Delft University of Technology		**Faculty of Aerospace Engineering**
Subject:	**AE4441 Value Engineering and Operations Research**	**Year: 4th**
Date:	**November 5th, 2012**	**Time: 09.00-12.00**

Complete the box in your answer sheets. Put your name and all your initials on each page of your answer sheets.

Also fill in your name and study number in the box below and hand in this exam together with your answer sheets. Scrap paper may not be added to your exam.

Name	Study number

To answer the questions use may be made of the Dutch and/or English language.
This exam comprises the questions: **1** (a and b), **2**, **3** (a, b, c, d, e), **4** (a, b, c, d, e, f), **5** (a, b, c, d), **6** (a, b and c), **7**, **8** (a and b)
Reference to literature as a way of answering questions is not allowed. The general way a numerical answer is obtained should be made clear. Mentioning the final answer only will not be rewarded. Marks will be lost for poor presentation.

Use of a pencil to write the exam is not permitted.

You can use the "IOR tutorial" tool as installed on the computers.

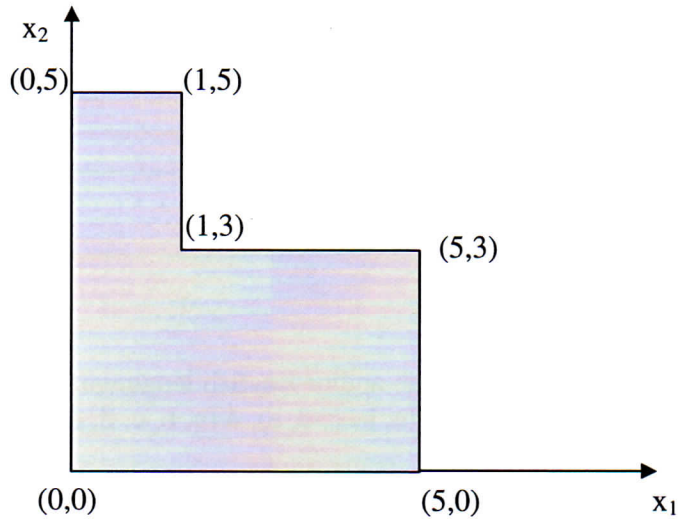
To log in use computer name (TUDxxxxx) and password 'Welkom01'

Question 1

The figure below presents the feasible region for a maximization problem. The objective function is a linear function of two variables, x_1 and x_2.

Read each of the following statements and label them as either false or true.

Also provide a motivation for your answer.



a) The point (0,0) cannot be an optimal solution.

b) If (1,5) produces a larger value for the objective function than (0,5) and (1,3), then (1,5) is a local optimum. However, it is not known whether it is also a global optimum.

Question 2

Perfect Seats is a company which features a product line consisting of three types of headrests for aircraft seats. The company is trying to decide what mix of these three headrest types to produce.

Perfect Seats' manufacturing labor force is unionized. Each full-time employee works a 40-hour week. In addition, by union contract, the number of full-time employees can never drop below 20. Nonunion, part-time workers can also be hired with the following union-imposed restrictions:

(1) each part-time worker works 20 hours per week, and
(2) there must be at least 2 full-time employees for each part-time employee.

All three types of headrests are made out of the same 100% genuine leather. The company has a long term contract with the supplier of the leather, and receives a 500 square meter shipment of the material each week. The material requirements and labor requirements, along with the *gross profit* per headrest sold (**i.e. not considering labor costs**) are given in the following table.

Headrest	Material Required (square meter per headrest)	Labor Required (minutes per headrest)	Gross Profit (per headrest)
Top	0.2	30	€8
DeLuxe	0.15	45	€10
Basic	0.1	40	€6

Each full-time employee costs €13 per hour, while each part-time employee costs €10 per hour. Management wishes to know what mix of each of the three types of headrests to produce per week, as well as how many full-time and how many part-time workers to employ. (All decision parameters are allowed to be non-integer.) They would like to maximize their *net profit*, i.e., their gross profit from sales minus their labor costs.

Formulate a linear programming model for this problem.

