AE1104 Physics 1

i

Summaries from, Thermodynamics: An Engineering Approach (Sixth Edition)

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Introduction and Basic Concepts

In this chapter, the basic concepts of thermodynamics are introduced and discussed. Thermodynamics is the science that primarily deals with energy. The *first law of thermodynamics* is simply an expression of the conservation of energy principle, and it asserts that *energy* is a thermodynamic property. The *second law of thermodynamics* asserts that energy has *quality* as well as *quantity*, and actual processes occur in the direction of decreasing quality of energy.

A system of fixed mass is called a *closed system*, or *control mass*, and a system that involves mass transfer across its boundaries is called an *open system*, or *control volume*. The mass-dependent properties of a system are called *extensive properties* and the others *intensive properties*. *Density* is mass per unit volume, and *specific volume* is volume per unit mass.

A system is said to be in *thermodynamic equilibrium* if it maintains thermal, mechanical, phase and chemical equilibrium. Any change from one state to another is called a *process*. A process with identical end states is called a *cycle*. During a *quasi-static* or *quasi-equilibrium process*, the system remains practically in equilibrium at all times. The state of a simple, compressible system is completely specified by two independent, intensive properties.

The *zeroth law of thermodynamics* states that two bodies are in thermal equilibrium if both have the same temperature reading even if they are not in contact.

The temperature scales used in the SI and English system today are the *Celsius scale* and the *Fahrenheit scale*, respectively. They are related to absolute temperature scales by

$$T(K) = T(^{\circ}C) + 273.15$$

 $T(R) = T(^{\circ}F) + 459.67$

The magnitudes of each division of 1 K and 1 °C are identical, and so are the magnitudes of each division of 1 R and 1 °F. Therefore,

$$\Delta T(K) = \Delta T(^{\circ}C)$$
 and $\Delta T(R) = \Delta T(^{\circ}F)$

The normal force exerted by a fluid per unit area is called *pressure*, and its unit is the *pascal*, $1 Pa = 1 N/m^2$. The pressure relative to absolute vacuum is called the *absolute pressure*, and the difference between the absolute pressure and the local atmospheric pressure is called the *gage pressure*. Pressures below atmospheric pressure are called *vacuum pressures*. The absolute, gage, and vacuum pressures are related by

$$P_{gage} = P_{abs} - P_{atm} \qquad \text{(for pressures above } P_{atm}\text{)}$$
$$P_{vac} = P_{atm} - P_{abs} \qquad \text{(for pressures below } P_{atm}\text{)}$$

The pressure at a point in a fluid has the same magnitude in all directions. the variation of pressure with elevation is given by

$$\frac{dP}{dz} = -\rho g$$

where the positive z direction is taken to be upward. When the density of the fluid is constant, the pressure difference across a fluid layer of thickness Δz is

$$\Delta P = P_2 - P_1 = \rho g \Delta z$$

The absolute and gage pressures in a liquid open to the atmosphere at a depth h from the free surface are

$$P = P_{atm} + \rho gh$$
 or $P_{gage} = \rho gh$

Small to moderate pressure differences are measured by a *manometer*. The pressure in a stationary fluid remains constant in the horizontal direction. *Pascal's principle* states that the pressure applied to a confined fluid increases the pressure thoughout by the same amount. the atmospheric pressure is measured by a *barometer* and is given by

$$P_{atm} = \rho g h$$

where h is the height of the liquid column.

Energy, Energy Transfer, and General Energy Analysis

The sum of all forms of energy of a system is called *total energy*, which consists of internal, kinetic, and potential energy for simple compressible systems. *Internal energy* represents the molecular energy of a system and may exist in sensible, latent, chemical, and nuclear forms.

Mass flow rate in is defined as the amount of mass flowing through a cross section per unit time. It is related to the *Volume flow rate* \dot{V} , which is the volume of a fluid flowing through a cross section per unit time, by

$$\dot{m} = \rho \dot{V} = \rho A_c V_{avg} \qquad \left(\frac{kg}{s}\right)$$

The energy flow rate associated with a fluid flowing at a rate of \dot{m} is

$$\dot{E} = \dot{m}e \qquad \left(\frac{kJ}{s} \quad \text{or} \quad kW\right)$$

which is analogous to E = me.

