Waves and Electromagnetism

Chapter 14

Oscillations:

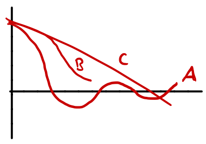
- Oscillations of a spring: If an object vibrates or oscillates back and forth taking the same time in doing the same distance we call it periodic oscillation
  - Force $\boxed{F = -k \cdot x}$ The minus sign indicates that it is a restoring force.
  - Vertical: the equilibrium position is when spring force is equal to the gravitational force.
- Simple harmonic motion: restoring force is proportional to the negative displacement.
  - $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$ • $x = A\cos(\omega t + \phi)$ • $\omega^2 = \frac{k}{m}$
  - Velocity: $v = \frac{dx}{dt} = \frac{d}{dt}[A\cos(\omega t + \phi)] = -\omega A\sin(\omega t + \phi) \longrightarrow$ max velocity dependent on amplitude.
  - Frequency: $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ • Period: $T = 2\pi\sqrt{\frac{m}{k}}$
  - Velocity using energy: $v = \pm v_{max}\sqrt{1 - \frac{x^2}{A^2}}$
  - $v_{max}^2 = \left(\frac{k}{m}\right)\cdot A^2$
  - Acceleration: $a = -\omega^2 \cdot A\cos(\omega t + \phi)$
  - Energy: $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ (conserved) $\longrightarrow$ dependent on amplitude: $\boxed{E = \frac{1}{2}k \cdot A^2}$

- Simple Pendulum
  - Period: $T = 2\pi\sqrt{\frac{L}{g}}$ • Frequency: $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

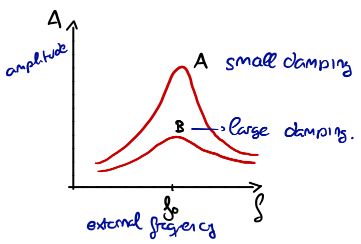
- Physical Pendulum
  - Period: $T = 2\pi\sqrt{\frac{I}{m \cdot g \cdot h}}$ • Angle: $\theta = \theta_{max}\cos(\omega t + \phi)$ • Torque: $\tau = -m \cdot g \cdot h \cdot \sin\theta$

- Torsional Pendulum
  - $\omega = \sqrt{k/I}$

- Damped harmonic motion harmonic motion with frictional or drag force.
  - $F_{damping} = -bv$ • $m \cdot a = -k \cdot x - b \cdot v$
  - (b small) $x = Ae^{-\gamma t}\cos(\omega' t)$
  - $\gamma = \frac{b}{2m}$ • $b = \frac{M}{T}$
  - $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
    - $b^2 > 4mk$ ω' imaginary, overdamped
    - $b^2 = 4mk$ critically damped
    - $b^2 < 4mk$ underdamped



- Resonance
  - Forced vibrations occur when there is a periodic driving force. This force may or may not have the same period as the natural frequency ω_0 of the system.
  - If the frequency is the same as the natural frequency, the amplitude can be quite large. (Resonance)



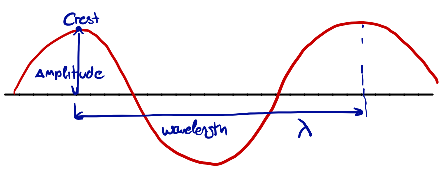
- Forced oscillators
  - $ma = -kx - bv + F_0\cos(\omega t)$
  - $x = A_0\sin(\omega t + \phi_0)$
  - $A_0 = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0^2)^2 + \frac{b^2\omega^2}{m^2}}}$
  - $\phi_0 = \tan^{-1}\left(\frac{\omega_0^2 - \omega^2}{\omega(b/m)}\right)$
  - $\omega_0^2 = \frac{k}{m}$
  - Width of the resonant peak

$$Q = \frac{m\omega_0}{b}$$

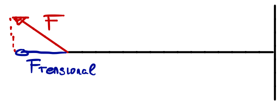
Chapter 15 | Wave Motion

- Traveling waves transport energy.
- Single wave pulse shows that it is begun with a vibration.

- Frequency: time in one oscillation.
- Velocity = $\lambda \cdot f$



- Transverse or longitudinal
- Velocity of a transverse: $V = \sqrt{\frac{F_T}{\mu}}$



(through solid)

- $\mu = \frac{m}{\ell}$

- Velocity of a longitudinal wave: $V = \sqrt{\frac{E}{\rho}}$ (through solid) $V = \sqrt{\frac{B}{\rho}}$ (throug liquid)

Energy transported by waves

- $E = \frac{1}{2}k \cdot A^2 = 2\pi^2 \cdot m \cdot f^2 \cdot A^2$ $\left(f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = f^2 = \frac{1}{4\pi^2} \cdot \frac{k}{m} \right)$
- Same density in the whole medium: $I = \frac{\bar{P}}{S} = 2\pi^2 v \rho f^2 A^2$ ($\bar{P} \rightarrow$ Power) (INTENSITY)
- Distance Relation: $I \propto \frac{1}{r^2}$
- After some time: $D(x,t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$
- Velocity: $V = \frac{w}{k}$

Supperposition principle

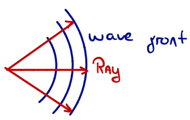
- Sum of waves can be decompose in simple waves

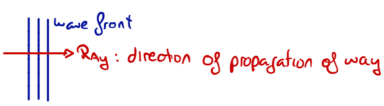
Reflection and transmission

- Wave reaching a obstacle: reflection inverted
- Wave reaching a free end: reflection
- When a wave encounters a denser medium part of the wave will be transmitted and the other part will be reflected (shorter wavelength in the dense medium)

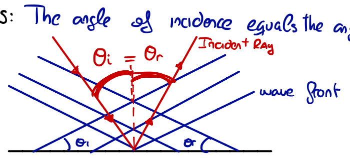
$\boxed{V = \sqrt{\frac{F_T}{\mu}}}$ $\boxed{v = \lambda f}$

- Wave Fronts: Curves of surfaces where all the waves have the same phase



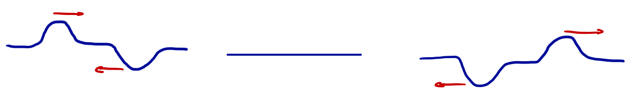


- Angles: The angle of incidence equals the angle of reflection

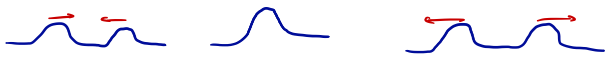


Interference

- Superposition principle: when two waves pass through the same point, the displacement is the arithmetic sum of the individual displacements.
  - Destructive interference:

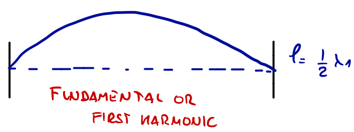


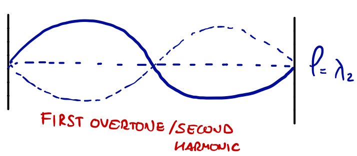
  - Constructive interference



Standing Waves: Resonance

- Standing waves occur when both ends of a string are fixed. In that case, only waves which are motionless at the ends of the string can persist. There are nodes: amplitude = 0 and antinodes: amplitude varies from 0 to maximum value.
- The frequencies of the standing waves on a particular string are called resonant frequencies.

