Propulsion and Power

Lecture 1

ELEMENTS OF A PROPULSION SYSTEM

1.	2.	3.
fuel cells batteries Hydrogen Gasoline	Different engines	Propeller Nozzle

3. thrust producing mechanism

1. Energy source 2. Energy to work converter

THERMODYNAMICS:

Zeroth Law: Thermal equilibrium and the concept of temperature

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First Law: Conservation of energy and its interconvertability

- $\Delta E = \Delta KE + \Delta PE + \Delta U$ = constant U = internal → independent of P and V
- enthalpy: $h = u + p \cdot v$ $U = f(T)$ and $dU = C_v \cdot \partial T$
- $C_v = (\partial u / \partial T)_v$ so $du = C_v \cdot \partial T$ constant volume only
- $C_p = (dh / \partial T)_p$ so $dh = C_p \cdot \partial T$ constant pressure only.

Second Law: Defines entropy [J/kg·K or J/K mole]

- Measure of randomness and disorder.
- State function
- Change of entropy $\Delta S = S_{final} - S_{initial}$ → negative means more order.
- Increases in a spontaneous process, decreases in an equilibrium process.
- The entropy change in a irreversible process is always positive.

Third Law: Entropy of a system is 0 at absolute 0 K.

BASIC EQUATIONS FOR IDEAL GAS:

- Ideal gas is $P \cdot v = R \cdot T$ P = pressure | v = specific volume | R = gas constant | T = abs temperature
- Heat supply at a constant pressure.
  - $dh = dQ_p = du + pdv = C_v dT + p dv$
  - $(dQ/\partial T)_p = C_v + p \cdot (\partial v / \partial T)_p$
  - $P \cdot v = R \cdot T$ so $p \cdot (\partial v / \partial T)_p = R$
  - $C_p = C_v + R$

Ratio between $C_p / C_v = k$ isentropic component

Ideal Gas law:

- Isothermal process: $p \cdot v =$ constant
- Isobaric process: $v/T =$ constant
- Isochloric process: $P/T =$ constant
- Isentropic process: $p \cdot V^k =$ constant
- Polytropic process: $p \cdot V^n =$ constant

Isentropic Process:

- $\Delta s = 0$ | $C_p =$ constant | $C_v =$ constant | $C_p / C_v = k$

- $p \cdot V^k =$ constant
  - $p \cdot v = R \cdot T \longrightarrow p = RT/v$
  - $RT/v \cdot v^k =$ constant
  - $RT \cdot v^{k-1} =$ constant $\quad R =$ constant

- $T \cdot v^{k-1} =$ constant
  - $P \cdot v = R \cdot T \rightarrow v = \frac{RT}{P}$
  - $P \cdot \left(\frac{RT}{P}\right)^k =$ constant
  - $P^{\frac{(1-k)}{k}} \cdot T =$ constant

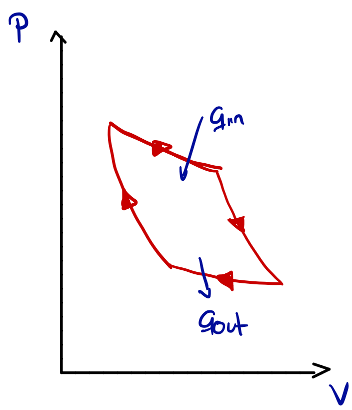
- $\left(\frac{T}{P}\right)^{\frac{(k-1)}{k}} =$ constant
- $T_2 / T_1 = \left(\frac{P_2}{P_1}\right)^{\frac{(k-1)}{k}}$

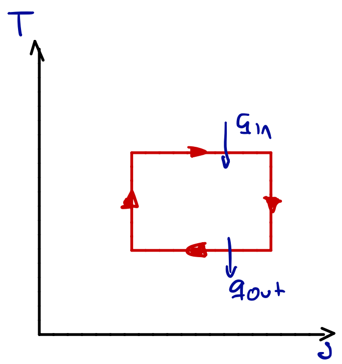
- $T_2 = T_1 \cdot \Pi^{(k-1)/k}$ $\quad \Pi = \frac{P_2}{P_1}$
- $T_2 - T_1 = T_1 \cdot \{\Pi^{(k-1)/k} - 1\}$

Power Cycles:

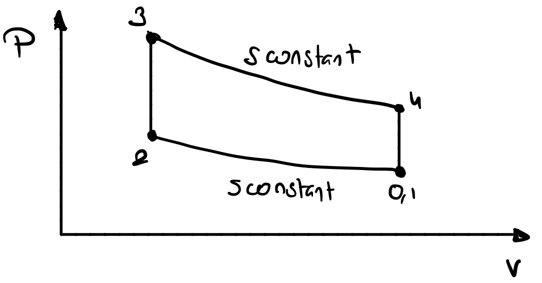
Carnot Cycle: Ideal Process.

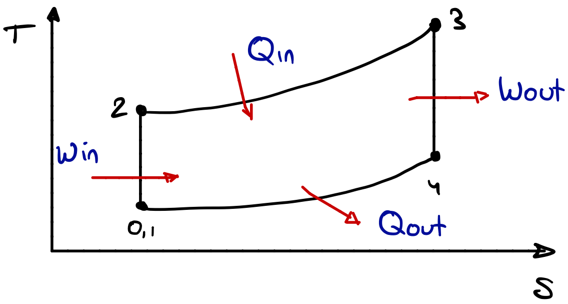
- Isothermal heat adition and expansion
- Isentropic expansion
- Isothermal compressions and heat rejection
- Isentropic compression





Second Lecture   Otto Cycle:





$$(q_{in} - q_{out}) + (W_{in} - W_{out}) = 0$$

$$(q_{in} + W_{in}) = q_{out} + W_{out}$$

$$\boxed{\eta_{th} = \frac{W_{net}}{q_{in}}}$$

$$\boxed{q_{in} = u_3 - u_2 = C_v (T_3 - T_2)} \qquad \boxed{q_{out} = u_4 - u_1 = C_v (T_4 - T_1)}$$

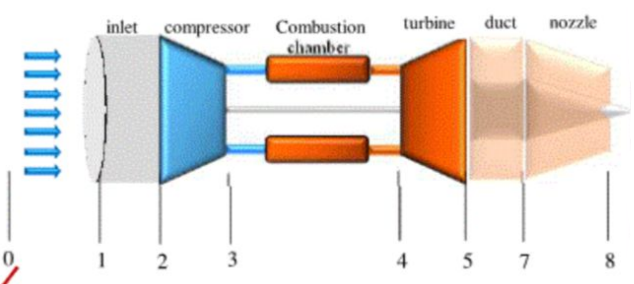
$$\eta_{th} = \frac{W_{out} - W_{in}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)}$$

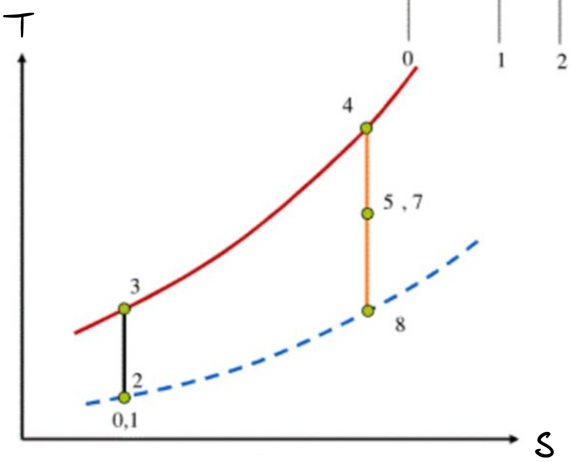
Process 1-2 and 3-4 are isentropic, and $V_2 = V_3$ and $V_4 = V_1$

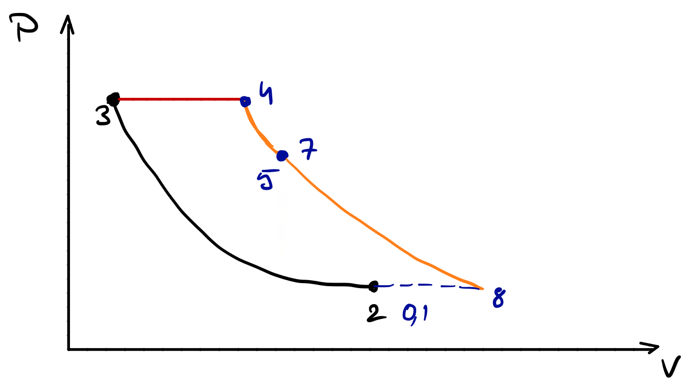
$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1} = \left(\frac{V_3}{V_4}\right)^{k-1} = \frac{T_4}{T_3} \longrightarrow \frac{T_1}{T_4} = \frac{T_2}{T_3} \quad \text{so} \quad \boxed{\eta_{th} = 1 - \frac{T_1}{T_2}}$$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{k-1} = 1 - \left(\frac{1}{CR}\right)^{k-1}$$

CR = compression ratio. CR > 1



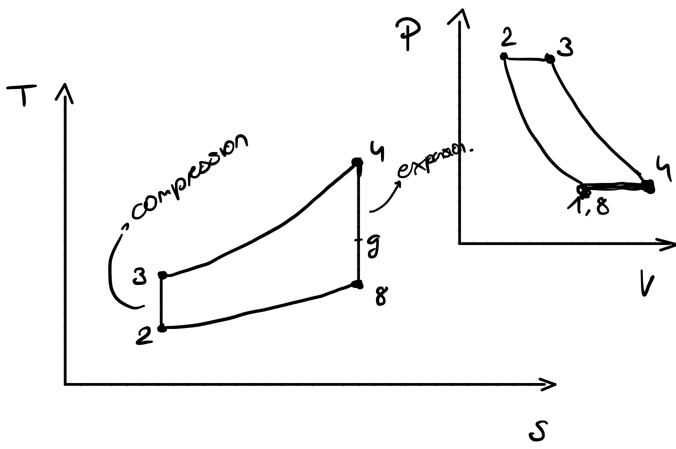




ASSUMPTIONS OF AN IDEAL CYCLE:

- Ideal gas with constant C_p and C_v and constant composition.
- Constant mass flow
- Compression and expansion adiabatic and reversible.
- No pressure losses
- Δ kinetic energy per component = 0

BRAYTON CYCLE



IDEAL CYCLE – PROCESS DATA

Compression 2-3: $W_{2-3} = mC_p(T_3 - T_2)$

Heat supply: $Q_{3-4} = mC_p(T_4 - T_3)$

Expansion 1: $W_{4-g} = mC_p(T_4 - T_g)$

Expansion 2: $W_{gs} = W_{g8} = mC_p(T_g - T_8)$

Heat Discharge: $Q_{8-2} = mC_p(T_8 - T_2)$

$$W_{2-3} + Q_{3-4} = W_{4-g} + W_{g8} + Q_{8-2}$$

$$W_{4-g} = W_{2-3}$$

$$\boxed{Q_{3-4} = W_{g8} + Q_{8-2}}$$

Π is the pressure ratio of the cycle

$\Pi = P_3/P_2 = P_4/P_8 \qquad k = C_p/C_v$

η_{th} = Thermal efficiency of the cycle:

$$\eta_{th} = \left(\frac{\text{work output}}{\text{heat input}}\right) = \left(\frac{W_{4-8} - W_{2-3}}{Q_{3-4}}\right) = \left(\frac{C_p(T_4 - T_8) - C_p(T_3 - T_2)}{C_p(T_4 - T_3)}\right)$$

$$\eta_{th} = \left(\frac{(T_4 - T_8) - (T_3 - T_2)}{(T_4 - T_3)}\right)$$

$$\boxed{\left(\frac{T_4}{T_8}\right) = \Pi^{\frac{k-1}{k}}} \quad \text{and} \quad \boxed{\frac{T_3}{T_2} = \Pi^{\frac{k-1}{k}}}$$

$$\eta_{th} = \left(\frac{T_4\left(1 - \frac{T_8}{T_4}\right) - T_3\left(1 - \frac{T_2}{T_3}\right)}{(T_4 - T_3)}\right) = \left(\frac{T_4\left(1 - \frac{1}{\Pi^{\frac{k-1}{k}}}\right) - T_3\left(1 - \frac{1}{\Pi^{\frac{k-1}{k}}}\right)}{(T_4 - T_3)}\right) = \left(\frac{(T_4 - T_3)\left(1 - \frac{1}{\Pi^{\frac{k-1}{k}}}\right)}{(T_4 - T_3)}\right)$$

$$\boxed{\eta_{th} = \left[1 - \frac{1}{\Pi^{\frac{k-1}{k}}}\right]}$$

GAS POWER

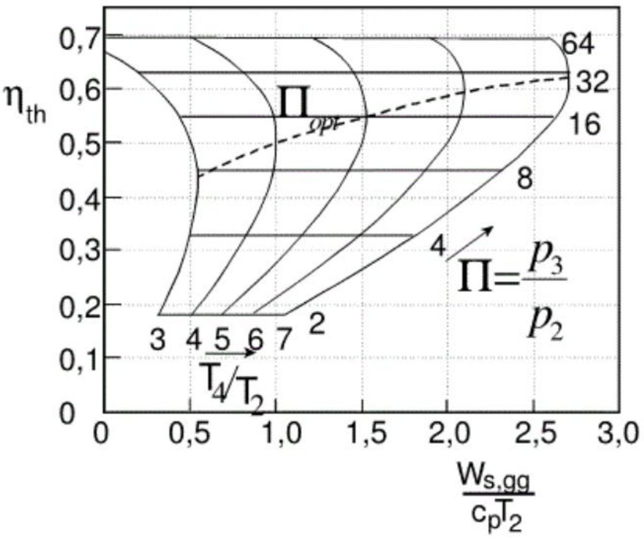
$$T_g = T_4 - (T_3 - T_2) = T_4 - T_2\left(\Pi^{\frac{k-1}{k}} - 1\right)$$

$$P_g = P_4\left(\frac{T_g}{T_4}\right)^{\frac{k}{k-1}} = P_2\,\Pi\left[1 - \frac{T_2}{T_4}\left(\Pi^{\frac{k-1}{k}} - 1\right)\right]^{\frac{k}{k-1}}$$

Specific Power:

$$\boxed{\frac{W_{s,gs}}{C_p T_2} = \frac{T_4}{T_2}\left[1 - \frac{1}{\Pi^{\frac{k-1}{k}}}\right] - \left[\Pi^{\frac{k-1}{k}} - 1\right]}$$

Specific Power and Efficiency:



Optimal Pressure Ratio

$$W_{gg} = W_{g-8} = m C_p (T_g - T_8)$$

$$\Pi^{\frac{k-1}{k}} = \frac{T_3}{T_2} = \frac{T_4}{T_8} \longrightarrow \boxed{T_8 = \frac{T_4}{T_3} T_2}$$

Differentiate W_{gg} w.r.t T_3 and equating to zero

$$W_{gg} = m C_p \left(T_g - \frac{T_4}{T_3} T_2\right) = -m C_p \cdot \frac{T_4 \cdot T_2}{T_3^2}$$

$$T_3 = \sqrt{T_2 \cdot T_4}$$

$$\Pi_{opt} = \left(\frac{T_3}{T_2}\right)^{\frac{k}{k-1}} = \left(\frac{T_4}{T_2}\right)^{\frac{k}{2(k-1)}}$$

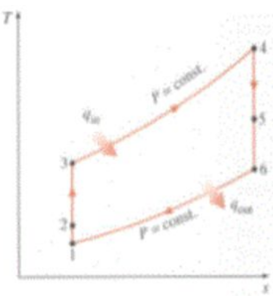
$$\boxed{T_3 = T_8}$$

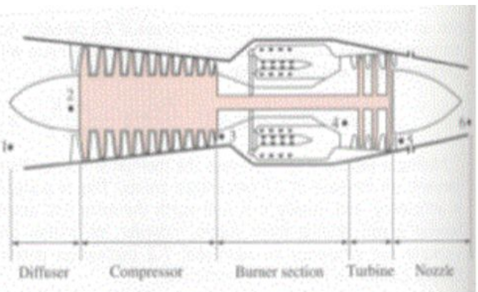
Thus: $\frac{W_{s,gg}}{C_p T_2} = max$ when

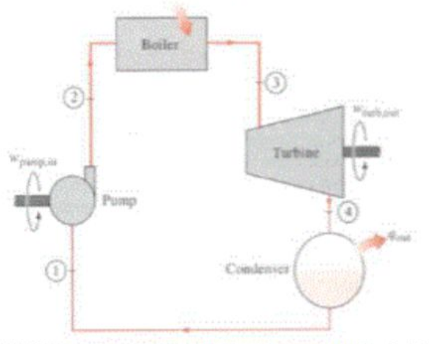
$$\boxed{\left(\frac{W_{s,gg}}{C_p T_2}\right)_{\Pi_{opt}} = \left(\sqrt{\frac{T_4}{T_2}} - 1\right)^2}$$

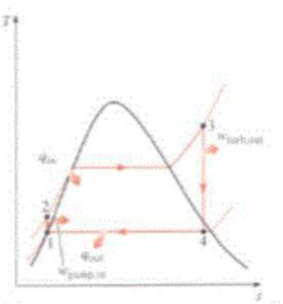
$$\boxed{(\eta_{th})_{\Pi_{opt}} = \left[1 - \sqrt{\frac{T_2}{T_4}}\right]}$$

Rankine Cycle

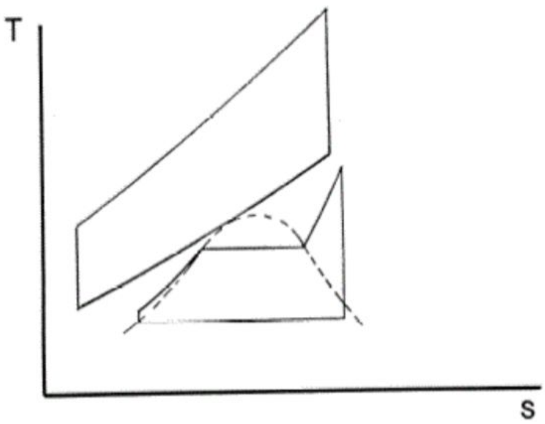




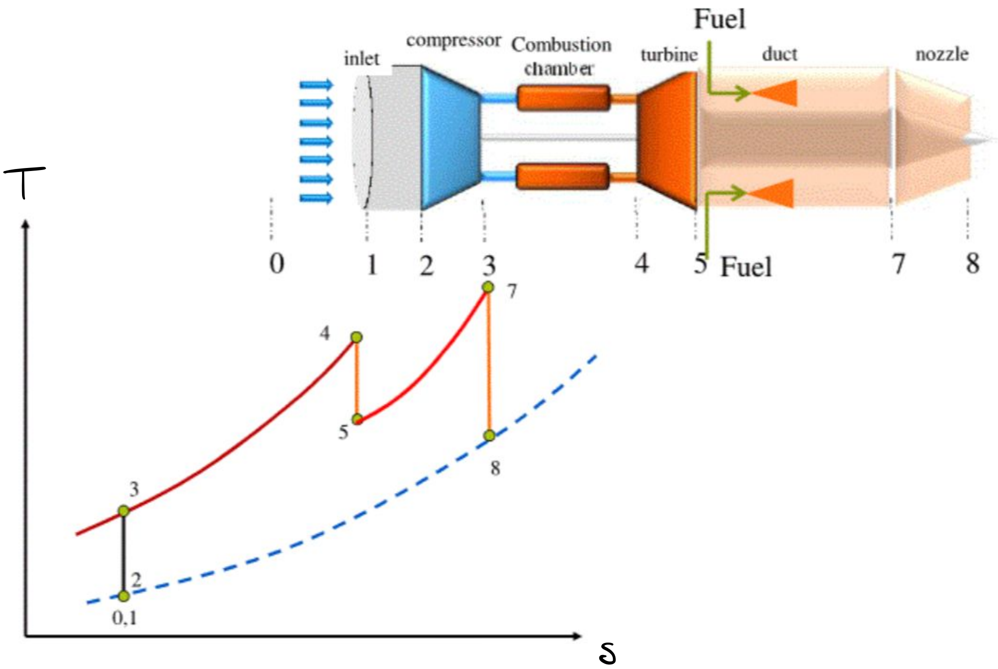




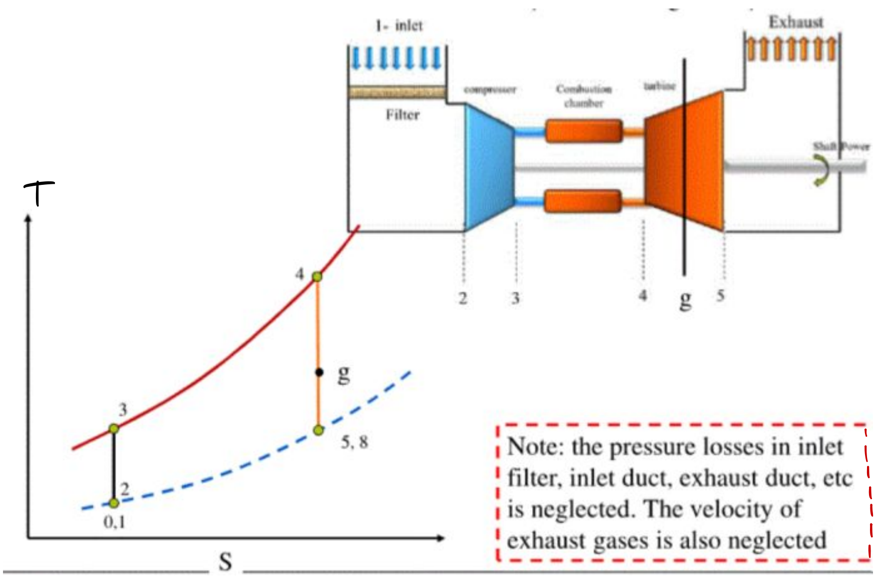
Combined: Brayton and Rankine



Variations of the Standard Brayton Cycle

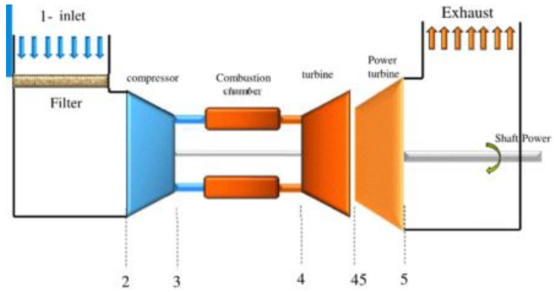


Land Based Gas Turbine

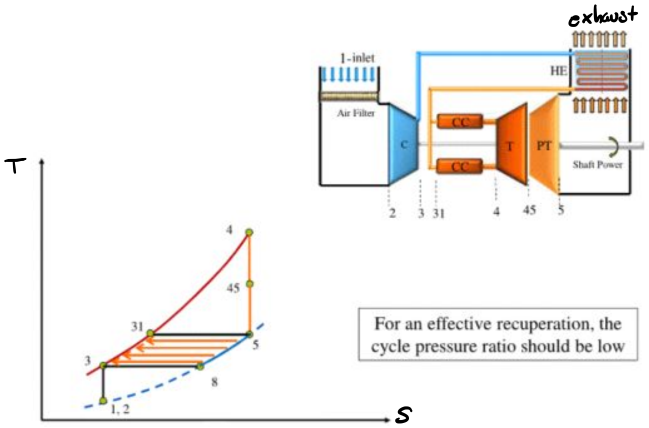


Note: the pressure losses in inlet filter, inlet duct, exhaust duct, etc is neglected. The velocity of exhaust gases is also neglected

Gas Turbine with Power Turbine

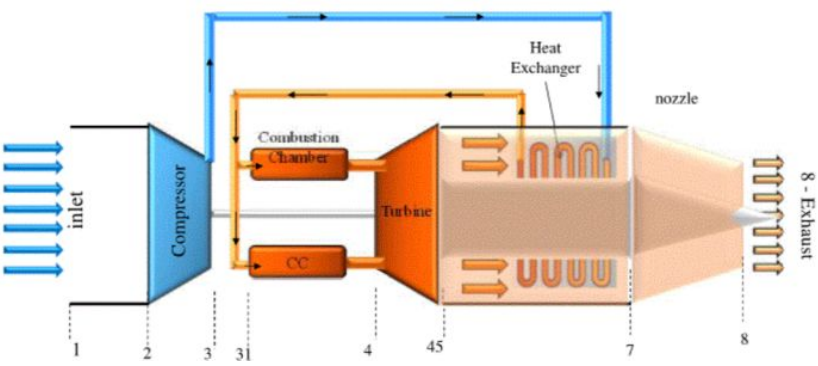


Gas turbine with Recuperator:



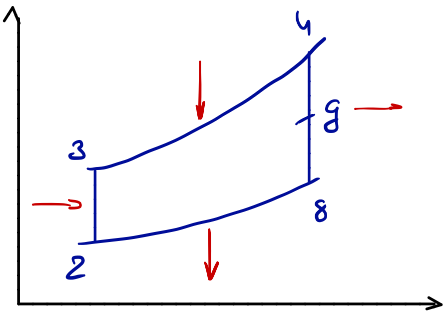
For an effective recuperation, the cycle pressure ratio should be low

Jet Engine with Recuperator : very dificult.



