Thermodynamics - ASE 231

SYSTEM: A Quantity of Matter.

SURROUNDINGS: "Everything Else"

BOUNDARY: Separates systems from surroundings. It may be real or "Abstract" also Control Surface.

CLOSED SYSTEM = Control Mass

No mass crosses the boundary.

OPEN SYSTEM = Control Volume

Mass crosses boundary

ISOLATED SYSTEM: Nothing crosses boundary.

CLASSICAL Vs STATISTICAL

THERMODYNAMICS

Macroscopic

Temperatures

Pressures

The measurable results of → Molecular Interactions

Microscopic

Atoms, Molecules

Molecular Interactions

Energy → **Total Energy** = E

$$E = U + K.E + P.E + \ldots\ldots$$

U = Interal energy associated with atomic + molecular energies.

K.E = Kinetic Energy

P.E = Potential Energy

★ Thermodynamic Properties

Characteristics that help describe the state of a system.

★ Thermodynamic State:

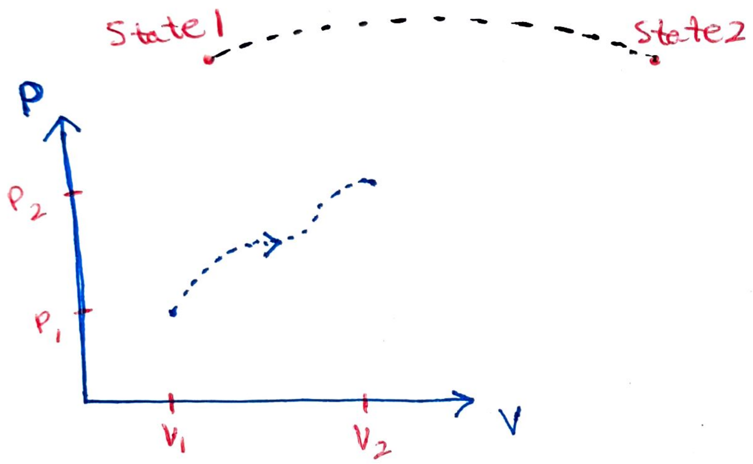
Specific condition a system is in determined by its properties.

★ Equilibrium:

A condition of balance characterized by the absence of driving potentials.

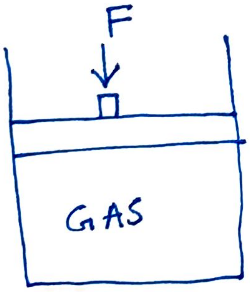
★ Process:

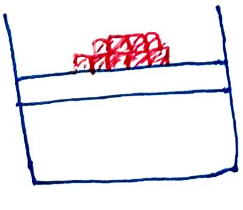
A transition from one state to another.



QUASI EQUILIBRIUM PROCESS:

A process in which each intermediate state is only infinitesimally removed from equil.





▨ Add infinitesimal weights one at a time.

TYPES OF PROPERTIES:

EXTENSIVE

Vary directly with the mass

mass
volume
energy

INTENSIVE

Idependent of the amount of mass.

Temperature
Pressure
Specific energy
Specific volume } divided by mass to make specific.

SPECIFIC:

per unit mass.

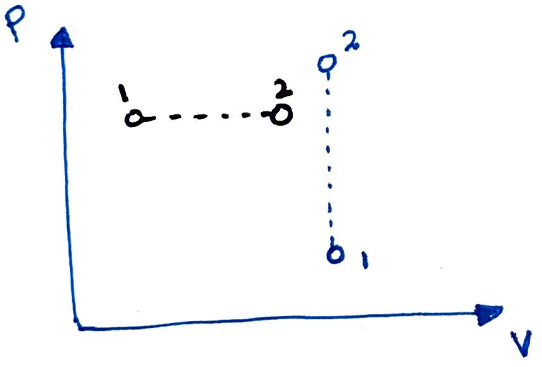
we write specific in lower case letters.

Specific Volume:

* $v = \frac{V}{m} = \left[\frac{m^3}{kg}\right]$

Specific Energy:

$e = \frac{E}{m} = \left[\frac{J}{kg}\right]$



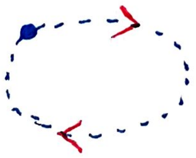
★ **ISOTHERMAL**: Process occuring at constant. temp.

★ **ISOBARIC**: Takes place at constant pressure.

★ **ISOCHORIC** or **ISOMETRIC**: Takes place at constant volume.

1....2 - Isobaric
1....2 - Isometric

★ **CYCLE**: System returns to initial state



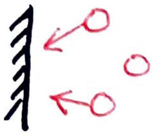
④

* Density:

$$\rho = \frac{m}{V} = \frac{1}{v} = \left[\frac{kg}{m^3}\right]$$

* Pressure:

$$P = \lim_{dA \to 0} \frac{dF_N}{dA} = \left[\frac{N}{m^2}\right]$$

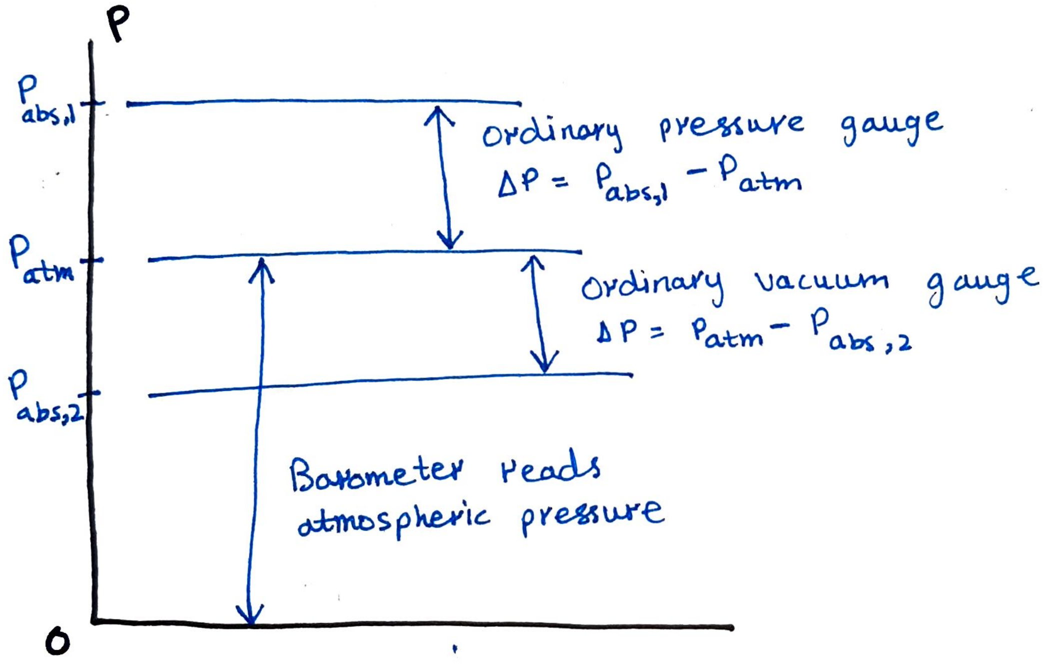


1 pascal $= 1 \frac{N}{m^2}$

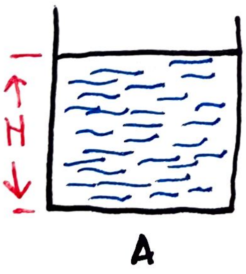
1 atm = ~~0.0101~~ 0.101 MPa = 14.7 Psi

1 bar = 10^5 Pa = 100 kPa

* Absolute Pressure Vs Gage Pressure:



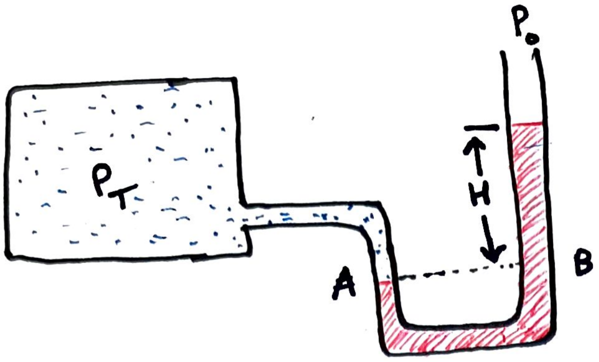
P_{abs} = Absolute pressure



$V = AH$

$W = \rho g \cancel{V} = \rho g A H$

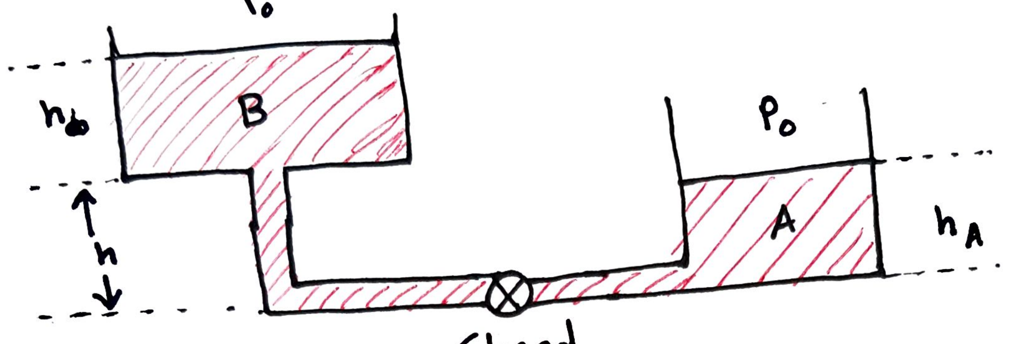
$P_A = \frac{W}{A} = \rho g H$



$$P_A = P_B \; ; \; P_T = PA$$

$$P_B = P_0 + \rho g H$$

$$\boxed{P_T = P_0 + \rho g H}$$



a) Determine pressure on both sides of valve.

Given: M_A, M_B, A_A, A_B, h, ρ

$$h_A = \frac{V_A}{A_A} = \frac{M_A/\rho}{A_A} \; ; \; h_B = \frac{M_B/\rho}{A_B}$$

$$P_A = P_0 + \rho g h_A = P_0 + \frac{M_A g}{A_A}$$

$$P_B = P_0 + \rho g (h_b + h) = P_0 + \rho g h + \frac{M_B g}{A_B}$$

b) If valve is opened, what is P at valve when equilibrium is reached?

Temperature:

°C, °F, $\underbrace{K, R}$

Absolute
Scales → start from 0.

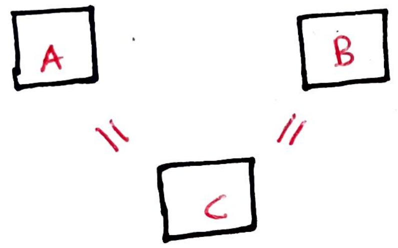
$K = °C + 273.15$

$R = °F + 459.67$

$°F = 1.8°C + 32$

$R = 1.8K$

★ Zeroth Law of Thermodynamics:



$T_A = T_C$

$T_B = T_C$

Then $T_A = T_B$

★ Energy:

⇒ **Internal energy**: U [J]

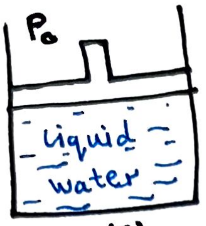
Molecules have energy

- Potential energy associated with intermolecular forces
- Kinetic energy associated with translational velocities.
- Vibrational energies associated with molecular structure.

All of these make up U.

Typically in terms of u

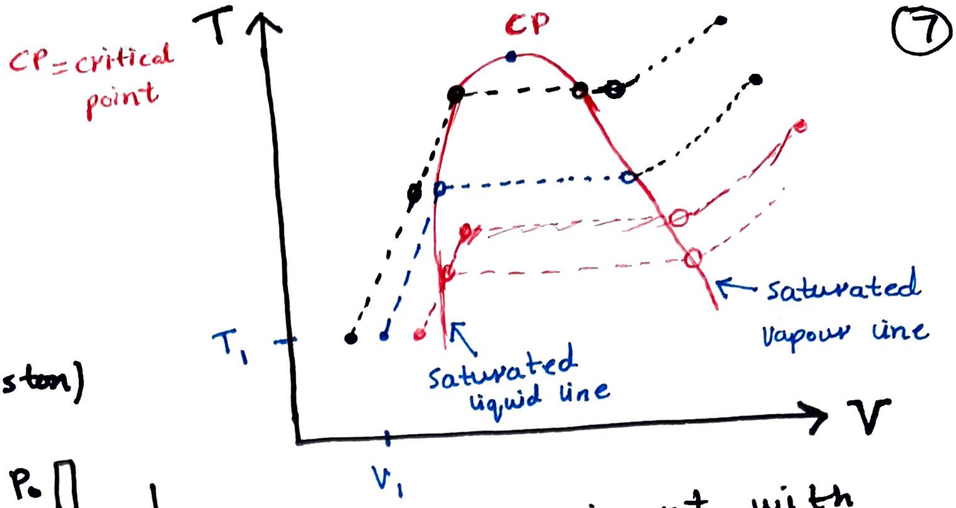
$u = \frac{U}{m} \left[\frac{kJ}{kg}\right]$

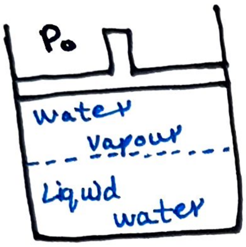


(a)

Heating very slowly.

(Ignore weight of piston)





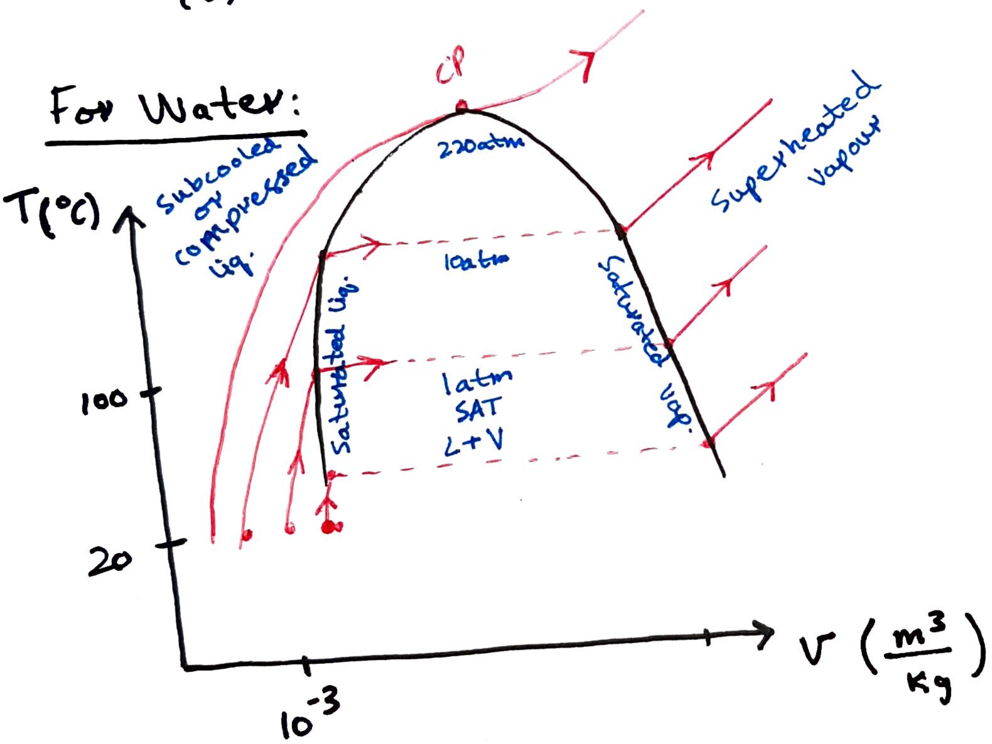
(b)

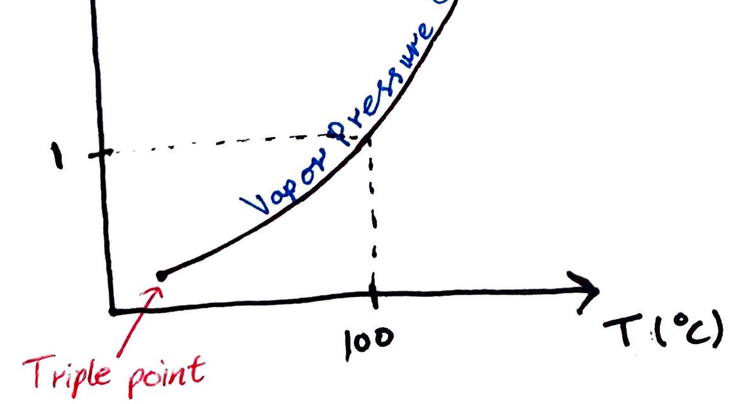
(c)

Repeating experiment with high pressure.

Repeating experiment with lower pressure

For Water:



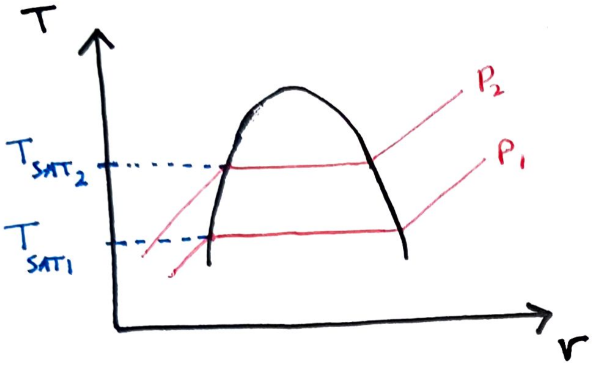


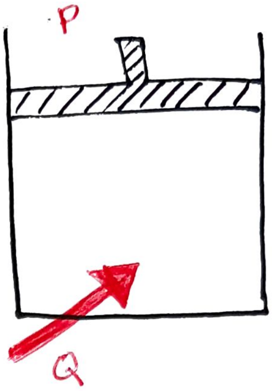
Properties Of Pure Substance

Pure Substance:

Homogeneous + invariable chemical composition.

Ex: Water, Air





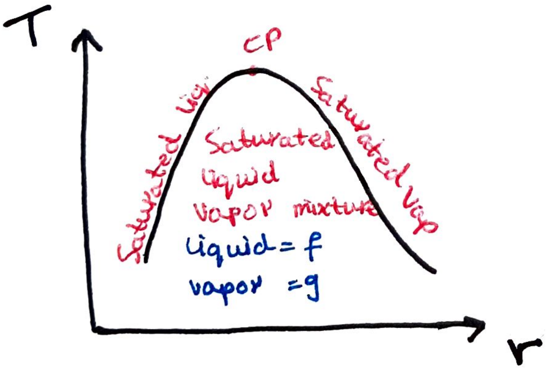
★ SATURATION TEMP:

Temp at which vaporization occurs at a given pressure.

★ SATURATION PRESSURE:

Pressure at which vaporization occurs at a given temperature.

★ Temp & Pressure are linked inside the dome★



★ QUALITY: X

In the dome region, SAT L+V

$$\boxed{X = \frac{m_g}{m_{ToT}} = \frac{m_g}{m_f + m_g}}$$

If $X = 0$ (saturated liquid)

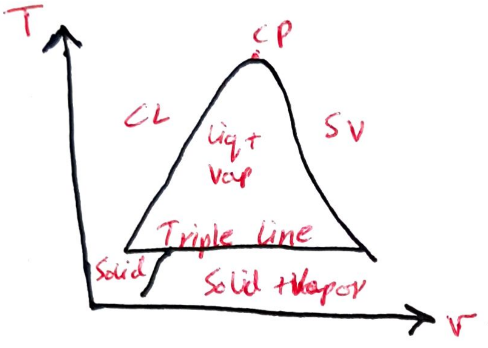
$X = 1$ or 100% (saturated vapor)

Then :

$$V = V_f + V_g = v_f m_f + v_g m_g \qquad \boxed{v_1 = v_f + x v_{fg}}$$

$$v = \frac{V}{m_T} = v_f \frac{m_f}{m_T} + v_g \frac{m_g}{m_T} = v_f (1-x) + v_g x$$

* $\boxed{v_{fg} = v_g - v_f}$ If $v_{fg} = 0$, critical point is reached



For Water

$T_{3L} = 0.01°C$

$P_{3L} = 0.61$ KPa

* STATE POSTULATE:

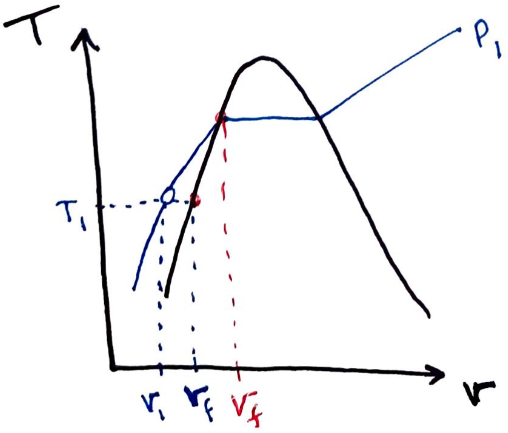
For a Pure Substance (simple compress), The state is defined by two independent, intensive properties.

independent of mass

* Note: Inside the dome, pressure and temp are not independent.

* NOTE: Regarding compressed-liquid table.

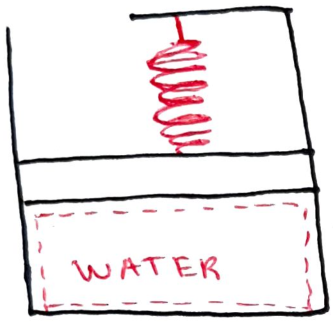
- Table has limited data
- Recall that liquid specific volume is much more sensitive to T then to P.



- same T, ignore P ✓ V_f
- same P, ignore T ✗ V_f

Example:

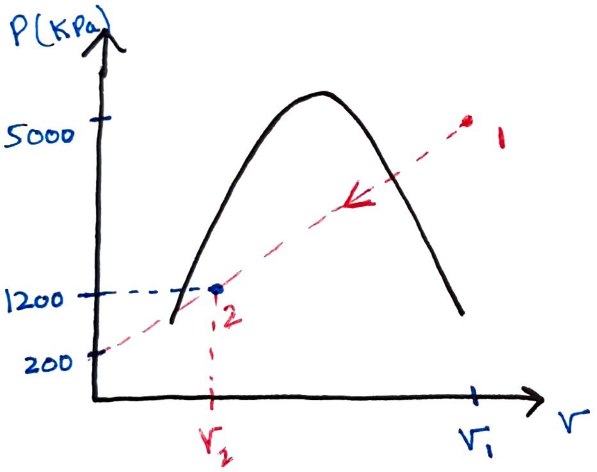
Q. A piston/cylinder arrangement is loaded with a linear spring and the outside atmosphere. It contains water at 5 MPa, 400°C with the volume being $0.1 m^3$. If the piston is at the bottom, the spring exerts a force such that $P_{lift} = 200$ kPa. The system now cools until the pressure reaches 1200 kPa. Find the mass of water, the final state (T_2, v_2) and plot the P-v diagram for process.