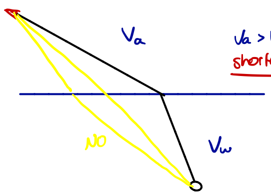




- $\lambda_n = \frac{2\ell}{n}$ $n = 1, 2, 3 \ldots$
- $f_n = \frac{v}{\lambda_n} = n\frac{v}{2\ell} = n f_1$, $n = 1, 2, 3$

Refraction

- If a wave enters a medium where the wave speed is different it will be refracted: wave fronts and rays will change direction.
- Calculate the angle of refraction $\frac{\sin\theta_2}{\sin\theta_1} = \frac{V_2}{V_1}$ } Law of refraction or Snell's law



Diffraction

- When waves encounter an obstacle, they bend around it leaving a shadow region.
- Depends on the size of the obstacle compared to the wavelength.
- If wavelength is smaller than object there will be a significant shadowed region.

Chapter 16 Sound

16.1 Characteristics of sound

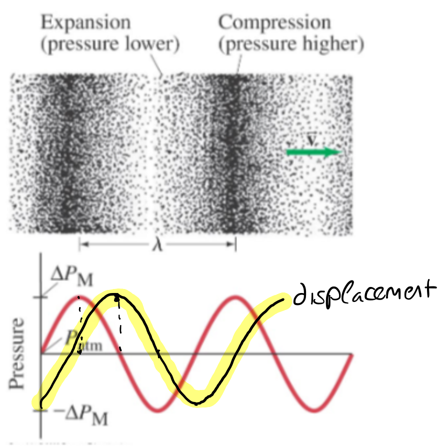
- Sound can travel through any kind of mater.
- Speed of sound is different in different materials, slowest in gases, faster in liquid fastest in solids.
- Speed depends on temperature.

$(V = 331 + 0.6\,T \quad T \text{ in } C^\circ)$ Air

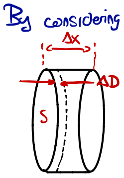
- Loudness: intensity of the sound wave
- Pitch: related to frequency
- Audible range: 20 Hz to 20000 Hz
- Ultrasound: above 20000 Hz
- Infrasound: below 20 Hz

16.2 Representation of Longitudinal Waves

- Longitudinal waves called pressure waves



- By considering a small cylinder B: bulk modulus.



- $\Delta P = -B\frac{\Delta V}{V} = -B\frac{S\Delta D}{S\Delta x}$ • or $\Delta P = -B\frac{\partial D}{\partial x}$
- $D = A\cdot\sin(kx - \omega t)$
- Sinusoidal displacement • where $v = \sqrt{\frac{B}{\rho}}$ $v = \frac{\omega}{k}$

$\Delta P = -(BAk)\cdot\cos(kx - \omega t)$

- Pressure amplitude:

$\Delta P_M = BAk = \rho v^2\cdot A\frac{\omega}{v}$ $[B = \rho v^2]$

$\Delta P_M = 2\pi f \rho v A$

16.3 Intensity of sound: Decibels

- Intensity: Energy transported per unit time across a unit area.
- Human can hear from 10^{-12} W/m² to 1 W/m²
- Perceived loudness is not proportional to the intensity.
- Loudness: measured in decibels (dB)

$\beta\,(\text{in dB}) = 10\log\left(\frac{I}{I_0}\right)$

$I_0 = 1.0\cdot10^{-12}$ W/m² (threshold of hearing)

- In open areas the intensity diminishes with distance

$I \propto \frac{1}{r^2}$

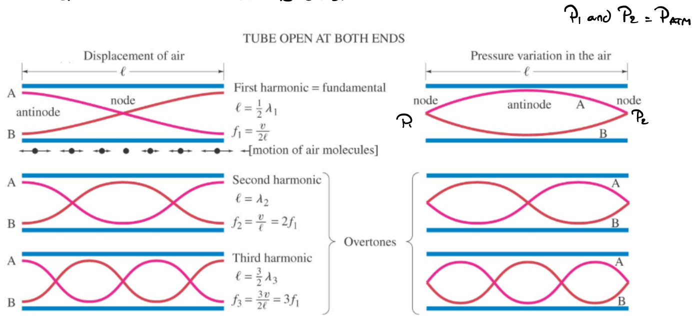
- Intensity can be written in terms of maximum pressure variation

$I = \frac{(\Delta P_M)^2}{2v\rho}$

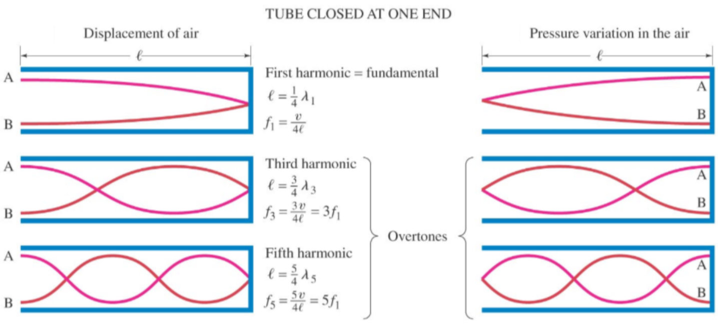
16.4 Sources of sound: Vibrating strings and air columns

- Vibrations of materials are transmitted to the air and then to our ears.
- Harmonics: $f_n = nf_1 = n\cdot\frac{v}{2\ell}$ $n = 1, 2, 3$ $v = \sqrt{\frac{F_T}{\mu}}$
- Modifying pitch: • the strings on a guitar can be shortened raising pitch
  - using a string of different density
- Wind instruments create sound through standing waves in a column of air
  - A tube opened at both ends has pressure nodes and therefore displacement antinodes at the ends.

P_1 and $P_2 = P_{ATM}$



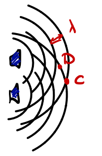
- Tube closed at 1 end has a displacement node at the closed end



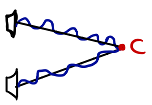
16.5 QUALITY OF SOUND, and NOISE SUPERPOSITION

- The sound wave is the superposition of the fundamental and all the overtones.

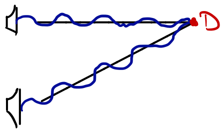
16.6 INTERFERENCE OF SOUND WAVES; BEATS



CONSTRUCTIVE INTERFERENCE



DESTRUCTIVE INTERFERENCE



- Waves can also interfere in time, causing a phenomenon called beats. Beats are the slow "envelope" around two waves that are relative close in frequency.

two waves with same amplitude and phase with two different frequencies.

$$D = \left[2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]\sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

represents a wave vibrating at the average frequency, with an "envelope" at the difference of the frequency.

$$f_{beat} = 2\,\frac{f_1 - f_2}{2} = f_1 - f_2$$

used to tune instruments.

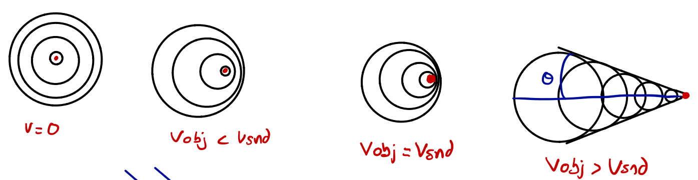
16.7 DOPLER EFFECT

- Occurs when a source of sound is moving with respect to an observer.
- A source moving towards the observer appears to have a higher frequency and a shorter wavelength.