The mechanical energy is defined as the form of energy that can be converted to mechanical work completely and directly by a mechanical device such as an ideal turbine. It is expressed on a unit mass basis and rate form as

$$e_{mech} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$

and

$$\dot{E}_{mech} = \dot{m}e_{mech} = \dot{m}\left(\frac{P}{\rho} + \frac{V^2}{2} + gz\right)$$

where P/ρ is the *flow energy*, $V^2/2$ is the *kinetic energy*, and *gz* is the *potential energy* of the fluid per unit mass.

Energy can cross the boundaries of a closed system in the form of heat or work. For control volumes, energy can also be transported by mass. If the energy transfer is due to a temperature difference between a closed system and its surroundings, it is *heat*; otherwise, it is *work*.

Work is the energy transfered as a force acts on a system through a distance. Various forms of work are expressed as follows:

Electrical work:
$$W_e = VI\Delta t$$
Shaft work: $W_{sh} = 2\pi nT$ Spring work: $W_{spring} = \frac{1}{2}k(x_2^2 - x_1^2)$

The *first law of thermodynamics* is essentially an expression of the conservation of energy principle, also called the *energy balance*. The general mass and energy balances for *any system* undergoing *any process* can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{but transfer hubbert work and more }} = \underbrace{\Delta E_{system}}_{\text{classified on the set of t$$

Net energy transfer by heat, work, and mass Changes in internal, kinetic, potential, etc., energies

It can also be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{orrunt transfer by best work, and mass}} = \underbrace{\frac{\Delta E_{system}}{dt}}_{\text{(kW)}}$$

Rate of net energy transfer by heat, work, and mass

Rate of changes in internal, kinetic, potential, etc., energies

The efficiencies of various devices are defined as

$$\begin{split} \eta_{pump} &= \frac{\Delta \dot{E}_{mech,fluid}}{\dot{W}_{shaft,in}} = \frac{\dot{W}_{pump,u}}{\dot{W}_{pump}} \\ \eta_{turbine} &= \frac{\dot{W}_{shaft,out}}{|\Delta \dot{E}_{mech,fluid}|} = \frac{\dot{W}_{turbine,e}}{\dot{W}_{turbine,e}} \\ \eta_{motor} &= \frac{\text{Mechanical power output}}{\text{Electrical power input}} = \frac{\dot{W}_{shaft,out}}{\dot{W}_{elect,in}} \\ \eta_{generator} &= \frac{\text{Electrical power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{elect,out}}{\dot{W}_{shaft,in}} \\ \eta_{pump-motor} &= \eta_{pump}\eta_{motor} = \frac{\dot{W}_{pump,u}}{\dot{W}_{elect,in}} = \frac{\Delta \dot{E}_{mech,fluid}}{\dot{W}_{elect,in}} \\ \eta_{turbine-generator} &= \eta_{turbine}\eta_{generator} = \frac{\dot{W}_{elect,out}}{\dot{W}_{turb,in}} = \frac{\dot{W}_{elect,out}}{|\Delta \dot{E}_{mech,fluid}|} \end{split}$$

The conversion of energy from one form to another is often associated with adverse effects on the environment, and environmental impact should be an important consideration in the conversion and utilization of energy.

Properties of Pure Substances

A substance that has a fixed chemical composition throughout is called a *pure substance*. A pure substance exists in different phases depending on its energy level. In the liquid phase, a substance that is not about to vaporize is called a *compressed* or *subcooled liquid*. In the gas phase, a substance that is not about to condense is called a *superheated vapor*. During a phase-change process, the temperature and pressure of a pure substance are dependent properties. At a given pressure, a substance changes phase at a fixed temperature, called the *saturation temperature*. Likewise, at a given temperature, the pressure at which a substance changes phase is called the *saturation pressure*. During a boiling process, both the liquid and the vapor phases coexist in equilibrium, and under this condition the liquid is called *saturated liquid* and the vapor *saturated vapor*.

In a saturated liquid-vapor mixture, the mass fraction of vapor is called the *quality* and is expressed as

$$x = \frac{m_{vapor}}{m_{total}}$$

Quality may have values between 0 (saturated liquid) and 1 (saturated vapor). It has no meaning in the compressed liquid or superheated vapor regions. In the saturated mixture region, the average value of any intensive property y is determined from

$$y = y_f + x y_{fg}$$

where f stands for saturated liquid and g for saturated vapor. In the absence of compressed liquid data, a general approximation is to treat a compressed liquid as a saturated liquid at the given *temperature*,

$$y \cong y_{f@T}$$

Where *y* is *v*, *u* or *h*.