$P_1 = 5$ MPa
$T_1 = 400°C$
$V_1 = 0.1 m^3$

$P_{lift} = 200$ kPa
with piston @ bottom

Process: Cooling to $P_2 = 1200$ kPa



$P = c_1 + c_2 v_2$

$1200 = 200 + 83030.6\, v_2$

$\frac{1000}{83030.6} = v_2$

$\boxed{0.0120 m^3/kg = v_2}$

$\boxed{188°C = T_2}$

Solution:

$v_1 = \frac{V}{m}$

$m = \frac{0.1}{0.05781}$

$\boxed{m = 1.729 \text{ kg}}$

Slope : $\frac{5000 - 200}{0.05781 - 0}$

$c_2 = 83030.6$

★ STATE EQUATIONS:

Given Prop 1 & Prop. 2 which are intensive and independent.

State Postulate:

★ $Prop_3 = f(Prop\,1, Prop\,2)$

This could then be expressed as tabulated data or as an equation.

$$P = \frac{RT}{v-b} - \frac{a}{v^2 - cbv + db^2}$$

parameters = a, b, c, d

★ IDEAL GAS BEHAVIOR:

Density low enough that intermolecular distance is large.

$$\boxed{PV = n\bar{R}T}$$

$R = 8.314 \frac{KJ}{Kmol\,K}$ | universal gas constant

$$n = \frac{m}{M}$$

$$PV = m\frac{\bar{R}}{M}T$$

$$\boxed{Pv = RT}$$

$R = \frac{\bar{R}}{M}$ = Gas constant (different for every gas)

$$P = \rho RT$$

$$\rho = \frac{P}{RT}$$

for ρ to be low $\begin{cases} T \text{ must be high} \\ \text{or} \\ P \text{ must be low} \end{cases}$

How high or ~~high to~~ how low?

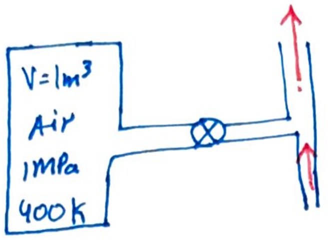
↳ Compare to critical values.

Example:

A $1 m^3$ rigid tank with air at 1 MPa, 400 K is connected to an air line. The valve is opened and air flows into the tank until the pressure reaches 5 MPa, at which point the valve is closed and the temp inside is 450 K.

a)- What is mass of air in tank before and after process?

b)- The tank eventually cools to room temp, 300 K. What is the pressure inside the tank then?



a)- For N_2 $P_c = 3.4 MPa$ $T_c = 126 K$ ✓ ok for ideal gas

$$PV = mRT$$

$$m_1 = \frac{P_1 V}{R T_1} = \frac{(1000 \text{ kPa})(1 m^3)}{\left(0.287 \frac{kJ}{kg\,K}\right)(400 K)} \boxed{= 8.71 \text{ kg}}$$

~~b)~~ ~~P~~ $$m_2 = \frac{P_2 V}{R T_2} = \frac{(5000 \text{ kPa})(1 m^3)}{\left(0.287 \frac{kJ}{kg\,K}\right)(450 K)} \boxed{= 38.71 \text{ kg}}$$

b)- $$Pv = RT$$

$$P = \frac{RT}{v}$$

$$P = \frac{\left(0.287 \frac{kJ}{kg\,K}\right)(300 K)}{(1 m^3)}$$

$$\boxed{P = 86.1 \text{ kPa}}$$

If Conditions for using Ideal Gas law are not satisfied we use:

★ Compressibility Factor:

$$\boxed{z = \frac{Pv}{RT}}$$

If you have an ideal gas $z = 1$.

Reduced Temp + Pressure

$$\boxed{P_R = \frac{P}{P_c}} \qquad \boxed{T_r = \frac{T}{T_c}}$$

Example:

$V = 5 m^3$

$T = -30°C$

$P = 3 MPa$

Find m.

$P_c = 150.8 K$

$T_c = 4.87 MPa$

$P_r = \frac{3}{4.87} = \underline{0.62}$ $\qquad T_r = \frac{-30+273}{151} = \underline{1.61}$

Using compressibility chart.

$z = 0.96$ error is 4%.

$PV = zRTm$

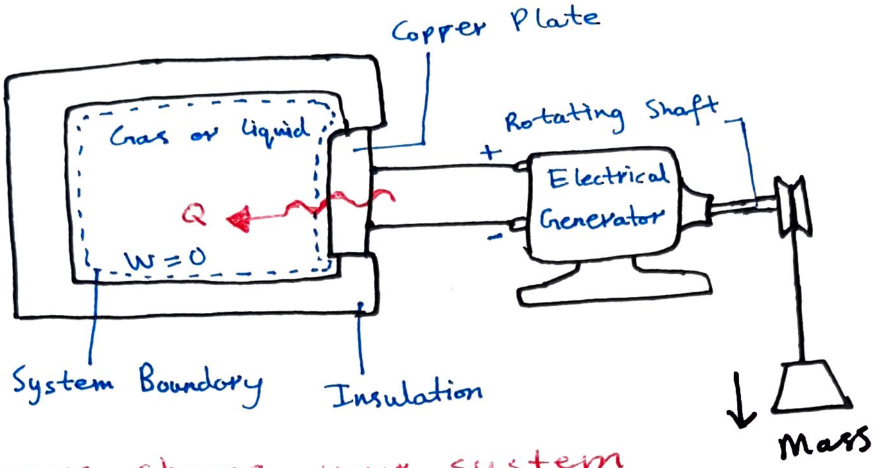
$$m = \frac{PV}{\cancel{z}RT}$$

$$m = \frac{(3000\,kPa)(5m^3)}{(0.96)\left(0.208\frac{kJ}{kg\,K}\right)(243K)}$$

$$\boxed{m = 309\ kg}$$

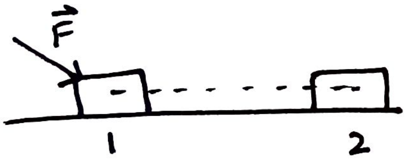
* First Law of Thermodynamics

Statement of Conservation of Energy



Always choose your system first as your equations will depend on the choice you make.

WORK + HEAT



$$W = \int_1^2 \overline{F}\cdot\overline{dx}$$

Work is a scalar with units of energy.

$[W] = [F][x] = N\cdot m = J$

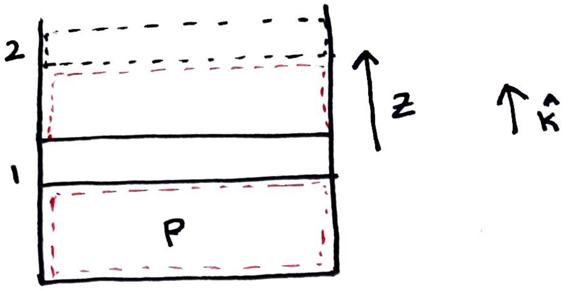
HEAT : Q also has units of energy.

WORK + HEAT may cross the System Boundary.

They are *not* thermodynamic properties.

$_1W_2$ depends on the path taken.

$_1Q_2$ depends on the path taken.

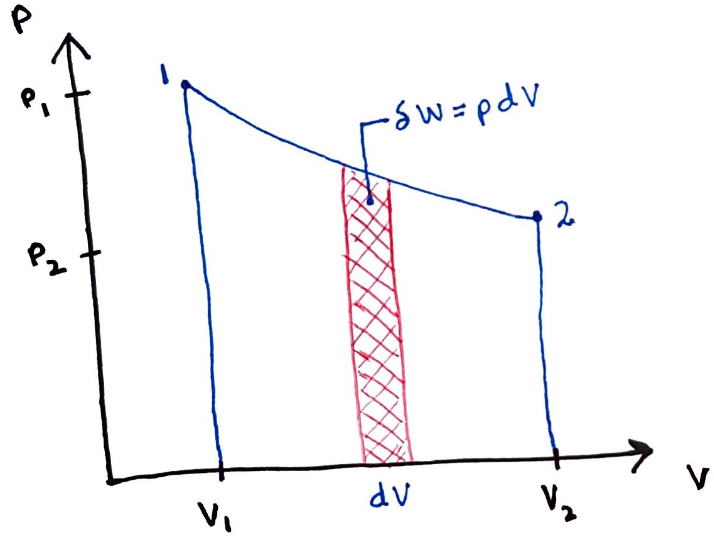


$\vec{F} = PA\,\hat{k}$, $\bar{d}z = dz\,\hat{k}$

$${}_1W_2 = \int_1^2 \vec{F}\cdot\bar{d}z = \int_1^2 P\,\underbrace{A\,dz}_{dV}$$

$$\boxed{{}_1W_2 = \int_1^2 P\,dV}$$

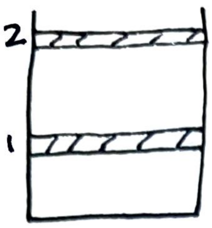
Area under P-V diagram = Work



Area $= \int_1^2 pdV$

$$\boxed{\text{WORK + HEAT are Path Dependent}}$$

- Thermodynamic Props are point functions.
- Work and Heat are not.



Volume Change

$$V_2 - V_1 = \Delta V = \int_1^2 dV$$

$$ {}_1W_2 = \int_1^2 \delta W \neq W_2 - W_1$$

$$ {}_1Q_2 = \int_1^2 \delta Q \neq Q_2 - Q_1$$

- δW + δQ are Inexact differentials.
- dV is an Exact differential.

$$\boxed{{}_1W_2 = \int_1^2 P\,dV}$$

- If V = const

$${}_1W_2 = 0$$

i). P = Constant

$${}_1W_2 = P\int_1^2 dV = P(V_2 - V_1)$$

ii). $P \neq$ Constant

Need to know how p varies with V.

★ One such case is the Polytropic process.

PV^n = constant $\qquad$ n = constant

$$P = \frac{\text{const}}{V^n}$$

$$ {}_1W_2 = \int_1^2 \frac{\text{const}}{V^n}\,dV = \int_1^2 \frac{c}{V^n}\,dV$$

$$ {}_1W_2 = \text{const}\int_1^2 \frac{dV}{V^n}$$

a)- If $n=1$

$${}_1W_2 = const \ln \frac{V_2}{V_1}$$

$const = P_1V_1 = P_2V_2$

$$\boxed{{}_1W_2 = P_1V_1 \ln \frac{V_2}{V_1} = P_2V_2 \ln \frac{V_2}{V_1}} \quad n=1$$

b). $n \neq 1$

$${}_1W_2 = const. \frac{V^{-n+1}}{-n+1}\Bigg|_1^2 = \frac{const.}{1-n}\left(V_2^{1-n} - V_1^{1-n}\right)$$

$const = P_1V_1^n = P_2V_2^n$

$$\boxed{{}_1W_2 = \frac{P_2V_2 - P_1V_1}{1-n}} \quad n \neq 1$$

★ Example:

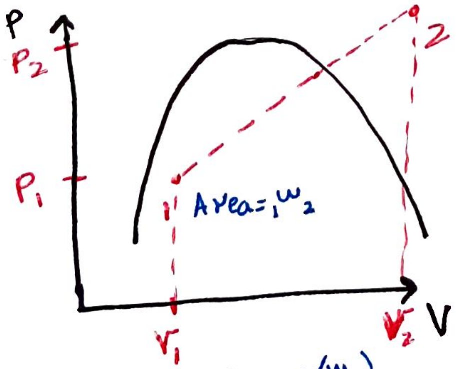
Ammonia (NH_3)

$m = 0.5$ kg

$T_1 = -20°C$

$x_1 = 25\%$

process: Heating to $T_2 = 20°C$
$V_2 = 1.41 V_1$



with $T_1, x_1 \Rightarrow$ Using tables

$P_1 = P_{SAT} = 190.2$ kPa

$v_1 = v_f + x\, v_{fg} = 0.001504 + 0.25 \times 0.62184$

$v_1 = 0.157$ m³/kg

${}_1W_2 = m({}_1w_2)$

$${}_1W_2 = \frac{1}{2}(P_1 + P_2)(v_2 - v_1)\, m$$

STATE 2

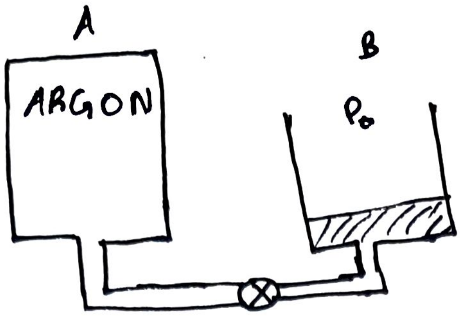
$T_2 = 20°C$

$v_2 = 1.41 v_1 = 0.221$ m³/kg

@ 20°C, $v_g = 0.149$ so superheated

@ 20°C, $v = 0.221 \Rightarrow P = 600$ kPa

Example:



Valve is opened,
Argon flows A → B until
$P_2 = 150$ kPa , $T_2 = 30°C$ everywhere.

Q. Find work done by Argon.

TANK A:

V = 400 L
250 kPa
30°C

Cylinder B:

$P_c = 150$ kPa to lift
Empty

Solution:

① SYSTEM: Argon

② Assume ideal gas since $P \ll P_c$ (4.9 MPa)

$PV = mRT$

$R = 0.208 \frac{kJ}{kgK}$

$m = \frac{PV}{RT}$

$m = 1586.7$ kg

At final state m is the same.

③ $P_2 V_2 = mRT_2$

$V_2 = \frac{PV}{\cancel{RT}} \times \frac{\cancel{R}\cancel{T_2}}{P_2}$

$T = T_2 = 30°C$

$V_2 = \frac{250 \times 400}{150}$

$V_2 = 666.7$ L

④ V_B = volume in cylinder

$V_B = V_2 - V$
$= 666.7 - 400$
$= 266.7$ L

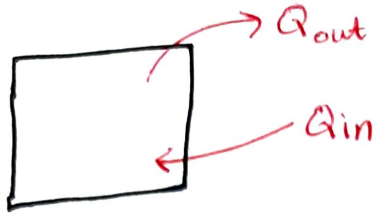
⑤ $_1W_2 = \int_1^2 P\,dV$

P = constant

$W = P_2 \Delta V$
$W = P_2 (V_2 - V_1)$
$W = P_2 (V_B - 0)$
$W = 150 (266.7)$
$W = 40000$ J

$\boxed{W = 40 \text{ kJ}}$

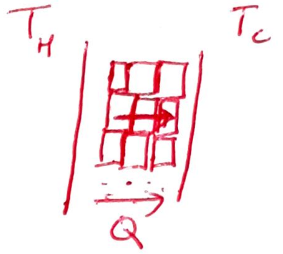
HEAT TRANSFER



★ Modes of Heat Transfer:

1. Conduction
2. Convection
3. Radiation

★ Conduction Heat Transfer:



Molecular (electron) activity

Lattice vibration (solids)

★ Fourier's Law:

$$q'' = -K\frac{\partial T}{\partial X} \simeq K\frac{T_H - T_c}{L}$$

q = Heat Transfer (Heat/unit area/unit time) $\left[\frac{\text{watts}}{m^2}\right]$

K = Thermal conductivity $\frac{\text{Watts}}{M \cdot K} = q$

L = length of material

★ Convection:

Forced Convection

BULK FLUID MOTION

Natural or

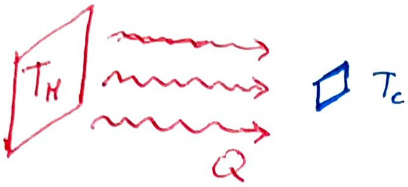
Free Convection

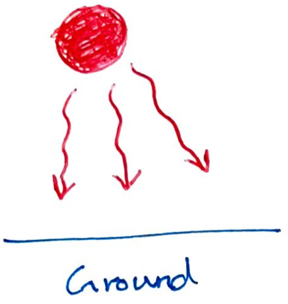
$$q'' = h\,(T_H - T_c) \quad \left[\frac{\text{watts}}{m^2}\right]$$

h = heat transfer coefficient

★Radiation:

Electromagnetic Waves





$q'' = 10^3 \frac{W}{m^2}$

$q'' = \epsilon A_{12} \sigma (T_H^4 - T_c^4) \quad [\frac{W}{m^2}]$

σ = Stefon-Boltzman constant

ϵ = emissivity

A_{12} = view factor

${}_1W_2$ work during process 1-2

${}_1Q_2$ heat transfer during process 1-2

Rates:

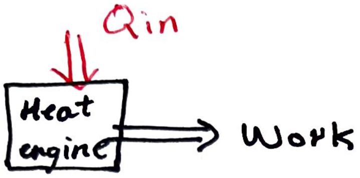
$\dot{W}$ = Power = $\frac{\text{Work done}}{\text{unit time}}$ $\frac{J}{s}$ = Watts

$\dot{Q}$ = Heat transfer = Watts

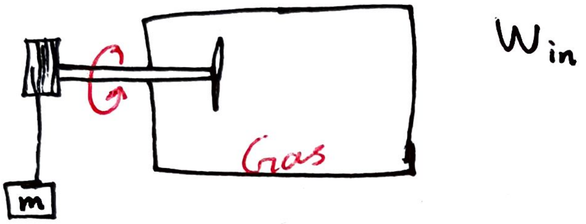
★ Sign Convention :

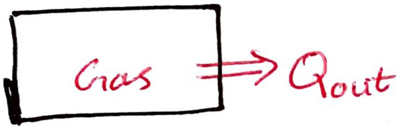
$_1W_2$ is (+) when done by system

$_1Q_2$ is (+) when recieved by system.



★ 1st Law Of Thermodynamics :





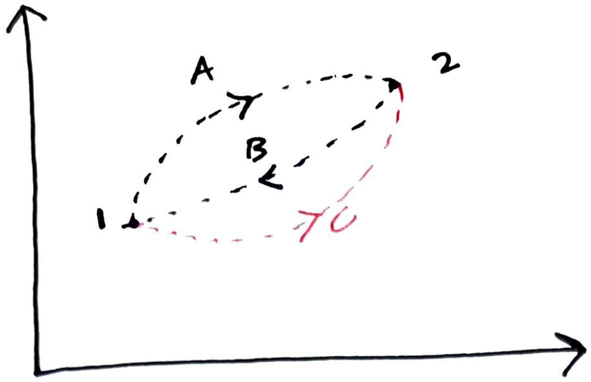
Initial and Final states are the same

$W_{in} = Q_{out}$ (cycle)

In general:

$\oint \delta W = \oint \delta Q$

$\oint$ cyclic integral



$$\oint \delta Q = \oint \delta W$$

$$\int_1^2 \delta Q_A + \int_2^1 \delta Q_B = \int_1^2 \delta W_A + \int_2^1 \delta W_B$$

$$\int_1^2 \delta Q_C + \int_2^1 \delta Q_B = \int_1^2 \delta W_C + \int_2^1 \delta W_B$$

Subtract:

$$\int_1^2 \delta Q_A - \int_1^2 \delta Q_C = \int_1^2 \delta W_A - \int_1^2 \delta W_C$$

$$\int_1^2 (\delta Q - \delta W)_A = \int_1^2 (\delta Q - \delta W)_C$$ → Path independent.

$\int_1^2 (\delta Q - \delta W)$ is independent of the path.

so it must be a property.

That Property is the Energy of the system.

$$\int_1^2 (\delta Q - \delta W) = \Delta E = E_2 - E_1$$

or

$$\boxed{\delta Q - \delta W = dE}$$

first law of thermodynamics!

OR

$$\boxed{{}_1Q_2 - {}_1W_2 = \Delta E}$$

Energy E = Sum of All Energies

Also called Total Energy.

It's an extensive property.

$E = U + KE + PE + \text{other}$

U = Internal Energy (molecular)

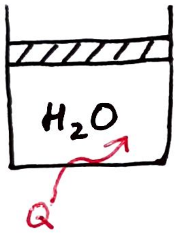
For a mixture of liquid + vapor

$U = U_f + U_g = u_f m_f + u_g m_g$

$mu = m_f V_f + m_g u_g$

$$\boxed{\begin{array}{l} u = (1-x)u_f + x u_g \\ u = u_f + x u_{fg} \end{array}}$$

Example:



$V_1 = 0.1\,m^3$

$m = 0.5\,kg$

$P_1 = 0.4\,MPa$

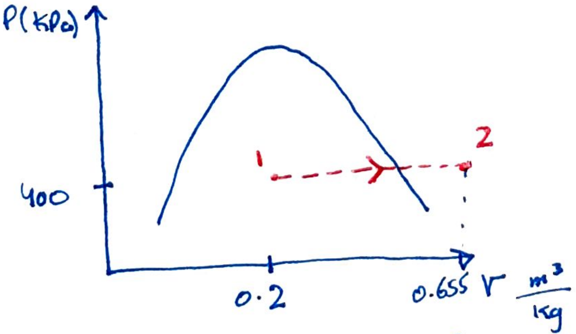
Piston is unrestrained (frictionless)

Heat until $T = 300°C$

Find ${}_1W_2$ & ${}_1Q_2$.

Solution:

As heat is added, piston moves up so that P = Constant.



$v_1 = \frac{V_1}{m} = \frac{0.1}{0.5} = .2\ \frac{m^3}{kg}$

$\left.\begin{array}{l} T_2 = 300°C \\ P_2 = 400\,KPa \end{array}\right] v_2 = 0.655\ m^3/kg$

(from super-heated vapor table)

$${}_1W_2 = P(V_2 - V_1)$$
$$= 400\,(0.655 - 0.2)\,m$$
$$= 91\ KJ$$

$\Delta E = E_2 - E_1$

$\Delta U = {}_1Q_2 - {}_1W_2 \quad (\overset{0}{\cancel{KE}}, \overset{0}{\cancel{PE}})$

$${}_1Q_2 = U_2 - U_1 + {}_1W_2$$
$$= m(u_2 - u_1) + {}_1W_2$$

$u_2 = 2805\ KJ/kg$ (superheat vapor table)

$u_1 = \underline{u_f} + x\,\underline{u_{fg}}$ (saturated table)

To find x:

$v = v_f(1-x) + v_g x$

★ ENTHALPY : $H = U + pV$

specific enthalpy: $h = u + pv$

For a liquid-vapor mixture

$$\boxed{h = (1-x)h_f + h_g \cancel{x} = h_f + x h_{fg}}$$ x = quality

⇒ Resolving ~~method~~ problem using enthalpy:

$$\Delta H = H_2 - H_1$$

$$\Delta H = U_2 + P_2V_2 - U_1 - P_1V_1 \qquad P_2 = P_1$$

$$= (U_2 - U_1) + P[V_2 - V_1]$$

$${}_1Q_2 = (U_2 - U_1) + {}_1W_2$$

$$\boxed{{}_1Q_2 = \Delta H} \quad \boxed{\text{For constant pressure process.}}$$

★ Specific Heats

Question: How much heat is needed to raise the temperature of 1 unit mass by 1 degree?

