{0.33\,\text{A}} \qquad I_B = P_B/V_B = 40\text{W}/12\,\text{V} = \boxed{3.3\,\text{A}}$$

(*b*) The resistance can be found from Eq. 25-7b.

$$R = \frac{V^2}{P} \qquad R_A = \frac{V_A^2}{P_A} = \frac{(120\,\text{V})^2}{40\,\text{W}} = \boxed{360\,\Omega} \qquad R_B = \frac{V_B^2}{P_B} = \frac{(12\,\text{V})^2}{40\,\text{W}} = \boxed{3.6\,\Omega}$$

(*c*) The charge is the current times the time.

$$Q = It \qquad Q_A = I_A t = (0.33\,\text{A})(3600\,\text{s}) = \boxed{1200\,\text{C}}$$

$$Q_B = I_B t = (3.3\,\text{A})(3600\,\text{s}) = \boxed{12{,}000\,\text{C}}$$

(*d*) The energy is the power times the time, and the power is the same for both bulbs.

$$E = Pt \qquad E_A = E_B = (40\,\text{W})(3600\,\text{s}) = \boxed{1.4\times10^5\,\text{J}}$$

(*e*) $\boxed{\text{Bulb B}}$ requires a larger current, and so should have larger diameter connecting wires to avoid overheating the connecting wires.

87. (*a*) The power is given by $P = IV$.

$$P = IV = (14\,\text{A})(220\,\text{V}) = 3080\text{W} \approx \boxed{3100\,\text{W}}$$

(*b*) The power dissipated is given by $P_R = I^2R$, and the resistance is $R = \dfrac{\rho\ell}{A}$.

$$P_R = I^2R = I^2\frac{\rho\ell}{A} = I^2\frac{\rho\ell}{\pi r^2} = I^2\frac{4\rho\ell}{\pi d^2} = (14\,\text{A})^2\frac{4(1.68\times10^{-8}\,\Omega\bullet\text{m})(15\,\text{m})}{\pi(1.628\times10^{-3}\,\text{m})^2} = 23.73\,\text{W}$$

$$\approx \boxed{24\,\text{W}}$$

(*c*) $$P_R = I^2\frac{4\rho L}{\pi d^2} = (14\,\text{A})^2\frac{4(1.68\times10^{-8}\,\Omega\bullet\text{m})(15\,\text{m})}{\pi(2.053\times10^{-3}\,\text{m})^2} = 14.92\,\text{W} \approx \boxed{15\,\text{W}}$$

(*d*) The savings is due to the power difference.

$$\text{Savings} = (23.73\,\text{W} - 14.92\,\text{W})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right)(30\,\text{d})\left(\frac{12\,\text{h}}{1\,\text{d}}\right)\left(\frac{\$0.12}{1\,\text{kWh}}\right)$$

$$= \$0.3806/\text{month} \approx \boxed{38\,\text{cents per month}}$$

88. The wasted power is due to losses in the wire. The current in the wire can be found by $I = P/V$.

(*a*) $$P_R = I^2R = \frac{P^2}{V^2}R = \frac{P^2}{V^2}\frac{\rho L}{A} = \frac{P^2}{V^2}\frac{\rho L}{\pi r^2} = \frac{P^2}{V^2}\frac{4\rho L}{\pi d^2}$$

$$= \frac{(1750\,\text{W})^2}{(120\,\text{V})^2}\frac{4(1.68\times10^{-8}\,\Omega\bullet\text{m})(25.0\,\text{m})}{\pi(2.59\times10^{-3}\,\text{m})^2} = 16.954\,\text{W} \approx \boxed{17.0\,\text{W}}$$

(*b*) $$P_R = \frac{P^2}{V^2}\frac{4\rho L}{\pi d^2} = \frac{(1750\,\text{W})^2}{(120\,\text{V})^2}\frac{4(1.68\times10^{-8}\,\Omega\bullet\text{m})(25.0\,\text{m})}{\pi(4.12\times10^{-3}\,\text{m})^2} = \boxed{6.70\,\text{W}}$$

89. (*a*) The D-cell provides 25 mA at 1.5 V for 820 h, at a cost of \$1.70.

$$\text{Energy} = Pt = VIt = (1.5\,\text{V})(0.025\,\text{A})(820\,\text{h})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right) = 0.03075\,\text{kWh}$$

$$\text{Cost/kWh} = \frac{\$1.70}{0.03075\,\text{kWh}} = \$55.28/\text{kWh} \approx \boxed{\$55/\text{kWh}}$$

(*b*) The AA-cell provides 25 mA at 1.5 V for 120 h, at a cost of \$1.25.

$$\text{Energy} = Pt = VIt = (1.5\,\text{V})(0.025\,\text{A})(120\,\text{h})\left(\frac{1\,\text{kW}}{1000\,\text{W}}\right) = 0.0045\,\text{kWh}$$

$$\text{Cost/kWh} = \frac{\$1.25}{0.0045\,\text{kWh}} = \$277.78/\text{kWh} \approx \boxed{\$280/\text{kWh}}$$

The D-cell is $\frac{\$55.28/\text{kWh}}{\$0.10/\text{kWh}} \approx \boxed{550 \times \text{as costly}}$. The AA-cell is $\frac{\$277.78/\text{kWh}}{\$0.10/\text{kWh}} \approx \boxed{2800 \times \text{as costly}}$.

90. The electrons are assumed to be moving with simple harmonic motion. During one cycle, an object in simple harmonic motion will move a distance equal to the amplitude from its equilibrium point. From Eq. 14-9a, we know that $v_{\text{max}} = A\omega$, where $\omega$ is the angular frequency of oscillation. From Eq. 25-13 in absolute value, we see that $I_{\text{max}} = neAv_{\text{max}}$. Finally, the maximum current can be related to the power by Eqs. 25-9 and 25-10. The charge carrier density, *n*, is calculated in Example 25-14.

$$\overline{P} = I_{\text{rms}}V_{\text{rms}} = \tfrac{1}{\sqrt{2}} I_{\text{max}}V_{\text{rms}}$$

$$A = \frac{v_{\text{max}}}{\omega} = \frac{I_{\text{max}}}{\omega neA} = \frac{\sqrt{2}\overline{P}}{\omega ne\frac{\pi d^2}{4}V_{\text{rms}}}$$

$$= \frac{4\sqrt{2}(550\,\text{W})}{2\pi(60\,\text{Hz})(8.4\times10^{28}\,\text{m}^{-3})(1.60\times10^{-19}\,\text{C})\pi(1.7\times10^{-3}\,\text{m})^2(120\,\text{V})} = \boxed{5.6\times10^{-7}\,\text{m}}$$

The electron will move this distance in both directions from its equilibrium point.

$\boxed{91.}$ Eq. 25-3 can be used. The area to be used is the cross-sectional area of the pipe.

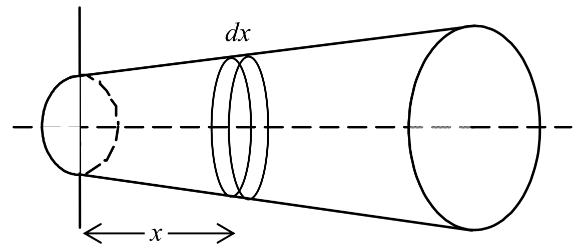
$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi\left(r_{\text{outside}}^2 - r_{\text{inside}}^2\right)} = \frac{(1.68\times10^{-8}\,\Omega\bullet\text{m})(10.0\,\text{m})}{\pi\left[(2.50\times10^{-2}\,\text{m})^2 - (1.50\times10^{-2}\,\text{m})^2\right]} = \boxed{1.34\times10^{-4}\,\Omega}$$

92. We assume that all of the current that enters at *a* leaves at *b*, so that the current is the same at each end. The current density is given by Eq. 25-11.

$$j_{\text{a}} = \frac{I}{A_{\text{a}}} = \frac{I}{\pi\left(\frac{1}{2}a\right)^2} = \frac{4I}{\pi a^2} = \frac{4(2.0\,\text{A})}{\pi(2.5\times10^{-3}\,\text{m})^2} = \boxed{4.1\times10^5\,\text{A/m}^2}$$

$$j_{\text{b}} = \frac{I}{A_{\text{b}}} = \frac{I}{\pi\left(\frac{1}{2}b\right)^2} = \frac{4I}{\pi b^2} = \frac{4(2.0\,\text{A})}{\pi(4.0\times10^{-3}\,\text{m})^2} = \boxed{1.6\times10^5\,\text{A/m}^2}$$

93. Using Eq. 25-3, we find the infinitesimal resistance first of a thin vertical slice at a horizontal distance $x$ from the center of the left side towards the center of the right side. Let the thickness of that slice be $dx$. That thickness corresponds to the variable $\ell$ in Eq. 25-3. The diameter of this slice is $a + \frac{x}{\ell}(b-a)$. Then integrate over all the slices to find the total resistance.




$$R = \rho\frac{\ell}{A} \rightarrow dR = \rho\frac{dx}{\pi\frac{1}{4}\left(a + \frac{x}{\ell}(b-a)\right)^2} \rightarrow$$

$$R = \int dR = \int_0^\ell \rho\frac{dx}{\pi\frac{1}{4}\left(a + \frac{x}{\ell}(b-a)\right)^2} = -\frac{4\rho}{\pi}\frac{\ell}{b-a}\frac{1}{\left(a + \frac{x}{\ell}(b-a)\right)}\Bigg|_0^\ell = \boxed{\frac{4\rho}{\pi}\frac{\ell}{ab}}$$

94. The resistance of the filament when the flashlight is on is $R = \frac{V}{I} = \frac{3.2\,\text{V}}{0.20\,\text{A}} = 16\,\Omega$. That can be used with a combination of Eqs. 25-3 and 25-5 to find the temperature.

$$R = R_0\left[1 + \alpha\left(T - T_0\right)\right] \rightarrow$$

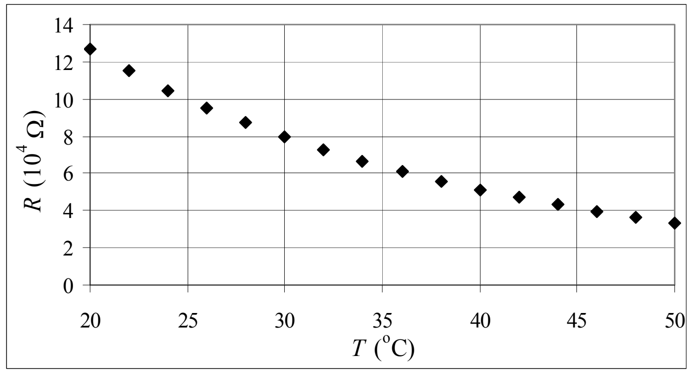
$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20^\circ\text{C} + \frac{1}{0.0045\left(\text{C}^\circ\right)^{-1}}\left(\frac{16\,\Omega}{1.5\,\Omega} - 1\right) = 2168^\circ\text{C} \approx \boxed{2200^\circ\text{C}}$$

95. When the tank is empty, the entire length of the wire is in a non-superconducting state, and so has a non-zero resistivity, which we call $\rho$. Then the resistance of the wire when the tank is empty is given by $R_0 = \rho\frac{\ell}{A} = \frac{V_0}{I}$. When a length $x$ of the wire is superconducting, that portion of the wire has 0 resistance. Then the resistance of the wire is only due to the length $\ell - x$, and so $R = \rho\frac{\ell - x}{A} = \rho\frac{\ell}{A}\frac{\ell - x}{\ell} = R_0\frac{\ell - x}{\ell}$. This resistance, combined with the constant current, gives $V = IR$.

$$V = IR = \left(\frac{V_0}{R_0}\right)R_0\frac{\ell - x}{\ell} = V_0\left(1 - \frac{x}{\ell}\right) = V_0\left(1 - f\right) \rightarrow \boxed{f = 1 - \frac{V}{V_0}}$$

Thus a measurement of the voltage can give the fraction of the tank that is filled with liquid helium.

96. We plot resistance vs. temperature. The graph is shown as follows, with no curve fitted to it. It is apparent that a linear fit will not be a good fit to this data. Both quadratic and exponential equations fit the data well, according to the R-squared coefficient as given by Excel. The equations and the predictions are given below.




$$R_{\text{exp}} = \left(30.1\times 10^4 e^{-0.0442T}\right)\Omega$$

$$R_{\text{quad}} = \left[\left(7.39\times 10^4\right)T^2 - 8200T + 25.9\times 10^4\right]\Omega$$

Solving these expressions for $R = 57,641\,\Omega$ (using the spreadsheet) gives $T_{\text{exp}} = 37.402°\text{C}$ and $T_{\text{quad}} = 37.021°\text{C}$. So the temperature is probably in the range between those two values: $\boxed{37.021°\text{C} < T < 37.402°\text{C}}$. The average of those two values is $\boxed{T = 37.21°\text{C}}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH25.XLS," on tab "Problem 25.96."

As an extra comment, how might you choose between the exponential and quadratic fits? While they both give almost identical predictions for this intermediate temperature, they differ significantly at temperatures near $0°\text{C}$. The exponential fit would give a resistance of about $301,000\,\Omega$ at $0°\text{C}$, while the quadratic fit would give a resistance of about 259,000 $\Omega$ at $0°\text{C}$. So a measurement of resistance near $0°\text{C}$ might be very useful.

# CHAPTER 26: DC Circuits

## Responses to Questions

1. Even though the bird's feet are at high potential with respect to the ground, there is very little potential difference between them, because they are close together on the wire. The resistance of the bird is much greater than the resistance of the wire between the bird's feet. These two resistances are in parallel, so very little current will pass through the bird as it perches on the wire. When you put a metal ladder up against a power line, you provide a direct connection between the high potential line and ground. The ladder will have a large potential difference between its top and bottom. A person standing on the ladder will also have a large potential difference between his or her hands and feet. Even if the person's resistance is large, the potential difference will be great enough to produce a current through the person's body large enough to cause substantial damage or death.

2. Series: The main disadvantage of Christmas tree lights connected in series is that when one bulb burns out, a gap is created in the circuit and none of the bulbs remains lit. Finding the burned-out bulb requires replacing each individual bulb one at a time until the string of bulbs comes back on. As an advantage, the bulbs are slightly easier to wire in series.

   Parallel: The main advantage of connecting the bulbs in parallel is that one burned-out bulb does not affect the rest of the strand, and is easy to identify and replace. As a disadvantage, wiring the bulbs in parallel is slightly more difficult.

3. Yes. You can put 20 of the 6-V lights in series, or you can put several of the 6-V lights in series with a large resistance.

4. When the bulbs are connected in series, they have the same current through them. $R_2$, the bulb with the greater resistance, will be brighter in this case, since $P = I^2R$. When the bulbs are connected in parallel, they will have the same voltage across them. In this case, $R_1$, the bulb with the lower resistance, will have a larger current flowing through it and will be brighter: $P = V^2/R$.

5. Double outlets are connected in parallel, since each has 120 V across its terminals and they can be used independently.

6. Arrange the two batteries in series with each other and the two bulbs in parallel across the combined voltage of the batteries. This configuration maximizes the voltage gain and minimizes the equivalent resistance, yielding the maximum power.

7. The battery has to supply less power when the two resistors are connected in series than it has to supply when only one resistor is connected. $P = IV = \dfrac{V^2}{R}$, so if $V$ is constant and $R$ increases, the power decreases.

8. The overall resistance decreases and more current is drawn from the source. A bulb rated at 60-W and 120-V has a resistance of 240 Ω. A bulb rated at 100-W and 120-V has a resistance of 144 Ω. When only the 60-W bulb is on, the total resistance is 240 Ω. When both bulbs are lit, the total resistance is the combination of the two resistances in parallel, which is only 90 Ω.

9. No. The sign of the battery's emf does not depend on the direction of the current through the battery. Yes, the terminal voltage of the battery does depend on the direction of the current through the

battery. Note that the sign of the battery's emf in the loop equation does depend on the direction the loop is traversed (+ in the direction of the battery's potential, – in the opposite direction), and the terminal voltage sign and magnitude depend on whether the loop is traversed with or against the current.

10. When resistors are connected in *series*, the equivalent resistance is the *sum* of the individual resistances, $R_{eq,series} = R_1 + R_2 + ....$ The current has to go through each additional resistance if the resistors are in series and therefore the equivalent resistance is greater than any individual resistance. In contrast, when capacitors are in *parallel* the equivalent capacitance is equal to the sum of the individual capacitors, $C_{eq,parallel} = C_1 + C_2 + ....$ Charge drawn from the battery can go down any one of the different branches and land on any one of the capacitors, so the overall capacitance is greater than that of each individual capacitor.

    When resistors are connected in *parallel*, the current from the battery or other source divides into the different branches and so the equivalent resistance is less than any individual resistor in the circuit. The corresponding expression is $1/R_{eq,parallel} = 1/R_1 + 1/R_2 + ....$ The formula for the equivalent capacitance of capacitors in *series* follows this same form, $1/C_{eq,series} = 1/C_1 + 1/C_2 + ....$ When capacitors are in series, the overall capacitance is less than the capacitance of any individual capacitor. Charge leaving the first capacitor lands on the second rather than going straight to the battery.

    Compare the expressions defining resistance ($R = V/I$) and capacitance ($C = Q/V$). Resistance is proportional to voltage, whereas capacitance is inversely proportional to voltage.

11. When batteries are connected in series, their emfs add together, producing a larger potential. The batteries do not need to be identical in this case. When batteries are connected in parallel, the currents they can generate add together, producing a larger current over a longer time period. Batteries in this case need to be nearly identical, or the battery with the larger emf will end up charging the battery with the smaller emf.

12. Yes. When a battery is being charged, current is forced through it "backwards" and then $V_{terminal}$ = emf + $Ir$, so $V_{terminal}$ > emf.

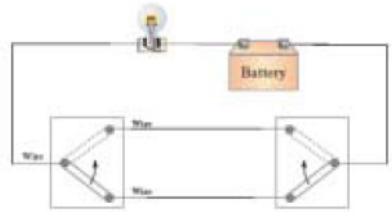
13. Put the battery in a circuit in series with a very large resistor and measure the terminal voltage. With a large resistance, the current in the circuit will be small, and the potential across the battery will be mainly due to the emf. Next put the battery in parallel with the large resistor (or in series with a small resistor) and measure the terminal voltage and the current in the circuit. You will have enough information to use the equation $V_{terminal}$ = emf – $Ir$ to determine the internal resistance $r$.

14. No. As current passes through the resistor in the $RC$ circuit, energy is dissipated in the resistor. Therefore, the total energy supplied by the battery during the charging is the combination of the energy dissipated in the resistor and the energy stored in the capacitor.

15. (*a*) Stays the same; (*b*) Increases; (*c*) Decreases; (*d*) Increases; (*e*) Increases; (*f*) Decreases; (*g*) Decreases; (*h*) Increases; (*i*) Remains the same.

16. The capacitance of a parallel plate capacitor is inversely proportional to the distance between the plates: ($C = \varepsilon_0 A/d$). As the diaphragm moves in and out, the distance between the plates changes and therefore the capacitance changes with the same frequency. This changes the amount of charge that can be stored on the capacitor, creating a current as the capacitor charges or discharges. The current oscillates with the same frequency as the diaphragm, which is the same frequency as the incident sound wave, and produces an oscillating $V_{output}$.

17. See the adjacent figure. If both switches are connected to the same wire, the circuit is complete and the light is on. If they are connected to opposite wires, the light will remain off.




18. In an analog ammeter, the internal resistor, or shunt resistor, has a small value and is in parallel with the galvanometer, so that the overall resistance of the ammeter is very small. In an analog voltmeter, the internal resistor has a large value and is in series with the galvanometer, and the overall resistance of the voltmeter is very large.

19. If you use an ammeter where you need to use a voltmeter, you will short the branch of the circuit. Too much current will pass through the ammeter and you will either blow the fuse on the ammeter or burn out its coil.

20. An ammeter is placed in series with a given circuit element in order to measure the current through that element. If the ammeter did not have very low (ideally, zero) resistance, its presence in the circuit would change the current it is attempting to measure by adding more resistance in series. An ideal ammeter has zero resistance and thus does not change the current it is measuring.

    A voltmeter is placed in parallel with a circuit element in order to measure the voltage difference across that element. If the voltmeter does not have a very high resistance, than its presence in parallel will lower the overall resistance and affect the circuit. An ideal voltmeter has infinite resistance so that when placed in parallel with circuit elements it will not change the value of the voltage it is reading.

21. When a voltmeter is connected across a resistor, the voltmeter is in parallel with the resistor. Even if the resistance of the voltmeter is large, the parallel combination of the resistor and the voltmeter will be slightly smaller than the resistor alone. If $R_{\text{eq}}$ decreases, then the overall current will increase, so that the potential drop across the rest of the circuit will increase. Thus the potential drop across the parallel combination will be less than the original voltage drop across the resistor.

22. A voltmeter has a very high resistance. When it is connected to the battery very little current will flow. A small current results in a small voltage drop due to the internal resistance of the battery, and the emf and terminal voltage (measured by the voltmeter) will be very close to the same value. However, when the battery is connected to the lower-resistance flashlight bulb, the current will be higher and the voltage drop due to the internal resistance of the battery will also be higher. As a battery is used, its internal resistance increases. Therefore, the terminal voltage will be significantly lower than the emf: $V_{\text{terminal}} = \text{emf} - Ir$. A lower terminal voltage will result in a dimmer bulb, and usually indicates a "used-up" battery.

23. (*a*) With the batteries in series, a greater voltage is delivered to the lamp, and the lamp will burn brighter.
    (*b*) With the batteries in parallel, the voltage across the lamp is the same as for either battery alone. Each battery supplies only half of the current going through the lamp, so the batteries will last twice as long.

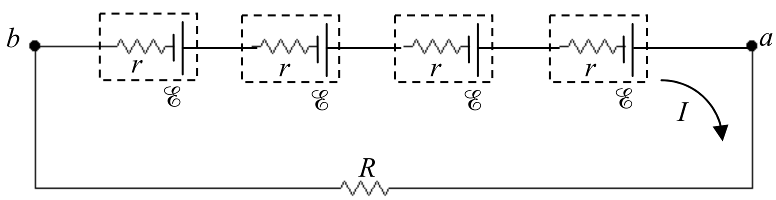
## Solutions to Problems

1. See Figure 26-2 for a circuit diagram for this problem. Using the same analysis as in Example 26-1, the current in the circuit is $I = \frac{\mathcal{E}}{R+r}$. Use Eq. 26-1 to calculate the terminal voltage.

(*a*) $V_{ab} = \mathcal{E} - Ir = \mathcal{E} - \left(\frac{\mathcal{E}}{R+r}\right)r = \frac{\mathcal{E}(R+r) - \mathcal{E}r}{R+r} = \mathcal{E}\frac{R}{R+r} = (6.00\,\text{V})\frac{81.0\,\Omega}{(81.0+0.900)\,\Omega} = \boxed{5.93\,\text{V}}$

(*b*) $V_{ab} = \mathcal{E}\frac{R}{R+r} = (6.00\,\text{V})\frac{810\,\Omega}{(810+0.900)\,\Omega} = \boxed{5.99\,\text{V}}$

2. See the circuit diagram below. The current in the circuit is $I$. The voltage $V_{ab}$ is given by Ohm's law to be $V_{ab} = IR$. That same voltage is the terminal voltage of the series EMF.

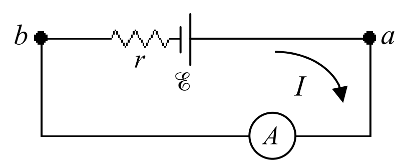



$$V_{ab} = (\mathcal{E} - Ir) + (\mathcal{E} - Ir) + (\mathcal{E} - Ir) + (\mathcal{E} - Ir) = 4(\mathcal{E} - Ir) \quad \text{and} \quad V_{ab} = IR$$

$$4(\mathcal{E} - Ir) = IR \;\rightarrow\; r = \frac{\mathcal{E} - \frac{1}{4}IR}{I} = \frac{(1.5\,\text{V}) - \frac{1}{4}(0.45\,\text{A})(12\,\Omega)}{0.45\,\text{A}} = 0.333\,\Omega \approx \boxed{0.3\,\Omega}$$

3. We take the low-resistance ammeter to have no resistance. The circuit is shown. The terminal voltage will be 0 volts.

$$V_{ab} = \mathcal{E} - Ir = 0 \;\rightarrow\; r = \frac{\mathcal{E}}{I} = \frac{1.5\,\text{V}}{25\,\text{A}} = \boxed{0.060\,\Omega}$$

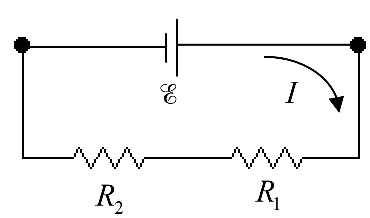



4. See Figure 26-2 for a circuit diagram for this problem. Use Eq. 26-1.

$$V_{ab} = \mathcal{E} - Ir \;\rightarrow\; r = \frac{\mathcal{E} - V_{ab}}{I} = \frac{12.0\,\text{V} - 8.4\,\text{V}}{95\,\text{A}} = \boxed{0.038\,\Omega}$$

$$V_{ab} = IR \;\rightarrow\; R = \frac{V_{ab}}{I} = \frac{8.4\,\text{V}}{95\,\text{A}} = \boxed{0.088\,\Omega}$$

5. The equivalent resistance is the sum of the two resistances: $R_{eq} = R_1 + R_2$. The current in the circuit is then the voltage divided by the equivalent resistance: $I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{R_1 + R_2}$. The voltage across the 2200-Ω resistor is given by Ohm's law.




$$V_{2200} = IR_2 = \frac{\mathcal{E}}{R_1 + R_2}R_2 = \mathcal{E}\frac{R_2}{R_1 + R_2} = (12.0\,\text{V})\frac{2200\,\Omega}{650\,\Omega + 2200\,\Omega} = \boxed{9.3\,\text{V}}$$

6. (*a*) For the resistors in series, use Eq. 26-3, which says the resistances add linearly.

$$R_{eq} = 3(45\Omega) + 3(65\Omega) = \boxed{330\Omega}$$

(*b*) For the resistors in parallel, use Eq. 26-4, which says the resistances add reciprocally.

$$\frac{1}{R_{eq}} = \frac{1}{45\Omega} + \frac{1}{45\Omega} + \frac{1}{45\Omega} + \frac{1}{65\Omega} + \frac{1}{65\Omega} + \frac{1}{65\Omega} = \frac{3}{45\Omega} + \frac{3}{65\Omega} = \frac{3(65\Omega) + 3(45\Omega)}{(65\Omega)(45\Omega)} \rightarrow$$

$$R_{eq} = \frac{(65\Omega)(45\Omega)}{3(65\Omega) + 3(45\Omega)} = \boxed{8.9\Omega}$$

7. (*a*) The maximum resistance is made by combining the resistors in series.

$$R_{eq} = R_1 + R_2 + R_3 = 680\,\Omega + 720\,\Omega + 1200\,\Omega = \boxed{2.60\,\text{k}\Omega}$$

(*b*) The minimum resistance is made by combining the resistors in parallel.

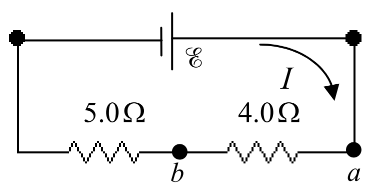
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \rightarrow$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{680\,\Omega} + \frac{1}{720\,\Omega} + \frac{1}{1200\,\Omega}\right)^{-1} = \boxed{270\,\Omega}$$

8. The equivalent resistance of five 100-Ω resistors in parallel is found, and then that resistance is divided by $10\Omega$ to find the number of 10-Ω resistors needed.

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)^{-1} = \left(\frac{5}{100\Omega}\right)^{-1} = 20\Omega = n(10\Omega) \rightarrow n = \frac{20\Omega}{10\Omega} = \boxed{2}$$

9. Connecting nine of the resistors in series will enable you to make a voltage divider with a 4.0 V output. To get the desired output, measure the voltage across four consecutive series resistors.




$$R_{eq} = 9(1.0\Omega) \qquad I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{9.0\Omega}$$

$$V_{ab} = (4.0\Omega)I = (4.0\Omega)\frac{\mathcal{E}}{9.0\Omega} = (4.0\Omega)\frac{9.0\,\text{V}}{9.0\Omega} = \boxed{4.0\,\text{V}}$$

10. The resistors can all be connected in series.

$$R_{eq} = R + R + R = 3(1.70\,\text{k}\Omega) = \boxed{5.10\,\text{k}\Omega}$$

The resistors can all be connected in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \rightarrow R_{eq} = \left(\frac{3}{R}\right)^{-1} = \frac{R}{3} = \frac{1.70\,\text{k}\Omega}{3} = \boxed{567\,\Omega}$$

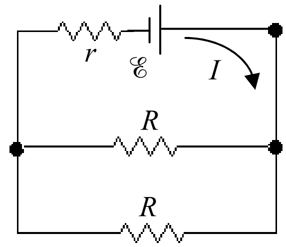
Two resistors in series can be placed in parallel with the third.

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R+R} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \rightarrow R_{eq} = \frac{2R}{3} = \frac{2(1.70\,\text{k}\Omega)}{3} = \boxed{1.13\,\text{k}\Omega}$$

Two resistors in parallel can be placed in series with the third.

$$R_{eq} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = R + \frac{R}{2} = \frac{3}{2}(1.70\,\text{k}\Omega) = \boxed{2.55\,\text{k}\Omega}$$

11. The resistance of each bulb can be found from its power rating.




$$P=\frac{V^2}{R} \rightarrow R=\frac{V^2}{P}=\frac{(12.0\,\text{V})^2}{4.0\,\text{W}}=36\,\Omega$$

Find the equivalent resistance of the two bulbs in parallel.

$$\frac{1}{R_{\text{eq}}}=\frac{1}{R}+\frac{1}{R}=\frac{2}{R} \rightarrow R_{\text{eq}}=\frac{R}{2}=\frac{36\,\Omega}{2}=18\,\Omega$$

The terminal voltage is the voltage across this equivalent resistance. Use that to find the current drawn from the battery.

$$V_{\text{ab}}=IR_{\text{eq}} \rightarrow I=\frac{V_{\text{ab}}}{R_{\text{eq}}}=\frac{V_{\text{ab}}}{R/2}=\frac{2V_{\text{ab}}}{R}$$

Finally, use the terminal voltage and the current to find the internal resistance, as in Eq. 26-1.

$$V_{\text{ab}}=\mathscr{E}-Ir \rightarrow r=\frac{\mathscr{E}-V_{\text{ab}}}{I}=\frac{\mathcal{E}-V_{\text{ab}}}{\left(\frac{2V_{\text{ab}}}{R}\right)}=R\frac{\mathscr{E}-V_{\text{ab}}}{2V_{\text{ab}}}=(36\,\Omega)\frac{12.0\,\text{V}-11.8\,\text{V}}{2(11.8\,\text{V})}=0.305\,\Omega\approx\boxed{0.3\,\Omega}$$

12. (*a*) Each bulb should get one-eighth of the total voltage, but let us prove that instead of assuming it. Since the bulbs are identical, the net resistance is $R_{\text{eq}}=8R$. The current flowing through the bulbs is then $V_{\text{tot}}=IR_{\text{eq}} \rightarrow I=\frac{V_{\text{tot}}}{R_{\text{eq}}}=\frac{V_{\text{tot}}}{8R}$. The voltage across one bulb is found from Ohm's law.

$$V=IR=\frac{V_{\text{tot}}}{8R}R=\frac{V_{\text{tot}}}{8}=\frac{110\,\text{V}}{8}=13.75\,\text{V}\approx\boxed{14\,\text{V}}$$

(*b*) $$I=\frac{V_{\text{tot}}}{8R} \rightarrow R=\frac{V_{\text{tot}}}{8I}=\frac{110\,\text{V}}{8(0.42\,\text{A})}=32.74\,\Omega\approx\boxed{33\,\Omega}$$

$$P=I^2R=(0.42\,\text{A})^2(32.74\,\Omega)=5.775\,\text{W}\approx\boxed{5.8\,\text{W}}$$

$\boxed{13.}$ We model the resistance of the long leads as a single resistor *r*. Since the bulbs are in parallel, the total current is the sum of the current in each bulb, and so $I=8I_R$. The voltage drop across the long leads is $V_{\text{leads}}=Ir=8I_Rr=8(0.24\,\text{A})(1.4\,\Omega)=2.688\,\text{V}$. Thus the voltage across each of the parallel resistors is $V_R=V_{\text{tot}}-V_{\text{leads}}=110\,\text{V}-2.688\,\text{V}=107.3\,\text{V}$. Since we have the current through each resistor, and the voltage across each resistor, we calculate the resistance using Ohm's law.

$$V_R=I_RR \rightarrow R=\frac{V_R}{I_R}=\frac{107.3\,\text{V}}{0.24\,\text{A}}=447.1\,\Omega=\boxed{450\,\Omega}$$

The total power delivered is $P=V_{\text{tot}}I$, and the "wasted" power is $I^2r$. The fraction wasted is the ratio of those powers.

$$\text{fraction wasted}=\frac{I^2r}{IV_{\text{tot}}}=\frac{Ir}{V_{\text{tot}}}=\frac{8(0.24\,\text{A})(1.4\,\Omega)}{110\,\text{V}}=\boxed{0.024}$$

So about 2.5% of the power is wasted.

14. The power delivered to the starter is equal to the square of the current in the circuit multiplied by the resistance of the starter. Since the resistors in each circuit are in series we calculate the currents as the battery emf divided by the sum of the resistances.

$$\frac{P}{P_0}=\frac{I^2R_S}{I_0^2R_S}=\left(\frac{I}{I_0}\right)^2=\left(\frac{\mathscr{E}/R_{eq}}{\mathscr{E}/R_{0eq}}\right)^2=\left(\frac{R_{0eq}}{R_{eq}}\right)^2=\left(\frac{r+R_S}{r+R_S+R_C}\right)^2$$
$$=\left(\frac{0.02\,\Omega+0.15\,\Omega}{0.02\,\Omega+0.15\,\Omega+0.10\,\Omega}\right)^2=\boxed{0.40}$$

15. To fix this circuit, connect another resistor in parallel with the 480-Ω resistor so that the equivalent resistance is the desired 370 Ω.

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}\quad\rightarrow\quad R_2=\left(\frac{1}{R_{eq}}-\frac{1}{R_1}\right)^{-1}=\left(\frac{1}{370\,\Omega}-\frac{1}{480\,\Omega}\right)^{-1}=1615\,\Omega\approx\boxed{1600\,\Omega}$$

So solder a 1600-Ω resistor in parallel with the 480-Ω resistor.

16. (*a*) The equivalent resistance is found by combining the 820 Ω and 680 Ω resistors in parallel, and then adding the 960 Ω resistor in series with that parallel combination.

$$R_{eq}=\left(\frac{1}{820\,\Omega}+\frac{1}{680\,\Omega}\right)^{-1}+960\,\Omega=372\,\Omega+960\,\Omega=1332\,\Omega\approx\boxed{1330\,\Omega}$$

(*b*) The current delivered by the battery is $I=\frac{V}{R_{eq}}=\frac{12.0\,\text{V}}{1332\,\Omega}=9.009\times10^{-3}\,\text{A}$. This is the current in the 960 Ω resistor. The voltage across that resistor can be found by Ohm's law.

$$V_{470}=IR=\left(9.009\times10^{-3}\,\text{A}\right)\left(960\,\Omega\right)=8.649\,\text{V}\approx\boxed{8.6\,\text{V}}$$

Thus the voltage across the parallel combination must be $12.0\,\text{V}-8.6\,\text{V}=\boxed{3.4\,\text{V}}$. This is the voltage across both the 820 Ω and 680 Ω resistors, since parallel resistors have the same voltage across them. Note that this voltage value could also be found as follows.

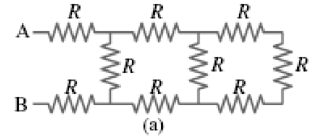
$$V_{parallel}=IR_{parallel}=\left(9.009\times10^{-3}\,\text{A}\right)\left(372\,\Omega\right)=3.351\text{V}\approx3.4\,\text{V}$$

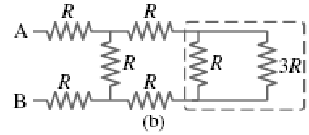
17. The resistance of each bulb can be found by using Eq. 25-7b, $P=V^2/R$. The two individual resistances are combined in parallel. We label the bulbs by their wattage.

$$P=V^2/R\quad\rightarrow\quad\frac{1}{R}=\frac{P}{V^2}$$

$$R_{eq}=\left(\frac{1}{R_{75}}+\frac{1}{R_{40}}\right)^{-1}=\left(\frac{75\,\text{W}}{(110\,\text{V})^2}+\frac{25\,\text{W}}{(110\,\text{V})^2}\right)^{-1}=121\,\Omega\approx\boxed{120\,\Omega}$$

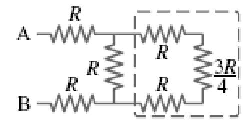
18. (*a*) The three resistors on the far right are in series, so their equivalent resistance is 3*R*. That combination is in parallel with the next resistor to the left, as shown in the dashed box in the second figure. The equivalent resistance of the dashed box is found as follows.




$$R_{eq1}=\left(\frac{1}{R}+\frac{1}{3R}\right)^{-1}=\tfrac{3}{4}R$$

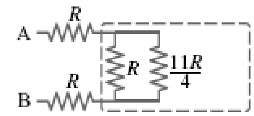
This equivalent resistance of $\frac{3}{4}R$ is in series with the next two resistors, as shown in the dashed box in the third figure (on the next page). The equivalent resistance of that dashed box is $R_{eq2}=2R+\frac{3}{4}R=\frac{11}{4}R.$ This $\frac{11}{4}R$ is in parallel with the next resistor to the left, as shown in the fourth figure. The equivalent resistance of that dashed box is found as follows.



(c)

$$R_{\text{eq2}} = \left(\frac{1}{R} + \frac{4}{11R}\right)^{-1} = \tfrac{11}{15}R.$$

This is in series with the last two resistors, the ones connected directly to A and B. The final equivalent resistance is given below.



(d)

$$R_{\text{eq}} = 2R + \tfrac{11}{15}R = \tfrac{41}{15}R = \tfrac{41}{15}(125\,\Omega) = 341.67\,\Omega \approx \boxed{342\,\Omega}$$

(*b*) The current flowing from the battery is found from Ohm's law.

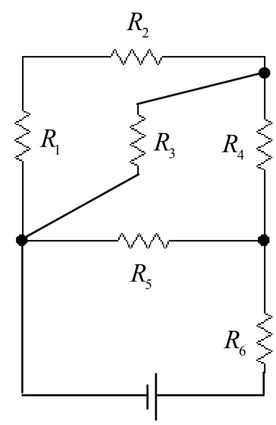
$$I_{\text{total}} = \frac{V}{R_{\text{eq}}} = \frac{50.0\,\text{V}}{341.67\,\Omega} = 0.1463\,\text{A} \approx \boxed{0.146\,\text{A}}$$

This is the current in the top and bottom resistors. There will be less current in the next resistor because the current splits, with some current passing through the resistor in question, and the rest of the current passing through the equivalent resistance of $\tfrac{11}{4}R$, as shown in the last figure. The voltage across $R$ and across $\tfrac{11}{4}R$ must be the same, since they are in parallel. Use this to find the desired current.

$$V_R = V_{\frac{11}{4}R} \;\rightarrow\; I_R R = I_{\frac{11}{4}R}\left(\tfrac{11}{4}R\right) = \left(I_{\text{total}} - I_R\right)\left(\tfrac{11}{4}R\right) \;\rightarrow$$

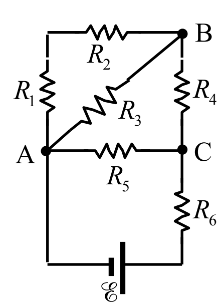
$$I_R = \tfrac{11}{15}I_{\text{total}} = \tfrac{11}{15}(0.1463\,\text{A})\,I_{\text{total}} = \boxed{0.107\,\text{A}}$$

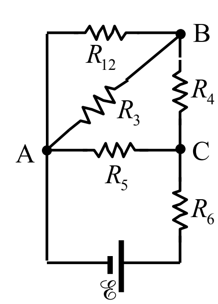
19. The resistors have been numbered in the accompanying diagram to help in the analysis. $R_1$ and $R_2$ are in series with an equivalent resistance of $R_{12} = R + R = 2R$. This combination is in parallel with $R_3$, with an equivalent resistance of $R_{123} = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \tfrac{2}{3}R$. This combination is in series with $R_4$, with an equivalent resistance of $R_{1234} = \tfrac{2}{3}R + R = \tfrac{5}{3}R$. This combination is in parallel with $R_5$, with an equivalent resistance of $R_{12345} = \left(\frac{1}{R} + \frac{3}{5R}\right)^{-1} = \tfrac{5}{8}R$. Finally, this combination is in series with $R_6$, and we calculate the final equivalent resistance.




$$R_{\text{eq}} = \tfrac{5}{8}R + R = \boxed{\tfrac{13}{8}R}$$

20. We reduce the circuit to a single loop by combining series and parallel combinations. We label a combined resistance with the subscripts of the resistors used in the combination. See the successive diagrams.



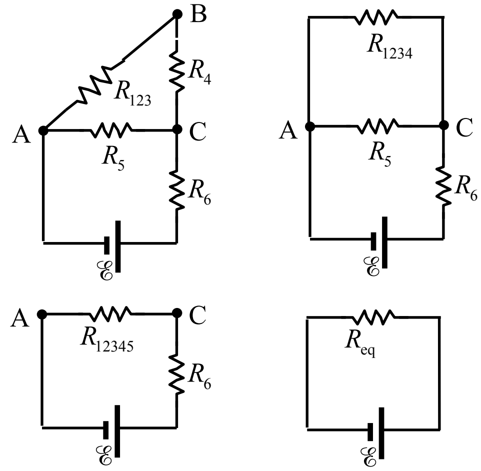




$R_1$ and $R_2$ are in series.

$$R_{12} = R_1 + R_2 = R + R = 2R$$

$R_{12}$ and $R_3$ are in parallel.

$$R_{123} = \left(\frac{1}{R_{12}} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{2R} + \frac{1}{R}\right)^{-1} = \tfrac{2}{3}R$$




$R_{123}$ and $R_4$ are in series.

$$R_{1234} = R_{123} + R_4 = \tfrac{2}{3}R + R = \tfrac{5}{3}R$$

$R_{1234}$ and $R_5$ are in parallel.

$$R_{12345} = \left(\frac{1}{R_{1234}} + \frac{1}{R_5}\right)^{-1} = \left(\frac{1}{\frac{5}{3}R} + \frac{1}{R}\right)^{-1} = \tfrac{5}{8}R$$

$R_{12345}$ and $R_6$ are in series, producing the equivalent resistance.

$$R_{eq} = R_{12345} + R_6 = \tfrac{5}{8}R + R = \tfrac{13}{8}R$$

Now work "backwards" from the simplified circuit. Resistors in series have the same current as their equivalent resistance, and resistors in parallel have the same voltage as their equivalent resistance. To avoid rounding errors, we do not use numeric values until the end of the problem.

$$I_{eq} = \frac{\mathscr{E}}{R_{eq}} = \frac{\mathscr{E}}{\frac{13}{8}R} = \boxed{\frac{8\mathscr{E}}{13R} = I_6} = I_{12345}$$

$$V_5 = V_{1234} = V_{12345} = I_{12345}R_{12345} = \left(\frac{8\mathscr{E}}{13R}\right)\left(\tfrac{5}{8}R\right) = \tfrac{5}{13}\mathscr{E} \ ; \ I_5 = \frac{V_5}{R_5} = \frac{\frac{5}{13}\mathscr{E}}{R} = \boxed{\frac{5\mathscr{E}}{13R} = I_5}$$

$$I_{1234} = \frac{V_{1234}}{R_{1234}} = \frac{\frac{5}{13}\mathscr{E}}{\frac{5}{3}R} = \boxed{\frac{3\mathscr{E}}{13R} = I_4} = I_{123} \ ; \ V_{123} = I_{123}R_{123} = \left(\frac{3\mathscr{E}}{13R}\right)\left(\tfrac{2}{3}R\right) = \tfrac{2}{13}\mathscr{E} = V_{12} = V_3$$

$$I_3 = \frac{V_3}{R_3} = \boxed{\frac{2\mathscr{E}}{13R} = I_3} \ ; \ I_{12} = \frac{V_{12}}{R_{12}} = \frac{\frac{2}{13}\mathscr{E}}{2R} = \boxed{\frac{\mathscr{E}}{13R} = I_1 = I_2}$$

Now substitute in numeric values.

$$I_1 = I_2 = \frac{\mathscr{E}}{13R} = \frac{12.0\,\text{V}}{13(1.20\,\text{k}\Omega)} = \boxed{0.77\,\text{mA}} \ ; \ I_3 = \frac{2\mathscr{E}}{13R} = \boxed{1.54\,\text{mA}} \ ; I_4 = \frac{3\mathscr{E}}{13R} = \boxed{2.31\,\text{mA}} \ ;$$

$$I_5 = \frac{5\mathscr{E}}{13R} = \boxed{3.85\,\text{mA}} \ ; \ I_6 = \frac{8\mathscr{E}}{13R} = \boxed{6.15\,\text{mA}} \ ; \ V_{AB} = V_3 = \tfrac{2}{13}\mathscr{E} = \boxed{1.85\,\text{V}}$$

21. The resistors *r* and *R* are in series, so the equivalent resistance of the circuit is $R + r$ and the current in the resistors is $I = \dfrac{\mathscr{E}}{R+r}$. The power delivered to load resistor is found from Eq. 25-7a. To find the value of *R* that maximizes this delivered power, set $\dfrac{dP}{dR} = 0$ and solve for *R*.

$$P = I^2R = \left(\frac{\mathscr{E}}{R+r}\right)R = \frac{\mathscr{E}^2 R}{(R+r)^2} \ ; \ \frac{dP}{dR} = \mathscr{E}^2\left[\frac{(R+r)^2 - R(2)(R+r)}{(R+r)^4}\right] = 0 \ \rightarrow$$

$$(R+r)^2 - R(2)(R+r) = 0 \ \rightarrow \ R^2 + 2Rr + r^2 - 2R^2 - 2Rr = 0 \ \rightarrow \ \boxed{R = r}$$

22. It is given that the power used when the resistors are in series is one-fourth the power used when the resistors are in parallel. The voltage is the same in both cases. Use Eq. 25-7b, along with the definitions of series and parallel equivalent resistance.

$$P_{\text{series}} = \tfrac{1}{4}P_{\text{parallel}} \rightarrow \frac{V^2}{R_{\text{series}}} = \tfrac{1}{4}\frac{V^2}{R_{\text{parallel}}} \rightarrow R_{\text{series}} = 4R_{\text{parallel}} \rightarrow (R_1 + R_2) = 4\frac{R_1R_2}{(R_1+R_2)} \rightarrow$$

$$(R_1+R_2)^2 = 4R_1R_2 \rightarrow R_1^2 + 2R_1R_2 + R_2^2 - 4R_1R_2 = 0 = (R_1 - R_2)^2 \rightarrow R_1 = R_2$$

Thus the two resistors must be the same, and so the "other" resistor is $\boxed{3.8\,\text{k}\Omega}$.

23. We label identical resistors from left to right as $R_{\text{left}}$, $R_{\text{middle}}$, and $R_{\text{right}}$. When the switch is opened, the equivalent resistance of the circuit increases from $\tfrac{3}{2}R + r$ to $2R + r$. Thus the current delivered by the battery decreases, from $\dfrac{\mathcal{E}}{\tfrac{3}{2}R + r}$ to $\dfrac{\mathcal{E}}{2R + r}$. Note that this is LESS than a 50% decrease.

(*a*) Because the current from the battery has decreased, the voltage drop across $R_{\text{left}}$ will decrease, since it will have less current than before. The voltage drop across $R_{\text{right}}$ decreases to 0, since no current is flowing in it. The voltage drop across $R_{\text{middle}}$ will increase, because even though the total current has decreased, the current flowing through $R_{\text{middle}}$ has increased since before the switch was opened, only half the total current was flowing through $R_{\text{middle}}$.

$\boxed{V_{\text{left}} \text{ decreases ; } V_{\text{middle}} \text{ increases ; } V_{\text{right}} \text{ goes to 0}}$.

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance.

$\boxed{I_{\text{left}} \text{ decreases ; } I_{\text{middle}} \text{ increases ; } I_{\text{right}} \text{ goes to 0}}$

(*c*) Since the current from the battery has decreased, the voltage drop across *r* will decrease, and thus the $\boxed{\text{terminal voltage increases}}$.

(*d*) With the switch closed, the equivalent resistance is $\tfrac{3}{2}R + r$. Thus the current in the circuit is $I_{\text{closed}} = \dfrac{\mathcal{E}}{\tfrac{3}{2}R + r}$, and the terminal voltage is given by Eq. 26-1.

$$V_{\substack{\text{terminal}\\\text{closed}}} = \mathcal{E} - I_{\text{closed}}r = \mathcal{E} - \frac{\mathcal{E}}{\tfrac{3}{2}R + r}r = \mathcal{E}\left(1 - \frac{r}{\tfrac{3}{2}R + r}\right) = (9.0\,\text{V})\left(1 - \frac{0.50\,\Omega}{\tfrac{3}{2}(5.50\,\Omega) + 0.50\,\Omega}\right)$$

$$= 8.486\,\text{V} \approx \boxed{8.5\,\text{V}}$$

(*e*) With the switch open, the equivalent resistance is $2R + r$. Thus the current in the circuit is $I_{\text{closed}} = \dfrac{\mathcal{E}}{2R + r}$, and again the terminal voltage is given by Eq. 26-1.

$$V_{\substack{\text{terminal}\\\text{closed}}} = \mathcal{E} - I_{\text{closed}}r = \mathcal{E} - \frac{\mathcal{E}}{2R + r}r = \mathcal{E}\left(1 - \frac{r}{2R + r}\right) = (9.0\,\text{V})\left(1 - \frac{0.50\,\Omega}{2(5.50\,\Omega) + 0.50\,\Omega}\right)$$

$$= 8.609\,\text{V} \approx \boxed{8.6\,\text{V}}$$

24. Find the maximum current and resulting voltage for each resistor under the power restriction.

$$P = I^2R = \frac{V^2}{R} \rightarrow I = \sqrt{\frac{P}{R}}, \; V = \sqrt{RP}$$

$$I_{1800} = \sqrt{\frac{0.5\,\text{W}}{1.8\times10^3\,\Omega}} = 0.0167\,\text{A} \qquad V_{1800} = \sqrt{(0.5\,\text{W})(1.8\times10^3\,\Omega)} = 30.0\,\text{V}$$

$$I_{2800} = \sqrt{\frac{0.5\,\text{W}}{2.8\times 10^3\,\Omega}} = 0.0134\,\text{A} \qquad V_{2800} = \sqrt{(0.5\,\text{W})(2.8\times 10^3\,\Omega)} = 37.4\,\text{V}$$

$$I_{3700} = \sqrt{\frac{0.5\,\text{W}}{3.7\times 10^3\,\Omega}} = 0.0116\,\text{A} \qquad V_{3700} = \sqrt{(0.5\,\text{W})(3.7\times 10^3\,\Omega)} = 43.0\,\text{V}$$

The parallel resistors have to have the same voltage, and so the voltage across that combination is limited to 37.4 V. That would require a current given by Ohm's law and the parallel combination of the two resistors.

$$I_{\text{parallel}} = \frac{V_{\text{parallel}}}{R_{\text{parallel}}} = V_{\text{parallel}}\left(\frac{1}{R_{2800}} + \frac{1}{R_{2100}}\right) = (37.4\,\text{V})\left(\frac{1}{2800\,\Omega} + \frac{1}{3700\,\Omega}\right) = 0.0235\,\text{A}$$

This is more than the maximum current that can be in $R_{1800}$. Thus the maximum current that $R_{1800}$ can carry, $0.0167\,\text{A}$, is the maximum current for the circuit. The maximum voltage that can be applied across the combination is the maximum current times the equivalent resistance. The equivalent resistance is the parallel combination of $R_{2800}$ and $R_{3700}$ added to $R_{1800}$.

$$V_{\max} = I_{\max}R_{\text{eq}} = I_{\max}\left[R_{1800} + \left(\frac{1}{R_{2800}} + \frac{1}{R_{3700}}\right)^{-1}\right] = (0.0167\,\text{A})\left[1800\,\Omega + \left(\frac{1}{2800\,\Omega} + \frac{1}{3700\,\Omega}\right)^{-1}\right]$$

$$= 56.68\text{V} \approx \boxed{57\,\text{V}}$$

25. (*a*) Note that adding resistors in series always results in a larger resistance, and adding resistors in parallel always results in a smaller resistance. Closing the switch adds another resistor in parallel with $R_3$ and $R_4$, which lowers the net resistance of the parallel portion of the circuit, and thus lowers the equivalent resistance of the circuit. That means that more current will be delivered by the battery. Since $R_1$ is in series with the battery, its voltage will increase. Because of that increase, the voltage across $R_3$ and $R_4$ must decrease so that the total voltage drops around the loop are equal to the battery voltage. Since there was no voltage across $R_2$ until the switch was closed, its voltage will increase. To summarize:

$$\boxed{V_1 \text{ and } V_2 \text{ increase}\,;\, V_3 \text{ and } V_4 \text{ decrease}}$$

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance. Thus

$$\boxed{I_1 \text{ and } I_2 \text{ increase}\,;\, I_3 \text{ and } I_4 \text{ decrease}}$$

(*c*) Since the battery voltage does not change and the current delivered by the battery increases, the power delivered by the battery, found by multiplying the voltage of the battery by the current delivered, $\boxed{\text{increases}}$.

(*d*) Before the switch is closed, the equivalent resistance is $R_3$ and $R_4$ in parallel, combined with $R_1$ in series.

$$R_{\text{eq}} = R_1 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 125\,\Omega + \left(\frac{2}{125\,\Omega}\right)^{-1} = 187.5\,\Omega$$

The current delivered by the battery is the same as the current through $R_1$.

$$I_{\text{total}} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{22.0\,\text{V}}{187.5\,\Omega} = 0.1173\,\text{A} = I_1$$

The voltage across $R_1$ is found by Ohm's law.

$$V_1 = IR_1 = (0.1173\,\text{A})(125\,\Omega) = 14.66\,\text{V}$$

The voltage across the parallel resistors is the battery voltage less the voltage across $R_1$.

$$V_\text{p} = V_\text{battery} - V_1 = 22.0\,\text{V} - 14.66\,\text{V} = 7.34\,\text{V}$$

The current through each of the parallel resistors is found from Ohm's law.

$$I_3 = \frac{V_\text{p}}{R_2} = \frac{7.34\,\text{V}}{125\,\Omega} = 0.0587\,\text{A} = I_4$$

Notice that the current through each of the parallel resistors is half of the total current, within the limits of significant figures. The currents before closing the switch are as follows.

$$\boxed{I_1 = 0.117\,\text{A} \quad I_3 = I_4 = 0.059\,\text{A}}$$

After the switch is closed, the equivalent resistance is $R_2$, $R_3$, and $R_4$ in parallel, combined with $R_1$ in series. Do a similar analysis.

$$R_\text{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 125\,\Omega + \left(\frac{3}{125\,\Omega}\right)^{-1} = 166.7\,\Omega$$

$$I_\text{total} = \frac{V_\text{battery}}{R_\text{eq}} = \frac{22.0\,\text{V}}{166.7\,\Omega} = 0.1320\,\text{A} = I_1 \qquad V_1 = IR_1 = (0.1320\,\text{A})(125\,\Omega) = 16.5\,\text{V}$$

$$V_\text{p} = V_\text{battery} - V_1 = 22.0\,\text{V} - 16.5\,\text{V} = 5.5\,\text{V} \qquad I_2 = \frac{V_\text{p}}{R_2} = \frac{5.5\,\text{V}}{125\,\Omega} = 0.044\,\text{A} = I_3 = I_4$$

Notice that the current through each of the parallel resistors is one third of the total current, within the limits of significant figures. The currents after closing the switch are as follows.

$$\boxed{I_1 = 0.132\,\text{A} \quad I_2 = I_3 = I_4 = 0.044\,\text{A}}$$

**Yes**, the predictions made in part (*b*) are all confirmed.

26. The goal is to determine *r* so that $\left.\frac{dP_R}{dR}\right|_{R=R_0} = 0$. This ensures that *R* produce very little change in $P_R$, since $\Delta P_R \approx \frac{dP_R}{dR}\Delta R$. The power delivered to the heater can be found by $P_\text{heater} = V_\text{heater}^2 / R$, and so we need to determine the voltage across the heater. We do this by calculating the current drawn from the voltage source, and then subtracting the voltage drop across *r* from the source voltage.

$$R_\text{eq} = r + \frac{Rr}{R+r} = \frac{2Rr + r^2}{R+r} = \frac{r(2R+r)}{R+r} \;;\; I_\text{total} = \frac{\mathscr{E}}{R_\text{eq}} = \frac{\mathscr{E}}{\dfrac{r(2R+r)}{R+r}} = \frac{\mathscr{E}(R+r)}{r(2R+r)}$$

$$V_\text{heater} = \mathscr{E} - I_\text{total}r = \mathscr{E} - \frac{\mathscr{E}(R+r)}{r(2R+r)}r = \mathscr{E} - \frac{\mathscr{E}(R+r)}{(2R+r)} = \frac{\mathscr{E}R}{(2R+r)} \;;\; P_\text{heater} = \frac{V_\text{heater}^2}{R} = \frac{\mathscr{E}^2 R}{(2R+r)^2}$$

$$\left.\frac{dP_\text{heater}}{dR}\right|_{R=R_0} = \mathscr{E}^2 \frac{(2R_0+r)^2 - R_0(2)(2R_0+r)(2)}{(2R_0+r)^4} = 0 \;\rightarrow\; (2R_0+r)^2 - R_0(2)(2R_0+r)(2) = 0 \;\rightarrow$$

$$4R_0^2 + 4R_0 r + r^2 - 8R_0^2 - 4R_0 r = 0 \;\rightarrow\; r^2 = 4R_0^2 \;\rightarrow\; \boxed{r = 2R_0}$$

27. All of the resistors are in series, so the equivalent resistance is just the sum of the resistors. Use Ohm's law then to find the current, and show all voltage changes starting at the negative pole of the battery and going counterclockwise.

$$I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{9.0\,\text{V}}{(9.5 + 12.0 + 2.0)\,\Omega} = 0.383\,\text{A} \approx \boxed{0.38\,\text{A}}$$

$$\sum \text{voltages} = 9.0\,\text{V} - (9.5\,\Omega)(0.383\,\text{A}) - (12.0\,\Omega)(0.383\,\text{A}) - (2.0\,\Omega)(0.383\,\text{A})$$

$$= 9.0\,\text{V} - 3.638\,\text{V} - 4.596\,\text{V} - 0.766\,\text{V} = \boxed{0.00\,\text{V}}$$

28. Apply Kirchhoff's loop rule to the circuit starting at the upper left corner of the circuit diagram, in order to calculate the current. Assume that the current is flowing clockwise.

$$-I(2.0\,\Omega) + 18\,\text{V} - I(6.6\,\Omega) - 12\,\text{V} - I(1.0\,\Omega) = 0 \;\rightarrow\; I = \frac{6\,\text{V}}{9.6\,\Omega} = 0.625\text{A}$$

The terminal voltage for each battery is found by summing the potential differences across the internal resistance and EMF from left to right. Note that for the 12 V battery, there is a voltage gain going across the internal resistance from left to right.

$$18\,\text{V battery: } V_{\text{terminal}} = -I(2.0\,\Omega) + 18\,\text{V} = -(0.625\,\text{A})(2.0\,\Omega) + 18\,\text{V} = 16.75\text{V} \approx \boxed{17\,\text{V}}$$

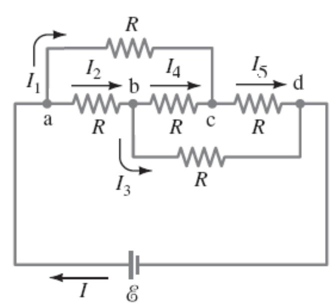
$$12\,\text{V battery: } V_{\text{terminal}} = I(1.0\,\Omega) + 12\,\text{V} = (0.625\,\text{A})(1.0\,\Omega) + 12\,\text{V} = 12.625\,\text{V} \approx \boxed{13\,\text{V}}$$

29. To find the potential difference between points a and b, the current must be found from Kirchhoff's loop law. Start at point a and go counterclockwise around the entire circuit, taking the current to be counterclockwise.

$$-IR + \mathcal{E} - IR - IR + \mathcal{E} - IR = 0 \;\rightarrow\; I = \frac{\mathcal{E}}{2R}$$

$$V_{\text{ab}} = V_{\text{a}} - V_{\text{b}} = -IR + \mathcal{E} - IR = \mathcal{E} - 2IR = \mathcal{E} - 2\frac{\mathcal{E}}{2R}R = \boxed{0\,\text{V}}$$

30. (*a*) We label each of the currents as shown in the accompanying figure. Using Kirchhoff's junction rule and the first three junctions (a-c) we write equations relating the entering and exiting currents.




$I = I_1 + I_2$ [1]

$I_2 = I_3 + I_4$ [2]

$I_1 + I_4 = I_5$ [3]

We use Kirchhoff's loop rule to write equations for loops abca, abcda, and bdcb.

$0 = -I_2R - I_4R + I_1R$ [4]

$0 = -I_2R - I_3R + \mathcal{E}$ [5]

$0 = -I_3R + I_5R + I_4R$ [6]

We have six unknown currents and six equations. We solve these equations by substitution. First, insert Eq. [3] into [6] to eliminate current $I_5$. Next insert Eq. [2] into Eqs. [1], [4], and [5] to eliminate $I_2$.

$$0=-I_3R+(I_1+I_4)R+I_4R \quad \rightarrow 0=-I_3R+I_1R+2I_4R \qquad [6^*]$$
$$I=I_1+I_3+I_4 \qquad [1^*]$$
$$0=-(I_3+I_4)R-I_4R+I_1R \rightarrow 0=-I_3R-2I_4\text{R}+I_1R \qquad [4^*]$$
$$0=-(I_3+I_4)R-I_3R+\mathcal{E} \rightarrow 0=-I_4R-2I_3R+\mathcal{E} \qquad [5^*]$$

Next we solve Eq. [4*] for $I_4$ and insert the result into Eqs. [1*], [5*], and [6*].

$$0=-I_3R-2I_4R+I_1R \rightarrow I_4=\tfrac{1}{2}I_1-\tfrac{1}{2}I_3$$
$$I=I_1+I_3+\tfrac{1}{2}I_1-\tfrac{1}{2}I_3 \rightarrow I=\tfrac{3}{2}I_1+\tfrac{1}{2}I_3 \qquad [1^{**}]$$
$$0=-I_3R+I_1R+2\left(\tfrac{1}{2}I_1-\tfrac{1}{2}I_3\right)R=-2I_3R+2I_1R \rightarrow I_1=I_3 \qquad [6^{**}]$$
$$0=-\left(\tfrac{1}{2}I_1-\tfrac{1}{2}I_3\right)R-2I_3R+\mathcal{E} \rightarrow 0=-\tfrac{1}{2}I_1R-\tfrac{3}{2}I_3R+\mathcal{E} \qquad [5^{**}]$$

Finally we substitute Eq. [6**] into Eq [5**] and solve for $I_1$. We insert this result into Eq. [1**] to write an equation for the current through the battery in terms of the battery emf and resistance.

$$0=-\tfrac{1}{2}I_1R-\tfrac{3}{2}I_1R+\mathcal{E} \rightarrow I_1=\frac{\mathcal{E}}{2R} \;;\; I=\tfrac{3}{2}I_1+\tfrac{1}{2}I_1 \;=2I_1 \rightarrow \boxed{I=\frac{\mathcal{E}}{R}}$$

(*b*) We divide the battery emf by the current to determine the effective resistance.

$$R_{eq}=\frac{\mathcal{E}}{I}=\frac{\mathcal{E}}{\mathcal{E}/R}=\boxed{R}$$

31. This circuit is identical to Example 26-9 and Figure 26-13 except for the numeric values. So we may copy the same equations as developed in that Example, but using the current values.

Eq. (*a*): $I_3=I_1+I_2$ ; Eq. (*b*): $-34I_1+45-48I_3=0$

Eq. (*c*): $-34I_1+19I_2-75=0$ Eq. (*d*): $I_2=\dfrac{75+34I_1}{19}=3.95+1.79I_1$

Eq. (*e*): $I_3=\dfrac{45-34I_1}{48}=0.938-0.708I_1$

$$I_3=I_1+I_2 \quad \rightarrow \quad 0.938-0.708I_1=I_1+3.95+1.79I_1 \quad \rightarrow \quad I_1=-0.861\,\text{A}$$
$$I_2=3.95+1.79I_1=2.41\,\text{A} \;;\; I_3=0.938-0.708I_1=1.55\,\text{A}$$

(*a*) To find the potential difference between points a and d, start at point a and add each individual potential difference until reaching point d. The simplest way to do this is along the top branch.

$$V_{\text{ad}}=V_{\text{d}}-V_{\text{a}}=-I_1(34\,\Omega)=-(-0.861\,\text{A})(34\,\Omega)=29.27\,\text{V}\approx\boxed{29\,\text{V}}$$

Slight differences will be obtained in the final answer depending on the branch used, due to rounding. For example, using the bottom branch, we get the following.

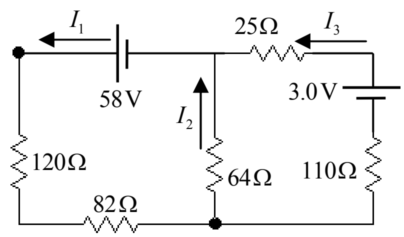
$$V_{\text{ad}}=V_{\text{d}}-V_{\text{a}}=\mathcal{E}_1-I_2(19\,\Omega)=75\,\text{V}-(2.41\,\text{A})(19\,\Omega)=29.21\,\text{V}\approx29\,\text{V}$$

(*b*) For the 75-V battery, the terminal voltage is the potential difference from point g to point e. For the 45-V battery, the terminal voltage is the potential difference from point d to point b.

75 V battery: $V_{\text{terminal}}=\mathcal{E}_1-I_2r=75\,\text{V}-(2.41\,\text{A})(1.0\,\Omega)=\boxed{73\,\text{V}}$

45 V battery: $V_{\text{terminal}}=\mathcal{E}_2-I_3r=45\,\text{V}-(1.55\,\text{A})(1.0\,\Omega)=\boxed{43\,\text{V}}$

32. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches at the top center of the circuit.




$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the left loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$58\,\text{V} - I_1(120\,\Omega) - I_1(82\,\Omega) - I_2(64\,\Omega) = 0 \;\rightarrow\; 58 = 202I_1 + 64I_2$$

The final equation comes from Kirchhoff's loop rule applied to the right loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$3.0\,\text{V} - I_3(25\,\Omega) + I_2(64\,\Omega) - I_3(110\,\Omega) = 0 \;\rightarrow\; 3 = -64I_2 + 135I_3$$

Substitute $I_1 = I_2 + I_3$ into the left loop equation, so that there are two equations with two unknowns.

$$58 = 202(I_2 + I_3) + 64I_2 = 266I_2 + 202I_3$$

Solve the right loop equation for $I_2$ and substitute into the left loop equation, resulting in an equation with only one unknown, which can be solved.

$$3 = -64I_2 + 135I_3 \;\rightarrow\; I_2 = \frac{135I_3 - 3}{64} \;;\; 58 = 266I_2 + 202I_3 = 266\left(\frac{135I_3 - 3}{64}\right) + 202I_3 \;\rightarrow$$

$$I_3 = 0.09235\,\text{A} \;;\; I_2 = \frac{135I_3 - 3}{64} = 0.1479\,\text{A} \;;\; I_1 = I_2 + I_3 = 0.24025\,\text{A}$$

The current in each resistor is as follows:

$$\boxed{120\,\Omega\text{: } 0.24\,\text{A} \quad 82\,\Omega\text{: } 0.24\,\text{A} \quad 64\,\Omega\text{: } 0.15\,\text{A} \quad 25\,\Omega\text{: } 0.092\,\text{A} \quad 110\,\Omega\text{: } 0.092\,\text{A}}$$

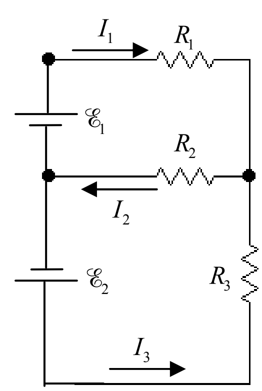
33. Because there are no resistors in the bottom branch, it is possible to write Kirchhoff loop equations that only have one current term, making them easier to solve. To find the current through $R_1$, go around the outer loop counterclockwise, starting at the lower left corner.

$$V_3 - I_1R_1 + V_1 = 0 \;\rightarrow\; I_1 = \frac{V_3 + V_1}{R_1} = \frac{6.0\,\text{V} + 9.0\,\text{V}}{22\,\Omega} = \boxed{0.68\,\text{A, left}}$$

To find the current through $R_2$, go around the lower loop counterclockwise, starting at the lower left corner.

$$V_3 - I_2R_2 = 0 \;\rightarrow\; I_2 = \frac{V_3}{R_2} = \frac{6.0\,\text{V}}{18\,\Omega} = \boxed{0.33\,\text{A, left}}$$

34. (a) There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.




$$I_2 = I_1 + I_3 \;\rightarrow\; I_1 = I_2 - I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$\mathcal{E}_1 - I_1R_1 - I_2R_2 = 0 \;\rightarrow\; 9 = 25I_1 + 48I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$\mathcal{E}_2 - I_3R_3 - I_2R_2 = 0 \ \rightarrow \ 12 = 35I_3 + 48I_2$$

Substitute $I_1 = I_2 - I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$9 = 25I_1 + 48I_2 = 25(I_2 - I_3) + 48I_2 = 73I_2 - 25I_3 \ ; \ 12 = 35I_3 + 48I_2$$

Solve the bottom loop equation for $I_2$ and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$12 = 35I_3 + 48I_2 \ \rightarrow \ I_2 = \frac{12 - 35I_3}{48}$$

$$9 = 73I_2 - 25I_3 = 73\left(\frac{12 - 35I_3}{48}\right) - 25I_3 \ \rightarrow \ 432 = 876 - 2555I_3 - 1200I_3 \ \rightarrow$$

$$I_3 = \frac{444}{3755} = 0.1182\,\text{A} \approx \boxed{0.12\,\text{A}\text{, up}} \ ; \ I_2 = \frac{12 - 35I_3}{48} = 0.1638\,\text{A} \approx \boxed{0.16\,\text{A}\text{, left}}$$

$$I_1 = I_2 - I_3 = 0.0456\,\text{A} \approx \boxed{0.046\,\text{A}\text{, right}}$$

(*b*) We can include the internal resistances simply by adding $1.0\,\Omega$ to $R_1$ and $R_3$. So let $R_1 = 26\,\Omega$ and let $R_3 = 36\,\Omega$. Now re-work the problem exactly as in part (*a*).

$$I_2 = I_1 + I_3 \ \rightarrow \ I_1 = I_2 - I_3$$

$$\mathcal{E}_1 - I_1R_1 - I_2R_2 = 0 \ \rightarrow \ 9 = 26I_1 + 48I_2$$

$$\mathcal{E}_2 - I_3R_3 - I_2R_2 = 0 \ \rightarrow \ 12 = 36I_3 + 48I_2$$

$$9 = 26I_1 + 48I_2 = 26(I_2 - I_3) + 48I_2 = 74I_2 - 26I_3 \ ; \ 12 = 36I_3 + 48I_2$$

$$12 = 36I_3 + 48I_2 \ \rightarrow \ I_2 = \frac{12 - 36I_3}{48} = \frac{1 - 3I_3}{4}$$

$$9 = 74I_2 - 26I_3 = 74\left(\frac{1 - 3I_3}{4}\right) - 26I_3 \ \rightarrow \ 36 = 74 - 222I_3 - 104I_3 \ \rightarrow$$

$$I_3 = \frac{38}{326} = 0.1166\,\text{A} \approx \boxed{0.12\,\text{A}\text{, up}} \ ; \ I_2 = \frac{1 - 3I_3}{4} = 0.1626\,\text{A} \approx \boxed{0.16\,\text{A, left}}$$

$$I_1 = I_2 - I_3 = \boxed{0.046\,\text{A}\text{, right}}$$

The currents are unchanged to 2 significant figures by the inclusion of the internal resistances.

35. We are to find the ratio of the power used when the resistors are in series, to the power used when the resistors are in parallel. The voltage is the same in both cases. Use Eq. 25-7b, along with the definitions of series and parallel equivalent resistance.

$$R_{\text{series}} = R_1 + R_2 + \cdots R_n = nR \ ; \ R_{\text{parallel}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}\right)^{-1} = \left(\frac{n}{R}\right)^{-1} = \frac{R}{n}$$

$$\frac{P_{\text{series}}}{P_{\text{parallel}}} = \frac{V^2/R_{\text{series}}}{V^2/R_{\text{parallel}}} = \frac{R_{\text{parallel}}}{R_{\text{series}}} = \frac{R/n}{nR} = \boxed{\frac{1}{n^2}}$$

36. (*a*) Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.

$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery, and progressing counterclockwise. We add series resistances.

$$12.0\,\text{V} - I_2(12\,\Omega) + 12.0\,\text{V} - I_1(35\,\Omega) = 0 \;\rightarrow\; 24 = 35I_1 + 12I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery, and progressing clockwise.

$$12.0\,\text{V} - I_2(12\,\Omega) - 6.0\,\text{V} + I_3(34\,\Omega) = 0 \;\rightarrow\; 6 = 12I_2 - 34I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$24 = 35I_1 + 12I_2 = 35(I_2 + I_3) + 12I_2 = 47I_2 + 35I_3$$

Solve the bottom loop equation for $I_2$ and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved for $I_3$.

$$6 = 12I_2 - 34I_3 \;\rightarrow\; I_2 = \frac{6 + 34I_3}{12} \;;\; 24 = 47I_2 + 35I_3 = 47\left(\frac{6 + 34I_3}{12}\right) + 35I_3 \;\rightarrow$$

$$I_3 = \boxed{2.97\,\text{mA}} \;;\; I_2 = \frac{6 + 34I_3}{12} = \boxed{0.508\,\text{A}} \;;\; I_1 = I_2 + I_3 = \boxed{0.511\,\text{A}}$$

(*b*) The terminal voltage of the 6.0-V battery is $6.0\,\text{V} - I_3 r = 6.0\,\text{V} - (2.97\times10^{-3}\,\text{A})(1.0\,\Omega)$

$= 5.997\,\text{V} \approx \boxed{6.0\,\text{V}}$.

37. This problem is the same as Problem 36, except the total resistance in the top branch is now $23\,\Omega$ instead of $35\,\Omega$. We simply reproduce the adjusted equations here without the prose.

$$I_1 = I_2 + I_3$$

$$12.0\,\text{V} - I_2(12\,\Omega) + 12.0\,\text{V} - I_1(23\,\Omega) = 0 \;\rightarrow\; 24 = 23I_1 + 12I_2$$

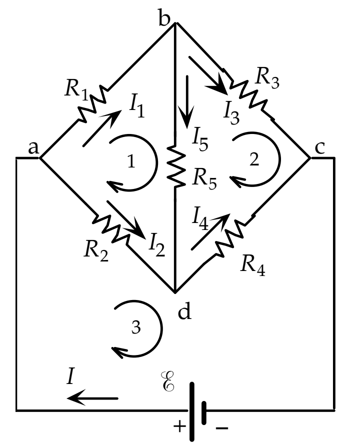
$$12.0\,\text{V} - I_2(12\,\Omega) - 6.0\,\text{V} + I_3(34\,\Omega) = 0 \;\rightarrow\; 6 = 12I_2 - 34I_3$$

$$24 = 23I_1 + 12I_2 = 23(I_2 + I_3) + 12I_2 = 35I_2 + 23I_3$$

$$6 = 12I_2 - 34I_3 \;\rightarrow\; I_2 = \frac{6 + 34I_3}{12} \;;\; 24 = 35I_2 + 23I_3 = 35\left(\frac{6 + 34I_3}{12}\right) + 23I_3 \;\rightarrow$$

$$I_3 = 0.0532\,\text{A} \;;\; I_2 = \frac{6 + 34I_3}{12} = 0.6508\,\text{A} \;;\; I_1 = I_2 + I_3 = 0.704\,\text{A} \approx \boxed{0.70\,\text{A}}$$

38. The circuit diagram has been labeled with six different currents. We apply the junction rule to junctions a, b, and c. We apply the loop rule to the three loops labeled in the diagram.




$$1)\ I = I_1 + I_2\ \ ;\ \ 2)\ I_1 = I_3 + I_5\ \ ;\ \ 3)\ I_3 + I_4 = I$$

$$4)\ -I_1R_1 - I_5R_5 + I_2R_2 = 0\ \ ;\ \ 5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$

$$6)\ \mathcal{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate $I$ using equations 1) and 3).

$$1)\ I_3 + I_4 = I_1 + I_2\ \ ;\ \ 2)\ I_1 = I_3 + I_5$$

$$4)\ -I_1R_1 - I_5R_5 + I_2R_2 = 0\ \ ;\ \ 5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$

$$6)\ \mathcal{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate $I_1$ using equation 2.

$$1)\ I_3 + I_4 = I_3 + I_5 + I_2 \ \rightarrow\ I_4 = I_5 + I_2$$

$$4)\ -(I_3 + I_5)R_1 - I_5R_5 + I_2R_2 = 0 \ \rightarrow\ -I_3R_1 - I_5(R_1 + R_5) + I_2R_2 = 0$$

$$5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$

$$6)\ \mathcal{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate $I_4$ using equation 1.

$$4)\ -I_3R_1 - I_5(R_1 + R_5) + I_2R_2 = 0$$

$$5)\ -I_3R_3 + (I_5 + I_2)R_4 + I_5R_5 = 0 \ \rightarrow\ -I_3R_3 + I_5(R_4 + R_5) + I_2R_4 = 0$$

$$6)\ \mathcal{E} - I_2R_2 - (I_5 + I_2)R_4 = 0 \ \rightarrow\ \mathcal{E} - I_2(R_2 + R_4) - I_5R_4 = 0$$

Eliminate $I_2$ using equation 4: $I_2 = \frac{1}{R_2}\left[I_3R_1 + I_5(R_1 + R_5)\right]$.

$$5)\ -I_3R_3 + I_5(R_4 + R_5) + \frac{1}{R_2}\left[I_3R_1 + I_5(R_1 + R_5)\right]R_4 = 0 \ \rightarrow$$

$$I_3(R_1R_4 - R_2R_3) + I_5(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4) = 0$$

$$6)\ \mathcal{E} - \frac{1}{R_2}\left[I_3R_1 + I_5(R_1 + R_5)\right](R_2 + R_4) - I_5R_4 = 0 \ \rightarrow$$

$$\mathcal{E}R_2 - I_3R_1(R_2 + R_4) - I_5(R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4) = 0$$

Eliminate $I_3$ using equation 5: $I_3 = -I_5\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}$

$$\mathcal{E}R_2 + \left[I_5\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}\right]R_1(R_2 + R_4) - I_5(R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4) = 0$$

$$\mathcal{E} = -\frac{I_5}{R_2}\left\{\left[\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}\right]R_1(R_2 + R_4) - (R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4)\right\}$$

$$= -\frac{I_5}{25\Omega}\left\{\begin{matrix}\left[\frac{(25\Omega)(14\Omega) + (25\Omega)(15\Omega) + (22\Omega)(14\Omega) + (15\Omega)(14\Omega)}{(22\Omega)(14\Omega) - (25\Omega)(12\Omega)}\right](22\Omega)(25\Omega + 14\Omega) \\ -\left[(22\Omega)(25\Omega) + (22)(14) + (15\Omega)(25\Omega) + (15\Omega)(14\Omega) + (25\Omega)(14\Omega)\right]\end{matrix}\right\}$$

$$= -I_5(5261\,\Omega) \;\rightarrow\; I_5 = -\frac{6.0\,\text{V}}{5261\,\Omega} = -1.140\,\text{mA (upwards)}$$

$$I_3 = -I_5\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}$$

$$= -(-1.140\,\text{mA})\frac{(25\,\Omega)(14\,\Omega)+(25\,\Omega)(15\,\Omega)+(22\,\Omega)(14\,\Omega)+(15\,\Omega)(14\,\Omega)}{(22\,\Omega)(14\,\Omega)-(25\,\Omega)(12\,\Omega)} = 0.1771\,\text{A}$$

$$I_2 = \frac{1}{R_2}\left[I_3R_1 + I_5(R_1+R_5)\right] = \frac{1}{25\,\Omega}\left[(0.1771\,\text{A})(22\,\Omega)+(-0.00114\,\text{A})(37\,\Omega)\right] = 0.1542\,\text{A}$$

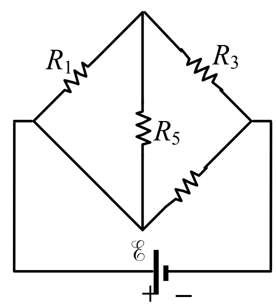
$$I_4 = I_5 + I_2 = -0.00114\,\text{A} + 0.1542\,\text{A} = 0.1531\,\text{A}$$

$$I_1 = I_3 + I_5 = 0.1771\,\text{A} - 0.00114\,\text{A} = 0.1760\,\text{A}$$

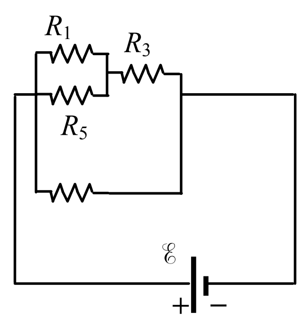
We keep an extra significant figure to show the slight difference in the currents.

$$\boxed{I_{22\Omega} = 0.176\text{A} \quad I_{25\Omega} = 0.154\,\text{A} \quad I_{12\Omega} = 0.177\,\text{A} \quad I_{14\Omega} = 0.153\,\text{A} \quad I_{15\Omega} = 0.001\,\text{A, upwards}}$$

39. The circuit diagram from Problem 38 is reproduced, with $R_2 = 0$. This circuit can now be simplified significantly. Resistors $R_1$ and $R_5$ are in parallel. Call that combination $R_{15}$. That combination is in series with $R_3$. Call that combination $R_{153}$. That combination is in parallel with $R_4$. See the second diagram. We calculate the equivalent resistance $R_{153}$, use that to find the current through the top branch in the second diagram, and then use that current to find the current through $R_5$.




$$R_{153} = \left(\frac{1}{R_1} + \frac{1}{R_5}\right)^{-1} + R_3 = \left(\frac{1}{22\,\Omega} + \frac{1}{15\,\Omega}\right)^{-1} + 12\,\Omega = 20.92\,\Omega$$




Use the loop rule for the outside loop to find the current in the top branch.

$$\mathcal{E} - I_{153}R_{153} = 0 \;\rightarrow\; I_{153} = \frac{\mathcal{E}}{R_{153}} = \frac{6.0\,\text{V}}{20.92\,\Omega} = 0.2868\,\text{A}$$

This current is the sum of the currents in $R_1$ and $R_5$. Since those two resistors are in parallel, the voltage across them must be the same.

$$V_1 = V_5 \;\rightarrow\; I_1R_1 = I_5R_5 \;\rightarrow\; (I_{153} - I_5)R_1 = I_5R_5 \;\rightarrow$$

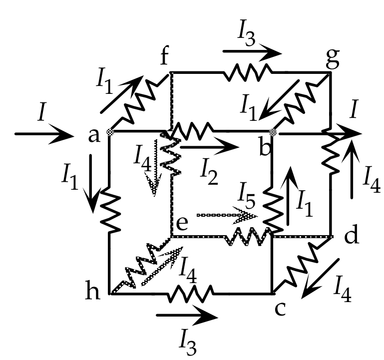
$$I_5 = I_{153}\frac{R_1}{(R_5 + R_1)} = (0.2868\,\text{A})\frac{22\,\Omega}{37\,\Omega} = \boxed{0.17\,\text{A}}$$

40. (*a*) As shown in the diagram, we use symmetry to reduce the number of independent currents to six. Using Kirchhoff's junction rule, we write equations for junctions a, c, and d. We then use Kirchhoff's loop rule to write the loop equations for loops afgba, hedch, and aba (through the voltage source).

$$I = 2I_1 + I_2 \;\; [1] \;;\; I_3 + I_4 = I_1 \;\; [2] \;;\; I_5 = 2I_4 \;\; [3]$$

$$0 = -2I_1R - I_3R + I_2R \;\; [4] \;;\; 0 = -2I_4R - I_5R + I_3R \;\; [5]$$

$$0 = \mathcal{E} - I_2R \;\; [6]$$



We have six equations with six unknown currents. We use the method of substitution to reduce the equations to a single equation relating the emf from the power source to the current through the power source. This resulting ratio is the effective resistance between points a and b. We insert Eqs. [2], [3], and [6] into the other three equations to eliminate $I_1$, $I_2$, and $I_5$.

$$I = 2(I_3 + I_4) + \frac{\mathscr{E}}{R} = 2I_3 + 2I_4 + \frac{\mathscr{E}}{R} \qquad [1*]$$

$$0 = -2(I_3 + I_4)R - I_3R + \frac{\mathscr{E}}{R}R = -2I_4R - 3I_3R + \mathscr{E} \qquad [4*]$$

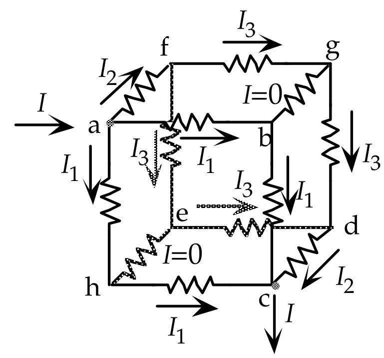
$$0 = -2I_4R - 2I_4R + I_3R = -4I_4R + I_3R \qquad [5*]$$

We solve Eq. [5*] for $I_3$ and insert that into Eq. [4*]. We then insert the two results into Eq. [1*] and solve for the effective resistance.

$$I_3 = 4I_4 \;\; ; \;\; 0 = -2I_4R - 3(4I_4)R + \mathscr{E} \to I_4 = \frac{\mathscr{E}}{14R}$$

$$I = 2(4I_4) + 2I_4 + \frac{\mathscr{E}}{R} = 10I_4 + \frac{\mathscr{E}}{R} = \frac{10\mathscr{E}}{14R} + \frac{\mathscr{E}}{R} = \frac{24\mathscr{E}}{14R} = \frac{12\mathscr{E}}{7R} \quad \to \quad R_{\text{eq}} = \frac{\mathscr{E}}{I} = \boxed{\tfrac{7}{12}R}$$

(*b*) As shown in the diagram, we use symmetry to reduce the number of currents to four. We use Kirchhoff's junction rule at junctions a and d and the loop rule around loops abca (through the voltage source) and afgdcha. This results in four equations with four unknowns. We solve these equations for the ratio of the voltage source to current *I*, to obtain the effective resistance.




$$I = 2I_1 + I_2 \;\; [1] \quad ; \;\; 2I_3 = I_2 \;\; [2]$$

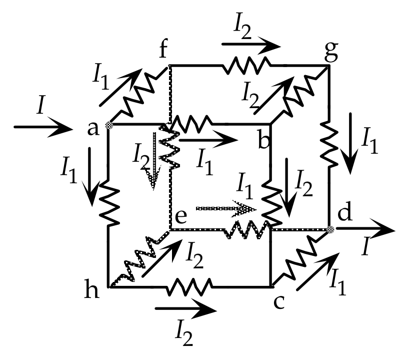
$$0 = -2I_2R + \mathscr{E} \;\; [3] \;\; ; \;\; 0 = -2I_2R - 2I_3R + 2I_1R \;\; [4]$$

We solve Eq. [3] for $I_2$ and Eq. [2] for $I_3$. These results are inserted into Eq. [4] to determine $I_1$. Using these results and Eq. [1] we solve for the effective resistance.

$$I_2 = \frac{\mathscr{E}}{2R} \;\; ; \;\; I_3 = \frac{I_2}{2} = \frac{\mathscr{E}}{4R} \;\; ; \;\; I_1 = I_2 + I_3 = \frac{\mathscr{E}}{2R} + \frac{\mathscr{E}}{4R} = \frac{3\mathscr{E}}{4R}$$

$$I = 2I_1 + I_2 = 2\left(\frac{3\mathscr{E}}{4R}\right) + \frac{\mathscr{E}}{2R} = \frac{2\mathscr{E}}{R} \;\; ; \;\; R_{\text{eq}} = \frac{\mathscr{E}}{I} = \boxed{\tfrac{1}{2}R}$$

(*c*) As shown in the diagram, we again use symmetry to reduce the number of currents to three. We use Kirchhoff's junction rule at points a and b and the loop rule around the loop abgda (through the power source) to write three equations for the three unknown currents. We solve these equations for the ratio of the emf to the current through the emf (I) to calculate the effective resistance.




$$I = 3I_1 \;\; [1] \;\; ; \;\; I_1 = 2I_2 \;\; [2]$$

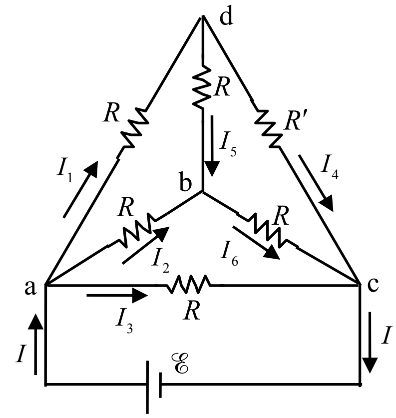
$$0 = -2I_1R - I_2R + \mathscr{E} \;\; [3]$$

We insert Eq. [2] into Eq. [3] and solve for $I_1$. Inserting $I_1$ into Eq. [1] enables us to solve for the effective resistance.

$$0 = -2I_1R - \tfrac{1}{2}I_1R + \mathscr{E} \to I_1 = \frac{2\mathscr{E}}{5R} \;\; ; \;\; I = 3I_1 = \frac{6\mathscr{E}}{5R} \to R_{\text{eq}} = \frac{\mathscr{E}}{I} = \boxed{\tfrac{5}{6}R}$$

41. (*a*) To find the equivalent resistance between points a and c, apply a voltage between points a and c, find the current that flows from the voltage source, and then calculate $R_{eq} = \mathscr{E}/I$.

There is no symmetry to exploit.




$$\begin{aligned}
&(\text{Bottom Loop}) && 1)\quad \mathscr{E} - RI_3 = 0 \\
&(\text{a - d - b}) && 2)\quad -RI_1 - RI_5 + RI_2 = 0 \\
&(\text{a - b - c}) && 3)\quad -RI_2 - RI_6 + RI_3 = 0 \\
&(\text{d - b - c}) && 4)\quad -RI_5 - RI_6 + R'I_4 = 0 \\
&(\text{junction a}) && 5)\quad I = I_1 + I_2 + I_3 \\
&(\text{junction d}) && 6)\quad I_1 = I_4 + I_5 \\
&(\text{junction b}) && 7)\quad I_2 + I_5 = I_6
\end{aligned}$$

From Eq. 1, substitute $I_3 = \mathscr{E}/R$.

2) $-RI_1 - RI_5 + RI_2 = 0 \ \rightarrow\ I_1 + I_5 = I_2$

3) $-RI_2 - RI_6 + R\dfrac{\mathscr{E}}{R} = 0 \ \rightarrow\ I_2 + I_6 = \dfrac{\mathscr{E}}{R}$

4) $-RI_5 - RI_6 + R'I_4 = 0 \ \rightarrow\ R(I_5 + I_6) = R'I_4$

5) $I = I_1 + I_2 + \dfrac{\mathscr{E}}{R}$ ; 6) $I_1 = I_4 + I_5$ ; 7) $I_2 + I_5 = I_6$

From Eq. 7, substitute $I_6 = I_2 + I_5$

2) $I_1 + I_5 = I_2$ ; 3) $I_2 + I_2 + I_5 = \dfrac{\mathscr{E}}{R} \ \rightarrow\ 2I_2 + I_5 = \dfrac{\mathscr{E}}{R}$

4) $R(2I_5 + I_2) = R'I_4$ ; 5) $I = I_1 + I_2 + \dfrac{\mathscr{E}}{R}$ ; 6) $I_1 = I_4 + I_5$

From Eq. 6, substitute $I_1 = I_4 + I_5 \ \rightarrow\ I_5 = I_1 - I_4$

2) $2I_1 - I_4 = I_2$ ; 3) $2I_2 + I_1 - I_4 = \dfrac{\mathscr{E}}{R}$

4) $R(2I_1 - 2I_4 + I_2) = R'I_4$ ; 5) $I = I_1 + I_2 + \dfrac{\mathscr{E}}{R}$

From Eq. 2, substitute $2I_1 - I_4 = I_2 \ \rightarrow\ I_4 = 2I_1 - I_2$

3) $2I_2 + I_1 - (2I_1 - I_2) = \dfrac{\mathscr{E}}{R} \ \rightarrow\ 3I_2 - I_1 = \dfrac{\mathscr{E}}{R}$

4) $R(2I_1 - 2(2I_1 - I_2) + I_2) = R'(2I_1 - I_2) \ \rightarrow\ R(3I_2 - 2I_1) = R'(2I_1 - I_2)$

5) $I = I_1 + I_2 + \dfrac{\mathscr{E}}{R}$

From Eq. 3, substitute $3I_2 - I_1 = \dfrac{\mathscr{E}}{R} \ \rightarrow\ I_1 = 3I_2 - \dfrac{\mathscr{E}}{R}$

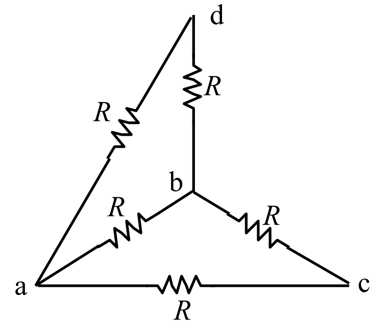
4) $R\left(3I_2 - 2\left(3I_2 - \dfrac{\mathscr{E}}{R}\right)\right) = R'\left(2\left(3I_2 - \dfrac{\mathscr{E}}{R}\right) - I_2\right) \ \rightarrow\ R\left(-3I_2 + 2\dfrac{\mathscr{E}}{R}\right) = R'\left(5I_2 - 2\dfrac{\mathscr{E}}{R}\right)$

5) $I = 3I_2 - \frac{\mathscr{E}}{R} + I_2 + \frac{\mathscr{E}}{R} \rightarrow I = 4I_2$

From Eq. 5, substitute $I_2 = \frac{1}{4}I$

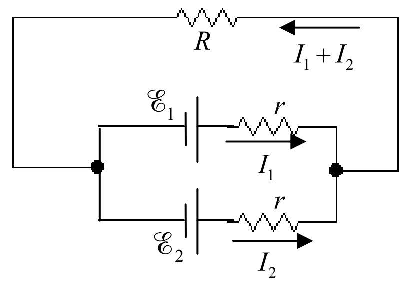
4) $R\left(-3\left(\frac{1}{4}I\right) + 2\frac{\mathscr{E}}{R}\right) = R'\left(5\left(\frac{1}{4}I\right) - 2\frac{\mathscr{E}}{R}\right) \rightarrow \frac{\mathscr{E}}{I} = \boxed{R_{\text{eq}} = \frac{R(5R' + 3R)}{8(R + R')}}$

(*b*) In this case, apply a voltage between points a and b. Now there is symmetry. In this case no current would flow through resistor $R'$, and so that branch can be eliminated from the circuit. See the adjusted diagram. Now the upper left two resistors (from a to d to b) are in series, and the lower right two resistors (from a to c to b) are in series. These two combinations are in parallel with each other, and with the resistor between a and b. The equivalent resistance is now relatively simple to calculate.




$$R_{\text{eq}} = \left(\frac{1}{2R} + \frac{1}{R} + \frac{1}{2R}\right)^{-1} = \left(\frac{4}{2R}\right)^{-1} = \boxed{\tfrac{1}{2}R}$$

42. Define $I_1$ to be the current to the right through the 2.00 V battery $(\mathscr{E}_1)$, and $I_2$ to be the current to the right through the 3.00 V battery $(\mathscr{E}_2)$. At the junction, they combine to give current $I = I_1 + I_2$ to the left through the top branch. Apply Kirchhoff's loop rule first to the upper loop, and then to the outer loop, and solve for the currents.




$$\mathscr{E}_1 - I_1 r - (I_1 + I_2)R = 0 \rightarrow \mathscr{E}_1 - (R + r)I_1 - RI_2 = 0$$

$$\mathscr{E}_2 - I_2 r - (I_1 + I_2)R = 0 \rightarrow \mathscr{E}_2 - RI_1 - (R + r)I_2 = 0$$

Solve the first equation for $I_2$ and substitute into the second equation to solve for $I_1$.

$$\mathscr{E}_1 - (R + r)I_1 - RI_2 = 0 \rightarrow I_2 = \frac{\mathscr{E}_1 - (R + r)I_1}{R} = \frac{2.00 - 4.450I_1}{4.00} = 0.500 - 1.1125I_1$$

$$\mathscr{E}_2 - RI_1 - (R + r)I_2 = 3.00\,\text{V} - (4.00\,\Omega)I_1 - (4.45\,\Omega)(0.500 - 1.1125I_1) = 0 \rightarrow$$

$$I_1 = -0.815\,\text{A} \;;\; I_2 = 0.500 - 1.1125I_1 = 1.407\,\text{A}$$

The voltage across *R* is its resistance times $I = I_1 + I_2$.

$$V_R = R(I_1 + I_2) = (4.00\,\Omega)(-0.815\,\text{A} + 1.407\,\text{A}) = 2.368\,\text{V} \approx \boxed{2.37\,\text{V}}$$

Note that the top battery is being charged – the current is flowing through it from positive to negative.

43. We estimate the time between cycles of the wipers to be from 1 second to 15 seconds. We take these times as the time constant of the *RC* combination.

$$\tau = RC \rightarrow R_{1\text{s}} = \frac{\tau}{C} = \frac{1\,\text{s}}{1 \times 10^{-6}\,\text{F}} = 10^6\,\Omega \;;\; R_{1\text{s}} = \frac{\tau}{C} = \frac{15\,\text{s}}{1 \times 10^{-6}\,\text{F}} = 15 \times 10^6\,\Omega$$

So we estimate the range of resistance to be $\boxed{1\,\text{M}\Omega - 15\,\text{M}\Omega}$.

44. (*a*) From Eq. 26-7 the product *RC* is equal to the time constant.

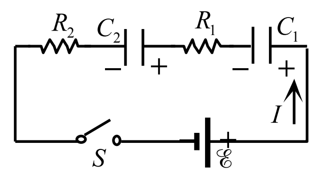
$$\tau = RC \rightarrow C = \frac{\tau}{R} = \frac{24.0\times10^{-6}\,\text{s}}{15.0\times10^{3}\,\Omega} = \boxed{1.60\times10^{-9}\,\text{F}}$$

(*b*) Since the battery has an EMF of 24.0 V, if the voltage across the resistor is 16.0 V, the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$V_C = \mathscr{E}\left(1-e^{-t/\tau}\right) \rightarrow e^{-t/\tau} = \left(1-\frac{V_C}{\mathscr{E}}\right) \rightarrow -\frac{t}{\tau} = \ln\left(1-\frac{V_C}{\mathscr{E}}\right) \rightarrow$$

$$t = -\tau\ln\left(1-\frac{V_C}{\mathscr{E}}\right) = -\left(24.0\times10^{-6}\,\text{s}\right)\ln\left(1-\frac{8.0\,\text{V}}{24.0\,\text{V}}\right) = \boxed{9.73\times10^{-6}\,\text{s}}$$

45. The current for a capacitor-charging circuit is given by Eq. 26-8, with *R* the equivalent series resistance and *C* the equivalent series capacitance.




$$I = \frac{\mathscr{E}}{R_{\text{eq}}}e^{-\left(\frac{t}{R_{\text{eq}}C_{\text{eq}}}\right)} \rightarrow$$

$$t = -R_{\text{eq}}C_{\text{eq}}\ln\left(\frac{IR_{\text{eq}}}{\mathscr{E}}\right) = -\left(R_1+R_2\right)\left(\frac{C_1C_2}{C_1+C_2}\right)\ln\left[\frac{I\left(R_1+R_2\right)}{\mathscr{E}}\right]$$

$$= -\left(4400\,\Omega\right)\left[\frac{\left(3.8\times10^{-6}\,\text{F}\right)^2}{7.6\times10^{-6}\,\text{F}}\right]\ln\left[\frac{\left(1.50\times10^{-3}\,\text{A}\right)\left(4400\,\Omega\right)}{\left(12.0\text{V}\right)}\right] = \boxed{5.0\times10^{-3}\,\text{s}}$$

46. Express the stored energy in terms of the charge on the capacitor, using Eq. 24-5. The charge on the capacitor is given by Eq. 26-6a.

$$U = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}\frac{\left[C\mathscr{E}\left(1-e^{-t/\tau}\right)\right]^2}{C} = \tfrac{1}{2}C\mathscr{E}^2\left(1-e^{-t/\tau}\right)^2 = U_{\text{max}}\left(1-e^{-t/\tau}\right)^2 \;;$$

$$U = 0.75U_{\text{max}} \rightarrow U_{\text{max}}\left(1-e^{-t/\tau}\right)^2 = 0.75U_{\text{max}} \rightarrow \left(1-e^{-t/\tau}\right)^2 = 0.75 \rightarrow$$

$$t = -\tau\ln\left(1-\sqrt{0.75}\right) = \boxed{2.01\tau}$$

47. The capacitance is given by Eq. 24-8 and the resistance by Eq. 25-3. The capacitor plate separation *d* is the same as the resistor length $\ell$. Calculate the time constant.

$$\tau = RC = \left(\frac{\rho d}{A}\right)\left(K\varepsilon_0\frac{A}{d}\right) = \boxed{\rho K\varepsilon_0} = \left(1.0\times10^{12}\,\Omega\text{•m}\right)\left(5.0\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N•m}^2\right) = \boxed{44\,\text{s}}$$

48. The voltage of the discharging capacitor is given by $V_\text{C} = V_0e^{-t/RC}$. The capacitor voltage is to be $0.0010V_0$.

$$V_\text{C} = V_0e^{-t/RC} \rightarrow 0.0010V_0 = V_0e^{-t/RC} \rightarrow 0.0010 = e^{-t/RC} \rightarrow \ln\left(0.010\right) = -\frac{t}{RC} \rightarrow$$

$$t = -RC\ln\left(0.010\right) = -\left(8.7\times10^3\,\Omega\right)\left(3.0\times10^{-6}\,\text{F}\right)\ln\left(0.0010\right) = \boxed{0.18\,\text{s}}$$

**49.** (*a*) At $t = 0$, the capacitor is uncharged and so there is no voltage difference across it. The capacitor is a "short," and so a simpler circuit can be drawn just by eliminating the capacitor. In this simpler circuit, the two resistors on the right are in parallel with each other, and then in series with the resistor by the switch. The current through the resistor by the switch splits equally when it reaches the junction of the parallel resistors.

$$R_{\text{eq}} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \tfrac{3}{2}R \rightarrow I_1 = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{\mathscr{E}}{\tfrac{3}{2}R} = \boxed{\frac{2\mathscr{E}}{3R}} \;;\; I_2 = I_3 = \tfrac{1}{2}I_1 = \boxed{\frac{\mathscr{E}}{3R}}$$

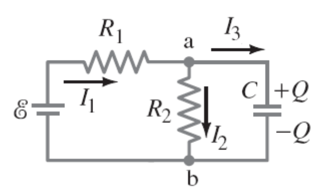
(*b*) At $t = \infty$, the capacitor will be fully charged and there will be no current in the branch containing the capacitor, and so a simpler circuit can be drawn by eliminating that branch. In this simpler circuit, the two resistors are in series, and they both have the same current.

$$R_{\text{eq}} = R + R = 2R \rightarrow I_1 = I_2 = \frac{\mathscr{E}}{R_{\text{eq}}} = \boxed{\frac{\mathscr{E}}{2R}} \;;\; I_3 = \boxed{0}$$

(*c*) At $t = \infty$, since there is no current through the branch containing the capacitor, there is no potential drop across that resistor. Therefore the voltage difference across the capacitor equals the voltage difference across the resistor through which $I_2$ flows.

$$V_C = V_{R_2} = I_2 R = \left(\frac{\mathscr{E}}{2R}\right)R = \boxed{\tfrac{1}{2}\mathscr{E}}$$

50. (*a*) With the currents and junctions labeled as in the diagram, we use point a for the junction rule and the right and left loops for the loop rule. We set current $I_3$ equal to the derivative of the charge on the capacitor and combine the equations to obtain a single differential equation in terms of the capacitor charge. Solving this equation yields the charging time constant.




$$I_1 = I_2 + I_3 \;\; [1] \;;\; \mathscr{E} - I_1R_1 - I_2R_2 = 0 \;\; [2] \;;\; -\frac{Q}{C} + I_2R_2 = 0 \;\; [3]$$

We use Eq. [1] to eliminate $I_1$ in Eq. [2]. Then we use Eq. [3] to eliminate $I_2$ from Eq. [2].

$$0 = \mathscr{E} - (I_2 + I_3)R_1 - I_2R_2 \;;\; 0 = \mathscr{E} - I_2(R_1 + R_2) - I_3R_1 \;;\; 0 = \mathscr{E} - \left(\frac{Q}{R_2C}\right)(R_1 + R_2) - I_3R_1$$

We set $I_3$ as the derivative of the charge on the capacitor and solve the differential equation by separation of variables.

$$0 = \mathscr{E} - \left(\frac{Q}{R_2C}\right)(R_1 + R_2) - \frac{dQ}{dt}R_1 \rightarrow \int_0^Q \frac{dQ'}{Q' - \left(\frac{R_2C\mathscr{E}}{R_1 + R_2}\right)} = \int_0^t \frac{-(R_1 + R_2)}{R_1R_2C}dt' \rightarrow$$

$$\ln\left[Q' - \left(\frac{R_2C\mathscr{E}}{R_1 + R_2}\right)\right]_0^Q = -\frac{(R_1 + R_2)}{R_1R_2C}t'\Bigg|_0^t \rightarrow \ln\left[\frac{Q - \left(\frac{R_2C\mathscr{E}}{R_1 + R_2}\right)}{\left(\frac{R_2C\mathscr{E}}{R_1 + R_2}\right)}\right] = -\frac{(R_1 + R_2)}{R_1R_2C}t \rightarrow$$

$$Q = \frac{R_2C\mathscr{E}}{R_1 + R_2}\left(1 - e^{-\frac{(R_1 + R_2)}{R_1R_2C}t}\right)$$

From the exponential term we obtain the time constant, $\tau = \boxed{\dfrac{R_1R_2C}{R_1 + R_2}}$.

(*b*) We obtain the maximum charge on the capacitor by taking the limit as time goes to infinity.

$$Q_{\max} = \lim_{t\to\infty} \frac{R_2 C\mathcal{E}}{R_1+R_2}\left(1-e^{-\frac{(R_1+R_2)}{R_1R_2C}t}\right) = \boxed{\frac{R_2C\mathcal{E}}{R_1+R_2}}$$

51. (*a*) With the switch open, the resistors are in series with each other, and so have the same current. Apply the loop rule clockwise around the left loop, starting at the negative terminal of the source, to find the current.

$$V - IR_1 - IR_2 = 0 \;\rightarrow\; I = \frac{V}{R_1+R_2} = \frac{24\,\text{V}}{8.8\,\Omega + 4.4\,\Omega} = 1.818\,\text{A}$$

The voltage at point a is the voltage across the $4.4\,\Omega$-resistor.

$$V_a = IR_2 = (1.818\,\text{A})(4.4\,\Omega) = \boxed{8.0\,\text{V}}$$

(*b*) With the switch open, the capacitors are in series with each other. Find the equivalent capacitance. The charge stored on the equivalent capacitance is the same value as the charge stored on each capacitor in series.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \;\rightarrow\; C_{\text{eq}} = \frac{C_1C_2}{C_1+C_2} = \frac{(0.48\,\mu\text{F})(0.36\,\mu\text{F})}{(0.48\,\mu\text{F}+0.36\,\mu\text{F})} = 0.2057\,\mu\text{F}$$

$$Q_{\text{eq}} = VC_{\text{eq}} = (24.0\,\text{V})(0.2057\,\mu\text{F}) = 4.937\,\mu\text{C} = Q_1 = Q_2$$

The voltage at point b is the voltage across the $0.24\,\mu\text{F}$-capacitor.

$$V_b = \frac{Q_2}{C_2} = \frac{4.937\,\mu\text{C}}{0.36\,\mu\text{F}} = 13.7\,\text{V} \approx \boxed{14\,\text{V}}$$

(*c*) The switch is now closed. After equilibrium has been reached a long time, there is no current flowing in the capacitors, and so the resistors are again in series, and the voltage of point a must be 8.0 V. Point b is connected by a conductor to point a, and so point b must be at the same potential as point a, $\boxed{8.0\,\text{V}}$. This also means that the voltage across $C_2$ is 8.0 V, and the voltage across $C_1$ is 16 V.

(*d*) Find the charge on each of the capacitors, which are no longer in series.

$$Q_1 = V_1C_1 = (16\,\text{V})(0.48\,\mu\text{F}) = 7.68\,\mu\text{C}$$

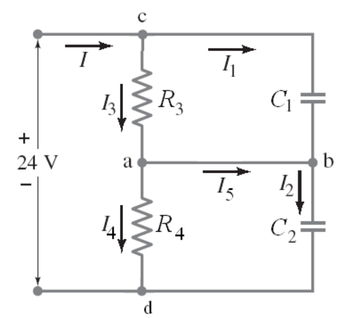
$$Q_2 = V_2C_2 = (8.0\,\text{V})(0.36\,\mu\text{F}) = 2.88\,\mu\text{C}$$

When the switch was open, point b had a net charge of 0, because the charge on the negative plate of $C_1$ had the same magnitude as the charge on the positive plate of $C_2$. With the switch closed, these charges are not equal. The net charge at point b is the sum of the charge on the negative plate of $C_1$ and the charge on the positive plate of $C_2$.

$$Q_b = -Q_1 + Q_2 = -7.68\,\mu\text{C} + 2.88\,\mu\text{C} = -4.80\,\mu\text{C} \approx -4.8\,\mu\text{C}$$

Thus $\boxed{4.8\,\mu\text{C}}$ of charge has passed through the switch, from right to left.

52. Because there are no simple series or parallel connections in this circuit, we use Kirchhoff's rules to write equations for the currents, as labeled in our diagram. We write junction equations for the junctions c and d. We then write loop equations for each of the three loops. We set the current through the capacitor equal to the derivative of the charge on the capacitor.




$$I = I_1 + I_3 \ \ [1] \ ; \ I = I_2 + I_4 \ \ [2] \ ; \ \mathscr{E} - \frac{Q_1}{C_1} - \frac{Q_2}{C_2} = 0 \ \ [3]$$

$$\frac{Q_1}{C_1} - I_3 R_3 = 0 \ \ [4] \ ; \ \frac{Q_2}{C_2} - I_4 R_4 = 0 \ \ [5]$$

We differentiate Eq. [3] with respect to time and set the derivative of the charge equal to the current.

$$0 = \frac{d\mathscr{E}}{dt} - \frac{dQ_1}{dt}\frac{1}{C_1} - \frac{dQ_2}{dt}\frac{1}{C_2} = 0 - \frac{I_1}{C_1} - \frac{I_2}{C_2} \to I_2 = -I_1\frac{C_2}{C_1}$$

We then substitute Eq. [1] into Eq. [2] to eliminate *I*. Then using Eqs. [4] and [5] we eliminate $I_3$ and $I_4$ from the resulting equation. We eliminate $I_2$ using the derivative equation above.

$$I_1 + I_3 = I_2 + I_4 \ ; \ I_1 + \frac{Q_1}{R_3 C_1} = -I_1\frac{C_2}{C_1} + \frac{Q_2}{R_4 C_2}$$

Finally, we eliminate $Q_2$ using Eq.[3].

$$I_1 + \frac{Q_1}{R_3 C_1} = -I_1\frac{C_2}{C_1} + \frac{1}{R_4}\left(\mathscr{E} - \frac{Q_1}{C_1}\right) \ \to \ \mathscr{E} = I_1 R_4\left(\frac{C_1 + C_2}{C_1}\right) + Q_1\left(\frac{R_4 + R_3}{R_3 C_1}\right) \ \to$$

$$\mathscr{E} = I_1 R + \frac{Q_1}{C} \qquad \text{where} \quad R = R_4\left(\frac{C_1 + C_2}{C_1}\right) \text{ and } C = C_1\left(\frac{R_3}{R_4 + R_3}\right)$$

This final equation represents a simple *RC* circuit, with time constant $\tau = RC$.

$$\tau = RC = R_4\left(\frac{C_1 + C_2}{C_1}\right)C_1\left(\frac{R_3}{R_4 + R_3}\right) = \frac{R_4 R_3 (C_1 + C_2)}{R_4 + R_3}$$

$$= \frac{(8.8\,\Omega)(4.4\,\Omega)(0.48\,\mu\text{F} + 0.36\,\mu\text{F})}{8.8\,\Omega + 4.4\,\Omega} = \boxed{2.5\,\mu\text{s}}$$

53. The full-scale current is the reciprocal of the sensitivity.

$$I_{\substack{\text{full-}\\\text{scale}}} = \frac{1}{35{,}000\,\Omega/\text{V}} = \boxed{2.9 \times 10^{-5}\,\text{A}} \ \text{ or } \ 29\,\mu\text{A}$$

54. The resistance is the full-scale voltage multiplied by the sensitivity.

$$R = V_{\substack{\text{full-}\\\text{scale}}}(\text{sensitivity}) = (250\,\text{V})(35{,}000\,\Omega/\text{V}) = 8.75 \times 10^6\,\Omega \approx \boxed{8.8 \times 10^6\,\Omega}$$

55. (*a*) The current for full-scale deflection of the galvanometer is

$$I_G = \frac{1}{\text{sensitivity}} = \frac{1}{45{,}000\,\Omega/\text{V}} = 2.222 \times 10^{-5}\,\text{A}$$

To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. The total current is to be 2.0 A. See Figure 26-28 for a circuit diagram.

$$I_G r_G = I_s R_s \rightarrow R_s = \frac{I_G}{I_s} r_G = \frac{I_G}{I_{full} - I_G} r_G = \frac{2.222\times10^{-5}\,\text{A}}{2.0\text{A} - 2.222\times10^{-5}\,\text{A}}(20.0\,\Omega)$$

$$= 2.222\times10^{-4}\,\Omega \approx \boxed{2.2\times10^{-4}\,\Omega \text{ in parallel}}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full scale voltage corresponds to the full scale current of the galvanometer. See Figure 26-29 for a circuit diagram. The total current must be the full-scale deflection current.

$$V_{full} = I_G\left(r_G + R\right) \rightarrow$$

$$R = \frac{V_{full}}{I_G} - r_G = \frac{1.00\text{V}}{2.222\times10^{-5}\,\text{A}} - 20.0\,\Omega = 44985\,\Omega \approx \boxed{45\,\text{k}\Omega \text{ in series}}$$

56. (*a*) To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. See Figure 26-28 for a circuit diagram.

$$V_{shunt} = V_G \rightarrow \left(I_{full} - I_G\right)R_{shunt} = I_G R_G \rightarrow$$

$$R_{shunt} = \frac{I_G R_G}{\left(I_{full} - I_G\right)} = \frac{\left(55\times10^{-6}\,\text{A}\right)\left(32\,\Omega\right)}{\left(25\,\text{A} - 55\times10^{-6}\,\text{A}\right)} = \boxed{7.0\times10^{-5}\,\Omega}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full-scale voltage corresponds to the full scale current of the galvanometer. See Figure 26-29 for a circuit diagram.

$$V_{full\ scale} = I_G\left(R_{ser} + R_G\right) \rightarrow R_{ser} = \frac{V_{full\ scale}}{I_G} - R_G = \frac{250\,\text{V}}{55\times10^{-6}\,\text{A}} - 30\,\Omega = \boxed{4.5\times10^{6}\,\Omega}$$

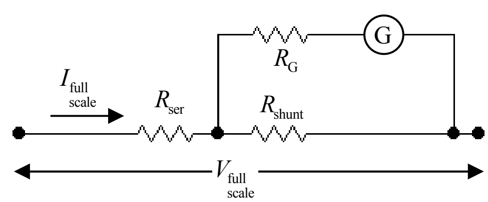
57. We divide the full-scale voltage of the electronic module by the module's internal resistance to determine the current through the module that will give full-scale deflection. Since the module and $R_2$ are in parallel they will have the same voltage drop across them (400 mV) and their currents will add to equal the current through $R_1$. We set the voltage drop across $R_1$ and $R_2$ equal to the 40 volts and solve the resulting equation for $R_2$.

$$I_{meter} = \frac{V_{meter}}{r} = \frac{400\text{ mV}}{100\text{ M}\Omega} = 4.00\text{ nA} \;;\; I_2 = \frac{V_{meter}}{R_2} \;;\; I_1 = I_2 + I_{meter} = \frac{V_{meter}}{R_2} + I_{meter}$$

$$V = I_1 R_1 + V_{meter} \rightarrow \left(V - V_{meter}\right) = \left(\frac{V_{meter}}{R_2} + I_{meter}\right)R_1 \rightarrow$$

$$R_2 = \frac{R_1 V_{meter}}{\left(V - V_{meter}\right) - I_{meter}R_1} = \frac{\left(10\times10^6\ \Omega\right)\left(0.400\ \text{V}\right)}{\left(40\ \text{V} - 0.400\ \text{V}\right) - \left(4.00\times10^{-9}\ \text{A}\right)\left(10\times10^6\ \Omega\right)} = \boxed{100\ \text{k}\Omega}$$

58. To make a voltmeter, a resistor $R_{ser}$ must be placed in series with the existing meter so that the desired full scale voltage corresponds to the full scale current of the galvanometer. We know that 25 mA produces full scale deflection of the galvanometer, so the voltage drop across the total meter must be 25 V when the current through the meter is 25 mA.



$$V_{\substack{\text{full}\\\text{scale}}} = I_{\substack{\text{full}\\\text{scale}}} R_{eq} = I_{\substack{\text{full}\\\text{scale}}} \left[ R_{ser} + \left( \frac{1}{R_G} + \frac{1}{R_{shunt}} \right)^{-1} \right] \rightarrow$$

$$R_{ser} = \frac{V_{\substack{\text{full}\\\text{scale}}}}{I_{\substack{\text{full}\\\text{scale}}}} - \left( \frac{1}{R_G} + \frac{1}{R_{shunt}} \right)^{-1} = \frac{25\,\text{V}}{25\times10^{-3}\,\text{A}} - \left( \frac{1}{33\,\Omega} + \frac{1}{0.20\,\Omega} \right)^{-1} = 999.8\,\Omega \approx \boxed{1000\,\Omega}$$

The sensitivity is $\frac{1000\,\Omega}{25\,\text{V}} = \boxed{40\,\Omega/\text{V}}$

59. If the voltmeter were ideal, then the only resistance in the circuit would be the series combination of the two resistors. The current can be found from the battery and the equivalent resistance, and then the voltage across each resistor can be found.

$$R_{tot} = R_1 + R_2 = 44\,\text{k}\Omega + 27\,\text{k}\Omega = 71\,\text{k}\Omega \ ; \ I = \frac{V}{R_{tot}} = \frac{45\,\text{V}}{71\times10^3\,\Omega} = 6.338\times10^{-4}\,\text{A}$$

$$V_{44} = IR_1 = \left(6.338\times10^{-4}\,\text{A}\right)\left(44\times10^3\,\Omega\right) = 27.89\,\text{V}$$

$$V_{27} = IR_2 = \left(6.338\times10^{-4}\,\text{A}\right)\left(27\times10^3\,\Omega\right) = 17.11\,\text{V}$$

Now put the voltmeter in parallel with the $44\,\text{k}\Omega$ resistor. Find its equivalent resistance, and then follow the same analysis as above.

$$R_{eq} = \left( \frac{1}{44\,\text{k}\Omega} + \frac{1}{95\,\text{k}\Omega} \right)^{-1} = 30.07\,\text{k}\Omega$$

$$R_{tot} = R_{eq} + R_2 = 30.07\,\text{k}\Omega + 27\text{k}\Omega = 57.07\,\text{k}\Omega \qquad I = \frac{V}{R_{tot}} = \frac{45\,\text{V}}{57.07\times10^3\,\Omega} = 7.885\times10^{-4}\,\text{A}$$

$$V_{44} = V_{eq} = IR_{eq} = \left(7.885\times10^{-4}\,\text{A}\right)\left(30.07\times10^3\,\Omega\right) = 23.71\,\text{V} \approx \boxed{24\,\text{V}}$$

$$\%\text{ error} = \frac{23.71\,\text{V} - 27.89\text{V}}{27.89\,\text{V}}\times100 = \boxed{-15\%\left(\text{reading too low}\right)}$$

And now put the voltmeter in parallel with the $27\,\text{k}\Omega$ resistor, and repeat the process.

$$R_{eq} = \left( \frac{1}{27\,\text{k}\Omega} + \frac{1}{95\,\text{k}\Omega} \right)^{-1} = 21.02\,\text{k}\Omega$$

$$R_{tot} = R_{eq} + R_1 = 21.02\,\text{k}\Omega + 44\,\text{k}\Omega = 65.02\,\text{k}\Omega \qquad I = \frac{V}{R_{tot}} = \frac{45\,\text{V}}{65.02\times10^3\,\Omega} = 6.921\times10^{-4}\,\text{A}$$

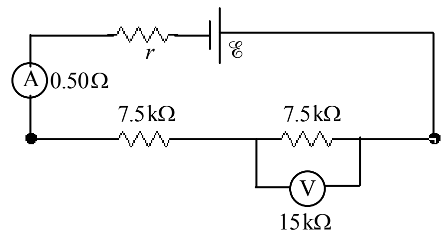
$$V_{27} = V_{eq} = IR_{eq} = \left(6.921\times10^{-4}\,\text{A}\right)\left(21.02\times10^3\,\Omega\right) = 14.55\text{V} \approx \boxed{15\,\text{V}}$$

$$\%\text{ error} = \frac{14.55\,\text{V} - 17.11\text{V}}{17.11\,\text{V}}\times100 = \boxed{-15\%\ \left(\text{reading too low}\right)}$$

60. The total resistance with the ammeter present is $R_{eq} = 650\,\Omega + 480\,\Omega + 53\,\Omega = 1183\,\Omega.$ The voltage supplied by the battery is found from Ohm's law to be $V_{battery} = IR_{eq} = \left(5.25\times10^{-3}\,\text{A}\right)\left(1183\,\Omega\right)$ $= 6.211\,\text{V}.$ When the ammeter is removed, we assume that the battery voltage does not change. The equivalent resistance changes to $R'_{eq} = 1130\,\Omega$, and the new current is again found from Ohm's law.

$$I = \frac{V_{\text{battery}}}{R'_{\text{eq}}} = \frac{6.211\,\text{V}}{1130\,\Omega} = \boxed{5.50 \times 10^{-3}\,\text{A}}$$

61. Find the equivalent resistance for the entire circuit, and then find the current drawn from the source. That current will be the ammeter reading. The ammeter and voltmeter symbols in the diagram below are each assumed to have resistance.



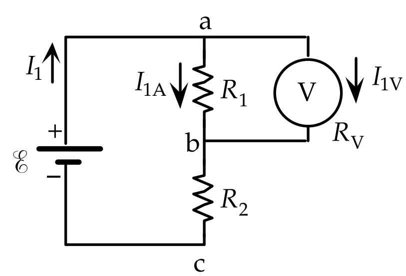

$$R_{\text{eq}} = 1.0\,\Omega + 0.50\,\Omega + 7500\,\Omega + \frac{(7500\,\Omega)(15000\,\Omega)}{(7500\,\Omega + 15000\,\Omega)}$$

$$= 12501.5\,\Omega \approx 12500\,\Omega \;;\; I_{\text{source}} = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{12.0\,\text{V}}{12500\,\Omega} = \boxed{9.60 \times 10^{-4}\,\text{A}}$$

The voltmeter reading will be the source current times the equivalent resistance of the resistor–voltmeter combination.

$$V_{\text{meter}} = I_{\text{source}} R_{\text{eq}} = \left(9.60 \times 10^{-4}\,\text{A}\right) \frac{(7500\,\Omega)(15000\,\Omega)}{(7500\,\Omega + 15000\,\Omega)} = \boxed{4.8\,\text{V}}$$

62. From the first diagram, write the sum of the currents at junction a, and then substitute in for those currents as shown.

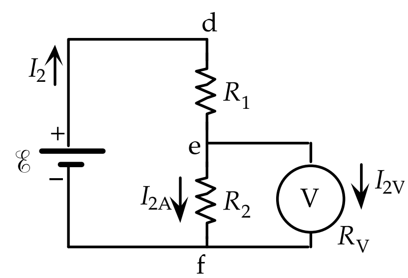



$$I_1 = I_{1\text{A}} + I_{1\text{V}}$$

$$\mathscr{E} - V_{R_1} - I_1 R_2 = 0 \;\rightarrow\; I_1 = \frac{\mathscr{E} - V_{R_1}}{R_2} \;;\; I_{1A} = \frac{V_{R_1}}{R_1} \;;\; I_{1V} = \frac{V_{1\text{V}}}{R_{\text{V}}}$$

$$\frac{\mathscr{E} - V_{R_1}}{R_2} = \frac{V_{R_1}}{R_1} + \frac{V_{1\text{V}}}{R_{\text{V}}}$$

Then do a similar procedure for the second diagram.




$$I_2 = I_{2\text{A}} + I_{2\text{V}}$$

$$\mathscr{E} - I_2 R_1 - V_{R_2} = 0 \;\rightarrow\; I_2 = \frac{\mathscr{E} - V_{R_2}}{R_1} \;;\; I_{2A} = \frac{V_{R_2}}{R_2} \;;\; I_{2V} = \frac{V_{2\text{V}}}{R_{\text{V}}}$$

$$\frac{\mathscr{E} - V_{R_2}}{R_1} = \frac{V_{R_2}}{R_2} + \frac{V_{2\text{V}}}{R_{\text{V}}}$$

Now there are two equations in the two unknowns of $R_1$ and $R_2$. Solve for the reciprocal values and then find the resistances. Assume that all resistances are measured in kilohms.

$$\frac{\mathscr{E} - V_{R_1}}{R_2} = \frac{V_{R_1}}{R_1} + \frac{V_{1\text{V}}}{R_{\text{V}}} \;\rightarrow\; \frac{12.0 - 5.5}{R_2} = \frac{5.5}{R_1} + \frac{5.5}{18.0} \;\rightarrow\; \frac{6.5}{R_2} = \frac{5.5}{R_1} + 0.30556$$

$$\frac{\mathscr{E} - V_{R_2}}{R_1} = \frac{V_{R_2}}{R_2} + \frac{V_{2\text{V}}}{R_{\text{V}}} \;\rightarrow\; \frac{12.0 - 4.0}{R_1} = \frac{4.0}{R_2} + \frac{4.0}{18.0} \;\rightarrow\; \frac{8.0}{R_1} = \frac{4.0}{R_2} + 0.22222$$

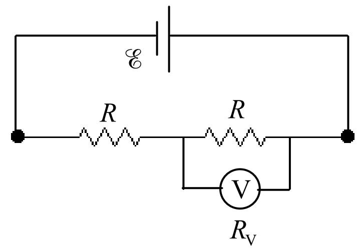
$$\frac{8.0}{R_1} = \frac{4.0}{R_2} + 0.22222 \;\rightarrow\; \frac{1}{R_2} = \frac{2}{R_1} - 0.05556$$

$$\frac{6.5}{R_2} = \frac{5.5}{R_1} + 0.30556 \;\rightarrow\; 6.5\left(\frac{2}{R_1} - 0.05556\right) = \frac{5.5}{R_1} + 0.30556 \;\rightarrow\; \frac{1}{R_1} = \frac{0.66667}{7.5} \;\rightarrow$$

$$R_1 = 11.25\,\text{k}\Omega \ ; \ \frac{1}{R_2} = \frac{2}{R_1} - 0.05556 \ \rightarrow \ R_2 = 8.18\,\text{k}\Omega$$

So the final results are $\boxed{R_1 = 11\,\text{k}\Omega \ ; \ R_2 = 8.2\,\text{k}\Omega}$

63. The sensitivity of the voltmeter is 1000 ohms per volt on the 3.0 volt scale, so it has a resistance of 3000 ohms. The circuit is shown in the diagram. Find the equivalent resistance of the meter–resistor parallel combination and the entire circuit.




$$R_p = \left(\frac{1}{R} + \frac{1}{R_V}\right)^{-1} = \frac{R_V R}{R_V + R} = \frac{(3000\,\Omega)(9400\,\Omega)}{3000\,\Omega + 9400\,\Omega} = 2274\,\Omega$$

$$R_{eq} = R + R_p = 2274\,\Omega + 9400\,\Omega = 11674\,\Omega$$

Using the meter reading of 2.3 volts, calculate the current into the parallel combination, which is the current delivered by the battery. Use that current to find the EMF of the battery.

$$I = \frac{V}{R_p} = \frac{2.3\,\text{V}}{2274\,\Omega} = 1.011 \times 10^{-3}\,\text{A}$$

$$\mathcal{E} = IR_{eq} = (1.011 \times 10^{-3}\,\text{A})(11674\,\Omega) = 11.80\,\text{V} \approx \boxed{12\,\text{V}}$$

64. By calling the voltmeter "high resistance," we can assume it has no current passing through it. Write Kirchhoff's loop rule for the circuit for both cases, starting with the negative pole of the battery and proceeding counterclockwise.

Case 1: $V_{meter} = V_1 = I_1 R_1 \quad \mathcal{E} - I_1 r - I_1 R_1 = 0 \ \rightarrow \ \mathcal{E} = I_1(r + R_1) = \dfrac{V_1}{R_1}(r + R_1)$

Case 2: $V_{meter} = V_2 = I_2 R_2 \quad \mathcal{E} - I_2 r - I_2 R_2 = 0 \ \rightarrow \ \mathcal{E} = I_2(r + R_2) = \dfrac{V_2}{R_2}(r + R_2)$

Solve these two equations for the two unknowns of $\mathcal{E}$ and $r$.

$$\mathcal{E} = \frac{V_1}{R_1}(r + R_1) = \frac{V_2}{R_2}(r + R_2) \ \rightarrow$$

$$r = R_1 R_2 \left(\frac{V_2 - V_1}{V_1 R_2 - V_2 R_1}\right) = (35\,\Omega)(14.0\,\Omega)\left(\frac{8.1\,\text{V} - 9.7\,\text{V}}{(9.7\,\text{V})(14.0\,\Omega) - (8.1\,\text{V})(35\,\Omega)}\right) = 5.308\,\Omega \approx \boxed{5.3\,\Omega}$$

$$\mathcal{E} = \frac{V_1}{R_1}(r + R_1) = \frac{9.7\,\text{V}}{35\,\Omega}(5.308\,\Omega + 35\,\Omega) = 11.17\,\text{V} \approx \boxed{11\,\text{V}}$$

65. We connect the battery in series with the body and a resistor. The current through this series circuit is the voltage supplied by the battery divided by the sum of the resistances. The voltage drop across the body is equal to the current multiplied by the body's resistance. We set the voltage drop across the body equal to 0.25 V and solve for the necessary resistance.

$$I = \frac{\mathcal{E}}{R + R_B}$$

$$V = IR_B = \frac{\mathcal{E} R_B}{R + R_B} \rightarrow R = \left(\frac{\mathcal{E}}{V} - 1\right) R_B = \left(\frac{1.5\ \text{V}}{0.25\ \text{V}} - 1\right)(1800\ \Omega) = 9000\ \Omega = \boxed{9.0\ \text{k}\Omega}$$

66. (*a*) Since $P = V^2/R$ and the voltage is the same for each combination, the power and resistance are inversely related to each other. So for the $\boxed{\text{50 W output, use the higher-resistance filament}}$. For the $\boxed{\text{100 W output, use the lower-resistance filament}}$. For the $\boxed{\text{150 W output, use the filaments in parallel}}$.

(*b*) $P = V^2/R \rightarrow$

$$R = \frac{V^2}{P} \quad R_{50\,\text{W}} = \frac{(120\,\text{V})^2}{50\,\text{W}} = 288\,\Omega \approx \boxed{290\,\Omega} \quad R_{100\,\text{W}} = \frac{(120\,\text{V})^2}{100\,\text{W}} = 144\,\Omega \approx \boxed{140\,\Omega}$$

As a check, the parallel combination of the resistors gives the following.

$$R_p = \frac{R_1R_2}{R_1 + R_2} = \frac{(288\,\Omega)(144\,\Omega)}{288\,\Omega + 144\,\Omega} = 96\,\Omega \quad P = \frac{V^2}{R} = \frac{(120\,\text{V})^2}{96\,\Omega} = 150\,\text{W}.$$

$\boxed{67.}$ The voltage drop across the two wires is the 3.0 A current times their total resistance.

$$V_{\text{wires}} = IR_{\text{wires}} = (3.0\,\text{A})(0.0065\,\Omega/\text{m})(130\,\text{m})\,R_p = 2.535\,\text{V} \approx \boxed{2.5\,\text{V}}$$

Thus the voltage applied to the apparatus is $V = V_{\text{source}} - V_{\text{wires}} = 120\,\text{V} - 2.535\,\text{V} = 117.465\,\text{V} \approx \boxed{117\,\text{V}}$.

68. The charge on the capacitor and the current in the resistor both decrease exponentially, with a time constant of $\tau = RC$. The energy stored in the capacitor is given by $U = \frac{1}{2}\frac{Q^2}{C}$, and the power dissipated in the resistor is given by $P = I^2R$.

$$Q = Q_0e^{-t/RC} \;;\; I = I_0e^{-t/RC} = \frac{V_0}{R}e^{-t/RC} = \frac{Q_0}{RC}e^{-t/RC}$$

$$U_{\text{decrease}} = -\Delta U = U_{t=0} - U_{t=\tau} = \frac{1}{2}\left(\frac{Q^2}{C}\right)_{t=0} - \frac{1}{2}\left(\frac{Q^2}{C}\right)_{t=\tau} = \frac{1}{2}\frac{Q_0^2}{C} - \frac{1}{2}\left(\frac{Q_0e^{-1}}{C}\right)^2 = \frac{1}{2}\frac{Q_0^2}{C}\left(1 - e^{-2}\right)$$

$$U_{\text{dissipated}} = \int P\,dt = \int_0^\tau I^2R\,dt = \int_0^\tau \left(\frac{Q_0}{RC}e^{-t/RC}\right)^2 R\,dt = \frac{Q_0^2}{RC^2}\int_0^\tau e^{-2t/RC}dt = \frac{Q_0^2}{RC^2}\left(-\frac{RC}{2}\right)\left(e^{-2t/RC}\right)_0^\tau$$

$$= -\frac{1}{2}\frac{Q_0^2}{C}\left(e^{-2} - 1\right) = \frac{1}{2}\frac{Q_0^2}{C}\left(1 - e^{-2}\right)$$

And so we see that $\boxed{U_{\text{decrease}} = U_{\text{dissipated}}}$.

69. The capacitor will charge up to 75% of its maximum value, and then discharge. The charging time is the time for one heartbeat.

$$t_{\text{beat}} = \frac{1\,\text{min}}{72\,\text{beats}} \times \frac{60\,\text{s}}{1\,\text{min}} = 0.8333\,\text{s}$$

$$V = V_0\left(1 - e^{-\frac{t}{RC}}\right) \rightarrow 0.75V_0 = V_0\left(1 - e^{-\frac{t_{\text{beat}}}{RC}}\right) \rightarrow e^{-\frac{t_{\text{beat}}}{RC}} = 0.25 \rightarrow \left(-\frac{t_{\text{beat}}}{RC}\right) = \ln(0.25) \rightarrow$$

$$R = -\frac{t_{\text{beat}}}{C\ln(0.25)} = -\frac{0.8333\,\text{s}}{(6.5\times10^{-6}\,\text{F})(-1.3863)} = \boxed{9.2\times10^4\,\Omega}$$

70. (*a*) Apply Ohm's law to find the current.

$$I = \frac{V_{\text{body}}}{R_{\text{body}}} = \frac{110\,\text{V}}{950\,\Omega} = 0.116\,\text{A} \approx \boxed{0.12\,\text{A}}$$

(*b*) The description of "alternative path to ground" is a statement that the $35\Omega$ path is in parallel with the body. Thus the full 110 V is still applied across the body, and so the current is the same: $\boxed{0.12\,\text{A}}$.

(*c*) If the current is limited to a total of 1.5 A, then that current will get divided between the person and the parallel path. The voltage across the body and the parallel path will be the same, since they are in parallel.

$$V_{\text{body}} = V_{\text{alternate}} \rightarrow I_{\text{body}}R_{\text{body}} = I_{\text{alternate}}R_{\text{alternate}} = \left(I_{\text{total}} - I_{\text{body}}\right)R_{\text{alternate}} \rightarrow$$

$$I_{\text{body}} = I_{\text{total}}\frac{R_{\text{alternate}}}{\left(R_{\text{body}} + R_{\text{alternate}}\right)} = \left(1.5\,\text{A}\right)\frac{35\,\Omega}{950\,\Omega + 35\,\Omega} = 0.0533\,\text{A} \approx \boxed{53\,\text{mA}}$$

This is still a very dangerous current.

71. (*a*) If the ammeter shows no current with the closing of the switch, then points B and D must be at the same potential, because the ammeter has some small resistance. Any potential difference between points B and D would cause current to flow through the ammeter. Thus the potential drop from A to B must be the same as the drop from A to D. Since points B and D are at the same potential, the potential drop from B to C must be the same as the drop from D to C. Use these two potential relationships to find the unknown resistance.

$$V_{\text{BA}} = V_{\text{DA}} \rightarrow I_3R_3 = I_1R_1 \rightarrow \frac{R_3}{R_1} = \frac{I_1}{I_3}$$

$$V_{\text{CB}} = V_{\text{CD}} \rightarrow I_3R_x = I_1R_2 \rightarrow R_x = R_2\frac{I_1}{I_3} = \boxed{R_2R_3/R_1}$$

(*b*) $R_x = R_2\dfrac{R_3}{R_1} = \left(972\,\Omega\right)\left(\dfrac{78.6\,\Omega}{630\,\Omega}\right) = \boxed{121\,\Omega}$

72. From the solution to problem 71, the unknown resistance is given by $R_x = R_2R_3/R_1$. We use that with Eq. 25-3 to find the length of the wire.

$$R_x = R_2\frac{R_3}{R_1} = \frac{\rho L}{A} = \frac{\rho L}{\pi\left(d/2\right)^2} = \frac{4\rho L}{\pi d^2} \rightarrow$$

$$L = \frac{R_2R_3\pi d^2}{4R_1\rho} = \frac{\left(29.2\,\Omega\right)\left(3.48\,\Omega\right)\pi\left(1.22\times10^{-3}\,\text{m}\right)^2}{4\left(38.0\,\Omega\right)\left(10.6\times10^{-8}\,\Omega\bullet\text{m}\right)} = \boxed{29.5\,\text{m}}$$

$\boxed{73.}$ Divide the power by the required voltage to determine the current drawn by the hearing aid.

$$I = \frac{P}{V} = \frac{2.5\text{ W}}{4.0\text{ V}} = 0.625\text{ A}$$

Use Eq. 26-1 to calculate the terminal voltage across the three batteries for mercury and dry cells.

$$V_{\text{Hg}} = 3\left(\mathscr{E} - Ir\right) = 3\left[1.35\text{ V} - \left(0.625\text{ A}\right)\left(0.030\ \Omega\right)\right] = 3.99\text{ V}$$

$$V_{\text{D}} = 3\left(\mathscr{E} - Ir\right) = 3\left[1.50\text{ V} - \left(0.625\text{ A}\right)\left(0.35\ \Omega\right)\right] = 3.84\text{ V}$$

The terminal voltage of the mercury cell batteries is closer to the required 4.0 V than the voltage from the dry cell.