The state beyond which there is no distinct vaporization process is called the *critical point*. At supercritical pressures, a substance gradually and uniformly expands from the liquid to vapor phase. All three phases of a substance coexist in equilibrium at states along the *triple line* characterized by triple-line temperatures and pressures. The compressed liquid has lower v, u, and h values than the saturated liquid at the same T or P. Likewise, superheated vapor has higher v, u, and h values than the saturated vapor at the same T or P.

Any relation among the pressure, temperature, and specific volume of a substance is called an *equation of state*. The simplest and best-known equation of state is the *ideal-gas equation of state*, given as

$$Pv = RT$$

where R is the gas constant. Caution should be exercised in using this relation since an ideal gas is a fictitious substance. Real gases exhibit ideal-gas behavior at relatively low pressures and high temperatures.

The deviation from ideal-gas behavior can be properly accounted for by using the compressibility factor Z, defined as

$$Z = \frac{Pv}{RT}$$
 or
$$Pv = ZRT$$

the Z factor is approximately the same for all gases at the same reduced temperature and reduced pressure, which are defined as

$$P_R = \frac{P}{P_{cr}}$$
 and $T_R = \frac{T}{T_{cr}}$

where P_{cr} and T_{cr} are the critical pressure and temperature, respectively. This is known as the principle of corresponding states. When either P or T is unknown, it can be determined from the compressibility chart with the help of the pseudo-reduced specific volume, defined as

$$v_R = \frac{v_{actual}}{RT_{cr}/P_{cr}}$$

The P - v - T behavior of substances can be represented more accurately by more complex equations of state. Three of the best known are Van der Waals:

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

$$a = \frac{2/R^2 I_{cr}^2}{64P_{cr}}$$
 and $b = \frac{RT_{cr}}{8P_{cr}}$

Beattie-Bridgeman:

$$P = \frac{R_u T}{\bar{v}^2} \left(1 - \frac{c}{\bar{v}T^3} \right) \left(\bar{v} + B \right) - \frac{A}{\bar{v}^2}$$

where

where

$$A = A_0 \left(1 - \frac{a}{\bar{v}}\right)$$
 and $B = B_0 \left(1 - \frac{b}{\bar{v}}\right)$

Benedict-Webb-Rubin:

$$P = \frac{R_u T}{\bar{v}} + \left(B_0 R_u T - A_0 - \frac{C_0}{T^2}\right) \frac{1}{\bar{v}^2} + \frac{b R_u T - a}{\bar{v}^3} + \frac{a \alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3 T^2} \left(1 + \frac{\gamma}{\bar{v}^2}\right) e^{-\frac{\gamma}{\bar{v}^2}}$$

where R_u is the universal gas constant and \bar{v} is the molar specific volume.

Energy Analysis of Closed Systems

Work is the energy transferred as a force acts on a system through a distance. The most common form of mechanical work is the *boundary work*, which is the work associated with the expansion and compression of substances. On a P - V diagram, the area under the process curve represents the boundary work for a quasi-equilibrium process. Various forms of boundary work are expressed as follows:

General

$$W_b = \int_1^2 P dV \qquad (kJ)$$

Isobaric process

$$W_b = P_0(V_2 - V_1)$$
 $(P_1 = P_2 = P_0 = \text{ constant})$

Polytropic process

$$W_b = \frac{P_2 V_2 - P_1 V_1}{1 - n} \quad (n \neq 1) \quad (PV^n = \text{ constant})$$

Isothermal process of an ideal gas

 $W_b = P_1 V_1 \ln \frac{V_2}{V_1} = mRT_0 \ln \frac{V_2}{V_1}$ (PV = mRT_0 = constant)

The first law of thermodynamics is essentially an expression of the conservation of energy principle, also called the energy balance. The general energy balances for *any system* undergoing *any process* can be expressed as

$$\underbrace{E_{in} - E_{out}}_{C_{in} - E_{out}} = \underbrace{\Delta E_{system}}_{KJ}$$

Net energy transfer by heat, work, and mass Changes in internal, kinetic, potential, etc., energies