Answer: It depends on the process.

⇒ At Constant Volume:

$$C_v = \frac{1}{m}\left(\frac{\delta Q}{dT}\right)_v$$

1st law:

$$dU = \delta Q - \cancel{\delta W}^{\,0}$$ (no work done)

$$C_v = \frac{1}{m}\left(\frac{dU}{dT}\right)_v$$

$$\boxed{C_v = \left(\frac{\partial u}{\partial T}\right)_v}$$

⇒ At Constant ~~Temp~~ Pressure:

$$C_p = \frac{1}{m}\left(\frac{\delta Q}{dT}\right)_p$$

1st law:

$$dU = \delta Q - p\,dV$$

$$\delta Q = dH$$

Specific Heat will be larger.

$$C_p = \frac{1}{m}\left(\frac{dH}{dT}\right)_p$$

$$\boxed{C_p = \left(\frac{\partial h}{\partial T}\right)_p}$$

★ Solids & liquids :

$h = u + pv$

$dh = du + pdv + vdp$

$dv \approx 0$ (incompressible)

$vdp << du$ (v is small)

$dh \approx du$

$C_p = C_v = C$

★ Ideal Gases:

In general $u(T,p)$ or $u(T,v)$

but for ideal gas $u(T)$ only.

Change in Internal Energy.

$$C_v = \left(\frac{\partial u}{\partial T}\right)_v = \frac{du}{dT}$$

$du = C_v dT$ and $dU = m C_v dT$

Also

$h = u + pv = u + RT$

Then $h = h(T)$ only

Change in Enthalpy

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p = \frac{dh}{dT}$$

$dH = C_p dT$ and $dH = m C_p dT$

Relationship b/w Cp and Cv for Ideal Gases:

$h \neq u + pv = u + RT$

$dh = du + RdT$

$$\frac{dh}{dT} = \frac{du}{dT} + R$$

$$\boxed{C_p = C_v + R}$$

For Ideal Gases:

$C_v = \frac{du}{dT}$ and $C_p = \frac{dh}{dT}$

Thus:

$du = C_v dT$ and $dh = C_p dT$

Integrating:

$\Delta u = u_2 - u_1 = \int_1^2 C_v dT$

$\Delta h = h_2 - h_1 = \int_1^2 C_p dT$

Can we assume C_v and C_p are constant?

1)- Constant for mono atomic gases, Ar, He, Ne.

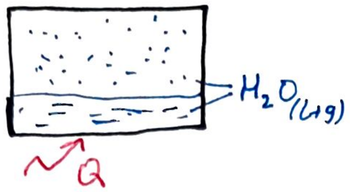
2)- If not, can assume constant if temp range is not too large.

Examples

$$\left.\begin{aligned} V &= 5\,m^3 \\ P &= 0.1\,MPa \\ V_f &= 0.05\,m^3 \\ V_g &= 4.95\,m^3 \end{aligned}\right\} ①$$

② no liquid left, only sat. vapor.

Find ${}_1Q_2$..



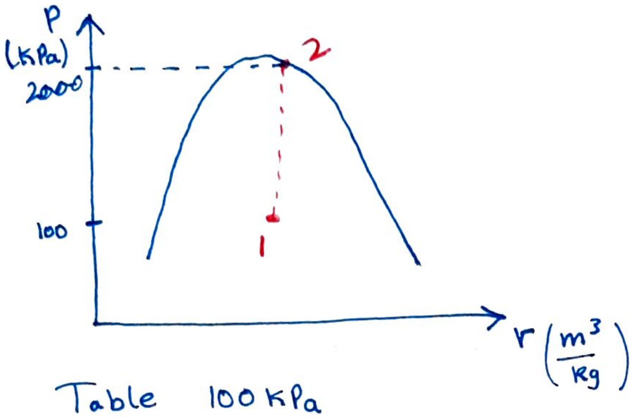


Table 100 KPa

$v_f = 0.001043\ m^3/kg$

$v_g = 1.694\ m^3/kg$

$$\left.\begin{aligned} m_f &= \frac{V_f}{v_f} = \frac{0.05}{0.001043} = 48\,kg \\ m_v &= \frac{V_g}{v_g} = \frac{4.95}{1.694} = 3\,kg \end{aligned}\right\rangle \quad x = \frac{3}{51}$$

$x = 0.0589$

State 2:

only sat. vapor ($x=1$)

$$v_1 = \frac{5\,m^3}{51\,kg}$$

$v_1 \approx 0.1\,m^3/kg$

$v_2 \approx 0.1\,m^3/kg$

from table $\underline{P_2 \approx 2000\,KPa}$

1^{st} law:

$\Delta U = {}_1Q_2 - \cancel{{}_1W_2}$ (rigid tank)

${}_1Q_2 = m\,(u_2 - u_1)$

$u_2 = 2600\ KJ/kg$

$u_1 = u_f + x\,u_{fg}$

u_f & u_{fg} from table.

$u_1 = 1785.47 + 0.0589\,(507.58)$

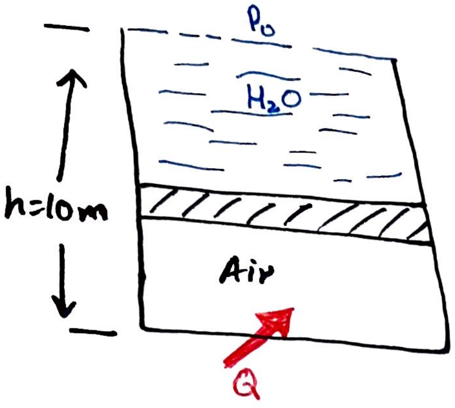
$u_1 = 1815\ KJ/kg$

${}_1Q_2 = 51\,(2600 - 1815)$

${}_1Q_2 \approx 40\,000\,KJ$

~~${}_1Q_2 \approx 40\,KJ$~~

$$\boxed{{}_1Q_2 \approx 40\,MJ}$$



$A = 0.1 m^2$

Piston is massless

Air = 300K

$0.3 m^3$

Air is heated, water spills out

Q. Find Q when all water is gone.

H_2O is compressed liquid. @ 0.1MPa, 300 K

Air : T ✓ V ✓

$$P = P_o + \frac{\rho V_{H_2O} g}{A}$$

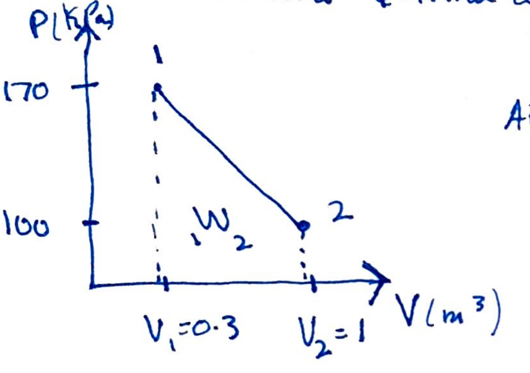
$$V_{H_2O} = V_T - V_{air}$$

$$= Ah - 0.3$$

$$= 0.1(10) - 0.3$$

$$= 0.7 m^3$$

P = 170 KPa (initial air pressure)



Air

$$P = P_o + \frac{\rho(Ah - V)g}{A}$$

$$\boxed{P = P_o + \frac{\rho A h g}{A} - \frac{\rho g}{A} V}$$

m = slope

$$\boxed{P = c_1 + c_2 V}$$

$${}_1W_2 = \frac{1}{2}(170+10)(0.7)$$

$${}_1W_2 = 95 \text{ KJ}$$

$$\Delta U = {}_1Q_2 - {}_1W_2$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

Ideal Gas law for Air:

$$PV = mRT$$

$$m = \frac{PV}{RT}$$

$$m = 0.59 \text{ kg}$$

Ideal Gas Table for Air:

$u_1 = 214.4$ KJ/kg $\Rightarrow T_1 = 300$ K

$$T_2 = \frac{P_2 V_2}{mR}$$

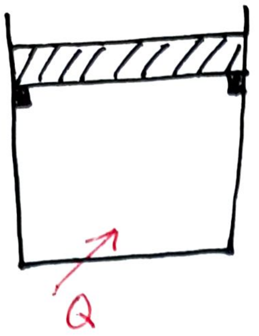
$$= \frac{100(1)}{0.59R} \approx 600 \text{ K}$$

$u_2 = 435.1$ KJ/kg $\Rightarrow T_2 =$ ~~435~~ 600K

$${}_1Q_2 = m(435.1 - 214.4) + 95$$

$$\boxed{{}_1Q_2 = 224 \text{ KJ}}$$

1 kg H_2O



① 100°C

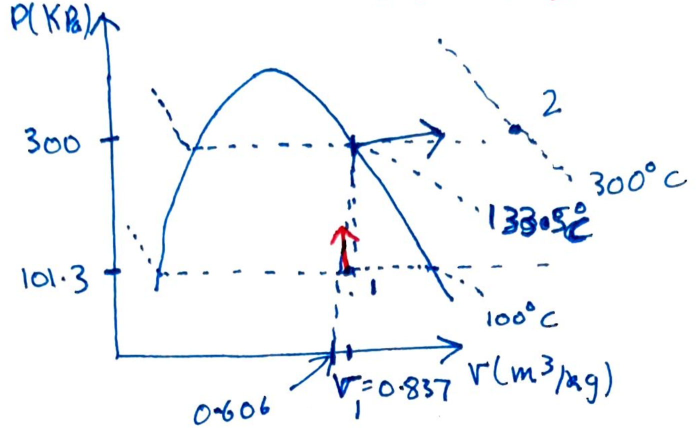
$x = 0.5$

Need: 300 KPa to lift piston.

Heat to 300°C.

a)- At what T is all liquid gone?

b)- Find $P_2, v_2, {}_1W_2, {}_1Q_2$.



$@100°C\ v_1 = v_f + x v_{fg} = 0.837\ m^3/kg$

a)- find v_g for $v = 0.837$ from table.

Ans. $\boxed{122°C}$

b)- $\boxed{P_2 = 300\ KPa}$

v_2 = superheated table @ 300 KPa, 300°C

$\boxed{v_2 = 0.875\ m^3/kg}$

$${}_1W_2 = P_2 (\Delta V)\, m$$

$$= 300(0.875 - 0.837)(1)$$

$$\boxed{= 11.4\ KJ}$$

$${}_1Q_2 = \Delta U + {}_1W_2$$

$$= m(u_2 - u_1) + 11.4$$

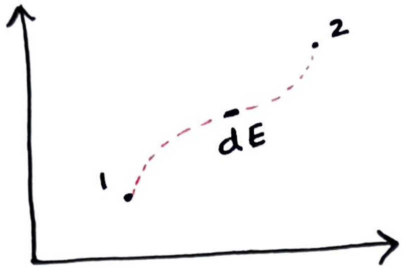
$$u_1 = u_f + x u_{fg}$$

$$= 561 + 0.5(1982) = 1552\ KJ/kg$$

$$u_2 = 2806\ KJ/kg$$

$${}_1Q_2 = 1(2806 - 1552) + 11.4$$

$$\boxed{{}_1Q_2 = 1265.4\ KJ}$$



$\Delta E = E_2 - E_1$

1st Law:

$$\boxed{\Delta E = {}_1Q_2 - {}_1W_2}$$

or

$$\boxed{dE = \delta Q - \delta W}$$

for an infinitesimal portion of the process during time dt

Divide by dt:

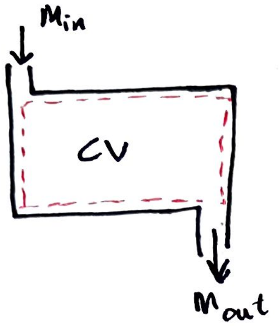
$$\frac{dE}{dt} = \frac{\delta Q}{dt} - \frac{\delta W}{dt}$$

$$\boxed{\frac{dE}{dt} = \dot{Q} - \dot{W}}$$

Rate form of 1st Law

1st Law For A Control Volume

Must keep track of mass



Recall:

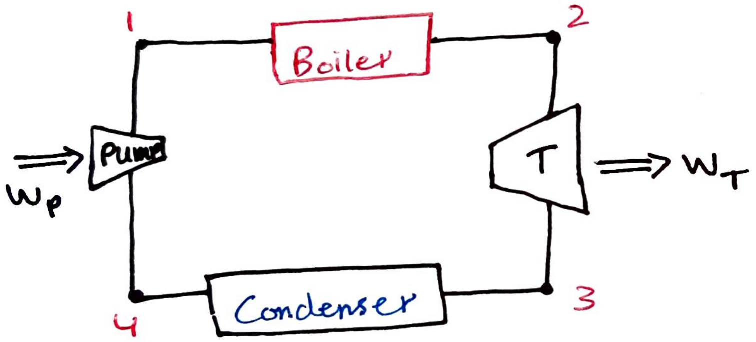
Control volume = Open System

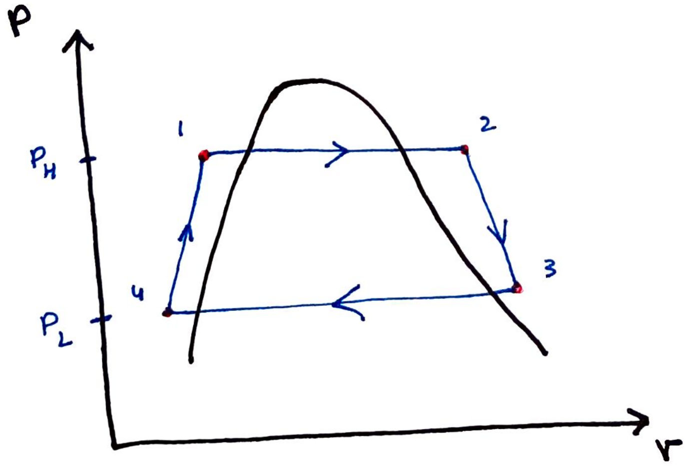
For the CV

$$\boxed{M_2 - M_1 = \Delta M_{cv} = M_{in} - M_{out}}$$

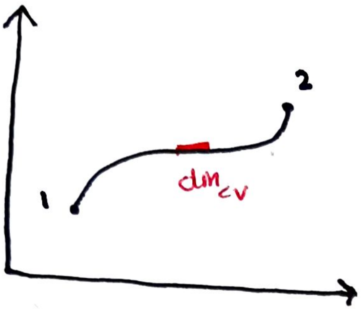
Conservation of Mass

★ RANKINE CYCLE:





★ Rate of Mass

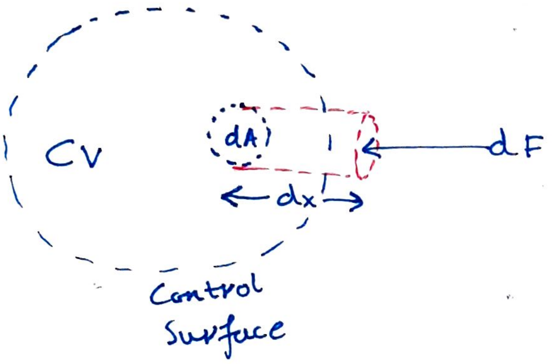


$$\frac{dM_{cv}}{dt} = \frac{dM_{in}}{dt} - \frac{dM_{out}}{dt}$$

$$\boxed{\frac{dM_{cv}}{dt} = \dot{M}_{in} - \dot{M}_{out}}$$

Rate form
kg/s

★ Mass Flow Rate :



$dV = dA\,dx$: differential volume enterin CV.

Volume Flow Rate: $\frac{dV}{dt} = dA\frac{dx}{dt} = dA\,V_n$ V_n = velocity normal to dA

Mass Flow Rate : $d\dot{m} = \rho\frac{dV}{dt} = \frac{1}{v}\frac{dV}{dt}$

$$\boxed{d\dot{m} = \rho\,dA\,V_n = \frac{1}{v}\,dA\,V_n}$$

★ Flow Work:

Work done by dF to push volume through CS (control surface).

$$\delta W = dF\,dx = P\,dA\,dx = p\,dV$$

Rate of flow Work:

$$\delta \dot{W}_{fw} = \frac{\delta W}{dt} = p\frac{dV}{dt} = p\,v\,d\dot{m}$$

$$\boxed{\delta \dot{W}_{FW} = p\,v\,d\dot{m}}$$ Flow Work

★ 1st Law:

For a Closed System (CM)

$$\boxed{\left(\frac{dE}{dt}\right)_{CM} = \dot{Q} - \dot{W}}$$ Rate Form

Needs to be modified for an open system.

⇒ Mass crossing the CS (system bound) carries energy with it.

$$\dot{E}_{cs} = \dot{m}_{cs}\,e_{cs} = \dot{m}_{cs}(u_{cs} + KE_{cs} + PE_{cs})$$

It helps to account separately for incoming (i) and exiting (e) mass.

Thus, for a CV:

$$\left(\frac{dE}{dt}\right)_{CV} = \dot{Q} - \dot{W} + \underbrace{\dot{m}_i e_i - \dot{m}_e e_e + \overbrace{\dot{m}_i p_i v_i - \dot{m}_e p_e v_e}^{\text{flow Work}}}_{\text{new terms}}$$

Using enthalpy to combine terms

$e_i = u_i + ke_i + pe_i + e_{other\,i}$ thus:

$$\dot{m}_i(e_i + p_i v_i) = \dot{m}_i(\underline{u_i} + ke_i + pe_i + e_{other\,i} + \underline{p_i v_i})$$

$h_i = u_i + p_i v_i$ enthalpy of incoming mass.

$$m_i(h_i + \underbrace{ke_i + pe_i + e_{other\,i}}_{\text{mostly neglected}})$$

1^{st} Law for CV:

$$\left(\frac{dE}{dt}\right)_{cv} = \dot{Q} - \dot{W} + \dot{m}_i\left(h_i + ke_i + pe_i + e_{other\,i}\right)$$
$$- \dot{m}_e\left(h_e + pe_e + e\ other\ i\right)$$

$$\boxed{\left(\frac{dU}{dt}\right)_{cv} = \dot{Q} - \dot{W} + \dot{m}_i h_i - \dot{m}_e h_e}$$

neglecting ke, pe and all other energies.

★ TYPES Of Processes For CV

1). Steady State Process

2). Transient Process

★ Steady State:

Process occurs continuously and does not change in time.

- Mass inside CV is always same amount
- Energy inside CV is always same amount.

Thus:

$$\left(\frac{dM}{dt}\right)_{cv} = 0 \qquad \left(\frac{dE}{dt}\right)_{cv} = 0$$

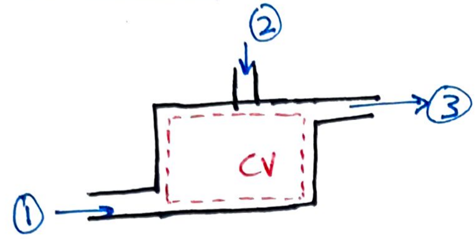
★ Steady Flow Devices:

- Heat Exchangers
- Turbines → produce power
- Pumps → increase pressure
- Compressors → increase pressure
- Nozzles → increase K.E
- Diffusers → reduce K.E
- Throttling Devices → reduce pressure

Conservation of Mass

$$\boxed{\dot{M}_i = \dot{M}_e} \Rightarrow \text{if } \left(\cancel{\frac{dM}{dt}}\right)_{cv}^{0} = \dot{m}_i - \dot{m}_e$$

Example:



- steady state
- insulated
- inlets : ①, ②
- exit : ③

① saturated liq (H_2O)
$T_1 = 90°C$
$\dot{m} = 5 kg/s$

② super heated vapor
$P_2 = 300$ kPa
$T_2 = 500°C$

③ want saturated vapor
@ $T_3 = 90°C$

Q. What is the needed mass flow rate $\dot{m}_2$ of superheated vapor?

State 1: From Table
$h_f = 377$ kJ/kg @ 90°

state: 2
$h = 3486$ kJ/kg

state 3:
$h_g = 2660$ kJ/kg @ 90°

mass conservation:

$$\frac{dM_{cv}}{dt} = \Sigma \dot{M}_i - \Sigma \dot{M}_e$$

$$0 = \dot{M}_1 + \dot{M}_2 - \dot{M}_3$$

$$\boxed{\dot{M}_3 = \dot{M}_1 + \dot{M}_2}$$

$$du = \dot{Q} - \dot{W} + \Sigma \dot{m}_i h_i - \Sigma \dot{m}_e h_e$$

$$0 = 0 - 0 + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$\dot{m}_2 = \frac{h_3 - h_1}{h_2 - h_3} \dot{m}_1 = \left(\frac{2660 - 377}{3486 - 2660}\right) 5$$

$$\dot{m}_2 = 2.76(5) = \boxed{13.8 \text{ kg/s}}$$

⋆TRANSIENT PROCESSES:

mass

$$\left(\frac{dm}{dt}\right)_{cv} = \dot{m}_i - \dot{m}_e \quad \text{Rate form}$$

$$m_2 - m_1 = m_i - m_e \quad \text{integrated form}$$

1st Law:

$$\left(\frac{dE}{dt}\right)_{cv} = \dot{Q} - \dot{W} + \dot{m}_i(h_i + ke_i + pe_i) - \dot{m}_e(h_e + ke_e + pe_e)$$

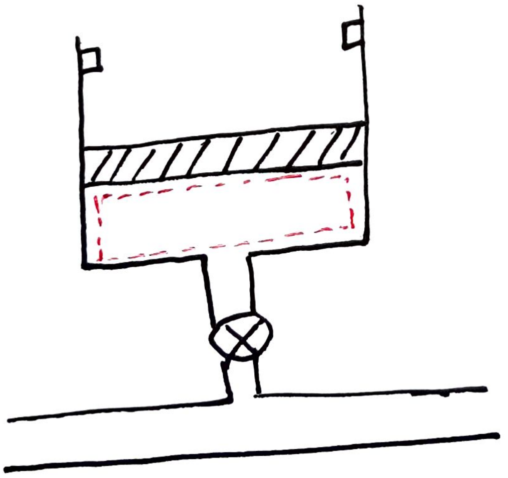
Integrating In Time:

$$E_2 - E_1 = {}_1Q_2 - {}_1W_2 + \int_1^2 \dot{m}_i(h_i + \dots)dt - \int_1^2 \dot{m}_e(h_e + \dots)dt$$

If $(h_i + \dots)$ AND $(h_e + \dots)$ vary during the process, the integrals can get complicated. However, if $(h_i + pe_i + ke_i)$ and $(h_e + ke_e + pe_e)$ do not change in time (during the process), they can be taken out of the time integrals.

$$\boxed{E_2 - E_1 = {}_1Q_2 - {}_1W_2 + m_i(h_i + \dots) - m_e(h_e + \dots)}$$

Example:



Air

P_1, V_1, T_1 Given

$P_1 = 300\text{kPa}$
$T_1 = 17°C$
$V_1 = 0.25\text{m}^3$

V_{stops} Given

$V_{stops} = 1\text{m}^3$
$P_2 = 500\text{kPa}$
$T_2 = 600\text{K}$

Air Line P_L, T_L Given

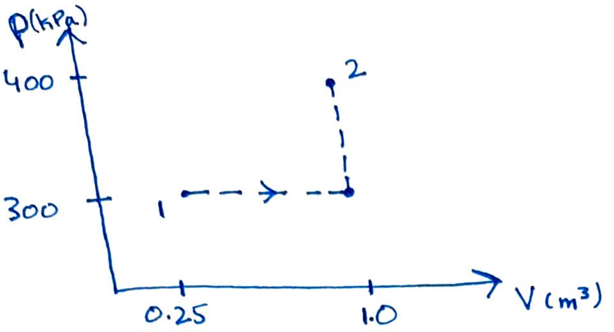
$P_L = 400\text{kPa}$
$T_L = 350\text{K}$

OPEN VALVE until P_2 and T_2 are reached.