- Change in frequency is given by: $f' = \dfrac{f}{\left(1 - \dfrac{v_{source}}{v_{snd}}\right)}$ (moving to) $\quad f' = \dfrac{f}{\left(1 + \dfrac{v_{source}}{v_{snd}}\right)}$ (moving away)
- If the observer is moving: towards sound: $f' = \left(1 + \dfrac{v_{obs}}{v_{snd}}\right)\cdot f$ moving away $f' = \left(1 - \dfrac{v_{obs}}{v_{snd}}\right) f$
- COMBINED EQUATION: $f' = f\left(\dfrac{v_{snd} \pm v_{obs}}{v_{snd} \mp v_{source}}\right)$

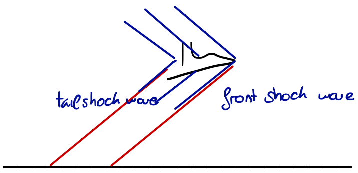
16.8 Shock waves and SONIC BOOM

- If a source is moving faster than the wave speed in a medium, waves cannot keep up and a shock wave is formed



$$\sin\theta = \frac{v_{snd}}{v_{obj}}$$

Same as waves produced by a boat going faster than the wave speed in water.



18.9 APLICATIONS:

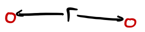
Sonar: used to locate objects underwater by measuring the time it takes a sound pulse to reflect back to the receiver.
- ultrasound: less likely to be difracted by obstacles.

Ultrasound: medical imaging: repeated traces are made as the transducer is moved and a complete picture is built.

CHAPTER 22 ELECTRIC CHARGE AND ELECTRIC FIELD

COULOMBS LAW

COULOMBS LAW: magnitude of the force is proportional to the product of the inverse of the distance.



$$\boxed{F = k \cdot \frac{Q_1 \cdot Q_2}{r_2}}$$

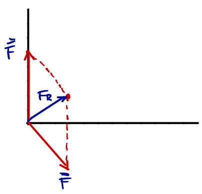
- this equation gives the magnitude of the force between two charges.
- precisse point charge

- Unit of charge: coulomb (C)
  $k = 8.99 \cdot 10^9 \ N \cdot m^2/C^2$
- Charges produced by rubbing are around μC
  $1 \mu C = 10 \ C$
- Charge on the electron
  $e = 1.602 \cdot 10^{-19}$ $1 \mu C \sim 1013$ electrons

- k can also be written in terms of ε_0

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2}$$

PRINCIPLE OF SUPERPOSITION



ELECTRIC FIELD:

Defined as the force on a small charge, divided by the magnitude of the charge.

$$\boxed{\vec{E} = \frac{\vec{F}}{q}}$$

- surrounds every charge

For a point charge:

$$\boxed{E = \frac{F}{q} = \frac{kqQ/r^2}{q} = k \cdot \frac{Q}{r^2}}$$

CONTINUOUS DISTRIBUTIONS OF CHARGE

- Succession of infinitesimal (point) charges. The total field is then the integral of the infinitesimal fields due to each bit of charge

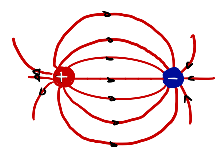
$$\boxed{dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dQ}{r^2}} \quad \boxed{\vec{E} = \int d\vec{E}}$$

- Electric field is a vector.

- Point charge $E \sim \frac{1}{r^2}$
- Line charge $E \sim \frac{1}{r}$
- Plane of charge E independent of r

FIELD LINES

- The electric field can be represented by field lines. Start on positive charge and end on a negative charge.
- PROPERTIES
  - Number of field lines, starting on a positive charge is proportional to the magnitude of the charge.
  - They indicate the direction of the electric field.
  - The electric field is stronger when the field lines are closer together.

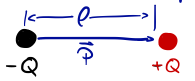


ELECTRIC FIELDS AND CONDUCTORS

- Electric field = 0 inside a conductor
- Any net charge on a conductor distributes itself on the surface.
- Electric field is always perpendicular to the surface outside of a conductor.
- Application: shielding.

ELECTRIC DIPOLES

- Two charges Q, equal in magnitude and opposite in sign separated by a distance r.
  The dipole moment points from the negative to the positive charge.



ELECTRIC DIPOLES

- An electric dipole in a uniform electric field will experience no net force, but it will, in general, experience a torque:

$$\boxed{\tau = QE\frac{\ell}{2}\sin\theta + QE\frac{\ell}{2}\sin\theta = pE\sin\theta} \qquad \vec{\tau} = \vec{p}\times\vec{E}$$

- The electric field created by a dipole is the sum of the fields created by the two charges; far from the dipole, the field shows a $1/r^3$ dependence

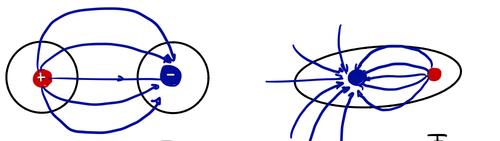
$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(r^2+\ell^2/4)^{3/2}}} \qquad \boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}}$$

CHAPTER 22 GAUSS'S LAW

- DIVERGENCE THEOREM

ELECTRIC FLUX

- $\Phi_E = E_\perp A = E A_\perp = EA\cos\theta$ Electric flux through an area is proportional to the total number of field lines crossing the area.
- E not uniform: $\Phi_E = \sum_i E_i \cdot \Delta A_i \rightarrow \int E\cdot dA$
- Through closed surface: $\Phi_E = \int E\cdot dA = 0$



$\Phi_E > 0$ $\Phi_E < 0$ $\Phi_E < 0$

GAUSS'S LAW

Net number of field lines through the surface is proportional to the charge enclosed, and also to the flux.

- $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclo}}{\varepsilon_0}$
- For a point charge $\oint E\,dA = E\oint dA = E\cdot 4\pi r^2$ solving for E $\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$
- If a gaussian surface encloses several point charges, the superposition principle shows that:

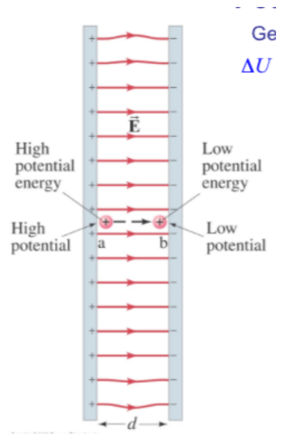
$$\boxed{\oint \vec{E}\cdot d\vec{A} = \oint\left(\Sigma \vec{E}_i\right)d\vec{A} = \Sigma\frac{Q_i}{\varepsilon_0} = \frac{Q_{enclosed}}{\varepsilon_0}}$$

- PROCEDURE FOR GAUSS'S LAW PROBLEMS

1. Identify the symmetry and choose a gaussian surface that takes advantage of it.
2. Draw surface
3. Use symmetry to find direction of $\vec{E}$
4. Evaluate flux by integrating
5. Calculate enclosed charge
6. Solve for the field.

CHAPTER 23 ELECTRIC POTENTIAL

- ELECTROSTATIC POTENTIAL ENERGY AND POTENTIAL DIFFERENCE



- $\Delta U = -W$, W is work done by conservative force.
- The electrostatic force is conservative – potential energy can be defined.
- Change in electric potential energy is negative of work done by electric force:

$$U_b - U_a = -W = -qEd < 0$$

- qEd = increase in kinetic energy $U_b < U_a$

DEFINITION: Electric potential is defined as potential energy per unit charge.