It can also be expressed in the *rate form* as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{dt} = \underbrace{\frac{\Delta E_{system}}{dt}}_{dt}$$
(kW)

Rate of net energy transfer by heat, work, and mass

Rate of changes in internal, kinetic, potential, etc., energies

Taking heat transfer *to* the system and work done *by* the system to be positive quantities, the energy balance for a closed system can also be expressed as

$$Q - W = \Delta U + \Delta KE + \Delta PE \qquad (kJ)$$

Where
$$W = W_{other} + W_b$$
$$\Delta U = m(u_2 - u_1)$$
$$\Delta KE = \frac{1}{2}m(V_2^2 - V_1^2)$$
$$\Delta PE = mg(z_2 - z_1)$$

For a *constant-pressure process*, $W_b + \Delta U = \Delta H$. Thus,

$$Q - W_{other} = \Delta H + \Delta K E + \Delta P E \qquad (kJ)$$

The amount of energy needed to raise the temperature of a unit mass of a substance by one degree is called the *specific heat at constant volume* c_v for a constant-volume process and the *specific heat at constant pressure* c_P for a constant-pressure process. They are defined as

$$c_v = \left(\frac{\delta u}{\delta T}\right)_v$$
 and $c_P = \left(\frac{\delta h}{\delta T}\right)_P$

For ideal gases u, h, c_v , and c_P are functions of temperature alone. The Δu and Δh of ideal gases are expressed as

$$\Delta u = u_2 - u_1 = \int_{1}^{2} c_v(T) dT \cong c_{v,avg}(T_2 - T_1)$$

and

$$\Delta h = h_2 - h_1 = \int_{1}^{2} c_P(T) dT \cong c_{P,avg}(T_2 - T_1)$$

For ideal gases, c_v and c_P are related by

$$c_P = c_v + R \qquad \left(\frac{kJ}{kg \cdot K}\right)$$

where R is the gas constant. The *specific heat ratio* k is defined as

$$k = \frac{c_P}{c_v}$$

For *incompressible substances* (liquids and solids), both the constant-pressure and constant-volume specific heats are identical and denoted by *c*:

$$c_P = c_v = c \qquad \left(\frac{kJ}{kg \cdot K}\right)$$

The Δu and Δh of incompressible substances are given by

$$\Delta u = u_2 - u_1 = \int_{1}^{2} c(T) dT \cong c_{avg}(T_2 - T_1) \qquad \left(\frac{kJ}{kg}\right)$$
$$\Delta h = \Delta u + v \Delta P \qquad \left(\frac{kJ}{kg}\right)$$

Mass and Energy Analysis of Control Volumes

The *conservation of mass principle* states that the net mass transfer to or from a system during a process is equal to the net change (increase or decrease) in the total mass of the system during that process, and is expressed as

$$m_{in} - m_{out} = \Delta m_{system}$$
 and $\dot{m}_{in} - \dot{m}_{oud} = \frac{dm_{system}}{dt}$

where $\Delta m_{system} = m_{final} - m_{initial}$ is the change in the mass of the system during the process, \dot{m}_{in} and \dot{m}_{out} are the total rates of mass flow into and out of the system, and dm_{system}/dt is the rate of change of mass within the system boundaries. The relations above are also referred to as the *mass balance* and are applicable to any system undergoing any kind of process.

The amount of mass flowing through a cross section per unit time is called the *mass flow rate*, and is expressed as

$$\dot{m} = \rho V A$$

where ρ = density of fluid, V = average fluid velocity normal to A, and A = cross-sectional area normal to flow direction. The volume of the fluid flowing through a cross section per unit time is called the *volume flow rate* and is expressed as

$$\dot{V} = VA = \frac{\dot{m}}{\rho}$$

The work required to push a unit mass of fluid into or out of a control volume is called *flow work* or *flow energy*, and is expressed as $w_{flow} = Pv$. In the analysis of control volumes, it is convenient to combine the flow energy and internal energy into *enthalpy*. Then the total energy of a flowing fluid is expressed as

$$\theta = h + ke + pe = h + \frac{V^2}{2} + gz$$

The total energy transported by a flowing fluid of mass *m* with uniform properties is $m\theta$. The rate of energy transport by a fluid with a mass flow rate of \dot{m} is $\dot{m}\theta$. When the kinetic and potential energies of a fluid stream are negligible, the amount and rate of energy transport become $E_{mass} = mh$ and $\dot{E}_{mass} = \dot{m}h$, respectively.