Lecture 3 Real Bryton Cycle

- Previous assumptions:



Constant C_p and C_v : ideal gas.

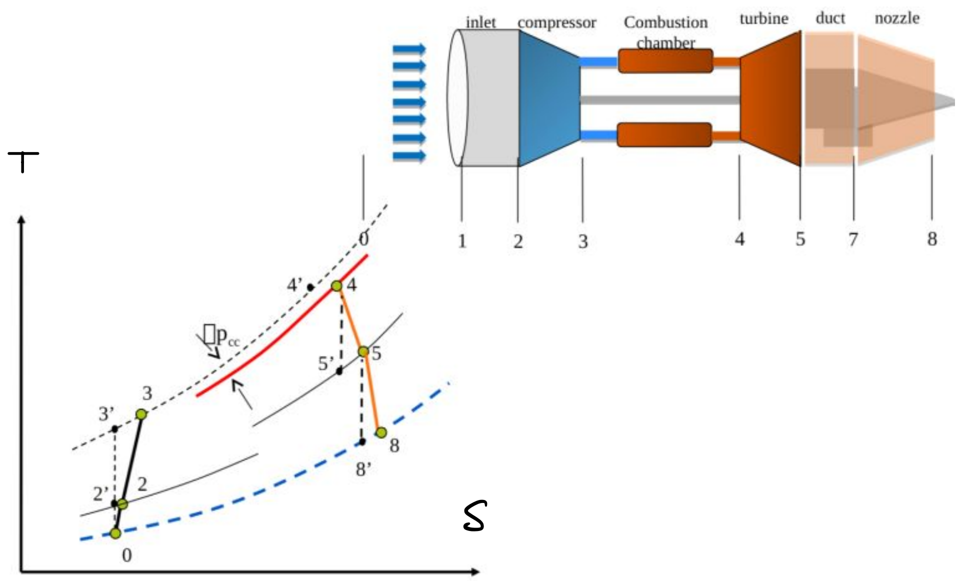
Constant mass flow.

Adiabatic compression and expansion

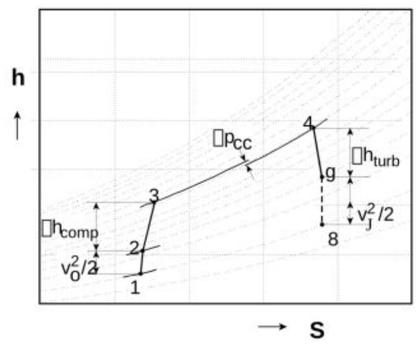
No pressure losses in ducts and heat supply and rejection.

Kinetic energy per component = 0

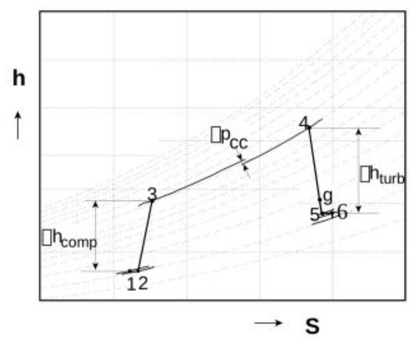
Real Process



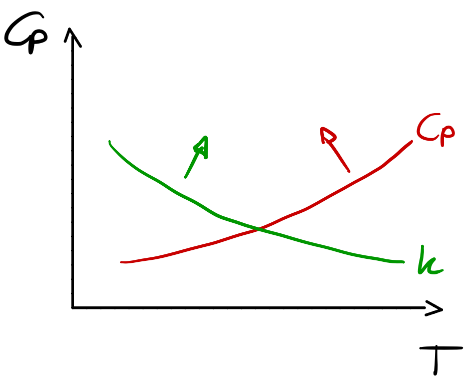
h-s diagram of jet engine



h-s in a stationary gas turbine



Cp and k reality:



$C_{p,a} = 1000$ J/kg·K

$C_{p,g} = 1150$ J/kg·K

$k_a = 1.4$ (before CC)

$k_g = 1.33$ (from CC onwards)

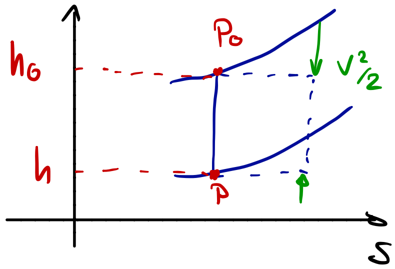
- As Temperature increases:

→ k decreases: increasing fuel to air ratio, → viscosity increases

→ Cp increases: increasing fuel to air ratio →

Total Properties

$$\Delta\left[m\left(h + \tfrac{1}{2}V^2 + gz\right)\right] = Q - W$$

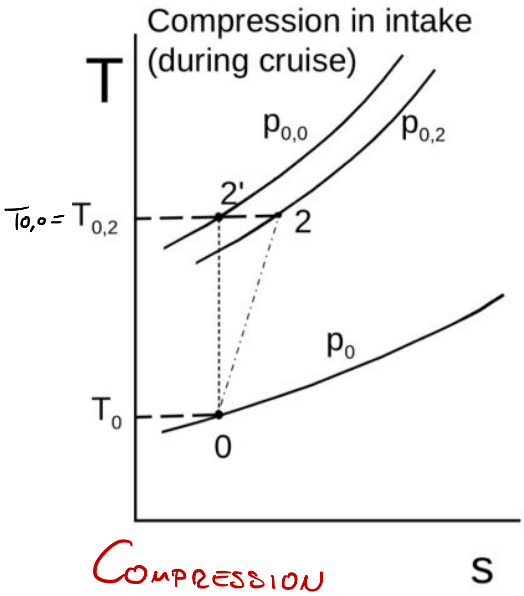


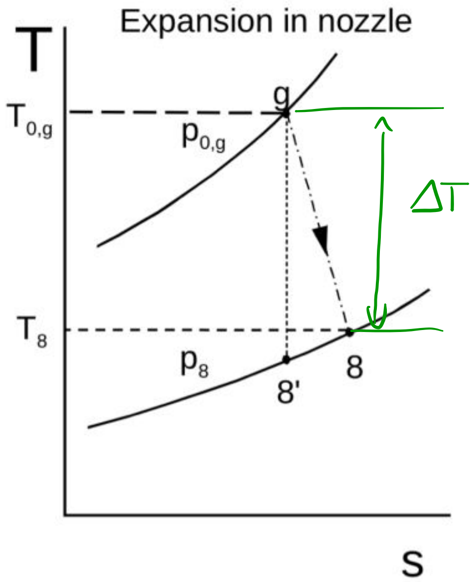
Total enthalpy: $h_0 = h + \tfrac{1}{2}V_0^2$

Total temperature: $T_0 = T + \dfrac{V_0^2}{2c_p}$

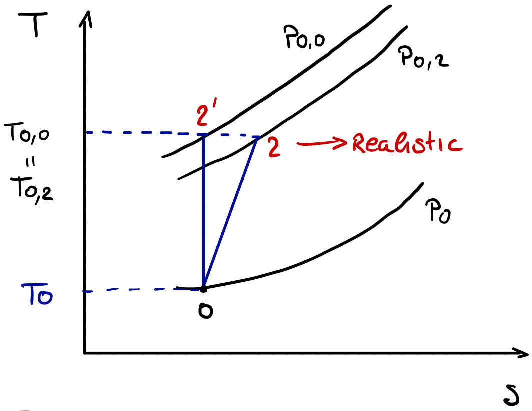
Total pressure: $p_0 = p\left[\dfrac{T_0}{T}\right]^{\frac{k}{k-1}} = p\left[1 + \dfrac{V_0^2}{2c_pT}\right]^{\frac{k}{k-1}}$

Non-Isentropic Compression and Expansion

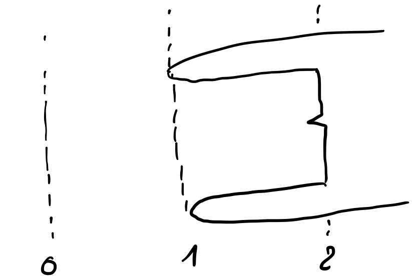




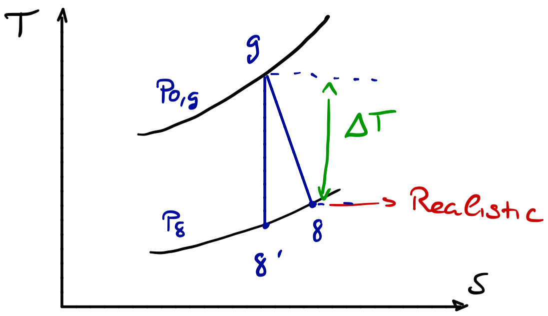
Compression

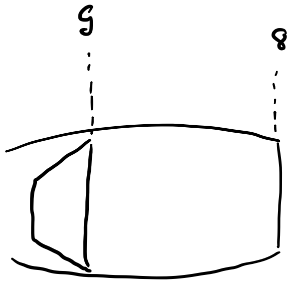


$T_{0,0} = T_{0,1} = T_{0,2}$



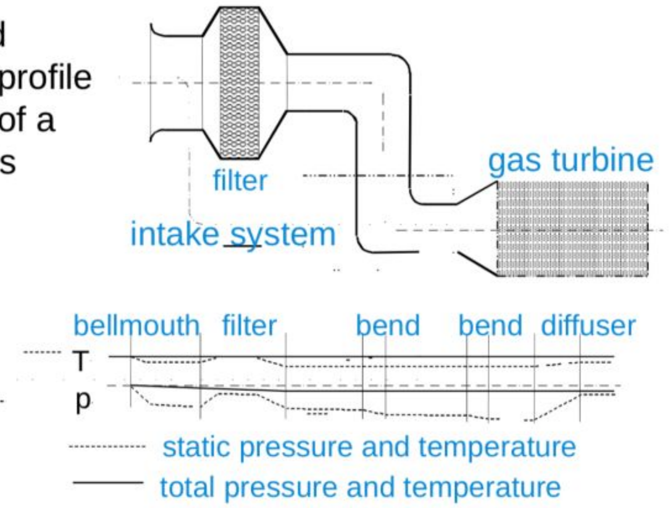
Expansion





Stationary Gas Turbine Intake

- Pressure and temperature profile in the intake of a stationary gas turbine

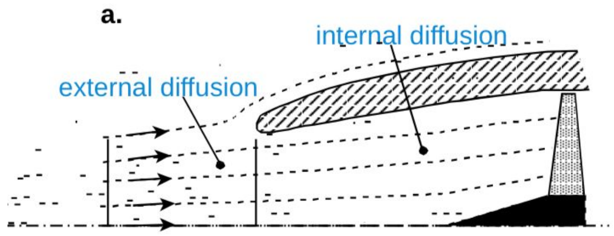


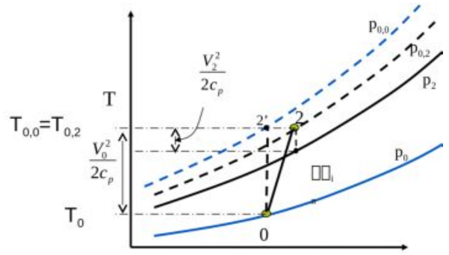
- Total pressure stays the same, no energy added to the flow.

Aircraft Gas Turbine Intake

- External and internal diffusion near an intake of an aircraft gas turbine.

a.) High flight speed and low engine load. (cruise)

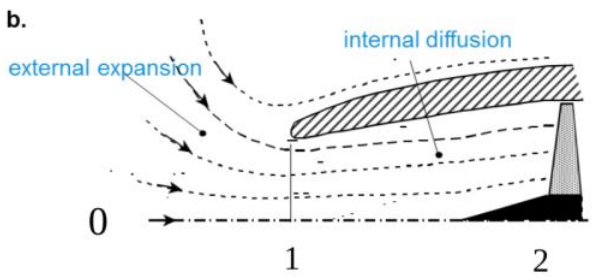


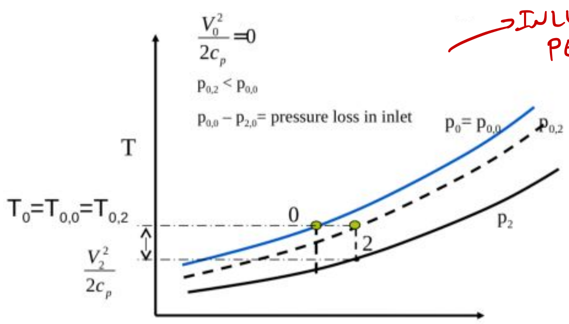


INLET PERFORMANCE

No rounded why?

b) Low flight speed and high engine load. (take-off)





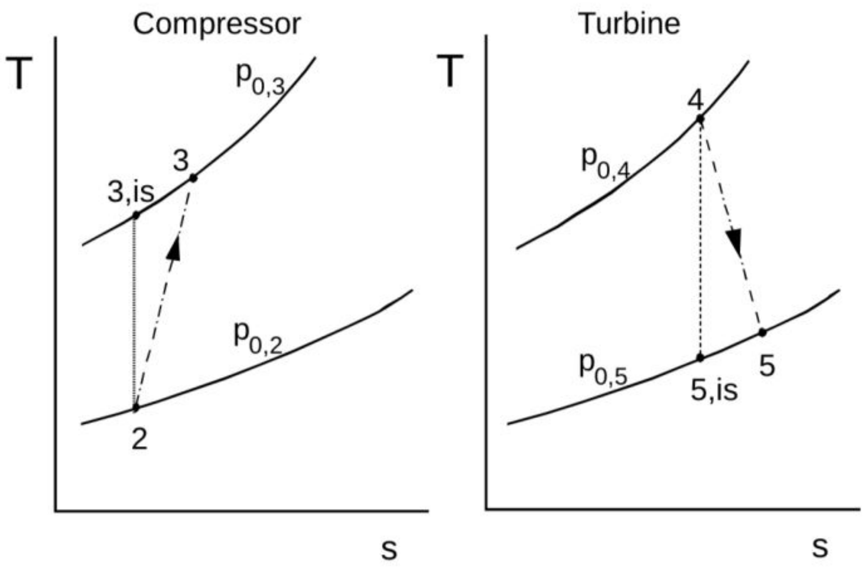
INLET PERFORMANCE

Inlet Efficiency

$$T_{0,2} = T_0 + \frac{V_0^2}{2C_{p,a}} = T_0\left(1 + \frac{k_a - 1}{2} M_0^2\right)$$

$$P_{0,2} = P_0\left[1 + \eta_{in}\cdot\frac{V_0^2}{2C_{p,a}T_0}\right]^{\frac{k_a}{k_a-1}} = P_0\left[1 + \eta_{in}\frac{k_a - 1}{2}M_0^2\right]^{\frac{k_a}{k_a-1}}$$

Real Process Rotating Components

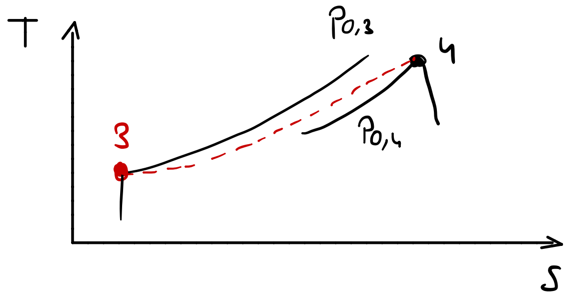


$\boxed{\text{Compressor}}$

$$\eta_{is,comp} = \frac{\dot{W}_{comp,is}}{\dot{W}_{comp,real}} =$$

$$= \frac{T_{0,3,is} - T_{0,2}}{T_{0,3} - T_{0,2}}$$

Real Combustion Chamber:



Pressure drop: $\Pi_{cc} = \dfrac{p_{0,4}}{p_{0,3}}$

Efficiency: $\eta_{cc} = \dfrac{\dot{m} \cdot c_{p,gas} \cdot (T_{0,4} - T_{0,3})}{\dot{m}_f \cdot LHV_f}$

$\boxed{\text{Turbine}}$

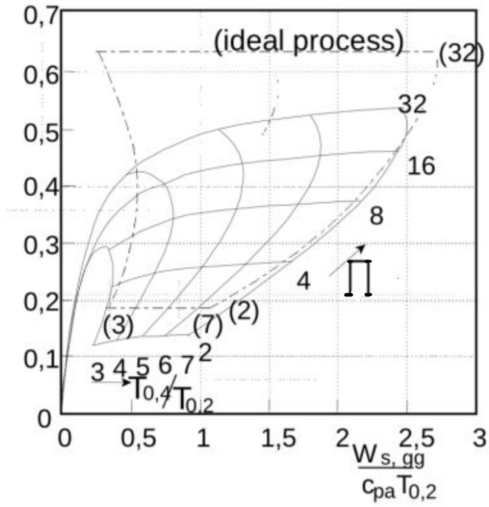
$$\eta_{is,turb} = \frac{W_{turb,real}}{W_{turb,is}} = \frac{T_{0,4} - T_{0,5}}{T_{0,4} - T_{0,5,is}}$$

Counting equations and parameters

Parameters 25 Relations 12	$T_o - T_{0,g}$ $p_o - p_{0,g}$ Π_{comp} - Π_{cc} v_o $\dot{m}$, $\dot{m}_f$, LHV_f η_{inlet}, η_{comp}, η_{cc}, η_{turb}, η_{mech}, η_{th} W_{comp}, W_{turb}, W_{gg}	5 5 2 1 3 6 3
		25
Design	T_o, p_o, v_o, Π_{comp}, T_{04}, $\dot{m}$, η_{inlet}, η_{comp}, η_{cc}, η_{turb}, η_{mech} Π_{cc}, LHV_f	(13)
Test bed	T_o, p_o, $T_{0,1}$, $T_{0,2}$, T_{0g}, $\dot{m}_f$, LHV_f, $p_{0,1}$, $p_{0,2}$, $p_{0,3}$, $p_{0,g}$, η_{mech}, η_{cc}	(13)

Gas Generator With Losses

η_{th}



Specific power and thermodynamic efficiency of a gas generator with losses.

Chapter 4 Aero - Engines

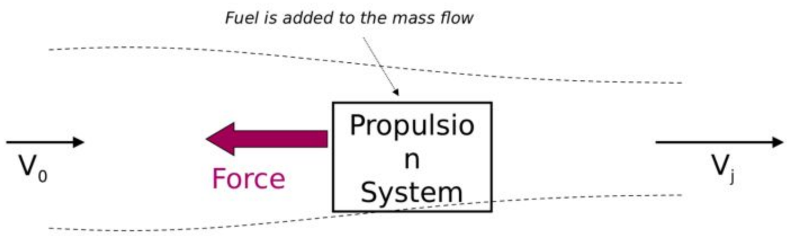
Propulsion

Linear momentum equation

$F = \Delta I = I_{out} - I_{in}$

$T = (m + m_f) V_j - m V_0$

$T = m (V_j - V_0)$



Force can be created:

Small amount of mass a large acceleration

Large amount of mass a small acceleration.

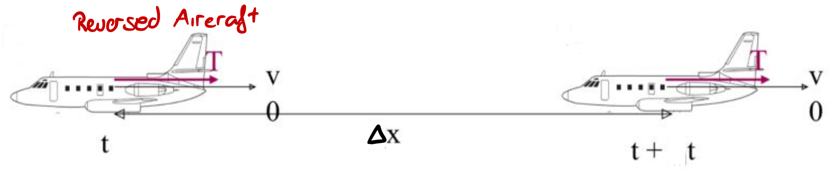
- A small acceleration to a large mass: Propeller.
- A large acceleration to a small mass: Jet.
- The mass can also be taken with you. Rocket.

Choice, what is the most efficient?