Find m_i, ${}_1W_2$, ${}_1Q_2$.

Treat as Ideal Gas:

Is piston going to reach the stops.



$PV = mRT$

$m_1 = \frac{P_1V_1}{RT_1}$ = ✓ $\qquad m_2 = \frac{P_2V_2}{RT_2}$ = ✓

mass conservation:

$m_2 - m_1 = m_i - \cancelto{0}{m_e}$

$\overset{\checkmark}{m_2} - \overset{\checkmark}{m_1} = m_i$ ✓

$E_2 - E_1 = {}_1Q_2 - {}_1W_2 + m_i(h_i) - \cancelto{0}{m_e(h_e)}$ (1st law)

$U_2 - U_1 = {}_1Q_2 - {}_1W_2 + m_i(h_i)$

$\underset{\checkmark}{m_2}\underset{\checkmark}{u_2} - \underset{\checkmark}{m_1}\underset{\checkmark}{u_1} = {}_1Q_2 - {}_1W_2 + \underset{\checkmark}{m_i}(\underset{\checkmark}{h_i})$

hi from ideal gas table for air with T_L.

u_1 " " " " " " " T_1.

u_2 " " " " " " " T_2.

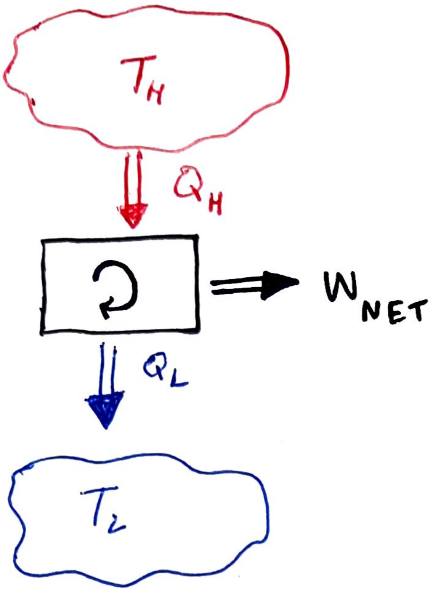
${}_1Q_2 = \underset{\checkmark}{m_2}\underset{\checkmark}{u_2} - \underset{\checkmark}{m_1}\underset{\checkmark}{u_1} + \underline{{}_1W_2}^{\checkmark} - \underset{\checkmark}{m_i}\underset{\checkmark}{h_i}$

${}_1W_2 = P(V_2 - V_1)$

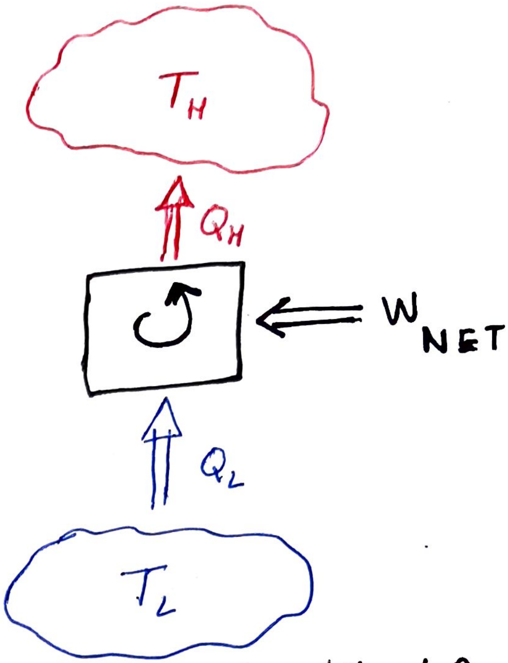
$= 300(1 - 0.25)$ ✓

We know everything now, solve to get ${}_1Q_2$ and problem is solved.

The 2nd Law Of Thermodynamics



Heat Engine



Refrigerator / Heat Pump

★ HEAT ENGINE :

- operates in a thermodynamic cycle.
- delivers a net amount of work.
- transfers heat from a high temp reservoir to a low temp reservoir

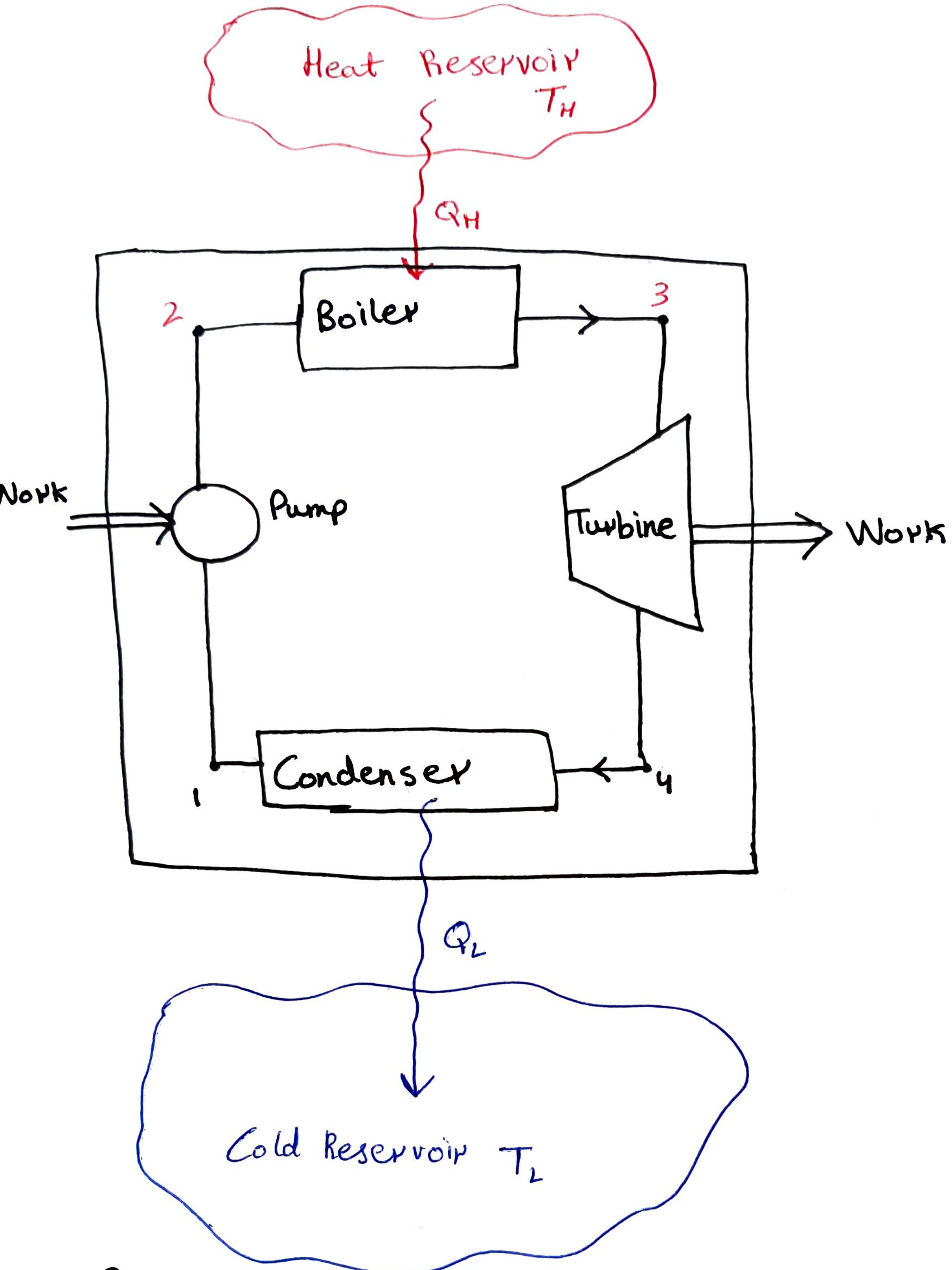
★ REFRIGERATOR OR HEAT PUMP :

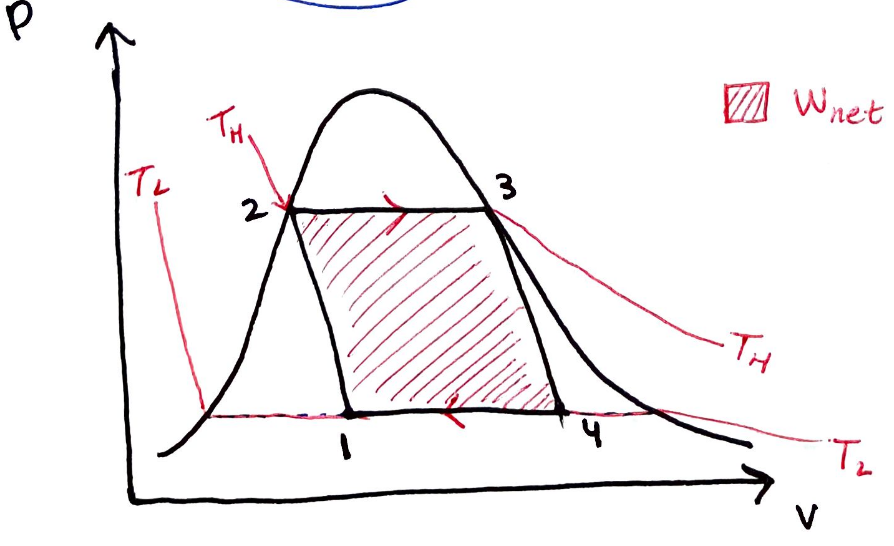
- does the opposite of a Heat Engine.

★ Thermal Reservoir :

A system large enough that it remains at constant temp. while it recieves or gives heat.

<u>Eg:</u>
- A large body of water
- The atmosphere


Heat Reservoir T_H
Q_H
2
Boiler
3
Work
Pump
Turbine
Work
Condenser
1
4
Q_L
Cold Reservoir T_L


P
W_{net}
T_H
T_L
2
3
T_H
1
4
T_L
V

★ Efficiency Of Heat Engine:

$$\boxed{\eta = \frac{W_{net}}{Q_H}} \qquad \frac{\text{want}}{\text{cost}}$$

1st Law: $Q_H - Q_L = W_{net}$ $(\Delta U = 0)$

$$\boxed{\eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}}$$

Ideal Efficiency = 1

★ ~~Efficiency Of~~ Refrigerator: (Coefficient Of Performance)

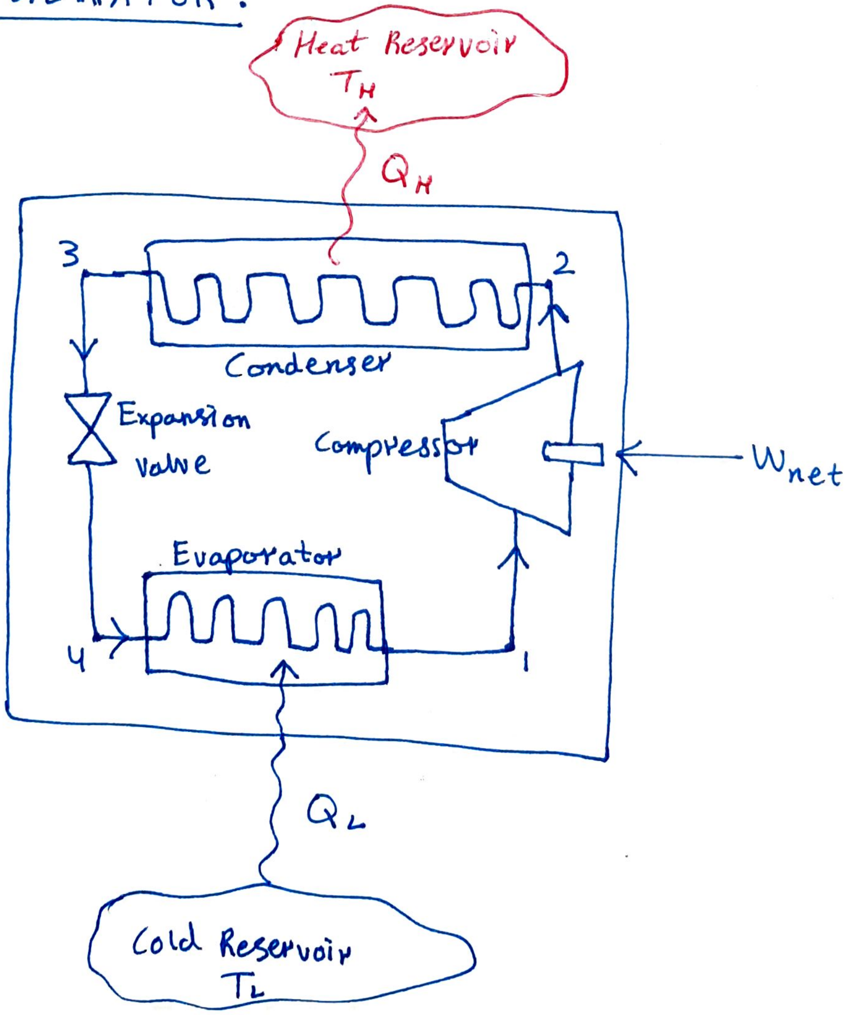
$$\boxed{COP_R = \beta_R = \frac{Q_L}{W_{net}}} \qquad \frac{\text{want}}{\text{cost}}$$

1st Law: $W_{net} = Q_H - Q_L$ $(\Delta U = 0)$

$$\boxed{\beta_R = \frac{Q_L}{Q_H - Q_L}}$$

Ideal efficiency = infinity

REFRIGERATOR:



* Coefficient Of Performance (Heat Pump)

$$COP_{HP} = \beta_{HP} = \frac{Q_H}{W_{net}}$$

$\frac{\text{want}}{\text{cost}}$

$$\boxed{\beta_{Hp} = \frac{Q_H}{Q_H - Q_L}}$$

* 2nd Law Statements:

1. Kelvin-Planck Statement (K-P)
2. Clausius Statement (C)

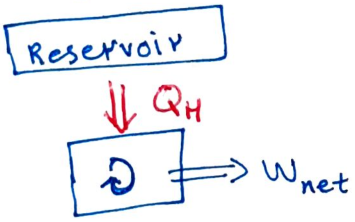
- K-P refers to Heat engine
- C statement refers to refrigerator.
- They are equivalent

* Kelvin-Planck Statement: (K-P)

Impossible to make a device which operates in a cycle and delivers a net amount of work while exchanging energy (Heat) with a single reservoir.

$W_{net} = Q_H$ (satisfies 1st Law)

but violates 2nd Law!



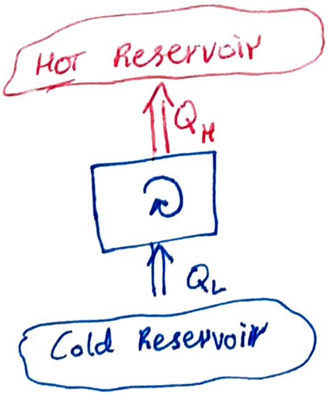
Impossible!

* Clausius Statement: (C)

Impossible to make a device which operates in a cycle and produces no effect other than the transfer of heat from a cold reservoir to a hot reservoir.

$Q_H = Q_L$ (satisfies 1st law)

but violates 2nd Law!



Impossible!

★ The K-P statement implies that

η_{HE} can not be 100%.

★ The C statement implies that

β_{REF} can not be ∞.
COP

$$\eta_{HE} = \frac{W_{net}}{Q_{in}} \qquad \beta_R = COP_R = \frac{Q_L}{W_{in}}$$

★ Perpetual Motion Machines:

- Of the first kind : violate the 1st law.
- Of the second kind : violate the 2nd law.

★ Reversible Process:

A process that can be reversed, returning both the system and the surrounding to their original states.

★ **All Real Processes are** Irreversible-

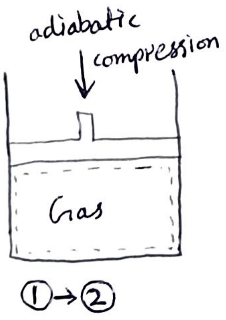
Some Reasons:

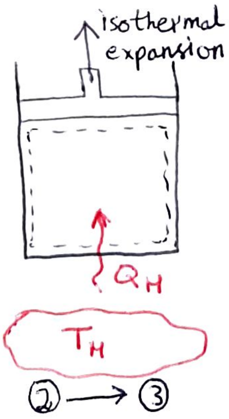
Friction, Heat transfer through a finite time diff, Finite-time process.

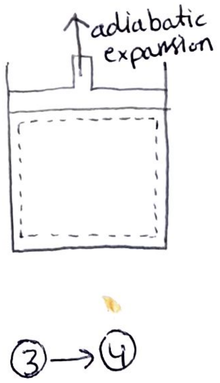
★ The Carnot Cycle:

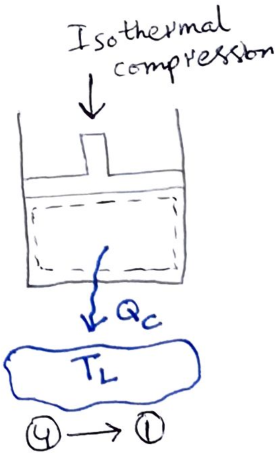
- Reversible
- Consists of 4 Processes
- 2 are adiabatic (not heat transfer) [$\delta Q = 0$]
- 2 are isothermal

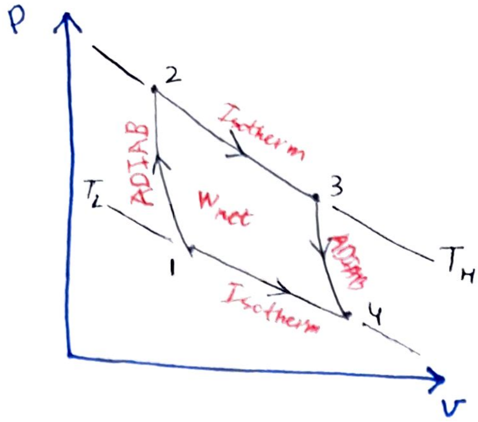
○ Between two reservoirs at T_H, T_L





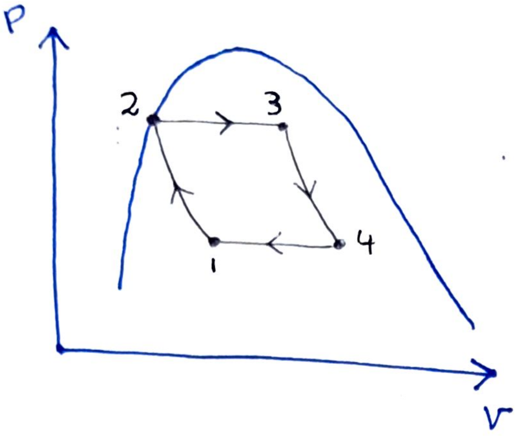






Carnot Cycle Of Ideal Gas

- We will compare this carnot cycle against any other gas cycle.



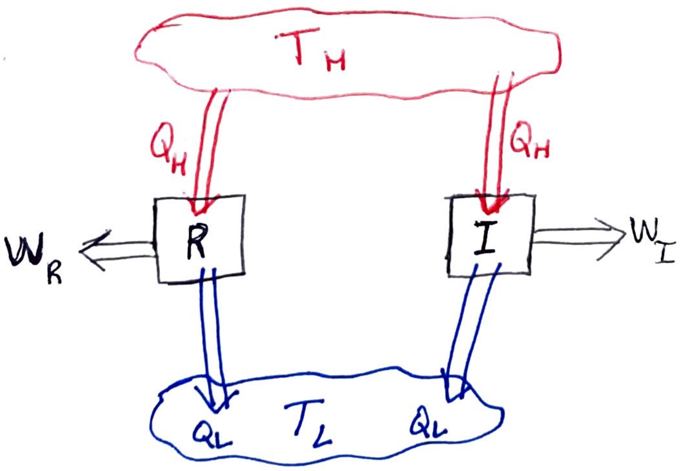
Carnot Cycle of Liquid-Vapor Mixture

★ Carnot Propositions:

1- For given T_H and T_L, a reversible HE is more efficient than any irreversible HE.

HE = Heat engine

2- For given T_H and T_L, all carnot HE have the same efficiency.



$W_I < W_R$

Absolute Temp Scale:

$$\eta_{HE} = \frac{W_{net}}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

For a Carnot HE, η only depends on T_H and T_L.

Thus:

$$\frac{Q_L}{Q_H} = f(T_H, T_L) = \frac{T_L}{T_H}$$

$$\boxed{\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \text{ for Carnot Cycle Only}}$$

AND

$$\boxed{\eta_{CHE} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}}$$

CHE = Carnot Heat Engine.

