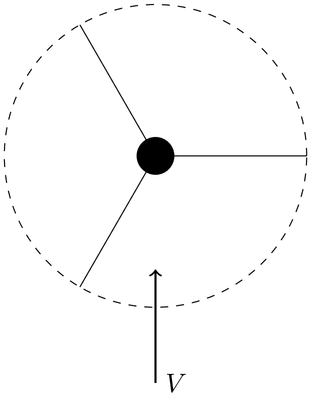
Question 1

1	F	8	F
2	F	9	T
3	T	10	F
4	F	11	T
5	F	12	F
6	F	13	T
7	T	14	F

Question 2

1) See sketch

2) Control volume fixed in space, x coordinate used whose reference does not change. 3) Idealizations:



- Speed V constant in magnitude and direction
- Potential energy changes of the system are negligible
- Constant air density ρ
- Constant efficiency

4) Time basis is a differential interval dt after which we evaluate on rate basis
5) See sketch
6) The energy balance is:

$$đW = dE_k$$

First find the amount of kinetic energy flowing through the windmill per second:

$$dE_k = \frac{1}{2}MV^3 = \frac{1}{2}\rho AV^2dx$$

Then taking the time derivative and using that $A = \pi R^2$:

$$\frac{dE_k}{dt} = \pi\frac{1}{2}\rho R^2V^3$$

The amount of this energy converted into electrical power is found by applying the given efficiency η:

$$\dot{W} = \eta\pi\frac{1}{2}\rho R^2V^3$$

Note that we are given the amount of power for 30 windmills and hence have the equality:

$$\frac{\dot{W}_{\text{wind farm}}}{30} = \eta\pi\frac{1}{2}\rho R^2V^3$$

Solve this equation for R:

$$R = \sqrt{\frac{\dot{W}_{\text{wind farm}}}{15\pi\eta\rho V^3}} = \sqrt{\frac{50\ \text{MW}}{15 \cdot \pi \cdot 0.63 \cdot 1.2\ \frac{\text{kg}}{\text{m}^3} \cdot (7\ \frac{\text{m}}{\text{s}})^3}} = \mathbf{63.97}\ \text{m}$$

Question 3

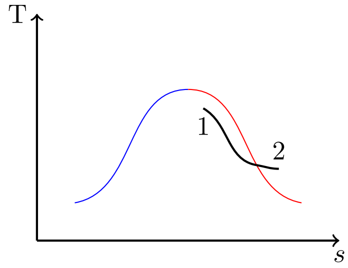


Figure 1: T-s diagram for question 3a

a. Out of saturation dome, along line of constant enthalpy. The process line should be close to the vapour line.

b. 1) Is on the exam sheet 2) Fixed control volume in space 3)

- Only inlets and outlets are the ones shown
- Steady state
- Kinetic and potential energy transfers are negligible
- System is adiabatic

4) Rate basis 5) Already on exam sheet 6) Energy balance for the valve only:

$$\dot{M}h_1 = \dot{M}h_2$$

7) The state is fixed as T_2 and P_2 are known. Finding the value of h_2 requires interpolation. First, the student should compose a table as shown in 1 by finding the enthalpy value at the saturation temperature of pressure $P = 0.2$ MPa for the pressure $P = 0.101235$ MPa. He should show that this is found by:

$$\begin{aligned} h(T = 393.4\ \text{K}, P = 0.101235\ \text{MPa}) &= \frac{h(T = 400\ \text{K}) - h(T = 373.1\ \text{K})}{400\ \text{K} - 373.1\ \text{K}}(393.4\ \text{K} - 373.1\ \text{K}) \\ &+ h(T = 373.1\ \text{K}) \\ &= \frac{2730.3 - 2675.5}{400 - 371.1} \cdot (393.4 - 373.1) + 2675.5 \\ &= 2716.85\ \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

Table 1: Intermediate specific enthalpy values for question 3b.