- Power available P_a → Total efficiency η_{tot}
- Jet Power P_j → Propulsive efficiency. η_{prop}
- Thermal Power $\dot{Q}$ → Thermal efficiency η_{th}

Power Available

Reversed Aircraft



Work:

$W = F \cdot \Delta V$

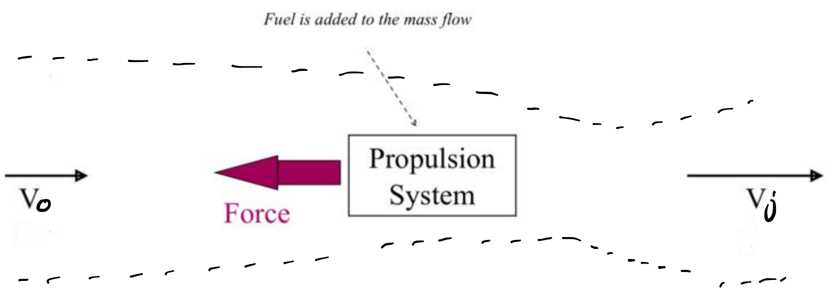
$W = T(x_2 - x_1)$

Power:

$$P = \frac{W}{\Delta t} = \frac{T(x_2 - x_1)}{\Delta t} = T\frac{\Delta x}{\Delta t} = T V_0$$

$P_a = T V_0$

Jet Power : increase in kinetic energy of the flow.



$$P_j = \frac{1}{2} m V_j^2 - \frac{1}{2} m V_0^2$$

Thermal Power Q

Heat energy supplied to the process (burning fuel) $\boxed{Q = \dot{m}_f \, LHV_f}$

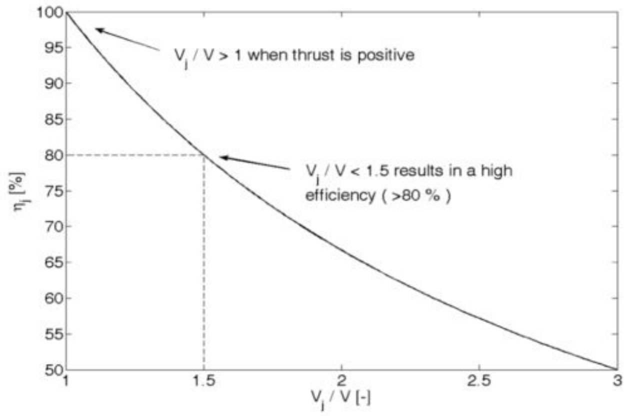
Total Efficiency

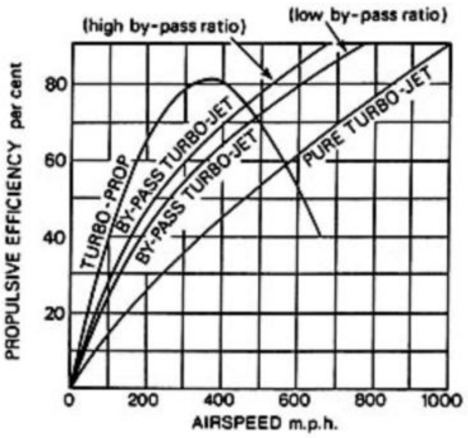
Ratio of Power available over Thermal power. $\eta_{tot} = \frac{P_a}{\dot{Q}}$ $\eta_{tot} = \frac{P_a}{P_j} \cdot \frac{P_j}{\dot{Q}} = \eta_{prop} \cdot \eta_{th}$

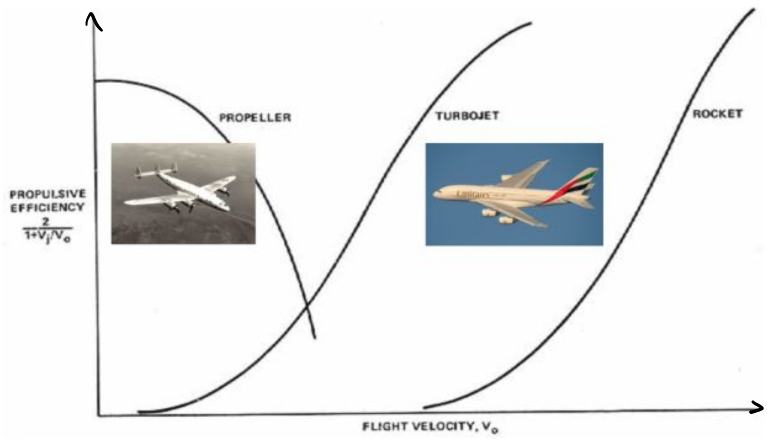
Propulsive Efficiency

$$\eta_{prop} = \frac{P_a}{P_j} = \frac{2}{1 + \frac{V_j}{V_0}}$$

- Conclusions:
  - Faster speeds become more efficient.
  - Lower speeds: propeller. (turboprop)
  - Higher speeds: jet engine.







Design Considerations

$$sfc \sim \frac{\text{airspeed}}{\eta_{th} \cdot \eta_{prop}}$$

Introduction of gears to reduce speed of fan blades.

η_{th}

- press ratio ↑
- component efficiency ↑
- TET ↑

→ higher mass flow needed for cooling; more wear / corrosion

η_{prop}

- TET ↓
- bypass ratio ↑

→ larger and heavier engine more nacelle drag

→ COMPROMISE

Useful Definitions

Thrust

$$T = m(V_j - V_0)$$

Power

$$P_a = T V_0 \quad \text{thrust power}$$

$$P_j = \frac{1}{2} m V_j^2 - \frac{1}{2} m V_0^2$$

$$Q = m_f \cdot LHV_f$$

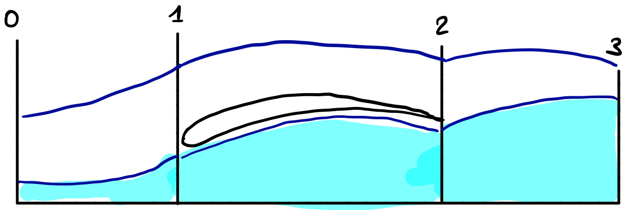
Efficiency

$$\eta_{tot} = \frac{P_a}{Q} = \eta_j \cdot \eta_{th}$$

$$\eta_{prop} = \frac{P_a}{P_j} = \frac{2}{1 + \frac{V_j}{V_0}}$$

$$\eta_{th} = \frac{P_j}{Q}$$

A closer look into Thrust



$$F_{0-1} = \dot{m}(V_1 - V_0) + A_1 \cdot (p_1 - p_0)$$

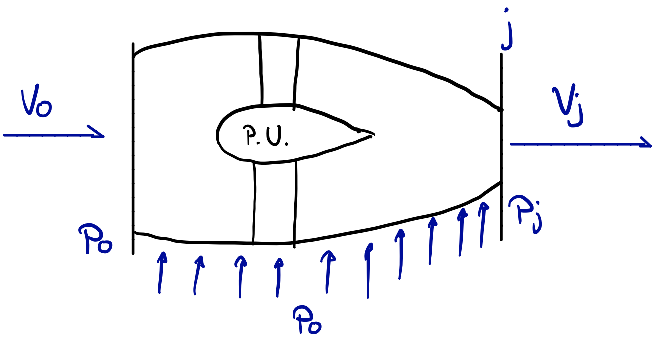
$$F_{1-2} = \dot{m}(V_2 - V_1) + A_2(p_2 - p_0) - A_1(p_1 - p_0)$$

$$F_{2-3} = \dot{m}(V_3 - V_2) - A_2(p_2 - p_0)$$

$$F_{0-2} = \dot{m}(V_2 - V_0) + A_2(p_2 - p_0) \quad \boxed{\text{NET THRUST}}$$

$$F_{0-3} = \dot{m}(V_3 - V_0)$$

Composition of Thrust



$$\boxed{\text{Thrust} = m(V_j - V_0) + A_j(p_j - p_0)}$$

Gross Thrust: $F_G = \Sigma\left[m V_j + A_j(p_j - p_0)\right]$

Net Thrust: $F_N = \Sigma\left[m(V_j - V_0) + A_j \cdot (p_j - p_0)\right]$

Specific Thrust: $F_s = F/m$

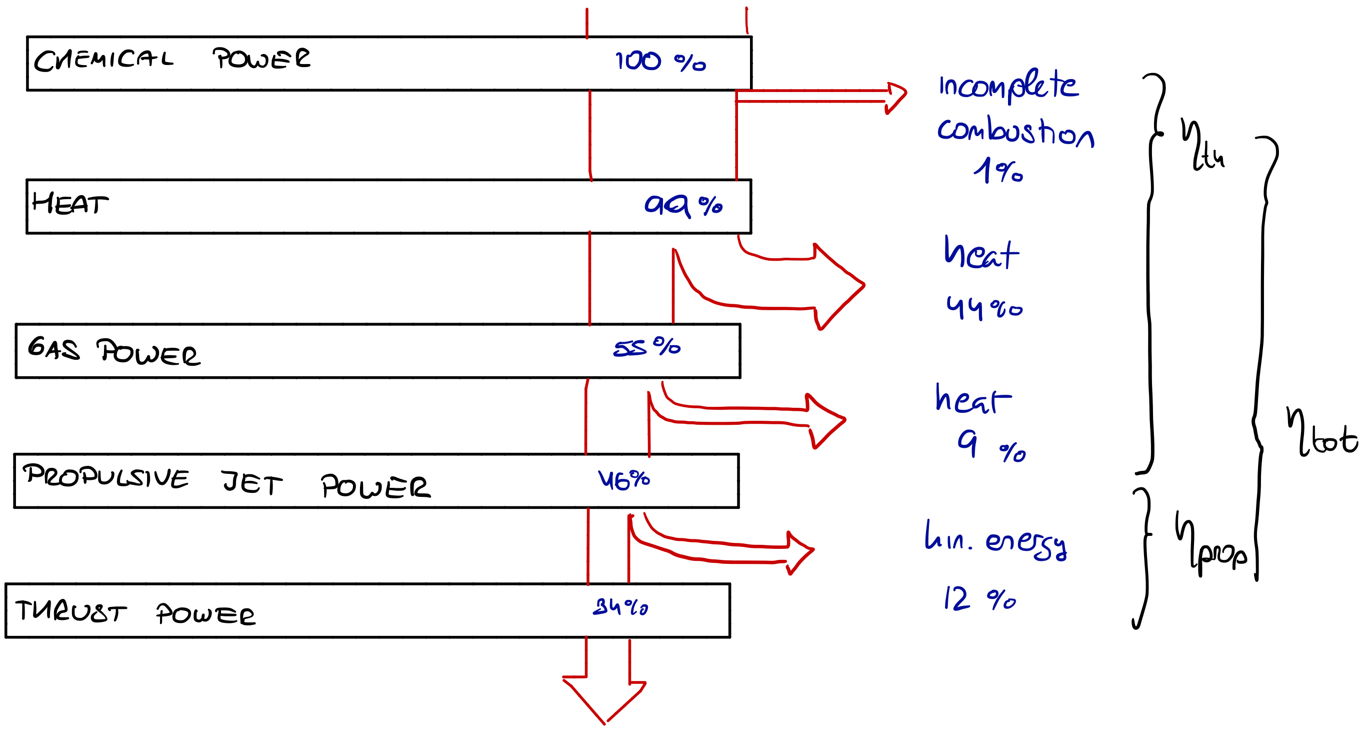
Specific fuel consumption: $sfc = \dfrac{m_f}{F_N}$

Jet Engine Power and losses

Jet Power: $W_{prop.jet} = \frac{1}{2} m (V_j^2 - V_0^2)$ Power loss: $P_{loss} = \frac{1}{2} m (V_j - V_0)^2$

Thrust Power: $W_{thrust} = m(V_j - V_0) V_0$

Jet Engine Sankey Diagram

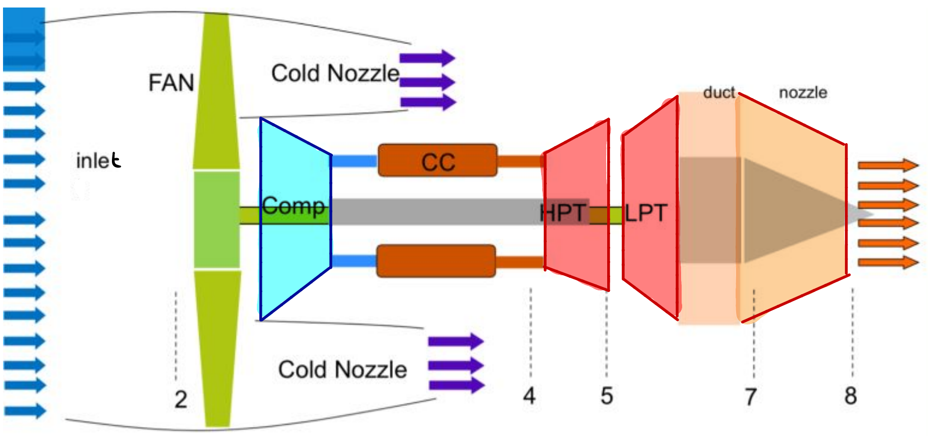


$$\eta_{thermal} = \frac{\Sigma\{\frac{1}{2}m(v_j^2 - v_0^2)\}}{m_f \cdot LHV_f}$$

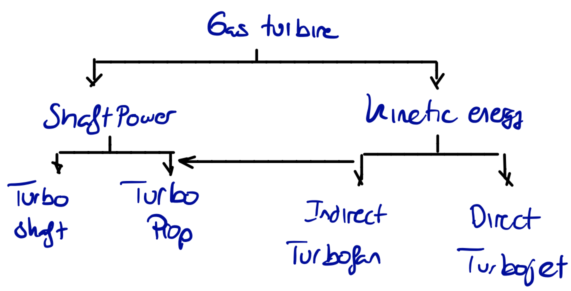
$$\eta_{prop} = \frac{\Sigma\{m(v_j - v_0)\}v_0}{\Sigma\{\frac{1}{2}m(v_j^2 - v_0^2)\}} = \frac{2}{1 + \frac{v_j}{v_0}}$$

$$\eta_{tot} = \eta_{prop} \cdot \eta_{th} = \frac{\Sigma\{m(v_j - v_0)\}v_0}{m_f \cdot LHV_f}$$

Increase Propulsive Efficiency



Gas Power Engines



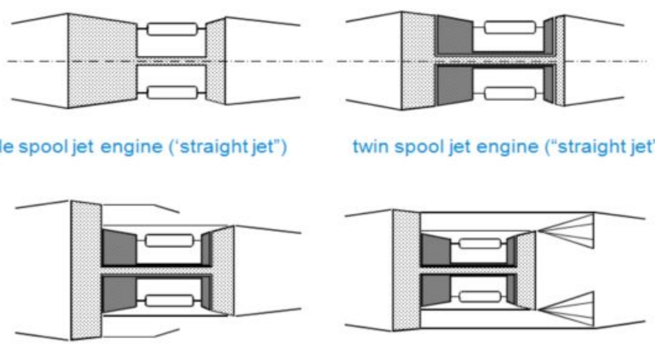
Turbojet: all air through combustion chamber

Turbofan: bypass, mixed vs. separate exhaust nozzles.

Turbo prop: propeller + exhaust jet thrust. free vs. fixed power turbine.

Single vs. multiple shaft engines.

Jet engine Configurations



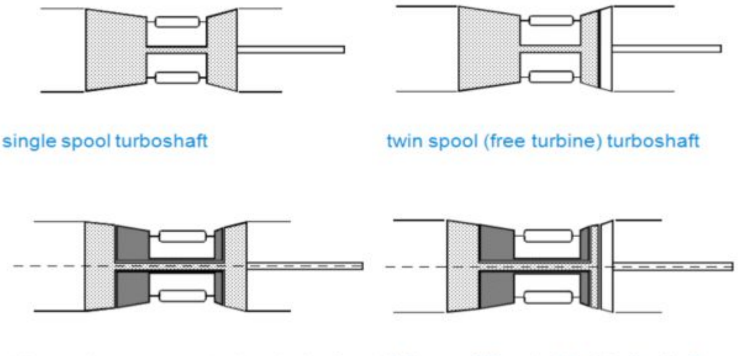
single spool jet engine ('straight jet")

twin spool jet engine ("straight jet")

twin spool turbofan (separate cold exhaust nozzle)

twin spool 'mixed' turbofan

Turboshaft (-prop) Configurations



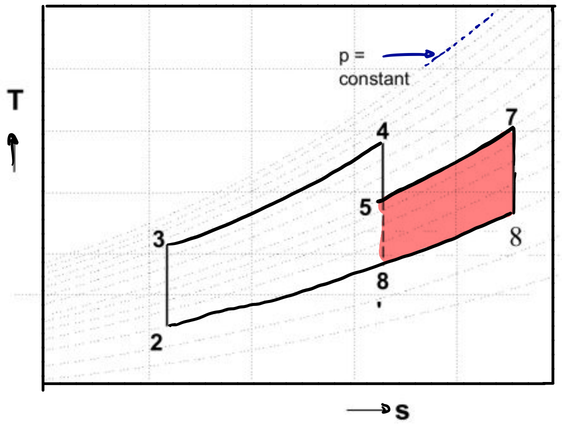
single spool turboshaft

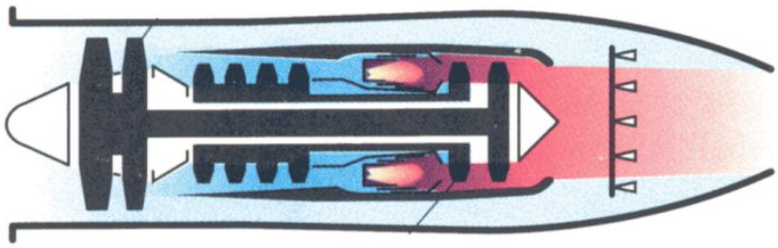
twin spool (free turbine) turboshaft

twin spool gas generator / turboshaft

triple spool (free turbine) turboshaft with twin spool gas generator

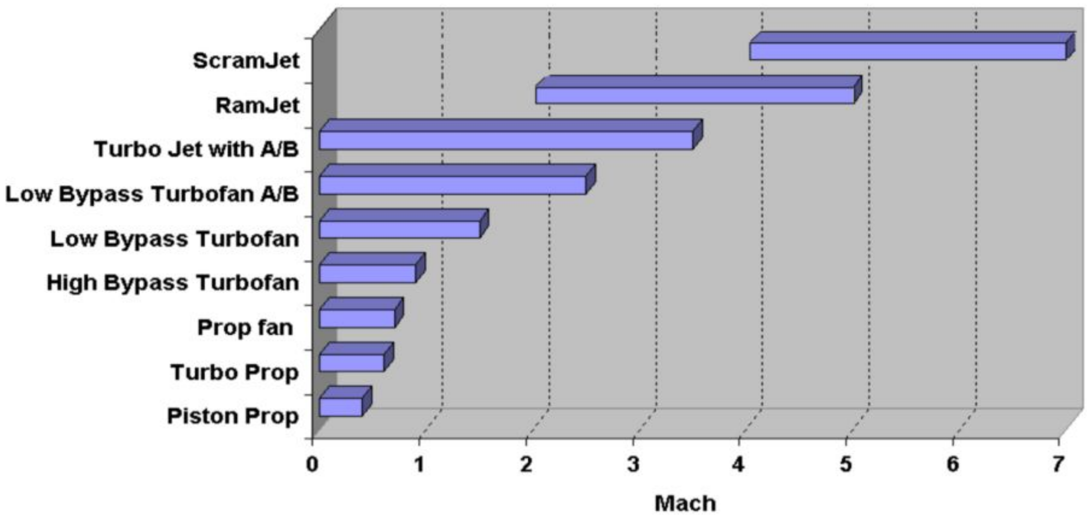
Afterburner:



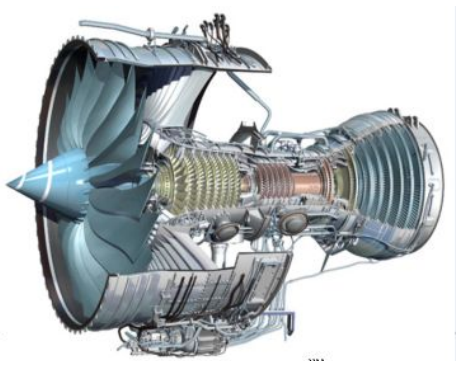


The engine burns much more fuel than in the whittle engine and occupies much less volume than the earlie combustion chambers

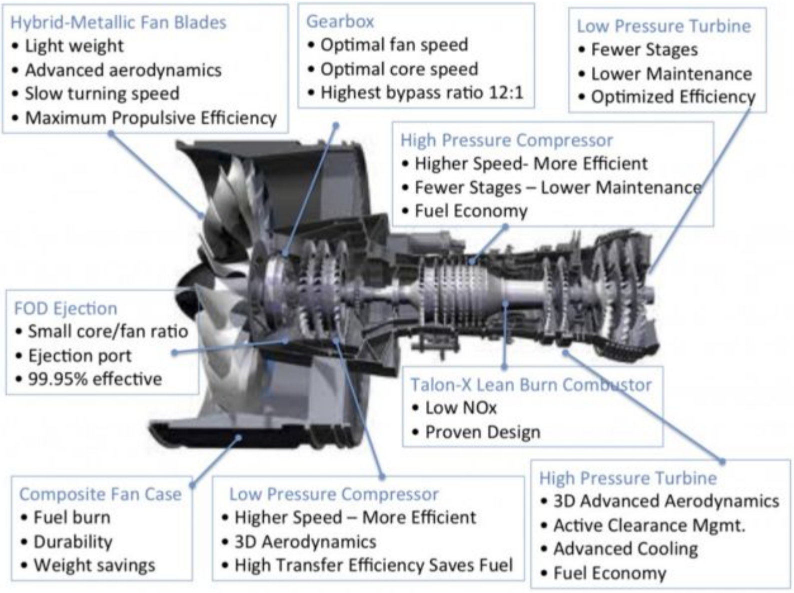
Why so many engines?



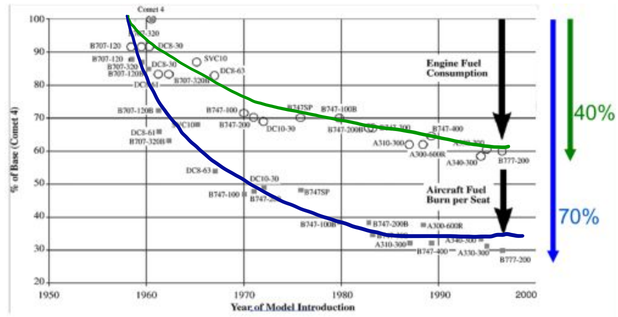
3 Spool Turbofan



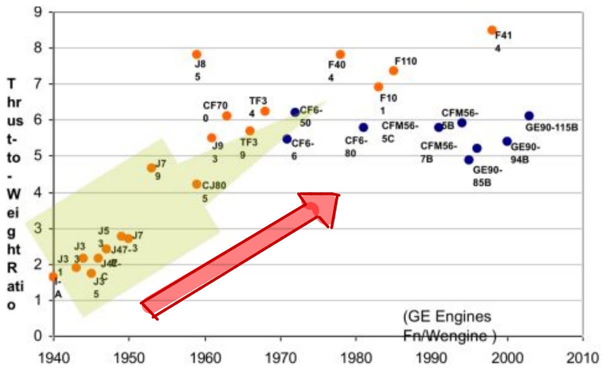
ENGINE IMPROVEMENTS



FUEL



THRUST



ENGINE INTAKE SYSTEM

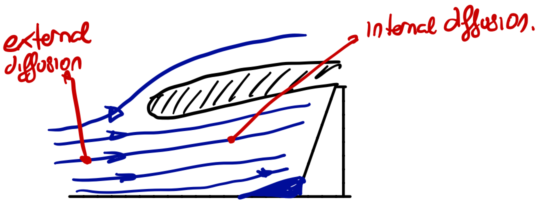
CONSTRAINS

- Engine Operation / Control
- FOD containment
- Noise
- Cost
- Weight, size, ground clearance
- Minimizing Pressure loss, drag
- RCS / IR

INFORMATION:

- Esentially a fluid flow duct whose task is to process airflow in a way that ensures that the engine functions properly.
- Provide adecuate amount of uniform airflow
- Cruising: conversion of Ekin into Epot (pressure)
- Ram compression / "Ram recovery"
- Supersonic vs. subsonic intake
- "Bellmouth" intake (test bed / stationary)
- Subsonic inlets are dominated by the boundary layer behaviour.
- Supersonic inlets are dominated by the shock structure.