The *first law of thermodynamics* is essentially an expression of the conservation of energy principle, also called the *energy balance*. The general mass and energy balances for *any system* undergoing *any process* can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Changes in internal, kinetic, potential, etc., energies}}$$

It can also be expressed in the *rate form* as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{het energy transfer by heat work and mass}} = \underbrace{\frac{\Delta E_{system}}{dt}}_{\text{transfer by heat work and mass}}$$

Rate of net energy transfer by heat, work, and n

Rate of changes in internal, kinetic, potential, etc., energies

Thermodynamic processes involving control volumes can be considered in two groups: steady-flow processes and unsteady-flow processes. During a steady-flow process, the fluid flows through the control volume steadily, experiencing no change with time at a fixed position. The mass and energy content of the control volume remain constant during a steady-flow process. Taking heat transfer to the system and work done by the system to be positive quantities, the conservation of mass and energy equations for steady-flow processes are expressed as

$$\dot{Q} - \dot{W} = \sum_{out} \frac{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}{\int_{for \ each \ exit}} - \sum_{in} \frac{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}{\int_{for \ each \ inlet}}$$

These are the most general forms of the equations for steady-flow processes. For single-stream (one-inlet-one-exit) systems such as nozzles, diffusers, turbines, compressors, and pumps, they simplify to

$$\dot{m}_1 = \dot{m}_2 \to \frac{1}{v_1} V_1 A_1 = \frac{1}{v_2} V_2 A_2$$
$$\dot{Q} - \dot{W} = \dot{m} \left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$

In these relations, subscripts 1 and 2 denote the inlet and exit states, respectively.

Most unsteady-flow processes can be modeled as a *uniform-flow process*, which requires that the fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties to not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process. When kinetic and potential energy changes associated with the control volume and the fluid streams are negligible, the mass and energy balance relations for a uniform-flow system are expressed as

$$m_{in} - m_{out} = \Delta m_{system}$$

 $Q - W = \sum_{out} mh - \sum_{in} mh + (m_2 u_2 - m_1 u_1)_{system}$

where $Q = Q_{net,in} = Q_{in} - Q_{out}$ is the net heat input and $W = W_{net,out} = W_{out} - W_{in}$ is the net work output.

When solving thermodynamic problems, it is recommended that the general form of the energy balance $E_{in} - E_{out} = \Delta E_{system}$ be used for all problems, and simplify it for the particular problem instead of using the specific relations given here for different processes.

The Second Law of Thermodynamics

The *second law of thermodynamics* states that processes occur in a certain direction, not any direction. A process does not occur unless it satisfies both the first and the second laws of thermodynamics. Bodies that can absorb or reject finite amounts of heat isothermally are called *thermal energy reservoirs* or *heat reservoirs*.

Work can be converted to heat directly, but heat can be converted to work only by some devices called *heat engines*. The *thermal efficiency* of a heat engine is defined as

$$\eta_{th} = \frac{W_{net,out}}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

where $W_{net,out}$ is the net work output of the heat engine, Q_H is the amount of heat supplied to the engine, and Q_L is the amount of heat rejected by the engine.

Refrigerators and heat pumps are devices that absorb heat from low-temperature media and reject it to higher-temperature ones. The performance of a refrigerator or a heat pump is expressed in terms of the *coefficient of performance*, which is defined as

$$COP_{R} = \frac{Q_{L}}{W_{net,in}} = \frac{1}{\frac{Q_{H}}{Q_{L}} - 1}$$
$$COP_{HP} = \frac{Q_{H}}{W_{net,in}} = \frac{1}{1 - \frac{Q_{L}}{Q_{H}}}$$

The *Kelvin-Planck statement* of the second law of thermodynamics states that no heat engine can produce a net amount of work while exchanging heat with a single reservoir only. The *Clausius statement* of the second law states that no device can transfer heat from a cooler body to a warmer one without leaving an effect on the surroundings.

Any device that violates the first or the second law of thermodynamics is called a *perpetual-motion machine*.

A process is said to be *reversible* if both the system and the surroundings can be restored to their original conditions. Any other process is *irreversible*. The effects such as friction, non-quasi-equilibrium expansion or compression, and heat transfer through a finite temperature difference render a process irreversible an are called *irreversibilities*.

