Statics problem solving strategies, hints and tricks

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1 Solving a problem in 7 steps

1.1 To read

So what is a good strategy to solve a statics problem? The first thing you should do, is read the problem carefully. It's a very common mistake to miss an important word in the problem, which creates an entirely different problem. So make sure you read the problem well.

1.2 To draw

The next thing you should do is draw a figure of the problem. Especially for 3D problems this may be difficult, because it's often hard to draw them in a clear way, but try to draw them anyway. When you've drawn the problem, try to draw ALL of the forces (and torques, if there are any torques without a force) acting on the object. This is a very important part. For every part in the figure you should ask yourself the question "*what forces can be applied on this point?*". If the answer is "*none*", you don't have to draw any forces. If you're still doubting whether to draw a force or not, just draw it. It might turn out to be 0 eventually, but it's better to draw a force that does not exist, than to not draw a force that does exist.

1.3 To assign names

So now you've got all of the forces. To solve the problem, you should probably find the magnitude or direction of some force or torque, because that's the case with a lot of statics problems. The first thing you ought to do, is to make sure the variable you ought to calculate has a name. Also all the other forces present, which play a role in the problem (which is often the case, otherwise the forces wouldn't be present), should be given a name. This name can, and should be rather simple, like F_A for the force in point A.

Another thing that you could do (this is helpful in most problems, but there are a few exceptions), is dissect all the forces along a coordinate system. Give all those dissected forces logical names too. For example, the dissection of force F_1 along the x-axis can be called $F_{1;x}$.

1.4 To plan

You shouldn't be calculating just yet. What you now should do, is look very carefully at the figure you've drawn (or has been drawn for you). Ask yourself the question "*What kind of formulas can I derive from this data?*". Don't derive the formula just yet, but just wonder what variables will be in the formula, and whether it will be an easy formula. If this is the case, remember it, so you might even write down the formula later. The reason why this is so important, is so you can start planning how you will get your solution in an easy way. You don't want to end up with 20 formulas and get lost in them. Small errors are often made, so the chance to make them should be minimized.

When you've looked at the problem for a little while, and got quite a good idea of the possible formulas you can use, you can continue. Try to ask yourself which variable you can derive from which formula (or group of formulas in difficult cases). Then go step by step towards the variable you were supposed to calculate to solve the problem. If you, for example, want to find F_C, and you know that you can use equation 1 to find F_A from known data, equation 2 to find F_B from F_A and known data, and equation 3 to find F_C from F_A and F_B, you have solved the problem, without still having written down any formulas!

1.5 To formulate

And now comes one of the most important parts: finding and writing down formulas. If the body is in equilibrium, you can always use the rules that the forces and torques are 0: $F_x = 0$, $F_y = 0$, $T = 0$ (for every center of rotation!). In 3D you even got more formula's (6 of them to be exact). You have to use these data, and any other data that's noted in the problem (that's why you were supposed to read it so well!) to find and write down all the formulas you need. Examples of these formula's are: $F_1 - 2 * F_2 + F_3 = 0$. Or $\frac{aF1}{\sqrt{a^2+2b^2}} - \frac{bF_2}{\sqrt{2a^2+b^2}} = 0$ (I never said they would be easy). Now you know why you had to give every force a name: so you can put the values of them in formulas and equations. If you run into any distance or angle that you need to use in a formula, but doesn't have a name yet, just give it a name.

Next to the amount of values you do know, there is a certain amount of unknown variables in the formulas you just stated. If the number of formulas is equal to the number of unknown variables in the formulas (or greater, but this doesn't occur often), then you can solve the problem by playing around with the formulas. However, if you do not have enough formulas, take a closer look at the problem, and at the picture you've drawn, to see if you can derive another different formula, possibly by choosing a different center of rotation.

An exception to the stated rule, that you need the same amount of formulas as unknown variables, is the following: When you need to express a certain value in another one, you have to end up with 1 equation, containing 2 unknown variables. In the previous case you always needed to end up with 1 equation, containing only 1 unknown variable. So when you need to end up with a certain variable expressed in another one, the amount of formulas should be (at least) the amount of unknown variables minus 1.

1.6 To calculate

And only in the end, we start calculating. We have a number n formulas and the same amount of variables (hopefully). We should solve that now. When you don't have the mathematical skills to use all kinds of fancy tricks, it's just a matter of isolating a single variable at a time, and substituting it in all other functions, to find the final answer, step by step. Take your time and always double check whether you solved the formulas in the right way.