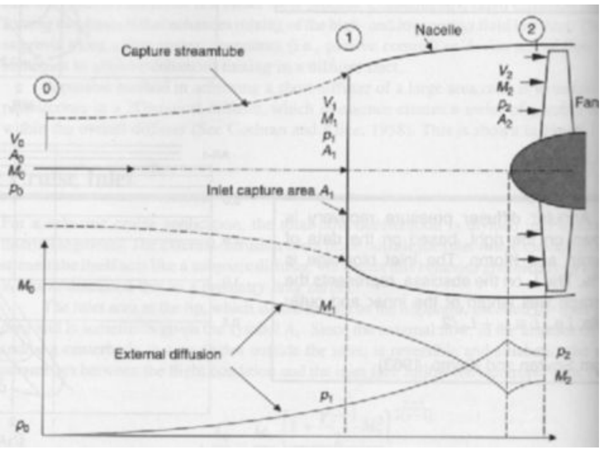
Ram Compression



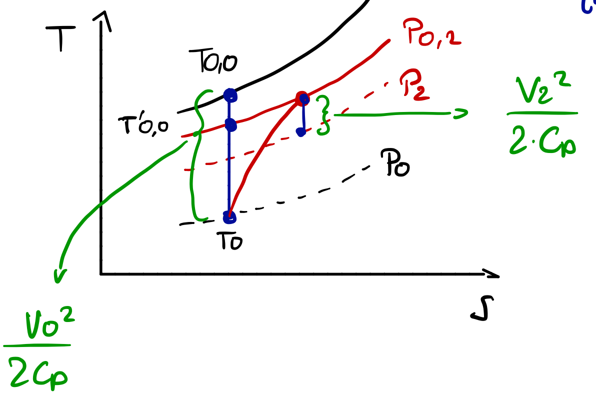
Without losses

$$T_{0,1} = T_0 + \frac{V_0^2}{2C_p} = T_0\left[1+\frac{k_a-1}{2}M_0^2\right]$$

$$P_{0,1} = P_0 + \left[\frac{T_1}{T_0}\right]^{\frac{k_a}{k_a-1}} = P_0\left[1+\frac{k_a-1}{2}M_0^2\right]^{\frac{k_a}{k_a-1}}$$



AT CRUISE



$$\eta_{is} = \left(\frac{T'_{0,0} - T_0}{T_{0,0} - T_0}\right)$$

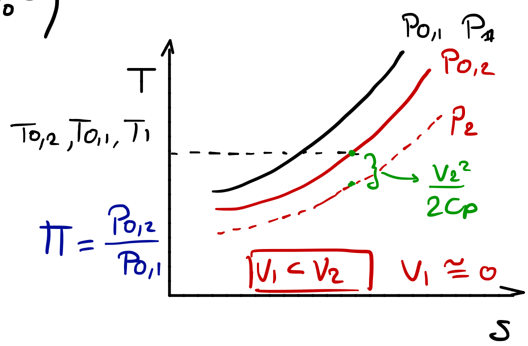
$$\eta_{ram} = \frac{P_{0,2} - P_0}{P_{0,0} - P_0}$$ easier to measure

η_{is} and η_{ram} can be used interchangeable till $M < 0.8$

$$P_{0,2} = P_0\left(1+\eta_{is}\cdot\frac{V_0^2}{2C_p T_0}\right)^{\frac{k_a}{k_a-1}} = P_0\left(1+\eta_{is}\frac{k_a-1}{2}M_0^2\right)^{\frac{k_a}{k_a-1}}$$

Subsonic Intake

a) Rest b) Static + crosswind c) low-speed flight.



$$\pi = \frac{P_{0,2}}{P_{0,1}}$$

Engine Exhaust System

- Minimizing Pressure loss, drag
- Cost
- Engine operation/control
- Thrust vectoring / Reversing
- Noise
- RCS / IR
- Minimizing weight, size

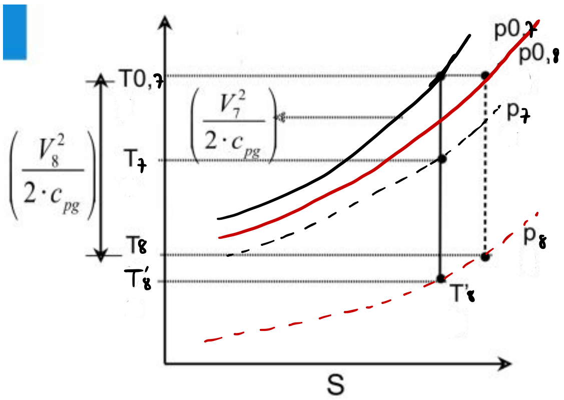
Information:

- Accelerate the flow to a high velocity with minimum pressure loss
- Match exit pressure to atmospheric, as closely as possible
- Suppress jet noise
- Permit control of the engine operating characteristics.

INFORMATION (CNTD)

- Permit afterburner operation
- Mix core and bypass streams of turbofan if necessary
- Allow for thrust reversing if desired
- Thrust Vectoring control
- Suppress radar cross-section
- Suppress infrared emision.
- Minimize cost, weight and drag while meeting reliability.

NOZZLE EFFICIENCY:



$$\eta_{is,noz} = \left(\frac{T_{0,7} - T_8}{T_{0,7} - T'_8} \right)$$

$$\eta_{is,noz} = \frac{T_{0,7} - T_8}{T_{0,7}\left[1 - \left(\frac{P_8}{P_{0,7}}\right)^{\frac{k_g-1}{k_g}}\right]}$$

No isentropic efficiency.

Represent losses:

LOSSES:

- CD : discharge coefficient = A_{eff} / A_{geom} ~ 0.95 - 0.97
- CV : velocity coefficient = $V_{jet} / V_{jet\ ideal}$ ~ 0.93 - 0.98
- CX : thrust coefficient = $F_6 / F_{6\ ideal}$ ~ 0.95 - 0.98

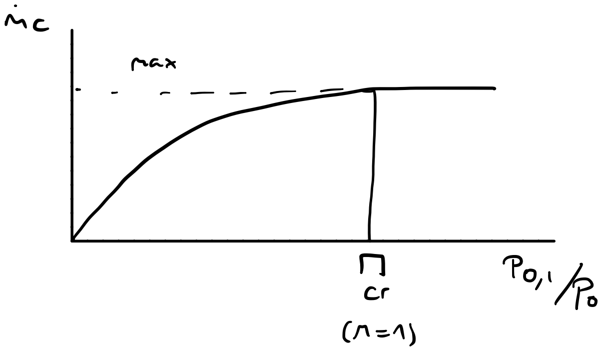
similar loss.

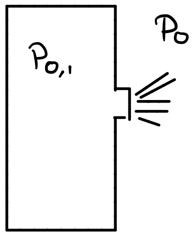
↳ gross thrust

THRUST LOSS:

- Due to the exhaust velocity angularity.
- Due to the exhaust swirl
- Due to loss of mass due to leakage.
- Due to reduction in the velocity magnitude caused by friction.

CRITICAL PRESSURE RATIO





COMPLETE EXPANSION — Calculation recipe for unchoched nozzle

$$\frac{P_{0,7}}{P_0} \le \Pi_{cr} \qquad P_{jet} = P_0 = P_{atm} \qquad T_{0,7} - T_8 = T_{0,7} \cdot \eta_{is,noz} \left[1 - \left(\frac{P_0}{P_{0,7}} \right)^{\frac{k_g - 1}{k_g}} \right]$$

$$P_{jet} = P_0 = P_{atm} \qquad V_{jet} = \sqrt{2 c_{p,g} \left(T_{0,7} - T_8 \right)} \qquad F_N = \dot m \cdot \left(V_8 - V_0 \right)$$

INCOMPLETE EXPANSION — Calculation recipe for chocked nozzle.

$$\frac{P_{0,7}}{P_{atm}} > \Pi_{cr} = \frac{1}{\left[1 - \frac{k_g - 1}{\eta_{is,noz} \left(k_g + 1 \right)} \right]^{\frac{k_g}{k_g - 1}}}$$

$$P_8 = P_{0,7} / \Pi_{cr}$$

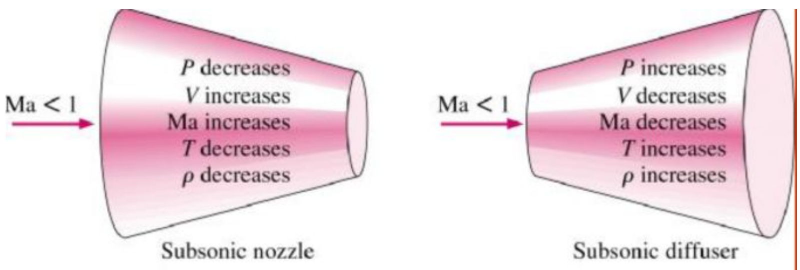
$$T_8 = T_{0,7} / TR_{cr}$$

$$TR_{cr} = \frac{k_g + 1}{2}$$

$$V_8 = \sqrt{k_g R T_8} \qquad \left(M_8 = 1 \right)$$

$$A_8 = \frac{\dot m R T_8}{P_8 \cdot V_8} \qquad F_N = \dot m \left(V_8 - V_0 \right) + A_8 \left(P_8 - P_0 \right)$$

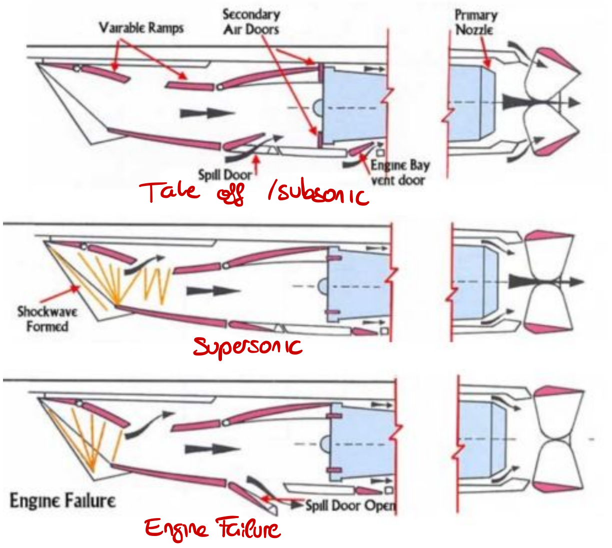
COMPRESSIBILITY:



(a) Subsonic flow

LECTURE 5 AERO - ENGINES

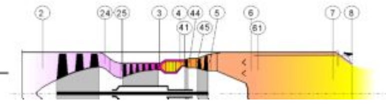
TURBOJET concorde → Rolls Royce - Snecma Olympus 593 / Variable geometry intake.



General characteristics at ISA take off conditions
Compressor= 2 spool axial, 7 low pressure stages, 7 high pressure stages
Turbine= Single stage high pressure, single stage low pressure
Nozzle= Convergent

LPC Pressure Ratio = 4.0 HPC Pressure Ratio = 4.0
Bypass Ratio = 0
Ambient Temperature = 288 K Combustor Exit Temperature ($T_{0,4}$) = 1450K

Intake Pressure ratio= 0.92 (at take off)
Combustion chamber Pressure Ratio = 0.97 Afterburner Pressure Ratio = 0.97
Afterburner efficiency = 0.95 Afterburner Exit Temperature ($T_{0,7}$) = 1850 K
Engine mass flow rate= 160 kg/s
Compressor isentropic efficiency = 0.85 Turbine isentropic efficiency = 0.9
Mechanical efficiency = 0.99 Combustion efficiency = 0.99
Nozzle efficiency = 0.95
$c_{p,air}$ = 1000 J/kg.K; κ_{air} = 1.4 $c_{p,gas}$ = 1150 J/kg.K; κ = 1.33
Gas constant= 287 J/kg.K
Fuel calorific value = 43 MJ/kg
Ambient Pressure = 101,325 Pa



- Inlet pressure ratio is 0.92 at takeoff

$P_{0,2} = 0.92 \cdot 101325$ $P_a = 93219$ $T_2 = 288\ K$

$P_{0,25} = P_{0,2} \cdot 4.0 = 372876\ Pa$

$T_{0,25} = 452.9\ K$, $\dot m_{25} = 160\ kg/s$

$P_{0,3} = P_{0,25} \cdot 4.0 = 1491504\ Pa$

$$\frac{T_{0,25}}{T_{0,2}} = 1 + \frac{1}{\eta_s}\left[\left(\frac{P_{0,25}}{P_{0,2}}\right)^{\left(\frac{k-1}{k}\right)} - 1\right]$$

$$\frac{T_{0,3}}{T_{0,25}} = 1 + \frac{1}{\eta_{is}}\left[\left(\frac{P_{0,3}}{P_{0,25}}\right)^{\left(\frac{k-1}{k}\right)} - 1\right]$$

- Combustion chamber:

$T_{0,4} = 1450\ K$

$$\dot m_{fuel} = \frac{\dot m_3 \cdot Cp_{gas} \cdot \Delta T}{LHV \cdot \eta_{cc}}$$

$\dot m_{fuel} = 3.19\ kg/s$

$P_{0,4} = 0.97 \cdot P_{0,3} = 1446758\ Pa$

$\dot m_4 = \dot m_3 + \dot m_{fuel} = 163.19\ kg/s$

Work done in LPC $= \dot m \cdot Cp_{air} \cdot (T_{0,25} - T_{0,2})$

Work done in HPC $= \dot m \cdot Cp_{air} (T_{0,3} - T_{0,25})$

$\dot W_{LPC} \rightarrow 26.352\ MW$
$\dot W_{HPC} \rightarrow 41.41\ MW$

$T_{0,45} - T_{0,5} = (\dot W_{LPC}) / (\dot m_{45} \cdot Cp_{gas} \cdot \eta_{mech}) = 141.9\ K$

$T_{0,5} = (1227\ K - 141.9\ K) = 1085.1\ K$

$$\frac{T_{0,5}}{T_{0,45}} = 1 - \eta_{is,LPT}\left[1 - \left(\frac{P_{0,5}}{P_{0,41}}\right)^{\left(\frac{k_s-1}{k_s}\right)}\right]$$

$P_{0,5} = 390565$

- Afterburner:

$P_{0,7} = P_{0,5} \cdot 0.97 = 379\,233.79 \text{ Pa}$ $\qquad T_{0,7} = 1850$ given

$$\dot{m}_{fuel,ab} = \frac{\dot{m}_5 \cdot C_{p,gas} \cdot \Delta T}{LHV \cdot \eta_{ab}} = 3.513 \text{ kg/s} \qquad \dot{m}_7 = \dot{m}_5 + \dot{m}_{ab}$$

- Nozzle:

$$\frac{P_{0,7}}{P_{crit}} = \left[\frac{1}{\left(1 - \left(\frac{1}{\eta_{noz}}\right) \cdot \left(\frac{k_g - 1}{k_g + 1}\right)\right)^{\frac{k_g}{k_g - 1}}} \right] = 1.916 \qquad \frac{P_{0,7}}{P_0} = 3.74$$

$(<) \rightarrow$ choked!

$$T_8 = T_{0,7} \cdot \left(\frac{2}{k_g+1}\right) = 1587.98 \text{K} \qquad P_8 = P_{0,7}\left(\frac{1}{P_{0,7}/P_{CR}}\right) = 19764$$

$$\rho_8 = \left(\frac{P_8}{R \cdot T_8}\right) = 0.434 \text{ kg/m}^3 \qquad V_8 = \sqrt{k_g \cdot R \cdot T_8} = 778.56$$

$$A_8 = A_{noz} = \left(\frac{\dot{m}}{\rho_8 \cdot V_8}\right) = 0.493 \qquad F = \dot{m}\,(V_8 - V_0) + A_8\,(P_8 - P_0) = 177.41 \text{ kN}$$

$$SFC = \left(\frac{\dot{m}_{fuel}}{F}\right) = 37.78 \text{ gm/kN}\cdot\text{s}$$

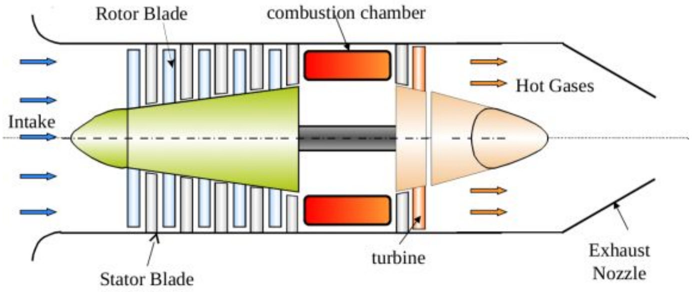
FOR TURBOPROP LOOK IN THE SLIDES

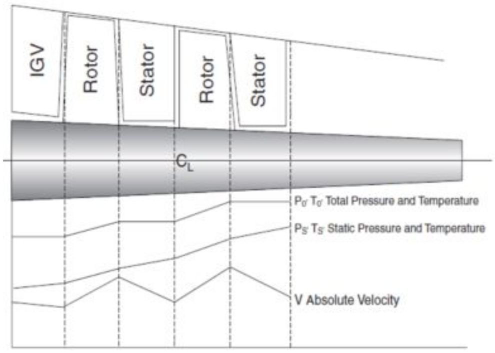
Lecture 6 Turbomachinery

Rotating equipment that performs work on a fluid or extracts work from a fluid.

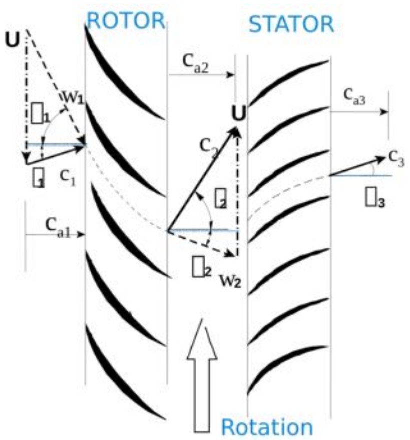
Types of Compressors:

Axial Compressor

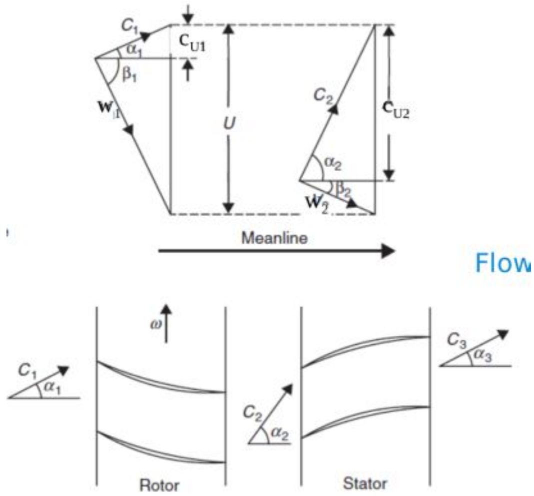


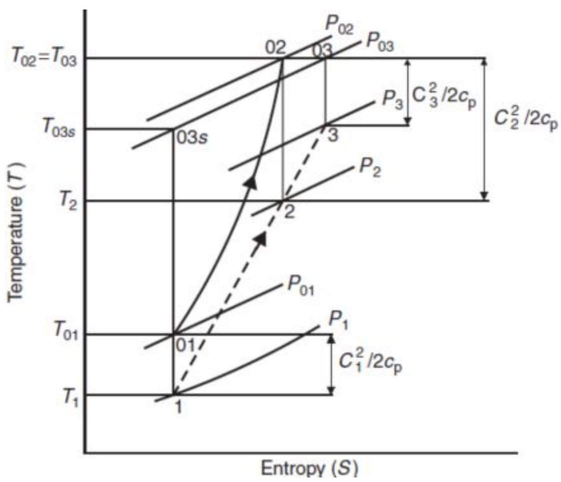


Flow pattern



→ velocity triangles





Euler Formula:

1. $F = m_2 \cdot C_{U2} - m_1 \cdot C_{U1}$ → force exerted on the fluid.

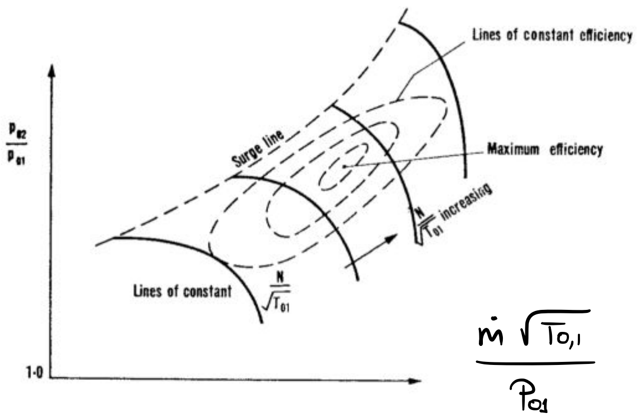
2. $t = m_2 \cdot C_{U2} \cdot r_2 - m_1 \cdot C_{U1} \cdot r_1$ → torque exerted on fluid.

3. $W = t \cdot \omega$ → Torque x Angular velocity
Power input to the fluid.

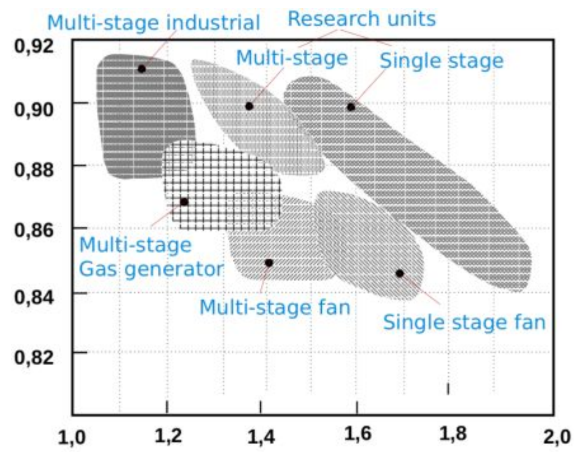
4. $W = H_{0,2} - H_{0,1} = m(C_{U2} U_2 - C_{U1} \cdot U_1)$

W_{sp} → ↳ without $m \rightarrow h_2 - h_1$

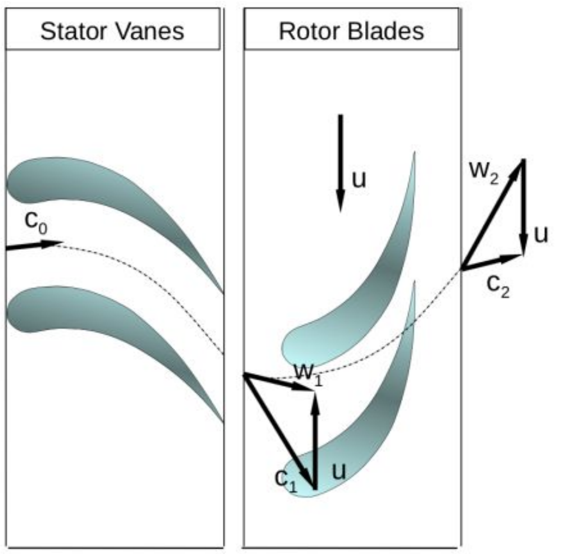
Performance Characteristic of a Compressor

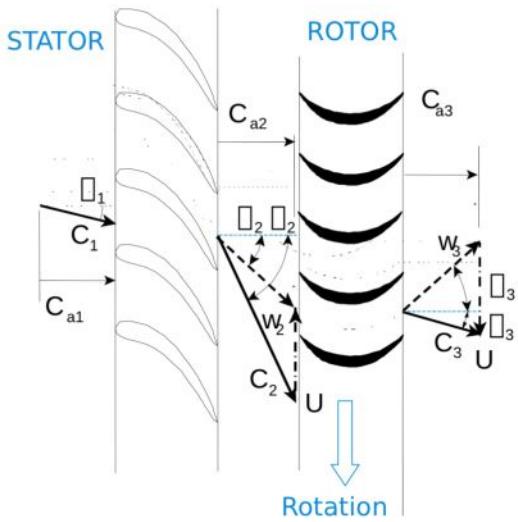


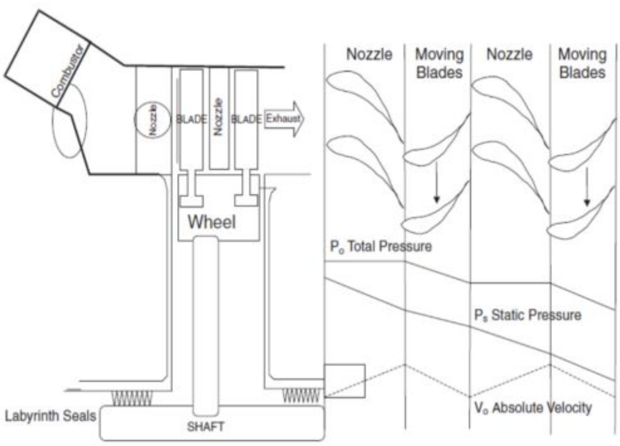
Pressure Ratios



Axial Turbines





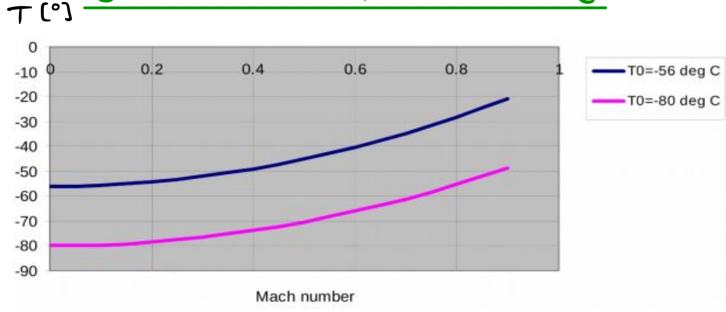


Combustion

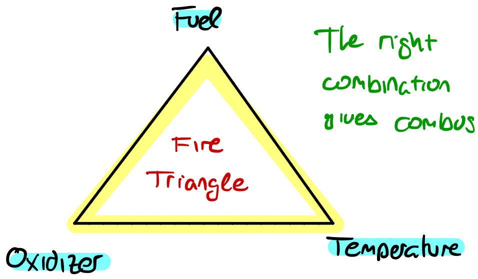
Kerosene as aviation fuel

- Diesel problem, it can freeze after 5 km altitude.
- Kerosene maximum freeze point is: -60°C good.
- Clean combustion
- Good energy density
- Good thermal stability

OPTIMAL FUEL FOR CIVIL: JET B



COMBUSTION: exothermic oxidation process (jet engines supposed a revolution)

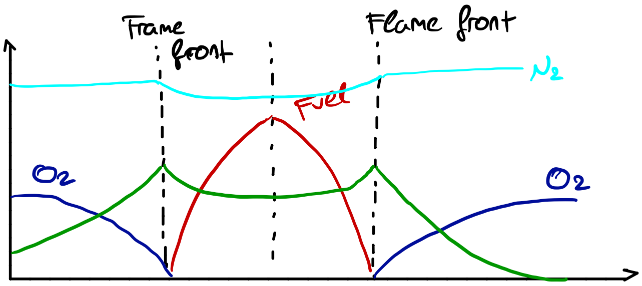


The right combination gives combustion.