$\boxed{V_a = \frac{U_a}{q}}$ • $1\,V = 1\,J/C$ (unit: volt)

REMARKS

1. V does not depend on q
2. We have to choose where the potential is zero
3. Batteries, electric generators are for maintaining potential differences

• POTENTIAL DIFFERENCE

Only changes in potential can be measured. The potential difference is:

$$V_{ba} = \Delta V = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q} = -Ed \quad E \text{ uniform field.}$$

The difference in potential energy, $U_b - U_a$ is equal to the negative of the work W_{ba}, done by the electric field as the charge moves from a to b.

• RELATION BETWEEN ELECTRIC POTENTIAL AND ELECTRIC FIELD

• The general relationship between a conservative force and potential energy:

$$\boxed{U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{\ell}}$$

• Non uniform

$$\boxed{V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}}$$

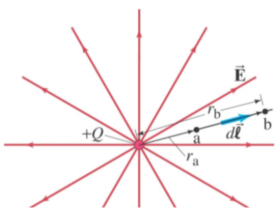
• Simplest case

$$\boxed{V_{ba} = -Ed}$$

• ELECTRIC POTENTIAL DUE TO POINT CHARGES

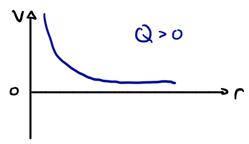
We integrate the field along a field line

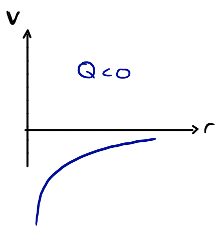
$$V_b - V_a = -\int_{r_a}^{r_b} \mathbf{E} \cdot d\vec{\ell} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{1}{r^2}\,dr = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right).$$



• Setting the potential to zero at $r = \infty$ gives the general form of the potential due to a point charge:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r}$$





• POTENTIAL DUE TO ANY CHARGE DISTRIBUTION

• The potential due to an arbitrary charge distribution can be expressed as a sum or integral

$$\boxed{V_a = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{Q_i}{r_{ia}}} \qquad \boxed{V = \frac{1}{4\pi\epsilon_0} \cdot \int \frac{dq}{r}}$$

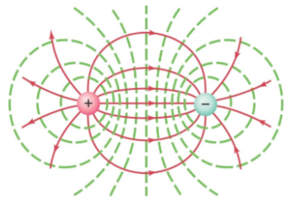
• E DETERMINED FROM V

If we know the field we can determine the potential by integrating.

$$V_{ba} = -\int_a^b \vec{E} \cdot d\vec{\ell} \rightarrow dV = -E\,d\vec{\ell} = -\vec{E}_\ell\, d\ell \rightarrow \boxed{E_\ell = \frac{dV}{d\ell}}$$

$$\boxed{E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = \frac{\partial V}{\partial z}}$$

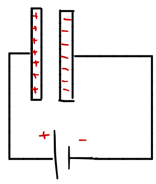
- ## Equipotential Surfaces



- Always perpendicular are always perpendicular to field lines; they are always closed surfaces
- Equipotential is a line or surface over which the potential is constant
- Electric field lines are perpendicular to equipotential
- The surface of a conductor is an equipotential.

Chapter 24 Capacitance, Dielectrics, Electric Energy Storage

Capacitor:



- consists of two conductors that are close but not touching
- When a capacitor is connected to a battery, the charge on this plates is proportional

$$Q = C \cdot V$$

- The quantity C is called the capacitance. Unit of capacitance: the farad (F): $1F = 1\ C/V$

Determination of capacitance

- Field between plates is $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 \cdot A}$
- Integrating along a path between the plates gives the potential difference $V_{ba} = Q \cdot \frac{d}{\varepsilon_0} \cdot A$
- Capacitance = $\boxed{C = \frac{Q}{V} = \varepsilon_0 \cdot \frac{A}{d}}$

Cylindrical: $E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{Q}{\ell \cdot R}$

Spherical: $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$

Capacitors in series and parallel

- In parallel have the same voltage across each one:

$$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$$

- Equivalent capacitor is one that stores the same charge when connected to the same battery.

- In series have the same charge

$$V = V_1 + V_2 + V_3$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Big\} \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Electric energy storage

$$dW = V\,dq$$

$$W = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

- Energy stored in a capacitor

$$U = \frac{1}{2}\cdot\frac{Q^2}{C} = \frac{1}{2}CV = \frac{1}{2}QV$$

Energy density:

$$u = \text{energy density} = \frac{1}{2}\varepsilon_0 \cdot E^2 \quad \text{Joule}/m^3$$

Dielectric:

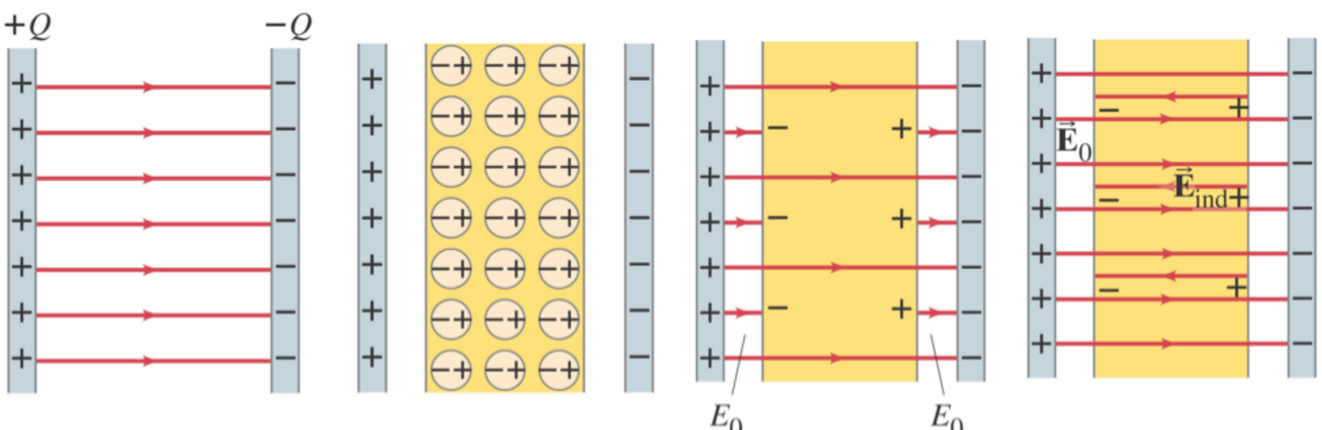
Insulator, and is characterized by a dielectric constant K

$$C = KC_0 = K\varepsilon_0 \cdot \frac{A}{d} = \varepsilon \cdot \frac{A}{d}$$

Permitivity $\varepsilon = K \cdot \varepsilon_0$

Molecular description:

The molecules in a dielectric, when in an external electric field, tend to become oriented in a way that reduces the external field.



Chapter 25 Electric Currents and Resistance

Electric Current

- Rate of flow of charge through a conductor: $\vec{I} = \frac{\Delta Q}{\Delta t}$
- Instantaneous current is given by: $I = \frac{dQ}{dt}$

Unit of electric current: the ampere A: $1A = 1C/s$

- A complete circuit: one where the current can flow all the way around
- By convention current flows from + to −. Electrons actually flow in the oposite direction.