★ Temperatures should be in absolute scale★

- According to Kelvin-Planck statement, there can not be a working (HE) with zero Q_L - It has to dissipate some heat energy, otherwise its efficiency would be 100% which is not possible. So, if $Q_L \neq 0$, $T_L \neq 0$, which means we can never ~~reach~~ reach absolute zero temp.

★ Carnot Refrigerator:

$$COP = \beta_R = \frac{Q_L}{W_{in}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

Since for a Carnot (reversible) cycle

$$\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$$

$$\boxed{COP = \beta_R = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{T_L}{T_H - T_L}}$$

★ Carnot Heat Pump:

$$\boxed{COP = \beta_{HP} = \frac{Q_H}{W_{in}} = \frac{T_H}{T_H - T_L}}$$

Some Ideal Cycles

All Reversible

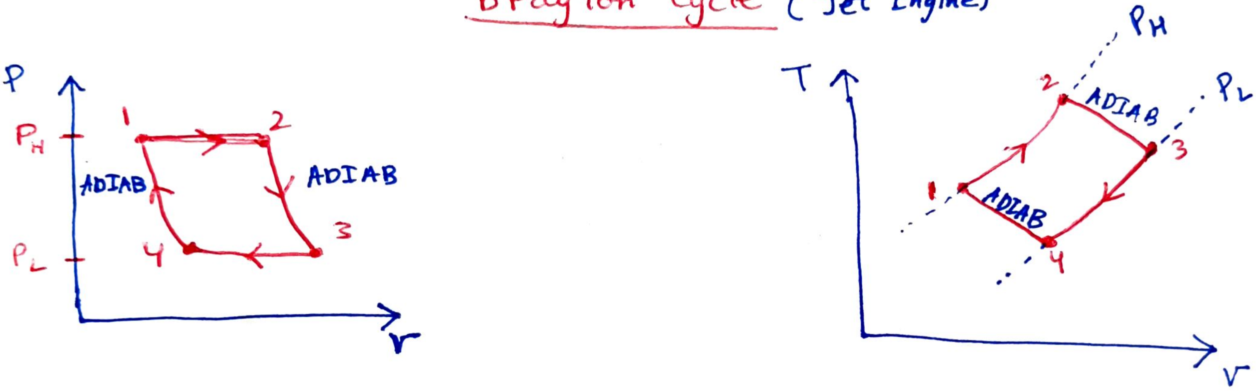
★ **CARNOT:**
- 2 isothermal
- 2 adiabatic

★ **RANKINE:** (Liquid-Vapor)
- 2 isobaric
- 2 adiabatic

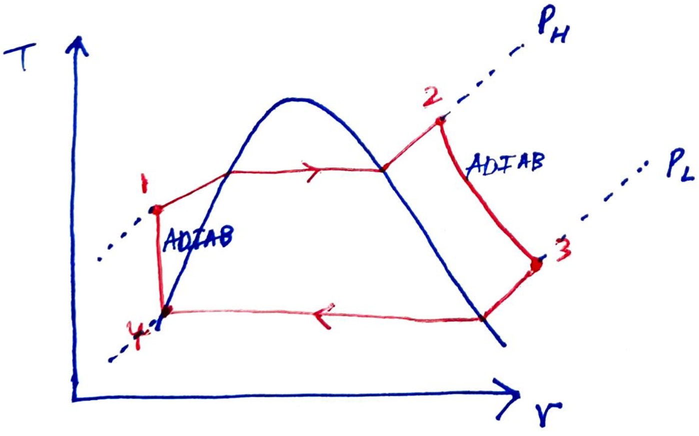
★ **BRAYTON:** (GAS)
- 2 isobaric
- 2 adiabatic

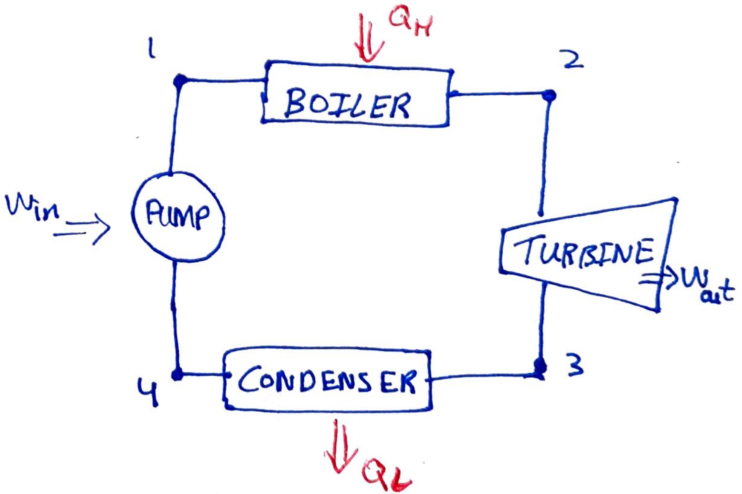
★ **REFRIGERATION:** (Vapor Compression)
- 2 Isobaric
- 2 adiabatic

Brayton Cycle (Jet Engine)



Rankine Cycle (Power Plant)

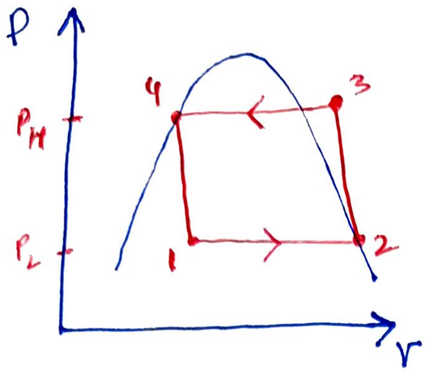


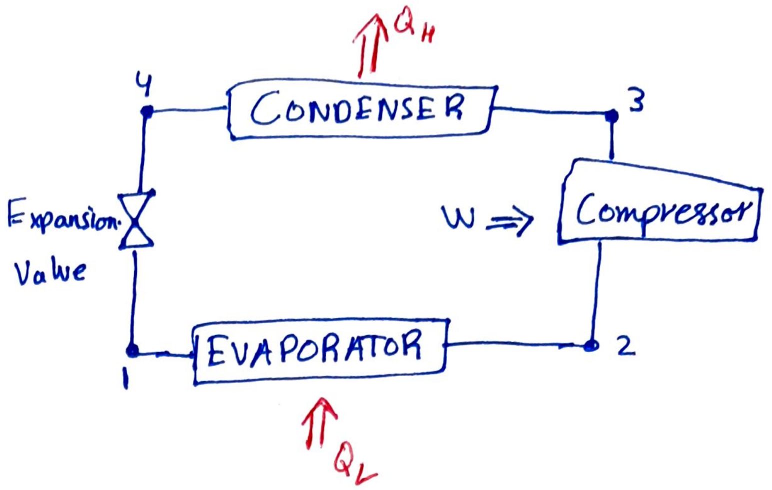


These are known as Power Cycles.

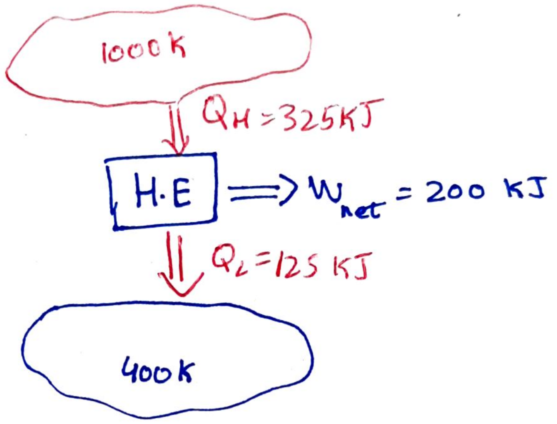
we will study these in detail later on!

*Refrigeration Cycle:





Possible?



Check first law: $Q_H - Q_L = W$ ✓

Check 2nd law:

$$\eta_{RE} = \frac{W}{Q_H} = \frac{200}{325} = 0.615$$

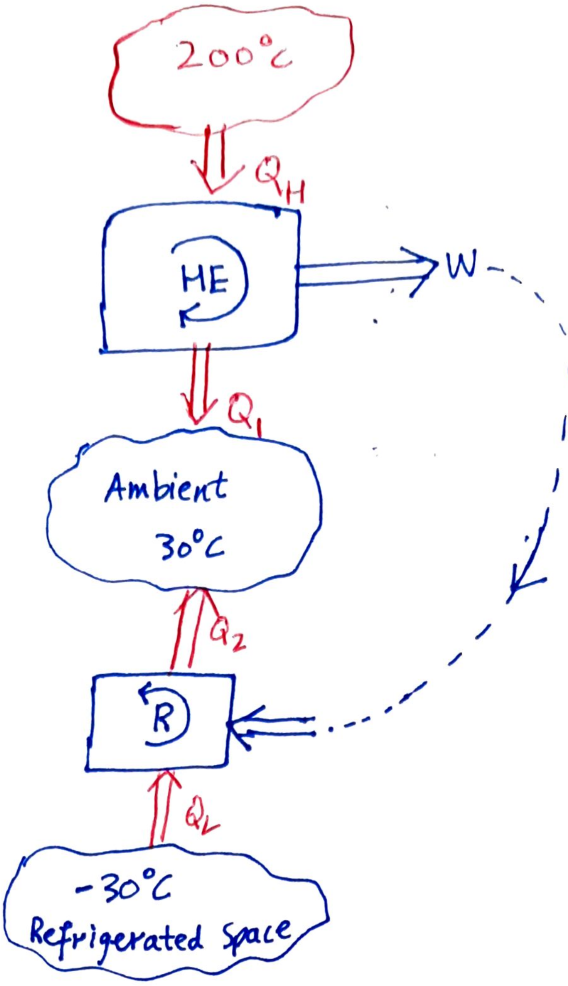
$$\eta_{CHE} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = 0.6$$

Ans. Not possible, since $\eta_{CHE} > \eta_{RE}$ for a possible heat engine. In this case it is not.

All reversible.

Find $\frac{Q_H}{Q_L}$

- Reversible means, HE is carnot H.E
- Same goes for refrigerator.



$$\eta_{CHE} = 1 - \frac{Q_1}{Q_H} = 1 - \frac{T_A}{T_H}$$

$$= 1 - \frac{30+273}{200+273} = 0.36$$

$$W = \eta Q_H$$
$$= 0.36\, Q_H$$

$$COP = \beta_R = \frac{Q_L}{W} = \frac{Q_L}{Q_2 - Q_L} = \frac{1}{\frac{Q_2}{Q_L} - 1}$$

$$= \frac{1}{\frac{30+273}{-30+273} - 1} = 4.05$$

~~$W = \beta_R Q_L$~~
~~$W = 4.05 Q_L$~~

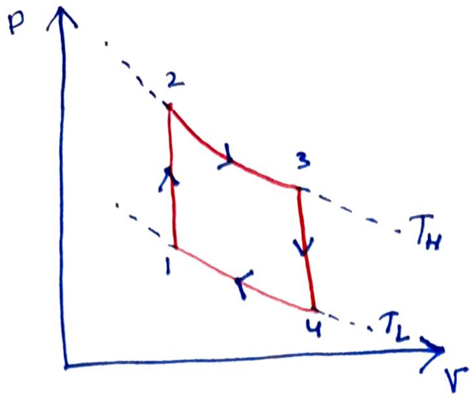
$$W = \frac{Q_L}{4.05}$$

$$W = W$$

$$0.36 Q_H = \frac{Q_L}{4.05}$$

$$\boxed{\frac{Q_H}{Q_L} = 0.685}$$

Carnot Cycle of an Ideal Gas



1st law: $du = \delta q - \delta w$

Ideal Gas: $pv = RT$

- Process 1-2 (Adiabatic)

$du = \cancel{\delta q} - \delta w$

$C_v dT = -p\,dv$

$C_v dT = -\frac{RT}{v}\,dv$

$C_v \frac{dT}{T} = -R\frac{dv}{v}$

$\int_1^2 C_v \frac{dT}{T} = \int_{T_L}^{T_H} C_v \frac{dT}{T} = -R\ln\left(\frac{v_2}{v_1}\right)$ — ①

- Process: 2-3 (isothermal)

$du = \delta q - \delta w$

$C_v \cancel{dT} = \delta q - \delta w$

$C_v = \delta q - p\,dv$

$\delta q = p\,dv$

$\delta q = \frac{RT}{v}\,dv$

$\int_2^3 \delta q = {}_2q_3 = RT_H \ln\frac{v_3}{v_2}$ — ②

- Process: 3-4 (Adiabatic)

$du = \cancel{\delta q} - \delta w$

$C_v dT = -p\,dv$

$C_v \frac{dT}{T} = -R\frac{dv}{v}$

$\int_3^4 C_v \frac{dT}{T} = -R\ln\left(\frac{v_4}{v_3}\right)$

$\int_{T_H}^{T_L} C_v \frac{dT}{T} = -R\ln\left(\frac{v_4}{v_3}\right)$

$\int_{T_L}^{T_H} C_v \frac{dT}{T} = -R\ln\left(\frac{v_3}{v_4}\right)$ — ③

- Process 4-1 (isothermal)

$du = \delta q - \delta w$

$C_v \cancel{dT} = \delta q - \delta w$

$C_v = \delta q - p\,dv$

$\delta q = \frac{RT}{v}\,dv$

${}_4q_1 = RT_L \ln\left(\frac{v_1}{v_4}\right)$ — ④

Since ① = ③

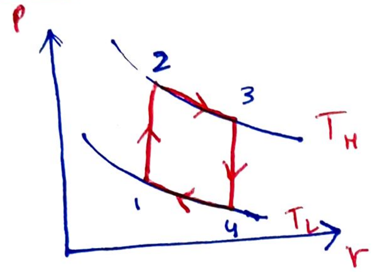
$\boxed{\frac{v_2}{v_1} = \frac{v_3}{v_4}}$ also $\boxed{\frac{v_3}{v_2} = \frac{v_4}{v_1}}$

④ ÷ ②

$$\frac{-{}_4q_1}{{}_2q_3} = \frac{q_L}{q_H} = \frac{T_L}{T_H}\,\frac{\ln v_4/v_1}{\ln v_3/v_2}$$

$$\boxed{\frac{q_L}{q_H} = \frac{T_L}{T_H}}$$ → True for a Carnot Cycle as mentioned earlier. pg (44)

Example :



Air Carnot cycle

$T_H = 600$ K $q_H = 250$ KJ/Kg

$T_L = 300$ K $P_4 = 75$ KPa

Find v_1, P_1, W_{net}.

$$\frac{q_H}{q_L} = \frac{T_H}{T_L}$$

$$\frac{q_H}{q_L} = \frac{600}{300}$$

$$q_L = 250/2$$

$$q_L = 125 \text{ KJ/Kg}$$

$$w_{net} = q_H - q_L \quad (1^{st} \text{ law})$$

$$w_{net} = 250 - 125$$

$$\boxed{w_{net} = 125 \text{ KJ/kg}}$$

$$-q_L = RT_L\left(\frac{v_1}{v_4}\right)$$

$P_4 v_4 = RT_4$ — Ideal Gas law

$$v_4 = \frac{RT_4}{P_4}$$

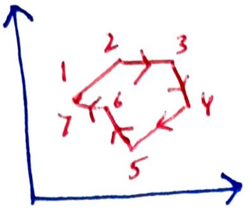
$v_4 =$ ✓

$$P_1 v_1 = RT_1$$

$$P_1 = \frac{RT_1}{v_1}$$

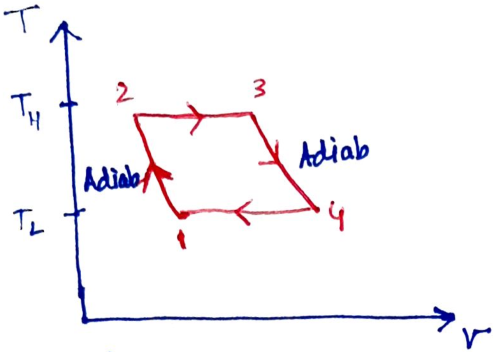
★ Entropy

$\oint$ cyclic Integral



- ${}_2W_3 = \int_2^3 P\,dV$
- $Q_{56} = \int_5^6 \delta Q$
- $Q_{cycle} = \oint \delta Q = \int_1^2 \delta Q + \int_2^3 \delta Q + \int_3^4 \delta Q + \dots + \int_7^1 \delta Q$

Carnot Cycle : (Heat Engine)



$$Q_{cycle} = \oint \delta Q$$

$$= \underset{0}{\cancel{\int_1^2 \delta Q}} + \int_2^3 \delta Q + \underset{0}{\cancel{\int_3^4 \delta Q}} + \int_4^1 \delta Q$$

$$= \int_2^3 \delta Q + \int_4^1 \delta Q$$

$$\boxed{\oint \delta Q = Q_H - Q_L}$$

$$\oint \frac{\delta Q}{T} = \underset{0}{\cancel{\int_1^2 \frac{\delta Q}{T}}} + \int_2^3 \frac{\delta Q}{T} + \underset{0}{\cancel{\int_3^4 \frac{\delta Q}{T}}} + \int_4^1 \frac{\delta Q}{T}$$

$$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}$$

$= 0$ (zero) because $\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$ for an ideal cycle.

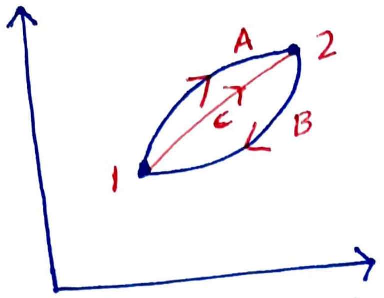
$\oint \frac{\delta Q}{T} = 0$ for Carnot Heat Engine

Now, its a real heat engine:

$$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} < 0$$

$\oint \frac{\delta Q}{T} < 0$ for a Real H.E.

$\boxed{\oint \frac{\delta Q}{T} \leq 0}$ Clausius Inequality



Reversible Cycle

$$\oint \frac{\delta Q}{T} = \int_{1,A}^{2} \frac{\delta Q}{T} + \int_{2,B}^{1} \frac{\delta Q}{T} = 0$$

$$\oint \frac{\delta Q}{T} = \int_{1,C}^{2} \frac{\delta Q}{T} + \int_{2,B}^{1} \frac{\delta Q}{T} = 0$$

Subtract: $\int_{1,A}^{2} \frac{\delta Q}{T} - \int_{1,C}^{2} \frac{\delta Q}{T} = 0$

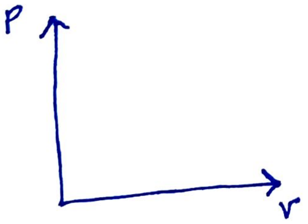
$$\int_{1,A}^{2} \frac{\delta Q}{T} = \int_{1,C}^{2} \frac{\delta Q}{T}$$

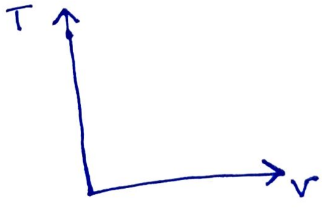
$\Rightarrow \int_{1}^{2} \frac{\delta Q}{T}$ is independent on the path.

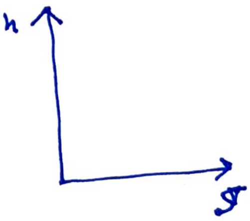
$$= \int_{1}^{2} dS = \Delta S = S_2 - S_1$$

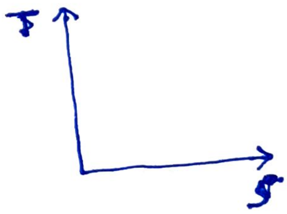
$S_2 - S_1 = \int_{1}^{2} \frac{\delta Q}{T}$ (reversible path) = entropy change

units: $\frac{KJ}{K}$



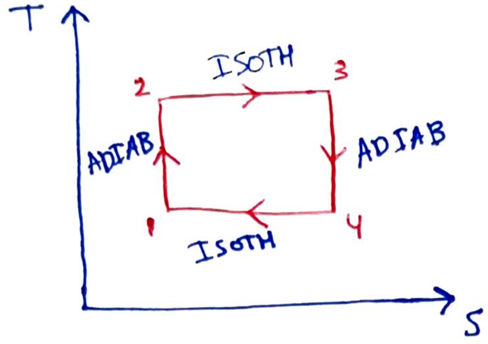






$\Delta s = \int_1^2 \frac{\delta Q}{T}$ reversible process

Carnot Cycle: 2 isothermals, 2 adiabatic on a T-S diagram.



$\Delta s = 0 \Rightarrow$ isentropic

↑

(Adiabatic + reversible)

Area □ $= Q_{cycle} = Q_{net} = Q_H - Q_L$

Ex: Carnot Ref-cycle R-410a.

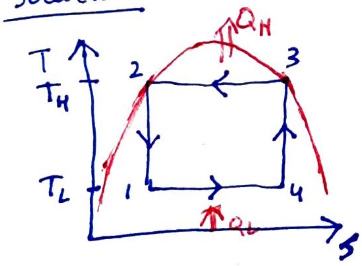
$T_H = 40°C$ During heat rejection process, R-410a goes from sat. vapor to sat. liquid.

$T_L = -5°C$

1- Find the X at beginning and end of the heat addition process.

2- Find COP.