CLASSIFICATION:

- MIXING OF FUEL AND OXIDIZER → NON PREMIXED (DIFFUSION) / PREMIXED
- NATURAL → LAMINAR / TURBULENT
- PHASES OF FUEL AND OXIDIZER → HOMOGENEOUS / NON HOMOGENEOUS

LAMINAR DIFFUSION FLAME



There is a difference between combustion and fire!

COMBUSTION STOICHIOMETRY: balance the equations to find how much oxigen we need, so how much air.

FUEL TO AIR RATIO

Rich mixture: more fuel than air } $\phi > 1$ → than necessary.

Lean mixture: more air than fuel. } $\phi < 1$

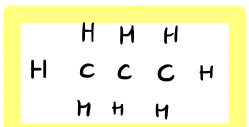
$$AF = \frac{m_{air}}{m_{fuel}} = \frac{1}{FA} \rightarrow \text{fuel to air ratio.}$$

$$AF_{mole} = \frac{n_{air}}{n_{fuel}}$$

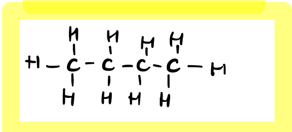
$$\phi = \frac{(F/A)_{actual}}{(F/A)_{stoich}}$$ } deviation from stoichiometric conditions.

GAS COMPOSITION

Propane:

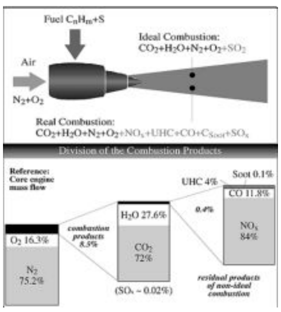


Butane:



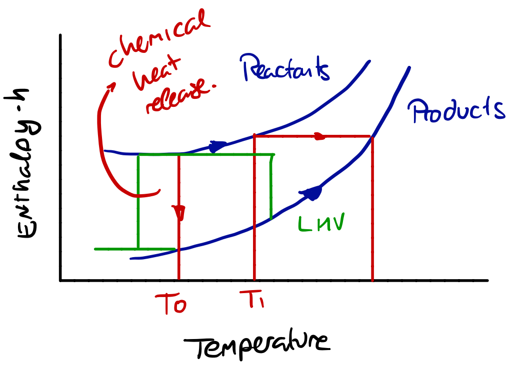
AFTER COMBUSTION:

Kerosene as aviation fuel

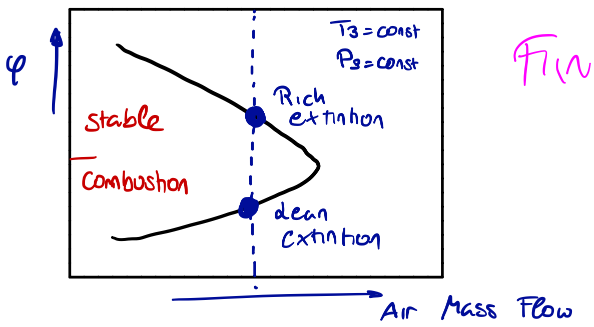


Engine Family	Aircraft	Take-Off Thrust, [kN]	Overall *AFR* at TO	Overall φ at TO
CFM56-7	B737 NG	91.6	54.0	0.27
RB211-535	B757	163.3	52.3	0.28
CF6-80E1	A330	297.4	49.3	0.30
PW4000-112"	B777	396.6	43.1	0.34

Heat of Reaction

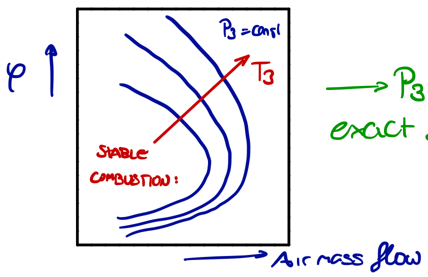


Flamability Limit



Finish

Effect of Temperature on the Flammability



→ P_3 has the exact same effect.

At the moment, we reduced the size of chambers a lot.

Combustion Chamber Requirements

- High combustion efficiency over wide operating conditions.
- Low pressure loss over the combustion system.
- Stable combustion over a wide range of inlet conditions and mass flow
- Wide range of equivalence ratio (operational reliability)
- Reliable starting capability (operational reliability)
- exhaust emmission consistent with regulation.
- minimum length
- low cost and good durability, maintainability reliability (cost)
- long operating life (engine life)
- Combustor exit temperature pattern (engine life)
- Multi-fuel capacity. (in future)

Conventional Combustor

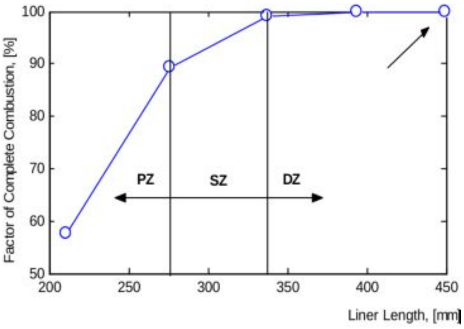
Primary zone:

- Anchors the flame
- Provides sufficient time, temperature and turbulence.

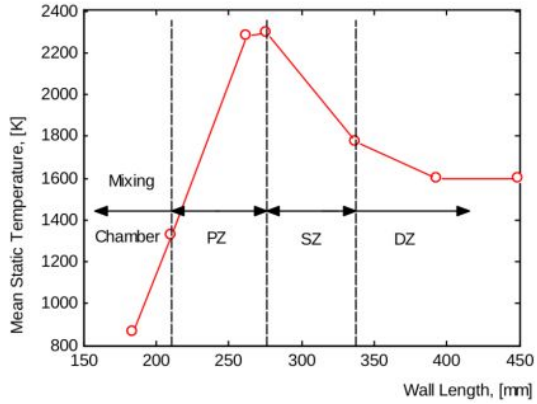
Three step process:

- Endothermic dissasociation of fuel
- Exothermic formation of CO & H_2O
- Exothermic conversion of CO to CO_2

Factor of complete combustion.



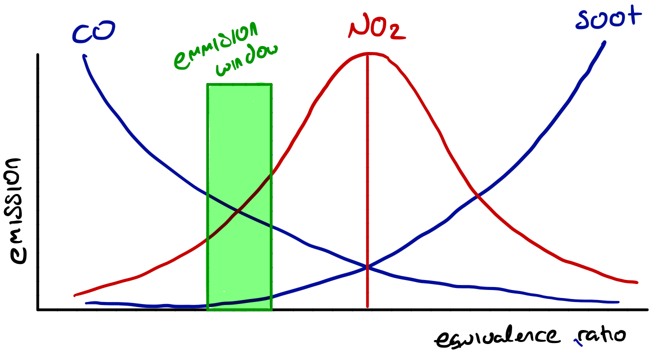
Temperature distribution:



Primary zone of the combustor:

The air around stabilizes the flame when its close to the end.

EMISSION



lean ← stoechiometric combustion → rich

COMBUSTION PERFORMANCE

efficiency: $\eta_{cc} = \dfrac{(\dot{m}_a + \dot{m}_f)c_p \Delta T}{\dot{m}_f \, LHV_f}$

$$\Pi_{cc} = \frac{P_{0,4}}{P_{0,3}}$$

pressure losses:

$$\Delta p_{3,4} = \Delta p_{cold} + \Delta p_{hot}$$

$$\Delta p_{hot} = \frac{\rho V^2}{2} \times \frac{T_4}{T_3} - 1$$

TYPES OF COMBUSTION CHAMBERS

- Elements and aspects
  - → diffuser
  - → wall and lining
  - → cooling
- Build in configurations
  - → can type
  - → annular type
  - → cannular type
  - → reverse flow

IMPACT OF AVIATION IN THE ENVIRONMENT

No effect on the Stratosphere, but in the troposphere the release of CO_2, NO_x, produces Climate Change.
At ground level, the most concerned problem is the sound.

Jet fuel consumption keeps growing.

ALTERNATIVES:

Electric motors, hydrogen, utilitary aircraft

Electrical Power Systems

- Provides power to several components in the vehicle.

Requirements:

Aircraft (combination)

Spacecraft

- Power types: $\underbrace{\text{Alternate current or } \underbrace{\text{Direct current}}_{\text{Spacecraft}}}_{\text{Aircraft (combination)}}$ non constant and constant voltage.
- Voltage and current level: For AC: root mean square (rms) are given.
- Frequency and number of phases: for AC only

Future DC systems with high voltage. reducts the size of vehicle.

Define system power of AC systems

Apparent Power $\quad P_{appa} = V \cdot I$ (V → voltage, I → current)

Actual Power $\quad P_{actual} = V \cdot I \cdot (PF) \rightarrow$ Power factor because of phase shift.

$PF = \cos(\varphi)$

- DC systems $PF = 1$
- AC systems $PF \leq 1$ $\quad 0.8 = PF$ early design.

Units: Actual power: kW or hp → 1hp = 0.746 kW
Apparent power: kVA → 1kVA = 1 kW
} to see what power is.

Exercise:

500 kVA at $PF = 0.8$

① Actual Power? 400 kW

$P_{actual} = P_{apparent} \cdot PF$

② Work of 8 hours, total energy?

$E_{TOTAL} = P_{ACTUAL} \cdot time$

Power on aircraft: new-generation spacecraft and aircraft heavily rely on electrical power. Spacecraft use very little power compared to aircraft.

How much electrical power is needed

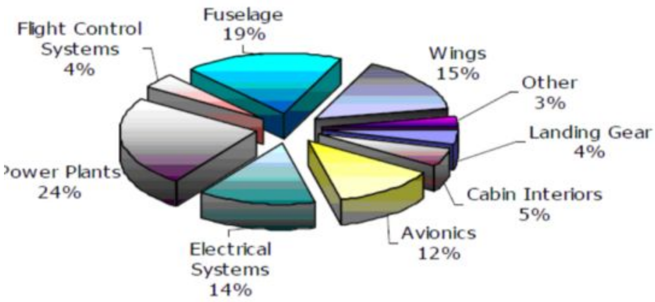
3 to 5 % total power is not used for propulsion
15 to 20 % of this power is used by electrical systems.
} in aircraft

Electrical Power system needs 0.45 to 1 %

FUTURE ↑

•

SYSTEMS OF AN AIRCRAFT



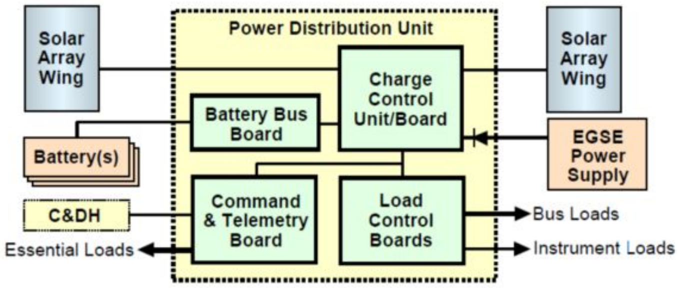
Power User	Comments	Typical Power Level
Air Conditioning	ECS	4 x 70 kW+
Flight Controls	Primary & Secondary	3 kW to 40 kW short duration at high loads
Fuel Pumps		About 10 kW
Wing Ice Protection	Thermal mats or similar	250 kW+
Landing Gear	Retraction, steering and braking	25 kW to 70 kW short duration
Engine Starting	May be used for additional applications	200 kW+ Short duration

RELIABILITY ISSUES:

- In aircrafts is good 0.9999?
  - In space launches 0.992
    - In spacecrafts 0.87

COMPONENTS

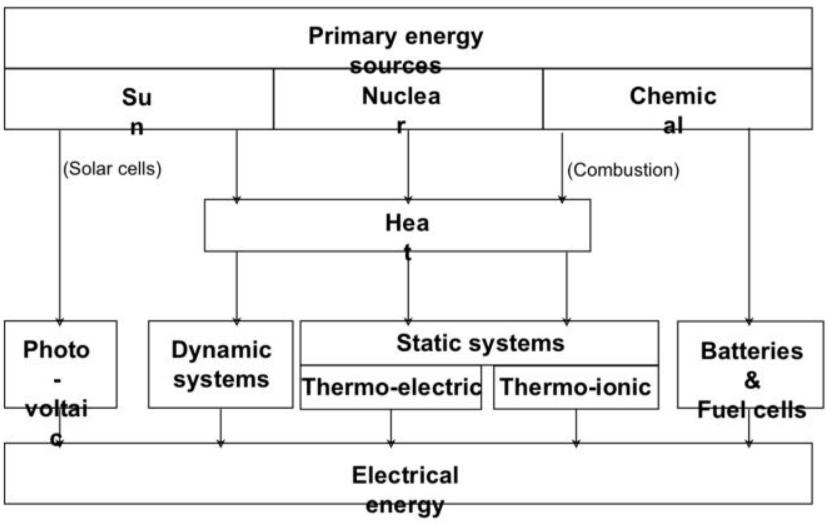
- Function is to generate, regulate and distribute the electrical power throughout the vehicle.



ENERGY SOURCES:

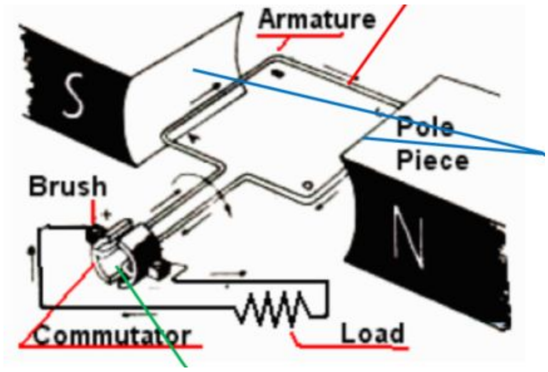
- External Sources: Outside vehicle. good Mass / cost / size budget out of the vehicle.
- Internal Sources: Stored or produced inside the vehicle. good Autonomous vehicle.

POWER GENERATIONS:



Dynamic Electric Generators

- Electromagnetic induction systems capable to convert kinetic energy into electrical energy.
- In aircraft, are driven by the engine through a belt.



(1) Rotating coil: input kinetic energy is used to rotate it at a given speed.

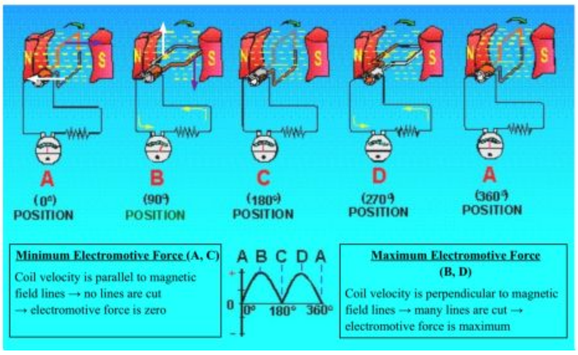
(2) Stationary magnetic field: produced by permanent magnets or electromagnets.

(3) Semi-cylindrical contacts: produce a constant-direction electromotive force

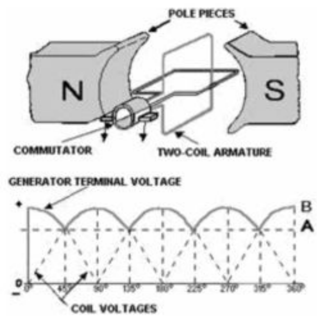
Rotating coil causes the Alternate current.

- Minimum force: coil velocity is parallel to magnetic field.
- Maximum force: coil velocity is perpendicular to magnetic field.

} Commutator makes that sinusoidal wave is always in the same direction.



Direct Current Generators



- Introduce a second commutator to reduce minimum
- Each coil is connected to the load circuit when the electromotive force is higher.
- Four coils produce reasonable constant voltge.

AC Generators:

- Same as a DC generator: without commutator.
- Two options:
  - → Rotating armature
  - → Rotating magnetic field.
- Belt driven or axis driven. simple, lighter and cheaper.
- Provide sufficient power at low rotating speed.
- Constant speed: synchronization with electrical frequency. or add controls.

Basic Equations (Performance)

Electromotive force produced: $\mathcal{E}$

$$\mathcal{E} = N \cdot B \cdot A \cdot \omega \cdot \sin(\omega t) \text{ (Volt)}$$

- N number of windings (turns) of coil
- B magnetic field strength. Tesla
- A Area enclosed by a single turn
- ω rotational speed. rad/s

Output voltage:

$$V = \mathcal{E} - I \cdot R_{arm} = N \cdot B \cdot A \cdot \omega \cdot \sin(\omega) - I R_{arm}$$

- I Current in the circuit
- R_{arm} Internal resistance of the armature

Power P:

$$P = I^2 R = \frac{V^2}{R}$$ → time dependent. → remove it using average power.

$$P_{av} = I_{rms}^2 \cdot R = \frac{V_{rms}^2}{R}$$ → root mean squared

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

Efficiency of Generator

$$\eta = \frac{P}{P_{in}}$$

At lower rotational speed → lower efficiency.

P_{in} → input power to generator.

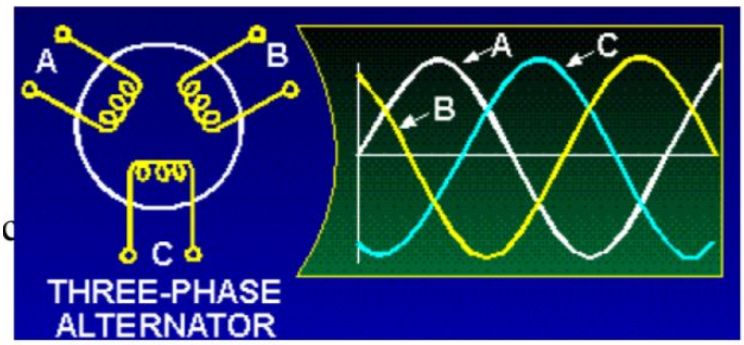
Output Frequency

$$f = \frac{n \cdot P}{120}$$

- n rotating speed (rpm)
- P number of magnetic poles.

Multi Phase Alternators

Two or more symmetrically spaced coils present. Each coil produces a phase shifted AC voltage. (3 are typically used) Produce a phase shift of 120°



Smaller and cheaper.

$$V_{rms} = \sqrt{3} \cdot (V_{rms})_{\text{single phase}}$$

Drive Systems

- Use the internal combustion engines or the ram air turbines.

$$P_{wind} = \frac{1}{2} \rho_{air} \cdot A_{blade} \cdot V_{wind}^3$$

- Also use a solar dinamic drive system. Sun heats gas, gas moves turbine.

- The drive system input power is higher than the generator input power.

$$(P_{in})_{generator} = \eta_{axis} \cdot \eta_{drive} \cdot (P_{in})_{drive}$$

└→ connection between generator and drive. (90 to 95 %)

Static Electric Power Generators

- Thermoelectric Converters (TEC) two different metals or semiconductors connected in a closed loop. If they are at different temperatures a potential is generated between them. (Seebeck effect) Peltier cells: opposite.

  Physical limit: never higher than Carnot efficiency

$$\eta_{carnot} = 1 - \frac{T_{cold}}{T_{hot}}$$ usually not more than 20%

- Thermo-Ionic Converters: (TIC) vessel containing two metal electrodes (plates) with an ionizable gas between them. The hot electrode is heated at 1400-1800 K and causes ionization of the gas between the plates. Gas generates voltage difference. 10 to 25% efficiency.

- Radio-Isotope Thermal Generator: (RTG) Power obtained from the radioactive decay of a radio-isotope material. Heat is converted to electricity. Deep space missions usually.

Photovoltaic Generators:

Primary energy source of spacecraft. Available at no cost, unlimited and external. Depends on sun.

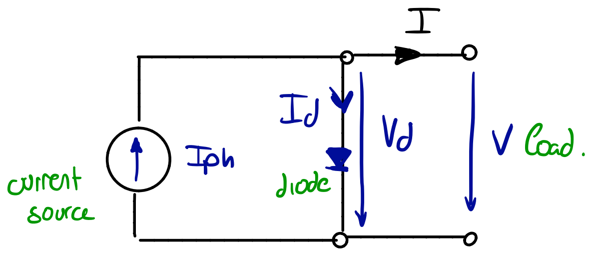
- Solar cell: semiconductor diode : electronic component that conducts current only in one direction.

  └→ Material that is an insulator at absolute zero temperature but conducts electricity at room temperature.

- A p-n junction is obtained by connecting two different types of semiconductors:
  - A positive charge carrier (p-type)
  - A negative charge carrier (n-type)

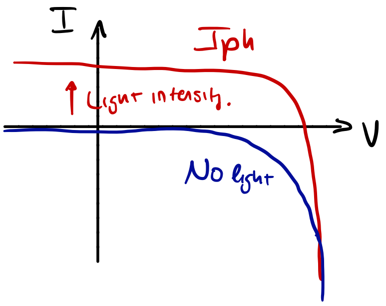
  When illuminated, electrons are released. If the circuit is closed, a current flows in it.

- Solar cell: current source in parallel with a diode.