Ohm's Law

- Experimentally, it is found that the current in a wire is proportional to the potential difference between its ends: $I \propto V$
- The ratio of voltage to current is called resistance

$$I = \frac{V}{R} \qquad V = I \cdot R$$

- Nonohmic materials do not follow ohm's law.

Resistivity

- Directly proportional to its length and inversely proportional to its cross-sectional area $R = \rho \cdot \frac{\ell}{A}$
- The constant ρ is characteristic of the material and resistivity increases with temperature.

$$\rho_T = \rho_0 \left[1 + \alpha (T - T_0)\right]$$

Electric Power

- Power: Energy transformed by a device per unit time

$$P = \frac{dU}{dt} = \frac{dq}{dt} \cdot V \quad \text{or} \quad P = IV$$

Ohmic devices: $P = I^2 \cdot R \qquad P = \frac{V^2}{R}$

- The unit of power is the Watt, W

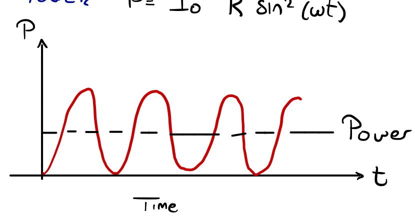
Alternating Current

- Current from a battery flows steadily in one direction. (Direct current)
- Current from a power plant varies sinusoidally (Alternating current)

$$\boxed{V = V_0 \cdot \sin(2\pi) f t = V_0 \cdot \sin(\omega t)}$$

$$\boxed{I = \frac{V}{R} = \frac{V_0}{R} \cdot \sin(\omega t) = I_0 \cdot \sin \omega t}$$

- Power $P = I_0{}^2 R \sin^2(\omega t)$



- Usually we are interested in the average power:

$$\boxed{\bar{P} = \frac{1}{2} I_0{}^2 R} \qquad \boxed{\bar{P} = \frac{1}{2} \frac{V_0{}^2}{R}}$$

- The current and voltage both have average values of zero, so we square them, take the average, then take the square root yielding root-mean-square:

$$\boxed{I_{rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0} \qquad \boxed{V_{rms} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = 0.707 V_0}$$

Microscopic View of Electric Current: Current Density and Drift Velocity

- Electrons in a conductor have large random speeds just due to their temperature. When a potential difference is applied, the electrons also acquire an average drift velocity, which is generally considerably smaller than the thermal velocity.
- We define the current density (current per unit area) - this is a convenient concept for relating the microscopic motions of electrons to the macroscopic current:

$$\boxed{j = \frac{I}{A} \quad \text{or} \quad I = j \cdot A}$$ → if current is not uniform $$\boxed{I = \int \vec{j} \cdot d\vec{A}}$$

- The drift speed is related to the current in the wire, and also number of electrons per unit volume.

$$\boxed{\Delta Q = (\text{no. of charges}, N) \times (\text{charge per particle}) = (n \cdot V) \cdot (-e) = -(n A v_d \Delta t)(e)}$$

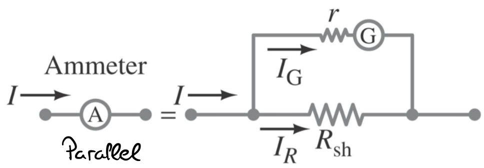
$$\boxed{I = \frac{\Delta Q}{\Delta t} = -neAv_d}$$

- The electric field inside a current-carrying wire can be found from the relationship between the current, voltage and resistance.

$$\boxed{j = \frac{1}{\rho} E = \sigma E}$$

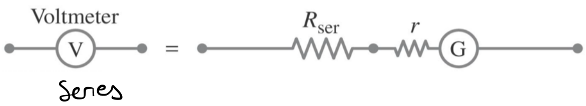
Ammeters and Voltmeters

- An ammeter measures current; a voltmeter mesures voltage. Both are based on galvanometers, unless they are digital.
- The current in a circuit passes through the ammeter; the ammeter should have low resistance so as not to affect the current.



- An ohmmeter measures resistance; it requires a battery to provide a current Parallel.

A voltmeter should not affect the voltage across the circuit element it is measuring; therefore its resistance should be very large.

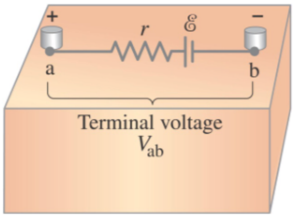


Chapter 26: DC Circuits

EMF and Terminal Voltage

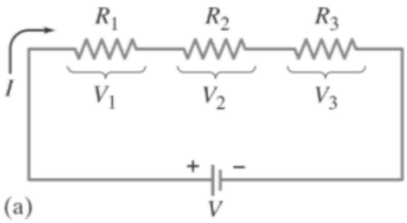
- Electric circuits needs battery or generator to produce current - called sources of electromotive force (emf)
- Battery is nearly constant voltage source, but does have a small internal resistance which reduces the actual voltage from the ideal emf

$$V_{ab} = \mathcal{E} - Ir$$



Resistors in Series and in Parallel

- A series connection has a single path from the battery, through each circuit element in turn, then back to the battery.

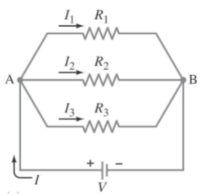


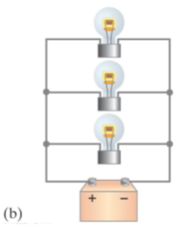
- current through each resistor is the same: the voltage depends on the resistance

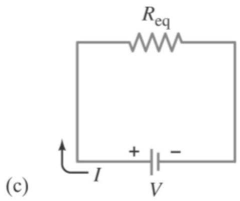
$$\boxed{V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3}$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

- A Parallel connection splits the current; the voltage across each resistor is the same







$$\boxed{I = I_1 + I_2 + I_3}$$

$$\boxed{\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}}$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

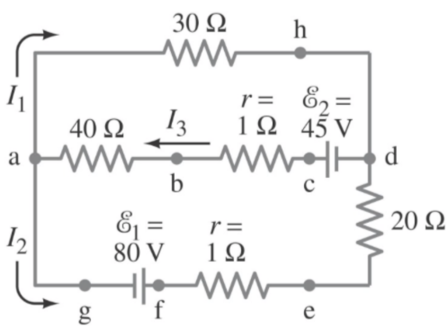
Kirchoff's Rules

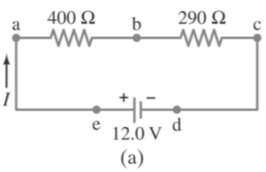
- Junction Rule: conservation of charge.

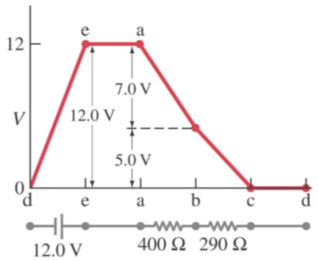
  The sum of the currents entering a junction equals the sum of the currents leaving it.

- Loop Rule: conservation of energy.

  The sum of the changes in potential around a closed loop is zero.







- **Problem Solving Rules**

1. Label each current and direction
2. Identify unknowns
3. Apply junction and loop rules
4. Solve the equations

Sign Convention

(a) For a resistor, apply Ohm's law; the potential difference is negative (a decrease) if your chosen loop direction is the same as the chosen current direction through that resistor; the potential difference is positive (an increase) if your chosen loop direction is opposite to the chosen current direction.