1.7 To evaluate

When you've done all the maths in the equations, you should have a final answer. A good idea is to take another look at the picture to see whether it's a possible and reasonable value. If it is, you probably have the right solution. If it isn't, try to find out why. Have you made a wrong calculation? Or did you draw the picture in the wrong way?

2 How to handle 3D problems

3D problems are rather tricky. You can't draw them well on a 2D sheet, which makes it a bit annoying. 3D problems can be very fun though. You just have to be able to imagine the problem in a good 3-dimensional way, however that is a quality some people have and others do not. But training can help.

2.1 Dissect forces (and of course naming them)

The first thing you often should do is name the forces and dissect them along the axes. But this is slightly more difficult than in 2D problems. There are 3 axes now, so how do you do this?

The easiest way is to make use of relations in sides of triangles. A very useful formula I often use is:

$$\frac{F_{r;x}}{l_{r;x}} = \frac{F_{r;y}}{l_{r;y}} = \frac{F_{r;z}}{l_{r;z}} = \frac{F_r}{l_r} \tag{2.1}$$

Looks great, but what are the meanings of those variables? Suppose there is a two-force member r with length l_r, and a force F_r in it. To know $F_{r;x}$ (the component of F_r pointing along the x-axis), you have to know $lr; x$, the length which the two-force member spans in the direction of the x-axis.

A small example might help. Suppose there is a rope (a two-force member) under a tension of $F_r = 100N$, and it goes from point (0,0,0) to point (2,3,6), thus it spans $2m$ in x-direction, $3m$ in y-direction and $6m$ in z-direction. So the total length of the rope is $l_r = \sqrt{2^2 + 3^2 + 6^2} = 7m$. Now applying the previous formula results in:

$$\frac{100}{7} = \frac{F_{r;x}}{2} = \frac{F_{r;y}}{3} = \frac{F_{r;z}}{4} \tag{2.2}$$

It is now quite easy to find the component of the two-force member in the direction of all axes. So dissecting forces along axes isn't so difficult really, not even in 3D or 4D, but 4D is something we won't be talking about. And when you've dissected forces, you can simply add them up in the way you're used to in 2D problems, as long as you've got the signs right.

2.2 But what about torques?

Now the next question arises, how do you use torques? That is quite tricky, until you get the hang of it. In 2D you rotate about a point. As you know a point is 1-dimensional. When we go to 3 dimension, we rotate about a 2-dimensional point, which is in fact a line. So we rotate about a line.

Suppose we have a line which we'll be rotating about. How do we do that? We first have to know which forces even cause a torque. In 2D there is a rule that says that a force of which the line of action goes through the point of rotation has an arm of 0. In 3D that's exactly the same, except for the fact that now the line of action of a force has to go through the line of rotation, for the force to have an arm of 0. There is just one "exception" to this rule (even though mathematicians don't find this an exception). A line which is parallel to the line of rotation has no arm too. (Mathematicians often say that two parallel lines cross each other in exactly one point infinitely far away. Don't ask them whether that point is to the left or to the right, because they don't know.) Anyway, forces with a line of action parallel to the line of rotation have no arm and thus cause no torque.

To calculate the torque caused by a force, you have to find the magnitude of the force and the arm of it. First make sure all the forces are dissected along axes perpendicular to the line of rotation (in most cases it's handy to choose a line of rotation parallel to 1 of the existing axes). The arm of the force is the minimal distance between the line of action of the force, and the line of rotation. This might sound a bit confusing, but if it does, just draw it on a paper and it'll make sense.

So, calculating torques isn't so difficult either. If you do calculate torques, do make sure that you choose a clever line of rotation. This is incredibly important, and might safe you a lot of work, so it might be well worth it to stare 5 minutes at a drawing looking for the right line of rotation, instead of immediately losing yourself in calculations leading to nothing.

2.3 Equilibrium equations in 3D

As you know (or should know), there are 3 equilibrium equations in 2D-situations. You that $F_x = 0$, $F_y = 0$ and $T = 0$. In 3D, there are more equilibrium equations. There are 6, to be exact. You can guess the first three: $F_x = 0$, $F_y = 0$ and $F_z = 0$.