Question 3

Consider the following problem:

Maximize $Z = 2x_1 + 4x_2 - 2x_3$,

subject to

$$\begin{aligned} 3x_2 - x_3 &\leq 30 && \text{(resource 1)} \\ 2x_1 - x_2 + x_3 &\leq 10 && \text{(resource 2)} \\ 4x_1 + 2x_2 - 2x_3 &\leq 40 && \text{(resource 3)} \end{aligned}$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

a) Construct and present for the above problem the complete first tableau for the simplex method.

b) Identify from the tableau the initial basic feasible solution, i.e., the initial values for the variables and the corresponding value for the objective function.

c) Use the simplex method to solve the problem, i.e., to determine the optimal value for the decision variables and the objective function.
(You do not need to show the tableaus for all iterations).

d) Identify the shadow prices for the resources from the final simplex tableau. Show how you got to your answer.
Describe the significance of these shadow prices.

e) What is the allowable range for b_1, i.e., the right hand side of the first constraint? Show your calculations.

Question 4

A company makes three types of tires. Its fiberglass belted tires (type 1) give €3 in profit per tire, its radials €2 a tire (type 2) and its solids €4 a tire (type 3). Each type of tire passes through two production stages as part of the entire production process.

The table below specifies the hours of available production time per day and the required production time needed for each of the three products (a batch consists of 100 tires).

Stage	Production time per batch (hours)			Production time available per day (hours)
	type1	type2	type3	
1	6	3	5	5
2	3	4	5	20
Profit per batch	€ 300	€ 200	€ 400	

a) Formulate the linear programming model for determining the product mix for each day's production that maximizes the total profit, assuming all tires are sold.

b) What is the optimal product mix and the corresponding profit?
(You do not need to show the tableaus for all iterations).

c) Identify **S*** and **y*** from the final simplex tableau.

Based on new decisions from management, the available production time for stage 1 is increased to 25 hours per day.

d) Calculate the revised final tableau for the modified constraint.

e) What is the solution corresponding to the revised tableau?

f) Is this solution feasible and/or optimal? Motivate your answer.

b) Apply the transportation simplex method to solve the problem. Use the table below for presenting your answer. Provide a value for all 20 decision variables x_{ij}, with i indicating the source number (i = 1,2,3,4) and j indicating the destination (j = 1,2,3,4,5).

	D1	D2	D3	D4	D5
S1					
S2					
S3					
S4					

c) What is the value of the objective function corresponding to the above solution?

d) Are there multiple optimal solutions for this problem? Motivate your answer.

Question 6

Consider the following unconstrained optimization problem

Maximize $f(x_1, x_2) = 10x_1 - 2x_1^2 - x_1^3 + 8x_2 - x_2^2$

a) Is this function concave? Motivate your answer.

Now the following constraints are added to the problem:

$x_1 + x_2 \leq 2$

$x_1 \geq 0$

$x_2 \geq 0$

b) The aim is to use the Frank-Wolfe algorithm for solving the problem. An initial trial solution is selected and, according to Frank-Wolfe, the problem is approximated at this point such that a linear programming problem is obtained.
Select (0,0) as the initial trial solution. What is the linear programming problem to be solved by the Frank-Wolfe algorithm at the initial trial solution?

c) Is there a possibility that the Frank-Wolfe algorithm will provide a local maximum instead of the global maximum for this constrained problem? Motivate your answer.

Question 7

For convex programming problems a local optimum is also the global optimum. For solving convex programming problems *without* constraints one could use, for example, Newton's method or the gradient search method.
For problems with constraints, alternative methods accounting for the constraints need to be selected.
These methods fall within a few main categories. Each category of methods is based on a specific approach to deal with the constraints. Describe one of such an approach (i.e., not a specific algorithm).

Question 8

Metaheuristic methods such as simulated annealing and genetic algorithms have a series of 'setting parameters'. The values of these setting parameters can have a huge effect on the performance of the method in locating the global optimum.
For genetic algorithms, one of these setting parameters is the number of generations.

a) Describe how this setting parameter affects the performance of a genetic algorithm.

b) Describe the approach you would take in selecting an appropriate value for this setting parameter. Motivate your answer.