Atmospheric Flight Dynamics Example Exam 2 – Problems

1 Question

Given the autocovariance function,

$$C_{\bar{x}\bar{x}}(\tau) = \frac{1}{2}\cos(2\pi\tau) \tag{1.1}$$

of stochastic variable $\bar{x}$. Calculate the autospectrum $S_{\bar{x}\bar{x}}(\omega)$.

<u>NOTE</u>

Assume that,

$$cos(2\pi\tau) = \frac{e^{-j2\pi\tau} + e^{j2\pi\tau}}{2} \tag{1.2}$$

and,

$$\int_{-\infty}^{+\infty} e^{-j\omega\tau}\, d\tau = 2\pi\delta(\omega) \tag{1.3}$$

2 Question

Proof that,

(a) the variance of the "stochastic" variable $\bar{y} = c$ equals $\sigma^2_{\bar{y}} = 0$

(b) $\mu^2_{\bar{x}} = \mathrm{E}\left\{\bar{x}^2\right\} - \sigma^2_{\bar{x}}$

(c) if $\bar{y} = b\bar{x} + c$ then $\sigma^2_{\bar{y}} = b^2\sigma^2_{\bar{x}}$

In the above mentioned questions b and c are <u>constants</u>.

3 Question

In figure 1 the product function $R_{\bar{u}\bar{u}}(\tau)$ of the stationary stochastic process $\bar{u}$ is given. What can be said about the properties of the stochastic variable $\bar{u}$?

(a) It is white noise.

(b) It is noise with a small bandwidth.

(c) It is white noise plus a sinus.

(d) It is a sinus.

4 Question

Given the probability density function of the stochastic variable $\bar{x}$ with parameter λ ($\lambda > 0$),

$$f_{\bar{x}}(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{cases} \tag{4.1}$$

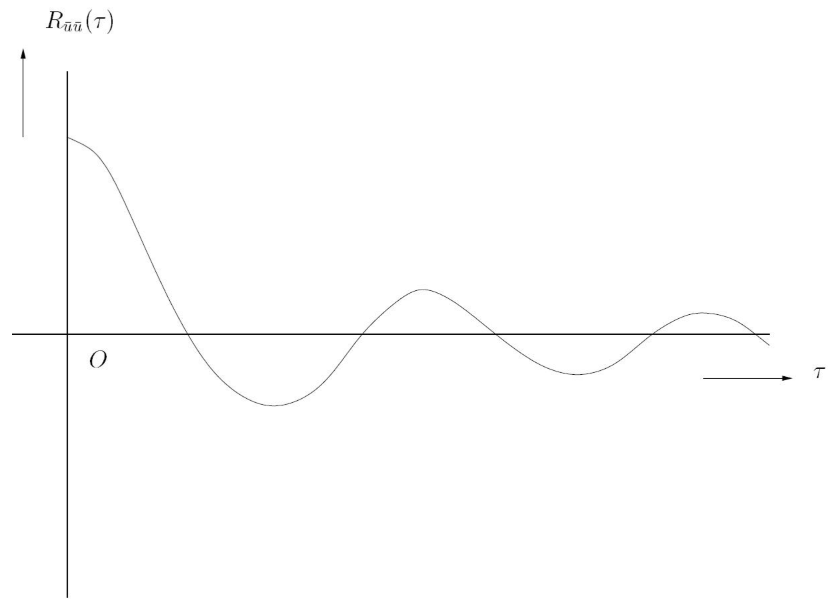

Figure 1: Product function $R_{\bar{u}\bar{u}}(\tau)$

Calculate the probability distribution function $F_{\bar{x}}(x)$, and prove that the mean value $\mu_{\bar{x}}$ and the variance $\sigma^2_{\bar{x}}$ are equal to,

$$\mu_{\bar{x}} = \frac{1}{\lambda} \qquad \text{and} \qquad \sigma^2_{\bar{x}} = \frac{1}{\lambda^2}. \tag{4.2}$$

5 Question

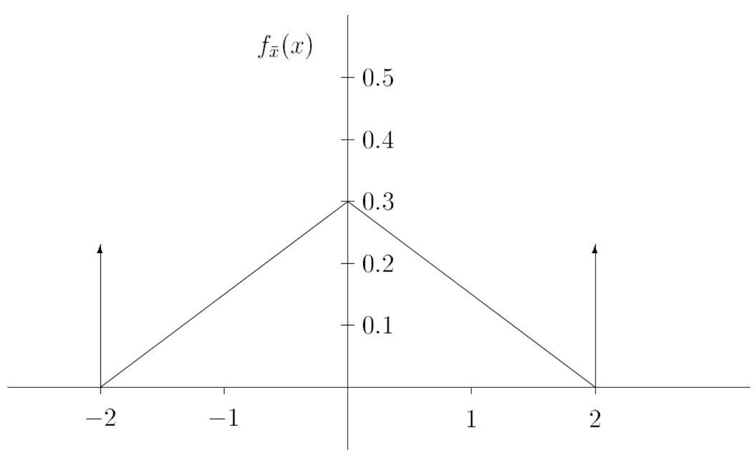

Figure 2: Probability density function $f_{\bar{x}}(x)$

The random variable $\bar{x}$ has a probability density function $f_{\bar{x}}(x)$ as depicted in figure 2.
What is the probability of $P(\bar{x} \geq -1)$?