Torques are something different though. Instead of rotating about a point, you rotate about a line. In 2D-situations, any point is just a point, but in 3D situations, lines have a direction (points don't). So instead of having 1 torque equation, you got 3: $T_x = 0$, $T_y = 0$ and $T_z = 0$. With M_x, we mean the torques about the x-axis or a line parallel to the x-axis. It is of course also possible to take the torque about a line not parallel to any of the axes. That will work out eventually (as long as you don't take torques about two lines that are parallel, since that will give the same equations), but it will make most statics problems rather complicated, so it is not advised.

3 Truss structures and frames

Truss structures can be incredibly difficult things to handle, and the only way to be able to handle them well, is to practice a lot. But we're going to try to state some general things about them anyway.

3.1 Cutting

When calculating with truss structures, it's always the trick to make the right cut. By doing this, you cut the structure in 2 or more parts, and every part must be in equilibrium. The advantage of this is that the internal forces of both parts can be ignored, and only the forces acting on the parts itself (which aren't many, as long as you've made the right cut) have to be calculated with.

So the important question usually is, where to make a cut? It all depends on the truss structure, which comes in a variety of shapes, and on the variable you need to calculate. So I can not give general rules about it. It is however important to make a cut in such a way, such that there are as few unknowns as possible.

3.2 Important things to remember

Something you should remember, is that you can only cut through two-force members! Other members have a normal force, a shear force, and a bending moment, but two-force members can only have a normal force, which makes them ideal for cutting through. Note that a two-force member is usually a bar between two hinges, certainly not containing any hinges or half-hinges itself.

However, another important and rather logical rule is that, if you need to know the tension/compression in a member, you should always make at least a cut through that member once, because otherwise, you won't have any equation with that unknown (which you must calculate) in it.

And the third important rule is to always assume there is tension in a two-force member. If the size of the tension turns out to be some negative number, you automatically know there is compression, and you also automatically have the right sign for the compression in that force. It's quite convenient. But more about this is said in the chapter "Using signs".

3.3 Looking at hinges

Next to cutting through two-force members, another way of calculating forces in members is by looking at the hinges. For every hinge, the equilibrium equations $F_x = 0$ and $F_y = 0$ apply (note that no bending moments can act on a hinge). So you have to look for a hinge, of which you know all the forces acting in a certain direction, except for 1. Then by using the equilibrium equation, that unknown value can be calculated. By doing this over and over again (sometimes even up to 30 times in a problem) it is possible to find the force in every member in the entire structure. This can be an especially handy method for structures containing only two-force members, despite the fact that it is a lot of work.

But if non-two-force members are connected to a hinge, the situation can get difficult. The best advise is to avoid such situations. But if this is impossible, it is important to know that non-two-force members exert both a force tangential to the member, as a force perpendicular to it. Those forces can sometimes be found by applying the 3 equilibrium equations ($F_x = 0$, $F_y = 0$ and $M = 0$) on the non-two-force member. But such situations are best to be avoided.

3.4 0-force members

Next to two-force members, also 0-force members can be present in a truss structure. Suppose that there are 3 two-force members (members A, B and C) connected to a hinge, of which members A and B point in the same direction. If you take the sum of the forces on the hinge in the direction perpendicular to A and B, you will find an equation saying something like $kF_C = 0$ (where k is any constant unequal to 0, depending on the angle between C and A or B). Thus F_C is zero, and C is a 0-force member. The first thing you can do, when looking at a truss, is detecting 0-force members. However, when there are hinges present to which non-two-force members are connected, the situation gets different, and this trick can often not be used anymore.

3.5 Planning

But what is most important while calculating with trusses (well, preferably before calculating) is to plan. This is so important, that I made this a separate paragraph. Don't write down any equation before you have a clue how you can solve the problem. Ask yourself questions like "*if I make a cut there, what kind of equation do I get? What variables does it contain? How many of them do I know? Can I easily find the variables I don't know?*". This is quite similar to the planning part in "Solving a problem in 7 steps", which you might want to read, if you haven't already.

4 Using signs: always a difficult thing

When you're calculating all kinds of formulas, there's always the matter whether to substract values from eachother, or to add them up. There is a method which saves you that trouble, which works in normal coordinate systems (positive x-axis pointing right, positive y-axis pointing up). If you have to define a coordinate system yourself, it is often handy to define such a coordinate system, since it is most often used. (In later chapters we will also assume such a coordinate system is used.)