	T = 393.4 K	T = 400 K
$P = 0.101325$ MPa	2716.85	2730.3
$P = 0.2$ MPa	2706.2	2720.5

Then another interpolation using the table gives the desired enthalpy value:

$$\begin{aligned}h_2(T = 393.4\text{ K}, P = 0.12\text{ MPa}) &= \frac{h(P = 0.2\text{ MPa}) - h(P = 0.101325\text{ MPa})}{0.2\text{ MPa} - 0.101325\text{ MPa}}(0.12\text{ MPa} - 0.101325\text{ MPa}) \\ &+ h(P = 0.101325\text{ MPa}) \\ &= \frac{2706.2 - 2716.85}{0.2\text{ MPa} - 0.101325\text{ MPa}} \cdot (0.12\text{ MPa} - 0.101325\text{ MPa}) + 2716.85 \\ &= 2714.83\ \frac{\text{kJ}}{\text{kg}}\end{aligned}$$

As the quality at state 1 is unknown, we solve it by using that:

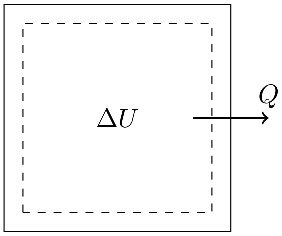
$$h_1 = h_2 = x_1 h_1^V + (1 - x_1)h_1^L$$

Hence:

$$x_1 = \frac{h_2 - h_1^L}{h_1^V - h_1^L} = \frac{2714.83 - 762.7}{2777.1 - 762.7} = \mathbf{0.969}$$

Question 4

1) 2) Control mass 3)



- Constant volume
- Changes in potential energy are negligible

4) Interval basis 5) See sketch 6) Interval based energy balance:

$$0 = \Delta U + Q$$

7) Rewrite energy balance as:

$$0 = M(u_2 - u_1) + Q$$

Hence:

$$Q = M(u_1 - u_2)$$

State 1 is fixed as it is given that it is the critical state. We find from the tables that:

$$v_1 = v^c = 0.004444\ \frac{\text{m}^3}{\text{kg}}$$

$$u_1 = u^c = h^c - Pv^c = 1262.38\ \frac{\text{kJ}}{\text{kg}} - (11.333 \cdot 10^3\text{kPa}) \cdot (0.004444\ \frac{\text{m}^3}{\text{kg}}) = 1212.02\ \frac{\text{kJ}}{\text{kg}}$$

As $T_2 = 310$ K and the tank is rigid meaning that $v_2 = v_1 = v^c$ we can determine the internal energy for state 2. Observation of the provided table (and chart) should allow the student to conclude that the ammonia is a saturated mixture at this temperature. First determine the quality from:

$$x_2 = \frac{v^c - v_2^L}{v_2^V - v_2^L} = \frac{0.004444\ \frac{\text{m}^3}{\text{kg}} - 0.001708\ \frac{\text{m}^3}{\text{kg}}}{0.09230\frac{\text{m}^3}{\text{kg}} - 0.001708\frac{\text{m}^3}{\text{kg}}} = 0.032$$

Then the specific internal energy follows from:

$$\begin{aligned} u_2 &= (1 - x_2)h_2^L + xh_2^V \\ &= (1 - 0.032)(515.36\ \frac{\text{kJ}}{\text{kg}} - (1.4 \cdot 10^3\text{kPa}) \cdot (0.001708\ \frac{\text{m}^3}{\text{kg}}) \\ &+ 0.032(1631.95\ \frac{\text{kJ}}{\text{kg}} - (1.4 \cdot 10^3\text{kPa}) \cdot (0.09230\ \frac{\text{m}^3}{\text{kg}}) \\ &= 544.64\ \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

Thus the energy transferred as heat *out* of the system is:

$$Q = M(u_1 - u_2) = 13\ \text{kg} \cdot (1212.02\frac{\text{kJ}}{\text{kg}} - 544.64\ \frac{\text{kJ}}{\text{kg}}) = \mathbf{8.67}\ \text{MJ}$$

b. The volume occupied by the liquid at state 2 is found by:

$$V_2^L = M_2^L v_2^L = (1 - x_2)Mv_2^L = (1 - 0.032) \cdot 13\ \text{kg} \cdot 0.001708\ \frac{\text{m}^3}{\text{kg}} = \mathbf{0.021\ m^3}$$

c. Process line should be close to liquidus line along an isochoric line.