74. One way is to connect $N$ resistors in series. If each resistor can dissipate 0.5 W, then it will take 7 resistors in series to dissipate 3.5 W. Since the resistors are in series, each resistor will be 1/7 of the total resistance.

$$R = \frac{R_{eq}}{7} = \frac{3200\,\Omega}{7} = 457\,\Omega \approx 460\,\Omega$$

So connect $\boxed{\text{7 resistors of } 460\,\Omega \text{ each, rated at ½ W, in series}}$.

Or, the resistors could be connected in parallel. Again, if each resistor watt can dissipate 0.5 W, then it will take 7 resistors in parallel to dissipate 3.5 W. Since the resistors are in parallel, the equivalent resistance will be 1/7 of each individual resistance.

$$\frac{1}{R_{eq}} = 7\left(\frac{1}{R}\right) \rightarrow R = 7R_{eq} = 7(3200\,\Omega) = 22.4\,\text{k}\Omega$$

So connect $\boxed{\text{7 resistors of } 22.4\,\text{k}\Omega \text{ each, rated at ½ W, in parallel}}$.

75. To build up a high voltage, the cells will have to be put in series. 120 V is needed from a series of 0.80 V cells. Thus $\frac{120\,\text{V}}{0.80\,\text{V/cell}} = 150 \text{ cells}$ are needed to provide the desired voltage. Since these cells are all in series, their current will all be the same at 350 mA. To achieve the higher current desired, banks made of 150 cells each can be connected in parallel. Then their voltage will still be at 120 V, but the currents would add making a total of $\frac{1.3\,\text{A}}{350\times10^{-3}\,\text{A/bank}} = 3.71 \text{ banks} \approx 4 \text{ banks}$. So the total number of cells is $\boxed{\text{600 cells}}$. The panel area is $600 \text{ cells}\left(9.0\times10^{-4}\,\text{m}^2/\text{cell}\right) = \boxed{0.54\,\text{m}^2}$.

The cells should be wired in $\boxed{\text{4 banks of 150 cells in series per bank, with the banks in parallel}}$. This will produce 1.4 A at 120 V. To optimize the output, $\boxed{\text{always have the panel pointed directly at the sun}}$.

76. (*a*) If the terminal voltage is to be 3.0 V, then the voltage across $R_1$ will be 9.0 V. This can be used to find the current, which then can be used to find the value of $R_2$.

$$V_1 = IR_1 \rightarrow I = \frac{V_1}{R_1} \qquad V_2 = IR_2 \rightarrow$$

$$R_2 = \frac{V_2}{I} = R_1\frac{V_2}{V_1} = (14.5\,\Omega)\frac{3.0\,\text{V}}{9.0\,\text{V}} = 4.833\,\Omega \approx \boxed{4.8\,\Omega}$$

(*b*) If the load has a resistance of $7.0\,\Omega$, then the parallel combination of $R_2$ and the load must be used to analyze the circuit. The equivalent resistance of the circuit can be found and used to calculate the current in the circuit. Then the terminal voltage can be found from Ohm's law, using the parallel combination resistance.

$$R_{2+\text{load}} = \frac{R_2R_{\text{load}}}{R_2 + R_{\text{load}}} = \frac{(4.833\,\Omega)(7.0\,\Omega)}{11.833\,\Omega} = 2.859\,\Omega \qquad R_{eq} = 2.859\,\Omega + 14.5\,\Omega = 17.359\,\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{12.0\,\text{V}}{17.359\,\Omega} = 0.6913\,\text{A} \qquad V_T = IR_{2+\text{load}} = (0.6913\,\text{A})(2.859\,\Omega) = 1.976\,\text{V} \approx \boxed{2.0\,\text{V}}$$

The presence of the load has affected the terminal voltage significantly.

77. There are two answers because it is not known which direction the given current is flowing through the $4.0\,\text{k}\Omega$ resistor. Assume the current is to the right. The voltage across the $4.0\,\text{k}\Omega$ resistor is given by Ohm's law as $V = IR = (3.10\times10^{-3}\,\text{A})(4000\,\Omega) = 12.4\,\text{V}$. The voltage drop across the $8.0\,\text{k}\Omega$ must be the same, and the current through it is $I = \frac{V}{R} = \frac{12.4\,\text{V}}{8000\,\Omega} = 1.55\times10^{-3}\,\text{A}$. The total current in the circuit is the sum of the two currents, and so $I_{\text{tot}} = 4.65\times10^{-3}\,\text{A}$. That current can be used to find the terminal voltage of the battery. Write a loop equation, starting at the negative terminal of the unknown battery and going clockwise.

$$V_{\text{ab}} - (5000\,\Omega)I_{\text{tot}} - 12.4\,\text{V} - 12.0\,\text{V} - (1.0\,\Omega)I_{\text{tot}} \rightarrow$$
$$V_{\text{ab}} = 24.4\text{V} + (5001\,\Omega)(4.65\times10^{-3}\,\text{A}) = 47.65\,\text{V} \approx \boxed{48\,\text{V}}$$

If the current is to the left, then the voltage drop across the parallel combination of resistors is still 12.4 V, but with the opposite orientation. Again write a loop equation, starting at the negative terminal of the unknown battery and going clockwise. The current is now to the left.

$$V_{\text{ab}} + (5000\,\Omega)I_{\text{tot}} + 12.4\,\text{V} - 12.0\,\text{V} + (1.0\,\Omega)I_{\text{tot}} \rightarrow$$
$$V_{\text{ab}} = -0.4\text{V} - (5001\,\Omega)(4.65\times10^{-3}\,\text{A}) = -23.65\,\text{V} \approx \boxed{-24\,\text{V}}$$

78. The terminal voltage and current are given for two situations. Apply Eq. 26-1 to both of these situations, and solve the resulting two equations for the two unknowns.

$$V_1 = \mathscr{E} - I_1 r \;;\; V_2 = \mathscr{E} - I_2 r \rightarrow \mathscr{E} = V_1 + I_1 r = V_2 + I_2 r \rightarrow$$
$$r = \frac{V_2 - V_1}{I_1 - I_2} = \frac{47.3\,\text{V} - 40.8\,\text{V}}{7.40\,\text{A} - 2.80\,\text{A}} = 1.413\,\Omega \approx \boxed{1.4\,\Omega}$$
$$\mathscr{E} = V_1 + I_1 r = 40.8\,\text{V} + (7.40\,\text{A})(1.413\,\Omega) = \boxed{51.3\,\text{V}}$$

$\boxed{79.}$ The current in the circuit can be found from the resistance and the power dissipated. Then the product of that current and the equivalent resistance is equal to the battery voltage.

$$P = I^2 R \rightarrow I = \sqrt{\frac{P_{33}}{R_{33}}} = \sqrt{\frac{0.80\,\text{W}}{33\,\Omega}} = 0.1557\,\text{A}$$
$$R_{\text{eq}} = 33\,\Omega + \left(\frac{1}{68\,\Omega} + \frac{1}{75\,\Omega}\right)^{-1} = 68.66\,\Omega \qquad V = IR_{\text{eq}} = (0.1557\,\text{A})(68.66\,\Omega) = 10.69\,\text{V} \approx \boxed{11\,\text{V}}$$

80. If the switches are both open, then the circuit is a simple series circuit. Use Kirchhoff's loop rule to find the current in that case.

$$6.0\,\text{V} - I(50\,\Omega + 20\,\Omega + 10\,\Omega) = 0 \rightarrow I = 6.0\,\text{V}/80\,\Omega = 0.075\,\text{A}$$

If the switches are both closed, the 20-Ω resistor is in parallel with $R$. Apply Kirchhoff's loop rule to the outer loop of the circuit, with the 20-Ω resistor having the current found previously.

$$6.0\,\text{V} - I(50\,\Omega) - (0.075\,\text{A})(20\,\Omega) = 0 \rightarrow I = \frac{6.0\,\text{V} - (0.075\,\text{A})(20\,\Omega)}{50\,\Omega} = 0.090\,\text{A}$$

This is the current in the parallel combination. Since 0.075 A is in the 20-Ω resistor, 0.015 A must be in $R$. The voltage drops across $R$ and the 20-Ω resistor are the same since they are in parallel.

$$V_{20} = V_R \rightarrow I_{20}R_{20} = I_R R \rightarrow R = R_{20}\frac{I_{20}}{I_R} = (20\,\Omega)\frac{0.075\,\text{A}}{0.015\,\text{A}} = \boxed{100\,\Omega}$$

81. (*a*) We assume that the ammeter is ideal and so has 0 resistance, but that the voltmeter has resistance $R_V$. Then apply Ohm's law, using the equivalent resistance. We also assume the voltmeter is accurate, and so it is reading the voltage across the battery.

$$V = IR_{eq} = I\frac{1}{\frac{1}{R}+\frac{1}{R_V}} \rightarrow V\left(\frac{1}{R}+\frac{1}{R_V}\right) = I \rightarrow \frac{1}{R}+\frac{1}{R_V} = \frac{I}{V} \rightarrow \boxed{\frac{1}{R} = \frac{I}{V} - \frac{1}{R_V}}$$

(*b*) We now assume the voltmeter is ideal, and so has an infinite resistance, but that the ammeter has resistance $R_A$. We also assume that the voltmeter is accurate and so is reading the voltage across the battery.

$$V = IR_{eq} = I\left(R + R_A\right) \rightarrow R + R_A = \frac{V}{I} \rightarrow \boxed{R = \frac{V}{I} - R_A}$$

82. (*a*) The 12-Ω and the 25-Ω resistors are in parallel, with a net resistance $R_{1\text{-}2}$ as follows.

$$R_{1\text{-}2} = \left(\frac{1}{12\,\Omega}+\frac{1}{25\,\Omega}\right)^{-1} = 8.108\,\Omega$$

$R_{1\text{-}2}$ is in series with the 4.5-Ω resistor, for a net resistance $R_{1\text{-}2\text{-}3}$ as follows.

$$R_{1\text{-}2\text{-}3} = 4.5\,\Omega + 8.108\,\Omega = 12.608\,\Omega$$

That net resistance is in parallel with the 18-Ω resistor, for a final equivalent resistance as follows.

$$R_{eq} = \left(\frac{1}{12.608\,\Omega}+\frac{1}{18\,\Omega}\right)^{-1} = 7.415\,\Omega \approx \boxed{7.4\,\Omega}$$

(*b*) Find the current in the 18-Ω resistor by using Kirchhoff's loop rule for the loop containing the battery and the 18-Ω resistor.

$$\mathcal{E} - I_{18}R_{18} = 0 \rightarrow I_{18} = \frac{\mathcal{E}}{R_{18}} = \frac{6.0\,\text{V}}{18\,\Omega} = \boxed{0.33\,\text{A}}$$

(*c*) Find the current in $R_{1\text{-}2}$ and the 4.5-Ω resistor by using Kirchhoff's loop rule for the outer loop containing the battery and the resistors $R_{1\text{-}2}$ and the 4.5-Ω resistor.

$$\mathcal{E} - I_{1\text{-}2}R_{1\text{-}2} - I_{1\text{-}2}R_{4.5} = 0 \rightarrow I_{1\text{-}2} = \frac{\mathcal{E}}{R_{1\text{-}2}+R_{4.5}} = \frac{6.0\,\text{V}}{12.608\,\Omega} = 0.4759\,\text{A}$$

This current divides to go through the 12-Ω and 25-Ω resistors in such a way that the voltage drop across each of them is the same. Use that to find the current in the 12-Ω resistor.

$$I_{1\text{-}2} = I_{12} + I_{25} \rightarrow I_{25} = I_{1\text{-}2} - I_{12}$$

$$V_{R_{12}} = V_{R_{25}} \rightarrow I_{12}R_{12} = I_{25}R_{25} = \left(I_{1\text{-}2} - I_{12}\right)R_{25} \rightarrow$$

$$I_{12} = I_{1\text{-}2}\frac{R_{25}}{\left(R_{12}+R_{25}\right)} = \left(0.4759\,\text{A}\right)\frac{25\,\Omega}{37\,\Omega} = \boxed{0.32\,\text{A}}$$

(*d*) The current in the 4.5-Ω resistor was found above to be $I_{1\text{-}2} = 0.4759\,\text{A}$. Find the power accordingly.

$$P_{4.5} = I_{1\text{-}2}^2 R_{4.5} = \left(0.4759\,\text{A}\right)^2\left(4.5\,\Omega\right) = 1.019\,\text{W} \approx \boxed{1.0\,\text{W}}$$

83. Write Kirchhoff's loop rule for the circuit, and substitute for the current and the bulb resistance based on the bulb ratings.

$$P_{bulb} = \frac{V_{bulb}^2}{R_{bulb}} \rightarrow R_{bulb} = \frac{V_{bulb}^2}{P_{bulb}} \qquad P_{bulb} = I_{bulb}V_{bulb} \rightarrow I_{bulb} = \frac{P_{bulb}}{V_{bulb}}$$

$$\mathscr{E} - I_{bulb}R - I_{bulb}R_{bulb} = 0 \rightarrow$$

$$R = \frac{\mathscr{E}}{I_{bulb}} - R_{bulb} = \frac{\mathscr{E}}{P_{bulb}/V_{bulb}} - \frac{V_{bulb}^2}{P_{bulb}} = \frac{V_{bulb}}{P_{bulb}}\left(\mathscr{E} - V_{bulb}\right) = \frac{3.0\,\text{V}}{2.0\,\text{W}}\left(9.0\,\text{V} - 3.0\,\text{V}\right) = \boxed{9.0\,\Omega}$$

84. The equivalent resistance of the circuit is the parallel combination of the bulb and the lower portion of the potentiometer, in series with the upper portion of the potentiometer. With the slide at position *x*, the resistance of the lower portion is $xR_{var}$, and the resistance of the upper portion is $(1-x)R_{var}$. From that equivalent resistance, we find the current in the loop, the voltage across the bulb, and then the power expended in the bulb.

$$R_{parallel} = \left(\frac{1}{R_{lower}} + \frac{1}{R_{bulb}}\right)^{-1} = \frac{R_{lower}R_{bulb}}{R_{lower} + R_{bulb}} = \frac{xR_{var}R_{bulb}}{xR_{var} + R_{bulb}}$$

$$R_{eq} = (1-x)R_{var} + R_{parallel} \ ; \ I_{loop} = \frac{\mathscr{E}}{R_{eq}} \ ; \ V_{bulb} = I_{loop}R_{parallel} \ ; \ P_{bulb} = \frac{V_{bulb}^2}{R_{bulb}}$$

(*a*) Consider the case in which $x = 1.00$. In this case, the full battery potential is across the bulb, and so it is obvious that $V_{bulb} = 120\,\text{V}$. Thus $P_{bulb} = \frac{V_{bulb}^2}{R_{bulb}} = \frac{(120\,\text{V})^2}{240\,\Omega} = \boxed{60\,\text{W}}$.

(*b*) Consider the case in which $x = 0.65$.

$$R_{parallel} = \frac{xR_{var}R_{bulb}}{xR_{var} + R_{bulb}} = \frac{(0.65)(150\,\Omega)(240\,\Omega)}{(0.65)(150\,\Omega) + 240\,\Omega} = 69.33\,\Omega$$

$$R_{eq} = (1-x)R_{var} + R_{parallel} = (0.35)(150\,\Omega) + 69.33\,\Omega = 121.83\,\Omega$$

$$I_{loop} = \frac{\mathscr{E}}{R_{eq}} = \frac{120\,\text{V}}{121.83\,\Omega} = 0.9850\,\text{A} \ ; \ V_{bulb} = (0.9850\,\text{A})(69.33\,\Omega) = 68.29\,\text{V}$$

$$P_{bulb} = \frac{(68.29\,\text{V})^2}{240\,\Omega} = 19.43\,\text{W} \approx \boxed{19\,\text{W}}$$

(*c*) Consider the case in which $x = 0.35$.

$$R_{parallel} = \frac{xR_{var}R_{bulb}}{xR_{var} + R_{bulb}} = \frac{(0.35)(150\,\Omega)(240\,\Omega)}{(0.35)(150\,\Omega) + 240\,\Omega} = 43.08\,\Omega$$

$$R_{eq} = (1-x)R_{var} + R_{parallel} = (0.65)(150\,\Omega) + 69.33\,\Omega = 140.58\,\Omega$$

$$I_{loop} = \frac{\mathscr{E}}{R_{eq}} = \frac{120\,\text{V}}{140.58\,\Omega} = 0.8536\,\text{A} \ ; \ V_{bulb} = (0.8536\,\text{A})(43.08\,\Omega) = 36.77\,\text{V}$$

$$P_{bulb} = \frac{(36.77\,\text{V})^2}{240\,\Omega} = 5.63\,\text{W} \approx \boxed{5.6\,\text{W}}$$

85. (*a*) When the galvanometer gives a null reading, no current is passing through the galvanometer or the emf that is being measured. All of the current is flowing through the slide wire resistance. Application of the loop rule to the lower loop gives $\mathcal{E} - IR = 0,$ since there is no current through the emf to cause voltage drop across any internal resistance. The amount of current flowing through the slide wire resistor will be the same no matter what emf is used since no current is flowing through the lower loop. Apply this relationship to the two emf's.

$$\mathcal{E}_x - IR_x = 0 \ ; \ \mathcal{E}_s - IR_s = 0 \ \rightarrow \ ; \ I = \frac{\mathcal{E}_x}{R_x} = \frac{\mathcal{E}_s}{R_s} \ \rightarrow \ \boxed{\mathcal{E}_x = \left(\frac{R_x}{R_s}\right)\mathcal{E}_s}$$

(*b*) Use the equation derived above. We use the fact that the resistance is proportional to the length of the wire, by Eq. 25-3, $R = \rho \ell / A$.

$$\mathcal{E}_x = \left(\frac{R_x}{R_s}\right)\mathcal{E}_s = \left(\frac{\rho\frac{\ell_x}{A}}{\rho\frac{\ell_s}{A}}\right)\mathcal{E}_s = \left(\frac{\ell_x}{\ell_s}\right)\mathcal{E}_s = \left(\frac{45.8\,\text{cm}}{33.6\,\text{cm}}\right)(1.0182\,\text{V}) = \boxed{1.39\,\text{V}}$$

(*c*) If there is current in the galvanometer, then the voltage between points A and C is uncertainty by the voltage drop across the galvanometer, which is $V_G = I_G R_G = \left(0.012\times10^{-3}\,\text{A}\right)\left(35\,\Omega\right)$ $= \boxed{4.2\times10^{-4}\,\text{V}}$. The uncertainty might of course be more than this, due to uncertainties compounding from having to measure distance for both the standard emf and the unknown emf. Measuring the distances also has some uncertainty associated with it.

(*d*) Using this null method means that the (unknown) internal resistance of the unknown emf does not enter into the calculation. No current passes through the unknown emf, and so there is no voltage drop across that internal resistance.

86. (*a*) In normal operation, the capacitor is fully charged by the power supply, and so the capacitor voltage is the same as the power supply voltage, and there will be no current through the resistor. If there is an interruption, the capacitor voltage will decrease exponentially – it will discharge. We want the voltage across the capacitor to be at 75% of the full voltage after 0.20 s. Use Eq. 26-9b for the discharging capacitor.

$$V = V_0 e^{-t/RC} \ ; \ 0.75V_0 = V_0 e^{-(0.20\text{s})/RC} \ \rightarrow \ 0.75 = e^{-(0.20\text{s})/RC} \ \rightarrow$$

$$R = \frac{-(0.20\,\text{s})}{C\ln(0.75)} = \frac{-(0.20\,\text{s})}{\left(8.5\times10^{-6}\,\text{F}\right)\ln(0.75)} = 81790\,\Omega \approx \boxed{82\,\text{k}\Omega}$$

(*b*) When the power supply is functioning normally, there is no voltage across the resistor, so the device should NOT be connected between terminals a and b. If the power supply is not functioning normally, there will be a larger voltage across the capacitor than across the capacitor–resistor combination, since some current might be present. This current would result in a voltage drop across the resistor. To have the highest voltage in case of a power supply failure, the device should be connected between terminals $\boxed{\text{b and c}}$.

87. Note that, based on the significant figures of the resistors, that the 1.0-Ω resistor will not change the equivalent resistance of the circuit as determined by the resistors in the switch bank.

Case 1: $n = 0$ switch closed. The effective resistance of the circuit is $16.0\,\text{k}\Omega$. The current in the circuit is $I = \dfrac{16\,\text{V}}{16.0\,\text{k}\Omega} = 1.0\,\text{mA}$. The voltage across the 1.0-Ω resistor is $V = IR$ $= (1.0\,\text{mA})(1.0\,\Omega) = \boxed{1.0\,\text{mV}}$.

Case 2: $n = 1$ switch closed. The effective resistance of the circuit is $8.0\,\text{k}\Omega$. The current in the circuit is $I = \dfrac{16\,\text{V}}{8.0\,\text{k}\Omega} = 2.0\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$ $= (2.0\,\text{mA})(1.0\,\Omega) = \boxed{2.0\,\text{mV}}$.

Case 3: $n = 2$ switch closed. The effective resistance of the circuit is $4.0\,\text{k}\Omega$. The current in the circuit is $I = \dfrac{16\,\text{V}}{4.0\,\text{k}\Omega} = 4.0\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$ $= (4.0\,\text{mA})(1.0\,\Omega) = \boxed{4.0\,\text{mV}}$.

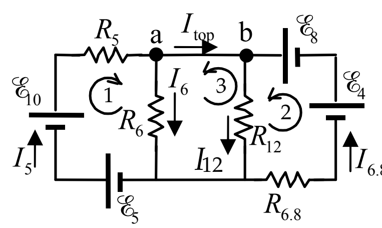
Case 4: $n = 3$ and $n = 1$ switches closed. The effective resistance of the circuit is found by the parallel combination of the $2.0\text{-k}\Omega$ and $8.0\text{-k}\Omega$ resistors.

$$R_{\text{eq}} = \left(\frac{1}{2.0\,\text{k}\Omega} + \frac{1}{8.0\,\text{k}\Omega}\right)^{-1} = 1.6\,\text{k}\Omega$$

The current in the circuit is $I = \dfrac{16\,\text{V}}{1.6\,\text{k}\Omega} = 10\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR = (10\,\text{mA})(1.0\,\Omega) = \boxed{10\,\text{mV}}$.

So in each case, the voltage across the $1.0\text{-}\Omega$ resistor, if taken in mV, is the expected analog value corresponding to the digital number set by the switches.

88. We have labeled the resistors and the currents through the resistors with the value of the specific resistance, and the emf's with the appropriate voltage value. We apply the junction rule to points a and b, and then apply the loop rule to loops 1, 2, and 3. This enables us to solve for all of the currents.




$$I_5 = I_6 + I_{\text{top}} \ ; \ I_{\text{top}} + I_{6.8} = I_{12} \ \rightarrow \ I_5 - I_6 = I_{12} - I_{6.8} \ \rightarrow$$

$$I_5 + I_{6.8} = I_{12} + I_6 \qquad [1]$$

$$\mathscr{E}_5 + \mathscr{E}_{10} - I_5R_5 - I_6R_6 = 0 \qquad [2] \ (\text{loop } 1)$$

$$\mathscr{E}_4 + \mathscr{E}_8 - I_{12}R_{12} - I_{6.8}R_{6.8} = 0 \qquad [3] \ (\text{loop } 2)$$

$$I_{12}R_{12} - I_6R_6 = 0 \qquad [4] \ (\text{loop } 3)$$

Use Eq. 4 to substitute $I_6R_6 = I_{12}R_{12}$ and $I_6 = I_{12}\dfrac{R_{12}}{R_6} = 2I_{12}$. Also combine the emf's by adding the voltages.

$$I_5 + I_{6.8} = 3I_{12} \ [1] \ ; \ \mathscr{E}_{15} - I_5R_5 - I_{12}R_{12} = 0 \ [2] \ ; \ \mathscr{E}_{12} - I_{12}R_{12} - I_{6.8}R_{6.8} = 0 \ [3]$$

Use Eq. 1 to eliminate $I_{6.8}$ by $I_{6.8} = 3I_{12} - I_5$.

$$\mathscr{E}_{15} - I_5R_5 - I_{12}R_{12} = 0 \qquad [2]$$

$$\mathscr{E}_{12} - I_{12}R_{12} - (3I_{12} - I_5)R_{6.8} = 0 \ \rightarrow \ \mathscr{E}_{12} - I_{12}(R_{12} + 3R_{6.8}) + I_5R_{6.8} = 0 \qquad [3]$$

Use Eq. 2 to eliminate $I_5$ by $I_5 = \dfrac{\mathscr{E}_{15} - I_{12}R_{12}}{R_5}$, and then solve for $I_{12}$.

$$\mathscr{E}_{12} - I_{12}(R_{12} + 3R_{6.8}) + \left[\frac{\mathscr{E}_{15} - I_{12}R_{12}}{R_5}\right]R_{6.8} = 0 \ \rightarrow$$

$$I_{12}=\frac{\mathscr{E}_{12}R_5+\mathscr{E}_{15}R_{6.8}}{R_{12}R_5+3R_{6.8}R_5+R_{12}R_{6.8}}=\frac{(12.00\,\text{V})(5.00\,\Omega)+(15.00\,\text{V})(6.800\,\Omega)}{(12.00\,\Omega)(5.00\,\Omega)+3(6.800\,\Omega)(5.00\,\Omega)+(12.00\,\Omega)(6.800\,\Omega)}$$

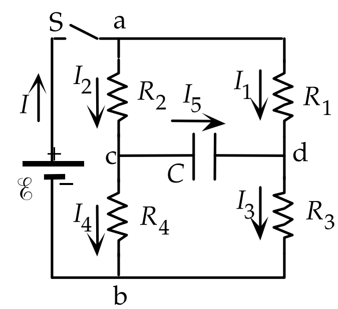
$$=0.66502\,\text{A}\approx\boxed{0.665\,\text{A}=I_{12}}$$

$$I_5=\frac{\mathscr{E}_{15}-I_{12}R_{12}}{R_5}=\frac{(15.00\,\text{V})-(0.66502\,\text{A})(12.00\,\Omega)}{(5.00\,\Omega)}=1.40395\,\text{A}\approx\boxed{1.40\,\text{A}=I_5}$$

$$I_{6.8}=3I_{12}-I_5=3(0.66502\,\text{A})-1.40395\,\text{A}=0.59111\,\text{A}\approx\boxed{0.591\,\text{A}=I_{6.8}}$$

$$I_6=2I_{12}=2(0.66502\,\text{A})\approx\boxed{1.33\,\text{A}=I_6}$$

89. (*a*) After the capacitor is fully charged, there is no current through it, and so it behaves like an "open" in the circuit. In the circuit diagram, this means that $I_5=0$, $I_1=I_3$, and $I_2=I_4$. Write loop equations for the leftmost loop and the outer loop in order to solve for the currents.




$$\mathscr{E}-I_2(R_2+R_4)=0\ \rightarrow\ I_2=\frac{\mathscr{E}}{R_2+R_4}=\frac{12.0\,\text{V}}{10.0\,\Omega}=1.20\,\text{A}$$

$$\mathscr{E}-I_1(R_1+R_3)=0\ \rightarrow\ I_1=\frac{\mathscr{E}}{R_1+R_3}=\frac{12.0\,\text{V}}{15.0\,\Omega}=0.800\,\text{A}$$

Use these currents to find the voltage at points c and d, which will give the voltage across the capacitor.

$$V_c=\mathscr{E}-I_2R_2=12.0\,\text{V}-(1.20\,\text{A})(1.0\,\Omega)=10.8\,\text{V}$$

$$V_d=\mathscr{E}-I_1R_1=12.0\,\text{V}-(0.800\,\text{A})(10.0\,\Omega)=4.00\,\text{V}$$

$$V_{cd}=10.8\,\text{V}-4.00\,\text{V}=\boxed{6.8\,\text{V}}\ ;\ Q=CV=(2.2\,\mu\text{F})(6.8\,\text{V})=14.96\,\mu\text{C}\approx\boxed{15\,\mu\text{C}}$$

(*b*) When the switch is opened, the emf is taken out of the circuit. Then we have the capacitor discharging through an equivalent resistance. That equivalent resistance is the series combination of $R_1$ and $R_2$, in parallel with the series combination of $R_3$ and $R_4$. Use the expression for discharging a capacitor, Eq. 26-9a.

$$R_{eq}=\left(\frac{1}{R_1+R_2}+\frac{1}{R_3+R_4}\right)^{-1}=\left(\frac{1}{11.0\,\Omega}+\frac{1}{14.0\,\Omega}\right)^{-1}=6.16\,\Omega$$

$$Q=Q_0e^{-t/R_{eq}C}=0.030Q_0\ \rightarrow$$

$$t=-R_{eq}C\ln(0.030)=-(6.16\,\Omega)(2.2\times10^{-6}\,\text{F})\ln(0.030)=\boxed{4.8\times10^{-5}\,\text{s}}$$

90. (*a*) The time constant of the *RC* circuit is given by Eq. 26-7.

$$\tau=RC=(33.0\ \text{k}\Omega)(4.00\ \mu\text{F})=132\ \text{ms}$$

During the charging cycle, the charge and the voltage on the capacitor increases exponentially as in Eq. 26-6b. We solve this equation for the time it takes the circuit to reach 90.0 V.

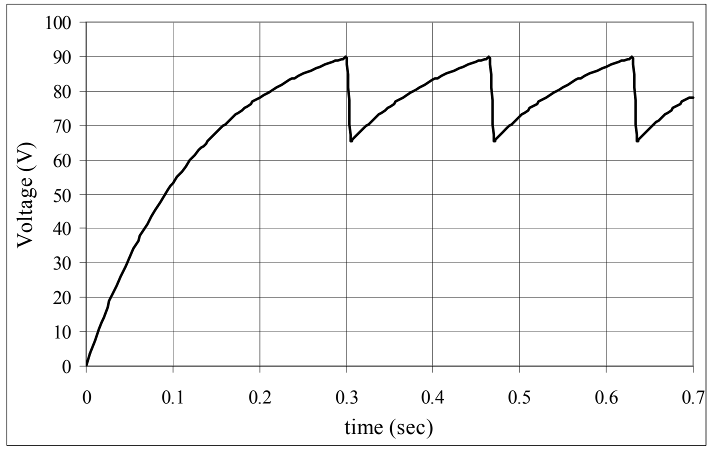
$$V=\mathscr{E}\left(1-e^{-t/\tau}\right)\rightarrow t=-\tau\ln\left(1-\frac{V}{\mathscr{E}}\right)=-(132\ \text{ms})\ln\left(1-\frac{90.0\ \text{V}}{100.0\ \text{V}}\right)=\boxed{304\ \text{ms}}$$

(*b*) When the neon bulb starts conducting, the voltage on the capacitor drops quickly to 65.0 V and then starts charging. We can find the recharging time by first finding the time for the capacitor to reach 65.0 V, and then subtract that time from the time required to reach 90.0 V.

$$t = -\tau \ln\left(1-\frac{V}{\mathcal{E}}\right) = -(132\text{ ms})\ln\left(1-\frac{65.0\text{ V}}{100.0\text{ V}}\right) = 139\text{ ms}$$

$$\Delta t = 304\text{ ms} - 139\text{ ms} = 165\text{ ms} \;;\; t_2 = 304\,\text{ms} + 165\,\text{ms} = \boxed{469\,\text{ms}}$$

(*c*) The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH26.XLS,” on tab “Problem 26.90c.”




91. We represent the 10.00-MΩ resistor by $R_{10}$, and the resistance of the voltmeter as $R_V$. In the first configuration, we find the equivalent resistance $R_{eqA}$, the current in the circuit $I_A$, and the voltage drop across $R$.

$$R_{eqA} = R + \frac{R_{10}R_V}{R_{10}+R_V} \;;\; I_A = \frac{\mathcal{E}}{R_{eqA}} \;;\; V_R = I_A R = \mathcal{E} - V_A \;\rightarrow\; \mathcal{E}\frac{R}{R_{eqA}} = \mathcal{E} - V_A$$

In the second configuration, we find the equivalent resistance $R_{eqB}$, the current in the circuit $I_B$, and the voltage drop across $R_{10}$.

$$R_{eqB} = R_{10} + \frac{RR_V}{R+R_V} \;;\; I_B = \frac{\mathcal{E}}{R_{eqB}} \;;\; V_{R_{10}} = I_B R_{10} = \mathcal{E} - V_B \;\rightarrow\; \mathcal{E}\frac{R_{10}}{R_{eqB}} = \mathcal{E} - V_B$$

We now have two equations in the two unknowns of $R$ and $R_V$. We solve the second equation for $R_V$ and substitute that into the first equation. We are leaving out much of the algebra in this solution.

$$\mathcal{E}\frac{R}{R_{eqA}} = \mathcal{E}\frac{R}{R + \dfrac{R_{10}R_V}{R_{10}+R_V}} = \mathcal{E} - V_A \;;$$

$$\mathcal{E}\frac{R_{10}}{R_{eqB}} = \mathcal{E}\frac{R_{10}}{R_{10} + \dfrac{RR_V}{R+R_V}} = \mathcal{E} - V_B \;\rightarrow\; R_V = \frac{V_B R_{10} R}{(\mathcal{E}R - V_B R_{10} - V_B R)}$$

$$\mathcal{E} - V_A = \mathcal{E}\frac{R}{R + \dfrac{R_{10}R_V}{R_{10}+R_V}} = \mathcal{E}\frac{R}{R + \dfrac{R_{10}\left[\dfrac{V_B R_{10} R}{(\mathcal{E}R - V_B R_{10} - V_B R)}\right]}{R_{10} + \left[\dfrac{V_B R_{10} R}{(\mathcal{E}R - V_B R_{10} - V_B R)}\right]}} \;\rightarrow$$

$$R = \frac{V_B}{V_A}R_{10} = \frac{7.317\,\text{V}}{0.366\,\text{V}}(10.00\,\text{M}\Omega) = 199.92\,\text{M}\Omega \approx \boxed{200\,\text{M}\Omega}\;\;(3\text{ sig. fig.})$$

92. Let the internal resistance of the voltmeter be indicated by $R_V$, and let the 15-MΩ resistance be indicated by $R_{15}$. We calculate the current through the probe and voltmeter as the voltage across the probe divided by the equivalent resistance of the problem and the voltmeter. We then set the voltage drop across the voltmeter equal to the product of the current and the parallel combination of $R_V$ and $R_{15}$. This can be solved for the unknown resistance.

$$I = \frac{V}{R + \frac{R_{15}R_V}{R_{15}+R_V}} \; ; \; V_V = I\frac{R_{15}R_V}{R_{15}+R_V} = \frac{V}{R + \frac{R_{15}R_V}{R_{15}+R_V}}\frac{R_{15}R_V}{R_{15}+R_V} = \frac{VR_{15}R_V}{R\left(R_{15}+R_V\right)+R_{15}R_V} \rightarrow$$

$$R = \frac{\frac{V}{V_V}R_{15}R_V - R_{15}R_V}{\left(R_{15}+R_V\right)} = \frac{R_{15}R_V}{\left(R_{15}+R_V\right)}\left(\frac{V}{V_V}-1\right) = \frac{(15\,\text{M}\Omega)(10\,\text{M}\Omega)}{(25\,\text{M}\Omega)}\left(\frac{50{,}000\,\text{V}}{50\,\text{V}}-1\right)$$

$$= 5994\,\text{M}\Omega \approx 6000\,\text{M}\Omega = \boxed{6\,\text{G}\Omega}$$

93. The charge and current are given by Eq. 26-6a and Eq. 26-8, respectively.

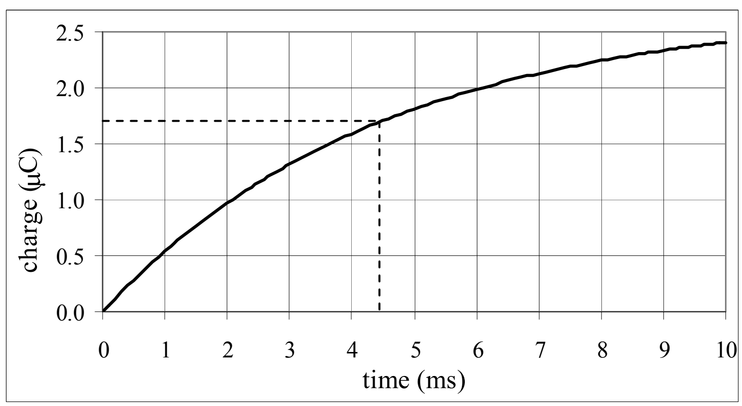
$$Q = C\mathscr{E}\left(1-e^{-t/RC}\right) \; ; \; I = \frac{\mathscr{E}}{R}e^{-t/RC} \; ; \; \tau = RC = \left(1.5\times10^4\,\Omega\right)\left(3.0\times10^{-7}\,\text{F}\right) = 4.5\times10^{-3}\,\text{s}$$

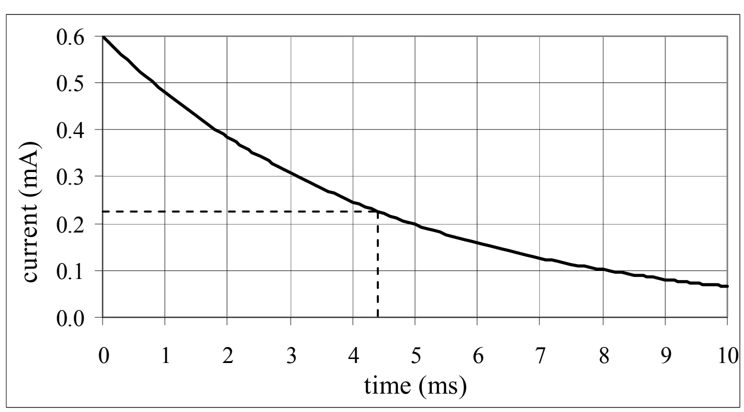
$$0.63Q_{\text{final}} = 0.63C\mathscr{E} = 0.63\left(3.0\times10^{-7}\,\text{F}\right)(9.0\,\text{V}) = 1.70\times10^{-6}\,\text{C}$$

$$0.37I_{\text{initial}} = 0.37\frac{\mathscr{E}}{R} = 0.37\left(\frac{9.0\,\text{V}}{1.5\times10^4\,\Omega}\right) = 2.22\times10^{-4}\,\text{A}$$

The graphs are shown. The times for the requested values are about 4.4 or 4.5 ms, about one time constant, within the accuracy of estimation on the graphs.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH26.XLS," on tab "Problem 26.93."



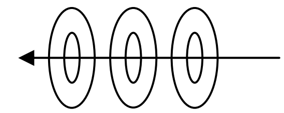



# CHAPTER 27: Magnetism

## Responses to Questions

1. The compass needle aligns itself with the local magnetic field of the Earth, and the Earth's magnetic field lines are not always parallel to the surface of the Earth.

2. The magnetic field lines are concentric circles around the wire. With the current running to the left, the field is directed counterclockwise when looking from the left end. So, the field goes into the page above the wire and comes out of the page below the wire.




3. The force is downward. The field lines point from the north pole to the south pole, or left to right. Use the right hand rule. Your fingers point in the direction of the current (away from you). Curl them in the direction of the field (to the right). Your thumb points in the direction of the force (downward).

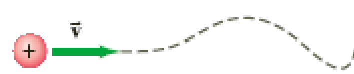
4. $\vec{\mathbf{F}}$ is always perpendicular to both $\vec{\mathbf{B}}$ and $\vec{\ell}$. $\vec{\mathbf{B}}$ and $\vec{\ell}$ can be at any angle with respect to each other.

5. Alternating currents will have little effect on the compass needle, due to the rapid change of the direction of the current and of the magnetic field surrounding it. Direct currents will deflect a compass needle. The deflection depends on the magnitude and direction of the current and the distance from the current to the compass. The effect on the compass decreases with increasing distance from the wire.

6. The kinetic energy of the particle will stay the same. The magnetic force on the particle will be perpendicular to the particle's velocity vector and so will do no work on the particle. The force will change the direction of the particle's velocity but not the speed.

7. Positive particle in the upper left: force is downward toward the wire. Negative particle in the upper right: force is to the left. Positive particle in the lower right: force is to the left. Negative particle in the lower left: force is upward toward the wire.

8. In the areas where the particle's path is curving up towards the top of the page, the magnetic field is directed into the page. Where the particle's path curves downward towards the bottom of the page, the magnetic field is directed out of the page. Where the particle is moving in a straight line, the magnetic field direction is parallel or anti-parallel to the particle's velocity. The strength of the magnetic field is greatest where the radius of curvature of the path is the smallest.




9. (*a*) Near one pole of a very long bar magnet, the magnetic field is proportional to $1/r^2$.
   (*b*) Far from the magnet as a whole, the magnetic field is proportional to $1/r^3$.

10. The picture is created when moving charged particles hit the back of the screen. A strong magnet held near the screen can deflect the particles from their intended paths, and thus distort the picture. If the magnet is strong enough, it is possible to deflect the particles so much that they do not even reach the screen, and the picture "goes black."

11. The negative particle will curve down (toward the negative plate) if $v > E/B$ because the magnetic force (down) will be greater than the electric force (up). If $v < E/B$ the negative particle will curve up toward the positive plate because the electric force will be greater than the magnetic force. The motion of a positive particle would be exactly opposite that of a negative particle.

12. No, you cannot set a resting electron into motion with a static magnetic field. In order for a charged particle to experience a magnetic force, it must already have a velocity with a component perpendicular to the magnetic field: $F = qvB\sin\theta$. If $v = 0$, $F = 0$. Yes, you can set an electron into motion with an electric field. The electric force on a charged particle does not depend on velocity: $F = qE$.

13. The particle will move in an elongating helical path in the direction of the electric field (for a positive charge). The radius of the helix will remain constant.

14. Consider a positive ion. It will experience a force downward due to the applied electric field. Once it begins moving downward, it will then experience a force out (in the direction of the red arrow) because of its motion in the magnetic field. A negative ion will experience a force up due to the electric field and then, because it is a negative particle moving up in the magnetic field directed to the right, it will experience a force out. The positive and negative ions therefore each feel a force in the same direction.

15. The beam is deflected to the right. The current in the wire creates a magnetic field into the page surrounding the beam of electrons. This results in a magnetic force on the negative particles that is to the right.

16. Yes. One possible situation is that the magnetic field is parallel or anti-parallel to the velocity of the charged particle. In this case, the magnetic force would be zero, and the particle would continue moving in a straight line. Another possible situation is that there is an electric field with a magnitude and direction (perpendicular to the magnetic field) such that the electric and magnetic forces on the particle cancel each other out. The net force would be zero and the particle would continue moving in a straight line.

17. No. A charged particle may be deflected sideways by an electric field if a component of its velocity is perpendicular to the field.

18. If the direction of the velocity of the electrons is changing but their speed is not, then they are being deflected by a magnetic field only, and their path will be circular or helical. If the speed of the electrons is changing but the direction is not, then they are being accelerated by an electric field only. If both speed and direction are changing, the particles are possibly being deflected by both magnetic and electric fields, or they are being deflected by an electric field that is not parallel to the initial velocity of the particles. In the latter case, the component of the electron velocity antiparallel to the field direction will continue to increase, and the component of the electron velocity perpendicular to the field direction will remain constant. Therefore, the electron will asymptotically approach a straight path in the direction opposite the field direction. If the particles continue with a circular component to their path, there must be a magnetic field present.

19. Use a small current-carrying coil or solenoid for the compass needle.

20. Suspend the magnet in a known magnetic field so that it is aligned with the field and free to rotate. Measure the torque necessary to rotate the magnet so that it is perpendicular to the field lines. The magnetic moment will be the torque divided by the magnetic field strength. $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$ so $\tau = \mu B$ when the magnetic moment and the field are perpendicular.

21. (*a*) If the plane of the current loop is perpendicular to the field such that the direction of $\vec{\mathbf{A}}$ is parallel to the field lines, the loop will be in stable equilibrium. Small displacements from this position will result in a torque that tends to return the loop to this position.
    (*b*) If the plane of the current loop is perpendicular to the field such that the direction of $\vec{\mathbf{A}}$ is anti-parallel to the field lines, the loop will be in unstable equilibrium.

22. The charge carriers are positive. Positive particles moving to the right in the figure will experience a magnetic force into the page, or toward point *a*. Therefore, the positive charge carriers will tend to move toward the side containing *a*; this side will be at a higher potential than the side with point *b*.

23. The distance 2*r* to the singly charged ions will be twice the distance to the doubly charged ions.

## Solutions to Problems

1. (*a*) Use Eq. 27-1 to calculate the force with an angle of 90° and a length of 1 meter.

$$F = I\ell B\sin\theta \;\rightarrow\; \frac{F}{\ell} = IB\sin\theta = (9.40\,\text{A})(0.90\,\text{T})\sin 90° = \boxed{8.5\,\text{N/m}}$$

(*b*) $\dfrac{F}{\ell} = IB\sin\theta = (9.40\,\text{A})(0.90\,\text{T})\sin 35.0° = \boxed{4.9\,\text{N/m}}$

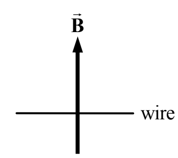
2. Use Eq. 27-1 to calculate the force.

$$F = I\ell B\sin\theta = (150\,\text{A})(240\,\text{m})(5.0\times10^{-5}\,\text{T})\sin 68° = \boxed{1.7\,\text{N}}$$

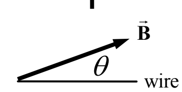
3. The dip angle is the angle between the Earth's magnetic field and the current in the wire. Use Eq. 27-1 to calculate the force.

$$F = I\ell B\sin\theta = (4.5\,\text{A})(1.6\,\text{m})(5.5\times10^{-5}\,\text{T})\sin 41° = \boxed{2.6\times10^{-4}\,\text{N}}$$

4. To have the maximum force, the current must be perpendicular to the magnetic field, as shown in the first diagram. Use $\frac{F}{\ell} = 0.25\frac{F_{max}}{\ell}$ to find the angle between the wire and the magnetic field, illustrated in the second diagram.




$$\frac{F}{\ell} = 0.25\frac{F_{max}}{\ell} \;\rightarrow\; IB\sin\theta = 0.25IB \;\rightarrow\; \theta = \sin^{-1}0.25 = \boxed{14°}$$




5. (*a*) By the right hand rule, the magnetic field must be pointing up, and so the top pole face must be a $\boxed{\text{South pole}}$.
    (*b*) Use Eq. 27-2 to relate the maximum force to the current. The length of wire in the magnetic field is equal to the diameter of the pole faces.

$$F_{max} = I\ell B \rightarrow I = \frac{F_{max}}{\ell B} = \frac{\left(7.50\times10^{-2}\,\text{N}\right)}{\left(0.100\,\text{m}\right)\left(0.220\,\text{T}\right)} = 3.4091\,\text{A} \approx \boxed{3.41\,\text{A}}$$

(*c*) Multiply the maximum force by the sine of the angle between the wire and the magnetic field.

$$F = F_{max}\sin\theta = \left(7.50\times10^{-2}\,\text{N}\right)\sin 80.0^\circ = \boxed{7.39\times10^{-2}\,\text{N}}$$

6. The magnetic force must be equal in magnitude to the force of gravity on the wire. The maximum magnetic force is applicable since the wire is perpendicular to the magnetic field. The mass of the wire is the density of copper times the volume of the wire.

$$F_B = mg \rightarrow I\ell B = \rho\pi\left(\tfrac{1}{2}d\right)^2 \ell g \rightarrow$$

$$I = \frac{\rho\pi d^2 g}{4B} = \frac{\left(8.9\times10^3\,\text{kg/m}^3\right)\pi\left(1.00\times10^{-3}\,\text{m}\right)^2\left(9.80\,\text{m/s}^2\right)}{4\left(5.0\times10^{-5}\,\text{T}\right)} = \boxed{1400\,\text{A}}$$

This answer does not seem feasible. The current is very large, and the resistive heating in the thin copper wire would probably melt it.

7. We find the force using Eq. 27-3, where the vector length is broken down into two parts: the portion along the z-axis and the portion along the line *y=2x*.

$$\vec{\boldsymbol{\ell}}_1 = -0.250\,\text{m}\,\hat{\mathbf{k}} \qquad \vec{\boldsymbol{\ell}}_2 = 0.250\,\text{m}\left(\frac{\hat{\mathbf{i}}+2\hat{\mathbf{j}}}{\sqrt{5}}\right)$$

$$\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = I\left(\vec{\boldsymbol{\ell}}_1+\vec{\boldsymbol{\ell}}_2\right)\times\vec{\mathbf{B}} = \left(20.0\,\text{A}\right)\left(0.250\,\text{m}\right)\left(-\hat{\mathbf{k}}+\frac{\hat{\mathbf{i}}+2\hat{\mathbf{j}}}{\sqrt{5}}\right)\times\left(0.318\hat{\mathbf{i}}\,\text{T}\right)$$

$$= \left(1.59\,\text{N}\right)\left(-\hat{\mathbf{k}}\times\hat{\mathbf{i}}+\frac{2}{\sqrt{5}}\hat{\mathbf{j}}\times\hat{\mathbf{i}}\right) = -\left(1.59\hat{\mathbf{j}}+1.42\hat{\mathbf{k}}\right)\text{N}$$

$$F = \left|\vec{\mathbf{F}}\right| = \sqrt{1.59^2+1.42^2}\,\text{N} = \boxed{2.13\,\text{N}}$$

$$\theta = \tan^{-1}\left(\frac{-1.42\,\text{N}}{-1.59\,\text{N}}\right) = \boxed{41.8^\circ\text{ below the negative y-axis}}$$

8. We find the force per unit length from Eq. 27-3. Note that while the length is not known, the direction is given, and so $\vec{\boldsymbol{\ell}} = \ell\hat{\mathbf{i}}$.

$$\vec{\mathbf{F}}_B = I\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = I\ell\hat{\mathbf{i}}\times\vec{\mathbf{B}} \rightarrow$$

$$\frac{\vec{\mathbf{F}}_B}{\ell} = I\hat{\mathbf{i}}\times\vec{\mathbf{B}} = \left(3.0\,\text{A}\right)\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 0 & 0\\ 0.20\,\text{T} & -0.36\,\text{T} & 0.25\,\text{T}\end{vmatrix} = \left(-0.75\hat{\mathbf{j}}-1.08\hat{\mathbf{k}}\right)\text{N/m}\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)$$

$$= \boxed{-\left(7.5\hat{\mathbf{j}}+11\hat{\mathbf{k}}\right)\times10^{-3}\,\text{N/cm}}$$

9. We find the net force on the loop by integrating the infinitesimal force on each infinitesimal portion of the loop within the magnetic field. The infinitesimal force is found using Eq. 27-4 with the current in an infinitesimal portion of the loop given by $Id\vec{\boldsymbol{\ell}} = I\left(-\cos\theta\hat{\mathbf{i}}+\sin\theta\hat{\mathbf{j}}\right)rd\theta$.

$$\vec{\mathbf{F}} = \int Id\vec{\boldsymbol{\ell}}\times\vec{B} = I\int_{\theta_0}^{2\pi-\theta_0}\left(-\cos\theta\hat{\mathbf{i}}+\sin\theta\hat{\mathbf{j}}\right)rd\theta\times B_0\hat{\mathbf{k}} = IB_0 r\int_{\theta_0}^{2\pi-\theta_0}\left(\cos\theta\hat{\mathbf{j}}+\sin\theta\hat{\mathbf{i}}\right)d\theta$$

$$= IB_0 r\left(\sin\theta\hat{\mathbf{j}} - \cos\theta\hat{\mathbf{i}}\right)\Big|_{\theta_0}^{2\pi-\theta_0} = IB_0 r\left[\sin(2\pi-\theta_0)\hat{\mathbf{j}} - \sin\theta_0\hat{\mathbf{j}} - \cos(2\pi-\theta_0)\hat{\mathbf{i}} + \cos\theta_0\hat{\mathbf{i}}\right]$$

$$= \boxed{-2IB_0 r\sin\theta_0\hat{\mathbf{j}}}$$

The trigonometric identities $\sin(2\pi-\theta) = -\sin\theta$ and $\cos(2\pi-\theta) = \cos\theta$ are used to simplify the solution.

10. We apply Eq. 27-3 to each circumstance, and solve for the magnetic field. Let $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$.

For the first circumstance, $\vec{\boldsymbol{\ell}} = \ell\hat{\mathbf{i}}$.

$$\vec{\mathbf{F}}_{\mathrm{B}} = I\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = (8.2\,\mathrm{A})\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 2.0\,\mathrm{m} & 0 & 0\\ B_x & B_y & B_z\end{vmatrix} = (-16.4\,\mathrm{A{\bullet}m})B_z\hat{\mathbf{j}} + (16.4\,\mathrm{A{\bullet}m})B_y\hat{\mathbf{k}} = (-2.5\hat{\mathbf{j}})\,\mathrm{N} \;\rightarrow$$

$$B_y = 0\,;\;(-16.4\,\mathrm{A{\bullet}m})B_z = -2.5\,\mathrm{N} \;\rightarrow\; B_z = \frac{2.5\,\mathrm{N}}{16.4\,\mathrm{A{\bullet}m}} = 0.1524\,\mathrm{T}\,;\;\; B_x \text{ unknown}$$

For the second circumstance, $\vec{\boldsymbol{\ell}} = \ell\hat{\mathbf{j}}$.

$$\vec{\mathbf{F}}_{\mathrm{B}} = I\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = (8.2\,\mathrm{A})\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 0 & 2.0\,\mathrm{m} & 0\\ B_x & 0 & 0.1524\,\mathrm{T}\end{vmatrix} = (2.5\,\mathrm{N})\hat{\mathbf{i}} + (-16.4\,\mathrm{A{\bullet}m})B_x\hat{\mathbf{k}} = (2.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{k}})\,\mathrm{N} \;\rightarrow$$

$$(-16.4\,\mathrm{A{\bullet}m})B_x = -5.0\,\mathrm{N} \;\rightarrow\; B_x = \frac{5.0\,\mathrm{N}}{16.4\,\mathrm{A{\bullet}m}} = 0.3049\,\mathrm{T}$$

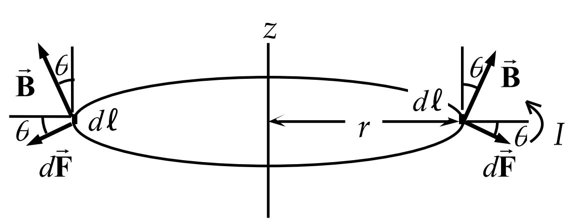
Thus $\vec{\mathbf{B}} = \boxed{(0.30\,\hat{\mathbf{i}} + 0.15\,\hat{\mathbf{k}})\,\mathrm{T}}$.

11. We find the force along the wire by integrating the infinitesimal force from each path element (given by Eq. 27-4) along an arbitrary path between the points *a* and *b*.

$$\vec{\mathbf{F}} = \int_a^b Id\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = I\int_a^b\left(\hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy\right)\times B_0\hat{\mathbf{k}} = IB_0\int_a^b\left(-\hat{\mathbf{j}}dx + \hat{\mathbf{i}}dy\right) = IB_0\left(-\Delta x\hat{\mathbf{j}} + \Delta y\hat{\mathbf{i}}\right)$$

The resultant magnetic force on the wire depends on the displacement between the points *a* and *b*, and not on the path taken by the wire. Therefore, the resultant force must be the same for the curved path, as for the straight line path between the points.

12. The net force on the current loop is the sum of the infinitesimal forces obtained from each current element. From the figure, we see that at each current segment, the magnetic field is perpendicular to the current. This results in a force with only radial and vertical components. By symmetry, we find that the radial force components from segments on opposite sides of the loop cancel. The net force then is purely vertical. Symmetry also shows us that each current element contributes the same magnitude of force.




$$\vec{\mathbf{F}} = \int Id\vec{\boldsymbol{\ell}}\times B = -IB_r\hat{\mathbf{k}}\int d\ell = -I(B\sin\theta)\hat{\mathbf{k}}(2\pi r) = \boxed{-2\pi IB\frac{r^2}{\sqrt{r^2+d^2}}\hat{\mathbf{k}}}$$

13. The maximum magnetic force as given in Eq. 27-5b can be used since the velocity is perpendicular to the magnetic field.

$$F_{\max} = qvB = \left(1.60\times10^{-19}\,\text{C}\right)\left(8.75\times10^{5}\,\text{m/s}\right)\left(0.45\,\text{T}\right) = \boxed{6.3\times10^{-14}\,\text{N}}$$

By the right hand rule, the force must be directed to the $\boxed{\text{North}}$.

14. The magnetic force will cause centripetal motion, and the electron will move in a clockwise circular path if viewed in the direction of the magnetic field. The radius of the motion can be determined.

$$F_{\max} = qvB = m\frac{v^2}{r} \quad\rightarrow\quad r = \frac{mv}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.70\times10^{6}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.480\,\text{T}\right)} = \boxed{2.02\times10^{-5}\,\text{m}}$$

15. In this scenario, the magnetic force is causing centripetal motion, and so must have the form of a centripetal force. The magnetic force is perpendicular to the velocity at all times for circular motion.

$$F_{\max} = qvB = m\frac{v^2}{r} \quad\rightarrow\quad B = \frac{mv}{qr} = \frac{\left(6.6\times10^{-27}\,\text{kg}\right)\left(1.6\times10^{7}\,\text{m/s}\right)}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(0.18\,\text{m}\right)} = \boxed{1.8\,\text{T}}$$

16. Since the charge is negative, the answer is the OPPOSITE of the result given from the right hand rule applied to the velocity and magnetic field.
(*a*) left
(*b*) left
(*c*) upward
(*d*) inward into the paper
(*e*) no force
(*f*) downward

17. The right hand rule applied to the velocity and magnetic field would give the direction of the force. Use this to determine the direction of the magnetic field given the velocity and the force.
(*a*) downward
(*b*) inward into the paper
(*c*) right

18. The force on the electron due to the electric force must be the same magnitude as the force on the electron due to the magnetic force.

$$F_{\text{E}} = F_{\text{B}} \quad\rightarrow\quad qE = qvB \quad\rightarrow\quad v = \frac{E}{B} = \frac{8.8\times10^{3}\,\text{V/m}}{7.5\times10^{-3}\,\text{T}} = 1.173\times10^{6}\,\text{m/s} \approx \boxed{1.2\times10^{6}\,\text{m/s}}$$

If the electric field is turned off, the magnetic force will cause circular motion.

$$F_{\text{B}} = qvB = m\frac{v^2}{r} \quad\rightarrow\quad r = \frac{mv}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.173\times10^{6}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(7.5\times10^{-3}\,\text{T}\right)} = \boxed{8.9\times10^{-4}\,\text{m}}$$

19. (*a*) The velocity of the ion can be found using energy conservation. The electrical potential energy of the ion becomes kinetic energy as it is accelerated. Then, since the ion is moving perpendicular to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path.

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad qV = \tfrac{1}{2}mv^2 \quad\rightarrow\quad v = \sqrt{\frac{2qV}{m}}$$

$$F_{max} = qvB = m\frac{v^2}{r} \quad \rightarrow$$

$$r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{1}{0.340\,\text{T}}\sqrt{\frac{2\left(6.6\times10^{-27}\,\text{kg}\right)\left(2700\,\text{V}\right)}{2\left(1.60\times10^{-19}\,\text{C}\right)}} = \boxed{3.1\times10^{-2}\,\text{m}}$$

(*b*) The period can be found from the speed and the radius. Use the expressions for the radius and the speed from above.

$$v = \frac{2\pi r}{T} \quad \rightarrow \quad T = \frac{2\pi r}{v} = \frac{2\pi\frac{1}{B}\sqrt{\frac{2mV}{q}}}{\sqrt{\frac{2qV}{m}}} = \frac{2\pi m}{qB} = \frac{2\pi\left(6.6\times10^{-27}\,\text{kg}\right)}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(0.340\,\text{T}\right)} = \boxed{3.8\times10^{-7}\,\text{s}}$$

20. The velocity of each charged particle can be found using energy conservation. The electrical potential energy of the particle becomes kinetic energy as it is accelerated. Then, since the particle is moving perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path, and the radius can be determined in terms of the mass and charge of the particle.

$$E_{\text{initial}} = E_{\text{final}} \quad \rightarrow \quad qV = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

$$F_{max} = qvB = m\frac{v^2}{r} \quad \rightarrow \quad r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$\frac{r_d}{r_p} = \frac{\frac{1}{B}\sqrt{\frac{2m_dV}{q_d}}}{\frac{1}{B}\sqrt{\frac{2m_pV}{q_p}}} = \frac{\sqrt{\frac{m_d}{m_p}}}{\sqrt{\frac{q_d}{q_p}}} = \frac{\sqrt{2}}{\sqrt{1}} = \sqrt{2} \quad \rightarrow \quad \boxed{r_d = \sqrt{2}r_p}$$

$$\frac{r_\alpha}{r_p} = \frac{\frac{1}{B}\sqrt{\frac{2m_\alpha V}{q_\alpha}}}{\frac{1}{B}\sqrt{\frac{2m_pV}{q_p}}} = \frac{\sqrt{\frac{m_\alpha}{m_p}}}{\sqrt{\frac{q_\alpha}{q_p}}} = \frac{\sqrt{4}}{\sqrt{2}} = \sqrt{2} \quad \rightarrow \quad \boxed{r_\alpha = \sqrt{2}r_p}$$

21. (*a*) From Example 27-7, we have that $r = \frac{mv}{qB}$, and so $v = \frac{rqB}{m}$. The kinetic energy is given by

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{rqB}{m}\right)^2 = \frac{r^2q^2B^2}{2m}$ and so we see that $\boxed{K \propto r^2}$.

(*b*) The angular momentum of a particle moving in a circular path is given by $L = mvr$. From Example 27-7, we have that $r = \frac{mv}{qB}$, and so $v = \frac{rqB}{m}$. Combining these relationships gives

$L = mvr = m\frac{rqB}{m}r = \boxed{qBr^2}$.

22. The force on the electron is given by Eq. 27-5a.

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = -e\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 7.0\times10^4\text{ m/s} & -6.0\times10^4\text{ m/s} & 0\\ -0.80\text{T} & 0.60\text{T} & 0\end{vmatrix} = -e(4.2-4.8)\times10^4\text{ T}\bullet\text{m/s}\,\hat{\mathbf{k}}$$

$$= -(1.60\times10^{-19}\text{C})(-0.6\times10^4\text{ T}\bullet\text{m/s}\,\hat{\mathbf{k}}) = 9.6\times10^{-16}\text{N}\,\hat{\mathbf{k}} \approx \boxed{1\times10^{-15}\text{N}\,\hat{\mathbf{k}}}$$

23. The kinetic energy of the proton can be used to find its velocity. The magnetic force produces centripetal acceleration, and from this the radius can be determined.

$$K = \tfrac{1}{2}mv^2 \;\rightarrow\; v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \;\rightarrow\; r = \frac{mv}{qB}$$

$$r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2K}{m}}}{qB} = \frac{\sqrt{2Km}}{qB} = \frac{\sqrt{2(6.0\times10^6\text{eV})(1.60\times10^{-19}\text{ J/eV})(1.67\times10^{-27}\text{kg})}}{(1.60\times10^{-19}\text{C})(0.20\text{T})} = \boxed{1.8\text{m}}$$

24. The magnetic field can be found from Eq. 27-5b, and the direction is found from the right hand rule. Remember that the charge is negative.

$$F_{max} = qvB \;\rightarrow\; B = \frac{F_{max}}{qv} = \frac{8.2\times10^{-13}\text{N}}{(1.60\times10^{-19}\text{C})(2.8\times10^6\text{ m/s})} = \boxed{1.8\text{T}}$$

The direction would have to be $\boxed{\text{East}}$ for the right hand rule, applied to the velocity and the magnetic field, to give the proper direction of force.

$\boxed{25.}$ The total force on the proton is given by the Lorentz equation, Eq. 27-7.

$$\vec{\mathbf{F}}_B = q(\vec{\mathbf{E}}+\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = e\left[(3.0\hat{\mathbf{i}}-4.2\hat{\mathbf{j}})\times10^3\text{ V/m} + \begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 6.0\times10^3\text{ m/s} & 3.0\times10^3\text{ m/s} & -5.0\times10^3\text{ m/s}\\ 0.45\text{T} & 0.38\text{T} & 0\end{vmatrix}\right]$$

$$= (1.60\times10^{-19}\text{C})\left[(3.0\hat{\mathbf{i}}-4.2\hat{\mathbf{j}}) + (1.9\hat{\mathbf{i}}-2.25\hat{\mathbf{j}}+0.93\hat{\mathbf{k}})\right]\times10^3\text{ N/C}$$

$$= (1.60\times10^{-19}\text{C})\left[(4.9\hat{\mathbf{i}}-6.45\hat{\mathbf{j}}+0.93\hat{\mathbf{k}})\right]\times10^3\text{ N/C}$$

$$= (7.84\times10^{-16}\hat{\mathbf{i}}-1.03\times10^{-15}\hat{\mathbf{j}}+1.49\times10^{-16}\hat{\mathbf{k}})\text{N/C}$$

$$= \boxed{\left[(0.78\hat{\mathbf{i}}-1.0\hat{\mathbf{j}}+0.15\hat{\mathbf{k}})\right]\times10^{-15}\text{N}}$$

26. The force on the electron is given by Eq. 27-5a. Set the force expression components equal and solve for the velocity components.

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} \;\rightarrow\; F_x\hat{\mathbf{i}}+F_y\hat{\mathbf{j}} = -e\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ v_x & v_y & v_z\\ 0 & 0 & B_z\end{vmatrix} = -ev_yB_z\hat{\mathbf{i}} - e(-v_xB_z)\hat{\mathbf{j}} \;\rightarrow$$

$$F_x = -ev_yB_z \;\rightarrow\; v_y = -\frac{F_x}{eB_z} = -\frac{3.8\times10^{-13}\text{N}}{(1.60\times10^{-19}\text{C})(0.85\text{T})} = -2.8\times10^6\text{ m/s}$$

$$F_y = ev_xB_z \rightarrow v_x = \frac{F_y}{eB_z} = \frac{-2.7\times10^{-13}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.85\,\text{T}\right)} = -2.0\times10^{6}\,\text{m/s}$$

$$\vec{\mathbf{v}} = \boxed{-\left(2.0\hat{\mathbf{i}} + 2.8\hat{\mathbf{j}}\right)\times10^{6}\,\text{m/s}}$$

Notice that we have not been able to determine the *z* component of the electron's velocity.

27. The kinetic energy of the particle can be used to find its velocity. The magnetic force produces centripetal acceleration, and from this the radius can be determined. Inserting the radius and velocity into the equation for angular momentum gives the angular momentum in terms of the kinetic energy and magnetic field.

$$K = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \rightarrow r = \frac{mv}{qB}$$

$$L = mvr = m\sqrt{\frac{2K}{m}}\left(\frac{m\sqrt{\frac{2K}{m}}}{qB}\right) = \frac{2mK}{qB}$$

From the equation for the angular momentum, we see that doubling the magnetic field while keeping the kinetic energy constant will cut the angular momentum in half.

$$\boxed{L_{\text{final}} = \tfrac{1}{2}L_{\text{initial}}}$$

28. The centripetal force is caused by the magnetic field, and is given by Eq. 27-5b.

$$F = qvB\sin\theta = qv_\perp B = m\frac{v_\perp^2}{r} \rightarrow$$

$$r = \frac{mv_\perp}{qB} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(3.0\times10^{6}\,\text{m/s}\right)\sin45^\circ}{\left(1.60\times10^{-19}\text{C}\right)\left(0.28\,\text{T}\right)} = 4.314\times10^{-5}\text{m} \approx \boxed{4.3\times10^{-5}\text{m}}$$

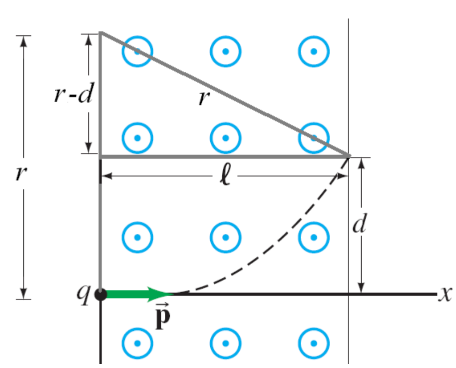
The component of the velocity that is parallel to the magnetic field is unchanged, and so the pitch is that velocity component times the period of the circular motion.

$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi\frac{mv_\perp}{qB}}{v_\perp} = \frac{2\pi m}{qB}$$

$$p = v_\parallel T = v\cos45^\circ\left(\frac{2\pi m}{qB}\right) = \left(3.0\times10^{6}\,\text{m/s}\right)\cos45^\circ\frac{2\pi\left(9.11\times10^{-31}\text{kg}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(0.28\,\text{T}\right)} = \boxed{2.7\times10^{-4}\text{m}}$$

29. (*a*) For the particle to move upward the magnetic force must point upward, by the right hand rule we see that the force on a positively charged particle would be downward. Therefore, the charge on the particle must be $\boxed{\text{negative}}$.

(*b*) In the figure we have created a right triangle to relate the horizontal distance ℓ, the displacement *d*, and the radius of curvature, *r*. Using the Pythagorean theorem we can write an expression for the radius in terms of the other two distances.



$$r^2 = (r-d)^2 + \ell^2 \rightarrow r = \frac{d^2+\ell^2}{2d}$$

Since the momentum is perpendicular to the magnetic field, we can solve for the momentum by relating the maximum force (Eq. 27-5b) to the centripetal force on the particle.

$$F_{\max} = qvB_0 = \frac{mv^2}{r} \rightarrow p = mv = qB_0 r = \boxed{\frac{qB_0\left(d^2+\ell^2\right)}{2d}}$$

30. In order for the path to be bent by 90° within a distance *d*, the radius of curvature must be less than or equal to *d*. The kinetic energy of the protons can be used to find their velocity. The magnetic force produces centripetal acceleration, and from this, the magnetic field can be determined.

$$K = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \rightarrow B = \frac{mv}{qr}$$

$$B \ge \frac{mv}{ed} = \frac{m\sqrt{\frac{2K}{m}}}{ed} = \boxed{\left(\frac{2Km}{e^2d^2}\right)^{1/2}}$$

31. The magnetic force will produce centripetal acceleration. Use that relationship to calculate the speed. The radius of the Earth is $6.38\times10^6\,\text{km}$, and the altitude is added to that.

$$F_B = qvB = m\frac{v^2}{r} \rightarrow v = \frac{qrB}{m} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(6.385\times10^6\,\text{m}\right)\left(0.50\times10^{-4}\,\text{T}\right)}{238\left(1.66\times10^{-27}\,\text{kg}\right)} = \boxed{1.3\times10^8\,\text{m/s}}$$

Compare the size of the magnetic force to the force of gravity on the ion.

$$\frac{F_B}{F_g} = \frac{qvB}{mg} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.3\times10^8\,\text{m/s}\right)\left(0.50\times10^{-4}\,\text{T}\right)}{238\left(1.66\times10^{-27}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)} = 2.3\times10^8$$

$\boxed{\text{Yes}}$, we may ignore gravity. The magnetic force is more than 200 million times larger than gravity.

32. The magnetic force produces an acceleration that is perpendicular to the original motion. If that perpendicular acceleration is small, it will produce a small deflection, and the original velocity can be assumed to always be perpendicular to the magnetic field. This leads to a constant perpendicular acceleration. The time that this (approximately) constant acceleration acts can be found from the original velocity *v* and the distance traveled $\ell$. The starting speed in the perpendicular direction will be zero.

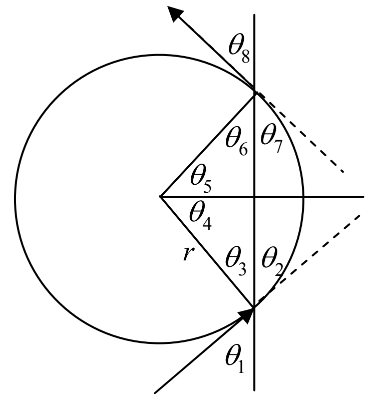
$$F_\perp = ma_\perp = qvB \rightarrow a_\perp = \frac{qvB}{m}$$

$$d_\perp = v_{0\perp}t + \tfrac{1}{2}a_\perp t^2 = \tfrac{1}{2}\frac{qvB}{m}\left(\frac{\ell}{v}\right)^2 = \frac{qB\ell^2}{2mv} = \frac{\left(18.5\times10^{-9}\,\text{C}\right)\left(5.00\times10^{-5}\,\text{T}\right)\left(1.00\times10^3\,\text{m}\right)^2}{2\left(3.40\times10^{-3}\,\text{kg}\right)\left(155\,\text{m/s}\right)}$$

$$= \boxed{8.8\times10^{-7}\,\text{m}}$$

This small distance justifies the assumption of constant acceleration.

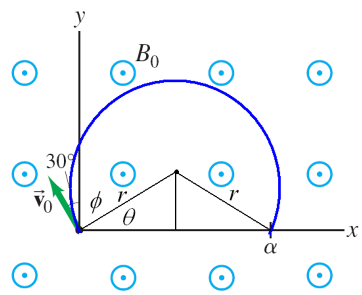
33. (*a*) In the magnetic field, the proton will move along an arc of a circle. The distance *x* in the diagram is a chord of that circle, and so the center of the circular path lies on the perpendicular bisector of the chord. That perpendicular bisector bisects the central angle of the circle which subtends the chord. Also recall that a radius is perpendicular to a tangent. In the diagram, $\theta_1 = \theta_2$ because they are vertical angles. Then $\theta_2 = \theta_4$, because they are both complements of $\theta_3$, so $\theta_1 = \theta_4$. We have $\theta_4 = \theta_5$ since the central angle is bisected by the perpendicular bisector of the chord. $\theta_5 = \theta_7$ because they are both complements of $\theta_6$, and $\theta_7 = \theta_8$ because they are vertical angles. Thus $\theta_1 = \theta_2 = \theta_4 = \theta_5 = \theta_7 = \theta_8$, and so in the textbook diagram, the angle at which the proton leaves is $\boxed{\theta = 45^\circ}$.




(*b*) The radius of curvature is given by $r = \dfrac{mv}{qB}$, and the distance *x* is twice the value of $r\cos\theta$.

$$x = 2r\cos\theta = 2\frac{mv}{qB}\cos\theta = 2\frac{\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.3\times10^{5}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.850\,\text{T}\right)}\cos 45^\circ = \boxed{2.3\times10^{-3}\,\text{m}}$$

34. (*a*) Since the velocity is perpendicular to the magnetic field, the particle will follow a circular trajectory in the *x*-*y* plane of radius *r*. The radius is found using the centripetal acceleration.




$$qvB = \frac{mv^2}{r} \quad \rightarrow \quad r = \frac{mv}{qB}$$

From the figure we see that the distance α is the chord distance, which is twice the distance $r\cos\theta$. Since the velocity is perpendicular to the radial vector, the initial direction and the angle $\phi$ are complementary angles. The angles $\phi$ and $\theta$ are also complementary angles, so $\theta = 30^\circ$.

$$\alpha = 2r\cos\theta = \frac{2mv_0}{qB_0}\cos 30^\circ = \boxed{\sqrt{3}\frac{mv_0}{qB_0}}$$

(*b*) From the diagram, we see that the particle travels a circular path, that is $2\phi$ short of a complete circle. Since the angles $\phi$ and $\theta$ are complementary angles, so $\phi = 60^\circ$. The trajectory distance is equal to the circumference of the circular path times the fraction of the complete circle. Dividing the distance by the particle speed gives $t_\alpha$.

$$t_\alpha = \frac{\ell}{v_0} = \frac{2\pi r}{v_0}\left(\frac{360^\circ - 2\left(60^\circ\right)}{360^\circ}\right) = \frac{2\pi}{v_0}\frac{mv_0}{qB_0}\left(\frac{2}{3}\right) = \boxed{\frac{4\pi m}{3qB_0}}$$

35. The work required by an external agent is equal to the change in potential energy. The potential energy is given by Eq. 27-12, $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}$.

(*a*) $W = \Delta U = \left(-\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{final}} - \left(-\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{initial}} = \left(\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{initial}} - \left(\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{final}} = NIAB\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right)$

$= NIAB\left(\cos 0^\circ - \cos 180^\circ\right) = \boxed{2NIAB}$

(*b*) $W = NIAB\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right) = NIAB\left(\cos 90^\circ - \cos\left(-90^\circ\right)\right) = \boxed{0}$

36. With the plane of the loop parallel to the magnetic field, the torque will be a maximum. We use Eq. 27-9.

$$\tau = NIAB\sin\theta \;\rightarrow\; B = \frac{\tau}{NIAB\sin\theta} = \frac{0.185\,\text{m}\bullet\text{N}}{(1)(4.20\,\text{A})\pi(0.0650\,\text{m})^2\sin 90^\circ} = \boxed{3.32\,\text{T}}$$

37. (*a*) The torque is given by Eq. 27-9. The angle is the angle between the B-field and the perpendicular to the coil face.

$$\tau = NIAB\sin\theta = 12(7.10\,\text{A})\left[\pi\left(\frac{0.180\,\text{m}}{2}\right)^2\right](5.50\times10^{-5}\,\text{T})\sin 24^\circ = \boxed{4.85\times10^{-5}\,\text{m}\bullet\text{N}}$$

(*b*) In Example 27-11 it is stated that if the coil is free to turn, it will rotate toward the orientation so that the angle is 0. In this case, that means the $\boxed{\text{north}}$ edge of the coil will rise, so that a perpendicular to its face will be parallel with the Earth's magnetic field.

38. The magnetic dipole moment is defined in Eq. 27-10 as $\mu = NIA$. The number of turns, $N$, is 1. The current is the charge per unit time passing a given point, which on the average is the charge on the electron divided by the period of the circular motion, $I = e/T$. If we assume the electron is moving in a circular orbit of radius *r*, then the area is $\pi r^2$. The period of the motion is the circumference of the orbit divided by the speed, $T = 2\pi r/v$. Finally, the angular momentum of an object moving in a circle is given by $L = mrv$. Combine these relationships to find the magnetic moment.

$$\mu = NIA = \frac{e}{T}\pi r^2 = \frac{e}{2\pi r/v}\pi r^2 = \frac{e\pi r^2 v}{2\pi r} = \frac{erv}{2} = \frac{emrv}{2m} = \frac{e}{2m}mrv = \frac{e}{2m}L$$

39. (*a*) The magnetic moment of the coil is given by Eq. 27-10. Since the current flows in the clockwise direction, the right hand rule shows that the magnetic moment is down, or in the negative *z*-direction.

$$\vec{\boldsymbol{\mu}} = NI\vec{\mathbf{A}} = 15(7.6\,\text{A})\pi\left(\frac{0.22\,\text{m}}{2}\right)^2(-\hat{\mathbf{k}}) = -4.334\,\hat{\mathbf{k}}\,\text{A}\bullet\text{m}^2 \approx \boxed{-4.3\,\hat{\mathbf{k}}\,\text{A}\bullet\text{m}^2}$$

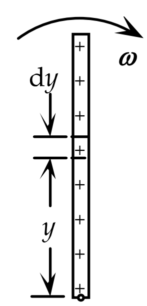
(*b*) We use Eq. 27-11 to find the torque on the coil.

$$\vec{\tau} = \vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}} = (-4.334\,\hat{\mathbf{k}}\,\text{A}\bullet\text{m}^2)\times(0.55\hat{\mathbf{i}} + 0.60\hat{\mathbf{j}} - 0.65\hat{\mathbf{k}})\text{T} = \boxed{(2.6\hat{\mathbf{i}} - 2.4\hat{\mathbf{j}})\text{m}\bullet\text{N}}$$

(*c*) We use Eq. 27-12 to find the potential energy of the coil.

$$U = -\vec{\boldsymbol{\mu}}\bullet\vec{\mathbf{B}} = -(-4.334\,\hat{\mathbf{k}}\,\text{A}\bullet\text{m}^2)(0.55\hat{\mathbf{i}} + 0.60\hat{\mathbf{j}} - 0.65\hat{\mathbf{k}})\text{T} = -(4.334\,\text{A}\bullet\text{m}^2)(0.65\,\text{T})$$
$$= \boxed{-2.8\,\text{J}}$$

40. To find the total magnetic moment, we divide the rod into infinitesimal pieces of thickness *dy*. As the rod rotates on its axis the charge in each piece, $(Q/d)dy$, creates a current loop around the axis of rotation. The magnitude of the current is the charge times the frequency of rotation, $\omega/2\pi$. By integrating the infinitesimal magnetic moments from each piece, we find the total magnetic moment.




$$\vec{\boldsymbol{\mu}} = \int d\vec{\boldsymbol{\mu}} = \int \vec{\mathbf{A}}dI = \int_0^d (\pi y^2)\left(\frac{\omega}{2\pi}\frac{Q}{d}dy\right) = \frac{Q\omega}{2d}\int_0^d y^2 dy = \boxed{\frac{Q\omega d^2}{6}}.$$

41. From Section 27-5, we see that the torque is proportional to the current, so if the current drops by 12%, the output torque will also $\boxed{\text{drop by 12\%}}$. Thus the final torque is 0.88 times the initial torque.

42. In Section 27-6, it is shown that the deflection of the galvanometer needle is proportional to the product of the current and the magnetic field. Thus if the magnetic field is decreased to 0.860 times its original value, the current must be increased by dividing the original value by 0.860 to obtain the same deflection.

$$(IB)_{\text{initial}} = (IB)_{\text{final}} \rightarrow I_{\text{final}} = \frac{I_{\text{initial}} B_{\text{initial}}}{B_{\text{final}}} = \frac{(63.0\,\mu\text{A})\,B_{\text{initial}}}{0.800 B_{\text{initial}}} = \boxed{78.8\,\mu\text{A}}$$

43. From the galvanometer discussion in Section 27-6, the amount of deflection is proportional to the ratio of the current and the spring constant: $\phi \propto \frac{I}{k}$. Thus if the spring constant decreases by 15%, the current can decrease by 15% to produce the same deflection. The new current will be 85% of the original current.

$$I_{\text{final}} = 0.85 I_{\text{initial}} = 0.85(46\,\mu\text{A}) = \boxed{39\,\mu\text{A}}$$

44. Use Eq. 27-13.

$$\frac{q}{m} = \frac{E}{B^2 r} = \frac{(260\,\text{V/m})}{(0.46\,\text{T})^2 (0.0080\,\text{m})} = \boxed{1.5\times10^5\,\text{C/kg}}$$

45. The force from the electric field must be equal to the weight.

$$|qE| = (ne)\left(\frac{V}{d}\right) = mg \rightarrow n = \frac{mgd}{eV} = \frac{(3.3\times10^{-15}\,\text{kg})(9.80\,\text{m/s}^2)(0.010\,\text{m})}{(1.60\times10^{-19})(340\,\text{V})} = 5.94 \approx \boxed{6\text{ electrons}}$$

46. (*a*) Eq. 27-14 shows that the Hall emf is proportional to the magnetic field perpendicular to the conductor's surface. We can use this proportionality to determine the unknown resistance. Since the new magnetic field is oriented 90° to the surface, the full magnetic field will be used to create the Hall potential.

$$\frac{\mathcal{E}_H'}{\mathcal{E}_H} = \frac{B_\perp'}{B_\perp} \rightarrow B_\perp' = \frac{\mathcal{E}_H'}{\mathcal{E}_H} B_\perp = \frac{63\,\text{mV}}{12\,\text{mV}}(0.10\,\text{T}) = \boxed{0.53\,\text{T}}$$

(*b*) When the field is oriented at 60° to the surface, the magnetic field, $B\sin 60°$, is used to create the Hall potential.

$$B_\perp' \sin 60° = \frac{\mathcal{E}_H'}{\mathcal{E}_H} B_\perp \rightarrow B_\perp' = \frac{63\text{ mV}}{12\text{ mV}} \frac{(0.10\,\text{T})}{\sin 60°} = \boxed{0.61\,\text{T}}$$

47. (*a*) We use Eq. 27-14 for the Hall Potential and Eq. 25-13 to write the current in terms of the drift velocity.

$$K_H = \frac{\mathcal{E}_H}{IB} = \frac{v_d B d}{\left[en(td)v_d\right]B} = \boxed{\frac{1}{ent}}$$

(*b*) We set the magnetic sensitivities equal and solve for the metal thickness.

$$\frac{1}{en_s t_s} = \frac{1}{en_m t_m} \rightarrow t_m = \frac{n_s}{n_m} t_s = \frac{3\times10^{22}\,\text{m}^{-3}}{1\times10^{29}\,\text{m}^{-3}}(0.15\times10^{-3}\,\text{m}) = \boxed{5\times10^{-11}\text{ m}}$$

This is less than one sixth the size of a typical metal atom.

(*c*) Use the magnetic sensitivity to calculate the Hall potential.

$$\mathcal{E}_H = K_H IB = \frac{IB}{ent} = \frac{(100\text{ mA})(0.1\text{T})}{(1.6\times10^{-19}\text{C})(3\times10^{22}\text{m}^{-3})(0.15\times10^{-3}\text{m})} = 14\,\text{mV} \approx \boxed{10\,\text{mV}}$$

48. (*a*) We find the Hall field by dividing the Hall emf by the width of the metal.

$$E_H = \frac{\mathcal{E}_H}{d} = \frac{6.5\ \mu\text{V}}{0.03\text{ m}} = 2.167\times10^{-4}\,\text{V/m} \approx \boxed{2.2\times10^{-4}\,\text{V/m}}$$

(*b*) Since the forces from the electric and magnetic fields are balanced, we can use Eq. 27-14 to calculate the drift velocity.

$$v_d = \frac{E_H}{B} = \frac{2.167\times10^{-4}\,\text{V/m}}{0.80\,\text{T}} = 2.709\times10^{-4}\text{ m/s} \approx \boxed{2.7\times10^{-4}\,\text{m/s}}$$

(*c*) We now find the density using Eq. 25-13.

$$n = \frac{I}{eAv_d} = \frac{42\,\text{A}}{(1.6\times10^{-19}\text{C})(6.80\times10^{-4}\text{m})(0.03\,\text{m})(2.709\times10^{-4}\text{m/s})} = \boxed{4.7\times10^{28}\,\text{electrons/m}^3}$$

49. We find the magnetic field using Eq. 27-14, with the drift velocity given by Eq. 25-13. To determine the electron density we divide the density of sodium by its atomic weight. This gives the number of moles of sodium per cubic meter. Multiplying the result by Avogadro's number gives the number of sodium atoms per cubic meter. Since there is one free electron per atom, this is also the density of free electrons.

$$B = \frac{\mathcal{E}_H}{v_d d} = \frac{\mathcal{E}_H}{\left(\dfrac{I}{ne(td)}\right)d} = \frac{\mathcal{E}_H net}{I} = \frac{\mathcal{E}_H et}{I}\left(\frac{\rho N_A}{m_A}\right)$$

$$= \frac{(1.86\times10^{-6}\text{V})(1.60\times10^{-19}\text{C})(1.30\times10^{-3}\text{m})}{12.0\text{ A}}\,\frac{(0.971)(1000\text{ kg/m}^3)(6.022\times10^{23}\text{e/mole})}{0.02299\text{ kg/mole}}$$

$$= \boxed{0.820\,\text{T}}$$

50. (*a*) The sign of the ions will not change the magnitude of the Hall emf, but will $\boxed{\text{determine the polarity of the emf}}$.

(*b*) The flow velocity corresponds to the drift velocity in Eq. 27-14.

$$\mathcal{E}_\text{H} = vBd \quad\rightarrow\quad\rightarrow\quad v = \frac{\mathcal{E}_\text{H}}{Bd} = \frac{(0.13\times10^{-3}\text{V})}{(0.070\text{T})(0.0033\text{m})} = \boxed{0.56\,\text{m/s}}$$

51. The magnetic force on the ions causes them to move in a circular path, so the magnetic force is a centripetal force. This results in the ion mass being proportional to the path's radius of curvature.

$$qvB = m v^2/r \quad\rightarrow\quad m = qBr/v \quad\rightarrow\quad m/r = qB/v = \text{constant} = 76\quad \text{u}/22.8\,\text{cm}$$

$$\frac{m_{21.0}}{21.0\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \rightarrow m_{21.0} = 70\,\text{u} \qquad \frac{m_{21.6}}{21.6\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \rightarrow m_{21.6} = 72\,\text{u}$$

$$\frac{m_{21.9}}{21.9\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \rightarrow m_{21.9} = 73\,\text{u} \qquad \frac{m_{22.2}}{22.2\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \rightarrow m_{22.2} = 74\,\text{u}$$

The other masses are $\boxed{\text{70 u, 72 u, 73 u, and 74 u}}$.

52. The velocity of the ions is found using energy conservation. The electrical potential energy of the ions becomes kinetic energy as they are accelerated. Then, since the ions move perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ions to move in a circular path.

$$qvB = \frac{mv^2}{R} \rightarrow v = \frac{qBR}{m} \qquad qV = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBR}{m}\right)^2 = \frac{q^2B^2R^2}{2m} \rightarrow m = \frac{qR^2B^2}{2V}$$

53. The location of each line on the film is twice the radius of curvature of the ion. The radius of curvature can be found from the expression given in Section 27-9.

$$m = \frac{qBB'r}{E} \rightarrow r = \frac{mE}{qBB'} \rightarrow 2r = \frac{2mE}{qBB'}$$

$$2r_{12} = \frac{2(12)(1.67\times10^{-27}\,\text{kg})(2.48\times10^{4}\,\text{V/m})}{(1.60\times10^{-19}\,\text{C})(0.58\,\text{T})^2} = 1.8467\times10^{-2}\,\text{m}$$

$$2r_{13} = 2.0006\times10^{-2}\,\text{m} \qquad 2r_{14} = 2.1545\times10^{-2}\,\text{m}$$

The distances between the lines are

$$2r_{13} - 2r_{12} = 2.0006\times10^{-2}\,\text{m} - 1.8467\times10^{-2}\,\text{m} = 1.539\times10^{-3}\,\text{m} \approx \boxed{1.5\times10^{-3}\,\text{m}}$$

$$2r_{14} - 2r_{13} = 2.1545\times10^{-2}\,\text{m} - 2.0006\times10^{-2}\,\text{m} = 1.539\times10^{-3}\,\text{m} \approx \boxed{1.5\times10^{-3}\,\text{m}}$$

If the ions are doubly charged, the value of $q$ in the denominator of the expression would double, and so the actual distances on the film would be halved. Thus the distances between the lines would also be halved.

$$2r_{13} - 2r_{12} = 1.0003\times10^{-2}\,\text{m} - 9.2335\times10^{-3}\,\text{m} = 7.695\times10^{-4}\,\text{m} \approx \boxed{7.7\times10^{-4}\,\text{m}}$$

$$2r_{14} - 2r_{13} = 1.07725\times10^{-2}\,\text{m} - 1.0003\times10^{-2}\,\text{m} = 7.695\times10^{-4}\,\text{m} \approx \boxed{7.7\times10^{-4}\,\text{m}}$$

54. The particles in the mass spectrometer follow a semicircular path as shown in Fig. 27-33. A particle has a displacement of $2r$ from the point of entering the semicircular region to where it strikes the film. So if the separation of the two molecules on the film is 0.65 mm, the difference in radii of the two molecules is 0.325 mm. The mass to radius ratio is the same for the two molecules.

$$qvB = mv^2/r \rightarrow m = qBr/v \rightarrow m/r = \text{constant}$$

$$\left(\frac{m}{r}\right)_{\text{CO}} = \left(\frac{m}{r}\right)_{\text{N}_2} \rightarrow \frac{28.0106\,\text{u}}{r} = \frac{28.0134\,\text{u}}{r + 3.25\times10^{-4}\,\text{m}} \rightarrow r = 3.251\,\text{m} \approx \boxed{3.3\,\text{m}}$$

$\boxed{55.}$ Since the particle is undeflected in the crossed fields, its speed is given by Eq. 27-8. Without the electric field, the particle will travel in a circle due to the magnetic force. Using the centripetal acceleration, we can calculate the mass of the particle. Also, the charge must be an integer multiple of the fundamental charge.

$$qvB = \frac{mv^2}{r} \rightarrow$$

$$m = \frac{qBr}{v} = \frac{qBr}{(E/B)} = \frac{neB^2r}{E} = \frac{n(1.60\times10^{-19}\,\text{C})(0.034\,\text{T})^2(0.027\,\text{m})}{1.5\times10^{3}\,\text{V/m}} = n(3.3\times10^{-27}\,\text{kg}) \approx n(2.0\,\text{u})$$

The particle has an atomic mass of a multiple of 2.0 u. The simplest two cases are that it could be a hydrogen-2 nucleus (called a deuteron), or a helium-4 nucleus (called an alpha particle): $\boxed{{}^{2}_{1}\text{H}, {}^{4}_{2}\text{He}}$.

56. The radius and magnetic field values can be used to find the speed of the protons. The electric field is then found from the fact that the magnetic force must be the same magnitude as the electric force for the protons to have straight paths.

$$qvB = mv^2/r \;\rightarrow\; v = qBr/m \qquad F_E = F_B \;\rightarrow\; qE = qvB \;\rightarrow$$

$$E = vB = qB^2r/m = \frac{(1.60\times10^{-19}\text{C})(0.625\text{T})^2(5.10\times10^{-2}\text{m})}{1.67\times10^{-27}\text{kg}} = \boxed{1.91\times10^6\ \text{V/m}}$$

The direction of the electric field must be perpendicular to both the velocity and the magnetic field, and must be in the opposite direction to the magnetic force on the protons.

57. The magnetic force produces centripetal acceleration.

$$qvB = mv^2/r \;\rightarrow\; mv = p = qBr \;\rightarrow\; B = \frac{p}{qr} = \frac{3.8\times10^{-16}\ \text{kg}\bullet\text{m/s}}{(1.60\times10^{-19}\text{C})(1.0\times10^3\text{m})} = \boxed{2.4\,\text{T}}$$

The magnetic field must point upward to cause an inward-pointing (centripetal) force that steers the protons clockwise.

58. The kinetic energy is used to determine the speed of the particles, and then the speed can be used to determine the radius of the circular path, since the magnetic force is causing centripetal acceleration.

$$K = \tfrac{1}{2}mv^2 \;\rightarrow\; v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \;\rightarrow\; r = \frac{mv}{qB} = \frac{m\sqrt{\dfrac{2K}{m}}}{qB} = \frac{\sqrt{2mK}}{qB}$$

$$\frac{r_p}{r_e} = \frac{\dfrac{\sqrt{2m_pK}}{qB}}{\dfrac{\sqrt{2m_eK}}{qB}} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67\times10^{-27}\text{kg}}{9.11\times10^{-31}\text{kg}}} = \boxed{42.8}$$

59. (*a*) There will be one force on the rod, due to the magnetic force on the charge carriers in the rod. That force is of magnitude $F_B = IdB$, and by Newton's second law is equal to the mass of the rod times its acceleration. That force is constant, so the acceleration will be constant, and constant acceleration kinematics can be used.

$$F_{net} = F_B = IdB = ma \;\rightarrow\; a = \frac{IdB}{m} = \frac{v-v_0}{t} = \frac{v}{t} \;\rightarrow\; v = \boxed{\frac{IdB}{m}t}$$

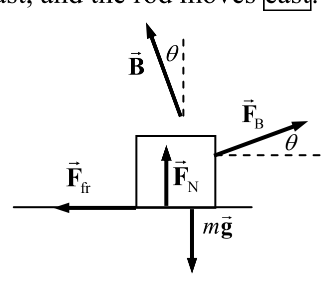
(*b*) Now the net force is the vector sum of the magnetic force and the force of kinetic friction.

$$F_{net} = F_B - F_{fr} = IdB - \mu_k F_N = IdB - \mu_k mg = ma \;\rightarrow$$

$$a = \frac{IdB}{m} - \mu_k g = \frac{v-v_0}{t} = \frac{v}{t} \;\rightarrow\; v = \boxed{\left(\frac{IdB}{m} - \mu_k g\right)t}$$

(*c*) Using the right hand rule, we find that the force on the rod is to the east, and the rod moves $\boxed{\text{east}}$.

60. Assume that the magnetic field makes an angle θ with respect to the vertical. The rod will begin to slide when the horizontal magnetic force $(IB\ell\cos\theta)$ is equal to the maximum static friction $(\mu_s F_N)$. Find the normal force by setting the sum of the vertical forces equal to zero. See the free body diagram.




$$F_B\sin\theta + F_N - mg = 0 \;\rightarrow\; F_N = mg - F_B\sin\theta = mg - I\ell B\sin\theta$$

$$IB\ell\cos\theta = \mu_s F_N = \mu_s\left(mg - I\ell B\sin\theta\right) \rightarrow B = \frac{\mu_s mg}{I\ell\left(\mu_s\sin\theta + \cos\theta\right)}$$

We find the angle for the minimum magnetic field by setting the derivative of the magnetic field with respect to the angle equal to zero and solving for the angle.

$$\frac{dB}{d\theta} = 0 = \frac{-\mu_s mg\left(\mu_s\cos\theta - \sin\theta\right)}{I\ell\left(\mu_s\sin\theta + \cos\theta\right)^2} \rightarrow \theta = \tan^{-1}\mu_s = \tan^{-1}0.5 = 26.6°$$

$$B = \frac{\mu_s mg}{I\ell\left(\mu_s\sin\theta + \cos\theta\right)} = \frac{0.5(0.40\,\text{kg})(9.80\,\text{m/s}^2)}{(36\,\text{A})(0.22\,\text{m})(0.5\sin 26.6° + \cos 26.6°)} = 0.22\,\text{T}$$

The minimum magnetic field that will cause the rod to move is $\boxed{0.22\text{ T at }27°\text{ from the vertical}}$.

61. The magnetic force must be equal in magnitude to the weight of the electron.

$$mg = qvB \rightarrow v = \frac{mg}{qB} = \frac{(9.11\times10^{-31}\text{kg})(9.80\,\text{m/s}^2)}{(1.60\times10^{-19}\text{C})(0.50\times10^{-4}\text{T})} = \boxed{1.1\times10^{-6}\,\text{m/s}}$$

The magnetic force must point upwards, and so by the right hand rule and the negative charge of the electron, the electron must be moving $\boxed{\text{west}}$ .

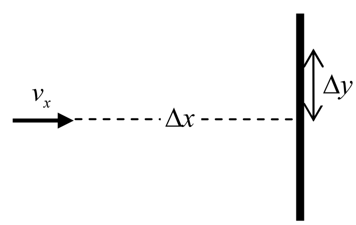
62. The airplane is a charge moving in a magnetic field. Since it is flying perpendicular to the magnetic field, Eq. 27-5b applies.

$$F_{\text{max}} = qvB = (1850\times10^{-6}\text{C})(120\,\text{m/s})(5.0\times10^{-5}\text{T}) = \boxed{1.1\times10^{-5}\text{N}}$$

63. The maximum torque is found using Eq. 27-9 with $\sin\theta = 1$. Set the current equal to the voltage divided by resistance and the area as the square of the side length.

$$\tau = NIAB = N\left(\frac{V}{R}\right)\ell^2 B = 20\left(\frac{9.0\,\text{V}}{24\,\Omega}\right)(0.050\,\text{m})^2(0.020\,\text{T}) = \boxed{3.8\times10^{-4}\text{m}\bullet\text{N}}$$

64. The speed of the electrons is found by assuming the energy supplied by the accelerating voltage becomes kinetic energy of the electrons. We assume that those electrons are initially directed horizontally, and that the television set is oriented so that the electron velocity is perpendicular to the Earth's magnetic field, resulting in the largest possible force. Finally, we assume that the magnetic force on the electrons is small enough that the electron velocity is essentially perpendicular to the Earth's field for the entire trajectory. This results in a constant acceleration for the electrons.




(*a*) Acceleration:

$$U_{\text{initial}} = K_{\text{final}} \rightarrow eV = \tfrac{1}{2}mv_x^2 \rightarrow v_x = \sqrt{\frac{2eV}{m}}$$

Deflection:

$$\text{time in field: } \Delta x_{\text{field}} = v_x t_{\text{field}} \rightarrow t_{\text{field}} = \frac{\Delta x_{\text{field}}}{v_x}$$

$$F_y = qv_x B_{\text{Earth}} = ma_y \rightarrow a_y = \frac{qv_x B_{\text{Earth}}}{m} = \frac{e\sqrt{\frac{2eV}{m}}B_{\text{Earth}}}{m} = \sqrt{\frac{2e^3V}{m^3}}B_{\text{Earth}}$$

$$\Delta y = \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}\sqrt{\frac{2e^3V}{m^3}}B_{\text{Earth}}\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \tfrac{1}{2}\sqrt{\frac{2e^3V}{m^3}}B_{\text{Earth}}\left(\Delta x\right)^2\frac{m}{2eV}$$

$$= \sqrt{\frac{e}{8mV}}B_{\text{Earth}}\left(\Delta x\right)^2 = \sqrt{\frac{1.60\times10^{-19}\,\text{C}}{8\left(9.11\times10^{-31}\,\text{kg}\right)\left(2.0\times10^{3}\,\text{V}\right)}}\left(5.0\times10^{-5}\,\text{T}\right)\left(0.18\,\text{m}\right)^2$$

$$= 5.37\times10^{-3}\,\text{m} \approx \boxed{5.4\,\text{mm}}$$

(*b*) $$\Delta y = \sqrt{\frac{e}{8mV}}B_{\text{Earth}}\left(\Delta x\right)^2 = \sqrt{\frac{1.60\times10^{-19}\,\text{C}}{8\left(9.11\times10^{-31}\,\text{kg}\right)\left(28{,}000\,\text{V}\right)}}\left(5.0\times10^{-5}\,\text{T}\right)\left(0.18\,\text{m}\right)^2$$

$$= \boxed{1.4\times10^{-3}\,\text{m}}$$

Note that the deflection is significantly smaller than the horizontal distance traveled, and so the assumptions made above are verified.

65. From Fig. 27-22 we see that when the angle $\theta$ is positive, the torque is negative. The magnitude of the torque is given by Eq. 27-9. For small angles we use the approximation $\sin\theta \approx \theta$. Using Eq. 10-14, we can write the torque in terms of the angular acceleration, showing that it is a harmonic oscillator.

$$\tau = -NIAB\sin\theta \approx -IabB\theta = I_M\alpha \;\rightarrow\; \alpha = -\left(\frac{IabB}{I_M}\right)\theta = -\omega^2\theta$$

We obtain the period of motion from the angular frequency, using $T = 2\pi/\omega$. First we determine the moment of inertia of the loop, as two wires rotating about their centers of mass and two wires rotating about an axis parallel to their lengths.

$$I_M = 2\left[\frac{1}{12}\left(\frac{b}{2a+2b}m\right)b^2\right] + 2\left(\frac{a}{2a+2b}m\right)\left(\frac{b}{2}\right)^2 = \frac{\left(3a+b\right)mb^2}{12\left(a+b\right)}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_M}{NIabB}} = 2\pi\sqrt{\frac{mb^2\left(3a+b\right)}{12\left(a+b\right)NIabB}} = \boxed{\pi\sqrt{\frac{mb\left(3a+b\right)}{3\left(a+b\right)NIaB}}}$$

66. (*a*) The frequency of the voltage must match the frequency of circular motion of the particles, so that the electric field is synchronized with the circular motion. The radius of each circular orbit is given in Example 27-7 as $r = \dfrac{mv}{qB}$. For an object moving in circular motion, the period is given by $T = \dfrac{2\pi r}{v}$, and the frequency is the reciprocal of the period.

$$T = \frac{2\pi r}{v} \;\rightarrow\; f = \frac{v}{2\pi r} = \frac{v}{2\pi\dfrac{mv}{qB}} = \boxed{\frac{Bq}{2\pi m}}$$

In particular we note that this frequency is independent of the radius, and so the same frequency can be used throughout the acceleration.

(*b*) For a small gap, the electric field across the gap will be approximately constant and uniform as the particles cross the gap. If the motion and the voltage are synchronized so that the maximum voltage occurs when the particles are at the gap, the particles receive an energy increase of

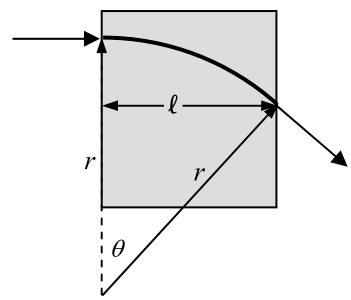
$K = qV_0$ as they pass each gap. The energy gain from one revolution will include the passing of 2 gaps, so the total kinetic energy increase is $\boxed{2qV_0}$.

(*c*) The maximum kinetic energy will occur at the outside of the cyclotron.

$$K_{\max} = \tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}m\left(\frac{r_{\max}qB}{m}\right)^2 = \tfrac{1}{2}\frac{r_{\max}^2 q^2 B^2}{m} = \tfrac{1}{2}\frac{(0.50\,\text{m})^2(1.60\times10^{-19}\,\text{C})^2(0.60\,\text{T})^2}{1.67\times10^{-27}\,\text{kg}}$$

$$= 6.898\times10^{-13}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)\left(\frac{1\,\text{MeV}}{10^6\,\text{eV}}\right) = \boxed{4.3\,\text{MeV}}$$

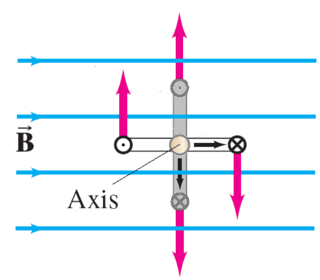
67. The protons will follow a circular path as they move through the region of magnetic field, with a radius of curvature given in Example 27-7 as $r = \dfrac{mv}{qB}$. Fast-moving protons will have a radius of curvature that is too large and so they will exit above the second tube. Likewise, slow-moving protons will have a radius of curvature that is too small and so they will exit below the second tube. Since the exit velocity is perpendicular to the radius line from the center of curvature, the bending angle can be calculated.




$$\sin\theta = \frac{\ell}{r} \quad \rightarrow$$

$$\theta = \sin^{-1}\frac{\ell}{r} = \sin^{-1}\frac{\ell qB}{mv} = \sin^{-1}\frac{(5.0\times10^{-2}\,\text{m})(1.60\times10^{-19}\,\text{C})(0.38\,\text{T})}{(1.67\times10^{-27}\,\text{kg})(0.85\times10^7\,\text{m/s})} = \sin^{-1}0.214 = \boxed{12^\circ}$$

68. (*a*) The force on each of the vertical wires in the loop is perpendicular to the magnetic field and is given by Eq. 27-1, with $\theta = 90^\circ$. When the face of the loop is parallel to the magnetic field, the forces point radially away from the axis. This provides a tension in the two horizontal sides. When the face of the loop is perpendicular to the magnetic field, the force on opposite vertical wires creates a shear force in the horizontal wires. From Table 12-2, we see that the tensile and shear strengths of aluminum are the same, so either can be used to determine the minimum strength. We set tensile strength multiplied by the cross-sectional area of the two wires equal the tensile strength multiplied by the safety factor and solve for the wire diameter.




$$\frac{F}{A}\pi\left(\frac{d}{2}\right)^2 = 10(I\ell B) \quad \rightarrow \quad d = 2\sqrt{\frac{10(I\ell B)}{\pi(F/A)}} = 2\sqrt{\frac{10(15.0\text{ A})(0.200\text{ m})(1.35\text{ T})}{\pi(200\times10^6\text{ N/m}^2)}}$$

$$= 5.0777\times10^{-4}\text{ m} \approx \boxed{0.508\text{ mm}}$$

(*b*) The resistance is found from the resistivity using Eq. 25-3.

$$R = \rho\frac{\ell}{A} = (2.65\times10^{-8}\,\Omega\bullet\text{m})\frac{4(0.200\,\text{m})}{\pi\left(\dfrac{5.0777\times10^{-4}}{2}\text{m}\right)^2} = \boxed{0.105\,\Omega}$$

69. The accelerating force on the bar is due to the magnetic force on the current. If the current is constant, the magnetic force will be constant, and so constant acceleration kinematics can be used.

$$v^2 = v_0^2 + 2a\Delta x \quad \to \quad a = \frac{v^2 - 0}{2\Delta x} = \frac{v^2}{2\Delta x}$$

$$F_{\text{net}} = ma = IdB \quad \to \quad I = \frac{ma}{dB} = \frac{m\left(\frac{v^2}{2\Delta x}\right)}{dB} = \frac{mv^2}{2\Delta x dB} = \frac{(1.5\times10^{-3}\,\text{kg})(25\,\text{m/s})^2}{2(1.0\,\text{m})(0.24\,\text{m})(1.8\,\text{T})} = \boxed{1.1\,\text{A}}$$

Using the right hand rule, for the force on the bar to be in the direction of the acceleration shown in Fig. 27-53, the magnetic field must be $\boxed{\text{down}}$ .

70. (*a*) For the beam of electrons to be undeflected, the magnitude of the magnetic force must equal the magnitude of the electric force. We assume that the magnetic field will be perpendicular to the velocity of the electrons so that the maximum magnetic force is obtained.

$$F_{\text{B}} = F_{\text{E}} \quad \to \quad qvB = qE \quad \to \quad B = \frac{E}{v} = \frac{8400\,\text{V/m}}{4.8\times10^6\,\text{m/s}} = 1.75\times10^{-3}\,\text{T} \approx \boxed{1.8\times10^{-3}\,\text{T}}$$

(*b*) Since the electric field is pointing up, the electric force is down. Thus the magnetic force must be up. Using the right hand rule with the negative electrons, the magnetic field must be $\boxed{\text{out}}$ of the plane of the plane formed by the electron velocity and the electric field.

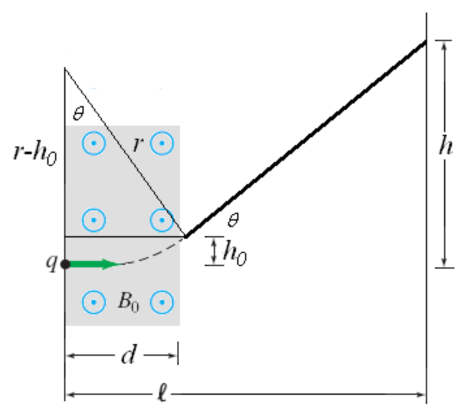
(*c*) If the electric field is turned off, then the magnetic field will cause a centripetal force, moving the electrons in a circular path. The frequency is the reciprocal of the period of the motion.

$$qvB = \frac{mv^2}{r} \quad \to \quad v = \frac{qBr}{m}$$

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\frac{qBr}{m}}{2\pi r} = \frac{qB}{2\pi m} = \frac{qE}{2\pi mv} = \frac{(1.60\times10^{-19}\,\text{C})(8400\,\text{V/m})}{2\pi(9.11\times10^{-31}\,\text{kg})(4.8\times10^6\,\text{m/s})} = \boxed{4.9\times10^7\,\text{Hz}}$$

71. We find the speed of the electron using conservation of energy. The accelerating potential energy becomes the kinetic energy of the electron.

$$eV = \tfrac{1}{2}mv^2 \quad \to \quad v = \sqrt{\frac{2eV}{m}}$$




Upon entering the magnetic field the electron is traveling horizontally. The magnetic field will cause the path of the electron to rotate an angle $\theta$ from the horizontal. While in the field, the electron will travel a horizontal distance $d$ and a vertical distance $h_0$. Using the Pythagorean theorem, and trigonometric relations, we can write three equations which relate the unknown parameters, $r$, $h_0$, and $\theta$.

$$\tan\theta = \frac{h - h_0}{\ell - d} \qquad \sin\theta = \frac{d}{r} \qquad r^2 = d^2 + (r - h_0)^2 \to h_0 = r - \sqrt{r^2 - d^2}$$

These three equations can be directly solved, for the radius of curvature. However, doing so requires solving a 3rd order polynomial. Instead, we can guess at a value for $h_0$, such as 1.0 cm. Then we use the tangent equation to calculate an approximate value for $\theta$. Then insert the approximate value into the sine equation to solve for $r$. Finally, inserting the value of $r$ into the third equation we solve for $h_0$. We then use the new value of $h_0$ as our guess and reiterated the process a couple of times until the value of $h_0$ does not significantly change.

$$\theta = \tan^{-1}\left(\frac{11\text{cm} - 1.0\text{cm}}{22\text{cm} - 3.5\text{cm}}\right) = 28.39° \to r = \frac{3.5\text{cm}}{\sin 28.39°} = 7.36\text{cm}$$

$$\to h_0 = 7.36\text{cm} - \sqrt{(7.36\text{cm})^2 - (3.5\text{cm})^2} = 0.885\text{cm}$$

$$\theta = \tan^{-1}\left(\frac{11\text{ cm} - 0.885\text{ cm}}{22\text{ cm} - 3.5\text{ cm}}\right) = 28.67° \to r = \frac{3.5\text{ cm}}{\sin 28.67°} = 7.30\text{ cm}$$

$$\to h_0 = 7.30\text{ cm} - \sqrt{(7.30\text{ cm})^2 - (3.5\text{ cm})^2} = 0.894\text{ cm}$$

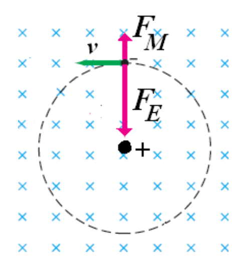
$$\theta = \tan^{-1}\left(\frac{11\text{ cm} - 0.894\text{ cm}}{22\text{ cm} - 3.5\text{ cm}}\right) = 28.65° \to r = \frac{3.5\text{ cm}}{\sin 28.67°} = 7.30\text{cm}$$

$$\to h_0 = 7.30\text{cm} - \sqrt{(7.30\text{cm})^2 - (3.5\text{cm})^2} = 0.894\text{cm}$$

The magnetic field can be determined from the trajectory's radius, as done in Example 27-7.

$$r = \frac{mv}{eB} \quad \to \quad B = \frac{mv}{er} = \frac{m}{er}\sqrt{\frac{2eV}{m}} = \sqrt{\frac{2mV}{er^2}} = \sqrt{\frac{2(9.11\times10^{-31}\text{kg})(25\times10^3\text{V})}{(1.60\times10^{-19}\text{ C})(0.0730\text{m})^2}} = \boxed{7.3\text{mT}}$$

72. (*a*) As the electron orbits the nucleus in the absence of the magnetic field, its centripetal acceleration is caused solely by the electrical attraction between the electron and the nucleus. Writing the velocity of the electron as the circumference of its orbit times its frequency, enables us to obtain an equation for the frequency of the electron's orbit.




$$\frac{ke^2}{r^2} = m\frac{v^2}{r} = m\frac{(2\pi r f_0)^2}{r} \to f_0^2 = \frac{ke^2}{4\pi^2 m r^3}$$

When the magnetic field is added, the magnetic force adds or subtracts from the centripetal acceleration (depending on the direction of the field) resulting in the change in frequency.

$$\frac{ke^2}{r^2} \pm q(2\pi r f)B = m\frac{(2\pi r f)^2}{r} \to f^2 \mp \frac{qB}{2\pi m} f - f_0^2 = 0$$

We can solve for the frequency shift by setting $f = f_0 + \Delta f$, and only keeping the lowest order terms, since $\Delta f \ll f_0$.

$$(f_0 + \Delta f)^2 \mp \frac{qB}{2\pi m}(f_0 + \Delta f) - f_0^2 = 0$$

$$\cancel{f_0^2} + 2f_0\Delta f + \cancel{\Delta f^2} \mp \frac{qB}{2\pi m} f_0 \mp \cancel{\frac{qB}{2\pi m}\Delta f} - \cancel{f_0^2} = 0 \quad \to \quad \boxed{\Delta f = \pm\frac{qB}{4\pi m}}$$

(*b*) The "±" indicates whether the magnetic force adds to or subtracts from the centripetal acceleration. If the magnetic force adds to the centripetal acceleration, the frequency increases. If the magnetic force is opposite in direction to the acceleration, the frequency decreases.

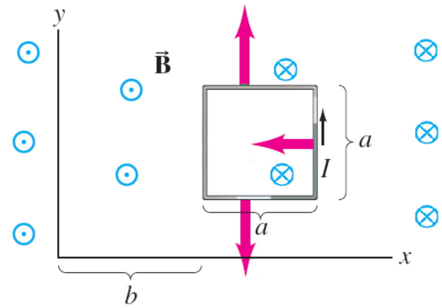
73. The speed of the proton can be calculated based on the radius of curvature of the (almost) circular motion. From that the kinetic energy can be calculated.

$$qvB = \frac{mv^2}{r} \quad \to \quad v = \frac{qBr}{m} \qquad K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2r^2}{2m}$$

$$\Delta K = \frac{q^2B^2}{2m}\left(r_2^2 - r_1^2\right) = \frac{(1.60\times10^{-19}\text{C})^2(0.018\text{T})^2}{2(1.67\times10^{-27}\text{kg})}\left[(8.5\times10^{-3}\text{m})^2 - (10.0\times10^{-3}\text{m})^2\right]$$

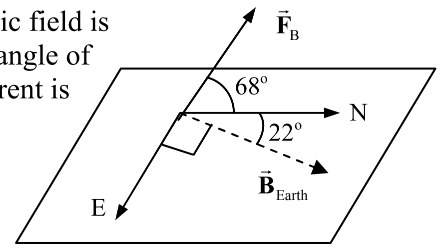
$$= \boxed{-6.9\times10^{-20}\text{J}} \text{ or } -0.43\text{eV}$$

74. The forces on each of the two horizontal sides of the loop have the same magnitude, but opposite directions, so these forces sum to zero. The left side of the loop is located at $x = b$, where the magnetic field is zero, and therefore the force is zero. The net force is the force acting on the right side of the loop. By the right hand rule, with the current directed toward the top of the page and the magnetic field into the page, the force will point in the negative $x$ direction with magnitude given by Eq. 27-2.




$$\vec{\mathbf{F}} = I\ell B\left(-\hat{\mathbf{i}}\right) = IaB_0\left(1-\frac{b+a}{b}\right)\hat{\mathbf{i}} = \boxed{-\frac{Ia^2B_0}{b}\hat{\mathbf{i}}}$$

75. We assume that the horizontal component of the Earth's magnetic field is pointing due north. The Earth's magnetic field also has the dip angle of 22°. The angle between the magnetic field and the eastward current is 90°. Use Eq. 27-1 to calculate the magnitude of the force.

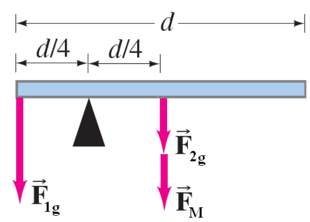



$$F = I\ell B\sin\theta = (330\,\text{A})(5.0\,\text{m})\left(5.0\times10^{-5}\,\text{T}\right)\sin 90^\circ$$

$$= \boxed{0.083\,\text{N}}$$

Using the right hand rule with the eastward current and the Earth's magnetic field, the force on the wire is $\boxed{\text{northerly and 68° above the horizontal}}$.

76. Since the magnetic and gravitational force along the entire rod is uniform, we consider the two forces acting at the center of mass of the rod. To be balanced, the net torque about the fulcrum must be zero. Using the usual sign convention for torques and Eq. 10-10a, we solve for the magnetic force on the rod.



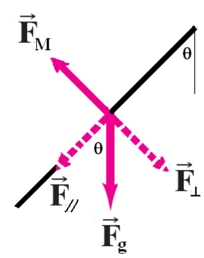

$$\sum\tau = 0 = Mg\left(\tfrac{1}{4}d\right) - mg\left(\tfrac{1}{4}d\right) - F_M\left(\tfrac{1}{4}d\right) \quad\rightarrow\quad F_M = (M-m)g$$

We solve for the current using Eq. 27-2.

$$I = \frac{F}{\ell B} = \frac{(M-m)g}{dB} = \frac{(8.0m-m)g}{dB} = \boxed{\frac{7.0mg}{dB}}$$

The right hand rule indicates that the current must flow $\boxed{\text{toward the left}}$ since the magnetic field is into the page and the magnetic force is downward.

77. (*a*) For the rod to be in equilibrium, the gravitational torque and the magnetic torque must be equal and opposite. Since the rod is uniform, the two torques can be considered to act at the same location (the center of mass). Therefore, components of the two forces perpendicular to the rod must be equal and opposite. Since the gravitational force points downward, its perpendicular component will point down and to the right. The magnetic force is perpendicular to the rod and must point towards the left to oppose the perpendicular component of the gravitational force. By the right hand rule, with a magnetic field pointing out of the page, the current must flow $\boxed{\text{downward from the pivot}}$ to produce this force.




(*b*) We set the magnitude of the magnetic force, using Eq. 27-2, equal to the magnitude of the perpendicular component of the gravitational force, $F_\perp = mg\sin\theta$, and solve for the magnetic field.

$$I\ell B = mg\sin\theta \;\rightarrow\; B = \frac{mg\sin\theta}{I\ell} = \frac{(0.150\,\text{kg})(9.80\,\text{m/s}^2)\sin 13^\circ}{(12\,\text{A})(1.0\,\text{m})} = \boxed{0.028\,\text{T}}$$

(*c*) The largest magnetic field that could be measured is when $\theta = 90^\circ$.

$$B_{\text{max}} = \frac{mg\sin 90^\circ}{I\ell} = \frac{(0.150\,\text{kg})(9.80\,\text{m/s}^2)\sin 90^\circ}{(12\,\text{A})(1.0\,\text{m})} = \boxed{0.12\,\text{T}}$$

# CHAPTER 28: Sources of Magnetic Field

## Responses to Questions

1. Alternating currents will have little effect on the compass needle, due to the rapid change of the direction of the current and of the magnetic field surrounding it. Direct currents will deflect a compass needle. The deflection depends on the magnitude and direction of the current and the distance from the current to the compass. The effect on the compass decreases with increasing distance from the wire.

2. The magnetic field due to a long straight current is proportional to the current strength. The electric field due to a long straight line of electric charge at rest is proportional to the charge per unit length. Both fields are inversely proportional to the distance from the wire or line of charge. The magnetic field lines form concentric circles around the wire; the electric field lines are directed radially outward if the line of charge is positive and radially inward if the line of charge is negative.

3. The magnetic forces exerted on one wire by the other try to align the wires. The net force on either wire is zero, but the net torque is not zero.

4. Yes. Assume the upper wire is fixed in position. Since the currents in the wires are in the same direction, the wires will attract each other. The lower wire will be held in equilibrium if this force of attraction (upward) is equal in magnitude to the weight of the wire (downward).

5. (*a*) The current in the lower wire is opposite in direction to the current in the upper wire.
   (*b*) The upper wire can be held in equilibrium due to the balance between the magnetic force from the lower wire and the gravitational force. The equilibrium will be stable for small vertical displacements, but not for horizontal displacements.

6. (*a*) Let $I_2 = I_1$. $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} = \mu_0 \left( I_1 + I_2 \right) = 2\mu_0 I_1$
   (*b*) Let $I_2 = -I_1$. $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} = \mu_0 \left( I_1 + I_2 \right) = 0$

7. Inside the cavity $\vec{\mathbf{B}} = 0$ since the geometry is cylindrical and no current is enclosed.

8. Construct a closed path similar to that shown in part (*a*) of the figure, such that sides *ab* and *cd* are perpendicular to the field lines and sides *bc* and *da* lie along the field lines. Unlike part (*a*), the path will not form a rectangle; the sides *ab* and *cd* will flare outward so that side *bc* is longer than side *da*. Since the field is stronger in the region of *da* than it is in the region of *bc*, but *da* is shorter than *bc*, the contributions to the integral in Ampère's law may cancel. Thus, $\mu_0 I_{\text{encl}} = \oint \vec{\mathbf{B}} \cdot d\vec{\ell} = 0$ is possible and the field is consistent with Ampère's law. The lines could not curve upward instead of downward, because then *bc* would be shorter than *da* and it would not be possible for the contributions to sum to zero.

9. The equation for the magnetic field strength inside a solenoid is given by $B = \mu_0 n I$.
   (*a*) The magnetic field strength is not affected if the diameter of the loops doubles.
   (*b*) If the spacing between the loops doubles, the number of loops per unit length decreases by a factor of 2, and the magnetic field strength then also decreases by a factor of 2.

(*c*) If the solenoid's length is doubled along with the doubling of the total number of loops, then the number of loops per unit length remains the same, and the magnetic field strength is not affected.

10. The Biot-Savart law states that the net field at a point in space is the vector sum of the field contributions due to each infinitesimal current element. As shown in Example 28-12, the magnetic field along the axis of a current loop is parallel to the axis because the perpendicular field contributions cancel. However, for points off the axis, the perpendicular contributions will not cancel. The net field for a point off the axis will be dominated by the current elements closest to it. For example, in Figure 28-21, the field lines inside the loop but below the axis curve downward, because these points in space are closer to the lower segment of the loop (where the current goes into the page) than they are to the upper segment (where the current comes out of the page).

11. No. The magnetic field varies in strength and direction for points in the plane of the loop. The magnetic field is strongest at the center of the loop.

12. The lead-in wires to electrical devices have currents running in opposite directions. The magnetic fields due to these currents are therefore also opposite in direction. If the wires are twisted together, then the distance from a point outside the wires to each of the individual wires is about the same, and the field contributions from the two wires will cancel. If the wires were not twisted and were separate from each other, then a point outside the wires would be a different distance from one of the wires than from the other, and there would be a net field due to the currents in the wires.

13. The Biot-Savart law and Coulomb's law are both inverse-square in the radius and both contain a proportionality constant. Coulomb's law describes a central force; the Biot-Savart law involves a cross product of vectors and so cannot describe a central force.

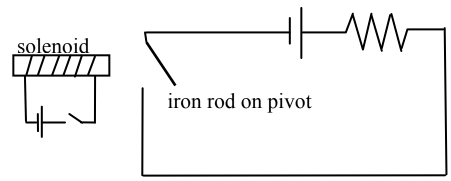
14. (*a*) The force between two identical electric charges is given by Coulomb's law: $F = \frac{kq^2}{r^2}$.

Magnetic pole strength of a bar magnet could be defined using an analogous expression for the magnetic force between the poles of two identical magnets: $F = \frac{\mu m^2}{4\pi r^2}$. Then, magnetic pole strength, *m*, would be given by $m = \sqrt{\frac{4\pi F r^2}{\mu}}$. To determine *m*, place two identical magnets with their poles facing each other a distance *r* apart and measure the force between them.

(*b*) The magnetic pole strength of a current loop could be defined the same way by using two identical current loops instead of two bar magnets.

15. Determine the magnetic field of the Earth at one of the magnetic poles (north or south), and use Equation 28-7b to calculate the magnetic moment. In this equation, *x* will be (approximately) the radius of the Earth.

16. To design a relay, place an iron rod inside a solenoid, with the solenoid oriented such that one end of it is facing a second iron rod on a pivot. The second iron rod functions as a switch for the large-current circuit and is normally held open by a spring. When current flows through the solenoid, the iron rod inside it becomes magnetized and attracts the second iron rod, closing the switch and allowing current to flow.



17. (*a*) The source of the kinetic energy is the attractive force produced by the magnetic field from the magnet acting on the magnetic moments of the atoms in the iron.
    (*b*) When the block strikes the magnet, some of the kinetic energy from the block is converted into kinetic energy in the iron atoms in the magnet, randomizing their magnetic moments and decreasing the overall field produced by the magnet. Some of the kinetic energy of the block as a whole is also converted into the kinetic energy of the individual atoms in the block, resulting in an increase in thermal energy.

18. No, a magnet with a steady field will only attract objects made of ferromagnetic materials. Aluminum is not ferromagnetic, so the magnetic field of the magnet will not cause the aluminum to become a temporary magnet and therefore there will be no attractive force. Iron is ferromagnetic, so in the presence of a magnet, the domains in a piece of iron will align such that it will be attracted to the magnet.

19. An unmagnetized nail has randomly oriented domains and will not generate an external magnetic field. Therefore, it will not attract an unmagnitized paper clip, which also has randomly oriented domains. When one end of the nail is in contact with a magnet, some of the domains in the nail align, producing an external magnetic field and turning the nail into a magnet. The magnetic nail will cause some of the domains in the paper clip to align, and it will be attracted to the nail.

20. Yes, an iron rod can attract a magnet and a magnet can attract an iron rod. Consider Newton's third law. If object A attracts object B then object B attracts object A.

21. Domains in ferromagnetic materials in molten form were aligned by the Earth's magnetic field and then fixed in place as the material cooled.

22. Yes. When a magnet is brought near an unmagnetized piece of iron, the magnet's field causes a temporary alignment of the domains of the iron. If the magnet's north pole is brought near the iron, then the domains align such that the temporary south pole of the iron is facing the magnet, and if the magnet's south pole is closest to the iron, then the alignment will be the opposite. In either case, the magnet and the iron will attract each other.

23. The two rods that have ends that repel each other will be the magnets. The unmagnetized rod will be attracted to both ends of the magnetized rods.

24. No. If they were both magnets, then they would repel one another when they were placed with like poles facing each other. However, if one is a magnet and the other isn't, they will attract each other no matter which ends are placed together. The magnet will cause an alignment of the domains of the non-magnet, causing an attraction.

25. (*a*) The magnetization curve for a paramagnetic substance is a straight line with slope slightly greater than 1. It passes through the origin; there is no hysteresis.
    (*b*) The magnetization curve for a diamagnetic substance is a straight line with slope slightly less than 1. It passes through the origin; there is no hysteresis.
    The magnetization curve for a ferromagnetic substance is a hysteresis curve (see Figure 28-29).

26. (*a*) Yes. Diamagnetism is present in all materials but in materials that are also paramagnetic or ferromagnetic, its effects will not be noticeable.
    (*b*) No. Paramagnetic materials are nonferromagnetic materials with a relative permeability greater than one.
    (*c*) No. Ferromagnetic materials are those that can be magnetized by alignment of their domains.

## Solutions to Problems

1. We assume the jumper cable is a long straight wire, and use Eq. 28-1.

$$B_{\text{cable}}=\frac{\mu_0 I}{2\pi r}=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(65\text{A}\right)}{2\pi\left(0.035\,\text{m}\right)}=3.714\times10^{-4}\,\text{T}\approx\boxed{3.7\times10^{-4}\,\text{T}}$$

Compare this to the Earth's field of $0.5\times10^{-4}\,\text{T}$.

$$B_{\text{cable}}/B_{\text{Earth}}=\frac{3.714\times10^{-4}\,\text{T}}{5.0\times10^{-5}\,\text{T}}=7.43\text{, so }\boxed{\text{the field of the cable is over 7 times that of the Earth.}}$$

2. We assume that the wire is long and straight, and use Eq. 28-1.

$$B_{\text{wire}}=\frac{\mu_0 I}{2\pi r}\quad\rightarrow\quad I=\frac{2\pi r B_{\text{wire}}}{\mu_0}=\frac{2\pi\left(0.15\,\text{m}\right)\left(0.50\times10^{-4}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}=37.5\,\text{A}\approx\boxed{38\,\text{A}}$$

3. Since the currents are parallel, the force on each wire will be attractive, toward the other wire. Use Eq. 28-2 to calculate the magnitude of the force.

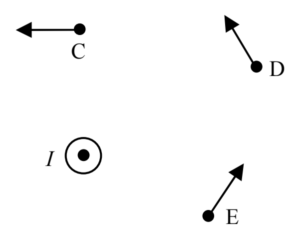
$$F_2=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi}\frac{\left(35\,\text{A}\right)^2}{\left(0.040\,\text{m}\right)}\left(25\,\text{m}\right)=\boxed{0.15\,\text{N, attractive}}$$

4. Since the force is attractive, the currents must be in the same direction, so the current in the second wire must also be upward. Use Eq. 28-2 to calculate the magnitude of the second current.

$$F_2=\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2\quad\rightarrow$$

$$I_2=\frac{2\pi}{\mu_0}\frac{F_2}{\ell_2}\frac{d}{I_1}=\frac{2\pi}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}\left(7.8\times10^{-4}\,\text{N/m}\right)\frac{0.070\,\text{m}}{28\,\text{A}}=9.75\,\text{A}\approx\boxed{9.8\,\text{A upward}}$$

5. To find the direction, draw a radius line from the wire to the field point. Then at the field point, draw a perpendicular to the radius line, directed so that the perpendicular line would be part of a counterclockwise circle.

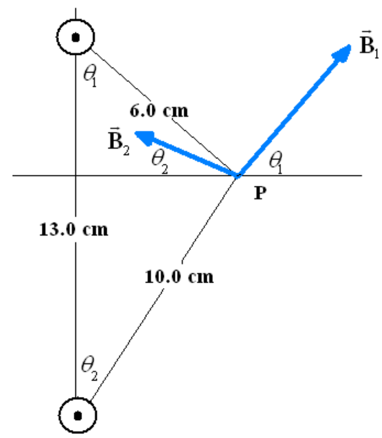



6. For the experiment to be accurate to $\pm2.0\%$, the magnetic field due to the current in the cable must be less than or equal to 2.0% of the Earth's magnetic field. Use Eq. 28-1 to calculate the magnetic field due to the current in the cable.

$$B_{\text{cable}}=\frac{\mu_0 I}{2\pi r}\le 0.020B_{\text{Earth}}\quad\rightarrow\quad I\le\frac{2\pi r\left(0.020B_{\text{Earth}}\right)}{\mu_0}=\frac{2\pi\left(1.00\,\text{m}\right)\left(0.020\right)\left(0.5\times10^{-4}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}=5.0\,\text{A}$$

Thus the maximum allowable current is $\boxed{5.0\,\text{A}}$.

7. Since the magnetic field from a current carrying wire circles the wire, the individual field at point P from each wire is perpendicular to the radial line from that wire to point P. We define $\vec{\mathbf{B}}_1$ as the field from the top wire, and $\vec{\mathbf{B}}_2$ as the field from the bottom wire. We use Eq. 28-1 to calculate the magnitude of each individual field.




$$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(35\,\text{A})}{2\pi(0.060\,\text{m})} = 1.17\times10^{-4}\,\text{T}$$

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(35\,\text{A})}{2\pi(0.100\,\text{m})} = 7.00\times10^{-5}\,\text{T}$$

We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the vertical. Since the field is perpendicular to the radial line, this is the same angle that the magnetic fields make with the horizontal.

$$\theta_1 = \cos^{-1}\left(\frac{(0.060\text{ m})^2 + (0.130\text{ m})^2 - (0.100\text{ m})^2}{2(0.060\text{ m})(0.130\text{ m})}\right) = 47.7°$$

$$\theta_2 = \cos^{-1}\left(\frac{(0.100\text{ m})^2 + (0.130\text{ m})^2 - (0.060\text{ m})^2}{2(0.100\text{ m})(0.130\text{ m})}\right) = 26.3°$$

Using the magnitudes and angles of each magnetic field we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle.

$$B_{\text{net}\,x} = B_1\cos(\theta_1) - B_2\cos\theta_2 = (1.174\times10^{-4}\,\text{T})\cos 47.7° - (7.00\times10^{-5}\,\text{T})\cos 26.3° = 1.626\times10^{-5}\,\text{T}$$

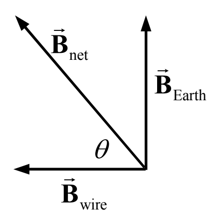
$$B_{\text{net}\,y} = B_1\sin(\theta_1) + B_2\sin\theta_1 = (1.17\times10^{-4}\,\text{T})\sin 47.7° + (7.00\times10^{-5}\,\text{T})\sin 26.3° = 1.18\times10^{-4}\,\text{T}$$

$$B = \sqrt{B_{net,x}^2 + B_{net,y}^2} = \sqrt{(1.626\times10^{-5}\text{ T})^2 + (1.18\times10^{-4}\text{ T})^2} = 1.19\times10^{-4}\text{ T}$$

$$\theta = \tan^{-1}\frac{B_{net,y}}{B_{net,y}} = \tan^{-1}\frac{1.18\times10^{-4}\text{ T}}{1.626\times10^{-5}\text{ T}} = 82.2°$$

$$\vec{\mathbf{B}} = 1.19\times10^{-4}\text{ T @ }82.2° \approx \boxed{1.2\times10^{-4}\,\text{T @ }82°}$$

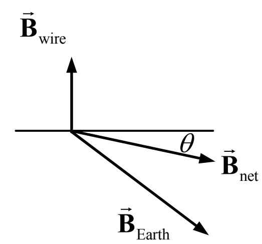
8. At the location of the compass, the magnetic field caused by the wire will point to the west, and the Earth's magnetic field points due North. The compass needle will point in the direction of the NET magnetic field.




$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(43\,\text{A})}{2\pi(0.18\,\text{m})} = 4.78\times10^{-5}\,\text{T}$$

$$\theta = \tan^{-1}\frac{B_{\text{Earth}}}{B_{\text{wire}}} = \tan^{-1}\frac{4.5\times10^{-5}\,\text{T}}{4.78\times10^{-5}\,\text{T}} = \boxed{43°\text{N of W}}$$

9. The magnetic field due to the long horizontal wire points straight up at the point in question, and its magnitude is given by Eq. 28-1. The two fields are oriented as shown in the diagram. The net field is the vector sum of the two fields.