The *Carnot cycle* is a reversible cycle that is composed of four reversible processes, two isothermal and two adiabatic. The *Carnot principles* state that the thermal efficiencies of all reversible heat engines operating between the same two reservoirs are the same, and that no heat engine is more efficient than a reversible one operating between the same two reservoirs. These statements form the basis for establishing a *thermodynamic temperature scale* related to the heat transfers between a

reversible device and the high- and low-temperature reservoirs by

$$\left(\frac{Q_H}{Q_L}\right)_{rev} = \frac{T_H}{T_H}$$

therefore, the Q_H/Q_L ratio can be replaced by T_H/T_L for reversible devices, where T_H and T_L are the absolute temperatures of the high- and low-temperature reservoirs, respectively.

A heat engine that operates on the reversible Carnot cycle is called a *Carnot heat engine*. The thermal efficiency of a Carnot heat engine, as well as all other reversible heat engines is given by

$$\eta_{th,rev} = 1 - \frac{T_L}{T_H}$$

This is the maximum efficiency a heat engine operating between two reservoirs at temperatures T_H and T_L can have.

The COPs of reversible refrigerators and heat pumps are given in a similar manner as

$$COP_{R,rev} = rac{1}{rac{T_{H}}{T_{L}}-1}$$
 and $COP_{HP,rev} = rac{1}{1-rac{T_{L}}{T_{H}}}$

Again, these are the highest *COP*s a refrigerator or a heat pump operating between the temperature limits of T_H and T_L can have.

Entropy

The second law of thermodynamics leads to the definition of a new property called *entropy*, which is a quantitative measure of microscopic disorder for a system. Any quantity whose cyclic integral is zero is called a property, and entropy is defined as

$$dS = \left(\frac{dQ}{T}\right)_{int,rev}$$

For the special case of an internally reversible, isothermal process, it gives

$$\Delta S = \frac{Q}{T_0}$$

The inequality part of the Clausius inequality combined with the definition of entropy yields an inequality known as the *increase of entropy principle*, expressed as

$$S_{gen} \geq 0$$

where S_{gen} is the *entropy generated* during the process. Entropy change is caused by heat transfer, mass flow, and irreversibilities. Heat transfer to a system increases the entropy, and heat transfer from a system decreases it. The effect of irreversibilities is always to increase the entropy. The *entropy-change* and *isentropic relations* for a process can be summarized as follows

1. Pure substances:

Any process:
$$\Delta s = s_2 - s_1$$

Isentropic process: $s_2 = s_1$

2. Incompressible substances:

Any process:
$$s_2 - s_1 = c_{avg} \ln \frac{T_2}{T_1}$$

Isentropic process: $T_2 = T_1$

- 3. Ideal gases:
 - a) Constant specific heats (approximate treatment):

 $s_2 - s_1 = c_{v,avg} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$ $s_2 - s_1 = c_{P,avg} \ln \frac{T_2}{T_1} + R \ln \frac{P_2}{P_1}$ Any process: $\left(\frac{T_2}{2}\right) = \left(\frac{v_1}{2}\right)^{k-1}$ Isentropic process: 1

$$\begin{pmatrix} T_1 \\ s = constant \end{pmatrix}_{s=constant} = \begin{pmatrix} v_2 \\ v_2 \end{pmatrix}^{\frac{k-1}{k}}$$
$$\begin{pmatrix} \frac{T_2}{T_1} \\ s = constant \\ \begin{pmatrix} \frac{P_2}{P_1} \\ s = constant \end{pmatrix}_{s=constant} = \begin{pmatrix} \frac{v_1}{v_2} \end{pmatrix}^k$$

b) Variable specific heats (exact treatment):

Any process:

$$s_{2} - s_{1} = s_{2}^{\circ} - s_{1}^{\circ} - R \ln \frac{P_{2}}{P_{1}}$$
Isentropic process:

$$s_{2}^{\circ} = s_{1}^{\circ} + R \ln \frac{P_{2}}{P_{1}}$$

$$\left(\frac{P_{2}}{P_{1}}\right)_{s=constant} = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{\nu_{2}}{\nu_{1}}\right)_{s=constant} = \frac{\nu_{r2}}{\nu_{r1}}$$

Where P_r is the *relative pressure* and v_r is the *relative specific volume*. The function s° depends on temperature only.