Solution:



Q_H is Q_{32}

Q_L is Q_{14}

$x_1 =$

$x_4 =$

$s_1 = s_2 = s_f @ 40°C$ } Table

$s_4 = s_3 = s_g @ 40°C$

$$COP = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L}$$

Since $\frac{T_L}{T_H} = \frac{Q_L}{Q_H}$

Thermodynamic Property Relations

1st law $\Delta U = Q - W$

$dU = \delta Q - \delta W$

$\delta W = p\,dV$

Reversible Process: $\delta Q = T\,ds$ (2nd Law)

$dU = T\,ds - p\,dV$

★ $\boxed{du = T\,ds - p\,dv}$ Always Valid (intensive properties)

$h = u + pv$

$u = h - pv$

$du = dh - p\,dv - v\,dp$ (applying product rule for derivative)

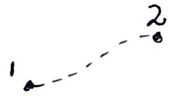
$dh - \cancel{p\,dv} - v\,dp = T\,ds - \cancel{p\,dv}$

★ $\boxed{dh = T\,ds + v\,dp}$ Always Valid (intensive properties)

Example: For an incompressible substance:

$du = T\,ds$ (no variation in volume)

$$ds = \frac{du}{T}$$



$$\Delta s = s_2 - s_1 = \int_1^2 \frac{du}{T} = \int_1^2 \frac{c\,dT}{T}$$

$du = c\,dT$ (for incompressible)

If $c \approx$ const.

$$\boxed{\Delta s = c \ln \frac{T_2}{T_1}}$$

★ For An Ideal Gas:

$Tds = du + pdv$ 1st relationship

$$ds = \frac{du}{T} + \frac{pdv}{T}$$

$$\left.\begin{aligned} du &= C_v dT \\ pv &= RT \end{aligned}\right\}$$ (for an ideal gas)

$$ds = C_v\frac{dT}{T} + R\frac{dv}{v}$$

$$\Delta s = s_2 - s_1 = \int_1^2 C_v\frac{dT}{T} + R\int\frac{dv}{v}$$

Assuming $C_v = const.$

$$\boxed{\Delta s = C_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}}$$

⇒ if we repeat the same process starting from 2nd relationship i.e $dh = Tds + vdp$ we get

$$\boxed{\Delta s = C_p\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}}$$

$dh = C_p dT$ (for an ideal gas)

$C_p = const.$

Taking this one step back

$$\Delta s = \int_{T_1}^{T_2} C_p\frac{dT}{T} - R\ln\frac{P_2}{P_1}$$

$\int_1^2 C_p\frac{dT}{T}$ is tabulated $= s_2^\circ - s_1^\circ$ ($s^\circ = s$ superzero)

This is done if we don't want to take C_p as a constant.

s° values can be found from ideal gas tables with respect to a given temp. $\boxed{\text{DO NOT FORGET}}$ s° is not s - It is **NOT** entropy.

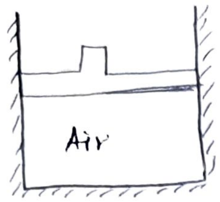
$$\Delta s^\circ = s_2^\circ - s_1^\circ = \int_1^2 C_p\frac{dT}{T}$$

$$\Delta s = s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1}$$

Get Value of s_2° & s_1° from tables, replace $\int_1^2 C_p\frac{dT}{T}$ with these values.

Plug into the equation of Δs and find change in entropy.

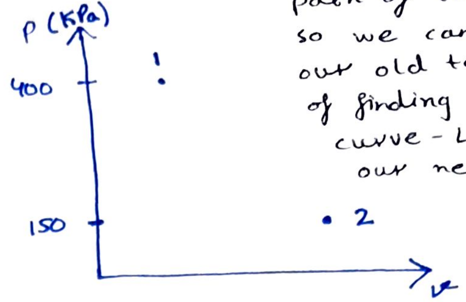
Example :



$m = 1\ \text{kg}$

$P_1 = 400\ \text{kPa}$

$T_1 = 600\ \text{K}$



* we dont know the path of the process, so we cannot use our old techniques of finding area under curve - Lets try our new method.

Expands adiabatically & reversibily. to $P_2 = 150$ kPa. Find ${}_1W_2$.

$\boxed{\text{Adiabatic + Reversible} = \Delta s = 0 \quad \text{(isentropic)}}$

1st law: $\Delta U = {}_1\cancel{Q}_2 - {}_1W_2$

$$-{}_1W_2 = \Delta U = U_2 - U_1 = m(u_2 - u_1)$$

$u_1\,(600\text{K}) = 435.1\ \text{kJ/kg}$ (from table data)

To find u_2, we need to know T_2.

$$\Delta s = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} = 0 \quad \text{(isentropic)}$$

$$s_2^\circ = s_1^\circ + R \ln \frac{P_2}{P_1}$$

$s_1^\circ\,(600\text{K}) = 7.576\ \text{kJ/kgK}$ (from table data)

$$s_2^\circ = 7.576 + R \ln \frac{150}{600}$$

$$s_2^\circ = 7.295\ \text{kJ/kgK}$$

$T_2\,(7.295)$ = Interpolate b/w 440 & 460 K (from tabledata)

$$T_2 = 457\text{K}$$

$$u_2 = 328\ \text{kJ/kgK}$$

$$-{}_1W_2 = m(u_2 - u_1)$$

$$\boxed{{}_1W_2 = -107\ \text{kJ}}$$

Isentropic Process Of An Ideal Gas:

Reversible Process $ds = \frac{\delta Q}{T}$

Reversible + Adiabatic $\Rightarrow ds = 0 \Rightarrow$ Isentropic

Ideal Gas with constant C_v, C_p

① $\Delta s = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} = 0$

Isentropic

② $\Delta s = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 0$

① $\ln \left(\frac{T_2}{T_1}\right)^{C_v} + \ln \left(\frac{v_2}{v_1}\right)^{R} = 0$

$$\left(\frac{T_2}{T_1}\right)^{C_v} \left(\frac{v_2}{v_1}\right)^{R} = 1$$

$R = C_p - C_v$ and let $k = \frac{C_p}{C_v}$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}}$$

Isentropic,
Ideal Gas
Const. C_p, C_v

$\boxed{k \text{ for air} = 1.4}$

② $\ln \left(\frac{T_2}{T_1}\right)^{C_p} - \ln \left(\frac{P_2}{P_1}\right)^{R} = 0$

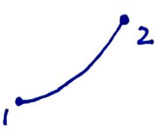
$$\left(\frac{T_2}{T_1}\right)^{C_p} \left(\frac{P_2}{P_1}\right)^{R} = 1$$

$$\boxed{\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}}$$

Isentropic,
Ideal Gas
Const. C_p, C_v

Setting ① = ② we get a new relationship

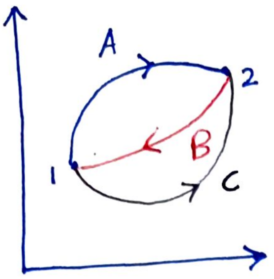
③ $$\boxed{\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k}}$$



$P_2 v_2^{k} = P_1 v_1^{k}$

$\boxed{P v^{k} = \text{const}}$ which is a polytropic process with $n = k$.

Irreversible (Real) Process :



A : revers.

B : revers.

C = irrevers.

$$\oint \frac{\delta Q}{T} = 0 \quad = \int_{1,A}^{2} \frac{\delta Q}{T} + \int_{2,B}^{1} \frac{\delta Q}{T} = 0$$

$$\oint \frac{\delta Q}{T} = \int_{1,C}^{2} \frac{\delta Q}{T} + \int_{2,B}^{1} \frac{\delta Q}{T} < 0$$

(Clausius inequality)

$$\int_{1,C}^{2} \frac{\delta Q}{T} < \int_{1,A}^{2} \frac{\delta Q}{T}$$

Irrev. Rev.

For Path A:

$$\int_{1,A}^{2} \frac{\delta Q}{T} = \Delta s = s_2 - s_1$$

$(\delta Q = T ds)$

$$\int_{1,C}^{2} \frac{\delta Q}{T} < \Delta s$$

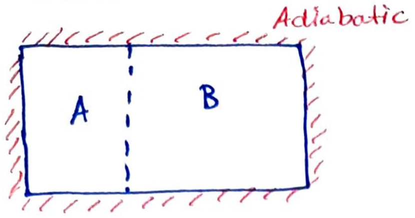
$$\Delta s > \int_{1,C}^{2} \frac{\delta Q}{T}$$

$$\boxed{\Delta s = \int_{1,C}^{2} \frac{\delta Q}{T} + S_{Generation}}$$

Any Real Process will absolutely have $S_{generation}$.

This explains why entropy of universe is always increasing, because all processes generate entropy.

Example:



Find T_f, P_f and determine if the process is Rev or Irrev.

A: $1.2\,m^3$

200 kPa

40°C

B: $2\,m^3$

100 KP

10°C

Solution:

$PV = mRT \Rightarrow m_A = 2.67\,kg$

$m_b = 2.46\,kg$

1st law: $\Delta U = \cancel{Q} - \cancel{W} = 0$

$U_2 = U_1 = m_A u_A + m_b u_b$

Two ways to solve:

① Via table

② Via $C_v \Rightarrow \boxed{T_f = 26^\circ C}$ $= m_A(C_v T_A) + m_b(C_v T_B)$

$P_f = \dfrac{m_T R T_f}{V_T}$ $\boxed{= 138\,KPa}$

To check if reversible or not, we need to look at entropy. Δs.

For an Irrev (Real) Process:

$$\Delta s = \cancel{\int_1^2 \frac{\delta Q}{T}}^{\,0} + S_{gen}$$

$$\Delta s = S_{gen}$$

$$\Delta s = S_2 - S_1 = m_2 s_2 - (m_A s_A + m_B s_B)$$

$$\Delta s = \underbrace{m_A(s_2 - s_A)}_{\Delta s_A} + \underbrace{m_B(s_2 - s_B)}_{\Delta s_B}$$

$$\Delta s = m_A\left[\left(C_p \ln \frac{26\cancel{+273}}{40}\right) + \left(R \ln \frac{138}{200}\right)\right] + m_B\left[\left(C_p \ln \frac{26}{10}\right) - \left(R \ln \frac{138}{100}\right)\right]$$

$\boxed{\Delta s = 0.072\,\frac{KJ}{K}}$ ⊕ ⇒ Irreversible

If $\Delta s = 0 \Rightarrow$ reversible

$\Delta s < 0 \Rightarrow$ recheck your calculations to find your mistake!

2nd Law for a Control Volume:

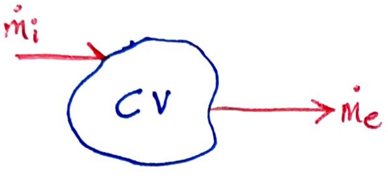
For a control Mass:

$$\Delta S = \int_1^2 \frac{\delta Q}{T} + S_{gen}.$$

No Mass crossing boundary

$$ds = \frac{\delta Q}{T} + \delta S_{gen}. \quad \text{(differential)}$$

$$\left(\frac{ds}{dt}\right)_{CM} = \frac{\dot{Q}}{T} + \dot{S}_{gen}. \quad \text{(rate form)}$$



$$\left(\frac{ds}{dt}\right)_{CV} = \frac{\dot{Q}}{T} + \dot{S}_{gen} + \dot{m}_i s_i - \dot{m}_e s_e \quad \text{(2nd law for a CV)}$$

★ Two Types of Problems:

1- Steady State Processes

2. Transient Processes

1- Steady State: no time variation.

$$\left(\frac{ds}{dt}\right)_{CV} = 0$$

$$0 = \frac{\dot{Q}}{T} + \dot{S}_{gen} + \dot{m}_i s_i - \dot{m}_e s_e$$

one inlet
one outlet

$$\dot{m}_e = \dot{m}_i = \dot{m}$$

$$s_e - s_i = \frac{q}{T} + s_{gen} \quad \text{per unit mass}$$

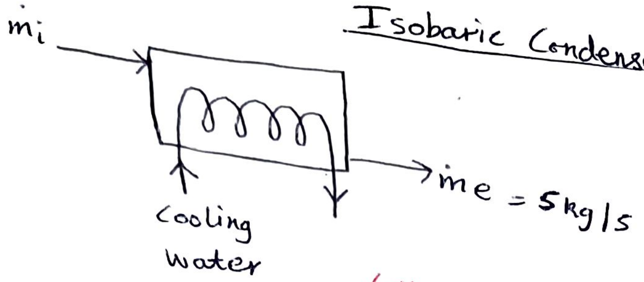
★ If a Process is reversible: ($s_{gen} = 0$)

★ If a Process is adiabatic: ($q = 0$)

★ If a Process is adiabatic + reversible: $s_e = s_i$

Example:

Isobaric Condenser

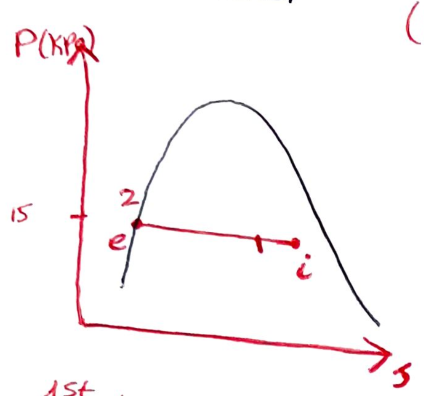


inlet: steam, 90% quality
15 KPa

outlet: sat. liquid

Find $\dot{Q}_{out} = ?$, $\dot{S}_{gen} = ?$

(Steady State)



1st law:

$$0 = \dot{Q} - \underset{0}{\cancel{\dot{W}}} + \dot{m}(h_i - h_e)$$

$\dot{W} = 0$ (because its a condenser)

$$\dot{Q} = \dot{m}(h_e - h_i)$$

$h_e = h_f$ @ 15KPa

h_i = using x.

Since 15KPa is not available in table; we need to interpolate,

Pressure (bars)	h_f (kJ/kg)
0.10	191.83
0.15	x
0.20	251.40

$$\frac{0.20 - 0.10}{251.40 - 191.83} = \frac{0.15 - 0.10}{x - 191.83}$$

$x = 221.615$ kJ/kg (h_f)

$h_i = h_f + x\,h_{fg}$

$= 221.615 + 0.9(2426.55)$

$= 2405.51$ kJ/kg

$\dot{Q} = 5(221.615 - 2405.51)$

$\boxed{\dot{Q} \simeq -10919\,KW}$ (out)

Do the same for h_{fg}, we get $= 2426.55$ kJ/kg

$$\dot{m}(s_e - s_i) = \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

$$\dot{S}_{gen} = \dot{m}(s_e - s_i) - \frac{\dot{Q}}{T}$$

$T_{sat} = 54°C$

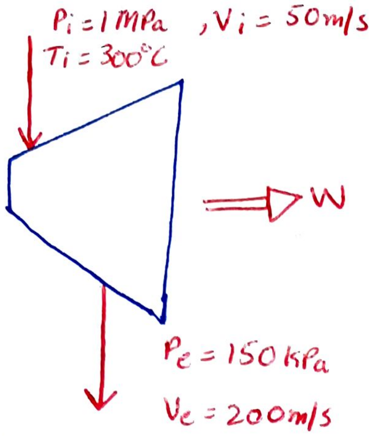
$s_i = s_f + x\,s_{fg}$ ($s_e = s_f$ @ 15KPa $= 0.7548$)

$s_i = 7.283$ KJ/kg K

$\dot{S}_{gen} = 5(0.7548 - 7.283) + 10919/(54+273)$

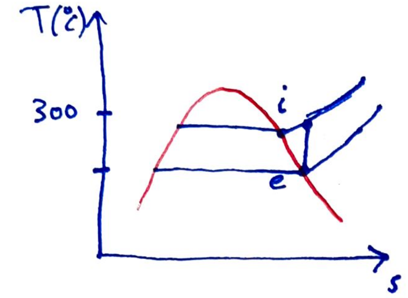
$\dot{S}_{gen} = 0.75$

Example : (Steam Turbine)



The Process is said to be reversible and adiabatic. Find work per kilogram of steam flowing.

Sol:



Continuity Equation gives us:

$\dot{m}_e = \dot{m}_i = \dot{m}$

1st Law: (adiabatic)

$$0 = \cancel{\dot{Q}}^{\,0} - \dot{w} + \left(h_i + \frac{V_i^2}{2}\right) - \left(h_e + \frac{V_e^2}{2}\right)$$

$$\dot{w} = \left(h_i + \frac{V_i^2}{2}\right) - \left(h_e + \frac{V_e^2}{2}\right)$$

2nd Law: Since process is rev & adiab.

$s_e = s_i$

From Steam Tables:

$h_i = 3051.2\ KJ/kg$

$s_i = 7.1228\ KJ/kg\,K$

we also know:

$P_e = 0.15\,MPa$

$s_e = s_i = 7.1228\ KJ/kgK$

Finding Quality (x_e) of steam:

$s_e = s_f + x_e s_{fg}$

$7.1228 = 1.4335 + 5.7897 x_e$

$x_e = 0.9827$

Find h_e using Quality (x_e):

$h_e = h_f + x_e h_{fg}$

$h_e = \cancel{225.91 + (0.9827)(2373.14}\,)$

$h_e = 467.08 + (0.9827)(2226.46)$

$h_e = 2655.00\ KJ/kg$

$$\dot{w} = \left(h_i + \frac{V_i^2}{2}\right) - \left(h_e + \frac{V_e^2}{2}\right)$$

$$\dot{w} = 3051.2 + \frac{(50)^2}{2\times1000} - 2655.0 - \frac{(200)^2}{2\times1000}$$

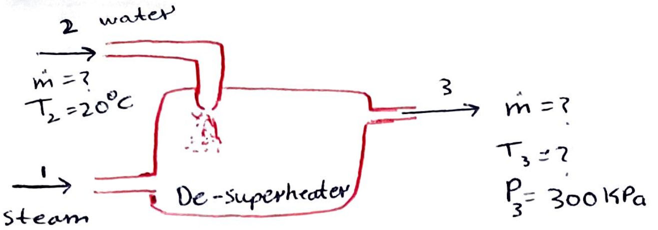
$$\boxed{\dot{w} = 377.5\ KJ/kg}$$

Example : (C.V. De-superheater)

Find what mass flow rate of liquid water at 20°C should be added to generate sat. vapor at 300 KPa. Also find out rate of entropy generation in process.

(no heat transfer, no work)

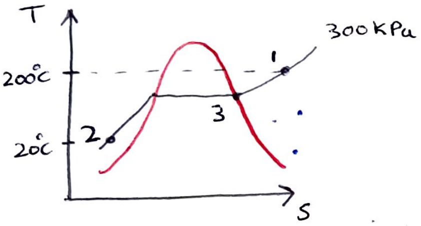
Isobaric Process.



$\dot{m} = 2 kg/s$

$P_1 = 300 KPa$

$T_1 = 200°C$



Continuity: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$

Entropy Eq: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3$

$P = const, \dot{W} = 0, \dot{Q} = 0$

Approximate state 2 with saturated liq @ 20°C.

$h_1 = 2865.54 \frac{KJ}{Kg}$, $s_1 = 7.3115 \frac{KJ}{KJK}$

$h_3 = 2725.3 \frac{KJ}{Kg}$, $s_3 = 6.9918 \frac{KJ}{KgK}$

$h_2 = 83.94 \frac{KJ}{Kg}$, $s_2 = 0.2966 \frac{KJ}{KgK}$

Solve for rate of low $\dot{m}_2$ from Energy Eq:

$$\dot{m}_2 = \dot{m}_1 \left(\frac{h_1 - h_3}{h_3 - h_2}\right) = 2\left(\frac{2865.54 - 2725.3}{2725.3 - 83.94}\right)$$

$$\boxed{\dot{m}_2 = 0.1062 \ kg/s}$$

$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 2.1062 \ kg/s$

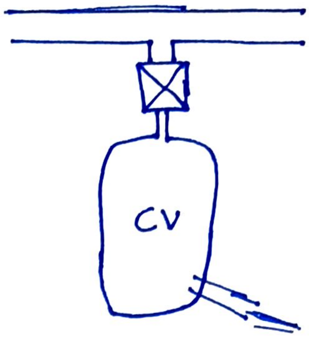
$\dot{S}_{gen} = \dot{m}_3 s_3 - \dot{m}_2 s_2 - \dot{m}_1 s_1$

$\dot{S}_{gen} = (2.1062)(6.9918) - (2 \times 7.3115) - (0.1062 \times 0.2966)$

$$\boxed{\dot{S}_{gen} = 0.072 \ KW/K}$$

★ Transient Process (CV):

2^{nd} law for a CV:



$$\left(\frac{dS}{dt}\right)_{CV} = \frac{\dot{Q}}{T} + \dot{S}_{gen} + \dot{m}_i \dot{s}_i - \dot{m}_e s_e$$

1 = Initial state

2 = final state

Integrate with time to see what changes occur in CV as process goes on:

$$\int dS = \int \frac{\dot{Q}\,dt}{T} + \int \dot{S}_{gen}\,dt + \int \dot{m}_i s_i\,dt - \int \dot{m}_e s_e\,dt$$

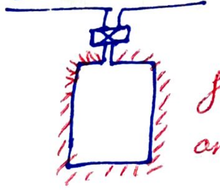
$$\Delta S = \int_1^2 \frac{\delta Q}{T} + S_{gen} + \int_1^2 \dot{m}_i s_i\,dt - \int_1^2 \dot{m}_e s_e\,dt$$

if s_i and s_e do not change in time:

$$\Delta S = \int_1^2 \frac{\delta Q}{T} + S_{gen} + s_i \int_1^2 \dot{m}_1\,dt - s_e \int_1^2 \dot{m}_e\,dt$$

$$\boxed{\Delta S = \int_1^2 \frac{\delta Q}{T} + S_{gen} + s_i m_i - s_e m_e}$$

Example: CO_2 @ 3 MPa, 60°C



$V = 0.2\,m^3$

find final Temp, and S_{gen}.

① = empty

② = "full"

Solution:

- $m_1 = 0 \qquad m_2 = m_i$
- $\Delta U = \cancel{Q} - \cancel{W} + m_i h_i$

$U_2 - \cancel{U_1} = m_i h_i$

$\cancel{m_2} u_2 = \cancel{m_i} h_i$

$u_2 = h_i$

from Table (B.3.2) $h_i = 400.2$ KJ/kg

$s_i = 1.477\ \frac{KJ}{kgK}$

$u_2 = 400.2$ KJ/Kg

Reaches full capacity when P reaches 3 MPa.

120°C – 140°C interpol for u_2.