$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(24.0\,\text{A})}{2\pi(0.200\,\text{m})} = 2.40\times10^{-5}\,\text{T}$$

$$B_{\text{Earth}} = 5.0\times10^{-5}\,\text{T}$$

$$B_{\text{net}\,x} = B_{\text{Earth}}\cos 44^\circ = 3.60\times10^{-5}\,\text{T} \qquad B_{\text{net}\,y} = B_{\text{wire}} - B_{\text{Earth}}\sin 44^\circ = -1.07\times10^{-5}\,\text{T}$$

$$B_{\text{net}} = \sqrt{B_{\text{net}\,x}^2 + B_{\text{net}\,y}^2} = \sqrt{\left(3.60\times10^{-5}\,\text{T}\right)^2 + \left(-1.07\times10^{-5}\,\text{T}\right)^2} = \boxed{3.8\times10^{-5}\,\text{T}}$$

$$\theta = \tan^{-1}\frac{B_{\text{net}\,y}}{B_{\text{net}\,x}} = \tan^{-1}\frac{-1.07\times10^{-5}\,\text{T}}{3.60\times10^{-5}\,\text{T}} = \boxed{17^\circ\text{ below the horizontal}}$$

10. The stream of protons constitutes a current, whose magnitude is found by multiplying the proton rate times the charge of a proton. Then use Eq. 28-1 to calculate the magnetic field.

$$B_{\text{stream}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(2.5\times10^{9}\,\text{protons/s}\right)\left(1.60\times10^{-19}\,\text{C/proton}\right)}{2\pi\left(2.0\,\text{m}\right)} = \boxed{4.0\times10^{-17}\,\text{T}}$$

11. (*a*) If the currents are in the same direction, the magnetic fields at the midpoint between the two currents will oppose each other, and so their magnitudes should be subtracted.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi\left(0.010\,\text{m}\right)}\left(I - 25\,\text{A}\right) = \boxed{\left(2.0\times10^{-5}\,\text{T/A}\right)\left(I - 25\,\text{A}\right)}$$

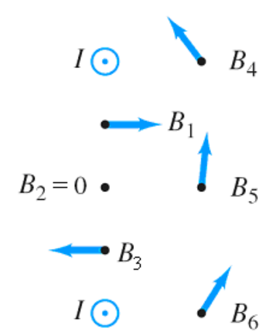
(*b*) If the currents are in the opposite direction, the magnetic fields at the midpoint between the two currents will reinforce each other, and so their magnitudes should be added.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi\left(0.010\,\text{m}\right)}\left(I + 25\,\text{A}\right) = \boxed{\left(2.0\times10^{-5}\,\text{T/A}\right)\left(I + 25\,\text{A}\right)}$$

12. Using the right-hand-rule we see that if the currents flow in the same direction, the magnetic fields will oppose each other between the wires, and therefore can equal zero at a given point. Set the sum of the magnetic fields from the two wires equal to zero at the point 2.2 cm from the first wire and use Eq. 28-1 to solve for the unknown current.

$$B_{net} = 0 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} \rightarrow I_2 = \left(\frac{r_2}{r_1}\right)I_1 = \left(\frac{6.0\,\text{cm} - 2.2\,\text{cm}}{2.2\,\text{cm}}\right)\left(2.0\,\text{A}\right) = \boxed{3.5\,\text{A}}$$

13. Use the right hand rule to determine the direction of the magnetic field from each wire. Remembering that the magnetic field is inversely proportional to the distance from the wire, qualitatively add the magnetic field vectors. The magnetic field at point #2 is zero.




14. The fields created by the two wires will oppose each other, so the net field is the difference of the magnitudes of the two fields. The positive direction for the fields is taken to be into the paper, and so the closer wire creates a field in the positive direction, and the farther wire creates a field in the negative direction. Let *d* be the separation distance of the wires.

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi r_{\text{closer}}} - \frac{\mu_0 I}{2\pi r_{\text{farther}}} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_{\text{closer}}} - \frac{1}{r_{\text{farther}}}\right) = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r - \frac{1}{2}d} - \frac{1}{r + \frac{1}{2}d}\right)$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{d}{\left(r-\tfrac{1}{2}d\right)\left(r+\tfrac{1}{2}d\right)}\right)$$
$$= \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)(28.0\,\mathrm{A})}{2\pi}\left(\frac{0.0028\,\mathrm{m}}{(0.10\,\mathrm{m}-0.0014\,\mathrm{m})(0.10\,\mathrm{m}+0.0014\,\mathrm{m})}\right)$$
$$= 1.568\times10^{-6}\,\mathrm{T} \approx \boxed{1.6\times10^{-6}\,\mathrm{T}}$$

Compare this to the Earth's field of $0.5\times10^{-4}\,\mathrm{T}$.

$$B_{\text{net}}/B_{\text{Earth}} = \frac{1.568\times10^{-6}\,\mathrm{T}}{0.5\times10^{-4}\,\mathrm{T}} = 0.031$$

The field of the wires is about 3% that of the Earth.

15. The center of the third wire is 5.6 mm from the left wire, and 2.8 mm from the right wire. The force on the near (right) wire will attract the near wire, since the currents are in the same direction. The force on the far (left) wire will repel the far wire, since the currents oppose each other. Use Eq. 28-2 to calculate the force per unit length.

$$F_{\text{near}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}}\ell_{\text{near}} \rightarrow$$
$$\frac{F_{\text{near}}}{\ell_{\text{near}}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}} = \frac{4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}}{2\pi}\frac{(25.0\,\mathrm{A})(28.0\,\mathrm{A})}{2.8\times10^{-3}\,\mathrm{m}} = \boxed{0.050\,\mathrm{N/m},\ \text{attractive}}$$
$$F_{\text{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}}\ell_{\text{far}} \rightarrow$$
$$\frac{F_{\text{far}}}{\ell_{\text{far}}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}} = \frac{4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}}{2\pi}\frac{(25.0\,\mathrm{A})(28.0\,\mathrm{A})}{5.6\times10^{-3}\,\mathrm{m}} = \boxed{0.025\,\mathrm{N/m},\ \text{repelling}}$$

16. (*a*) We assume that the power line is long and straight, and use Eq. 28-1.

$$B_{\text{line}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)(95\,\mathrm{A})}{2\pi(8.5\,\mathrm{m})} = 2.235\times10^{-6}\,\mathrm{T} \approx \boxed{2.2\times10^{-6}\,\mathrm{T}}$$

The direction at the ground, from the right hand rule, is south. Compare this to the Earth's field of $0.5\times10^{-4}\,\mathrm{T}$, which points approximately north.

$$B_{\text{line}}/B_{\text{Earth}} = \frac{2.235\times10^{-6}\,\mathrm{T}}{0.5\times10^{-4}\,\mathrm{T}} = 0.0447$$

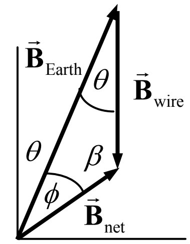
The field of the cable is about 4% that of the Earth.

(*b*) We solve for the distance where $B_{\text{line}} = B_{\text{Earth}}$.

$$B_{\text{line}} = \frac{\mu_0 I}{2\pi r} = B_{\text{Earth}} \rightarrow r = \frac{\mu_0 I}{2\pi B_{\text{Earth}}} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)(95\,\mathrm{A})}{2\pi\left(0.5\times10^{-4}\,\mathrm{T}\right)} = 0.38\,\mathrm{m} \approx \boxed{0.4\,\mathrm{m}}$$

So about 0.4 m below the wire, the net B-field would be 0, assuming the Earth's field points straight north at this location.

17. The Earth's magnetic field is present at both locations in the problem, and we assume it is the same at both locations. The field east of a vertical wire must be pointing either due north or due south. The compass shows the direction of the net magnetic field, and it changes from 28° E of N to 55° E of N when taken inside. That is a "southerly" change (rather than a "northerly" change), and so the field due to the wire must be pointing due south. See the diagram. For the angles, $\theta = 28°$, $\theta + \phi = 55°$, and $\beta + \theta + \phi = 180°$ and so $\phi = 27°$ and $\beta = 125°$. Use the law of sines to find the magnitude of $\vec{\mathbf{B}}_{\text{wire}}$, and then use Eq. 28-1 to find the magnitude of the current.




$$\frac{B_{\text{wire}}}{\sin\phi} = \frac{B_{\text{Earth}}}{\sin\beta} \rightarrow B_{\text{wire}} = B_{\text{Earth}}\frac{\sin\phi}{\sin\beta} = \frac{\mu_0}{2\pi}\frac{I}{r} \rightarrow$$

$$I = B_{\text{Earth}}\frac{\sin\phi}{\sin\beta}\frac{2\pi}{\mu_0}r = \left(5.0\times10^{-5}\,\text{T}\right)\frac{\sin 27^\circ}{\sin 125^\circ}\frac{2\pi}{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}\left(0.120\,\text{m}\right) = \boxed{17\,\text{A}}$$

Since the field due to the wire is due south, the current in the wire must be $\boxed{\text{down}}$.

18. The magnetic field at the loop due to the long wire is into the page, and can be calculated by Eq. 28-1. The force on the segment of the loop closest to the wire is towards the wire, since the currents are in the same direction. The force on the segment of the loop farthest from the wire is away from the wire, since the currents are in the opposite direction.

Because the magnetic field varies with distance, it is more difficult to calculate the total force on the left and right segments of the loop. Using the right hand rule, the force on each small piece of the left segment of wire is to the left, and the force on each small piece of the right segment of wire is to the right. If left and right small pieces are chosen that are equidistant from the long wire, the net force on those two small pieces is zero. Thus the total force on the left and right segments of wire is zero, and so only the parallel segments need to be considered in the calculation. Use Eq. 28-2.

$$F_{\text{net}} = F_{\text{near}} - F_{\text{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}}\ell_{\text{near}} - \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}}\ell_{\text{far}} = \frac{\mu_0}{2\pi}I_1 I_2 \ell\left(\frac{1}{d_{\text{near}}} - \frac{1}{d_{\text{far}}}\right)$$

$$= \frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi}\left(3.5\,\text{A}\right)^2\left(0.100\,\text{m}\right)\left(\frac{1}{0.030\,\text{m}} - \frac{1}{0.080\,\text{m}}\right) = \boxed{5.1\times10^{-6}\,\text{N, towards wire}}$$

$\boxed{19.}$ The left wire will cause a field on the *x* axis that points in the *y* direction, and the right wire will cause a field on the *x* axis that points in the negative *y* direction. The distance from the left wire to a point on the *x* axis is *x*, and the distance from the right wire is $d - x$.

$$\vec{\mathbf{B}}_{\text{net}} = \frac{\mu_0 I}{2\pi x}\hat{\mathbf{j}} - \frac{\mu_0 I}{2\pi(d-x)}\hat{\mathbf{j}} = \boxed{\frac{\mu_0 I}{2\pi}\left(\frac{1}{x} - \frac{1}{d-x}\right)\hat{\mathbf{j}}} = \boxed{\frac{\mu_0 I}{2\pi}\left(\frac{d-2x}{x(d-x)}\right)\hat{\mathbf{j}}}$$

20. The left wire will cause a field on the *x* axis that points in the negative *y* direction, and the right wire will also cause a field on the *x* axis that points in the negative *y* direction. The distance from the left wire to a point on the *x* axis is *x*, and the distance from the right wire is $d - x$.

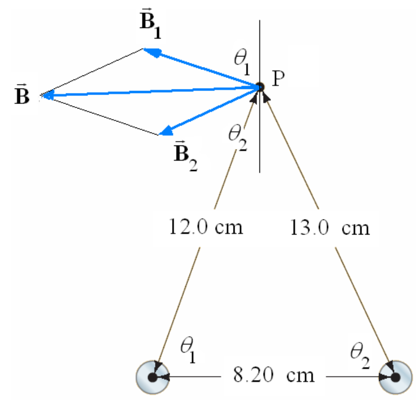
$$\vec{\mathbf{B}}_{\text{net}} = -\frac{\mu_0(2I)}{2\pi x}\hat{\mathbf{j}} - \frac{\mu_0 I}{2\pi(d-x)}\hat{\mathbf{j}} = \boxed{-\frac{\mu_0 I}{2\pi}\left(\frac{2}{x} + \frac{1}{d-x}\right)\hat{\mathbf{j}}}$$

21. The magnetic fields created by the individual currents will be at right angles to each other. The field due to the top wire will be to the right, and the field due to the bottom wire will be out of the page. Since they are at right angles, the net field is the hypotenuse of the two individual fields.

$$B_{\text{net}} = \sqrt{\left(\frac{\mu_0 I_{\text{top}}}{2\pi r_{\text{top}}}\right)^2 + \left(\frac{\mu_0 I_{\text{bottom}}}{2\pi r_{\text{bottom}}}\right)^2} = \frac{\mu_0}{2\pi r}\sqrt{I_{\text{top}}^2 + I_{\text{bottom}}^2} = \frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi(0.100\,\text{m})}\sqrt{(20.0\,\text{A})^2+(12.0\,\text{A})^2}$$

$$= \boxed{4.66\times10^{-5}\,\text{T}}$$

22. The net magnetic field is the vector sum of the magnetic fields produced by each current carrying wire. Since the individual magnetic fields encircle the wire producing it, the field is perpendicular to the radial line from the wire to point P. We let $\vec{\mathbf{B}}_1$ be the field from the left wire, and $\vec{\mathbf{B}}_2$ designate the field from the right wire. The magnitude of the magnetic field vectors is calculated from Eq. 28-1.




$$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(16.5\,\text{A})}{2\pi(0.12\,\text{m})} = 2.7500\times10^{-5}\,\text{T}$$

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(16.5\,\text{A})}{2\pi(0.13\,\text{m})} = 2.5385\times10^{-5}\,\text{T}$$

We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the horizontal. Since the magnetic fields are perpendicular to the radial lines, these angles are the same as the angles the magnetic fields make with the vertical.

$$\theta_1 = \cos^{-1}\left(\frac{(0.12\text{ m})^2 + (0.082\text{ m})^2 - (0.13\text{ m})^2}{2(0.12\text{ m})(0.082\text{ m})}\right) = 77.606^\circ$$

$$\theta_2 = \cos^{-1}\left(\frac{(0.13\text{ m})^2 + (0.082\text{ m})^2 - (0.12\text{ m})^2}{2(0.13\text{ m})(0.082\text{ m})}\right) = 64.364^\circ$$

Using the magnitudes and angles of each magnetic field we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle.

$$B_{\text{net}\,x} = -B_1\sin(\theta_1) - B_2\sin\theta_2 = -(2.7500\times10^{-5}\text{T})\sin77.606^\circ - (2.5385\times10^{-5}\text{T})\sin64.364^\circ$$

$$= -49.75\times10^{-6}\text{T}$$

$$B_{\text{net}\,y} = B_1\cos(\theta_1) - B_2\cos\theta_1 = (2.7500\times10^{-5}\text{T})\cos77.606^\circ - (2.5385\times10^{-5}\text{T})\cos64.364^\circ$$

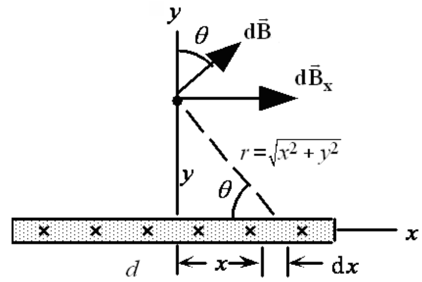
$$= -5.080\times10^{-6}\text{T}$$

$$B = \sqrt{B_{net,x}^2 + B_{net,y}^2} = \sqrt{(-49.75\times10^{-6}\text{ T})^2 + (-5.080\times10^{-6}\text{ T})^2} = 5.00\times10^{-5}\text{ T}$$

$$\theta = \tan^{-1}\frac{B_{net,y}}{B_{net,y}} = \tan^{-1}\frac{-5.08\times10^{-6}\text{ T}}{-49.75\times10^{-6}\text{ T}} = 5.83^\circ$$

$$\boxed{\vec{\mathbf{B}} = 5.00\times10^{-5}\text{ T @ } 5.83^\circ \text{ below the negative } x\text{-axis}}$$

23. (*a*) The net magnetic field a point $y$ above the center of the strip can be found by dividing the strip into infinitely thin wires and integrating the field contribution from each wire. Since the point is directly above the center of the strip, we see that the vertical contributions to the magnetic field from symmetric points on either side of the center cancel out. Therefore, we only need to integrate the horizontal component of the magnetic field. We use Eq. 28-1 for the magnitude of the magnetic field, with the current given by




$$dI = \frac{I}{d}dx.$$

$$B_x = \int \frac{\mu_0 \sin\theta}{2\pi r} dI = \frac{\mu_0 I}{2\pi d}\int_{-\frac{1}{2}d}^{\frac{1}{2}d} \frac{dx}{\sqrt{x^2+y^2}}\left(\frac{y}{\sqrt{x^2+y^2}}\right) = \frac{\mu_0 I y}{2\pi d}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dx}{x^2+y^2}$$

$$= \frac{\mu_0 I y}{2\pi d}\frac{1}{y}\tan^{-1}\frac{x}{y}\bigg|_{-d/2}^{d/2} = \boxed{\frac{\mu_0 I}{\pi d}\tan^{-1}\left(\frac{d}{2y}\right)}$$

(*b*) In the limit of large $y$, $\tan^{-1} d/2y \approx d/2y.$

$$B_x = \frac{\mu_0 I}{\pi d}\tan^{-1}\left(\frac{d}{2y}\right) \approx \frac{\mu_0 I}{\pi d}\frac{d}{2y} = \boxed{\frac{\mu_0 I}{2\pi y}}$$

This is the same as the magnetic field for a long wire.

24. We break the current loop into the three branches of the triangle and add the forces from each of the three branches. The current in the parallel branch flows in the same direction as the long straight wire, so the force is attractive with magnitude given by Eq. 28-2.

$$F_1 = \frac{\mu_0 I I'}{2\pi d}a$$

By symmetry the magnetic force for the other two segments will be equal. These two wires can be broken down into infinitesimal segments, each with horizontal length $dx$. The net force is found by integrating Eq. 28-2 over the side of the triangle. We set $x$=0 at the left end of the left leg. The distance of a line segment to the wire is then given by $r = d + \sqrt{3}x$ . Since the current in these segments flows opposite the direction of the current in the long wire, the force will be repulsive.

$$F_2 = \int_0^{a/2} \frac{\mu_0 I I'}{2\pi\left(d+\sqrt{3}x\right)}dx = \frac{\mu_0 I I'}{2\pi\sqrt{3}}\ln\left(d+\sqrt{3}x\right)\bigg|_0^{a/2} = \frac{\mu_0 I I'}{2\pi\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2d}\right)$$

We calculate the net force by summing the forces from the three segments.

$$F = F_1 - 2F_2 = \frac{\mu_0 I I'}{2\pi d}a - 2\frac{\mu_0 I I'}{2\pi\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2d}\right) = \boxed{\frac{\mu_0 I I'}{\pi}\left[\frac{a}{2d} - \frac{\sqrt{3}}{3}\ln\left(1+\frac{\sqrt{3}a}{2d}\right)\right]}$$

25. Use Eq. 28-4 for the field inside a solenoid.

$$B = \frac{\mu_0 I N}{\ell} \quad \rightarrow \quad I = \frac{B\ell}{\mu_0 N} = \frac{\left(0.385\times10^{-3}\,\text{T}\right)\left(0.400\,\text{m}\right)}{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(765\right)} = \boxed{0.160\,\text{A}}$$

26. The field inside a solenoid is given by Eq. 28-4.

$$B = \frac{\mu_0 IN}{\ell} \quad \rightarrow \quad N = \frac{B\ell}{\mu_0 I} = \frac{(0.30\,\text{T})(0.32\,\text{m})}{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(4.5\,\text{A})} = \boxed{1.7\times10^{4}\,\text{turns}}$$

27. (*a*) We use Eq. 28-1, with *r* equal to the radius of the wire.

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(33\ \text{A})}{2\pi(1.25\times10^{-3}\ \text{m})} = \boxed{5.3\ \text{mT}}$$

(*b*) We use the results of Example 28-6, for points inside the wire. Note that $r = (1.25 - 0.50)\,\text{mm}$ $= 0.75\,\text{mm}$.

$$B = \frac{\mu_0 Ir}{2\pi R^2} = \frac{(4\pi\times10^{-7}\text{T}\bullet\text{m/A})(33\,\text{A})(0.75\times10^{-3}\,\text{m})}{2\pi(1.25\times10^{-3}\,\text{m})^2} = \boxed{3.2\,\text{mT}}$$

(*c*) We use Eq. 28-1, with *r* equal to the distance from the center of the wire.

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(33\ \text{A})}{2\pi(1.25\times10^{-3}\ \text{m} + 2.5\times10^{-3}\ \text{m})} = \boxed{1.8\ \text{mT}}$$

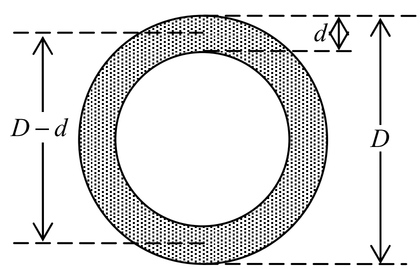
28. We use the results of Example 28-10 to find the maximum and minimum fields.

$$B_{\text{min}} = \frac{\mu_0 NI}{2\pi r_{\text{max}}} = \frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(687)(25.0\ \text{A})}{2\pi(0.270\ \text{m})} = 12.7\ \text{mT}$$

$$B_{\text{max}} = \frac{\mu_0 NI}{2\pi r_{\text{min}}} = \frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(687)(25.0\ \text{A})}{2\pi(0.250\ \text{m})} = 13.7\ \text{mT}$$

$$\boxed{12.7\ \text{mT} < B < 13.7\ \text{mT}}$$

29. (*a*) The copper wire is being wound about an average diameter that is approximately equal to the outside diameter of the solenoid minus the diameter of the wire, or $D - d$. See the (not to scale) end-view diagram. The length of each wrapping is $\pi(D-d)$. We divide the length of the wire $L$ by the length of a single winding to determine the number of loops. The length of the solenoid is the number of loops multiplied by the outer diameter of the wire, *d*.




$$\ell = d\frac{L}{\pi(D-d)} = (2.00\times10^{-3}\text{m})\frac{20.0\,\text{m}}{\pi\left[2.50\times10^{-2}\text{m} - (2.00\times10^{-3}\text{m})\right]} = \boxed{0.554\,\text{m}}$$

(*b*) The field inside the solenoid is found using Eq. 28-4. Since the coils are wound closely together, the number of turns per unit length is equal to the reciprocal of the wire diameter.

$$n = \frac{\#\ \text{turns}}{\ell} = \frac{\overline{\dfrac{L}{\pi(D-d)}}}{\ell} = \frac{\ell/d}{\ell} = \frac{1}{d}$$

$$B = \mu_0 nI = \frac{\mu_0 I}{d} = \frac{(4\pi\times10^{-7}\text{T}\bullet\text{m/A})(16.7\,\text{A})}{2.00\times10^{-3}\text{m}} = \boxed{10.5\,\text{mT}}$$

30. (*a*) The magnitude of the magnetic field from each wire is found using Eq. 28-1. The direction of the magnetic field is perpendicular to the radial vector from the current to the point of interest. Since the currents are both coming out of the page, the magnetic fields will point counterclockwise from the radial line. The total magnetic field is the vector sum of the individual fields.

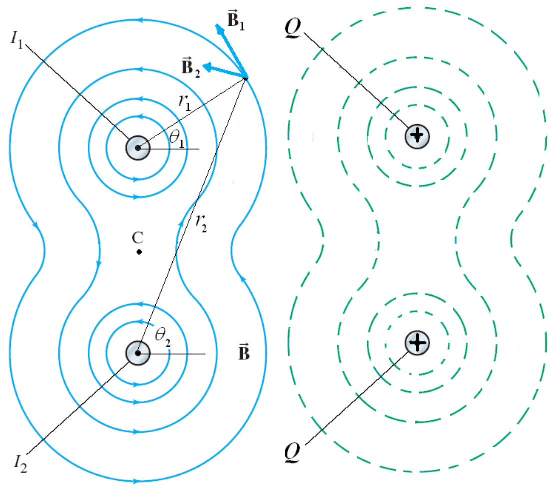
$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 I}{2\pi r_1}\left(-\sin\theta_1\hat{\mathbf{i}} + \cos\theta_1\hat{\mathbf{j}}\right) + \frac{\mu_0 I}{2\pi r_2}\left(-\sin\theta_2\hat{\mathbf{i}} + \cos\theta_2\hat{\mathbf{j}}\right)$$

$$= \frac{\mu_0 I}{2\pi}\left[\left(-\frac{\sin\theta_1}{r_1} - \frac{\sin\theta_1}{r_2}\right)\hat{\mathbf{i}} + \left(\frac{\cos\theta_1}{r_1} + \frac{\cos\theta_2}{r_2}\right)\hat{\mathbf{j}}\right]$$

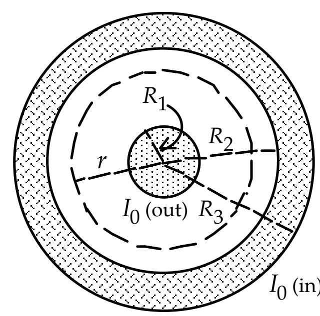
This equation for the magnetic field shows that the *x*-component of the magnetic field is symmetric and the *y*-component is anti-symmetric about $\theta = 90°$.

(*b*) See sketch.

(*c*) The two diagrams are similar in shape, as both form loops around the central axes. However, the magnetic field lines form a vector field, showing the direction, not necessarily the magnitude of the magnetic field. The equipotential lines are from a scalar field showing the points of constant magnitude. The equipotential lines do not have an associated direction.




31. Because of the cylindrical symmetry, the magnetic fields will be circular. In each case, we can determine the magnetic field using Ampere's law with concentric loops. The current densities in the wires are given by the total current divided by the cross-sectional area.




$$J_{\text{inner}} = \frac{I_0}{\pi R_1^2} \qquad J_{\text{outer}} = -\frac{I_0}{\pi\left(R_3^2 - R_2^2\right)}$$

(*a*) Inside the inner wire the enclosed current is determined by the current density of the inner wire.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0\left(J_{\text{inner}}\pi R^2\right)$$

$$B(2\pi R) = \mu_0 \frac{I_0 \pi R^2}{\pi R_1^2} \rightarrow \boxed{B = \frac{\mu_0 I_0 R}{2\pi R_1^2}}$$

(*b*) Between the wires the current enclosed is the current on the inner wire.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} \rightarrow B(2\pi R) = \mu_0 I_0 \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}}$$

(*c*) Inside the outer wire the current enclosed is the current from the inner wire and a portion of the current from the outer wire.

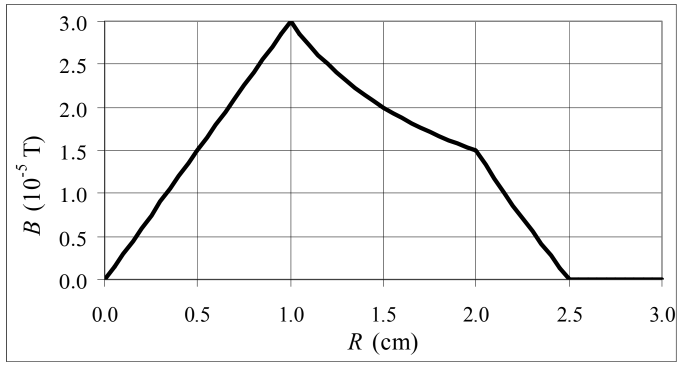
$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0\left[I_0 + J_{\text{outer}}\pi\left(R^2 - R_2^2\right)\right]$$

$$B(2\pi r) = \mu_0\left[I_0 - I_0\frac{\pi\left(R^2 - R_2^2\right)}{\pi\left(R_3^2 - R_2^2\right)}\right] \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}\frac{\left(R_3^2 - R^2\right)}{\left(R_3^2 - R_2^2\right)}}$$

(*d*) Outside the outer wire the net current enclosed is zero.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = 0 \rightarrow B(2\pi R) = 0 \rightarrow \boxed{B = 0}$$

(*e*) See the adjacent graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.31e."




32. We first find the constants $C_1$ and $C_2$ by integrating the currents over each cylinder and setting the integral equal to the total current.

$$I_0 = \int_0^{R_1} C_1 R 2\pi R dR = 2\pi C_1 \int_0^{R_1} R^2 dR = \frac{2}{3}\pi R_1^3 C_1 \rightarrow C_1 = \frac{3I_0}{2\pi R_1^3}$$

$$-I_0 = 2\pi C_2 \int_{R_2}^{R_3} R^2 dR = \frac{2}{3}\pi\left(R_3^3 - R_2^3\right)C_2 \rightarrow C_2 = \frac{-3I_0}{2\pi\left(R_3^3 - R_2^3\right)}$$

(*a*) Inside the inner wire the enclosed current is determined by integrating the current density inside the radius R.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0 \int_0^R (C_1 R') 2\pi R' dR' = \frac{2}{3}\mu_0 \pi C_1 R^3 = \frac{2}{3}\mu_0 \pi \left(\frac{3I_0}{2\pi R_1^3}\right) R^3$$

$$B(2\pi R) = \mu_0 \frac{I_0 \pi R^3}{\pi R_1^3} \rightarrow \boxed{B = \frac{\mu_0 I_0 R^2}{2\pi R_1^3}}$$

(*b*) Between the wires the current enclosed is the current on the inner wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} \rightarrow B(2\pi R) = \mu_0 I_0 \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}}$$

(*c*) Inside the outer wire the current enclosed is the current from the inner wire and a portion of the current from the outer wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0 \left[ I_0 + \int_{R_2}^{R} (C_2 R) 2\pi R dR \right] = \mu_0 \left[ I_0 + \int_{R_2}^{R} (C_2 R) 2\pi R dR \right]$$

$$= \mu_0 I_0 \left[ 1 - \frac{2}{3}\pi C_2 \left(R^3 - R_2^3\right) \right] = \mu_0 \left[ I_0 - \frac{\left(R^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)} \right]$$

$$B(2\pi r) = \mu_0 I_0 \left[ \frac{\left(R_3^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)} - \frac{\left(R^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)} \right] \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R} \frac{\left(R_3^3 - R^3\right)}{\left(R_3^3 - R_2^3\right)}}$$

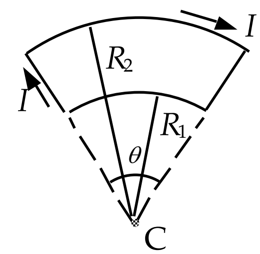
(*d*) Outside the outer wire the net current enclosed is zero.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = 0 \rightarrow B(2\pi R) = 0 \rightarrow \boxed{B = 0}$$

33. Use Eq. 28-7b to write a ratio of the magnetic fields at the surface of the earth and 13,000 km above the surface. Use the resulting ratio to determine the magnetic field above the surface.

$$\frac{B_2}{B_1}=\frac{\dfrac{\mu_0}{2\pi}\dfrac{\mu}{x_2^3}}{\dfrac{\mu_0}{2\pi}\dfrac{\mu}{x_1^3}}=\frac{x_1^3}{x_2^3}\ \rightarrow\ B_2=B_1\frac{x_1^3}{x_2^3}=\left(1.0\times10^{-4}\text{T}\right)\left(\frac{6.38\times10^3\text{km}}{19.38\times10^3\text{km}}\right)^3=\boxed{3.6\times10^{-6}\text{T}}$$

34. Since the point C is along the line of the two straight segments of the current, these segments do not contribute to the magnetic field at C. We calculate the magnetic field by integrating Eq. 28-5 along the two curved segments. Along each integration the line segment is perpendicular to the radial vector and the radial distance is constant.



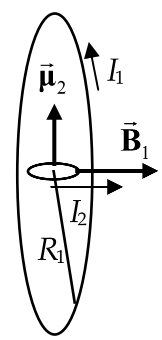

$$\vec{\mathbf{B}}=\frac{\mu_0 I}{4\pi}\int_0^{R_1\theta}\frac{d\vec{\ell}\times\hat{r}}{R_1^2}+\frac{\mu_0 I}{4\pi}\int_{R_2\theta}^{0}\frac{d\vec{\ell}\times\hat{r}}{R_2^2}=\frac{\mu_0 I}{4\pi R_1^2}\hat{\mathbf{k}}\int_0^{R_1\theta}ds+\frac{\mu_0 I}{4\pi R_2^2}\hat{\mathbf{k}}\int_{R_2\theta}^{0}ds$$

$$=\frac{\mu_0 I\theta}{4\pi R_1}\hat{\mathbf{k}}-\frac{\mu_0 I\theta}{4\pi R_2}\hat{\mathbf{k}}=\boxed{\frac{\mu_0 I\theta}{4\pi}\left(\frac{R_2-R_1}{R_1R_2}\right)\hat{\mathbf{k}}}$$

35. Since the current in the two straight segments flows radially toward and away from the center of the loop, they do not contribute to the magnetic field at the center. We calculate the magnetic field by integrating Eq. 28-5 along the two curved segments. Along each integration segment, the current is perpendicular to the radial vector and the radial distance is constant. By the right-hand-rule the magnetic field from the upper portion will point into the page and the magnetic field from the lower portion will point out of the page.

$$\vec{\mathbf{B}}=\frac{\mu_0 I_1}{4\pi}\int_{\text{upper}}\frac{ds}{R^2}\hat{\mathbf{k}}+\frac{\mu_0 I_2}{4\pi}\int_{\text{lower}}\frac{ds}{R^2}\left(-\hat{\mathbf{k}}\right)=\frac{\mu_0(\pi R)}{4\pi R^2}\hat{\mathbf{k}}\left(I_1-I_2\right)=\frac{\mu_0}{4R}\hat{\mathbf{k}}\left(0.35I-0.65I\right)=\boxed{-\frac{3\mu_0 I}{40R}}$$

36. We assume that the inner loop is sufficiently small that the magnetic field from the larger loop can be considered to be constant across the surface of the smaller loop. The field at the center of the larger loop is illustrated in Example 28-12. Use Eq. 27-10 to calculate the magnetic moment of the small loop, and Eq. 27-11 to calculate the torque.



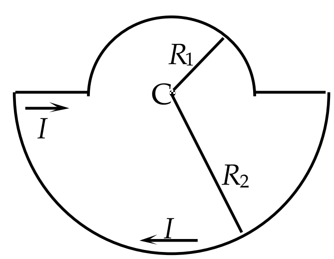

$$\vec{\mathbf{B}}=\frac{\mu_0 I}{2R}\hat{\mathbf{i}}\qquad\qquad \vec{\boldsymbol{\mu}}=I\vec{\mathbf{A}}=I\pi R_2^2\hat{\mathbf{j}}$$

$$\vec{\boldsymbol{\tau}}=\vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}}=I\pi R_2^2\hat{\mathbf{j}}\times\frac{\mu_0 I}{2R}\hat{\mathbf{i}}=-\frac{\mu_0\pi I^2R_2^2}{2R}\hat{\mathbf{k}}$$

$$=-\frac{\left(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A}\right)\pi\left(7.0\ \text{A}\right)^2\left(0.018\ \text{m}\right)^2}{2\left(0.25\ \text{m}\right)}\hat{\mathbf{k}}=\boxed{-1.3\times10^{-7}\hat{\mathbf{k}}\ \text{m}\bullet\text{N}}$$

This torque would cause the inner loop to rotate into the same plane as the outer loop with the currents flowing in the same direction.

37. (*a*) The magnetic field at point C can be obtained using the Biot-Savart law (Eq. 28-5, integrated over the current). First break the loop into four sections: 1) the upper semi-circle, 2) the lower semi-circle, 3) the right straight segment, and 4) the left straight segment. The two straight segments do not contribute to the magnetic field as the point C is in the same direction that the



current is flowing. Therefore, along these segments $\hat{r}$ and $d\hat{\ell}$ are parallel and $d\hat{\ell}\times\hat{r}=0$. For the upper segment, each infinitesimal line segment is perpendicular to the constant magnitude radial vector, so the magnetic field points downward with constant magnitude.

$$\vec{\mathbf{B}}_{\text{upper}}=\int\frac{\mu_0 I}{4\pi}\frac{d\hat{\ell}\times\hat{r}}{r^2}=\frac{\mu_0 I}{4\pi}\frac{-\hat{k}}{R_1{}^2}\left(\pi R_1\right)=-\frac{\mu_0 I}{4R_1}\hat{k}\,.$$

Along the lower segment, each infinitesimal line segment is also perpendicular to the constant radial vector.

$$\vec{\mathbf{B}}_{\text{lower}}=\int\frac{\mu_0 I}{4\pi}\frac{d\hat{\ell}\times\hat{r}}{r^2}=\frac{\mu_0 I}{4\pi}\frac{-\hat{k}}{R_2{}^2}\left(\pi R_2\right)=-\frac{\mu_0 I}{4R_2}\hat{k}$$

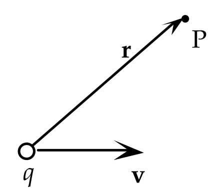
Adding the two contributions yields the total magnetic field.

$$\vec{\mathbf{B}}=\vec{\mathbf{B}}_{\text{upper}}+\vec{\mathbf{B}}_{\text{lower}}=-\frac{\mu_0 I}{4R_1}\hat{k}-\frac{\mu_0 I}{4R_2}\hat{k}=\boxed{-\frac{\mu_0 I}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\hat{k}}$$

(*b*) The magnetic moment is the product of the area and the current. The area is the sum of the two half circles. By the right-hand-rule, curling your fingers in the direction of the current, the thumb points into the page, so the magnetic moment is in the $-\hat{k}$ direction.

$$\vec{\boldsymbol{\mu}}=-\left(\frac{\pi R_1^2}{2}+\frac{\pi R_2^2}{2}\right)I\hat{k}=\boxed{\frac{-\pi I}{2}\left(R_1^2+R_2^2\right)\hat{k}}$$

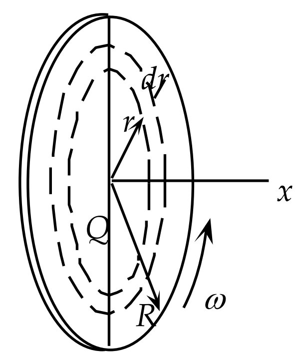
38. Treat the moving point charge as a small current segment. We can write the product of the charge and velocity as the product of a current and current segment. Inserting these into the Biot-Savart law gives us the magnetic field at point P.




$$q\vec{\mathbf{v}}=q\frac{d\vec{\boldsymbol{\ell}}}{dt}=\frac{dq}{dt}d\vec{\boldsymbol{\ell}}=Id\vec{\boldsymbol{\ell}}$$

$$\vec{\mathbf{B}}=\frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\hat{r}}{r^2}=\frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{r}}{r^2}=\boxed{\frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\vec{\mathbf{r}}}{r^3}}$$

39. (*a*) The disk can be broken down into a series of infinitesimal thick rings. As the charge in each of these rings rotates it produces a current of magnitude $dI=\left(\omega/2\pi\right)dq$, where *dq* is the surface charge density multiplied by the area of the ring. We use Eq. 27-10 to calculate the magnetic dipole moment of each current loop and integrate the dipole moments to obtain the total magnetic dipole moment.




$$d\vec{\boldsymbol{\mu}}=dI\vec{\mathbf{A}}=\left(\frac{Q}{\pi R^2}2\pi r dr\frac{\omega}{2\pi}\right)\left(\pi r^2\right)=\frac{Q\omega}{R^2}r^3 dr\hat{\mathbf{i}}$$

$$\vec{\boldsymbol{\mu}}=\int_0^R\frac{Q\omega}{R^2}r^3 dr\,\hat{\mathbf{i}}=\boxed{\frac{Q\omega R^2}{4}\hat{\mathbf{i}}}$$

(*b*) To find the magnetic field a distance *x* along the axis of the disk, we again consider the disk as a series of concentric currents. We use the results of Example 28-12 to determine the magnetic field from each current loop in the disk, and then integrate to obtain the total magnetic field.

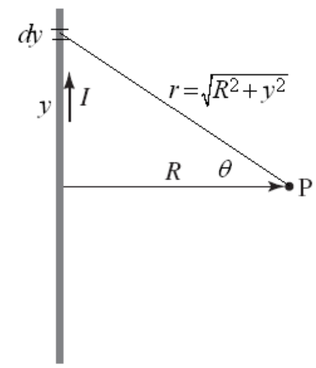
$$d\vec{\mathbf{B}}=\frac{\mu_0 r^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}dI=\frac{\mu_0 r^2}{2\left(r^2+x^2\right)^{\frac{3}{2}}}\frac{Q\omega}{\pi R^2}r dr$$

$$\vec{\mathbf{B}} = \frac{\mu_0 Q\omega \hat{\mathbf{i}}}{2\pi R^2}\int_0^R \frac{r^3}{\left(r^2+x^2\right)^{\frac{3}{2}}}dr = \frac{\mu_0 Q\omega \hat{\mathbf{i}}}{2\pi R^2}\frac{\left(r^2+2x^2\right)}{\sqrt{r^2+x^2}}\Bigg|_0^R = \boxed{\frac{\mu_0 Q\omega \hat{\mathbf{i}}}{2\pi R^2}\left[\frac{\left(R^2+2x^2\right)}{\sqrt{R^2+x^2}}-2x\right]}$$

(*c*) When we take the limit $x \gg R$ our equation reduces to Eq. 28-7b.

$$\vec{\mathbf{B}} \approx \frac{\mu_0 Q\omega \hat{\mathbf{i}}}{2\pi R^2}\left[2x\left(1+\frac{R^2}{2x^2}\right)\left(1-\frac{R^2}{2x^2}+\frac{3R^4}{8x^4}+...\right)-2x\right] \approx \frac{\mu_0 Q\omega \hat{\mathbf{i}}}{2\pi R^2}\left(\frac{R^4}{4x^3}\right) = \boxed{\frac{\mu_0}{2\pi}\frac{\vec{\boldsymbol{\mu}}}{x^3}}$$

40. (*a*) Choose the *y* axis along the wire and the *x* axis passing from the center of the wire through the point P. With this definition we calculate the magnetic field at P by integrating Eq. 28-5 over the length of the wire. The origin is at the center of the wire.




$$\vec{\mathbf{B}} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\hat{r}}{r^2} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\vec{\mathbf{r}}}{r^3} = \frac{\mu_0 I}{4\pi}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dy\hat{\mathbf{j}}\times\left(R\hat{\mathbf{i}}-y\hat{\mathbf{j}}\right)}{\left(R^2+y^2\right)^{3/2}}$$

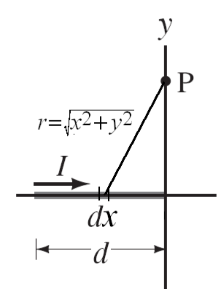
$$= -\frac{\mu_0 IR}{4\pi}\hat{\mathbf{k}}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dy}{\left(R^2+y^2\right)^{3/2}}$$

$$= -\frac{\mu_0 IR}{4\pi}\hat{\mathbf{k}}\frac{y}{R^2\left(R^2+y^2\right)^{1/2}}\Bigg|_{-d/2}^{d/2} = \boxed{-\frac{\mu_0 I}{2\pi R}\frac{d}{\left(4R^2+d^2\right)^{1/2}}\hat{\mathbf{k}}}$$

(*b*) If we take the limit as $d \to \infty$, this equation reduces to Eq. 28-1.

$$B = \lim_{d\to\infty}\left(\frac{\mu_0 I}{2\pi R}\frac{d}{\left(4R^2+d^2\right)^{1/2}}\right) \approx \frac{\mu_0 I}{2\pi R}$$

41. (*a*) The magnetic field at point Q can be obtained by integrating Eq. 28-5 over the length of the wire. In this case, each infinitesimal current segment d$\ell$ is parallel to the *x* axis, as is each radial vector. Since the magnetic field is proportional to the cross-product of the current segment and the radial vector, each segment contributes zero field. Thus the magnetic field at point Q is $\boxed{\text{zero.}}$

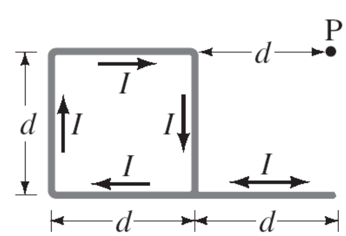



(*b*) The magnetic field at point P is found by integrating Eq. 28-5 over the length of the current segment.

$$\vec{\mathbf{B}} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\hat{r}}{r^2} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{\ell}}\times\vec{\mathbf{r}}}{r^3} = \frac{\mu_0 I}{4\pi}\int_{-d}^{0}\frac{dx\hat{\mathbf{i}}\times\left(-x\hat{\mathbf{i}}+y\hat{\mathbf{j}}\right)}{\left(x^2+y^2\right)^{3/2}} = \frac{\mu_0 Iy}{4\pi}\hat{\mathbf{k}}\int_{-d}^{0}\frac{dx}{\left(x^2+y^2\right)^{3/2}}$$

$$= \frac{\mu_0 Iy}{4\pi}\hat{\mathbf{k}}\frac{x}{y^2\left(x^2+y^2\right)^{1/2}}\Bigg|_{-d}^{0} = \boxed{\frac{\mu_0 I}{4\pi y}\frac{d}{\left(y^2+d^2\right)^{1/2}}\hat{\mathbf{k}}}$$

42. We treat the loop as consisting of 5 segments, The first has length *d*, is located a distance *d* to the left of point P, and has current flowing toward the right. The second has length *d*, is located a distance 2*d* to left of point P, and has current flowing upward. The third has length *d*, is located a distance *d* to the left of point P, and has current flowing downward. The fourth has length 2*d*, is located a distance *d*



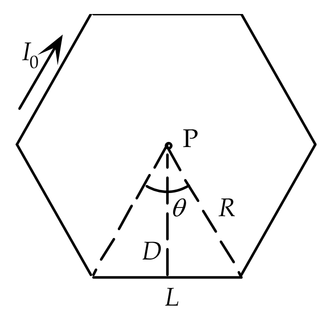
below point P, and has current flowing toward the left. Note that the fourth segment is twice as long as the actual fourth current. We therefore add a fifth line segment of length *d*, located a distance *d* below point P with current flowing to the right. This fifth current segment cancels the added portion, but allows us to use the results of Problem 41 in solving this problem. Note that the first line points radially toward point P, and therefore by Problem 41(*a*) does not contribute to the net magnetic field. We add the contributions from the other four segments, with the contribution in the positive *z*-direction if the current in the segment appears to flows counterclockwise around the point P.

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 + \vec{\mathbf{B}}_4 + \vec{\mathbf{B}}_5$$

$$= -\frac{\mu_0 I}{4\pi(2d)}\frac{d}{\left(4d^2+d^2\right)^{1/2}}\hat{\mathbf{k}} + \frac{\mu_0 I}{4\pi d}\frac{d}{\left(d^2+d^2\right)^{1/2}}\hat{\mathbf{k}} - \frac{\mu_0 I}{4\pi d}\frac{2d}{\left(d^2+4d^2\right)^{1/2}}\hat{\mathbf{k}} + \frac{\mu_0 I}{4\pi d}\frac{d}{\left(d^2+d^2\right)^{1/2}}\hat{\mathbf{k}}$$

$$= \boxed{\frac{\mu_0 I}{4\pi d}\left(\sqrt{2} - \frac{\sqrt{5}}{2}\right)\hat{\mathbf{k}}}$$

43. (*a*) The angle subtended by one side of a polygon, $\theta$, from the center point P is $2\pi$ divided by the number of sides, *n*. The length of the side *L* and the distance from the point to the center of the side, *D*, are obtained from trigonometric relations.




$$L = 2R\sin(\theta/2) = 2R\sin(\pi/n)$$

$$D = R\cos(\theta/2) = R\cos(\pi/n)$$

The magnetic field contribution from each side can be found using the result of Problem 40.

$$B = \frac{\mu_0 I}{2\pi D}\frac{L}{\left(L^2+4D^2\right)^{\frac{1}{2}}} = \frac{\mu_0 I}{2\pi\left(R\cos(\pi/n)\right)}\frac{2R\sin(\pi/n)}{\left(\left(2R\sin(\pi/n)\right)^2 + 4\left(R\cos(\pi/n)\right)^2\right)^{\frac{1}{2}}}$$

$$= \frac{\mu_0 I}{2\pi R}\tan(\pi/n)$$

The contributions from each segment add, so the total magnetic field is *n* times the field from one side.

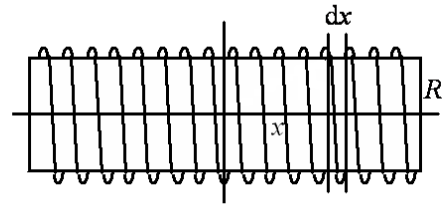
$$\boxed{B_{\text{total}} = \frac{\mu_0 I n}{2\pi R}\tan(\pi/n)}$$

(*b*) In the limit of large *n*, $\pi/n$, becomes very small, so $\tan(\pi/n) \approx \pi/n$.

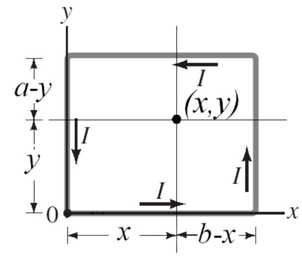
$$B_{\text{total}} = \frac{\mu_0 I n}{2\pi R}\frac{\pi}{n} = \boxed{\frac{\mu_0 I}{2R}}$$

This is the magnetic field at the center of a circle.

44. The equation derived in Eq. 28-12 gives the magnetic field a distance *x* from a single loop. We expand this single loop to the field of an infinite solenoid by multiplying the field from a single loop by *n dx*, the density of loops times the infinitesimal thickness, and integrating over all values of *x*. Use the table in Appendix B-4 to evaluate the integral.




$$B = \int_{-\infty}^{\infty}\frac{\mu_0 I R^2 n\,dx}{2\left(R^2+x^2\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2 n}{2}\int_{-\infty}^{\infty}\frac{dx}{\left(R^2+x^2\right)^{\frac{3}{2}}} = \frac{\mu_0 I R^2 n}{2}\left.\frac{x}{R^2\left(R^2+x^2\right)^{\frac{1}{2}}}\right|_{-\infty}^{\infty} = \boxed{\mu_0 I n}$$




45. To find the magnetic field at point (*x*,*y*) we break each current segment into two segments and sum fields from each of the eight segments to determine the magnetic field at the center. We use the results of Problem 41(*b*) to calculate the magnetic field of each segment.

$$\vec{\mathbf{B}}=\frac{\mu_0 I}{4\pi y}\frac{x}{\left(y^2+x^2\right)^{1/2}}\hat{\mathbf{k}}+\frac{\mu_0 I}{4\pi y}\frac{(b-x)}{\left(y^2+(b-x)^2\right)^{1/2}}\hat{\mathbf{k}}$$
$$+\frac{\mu_0 I}{4\pi(b-x)}\frac{y}{\left((b-x)^2+y^2\right)^{1/2}}\hat{\mathbf{k}}+\frac{\mu_0 I}{4\pi(b-x)}\frac{(a-y)}{\left((a-y)^2+(b-x)^2\right)^{1/2}}\mathbf{k}$$
$$+\frac{\mu_0 I}{4\pi(a-y)}\frac{(b-x)}{\left((a-y)^2+(b-x)^2\right)^{1/2}}\hat{\mathbf{k}}+\frac{\mu_0 I}{4\pi(a-y)}\frac{x)}{\left((a-y)^2+x^2\right)^{1/2}}\hat{\mathbf{k}}$$
$$+\frac{\mu_0 I}{4\pi x}\frac{(a-y)}{\left((a-y)^2+x^2\right)^{1/2}}\hat{\mathbf{k}}+\frac{\mu_0 I}{4\pi x}\frac{y}{\left(y^2+x^2\right)^{1/2}}\hat{\mathbf{k}}$$

We simplify this equation by factoring out common constants and combining terms with similar roots.

$$\boxed{\vec{\mathbf{B}}=\frac{\mu_0 I}{4\pi}\left(\frac{\sqrt{y^2+x^2}}{xy}+\frac{\sqrt{y^2+(b-x)^2}}{(b-x)y}+\frac{\sqrt{(a-y)^2+(b-x)^2}}{(a-y)(b-x)}+\frac{\sqrt{(a-y)^2+x^2}}{x(a-y)}\right)\hat{\mathbf{k}}}$$

46. (*a*) By symmetry we see that on the *x* axis the magnetic field can only have an *x* component. To justify this assertion, imagine that the magnetic field had a component off the axis. If the current loop were rotated by 90° about the *x* axis, the loop orientation would be identical to the original loop, but the off-axis magnetic field component would have changed. This is not possible, so the field only has an *x* component. The contribution to this field is the same for each loop segment, and so the total magnetic field is equal to 4 times the *x* component of the magnetic field from one segment. We integrate Eq. 28-5 to find this magnetic field.

$$\vec{\mathbf{B}}=4\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{\mu_0 I}{4\pi}\frac{dy\,\hat{\mathbf{j}}\times\left(\frac{1}{2}d\,\hat{\mathbf{k}}\right)}{\left[\left(\frac{1}{2}d\right)^2+x^2+y^2\right]^{3/2}}=\frac{\mu_0 Id\,\hat{\mathbf{i}}}{2\pi}\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\frac{dy}{\left[\left(\frac{1}{2}d\right)^2+x^2+y^2\right]^{3/2}}$$
$$=\frac{\mu_0 Id\,\hat{\mathbf{i}}}{2\pi}\frac{y}{\left[\left(\frac{1}{2}d\right)^2+x^2\right]\left[\left(\frac{1}{2}d\right)^2+x^2+y^2\right]^{1/2}}\Bigg|_{-\frac{1}{2}d}^{\frac{1}{2}d}=\boxed{\frac{2\sqrt{2}d^2\mu_0 I\,\hat{\mathbf{i}}}{\pi\left(d^2+4x^2\right)\left(d^2+2x^2\right)^{1/2}}}$$

(*b*) Let $x\gg d$ to show that the magnetic field reduces to a dipole field of Eq. 28-7b.

$$\vec{\mathbf{B}}\approx\frac{2\sqrt{2}d^2\mu_0 I\,\hat{\mathbf{i}}}{\pi\left(4x^2\right)\left(2x^2\right)^{1/2}}=\frac{d^2\mu_0 I\,\hat{\mathbf{i}}}{2\pi x^3}$$

Comparing our magnetic field to Eq. 28-7b we see that it is a dipole field with the magnetic moment $\boxed{\vec{\boldsymbol{\mu}}=d^2 I\,\hat{\mathbf{i}}}$

47. (*a*) If the iron bar is completely magnetized, all of the dipoles are aligned. The total dipole moment is equal to the number of atoms times the dipole moment of a single atom.

$$\mu=N\mu_1=\frac{N_A\rho V}{M_m}\mu_1$$

$$= \frac{(6.022\times10^{23}\text{atoms/mole})(7.80\,\text{g/cm}^3)(9.0\,\text{cm})(1.2\,\text{cm})(1.0\,\text{cm})}{55.845\,\text{g/mole}}\left(1.8\times10^{-23}\,\frac{\text{A}\bullet\text{m}^2}{\text{atom}}\right)$$

$$= 16.35\ \text{A}\bullet\text{m}^2 \approx \boxed{16\ \text{A}\bullet\text{m}^2}$$

(*b*) We use Eq. 27-9 to find the torque.

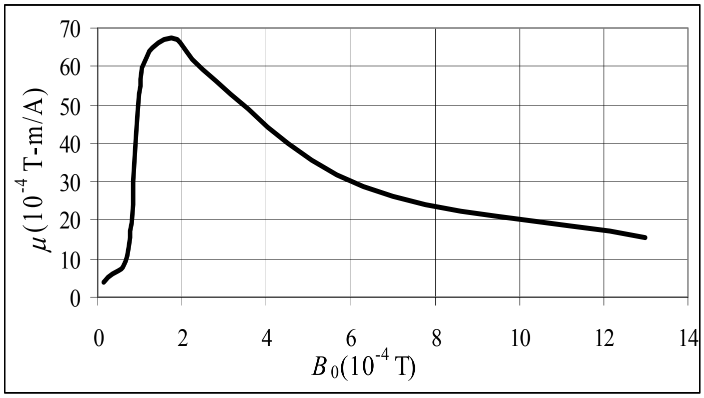
$$\tau = \mu B\sin\theta = (16.35\ \text{A}\bullet\text{m}^2)(0.80\ \text{T})\sin 90^\circ = \boxed{13\ \text{m}\bullet\text{N}}$$

48. The magnetic permeability is found from the two fields.

$$B_0 = \mu_0 nI \ ;\ B = \mu nI \ ;$$

$$\frac{B}{B_0} = \frac{\mu}{\mu_0} \rightarrow \mu = \mu_0\frac{B}{B_0}$$

For the graph, we have not plotted the last three data points so that the structure for low fields is seen. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.48."




$\boxed{49.}$ The magnetic field of a long, thin torus is the same as the field given by a long solenoid, as in Eq. 28-9.

$$B = \mu nI = (2200)(4\pi\times10^{-7}\ \text{Tm/A})(285\ \text{m}^{-1})(3.0\ \text{A}) = \boxed{2.4\ \text{T}}$$

50. The field inside the solenoid is given by Eq. 28-4 with $\mu_0$ replaced by the permeability of the iron.

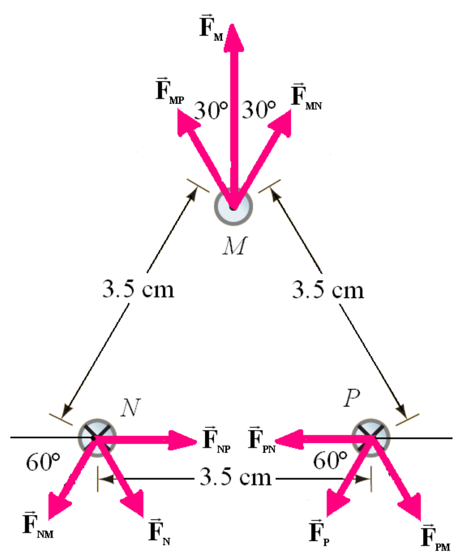
$$B = \frac{\mu NI}{\ell} \rightarrow \mu = \frac{B\ell}{NI} = \frac{(2.2\,\text{T})(0.38\,\text{m})}{(640)(48\,\text{A})} = \boxed{2.7\times10^{-5}\,\text{T}\bullet\text{m/A}} \approx 22\mu_0$$

51. Since the wires all carry the same current and are equidistant from each other, the magnitude of the force per unit length between any two wires is the same and is given by Eq. 28-2.

$$\frac{F}{\ell} = \frac{\mu_0 I^2}{2\pi d} = \frac{(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A})(8.00\ \text{A})^2}{2\pi(0.035\ \text{m})}$$

$$= 3.657\times10^{-4}\,\text{N/m}$$

The direction of the force between two wires is along the radial line and attractive for currents traveling in the same direction and repulsive for currents traveling in opposite directions. The forces acting on wire M are radially away from the other two wires. By symmetry, the horizontal components of these forces cancel and the net force is the sum of the vertical components.




$$F_M = F_{MP}\cos 30^\circ + F_{MN}\cos 30^\circ$$

$$= 2(3.657\times10^{-4}\,\text{N/m})\cos 30^\circ = \boxed{6.3\times10^{-4}\,\text{N/m at } 90^\circ}$$

The force on wire N is found by adding the components of the forces from the other two wires. By symmetry we see that this force is directed at an angle of 300°. The force on wire P, will have the same magnitude but be directed at 240°.

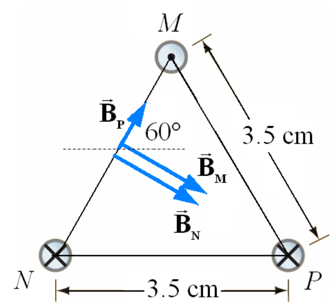
$$F_{N,x} = F_{NP} - F_{NM}\cos 60° = 3.657\times10^{-4}\,\text{N/m} - \left(3.657\times10^{-4}\,\text{N/m}\right)\cos 60° = 1.829\times10^{-4}\,\text{N/m}$$

$$F_{N,y} = -F_{NM}\sin 60° = -\left(3.657\times10^{-4}\,\text{N/m}\right)\sin 60° = -3.167\times10^{-4}\,\text{N/m}$$

$$F_N = \sqrt{\left(1.829\times10^{-4}\,\text{N/m}\right)^2 + \left(-3.167\times10^{-4}\,\text{N/m}\right)^2} = \boxed{3.7\times10^{-4}\,\text{N/m at } 300°}$$

$$F_P = \boxed{3.7\times10^{-4}\,\text{N/m at } 240°}$$

52. The magnetic field at the midpoint between currents M and N is the vector sum of the magnetic fields from each wire, given by Eq. 28-1. Each field points perpendicularly to the line connecting the wire to the midpoint.




$$\vec{\mathbf{B}}_{net} = \vec{\mathbf{B}}_M + \vec{\mathbf{B}}_N + \vec{\mathbf{B}}_P$$

$$B_M = B_N = \frac{\mu_0}{2\pi}\frac{I}{r_M} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)}{2\pi}\frac{8.00\,\text{A}}{0.0175\,\text{m}} = 9.143\times10^{-5}\,\text{T}$$

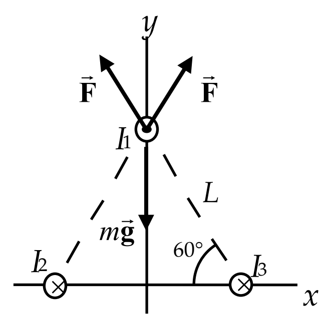
$$B_P = \frac{\mu_0}{2\pi}\frac{I}{r_P} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)}{2\pi}\frac{8.00\,\text{A}}{\sqrt{3}\left(0.0175\,\text{m}\right)} = 5.279\times10^{-5}\,\text{T}$$

$$B_{net} = \sqrt{B_{net\,x}^2 + B_{net\,y}^2} = \sqrt{\left(1.849\times10^{-4}\,\text{T}\right)^2 + \left(-4.571\times10^{-5}\,\text{T}\right)} = \boxed{4.93\times10^{-4}\,\text{T}}$$

$$\theta_{net} = \tan^{-1}\frac{B_{net\,y}}{B_{net\,y}} = \tan^{-1}\frac{-4.210\times10^{-5}\,\text{T}}{1.702\times10^{-4}\,\text{T}} = \boxed{-14°}$$

The net field points slightly below the horizontal direction.

53. For the wire to be suspended the net magnetic force must equal the gravitational force. Since the same current flows through the two lower wires, the net magnetic force is the sum of the vertical components of the force from each wire, given by Eq. 28-2. We solve for the unknown current by setting this force equal to the weight of the wire.




$$F_M = 2\frac{\mu_0 I_M I_{NP}}{2\pi r}\ell\cos 30° = \rho g\left(\tfrac{1}{4}\pi d^2 \ell\right)$$

$$I_M = \frac{\rho g \pi^2 r d^2}{4\mu_0 I_{NP}\cos 30°}$$

$$= \frac{\left(8900\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\pi^2\left(0.035\,\text{m}\right)\left(1.00\times10^{-3}\,\text{m}\right)^2}{4\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(40.0\,\text{A}\right)\cos 30°} = \boxed{170\,\text{A}}$$

54. The centripetal force is caused by the magnetic field, and is given by Eq. 27-5b. From this force we can calculate the radius of curvature.

$$F = qvB\sin\theta = qv_\perp B = m\frac{v_\perp^2}{r} \quad \rightarrow$$

$$r = \frac{mv_\perp}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.3\times10^7\,\text{m/s}\right)\sin 7°}{\left(1.60\times10^{-19}\,\text{C}\right)\left(3.3\times10^{-2}\,\text{T}\right)} = 2.734\times10^{-4}\,\text{m} \approx \boxed{0.27\text{ mm}}$$

The component of the velocity that is parallel to the magnetic field is unchanged, and so the pitch is that velocity component times the period of the circular motion.

$$T=\frac{2\pi r}{v_\perp}=\frac{2\pi\frac{mv_\perp}{qB}}{v_\perp}=\frac{2\pi m}{qB}$$

$$p=v_{\|}T=v\cos 7^\circ\left(\frac{2\pi m}{qB}\right)=\left(1.3\times10^{7}\,\text{m/s}\right)\cos 7^\circ\frac{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(3.3\times10^{-2}\,\text{T}\right)}=\boxed{1.4\text{ cm}}$$

55. (*a*) Use Eq. 28-1 to calculate the field due to a long straight wire.

$$B_{\text{A at B}}=\frac{\mu_0 I_{\text{A}}}{2\pi r_{\text{A to B}}}=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(2.0\,\text{A}\right)}{2\pi\left(0.15\,\text{m}\right)}=2.667\times10^{-6}\,\text{T}\approx\boxed{2.7\times10^{-6}\,\text{T}}$$

(*b*) $$B_{\text{B at A}}=\frac{\mu_0 I_{\text{B}}}{2\pi r_{\text{B to A}}}=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(4.0\,\text{A}\right)}{2\pi\left(0.15\,\text{m}\right)}=5.333\times10^{-6}\,\text{T}\approx\boxed{5.3\times10^{-6}\,\text{T}}$$

(*c*) The two fields are not equal and opposite. Each individual field is due to a single wire, and has no dependence on the other wire. The magnitude of current in the second wire has nothing to do with the value of the field caused by the first wire.

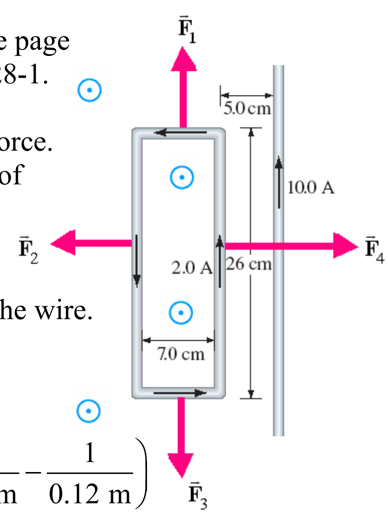
(*d*) Use Eq. 28-2 to calculate the force due to one wire on another. The forces are attractive since the currents are in the same direction.

$$\frac{F_{\text{on A due to B}}}{\ell_{\text{A}}}=\frac{F_{\text{on B due to A}}}{\ell_{\text{B}}}=\frac{\mu_0}{2\pi}\frac{I_{\text{A}}I_{\text{B}}}{d_{\text{A to B}}}=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}{2\pi}\frac{\left(2.0\,\text{A}\right)\left(4.0\,\text{A}\right)}{\left(0.15\,\text{m}\right)}$$

$$=1.067\times10^{-5}\,\text{N/m}\approx\boxed{1.1\times10^{-5}\,\text{N/m}}$$

These two forces per unit length are equal and opposite because they are a Newton's third law pair of forces.

56. (*a*) The magnetic field from the long straight wire will be out of the page in the region of the wire loop with its magnitude given by Eq. 28-1. By symmetry, the forces from the two horizontal segments are equal and opposite, therefore they do not contribute to the net force. We use Eq. 28-2 to find the force on the two vertical segments of the loop and sum the results to determine the net force. Note that the segment with the current parallel to the straight wire will be attracted to the wire, while the segment with the current flowing in the opposite direction will be repelled from the wire.



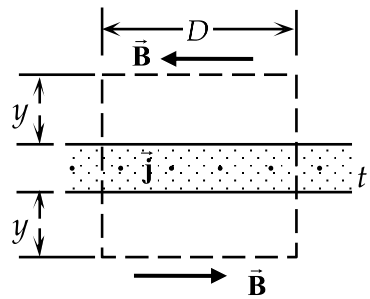

$$F_{net}=F_2+F_4=-\frac{\mu_0 I_1 I_2}{2\pi d_2}\ell+\frac{\mu_0 I_1 I_2}{2\pi d_1}\ell=\frac{\mu_0 I_1 I_2\ell}{2\pi}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)$$

$$=\frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(2.0\text{ A}\right)\left(10.0\text{ A}\right)\left(0.26\text{ m}\right)}{2\pi}\left(\frac{1}{0.05\text{ m}}-\frac{1}{0.12\text{ m}}\right)$$

$$=\boxed{1.2\times10^{-5}\text{ N toward the wire}}$$

(*b*) Since the forces on each segment lie in the same plane, the net torque on the loop is $\boxed{\text{zero}}$.

57. The sheet may be treated as an infinite number of parallel wires. The magnetic field at a location *y* above the wire will be the sum of the magnetic fields produced by each of the wires. If we consider the magnetic field from two wires placed symmetrically on either side of where we are measuring the magnetic field, we see that the vertical magnetic field components cancel each other out. Therefore, the field above the wire must be horizontal and to the left. By symmetry, the field a location *y* below the wire must have the same magnitude, but point in the opposite direction. We calculate the magnetic field using Ampere's law with a rectangular loop that extends a distance *y* above and below the current sheet, as shown in the figure.




$$\oint \vec{\mathbf{B}}\cdot d\vec{\ell} = \int_{\text{sides}} \vec{\mathbf{B}}\cdot d\vec{\ell} + \int_{\text{top}} \vec{\mathbf{B}}\cdot d\vec{\ell} + \int_{\text{bottom}} \vec{\mathbf{B}}\cdot d\vec{\ell} = 0 + 2B_{\parallel}D = \mu_0 I_{\text{encl}} = \mu_0\left(jtD\right)$$

$$\rightarrow B_{\parallel} = \boxed{\tfrac{1}{2}\mu_0 jt, \text{ to the left above the sheet}}$$

58. (*a*) We set the magnetic force, using Eq. 28-2, equal to the weight of the wire and solve for the necessary current. The current must flow in the same direction as the upper current, for the magnetic force to be upward.

$$F_M = \frac{\mu_0 I_1 I_2}{2\pi r}\ell = \rho g\left(\frac{\pi d^2}{4}\ell\right) \rightarrow$$

$$I_2 = \frac{\rho g \pi^2 r d^2}{4\mu_0 I_1} = \frac{\left(8900\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\pi^2\left(0.050\,\text{m}\right)\left(1.00\times10^{-3}\,\text{m}\right)^2}{2\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(48.0\,\text{A}\right)} = \boxed{360\,\text{A, right}}$$

(*b*) The lower wire is in **[unstable equilibrium]**, since if it is raised slightly from equilibrium, the magnetic force would be increased, causing the wire to move further from equilibrium.

(*c*) If the wire is suspended above the first wire at the same distance, the same current is needed, but in the opposite direction, as the wire must be repelled from the lower wire to remain in equilibrium. Therefore the current must be **[360 A to the left]**. This is a **[stable equilibrium]** for vertical displacement since if the wire is moved slightly off the equilibrium point the magnetic force will increase or decrease to push the wire back to the equilibrium height.

59. The magnetic field at the center of the square loop is four times the magnetic field from one of the sides. It will be directed out of the page. We can use the result of Problem 40 for the magnitude of the field from one side, with $R = \frac{1}{2}d$. If the current is flowing counterclockwise around the square loop, the magnetic field due to each piece will point upwards.

$$\vec{\mathbf{B}}_{\substack{\text{one}\\\text{wire}}} = \frac{\mu_0 I}{2\pi R}\frac{d\,\hat{\mathbf{k}}}{\left(4R^2+d^2\right)^{1/2}} = \frac{\mu_0 I}{2\pi\left(\frac{1}{2}d\right)}\frac{d\,\hat{\mathbf{k}}}{\left(4\left(\frac{1}{2}d\right)^2+d^2\right)^{1/2}} = \frac{\mu_0 I}{\sqrt{2}\pi d}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}}_{\text{total}} = 4\vec{\mathbf{B}}_{\substack{\text{one}\\\text{wire}}} = \boxed{\frac{2\sqrt{2}\mu_0 I}{\pi d}\hat{\mathbf{k}}}$$

60. The magnetic field at the center of a circular loop was calculated in Example 28-12. To determine the radius of the loop, we set the circumferences of the loops equal.

$$2\pi R = 4d \rightarrow R = \frac{2d}{\pi} \;;\; B_{\text{circle}} = \frac{\mu_0 I}{2R} = \frac{\mu_0 I\pi}{4d} < \frac{2\sqrt{2}\mu_0 I}{\pi d} = B_{\text{square}}$$

Therefore, changing the shape to a circular loop will **[decrease]** the magnetic field.

61. (*a*) Choose $x = 0$ at the center of one coil. The center of the other coil will then be at $x = R$. Since the currents flow in the same direction in both coils, the right-hand-rule shows that the magnetic fields from the two coils will point in the same direction along the axis. The magnetic field from a current loop was found in Example 28-12. Adding the two magnetic fields together yields the total field.

$$B(x) = \boxed{\frac{\mu_0 NIR^2}{2\left[R^2+x^2\right]^{3/2}} + \frac{\mu_0 NIR^2}{2\left[R^2+(x-R)^2\right]^{3/2}}}$$

(*b*) Evaluate the derivative of the magnetic field at $x = \tfrac{1}{2}R$.

$$\frac{dB}{dx} = -\frac{3\mu_0 NIR^2 x}{2\left[R^2+x^2\right]^{5/2}} - \frac{3\mu_0 NIR^2(x-R)}{2\left[R^2+(x-R)^2\right]^{5/2}} = -\frac{3\mu_0 NIR^3}{4\left[R^2+R^2/4\right]^{5/2}} - \frac{-3\mu_0 NIR^3}{4\left[R^2+R^2/4\right]^{5/2}} = \boxed{0}$$

Evaluate the second derivative of the magnetic field at $x = \tfrac{1}{2}R$.

$$\frac{d^2B}{dx^2} = -\frac{3\mu_0 NIR^2}{2\left[R^2+x^2\right]^{5/2}} + \frac{15\mu_0 NIR^2 x^2}{2\left[R^2+x^2\right]^{7/2}} - \frac{3\mu_0 NIR^2}{2\left[R^2+(x-R)^2\right]^{5/2}} + \frac{15\mu_0 NIR^2(x-R)^2}{2\left[R^2+(x-R)^2\right]^{7/2}}$$

$$= -\frac{3\mu_0 NIR^2}{2\left[5R^2/4\right]^{5/2}} + \frac{15\mu_0 NIR^4}{8\left[5R^2/4\right]^{7/2}} - \frac{3\mu_0 NIR^2}{2\left[5R^2/4\right]^{5/2}} + \frac{15\mu_0 NIR^4}{8\left[5R^2/4\right]^{7/2}}$$

$$= \frac{\mu_0 NIR^2}{\left[5R^2/4\right]^{5/2}}\left(-\frac{3}{2} + \frac{15\cdot 4}{8\cdot 5} - \frac{3}{2} + \frac{15\cdot 4}{8\cdot 5}\right) = \boxed{0}$$

Therefore, at the midpoint $\dfrac{dB}{dx} = 0$ and $\dfrac{d^2B}{dx^2} = 0$.

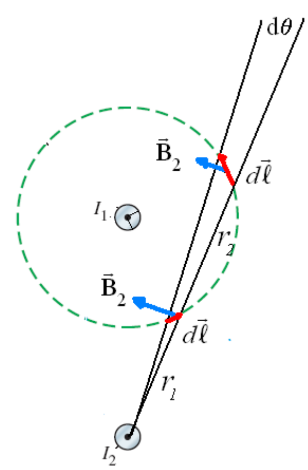
(*c*) We insert the given data into the magnetic field equation to calculate the field at the midpoint.

$$B\left(\tfrac{1}{2}R\right) = \frac{\mu_0 NIR^2}{2\left[R^2+\left(\tfrac{1}{2}R\right)^2\right]^{3/2}} + \frac{\mu_0 NIR^2}{2\left[R^2+\left(\tfrac{1}{2}R\right)^2\right]^{3/2}} = \frac{\mu_0 NIR^2}{\left[R^2+\left(\tfrac{1}{2}R\right)^2\right]^{3/2}}$$

$$= \frac{\left(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A}\right)(250)(2.0\,\mathrm{A})(0.10\,\mathrm{m})^2}{\left[(0.10\,\mathrm{m})^2+(0.05\,\mathrm{m})^2\right]^{3/2}} = \boxed{4.5\,\mathrm{mT}}$$

62. The total field is the vector sum of the fields from the two currents. We can therefore write the path integral as the sum of two such integrals.

$$\oint \vec{\mathbf{B}}\bullet d\vec{\ell} = \oint \vec{\mathbf{B}}_1\bullet d\vec{\ell} + \oint \vec{\mathbf{B}}_2\bullet d\vec{\ell}$$




To evaluate the integral for current 1, we use Eq. 28-1, with the magnetic field constant and parallel to the loop at each line segment.

$$\oint \vec{\mathbf{B}}_1\bullet d\vec{\ell} = \frac{\mu_0 I_1}{2\pi r}\int_0^{2\pi} r\,d\theta = \mu_0 I_1$$

To evaluate the integral for current 2, we consider a different angle $d\theta$ centered at $I_2$ and crossing the path of the loop at two locations, as shown in the diagram. If we integrate clockwise around the path, the components of $d\ell$ parallel to the field will be $-r_1 d\theta$ and $r_2 d\theta$. Multiplying these components by the magnetic field at both locations gives the contribution to the integral from the sum of these segments.

$$B_1 d\ell_1 + B_2 d\ell_2 = \frac{\mu_0 I_2}{2\pi r_1}\left(-r_1 d\theta\right) + \frac{\mu_0 I_2}{2\pi r_2}\left(r_2 d\theta\right) = 0$$

The total integral will be the sum of these pairs resulting in a zero net integral.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \oint \vec{\mathbf{B}}_1 \cdot d\vec{\ell} + \oint \vec{\mathbf{B}}_2 \cdot d\vec{\ell} = \mu_0 I_1 + 0 = \boxed{\mu_0 I_1}$$

63. From Example 28-12, the magnetic field on the axis of a circular loop of wire of radius R carrying current I is $B = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}}$, where $x$ is the distance along the axis from the center of the loop.

For the loop described in this problem, we have $R = x = R_{\text{Earth}}$.

$$B = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}} \rightarrow I = \frac{2B\left(R^2 + x^2\right)^{3/2}}{\mu_0 R^2} = \frac{2B\left(R_{\text{Earth}}^2 + R_{\text{Earth}}^2\right)^{3/2}}{\mu_0 R_{\text{Earth}}^2} = \frac{2(2)^{3/2} B R_{\text{Earth}}}{\mu_0}$$

$$= \frac{2(2)^{3/2}\left(1 \times 10^{-4}\,\text{T}\right)\left(6.38 \times 10^6\,\text{m}\right)}{4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}} = \boxed{3 \times 10^9\,\text{A}}$$

64. The magnetic field from the wire at the location of the plane is perpendicular to the velocity of the plane since the plane is flying parallel to the wire. We calculate the force on the plane, and thus the acceleration, using Eq. 27-5b, with the magnetic field of the wire given by Eq. 28-1.

$$F = qvB = qv\frac{\mu_0 I}{2\pi r}$$

$$a = \frac{F}{m} = \frac{qv}{m}\frac{\mu_0 I}{2\pi r} = \frac{\left(18 \times 10^{-3}\ \text{C}\right)\left(2.8\ \text{m/s}\right)\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(25\ \text{A}\right)}{2\pi\left(0.175\ \text{kg}\right)\left(0.086\ \text{m}\right)}$$

$$= 1.67 \times 10^{-5}\ \text{m/s}^2 = \boxed{1.7 \times 10^{-6}\, g\text{'s}}$$

65. (*a*) To find the length of wire that will give the coil sufficient resistance to run at maximum power, we write the power equation (Eq. 25-7b) with the resistance given by Eq. 25-3. We divide the length by the circumference of one coil to determine the number of turns.

$$P_{\text{max}} = \frac{V^2}{R} = \frac{V^2}{\rho\ell/\left(d^2\right)} \rightarrow \ell = \frac{V^2 d^2}{\rho P_{\text{max}}}$$

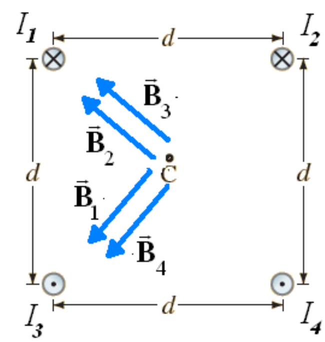
$$N = \frac{\ell}{\pi D} = \frac{V^2 d^2}{\pi D \rho P_{\text{max}}} = \frac{\left(35\ \text{V}\right)^2\left(2.0 \times 10^{-3}\ \text{m}\right)^2}{\pi\left(2.0\ \text{m}\right)\left(1.68 \times 10^{-8}\ \Omega\text{m}\right)\left(1.0 \times 10^3\ \text{W}\right)} = \boxed{46\ \text{turns}}$$

(*b*) We use the result of Example 28-12 to determine the magnetic field at the center of the coil, with the current obtained from Eq. 25-7b.

$$B = \frac{\mu_0 N I}{D} = \frac{\mu_0 N}{D}\frac{P_{\text{max}}}{V} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(46\right)\left(1.0 \times 10^3\ \text{W}\right)}{\left(2.0\ \text{m}\right)\left(35\ \text{V}\right)} = \boxed{0.83\ \text{mT}}$$

(*c*) Increasing the number of turns will proportionately increase the resistance and therefore decrease the current. The net result is no change in the magnetic field.

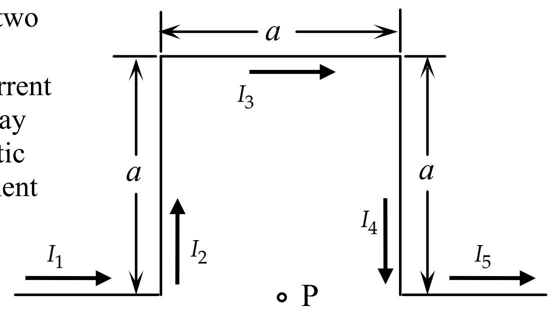
66. The magnetic field at the center of the square is the vector sum of the magnetic field created by each current. Since the magnitudes of the currents are equal and the distance from each corner to the center is the same, the magnitude of the magnetic field from each wire is the same and is given by Eq. 28-1. The direction of the magnetic field is directed by the right-hand-rule and is shown in the diagram. By symmetry, we see that the vertical components of the magnetic field cancel and the horizontal components add.




$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 + \vec{\mathbf{B}}_4 = -4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45°\hat{\mathbf{i}}$$

$$= -4\left(\frac{\mu_0 I}{2\pi\frac{\sqrt{2}}{2}d}\right)\frac{\sqrt{2}}{2}\hat{\mathbf{i}} = \boxed{-\frac{2\mu_0 I}{\pi d}\hat{\mathbf{i}}}$$

67. The wire can be broken down into five segments: the two long wires, the left vertical segment, the right vertical segment, and the top horizontal segment. Since the current in the two long wires either flow radially toward or away from the point P, they will not contribute to the magnetic field. The magnetic field from the top horizontal segment points into the page and is obtained from the solution to Problem 40.




$$B_{top} = \frac{\mu_0 I}{2\pi a}\frac{a}{\left(a^2 + 4a^2\right)^{\frac{1}{2}}} = \frac{\mu_0 I}{2\pi a\sqrt{5}}$$

The magnetic fields from the two vertical segments both point into the page with magnitudes obtained from the solution to Problem 41.

$$B_{vert} = \frac{\mu_0 I}{4\pi(a/2)}\frac{a}{\left(a^2 + (a/2)^2\right)^{\frac{1}{2}}} = \frac{\mu_0 I}{\pi a\sqrt{5}}$$

Summing the magnetic fields from all the segments yields the net field.

$$B = B_{top} + 2B_{vert} = \frac{\mu_0 I}{2\pi a\sqrt{5}} + 2\frac{\mu_0 I}{\pi a\sqrt{5}} = \boxed{\frac{\mu_0 I\sqrt{5}}{2\pi a}\text{, into the page.}}$$

68. Use Eq. 28-4 for the field inside a solenoid.

$$B = \frac{\mu_0 IN}{\ell} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(2.0\,\text{A})(420)}{0.12\,\text{m}} = \boxed{8.8\times10^{-3}\,\text{T}}$$

69. The field due to the solenoid is given by Eq. 28-4. Since the field due to the solenoid is perpendicular to the current in the wire, Eq. 27-2 can be used to find the force on the wire segment.

$$F = I_{\text{wire}}\ell_{\text{wire}}B_{\text{solenoid}} = I_{\text{wire}}\ell_{\text{wire}}\frac{\mu_0 I_{\text{solenoid}}N}{\ell_{\text{solenoid}}} = (22\,\text{A})(0.030\text{m})\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(33\,\text{A})(550)}{(0.15\,\text{m})}$$

$$= \boxed{0.10\,\text{N to the south}}$$

70. Since the mass of copper is fixed and the density is fixed, the volume is fixed, and we designate it as $V_{\text{Cu}} = m_{\text{Cu}}\rho_{\text{Cu}} = \ell_{\text{Cu}}A_{\text{Cu}}$. We call the fixed voltage $V_0$. The magnetic field in the solenoid is given by Eq. 28-4.

$$B = \frac{\mu_0 IN}{\ell_{\text{sol}}} = \mu_0 V_0 \frac{N}{R_{\text{Cu}}\ell_{\text{sol}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}}\frac{N}{\frac{\ell_{\text{Cu}}}{A_{\text{Cu}}}\ell_{\text{sol}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}}\frac{N}{\ell_{\text{sol}}}\frac{A_{\text{Cu}}}{\ell_{\text{Cu}}} = \frac{\mu_0 V_0}{\rho_{\text{R Cu}}}\frac{N}{\ell_{\text{sol}}}\frac{m_{\text{Cu}}\rho_{\text{Cu}}}{\ell_{\text{Cu}}^2}$$

$$= \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}}\frac{N}{\ell_{\text{sol}}\ell_{\text{Cu}}^2}$$

The number of turns of wire is the length of wire divided by the circumference of the solenoid.

$$N = \frac{\ell_{\text{Cu}}}{2\pi r_{\text{sol}}} \rightarrow B = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}}\frac{N}{\ell_{\text{sol}}\ell_{\text{Cu}}^2} = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{\rho_{\text{R Cu}}}\frac{\frac{\ell_{\text{Cu}}}{2\pi r_{\text{sol}}}}{\ell_{\text{sol}}\ell_{\text{Cu}}^2} = \frac{\mu_0 V_0 m_{\text{Cu}}\rho_{\text{Cu}}}{2\pi\rho_{\text{R Cu}}}\frac{1}{\ell_{\text{sol}}r_{\text{sol}}\ell_{\text{Cu}}}$$

The first factor in the expression for *B* is made of constants, so we have $B \propto \frac{1}{\ell_{\text{sol}}r_{\text{sol}}\ell_{\text{Cu}}}$. Thus we want the wire to be $\boxed{\text{short and fat}}$. Also the radius of the solenoid should be small and the length of the solenoid small.

71. The magnetic field inside the smaller solenoid will equal the sum of the fields from both solenoids. The field outside the inner solenoid will equal the field produced by the outer solenoid only. We set the sum of the two fields given by Eq. 28-4 equal to $-\frac{1}{2}$ times the field of the outer solenoid and solve for the ratio of the turn density.

$$\mu_0(-I)n_a + \mu_0 I n_b = -\tfrac{1}{2}(\mu_0 I n_b) \rightarrow \boxed{\frac{n_b}{n_a} = \frac{2}{3}}$$

72. Take the origin of coordinates to be at the center of the semicircle.The magnetic field at the center of the semicircle is the vector sum of the magnetic fields from each of the two long wires and from the semicircle. By the right-hand-rule each of these fields point into the page, so we can sum the magnitudes of the fields. The magnetic field for each of the long segments is obtained by integrating Eq. 28-5 over the straight segment.

$$\vec{\mathbf{B}}_{\text{straight}} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{\ell}\times\hat{R}}{R^2} = \int \frac{\mu_0 I}{4\pi}\frac{d\vec{\ell}\times\vec{\mathbf{R}}}{R^3} = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{0}\frac{dx\,\hat{\mathbf{i}}\times(-x\hat{\mathbf{i}} - r\hat{\mathbf{j}})}{(x^2+r^2)^{3/2}} = -\frac{\mu_0 Ir}{4\pi}\hat{\mathbf{k}}\int_{-\infty}^{0}\frac{dx}{(x^2+r^2)^{3/2}}$$

$$= -\frac{\mu_0 Ir}{4\pi}\hat{\mathbf{k}}\left.\frac{x}{r^2(x^2+r^2)^{1/2}}\right|_{-\infty}^{0} = -\frac{\mu_0 I}{4\pi r}\hat{\mathbf{k}}$$

The magnetic field for the curved segment is obtained by integrating Eq. 28-5 over the semicircle.

$$\vec{\mathbf{B}}_{\text{curve}} = \frac{\mu_0 I}{4\pi}\int_0^{\pi r}\frac{d\vec{\ell}\times\hat{r}}{r^2} = -\frac{\mu_0 I}{4\pi r^2}\hat{\mathbf{k}}\int_0^{\pi r} ds = -\frac{\mu_0 I}{4r}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = 2\vec{\mathbf{B}}_{\text{straight}} + \vec{\mathbf{B}}_{\text{curve}} = -2\frac{\mu_0 I}{4\pi r}\hat{\mathbf{k}} - \frac{\mu_0 I}{4r}\hat{\mathbf{k}} = \boxed{-\frac{\mu_0 I}{4\pi r}(2+\pi)\hat{\mathbf{k}}}$$

73. (*a*) Set $x = 0$ at the midpoint on the axis between the two loops. Since the loops are a distance $R$ apart, the center of one loop will be at $x = -\frac{1}{2}R$ and the center of the other at $x = \frac{1}{2}R$. The currents in the loops flow in opposite directions, so by the right-hand-rule the magnetic fields from the two wires will subtract from each other. The magnitude of each field can be obtained from Example 28-12.

$$B(x) = \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R - x\right)^2\right]^{3/2}} - \frac{\mu_0 NIR^2}{2\left[R^2 + \left(\frac{1}{2}R + x\right)^2\right]^{3/2}}$$

Factoring out $\frac{1}{8}R^3$ from each of the denominators yields the desired equation.

$$B(x) = \frac{4\mu_0 NI}{R\left[4 + (1 - 2x/R)^2\right]^{3/2}} - \frac{4\mu_0 NI}{R\left[4 + (1 + 2x/R)^2\right]^{3/2}}$$

$$= \boxed{\frac{4\mu_0 NI}{R}\left\{\left[4 + \left(1 - \frac{2x}{R}\right)^2\right]^{-3/2} - \left[4 + \left(1 + \frac{2x}{R}\right)^2\right]^{-3/2}\right\}}$$

(*b*) For small values of $x$, we can use the approximation $\left(1 \pm \frac{2x}{R}\right)^2 \approx 1 \pm \frac{4x}{R}$.

$$B(x) = \frac{4\mu_0 NI}{R}\left\{\left[4 + 1 - \frac{4x}{R}\right]^{-3/2} - \left[4 + 1 + \frac{4x}{R}\right]^{-3/2}\right\}$$

$$= \frac{4\mu_0 NI}{5R\sqrt{5}}\left\{\left[1 - \frac{4x}{5R}\right]^{-3/2} - \left[1 + \frac{4x}{5R}\right]^{-3/2}\right\}$$

Again we can use the expansion for small deviations $\left(1 \pm \frac{4x}{5R}\right)^{-3/2} \approx 1 \mp \frac{6x}{5R}$

$$B(x) = \frac{4\mu_0 NI}{5R\sqrt{5}}\left[\left(1 + \frac{6x}{5R}\right) - \left(1 - \frac{6x}{5R}\right)\right] = \boxed{\frac{48\mu_0 NIx}{25R^2\sqrt{5}}}$$

This magnetic field has the expected linear dependence on $x$ with a coefficient of $C = 48\mu_0 NI/\left(25R^2\sqrt{5}\right)$.

(*c*) Set $C$ equal to 0.15 T/m and solve for the current.

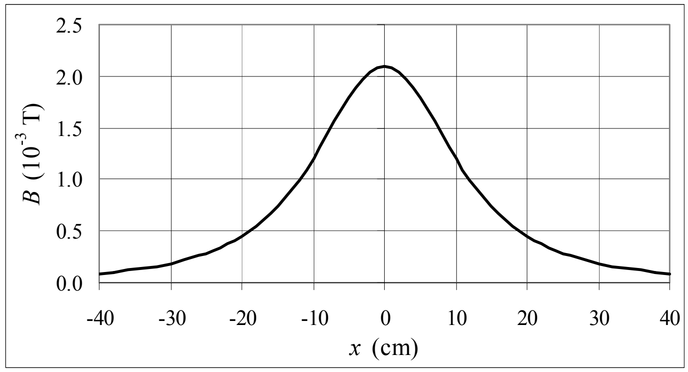
$$I = \frac{25CR^2\sqrt{5}}{48\mu_0 N} = \frac{25(0.15\text{ T/m})(0.04\text{ m})^2\sqrt{5}}{48\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}\right)(150)} = \boxed{1.5\text{ A}}$$

74. We calculate the peak current using Eqs. 25-7 and 25-9. Then we use the peak current in Eq. 28-1 to calculate the maximum magnetic field.

$$I_{\text{max}} = \sqrt{2}I_{\text{rms}} = \sqrt{2}\frac{P_{\text{avg}}}{V_{\text{rms}}} \rightarrow B_{\text{max}} = \frac{\mu_0 I_{\text{max}}}{2\pi r} = \frac{\sqrt{2}\mu_0}{2\pi r}\frac{P_{\text{avg}}}{V_{\text{rms}}}$$

$$= \frac{\sqrt{2}\left(4\pi \times 10^{-7}\text{ T}\bullet\text{m/A}\right)}{2\pi(12\text{ m})}\frac{\left(45 \times 10^6\text{ W}\right)}{\left(15 \times 10^3\text{ V}\right)} = \boxed{71\ \mu\text{T}}$$

75. We use the results of Example 28-12 to calculate the magnetic field as a function of position. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.75."

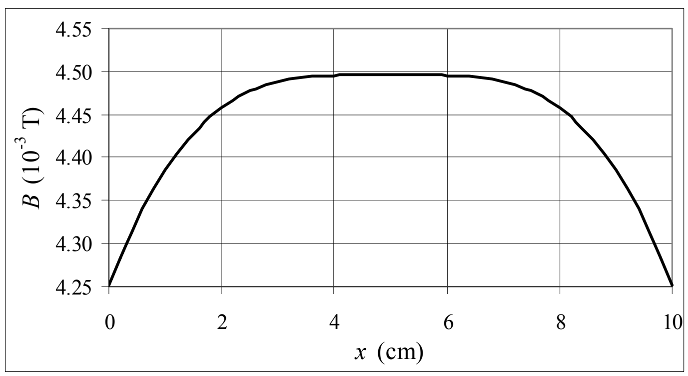



$$B=\frac{N\mu_0IR^2}{2\left(R^2+x^2\right)^{3/2}}=\frac{(250)\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(2.0\,\text{A})(0.15\,\text{m})^2}{2\left[(0.15\,\text{m})^2+x^2\right]^{3/2}}=\frac{7.0686\times10^{-6}\,\text{T}\bullet\text{m}^3}{\left[(0.15\,\text{m})^2+x^2\right]^{3/2}}.$$

76. (*a*) Use the results of Problem 61(a) to write the magnetic field.

$$B(x)=\frac{\mu_0NIR^2}{2\left[R^2+x^2\right]^{3/2}}+\frac{\mu_0NIR^2}{2\left[R^2+(x-R)^2\right]^{3/2}}$$

(*b*) See the graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.76b."




(*c*) Use the values from the spreadsheet to find the % difference.

$$\%\,\text{diff}=\frac{B(x=6.0\,\text{cm})-B(x=5.0\,\text{cm})}{B(x=5.0\,\text{cm})}(100)=\frac{4.49537\,\text{mT}-4.49588\,\text{mT}}{4.49588\,\text{mT}}(100)$$

$$=\boxed{-1.1\times10^{-2}\%}$$

# CHAPTER 29: Electromagnetic Induction and Faraday's Law

## Responses to Questions

1. Using coils with many ($N$) turns increases the values of the quantities to be experimentally measured, because the induced emf and therefore the induced current are proportional to $N$.

2. Magnetic flux is a quantitative measure of the number of magnetic field lines passing through a given area. It depends not only on the field itself, but also on the area and on the angle between the field and the area.

3. Yes, the current is induced clockwise. No, there is no induced current if the magnet is steady, because there is no changing flux through the ring. Yes, the current is induced counterclockwise.

4. There is no induced current in the loop that is moving parallel to the wire because there is no change of magnetic flux through the loop. The induced current in the loop moving away from the wire is clockwise. The magnetic field through the loop due to the current is directed into the page, and the loop is moving such that its distance from the wire is increasing, resulting in a decrease in magnetic field strength and therefore a decrease in magnetic flux through the loop. By Lenz's law, a decreasing magnetic flux into the page results in a clockwise induced current.

5. Yes. The force is attractive. The induced clockwise current in the right loop will induce a counterclockwise current in the left loop which will slow the relative motion of the loops.

6. (*a*) Yes.
   (*b*) The current starts as soon as the battery is connected and current begins to flow in the first loop.
   (*c*) The induced current stops as soon as the current in the first loop has reached its steady value.
   (*d*) The induced current in the second loop will be counterclockwise, in order to oppose the change.
   (*e*) While there is an induced current, there will be a force between the two loops.
   (*f*) The force will be repulsive, since the currents are in opposite directions.

7. Yes, a current will be induced in the second coil. It will start when the battery is disconnected from the first coil and stop when the current falls to zero in the first coil. The current in the second loop will be clockwise.

8. Counterclockwise. If the area of the loop decreases, the flux through the loop (directed out of the page) decreases. By Lenz's law, the resulting induced current will be counterclockwise to oppose the change. Another way to approach this question is to use the right-hand rule. As the bar moves to the left, the negative electrons in the bar will experience a force down, which results in a counterclockwise current.

9. (*a*) The current through $R_A$ will be to the right. The field due to the current in coil B will be to the left. As coil B is moved toward coil A, the flux through A will increase, so the induced field in coil A will be to the right, to oppose the change. This field corresponds to an induced current flowing from left to right in $R_A$.
   (*b*) The current through $R_A$ will be to the left. When coil B is moved away from coil A, the flux through coil A will decrease, so the induced field will be to the left, to oppose the change. This field corresponds to an induced current flowing from right to left in $R_A$.
   (*c*) If $R_B$ is increased, the current in the circuit will decrease, decreasing the flux through coil A, resulting in a current through $R_A$ to the left.

10. The shielding prevents external fields from inducing a current which would cause a false signal in the inner signal wire.

11. The currents in the two wires will be 180º out of phase. If they are very close together, or wrapped around each other, then the magnetic fields created by the currents in the wires will very nearly cancel each other.

12. The straight wire will fall faster. Since the magnetic field is non-uniform, the flux through the loop will change as the loop falls, inducing a current which will oppose the change and therefore resist the downward motion. Eddy currents will also be induced in the straight wire, but they will be much smaller since the straight wire does not form a closed loop.

13. (*a*) Yes. If a rapidly changing magnetic field exists outside, then currents will be induced in the metal sheet. These currents will create magnetic fields which will partially cancel the external fields.
    (*b*) Yes. Since the metal sheet is permeable, it will partially shield the interior from the exterior static magnetic field; some of the magnetic field lines will travel through the metal sheet.
    (*c*) The superconducting sheet will shield the interior from magnetic fields.

14. Each of the devices mentioned has a different operating current and voltage, and each needs its own transformer with its own ratio of primary to secondary turns designed to convert normal household current and voltage into the required current and voltage. If the devices were designed to operate with the same current and voltage, they could all run on identical transformers.

15. You could hook the transformer up to a known ac voltage source. The ratio of the output voltage to the input voltage will give the ratio of turns on the two coils. If you pair up the leads incorrectly (one lead from each coil, rather than both leads from the same coil), there will be no output voltage. Alternatively, you could attach an ohmmeter to two of the leads. The resistance will be infinite if you have one lead from each pair, and nearly zero if you have both leads from the same pair.

16. Higher voltages are inherently more dangerous because of the increased risk of establishing large currents and large electromagnetic fields. The large potential differences between the wires and the ground could cause arcing and short circuits, leading to accidental electrocutions. In addition, higher-voltage power lines will have higher electromagnetic fields associated with them than lower-voltage power lines. Biological effects of exposure to high electromagnetic fields are not well understood, but there is evidence of increased health risks to people who live close to high voltage power lines.

17. When the transformer is connected to the 120-V dc source no back emf is generated, as would happen with an ac source. Therefore, the current in the transformer connected to the dc source will be very large. Because transformers generally are made with fine, low resistance wires, the large current could cause the wires to overheat, melt the insulation, and burn out.

18. A motor uses electric energy to create mechanical energy. When a large electric motor is running, the current in the motor's coil creates a back emf. When the motor is first turned on, the back emf is small, allowing the motor to draw maximum current. The back emf has a maximum value when the motor is running at full speed, reducing the amount of current required to run the motor. As the current flow in the motor's coil stabilizes, the motor will operate at its lower, normal current. The lights will dim briefly when the refrigerator motor starts due to the increased current load on the house circuit. Electric heaters operate by sending a large current through a large resistance, generating heat. When an electric heater is turned on, the current will increase quickly to its

maximum value (no coil, so no back emf) and will stay at its maximum value as long as the heater is on. Therefore, the lights will stay dim as long as the heater is on.

19. At the moment shown in Figure 29-15, the armature is rotating clockwise and so the current in length *b* of the wire loop on the armature is directed outward. (Use the right-hand rule: the field is north to south and the wire is moving with a component downward, therefore force on positive charge carriers is out.) This current is increasing, because as the wire moves down, the downward component of the velocity increases. As the current increases, the flux through the loop also increases, and therefore there is an induced emf to oppose this change. The induced emf opposes the current flowing in section *b* of the wire, and therefore creates a counter-torque.

20. Eddy currents exist in any conducting material, so eddy current brakes could work with wheels made of copper or aluminum.

21. The nonferrous materials are not magnetic but they are conducting. As they pass by the permanent magnets, eddy currents will be induced in them. The eddy currents provide a "braking" mechanism which will cause the metallic materials to slide more slowly down the incline than the nonmetallic materials. The nonmetallic materials will reach the bottom with larger speeds. The nonmetallic materials can therefore be separated from the metallic, nonferrous materials by placing bins at different distances from the bottom of the incline. The closest bin will catch the metallic materials, since their projectile velocities off the end of the incline will be small. The bin for the nonmetallic materials should be placed farther away to catch the higher-velocity projectiles.

22. The slots in the metal bar prevent the formation of large eddy currents, which would slow the bar's fall through the region of magnetic field.

23. As the aluminum sheet is moved through the magnetic field, eddy currents are created in the sheet. The magnetic force on these induced currents opposes the motion. Thus it requires some force to pull the sheet out. (See Figure 29-21.)

24. As the bar magnet falls, it sets up eddy currents in the metal tube which will interact with the magnet and slow its fall. The magnet will reach terminal velocity (due to the interactions with the magnetic dipoles set up by the eddy currents, not air resistance) when the weight of the magnet is balanced by the upward force from the eddy currents.

25. As the bar moves in the magnetic field, induced eddy currents are created in the bar. The magnetic field exerts a force on these currents that opposes the motion of the bar. (See Figure 29-21.)

26. Although in principle you could use a loudspeaker in reverse as a microphone, it would probably not work in actual practice. The membrane of the microphone is very lightweight and sensitive to the sound waves produced by your voice. The cardboard cone of a loudspeaker is much stiffer and would significantly dampen the vibrations so that the frequency of the impinging sound waves would not be translated into an induced emf with the same frequency.

## Solutions to Problems

1. The average induced emf is given by Eq. 29-2b.

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} = -N\frac{\Delta\Phi_B}{\Delta t} = -2\frac{38\,\text{Wb} - (-58\,\text{Wb})}{0.42\,\text{s}} = \boxed{-460\,\text{V}}$$

2. As the magnet is pushed into the coil, the magnetic flux increases to the right. To oppose this increase, flux produced by the induced current must be to the left, so the induced current in the resistor will be $\boxed{\text{from right to left}}$.

3. As the coil is pushed into the field, the magnetic flux through the coil increases into the page. To oppose this increase, the flux produced by the induced current must be out of the page, so the induced current is $\boxed{\text{counterclockwise}}$.

4. The flux changes because the loop rotates. The angle between the field and the normal to the loop changes from 0° to 90°. The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\mathscr{E}_{\text{avg}} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{AB\Delta\cos\theta}{\Delta t} = -\frac{\pi(0.110\,\text{m})^2\,1.5\,\text{T}(\cos 90^\circ - \cos 0^\circ)}{0.20\,\text{s}}$$

$$= -\frac{\pi(0.110\,\text{m})^2\,1.5\,\text{T}(0-1)}{0.20\,\text{s}} = \boxed{0.29\,\text{V}}$$

5. Use Eq. 29-2a to calculate the emf. Setting the flux equal to the magnetic field multiplied by the area of the loop, $A = \pi r^2$, and the emf equal to zero, we can solve for the rate of change in the coil radius.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(B\pi r^2\right) = -\frac{dB}{dt}\pi r^2 - 2\pi Br\frac{dr}{dt} = 0$$

$$\frac{dr}{dt} = -\frac{dB}{dt}\frac{r}{2B} = -(-0.010\text{ T/s})\frac{0.12\text{ m}}{2(0.500\text{ T})} = 0.0012\text{ m/s} = \boxed{1.2\text{ mm/s}}$$

6. We choose up as the positive direction. The average induced emf is given by the "difference" version of Eq. 29-2a.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi(0.054\,\text{m})^2(-0.25\,\text{T} - 0.68\,\text{T})}{0.16\,\text{s}} = \boxed{5.3\times10^{-2}\,\text{V}}$$

$\boxed{7.}$ (*a*) When the plane of the loop is perpendicular to the field lines, the flux is given by the maximum of Eq. 29-1a.

$$\Phi_B = BA = B\pi r^2 = (0.50\,\text{T})\pi(0.080\,\text{m})^2 = \boxed{1.0\times10^{-2}\,\text{Wb}}$$

(*b*) The angle is $\theta = \boxed{55^\circ}$

(*c*) Use Eq. 29-1a.

$$\Phi_B = BA\cos\theta = B\pi r^2 = (0.50\,\text{T})\pi(0.080\,\text{m})^2\cos 55^\circ = \boxed{5.8\times10^{-3}\,\text{Wb}}$$

8. (*a*) As the resistance is increased, the current in the outer loop will decrease. Thus the flux through the inner loop, which is out of the page, will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux out of the page, so the direction of the induced current will be $\boxed{\text{counterclockwise}}$.

(*b*) If the small loop is placed to the left, the flux through the small loop will be into the page and will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

9. As the solenoid is pulled away from the loop, the magnetic flux to the right through the loop decreases. To oppose this decrease, the flux produced by the induced current must be to the right, so the induced current is $\boxed{\text{counterclockwise}}$ as viewed from the right end of the solenoid.

10. (*a*) The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi(0.040\,\text{m})^2(-0.45\,\text{T} - 0.52\,\text{T})}{0.18\,\text{s}} = \boxed{2.7\times10^{-2}\,\text{V}}$$

(*b*) The positive result for the induced emf means the induced field is away from the observer, so the induced current is $\boxed{\text{clockwise}}$.

11. (*a*) The magnetic flux through the loop is into the paper and decreasing, because the area is decreasing. To oppose this decrease, the induced current in the loop will produce a flux into the paper, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

(*b*) The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\left|\mathscr{E}_{\text{avg}}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B|\Delta A|}{\Delta t} = \frac{(0.75\,\text{T})\pi\left[(0.100\,\text{m})^2 - (0.030\,\text{m})^2\right]}{0.50\,\text{s}}$$

$$= 4.288\times10^{-2}\,\text{V} \approx \boxed{4.3\times10^{-2}\,\text{V}}$$

(*c*) We find the average induced current from Ohm's law.

$$I = \frac{\mathscr{E}}{R} = \frac{4.288\times10^{-2}\,\text{V}}{2.5\,\Omega} = \boxed{1.7\times10^{-2}\,\text{A}}$$

12. As the loop is pulled from the field, the flux through the loop decreases, causing an induced EMF whose magnitude is given by Eq. 29-3, $\mathscr{E} = B\ell v$. Because the inward flux is decreasing, the induced flux will be into the page, so the induced current is clockwise, given by $I = \mathscr{E}/R$. Because this current in the left-hand side of the loop is in a downward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by $F = I\ell B$.

$$F = I\ell B = \frac{\mathscr{E}}{R}\ell B = \frac{B\ell v}{R}\ell B = \frac{B^2\ell^2 v}{R} = \frac{(0.650\,\text{T})^2(0.350\,\text{m})^2(3.40\,\text{m/s})}{0.280\,\Omega} = \boxed{0.628\,\text{N}}$$

13. (*a*) Use Eq. 29-2a to calculate the emf induced in the ring, where the flux is the magnetic field multiplied by the area of the ring. Then using Eq. 25-7, calculate the average power dissipated in the ring as it is moved away. The thermal energy is the average power times the time.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta BA}{\Delta t} = -\frac{\Delta B\left(\frac{1}{4}\pi d^2\right)}{\Delta t}$$

$$Q = P\Delta t = \left(\frac{\mathscr{E}^2}{R}\right)\Delta t = \left(\frac{\Delta B\left(\frac{1}{4}\pi d^2\right)}{\Delta t}\right)^2\left(\frac{\Delta t}{R}\right) = \frac{(\Delta B)^2\pi^2 d^4}{16R\Delta t}$$

$$= \frac{(0.80\,\text{T})^2\pi^2(0.015\,\text{m})^4}{16(55\times10^{-6}\,\Omega)(45\times10^{-3}\,\text{s})} = 8.075\times10^{-3}\,\text{J} \approx \boxed{8.1\,\text{mJ}}$$

(*b*) The temperature change is calculated from the thermal energy using Eq. 19-2.

$$\Delta T = \frac{Q}{mc} = \frac{8.075\times10^{-3}\,\text{J}}{(15\times10^{-3}\,\text{kg})(129\,\text{J/kg}\bullet{}^\circ\text{C})} = \boxed{4.2\times10^{-3}\,\text{C}^\circ}$$

14. The average emf induced in the short coil is given by the "difference" version of Eq. 29-2b. $N$ is the number of loops in the short coil, and the flux change is measured over the area of the short coil. The magnetic flux comes from the field created by the solenoid. The field in a solenoid is given by Eq. 28-4, $B = \mu_0 I N_{\text{solenoid}} / \ell_{\text{solenoid}}$, and the changing current in the solenoid causes the field to change.

$$|\mathcal{E}| = \frac{N_{\text{short}} A_{\text{short}} \Delta B}{\Delta t} = \frac{N_{\text{short}} A_{\text{short}} \Delta\left(\dfrac{\mu_0 I N_{\text{solenoid}}}{\ell_{\text{solenoid}}}\right)}{\Delta t} = \frac{\mu_0 N_{\text{short}} N_{\text{solenoid}} A_{\text{short}}}{\ell_{\text{solenoid}}} \frac{\Delta I}{\Delta t}$$

$$= \frac{\left(4\pi \times 10^{-7}\ \text{T•m/A}\right)(15)(420)\pi\left(0.0125\,\text{m}\right)^2}{\left(0.25\,\text{m}\right)} \frac{\left(5.0\,\text{A}\right)}{\left(0.60\,\text{s}\right)} = \boxed{1.3 \times 10^{-4}\,\text{V}}$$

15. (*a*) There is an emf induced in the coil since the flux through the coil changes. The current in the coil is the induced emf divided by the resistance of the coil. The resistance of the coil is found from Eq. 25-3.

$$|\mathcal{E}| = N A_{\text{coil}} \frac{dB}{dt} \qquad R = \frac{\rho \ell}{A_{\text{wire}}}$$

$$I = \frac{\mathcal{E}}{R} = \frac{N A_{\text{coil}} \dfrac{dB}{dt}}{\dfrac{\rho \ell}{A_{\text{wire}}}} = \frac{N A_{\text{coil}} A_{\text{wire}}}{\rho \ell} \frac{dB}{dt}$$

$$= \frac{28\left[\pi\left(0.110\,\text{m}\right)^2\right]\left[\pi\left(1.3 \times 10^{-3}\,\text{m}\right)^2\right]\left(8.65 \times 10^{-3}\ \text{T/s}\right)}{\left(1.68 \times 10^{-8}\,\Omega\text{•m}\right)28\left(2\pi\right)\left(0.110\,\text{m}\right)} = 0.1504\,\text{A} \approx \boxed{0.15\,\text{A}}$$

(*b*) The rate at which thermal energy is produced in the wire is the power dissipated in the wire.

$$P = I^2 R = I^2 \frac{\rho \ell}{A_{\text{wire}}} = \left(0.1504\,\text{A}\right)^2 \frac{\left(1.68 \times 10^{-8}\,\Omega\text{•m}\right)28\left(2\pi\right)\left(0.11\right)}{\pi\left(1.3 \times 10^{-3}\,\text{m}\right)^2} = \boxed{1.4 \times 10^{-3}\,\text{W}}$$

16. The sinusoidal varying current in the power line creates a sinusoidal varying magnetic field encircling the power line, given by Eq. 28-1. Using Eq. 29-1b we integrate this field over the area of the rectangle to determine the flux through it. Differentiating the flux as in Eq. 29-2b gives the emf around the rectangle. Finally, by setting the maximum emf equal to 170 V we can solve for the necessary length of the rectangle.

$$B(t) = \frac{\mu_0 I_0}{2\pi r} \cos\left(2\pi f t\right) \ ;$$

$$\Phi_B(t) = \int B\,dA = \int_{5.0\text{ m}}^{7.0\text{ m}} \frac{\mu_0 I_0}{2\pi r} \cos\left(2\pi f t\right) \ell\, dr = \frac{\mu_0 I_0}{2\pi} \ell \cos\left(2\pi f t\right) \int_{5.0\text{ m}}^{7.0\text{ m}} \frac{dr}{r} = \frac{\mu_0 I_0}{2\pi} \ln\left(1.4\right) \ell \cos\left(2\pi f t\right)$$

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} = -\frac{N \mu_0 I_0}{2\pi} \ln\left(1.4\right) \ell \left[\frac{d}{dt} \cos\left(2\pi f t\right)\right] = N \mu_0 I_0 f \ \ln\left(1.4\right) \ell \sin\left(2\pi f t\right) \ ;$$

$$\mathcal{E}_0 = N \mu_0 I_0 f \ \ln\left(1.4\right) \ell \ \rightarrow$$

$$\ell = \frac{\mathcal{E}_0}{N \mu_0 I_0 f \ \ln\left(1.4\right)} = \frac{170\,\text{V}}{10\left(4\pi \times 10^{-7}\ \text{T•m/A}\right)\left(55{,}000\,\text{A}\right)\left(60\,\text{Hz}\right)\ln\left(1.4\right)} = \boxed{12\,\text{m}}$$

This is unethical because the current in the rectangle creates a back emf in the initial wire. This results in a power loss to the electric company, just as if the wire had been physically connected to the line.

17. The charge that passes a given point is the current times the elapsed time, $Q = I\Delta t$. The current will be the emf divided by the resistance, $I = \frac{\mathscr{E}}{R}$. The resistance is given by Eq. 25-3, $R = \frac{\rho \ell}{A_{\text{wire}}}$, and the emf is given by the "difference" version of Eq. 29-2a. Combine these equations to find the charge during the operation.

$$|\mathscr{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{A_{\text{loop}}|\Delta B|}{\Delta t} \; ; \; R = \frac{\rho\ell}{A_{\text{wire}}} \; ; \; I = \frac{\mathscr{E}}{R} = \frac{\frac{A_{\text{loop}}|\Delta B|}{\Delta t}}{\frac{\rho\ell}{A_{\text{wire}}}} = \frac{A_{\text{loop}}A_{\text{wire}}|\Delta B|}{\rho\ell\Delta t}$$

$$Q = I\Delta t = \frac{A_{\text{loop}}A_{\text{wire}}|\Delta B|}{\rho\ell} = \frac{\pi r_{\text{loop}}^2 \pi r_{\text{wire}}^2 |\Delta B|}{\rho(2\pi)r_{\text{loop}}} = \frac{r_{\text{loop}}\pi r_{\text{wire}}^2|\Delta B|}{2\rho}$$

$$= \frac{(0.091\,\text{m})\pi\left(1.175\times10^{-3}\,\text{m}\right)^2(0.750\,\text{T})}{2\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)} = \boxed{8.81\,\text{C}}$$

18. (*a*) Use Eq. 29-2b to calculate the emf.

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} = (-75)\frac{d}{dt}\left[\left(8.8t - 0.51t^3\right)\times10^{-2}\,\text{T}\bullet\text{m}^2\right] = \left(-6.6 + 1.1475t^2\right)\text{V}$$

$$\approx \boxed{\left(-6.6 + 1.1t^2\right)\text{V}}$$

(*b*) Evaluate at the specific times.

$$\mathscr{E}(t = 1.0\,\text{s}) = \left(-6.6 + 1.1475(1.0)^2\right)\text{V} = \boxed{-5.5\,\text{V}}$$

$$\mathscr{E}(t = 4.0\,\text{s}) = \left(-6.6 + 1.1475(4.0)^2\right)\text{V} = \boxed{12\,\text{V}}$$

19. The energy dissipated in the process is the power dissipated by the resistor, times the elapsed time that the current flows. The average induced emf is given by the "difference" version of Eq. 29-2a.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} \; ; \; P = \frac{\mathscr{E}^2}{R} \; ;$$

$$E = P\Delta t = \frac{\mathscr{E}^2}{R}\Delta t = \left(\frac{\Delta\Phi_B}{\Delta t}\right)^2\frac{\Delta t}{R} = \frac{A^2(\Delta B)^2}{R\Delta t} = \frac{\left[\pi(0.125\text{m})^2\right]^2(0.40\,\text{T})^2}{(150\,\Omega)(0.12\,\text{s})} = \boxed{2.1\times10^{-5}\,\text{J}}$$

20. The induced emf is given by Eq. 29-2a. Since the field is uniform and is perpendicular to the area, the flux is simply the field times the area.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -(0.28\,\text{T})\left(-3.50\times10^{-2}\,\text{m}^2/\text{s}\right) = \boxed{9.8\,\text{mV}}$$

Since the area changes at a constant rate, and the area has not shrunk to 0 at $t$ = 2.00 s, the emf is the same for both times.

21. The induced emf is given by Eq. 29-2a. Since the field is uniform and is perpendicular to the area, the flux is simply the field times the area of a circle. We calculate the initial radius from the initial area. To calculate the radius after one second we add the change in radius to the initial radius.

$$\mathscr{E}(t) = \frac{d\Phi_B}{dt} = B\frac{d\left(\pi r^2\right)}{dt} = 2\pi Br\frac{dr}{dt} \qquad A_0 = \pi r_0^2 \rightarrow r_0 = \sqrt{\frac{A_0}{\pi}}$$

$$\mathscr{E}(0) = 2\pi\left(0.28\,\text{T}\right)\sqrt{\frac{0.285\,\text{m}^2}{\pi}}\left(0.043\,\text{m/s}\right) = \boxed{23\,\text{mV}}$$

$$\mathscr{E}(1.00\,\text{s}) = 2\pi\left(0.28\,\text{T}\right)\left[\sqrt{\frac{0.285\,\text{m}^2}{\pi}} + \left(0.043\,\text{m/s}\right)\left(1.00\,\text{s}\right)\right]\left(0.043\,\text{m/s}\right) = \boxed{26\,\text{mV}}$$

22. The magnetic field inside the solenoid is given by Eq. 28-4, $B = \mu_0 nI$. Use Eq. 29-2a to calculate the induced emf. The flux causing the emf is the flux through the small loop.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = -A_1\frac{dB_{\text{solenoid}}}{dt} = -A_1\mu_0 n\frac{dI}{dt} = -A_1\mu_0 n\left(-\omega I_0 \sin\omega t\right) = \boxed{A_1\mu_0 n\omega I_0 \sin\omega t}$$

23. (*a*) If the magnetic field is parallel to the plane of the loop, no magnetic flux passes through the loop at any time. Therefore, the emf and the current in the loop are zero.

(*b*) When the magnetic field is perpendicular to the plane of the loop, we differentiate Eq. 29-1a with respect to time to obtain the emf in the loop. Then we divide the emf by the resistance to calculate the current in the loop.

$$I = \frac{\mathscr{E}}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right) = -\frac{1}{R}\frac{d}{dt}\left[\left(\alpha t\right)\left(A_0 + \beta t\right)\right] = -\frac{\alpha}{R}\left[A_0 + 2\beta t\right]$$

$$= -\frac{\left(0.60\text{ T/s}\right)\left[\left(0.50\text{ m}^2\right) + 2\left(0.70\text{ m}^2\text{/s}\right)\left(2.0\text{ s}\right)\right]}{2.0\ \Omega} = -0.99\text{ A}$$

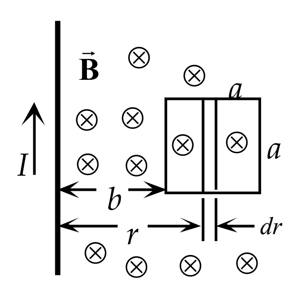
Since the magnetic field is pointing down into the page, the downward flux is increasing. The current then flows in a direction to create an upward flux. The resulting current is then 0.99 A in the counterclockwise direction.

24. The magnetic field across the primary coil is constant and is that of a solenoid (Eq. 28-4). We multiply this magnetic field by the area of the secondary coil to calculate the flux through the secondary coil. Then using Eq. 29-2b we differentiate the flux to calculate the induced emf.

$$\Phi_B = BA = \mu_0 n_p I_0 \sin\left(2\pi ft\right)\left(\pi d^2/4\right)$$

$$\mathscr{E}_2 = N\frac{d\Phi_B}{dt} = N\mu_0 n_p I_0\left(\pi d^2/4\right)\left[\frac{d}{dt}\sin\left(2\pi ft\right)\right] = \boxed{-\tfrac{1}{2}\pi^2 d^2 fN\mu_0 n_p I_0 \cos\left(2\pi ft\right)}$$

25. (*a*) The magnetic field a distance *r* from the wire is perpendicular to the wire and given by Eq. 28-1. Integrating this magnetic field over the area of the loop gives the flux through the loop.




$$\Phi_B = \int BdA = \int_b^{b+a}\frac{\mu_0 I}{2\pi r}adr = \boxed{\frac{\mu_0 Ia}{2\pi}\ln\left(1+\frac{a}{b}\right)}$$

(*b*) Since the loop is being pulled away, $v = \frac{db}{dt}$. Differentiate the magnetic flux with respect to time to calculate the emf in the loop.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 I a}{2\pi}\ln\left(1+\frac{a}{b}\right)\right] = -\frac{\mu_0 I a}{2\pi}\frac{d}{db}\left[\ln\left(1+\frac{a}{b}\right)\right]\frac{db}{dt} = \boxed{\frac{\mu_0 I a^2 v}{2\pi b(b+a)}}$$

Note that this is the emf at the instant the loop is a distance *b* from the wire. The value of *b* is changing with time.

(*c*) Since the magnetic field at the loop points into the page, and the flux is decreasing, the induced current will create a downward magnetic field inside the loop. The current in the loop then flows clockwise.

(*d*) The power dissipated in the loop as it is pulled away is related to the emf and resistance by Eq. 25-7b. This power is provided by the force pulling the loop away. We calculate this force from the power using Eq. 8-21. As in part (*b*), the value of *b* is changing with time.

$$F = \frac{P}{v} = \frac{\mathcal{E}^2}{Rv} = \boxed{\frac{\mu_0^2 I^2 a^4 v}{4\pi^2 R b^2 (b+a)^2}}$$

26. From Problem 25, the flux through the loop is given by $\Phi_B = \frac{\mu_0 I a}{2\pi}\ln\left(1+\frac{a}{b}\right)$. The emf is found from Eq. 29-2a.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 I a}{2\pi}\ln\left(1+\frac{a}{b}\right)\right] = -\frac{\mu_0 a}{2\pi}\ln\left(1+\frac{a}{b}\right)\frac{dI}{dt}$$

$$= -\frac{(4\pi\times10^{-7}\,\mathrm{T{\bullet}m/A})(0.120\,\mathrm{m})}{2\pi}\ln\left(1+\frac{12.0}{15.0}\right)(15.0\,\mathrm{A})(2500\,\mathrm{rad/s})\cos(2500t)$$

$$= \boxed{(5.3\times10^{-4}\,\mathrm{V})\cos(2500t)}$$

27. The velocity is found from Eq. 29-3.

$$\mathcal{E} = B\ell v \quad\rightarrow\quad v = \frac{\mathcal{E}}{B\ell} = \frac{0.12\,\mathrm{V}}{(0.90\,\mathrm{T})(0.132\,\mathrm{m})} = \boxed{1.0\,\mathrm{m/s}}$$

28. Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from Eq. 29-3.

$$\mathcal{E} = B\ell v = (0.800\,\mathrm{T})(0.120\,\mathrm{m})(0.150\,\mathrm{m/s}) = \boxed{1.44\times10^{-2}\,\mathrm{V}}$$

29. (*a*) Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from Eq. 29-3.

$$\mathcal{E} = B\ell v = (0.35\,\mathrm{T})(0.250\,\mathrm{m})(1.3\,\mathrm{m/s}) = 0.1138\,\mathrm{V} \approx \boxed{0.11\,\mathrm{V}}$$

(*b*) Find the induced current from Ohm's law, using the **total** resistance.

$$I = \frac{\mathcal{E}}{R} = \frac{0.1138\,\mathrm{V}}{25.0\,\Omega + 2.5\,\Omega} = 4.138\times10^{-3}\,\mathrm{A} \approx \boxed{4.1\,\mathrm{mA}}$$

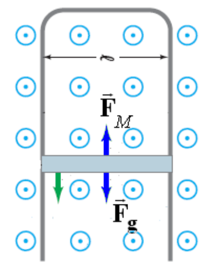
(*c*) The induced current in the rod will be down. Because this current is in an upward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by Eq. 27-1.

$$F = I\ell B = (4.138\times10^{-3}\,\mathrm{A})(0.250\,\mathrm{m})(0.35\,\mathrm{T}) = 3.621\times10^{-4}\,\mathrm{N} \approx \boxed{0.36\,\mathrm{mN}}$$

30. The emf is given by Eq. 29-3 as $\mathscr{E} = B\ell v$. The resistance of the conductor is given by Eq. 25-3. The length in Eq. 25-3 is the length of resistive material. Since the movable rod starts at the bottom of the U at time $t = 0$, in a time $t$ it will have moved a distance $vt$.

$$I = \frac{\mathscr{E}}{R} = \frac{B\ell v}{\frac{\rho L}{A}} = \frac{B\ell v}{\frac{\rho(2vt+\ell)}{A}} = \boxed{\frac{B\ell vA}{\rho(2vt+\ell)}}$$

31. The rod will descend at its terminal velocity when the magnitudes of the magnetic force (found in Example 29-8) and the gravitational force are equal. We set these two forces equal and solve for the terminal velocity.



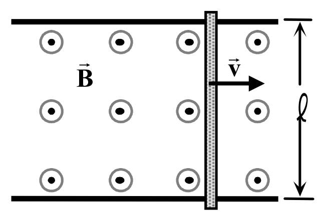

$$\frac{B^2\ell^2 v_t}{R} = mg \rightarrow$$

$$v_t = \frac{mgR}{B^2\ell^2} = \frac{(3.6\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.0013\ \Omega)}{(0.060\text{ T})^2(0.18\text{ m})^2} = \boxed{0.39\text{ m/s}}$$

32. Since the antenna is vertical, the maximum emf will occur when the car is traveling perpendicular to the horizontal component of the Earth's magnetic field. This occurs when the car is traveling in the east or west direction. We calculate the magnitude of the emf using Eq. 29-3, where $B$ is the horizontal component of the Earth's magnetic field.

$$\mathscr{E} = B_x\ell v = (5.0\times10^{-5}\text{T}\cos45^\circ)(0.750\text{m})(30.0\text{m/s}) = 8.0\times10^{-4}\text{V} = \boxed{0.80\text{mV}}$$

33. (*a*) As the rod moves through the magnetic field an emf will be built up across the rod, but no current can flow. Without the current, there is no force to oppose the motion of the rod, so yes, the rod travels at constant speed.



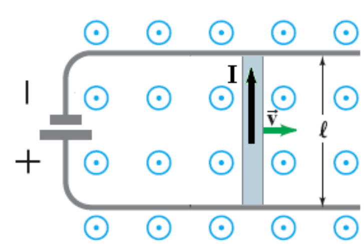

(*b*) We set the force on the moving rod, obtained in Example 29-8, equal to the mass times the acceleration of the rod. We then write the acceleration as the derivative of the velocity, and by separation of variables we integrate the velocity to obtain an equation for the velocity as a function of time.

$$F = ma = m\frac{dv}{dt} = -\frac{B^2\ell^2}{R}v \rightarrow \frac{dv}{v} = -\frac{B^2\ell^2}{mR}dt$$

$$\int_{v_0}^{v}\frac{dv'}{v'} = -\frac{B^2\ell^2}{mR}\int_0^t dt' \rightarrow \ln\frac{v}{v_0} = -\frac{B^2\ell^2}{mR}t \rightarrow \boxed{v(t) = v_0 e^{-\frac{B^2\ell^2}{mR}t}}$$

The magnetic force is proportional to the velocity of the rod and opposes the motion. This results in an exponentially decreasing velocity.

34. (*a*) For a constant current, of polarity shown in the figure, the magnetic force will be constant, given by Eq. 27-2. Using Newton's second law we can integrate the acceleration to calculate the velocity as a function of time.




$$F = m\frac{dv}{dt} = I\ell B \rightarrow \int_0^v dv = \frac{I\ell B}{m}\int_0^t dt \rightarrow \boxed{v(t) = \frac{I\ell B}{m}t}$$

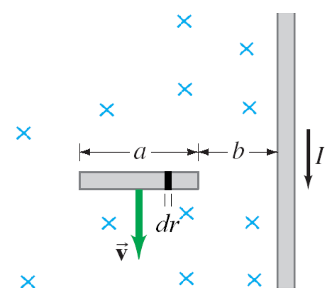
(*b*) For a constant emf, the current will vary with the speed of the rod, as motional emf opposes the motion of the rod. We again use Eq. 27-2 for the force on the rod, with the current given by Ohm's law, and the induced motional emf given by Eq 29-3. The current produced by the induced emf opposes the current produced by the battery.

$$F = m\frac{dv}{dt} = I\ell B = \left(\frac{\mathcal{E}_0 - B\ell v}{R}\right)\ell B \rightarrow \frac{dv}{\mathcal{E}_0 - B\ell v} = \frac{\ell B}{mR}dt \rightarrow \frac{dv}{v - \mathcal{E}_0/B\ell} = -\frac{B^2\ell^2}{mR}dt \rightarrow$$

$$\int_0^v \frac{dv}{v - \mathcal{E}_0/B\ell} = -\frac{B^2\ell^2}{mR}\int_0^t dt \rightarrow \ln\left(\frac{v - \mathcal{E}_0/B\ell}{-\mathcal{E}_0/B\ell}\right) = -\frac{B^2\ell^2}{mR}t \rightarrow \boxed{v(t) = \frac{\mathcal{E}_0}{B\ell}\left(1 - e^{-\frac{B^2\ell^2}{mR}t}\right)}$$

(*c*) With constant current, the acceleration is constant and so the velocity does not reach a terminal velocity. However, with constant emf, the increasing motional emf decreases the applied force. This results in a limiting, or terminal velocity of $\boxed{v_t = \mathcal{E}_0/B\ell}$.

35. (*a*) The magnetic field is perpendicular to the rod, with the magnetic field decreasing with distance from the rod, as in Eq. 28-1. The emf, $d\mathcal{E}$, across a short segment, *dr*, of the rod is given by the differential version of Eq. 29-3. Integrating this emf across the length of the wire gives the total emf.




$$d\mathcal{E} = Bvdr \rightarrow$$

$$\mathcal{E} = \int d\mathcal{E} = \int_b^{b+a} \frac{\mu_0 I}{2\pi r}vdr = \boxed{\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b+a}{b}\right)}$$

This emf points $\boxed{\text{toward the wire}}$, as positive charges are attracted toward the current.

(*b*) The only change is the direction of the current, so the magnitude of the emf remains the same, but points $\boxed{\text{away from the wire}}$, since positive charges are repelled from the current.

36. From Eq. 29-4, the induced voltage is proportional to the angular speed. Thus their quotient is a constant.

$$\frac{\mathcal{E}_1}{\omega_1} = \frac{\mathcal{E}_2}{\omega_2} \rightarrow \mathcal{E}_2 = \mathcal{E}_1\frac{\omega_2}{\omega_1} = (12.4\,\text{V})\frac{1550\,\text{rpm}}{875\,\text{rpm}} = \boxed{22.0\,\text{V}}$$

$\boxed{37.}$ We find the number of turns from Eq. 29-4. The factor multiplying the sine term is the peak output voltage.

$$\mathcal{E}_{\text{peak}} = NB\omega A \rightarrow N = \frac{\mathcal{E}_{\text{peak}}}{B\omega A} = \frac{24.0\,\text{V}}{(0.420\,\text{T})(2\pi\,\text{rad/rev})(60\,\text{rev/s})(0.0515\,\text{m})^2} = \boxed{57.2\text{ loops}}$$

38. From Eq. 29-4, the peak voltage is $\mathcal{E}_{\text{peak}} = NB\omega A$. Solve this for the rotation speed.

$$\mathcal{E}_{\text{peak}} = NB\omega A \rightarrow \omega = \frac{\mathcal{E}_{\text{peak}}}{NBA} = \frac{120\,\text{V}}{480(0.550\,\text{T})(0.220\,\text{m})^2} = 9.39\,\text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{9.39\,\text{rad/s}}{2\pi\,\text{rad/rev}} = \boxed{1.49\,\text{rev/s}}$$

39. From Eq. 29-4, the peak voltage is $\mathcal{E}_{\text{peak}} = NAB\omega$. The rms voltage is the peak voltage divided by $\sqrt{2}$, and so $V_{\text{rms}} = \mathcal{E}_{\text{peak}}/\sqrt{2} = NAB\omega/\sqrt{2}$.

40. Rms voltage is found from the peak induced emf. Peak induced emf is calculated from Eq. 29-4.

$$\mathcal{E}_{\text{peak}} = NB\omega A \rightarrow$$

$$V_{rms} = \frac{\mathcal{E}_{peak}}{\sqrt{2}} = \frac{NB\omega A}{\sqrt{2}} = \frac{(250)(0.45\,\text{T})(2\pi\,\text{rad/rev})(120\,\text{rev/s})\pi(0.050\text{m})^2}{\sqrt{2}}$$

$$= 471.1\,\text{V} \approx \boxed{470\,\text{V}}$$

To double the output voltage, you must double the rotation frequency to 240 rev/s.

41. From Eq. 29-4, the induced voltage (back emf) is proportional to the angular speed. Thus their quotient is a constant.

$$\frac{\mathcal{E}_1}{\omega_1} = \frac{\mathcal{E}_2}{\omega_2} \rightarrow \mathcal{E}_2 = \mathcal{E}_1\frac{\omega_2}{\omega_1} = (72\,\text{V})\frac{2500\,\text{rpm}}{1200\,\text{rpm}} = \boxed{150\,\text{V}}$$

42. When the motor is running at full speed, the back emf opposes the applied emf, to give the net across the motor.

$$\mathcal{E}_{applied} - \mathcal{E}_{back} = IR \rightarrow \mathcal{E}_{back} = \mathcal{E}_{applied} - IR = 120\,\text{V} - (7.20\,\text{A})(3.05\,\Omega) = \boxed{98\,\text{V}}$$

43. The back emf is proportional to the rotation speed (Eq. 29-4). Thus if the motor is running at half speed, the back emf is half the original value, or 54 V. Find the new current from writing a loop equation for the motor circuit, from Figure 29-20.

$$\mathcal{E} - \mathcal{E}_{back} - IR = 0 \rightarrow I = \frac{\mathcal{E} - \mathcal{E}_{back}}{R} = \frac{120\,\text{V} - 54\,\text{V}}{5.0\,\Omega} = \boxed{13\,\text{A}}$$

44. The magnitude of the back emf is proportional to both the rotation speed and the magnetic field, from Eq. 29-4. Thus $\frac{\mathcal{E}}{B\omega}$ is constant.

$$\frac{\mathcal{E}_1}{B_1\omega_1} = \frac{\mathcal{E}_2}{B_2\omega_2} \rightarrow B_2 = \frac{\mathcal{E}_2}{\omega_2}\frac{B_1\omega_1}{\mathcal{E}_1} = \frac{(75\,\text{V})}{(2300\,\text{rpm})}\frac{B_1(1100\,\text{rpm})}{(85\,\text{V})} = 0.42B_1$$

So reduce the magnetic field to 42% of its original value.

45. (*a*) The generator voltage rating is the generator emf less the back emf. The ratio of the generator voltage rating to the generator emf is equal to the ratio of the effective resistance to the armature resistance. We solve this ratio for the generator emf, which is the same as the "no load" voltage.

$$V_{nl} = \mathcal{E} = V_{load}\frac{R_{load}}{R_{nl}} = V_{load}\frac{V_{load}/I_{load}}{R_{nl}} = 250\,\text{V}\left(\frac{250\,\text{V}/64\,\text{A}}{0.40\,\Omega}\right) = 2441\,\text{V} \approx \boxed{2.4\,\text{kV}}$$

(*b*) The generator voltage is proportional to the rotation frequency. From this proportionality we solve for the new generator voltage.

$$\frac{V_2}{V_1} = \frac{\omega_2}{\omega_1} \rightarrow V_2 = V_1\frac{\omega_2}{\omega_1} = (250\,\text{V})\frac{750\,\text{rpm}}{1000\,\text{rpm}} = \boxed{190\,\text{V}}$$

46. Because $N_S < N_P$, this is a step-down transformer. Use Eq. 29-5 to find the voltage ratio, and Eq. 29-6 to find the current ratio.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{85\,\text{turns}}{620\,\text{turns}} = \boxed{0.14} \qquad \frac{I_S}{I_P} = \frac{N_P}{N_S} = \frac{620\,\text{turns}}{85\,\text{turns}} = \boxed{7.3}$$

47. We find the ratio of the number of turns from Eq. 21-6.

$$\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{12000\,\text{V}}{240\,\text{V}} = \boxed{50}$$

If the transformer is connected backward, the role of the turns will be reversed:

$$\frac{N_S}{N_P} = \frac{V_S}{V_P} \rightarrow \frac{1}{50} = \frac{V_S}{240\,\text{V}} \rightarrow V_S = \frac{1}{50}(240\,\text{V}) = \boxed{4.8\,\text{V}}$$

48. (*a*) Use Eqs. 29-5 and 29-6 to relate the voltage and current ratios.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}\ ;\ \frac{I_S}{I_P} = \frac{N_P}{N_S} \rightarrow \frac{V_S}{V_P} = \frac{I_P}{I_S} \rightarrow V_S = V_P\frac{I_P}{I_S} = (120\,\text{V})\frac{0.35\,\text{A}}{7.5\,\text{A}} = \boxed{5.6\,\text{V}}$$

(*b*) Because $V_S < V_P$, this is a $\boxed{\text{step-down}}$ transformer.

$\boxed{49.}$ (*a*) We assume 100% efficiency, and find the input voltage from $P = IV$.

$$P = I_P V_P \rightarrow V_P = \frac{P}{I_P} = \frac{75\,\text{W}}{22\,\text{A}} = 3.409\,\text{V}$$

Since $V_P < V_S$, this is a $\boxed{\text{step-up}}$ transformer.