The steady-flow work for a reversible process can be expressed in terms of the fluid properties as

$$w_{rev} = -\int_{1}^{2} v dP - \Delta ke - \Delta pe$$

For incompressible substances (v = constant) it simplifies to

$$w_{rev} = -v(P_2 - P_1) - \Delta ke - \Delta pe$$

The work done during a steady-flow process is proportional to the specific volume. Therefore, vshould be kept as small as possible during a compression process to minimize the work input and as large as possible during an expansion process to maximize the work output.

The reversible work inputs to a compressor compressing an ideal gas from T_1 , P_1 to P_2 in an isentropic $(Pv^k = \text{constant})$, polytropic $(Pv^n = \text{constant})$ or isothermal (Pv = constant) manner, are determined by integration for each case with the following results:

Isentropic:
$$w_{comp,in} = \frac{kR(T_2 - T_1)}{k-1} = \frac{kRT_1}{k-1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right]$$

Polytropic: $w_{comp,in} = \frac{nR(T_2 - T_1)}{n-1} = \frac{nRT_1}{n-1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$
Isothermal: $w_{comp,in} = RT \ln \frac{P_2}{P_1}$

The work input to a compressor can be reduced by using multistage compression with intercooling. For maximum savings from the work input, the pressure ratio across each stage of the compressor must be the same.

Most steady-flow devices operate under adiabatic conditions, and the ideal process for these devices is the isentropic process. The parameter that describes how efficiently a device approximates a corresponding isentropic device is called *isentropic* or *adiabatic efficiency*. It is expressed for turbines, compressors, and nozzles as follows:

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$
$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$
$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2} \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

In the relations above, h_{2a} and h_{2s} are the enthalpy values at the exit state for actual and isentropic *processes*, respectively.

The entropy balance for any system undergoing any process can be expressed in the general form as

$$\underbrace{S_{in} - S_{out}}_{\text{transfer by heat and mass}} + \underbrace{S_{gen}}_{\text{transfer by heat and mass}} = \underbrace{\Delta S_{system}}_{\text{transfer by heat and mass}}$$

Net entropy transfer by heat and mass Entropy generation Change in entropy

or, in rate form, as

$$\frac{\dot{S}_{in} - \dot{S}_{out}}{dt} + \frac{\dot{S}_{gen}}{dt} = \frac{dS_{system}}{dt}$$

Rate of net entropy transfer by heat and mass Rate of entropy generation Rate of change in entropy

For a general stead-flow process it simplifies to

$$\dot{S}_{gen} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

Exergy: A Measure of Work Potentional

Not mandatory for the exam

Gas Power Cycles

A cycle during which a net amount of work is produced is called a *power cycle*, and a power cycle during which the working fluid remains a gas throughout is called a *gas power cycle*. The most efficient cycle operating between a heat source at temperature T_H and a sink at temperature T_L is the Carnot cycle, and its thermal efficiency is given by

$$\eta_{th,Carnot} = 1 - \frac{T_L}{T_H}$$

The actual gas cycles are rather complex. The approximations used to simplify the analysis are known as the *air-standard assumptions*. Under these assumptions, all the processes are assumed to be internally reversible; the working fluid is assumed to be air, which behaves as an ideal gas; and the combustion and exhaust processes are replaced by heat-addition and heat-rejection processes, respectively. The air-standard assumptions are called *cold-air-standard assumptions* if air is also assumed to have constant specific heats at room temperature.

In reciprocating engines, the *compression ratio r* and the *mean effective pressure MEP* are defined as

$$r = \frac{V_{max}}{V_{min}} = \frac{V_{BDC}}{V_{TDC}}$$
$$MEP = \frac{w_{net}}{v_{max} - v_{min}}$$

The *Otto cycle* is the ideal cycle for the spark-ignition reciprocating engines, and it consists of four internally reversible processes: isentropic compression, constant-volume heat-addition, isentropic expansion, and constant-volume heat-rejection. Under cold-air-standard assumptions, the thermal efficiency of the ideal Otto cycle is

$$\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}}$$

where *r* is the compression ratio and *k* is the specific heat ratio c_P/c_v .