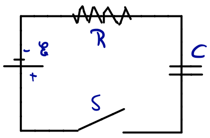
(b) For a battery, the potential difference is positive if your chosen loop direction is from the negative terminal toward the positive terminal; the potential difference is negative if the loop direction is from the positive terminal toward the negative terminal.

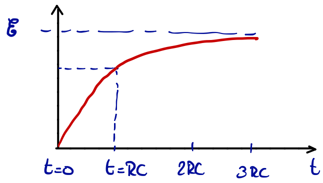
Circuits Containing Resistors and Capacitor

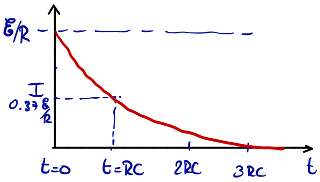
- When the switch is closed, the capacitor will begin to charge. As it does, the voltage across it increases and the current through the resistor decreases.

- To find the voltage as a function of time, we write the equation for the voltage changes around the loop: $\boxed{\mathcal{E} = IR + \frac{Q}{C}}$

- Since $I = \frac{dQ}{dt}$, we can integrate to find the charge as a function of time: $\boxed{Q = C\mathcal{E}\left(1 - e^{\frac{-t}{RC}}\right)}$







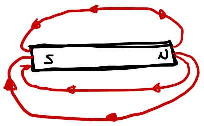
- Voltage across capacitor $\boxed{V_C = Q/C}$ $\boxed{V_C = \mathcal{E}\cdot\left(1 - e^{-t/RC}\right)}$
- RC is called the time constant of the circuit. $\boxed{\tau = RC}$
- Current at any time t can be found by differentiating the charge: $\boxed{I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}}$

- If an isolated charge capacitor is connected across a resistor, it discharges:

Chapter 27 Magnetism

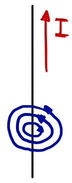
27.1 Magnets and Magnetic Fields

- Like poles repel
- Unlike poles attract



27.2 Electric Currents Produce Magnetic Fields

- The direction of the field is given by a right-hand rule.



27.3 Force on an Electric Current in a Magnetic Field; Definition of $\vec{B}$

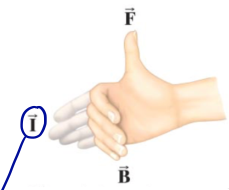
- A magnet exerts a force on a current-carrying wire. The direction of the force is given by a right-hand rule.
- The force on the wire depends on the current, the length of the wire, the magnetic field and its orientation.

$$\boxed{F = I \cdot l \cdot B \cdot \sin\theta}$$

- This equation defines the magnetic field $\vec{B}$. [Unit of B is the Tesla, T]

$$\vec{F} = I\vec{l} \times \vec{B}$$

$1T = 1\ N/A \cdot m$



can also be direction

27.4 Force on an Electric Charge Moving in a Magnetic Field

- Force on a moving charge is related to the force of a current:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Problem Solving

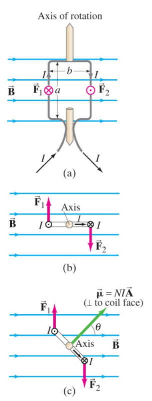
1. Magnetic force perpendicular to the magnetic field direction.
2. Right hand rule for directions
3. Equations give magnitudes.

- Lorentz Equation

If a particle of charge q moves with velocity $\vec{v}$ in the presence of both a magnetic field $\vec{B}$ and an electric field $\vec{E}$, it will feel a force

$$F = q(\vec{E} + \vec{v} \times \vec{B})$$

27.5 Torque on a Current Loop; Magnetic Dipole Moment



Total torque:

$$\tau = I \cdot a \cdot B \cdot \frac{b}{2} + I \cdot a \cdot B \cdot \frac{b}{2} = I \cdot a \cdot b \cdot B = I \cdot A \cdot B$$

Magnetic dipole moment: $\vec{\mu}$

$$\vec{\mu} = N \cdot I \cdot \vec{A}$$

$$\vec{\tau} = NI\vec{A} \times \vec{B} \longrightarrow \vec{\tau} = \vec{\mu} \cdot \vec{B}$$

Potential Energy

$$U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$$

27.6 Galvanometer

Measuring device that takes advantage of the torque on a current loop to measure current.

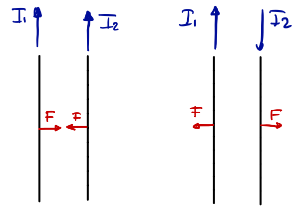
Chapter 28 Sources of Magnetic Field

28.1 Magnetic Field due to a Straight Wire

- Inversely proportional to the distance from the wire.

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r} \qquad \mu_0 = 4\pi \cdot 10^{-7} \text{ permeability of free space}$$

28.2 Force Between Two Parallel Wires



- The magnetic field produced at the position of the wire 2 due to the current in wire 1 is

$$B_1 = \frac{\mu_0}{2\pi} \frac{I_1}{d}$$

- The force this field exerts on a length l_2 of wire 2 is

$$F_2 = I_2 \cdot B_1 \cdot l_2 = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{d} \cdot l_2$$

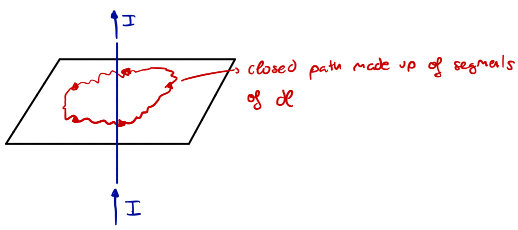
28.3 Definition of the Ampere and the Coulomb

- Ampere: Current flowing in each of two long parallel wires 1m apart which results in a force of exactly 2×10^{-7} N per meter of length
- Coulomb: one ampere per second.

28.4 Ampère's Law

- Relates magnetic field around a closed loop to the total current flowing through the loop

$$\oint \vec{B} \cdot dl = \mu_0 \cdot I_{encl}$$



- The integral is taken around the edge of the closed loop.
- Magnetic field is parallel to the loop

$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = \oint B \cdot dl = B \oint dl = B(2\pi r) \qquad B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$$

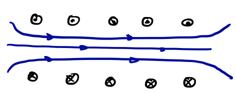
Solving Problems

1. Identify symmetry
2. Integration path that relates to symmetry
3. Use symmetry to determine direction of field
4. Determine enclosed current.

28.5 Magnetic Field of a Solenoid and a Toroid

- Solenoid: coil of wire containing many loops N (n number of loops per length)

$$B = \mu_0 \cdot n \cdot I$$



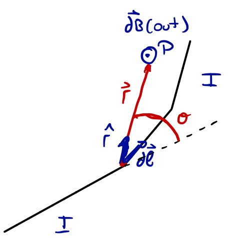
- field is zero outside the solenoid, and the path integral is zero along the vertical lines.