(*b*) $$\frac{V_S}{V_P} = \frac{12\,\text{V}}{3.409\,\text{V}} = \boxed{3.5}$$

50. (*a*) The current in the transmission lines can be found from Eq. 25-10a, and then the emf at the end of the lines can be calculated from Kirchhoff's loop rule.

$$P_{\text{town}} = V_{\text{rms}} I_{\text{rms}} \rightarrow I_{\text{rms}} = \frac{P_{\text{town}}}{V_{\text{rms}}} = \frac{65\times10^6\,\text{W}}{45\times10^3\,\text{V}} = 1444\,\text{A}$$

$$\mathcal{E} - IR - V_{\text{output}} = 0 \rightarrow$$

$$\mathcal{E} = IR + V_{\text{output}} = \frac{P_{\text{town}}}{V_{\text{rms}}}R + V_{\text{rms}} = \frac{65\times10^6\,\text{W}}{45\times10^3\,\text{V}}(3.0\,\Omega) + 45\times10^3\,\text{V} = 49333\,\text{V} = \boxed{49\,\text{kV}(\text{rms})}$$

(*b*) The power loss in the lines is given by $P_{\text{loss}} = I_{\text{rms}}^2 R$.

$$\text{Fraction wasted} = \frac{P_{\text{loss}}}{P_{\text{total}}} = \frac{P_{\text{loss}}}{P_{\text{town}} + P_{\text{loss}}} = \frac{I_{\text{rms}}^2 R}{P_{\text{town}} + I_{\text{rms}}^2 R} = \frac{(1444\,\text{A})^2(3.0\,\Omega)}{65\times10^6\,\text{W} + (1444\,\text{A})^2(3.0\,\Omega)}$$

$$= \boxed{0.088} = 8.8\%$$

51. (*a*) If the resistor $R$ is connected between the terminals, then it has a voltage $V_0$ across it and current $I_0$ passing through it. Then by Ohm's law the equivalent resistance is equal to the resistance of the resistor.

$$R_{\text{eq}} = \frac{V_0}{I_0} = \boxed{R}$$

(*b*) We use Eqs. 29-5 and 29-6 to write the voltage drop and current through the resistor in terms of the source voltage and current to calculate the effective resistance.

$$R = \frac{V_s}{I_s} = \frac{\frac{N_S}{N_P}V_0}{\frac{N_P}{N_S}I_0} \rightarrow R_{\text{eq}} = \frac{V_0}{I_0} = \boxed{\left(\frac{N_P}{N_S}\right)^2 R}$$

52. We set the power loss equal to 2% of the total power. Then using Eq. 25-7a we write the power loss in terms of the current (equal to the power divided by the voltage drop) and the resistance. Then, using Eq. 25-3, we calculate the cross-sectional area of each wire and the minimum wire diameter. We assume there are two lines to have a complete circuit.

$$P_{\text{loss}} = 0.020P = I^2R = \left(\frac{P}{V}\right)^2\left(\frac{\rho\ell}{A}\right) \rightarrow A = \frac{P\rho\ell}{0.020V^2} = \frac{\pi d^2}{4} \rightarrow$$

$$d = \sqrt{\frac{4}{0.020\pi}\frac{P\rho\ell}{V^2}} = \sqrt{\frac{4\left(225\times10^6\,\text{W}\right)\left(2.65\times10^{-8}\,\Omega\bullet\text{m}\right)2\left(185\times10^3\text{ m}\right)}{0.020\pi\left(660\times10^3\,\text{V}\right)^2}} = 0.01796\,\text{m} \approx \boxed{1.8\,\text{cm}}$$

The transmission lines must have a diameter greater than or equal to 1.8 cm.

53. Without the transformers, we find the delivered current, which is the current in the transmission lines, from the delivered power, and the power lost in the transmission lines.

$$P_{\text{out}} = V_{\text{out}}I_{\text{line}} \rightarrow I_{\text{line}} = \frac{P_{\text{out}}}{V_{\text{out}}} = \frac{85000\,\text{W}}{120\,\text{V}} = 708.33\,\text{A}$$

$$P_{\text{lost}} = I_{\text{line}}^2R_{\text{line}} = \left(708.33\,\text{A}\right)^2 2\left(0.100\,\Omega\right) = 100346\,\text{W}$$

Thus there must be $85000\,\text{W} + 100346\,\text{W} = 185346\text{W} \approx 185\,\text{kW}$ of power generated at the start of the process.

With the transformers, to deliver the same power at 120 V, the delivered current from the step-down transformer must still be 708.33 A. Using the step-down transformer efficiency, we calculate the current in the transmission lines, and the loss in the transmission lines.

$$P_{\text{out}} = 0.99P_{\substack{\text{line}\\\text{end}}} \rightarrow V_{\text{out}}I_{\text{out}} = 0.99V_{\text{line}}I_{\text{line}} \rightarrow I_{\text{line}} = \frac{V_{\text{out}}I_{\text{out}}}{0.99V_{\text{line}}} = \frac{\left(120\,\text{V}\right)\left(708.33\,\text{A}\right)}{\left(0.99\right)\left(1200\,\text{V}\right)} = 71.548\,\text{A}$$

$$P_{\text{lost}} = I_{\text{line}}^2R_{\text{line}} = \left(71.548\,\text{A}\right)^2 2\left(0.100\,\Omega\right) = 1024\,\text{W}$$

The power to be delivered is 85000 W. The power that must be delivered to the step-down transformer is $\frac{85000\,\text{W}}{0.99} = 85859\,\text{W}$. The power that must be present at the start of the transmission must be $85859\,\text{W} + 1024\,\text{W} = 86883\,\text{W}$ to compensate for the transmission line loss. The power that must enter the transmission lines from the 99% efficient step-up transformer is $\frac{86883\,\text{W}}{0.99} = 87761 \approx 88\,\text{kW}$. So the power saved is $185346\,\text{W} - 87761\,\text{W} = 97585\,\text{W} \approx \boxed{98\,\text{kW}}$.

54. We choose a circular path centered at the origin with radius 10 cm. By symmetry the electric field is uniform along this path and is parallel to the path. We then use Eq. 29-8 to calculate the electric field at each point on this path. From the electric field we calculate the force on the charged particle.

$$\oint \vec{E}\bullet d\vec{\ell} = E\left(2\pi r\right) = -\frac{d\Phi_B}{dt} = -\left(\pi r^2\right)\frac{dB}{dt}$$

$$F = QE = -Q\frac{r}{2}\frac{dB}{dt} = -\left(1.0\times10^{-6}\text{ C}\right)\frac{0.10\text{ m}}{2}\left(-0.10\text{ T/s}\right) = \boxed{5.0\text{ nN}}$$

Since the magnetic field points into the page and is decreasing, Lenz's law tells us that an induced circular current centered at the origin would flow in the clockwise direction. Therefore, the force on a positive charge along the positive x-axis would be down, or $\boxed{\text{in the } -\hat{\mathbf{j}} \text{ direction.}}$

55. (*a*) The increasing downward magnetic field creates a circular electric field along the electron path. This field applies an electric force to the electron causing it to accelerate.

(*b*) For the electrons to move in a circle, the magnetic force must provide a centripetal acceleration. With the magnetic field pointing downward, the right-hand-rule requires the electrons travel in a clockwise direction for the force to point inward.

(*c*) For the electrons to accelerate, the electric field must point in the counterclockwise direction. A current in this field would create an upward magnetic flux. So by Lenz's law, the downward magnetic field must be increasing.

(*d*) For the electrons to move in a circle and accelerate, the field must be pointing downward and increasing in magnitude. For a sinusoidal wave, the field is downward half of the time and upward the other half. For the half that it is downward its magnitude is decreasing half of the time and increasing the other half. Therefore, the magnetic field is pointing downward and increasing for only one fourth of every cycle.

56. In Example 29-14 we found the electric field along the electron's path from Faraday's law. Multiplying this field by the electron charge gives the force on the electron, and from the force, we calculate the change in tangential velocity.

$$\frac{dv}{dt} = \frac{F}{m} = E\frac{q}{m} = \frac{q}{m}\frac{r}{2}\frac{dB_{\text{avg}}}{dt}$$

We set the centripetal force on the electron equal to the magnetic force (using Eq. 27-5b) and solve for the velocity. Differentiating the velocity with respect to time (keeping the radius constant) yields a relation for the acceleration in terms of the changing magnetic field.

$$qvB = m\frac{v^2}{r} \quad\to\quad v = \frac{qBr}{m} \quad\to\quad \frac{dv}{dt} = \frac{q}{m}r\frac{dB_0}{dt}$$

Equating these two equations for the electron acceleration, we see that the change in magnetic field at the electron must equal $\frac{1}{2}$ of the average change in magnetic field. This relation is satisfied if at all times $B_0 = \frac{1}{2}B_{\text{avg}}$.

57. (*a*) The electric field is the change in potential across the rod (obtained from Ohm's law) divided by the length of the rod.

$$E = \frac{\Delta V}{\ell} = \boxed{\frac{IR}{\ell}}$$

(*b*) Again the electric field is the change in potential across the rod divided by the length of the rod. The electric potential is the supplied potential less the motional emf found using Eq. 29-3 and the results of Problem 34(b).

$$E = \frac{\Delta V}{\ell} = \frac{\mathcal{E}_0 - B\ell v}{\ell} = \frac{\mathcal{E}_0 - B\ell\,\mathcal{E}_0/B\ell\left(1 - e^{-\frac{B^2\ell^2}{mR}t}\right)}{\ell} = \boxed{\frac{\mathcal{E}_0}{\ell}e^{-\frac{B^2\ell^2}{mR}t}}$$

58. (*a*) The clockwise current in the left-hand loop produces a magnetic field which is into the page within the loop and out of the page outside the loop. Thus the right-hand loop is in a magnetic field that is directed out of the page. Before the current in the left-hand loop reaches its steady state, there will be an induced current in the right-hand loop that will produce a magnetic field into the page to oppose the increase of the field from the left-hand loop. Thus the induced current will be clockwise.

(*b*) After a long time, the current in the left-hand loop is constant, so there will be no induced current in the right-hand coil.

(*c*) If the second loop is pulled to the right, the magnetic field out of the page from the left-hand loop through the second loop will decrease. During the motion, there will be an induced current in the right-hand loop that will produce a magnetic field out of the page to oppose the decrease of the field from the left-hand loop. Thus the induced current will be counterclockwise.

59. The electrical energy is dissipated because there is current flowing in a resistor. The power dissipation by a resistor is given by $P = I^2R$, and so the energy dissipated is $E = P\Delta t = I^2R\Delta t$. The current is created by the induced emf caused by the changing B-field. The average induced emf is given by the "difference" version of Eq. 29-2b.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} \qquad I = \frac{\mathscr{E}}{R} = -\frac{A\Delta B}{R\Delta t}$$

$$E = P\Delta t = I^2R\Delta t = \frac{A^2\left(\Delta B\right)^2}{R^2\left(\Delta t\right)^2}R\Delta t = \frac{A^2\left(\Delta B\right)^2}{R\left(\Delta t\right)} = \frac{\left[\left(0.270\,\text{m}\right)^2\right]^2\left[\left(0-0.755\,\text{T}\right)\right]^2}{\left(7.50\,\Omega\right)\left(0.0400\,\text{s}\right)}$$

$$= \boxed{1.01\times10^{-2}\,\text{J}}$$

60. Because there are perfect transformers, the power loss is due to resistive heating in the transmission lines. Since the town requires 65 MW, the power at the generating plant must be $\frac{65\,\text{MW}}{0.985} = 65.99$ MW. Thus the power lost in the transmission is 0.99 MW. This can be used to determine the current in the transmission lines.

$$P = I^2R \quad\rightarrow\quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{0.99\times10^6\,\text{W}}{2\left(85\,\text{km}\right)0.10\,\Omega/\text{km}}} = 241.3\,\text{A}$$

To produce 65.99 MW of power at 241.3 A requires the following voltage.

$$V = \frac{P}{I} = \frac{65.99\times10^6\,\text{W}}{241.3\,\text{A}} = 2.73\times10^5\,\text{V} \approx \boxed{270\,\text{kV}}$$

61. The charge on the capacitor can be written in terms of the voltage across the battery and the capacitance using Eq. 24-1. When fully charged the voltage across the capacitor will equal the emf of the loop, which we calculate using Eq. 29-2b.

$$Q = CV = C\frac{d\Phi_B}{dt} = CA\frac{dB}{dt} = \left(5.0\times10^{-12}\ \text{F}\right)\left(12\ \text{m}^2\right)\left(10\ \text{T/s}\right) = \boxed{0.60\ \text{nC}}$$

62. (*a*) From the efficiency of the transformer, we have $P_S = 0.85P_P$. Use this to calculate the current in the primary.

$$P_S = 0.85P_P = 0.85I_PV_P \quad\rightarrow\quad I_P = \frac{P_S}{0.85V_P} = \frac{75\,\text{W}}{0.85\left(110\,\text{V}\right)} = 0.8021\,\text{A} \approx \boxed{0.80\,\text{A}}$$

(*b*) The voltage in both the primary and secondary is proportional to the number of turns in the respective coil. The secondary voltage is calculated from the secondary power and resistance since $P = V^2/R$.

$$\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{V_P}{\sqrt{P_SR_S}} = \frac{110\,\text{V}}{\sqrt{\left(75\,\text{W}\right)\left(2.4\,\Omega\right)}} = \boxed{8.2}$$

63. (*a*) The voltage drop across the lines is due to the resistance.

$$V_{out} = V_{in} - IR = 42000\,V - (740\,A)(2)(0.80\,\Omega) = 40816\,V \approx \boxed{41\,kV}$$

(*b*) The power input is given by $P_{in} = IV_{in}$.

$$P_{in} = IV_{in} = (740\,A)(42000\,V) = 3.108\times10^7\,W \approx \boxed{3.1\times10^7\,W}$$

(*c*) The power loss in the lines is due to the current in the resistive wires.

$$P_{loss} = I^2R = (740\,A)^2(1.60\,\Omega) = 8.76\times10^5\,W \approx \boxed{8.8\times10^5\,W}$$

(*d*) The power output is given by $P_{out} = IV_{out}$.

$$P_{out} = IV_{out} = (740\,A)(40816\,V) = 3.020\times10^7\,W \approx \boxed{3.0\times10^7\,W}.$$

This could also be found by subtracting the power lost from the input power.

$$P_{out} = P_{in} - P_{loss} = 3.108\times10^7\,W - 8.76\times10^5\,W = 3.020\times10^7\,W \approx \boxed{3.0\times10^7\,W}$$

64. We find the current in the transmission lines from the power transmitted to the user, and then find the power loss in the lines.

$$P_T = I_LV \rightarrow I_L = \frac{P_T}{V} \qquad P_L = I_L^2R_L = \left(\frac{P_T}{V}\right)^2 R_L = \boxed{\frac{P_T^2R_L}{V^2}}$$

65. (*a*) Because $V_S < V_P$, this is a $\boxed{\text{step-down}}$ transformer.

(*b*) Assuming 100% efficiency, the power in both the primary and secondary is 35 W. Find the current in the secondary from the relationship $P = IV$.

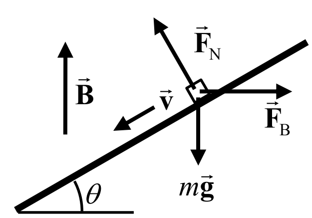
$$P_S = I_SV_S \rightarrow I_S = \frac{P_S}{V_S} = \frac{35\,W}{12\,V} = \boxed{2.9\,A}$$

(*c*) $P_P = I_PV_P \rightarrow I_P = \frac{P_P}{V_P} = \frac{35\,W}{120\,V} = \boxed{0.29\,A}$

(*d*) Find the resistance of the bulb from Ohm's law. The bulb is in the secondary circuit.

$$V_S = I_SR \rightarrow R = \frac{V_S}{I_S} = \frac{12\,V}{2.9\,A} = \boxed{4.1\,\Omega}$$

66. A side view of the rail and bar is shown in the figure. From Section 21-3, the emf in the bar is produced by the components of the magnetic field, the length of the bar, and the velocity of the bar, which are all mutually perpendicular. The magnetic field and the length of the bar are already perpendicular. The component of the velocity of the bar that is perpendicular to the magnetic field is $v\cos\theta$, and so the induced emf is given by the following.




$$\mathcal{E} = B\ell v\cos\theta$$

This produces a current in the wire, which can be found by Ohm's law. That current is pointing into the page on the diagram.

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v\cos\theta}{R}$$

Because the current is perpendicular to the magnetic field, the force on the wire from the magnetic field can be calculated from Eq. 27-2, and will be horizontal, as shown in the diagram.

$$F_B = I\ell B = \frac{B\ell v\cos\theta}{R}\ell B = \frac{B^2\ell^2 v\cos\theta}{R}$$

For the wire to slide down at a steady speed, the net force along the rail must be zero. Write Newton's second law for forces along the rail, with up the rail being positive.

$$F_{\text{net}} = F_B\cos\theta - mg\sin\theta = 0 \quad\rightarrow\quad \frac{B^2\ell^2 v\cos^2\theta}{R} = mg\sin\theta \quad\rightarrow$$

$$v = \frac{Rmg\sin\theta}{B^2\ell^2\cos^2\theta} = \frac{(0.60\,\Omega)(0.040\,\text{kg})(9.80\,\text{m/s}^2)\sin 6.0^\circ}{(0.55\,\text{T})^2(0.32\,\text{m})^2\cos^2 6.0^\circ} = \boxed{0.80\,\text{m/s}}$$

67. The induced current in the coil is the induced emf divided by the resistance. The induced emf is found from the changing flux by Eq. 29-2a. The magnetic field of the solenoid, which causes the flux, is given by Eq. 28-4. For the area used in Eq. 29-2a, the cross-sectional area of the solenoid (not the coil) must be used, because all of the magnetic flux is inside the solenoid.

$$I = \frac{\mathcal{E}_{\text{ind}}}{R} \qquad \left|\mathcal{E}_{\text{ind}}\right| = N_{\text{coil}}\frac{d\Phi}{dt} = N_{\text{coil}}A_{\text{sol}}\frac{dB_{\text{sol}}}{dt} \qquad B_{\text{sol}} = \mu_0\frac{N_{\text{sol}}I_{\text{sol}}}{\ell_{\text{sol}}}$$

$$I = \frac{N_{\text{coil}}A_{\text{sol}}\mu_0\dfrac{N_{\text{sol}}}{\ell_{\text{sol}}}\dfrac{dI_{\text{sol}}}{dt}}{R} = \frac{N_{\text{coil}}A_{\text{sol}}\mu_0}{R}\frac{N_{\text{sol}}}{\ell_{\text{sol}}}\frac{dI_{\text{sol}}}{dt}$$

$$= \frac{(150\,\text{turns})\pi(0.045\,\text{m})^2(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})}{12\,\Omega}\frac{(230\,\text{turns})}{(0.01\,\text{m})}\frac{2.0\,\text{A}}{0.10\,\text{s}} = \boxed{4.6\times10^{-2}\,\text{A}}$$

As the current in the solenoid increases, a magnetic field from right to left is created in the solenoid and the loop. The induced current will flow in such a direction as to oppose that field, and so must flow from [left to right] through the resistor.

68. The average induced emf is given by the "difference" version of Eq. 29-2b. Because the coil orientation changes by 180°, the change in flux is the opposite of twice the initial flux. The average current is the induced emf divided by the resistance, and the charge that flows in a given time is the current times the elapsed time.

$$\mathcal{E}_{\text{avg}} = -N\frac{\Delta\Phi_B}{\Delta t} = -NA\frac{\Delta B}{\Delta t} = -NA\frac{[(-B)-(+B)]}{\Delta t} = \frac{2NAB}{\Delta t}$$

$$Q = I\Delta t = \frac{\mathcal{E}_{\text{avg}}}{R}\Delta t = \frac{\left(\dfrac{2NAB}{\Delta t}\right)}{R}\Delta t = \frac{2NAB}{R} \quad\rightarrow\quad \boxed{B = \frac{RQ}{2NA}}$$

69. Calculate the current in the ring from the magnitude of the emf (from Eq. 29-2a) divided by the resistance. Setting the current equal to the derivative of the charge, we integrate the charge and flux over the 90° rotation, with the flux given by Eq. 29-1a. This results in the total charge flowing past a given point in the ring. Note that the initial orientation of the ring area relative to the magnetic field is not given.

$$\mathscr{E} = \frac{d\Phi_B}{dt} \;;\; I = \frac{\mathscr{E}}{R} = \frac{dQ}{dt} \;\rightarrow\; dQ = \frac{\mathscr{E}}{R}dt \;\rightarrow$$

$$Q = \int dQ = \int \frac{\mathscr{E}}{R}dt = \frac{1}{R}\int \frac{d\Phi}{dt}dt = \frac{1}{R}\int d\Phi = \frac{1}{R}\int_{BA\cos(\theta)}^{BA\cos(\theta+90°)} d\Phi = \frac{BA\left[\cos(\theta+90°)-\cos\theta\right]}{R}$$

$$= \frac{(0.23\,\text{T})\pi(0.030\,\text{m})^2}{0.025\,\Omega}\left[\cos(\theta+90°)-\cos\theta\right] = 0.02601\,\text{C}\left[\cos(\theta+90°)-\cos\theta\right]$$

To find the maximum charge, we set the derivative of the charge with respect to the starting angle, $\theta$, equal to zero to find the extremes. Inserting the maximum angle into our equation, we find the maximum charge passing through the ring. Finally, we divide the maximum charge by the charge of a single electron to obtain the number of electrons passing the point in the ring.

$$\frac{dQ}{d\theta} = 0.02601\text{ C}\left[-\sin(\theta+90°)+\sin\theta\right] = 0.02601\text{ C}\left[-\cos\theta+\sin\theta\right] = 0 \;\rightarrow\; \tan\theta = 1 \;\rightarrow$$

$$\theta = 45° \text{ or } 225°$$

$$Q_{\max} = 0.02601\text{ C}\left[\cos(225°+90°)-\cos 225°\right] = 0.03678\text{ C}$$

$$N_{\max} = \frac{Q_{\max}}{q} = \frac{0.03678\text{ C}}{1.60\times10^{-19}\text{ C/e}} = \boxed{2.3\times10^{17}\,\text{electrons}}$$

70. The coil should have a diameter about equal to the diameter of a standard flashlight D-cell so that it will be simple to hold and use. This would give the coil a radius of about 1.5 cm. As the magnet passes through the coil the field changes direction, so the change in flux for each pass is twice the maximum flux. Let us assume that the magnet is shaken with a frequency of about two shakes per second, so the magnet passes through the coil four times per second. We obtain the number of turns in the coil using Eq. 29-2b.

$$N = \frac{\mathscr{E}}{\Delta\Phi/\Delta t} = \frac{\mathscr{E}\Delta t}{\Delta\Phi} = \frac{\mathscr{E}\Delta t}{2B_0A} = \frac{(3.0\,\text{V})(0.25\,\text{s})}{2(0.050\,\text{T})\pi(0.015\,\text{m})^2} \approx \boxed{11{,}000\text{ turns}}$$

71. (*a*) Since the coils are directly connected to the wheels, the torque provided by the motor (Eq. 27-9) balances the torque caused by the frictional force.

$$NIAB = Fr \rightarrow I = \frac{Fr}{NAB} = \frac{(250\text{ N})(0.29\text{ m})}{270(0.12\text{ m})(0.15\text{ m})(0.60\text{ T})} = 24.86\text{ A} \approx \boxed{25\text{ A}}$$

(*b*) To maintain this speed the power loss due to the friction (Eq. 8-21) must equal the net power provided by the coils. The power provided by the coils is the current through the coils multiplied by the back emf.

$$P = Fv = I\mathscr{E}_{\text{back}} \rightarrow \mathscr{E}_{\text{back}} = \frac{Fv}{I} = \frac{(250\,\text{N})(35\,\text{km/h})}{24.86\,\text{A}}\left(\frac{1000\,\text{m/km}}{3600\text{ s/h}}\right) = 97.76\,\text{V} \approx \boxed{98\,\text{V}}$$

(*c*) The power dissipated in the coils is the difference between the power produced by the coils and the net power provided to the wheels.

$$P_{\text{loss}} = P - P_{\text{net}} = I\mathscr{E} - I\mathscr{E}_{\text{back}} = (24.86\,\text{A})(120\,\text{V} - 97.76\,\text{V}) = 553\,\text{W} \approx \boxed{600\,\text{W}}$$

(*d*) We divide the net power by the total power to determine the percent used to drive the car.

$$\frac{P_{\text{net}}}{P} = \frac{I\mathcal{E}_{\text{back}}}{I\mathcal{E}} = \frac{97.76\,\text{V}}{120\,\text{V}} = 0.8147 \approx \boxed{81\%}$$

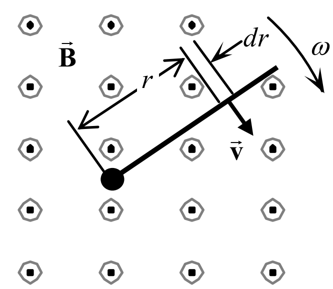
72. The energy is dissipated by the resistance. The power dissipated by the resistor is given by Eq. 25-7b, and the energy is the integral of the power over time. The induced emf is given by Eq. 29-2a.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NA\frac{dB}{dt} = \frac{NAB_0}{\tau}e^{-t/\tau} \;\; ; \;\; P = I^2R = \frac{\mathcal{E}^2}{R} = \left(\frac{N^2A^2B_0^2}{R\tau^2}\right)e^{-2t/\tau}$$

$$E = \int P\,dt = \int_0^t \left(\frac{N^2A^2B_0^2}{R\tau^2}\right)e^{-2t/\tau}dt = \left(\frac{N^2A^2B_0^2}{R\tau^2}\right)\left(-\frac{\tau}{2}e^{-2t/\tau}\right)_0^t = \frac{(NAB_0)^2}{2R\tau}\left(1-e^{-2t/\tau}\right)$$

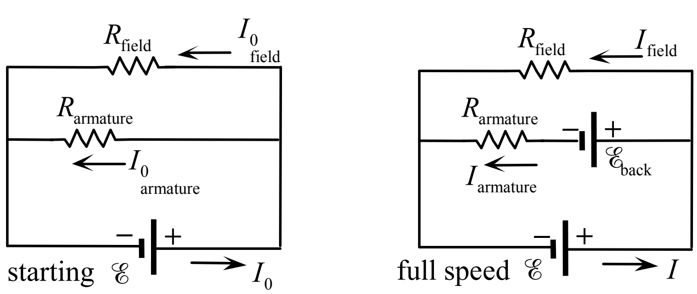
$$= \frac{\left[18\pi(0.100\,\text{m})^2(0.50\,\text{T})\right]^2}{2(2.0\,\Omega)(0.10\,\text{s})}\left(1-e^{-2t/(0.10\,\text{s})}\right) = \boxed{(0.20\,\text{J})\left(1-e^{-20t}\right)}$$

$\boxed{73.}$ The total emf across the rod is the integral of the differential emf across each small segment of the rod. For each differential segment, *dr*, the differential emf is given by the differential version of Eq. 29-3. The velocity is the angular speed multiplied by the radius. The figure is a top view of the spinning rod.




$$d\mathcal{E} = Bv\,d\ell = B\omega r\,dr \;\rightarrow\; \mathcal{E} = \int d\mathcal{E} = \int_0^\ell B\omega r\,dr = \boxed{\tfrac{1}{2}B\omega\ell^2}$$

74. (*a*)



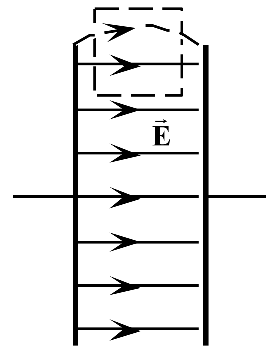

(*b*) At startup there is no back emf. We therefore treat the circuit as two parallel resistors, each with the same voltage drop. The current through the battery is the sum of the currents through each resistor.

$$I = I_{0\,\text{field}} + I_{0\,\text{armature}} = \frac{\mathcal{E}}{R_{\text{field}}} + \frac{\mathcal{E}}{R_{\text{armature}}} = \frac{115\text{ V}}{36.0\ \Omega} + \frac{115\text{ V}}{3.00\ \Omega} = \boxed{41.5\text{ A}}$$

(*c*) At full speed the back emf decreases the voltage drop across the armature resistor.

$$I = I_{\text{field}} + I_{\text{armature}} = \frac{\mathcal{E}}{R_{\text{field}}} + \frac{\mathcal{E}-\mathcal{E}_{\text{back}}}{R_{\text{armature}}} = \frac{115\,\text{V}}{36.0\,\Omega} + \frac{115\,\text{V}-105\,\text{V}}{3.00\,\Omega} = \boxed{6.53\,\text{A}}$$

75. Assume that the electric field does not fringe, but only has a horizontal component between the plates and zero field outside the plates. Apply Faraday's law (Eq. 29-8) to this situation for a rectangular loop with one horizontal leg inside the plates and the second horizontal leg outside the plates. We integrate around this path in the counterclockwise direction. Since the field only has a horizontal component between the plates, only the horizontal leg will contribute to the electric field integral. Since the field is constant in this region, the integral is the electric field times the length of the leg.

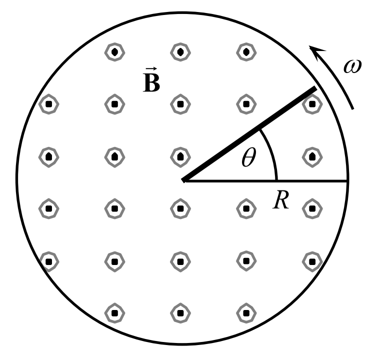



$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = \int_0^{\ell} E\,d\ell = E\ell$$

For a static electric field, the magnetic flux is unchanging. Therefore $\dfrac{d\Phi_B}{dt} = 0.$

Using Faraday's law, we have $\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\dfrac{d\Phi_B}{dt} \;\rightarrow\; E\ell = 0,$ which is not possible. Thus one of the initial assumptions must be false. We conclude that the field must have some fringing at the edges.

76. The total emf across the disk is the integral of the differential emf across each small segment of the radial line passing from the center of the disk to the edge. For each differential segment, $dr$, the emf is given by the differential version of Eq. 29-3. The velocity is the angular speed multiplied by the radius. Since the disk is rotating in the counterclockwise direction, and the field is out of the page, the emf is increasing with increasing radius. Therefore the rim is at the higher potential.




$$d\mathscr{E} = Bv\,d\ell = B\omega r\,dr \;\rightarrow$$

$$\mathscr{E} = \int d\mathscr{E} = \int_0^R B\omega r\,dr = \boxed{\tfrac{1}{2}B\omega R^2}$$

77. We set the electric field equal to the negative gradient of the electric potential (Eq. 23-8), with the differential potential given by Eq. 29-3, as in Problem 76.

$$\vec{\mathbf{E}} = -\frac{d\mathscr{E}}{dr}\hat{\mathbf{r}} = -\frac{Br\omega\,dr}{dr}\hat{\mathbf{r}} = \boxed{-Br\omega\hat{\mathbf{r}}}$$

The electric field has magnitude $Br\omega$ and points radially inwards, toward the center of the disk.

78. The emf around the loop is equal to the time derivative of the flux, as in Eq. 29-2a. Since the area of the coil is constant, the time derivative of the flux is equal to the derivative of the magnetic field multiplied by the area of the loop. To calculate the emf in the loop we add the voltage drop across the capacitor to the voltage drop across the resistor. The current in the loop is the derivative of the charge on the capacitor (Eq. 24-1).

$$I = \frac{dQ}{dt} = \frac{dCV}{dt} = \frac{d}{dt}\left[CV_0\left(1 - e^{-t/\tau}\right)\right] = \frac{CV_0}{\tau}e^{-t/\tau} = \frac{V_0}{R}e^{-t/\tau}$$

$$\mathscr{E} = IR + V_C = \left(\frac{V_0}{R}e^{-t/\tau}\right)R + V_0\left(1 - e^{-t/\tau}\right) = V_0 = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r^2\frac{dB}{dt} \;\rightarrow\; \boxed{\frac{dB}{dt} = \frac{V_0}{\pi r^2}}$$

Since the charge is building up on the top plate of the capacitor, the induced current is flowing clockwise. By Lenz's law this produces a downward flux, so the external downward magnetic field must be $\boxed{\text{decreasing}}$.

79. (*a*) As the loop falls out of the magnetic field, the flux through the loop decreases with time creating an induced emf in the loop. The current in the loop is equal to the emf divided by the resistance, which can be written in terms of the resistivity using Eq. 25-3.

$$I = \frac{\mathcal{E}}{R} = \left(\frac{\pi d^2/4}{\rho 4\ell}\right)\frac{d\Phi_B}{dt} = \left(\frac{\pi d^2}{16\rho\ell}\right)B\frac{dA}{dt} = \frac{\pi d^2}{16\rho\ell}B\ell v$$

This current induces a force on the three sides of the loop in the magnetic field. The forces on the two vertical sides are equal and opposite and therefore cancel.

$$F = I\ell B = \frac{\pi d^2}{16\rho\ell}B\ell v\ell B = \boxed{\frac{\pi d^2 B^2 \ell v}{16\rho}}$$

By Lenz's law this force is upward to slow the decrease in flux.

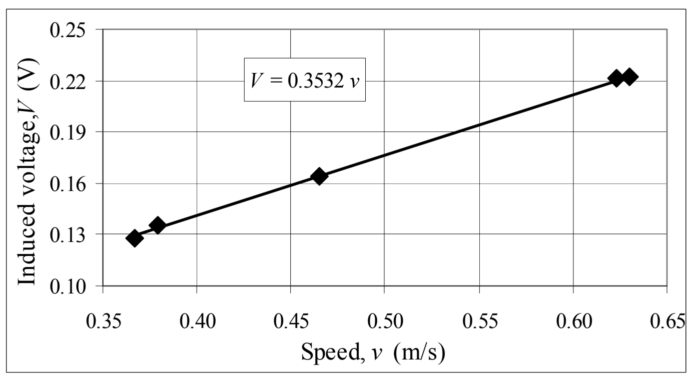
(*b*) Terminal speed will occur when the gravitational force is equal to the magnetic force.

$$F_g = \rho_m\left(4\pi\ell\frac{d^2}{4}\right)g = \frac{\pi d^2 B^2 \ell v_T}{16\rho} \rightarrow v_T = \boxed{\frac{16\rho\rho_m g}{B^2}}$$

(*c*) We calculate the terminal velocity using the given magnetic field, the density of copper from Table 13-1, and the resistivity of copper from Table 25-1

$$v_T = \frac{16\left(8.9\times10^3\ \text{kg/m}^3\right)\left(1.68\times10^{-8}\ \Omega\text{m}\right)\left(9.80\ \text{m/s}^2\right)}{\left(0.80\ \text{T}\right)^2} = \boxed{3.7\ \text{cm/s}}$$

80. (*a*) See the graph, with best fit linear trend line (with the *y* intercept forced to be 0).




(*b*) The theoretical slope is the induced voltage divided by the velocity. Take the difference between the experimental value found in part (*a*) and the theoretical value and divide the result by the theoretical value to obtain the percent difference.

$$\%\ \text{diff} = \left(\frac{m_{\text{exp}} - m_{\text{theory}}}{m_{\text{theory}}}\right)100 = \left(\frac{m_{\text{exp}}}{BN\ell} - 1\right)100 = \left(\frac{0.3532\ \text{V}\bullet\text{s/m}}{(0.126\text{T})(50)(0.0561\text{m})} - 1\right)100$$

$$= \boxed{-0.065\%}$$

(*c*) Use the theoretical equation to calculate the voltage at each experimental speed. Then calculate the percent difference at each speed.

| Speed (m/s) | Induced Voltage (V) | Theoretical Induced Voltage (V) | % diff. |
|---|---|---|---|
| 0.367 | 0.128 | 0.130 | – 1.32% |
| 0.379 | 0.135 | 0.134 | 0.78% |
| 0.465 | 0.164 | 0.164 | – 0.21% |
| 0.623 | 0.221 | 0.220 | 0.37% |
| 0.630 | 0.222 | 0.223 | – 0.30% |

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH29.XLS," on tab "Problem 29.80."

# CHAPTER 30: Inductance, Electromagnetic Oscillations, and AC Circuits

## Responses to Questions

1. (*a*) For the maximum value of the mutual inductance, place the coils close together, face to face, on the same axis.
   (*b*) For the least possible mutual inductance, place the coils with their faces perpendicular to each other.

2. The magnetic field near the end of the first solenoid is less than it is in the center. Therefore the flux through the second coil would be less than that given by the formula, and the mutual inductance would be lower.

3. Yes. If two coils have mutual inductance, then they each have the capacity for self-inductance. Any coil that experiences a changing current will have a self-inductance.

4. The energy density is greater near the center of a solenoid, where the magnetic field is greater.

5. To create the greatest self-inductance, bend the wire into as many loops as possible. To create the least self-inductance, leave the wire as a straight piece of wire.

6. (*a*) No. The time needed for the *LR* circuit to reach a given fraction of its maximum possible current depends on the time constant, $\tau = L/R$, which is independent of the emf.
   (*b*) Yes. The emf determines the maximum value of the current ($I_{max} = V_0/R$,) and therefore will affect the time it takes to reach a particular value of current.

7. A circuit with a large inductive time constant is resistant to changes in the current. When a switch is opened, the inductor continues to force the current to flow. A large charge can build up on the switch, and may be able to ionize a path for itself across a small air gap, creating a spark.

8. Although the current is zero at the instant the battery is connected, the rate at which the current is changing is a maximum and therefore the rate of change of flux through the inductor is a maximum. Since, by Faraday's law, the induced emf depends on the rate of change of flux and not the flux itself, the emf in the inductor is a maximum at this instant.

9. When the capacitor has discharged completely, energy is stored in the magnetic field of the inductor. The inductor will resist a change in the current, so current will continue to flow and will charge the capacitor again, with the opposite polarity.

10. Yes. The instantaneous voltages across the different elements in the circuit will be different, but the current through each element in the series circuit is the same.

11. The energy comes from the generator. (A generator is a device that converts mechanical energy to electrical energy, so ultimately, the energy came from some mechanical source, such as falling water.) Some of the energy is dissipated in the resistor and some is stored in the fields of the capacitor and the inductor. An increase in *R* results in an increase in energy dissipated by the circuit. *L*, *C*, *R*, and the frequency determine the current flow in the circuit, which determines the power supplied by generator.

12. $X_L = X_C$ at the resonant frequency. If the circuit is predominantly inductive, such that $X_L > X_C$, then the frequency is greater than the resonant frequency and the voltage leads the current. If the circuit is predominantly capacitive, such that $X_C > X_L$, then the frequency is lower than the resonant frequency and the current leads the voltage. Values of $L$ and $C$ cannot be meaningfully compared, since they are in different units. Describing the circuit as "inductive" or "capacitive" relates to the values of $X_L$ and $X_C$, which are both in ohms and which both depend on frequency.

13. Yes. When $\omega$ approaches zero, $X_L$ approaches zero, and $X_C$ becomes infinitely large. This is consistent with what happens in an ac circuit connected to a dc power supply. For the dc case, $\omega$ is zero and $X_L$ will be zero because there is no changing current to cause an induced emf. $X_C$ will be infinitely large, because steady direct current cannot flow across a capacitor once it is charged.

14. The impedance in an *LRC* circuit will be a minimum at resonance, when $X_L = X_C$. At resonance, the impedance equals the resistance, so the smallest $R$ possible will give the smallest impedance.

15. Yes. The power output of the generator is $P = IV$. When either the instantaneous current or the instantaneous voltage in the circuit is negative, and the other variable is positive, the instantaneous power output can be negative. At this time either the inductor or the capacitor is discharging power back to the generator.

16. Yes, the power factor depends on frequency because $X_L$ and $X_C$, and therefore the phase angle, depend on frequency. For example, at resonant frequency, $X_L = X_C$, the phase angle is 0º, and the power factor is one. The average power dissipated in an *LRC* circuit also depends on frequency, since it depends on the power factor: $P_{\text{avg}} = I_{\text{rms}} V_{\text{rms}} \cos\varphi$. Maximum power is dissipated at the resonant frequency. The value of the power factor decreases as the frequency gets farther from the resonant frequency.

17. (*a*) The impedance of a pure resistance is unaffected by the frequency of the source emf.
(*b*) The impedance of a pure capacitance decreases with increasing frequency.
(*c*) The impedance of a pure inductance increases with increasing frequency.
(*d*) In an *LRC* circuit near resonance, small changes in the frequency will cause large changes in the impedance.
(*e*) For frequencies far above the resonance frequency, the impedance of the *LRC* circuit is dominated by the inductive reactance and will increase with increasing frequency. For frequencies far below the resonance frequency, the impedance of the *LRC* circuit is dominated by the capacitive reactance and will decrease with increasing frequency.

18. In all three cases, the energy dissipated decreases as $R$ approaches zero. Energy oscillates between being stored in the field of the capacitor and being stored in the field of the inductor.
(*a*) The energy stored in the fields (and oscillating between them) is a maximum at resonant frequency and approaches an infinite value as $R$ approaches zero.
(*b*) When the frequency is near resonance, a large amount of energy is stored in the fields but the value is less than the maximum value.
(*c*) Far from resonance, a much lower amount of energy is stored in the fields.

19. In an *LRC* circuit, the current and the voltage in the circuit both oscillate. The energy stored in the circuit also oscillates and is alternately stored in the magnetic field of the inductor and the electric field of the capacitor.

20. In an *LRC* circuit, energy oscillates between being stored in the magnetic field of the inductor and being stored in the electric field of the capacitor. This is analogous to a mass on a spring, with energy alternating between kinetic energy of the mass and spring potential energy as the spring compresses and extends. The energy stored in the magnetic field is analogous to the kinetic energy of the moving mass, and *L* corresponds to the mass, *m*, on the spring. The energy stored in the electric field of the capacitor is analogous to the spring potential energy, and *C* corresponds to the reciprocal of the spring constant, 1/*k*.

## Solutions to Problems

1. (*a*) The mutual inductance is found in Example 30-1.

$$M = \frac{\mu N_1 N_2 A}{\ell} = \frac{1850\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(225)(115)\pi(0.0200\,\text{m})^2}{2.44\,\text{m}} = \boxed{3.10\times10^{-2}\,\text{H}}$$

(b) The emf induced in the second coil can be found from Eq. 30-3b.

$$\mathscr{E}_2 = -M\frac{dI_1}{dt} = -M\frac{\Delta I_1}{\Delta t} = \left(-3.10\times10^{-2}\,\text{H}\right)\frac{(-12.0\,\text{A})}{0.0980\,\text{ms}} = \boxed{3.79\,\text{V}}$$

2. If we assume the outer solenoid is carrying current $I_1$, then the magnetic field inside the outer solenoid is $B = \mu_0 n_1 I_1$. The flux in each turn of the inner solenoid is $\Phi_{21} = B\pi r_2^2 = \mu_0 n_1 I_1 \pi r_2^2$. The mutual inductance is given by Eq. 30-1.

$$M = \frac{N_2\Phi_{21}}{I_1} = \frac{n_2\ell\mu_0 n_1 I_1 \pi r_2^2}{I_1} \quad\rightarrow\quad \frac{M}{\ell} = \boxed{\mu_0 n_1 n_2 \pi r_2^2}$$

3. We find the mutual inductance of the inner loop. If we assume the outer solenoid is carrying current $I_1$, then the magnetic field inside the outer solenoid is $B = \mu_0 \frac{N_1}{\ell} I_1$. The magnetic flux through each loop of the small coil is the magnetic field times the area perpendicular to the field. The mutual inductance is given by Eq. 30-1.

$$\Phi_{21} = BA_2\sin\theta = \mu_0\frac{N_1 I_1}{\ell}A_2\sin\theta \;;\; M = \frac{N_2\Phi_{21}}{I_1} = \frac{N_2\mu_0\frac{N_1 I_1}{\ell}A_2\sin\theta}{I_1} = \boxed{\frac{\mu_0 N_1 N_2 A_2\sin\theta}{\ell}}$$

4. We find the mutual inductance of the system using Eq. 30-1, with the flux equal to the integral of the magnetic field of the wire (Eq. 28-1) over the area of the loop.

$$M = \frac{\Phi_{12}}{I_1} = \frac{1}{I_1}\int_{\ell_1}^{\ell_2}\frac{\mu_0 I_1}{2\pi r}w\,dr = \boxed{\frac{\mu_0 w}{2\pi}\ln\left(\frac{\ell_2}{\ell_1}\right)}$$

5. Find the induced emf from Eq. 30-5.

$$\mathscr{E} = -L\frac{dI}{dt} = -L\frac{\Delta I}{\Delta t} = -(0.28\,\text{H})\frac{(10.0\,\text{A}-25.0\,\text{A})}{0.36\,\text{s}} = \boxed{12\,\text{V}}$$

6. Use the relationship for the inductance of a solenoid, as given in Example 30-3.

$$L=\frac{\mu_0 N^2 A}{\ell} \quad\rightarrow\quad N=\sqrt{\frac{L\ell}{\mu_0 A}}=\sqrt{\frac{(0.13\,\text{H})(0.300\,\text{m})}{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})\pi(0.021\,\text{m})^2}}\approx\boxed{4700\,\text{turns}}$$

7. Because the current in increasing, the emf is negative. We find the self-inductance from Eq. 30-5.

$$\mathscr{E}=-L\frac{dI}{dt}=-L\frac{\Delta I}{\Delta t} \quad\rightarrow\quad L=-\mathscr{E}\frac{\Delta t}{\Delta I}=-(-2.50\,\text{V})\frac{0.0120\,\text{s}}{[0.0250\,\text{A}-(-0.0280\,\text{A})]}=\boxed{0.566\,\text{H}}$$

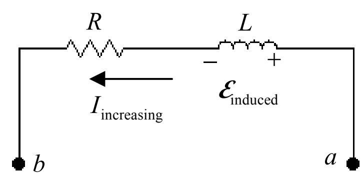
8. (*a*) The number of turns can be found from the inductance of a solenoid, which is derived in Example 30-3.

$$L=\frac{\mu_0 N^2 A}{\ell}=\frac{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})(2800)^2\pi(0.0125\,\text{m})^2}{(0.217\,\text{m})}=0.02229\,\text{H}\approx\boxed{0.022\,\text{H}}$$

(*b*) Apply the same equation again, solving for the number of turns.

$$L=\frac{\mu_0 N^2 A}{\ell} \quad\rightarrow\quad N=\sqrt{\frac{L\ell}{\mu_0 A}}=\sqrt{\frac{(0.02229\,\text{H})(0.217\,\text{m})}{(1200)(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})\pi(0.0125\,\text{m})^2}}\approx\boxed{81\,\text{turns}}$$

9. We draw the coil as two elements in series, and pure resistance and a pure inductance. There is a voltage drop due to the resistance of the coil, given by Ohm's law, and an induced emf due to the inductance of the coil, given by Eq. 30-5. Since the current is increasing, the inductance will create a potential difference to oppose the increasing current, and so there is a drop in the potential due to the inductance. The potential difference across the coil is the sum of the two potential drops.




$$V_{ab}=IR+L\frac{dI}{dt}=(3.00\,\text{A})(3.25\,\Omega)+(0.44\,\text{H})(3.60\,\text{A/s})=\boxed{11.3\,\text{V}}$$

10. We use the result for inductance per unit length from Example 30-5.

$$\frac{L}{\ell}=\frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}\le 55\times10^{-9}\,\text{H/m} \quad\rightarrow\quad r_1\ge r_2 e^{-\frac{2\pi}{\mu_0}(55\times10^{-9}\,\text{H/m})}=(0.0030\,\text{m})e^{-\frac{2\pi(55\times10^{-9}\,\text{H/m})}{(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A})}}=0.00228\,\text{m}$$

$$\boxed{r_1\ge 0.0023\,\text{m}}$$

11. The self-inductance of an air-filled solenoid was determined in Example 30-3. We solve this equation for the length of the tube, using the diameter of the wire as the length per turn.

$$L=\frac{\mu_0 N^2 A}{\ell}=\mu_0 n^2 A\ell=\frac{\mu_0 A\ell}{d^2}$$

$$\ell=\frac{Ld^2}{\mu_0\pi r^2}=\frac{(1.0\text{H})(0.81\times10^{-3}\text{m})^2}{(4\pi\times10^{-7}\text{T}\bullet\text{m/A})\pi(0.060\,\text{m})^2}=46.16\,\text{m}\approx\boxed{46\,\text{m}}$$

The length of the wire is equal to the number of turns (the length of the solenoid divided by the diameter of the wire) multiplied by the circumference of the turn.

$$L=\frac{\ell}{d}\pi D=\frac{46.16\,\text{m}}{0.81\times10^{-3}\text{m}}\pi(0.12\,\text{m})=21{,}490\,\text{m}\approx\boxed{21\text{km}}$$

The resistance is calculated from the resistivity, area, and length of the wire.

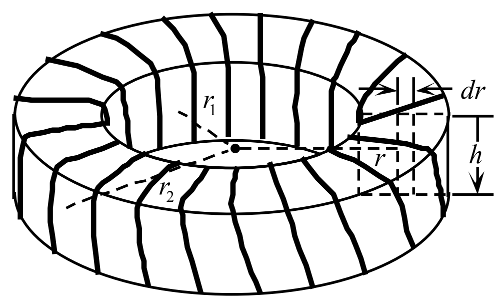
$$R=\frac{\rho\ell}{A}=\frac{\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)\left(21{,}490\,\text{m}\right)}{\pi\left(0.405\times10^{-3}\,\text{m}\right)^2}=\boxed{0.70\,\text{k}\Omega}$$

12. The inductance of the solenoid is given by $L=\frac{\mu_0 N^2 A}{\ell}=\frac{\mu_0 N^2}{\ell}\frac{\pi d^2}{4}$. The (constant) length of the wire is given by $\ell_{\text{wire}}=N\pi d_{\text{sol}}$, and so since $d_{\text{sol 2}}=2.5\,d_{\text{sol 1}}$, we also know that $N_1=2.5N_2$. The fact that the wire is tightly wound gives $\ell_{\text{sol}}=Nd_{\text{wire}}$. Find the ratio of the two inductances.

$$\frac{L_2}{L_1}=\frac{\frac{\mu_0\pi}{4}\frac{N_2^2}{\ell_{\text{sol 2}}}d_{\text{sol 2}}^2}{\frac{\mu_0\pi}{4}\frac{N_1^2}{\ell_{\text{sol 1}}}d_{\text{sol 1}}^2}=\frac{\frac{N_2^2}{\ell_{\text{sol 2}}}d_{\text{sol 2}}^2}{\frac{N_1^2}{\ell_{\text{sol 1}}}d_{\text{sol 1}}^2}=\frac{\frac{\ell_{\text{wire}}^2/\pi^2}{\ell_{\text{sol 2}}}}{\frac{\ell_{\text{wire}}^2/\pi^2}{\ell_{\text{sol 1}}}}=\frac{\ell_{\text{sol 1}}}{\ell_{\text{sol 2}}}=\frac{N_1 d_{\text{wire}}}{N_2 d_{\text{wire}}}=\frac{N_1}{N_2}=\boxed{2.5}$$

13. We use Eq. 30-4 to calculate the self-inductance, where the flux is the integral of the magnetic field over a cross-section of the toroid. The magnetic field inside the toroid was calculated in Example 28-10.

$$L=\frac{N}{I}\Phi_B=\frac{N}{I}\int_{r_1}^{r_2}\frac{\mu_0 NI}{2\pi r}h\,dr=\boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{r_2}{r_1}\right)}$$




14. (*a*) When connected in series the voltage drops across each inductor will add, while the currents in each inductor are the same.

$$\mathscr{E}=\mathscr{E}_1+\mathscr{E}_2=-L_1\frac{dI}{dt}-L_2\frac{dI}{dt}=-\left(L_1+L_2\right)\frac{dI}{dt}=-L_{\text{eq}}\frac{dI}{dt}\;\rightarrow\;\boxed{L_{\text{eq}}=L_1+L_2}$$

(*b*) When connected in parallel the currents in each inductor add to the equivalent current, while the voltage drop across each inductor is the same as the equivalent voltage drop.

$$\frac{dI}{dt}=\frac{dI_1}{dt}+\frac{dI_2}{dt}\;\rightarrow\;\frac{\mathscr{E}}{L_{\text{eq}}}=\frac{\mathscr{E}}{L_1}+\frac{\mathscr{E}}{L_2}\;\rightarrow\;\boxed{\frac{1}{L_{\text{eq}}}=\frac{1}{L_1}+\frac{1}{L_2}}$$

Therefore, inductors in series and parallel add the same as resistors in series and parallel.

15. The magnetic energy in the field is derived from Eq. 30-7.

$$u=\frac{\text{Energy stored}}{\text{Volume}}=\frac{1}{2}\frac{B^2}{\mu_0}\;\rightarrow$$

$$\text{Energy}=\frac{1}{2}\frac{B^2}{\mu_0}\left(\text{Volume}\right)=\frac{1}{2}\frac{B^2}{\mu_0}\pi r^2\ell=\frac{1}{2}\frac{\left(0.600\,\text{T}\right)^2}{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}\pi\left(0.0105\,\text{m}\right)^2\left(0.380\,\text{m}\right)=\boxed{18.9\,\text{J}}$$

16. (*a*) We use Eq. 24-6 to calculate the energy density in an electric field and Eq. 30-7 to calculate the energy density in the magnetic field.

$$u_E=\frac{1}{2}\varepsilon_0E^2=\frac{1}{2}\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)\left(1.0\times10^4\ \text{N/C}\right)^2=\boxed{4.4\times10^{-4}\ \text{J/m}^3}$$

$$u_B = \frac{B^2}{2\mu_0} = \frac{(2.0\text{ T})^2}{2(4\pi \times 10^{-7}\text{ T}\bullet\text{m/A})} = 1.592 \times 10^6\text{ J/m}^3 \approx \boxed{1.6 \times 10^6\text{ J/m}^3}$$

(*b*) Use Eq. 24-6 to calculate the electric field from the energy density for the magnetic field given in part (*a*).

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = u_B \quad \rightarrow \quad E = \sqrt{\frac{2u_B}{\varepsilon_0}} = \sqrt{\frac{2(1.592\times10^6\text{ J/m}^3)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\bullet\text{m}^2)}} = \boxed{6.0\times10^8\text{ N/C}}$$

17. We use Eq. 30-7 to calculate the energy density with the magnetic field calculated in Example 28-12.

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2R}\right)^2 = \frac{\mu_0 I^2}{8R^2} = \frac{(4\pi\times10^{-7}\text{ T}\bullet\text{m/A})(23.0\text{ A})^2}{8(0.280\text{ m})^2} = \boxed{1.06\times10^{-3}\text{ J/m}^3}$$

18. We use Eq. 30-7 to calculate the magnetic energy density, with the magnetic field calculated using Eq. 28-1.

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi R}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 R^2} = \frac{(4\pi\times10^{-7}\text{ T}\bullet\text{m/A})(15\text{ A})^2}{8\pi^2(1.5\times10^{-3}\text{ m})^2} = \boxed{1.6\text{ J/m}^3}$$

To calculate the electric energy density with Eq. 24-6, we must first calculate the electric field at the surface of the wire. The electric field will equal the voltage difference along the wire divided by the length of the wire. We can calculate the voltage drop using Ohm's law and the resistance from the resistivity and diameter of the wire.

$$E = \frac{V}{\ell} = \frac{IR}{\ell} = \frac{I\rho\ell}{\ell\pi r^2} = \frac{I\rho}{\pi r^2}$$

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0\left(\frac{I\rho}{\pi r^2}\right)^2 = \tfrac{1}{2}(8.85\times10^{-12}\text{C}^2/\text{N}\bullet\text{m}^2)\left[\frac{(15\text{A})(1.68\times10^{-8}\Omega\bullet\text{m})}{\pi(1.5\times10^{-3}\text{m})^2}\right]^2$$

$$= \boxed{5.6\times10^{-15}\text{J/m}^3}$$

19. We use Eq. 30-7 to calculate the energy density in the toroid, with the magnetic field calculated in Example 28-10. We integrate the energy density over the volume of the toroid to obtain the total energy stored in the toroid. Since the energy density is a function of radius only, we treat the toroid as cylindrical shells each with differential volume $dV = 2\pi rhdr$.

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 NI}{2\pi r}\right)^2 = \boxed{\frac{\mu_0 N^2 I^2}{8\pi^2 r^2}}$$

$$U = \int u_B dV = \int_{r_1}^{r_2}\frac{\mu_0 N^2 I^2}{8\pi^2 r^2}2\pi rhdr = \frac{\mu_0 N^2 I^2 h}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \boxed{\frac{\mu_0 N^2 I^2 h}{4\pi}\ln\left(\frac{r_2}{r_1}\right)}$$

20. The magnetic field between the cables is given in Example 30-5. Since the magnetic field only depends on radius, we use Eq. 30-7 for the energy density in the differential volume $dV = 2\pi r\ell dr$ and integrate over the radius between the two cables.

$$\frac{U}{\ell} = \frac{1}{\ell}\int u_B dV = \int_{r_1}^{r_2}\frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 2\pi rdr = \frac{\mu_0 I^2}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \boxed{\frac{\mu_0 I^2}{4\pi}\ln\left(\frac{r_2}{r_1}\right)}$$

21. We create an Amperean loop of radius *r* to calculate the magnetic field within the wire using Eq. 28-3. Since the resulting magnetic field only depends on radius, we use Eq. 30-7 for the energy density in the differential volume $dV = 2\pi r\ell dr$ and integrate from zero to the radius of the wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{enc} \rightarrow B(2\pi r) = \mu_0 \left(\frac{I}{\pi R^2}\right)(\pi r^2) \rightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

$$\frac{U}{\ell} = \frac{1}{\ell}\int u_B dV = \int_0^R \frac{1}{2\mu_0}\left(\frac{\mu_0 I r}{2\pi R^2}\right)^2 2\pi r dr = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3 dr = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

22. For an *LR* circuit, we have $I = I_{\max}\left(1-e^{-t/\tau}\right)$. Solve for $t$.

$$I = I_{\max}\left(1-e^{-t/\tau}\right) \rightarrow e^{-t/\tau} = 1-\frac{I}{I_{\max}} \rightarrow t = -\tau \ln\left(1-\frac{I}{I_{\max}}\right)$$

(*a*) $I = 0.95 I_{\max} \rightarrow t = -\tau \ln\left(1-\frac{I}{I_{\max}}\right) = -\tau \ln(1-0.95) = \boxed{3.0\,\tau}$

(*b*) $I = 0.990 I_{\max} \rightarrow t = -\tau \ln\left(1-\frac{I}{I_{\max}}\right) = -\tau \ln(1-0.990) = \boxed{4.6\,\tau}$

(*c*) $I = 0.9990 I_{\max} \rightarrow t = -\tau \ln\left(1-\frac{I}{I_{\max}}\right) = -\tau \ln(1-0.9990) = \boxed{6.9\,\tau}$

23. We set the current in Eq. 30-11 equal to $0.03I_0$ and solve for the time.

$$I = 0.03 I_0 = I_0 e^{-t/\tau} \rightarrow t = -\tau \ln(0.03) \approx \boxed{3.5\tau}$$

24. (*a*) We set *I* equal to 75% of the maximum value in Eq. 30-9 and solve for the time constant.

$$I = 0.75 I_0 = I_0\left(1-e^{-t/\tau}\right) \rightarrow \tau = -\frac{t}{\ln(0.25)} = -\frac{(2.56\text{ ms})}{\ln(0.25)} = 1.847\text{ ms} \approx \boxed{1.85\text{ ms}}$$

(*b*) The resistance can be calculated from the time constant using Eq. 30-10.

$$R = \frac{L}{\tau} = \frac{31.0\text{ mH}}{1.847\text{ ms}} = \boxed{16.8\ \Omega}$$

25. (*a*) We use Eq. 30-6 to determine the energy stored in the inductor, with the current given by Eq. Eq 30-9.

$$U = \tfrac{1}{2}LI^2 = \boxed{\frac{LV_0^2}{2R^2}\left(1-e^{-t/\tau}\right)^2}$$

(*b*) Set the energy from part (*a*) equal to 99.9% of its maximum value and solve for the time.

$$U = 0.999\frac{V_0^2}{2R^2} = \frac{V_0^2}{2R^2}\left(1-e^{-t/\tau}\right)^2 \rightarrow t = \tau \ln\left(1-\sqrt{0.999}\right) \approx \boxed{7.6\tau}$$

26. (*a*) At the moment the switch is closed, no current will flow through the inductor. Therefore, the resistors $R_1$ and $R_2$ can be treated as in series.

$$\mathscr{E} = I(R_1+R_2) \rightarrow \boxed{I_1 = I_2 = \frac{\mathscr{E}}{R_1+R_2},\ I_3 = 0}$$

(*b*) A long time after the switch is closed, there is no voltage drop across the inductor so resistors $R_2$ and $R_3$ can be treated as parallel resistors in series with $R_1$.

$$I_1 = I_2 + I_3, \quad \mathscr{E} = I_1 R_1 + I_2 R_2, \quad I_2 R_2 = I_3 R_3$$

$$\frac{\mathscr{E} - I_2 R_2}{R_1} = I_2 + \frac{I_2 R_2}{R_3} \rightarrow \boxed{I_2 = \frac{\mathscr{E} R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}}$$

$$I_3 = \frac{I_2 R_2}{R_3} = \boxed{\frac{\mathscr{E} R_2}{R_2 R_3 + R_1 R_3 + R_1 R_2}} \qquad I_1 = I_2 + I_3 = \boxed{\frac{\mathscr{E}(R_3 + R_2)}{R_2 R_3 + R_1 R_3 + R_1 R_2}}$$

(*c*) Just after the switch is opened the current through the inductor continues with the same magnitude and direction. With the open switch, no current can flow through the branch with the switch. Therefore the current through $R_2$ must be equal to the current through $R_3$, but in the opposite direction.

$$I_3 = \boxed{\frac{\mathscr{E} R_2}{R_2 R_3 + R_1 R_3 + R_1 R_2}}, \quad I_2 = \boxed{\frac{-\mathscr{E} R_2}{R_2 R_3 + R_1 R_3 + R_1 R_2}}, \quad I_1 = \boxed{0}$$

(*d*) After a long time, with no voltage source, the energy in the inductor will dissipate and no current will flow through any of the branches.

$$I_1 = I_2 = I_3 = \boxed{0}$$

27. (*a*) We use Eq. 30-5 to determine the emf in the inductor as a function of time. Since the exponential term decreases in time, the maximum emf occurs when $t = 0$.

$$\mathscr{E} = -L\frac{dI}{dt} = -L\frac{d}{dt}\left[I_0 e^{-tR/L}\right] = \frac{LI_0 R}{L} e^{-t/\tau} = V_0 e^{-t/\tau} \rightarrow \boxed{\mathscr{E}_{max} = V_0}.$$

(*b*) The current is the same just before and just after the switch moves from A to B. We use Ohm's law for a steady state current to determine $I_0$ before the switch is thrown. After the switch is thrown, the same current flows through the inductor, and therefore that current will flow through the resistor *R'*. Using Kirchhoff's loop rule we calculate the emf in the inductor. This will be a maximum at $t = 0$.

$$I_0 = \frac{V_0}{R}, \qquad \mathscr{E} - IR' = 0 \rightarrow \mathscr{E} = R'\frac{V_0}{R} e^{-t/\tau'} \rightarrow \mathscr{E}_{max} = \left(\frac{R'}{R}\right) V_0 = \left(\frac{55R}{R}\right)(120\,\text{V}) = \boxed{6.6\,\text{kV}}$$

28. The steady state current is the voltage divided by the resistance while the time constant is the inductance divided by the resistance, Eq. 30-10. To cut the time constant in half, we must double the resistance. If the resistance is doubled, we must double the voltage to keep the steady state current constant.

$$R' = 2R = 2(2200\ \Omega) = \boxed{4400\ \Omega} \qquad V_0' = 2V_0 = 2(240\ \text{V}) = \boxed{480\ \text{V}}$$

29. We use Kirchhoff's loop rule in the steady state (no voltage drop across the inductor) to determine the current in the circuit just before the battery is removed. This will be the maximum current after the battery is removed. Again using Kirchhoff's loop rule, with the current given by Eq. 30-11, we calculate the emf as a function of time.

$$V - I_0 R = 0 \quad \rightarrow \quad I_0 = \frac{V}{R}$$

$$\mathscr{E} - IR = 0 \quad \rightarrow \quad \mathscr{E} = I_0 R e^{-t/\tau} = V\,\text{e}^{-tR/L} = (12\,\text{V})e^{-t(2.2\,\text{k}\Omega)/(18\,\text{mH})} = \boxed{(12\,\text{V})e^{-\left(1.22\times10^5\,\text{s}^{-1}\right)t}}$$

The emf across the inductor is greatest at $\boxed{t = 0}$ with a value of $\boxed{\mathscr{E}_{max} = 12\,\text{V}}$.

30. We use the inductance of a solenoid, as derived in Example 30-3: $L_{\text{sol}} = \dfrac{\mu_0 N^2 A}{\ell}$.

(*a*) Both solenoids have the same area and the same length. Because the wire in solenoid 1 is 1.5 times as thick as the wire in solenoid 2, solenoid 2 will have 1.5 times the number of turns as solenoid 1.

$$\frac{L_2}{L_1} = \frac{\dfrac{\mu_0 N_2^2 A}{\ell}}{\dfrac{\mu_0 N_1^2 A}{\ell}} = \frac{N_2^2}{N_1^2} = \left(\frac{N_2}{N_1}\right)^2 = 1.5^2 = 2.25 \rightarrow \boxed{\frac{L_2}{L_1} = 2.25}$$

(*b*) To find the ratio of the time constants, both the inductance and resistance ratios need to be known. Since solenoid 2 has 1.5 times the number of turns as solenoid 1, the length of wire used to make solenoid 2 is 1.5 times that used to make solenoid 1, or $\ell_{\text{wire 2}} = 1.5\ell_{\text{wire 1}}$, and the diameter of the wire in solenoid 1 is 1.5 times that in solenoid 2, or $d_{\text{wire 1}} = 1.5 d_{\text{wire 2}}$. Use this to find their relative resistances, and then the ratio of time constants.

$$\frac{R_1}{R_2} = \frac{\dfrac{\rho\ell_{\text{wire 1}}}{A_{\text{wire 1}}}}{\dfrac{\rho\ell_{\text{wire 2}}}{A_{\text{wire 2}}}} = \frac{\dfrac{\ell_{\text{wire 1}}}{\pi\left(d_{\text{wire 1}}/2\right)^2}}{\dfrac{\ell_{\text{wire 2}}}{\pi\left(d_{\text{wire 2}}/2\right)^2}} = \frac{\dfrac{\ell_{\text{wire 1}}}{d_{\text{wire 1}}^2}}{\dfrac{\ell_{\text{wire 2}}}{d_{\text{wire 2}}^2}} = \frac{\ell_{\text{wire 1}}}{\ell_{\text{wire 2}}}\left(\frac{d_{\text{wire 2}}}{d_{\text{wire 1}}}\right)^2 = \left(\frac{1}{1.5}\right)\left(\frac{1}{1.5}\right)^2 = \frac{1}{1.5^3} \rightarrow$$

$$\frac{R_1}{R_2} = \frac{1}{1.5^3} \; ; \; \frac{\tau_1}{\tau_2} = \frac{L_1/R_1}{L_2/R_2} = \frac{L_1}{L_2}\frac{R_2}{R_1} = \left(\frac{1}{2.25}\right)\left(1.5^3\right) = 1.5 \rightarrow \boxed{\frac{\tau_1}{\tau_2} = 1.5}$$

31. (*a*) The AM station received by the radio is the resonant frequency, given by Eq. 30-14. We divide the resonant frequencies to create an equation relating the frequencies and capacitances. We then solve this equation for the new capacitance.

$$\frac{f_1}{f_2} = \frac{\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC_1}}}{\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC_2}}} = \sqrt{\frac{C_2}{C_1}} \rightarrow C_2 = C_1\left(\frac{f_1}{f_2}\right)^2 = (1350\,\text{pF})\left(\frac{550\,\text{kHz}}{1600\,\text{kHz}}\right)^2 = \boxed{0.16\,\text{nF}}$$

(*b*) The inductance is obtained from Eq. 30-14.

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC_1}} \rightarrow L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2\left(550\times10^3\,\text{Hz}\right)^2\left(1350\times10^{-12}\,\text{F}\right)} = \boxed{62\,\mu\text{H}}$$

32. (*a*) To have maximum current and no charge at the initial time, we set $t = 0$ in Eqs. 30-13 and 30-15 to solve for the necessary phase factor $\phi$.

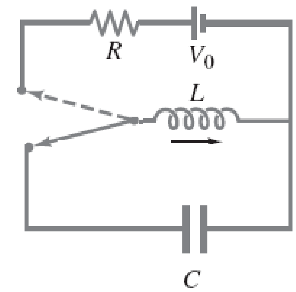
$$I_0 = I_0 \sin\phi \rightarrow \phi = \frac{\pi}{2} \rightarrow I(t) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \cos\omega t$$

$$Q(0) = Q_0 \cos\left(\frac{\pi}{2}\right) = 0 \rightarrow Q = Q_0 \cos\left(\omega t + \frac{\pi}{2}\right) = -Q_0 \sin(\omega t)$$

Differentiating the charge with respect to time gives the negative of the current. We use this to write the charge in terms of the known maximum current.

$$I = -\frac{dQ}{dt} = -Q_0\omega\cos(\omega t) = I_0\cos(\omega t) \rightarrow Q_0 = \frac{I_0}{\omega} \rightarrow \boxed{Q(t) = \frac{I_0}{\omega}\sin(\omega t)}$$

(*b*) As in the figure, attach the inductor to a battery and resistor for an extended period so that a steady state current flows through the inductor. Then at time $t = 0$, flip the switch connecting the inductor in series to the capacitor.




33. (*a*) We write the oscillation frequency in terms of the capacitance using Eq. 30-14, with the parallel plate capacitance given by Eq. 24-2. We then solve the resulting equation for the plate separation distance.

$$2\pi f = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{L\left(\varepsilon_0 A/x\right)}} \rightarrow x = \boxed{4\pi^2 A\varepsilon_0 f^2 L}$$

(*b*) For small variations we can differentiate *x* and divide the result by *x* to determine the fractional change.

$$dx = 4\pi^2 A\varepsilon_0\left(2fdf\right)L \;\;;\;\; \frac{dx}{x} = \frac{4\pi^2 A\varepsilon_0\left(2fdf\right)L}{4\pi^2 A\varepsilon_0 f^2 L} = \frac{2df}{f} \rightarrow \boxed{\frac{\Delta x}{x} \approx \frac{2\Delta f}{f}}$$

(*c*) Inserting the given data, we can calculate the fractional variation on *x*.

$$\frac{\Delta x}{x} \approx \frac{2\left(1\text{ Hz}\right)}{1\text{ MHz}} = 2\times 10^{-6} = \boxed{0.0002\%}$$

34. (*a*) We calculate the resonant frequency using Eq. 30-14.

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{\left(0.175\text{ H}\right)\left(425\times 10^{-12}\text{ F}\right)}} = 18,450\text{ Hz} \approx \boxed{18.5\text{ kHz}}$$

(*b*) As shown in Eq. 30-15, we set the peak current equal to the maximum charge (from Eq. 24-1) multiplied by the angular frequency.

$$I = Q_0\omega = CV\left(2\pi f\right) = \left(425\times 10^{-12}\text{ F}\right)\left(135\text{ V}\right)\left(2\pi\right)\left(18,450\text{ Hz}\right)$$

$$= 6.653\times 10^{-3}\text{ A} \approx \boxed{6.65\text{ mA}}$$

(*c*) We use Eq. 30-6 to calculate the maximum energy stored in the inductor.

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}\left(0.175\text{ H}\right)\left(6.653\times 10^{-3}\text{ A}\right)^2 = \boxed{3.87\;\mu\text{J}}$$

35. (*a*) When the energy is equally shared between the capacitor and inductor, the energy stored in the capacitor will be one half of the initial energy in the capacitor. We use Eq. 24-5 to write the energy in terms of the charge on the capacitor and solve for the charge when the energy is equally shared.

$$\frac{Q^2}{2C} = \frac{1}{2}\frac{Q_0^2}{2C} \rightarrow Q = \boxed{\frac{\sqrt{2}}{2}Q_0}$$

(*b*) We insert the charge into Eq. 30-13 and solve for the time.

$$\frac{\sqrt{2}}{2}Q_0 = Q_0\cos\omega t \rightarrow t = \frac{1}{\omega}\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{T}{2\pi}\left(\frac{\pi}{4}\right) = \boxed{\frac{T}{8}}$$

36. Since the circuit loses 3.5% of its energy per cycle, it is an underdamped oscillation. We use Eq. 24-5 for the energy with the charge as a function of time given by Eq. 30-19. Setting the change in energy equal to 3.5% and using Eq. 30-18 to determine the period, we solve for the resistance.

$$\frac{\Delta E}{E}=\frac{\dfrac{Q_0^2e^{-\frac{R}{L}T}\cos^2(2\pi)}{2C}-\dfrac{Q_0^2\cos^2(0)}{2C}}{\dfrac{Q_0^2\cos^2(0)}{2C}}=e^{-\frac{R}{L}T}-1=-0.035\rightarrow\frac{RT}{L}=\ln(1-0.035)=0.03563$$

$$0.03563=\frac{R}{L}\left(\frac{2\pi}{\omega'}\right)=\frac{R}{L}\frac{2\pi}{\sqrt{1/LC-R^2/4L^2}}\rightarrow R=\sqrt{\frac{4L(0.03563)^2}{C\left[16\pi^2+(0.03563)^2\right]}}$$

$$R=\sqrt{\frac{4(0.065\,\text{H})(0.03563)^2}{(1.00\times10^{-6}\,\text{F})\left[16\pi^2+(0.03563)^2\right]}}=1.4457\,\Omega\approx\boxed{1.4\,\Omega}$$

37. As in the derivation of 30-16, we set the total energy equal to the sum of the magnetic and electric energies, with the charge given by Eq. 30-19. We then solve for the time that the energy is 75% of the initial energy.

$$U=U_E+U_B=\frac{Q^2}{2C}+\frac{LI^2}{2}=\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}\cos^2(\omega't+\phi)+\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}\sin^2(\omega't+\phi)=\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}$$

$$0.75\frac{Q_0^2}{2C}=\frac{Q_0^2}{2C}e^{-\frac{R}{L}t}\rightarrow t=-\frac{L}{R}\ln(0.75)=-\frac{L}{R}\ln(0.75)\approx\boxed{0.29\frac{L}{R}}$$

38. As shown by Eq. 30-18, adding resistance will decrease the oscillation frequency. We use Eq. 30-14 for the pure LC circuit frequency and Eq. 30-18 for the frequency with added resistance to solve for the resistance.

$$\omega'=(1-.0025)\omega\quad\rightarrow\quad\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}=0.9975\sqrt{\frac{1}{LC}}\quad\rightarrow$$

$$R=\sqrt{\frac{4L}{C}\left(1-0.9975^2\right)}=\sqrt{\frac{4(0.350\,\text{H})}{(1.800\times10^{-9}\,\text{F})}\left(1-0.9975^2\right)}=\boxed{2.0\,\text{k}\Omega}$$

39. We find the frequency from Eq. 30-23b for the reactance of an inductor.

$$X_L=2\pi fL\quad\rightarrow\quad f=\frac{X_L}{2\pi L}=\frac{660\,\Omega}{2\pi(0.0320\,\text{H})}=3283\,\text{Hz}\approx\boxed{3300\,\text{Hz}}$$

40. The reactance of a capacitor is given by Eq. 30-25b, $X_C=\dfrac{1}{2\pi fC}$.

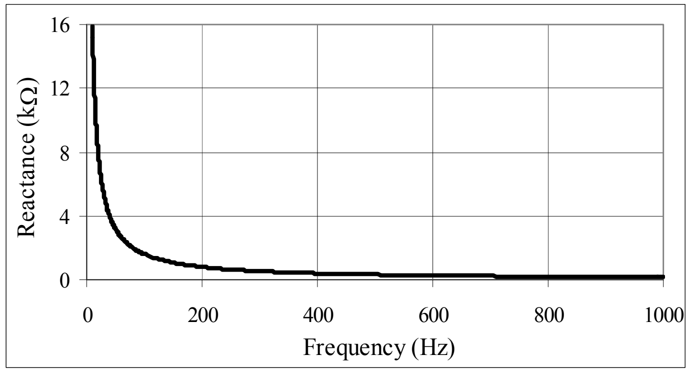
(*a*) $$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(60.0\,\text{Hz})(9.2\times10^{-6}\,\text{F})}=\boxed{290\,\Omega}$$

(*b*) $$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(1.00\times10^6\,\text{Hz})(9.2\times10^{-6}\,\text{F})}=\boxed{1.7\times10^{-2}\,\Omega}$$

41. The impedance is $X_C = \dfrac{1}{2\pi fC}$. The extreme values are as follows.

$$X_{max} = \frac{1}{2\pi(10\,\text{Hz})(1.0\times10^{-6}\,\text{F})} = 16{,}000\,\Omega$$

$$X_{min} = \frac{1}{2\pi(1000\,\text{Hz})(1.0\times10^{-6}\,\text{F})} = 160\,\Omega$$




The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.41."

42. We find the reactance from Eq. 30-23b, and the current from Ohm's law.

$$X_L = 2\pi fL = 2\pi(33.3\times10^3\,\text{Hz})(0.0360\,\text{H}) = 7532\,\Omega \approx \boxed{7530\,\Omega}$$

$$V = IX_L \rightarrow I = \frac{V}{X_L} = \frac{250\,\text{V}}{7532\,\Omega} = 0.03319\,\text{A} \approx \boxed{3.3\times10^{-2}\,\text{A}}$$

43. (*a*) At $\omega = 0$, the impedance of the capacitor is infinite. Therefore the parallel combination of the resistor *R* and capacitor *C* behaves as the resistor only, and so is *R*. Thus the impedance of the entire circuit is equal to the resistance of the two series resistors.

$$Z = \boxed{R + R'}$$

(*b*) At $\omega = \infty$,, the impedance of the capacitor is zero. Therefore the parallel combination of the resistor *R* and capacitor *C* is equal to zero. Thus the impedance of the entire circuit is equal to the resistance of the series resistor only.

$$Z = \boxed{R'}$$

44. We use Eq. 30-22a to solve for the impedance.

$$V_{rms} = I_{rms}\omega L \rightarrow L = \frac{V_{rms}}{I_{rms}\omega} = \frac{110\,\text{V}}{(3.1\,\text{A})2\pi(60\,\text{Hz})} = \boxed{94\,\text{mH}}$$

45. (*a*) We find the reactance from Eq. 30-25b.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(660\,\text{Hz})(8.6\times10^{-8}\,\text{F})} = 2804\,\Omega \approx \boxed{2800\,\Omega}$$

(*b*) We find the peak value of the current from Ohm's law.

$$I_{peak} = \sqrt{2}I_{rms} = \sqrt{2}\frac{V_{rms}}{X_C} = \sqrt{2}\frac{22{,}000\,\text{V}}{2804\,\Omega} = \boxed{11\,\text{A at }660\,\text{Hz}}$$

46. (*a*) Since the resistor and capacitor are in parallel, they will have the same voltage drop across them. We use Ohm's law to determine the current through the resistor and Eq. 30-25 to determine the current across the capacitor. The total current is the sum of the currents across each element.

$$I_R = \frac{V}{R} \;;\; I_C = \frac{V}{X_C} = V(2\pi fC)$$

$$\frac{I_C}{I_R + I_C} = \frac{V(2\pi fC)}{V(2\pi fC) + V/R} = \frac{R(2\pi fC)}{R(2\pi fC)+1} = \frac{(490\ \Omega)2\pi(60\ \text{Hz})(0.35\times10^{-6}\ \text{F})}{(490\ \Omega)2\pi(60\ \text{Hz})(0.35\times10^{-6}\ \text{F})+1}$$

$$= 0.0607 \approx \boxed{6.1\%}$$

(*b*) We repeat part (*a*) with a frequency of 60,000 Hz.

$$\frac{I_C}{I_R + I_C} = \frac{(490\ \Omega)2\pi(60{,}000\ \text{Hz})(0.35\times10^{-6}\ \text{F})}{(490\ \Omega)2\pi(60{,}000\ \text{Hz})(0.35\times10^{-6}\ \text{F})+1} = 0.9847 \approx \boxed{98\%}$$

47. The power is only dissipated in the resistor, so we use the power dissipation equation obtained in section 25-7.

$$P_{\text{avg}} = \tfrac{1}{2}I_0^2R = \tfrac{1}{2}(1.80\,\text{A})^2(1350\,\Omega) = 2187\,\text{W} \approx \boxed{2.19\,\text{kW}}$$

48. The impedance of the circuit is given by Eq. 30-28a without a capacitive reactance. The reactance of the inductor is given by Eq. 30-23b.

(*a*) $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2 f^2L^2} = \sqrt{(10.0\times10^3\Omega)^2 + 4\pi^2(55.0\,\text{Hz})^2(0.0260\,\text{H})^2}$

$= \boxed{1.00\times10^4\Omega}$

(*b*) $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2 f^2L^2} = \sqrt{(10.0\times10^3\Omega)^2 + 4\pi^2(5.5\times10^4\,\text{Hz})^2(0.0260\,\text{H})^2}$

$= \boxed{1.34\times10^4\Omega}$

$\boxed{49.}$ The impedance of the circuit is given by Eq. 30-28a without an inductive reactance. The reactance of the capacitor is given by Eq. 30-25b.

(*a*) $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{4\pi^2 f^2C^2}} = \sqrt{(75\Omega)^2 + \frac{1}{4\pi^2(60\,\text{Hz})^2(6.8\times10^{-6}\text{F})^2}} = 397\,\Omega$

$\approx \boxed{400\,\Omega}$ (2 sig. fig.)

(*b*) $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{4\pi^2 f^2C^2}} = \sqrt{(75\Omega)^2 + \frac{1}{4\pi^2(60000\,\text{Hz})^2(6.8\times10^{-6}\text{F})^2}} = \boxed{75\,\Omega}$

50. We find the impedance from Eq. 30-27.

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\,\text{V}}{70\times10^{-3}\,\text{A}} = \boxed{1700\,\Omega}$$

51. The impedance is given by Eq. 30-28a with no capacitive reactance.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2}$$

$$Z_f = 2Z_{60} \;\rightarrow\; \sqrt{R^2 + 4\pi^2 f^2L^2} = 2\sqrt{R^2 + 4\pi^2(60\,\text{Hz})^2L^2} \;\rightarrow$$

$$R^2 + 4\pi^2 f^2L^2 = 4\left[R^2 + 4\pi^2(60\,\text{Hz})^2L^2\right] = 4R^2 + 16\pi^2(60\,\text{Hz})^2L^2 \;\rightarrow$$

$$f=\sqrt{\frac{3R^2+16\pi^2\left(60\,\text{Hz}\right)^2L^2}{4\pi^2L^2}}=\sqrt{\frac{3R^2}{4\pi^2L^2}+4\left(60\,\text{Hz}\right)^2}=\sqrt{\frac{3\left(2500\,\Omega\right)^2}{4\pi^2\left(0.42\,\text{H}\right)^2}+4\left(60\,\text{Hz}\right)^2}$$

$$=1645\,\text{Hz}\approx\boxed{1.6\,\text{kHz}}$$

52. (*a*) The rms current is the rmv voltage divided by the impedance. The impedance is given by Eq. 30-28a with no inductive reactance,

$$Z=\sqrt{R^2+X_C^2}=\sqrt{R^2+\frac{1}{\left(2\pi fC\right)^2}}\,.$$

$$I_{\text{rms}}=\frac{V_{\text{rms}}}{Z}=\frac{V_{\text{rms}}}{\sqrt{R^2+\dfrac{1}{4\pi^2f^2L^2}}}=\frac{120\,\text{V}}{\sqrt{\left(3800\,\Omega\right)^2+\dfrac{1}{4\pi^2\left(60.0\,\text{Hz}\right)^2\left(0.80\times10^{-6}\,\text{F}\right)^2}}}$$

$$=\frac{120\,\text{V}}{5043\,\Omega}=2.379\times10^{-2}\,\text{A}\approx\boxed{2.4\times10^{-2}\,\text{A}}$$

(*b*) The phase angle is given by Eq. 30-29a with no inductive reactance.

$$\phi=\tan^{-1}\frac{-X_C}{R}=\tan^{-1}\frac{-\dfrac{1}{2\pi fC}}{R}=\tan^{-1}\frac{-\dfrac{1}{2\pi\left(60.0\,\text{Hz}\right)\left(0.80\times10^{-6}\,\text{F}\right)}}{3800\,\Omega}=\boxed{-41^\circ}$$

The current is leading the source voltage.

(*c*) The power dissipated is given by $P=I_{\text{rms}}^2R=\left(0.02379\text{A}\right)^2\left(6.0\times10^3\Omega\right)=\boxed{2.2\,\text{W}}$

(*d*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\substack{\text{rms}\\R}}=I_{\text{rms}}R=\left(2.379\times10^{-2}\,\text{A}\right)\left(3800\,\Omega\right)=90.4\,\text{V}\approx\boxed{90\,\text{V}}\quad\left(2\text{ sig. fig.}\right)$$

$$V_{\substack{\text{rms}\\C}}=I_{\text{rms}}X_C=\frac{I_{\text{rms}}}{2\pi fC}=\frac{\left(2.379\times10^{-2}\,\text{A}\right)}{2\pi\left(60.0\,\text{Hz}\right)\left(0.80\times10^{-6}\,\text{F}\right)}=78.88\,\text{V}\approx\boxed{79\,\text{V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

53. We use the rms voltage across the resistor to determine the rms current through the circuit. Then, using the rms current and the rms voltage across the capacitor in Eq. 30-25 we determine the frequency.

$$I_{\text{rms}}=\frac{V_{R,\text{rms}}}{R}\qquad V_{C,\text{rms}}=\frac{I_{\text{rms}}}{2\pi fC}$$

$$f=\frac{I_{\text{rms}}}{2\pi CV_{C,\text{rms}}}=\frac{V_{R,\text{rms}}}{2\pi CRV_{C,\text{rms}}}=\frac{\left(3.0\text{ V}\right)}{2\pi\left(1.0\times10^{-6}\text{ C}\right)\left(750\ \Omega\right)\left(2.7\text{ V}\right)}=\boxed{240\text{ Hz}}$$

Since the voltages in the resistor and capacitor are not in phase, the rms voltage across the power source will not be the sum of their rms voltages.

54. The total impedance is given by Eq. 30-28a.

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{\left(8.70\times10^3\,\Omega\right)^2 + \left[2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(3.20\times10^{-2}\,\text{H}\right) - \frac{1}{2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(6.25\times10^{-9}\,\text{F}\right)}\right]^2}$$

$$= 8716.5\,\Omega \approx \boxed{8.72\,\text{k}\Omega}$$

The phase angle is given by Eq. 30-29a.

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{2\pi fL - \dfrac{1}{2\pi fC}}{R}$$

$$= \tan^{-1}\frac{2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(3.20\times10^{-2}\,\text{H}\right) - \dfrac{1}{2\pi\left(1.00\times10^4\,\text{Hz}\right)\left(6.25\times10^{-9}\,\text{F}\right)}}{R}$$

$$= \tan^{-1}\frac{-535.9\,\Omega}{8.70\times10^3\,\Omega} = \boxed{-3.52^\circ}$$

The voltage is lagging the current, or the current is leading the voltage.
The rms current is given by Eq. 30-27.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{725\,\text{V}}{8716.5\,\Omega} = \boxed{8.32\times10^{-2}\,\text{A}}$$

55. (*a*) The rms current is the rms voltage divided by the impedance. The impedance is given by Eq. 30-28a with no capacitive reactance.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \left(2\pi fL\right)^2}\ .$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + 4\pi^2 f^2 L^2}} = \frac{120\,\text{V}}{\sqrt{\left(965\,\Omega\right)^2 + 4\pi^2\left(60.0\,\text{Hz}\right)^2\left(0.225\,\text{H}\right)^2}}$$

$$= \frac{120\,\text{V}}{968.7\,\Omega} = \boxed{0.124\,\text{A}}$$

(*b*) The phase angle is given by Eq. 30-29a with no capacitive reactance.

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{2\pi fL}{R} = \tan^{-1}\frac{2\pi\left(60.0\,\text{Hz}\right)\left(0.225\,\text{H}\right)}{965\,\Omega} = \boxed{5.02^\circ}$$

The current is lagging the source voltage.

(*c*) The power dissipated is given by $P = I_{\text{rms}}^2 R = \left(0.124\,\text{A}\right)^2\left(965\,\Omega\right) = \boxed{14.8\,\text{W}}$

(*d*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\text{rms}\atop R} = I_{\text{rms}}R = \left(0.124\,\text{A}\right)\left(965\,\Omega\right) = 119.7\,\text{V} \approx \boxed{120\,\text{V}}$$

$$V_{\text{rms}\atop L} = I_{\text{rms}}X_L = I_{\text{rms}}2\pi fL = \left(0.124\,\text{A}\right)2\pi\left(60.0\,\text{Hz}\right)\left(0.25\,\text{H}\right) = \boxed{10.5\,\text{V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

56. (*a*) The current is found from the voltage and impedance. The impedance is given by Eq. 30-28a.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{(2.0\,\Omega)^2 + \left[2\pi(60\,\text{Hz})(0.035\,\text{H}) - \frac{1}{2\pi(60\,\text{Hz})(26\times10^{-6}\,\text{F})}\right]^2} = 88.85\,\Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{45\,\text{V}}{88.85\,\Omega} = 0.5065\,\text{A} \approx \boxed{0.51\,\text{A}}$$

(*b*) Use Eq. 30-29a to find the phase angle.

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{2\pi fL - \frac{1}{2\pi fC}}{R}$$

$$= \tan^{-1}\frac{2\pi(60\,\text{Hz})(0.035\,\text{H}) - \frac{1}{2\pi(60\,\text{Hz})(26\times10^{-6}\,\text{F})}}{2.0\,\Omega} = \tan^{-1}\frac{-88.83\,\Omega}{2.0\,\Omega} = \boxed{-88^\circ}$$

(*c*) The power dissipated is given by $P = I_{\text{rms}}^2 R = (0.5065\,\text{A})^2(2.0\,\Omega) = \boxed{0.51\,\text{W}}$

57. For the current and voltage to be in phase, the reactances of the capacitor and inductor must be equal. Setting the two reactances equal enables us to solve for the capacitance.

$$X_L = 2\pi fL = X_C = \frac{1}{2\pi fC} \rightarrow C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2(360\,\text{Hz})^2(0.025\,\text{H})} = \boxed{7.8\,\mu\text{F}}$$

58. The light bulb acts like a resistor in series with the inductor. Using the desired rms voltage across the resistor and the power dissipated by the light bulb we calculate the rms current in the circuit and the resistance. Then using this current and the rms voltage of the circuit we calculate the impedance of the circuit (Eq. 30-27) and the required inductance (Eq. 30-28b).

$$I_{\text{rms}} = \frac{P}{V_{R,\text{rms}}} = \frac{75\,\text{W}}{120\,\text{V}} = 0.625\,\text{A} \qquad R = \frac{V_{R,\text{rms}}}{I_{\text{rms}}} = \frac{120\,\text{V}}{0.625\,\text{A}} = 192\,\Omega$$

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \sqrt{R^2 + (2\pi fL)^2}$$

$$L = \frac{1}{2\pi f}\sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2} = \frac{1}{2\pi(60\,\text{Hz})}\sqrt{\left(\frac{240\,\text{V}}{0.625\,\text{A}}\right)^2 - (192\,\Omega)^2} = \boxed{0.88\,\text{H}}$$

59. We multiply the instantaneous current by the instantaneous voltage to calculate the instantaneous power. Then using the trigonometric identity for the summation of sine arguments (inside back cover of text) we can simplify the result. We integrate the power over a full period and divide the result by the period to calculate the average power.

$$P = IV = (I_0 \sin\omega t)V_0 \sin(\omega t + \phi) = I_0 V_0 \sin\omega t(\sin\omega t\cos\phi + \sin\phi\cos\omega t)$$

$$= I_0 V_0(\sin^2\omega t\cos\phi + \sin\omega t\cos\omega t\sin\phi)$$

$$\overline{P} = \frac{1}{T}\int_0^T PdT = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} I_0V_0\left(\sin^2\omega t\cos\phi + \sin\omega t\cos\omega t\sin\phi\right)dt$$

$$= \frac{\omega}{2\pi}I_0V_0\cos\phi\int_0^{\frac{2\pi}{\omega}}\sin^2\omega t\ dt + \frac{\omega}{2\pi}I_0V_0\sin\phi\int_0^{\frac{2\pi}{\omega}}\sin\omega t\cos\omega t\ dt$$

$$= \frac{\omega}{2\pi}I_0V_0\cos\phi\left(\frac{1}{2}\frac{2\pi}{\omega}\right) + \frac{\omega}{2\pi}I_0V_0\sin\phi\left(\frac{1}{\omega}\sin^2\omega t\Big|_0^{\frac{2\pi}{\omega}}\right) = \boxed{\tfrac{1}{2}I_0V_0\cos\phi}$$

60. Given the resistance, inductance, capacitance, and frequency, we calculate the impedance of the circuit using Eq. 30-28b.

$$X_L = 2\pi fL = 2\pi(660\text{ Hz})(0.025\text{ H}) = 103.67\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(660\text{ Hz})(2.0\times10^{-6}\text{ F})} = 120.57\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(150\ \Omega)^2 + (103.67\ \Omega - 120.57\ \Omega)^2} = 150.95\ \Omega$$

(*a*) From the impedance and the peak voltage we calculate the peak current, using Eq. 30-27.

$$I_0 = \frac{V_0}{Z} = \frac{340\text{ V}}{150.95\ \Omega} = 2.252\text{ A} \approx \boxed{2.3\text{ A}}$$

(*b*) We calculate the phase angle of the current from the source voltage using Eq. 30-29a.

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{103.67\ \Omega - 120.57\ \Omega}{150\ \Omega} = \boxed{-6.4^\circ}$$

(*c*) We multiply the peak current times the resistance to obtain the peak voltage across the resistor. The voltage across the resistor is in phase with the current, so the phase angle is the same as in part (*b*).

$$V_{0,R} = I_0R = (2.252\text{ A})(150\ \Omega) = \boxed{340\text{ V}}\ ;\ \boxed{\phi = -6.4^\circ}$$

(*d*) We multiply the peak current times the inductive reactance to calculate the peak voltage across the inductor. The voltage in the inductor is 90º ahead of the current. Subtracting the phase difference between the current and source from the 90º between the current and inductor peak voltage gives the phase angle between the source voltage and the inductive peak voltage.

$$V_{0,L} = I_0X_L = (2.252\text{ A})(103.67\ \Omega) = \boxed{230\text{ V}}$$

$$\phi_L = 90.0^\circ - \phi = 90.0^\circ - (-6.4)^\circ = \boxed{96.4^\circ}$$

(*e*) We multiply the peak current times the capacitive reactance to calculate the peak voltage across the capacitor. Subtracting the phase difference between the current and source from the -90º between the current and capacitor peak voltage gives the phase angle between the source voltage and the capacitor peak voltage.

$$V_{0,C} = I_0X_C = (2.252\text{ A})(120.57\ \Omega) = \boxed{270\text{ V}}$$

$$\phi_C = -90.0^\circ - \phi = -90.0^\circ - (-6.4^\circ) = \boxed{-83.6^\circ}$$

61. Using Eq. 30-23b we calculate the impedance of the inductor. Then we set the phase shift in Eq. 30-29a equal to 25º and solve for the resistance. We calculate the output voltage by multiplying the current through the circuit, from Eq. 30-27, by the inductive reactance (Eq. 30-23b).

$$X_L = 2\pi fL = 2\pi(175\text{ Hz})(0.055\text{ H}) = 60.48\ \Omega$$

$$\tan\phi = \frac{X_L}{R} \Longrightarrow R = \frac{X_L}{\tan\phi} = \frac{60.48\ \Omega}{\tan\ 25^\circ} = 129.7\,\Omega \approx \boxed{130\ \Omega}$$

$$\frac{V_{\text{output}}}{V_0}=\frac{V_R}{V_0}=\frac{IR}{IZ}=\frac{R}{Z}=\frac{129.70\,\Omega}{\sqrt{(129.70\,\Omega)^2+(60.48\,\Omega)^2}}=\boxed{0.91}$$

62. The resonant frequency is found from Eq. 30-32. The resistance does not influence the resonant frequency.

$$f_0=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}=\frac{1}{2\pi}\sqrt{\frac{1}{(26.0\times10^{-6}\,\text{H})(3800\times10^{-12}\,\text{F})}}=\boxed{5.1\times10^5\,\text{Hz}}$$

63. We calculate the resonant frequency using Eq. 30-32 with the inductance and capacitance given in the example. We use Eq. 30-30 to calculate the power dissipation, with the impedance equal to the resistance.

$$f_0=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{(0.0300\text{ H})(12.0\times10^{-6}\text{ F})}}=\boxed{265\text{ Hz}}$$

$$\bar{P}=I_{\text{rms}}V_{\text{rms}}\cos\phi=\left(\frac{V_{\text{rms}}}{R}\right)V_{\text{rms}}\left(\frac{R}{R}\right)=\frac{V^2_{\text{rms}}}{R}=\frac{(90.0\,\text{V})^2}{25.0\,\Omega}=\boxed{324\,\text{W}}$$

64. (*a*) We find the capacitance from the resonant frequency, Eq. 30-32.

$$f_0=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}\quad\rightarrow\quad C=\frac{1}{4\pi^2Lf_0^2}=\frac{1}{4\pi^2(4.15\times10^{-3}\,\text{H})(33.0\times10^3\,\text{Hz})^2}=\boxed{5.60\times10^{-9}\,\text{F}}$$

(*b*) At resonance the impedance is the resistance, so the current is given by Ohm's law.

$$I_{\text{peak}}=\frac{V_{\text{peak}}}{R}=\frac{136\,\text{V}}{3800\,\Omega}=\boxed{35.8\,\text{mA}}$$

65. (*a*) The peak voltage across the capacitor is the peak current multiplied by the capacitive reactance. We calculate the current in the circuit by dividing the source voltage by the impedance, where at resonance the impedance is equal to the resistance.

$$V_{C0}=X_CI_0=\frac{1}{2\pi f_0C}\frac{V_0}{R}=\frac{V_0}{2\pi(RC)}\frac{1}{f_0}=\boxed{\frac{V_0}{2\pi\tau}T_0}$$

(*b*) We set the amplification equal to 125 and solve for the resistance.

$$\beta=\frac{T_0}{2\pi\tau}=\frac{1}{2\pi f_0RC}\rightarrow R=\frac{1}{2\pi f_0\beta C}=\frac{1}{2\pi(5000\,\text{Hz})(125)(2.0\times10^{-9}\,\text{F})}=\boxed{130\,\Omega}$$

66. (*a*) We calculate the resonance frequency from the inductance and capacitance using Eq.30-32.

$$f_0=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{(0.055\text{ H})(1.0\times10^{-9}\text{ F})}}=21460\text{ Hz}\approx\boxed{21\text{ kHz}}$$

(*b*) We use the result of Problem 65 to calculate the voltage across the capacitor.

$$V_{C0}=\frac{V_0}{2\pi(RC)}\frac{1}{f_0}=\frac{2.0\text{ V}}{2\pi(35\ \Omega)(1.0\times10^{-9}\text{ F})(21460\text{ Hz})}=\boxed{420\text{ V}}$$

(*c*) We divide the voltage across the capacitor by the voltage source.

$$\frac{V_{C0}}{V_0}=\frac{420\text{ V}}{2.0\text{ V}}=\boxed{210}$$

67. (*a*) We write the average power using Eq. 30-30, with the current in terms of the impedance (Eq. 30-27) and the power factor in terms of the resistance and impedance (Eq. 30-29b). Finally we write the impedance using Eq. 30-28b.

$$\bar{P} = I_{\text{rms}}V_{\text{rms}}\cos\phi = \frac{V_{\text{rms}}}{Z}V_{\text{rms}}\frac{R}{Z} = \frac{V_{\text{rms}}^2 R}{Z^2} = \boxed{\frac{V_0^2 R}{2\left[R^2 + \left(\omega L - 1/\omega C\right)^2\right]}}$$

(*b*) The power dissipation will be a maximum when the inductive reactance is equal to the capacitive reactance, which is the resonant frequency.

$$f = \boxed{\frac{1}{2\pi\sqrt{LC}}}$$

(*c*) We set the power dissipation equal to ½ of the maximum power dissipation and solve for the angular frequencies.

$$\bar{P} = \frac{1}{2}\bar{P}_{\max} = \frac{V_0^2 R}{2\left[R^2 + \left(\omega L - 1/\omega C\right)^2\right]} = \frac{1}{2}\left(\frac{V_0^2 R}{2R^2}\right) \rightarrow \left(\omega L - 1/\omega C\right) = \pm R$$

$$\rightarrow\ 0 = \omega^2 LC \pm RC\omega - 1\ \rightarrow\ \omega = \frac{\pm RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$$

We require the angular frequencies to be positive and for a sharp peak, $R^2C^2 \ll 4LC$. The angular width will then be the difference between the two positive frequencies.

$$\omega = \frac{2\sqrt{LC} \pm RC}{2LC} = \frac{1}{\sqrt{LC}} \pm \frac{R}{2L}\ \rightarrow\ \Delta\omega = \left(\frac{1}{\sqrt{LC}} + \frac{R}{2L}\right) - \left(\frac{1}{\sqrt{LC}} - \frac{R}{2L}\right) = \boxed{\frac{R}{L}}$$

68. (*a*) We write the charge on the capacitor using Eq. 24-1, where the voltage drop across the capacitor is the inductive capacitance multiplied by the circuit current (Eq. 30-25a) and the circuit current is found using the source voltage and circuit impedance (Eqs. 30-27 and 30-28b).

$$Q_0 = CV_{C0} = CI_0X_C = C\left(\frac{V_0}{Z}\right)X_C = \frac{CV_0}{\omega C\sqrt{R^2 + \left(\omega L - 1/\omega C\right)^2}} = \boxed{\frac{V_0}{\sqrt{\omega^2R^2 + \left(\omega^2 L - 1/C\right)^2}}}$$

(*b*) We set the derivative of the charge with respect to the frequency equal to zero to calculate the frequency at which the charge is a maximum.

$$\frac{dQ_0}{d\omega} = \frac{d}{d\omega}\frac{V_0}{\sqrt{\omega'^2R^2 + \left(\omega'^2 L - 1/C\right)^2}} = \frac{-V_0\left(2\omega' R^2 + 4\omega'^3 L^2 - 4\omega' L/C\right)}{\left[\omega'^2R^2 + \left(\omega'^2L - 1/C\right)^2\right]^{\frac{3}{2}}} = 0$$

$$\rightarrow \omega' = \boxed{\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$

(*c*) The amplitude in a forced damped harmonic oscillation is given by Eq. 14-23. This is equivalent to the *LRC* circuit with $F_0 \leftrightarrow V_0,\ k \leftrightarrow 1/C,\ \ m \leftrightarrow L,$ and $b \leftrightarrow R.$

69. Since the circuit is in resonance, we use Eq. 30-32 for the resonant frequency to determine the necessary inductance. We set this inductance equal to the solenoid inductance calculated in Example 30-3, with the area equal to the area of a circle of radius *r*, the number of turns equal to the length of the wire divided by the circumference of a turn, and the length of the solenoid equal to the diameter of the wire multiplied by the number of turns. We solve the resulting equation for the number of turns.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \to \quad L = \frac{1}{4\pi^2 f_o^2 C} = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 \left(\frac{\ell_{wire}}{2\pi r}\right)^2 \pi r^2}{Nd} \quad \to$$

$$N = \frac{\pi f_o^2 C \mu_0 \ell^2_{wire}}{d} = \frac{\pi\left(18.0\times10^3\,\text{Hz}\right)^2\left(2.20\times10^{-7}\,\text{F}\right)\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)\left(12.0\,\text{m}\right)^2}{1.1\times10^{-3}\,\text{m}} = \boxed{37\text{ loops}}$$

70. The power on each side of the transformer must be equal. We replace the currents in the power equation with the number of turns in the two coils using Eq. 29-6. Then we solve for the turn ratio.

$$P_p = I_p^2 Z_p = P_s = I_s^2 Z_s \to \frac{Z_p}{Z_s} = \left(\frac{I_s}{I_p}\right)^2 = \left(\frac{N_p}{N_s}\right)^2$$

$$\to \frac{N_p}{N_s} = \sqrt{\frac{Z_p}{Z_s}} = \sqrt{\frac{45\times10^3\ \Omega}{8.0\ \Omega}} = \boxed{75}$$

71. (*a*) We calculate the inductance from the resonance frequency.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \to$$

$$L = \frac{1}{4\pi^2 f_o^2 C} = \frac{1}{4\pi^2\left(17\times10^3\,\text{Hz}\right)^2\left(2.2\times10^{-9}\,\text{F}\right)} = 0.03982\,\text{H} \approx \boxed{0.040\,\text{H}}$$

(*b*) We set the initial energy in the electric field, using Eq. 24-5, equal to the maximum energy in the magnetic field, Eq. 30-6, and solve for the maximum current.

$$\tfrac{1}{2}CV_0^2 = \tfrac{1}{2}LI_{\max}^2 \quad \to \quad I_{\max} = \sqrt{\frac{CV_0^2}{L}} = \sqrt{\frac{\left(2.2\times10^{-9}\,\text{F}\right)\left(120\,\text{V}\right)^2}{\left(0.03984\,\text{H}\right)}} = \boxed{0.028\,\text{A}}$$

(*c*) The maximum energy in the inductor is equal to the initial energy in the capacitor.

$$U_{L,\max} = \tfrac{1}{2}CV_0^2 = \tfrac{1}{2}\left(2.2\times10^{-9}\,\text{F}\right)\left(120\,\text{V}\right)^2 = \boxed{16\,\mu\text{J}}$$

72. We use Eq. 30-6 to calculate the initial energy stored in the inductor.

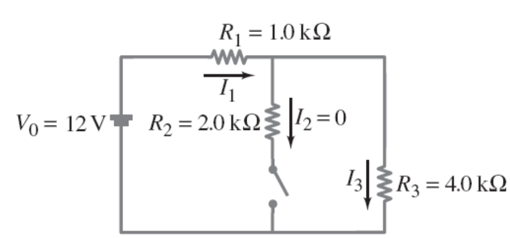
$$U_0 = \tfrac{1}{2}LI_0^2 = \tfrac{1}{2}\left(0.0600\,\text{H}\right)\left(0.0500\,\text{A}\right)^2 = \boxed{7.50\times10^{-5}\,\text{J}}$$

We set the energy in the inductor equal to five times the initial energy and solve for the current. We set the current equal to the initial current plus the rate of increase multiplied by time and solve for the time.

$$U = \tfrac{1}{2}LI^2 \quad \to \quad I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(5.0\times7.50\times10^{-5}\,\text{J}\right)}{0.0600\,\text{H}}} = 111.8\,\text{mA}$$

$$I = I_0 + \beta t \quad \to \quad t = \frac{I - I_0}{\beta} = \frac{111.8\,\text{mA} - 50.0\,\text{mA}}{78.0\,\text{mA/s}} = \boxed{0.79\,\text{s}}$$

73. When the currents have acquired their steady-state values, the capacitor will be fully charged, and so no current will flow through the capacitor. At this time, the voltage drop across the inductor will be zero, as the current flowing through the inductor is constant. Therefore, the current through $R_1$ is zero, and the resistors $R_2$ and $R_3$ can be treated as in series.



$$I_1 = I_3 = \frac{V_0}{R_1 + R_3} = \frac{12\,\text{V}}{5.0\,\text{k}\Omega} = \boxed{2.4\,\text{mA}} \;;\; I_2 = \boxed{0}$$

74. (*a*) The self inductance is written in terms of the magnetic flux in the toroid using Eq. 30-4. We set the flux equal to the magnetic field of a toroid, from Example 28-10. The field is dependent upon the radius of the solenoid, but if the diameter of the solenoid loops is small compared with the radius of the solenoid, it can be treated as approximately constant.

$$L = \frac{N\Phi_B}{I} = \frac{N\left(\pi d^2/4\right)\left(\mu_0 NI/2\pi r_0\right)}{I} = \boxed{\frac{\mu_0 N^2 d^2}{8r_0}}$$

This is consistent with the inductance of a solenoid for which the length is $\ell = 2\pi r_0$.

(*b*) We calculate the value of the inductance from the given data, with $r_0$ equal to half of the diameter.

$$L = \frac{\mu_0 N^2 d^2}{8r_0} = \frac{\left(4\pi\times10^{-7}\ \text{T}\bullet\text{m/A}\right)\left(550\right)^2\left(0.020\ \text{m}\right)^2}{8\left(0.33\ \text{m}\right)} = \boxed{58\ \mu\text{H}}$$

75. We use Eq. 30-4 to calculate the self inductance between the two wires. We calculate the flux by integrating the magnetic field from the two wires, using Eq. 28-1, over the region between the two wires. Dividing the inductance by the length of the wire gives the inductance per unit length.

$$L = \frac{\Phi_B}{I} = \frac{1}{I}\int_r^{\ell-r}\left[\frac{\mu_0 I}{2\pi r'} + \frac{\mu_0 I}{2\pi\left(\ell - r'\right)}\right]h\,dr' = \frac{\mu_0 h}{2\pi}\int_r^{\ell-r}\left[\frac{1}{r'} + \frac{1}{\left(\ell - r'\right)}\right]dr'$$

$$\frac{L}{h} = \frac{\mu_0}{2\pi}\left[\ln\left(r'\right) - \ln\left(\ell - r'\right)\right]_r^{\ell-r} = \frac{\mu_0}{2\pi}\left[\ln\left(\frac{\ell - r}{r}\right) - \ln\left(\frac{r}{\ell - r}\right)\right] = \boxed{\frac{\mu_0}{\pi}\ln\left(\frac{\ell - r}{r}\right)}$$

76. The magnetic energy is the energy density (Eq. 30-7) multiplied by the volume of the spherical shell enveloping the earth.

$$U = u_B V = \frac{B^2}{2\mu_0}\left(4\pi r^2 h\right) = \frac{\left(0.50\times10^{-4}\text{T}\right)^2}{2\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)}\left[4\pi\left(6.38\times10^6\text{m}\right)^2\left(5.0\times10^3\text{m}\right)\right] = \boxed{2.5\times10^{15}\text{J}}$$

77. (*a*) For underdamped oscillation, the charge on the capacitor is given by Eq. 30-19, with $\phi = 0$. Differentiating the current with respect to time gives the current in the circuit.

$$Q(t) = Q_0 e^{-\frac{R}{2L}t}\cos\omega' t \;;\; I(t) = \frac{dQ}{dt} = -Q_0 e^{-\frac{R}{2L}t}\left(\frac{R}{2L}\cos\omega' t + \omega'\sin\omega' t\right)$$

The total energy is the sum of the energies stored in the capacitor (Eq. 24-5) and the energy stored in the inductor (Eq. 30-6). Since the oscillation is underdamped ($\omega' \gg R/2L$), the cosine term in the current is much smaller than the sine term and can be ignored. The frequency of oscillation is approximately equal to the undamped frequency of Eq. 30-14.

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{LI^2}{2} = \frac{\left(Q_0 e^{-\frac{R}{2L}t}\cos\omega' t\right)^2}{2C} + \frac{L\left(Q_0 e^{-\frac{R}{2L}t}\right)^2\left(\omega'\sin\omega' t\right)^2}{2}$$

$$= \frac{Q_0^2 e^{-\frac{R}{L}t}}{2C}\left(\cos^2\omega' t + \omega'^2 LC\sin^2\omega' t\right) \approx \boxed{\frac{Q_0^2 e^{-\frac{R}{L}t}}{2C}}$$

(*b*) We differentiate the energy with respect to time to show the average power dissipation. We then set the power loss per cycle equal to the resistance multiplied by the square of the current. For a lightly damped oscillation, the exponential term does not change much in one cycle, while the sine squared term averages to ½ .

$$\frac{dU}{dt}=\frac{d}{dt}\left(\frac{Q_0^2 e^{-\frac{R}{L}t}}{2C}\right)=-\frac{RQ_0^2 e^{-\frac{R}{L}t}}{2LC}$$

$$P=-I^2R=-Q_0^2e^{-\frac{R}{L}t}\left(\omega'^2\sin^2\omega' t\right)\approx -Q_0^2e^{-\frac{R}{L}t}\left(\frac{1}{LC}\right)\left(\frac{1}{2}\right)=\boxed{-\frac{RQ_0^2e^{-\frac{R}{L}t}}{2LC}}$$

The change in power in the circuit is equal to the power dissipated by the resistor.

78. Putting an inductor in series with the device will protect it from sudden surges in current. The growth of current in an *LR* circuit is given is Eq. 30-9.

$$I=\frac{V}{R}\left(1-e^{-tR/L}\right)=I_{max}\left(1-e^{-tR/L}\right)$$

The maximum current is 33 mA, and the current is to have a value of 7.5 mA after a time of 75 microseconds. Use this data to solve for the inductance.

$$I=I_{max}\left(1-e^{-tR/L}\right)\ \rightarrow\ e^{-tR/L}=1-\frac{I}{I_{max}}\ \rightarrow$$

$$L=-\frac{tR}{\ln\left(1-\frac{I}{I_{max}}\right)}=-\frac{\left(75\times10^{-6}\text{sec}\right)\left(150\,\Omega\right)}{\ln\left(1-\frac{7.5\,\text{mA}}{33\,\text{mA}}\right)}=4.4\times10^{-2}\,\text{H}$$

Put an inductor of value $4.4\times10^{-2}\,\text{H}$ in series with the device.

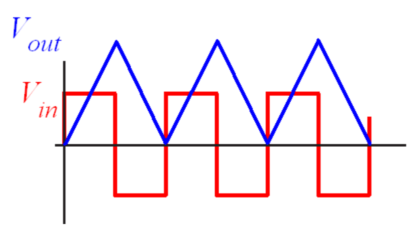
79. We use Kirchhoff's loop rule to equate the input voltage to the voltage drops across the inductor and resistor. We then multiply both sides of the equation by the integrating factor $e^{\frac{Rt}{L}}$ and integrate the right-hand side of the equation using a *u* substitution with $u=IRe^{\frac{Rt}{L}}$ and $du=dIRe^{\frac{Rt}{L}}+Ie^{\frac{Rt}{L}}dt/L$

$$V_{in}=L\frac{dI}{dt}+IR\ \rightarrow$$

$$\int V_{in}e^{\frac{Rt}{L}}dt=\int\left(L\frac{dI}{dt}+IR\right)e^{\frac{Rt}{L}}dt=\frac{L}{R}\int du=IR\frac{L}{R}e^{\frac{Rt}{L}}=V_{out}\frac{L}{R}e^{\frac{Rt}{L}}$$

For $L/R<<t$, $e^{\frac{Rt}{L}}\approx1$. Setting the exponential term equal to unity on both sides of the equation gives the desired results.

$$\int V_{in}dt=V_{out}\frac{L}{R}$$




80. (*a*) Since the capacitor and resistor are in series, the impedance of the circuit is given by Eq. 30-28a. Divide the source voltage by the impedance to determine the current in the circuit. Finally, multiply the current by the resistance to determine the voltage drop across the resistor.

$$V_R=IR=\frac{V_{in}}{Z}R=\frac{V_{in}R}{\sqrt{R^2+1/\left(2\pi fC\right)^2}}$$

$$= \frac{(130\text{ mV})(550\ \Omega)}{\sqrt{(550\ \Omega)^2 + 1\big/\left[2\pi(60\text{ Hz})(1.2\times10^{-6}\text{ F})\right]^2}} = \boxed{31\text{ mV}}$$

(*b*) Repeat the calculation with a frequency of 6.0 kHz.

$$V_R = \frac{(130\text{ mV})(550\ \Omega)}{\sqrt{(550\ \Omega)^2 + 1\big/\left[2\pi(6000\text{ Hz})(1.2\times10^{-6}\text{ F})\right]^2}} = \boxed{130\text{ mV}}$$

Thus the capacitor allows the higher frequency to pass, but attenuates the lower frequency.

81. (*a*) We integrate the power directly from the current and voltage over one cycle.

$$\overline{P} = \frac{1}{T}\int_0^T IV dt = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} I_0 \sin(\omega t) V_0 \sin(\omega t + 90°)\,dt = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} I_0 \sin(\omega t) V_0 \cos(\omega t)\,dt$$

$$= \frac{\omega}{2\pi} I_0 V_0 \frac{\sin^2(\omega t)}{2\omega}\Bigg|_0^{\frac{2\pi}{\omega}} = \frac{I_0 V_0}{4\pi}\left[\sin^2\left(\omega\frac{2\pi}{\omega}\right) - \sin^2(0)\right] = \boxed{0}$$

(*b*) We apply Eq. 30-30, with $\phi = 90°$.

$$\overline{P} = I_{\text{rms}} V_{\text{rms}} \cos 90° = \boxed{0}$$

As expected the average power is the same for both methods of calculation.

82. Since the current lags the voltage one of the circuit elements must be an inductor. Since the angle is less than 90º, the other element must be a resistor. We use 30-29a to write the resistance in terms of the impedance. Then using Eq. 30-27 to determine the impedance from the voltage and current and Eq. 30-28b, we solve for the unknown inductance and resistance.

$$\tan\phi = \frac{2\pi fL}{R} \to R = 2\pi fL\cot\phi$$

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \sqrt{R^2 + (2\pi fL)^2} = \sqrt{(2\pi fL\cot\phi)^2 + (2\pi fL)^2} = 2\pi fL\sqrt{1+\cot^2\phi}$$

$$L = \frac{V_{\text{rms}}}{2\pi f\, I_{\text{rms}}\sqrt{1+\cot^2\phi}} = \frac{120\text{ V}}{2\pi(60\text{ Hz})(5.6\text{ A})\sqrt{1+\cot^2 65°}} = 51.5\text{ mH} \approx \boxed{52\text{ mH}}$$

$$R = 2\pi f\,L\cot\phi = 2\pi(60\text{ Hz})(51.5\text{ mH})\cot 65° = \boxed{9.1\Omega}$$

83. We use Eq. 30-28b to calculate the impedance at 60 Hz. Then we double that result and solve for the required frequency.

$$Z_0 = \sqrt{R^2 + (2\pi f_0 L)^2} = \sqrt{(3500\Omega)^2 + \left[2\pi(60\text{ Hz})(0.44\text{ H})\right]^2} = 3504\Omega$$

$$2Z_0 = \sqrt{R^2 + (2\pi fL)^2} \to f = \frac{\sqrt{4Z_0^2 - R^2}}{2\pi L} = \frac{\sqrt{4(3504\Omega)^2 - (3500\Omega)^2}}{2\pi(0.44\text{ H})} = \boxed{2.2\text{ kHz}}$$

84. (*a*) We calculate capacitive reactance using Eq. 30-25b. Then using the resistance and capacitive reactance we calculate the impedance. Finally, we use Eq. 30-27 to calculate the rms current.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60.0\text{ Hz})(1.80\times10^{-6}\text{ F})} = 1474\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(5700\Omega)^2 + (1474\Omega)^2} = 5887\Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120\,\text{V}}{5887\,\Omega} = 20.38\,\text{mA} \approx \boxed{20.4\,\text{mA}}$$

(*b*) We calculate the phase angle using Eq. 30-29a.

$$\phi = \tan^{-1}\frac{-X_C}{R} = \tan^{-1}\frac{-1474\,\Omega}{5700\,\Omega} = \boxed{-14.5^\circ}$$

(*c*) The average power is calculated using Eq. 30-30.

$$\bar{P} = I_{\text{rms}}V_{\text{rms}}\cos\phi = (0.0204\,\text{A})(120\,\text{V})\cos(-14.5^\circ) = \boxed{2.37\,\text{W}}$$

(*d*) The voltmeter will read the rms voltage across each element. We calculate the rms voltage by multiplying the rms current through the element by the resistance or capacitive reactance.

$$V_R = I_{\text{rms}}R = (20.38\,\text{mA})(5.70\,\text{k}\Omega) = \boxed{116\,\text{V}}$$

$$V_C = I_{\text{rms}}X_C = (20.38\,\text{mA})(1474\,\Omega) = \boxed{30.0\,\text{V}}$$

Note that since the voltages are out of phase they do not sum to the applied voltage. However, since they are 90º out of phase their squares sum to the square of the input voltage.

$\boxed{85.}$ We find the resistance using Ohm's law with the dc voltage and current. When then calculate the impedance from the ac voltage and current, and using Eq. 30-28b.

$$R = \frac{V}{I} = \frac{45\,\text{V}}{2.5\,\text{A}} = \boxed{18\,\Omega} \;;\; Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{120\,\text{V}}{3.8\,\text{A}} = 31.58\,\Omega$$

$$\sqrt{R^2 + (2\pi fL)^2} \to L = \frac{\sqrt{Z^2 - R^2}}{2\pi f} = \frac{\sqrt{(31.58\,\Omega)^2 - (18\,\Omega)^2}}{2\pi(60\,\text{Hz})} = \boxed{69\,\text{mH}}$$

86. (*a*) From the text of the problem, the *Q* factor is the ratio of the voltage across the capacitor or inductor to the voltage across the resistor, at resonance. The resonant frequency is given by Eq. 30-32.

$$Q = \frac{V_L}{V_R} = \frac{I_{\text{res}}X_L}{I_{\text{res}}R} = \frac{2\pi f_0 L}{R} = \frac{2\pi\frac{1}{2\pi}\sqrt{\frac{1}{LC}}L}{R} = \boxed{\frac{1}{R}\sqrt{\frac{L}{C}}}$$

(*b*) Find the inductance from the resonant frequency, and the resistance from the *Q* factor.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad \to$$

$$L = \frac{1}{4\pi^2 C f_0^2} = \frac{1}{4\pi^2(1.0\times10^{-8}\,\text{F})(1.0\times10^{6}\,\text{Hz})^2} = 2.533\times10^{-6}\,\text{H} \approx \boxed{2.5\times10^{-6}\,\text{H}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \to \quad R = \frac{1}{Q}\sqrt{\frac{L}{C}} = \frac{1}{350}\sqrt{\frac{2.533\times10^{-6}\,\text{H}}{1.0\times10^{-8}\,\text{F}}} = \boxed{4.5\times10^{-2}\,\Omega}$$

87. We calculate the period of oscillation as 2π divided by the angular frequency. Then set the total energy of the system at the beginning of each cycle equal to the charge on the capacitor as given by Eq. 24-5, with the charge given by Eq. 30-19, with $\cos(\omega' t + \phi) = \cos[\omega'(t+T)+\phi] = 1$. We take the difference in energies at the beginning and end of a cycle, divided by the initial energy. For small damping, the argument of the resulting exponential term is small and we replace it with the first two terms of the Taylor series expansion.

$$T=\frac{2\pi}{\omega'}\approx\frac{2\pi}{\omega}\qquad U_{\max}=\frac{Q_0^2e^{-\frac{R}{L}t}\cos^2\left(\omega' t+\phi\right)}{2C}=\frac{Q_0^2e^{-\frac{R}{L}t}}{2C}$$

$$\frac{\Delta U}{U}=\frac{Q_0^2e^{-\frac{R}{L}t}-Q_0^2e^{-\frac{R}{L}\left(t+\frac{2\pi}{\omega}\right)}}{Q_0^2e^{-\frac{R}{L}t}}=1-e^{-\frac{2\pi R}{\omega L}}\approx 1-\left(1-\frac{2\pi R}{\omega L}\right)=\frac{2\pi R}{\omega L}=\boxed{\frac{2\pi}{Q}}$$

88. We set the power factor equal to the resistance divided by the impedance (Eq. 30-28a) with the impedance written in terms of the angular frequency (Eq. 30-28b). We rearrange the resulting equation to form a quadratic equation in terms of the angular frequency. We divide the positive angular frequencies by 2π to determine the desired frequencies.

$$\cos\phi=\frac{R}{Z}=\frac{R}{\sqrt{R^2+\left(\omega L-1/\omega C\right)^2}}\quad\rightarrow\quad\omega^2LC\pm\omega C\sqrt{R^2\left(\frac{1}{\cos^2\phi}-1\right)}-1=0$$

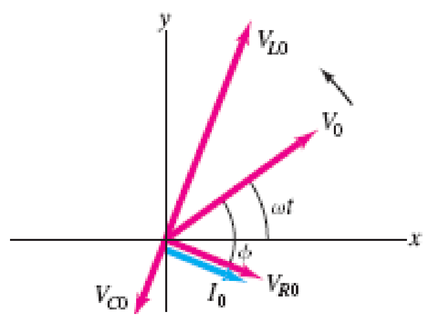
$$\omega^2\left(0.033\,\text{H}\right)\left(55\times10^{-9}\,\text{F}\right)\pm\omega\left(55\times10^{-9}\,\text{F}\right)\sqrt{\left(1500\,\Omega\right)^2\left(\frac{1}{0.17^2}-1\right)}-1=0$$

$$\left(1.815\times10^{-9}\,\text{F}\bullet\text{H}\right)\omega^2\pm\left(4.782\times10^{-4}\,\Omega\bullet\text{F}\right)\omega-1=0$$

$$\omega=\frac{\pm4.78225\times10^{-4}\,\Omega\bullet\text{F}\pm4.85756\times10^{-4}\,\Omega\bullet\text{F}}{3.63\times10^{-9}\,\text{F}\bullet\text{H}}=\pm2.65\times10^5\,\text{rad/s},\ \pm2.07\times10^3\,\text{rad/s}$$

$$f=\frac{\omega}{2\pi}=\frac{2.65\times10^5\,\text{rad/s}}{2\pi}=\boxed{42\,\text{kHz}}\quad\text{and}\quad\frac{2.07\times10^3\,\text{rad/s}}{2\pi}=\boxed{330\,\text{Hz}}$$

89. (*a*) We set $V=V_0\sin\omega t$ and assume the inductive reactance is greater than the capacitive reactance. The current will lag the voltage by an angle $\phi$. The voltage across the resistor is in phase with the current and the voltage across the inductor is 90º ahead of the current. The voltage across the capacitor is smaller than the voltage in the inductor, and antiparallel to it.




(*b*) From the diagram, the current is the projection of the maximum current onto the *y* axis, with the current lagging the voltage by the angle $\phi$. This is the same angle obtained in Eq. 30-29a. The magnitude of the maximum current is the voltage divided by the impedance, Eq. 30-28b.

$$I(t)=I_0\sin\left(\omega t-\phi\right)=\boxed{\frac{V_0}{\sqrt{R^2+\left(\omega L-1/\omega C\right)^2}}\sin\left(\omega t-\phi\right)}\ ;\ \phi=\tan^{-1}\frac{\omega L-1/\omega C}{R}$$

90. (*a*) We use Eq. 30-28b to calculate the impedance and Eq. 30-29a to calculate the phase angle.

$$X_L=\omega L=\left(754\,\text{rad/s}\right)\left(0.0220\,\text{H}\right)=16.59\,\Omega$$

$$X_C=1/\omega C=1\Big/\left(754\,\text{rad/s}\right)\left(0.42\times10^{-6}\,\text{F}\right)=3158\,\Omega$$

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}=\sqrt{\left(23.2\times10^3\,\Omega\right)^2+\left[16.59\,\Omega-3158\,\Omega\right]^2}=\boxed{23.4\,\text{k}\Omega}$$

$$\phi=\tan^{-1}\frac{X_L-X_C}{R}=\tan^{-1}\frac{16.59\,\Omega-3158\,\Omega}{23.2\times10^3\,\Omega}=\boxed{-7.71^\circ}$$

(*b*) We use Eq. 30-30 to obtain the average power. We obtain the rms voltage by dividing the maximum voltage by $\sqrt{2}$. The rms current is the rms voltage divided by the impedance.

$$\bar{P} = I_{\rm rms}V_{\rm rms}\cos\phi = \frac{V_{\rm rms}^2}{Z}\cos\phi = \frac{V_0^2}{2Z}\cos\phi = \frac{(0.95\,\text{V})^2}{2(23.4\times10^3\,\Omega)}\cos(-7.71^\circ) = \boxed{19\,\mu\text{W}}$$

(*c*) The rms current is the peak voltage, divided by $\sqrt{2}$, and then divided by the impedance.

$$I_{\rm rms} = \frac{V_0/\sqrt{2}}{Z} = \frac{0.95\,\text{V}/\sqrt{2}}{23.4\times10^3\,\Omega} = 2.871\times10^{-5}\,\text{A} \approx \boxed{29\,\mu\text{A}}$$

The rms voltage across each element is the rms current times the resistance or reactance of the element.

$$V_R = I_{\rm rms}R = (2.871\times10^{-5}\,\text{A})(23.2\times10^3\,\Omega) = \boxed{0.67\,\text{V}}$$

$$V_C = I_{\rm rms}X_C = (2.871\times10^{-5}\,\text{A})(3158\,\Omega) = \boxed{0.091\,\text{V}}$$

$$V_L = I_{\rm rms}X_L = (2.871\times10^{-5}\,\text{A})(16.59\,\Omega) = \boxed{4.8\times10^{-4}\,\text{V}}$$

91\. (*a*) The impedance of the circuit is given by Eq. 30-28b with $X_L > X_C$ and $R = 0$. We divide the magnitude of the ac voltage by the impedance to get the magnitude of the ac current in the circuit. Since $X_L > X_C$, the voltage will lead the current by $\phi = \pi/2$. No dc current will flow through the capacitor.

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \omega L - 1/\omega C \qquad I_0 = \frac{V_{20}}{Z} = \frac{V_{20}}{\omega L - 1/\omega C}$$

$$I(t) = \boxed{\frac{V_{20}}{\omega L - 1/\omega C}\sin(\omega t - \pi/2)}$$

(*b*) The voltage across the capacitor at any instant is equal to the charge on the capacitor divided by the capacitance. This voltage is the sum of the ac voltage and dc voltage. There is no dc voltage drop across the inductor so the dc voltage drop across the capacitor is equal to the input dc voltage.

$$V_{\rm out,ac} = V_{\rm out} - V_1 = \frac{Q}{C} - V_1$$

We treat the emf as a superposition of the ac and dc components. At any instant of time the sum of the voltage across the inductor and capacitor will equal the input voltage. We use Eq. 30-5 to calculate the voltage drop across the inductor. Subtracting the voltage drop across the inductor from the input voltage gives the output voltage. Finally, we subtract off the dc voltage to obtain the ac output voltage.

$$V_L = L\frac{dI}{dt} = L\frac{d}{dt}\left[\frac{V_{20}}{\omega L - 1/\omega C}\sin(\omega t - \pi/2)\right] = \frac{V_{20}L\omega}{\omega L - 1/\omega C}\cos(\omega t - \pi/2)$$

$$= \frac{V_{20}L\omega}{\omega L - 1/\omega C}\sin(\omega t)$$

$$V_{\rm out} = V_{\rm in} - V_L = V_1 + V_{20}\sin\omega t - \left(\frac{V_{20}L\omega}{\omega L - 1/\omega C}\sin(\omega t)\right)$$

$$= V_1 + V_{20}\left(1 - \frac{L\omega}{\omega L - 1/\omega C}\right)\sin(\omega t) = V_1 - V_{20}\left(\frac{1/\omega C}{\omega L - 1/\omega C}\right)\sin(\omega t)$$

$$V_{\text{out,ac}} = V_{\text{out}} - V_1 = -V_{20}\left(\frac{1/\omega C}{\omega L - 1/\omega C}\right)\sin(\omega t) = \boxed{\left(\frac{V_{20}}{\omega^2 LC - 1}\right)\sin(\omega t - \pi)}$$

(*c*) The attenuation of the ac voltage is greatest when the denominator is large.

$$\omega^2 LC \gg 1 \to \omega L \gg \frac{1}{\omega C} \to X_L \gg X_C$$

We divide the output ac voltage by the input ac voltage to obtain the attenuation.

$$\frac{V_{2,\text{out}}}{V_{2,\text{in}}} = \frac{\frac{V_{20}}{\omega^2 LC - 1}}{V_{20}} = \frac{1}{\omega^2 LC - 1} \approx \boxed{\frac{1}{\omega^2 LC}}$$

(*d*) The dc output is equal to the dc input, since there is no dc voltage drop across the inductor.

$$\boxed{V_{1,\text{out}} = V_1}$$

92. Since no dc current flows through the capacitor, there will be no dc current through the resistor. Therefore the dc voltage passes through the circuit with little attenuation. The ac current in the circuit is found by dividing the input ac voltage by the impedance (Eq. 30-28b) We obtain the output ac voltage by multiplying the ac current by the capacitive reactance. Dividing the result by the input ac voltage gives the attenuation.

$$V_{2,\text{out}} = IX_C = \frac{V_{20}X_C}{\sqrt{R^2 + X_C^2}} \quad \to \quad \frac{V_{2,\text{out}}}{V_{20}} = \frac{1}{\sqrt{R^2\omega^2C^2 + 1}} \approx \boxed{\frac{1}{R\omega C}}$$

93. (*a*) Since the three elements are connected in parallel, at any given instant in time they will all three have the same voltage drop across them. That is the voltages across each element will be in phase with the source. The current in the resistor is in phase with the voltage source with magnitude given by Ohm's law.

$$I_R(t) = \boxed{\frac{V_0}{R}\sin\omega t}$$

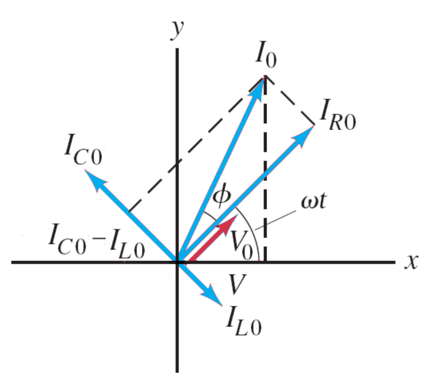
(*b*) The current through the inductor will lag behind the voltage by π/2, with magnitude equal to the voltage source divided by the inductive reactance.

$$I_L(t) = \boxed{\frac{V_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)}$$

(*c*) The current through the capacitor leads the voltage by π/2, with magnitude equal to the voltage source divided by the capacitive reactance.

$$I_C(t) = \boxed{\frac{V_0}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)}$$

(*d*) The total current is the sum of the currents through each element. We use a phasor diagram to add the currents, as was used in Section 30-8 to add the voltages with different phases. The net current is found by subtracting the current through the inductor from the current through the capacitor. Then using the Pythagorean theorem to add the current through the resistor. We use the tangent function to find the phase angle between the current and voltage source.




$$I_0 = \sqrt{I_{R0}^2 + (I_{C0} - I_{L0})^2} = \sqrt{\left(\frac{V_0}{R}\right)^2 + \left(\frac{V_0}{X_C} - \frac{V_0}{X_L}\right)^2} = \frac{V_0}{R}\sqrt{1 + \left(R\omega C - \frac{1}{R\omega L}\right)^2}$$

$$I(t)=\boxed{\frac{V_0}{R}\sqrt{1+\left(R\omega C-\frac{R}{\omega L}\right)^2}\sin(\omega t+\phi)}$$

$$\tan\phi=\frac{\frac{V_0}{X_C}-\frac{V_0}{X_L}}{\frac{V_0}{R}}\rightarrow\phi=\tan^{-1}\left(\frac{R}{X_C}-\frac{R}{X_L}\right)=\boxed{\tan^{-1}\left(R\omega C-\frac{R}{\omega L}\right)}$$

(*e*) We divide the magnitude of the voltage source by the magnitude of the current to find the impedance.

$$Z=\frac{V_0}{I_0}=\frac{V_0}{\frac{V_0}{R}\sqrt{1+\left(R\omega C-\frac{R}{\omega L}\right)^2}}=\boxed{\frac{R}{\sqrt{1+\left(R\omega C-\frac{R}{\omega L}\right)^2}}}$$

(*f*) The power factor is the ratio of the power dissipated in the circuit divided by the product of the rms voltage and current.

$$\frac{I_{R,\text{rms}}^2R}{V_{\text{rms}}I_{\text{rms}}}=\frac{I_R^2R}{V_0I_0}=\frac{\left(\frac{V_0}{R}\right)^2R}{V_0\frac{V_0}{R}\sqrt{1+\left(R\omega C-\frac{R}{\omega L}\right)^2}}=\boxed{\frac{1}{\sqrt{1+\left(R\omega C-\frac{R}{\omega L}\right)^2}}}$$

94. We find the equivalent values for each type of element in series. From the equivalent values we calculate the impedance using Eq. 30-28b.

$$R_{\text{eq}}=R_1+R_2\qquad\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}\qquad L_{\text{eq}}=L_1+L_2$$

$$Z=\sqrt{R_{\text{eq}}^2+\left(\omega L_{\text{eq}}-\frac{1}{\omega C_{\text{eq}}}\right)^2}=\boxed{\sqrt{(R_1+R_2)^2+\left(\omega L_1+\omega L_2-\frac{1}{\omega C_1}-\frac{1}{\omega C_2}\right)^2}}$$

95. If there is no current in the secondary, there will be no induced emf from the mutual inductance. Therefore, we set the ratio of the voltage to current equal to the inductive reactance and solve for the inductance.

$$\frac{V_{\text{rms}}}{I_{\text{rms}}}=X_L=2\pi fL\rightarrow L=\frac{V_{\text{rms}}}{2\pi f\,I_{\text{rms}}}=\frac{220\text{ V}}{2\pi(60\text{ Hz})(4.3\text{ A})}=\boxed{0.14\text{ H}}$$

96. (*a*) We use Eq. 24-2 to calculate the capacitance, assuming a parallel plate capacitor.

$$C=\frac{K\varepsilon_o A}{d}=\frac{(5.0)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.0\times10^{-4}\text{m}^2\right)}{2.0\times10^{-3}\text{m}}=2.213\times10^{-12}\,\text{F}\approx\boxed{2.2\,\text{pF}}$$

(*b*) We use Eq. 30-25b to calculate the capacitive reactance.

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi(12000\,\text{Hz})\left(2.2\times10^{-12}\,\text{F}\right)}=5.995\times10^6\,\Omega\approx\boxed{6.0\,\text{M}\Omega}$$

(*c*) Assuming that the resistance in the plasma and in the person is negligible compared with the capacitive reactance, calculate the current by dividing the voltage by the capacitive reactance.

$$I_o\approx\frac{V_o}{X_C}=\frac{2500\text{ V}}{5.995\times10^6\,\Omega}=4.17\times10^{-4}\,\text{A}\approx\boxed{0.42\,\text{mA}}$$

This is not a dangerous current level.

(*d*) We replace the frequency with 1.0 MHz and recalculate the current.

$$I_0 \approx \frac{V_0}{X_C} = 2\pi fCV_0 = 2\pi\left(1.0\times10^6\,\text{Hz}\right)\left(2.2\times10^{-12}\,\text{F}\right)\left(2500\,\text{V}\right) = \boxed{35\,\text{mA}}$$

This current level is dangerous.

97. We calculate the resistance from the power dissipated and the current. Then setting the ratio of the voltage to current equal to the impedance, we solve for the inductance.

$$\overline{P} = I_{\text{rms}}^2 R \rightarrow R = \frac{\overline{P}}{I_{\text{rms}}^2} = \frac{350\,\text{W}}{\left(4.0\,\text{A}\right)^2} = 21.88\,\Omega \approx \boxed{22\,\Omega}$$

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \sqrt{R^2 + \left(2\pi fL\right)^2} \quad \rightarrow$$

$$L = \frac{\sqrt{\left(V_{\text{rms}}/I_{\text{rms}}\right)^2 - R^2}}{2\pi f} = \frac{\sqrt{\left(120\,\text{V}/4.0\,\text{A}\right)^2 - \left(21.88\,\Omega\right)^2}}{2\pi\left(60\,\text{Hz}\right)} = \boxed{54\,\text{mH}}$$

98. We insert the proposed current into the differential equation and solve for the unknown peak current and phase.

$$\begin{aligned} V_0 \sin\omega t &= L\frac{d}{dt}\left[I_0 \sin\left(\omega t - \phi\right)\right] + RI_0 \sin\left(\omega t - \phi\right) \\ &= L\omega I_0 \cos\left(\omega t - \phi\right) + RI_0 \sin\left(\omega t - \phi\right) \\ &= L\omega I_0\left(\cos\omega t\cos\phi + \sin\omega t\sin\phi\right) + RI_0\left(\sin\omega t\cos\phi - \cos\omega t\sin\phi\right) \\ &= \left(L\omega I_0\cos\phi - RI_0\sin\phi\right)\cos\omega t + \left(L\omega I_0\sin\phi + RI_0\cos\phi\right)\sin\omega t \end{aligned}$$

For the given equation to be a solution for all time, the coefficients of the sine and cosine terms must independently be equal.