4.1 Which direction is positive, and which is negative?

First, I just state the following things: To the right (along the x-axis) is positive. Upward (along the y-axis) is positive. To the left (along the x-axis) is negative. Downward (along the y-axis) is negative. A counter-clockwise torque is positive. A clockwise torque is negative. All these rules are important, otherwise I wouldn't write them.

4.2 What's the use of it?

So what's the use of this rule? As long as you always use these values for any forces along the axes (when you dissect a force along axes, you always have to check the sign again, but you don't have to do it during calculations, which is the huge advantage of this method), you don't have to worry about signs or directions of forces or torques, simply because you stated the rules above.

Adding forces up is now a lot easier. You just have to put a plus between all the forces. When the force is negative, it will automatically get subtracted, because of its sign, so there's no need to worry about that. For example, just write: $F_5 = F_1 + F_2 + F_3$. When, for example, F_2 points downward, you know its value is negative, and you don't have to write: $F_4 = F_1 - F_2 + F_3$.

4.3 But what about torques?

Even when calculating torques, this method works. As you know, you calculate a torque T with the formula $T = d\,F$, where F is the force and d is the distance between the force and the center of rotation. But for this method to work, you also have to make sure the distance d is negative if it's pointed downward or to the left. This is quite important, and works really well if you're too lazy to ask yourself whether a force is directed clockwise or counterclockwise.

But I still haven't told you another technique to calculate torques. You simply have to use the cross product, which you might know from vector mathematics. For every force F, the torque T of the force is equal to:

$$M = d \times F = \begin{bmatrix} d_x \\ d_y \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \end{bmatrix} = d_x F_y - d_y F_x \tag{4.1}$$

The minus here is one of the few minus's you see in this method, which makes it ever more important. You also should keep in mind the order in which to multiply d and F. This is because $d \times F = -F \times d$. Of course if you already dissected all the forces along the axes, you don't have to use the cross product.

But you still might be wondering, why is the minus there? Suppose d_y is positive, and F_x is negative. Then the torque is directed counter-clockwise, so it must be positive. But $d_y\,F_x$ is negative, so there must be a minus sign. In any other case you will also conclude that the minus sign ought to be present.

4.4 Two-force members

Two-force members are an important thing in statics when trusses are present. But how to handle signs in that case? There is also a simple rule for that: assume tension. Tension is positive and compression negative, so if you assume there is tension, you always wind up with the right sign.

But what if you have a two-force member F_A, and want to dissect the force along the axes to find $F_{A;x}$ and $F_{A;y}$? There is a quite simple rule for that. Assume that the two-force member you're dissecting is in tension. If the tensile force points upward (or left-upward or right-upward), then the vertical component $F_{A;y}$ has the same sign as the two-force member F_A. If it points downward, $F_{A;y}$ has a different sign than F_A. The situation is analog for the horizontal component $F_{A;x}$.

But do remember that tensile forces, when present in two-force members, point inward. Most people find this strange, because an outward force is necessary to put a two-force member in tension. This is true, but the force we're talking about is not the force an object is acting on the two-force member, but the force that the two-force member is acting on an object. And according to Newton's third law, this force is oppositely directed, and thus points inward.

4.5 The conclusion

So by applying this rule in a very consistent way (forget it once, and you'll screw up your entire solution, pardon me for saying it), you don't have to worry about signs at all. At the end of a difficult series of calculations, you, for example, only have to conclude: "*oh, the sign is negative, so the torque is clockwise!*". Isn't it easy?

By the way, this is just a method of getting rid of those awful problems with signs and such. If you already have an own method which serves you really well, please use it! This is just a useful method for those people who just keep having trouble with the signs, and have no idea how to solve it.

5 Distributed Loads

Distributed loads don't have to be difficult, as long as you know how to deal with them. It's all about replacing them by normal forces, so you can use the normal statics rules on them. There are multiple kinds of distributed loads. We'll handle the simple ones first.

5.1 Constant distributed loads

Constant lineair loads are quite easy to handle. You can simply replace them by a normal force. As you probably should know, the resultant force caused by a distributed load is equal to the area under the distributed load. If the size of the load is q (N/m) and the length is a (m), then the size of the load is $a\,q$ (N). Interesting here, is to note the units of the values. The position of the resultant force of the load is simple too: right in the middle of the distributed load.

However, when there are hinges in the field of a distributed load, you may not always replace the entire distributed load by a resultant force, but have to split the distributed load up in two parts, and find a resultant force for every separate part. This is only necessary when you make a cut through the hinge though, but there may be a very few small exceptions to this rule.