$I = I_{ph} - I_d$

- I_{ph}: photovoltaic current also, I_{sc} (short-circuit current) → 0 when NO light
- I_d: diode current



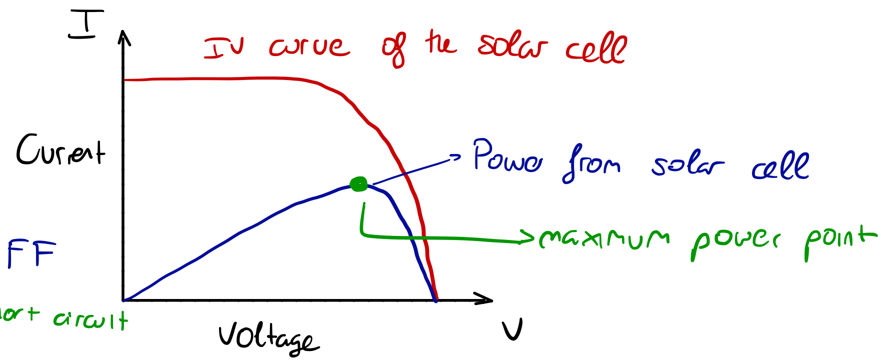
$$I_d = I_0 \left(e^{\frac{qV}{kT}} - 1 \right)$$

- I_0 reverse situation current
- k Boltzmann constant $1.38 \cdot 10^{-23}$
- q elementary electron charge $1.6 \cdot 10^{-19}$ C
- T Temperature.

Power Delivered

- $P = V \cdot I$
- $P_{max} = V_{mpp} \cdot I_{mpp} = V_{oc} \cdot I_{sc} \cdot FF$

($I=0$) open circuit; ↳ short circuit



FF: Fill factor: $FF = 1$ when I-V curve shape is completely rectangular.

EFFICIENCY: $\boxed{\eta = \frac{P_{max}}{P_{in}}}$ → It can still be improved, max of 44%

Photovoltaic Module

Made of solar cells connected in series and in parallel.

- N cells in series form a string.
- M cells in parallel form a module.

} Connected cells have the same IV curves

Various modules together form a solar array.

Single cell voltage V_{cell}
Single cell current I_{cell}
} → N cells in series: $V_a = N \cdot V_{cell}$ } M cells in parallel $I_a = M \cdot I_{cell}$

EFFECTS OF CELLS AND STRINGS FAILURES:

- Open-circuit failure of a single cell:
  - → The entire string fails.
  - → Avoid by including shunt diodes.
- Short-circuit failure of a string:
  - → Lower string voltage
  - → Reverse current might occur, higher temperatures in the cells.
  - → Avoid by including blocking diodes.

PERFORMANCE AND SIZING

- The efficiency of the solar cells in space is 1 to 3 % lower. Efficiency is based on a reference temperature of 25°C
- Variations of the I-V curve with temperature are expressed by the Temperature Coefficient.

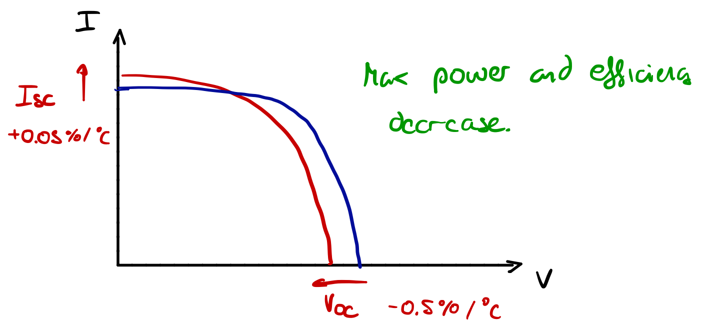
EFFECTS OF TEMPERATURE

- Increase of Temperature:

  Short circuit current slightly increases.

  Open circuit voltage significantly decreases.

  $$V_{oc}(T) = V_{oc}(T_0) + \alpha(T - T_0)$$



- Operational range $-60°C$ to $55°C$
- Thermal control is needed. Back surface reflectors, Filters cut out the energy of wavelengths.

EFFECTS OF AGEING

Caused by high energy radiations. Also by atomic oxygen, thermal cycling, micrometeoroid strikes.

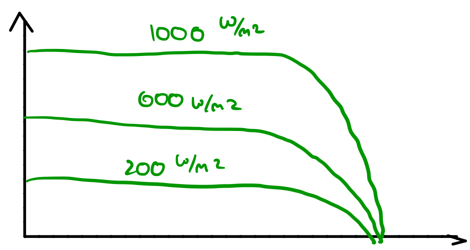
Total life degradation: $L_d = (1-\delta)^x \rightarrow P_{x\text{-years}} = L_d \cdot P_{BOL}$

δ is the yearly power degradation

→ maximum cell power at beginning of life

EFFECTS OF SOLAR FLUX:

Depends on distance from the sun.



Influence of Incidence Angle

Depends on influence Area

$$A_e = A \cdot \cos(\theta) \longrightarrow \text{maximum for perpendicular.}$$

Effect of Solar Radiation Spectrum:

- Solar radiation depends on height, decreases when in the atmosphere.
- Efficieny of the cells on Space is higher than in space.
- The air mass coefficient AM is defined as $1/\cos\theta$ θ: angle between the solar radiation direction and the earth surface.

Available Solar Array Power

$$\boxed{P_{sa} = \underbrace{S_{in}}_{\substack{\text{incident}\\ \text{solar}\\ \text{flux}}} \cdot \overset{\text{solar arrays area}}{A} \cdot \underbrace{\eta}_{\substack{\text{efficieny}\\ \text{of solar}\\ \text{cells}}} \cdot I_d \cdot \underbrace{L_d}_{\substack{\text{life}\\ \text{degradation}}} \cdot \cos(\theta)}$$

$\cos(\theta) \rightarrow$ incidence angle

$I_d \rightarrow$ Inherent degradation: vary from 0.5 to 0.9 due to shadow effects, temperature effects, packing factors. $\rightarrow$ 0.6 to 0.9

Solar Array Duty Cycle and Required Power

Nominal Power

$$P_{sa} \cdot \overset{\text{day time}}{t_d} = \frac{P_d \cdot t_d}{\eta_d} + \frac{P_n \cdot t_n}{\eta_n}$$

$P_d \rightarrow$ power during day

$P_n \rightarrow$ power during night

$t_n \rightarrow$ night time

$\eta_d \rightarrow$ efficiency day

$\eta_n \rightarrow$ efficiency night

$P_{sa} \cdot t_d \rightarrow$ total available power

Lecture 10 Electrical Power Systems

Batteries: series of voltaic cells

voltaic cell: two electrodes made of different materials inmersed in a conductor.

- cathode: positive electrode
- anode: negative electrode

also called a constant voltage source.

Chemical reactions in the cell generate a voltage difference between the electrodes.

When connected to a load, current flows from the cathode to the anode.

BATTERY CHARACTERISTICS - CELL VOLTAGE

- Voltage decreases during discharge.
- Voltage is strongly dependent on the chemistry and the materials.
- Nominal voltage: voltage at 50% discharge. (MPV) mid point voltage
- End of discharge voltage.
- The voltage of a battery while charging is different to its voltage during discharge.

TYPES OF BATTERIES:

- Wet Cells: liquid electrolyte
- Dry cells: paste electrolyte.
- Primary: non rechargeable
- Secondary: rechargeable

- Cell capacity: total ammount of energy that a cell can deliver.

$$C = \frac{E}{V}$$

E → total amount of energy.

V → nominal cell voltage 50%

[A/h]

Nominal capacity: conditions

- Total discharge time
- Discharge temperature
- End-of-discharge voltage

- RATE DISCHARGE CURRENT: conditions

$$I = \frac{C}{t_D}$$

t_D → discharge time

for a different discharge time, capacity will not be nominal.

- SPECIFIC ENERGY = $\frac{\text{TOTAL ENERGY}}{\text{CELL MASS}}$

ENERGY DENSITY = $\frac{\text{TOTAL ENERGY}}{\text{CELL VOLUME}}$

C-RATE OF A CELL

→ Indication of its rated discharge time.

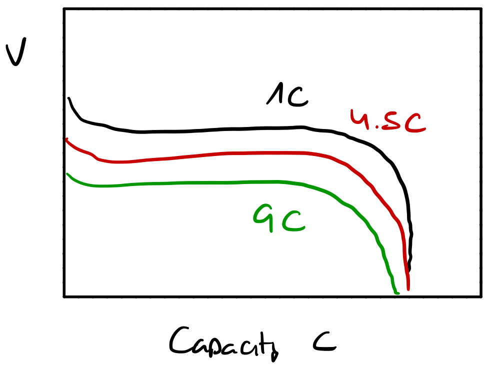
- $C = 1 \rightarrow$ discharge rate of 1 hour
- $\frac{C}{2} = 2$ hours
- $2C = \frac{1}{2}$ hours.

→ Indication of the rated discharge current.

$I = C/t_D \longrightarrow$ C-RATE $= C/3 \rightarrow 3$ hours $\longrightarrow$ for $C = 6 \rightarrow \boxed{2\,A}$

EFFECTS OF DISCHARGE CURRENT:

- If a cell is discharged at a current higher than its rated discharge current, the actual capacity of the cell is lower than the nominal capacity.



Peukert's law:

$$\boxed{C_{actual} = C \cdot \left[\frac{C}{I_{actual} \cdot t_D}\right]^{k-1}}$$

$k > 1$ Peukert Cnst.

$$\boxed{t_{D\text{-}ACTUAL} = \frac{C_{ACTUAL}}{I_{actual}}}$$

- C = nominal cell capacity
- C_{actual} = actual cell capacity.
- I_{actual} = actual disch current
- t_D = nominal disch time

EFFECTS OF TEMPERATURE

- Cell capacity decreases when temperature decreases.
- Tipically: 1% per each ºC below nominal rated temperature.

} Thermal control is important!

CELLS IN SERIES AND PARALLEL

- Battery is the connection of cells in series.
  - Series: C = same $\quad V = N \cdot V_{cell}$
- Connections in parallel multiple batteries forms an array
  - Parallel: V = same $\quad C = N \cdot C_{batt}$

CHARGE / DISCHARGE EFFICIENCY

$$\eta_{batt} = \frac{E_{dis} \rightarrow \text{discharge}}{E_{ch} \rightarrow \text{charge}} \qquad E_{dis} < E_{ch}$$

CYCLE LIFE OF A BATTERY

- Each charge/discharge cycle reduces the capacity of a battery.
- CYCLE LIFE: number of complete c and d that can be performed before the capacity falls below 80% of the initial value.
- Typical around 500 - 1200 cycles.
- To extend the cycle life it is possible to reduce its $\underbrace{\text{Depth of Discharge}}$.

how much the battery is discharged at each cycle.

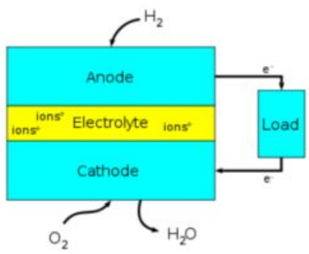
BATTERY SIZING

$$C_{actual} = \frac{P_{load} \cdot t_{op}}{\eta_{tot} \cdot V_{load} \cdot DOD}$$

- P_{load}: Average power required by the load.
- t_{op}: max operational time before recharging.
- η_{tot}: Total efficiency (discharge efficiency × transmission)
- V_{load}: load voltage
- DoD: Depth of Discharge

FUEL CELLS

- Fuel cells produce electrical energy from chemical reaction
- One reactant (hydrogen) flows through the anode and gives electrons, the other reactant (oxigen) flows through the cathode and receives electrons.
- A fuel cell can not be reusable.



CHARACTERISTICS

- Typical efficiency of 40-60% Higher power → lower efficiency. → space: reactants mass is 10-20 times higher than dry mass
- Tipically used for high-power loads (1-10 kW)
- Higher specific energy than batteries. same energy with lower mass. $SE = \frac{\text{Total energy}}{\text{Cell mass}}$

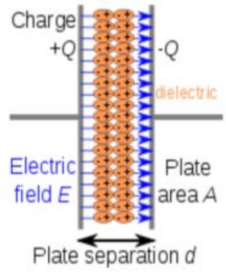
SIZING: consider two parameters.

- Specific power: cell dry mass.
- Specific energy: cell reactants only.

CAPACITORS: characterized by similar characteristics as batteries.

- Made of two conductors insulated from each other by a dielectric.
- Store electrical energy and provide it back when connected to a load
- No chemical energy conversion takes place
- Faster discharge than on batteries.
- CAPACITANCE: amount of charge.

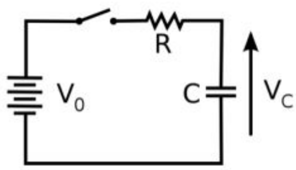
$$C = \frac{Q \to \text{electrical charge}}{V \to \text{voltage difference.}}$$



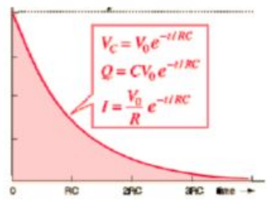
- ENERGY:

$$E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

- DISCHARGE



- $I(t) = \frac{V_0}{R} e^{-\frac{t}{RC}}$
- $V_C(t) = V_0 \cdot e^{-\frac{t}{RC}}$
- $Q(t) = C V_0 e^{-\frac{t}{RC}}$



- Capacitance depends on:
  - Ageing and cycling
  - Temperature
  - Discharge current

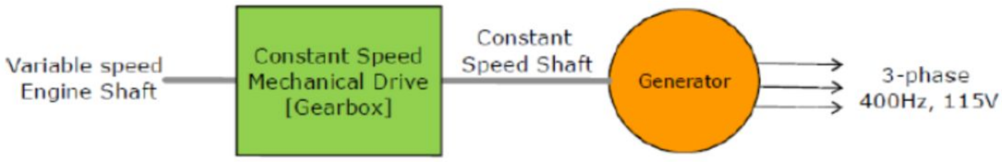
- Connecting N capacitors:
  - Series: $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$
  - Parallel: $C_T = C_1 + C_2 + \dots + C_N$

POWER MANAGEMENT

- Used to adapt the electrical power to the type of load that it uses it.
- Because of different voltage/current
- Extends life of a device by forcing it to work under more "comfortable" conditions

ALTERNATORS

- An usual requirement of AC devices is to receive power at constant frequency.
- Energy source, operates at variable speed
- USE | INTEGRATED DRIVE GENERATOR | (IDG)
  - → Constant Speed Drive, installed between the engine and generator
  - → The CSD, hydro mechanical device.



- USE | VARIABLE SPEED CONSTANT FREQUENCY | converters.

  Made of electronic components (AC/DC converter + DC/AC one)

SOLAR ARRAYS:

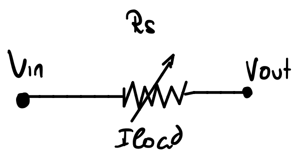
- The I-V curve and the maximum power change over time.
- A voltage regulator is used to control this.

ON/OFF SWITCHES

- The only available control is on the number of array strings that are connected to the load.
- Current can not be controlled.

SERIES REGULATORS:

An adjustable resistance is installed in series between the solar array and the load.



$V_{out} = V_{in} - R_s \cdot I_{load}$

R_s → adjustable resistance.

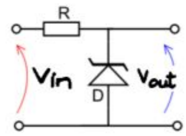
I_{load} can also be adjusted.

Low efficiency.

$P_{in} = V_{in} \cdot I_{LOAD}$

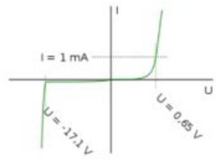
$P_{diss} = R_s \cdot I_{LOAD}^2$

Shunt Regulators:



- Smaller excess power is dissipated in the regulator. → higher efficiency.
- Shunt regulators make use of Zener diodes (allow for current in reverse direction)
- $V_{out} = V_{in} - R \cdot (I_{LOAD} + I_D)$ (R: fixed regulator resistance) (I_D: current in the diode)

  I_D allows for dissipation of excess power in the low resistance diode circuit.



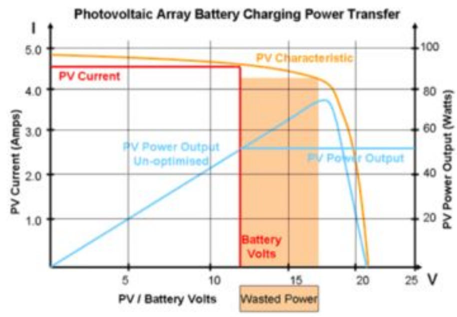
Zener diode characteristic

Shunt regulators are used at low power. To limit the heat dissipated through the regulator.

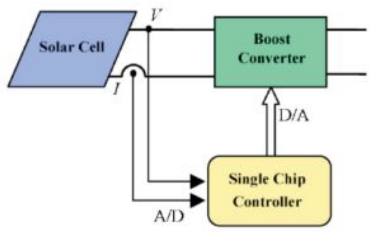
PROBLEM $I_{LOAD} \neq I_{mpp}$

↓

Switched Mode Regulators



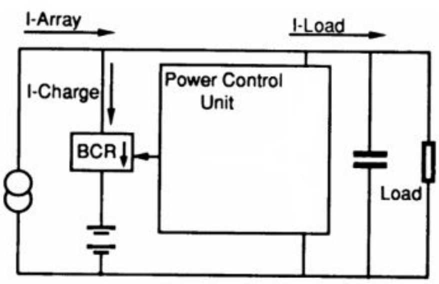
- An electronic controller is used to decouple the solar array current from the load current.
- Both solar array AND load can work at their optimum current level.
- Used to manage high power levels (> 1 kW)
- High efficiency but high cost and lower reliability



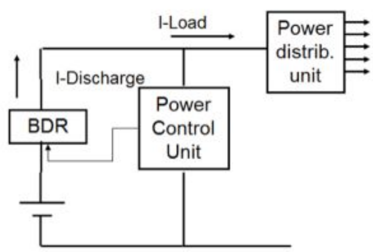
Batteries

APPLICATION

- BATTERY CHARGE REGULATOR: adjust the voltage provided by the power source to the optimum point for battery charging.
  Remain power goes to the load.



- BATTERY DISCHARGE REGULATOR: ensures constant voltage to the load while discharge.

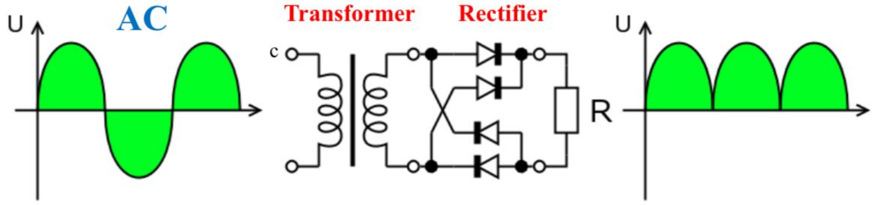


- Longer life of battery and loads at a higher cost and decrease in efficiency.

Power Conversion And Distribution:

Conversion: AC to DC

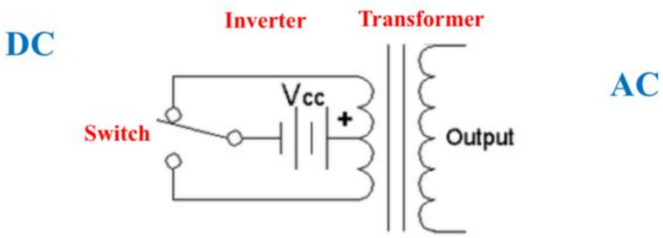
- A transformer Rectifier Unit
  1. Transformer reduces the AC peak voltage
  2. Rectifier forces the voltage to be always in the same direction
  3. A smoothing circuit makes the voltage constant.



smoothing circuit DC

Conversion: DC to AC:

- Inverter:
  - 3 way switches are switched back and forth with the required frequency to change the direction of the DC current → also transistors can be used.
  - Peak voltage can be modified with a transformer.

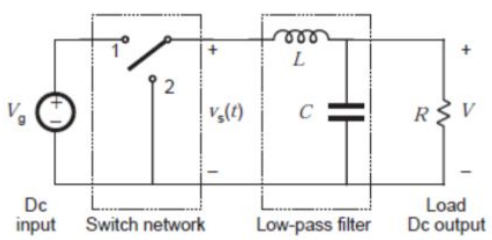


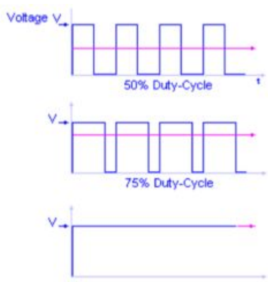
Conversion: DC to DC

- Pulse width modulator: changes voltage.

  Switch network converts the DC input into a square wave. time % when 0 voltage.

  A low-pass filter converts the square wave into a DC output (higher duty cycle)



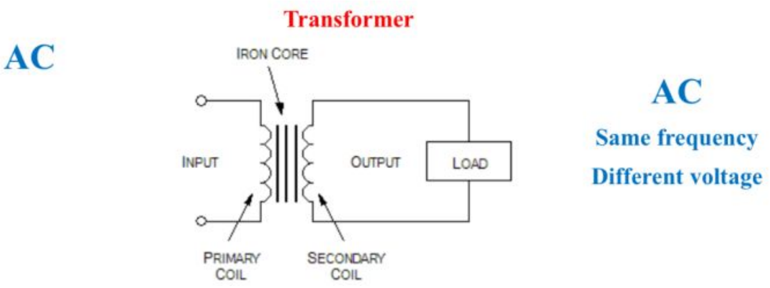


CONVERSION: AC to AC

Transformer: same frequency, different peak voltage.

If we want to change frequency. AC - DC - AC conversion required.

Transformer rectifier unit + Pulse width modulator + Inverter.



SIZING FOR ELECTRICAL WIRING

- Define the length and diameter of all wires.
- It should keep the wire loss lower than 5% in the complete circuit.
- Wire gauge: measure of diameter of a wire
  - → Lower AWG → higher diameter → higher mass → lower resistance
  - → Higher wire current → higher diameter → higher mass → lower AWG.

Rocket Propulsion

Rockets work with the principle of action-reaction, a mass is propelled in the opposite direction of motion.

1. The energy of the fluid is increased.
2. The energy is converted into kinetic energy.

External fluid → aircraft
internal fluid → rocket engine.

Positives:

- Almost Independent of ambient conditions and flight velocity.

Negatives:

- Higher propellant consumption.

Components and Functions

1. Provide the fluid to be expelled (Propellant)
2. Store the propellant (tanks)
3. Feed and distribute the propellant (pipes, valves...)
4. Accelerate the propellant (thruster)
5. Provide the required power to the system components. (power plant.)

Power plant.

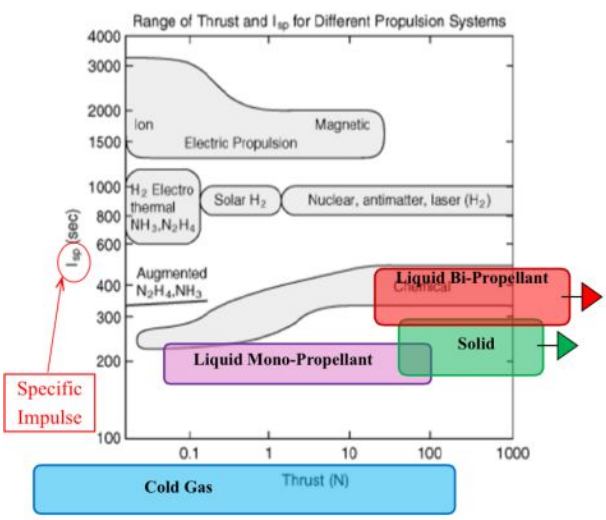
Classification:

Based on: how the propellant is accelerated

Thermal expansion, electrostatic forces, electromagnetic forces.