For the $\cos\omega t$ term:

$$0 = L\omega I_0\cos\phi - RI_0\sin\phi \rightarrow \tan\phi = \frac{\omega L}{R} \rightarrow \boxed{\phi = \tan^{-1}\frac{\omega L}{R}}$$

For the $\sin\omega t$ term:

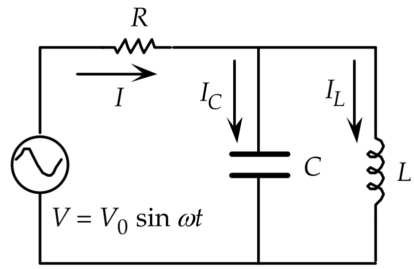
$$V_0 = L\omega I_0\sin\phi + RI_0\cos\phi$$

$$I_0 = \frac{V_0}{L\omega\sin\phi + R\cos\phi} = \frac{V_0}{L\omega\frac{\omega L}{\sqrt{R^2+\omega^2L^2}} + R\frac{R}{\sqrt{R^2+\omega^2L^2}}} = \boxed{\frac{V_0}{\sqrt{R^2+\omega^2L^2}}}$$

99. The peak voltage across either element is the current through the element multiplied by the reactance. We set the voltage across the inductor equal to six times the voltage across the capacitor and solve for the frequency in terms of the resonant frequency, Eq. 30-14.

$$V_L = I_0 2\pi fC = 6V_C = \frac{6I_0}{2\pi fC} \quad \rightarrow \quad \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6}{LC}} = \boxed{\sqrt{6}f_0}$$

100. We use Kirchhoff's junction rule to write an equation relating the currents in each branch, and the loop rule to write two equations relating the voltage drops around each loop. We write the voltage drops across the capacitor and inductor in terms of the charge and derivative of the current.




$$I_R = I_L + I_C$$

$$V_0 \sin \omega t - I_R R - \frac{Q_C}{C} = 0 \ ; \ V_0 \sin \omega t - I_R R - L\frac{dI_L}{dt} = 0$$

We combine these equations to eliminate the charge in the capacitor and the current in the inductor to write a single differential equation in terms of the current through the resistor.

$$\frac{dI_L}{dt} = \frac{V_0}{L}\sin \omega t - \frac{I_R R}{L}$$

$$\frac{dI_C}{dt} = \frac{d^2 Q_C}{dt^2} = \frac{d^2}{dt^2}\left(CV_0 \sin \omega t - I_R RC\right) = -CV_0\omega^2 \sin \omega t - RC\frac{dI_R^{\ 2}}{dt^2}$$

$$\frac{dI_R}{dt} = \frac{dI_L}{dt} + \frac{dI_C}{dt} = -\frac{V_0}{L}\sin \omega t + \frac{I_R R}{L} - CV_0\omega^2 \sin \omega t - RC\frac{dI_R^{\ 2}}{dt^2}$$

We set the current in the resistor, $I_R = I_0 \sin\left(\omega t + \phi\right) = I_0\left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right)$, equal to the current provided by the voltage source and take the necessary derivatives.

$$I_0 \frac{d}{dt}\left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right) = \frac{V_0}{L}\sin \omega t - \left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right)\frac{I_0 R}{L} - CV_0\omega^2 \sin \omega t$$
$$- RCI_0 \frac{d^2}{dt^2}\left(\sin \omega t \cos \phi + \cos \omega t \sin \phi\right)$$

$$I_0\omega \cos \omega t \cos \phi - I_0\omega \sin \omega t \sin \phi = \frac{V_0}{L}\sin \omega t - \frac{I_0 R}{L}\sin \omega t \cos \phi + \frac{I_0 R}{L}\cos \omega t \sin \phi - CV_0\omega^2 \sin \omega t$$
$$+ RCI_0\omega^2 \sin \omega t \cos \phi + RCI_0\omega^2 \cos \omega t \sin \phi$$

Setting the coefficients of the time dependent sine and cosine terms separately equal to zero enables us to solve for the magnitude and phase of the current through the voltage source. We also use Eq. 30-23b and Eq. 30-25b to write the inductance and capacitance in terms of their respective reactances.

From the $\cos(\omega t)$ term:

$$I_0\omega \cos \phi = \frac{I_0\omega R}{X_L}\sin \phi - \frac{I_0 R\omega}{X_C}\sin \phi \rightarrow \tan \phi = \frac{X_L X_C}{R\left(X_L - X_C\right)} \rightarrow \phi = \tan^{-1}\left[\frac{X_L X_C}{R\left(X_L - X_C\right)}\right]$$

From the $\sin(\omega t)$ term:

$$-I_0\omega \sin \phi = \frac{V_0\omega}{X_L} - \frac{I_0\omega R}{X_L}\cos \phi - \frac{V_0\omega}{X_C} + \frac{RI_0\omega}{X_C}\cos \phi$$

$$I_0 = \frac{V_0\left(X_C - X_L\right)}{X_C X_L \sin \phi + R\left(X_C - X_L\right)\cos \phi}$$

$$= \frac{V_0\left(X_C - X_L\right)}{X_C X_L \frac{X_C X_L}{\sqrt{\left(X_C X_L\right)^2 + R^2\left(X_C - X_L\right)^2}} + R\left(X_C - X_L\right)\frac{R\left(X_C - X_L\right)}{\sqrt{\left(X_C X_L\right)^2 + R^2\left(X_C - X_L\right)^2}}}$$

$$= \frac{V_0\left(X_C - X_L\right)}{\sqrt{\left(X_C X_L\right)^2 + R^2\left(X_C - X_L\right)^2}}$$

This gives us the current through the power source and resistor. We insert these values back into the junction and loop equations to determine the current in each element as a function of time. We calculate the impedance of the circuit by dividing the peak voltage by the peak current through the voltage source.

$$Z=\frac{V_0}{I_0}=\boxed{\frac{\sqrt{\left(X_C X_L\right)^2+R^2\left(X_C-X_L\right)^2}}{\left(X_C-X_L\right)}}\ ;\ \phi=\boxed{\tan^{-1}\left[\frac{X_L X_C}{R\left(X_L-X_C\right)}\right]}\ ;\ I_R=\boxed{\frac{V_0}{Z}\sin\left(\omega t+\phi\right)}$$

$$I_C=\frac{dQ_C}{dt}=\frac{d}{dt}\left(CV_0\sin\omega t-I_R RC\right)=CV_0\omega\cos\omega t-RC\frac{dI_R}{dt}$$

$$=\boxed{\frac{V_0}{X_C}\left[\cos\omega t-\frac{R}{Z}\cos\left(\omega t+\phi\right)\right]}$$

$$I_L=I_R-I_C=\frac{V_0}{Z}\sin\left(\omega t+\phi\right)-\frac{V_0}{X_C}\left[\cos\omega t-\frac{R}{Z}\cos\left(\omega t+\phi\right)\right]$$

$$=\boxed{\frac{V_0}{Z}\left[\sin\left(\omega t+\phi\right)+\frac{R}{X_C}\cos\left(\omega t+\phi\right)\right]-\frac{V_0}{X_C}\cos\omega t}$$

101. (*a*) The resonant frequency is given by Eq. 30-32. At resonance, the impedance is equal to the resistance, so the rms voltage of the circuit is equal to the rms voltage across the resistor.

$$f=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{\left(0.0050\,\text{H}\right)\left(0.10\times10^{-6}\,\text{F}\right)}}=7118\,\text{Hz}\approx\boxed{7.1\,\text{kHz}}$$

$$\left(V_R\right)_{\text{rms}}=\boxed{V_{\text{rms}}}$$

(*b*) We set the inductance equal to 90% of the initial inductance and use Eq. 30-28b to calculate the new impedance. Dividing the rms voltage by the impedance gives the rms current. We multiply the rms current by the resistance to determine the voltage drop across the resistor.

$$X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi\left(7118\,\text{Hz}\right)\left(0.10\times10^{-6}\,\text{F}\right)}=223.6\,\Omega$$

$$X_L=2\pi fL=2\pi\left(7118\,\text{Hz}\right)\left(0.90\right)\left(0.0050\,\text{H}\right)=201.3\,\Omega$$

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}=\sqrt{\left(45\,\Omega\right)^2+\left(201.3\,\Omega-223.6\,\Omega\right)^2}=50.24\,\Omega$$

$$\left(V_R\right)_{\text{rms}}=\left(\frac{R}{Z}\right)V_{\text{rms}}=\left(\frac{45\,\Omega}{50.24\,\Omega}\right)V_{\text{rms}}=\boxed{0.90V_{\text{rms}}}$$

102. With the given applied voltage, calculate the rms current through each branch as the rms voltage divided by the impedance in that branch.

$$I_{C,\text{rms}}=\frac{V_{\text{rms}}}{\sqrt{R_1^2+X_C^2}}\qquad I_{L,\text{rms}}=\frac{V_{\text{rms}}}{\sqrt{R_2^2+X_L^2}}$$

Calculate the potential difference between points a and b in two ways. First pass through the capacitor and then through $R_2$. Then pass through $R_1$ and the inductor.

$$V_{\text{ab}}=I_C X_C-I_L R_2=\frac{V_{\text{rms}}X_C}{\sqrt{R_1^2+X_C^2}}-\frac{V_{\text{rms}}R_2}{\sqrt{R_2^2+X_L^2}}$$

$$V_{ab} = -I_C R_1 + X_L I_L = -\frac{V_{rms} R_1}{\sqrt{R_1^2 + X_C^2}} + \frac{V_{rms} X_L}{\sqrt{R_2^2 + X_L^2}}$$

Set these voltage differences equal to zero, and rearrange the equations.

$$\frac{V_{rms} X_C}{\sqrt{R_1^2 + X_C^2}} - \frac{V_{rms} R_2}{\sqrt{R_2^2 + X_L^2}} = 0 \rightarrow X_C \sqrt{R_2^2 + X_L^2} = R_2 \sqrt{R_1^2 + X_C^2}$$

$$-\frac{V_{rms} R_1}{\sqrt{R_1^2 + X_C^2}} + \frac{V_{rms} X_L}{\sqrt{R_2^2 + X_L^2}} = 0 \rightarrow R_1 \sqrt{R_2^2 + X_L^2} = X_L \sqrt{R_1^2 + X_C^2}$$

Divide the resulting equations and solve for the product of the resistances. Write the reactances in terms of the capacitance and inductance to show that the result is frequency independent.

$$\frac{X_C \sqrt{R_2^2 + X_L^2}}{R_1 \sqrt{R_2^2 + X_L^2}} = \frac{R_2 \sqrt{R_1^2 + X_C^2}}{X_L \sqrt{R_1^2 + X_C^2}} \rightarrow R_1 R_2 = X_L X_C = \frac{\omega L}{\omega C} \rightarrow \boxed{R_1 R_2 = \frac{L}{C}}$$

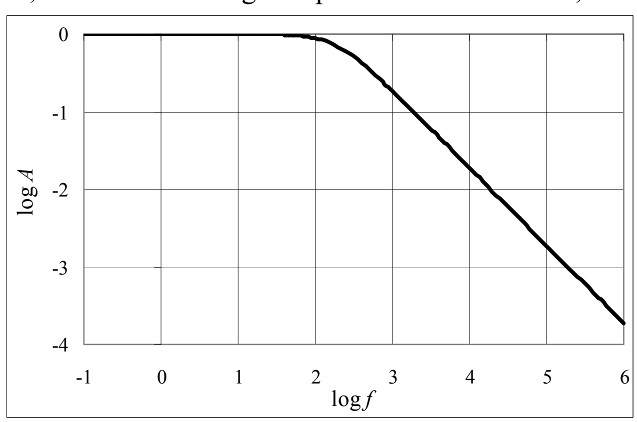
103. (*a*) The output voltage is the voltage across the capacitor, which is the current through the circuit multiplied by the capacitive reactance. We calculate the current by dividing the input voltage by the impedance. Finally, we divide the output voltage by the input voltage to calculate the gain.

$$V_{out} = IX_C = \frac{V_{in} X_C}{\sqrt{R^2 + X_C^2}} = \frac{V_{in}}{\sqrt{(R/X_C)^2 + 1}} = \frac{V_{in}}{\sqrt{(2\pi fCR)^2 + 1}}$$

$$A = \frac{V_{out}}{V_{in}} = \boxed{\frac{1}{\sqrt{4\pi^2 f^2 C^2 R^2 + 1}}}$$

(*b*) As the frequency goes to zero, the gain becomes one. In this instance the capacitor becomes fully charged, so no current flows across the resistor. Therefore the output voltage is equal to the input voltage. As the frequency becomes very large, the capacitive reactance becomes very small, allowing a large current. In this case, most of the voltage drop is across the resistor, and the gain goes to zero.

(*c*) See the graph of the log of the gain as a function of the log of the frequency. Note that for frequencies less than about 100 Hz the gain is ~ 1. For higher frequencies the gain drops off proportionately to the frequency. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.103c."



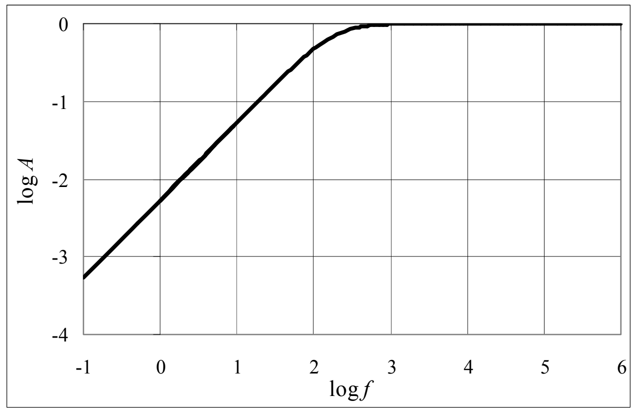

104. (*a*) The output voltage is the voltage across the resistor, which is the current through the circuit multiplied by the resistance. We calculate the current by dividing the input voltage by the impedance. Finally, we divide the output voltage by the input voltage to calculate the gain.

$$V_{out} = IR = \frac{V_{in} R}{\sqrt{R^2 + X_C^2}} = \frac{V_{in} R}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}} = \frac{2\pi fCRV_{in}}{\sqrt{(2\pi fCR)^2 + 1}}$$

$$A = \frac{V_{\text{out}}}{V_{\text{in}}} = \boxed{\frac{2\pi fCR}{\sqrt{4\pi^2 f^2 C^2 R^2 + 1}}}$$

(*b*) As the frequency goes to zero, the gain drops to zero. In this instance the capacitor becomes fully charged, so no current flows across the resistor. Therefore the output voltage drops to zero. As the frequency becomes very large, the capacitive reactance becomes very small, allowing a large current. In this case, most of the voltage drop is across the resistor, and the gain is equal to unity.

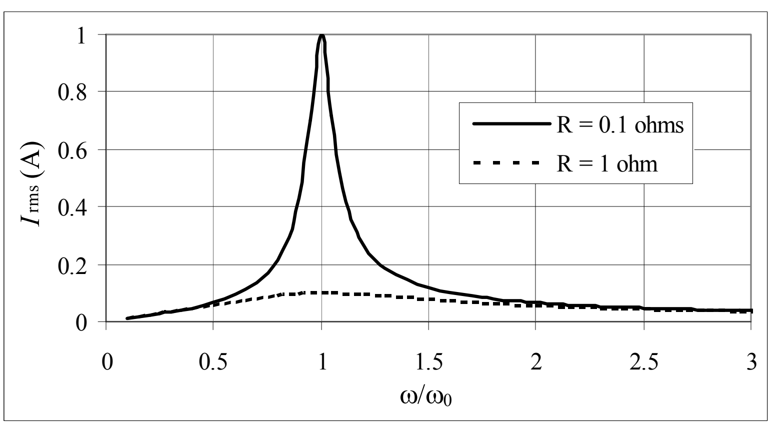
(*c*) See the graph of the log of the gain as a function of the log of the frequency. Note that for frequencies greater than about 1000 Hz the gain is ~ 1. For lower frequencies the gain drops off proportionately to the inverse of the frequency. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.104c."




105. We calculate the resonant frequency using Eq. 30-32.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(50\times10^{-6}\,\text{H})(50\times10^{-6}\,\text{F})}} = 20{,}000\,\text{rad/s}$$

Using a spreadsheet, we calculate the impedance as a function of frequency using Eq. 30-28b. We divide the rms voltage by the impedance to plot the rms current as a function of frequency. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH30.XLS," on tab "Problem 30.105."



# CHAPTER 31: Maxwell's Equations and Electromagnetic Waves

## Responses to Questions

1. The magnetic field will be clockwise in both cases. In the first case, the electric field is away from you and is increasing. The direction of the displacement current (proportional to $\frac{d\Phi_E}{dt}$) is therefore away from you and the corresponding magnetic field is clockwise. In the second case, the electric field is directed towards you and is decreasing; the displacement current is still away from you, and the magnetic field is still clockwise.

2. The displacement current is to the right.

3. The displacement current is spread out over a larger area than the conduction current. Thus, the displacement current produces a less intense field at any given point.

4. One possible reason the term $\varepsilon_0 \frac{d\Phi_E}{dt}$ can be called a "current" is because it has units of amperes.

5. The magnetic field vector will oscillate up and down, perpendicular to the direction of propagation and to the electric field vector.

6. No. Sound is a longitudinal mechanical wave. It requires the presence of a medium; electromagnetic waves do not require a medium.

7. EM waves are self-propagating and can travel through a perfect vacuum. Sound waves are mechanical waves which require a medium, and therefore cannot travel through a perfect vacuum.

8. No. Electromagnetic waves travel at a very large but finite speed. When you flip on a light switch, it takes a very small amount of time for the electrical signal to travel along the wires.

9. The wavelengths of radio and television signals are longer than those of visible light.

10. The wavelength of the current is 5000 km; the house is only 200 km away. The phase of the current at the position of the house is $2\pi/25$ radians different from the phase at the source due to the position of the house.

11. The signals travel through the wires at close to the speed of light, so the length of the wires in a normal room will have an insignificant effect.

12. $10^3$ km: radio wave; 1 km: radio wave; 1 m: microwave; 1 cm: microwave; 1 mm: microwave or infrared; 1 μm: infrared.

13. Yes, although the wavelengths for radio waves will be much longer than for sound waves, since the radio waves travel at the speed of light.

14. Both cordless phones and cellular phones are radio receivers and transmitters. When you speak, the phone converts the sound waves into electrical signals which are amplified, modulated, and transmitted. The receiver picks up the EM waves and converts them back into sound. Cordless phones and cell phones use different frequency ranges and different intensities.

15. Yes. If one signal is sent by amplitude modulation and the other signal is sent by frequency modulation, both could be carried over the same carrier frequency. There are other ways two signals can be sent on the same carrier frequency which are more complex.

16. The receiver's antenna should also be vertical for the best reception.

17. Diffraction is significant when the order of magnitude of the wavelength of the waves is the same as the size of the obstacles. AM waves have longer wavelengths than FM waves and will be more likely to diffract around hills and other landscape barriers.

18. It is amplitude modulated, or AM. The person flashing the light on and off is changing the amplitude of the light ("on" is maximum amplitude and "off" is zero). The frequency of the carrier wave is just the frequency of the visible light, approximately $10^{14}$ to $10^{15}$ Hz.

## Solutions to Problems

1. The electric field between the plates is given by $E = \frac{V}{d}$, where $d$ is the distance between the plates.

$$E = \frac{V}{d} \quad \rightarrow \quad \frac{dE}{dt} = \frac{1}{d}\frac{dV}{dt} = \left(\frac{1}{0.0011\,\text{m}}\right)(120\,\text{V/s}) = \boxed{1.1\times10^5\,\frac{\text{V/m}}{\text{s}}}$$

2. The displacement current is shown in section 31-1 to be $I_\text{D} = \varepsilon_0 A\frac{dE}{dt}$.

$$I_\text{D} = \varepsilon_0 A\frac{dE}{dt} = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(0.058\,\text{m})^2\left(2.0\times10^6\,\frac{\text{V}}{\text{m•s}}\right) = \boxed{6.0\times10^{-8}\,\text{A}}$$

3. The current in the wires must also be the displacement current in the capacitor. Use the displacement current to find the rate at which the electric field is changing.

$$I_D = \varepsilon_0 A\frac{dE}{dt} \quad \rightarrow \quad \frac{dE}{dt} = \frac{I_\text{D}}{\varepsilon_0 A} = \frac{(2.8\,\text{A})}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(0.0160\,\text{m})^2} = \boxed{1.2\times10^{15}\,\frac{\text{V}}{\text{m•s}}}$$

4. The current in the wires is the rate at which charge is accumulating on the plates and also is the displacement current in the capacitor. Because the location in question is outside the capacitor, use the expression for the magnetic field of a long wire.

$$B = \frac{\mu_0 I}{2\pi R} = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{R} = \frac{\left(10^{-7}\,\text{T•m/A}\right)2\left(38.0\times10^{-3}\,\text{A}\right)}{(0.100\,\text{m})} = \boxed{7.60\times10^{-8}\,\text{T}}$$

After the capacitor is fully charged, all currents will be zero, so the magnetic field will be $\boxed{\text{zero.}}$

5. The electric field between the plates is given by $E=\frac{V}{d}$, where *d* is the distance between the plates.

The displacement current is shown in section 31-1 to be $I_D=\varepsilon_0 A\frac{dE}{dt}$.

$$I_D=\varepsilon_0 A\frac{dE}{dt}=\varepsilon_0 A\frac{1}{d}\frac{dV}{dt}=\frac{\varepsilon_0 A}{d}\frac{dV}{dt}=\boxed{C\frac{dV}{dt}}$$

6. (*a*) The footnote on page 816 indicates that Kirchhoff's junction rule is valid at a capacitor plate, and so the conduction current is the same value as the displacement current. Thus the maximum conduction current is $\boxed{35\mu\text{A}}$.

(*b*) The charge on the pages is given by $Q=CV=C\mathscr{E}_0\cos\omega t$. The current is the derivative of this.

$$I=\frac{dQ}{dt}=-\omega C\mathscr{E}_0\sin\omega t\ ;\ I_{\max}=\omega C\mathscr{E}_0\ \rightarrow$$

$$\mathscr{E}_0=\frac{I_{\max}}{\omega C}=\frac{I_{\max}d}{2\pi f\varepsilon_0 A}=\frac{\left(35\times10^{-6}\text{A}\right)\left(1.6\times10^{-3}\text{m}\right)}{2\pi\left(76.0\,\text{Hz}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\pi\left(0.025\text{m}\right)^2}$$

$$=6749\,\text{V}\approx\boxed{6700\,\text{V}}$$

(*c*) From Eq. 31-3, $I_D=\varepsilon_0\frac{d\Phi_E}{dt}$.

$$I_D=\varepsilon_0\frac{d\Phi_E}{dt}\ \rightarrow\ \left(\frac{d\Phi_E}{dt}\right)_{\max}=\frac{\left(I_D\right)_{\max}}{\varepsilon_0}=\frac{35\times10^{-6}\text{A}}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2}=\boxed{4.0\times10^6\ \text{V}\bullet\text{m/s}}$$

7. (*a*) We follow the development and geometry given in Example 31-1, using R for the radial distance. The electric field between the plates is given by $E=\frac{V}{d}$, where *d* is the distance between the plates.

$$\oint\vec{\mathbf{B}}\bullet d\vec{\boldsymbol{\ell}}=\mu_0\varepsilon_0\frac{d\Phi_E}{dt}\ \rightarrow\ B\left(2\pi R_{\text{path}}\right)=\mu_0\varepsilon_0\frac{d\left(\pi R_{\text{flux}}^2E\right)}{dt}$$

The subscripts are used on the radial variable because there might not be electric field flux through the entire area bounded by the amperian path. The electric field between the plates is given by $E=\frac{V}{d}=\frac{V_0\sin\left(2\pi ft\right)}{d}$, where *d* is the distance between the plates.

$$B\left(2\pi R_{\text{path}}\right)=\mu_0\varepsilon_0\frac{d\left(\pi R_{\text{flux}}^2E\right)}{dt}\ \rightarrow$$

$$B=\frac{\mu_0\varepsilon_0\pi R_{\text{flux}}^2}{2\pi R_{\text{path}}}\frac{d\left(E\right)}{dt}=\frac{\mu_0\varepsilon_0\pi R_{\text{flux}}^2}{2\pi R_{\text{path}}}\frac{2\pi fV_0}{d}\cos\left(2\pi ft\right)=\frac{\mu_0\varepsilon_0 R_{\text{flux}}^2}{R_{\text{path}}}\frac{\pi fV_0}{d}\cos\left(2\pi ft\right)$$

We see that the functional form of the magnetic field is $\boxed{B=B_0\left(R\right)\cos\left(2\pi ft\right)}$.

(*b*) If $R \le R_0$, then there is electric flux throughout the area bounded by the amperian loop, and so $R_{\text{path}} = R_{\text{flux}} = R.$

$$B_0(R \le R_0) = \frac{\mu_0\varepsilon_0 R_{\text{flux}}^2}{R_{\text{path}}}\frac{\pi f V_0}{d} = \mu_0\varepsilon_0\frac{\pi f V_0}{d}R = \frac{\pi(60\,\text{Hz})(150\,\text{V})}{(3.00\times10^8\,\text{m/s})^2(5.0\times10^{-3}\,\text{m})}R$$

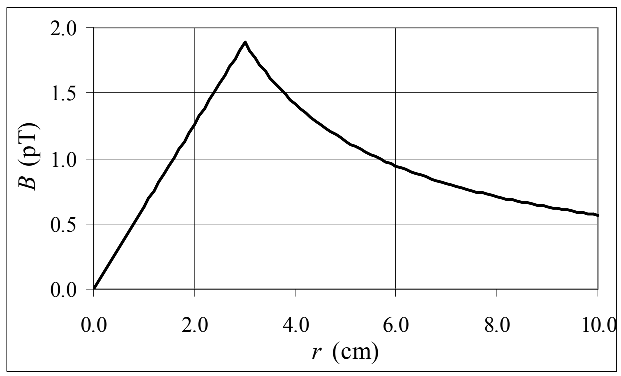
$$= (6.283\times10^{-11}\,\text{T/m})R \approx \boxed{(6.3\times10^{-11}\,\text{T/m})R}$$

If $R > R_0$, then there is electric flux only out a radial distance of $R_0$. Thus $R_{\text{path}} = R$ and $R_{\text{flux}} = R_0$.

$$B_0(R > R_0) = \frac{\mu_0\varepsilon_0 R_{\text{flux}}^2}{R_{\text{path}}}\frac{\pi f V_0}{d} = \mu_0\varepsilon_0\frac{\pi f V_0 R_0^2}{d}\frac{1}{R} = \frac{\pi(60\,\text{Hz})(150\,\text{V})(0.030\,\text{m})^2}{(3.00\times10^8\,\text{m/s})^2(5.0\times10^{-3}\,\text{m})}\frac{1}{R}$$

$$= (5.655\times10^{-14}\,\text{T}\bullet\text{m})\frac{1}{R} \approx \boxed{(5.7\times10^{-14}\,\text{T}\bullet\text{m})\frac{1}{R}}$$

(*c*) See the adjacent graph. Note that the magnetic field is continuous at the transition from "inside" to "outside" the capacitor radius. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH31.XLS," on tab "Problem 31.7c."




8. Use Eq. 31-11 with $v = c$.

$$\frac{E_0}{B_0} = c \quad\rightarrow\quad B_0 = \frac{E_0}{c} = \frac{0.57\times10^{-4}\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = \boxed{1.9\times10^{-13}\,\text{T}}$$

9. Use Eq. 31-11 with $v = c$.

$$\frac{E_0}{B_0} = c \quad\rightarrow\quad E_0 = B_0 c = (12.5\times10^{-9}\,\text{T})(3.00\times10^8\,\text{m/s}) = \boxed{3.75\,\text{V/m}}$$

10. The frequency of the two fields must be the same: $\boxed{80.0\,\text{kHz}}$. The rms strength of the electric field can be found from Eq. 31-11 with $v = c$.

$$E_{\text{rms}} = cB_{\text{rms}} = (3.00\times10^8\,\text{m/s})(7.75\times10^{-9}\,\text{T}) = \boxed{2.33\,\text{V/m}}$$

The electric field is perpendicular to both the direction of travel and the magnetic field, so the electric field oscillates along the $\boxed{\text{horizontal north-south line.}}$

11. (*a*) If we write the argument of the cosine function as $kz + \omega t = k(z + ct)$, we see that the wave is traveling in the $\boxed{-z \text{ direction}}$, or $\boxed{-\hat{\mathbf{k}}}$.

(*b*) $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other and to the direction of propagation. At the origin, the electric field is pointing in the positive *x* direction. Since $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ must point in the negative *z* direction, $\vec{\mathbf{B}}$ must point in the the $\boxed{-y \text{ direction}}$, or $\boxed{-\hat{\mathbf{j}}}$. The magnitude of the magnetic field is found from Eq. 31-11 as $B_0 = \boxed{E_0/c}$.

12. The wave equation to be considered is $v^2\dfrac{\partial^2 E}{\partial x^2}=\dfrac{\partial^2 E}{\partial t^2}$.

(*a*) Given $E(x,t)=Ae^{-\alpha(x-vt)^2}$.

$$\frac{\partial E}{\partial x}=Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)\right]$$

$$\frac{\partial^2 E}{\partial x^2}=Ae^{-\alpha(x-vt)^2}(-2\alpha)+Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)\right]^2=-2\alpha Ae^{-\alpha(x-vt)^2}\left[1-2\alpha(x-vt)^2\right]$$

$$\frac{\partial E}{\partial t}=Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)(-v)\right]=Ae^{-\alpha(x-vt)^2}\left[2\alpha v(x-vt)\right]$$

$$\frac{\partial^2 E}{\partial t^2}=Ae^{-\alpha(x-vt)^2}(-2\alpha v^2)+Ae^{-\alpha(x-vt)^2}\left[2\alpha v(x-vt)\right]^2=-2\alpha v^2Ae^{-\alpha(x-vt)^2}\left[1-2\alpha(x-vt)^2\right]$$

We see that $v^2\dfrac{\partial^2 E}{\partial x^2}=\dfrac{\partial^2 E}{\partial t^2}$, and so the wave equation is satisfied.

(*b*) Given $E(x,t)=Ae^{-(\alpha x^2-vt)}$.

$$\frac{\partial E}{\partial x}=Ae^{-(\alpha x^2-vt)}(-2\alpha x)$$

$$\frac{\partial^2 E}{\partial x^2}=Ae^{-(\alpha x^2-vt)}(-2\alpha)+Ae^{-(\alpha x^2-vt)}(-2\alpha x)^2=-2\alpha Ae^{-(\alpha x^2-vt)}\left[1-2\alpha x^2\right]$$

$$\frac{\partial E}{\partial t}=Ave^{-(\alpha x^2-vt)}\ ;\ \frac{\partial^2 E}{\partial t^2}=Av^2e^{-(\alpha x^2-vt)}$$

This does NOT satisfy $v^2\dfrac{\partial^2 E}{\partial x^2}=\dfrac{\partial^2 E}{\partial t^2}$, since $-2\alpha v^2Ae^{-(\alpha x^2-vt)}\left[1-2\alpha x^2\right]\neq Av^2e^{-(\alpha x^2-vt)}$ in general.

13. Use Eq. 31-14 to find the frequency of the microwave.

$$c=\lambda f \rightarrow f=\frac{c}{\lambda}=\frac{(3.00\times10^8\,\text{m/s})}{(1.50\times10^{-2}\,\text{m})}=\boxed{2.00\times10^{10}\,\text{Hz}}$$

14. Use Eq. 31-14 to find the wavelength and frequency.

(*a*) $$c=\lambda f \rightarrow \lambda=\frac{c}{f}=\frac{(3.000\times10^8\,\text{m/s})}{(25.75\times10^9\,\text{Hz})}=\boxed{1.165\times10^{-2}\,\text{m}}$$

(*b*) $$c=\lambda f \rightarrow f=\frac{c}{\lambda}=\frac{(3.00\times10^8\,\text{m/s})}{(0.12\times10^{-9}\,\text{m})}=\boxed{2.5\times10^{18}\,\text{Hz}}$$

15. Use the relationship that $d = vt$ to find the time.

$$d = vt \quad \rightarrow \quad t = \frac{d}{v} = \frac{\left(1.50 \times 10^{11}\,\text{m}\right)}{\left(3.00 \times 10^{8}\,\text{m/s}\right)} = 5.00 \times 10^{2}\,\text{s} = \boxed{8.33\,\text{min}}$$

16. Use Eq. 31-14 to find the wavelength.

$$c = \lambda f \quad \rightarrow \quad \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)}{\left(8.56 \times 10^{14}\,\text{Hz}\right)} = \boxed{3.50 \times 10^{-7}\,\text{m}} = 311\,\text{nm}$$

This wavelength is just outside the violet end of the visible region, so it is $\boxed{\text{ultraviolet.}}$

17. (*a*) Use Eq. 31-14 to find the wavelength.

$$c = \lambda f \quad \rightarrow \quad \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)}{\left(1.00 \times 10^{3}\,\text{Hz}\right)} = \boxed{3.00 \times 10^{5}\,\text{m}}$$

(*b*) Again use Eq. 31-14, with the speed of sound in place of the speed of light.

$$v = \lambda f \quad \rightarrow \quad \lambda = \frac{v}{f} = \frac{\left(341\,\text{m/s}\right)}{\left(1.00 \times 10^{3}\,\text{Hz}\right)} = \boxed{0.341\,\text{m}}$$

(*c*) $\boxed{\text{No}}$, you cannot hear a 1000-Hz EM wave. It takes a pressure wave to excite the auditory system. However, if you applied the 1000-Hz EM wave to a speaker, you could hear the 1000-Hz pressure wave.

18. The length of the pulse is $\Delta d = c\Delta t$. Use this to find the number of wavelengths in a pulse.

$$N = \frac{\left(c\Delta t\right)}{\lambda} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)\left(38 \times 10^{-12}\,\text{s}\right)}{\left(1062 \times 10^{-9}\,\text{m}\right)} = 10734 \approx \boxed{11{,}000 \text{ wavelengths}}$$

If the pulse is to be only one wavelength long, then its time duration is the period of the wave, which is the reciprocal of the wavelength.

$$T = \frac{1}{f} = \frac{\lambda}{c} = \frac{\left(1062 \times 10^{-9}\,\text{m}\right)}{\left(3.00 \times 10^{8}\,\text{m/s}\right)} = \boxed{3.54 \times 10^{-15}\,\text{s}}$$

$\boxed{19.}$ (*a*) The radio waves travel at the speed of light, and so $\Delta d = v\Delta t$. The distance is found from the radii of the orbits. For Mars when nearest the Earth, the radii should be subtracted.

$$\Delta t = \frac{\Delta d}{c} = \frac{\left(227.9 \times 10^{9}\,\text{m} - 149.6 \times 10^{9}\,\text{m}\right)}{\left(3.000 \times 10^{8}\,\text{m/s}\right)} = \boxed{261\,\text{s}}$$

(*b*) For Mars when farthest from Earth, the radii should be subtracted.

$$\Delta t = \frac{\Delta d}{c} = \frac{\left(227.9 \times 10^{9}\,\text{m} + 149.6 \times 10^{9}\,\text{m}\right)}{\left(3.000 \times 10^{8}\,\text{m/s}\right)} = \boxed{1260\,\text{s}}$$

20. (*a*) The general form of a plane wave is given in Eq. 31-7. For this wave, $E_x = E_0 \sin\left(kz - \omega t\right)$.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.077\,\text{m}^{-1}} = 81.60\,\text{m} \approx \boxed{82\,\text{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{2.3\times10^{7}\,\text{rad/s}}{2\pi} = 3.661\times10^{6}\,\text{Hz} \approx \boxed{3.7\,\text{MHz}}$$

Note that $\lambda f = \left(81.60\,\text{m}\right)\left(3.661\times10^{6}\,\text{Hz}\right) = 2.987\times10^{8}\,\text{m/s} \approx c$.

(*b*) The magnitude of the magnetic field is given by $B_0 = E_0/c$. The wave is traveling in the $\hat{\mathbf{k}}$ direction, and so the magnetic field must be in the $\hat{\mathbf{j}}$ direction, since the direction of travel is given by the direction of $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$.

$$B_0 = \frac{E_0}{c} = \frac{225\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = 7.50\times10^{-7}\,\text{T} \quad\rightarrow$$

$$\vec{\mathbf{B}} = \boxed{\hat{\mathbf{j}}\left(7.50\times10^{-7}\,\text{T}\right)\sin\left[\left(0.077\,\text{m}^{-1}\right)z - \left(2.3\times10^7\,\text{rad/s}\right)t\right]}$$

21. The eight-sided mirror would have to rotate 1/8 of a revolution for the succeeding mirror to be in position to reflect the light in the proper direction. During this time the light must travel to the opposite mirror and back.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\frac{1}{8}(2\pi\,\text{rad})}{(2\Delta x/c)} = \frac{(\pi\,\text{rad})c}{8\Delta x} = \frac{(\pi\,\text{rad})\left(3.00\times10^8\,\text{m/s}\right)}{8\left(35\times10^3\,\text{m}\right)} = \boxed{3400\,\text{rad/s}}\ \left(3.2\times10^4\,\text{rev/min}\right)$$

22. The average energy transferred across unit area per unit time is the average magnitude of the Poynting vector, and is given by Eq. 31-19a.

$$\bar{S} = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \tfrac{1}{2}\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(3.00\times10^8\,\text{m/s}\right)\left(0.0265\,\text{V/m}\right) = \boxed{9.32\times10^{-7}\,\text{W/m}^2}$$

23. The energy per unit area per unit time is given by the magnitude of the Poynting vector. Let $\Delta U$ represent the energy that crosses area $A$ in a time $\Delta T$.

$$S = \frac{cB_{\text{rms}}^2}{\mu_0} = \frac{\Delta U}{A\Delta t} \quad\rightarrow$$

$$\Delta t = \frac{\mu_0\Delta U}{AcB_{\text{rms}}^2} = \frac{\left(4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}\right)(335\,\text{J})}{\left(1.00\times10^{-4}\,\text{m}^2\right)\left(3.00\times10^8\,\text{m/s}\right)\left(22.5\times10^{-9}\,\text{T}\right)^2} = 0.194\,\text{W/m}^2$$

$$= \boxed{2.77\times10^7\,\text{s}} \approx 321\text{ days}$$

24. The energy per unit area per unit time is given by the magnitude of the Poynting vector. Let $\Delta U$ represent the energy that crosses area $A$ in a time $\Delta t$.

$$S = c\varepsilon_0 E_{\text{rms}}^2 = \frac{\Delta U}{A\Delta t} \quad\rightarrow$$

$$\frac{\Delta U}{\Delta t} = c\varepsilon_0 E_{\text{rms}}^2 A$$

$$= \left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.0328\,\text{V/m}\right)^2\left(1.00\times10^{-4}\,\text{m}^2\right)\left(3600\,\text{s/h}\right)$$

$$= \boxed{1.03\times10^{-6}\,\text{J/h}}$$

$\boxed{25.}$ The intensity is the power per unit area, and also is the time averaged value of the Poynting vector. The area is the surface area of a sphere, since the wave is spreading spherically.

$$\bar{S} = \frac{P}{A} = \frac{(1500\,\text{W})}{\left[4\pi(5.0\,\text{m})^2\right]} = 4.775\,\text{W/m}^2 \approx \boxed{4.8\,\text{W/m}^2}$$

$$\bar{S} = c\varepsilon_0 E_{\text{rms}}^2 \quad\rightarrow\quad E_{\text{rms}} = \sqrt{\frac{\bar{S}}{c\varepsilon_0}} = \sqrt{\frac{4.775\,\text{W/m}^2}{\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}} = \boxed{42\,\text{V/m}}$$

26. (*a*) We find $E$ using Eq. 31-11 with $v = c$.

$$E = cB = \left(3.00 \times 10^{8}\ \text{m/s}\right)\left(2.5 \times 10^{-7}\ \text{T}\right) = \boxed{75\ \text{V/m}}$$

(*b*) The average power per unit area is given by the Poynting vector, from Eq. 31-19a.

$$\overline{I} = \frac{E_0 B_0}{\left(2\mu_0\right)} = \frac{\left(75\ \text{V/m}\right)\left(2.5 \times 10^{-7}\ \text{T}\right)}{\left[2\left(4\pi \times 10^{-7}\ \text{N}\bullet\text{s}^2/\text{C}^2\right)\right]} = \boxed{7.5\ \text{W/m}^2}$$

27. From Eq. 31-16b, the instantaneous energy density is $u = \varepsilon_0 E^2$. From Eq. 31-17, we see that this instantaneous energy density is also given by $S/c$. The time-averaged value is therefore $\overline{S}/c$. Multiply that times the volume to get the energy.

$$U = uV = \frac{\overline{S}}{c}V = \frac{1350\ \text{W/m}^2}{3.00 \times 10^{8}\ \text{m/s}}\left(1.00\ \text{m}^3\right) = \boxed{4.50 \times 10^{-6}\ \text{J}}$$

28. The power output per unit area is the intensity, and also is the magnitude of the Poynting vector. Use Eq. 31-19a with rms values.

$$S = \frac{P}{A} = c\varepsilon_0 E_{\text{rms}}^2 \quad \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{0.0158\ \text{W}}{\pi\left(1.00 \times 10^{-3}\ \text{m}\right)^2\left(3.00 \times 10^{8}\ \text{m/s}\right)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}}$$

$$= 1376.3\ \text{V/m} \approx \boxed{1380\ \text{V/m}}$$

$$B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{1376.3\ \text{V/m}}{3.00 \times 10^{8}\ \text{m/s}} = \boxed{4.59 \times 10^{-6}\ \text{T}}$$

29. The radiation from the Sun has the same intensity in all directions, so the rate at which it reaches the Earth is the rate at which it passes through a sphere centered at the Sun with a radius equal to the Earth's orbit radius. The $1350\ \text{W/m}^2$ is the intensity, or the magnitude of the Poynting vector.

$$S = \frac{P}{A} \quad \rightarrow \quad P = SA = 4\pi R^2 S = 4\pi\left(1.496 \times 10^{11}\ \text{m}\right)^2\left(1350\ \text{W/m}^2\right) = \boxed{3.80 \times 10^{26}\ \text{W}}$$

30. (*a*) The energy emitted in each pulse is the power output of the laser times the time duration of the pulse.

$$P = \frac{\Delta W}{\Delta t} \quad \rightarrow \quad \Delta W = P\Delta t = \left(1.8 \times 10^{11}\ \text{W}\right)\left(1.0 \times 10^{-9}\ \text{s}\right) = \boxed{180\ \text{J}}$$

(*b*) We find the rms electric field from the intensity, which is the power per unit area. That is also the magnitude of the Poynting vector. Use Eq. 31-19a with rms values.

$$S = \frac{P}{A} = c\varepsilon_0 E_{\text{rms}}^2 \quad \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{\left(1.8 \times 10^{11}\ \text{W}\right)}{\pi\left(2.2 \times 10^{-3}\ \text{m}\right)^2\left(3.00 \times 10^{8}\ \text{m/s}\right)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}}$$

$$= \boxed{2.1 \times 10^{9}\ \text{V/m}}$$

31. In each case, the required area is the power requirement of the device divided by 10% of the intensity of the sunlight.

(*a*) $A = \frac{P}{I} = \frac{50\times10^{-3}\,\text{W}}{100\,\text{W/m}^2} = 5\times10^{-4}\,\text{m}^2 = \boxed{5\,\text{cm}^2}$

A typical calculator is about 17 cm x 8 cm, which is about $140\,\text{cm}^2$. So yes, the solar panel can be mounted directly on the calculator.

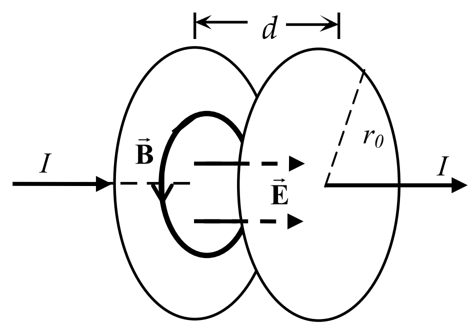
(*b*) $A = \frac{P}{I} = \frac{1500\,\text{W}}{100\,\text{W/m}^2} = 15\,\text{m}^2 \approx \boxed{20\,\text{m}^2}$ (to one sig. fig.)

A house of floor area $1000\,\text{ft}^2$ would have on the order of $100\,\text{m}^2$ of roof area. So yes, a solar panel on the roof should be able to power the hair dryer.

(*c*) $A = \frac{P}{I} = \frac{20\,\text{hp}(746\,\text{W/hp})}{100\,\text{W/m}^2} = 149\,\text{m}^2 \approx \boxed{100\,\text{m}^2}$ (to one sig. fig.)

This would require a square panel of side length about 12 m. So no, this panel could not be mounted on a car and used for real-time power.

32. (*a*) Example 31-1 refers back to Example 21-13 and Figure 21-31. In that figure, and the figure included here, the electric field between the plates is to the right. The magnetic field is shown as counterclockwise circles. Take any point between the capacitor plates, and find the direction of $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$. For instance, at the top of the circle shown in Figure 31-4, $\vec{\mathbf{E}}$ is toward the viewer, and $\vec{\mathbf{B}}$ is to the left. The cross product $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ points down, directly to the line connecting the center of the plates. Or take the rightmost point on the circle. $\vec{\mathbf{E}}$ is again toward the viewer, and $\vec{\mathbf{B}}$ is upwards. The cross product $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ points to the left, again directly to the line connecting the center of the plates. In cylindrical coordinates, $\vec{\mathbf{E}} = E\,\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B\hat{\boldsymbol{\varphi}}$. The cross product $\hat{\mathbf{k}}\times\hat{\boldsymbol{\varphi}} = -\hat{\mathbf{r}}$.




(*b*) We evaluate the Poynting vector, and then integrate it over the curved cylindrical surface between the capacitor plates. The magnetic field (from Example 31-1) is $B = \frac{1}{2}\mu_0\varepsilon_0 r_0 \frac{dE}{dt}$, evaluated at $r = r_0$. $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other, so $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{2}\varepsilon_0 r_0 E\frac{dE}{dt}$, inward. In calculating $\iint \vec{\mathbf{S}}\bullet d\vec{\mathbf{A}}$ for energy flow into the capacitor volume, note that both $\vec{\mathbf{S}}$ and $d\vec{\mathbf{A}}$ point inward, and that $S$ is constant over the curved surface of the cylindrical volume.

$$\iint \vec{\mathbf{S}}\bullet d\vec{\mathbf{A}} = \iint S\,dA = S\iint dA = SA = S2\pi r_0 d = \tfrac{1}{2}\varepsilon_0 r_0 E\frac{dE}{dt}2\pi r_0 d = \varepsilon_0 d\pi r_0^2 E\frac{dE}{dt}$$

The amount of energy stored in the capacitor is the energy density times the volume of the capacitor. The energy density is given by Eq. 24-6, $u = \frac{1}{2}\varepsilon_0 E^2$, and the energy stored is the energy density times the volume of the capacitor. Take the derivative of the energy stored with respect to time.

$$U = u(\text{Volume}) = \tfrac{1}{2}\varepsilon_0 E^2\pi r_0^2 d \quad\rightarrow\quad \frac{dU}{dt} = \varepsilon_0 E\pi r_0^2 d\frac{dE}{dt}$$

We see that $\iint \vec{\mathbf{S}}\bullet d\vec{\mathbf{A}} = \frac{dU}{dt}$.

33. (*a*) The intensity from a point source is inversely proportional to the distance from the source.

$$\frac{I_{\text{Sun}}}{I_{\text{Star}}}=\frac{r^2_{\text{Star-Earth}}}{r^2_{\text{Sun-Earth}}}\rightarrow r_{\text{Star-Earth}}=r_{\text{Sun-Earth}}\sqrt{\frac{I_{\text{Sun}}}{I_{\text{Star}}}}=\left(1.496\times10^{11}\,\text{m}\right)\sqrt{\frac{1350\,\text{W/m}^2}{1\times10^{-23}\,\text{W/m}^2}}\left(\frac{1\,\text{ly}}{9.46\times10^{15}\,\text{m}}\right)$$

$$=1.84\times10^{8}\,\text{ly}\approx\boxed{2\times10^{8}\,\text{ly}}$$

(*b*) Compare this distance to the galactic size.

$$\frac{r_{\text{Star-Earth}}}{\text{galactic size}}=\frac{1.84\times10^{8}\,\text{ly}}{1\times10^{5}\,\text{ly}}=1840\approx\boxed{2000}$$

The distance to the star is about 2000 times the size of our galaxy.

34. We assume the light energy is all absorbed, and so use Eq. 31-21a.

$$P=\frac{\bar{S}}{c}=\frac{\dfrac{75\,\text{W}}{4\pi\left(8.0\times10^{-2}\,\text{m}\right)^2}}{\left(3.00\times10^{8}\,\text{m/s}\right)}=3.108\times10^{-6}\,\text{N/m}^2\approx\boxed{3.1\times10^{-6}\,\text{N/m}^2.}$$

The force is pressure times area. We approximate the area of a fingertip to be 1.0 cm$^2$.

$$F=PA=\left(3.108\times10^{-6}\,\text{N/m}^2\right)\left(1.0\times10^{-4}\,\text{m}^2\right)=\boxed{3.1\times10^{-10}\,\text{N}}$$

35. The acceleration of the cylindrical particle will be the force on it (due to radiation pressure) divided by its mass. The light is delivering electromagnetic energy to an area $A$ at a rate of $\frac{dU}{dt}=1.0\,\text{W}$. That power is related to the average magnitude of the Poynting vector by $\bar{S}=\frac{dU/dt}{A}$. From Eq. 31-21a, that causes a pressure on the particle of $P=\frac{\bar{S}}{c}$, and the force due to that pressure is $F_{\text{laser}}=PA$. Combine these relationships with Newton's second law to calculate the acceleration. The mass of the particle is its volume times the density of water.

$$F_{\text{laser}}=PA=\frac{\bar{S}}{c}A=\frac{1}{c}\frac{dU}{dt}=ma=\rho_{H_2O}\pi r^2 ra\quad\rightarrow$$

$$a=\frac{dU/dt}{c\rho_{H_2O}\pi r^3}=\frac{(1.0\,\text{W})}{\left(3.00\times10^{8}\,\text{m/s}\right)\left(1000\,\text{kg/m}^2\right)\pi\left(5\times10^{-7}\,\text{m}\right)^3}=\boxed{8\times10^{6}\,\text{m/s}^2}$$

36. (*a*) The light is delivering electromagnetic energy to an area $A$ of the suit at a rate of $\frac{dU}{dt}=3.0\,\text{W}$. That power is related to the average magnitude of the Poynting vector by $\bar{S}=\frac{dU/dt}{A}$. From Eq. 31-21b, that causes a pressure on the suit of $P=\frac{2\bar{S}}{c}$, and the force due to that pressure is $F_{\text{laser}}=PA$. Combine these relationships to calculate the force.

$$F_{\text{laser}}=PA=\frac{2\bar{S}}{c}A=\frac{2}{c}\frac{dU}{dt}=\frac{2(3.0\,\text{W})}{3.00\times10^{8}\,\text{m/s}}=\boxed{2.0\times10^{-8}\,\text{N}}$$

(*b*) Use Newton's law of universal gravitation, Eq. 6-1, to estimate the gravitational force. We take the 20 m distance as having 2 significant figures.

$$F_{\text{grav}} = G\frac{m_{\text{shuttle}}m_{\text{astronaut}}}{r^2} = \left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\frac{\left(1.03\times10^5\,\text{kg}\right)\left(120\,\text{kg}\right)}{\left(20\,\text{m}\right)^2}$$

$$= 2.061\times10^{-6}\,\text{N} \approx \boxed{2.1\times10^{-6}\,\text{N}}$$

(*c*) The gravity force is larger, by a factor of approximately 100.

37. The intensity from a point source is inversely proportional to the distance from the source.

$$\frac{I_{\text{Earth}}}{I_{\text{Jupiter}}} = \frac{r^2_{\text{Sun-Jupiter}}}{r^2_{\text{Sun-Earth}}} = \frac{\left(7.78\times10^{11}\,\text{m}\right)^2}{\left(1.496\times10^{11}\,\text{m}\right)^2} = 27.0$$

So it would take an area of $\boxed{27\text{m}^2}$ at Jupiter to collect the same radiation as a $1.0\text{-m}^2$ solar panel at the Earth.

38. Use Eq. 31-14. Note that the higher frequencies have the shorter wavelengths.

(*a*) For FM radio we have the following.

$$\lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.08\times10^8\,\text{Hz}\right)} = \boxed{2.78\,\text{m}} \text{ to } \lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(8.8\times10^7\,\text{Hz}\right)} = \boxed{3.41\,\text{m}}$$

(*b*) For AM radio we have the following.

$$\lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.7\times10^6\,\text{Hz}\right)} = \boxed{180\,\text{m}} \text{ to } \lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(5.35\times10^5\,\text{Hz}\right)} = \boxed{561\,\text{m}}$$

39. Use Eq. 31-14.

$$\lambda = \frac{c}{f} = \frac{\left(3.00\times10^8\,\text{m/s}\right)}{\left(1.9\times10^9\,\text{Hz}\right)} = \boxed{0.16\,\text{m}}$$

40. The resonant frequency of an $LC$ circuit is given by $f = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{LC}}$. We assume the inductance is constant, and form the ratio of the two frequencies.

$$\frac{f_1}{f_2} = \frac{\dfrac{2\pi}{\sqrt{LC_1}}}{\dfrac{2\pi}{\sqrt{LC_2}}} = \sqrt{\frac{C_2}{C_1}} \rightarrow C_2 = \left(\frac{f_1}{f_2}\right)^2 C_1 = \left(\frac{550\,\text{kHz}}{1610\,\text{kHz}}\right)^2\left(2200\,\text{pF}\right) = \boxed{260\,\text{pF}}$$

41. The resonant frequency of an $LC$ circuit is given by $f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$. Solve for the inductance.

$$f = \frac{1}{2\pi\sqrt{LC}} \rightarrow L = \frac{1}{4\pi^2 f^2 C}$$

$$L_1 = \frac{1}{4\pi^2 f_1^2 C} = \frac{1}{4\pi^2\left(88\times10^6\,\text{Hz}\right)^2\left(620\times10^{-12}\,\text{F}\right)} = 5.3\times10^{-9}\,\text{H}$$

$$L_2 = \frac{1}{4\pi^2 f_2^2 C} = \frac{1}{4\pi^2\left(108\times10^6\,\text{Hz}\right)^2\left(620\times10^{-12}\,\text{F}\right)} = 3.5\times10^{-9}\,\text{H}$$

The range of inductances is $\boxed{3.5\times10^{-9}\,\text{H} \le L \le 5.3\times10^{-9}\,\text{H}}$

42. The rms electric field strength of the beam can be found from the Poynting vector.

$$S = \frac{P}{A} = c\varepsilon_0 E_{\text{rms}}^2 \quad \to$$

$$E_{\text{rms}} = \sqrt{\frac{P}{Ac\varepsilon_0}} = \sqrt{\frac{1.2\times10^4\,\text{W}}{\pi(750\,\text{m})^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}} = \boxed{1.6\,\text{V/m}}$$

43. The electric field is found from the desired voltage and the length of the antenna. Then use that electric field to calculate the magnitude of the Poynting vector.

$$E_{\text{rms}} = \frac{V_{\text{rms}}}{d} = \frac{1.00\times10^{-3}\,\text{V}}{1.60\,\text{m}} = \boxed{6.25\times10^{-4}\,\text{V/m}}$$

$$S = c\varepsilon_0 E_{\text{rms}}^2 = c\varepsilon_0\frac{V_{\text{rms}}^2}{d^2} = \left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(1.00\times10^{-3}\,\text{V}\right)^2}{(1.60\,\text{m})^2}$$

$$= \boxed{1.04\times10^{-9}\,\text{W/m}^2}$$

44. We ignore the time for the sound to travel to the microphone. Find the difference between the time for sound to travel to the balcony and for a radio wave to travel 3000 km.

$$\Delta t = t_{\text{radio}} - t_{\text{sound}} = \left(\frac{d_{\text{radio}}}{c}\right) - \left(\frac{d_{\text{sound}}}{v_{\text{sound}}}\right) = \left(\frac{3\times10^6\,\text{m}}{3.00\times10^8\,\text{m/s}}\right) - \left(\frac{50\,\text{m}}{343\,\text{m/s}}\right) = -0.14\,\text{s},$$

so the $\boxed{\text{person at the radio hears the voice 0.14 s sooner.}}$

45. The length is found from the speed of light and the duration of the burst.

$$d = ct = \left(3.00\times10^8\,\text{m/s}\right)\left(10^{-8}\text{s}\right) = \boxed{3\,\text{m}}$$

46. The time travel delay is the distance divided by the speed of radio waves (which is the speed of light).

$$t = \frac{d}{c} = \frac{3\times10^6\,\text{m}}{3.00\times10^8\,\text{m/s}} = \boxed{0.01\,\text{s}}$$

47. The time consists of the time for the radio signal to travel to Earth and the time for the sound to travel from the loudspeaker. We use 343 m/s for the speed of sound.

$$t = t_{\text{radio}} + t_{\text{sound}} = \left(\frac{d_{\text{radio}}}{c}\right) + \left(\frac{d_{\text{sound}}}{v_{\text{sound}}}\right) = \left(\frac{3.84\times10^8\,\text{m}}{3.00\times10^8\,\text{m/s}}\right) + \left(\frac{25\,\text{m}}{343\,\text{m/s}}\right) = \boxed{1.35\,\text{s}}$$

Note that about 5% of the time is for the sound wave.

48. (*a*) The rms value of the associated electric field is found from Eq. 24-6.

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \varepsilon_0 E_{\text{rms}}^2 \quad \to \quad E_{\text{rms}} = \sqrt{\frac{u}{\varepsilon_0}} = \sqrt{\frac{4\times10^{-14}\,\text{J/m}^3}{8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2}} = 0.0672\,\text{V/m} \approx \boxed{0.07\,\text{V/m}}$$

(*b*) A comparable value can be found using the magnitude of the Poynting vector.

$$\overline{S}=\varepsilon_0 c E_{\rm rms}^2=\frac{P}{4\pi r^2}\quad\rightarrow$$

$$r=\frac{1}{E_{\rm rms}}\sqrt{\frac{P}{4\pi\varepsilon_0 c}}=\frac{1}{0.0672\,{\rm V/m}}\sqrt{\frac{7500\,{\rm W}}{4\pi\left(8.85\times10^{-12}{\rm C^2/n\bullet m^2}\right)\left(3.00\times10^{8}\,{\rm m/s}\right)}}$$

$$=7055\,{\rm m}\approx\boxed{7\,{\rm km}}$$

49. The light has the same intensity in all directions, so use a spherical geometry centered on the source to find the value of the Poynting vector. Then use Eq. 31-19a to find the magnitude of the electric field, and Eq. 31-11 with $v=c$ to find the magnitude of the magnetic field.

$$S=\frac{P_0}{A}=\frac{P_0}{4\pi r^2}=\tfrac{1}{2}c\varepsilon_0 E_0^2\quad\rightarrow$$

$$E_0=\sqrt{\frac{P_0}{2\pi r^2 c\varepsilon_0}}=\sqrt{\frac{(75\,{\rm W})}{2\pi(2.00\,{\rm m})^2\left(3.00\times10^{8}\,{\rm m/s}\right)\left(8.85\times10^{-12}{\rm C^2/n\bullet m^2}\right)}}=33.53\,{\rm V/m}$$

$$\approx\boxed{34\,{\rm V/m}}$$

$$B_0=\frac{E_0}{c}=\frac{(33.53\,{\rm V/m})}{\left(3.00\times10^{8}\,{\rm m/s}\right)}=\boxed{1.1\times10^{-7}\,{\rm T}}$$

50. The radiation from the Sun has the same intensity in all directions, so the rate at which energy passes through a sphere centered at the Sun is $P=S4\pi R^2$. This rate must be the same at any distance from the Sun. Use this fact to calculate the magnitude of the Poynting vector at Mars, and then use the Poynting vector to calculate the rms magnitude of the electric field at Mars.

$$S_{\rm Mars}\left(4\pi R_{\rm Mars}^2\right)=S_{\rm Earth}\left(4\pi R_{\rm Earth}^2\right)\quad\rightarrow\quad S_{\rm Mars}=S_{\rm Earth}\left(\frac{R_{\rm Earth}^2}{R_{\rm Mars}^2}\right)=c\varepsilon_0 E_{\substack{\rm rms\\ \rm Mars}}^2\quad\rightarrow$$

$$E_{\substack{\rm rms\\ \rm Mars}}=\sqrt{\frac{S_{\rm Earth}}{c\varepsilon_0}}\left(\frac{R_{\rm Earth}}{R_{\rm Mars}}\right)=\sqrt{\frac{1350\,{\rm W/m^2}}{\left(3.00\times10^{8}\,{\rm m/s}\right)\left(8.85\times10^{-12}{\rm C^2/n\bullet m^2}\right)}}\left(\frac{1}{1.52}\right)=\boxed{469\,{\rm V/m}}$$

51. The direction of the wave velocity is the direction of the cross product $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$. "South" crossed into "west" gives the direction $\boxed{\text{downward}}$. The electric field is found from the Poynting vector, Eq. 31-19a, and then the magnetic field is found from Eq. 31-11 with $v=c$.

$$S=\tfrac{1}{2}c\varepsilon_0 E_0^{\,2}\quad\rightarrow$$

$$E_0=\sqrt{\frac{2S}{c\varepsilon_0}}=\sqrt{\frac{2\left(560\,{\rm W/m^2}\right)}{\left(3.00\times10^{8}\,{\rm m/s}\right)\left(8.85\times10^{-12}{\rm C^2/n\bullet m^2}\right)}}=649\,{\rm V/m}\approx\boxed{650\,{\rm V/m}}$$

$$B_0=\frac{E_0}{c}=\frac{(649\,{\rm V/m})}{\left(3.00\times10^{8}\,{\rm m/s}\right)}=\boxed{2.2\times10^{-6}\,{\rm T}}$$

52. From the hint, we use Eq. 29-4, which says $\mathscr{E}=\mathscr{E}_0\sin\omega t=NBA\omega\sin\omega t$. The intensity is given, and this can be used to find the magnitude of the magnetic field.

$$\overline{S}=\frac{E_{\rm rms}B_{\rm rms}}{\mu_0}=\frac{cB_{\rm rms}^2}{\mu_0}\quad\rightarrow\quad B_{\rm rms}=\sqrt{\frac{\mu_0\overline{S}}{c}}\quad;\quad\mathscr{E}=\mathscr{E}_0\sin\omega t=NBA\omega\sin\omega t\quad\rightarrow$$

$$\mathcal{E}_{\text{rms}} = NA\omega B_{\text{rms}} = NA\omega\sqrt{\frac{\mu_0 \bar{S}}{c}}$$

$$= (320)\pi(0.011\,\text{m})^2\, 2\pi(810\times10^3\,\text{Hz})\sqrt{\frac{(4\pi\times10^{-7}\,\text{N}\bullet\text{s}^2/\text{C}^2)(1.0\times10^{-4}\,\text{W/m}^2)}{3.00\times10^8\,\text{m/s}}}$$

$$= \boxed{4.0\times10^{-4}\,\text{V}}$$

53. (*a*) Since intensity is energy per unit time per unit area, the energy is found by multiplying the intensity times the area of the antenna times the elapsed time.

$$U = IAt = (1.0\times10^{-13}\,\text{W/m}^2)\pi\left(\frac{0.33\,\text{m}}{2}\right)^2(6.0\,\text{h})(3600\,\text{s/h}) = \boxed{1.8\times10^{-10}\,\text{J}}$$

(*b*) The electric field amplitude can be found from the intensity, which is the magnitude of the Poynting vector. The magnitude of the magnetic field is then found from Eq. 31-11 with $v = c$.

$$\bar{I} = \tfrac{1}{2}\varepsilon_0 c E_0^2 \quad \rightarrow$$

$$E_0 = \sqrt{\frac{2\bar{I}}{\varepsilon_0 c}} = \sqrt{\frac{2(1.0\times10^{-13}\,\text{W/m}^2)}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)(3.00\times10^8\,\text{m/s})}} = 8.679\times10^{-6}\,\text{V/m}$$

$$\approx \boxed{8.7\times10^{-6}\,\text{V/m}}$$

$$B_0 = \frac{E_0}{c} = \frac{8.679\times10^{-6}\,\text{V/m}}{3.00\times10^8\,\text{m/s}} = \boxed{2.9\times10^{-14}\,\text{T}}$$

54. Use the relationship between average intensity (the magnitude of the Poynting vector) and electric field strength, as given by Eq. 31-19a. Also use the fact that intensity is power per unit area. We assume that the power is spherically symmetric about source.

$$\bar{S} = \tfrac{1}{2}\varepsilon_0 c E_0^{\,2} = \frac{P}{A} = \frac{P}{4\pi r^2} \quad \rightarrow$$

$$r = \sqrt{\frac{P}{2\pi\varepsilon_0 c E_0^{\,2}}} = \sqrt{\frac{25{,}000\,\text{W}}{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)(3.00\times10^8\,\text{m/s})(0.020\,\text{V/m})^2}} = 61{,}200\,\text{m}$$

$$\approx \boxed{61\,\text{km}}$$

Thus, to receive the transmission one should be within 61 km of the station.

$\boxed{55.}$ The light has the same intensity in all directions. Use a spherical geometry centered at the source with the definition of the Poynting vector.

$$S = \frac{P_0}{A} = \frac{P_0}{4\pi r^2} = \tfrac{1}{2}c\varepsilon_0 E_0^{\,2} = \tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0 \quad\rightarrow\quad \tfrac{1}{2}c\left(\frac{1}{c^2\mu_0}\right)E_0 = \frac{P_0}{4\pi r^2} \quad\rightarrow\quad \boxed{E_0 = \sqrt{\frac{\mu_0 c P_0}{2\pi r^2}}}$$

56. (*a*) The radio waves have the same intensity in all directions. The power crossing a given area is the intensity times the area. The intensity is the total power through the area of a sphere centered at the source.

$$P = IA = \frac{P_0}{A_{\text{total}}}A = \frac{35{,}000\,\text{W}}{4\pi(1.0\times10^3\,\text{m})^2}(1.0\,\text{m}^2) = 2.785\times10^{-3}\,\text{W} \approx \boxed{2.8\,\text{mW}}$$

(*b*) We find the rms value of the electric field from the intensity, which is the magnitude of the Poynting vector.

$$S = c\varepsilon_0 E_{\text{rms}}^2 = \frac{P_0}{4\pi r^2} \quad \rightarrow$$

$$E_{\text{rms}} = \sqrt{\frac{P_0}{4\pi r^2 c\varepsilon_0}} = \sqrt{\frac{35{,}000\,\text{W}}{4\pi\left(1.0\times10^3\,\text{m}\right)^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)}}$$

$$= 1.024\,\text{V/m} \approx \boxed{1.0\,\text{V/m}}$$

(*c*) The voltage over the length of the antenna is the electric field times the length of the antenna.

$$V_{\text{rms}} = E_{\text{rms}} d = \left(1.024\,\text{V/m}\right)\left(1.0\,\text{m}\right) = \boxed{1.0\,\text{V}}$$

(*d*) We calculate the electric field at the new distance, and then calculate the voltage.

$$E_{\text{rms}} = \sqrt{\frac{P_0}{4\pi r^2 c\varepsilon_0}} = \sqrt{\frac{35{,}000\,\text{W}}{4\pi\left(5.0\times10^4\,\text{m}\right)^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)}}$$

$$= 2.048\times10^{-2}\,\text{V/m} \;\; ; \;\; V_{\text{rms}} = E_{\text{rms}} d = \left(2.048\times10^{-2}\,\text{V/m}\right)\left(1.0\,\text{m}\right) = \boxed{2.0\times10^{-2}\,\text{V}}$$

57. The power output of the antenna would be the intensity at a given distance from the antenna, times the area of a sphere surrounding the antenna. The intensity is the magnitude of the Poynting vector.

$$S = \tfrac{1}{2} c\varepsilon_0 E_0^2$$

$$P = 4\pi r^2 S = 2\pi r^2 c\varepsilon_0 E_0^2 = 2\pi\left(0.50\,\text{m}\right)^2\left(3.00\times10^8\,\text{m/s}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(3\times10^6\,\text{V/m}\right)^2$$

$$\approx \boxed{4\times10^{10}\,\text{W}}$$

This is many orders of magnitude higher than the power output of commercial radio stations, which are no higher than the 10's of kilowatts.

58. We calculate the speed of light in water according to the relationship given.

$$v_{\text{water}} = \frac{1}{\sqrt{K\varepsilon_0\mu_0}} = \frac{1}{\sqrt{K}}\frac{1}{\sqrt{\varepsilon_0\mu_0}} = \frac{1}{\sqrt{K}}c = \frac{1}{\sqrt{1.77}}\left(3.00\times10^8\,\text{m/s}\right) = \boxed{2.25\times10^8\,\text{m/s}}$$

$$\frac{v_{\text{water}}}{c} = \frac{\frac{1}{\sqrt{K}}c}{c} = \frac{1}{\sqrt{K}} = 0.752 = \boxed{75.2\%}$$

59. A standing wave has a node every half-wavelength, including the endpoints. For this wave, the nodes would occur at the spacing calculated here.

$$\tfrac{1}{2}\lambda = \tfrac{1}{2}\frac{c}{f} = \tfrac{1}{2}\frac{3.00\times10^8\,\text{m/s}}{2.45\times10^9\,\text{Hz}} = \boxed{0.0612\,\text{m}}$$

Thus there would be nodes at the following distances from a wall:

0, 6.12 cm, 12.2 cm, 18.4 cm, 24.5 cm, 30.6 cm, and 36.7 cm (approximately the other wall).

So there are $\boxed{\text{5 nodes}}$, not counting the ones at (or near) the walls.

60. (*a*) Assume that the wire is of length $\ell$ and cross-sectional area $A$. There must be a voltage across the ends of the wire to make the current flow $(V = IR)$, and there must be an electric field associated with that voltage $(E = V/\ell)$. Use these relationships with the definition of displacement current, Eq. 31-3.

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d(V/\ell)}{dt} = \varepsilon_0 \frac{A}{\ell}\frac{dV}{dt} = \varepsilon_0 \rho \frac{A}{\rho\ell}\frac{d(IR)}{dt}$$
$$= \varepsilon_0 \rho \frac{1}{R} R \frac{dI}{dt} = \boxed{\varepsilon_0 \rho \frac{dI}{dt}}$$

(*b*) Calculate the displacement current found in part (*a*).

$$I_D = \varepsilon_0 \rho \frac{dI}{dt} = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(1.68\times10^{-8}\,\Omega\bullet\text{m}\right)\left(\frac{1.0\,\text{A}}{1.0\times10^{-3}\,\text{s}}\right)$$
$$= 1.4868\times10^{-16}\,\text{A} \approx \boxed{1.5\times10^{-16}\,\text{A}}$$

(*c*) From example 28-6, Ampere's law gives the magnetic field created by a cylinder of current as $B = \frac{\mu_0 I}{2\pi r}$ at a distance of *r* from the axis of the cylindrical wire. This is true whether the current is displacement current or steady current.

$$B_D = \frac{\mu_0 I_D}{2\pi r} = \frac{\left(4\pi\times10^{-7}\,\text{N}\bullet\text{s}^2/\text{C}^2\right)\left(1.486\times10^{-16}\,\text{A}\right)}{2\pi\left(1.0\times10^{-3}\,\text{m}\right)} = 2.97\times10^{-20}\,\text{T} \approx \boxed{3.0\times10^{-20}\,\text{T}}$$

$$\frac{B_D}{B_{\text{steady}}} = \frac{\frac{\mu_0 I_D}{2\pi r}}{\frac{\mu_0 I_{\text{steady}}}{2\pi r}} = \frac{I_D}{I_{\text{steady}}} = \frac{1.486\times10^{-16}\,\text{A}}{1.0\,\text{A}} = 1.486\times10^{-16} \approx \boxed{1.5\times10^{-16}}$$

61. (*a*) We note that $-\alpha x^2 - \beta^2 t^2 + 2\alpha\beta x t = -(\alpha x - \beta t)^2$ and so $E_y = E_0 e^{-(\alpha x - \beta t)^2} = E_0 e^{-\alpha^2\left(x - \frac{\beta}{\alpha}t\right)^2}$. Since the wave is of the form $f(x - vt)$, with $v = \beta/\alpha$, the wave is moving in the $\boxed{+x \text{ direction}}$.

(*b*) The speed of the wave is $v = \beta/\alpha = c$, and so $\boxed{\beta = \alpha c}$.

(*c*) The electric field is in the *y* direction, and the wave is moving in the *x* direction. Since $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ must be in the direction of motion, the magnetic field must be in the *z* direction. The magnitudes are related by $|\vec{\mathbf{B}}| = |\vec{\mathbf{E}}|/c$.

$$\boxed{B_z = \frac{E_0}{c} e^{-(\alpha x - \beta t)^2}}$$

62. (*a*) Use the $\sin A \pm \sin B = 2\sin\left(\frac{A\pm B}{2}\right)\cos\left(\frac{A\mp B}{2}\right)$ from page A-4 in Appendix A.

$$E_y = E_0\left[\sin(kx-\omega t) + \sin(kx+\omega t)\right]$$
$$= 2E_0 \sin\left(\frac{(kx-\omega t)+(kx+\omega t)}{2}\right)\cos\left(\frac{(kx-\omega t)-(kx+\omega t)}{2}\right) = 2E_0\sin(kx)\cos(-\omega t)$$
$$= \boxed{2E_0\sin(kx)\cos(\omega t)}$$

$$B_z = B_0\left[\sin(kx-\omega t) - \sin(kx+\omega t)\right]$$
$$= 2B_0 \sin\left(\frac{(kx-\omega t)-(kx+\omega t)}{2}\right)\cos\left(\frac{(kx-\omega t)+(kx+\omega t)}{2}\right) = 2B_0\sin(-\omega t)\cos(kx)$$
$$= \boxed{-2B_0\cos(kx)\sin(\omega t)}$$

(*b*) The Poynting vector is given by $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}$.

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2E_0\sin(kx)\cos(\omega t) & 0 \\ 0 & 0 & -2B_0\cos(kx)\sin(\omega t) \end{vmatrix}$$

$$= \frac{1}{\mu_0}\hat{\mathbf{i}}\left[-4E_0B_0\sin(kx)\cos(kx)\sin(\omega t)\cos(\omega t)\right] = \boxed{-\frac{1}{\mu_0}\hat{\mathbf{i}}\,E_0B_0\sin(2kx)\sin(2\omega t)}$$

This is 0 for all times at positions where $\sin(2kx) = 0$.

$$\sin(2kx) = 0 \;\rightarrow\; 2kx = n\pi \;\rightarrow\; \boxed{x = \frac{n\pi}{2k},\; n = 0, \pm1, \pm2, \ldots}$$

63. (*a*) To show that $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular, calculate their dot product.

$$\vec{\mathbf{E}}\bullet\vec{\mathbf{B}} = \left[E_0\sin(kx-\omega t)\hat{\mathbf{j}} + E_0\cos(kx-\omega t)\hat{\mathbf{k}}\right]\bullet\left[B_0\cos(kx-\omega t)\hat{\mathbf{j}} - B_0\sin(kx-\omega t)\hat{\mathbf{k}}\right]$$

$$= E_0\sin(kx-\omega t)B_0\cos(kx-\omega t) - E_0\cos(kx-\omega t)B_0\sin(kx-\omega t) = 0$$

Since $\vec{\mathbf{E}}\bullet\vec{\mathbf{B}} \equiv 0$, $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other at all times.

(*b*) The wave moves in the direction of the Poynting vector, which is given by $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}$.

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & E_0\sin(kx-\omega t) & E_0\cos(kx-\omega t) \\ 0 & B_0\cos(kx-\omega t) & -B_0\sin(kx-\omega t) \end{vmatrix}$$

$$= \frac{1}{\mu_0}\hat{\mathbf{i}}\left[-E_0B_0\sin^2(kx-\omega t) - E_0B_0\cos^2(kx-\omega t)\right] + \hat{\mathbf{j}}(0) + \hat{\mathbf{k}}(0) = -\frac{1}{\mu_0}E_0B_0\hat{\mathbf{i}}$$

We see that the Poynting vector points in the negative $\underline{x}$ direction, and so the wave moves in the negative $x$ direction, which is perpendicular to both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$.

(*c*) We find the magnitude of the electric field vector and the magnetic field vector.

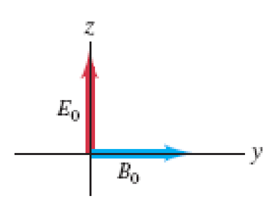
$$\left|\vec{\mathbf{E}}\right| = E = \left(\left[E_0\sin(kx-\omega t)\right]^2 + \left[E_0\cos(kx-\omega t)\right]^2\right)^{1/2}$$

$$= \left[E_0^2\sin^2(kx-\omega t) + E_0^2\cos^2(kx-\omega t)\right]^{1/2} = E_0$$

$$\left|\vec{\mathbf{B}}\right| = B = \left(\left[B_0\cos(kx-\omega t)\right]^2 + \left[B_0\sin(kx-\omega t)\right]^2\right)^{1/2}$$

$$= \left[B_0^2\cos^2(kx-\omega t) + B_0^2\sin^2(kx-\omega t)\right]^{1/2} = B_0$$

(*d*) At $x = 0$ and $t = 0$, $\vec{\mathbf{E}} = E_0\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B_0\hat{\mathbf{j}}$. See the figure. The $x$ axis is coming out of the page toward the reader. As time increases, the component of the electric field in the $z$ direction electric field begins to get smaller and the component in the negative $y$ direction begins to get larger. At the same time, the component of the magnetic field in the $y$ direction begins to get smaller, and the component in the $z$ direction begins to get larger. The net effect is that $\boxed{\text{both vectors rotate counterclockwise}}$.



# CHAPTER 32: Light: Reflection and Refraction

## Responses to Questions

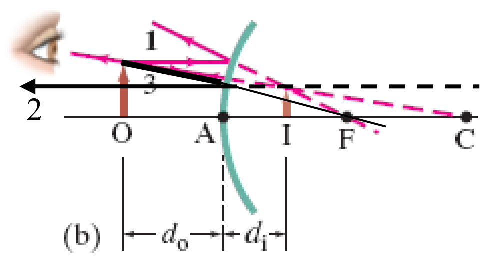
1. (*a*) The Moon would look just like it does now, since the surface is rough. Reflected sunlight is scattered by the surface of the Moon in many directions, making the surface appear white.
   (*b*) With a polished, mirror-like surface, the Moon would reflect an image of the Sun, the stars, and the Earth. The appearance of the Moon would be different as seen from different locations on the Earth.

2. Yes, it would have been possible, although certainly difficult. Several attempts have been made to reenact the event in order to test its feasibility. Two of the successful attempts include a 1975 experiment directed by Greek scientist Dr. Ioannis Sakkas and a 2005 experiment performed by a group of engineering students at MIT. (See www.mit.edu for links to both these and other similar experiments.) In both these cases, several individual mirrors operating together simulated a large spherical mirror and were used to ignite a wooden boat. If in fact the story is true, Archimedes would have needed good weather and an enemy fleet that cooperated by staying relatively still while the focused sunlight heated the wood.

3. The focal length of a plane mirror is infinite. The magnification of a plane mirror is 1.

4. The image is real and inverted, because the magnification is negative. The mirror is concave, because convex mirrors can only form virtual images. The image is on the same side of the mirror as the object; real images are formed by converging light rays and light rays cannot actually pass through a mirror.

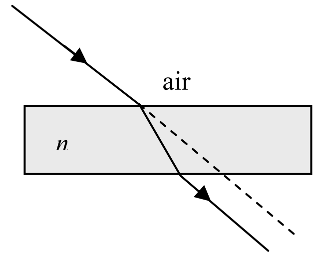
5. Ray 2 is directed as if it were going through the focal point and is reflected from the convex mirror parallel to the principal axis.




6. Yes. For a plane mirror, $d_o = -d_i$, since the object and image are equidistant from the mirror and the image is virtual, or behind the mirror. The focal length of a plane mirror is infinite, so the result of the mirror equation, Eq. 32-2, is $\frac{1}{d_o} + \frac{1}{d_i} = 0$, or $d_o = -d_i$, as expected.

7. Yes. When a concave mirror produces a real image of a real object, both $d_o$ and $d_i$ are positive. The magnification equation, $m = -\frac{d_i}{d_o}$, results in a negative magnification, which indicates that the image is inverted.

8. A light ray entering the solid rectangular object will exit the other side following a path that is parallel to its original path but displaced slightly from it. The angle of refraction in the glass can be determined geometrically from this displacement and the thickness of the object. The index of refraction can then be determined using Snell's Law with this angle of refraction and the original angle of incidence. The speed of light in the material follows from the definition of the index of refraction: $n = c/v$.

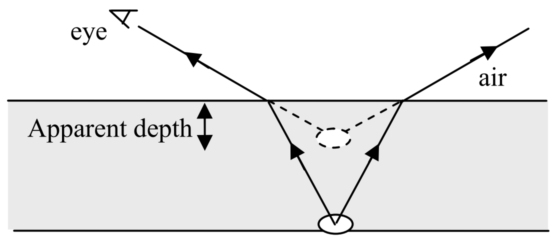



9. This effect is similar to diffuse reflection off of a rough surface. A ripply sea has multiple surfaces which are at an angle to reflect the image of the Moon into your eyes. This makes the image of the Moon appear elongated.

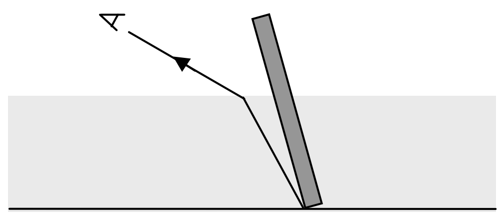
10. A negative object distance corresponds to a virtual object. This could occur if converging rays from another mirror or lens were intercepted by the mirror before actually forming an image. This image would be the object for the mirror.

11. The angle of refraction and the angle of incidence are both zero in this case.

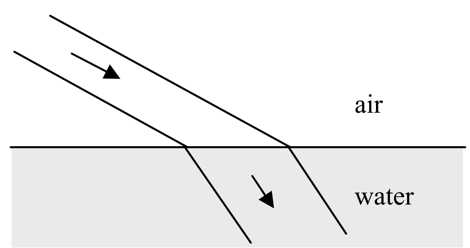
12. Underestimate. The light rays leaving the bottom of the pool bend away from the normal as they enter the air, so their source appears to be more shallow than it actually is. The greater the viewing angle, the more the bending of the light and therefore the less the apparent depth.



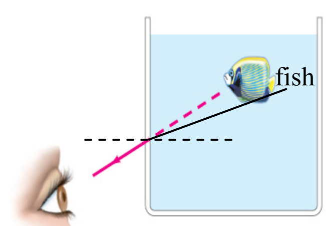

13. Your brain interprets the refracted rays as if the part of the stick that is under water is closer to the surface than it actually is, so the stick appears bent.



14. Because the broad beam hits the surface of the water at an angle, it illuminates an area of the surface that is wider than the beam width. Light from the beam bends towards the normal. The refracted beam is wider than the incident beam because one edge of the beam strikes the surface first, while the other edge travels farther in the air. (See the adjacent diagram.)




15. The light rays from the fish are bent away from the normal as they leave the tank. The fish will appear closer to the side of the tank than it really is.



16. The water drop acts like a lens, and refracts light as the light passes through it. Also, some of the light incident on the air/water boundary is reflected at the surface, so the drop can be seen in reflected light.

17. When the light ray passes from the blue material to the green material, the ray bends toward the normal. This indicates that the index of refraction of the blue material is less than that of the green material. When the light ray passes from the green material to the yellow material, the ray bends away from the normal, but not far enough to make the ray parallel to the initial ray, indicating that the index of refraction of the yellow material is less than that of the green material but larger than the index of refraction of the blue material. The ranking of the indices of refraction is, least to greatest, blue, yellow, and green.

18. No. Total internal reflection can only occur when light travels from a medium of higher index of refraction to a medium of lower index of refraction.

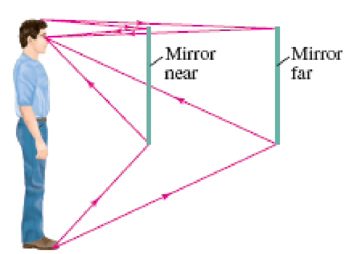
19. No. The refraction of light as it enters the pool will make the object look smaller. See Figure 32-32 and Conceptual Example 32-11.

20. The mirror is concave, and the person is standing inside the focal point so that a virtual, upright image is formed. (A convex mirror would also form a virtual, upright image but the image would be smaller than the object.) In addition, an image is also present at the far right edge of the mirror, which is only possible if the mirror is concave.

21. (*a*) Since the light is coming from a vacuum into the atmosphere, which has a larger index of refraction, the light rays should bend toward the normal (toward the vertical direction).
(*b*) The stars are closer to the horizon than they appear to be from the surface of the Earth.

## Solutions to Problems

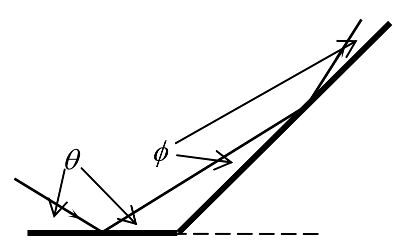
1. Because the angle of incidence must equal the angle of reflection, we see from the ray diagrams that the ray that reflects to your eye must be as far below the horizontal line to the reflection point on the mirror as the top is above the line, regardless of your position.




2. For a flat mirror the image is as far behind the mirror as the object is in front, so the distance from object to image is twice the distance from the object to the mirror, or $\boxed{5.6\,\text{m}}$.

3. The law of reflection can be applied twice. At the first reflection, the angle is $\theta$, and at the second reflection, the angle is $\phi$. Consider the triangle formed by the mirrors and the first reflected ray.

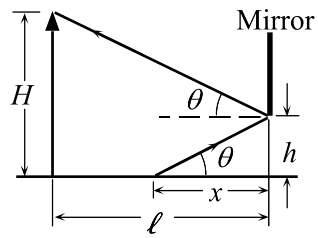
$$\theta + \alpha + \phi = 180° \quad \rightarrow \quad 38° + 135° + \phi = 180° \quad \rightarrow \quad \boxed{\phi = 7°}$$



4. The angle of incidence is the angle of reflection. See the diagram for the appropriate lengths.

$$\tan\theta = \frac{(H-h)}{\ell} = \frac{h}{x} \rightarrow$$

$$\frac{(1.64\,\text{m} - 0.38\,\text{m})}{(2.30\,\text{m})} = \frac{(0.38\,\text{m})}{x} \rightarrow x = \boxed{0.69\,\text{m}}$$




5. The incoming ray is represented by line segment DA. For the first reflection at *A* the angles of incidence and reflection are $\theta_1$. For the second reflection at *B* the angles of incidence and reflection are $\theta_2$. We relate $\theta_1$ and $\theta_2$ to the angle at which the mirrors meet, $\phi$, by using the sum of the angles of the triangle *ABC*.

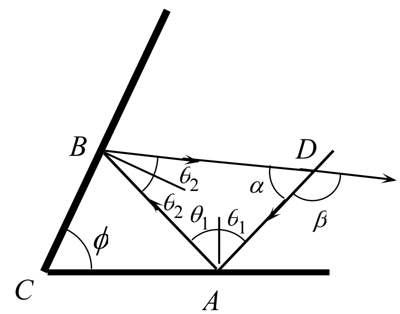
$$\phi + (90^\circ - \theta_1) + (90^\circ - \theta_2) = 180^\circ \rightarrow \phi = \theta_1 + \theta_2$$

Do the same for triangle *ABD*.

$$\alpha + 2\theta_1 + 2\theta_2 = 180^\circ \rightarrow \alpha = 180^\circ - 2(\theta_1 + \theta_2) = 180^\circ - 2\phi$$

At point *D* we see that the deflection is as follows.

$$\beta = 180^\circ - \alpha = 180^\circ - (180^\circ - 2\phi) = \boxed{2\phi}$$

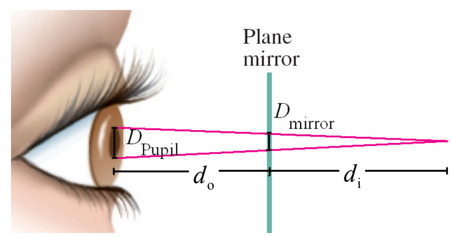



6. The rays entering your eye are diverging from the virtual image position behind the mirror. Thus the diameter of the area on the mirror and the diameter of your pupil must subtend the same angle from the image.

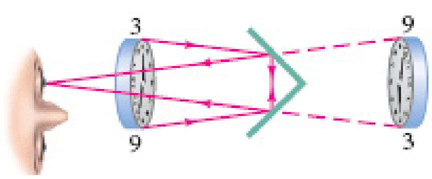
$$\frac{D_{\text{mirror}}}{d_i} = \frac{D_{\text{pupil}}}{(d_o + d_i)} \rightarrow D_{\text{mirror}} = D_{\text{pupil}}\frac{d_i}{(d_o + d_i)} = \tfrac{1}{2}D_{\text{pupil}}$$

$$A_{\text{mirror}} = \tfrac{1}{4}\pi D_{\text{mirror}}^2 = \tfrac{1}{4}\pi\tfrac{1}{4}D_{\text{pupil}}^2 = \frac{\pi}{16}(4.5\times10^{-3}\,\text{m})^2$$

$$= \boxed{4.0\times10^{-6}\,\text{m}^2}$$




7. See the "top view" ray diagram.




8. (*a*) The velocity of the incoming light wave is in the direction of the initial light wave. We can write this velocity in component form, where the three axes of our coordinate system are chosen to be perpendicular to the plane of each of the three mirrors. As the light reflects off any of the three mirrors, the component of the velocity perpendicular to that mirror reverses direction. The other two velocity components will remain unchanged. After the light has reflected off of each of the three mirrors, each of the three velocity components will be reversed and the light will be traveling directly back from where it came.

(*b*) If the mirrors are assumed to be large enough, the light can only reflect off two of the mirrors if the velocity component perpendicular to the third mirror is zero. Therefore, in this case the light is still reflected back directly to where it came.

9. The rays from the Sun will be parallel, so the image will be at the focal point, which is half the radius of curvature.

$$r = 2f = 2(18.8\,\text{cm}) = \boxed{37.6\,\text{cm}}$$

10. To produce an image at infinity, the object must be at the focal point, which is half the radius of curvature.

$$d_o = f = \tfrac{1}{2}r = \tfrac{1}{2}(24.0\,\text{cm}) = \boxed{12.0\,\text{cm}}$$

11. The image flips at the focal point, which is half the radius of curvature. Thus the radius is $\boxed{1.0\text{ m}}$.

12. (*a*) The focal length is half the radius of curvature, so $f = \tfrac{1}{2}r = \tfrac{1}{2}(24\,\text{cm}) = \boxed{12\,\text{cm}}$.

(*b*) Use Eq. 32-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad\rightarrow\quad d_i = \frac{d_o f}{d_o - f} = \frac{(35\,\text{cm})(24\,\text{cm})}{35\,\text{cm} - 24\,\text{cm}} = \boxed{76\,\text{cm}}$$

(*c*) The image is $\boxed{\text{inverted}}$, since the magnification is negative.

$\boxed{13.}$ The ball is a convex mirror with a focal length $f = \tfrac{1}{2}r = \tfrac{1}{2}(-4.6\,\text{cm}) = -2.3\,\text{cm}$. Use Eq. 32-3 to locate the image.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad\rightarrow\quad d_i = \frac{d_o f}{d_o - f} = \frac{(25.0\,\text{cm})(-2.3\,\text{cm})}{25.0\,\text{cm} - (-2.3\,\text{cm})} = -2.106\,\text{cm} \approx -2.1\,\text{cm}$$

The image is $\boxed{\text{2.1 cm behind the surface of the ball, virtual, and upright}}$. Note that the magnification is $m = -\dfrac{d_i}{d_o} = \dfrac{-(-2.106\,\text{cm})}{(25.0\,\text{cm})} = +0.084$.

14. The image distance can be found from the object distance of 1.7 m and the magnification of +3. With the image distance and object distance, the focal length and radius of curvature can be found.

$$m = \frac{-d_i}{d_o} \quad\rightarrow\quad d_i = -md_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad\rightarrow\quad f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1} = \frac{3(1.7\,\text{m})}{3-1} = 2.55\,\text{m}$$

$$r = 2f = 2(2.55\,\text{m}) = \boxed{5.1\,\text{m}}$$

15. The object distance of 2.00 cm and the magnification of +4.0 are used to find the image distance. The focal length and radius of curvature can then be found.

$$m = \frac{-d_i}{d_o} \quad\rightarrow\quad d_i = -md_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad\rightarrow\quad f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1} = \frac{4(2.00\,\text{cm})}{4-1} = 2.677\,\text{cm}$$

$$r = 2f = 2(2.667\,\text{cm}) = \boxed{5.3\,\text{cm}}$$

Because the focal length is positive, the mirror is $\boxed{\text{concave}}$.

16. The mirror must be $\boxed{\text{convex}}$. Only convex mirrors produce images that are upright and smaller than the object. The object distance of 18.0 m and the magnification of +0.33 are used to find the image distance. The focal length and radius of curvature can then be found.

$$m = \frac{-d_i}{d_o} \rightarrow d_i = -md_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1} = \frac{0.33(18.0\,\text{m})}{0.33-1} = -8.866\,\text{m}$$

$$r = 2f = 2(-8.866\,\text{m}) = \boxed{-17.7\,\text{m}}$$

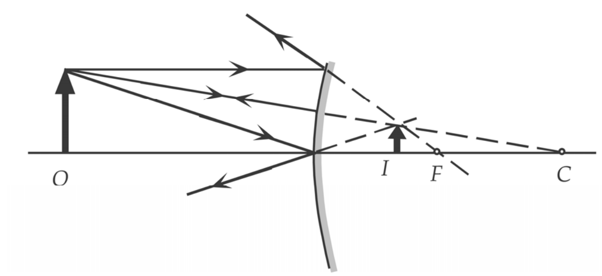
17. The object distance of 3.0 m and the magnification of +0.5 are used to find the image distance. The focal length and radius of curvature can then be found.

$$m = \frac{-d_i}{d_o} \rightarrow d_i = -md_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1} = \frac{0.5(3.0\,\text{m})}{0.5-1} = -3.0\,\text{m}$$

$$r = 2f = 2(-3.0\,\text{m}) = \boxed{-6.0\,\text{m}}$$

18. (*a*) From the ray diagram it is seen that the image is virtual. We estimate the image distance as $\boxed{-6\,\text{cm}.}$




(*b*) Use a focal length of $-9.0\,\text{cm}$ with the object distance of $18.0\,\text{cm}$.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow$$

$$d_i = \frac{d_o f}{d_o - f} = \frac{(18.0\,\text{cm})(-9.0\,\text{cm})}{18.0\,\text{cm} - (-9.0\,\text{cm})} = \boxed{-6.0\,\text{cm}}$$

(*c*) We find the image size from the magnification:

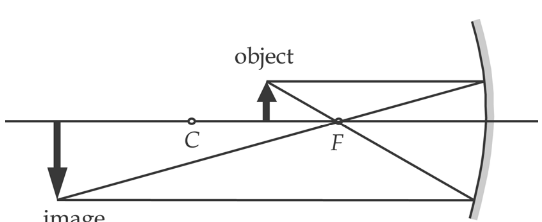
$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o} \rightarrow h_i = h_o\left(\frac{-d_i}{d_o}\right) = (3.0\,\text{mm})\left(\frac{6.0\,\text{cm}}{18.0\,\text{cm}}\right) = \boxed{1.0\,\text{mm}}$$

$\boxed{19.}$ Take the object distance to be ∞, and use Eq. 32-3. Note that the image distance is negative since the image is behind the mirror.

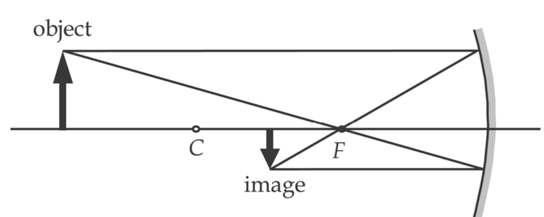
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow \frac{1}{\infty} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = d_i = -16.0\,\text{cm} \rightarrow r = 2f = \boxed{-32.0\,\text{cm}}$$

Because the focal length is negative, the mirror is $\boxed{\text{convex.}}$

20. (*a*)




$|h_i| > |h_o|$ ; $m > 1$ ; $d_o < r$




$|h_i| < |h_o|$ ; $m < 1$ ; $d_o > r$

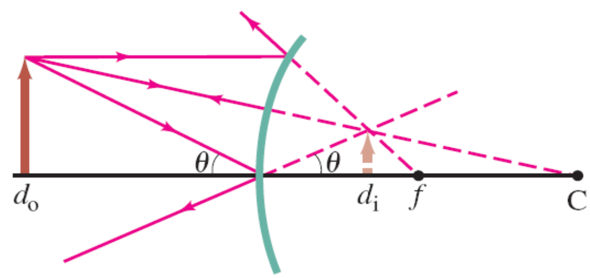
(*b*) Apply Eq. 32-3 and Eq. 32-4.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=\frac{2}{r} \rightarrow d_i=\frac{rd_o}{(2d_o-r)} \; ; \; m=\frac{-d_i}{d_o}=\frac{-r}{(2d_o-r)}$$

If $d_o>r$, then $(2d_o-r)>r$, so $|m|=\frac{r}{(2d_o-r)}=\frac{r}{(>r)}<1.$

If $d_o<r$, then $(2d_o-r)<r$, so $|m|=\frac{r}{(2d_o-r)}=\frac{r}{(<r)}>1.$

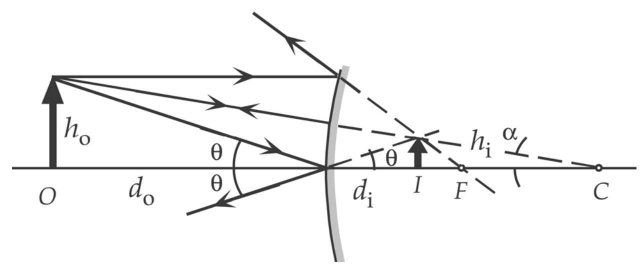
21. Consider the ray that reflects from the center of the mirror, and note that $d_i<0$.




$$\tan\theta=\frac{h_o}{d_o}=\frac{h_i}{-d_i} \rightarrow \frac{-d_i}{d_o}=\frac{h_i}{h_o}$$

$$m=\frac{h_i}{h_o}=\boxed{-\frac{d_i}{d_o}}$$

22. From the ray diagram, we see that with a negative image distance, we have the following.




$$\tan\theta=\frac{h_o}{d_o}=\frac{h_i}{-d_i}$$

$$\tan\alpha=\frac{h_o}{(d_o+r)}=\frac{h_i}{(r+d_i)}$$

When we divide the two equations, we get

$$\frac{(d_o+r)}{d_o}=-\frac{(r+d_i)}{d_i} \rightarrow 1+\frac{r}{d_o}=-1-\frac{r}{d_i} \rightarrow \frac{r}{d_o}+\frac{r}{d_i}=-2 \rightarrow \frac{1}{d_o}+\frac{1}{d_i}=-\frac{2}{r}$$

If we define $f=\frac{r}{2}$ and consider the radius of curvature and focal length to be negative, then we have Eq. 32-2, $\boxed{\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}}$..

23. Use Eq. 32-2 and 32-3.

$$m=\frac{-d_i}{d_o} \rightarrow d_i=-md_o$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow f=\frac{d_od_i}{d_o+d_i}=\frac{d_o(-md_o)}{d_o-md_o}=\frac{md_o}{m-1}=\frac{(0.55)(3.2\,\text{m})}{0.55-1}=\boxed{-3.9\,\text{m}}$$

24. (*a*) We are given that $d_i=d_o$. Use Eq. 32-3.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{2}{d_o}=\frac{1}{f} \rightarrow \boxed{d_o=2f=r}$$

The object should be placed at the $\boxed{\text{center of curvature.}}$

(*b*) Because the image is in front of the mirror, $d_i>0$, it is $\boxed{\text{real.}}$

(*c*) The magnification is $m=\frac{-d_i}{d_o}=\frac{-d_o}{d_o}=-1$. Because the magnification is negative, the image is $\boxed{\text{inverted.}}$

(*d*) As found in part (*c*), $m=\boxed{-1.}$

$\boxed{25.}$ (*a*) To produce a smaller image located behind the surface of the mirror requires a $\boxed{\text{convex mirror.}}$

(*b*) Find the image distance from the magnification.

$$m=\frac{h_i}{h_o}=\frac{-d_i}{d_o}\rightarrow d_i=-\frac{d_oh_i}{h_o}=-\frac{(26\,\text{cm})(3.5\,\text{cm})}{(4.5\,\text{cm})}=-20.2\,\text{cm}\approx\boxed{-20\,\text{cm}}\ \text{(2 sig. fig.)}$$

As expected, $d_i<0$. The image is located $\boxed{\text{20 cm behind the surface.}}$

(*c*) Find the focal length from Eq. 32.3.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\rightarrow f=\frac{d_od_i}{d_o+d_i}=\frac{(26\,\text{cm})(-20.2\,\text{cm})}{(26\,\text{cm})+(-20.2\,\text{cm})}=-90.55\,\text{cm}\approx\boxed{-91\,\text{cm}}$$

(*d*) The radius of curvature is twice the focal length.

$$r=2f=2(-90.55\,\text{cm})=-181.1\,\text{cm}\approx\boxed{-180\,\text{cm}}$$

26. (*a*) To produce a larger upright image requires a $\boxed{\text{concave mirror.}}$

(*b*) The image will be $\boxed{\text{upright and virtual.}}$

(*c*) We find the image distance from the magnification:

$$m=\frac{h_i}{h_o}=\frac{-d_i}{d_o}\rightarrow d_i=-md_o$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\rightarrow f=\frac{d_od_i}{d_o+d_i}=\frac{d_o(-md_o)}{d_o-md_o}=\frac{md_o}{m-1}\rightarrow$$

$$r=2f=\frac{2md_o}{m-1}=\frac{2(1.35)(20.0\,\text{cm})}{1.35-1}=\boxed{154\,\text{cm}}$$

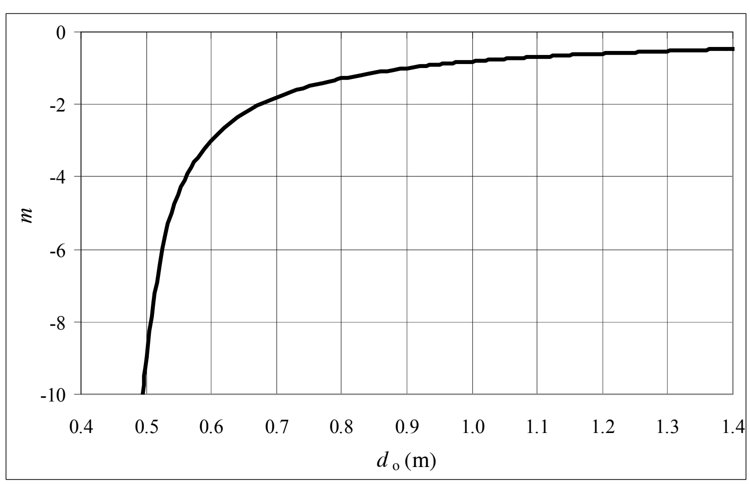
27. (*a*) We use the magnification equation, Eq. 32-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the magnification in terms of the object distance and the focal length.

$$m=-d_i/d_o\rightarrow d_i=-md_o$$

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\rightarrow$$

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{-md_o}\rightarrow$$

$$\boxed{m=\frac{f}{f-d_o}}$$




(*b*) We set $f=0.45$ m and draw a graph of the magnification as a function of the object distance. The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH32.XLS,” on tab “Problem 32.27b.”

(*c*) The image and object will have the same lateral size when the magnification is equal to negative one. Setting the magnification equal to negative one, we solve the equation found in part (a) for the object distance.

$$m = \frac{f}{f - d_o} = -1 \quad \rightarrow \quad d_o = 2f = \boxed{0.90\,\text{m}}$$

(*d*) From the graph we see that for the image to be much larger than the object, the object should be placed at a point $\boxed{\text{just beyond the focal point.}}$

28. We use the magnification equation, Eq. 32-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the magnification in terms of the object distance and the focal length, with the focal length given as $f = -|f|$.

$$m = -\frac{d_i}{d_o} \quad \rightarrow \quad d_i = -md_o \quad \rightarrow \quad \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{-|f|} = \frac{1}{d_o} + \frac{1}{-md_o} \rightarrow \boxed{m = \frac{|f|}{|f| + d_o}}$$

From this relation, the closer the object is to the mirror (i.e., smaller object distance) the greater the magnification. Since a person's nose is closer to the mirror than the rest of the face, its image appears larger.

29. (*a*) We use the magnification equation, Eq. 32-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the object distance in terms of the magnification and the focal length.

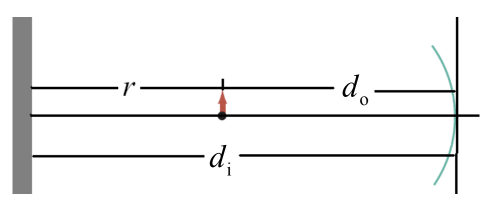
$$m = -\frac{d_i}{d_o} \rightarrow d_i = -md_o$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{f} = \frac{1}{d_o} + \frac{1}{-md_o} = \frac{1}{d_o}\left(1 - \frac{1}{m}\right) \rightarrow \boxed{d_o = f\left(1 - \frac{1}{m}\right)}$$

(*b*) We set the object distance equal to the range of all positive numbers. Since the focal length of a convex lens is negative, the term in parentheses in the above equation must be the range of all negative numbers for the object distance to include the range of all positive numbers. We solve the resulting equation for all possible values of the magnification.

$$\left(1 - \frac{1}{m}\right) \le 0 \rightarrow 1 \le \frac{1}{m} \rightarrow \boxed{0 \le m \le 1}$$

30. The distance between the mirror and the wall is equal to the image distance, which we can calculate using Eq. 32-2. The object is located a distance *r* from the wall, so the object distance will be *r* less than the image distance. The focal length is given by Eq. 32-1. For the object distance to be real, the image distance must be greater than *r*.

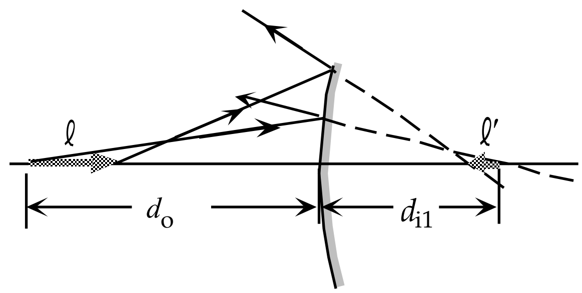



$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{2}{r} = \frac{1}{d_i - r} + \frac{1}{d_i} \rightarrow 2d_i^2 - 4d_i r + r^2 = 0$$

$$d_i = \frac{4r \pm \sqrt{16r^2 - 8r^2}}{4} = r\left(1 \pm \frac{\sqrt{2}}{2}\right) \approx 0.292r \text{ or } \boxed{1.71r}$$

Use Eq. 32-3 to calculate the magnification: $m = -\frac{d_i}{d_o} = \frac{1.71r}{1.71r - r} = \boxed{-2.41}$

31. The lateral magnification of an image equals the height of the image divided by the height of the object. This can be written in terms of the image distance and focal length with Eqs. 32-2 and 32-3.




$$\frac{1}{f}=\frac{1}{d_i}+\frac{1}{d_o} \rightarrow d_i=\left(\frac{fd_o}{d_o-f}\right)$$

$$m=\frac{-d_i}{d_o}=-\frac{f}{d_o-f}$$

The longitudinal magnification will be the difference in image distances of the two ends of the object divided by the length of the image. Call the far tip of the wire object 1 with object distance $d_o$. The close end of the wire will be object 2 with object distance $d_o-\ell$. Using Eq. 32-2 we can find the image distances for both ends.

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_{i1}} \rightarrow d_{i1}=\frac{d_o f}{d_o-f} \;;\; \frac{1}{f}=\frac{1}{d_o-\ell}+\frac{1}{d_{i2}} \rightarrow d_{i2}=\frac{(d_o-\ell)f}{d_o-\ell-f}$$

Taking the difference in image distances and dividing by the object length gives the longitudinal magnification.

$$m_\ell=\frac{d_{i1}-d_{i2}}{\ell}=\frac{1}{\ell}\left(\frac{d_o f}{d_o-f}-\frac{(d_o-\ell)f}{d_o-\ell-f}\right)=\frac{d_o f(d_o-\ell-f)-(d_o-f)(d_o-\ell)f}{\ell(d_o-f)(d_o-\ell-f)}$$

$$=\frac{-f^2}{(d_o-f)(d_o-\ell-f)}$$

Set $\ell \ll d_o$, so that the $\ell$ drops out of the second factor of the denominator. Then rewrite the equation in terms of the lateral magnification, using the expression derived at the beginning of the problem.

$$m_\ell=\frac{-f^2}{(d_o-f)^2}=-\left[\frac{f}{(d_o-f)}\right]^2=\boxed{-m^2}$$

The negative sign indicates that the image is reversed front to back, as shown in the diagram.

32. We find the index of refraction from Eq. 32-1.

$$n=\frac{c}{v}=\frac{3.00\times10^8\ \text{m/s}}{2.29\times10^8\ \text{m/s}}=\boxed{1.31}$$

33. In each case, the speed is found from Eq. 32-1 and the index of refraction.

(*a*) Ethyl alcohol: $v=\frac{c}{n}=\frac{3.00\times10^8\ \text{m/s}}{1.36}=\boxed{2.21\times10^8\ \text{m/s}}$

(*b*) Lucite: $v=\frac{c}{n}=\frac{3.00\times10^8\ \text{m/s}}{1.51}=\boxed{1.99\times10^8\ \text{m/s}}$

(*c*) Crown glass: $v=\frac{c}{n}=\frac{3.00\times10^8\ \text{m/s}}{1.52}=\boxed{1.97\times10^8\ \text{m/s}}$

34. Find the distance traveled by light in 4.2 years.

$$d=c\Delta t=(3.00\times10^8\ \text{m/s})(4.2\ \text{yr})(3.16\times10^7\ \text{s/yr})=\boxed{4.0\times10^{16}\ \text{m}}$$

35. The time for light to travel from the Sun to the Earth is found from the distance between them and the speed of light.

$$\Delta t = \frac{d}{c} = \frac{1.50\times10^{11}\,\text{m}}{3.00\times10^{8}\,\text{m/s}} = 5.00\times10^{2}\,\text{s} = \boxed{8.33\,\text{min}}$$

36. We find the index of refraction from Eq. 32-1.

$$n = \frac{c}{v} = \frac{c}{0.88v_{\text{water}}} = \frac{c}{0.88\left(\frac{c}{n_{\text{water}}}\right)} = \frac{n_{\text{water}}}{0.88} = \frac{1.33}{0.88} = \boxed{1.51}$$

$\boxed{37.}$ The length in space of a burst is the speed of light times the elapsed time.

$$d = ct = \left(3.00\times10^{8}\,\text{m/s}\right)\left(10^{-8}\,\text{s}\right) = \boxed{3\,\text{m}}$$

38. Find the angle of refraction from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \;\rightarrow\; \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.33}{1.00}\sin 38.5^\circ\right) = \boxed{55.9^\circ}$$

39. Find the angle of refraction from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \;\rightarrow\; \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.00}{1.56}\sin 63^\circ\right) = \boxed{35^\circ}$$

40. We find the incident angle in the air (relative to the normal) from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \;\rightarrow\; \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.33}{1.00}\sin 33.0^\circ\right) = 46.4^\circ$$

Since this is the angle relative to the horizontal, the angle as measured from the horizon is $90.0^\circ - 46.4^\circ = \boxed{43.6^\circ}$.

41. We find the incident angle in the water from Snell's law.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \;\rightarrow\; \theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right) = \sin^{-1}\left(\frac{1.00}{1.33}\sin 56.0^\circ\right) = \boxed{38.6^\circ}$$

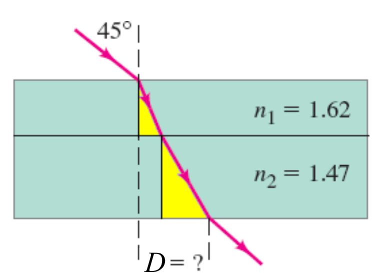
42. The angle of reflection is equal to the angle of incidence: $\theta_{\text{refl}} = \theta_1 = 2\theta_2$. Use Snell's law

$$n_{\text{air}}\sin\theta_1 = n_{\text{glass}}\sin\theta_2 \;\rightarrow\; (1.00)\sin 2\theta_2 = (1.56)\sin\theta_2$$

$$\sin 2\theta_2 = 2\sin\theta_2\cos\theta_2 = (1.56)\sin\theta_2 \;\rightarrow\; \cos\theta_2 = 0.780 \;\rightarrow\; \theta_2 = 38.74^\circ$$

$$\theta_1 = 2\theta_2 = \boxed{77.5^\circ}$$

$\boxed{43.}$ The beam forms the hypotenuse of two right triangles as it passes through the plastic and then the glass. The upper angle of the triangle is the angle of refraction in that medium. Note that the sum of the opposite sides is equal to the displacement *D*. First, we calculate the angles of refraction in each medium using Snell's Law (Eq. 32-5).



$$\sin 45 = n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_1 = \sin^{-1}\left(\frac{\sin 45}{n_1}\right) = \sin^{-1}\left(\frac{\sin 45}{1.62}\right) = 25.88°$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 45}{n_2}\right) = \sin^{-1}\left(\frac{\sin 45}{1.47}\right) = 28.75°$$

We then use the trigonometric identity for tangent to calculate the two opposite sides, and sum to get the displacement.

$$D = D_1 + D_2 = h_1 \tan\theta_1 + h_1 \tan\theta_1 = (2.0\,\text{cm})\tan 25.88° + (3.0\,\text{cm})\tan 28.75° = \boxed{2.6\,\text{cm}}$$

44. (*a*) We use Eq. 32-5 to calculate the refracted angle as the light enters the glass ($n$=1.56) from the air ($n$=1.00).

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \to \theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.56}\sin\ 43.5°\right] = 26.18° \approx \boxed{26.2°}$$

(*b*) We again use Eq. 32-5 using the refracted angle in the glass and the indices of refraction of the glass and water.

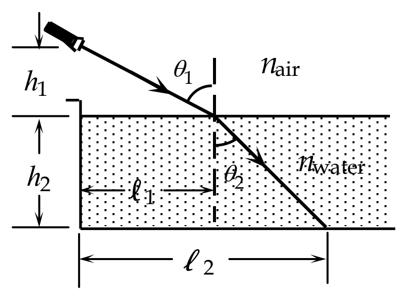
$$\theta_3 = \sin^{-1}\left[\frac{n_2}{n_3}\sin\theta_2\right] = \sin^{-1}\left[\frac{1.56}{1.33}\sin\ 26.18°\right] = 31.17° \approx \boxed{31.2°}$$

(*c*) We repeat the same calculation as in part (a), but using the index of refraction of water.

$$\theta_3 = \sin^{-1}\left[\frac{n_1}{n_3}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.33}\sin\ 43.5°\right] = 31.17° \approx \boxed{31.2°}$$

As expected the refracted angle in the water is the same whether the light beam first passes through the glass, or passes directly into the water.

45. We find the angle of incidence from the distances.




$$\tan\theta_1 = \frac{\ell_1}{h_1} = \frac{(2.5\,\text{m})}{(1.3\,\text{m})} = 1.9231 \quad \to \quad \theta_1 = 62.526°$$

For the refraction from air into water, we have

$$n_{\text{air}} \sin\theta_1 = n_{\text{water}} \sin\theta_2;$$

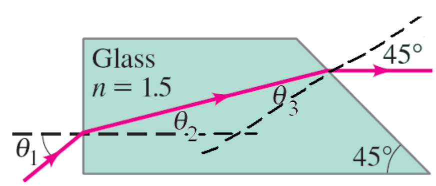
$$(1.00)\sin 62.526° = (1.33)\sin\theta_2 \quad \to \quad \theta_2 = 41.842°$$

We find the horizontal distance from the edge of the pool from

$$\ell = \ell_1 + \ell_2 = \ell_1 + h_2 \tan\theta_2$$

$$= 2.5\,\text{m} + (2.1\,\text{m})\tan 41.842° = 4.38\,\text{m} \approx \boxed{4.4\,\text{m}}$$

46. Since the light ray travels parallel to the base when it exits the glass, and the back edge of the glass makes a 45° angle to the horizontal, the exiting angle of refraction is 45°. We use Snell's law, Eq. 32-5, to calculate the incident angle at the back pane.




$$\theta_3 = \sin^{-1}\left[\frac{n_4}{n_3}\sin\theta_4\right] = \sin^{-1}\left[\frac{1.0}{1.5}\sin\ 45°\right] = 28.13°$$

We calculate the refracted angle at the front edge of the glass by noting that the angles $\theta_2$ and $\theta_3$ in the figure form two angles of a triangle. The third angle, as determined by the perpendiculars to the surface, is 135°.

$$\theta_2 + \theta_3 + 135° = 180° \rightarrow \theta_2 = 45° - \theta_3 = 45° - 28.13° = 16.87°$$

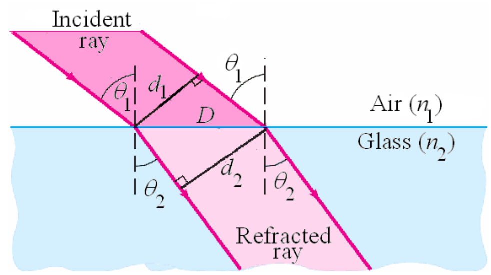
Finally, we use Snell's law at the front face of the glass to calculate the incident angle.

$$\theta_1 = \sin^{-1}\left[\frac{n_2}{n_1}\sin\theta_2\right] = \sin^{-1}\left[\frac{1.5}{1.0}\sin\ 16.87°\right] = 25.81° \approx \boxed{26°}$$

47. As the light ray passes from air into glass with an angle of incidence of 25°, the beam will refract. Determine the angle of refraction by applying Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow$$

$$\theta_2 = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_1\right] = \sin^{-1}\left[\frac{1.00}{1.5}\sin\ 25°\right] = 16.36°$$



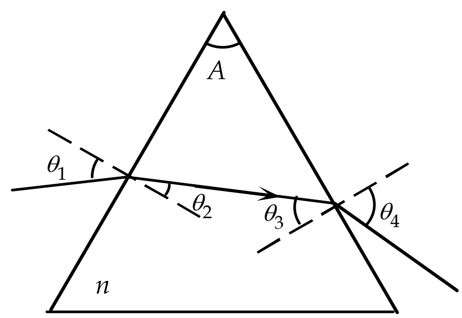

We now consider the two right triangles created by the diameters of the incident and refracted beams with the air–glass interface, as shown in the figure. The diameters form right angles with the ray direction and using complementary angles we see that the angle between the diameter and the interface is equal to the incident and refracted angles. Since the air–glass interface creates the hypotenuse for both triangles we use the definition of the cosine to solve for this length in each triangle and set the lengths equal. The resulting equation is solved for the diameter of the refracted ray.

$$D = \frac{d_1}{\cos\theta_1} = \frac{d_2}{\cos\theta_2} \quad \rightarrow \quad d_2 = d_1\frac{\cos\theta_2}{\cos\theta_1} = (3.0\,\text{mm})\frac{\cos 16.36°}{\cos 25°} = \boxed{3.2\,\text{mm}}$$

48. Find the angle $\theta_2$ for the refraction at the first surface.

$$n_{\text{air}} \sin\theta_1 = n \sin\theta_2$$

$$(1.00)\sin 45.0° = (1.54)\sin\theta_2 \quad \rightarrow \quad \theta_2 = 27.33°$$




Find the angle of incidence at the second surface from the triangle formed by the two sides of the prism and the light path.

$$(90° - \theta_2) + (90° - \theta_3) + A = 180° \quad \rightarrow$$

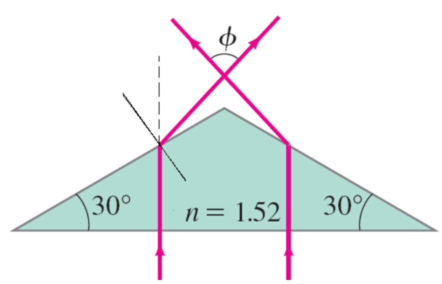
$$\theta_3 = A - \theta_2 = 60° - 27.33° = 32.67°$$

Use refraction at the second surface to find $\theta_4$.

$$n \sin\theta_3 = n_{\text{air}} \sin\theta_4 \quad \rightarrow \quad (1.54)\sin 32.67° = (1.00)\sin\theta_4 \quad \rightarrow \quad \theta_4 = \boxed{56.2° \text{ from the normal}}$$

$\boxed{49.}$ Since the angle of incidence at the base of the prism is $0°$, the rays are undeflected there. The angle of incidence at the upper face of the prism is $30°$. Use Snell's law to calculate the angle of refraction as the light exits the prism.

$$n_1 \sin\theta_1 = \sin\theta_r \rightarrow \theta_r = \sin^{-1}(1.52 \sin 30°) = 49.46°$$




From the diagram, note that a normal to either top surface makes a 30° angle from the vertical. Subtracting 30° from the refracted angle will give the angle of the beam with respect to the vertical. By symmetry, the angle $\phi$ is twice the angle of the refracted beam from the vertical.

$$\phi = 2(\theta_r - 30°) = 2(49.46° - 30°) = \boxed{38.9°}$$

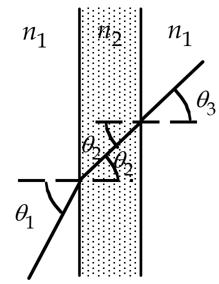
50. Because the surfaces are parallel, the angle of refraction from the first surface is the angle of incidence at the second. Thus for the two refractions, we have the following.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \ \ ; \ \ n_2 \sin\theta_2 = n_1 \sin\theta_3$$

Substitute the second equation into the first.

$$n_1 \sin\theta_1 = n_1 \sin\theta_3 \quad \rightarrow \quad \boxed{\theta_3 = \theta_1}$$

Because the ray emerges in the same index of refraction, it is undeviated.

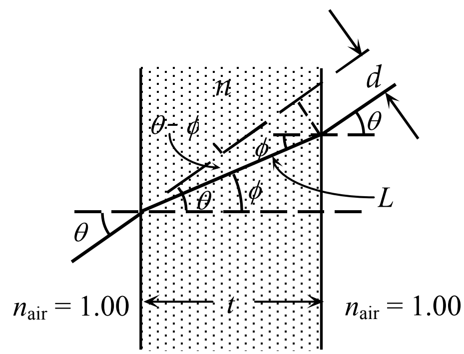



51. Because the glass surfaces are parallel, the exit beam will be traveling in the same direction as the original beam.
Find the angle inside the glass from Snell's law, $n_{\text{air}} \sin\theta = n \sin\phi$. Since the angles are small, $\cos\phi \approx 1$ and $\sin\phi \approx \phi$, where $\phi$ is in radians.

$$(1.00)\ \theta = n\phi \quad \rightarrow \quad \phi = \frac{\theta}{n}$$

Find the distance along the ray in the glass from $L = \dfrac{t}{\cos\phi} \approx t$, and then find the perpendicular displacement from the original direction.

$$d = L\sin(\theta - \phi) \approx t(\theta - \phi) = t\left[\theta - \left(\frac{\theta}{n}\right)\right] = \boxed{\frac{t\theta(n-1)}{n}}$$




52. We find the speed of light from the speed of light in a vacuum divided by the index of refraction. Examining the graph we estimate that the index of refraction of 450 nm light in silicate flint glass is 1.643 and of 680 nm light is 1.613. There will be some variation in the answers due to estimation from the graph.

$$\frac{v_{\text{red}} - v_{\text{blue}}}{v_{\text{red}}} = \frac{c/n_{680} - c/n_{450}}{c/n_{680}} = \frac{1/1.613 - 1/1.643}{1/1.613} = 0.01826 \approx \boxed{1.8\%}$$

53. We find the angles of refraction in the glass from Snell's law, Eq. 32-5.

$$(1.00)\sin 60.00^\circ = (1.4831)\sin\theta_{2,\text{blue}} \quad \rightarrow \quad \theta_{2,\text{blue}} = 35.727^\circ$$

$$(1.00)\sin 60.00^\circ = (1.4754)\sin\theta_{2,\text{red}} \quad \rightarrow \quad \theta_{2,\text{red}} = 35.943^\circ \text{ which gives } \theta_{2,700} = 35.943^\circ.$$

Thus the angle between the refracted beams is

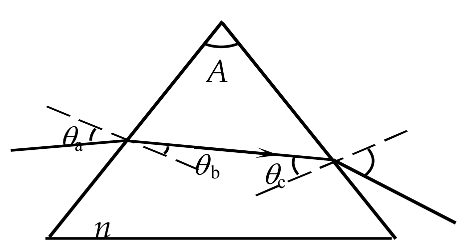
$$\theta_{2,\text{red}} - \theta_{2,\text{blue}} = 35.943^\circ - 35.727^\circ = 0.216^\circ \approx \boxed{0.22^\circ.}$$

54. The indices of refraction are estimated from Figure 32-28 as 1.642 for 465 nm and 1.619 for 652 nm. Consider the refraction at the first surface.

$$n_{\text{air}} \sin\theta_{\text{a}} = n \sin\theta_{\text{b}} \quad \rightarrow$$

$$(1.00)\sin 45^\circ = (1.642)\sin\theta_{\text{b1}} \quad \rightarrow \quad \theta_{\text{b1}} = 25.51^\circ$$

$$(1.00)\sin 45^\circ = (1.619)\sin\theta_{\text{b2}} \quad \rightarrow \quad \theta_{\text{b2}} = 25.90^\circ$$




We find the angle of incidence at the second surface from the upper triangle.

$$(90^\circ - \theta_{\text{b}}) + (90^\circ - \theta_{\text{c}}) + A = 180^\circ \quad \rightarrow$$

$\theta_{c1} = A - \theta_{b1} = 60.00° - 25.51° = 34.49°$ ; $\theta_{c2} = A - \theta_{b2} = 60.00° - 25.90° = 34.10°$

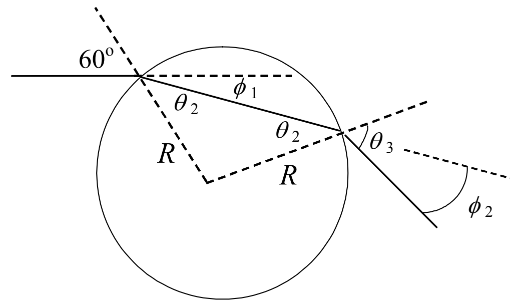
Apply Snell's law at the second surface.

$$n \sin\theta_c = n_{air} \sin\theta_d$$

$$(1.642)\sin 34.49° = (1.00)\sin\theta_{d1} \rightarrow \boxed{\theta_{d1} = 68.4° \text{ from the normal}}$$

$$(1.619)\sin 34.10° = (1.00)\sin\theta_{d2} \rightarrow \boxed{\theta_{d2} = 65.2° \text{ from the normal}}$$

55. At the first surface, the angle of incidence $\theta_1 = 60^o$ from air $(n_1 = 1.000)$ and the angle of refraction $\theta_2$ into water $(n_2 = n)$ is found using Snell's law.




$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow$$

$$(1.000)\sin 60° = (n)\sin\theta_2 \rightarrow$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 60°}{n}\right)$$

Note that at this surface the ray has been deflected from its initial direction by angle $\phi_1 = 60° - \theta_2$. From the figure we see that the triangle that is interior to the drop is an isosceles triangle, so the angle of incidence from water $(n_2 = n)$ at the second surface is $\theta_2$ and angle of refraction is $\theta_3$ into air $(n_3 = 1.000)$. This relationship is identical to the relationship at the first surface, showing that the refracted angle as the light exits the drop is again 60°.

$$n_2 \sin\theta_2 = n_3 \sin\theta_3 \rightarrow (n)\sin\theta_2 = (1.000)\sin\theta_3 \rightarrow \sin\theta_3 = n\sin\theta_2 \rightarrow$$

$$\sin\theta_3 = n\left(\frac{\sin 60^o}{n}\right) = \sin 60^o \rightarrow \theta_3 = 60^o$$

Note that at this surface the ray has been deflected from its initial direction by the angle $\phi_2 = \theta_3 - \theta_2 = 60^o - \theta_2$. The total deflection of the ray is equal to the sum of the deflections at each surface.

$$\phi = \phi_1 + \phi_2 = (60^o - \theta_2) + (60^o - \theta_2) = 120^o - 2\theta_2 = 120^o - 2\sin^{-1}\left(\frac{\sin 60^o}{n}\right)$$

Inserting the indices of refraction for the two colors and subtracting the angles gives the difference in total deflection.

$$\Delta\phi = \phi_{violet} - \phi_{red} = \left\{120^o - 2\sin^{-1}\left[\frac{\sin 60^o}{n_{violet}}\right]\right\} - \left\{120^o - 2\sin^{-1}\left[\frac{\sin 60^o}{n_{red}}\right]\right\}$$

$$= 2\left\{\sin^{-1}\left[\frac{\sin 60^o}{n_{red}}\right] - \sin^{-1}\left[\frac{\sin 60^o}{n_{violet}}\right]\right\} = 2\left\{\sin^{-1}\left[\frac{\sin 60^o}{1.330}\right] - \sin^{-1}\left[\frac{\sin 60^o}{1.341}\right]\right\} = \boxed{0.80°}$$

56. (*a*) We solve Snell's law for the refracted angle. Then, since the index varies by only about 1%, we differentiate the angle with respect to the index of refraction to determine the spread in angle.

$$\sin\theta_1 = n\sin\theta_2 \rightarrow \theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n}\right) \rightarrow$$

$$\frac{\Delta\theta_2}{\Delta n} \approx \frac{d\theta_2}{dn} = \frac{\sin\theta_1}{n^2\sqrt{1 - \frac{\sin^2\theta_1}{n^2}}} \rightarrow \boxed{\Delta\theta_2 = \frac{\Delta n}{n}\frac{\sin\theta_1}{\sqrt{n^2 - \sin^2\theta_1}}}$$

(*b*) We set $n = 1.5$ and $\theta_1 = 0° = 0\,\text{rad}$ and solve for the spread in refracted angle.

$$\Delta\theta_2 = \frac{\Delta n}{n}\frac{\sin\theta_1}{\sqrt{n^2 - \sin^2\theta_1}} = (0.01)\frac{\sin\ 0}{\sqrt{1.5^2 - \sin^2 0}} = \boxed{0}$$

(*c*) We set $n = 1.5$ and $\theta_1 = 90°$ and solve for the spread in refracted angle. We must convert the spread from radians back to degrees.

$$\Delta\theta_2 = (0.01)\frac{\sin\ 90°}{\sqrt{1.5^2 - \sin^2 90°}} = 0.0089\ \text{rad}\left(\frac{180°}{\pi\ \text{rad}}\right) = \boxed{0.5°}$$

57. When the light in the material with a higher index is incident at the critical angle, the refracted angle is 90°. Use Snell's law.

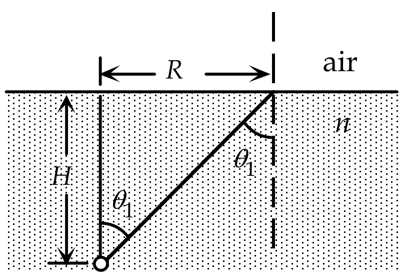
$$n_{\text{diamond}}\sin\theta_1 = n_{\text{water}}\sin\theta_2 \ \rightarrow\ \theta_1 = \sin^{-1}\left(\frac{n_{\text{water}}}{n_{\text{diamond}}}\right) = \sin^{-1}\frac{1.33}{2.42} = \boxed{33.3°}$$

Because diamond has the higher index, the light must start in $\boxed{\text{diamond.}}$

58. When the light in the liquid is incident at the critical angle, the refracted angle is 90°. Use Snell's law.

$$n_{\text{liquid}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \ \rightarrow\ n_{\text{liquid}} = n_{\text{air}}\frac{\sin\theta_2}{\sin\theta_1} = (1.00)\frac{1}{\sin 49.6°} = \boxed{1.31}$$

59. We find the critical angle for light leaving the water:



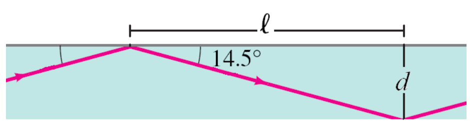

$$n_{\text{water}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \ \rightarrow$$

$$\theta_1 = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\frac{1.00}{1.33} = 48.75°$$

If the light is incident at a greater angle than this, it will totally reflect. Find $R$ from the diagram.

$$R > H\tan\theta_1 = (72.0\,\text{cm})\tan 48.75° = \boxed{82.1\,\text{cm}}$$

60. The ray reflects at the same angle, so each segment makes a 14.5° angle with the side. We find the distance $\ell$ between reflections from the definition of the tangent function.




$$\tan\theta = \frac{d}{\ell} \rightarrow \ell = \frac{d}{\tan\theta} = \frac{1.40\times10^{-4}\ \text{m}}{\tan\ 14.5°} = \boxed{5.41\times10^{-4}\ \text{m}}$$

$\boxed{61.}$ We find the angle of incidence from the distances.

$$\tan\theta_1 = \frac{\ell}{h} = \frac{(7.6\,\text{cm})}{(8.0\,\text{cm})} = 0.95 \ \rightarrow\ \theta_1 = 43.53°$$

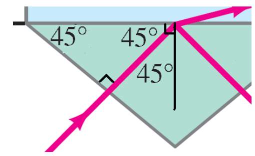
The relationship for the maximum incident angle for refraction from liquid into air gives this.

$$n_{\text{liquid}}\sin\theta_1 = n_{\text{air}}\sin\theta_2 \ \rightarrow\ n_{\text{liquid}}\sin\theta_{1\,\text{max}} = (1.00)\sin 90° \ \rightarrow\ \sin\theta_{1\,\text{max}} = \frac{1}{n_{\text{liquid}}}$$

Thus we have the following.

$$\sin\theta_1 \ge \sin\theta_{1\max} = \frac{1}{n_{\text{liquid}}} \rightarrow \sin 43.53° = 0.6887 \ge \frac{1}{n_{\text{liquid}}} \rightarrow \boxed{n_{\text{liquid}} \ge 1.5}$$

62. For the device to work properly, the light should experience total internal reflection at the top surface of the prism when it is a prism to air interface, but not total internal reflection when the top surface is a prism to water interface. Since the incident ray is perpendicular to the lower surface of the prism, light does not experience refraction at that surface. As shown in the diagram, the incident angle for the upper surface will be 45°. We then use Eq. 32-7 to determine the minimum index of refraction for total internal reflection with an air interface, and the maximum index of refraction for a water interface. The usable indices of refraction will lie between these two values.




$$\frac{n_2}{n_1} = \sin\theta_c \rightarrow n_{1,\min} = \frac{n_{\text{air}}}{\sin\theta_c} = \frac{1.00}{\sin 45°} = 1.41 \rightarrow n_{1,\max} = \frac{n_{\text{water}}}{\sin\theta_c} = \frac{1.33}{\sin 45°} = 1.88$$

The index of refraction must fall within the range $1.41 < n < 1.88$. $\boxed{\text{A Lucite prism will work.}}$

63. (*a*) We calculate the critical angle using Eq. 32-7. We calculate the time for each ray to pass through the fiber by dividing the length the ray travels by the speed of the ray in the fiber. The length for ray A is the horizontal length of the fiber. The length for ray B is equal to the length of the fiber divided by the critical angle, since ray B is always traveling along a diagonal line at the critical angle relative to the horizontal. The speed of light in the fiber is the speed of light in a vacuum divided by the index of refraction in the fiber.

$$\sin\theta_c = \frac{n_2}{n_1} \;;\; \Delta t = t_B - t_A = \frac{\ell_B}{v} - \frac{\ell_A}{v} = \frac{\ell_A}{v\sin\theta_c} - \frac{\ell_A}{v} = \frac{\ell_A}{c/n_1}\left(\frac{n_1}{n_2} - 1\right)$$

$$= \frac{(1.0\,\text{km})(1.465)}{(3.00\times10^5\,\text{km/s})}\left(\frac{1.465}{1.000} - 1\right) = \boxed{2.3\times10^{-6}\,\text{s}}$$

(*b*) We now replace the index of refraction of air ($n$ = 1.000) with the index of refraction of the glass "cladding" ($n$ = 1.460).

$$\Delta t = \frac{\ell_A n_1}{c}\left(\frac{n_1}{n_2} - 1\right) = \frac{(1.0\,\text{km})(1.465)}{3.00\times10^5\,\text{km/s}}\left(\frac{1.465}{1.460} - 1\right) = \boxed{1.7\times10^{-8}\,\text{s}}$$

64. (*a*) The ray enters normal to the first surface, so there is no deviation there. The angle of incidence is 45° at the second surface. When there is air outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow n_1\sin 45° = (1.00)\sin\theta_2$$

For total internal reflection to occur, $\sin\theta_2 \ge 1$, and so $n_1 \ge \dfrac{1}{\sin 45°} = \boxed{1.41}$.