5.2 Lineair distributed loads

Sometimes distributed loads are lineair, and usually they have the shape of a triangle. It is a bit more difficult to replace them by a resultant force, but it's nevertheless possible. The size of a distributed load, which ranges from 0 to q (N/m) and has length a, is simply $\frac{1}{2}a\,q$, which is the area of the triangle. The resultant force isn't in the middle of the load, but it's on $\frac{2}{3}$ of it, or of course at $\frac{1}{3}$ if you turn the triangle around, and have a value of q on the left and 0 on the right of the distributed load.

5.3 Combined lineair and constant loads

Sometimes lineair distributed loads don't have the shape of a triangle, but go from q_1 to q_2. If this is the case, you can simply cut the distributed load up in 2 parts, of which one has the shape of a rectangle (and thus is a constant distributed load with size q_1 (assuming $q_1 < q_2$, otherwise it's q_2)) and the other one has the shape of a triangle, which we also have previously discussed. You only have to find the resultant force of each of the 2 distributed loads. After that you may simplify it as mush as you wish.

5.4 Other distributed loads

There are also other distributed loads. These are quite difficult to handle. If there is no data about the shape/size of the distributed load, you can't calculate with it, of course. But sometimes there is a formula which gives the size of the distributed load on a point x, or sometimes you have to find that formula yourself. Let's do an example. Suppose $D(x) = -x^2 + 6x$, where $D(x)$ is the size of the distributed load, and x ranges from 0 to 6.

Since the resultant force of a distributed load is equal to the area under the distributed load, the resultant force can be calculated using an integral. So:

$$F_R = \int_0^6 D(x)dx = \left[-\frac{1}{3}x^3 + 3x^2\right]_0^6 = 3 \cdot 36 - \frac{1}{3} \cdot 216 = 36 \tag{5.1}$$

Now we have found the resultant force, so we need to know where it applies. For that, we take the torque about point 0. Since $T = F\,d$ where F is the force and d is the arm, we know that the torque caused by the distributed load at point x is $x\,D(x)$. So we can calculate T of the entire distributed load, around point 0:

$$T_0 = \int_0^6 D(x)\,x\,dx = [-\frac{1}{4}x^4 + 2x^3]_0^6 = 432 - 324 = 108 \tag{5.2}$$

So the torque around point 0 is 108, while the resultant force itself is 36. The resultant force must therefore apply at a distance $\frac{T}{F} = \frac{108}{36} = 3$ from point 0. In this case it is in the middle of the distributed load, which is quite logical because the shape of it was a symmetric parabola. Of course this isn't always the case. But this example does show how to calculate the resultant force of a strange distributed load.

6 Sections

Shear forces and bending moments aren't difficult, as long as you got an easy method to calculate them. In this chapter I just explain a method. Use it if you find it handy, and otherwise just use the method you're used to.

6.1 The convention

The most important thing is the convention we're using. First we'll discuss the normal force convention. But you probably already know that. Tension is positive and compression is negative.

The shear force sign convention is not that difficult either. There are 2 kinds of shear forces. The first one (which I'll call Z) has a downward force on the left, and an upward force on the right. The other one (which I'll call S) is exactly the opposite. The convection is: the Z-type shear force is positive and the S-type is negative. In diagrams, positive is drawn above the x-axis.

We also have a convention for bending moments. Suppose a bar bents in such a way that it is shaped as a hill, then it is negative. If it bends in the shape of a valley, then it is positive. To keep things simple, we still draw positive bending moments upward in diagrams.

6.2 Normal forces

Suppose we want to know the normal force in a point B. The most simple way to calculate a normal force in a point B, is to look at all the horizontal forces acting on the object on one side of point B. Add them up, while taking forces pointing away from B (thus tensing the object) positive, and forces pointing towards B negative.

When there is a structure which is not a straight line, things can get a little bit more complicated. Just draw a line tangential to the structure at point B. When calculating the shear force, only the forces parallel to that line should be added up. (This does mean you might have to dissect some forces.)

6.3 Shear forces

Shear forces aren't difficult to calculate either. When calculating a shear force in a point B, simply look at all the vertical forces on one side of point B. If we look at the left side, we can simply add up all the forces, which gives our shear force directly (note that negative forces automatically should get subtracted). If we look at the right side, we should take upward forces negative and downward forces positive to get the shear force. In the end, a positive shear force indicates a Z-type shear force, and a negative shear force indicates an S-type shear force, as discussed in the previous paragraph.