$\boxed{T_2 \approx 130°C}$

2^{nd} law:

$S_2 - \underset{0}{\cancel{S_1}} = 0 + S_{gen} + m_i s_i$

$m_2 s_2 = S_{gen} + m_i s_i \qquad m_2 = m_i$

$S_{gen} = m_2 (s_2 - s_i)$

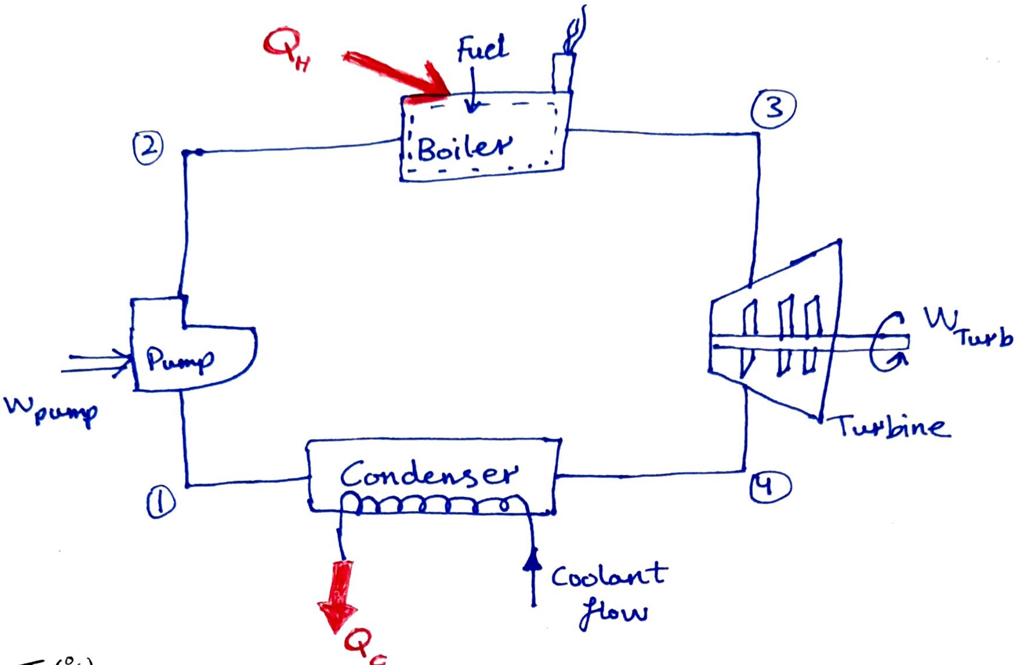
read v at T_2 (130°C)

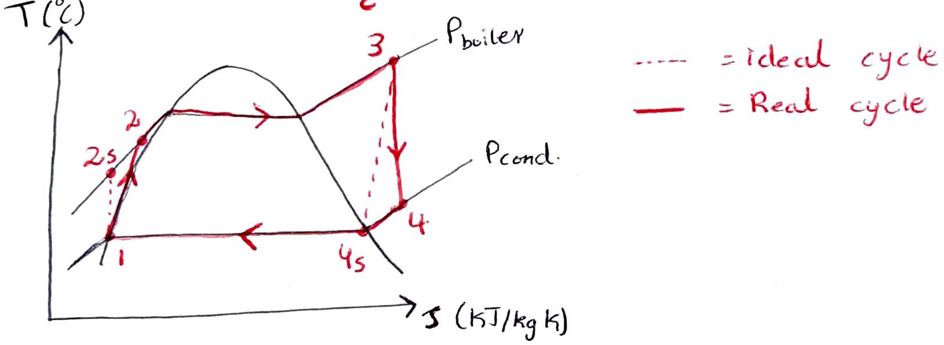
$m_2 = \frac{V}{v_2}$

read s_2 at T_2, P_2

$S_{gen} > 0$

Simple Power Plant





★ Turbine: (steady state)

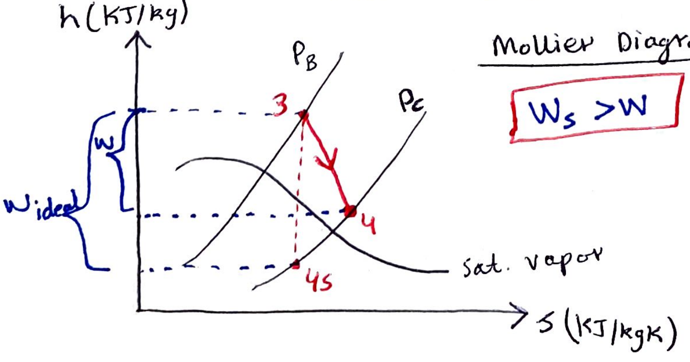
1st Law: $\cancelto{0}{\left(\frac{dU}{dt}\right)_{cv}} = \cancelto{0\,(\text{adiabatic})}{{}_3\dot{Q}_4} - {}_3W_4 + \dot{m}(h_3 - h_4)$

$\dot{W}_T = {}_3\dot{W}_4 = \dot{m}(h_3 - h_4)$ Real

$\dot{W}_{T\,ideal} = {}_3\dot{W}_4 = \dot{m}(h_3 - h_{4s})$ Ideal

Mollier Diagram

$\boxed{W_s > W}$



$w = \frac{\dot{W}}{\dot{m}} = h_3 - h_4$ (real)

$w_s = h_3 - h_{4s}$ (ideal)

Isentropic Efficiency of Turbine:

$$\boxed{\eta_T = \frac{w}{w_s} = \frac{h_3 - h_4}{h_3 - h_{4s}}}$$

2^{nd}- law (Steady): Turbine

$$\cancel{\left(\frac{ds}{dt}\right)_{CV}}^{0} = \cancel{\frac{\dot{Q}}{T}}^{0} + \dot{S}_{gen} + \dot{m}(s_3 - s_4)$$

(steady) (adiab)

Ideal Turbine:

$\dot{S}_{gen} = 0 \Rightarrow S_{4s} = S_3$

Real Turbine:

$\dot{S}_{gen} > 0 \Rightarrow S_4 > S_3$

★ Pump: (Steady flow, adiab)

★Opposite of Turbine★

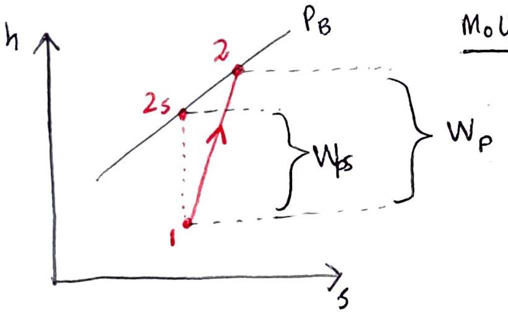
1^{st}-Law:

$$0 = -{}_1\dot{W}_2 + \dot{m}(h_1 - h_2)$$

2^{nd}-law:

$$0 = \dot{S}_{gen} + \dot{m}(s_1 - s_2)$$

$W_p = \frac{{}_1\dot{W}_2}{\dot{m}} = h_1 - h_2$ (Real) $\dot{W}_{p_s} = h_1 - h_{2s}$ (Ideal) $\boxed{W_p > W_{ps}}$

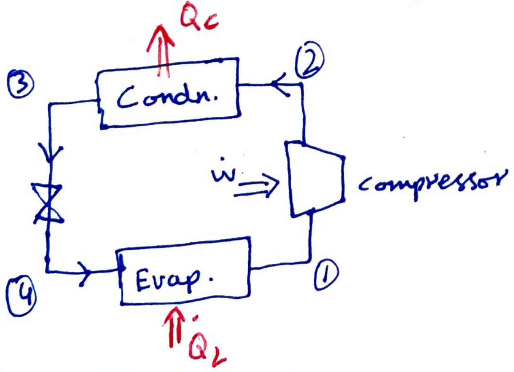


Mollier Diagram (Liquid)

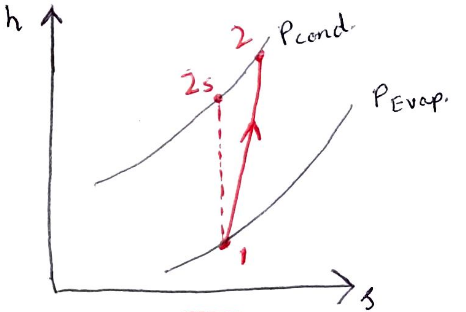
Isentropic Efficiency of Pump:

$$\boxed{\eta_p = \frac{W_s}{W} = \frac{h_1 - h_{2s}}{h_1 - h_2}}$$

Refrigeration



Compressor Mollier Diag. (Vapor)



$$\boxed{\eta_c = \frac{W_s}{W} = \frac{h_1 - h_{2s}}{h_1 - h_2}}$$

★ Nozzle: (To increase K.E of flow)

(Steady & Adiab)

1^{st} Law:

$$\cancel{\left(\frac{du}{dt}\right)_{CV}}^{0} = \cancel{{}_1\dot{Q}_2}^{0} - \cancel{{}_1\dot{W}_2}^{0} + \dot{m}\left(h_1 + \overset{K.E \approx 0}{\frac{V_1^2}{2}} - h_2 - \overset{K.E}{\frac{V_2^2}{2}}\right)$$

$$\boxed{\frac{V_2^2}{2} = h_1 - h_2}$$

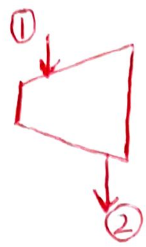
2^{nd} law:

$s_2 = s_1$ (Ideal)

$s_2 > s_1$ (Real)

$$\eta_N = \frac{\frac{V_2^2}{2}}{\frac{V_{2s}^2}{2}} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Example : (Steam Turbine)



inlet :

$T_1 = 300^\circ c$

$P_1 = 600 KPa$

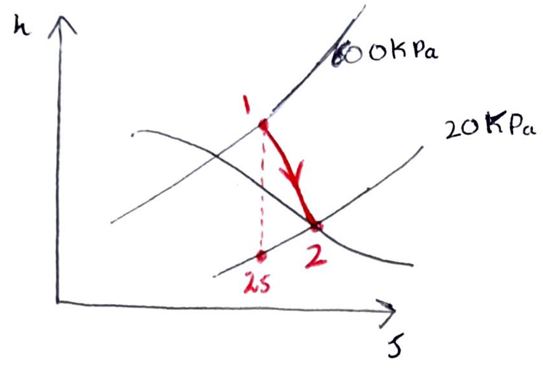
Outlet :

Sat. vap

$P_2 = 20 KPa$

Find $\eta_T = ?$

Solution :



State 1 :

$h_1 = 3062$ KJ/kg (from table)

$s_1 = 7.372$ KJ/kgK (from table)

State 2 :

$h_2 = h_g$ @ 20 KPa = 2610 KJ/kg

State 2s : (isentropic)

$s_{2s} = s_1 = 7.372$ KJ/kgK @ 20KPa

$s_f = 0.832$ KJ/kgK

$s_{fg} = 7.908$ KJ/kgK

$$X_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = 0.92$$

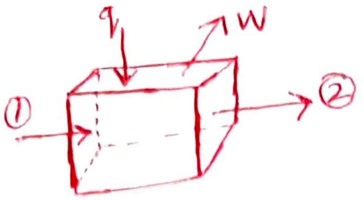
$h_{2s} = h_f + X_{2s} h_{fg}$ @ 20KPa

$h_{2s} = 2431$ KJ/kg

$$\eta_T = \frac{W}{W_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

$$\eta_T = \frac{3062 - 2610}{3062 - 2431}$$

$$\boxed{\eta_T = 0.72 \Rightarrow 72\%}$$



- Steady State
- neglect K.E, P.E
- Reversible

1st Law:

$0 = q - w + \cancel{h_1} h_1 - h_2$ — ①

2nd Law:

$ds = \frac{\delta q}{T} + \cancel{\delta s_{gen}}^{0}$

$\delta q = Tds$ — ②

Property Relation:

$Tds = dh - v dP$ — ③

$\delta q = dh - v dP$

$q = h_2 - h_1 - \int_1^2 v dp \Rightarrow ①$

$0 = (h_2 - h_1 - \int_1^2 v dp) - w + h_1 - h_2$

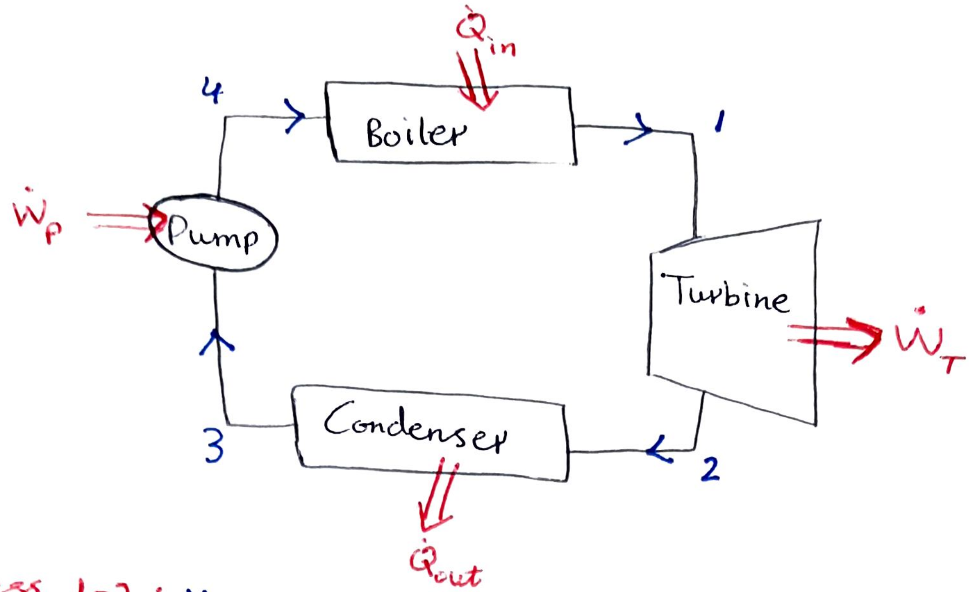
$0 = -\int_1^2 v dp - w$

$$\boxed{w_s = -\int_1^2 v dp}$$

★ If liquid: (Pump)

$$\boxed{w_s = v (P_1 - P_2)}$$

The Rankine Cycle - (Vapor Power Plant)



- **Process 1-2:** Vapor expands through turbine developing work.
- **Process 2-3:** Vapor condenses to liquid through heat transfer to cooling water.
- **Process 3-4:** Liquid is pumped into boiler requiring work input.
- **Process 4-1:** Liquid is heated to saturation and evaporated in boiler through heat transfer from energy source.

★ Each component is analyzed as control volume at steady state.

★ Turbine and Pump operate adiabatically.

★ **Mass & Energy rate balances:**

Turbine: $\frac{\dot{W}_T}{\dot{m}} = h_1 - h_2$

Condenser: $\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3$

Pump: $\frac{\dot{W}_p}{\dot{m}} = h_4 - h_3$

Boiler: $\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4$

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\frac{\dot{W}_T}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}}}{\frac{\dot{Q}_{in}}{\dot{m}}}$$

$$= \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

(Review pg 64 & 65 for calculations of turbine and pump)

Air Standard Analysis

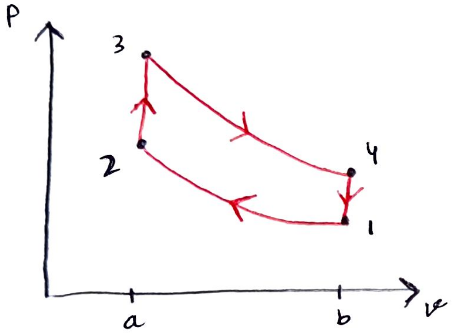
① Otto Cycle :

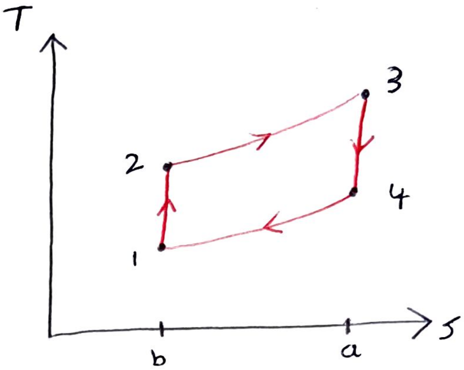
- Heat addition at constant Volume.

② Diesel Cycle :

- Heat addition at constant Pressure

★ Otto Cycle :





★ Processes

- 1 – 2 : isentropic compression
- 2 – 3 : constant-volume heat addition to air
- 3 – 4 : isentropic expansion
- 4 – 1 : constant-volume heat transfer from air.

★ Compression ratio :

$$\boxed{r = \frac{V_1}{V_2} = \frac{V_4}{V_3}}$$

$$\frac{{}_{1}W_{2}}{m} = u_2 - u_1 \qquad {}_{3}W_{4} = u_3 - u_4$$

$$\frac{{}_{2}Q_{3}}{m} = u_3 - u_2 \qquad \frac{{}_{4}Q_{1}}{m} = u_4 - u_1$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{(u_3 - u_2) - (u_4 - u_1)}{(u_3 - u_2)}$$

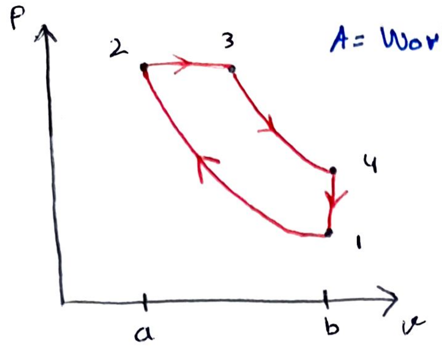
$$\boxed{\eta = 1 - \frac{u_4 - u_1}{u_3 - u_2}}$$

★ Efficiency of Otto cycle increases as Compression Ratio increases.

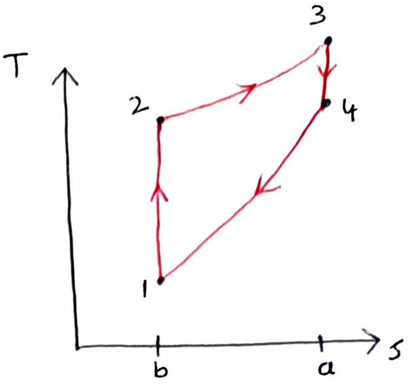
★ Area under T-s diagram = Heat

★ Area under p-v diagram = Work.

★ Diesel Cycle:



A= Work



A= heat transfer

Processes:

- 1-2: isentropic compression
- 2-3: constant-pressure heat addition to air
- 3-4: isentropic expansion
- 4-1: constant-volume heat transfer from air.

★ Compression Ratio:

$$\boxed{r = \frac{V_1}{V_2}}$$

★ Cut-off Ratio:

$$\boxed{r_c = \frac{V_3}{V_2}}$$

$$\frac{{}_2W_3}{m} = \int_2^3 p\,dv = p_2(v_3 - v_2)$$

$$\frac{{}_2Q_3}{m} = (u_3 - u_2) + p(v_3 - v_2)$$

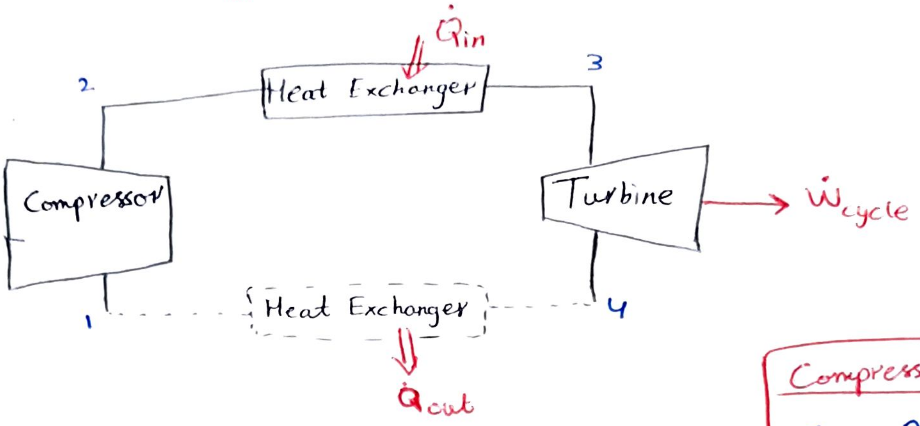
$$= (u_3 + pv_3) - (u_2 + pv_2)$$

$$= h_3 - h_2$$

$$\eta = \frac{W_{cycle}}{{}_2Q_3} = 1 - \frac{{}_4Q_1}{{}_2Q_3} \boxed{= 1 - \frac{u_4 - u_1}{h_3 - h_2}}$$

★ Like otto cycle, efficiency increases with increasing compression ratio.

Brayton Cycle - (Jet Engine)



Compressor Pressure Ratio:

$$C_{pr} = \frac{P_2}{P_1}$$

Turbine:

$$\frac{\dot{W}_T}{\dot{m}} = h_3 - h_4$$

Heat Addition:

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2$$

Compressor:

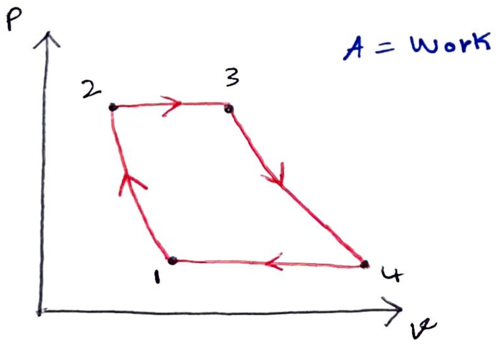
$$\frac{\dot{W}_c}{\dot{m}} = h_2 - h_1$$

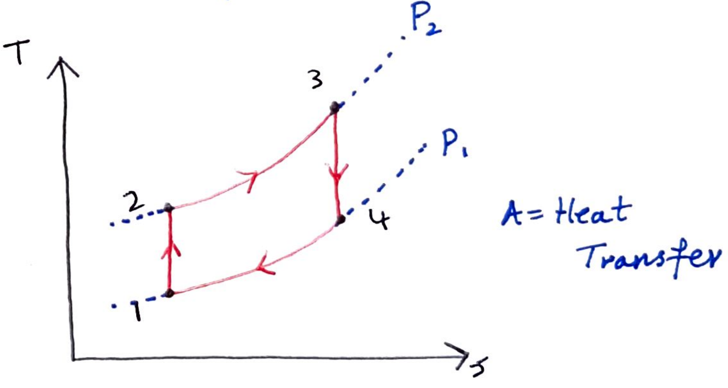
Heat Rejection:

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_1$$

$$\eta = \frac{\dot{W}_T - \dot{W}_c}{\dot{Q}_{in}} = \boxed{\frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}}$$ Efficiency

★ Ideal Brayton Cycle: (rev + adiab = isentropic)





Processes:

1-2: Isentropic Compression of air flowing through compressor.

2-3: Heat transfer to air at constant pressure.

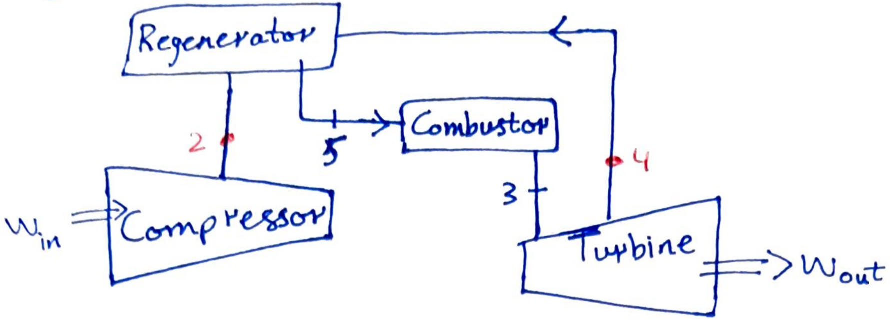
3-4: Isentropic expansion of air through turbine

4-1: Heat transfer from air at constant pressure.