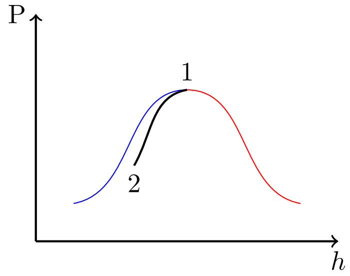
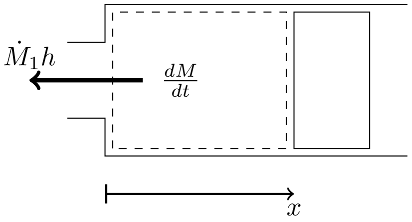


Figure 2: P-h diagram for question 4c

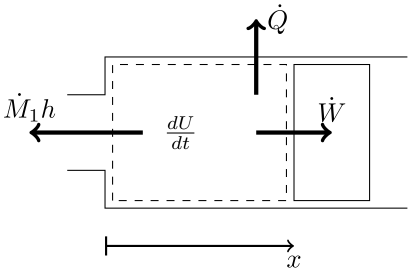
Question 5

1)



2) Time-varying control volume. All boundaries are fixed to the cylinder except one, which moves along with the piston face. The inertial coordinate x is used to quantify work.
3) Idealizations:

- Only one outlet
- The temperature at the outlet is equal to the temperature of the entire system
- Kinetic and potential energy transfer of the working fluid is negligible.



4) Rate basis 5) See sketch 6) The mass balance is:

$$\dot{M}_1 = \frac{dM}{dt}$$

The energy balance is:

$$\dot{M}_1 h = \frac{dU}{dt} + \dot{W} + \dot{Q}(T)$$

7) Individually the terms of the energy balance can be stated as:

$$\dot{M}_1 h = \frac{dM}{dt} c_p T$$

$$\frac{dU}{dt} = M\frac{du}{dt} + u\frac{dM}{dt} = Mc_v\frac{dT}{dt} + c_v T\frac{dM}{dt}$$

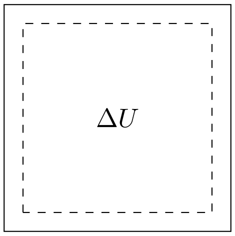
$$\dot{W} = \frac{dW}{dt} = \frac{d}{dt}(PAdx) = PA\frac{dx}{dt} = \frac{MRT}{V}\frac{dV}{dt}$$

Re-substitute in the energy balance:

$$\frac{dM}{dt} c_p T = Mc_v\frac{dT}{dt} + c_v T\frac{dM}{dt} + \frac{MRT}{V}\frac{dV}{dt} + \dot{Q}(T)$$

Question 6

1) See sketch 2) Control mass 3) Idealizations



- Adiabatic container
- Constant specific heats for the computer chips
- Fluid remains saturated over the process
- Pressure remains constant

4) Interval basis 5) See sketch 6)The energy balance is:

$$0 = \Delta U$$

The entropy balance is:

$$\mathcal{P}_s = \Delta S$$

7) **a.** Using the Gibbs equation:

$$ds_{\text{chips}} = \frac{du_{\text{chips}}}{T} + \frac{Pdv_{\text{chips}}}{T} = \frac{dh_{\text{chips}}}{T} - \frac{v_{\text{chips}}dP}{T}$$

Given is that $dP = 0$. We also know that $dh = c_p dT$, hence integration of above equation now gives:

$$\Delta s_{\text{chips}} = c_p \ln(\frac{T_2}{T_1})$$

For solving this problem, it is required to assume that the R134a is a saturated mixture at the final state. This is reasonable, as it is a saturated liquid at the initial state and can only become a mixture as energy will be added in order to reach thermal equilibrium with the chips. Hence, the final temperature of the whole system is $T_2 = -40°C$. Thus we can evaluate the entropy change:

$$\Delta s_{\text{chips}} = 0.3\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \ln\left(\frac{(273.15-40)\ \text{K}}{(273.15+20)\ \text{K}}\right) = -0.0687\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

And in extensive form:

$$\Delta S_{\text{chips}} = M_{\text{chips}}\Delta s_{\text{chips}} = 0.010\ \text{kg}\cdot -0.0687\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} = \mathbf{-0.69}\ \frac{\text{J}}{\text{K}}$$

b. The energy balance can be rewritten as:

$$0 = M_{\text{chips}}c_v(T_2 - T_1) + M_{\text{R134a}}(u_2 - u_1)$$

Solve for the specific internal energy of the R134a at the final state:

$$u_2 = u_1 - \frac{M_{\text{chips}}c_v(T_2 - T_1)}{M_{\text{R134a}}}$$

The specific internal energy at state 1 for the R134a follows from (approximation is used) :

$$u_1(T = 233.15\ \text{K}) \approx u^L(T = 232\ \text{K}) = -277.95\ \frac{\text{kJ}}{\text{kg}} - (0.04869\cdot 10^3\ \text{kPa})\cdot 0.000715\ \frac{\text{m}^3}{\text{kg}} = -277.98\ \frac{\text{kJ}}{\text{kg}}$$