When there is a structure which is not a straight line, things can get a little bit more complicated. Just draw a line perpendicular to the structure at point B. When calculating the shear force, only the forces parallel to that line should be added up.

6.4 Bending moments

Calculating bending moments isn't very difficult either. When calculating a bending moment in a point B, we look at all the forces and torques acting on the object on one side of it. The bending moment can then be easily calculated by adding up all the external torques on one side, and the torques caused by the forces on that side. One very important rule here, is that upward forces cause positive torques and downward forces cause negative torque. This implies for both the left and the right side of point B. (To be exact, when you're looking to forces on the left, clockwise counts as positive, and when you're looking on the right, counter-clockwise counts as positive.) A positive bending moment number indicates a valley-type bending moment, while a negative bending moment indicates a hill-type bending moment.

7 Diagrams

A diagram is just a visual representation of the normal force/bending moment/shear force in the entire structure. Drawing them doesn't have to be difficult. But before you start drawing, you first have to make sure that you have a clear drawing of the object, in which all the forces and torques are drawn. If you miss one force, it will mess up your entire diagram. We will discuss a few methods to draw the three diagrams now.

7.1 Normal force diagram

To draw a normal force diagram, you first ought to make sure you know the normal force at one point in the diagram. (On the edges of an object, the normal force is usually 0.) Draw that point in the diagram. Then, starting from that point, you "travel" over the object. It is often wisest to travel from left to right. The normal force in the object stays constant (so draw that too!), until you run into a force (or when the structure bends, but we'll come to that later, we assume a straight rigid bar now). Dissect the force into 2 components, one perpendicular to the structure, and one tangential to it. The perpendicular component doesn't effect the normal force diagram (it does effect the shear force diagram), but the component tangential to the structure does effect the normal force. You should look at the effect this component has on the part of the structure you're "traveling" to. If the force is pointing in the direction you came from, it is tensing the part of the structure you're traveling to, thus it's positive, and the normal force diagram should go up with the magnitude of the component. If the component is compressing the part of the structure you're traveling to, it is negative, and the normal force diagram should go down. In table **??** is an overview of the sign convention.

If you run into a distributed load tangential to the structure (note that most distributed loads are perpendicular to the structure, and therefore do not effect the normal force diagram), you should draw that in the normal force diagram too. The sign convention is identical as discussed previously, but the diagram doesn't "jump" up. Instead, the normal force diagram slowly goes up, and the slope of the line with which it goes up is equal to the distributed load.

7.2 Shear force diagram

Just like with the normal force diagram, you ought to make sure you know the shear force at one point. Then you once more travel along the structure and look at how the value of the shear force changes.

	Normal force diagram		Shear force diagram	
Traveling	Force pointing left	Force pointing right	Force pointing up	Force pointing down
Left to right	+	-	+	-
Right to left	-	+	-	+

Table 7.1: Normal/Shear force diagram traveling overview (A plus indicates the diagram goes up).

It's easiest to travel from left to right along the structure. Every time you run into a force, dissect it, as discussed in the previous paragraph. The component perpendicular to the structure effects the shear force diagram. If it points up, the shear force diagram should go up, and if it points down, the shear force diagram should go down. That's rather easy. However, if you're traveling along the structure from right to left, exactly the opposite applies. It is important to check whether the diagram eventually gets back to a 0 value. If it doesn't, you've probably done something wrong, and might want to calculate the shear force at certain positions, to see whether they match the diagram.

Distributed loads are slightly more difficult, but nothing to worry about either. We're once more traveling along the structure from left to right. If the distributed load points upward, the shear force diagram should go up in a line with a slope equal to the size of the load. If the distributed load points downward, the shear force diagram should go down. However, when you're traveling from right to left, this is exactly the other way around. In table 7.1 is an overview of the sign convention.

7.3 Bending moment diagram

All that's left, is the bending moment diagram. But the bending moment diagram is the derivative of the shear force diagram. So you should use the shear force diagram to draw the bending moment diagram. Start at a point of which you know the value of the bending moment diagram (usually on the left edge where it's 0). Then travel from left to right, and make sure the slope of the bending moment diagram is equal to the value of the shear force diagram at every point. If you run into an external torque in the structure, a vertical line will appear in the bending moment diagram. A clockwise torque is positive, and a counter-clockwise torque is negative. If you travel through the structure from left to right, positive means the diagram jumps up, and negative means it jumps down. If you travel through the structure from right to left, this is exactly opposite. In table 7.3 is an overview of the sign convention.