(*b*) When there is water outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow (1.58)\sin 45° = (1.33)\sin\theta_2 \rightarrow \sin\theta_2 = 0.84$$

Because $\sin\theta_2 < 1$, $\boxed{\text{the prism will not be totally reflecting.}}$

(*c*) For total reflection when there is water outside the surface, we have the following.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \rightarrow n_1\sin 45° = (1.33)\sin\theta_2$$

$$n_1\sin 45° = (1.33)\sin\theta_2.$$

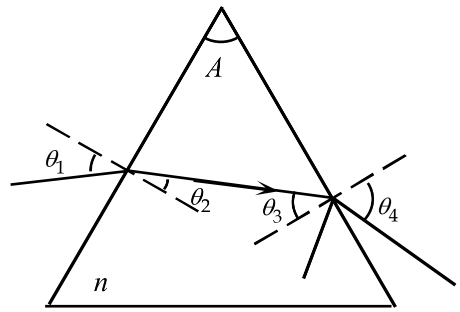
For total internal reflection to occur, $\sin\theta_2 \ge 1$.

$$n_1 \ge \frac{1.33}{\sin 45°} = \boxed{1.88}$$

65. For the refraction at the first surface, we have the following.

$$n_{\text{air}} \sin\theta_1 = n\sin\theta_2 \quad\rightarrow\quad (1.00)\sin\theta_1 = n\sin\theta_2 \quad\rightarrow$$

$$\sin\theta_2 = \sin\frac{\theta_1}{n}.$$




Find the angle of incidence at the second surface.

$$(90° - \theta_2) + (90° - \theta_3) + A = 180° \quad\rightarrow$$

$$\theta_3 = A - \theta_2 = 60.0° - \theta_2$$

For the refraction at the second surface, we have this.

$$n\sin\theta_3 = n_{\text{air}}\sin\theta_4 = (1.00)\sin\theta_4$$

The maximum value of $\theta_4$ before internal reflection takes place at the second surface is 90°. For internal reflection to occur, we have the following.

$$n\sin\theta_3 = n\sin(A - \theta_2) \ge 1 \quad\rightarrow\quad n(\sin A\cos\theta_2 - \cos A\sin\theta_2) \ge 1$$

Use the result from the first surface to eliminate $n$.

$$\frac{\sin\theta_1(\sin A\cos\theta_2 - \cos A\sin\theta_2)}{(\sin\theta_2)} = \sin\theta_1\left(\frac{\sin A}{\tan\theta_2 - \cos A}\right) \ge 1 \quad\rightarrow$$

$$\frac{1}{\tan\theta_2} \ge \frac{\left[\left(\frac{1}{\sin\theta_1}\right) + \cos A\right]}{\sin A} = \frac{\left[\left(\frac{1}{\sin 45.0°}\right) + \cos 60.0°\right]}{\sin 60.0°} = 2.210 \quad\rightarrow \text{ or}$$

$$\tan\theta_2 \le 0.452 \quad\rightarrow\quad \theta_2 \le 24.3°$$

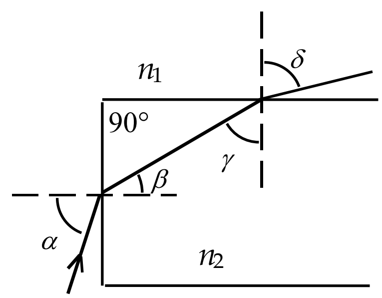
Use the result from the first surface.

$$n_{\text{min}} = \frac{\sin\theta_1}{\sin\theta_{2\,\text{max}}} = \frac{\sin 45.0°}{\sin 24.3°} = 1.715 \quad\rightarrow\quad \boxed{n \ge 1.72}$$

66. For the refraction at the side of the rod, we have $n_2 \sin\gamma = n_1 \sin\delta$.

The minimum angle for total reflection $\gamma_{\text{min}}$ occurs when $\delta = 90°$.

$$n_2 \sin\gamma_{\text{min}} = (1.00)(1) = 1 \quad\rightarrow\quad \sin\gamma_{\text{min}} = \frac{1}{n_2}$$




Find the maximum angle of refraction at the end of the rod.

$$\beta_{\text{max}} = 90° - \gamma_{\text{min}}$$

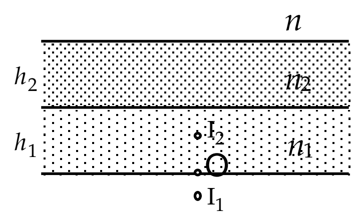
Because the sine function increases with angle, for the refraction at the end of the rod, we have the following.

$$n_1 \sin\alpha_{\text{max}} = n_2 \sin\beta_{\text{max}} \quad\rightarrow\quad (1.00)\sin\alpha_{\text{max}} = n_2 \sin(90° - \gamma_{\text{min}}) = n_2 \cos\gamma_{\text{min}}$$

If we want total internal reflection to occur for any incident angle at the end of the fiber, the maximum value of $\alpha$ is 90°, so $n_2 \cos\gamma_{\text{min}} = 1$. When we divide this by the result for the refraction at the side, we get $\tan\gamma_{\text{min}} = 1 \rightarrow \gamma_{\text{min}} = 45°$. Thus we have the following.

$$\boxed{n_2 \ge \frac{1}{\sin\gamma_{\text{min}}} = \frac{1}{\sin 45°} = 1.414}$$

67. We find the location of the image of a point on the bottom from the refraction from water to glass, using Eq. 32-8, with $R = \infty$.




$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R} = 0 \rightarrow$$

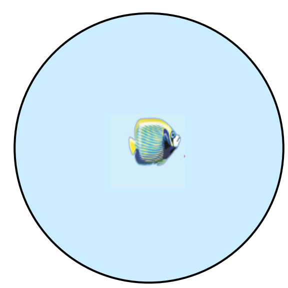
$$d_i = -\frac{n_2 d_o}{n_1} = -\frac{1.58(12.0\,\text{cm})}{1.33} = -14.26\,\text{cm}$$

Using this image distance from the top surface as the object for the refraction from glass to air gives the final image location, which is the apparent depth of the water.

$$\frac{n_2}{d_{o2}} + \frac{n_3}{d_{i2}} = \frac{n_3 - n_2}{R} = 0 \rightarrow d_{i2} = -\frac{n_3 d_{o2}}{n_2} = -\frac{1.00(13.0\,\text{cm} + 14.26\,\text{cm})}{1.58} = -17.25\,\text{cm}$$

Thus the bottom appears to be 17.3 cm below the surface of the glass. In reality it is 25 cm.

68. (*a*) We use Eq. 32-8 to calculate the location of the image of the fish. We assume that the observer is outside the circle in the diagram, to the right of the diagram. The fish is located at the center of the sphere so the object distance is 28.0 cm. Since the glass is thin we use the index of refraction of the water and of the air. Index 1 refers to the water, and index 2 refers to the air. The radius of curvature of the right side of the bowl is negative.



$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R} \rightarrow$$

$$d_i = n_2\left[\frac{n_2 - n_1}{R} - \frac{n_1}{d_o}\right]^{-1} = 1.00\left[\frac{1.00 - 1.33}{-28.0\,\text{cm}} - \frac{1.33}{28.0\,\text{cm}}\right]^{-1} = \boxed{-28.0\,\text{cm}}$$

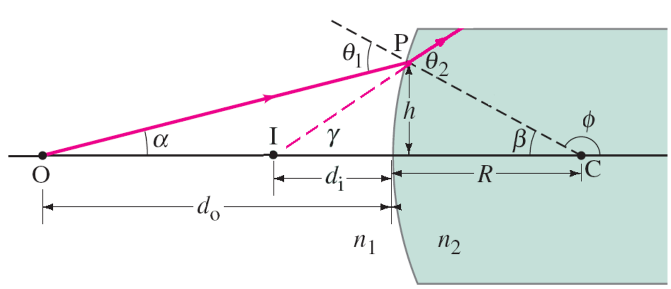
The image is also at the center of the bowl. When the fish is at the center of the bowl, all small-angle light rays traveling outward from the fish are approximately perpendicular to the surface of the bowl, and therefore do not refract at the surface. This causes the image of the fish to also be located at the center of the bowl.

(*b*) We repeat the same calculation as above with the object distance 20.0 from the right side of the bowl, so $d_o = 20.0\,\text{cm}$.

$$d_i = n_2\left[\frac{n_2 - n_1}{R} - \frac{n_1}{d_o}\right]^{-1} = 1.00\left[\frac{1.00 - 1.33}{-28.0\,\text{cm}} - \frac{1.33}{20.0\,\text{cm}}\right]^{-1} = \boxed{-18.3\,\text{cm}}$$

The fish appears closer to the center of the bowl than it actually is.

69. (*a*) The accompanying figure shows a light ray originating at point O and entering the convex spherical surface at point P. In this case $n_2 < n_1$. The ray bends away from the normal and creates a virtual image at point I. From the image and supplementary angles we obtain the relationships between the angles.




$$\theta_1 = \alpha + \beta \qquad \theta_2 = \gamma + \beta$$

We then use Snell's law to relate the incident and refracted angle. For this derivation we assume these are small angles.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \rightarrow \quad n_1\theta_1 = n_2\theta_2$$

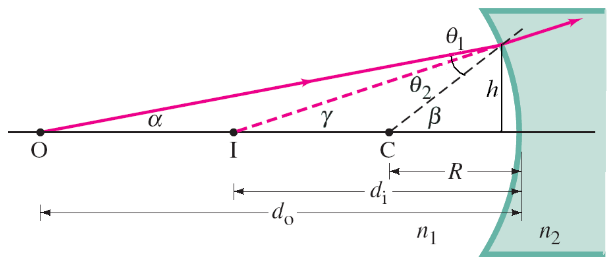
From the diagram we can create three right triangles, each with height *h* and lengths $d_o$, $d_i$, and *R*. Again, using the small angle approximation we obtain a relationship between the angles and lengths. Combining these definitions to eliminate the angles we obtain Eq. 32-8, noting that by our definition $d_i$ is a negative value.

$$\alpha = \frac{h}{d_o} \;;\; \gamma = \frac{h}{R} \;;\; \beta = \frac{h}{(-d_i)}$$

$$n_1\theta_1 = n_2\theta_2 \quad \rightarrow \quad n_1(\alpha+\beta) = n_2(\gamma+\beta) = n_1\alpha + n_1\beta = n_2\gamma + n_2\beta \quad \rightarrow$$

$$n_1\frac{h}{d_o} + n_1\frac{h}{R} = n_2\frac{h}{(-d_i)} + n_2\frac{h}{R} \quad \rightarrow \quad \boxed{\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}}$$

(*b*) This image shows a concave surface with $n_2 > n_1$. Again, we use the approximation of small angles and sign convention that $R < 0$ and $d_i < 0$. We write relationships between the angles using supplementary angles, Snell's law, and right triangles. Combining these equations to eliminate the angles we arrive at Eq. 32-8.

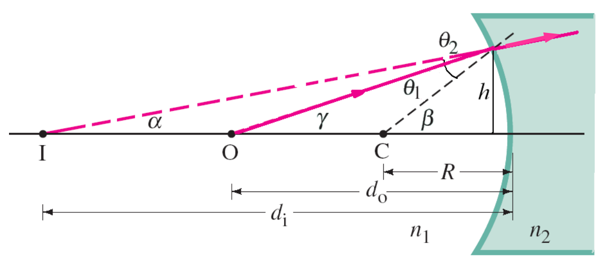



$$\theta_1 = \beta - \alpha \;;\; \theta_2 = \beta - \gamma \;;\; \alpha = \frac{h}{d_o} \;;\; \gamma = \frac{h}{(-d_i)} \;;\; \beta = \frac{h}{(-R)}$$

$$n_1\theta_1 = n_2\theta_2 \quad \rightarrow \quad n_1(\beta-\alpha) = n_2(\beta-\gamma) = n_1\beta - n_1\alpha = n_2\beta - n_2\gamma \quad \rightarrow$$

$$n_1\frac{h}{(-R)} - n_1\frac{h}{d_o} = n_2\frac{h}{(-R)} - n_2\frac{h}{(-d_i)} \quad \rightarrow \quad \boxed{\frac{n_1}{d_0} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}}$$

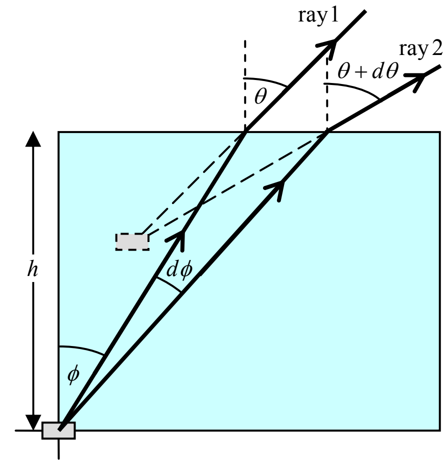
(*c*) This image shows a concave surface with $n_2 < n_1$. Again, we use the approximation of small angles and sign convention that $R < 0$ and $d_i < 0$. We write relationships between the angles using supplementary angles, Snell's law, and right triangles. Combining these equations to eliminate the angles we arrive at Eq. 32-8.




$$\theta_1 = \beta - \gamma \;;\; \theta_2 = \beta - \alpha \;;\; \alpha = \frac{h}{(-d_i)} \;;\; \gamma = \frac{h}{d_o} \;;\; \beta = \frac{h}{(-R)}$$

$$n_1\theta_1 = n_2\theta_2 \quad \rightarrow \quad n_1(\beta-\gamma) = n_2(\beta-\alpha) = n_1\beta - n_1\gamma = n_2\beta - n_2\alpha \quad \rightarrow$$

$$n_1\frac{h}{(-R)} - n_1\frac{h}{d_o} = n_2\frac{h}{(-R)} - n_2\frac{h}{(-d_i)} \quad \rightarrow \quad \boxed{\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}}$$




70. We consider two rays leaving the coin. These rays refract upon leaving the surface and reach the observer's eye with angles of refraction all very near $\theta = 45°$. Let the origin of coordinates be at the actual location of the coin. We will write straight-line equations for each of two refracted rays, one with a refraction angle of $\theta$ and the other with a refraction angle of $\theta + d\theta$, and extrapolate them back to where they intersect to find the location of the image. We utilize the relationship $f(x+dx) = f(x) + \left(\frac{df}{dx}\right)dx.$

First, apply Snell's law to both rays.

Ray # 1, leaving the coin at angle $\phi$.

$$n \sin\phi = \sin\theta$$

Ray # 2, leaving the coin at angle $\phi + d\phi$.

$$n \sin(\phi + d\phi) = \sin(\theta + d\theta)$$

Note the following relationship involving the differential angles.

$$\sin(\phi + d\phi) = \sin\phi + \frac{d(\sin\phi)}{d\phi}d\phi = \sin\phi + \cos\phi d\phi \;\; ; \;\; \sin(\theta + d\theta) = \sin\theta + \cos\theta d\theta$$

So for Ray # 2, we would have the following Snell's law relationship.

$$n[\sin\phi + \cos\phi d\phi] = [\sin\theta + \cos\theta d\theta] \;\; \rightarrow \;\; n\sin\phi + n\cos\phi d\phi = \sin\theta + \cos\theta d\theta \;\; \rightarrow$$

$$n\cos\phi d\phi = \cos\theta d\theta \;\; \rightarrow \;\; d\phi = \frac{\cos\theta}{n\cos\phi}d\theta$$

This relationship between $d\phi$ and $d\theta$ will be useful later in the solution.

Ray # 1 leaves the water at coordinates $x_1 = h\tan\phi,\ y_1 = h$ and has a slope after it leaves the water of $m_1 = \tan(90° - \theta) = \cot\theta.$ Thus a straight-line equation describing ray # 1 after it leaves the water is as follows.

$$y - y_1 = (x - x_1)m_1 \;\; \rightarrow \;\; y = h + (x - h\tan\phi)\cot\theta$$

Ray # 2 leaves the water at the following coordinates.

$$x_2 = h\tan(\phi + d\phi) = h\left[\tan\phi + \frac{d(\tan\phi)}{d\phi}d\phi\right] = h\left[\tan\phi + \sec^2\phi d\phi\right],\ y_2 = h$$

Ray # 2 has the following slope after it leaves the water.

$$m_2 = \tan\left[90° - (\theta + d\theta)\right] = \cot(\theta + d\theta) = \cot\theta + \frac{d(\cot\theta)}{d\theta}d\theta = \cot\theta - \csc^2\theta d\theta$$

Thus a straight-line equation describing ray # 2 after it leaves the water is as follows.

$$y - y_2 = (x - x_2)m_2 \;\; \rightarrow \;\; y = h + \left(x - h\left[\tan\phi + \sec^2\phi d\phi\right]\right)\left[\cot\theta - \csc^2\theta d\theta\right]$$

To find where these rays intersect, which is the image location, set the two expressions for $y$ equal to each other.

$$h + (x - h\tan\phi)\cot\theta = h + \left(x - h\left[\tan\phi + \sec^2\phi d\phi\right]\right)\left[\cot\theta - \csc^2\theta d\theta\right] \;\; \rightarrow$$

Expanding the terms and subtracting common terms gives us the following.

$$x\csc^2\theta d\theta = h\tan\phi\csc^2\theta d\theta - h\sec^2\phi d\phi\cot\theta + h\sec^2\phi d\phi\csc^2\theta d\theta$$

The first three terms each have a differential factor, but the last term has two differential factors.

That means the last term is much smaller than the other terms, and so can be ignored. So we delete the last term, and use the relationship between the differentials derived earlier.

$$x\csc^2\theta d\theta = h\tan\phi\csc^2\theta d\theta - h\sec^2\phi d\phi\cot\theta \quad ; \quad d\phi = \frac{\cos\theta}{n\cos\phi}d\theta \quad \rightarrow$$

$$x\csc^2\theta d\theta = h\tan\phi\csc^2\theta d\theta - h\sec^2\phi\frac{\cos\theta}{n\cos\phi}d\theta\cot\theta$$

$$x = h\left[\tan\phi - \sec^2\phi\frac{\cos\theta}{n\cos\phi}\frac{\cot\theta}{\csc^2\theta}\right] = h\left[\tan\phi - \frac{\cos^2\theta\sin\theta}{n\cos^3\phi}\right]$$

Now we may substitute in values. We know that $\theta = 45°$ and $h = 0.75\,\text{m}$. We use the original relationship for ray # 1 to solve for $\phi$. And once we solve for $x$, we use the straight-line equation for ray # 1 to solve for $y$.

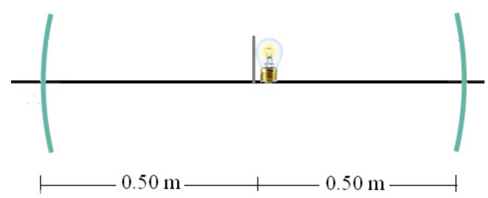
$$n\sin\phi = \sin\theta \quad \rightarrow \quad \phi = \sin^{-1}\frac{\theta}{n} = \sin^{-1}\frac{\sin 45°}{1.33} = 32.12°$$

$$x = h\left[\tan\phi - \frac{\cos^2\theta\sin\theta}{n\cos^3\phi}\right] = 0.75\left[\tan 32.12 - \frac{\cos^2 45\sin 45}{1.33\cos^3 32.12}\right] = 0.1427\,\text{m}$$

$$y = h + (x - h\tan\phi)\cot\theta = 0.75 + (0.1427 - 0.75\tan 32.12)\cot 45 = 0.4264\text{m}$$

The image of the coin is located 0.14 m toward the viewer and 0.43 m above the actual coin.

71. Use Eq. 32-2 to determine the location of the image from the right mirror, in terms of the focal length. Since this distance is measured from the right mirror, we subtract that distance from the separation distance between the two mirrors to obtain the object distance for the left mirror. We then insert this object distance back into Eq. 32-2, with the known image distance and combine terms to write a quadratic equation for the focal length.




$$\frac{1}{f} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \quad \rightarrow \quad d_{i1} = \left(\frac{1}{f} - \frac{1}{d_{o1}}\right)^{-1} = \frac{fd_{o1}}{d_{o1} - f}$$

$$d_{o2} = D - d_{i1} = D - \frac{fd_{o1}}{d_{o1} - f} = \frac{Dd_{o1} - fD - fd_{o1}}{d_{o1} - f}$$

$$\frac{1}{f} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{d_{o1} - f}{Dd_{o1} - fD - fd_{o1}} + \frac{1}{d_{i2}} = \frac{d_{i2}d_{o1} - fd_{i2} + Dd_{o1} - fD - fd_{o1}}{d_{i2}(Dd_{o1} - fD - fd_{o1})}$$

$$d_{i2}(Dd_{o1} - fD - fd_{o1}) = d_{i2}d_{o1}f - f^2d_{i2} + fDd_{o1} - f^2D - f^2d_{o1}$$

$$f^2[d_{i2} + D + d_{o1}] - f[2d_{i2}d_{o1} + Dd_{o1} + Dd_{i2}] + Dd_{o1}d_{i2} = 0$$

We insert the values for the initial object distance, final image distance, and mirror separation distance and then solve the quadratic equation.

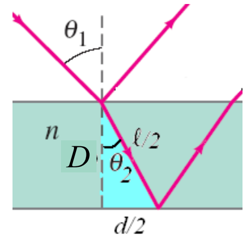
$$f^2[0.50\,\text{m} + 1.00\,\text{m} + 0.50\,\text{m}] - f\left[2(0.50\,\text{m})^2 + 2(1.00\,\text{m})(0.50\,\text{m})\right] + (1.00\,\text{m})(0.50\,\text{m})^2 = 0$$

$$(2.00\,\text{m})f^2 - (1.50\,\text{m}^2)f + 0.25\,\text{m}^3 = 0$$

$$f = \frac{1.50\,\text{m}^2 \pm \sqrt{(1.50\,\text{m}^2)^2 - 4(2.00\,\text{m})(0.25\,\text{m}^3)}}{2(2.00\,\text{m})} = \boxed{0.25\,\text{m} \text{ or } 0.50\,\text{m}}$$

If the focal length is 0.25 m, the right mirror creates an image at the location of the object. With the paper in place, this image would be blocked out. With a focal length of 0.50 m, the light from the right mirror comes out as parallel light. No image is formed from the right mirror. When this parallel light enters the second mirror it is imaged at the focal point (0.50 m) of the second mirror.

72. (*a*) We use Snell's law to calculate the refracted angle within the medium. Then using the right triangle formed by the ray within the medium, we can use the trigonometric identities to write equations for the horizontal displacement and path length.




$$\sin\theta_1 = n\sin\theta_2 \rightarrow \sin\theta_2 = \frac{\sin\theta_1}{n}$$

$$\cos\theta_2 = \frac{D}{\ell/2} \rightarrow \ell = \frac{2D}{\cos\theta_2} = \frac{2D}{\sqrt{1-\sin^2\theta_2}} = \boxed{\frac{2nD}{\sqrt{n^2-\sin^2\theta_1}}}$$

$$\sin\theta_2 = \frac{d/2}{\ell/2} \rightarrow d = \ell\sin\theta_2 = \frac{2nD}{\sqrt{n^2-\sin^2\theta_1}}\frac{\sin\theta_1}{n} = \boxed{\frac{2D\sin\theta_1}{\sqrt{n^2-\sin^2\theta_1}}}$$

(*b*) Evaluate the above expressions for $\theta_1 = 0°$.

$$\ell = \frac{2nD}{\sqrt{n^2-\sin^2\theta_1}} = \frac{2nD}{\sqrt{n^2}} = 2D \ ; \ \sin\theta_2 = \frac{d/2}{\ell/2} \rightarrow d = \frac{2D\sin\theta_1}{\sqrt{n^2-\sin^2\theta_1}} = 0$$

These are the expected values.

73. (*a*) The first image seen will be due to a single reflection off the front glass. This image will be equally far behind the mirror as you are in front of the mirror.

$$D_1 = 2\times 1.5\text{ m} = \boxed{3.0\text{ m}}$$

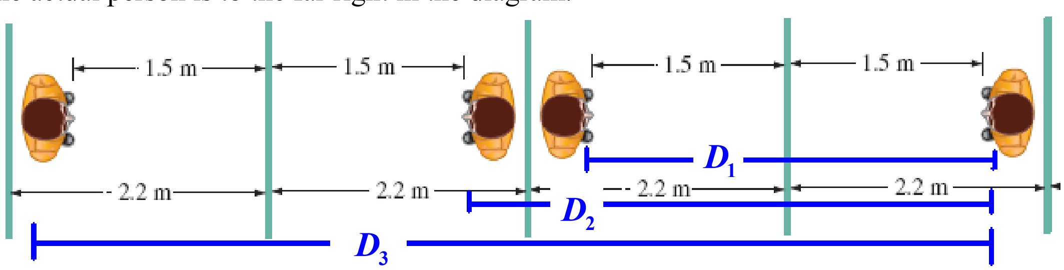
The second image seen will be the image reflected once off the front mirror and once off the back mirror. As seen in the diagram, this image will appear to be twice the distance between the mirrors.

$$D_2 = 1.5\text{ m} + 2.2\text{ m} + (2.2\text{ m} - 1.5\text{ m}) = 2\times 2.2\text{ m} = \boxed{4.4\text{ m}}$$

The third image seen will be the image reflected off the front mirror, the back mirror, and off the front mirror again. As seen in the diagram this image distance will be the sum of twice your distance to the mirror and twice the distance between the mirrors.

$$D_3 = 1.5\text{ m} + 2.2\text{ m} + 2.2\text{ m} + 1.5\text{ m} = 2\times 1.5\text{ m} + 2\times 2.2\text{ m} = \boxed{7.4\text{ m}}$$

The actual person is to the far right in the diagram.




(*b*) We see from the diagram that the first image is facing **toward you**; the second image is facing **away from you**; and the third image is facing **toward you**.

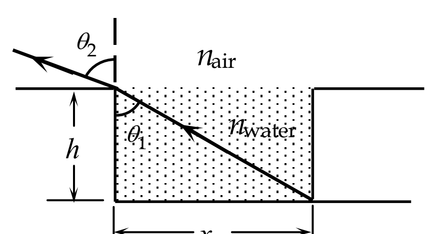
74. Find the angle of incidence for refraction from water into air.

$$n_{water}\sin\theta_1 = n_{air}\sin\theta_2 \rightarrow$$

$$(1.33)\sin\theta_1 = (1.00)\sin(90.0° - 13.0°) \rightarrow \theta_1 = 47.11°$$

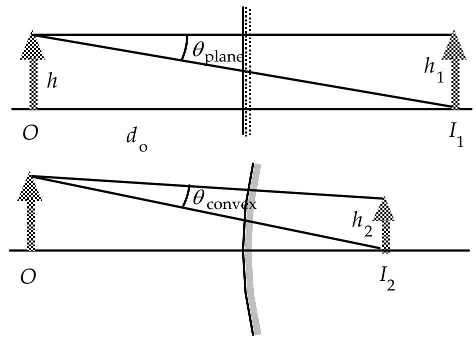
$$(1.33)\sin\theta_1 = (1.00)\sin(90.0° - 13.0°),$$

We find the depth of the pool from $\tan\theta_1 = x/h$.



$$\tan 47.11° = (5.50\,\text{m})/h \quad \rightarrow \quad h = \boxed{5.11\,\text{m}}$$

75. The apparent height of the image is related to the angle subtended by the image. For small angles, this angle is the height of the image divided by the distance between the image and viewer. Since both images are virtual, which gives a negative image distance, the image to viewer (object) distance will be the object distance minus the image distance. For the plane mirror the object and image heights are the same, and the image distance is the negative of the object distance.




$$h_i = h_o \;;\; d_i = -d_o \;;\; \theta_{plane} = \frac{h_i}{d_o - d_i} = \frac{h_o}{2d_o}$$

We use Eq. 32-2 and 32-3 to write the angle of the image in the convex mirror in terms of the object size and distance.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \;\rightarrow\; d_i = \frac{d_o f}{d_o - f} \;\rightarrow\; d_o - d_i = \frac{d_o^2 - 2d_o f}{d_o - f}$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o} \;\rightarrow\; h_i = -\frac{h_o d_i}{d_o} = -\frac{h_o f}{d_o - f}$$

$$\theta_{convex} = \frac{h_i}{d_o - d_i} = -\left(\frac{h_o f}{d_o - f}\right)\left(\frac{d_o - f}{d_o^2 - 2d_o f}\right) = \frac{-h_o f}{d_o^2 - 2d_o f}$$

We now set the angle in the convex mirror equal to ½ of the angle in the plane mirror and solve for the focal length.

$$\theta_{convex} = \tfrac{1}{2}\theta_{plane} \;\rightarrow\; \frac{-h_o f}{d_o^2 - 2d_o f} = \frac{h_o}{4d_o} \;\rightarrow\; -4d_o f = d_o^2 - 2d_o f \;\rightarrow\; f = -\tfrac{1}{2}d_o$$

We use Eq. 32-1 to calculate the radius of the mirror.

$$r = 2f = 2\left(-\tfrac{1}{2}d_o\right) = -d_o = \boxed{-3.80\,\text{m}}$$

76. For the critical angle, the refracted angle is 90°. For the refraction from plastic to air, we have the following.

$$n_{plastic}\sin\theta_{plastic} = n_{air}\sin\theta_{air} \;\rightarrow\; n_{plastic}\sin 39.3° = (1.00)\sin 90° \;\rightarrow\; n_{plastic} = 1.5788$$

For the refraction from plastic to water, we have the following.

$$n_{plastic}\sin\theta'_{plastic} = n_{water}\sin\theta_{water} \;\rightarrow\; (1.5788)\sin\theta'_{plastic} = (1.33)\sin 90° \;\rightarrow\; \theta'_{plastic} = \boxed{57.4°}$$

77. The two students $\boxed{\text{chose different signs for the magnification,}}$ i.e., one upright and one inverted. The focal length of the concave mirror is $f = \tfrac{1}{2}R = \tfrac{1}{2}(46\,\text{cm}) = 23\,\text{cm}$. We relate the object and image distances from the magnification.

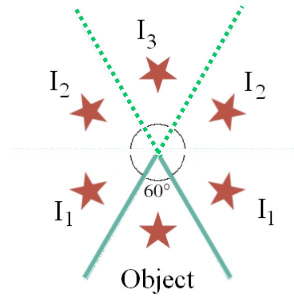
$$m = -\frac{d_i}{d_o} \;\rightarrow\; \pm 3 = -\frac{d_i}{d_o} \;\rightarrow\; d_i = \mp 3d_o$$

Use this result in the mirror equation.

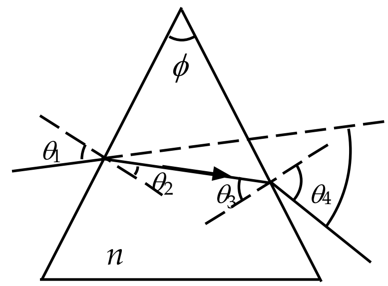
$$\left(\frac{1}{d_o}\right) + \left(\frac{1}{d_i}\right) = \frac{1}{f} \;\rightarrow\; \left(\frac{1}{d_o}\right) + \left[\frac{1}{(\mp 3d_o)}\right] = \frac{1}{f} \;\rightarrow\; d_o = \frac{2f}{3}, \frac{4f}{3} = 15.3\,\text{cm}, 30.7\,\text{cm}$$

So the object distances are $\boxed{\text{15 cm (produces virtual image)}}$ , and $\boxed{\text{+31 cm (produces real image)}}$.

78. The object "creates" the $I_1$ images as reflections from the actual mirrors. The $I_2$ images can be considered as images of the $I_1$ "objects," formed by the original mirrors. A specific $I_2$ image is the image of the $I_1$ "object" that is diametrically opposite it. Then the $I_3$ image can be considered as an image of the $I_2$ "objects." Each $I_2$ "object" would make the $I_3$ "image" at the same location. We can consider the extension of the actual mirrors, shown as dashed lines, to help understand the image formation.




79. The total deviation of the beam is the sum of the deviations at each surface. The deviation at the first surface is the refracted angle $\theta_2$ subtracted from the incident angle $\theta_1$. The deviation at the second surface is the incident angle $\theta_3$ subtracted from the refracted angle $\theta_4$. This gives the total deviation.




$$\delta = \delta_1 + \delta_2 = \theta_1 - \theta_2 + \theta_4 - \theta_3$$

We will express all of the angles in terms of $\theta_2$. To minimize the deviation, we will take the derivative of the deviation with respect to $\theta_2$, and then set that derivative equal to zero. Use Snell's law at the first surface to write the incident angle in terms of the refracted angle.

$$\sin\theta_1 = n\sin\theta_2 \quad\rightarrow\quad \theta_1 = \sin^{-1}(n\sin\theta_2)$$

The angle of incidence at the second surface is found using complementary angles, such that the sum of the refracted angle from the first surface and the incident angle at the second surface must equal the apex angle.

$$\phi = \theta_2 + \theta_3 \quad\rightarrow\quad \theta_3 = \phi - \theta_2$$

The refracted angle from the second surface is again found using Snell's law with the deviation in angle equal to the difference between the incident and refracted angles at the second surface.

$$n\sin\theta_3 = \sin\theta_4 \quad\rightarrow\quad \theta_4 = \sin^{-1}(n\sin\theta_3) = \sin^{-1}\left(n\sin(\phi-\theta_2)\right)$$

Inserting each of the angles into the deviation and setting the derivative equal to zero allows us to solve for the angle at which the deviation is a minimum.

$$\delta = \sin^{-1}(n\sin\theta_2) - \theta_2 + \sin^{-1}\left(n\sin(\phi-\theta_2)\right) - (\phi-\theta_2)$$
$$= \sin^{-1}(n\sin\theta_2) + \sin^{-1}\left(n\sin(\phi-\theta_2)\right) - \phi$$

$$\frac{d\delta}{d\theta_2} = \frac{n\cos\theta_2}{\sqrt{1-n^2\sin^2\theta_2}} - \frac{n\cos(\phi-\theta_2)}{\sqrt{1-n^2\sin^2(\phi-\theta_2)}} = 0 \quad\rightarrow\quad \theta_2 = \phi-\theta_2 \quad\rightarrow\quad \theta_2 = \theta_3 = \tfrac{1}{2}\phi$$

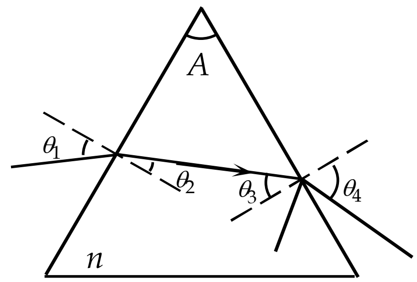
In order for $\theta_2 = \theta_3$, the ray must pass through the prism horizontally, which is perpendicular to the bisector of the apex angle $\phi$. Set $\theta_2 = \frac{1}{2}\phi$ in the deviation equation (for the minimum deviation, $\delta_m$) and solve for the index of refraction.

$$\delta_m = \sin^{-1}(n\sin\theta_2) + \sin^{-1}\left(n\sin(\phi-\theta_2)\right) - \phi$$
$$= \sin^{-1}\left(n\sin\tfrac{1}{2}\phi\right) + \sin^{-1}\left(n\sin\tfrac{1}{2}\phi\right) - \phi = 2\sin^{-1}\left(n\sin\tfrac{1}{2}\phi\right) - \phi$$

$$\rightarrow \boxed{n = \frac{\sin\left(\frac{1}{2}(\delta_m+\phi)\right)}{\sin\frac{1}{2}\phi}}$$

80. For the refraction at the second surface, we have this.

$$n\sin\theta_3 = n_{\text{air}}\sin\theta_4 \rightarrow (1.58)\sin\theta_3 = (1.00)\sin\theta_4$$




The maximum value of $\theta_4$ before internal reflection takes place at the second surface is 90°. Thus for internal reflection not to occur, we have

$$(1.58)\sin\theta_3 \le 1.00 \rightarrow \sin\theta_3 \le 0.6329 \rightarrow \theta_3 \le 39.27°$$

We find the refraction angle at the second surface.

$$(90° - \theta_2) + (90° - \theta_3) + A = 180° \rightarrow$$

$$\theta_2 = A - \theta_3 = 72° - \theta_3$$

Thus $\theta_2 \ge 72° - 39.27° = 32.73°$.

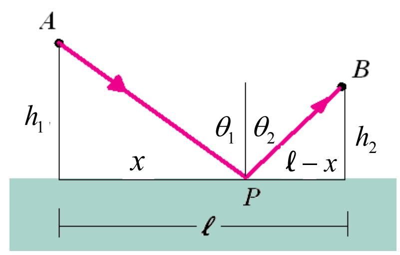
For the refraction at the first surface, we have the following.

$$n_{\text{air}}\sin\theta_1 = n\sin\theta_2 \rightarrow (1.00)\sin\theta_1 = (1.58)\sin\theta_2 \rightarrow \sin\theta_1 = (1.50)\sin\theta_2$$

Now apply the limiting condition.

$$\sin\theta_1 \ge (1.58)\sin 32.73° = 0.754 \rightarrow \boxed{\theta_1 \ge 58.69°}$$

81. (*a*) Consider the light ray shown in the figure. A ray of light starting at point A reflects off the surface at point P before arriving at point B, a horizontal distance ℓ from point A. We calculate the length of each path and divide the length by the speed of light to determine the time required for the light to travel between the two points.



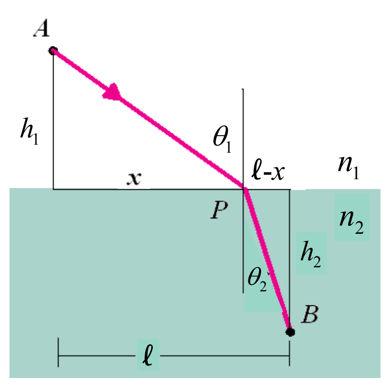

$$t = \frac{\sqrt{x^2 + h_1^2}}{c} + \frac{\sqrt{(\ell - x)^2 + h_2^2}}{c}$$

To minimize the time we set the derivative of the time with respect to *x* equal to zero. We also use the definition of the sine as opposite side over hypotenuse to relate the lengths to the angles of incidence and reflection.

$$0 = \frac{dt}{dx} = \frac{x}{c\sqrt{x^2 + h_1^2}} + \frac{-(\ell - x)}{c\sqrt{(\ell - x)^2 + h_2^2}} \rightarrow$$

$$\frac{x}{\sqrt{x^2 + h_1^2}} = \frac{(\ell - x)}{\sqrt{(\ell - x)^2 + h_2^2}} \rightarrow \sin\theta_1 = \sin\theta_2 \rightarrow \boxed{\theta_1 = \theta_2}$$

(*b*) Now we consider a light ray traveling from point A to point B in media with different indices of refraction, as shown in the figure. The time to travel between the two points is the distance in each medium divided by the speed of light in that medium.




$$t = \frac{\sqrt{x^2 + h_1^2}}{c/n_1} + \frac{\sqrt{(\ell - x)^2 + h_2^2}}{c/n_2}$$

To minimize the time we set the derivative of the time with respect to *x* equal to zero. We also use the definition of the sine as opposite side over hypotenuse to relate the lengths to the angles of incidence and reflection.

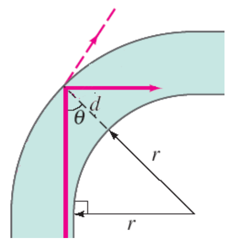
$$0=\frac{dt}{dx}=\frac{n_1x}{c\sqrt{x^2+h_1^2}}+\frac{-n_2(\ell-x)}{c\sqrt{(\ell-x)^2+h_2^2}}\rightarrow\frac{n_1x}{\sqrt{x^2+h_1^2}}=\frac{n_2(\ell-x)}{\sqrt{(\ell-x)^2+h_2^2}}\rightarrow\boxed{n_1\sin\theta_1=n_2\sin\theta_2}$$

82. We use Eq. 32-8 to calculate the location of the image and Eq. 32-3 to calculate the height of the image.

$$\frac{n_1}{d_o}+\frac{n_2}{d_i}=\frac{n_2-n_1}{R}\rightarrow d_i=n_2\left[\frac{n_2-n_1}{R}-\frac{n_1}{d_o}\right]^{-1}=1.53\left[\frac{1.53-1.33}{2.00\text{cm}}-\frac{1.33}{23\text{cm}}\right]^{-1}=\boxed{36.3\text{cm}}$$

$$\frac{h_i}{h_o}=-\frac{d_i}{d_o}\rightarrow h_i=-h_o\frac{d_i}{d_o}=-(2.0\text{mm})\frac{36.3\text{cm}}{23\text{cm}}=\boxed{3.2\text{mm}}$$

83. A ray of light initially on the inside of the beam will strike the far surface at the smallest angle, as seen in the associated figure. The angle is found using the triangle shown in the figure, with side *r* and hypotenuse *r*+*d*. We set this angle equal to the critical angle, using Eq. 32-7, and solve for the minimum radius of curvature.




$$\sin\theta_C=\frac{r}{r+d}=\frac{n_2}{n_1}=\frac{1}{n}\rightarrow\boxed{r=\frac{d}{n-1}}$$

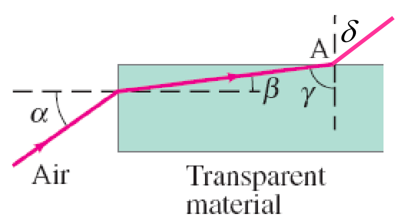
84. A relationship between the image and object distances can be obtained from the given information.

$$m=-\frac{1}{2}=-\frac{d_i}{d_o}\quad\rightarrow\quad d_i=\tfrac{1}{2}d_o=\boxed{7.5\text{cm}}$$

Now we find the focal length and the radius of curvature.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\quad\rightarrow\quad\frac{1}{15\text{cm}}+\frac{1}{7.5\text{cm}}=\frac{1}{f}\quad\rightarrow\quad f=5.0\text{cm}\quad\rightarrow\quad\boxed{r=10\text{cm}}$$

85. If total internal reflection fails at all, it fails for $\alpha\approx90°$. Assume $\alpha=90°$ and use Snell's law to determine the maximum *β*.




$$n_2\sin\beta=n_1\sin\alpha=n_1\sin90°=n_1\quad\rightarrow\quad\sin\beta=\frac{n_1}{n_2}$$

Snell's law can again be used to determine the angle $\delta$ for which light (if not totally internally reflected) would exit the top surface, using the relationship $\beta+\gamma=90°$ since they form two angles of a right triangle.

$$n_1\sin\delta=n_2\sin\gamma=n_2\sin(90°-\beta)=n_2\cos\beta\quad\rightarrow\quad\sin\delta=\frac{n_2}{n_1}\cos\beta$$

Using the trigonometric relationship $\cos\beta=\sqrt{1-\sin^2\beta}$ we can solve for the exiting angle in terms of the indices of refraction.

$$\sin\delta=\frac{n_2}{n_1}\sqrt{1-\sin^2\beta}=\frac{n_2}{n_1}\sqrt{1-\left(\frac{n_1}{n_2}\right)^2}$$

Insert the values for the indices ($n_1=1.00$ and $n_2=1.51$) to determine the sine of the exit angle.

$$\sin\delta=\frac{1.51}{1.00}\sqrt{1-\left(\frac{1.00}{1.51}\right)^2}=1.13$$

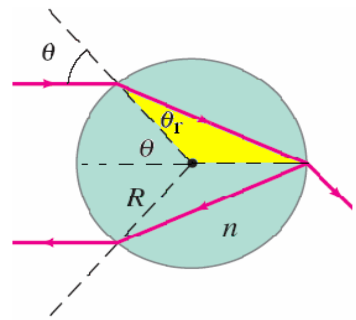
Since the sine function has a maximum value of 1, the light totally internally reflects at the glass–air interface for any incident angle of light.

If the glass is immersed in water, then $n_1 = 1.33$ and $n_2 = 1.51$.

$$\sin\delta = \frac{1.51}{1.33}\sqrt{1-\left(\frac{1.33}{1.51}\right)^2} = 0.538 \quad\rightarrow\quad \delta = \sin^{-1}0.538 = 32.5^\circ$$

Light entering the glass from water at 90° can escape out the top at 32.5°, therefore total internal reflection only occurs for incident angles $\boxed{\leq 32.5^\circ}$.

86. The path of the ray in the sphere forms an isosceles triangle with two radii. The two identical angles of the triangle are equal to the refracted angle. Since the incoming ray is horizontal, the third angle is the supplementary angle of the incident angle. We set the sum of these angles equal to 180° and solve for the ratio of the incident and refracted angles. Finally we use Snell's law in the small angle approximation to calculate the index of refraction.

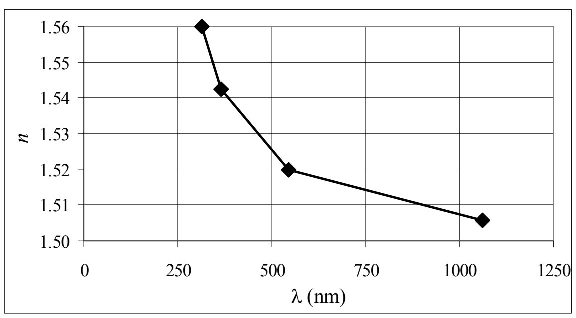


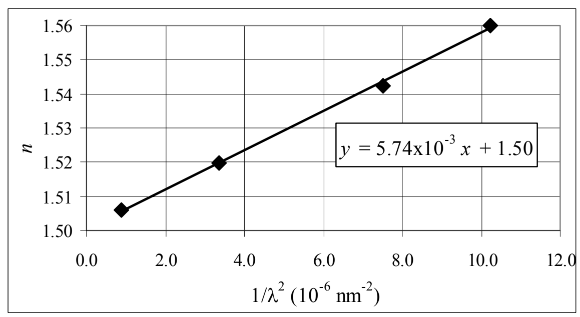

$$2\theta_r + (180^\circ - \theta) = 180^\circ \quad\rightarrow\quad \theta = 2\theta_r$$

$$n_1 \sin\theta = n_2 \sin\theta_r \quad\rightarrow\quad \theta = n\theta_r = 2\theta_r \quad\rightarrow\quad \boxed{n = 2}$$

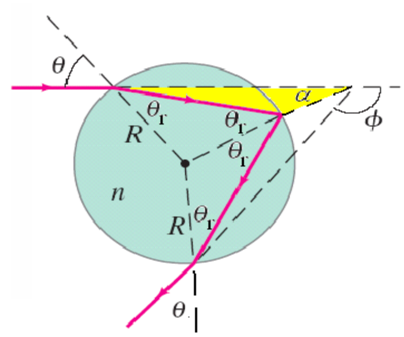
87. The first graph is a graph of *n* vs. λ. The second graph is a graph *n* vs. of $1/\lambda^2$. By fitting a line of the form $n = A + B/\lambda^2$, we have $A = 1.50$ and $B = (5.74\times10^{-3})/10^{-6}\,\text{nm}^{-2} = \boxed{5740\,\text{nm}^2}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH32.XLS," on tab "Problem 32.87."




88. (*a*) As the light ray enters the water drop, its path changes by the difference between the incident and refracted angles. We use Snell's law to calculate the refracted angle. The light ray then reflects off the back surface of the droplet. At this surface its path changes by $180^\circ - 2\theta_r$, as seen in the diagram. As the light exits the droplet it refracts again, changing its path by the difference between the incident and refracted angles. Summing these three angles gives the total path change.




$$\sin\theta = n\sin\theta_r \rightarrow \theta_r = \sin^{-1}\left(\frac{\sin\theta}{n}\right)$$

$$\phi = (\theta-\theta_r) + (180^\circ - 2\theta_r) + (\theta - \theta_r) = 180^\circ + 2\theta - 4\theta_r = \boxed{180^\circ + 2\theta - 4\sin^{-1}\left(\frac{\sin\theta}{n}\right)}$$

(*b*) Here is the graph of $\phi$ vs $\theta$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH32.XLS," on tab "Problem 32.88."


(*c*) On the spreadsheet, the incident angles that give scattering angles from 138° to 140° are approximately $48.5° \le \theta \le 54.5°$ and $64.5° \le \theta \le 69.5°$. This is 11/90 of the possible incident angles, or about $\boxed{12\%}$.

# CHAPTER 33: Lenses and Optical Instruments

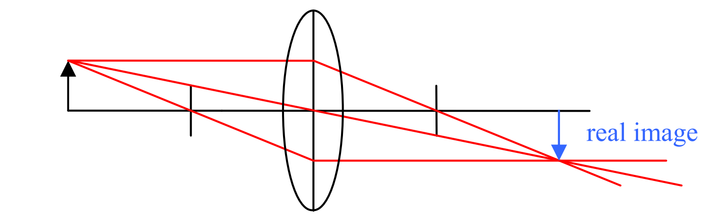
## Responses to Questions

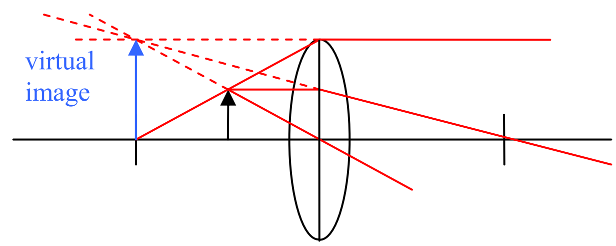
1. The film must be placed behind the lens at the focal length of the lens.

2. The lens moves farther away from the film. When the photographer moves closer to his subject, the object distance decreases. The focal length of the lens does not change, so the image distance must increase, by Eq. 33-2, $\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}$.

3. Yes. Diverging lenses, by definition, cause light rays to diverge and will not bring rays from a real object to a focal point as required to form a real image. However, if another optical element (for example, a converging lens) forms a virtual object for the diverging lens, it is possible for the diverging lens to form a real image.

4. A real image formed by a thin lens is on the opposite side of the lens as the object, and will always be inverted as shown in the top diagram. A virtual image is formed on the same side of the lens as the real object, and will be upright, as shown in the bottom diagram.



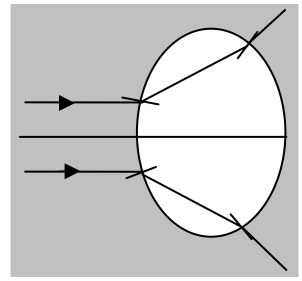

5. Yes. In the thin-lens equation, the variables for object distance and image distance can be interchanged and the formula remains the same.

6. Yes, real images can be projected on a screen. No, virtual images cannot, because they are formed by diverging rays, which do not come to a focus on the screen. Both kinds of images can be photographed. The lenses in a camera are designed to focus either converging or diverging light rays down onto the film.

7. (*a*) Yes. The image moves farther from the lens.
   (*b*) Yes. The image also gets larger.

8. The mirror equation and the lens equation are identical. According to the sign conventions, $d > 0$ indicates a real object or image and $d < 0$ indicates a virtual object or image, for both mirrors and lenses. But the positions of the objects and images are different for a mirror and a lens. For a mirror, a real object or image will be in front of the mirror and a virtual object or image will be behind the mirror. For a lens, a real image will be on the opposite side of the lens from a real object, and a virtual image will be on the same side of the lens as the real object.

9. No. The lens will be a diverging lens when placed in water because the index of refraction of the lens is less than the index of refraction of the medium surrounding it. Rays going from water to lens material will bend away from the normal instead of toward the normal, and rays going from the lens back to the water will bend towards the normal.



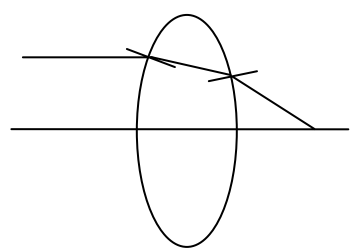
10. A virtual image created by a previous lens can serve as a virtual object for a second lens. If the previous lens creates an image behind the position of the second lens, that image will also serve as a virtual object for the second lens.

11. Assuming that the lens remains fixed and the screen is moved, the dog's head will have the greater magnification. The object distance for the head is less than the object distance for the tail, because the dog is facing the mirror. The image distance for the head will therefore be greater than the image distance for the tail. Magnification is the ratio of the image distance to the object distance, so will be greater for the head.

12. If the cat's nose is closer to the lens than the focal point and the tail is farther from the lens than the focal point, the image of the nose will be virtual and the image of the tail will be real. The virtual image of the front part of the cat will be spread out from the image of the nose to infinity on the same side of the lens as the cat. The real image of the back part of the cat will be spread out from the image of the tail to infinity on the opposite side of the lens.

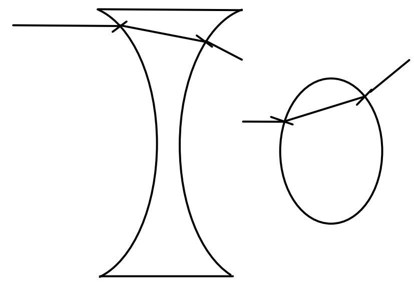
13. The technique for determining the focal length of the diverging lens in Example 33-6 requires the combination of the two lenses together to project a real image of the sun onto a screen. The focal length of the lens combination can be measured. If the focal length of the converging lens is longer than the focal length of the diverging lens (the converging lens is weaker than the diverging lens), then the lens combination will be diverging, and will not form a real image of the sun. In this case the focal length of the combination of lenses cannot be measured, and the focal length of the diverging lens alone cannot be determined.

14. A double convex lens causes light rays to converge because the light bends towards the normal as it enters the lens and away from the normal as it exits the lens. The result, due to the curvature of the sides of the lens, is that the light bends towards the principal axis at both surfaces. The more strongly the sides of the lens are curved, the greater the bending, and the shorter the focal length.



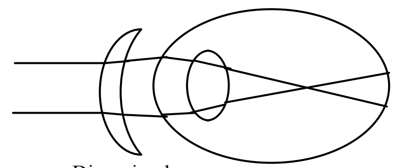
15. Yes. The relative values of the index of refraction of the fluid and the index of refraction of the lens will determine the refraction of light as it passes from the fluid through the lens and back into the fluid. The amount of refraction of light determines the focal length of the lens, so the focal length will change if the lens is immersed in a fluid. No, the image formation of the spherical mirror is determined by reflection, not refraction, and is independent of the medium in which the mirror is immersed.

16. The lens material is air and the medium in which the lens is placed is water. Air has a lower index of refraction than water, so the light rays will bend away from the normal when entering the lens and towards the normal when leaving the lens.
    (*a*) A converging lens can be made by a shape that is thinner in the middle than it is at the edges.
    (*b*) A diverging lens will be thicker in the middle than it is at the edges.



17. If the object of the second lens (the image from the first lens) is exactly at the focal point, then a virtual image will be formed at infinity and can be viewed with a relaxed eye.

18. The corrective lenses will not work the same underwater as in air, and so the nearsighted person will probably not be able to see clearly underwater. The difference in the index of refraction of water and glass is much smaller than the difference in the indices for air and glass, so the lenses will not cause the incoming rays to diverge sufficiently.



Diverging lens

19. Nearsighted. Diverging lenses are used to correct nearsightedness and converging lenses are used to correct farsightedness. If the person's face appears narrower through the glasses, then the image of the face produced by the lenses is smaller than the face, virtual, and upright. Thus, the lenses must be diverging, and therefore the person is nearsighted.

20. All light entering the camera lens while the shutter is open contributes to a single picture. If the camera is moved while the shutter is open, the position of the image on the film moves. The new image position overlaps the previous image position, causing a blurry final image. With the eye, new images are continuously being formed by the nervous system, so images do not "build up" on the retina and overlap with each other.

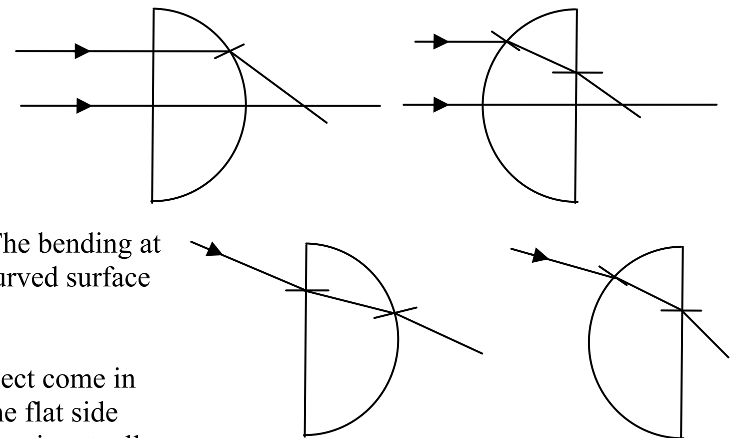
21. Squinting limits the off-axis rays that enter the eye and results in an image that is formed primarily by the center part of the lens, reducing spherical aberration and spreading of the image.

22. The image formed on the retina is inverted. The human brain then processes the image so that we interpret the world we see correctly.

23. Both reading glasses and magnifiers are converging lenses used to produce magnified images. A magnifier, generally a short focal length lens, is typically used by adjusting the distance between the lens and the object so that the object is exactly at or just inside the focal point. An object exactly at the focal point results in an image that is at infinity and can be viewed with a relaxed eye. If the lens is adjusted so that it focuses the image at the eye's near point, the magnification is slightly greater. The lenses in reading glasses typically are a fixed distance from the eye. These lenses cause the rays from a nearby object to converge somewhat before they reach the eye, allowing the eye to focus on an object that is inside the near point. The focal length of the lens needed for reading glasses will depend on the individual eye. The object does not have to be inside the focal point of the lens. For both reading glasses and magnifiers, the lenses allow the eye to focus on an object closer than the near point.

24. The relationship between $d_i$ and $d_o$ for a given lens of focal length $f$ is given by Eq. 33-2, $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$. The focal length is fixed for a camera lens, so if the lens focuses on a closer object, $d_o$ decreases and therefore $d_i$ must increase. An increase in $d_i$ means that the lens must be farther from the film.

25. The curved surface should face the object. If the flat surface faces the object and the rays come in parallel to the optical axis, then no bending will occur at the first surface and all the bending will occur at the second surface. Bending at the two surfaces will clearly not be equal in this case. The bending at the two surfaces may be equal if the curved surface faces the object.

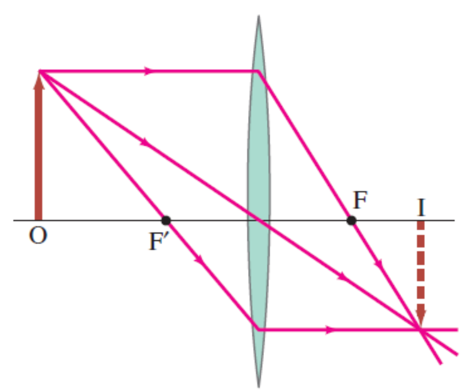


If the parallel rays from the distant object come in above or below the optical axis with the flat side towards the object, then the first bending is actually away from the axis. In this case also, bending at both surfaces can be equal if the curved side of the lens faces the object.

26. For both converging and diverging lenses, the focal point for violet light is closer to the lens than the focal point for red light. The index of refraction for violet light is slightly greater than for red light for glass, so the violet light bends more, resulting in a smaller magnitude focal length.

## Solutions to Problems

1. (*a*) From the ray diagram, the object distance is about $\boxed{480\text{ cm}}$.




(*b*) We find the object distance from Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad\rightarrow$$

$$d_o=\frac{fd_i}{d_i-f}=\frac{(215\,\text{mm})(373\,\text{mm})}{373\,\text{mm}-215\,\text{mm}}=\boxed{508\,\text{mm}}$$

NOTE: In the first printing of the textbook, a different set of values was given: $f=75.0\,\text{mm}$ and $d_i=88.0\,\text{mm}$. Using that set of values gives the same object distance as above. But the ray diagram would be much more elongated, with the object distance almost 7 times the focal length.

2. (*a*) To form a real image from parallel rays requires a $\boxed{\text{converging lens.}}$

(*b*) We find the power of the lens from Eqs. 33-1 and 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P=\frac{1}{\infty}+\frac{1}{0.185\,\text{m}}=\boxed{5.41\,\text{D}}$$

3. (*a*) The power of the lens is given by Eq. 33.1

$$P=\frac{1}{f}=\frac{1}{0.235\,\text{m}}=\boxed{4.26\,\text{D}}$$

This lens is $\boxed{\text{converging}}$.

(*b*) We find the focal length of the lens from Eq. 33.1

$$P=\frac{1}{f}\;\rightarrow\; f=\frac{1}{D}=-\frac{1}{6.75\,\text{D}}=\boxed{-0.148\,\text{m}}$$

This lens is $\boxed{\text{diverging}}$.

4. To form a real image from a real object requires a $\boxed{\text{converging lens}}$. We find the focal length of the lens from Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\;\rightarrow\; f=\frac{d_o d_i}{d_o+d_i}=\frac{(1.85\,\text{m})(0.483\,\text{m})}{1.85\,\text{m}+0.483\,\text{m}}=\boxed{0.383\,\text{m}}$$

Because $d_i>0$, the image is $\boxed{\text{real.}}$

5. (*a*) We find the image distance from Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\;\rightarrow\; d_i=\frac{d_o f}{d_o-f}=\frac{(10.0\,\text{m})(0.105\,\text{m})}{10.0\,\text{m}-0.105\,\text{m}}=0.106\,\text{m}=\boxed{106\,\text{mm}}$$

(*b*) Use the same general calculation.

$$d_i=\frac{d_o f}{d_o-f}=\frac{(3.0\,\text{m})(0.105\,\text{m})}{3.0\,\text{m}-0.105\,\text{m}}=0.109\,\text{m}=\boxed{109\,\text{mm}}$$

(*c*) Use the same general calculation.

$$d_i=\frac{d_o f}{d_o-f}=\frac{(1.0\,\text{m})(0.105\,\text{m})}{1.0\,\text{m}-0.105\,\text{m}}=0.117\,\text{m}=\boxed{117\,\text{mm}}$$

(*d*) We find the smallest object distance from the maximum image distance.

$$\frac{1}{d_{o_{\text{min}}}}+\frac{1}{d_{i_{\text{max}}}}=\frac{1}{f}\;\rightarrow\; d_{o_{\text{min}}}=\frac{d_{i_{\text{max}}}-f}{d_{i_{\text{max}}}\,f}=\frac{(132\,\text{mm})(105\,\text{mm})}{132\,\text{mm}-105\,\text{mm}}=513\,\text{mm}=\boxed{0.513\,\text{m}}$$

6. (*a*) We locate the image using Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\;\rightarrow\; d_i=\frac{d_o f}{d_o-f}=\frac{(18\,\text{cm})(28\,\text{cm})}{18\,\text{cm}-28\,\text{cm}}=-50.4\,\text{cm}\approx-50\,\text{cm}$$

The negative sign means the image is $\boxed{\text{50 cm behind the lens (virtual).}}$

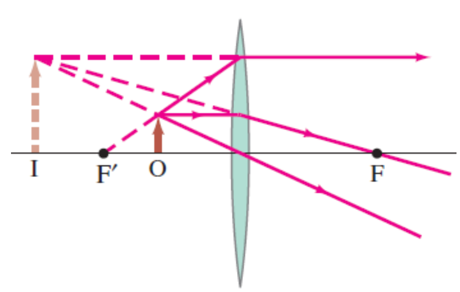
(*b*) We find the magnification from Eq. 33-3.

$$m=-\frac{d_i}{d_o}=-\frac{(-50.4\,\text{cm})}{(18\,\text{cm})}=\boxed{+2.8}$$

$\boxed{7.}$ (*a*) The image should be upright for reading. The image will be $\boxed{\text{virtual, upright, and magnified}}$.

(*b*) To form a virtual, upright magnified image requires a $\boxed{\text{converging lens}}$.

(*c*) We find the image distance, then the focal length, and then the power of the lens. The object distance is given.



$$m = -\frac{d_i}{d_o} \rightarrow d_i = -md_o$$

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{d_i + d_o}{d_o d_i} = \frac{-md_o + d_o}{d_o(-md_o)} = \frac{m-1}{md_o} = \frac{2.5-1}{(2.5)(0.090\,\text{m})} = \boxed{6.7\,\text{D}}$$

8. Use Eqs. 33-1 and 33-2 to find the image distance, and Eq. 33-3 to find the image height.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = P \rightarrow d_i = \frac{d_o}{Pd_o - 1} = \frac{(0.125\,\text{m})}{(-8.00\,\text{D})(0.125\,\text{m}) - 1} = -0.0625\,\text{m} = \boxed{-6.25\,\text{cm}}$$

Since the image distance is negative, the image is $\boxed{\text{virtual and behind the lens}}$.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \rightarrow h_i = -\frac{d_i}{d_o}h_o = -\frac{(-6.25\,\text{cm})}{12.5\,\text{cm}}(1.00\,\text{mm}) = \boxed{0.500\,\text{mm}\,(\text{upright})}$$

9. First, find the original image distance from Eqs. 33-1 and 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = P \rightarrow d_i = \frac{d_o}{Pd_o - 1} = \frac{(1.50\,\text{m})}{(8.00\,\text{D})(1.50\,\text{m}) - 1} = 0.1364\,\text{m}$$

(*a*) With $d_o = 0.60\,\text{m}$, find the new image distance.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = P \rightarrow d_i = \frac{d_o}{Pd_o - 1} = \frac{(0.60\,\text{m})}{(8.00\,\text{D})(0.60\,\text{m}) - 1} = 0.1579\,\text{m}$$

Thus the image has moved $0.1579\,\text{m} - 0.1364\,\text{m} = 0.0215\,\text{m} \approx \boxed{0.02\,\text{m}}$ away from the lens.

(*b*) With $d_o = 2.40\,\text{m}$, find the new image distance.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = P \rightarrow d_i = \frac{d_o}{Pd_o - 1} = \frac{(2.40\,\text{m})}{(8.00\,\text{D})(2.40\,\text{m}) - 1} = 0.1319\,\text{m}$$

The image has moved $0.1319\,\text{m} - 0.1364\,\text{m} = -0.0045\,\text{m} \approx \boxed{0.004\,\text{m}}$ toward the lens.

10. (*a*) If the image is real, the focal length must be positive, the image distance must be positive, and the magnification is negative. Thus $d_i = 2.50d_o$. Use Eq. 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{2.50d_o} = \frac{1}{f} \rightarrow d_o = \left(\frac{3.50}{2.50}\right)f = \left(\frac{3.50}{2.50}\right)(50.0\,\text{mm}) = \boxed{70.0\,\text{mm}}$$

(*b*) If the image is magnified, the lens must have a positive focal length, because negative lenses always form reduced images. Since the image is virtual the magnification is positive. Thus $d_i = -2.50d_o$. Again use Eq. 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} - \frac{1}{2.5d_o} = \frac{1}{f} \rightarrow d_o = \left(\frac{1.50}{2.50}\right)f = \left(\frac{1.50}{2.50}\right)(50.0\,\text{mm}) = \boxed{30.0\,\text{mm}}$$

11. From Eq. 33-3, $|h_i| = |h_o|$ when $d_i = d_o$. So find $d_o$ from Eq. 33-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{d_o} = \frac{1}{f} \rightarrow d_o = 2f = \boxed{50\,\text{cm}}$$

12. (*a*) Use Eqs. 33-2 and 33-3.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow d_i = \frac{d_o f}{d_o - f} = \frac{(1.30\,\text{m})(0.135\,\text{m})}{1.30\,\text{m} - 0.135\,\text{m}} = 0.1506\,\text{m} \approx \boxed{151\,\text{mm}}$$

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \rightarrow h_i=-\frac{d_i}{d_o}h_o=-\frac{0.1506\,\text{m}}{1.30\,\text{m}}(2.80\,\text{cm})=\boxed{-0.324\,\text{m}}$$

The image is behind the lens a distance of 151 mm, is real, and is inverted.

(*b*) Again use Eqs. 33-2 and 33-3.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_o f}{d_o-f}=\frac{(1.30\,\text{m})(-0.135\,\text{m})}{1.30\,\text{m}-(-0.135\,\text{m})}=-0.1223\,\text{m}\approx\boxed{-122\,\text{mm}}$$

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \rightarrow h_i=-\frac{d_i}{d_o}h_o=-\frac{(-0.1223\,\text{m})}{1.30\,\text{m}}(2.80\,\text{cm})=\boxed{0.263\,\text{m}}$$

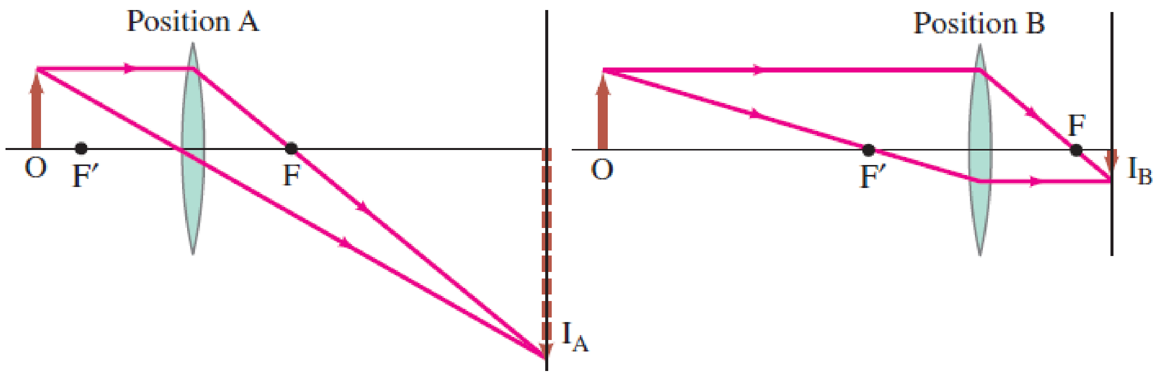
The image is in front of the lens a distance of 122 mm, is virtual, and is upright.

13. The sum of the object and image distances must be the distance between object and screen, which we label as $d_T$. We solve this relationship for the image distance, and use that expression in Eq. 33-2 in order to find the object distance.

$$d_o+d_i=d_T \rightarrow d_i=d_T-d_o \;;\; \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{(d_T-d_o)}=\frac{1}{f} \rightarrow d_o^2-d_Td_o+fd_T=0 \rightarrow$$

$$d_o=\frac{d_T\pm\sqrt{d_T^2-4fd_T}}{2}=\frac{(86.0\,\text{cm})\pm\sqrt{(86.0\,\text{cm})^2-4(16.0\,\text{cm})(86.0\,\text{cm})}}{2}=\boxed{21.3\,\text{cm},\ 64.7\,\text{cm}}$$

Note that to have real values for $d_o$, we must in general have $d_T^2-4fd_T>0 \rightarrow d_T>4f$.




14. For a real image both the object distance and image distances are positive, and so the magnification is negative. Use Eqs. 33-2 and 33-3 to find the object and image distances. Since they are on opposite sides of the lens, the distance between them is their sum.

$$m=\frac{-d_i}{d_o} \rightarrow d_i=-md_o=2.95d_o$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{2.95d_o}=\frac{1}{f} \rightarrow d_o=\left(\frac{3.95}{2.95}\right)f=\left(\frac{3.95}{2.95}\right)(85\,\text{cm})=113.8\,\text{cm}$$

$$d_i=2.95d_o=2.95(113.8\,\text{cm})=335.7\,\text{cm}$$

$$d_o+d_i=113.8\,\text{cm}+335.7\,\text{cm}=449.5\,\text{cm}\approx\boxed{450\,\text{cm}}$$

15. (*a*) Use Eq. 33-2 to write an expression for the image distance in terms of the object distance and focal length. We then use Eq. 33-3 to write an expression for the magnification.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_o f}{d_o-f} \;;\; m=-\frac{d_i}{d_o}=-\frac{f}{d_o-f}$$

These expressions show that when $d_o > f$, the image distance is positive, producing a real image, and the magnification is negative, which gives an inverted image.

(*b*) From the above equations, when $d_o < f$, the image distance is negative, producing a virtual image, and the magnification is positive, which gives an upright image.

(*c*) We set $-d_o = f$ and calculate the limiting image distance and magnification.

$$d_i = \frac{(-f)f}{-f-f} = \frac{f}{2} \qquad m = -\frac{d_i}{d_o} = -\frac{f}{-f-f} = \frac{1}{2}$$

We also take the limit of large negative object distance.

$$d_i = \frac{(-\infty)f}{-\infty - f} = f \qquad m = -\frac{d_i}{d_o} = -\frac{f}{-\infty - f} = 0$$

From these limiting cases, we see that when $-d_o > f$, the image is $\boxed{\text{real and upright}}$ with $\frac{1}{2}f < d_i < f$ and $0 < m < \frac{1}{2}$.

(*d*) We take the limiting condition $d_o \to 0$, and determine the resulting image distance and magnification.

$$d_i = \frac{(0)f}{0-f} = 0 \qquad m = -\frac{d_i}{d_o} = -\frac{f}{0-f} = 1$$

From this limit and that found in part (*c*), we see that when $0 < -d_o < f$, the image is $\boxed{\text{real and upright}}$, with $0 < d_i < \frac{1}{2}f$ and $\frac{1}{2} < m < 1$.

16. (*a*) We use the magnification equation, Eq. 33-3, to write the image distance in terms of the magnification and object distance. We then replace the image distance in the mirror equation, Eq. 32-2, and solve for the magnification in terms of the object distance and the focal length.

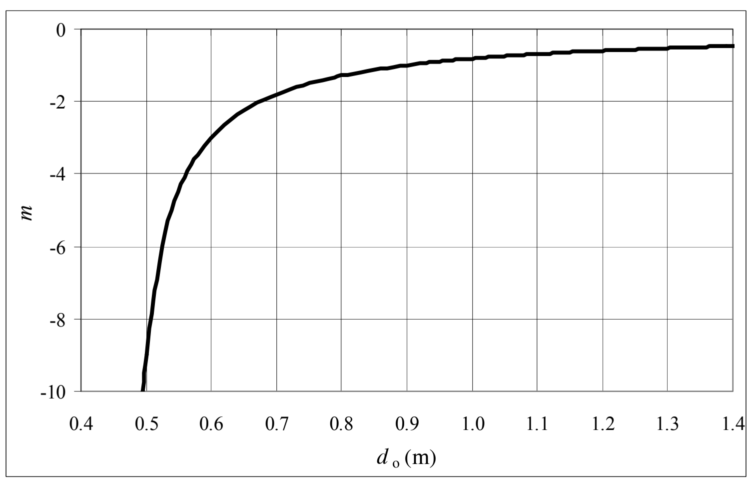
$$m = -d_i/d_o \quad \to \quad d_i = -md_o$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \to$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{-md_o} \quad \to$$

$$\boxed{m = \frac{f}{f - d_o}}$$

(*b*) We set $f = 0.45$ m and draw a graph of the magnification as a function of the object distance. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH33.XLS," on tab "Problem 33.16b."




(*c*) The image and object will have the same lateral size when the magnification is equal to negative one. Setting the magnification equal to negative one, we solve the equation found in part (a) for the object distance.

$$m = \frac{f}{f - d_o} = -1 \quad \to \quad d_o = 2f = \boxed{0.90\,\text{m}}$$

(*d*) From the graph we see that for the image to be much larger than the object, the object should be placed at a point $\boxed{\text{just beyond the focal point.}}$

17. Find the object distance from Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_o=\frac{fd_i}{d_i-f}=\frac{(0.105\,\text{m})(6.50\,\text{m})}{6.50\,\text{m}-0.105\,\text{m}}=\boxed{0.107\,\text{m}}$$

Find the size of the image from Eq. 33-3.

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \rightarrow |h_i|=\frac{d_i}{d_o}h_o=\frac{6.50\,\text{m}}{0.107\,\text{m}}(36\,\text{mm})=2187\,\text{mm}\approx\boxed{2.2\,\text{m}}$$

18. (*a*) Use Eq. 33-2 with $d_o+d_i=d_T \rightarrow d_i=d_T-d_o$.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{(d_T-d_o)}=\frac{1}{f} \rightarrow d_o^2-d_Td_o+fd_T=0 \rightarrow d_o=\frac{d_T\pm\sqrt{d_T^2-4fd_T}}{2}$$

There are only real solutions for $d_o$ if $d_T^2-4fd_T>0 \rightarrow d_T>4f$. If that condition is met, then there will be two locations for the lens, at distances $d_o=\frac{1}{2}\left(d_T\pm\sqrt{d_T^2-4fd_T}\right)$ from the object, that will form sharp images on the screen.

(*b*) If $d_T<4f$, then Eq. 33-2 cannot be solved for real values of $d_o$ or $d_i$.

(*c*) If $d_T>4f$, the lens locations relative to the object are given by $d_{o1}=\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)$ and $d_{o2}=\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)$.

$$\Delta d=d_{o1}-d_{o2}=\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)-\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)=\boxed{\sqrt{d_T^2-4fd_T}}$$

Find the ratio of image sizes using Eq. 33-3.

$$\frac{h_{i2}}{h_{i1}}=\frac{-h_o\frac{d_{i2}}{d_{o2}}}{-h_o\frac{d_{i1}}{d_{o1}}}=\frac{d_{i2}}{d_{o2}}\frac{d_{o1}}{d_{i1}}=\frac{d_T-d_{o2}}{d_{o2}}\frac{d_{o1}}{d_T-d_{o1}}$$

$$=\left[\frac{d_T-\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)}{\frac{1}{2}\left(d_T-\sqrt{d_T^2-4fd_T}\right)}\right]\left[\frac{\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)}{d_T-\frac{1}{2}\left(d_T+\sqrt{d_T^2-4fd_T}\right)}\right]=\boxed{\left(\frac{d_T+\sqrt{d_T^2-4fd_T}}{d_T-\sqrt{d_T^2-4fd_T}}\right)^2}$$

$\boxed{19.}$ (*a*) With the definitions as given in the problem, $x=d_o-f \rightarrow d_o=x+f$ and $x'=d_i-f \rightarrow d_i=x'+f$. Use Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{x+f}+\frac{1}{x'+f}=\frac{1}{f} \rightarrow \frac{(x'+f)+(x+f)}{(x+f)(x'+f)}=\frac{1}{f} \rightarrow$$

$$(2f+x+x')f=(x+f)(x'+f) \rightarrow 2f^2+xf+x'f=x'x+xf+fx'+f^2 \rightarrow \boxed{f^2=x'x}$$

(*b*) Use Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_of}{d_o-f}=\frac{(48.0\,\text{cm})(38.0\,\text{cm})}{48.0\,\text{cm}-38.0\,\text{cm}}=\boxed{182\,\text{cm}}$$

(*c*) Use the Newtonian form.

$$xx'=f^2 \rightarrow x'=\frac{f^2}{x}=\frac{(38.0\,\text{cm})^2}{(48.0\,\text{cm}-38.0\,\text{cm})}=144.2\,\text{cm}$$

$$d_i=x'+f=144.2\,\text{cm}+38.0\,\text{cm}=\boxed{182\,\text{cm}}$$

20. The first lens is the converging lens. An object at infinity will form an image at the focal point of the converging lens, by Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1}=\frac{1}{\infty}+\frac{1}{d_{i1}} \rightarrow d_{i1}=f_1=20.0\,\text{cm}$$

This image is the object for the second lens. Since this image is behind the second lens, the object distance for the second lens is negative, and so $d_{o2}=-6.0\,\text{cm}$. Again use Eq. 33-2.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(-6.0\,\text{cm})(-33.5\,\text{cm})}{(-6.0\,\text{cm})-(-33.5\,\text{cm})}=7.3\,\text{cm}$$

Thus the final image is real, 7.3 cm beyond the second lens.

21. Find the image formed by the first lens, using Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow d_{i1}=\frac{d_{o1}f_1}{d_{o1}-f_1}=\frac{(35.0\,\text{cm})(25.0\,\text{cm})}{(35.0\,\text{cm})-(25.0\,\text{cm})}=87.5\,\text{cm}$$

This image is the object for the second lens. Because it is beyond the second lens, it has a negative object distance.

$$d_{o2}=16.5\,\text{cm}-87.5\,\text{cm}=-71.0\,\text{cm}$$

Find the image formed by the second lens, again using Eq. 33-2.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(-71.0\,\text{cm})(25.0\,\text{cm})}{(-71.0\,\text{cm})-(25.0\,\text{cm})}=18.5\,\text{cm}$$

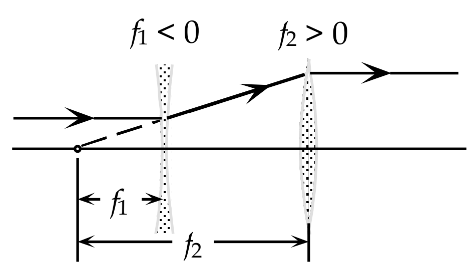
Thus the final image is real, 18.5 cm beyond second lens.

The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{d_{i1}d_{i2}}{d_{o1}d_{o2}}$$

$$=\frac{(+87.5\,\text{cm})(+18.5\,\text{cm})}{(+35.0\,\text{cm})(-71.0\,\text{cm})}=\boxed{-0.651\times(\text{inverted})}$$

22. From the ray diagram, the image from the first lens is a virtual image at the focal point of the first lens. This is a real object for the second lens. Since the light is parallel after leaving the second lens, the object for the second lens must be at its focal point. Let the separation of the lenses be $\ell$. Note that the focal length of the diverging lens is negative.




$$|f_1|+\ell=f_2 \rightarrow$$

$$|f_1|=f_2-\ell=34.0\,\text{cm}-24.0\,\text{cm}=10.0\,\text{cm} \rightarrow$$

$$f_1=\boxed{-10.0\,\text{cm}}$$

23. (*a*) The first image is formed as in Example 33-5, and so $d_{iA}=30.0\,\text{cm}$. This image becomes the object for the lens B, at a distance $d_{oB}=20.0\,\text{cm}-30.0\,\text{cm}=-10.0\,\text{cm}$. This is a virtual object since it is behind lens N. Use Eq. 33-2 to find the image formed by lens B, which is the final image.

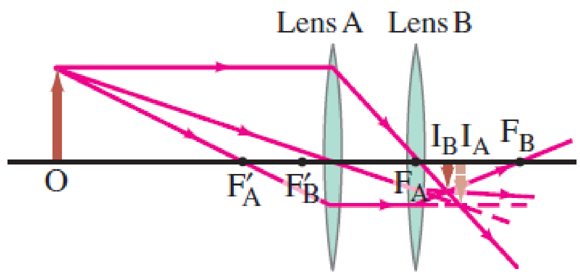
$$\frac{1}{d_{oB}}+\frac{1}{d_{iB}}=\frac{1}{f_B} \rightarrow d_{iB}=\frac{d_{oB}f_B}{d_{oB}-f_B}=\frac{(-10.0\,\text{cm})(25.0\,\text{cm})}{-10.0\,\text{cm}-25.0\,\text{cm}}=7.14\,\text{cm}$$

So the final image is 7.14 cm beyond lens B.

(*b*) The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(-\frac{d_{iA}}{d_{oA}}\right)\left(-\frac{d_{iB}}{d_{oB}}\right) = \frac{d_{iA} d_{iB}}{d_{oA} d_{oB}} = \frac{(30.0\,\text{cm})(7.14\,\text{cm})}{(60.0\,\text{cm})(-10.0\,\text{cm})} = \boxed{-0.357}$$

(*c*) See the ray diagram here.




24. (*a*) Find the image formed by the converging lens, using Eq. 33-2.

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \;\rightarrow\; d_{i1} = \frac{d_{o1} f_1}{d_{o1} - f_1} = \frac{(33\,\text{cm})(18\,\text{cm})}{(33\,\text{cm}) - (18\,\text{cm})} = 39.6\,\text{cm}$$

This image is the object for the second lens. The image is to the right of the second lens, and so is virtual. Use that image to find the final image.

$$d_{o2} = 12\,\text{cm} - 39.6\,\text{cm} = -27.6\,\text{cm} \;;\; \frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \;\rightarrow$$

$$d_{i2} = \frac{d_{o2} f_2}{d_{o2} - f_2} = \frac{(-27.6\,\text{cm})(-14\,\text{cm})}{(-27.6\,\text{cm}) - (-14\,\text{cm})} = -28.4\,\text{cm}$$

So the final image is 28 cm to the left of the diverging lens, or **16 cm to the left of the converging lens**.

(*b*) The initial image is unchanged. With the change in the distance between the lenses, the image distance for the second lens has changed.

$$d_{o2} = 38\,\text{cm} - 39.6\,\text{cm} = -1.6\,\text{cm} \;;\; \frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \;\rightarrow$$

$$d_{i2} = \frac{d_{o2} f_2}{d_{o2} - f_2} = \frac{(-1.6\,\text{cm})(-14\,\text{cm})}{(-1.6\,\text{cm}) - (-14\,\text{cm})} = 1.8\,\text{cm}$$

Now the final image is **1.8 cm to the right of the diverging lens.**

25. (*a*) The first lens is the converging lens. Find the image formed by the first lens.

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \;\rightarrow\; d_{i1} = \frac{d_{o1} f_1}{d_{o1} - f_1} = \frac{(60.0\,\text{cm})(20.0\,\text{cm})}{(60.0\,\text{cm}) - (20.0\,\text{cm})} = 30.0\,\text{cm}$$

This image is the object for the second lens. Since this image is behind the second lens, the object distance for the second lens is negative, and so $d_{o2} = 25.0\,\text{cm} - 30.0\,\text{cm} = -5.0\,\text{cm}$. Use Eq. 33-2.

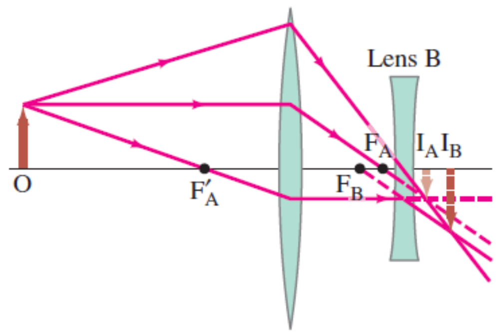
$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \;\rightarrow\; d_{i2} = \frac{d_{o2} f_2}{d_{o2} - f_2} = \frac{(-5.0\,\text{cm})(-10.0\,\text{cm})}{(-5.0\,\text{cm}) - (-10.0\,\text{cm})} = 10\,\text{cm}$$

Thus the final image is real, **10 cm beyond the second lens**. The distance has two significant figures.

(*b*) The total magnification is the product of the magnifications for the two lenses:

$$m = m_1 m_2 = \left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right) = \frac{d_{i1} d_{i2}}{d_{o1} d_{o2}} = \frac{(30.0\,\text{cm})(10.0\,\text{cm})}{(60.0\,\text{cm})(-5.0\,\text{cm})} = \boxed{-1.0\times}$$

(*c*) See the diagram here.




26. We find the focal length of the combination by finding the image distance for an object very far away. For the converging lens, we have the following from Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_C}=\frac{1}{\infty}+\frac{1}{d_{i1}} \rightarrow d_{i1}=f_C$$

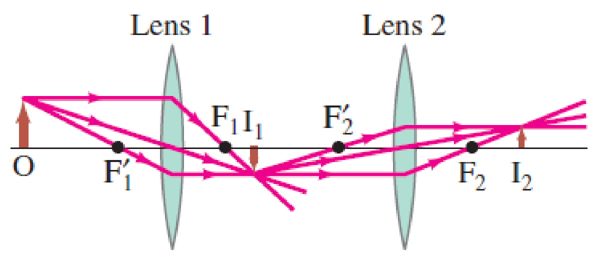
The first image is the object for the second lens. Since the first image is real, the second object distance is negative. We also assume that the lenses are thin, and so $d_{o2}=-d_{i1}=-f_C$.
For the second diverging lens, we have the following from Eq. 33-2.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_D}=-\frac{1}{f_C}+\frac{1}{d_{i2}}$$

Since the original object was at infinity, the second image must be at the focal point of the combination, and so $d_{i2}=f_T$.

$$\frac{1}{f_D}=-\frac{1}{f_C}+\frac{1}{d_{i2}}=-\frac{1}{f_C}+\frac{1}{f_T}$$

27. (*a*) We see that the image is real and upright. We estimate that it is 30 cm beyond the second lens, and that the final image height is half the original object height.




(*b*) Find the image formed by the first lens, using Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow$$

$$d_{i1}=\frac{d_{o1}f_1}{d_{o1}-f_1}=\frac{(36\text{cm})(13\text{cm})}{(36\text{cm})-(13\text{cm})}=20.35\text{cm}$$

This image is the object for the second lens. Because it is between the lenses, it has a positive object distance.

$$d_{o2}=56\text{cm}-20.35\text{cm}=35.65\text{cm}$$

Find the image formed by the second lens, again using Eq. 33-2.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{d_{o2}f_2}{d_{o2}-f_2}=\frac{(35.65\text{cm})(16\text{cm})}{(35.65\text{cm})-(16\text{cm})}=29.25\text{cm}$$

Thus the final image is real, 29 cm beyond the second lens.
The total magnification is the product of the magnifications for the two lenses:

$$m=m_1m_2=\left(-\frac{d_{i1}}{d_{o1}}\right)\left(-\frac{d_{i2}}{d_{o2}}\right)=\frac{(20.35\text{cm})(29.25\text{cm})}{(36\text{cm})(35.65\text{cm})}=\boxed{0.46\times}$$

28. Use Eq. 33-4, the lensmaker's equation.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$

$$f=\frac{1}{(n-1)}\left(\frac{R_1R_2}{R_1+R_2}\right)=\frac{1}{(1.58-1)}\left(\frac{(-33.4\,\text{cm})(-28.8\,\text{cm})}{(-33.4\,\text{cm})+(-28.8\,\text{cm})}\right)=-26.66\,\text{cm}\approx\boxed{-27\,\text{cm}}$$

29. Find the index from Eq. 33-4, the lensmaker's equation.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow n=1+\frac{1}{f}\left(\frac{R_1R_2}{R_1+R_2}\right)=1+\left(\frac{1}{28.9\,\text{cm}}\right)\left(\tfrac{1}{2}(31.4\,\text{cm})\right)=\boxed{1.54}$$

30. With the surfaces reversed, we have $R_1=-46.2\,\text{cm}$ and $R_2=+22.4\,\text{cm}$. Use Eq. 33-4 to find the focal length.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$

$$f=\frac{1}{(n-1)}\left(\frac{R_1R_2}{R_1+R_2}\right)=\frac{1}{(1.50-1)}\left(\frac{(-46.2\,\text{cm})(+22.4\,\text{cm})}{(-46.2\,\text{cm})+(+22.4\,\text{cm})}\right)=\boxed{87.0\,\text{cm}}$$

31. The plane surface has an infinite radius of curvature. Let the plane surface be surface 2, so $R_2=\infty$. The index of refraction is found in Table 32-1.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{\infty}\right)=\frac{(n-1)}{R_1} \rightarrow$$

$$R_1=(n-1)f=(1.46-1)(18.7\,\text{cm})=\boxed{8.6\,\text{cm}}$$

32. First we find the focal length from Eq. 33-3, the lensmaker's equation. Then we use Eq. 33-2 to find the image distance, and Eq. 33-3 to find the magnification.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$

$$f=\frac{1}{(n-1)}\left(\frac{R_1R_2}{R_1+R_2}\right)=\frac{1}{(1.52-1)}\left(\frac{(-22.0\,\text{cm})(+18.5\,\text{cm})}{(-22.0\,\text{cm})+(+18.5\,\text{cm})}\right)=223.6\,\text{cm}$$

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_i=\frac{d_o f}{d_o-f}=\frac{(90.0\,\text{cm})(223.6\,\text{m})}{90.0\,\text{cm}-223.6\,\text{cm}}=-150.6\,\text{cm}\approx\boxed{-151\,\text{cm}}$$

$$m=-\frac{d_i}{d_o}=-\frac{-150.6\,\text{cm}}{90.0\,\text{cm}}=\boxed{+1.67}$$

The image is virtual, in front of the lens, and upright.

33. Find the radius from the lensmaker's equation, Eq. 33-4.:

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow P=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \rightarrow$$

$$R_2=\frac{(n-1)R_1}{PR_1-(n-1)}=\frac{(1.56-1)(0.300\,\text{m})}{(3.50\,\text{D})(0.300\,\text{m})-(1.56-1)}=\boxed{0.34\,\text{m}}$$

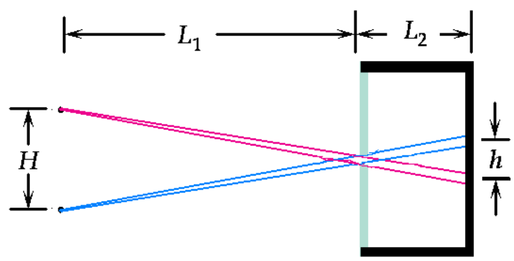
34. The exposure is proportional to the product of the lens opening area and the exposure time, with the square of the *f*-stop number inversely proportional to the lens opening area. Setting the exposures equal for both exposure times we solve for the needed *f*-stop number.

$$t_1\left(f\text{-stop}_1\right)^{-2}=t_2\left(f\text{-stop}_2\right)^{-2}\ \rightarrow\ f\text{-stop}_2=f\text{-stop}_1\sqrt{\frac{t_2}{t_1}}=16\sqrt{\frac{1/1000\text{ s}}{1/120\text{s}}}=5.54\text{ or }\boxed{\frac{f}{5.6}}$$

35. We find the *f*-number from $f\text{-stop}=f/D$.

$$f\text{-stop}=\frac{f}{D}=\frac{(17\,\text{cm})}{(6.0\,\text{cm})}=\boxed{\frac{f}{2.8}}$$

36. We use similar triangles, created from the distances between the centers of the two objects ($H$) and their ray traces to the hole ($L_1$) and the distance between the centers of the two images ($h$) and the distance of the screen to the hole ($L_2$) to determine the distance between the center of the two image circles. We then create similar triangles from the two ray traces for a single source with the base of one triangle equal to the diameter of the hole ($d$) , and the base of the second triangle equal to the diameter of the image circle ($D$). The heights for these two triangles are the distance from object to hole ($L_1$) and the distance from object to image ($L_1 + L_2$).




$$\frac{H}{L_1}=\frac{h}{L_2}\ \rightarrow h=H\frac{L_2}{L_1}=(2.0\text{ cm})\frac{7.0\text{ cm}}{100\text{ cm}}=0.14\text{ cm}=1.4\text{ mm}$$

$$\frac{d}{L_1}=\frac{D}{L_1+L_2}\ \rightarrow D=d\frac{L_1+L_2}{L_1}=(1.0\,\text{mm})\frac{100\,\text{cm}+7.0\,\text{cm}}{100\,\text{cm}}=1.07\,\text{mm}$$

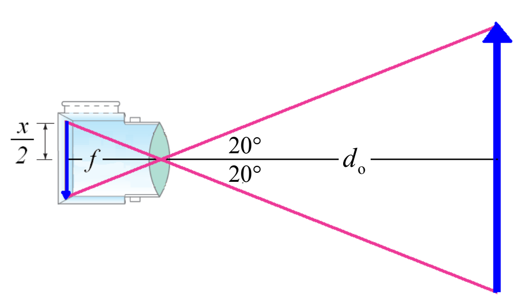
Since the separation distance of the two images is greater than their diameters, the two circles do not overlap.

37. We calculate the effective *f*-number for the pinhole camera by dividing the focal length by the diameter of the pinhole. The focal length is equal to the image distance. Setting the exposures equal for both cameras, where the exposure is proportional to the product of the exposure time and the area of the lens opening (which is inversely proportional to the square of the *f*-stop number), we solve for the exposure time.

$$f\text{-stop}_2=\frac{f}{D}=\frac{(70\,\text{mm})}{(1.0\,\text{mm})}=\frac{f}{70}.$$

$$t_1\left(f\text{-stop}_1\right)^{-2}=t_2\left(f\text{-stop}_2\right)^{-2}\ \rightarrow\ t_2=t_1\left(\frac{f\text{-stop}_2}{f\text{-stop}_1}\right)^2=\frac{1}{250\text{ s}}\left(\frac{70}{11}\right)^2=0.16\text{ s}\approx\boxed{\tfrac{1}{6}\text{s}}$$

38. Consider an object located a distance $d_o$ from a converging lens of focal length $f$ and its real image formed at distance $d_i$. If the distance $d_o$ is much greater than the focal length, the lens equation tells us that the focal length and image distance are equal.




$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\ \rightarrow\ d_i=\frac{fd_o}{d_o-f}\approx\frac{fd_o}{d_o}=f$$

Thus, in a camera, the recording medium of spatial extent $x$ is placed a distance equal to $f$ behind the lens to form a focused image of a distant object. Assume the distant object subtends an angle of $40°$ at the position of the lens, so that the half-angle subtended is $20°$, as shown in the figure. We then use the tangent of this angle to determine the relationship between the focal length and half the image height.

$$\tan 20^{\circ} = \frac{\frac{1}{2}x}{f} \rightarrow f = \frac{x}{2\tan 20^{\circ}}$$

(*a*) For a 35-mm camera, we set $x = 36\text{ mm}$ to calculate the focal length.

$$f = \frac{36\text{ mm}}{2\tan 20^{\circ}} = \boxed{49\text{ mm}}$$

(*b*) For a digital camera, we set $x = 1.0\text{ cm} = 10\text{ mm}$.

$$f = \frac{10\text{ mm}}{2\tan 20^{\circ}} = \boxed{14\text{ mm}}$$

39. The image distance is found from Eq. 33-3, and then the focal length from Eq. 33-2. The image is inverted.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \rightarrow d_i = -d_o\frac{h_i}{h_o} = -(65\text{m})\frac{(-24\text{mm})}{(38\text{m})} = 41\text{mm}$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{d_o d_i}{d_o + d_i} = \frac{(65\text{m})(0.041\text{m})}{65\text{m} - 0.041\text{m}} = 0.041\text{m} = \boxed{41\text{mm}}$$

The object is essentially at infinity, so the image distance is equal to the focal length.

40. The length of the eyeball is the image distance for a far object, i.e., the focal length of the lens. We find the *f*-number from $f\text{-stop} = f/D$.

$$f\text{-stop} = \frac{f}{D} = \frac{(20\text{mm})}{(8.0\text{mm})} = \boxed{2.5 \text{ or } \frac{f}{2.5}}.$$

41. The actual near point of the person is 55 cm. With the lens, an object placed at the normal near point, 25 cm, or 23 cm from the lens, is to produce a virtual image 55 cm from the eye, or 53 cm from the lens. We find the power of the lens from Eqs. 33-1 and 33-3.

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.23\text{m}} + \frac{1}{-0.53\text{m}} = \boxed{2.5\text{D}}$$

42. The screen placed 55 cm from the eye, or 53.2 cm from the lens, is to produce a virtual image 105 cm from the eye, or 103.2 cm from the lens. Find the power of the lens from Eqs. 33-1 and 33-2.

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.532\text{m}} + \frac{1}{-1.032\text{m}} = \boxed{0.91\text{D}}$$

43. With the contact lens, an object at infinity should form a virtual image at the far point of the eye, 17 cm from the contact lens. Use that with Eq. 33-2 to find the focal length of the contact lens. We find the power of the lens from

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow \frac{1}{\infty} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = d_i = -17\text{cm}$$

Find the new near point as the object location that forms a virtual image at the actual near point of 12 cm from the contact lens. Again use Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_o=\frac{fd_i}{d_i-f}=\frac{(-17\,\text{cm})(-12\,\text{cm})}{(-12\,\text{cm})-(-17\,\text{cm})}=\boxed{41\,\text{cm}}$$

So the person would have to hold the object 41 cm from their eye to see it clearly. With glasses, they only had to hold the object 32 cm from the eye. So glasses would be better.

44. (*a*) Since the lens power is negative, the lens is diverging, so it produces images closer than the object. Thus the person is nearsighted.

(*b*) We find the far point by finding the image distance for an object at infinity. Since the lens is 2.0 cm in front of the eye, the far point is 2.0 cm farther than the absolute value of the image distance.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow \frac{1}{\infty}+\frac{1}{d_i}=-4.50\,\text{D} \rightarrow d_i=-\frac{1}{4.50\text{ D}}=-0.222\text{ m}=-22.2\text{ cm}$$

$$\text{FP}=\left|-22.2\text{ cm}\right|+2.0\text{ cm}=\boxed{24.2\text{ cm}}\text{ from eye}$$

45. (*a*) The lens should put the image of an object at infinity at the person's far point of 78 cm. Note that the image is still in front of the eye, so the image distance is negative. Use Eqs. 33-2 and 33-1.

$$P=\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{\infty}+\frac{1}{(-0.78\,\text{m})}=-1.282\,\text{D}\approx\boxed{-1.3\,\text{D}}$$

(*b*) To find the near point with the lens in place, we find the object distance to form an image 25 cm in front of the eye.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \rightarrow d_o=\frac{d_i}{Pd_i-1}=\frac{(-0.25\,\text{m})}{(-1.282\,\text{D})(-0.25\,\text{m})-1}=0.37\,\text{m}=\boxed{37\,\text{cm}}$$

46. The image of an object at infinity is to be formed 14 cm in front of the eye. So for glasses, the image distance is to be $d_i=-12\,\text{cm}$, and for contact lenses, the image distance is to be $d_i=-14\,\text{cm}$.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=\frac{1}{\infty}+\frac{1}{d_i} \rightarrow f=d_i \rightarrow P=\frac{1}{f}=\frac{1}{d_i}$$

$$P_{\text{glasses}}=\frac{1}{-0.12\,\text{m}}=\boxed{-8.3\,\text{D}}\ ;\ P_{\text{contacts}}=\frac{1}{-0.14\,\text{m}}=\boxed{-7.1\,\text{D}}$$

47. Find the far point of the eye by finding the image distance FROM THE LENS for an object at infinity, using Eq. 33-2.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow \frac{1}{\infty}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow d_{i1}=f_1=-23.0\,\text{cm}$$

Since the image is 23.0 in front of the lens, the image is 24.8 cm in front of the eye. The contact lens must put the image of an object at infinity at this same location. Use Eq. 33-2 for the contact lens with an image distance of -24.8 cm and an object distance of infinity.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow \frac{1}{\infty}+\frac{1}{d_{i2}}=\frac{1}{f_1} \rightarrow f_1=d_{i2}=\boxed{-24.8\,\text{cm}}$$

48. (*a*) We find the focal length of the lens for an object at infinity and the image on the retina. The image distance is thus 2.0 cm. Use Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{1}{\infty}+\frac{1}{2.0\,\text{cm}}=\frac{1}{f} \rightarrow f=\boxed{2.0\,\text{cm}}$$

(*b*) We find the focal length of the lens for an object distance of 38 cm and an image distance of 2.0 cm. Again use Eq. 33.2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad\rightarrow\quad f=\frac{d_o d_i}{d_o+d_i}=\frac{(38\,\text{cm})(2.0\,\text{cm})}{(38\,\text{cm})+(2.0\,\text{cm})}=\boxed{1.9\,\text{cm}}$$

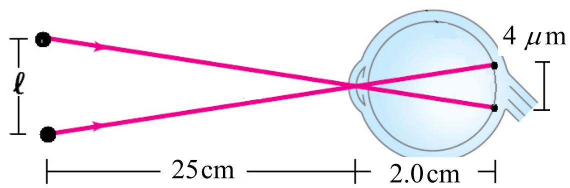
49. Find the object distance for the contact lens to form an image at the eye's near point, using Eqs. 33-2 and 33-1.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=P \quad\rightarrow\quad d_o=\frac{d_i}{Pd_i-1}=\frac{-0.106\,\text{m}}{(-4.00\,\text{D})(-0.106\,\text{m})-1}=0.184\,\text{m}=\boxed{18.4\,\text{cm}}$$

Likewise find the object distance for the contact lens to form an image at the eye's far point.

$$d_o=\frac{d_i}{Pd_i-1}=\frac{-0.200\,\text{m}}{(-4.0\,\text{D})(-0.200\,\text{m})-1}=1.00\,\text{m}=\boxed{100\,\text{cm}}\ \text{(3 sig. fig.)}$$

50. In the image we show the principal rays from each of the two points as they pass directly through the cornea and onto the lens. These two rays and the distance between the two objects, ℓ, and the distance between the two images (4 μm) create similar triangles. We set the ratio of the bases and heights of these two triangles equal to solve for ℓ.




$$\frac{\ell}{25\text{ cm}}=\frac{4\ \mu\text{m}}{2.0\text{ cm}}\rightarrow \ell=25\text{ cm}\frac{4\ \mu\text{m}}{2.0\text{ cm}}=\boxed{50\,\mu\text{m}}$$

51. We find the focal length from Eq. 33-6

$$M=\frac{N}{f}\quad\rightarrow\quad f=\frac{N}{M}=\frac{25\,\text{cm}}{3.8}=\boxed{6.6\,\text{cm}}$$

52. Find the magnification from Eq. 33-6.

$$M=\frac{N}{f}=\frac{(25\,\text{cm})}{(13\,\text{cm})}=\boxed{1.9\times}$$

53. (*a*) We find the focal length with the image at the near point from Eq. 33-6b.

$$M=1+\frac{N}{f}\quad\rightarrow\quad f=\frac{N}{M-1}=\frac{25\,\text{cm}}{3.0-1}=12.5\,\text{cm}\approx\boxed{13\,\text{cm}}$$

$$3.0=1+\frac{(25\,\text{cm})}{f_1},\ \text{which gives}\ f_1=12.5\,\text{cm}\approx\boxed{13\,\text{cm.}}$$

(*b*) If the eye is relaxed, the image is at infinity, and so use Eq. 33-6a.

$$M=\frac{N}{f}\quad\rightarrow\quad f=\frac{N}{M}=\frac{25\,\text{cm}}{3.0}=\boxed{8.3\,\text{cm}}$$

54. Maximum magnification is obtained with the image at the near point (which is negative). We find the object distance from Eq. 33-2, and the magnification from Eq. 33-6b.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\quad\rightarrow\quad d_o=\frac{d_i f}{d_i-f}=\frac{(-25.0\,\text{cm})(8.80\,\text{cm})}{(-25.0\,\text{cm})-(8.80\,\text{cm})}=\boxed{6.51\,\text{cm}}$$

$$M=1+\frac{N}{f}=1+\frac{25.0\,\text{cm}}{8.80\,\text{cm}}=\boxed{3.84\times}$$

55. (*a*) We find the image distance from Eq. 33-2.

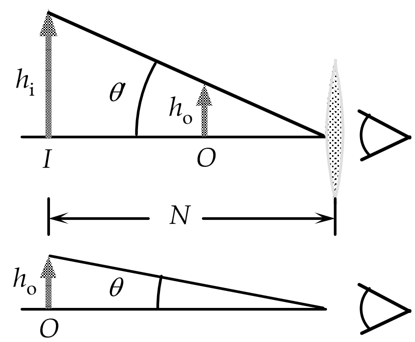
$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\;\;\rightarrow\;\; d_i=\frac{fd_o}{d_o-f}=\frac{(6.00\,\text{cm})(5.85\,\text{cm})}{5.85\,\text{cm}-6.00\,\text{cm}}=\boxed{-234\,\text{cm}}$$

(*b*) The angular magnification is given by Eq. 33-6a, since the eye will have to focus over 2 m away.

$$M=\frac{N}{f}=\frac{25.0\,\text{cm}}{6.00\,\text{cm}}=\boxed{4.17\times}$$

56. (*a*) We use Eq. 33-6b to calculate the angular magnification.

$$M=1+\frac{N}{f}=1+\frac{(25.0\,\text{cm})}{(9.60\,\text{cm})}=\boxed{3.60\times}$$




(*b*) Because the object without the lens and the image with the lens are at the near point, the angular magnification is also the ratio of widths. Using this relationship we calculate the image width.

$$M=\frac{h_i}{h_o}\rightarrow h_i=Mh_o=3.60(3.40\,\text{mm})=\boxed{12.3\,\text{mm}}$$

(*c*) We use Eq. 33-2 to calculate the object distance, with the image distance at -25.0 cm.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\;\;\rightarrow\;\; d_o=\frac{fd_i}{d_i-f}=\frac{(9.60\,\text{cm})(-25.0\,\text{cm})}{-25.0\,\text{cm}-9.60\,\text{cm}}=\boxed{6.94\,\text{cm}}$$

57. (*a*) We find the image distance using Eq. 33-2.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}\;\;\rightarrow\;\; d_i=\frac{fd_o}{d_o-f}=\frac{(9.5\,\text{cm})(8.3\,\text{cm})}{8.3\,\text{cm}-9.5\,\text{cm}}=\boxed{-66\,\text{cm}}$$

(*b*) The angular magnification is found using Eq. 33-5, with the angles given as defined in Figure 33-33.

$$M=\frac{\theta'}{\theta}=\frac{(h_o/d_o)}{(h_o/N)}=\frac{N}{d_o}=\frac{25\text{ cm}}{8.3\text{ cm}}=\boxed{3.0\times}$$

58. First, find the focal length of the magnifying glass from Eq. 33-6a, for a relaxed eye (focused at infinity).

$$M=\frac{N}{f}\;\;\rightarrow\;\; f=\frac{N}{M}=\frac{25.0\,\text{cm}}{3.0}=8.33\,\text{cm}$$

(*a*) Again use Eq. 33-6a for a different near point.

$$M_1=\frac{N_1}{f}=\frac{(65\,\text{cm})}{(8.33\,\text{cm})}=\boxed{7.8\times}$$

(*b*) Again use Eq. 33-6a for a different near point.

$$M_2=\frac{N_2}{f}=\frac{(17\,\text{cm})}{(8.33\,\text{cm})}=\boxed{2.0\times}$$

Without the lens, the closest an object can be placed is the near point. A farther near point means a smaller angle subtended by the object without the lens, and thus greater magnification.

59. The focal length is 10 cm. First, find the object distance for an image at infinity. Then, find the object distance for an image 25 cm in front of the eye.

Initial: $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow \frac{1}{d_o}+\frac{1}{\infty}=\frac{1}{f} \rightarrow d_o=f=12\,\text{cm}$

Final: $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \rightarrow d_o=\frac{d_i f}{d_i-f}=\frac{(-25\,\text{cm})(12\,\text{cm})}{(-25\,\text{cm})-(12\,\text{cm})}=8.1\,\text{cm}$

The lens was moved $12\,\text{cm}-8.1\,\text{cm}=3.9\,\text{cm}\approx$ 4 cm toward the fine print.

60. The magnification of the telescope is given by Eq. 33-7.

$$M=-\frac{f_o}{f_e}=-\frac{(78\,\text{cm})}{(2.8\,\text{cm})}=\boxed{-28\times}$$

For both object and image far away, the separation of the lenses is the sum of the focal lengths.

$$f_o+f_e=78\,\text{cm}+2.8\,\text{cm}=\boxed{81\,\text{cm}}$$

61. We find the focal length of the eyepiece from the magnification by Eq. 33-7.

$$M=-\frac{f_o}{f_e} \rightarrow f_e=-\frac{f_o}{M}=-\frac{88\,\text{cm}}{35\times}=\boxed{2.5\,\text{cm}}$$

For both object and image far away, the separation of the lenses is the sum of the focal lengths.

$$f_o+f_e=88\,\text{cm}+2.5\,\text{cm}=\boxed{91\,\text{cm}}$$

62. We find the focal length of the objective from Eq. 33-7.

$$M=f_o/f_e \rightarrow f_o=Mf_e=(7.0)(3.0\,\text{cm})=\boxed{21\,\text{cm}}$$

63. The magnification is given by Eq. 33-7.

$$M=-f_o/f_e==-f_oP_e=-(0.75\,\text{m})(35\,\text{D})=\boxed{-26\times}$$

64. For a distant object and a relaxed eye (which means the image is at infinity), the separation of the eyepiece and objective lenses is the sum of their focal lengths. Use Eq. 33-7 to find the magnification.

$$\ell=f_o+f_e\ ;\ M=-\frac{f_o}{f_e}=-\frac{f_o}{\ell-f_o}=-\frac{75.5\,\text{cm}}{78.0\,\text{cm}-75.5\,\text{cm}}=\boxed{-30\times}$$

65. For a distant object and a relaxed eye (which means the image is at infinity), the separation of the eyepiece and objective lenses is the sum of their focal lengths. Use Eq. 33-7 to find the magnification.

$$\ell=f_o+f_e\ ;\ M=-\frac{f_o}{f_e}=-\frac{f_o}{\ell-f_o}=-\frac{36.0\,\text{cm}}{33.8\,\text{cm}-36.0\,\text{cm}}=\boxed{+16\times}$$

66. The focal length of the objective is just half the radius of curvature. Use Eq. 33-7 for the magnification.

$$M=-\frac{f_o}{f_e}=-\frac{\frac{1}{2}r}{f_e}=-\frac{3.2\,\text{m}}{0.028\,\text{m}}=-114\times\approx\boxed{-110\times}$$

67. The focal length of the mirror is found from Eq. 33-7. The radius of curvature is twice the focal length.

$$M = -\frac{f_o}{f_e} \rightarrow f_o = -Mf_e = -(120)(0.031\,\text{m}) = 3.72\,\text{m} \approx \boxed{3.7\,\text{m}} \;;\; r = 2f = \boxed{7.4\,\text{m}}$$

68. The relaxed eye means that the image is at infinity, and so the distance between the two lenses is 1.25 m. Use that relationship with Eq. 33-7 to solve for the focal lengths. Note that the magnification for an astronomical telescope is negative.

$$\ell = f_o + f_e \;;\; M = -\frac{f_o}{f_e} = -\frac{f_o}{\ell - f_o} \rightarrow f_o = \frac{M\ell}{M-1} = \frac{-120(1.25\,\text{m})}{-120-1} = \boxed{1.24\,\text{m}}$$

$$f_e = \ell - f_o = 1.25\,\text{m} - 1.24\,\text{m} = 0.01\,\text{m} = \boxed{1\text{ cm}}$$

69. We use Eq. 33-6a and the magnification of the eyepiece to calculate the focal length of the eyepiece. We set the sum of the focal lengths equal to the length of the telescope to calculate the focal length of the objective. Then using both focal lengths in Eq. 33-7 we calculate the maximum magnification.

$$f_e = \frac{N}{M} = \frac{25\,\text{cm}}{5} = 5\,\text{cm} \;;\; \ell = f_e + f_o \rightarrow f_o = \ell - f_e = 50\,\text{cm} - 5\,\text{cm} = 45\,\text{cm}$$

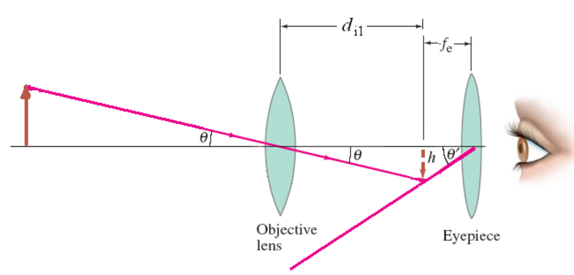
$$M = -\frac{f_o}{f_e} = -\frac{45\,\text{cm}}{5\,\text{cm}} = \boxed{-9\times}$$

70. Since the star is very far away, the image of the star from the objective mirror will be at the focal length of the objective, which is equal to one-half its radius of curvature (Eq. 32-1). We subtract this distance from the separation distance to determine the object distance for the second mirror. Then, using Eq. 33-2, we calculate the final image distance, which is where the sensor should be placed.

$$d_{i1} = f_o = \frac{R_o}{2} = \frac{3.00\,\text{m}}{2} = 1.50\,\text{m} \;;\; d_{o2} = \ell - d_{i1} = 0.90\,\text{m} - 1.50\,\text{m} = -0.60\,\text{m}$$

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_e} = \frac{2}{R_e} \rightarrow d_i = \frac{R_e d_{o2}}{2d_{o2} - R_e} = \frac{(-1.50\,\text{m})(-0.60\,\text{m})}{2(-0.60\,\text{m}) - (-1.50\,\text{m})} = \boxed{3.0\,\text{m}}$$

71. We assume a prism binocular so the magnification is positive, but simplify the diagram by ignoring the prisms. We find the focal length of the eyepiece using Eq. 33-7, with the design magnification.




$$f_e = \frac{f_o}{M} = \frac{26\,\text{cm}}{7.5} = 3.47\,\text{cm}$$

Using Eq. 33-2 and the objective focal length, we calculate the intermediate image distance. With the final image at infinity (relaxed eye), the secondary object distance is equal to the focal length of the eyepiece. We calculate the angular magnification using Eq. 33-5, with the angles shown in the diagram.

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_o} \rightarrow d_{i1} = \frac{f_o d_{o`}}{d_{o1} - f_o} = \frac{(26\,\text{cm})(400\,\text{cm})}{400\,\text{cm} - 26\,\text{cm}} = 27.81\,\text{cm}$$

$$M = \frac{\theta'}{\theta} = \frac{h/f_e}{h/d_{i1}} = \frac{d_{i1}}{f_e} = \frac{27.81\,\text{cm}}{3.47\,\text{cm}} = \boxed{8.0\times}$$

72. The magnification of the microscope is given by Eq. 33-10b.

$$M = \frac{N\ell}{f_o f_e} = \frac{(25\,\text{cm})(17.5\,\text{cm})}{(0.65\,\text{cm})(1.50\,\text{cm})} = 448.7\times \approx \boxed{450\times}$$

$\boxed{73.}$ We find the focal length of the eyepiece from the magnification of the microscope, using the approximate results of Eq. 33-10b. We already know that $f_o \ll \ell$.

$$M \approx \frac{N\ell}{f_o f_e} \rightarrow f_e = \frac{N\ell}{Mf_o} = \frac{(25\,\text{cm})(17.5\,\text{cm})}{(680)(0.40\,\text{cm})} = \boxed{1.6\,\text{cm}}$$

Note that this also satisfies the assumption that $f_e \ll \ell$.

74. We use Eq. 33-10b.

$$M \approx \frac{N\ell}{f_e f_o} = \frac{(25\,\text{cm})(17\,\text{cm})}{(2.5\,\text{cm})(0.28\,\text{cm})} = 607.1\times \approx \boxed{610\times}$$

75. (*a*) The total magnification is found from Eq. 33-10a.

$$M = M_o M_e = (58.0)(13.0) = \boxed{754\times}$$

(*b*) With the final image at infinity, we find the focal length of the eyepiece using Eq. 33-9.

$$M_e = \frac{N}{f_e} \rightarrow f_e = \frac{N}{M_e} = \frac{25.0\,\text{cm}}{13.0} = 1.923\,\text{cm} \approx \boxed{1.92\,\text{cm}}$$

Since the image from the objective is at the focal point of the eyepiece, we set the image distance from the objective as the distance between the lenses less the focal length of the eyepiece. Using the image distance and magnification in Eq. 33-3, we calculate the initial object distance. Then using the image and object distance in Eq. 33-2 we calculate the objective focal length.

$$d_i = \ell - f_e = 20.0\,\text{cm} - 1.92\,\text{cm} = 18.08\,\text{cm}$$

$$m = \frac{d_i}{d_o} \rightarrow d_o = \frac{d_i}{m} = \frac{18.08\,\text{cm}}{58.0} = 0.312\,\text{cm}$$

$$\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow f_o = \frac{d_o d_i}{d_o + d_i} = \frac{(0.312\,\text{cm})(18.08\,\text{cm})}{0.312\,\text{cm} + 18.08\,\text{cm}} = \boxed{0.307\,\text{cm}}$$

(*c*) We found the object distance, in part (*b*), $d_o = \boxed{0.312\,\text{cm}}$.

76. (*a*) The total magnification is the product of the magnification of each lens, with the magnification of the eyepiece increased by one, as in Eq. 33-6b.

$$M = M_o(M_e + 1) = (58.0)(13.0 + 1.0) = \boxed{812\times}$$

(*b*) We find the focal length of the eyepiece using Eq. 33-6b.

$$(M_e + 1) = \frac{N}{f_e} + 1 \rightarrow f_e = \frac{N}{M_e} = \frac{25\,\text{cm}}{13.0} = \boxed{1.92\,\text{cm}}$$

Since the image from the eyepiece is at the near point, we use Eq. 33-2 to calculate the location of the object. This object distance is the location of the image from the objective. Subtracting this object distance from the distance between the lenses gives us the image distance from the objective. Using the image distance and magnification in Eq. 33-3, we calculate the initial object distance. Then using the image and object distance in Eq. 33-2 we calculate the objective focal length.

$$\frac{1}{f_e}=\frac{1}{d_{o2}}+\frac{1}{d_{i2}} \rightarrow d_{o2}=\frac{f_e d_{i2}}{d_{i2}-f_e}=\frac{(1.92\,\text{cm})(-25.0\,\text{cm})}{-25.0\,\text{cm}-1.92\,\text{cm}}=1.78\,\text{cm}$$

$$d_{i1}=\ell-d_{o2}=20.0\,\text{cm}-1.78\,\text{cm}=18.22\,\text{cm}$$

$$m=\frac{d_i}{d_o} \rightarrow d_o=\frac{d_i}{m}=\frac{18.22\,\text{cm}}{58.0}=0.314\,\text{cm}$$

$$\frac{1}{f_o}=\frac{1}{d_o}+\frac{1}{d_i} \rightarrow f_o=\frac{d_o d_i}{d_o+d_i}=\frac{(0.314\,\text{cm})(18.22\,\text{cm})}{0.314\,\text{cm}+18.22\,\text{cm}}=\boxed{0.308\,\text{cm}}$$

(*c*) We found the object distance, in part (*b*), $d_o=\boxed{0.314\,\text{cm}}$.

77. (*a*) Since the final image is at infinity (relaxed eye) the image from the objective is at the focal point of the eyepiece. We subtract this distance from the distance between the lenses to calculate the objective image distance. Then using Eq. 33-2, we calculate the object distance.

$$d_{i1}=\ell-f_e=16.8\,\text{cm}-1.8\,\text{cm}=15.0\,\text{cm}$$

$$\frac{1}{f_o}=\frac{1}{d_{o1}}+\frac{1}{d_{i1}} \rightarrow d_{o1}=\frac{f_o d_{i1}}{d_{i1}-f_o}=\frac{(0.80\,\text{cm})(15.0\,\text{cm})}{15.0\,\text{cm}-0.80\,\text{cm}}=\boxed{0.85\,\text{cm}}$$

(*b*) With the final image at infinity, the magnification of the eyepiece is given by Eq. 33-10a.

$$M=\frac{N}{f_e}\left(\frac{\ell-f_e}{d_o}\right)=\frac{(25.0\,\text{cm})}{(1.8\,\text{cm})}\left(\frac{16.8\,\text{cm}-1.8\,\text{cm}}{0.85\,\text{cm}}\right)=247\times\approx\boxed{250\times}$$

78. (*a*) We find the image distance from the objective using Eq. 33-2. For the final image to be at infinity (viewed with a relaxed eye), the objective image distance must be at the focal distance of the eyepiece. We calculate the distance between the lenses as the sum of the objective image distance and the eyepiece focal length.

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_o} \rightarrow d_{i1}=\frac{f_o d_{o1}}{d_{o1}-f_o}=\frac{(0.740\,\text{cm})(0.790\,\text{cm})}{0.790\,\text{cm}-0.740\,\text{cm}}=11.7\,\text{cm}$$

$$\ell=d_{i1}+f_e=11.7\,\text{cm}+2.80\,\text{cm}=\boxed{14.5\,\text{cm}}$$

(*b*) We use Eq. 33-10a to calculate the total magnification.

$$M=\frac{N}{f_e}\left(\frac{\ell-f_e}{d_o}\right)=\frac{(25.0\,\text{cm})}{(2.80\,\text{cm})}\left(\frac{14.5\,\text{cm}-2.80\,\text{cm}}{0.790\,\text{cm}}\right)=\boxed{132\times}$$

79. For each objective lens we set the image distance equal to the sum of the focal length and 160 mm. Then, using Eq. 33-2 we write a relation for the object distance in terms of the focal length. Using this relation in Eq. 33-3 we write an equation for the magnification in terms of the objective focal length. The total magnification is the product of the magnification of the objective and focal length.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f_o} \rightarrow \frac{1}{d_o}=\frac{1}{f_o}-\frac{1}{d_i} \rightarrow \frac{1}{d_o}=\frac{1}{f_o}-\frac{1}{f_o+160\text{ mm}} \rightarrow d_o=\frac{f_o(f_o+160\text{ mm})}{160\text{ mm}}$$

$$m_o=\frac{d_i}{d_o}=\frac{f_o+160\,\text{mm}}{\left[\dfrac{f_o(f_o+160\,\text{mm})}{160\text{ mm}}\right]}=\frac{160\text{ mm}}{f_o}$$

Since the objective magnification is inversely proportional to the focal length, the objective with the smallest focal length $(f_o=3.9\text{ mm})$ combined with the largest eyepiece magnification $(M_e=10)$ yields the largest overall magnification. The objective with the largest focal length $(f_o=32\text{ mm})$

coupled with the smallest eyepiece magnification $(M_e = 5)$ yields the smallest overall magnification.

$$M_{\text{largest}} = \frac{160\,\text{mm}}{3.9\,\text{mm}}(10\times) = \boxed{410\times} \; ; \; M_{\text{smallest}} = \frac{160\,\text{mm}}{32\,\text{mm}}(5\times) = \boxed{25\times}$$

80. (*a*) For this microscope both the objective and eyepiece have focal lengths of 12 cm. Since the final image is at infinity (relaxed eye) the image from the objective must be at the focal length of the eyepiece. The objective image distance must therefore be equal to the distance between the lenses less the focal length of the objective. We calculate the object distance by inserting the objective focal length and image distance into Eq. 33-2.

$$d_{i1} = \ell - f_e = 55\,\text{cm} - 12\,\text{cm} = 43\,\text{cm}$$

$$\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_{i1}} \rightarrow d_o = \frac{f_o d_{i1}}{d_{i1} - f_o} = \frac{(12\,\text{cm})(43\,\text{cm})}{43\,\text{cm} - 12\,\text{cm}} = 16.65\,\text{cm} \approx \boxed{17\,\text{cm}}$$

(*b*) We calculate the magnification using Eq. 33-10a.

$$M = \frac{N}{f_e}\left(\frac{\ell - f_e}{d_o}\right) = \frac{(25\,\text{cm})}{(12\,\text{cm})}\left(\frac{55\,\text{cm} - 12\,\text{cm}}{16.65\,\text{cm}}\right) = 5.38\times \approx \boxed{5.4\times}$$

(*c*) We calculate the magnification using Eq. 33-10b, and divide the result by the answer to part (*b*) to determine the percent difference.

$$M_{\text{approx}} \approx \frac{N\ell}{f_e f_o} = \frac{(25\,\text{cm})(55\,\text{cm})}{(12\,\text{cm})(\,12\,\text{cm})} = 9.55\times \; ; \; \frac{M_{\text{approx}} - M}{M} = \frac{9.55 - 5.38}{5.38} = 0.775 \approx \boxed{78\%}$$

81. We use Eq. 33-4 to find the focal length for each color, and then Eq. 33-2 to find the image distance. For the plano-convex lens, $R_1 > 0$ and $R_2 = \infty$.

$$\frac{1}{f_{\text{red}}} = (n_{\text{red}} - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.5106 - 1)\left[\left(\frac{1}{18.4\,\text{cm}}\right) + \left(\frac{1}{\infty}\right)\right] \rightarrow f_{\text{red}} = 36.036\,\text{cm}$$

$$\frac{1}{f_{\text{yellow}}} = (n_{\text{yellow}} - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.5226 - 1)\left[\left(\frac{1}{18.4\,\text{cm}}\right) + \left(\frac{1}{\infty}\right)\right] \rightarrow f_{\text{orange}} = 35.209\,\text{cm}$$

We find the image distances from

$$\frac{1}{d_o} + \frac{1}{d_{i_{\text{red}}}} = \frac{1}{f_{\text{red}}} \rightarrow d_{i_{\text{red}}} = \frac{d_o f_{\text{red}}}{d_o - f_{\text{red}}} = \frac{(66.0\,\text{cm})(36.036\,\text{cm})}{(66.0\,\text{cm}) - (36.036\,\text{cm})} = 79.374\,\text{cm} \approx \boxed{79.4\,\text{cm}}$$

$$\frac{1}{d_o} + \frac{1}{d_{i_{\text{yellow}}}} = \frac{1}{f_{\text{yellow}}} \rightarrow d_{i_{\text{yellow}}} = \frac{d_o f_{\text{yellow}}}{d_o - f_{\text{yellow}}} = \frac{(66.0\,\text{cm})(35.209\,\text{cm})}{(66.0\,\text{cm}) - (35.209\,\text{cm})} = 75.469\,\text{cm} \approx \boxed{75.5\,\text{cm}}$$

The images are 3.9 cm apart, an example of chromatic aberration.

82. From Problem 26 we have a relationship between the individual focal lengths and the focal length of the combination.

$$\frac{1}{f_D} = -\frac{1}{f_C} + \frac{1}{f_T} \rightarrow \frac{1}{f_T} = \frac{1}{f_D} + \frac{1}{f_C} \rightarrow f_T = \frac{f_C f_D}{f_C + f_D} = \frac{(25\,\text{cm})(-28\,\text{cm})}{(25\,\text{cm}) + (-28\,\text{cm})} = 233\,\text{cm}$$

(*a*) The combination is $\boxed{\text{converging}}$, since the focal length is positive. Also, the converging lens is "stronger" than the diverging lens since it has a smaller absolute focal length (or higher absolute power).

(*b*) From above, $f_T \approx \boxed{230\,\text{cm}}$.

83. We calculate the range object distances from Eq. 33-2 using the given focal length and maximum and minimum image distances.

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i} \rightarrow d_{o,min}=\frac{fd_{i,max}}{d_{i,max}-f}=\frac{(200.0\,mm)(206.4\,mm)}{206.4\,mm-200.0\,mm}=6450\,mm=6.45\,m$$

$$d_{o,max}=\frac{fd_{i,min}}{d_{i,min}-f}=\frac{(200.0\,mm)(200.0\,mm)}{200.0\,mm-200.0\,mm}=\infty$$

Thus the range of object distances is $\boxed{6.45\,m \le d_o < \infty}$.

84. We calculate the maximum and minimum image distances from Eq. 33-2, using the given focal length and maximum and minimum object distances. Subtracting these two distances gives the distance over which the lens must move relative to the plane of the sensor or film.

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i} \rightarrow d_{i,max}=\frac{fd_{o,min}}{d_{o,min}-f}=\frac{(135\,mm)(1.30\,m)}{1300\,mm-135\,mm}=0.151\,m=151\,mm$$

$$d_{i,min}=\frac{fd_{o,max}}{d_{o,max}-f}=\frac{(135\,mm)(\infty)}{\infty-135\,mm}=135\,mm$$

$$\Delta d=d_{i,max}-d_{i,min}=151\,mm-135\,mm=\boxed{16\,mm}$$

85. Since the object height is equal to the image height, the magnification is –1. We use Eq. 33-3 to obtain the image distance in terms of the object distance. Then we use this relationship with Eq. 33-2 to solve for the object distance.

$$m=-1=-\frac{d_i}{d_o} \rightarrow d_i=d_o$$

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{d_o}=\frac{2}{d_o} \rightarrow d_o=2f=2(58\,mm)=\boxed{116\,mm}$$

The distance between the object and the film is the sum of the object and image distances.

$$d=d_o+d_i=d_o+d_o=2d_o=2(116\text{ mm})=\boxed{232\,mm}$$

86. When an object is very far away, the image will be at the focal point. We set the image distance in Eq. 33-3 equal to the focal length to show that the magnification is proportional to the focal length.

$$m=-\frac{d_i}{d_o}=-\frac{f}{d_o}=\left(-\frac{1}{d_o}\right)f=(\text{constant})f \rightarrow \boxed{m\propto f}$$

87. We use Eq. 33-2 with the final image distance and focal length of the converging lens to determine the location of the object for the second lens. Subtracting this distance from the separation distance between the lenses gives us the image distance from the first lens. Inserting this image distance and object distance into Eq. 33-2, we calculate the focal length of the diverging lens.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{o2}=\frac{d_{i2}f_2}{d_{i2}-f_2}=\frac{(17.0\,cm)(12.0\,cm)}{17.0\,cm-12.0\,cm}=40.8\,cm$$

$$d_{i1}=\ell-d_{o2}=30.0\,cm-40.8\,cm=-10.8\,cm$$

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f_1} \rightarrow f_1=\frac{d_{i1}d_{o1}}{d_{i1}+d_{o1}}=\frac{(-10.8\,cm)(25.0\,cm)}{-10.8\,cm+25.0\,cm}=\boxed{-19.0\,cm}$$

88. The relationship between two lenses in contact was found in Problem 26. We use this resulting equation to solve for the combination focal length.

$$\frac{1}{f_T}=\frac{1}{f_D}+\frac{1}{f_C} \rightarrow f_T=\frac{f_D f_C}{f_D+f_C}=\frac{(-20.0\,\text{cm})(13.0\,\text{cm})}{-20.0\,\text{cm}+13.0\,\text{cm}}=\boxed{37.1\,\text{cm}}$$

Since the focal length is positive, the combination is a $\boxed{\text{converging lens}}$.

89. We use Eq. 33-7, which relates the magnification to the focal lengths, to write the focal length of the objective lens in terms of the magnification and focal length of the eyepiece. Then setting the sum of the focal lengths equal to the length of the telescope we solve for the focal length of the eyepiece and the focal length of the objective.

$$M=-\frac{f_o}{f_e} \rightarrow f_o=-Mf_e \ ; \ \ell=f_e+f_o=f_e(1-M) \rightarrow f_e=\frac{\ell}{1-M}=\frac{28\,\text{cm}}{1-(-8.0)}=\boxed{3.1\,\text{cm}}$$

$$f_o=\ell-f_e=28\,\text{cm}-3.1\,\text{cm}=\boxed{25\,\text{cm}}$$

90. (*a*) When two lenses are placed in contact, the negative of the image of the first lens is the object distance of the second. Using Eq. 33-2, we solve for the image distance of the first lens. Inserting the negative of this image distance into the lens equation for the second lens we obtain a relationship between the initial object distance and final image distance. Again using the lens equation with this relationship, we obtain the focal length of the lens combination.

$$\frac{1}{f_1}=\frac{1}{d_{o1}}+\frac{1}{d_{i1}} \rightarrow \frac{1}{d_{i1}}=\frac{1}{f_1}-\frac{1}{d_{o1}}=-\frac{1}{d_{o2}}$$

$$\frac{1}{f_2}=\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{d_{o2}}-\left(\frac{1}{f_1}-\frac{1}{d_{o1}}\right)\Rightarrow\frac{1}{f_2}+\frac{1}{f_1}=\frac{1}{d_{o2}}+\frac{1}{d_{o1}}=\frac{1}{f_T}$$

$$\frac{1}{f_T}=\frac{1}{f_1}+\frac{1}{f_2} \rightarrow \boxed{f_T=\frac{f_1 f_2}{f_1+f_2}}$$

(*b*) Setting the power equal to the inverse of the focal length gives the relationship between powers of adjacent lenses.

$$\frac{1}{f_T}=\frac{1}{f_1}+\frac{1}{f_2}\Rightarrow\boxed{P_T=P_1+P_2}$$

$\boxed{91.}$ (*a*) Because the Sun is very far away, the image will be at the focal point, or $d_i=f$. We find the magnitude of the size of the image using Eq. 33-3, with the image distance equal to 28 mm.

$$\frac{h_i}{h_o}=\frac{-d_i}{d_o} \rightarrow |h_i|=\frac{h_o d_i}{d_o}=\frac{(1.4\times10^6\,\text{km})(28\,\text{mm})}{1.5\times10^8\,\text{km}}=\boxed{0.26\,\text{mm}}$$

(*b*) We repeat the same calculation with a 50 mm image distance.

$$|h_i|=\frac{(1.4\times10^6\,\text{km})(50\,\text{mm})}{1.5\times10^8\,\text{km}}=\boxed{0.47\,\text{mm}}$$

(*c*) Again, with a 135 mm image distance.

$$|h_i|=\frac{(1.4\times10^6\,\text{km})(135\,\text{mm})}{1.5\times10^8\,\text{km}}=\boxed{1.3\text{ mm}}$$

(*d*) The equations show that image height is directly proportional to focal length. Therefore the relative magnifications will be the ratio of focal lengths.

$$\frac{28\,\text{mm}}{50\,\text{mm}}=\boxed{0.56\times} \text{ for the 28 mm lens ; } \frac{135\,\text{mm}}{50\,\text{mm}}=\boxed{2.7\times} \text{ for the 135 mm lens.}$$

92. We solve this problem by working through the lenses "backwards." We use the image distances and focal lengths to calculate the object distances. Since the final image from the right lens is halfway between the lenses, we set the image distance of the second lens equal to the negative of half the distance between the lenses. Using Eq. 33-2, we solve for the object distance of this lens. By subtracting this object distance from the distance between the two lenses, we find the image distance from the first lens. Then using Eq. 33-2 again, we solve for the initial object distance.

$$d_{i2} = -\tfrac{1}{2}\ell = -\tfrac{1}{2}(30.0\,\text{cm}) = -15.0\,\text{cm}$$

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2} \rightarrow d_{o2} = \frac{d_{i2}f_2}{d_{i2} - f_2} = \frac{(-15.0\,\text{cm})(20.0\,\text{cm})}{-15.0\,\text{cm} - 20.0\,\text{cm}} = 8.57\,\text{cm}$$

$$d_{i1} = \ell - d_{o2} = 30.0\text{ cm} - 8.57\text{ cm} = 21.4\text{ cm}$$

$$\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1} \rightarrow d_{o1} = \frac{d_{i1}f_1}{d_{i1} - f_1} = \frac{(21.4\,\text{cm})(15.0\,\text{cm})}{21.4\,\text{cm} - 15.0\,\text{cm}} = \boxed{50.0\,\text{cm}}$$

93. We set $d_i$ as the original image distance and $d_i + 10.0\,\text{cm}$ as the new image distance. Then using Eq. 33-2 for both cases, we eliminate the focal length and solve for the image distance. We insert the real image distance into the initial lens equation and solve for the focal length.

$$\frac{1}{d_{o1}} + \frac{1}{d_i} = \frac{1}{f} = \frac{1}{d_{o2}} + \frac{1}{d_i + 10.0\,\text{cm}} \rightarrow \frac{1}{d_{o1}} - \frac{1}{d_{o2}} = \frac{1}{d_i + 10.0\,\text{cm}} - \frac{1}{d_i} = \frac{-10.0\,\text{cm}}{d_i(d_i + 10.0\,\text{cm})}$$

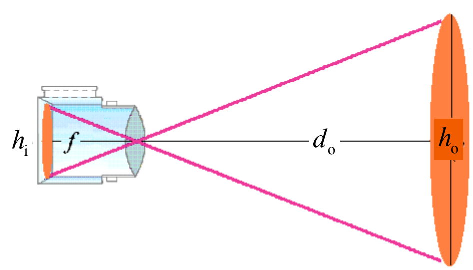
$$\frac{1}{60.0\,\text{cm}} - \frac{1}{40.0\,\text{cm}} = \frac{-10.0\,\text{cm}}{d_i(d_i + 10.0\,\text{cm})} \rightarrow d_i^2 + (10.0\,\text{cm})d_i - 1200\,\text{cm}^2 = 0$$

$$d_i = -40.0\,\text{cm} \text{ or } 30.0\,\text{cm}$$

Only the positive image distance will produce the real image.

$$\frac{1}{f} = \frac{1}{d_{o1}} + \frac{1}{d_i} \Rightarrow f = \frac{d_i d_{o1}}{d_i + d_{o1}} = \frac{(30.0\,\text{cm})(60.0\,\text{cm})}{30.0\,\text{cm} + 60.0\,\text{cm}} = \boxed{20.0\,\text{cm}}$$

94. Since the distance to the sun is much larger than the telescope's focal length, the image distance is about equal to the focal length. Rays from the top and bottom edges of the sun pass through the lens unrefracted. These rays with the object and image heights form similar triangles. We calculate the focal length of the telescope by setting the ratio of height to base for each triangle equal.




$$\frac{f}{h_i} = \frac{d_o}{h_o} \rightarrow$$

$$f = h_i \frac{d_o}{h_o} = (15\text{ mm})\frac{1.5\times10^8\text{ km}}{1.4\times10^6\text{ km}} = 1607\text{ mm} \approx \boxed{1.6\text{ m}}$$

95. We use Eq. 33-3 to write the image distance in terms of the object distance, image height, and object height. Then using Eq. 33-2 we solve for the object distance, which is the distance between the photographer and the subject.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \rightarrow \frac{1}{d_i} = -\frac{h_o}{h_i}\frac{1}{d_o}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \left(-\frac{h_o}{h_i}\frac{1}{d_o}\right) = \left(1 - \frac{h_o}{h_i}\right)\frac{1}{d_o} \rightarrow$$

$$d_o = \left(1 - \frac{h_o}{h_i}\right) f = \left(1 - \frac{1750\,\text{mm}}{-8.25\,\text{mm}}\right)(220\,\text{mm}) = 46{,}900\,\text{mm} \approx \boxed{47\,\text{m}}$$

96. The exposure is proportional to the intensity of light, the area of the shutter, and the time. The area of the shutter is proportional to the square of the diameter or inversely proportional to the square of the *f*-stop. Setting the two proportionalities equal, with constant time, we solve for the change in intensity.

$$\frac{I_1 t}{(f\text{-stop}_1)^2} = \frac{I_2 t}{(f\text{-stop}_2)^2} \quad \rightarrow \quad \frac{I_2}{I_1} = \left(\frac{f\text{-stop}_2}{f\text{-stop}_1}\right)^2 = \left(\frac{16}{5.6}\right)^2 = \boxed{8.2}$$

97. The maximum magnification is achieved with the image at the near point, using Eq. 33-6b.

$$M_1 = 1 + \frac{N_1}{f} = 1 + \frac{(15.0\,\text{cm})}{(8.5\,\text{cm})} = \boxed{2.8\times}$$

For an adult we set the near point equal to 25.0 cm.

$$M_2 = 1 + \frac{N_2}{f} = 1 + \frac{(25.0\,\text{cm})}{(8.5\,\text{cm})} = \boxed{3.9\times}$$

The person with the normal eye (adult) sees more detail.