Hence the specific internal energy at state 2 is:

$$u_2 = -277.98\ \frac{\text{kJ}}{\text{kg}} - \frac{0.01\ \text{kg}\cdot 0.3\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}(-40-20)\ \text{K}}{0.005\ \text{kg}} = -241.98\ \frac{\text{kJ}}{\text{kg}}$$

Knowing this u_2 we can find the quality of the saturated R134a at the second state by:

$$x_2 = \frac{u_2 - u_2^L}{u_2^V - u_2^L} = \frac{-241.98\ \frac{\text{kJ}}{\text{kg}} - (-277.98\ \frac{\text{kJ}}{\text{kg}})}{-70.75\ \frac{\text{kJ}}{\text{kg}} - (-277.98\ \frac{\text{kJ}}{\text{kg}})} = 0.174$$

Where we've used that $u_2^L = u_1^L$ and idem for the vapor values. The entropy change of the R134a is now found by:

$$\begin{aligned}\Delta S_{\text{R134a}} &= M_{\text{R134a}}(s_2 - s_2) = M_{\text{R134a}}(((1-x)s_2^L + xs_2^V) - s_2^L) = M_{\text{R134a}}x(s_2^V - s_2^L)\\ &= 0.005\ \text{kg}\cdot 0.174(-0.1370\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} - (-1.1108\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}})) = \mathbf{0.85}\ \frac{\text{J}}{\text{K}}\end{aligned}$$

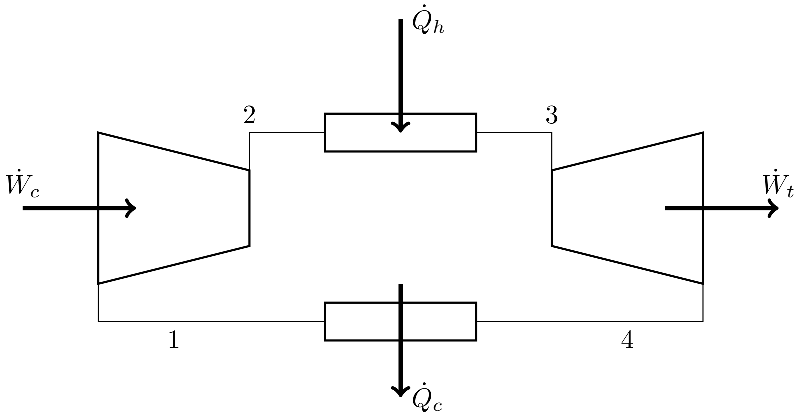
c. Sum the change of entropy for both substances (system is adiabatic so no transfer of entropy):