	Bending moment diagram	
Traveling	Clockwise moment	Counter-clockwise moment
Left to right	Diagram goes up	Diagram goes down
Right to left	Diagram goes down	Diagram goes up

Table 7.2: Bending moment diagram overview.

7.4 Non-lineair structures

The past paragraphs all have assumed that the structure consisted of just one (horizontal) bar. This is of course not always the case. There may be structures with an angle in it (for example this shape: $\wedge$). Other structures sometimes even split up (for example: $\top$). If the angle in which a structure bends is 90 degrees, a simple rule applies. The normal force diagram turns into the shear force diagram, and the shear force diagram turns into the normal force diagram. The bending moment diagram stays the same.

However, such a bend is not always the case. In more complicated situations, it is easier to cut the structure up in parts, making the cut at the point where the structure bends or splits up. Draw all the parts separately. On every part, at every position you made a cut, you should draw the normal force, shear force and bending moment (if present) the other parts are acting on that part. Now the diagrams can be drawn in the normal way for every part, keeping in mind the internal forces and moments you've just drawn. If you've done everything in the right way, every part should be in equilibrium. This is often worth checking.

8 Virtual Work

Virtual work, if mastered well, can make it very easy to solve certain problems. While normal solving methods can involve numerous calculations, virtual work sometimes enables you to solve a problem with just one equation, and that's the benefit. The downside is that it's easy to make errors.

8.1 Increasing the degree of freedom

Most structures that occur in problems are statically determinant. They therefore can not move. To use virtual work, you must increase the degree of freedom in such a way, that the structure can move. But how to do that is sometimes complicated.

Every part in the structure can pass on forces. But which forces depends on the type of structure. In table 8.1 is an overview of all the structure types, and which forces they can transfer. As you can see, there are a lot of different connection types, some of which are familiar, some of which are not, but for which you just have to use your imagination to find out what they look like. The important part is that they can move in certain directions, and can not move in other directions.

Structure type	**Normal forces**	**Shear forces**	**Bending moments**
Normal bar	X	X	X
Horizontal sliding bar		X	X
Vertical sliding bar	X		X
Hinge	X	X	
Hinge on wheel support (tangential to the bar)	X		
Hinge on wheel support (perpendicular to the bar)		X	
Sliding connection (unable to rotate)			X
No connection			

Table 8.1: Overview of structure types, and their degree of freedom.

To use virtual work, you need to replace a certain connection by a connection that has just one more degree of freedom (one less X). The additional degree of freedom given should be replaced by an external force or torque. Suppose you want to know the normal force caused by a normal hinge on wheel support, you have to replace it by just a force (no connection), acting in the same direction as the reaction force caused by the support was acting. Suppose you want to know the bending moment in a bar, you have to replace part of the bar by a hinge and two moments (one on each side of the hinge, oppositely directed).

8.2 Making the structure move

After the degree of freedom has been created, the structure should move. Just imagine that the structure moves (rotates/slides) at the point at which you've replaced a connection, and draw it in the picture. Give the distance/angle that this point has moved a name (for example δu for distances or $\delta\theta$ for angles).

Now draw the rest of the "new" structure, with the corresponding movements. But do keep in mind that bars remain straight, fixed hinges can not move, and hinges on wheel supports can only move in 1 direction.

Now look at every significant point in the structure that has moved/rotated in any way. If it has moved, express the movement in δu (if δu hasn't been defined yet, just define it as a certain movement). If it has rotated, express the rotation in $\delta\theta$ (define it if necessary). Now you should have a drawing on which is clearly visible what point moved what distance in what direction.

8.3 Setting up the equation

The most important thing now, is to set up the virtual work equation. As you should know, work is force times distance traveled ($\delta A = F\,\delta u$). Also, work is torque times angle rotated ($\delta A = M\,\delta\theta$) ($\delta\theta$ in radians!). It is very important to remember the following rule: If a force points in the same direction as the movement (thus the force partially causes the movement), the work done is positive. If a force points in the opposite direction as the movement (thus the force partially counteracts the movement), the work done is negative. And if the force is perpendicular to the movement, then the work done is 0. In formula, this can be written as $\delta A = F\,\delta u\,\cos\alpha$, where α is the angle between the force and the movement.