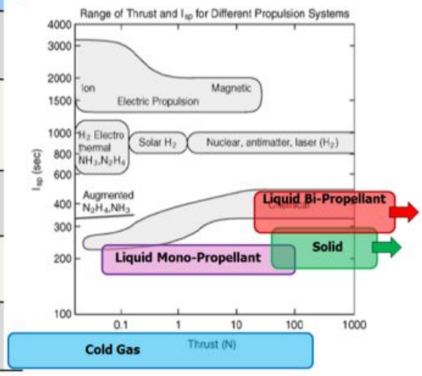
Based on: what type of energy source is used

Cold gas, Chemical energy, Nuclear energy, Solar energy, Electric energy.



APPLICATIONS:

		Cold Gas	Solid	Liquid Mono-propellant	Liquid Bi-Propellant	Electric
Launchers Missiles	High acceleration Short operation		●		●	
Amateur rockets Ascent/Lander vehicles	Moderate-High acceleration Short operation		●	●	●	
Orbit insertion & transfer	Moderate-long operation		●		●	●
Orbit maintenance Attitude control	Low-Moderate acceleration	●		●	●	●
Propulsion belts	Moderate acceleration Short operation			●		



WORKING PRINCIPLE

MOMENTUM:

$$\vec{I} = M \cdot \vec{V}$$

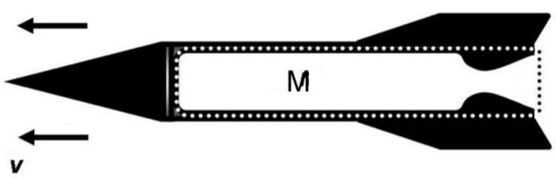
the momentum changes when a force acts on the body remains constant if no force applies.

TOTAL MOMENTUM:

Sum of momentums of the different bodies in the system.

System : $m_1 v_1 = m_2 v_2$ The faster we expell the propellant the more velocity we can attain.

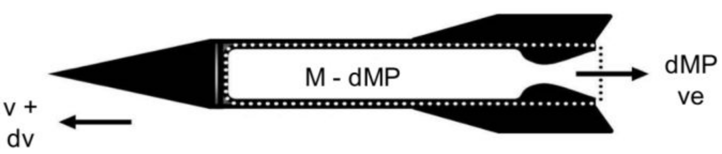
THE ROCKET EQUATION:



Momentum at time t:

$I = M \cdot V$, no external forces.

PROPELLANT IS EXPELLED: dM_p with a jet velocity V_e : V_e is in the oposite direction to the flight direction.



MOMENTUM:

absolute velocity of propellant

$$\underbrace{I + dI = (M - dM_p)(V + dV)}_{\text{Rocket momentum}} - \underbrace{dM_p (V_e - V)}_{\text{Expelled propellant momentum}}$$

MULTIPLY OUT

$$I + dI = MV + M \cdot dV - dM_p \cdot V - dM_p \cdot dV + dM_p \cdot V - dM_p \cdot V_e$$

$I \to MV$

low order.

$$dI = M \cdot dV - \cancel{dM_p \cdot V} - dM_p \cdot dV + \cancel{dM_p \cdot V} - dM_p \cdot V_e$$

$dI = M \cdot dV - dM_p \cdot V_e \longrightarrow$ no external forces. $\boxed{dI = 0}$ $\quad M \cdot dV = dM_p \cdot V_e$

- separate variables and integrate

$$M \cdot dv = dM_p \cdot V_e \longrightarrow \frac{dv}{V_e} = \frac{dM_p}{M} = -\frac{dM}{M}$$

Tsiolkovsky Equation

$$\frac{1}{V_e}\int dv = -\int \frac{dM}{M} \longrightarrow \boxed{\frac{1}{V_e}\Delta V = \ln\left(\frac{M_{in}}{M_{fin}}\right)}$$

$$\boxed{\Delta V = V_e \cdot \ln\left(\frac{M_{in}}{M_{in} - M_p}\right)}$$

- we have assumed V_e = constant → NOT VALID IF CHANGES OVER TIME
- $dM_p = -dM$
- M_{in} = initial mass of the rocket, M_{fin} = final mass of the rocket
- M_p total mass of propellant expelled. $M_p = M_{in} - M_{fin}$.

- No gravity
- No drag
- No forces

} NOT VALID ASCENT FROM PLANET SURFACE

- propellant expelled straight NOT WHEN AXIS DOES NOT ALIGN WITH FLIGHT PATH.

Indicator of ammount of energy needed, even if some assumptions are not true.

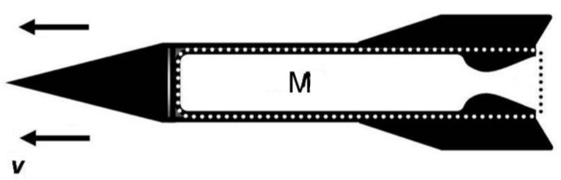
ASSUME GRAVITY FORCE

$$\Delta V = V_e \cdot \ln\left(\frac{M_{in}}{M_{in} - M_p}\right) - \underbrace{g \cdot t_b}_{\text{gravity loss}}$$

t_b = burntime

g = acceleration of gravity.

Rocket Thrust Equation



Before $dI = M \cdot dv - dM_p \cdot V_e$ → divide by dt

$$\frac{dI}{dt} = M \cdot \frac{dv}{dt} - V_e \frac{dM_p}{dt} = F_p + F_{other}$$

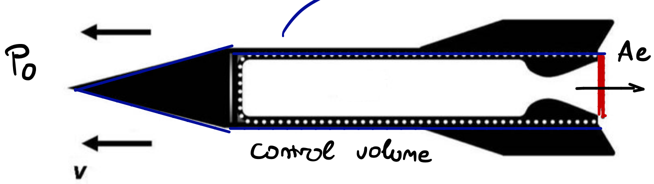
F_p → pressure forces

F_{other} → gravity...

$\frac{dI}{dt}$ → equal to the external force acting on the system

F_p is generated by the pressure of the fluid around the rocket.

→ consider P_0 on the rest of the rocket.



P_e : pressure of propellant. constant over A_e

CONTROL VOLUME

$$F_p = \int_{A_0} P_0 \cdot dA_0 + P_e \cdot A_e \quad (1)$$

when a uniform pressure acts on a closed surface the total pressure force shall be zero.

$$\int_{A_0} P_a \cdot dA_0 + P_a \cdot A_e = 0 \quad (2)$$

$$F_p - 0 = \int_{A_0} P_0\, dA_0 + P_e \cdot A_e - \int_{A_0} P_a \cdot dA_0 - P_a \cdot A_e$$

$$F_p = \underbrace{\int_{A_0} (P_0 - P_a)\, dA_0}_{\text{Aerodynamic drag. } F_{drag}} + (P_e - P_a) \cdot A_e = F_{drag} + (P_e - P_a) \cdot A_e$$

Rocket Thrust Equation (cntd)

$$\frac{dI}{dt} = M\cdot\frac{dV}{dt} - V_e\cdot\frac{dM_p}{dt} = F_p + F_{other} = F_{drag} + F_{other} + (p_e - p_a)\cdot A_e$$

Propellant mass flow.

$$\boxed{M\cdot\frac{dV}{dt} = F_{drag} + F_{other} + \underbrace{V_e\cdot\dot{m} + (p_e - p_a)\cdot A_e}_{\text{this must be also a force.}}}$$

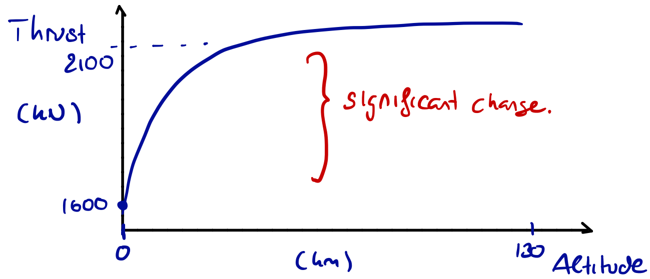
this must be also a force. → F_T : rocket thrust

$$\boxed{F_T = \underbrace{\dot{m}\cdot V_e}_{\text{action reaction}} + \underbrace{(p_e - p_a)\cdot A_e}_{\text{Pressure.}}}$$

Rocket Thrust vs Altitude

Important to specify altitude.

- Sea level: $p_a = 101325$ Pa
- Vacuum $p_a = 0$ Pa



Equivalent Jet Velocity

$$\boxed{\underbrace{V_{eq}}_{} = \frac{F_T}{\dot{m}} = V_e + \frac{(p_e - p_a)\cdot A_e}{\dot{m}}}$$

$$\boxed{F_T = \dot{m}V_e + (p_e - p_a)\cdot A_e = \dot{m}V_{eq}}$$

No physical meaning, used to simplify.

when $V_e = V_{eq}$, $p_e = p_a$ → $\boxed{\Delta V = V_{eq}\cdot \ln\left(\frac{M_{in}}{M_{in} - M_p}\right)}$

Total Impulse

When a force acts on a body for a longer time a larger momentum change is obtained.

To take this effect into account, define total impulse.

$$I_{tot} = \int_0^{t_b} F_t\cdot dt \xrightarrow{F_t = \text{constant}} F_t\cdot t_b \xrightarrow{V_{eq} = \text{constant}} I_{tot} = \int_0^{t_b}\dot{m}V_{eq}\cdot dt = V_{eq}\int_0^{t_b} dM_p = V_{eq}\cdot M_p$$

Specific Impulse

Proportional to the total impulse divided by the total mass of propellant used

$$I_{sp} = \frac{1}{g_0}\cdot\frac{\int_0^{t_b} F_T\cdot dt}{\int_0^{t_b}\dot{m}\cdot dt} \xrightarrow{V_{eq} = \text{constant}} = \frac{V_{eq}}{g_0}\cdot\frac{\int_0^{t_b}\dot{m}\,dt}{\int_0^{t_b}\dot{m}\,dt} = \boxed{\frac{V_{eq}}{g_0}}$$

Specific Impulse: (CNTD)

- A higher specific impulse means that a larger momentum change can be generated by a smaller mass of propellant.

$$I_{sp} = \frac{V_{eq}}{g_0}$$

- Specific impulse is proportional to the equivalent jet velocity.
- Velocity change ΔV and rocket thrust F_T are also proportional

- Higher I_{sp} = Higher F and Higher ΔV

I_{sp} increases when altitude increases.

- g_0 is in I_{sp} because expresses I_{sp} as time which is a universal measure.

Volumetric Specific Impulse

Propellant volume is used instead of propellant mass.

$$I_p = \frac{1}{g_0} \cdot \frac{\int_0^{t_b} F_T \cdot dt}{\int_0^{t_b} \frac{\dot{m}}{\rho} \cdot dt}$$

ρ → propellant density

- Higher I_p : larger momentum with less volume.
- Important to define size of a rocket.

Jet Power and Cycle Efficiency

P_{jet} : measure of available amount of power in the jet of expelled propellant

$$\boxed{P_{jet} = \frac{1}{2} F_T \cdot V_{eq} = \frac{1}{2} \dot{m} \cdot V_{eq}^2}$$

P_{source}: provided by the energy source → compare to P_{jet}

$$\eta_c = \frac{P_{jet}}{P_{source}}$$

→ cycle efficiency

(70%)

how much power provided by source is converted to kinetic energy.

Thrust Power and Propulsive Efficiency

P_T : measure of the ammount of power effectively used to propel the rocket

$$P_T = F_T \cdot V_0 = \dot{m} \cdot V_{eq} \cdot V_0$$

V_0 ↓ rocket velocity component in the direction of thrust.

OVERALL ENERGY EFF

$$\eta = \eta_p \cdot \eta_c$$

Absolute jet Power: amount of jet Power not effectively used for thrust.

$$\boxed{P_{jet\_abs} = \frac{1}{2} \dot{m} (V_{eq} - V_0)^2}$$

$(V_{eq} - V_0)$ ↓ jet absolute velocity.

Propulsive efficiency: $$\eta_p = \frac{P_T}{P_T + P_{JET\_ABS}} = \frac{2 \frac{V_0}{V_{eq}}}{1 + \left(\frac{V_0}{V_{eq}}\right)^2}$$

In rockets $V_0 > V_{eq}$ is possible so $V_0 = V_{eq}$ is the optimal.

Ideal Rocket Theory:

Assumptions and Building Blocks

$$F_T = \dot{m} \cdot V_e - (P_e - P_a)\, A_e$$

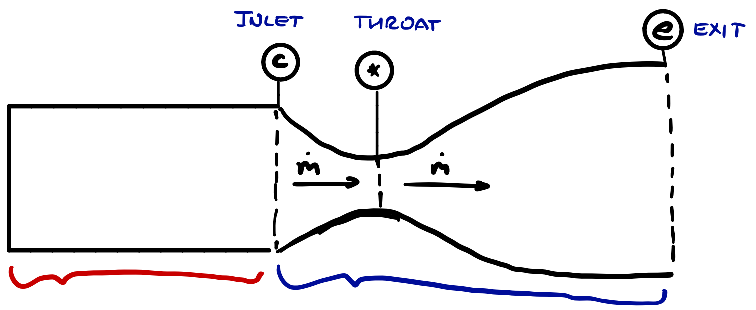
$$V_{eq} = V_e + \frac{(P_e - P_a)\, A_e}{\dot{m}}$$

$$\Delta V = V_{eq} \cdot \ln\left(\frac{M_{in}}{M_{in} - M_p}\right)$$

$$I_{sp} = \frac{V_{eq}}{g_0}$$

depend on propellant heating and acceleration process.

Evaluation of parameters: V_e, P_e, $\dot{m}$



EXPANSION RATIO

$$\boxed{\varepsilon = \frac{A_e}{A^*}}$$

Combustion chamber
- High pressure
- (High temperature)
- Low speed

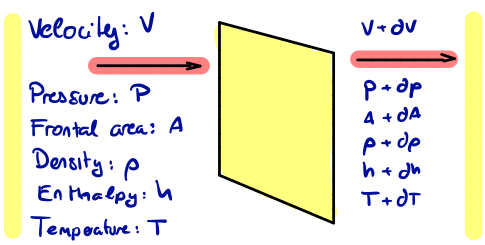
Convergent-Divergent Nozzle
- Propellant is accelerated
- No external energy is provided.

Assumptions:

1.) Propellant in the chamber + nozzle is a perfect gas.
2.) Propellant in the chamber + nozzle is a callorically ideal gas (specific heats are not dependent on temperature).
3.) Propellant in the chamber + nozzle has homogeneous and constant chemical composition.
4.) Flow in the nozzle is steady (not dependent on time)
5.) Flow in the nozzle is isentropic (no external energy applied, no lost)
6.) Flow in the nozzle is 1.-dimensional (quantities vary along nozzle axis)
7.) Flow velocity is purely axial.
8.) No external forces act on the propellant flowing in the nozzle.
9.) Propellant in the chamber has negligible velocity. ($V_c = 0$)

Conservation Equations:

Mass, momentum and energy: variations are very small, because we are considering a small nozzle portion.



MASS CONSERVATION EQUATION:

$\boxed{\dot{m} = \rho \cdot V \cdot A}$ no mass generated or extracted $\longrightarrow$ $d(\rho V \cdot A) = 0$

$\rho V A = \text{constant}$

MOMENTUM CONSERVATION EQUATION:

$I = \dot{m} \cdot V$ momentum equations can only be balanced by pressure forces.

$d(\dot{m} \cdot V) = p \cdot A - (p + dp) \cdot A \longrightarrow \dot{m} \cdot dv = -A \cdot dp$

$\dot{m} = \rho \cdot v \cdot A \longrightarrow \dot{m} \cdot dv = \rho \cdot v \cdot A \cdot dv = -A \cdot dp$

$\boxed{dp + \rho \cdot v \cdot dv = 0}$ $\boxed{p + \frac{1}{2} \rho v^2 = \text{constant}}$

ENERGY CONSERVATION EQUATION

The propellant does not exchange any energy, no total enthalpy variations are possible:

$d\left(h + \frac{1}{2} v^2\right) = 0$ $\boxed{dh + v\, dv = 0}$ INTEGRATION $\boxed{h + \frac{1}{2} v^2 = \text{constant}}$

IDEAL GAS EQUATIONS

EQUATION OF STATE

$\boxed{p = \rho \cdot \frac{R_A}{M_w} T}$

- p: gass pressure
- ρ: gas density
- R_A: universal gas constant 8314 J/(K × kmol)
- T: gas Temperature
- M_w: gas molecular mass

$\boxed{\frac{R_A}{M_w} = C_p - C_v}$ $\boxed{\gamma = \frac{C_p}{C_v}}$ $\boxed{C_p = \frac{\gamma}{\gamma - 1} \cdot \frac{R_A}{M_w}}$

- C_p: constant pressure specific heat
- C_v: constant volume specific heat.

$\boxed{h = C_p \cdot T}$ $\boxed{dh = C_p \cdot dT}$ } ENTHALPY

ISENTROPIC FLOW:

$\boxed{\frac{p}{\rho^{\gamma}} = \text{constant}}$ $\boxed{p \cdot T^{\left(\frac{\gamma}{1-\gamma}\right)} = \text{constant}}$

MACH NUMBER

$\boxed{M = \frac{V}{a}}$ $\boxed{a^2 = \left(\frac{dp}{d\rho}\right)_{\text{constant entropy}}}$

$\boxed{a^2 = \gamma \cdot \frac{R_A}{M_w} \cdot T = \gamma \cdot \frac{p}{\rho}}$

Convergent – Divergent Nozzle:

Differentiating the mass conservation equation and dividing by the (constant) mass flow rate:

$$d(\rho \cdot V \cdot A) = 0 \longrightarrow \frac{\cancel{\rho}\cdot\cancel{V}\cdot dA}{\cancel{\rho}\cdot\cancel{V}\cdot A} + \frac{\cancel{V}\cdot\cancel{A}\cdot d\rho}{\rho\cdot\cancel{V}\cdot\cancel{A}} + \frac{\cancel{\rho}\cdot\cancel{A}\cdot dV}{\cancel{\rho}\cdot V\cdot\cancel{A}} = 0 \quad \Big| \quad \frac{dA}{A} + \frac{d\rho}{\rho} + \frac{dV}{V} = 0$$

$\underbrace{\text{Momentum conservation}}$ equation and $\underbrace{\text{speed of sound}}$:

$$dp + \rho V \cdot dV = 0 \longrightarrow \rho = -\frac{dp}{V \cdot dV} \qquad a^2 = \left(\frac{dp}{d\rho}\right)_{\text{constant entropy}}$$

$$\frac{d\rho}{\rho} = -\frac{d\rho}{dp}\cdot V \cdot dV = -\frac{V^2}{a^2}\cdot\frac{dV}{V} = \boxed{-M^2 \cdot \frac{dV}{V}}$$

$$\frac{dA}{A} - M^2 \cdot \frac{dV}{V} + \frac{dV}{V} = 0$$

$$\boxed{\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}}$$

		$M<1$	$M>1$
Convergent	$dA<0$	$dV>0$	$dV<0$
Divergent	$dA>0$	$dV<0$	$dV>0$

- Convergent when $M<1$
- Divergent when $M>1$

When $M=1$ $\quad \underbrace{dA=0}_{\text{throat}}$

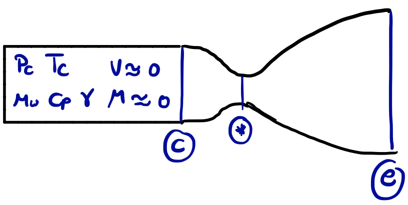
THE NOZZLE SHOULD BE DESIGNED TO HAVE M=1 AT NOZZLE

Nozzle Flow Equations

ASSUMPTIONS

Conditions of propellant in the chamber are known. T_c, P_c

propellant composition and characteristics are known. $\gamma, C_p, M_W \rightarrow$ constant.



ENERGY EQUATION:

$$\left.\begin{array}{l} h + \frac{1}{2}V^2 = \text{constant} \\ h = C_p \cdot T \end{array}\right\} \quad \underbrace{C_p T_c}_{\text{chamber}} + 0 = \underbrace{C_p \cdot T + \frac{1}{2}V^2}_{\text{generic nozzle.}}$$

REWRITE AS: $\longrightarrow$ SPECIFIC HEAT AND ISENTROPIC FLOW

$$V = \sqrt{2C_p \cdot (T_c - T)}$$

$$C_p = \frac{\gamma}{\gamma-1}\cdot\frac{R_A}{M_W} \longrightarrow V = \sqrt{\frac{2\gamma}{\gamma-1}\cdot\frac{R_A}{M_W}\cdot(T_c - T)} = \sqrt{\frac{2\gamma}{\gamma-1}\cdot\frac{R_A}{M_W}\cdot T_c \cdot \left(1 - \frac{T}{T_c}\right)}$$

$$P T^{\left(\frac{\gamma}{1-\gamma}\right)} = \text{constant} \qquad \frac{T}{T_c} = \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{V = \sqrt{\frac{2\gamma}{\gamma-1}\cdot\frac{R_A}{M_W}\cdot T_c\left[1 - \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

Nozzle Exit Velocity   Jet Velocity

$$\boxed{V_e = \sqrt{\frac{2\gamma}{\gamma-1} \cdot \frac{R_A}{M_w} \cdot T_c \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

Higher jet velocity:

- Increasing T_c
- Reducing M_w
- Reducing exit pressure

V_{e_limit} is attained when the exit pressure $P_e = 0$.