The *Diesel cycle* is the ideal cycle for the compression-ignition reciprocating engines. It is very similar to the Otto cycle, except that the constant-volume heat-addition process is replaced by a constant-pressure heat-addition process. Its thermal efficiency under cold-air-standard assumptions is

$$\eta_{th,Diesel} = 1 - \frac{1}{r^{k-1}} \left[\frac{r_c^k - 1}{k(r_c - 1)} \right]$$

where r_c is the *cutoff ratio*, defined as the ratio of the cylinder volumes after and before the combustion process.

Stirling and *Ericssons cycles* are two totally reversible cycles that involve an isothermal heat-addition process at T_H and an isothermal heat-rejection process at T_L . They differ from the Carnot cycle in

that the two isentropic processes are replaced by two constant-volume regeneration processes in the Stirling cycle and by two constant-pressure regeneration processes in the Ericsson cycle. Both cycles utilize *regeneration*, a process during which heat is transferred to a thermal energy storage device (called a *regenerator*) during one part of the cycle that is then transferred back to the working fluid during another part of the cycle.

The ideal cycle for modern gas-turbine engines is the *Brayton cycle*, which is made up of four internally reversible processes: isentropic compression, constant-pressure heat-addition, isentropic expansion, and constant-pressure heat-rejection. Under cold-air-standard assumptions, its thermal efficiency is

$$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

where $r_P = P_{max}/P_{min}$ is the pressure ratio and k is the specific heat ratio. The thermal efficiency of the simple Brayton cycle increases with the pressure ratio.

The deviation of the actual compressor and the turbine from the idealize isentropic ones can be accurately accounted for by utilizing their isentropic efficiencies, defined as

$$\eta_C = rac{w_s}{w_a} \cong rac{h_{2s}-h_1}{h_{2a}-h_1}$$
 and $\eta_T = rac{w_a}{w_s} \cong rac{h_3-h_{4a}}{h_3-h_{4s}}$

where states 1 and 3 are the inlet states, 2a and 4a are the actual exit states, and 2s and 4s are the isentropic exit states.

In gas-turbine engines, the temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor. Therefore, the high-pressure air leaving the compressor can be heated by transferring heat to it from the hot exhaust gases in a counter-flow heat exchanger, which is also known as a *regenerator*. The extent to which a regenerator approaches an ideal regenerator is called the *effectiveness* ε and is defined as

$$\varepsilon = \frac{q_{regen,act}}{q_{regen,max}}$$

Under cold-air-standard assumptions, the thermal efficiency of an ideal Brayton cycle with regeneration becomes

$$\eta_{th,regen} = 1 - \left(\frac{T_1}{T_3}\right) (r_P)^{\frac{k-1}{k}}$$

where T_1 and T_3 are the minimum and maximum temperatures, respectively, in the cycle.

The thermal efficiency of the Brayton cycle can also be increased by utilizing *multistage compression with intercooling, regeneration, and multistage expansion with reheating*. The work input to the compressor is minimized when equal pressure ratios are maintained across each stage. This procedure also maximizes the turbine work output.

Gas-turbine engines are widely used to power aircraft because they are light and compact and have a high power-to-weight ratio. The ideal *jet-propulsion cycle* differs from the simple ideal Brayton cycle in that the gases are partially expanded in the turbine. The gases that exit the turbine at a relatively high pressure are subsequently accelerated in a nozzle to provide the thrust needed to propel the aircraft.

The net thrust developed by the engine is

$$F = \dot{m}(V_{exit} - V_{inlet})$$

where \dot{m} is the mass flow rate of gases, V_{exit} is the exit velocity of the exhaust gases, and V_{inlet} is the inlet velocity of the air, both relative to the aircraft.

The power developed from the thrust of the engine is called the *propulsive power* \dot{W}_p , and it is given by

$$W_P = \dot{m}(V_{exit} - V_{inlet})V_{aircraft}$$

Propulsive efficiency is a measure of how efficiently the energy released during the combustion process is converted to propulsive energy, and it is defined as

$$\eta_P = \frac{\text{Propulsive power}}{\text{Energy input rate}} = \frac{\dot{W}_P}{\dot{Q}_{in}}$$

For an ideal cycle that involves heat transfer only with a source at T_H and a sink at T_L , the exergy destruction is

$$x_{dest} = T_0 \left(\frac{q_{out}}{T_L} - \frac{q_{in}}{T_H} \right)$$