(a) 0.125

(b) 0.275

(c) 0.725

(d) 0.750

(e) 0.875

(f) Not enough data available

6 Question

Prove that the Fourier transform of the signal $x(t - t_0)$ equals,

$$\mathcal{F}\{x(t-t_0)\} = \left(e^{-j\omega t_0}\right) X(\omega) \tag{6.1}$$

7 Question

Which of the following statements are true?

(a) $R_{\bar{x}\bar{y}}(\tau) = R_{\bar{y}\bar{x}}(\tau)$

(b) $C_{\bar{x}\bar{y}}(\tau) = C_{\bar{y}\bar{x}}(\tau)$

(c) $K_{\bar{x}\bar{x}}(\tau) = K_{\bar{x}\bar{x}}(-\tau)$

(d) $K_{\bar{x}\bar{x}}(0) = 1$

(e) $S_{\bar{x}\bar{y}}(\omega) = S_{\bar{y}\bar{x}}(\omega)$

(f) $S_{\bar{x}\bar{x}}(\omega) = S_{\bar{x}\bar{x}}(-\omega)$

8 Question

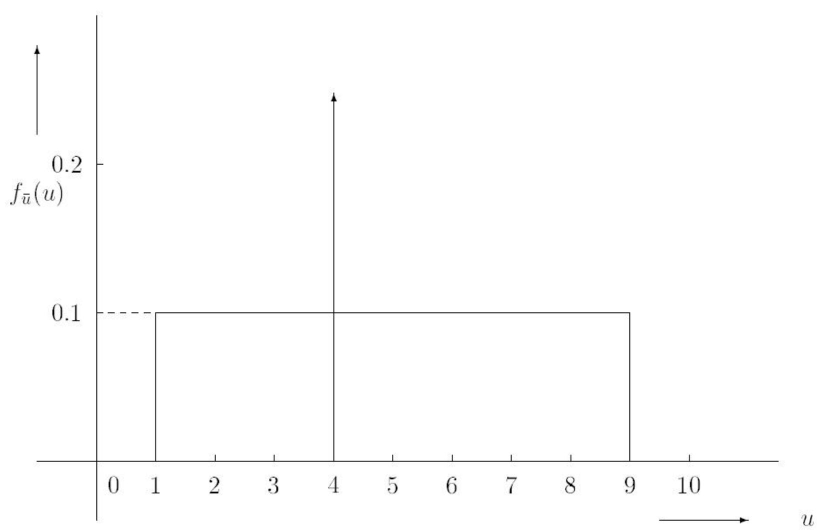

Figure 3: Probability density function $f_{\bar{x}}(x)$

(a) The random variable $\bar{x}$ has a probability density function $f_{\bar{x}}(x)$ as depicted in figure 3. Calculate the probability $P(u = 4)$.

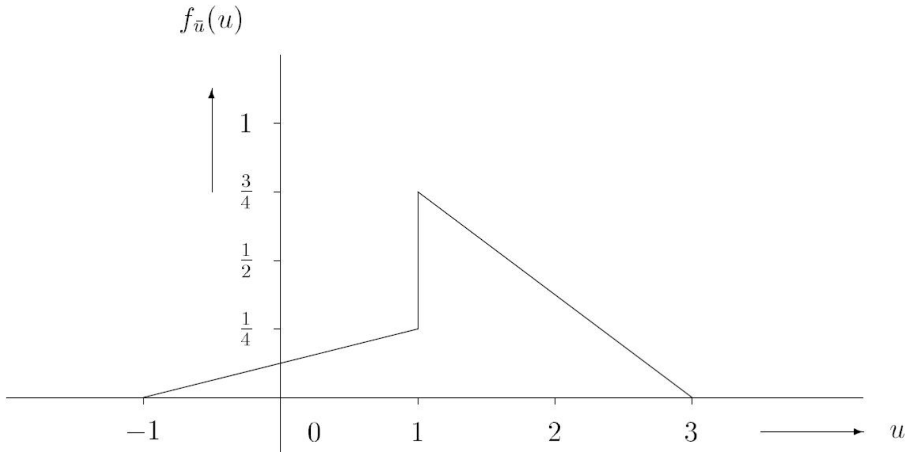

Figure 4: Probability density function $f_{\bar{x}}(x)$

(b) The random variable $\bar{x}$ has a probability density function $f_{\bar{x}}(x)$ as depicted in figure 4. Calculate the probability $P(u = 1)$.

9 Question

Proof that the periodogram $I_{\bar{y}\bar{y}}[k]$ of the signal $y[n] = ax[n] + b$ equals,

$$I_{\bar{y}\bar{y}}[k] = a^2 I_{\bar{x}\bar{x}}[k] + (2a\mathrm{Re}\{X[k]\} + b)b\delta[k] \tag{9.1}$$

with,

$$I_{\bar{x}\bar{x}}[k] = X^*[k]X[k] \tag{9.2}$$

and $\mathrm{Re}\{X[k]\}$ the real part of the Fourier transform of $x[n]$.

Note: the Discrete Fourier Transform (FFT) of a constant b equals,

$$FFT\{b\} = \left(\frac{1}{N} \sum_{n=0}^{N-1} be^{-j\frac{2\pi k}{N}n} \right) = b\delta[k] \tag{9.3}$$

with $\delta[k]$ the Kronecker delta function. Use the result $FFT\{b\} = b\delta[k]$ in your proof. Remember that $\delta[k]$ equals 0 for $k \neq 0$ and $\delta[k]$ equals 1 for $k = 0$.