Regenerative Gas Turbines:

Exhaust gas from turbine has considerable thermodynamic utility (exergy) that would be lost to ambient.

Regenerative gas turbines aim to avoid such a significant loss by using hot exhaust gas. Let's see how.



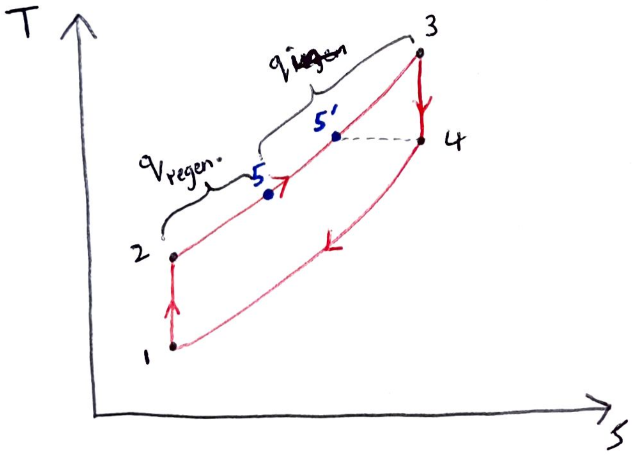
With Regeneration:

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_3 - h_x)$$

Without Regeneration:

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_3 - h_2)$$

★ Remember! : Net work/unit mass flowing is not changed with inclusion of a regenerator. Only $\dot{Q}_{in}$ is affected. Since heat added is reduced, thermal efficiency increases.



$$q_{regen.} = h_5 - h_2$$

$$q_{regen,max} = h_{5'} - h_2 = h_4 - h_2$$

$$\eta_{regen} = \frac{q_{regen}}{q_{regen,max}} = \frac{h_5 - h_2}{h_{5'} - h_2}$$

$$\boxed{\eta_{regen} = \frac{h_5 - h_2}{h_4 - h_2}}$$

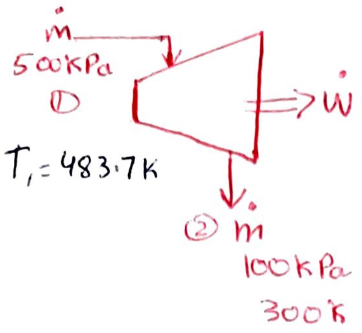
If specific heat = constant;

$$\boxed{\eta_{regen} = \frac{T_5 - T_2}{T_4 - T_2}}$$

★ Regeneration is most effective at lower pressure-ratios (pressure diff is min) and low minimum-to-maximum temperature ratios.

THE END

Q. Turbine at Stead State, adiab. $\eta_T = 80\%$



fluid = monoatomic gas

$C_p = 5\ KJ/kg\,K$

$C_v = 3\ KJ/kg\,K$

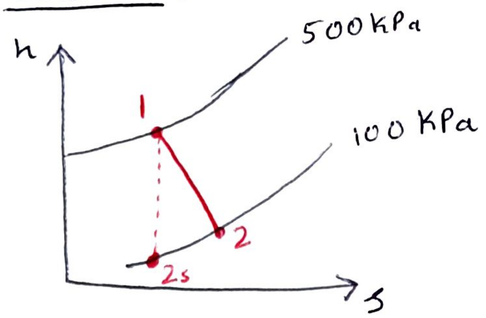
find

a)- Inlet temp. (T_1)

b)- W per unit mass.

c)- S generation.

Solution :



$$\eta_T = \frac{{}_1W_2}{{}_1W_{2s}} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{\cancel{C_p}(T_1 - T_2)}{\cancel{C_p}(T_1 - T_{2s})}$$

$$\eta_T = \frac{T_1 - T_2}{T_1 - T_{2s}}$$

To find T_{2s}:

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$T_{2s} = T_1\left(\frac{100}{500}\right)^{2/5}$$

$$T_{2s} = 0.5253\,T_1$$

$$0.8 = \frac{T_1 - 300}{T_1 - 0.5253T_1}$$

$$300 = T_1 - 0.37976T_1$$

$$\boxed{483.7K = T_1}$$

$${}_1\dot{W}_2 = \frac{\cancel{W_2}}{\cancel{m}} = C_p(T_1 - T_2)$$

$$= 5(483.7 - 300)$$

$$\boxed{= 918.5\ KW}$$

$$\Delta s = \cancel{\int_1^2 \frac{\delta Q}{T}}^{\,0} + S_{gen}$$

$$s_2 - s_1 = S_{gen}$$

[illegible]

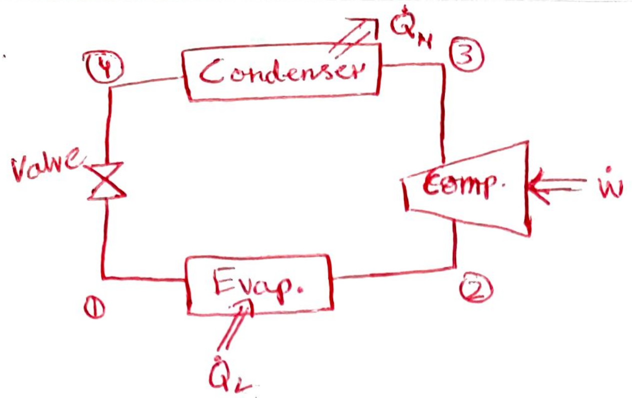
$$C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = S_{gen}$$

$\boxed{R = C_p - C_v}$

$$\dot{S}_{gen} = 5\ln\frac{300}{483.7} - 2\ln\frac{100}{500}$$

$$\boxed{\dot{S}_{gen} = 0.83\ KJ/kg\,K}$$

Q.



R-134a

Comp = adiab. + Rev.

$\dot{m} = 0.05$ kg/s

Cond & Evap = Isobaric

Complete the table.

a)- $\dot{W}$

b)- $\dot{Q}_L$

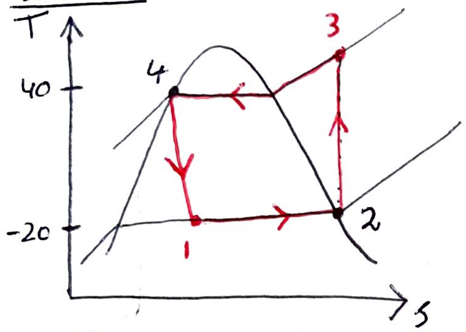
c)- $\dot{Q}_H$

d)- COP.

e)- Plot T-s diag.

State	P (KPa)	T (°C)	X	h (KJ/kg)	S (KJ/kgK)
1	133.7	-20	0.39	257	1.23
2	133.7	-20	1	386	1.74
3	1017	50	NA	431.24	1.74
4	1017	40	0	257	1.19

Solution:



1st Law for Valve:

$$0 = \cancel{\dot{Q}_{41}} - \cancel{\dot{W}_{41}} + \dot{m}(h_4 - h_1)$$

$h_4 = h_1$

$h_1 = h_f + x_3 h_{fg}$ @ -20°C from table

$257 = 173 + x_3 212.34$

$x_1 = 0.39$

$s_1 = s_f + x_1 s_{fg}$ @ -20°C from table

$s_1 = 0.9 + 0.39(0.8388)$

$s_1 = 1.23$ KJ/kgK

1st law: Compressor

$\dot{W} = (h_2 - h_3)\dot{m}$

$\dot{W} = 0.05(386 - 431.24)$

$$\boxed{\dot{W} = -2.3 \text{ KW}}$$

1st law: Evaporator

$\dot{Q}_L = \dot{m}(h_2 - h_1)$

$0 = \dot{Q}_{12} - \cancel{\dot{W}_{12}} + \dot{m}(h_1 - h_2)$

$\dot{Q}_L = 0.05(386-257)$, $\dot{Q}_2 = \dot{m}(h_2 - h_1)$

$$\boxed{\dot{Q}_L = 6.45 \text{ KW}}$$

1st law: Condenser

$\dot{Q}_H = \dot{m}(h_4 - h_3)$

$\dot{Q}_H = 0.05(257 - 431.24)$

$$\boxed{\dot{Q}_H = -8.71 \text{ KW}}$$

$$COP_{RE} = \frac{\dot{Q}_L}{\dot{W}}$$

$$= \frac{6.45}{2.3}$$

$$\boxed{= 2.8}$$

Q- Heat Pump used to heat a house in winters. In summers, reversed for cooling.

Winter:

$T_{House} = 20°C$

$T_{out} = 0°C$

$\to \dot{Q} = 0.5\,KW/degree$ of ΔT

Summers:

$T_{HOUSE} = 25°C$

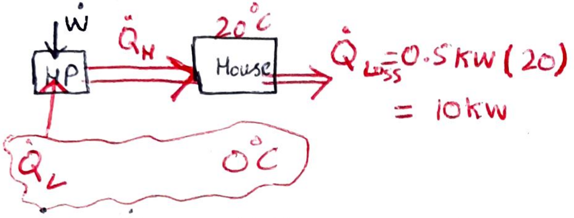
$\dot{Q} = 0.5\,KW/degree$ of ΔT

a)- Find $\dot{W}_{min}$?

b)- $2\dot{W}$, what is Tout max in summers?

Solution:

Winters:



$\dot{Q}_H = \dot{Q}_{Loss} = 10\,KW$

$\dot{W}_{min} \Rightarrow$ ideal HP

$$\frac{\dot{Q}_L}{Q_H} = \frac{T_L}{T_H}$$

$$\dot{Q}_L = \left(\frac{0+273}{20+273}\right)(10)$$

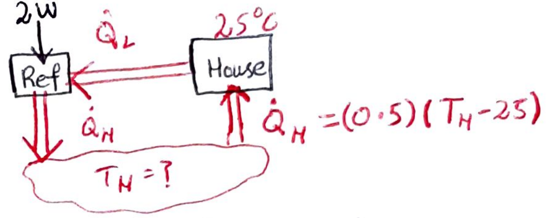
$$\dot{Q}_L = 9.32\,KW$$

$$\dot{W}_{min} = \dot{Q}_H - \dot{Q}_L$$

$$\dot{W}_{min} = 10 - 9.32$$

$$\boxed{\dot{W}_{min} = 0.683\,KW = 683\,W}$$

Summers:



$$COP_{Ref} = \frac{\dot{Q}_L}{2\dot{W}} = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} \quad \text{(Ideal)}$$

$$\frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} = \frac{T_L}{T_H - T_L}$$

$$\frac{(0.5)(T_H - 25)}{1366} = \frac{25+273}{T_H - 25}$$

~~$0.5T_H - 12.5 = \frac{407068}{T_H - 25}$~~

~~$0.5T_H^2 - (12.5T_H)2 + 312.5 = 407068$~~

~~$0.5T_H^2 - 25T_H - 406755.5 = 0$~~

~~$= 25 \pm \sqrt{}$~~

$$(T_H - 25)(T_H - 25) = \frac{298}{2732}$$

$$T_H^2 - 25T_H - 25T_H + 625 = \frac{298}{2732}$$

$$T_H^2 - 50T_H + 624.89 = 0$$

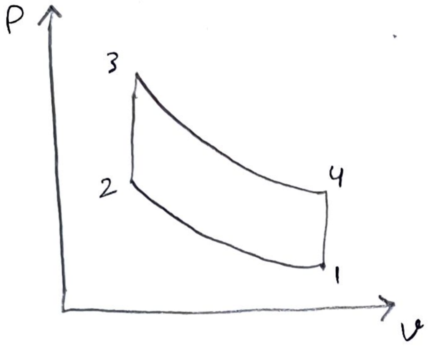
25.33°C & 24.66°C

Q-1. Polytropic compression & expansion. $n = 1.3$

Closed system so use u instead of h.

$\frac{v_1}{v_2} = r = 9$ $P_1 = 1$ bar $T_1 = 300K$ $V = 2270\ cm^3$

$T_3 = T_{max} = 2000K$



Polytropic:

$$\frac{T_2}{T_1} = \left[\frac{V_1}{V_2}\right]^{n-1} \quad \& \quad \int p\,dV = \frac{P_2V_2 - P_1V_1}{1-n}$$

So

$$\frac{T_2}{T_1} = \left[\frac{V_1}{V_2}\right]^{1.3-1}$$

$$T_2 = T_1 r^{0.3} = (300)(9)^{0.3}$$
$$= 580K$$

$$\frac{T_4}{T_3} = \left[\frac{V_3}{V_4}\right]^{0.3} = T_3\left(\frac{1}{r}\right)^{0.3} = 2000\left(\frac{1}{9}\right)^{0.3}$$

$$T_4 = 1035K$$

Using Air table: A.22

$u_1 = 214.07$ KJ/K @ 300K

u_2 419.55 KJ/K @ 580K

$u_3 = 1678.7$ KJ/K @ 2000K

$u_4 = 789.05$ KJ/K @ 1035K

Process 1-2:

$\Delta U = Q - W_B$

$$W_B = \int p\,dV = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{RT_2 - RT_1}{1-n} = \frac{(8324/28.97)(580-300)}{1-1.3}$$

$$\boxed{{}_1W_B = -267.9\ KJ/Kg}$$

$u_2 - u_1 + W_B = Q$

$(419.55 - 214.07) - 267.9 = Q$

$$\boxed{{}_1Q_2 = -62.42\ KJ/Kg}$$

Process 2-3:

$\Delta U = Q - \cancel{W}^{\,0}$

$Q = u_3 - u_2 = 1678.7 - 419.55$

$$\boxed{{}_2Q_3 = 1259.15\ \frac{KJ}{kg}}$$

Process 3-4:

$$\Delta U = Q - W \qquad W_B = \int p\,dV = \frac{R(T_4 - T_3)}{1-n} = \frac{\frac{8.314}{28.97}(1035-2000)}{-0.3}$$

$$W_B = 923.1 \text{ kJ/kg}$$

$$Q = \Delta U + W$$

$$Q = (\overset{\checkmark}{u_4} - \overset{\checkmark}{u_3}) + 923.1$$

$$\boxed{{}_3Q_4 = 33.45 \text{ kJ/kg}}$$

Process 4→1:

$$\Delta U = Q - \cancelto{0}{W}$$

$${}_4Q_1 = u_1 - u_4$$

$$\boxed{{}_4Q_1 = -574.98 \text{ kJ/kg}}$$

b)- $\eta_T = 1 - \frac{Q_{out}}{Q_{in}} = \frac{W_{cycle}}{Q_{in}} = \frac{\overset{\checkmark}{{}_1W_2} + \cancelto{0}{{}_2W_3} + \overset{\checkmark}{{}_3W_4} + \cancelto{0}{{}_4W_1}}{\overset{\checkmark}{{}_2Q_3} + \overset{\checkmark}{{}_3Q_4}}$

$$= 0.507 \text{ or } 50.7\%$$

c)- $m.e.p = \frac{W_{cycle}}{v_1 - v_2} = \frac{W_{cycle}}{v_1\left(1 - \frac{v_2}{v_1}\right)}$

$$= \frac{W_{cycle}}{v_1\left(1 - \frac{1}{r}\right)}$$

$$v_1 = \frac{RT_1}{P_1}$$

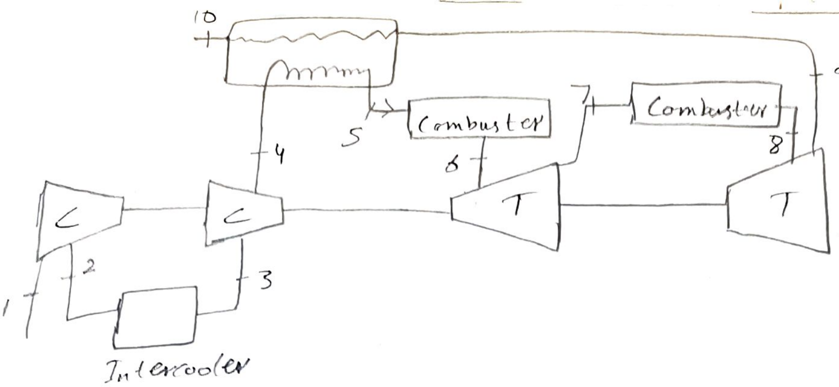
$$= \frac{0.287(300)}{10^5 \text{ Pa}}$$

$$= 0.861 \text{ m}^3/\text{kg}$$

$$m.e.p = \frac{-267.9 + 923.1}{0.861\left(1 - \frac{1}{9}\right)}$$

$$= 8.56 \times 10^5 \text{ Pa}$$

$$\boxed{= 8.56 \text{ bars}}$$



$T_1 = 300\,K$

$P_1 = 100\,KPa$

$\dot{m} = 6\,kg/s$

$\frac{P_4}{P_3} = \frac{P_2}{P_1} = 10 = c.p.r$

$T_3 = 300K = T_1$

$T_6 = T_8 = 1400K$

$\eta_t = \eta_{com} = \eta_{regen} = 0.8$

$K = 1.4$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}$$

$$T_{2s} = (10)^{\frac{1.4-1}{1.4}}(300)$$

$$T_{2s} = 416.85\,K$$

$$\eta_{comp} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{\cancel{\dot{m}c_p}(T_{2s} - T_1)}{\cancel{\dot{m}c_p}(T_2 - T_1)} = \frac{T_{2s} - T_1}{T_2 - T_1} = 0.8$$

$$\boxed{T_2 = 446.1\,K}$$

$$T_1 = T_3 \quad , \quad \frac{P_2}{P_1} = \frac{P_4}{P_3}$$

$$\boxed{T_2 = T_4 = 446.1K}$$

$$\frac{T_{7s}}{T_6} = \left(\frac{P_7}{P_6}\right)^{\frac{K-1}{K}}$$

$$T_{7s} = T_6\left(\frac{P_7}{P_6}\right)^{\frac{1.4-1}{1.4}}$$

$$T_{7s} = (1400)\left(\frac{1}{10}\right)^{0.4/1.4}$$

$$\boxed{T_{7s} = 1007.6\,K}$$

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = \frac{\cancel{\dot{m}c_p}(T_6 - T_7)}{\cancel{\dot{m}c_p}(T_6 - T_{67})}$$

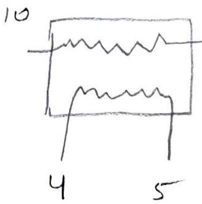
$$\boxed{T_7 = 1086.1K} = T_9$$

$$\eta_{regenerator} = \frac{h_x - h_2}{h_4 - h_2} = \frac{h_5 - h_4}{h_9 - h_4}$$

$$\eta_{regen.} = \frac{T_5 - T_4}{T_9 - T_4}$$

$$T_5 = T_4 + \eta_{regen}(T_9 - T_4)$$

$$\boxed{T_5 = 958.1\,K}$$



$$h_{10} - h_9 = h_5 - h_4$$

$$T_{10} - T_9 = T_5 - T_4$$

$$\boxed{T_{10} = 574.1\,K}$$

$$\eta_t = 1 - \frac{Q_{out}}{Q_{in}} \Rightarrow Q_{in} = Q_{56} + Q_{78}$$

c_p from table

$$= \dot{m}c_p[(T_6 - T_5) + (T_8 - T_7)]$$

$$= 4557.5\,kJ/s$$

$$= 1 - \frac{2533.8}{4557.5}$$

$$\boxed{= 44.4\%}$$

$$Q_{out} = \dot{m}c_p[(T_{10} - T_1) + (T_2 - T_3)]$$

$$= 2533.8\,kJ/s$$

b)- Back work ratio $= \dfrac{\dot{W}_{comp}}{\dot{W}_{Turbine}}$

$\dot{W}_T = 2 W_T$ since identical Turbines

$= 2 \dot{m} C_p (T_8 - T_7)$

$= 2(6)(1.005)(1400 - 1088.1)$

$= 3785.6$ KJ/s

$\dot{w}_{comp} = 2 \dot{m} C_p (T_2 - T_1)$

$= 1762.0$

bwr $= \dfrac{|1762|}{|3785.6|} = 0.4654$

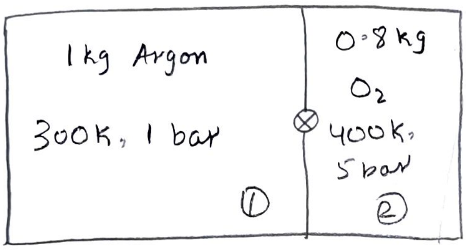
c)- Net power $= \dot{W}_T - \dot{W}_c$

$= 3785.6 - 1762$

$= 2023.6 \frac{KJ}{s}$ [KW]

Ch 12:

3)-



$T_f = 87°C = 360K$

a) $P_1 V_1 = mRT$

$V_1 = \dfrac{mRT}{P_1} = \dfrac{(1\ kg)\left(\frac{R}{Mr_{Ar}}\right)T}{P_1}$

$= \dfrac{1\left(\frac{8.314}{39.94}\right)(300)}{10^5\ Pa} = 0.624\ m^3$

KN.m → KJ

KPa → $\frac{KN}{m^2}$

$V_2 = \dfrac{m\left(\frac{8.314}{32}\right)(400)}{5 \times 10^5\ Pa} = 0.166 m^3$

b)- $P_f V_f = n_f R T_f$

$n_f = n_{O_2} + n_{Ar}$

$= \dfrac{0.8}{32} + \dfrac{1}{39.94}$

$= 0.05$ kmol

$P_f = \dfrac{n_f R T_f}{V_f}$

$= \dfrac{0.05(8.314)(360)}{0.166 + 0.624}$

$= 1.89 \times 10^5 \frac{N}{m^2}$ $\boxed{= 1.89\ bar}$

c) - $\Delta U = Q - \cancelto{0}{W}$

$C_p = h$

$C_v = u$

$Q = \Delta u$

$Q = \Delta U_{O_2} + \Delta U_{Ar}$

$\Delta U_{O_2} = n_{O_2} \bar{C}_{v_{O_2}} [T_f - T_{O_2}]$ or $m_{O_2} C_v \Delta T \rightarrow$ Table A.20

$\Delta U_{Ar} = n_{Ar} \bar{C}_{v_{Ar}} [T_f - T_{Ar}]$ or $m_{Ar} C_v \Delta T$ (inert noble gas) $\boxed{\bar{C}_p = \frac{5}{2} \bar{R}}$

$\boxed{\bar{C}_v = \bar{C}_p - \bar{R}}$

$\boxed{Q = -2.94 \text{ KJ}}$

$C_v = \frac{5}{2} \bar{R} - \bar{R}$

$C_v = 1.5 \bar{R}$