- Toroid formula
  - inside: $B = \frac{\mu_0 \cdot N \cdot I}{2\pi r}$
  - outside $B = 0$

28.6 Biot-Savart Law

gives the magnetic field due to an infinitesimal length of current; the total field can then be found by integrating over the total length of all currents.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{l} \times \hat{r}}{r^2}$$



$$\vec{B} = \frac{\mu_0 \cdot I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2}$$

Electric Dipole

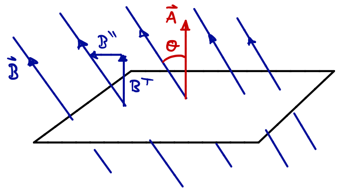
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{r^3}$$

Chapter 29 Electromagnetic Induction and Faraday's Law

29.1 Faraday's Law of Induction

Induced EMF in a wire loop is proportional to the rate of change of magnetic flux through the loop

$\Phi_B = \int \vec{B} \cdot d\vec{A}$ $\quad 1\ Wb = 1 \cdot T \cdot m^2$



$$\mathcal{E} = -\frac{d\Phi_B}{dt} \cdot N$$

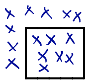
N: number of loops.

Lenz's law:

A current produced by an induced EMF moves in a direction so that the magnetic field it produces tends to restore field.

Example

B = 0.6 T



0,05m

ⓐ rate of change in flux

$\Phi_B = B \cdot A = 0.6 \cdot (5 \cdot 10^{-2})^2 = 1.5 \cdot 10^{-3}\ Wb$

$\Phi_B = 0$ after $\Delta T = 0.1\,s$ $\quad \frac{\Delta \Phi_B}{\Delta t} = -1.5 \cdot 10^{-2}\ Wb \cdot s^{-1}$

ⓑ $\mathcal{E} = -N \cdot \frac{\Delta \Phi_B}{\Delta t} = -100 \cdot -1.5 \cdot 10^{-2} = 1.5\ V$

$I_{(oc)} = \frac{\mathcal{E}}{R} = \frac{1.5\ V}{100\ \Omega} = 15\ mA$

ⓒ $I^2 \cdot R \cdot \Delta t = 2.25 \cdot 10^{-3}\ J = E$

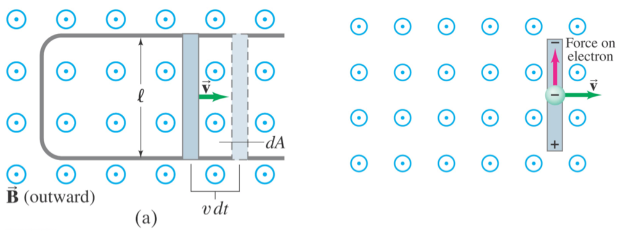
ⓓ $W = F \cdot d = E$

$F = \frac{2.25 \cdot 10^{-3}}{5 \cdot 10^{-2}} = \boxed{0.045\ N}$

29.3 EMF Induced in a Moving Conductor

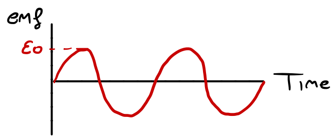
$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{B \cdot dA}{dt} = \frac{B \cdot \ell v\, dt}{dt} = B \cdot \ell \cdot v$$

valid as long as $B \perp \ell \perp v$



29.4 Electric Generator

- Transform mechanical energy into electrical energy.
- Constant angular velocity: emf sinusoidal.



- $\mathcal{E} = -\frac{d}{dt}[B \cdot A \cdot \cos\theta]$
- $\mathcal{E} = BA \sin(\omega t)$
- $\mathcal{E} = \mathcal{E}_0 \cdot \sin(\omega t)$ $\quad N \cdot B \cdot A$ $\quad$ N: number of loops.

29.7 A Changing Magnetic Flux Produces an Electric Field

- Generalization of Faraday's law

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

29.6 Transformers and Transmission of Power

- Transformers: consist in two coils, interwoven or linked by an iron core. A change emf in one induces an emf in the other.
- The ratio of the emfs is equal to the ratio of the number of turns in each coil.

$$\left.\begin{aligned} V_S &= N_S \frac{d\Phi_B}{dt} \\ V_P &= N_P \cdot \frac{d\Phi_B}{dt} \end{aligned}\right\} \longrightarrow \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

- Energy must be conserved so

$I_P \cdot V_P = I_S \cdot V_S$ $\quad \frac{I_S}{I_P} = \frac{N_P}{N_S}$

Chapter 30 Inductance, Electromagnetic Oscillations, and AC Circuits

30.1 Mutual Inductance

- A changing current in one coil will induce a current in the second coil.

$$\mathcal{E}_2 = -M_{21} \cdot \frac{dI_1}{dt}$$

- Unit of inductance: the henry, H

$1\,H = 1\,V \cdot s/A = 1\,\Omega \cdot s$

- $M_{21} = \frac{N_2 \cdot \Phi_{21}}{I_1}$

30.2 Self-Inductance

- A changing current in a coil will also induce an emf in itself.

$$\mathcal{E} = -N \cdot \frac{d\Phi_B}{dt} = -L \cdot \frac{dI}{dt}$$

L = self inductance $\quad L = \frac{N\,\Phi_B}{I}$

- Inductor

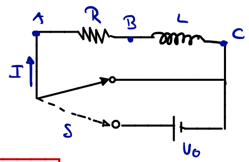
30.3 Energy Stored in a Magnetic Field

- $P = I \cdot \mathcal{E} = L \cdot I \cdot \frac{dI}{dt}$ $\quad \boxed{U = \frac{1}{2} L I^2}$

- Energy density: $u = \frac{1}{2} \frac{B^2}{\mu_0}$

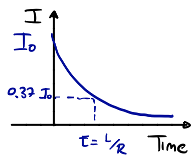
30.4 LR Circuits

- Circuit consisting of an inductor and a resistor will begin with most of the voltage drop across the inductor. With time, the current will increase less and less, until all the voltage is across the resistor.

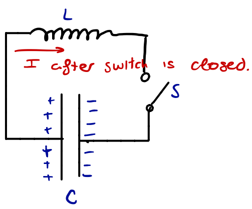


- $L \cdot \frac{dI}{dt} + RI = V_0 \longrightarrow \boxed{I = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right)}$

- If the circuit is shorted across the battery, the current will gradually decay $\boxed{I = I_0\, e^{t/\tau}}$



30.5 LC Circuits and Electromagnetic Oscillations



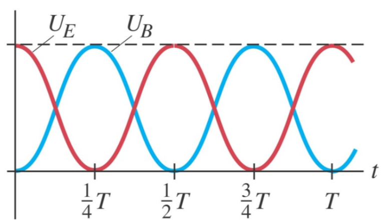
- Circuit charged capacitor shorted through an inductor.
- Summing the potential drops around the circuit gives a differential equation for Q:

$$\left.\begin{array}{l} -L\frac{dI}{dt} + \frac{Q}{C} = 0 \\ I = -\frac{dQ}{dt} \end{array}\right\} \longrightarrow \left.\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0\right\} \quad Q = Q_0 \cos(\omega t + \phi)$$

frequency $= \omega = 2\pi f = \sqrt{\frac{1}{LC}}$

- Current: $I = I_0 \cdot \sin(\omega t + \phi)$

- The total energy is constant, it oscillates between the capacitor and the inductor



30.6 Oscillations with Resistance (LRC circuit)

- Voltage drops around circuit

$$L \cdot \frac{d^2Q}{dt^2} + R \cdot \frac{dQ}{dt} + \frac{1}{C} \cdot Q = 0$$

- Underdamped: $R^2 < 4L/C$

$$\boxed{\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}$$

- Overdamped: $R^2 > 4L/C$
- Critical damping: $R^2 = 4L/C$

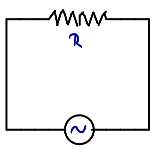
- Charge in circuit as a function of time

$$Q = Q_0 e^{-\frac{R}{2L}t} \cos(\omega' t + \phi)$$

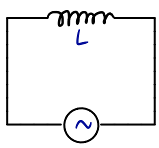
30.7 AC Circuits with AC Source

- Resistors, capacitors and inductors have different phase relationships between current and voltage when placed in an ac circuit

- The current is in phase with the voltage.