8.4 Working out the equation

When the work done by all the forces and the torques has been written down (with the right sign), it should be equal to 0, since the structure is in equilibrium. Now the equation should be solved. However, next to the unknown force/torque, there are often 2 unknown variables: δu and $\delta\theta$. To solve the equation, you first have to express one of those variables in the other one. When doing this, the small angle approximation should be used: $\tan\theta = \theta$. This makes solving the equation a lot simpler.

When one of the two unknown variables has been filled in, the equation should be solved. While solving it, the other unknown variable will also disappear (if you've done everything right), so that the only unknown left is the unknown force/torque, expressed in other known forces and torques. Of course you need to evaluate your answer (check whether it has the right unit, etc.), but if you have followed the steps, the answer should be right.

9 Cables and pressure vessels

Next to rigid bars, structures can also consist of cables and pressure valves. This is often a difficult subject, since cables are rarely straight. Of course the normal equilibrium equations still apply, but these are often not enough to solve the problem. But there are a few tricks that can be used to solve problems involving these subjects.

9.1 Cables without distributed loads

Cables bend, and bars don't. That's why cables are sometimes a bit more difficult to calculate with. However, cables are often a bit like structures containing normal bars. Suppose there is a cable, on which one or more normal forces (no distributed loads) apply. If you replace the cable by bars, and place hinges at the points where the forces apply, the structures are more or less equivalent, and the normal solving methods for trusses can be applied. However, when distributed loads are present, it is not possible to put an infinite amount of hinges in the bar, so this trick won't work.

9.2 Cables without horizontal forces

In almost all problems involving cables, all the forces that are acting on the cable are downwards (except at the two ends of the cable). Thus the horizontal component of the cable tension does not change in the entire cable, and is therefore constant. So if you at every point know in which direction the cable points towards, and if you know the horizontal component of the cable tension, you can easily calculate the cable tension at any given point in the cable.

9.3 The rigid bar trick for cables

If all the forces acting on the cable are directed downward (or sometimes upward, just as long as there's no horizontal component), there is another trick you can use. Suppose there is a cable that spans from point A to point B, which is subjected to a number of vertical forces. Assume that there is a rigid and straight bar AB between points A and B (which don't have to be at the same height), and that the vertical forces acting on the rope act on the bar. Draw the bending moment diagram of the bar, and turn it upside down (positive downward). Now divide the entire diagram by the horizontal component of the cable tension H_{cable}. The diagram that appears shows the distance between the bar and the cable at any given point. In formula, this is:

$$y_{bar} - y_{cable} = -\frac{M_{bar}}{H_{cable}} \tag{9.1}$$

The variable y indicates the height, and M_{cable} the bending moment in the cable. But the bending moment is the integral of the shear force diagram, and the shear force diagram is the integral of the "force diagram". So also the following function applies:

$$(y_{bar} - y_{cable})'' = -\frac{q_x}{H_{cable}} \tag{9.2}$$

Where q_x is the force (often the magnitude of the distributed load) acting on the cable at any point x. Note that the second derivative of the height is taken. If you choose a nice and simple root for your coordinate system, it is often easy to find a function for the height of the cable at any given point.

9.4 Pressure vessels

Pressure vessels are just like cables. However, instead of a distributed load directed downward, there is a distributed load directed perpendicular to the vessel hull. This might seem difficult to calculate with (if you want, you can use integrals to find resultant forces and such, but I wouldn't recommend this), but there is a trick which makes it simple. It is called Pascal's law.

Suppose you have a pressure vessel with the shape of a circle (radius r). Now cut it in half by a vertical line right through the middle. The pressure being exerted on the cut is of course $2r \cdot p$, where p is the pressure inside the vessel. The pressure exerted on the other side is $\pi r \cdot p$. However, this pressure consists of forces directed in multiple directions. Pascal's law states that the resultant force of the latter pressure is of equal size and opposite direction with respect to the resultant force of the pressure exerted on the cut. Also the lines of action of these two resultant forces are equal.

So by using this law, it can be proven that the tension in the pressure vessel is $r \cdot p$. The 2 disappeared, because we cut twice through the pressure vessel (once on top of the circle, once on the bottom). However, this law holds not only for circular shapes but for any shape. And if you know this law, you just have to cut the pressure vessel in the right way to find what you need to know.