Delft University of Technology	**Faculty of Aerospace Engineering**
Subject: AE4425 Value Engineering and Operations Research	**Year: 4th**
Date: November 9th, 2011	**Time: 14.00-17.00**

Complete the box in the upper right corner of your exam. Put your name and all your initials on each page of your exam and answer all questions.
To answer the questions use may be made of the Dutch and/or English language.
This exam comprises the questions: 1, 2 (**a** and **b**), 3 (**a**, **b**, and **c**), 4 (**a**, **b**, **c**, **d**, and **e**), 5 (**a**, **b**, and **c**), 6 (**a**, **b**, and **c**), 7, and 8

Reference to literature as a way of answering questions is not allowed.

The way a numerical answer is obtained, should be clearly indicated by a visible substitution of numbers in the formula(e) in question. **Mentioning the final answer only will not be rewarded. Marks will be lost for poor presentation.**

Use of a pencil to write the exam is not permitted.

Scrap paper may not be added to your exam.

You can use the "IOR tutorial" tool as installed on the computers.

To log in use computer name (TUDxxxxx) and password 'Welkom01'

Question 1

Consider the following problem:

Minimize $Z = 3x_1 + 2x_2$,

Subject to

$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$

, and

$$x_1 \geq 0, \quad x_2 \geq 0.$$

Use the graphical approach to solve for the decision variables x_1 and x_2 and the corresponding value for the objective function Z. Indicate in your graph the feasible region and objective function.

Question 2

An airline company operating at a small airport is renting office space for employees. For permanent housing of employees, a dedicated building is planned to be completed in three years. The rented space will not be needed after completion of this new building. Several locations are available for renting. A significant growth is foreseen for the airport. Consequently, the price for renting is dependent on the renting period and the price increases with increasing duration. Consequently, it may be economical to move to a new location after one or two years. However, taking a short-term contract and moving to another location also induces additional costs.

The following table gives the total net costs in thousands or dollars associated with renting the required space at the end of year i and leaving it at the end of year j (where year 0 is now, i.e., end of current year). The problem is to determine at what times (if any) employees should move to other offices to minimize the total cost over three years. It is possible to rent only a part of a certain location.

Total net costs in thousands of dollars associated with moving to and renting the required office space at the end of year *i* and leaving it at the end of year *j*.

		j		
		1	2	3
i	0	**13**	**28**	**48**
	1		**17**	**33**
	2			**20**

a) Formulate the minimization (of cost) problem, consisting of the objective function and the constraints. Consider the following decision variables (x_{01}, x_{02}, x_{03}, x_{12}, x_{13}, x_{23}), with x_{ij} the fraction of the required space rented at the end of year i and left at the end of year j.

b) What is the optimal renting strategy?

Question 3

Consider the following problem:

Maximize $Z = 2x_1 + x_2$

Subject to

$3x_1 + 2x_2 \leq 22$

$2x_1 - x_2 \geq 12$

a) Use the big M method to construct the complete first tableau for the simplex method.

b) Identify the corresponding initial (artificial) basic feasible solution.

c) Identify the columns that will contain **S*** in the final tableau. Explain why these are the appropriate columns.

Question 4

We consider a company that produces parts of a jet exhaust. The table below presents for three of their products (P1, P2 and P3) the required hours for producing them and their unit profit. It also presents the amount of hours that are available for the production of these three products in each of the four plants involved.

Plant	Production time per product (hours)			Production time available per week (Hours)
	P1	P2	P3	
1	30	0	0	10
2	0	10	10	50
3	10	10	5	55
4	5	10	15	70
Profit per product	$ 300	$ 400	$ 300	

a) Formulate the linear programming model for determining the optimum production rate, i.e., number of products produced per week, for all three products.

The final simplex tableau for this linear programming problem is found to be:

Bas Var	Eq No	Z	X1	X2	X3	X4	X5	X6	X7	Right side
Z	0	1	0	0	100	10	40	0	0	2100
X1	1	0	1	0	0	0.033	0	0	0	0.333
X2	2	0	0	1	1	0	0.1	0	0	5
X6	3	0	0	0	-5	-0.33	-1	1	0	1.667
X7	4	0	0	0	5	-0.17	-1	0	1	18.33

where decision variable x_1 is the production rate of product P1, x_2 the production rate of product P2 and x_3 the production rate of product P3.

b) What are the defining constraints? Explain your answer.

Based on the outcome shown in the above tableau, indicating that it is not opportune to produce product P3, management is reconsidering the available production times.
Based on discussions with employees of plant 1 it becomes clear that the original estimate for the production time available in plant 1 is very conservative. Carrying out a more detailed study shows that the actual available production time is 30 hours.

c) Calculate **from the tableau provided above** the **revised** final tableau reflecting this change from 10 to 30 hours in available production time. Show your calculations.

d) What is the solution corresponding to the revised tableau? Is this solution feasible? Motivate your answer.

e) For strategic reasons it is of high importance to produce all three products. Is this aim realized by the increase in plant 1 production time from 10 to 30 hours? Motivate your answer.

Question 5

A company has two plants for producing and assembling bikes. After production, the bikes are shipped to three distribution centers. The costs of producing the bikes are the same for the two plants. The cost of shipping are dependent on the combination of plant and distribution center, and are indicated in the table below per bicycle.

		Distribution center		
		1	**2**	**3**
Plant	**1**	**$8**	**$7**	**$4**
	2	**$6**	**$8**	**$5**

A total of 180 bikes are produced and shipped per week.
Each plant can produce and ship up to a maximum of 150 bikes per week.
Each distribution centre must receive exactly 60 bikes per week.

The objective of the management is to determine how many bikes need to be produced at each of the two plants, and what the shipping pattern should be in order to minimize the total shipping cost.

a) Formulate this problem as a transportation problem by constructing the appropriate parameter table.

b) Find an initial solution to this problem using the North-West corner rule.

c) Apply the optimality test to investigate if this solution is already optimal. Show your calculations.

Question 6

Consider the following unconstrained optimization problem:

Maximize $f(x_1, x_2) = 2x_1 x_2 - 4x_1^2 - 2x_2^2$

a) Is this function concave? Motivate you answer.

b) Mention two methods that you consider suitable for locating the optimum of this function. Explain your answer.

c) What is the solution to this problem, i.e., the values of x_1 and x_2 at which f has its maximum?

Question 7

Discuss the pros and cons of metaheuristic methods (such a simulated annealing and genetic algorithms), compared to Newton's method. Mention for each of the two methods at least one advantage and one disadvantage.

Question 8

The performance of the genetic algorithm is heavily dependent on a series of setting parameters such as the crossover probability. Mention three other Genetic Algorithm setting parameters and describe their role in the optimization.