Mass Flow Rate And Expansion Ratio

Mach number relations:

$$C_p T_c = C_p T + \frac{1}{2} V^2 \rightarrow \boxed{\frac{T_c}{T} = 1 + \frac{V^2}{2 C_p T}}$$

$$C_p = \frac{\gamma}{\gamma-1} \cdot \frac{R_A}{M_w}$$

$$C_p T = \frac{1}{\gamma-1} \cdot \underbrace{\gamma \cdot \frac{R_A}{M_w} \cdot T}_{a^2} = \frac{a^2}{\gamma-1}$$

$$\boxed{\frac{T_c}{T}} = 1 + \frac{\gamma-1}{2} \cdot \frac{V^2}{a^2} = \boxed{1 + \frac{\gamma-1}{2} \cdot M^2}$$

$$P T^{\left(\frac{\gamma}{1-\gamma}\right)} = \text{constant}$$

$$\frac{P_c}{P}\left(\frac{T_c}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma-1}{2} \cdot M^2\right)^{\frac{\gamma}{\gamma-1}}$$

Sonic Throat

$$\frac{T_c}{T^*} = 1 + \frac{\gamma-1}{2} = \frac{1+\gamma}{2} \qquad \frac{P_c}{P^*} = \left(\frac{1+\gamma}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

Mass Flow Rate Equations

- $$\frac{\dot{m}}{A} = \rho \cdot V = \rho_c \cdot \frac{\rho}{\rho_c} \cdot \sqrt{\frac{2\gamma}{\gamma-1} \cdot \frac{R_A}{M_w} \cdot T_c \left[1 - \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

- $$\frac{\dot{m}}{A} = \rho \cdot V = \rho_c \cdot \frac{R_A}{M_w} \cdot T_c \cdot \sqrt{\frac{2\gamma}{\gamma-1} \cdot \left(\frac{\rho}{\rho_c}\right)^2 \left[1 - \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{P}{\rho^\gamma} = \text{constant}$$

$$\frac{\rho}{\rho_c} = \left(\frac{P}{P_c}\right)^{1/\gamma}$$

$$P_c = \rho_c \cdot \frac{R_A}{M_w} T_c$$

$$\rho_c = \frac{P_c}{\frac{R_A}{M_w} T_c}$$

$$\boxed{\frac{\dot{m}}{A} = \frac{P_c}{\sqrt{\frac{R_A}{M_w} \cdot T_c}} \cdot \sqrt{\frac{2\gamma}{\gamma-1} \left(\frac{P}{P_c}\right)^{\frac{2}{\gamma}} \cdot \left[1 - \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

→ constant through nozzle

SONIC THROAT

$$\frac{P_c}{P^*}=\left(\frac{1+\gamma}{2}\right)^{\frac{\gamma}{\gamma-1}} \longrightarrow \frac{P^*}{P_c}=\left(\frac{1+\gamma}{2}\right)^{\frac{\gamma}{1-\gamma}}$$

MASS FLOW RATE

$$\boxed{\frac{\dot m}{A^*}=\frac{P_c}{\sqrt{\frac{R_A}{M_w}\cdot T_c}}\cdot\sqrt{\frac{2\gamma}{\gamma-1}\left(\frac{P^*}{P_c}\right)^{\frac{2}{\gamma}}\cdot\left[1-\left(\frac{P^*}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}} = \frac{P_c}{\sqrt{\frac{R_A}{M_w}\cdot T_c}}\cdot\sqrt{\frac{2\gamma}{\gamma-1}\cdot\left(\frac{1+\gamma}{2}\right)^{\frac{2}{1-\gamma}}\cdot\left[1-\left(\frac{1+\gamma}{2}\right)^{-1}\right]}$$

$$\dot m=\frac{P_c\cdot A^*}{\sqrt{\frac{R_A}{M_w}\cdot T_c}}\cdot\Gamma(\gamma) \qquad \Gamma(\gamma)=\sqrt{\gamma\cdot\left(\frac{1+\gamma}{2}\right)^{\frac{1+\gamma}{1-\gamma}}}$$

Vandenkerckhove Functions

→ only one value makes the throat conditions possible (chocked flow)

AREA RATIO EQUATION

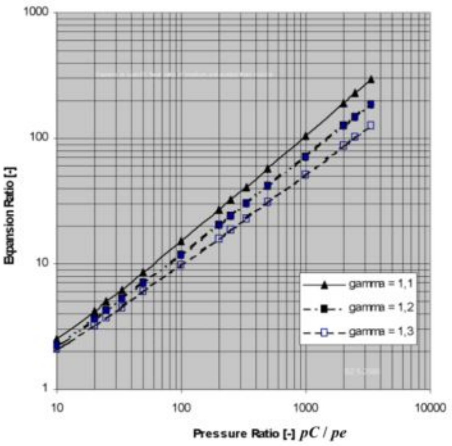
$$\frac{A}{A^*}=\frac{\Gamma(\gamma)}{\sqrt{\frac{2\gamma}{\gamma-1}\cdot\left(\frac{P}{P_c}\right)^{2/\gamma}\cdot\left[1-\left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

- for a given chamber conditions (p_c, γ) and throat area A^*, this equation gives the flow pressure corresponding to each nozzle section of area A.
- Two solutions for each area, subsonic, supersonic.

EXIT AREA RATIO:

$$\varepsilon=\frac{A_e}{A^*}=\frac{\Gamma(\gamma)}{\sqrt{\frac{2\gamma}{\gamma-1}\cdot\left(\frac{P_e}{P_c}\right)^{2/\gamma}\cdot\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

- Direct relationship between expansion ratio ε and the pressure ratio P_e/P_c
- Nozzle pressure ratio P_e/P_c can be calculated as a function of γ and (A_e, A^*)

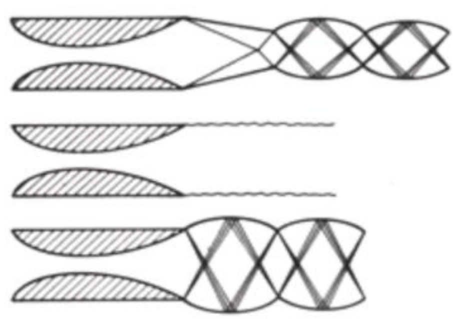


EXPANSION CONDITIONS:

(1) $P_e < P_a$ over expanded

(2) $P_e = P_a$ adapted expanded

(3) $P_e > P_a$ under-expanded



(1), (3) Shock waves are created to adapt to P_a.

CHARACTERISTIC VELOCITY AND THRUST COEFFICIENT

THRUST EQUATION:

$$F_T = \dot{m} \cdot V_e + (P_e - P_a) \cdot A_e$$

COMBINING THE PREVIOUS EQUATIONS

$$F_T = p_C \cdot A^* \cdot \Gamma(\gamma) \cdot \sqrt{\frac{2\gamma}{\gamma-1} \cdot \left[1 - \left(\frac{p_e}{p_C}\right)^{\frac{\gamma-1}{\gamma}}\right]} + (p_e - p_a) \cdot \frac{A^* \cdot \Gamma(\gamma)}{\sqrt{\frac{2\gamma}{\gamma-1} \cdot \left(\frac{p_e}{p_C}\right)^{\frac{2}{\gamma}} \cdot \left[1 - \left(\frac{p_e}{p_C}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

- FIXED NOZZLE GEOMETRY MAXIMUM THRUST WHEN ($P_a = 0$)
- FIXED ATMOSPHERIC PRESSURE MAXIMUM THRUST ($P_a = P_e$)

CHARACTERISTIC VELOCITY:

$$C^* = \frac{P_C \cdot A^*}{\dot{m}}$$

THRUST COEFFICIENT:

$$C_F = \frac{F_T}{P_C \cdot A^*}$$

combined:

$$F_T = \dot{m} \cdot C^* \cdot C_F$$

CHARACTERISTIC VELOCITY

$$C^* = \frac{1}{\Gamma(\gamma)} \cdot \sqrt{\frac{R_A}{M_\omega} \cdot T_C}$$

Depends only of chamber conditions. } measure performance of propellant. ↑ BETTER.

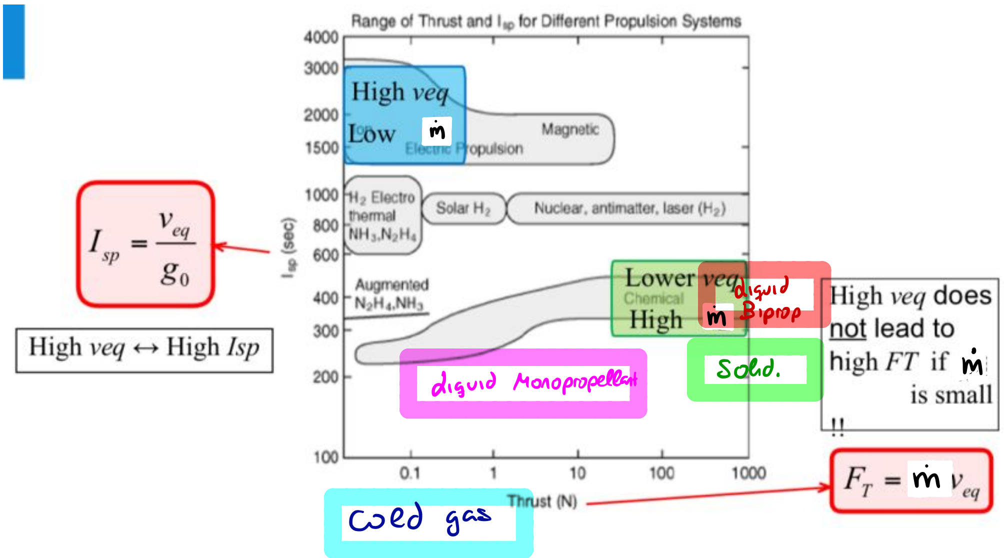
THRUST COEFFICIENT

$$C_F = \frac{F_T}{p_C \cdot A^*} = \Gamma(\gamma) \cdot \sqrt{\frac{2\gamma}{\gamma-1} \cdot \left[1 - \left(\frac{p_e}{p_C}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{p_e}{p_C} - \frac{p_a}{p_C}\right) \cdot \frac{\Gamma(\gamma)}{\sqrt{\frac{2\gamma}{\gamma-1} \cdot \left(\frac{p_e}{p_C}\right)^{\frac{2}{\gamma}} \cdot \left[1 - \left(\frac{p_e}{p_C}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

Depends mainly on nozzle geometry. (P_e/P_c)

measure of effects on nozzle geometry.

Classification of Rocket Engines



Cold Gas Rockets : Attitude control.

- Not heated, accelerated from the chamber at ambient temperature.
- Thrust from propellant pressurization

PERFORMANCE

Low Isp, low Thrust.

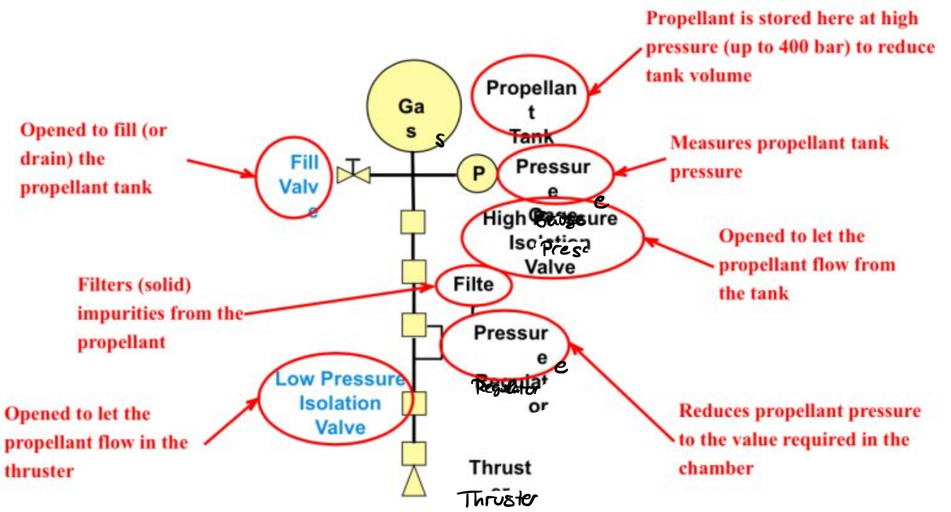
DISADVANTAGES

Tc low, limited performance (jet velocity and specific impulse)

ADVANTAGES

Simplicity and safety.

- Compact, small satellites.



PROPELLANTS

- Gaseous at ambient conditions
- Chemically inert, safe

Helium and Nitrogen.

- Lowest Mw generate Highest Isp (Helium)
- Highest Mw generate Highest Ip (xenon) less volume used.

Blow-Down Systems

No pressure regulator is used $P_t = P_c$

15 to 30 bar $= P_t$

Tank pressure P_T decreases with time

$$\frac{P_T}{\rho_T^{\,k}} = \text{constant}$$

- Isentropic: $k = \gamma$
- Isothermal: $k = 1$

tank volume constant

$$\frac{P_T}{M_T^{\,k}} = \text{constant}$$

LIQUID MONO-PROPELLANT ROCKETS

attitude controls, propulsion belts

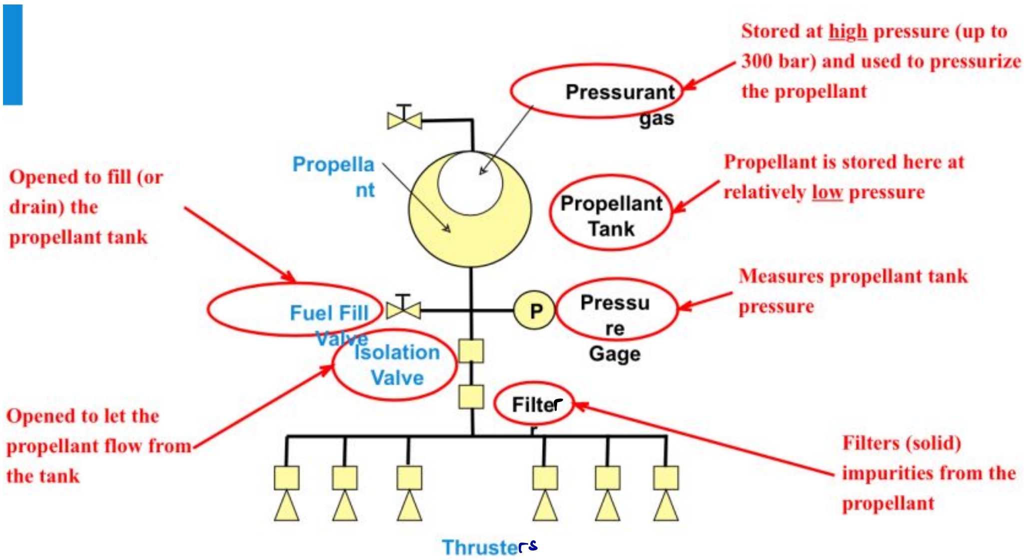
- Propellants are heated by a chemical reaction in the combustion chamber.
- Energy used to generate thrust comes from propellant pressurization and from chemical reaction.
- The chemical reaction is a decomposition of the propellant.

PERFORMANCE

Low I_{sp} (200) Low Thrust (20N)

PROPELLANTS:

- Hydrazine
- Hydrogen peroxide



CHAMBER TEMPERATURE

- The decomposition power P_{dec} of a mono propellant can be expressed as a function of:

$$P_{dec} = \dot{m} \cdot H_D \longrightarrow \text{Heat of decomposition (kJ/mol)}$$

- Power needed to increase the temperature of decomposition products from T_0 to T_c is:

$$P_{heat} = \dot{m} \cdot C_p \cdot (T_c - T_0)$$

- Assume that P_{dec} is used to heat the products of the decomposition

$$P_{heat} = P_{dec} \longrightarrow \dot{m} \cdot H_D = \dot{m} \cdot C_p \cdot (T_c - T_0) \longrightarrow T_c = T_0 + \frac{H_D}{C_p}$$

Liquid Bi-Propellant Rockets

launchers, missiles, orbit insertion, transfer orbits.

- Two Propellants are used (oxidizer and fuel) combustion
- Hypergolic propellants if combustion takes place spontaneously when they enter in contact.
- Oxidizer/fuel ratio (O/F) is the ratio of oxidizer to fuel mass flow rates:

$$O/F = \frac{\dot{m}_{oxidizer}}{\dot{m}_{fuel}}$$

Performance

- Relatively high Isp
- High or very high Thrust

Avoid gas to flow back from the propellant tanks

Reduces gas pressure to the required value

Propellants are stored here at relatively low pressure

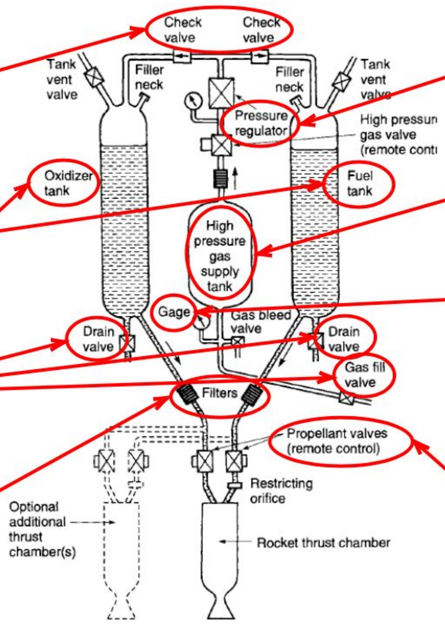
High pressure inert gas is used to pressurize the propellants

Measures tank pressure

Opened to fill (or drain) the tanks

Filter (solid) impurities from the propellants

Opened to let the propellants flow from the tanks



Pressure-fed propulsion system

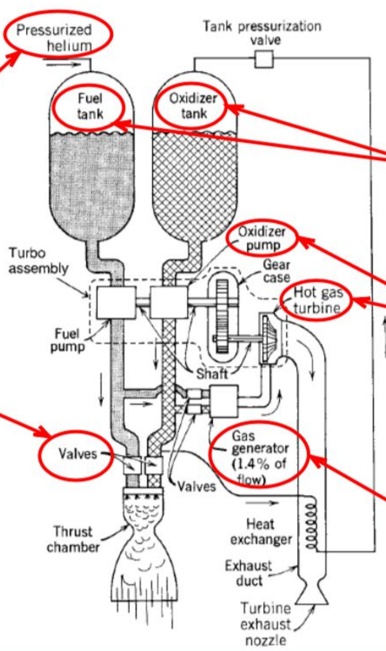
A high pressure inert gas may be used to initiate the propellant flow

Propellants are stored here at relatively low pressure

Propellants are pressurized by a turbo-pump system (pump + turbine)

Opened to let the propellants flow into the combustion chamber

Generates the hot gas that drives the turbine (through combustion of a small quantity of propellants)



Pump-fed propulsion system

Propellants

- **Oxidizers:** Liquid Oxigen, Nitrogen Tetroxide, Hidrogen Peroxide, Nitrous Oxide
- **Fuels:** Liquid Hidrogen, Liquid Methane, kerosene, Hydrozine

ROCKET PROPELLANT PERFORMANCE

Combustion chamber pressure, P_c = 68 atm (1000 PSI) ... Nozzle exit pressure, P_e = 1 atm

Oxidizer	Fuel	Hypergolic	Mixture Ratio	Specific Impulse (s, sea level)	Density Impulse (kg-s/l, S.L.)
criogenic	Liquid Hydrogen	No	5.00	381 n. 2	124
	~~Liquid Methane~~	No	2.77	299	235
	Ethanol + 25% water	No	1.29	269	264
Liquid Oxygen	Kerosene	No	2.29	289	294
	Hydrazine	No	0.74	303	321
	MMH	No	1.15	300	298
very low temp. to keep them liquid.	UDMH	No	1.38	297	286
	50-50	No	1.06	300	300
Liquid Fluorine	Liquid Hydrogen	Yes	6.00	400 n. 1	155
	Hydrazine	Yes	1.82	338	432
FLOX-70	Kerosene	Yes	3.80	320	385
Nitrogen Tetroxide	Kerosene	No	3.53	267	330
	Hydrazine	Yes	1.08	286	342
	MMH	Yes	1.73	280	325
	UDMH	Yes	2.10	277	316
	50-50	Yes	1.59	280	326
Hydrogen Peroxide (85% concentration)	Kerosene	No	7.84	258	324
	Hydrazine	Yes	2.15	269	328
Nitrous Oxide	HTPB (solid)	No	6.48	248	290
Chlorine Pentafluoride	Hydrazine	Yes	2.12	297	439
Ammonium Perchlorate (solid)	Aluminum + HTPB (a)	No	2.12	266	469
	Aluminum + PBAN (b)	No	2.33	267	472
Red-Fuming Nitric Acid (14% N_2O_4)	Kerosene	No	4.42	256	335
	Hydrazine	Yes	1.28	276	341
	MMH	Yes	2.13	269	328
	UDMH	Yes	2.60	266	321
	50-50	Yes	1.94	270	329

Temperature

Combustion Power P_{comb} can be expressed as:

$$P_{comb} = \dot{m}_{fuel} \cdot H_v \rightarrow \text{Heating value of fuel at 1atm and 298 K}$$

Power P_{heat} needed to increase the temperature of combustion products from their initial value T_0 to a final value T_c must take into account the entire amount of propellant (Fuel + oxid.)

$$P_{heat} = (\dot{m}_{fuel} + \dot{m}_{oxidizer}) \cdot C_p \cdot (T_c - T_0) = \dot{m}_{fuel} (1 + O/F) \cdot C_p \cdot (T_c - T_0)$$

Equating P_{comb} and P_{heat} we get the chamber temperature

$$P_{heat} = P_{dec} \rightarrow \dot{m}_{fuel} \cdot H_v = \dot{m}_{fuel} (1 + O/F) \cdot C_p \cdot (T_c - T_0) \rightarrow \boxed{T_c = T_0 + \frac{H_v}{C_p (1 + O/F)}}$$

Combustion Products Properties

The combustion products properties are evaluated as molar average of the properties of the single components.

$$C_{12}H_{26} + \frac{37}{2} O_2 \rightarrow 12 CO_2 + 13 H_2O$$

$$\left.\begin{array}{l} n_{CO_2} = 12 \text{ moles} \\ n_{H_2O} = 13 \end{array}\right\} \text{molar averaged values of } C_p \text{ and } M_w$$

$$C_p = \frac{12 \cdot C_{p\,CO_2} + 13\, C_{p\,H_2O}}{12 + 13}$$

M_w = same

Pump-Fed Rockets

Large engines, not convenient to use pressure gas (too heavy).

Propellants are pressurized by a pump.

More convenient when thrust is higher than 20 kN

Power required by the pump:

$$\boxed{P_{pump} = \frac{\dot{m} \cdot \Delta p}{\eta \cdot \rho}}$$

Δp — Required pressure increase.

η — Pump efficiency (50 %)

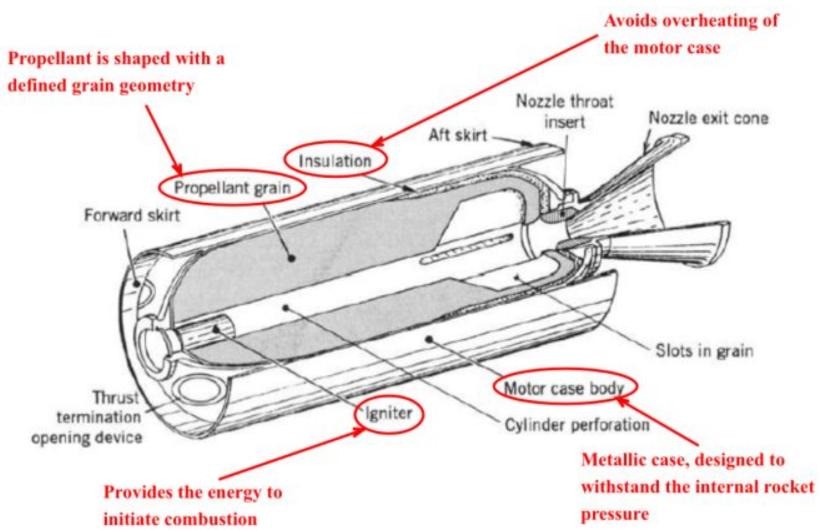
Solid Propellant Rockets

Oxidizer and fuel are mixed together in a solid-state grain.

The combustion is initiated by an igniter and can not be stopped once started.

No propellant feed system is needed: simpler

Mass flow rate is much higher than liquid. More or less same Isp but higher thrust.



Regression Rate

Mass flow rate:

$$\dot{m} = \rho_s \cdot r \cdot A_b$$

A_b → burning grain surface

r → regression rate. measure of burning speed of the solid grain.

Regression rate:

$$r = a \cdot P_c^{\,n}$$

depend on solid propellant

a → chamber temperature

n → stability: $n < 1$ stable, $n \geq 1$ unstable.

Chamber Pressure

Mass flow rate is also

$$\dot{m} = \frac{P_c \cdot A^*}{C^*}$$

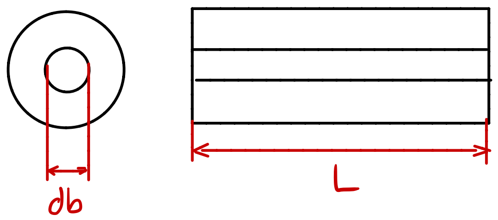
Combining the two equations:

$$\dot{m} = \rho_s \cdot a \cdot P_c^{\,n} \cdot A_b = \frac{P_c \cdot A^*}{C^*}$$

$$P_c = \left(\frac{a \cdot \rho_s \cdot C^*}{A^*} \cdot A_b \right)^{\frac{1}{1-n}}$$

A_b → change with time.

GRAIN SHAPES:



$$A_b = \pi \cdot d_b \cdot L$$

- Since diameter d_b increases during burning the burning-surface also increases.