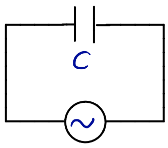
- Voltage across the inductor: $\boxed{V = L \cdot \frac{dI}{dt} = -\omega L I_0 \cdot \sin(\omega t)}$



$$\boxed{V = V_0 \cos(\omega t + 90°)}$$

- $V_0 = I_0 \cdot X_L$ X_L: inductive reactance Ω $\boxed{X_L = \omega L = 2\pi f \cdot L}$

- Votage across capacitor

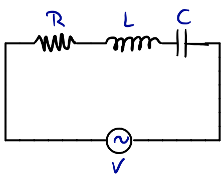


- $V = I_0 \left(\frac{1}{\omega C}\right) \cos(\omega t - 90°)$

  $= V_0 \cos(\omega t - 90°)$

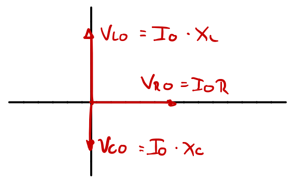
- X_C: capacitance reactance Ω $\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$

30.8 LRC Series AC Circuit

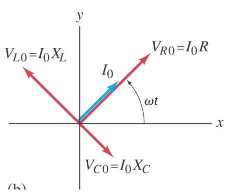


- Analyze is complicated: as the voltages are not in phase.
- Reactances depend on frequency.
- Calculate voltage with phasors: vectors representing individual voltages.

at $t = 0$



- Some time later



- Voltage accros each is given by the x component of each.
- Impedance of circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$ $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$
- Phase angle between voltage and current

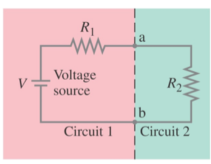
$$\boxed{\tan\phi = \frac{X_L - X_C}{R}} \quad \boxed{\cos\phi = \frac{R}{Z}}$$

30.9 Resonance in AC Circuits

- I_{rms} current $I_{rms} = \frac{V_{rms}}{Z}$
- $f_0 = \omega_0 / 2\pi$ resonant frequency
- I_{rms} at max when $X_C = X_L \rightarrow \boxed{\omega_0 = \sqrt{\frac{1}{LC}}}$

30.10 Impedance Matching

When one electrical circuit is connected to another, maximum power is transmitted when the output impedance of the first equals the input impedance of the second.



The power delivered to circuit 2 is

$$P = I^2 R_2 = \frac{V^2 R_2}{(R_1 + R_2)^2}$$

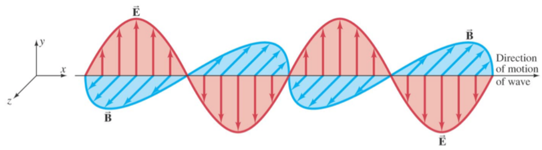
Chapter 31 Maxwell's Equations and Electromagnetic Waves

31.3 Maxwell's Equations

- Modification to Ampere's Law $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \cdot I + \mu_0 \cdot \varepsilon_0 \cdot \frac{d\Phi_E}{dt}$

31.4 Production of Electromagnetic Waves

- Electric and magnetic waves are perpendicular to each other



$E = E_y = E_0 \sin(kx - \omega t)$

$E = B_z = B_0 \sin(kx - \omega t)$

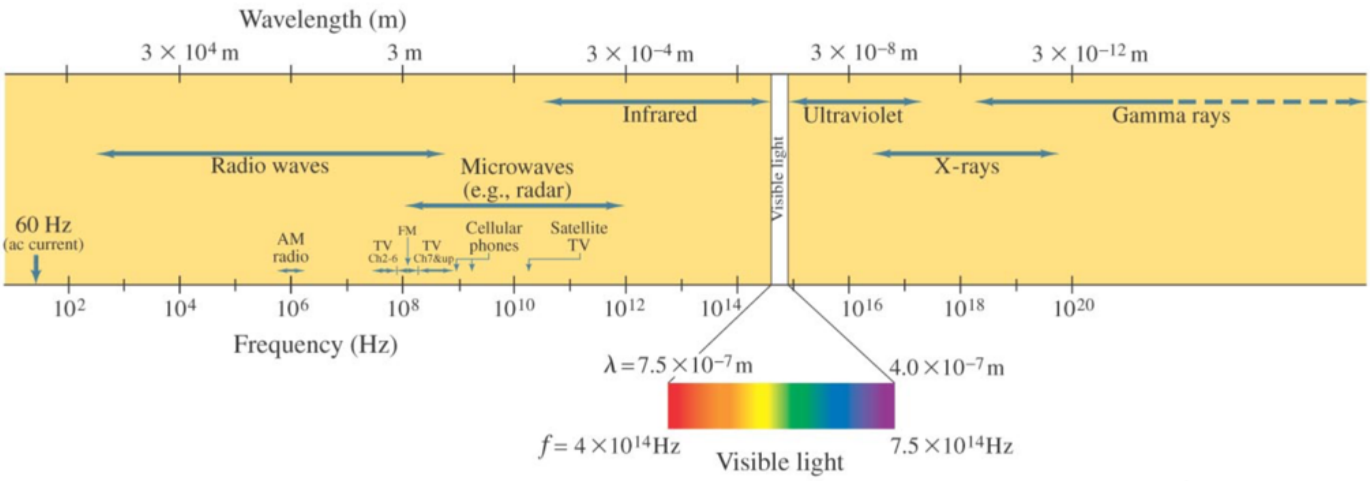
31.5 Electromagnetic Waves

c: speed of electromagnetic waves in space

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0 \cdot 10^8 \ m/s \text{ speed of light}$$

31.6 Light as an Electromagnetic Wave and the Electromagnetic Spectrum

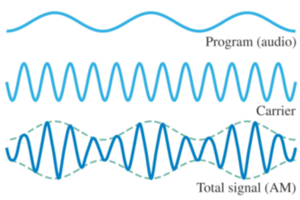
- Names to different wavelengths.



31.8 Energy in EM Waves

- $u = u_E + u_B = \frac{1}{2}\varepsilon_0 \cdot E^2 + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\varepsilon_0 \cdot E^2 + \frac{1}{2} \cdot \frac{\varepsilon_0 \cdot \mu_0 \cdot E^2}{\mu_0} = \varepsilon_0 E^2$
- $\vec{S} = \frac{1}{\mu_0} \cdot (\vec{E} \times \vec{B})$ $\quad \vec{S} = \frac{E_0 \cdot B_0}{2\mu_0}$
- $S = \varepsilon_0 c E^2 = \frac{cB^2}{\mu_0} = \frac{EB}{\mu_0}$

31.10 Radio and Television; Wireless Communication



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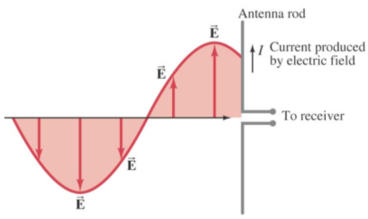

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