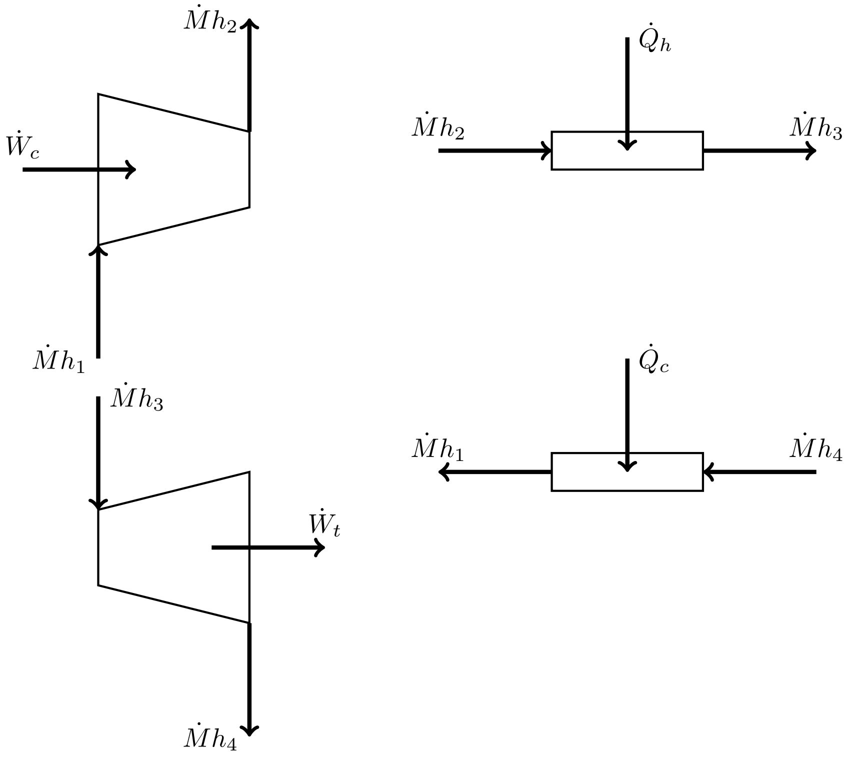
$$\Delta S_{\text{system}} = \Delta S_{\text{R134a}} + \Delta S_{\text{chips}} = 0.85\ \frac{\text{J}}{\text{K}} + (-0.69\ \frac{\text{J}}{\text{K}}) = \mathbf{0.16}\ \frac{\text{J}}{\text{K}}$$

d. As the change in entropy of the entire system is positive, the process is possible. Re-substitution of the above result in the entropy balance would lead to the conclusion that the entropy production is positive, which corresponds to our conceptualization of 'possible'.

Question 7

1)





2) For all inter-component balances: fixed control volume 3) Idealizations:

- Compression and expansion are isentropic
- Heating and cooling is isobaric
- Transfer of kinetic and potential energy is negligible.
- Constant specific heats

4) See sketch 5) Rate basis 6) Inter-component energy balances lead to:

$$\dot{W}_c = \dot{M}c_p(T_2 - T_1)$$

$$\dot{W}_t = \dot{M}c_p(T_3 - T_4)$$

$$\dot{Q}_h = \dot{M}c_p(T_3 - T_2)$$
$$\dot{Q}_c = \dot{M}c_p(T_4 - T_1)$$

7) **a.** The efficiency for the ideal case then is:

$$\begin{aligned}
\eta &= \frac{\text{Net work}}{\text{Energy input as heat}} \\
&= \frac{\dot{W}_t - \dot{W}_c}{\dot{Q}_h} \\
&= \frac{\dot{M}c_p(T_3 - T_4) - \dot{M}c_p(T_2 - T_1)}{\dot{M}c_p(T_3 - T_2)} \\
&= \frac{(T_3 - T_2) - (T_4 - T_1)}{T_3 - T_2} \\
&= 1 - \frac{T_4 - T_1}{T_3 - T_2} \\
&= 1 - \frac{T_1(\frac{T_4}{T_1} - 1)}{T_2(\frac{T_3}{T_2} - 1)}
\end{aligned}$$

Let us use the fact that both the compression and expansion are isentropic. The Gibbs equation for an ideal gas can be integrated to yield the isentropic relations as follows:

$$\begin{aligned}
ds &= \frac{du}{T} + \frac{Pdv}{T} \\
&= \frac{dh}{T} - \frac{vdP}{T} \\
&= \frac{c_p dT}{T} - \frac{RdP}{P}
\end{aligned}$$

Integrate and using that $\Delta s = 0$:

$$0 = c_p \ln(\frac{T_f}{T_i}) - R\ln(\frac{P_f}{P_i})$$

This can be algebraically manipulated to show that:

$$\frac{T_f}{T_i} = \left(\frac{P_f}{P_i}\right)^{c_p/R} = \left(\frac{P_f}{P_i}\right)^{\frac{\gamma}{\gamma-1}}$$

Where subscripts f and i denote final and initial state respectively, to avoid confusion with any states of the problem in question.

The student has to note that the compressor and turbine work between the same pressures (and thus same pressure ratio) as the cycle is closed and the heating and cooling processes are isobaric (and we neglect pressure losses). Thus, we can show that:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma}{\gamma-1}} = \frac{T_3}{T_4}$$

Hence:

$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Thus the expression for the efficiency becomes:

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{1}{\Pi^{\frac{\gamma-1}{\gamma}}}$$

b. Question b was not included in grading