98. The actual far point of the person is 155 cm. With the lens, an object far away is to produce a virtual image 155 cm from the eye, or 153 cm from the lens. We calculate the power of the upper part of the bifocals using Eq. 33-2 with the power equal to the inverse of the focal length in meter.

$$P_1 = \frac{1}{f_1} = \left(\frac{1}{d_{o1}}\right) + \left(\frac{1}{d_{i1}}\right) = \left(\frac{1}{\infty}\right) + \left(\frac{1}{-1.53\text{ m}}\right) = \boxed{-0.65\text{ D (upper part)}}$$

The actual near point of the person is 45 cm. With the lens, an object placed at the normal near point, 25 cm, or 23 cm from the lens, is to produce a virtual image 45 cm from the eye, or 43 cm from the lens. We again calculate the power using Eq. 33-2.

$$P_2 = \frac{1}{f_2} = \left(\frac{1}{d_{o2}}\right) + \left(\frac{1}{d_{i2}}\right) = \left(\frac{1}{0.23\text{ m}}\right) + \left(\frac{1}{-0.43\text{ m}}\right) = \boxed{+2.0\text{ D (lower part)}}$$

99. The magnification for a relaxed eye is given by Eq. 33-6a.

$$M = N/f = NP = (0.25\,\text{m})(+4.0\,\text{D}) = \boxed{1.0\times.}$$

100. (*a*) The magnification of the telescope is given by Eq. 33-7. The focal lengths are expressed in terms of their powers.

$$M = -\frac{f_o}{f_e} = -\frac{P_e}{P_o} = -\frac{(4.5\,\text{D})}{(2.0\,\text{D})} = -2.25\times \approx \boxed{-2.3\times}$$

(*b*) To get a magnification greater than 1, for the eyepiece we use the lens with the smaller focal length, or greater power: $\boxed{4.5\text{ D.}}$

101. We calculate the man's near point $(d_i)$ using Eq. 33-2, with the initial object at 0.32 m with a 2.5 D lens. To give him a normal near point, we set the final object distance as 0.25 m and calculate the power necessary to have the image at his actual near point.

$$P_1 = \frac{1}{d_i} + \frac{1}{d_{o1}} \rightarrow \frac{1}{d_i} = P_1 - \frac{1}{d_{o1}} \rightarrow d_i = \frac{d_{o1}}{P_1 d_{o1} - 1} = \frac{0.32\,\text{m}}{(2.5\,\text{D})(0.32\,\text{m}) - 1} = -1.6\,\text{m}$$

$$P_2 = \frac{1}{d_i} + \frac{1}{d_{o2}} = \left(P_1 - \frac{1}{d_{o1}}\right) + \frac{1}{d_{o2}} = \left(+2.5\text{ D} - \frac{1}{0.32\text{ m}}\right) + \frac{1}{0.25\text{ m}} = \boxed{+3.4\text{ D}}$$

102. (*a*) We solve Eq. 33-2 for the image distance. Then taking the time derivative of the image distance gives the image velocity. If the velocity of the object is taken to be positive, then the image distance is decreasing, and so $v_o = -\frac{d}{dt}(d_o)$.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \rightarrow d_i = \frac{fd_o}{d_o - f}$$

$$v_i = \frac{d}{dt}(d_i) = \frac{d}{dt}\left(\frac{fd_o}{d_o - f}\right) = \frac{f}{d_o - f}(-v_o) - \frac{fd_o}{(d_o - f)^2}(-v_o) = \frac{f(d_o - f) - fd_o}{(d_o - f)^2}(-v_o)$$

$$= \boxed{\frac{f^2 v_o}{(d_o - f)^2}}$$

(*b*) The velocity of the image is positive, which means the image is moving the same direction as the object. But since the image is on the opposite side of the lens as the object, the image must be moving away from the lens.

(*c*) We set the image and object velocities equal and solve for the image distance.

$$v_i = v_o \rightarrow \frac{f^2 v_o}{(d_o - f)^2} = v_o \rightarrow (d_o - f)^2 = f^2 \rightarrow d_o - f = f \rightarrow \boxed{d_o = 2f}$$

103. The focal length of the eyepiece is found using Eq. 33-1.

$$f_e = \frac{1}{P_e} = \frac{1}{23\text{ D}} = 4.3 \times 10^{-2}\text{ m} = 4.3\,\text{cm}.$$

For both object and image far away, we find the focal length of the objective from the separation of the lenses.

$$\ell = f_o + f_e \rightarrow f_o = \ell - f_e = 85\,\text{cm} - 4.3\,\text{cm} = 80.7\,\text{cm}$$

The magnification of the telescope is given by Eq. 33-7.

$$M = -\frac{f_o}{f_e} = -\frac{(80.7\,\text{cm})}{(4.3\,\text{cm})} = \boxed{-19\times}$$

104. (*a*) The length of the telescope is the sum of the focal lengths. The magnification is the ratio of the focal lengths (Eq. 33-7). For a magnification greater than one, the lens with the smaller focal length should be the eyepiece. Therefore the 4.0 cm lens should be the eyepiece.

$$\ell = f_o + f_e = 4.0\,\text{cm} + 44\,\text{cm} = \boxed{48\,\text{cm}}$$

$$M = -\frac{f_o}{f_e} = -\frac{(44\,\text{cm})}{(4.0\,\text{cm})} = \boxed{-11\times}$$

(*b*) We use Eq. 33-10b to solve for the length, $\ell$, of the microscope.

$$M = -\frac{N\ell}{f_e f_o} \Rightarrow \ell = \frac{-Mf_e f_o}{N} = \frac{-(25)(4.0\text{ cm})(44\text{ cm})}{25\text{ cm}} = 180\text{ cm} = \boxed{1.8\text{ m}}$$

This is far too long to be practical.

105. (*a*) The focal length of the lens is the inverse of the power.

$$f = \frac{1}{P} = \frac{1}{3.50\,\text{D}} = 0.286\,\text{m} = \boxed{28.6\,\text{cm.}}$$

(*b*) The lens produces a virtual image at his near point. We set the object distance at 23 cm from the glass (25 cm from the eyes) and solve for the image distance. We add the two centimeters between the glass and eyes to determine the near point.

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow d_i = \left(P - \frac{1}{d_o}\right)^{-1} = \left(3.50\,\text{D} - \frac{1}{0.23\,\text{m}}\right)^{-1} = -1.18\,\text{m}$$

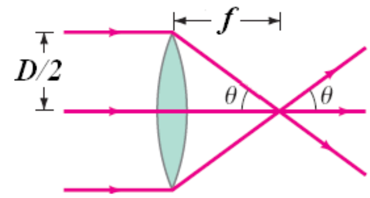
$$N = |d_i| + 0.02\,\text{m} = 1.18\,\text{m} + 0.02\,\text{m} = \boxed{1.20\,\text{m}}$$

(*c*) For Pam, find the object distance that has an image at her near point, –0.23 m from the lens.

$$P = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow d_o = \left(P - \frac{1}{d_i}\right)^{-1} = \left(3.50\ \text{D} - \frac{1}{-0.23\ \text{m}}\right)^{-1} = 0.13\,\text{m}$$

Pam's near point with the glasses is 13 cm from the glasses or $\boxed{15\text{ cm}}$ from her eyes.

106. As shown in the image, the parallel rays will pass through a single point located at the focal distance from the lens. The ray passing through the edge of the lens (a distance D/2 from the principal axis) makes an angle $\theta$ with the principal axis. We set the tangent of this angle equal to the ratio of the opposite side (*D*/2) to the adjacent side (*f*) and solve for the focal length.




$$\tan\theta = \frac{D/2}{f} \rightarrow f = \frac{D}{2\tan\theta} = \frac{5.0\text{ cm}}{2\tan 3.5^\circ} = \boxed{41\text{ cm}}$$

107. We use Eq. 33-6b to calculate the necessary focal length for a magnifying glass held at the near point ($N = 25\text{ cm}$) to have a magnification of $M = 3.0$.

$$M = \frac{N}{f} + 1 \rightarrow f = \frac{N}{M-1} = \frac{25\text{ cm}}{3.0-1} = 12.5\text{ cm}$$

In the text, the lensmaker's equation (Eq. 33-4) is derived assuming the lens is composed of material with index of refraction $n$ and is surrounded by air, whose index of refraction is $n_a = 1$. We now modify this derivation, with the lens composed of air with index of refraction $n_a = 1$ surrounded by water, whose index of refraction is $n_w = 1.33$. In the proof of the lensmaker's equation, Snell's law at small angles is first applied at both surfaces of the lens.

$$n_w \sin\theta_1 = n\sin\theta_2 \rightarrow n_w\theta_1 \approx \theta_2 \rightarrow \theta_1 \approx \frac{1}{n_w}\theta_2$$

$$n\sin\theta_3 = n_w\sin\theta_4 \rightarrow \theta_3 \approx n_w\theta_4 \rightarrow \frac{1}{n_w}\theta_3 \approx \theta_4$$

These equations are the same as those following Fig. 33-16, but with *n* replaced by $1/n_w$. The rest of the derivation is the same, so we can rewrite the lensmaker's equation with this single modification. We assume the radii are equal, insert the necessary focal length, and solve for the radius of curvature

$$\frac{1}{f} = \left(\frac{1}{n_w} - 1\right)\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \left(\frac{1}{n_w} - 1\right)\left[\frac{1}{R} + \frac{1}{R}\right] = \left(\frac{1}{n_w} - 1\right)\left[\frac{2}{R}\right]$$

$$R = 2f\left(\frac{1}{n_w} - 1\right) = 2(12.5\text{ cm})\left(\frac{1}{1.33} - 1\right) = -6.20\,\text{cm} \approx \boxed{-6.2\,\text{cm}}$$

The lens is therefore a $\boxed{\text{concave lens}}$ with radii of curvature –6.2 cm.

108. (*a*) We use Eq. 32-2 to calculate the image distance and then use the object and image distances in Eq. 32-3 to calculate the magnification. We finally make the approximation that the object distance is much larger than the focal length.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f_1} \rightarrow d_i=\left(\frac{1}{f_1}-\frac{1}{d_o}\right)^{-1}=\frac{f_1 d_o}{d_o-f_1}$$

$$m_1=-\frac{d_i}{d_o}=-\frac{1}{d_o}\left(\frac{f_1 d_o}{d_o-f_1}\right)=-\frac{f_1}{d_o-f_1}\approx \boxed{-\frac{f_1}{d_o}}$$

This real image, located near the focal distance from lens 1, becomes the object for the second lens. We subtract the focal length from the separation distance to determine the object distance for lens 2. Using Eq. 32-2, we calculate the second image distance and Eq. 32-3 to calculate the second magnification. Multiplying the two magnifications gives the total magnification.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{f_2 d_{o2}}{d_{o2}-f_2}$$

$$m_2=-\frac{d_{i2}}{d_{o2}}=-\frac{1}{d_{o2}}\left(\frac{f_2 d_{o2}}{d_{o2}-f_2}\right)=-\frac{f_2}{d_{o2}-f_2}=-\frac{\left(-\frac{1}{2}f_1\right)}{\left(\frac{3}{4}f_1-f_1\right)-\left(-\frac{1}{2}f_1\right)}=2$$

$$m_1 m_2=\left(-\frac{f_1}{d_o}\right)(2)=\boxed{-\frac{2f_1}{d_o}}$$

(*b*) If the object is at infinity, the image from the first lens will form a focal length behind that lens. Subtracting this distance from the separation distance gives the object distance for the second lens. We use Eq. 32-2 to calculate the image distance from the second lens. Adding this distance to the separation distance between the lenses gives the distance the image is from the first lens.

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f_2} \rightarrow d_{i2}=\frac{f_2 d_{o2}}{d_{o2}-f_2}=\frac{\left(-\frac{1}{2}f_1\right)\left(\frac{3}{4}f_1-f_1\right)}{\left(\frac{3}{4}f_1-f_1\right)-\left(-\frac{1}{2}f_1\right)}=\frac{1}{2}f_1$$

$$d=\ell+d_{i2}=\frac{3}{4}f_1+\frac{1}{2}f_1=\boxed{\frac{5}{4}f_1}$$

(*c*) We set the magnification equal to the total magnification found in part (*a*) and solve for the focal length.

$$m=-\frac{250\,\text{mm}}{d_o}=-\frac{2f_1}{d_o}\Rightarrow f_1=\frac{250\,\text{mm}}{2}=\boxed{125\,\text{mm}}$$

We use the results of part (*b*) to determine the distance of the lens to the film. We subtract this distance from 250 mm to determine how much closer the lens can be to the film in the two lens system.

$$d=\frac{5}{4}f_1=\frac{5}{4}(125\,\text{mm})=\boxed{156\,\text{mm}}\ ;\ \Delta d=250\,\text{mm}-156\,\text{mm}=\boxed{94\,\text{mm}}$$

109. (*a*) We use Eqs. 33-2 and 33-3.

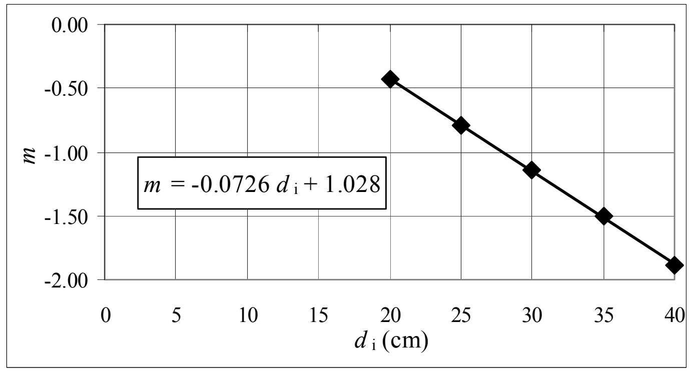
$$m=-\frac{d_i}{d_o} \rightarrow d_o=-\frac{d_i}{m}\ ;\ \frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}=-\frac{m}{d_i}+\frac{1}{d_i} \rightarrow m=-\frac{d_i}{f}+1$$

This is a straight line with $\boxed{\text{slope}=-\frac{1}{f}\ \text{and}\ y\text{-intercept}=1}$.

(*b*) A plot of $m$ vs. $d_i$ is shown here.

$$f = -\frac{1}{\text{slope}} = -\frac{1}{-.0726\,\text{cm}^{-1}} = 13.8\,\text{cm} \approx \boxed{14\,\text{cm}}$$

The $y$-intercept is 1.028. $\boxed{\text{Yes}}$, it is close to the expected value of 1. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH33.XLS," on tab "Problem 33.109b."




(*c*) Use the relationship derived above.

$$m = -\frac{d_i}{f} + 1 = -\frac{d' + \ell_i}{f} + 1 = -\frac{d'_i}{f} + \left(1 - \frac{\ell_i}{f}\right)$$

A plot of $m$ vs. $d'_i$ would still have a slope of $-\frac{1}{f}$, so $\boxed{f = -\frac{1}{\text{slope}}}$ as before. The $y$-intercept will have changed, to $1 - \frac{\ell_i}{f}$.

# CHAPTER 34: The Wave Nature of Light; Interference

## Responses to Questions

1. Yes, Huygens' principle applies to all waves, including sound and water waves.

2. Light from the Sun can be focused by a converging lens on a piece of paper and burn a hole in the paper. This provides evidence that light is energy. Also, you can feel the heat from the Sun beating down on you on a hot summer day. When you move into the shade you may still feel hot, but you don't feel the Sun's energy directly.

3. A ray shows the direction of propagation of a wave front. If this information is enough for the situation under discussion, then light can be discussed as rays. Sometimes, however, the wave nature of light is essential to the discussion. For instance, the double slit interference pattern depends on the interference of the waves, and could not be explained by examining light as only rays.

4. The bending of waves around corners or obstacles is called diffraction. Diffraction is most prominent when the size of the obstacle is on the order of the size of the wavelength. Sound waves have much longer wavelengths than do light waves. As a result, the diffraction of sound waves around a corner is noticeable and we can hear the sound in the "shadow region," but the diffraction of light waves around a corner is not noticeable.

5. The wavelength of light cannot be determined from reflection measurements alone, because the law of reflection is the same for all wavelengths. However, thin film interference, which involves interference of the rays reflecting from the front and back surfaces of the film, can be used to determine wavelength. Refraction can also be used to determine wavelength because the index of refraction for a given medium is different for different wavelengths.

6. For destructive interference, the path lengths must differ by an odd number of half wavelengths, such as $\lambda/2$, $3\lambda/2$, $5\lambda/2$, $7\lambda/2$, etc. In general, the path lengths must differ by $\lambda(m + ½)$, where $m$ is an integer.

7. Blue light has a shorter wavelength than red light. The angles to each of the bright fringes for the blue light would be smaller than for the corresponding orders for the red light, so the bright fringes would be closer together for the blue light.

8. The fringes would be closer together because the wavelength of the light underwater is less than the wavelength in air.

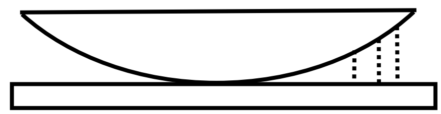
9. The two experiments are the same in principle. Each requires coherent sources and works best with a single frequency source. Each produces a pattern of alternating high and low intensity. Sound waves have much longer wavelengths than light waves, so the appropriate source separation for the sound experiment would be larger. Also, sound waves are mechanical waves which require a medium through which to travel, so the sound experiment could not be done in a vacuum and the light experiment could.

10. The red light and the blue light coming from the two different slits will have different wavelengths (and different frequencies) and will not have a constant phase relationship. In order for a double-slit pattern to be produced, the light coming from the slits must be coherent. No distinct double-slit interference pattern will appear. However, each slit will individually produce a "single-slit diffraction" pattern, as will be discussed in Chapter 35.

11. Light from the two headlights would not be coherent, so would not maintain a consistent phase relationship and therefore no stable interference pattern would be produced.

12. As the thickness of the film increases, the number of different wavelengths in the visible range that meet the constructive interference criteria increases. For a thick piece of glass, many different wavelengths will undergo constructive interference and these will all combine to produce white light.

13. Bright colored rings will occur when the path difference between the two interfering rays is $\lambda/2$, $3\lambda/2$, $5\lambda/2$, and so forth. A given ring, therefore, has a path difference that is exactly one wavelength longer than the path difference of its neighboring ring to the inside and one wavelength shorter than the path difference of its neighboring ring to the outside. Newton's rings are created by the thin film of air between a glass lens and the flat glass surface on which it is placed. Because the glass of the lens is curved, the thickness of this air film does not increase linearly. The farther a point is from the center, the less the horizontal distance that corresponds to an increase in vertical thickness of one wavelength. The horizontal distance between two neighboring rings therefore decreases with increasing distance from the center.

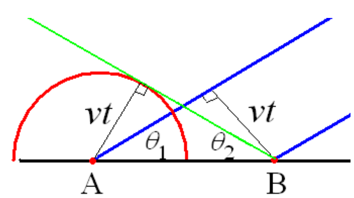


14. These lenses probably are designed to eliminate wavelengths at both the red and the blue ends of the spectrum. The thickness of the coating is designed to cause destructive interference for reflected red and blue light. The reflected light then appears yellow-green.

15. The index of refraction of the oil must be less than the index of refraction of the water. If the oil film appears bright at the edge, then the interference between the light reflected from the top of the oil film and from the bottom of the oil film at that point must be constructive. The light reflecting from the top surface (the air/oil interface) undergoes a 180º phase shift since the index of refraction of the oil is greater than that of air. The thickness of the oil film at the edge is negligible, so for there to be constructive interference, the light reflecting from the bottom of the oil film (the oil/water interface) must also undergo a 180º phase shift. This will occur only if the index of refraction of the oil is less than that of the water.

## Solutions to Problems

1. Consider a wave front traveling at an angle $\theta_1$ relative to a surface. At time $t = 0$, the wave front touches the surface at point A, as shown in the figure. After a time *t*, the wave front, moving at speed *v*, has moved forward such that the contact position has moved to point B. The distance between the two contact points is calculated using simple geometry: $\mathrm{AB} = \dfrac{vt}{\sin\theta_1}$.




By Huygens' principle, at each point the wave front touches the surface, it creates a new wavelet. These wavelets expand out in all directions at speed *v*. The line passing through the surface of each of these wavelets is the reflected wave front. Using the radius of the wavelet created at $t = 0$, the center of the wavelet created at time *t*, and the distance between the two contact points (AB) we create a right triangle. Dividing the radius of the wavelet centered at AB (*vt*) by distance between the contact points gives the sine of the angle between the contact surface and the reflected wave, $\theta_2$.

$$\sin\theta_2 = \frac{vt}{AB} = \frac{vt}{\dfrac{vt}{\sin\theta_1}} = \sin\theta_1 \rightarrow \boxed{\theta_2 = \theta_1}$$

Since these two angles are equal, their complementary angles (the incident and reflected angles) are also equal.

2. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. Apply this to the fifth order.

$$d\sin\theta = m\lambda \rightarrow \lambda = \frac{d\sin\theta}{m} = \frac{(1.8\times10^{-5}\,\text{m})\sin 9.8^\circ}{5} = \boxed{6.1\times10^{-7}\,\text{m}}$$

3. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. Apply this to the third order.

$$d\sin\theta = m\lambda \rightarrow d = \frac{m\lambda}{\sin\theta} = \frac{3(610\times10^{-9}\,\text{m})}{\sin 28^\circ} = \boxed{3.9\times10^{-6}\,\text{m}}$$

4. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Adjacent fringes will have $\Delta m = 1$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d}$$

$$x_1 = \frac{\lambda m_1\ell}{d} \;;\; x_2 = \frac{\lambda(m+1)\ell}{d} \rightarrow \Delta x = x_2 - x_1 = \frac{\lambda(m+1)\ell}{d} - \frac{\lambda m\ell}{d} = \frac{\lambda\ell}{d}$$

$$\lambda = \frac{d\Delta x}{\ell} = \frac{(4.8\times10^{-5}\,\text{m})(0.085\,\text{m})}{6.00\,\text{m}} = \boxed{6.8\times10^{-7}\,\text{m}} \;;\; f = \frac{c}{\lambda} = \frac{3.00\times10^8\,\text{m/s}}{6.8\times10^{-7}\,\text{m}} = \boxed{4.4\times10^{14}\,\text{Hz}}$$

5. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Second order means $m = 2$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d} \;;\; x_1 = \frac{\lambda_1 m\ell}{d} \;;\; x_2 = \frac{\lambda_2 m\ell}{d} \rightarrow$$

$$\Delta x = x_2 - x_1 = \frac{(\lambda_2-\lambda_1)m\ell}{d} = \frac{\left[(720-660)\times10^{-9}\,\text{m}\right](2)(1.0\,\text{m})}{(6.8\times10^{-4}\,\text{m})} = 1.76\times10^{-4}\,\text{m} \approx \boxed{0.2\,\text{mm}}$$

This justifies using the small angle approximation, since $x \ll \ell$.

6. The slit spacing and the distance from the slits to the screen is the same in both cases. The distance between bright fringes can be taken as the position of the first bright fringe (m = 1) relative to the central fringe. We indicate the lab laser with subscript 1, and the laser pointer with subscript 2. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d} \;;\; x_1 = \frac{\lambda_1\ell}{d} \;;\; x_2 = \frac{\lambda_2\ell}{d} \rightarrow$$

$$\lambda_2 = \frac{d}{\ell}x_2 = \frac{\lambda_1}{x_1}x_2 = (632.8\,\text{nm})\frac{5.14\,\text{mm}}{5.00\,\text{mm}} = 650.52\,\text{nm} \approx \boxed{651\,\text{nm}}$$

7. Using a ruler on Fig. 35-9a, the distance from the $m = 0$ fringe to the $m = 10$ fringe is found to be about 13.5 mm. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow \lambda = \frac{dx}{m\ell} = \frac{dx}{m\ell} = \frac{(1.7\times10^{-4}\text{m})(0.0135\text{m})}{(10)(0.35\text{m})} = \boxed{6.6\times10^{-7}\text{m}}$$

8. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow d = \frac{\lambda m\ell}{x} = \frac{(680\times10^{-9}\text{m})(3)(2.6\text{m})}{38\times10^{-3}\text{m}} = \boxed{1.4\times10^{-4}\text{m}}$$

9. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d} \rightarrow$$

$$\Delta x = \Delta m\frac{\lambda\ell}{d} = (1)\frac{(633\times10^{-9}\text{m})(3.8\text{m})}{(6.8\times10^{-5}\text{m})} = 0.035\text{m} = \boxed{3.5\text{cm}}$$

10. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow d = \frac{\lambda m\ell}{x} = \frac{(633\times10^{-9}\text{m})(1)(5.0\text{m})}{(0.25\text{m})} = \boxed{1.3\times10^{-5}\text{m}}$$

11. The 180° phase shift produced by the glass is equivalent to a path length of $\frac{1}{2}\lambda$. For constructive interference on the screen, the total path difference is a multiple of the wavelength:

$$\tfrac{1}{2}\lambda + d\sin\theta_{\text{max}} = m\lambda,\ m = 0, 1, 2, \cdots \rightarrow d\sin\theta_{\text{max}} = \left(m - \tfrac{1}{2}\right)\lambda,\ m = 1, 2, \cdots$$

We could express the result as $d\sin\theta_{\text{max}} = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, \cdots$.

For destructive interference on the screen, the total path difference is

$$\tfrac{1}{2}\lambda + d\sin\theta_{\text{min}} = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, \cdots \rightarrow d\sin\theta_{\text{min}} = m\lambda,\ m = 0, 1, 2, \cdots$$

Thus the pattern is just the $\boxed{\text{reverse of the usual double-slit pattern}}$. There will be a dark central line. Every place there was a bright fringe will now have a dark line, and vice versa.

12. We equate the expression from Eq. 34-2a for the second order blue light to Eq. 34-2b, since the slit separation and angle must be the same for the two conditions to be met at the same location.

$$d\sin\theta = m\lambda_b = (2)(480\text{nm}) = 960\text{nm} \ ;\ d\sin\theta = \left(m' + \tfrac{1}{2}\right)\lambda,\ m' = 0, 1, 2, \cdots$$

$$\left(m' + \tfrac{1}{2}\right)\lambda = 960\text{nm} \quad m' = 0 \rightarrow \lambda = 1920\text{nm} \ ;\ m' = 1 \rightarrow \lambda = 640\text{nm}$$

$$m' = 2 \rightarrow \lambda = 384\text{nm}$$

The only one visible is $\boxed{640\text{ nm}}$. 384 nm is near the low-wavelength limit for visible light.

13. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow x = \frac{\lambda m\ell}{d} \rightarrow$$

$$\Delta x = \Delta m\frac{\lambda\ell}{d} = (1)\frac{(544\times10^{-9}\,\text{m})(5.0\,\text{m})}{(1.0\times10^{-3}\,\text{m})} = \boxed{2.7\times10^{-3}\,\text{m}}$$

14. An expression is derived for the slit separation from the data for the 500 nm light. That expression is then used to find the location of the maxima for the 650 nm light. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{\ell} = m\lambda \rightarrow d = \frac{\lambda m\ell}{x} = \frac{\lambda_1 m_1\ell}{x_1} \rightarrow x = \frac{\lambda m\ell}{d} \rightarrow$$

$$x_2 = \frac{\lambda_2 m_2\ell}{\frac{\lambda_1 m_1\ell}{x_1}} = x_1\frac{\lambda_2 m_2}{\lambda_1 m_1} = (12\,\text{mm})\frac{(650\,\text{nm})(2)}{(500\,\text{nm})(3)} = 10.4\,\text{mm} \approx \boxed{10\,\text{mm}}\ (2\text{ sig. fig.})$$

15. The presence of the water changes the wavelength according to Eq. 34-1, and so we must change $\lambda$ to $\lambda_n = \lambda/n$. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Adjacent fringes will have $\Delta m = 1$.

$$d\sin\theta = m\lambda_n \rightarrow d\frac{x}{\ell} = m\lambda_n \rightarrow x = \frac{\lambda_n m\ell}{d}\ ;\ x_1 = \frac{\lambda m_1\ell}{d}\ ;\ x_2 = \frac{\lambda(m+1)\ell}{d} \rightarrow$$

$$\Delta x = x_2 - x_1 = \frac{\lambda_n(m+1)\ell}{d} - \frac{\lambda_n m\ell}{d} = \frac{\lambda_n\ell}{d} = \frac{\lambda\ell}{nd} = \frac{(470\times10^{-9}\,\text{m})(0.500\,\text{m})}{(1.33)(6.00\times10^{-5}\,\text{m})} = \boxed{2.94\times10^{-3}\,\text{m}}$$

16. To change the center point from constructive interference to destructive interference, the phase shift produced by the introduction of the plastic must be equivalent to half a wavelength. The wavelength of the light is shorter in the plastic than in the air, so the number of wavelengths in the plastic must be ½ greater than the number in the same thickness of air. The number of wavelengths in the distance equal to the thickness of the plate is the thickness of the plate divided by the appropriate wavelength.

$$N_{\text{plastic}} - N_{\text{air}} = \frac{t}{\lambda_{\text{plastic}}} - \frac{t}{\lambda} = \frac{tn_{\text{plastic}}}{\lambda} - \frac{t}{\lambda} = \frac{t}{\lambda}(n_{\text{plastic}} - 1) = \tfrac{1}{2} \rightarrow$$

$$t = \frac{\lambda}{2(n_{\text{plastic}} - 1)} = \frac{680\,\text{nm}}{2(1.60 - 1)} = \boxed{570\,\text{nm}}$$

17. The intensity is proportional to the square of the amplitude. Let the amplitude at the center due to one slit be $E_0$. The amplitude at the center with both slits uncovered is $2E_0$.

$$\frac{I_{1\text{ slit}}}{I_{2\text{ slits}}} = \left(\frac{E_0}{2E_0}\right)^2 = \boxed{\frac{1}{4}}$$

Thus the amplitude due to a single slit is one-fourth the amplitude when both slits are open.

18. The intensity as a function of angle from the central maximum is given by Eq. 34-6.

$$I_\theta = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) = \tfrac{1}{2} I_0 \;\rightarrow\; \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) = \tfrac{1}{2} \;\rightarrow\; \cos\left(\frac{\pi d \sin\theta}{\lambda}\right) = \pm\frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi d \sin\theta}{\lambda}\right) = \pm\frac{1}{\sqrt{2}} \;\rightarrow\; \frac{\pi d \sin\theta}{\lambda} = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = 45^\circ \pm n(90^\circ) = \frac{\pi}{4} \pm n\frac{\pi}{2} \;\rightarrow$$

$$2d\sin\theta = \left(\tfrac{1}{2} \pm n\right)\lambda$$

To only consider $\theta \geq 0$, we take just the plus sign.

$$\boxed{2d\sin\theta = \left(n + \tfrac{1}{2}\right)\lambda,\; n = 0, 1, 2, \cdots}$$

19. The intensity of the pattern is given by Eq. 34-6. We find the angle where the intensity is half its maximum value.

$$I_\theta = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) = \tfrac{1}{2} I_0 \;\rightarrow\; \cos^2\left(\frac{\pi d \sin\theta_{1/2}}{\lambda}\right) = \tfrac{1}{2} \;\rightarrow\; \cos\left(\frac{\pi d \sin\theta_{1/2}}{\lambda}\right) = \frac{1}{\sqrt{2}} \;\rightarrow$$

$$\frac{\pi d \sin\theta_{1/2}}{\lambda} = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4} \;\rightarrow\; \sin\theta_{1/2} = \frac{\lambda}{4d}$$

If $\lambda \ll d$, then $\sin\theta = \dfrac{\lambda}{4d} \ll 1$ and so $\sin\theta \approx \theta$. This is the angle from the central maximum to the location of half intensity. The angular displacement from the half-intensity position on one side of the central maximum to the half-intensity position on the other side would be twice this.

$$\Delta\theta = 2\theta_{1/2} = 2\frac{\lambda}{4d} = \boxed{\frac{\lambda}{2d}}$$

20. (*a*) The phase difference is given in Eq. 34-4. We are given the path length difference, $d\sin\theta$.

$$\frac{\delta}{2\pi} = \frac{d\sin\theta}{\lambda} \;\rightarrow\; \delta = 2\pi\frac{1.25\lambda}{\lambda} = \boxed{2.50\pi}$$

(*b*) The intensity is given by Eq. 34-6.

$$I = I_0\cos^2\left(\frac{\delta}{2}\right) = I_0\cos^2(1.25\pi) = \boxed{0.500 I_0}$$

21. A doubling of the intensity means that the electric field amplitude has increased by a factor of $\sqrt{2}$. We set the amplitude of the electric field of one slit equal to $E_0$ and of the other equal to $\sqrt{2}E_0$. We use Eq. 34-3 to write each of the electric fields, where the phase difference, $\delta$, is given by Eq. 34-4. Summing these two electric fields gives the total electric field.

$$E_\theta = E_0\sin\omega t + \sqrt{2}E_0\sin(\omega t + \delta) = E_0\sin\omega t + \sqrt{2}E_0\sin\omega t\cos\delta + \sqrt{2}E_0\cos\omega t\sin\delta$$

$$= E_0\left(1 + \sqrt{2}\cos\delta\right)\sin\omega t + \sqrt{2}E_0\cos\omega t\sin\delta$$

We square the total electric field intensity and integrate over the period to determine the average intensity.

$$\overline{E}_\theta^2 = \frac{1}{T}\int_0^T E_\theta^2 dt = \frac{1}{T}\int_0^T\left[E_0\left(1+\sqrt{2}\cos\delta\right)\sin\omega t + \sqrt{2}E_0\cos\omega t\sin\delta\right]^2 dt$$

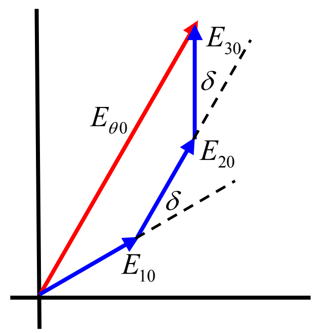
$$= \frac{E_0^2}{T}\int_0^T\left[\left(1+\sqrt{2}\cos\delta\right)^2\sin^2\omega t + 2\cos^2\omega t\sin^2\delta + 2\sqrt{2}\left(1+\sqrt{2}\cos\delta\right)\sin\delta\sin\omega t\cos\omega t\right]dt$$

$$=\frac{E_0^2}{2}\left[\left(1+\sqrt{2}\cos\delta\right)^2+2\sin^2\delta\right]=\frac{E_0^2}{2}\left[3+2\sqrt{2}\cos\delta\right]$$

Since the intensity is proportional to this average square of the electric field, and the intensity is maximum when $\delta=0,$ we obtain the relative intensity by dividing the square of the electric field by the maximum square of the electric field.

$$\frac{I_\theta}{I_0}=\frac{\bar{E}_\theta^2}{E_{\delta=0}^2}=\boxed{\frac{3+2\sqrt{2}\cos\delta}{3+2\sqrt{2}},\text{ with }\delta=\frac{2\pi}{\lambda}d\sin\theta}$$

22\. (*a*) If the sources have equal intensities, their electric fields will have the same magnitudes. We show a phasor diagram with each of the electric fields shifted by an angle $\delta$. As shown in the sketch, the three electric fields and their sum form a symmetric trapezoid. Since $E_{20}$ and $E_{\theta 0}$ are parallel, and $E_{20}$ is rotated from $E_{10}$ and $E_{30}$ by the angle $\delta,$ the magnitude of $E_{\theta 0}$ is the sum of the components of $E_{10},$ $E_{20},$ and $E_{30}$ that are parallel to $E_{20}$.




$$E_{\theta 0}=E_{10}\cos\delta+E_{20}+E_{30}\cos\delta=E_{10}\left(1+2\cos\delta\right)$$

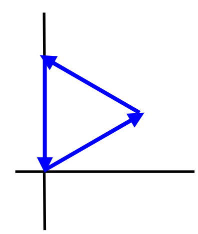
We set the intensity proportional to the square of the electric field magnitude and divide by the maximum intensity (at $\delta=0$) to determine the relative intensity.

$$\frac{I_\theta}{I_0}=\frac{E_{\theta 0}^2}{E_{\delta=0}^2}=\frac{\left[E_{10}\left(1+2\cos\delta\right)\right]^2}{\left[E_{10}\left(1+2\cos 0\right)\right]^2}=\boxed{\frac{\left(1+2\cos\delta\right)^2}{9},\ \delta=\frac{2\pi}{\lambda}d\sin\theta}$$

(*b*) The intensity will be at its maximum when $\cos\delta=1$. In this case the three phasors are all in line.

$$\cos\delta_{\max}=1\ \rightarrow\ \delta_{\max}=2m\pi=\frac{2\pi}{\lambda}d\sin\theta_{\max}\ \rightarrow\ \boxed{\sin\theta_{\max}=\frac{m\lambda}{d},\quad m=0,1,2,\cdots}$$

The intensity will be a minimum when $1+2\cos\delta=0$. In this case the three phasors add to 0 and form an equilateral triangle as shown in the second diagram, for the case of $k$ = 1, where $k$ is defined below.



$$1+2\cos\delta_{\min}=0\ \rightarrow$$

$$\delta_{\min}=\cos^{-1}\left(-\tfrac{1}{2}\right)=\begin{cases}\tfrac{2}{3}\pi+2m\pi=2\pi\left(m+\tfrac{1}{3}\right)\\ \tfrac{4}{3}\pi+2m\pi=2\pi\left(m+\tfrac{2}{3}\right)\end{cases},\ m=0,1,2,\cdots$$

This can be written as one expression with two parameters.

$$\delta_{\min}=2\pi\left(m+\tfrac{1}{3}k\right)=\frac{2\pi}{\lambda}d\sin\theta_{\min},\ k=1,2;\ m=0,1,2,\cdots\ \rightarrow$$

$$\boxed{\sin\theta_{\min}=\frac{\lambda}{d}\left(m+\tfrac{1}{3}k\right)=,\ k=1,2;\ m=0,1,2,\cdots}$$

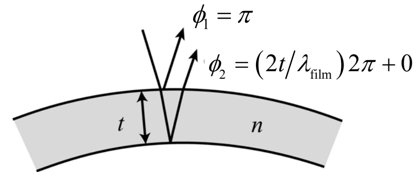
23\. From Example 34-7, we see that the thickness is related to the bright color wavelength by $t=\lambda/4n$.

$$t=\lambda/4n\ \rightarrow\ \lambda=4nt=4(1.32)(120\,\text{nm})=\boxed{634\,\text{nm}}$$

24\. Between the 25 dark lines there are 24 intervals. When we add the half-interval at the wire end, we have 24.5 intervals over the length of the plates.

$$\frac{28.5\,\text{cm}}{24.5\,\text{intervals}}=\boxed{1.16\,\text{cm}}$$

25. (*a*) An incident wave that reflects from the outer surface of the bubble has a phase change of $\phi_1 = \pi$. An incident wave that reflects from the inner surface of the bubble has a phase change due to the additional path length, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi$. For destructive interference with a minimum non-zero thickness of bubble, the net phase change must be $\pi$.




$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = \pi \quad \rightarrow \quad t = \tfrac{1}{2}\lambda_{\text{film}} = \frac{\lambda}{2n} = \frac{480\,\text{nm}}{2(1.33)} = \boxed{180\,\text{nm}}$$

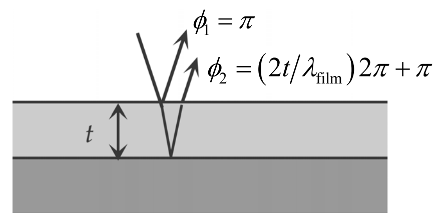
(*b*) For the next two larger thicknesses, the net phase change would be $3\pi$ and $5\pi$.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = 3\pi \quad \rightarrow \quad t = \lambda_{\text{film}} = \frac{\lambda}{n} = \frac{480\,\text{nm}}{(1.33)} = \boxed{361\,\text{nm}}$$

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi\right] - \pi = 5\pi \quad \rightarrow \quad t = \lambda_{\text{film}} = \frac{\lambda}{n} = \tfrac{3}{2}\frac{480\,\text{nm}}{(1.33)} = \boxed{541\,\text{nm}}$$

(*c*) If the thickness were much less than one wavelength, then there would be very little phase change introduced by additional path length, and so the two reflected waves would have a phase difference of about $\phi_1 = \pi$. This would produce destructive interference.

26. An incident wave that reflects from the top surface of the coating has a phase change of $\phi_1 = \pi$. An incident wave that reflects from the glass $(n \approx 1.5)$ at the bottom surface of the coating has a phase change due to both the additional path length and a phase change of $\pi$ on reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For constructive interference with a minimum non-zero thickness of coating, the net phase change must be $2\pi$.




$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = 2\pi \quad \rightarrow \quad t = \tfrac{1}{2}\lambda_{\text{film}} = \tfrac{1}{2}\left(\frac{\lambda}{n_{\text{film}}}\right).$$

The lens reflects the most for $\lambda = 570\,\text{nm}$. The minimum non-zero thickness occurs for $m = 1$:

$$t_{\min} = \frac{\lambda}{2n_{\text{film}}} = \frac{(570\,\text{nm})}{2(1.25)} = \boxed{228\,\text{nm}}$$

Since the middle of the spectrum is being selectively reflected, the transmitted light will be stronger in the red and blue portions of the visible spectrum.

27. (*a*) When illuminated from above at A, a light ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray reflected at the oil-water interface undergoes no phase shift. If the oil thickness at A is negligible compared to the wavelength of the light, then there is no significant shift in phase due to a path distance traveled by a ray in the oil. Thus the light reflected from the two surfaces will destructively interfere for all visible wavelengths, and the oil will appear black when viewed from above.

(*b*) From the discussion in part (*a*), the ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray that reflects from the oil-water interface has no phase change due to reflection, but has a phase change due to the additional path length of $\phi_2 = \left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi$. For constructive interference, the net phase change must be a multiple of $2\pi$.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi\right] - \pi = m(2\pi) \rightarrow t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{oil}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}}$$

From the diagram, we see that point B is the second thickness that yields constructive interference for 580 nm, and so we use $m = 1$. (The first location that yields constructive interference would be for $m = 0$.)

$$t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}} = \tfrac{1}{2}\left(1+\tfrac{1}{2}\right)\frac{580\,\text{nm}}{1.50} = \boxed{290\,\text{nm}}$$

28. When illuminated from above, the light ray reflected from the air-oil interface undergoes a phase shift of $\phi_1 = \pi$. A ray reflected at the oil-water interface undergoes no phase shift due to reflection, but has a phase change due to the additional path length of $\phi_2 = \left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi$. For constructive interference to occur, the net phase change must be a multiple of $2\pi$.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{oil}}}\right)2\pi\right] - \pi = m(2\pi) \rightarrow t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{oil}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{o}}}$$

For $\lambda = 650\,\text{nm}$, the possible thicknesses are as follows.

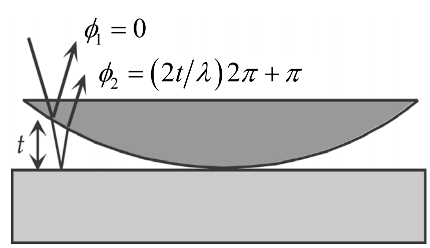
$$t_{650} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{650\,\text{nm}}{1.50} = 108\,\text{nm},\ 325\,\text{nm},\ 542\,\text{nm},\ \cdots$$

For $\lambda = 390\,\text{nm}$, the possible thicknesses are as follows.

$$t_{390} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{390\,\text{nm}}{1.50} = 65\,\text{nm},\ 195\,\text{nm},\ 325\,\text{nm},\ 455\,\text{nm},\ \cdots$$

The minimum thickness of the oil slick must be $\boxed{325\,\text{nm}}$.

29. An incident wave that reflects from the convex surface of the lens has no phase change, so $\phi_1 = 0$. An incident wave that reflects from the glass underneath the lens has a phase change due to both the additional path length and a phase change of $\pi$ on reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For destructive interference (dark rings), the net phase change must be an odd-integer multiple of $\pi$, so $\phi_{\text{net}} = \phi_2 - \phi_1 = (2m+1)\pi,\ m = 0, 1, 2, \cdots$. Because $m = 0$ corresponds to the dark center, $m$ represents the number of the ring.

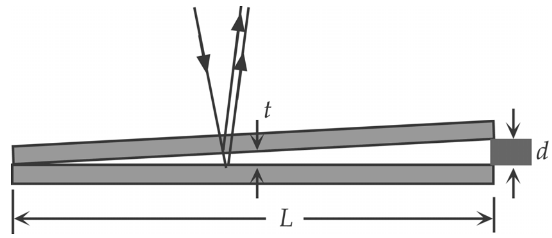



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi,\ m = 0, 1, 2, \cdots \rightarrow$$

$$t = \tfrac{1}{2}m\lambda_{\text{air}} = \tfrac{1}{2}(31)(560\,\text{nm}) = 8680\,\text{nm} = \boxed{8.68\,\mu\text{m}}$$

The thickness of the lens is the thickness of the air at the edge of the lens:

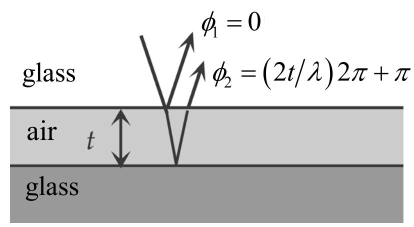
30. An incident wave that reflects from the second surface of the upper piece of glass has no phase change, so $\phi_1 = 0$. An incident wave that reflects from the first surface of the second piece of glass has a phase change due to both the additional path length and a phase change of $\pi$ on reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For destructive interference (dark lines), the net phase change must be an odd-integer multiple of $\pi$, so $\phi_{net} = \phi_2 - \phi_1 = (2m+1)\pi,\ m = 0, 1, 2, \cdots$. Because $m = 0$ corresponds to the left edge of the diagram, the 28th dark line corresponds to $m$ = 27. The 28th dark line also has a gap thickness of $d$.




$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi \ \rightarrow\ t = \tfrac{1}{2}m\lambda \ \rightarrow$$

$$d = \tfrac{1}{2}(27)(670\,\text{nm}) = 9045\,\text{nm} \approx \boxed{9.0\,\mu\text{m}}$$

$\boxed{31.}$ With respect to the incident wave, the wave that reflects from the air at the top surface of the air layer has a phase change of $\phi_1 = 0$. With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the air layer has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\frac{2t}{\lambda}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even non-zero integer multiple of $\pi$.




$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = 2m\pi \ \rightarrow\ t = \tfrac{1}{2}\left(m - \tfrac{1}{2}\right)\lambda,\ m = 1, 2, \cdots.$$

The minimum thickness is with $m = 1$.

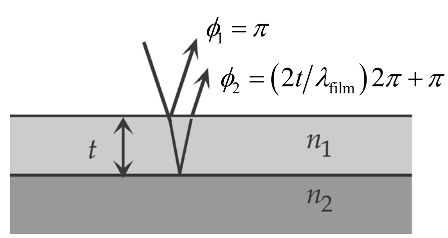
$$t_{min} = \tfrac{1}{2}(450\,\text{nm})\left(1 - \tfrac{1}{2}\right) = \boxed{113\,\text{nm}}$$

For destructive interference, the net phase change must be an odd-integer multiple of $\pi$.

$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda}\right)2\pi + \pi\right] - 0 = (2m+1)\pi \ \rightarrow\ t = \tfrac{1}{2}m\lambda,\ = 0, 1, 2, \cdots$$

The minimum non-zero thickness is $t_{min} = \tfrac{1}{2}(450\,\text{nm})(1) = \boxed{225\,\text{nm}}$.

32. With respect to the incident wave, the wave that reflects from the top surface of the alcohol has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass at the bottom surface of the alcohol has a phase change due to both the additional path length and a phase change of $\pi$ on reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{film}}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even non-zero integer multiple of $\pi$.




$$\phi_{net} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{1film}}\right)2\pi + \pi\right] - \pi = m_1 2\pi \ \rightarrow\ t = \tfrac{1}{2}\lambda_{1film}m_1 = \tfrac{1}{2}\frac{\lambda_1}{n_{film}}m_1,\ m_1 = 1, 2, 3, \ldots.$$

For destructive interference, the net phase change must be an odd-integer multiple $\pi$.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{2\,\text{film}}}\right)2\pi + \pi\right] - \pi = (2m_2+1)\pi \;\rightarrow\; t = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1),\; m_2 = 0, 1, 2, \ldots .$$

Set the two expressions for the thickness equal to each other.

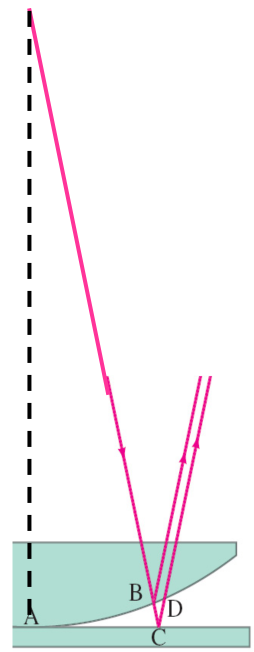
$$\tfrac{1}{2}\frac{\lambda_1}{n_{\text{film}}}m_1 = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1) \;\rightarrow\; \frac{2m_2+1}{2m_1} = \frac{\lambda_1}{\lambda_2} = \frac{(635\,\text{nm})}{(512\,\text{nm})} = 1.24 \approx 1.25 = \frac{5}{4}$$

Thus we see that $m_1 = m_2 = 2,$ and the thickness of the film is

$$t = \tfrac{1}{2}\frac{\lambda_1}{n_{\text{film}}}m_1 = \tfrac{1}{2}\left(\frac{635\,\text{nm}}{1.36}\right)(2) = \boxed{467\,\text{nm}} \text{ or } t = \tfrac{1}{4}\frac{\lambda_2}{n_{\text{film}}}(2m_2+1) = \tfrac{1}{4}\left(\frac{512\,\text{nm}}{1.36}\right)(5) = \boxed{471\,\text{nm}}$$

With 2 sig.fig., the thickness is 470 nm. The range of answers is due to rounding $\lambda_1/\lambda_2$.

33. With respect to the incident wave, the wave that reflects from point B in the first diagram will not undergo a phase change, and so $\phi_B = 0$. With respect to the incident wave, the wave that reflects from point C in the first diagram has a phase change due to both the additional path length in air, and a phase change of $\pi$ on reflection, and so we say that $\phi_D = \frac{2y}{\lambda}(2\pi) + \pi,$ where $y$ is the thickness of the air gap from B to C (or C to D). For dark rings, the net phase difference of the waves that recombine as they leave the glass moving upwards must be an odd-integer multiple of $\pi$.

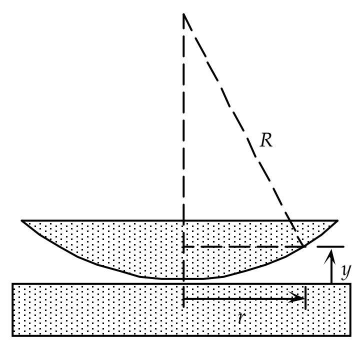



$$\phi_{\text{net}} = \phi_D - \phi_B = \frac{2y}{\lambda}(2\pi) + \pi = (2m+1)\pi \;\rightarrow$$

$$y_{\text{dark}} = \tfrac{1}{2}m\lambda,\; m = 0, 1, 2, \cdots$$

Because $m$ = 0 corresponds to the dark center, $m$ represents the number of the dark ring.

Let the air gap of $y$ be located a horizontal distance $r$ from the center of the lens, as seen in the second diagram. Consider the dashed right triangle in the second diagram.




$$R^2 = r^2 + (R-y)^2 \;\rightarrow$$

$$R^2 = r^2 + R^2 - 2Ry + y^2 \;\rightarrow$$

$$r^2 = 2Ry - y^2$$

If we assume that $y \ll R$, then $r^2 \approx 2Ry$.

$$r^2 = 2Ry \;\rightarrow\; r_{\text{dark}}^2 = 2Ry_{\text{dark}} = 2R\left(\tfrac{1}{2}m\lambda\right) \;\rightarrow\; \boxed{r_{\text{dark}} = \sqrt{m\lambda R},\; m = 0, 1, 2, \cdots}$$

34. From Problem 33, we have $r = \sqrt{m\lambda R} = (m\lambda R)^{1/2}$. To find the distance between adjacent rings, we assume $m >> 1 \;\rightarrow\; \Delta m = 1 \ll m$. Since $\Delta m \ll m$, $\Delta r \approx \frac{dr}{dm}\Delta m.$

$$r = (m\lambda R)^{1/2} \;;\; \frac{dr}{dm} = \tfrac{1}{2}(m\lambda R)^{-1/2}\lambda R$$

$$\Delta r \approx \frac{dr}{dm}\Delta m = \left[\tfrac{1}{2}(m\lambda R)^{-1/2}\lambda R\right](1) = \left[\frac{\lambda^2 R^2}{4m\lambda R}\right]^{1/2} = \boxed{\sqrt{\frac{\lambda R}{4m}}}$$

35. The radius of the *m*-th ring in terms of the wavelength of light and the radius of curvature is derived in Problem 33 as $r = \sqrt{m\lambda R}$. Using this equation, with the wavelength of light in the liquid given by Eq. 34-1, we divide the two radii and solve for the index of refraction.

$$\frac{r_{\text{air}}}{r_{\text{liquid}}} = \frac{\sqrt{m\lambda R}}{\sqrt{m(\lambda/n)R}} = \sqrt{n} \rightarrow n = \left(\frac{r_{\text{air}}}{r_{\text{liquid}}}\right)^2 = \left(\frac{2.92 \text{ cm}}{2.54 \text{ cm}}\right)^2 = \boxed{1.32}$$

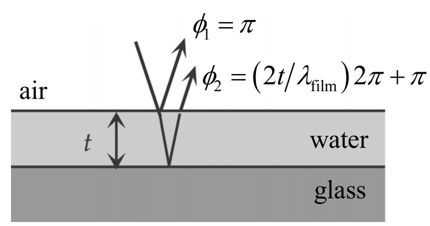
36. We use the equation derived in Problem 33, where *r* is the radius of the lens (1.7 cm) to solve for the radius of curvature. Since the outer edge is the 44th bright ring, which would be halfway between the 44th and 45th dark fringes, we set *m*=44.5

$$r = \sqrt{m\lambda R} \rightarrow R = \frac{r^2}{m\lambda} = \frac{(0.017\,\text{m})^2}{(44.5)(580\times10^{-9}\,\text{m})} = 11.20\,\text{m} \approx \boxed{11\,\text{m}}$$

We calculate the focal length of the lens using Eq. 33-4 (the lensmaker's equation) with the index of refraction of lucite taken from Table 32-1.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = (1.51-1)\left(\frac{1}{11.2\,\text{m}}+\frac{1}{\infty}\right) = 0.0455\,\text{m}^{-1} \rightarrow f = \frac{1}{0.0455\,\text{m}^{-1}} = \boxed{22\,\text{m}}$$

37. (*a*) Assume the indices of refraction for air, water, and glass are 1.00, 1.33, and 1.50, respectively. When illuminated from above, a ray reflected from the air-water interface undergoes a phase shift of $\phi_1 = \pi$, and a ray reflected at the water-glass interface also undergoes a phase shift of $\pi$. Thus, the two rays are unshifted in phase relative to each other due to reflection. For constructive interference, the path difference $2t$ must equal an integer number of wavelengths in water.




$$2t = m\lambda_{\text{water}} = m\frac{\lambda}{n_{\text{water}}},\ m = 0, 1, 2, \cdots \rightarrow \boxed{\lambda = \frac{2n_{\text{water}}t}{m}}$$

(*b*) The above relation can be solved for the *m*-value associated with the reflected color. If this *m*-value is an integer the wavelength undergoes constructive interference upon reflection.

$$\lambda = \frac{2n_{\text{water}}t}{m} \rightarrow m = \frac{2n_{\text{water}}t}{\lambda}$$

For a thickness $t = 200\,\mu\text{m} = 2\times10^5\,\text{nm}$ the *m*-values for the two wavelengths are calculated.

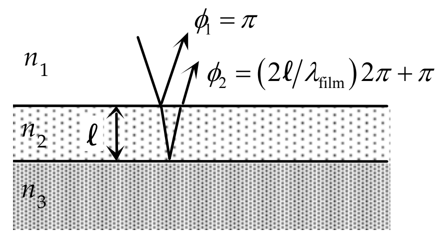
$$m_{700\,\text{nm}} = \frac{2n_{\text{water}}t}{\lambda} = \frac{2(1.33)(2\times10^5\,\text{nm})}{700\,\text{nm}} = 760$$

$$m_{400\,\text{nm}} = \frac{2n_{\text{water}}t}{\lambda} = \frac{2(1.33)(2\times10^5\,\text{nm})}{400\,\text{nm}} = 1330$$

Since both wavelengths yield integers for $m$, they are both reflected.

(*c*) All *m*-values between *m* = 760 and *m* = 1330 will produce reflected visible colors. There are 1330 – (760 – 1) = $\boxed{571}$ such values.

(*d*) This mix of a large number of wavelengths from throughout the visible spectrum will give the thick layer a white or grey appearance.




38. We assume $n_1 < n_2 < n_3$ and that most of the incident light is transmitted. If the amplitude of an incident ray is taken to be $E_0$, then the amplitude of a reflected ray is $rE_0$, with $r << 1$. The light reflected from the top surface of the film therefore has an amplitude of $rE_0$ and is phase shifted by $\phi_1 = \pi$ from the incident wave, due to the higher index of refraction. The light transmitted at that top surface has an amplitude of $(1-r)E_0$. That light is then reflected off the bottom surface of the film, and we assume that it has the same reflection coefficient. Thus the amplitude of that second reflected ray is $r(1-r)E_0 = (r-r^2)E_0 \approx rE_0$, the same amplitude as the first reflected ray. Due to traveling through the film and reflecting from the glass, the second ray has a phase shift of $\phi_2 = \pi + 2\pi(2\ell/\lambda_{film}) = \pi + 4\pi\ell n_2/\lambda$, where $\ell$ is the thickness of the film. Summing the two reflected rays gives the net reflected wave.

$$E = rE_0\cos(\omega t + \pi) + rE_0\cos(\omega t + \pi + 4\pi\ell/\lambda_n)$$
$$= rE_0\left[(1+\cos 4\pi\ell n_2/\lambda)\cos(\omega t+\pi) - \sin(4\pi\ell n_2/\lambda)\sin(\omega t+\pi)\right]$$

As with the double slit experiment, we set the intensity proportional to the square of the wave amplitude and integrate over one period to calculate the average intensity.

$$I \propto \frac{1}{T}\int_0^T E^2 dt = \frac{1}{T}\int_0^T \left[rE_0\left[(1+\cos 4\pi\ell n_2/\lambda)\cos(\omega t+\pi) - \sin(4\pi\ell n_2/\lambda)\sin(\omega t+\pi)\right]\right]^2 dt$$
$$= \frac{r^2E_0^2}{T}\int_0^T \begin{bmatrix}(1+\cos 4\pi\ell n_2/\lambda)^2\cos^2(\omega t+\pi) + \sin^2(4\pi\ell n_2/\lambda)\sin^2(\omega t+\pi) \\ -2(1+\cos 4\pi\ell n_2/\lambda)\sin(4\pi\ell n_2/\lambda)\cos(\omega t+\pi)\sin(\omega t+\pi)\end{bmatrix} dt$$
$$= \frac{r^2E_0^2}{2}\left[(1+\cos 4\pi\ell n_2/\lambda)^2 + \sin^2(4\pi\ell n_2/\lambda)\right] = r^2E_0^2(1+\cos 4\pi\ell n_2/\lambda)$$

The reflected intensity without the film is proportional to the square of the intensity of the single reflected electric field.

$$I_0 \propto \frac{1}{T}\int_0^T E_{\text{no film}}^2 dt = \frac{1}{T}\int_0^T \left[rE_0\cos(\omega t+\pi)\right]^2 dt = \frac{r^2E_0^2}{T}\int_0^T\left[\cos^2(\omega t+\pi)\right]dt = \frac{r^2E_0^2}{2}$$

Dividing the intensity with the film to that without the film gives the factor by which the intensity is reduced.

$$\frac{I}{I_0} = \frac{r^2E_0^2(1+\cos 4\pi\ell n_2/\lambda)}{\frac{1}{2}r^2E_0^2} = 2(1+\cos 4\pi\ell n_2/\lambda)$$

To determine the thickness of the film, the phase difference between the two reflected waves with $\lambda = 550\,\text{nm}$ must be an odd integer multiple of $\pi$ so that there is destructive interference. The minimum thickness will be for $m = 0$.

$$\phi_2 - \phi_1 = [\pi + 4\pi\ell n_2/\lambda] - \pi = (2m+1)\pi \;\rightarrow\; \ell = \frac{\lambda_n}{4} = \frac{\lambda}{4n} = \frac{550\,\text{nm}}{4n}$$

It is interesting to see that the same result is obtained if we set the reflected intensity equal to zero for a wavelength of 550 nm.

$$\frac{I}{I_0} = 2(1+\cos 4\pi\ell n_2/\lambda) = 0 \;\rightarrow\; \cos 4\pi\ell n_2/\lambda = -1 \;\rightarrow\; 4\pi\ell n_2/\lambda = \pi \;\rightarrow\; \ell = \frac{550\,\text{nm}}{4n}$$

Finally, we insert the two given wavelengths (430 nm and 670 nm) into the intensity equation to determine the reduction in intensities.

For $\lambda = 430$ nm, $\dfrac{I}{I_0} = 2\left(1+\cos 4\pi \dfrac{550\text{ nm}/4n}{430\text{ nm}/n}\right) = 2\left(1+\cos 4\pi \dfrac{550\text{ nm}}{4(430\text{ nm})}\right) = 0.721 \approx \boxed{72\%}$

For $\lambda = 670$ nm, $\dfrac{I}{I_0} = 2\left(1+\cos 4\pi \dfrac{550\text{ nm}}{4(670\text{ nm})}\right) = 0.308 \approx \boxed{31\%}$

39. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of $\lambda$, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let *N* be the number of fringe shifts produced by a mirror movement of $\Delta x$.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \rightarrow \Delta x = \tfrac{1}{2}N\lambda = \tfrac{1}{2}(650)(589\times10^{-9}\text{ m}) = \boxed{1.91\times10^{-4}\text{ m}}$$

40. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of $\lambda$, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let *N* be the number of fringe shifts produced by a mirror movement of $\Delta x$.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \rightarrow \lambda = \frac{2\Delta x}{N} = \frac{2(1.25\times10^{-4}\text{ m})}{384} = 6.51\times10^{-7}\text{ m} = \boxed{651\text{ nm}}$$

41. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of $\lambda$, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let *N* be the number of fringe shifts produced by a mirror movement of $\Delta x$. The thickness of the foil is the distance that the mirror moves during the 272 fringe shifts.

$$N = \frac{\Delta x}{\frac{1}{2}\lambda} \rightarrow \Delta x = \tfrac{1}{2}N\lambda = \tfrac{1}{2}(272)(589\times10^{-9}\text{ m}) = \boxed{8.01\times10^{-5}\text{ m}}$$

42. One fringe shift corresponds to an effective change in path length of $\lambda$. The actual distance has not changed, but the number of wavelengths in the depth of the cavity has. If the cavity has a length *d*, the number of wavelengths in vacuum is $\dfrac{d}{\lambda}$, and the (greater) number with the gas present is $\dfrac{d}{\lambda_{\text{gas}}} = \dfrac{n_{\text{gas}}d}{\lambda}$. Because the light passes through the cavity twice, the number of fringe shifts is twice the difference in the number of wavelengths in the two media.

$$N = 2\left(\frac{n_{\text{gas}}d}{\lambda} - \frac{d}{\lambda}\right) = 2\frac{d}{\lambda}(n_{\text{gas}}-1) \rightarrow n_{\text{gas}} = \frac{N\lambda}{2d} + 1 = \frac{(176)(632.8\times10^{-9}\text{ m})}{2(1.155\times10^{-2}\text{ m})} + 1 = \boxed{1.00482}$$

$\boxed{43.}$ There are two interference patterns formed, one by each of the two wavelengths. The fringe patterns overlap but do not interfere with each other. Accordingly, when the bright fringes of one pattern occurs at the same locations as the dark fringes of the other patterns, there will be no fringes seen, since there will be no dark bands to distinguish one fringe from the adjacent fringes.

To shift from one "no fringes" occurrence to the next, the mirror motion must produce an integer number of fringe shifts for each wavelength, and the number of shifts for the shorter wavelength must be one more than the number for the longer wavelength. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. One fringe shift corresponds to a change in path length of $\lambda$, and so corresponds to a mirror motion of $\frac{1}{2}\lambda$. Let $N$ be the number of fringe shifts produced by a mirror movement of $\Delta x$.

$$N_1 = 2\frac{\Delta x}{\lambda_1} \;;\; N_2 = 2\frac{\Delta x}{\lambda_2} \;;\; N_2 = N_1 + 1 \;\rightarrow\; 2\frac{\Delta x}{\lambda_2} = 2\frac{\Delta x}{\lambda_1} + 1 \;\rightarrow$$

$$\Delta x = \frac{\lambda_1 \lambda_2}{2(\lambda_1 - \lambda_2)} = \frac{(589.6\,\text{nm})(589.0\,\text{nm})}{2(0.6\,\text{nm})} = 2.89\times 10^5\,\text{nm} \approx \boxed{2.9\times 10^{-4}\,\text{m}}$$

44. We assume the luminous flux is uniform, and so is the same in all directions.

$$F_\ell = E_\ell A = E_\ell 4\pi r^2 = \left(10^5\,\text{lm/m}^2\right)4\pi\left(1.496\times 10^{11}\,\text{m}\right)^2 = 2.81\times 10^{28}\,\text{lm} \approx \boxed{3\times 10^{28}\,\text{lm}}$$

$$I_\ell = \frac{F_\ell}{4\pi\,\text{sr}} = \frac{2.81\times 10^{28}\,\text{lm}}{4\pi\,\text{sr}} = 2.24\times 10^{27}\,\text{cd} \approx \boxed{2\times 10^{27}\,\text{cd}}$$

45. (*a*) The wattage of the bulb is the electric power input to the bulb.

$$\text{luminous efficiency} = \frac{F_\ell}{P} = \frac{1700\,\text{lm}}{100\,\text{W}} = \boxed{17\,\text{lm/W}}$$

(*b*) The illuminance is the luminous flux incident on a surface, divided by the area of the surface. Let $N$ represent the number of lamps, each contributing an identical amount of luminous flux.

$$E_\ell = \frac{F_\ell}{A} = \frac{N\left[\frac{1}{2}(\text{luminous efficiency})P\right]}{A} \;\rightarrow$$

$$N = \frac{2E_\ell A}{(\text{luminous efficiency})P} = \frac{2\left(250\,\text{lm/m}^2\right)(25\,\text{m})(30\,\text{m})}{(60\,\text{lm/W})(40\,\text{W})} = 156\,\text{lamps} \approx \boxed{160\,\text{lamps}}$$

46. (*a*) For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; x = \frac{\lambda m \ell}{d} \;\rightarrow\; \Delta x = \Delta m\frac{\lambda\ell}{d} \;\rightarrow$$

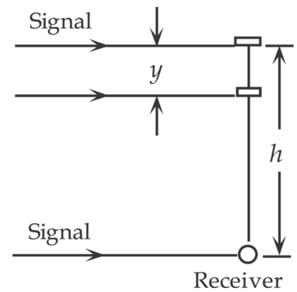
$$d = \frac{\lambda\ell\Delta m}{\Delta x} = \frac{\left(5.0\times 10^{-7}\,\text{m}\right)(4.0\,\text{m})(1)}{\left(2.0\times 10^{-2}\,\text{m}\right)} = \boxed{1.0\times 10^{-4}\,\text{m}}$$

(*b*) For minima, we use Eq. 34-2b. The fourth-order minimum corresponds to $m = 3$, and the fifth-order minimum corresponds to $m = 4$. The slit separation, screen distance, and location on the screen are the same for the two wavelengths.

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \;\rightarrow\; d\frac{x}{\ell} = \left(m + \tfrac{1}{2}\right)\lambda \;\rightarrow\; \left(m_A + \tfrac{1}{2}\right)\lambda_A = \left(m_B + \tfrac{1}{2}\right)\lambda_B \;\rightarrow$$

$$\lambda_B = \lambda_A\frac{\left(m_A + \frac{1}{2}\right)}{\left(m_B + \frac{1}{2}\right)} = \left(5.0\times 10^{-7}\,\text{m}\right)\frac{3.5}{4.5} = \boxed{3.9\times 10^{-7}\,\text{m}}$$

47. The wavelength of the signal is $\lambda = \frac{v}{f} = \frac{(3.00\times10^8\,\text{m/s})}{(75\times10^6\,\text{Hz})} = 4.00\,\text{m}.$




(*a*) There is a phase difference between the direct and reflected signals from both the path difference, $\left(\frac{h}{\lambda}\right)2\pi$, and the reflection, $\pi$.

The total phase difference is the sum of the two.

$$\phi = \left(\frac{h}{\lambda}\right)2\pi + \pi = \frac{(122\,\text{m})}{(4.00\,\text{m})}2\pi + \pi = 62\pi = 31(2\pi)$$

Since the phase difference is an integer multiple of $2\pi$, the interference is $\boxed{\text{constructive.}}$

(*b*) When the plane is 22 m closer to the receiver, the phase difference is as follows.

$$\phi = \left[\frac{(h-y)}{\lambda}\right]2\pi + \pi = \left[\frac{(122\,\text{m} - 22\,\text{m})}{(4.00\,\text{m})}\right]2\pi + \pi = 51\pi = \frac{51}{2}(2\pi)$$

Since the phase difference is an odd-half-integer multiple of $2\pi$, the interference is $\boxed{\text{destructive.}}$

48. Because the measurements are made far from the antennas, we can use the analysis for the double slit. Use Eq. 34-2a for constructive interference, and 34-2b for destructive interference. The wavelength of the signal is $\lambda = \frac{v}{f} = \frac{(3.00\times10^8\,\text{m/s})}{(88.5\times10^6\,\text{Hz})} = 3.39\,\text{m}.$

For constructive interference, the path difference is a multiple of the wavelength:

$$d\sin\theta = m\lambda,\ m = 0, 1, 2, 3, \ldots;\ \rightarrow\ \theta = \sin^{-1}\frac{m\lambda}{d}$$

$$\theta_{1\,\text{max}} = \sin^{-1}\frac{(1)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{22^\circ}\ ;\ \theta_{2\,\text{max}} = \sin^{-1}\frac{(2)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{49^\circ}\ ;$$

$$\theta_{3\,\text{max}} = \sin^{-1}\frac{(3)(3.39\,\text{m})}{9.0\,\text{m}} = \text{impossible}$$

For destructive interference, the path difference is an odd multiple of half a wavelength:

$$d\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, 3, \ldots;\ \rightarrow\ \theta = \sin^{-1}\frac{\left(m+\frac{1}{2}\right)\lambda}{d}$$

$$\theta_{0\,\text{max}} = \sin^{-1}\frac{\left(\frac{1}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{11^\circ}\ ;\ \theta_{1\,\text{max}} = \sin^{-1}\frac{\left(\frac{3}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{34^\circ}\ ;$$

$$\theta_{2\,\text{max}} = \sin^{-1}\frac{\left(\frac{5}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \boxed{70^\circ}\ ;\ \theta_{3\,\text{max}} = \sin^{-1}\frac{\left(\frac{7}{2}\right)(3.39\,\text{m})}{9.0\,\text{m}} = \text{impossible}$$

These angles are applicable both above and below the midline, and both to the left and the right of the antennas.

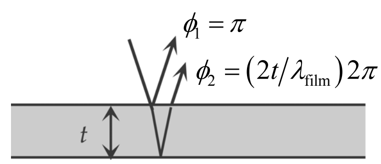
$\boxed{49.}$ For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Second order means $m = 2$.

$$d\sin\theta = m\lambda\ \rightarrow\ d\frac{x}{\ell} = m\lambda\ \rightarrow\ x = \frac{\lambda m\ell}{d}\ ;\ x_1 = \frac{\lambda_1 m\ell}{d}\ ;\ x_2 = \frac{\lambda_2 m\ell}{d}\ \rightarrow$$

$$\Delta x = x_1 - x_2 = \frac{\lambda_1 m\ell}{d} - \frac{\lambda_2 m\ell}{d} \rightarrow$$

$$\lambda_2 = \lambda_1 - \frac{d\Delta x}{m\ell} = 690\times10^{-9}\,\text{m} - \frac{(6.6\times10^{-4}\,\text{m})(1.23\times10^{-3}\,\text{m})}{2(1.60\,\text{m})} = 4.36\times10^{-7}\,\text{m} \approx \boxed{440\,\text{nm}}$$

50. PLEASE NOTE: In early versions of the textbook, in which the third line of this problem states that "… light is a minimum only for …," the resulting answer does not work out properly. It yields values of $m = 6$ and $m = 4$ for the integers in the interference relationship. Accordingly, the problem was changed to read "… light is a maximum only for … ." The solution here reflects that change.




With respect to the incident wave, the wave that reflects at the top surface of the film has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the bottom surface of the film has a phase change due to the additional path length and no phase change due to reflection, so $\phi_2 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + 0$. For constructive interference, the net phase change must be an integer multiple of $2\pi$.

$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi - \pi = 2\pi m \rightarrow t = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\lambda_{\text{film}} = \tfrac{1}{2}\left(m+\tfrac{1}{2}\right)\frac{\lambda}{n_{\text{film}}},\ m = 0, 1, 2, \ldots.$$

Evaluate the thickness for the two wavelengths.

$$t = \tfrac{1}{2}\left(m_1+\tfrac{1}{2}\right)\frac{\lambda_1}{n_{\text{film}}} = \tfrac{1}{2}\left(m_2+\tfrac{1}{2}\right)\frac{\lambda_2}{n_{\text{film}}} \rightarrow \frac{(m_2+\frac{1}{2})}{(m_1+\frac{1}{2})} = \frac{\lambda_1}{\lambda_2} = \frac{688.0\,\text{nm}}{491.4\,\text{nm}} = 1.40 = \frac{7}{5}$$

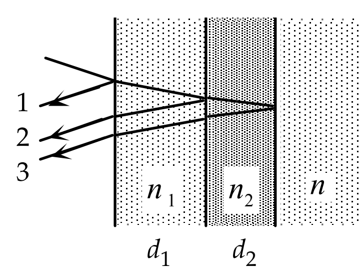
Thus $m_2 = 3$ and $m_1 = 2$. Evaluate the thickness with either value and the corresponding wavelength.

$$t = \tfrac{1}{2}\left(m_1+\tfrac{1}{2}\right)\frac{\lambda_1}{n_{\text{film}}} = \tfrac{1}{2}\left(\tfrac{5}{2}\right)\frac{688.0\,\text{nm}}{1.58} = \boxed{544\,\text{nm}}\ ;\ t = \tfrac{1}{2}\left(m_2+\tfrac{1}{2}\right)\frac{\lambda_2}{n_{\text{film}}} = \tfrac{1}{2}\left(\tfrac{7}{2}\right)\frac{491.4\,\text{nm}}{1.58} = \boxed{544\,\text{nm}}$$

51. From the discussion in section 34-6, we see that the path length change is twice the distance that the mirror moves. The phase shift is $2\pi$ for every wavelength of path length change. The intensity as a function of phase shift is given by Eq. 34-6.

$$\frac{\delta}{2\pi} = \frac{\text{path change}}{\lambda} = \frac{2x}{\lambda} \rightarrow \delta = \frac{4\pi x}{\lambda}\ ;\ I = I_0\cos^2\frac{\delta}{2} = \boxed{I_0\cos^2\left(\frac{2\pi x}{\lambda}\right)}$$

52. To maximize reflection, the three rays shown in the figure should be in phase. We first compare rays 2 and 3. Ray 2 reflects from $n_2 > n_1$, and so has a phase shift of $\phi_2 = \pi$. Ray 3 will have a phase change due to the additional path length in material 2, and a phase shift of $\pi$ because of reflecting from $n > n_2$. Thus $\phi_3 = \left(\frac{2d_2}{\lambda_2}\right)2\pi + \pi$. For constructive interference the net phase change for rays 2 and 3 must be a non-zero integer multiple of $2\pi$.

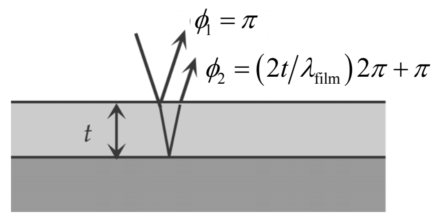



$$\Delta\phi_{2-3} = \phi_3 - \phi_2 = \left[\left(\frac{2d_2}{\lambda_2}\right)2\pi + \pi\right] - \pi = 2m\pi \rightarrow d_2 = \tfrac{1}{2}m\lambda_2,\ m = 1, 2, 3\cdots$$

The minimum thickness is for $m = 1$, and so $d_2 = \frac{1}{2}m\lambda_2 = \boxed{\dfrac{\lambda}{2n_2}}$.

Now consider rays 1 and 2. The exact same analysis applies, because the same relationship exists between the indices of refraction: $n_1 > n$ and $n_2 > n_1$. Thus $d_1 = \boxed{\dfrac{\lambda}{2n_1}}$.

53. With respect to the incident wave, the wave that reflects from the top surface of the coating has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass $(n \approx 1.5)$ at the bottom surface of the coating has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\dfrac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For destructive interference, the net phase change must be an odd-integer multiple of $\pi$.




$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = (2m+1)\pi \quad \rightarrow$$

$$t = \tfrac{1}{4}(2m+1)\lambda_{\text{film}} = \tfrac{1}{4}(2m+1)\frac{\lambda}{n_{\text{film}}},\ m = 0, 1, 2, ...$$

The minimum thickness has $m = 0$, and so $t_{\min} = \frac{1}{4}\dfrac{\lambda}{n_{\text{film}}}$.

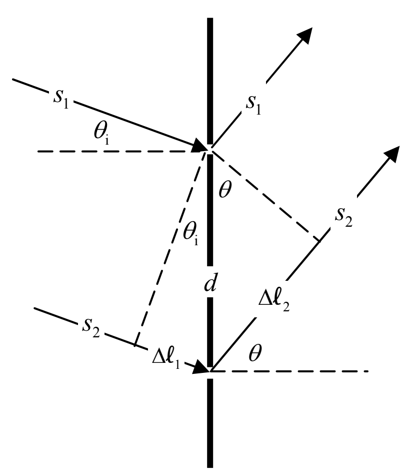
(*a*) For the blue light: $t_{\min} = \frac{1}{4}\dfrac{(450\,\text{nm})}{(1.38)} = 81.52\,\text{nm} \approx \boxed{82\,\text{nm}}$.

(*b*) For the red light: $t_{\min} = \frac{1}{4}\dfrac{(700\,\text{nm})}{(1.38)} = 126.8\,\text{nm} \approx \boxed{130\,\text{nm}}$.

54. The phase difference caused by the path difference back and forth through the coating must correspond to half a wavelength in order to produce destructive interference.

$$2t = \tfrac{1}{2}\lambda \quad \rightarrow \quad t = \tfrac{1}{4}\lambda = \tfrac{1}{4}(2\,\text{cm}) = \boxed{0.5\,\text{cm}}$$

55. We consider a figure similar to Figure 34-12, but with the incoming rays at an angle of $\theta_i$ to the normal. Ray $s_2$ will travel an extra distance $\Delta\ell_1 = d\sin\theta_i$ before reaching the slits, and an extra distance $\Delta\ell_2 = d\sin\theta$ after leaving the slits. There will be a phase difference between the waves due to the path difference $\Delta\ell_1 + \Delta\ell_2$. When this total path difference is a multiple of the wavelength, constructive interference will occur.




$$\Delta\ell_1 + \Delta\ell_2 = d\sin\theta_i + d\sin\theta = m\lambda \quad \rightarrow$$

$$\sin\theta = \frac{m\lambda}{d} - \sin\theta_i,\ \ m = 0, 1, 2, \cdots$$

Since the rays leave the slits at all angles in the forward direction, we could have drawn the leaving rays with a downward tilt instead of an upward tilt. This would make the ray $s_2$ traveling a longer distance from the slits to the screen. In this case the path difference would be $\Delta\ell_2 - \Delta\ell_1$, and would result in the following expression.

$$\Delta\ell_2 - \Delta\ell_1 = d\sin\theta - d\sin\theta_i = m\lambda \;\rightarrow\; \sin\theta = \frac{m\lambda}{d} + \sin\theta_i,\; m = 0, 1, 2, \cdots$$

$$\Delta\ell_1 - \Delta\ell_2 = d\sin\theta_i - d\sin\theta = m\lambda \;\rightarrow\; \sin\theta = -\frac{m\lambda}{d} + \sin\theta_i,\; m = 0, 1, 2, \cdots$$

We combine the statements as follows.

$$\boxed{\sin\theta = \frac{m\lambda}{d} \pm \sin\theta_i,\; m = 0, 1, 2, \cdots}$$

Because of an arbitrary choice of taking $\Delta\ell_2 - \Delta\ell_1$, we could also have formulated the problem so that the result would be expressed as $\boxed{\sin\theta = \sin\theta_i \pm \frac{m\lambda}{d},\; m = 0, 1, 2, \cdots}$.

56. The signals will be out of phase when the path difference equals an odd number of half-wavelengths. Let the 175-m distance be represented by *d*.

$$\sqrt{y^2 + d^2} - y = \left(m + \tfrac{1}{2}\right)\lambda,\; m = 0, 1, 2, 3, \ldots \;\rightarrow\; \sqrt{y^2 + d^2} = y + \left(m + \tfrac{1}{2}\right)\lambda \;\rightarrow$$

$$y^2 + d^2 = y^2 + 2y\left(m + \tfrac{1}{2}\right)\lambda + \left(m + \tfrac{1}{2}\right)^2\lambda^2 \;\rightarrow\; y = \frac{d^2 - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda}$$

We evaluate this for the first three values of *m*. The wavelength is $\lambda = \frac{c}{f} = \frac{3.00\times10^8\,\text{m/s}}{6.0\times10^6\,\text{Hz}} = 50\,\text{m}$.

$$y = \frac{d^2 - \left(m + \tfrac{1}{2}\right)^2\lambda^2}{2\left(m + \tfrac{1}{2}\right)\lambda} = \frac{(175\,\text{m})^2 - \left(m + \tfrac{1}{2}\right)^2(50\,\text{m})^2}{2\left(m + \tfrac{1}{2}\right)(50\,\text{m})} = 600\,\text{m},\, 167\,\text{m},\, 60\,\text{m},\, 0\,\text{m}$$

The first three points on the *y* axis where the signals are out of phase are at $y = \boxed{0, 60\,\text{m, and } 167\,\text{m}}$.

57. As explained in Example 34-6 the $\frac{1}{2}$-cycle phase change at the lower surface means that destructive interference occurs when the thickness *t* is such that $2t = m\lambda,\; m = 0, 1, 2, \ldots$. Set $m = 1$ to find the smallest nonzero value of *t*.

$$t = \tfrac{1}{2}\lambda = \tfrac{1}{2}(680\,\text{nm}) = \boxed{340\,\text{nm}}$$

As also explained in Example 34-6, constructive interference will occur when $2t = \left(m + \tfrac{1}{2}\right)\lambda$, $m = 0, 1, 2, \ldots$ . We set $m = 0$ to find the smallest value of *t*:

$$t = \tfrac{1}{4}\lambda = \tfrac{1}{4}(680\,\text{nm}) = \boxed{170\,\text{nm}}$$

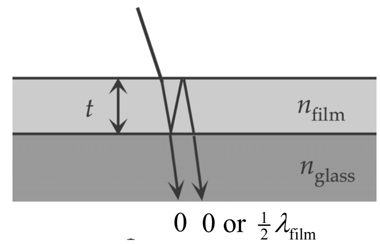
58. The reflected wave appears to be coming from the virtual image, so this corresponds to a double slit, with the separation being *d* = 2S. The reflection from the mirror produces a $\pi$ phase shift, however, so the maxima and minima are interchanged, as described in Problem 11.

$$\sin\theta_{\text{max}} = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2\text{S}},\; m = 0, 1, 2, \cdots \;;\; \sin\theta_{\text{min}} = m\frac{\lambda}{2\text{S}},\; m = 0, 1, 2, \cdots$$

59. Since the two sources are 180° out of phase, destructive interference will occur when the path length difference between the two sources and the receiver is 0, or an integer number of wavelengths. Since the antennae are separated by a distance of $d = \lambda/2$, the path length difference can never be greater than $\lambda/2$, so the only points of destructive interference occur when the receiver is equidistant from each antenna, that is, at $\boxed{\theta_{\text{destructive}} = 0° \text{ and } 180°}$. Constructive interference occurs when the path difference is a half integer wavelength. Again, since the separation distance between the two

antennas is $d = \lambda/2$, the maximum path length difference is $\lambda/2$, which occurs along the line through the antennae, therefore the constructive interference only occurs at $\boxed{\theta_{\text{constructive}} = 90^\circ \text{ and } 270^\circ}$. As expected, these angles are reversed from those in phase, found in Example 34-5c.

60. If we consider the two rays shown in the diagram, we see that the first ray passes through with no reflection, while the second ray has reflected twice. If $n_{\text{film}} < n_{\text{glass}}$, the first reflection from the glass produces a phase shift equivalent to $\frac{1}{2}\lambda_{\text{film}}$, while the second reflection from the air produces no shift. When we compare the two rays at the film-glass surface, we see that the second ray has a total shift in phase, due to its longer path length ($2t$) and reflection $\left(\frac{1}{2}\lambda_{\text{film}}\right)$. We set this path difference equal to an integer number of wavelengths for maximum intensity and equal to a half-integer number of wavelengths for minimum intensity.




max: $2t + \frac{1}{2}\lambda_{\text{film}} = m\lambda_{\text{film}},\ m = 1, 2, 3, \ldots \ \rightarrow\ \boxed{t = \dfrac{\frac{1}{2}\left(m - \frac{1}{2}\right)\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots}$

min: $2t + \frac{1}{2}\lambda_{\text{film}} = \left(m + \frac{1}{2}\right)\lambda_{\text{film}},\ m = 0, 1, 2, 3, \ldots \ \rightarrow\ \boxed{t = \dfrac{\frac{1}{2}m\lambda}{n_{\text{film}}},\ m = 0, 1, 2, 3, \ldots}$

At $t = 0$, or in the limit $t \ll \lambda/n_{\text{film}}$, the transmitted beam will be at a minimum. Each time the thickness increases by a quarter wavelength the intensity switches between a maximum and a minimum.

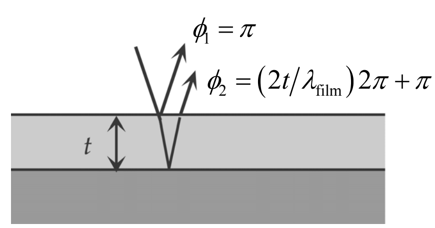
If $n_{\text{film}} > n_{\text{glass}}$, the first reflection from the glass produces no shift, while the second reflection from the air also produces no shift. When we compare the two rays at the film-glass surface, we see that the second ray has a total shift due solely to the difference in path lengths, $2t$.
For maxima, we have

max: $2t = m\lambda_{\text{film}},\ m = 0, 1, 2, 3, \ldots \ \rightarrow\ \boxed{t = \dfrac{\frac{1}{2}m\lambda}{n_{\text{film}}},\ m = 0, 1, 2, 3, \ldots}$

min: $2t = \left(m - \frac{1}{2}\right)\lambda_{\text{film}},\ m = 1, 2, 3, \ldots \ \rightarrow\ \boxed{t = \dfrac{\frac{1}{2}\left(m - \frac{1}{2}\right)\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots}$

At $t = 0$, or in the limit $t \ll \lambda/n_{\text{film}}$, the transmitted beam will be at a maximum. Each time the thickness increases by a quarter wavelength the intensity switches between a maximum and a minimum.

$\boxed{61.}$ With respect to the incident wave, the wave that reflects from the top surface of the film has a phase change of $\phi_1 = \pi$. With respect to the incident wave, the wave that reflects from the glass $(n = 1.52)$ at the bottom surface of the film has a phase change due to both the additional path length and reflection, so $\phi_2 = \left(\dfrac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi$. For constructive interference, the net phase change must be an even non-zero integer multiple of $\pi$.



$$\phi_{\text{net}} = \phi_2 - \phi_1 = \left[\left(\frac{2t}{\lambda_{\text{film}}}\right)2\pi + \pi\right] - \pi = m2\pi \quad \to \quad t = \tfrac{1}{2}m\lambda_{\text{film}} = \tfrac{1}{2}m\frac{\lambda}{n_{\text{film}}},\ m = 1, 2, 3, \ldots .$$

The minimum non-zero thickness occurs for $m = 1$.

$$t_{\min} = \frac{\lambda}{2n_{\text{film}}} = \frac{643\,\text{nm}}{2(1.34)} = \boxed{240\,\text{nm}}$$

62. The path difference to a point on the *x* axis from the two sources is $\Delta d = d_2 - d_1 = \sqrt{x^2 + d^2} - x$. For the two signals to be out of phase, this path difference must be an odd number of half-wavelengths, so $\Delta d = \left(m + \frac{1}{2}\right)\lambda,\ m = 0, 1, 2, \cdots$. Also, the maximum path difference is $d = 3\lambda$. Thus the path difference must be $\frac{1}{2}\lambda,\ \frac{3}{2}\lambda$, or $\frac{5}{2}\lambda$ for the signals to be out of phase (*m* = 0, 1, or 2). We solve for *x* for the three path differences.

$$\Delta d = \sqrt{x^2 + d^2} - x = \left(m + \tfrac{1}{2}\right)\lambda \quad \to \quad \sqrt{x^2 + d^2} = x + \left(m + \tfrac{1}{2}\right)\lambda \quad \to$$

$$x^2 + d^2 = x^2 + 2x\left(m + \tfrac{1}{2}\right)\lambda + \left(m + \tfrac{1}{2}\right)^2\lambda^2 \quad \to$$

$$x = \frac{d^2 - \left(m + \frac{1}{2}\right)^2\lambda^2}{2\left(m + \frac{1}{2}\right)\lambda} = \frac{9\lambda^2 - \left(m + \frac{1}{2}\right)^2\lambda^2}{2\left(m + \frac{1}{2}\right)\lambda} = \frac{9 - \left(m + \frac{1}{2}\right)^2}{2\left(m + \frac{1}{2}\right)}\lambda$$

$$m = 0: x = \frac{9 - \left(0 + \frac{1}{2}\right)^2}{2\left(0 + \frac{1}{2}\right)}\lambda = \boxed{8.75\lambda}\ ;\ m = 1: x = \frac{9 - \left(1 + \frac{1}{2}\right)^2}{2\left(1 + \frac{1}{2}\right)}\lambda = \boxed{2.25\lambda}$$

$$m = 2: x = \frac{9 - \left(2 + \frac{1}{2}\right)^2}{2\left(2 + \frac{1}{2}\right)}\lambda = \boxed{0.55\lambda}$$

63. For both configurations, we have $d\sin\theta = m\lambda$. The angles and the orders are to be the same. The slit separations and wavelengths will be different. Use the fact that frequency and wavelength are related by $v = f\lambda$. The speed of sound in room-temperature air is given in Chapter 16 as 343 m/s.

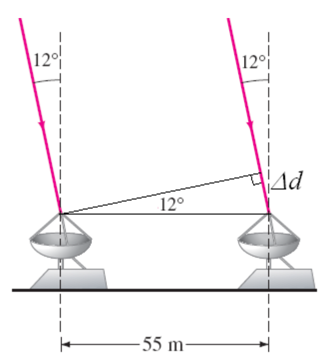
$$d\sin\theta = m\lambda \quad \to \quad \frac{\sin\theta}{m} = \frac{\lambda}{d} = \frac{\lambda_{\text{L}}}{d_{\text{L}}} = \frac{\lambda_{\text{S}}}{d_{\text{S}}} \quad \to$$

$$d_{\text{S}} = d_{\text{L}}\frac{\lambda_{\text{S}}}{\lambda_{\text{L}}} = d_{\text{L}}\frac{\frac{v_{\text{S}}}{f_{\text{S}}}}{\frac{v_{\text{L}}}{f_{\text{L}}}} = d_{\text{L}}\frac{v_{\text{S}}f_{\text{L}}}{v_{\text{L}}f_{\text{S}}} = \left(1.0\times10^{-4}\,\text{m}\right)\left(\frac{343\,\text{m/s}}{3.00\times10^{8}\,\text{m/s}}\right)\left(\frac{4.6\times10^{14}\,\text{Hz}}{262\,\text{Hz}}\right) = \boxed{200\,\text{m}}$$

The answer has 2 significant figures.

64. Light traveling from a region 12° from the vertical would have to travel a slightly longer distance to reach the far antenna. Using trigonometry we calculate this distance, as was done in Young's double slit experiment. Dividing this additional distance by the speed of light gives us the necessary time shift.

$$\Delta t = \frac{\Delta d}{c} = \frac{\ell\sin\theta}{c} = \frac{(55\text{ m})\sin 12^\circ}{3.00\times10^{8}\text{ m/s}} = 3.81\times10^{-8}\text{ s} = \boxed{38.1\text{ ns}}$$



65. In order for the two reflected halves of the beam to be $180^\circ$ out of phase with each other, the minimum path difference (2*t*) should be $\frac{1}{2}\lambda$ in the plastic. Notice that there is no net phase difference between the two halves of the beam due to reflection, because both halves reflect from the same material.

$$2t = \frac{1}{2}\frac{\lambda}{n} \quad \rightarrow \quad t = \frac{\lambda}{4n} = \frac{780\,\text{nm}}{4(1.55)} = \boxed{126\,\text{nm}}$$

66. We determine *n* for each angle using a spreadsheet. The results are shown below.

| $N$ | 25 | 50 | 75 | 100 | 125 | 150 |
|---|---|---|---|---|---|---|
| $\theta$(degree) | 5.5 | 6.9 | 8.6 | 10.0 | 11.3 | 12.5 |
| $n$ | 1.75 | 2.19 | 2.10 | 2.07 | 2.02 | 1.98 |

The average value is $n_{avg} = \boxed{2.02}$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH34.XLS," on tab "Problem 34.66."

# CHAPTER 35: Diffraction and Polarization

## Responses to Questions

1. Radio waves have a much longer wavelength than visible light and will diffract around normal-sized objects (like hills). The wavelengths of visible light are very small and will not diffract around normal-sized objects.

2. You see a pattern of dark and bright lines parallel to your fingertips in the narrow opening between your fingers.

3. Light from all points of an extended source produces diffraction patterns, and these many different diffraction patterns overlap and wash out each other so that no distinct pattern can be easily seen. When using white light, the diffraction patterns of the different wavelengths will overlap because the locations of the fringes depend on wavelength. Monochromatic light will produce a more distinct diffraction pattern.

4. (*a*) If the slit width is increased, the diffraction pattern will become more compact.
   (*b*) If the wavelength of the light is increased, the diffraction pattern will spread out.

5. (*a*) A slit width of 50 nm would produce a central maximum so spread out that it would cover the entire width of the screen. No minimum (and therefore no diffraction pattern) will be seen. The different wavelengths will all overlap, so the light on the screen will be white. It will also be dim, compared to the source, because it is spread out.
   (*b*) For the 50,000 nm slit, the central maximum will be very narrow, about a degree in width for the blue end of the spectrum and about a degree and a half for the red. The diffraction pattern will not be distinct, because most of the intensity will be in the small central maximum and the fringes for the different wavelengths of white light will not coincide.

6. (*a*) If the apparatus is immersed in water, the wavelength of the light will decrease $\left(\lambda' = \frac{\lambda}{n}\right)$ and the diffraction pattern will become more compact.
   (*b*) If the apparatus is placed in a vacuum, the wavelength of the light will increase slightly, and the diffraction pattern will spread out very slightly.

7. The intensity pattern is actually a function of the form $\left(\frac{\sin x}{x}\right)^2$ (see equations 35-7 and 35-8). The maxima of this function do not coincide exactly with the maxima of $\sin^2 x$. You can think of the intensity pattern as the combination of a $\sin^2 x$ function and a $1/x^2$ function, which forces the intensity function to zero and shifts the maxima slightly.

8. Similarities: Both have a regular pattern of light and dark fringes. The angular separation of the fringes is proportional to the wavelength of the light, and inversely proportional to the slit size or slit separation. Differences: The single slit diffraction maxima decrease in brightness from the center. Maxima for the double slit interference pattern would be equally bright (ignoring single slit effects) and are equally spaced.

9. No. $D$ represents the slit width and $d$ the distance between the centers of the slits. It is possible for the distance between the slit centers to be greater than the width of the slits; it is not possible for the distance between the slit centers to be less than the width of the slits.

10. (*a*) Increasing the wavelength, $\lambda$, will spread out the diffraction pattern, since the locations of the minima are given by $\sin\theta = m\lambda/D$. The interference pattern will also spread out; the interference maxima are given by $\sin\theta = m\lambda/d$. The number of interference fringes in the central diffraction maximum will not change.
    (*b*) Increasing the slit separation, $d$, will decrease the spacing between the interference fringes without changing the diffraction, so more interference maxima will fit in the central maximum of the diffraction envelope.
    (*c*) Increasing the slit width, $D$, will decrease the angular width of the diffraction central maximum without changing the interference fringes, so fewer bright fringes will fit in the central maximum.

11. Yes. As stated in Section 35-5, "It is not possible to resolve detail of objects smaller than the wavelength of the radiation being used."

12. Yes. Diffraction effects will occur for both real and virtual images.

13. A large mirror has better resolution and gathers more light than a small mirror.

14. No. The resolving power of a lens is on the order of the wavelength of the light being used, so it is not possible to resolve details smaller than the wavelength of the light. Atoms have diameters of about $10^{-8}$ cm and the wavelength of visible light is on the order of $10^{-5}$ cm.

15. Violet light would give the best resolution in a microscope, because the wavelengths are shortest.

16. Yes. (See the introduction to Section 35-7.) The analysis for a diffraction grating of many slits is essentially the same as for Young's double slit interference. However, the bright maxima of a multiple-slit grating are much sharper and narrower than those in a double-slit pattern.

17. The answer depends on the slit spacing of the grating being used. If the spacing is small enough, only the first order will appear so there will not be any overlap. For wider slit spacing there can be overlap. If there is overlap, it will be the higher orders of the shorter wavelength light overlapping with lower orders of the longer wavelength light. See, for instance, Example 35-9, which shows the overlap of the third order blue light with the second order red light.

18. The bright lines will coincide, but those for the grating will be much narrower with wider dark spaces in between. The grating will produce a much sharper pattern.

19. (*a*) Violet light will be at the top of the rainbow created by the diffraction grating. Principal maxima for a diffraction grating are at positions given by $\sin\theta = \frac{m\lambda}{d}$. Violet light has a shorter wavelength than red light and so will appear at a smaller angle away from the direction of the horizontal incident beam.
    (*b*) Red light will appear at the top of the rainbow created by the prism. The index of refraction for violet light in a given medium is slightly greater than for red light in the same medium, and so the violet light will bend more and will appear farther from the direction of the horizontal incident beam.

20. The tiny peaks are produced when light from some but not all of the slits interferes constructively. The peaks are tiny because light from only some of the slits interferes constructively.

21. Polarization demonstrates the transverse wave nature of light, and cannot be explained if light is considered only as particles.

22. Take the sunglasses outside and look up at the sky through them. Rotate the sunglasses (about an axis perpendicular to the lens) through at least 180°. If the sky seems to lighten and darken as you rotate the sunglasses, then they are polarizing. You could also look at a liquid crystal display or reflections from the floor while rotating the glasses, or put one pair of glasses on top of the other and rotate them. If what you see through the glasses changes as you rotate them, then the glasses are polarizing.

23. Black. If there were no atmosphere, there would be no scattering of the sunlight coming to Earth.

## Solutions to Problems

1. We use Eq. 35-1 to calculate the angular distance from the middle of the central peak to the first minimum. The width of the central peak is twice this angular distance.

$$\sin\theta_1 = \frac{\lambda}{D} \to \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{680\times10^{-9}\,\text{m}}{0.0365\times10^{-3}\,\text{m}}\right) = 1.067°$$

$$\Delta\theta = 2\theta_1 = 2(1.067°) = \boxed{2.13°}$$

2. The angle from the central maximum to the first dark fringe is equal to half the width of the central maximum. Using this angle and Eq. 35-1, we calculate the wavelength used.

$$\theta_1 = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(32°) = 16°$$

$$\sin\theta_1 = \frac{\lambda}{D} \to \lambda = D\sin\theta_1 = \left(2.60\times10^{-3}\text{ mm}\right)\sin(16°) = 7.17\times10^{-4}\text{ mm} = \boxed{717\text{ nm}}$$

3. The angle to the first maximum is about halfway between the angles to the first and second minima. We use Eq. 35-2 to calculate the angular distance to the first and second minima. Then we average these to values to determine the approximate location of the first maximum. Finally, using trigonometry, we set the linear distance equal to the distance to the screen multiplied by the tangent of the angle.

$$D\sin\theta_m = m\lambda \to \theta_m = \sin^{-1}\left(\frac{m\lambda}{D}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{1\times580\times10^{-9}\text{ m}}{3.8\times10^{-6}\text{ m}}\right) = 8.678° \qquad \theta_2 = \sin^{-1}\left(\frac{2\times580\times10^{-9}\text{ m}}{3.8\times10^{-6}\text{ m}}\right) = 17.774°$$

$$\theta = \frac{\theta_1+\theta_2}{2} = \frac{8.678°+17.774°}{2} = 13.23°$$

$$y = \ell\tan\theta_1 = (10.0\text{ m})\tan(13.23°) = \boxed{2.35\text{ m}}$$

4. (*a*) We use Eq. 35-2, using m=1,2,3,… to calculate the possible diffraction minima, when the wavelength is 0.50 cm.

$$D\sin\theta_m = m\lambda \to \theta_m = \sin^{-1}\left(\frac{m\lambda}{D}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{1\times0.50\text{ cm}}{1.6\text{ cm}}\right) = 18.2° \qquad \theta_2 = \sin^{-1}\left(\frac{2\times0.50\text{ cm}}{1.6\text{ cm}}\right) = 38.7°$$

$$\theta_3 = \sin^{-1}\left(\frac{3\times 0.50\text{ cm}}{1.6\text{ cm}}\right) = 69.6^\circ \qquad \theta_4 = \sin^{-1}\left(\frac{4\times 0.50\text{ cm}}{1.6\text{ cm}}\right) \rightarrow \text{ no solution}$$

There are three diffraction minima: $\boxed{18^\circ, 39^\circ, \text{and } 70^\circ.}$

(*b*) We repeat the process from part (*a*) using a wavelength of 1.0 cm.

$$\theta_1 = \sin^{-1}\left(\frac{1\times 1.0\text{ cm}}{1.6\text{ cm}}\right) = 38.7^\circ \qquad \theta_2 = \sin^{-1}\left(\frac{2\times 1.0\text{ cm}}{1.6\text{ cm}}\right) = \text{no real solution}$$

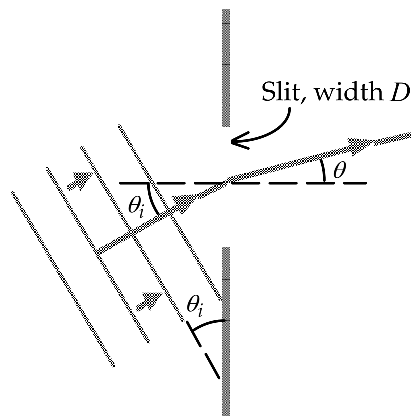
The only diffraction minima is at $\boxed{39^\circ}$

(*c*) We repeat the process from part (*a*) using a wavelength of 3.0 cm.

$$\theta_1 = \sin^{-1}\left(\frac{1\times 3.0\text{ cm}}{1.6\text{ cm}}\right) = \text{no real solution}$$

There are $\boxed{\text{no diffraction minima}}$.

5. The path-length difference between the top and bottom of the slit for the incident wave is $D \sin \theta_i$. The path-length difference between the top and bottom of the slit for the diffracted wave is $D \sin \theta$. When the net path-length difference is equal to a multiple of the wavelength, there will be an even number of segments of the wave having a path-length difference of $\lambda/2$. We set the path-length difference equal to $m$ (an integer) times the wavelength and solve for the angle of the diffraction minimum.




$$D\sin\theta_i - D\sin\theta = m\lambda \quad \rightarrow$$

$$\boxed{\sin\theta = \sin\theta_i - \frac{m\lambda}{D}, \quad m = \pm 1, \pm 2, ...}$$

From this equation we see that when $\theta = 23.0^\circ$, the minima will be symmetrically distributed around a central maximum at 23.0°

6. The angle from the central maximum to the first bright maximum is half the angle between the first bright maxima on either side of the central maximum. The angle to the first maximum is about halfway between the angles to the first and second minima. We use Eq. 35-2, setting $m = 3/2$, to calculate the slit width, $D$.

$$\theta_1 = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(35^\circ) = 17.5^\circ$$

$$D\sin\theta_m = m\lambda \rightarrow D = \frac{m\lambda}{\sin\theta_1} = \frac{(3/2)(633\text{ nm})}{\sin 17.5^\circ} = 3157.6\text{ nm} \approx \boxed{3.2\,\mu\text{m}}$$

$\boxed{7.}$ We use the distance to the screen and half the width of the diffraction maximum to calculate the angular distance to the first minimum. Then using this angle and Eq. 35-1 we calculate the slit width. Then using the slit width and the new wavelength we calculate the angle to the first minimum and the width of the diffraction maximum.

$$\tan\theta_1 = \frac{\left(\frac{1}{2}\Delta y_1\right)}{\ell} \quad\rightarrow\quad \theta_1 = \tan^{-1}\frac{\left(\frac{1}{2}\Delta y_1\right)}{\ell} = \tan^{-1}\frac{\left(\frac{1}{2}\times 0.06\,\text{m}\right)}{2.20\,\text{m}} = 0.781^\circ$$

$$\sin\theta_1 = \frac{\lambda_1}{D} \quad\rightarrow\quad D = \frac{\lambda_1}{\sin\theta_1} = \frac{580\,\text{nm}}{\sin 0.781^\circ} = 42{,}537\,\text{nm}$$

$$\sin\theta_2 = \frac{\lambda_2}{D} \quad\to\quad \theta_2 = \sin^{-1}\left(\frac{\lambda_2}{D}\right) = \sin^{-1}\left(\frac{460\,\text{nm}}{42{,}537\,\text{nm}}\right) = 0.620^\circ$$

$$\Delta y_2 = 2\ell\tan\theta_2 = 2(2.20\,\text{m})\tan(0.620^\circ) = 0.0476\,\text{m} \approx \boxed{4.8\,\text{cm}}$$

8. (*a*) There will be no diffraction minima if the angle for the first minimum is greater than $90^\circ$. We set the angle in Eq. 35-1 equal to 90° and solve for the slit width.

$$\sin\theta = \frac{\lambda}{D} \to D = \frac{\lambda}{\sin 90^\circ} = \boxed{\lambda}$$

(*b*) For no visible light to exhibit a diffraction minimum, the slit width must be equal to the shortest visible wavelength.

$$D = \lambda_{\text{min}} = \boxed{400\text{ nm}}.$$

9. We set the angle to the first minimum equal to half of the separation angle between the dark bands. We insert this angle into Eq. 35-1 to solve for the slit width.

$$\theta = \tfrac{1}{2}\Delta\theta = \tfrac{1}{2}(55.0^\circ) = 27.5^\circ$$

$$\sin\theta = \frac{\lambda}{D} \quad\to\quad D = \frac{\lambda}{\sin\theta} = \frac{440\,\text{nm}}{\sin 27.5^\circ} = \boxed{953\,\text{nm}}$$

10. We find the angle to the first minimum using Eq. 35-1. The distance on the screen from the central maximum is found using the distance to the screen and the tangent of the angle. The width of the central maximum is twice the distance from the central maximum to the first minimum.

$$\sin\theta_1 = \frac{\lambda}{D} \quad\to\quad \theta_1 = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{450\times10^{-9}\,\text{m}}{1.0\times10^{-3}\,\text{m}}\right) = \underline{\underline{0.02578^\circ}}$$

$$y_1 = \ell\tan\theta_1 = (5.0\text{ m})\tan 0.02578^\circ = 0.00225\text{ m}$$

$$\Delta y = 2y_1 = 2(0.00225\text{ m}) = 0.0045\text{ m} = \boxed{0.45\text{ cm}}$$

11. (*a*) For vertical diffraction we use the height of the slit (1.5 μm) as the slit width in Eq. 35-1 to calculate the angle between the central maximum to the first minimum. The angular separation of the first minima is equal to twice this angle.

$$\sin\theta_1 = \frac{\lambda}{D} \quad\to\quad \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{780\times10^{-9}\,\text{m}}{1.5\times10^{-6}\,\text{m}} = 31.3^\circ$$

$$\Delta\theta = 2\theta_1 = 2(31.3^\circ) \approx \boxed{63^\circ}$$

(*b*) To find the horizontal diffraction we use the width of the slit (3.0 μm) in Eq. 35-1.

$$\sin\theta_1 = \frac{\lambda}{D} \quad\to\quad \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{780\times10^{-9}\,\text{m}}{3.0\times10^{-6}\,\text{m}} = 15.07^\circ$$

$$\Delta\theta = 2\theta_1 = 2(15.07^\circ) \approx \boxed{30^\circ}$$

12. (*a*) If we consider the slit made up of *N* wavelets each of amplitude $E_0$, the total amplitude at the central maximum, where they are all in phase, is $NE_0$. Doubling the size of the slit doubles the number of wavelets and thus the total amplitude of the electric field. Because the intensity is proportional to the square of the electric field amplitude, the intensity at the central maximum is increased by a factor of 4.

$$I \propto E^2 = (2E_0)^2 = 4E_0^2 \propto \boxed{4I_0}$$

(*b*) From Eq. 35-1 we see that, for small angles, the width of the central maximum is inversely proportional to the slit width. Therefore doubling the slit width will cut the area of the central peak in half. Since the intensity is spread over only half the area, where the intensity is four times the initial intensity, the average intensity (or energy) over the central maximum has doubled. This is true for all fringes, so when the slit width is doubled, allowing twice the energy to pass through the slit, the average energy within each slit will also double, in accord with the conservation of energy.

13. We use Eq. 35-8 to calculate the intensity, where the angle $\theta$ is found from the displacement from the central maximum (15 cm) and the distance to the screen.

$$\tan\theta = \frac{y}{\ell} \rightarrow \theta = \tan^{-1}\left(\frac{15\,\text{cm}}{25\,\text{cm}}\right) = 31.0°$$

$$\beta = \frac{2\pi}{\lambda} D\sin\theta = \frac{2\pi}{\left(750\times10^{-9}\,\text{m}\right)}\left(1.0\times10^{-6}\,\text{m}\right)\sin 31.0° = 4.31\,\text{rad}$$

$$\frac{I}{I_0} = \left(\frac{\sin\beta/2}{\beta/2}\right)^2 = \left(\frac{\sin\left(4.31\,\text{rad}/2\right)}{4.31\,\text{rad}/2}\right)^2 = 0.1498 \approx \boxed{0.15}$$

So the light intensity at 15 cm is about 15% of the maximum intensity.

14. (*a*) The secondary maxima do not occur precisely where $\sin(\beta/2)$ is a maximum, that is at $\beta/2 = (m+\frac{1}{2})\pi$ where $m = 1,2,3,...,$ because the diffraction intensity (Eq. 35-7) is the ratio of the sine function and $\beta/2$. Near the maximum of the sine function, the denominator of the intensity function causes the intensity to decrease more rapidly than the sine function causes it to increase. This results in the intensity reaching a maximum slightly before the sine function reaches its maximum.

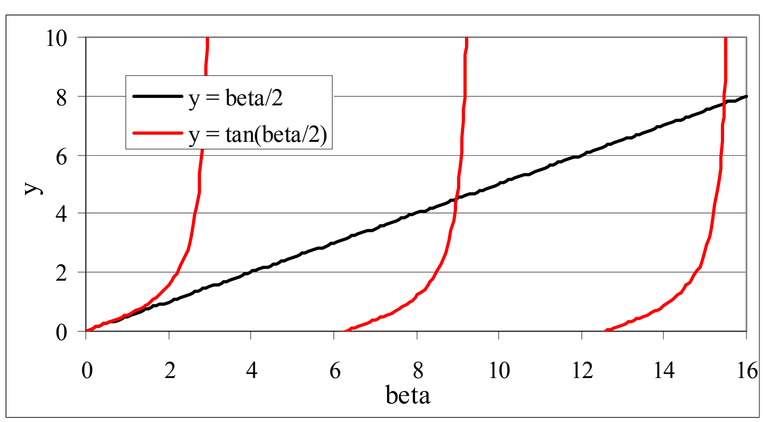
(*b*) We set the derivative of Eq. 35-7 with respect to $\beta$ equal to zero to determine the intensity extrema.

$$0 = \frac{dI}{d\beta} = \frac{d}{d\beta} I_o \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = 2I_o\left[\frac{\sin(\beta/2)}{\beta/2}\right]\left[\frac{\cos(\beta/2)}{\beta} - \frac{\sin(\beta/2)}{\beta^2/2}\right]$$

When the first term in brackets is zero, the intensity is a minimum, so the intensity is a maximum when the second term in brackets is zero.

$$0 = \frac{\cos(\beta/2)}{\beta} - \frac{\sin(\beta/2)}{\beta^2/2} \rightarrow \boxed{\beta/2 = \tan(\beta/2)}$$

(*c*) The first and secondary maxima are found where these two curves intersect, or $\boxed{\beta_1 = 8.987}$ and $\boxed{\beta_2 = 15.451.}$ We calculate the percent difference between these and the maxima of the sine curve, $\beta_1' = 3\pi$ and $\beta_2' = 5\pi$.



$$\left.\frac{\Delta\beta}{\beta}\right|_1 = \frac{\beta_1 - \beta_1'}{\beta_1'} = \frac{8.987 - 3\pi}{3\pi} = -0.0464 = \boxed{-4.64\%}$$

$$\left.\frac{\Delta\beta}{\beta}\right|_2 = \frac{15.451 - 5\pi}{5\pi} = -0.0164 = \boxed{-1.64\%}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.14."

15. If the central diffraction peak contains nine fringes, there will be four fringes on each side of the central peak. Thus the fifth maximum of the double slit must coincide with the first minimum of the diffraction pattern. We use Eq. 34-2a with *m* = 5 to find the angle of the fifth interference maximum and set that angle equal to the first diffraction minimum, given by Eq. 35-1, to solve for the ratio of the slit separation to slit width.

$$d\sin\theta = m\lambda \;\rightarrow\; \sin\theta = \frac{5\lambda}{d} \;;\; \sin\theta = \frac{\lambda}{D} = \frac{5\lambda}{d} \;\rightarrow\; \boxed{d = 5D}$$

16. (*a*) If the central diffraction peak is to contain seventeen fringes, there will be eight fringes on each side of the central peak. Thus, the ninth minimum of the double slit must coincide with the first minimum of the diffraction pattern. We use Eq. 34-2b with *m* = 8 to find the angle of the ninth interference minimum and set that angle equal to the first diffraction minimum, given by Eq. 35-1, to solve for the ratio of the slit separation to slit width.

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \;\rightarrow\; \sin\theta = \frac{\left(8 + \frac{1}{2}\right)\lambda}{d} = \frac{8.5\lambda}{d}$$

$$\sin\theta = \frac{\lambda}{D} = \frac{8.5\lambda}{d} \;\rightarrow\; \boxed{d = 8.5D}$$

Therefore, for the first diffraction minimum to be at the ninth interference minimum, the separation of slits should be 8.5 times the slit width.

(*b*) If the first diffraction minimum is to occur at the ninth interference maximum, we use Eq. 34-2a with *m* = 9 to find the angle of the ninth interference maximum and set that angle equal to the first diffraction minimum, given by Eq. 35-1, to solve for the ratio of the slit separation to slit width.

$$d\sin\theta = m\lambda \;\rightarrow\; \sin\theta = \frac{9\lambda}{d} = \frac{9\lambda}{d} \;;\; \sin\theta = \frac{\lambda}{D} = \frac{9\lambda}{d} \;\rightarrow\; \boxed{d = 9D}$$

Therefore, for the first diffraction minimum to be at the ninth interference maximum, the separation of slits should be 9 times the slit width.

17. Given light with $\lambda = 605\text{ nm}$ passing through double slits with separation $d = 0.120\text{ mm}$, we use Eq. 34-2a to find the highest integer $m$ value for the interference fringe that occurs before the angle $\theta = 90^\circ$.

$$d\sin\theta = m\lambda \;\rightarrow\; m = \frac{\left(0.120\times10^{-3}\text{m}\right)\sin 90^{\circ\circ}}{605\times10^{-9}\text{m}} = 198$$

So, including the $m = 0$ fringe, and the symmetric pattern of interference fringes on each side of $\theta = 0$, there are potentially a total of $198 + 198 + 1 = 397$ fringes. However, since slits have width $a = 0.040$ mm, the potential interference fringes that coincide with the slits' diffraction minima will be absent. Let the diffraction minima be indexed by $m' = 1, 2, 3,$ etc. We then set the diffraction angles in Eq. 34-2a and Eq. 35-2 equal to solve for the *m* values of the absent fringes.

$$\sin\theta = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \rightarrow \frac{m}{m'} = \frac{d}{D} = \frac{0.120 \text{ mm}}{0.040 \text{ mm}} = 3 \rightarrow m = 3m'$$

Using $m' = 1, 2, 3, \text{etc.}$, the 66 interference fringes on each side of $\theta = 0$ with $m = 3, 6, 9, \ldots, 198$ will be absent. Thus the number of fringes on the screen is $397 - 2(66) = \boxed{265}$.

18. In a double-slit experiment, if the central diffraction peak contains 13 interference fringes, there is the $m = 0$ fringe, along with fringes up to $m = 6$ on each side of $\theta = 0$. Then, at angle $\theta$, the $m = 7$ interference fringe coincides with the first diffraction minima. We set this angle in Eq. 34-2a and 35-2 equal to solve for the relationship between the slit width and separation.

$$\sin\theta_1 = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \rightarrow \frac{d}{D} = \frac{m}{m'} = \frac{7}{1} = 7 \rightarrow d = 7D$$

Now, we use these equations again to find the *m* value at the second diffraction minimum, $m' = 0$.

$$\sin\theta_2 = \frac{m'\lambda}{D} = \frac{m\lambda}{d} \rightarrow m = m'\frac{d}{D} = 2\frac{7D}{D} = 14$$

Thus, the $\boxed{\text{six fringes}}$ corresponding to $m = 8$ to $m = 13$ will occur within the first and second diffraction minima.

$\boxed{19.}$ (*a*) The angle to each of the maxima of the double slit are given by Eq. 34-2a. The distance of a fringe on the screen from the center of the pattern is equal to the distance between the slit and screen multiplied by the tangent of the angle. For small angles, we can set the tangent equal to the sine of the angle. The slit spacing is found by subtracting the distance between two adjacent fringes.

$$\sin\theta_m = \frac{m\lambda}{d} \qquad y_m = \ell\tan\theta_m \approx \ell\sin\theta_m = \ell\frac{m\lambda}{d}$$

$$\Delta y = y_{m+1} - y_m = \ell\frac{(m+1)\lambda}{d} - \ell\frac{m\lambda}{d} = \frac{\ell\lambda}{d} = \frac{(1.0\,\text{m})(580\times10^{-9}\,\text{m})}{0.030\times10^{-3}\text{m}} = 0.019\,\text{m} = \boxed{1.9\,\text{cm}}$$

(*b*) We use Eq. 35-1 to determine the angle between the center and the first minimum. Then by multiplying the distance to the screen by the tangent of the angle we find the distance from the center to the first minima. The distance between the two first order diffraction minima is twice the distance from the center to one of the minima.

$$\sin\theta_1 = \frac{\lambda}{D} \rightarrow \theta_1 = \sin^{-1}\frac{\lambda}{D} = \sin^{-1}\frac{580\times10^{-9}\,\text{m}}{0.010\times10^{-3}\text{m}} = 3.325°$$

$$y_1 = \ell\tan\theta_1 = (1.0\,\text{m})\tan 3.325° = 0.0581\text{ m}$$

$$\Delta y = 2y_1 = 2(0.0581\,\text{m}) = 0.116\,\text{m} \approx \boxed{12\,\text{cm}}$$

20. We set $d = D$ in Eqs. 34-4 and 35-6 to show $\beta = \delta$. Replacing $\delta$ with $\beta$ in Eq. 35-9, and using the double angle formula we show that Eq. 35-9 reduces to Eq. 35-7, with $\beta' = 2\beta$. Finally using Eq. 35-6 again, we show that $\beta' = 2\beta$ implies that the new slit width $D'$ is simply double the initial slit width.

$$\delta = \frac{2\pi}{\lambda}d\sin\theta = \frac{2\pi}{\lambda}D\sin\theta = \beta$$

$$I_\theta = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2\cos^2(\delta/2) = I_0\frac{\sin^2(\beta/2)\cos^2(\beta/2)}{(\beta/2)^2} = I_0\frac{\frac{1}{4}\sin^2[2(\beta/2)]}{(\beta/2)^2}$$

$$= I_0 \frac{\sin^2 \beta}{\beta^2} = \boxed{I_0 \frac{\sin^2(\beta'/2)}{(\beta'/2)^2}}, \text{ where } \beta' = 2\beta.$$

$$\beta' = \frac{2\pi}{\lambda} D' \sin\theta = 2\left(\frac{2\pi}{\lambda} D \sin\theta\right) \rightarrow \boxed{D' = 2D}$$

21. Using Eq. 34-2a we determine the angle at which the third-order interference maximum occurs. Then we use Eq. 35-9 to determine the ratio of the intensity of the third-order maximum, where $\beta$ is given by Eq. 35-6 and $\delta$ is given by Eq. 34-4.

$$d\sin\theta = m\lambda \;\Rightarrow\; \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3\lambda}{40.0\lambda}\right) = 4.301°$$

$$\frac{\beta}{2} = \frac{2\pi}{2\lambda} D\sin\theta = \frac{\pi(40.0\lambda/5)}{\lambda}\sin(4.301°) = 1.885 \text{ rad}$$

$$\frac{\delta}{2} = \frac{2\pi d}{2\lambda}\sin\theta = \frac{\pi(40.0\lambda)}{\lambda}\sin(4.301°) = 9.424 \text{ rad}$$

$$I = I_o\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2\left[\cos\left(\frac{\delta}{2}\right)\right]^2 = I_o\left[\frac{\sin(1.885 \text{ rad})}{1.885 \text{ rad}}\right]^2\left[\cos(9.424)\right]^2 = \boxed{0.255 I_o}$$

22. We use Eq. 34-2a to determine the order of the double slit maximum that corresponds to the same angle as the first order single slit minimum, from Eq. 35-1. Since this double slit maximum is darkened, inside the central diffraction peak, there will be the zeroth order fringe and on either side of the central peak a number of maximum equal to one less than the double slit order. Therefore, there will be $2(m-1)+1$, or $2m-1$ fringes.

$$d\sin\theta = m\lambda \;\rightarrow\; m = \frac{d\sin\theta}{\lambda} = \frac{d}{\lambda}\left(\frac{\lambda}{D}\right) = \frac{d}{D} \;;\; N = 2m - 1 = 2\frac{d}{D} - 1$$

(*a*) We first set the slit separation equal to twice the slit width, $d = 2.00\,D$.

$$N = 2\frac{2.00D}{D} - 1 = \boxed{3}$$

(*b*) Next we set $d = 12.0\,D$.

$$N = 2\frac{12.00D}{D} - 1 = \boxed{23}$$

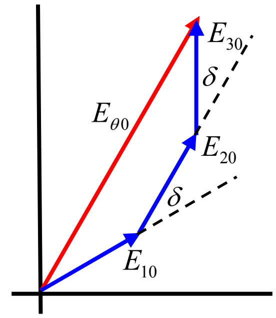
(*c*) For the previous two parts, the ratio of slits had been an integer value. This corresponded to the single slit minimum overlapping the double slit maximum. Now that $d = 4.50\,D$, the single slit minimum overlaps a double slit minimum. Therefore, the last order maximum, $m = 4$, is not darkened and $N = 2m + 1$.

$$N = 2m + 1 = 2(4) + 1 = \boxed{9}$$

(*d*) In this case the ratio of the slit separation to slit width is not an integer, nor a half-integer value. The first order single-slit minimum falls between the seventh order maximum and the seventh order minimum. Therefore, the seventh order maximum will partially be seen as a fringe.

$$N = 2m + 1 = 2(7) + 1 = \boxed{15}$$

23. (*a*) If $D \approx \lambda$, the central maximum of the diffraction pattern will be very wide. Thus we need consider only the interference between slits. We construct a phasor diagram for the interference, with $\delta = \frac{2\pi}{\lambda} d \sin\theta$ as the phase difference between adjacent slits. The magnitude of the electric fields of the slits will have the same magnitude, $E_{10} = E_{20} = E_{30} = E_0$. From the symmetry of the phasor diagram we see that $\phi = \delta$. Adding the three electric field vectors yields the net electric field.




$$E_{\theta 0} = E_{10}\cos\delta + E_{20} + E_{30}\cos\delta = E_0\left(1 + 2\cos\delta\right)$$

The central peak intensity occurs when $\delta = 0$. We set the intensity proportional to the square of the electric field and calculate the ratio of the intensities.

$$\frac{I_\theta}{I_0} = \frac{E_{\theta 0}^2}{E_{00}^2} = \frac{E_0^2\left(1+2\cos\delta\right)^2}{E_0^2\left(1+2\cos 0\right)^2} = \boxed{\frac{\left(1+2\cos\delta\right)^2}{9}}$$

(*b*) We find the locations of the maxima and minima by setting the first derivative of the intensity equal to zero.

$$\frac{dI_\theta}{d\delta} = \frac{d}{d\delta}\frac{I_0}{9}\left(1+2\cos\delta\right)^2 = \frac{2I_0}{9}\left(1+2\cos\delta\right)\left(-2\sin\delta\right) = 0$$

This equation is satisfied when either of the terms in parentheses is equal to zero. When $1+2\cos\delta = 0$, the intensity equals zero and is a minimum.

$$1 + 2\cos\delta = 0 \rightarrow \delta = \cos^{-1}\left(-\tfrac{1}{2}\right) = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \ldots$$

Maxima occur for $\sin\delta = 0$, which also says $\cos\delta = \pm 1$.

$$\sin\delta = 0 \rightarrow \delta = \sin^{-1} 0 = 0, \pi, 2\pi, 3\pi, \ldots$$

When $\cos\delta = 1$, the intensity is a principal maximum. When $\cos\delta = -1$, the intensity is a secondary maximum.

$$I_\theta(0) = I_0\frac{\left(1+2\cos\delta\right)^2}{9} = I_0\frac{\left(1+2\cos 0\right)^2}{9} = I_0$$

$$I_\theta(\pi) = I_0\frac{\left(1+2\cos\pi\right)^2}{9} = I_0\frac{\left(1+2\left(-1\right)\right)^2}{9} = \frac{I_0}{9}$$

$$I_\theta(2\pi) = I_0\frac{\left(1+2\cos 2\pi\right)^2}{9} = I_0\frac{\left(1+2\right)^2}{9} = I_0$$

Thus we see that, since $\cos\delta$ alternates between +1 and –1, there is only a single secondary maximum between each principal maximum.

24. The angular resolution is given by Eq. 35-10.

$$\theta = 1.22\frac{\lambda}{D} = 1.22\frac{560\times10^{-9}\text{ m}}{254\times10^{-2}\text{ m}} = 2.69\times10^{-7}\text{ rad}\left(\frac{180^\circ}{\pi\text{ rad}}\right)\left(\frac{3600''}{1^\circ}\right) = \boxed{0.055''}$$

25. The angular resolution is given by Eq. 35-10. The distance between the stars is the angular resolution times the distance to the stars from the Earth.

$$\theta = 1.22\frac{\lambda}{D}\ ;\ \ell = r\theta = 1.22\frac{r\lambda}{D} = 1.22\frac{\left(16\text{ly}\right)\left(\frac{9.46\times10^{15}\text{m}}{1\text{ly}}\right)\left(550\times10^{-9}\text{m}\right)}{\left(0.66\text{m}\right)} = \boxed{1.5\times10^{11}\text{m}}$$

26. We find the angle $\theta$ subtended by the planet by dividing the orbital radius by the distance of the star to the earth. Then using Eq. 35-10 we calculate the minimum diameter aperture needed to resolve this angle.

$$\theta = \frac{r}{d} = \frac{1.22\lambda}{D} \quad \rightarrow$$

$$D = \frac{1.22\lambda d}{r} = \frac{1.22\left(550\times10^{-9}\,\text{m}\right)\left(4\,\text{ly}\right)\left(9.461\times10^{15}\,\text{m/ly}\right)}{\left(1\,\text{AU}\right)\left(1.496\times10^{11}\,\text{m/AU}\right)} = 0.17\,\text{m} \approx \boxed{20\,\text{cm}}$$

27. We find the angular half-width of the flashlight beam using Eq. 35-10 with $D = 5\text{ cm}$ and $\lambda = 550\text{ nm}$. We set the diameter of the beam equal to twice the radius, where the radius of the beam is equal to the angular half-width multiplied by the distance traveled, $3.84\times10^{8}\,\text{m}$.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22\left(550\times10^{-9}\,\text{m}\right)}{0.050\text{ m}} = 1.3\times10^{-5}\,\text{rad}$$

$$d = 2\left(r\theta\right) = 2\left(3.84\times10^{8}\,\text{m}\right)\left(1.3\times10^{-5}\,\text{rad}\right) = \boxed{1.0\times10^{4}\,\text{m}}$$

28. To find the focal length of the eyepiece we use Eq. 33-7, where the objective focal length is 2.00 m, $\theta'$ is the ratio of the minimum resolved distance and 25 cm, and $\theta$ is the ratio of the object on the moon and the distance to the moon. We ignore the inversion of the image.

$$\frac{f_o}{f_e} = \frac{\theta'}{\theta} \rightarrow f_e = f_o\frac{\theta}{\theta'} = f_o\frac{\left(d_o/\ell\right)}{\left(d/N\right)} = \left(2.0\text{ m}\right)\frac{\left(7.5\text{ km}/384{,}000\text{ km}\right)}{\left(0.10\text{ mm}/250\text{ mm}\right)} = 0.098\text{ m} = \boxed{9.8\text{ cm}}$$

We use Eq. 35-10 to determine the resolution limit.

$$\theta = 1.22\frac{\lambda}{D} = 1.22\frac{560\times10^{-9}\text{ m}}{0.11\text{ m}} = \boxed{6.2\times10^{-6}\text{ rad}}$$

This corresponds to a minimum resolution distance, $r = \left(384{,}000\,\text{km}\right)\left(6.2\times10^{-6}\,\text{rad}\right) = \boxed{2.4\,\text{km}}$, which is smaller than the 7.5 km object we wish to observe.

29. We set the resolving power as the focal length of the lens multiplied by the angular resolution, as in Eq. 35-11. The resolution is the inverse of the resolving power.

$$\frac{1}{RP(f/2)} = \left[\frac{1.22\lambda f}{D}\right]^{-1} = \frac{D}{1.22\lambda f} = \frac{25\text{ mm}}{1.22\left(560\times10^{-6}\text{mm}\right)\left(50.0\text{ mm}\right)} = \boxed{730\text{ lines/mm}}$$

$$\frac{1}{RP(f/16)} = \frac{3.0\text{ mm}}{1.22\left(560\times10^{-6}\text{mm}\right)\left(50.0\text{ mm}\right)} = \boxed{88\text{ lines/mm}}$$

30. We use Eq. 35-13 to calculate the angle for the second order maximum.

$$d\sin\theta = m\lambda \rightarrow \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{2\left(480\times10^{-9}\text{ m}\right)}{1.35\times10^{-5}\text{ m}}\right) = \boxed{4.1^\circ}$$

31. We use Eq. 35-13 to calculate the wavelengths from the given angles. The slit separation, *d*, is the inverse of the number of lines per cm, *N*. We assume that 12,000 is good to 3 significant figures.

$$d\sin\theta = m\lambda \rightarrow \lambda = \frac{\sin\theta}{Nm}$$

$$\lambda_1 = \frac{\sin 28.8°}{12{,}000\text{ /cm}} = 4.01\times10^{-5}\text{ cm} = \boxed{401\text{ nm}} \qquad \lambda_2 = \frac{\sin 36.7°}{12{,}000\text{ /cm}} = 4.98\times10^{-5}\text{ cm} = \boxed{498\text{ nm}}$$

$$\lambda_3 = \frac{\sin 38.6°}{12{,}000\text{ /cm}} = 5.201\times10^{-5}\text{ cm} = \boxed{520\text{ nm}} \qquad \lambda_4 = \frac{\sin 47.9°}{12{,}000\text{ /cm}} = 6.18\times10^{-5}\text{ cm} = \boxed{618\text{ nm}}$$

32. We use Eq. 35-13 to find the wavelength, where the number of lines, *N,* is the inverse of the slit separation, or $d=1/N$.

$$d\sin\theta = m\lambda \to \lambda = \frac{\sin\theta}{mN} = \frac{\sin 26.0°}{3(3500\text{ /cm})} = 4.17\times10^{-5}\text{cm} \approx \boxed{420\text{nm}}$$

33. Because the angle increases with wavelength, to have a complete order we use the largest wavelength. We set the maximum angle is 90° to determine the largest integer *m* in Eq. 35-13.

$$d\sin\theta = m\lambda \to m = \frac{\sin\theta}{\lambda N} = \frac{\sin 90°}{(700\times10^{-9}\text{ m})(6800\text{ /cm})(100\text{ cm/m})} = 2.1$$

Thus, $\boxed{\text{two}}$ full spectral orders can be seen on each side of the central maximum, and a portion of the third order.

34. We find the slit separation from Eq. 35-13. Then set the number of lines per centimeter equal to the inverse of the slit separation, $N=1/d$.

$$d\sin\theta = m\lambda \to N = \frac{1}{d} = \frac{\sin\theta}{m\lambda} = \frac{\sin 15.0°}{3(650\times10^{-7}\text{cm})} = \boxed{1300\text{lines/cm}}$$

35. Since the same diffraction grating is being used for both wavelengths of light, the slit separation will be the same. We solve Eq. 35-13 for the slit separation for both wavelengths and set the two equations equal. The resulting equation is then solved for the unknown wavelength.

$$d\sin\theta = m\lambda \;\Rightarrow\; d = \frac{m_1\lambda_1}{\sin\theta_1} = \frac{m_2\lambda_2}{\sin\theta_2} \;\Rightarrow\; \lambda_2 = \frac{m_1}{m_2}\frac{\sin\theta_2}{\sin\theta_1}\lambda_1 = \frac{2}{1}\frac{\sin 20.6°}{\sin 53.2°}(632.8\text{nm}) = \boxed{556\text{nm}}$$

36. We find the first order angles for the maximum and minimum wavelengths using Eq. 35-13, where the slit separation distance is the inverse of the number of lines per centimeter. Then we set the distance from the central maximum of the maximum and minimum wavelength equal to the distance to the screen multiplied by the tangent of the first order angle. The width of the spectrum is the difference in these distances.

$$d\sin\theta = m\lambda \to \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(m\lambda N)$$

$$\theta_1 = \sin^{-1}\left[(410\times10^{-7}\text{cm})(7800\text{ lines/cm})\right] = 18.65°$$

$$\theta_2 = \sin^{-1}\left[(750\times10^{-7}\text{cm})(7800\text{ lines/cm})\right] = 35.80°$$

$$\Delta y = y_2 - y_1 = \ell(\tan\theta_2 - \tan\theta_1) = (2.80\text{ m})(\tan 35.80° - \tan 18.65°) = \boxed{1.1\text{m}}$$

37. We find the second order angles for the maximum and minimum wavelengths using Eq. 35-13, where the slit separation distance is the inverse of the number of lines per centimeter. Subtracting these two angles gives the angular width.

$$d\sin\theta = m\lambda \to \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}(m\lambda N)$$

$$\theta_1 = \sin^{-1}\left[2\left(4.5\times10^{-7}\text{ m}\right)\left(6.0\times10^{5}\text{ /m}\right)\right] = 32.7^\circ$$
$$\theta_2 = \sin^{-1}\left[2\left(7.0\times10^{-7}\text{ m}\right)\left(6.0\times10^{5}\text{ /m}\right)\right] = 57.1^\circ$$
$$\Delta\theta = \theta_2 - \theta_1 = 57.1^\circ - 32.7^\circ = \boxed{24^\circ}$$

38. The $m=1$ brightness maximum for the wavelength of 1200 nm occurs at angle $\theta$. At this same angle $m=2$, $m=3$, etc. brightness maximum will form for other wavelengths. To find these wavelengths, we use Eq. 35-13, where the right hand side of the equation remains constant, and solve for the wavelengths of higher order.

$$d\sin\theta = m_1\lambda_1 = m\lambda_m \Rightarrow \lambda_m = \frac{m_1\lambda_1}{m} = \frac{\lambda_1}{m}$$
$$\lambda_2 = \frac{1200\text{ nm}}{2} = 600\text{ nm} \qquad \lambda_3 = \frac{1200\text{ nm}}{3} = 400\text{ nm} \qquad \lambda_4 = \frac{1200\text{ nm}}{4} = 300\text{ nm}$$

Higher order maxima will have shorter wavelengths. Therefore in the range 360 nm to 2000 nm, the only wavelengths that have a maxima at the angle $\theta$ are $\boxed{\text{600 nm and 400 nm}}$ besides the 1200 nm.

39. Because the angle increases with wavelength, we compare the maximum angle for the second order with the minimum angle for the third order, using Eq. 35-13, by calculating the ratio of the sines for each angle. Since this ratio is greater than one, the maximum angle for the second order is larger than the minimum angle for the first order and the spectra overlap.

$$d\sin\theta = m\lambda \quad\rightarrow\quad \sin\theta = \left(\frac{m\lambda}{d}\right) \;;\quad \frac{\sin\theta_2}{\sin\theta_3} = \frac{2\lambda_2/d}{3\lambda_3/d} = \frac{2\lambda_2}{3\lambda_3} = \frac{2\left(700\text{ nm}\right)}{3\left(400\text{ nm}\right)} = 1.2$$

To determine which wavelengths overlap, we set this ratio of sines equal to one and solve for the second order wavelength that overlaps with the shortest wavelength of the third order. We then repeat this process to find the wavelength of the third order that overlaps with the longest wavelength of the second order.

$$\frac{\sin\theta_2}{\sin\theta_3} = 1 = \frac{2\lambda_2/d}{3\lambda_3/d} = \frac{2\lambda_2}{3\lambda_3} \quad\rightarrow\quad \lambda_3 = \frac{2}{3}\lambda_{2,\max} = \frac{2}{3}\left(700\text{ nm}\right) = 467\text{ nm}$$
$$\rightarrow\quad \lambda_2 = \frac{3}{2}\lambda_{3,\min} = \frac{3}{2}\left(400\text{ nm}\right) = 600\text{ nm}$$

Therefore, the wavelengths $\boxed{\text{600 nm – 700 nm of the second order}}$ overlap with the wavelengths $\boxed{\text{400 nm – 467 nm of the third order.}}$ Note that these wavelengths are independent of the slit spacing.

40. We set the diffraction angles as one half the difference between the angles on opposite sides of the center. Then we solve Eq. 35-13 for the wavelength, with *d* equal to the inverse of the number of lines per centimeter.

$$\theta_1 = \frac{\theta_r - \theta_\ell}{2} = \frac{26^\circ38' - \left(-26^\circ18'\right)}{2} = 26^\circ28' = 26 + 28/60 = 26.47^\circ$$
$$\lambda_1 = d\sin\theta = \frac{\sin\theta}{N} = \frac{\sin 26.47^\circ}{9650\text{ line/cm}} = 4.618\times10^{-5}\text{ cm} = \boxed{462\text{ nm}}$$
$$\theta_2 = \frac{\theta_{2r} - \theta_{2\ell}}{2} = \frac{41^\circ02' - \left(-40^\circ27'\right)}{2} = 40^\circ44.5' = 40 + 44.5/60 = 40.742^\circ$$
$$\lambda_2 = \frac{\sin 40.742^\circ}{9650\text{ line/cm}} = 6.763\times10^{-5}\text{ cm} = \boxed{676\text{ nm}}$$

41. If the spectrometer were immersed in water, the wavelengths calculated in Problem 40 would be wavelengths in water. To change those wavelengths into wavelengths in air, we must multiply by the index of refraction.

$$\lambda_{\substack{1\\\text{air}}} = \left(4.618\times10^{-5}\,\text{cm}\right)\left(1.33\right) = \boxed{614\,\text{nm}} \;;\; \lambda_{\substack{2\\\text{air}}} = \left(6.763\times10^{-5}\,\text{cm}\right)\left(1.33\right) = \boxed{899\,\text{nm}}$$

Note that the second wavelength is not in the visible range.

42. We solve Eq. 35-13 for the slit separation width, *d,* using the given information. Then setting *m*=3, we solve for the angle of the third order maximum.

$$\sin\theta = \frac{m\lambda}{d} \;\rightarrow\; d = \frac{m\lambda}{\sin\theta} = \frac{1\left(589\,\text{nm}\right)}{\sin 16.5^\circ} = 2074\,\text{nm} = \boxed{2.07\,\mu\text{m}}$$

$$\theta_3 = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3\times589\,\text{nm}}{2074\,\text{nm}}\right) = \boxed{58.4^\circ}$$

43. We find the angle for each "boundary" color from Eq. 35-13, and then use the fact that the displacement on the screen is given by $\tan\theta = \frac{y}{L}$, where *y* is the displacement on the screen from the central maximum, and *L* is the distance from the grating to the screen.

$$\sin\theta = \frac{m\lambda}{d} \;;\; d = \frac{1}{610\,\text{lines/mm}}\left(\frac{1\text{m}}{10^3\text{mm}}\right) = \left(1/6.1\times10^5\right)\text{m} \;;\; y = L\tan\theta = L\tan\left[\sin^{-1}\frac{m\lambda}{d}\right]$$

$$\ell_1 = L\tan\left[\sin^{-1}\frac{m\lambda_{\text{red}}}{d}\right] - L\tan\left[\sin^{-1}\frac{m\lambda_{\text{violet}}}{d}\right]$$

$$= \left(0.32\,\text{m}\right)\left\{\tan\left[\sin^{-1}\frac{(1)\left(700\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right] - \tan\left[\sin^{-1}\frac{(1)\left(400\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right]\right\}$$

$$= 0.0706\,\text{m} \approx \boxed{7\,\text{cm}}$$

$$\ell_2 = L\tan\left[\sin^{-1}\frac{m\lambda_{\text{red}}}{d}\right] - L\tan\left[\sin^{-1}\frac{m\lambda_{\text{violet}}}{d}\right]$$

$$= \left(0.32\,\text{m}\right)\left\{\tan\left[\sin^{-1}\frac{(2)\left(700\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right] - \tan\left[\sin^{-1}\frac{(2)\left(400\times10^{-9}\,\text{m}\right)}{\left(1/6.1\times10^5\right)\text{m}}\right]\right\}$$

$$= 0.3464\,\text{m} \approx \boxed{35\,\text{cm}}$$

The second order rainbow is dispersed over a larger distance.

44. (*a*) Missing orders occur when the angle to the interference maxima (Eq. 34-2a) is equal to the angle of a diffraction minimum (Eq. 35-2). We set $d = 2D$ and show that the even interference orders are missing.

$$\sin\theta = \frac{m_1\lambda}{d} = \frac{m_2\lambda}{D} \;\rightarrow\; \frac{m_1}{m_2} = \frac{d}{D} = \frac{2D}{D} = 2 \;\rightarrow\; m_1 = 2m_2$$

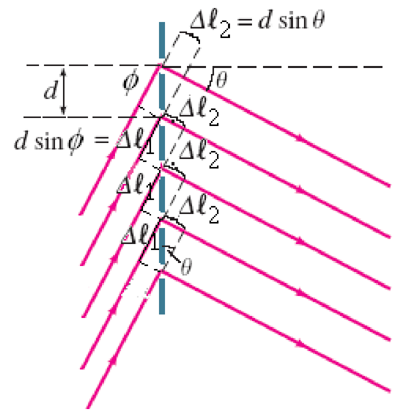
Since $m_2 = 1, 2, 3, 4, \ldots,$ all even orders of $m_1$ correspond to the diffraction minima and will be missing from the interference pattern.

(*b*) Setting the angle of interference maxima equal to the angle of diffraction minimum, with the orders equal to integers we determine the relationship between the slit size and separation that will produce missing orders.

$$\sin\theta = \frac{m_1\lambda}{d} = \frac{m_2\lambda}{D} \quad \rightarrow \quad \boxed{\frac{d}{D} = \frac{m_1}{m_2}}$$

(*c*) When $d = D$, all interference maxima will overlap with diffraction minima so that no fringes will exist. This is expected because if the slit width and separation distance are the same, the slits will merge into one single opening.

45. (*a*) Diffraction maxima occur at angles for which the incident light constructive interferes. That is, when the path length difference between two rays is equal to an integer number of wavelengths. Since the light is incident at an angle $\phi$ relative to the grating, each succeeding higher ray, as shown in the diagram, travels a distance $\Delta\ell_1 = d\sin\phi$ farther to reach the grating. After passing through the grating the higher rays travel a distance to the screen that is again longer by $\Delta\ell_2 = d\sin\theta$. By setting the total path length difference equal to an integer number of wavelengths, we are able to determine the location of the bright fringes.




$$\Delta\ell = \Delta\ell_1 + \Delta\ell_1 = d\left(\sin\phi + \sin\theta\right) = \pm m\lambda, \qquad m = 0,1,2,....$$

(*b*) The ± allows for the incident angle and the diffracted angle to have positive and negative values.

(*c*) We insert the given data, with *m*=1, to solve for the angles $\theta$.

$$\theta = \sin^{-1}\left(-\sin\phi \pm \frac{m\lambda}{d}\right) = \sin^{-1}\left(-\sin 15° \pm \frac{550\times10^{-9}\,\text{m}}{0.01\,\text{m}/5000\,\text{lines}}\right) = \boxed{0.93° \text{ and } -32°}$$

46. Using Eq. 35-13 we calculate the maximum order possible for this diffraction grating, by setting the angle equal to 90°. Then we set the resolving power equal to the product of the number of grating lines and the order, where the resolving power is the wavelength divided by the minimum separation in wavelengths (Eq. 35-19) and solve for the separation.

$$\sin\theta = \frac{m\lambda}{d} \quad \rightarrow \quad m = \frac{d\sin\theta}{\lambda} = \frac{\left(0.01\,\text{m}/6500\,\text{lines}\right)\sin 90°}{624\times10^{-9}\text{m}} = 2.47 \approx 2$$

$$\frac{\lambda}{\Delta\lambda} = Nm \;\Rightarrow\; \Delta\lambda = \frac{\lambda}{Nm} = \frac{624\text{ nm}}{\left(6500\text{ lines/cm}\right)\left(3.18\text{ cm}\right)\left(2\right)} = \boxed{0.015\text{ nm}}$$

The resolution is best for the $\boxed{\text{second order}}$, since it is more spread out than the first order.

47. (*a*) The resolving power is given by Eq. 35-19.

$$\text{R} = Nm \quad \rightarrow \quad \text{R}_1 = (16{,}000)(1) = \boxed{16{,}000} \;\; ; \;\; \text{R}_1 = (16{,}000)(2) = \boxed{32{,}000}$$

(*b*) The wavelength resolution is also given by Eq. 35-19.

$$\text{R} = \frac{\lambda}{\Delta\lambda} = Nm \quad \rightarrow \quad \Delta\lambda = \frac{\lambda}{Nm}$$

$$\Delta\lambda_1 = \frac{410\,\text{nm}}{(16{,}000)(1)} = 2.6\times10^{-2}\,\text{nm} = \boxed{26\,\text{pm}} \;\; ; \;\; \Delta\lambda_1 = \frac{410\,\text{nm}}{(32{,}000)(1)} = 1.3\times10^{-2}\,\text{nm} = \boxed{13\,\text{pm}}$$

48. (*a*) We use Eq. 35-13, with the angle equal to 90° to determine the maximum order.

$$\sin\theta = \frac{m\lambda}{d} \quad\rightarrow\quad m = \frac{d\sin\theta}{\lambda} = \frac{(1050\,\text{nm})\sin 90^\circ}{580\,\text{nm}} = 1.81$$

Since the order must be an integer number there will only be one principal maximum on either side of the central maximum. Counting the central maximum and the two other principal maxima there will be a total of $\boxed{\text{three principal maxima.}}$

(*b*) We use Eq. 35-17 to calculate the peak width, where the full peak width is double the half-peak width and the angle to the peak is given by Eq. 35-13.

$$\theta_0 = 0$$

$$\Delta\theta_0 = 2\frac{\lambda}{Nd\cos\theta_0} = \frac{2\lambda}{\ell\cos\theta_0} = \frac{2(580\,\text{nm})}{(1.80\times10^{-2}\,\text{m})\cos 0^\circ} = 6.4\times10^{-5}\,\text{rad} = \boxed{0.0037^\circ}$$

$$\theta_{\pm1} = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{\pm1\times580\,\text{nm}}{1050\,\text{nm}}\right) = \pm33.5^\circ$$

$$\Delta\theta_{\pm1} = \frac{2\lambda}{\ell\cos\theta_{\pm1}} = \frac{2(580\,\text{nm})}{(1.80\times10^{-2}\,\text{m})\cos(\pm33.5^\circ)} = 7.7\times10^{-5}\,\text{rad} = \boxed{0.0044^\circ}$$

$\boxed{49.}$ We use Eq. 35-20, with $m = 1$.

$$m\lambda = 2d\sin\phi \quad\rightarrow\quad \phi = \sin^{-1}\frac{m\lambda}{2d} = \sin^{-1}\frac{(1)(0.138\,\text{nm})}{2(0.285\,\text{nm})} = \boxed{14.0^\circ}$$

50. We use Eq. 35-20 for X-ray diffraction.

(*a*) Apply Eq. 35-20 to both orders of diffraction.

$$m\lambda = 2d\sin\phi \quad\rightarrow\quad \frac{m_1}{m_2} = \frac{\sin\phi_1}{\sin\phi_2} \quad\rightarrow\quad \phi_2 = \sin^{-1}\left(\frac{m_2}{m_1}\sin\phi_1\right) = \sin^{-1}\left(\frac{2}{1}\sin 26.8^\circ\right) = \boxed{64.4^\circ}$$

(*b*) Use the first order data.

$$m\lambda = 2d\sin\phi \quad\rightarrow\quad \lambda = \frac{2d\sin\phi}{m} = \frac{2(0.24\,\text{nm})\sin 26.8^\circ}{1} = \boxed{0.22\,\text{nm}}$$

51. For each diffraction peak, we can measure the angle and count the order. Consider Eq. 35-20.

$$m\lambda = 2d\sin\phi \quad\rightarrow\quad \lambda = 2d\sin\phi_1 \;;\; 2\lambda = 2d\sin\phi_2 \;;\; 3\lambda = 2d\sin\phi_3$$

From each equation, all we can find is the ratio $\frac{\lambda}{d} = 2\sin\phi = \sin\phi_2 = \frac{2}{3}\sin\phi_3$. $\boxed{\text{No}}$, we cannot separately determine the wavelength or the spacing.

52. Use Eq. 35-21. Since the initial light is unpolarized, the intensity after the first polarizer will be half the initial intensity. Let the initial intensity be $I_0$.

$$I_1 = \tfrac{1}{2}I_0 \;;\; I_2 = I_1\cos^2\theta = \tfrac{1}{2}I_0\cos^2\theta \quad\rightarrow\quad \frac{I_2}{I_0} = \frac{\cos^2 65^\circ}{2} = \boxed{0.089}$$

53. If $I_0$ is the intensity passed by the first Polaroid, the intensity passed by the second will be $I_0$ when the two axes are parallel. To calculate a reduction to half intensity, we use Eq. 35-21.

$$I = I_0\cos^2\theta = \tfrac{1}{2}I_0 \quad\rightarrow\quad \cos^2\theta = \tfrac{1}{2} \quad\rightarrow\quad \theta = \boxed{45^\circ}$$

54. We assume that the light is coming from air to glass, and use Eq. 35-22b.

$$\tan\theta_p = n_{glass} = 1.58 \quad\rightarrow\quad \theta_p = \tan^{-1}1.58 = \boxed{57.7^\circ}$$

$\boxed{55.}$ The light is traveling from water to diamond. We use Eq. 35-22a.

$$\tan\theta_p = \frac{n_{diamond}}{n_{water}} = \frac{2.42}{1.33} = 1.82 \quad\rightarrow\quad \theta_p = \tan^{-1}1.82 = \boxed{61.2^\circ}$$

56. The critical angle exists when light passes from a material with a higher index of refraction $(n_1)$ into a material with a lower index of refraction $(n_2)$. Use Eq. 32-7.

$$\frac{n_2}{n_1} = \sin\theta_C = \sin 55^\circ$$

To find the Brewster angle, use Eq. 35-22a. If light is passing from high index to low index, we have the following.

$$\frac{n_2}{n_1} = \tan\theta_p = \sin 55^\circ \quad\rightarrow\quad \theta_p = \tan^{-1}(\sin 55^\circ) = \boxed{39^\circ}$$

If light is passing from low index to high index, we have the following.

$$\frac{n_1}{n_2} = \tan\theta_p = \frac{1}{\sin 55^\circ} \quad\rightarrow\quad \theta_p = \tan^{-1}\left(\frac{1}{\sin 55^\circ}\right) = \boxed{51^\circ}$$

57. Let the initial intensity of the unpolarized light be $I_0$. The intensity after passing through the first Polaroid will be $I_1 = \frac{1}{2}I_0$. Then use Eq. 35-21.

$$I_2 = I_1\cos^2\theta = \tfrac{1}{2}I_0\cos^2\theta \quad\rightarrow\quad \theta = \cos^{-1}\sqrt{\frac{2I_2}{I_0}}$$

(*a*) $\theta = \cos^{-1}\sqrt{\frac{2I_2}{I_0}} = \cos^{-1}\sqrt{\frac{2}{3}} = \boxed{35.3^\circ}$

(*b*) $\theta = \cos^{-1}\sqrt{\frac{2I_2}{I_0}} = \cos^{-1}\sqrt{\frac{2}{10}} = \boxed{63.4^\circ}$

58. For the first transmission, the angle between the light and the polarizer is $18.0^\circ$. For the second transmission, the angle between the light and the polarizer is $36.0^\circ$. Use Eq. 35-21 twice.

$$I_1 = I_0\cos^2 18.0^\circ \;;\; I_2 = I_1\cos^2 36.0^\circ = I_0\cos^2 18.0^\circ\cos^2 36.0^\circ = 0.592I_0$$

Thus the transmitted intensity is $\boxed{59.2\%}$ of the incoming intensity.

59. First case: the light is coming from water to air. Use Eq. 35-22a.

$$\tan\theta_p = \frac{n_{air}}{n_{water}} \quad\rightarrow\quad \theta_p = \tan^{-1}\frac{n_{air}}{n_{water}} = \tan^{-1}\frac{1.00}{1.33} = \boxed{36.9^\circ}$$

Second case: for total internal reflection, the light must also be coming from water into air. Use Eq. 32-7.

$$\sin\theta_C = \frac{n_{air}}{n_{water}} \quad\rightarrow\quad \theta_p = \sin^{-1}\frac{n_{air}}{n_{water}} = \sin^{-1}\frac{1.00}{1.33} = \boxed{48.8^\circ}$$

Third case: the light is coming from air to water. Use Eq. 35-22b.

$$\tan\theta_p = n_{water} \quad\rightarrow\quad \theta_p = \tan^{-1}n_{water} = \tan^{-1}1.33 = \boxed{53.1^\circ}$$

Note that the two Brewster's angles add to give $90.0^\circ$.

60. When plane-polarized light passes through a sheet oriented at an angle $\theta$, the intensity decreases according to Eq. 35-21, $I = I_0\cos^2\theta$. For $\theta = 45°$, $\cos^2\theta = \frac{1}{2}$. Thus sheets 2 through 6 will each reduce the intensity by a factor of $\frac{1}{2}$. The first sheet reduces the intensity of the unpolarized incident light by $\frac{1}{2}$ as well. Thus we have the following.

$$I = I_0\left(\tfrac{1}{2}\right)^6 = \boxed{0.016 I_0}$$

61. We assume vertically polarized light of intensity $I_0$ is incident upon the first polarizer. The angle between the polarization direction and the polarizer is $\theta$. After the light passes that first polarizer, the angle between that light and the next polarizer will be $90° - \theta$. Apply Eq. 35-21.

$$I_1 = I_0\cos^2\theta \;;\; I = I_1\cos^2(90° - \theta) = I_0\cos^2\theta\cos^2(90° - \theta) = \boxed{I_0\cos^2\theta\sin^2\theta}$$

We can also use the trigonometric identity $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ to write the final intensity as

$$I = I_0\cos^2\theta\sin^2\theta = \boxed{\tfrac{1}{4}I_0\sin^2 2\theta}.$$

$$\frac{dI}{d\theta} = \frac{d}{d\theta}\left(\tfrac{1}{4}I_0\sin^2 2\theta\right) = \tfrac{1}{4}I_0(2\sin 2\theta)(\cos 2\theta)2 = I_0\sin 2\theta\cos 2\theta = \boxed{\tfrac{1}{2}I_0\sin 4\theta}$$

$$\tfrac{1}{2}I_0\sin 4\theta = 0 \;\rightarrow\; 4\theta = 0,\, 180°,\, 360° \;\rightarrow\; \theta = 0, 45°, 90°$$

Substituting the three angles back into the intensity equation, we see that the angles $0°$ and $90°$ both give minimum intensity. The angle $\boxed{45° \text{ gives the maximum intensity}}$ of $\frac{1}{4}I_0$.

62. We set the intensity of the beam as the sum of the maximum and minimum intensities. Using Eq. 35-21, we determine the intensity of the beam after it has passed through the polarizer. Since $I_{min}$ is polarized perpendicular to $I_{max}$ and the polarizer is rotated at an angle $\phi$ from the polarization of $I_{max}$, the polarizer is oriented at an angle of $(90° - \phi)$ from $I_{min}$.

$$I_0 = I_{max} + I_{min}$$

$$I = I_0\cos^2\phi = I_{max}\cos^2\phi + I_{min}\cos^2(90 - \phi) = I_{max}\cos^2\phi + I_{min}\sin^2\phi$$

We solve the percent polarization equation for $I_{min}$ and insert the result into our intensity equation.

$$p = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \;\rightarrow\; I_{min} = \frac{1-p}{1+p}I_{max}$$

$$I = I_{max}\cos^2\phi + \left(\frac{1-p}{1+p}I_{max}\right)\sin^2\phi = I_{max}\left[\frac{(1+p)\cos^2\phi + (1-p)\sin^2\phi}{1+p}\right]$$

$$= I_{max}\left[\frac{(\cos^2\phi + \sin^2\phi) + p(\cos^2\phi - \sin^2\phi)}{1+p}\right] = \boxed{I_{max}\left[\frac{1 + p\cos 2\phi}{1+p}\right]}$$

63. Because the width of the pattern is much smaller than the distance to the screen, the angles from the diffraction pattern for this first order will be small. Thus we may make the approximation that $\sin\theta = \tan\theta$. We find the angle to the first minimum from the distances, using half the width of the full first order pattern. Then we use Eq. 35-2 to find the slit width.

$$\tan\theta_{1\,min} = \tfrac{1}{2}\frac{(8.20\,\text{cm})}{(285\,\text{cm})} = 0.01439 = \sin\theta_{1\,min}$$

$$D\sin\theta = m\lambda \;\rightarrow\; D = \frac{m\lambda}{\sin\theta} = \frac{(1)(415\,\text{nm})}{0.01439} = 2.88\times10^4\,\text{nm} = \boxed{2.88\times10^{-5}\,\text{m}}$$

64. If the original intensity is $I_0$, the first polarizers will reduce the intensity to one half the initial intensity, or $I_1 = \frac{1}{2} I_0$. Each subsequent polarizer oriented at an angle $\theta$ to the preceding one will reduce the intensity by $\cos^2\theta$, as given by Eq. 35-21. We set the final intensity equal to one quarter of the initial intensity, with $\theta = 10°$ for each polarizer and solve for the minimum number of polarizers.

$$I = \tfrac{1}{2} I_0 \left(\cos^2\theta\right)^{n-1} \quad \Rightarrow \quad n = 1 + \frac{\ln\left(2I/I_0\right)}{\ln\left(\cos^2\theta\right)} = 1 + \frac{\ln\left(2\times 0.25\right)}{\ln\left(\cos^2 10°\right)} = 23.6 \approx \boxed{24 \text{ polarizers}}$$

We round the number of lenses up to the integer number of polarizers, so that the intensity will be less than 25% of the initial intensity.

65. The lines act like a grating. We assume that we see the first diffractive order, so *m* = 1. Use Eq. 35-13.

$$d\sin\theta = m\lambda \quad \rightarrow \quad d = \frac{m\lambda}{\sin\theta} = \frac{(1)(480\,\text{nm})}{\sin 56°} = \boxed{580\,\text{nm}}$$

66. We assume the sound is diffracted when it passes through the doorway, and find the angles of the minima from Eq. 35-2.

$$\lambda = \frac{v}{f} \;;\; D\sin\theta = m\lambda = \frac{mv}{f} \quad \rightarrow \quad \theta = \sin^{-1}\frac{mv}{Df},\; m = 1, 2, 3, \ldots$$

$$m = 1:\; \theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(1)(340\,\text{m/s})}{(0.88\,\text{m})(850\,\text{Hz})} = \boxed{27°}$$

$$m = 2:\; \theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(2)(340\,\text{m/s})}{(0.88\,\text{m})(850\,\text{Hz})} = \boxed{65°}$$

$$m = 3:\; \theta = \sin^{-1}\frac{mv}{Df} = \sin^{-1}\frac{(3)(340\,\text{m/s})}{(0.88\,\text{m})(850\,\text{Hz})} = \sin^{-1}1.36 = \text{impossible}$$

Thus the whistle would not be heard clearly at angles of $\boxed{27° \text{ and } 65° \text{ on either side of the normal.}}$

$\boxed{67.}$ We find the angles for the first order from Eq. 35-13.

$$\theta_1 = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)\left(4.4\times 10^{-7}\ \text{m}\right)}{0.01\ \text{m}/7600} = 19.5°$$

$$\theta_2 = \sin^{-1}\frac{(1)\left(6.8\times 10^{-7}\ \text{m}\right)}{0.01\ \text{m}/7600} = 31.1°$$

The distances from the central white line on the screen are found using the tangent of the angle and the distance to the screen.

$$y_1 = L\tan\theta_1 = (2.5\,\text{m})\tan 19.5° = 0.89\,\text{m}$$

$$y_2 = L\tan\theta_2 = (2.5\,\text{m})\tan 31.1° = 1.51\,\text{m}$$

Subtracting these two distances gives the linear separation of the two lines.

$$y_2 - y_1 = 1.51\,\text{m} - 0.89\,\text{m} = \boxed{0.6\,\text{m}}$$

68. Because the angle increases with wavelength, to miss a complete order we use the smallest visible wavelength, 400 nm. The maximum angle is 90°. With these parameters we use Eq. 35-13 to find the slit separation, *d*. The inverse of the slit separation gives the number of lines per unit length.

$$d\sin\theta = m\lambda \quad\rightarrow\quad d = \frac{m\lambda}{\sin\theta} = \frac{2(400\,\text{nm})}{\sin 90^\circ} = \boxed{800\,\text{nm}}$$

$$\frac{1}{d} = \frac{1}{800\times10^{-7}\,\text{cm}} = \boxed{12{,}500\,\text{lines/cm}}$$

69. We find the angles for the two first-order peaks from the distance to the screen and the distances along the screen to the maxima from the central peak.

$$\tan\theta_1 = \frac{y_1}{\ell} \quad\rightarrow\quad \theta_1 = \tan^{-1}\frac{y_1}{\ell} = \tan^{-1}\frac{(3.32\,\text{cm})}{(66.0\,\text{cm})} = 2.88^\circ$$

$$\tan\theta_2 = \frac{y_2}{\ell} \quad\rightarrow\quad \theta_2 = \tan^{-1}\frac{y_2}{\ell} = \tan^{-1}\frac{(3.71\,\text{cm})}{(66.0\,\text{cm})} = 3.22^\circ$$

Inserting the wavelength of yellow sodium light and the first order angle into Eq. 35-13, we calculate the separation of lines. Then, using the separation of lines and the second angle, we calculate the wavelength of the second source. Finally, we take the inverse of the line separation to determine the number of lines per centimeter on the grating.

$$d\sin\theta_1 = m\lambda_1 \quad\rightarrow\quad d = \frac{m\lambda_1}{\sin\theta_1} = \frac{1(589\text{ nm})}{\sin 2.88^\circ} = 11{,}720\,\text{nm}$$

$$\lambda_2 = \frac{d\sin\theta_1}{m} = (11{,}720\,\text{nm})\sin 3.22^\circ = \boxed{658\,\text{nm}}$$

$$\frac{1}{d} = \frac{1\text{ line}}{11{,}720\times10^{-7}\,\text{cm}} = \boxed{853\,\text{lines/cm}}$$

70. We find the angles for the first order from Eq. 35-13, with $m = 1$. The slit spacing is the inverse of the lines/cm of the grating.

$$d = \frac{1}{8100\,\text{lines/cm}}\times\frac{1\,\text{m}}{100\,\text{cm}} = \frac{1}{8.1\times10^5}\,\text{m} \;;\; d\sin\theta = m\lambda \quad\rightarrow\quad \theta = \sin^{-1}\frac{m\lambda}{d} \quad\rightarrow$$

$$\Delta\theta = \sin^{-1}\frac{\lambda_1}{d} - \sin^{-1}\frac{\lambda_2}{d} = \sin^{-1}\frac{656\times10^{-9}\,\text{m}}{\left(\frac{1}{8.1\times10^5}\,\text{m}\right)} - \sin^{-1}\frac{410\times10^{-9}\,\text{m}}{\left(\frac{1}{8.1\times10^5}\,\text{m}\right)} = \boxed{13^\circ}$$

71. (*a*) This is very similar to Example 35-6. We use the same notation as in that Example, and solve for the distance $\ell$.

$$s = \ell\theta = \ell\frac{1.22\lambda}{D} \quad\rightarrow\quad \ell = \frac{Ds}{1.22\lambda} = \frac{(6.0\times10^{-3}\,\text{m})(2.0\,\text{m})}{1.22(560\times10^{-9}\,\text{m})} = \boxed{1.8\times10^4\,\text{m}} = 18\,\text{km}$$

(*b*) We use the same data for the eye and the wavelength.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(560\times10^{-9}\,\text{m})}{(6.0\times10^{-3}\,\text{m})} = 1.139\times10^{-4}\,\text{rad}\left(\frac{180^\circ}{\pi\,\text{rad}}\right)\left(\frac{3600''}{1^\circ}\right) = \boxed{23''}$$

Our answer is less than the real resolution, because of atmospheric effects and aberrations in the eye.

72. We first find the angular half-width for the first order, using Eq. 35-1, $\sin\theta = \dfrac{\lambda}{D}$. Since this angle is small, we may use the approximation that $\sin\theta \approx \tan\theta$. The width from the central maximum to the first minimum is given by $y = L\tan\theta$. That width is then doubled to find the width of the beam, from the first diffraction minimum on one side to the first diffraction minimum on the other side.

$$y = L\tan\theta = L\sin\theta$$

$$\Delta y = 2y = 2L\sin\theta = 2L\frac{\lambda}{D} = \frac{2\left(3.8\times10^{8}\,\text{m}\right)\left(633\times10^{-9}\,\text{m}\right)}{0.010\,\text{m}} = \boxed{4.8\times10^{4}\,\text{m}}$$

73. The distance between lines on the diffraction grating is found by solving Eq. 35-13 for *d*, the grating spacing. The number of lines per meter is the reciprocal of *d*.

$$d = \frac{m\lambda}{\sin\theta} \quad\rightarrow\quad \frac{1}{d} = \frac{\sin\theta}{m\lambda} = \frac{\sin 21.5^\circ}{(1)6.328\times10^{-7}\,\text{m}} = \boxed{5.79\times10^{5}\ \text{lines/m}}$$

74. (*a*) We calculate the wavelength of the mother's sound by dividing the speed of sound by the frequency of her voice. We use Eq. 34-2b to determine the double slit interference minima with $d = 3.0$ m.

$$\lambda = v/f = (340\ \text{m/s})/(400\ \text{Hz}) = 0.85\ \text{m}$$

$$\theta = \sin^{-1}\left[\frac{(m+\frac{1}{2})\lambda}{d}\right] = \sin^{-1}\left[\frac{(m+\frac{1}{2})(0.85\ \text{m})}{(3.0\ \text{m})}\right] = \sin^{-1}\left[0.2833\left(m+\tfrac{1}{2}\right)\right],\ m = 0,1,2,...$$

$$= \boxed{8.1^\circ,\ 25^\circ,\ 45^\circ,\ \text{and } 83^\circ}$$

We use Eq. 35-2 to determine the angles for destructive interference from single slit diffraction, with $D = 1.0$ m.

$$\theta = \sin^{-1}\left[\frac{m\lambda}{D}\right] = \sin^{-1}\left[\frac{m(0.85\ \text{m})}{(1.0\ \text{m})}\right] = \sin^{-1}\left[0.85m\right],\ m = 1,2,...$$

$$\theta = \boxed{58^\circ}$$

(*b*) We use the depth and length of the room to determine the angle the sound would need to travel to reach the son.

$$\theta = \tan^{-1}\left(\frac{8.0\ \text{m}}{5.0\ \text{m}}\right) = 58^\circ$$

This angle is close to the single slit diffraction minimum, so the son has a good explanation for not hearing her.

75. We use the Brewster angle, Eq. 35-22b, for light coming from air to water.

$$\tan\theta_{\text{p}} = n \quad\rightarrow\quad \theta_{\text{p}} = \tan^{-1} n = \tan^{-1} 1.33 = 53.1^\circ$$

This is the angle from the normal, as seen in Fig. 35-41, so the angle above the horizontal is the complement of $90.0^\circ - 53.1^\circ = \boxed{36.9^\circ}$.

76. (*a*) Let the initial unpolarized intensity be $I_0$. The intensity of the polarized light after passing the first polarizer is $I_1 = \frac{1}{2}I_0$. Apply Eq. 35-21 to find the final intensity.

$$I_2 = I_1\cos^2\theta = I_1\cos^2 90^\circ = \boxed{0}.$$

(*b*) Now the third polarizer is inserted. The angle between the first and second polarizers is $66^\circ$, so the angle between the second and third polarizers is $24^\circ$. It is still true that $I_1 = \frac{1}{2}I_0$.

$$I_2 = I_1 \cos^2 66° = \tfrac{1}{2} I_0 \cos^2 66° \;\; ; \;\; I_3 = I_2 \cos^2 24° = \tfrac{1}{2} I_0 \cos^2 66° \cos^2 24° = 0.069 \;\; \rightarrow$$

$$\frac{I_3}{I_1} = \boxed{0.069}$$

(*c*) The two crossed polarizers, which are now numbers 2 and 3, will still not allow any light to pass through them if they are consecutive to each other. Thus $\frac{I_3}{I_1} = \boxed{0}$.

77. The reduction being investigated is that which occurs when the polarized light passes through the second Polaroid. Let $I_1$ be the intensity of the light that emerges from the first Polaroid, and $I_2$ be the intensity of the light after it emerges from the second Polaroid. Use Eq. 35-21.

(*a*) $I_2 = I_1 \cos^2 \theta = 0.25 I_1 \;\; \rightarrow \;\; \theta = \cos^{-1} \sqrt{0.25} = \boxed{60°}$

(*b*) $I_2 = I_1 \cos^2 \theta = 0.10 I_1 \;\; \rightarrow \;\; \theta = \cos^{-1} \sqrt{0.10} = \boxed{72°}$

(*c*) $I_2 = I_1 \cos^2 \theta = 0.010 I_1 \;\; \rightarrow \;\; \theta = \cos^{-1} \sqrt{0.010} = \boxed{84°}$

78. (*a*) We apply Eq. 35-21 through the successive polarizers. The initial light is unpolarized. Each polarizer is then rotated $30°$ from the previous one.

$$I_1 = \tfrac{1}{2} I_0 \;\; ; \;\; I_2 = I_1 \cos^2 \theta_2 = \tfrac{1}{2} I_0 \cos^2 \theta_2 \;\; ; \;\; I_3 = I_2 \cos^2 \theta_3 = \tfrac{1}{2} I_0 \cos^2 \theta_2 \cos^2 \theta_3 \;\; ;$$

$$I_4 = I_3 \cos^2 \theta_4 = \tfrac{1}{2} I_0 \cos^2 \theta_2 \cos^2 \theta_3 \cos^2 \theta_4 = \tfrac{1}{2} I_0 \cos^2 30° \cos^2 30° \cos^2 30° = \boxed{0.21 I_0}$$

(*b*) If we remove the second polarizer, then the angle between polarizers # 1 and # 3 is now $60°$.

$$I_1 = \tfrac{1}{2} I_0 \;\; ; \;\; I_3 = I_1 \cos^2 \theta_3 = \tfrac{1}{2} I_0 \cos^2 \theta_3 \;\; ;$$

$$I_4 = I_3 \cos^2 \theta_4 = \tfrac{1}{2} I_0 \cos^2 \theta_3 \cos^2 \theta_4 = \tfrac{1}{2} I_0 \cos^2 60° \cos^2 30° = 0.094 I_0$$

The same value would result by removing the third polarizer, because then the angle between polarizers # 2 and # 4 would be $60°$. Thus we can decrease the intensity by removing either the $\boxed{\text{second or third polarizer.}}$

(*c*) If we remove both the $\boxed{\text{second and third polarizers,}}$ we will have two polarizers with their axes perpendicular, so no light will be transmitted.

$\boxed{79.}$ For the minimum aperture the angle subtended at the lens by the smallest feature is the angular resolution, given by Eq. 35-10. We let $\ell$ represent the spatial separation, and *r* represent the altitude of the camera above the ground.

$$\theta = \frac{1.22\lambda}{D} = \frac{\ell}{r} \;\; \rightarrow \;\; D = \frac{1.22\lambda r}{\ell} = \frac{1.22\left(580 \times 10^{-9}\,\text{m}\right)\left(25000\,\text{m}\right)}{\left(0.05\,\text{m}\right)} = 0.3538\,\text{m} \approx \boxed{0.4\,\text{m}}$$

80. Let $I_0$ be the initial intensity. Use Eq. 35-21 for both transmissions of the light.

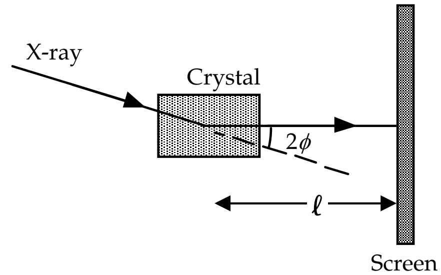
$$I_1 = I_0 \cos^2 \theta_1 \;\; ; \;\; I_2 = I_1 \cos^2 \theta_2 = I_0 \cos^2 \theta_1 \cos^2 \theta_2 = 0.25 I_0 \;\; \rightarrow$$

$$\theta_1 = \cos^{-1}\left(\frac{\sqrt{0.25}}{\cos\theta_2}\right) = \cos^{-1}\left(\frac{\sqrt{0.25}}{\cos 48°}\right) = \boxed{42°}$$

81. We find the spacing from Eq. 35-20.

$$m\lambda = 2d \sin\phi \;\; \rightarrow \;\; d = \frac{m\lambda}{2\sin\phi} = \frac{(2)\left(9.73 \times 10^{-11}\,\text{m}\right)}{2 \sin 23.4°} = \boxed{2.45 \times 10^{-10}\,\text{m}}$$

82. The angles for Bragg scattering are found from Eq. 35-20, for $m = 1$ and $m = 2$. If the distance from the crystal to the screen is $\ell$, the radius of the diffraction ring is given by $r = \ell \tan 2\phi$.




$$2d \sin\phi = m\lambda \; ; \; r = \ell \tan 2\phi = \ell \tan\left[2\sin^{-1}\left(\frac{m\lambda}{2d}\right)\right]$$

$$r_1 = \ell \tan\left[2\sin^{-1}\left(\frac{m\lambda}{2d}\right)\right]$$

$$= (0.12\,\text{m})\tan\left[2\sin^{-1}\left(\frac{(1)(0.10\times10^{-9}\,\text{m})}{2(0.22\times10^{-9}\,\text{m})}\right)\right] = \boxed{0.059\,\text{m}}$$

$$r_2 = \ell \tan\left[2\sin^{-1}\left(\frac{m\lambda}{2d}\right)\right] = (0.12\,\text{m})\tan\left[2\sin^{-1}\left(\frac{(2)(0.10\times10^{-9}\,\text{m})}{2(0.22\times10^{-9}\,\text{m})}\right)\right] = \boxed{0.17\,\text{m}}$$

83. From Eq. 35-10 we calculate the minimum resolvable separation angle. We then multiply this angle by the distance between the Earth and Moon to obtain the minimum distance between two objects on the Moon that the Hubble can resolve.

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(550\times10^{-9}\,\text{m})}{2.4\,\text{m}} = 2.796\times10^{-7}\,\text{rad}$$

$$\ell = s\theta = (3.84\times10^{8}\,\text{m})(2.796\times10^{-7}\,\text{rad}) = \boxed{110\,\text{m}}$$

84. From Eq. 35-10 we calculate the minimum resolvable separation angle. We then multiply this angle by the distance between Mars and Earth to obtain the minimum distance between two objects that can be resolved by a person on Mars

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22(550\times10^{-9}\ \text{m})}{0.005\ \text{m}} = 1.34\times10^{-4}\ \text{rad}$$

$$\ell = s\theta = (8\times10^{10}\ \text{m})(1.34\times10^{-4}\ \text{rad}) = \boxed{1.07\times10^{7}\ \text{m}}$$

Since the minimum resolvable distance is much less than the Earth-Moon distance, a person standing on Mars could resolve the Earth and Moon as two separate objects without a telescope.

85. The distance $x$ is twice the distance to the first minima. We can write $x$ in terms of the slit width $D$ using Eq. 35-2, with $m = 1$. The ratio $\frac{\lambda}{D}$ is small, so we may approximate $\sin\theta \approx \tan\theta \approx \theta$.

$$\sin\theta = \frac{\lambda}{D} \approx \theta \; ; \; x = 2y = 2\ell \tan\theta = 2\ell\theta = 2\ell\frac{\lambda}{D}$$

When the plate is heated up the slit width increases due to thermal expansion. Eq. 17-1b is used to determine the new slit width, with the coefficient of thermal expansion, $\alpha$, given in Table 17-1. Each slit width is used to determine a value for $x$. Subtracting the two values for $x$ gives the change $\Delta x$. We use the binomial expansion to simplify the evaluation.

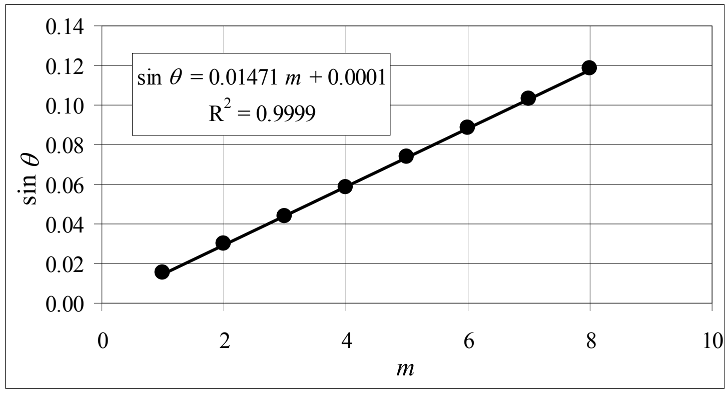
$$\Delta x = x - x_0 = 2\ell\left(\frac{\lambda}{D_0(1+\alpha\Delta T)}\right) - 2\ell\left(\frac{\lambda}{D_0}\right) = \frac{2\ell\lambda}{D_0}\left(\frac{1}{(1+\alpha\Delta T)} - 1\right) = \frac{2\ell\lambda}{D_0}\left((1+\alpha\Delta T)^{-1} - 1\right)$$

$$= \frac{2\ell\lambda}{D_0}(1 - \alpha\Delta T - 1) = -\frac{2\ell\lambda}{D_0}\alpha\Delta T = -\frac{2(2.0\,\text{m})(650\times10^{-9}\,\text{m})}{(22\times10^{-6}\,\text{m})}\left[25\times10^{-6}(\text{C}^\circ)^{-1}\right](55\text{C}^\circ)$$

$$= \boxed{-1.7\times10^{-4}\,\text{m}}$$

86. The tangent of the angle for each order is the distance in the table divided by the distance to the screen. If we call the distance in the table $y$ and the distance to the screen $\ell$, then we have this relationship.

$$\tan\theta = \frac{y}{\ell} \quad \rightarrow \quad \theta = \tan^{-1}\frac{y}{\ell}$$




The relationship between the angle and the wavelength is given by Eq. 35-2, $D\sin\theta = m\lambda$, which can be written as $\sin\theta = \frac{\lambda}{D}m$. A plot of $\sin\theta$ vs. $m$ should have a slope of $\frac{\lambda}{D}$, and so the wavelength can be determined from the slope and the slit width. The graph is shown, and the slope used to calculate the wavelength.

$$\frac{\lambda}{D} = \text{slope} \quad \rightarrow \quad \lambda = (\text{slope})D = (0.01471)(4.000\times10^{-5}\,\text{m}) = \boxed{588.4\,\text{nm}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.86."

87. We have N polarizers providing a rotation of $90°$. Thus, each polarizer must rotate the light by an angle of $\theta_N = (90/N)°$. As the light passes through each polarizer, the intensity will be reduced by a factor of $\cos^2\theta_N$. Let the original intensity be $I_0$.

$$I_1 = I_0\cos^2\theta_N \ ; \ I_2 = I_1\cos^2\theta_N = I_0\cos^4\theta_N \ ; \ I_3 = I_2\cos^2\theta_N = I_0\cos^6\theta_N$$

$$I_N = I_0(\cos\theta_N)^{2N} = 0.90 I_0 \quad \rightarrow \quad \left[\cos(90°/N)\right]^{2N} = 0.90$$

We evaluate $\left[\cos(90°/N)\right]^{2N}$ for various values of N. A table for a few values of N is shown here. We see that N = 24 satisfies the criteria, and so $\theta_N = (90/24N)° = (90/24N)° = 3.75°$. So we need to $\boxed{\text{put 24 polarizers in the path of the original polarized light, each rotated } 3.75° \text{ from the previous one.}}$ The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.87."

| $N$ | $[\cos(90/N)]^{2N}$ |
|---|---|
| 21 | 0.8890 |
| 22 | 0.8938 |
| 23 | 0.8982 |
| 24 | 0.9022 |
| 25 | 0.9060 |

88. (*a*) The intensity of the diffraction pattern is given by Eqs. 35-6 and 35-7. We want to find the angle where $I = \frac{1}{2}I_0$. Doubling this angle will give the desired $\Delta\theta$.

$$I_\theta = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = \tfrac{1}{2}I_0 \quad \rightarrow \quad \sin\beta/2 = \frac{\beta/2}{\sqrt{2}} \ \text{or} \ \sin\alpha = \frac{\alpha}{\sqrt{2}}, \ \text{with} \ \alpha = \tfrac{1}{2}\beta$$

This equation must be solved numerically. A spreadsheet was developed to find the non-zero values of $\alpha$ that satisfy $\sin\alpha - \frac{\alpha}{\sqrt{2}} = 0$. It is apparent from this expression that there will be no solutions for $\alpha > \sqrt{2}$. The only non-zero value is $\alpha = 1.392$. Now use Eq. 35-6 to find $\theta$.

$$\beta = \frac{2\pi}{\lambda}D\sin\theta \quad \rightarrow \quad \theta = \sin^{-1}\frac{\lambda\beta}{2\pi D} = \sin^{-1}\frac{2\lambda\alpha}{2\pi D} = \sin^{-1}\frac{\lambda(1.392)}{\pi D} \ ;$$

$$\Delta\theta = 2\theta = \boxed{2\sin^{-1}\frac{\lambda(1.392)}{\pi D}}$$

(*b*) For $D=\lambda$:

$$\Delta\theta = 2\sin^{-1}\frac{\lambda(1.392)}{\pi D} = 2\sin^{-1}\frac{(1.392)}{\pi} = \boxed{52.6^\circ}$$

For $D=100\lambda$:

$$\Delta\theta = 2\sin^{-1}\frac{\lambda(1.392)}{\pi D} = 2\sin^{-1}\frac{(1.392)}{100\pi} = \boxed{0.508^\circ}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH35.XLS," on tab "Problem 35.88."

# CHAPTER 37: Early Quantum Theory and Models of the Atom

## Responses to Questions

1. A reddish star is the coolest, followed by a whitish-yellow star. Bluish stars have the highest temperatures. The temperature of the star is related to the frequency of the emitted light. Since red light has a lower frequency than blue light, red stars have a lower temperature than blue stars.

2. The energy radiated by an object may not be in the visible part of the electromagnetic spectrum. The spectrum of a blackbody with a temperature of 1000 K peaks in the IR and the object appears red, since it includes some radiation at the red end of the visible spectrum. Cooler objects will radiate less overall energy and peak at even longer wavelengths. Objects that are cool enough will not radiate any energy at visible wavelengths.

3. The lightbulb will not produce light as white as the Sun, since the peak of its emitted light is in the infrared. The lightbulb will appear more yellowish than the Sun, which has a spectrum that peaks in the visible range.

4. A bulb which appears red would emit very little radiant energy at higher visible frequencies and therefore would not expose black and white photographic paper. This strategy would not work in a darkroom for developing color photographs since the photographic paper would be sensitive to light at all visible frequencies, including red.

5. If the threshold wavelength increases for the second metal, then it has a smaller work function than the first metal. Longer wavelength corresponds to lower energy. It will take less energy for the electron to escape the surface of the second metal.

6. According to the wave theory, light of any frequency can cause electrons to be ejected as long as the light is intense enough. A higher intensity corresponds to a greater electric field magnitude and more energy. Therefore, there should be no frequency below which the photoelectric effect does not occur. According to the particle theory, however, each photon carries an amount of energy which depends upon its frequency. Increasing the intensity of the light increases the number of photons but does not increase the energy of the individual photons. The cutoff frequency is that frequency at which the energy of the photon equals the work function. If the frequency of the incoming light is below the cutoff, the electrons will not be ejected because no individual photon has enough energy to impart to an electron.

7. Individual photons of ultraviolet light are more energetic than photons of visible light and will deliver more energy to the skin, causing burns. UV photons also can penetrate farther into the skin, and, once at the deeper level, can deposit a large amount of energy that can cause damage to cells.

8. Cesium will give a higher maximum kinetic energy for the electrons. Cesium has a lower work function, so more energy is available for the kinetic energy of the electrons.

9. (*a*) No. The energy of a beam of photons depends not only on the energy of each individual photon but also on the total number of photons. If there are enough infrared photons, the infrared beam may have more energy than the ultraviolet beam.
   (*b*) Yes. The energy of a single photon depends on its frequency: $E = hf$. Since infrared light has a lower frequency than ultraviolet light, a single IR photon will always have less energy than a single UV photon.

10. Fewer electrons are emitted from the surface struck by the 400 nm photons. Each 400 nm photon has a higher energy than each 450 nm photon, so it will take fewer 400 nm photons to produce the same intensity (energy per unit area per unit time) as the 450 nm photon beam. The maximum kinetic energy of the electrons emitted from the surface struck by the 400 nm photons will be greater than the maximum kinetic energy of the electrons emitted from the surface struck by the 450 nm photons, again because each 400 nm photon has a higher energy.

11. (*a*) In a burglar alarm, when the light beam is interrupted (by an intruder, or a door or window opening), the current stops flowing in the circuit. An alarm could be set to go off when the current stops.
    (*b*) In a smoke detector, when the light beam is obscured by smoke, the current in the circuit would decrease or stop. An alarm could be set to go off when the current decreased below a certain level.
    (*c*) The amount of current in the circuit depends on the intensity of the light, as long as the frequency of the light is above the threshold frequency. The ammeter in the circuit could be calibrated to reflect the light intensity.

12. Yes, the wavelength increases. In the scattering process, some of the energy of the incident photon is transferred to the electron, so the scattered photon has less energy, and therefore a lower frequency and longer wavelength, than the incident photon. ($E = hf = hc/\lambda$.)

13. In the photoelectric effect the photon energy is completely absorbed by the electron. In the Compton effect, the photon is scattered from the electron and travels off at a lower energy.

14. According to both the wave theory and the particle theory the intensity of a point source of light decreases as the inverse square of the distance from the source. In the wave theory, the intensity of the waves obeys the inverse square law. In the particle theory, the surface area of a sphere increases with the square of the radius, and therefore the density of particles decreases with distance, obeying the inverse square law. The variation of intensity with distance cannot be used to help distinguish between the two theories.

15. The proton will have the shorter wavelength, since it has a larger mass than the electron and therefore a larger momentum $\left(\lambda = h/p\right)$.

16. Light demonstrates characteristics of both waves and particles. Diffraction and interference are wave characteristics, and are demonstrated, for example, in Young's double-slit experiment. The photoelectric effect and Compton scattering are examples of experiments in which light demonstrates particle characteristics. We can't say that light IS a wave or a particle, but it has properties of each.

17. Electrons demonstrate characteristics of both waves and particles. Electrons act like waves in electron diffraction and like particles in the Compton effect and other collisions.

18. Both a photon and an electron have properties of waves and properties of particles. They can both be associated with a wavelength and they can both undergo scattering. An electron has a negative charge and a rest mass, obeys the Pauli exclusion principle, and travels at less than the speed of light. A photon is not charged, has no rest mass, does not obey the Pauli exclusion principle, and travels at the speed of light.

19. Opposite charges attract, so the attractive Coulomb force between the positive nucleus and the negative electrons keeps the electrons from flying off into space.

20. Look at a solar absorption spectrum, measured above the Earth's atmosphere. If there are dark (absorption) lines at the wavelengths corresponding to oxygen transitions, then there is oxygen near the surface of the Sun.

21. At room temperature, nearly all the atoms in hydrogen gas will be in the ground state. When light passes through the gas, photons are absorbed, causing electrons to make transitions to higher states and creating absorption lines. These lines correspond to the Lyman series since that is the series of transitions involving the ground state or n = 1 level. Since there are virtually no atoms in higher energy states, photons corresponding to transitions from $n \geq 2$ to higher states will not be absorbed.

22. The closeness of the spacing between energy levels near the top of Figure 37-26 indicates that the energy differences between these levels are small. Small energy differences correspond to small wavelength differences, leading to the closely spaced spectral lines in Figure 37-21.

23. There is no direct connection between the size of a particle and its de Broglie wavelength. It is possible for the wavelength to be smaller or larger than the particle.

24. On average the electrons of helium are closer to the nucleus than the electrons of hydrogen. The nucleus of helium contains two protons (positive charges), and so attracts each electron more strongly than the single proton in the nucleus of hydrogen. (There is some shielding of the nuclear charge by the second electron, but each electron still feels the attractive force of more than one proton's worth of charge.)

25. The lines in the spectrum of hydrogen correspond to all the possible transitions that the electron can make. The Balmer lines, for example, correspond to an electron moving from all higher energy levels to the n = 2 level. Although an individual hydrogen atom only contains one electron, a sample of hydrogen gas contains many atoms and all the different atoms will be undergoing different transitions.

26. The Balmer series spectral lines are in the visible light range and could be seen by early experimenters without special detection equipment.

27. The photon carries momentum, so according to conservation of momentum, the hydrogen atom will recoil as the photon is ejected. Some of the energy emitted in the transition of the atom to a lower energy state will be the kinetic energy of the recoiling atom, so the photon will have slightly less energy than predicted by the simple difference in energy levels.

28. No. At room temperature, virtually all the atoms in a sample of hydrogen gas will be in the ground state. Thus, the absorption spectrum will contain primarily just the Lyman lines, as photons corresponding to transitions from the n = 1 level to higher levels are absorbed. Hydrogen at very high temperatures will have atoms in excited states. The electrons in the higher energy levels will fall to all lower energy levels, not just the n = 1 level. Therefore, emission lines corresponding to transitions to levels higher than n = 1 will be present as well as the Lyman lines. In general, you would expect to see only Lyman lines in the absorption spectrum of room temperature hydrogen, but you would find Lyman, Balmer, Paschen, and other lines in the emission spectrum of high-temperature hydrogen.

## Solutions to Problems

In several problems, the value of *hc* is needed. We often use the result of Problem 96, $hc = 1240\,\text{eV}\bullet\text{nm}$.

1. We use Wien's law, Eq. 37-1.

(*a*) $$\lambda_P = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{T} = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(273\,\text{K})} = 1.06\times10^{-5}\,\text{m} = \boxed{10.6\,\mu\text{m}}$$

This wavelength is in the $\boxed{\text{far infrared.}}$

(*b*) $$\lambda_P = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{T} = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(3500\,\text{K})} = 8.29\times10^{-7}\,\text{m} = \boxed{829\,\text{nm}}$$

This wavelength is in the $\boxed{\text{infrared.}}$

(*c*) $$\lambda_P = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{T} = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(4.2\,\text{K})} = 6.90\times10^{-4}\,\text{m} = \boxed{0.69\,\text{mm}}$$

This wavelength is in the $\boxed{\text{microwave}}$ region.

(*d*) $$\lambda_P = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{T} = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(2.725\,\text{K})} = 1.06\times10^{-3}\,\text{m} = \boxed{1.06\,\text{mm}}$$

This wavelength is in the $\boxed{\text{microwave}}$ region.

2. We use Wien's law to find the temperature for a peak wavelength of 460 nm.

$$T = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{\lambda_P} = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(460\times10^{-9}\,\text{m})} = \boxed{6300\,\text{K}}$$

3. Because the energy is quantized according to Eq. 37-2, the difference in energy between adjacent levels is simply *E* = *nhf*.

$$\Delta E = hf = (6.63\times10^{-34}\,\text{J}\bullet\text{s})(8.1\times10^{13}\,\text{Hz}) = \boxed{5.4\times10^{-20}\,\text{J} = 0.34\,\text{eV}}$$

4. We use Eq. 37-1 with a temperature of $98°\text{F} = 37°\text{C} = 310\,\text{K}$.

$$\lambda_P = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{T} = \frac{(2.90\times10^{-3}\,\text{m}\bullet\text{K})}{(310\,\text{K})} = 9.4\times10^{-6}\,\text{m} = \boxed{9.4\,\mu\text{m}}$$

5. (*a*) Wien's displacement law says that $\lambda_P T =$ constant. We must find the wavelength at which $I(\lambda, T)$ is a maximum for a given temperature. This can be found by setting $\partial I/\partial\lambda = 0$.

$$\frac{\partial I}{\partial\lambda} = \frac{\partial}{\partial\lambda}\left(\frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}\right) = 2\pi hc^2\frac{\partial}{\partial\lambda}\left(\frac{\lambda^{-5}}{e^{hc/\lambda kT}-1}\right)$$

$$= 2\pi hc^2\left[\frac{(e^{hc/\lambda kT}-1)(-5\lambda^{-6}) - \lambda^{-5}e^{hc/\lambda kT}\left(-\dfrac{hc}{kT\lambda^2}\right)}{(e^{hc/\lambda kT}-1)^2}\right]$$

$$= \frac{2\pi hc^2}{\lambda^6(e^{hc/\lambda kT}-1)^2}\left[5 + e^{hc/\lambda kT}\left(\frac{hc}{kT\lambda}-5\right)\right] = 0 \;\rightarrow\; 5 = e^{hc/\lambda kT}\left(5-\frac{hc}{kT\lambda}\right) \;\rightarrow$$

$$e^x(5-x)=5\ ;\ x=\frac{hc}{\lambda_{\text{P}}kT}$$

This transcendental equation will have some solution $x$ = constant, and so $\frac{hc}{\lambda_{\text{P}}kT}$ = constant, and so $\boxed{\lambda_{\text{P}}T=\text{constant}}$. The constant could be evaluated from solving the transcendental equation,

(*b*) To find the value of the constant, we solve $e^x(5-x)=5$, or $5-x=5e^{-x}$. This can be done graphically, by graphing both $y=5-x$ and $y=5e^{-x}$ on the same set of axes and finding the intersection point. Or, the quantity $5-x-5e^{-x}$ could be calculated, and find for what value of $x$ that expression is 0. The answer is $x$ = 4.966. We use this value to solve for $h$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.5."

$$\frac{hc}{\lambda_{\text{P}}kT}=4.966\ \rightarrow$$

$$h=4.966\frac{\lambda_{\text{P}}Tk}{c}=4.966\frac{\left(2.90\times10^{-3}\,\text{m}\bullet\text{K}\right)\left(1.38\times10^{-23}\,\text{J/K}\right)}{3.00\times10^{8}\,\text{m/s}}=\boxed{6.62\times10^{-34}\,\text{J}\bullet\text{s}}$$

(*c*) We integrate Planck's radiation formula over all wavelengths.

$$\int_0^\infty I(\lambda,T)\,d\lambda=\int_0^\infty\left(\frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}\right)d\lambda\ ;\ \text{let}\ \frac{hc}{\lambda kT}=x\ ;\ \lambda=\frac{hc}{xkT}\ ;\ d\lambda=-\frac{hc}{x^2kT}dx$$

$$\int_0^\infty I(\lambda,T)\,d\lambda=\int_0^\infty\left(\frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}\right)d\lambda=\int_\infty^0\left(\frac{2\pi hc^2\left(\frac{hc}{xkT}\right)^{-5}}{e^x-1}\right)\left(-\frac{hc}{x^2kT}dx\right)=\frac{2\pi k^4T^4}{h^3c^2}\int_0^\infty\left(\frac{x^3}{e^x-1}\right)dx$$

$$=\frac{2\pi k^4}{h^3c^2}\left[\int_0^\infty\left(\frac{x^3}{e^x-1}\right)dx\right]T^4\propto T^4$$

Thus the total radiated power per unit area is proportional to $T^4$. Everything else in the expression is constant with respect to temperature.

6. We use Eq. 37-3.

$$E=hf=\left(6.626\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(104.1\times10^{6}\,\text{Hz}\right)=\boxed{6.898\times10^{-26}\,\text{J}}$$

$\boxed{7.}$ We use Eq. 37-3 along with the fact that $f=c/\lambda$ for light. The longest wavelength will have the lowest energy.

$$E_1=hf_1=\frac{hc}{\lambda_1}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(410\times10^{-9}\,\text{m}\right)}=4.85\times10^{-19}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)=3.03\,\text{eV}$$

$$E_2=hf_2=\frac{hc}{\lambda_2}=\frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(750\times10^{-9}\,\text{m}\right)}=2.65\times10^{-19}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)=1.66\,\text{eV}$$

Thus the range of energies is $\boxed{2.7\times10^{-19}\,\text{J}<E<4.9\times10^{-19}\,\text{J}}$ or $\boxed{1.7\,\text{eV}<E<3.0\,\text{eV}}$.

8. We use Eq. 37-3 with the fact that $f = c/\lambda$ for light.

$$\lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(380\times10^{3}\,\text{eV})} = 3.27\times10^{-12}\,\text{m} \approx \boxed{3.3\times10^{-3}\,\text{nm}}$$

Significant diffraction occurs when the opening is on the order of the wavelength. Thus there would be $\boxed{\text{insignificant diffraction}}$ through the doorway.

9. We use Eq. 37-3 with the fact that $f = c/\lambda$ for light.

$$E_{\min} = hf_{\min} \rightarrow f_{\min} = \frac{E_{\min}}{h} = \frac{(0.1\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(6.63\times10^{-34}\,\text{J}\bullet\text{s})} = 2.41\times10^{13}\,\text{Hz} \approx \boxed{2\times10^{13}\,\text{Hz}}$$

$$\lambda_{\max} = \frac{c}{f_{\min}} = \frac{(3.00\times10^{8}\,\text{m/s})}{(2.41\times10^{13}\,\text{Hz})} = 1.24\times10^{-5}\,\text{m} \approx \boxed{1\times10^{-5}\,\text{m}}$$

10. We use Eq. 37-5.

$$p = \frac{h}{\lambda} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{(6.20\times10^{-7}\,\text{m})} = \boxed{1.07\times10^{-27}\,\text{kg}\bullet\text{m/s}}$$

11. At the minimum frequency, the kinetic energy of the ejected electrons is 0. Use Eq. 37-4a.

$$K = hf_{\min} - W_0 = 0 \rightarrow f_{\min} = \frac{W_0}{h} = \frac{4.8\times10^{-19}\,\text{J}}{6.63\times10^{-34}\,\text{J}\bullet\text{s}} = \boxed{7.2\times10^{14}\,\text{Hz}}$$

12. The longest wavelength corresponds to the minimum frequency. That occurs when the kinetic energy of the ejected electrons is 0. Use Eq. 37-4a.

$$K = hf_{\min} - W_0 = 0 \rightarrow f_{\min} = \frac{c}{\lambda_{\max}} = \frac{W_0}{h} \rightarrow$$

$$\lambda_{\max} = \frac{ch}{W_0} = \frac{(3.00\times10^{8}\,\text{m/s})(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{(3.70\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})} = \boxed{3.36\times10^{-7}\,\text{m}} = 336\,\text{nm}$$

$\boxed{13.}$ The energy of the photon will equal the kinetic energy of the baseball. We use Eq. 37-3.

$$K = hf \rightarrow \tfrac{1}{2}mv^2 = h\frac{c}{\lambda} \rightarrow \lambda = \frac{2hc}{mv^2} = \frac{2(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(0.145\,\text{kg})(30.0\,\text{m/s})^2} = \boxed{3.05\times10^{-27}\,\text{m}}$$

14. We divide the minimum energy by the photon energy at 550 nm to find the number of photons.

$$E = nhf = E_{\min} \rightarrow n = \frac{E_{\min}}{hf} = \frac{E_{\min}\lambda}{hc} = \frac{(10^{-18}\,\text{J})(550\times10^{-9}\,\text{m})}{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})} = 2.77 \approx \boxed{3\text{ photons}}$$

15. The photon of visible light with the maximum energy has the least wavelength. We use 410 nm as the lowest wavelength of visible light.

$$hf_{\max} = \frac{hc}{\lambda_{\min}} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(410\times10^{-9}\,\text{m})} = 3.03\,\text{eV}$$

Electrons will not be emitted if this energy is less than the work function.
The metals with work functions greater than 3.03 eV are $\boxed{\text{copper and iron.}}$

16. (*a*) At the threshold wavelength, the kinetic energy of the photoelectrons is zero, so the work function is equal to the energy of the photon.

$$W_0 = hf - K_{max} = hf = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{520\,\text{nm}} = \boxed{2.4\,\text{eV}}$$

(*b*) The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy. We use Eq. 37-4b to calculate the maximum kinetic energy.

$$K_{max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\,\text{eV}\cdot\text{nm}}{470\,\text{nm}} - 2.38\,\text{eV} = 0.25\,\text{eV}$$

$$V_0 = \frac{K_{max}}{e} = \frac{0.25\,\text{eV}}{e} = \boxed{0.25\,\text{V}}$$

17. The photon of visible light with the maximum energy has the minimum wavelength. We use Eq. 37-4b to calculate the maximum kinetic energy.

$$K_{max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\text{ eV}\cdot\text{nm}}{410\text{ nm}} - 2.48\text{ eV} = \boxed{0.54\text{ eV}}$$

18. We use Eq. 37-4b to calculate the maximum kinetic energy. Since the kinetic energy is much less than the rest energy, we use the classical definition of kinetic energy to calculate the speed.

$$K_{max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\,\text{eV}\cdot\text{nm}}{365\,\text{nm}} - 2.48\,\text{eV} = \boxed{0.92\,\text{eV}}$$

$$K_{max} = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2K_{max}}{m}} = \sqrt{\frac{2(0.92\,\text{eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{kg}}} = \boxed{5.7\times10^5\,\text{m/s}}$$

19. We use Eq. 37-4b to calculate the work function.

$$W_0 = hf - K_{max} = \frac{hc}{\lambda} - K_{max} = \frac{1240\text{ eV}\cdot\text{nm}}{285\text{ nm}} - 1.70\text{ eV} = \boxed{2.65\text{ eV}}$$

20. Electrons emitted from photons at the threshold wavelength have no kinetic energy. We use Eq. 37-4b with the threshold wavelength to determine the work function.

$$W_0 = \frac{hc}{\lambda} - K_{max} = \frac{hc}{\lambda_{max}} = \frac{1240\text{ eV}\cdot\text{nm}}{320\text{ nm}} = 3.88\text{ eV}.$$

(*a*) We now use Eq. 36-4b with the work function determined above to calculate the kinetic energy of the photoelectrons emitted by 280 nm light.

$$K_{max} = \frac{hc}{\lambda} - W_0 = \frac{1240\text{ eV}\cdot\text{nm}}{280\text{ nm}} - 3.88\text{ eV} = \boxed{0.55\text{ eV}}$$

(*b*) Because the wavelength is greater than the threshold wavelength, the photon energy is less than the work function, so there will be $\boxed{\text{no ejected electrons.}}$

21. The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy of the photoelectrons. We use Eq. 37-4b to calculate the work function where the maximum kinetic energy is the product of the stopping voltage and electron charge.

$$W_0 = \frac{hc}{\lambda} - K_{max} = \frac{hc}{\lambda} - eV_0 = \frac{1240\text{ eV}\cdot\text{nm}}{230\text{ nm}} - (1.84\text{ V})e = \boxed{3.55\text{ eV}}$$

22. The energy required for the chemical reaction is provided by the photon. We use Eq. 37-3 for the energy of the photon, where $f = c/\lambda$.

$$E = hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV•nm}}{630 \text{ nm}} = \boxed{2.0 \text{ eV}}$$

Each reaction takes place in a molecule, so we use the appropriate conversions to convert eV/molecule to kcal/mol.

$$E = \left(\frac{2.0 \text{ eV}}{\text{molecule}}\right)\left(\frac{1.60\times10^{-19} \text{ J}}{\text{eV}}\right)\left(\frac{6.02\times10^{23} \text{ molecules}}{\text{mol}}\right)\left(\frac{\text{kcal}}{4186 \text{ J}}\right) = \boxed{45 \text{ kcal/mole}}$$

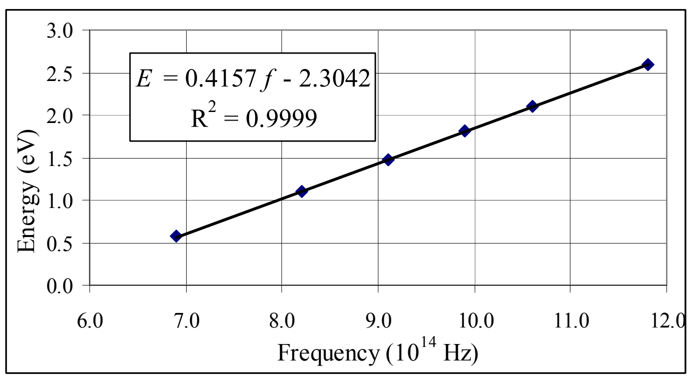
23. (*a*) Since $f = c/\lambda$, the photon energy given by Eq. 37-3 can be written in terms of the wavelength as $E = hc/\lambda$. This shows that the photon with the largest wavelength has the smallest energy. The 750-nm photon then delivers the minimum energy that will excite the retina.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J•s})(3.00\times10^{8} \text{ m/s})}{(750\times10^{-9} \text{ m})}\left(\frac{1 \text{ eV}}{1.60\times10^{-19} \text{ J}}\right) = \boxed{1.66 \text{ eV}}$$

(*b*) The eye cannot see light with wavelengths less than 410 nm. Obviously, these wavelength photons have more energy than the minimum required to initiate vision, so they must not arrive at the retina. That is, wavelength less than 410 nm are absorbed near the front portion of the eye. The threshold photon energy is that of a 410-nm photon.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J•s})(3.00\times10^{8} \text{ m/s})}{(410\times10^{-9} \text{ m})}\left(\frac{1 \text{ eV}}{1.60\times10^{-19} \text{ J}}\right) = \boxed{3.03 \text{ eV}}$$

24. We plot the maximum (kinetic) energy of the emitted electrons vs. the frequency of the incident radiation. Eq. 37-4b says $K_{max} = hf - W_0$. The best-fit straight line is determined by linear regression in Excel. The slope of the best-fit straight line to the data should give Planck's constant, the *x*-intercept is the cutoff frequency, and the *y*-intercept is the opposite of the work function. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.24."




(*a*) $h = (0.4157 \text{ eV}/10^{14} \text{Hz})(1.60\times10^{-19} \text{ J/eV}) = \boxed{6.7\times10^{-34} \text{ J•s}}$

(*b*) $hf_{cutoff} = W_0 \rightarrow f_{cutoff} = \frac{W_0}{h} = \frac{2.3042 \text{ eV}}{(0.4157 \text{ eV}/10^{14} \text{Hz})} = \boxed{5.5\times10^{14} \text{ Hz}}$

(*c*) $W_0 = \boxed{2.3 \text{ eV}}$

25. (*a*) Since $f = c/\lambda$, the photon energy is $E = hc/\lambda$ and the largest wavelength has the smallest energy. In order to eject electrons for all possible incident visible light, the metal's work function must be less than or equal to the energy of a 750-nm photon. Thus the maximum value for the metal's work function $W_o$ is found by setting the work function equal to the energy of the 750-nm photon.

$$W_o = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(750\times10^{-9}\,\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = \boxed{1.66\,\text{eV}}$$

(*b*) If the photomultiplier is to function only for incident wavelengths less than 410-nm, then we set the work function equal to the energy of the 410-nm photon.

$$W_o = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(410\times10^{-9}\,\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = \boxed{3.03\,\text{eV}}$$

26. Since $f = c/\lambda$, the energy of each emitted photon is $E = hc/\lambda$. We multiply the energy of each photon by $1.0\times10^{6}/\text{s}$ to determine the average power output of each atom. At distance of $r = 25\,\text{cm}$, the light sensor measures an intensity of $I = 1.6\,\text{nW}/1.0\,\text{cm}^2$. Since light energy emitted from atoms radiates equally in all directions, the intensity varies with distance as a spherical wave. Thus, from Section 15–3 in the text, the average power emitted is $\overline{P} = 4\pi r^2 I$. Dividing the total average power by the power from each atom gives the number of trapped atoms.

$$N = \frac{\overline{P}}{\overline{P}_{\text{atom}}} = \frac{4\pi r^2 I}{nhc/\lambda} = \frac{4\pi\left(25\,\text{cm}\right)^2\left(1.6\times10^{-9}\,\text{W/cm}^2\right)}{\left(1.0\times10^{6}/\text{s}\right)\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)/\left(780\times10^{-9}\,\text{m}\right)}$$

$$= \boxed{4.9\times10^{7}\,\text{atoms}}$$

27. We set the kinetic energy in Eq. 37-4b equal to the stopping voltage, $eV_0$, and write the frequency of the incident light in terms of the wavelength, $f = c/\lambda$. We differentiate the resulting equation and solve for the fractional change in wavelength, and we take the absolute value of the final expression.

$$eV_0 = \frac{hc}{\lambda} - W_0 \;\rightarrow\; e\,dV_0 = -\frac{hc}{\lambda^2}d\lambda \;\rightarrow\; \frac{d\lambda}{\lambda} = -\frac{e\,dV_0\lambda}{hc} \;\approx\; \boxed{\frac{\Delta\lambda}{\lambda} = \frac{e\lambda}{hc}\Delta V_0}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(550\times10^{-9}\,\text{m}\right)}{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}\left(0.01\,\text{V}\right) = \boxed{0.004}$$

28. We use Eq. 37-6b. Note that the answer is correct to two significant figures.

$$\Delta\lambda = \frac{h}{m_e c}\left(1-\cos\phi\right) \;\rightarrow$$

$$\phi = \cos^{-1}\left(1 - \frac{m_e c\Delta\lambda}{h}\right) = \cos^{-1}\left(1 - \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)\left(1.5\times10^{-13}\,\text{m}\right)}{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)}\right) = \boxed{20^\circ}$$

29. The Compton wavelength for a particle of mass *m* is $h/mc$.

(*a*) $$\frac{h}{m_e c} = \frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)} = \boxed{2.43\times10^{-12}\,\text{m}}$$

(*b*) $$\frac{h}{m_p c} = \frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)} = \boxed{1.32\times10^{-15}\,\text{m}}$$

(*c*) The energy of the photon is given by Eq. 37-3.

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{hc}{\left(h/mc\right)} = mc^2 = \text{rest energy}$$

30. We find the Compton wavelength shift for a photon scattered from an electron, using Eq. 37-6b. The Compton wavelength of a free electron is given in the text right after Eq. 37-6b.

$$\lambda' - \lambda = \left(\frac{h}{m_e c}\right)(1 - \cos\theta) = \lambda_C(1 - \cos\theta) = \left(2.43\times10^{-3}\,\text{nm}\right)(1 - \cos\theta)$$

(*a*) $\lambda'_a - \lambda = \left(2.43\times10^{-3}\,\text{nm}\right)(1-\cos 60°) = \boxed{1.22\times10^{-3}\,\text{nm}}$

(*b*) $\lambda'_b - \lambda = \left(2.43\times10^{-3}\,\text{nm}\right)(1-\cos 90°) = \boxed{2.43\times10^{-3}\,\text{nm}}$

(*c*) $\lambda'_c - \lambda = \left(2.43\times10^{-3}\,\text{nm}\right)(1-\cos 180°) = \boxed{4.86\times10^{-3}\,\text{nm}}$

31. (*a*) In the Compton effect, the maximum change in the photon's wavelength is when scattering angle $\phi = 180°$. We use Eq. 37-6b to determine the maximum change in wavelength. Dividing the maximum change by the initial wavelength gives the maximum fractional change.

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) \rightarrow$$

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{m_e c\lambda}(1-\cos\theta) = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)(1-\cos180°)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)\left(550\times10^{-9}\,\text{m}\right)} = \boxed{8.8\times10^{-6}}$$

(*b*) We replace the initial wavelength with $\lambda = 0.10$ nm.

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{m_e c\lambda}(1-\cos\theta) = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)(1-\cos180°)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)\left(0.10\times10^{-9}\,\text{m}\right)} = \boxed{0.049}$$

32. We find the change in wavelength for each scattering event using Eq. 37-6b, with a scattering angle of $\phi = 0.50°$. To calculate the total change in wavelength, we subtract the initial wavelength, obtained from the initial energy, from the final wavelength. We divide the change in wavelength by the wavelength change from each event to determine the number of scattering events.

$$\Delta\lambda = \frac{h}{m_e c}\left(1-\cos0.5°\right) = \frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(1-\cos0.5°\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.00\times10^{8}\,\text{m/s}\right)} = 9.24\times10^{-17}\,\text{m} = 9.24\times10^{-8}\,\text{nm}$$

$$\lambda_0 = \frac{hc}{E_0} = \frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.0\times10^{6}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = 1.24\times10^{-12}\,\text{m} = 0.00124\,\text{nm}.$$

$$n = \frac{\lambda-\lambda_0}{\Delta\lambda} = \frac{(555\,\text{nm})-(0.00124\,\text{nm})}{9.24\times10^{-8}\,\text{nm}} = \boxed{6\times10^{9}\,\text{events}}$$

33. (*a*) We use conservation of momentum to set the initial momentum of the photon equal to the sum of the final momentum of the photon and electron, where the momentum of the photon is given by Eq. 37-5 and the momentum of the electron is written in terms of the total energy (Eq. 36-13). We multiply this equation by the speed of light to simplify.

$$\frac{h}{\lambda} + 0 = -\left(\frac{h}{\lambda'}\right) + p_e \rightarrow \frac{hc}{\lambda} = -\left(\frac{hc}{\lambda'}\right) + \sqrt{E^2 - E_0^2}$$

Using conservation of energy we set the initial energy of the photon and rest energy of the electron equal to the sum of the final energy of the photon and the total energy of the electron.

$$\left(\frac{hc}{\lambda}\right) + E_0 = \left(\frac{hc}{\lambda'}\right) + E$$

By summing these two equations, we eliminate the final wavelength of the photon. We then solve the resulting equation for the kinetic energy of the electron, which is the total energy less the rest energy.

$$2\left(\frac{hc}{\lambda}\right)+E_0=\sqrt{E^2-E_0^{\,2}}+E \;\rightarrow\; \left[2\left(\frac{hc}{\lambda}\right)+E_0-E\right]^2=E^2-E_0^{\,2}$$

$$\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2-2E\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]+E^2=E^2-E_0^{\,2} \;\rightarrow\; E=\frac{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2+E_0^{\,2}}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}$$

$$K=E-E_0=\frac{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2+E_0^{\,2}}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}-\frac{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]E_0}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}=\frac{2\left(\frac{hc}{\lambda}\right)^2}{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}$$

$$=\frac{2\left(\frac{1240\text{ eV}\bullet\text{nm}}{0.160\text{ nm}}\right)^2}{\left[2\left(\frac{1240\text{ eV}\bullet\text{nm}}{0.160\text{ nm}}\right)+5.11\times10^5\text{eV}\right]}=\boxed{228\,\text{eV}}$$

(*b*) We solve the energy equation for the final wavelength.

$$\left(\frac{hc}{\lambda}\right)+E_0=\left(\frac{hc}{\lambda'}\right)+E$$

$$\lambda'=\frac{hc}{\left(\frac{hc}{\lambda}\right)+E_0-E}=\left[\frac{1}{\lambda}-\frac{K}{hc}\right]^{-1}=\left[\frac{1}{0.160\,\text{nm}}-\frac{228\,\text{eV}}{1240\,\text{eV}\bullet\text{nm}}\right]^{-1}=\boxed{0.165\,\text{nm}}$$

34. First we use conservation of energy, where the energy of the photon is written in terms of the wavelength, to relate the initial and final energies. Solve this equation for the electron's final energy.

$$\left(\frac{hc}{\lambda}\right)+mc^2=\left(\frac{hc}{\lambda'}\right)+E \;\Rightarrow\; E=\left(\frac{hc}{\lambda}\right)-\left(\frac{hc}{\lambda'}\right)+mc^2$$

Next, we define the *x*-direction as the direction of the initial motion of the photon. We write equations for the conservation of momentum in the horizontal and vertical directions, where $\theta$ is the angle the photon makes with the initial direction of the photon and $\phi$ is the angle the electron makes.

$$p_x:\quad \frac{h}{\lambda}=p_e\cos\phi+\frac{h}{\lambda'}\cos\theta \qquad\qquad p_y:\quad 0=p_e\sin\phi+\frac{h}{\lambda'}\sin\theta$$

To eliminate the variable $\phi$ we solve the momentum equations for the electron's momentum, square the resulting equations and add the two equations together using the identity $\cos^2\theta+\sin^2\theta=1$.

$$\left(\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\theta\right)^2=\left(p_e\cos\phi\right)^2 \qquad\qquad \left(\frac{h}{\lambda'}\sin\theta\right)^2=\left(p_e\sin\phi\right)^2$$

$$\left(p_e\cos\phi\right)^2+\left(p_e\sin\phi\right)^2=\left(\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\theta\right)^2+\left(\frac{h}{\lambda'}\sin\theta\right)^2$$

$$p_e^2=\left(\frac{h}{\lambda}\right)^2-\frac{2h^2}{\lambda\lambda'}\cos\theta+\left(\frac{h}{\lambda'}\right)^2$$

We now apply the relativistic invariant equation, Eq. 36-13, to write the electron momentum in terms of the electron energy. Then using the electron energy obtained from the conservation of energy equation, we eliminate the electron energy and solve for the change in wavelength.

$$\left(\frac{h}{\lambda}\right)^2 - \frac{2h^2}{\lambda\lambda'}\cos\theta + \left(\frac{h}{\lambda'}\right)^2 = \frac{E^2 - m^2c^4}{c^2} = \left[\left(\frac{h}{\lambda}\right) - \left(\frac{h}{\lambda'}\right) + mc\right]^2 - m^2c^2$$

$$= \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 + m^2c^2 + 2hmc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) - \frac{h^2}{\lambda\lambda'} - m^2c^2$$

$$-\frac{2h^2}{\lambda\lambda'}\cos\theta = 2hmc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) - \frac{h^2}{\lambda\lambda'}$$

$$-h\cos\theta = mc(\lambda' - \lambda) - h \quad \to \quad \boxed{\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)}$$

35. The photon energy must be equal to the kinetic energy of the products plus the mass energy of the products. The mass of the positron is equal to the mass of the electron.

$$E_{\text{photon}} = K_{\text{products}} + m_{\text{products}}c^2 \quad \to$$

$$K_{\text{products}} = E_{\text{photon}} - m_{\text{products}}c^2 = E_{\text{photon}} - 2m_{\text{electron}}c^2 = 2.67\,\text{MeV} - 2(0.511\,\text{MeV}) = \boxed{1.65\,\text{MeV}}$$

36. The photon with the longest wavelength has the minimum energy in order to create the masses with no additional kinetic energy. Use Eq. 37-5.

$$\lambda_{\text{max}} = \frac{hc}{E_{\text{min}}} = \frac{hc}{2mc^2} = \frac{h}{2mc} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{2(1.67\times10^{-27}\,\text{kg})(3.00\times10^8\,\text{m/s})} = \boxed{6.62\times10^{-16}\,\text{m}}$$

This must take place in the presence of some other object in order for momentum to be conserved.

37. The minimum energy necessary is equal to the rest energy of the two muons.

$$E_{\text{min}} = 2mc^2 = 2(207)(0.511\,\text{MeV}) = \boxed{212\,\text{MeV}}$$

The wavelength is given by Eq. 37-5.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(212\times10^6\,\text{eV})} = \boxed{5.86\times10^{-15}\,\text{m}}$$

38. Since $v < 0.001c$, the total energy of the particles is essentially equal to their rest energy. Both particles have the same rest energy of 0.511 MeV. Since the total momentum is 0, each photon must have half the available energy and equal momenta.

$$E_{\text{photon}} = m_{\text{electron}}c^2 = \boxed{0.511\,\text{MeV}} \quad ; \quad p_{\text{photon}} = \frac{E_{\text{photon}}}{c} = \boxed{0.511\,\text{MeV}/c}$$

39. The energy of the photon is equal to the total energy of the two particles produced. Both particles have the same kinetic energy and the same mass.

$$E_{\text{photon}} = 2(K + mc^2) = 2(0.375\,\text{MeV} + 0.511\,\text{MeV}) = \boxed{1.772\,\text{MeV}}$$

The wavelength is found from Eq. 37-5.

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(1.772\times10^6\,\text{eV})} = \boxed{7.02\times10^{-13}\,\text{m}}$$

40. We find the wavelength from Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{(0.23\,\text{kg})(0.10\,\text{m/s})} = \boxed{2.9\times10^{-32}\,\text{m}}$$

41. The neutron is not relativistic, so we can use $p = mv$. We also use Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(8.5\times10^{4}\,\text{m/s}\right)} = \boxed{4.7\times10^{-12}\,\text{m}}$$

42. We assume the electron is non-relativistic, and check that with the final answer. We use Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad\rightarrow\quad v = \frac{h}{m\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(0.21\times10^{-9}\,\text{m}\right)} = 3.466\times10^{6}\,\text{m/s} = 0.01155c$$

Our use of classical expressions is justified. The kinetic energy is equal to the potential energy change.

$$eV = K = \tfrac{1}{2}mv^2 = \frac{\tfrac{1}{2}\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.466\times10^{6}\,\text{m/s}\right)^2}{\left(1.60\times10^{-19}\,\text{J/eV}\right)} = 34.2\,\text{eV}$$

Thus the required potential difference is $\boxed{34\ \text{V}}$.

$\boxed{43.}$ The theoretical resolution limit is the wavelength of the electron. We find the wavelength from the momentum, and find the momentum from the kinetic energy and rest energy. We use the result from Problem 94. The kinetic energy of the electron is 85 keV.

$$\lambda = \frac{hc}{\sqrt{K^2 + 2mc^2K}} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{J/eV}\right)\sqrt{\left(85\times10^{3}\text{eV}\right)^2 + 2\left(0.511\times10^{6}\text{eV}\right)\left(85\times10^{3}\text{eV}\right)}}$$

$$= \boxed{4.1\times10^{-12}\,\text{m}}$$

44. We use the relativistic expression for momentum, Eq. 36-8.

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \frac{mv}{\sqrt{1 - v^2/c^2}} = \frac{h}{\lambda} \quad\rightarrow$$

$$\lambda = \frac{h\sqrt{1 - v^2/c^2}}{mv} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)\sqrt{1 - (0.98)^2}}{\left(9.11\times10^{-31}\,\text{kg}\right)(0.98)\left(3.00\times10^{8}\,\text{m/s}\right)} = \boxed{4.9\times10^{-13}\,\text{m}}$$

45. Since the particles are not relativistic, we may use $K = p^2/2m$. We then form the ratio of the kinetic energies, using Eq. 37-7.

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \quad ; \quad \frac{\lambda_e}{\lambda_p} = \frac{\dfrac{h^2}{2m_e\lambda^2}}{\dfrac{h^2}{2m_p\lambda^2}} = \frac{m_p}{m_e} = \frac{1.67\times10^{-27}\,\text{kg}}{9.11\times10^{-31}\,\text{kg}} = \boxed{1840}$$

46. We assume the neutron is not relativistic. If the resulting velocity is small, our assumption will be valid. We use Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \rightarrow v = \frac{h}{m\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(0.3\times10^{-9}\,\text{m}\right)} = 1300\,\text{m/s} \approx \boxed{1000\,\text{m/s}}$$

This is not relativistic, so our assumption was valid.

47. (*a*) We find the momentum from Eq. 37-7.

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{6.0\times10^{-10}\,\text{m}} = \boxed{1.1\times10^{-24}\,\text{kg}\bullet\text{m/s}}$$

(*b*) We assume the speed is non-relativistic.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \rightarrow v = \frac{h}{m\lambda} == \frac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(6.0\times10^{-10}\,\text{m}\right)} = \boxed{1.2\times10^{6}\,\text{m/s}}$$

Since $v/c = 4.04\times10^{-3}$, our assumption is valid.

(*c*) We calculate the kinetic energy classically.

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(mc^2\right)\left(v/c\right)^2 = \tfrac{1}{2}\left(0.511\,\text{MeV}\right)\left(4.04\times10^{-3}\right)^2 = 4.17\times10^{-6}\,\text{MeV} = 4.17\,\text{eV}$$

This is the energy gained by an electron if accelerated through a potential difference of $\boxed{4.2\text{ V}}$.

48. Because all of the energies to be considered are much less than the rest energy of an electron, we can use non-relativistic relationships. We use Eq. 37-7 to calculate the wavelength.

$$K = \frac{p^2}{2m} \rightarrow p = \sqrt{2mK} \;;\; \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

(*a*) $$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{\sqrt{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(20\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}} = 2.7\times10^{-10}\,\text{m} \approx \boxed{3\times10^{-10}\,\text{m}}$$

(*b*) $$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{\sqrt{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(200\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}} = 8.7\times10^{-11}\,\text{m} \approx \boxed{9\times10^{-11}\,\text{m}}$$

(*c*) $$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{\sqrt{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(2.0\times10^{3}\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}} = \boxed{2.7\times10^{-11}\,\text{m}}$$

$\boxed{49.}$ Since the particles are not relativistic, we may use $K = p^2/2m$. We then form the ratio of the wavelengths, using Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \;;\; \frac{\lambda_{\text{p}}}{\lambda_{\text{e}}} = \frac{\dfrac{h}{\sqrt{2m_{\text{p}}K}}}{\dfrac{h}{\sqrt{2m_{\text{e}}K}}} = \sqrt{\frac{m_{\text{e}}}{m_{\text{p}}}} < 1$$

Thus we see the proton has the shorter wavelength, since $m_{\text{e}} < m_{\text{p}}$.

50. The final kinetic energy of the electron is equal to the negative change in potential energy of the electron as it passes through the potential difference. We compare this energy to the rest energy of the electron to determine if the electron is relativistic.

$$K = -q\Delta V = \left(1\text{e}\right)\left(33\times10^{3}\,\text{V}\right) = 33\times10^{3}\,\text{eV}$$

Because this is greater than 1% of the electron rest energy, $\boxed{\text{the electron is relativistic}}$. We use Eq. 36-13 to determine the electron momentum and then Eq. 37-5 to determine the wavelength.

$$E^2 = \left[K + mc^2\right]^2 = p^2c^2 + m^2c^4 \;\Rightarrow p = \frac{\sqrt{K^2 + 2Kmc^2}}{c}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2Kmc^2}} = \frac{1240\,\text{eV}\bullet\text{nm}}{\sqrt{\left(33\times10^3\text{eV}\right)^2 + 2\left(33\times10^3\text{eV}\right)\left(511\times10^3\text{eV}\right)}} = 0.0066\,\text{nm}$$

Because $\lambda$ « 5 cm, $\boxed{\text{diffraction effects are negligible.}}$

51. We will assume that the electrons are non-relativistic, and then examine the result in light of that assumption. The wavelength of the electron can be found from Eq. 34-2a. The speed can then be found from Eq. 37-7.

$$d\sin\theta = m_{\text{order}}\lambda \;\rightarrow\; \lambda = \frac{d\sin\theta}{m_{\text{order}}} \;;\; \lambda = \frac{h}{p} = \frac{h}{m_e v} \;\rightarrow$$

$$v = \frac{hm_{\text{order}}}{m_e d\sin\theta} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)(2)}{\left(9.11\times10^{-31}\text{kg}\right)\left(3.0\times10^{-6}\text{m}\right)\left(\sin55^\circ\right)} = \boxed{590\,\text{m/s}}$$

This is far from being relativistic, so our original assumption was fine.

52. We relate the kinetic energy to the momentum with a classical relationship, since the electrons are non-relativistic. We also use Eq. 37-7. We then assume that the kinetic energy was acquired by electrostatic potential energy.

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = eV \;\rightarrow$$

$$V = \frac{h^2}{2me\lambda^2} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.60\times10^{-19}\text{C}\right)\left(0.28\times10^{-9}\text{m}\right)^2} = \boxed{19\,\text{V}}$$

53. The kinetic energy is 3450 eV. That is small enough compared to the rest energy of the electron for the electron to be non-relativistic. We use Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{\left(2mK\right)^{1/2}} = \frac{hc}{\left(2mc^2K\right)^{1/2}} = \frac{\left(6.63\times10^{-34}\text{J}\bullet\text{s}\right)\left(3.00\times10^8\text{m/s}\right)}{\left(1.60\times10^{-19}\text{J/eV}\right)\left[2\left(0.511\times10^6\,\text{eV}\right)\left(3450\text{ eV}\right)\right]^{1/2}}$$

$$= 2.09\times10^{-11}\text{m} = \boxed{20.9\,\text{pm}}$$

54. The energy of a level is $E_n = -\dfrac{(13.6\text{ eV})}{n^2}$.

(*a*) The transition from $n$ = 1 to $n'$ = 3 is an $\boxed{\text{absorption,}}$ because the $\boxed{\text{final state,}}$ $n'$ = 3, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = E_{n'} - E_n = -(13.6\text{ eV})\left[\left(\frac{1}{3^2}\right) - \left(\frac{1}{1^2}\right)\right] = 12.1\text{ eV}$$

(*b*) The transition from $n$ = 6 to $n'$ = 2 is an $\boxed{\text{emission,}}$ because the $\boxed{\text{initial state,}}$ $n'$ = 2, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = -\left(E_{n'} - E_n\right) = (13.6\text{ eV})\left[\left(\frac{1}{2^2}\right) - \left(\frac{1}{6^2}\right)\right] = 3.0\text{ eV}$$

(*c*) The transition from $n$ = 4 to $n'$ = 5 is an $\boxed{\text{absorption,}}$ because the $\boxed{\text{final state,}}$ $n'$ = 5, has a higher energy. The photon energy is the difference between the energies of the two states.

$$hf = E_{n'} - E_n = -(13.6\text{ eV})\left[\left(\frac{1}{5^2}\right) - \left(\frac{1}{4^2}\right)\right] = 0.31\text{ eV}$$

The photon for the transition from $\boxed{n = 1 \text{ to } n' = 3}$ has the largest energy.

55. To ionize the atom means removing the electron, or raising it to zero energy.

$$E_{\text{ionization}} = 0 - E_n = 0 - \frac{(-13.6\text{eV})}{n^2} = \frac{(13.6\text{eV})}{3^2} = \boxed{1.51\text{eV}}$$

56. We use the equation that appears above Eq. 37-15 in the text.

(*a*) The second Balmer line is the transition from $n = 4$ to $n = 2$.

$$\lambda = \frac{hc}{(E_4 - E_2)} = \frac{1240\text{eV}\bullet\text{nm}}{[-0.85\text{eV} - (-3.4\text{eV})]} = \boxed{490\text{nm}}$$

(*b*) The third Lyman line is the transition from $n = 4$ to $n = 1$.

$$\lambda = \frac{hc}{(E_4 - E_1)} = \frac{1240\text{eV}\bullet\text{nm}}{[-0.85\text{ eV} - (-13.6\text{ eV})]} = \boxed{97.3\text{nm}}$$

(*c*) The first Balmer line is the transition from $n = 3$ to $n = 2$.
For the jump from $n = 5$ to $n = 2$, we have

$$\lambda = \frac{hc}{(E_3 - E_2)} = \frac{1240\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-3.4\text{ eV})]} = \boxed{650\text{nm}}$$

57. Doubly ionized lithium is similar to hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace $e^2$ by $Ze^2$:

$$E_n = -\frac{Z^2(13.6\text{eV})}{n^2} = -\frac{3^2(13.6\text{eV})}{n^2} = -\frac{(122\text{eV})}{n^2}$$

$$E_{\text{ionization}} = 0 - E_1 = 0 - \left[-\frac{(122\text{eV})}{(1)^2}\right] = \boxed{122\text{eV}}$$

58. We evaluate the Rydberg constant using Eq. 37-8 and 37-15. We use hydrogen so $Z = 1$.

$$\frac{1}{\lambda} = R\left(\frac{1}{(n')^2} - \frac{1}{(n)^2}\right) = \frac{Z^2e^4m}{8\varepsilon_0^2h^3c}\left(\frac{1}{(n')^2} - \frac{1}{(n)^2}\right) \rightarrow$$

$$R = \frac{Z^2e^4m}{8\varepsilon_0^2h^3c} = \frac{(1)^2(1.602176\times10^{-19}\text{C})^4(9.109382\times10^{-31}\text{kg})}{8(8.854188\times10^{-12}\text{ C}^2/\text{N}\bullet\text{m}^2)^2(6.626069\times10^{-34}\text{ J}\bullet\text{s})^3(2.997925\times10^8\text{ m/s})}$$

$$= 1.0974\times10^7\frac{\text{C}^4\bullet\text{kg}}{\frac{\text{C}^4}{\text{N}^2\bullet\text{m}^4}\text{J}^3\text{s}^3\text{ m/s}} = \boxed{1.0974\times10^7\text{m}^{-1}}$$

59. The longest wavelength corresponds to the minimum energy, which is the ionization energy:

$$\lambda = \frac{hc}{E_{\text{ion}}} = \frac{(6.63\times10^{-34}\text{J}\bullet\text{s})(3.00\times10^8\text{m/s})}{(1.60\times10^{-19}\text{ J/eV})(13.6\text{eV})} = 9.14\times10^{-8}\text{m} = \boxed{91.4\text{nm}}$$

60. Singly ionized helium is like hydrogen, except that there are two positive charges ($Z = 2$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace $e^2$ by $Ze^2$.

$$E_n = -\frac{Z^2(13.6\text{ eV})}{n^2} = -\frac{2^2(13.6\text{ eV})}{n^2} = -\frac{(54.4\text{ eV})}{n^2}$$

We find the energy of the photon from the $n = 5$ to $n = 2$ transition in singly-ionized helium.

$$\Delta E = E_5 - E_2 = -(54.4\text{ eV})\left[\left(\frac{1}{5^2}\right) - \left(\frac{1}{2^2}\right)\right] = 11.4\text{ eV}$$

Because this is NOT the energy difference between any two specific energy levels for hydrogen, the photon CANNOT be absorbed by hydrogen.

61. The energy of the photon is the sum of the ionization energy of 13.6 eV and the kinetic energy of 20.0eV. The wavelength is found from Eq. 37-3.

$$hf = \frac{hc}{\lambda} = E_{\text{total}} \rightarrow \lambda = \frac{hc}{E_{\text{total}}} = \frac{(6.63\times10^{-34}\text{ J}\bullet\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ J/eV})(33.6\text{ eV})} = 3.70\times10^{-8}\text{ m} = \boxed{37.0\text{ nm}}$$

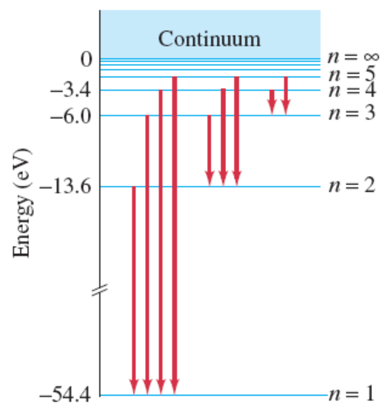
62. A collision is elastic if the kinetic energy before the collision is equal to the kinetic energy after the collision. If the hydrogen atom is in the ground state, then the smallest amount of energy it can absorb is the difference in the $n = 1$ and $n = 2$ levels. So as long as the kinetic energy of the incoming electron is less than that difference, the collision must be elastic.

$$K < E_2 - E_1 = \left(-\frac{13.6\text{eV}}{4}\right) - (-13.6\text{eV}) = \boxed{10.2\text{eV}}$$

63. Singly ionized helium is like hydrogen, except that there are two positive charges ($Z = 2$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace $e^2$ by $Ze^2$:

$$E_n = -\frac{Z^2(13.6\text{eV})}{n^2} = -\frac{2^2(13.6\text{eV})}{n^2} = -\frac{(54.4\text{eV})}{n^2}$$

$$E_1 = -54.5\text{eV},\ E_2 = -13.6\text{eV},\ E_3 = -6.0\text{eV},\ E_4 = -3.4\text{eV}$$

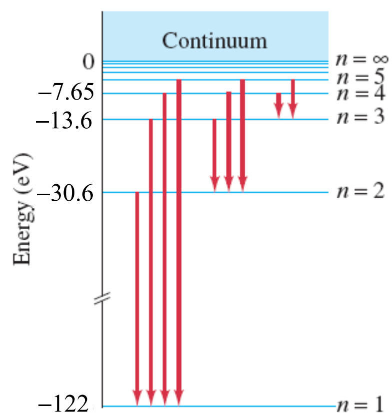



64. Doubly ionized lithium is like hydrogen, except that there are three positive charges ($Z = 3$) in the nucleus. The square of the product of the positive and negative charges appears in the energy term for the energy levels. We can use the results for hydrogen, if we replace $e^2$ by $Ze^2$:

$$E_n = -\frac{Z^2(13.6\text{eV})}{n^2} = -\frac{3^2(13.6\text{eV})}{n^2} = -\frac{(122.4\text{eV})}{n^2}$$

$$E_1 = -122\text{eV},\ E_2 = -30.6\text{eV},\ E_3 = -13.6\text{eV},$$

$$E_4 = -7.65\text{eV}$$



65. The potential energy for the ground state is given by the charge of the electron times the electric potential caused by the proton.

$$U = (-e)V_{\text{proton}} = (-e)\frac{1}{4\pi\varepsilon_0}\frac{e}{r_1} = -\frac{(9.00\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2(1\,\text{eV}/1.60\times10^{-19}\,\text{J})}{(0.529\times10^{-10}\,\text{m})}$$

$$= \boxed{-27.2\,\text{eV}}$$

The kinetic energy is the total energy minus the potential energy.

$$K = E_1 - U = -13.6\,\text{eV} - (-27.2\,\text{eV}) = \boxed{+13.6\,\text{eV}}$$

66. The value of $n$ is found from $r_n = n^2 r_1$, and then find the energy from Eq. 37-14b.

$$r_n = n^2 r_1 \quad\rightarrow\quad n = \sqrt{\frac{r_n}{r_1}} = \sqrt{\frac{\frac{1}{2}(0.10\times10^{-3}\,\text{m})}{0.529\times10^{-10}\,\text{m}}} = \boxed{972}$$

$$E = -\frac{(13.6\ \text{eV})}{n^2} = -\frac{(13.6\ \text{eV})}{972^2} = -\frac{(13.6\ \text{eV})}{1375^2} = \boxed{-1.4\times10^{-5}\,\text{eV}}$$

67. The velocity is found from Eq. 37-10 evaluated for $n = 1$.

$$mvr_n = \frac{nh}{2\pi} \quad\rightarrow$$

$$v = \frac{h}{2\pi r_1 m_e} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{2\pi(0.529\times10^{-10}\,\text{m})(9.11\times10^{-31}\,\text{kg})} = 2.190\times10^6\ \text{m/s} = \boxed{7.30\times10^{-3}c}$$

We see that $v \ll c$, and so $\boxed{\text{yes}}$, non-relativistic formulas are justified.
The relativistic factor is as follows.

$$\left[1-\left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} \approx 1 - \tfrac{1}{2}\left(\frac{v}{c}\right)^2 = 1 - \tfrac{1}{2}\left(\frac{2.190\times10^6\ \text{m/s}}{3.00\times10^8\ \text{m/s}}\right)^2 = \boxed{1 - 2.66\times10^{-5}} \approx 0.99997$$

We see that $\sqrt{1-v^2/c^2}$ is essentially 1, and so again the answer is $\boxed{\text{yes}}$, non-relativistic formulas are justified.

68. The angular momentum can be used to find the quantum number for the orbit, and then the energy can be found from the quantum number. Use Eqs. 37-10 and 37-14b.

$$L = n\frac{h}{2\pi} \quad\rightarrow\quad n = \frac{2\pi L}{h} = \frac{2\pi(5.273\times10^{-34}\ \text{kg}\bullet\text{m}^2/\text{s})}{(6.626\times10^{-34}\,\text{J}\bullet\text{s})} = 5.000 \approx 5$$

$$E_n = -(13.6\text{eV})\frac{Z^2}{n^2} = -\frac{13.6\text{eV}}{25} = \boxed{0.544\,\text{eV}}$$

69. Hydrogen atoms start in the $n = 1$ orbit ("ground state"). Using Eq. 37-9 and Eq. 37-14b, we determine the orbit to which the atom is excited when it absorbs a photon of 12.75 Ev via collision with an electron. Then, using Eq. 37-15, we calculate all possible wavelengths that can be emitted as the electron cascades back to the ground state.

$$\Delta E = E_U - E_L \quad\rightarrow\quad E_U = -\frac{13.6\ \text{eV}}{n^2} = E_L + \Delta E \quad\rightarrow$$

$$n = \sqrt{\frac{-13.6\ \text{eV}}{E_L + \Delta E}} = \sqrt{\frac{-13.6\ \text{eV}}{-13.6\ \text{eV} + 12.75\ \text{eV}}} = 4$$

Starting with the electron in the $n=4$ orbit, the following transitions are possible: $n=4$ to $n=3$; $n=4$ to $n=2$; $n=4$ to $n=1$; $n=3$ to $n=2$; $n=3$ to $n=1$; $n=2$ to $n=1$.

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=5.333\times10^{5}\text{ m}^{-1}\Rightarrow\lambda=\boxed{1875\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=2.057\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{486.2\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2}-\frac{1}{4^2}\right)=1.028\times10^{7}\text{ m}^{-1}\Rightarrow\lambda=\boxed{97.23\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=1.524\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{656.3\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2}-\frac{1}{3^2}\right)=9.751\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{102.6\text{ nm}}$$

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\text{ m}^{-1}\right)\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=8.228\times10^{6}\text{ m}^{-1}\Rightarrow\lambda=\boxed{121.5\text{ nm}}$$

70. When we compare the gravitational and electric forces we see that we can use the same expression for the Bohr orbits, Eq. 37-11 and 37-14a, if we replace $Ze^2/4\pi\varepsilon_0$ with $Gm_em_p$.

$$r_1=\frac{h^2\varepsilon_0}{\pi m_eZe^2}=\frac{h^2}{4\pi^2m_e}\frac{4\pi\varepsilon_0}{Ze^2}\quad\rightarrow$$

$$r_1=\frac{h^2}{4\pi^2Gm^2{}_em_p}=\frac{\left(6.626\times10^{-34}\,\text{J•s}\right)^2}{4\pi^2\left(6.67\times10^{-11}\,\text{N•m}^2/\text{kg}^2\right)\left(9.11\times10^{-31}\,\text{kg}\right)^2\left(1.67\times10^{-27}\,\text{kg}\right)}$$

$$=\boxed{1.20\times10^{29}\,\text{m}}$$

$$E_1=-\frac{Z^2e^4m_e}{8\varepsilon_0^2h^2}=-\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\frac{2\pi^2m_e}{h^2}\quad\rightarrow\quad E_1=-\frac{2\pi^2G^2m_e{}^3m_p{}^2}{h^2}$$

$$=-\frac{2\pi^2\left(6.67\times10^{-11}\,\text{N•m}^2/\text{kg}^2\right)^2\left(9.11\times10^{-31}\,\text{kg}\right)^3\left(1.67\times10^{-27}\,\text{kg}\right)^2}{\left(6.626\times10^{-34}\,\text{J•s}\right)^2}=\boxed{-4.22\times10^{-97}\,\text{J}}$$

71. We know that the radii of the orbits are given by $r_n=n^2r_1$. Find the difference in radius for adjacent orbits.

$$\Delta r=r_n-r_{n-1}=n^2r_1-\left(n-1\right)^2r_1=n^2r_1-\left(n^2-2n+1\right)r_1=\left(2n-1\right)r_1$$

If $n\gg1$, we have $\Delta r\approx2nr_1=2n\frac{r_n}{n^2}=\frac{2r_n}{n}$.

In the classical limit, the separation of radii (and energies) should be very small. We see that letting $n\rightarrow\infty$ accomplishes this. If we substitute the expression for $r_1$ from Eq. 37-11, we have this.

$$\Delta r\approx2nr_1=\frac{2nh^2\varepsilon_0}{\pi me^2}$$

We see that $\Delta r\propto h^2$, and so letting $h\rightarrow0$ is equivalent to considering $n\rightarrow\infty$.

72. We calculate the energy from the light bulb that enters the eye by calculating the intensity of the light at a distance of 250 m by dividing the power in the visible spectrum by the area of a sphere of radius 250 m. We multiply the intensity of the light by the area of the pupil to determine the energy entering the eye per second. We divide this energy by the energy of a photon (Eq. 37-3) to calculate the number of photons entering the eye per second.

$$I = \frac{P}{4\pi\ell^2} \qquad P_e = I\left(\pi D^2/4\right) = \frac{P}{16}\left(\frac{D}{\ell}\right)^2$$

$$n = \frac{P_e}{hc/\lambda} = \frac{P\lambda}{16hc}\left(\frac{D}{\ell}\right)^2 = \frac{0.030(75\,\text{W})(550\times10^{-9}\,\text{m})}{16(6.626\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})}\left(\frac{4.0\times10^{-3}\,\text{m}}{250\,\text{m}}\right)^2$$

$$= \boxed{1.0\times10^8\,\text{photons/sec}}$$

[73.] To produce a photoelectron, the hydrogen atom must be ionized, so the minimum energy of the photon is 13.6 eV. We find the minimum frequency of the photon from Eq. 37-3.

$$E = hf \;\rightarrow\; f = \frac{E}{h} \;\rightarrow\; f_{\text{min}} = \frac{E_{\text{min}}}{h} = \frac{(13.6\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(6.63\times10^{-34}\,\text{J}\bullet\text{s})} = \boxed{3.28\times10^{15}\,\text{Hz}}$$

74. From Section 35-10, the spacing between planes, $d$, for the first-order peaks is given by Eq. 35-20, $\lambda = 2d\sin\theta$. The wavelength of the electrons can be found from their kinetic energy. The electrons are not relativistic at the energy given.

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \;\rightarrow\; \lambda = \frac{h}{\sqrt{2mK}} = 2d\sin\theta \;\rightarrow$$

$$d = \frac{h}{2\sin\theta\sqrt{2mK}} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{2(\sin 38^\circ)\sqrt{2(9.11\times10^{-31}\,\text{kg})(125\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}} = \boxed{8.9\times10^{-11}\,\text{m}}$$

75. The power rating is the amount of energy produced per second. If this is divided by the energy per photon, then the result is the number of photons produced per second.

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} \;;\; \frac{P}{E_{\text{photon}}} = \frac{P\lambda}{hc} = \frac{(860\,\text{W})(12.2\times10^{-2}\,\text{m})}{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})} = \boxed{5.3\times10^{26}\text{ photons/s}}$$

76. The intensity is the amount of energy per second per unit area reaching the Earth. If that intensity is divided by the energy per photon, the result will be the photons per second per unit area reaching the Earth. We use Eq. 37-3.

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}$$

$$I_{\text{photons}} = \frac{I_{\text{sunlight}}}{E_{\text{photon}}} = \frac{I_{\text{sunlight}}\lambda}{hc} = \frac{(1350\,\text{W/m}^2)(550\times10^{-9}\,\text{m})}{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^8\,\text{m/s})} = \boxed{3.7\times10^{21}\text{ photons/s}\bullet\text{m}^2}$$

77. The impulse on the wall is due to the change in momentum of the photons. Each photon is absorbed, and so its entire momentum is transferred to the wall.

$$F_{\text{on wall}}\Delta t = \Delta p_{\text{wall}} = -\Delta p_{\text{photons}} = -\left(0 - np_{\text{photon}}\right) = np_{\text{photon}} = \frac{nh}{\lambda} \quad \rightarrow$$

$$\frac{n}{\Delta t} = \frac{F\lambda}{h} = \frac{\left(6.5 \times 10^{-9}\,\text{N}\right)\left(633 \times 10^{-9}\,\text{m}\right)}{\left(6.63 \times 10^{-34}\,\text{J}\bullet\text{s}\right)} = \boxed{6.2 \times 10^{18}\text{ photons/s}}$$

78. We find the peak wavelength from Wien's law, Eq. 37-1.

$$\lambda_{\text{P}} = \frac{\left(2.90 \times 10^{-3}\,\text{m}\bullet\text{K}\right)}{T} = \frac{\left(2.90 \times 10^{-3}\,\text{m}\bullet\text{K}\right)}{\left(2.7\,\text{K}\right)} = 1.1 \times 10^{-3}\,\text{m} = \boxed{1.1\,\text{mm}}$$

79. The total energy of the two photons must equal the total energy (kinetic energy plus mass energy) of the two particles. The total momentum of the photons is 0, so the momentum of the particles must have been equal and opposite. Since both particles have the same mass and the same momentum, they each have the same kinetic energy.

$$E_{\text{photons}} = E_{\text{particles}} = 2\left(m_{\text{e}}c^2 + K\right) \quad \rightarrow$$

$$K = \tfrac{1}{2}E_{\text{photons}} - m_{\text{e}}c^2 = 0.755\,\text{MeV} - 0.511\,\text{MeV} = \boxed{0.244\,\text{MeV}}$$

80. We calculate the required momentum from de Broglie's relation, Eq. 37-7.

$$p = \frac{h}{\lambda} = \frac{\left(6.63 \times 10^{-34}\,\text{J}\bullet\text{s}\right)}{\left(6.0 \times 10^{-12}\,\text{m}\right)} = 1.11 \times 10^{22}\ \text{kg}\bullet\text{m/s}$$

(*a*) For the proton, we use the classical definition of momentum to determine the speed of the electron, and then the kinetic energy. We divide the kinetic energy by the charge of the proton to determine the required potential difference.

$$v = \frac{p}{m} = \frac{1.11 \times 10^{-22}\,\text{kg}\bullet\text{m/s}}{1.67 \times 10^{-27}\,\text{kg}} = 6.65 \times 10^4\,\text{m/s} \ll c$$

$$V = \frac{K}{e} = \frac{mv^2}{2e} = \frac{\left(1.67 \times 10^{-27}\,\text{kg}\right)\left(6.65 \times 10^4\,\text{m/s}\right)^2}{2\left(1.60 \times 10^{-19}\,\text{C}\right)} = \boxed{23\,\text{V}}$$

(*b*) For the electron, if we divide the momentum by the electron mass we obtain a speed greater than 10% of the speed of light. Therefore, we must use the relativistic invariant equation to determine the energy of the electron. We then subtract the rest energy from the total energy to determine the kinetic energy of the electron. Finally, we divide the kinetic energy by the electron charge to calculate the potential difference.

$$E = \left[\left(pc\right)^2 + \left(m_0c^2\right)^2\right]^{\frac{1}{2}}$$

$$= \left[\left(1.11 \times 10^{-22}\,\text{kg}\bullet\text{m/s}\right)^2\left(3.00 \times 10^8\,\text{m/s}\right)^2 + \left(9.11 \times 10^{-31}\,\text{kg}\right)^2\left(3.00 \times 10^8\,\text{m/s}\right)^4\right]^{\frac{1}{2}}$$

$$= 8.85 \times 10^{-14}\,\text{J}$$

$$K = E - m_0c^2 = 8.85 \times 10^{-14}\,\text{J} - \left(9.11 \times 10^{-31}\,\text{kg}\right)\left(3.00 \times 10^8\,\text{m/s}\right)^2 = 6.50 \times 10^{-15}\,\text{J}$$

$$V = \frac{K}{e} = \frac{6.50 \times 10^{-15}\,\text{J}}{1.60 \times 10^{-19}\,\text{C}} = \boxed{41\text{ kV}}$$

81. If we ignore the recoil motion, at the closest approach the kinetic energy of both particles is zero. The potential energy of the two charges must equal the initial kinetic energy of the $\alpha$ particle:

$$K_\alpha = U = \frac{1}{4\pi\varepsilon_0}\frac{(Z_\alpha e)(Z_{\text{Ag}} e)}{r_{\min}} \rightarrow$$

$$r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{(Z_\alpha e)(Z_{\text{Ag}} e)}{K_\alpha} = \frac{(9.00\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2)(2)(47)(1.60\times10^{-19}\,\text{C})^2}{(4.8\,\text{MeV})(1.60\times10^{-13}\,\text{J/MeV})} = \boxed{2.8\times10^{-14}\,\text{m}}$$

82. The electrostatic potential energy is given by Eq. 23-5. The kinetic energy is given by the total energy, Eq. 37-14a, minus the potential energy. The Bohr radius is given by Eq. 37-11.

$$U = -eV = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2\pi m Ze^2}{n^2h^2\varepsilon_0} = -\frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}$$

$$K = E - U = -\frac{Z^2e^4m}{8\varepsilon_0^2h^2n^2} - \left(-\frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}\right) = \frac{Z^2e^4m}{8n^2h^2\varepsilon_0^2} \;;\; \frac{|U|}{K} = \frac{\dfrac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}}{\dfrac{Z^2e^4m}{8n^2h^2\varepsilon_0^2}} = \frac{Z^2e^4m}{4n^2h^2\varepsilon_0^2}\frac{8n^2h^2\varepsilon_0^2}{Z^2e^4m} = \boxed{2}$$

83. We calculate the ratio of the forces.

$$\frac{F_{\text{gravitational}}}{F_{\text{electric}}} = \frac{\left(\dfrac{Gm_em_p}{r^2}\right)}{\left(\dfrac{ke^2}{r^2}\right)} = \frac{Gm_em_p}{ke^2} = \frac{(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2)(9.11\times10^{-31}\,\text{kg})^2(1.67\times10^{-27}\,\text{kg})}{(9.00\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{C})^2}$$

$$= \boxed{4.4\times10^{-40}}$$

$\boxed{\text{Yes,}}$ the gravitational force may be safely ignored.

84. The potential difference gives the electrons a kinetic energy of 12.3 eV, so it is possible to provide this much energy to the hydrogen atom through collisions. From the ground state, the maximum energy of the atom is $-13.6\text{ eV} + 12.3\text{ eV} = -1.3\text{ eV}$. From the energy level diagram, Figure 37-26, we see that this means the atom could be excited to the $n = 3$ state, so the possible transitions when the atom returns to the ground state are $n = 3$ to $n = 2$, $n = 3$ to $n = 1$, and $n = 2$ to $n = 1$. We calculate the wavelengths from the equation above Eq. 37-15.

$$\lambda_{3\to2} = \frac{hc}{(E_3 - E_2)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-3.4\text{ eV})]} = \boxed{650\,\text{nm}}$$

$$\lambda_{3\to1} = \frac{hc}{(E_3 - E_1)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-1.5\text{ eV} - (-13.6\text{ eV})]} = \boxed{102\,\text{nm}}$$

$$\lambda_{2\to1} = \frac{hc}{(E_2 - E_1)} = \frac{1240\,\text{eV}\bullet\text{nm}}{[-3.4\text{ eV} - (-13.6\text{ eV})]} = \boxed{122\,\text{nm}}$$

$\boxed{85.}$ The stopping potential is the voltage that gives a potential energy change equal to the maximum kinetic energy. We use Eq. 37-4b to first find the work function, and then find the stopping potential for the higher wavelength.

$$K_{\max} = eV_0 = \frac{hc}{\lambda} - W_0 \rightarrow W_0 = \frac{hc}{\lambda_0} - eV_0$$

$$eV_1 = \frac{hc}{\lambda_1} - W_0 = \frac{hc}{\lambda_1} - \left(\frac{hc}{\lambda_0} - eV_0\right) = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_0}\right) + eV_0$$

$$= \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})}\left(\frac{1}{440\times10^{-9}\,\text{m}} - \frac{1}{380\times10^{-9}\,\text{m}}\right) + 2.70\,\text{eV} = 2.25\,\text{eV}$$

The potential difference needed to cancel an electron kinetic energy of 2.25 eV is $\boxed{2.25\text{ V.}}$

86. (*a*) The electron has a charge *e*, so the potential difference produces a kinetic energy of *eV*. The shortest wavelength photon is produced when all the kinetic energy is lost and a photon is emitted.

$$hf_{\text{max}} = \frac{hc}{\lambda_0} = eV \rightarrow \lambda_0 = \boxed{\frac{hc}{eV}} \text{ which gives } \lambda_0 = \frac{hc}{eV}.$$

(*b*) $\lambda_0 = \frac{hc}{eV} = \frac{1240\,\text{eV}\bullet\text{nm}}{33\times10^{3}\,\text{eV}} = \boxed{0.038\,\text{nm}}$

87. The average force on the sail is equal to the impulse on the sail divided by the time (Eq. 9-2). Since the photons bounce off the mirror the impulse is equal to twice the incident momentum. We use Eq. 37-5 to write the momentum of the photon in terms of the photon energy. The total photon energy is the intensity of the sunlight multiplied by the area of the sail

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{2(E/c)}{\Delta t} = \frac{2(E/\Delta t)}{c} = \frac{2IA}{c} = \frac{2(1350\text{ W/m}^2)(1000\text{ m})^2}{3.00\times10^{8}\text{ m/s}} = \boxed{9.0\text{ N}}$$

88. We first find the work function from the given data. A photon energy of 9.0 eV corresponds with a stopping potential of 4.0 V.

$$eV_0 = hf - W_0 \rightarrow W_0 = hf - eV_0 = 9.0\,\text{eV} - 4.0\,\text{eV} = 5.0\,\text{eV}$$

If the photons' wavelength is doubled, the energy is halved, from 9.0 eV to 4.5 eV. This is smaller than the work function, and so no current flows. Thus the maximum kinetic energy is $\boxed{0}$. Likewise, if the photon's wavelength is tripled, the energy is only 3.0 eV, which is still less than the work function, and so $\boxed{\text{no current flows}}$.

89. The electrons will be non-relativistic at that low energy. The maximum kinetic energy of the photoelectrons is given by Eq. 37-4b. The kinetic energy determines the momentum, and the momentum determines the wavelength of the emitted electrons. The shortest electron wavelength corresponds to the maximum kinetic energy.

$$K_{\text{electron}} = \frac{hc}{\lambda} - W_0 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda_{\text{electron}}^2} \rightarrow \lambda_{\text{electron}} = \frac{h}{\sqrt{2m\left(\frac{hc}{\lambda} - W_0\right)}}$$

$$= \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})}{\sqrt{2(9.11\times10^{-31}\,\text{kg})\left(\frac{1240\,\text{eV}\bullet\text{nm}}{360\,\text{nm}} - 2.4\,\text{eV}\right)(1.60\times10^{-19}\,\text{J/eV})}} = \boxed{1.2\times10^{-9}\,\text{m}}$$

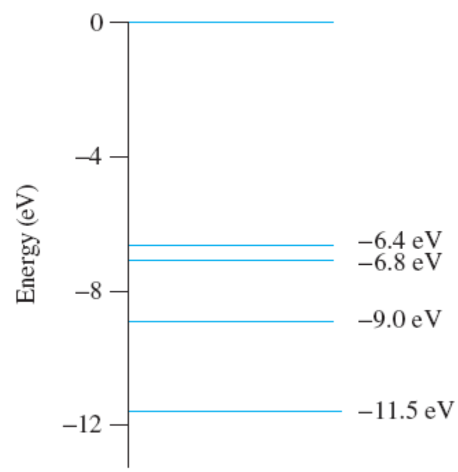
90. The wavelength is found from Eq. 35-13. The velocity of electrons with the same wavelength (and thus the same diffraction pattern) is found from their momentum, assuming they are not relativistic. We use Eq. 37-7 to relate the wavelength and momentum.

$$d\sin\theta = n\lambda \rightarrow \lambda = \frac{d\sin\theta}{n} = \frac{h}{p} = \frac{h}{mv} \rightarrow$$

$$v = \frac{hn}{md\sin\theta} = \frac{(6.63\times10^{-34}\,\text{J}\bullet\text{s})(1)}{(9.11\times10^{-31}\,\text{kg})(0.012\times10^{-3}\,\text{m})(\sin 3.5°)} = \boxed{990\,\text{m/s}}$$

91. (*a*) See the adjacent figure.

(*b*) Absorption of a 5.1 eV photon represents a transition from the ground state to the state 5.1 eV above that, the third excited state. Possible photon emission energies are found by considering all the possible downward transitions that might occur as the electron makes its way back to the ground state.




$$-6.4\,\text{eV} - (-6.8\,\text{eV}) = \boxed{0.4\,\text{eV}}$$

$$-6.4\,\text{eV} - (-9.0\,\text{eV}) = \boxed{2.6\,\text{eV}}$$

$$-6.4\,\text{eV} - (-11.5\,\text{eV}) = \boxed{5.1\,\text{eV}}$$

$$-6.8\,\text{eV} - (-9.0\,\text{eV}) = \boxed{2.2\,\text{eV}}$$

$$-6.8\,\text{eV} - (-11.5\,\text{eV}) = \boxed{4.7\,\text{eV}}$$

$$-9.0\,\text{eV} - (-11.5\,\text{eV}) = \boxed{2.5\,\text{eV}}$$

92. (*a*) We use Eq. 37-4b to calculate the maximum kinetic energy of the electron and set this equal to the product of the stopping voltage and the electron charge.

$$K_{\text{max}} = hf - W_0 = eV_0 \rightarrow V_0 = \frac{hf - W_0}{e} = \frac{hc/\lambda - W_0}{e}$$

$$V_0 = \frac{(1240\,\text{eV}\bullet\text{nm})/(424\,\text{nm}) - 2.28\,\text{eV}}{\text{e}} = \boxed{0.65\,\text{V}}$$

(*b*) We calculate the speed from the non-relativistic kinetic energy equation and the maximum kinetic energy found in part (*a*).

$$K_{\text{max}} = \tfrac{1}{2}mv_{\text{max}}^2 \rightarrow v_{\text{max}} = \sqrt{\frac{2K_{\text{max}}}{m}} = \sqrt{\frac{2(0.65\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = \boxed{4.8\times10^5\,\text{m/s}}$$

(*c*) We use Eq. 37-7 to calculate the de Broglie wavelength.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,\text{J}\bullet\text{s}}{(9.11\times10^{-31}\,\text{kg})(4.8\times10^5\,\text{m/s})} = 1.52\times10^{-9}\,\text{m} = \boxed{1.5\,\text{nm}}$$

93. (*a*) We use Bohr's analysis of the hydrogen atom, where we replace the proton mass with Earth's mass, the electron mass with the Moon's mass, and the electrostatic force $F_e = \frac{ke^2}{r^2}$ with the gravitational force $F_g = \frac{Gm_E m_M}{r^2}$. To account for the change in force, we replace $ke^2$ with $Gm_E m_M$. With these replacements, we write expressions similar to Eq. 37-11 and Eq. 37-14a for the Bohr radius and energy.

$$r_n = \frac{h^2 n^2}{4\pi^2 mke^2} \rightarrow$$

$$r_n = \frac{h^2 n^2}{4\pi^2 G m_M^2 m_E} = \frac{\left(6.626\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}{4\pi^2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\,\text{kg}\right)^2\left(5.98\times10^{24}\,\text{kg}\right)} n^2$$

$$= \boxed{n^2\left(5.16\times10^{-129}\,\text{m}\right)}$$

$$E_n = -\frac{2\pi^2 e^4 mk^2}{n^2 h^2} \rightarrow$$

$$E_n = -\frac{2\pi^2 G^2 m_E^2 m_M^3}{n^2 h^2} = -\frac{2\pi^2\left(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2\right)^2\left(5.98\times10^{24}\,\text{kg}\right)^2\left(7.35\times10^{22}\,\text{kg}\right)^3}{n^2\left(6.626\times10^{-34}\,\text{J}\bullet\text{s}\right)^2}$$

$$= \boxed{-\frac{2.84\times10^{165}\,\text{J}}{n^2}}$$

(*b*) We insert the known masses and Earth–Moon distance into the Bohr radius equation to determine the Bohr state.

$$n = \sqrt{\frac{4\pi^2 G m_M^2 m_E r_n}{h^2}}$$

$$= \sqrt{\frac{4\pi^2\left(6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2\right)\left(7.35\times10^{22}\ \text{kg}\right)^2\left(5.98\times10^{24}\ \text{kg}\right)\left(3.84\times10^{8}\ \text{m}\right)}{\left(6.626\times10^{-34}\ \text{Js}\right)^2}}$$

$$= 2.73\times10^{68}$$

Since $n \approx 10^{68}$, a value of $\Delta n = 1$ is negligible compared to *n*. Hence the quantization of energy and radius is $\boxed{\text{not apparent.}}$

94. We use Eqs. 36-13, 36-11, and 37-7 to derive the expression.

$$p^2c^2 + m^2c^4 = E^2 \ ;\ E = K + mc^2 \rightarrow p^2c^2 + m^2c^4 = \left(K + mc^2\right)^2 = K^2 + 2mc^2K + m^2c^4 \rightarrow$$

$$K^2 + 2mc^2K = p^2c^2 = \frac{h^2c^2}{\lambda^2} \rightarrow \lambda^2 = \frac{h^2c^2}{\left(K^2 + 2mc^2K\right)} \rightarrow \boxed{\lambda = \frac{hc}{\sqrt{K^2 + 2mc^2K}}}$$

95. As light leaves the flashlight it gains momentum. This change in momentum is given by Eq. 31-20. Dividing the change in momentum by the elapsed time gives the force the flashlight must apply to the light to produce this momentum. This is equal to the reaction force that light applies to the flashlight.

$$\frac{\Delta p}{\Delta t} = \frac{\Delta U}{c\Delta t} = \frac{P}{c} = \frac{3.0\,\text{W}}{3.00\times10^8\,\text{m/s}} = \boxed{1.0\times10^{-8}\,\text{N}}$$

96. (*a*) Since $f = c/\lambda$, the energy of each emitted photon is $E = hc/\lambda$. We insert the values for *h* and *c* and convert the resulting units to $\text{eV}\bullet\text{nm}$.

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\,\text{J}\bullet\text{s}\right)\left(2.998\times10^8\,\text{m/s}\right)}{\lambda}\frac{\left(1\,\text{eV}/1.602\times10^{-19}\,\text{J}\right)}{\left(10^{-9}\text{m}/1\,\text{nm}\right)} = \boxed{\frac{1240\ \text{eV}\bullet\text{nm}}{\lambda\left(\text{in nm}\right)}}$$

(*b*) Insert 650 nm into the above equation.

$$E = \frac{1240\ \text{eV}\bullet\text{nm}}{650\ \text{nm}} = \boxed{1.9\ \text{eV}}$$

97. (*a*) We write the Planck time as $t_P = G^\alpha h^\beta c^\gamma$, and the units of $t_P$ must be $[T]$.

$$t_P = G^\alpha h^\beta c^\gamma \quad \rightarrow \quad [T] = \left[\frac{L^3}{MT^2}\right]^\alpha \left[\frac{ML^2}{T}\right]^\beta \left[\frac{L}{T}\right]^\gamma = [L]^{3\alpha+2\beta+\gamma}[M]^{\beta-\alpha}[T]^{-2\alpha-\beta-\gamma}$$

There are no mass units in $[T]$, and so $\beta = \alpha$, and $[T] = [L]^{5\alpha+\gamma}[T]^{-3\alpha-\gamma}$. There are no length units in $[T]$, and so $\gamma = -5\alpha$ and $[T] = [T]^{-3\alpha+5\alpha} = [T]^{2\alpha}$. Thus $\alpha = \frac{1}{2} = \beta$ and $\gamma = -\frac{5}{2}$.

$$\boxed{t_P = G^{1/2}h^{1/2}c^{-5/2} = \sqrt{\frac{Gh}{c^5}}}$$

(*b*) $$t_P = \sqrt{\frac{Gh}{c^5}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^5}} = \boxed{1.35\times10^{-43}\text{s}}$$

(*c*) We write the Planck length as $\lambda_P = G^\alpha h^\beta c^\gamma$, and the units of $\lambda_P$ must be $[L]$.

$$\lambda_P = G^\alpha h^\beta c^\gamma \quad \rightarrow \quad [L] = \left[\frac{L^3}{MT^2}\right]^\alpha \left[\frac{ML^2}{T}\right]^\beta \left[\frac{L}{T}\right]^\gamma = [L]^{3\alpha+2\beta+\gamma}[M]^{\beta-\alpha}[T]^{-2\alpha-\beta-\gamma}$$

There are no mass units in $[L]$, and so $\beta = \alpha$, and $[L] = [L]^{5\alpha+\gamma}[T]^{-3\alpha-\gamma}$. There are no time units in $[L]$, and so $\gamma = -3\alpha$ and $[L] = [L]^{5\alpha-3\alpha} = [L]^{2\alpha}$. Thus $\alpha = \frac{1}{2} = \beta$ and $\gamma = -\frac{3}{2}$.

$$\boxed{t_P = G^{1/2}h^{1/2}c^{-3/2} = \sqrt{\frac{Gh}{c^3}}}$$

(*d*) $$\lambda_P = \sqrt{\frac{Gh}{c^3}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(6.63\times10^{-34}\ \text{J}\bullet\text{s}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^5}} = \boxed{4.05\times10^{-35}\text{m}}$$

98. For standing matter waves, there are nodes at the two walls. For the ground state (first harmonic), the wavelength is twice the distance between the walls, or $\ell = \frac{1}{2}\lambda$ (see Figure 15-26b). We use Eq. 37-7 to find the velocity and then the kinetic energy.

$$\ell = \tfrac{1}{2}\lambda \quad \rightarrow \quad \lambda = 2\ell \ ; \ p = \frac{h}{\lambda} = \frac{h}{2\ell} \ ; \ K = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{h}{2\ell}\right)^2 = \boxed{\frac{h^2}{8m\ell^2}}$$

For the second harmonic, the distance between the walls is a full wavelength, and so $\ell = \lambda$.

$$\ell = \lambda \quad \rightarrow \quad p = \frac{h}{\lambda} = \frac{h}{\ell} \ ; \ K = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{h}{\ell}\right)^2 = \boxed{\frac{h^2}{2m\ell^2}}$$

99. (*a*) Apply conservation of momentum before and after the emission of the photon to determine the recoil speed of the atom, where the momentum of the photon is given by Eq. 37-7.

$$0 = \frac{h}{\lambda} - mv \quad \rightarrow \quad v = \frac{h}{m\lambda} = \frac{\left(6.63\times10^{-34}\,\text{J}\bullet\text{s}\right)}{85\left(1.66\times10^{-27}\,\text{kg}\right)\left(780\times10^{-9}\,\text{m}\right)} = \boxed{6.0\times10^{-3}\,\text{m/s}}$$

(*b*) We solve Eq. 18-5 for the lowest achievable temperature, where the recoil speed is the rms speed of the rubidium gas.

$$v = \sqrt{\frac{3kT}{m}} \quad \rightarrow \quad T = \frac{mv^2}{3k} = \frac{85\left(1.66\times10^{-27}\,\text{kg}\right)\left(6.0\times10^{-3}\,\text{m/s}\right)^2}{3\left(1.38\times10^{-23}\,\text{J/K}\right)} = 1.2\times10^{-7}\,\text{K} = \boxed{0.12\,\mu\text{K}}$$

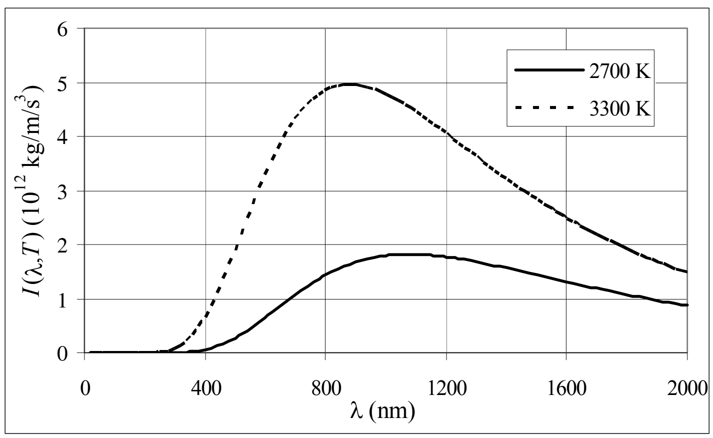
100. Each time the rubidium atom absorbs a photon its momentum decreases by the momentum of the photon. Dividing the initial momentum of the rubidium atom by the momentum of the photon, Eq. 37-7, gives the number of collisions necessary to stop the atom. Multiplying the number of collisions by the absorption time, 25 ns per absorption, provides the time to completely stop the atom.

$$n = \frac{mv}{h/\lambda} = \frac{mv\lambda}{h} = \frac{\left(8\,\text{u}\right)\left(1.66\times10^{-27}\,\text{kg/u}\right)\left(290\,\text{m/s}\right)\left(780\times10^{-9}\,\text{m}\right)}{6.63\times10^{-34}\,\text{J}\bullet\text{s}} = 48{,}140$$

$$T = 48{,}140\left(25\,\text{ns}\right) = \boxed{1.2\,\text{ms}}$$

101. (*a*) See the adjacent graphs.

(*b*) To compare the intensities, the two graphs are numerically integrated from 400 nm to 760 nm, which is approximately the range of wavelengths for visible light. The result of those integrations is that the higher temperature bulb is about $\boxed{\text{4.8 times more intense}}$ than the lower temperature bulb.




The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.101."

102. Planck's radiation formula $I(\lambda,T)$ was calculated for a temperature of 6000 K, for wavelengths from 20 nm to 2000 nm. A plot of those calculations is in the spreadsheet for this problem. To estimate the % of emitted sunlight that is in the visible, this ratio was calculated by numeric integration. The details are in the spreadsheet.

$$\%\,\text{visible} = \frac{\displaystyle\int_{400\,\text{nm}}^{700\,\text{nm}} I(\lambda,T)\,d\lambda}{\displaystyle\int_{20\,\text{nm}}^{2000\,\text{nm}} I(\lambda,T)\,d\lambda} = \boxed{0.42}$$

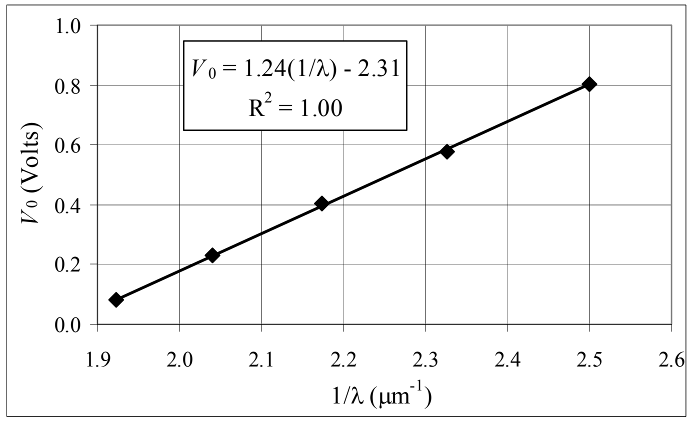
So our estimate is that 42% of emitted sunlight is in the visible wavelengths. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.102."

103. (*a*) For the photoelectric effect experiment, Eq. 37-4b can be expressed as $K_{max} = hf - W_0$. The maximum kinetic energy is equal to the potential energy associated with the stopping voltage, so $K_{max} = eV_0$. We also have $f = c/\lambda$. Combine those relationships as follows.

$$K_{max} = hf - W_0 \rightarrow eV_0 = \frac{hc}{\lambda} - W_0 \rightarrow V_0 = \frac{hc}{e}\frac{1}{\lambda} - \frac{W_0}{e}$$

A plot of $V_0$ vs. $\frac{1}{\lambda}$ should yield a straight line with a slope of $\boxed{\frac{hc}{e}}$ and a $y$-intercept of $\boxed{-\frac{W_0}{e}}$.

(*b*) The graph is shown, with a linear regression fit as given by Excel.




(*c*) The slope is $a = \frac{hc}{e} = 1.24\,\text{V}\bullet\mu\text{m}$, and the $y$-intercept is $b = -2.31\,\text{V}$.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH37.XLS," on tab "Problem 37.103."

(*d*) $b = -\frac{W_0}{e} = -2.31\,\text{V} \rightarrow W_0 = \boxed{2.31\,\text{eV}}$

(*e*) $h = \frac{ea}{c} = \frac{(1.60\times10^{-19}\,\text{C})(1.24\times10^{-6}\,\text{V}\bullet\text{m})}{3.00\times10^{8}\,\text{m/s}} = \boxed{6.61\times10^{-34}\,\text{J}\bullet\text{s}}$